ABSTRACT

THE APPLICABILITY OF SELECTED RATIO AND
LEAST-SQUARES REGRESSION ANALYSIS TECHNIQUES
TO THE PREDICTION OF FUTURE EDUCATIONAL
ATTENDANCE PATTERNS

By

William John Webster

This study is concerned with the exploration of three major questions.

1. Can educational attendance prediction problems be formulated in such a manner that the least-squares regression model can be used in the analysis thereof?
2. Are different variables and diverse projection methodologies relevant to school districts with different growth characteristics, or is there one projection methodology that provides the best estimates, ie. estimates with the least amount of error, across all school districts.
3. Is it worthwhile to make the necessary data transformations to estimate local population statistics for projection purposes, or will county population statistics, which are readily available from relatively reliable sources, provide more reliable estimates on which to base future school population projections?

The population identified in this study consists of those public school districts in the State of Michigan that are largely coterminous

with cities of 10,000 population or more. It is divided into five strata on the basis of past community growth characteristics. Twenty-five school districts, five from each stratum, were randomly selected for study. Seventeen separate and distinct ratio and regression methodologies were used to obtain estimates of elementary and secondary enrollment for each of the twenty-five districts in the sample. Pre-1960 observations on various diverse demographic variables were used as base data from which projections were made to 1965 and 1968. Actual 1965 and 1968 enrollment figures were used as the criteria against which to judge the goodness of fit of the estimates produced by each of the five ratio and twelve regression approaches examined in the study.

The major findings of the study suggest that educational attendance prediction problems can in fact be formulated in such a manner that the least-squares regression model can be used to advantage in the analysis thereof; that different predictor variables and diverse ratio and regression approaches are relevant to districts with different growth characteristics; and, that most population variables, whether they derive from local or county data, provide estimates that are inferior (in terms of goodness of fit) to least-squares regression and ratio approaches that use time, births, and the tendencies of students to advance from one grade to another as the predictor variables. One such approach, a regression approach that is called "Transition Analysis", represents a unique approach to the projection of public school enrollment. This method provided the most accurate projections of all of the seventeen ratio and regression projection methodologies identified in this study, including those ratio approaches that are currently the most used in estimating future public school enrollment.

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William John Webster

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DEDICATION

This document is dedicated to my Mother and Father in recognition of their unending support and encouragement.

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CHAPTER I

INTRODUCTION

Statement of the Problem

The acute need for educational planning has long been realized. An essential facet of this planning involves the estimation of the number of pupils that are to be housed at relevant future dates. Accurate enrollment forecasts would greatly assist in the making of educational decisions affecting both educational and fiscal policies. Such decisions must be made at every educational level, although the necessary distance into the future and degree of precision may vary. A list of such decision problems would include the following.

1. Short-range District Financing. How much money will be required in the next year, and for what purposes? Are expected increases in attendance in the near future sufficient to warrant new building appropriations now?
2. Long-range District Financing. Is such an expansion of school enrollment foreseen that funds for building and operation will need to be radically increased? Will future current taxes, if not radically revised, be able to take care of the operating budget of the school district? Given the answer to the thorny philosophical problem of how large a school unit of a certain type should be, it is from numbers of expected

pupils that deductions can be made as to the numbers of sites and buildings required.

3. State Short and Long-range Financing. How much money will be needed for public education at a state level? Is it sufficient to plan for public education collectively, or is it more feasible to consider each individual district separately?

4. Short and Moderate-range District Planning. What increases in staff should be incorporated immediately into the budget in view of current and anticipated increases in attendance? How heavily must the district plan on recruiting, ie., approximately how many man hours must be devoted to recruiting?

5. Long-range District Planning. When should major physical plant changes begin? Does projected enrollment warrant major physical plant changes? From enrollment projections the community must make definite decisions with respect to space requirements, time for and placement of new buildings, length of bond issues, and even the future functions of the school plant.

6. Moderate and Long-range State Planning. What is the state-wide demand for educational personnel going to be? Accurate enrollment projections would lend valuable insights into the question of whether or not state universities are capable of furnishing necessary personnel, and the related

decisions concerning an out-of-state recruitment program. Should modifications in educational policy be imposed at a state level so that undesirable outcomes can be avoided. e.g. Should public aid to parochial education be supported in order to curtail overcrowding of facilities, or is the school-age population in the state leveling off or decreasing to the point where present facilities will be sufficient to meet future demand?

In planning for the operation of a school district, knowledge of attendance patterns alone is not sufficient, for the cost of operating a public school is not proportional to attendance alone. Expansion of a vocational education program by fifty students is not comparable to expansion of a college prep program by the same number. Other activities of the public schools may be insensitive to attendance, for example, the auditorium and gymnasium actually service the local community as well as the student body. Hence the need exists, particularly at a local level, for detailed predictions of attendance as well as for a knowledge of the relationships between attendance and other variables.

Despite the many uses for accurate enrollment projections, the formulae currently in vogue in education are lacking in the necessary precision.[1] The reasons for this lack of precision are many. It must be recognized that never before in the history of American Education has the estimation of future school population been fraught with greater problems than at the present time.

Efforts to estimate future school population must reckon with, among other things, gargantuan population and community changes. These changes include resurgent national growth; the increased population concentration in large metropolitan areas; great waves of in-migrant peoples of diverse races, cultural backgrounds, and school enrollment practices, into the large metropolitan areas of the country; accelerating decentralization within some metropolitan areas and centralization in others; and, a rapidly changing age structure in our society. Simultaneously, unprecedented community changes are under way. Included are such phenomena as changes in patterns of land use resulting from urban renewal programs, public housing projects, federal and local highway construction, increased utilization of physical planning techniques, and the ever increasing transportation flow. The problem of estimating school enrollment for local districts is further complicated by differential rates of expansion of public, private, and parochial schools, and by the need to replace obsolete school structures while, at the same time, grappling with the many problems of rapid growth.

Population growth, or lack of growth, is the net result of births, deaths, and migration. Most projections that are made of population are based on the assumption that there will be no large-scale war, no widespread epidemic, no major economic depression, or any similar catastrophe. Within the limits of this assumption, death rates become relatively easy to estimate. For example, in the United States as a whole there has been little change in death rate since 1954.[2] Available statistics indicate that the death rate in the State of Michigan has also remained largely constant during that period of time.[3] Birth rates and migration present more difficult problems. Births have fluctuated widely throughout the United States during the past fifty years and could easily do so again.[4] Migration to or from a given area is effected by such factors as the availability of jobs, which in turn fluctuates with industrial expansion, the availability of housing, and, the quantity and quality of available shopping, recreational, and educational facilities.

The interplay of the aforementioned factors is by no means uniform and therefore requires a separate enumeration and evaluation, district by district, to adequately estimate future school enrollment. Most American communities are faced with problems of rapid growth, however, some face actual decline in both total population and school enrollment. One projection procedure is probably not adequate for use in all situations. The best method for projecting school needs in a particular community depends to a great extent on the characteristics of the community itself, including its age, size, location, and the rate of net migration to the area. Of course, the kind and quality of available data must also be taken into account. There follows a classification of community types that may prove useful.[5]

A. Exploding Community. Most of the areas falling into this category are located in the suburban part or outlying fringe of large metropolitan areas. Most of them are either new communities or areas of rapidly expanding residential development in older communities with large parcels of vacant land. Projection of school needs for communities of this type is exceedingly difficult without a special survey of the local area since past enrollment figures generally cannot be used to predict future enrollment.

B. Older But Still Growing Community. Areas falling into this category are characterized by parcels of vacant land that may be or are in the process of being utilized for the construction of housing units. In this type of community migration is still a definite problem in that it is difficult to predict, ie., one may encounter difficulty in attempting to predict future trends from past performance.

C. Stable, Well-established Community. Areas falling into this category are characterized by either a lack of vacant area for expansion or a lack of reason to expand. Large metropolitan areas and many small rural communities would fall into this category. Difficulty in projecting enrollments for this community type stems mainly from changing population characteristics and the resultant increase or decrease in school-age youth relative to total population.

