ON THE NUMERICAL ANALYSIS OF MAGNETIC AMPLIFIER CIRCUITS

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ABSTRACT

ON THE NUMBERICAL ANALYSIS OF MAGNETIC AMPLIFIER CIRCUITS

by Burton Howard Wayne

In the years 1948-52 a great deal of work was done by many people to qualitatively explain the operational principles of the magnetic amplifiers. Many articles have been written on this subject. During this period the empirical design of magnetic amplifiers was developed extensively. However, in recent, most of the published papers have dealt with the application of magnetic amplifiers to specific circuitry and the analysis has been limited to a few people.

In this thesis, a new formulation technique is developed, by considering the core characteristic as a function of the ampere-turns variables. The core characteristic is considered to be the saturation curve, with finite slope, of a ferromagnetic inductor. The formulation procedure in the example yields a pair of first order simultaneous differential equations which are explicit in the derivatives. The coefficients of the derivatives are functions of only the ampere-turns variables.

A review of linear transformations of variables for two-winding inductors and the extensions to inductors with k-windings is given in Section II.

In Sections III and V theorems are developed to give a criterion for the locations of the inductors in the network such that the ampere-turns variable will appear explicitly in the system equations.

The arrangement of the elements of terminal and transformation equations, such that they may be written systematically is the subject of Section IV.

The details of formulating the system equations such that they are adaptable to a computer solution is given by an example, in addition to the computer program used.

ON THE NUMERICAL ANALYSIS OF MAGNETIC AMPLIFIER CIRCUITS

By

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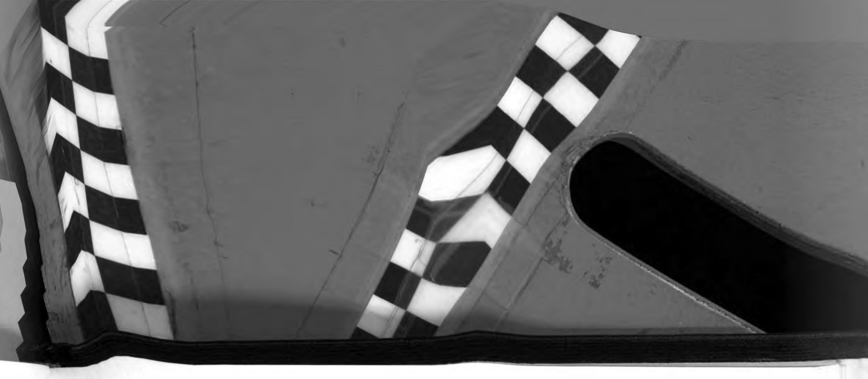

I. INTRODUCTION

The term 'magnetic amplifier' was introduced by E. F. W. Alexanderson in 1916.[1] Today magnetic amplifiers are found in nearly all branches of industry, especially in the control systems area. Although the magnetic amplifier is relatively simple in its external appearance, the nonlinearity and double-valuedness of the hysteresis loop of the core material makes the analysis of the magnetic amplifier very complex, unless gross approximations are used. Perhaps the most frequently used approximation is to represent the core characteristic by a series of straight lines, one with infinite slope in the "unsaturated region," and the others with zero slope in the "saturated region."[2,3,4,5,6,7] A serious difficulty of this approximation is that the flux is independent of the current in the unsaturated region, while in the saturated region it is independent of the voltage. Hence to obtain a solution over the entire range of operation requires two different variables which must be interrelated at the intersection of the straight line segments.

Milnes and Law[8] have analyzed the magnetic amplifier using straight lines with infinite slope in the "unsaturated region" and zero slope in the "saturated region." However, only the steady-state component of the solution is discussed, and the analysis is based on linear approximations in the two regions of the saturation curve.

Ettinger[9] assumes the core hysteresis loop to be rectangular with infinite and zero slopes. Two distinct variables, one for each mode of operation, are again used.

With the advent of the computer, many problems once considered impractical, become practical. The objective of this thesis is to

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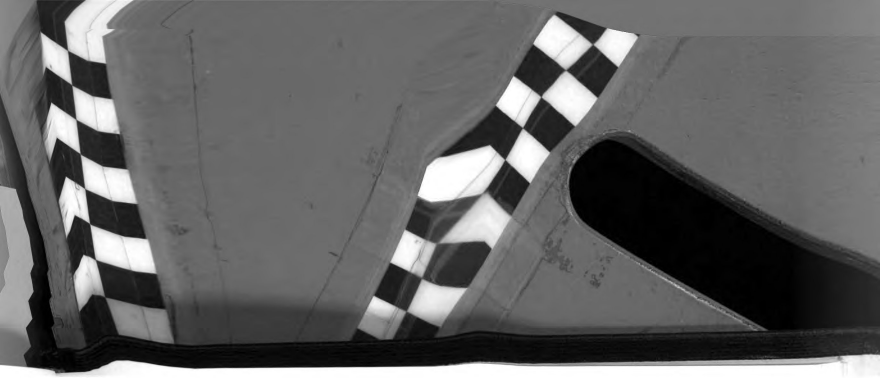
formulate a set of equations describing the magnetic amplifier, such that they are adaptable to computer solution. In following this objective a change of variables discussed in Section II has been introduced such that the slope of the B-H curve may be conveniently incorporated into the equations.

By applying a change of variable it is possible to establish a set of terminal equations for the inductors in which all coefficients are constant, with the exception of one. Furthermore this coefficient is a function of one variable only, namely the ampere-turns of the core.

In formulating the mathematical model of systems containing ferromagnetic inductors, it is desirable to have the ampere-turns of each inductor appear explicitly in the system equations. The objectives of this thesis is to present techniques for formulating the mathematical model in this form and show an example of a numerical solution to a typical magnetic amplifier circuit.

In this solution the saturation curve is approximated by segments of straight lines with finite slope. To the author's knowledge the approximation of the saturation curve by more than three straight lines has not been used. However the formulation techniques in this thesis permit the use of a computer such that the saturation curve may now be approximated by any number of straight line segments.



II. BACKGROUND

2.1. Terminal Equations for Two-Winding Inductors.

The problem of numerical accuracy for a two-winding ferromagnetic inductor is discussed by Koenig.[10] To review the basic problem, let the terminal equations of a two-winding inductor be written as

$$\begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix} \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix} + \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix} \qquad (2.1.1)$$

or symbolically,

$$\mathcal{V}(t) = \mathcal{R}\,\mathcal{I}(t) + \mathcal{M}\frac{d}{dt}\mathcal{I}(t)$$

In the above equations, the coefficients in the matrix $\mathcal{M}$ are found from a series of open circuit tests. For high permeability cores such tests are impractical in that acceptable numerical accuracy is difficult to realize. That is, the determinant of the coefficient matrix $\mathcal{M}$ is nearly zero. In order to alleviate this difficulty, let a new set of variables be defined by the following non-singular linear transformations:

$$\begin{bmatrix} v_1^n(t) \\ v_2^n(t) \end{bmatrix} = \begin{bmatrix} -n_{21} & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} \qquad (2.1.2)$$

which may be written symbolically,

$$\mathcal{V}^n(t) = \mathcal{A}_v\,\mathcal{V}(t)$$

$$\begin{bmatrix} i_1^n(t) \\ i_2^n(t) \end{bmatrix} = \begin{bmatrix} -n_{12} & 0 \\ n_{12} & 1 \end{bmatrix} \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix} \qquad (2.1.3)$$

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which may also be written symbolically,

$$\mathcal{I}^{n}(t) = \mathcal{A}_i \, \mathcal{I}(t)$$

where $n_{12} = N_1/N_2$ and $n_{21} = N_2/N_1$: N_1 is the number of turns on winding the number one and N_2 is the number of turns on winding number two.

The linear transformations on the current and voltage variables have been chosen such that

$$\mathcal{A}_v = (\mathcal{A}_i^{-1})'.$$

Substituting 2.1.2 and 2.1.3 into 2.1.1, a new form of the terminal equations results.

$$\begin{bmatrix} v_1^n(t) \\ v_2^n(t) \end{bmatrix} = \begin{bmatrix} R_2 + n_{21}^2 R_1 & R_2 \\ R_2 & R_2 \end{bmatrix} \begin{bmatrix} i_1^n(t) \\ i_2^n(t) \end{bmatrix} + \begin{bmatrix} (n_{21}^2 M_{11} - n_{21}M_{21} - n_{21}M_{12} + M_{22}) & (M_{22} - n_{21}M_{12}) \\ (M_{22} - n_{21}M_{21}) & M_{22} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_1^n(t) \\ i_2^n(t) \end{bmatrix} \quad (2.1.4)$$

Definition 2.1.1. Leakage Inductance

The leakage inductance between the j^{th} and the k^{th} element of a mutually coupled component is defined as $L_{jk} = M_{jj} - n_{jk} M_{jk}$.

Assuming $M_{jk} = M_{kj}$ and applying Definition 2.1.1, 2.1.4 may be written as

$$\begin{bmatrix} v_1^n(t) \\ v_2^n(t) \end{bmatrix} = \begin{bmatrix} R_1 + n_{21}^2 R_2 & R_2 \\ R_2 & R_2 \end{bmatrix} \begin{bmatrix} i_1^n(t) \\ i_2^n(t) \end{bmatrix} + \begin{bmatrix} L_{21} + n_{21}^2 L_{12} & L_{21} \\ L_{21} & M_{22} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_1^n(t) \\ i_2^n(t) \end{bmatrix} \quad (2.1.5)$$

The leakage inductance terms are usually very small compared to M_{22}. In addition one of the new variables is proportional to the total ampere turns of the core, which is of equal importance to the study presented in this thesis.

Specifically,

$$i_2^n(t) = i_2(t) + n_{12}i_1(t).$$

2.2. Terminal Equations for Three-winding Inductors

Let the terminal equations of a three-winding inductor be written as

$$\begin{bmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \end{bmatrix} = \begin{bmatrix} R_1 & 0 & 0 \\ 0 & R_2 & 0 \\ 0 & 0 & R_3 \end{bmatrix} \begin{bmatrix} i_1(t) \\ i_2(t) \\ i_3(t) \end{bmatrix} + \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_1(t) \\ i_2(t) \\ i_3(t) \end{bmatrix} \qquad (2.2.1)$$

In the study of systems containing inductors with high permeability cores it is desirable to formulate the equations in such a way that the total ampere turns appears as one of the variables. Let the transformations on the current and voltage variables in the system equations be defined as:

$$\begin{bmatrix} i_1^n(t) \\ i_2^n(t) \\ i_3^n(t) \end{bmatrix} = \begin{bmatrix} -n_{13} & 0 & 0 \\ 0 & -n_{23} & 0 \\ n_{13} & n_{23} & 1 \end{bmatrix} \begin{bmatrix} i_1(t) \\ i_2(t) \\ i_3(t) \end{bmatrix} \qquad (2.2.2)$$

$$\begin{bmatrix} v_1^n(t) \\ v_2^n(t) \\ v_3^n(t) \end{bmatrix} = \begin{bmatrix} -n_{31} & 0 & 1 \\ 0 & -n_{32} & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \end{bmatrix} \qquad (2.2.3)$$

Note that the voltage coefficient matrix is the transposed inverse of the current coefficient matrix.

The form of the terminal equations resulting from substituting 2.2.2 and 2.2.3 into 2.2.1 is

$$\begin{bmatrix} v_1^n(t) \\ v_2^n(t) \\ v_3^n(t) \end{bmatrix} = \begin{bmatrix} R_3+n_{31}^2R_1 & R_3 & R_3 \\ R_3 & R_3+n_{32}^2R_2 & R_3 \\ R_3 & R_3 & R_3 \end{bmatrix} \begin{bmatrix} i_1^n(t) \\ i_2^n(t) \\ i_3^n(t) \end{bmatrix} \qquad (2.2.4)$$

$$\begin{bmatrix} L_{31}+n_{31}^2 L_{13} & L_{32}+n_{31}^2(L_{13}-L_{12}) & L_{31} \\ L_{31}+n_{32}^2(L_{23}-L_{21}) & L_{32}+n_{32}^2L_{23} & L_{32} \\ L_{31} & L_{32} & M_{33} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_1^n(t) \\ i_2^n(t) \\ i_3^n(t) \end{bmatrix}$$

where, using the definition of leakage inductance given as Definition 2.1.1,

$$L_{31} + n_{31}^2 L_{13} = M_{33} - n_{31} M_{31} + n_{31}^2 M_{11} - n_{31} M_{13}$$

$$L_{32} + n_{32}^2 L_{23} = M_{33} - n_{32} M_{32} + n_{32}^2 M_{22} - n_{32} M_{23}$$

$$L_{31} + n_{32}^2 (L_{23} - L_{21}) = M_{33} - n_{31} M_{31} + n_{32}n_{31} M_{21} - n_{32}M_{23}$$

$$L_{32} + n_{31}^2 (L_{13} - L_{12}) = M_{33} - n_{32}M_{32} + n_{32}n_{31} M_{12} - n_{31} M_{13}$$

For convenience let the (i, j) coefficient in 2.2.4 be symbolized as L_{ij}^n, and write 2.2.4 as

$$\begin{bmatrix} v_1^n(t) \\ v_2^n(t) \\ v_3^n(t) \end{bmatrix} = \begin{bmatrix} R_3 + n_{31}^2 R_1 & R_3 & R_3 \\ R_3 & R_3+n_{32}^2R_2 & R_3 \\ R_3 & R_3 & R_3 \end{bmatrix} \begin{bmatrix} i_1^n(t) \\ i_2^n(t) \\ i_3^n(t) \end{bmatrix}$$

$$+ \begin{bmatrix} L_{11}^n & L_{12}^n & L_{13}^n \\ L_{21}^n & L_{22}^n & L_{23}^n \\ L_{31}^n & L_{32}^n & M_{33}^n \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_1^n(t) \\ i_2^n(t) \\ i_3^n(t) \end{bmatrix} \qquad (2.2.5)$$

Since the mutually coupled component is a bilateral element, i.e. $M_{ij} = M_{ji}$, the off-diagonal elements of 2.2.5 are equal, i.e. $L_{ij}^{n} = L_{ji}^{n}$.