D. Declining Community. Areas included in this category are characterized by a declining population. They are most likely to be rural farming and mining areas, although, they may also be communities whose rolls were filled with tuition students from "non-high school" districts that are building high schools and siphoning students off from the host district.[6]

A number of methods for projecting school population have been devised. However, one mathematical model which seems to this writer to be uniquely appropriate for this type of problem, the linear, least-squares model, has not been given an adequate trial by educators.[7] As early as 1932, a study of the available methods for estimating student enrollment in education revealed gross inadequacies in current methods.[8] Projection methodology has largely remained the same since that time, and there is little evidence to suggest that results have become more accurate.[9] It seems apparent that the time to try new projection methods in education has long since passed. It is to this task that this dissertation is addressed.

Definition of Terms

Throughout this manuscript several terms are used repeatedly. In the interest of clarity those terms are defined below.

1. Projection

   The term "projection" is used to describe the methodology involved in the forecasting of future enrollments. The term "prediction" is avoided because, considering the nature of the data and methodology, it presumes too much, implying that a statement made about enrollment at some future date does not admit of possible variation in the actual number that will be found at that date. The term "prediction" is used in the title as a retrieval term, potential readers being more familiar with the term "prediction" than with the term "projection."

2. Predictor

The term "predictor" is used to describe the independent variable, that is the variable or variables that are manipulated in the regression and ratio equations. The predictor variable is the one whose changes or differences are associated with changes or differences in the dependent or criterion variable. Values of a predictor variable thus afford a basis for projection.

3. Criterion

The term "criterion" is used to describe the dependent variable, that is the variable whose behavior is presumed to be predictable from the predictor variable. The dependent or criterion variable in this study is always some form of school enrollment.

Purpose of the Study

This study is concerned with the exploration of three major questions.

1. Can educational attendance prediction problems be formulated in such a manner that the least squares regression model can be used in the analysis thereof? Ultimately we are asking if simple or multiple linear regression, or a transformation thereof, will yield more accurate projections of future school attendance than will the most popular

educational projection methodologies currently in use at the district level.

2. Are different variables and diverse projection methodologies relevant to different types of school districts, or is there one projection methodology that provides the best estimates, ie. estimates with the least amount of error, across all school districts?

3. Is it worthwhile to make the necessary data transformations to estimate local population statistics for projection purposes, or will county population statistics, which are readily available from relatively reliable sources, provide more reliable estimates on which to base future school population projections?

Limitations of the Study

There are two major conditions which place important limitations on this study and which, therefore, must be taken into consideration when discussing any results that are obtained. These limitations are listed in their probable order of importance to the outcome of the study.

1. There is a serious lack of adequate data for a study of this type. Much of the data that is used must be extrapolated or interpolated from the little data that is available.

2. Statements or conclusions derived from the statistical projection methods used in this study must be interpreted in the light of a number of qualifying phrases.

Data Limitations

This study is specifically concerned with the problems encountered in short-term enrollment projection. Generally six data points, one for each of the years 1955-1960, are used as the basis for establishing equations to project future enrollment figures to 1965 and 1968. There are definite limitations inherent in the kind and amount of data that is available. Projections based on incomplete and unreliable statistics are subject to the inadequacies and limitations of those statistics. Considered in sum, the data used in this study illustrates all too clearly that there is ample justification for the view that the development of enrollment projections of measurable accuracy and dependability is retarded by the lack of adequate and dependable data.

Qualifications Inherent In The Statistical Projection Methods Utilized

The major qualifying phrase which must be stated for every enrollment projection method utilized in this study involves the basic tenet that the environment must remain similar in the period for which projections are made to the period from which projections were drawn. Stated more succinctly, the assumption of stable conditions, or, at the very least, regular rates of change, must be made for every enrollment projection method examined in this study. To the extent that this assumption is violated, the enrollment projections will be in error.

This study recognizes two classifications of projection methodologies, ratio methods and regression or least-squares analysis methods. Virtually all of the projection methods described in the literature are ratio methods. The distinction between ratio and regression analysis methods is made on the basis of the measure of relationship used in the specific computation of projected enrollments. The classification of a projection method as a ratio method rests on the fact that the ratio, or proportion of a given predictor to enrollment, is the basis for the computation of the projected enrollment. The various ratio methods are in turn sub-divided into classifications determined by the predictor selected for study. The classification of a projection method as a regression analysis method means that the measures of relationship used as the basis for calculation are the coefficients of correlation and multiple correlation, statistics which purport to measure the degree of association between variables.

The ratio methods require only one major assumption, that being the previously mentioned one that the ratios of predictor to criterion may be established historically and will remain constant or alter at a stable rate in the future. They represent a composite, or average, of past trends and occurences.

The regression methods assume that the criterion variable can be expressed in the form of the equation "$Y = A + BX + E$," where "Y" is the criterion variable, "A" and "B" are unknown coefficients to be determined by the least squares procedure, "X" is the independent or predictor variable, and "E" is the error inherent in the model. The predictor variable is generally chosen because it is a variable that is known, or can be estimated, with greater precision than the criterion variable. If more than one predictor variable is utilized, then multiple linear regression is required. In multiple linear regression a restriction is placed upon the number of predictor variables that can be included by the number of cases available for study. Generally speaking, high correlations and high multiple correlations appear as artifacts of the computational process as the number of variables approach the number of cases available for study.

The regression methods also require that the ratios of predictor to criterion may be established historically and will remain constant or alter at a stable rate in the future. In addition, there are several more important assumptions that must be made. The most important of these assumptions states that the line of best fit which specifies the relationship between the predictor and criterion variables be a straight line. To the extent that the data deviates from linearity, projections derived from the linear model will be in error. This assumption is not seen as a problem since most functions can be best approximated by a straight line over a short period of time or small number of data points.

A second basic assumption underlying the application of regression methods is that the error terms in the regression model are independent. A great deal of use has been made in the behavioral sciences of least squares regression methods in circumstances where they are thought to be inapplicable due to the violation of this assumption.[10] The classical example of autocorrelated error terms occurs when an incorrect specification of the form of a relationship between variables is made. If the straight forward least-squares formulae are applied directly to relationships where the preceding mistake is made, there are three major consequences. First, although the procedure will yield unbiased estimates of α and β, it is possible that the sampling variances of those estimates will be unduly large. Second, if the usual least squares formulae are applied for the sampling variances of the regression coefficients, serious underestimates of those variances are likely to occur. Finally, the predictions obtained from such a procedure will be inefficient in that they will have needlessly large sampling variances.

Cochrane and Orcutt have demonstrated that the usual application of the method of least squares to relationships containing highly positively autocorrelated error terms results in a marked decline of the variances of both the correlation coefficient and the regression coefficient as the error term becomes more random.[11] Least squares methodology was applied to five different cases; a random case, a first-order Markov scheme, a limiting first order case when $\rho = 1$, a second-order autoregressive scheme, and the first differencing of a random series. Even in the case of the random series there is a slight underestimate of the true error variance, but this becomes very serious in the autocorrelated cases.[12]

It should be obvious from the preceding discussion that the violation of this assumption does not affect the estimates as such, but rather the confidence intervals that can be placed around those estimates. Given the fact that there is a high probability of autocorrelated error terms in time-series data, it is extremely questionable as to whether or not conficence intervals computed in the traditional least-squares manner will add much to our knowledge about the data. This problem is not examined further in this study because, given the nature of the data, the nature of the projection problem, and the small number of data points, it is questionable as to the amount of information that the confidence intervals would supply anyway. The Standard Errors of the Regression Coefficients are merely used as relative indicants of the amount of error present in each of the various projections. The regression approach is used in an algebraic manner as a method of projecting a curve into the future. The problem of setting confidence intervals around that curve must be reserved for future study.

The other assumptions of the least-squares regression model, namely those of normality and common variance, are of little consequence since moderate variations in those entities have little affect on the results obtained from the model.

Footnotes for Chapter I

[1] Albert Bryon Crawford and Leo Martin Chamberlain, Prediction of Population and School Enrollment in the School Survey, (Univ. of Kentucky: Bureau of School Services Bulletin, v.4, no.3, 1932 27 p.). Dean Waldfagel and Frank R. Krajewski, A Study of School Enrollment Projection Formulae, unpublished manuscript, Michigan State University, 1964. Knute G. Larson and Wallace H. Strevell, "How Reliable are Enrollment Forecasts, " School Executive, LXXI, (February, 1952), pp. 65-68.