2.3. Leakage Inductance Form of Terminal Equations for a k-Winding Inductor.

The current and voltage transformation matrices for a k-winding inductor are

$$\begin{bmatrix} i_1^n(t) \\ i_2^n(t) \\ - \\ - \\ i_{k-1}^n(t) \\ i_k^n(t) \end{bmatrix} = \begin{bmatrix} -n_{1k} & 0 & - & - & 0 & 0 \\ 0 & -n_{2k} & - & - & 0 & 0 \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ 0 & 0 & - & - & -n_{k-1,k} & 1 \\ n_{1k} & n_{2k} & - & - & n_{k-1,k} & 1 \end{bmatrix} \begin{bmatrix} i_1(t) \\ i_2(t) \\ - \\ - \\ i_{k-1}(t) \\ i_k(t) \end{bmatrix} \quad (2.3.1)$$

$$\begin{bmatrix} v_1^n(t) \\ v_2^n(t) \\ - \\ - \\ v_{k-1}^n(t) \\ v_k^n(t) \end{bmatrix} = \begin{bmatrix} -n_{k1} & 0 & - & - & 0 & 1 \\ 0 & -n_{k2} & - & - & 0 & 1 \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ 0 & 0 & - & - & -n_{k,k-1} & 1 \\ 0 & 0 & - & - & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1(t) \\ v_2(t) \\ - \\ - \\ v_{k-1}(t) \\ v_k(t) \end{bmatrix} \quad (2.3.2)$$

where $i_k(t)$, specifically, is proportional to the total ampere-turns of the k-winding inductor.

The transformed terminal equations (leakage inductance form) for a k-winding inductor, as determined by substituting 2.3.1 and 2.3.2 into the coupled circuit form* of the terminal equations, are

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*The coupled circuit form of the terminal equations is when the voltages are written as explicit function of the currents.

$$
\begin{bmatrix} v_1^n(t) \\ v_2^n(t) \\ - \\ v_{k-1}^n(t) \\ v_k^n(t) \end{bmatrix}
=
\begin{bmatrix}
(R_k+n_{k1}^2R_1) & R_k & - & - & R_k \\
R_k & (R_k+n_{k2}^2\,R_2) & - & - & - \\
- & - & - & - & - \\
- & - & - & (R_k+n_{k,\,k-1}^2R_{k-1}) & R_k \\
R_k & - & - & R_k & R_k
\end{bmatrix}
\begin{bmatrix} i_1^n(t) \\ i_2^n(t) \\ - \\ i_{k-1}^n(t) \\ i_k^n(t) \end{bmatrix}
$$

$$
+\begin{bmatrix}
L_{11}^n & L_{12}^n & - & - & L_{1k}^n \\
L_{21}^n & L_{22}^n & - & - & - \\
- & - & - & - & - \\
- & - & - & L_{k-1,\,k-1}^n & L_{k-1,\,k}^n \\
L_{k_1}^n & - & - & L_{k,\,k-1}^n & M_{kk}
\end{bmatrix}
\frac{d}{dt}
\begin{bmatrix} i_1^n(t) \\ i_2^n(t) \\ - \\ i_{k-1}^n(t) \\ i_k^n(t) \end{bmatrix}
\qquad (2.3.3)
$$

where

$$
\begin{aligned}
L_{nn}^n &= n_{kn}^2\,L_{nk_2} + L_{kn} && n \neq k \\
L_{nj}^n &= L_{kn} + n_{kn}\,(L_{nk} - L_{nj}) && n \neq j \neq k \\
L_{nj}^n &= L_{nk} && n \neq k,\ j = k \qquad (2.3.4) \\
L_{jn}^n &= L_{jk} && j \neq k,\ n = k
\end{aligned}
$$

and

$$
\begin{aligned}
v_n^n(t) &= -n_{kn}v_n(t) + v_k(t) && n \neq k \\
v_i^n(t) &= v_k(t) && \qquad (2.3.5) \\
i_n^n(t) &= -n_{nk}\,i_n(t) && n \neq k \\
i_k^n(t) &= \sum_{j=1}^{k-1} n_{jk}i_j(t)
\end{aligned}
$$

The self-and mutual-inductance coefficients of any two windings, j and k, can be related by an expression of the form

$$\frac{C_{nj} M_{nn}}{N_n^2} = \frac{C_{jn} M_{jj}}{N_j^2} = \frac{M_{nj}}{N_n N_j} \qquad (2.3.6)$$

where $\sqrt{C_{nj} C_{jn}} = k_{nj}$ and is called the coupling coefficient. The coupling coefficient is less than or equal to one. Hence, if $k_{nj} = 1$, then $C_{nj} = C_{jn} = 1$ and if $k_{nj} < 1$, $C_{nj} < 1$ and $C_{jn} < 1$.

From 2.3.6 and Definition 2.1.1 the leakage inductance coefficients in 2.3.4 can be expressed in the form

$$\begin{aligned}
L_{nn}^{n} &= M_{kk}\left(1 + \frac{C_{kn}}{C_{nk}} - 2C_{kn}\right) && n \neq k \\
L_{nj}^{n} &= M_{kk}\left(1 + \frac{C_{nj}\, C_{kn}}{C_{nk}} - C_{kj} - C_{kn}\right) && n \neq j \neq k \qquad (2.3.7) \\
L_{nk}^{n} &= M_{kk}\left(1 - C_{kn}\right) \\
L_{nk}^{n} &= M_{kk}\left(1 - C_{nk}\right)
\end{aligned}$$

Since the k-winding inductor is a bilateral element, i.e. $M_{nj} = M_{jn}$, the off diagonal elements are equal, i.e. $L_{jn}^{n} = L_{nj}^{n}$ for $j \neq n$. Under these conditions we see that upon comparing L_{nn}^{n} and L_{nk}^{n} that as the coefficient of coupling approaches unity, $L_{nk}^{n} \longrightarrow \frac{L_{nn}^{n}}{2}$.

Furthermore, if the core has very high permeability, such as those found in magnetic amplifiers and/or if the coils are wound on a toroid the coefficient of coupling approaches unity and as a very close approximation $L_{nn}^{n} = L_{nj}^{n} = 0$. Under these conditions the inductance matrix of 2.3.3 contains zeros in every position except the M_{kk} position and the value of the M_{kk} coefficient is proportional to the slope of the B-H characteristics of the core material. Even when the coefficients of coupling is appreciably less than unity all leakage inductance coefficients can be taken as constants, leaving M_{kk} as the only nonlinear term. The important advantage of the leakage inductance form is that M_{kk} is a function of one variable, namely $i_k^n(t)$. The objective of the following

section is to establish the requirements sufficient for formulating the system equations to include this variable explicitly.

All coefficients in 2.3.3, except those in the k^{th} row and column, can be determined from steady-state impedance measurements.

The terminal equations in 2.3.3 is the more useful and will be used throughout the analysis. Hence the measurement of the coefficients of 2.3.3 is discussed briefly.

The transformed equation variables must be expressed in terms of measurable voltages and currents, thus the transformed variables should be expressed in terms of the original variables as given in 2.3.5.

Note that, in 2.3.5, except when the terminals n or k are shorted the voltage matrix contains terms of the form $(V_k - n_{kn} V_n)$ for $n \neq k$. The term $(V_k - n_{kn} V_n)$ is a very small number and usually cannot be determined accurately by taking the difference between the measured values of V_n and V_k. Also, $i_k^n(t)$ represents the ampere turns of the core which can be made zero by opening all of the windings. It is impossible to determine any of the coefficients under this condition. However, it is usually sufficiently accurate to assume the exciting current is zero under short-circuit conditions, provided that this term is not multiplied by a large impedance. The short-circuit condition also removes the small difference voltage for the determination of the Z_{nn}^n, $Z_{nn}^n (n \neq k)$ and $Z_{jn}^n (n \neq k)$ coefficients. Hence these coefficients may be evaluated by open- and short-circuit measurements.

The determination of $Z_{nk}^n (n \neq k)$ involves the small difference terms of voltage hence cannot be measured with sufficient accuracy. The determination of Z_{kn}^n $(n \neq k)$ requires all but one winding be opened which is an impossible situation. However, these terms need not be measured in view of the conditions established in this section, i.e.

$$Z_{nk}^n = Z_{kn}^n \approx \frac{Z_{nn}^n}{2} .$$

III. GRAPH EQUATIONS

3.1. Circuit Matrix Formulation

In a later portion of this thesis, the following definition and theorems are useful in establishing the requirements sufficient for formulating the system equation with the ampere-turn variable appearing explicitly.

If a graph G' is not connected and consists of parts $p_1, p_2 \ldots .$ p_k, then at least one vertex of p_m may be identified with a vertex of p_n such that the new graph G is separable into parts $p_1, p_2, \ldots . p_k$ and is connected. The preceding identification of vertices does not alter the segregate or circuit equations associated with the graph G'.

Definition 3.1.1. An f-circuit is a fundamental circuit that is defined by a chord and its unique path through the tree.[12]

Definition 3.1.2. A subtree T_{j-1}, of a tree T_j, is a connected subgraph of T_j, where $j = 1, 2, 3, \ldots .$

Definition 3.1.3. The complement, C_{j-1}, of a subtree T_{j-1}, contains all of the elements of graph G in the complement of T_j having both vertices in T_{j-1}.

Theorem 3.1.1. If there exists a subtree T_{j-1}, of a tree T_j, in the system graph G such that the complement of T_{j-1} is nonempty, then the fundamental circuit matrix for the graph G can be written in the form

$$\mathcal{B}_f = \begin{bmatrix} \mathcal{B}_{11} & 0 & \mathcal{U} & 0 \\ \mathcal{B}_{21} & \mathcal{B}_{22} & 0 & \mathcal{U} \end{bmatrix} \qquad (3.1.1)$$

Proof: Let e_c be any element of C_{j-1}, e_c is a chord. There exists one and only one path in T_j spanning e_c. Both vertices of e_c are in T_{j-1} by definition 3.1.3. T_{j-1} is in T_j. Hence there is a path in T_{j-1} spanning e_c.

The elements of $\mathcal{B}_{21}$ and $\mathcal{B}_{22}$ correspond to branches of the tree T_j and the elements of $\mathcal{B}_{11}$ correspond to branches of T_{j-1}, and the first row corresponds to the f-circuits defined by C_{j-1}.

Theorem 3.1.2. If there exists a set of subtrees, T_{j-1}, T_{j-2}, . . . , T_0 of a graph G with T_{j-1} in T_j, T_{j-2} in T_{j-1}, . . . , T_0 in T_1, then the f-circuit equations for the graph G can be written in the form of 3.1.2.

$$\begin{bmatrix} \mathcal{B}_{11} & \mathcal{B}_{12} & 0 & 0 & \cdots & 0 \\ \mathcal{B}_{21} & \mathcal{B}_{22} & \mathcal{B}_{23} & 0 & \cdots & \cdot \\ \mathcal{B}_{31} & \mathcal{B}_{32} & \mathcal{B}_{33} & \mathcal{B}_{34} & & \cdot \\ \vdots & & & & \ddots & \vdots \\ \vdots & & & & \mathcal{B}_{j-1,j-2} & 0 \\ \mathcal{B}_{j-1,1} & \cdots & & & \mathcal{B}_{j-1,j-1} & \mathcal{B}_{j-1,j} \end{bmatrix} \begin{bmatrix} \mathcal{V}_1(t) \\ \mathcal{V}_2(t) \\ \vdots \\ \vdots \\ \mathcal{V}_{j-1}(t) \\ \mathcal{V}_j(t) \end{bmatrix}$$

$$= \begin{bmatrix} \mathcal{U} & 0 & \cdots & & 0 \\ 0 & \mathcal{U} & & & \cdot \\ \vdots & & & & \vdots \\ \vdots & & & \mathcal{U} & 0 \\ 0 & \cdots & & 0 & \mathcal{U} \end{bmatrix} \begin{bmatrix} \mathcal{V}_{c1}(t) \\ \mathcal{V}_{c2}(t) \\ \vdots \\ \mathcal{V}_{cj-1}(t) \\ \mathcal{V}_{cj}(t) \end{bmatrix} \qquad (3.2.2)$$

where $\mathcal{V}_j(t)$ correspond to the variables associated with the branches of T_j and $\mathcal{V}_{cj}(t)$ correspond to the variables associated with the chords C_j of T_j (j = 1, 2, 3 . . . .).