[2] U.S. Bureau of the Census, Current Population Reports, Series P-25, No. 388, p.3. The present death rate (1968) is 9.5 per 1000 population and, according to the Bureau of the Census, is likely to drop somewhat to approximately 9.0 per 1000 population by 1990.

[3] It is certainly true for the State of Michigan. See R. Raja Indra, ed., Michigan Population Handbook, Michigan Department of Public Health, Nov., 1965, p.7.

[4] Current Population Reports, Series P-25, No. 388, op. cit., p.4. According to the U.S. Census Bureau birth rate is not likely to drop below 17 per 1000 population, but might increase and approach 25 per 1000 population within the next twenty years. Even the lower rate, being approximately twice the death rate, would guarantee a continuing increase in population. There are two reasons for believing that there will be a new spurt in births during the next few years. First, the number of women of the age of childbearing is increasing rapidly as the children of the postwar baby boom reach maturity. Second, there is evidence that the recent fall in the birth rate is partly due to delays in family formation.

[5] The original classification scheme was taken from: Philip M. Hauser and Evelyn M. Kitagawa, "On Estimation of School Population," American Statistical Association Proceedings, (Washington: ASA), 1961, pp.69-70. It has been revised somewhat for purposes of clarification.

[6] In this category are the so-called "dead" communities. One such community of this type would be a community where there once existed a "live" mine which has since been worked out. This type of community has some unique projection needs. Provisions must be made, for example, for dropping tenured personnel as a result of declining school enrollment and the resulting drop in State Aid.

[7]At least such a trial has not been reported in the educational literature. For example, writing in 1954, and having in mind the approaches used in public school enrollment forecasting, Strevell identified four methods of enrollment projection, all of which were ratio methods. Wallace H. Strevell, "Techniques of Estimating Future Enrollment," American School Board Journal, March, 1952. This over-sight is prevalent in the educational literature, although the linear, least-squares model has been occasionally experimented with in higher education. This will be discussed at length in the review of the literature.

[8]A.B. Crawford and L.M. Chamberlain, op.cit., 27p.

[9]Chamberlain and Crawford's work has been largely ignored in educational circles. The results of an analysis of existing prediction formulae carried on at Michigan State University in an unpublished paper by Waldfogel and Krajewski, op.cit., tend to confirm Chamberlain and Crawford's hypothesis. This work was completed as recently as 1964.

[10]J. Durbin and G.S. Watson, "Testing for Serial Correlation in Least Squares Regression," I., Biometrika, 37, 1950, p.409.

[11]D. Cochrane and G.H. Orcutt, "Application of Least Squares Regression to Relationships Containing Autocorrelated Error Terms," Journal of the American Statistical Association, 44, 1949, pp.48-49.

[12]Cochrane and Orcutt, op.cit., p.51.

CHAPTER II

REVIEW OF THE LITERATURE

This review of the literature will begin with a presentation of the efforts of the United States Census Bureau to project both total population and student enrollment. It is felt that a review of the projection methodology utilized by the Census Bureau will serve to highlight some of the problems inherent in population projection. A consideration of the methods utilized to project total population can also be justified by the realization that many of the enrollment projection formulae currently utilized by educators rely on Census projections of total population in making their projections. Following the brief presentation of the methodology utilized by the United States Census Bureau, a review of the major projection formulae used in public education will be discussed. Each formula will be presented with a brief explanation of the assumptions involved in its use, its application, its strengths and weaknesses, and the frequency of its use. Finally, a consideration of the projection methods utilized in higher education will be presented. At this point the strengths and weaknesses, assumptions, and possible applicability of simple and multiple linear regression to educational projection problems will be discussed.

Projection Methodology Used By The United States Census Bureau

The United States Census Bureau concerns itself with three types of population estimates; inter-censal, post-censal, and future. Future estimates are defined as those made for any period of time after the present moment, and, therefore, are the only type of estimate that will be considered here. Two basic procedures are used by the Census Bureau for the construction of estimates of future population. The first, and most rarely used method, involves the application of one of a variety of mathematical curves which presumably describe the manner in which the population has grown in the past and therefore is likely to grow in the future. The second involves the use of vital statistics which, in conjunction with census enumerations, permit the construction of estimates of future populations.[13]

The simplest mathematical procedure suggested is the use of a linear interpolation between two consecutive censuses, and subsequent extrapolation beyond the last one.[14] This procedure assumes that the rate of growth over the past ten years will remain constant for the period to which the projection is being made. In the event that three or more observations are available, it is suggested that curved lines be fitted to the data and extrapolated. Such curves, specified by quadratic equations, assume no particular pattern of population growth other than that of a smooth and continuous rate of change. The quadratic approach is thought to be quite inadequate by government demographers because it pre-supposes increasingly larger gains or losses to the population.[15] The population estimate, if a quadratic equation is used as an approximation, will increase indefinitely into the future at a positively accelerating rate.

Another mathematical procedure considered is estimation through the use of an exponential curve. In the event that a population is increasing in size, an exponential curve will predict that the absolute increment increases yearly and that population growth takes the shape of a "j curve." (The reverse holds true if the population is decreasing in size.) The further calculations are projected into the future, the greater is the absolute increment added yearly. Therefore, if one is interested in long-range estimates of population the results yielded by the use of an exponential curve might prove to be quite ludicrous.[16]

The logistic curve is the curve that government demographers see as being the most applicable of the mathematical approaches mentioned to the problem of long-range population projection. Unlike the curves referred to above, the logistic curve becomes asymptotic, indicating a plateau as far as population size is concerned. Since it is known from past observation that populations do not continue to increase indefinitely at constant, or increasing rates of growth, the Census Bureau prefers this approach.[17]

At the time of the publication of the Handbook of Statistical Methods For Demographers, the United States Census Bureau and the

Office of the Coordinator of International Statistics were not extremely enthusiastic about the possible applicability of mathematical curves to population projection.[18] The main criticism of mathematical curves is that they present a theoretical pattern of growth and, therefore, cannot depict the year to year variations which result from fluctuations in the numbers of births and deaths as well as the number of immigrants and emigrants. This view is supported by another researcher who bemoans the fact that a curve may fit enrollment data for the past fifty years with a high degree of accuracy, yet fail to project future enrollment for the next year or two.[19] All methods of population projection extrapolate past trends into the future, although some make different assumptions than others. Mathematical curves perhaps do not offer the flexibility of some other methods, but, where complete and reliable vital statistics and migration data are not available, as is the case when dealing with local areas, certain varieties of mathematical curves seem quite appropriate. Perhaps the dissatisfaction of government demographers with the usefulness of mathematical curves stems from their apparent failure to utilize some appropriate models, their apparent failure to consider any variable but time as the independent variable, and the nature and complexity of the projection problem.[20]

The main set of methods utilized by the United States Census Bureau for the projection of the total population is based on the notion that the population of any given area at a given time can be specified by subtracting the number of deaths plus the number of emigrants from the number of births plus the number of immigrants. Two methods are used in the production of projections. Projections of total population by age and sex are made through the use of the so-called "cohort-component" method, and projections of population characteristics by that same method and by the so-called "participation-rate" method.