Proof: Partitioning the matrix j-1 times and applying theorem 3.2.1 j-1 times, the theorem is proven.

IV. PROPERTIES OF SYSTEMS CONTAINING MULTIWINDING INDUCTORS

4.1. Terminal Equations of a System Containing Multi-terminal Inductors.

In a preceding section the terminal equations and the transformation equations for the currents and voltages are given for a k winding inductor. In general a system may consist of many inductors each having different numbers of windings. In anticipation of future development, it is desirable to partition these equations into four groups.

The general form of the terminal equations for a system containing any arbitrary set of k-winding inductors can be arranged in the following general form:

$$\begin{bmatrix} \mathcal{V}_{t_1}(t) \\ \mathcal{V}_{t_2}(t) \\ \mathcal{V}_{c_1}(t) \\ \mathcal{V}_{c_2}(t) \end{bmatrix} = \begin{bmatrix} \mathcal{R}_{11} & 0 & 0 & 0 \\ 0 & \mathcal{R}_{22} & 0 & 0 \\ 0 & 0 & \mathcal{R}_{33} & 0 \\ 0 & 0 & 0 & \mathcal{R}_{44} \end{bmatrix} \begin{bmatrix} \mathcal{I}_{t_1}(t) \\ \mathcal{I}_{t_2}(t) \\ \mathcal{I}_{c_1}(t) \\ \mathcal{I}_{c_2}(t) \end{bmatrix}$$

$$+ \begin{bmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} & \mathcal{M}_{13} & \mathcal{M}_{14} \\ \mathcal{M}_{21} & \mathcal{M}_{22} & \mathcal{M}_{23} & \mathcal{M}_{23} \\ \mathcal{M}_{31} & \mathcal{M}_{32} & \mathcal{M}_{33} & \mathcal{M}_{34} \\ \mathcal{M}_{41} & \mathcal{M}_{42} & \mathcal{M}_{43} & \mathcal{M}_{44} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} \mathcal{I}_{t_1}(t) \\ \mathcal{I}_{t_2}(t) \\ \mathcal{I}_{c_1}(t) \\ \mathcal{I}_{c_2}(t) \end{bmatrix} \qquad (4.1.1)$$

It is shown in the following that under certain topologies the subscripts t and c designate tree and chord variables respectively.

The detailed form of the submatrices in equation 4.1.1 are best exemplified by an example.

Example 4.1.1. Let the system graph contain two two-winding, two three-winding, and two four-winding mutually-coupled components as shown in Figure 4.2.1.

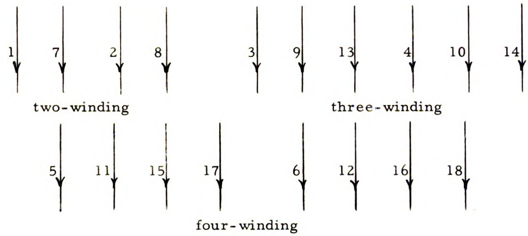

Figure 4.1.1

The resistance coefficient matrix is diagonal, hence for the purpose of this illustration, only the inductance coefficient matrix is considered. Let it be written as in 4.1.2.

Comparing 4.1.1 and 4.1.2, the entries in the voltage and current matrices are ordered as follows:

Each vector $\mathcal{V}_{t1}(t)$ and $\begin{bmatrix}\mathcal{V}_{c1}(t)\\ \mathcal{V}_{c2}(t)\end{bmatrix}$ contains one element of each of the mutually coupled components with the elements ordered such that the first entries are the variables associated with the components with the least number of windings, the next entries are the variables associated with the components with the next to the least number of windings, etc.

The vector $\mathcal{V}_{c1}(t)$ consists of the variables associated with the components with the least number of windings, except when the system contains only components with the same number of windings, then at least one but not all of the variables are placed in $\mathcal{V}_{c1}(t)$.

$$
\left[\begin{array}{c}
v_7(t)\\ v_8(t)\\ v_9(t)\\ v_{10}(t)\\ v_{11}(t)\\ v_{12}(t)\\ \hline
v_{13}(t)\\ v_{14}(t)\\ v_{15}(t)\\ v_{16}(t)\\ v_{17}(t)\\ v_{18}(t)\\ \hline
v_1(t)\\ v_2(t)\\ \hline
v_3(t)\\ v_4(t)\\ v_5(t)\\ v_6(t)
\end{array}\right]
=
\left[\begin{array}{cccccc|cccccc|cc|cccc}
M_{77} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_{71} & 0 & 0 & 0 & 0 & 0\\
0 & M_{88} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_{82} & 0 & 0 & 0 & 0\\
0 & 0 & M_{99} & 0 & 0 & 0 & M_{913} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_{93} & 0 & 0 & 0\\
0 & 0 & 0 & M_{1010} & 0 & 0 & 0 & M_{1014} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_{104} & 0 & 0\\
0 & 0 & 0 & 0 & M_{1111} & 0 & 0 & 0 & M_{1115} & 0 & M_{1117} & 0 & 0 & 0 & 0 & 0 & M_{115} & 0\\
0 & 0 & 0 & 0 & 0 & M_{1212} & 0 & 0 & 0 & M_{1216} & 0 & M_{1218} & 0 & 0 & 0 & 0 & 0 & M_{126}\\ \hline
0 & 0 & M_{139} & 0 & 0 & 0 & M_{1313} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_{133} & 0 & 0 & 0\\
0 & 0 & 0 & M_{1410} & 0 & 0 & 0 & M_{1414} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_{144} & 0 & 0\\
0 & 0 & 0 & 0 & M_{1511} & 0 & 0 & 0 & M_{1515} & 0 & M_{1517} & 0 & 0 & 0 & 0 & 0 & M_{155} & 0\\
0 & 0 & 0 & 0 & 0 & M_{1612} & 0 & 0 & 0 & M_{1616} & 0 & M_{1618} & 0 & 0 & 0 & 0 & 0 & M_{166}\\
0 & 0 & 0 & 0 & M_{1711} & 0 & 0 & 0 & M_{1715} & 0 & M_{1717} & 0 & 0 & 0 & 0 & 0 & M_{175} & 0\\
0 & 0 & 0 & 0 & 0 & M_{1812} & 0 & 0 & 0 & M_{1816} & 0 & M_{1818} & 0 & 0 & 0 & 0 & 0 & M_{186}\\ \hline
M_{17} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_{11} & 0 & 0 & 0 & 0 & 0\\
0 & M_{28} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_{22} & 0 & 0 & 0 & 0\\ \hline
0 & 0 & M_{39} & 0 & 0 & 0 & M_{313} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_{33} & 0 & 0 & 0\\
0 & 0 & 0 & M_{410} & 0 & 0 & 0 & M_{414} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_{44} & 0 & 0\\
0 & 0 & 0 & 0 & M_{511} & 0 & 0 & 0 & M_{515} & 0 & M_{517} & 0 & 0 & 0 & 0 & 0 & M_{55} & 0\\
0 & 0 & 0 & 0 & 0 & M_{612} & 0 & 0 & 0 & M_{616} & 0 & M_{618} & 0 & 0 & 0 & 0 & 0 & M_{66}
\end{array}\right]
\frac{d}{dt}
\left[\begin{array}{c}
i_7(t)\\ i_8(t)\\ i_9(t)\\ i_{10}(t)\\ i_{11}(t)\\ i_{12}(t)\\ \hline
i_{13}(t)\\ i_{14}(t)\\ i_{15}(t)\\ i_{16}(t)\\ i_{17}(t)\\ i_{18}(t)\\ \hline
i_1(t)\\ i_2(t)\\ \hline
i_3(t)\\ i_4(t)\\ i_5(t)\\ i_6(t)
\end{array}\right]
\qquad (4.1.2)
$$

The vector $\mathcal{V}_{t_1}(t)$ consists of the remainder of the variables associated with the mutually coupled components. These variables are ordered such that one variable of each component is placed in the first group, then one additional variable from each component in the second group, etc., until all the variables have been included.

There are several important properties of the terminal equations.

Property 4.1.1. The terminal equations form a symmetric matrix if $M_{nj} = M_{jn}$.

Property 4.1.2. $\mathcal{M}_{43} = \mathcal{M}_{34} = 0$, when the variables are ordered as shown in 4.1.2.

Property 4.1.3. If the system contains only two-winding inductors then $\mathcal{M}_{23}$, $\mathcal{M}_{32}$, $\mathcal{M}_{12}$, $\mathcal{M}_{21}$, $\mathcal{M}_{41}$, $\mathcal{M}_{14}$, are all zero matrices and all nonzero submatrices are diagonal.

Property 4.1.4. If the system contains no two-winding components then in 4.1.1 $\mathcal{M}_{43} = \mathcal{M}_{34} = 0$ and $\mathcal{M}_{11}$, $\mathcal{M}_{33}$, $\mathcal{M}_{44}$ are diagonal matrices.

4.2. The Current Transformations for Systems Containing Multiterminal Inductors.

The detailed form of the current transformation for an arbitrary collection of components can not be shown explicitly. However, the procedure can be established by an example. It is hoped the extension of this procedure to other systems is evident. Referring to 2.3.1 and retaining the ordering used in example 4.1.1, the current transformation equations are as shown in 4.2.1. For convenience, in general the current transformation for an arbitrary system can be in the partitioned matrix form.

$$\begin{bmatrix} \mathcal{I}^n_{t1}(t) \\ \mathcal{I}^n_{t2}(t) \\ \mathcal{I}^n_{c1}(t) \\ \mathcal{I}^n_{c2}(t) \end{bmatrix} = \begin{bmatrix} -\mathcal{N}_{11} & 0 & 0 & 0 \\ 0 & -\mathcal{N}_{22} & 0 & 0 \\ \mathcal{N}_{13} & \mathcal{N}_{23} & \mathcal{U} & 0 \\ \mathcal{N}_{14} & \mathcal{N}_{24} & 0 & \mathcal{U} \end{bmatrix} \begin{bmatrix} \mathcal{I}_{t1}(t) \\ \mathcal{I}_{t2}(t) \\ \mathcal{I}_{c1}(t) \\ \mathcal{I}_{c2}(t) \end{bmatrix} \tag{4.2.2}$$

$$
\left[\begin{array}{c}
\dot{i}^n_7(t)\\ \dot{i}^n_8(t)\\ \dot{i}^n_9(t)\\ \dot{i}^n_{10}(t)\\ \dot{i}^n_{11}(t)\\ \dot{i}^n_{12}(t)\\ \hline
\dot{i}^n_{13}(t)\\ \dot{i}^n_{14}(t)\\ \dot{i}^n_{15}(t)\\ \dot{i}^n_{16}(t)\\ \dot{i}^n_{17}(t)\\ \dot{i}^n_{18}(t)\\ \hline
\dot{i}^n_1(t)\\ \dot{i}^n_2(t)\\ \hline
\dot{i}^n_3(t)\\ \dot{i}^n_4(t)\\ \dot{i}^n_5(t)\\ \dot{i}^n_6(t)
\end{array}\right]
=
\left[\begin{array}{cccccc|cccccc|cc|cccc}
-n_{71} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & -n_{82} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & -n_{93} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & -n_{104} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & -n_{115} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -n_{126} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ \hline
0 & 0 & 0 & 0 & 0 & 0 & -n_{133} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -n_{144} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -n_{155} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -n_{166} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -n_{175} & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -n_{186} & 0 & 0 & 0 & 0 & 0 & 0\\ \hline
n_{71} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\
0 & n_{82} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0\\ \hline
0 & 0 & n_{93} & 0 & 0 & 0 & n_{133} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\
0 & 0 & 0 & n_{104} & 0 & 0 & 0 & n_{144} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\
0 & 0 & 0 & 0 & n_{115} & 0 & 0 & 0 & n_{155} & 0 & n_{175} & 0 & 0 & 0 & 0 & 0 & 1 & 0\\
0 & 0 & 0 & 0 & 0 & n_{126} & 0 & 0 & 0 & n_{166} & 0 & n_{186} & 0 & 0 & 0 & 0 & 0 & 1
\end{array}\right]
\left[\begin{array}{c}
i_7(t)\\ i_8(t)\\ i_9(t)\\ i_{10}(t)\\ i_{11}(t)\\ i_{12}(t)\\ \hline
i_{13}(t)\\ i_{14}(t)\\ i_{15}(t)\\ i_{16}(t)\\ i_{17}(t)\\ i_{18}(t)\\ \hline
i_1(t)\\ i_2(t)\\ \hline
i_3(t)\\ i_4(t)\\ i_5(t)\\ i_6(t)
\end{array}\right]
\tag{4.2.1}
$$

There are several important properties of the current transformation matrix to be used.