The "cohort-component" method involves dividing the population into meaningful groups based on relevant variables, and keeping track of those groups of individuals through life. Each of the components of population change - births, deaths, and interstate and international migration - is projected separately. After projecting each component of change, the projections are added to the population at a relevant base date to measure change in total population over time. The "cohort-component" method must obviously take into account the aging of the population over time. For example, the population age 20 to 24 in 1965 will be ages 25 to 29 in 1970, and, as a result will perhaps require a different mortality assumption in 1970 than it did in 1965. To keep the computations straight, the populations and the components of change are classified by the year of birth of the population. The population which was age 20 to 24 in 1965 and will be 25 to 29 in 1970, was born between 1940 and 1944. This population is thus called the cohort of 1940-44.[21]

When utilizing the "cohort-component" method to make national population projections, the Census Bureau makes a number of estimates of future population. This stems from the fact that demographers have found it extremely difficult, if not impossible to make precise projections of future population. Therefore, the device of presenting a number of estimates based on a variety of assumptions about the components of population change, instead of a single estimate based on a single set of assumptions, has been adopted. Since the assignment of mathematical probabilities to the various series of projections, analogous to sampling errors of population estimates based on probability samples, is not feasible, the minimum and maximum estimates are often used as an approximation to the upper and lower bounds of a crude confidence interval, thus facilitating interpretation of the projections.[22]

When projecting population on a national scale the Census Bureau has, in the past, made four series of projections. These projections differ from one another wholly with respect to the assumptions made relative to fertility; Series A, the most liberal estimate, places the fertility rate approximately 37% higher than does Series D, the conservative estimate.[23] Mortality and migration are allotted only one set of assumptions each, because it was reasoned that the possible range of those two components of population change is small.[24]

In summary, the "cohort-component" method, as applied to projection problems by the United States Census Bureau, involves the projection of each component of change for each successive cohort on each relevant variable. Such projections involve the use of further projections of such things as age-specific life table survival rates for mortality, cumulative age-specific fertility rates for cohorts of women, and assumptions as to the total age-sex distribution of net migration. These projections involve a relatively complex procedure and require a vast amount of reliable data.

In the "participation-rate" method, first, the rates at which the population participates in some activity or possesses some characteristic such as school attendance, employment, or household status, are projected, and then the rates for a given year are applied to the projections of the population for that year. As far as this paper is concerned, the most relevant use that the Census Bureau makes of the "participation-rate" method is in the projection of future national school enrollment. Projections are made of enrollment rates by age and sex, and these rates are then applied to the projections of total population in order to obtain an estimate of the number enrolled. Once again several estimates are utilized. The relevant variables that are varied are fertility rate and percent of population enrolled in school. There are two assumptions under each variable, thus contributing to a total of four series of estimates. The projections distribute enrollment among three levels of school: elementary including kindegarten, high school, and college and professional school.[25]

This procedure requires a relatively accurate estimate of total population because this is the projection to which the projected rates are applied. Therefore, one is likely to encounter data collection difficulties if he attempts to utilize a procedure similar to this in areas smaller than the state level.

Projection Methodology Used By the State of Michigan

States also have need for accurate population projections. These projections are made by both the United States Census Bureau and the individual states. The state population of Michigan was recently projected to 1980 by the Population Studies Center at the University of Michigan.[26] The procedure used was a component procedure, similar to the methods utilized by the U.S. Census Bureau and previously discussed in this paper. In the interest of clarifying any problems that the reader may have in understanding the component procedure of population estimation, as applied to the state or national level, the assumptions and methodology of the Michigan procedure are described in detail in Appendix A.

County population projections make use of all of the techniques presented in Appendix A, plus some additional procedures which adjust the county totals to equal the state total. Unique demographic characteristics of counties, as well as variations in county migration and fertility patterns, are also taken into account.[27] Projections of total county populations in the state of Michigan are generally carried out in the following manner. First, average annual growth rates are determined for each relevant variable in each county for specified time periods. Next, the share of the state total population found in a given county is projected forward by using the growth rates in the county shares obtained in the first step. Third, since it is evident that this projection procedure will not necessarily sum to 100 per cent of the state projection, the projected shares of state total population are adjusted so that they do sum to 100 per cent. Finally, the adjusted shares are multiplied by the state total population to obtain the projected population for each county.[28]

Realizing the intense need for accurate population projections for small geographic units within the state, the Michigan Department of Commerce prepared a report entitled Preliminary Population Projections for Small Areas in Michigan.[29] The principal objective of this paper was stated as the development of an initial population projection method, suitable for statewide application on a small area basis, with sufficient flexibility for the later incorporation of additional variables and appropriate methodological refinements. It is significant that the State Resource Planning Division and their advisors chose a refinement of the least squares regression model to attack this problem.

The projection technique utilized in the paper projects local units' shares of total county population. Each local unit's population is expressed as a fractional share of its county population.[30] A regression line of the following form is then fitted to the shares for each local unit.

$$P = a + b \ X \log \left(\frac{B-A}{N}\right)$$ [31]

Next, the local unit shares projected for each of the relevant years are multiplied by the projected county populations for each relevant year to arrive at the projected local unit populations. In order to avoid negative share projections, which could result in some cases of declining shares, a constraint is incorporated that no local data unit can fall below 1/10 of 1% of its parent county.[32] Since the sum of the projected shares would not necessarily total exactly 100% of the county total, the sum of the projected data units also would not necessarily exactly total the county projected population. To adjust for this difference, the projected population of each local unit was multiplied by the factor:

$$\frac{\text{County Projected Total}}{\text{Sum of the Local Unit Projections}}$$ [33]

The local unit projections, and the county projections to which they are tied, represent extrapolations of past trends which show, since 1940, a rapid increase in population in the southern part of Michigan and a slowly declining population in the northern part. Superimposed on this trend is the general increase of city populations, with the largest increases occurring in the metropolitan areas of southern Michigan, and the highest percentage increases occurring primarily in suburban cities and townships.

The projected local unit shares, when applied to projected county populations, may yield any one of the following four results.[34]

CASE	COUNTY POPULATION PROJECTION	LOCAL UNIT SHARE PROJECTION	PROJECTED LOCAL UNIT POPULATION
1	increase	increase	increase
2	increase	decrease	increase or decrease
3	decrease	increase	increase or decrease
4	decrease	decrease	decrease

As you can see, when the change in county population and the change in local unit's share of county population are opposite in sign, the projected change in the unit's population may be either an increase or decrease, depending on the relative magnitude of the rates of change of the county population and local unit's share.

The most significant limitation of this type of projection comes to light particularly in suburban fringe areas where population growth is occurring in cities and townships at successively greater distances from major urban centers. Since this projection technique is based on historical trends of population change for individual minor civil divisions, it tends to show increases for units which have increased in the past, and decreases for those which have decreased in the past. This is no great problem except in the situation where a unit has shown substantial population increase in the past, but which has finally reached its "holding capacity." A model is needed that takes this "holding capacity" into consideration.

Projection Methodology Used In Public Education

Educators have modified the techniques utilized by the United States Census Bureau, apparently failing to appreciate the significance of the differences in projection problems. As a result, many of the enrollment projection methods currently used in education forfeit many of the advantages of the Census Bureau's techniques while retaining most of the disadvantages. Federal demographers working with large population and geographic areas about which large amounts of reliable data are available are certainly faced with different kinds of problems than are educators who must work with small geographic and population areas about which little reliable data is available.

The most popular enrollment projection formula currently used for projecting public school enrollment is the so-called "cohort-survival" or "grade-cohort" method. This method depends upon the relationships of grade to grade enrollment throughout a school system, as indicated by grade statistics for a number of years. The method operates on the premise that by examining the enrollment data of a particular district by grade for a number of years, a trend can be established as to the proportion of students enrolled in each grade in a given year who progress to the next higher grade the following year. The essential steps involved in the procedure are as follows:

1. Begin with the actual number of births in a given school district during any six year period. (Six years is used because this is the age at which most children enter the first grade. If kindegarten enrollment is of interest, five years would be used.)

2. For each of the years of a given six year period, say the years "n" to "n+6", find the actual enrollment in the first grade.

3. Compute the percentage of survivorship between the number of births six years earlier, and the enrollment in the first grade for each of the known years, "n" to "n+6". Average the proportions.

Thus we have:

$$A = \frac{\frac{E_n}{B_{n-6}} + \frac{E_{n+1}}{B_{n-5}} + \frac{E_{n+2}}{B_{n-4}} + \frac{E_{n+3}}{B_{n-3}} + \frac{E_{n+4}}{B_{n-2}} + \frac{E_{n+5}}{B_{n-1}}}{N}$$

where:

A = The average percent of survival from birth to first grade for any period of six years. (This could of course be a greater or lesser number of years.)

B = The number of births in a given year. As can be seen by looking at the formula, births appear in the denominator of each ratio in the numerator, and are recorded for six years earlier than is enrollment.

E = Public school enrollment in the first grade in a given year.

N = The number of years for which the ratio of births to is computed.

4. Making the assumption that the average of the computed proportions applies to each future year, apply this average to the actual number of births which occurred during the years "n" to "n+6" to obtain estimated first grade enrollment for the years "n+6" to "n+12", the period during which those children born between the years "n" and "n+6" will enter school.

Thus we have:

$$EE_n = B_{n-6}\ (A)$$

where:

EE_n = Estimated first grade enrollment in the year "n".

B_{n-6} = Births in a given year six years earlier.

A = The average percent of survival from birth to the first grade.

5. Calculate the percentage of survivorship from grade one to grade two and so apply these to the known figures to get estimates for the years immediately ahead.

Thus we have:

$$ES_{n+1} = B_{n-6}\,(A)\,(P_{1\cdot 2})$$

where:

ES_{n+1} = Estimated enrollment in the second grade in year "n+1".

B_{n-6} = The number of births in a given year, "n-6".

A = The average percent of survival from birth to the first grade.

$P_{1\cdot 2}$ = The average percent of survival between the first and second grade.

where:

$$P_{1\cdot 2} = \frac{\frac{S_{n+1}}{E_n} + \frac{S_{n+2}}{E_{n+1}} + \frac{S_{n+3}}{E_{n+2}} + \frac{S_{n+4}}{E_{n+3}} + \frac{S_{n+5}}{E_{n+4}}}{N}$$

where:

S = the enrollment in the second grade in a given year.

E = The enrollment in the first grade one year earlier.

N = The number of years for which ratios were computed.

This procedure is carried on for each grade, then summed for a total enrollment estimate. Obviously the degree of detail depends upon the demands of the particular situation, ie., whether an estimate of how many students will be taking chemistry is desired or merely a total enrollment estimate.[35]

The procedure in its common form has a number of weaknesses. First, it requires an estimate of births for a number of years into the future if projections of elementary enrollments of more than five or six years are desired. This is a task which demographers have not been able to accomplish with any great deal of success. Second, and perhaps of a more serious nature, birth statistics in Michigan are often not available for school districts as such. School district boundaries in Michigan do not necessarily correspond to any other govermental division. Therefore, if one is to use this approach some sort of interpolation from county or state statistics is required to estimate births by district. In an attempt to avoid estimating births by district, other formulae have been derived which are based on similar rationale but do not require birth statistics. One such formula is presented below.

$$EE_n = PSE_{n-5} \text{ (API x 5)}$$

where:

EE_n = Estimated public school enrollment in a given year. This can of course also be done by grade.

PSE_{n-5} = Public school enrollment five years prior to the year which is being estimated. The interval could of course be greater or less than five, depending upon how far into the future you are interested in projecting, as well as the nature of the available statistics.

API = The average percent increase in public school enrollment over a previous five year period.[36]

Third, this method relies quite heavily upon the averaging of percentages, a procedure that makes statisticians shudder. At least one report on the use of this approach demonstrates an awareness of this problem. Liu declares that ratios may be averaged over a number of years if no clear trend is discernible. However, if a trend is discernible, Liu suggests that it be taken into definite consideration when projecting enrollments. Unfortunately, no standardized method of accomplishing this has been reported in the literature.[37]

Finally, the projections which the procedure yields are often inaccurate. Thomas C. Holy stated back in 1947 that the estimates were consistently somewhat high.[38] No new developments since that time would lead this reviewer to believe that the formula has become any more accurate. Other weaknesses of this method will be discussed in the section that discusses common weaknesses of the various methods currently popular in education.[39]

Another method that has seen wide use in making educational enrollment projections is the so-called "enrollment-ratio" method. This method assumes that an observable ratio between school enrollment and school-age population or total population of some defined area has existed in the past and will continue to hold good for the future. The essential steps to the procedure are as follows:

1. Compute the average of the percents which total school enrollments were of total population, total school enrollments were of school-age population, or, grade enrollments were of grade-age population, for a relevant geographic area for each of a number of previous years.[40]

2. Apply that average to the future estimated total population of that relevant geographic area.

The formula can be expressed as follows:

$$EE_n = \frac{\frac{PSE_{n-10}}{TP_{n-10}} + \frac{PSE_{n-9}}{TP_{n-9}} + \frac{PSE_{n-8}}{TP_{n-8}} + \frac{PSE_{n-7}}{TP_{n-7}} + \frac{PSE_{n-6}}{TP_{n-6}}}{N} \quad X \quad EP_n$$

where:

EE_n = Estimated enrollment in a given year.

PSE = Public school enrollment in a given year. In the example enrollment is being projected five years into the future.

TP = The total population of a relevant geographic area in a given year

N = The number of ratios involved in obtaining the average. In the example, this is five.

EP = The estimated total population of a relevant geographic area in the year to which enrollment is being projected.[41]

It is obvious from the above presentation that projection procedures of this variety require a reliable estimation of the future population of relevant geographic areas. In Michigan, estimates of this type are made for cities, townships, and counties, but not for school districts. The three most common methods for projecting the population of small areas are the already discussed "cohort-survival" method, a simpler type of component method in which separate allowance is made for births, deaths, and migration without taking age into account, and the so-called "ratio" method.[42] Other methods include graphic and mathematical extrapolations.[43]

Since the "ratio" method is going to be used quite often in this study in order to translate available data into useable form for some of the formulae to be tested, there follows a discussion of it. Briefly, the "ratio" method derives population projections of the future population of an area on the basis of a series of observed historical ratios between the population of that area and the population of a larger area for which projections are already available. Such a procedure was used by the Philadelphia Planning Commission to project the population of the Philadelphia-Camden Industrial Area to the year 2000. In this case the projection was carried through the following four stages, using the already available total United States projections as a starting point.

1. United States urban population projections were obtained by extrapolating the historical ratios of the U.S. urban to the U.S. total population and multiplying the extrapolated ratio by the total population projections of the U.S.

2. The Northeastern Industrial Region's urban population was then projected by extrapolating ratios of its urban population to the U.S. urban population, and applying them to the U.S. urban population projections.

3. The Philadelphia-Camden Industrial Area's urban population was next projected by extrapolating from ratios of its urban population to the urban population of the Northeastern Industrial Region, and applying them to the projected urban population of this region.

4. Projections for the total population of the Philadelphia-Camden area were obtained by extrapolating ratios of its total population to its urban population and applying them to the projected urban population of the Philadelphia-Camden Area.[44]

When attempting to estimate district enrollment, the "enrollment-ratio" method encounters many of the same difficulties considered when the "cohort-survival" method was presented. There is evidence that suggests that the "cohort-survival" method yields better projections across the board than does the "enrollment-ratio" method.[45]

A relatively recent study conducted in Detroit developed a technique for projecting future enrollment based on both the "cohort-survival" and the "enrollment-ratio" techniques.[46] Different assumptions were used to estimate future births in different types of housing areas. Elementary service areas, that is, areas from which children may attend given elementary schools, were established as the base unit for estimating future school enrollments. The number of estimated births, year by year, for each service area, in conjunction with the age distribution of the population of that service area for the latest census period, permit the determination of future age distributions. These future age distributions are then used to estimate the future school population of each service area. The results of this procedure were sufficiently good that it is currently the procedure used for projecting future school enrollment in the Detroit Schools.

A third approach to the projection of school enrollments is based on the projection of dwelling units and is called the "pupil-home unit ratio" method. According to the literature, this approach is most feasible when one is projecting future school enrollment for rapidly expanding communities which are in their early period of growth and which still have relatively large parcels of unused land available for residential development.[47] This approach usually involves a local survey which collects information concerning such variables as the composition of households living in various types of dwelling places at the present time; the type of housing recently constructed in the area; the types of families moving into the new housing, including the number of persons per family and the number of elementary and high school students per family; the amount of land available for residential construction and the type of construction likely to be undertaken on this land; and any other information that seems relevant to the future development of the particular area.[48]

The information obtained from the local survey should provide a basis for:

1. Estimates of the rate of land utilization and the rate of construction of dwelling units, which can then be converted into projections of numbers of dwelling units on specified future dates.