Property 4.2.1. If the system contains only two-winding inductors $\mathcal{N}_{14}$, and $\mathcal{N}_{23}$ are both zero and $\mathcal{N}_{13}$ and $\mathcal{N}_{24}$ have the same entries as $\mathcal{N}_{11}$ and $\mathcal{N}_{22}$, respectively.

Property 4.2.2. If the system contains at least one two-winding inductor in addition to other multiwinding inductors, then $\mathcal{N}_{23} = 0$.

As a solution to 4.2.2, we have

$$\begin{bmatrix} \mathcal{I}_{t1}(t) \\ \mathcal{I}_{t2}(t) \\ \mathcal{I}_{c1}(t) \\ \mathcal{I}_{c2}(t) \end{bmatrix} = \begin{bmatrix} -\mathcal{N}_{11}^{-1} & 0 & 0 & 0 \\ 0 & -\mathcal{N}_{22}^{-1} & 0 & 0 \\ \mathcal{N}_{13}\mathcal{N}_{11}^{-1} & \mathcal{N}_{23}\mathcal{N}_{22}^{-1} & \mathcal{U} & 0 \\ \mathcal{N}_{14}\mathcal{N}_{11}^{-1} & \mathcal{N}_{24}\mathcal{N}_{22}^{-1} & 0 & \mathcal{U} \end{bmatrix} \begin{bmatrix} \mathcal{I}^{n}_{t1}(t) \\ \mathcal{I}^{n}_{t2}(t) \\ \mathcal{I}^{n}_{c1}(t) \\ \mathcal{I}^{n}_{c2}(t) \end{bmatrix} \quad (4.2.3$$

Property 4.2.3. If the system contains only two-winding components then $\mathcal{N}_{13}\mathcal{N}_{11}^{-1}$ and $\mathcal{N}_{24}\mathcal{N}_{22}^{-1}$ are negative unit matrices and $\mathcal{N}_{14}\mathcal{N}_{11}^{-1}$ and $\mathcal{N}_{23}\mathcal{N}_{22}^{-1}$ are zero.

4.3. The Voltage Transformations for Systems Containing Multiterminal Components.

The coefficient matrix for voltage transformations is chosen as the inverse transpose of the coefficient matrix in current transformation equations.

$$\begin{bmatrix} \mathcal{V}^{n}_{t1}(t) \\ \mathcal{V}^{n}_{t2}(t) \\ \mathcal{V}^{n}_{c_1}(t) \\ \mathcal{V}^{n}_{c2}(t) \end{bmatrix} = \begin{bmatrix} -\mathcal{N}_{11}^{-1} & 0 & \mathcal{N}_{13}\mathcal{N}_{11}^{-1} & \mathcal{N}_{14}\mathcal{N}_{11}^{-1} \\ 0 & -\mathcal{N}_{22}^{-1} & \mathcal{N}_{23}\mathcal{N}_{22}^{-1} & \mathcal{N}_{24}\mathcal{N}_{22}^{-1} \\ 0 & 0 & \mathcal{U} & 0 \\ 0 & 0 & 0 & \mathcal{U} \end{bmatrix} \begin{bmatrix} \mathcal{V}_{t1}(t) \\ \mathcal{V}_{t2}(t) \\ \mathcal{V}_{c1}(t) \\ \mathcal{V}_{c2}(t) \end{bmatrix} \quad (4.3.1)$$

$$\begin{bmatrix} \mathcal{V}_{t1}(t) \\ \mathcal{V}_{t2}(t) \\ \mathcal{V}_{c1}(t) \\ \mathcal{V}_{c2}(t) \end{bmatrix} = \begin{bmatrix} -\mathcal{N}_{11} & 0 & \mathcal{N}_{13} & \mathcal{N}_{14} \\ 0 & -\mathcal{N}_{22} & \mathcal{N}_{23} & \mathcal{N}_{24} \\ 0 & 0 & \mathcal{U} & 0 \\ 0 & 0 & 0 & \mathcal{U} \end{bmatrix} \begin{bmatrix} \mathcal{V}^n_{t1}(t) \\ \mathcal{V}^n_{t2}(t) \\ \mathcal{V}^n_{c1}(t) \\ \mathcal{V}^n_{c2}(t) \end{bmatrix} \qquad (4.3.2)$$

4.4. The Transformed Terminal Equations.

In Section 2.3 the transformed terminal equations for a k-winding inductor are established. Instead of pre and post multiplying the original terminal equations by the voltage and current transformation matrices respectively, which may be tedious, it is convenient to regard the terminal equations written in terms of the ampere turns variables (the new variables) as the given terminal equations. The coefficients of these equations can be determined directly in the laboratory by the methods discussed in Section 2.3.

It is next shown that it is possible to formulate the equations for the system from the component terminal equations given in terms of the ampere-turns variables.

Let the ampere-turns form of the terminal equation for the components be arranged in the following order.

$$\begin{bmatrix} \mathcal{V}^n_{t1}(t) \\ \mathcal{V}^n_{t2}(t) \\ \mathcal{V}^n_{c1}(t) \\ \mathcal{V}^n_{c2}(t) \end{bmatrix} = \begin{bmatrix} \mathcal{R}^n_{11} & \mathcal{R}^n_{12} & \mathcal{R}^n_{13} & \mathcal{R}^n_{14} \\ \mathcal{R}^n_{21} & \mathcal{R}^n_{22} & \mathcal{R}^n_{23} & \mathcal{R}^n_{24} \\ \mathcal{R}^n_{31} & \mathcal{R}^n_{32} & \mathcal{R}^n_{33} & \mathcal{R}^n_{34} \\ \mathcal{R}^n_{41} & \mathcal{R}^n_{42} & \mathcal{R}^n_{43} & \mathcal{R}^n_{44} \end{bmatrix} \begin{bmatrix} \mathcal{I}^n_{t1}(t) \\ \mathcal{I}^n_{t2}(t) \\ \mathcal{I}^n_{c1}(t) \\ \mathcal{I}^n_{c2}(t) \end{bmatrix}$$

$$+ \begin{bmatrix} \mathcal{L}^n_{11} & \mathcal{L}^n_{12} & \mathcal{L}^n_{13} & \mathcal{L}^n_{14} \\ \mathcal{L}^n_{21} & \mathcal{L}^n_{22} & \mathcal{L}^n_{23} & \mathcal{L}^n_{24} \\ \mathcal{L}^n_{31} & \mathcal{L}^n_{32} & \mathcal{L}^n_{33} & \mathcal{L}^n_{34} \\ \mathcal{L}^n_{41} & \mathcal{L}^n_{42} & \mathcal{L}^n_{43} & \mathcal{L}^n_{44} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} \mathcal{I}^n_{t1}(t) \\ \mathcal{I}^n_{t2}(t) \\ \mathcal{I}^n_{c1}(t) \\ \mathcal{I}^n_{c2}(t) \end{bmatrix} \qquad (4.4.1)$$

Again it is to be emphasized that in any practical problem the terminal equations for the high permeability, nonlinear transformers would be given in this form. The detailed form of the coefficient matrices in the above equations are best examplified by an example. For comparison, the ampere-turns terminal equations are established for Example 4.1.1, and given by 4.4.2, where $R_{ii} = R_j + n_{ji}^2 R_i$. The subscripts i and j refer to the variables associated with the elements classified as branches and chords, respectively, when the components are included in a system.

There are several important properties of these ampere-turns terminal equations.

Property 4.3.1. The terminal equations form a symmetric matrix if $L^n_{nj} = L^n_{jn}$, i.e. $M_{nj} = M_{jn}$.

Property 4.3.2. $\mathcal{L}^n_{34}$, $\mathcal{L}^n_{43}$, $\mathcal{L}^n_{34}$, and $\mathcal{L}^n_{43}$ are zero when the variables are ordered as in 4.3.2.

Property 4.3.3. If the system contains at least one two-winding inductor in addition to multiwinding inductors, then $\mathcal{L}^n_{23}$, $\mathcal{L}^n_{32}$, $\mathcal{L}^n_{23}$, and $\mathcal{L}^n_{32}$ are zeros.

Property 4.5.4. If all the inductors in the system contain only two-windings, then $\mathcal{L}^n_{23}$, $\mathcal{L}^n_{32}$, $\mathcal{L}^n_{12}$, $\mathcal{L}^n_{21}$, $\mathcal{L}^n_{14}$, $\mathcal{L}^n_{41}$, $\mathcal{R}^n_{23}$, $\mathcal{R}^n_{32}$, $\mathcal{R}^n_{12}$, $\mathcal{R}^n_{21}$, $\mathcal{R}^n_{14}$, and $\mathcal{R}^n_{41}$ are zero and the remaining sub-matrices are diagonal.

$$
\begin{bmatrix} v_7^n(t) \\ v_8^n(t) \\ v_9^n(t) \\ v_{10}^n(t) \\ v_{11}^n(t) \\ v_{12}^n(t) \\ \hline v_{13}^n(t) \\ v_{14}^n(t) \\ v_{15}^n(t) \\ v_{16}^n(t) \\ v_{17}^n(t) \\ v_{18}^n(t) \\ \hline v_1^n(t) \\ v_2^n(t) \\ \hline v_3^n(t) \\ v_4^n(t) \\ v_5^n(t) \\ v_6^n(t) \end{bmatrix}
=
\left[\begin{array}{cccccc|cccccc|cc|cccc}
R_{77} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_1 & 0 & 0 & 0 & 0 & 0 \\
0 & R_{88} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_2 & 0 & 0 & 0 & 0 \\
0 & 0 & R_{99} & 0 & 0 & 0 & R_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_3 & 0 & 0 & 0 \\
0 & 0 & 0 & R_{1010} & 0 & 0 & 0 & R_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_4 & 0 & 0 \\
0 & 0 & 0 & 0 & R_{1111} & 0 & 0 & 0 & R_5 & 0 & R_5 & 0 & 0 & 0 & 0 & 0 & R_5 & 0 \\
0 & 0 & 0 & 0 & 0 & R_{1212} & 0 & 0 & 0 & R_6 & 0 & R_6 & 0 & 0 & 0 & 0 & 0 & R_6 \\
\hline
0 & 0 & R_3 & 0 & 0 & 0 & R_{1313} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_3 & 0 & 0 & 0 \\
0 & 0 & 0 & R_4 & 0 & 0 & 0 & R_{1414} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_4 & 0 & 0 \\
0 & 0 & 0 & 0 & R_5 & 0 & 0 & 0 & R_{1515} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_5 & 0 \\
0 & 0 & 0 & 0 & 0 & R_6 & 0 & 0 & 0 & R_{1616} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_6 \\
0 & 0 & 0 & 0 & R_5 & 0 & 0 & 0 & 0 & 0 & R_{1717} & 0 & 0 & 0 & 0 & 0 & R_5 & 0 \\
0 & 0 & 0 & 0 & 0 & R_6 & 0 & 0 & 0 & 0 & 0 & R_{1818} & 0 & 0 & 0 & 0 & 0 & R_6 \\
\hline
R_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_1 & 0 & 0 & 0 & 0 & 0 \\
0 & R_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_2 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & R_3 & 0 & 0 & 0 & R_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_3 & 0 & 0 & 0 \\
0 & 0 & 0 & R_4 & 0 & 0 & 0 & R_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & R_4 & 0 & 0 \\
0 & 0 & 0 & 0 & R_5 & 0 & 0 & 0 & R_5 & 0 & R_5 & 0 & 0 & 0 & 0 & 0 & R_5 & 0 \\
0 & 0 & 0 & 0 & 0 & R_6 & 0 & 0 & 0 & R_6 & 0 & R_6 & 0 & 0 & 0 & 0 & 0 & R_6
\end{array}\right]
\begin{bmatrix} i_7^n(t) \\ i_8^n(t) \\ i_9^n(t) \\ i_{10}^n(t) \\ i_{11}^n(t) \\ i_{12}^n(t) \\ \hline i_{13}^n(t) \\ i_{14}^n(t) \\ i_{15}^n(t) \\ i_{16}^n(t) \\ i_{17}^n(t) \\ i_{18}^n(t) \\ \hline i_1^n(t) \\ i_2^n(t) \\ \hline i_3^n(t) \\ i_4^n(t) \\ i_5^n(t) \\ i_6^n(t) \end{bmatrix}
\quad (4.4.2)
$$