2. Estimates of population per dwelling unit and pupil-population ratios which, when applied to the projected number of dwelling units will give the projections of total population and school enrollment.[49]

The major disadvantage of this method is the large amount of data which is necessary in order to successfully utilize it. The collection of this data involves the carrying through of a local survey for each community in which one is interested in projecting enrollment.[50]

A recent study examined the relationships between four characteristics of new homes and pupil yield. The four variables considered were: number of rooms, zoning ordinances, building permit valuation, and location of the home. From the cross tabulating of pupil yield with each of the four variables at each level of school, indices were derived based upon the mean yield of each category. The indices were tested by applying them to two areas of Jefferson County, Colorado, where large building developments were recently completed and children were attending the Jefferson County Public Schools. The projected enrollments were compared with actual enrollments and also with enrollment projections made by the school district through the use of traditional ratios. It was found that the indices derived from the study were able to project the number and grade distribution of pupils from newly occupied homes more accurately than those methods used by the school system.[51]

A number of other projection methods have been tried in public education, although apparently without a great deal of success. One such method averages the percentage of population growth in the past and applies this average to the future, using time as the independent variable. The method involves plotting a curve showing how the percentage increase changed in the past and extending that curve to show future percentage increases. Experience has shown that this method does not yield very reliable projections.[52]

Another approach consists of studying the growth of enrollment in similar school districts from a period at which many of their demographic variables were similar to those of the district of interest, and deducing their probable future enrollment from these studies. This method, forecasting by analogy, has not proven very reliable.[53]

Perhaps the major disadvantage of the methods currently in use for projecting public school enrollment is the assumption inherent in their methodology that the future represents a composite or average description of the past. In cases of ever increasing, or decreasing, total and school-age population and enrollment, this assumption could be invalid. The techniques to be discussed shortly assume a continued linear increase in the criterion variable, perhaps a more valid assumption when dealing with problems of enrollment projection.

Projection Methodology Used In Higher Education

Researchers working in higher education have used many of the techniques that have been used in the projection of public school enrollments.[54] Writing in 1960 in a methodology handbook on the subject of enrollment projections for colleges and universities, Lins identified five methods:[55]

(1) curve fitting methods
(2) ratio methods
(3) cohort survival methods
(4) combined ratio and cohort survival methods
(5) correlational analysis methods.

All of these methods, with the exception of curve fitting and correlational analysis, as well as practically every method described in the literature, are ratio methods. However, some researchers in higher education have experimented with regression analysis as a projective tool. The most comprehensive research in this area has been carried out by Savage and Brown at the University of Minnesota.[56] These authors concerned themselves with two types of projection problems which are relevant to this dissertation; the projection of numbers of high school graduates in a given year, and the projection of university attendance.[57]

Several methods of projection of numbers of public high school graduates for the state of Minnesota are presented. The data utilized are compiled from births and from children's tendencies to advance from one grade to the next. The projection models consist of combinations of linear regressions of various ratios on time. The authors believe that the compound use of linear regressions in a pseudo-cohort method is a new approach to the projection of future numbers of high school graduates.[58]

Method I involves the estimation of numbers of public high school graduates from the ratio of number of graduates to births eighteen years earlier. A straight line is fitted to the data by the least squares method. Using this straight line and appropriate birth rates, the numbers of high school graduates can be projected eighteen years beyond the latest data available on births. An added advantage of this approach is that confidence intervals can easily be computed for the estimates. These limits are based on the assumptions that the live-birth ratio is linearly related to time over the relevant time interval, and that deviations from linearity over this interval are unbiased, independent, and normally and identically distributed variables.[59] This method can also be used to project enrollment by grade through the examination of past enrollments in each grade relative to numbers of births for the appropriate earlier years.

A more sensitive and comprehensive description of changing school enrollment patterns was attained through the use of information on the changing tendencies of children to advance from one grade to the next. Transition ratios were computed for each of the twelve grades and the graduating class over a period of past years. Straight lines were then fitted to the data by the least squares procedure, and extrapolated to yield future estimates. These estimates of retention ratios were then applied to the number of births of "i+6" years previously, where "i" denotes the particular grade in which one is interested in projecting the enrollment. Confidence limits were computed for the ratios, but not for the actual enrollment estimates. The problem of computing confidence limits for the enrollment estimates is a complex one, since each estimate is based on the product of thirteen dependent estimates by linear regressions.[60]

Unfortunately no attempt was made by the authors to compare the results obtained through the use of the methods described above with actual enrollment statistics. Method II, the one using the transition ratios, consistently projected about 3.3% (with standard error of $\pm$0.4%) higher than Method I, and both methods gave estimates consistently above estimates made by the Minnesota Department of Education.[61] This is probably due to the assumption inherent in the regression methodology of a continued linear increase in the criterion variable rather than the customary assumption that the future resembles a composite of the past.

Savage and Brown also adapted regression procedures to projecting University attendance. They used a series of regression equations which range from simple extrapolation of attendance over time to a multivariate approach utilizing lags in numbers of high school graduates and net changes in military personnel.[62] As one might suspect, the multivariate procedures produced better estimates when compared with actual enrollment figures.[63]

In a study of enrollment projection methodology for Texas public junior colleges, multiple linear regression was tried.[64] Besides the experimental use of regression, standard ratio methods of projection, including cohort-survival methods, census class projections, and total population projections were explored. It was concluded that the ratio methods vary widely in their usefulness from community to community, make very limited use of the knowledge of relationships between enrollment and community characteristics, and require assumptions that are difficult to justify except on the basis of extensive local experience. It was also concluded that multiple linear regression, as used in the study, is best justified, and implemented, at the state level.[65]

Linear regression, as utilized by the authors mentioned in the preceding review of literature, and as will be used in this study, is a non-analytic tool in the sense that there is no underlying theory of human behavior which suggests such linear functions. The rationale is based on the principle that, next to a static pattern of behavior, linear changes represent the simplest case. So long as dramatic changes in educational and population patterns are not anticipated, linear approximations should yield good short-term estimates of future enrollment.[66]

Footnotes for Chapter II

[13]The basis for much of the discussion presented in the next few pages can be found in the Handbook of Statistical Methods For Demographers by A.J. Jaffe, Demographer, Office of the Coordinator of International Statistics, (Washington: U.S. Bureau of the Census), United States Government Printing Office: 1951, 278p.

[14]The approach suggested here is through the use of the equation: $P = A + X \left(\frac{B-A}{N}\right)$ where P is the population to be calculated, A is the population enumerated at an earlier census, B is the population enumerated at a later census, X is the number of years beyond the earlier census for which the population is being calculated, and N is the number of years between censuses. Apparently, no attempt has been made to apply the least squares model to the projection problem, or to use any independent variable other than time.

[15]Jaffe, Handbook, op.cit., p.212.

[16]Ibid., p.213. Occasionally in the discussion the interval "ten years" is referred to. Government demographers generally work with data that is collected once every ten years, the interval between Federal Censuses.

[17]Jaffe, Handbook, op.cit., p.213. It seems that this objection could be gotten around simply by putting an upper bound on a polynomial or exponential function. For a good discussion of the logistic curve see: W.A. Spurr and D.R. Arnold, "A Short-Cut Method Of Fitting a Logistic Curve," Journal of the American Statistical Association, (March, 1948), pp. 127-34. v.43.

[18]The Handbook was published in 1951. Publications of the Census Bureau up through 1968 would not lead one to believe that this position has changed.

[19]Huan D. Kuang, "Forecasting Future Enrollments by the Curve Fitting Techniques," Journal of Experimental Education, XXIII (1955), p.273.

[20]The U.S. Census Bureau is making population projections for large georgraphic and demographic classifications that are specified by relatively complete and accurate data. These projections are often made far into the future. Mathematical curves are perhaps not as applicable to this type of problem. However, for moderately short estimates, say 5 to 15 years into the future, with relatively imprecise data and a relatively homogenous population, as is the situation when working at the local level, some varieties of mathematical curves might prove quite valuable for projection purposes.

[21]U.S. Bureau of the Census, Current Population Reports, Series P-25, No.388, "Summary of Demographic Projections," (Washington: U.S. Government Printing Office, 1968), p.25.