(cont'd on next page)

$$
+\left[\begin{array}{cccccc|cccccc|cc|cccc}
L^n_{77} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & L^n_{17} & 0 & 0 & 0 & 0 & 0 \\
0 & L^n_{88} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & L^n_{28} & 0 & 0 & 0 & 0 \\
0 & 0 & L^n_{99} & 0 & 0 & 0 & L^n_{913} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & L^n_{93} & 0 & 0 & 0 \\
0 & 0 & 0 & L^n_{1010} & 0 & 0 & 0 & L^n_{1014} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & L^n_{104} & 0 & 0 \\
0 & 0 & 0 & 0 & L^n_{1111} & 0 & 0 & 0 & L^n_{1115} & 0 & L^n_{1117} & 0 & 0 & 0 & 0 & 0 & L^n_{115} & 0 \\
0 & 0 & 0 & 0 & 0 & L^n_{1212} & 0 & 0 & 0 & L^n_{1216} & 0 & L^n_{1218} & 0 & 0 & 0 & 0 & 0 & L^n_{116} \\
\hline
0 & 0 & L^n_{913} & 0 & 0 & 0 & L^n_{1313} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & L^n_{134} & 0 & 0 & 0 \\
0 & 0 & 0 & L^n_{1014} & 0 & 0 & 0 & L^n_{1414} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & L^n_{144} & 0 & 0 \\
0 & 0 & 0 & 0 & L^n_{1115} & 0 & 0 & 0 & L^n_{1515} & 0 & L^n_{1517} & 0 & 0 & 0 & 0 & 0 & L^n_{155} & 0 \\
0 & 0 & 0 & 0 & 0 & L^n_{1216} & 0 & 0 & 0 & L^n_{1616} & 0 & L^n_{1618} & 0 & 0 & 0 & 0 & 0 & L^n_{166} \\
0 & 0 & 0 & 0 & L^n_{1117} & 0 & 0 & 0 & L^n_{1517} & 0 & L^n_{1717} & 0 & 0 & 0 & 0 & 0 & L^n_{175} & 0 \\
0 & 0 & 0 & 0 & 0 & L^n_{1218} & 0 & 0 & 0 & L^n_{1618} & 0 & L^n_{1818} & 0 & 0 & 0 & 0 & 0 & L^n_{186} \\
\hline
L^n_{71} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_{11} & 0 & 0 & 0 & 0 & 0 \\
0 & L^n_{82} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_{22} & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & L^n_{93} & 0 & 0 & 0 & L^n_{134} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & L^n_{104} & 0 & 0 & 0 & L^n_{144} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & M_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & L^n_{1115} & 0 & 0 & 0 & L^n_{155} & 0 & L^n_{175} & 0 & 0 & 0 & 0 & 0 & M_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & L^n_{116} & 0 & 0 & 0 & L^n_{166} & 0 & L^n_{186} & 0 & 0 & 0 & 0 & 0 & M_{66}
\end{array}\right]
\frac{d}{dt}
\left[\begin{array}{c}
i^n_7(t) \\ i^n_8(t) \\ i^n_9(t) \\ i^n_{10}(t) \\ i^n_{11}(t) \\ i^n_{12}(t) \\ \hline
i^n_{13}(t) \\ i^n_{14}(t) \\ i^n_{15}(t) \\ i^n_{16}(t) \\ i^n_{17}(t) \\ i^n_{18}(t) \\ \hline
i^n_1(t) \\ i^n_2(t) \\ \hline
i^n_3(t) \\ i^n_4(t) \\ i^n_5(t) \\ i^n_6(t)
\end{array}\right]
\qquad (4.4.2)
$$

V. SUFFICIENCY CRITERIA

5.1. The Original and Transformed Circuit Equations

Let the system graph be such that the fundamental circuit equations can be partitioned in the form

$$\begin{bmatrix} \mathcal{B}_{11} & \mathcal{B}_{12} & \mathcal{B}_{13} & \mathcal{B}_{14} & \mathcal{U} & 0 & 0 & 0 \\ \mathcal{B}_{21} & \mathcal{B}_{22} & \mathcal{B}_{23} & \mathcal{B}_{24} & 0 & \mathcal{U} & 0 & 0 \\ \mathcal{B}_{31} & \mathcal{B}_{32} & \mathcal{B}_{33} & \mathcal{B}_{34} & 0 & 0 & \mathcal{U} & 0 \\ \mathcal{B}_{41} & \mathcal{B}_{42} & \mathcal{B}_{43} & \mathcal{B}_{44} & 0 & 0 & 0 & \mathcal{U} \end{bmatrix} \begin{bmatrix} \mathcal{V}_{ne}(t) \\ \mathcal{V}_{rt}(t) \\ \mathcal{V}_{t1}(t) \\ \mathcal{V}_{t2}(t) \\ \mathcal{V}_{c1}(t) \\ \mathcal{V}_{c2}(t) \\ \mathcal{V}_{rc}(t) \\ \mathcal{V}_{nh}(t) \end{bmatrix} = 0 \tag{5.1.1}$$

where the indicated partitioning is the same as in 4.1.1.

Applying the transformation of variable in 4.3.2 to 5.1.1, the transformed circuit equations become:

$$\begin{bmatrix} \mathcal{B}_{11} & \mathcal{B}_{12} & \mathcal{B}_{13}\,\mathcal{N}_{11} & \mathcal{B}_{14}\,\mathcal{N}_{22} \\ \mathcal{B}_{21} & \mathcal{B}_{22} & \mathcal{B}_{23}\,\mathcal{N}_{11} & \mathcal{B}_{24}\,\mathcal{N}_{22} \\ \mathcal{B}_{31} & \mathcal{B}_{32} & \mathcal{B}_{33}\,\mathcal{N}_{11} & \mathcal{B}_{34}\,\mathcal{N}_{22} \\ \mathcal{B}_{41} & \mathcal{B}_{42} & \mathcal{B}_{43}\,\mathcal{N}_{11} & \mathcal{B}_{44}\,\mathcal{N}_{22} \end{bmatrix} \begin{bmatrix} \mathcal{V}_{ne}(t) \\ \mathcal{V}_{rt}(t) \\ \mathcal{V}_{t1}(t) \\ \mathcal{V}_{t2}(t) \end{bmatrix} \tag{5.1.2}$$

$$= - \begin{bmatrix} \mathcal{B}_{13}\mathcal{N}_{13} + \mathcal{B}_{14}\mathcal{N}_{23} + \mathcal{U} & \mathcal{B}_{13}\mathcal{N}_{14} + \mathcal{B}_{14}\mathcal{N}_{24} & 0 & 0 \\ \mathcal{B}_{23}\mathcal{N}_{13} + \mathcal{B}_{24}\mathcal{N}_{23} & \mathcal{B}_{23}\mathcal{N}_{14} + \mathcal{B}_{24}\mathcal{N}_{24} + \mathcal{U} & 0 & 0 \\ \mathcal{B}_{33}\mathcal{N}_{13} + \mathcal{B}_{34}\mathcal{N}_{23} & \mathcal{B}_{33}\mathcal{N}_{14} + \mathcal{B}_{34}\mathcal{N}_{24} & \mathcal{U} & 0 \\ \mathcal{B}_{43}\mathcal{N}_{13} + \mathcal{B}_{44}\mathcal{N}_{23} & \mathcal{B}_{43}\mathcal{N}_{14} + \mathcal{B}_{44}\mathcal{N}_{24} & 0 & \mathcal{U} \end{bmatrix} \begin{bmatrix} \mathcal{V}^{n}_{c1}(t) \\ \mathcal{V}^{n}_{c2}(t) \\ \mathcal{V}_{rc}(t) \\ \mathcal{V}_{nh}(t) \end{bmatrix}$$

5.2. The Original and Transformed Segregate Equations

Using the same formulation tree as in 5.1.1 the fundamental segregate equations may be partitioned in the form[12]

$$
\begin{bmatrix}
\mathcal{U} & 0 & 0 & 0 & \mathcal{S}_{11} & \mathcal{S}_{12} & \mathcal{S}_{13} & \mathcal{S}_{14} \\
0 & \mathcal{U} & 0 & 0 & \mathcal{S}_{21} & \mathcal{S}_{22} & \mathcal{S}_{23} & \mathcal{S}_{24} \\
0 & 0 & \mathcal{U} & 0 & \mathcal{S}_{31} & \mathcal{S}_{32} & \mathcal{S}_{33} & \mathcal{S}_{34} \\
0 & 0 & 0 & \mathcal{U} & \mathcal{S}_{41} & \mathcal{S}_{42} & \mathcal{S}_{43} & \mathcal{S}_{44}
\end{bmatrix}
\begin{bmatrix}
\mathcal{I}_{ne}(t) \\ \mathcal{I}_{rt}(t) \\ \mathcal{I}_{t1}(t) \\ \mathcal{I}_{t2}(t) \\ \mathcal{I}_{c1}(t) \\ \mathcal{I}_{c2}(t) \\ \mathcal{I}_{rc}(t) \\ \mathcal{H}(t)
\end{bmatrix}
= 0 \qquad (5.2.1)
$$

where the indicated partitioning is the same as in 4.1.1. Substituting 4.2.3 into 5.2.1 and partitioning, the transformed segregate equations become:

$$
\begin{bmatrix}
\mathcal{U} & 0 & (\mathcal{S}_{11}\mathcal{N}_{13} + \mathcal{S}_{12}\mathcal{N}_{14})\mathcal{N}_{11}^{-1} & (\mathcal{S}_{11}\mathcal{N}_{23} + \mathcal{S}_{12}\mathcal{N}_{24})\mathcal{N}_{22}^{-1} \\
0 & \mathcal{U} & (\mathcal{S}_{21}\mathcal{N}_{13} + \mathcal{S}_{22}\mathcal{N}_{14})\mathcal{N}_{11}^{-1} & (\mathcal{S}_{21}\mathcal{N}_{23} + \mathcal{S}_{22}\mathcal{N}_{24})\mathcal{N}_{22}^{-1} \\
0 & 0 & (\mathcal{S}_{31}\mathcal{N}_{13} + \mathcal{S}_{32}\mathcal{N}_{14} + \mathcal{U})\mathcal{N}_{11}^{-1} & (\mathcal{S}_{31}\mathcal{N}_{23} + \mathcal{S}_{32}\mathcal{N}_{24})\mathcal{N}_{22}^{-1} \\
0 & 0 & (\mathcal{S}_{41}\mathcal{N}_{13} + \mathcal{S}_{42}\mathcal{N}_{14})\mathcal{N}_{11}^{-1} & (\mathcal{S}_{41}\mathcal{N}_{23} + \mathcal{S}_{42}\mathcal{N}_{24} + \mathcal{U})\mathcal{N}_{22}^{-1}
\end{bmatrix}
\begin{bmatrix}
\mathcal{I}_{ne}(t) \\ \mathcal{I}_{rt}(t) \\ \mathcal{I}_{t1}(t) \\ \mathcal{I}_{t2}(t)
\end{bmatrix}
$$

$$
= - \begin{bmatrix}
\mathcal{S}_{11} & \mathcal{S}_{12} & \mathcal{S}_{13} & \mathcal{S}_{14} \\
\mathcal{S}_{21} & \mathcal{S}_{22} & \mathcal{S}_{23} & \mathcal{S}_{24} \\
\mathcal{S}_{31} & \mathcal{S}_{32} & \mathcal{S}_{33} & \mathcal{S}_{34} \\
\mathcal{S}_{41} & \mathcal{S}_{42} & \mathcal{S}_{43} & \mathcal{S}_{44}
\end{bmatrix}
\begin{bmatrix}
\mathcal{I}^{n}_{c1}(t) \\ \mathcal{I}^{n}_{c2}(t) \\ \mathcal{I}_{rc}(t) \\ \mathcal{H}(t)
\end{bmatrix}
\qquad (5.2.2.)
$$

5.3. The Location in the Network of the Inductor Elements.

A sufficient condition for the ampere-turns of each inductor to appear explicitly in the system equations is

$$\begin{vmatrix} (\mathcal{S}_{31}\mathcal{N}_{13} + \mathcal{S}_{42}\mathcal{N}_{14} - \mathcal{U})\mathcal{N}_{11}^{-1} & (\mathcal{S}_{31}\mathcal{N}_{23} + \mathcal{S}_{32}\mathcal{N}_{24})\mathcal{N}_{22}^{-1} \\ (\mathcal{S}_{41}\mathcal{N}_{13} + \mathcal{S}_{42}\mathcal{N}_{14})\mathcal{N}_{11}^{-1} & (\mathcal{S}_{41}\mathcal{N}_{23} + \mathcal{S}_{42}\mathcal{N}_{24} - \mathcal{U})\mathcal{N}_{22}^{-1} \end{vmatrix} \neq 0$$

(5.3.1)

Referring to 5.2.2, the coefficient matrix on the left is non-singular, hence the variables on the left may be expressed in terms of the variables on the right. Therefore the total ampere turns variable of each inductor appear in the equations of the system graph explicitly.

The location of the elements of these inductors must be investigated in order to determine whether the determinant of 5.3.1 is non-singular.

Definition 5.3.1. Let S_j be defined as the set of inductor elements in the system.

Definition 5.3.2. Let S_{j-1} be defined as a subset of S_j, j = 1, 2, ...

Theorem 5.3.1. If there exists a subtree T_1 of a tree T, in the system graph, G, which contains no elements of S_j and if the complement of T_1 contains at least one element of each inductor, then the ampere-turns variable for each inductor in S_j will appear explicitly in the system equations.

Proof. Let the elements of $\mathcal{V}_{ne}(t)$, $\mathcal{V}_{rt}(t)$, $\mathcal{V}_{t1}(t)$, and $\mathcal{V}_{t2}(t)$ be in tree T. Also let $\mathcal{V}_{ne}(t)$ and $\mathcal{V}_{rt}(t)$ be in T_1. By Theorem 3.1.1 $\mathcal{B}_{13}$, $\mathcal{B}_{14}$, $\mathcal{B}_{23}$, and $\mathcal{B}_{24}$ in 5.1.1 contain only zeros. If the same tree is used to formulate the circuit and segregate equations, then $\mathcal{B}_{ij} = \mathcal{S}_{ji}'$ [11] Then $\mathcal{S}_{31}$, $\mathcal{S}_{41}$, $\mathcal{S}_{32}$, and $\mathcal{S}_{42}$ in 5.3.1 contain only zeros, hence the determinant 5.3.1 is non-singular, as $\mathcal{N}_{11}^{-1}$ and $\mathcal{N}_{22}^{-1}$ are non-singular by definition. The theorem follows.

The preceding theorem requires that at least one element of each inductor is in a circuit which contains no other elements of any inductor.

If the system contains only two-winding inductors the hypothesis of Theorem 5.3.1 can be relaxed.

Definition 5.3.3. The set of elements S_{cj-1}, is defined as a subset of the elements of S_{j-1} contained in the complement of the tree T_{j-1} of the system graph G.

Theorem 5.3.2. If all inductors in the system contain only two-windings and if there exists a set of subtrees T_{j-1}, T_{j-2}, . . . T_0 with T_{j-1} in T_j, T_{j-2} in T_{j-1}, . . . , T_0 in T_1, such that the f-circuits defined by S_{cj-1}, S_{cj-2}, . . . S_{c0} can be arranged such that each contains one less branch element of S_j, than the preceding, then the ampere-turns variables appear explicitly in the system equations.

Proof. Referring to 5.1.1 and considering the first two rows, by theorem 3.1.2, $\mathcal{B}_{13} = \mathcal{B}_{14} = \mathcal{B}_{24} = 0$. Hence $\mathcal{L}_{31} = \mathcal{L}_{41} = \mathcal{L}_{42} = 0$. According to property 4.2.1, $\mathcal{M}_{23} = \mathcal{M}_{14} = 0$. Thus, 5.3.1 is non-singular.

VI. EXAMPLE

6.1. Equations to be Solved

The previous sections have been devoted to the development of sufficient criteria on the location of the elements of k-winding inductors in the system graph, such that the ampere-turns variable of the inductors will appear explicitly in the system equations. The application of these criteria in formulating the equations for a typical magnetic amplifier is demonstrated in the first part of this section. The system equations are then prepared for solution on the digital computer to obtain the output current of the magnetic amplifier, for various input voltages. Many of the steps involved in the preparation of the equations, such that they are adaptable to computer solution are omitted and left to the reader to fill in.

The circuit diagram of the magnetic amplifier to be considered is shown in Figure 6.1.1 and its linear graph in Figure 6.1.2.

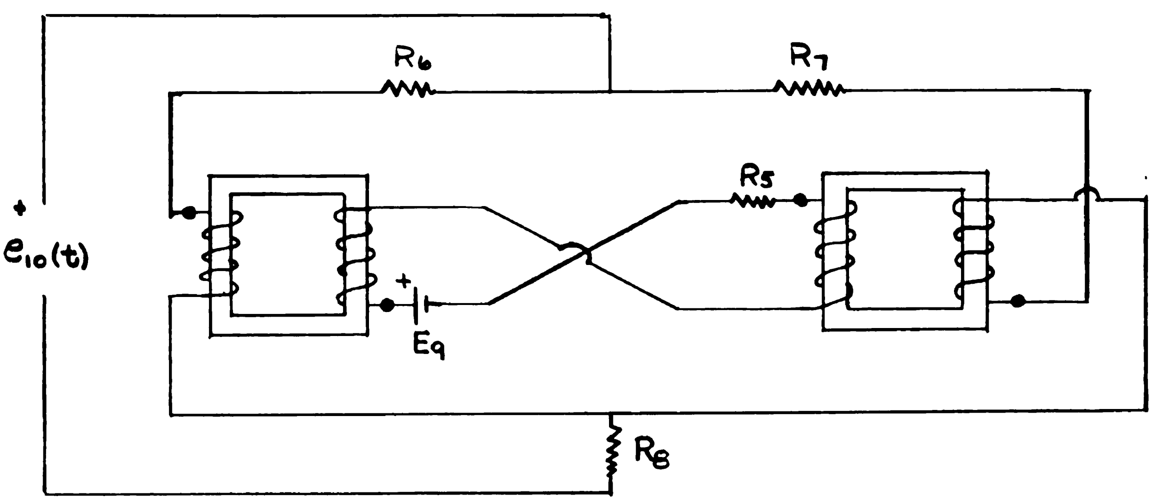

Figure 6.1.1

6.1. Equ[illegible]

The [illegible] of a sufficient [illegible] saturating inductors in the [illegible] of the inductors [illegible]. For application of these [illegible] magnetic amplifier [illegible]. This system equations [illegible] computer to obtain the [illegible] input voltage [illegible] the equations such that [illegible] omitted and left to the reader.

(1) [illegible] to be considered is shown [illegible] Figure 6.1.2.

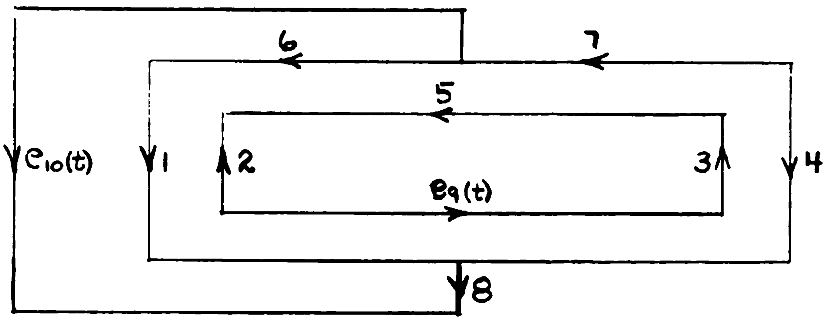

Figure 6.1.2

The "dots" on Figure 6.1.1 signify the winding orientation.

Choosing the elements 4, 1, and 2 as chords, the fundamental circuit equations are

$$\begin{bmatrix} +1 & 0 \\ +1 & 0 \\ 0 & +1 \end{bmatrix} \begin{bmatrix} e_{10}(t) \\ e_9(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_5(t) \\ v_6(t) \\ v_7(t) \\ v_8(t) \\ v_3(t) \\ v_4(t) \\ v_1(t) \\ v_2(t) \end{bmatrix} \qquad (6.1.1)$$

The voltage transformation used is

$$\begin{bmatrix} v_3(t) \\ v_4(t) \\ v_1(t) \\ v_2(t) \end{bmatrix} = \begin{bmatrix} -n_{34} & n_{34} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & n_{21} & -n_{21} \end{bmatrix} \begin{bmatrix} v_3^n(t) \\ v_4^n(t) \\ v_1^n(t) \\ v_2^n(t) \end{bmatrix} \qquad (6.1.2)$$

Substituting 6.1.2 into 6.1.1, the transformed circuit equations become:

$$\begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} e_{10}(t) \\ e_9(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 & n_{34} & -n_{34} & n_{21} & -n_{21} \end{bmatrix} \begin{bmatrix} v_5(t) \\ v_6(t) \\ v_7(t) \\ v_8(t) \\ v_3(t) \\ v_4(t) \\ v_1(t) \\ v_2(t) \end{bmatrix} \qquad (6.1.3)$$

The fundamental segregate equations are:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_{10}(t) \\ i_9(t) \\ i_5(t) \\ i_6(t) \\ i_7(t) \\ i_8(t) \\ i_3(t) \\ i_4(t) \\ i_1(t) \\ i_2(t) \end{bmatrix} = 0 \qquad (6.1.4)$$

The current transformation used is

$$\begin{bmatrix} i_3(t) \\ i_4(t) \\ i_1(t) \\ i_2(t) \end{bmatrix} = \begin{bmatrix} -n_{43} & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & -n_{12} \end{bmatrix} \begin{bmatrix} i_3^n(t) \\ i_4^n(t) \\ i_1^n(t) \\ i_2^n(t) \end{bmatrix} \qquad (6.1.5)$$

Substituting 6.1.5 into 6.1.4, the transformed segregate equations become

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -n_{12} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & -n_{12} \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -n_{43} & 0 & 0 & -n_{12} \end{bmatrix} \begin{bmatrix} i_{10}(t) \\ i_9(t) \\ i_5(t) \\ i_6(t) \\ i_7(t) \\ i_8(t) \\ \overset{n}{i_3}(t) \\ \overset{n}{i_4}(t) \\ \overset{n}{i_1}(t) \\ \overset{n}{i_2}(t) \end{bmatrix} = 0 \qquad (6.1.6)$$

Removing the first two equations, which contain the voltage source current, the reduced set of transformed segregate equations may be written in the following form:

$$\begin{bmatrix} i_5(t) \\ i_6(t) \\ i_7(t) \\ i_8(t) \\ \overset{n}{i_3}(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & n_{12} \\ 0 & 1 & 1 \\ -1 & 0 & n_{12}n_{34} \\ 1 & 1 & (1-n_{12}n_{34}) \\ 0 & 0 & -n_{12}n_{34} \end{bmatrix} \begin{bmatrix} \overset{n}{i_4}(t) \\ \overset{n}{i_1}(t) \\ \overset{n}{i_2}(t) \end{bmatrix} \qquad (6.1.7)$$

where the two variables, $\overset{n}{i_4}(t) = n_{34}i_3(t) + i_4(t)$, $\overset{n}{i_1}(t) = n_{21}i_2(t) + i_1(t)$, are the ampere-turns variables, one associated with each inductor.

The leakage inductance form of the inductor component terminal equations along with the terminal equations for the other components in the system are

$$\begin{bmatrix} v_5(t) \\ v_6(t) \\ v_7(t) \\ v_8(t) \\ \overset{n}{v_3}(t) \\ \overset{n}{v_4}(t) \\ \overset{n}{v_1}(t) \\ \overset{n}{v_2}(t) \end{bmatrix} = \begin{bmatrix} R_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & R_6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & R_7 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & R_8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & (R_4+n_{43}^2R_3) & R_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & R_4 & R_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & R_1 & R_1 \\ 0 & 0 & 0 & 0 & 0 & 0 & R_1 & (R_1+n_{12}^2R_2) \end{bmatrix} \begin{bmatrix} i_5(t) \\ i_6(t) \\ i_7(t) \\ i_8(t) \\ \overset{n}{i_3}(t) \\ \overset{n}{i_4}(t) \\ \overset{n}{i_1}(t) \\ \overset{n}{i_2}(t) \end{bmatrix} \qquad (6.1.8)$$

$$+\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & (L_{43}+n_{43}^2L_{34}) & L_{43} & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{43} & M_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & M_{11} & L_{12} \\ 0 & 0 & 0 & 0 & 0 & 0 & L_{12} & (L_{12}+n_{12}^2L_{21}) \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_5(t) \\ i_6(t) \\ i_7(t) \\ i_8(t) \\ i_3^n(t) \\ i_4^n(t) \\ i_1^n(t) \\ i_2^n(t) \end{bmatrix}$$

Substituting the transformed circuit and segregate equations into 6.1.8, the system equations result:

$$\begin{bmatrix} e_{10}(t) \\ e_{10}(t) \\ e_9(t) \end{bmatrix} = \begin{bmatrix} R_4+R_7+R_8 & R_8 & -n_{12}n_{34}(R_4+R_7)+R_8(1-n_{12}n_{34}) \\ R_8 & R_1+R_6+R_8 & R_1+R_6+R_8(1-n_{12}n_{34}) \\ 0 & 0 & -n_{12}(R_2+R_3+R_5) \end{bmatrix} \begin{bmatrix} i_4^n(t) \\ i_1^n(t) \\ i_2^n(t) \end{bmatrix}$$

$$\begin{bmatrix} M_{44} & 0 & -n_{12}n_{34}L_{34} \\ 0 & M_{11} & L_{12} \\ -n_{34}(M_{44}-L_{43}) & n_{21}(M_{11}-L_{12}) & -n_{12}(L_{34}+L_{21}) \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_4^n(t) \\ i_1^n(t) \\ i_2^n(t) \end{bmatrix} \qquad (6.1.9)$$

In section 2.3, it was indicated that the leakage inductance coefficients are small in magnetic amplifier circuits, and will hereafter be assumed zero. It is also assumed that the two inductors are identical. Thus, $n_{21} = n_{34}$, $n_{12} = n_{43}$, $n_{21}n_{43} = n_{12}n_{34} = 1$, $n_{21}n_{34} = n_{12}^2$, and $R_1 = R_4$, $R_2 = R_3$.