[22]U.S. Bureau of the Census, Current Population Reports, Series P-25, No.381, "Projections of the Population of the United States by Age, Sex, and Color, to 1990, with Extensions of Population by Age and Sex to 2015," (Washington: U.S. Government Printing Office, 1967), p.45.

[23]See Current Population Reports, Series P-25, No.388, op.cit., pp.2-10.

[24]The presentation here is only designed to give the reader some insights into the methodology of population projection. Different assumptions are made for white and non-white populations, different age cohorts, etc. This is why assumptions are referred to as sets of assumptions in the text of this paper. For a detailed presentation of the methods and assumptions see: Jacob S. Siegel, Current Population Reports, Series P-25, No.381, op.cit., Francisco Bayo, Social Security Administration, Office of the Actuary, Actuarial Study No.62, "United States Population Projections for OASDHI Cost Estimates," 1966. When projecting populations for states, a more intensive consideration of migration is needed. See Current Population Reports, Series P-25, No.375, "Revised Projections of the Population of States: 1970 to 1985," 1967.

[25]Projections of future enrollment as well as a brief discussion of the projection methodology can be found in Current Population Reports, Series P-25, No.365, "Revised Projections of School and College Enrollment in the United States to 1985," Jacob S. Siegel, (Washington: U.S. Government Printing Office), 1967.

[26]Michigan Population - 1960 to 1980. The Population Studies Center at the University of Michigan, in cooperation with the State Resource Planning Division, Michigan Office of Economic Expansion, Michigan Department of Commerce. (Working Paper No.1, January, 1966). David Goldberg and Allan Schnaiberg were the major contributors.

[27]Estimates are prepared using a ratio correlation method based on data collected during the 1950-1960 period.

[28]Michigan Office of Economic Expansion, Working Paper No.1, op.cit., pp.9-14.

[29]Michigan Department of Commerce, Preliminary Population Projections for Small Areas in Michigan, State Resource Planning Program, Office of Economic Expansion, (Working Paper No.9, November, 1966).

[30]Projections of total population by counties presented in Michigan Population - 1960 to 1980, and an unpublished extension of these projections to 2000, were used as control figures. The sum of the projected populations for all local units in each county are adjusted to equal the projected county totals for each date.

[31]Where: P=projected population; a=the y intercept of the regression line; b=the regression coefficient; X=the independent variable; B=the year of the later census enumeration; A=the year of the earliest census enumeration; N=the interval between censuses, in this case 10. The same type of regression line was fitted to each local unit. Incorporating the log of time in the equation has the effect of reducing an indicated past rate of increase or decrease in each local unit's share of its parent county population.

[32]In the few cases where this constraint applied, the projected shares of other local units in the county are below the values which would have occurred without the constraint.

[33]Michigan Office of Economic Expansion, Working Paper No.9, op.cit., pp.3-6.

[34]Ibid., p.7.

[35]The information for the preceding two pages was collected from a number of sources, too numerous to cite here. However, some of the more important sources were James D. MacConnell, Planning for School Buildings, Prentice-Hall, Englewood Cliffs, N.J.: 1957, pp. 32-38; Alfred Liu Bangnee, Estimating Future School Enrollment in Developing Countries-A Manual of Methodology, UNESCO, Population Studies, ST/SOA/Series A, No. 40, 1966; Harold J. Bowers, "Projecting School Enrollments for One State," Journal of Teacher Education, Vol. V, No.1, pp.64-66. Harold J. Boles, Step to Step To Better School Facilities, New York: Hold, Rinehart and Winston, 1965, pp.40-48; Hauser and Kitagawa, op.cit., pp.64-65.

[36]Waldfogel, op.cit., p.4.

[37]Liu Bangnee, op.cit., p.19.

[38]Thomas C. Holy, "What Future Needs Are Revealed by School Population Studies," Education Digest, May, 1947, V.9: pp.24-26. Even so there is some evidence to suggest that, of the various common techniques for projecting school enrollment, this method is the most reliable except in the case when a major change in the rate of increase or decrease of school enrollment occurs. Donald Lee Peterson, An Investigation of Techniques for Predicting School District Enrollments in Florida, Ed.D. Dissertation, University of Florida, 1959.

[39]This is not to imply that there is nothing good about this projection method. It has generally proven adequate for estimating enrollments at a county or state level, where relevant data are available.

[40]The particular form in which school enrollment rates are computed depends upon the detail of the enrollment projections desired and the amount of age detail available in the population projections to which the enrollment rates are to be applied. For example, if enrollment projections for single grades are desired, grade-specific enrollment rates for each year of age should be computed and multiplied by the projected population in each year of age. So detailed a projection is probably only warranted when the projection is for a short span of years, no more than five years, perhaps, and if reasonably reliable projections by single years of age can be obtained. When enrollment projections are desired for longer periods of time, or when population projections are not available by single years of age, projections of total elementary and high school enrollments may be obtained in the following manner. (1) compute an enrollment rate expressing the ratio of elementary enrollment in grades 1-8 to the number of persons 6-13 years of age and one of high school enrollment to the number of persons 14-17 years of age, from statistics for the last census. (2) multiply the elementary enrollment rate by the estimated population 6-13 years of age on the projection date, and multiply the adjusted high school enrollment rate by the estimated population 14-17 years of age on the projection date. The resulting products are the estimated elementary and high school enrollments on the projection date.

[41]Information collected from a number of sources cited in the Bibliography. This approach is suggested in; American Association of School Administrators, 27th Yearbook, American School Buildings, School Planning Lab, Stanford University, pp.55.

[42]As well as some experimental methods that have already been discussed.

[43]For a discussion of the various methods and a citation of relevant source material see; Jacob S. Siegel, "Forecasting the Population of Small Areas," Land Economics, February, 1953, pp.72-87. In most cases population projections are obtained through the use of the "cohort-survival" method as previously described.

[44]Philadelphia Planning Commission, Population Estimates 1950-2000, Philadelphia-Camden Area, (April, 1948).

[45]Peterson, op.cit.

[46]Norman S. Wheeler, Techniques Used For Predicting Public School Enrollments in Detroit, Michigan, Using the Elementary School Service Area as a Base, Ed.D. Dissertation, Wayne State University, 1955.

[47]Abe Gottlieb, "A Planning Approach To School Enrollment Forecasts," American School Board Journal, February, 1954, p.110. See also Hauser and Kitagawa, op.cit., p.69.

[48]Hauser and Kitagawa, op.cit., p.69.

[49]In the second step, estimates of pupils per dwelling unit may be made directly and applied to the projections of dwelling units without the intermediate step of estimating total population.

[50]Despite the effort that must be extended in this approach, there is little evidence to suggest that it yields accurate results. It is generally felt that it yields the best results in "exploding communities." Hauser and Kitagawa, op.cit., p.69.

[51]William N. Driscoll, The Prediction of School Enrollment From New Home Developments, (Research Study No.1), Ed.D. Dissertation, Colorado State University, 1965.

[52]MacConnell, op.cit., pp.32-38.

[53]Larson and Strevell, op.cit., pp.65-68.

[54]L.J. Linis, Methodology of Enrollment Predictions for Colleges and Universities, American Association of Collegiate Registrars and Admissions Officers, March, 1960. C.F. Schmid and F.J. Shanley, "Techniques of Forecasting University Enrollment," Journal of Higher Education, Vol. XXIII, No.9, 1952, pp.483-489.

[55]L.J. Lins, Methodology of Enrollment Projections for Colleges and Universities, American Association of Collegiate Registrars and Admissions Officers, March, 1960.

[56]Bryon W. Brown and I. Richard Savage, Methodological Studies in Educational Attendance Prediction, University of Minnesota, Dept. of Statistics, Sept., 1960.

[57]The methodological aspects of university attendance projection are of interest.

[58]Brown and Savage, op.cit., p.11.

[59]Brown and Savage, op.cit., p.12.

[60] Ibid., p.20.

[61] Brown and Savage, op.cit., p.19.

[62] Ibid., pp.23-41.

[63] Ibid., p.36.

[64] David G. Hunt, The Development of Enrollment Projection and Prediction Methods for Texas Public Junior Colleges, The University of Texas, Ph.D. Dissertation, 1962.