Solving for the highest order derivatives reduces (6.1.9) to a system of two differential and one algebraic equation.

$$\begin{bmatrix} D'M_{44}\frac{d}{dt}\, i_4^n(t) \\ D'M_{11}\frac{d}{dt}\, i_1^n(t) \end{bmatrix} = \begin{bmatrix} D' & K_5 \\ D' & K_6 \end{bmatrix} \begin{bmatrix} e_{10}(t) \\ e_9(t) \end{bmatrix}$$

$$- \begin{bmatrix} (R_0K_1+K_1K_3+R_8K_2) & (R_0R_8+K_2K_4+R_8K_3) \\ (R_0K_8+K_2R_8+K_1K_3) & (R_0K_4+K_2K_4+R_8K_3) \end{bmatrix} \begin{bmatrix} i_4^n(t) \\ i_1^n(t) \end{bmatrix} \qquad (6.1.10)$$

$$i_2^n(t) = \frac{-n_{21}e_9(t) + K_2 i_4^n(t) - K_3 i_1^n(t)}{R_0 + K_2 + K_3} \qquad (6.1.10a)$$

Where: $K_1 = R_1 + R_7 + R_8$ $\qquad K_4 = R_1 + R_6 + R_8$

$K_2 = n_{21}^2 (R_1 + R_7$_ $\qquad K_5 = -n_{21} (R_1 + R_7)$

$K_3 = n_{21}^2 (R_1 + R_6)$ $\qquad K_6 = n_{21} (R_1 + R_6)$

$$D' = R_0 + n_{21}^2 (R_1 + R_6 + R_1 + R_7)$$

$$R_0 = 2R_2 + R_5$$

If elements R_6 and R_7 are equal, then 6.1.10 and 6.1.10a become

$$\begin{bmatrix} (R_0+2K_2)M_{44}\frac{d}{dt}\, i_4^n(t) \\ (R_0+2K2)M_{11}\frac{d}{dt}\, i_1^n(t) \end{bmatrix} = \begin{bmatrix} R_0+K_2 & -K_5 \\ R_0+K_2 & K_5 \end{bmatrix} \begin{bmatrix} e_{10}(t) \\ e_9(t) \end{bmatrix}$$

$$- \begin{bmatrix} T_1 & T_2 \\ T_2 & T_1 \end{bmatrix} \begin{bmatrix} i_4^n(t) \\ i_1^n(t) \end{bmatrix} \qquad (6.1.11)$$

$$i_2^n(t) = \frac{-n_{21}e_9(t) + K_2(i_4^n(t) - i_1^n(t)}{R_0 + 2K_2} \qquad (6.1.11a)$$

where $T_1 = R_0K_1 + K_1K_2 + R_8K_2$

$T_2 = R_0R_8 + K_1K_2 + R_8K_2$

6.2. The Computer Program.

The 'flow diagram' of the computer program of 6.1.11 is given in Figure 6.2.1. The complete program has been broken down into smaller sections than is necessary to obtain a solution to the particular problem in this thesis. With this added flexability it is possible to use the same program for the solution of other problems by changing only the data tape. Also, two additional sections have been included which are not necessary for the solution of 6.1.11. However, if the solution of 6.1.10 is desired then these two sections are required. They are: (1) Calculation of R_6 and R_7 if they are nonlinear, and (2) Calculation of $i_2^n(t)$.

The Address routine permits the user to specify the number of points to be used to calculate the value of the nonlinear elements by the

Where

If elem

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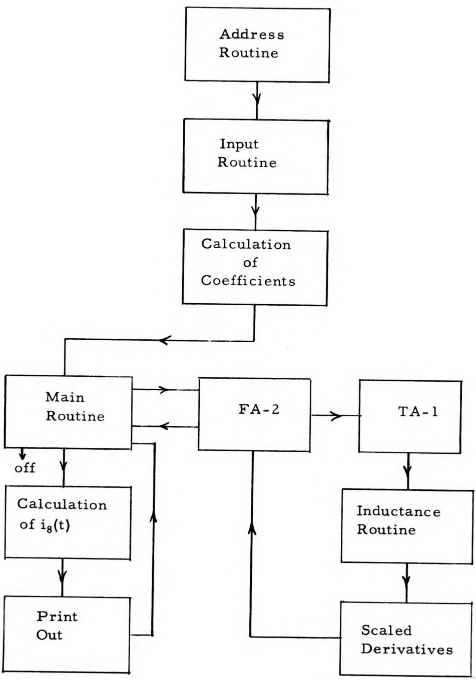

Address Routine
Input Routine
Calculation of Coefficients
Main Routine
FA-2
TA-1
off
Calculation of $i_8(t)$
Inductance Routine
Print Out
Scaled Derivatives

Figure 6.2.1

use of the linear interpolator routine (see Appendix A). It also permits the user to set the transfer order at the end of each routine to the address of the next routine to be entered.

The Input Routine permits the data tape to be prepared in three sections. With this arrangement it is not necessary to alter the entire tape in order to change a selected group of coefficients. The various entries on the data tape are shown explicitly in Appendix B.

The routine for calculating the coefficients calculates the entries in the coefficient matrix of 6.1.10. This routine is written in two sections for added flexibility.

The Scaled-Derivatives Routine solves for the derivatives of the dependent variables as required by the MISTIC Library Routine FA-2.

The MISTIC Library Routine FA-2 is named 'Floating Decimal Solution of a System of Ordinary Differential Equations.' This routine will handle a set of 'n' simultaneous first order differential equations explicit in the derivatives. The Runge-Kutta method of solution is used.

The Routine TA-1 is a regular subroutine in the MISTIC Library which calculates the sin kt.

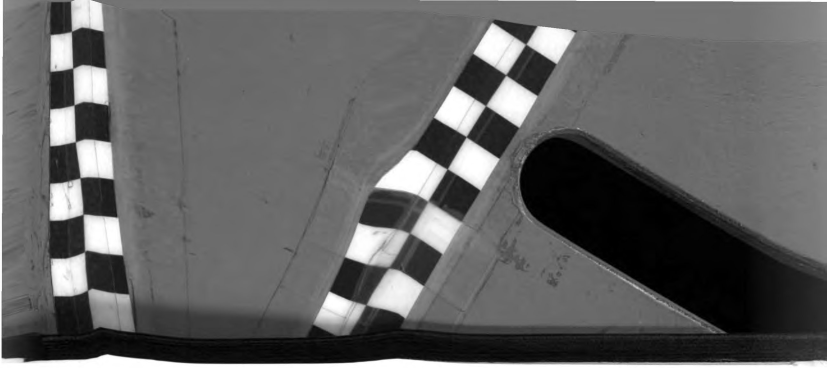
The calculation of $i_8(t)$ simply calculates $i_8(t)$ in 6.1.7.

The Output Routine in this case places on tape the variables t, $i_1^n(t)$, $i_4^n(t)$, $i_8(t)$, M_{11}, M_{44}, and sin ωt. However, this routine can be altered to read out any of the element variables desired.

The Inductance Routine calculates the value of the nonlinear elements M_{11} and M_{44} from the curves showing M_{11} and M_{44} as a function of the ampere-turns. This routine is a special case of the routine in Appendix A.

The main routine directs the progress throughout the entire calculation, such as the number of iterations, number of iterations without print, and the number of iterations between the dump of memory. The user is at liberty to specify the above. The reason for the dump of the

memory is due to the length of the calculation. Should a computer error occur in the middle of the program, it is not necessary to start all over. An additional program has been written which enables the calculation to be started again prior to the point where the error occurred.



VII. CONCLUSIONS

The restrictions on the locations of inductors in a system as stated in the sufficiency criteria of Theorems 5.3.1 and 5.3.2, such that the system equations can be formulated with the ampere-turns variables of the inductors appearing explicitly, are very restrictive. However, a great many magnetic amplifier circuits will fulfill these requirements. The restrictions that Theorem 5.2.2 places on the location of inductors in the network could possibly be relaxed by further partitioning of the segregate, circuit and terminal equations.

The saturation curves used in the example solution are shown in Figure 7.1. The curves showing the coefficients M as a function of the ampere-turns are also given in the same figure.

The curve C represents the "ideal" saturation characteristic usually used for a qualitative analysis. When this curve is used as a basis of the steady-state analysis, it is assumed that during the "exciting interval" (the unsaturated region) the core has a very high "reactance" (actually $M \rightarrow \infty$) compared to the load resistance. During the "saturation interval" the core is assumed to have low "reactance" (actually $M = 0$), such that the current is limited by the resistances of the circuit. The form of the curves resulting from this qualitative analysis are shown in Figure 7.2.

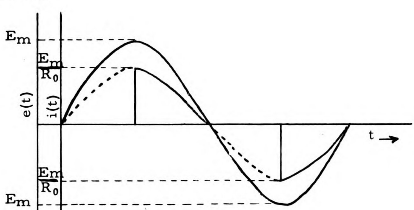


Figure 7.2

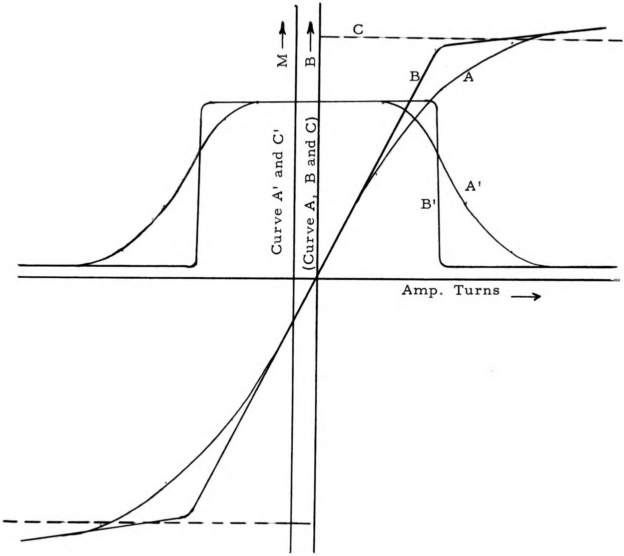


Figure 7.1

However, if an approximation to the solution curve similar to curve A of Figure 7.1 is used, one would expect the general wave form to be as shown in Figure 7.3.

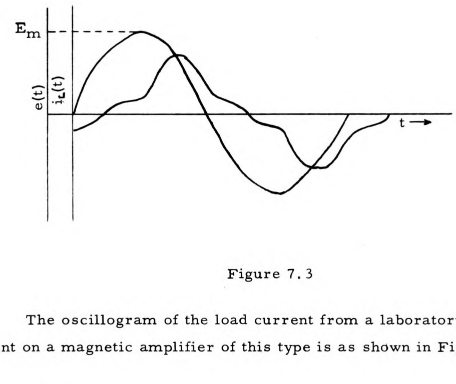


Figure 7.3

The oscillogram of the load current from a laboratory measurement on a magnetic amplifier of this type is as shown in Figure 7.4.[2]

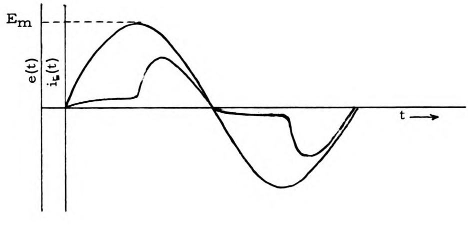


Figure 7.4.

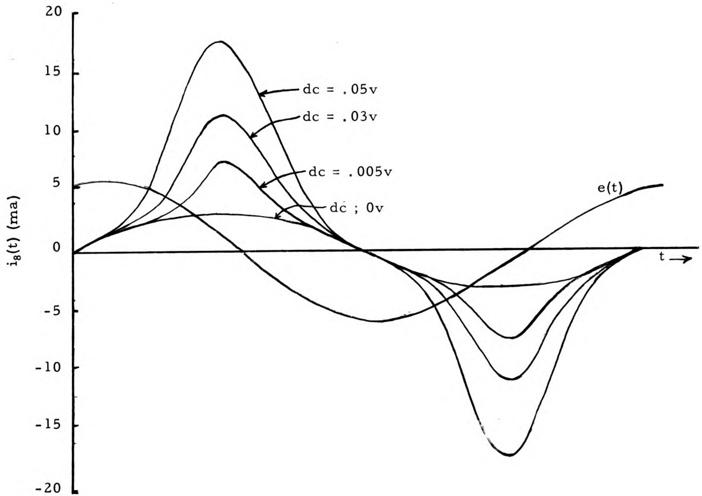


Figure 7.5

The steady-state solution of $i_8(t)$ obtained as a computer solution in the example in this thesis is given in Figure 7.5. These solutions were obtained using the curve 'A' in Figure 7.1 as the value of the coefficient M.

A typical hysteresis loop for the type of material used in magnetic amplifier cores is shown in Figure 7.6. A more accurate representation of this curve is realized when the width of the loop is not neglected.

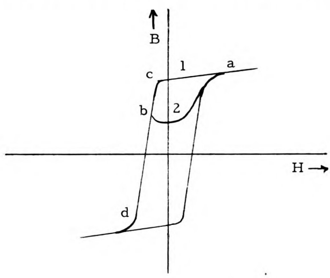


Figure 7.6

An investigation was made concerning the possibility of incorporating this type of non-linear characteristic in a computer solution. One of the problems encountered is the specification of the curve the flux should follow at the point where the slope of the hysteresis loop changes abruptly. The problem is further complicated by the fact that during the transient period, the operation is on a minor hysteresis loop. This requires that as the flux follow curve 1 from "a" to "b, " and back to "a" over curve 2 in Figure 7.6. However, the point "b" can not be predetermined by the programmer as it may lie anywhere on the c-d portion of the hysteresis loop. A steady-state solution could possibly be

realized for the magnetic amplifier with a hysteresis loop of finite width, if the minor hysteresis loop on which the magnetic amplifier is operating is known. In many of the qualitative analyses only the steady-state solution is considered.

Since the above seem insurmountable, the core characteristics used in this thesis is shown in Figure 7.1.

The size of the increment of time required is a function of the slope of the M coefficient curve. In particular of the M coefficient curve is discontinuous, then the solutions of the equations developed in this thesis do not exist. The "computer solutions" obtained when the M coefficient curve B is used differs greatly for different values of increment. The size of the increment that could be used in this program is limited by the length of time required for a solution. This limitation is due to two factors: (1) the program is written in interpretive language which requires more time per operation than regular machine language, (2) the time required for the solution to reach a steady-state condition is long for the solution of these equations (Figure 7.7).

Also the possibility of computer failure increases greatly with time especially after several hours of continuous operation. This problem was alleviated by writing the program such that it could be restarted at specific intervals. However, even then the time is a very serious problem.

As a first correction to these difficulties, it is suggested that a variable increment based on the rate of change of the ampere-turns variable be incorporated into this program. If this still does not alleviate the difficulties the entire program should be written in regular machine language. Faster computers are becoming available which will help to alleviate this difficulty.

Figure 7.8 shows the effect of the dc voltage of the control winding on the ampere-turns variables of each inductor. Note that the axis of

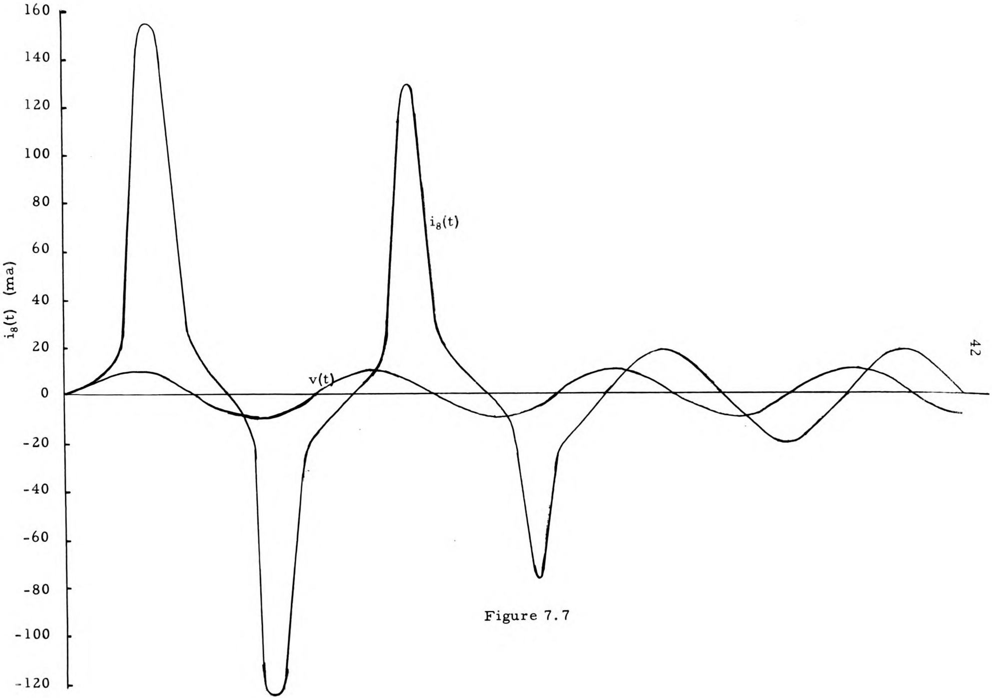


Figure 7.7

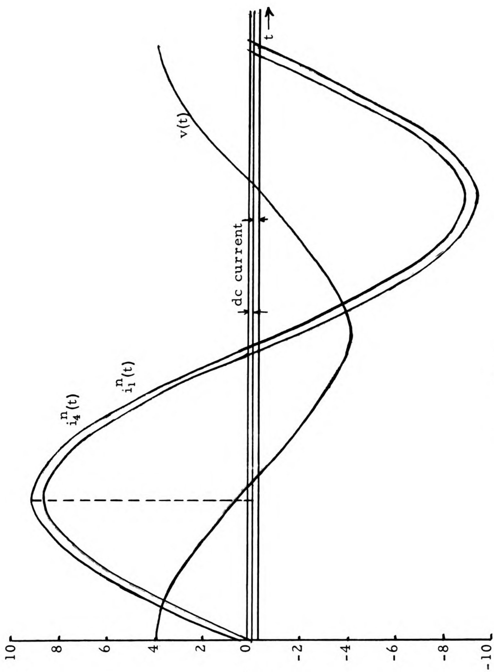


Figure 7.8

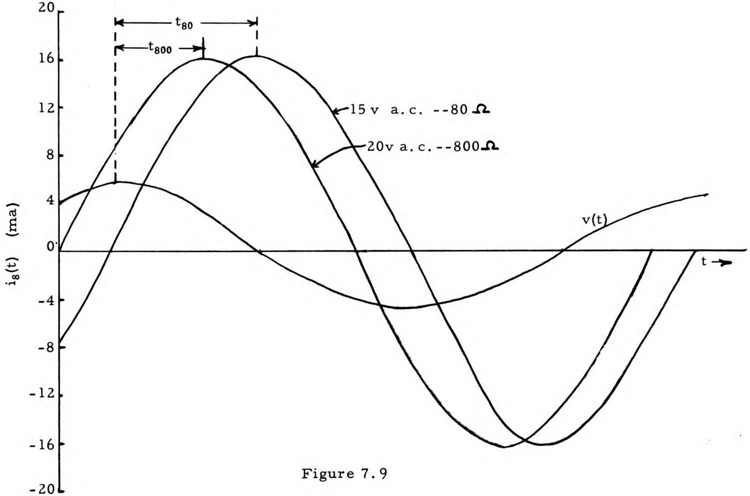


Figure 7.9

$i_1^n(t)$ is shifted in the negative direction where as the axis of $i_4^n(t)$ is shifted in the positive direction. Neither inductor however, is driven into saturation.

The effect of the load resistance on the steady-state linear solution resulting when the inductors are not driven into saturation is shown in Figure 7.9.

It is hoped that the techniques investigated in this thesis will provide a basis for future development.

APPENDIX A

The Linear Interpolator Routine

This routine is designed to interpolate a curve given by a table which consists of a sequential set of function values, $f(x_i)$, for arguments x_i. Let x_0 be the smallest argument, x_p be the largest argument and x_a any arbitrary argument. This routine is written such that if $x_a < x_0$, $f(x_a) = f(x_0)$ or if $x_a > x_p$, $f(x_a) = f(x_p)$. The number of points and the spacing between points is left to the programmer.

The mathematical formulation is:

$$f(x_a) = f(x_n) - \frac{[f(x_n) - f(x_{n-1})]\,(x_n - x_a)}{x_n - x_{n-1}}$$

The following routine is written in A-1 (floating point) and requires the following preset parameters:

SK - location of x_a

SN - location of $f(x_i)$

SJ - location of x_i

In addition the addresses of the following orders must be set:

1. The right hand order of 4 to p-1 +SN
2. The right hand order of 5 to number of points
3. The right hand order of 20 to the location of the next routine
4. the left hand order of 18 to (p-1) +SN
5. The right hand order of 1 to the number of times through the routine

The individual orders of this program are included here for the benefit of the reader who is not familiar with computer coding.

0.	50	F	
	50	L	
1.	26	20F	Transfer to A-1
	0K	qF	Loop 'q' times
2.	85	SJ	Place x_0 in the accumulator
	00	19SK	Subtract x_a
3.	83	16L	if $x_0 > x_a$, transfer to 16
	8K	F	waste
4.	05	19SK	Place x_a in the accumulator
	80	_F	Subtract x_p
5.	83	18L	if $x_a > x_p$, transfer to 18
	1K	pF	Loop p times, if necessary
6.	15	1SJ	Place x_n in the accumulator
	00	19SK	form $x_n - x_a$
7.	83	8L	if $x_n > x_a$, transfer to 8
	13	6L	if $x_n < x_a$, transfer to 6
8.	15	1SJ	Place x_n in the accumulator
	10	SJ	form $x_n - x_{n-1}$
9.	8S	21L	Store $x_n - x_{n-1}$ in 21
	15	1SJ	Place x_n in the accumulator
10.	00	19SK	form $x_n - x_a$
	86	21L	Divide $x_n - x_a$ by $x_n - x_{n-1}$, let this be 'm'
11.	8S	21L	Store 'm' in 21
	15	1SN	Place $f(x_n)$ in the accumulator
12.	10	SN	form $f(x_n) - f(x_{n-1})$
	87	21L	multiply by 'm'
13.	8S	21L	store $m[f(x_n) - f(x_{n-1})]$ in 21
	15	1SN	Place $f(x_n)$ in the accumulator
14.	80	21L	subtract $m[f(x_n) - f(x_{n-1})]$
	8S	21L	store $f(x_n) - m[f(x_n) - f(x_{n-1})]$

15.	8K	1F		
	83	19L	transfer to 19	
16.	85	SN	Place $f(x_0)$ in the accumulator	(from 3)
	8S	21L	store $f(x_0)$ in 21	
17.	8K	1F	Transfer to 19	
	83	19L		
18.	85	__F	Place $f(x_p)$ in the accumulator	(from 5)
	8S	21L	Store $f(x_p)$ in 21	
19.	85	21L	Place $f(x_a)$ in the accumulator	(from 15 or 17)
	0S	7SK	Store $f(x_a)$ in SK	
20.	03	2L	loop to 2 'q' times	
	8J	b F	Transfer to 'b'	
21.	Temporary storage			

APPENDIX B

DATA TAPE

Section 1

a. Three Sexadecimal Characters to specify:

1. The number of points on the curves showing R_6 and R_7 as a function of i_6 and i_7.
2. The number of points on the curves showing M_{11} and M_{44} as a function of the ampere turns.

b. Three Sexadecimal Characters to set the address of the transfer order at the end of:

1. Input routine.
2. Coefficient routine (Section 1).
3. Coefficient routine (Section 2).
4. Scaled-Derivative routine.
5. Sine-Function routine (TA-1).
6. Address routine.
7. Routine for calculating R_6 and R_7.
8. Routine for calculating $i_8(t)$.
9. Output routine.
10. Routine for calculating $i_2^n(t)$.

c. The points of the nonlinear elements, with each $f(x_i)$ followed by its argument x_i in sequence starting with the smallest argument, written in floating decimal form (sign followed by nine significant figures followed by a sign plus two figures, i.e. + 1 + 00) for:

1. The nonlinear elements R_6 and R_7.
2. The nonlinear elements M_{11} and M_{44}.

Section 2

In floating decimal form.

R_1

R_2

R_5

R_8

n_{21} - the turns ratio

l

ω - angular velocity

Section 3

In floating decimal form.

h - increment

$e_9(t)$

$e_{10}(t)$

t

$i_1^n(t)$ - the initial value

$i_4^n(t)$ - the initial value

$i_2^n(t)$ - Not needed in the problem solved in this thesis, hence 0 is used.

$\left.\begin{matrix} R_6 \\ R_7 \end{matrix}\right\}$ If considered nonlinear elements place 0 in each position, if constant place the desired value in each position.

Three Sexadecimal characters, read in by the Main routine, to specify:

1. Number of loops with memory dumps in between.
2. Number of iterations, with no output of memory.
3. Number of loops through FA-2 between outputs.
4. Number of times through the entire calculation, with a change of data each time.

The product of 1 and 2 above is the total number of iterations.

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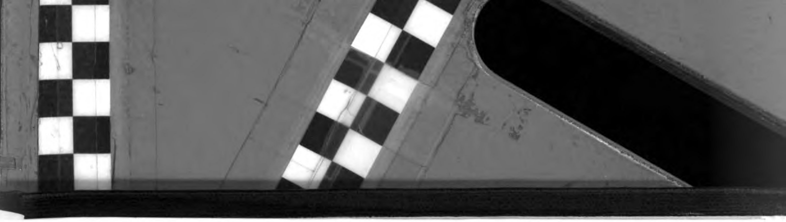

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