[65] This is strictly an empirical conclusion based on data collected from Texas Public Junior Colleges and cannot be interpreted as a condemnation of the use of multiple linear regression for projecting school district enrollment at a local level.

[66] Simple linear models are not appropriate for long-range projections. For example, if the birth-ratio method were pushed indefinitely into the future, more graduates from high school would be projected than babies were born eighteen years previously. Brown and Savage, op.cit., p.21.

CHAPTER III

METHODOLOGY

Population and Sample

The population identified in this study consists of every public school district in Michigan, the major part of which is coterminous with a city of 10,000 population or more. (According to the 1960 Census.) Building on the classification scheme proposed by Hauser and Kitagawa,[67] the population has been divided into five strata on the basis of:

1. The percent of increase of the general population, during the decade 1950-1960, of the urban place with which the majority of the district is coterminous.
2. The percent of increase of the number of households, during the decade 1950-1960, of the urban place with which the majority of the district is coterminous.

A sample of five school districts was randomly drawn from within each of the five strata, yielding a total of twenty-five school districts. Each school district was treated as a separate case study.

Definition of Terms

1. Urban Place

   The term "place" as used in reports of the decennial census refers to a concentration of population regardless of the existence of legally prescribed limits, powers, or functions. Most of the places listed in the Census Reports are incorporated as cities, towns, villages, or buroughs, however, and all of the communities in which the school districts that make up the population for this study are located are incorporated as cities.

2. City

   The term "city" as used in this study refers to urban places of 10,000 population or more that are incorporated as cities.

3. County

   The term "county" refers to the primary political divisions of the State.

4. Households

   Changes in definitions can have important implications for studies of population groups. In the 1950 Census, "households" did not include persons living in institutional dwelling units such as hospitals, college dormitories, or military posts. In the 1960 Census, persons living in such units were counted as households.

Classification of Districts into Strata

School districts were classified into strata on the basis of certain decision rules involving the population growth variables of the urban place with which the major portion of each district is coterminous. A discussion of those decision rules by stratum follows.

Stratum A

School districts classified into this stratum are located in communities that increased by more than 100%, during the decade 1950-1960, in both general population and in the number of households. Communities that have experienced such rapid growth during the relatively brief period of ten years will be called "Exploding Communities."

TABLE 1

SAMPLE DISTRICTS DRAWN FROM EXPLODING COMMUNITIES (STRATUM A)

District	% of Pop. Increase 1950-1960	% of Household Increase 1950-1960
Allen Park	200.5	201.7
Garden City	321.8	291.1
Inkster	133.7	136.8
Livonia	280.4	252.1
Oak Park	595.5	569.3

Stratum B

School districts classified in this stratum are located in communities that increased by 50% or more in either or both of the classification variables, but 100% or less in one of the classification variables, during the decade 1950-1960. Communities that can be classified into this category will be called "Rapidly Growing Communities."

TABLE 2

SAMPLE DISTRICTS DRAWN FROM RAPIDLY GROWING COMMUNITIES (STRATUM B)

District	% of Pop. Increase 1950-1960	% of Household Increase 1950-1960
Ann Arbor	39.6	71.4
Birmingham	65.0	63.7
Lincoln Park	84.0	79.7
Midland	94.5	86.3
Royal Oak	71.9	71.7

Stratum C

School districts classified in this stratum are located in communities that have experienced an increase of 10% or more, but less than 50%, in both the number of inhabitants and the number of households during the decade 1950-1960. This type of community will be called an "Older But Still Growing Community."

TABLE 3

SAMPLE DISTRICTS DRAWN FROM OLDER BUT STILL GROWING COMMUNITIES (STRATUM C)

District	% of Pop. Increase 1950-1960	% of Household Increase 1950-1960
Alpena	11.8	11.8
Dearborn	17.9	27.0
Kalamazoo	42.3	40.5
Pontiac	11.6	15.8
Wyandotte	18.1	20.7

Statum D

School Districts classified in this stratum are located in communities that have experienced an increase of less than 10% but equal to or greater than 0% in both the number of households and the number of inhabitants, or, an increase of less than 10% but equal to or greater than 0% in one of the classification variables, and an increase of 10% or more in the other, during the decade 1950-1960. Communities of this type will be referred to as "Stable, Well-Established Communities."

TABLE 4

SAMPLE DISTRICTS DRAWN FROM STABLE WELL-ESTABLISHED COMMUNITIES (STRATUM D)

District	% of Pop. Increase 1950-1960	% of Household Increase 1950-1960
Bay City	2.1	4.5
Grand Rapids	0.5	2.6
Ferndale	5.6	13.7
Monroe	7.0	10.2
Saginaw	5.8	7.0

Stratum E

School districts classified in this stratum are located in communities that have experienced a decrease in either or both general population and the number of households during the decade 1950-1960. This type of community will be called a "Declining Community."

TABLE 5

SAMPLE DISTRICTS DRAWN FROM DECLINING COMMUNITIES (STRATUM E)

District	% of Pop. Increase 1950-1960	% of Household Increase 1950-1960
Battle Creek	-9.2	-3.1
Hamtramck	-21.3	-9.8
Highland Park	-18.0	-4.2
Ironwood	-10.5	-1.6
Muskegon	-4.0	-0.4

At another level of sampling, six data points were obtained on each of the predictor variables used in the study. The major reason for the small number of data points was simply the lack of availability of adequate data. The small number of data points severely limits the use of multiple linear regression, placing an upper limit of four on the number of independent or predictor variables that can justifiably be used at any given time.

Data

Probably the most thorough statistical description of factors related to the projection of enrollment, although admittedly junior college enrollment, was compiled by Rodgers.[68] Although most of the factors that he investigated were intended merely for descriptive purposes, he did eventually select ten factors which his data suggested would form the best basis for enrollment projections. Those ten factors were:

1. Total population of scholastic age.
2. Public school enrollment.
3. Average daily attendance relative to school-age population.
4. High school enrollment.
5. High school average daily attendance.
6. High school graduates.
7. High school seniors planning to attend college.
8. College Age youth in the general population.
9. Total population in census years.
10. Estimated total population.

Rodgers also described the relationship between enrollment and such other factors as:

1. Population per square mile.
2. Number of farm owners.
3. Number of farm tenants.
4. Automobile registrations.
5. Poll taxes paid.
6. Number of employed persons.
7. Wages paid.
8. Income.
9. Retail sales.
10. Wholesales.
11. Value of manufactures.
12. Bank deposits.
13. County and State tax values.

These factors were related to enrollment by the means of ratios, however, it is evident from the data that these factors were not as highly related to enrollment and therefore would not provide as reliable an estimate as the aforementioned ten factors that were eventually used.

Keeping both Rodgers' study and traditional projection methods in mind, the following independent or predictor variables were chosen as the most appropriate for use in this study.

1. Past District elementary enrollment.
2. Past District secondary enrollment.
3. Elementary school-age births.
4. Secondary school-age births.
5. Total county population.
6. Total city population.
7. City elementary school-age population.
8. City secondary school-age population.
9. Extrapolated city elementary school-age population.
10. Extrapolated city secondary school-age population.
11. Extrapolated county elementary school-age population.
12. Extrapolated county secondary school-age population.
13. City occupied housing.
14. Mean number of persons per occupied unit.
15. State Equalized Valuation of Districts.
16. Annexations.

Data Sources and Projection Methodology

Much of the data that is used to test the various projection formulae which are dealt with in this study was extrapolated or interpolated from the little data that was available in raw form. Therefore, a major assumption of this study must be that the methods that are utilized to obtain much of the data are valid means of doing so. There follows a discussion of the data sources and of the methods used to obtain estimates of those values that were not directly available. Much thought was given to the development and utilization of appropriate methodology for obtaining estimates of important predictor variables since the researcher is well aware that the outcome of the study is dependent in part upon the accuracy of projections of values for those predictor variables whose values were not directly available.

1. Elementary Enrollment.

Elementary enrollment is defined as those full time, non-special education, students enrolled in grades K through 8 of the various school districts that comprise the sample. The K through 8 breakdown was used in order to make it possible to utilize United States Census data on the school-age population of the various cities of the sample. Non-public school children were not considered because of the extreme difficulty in obtaining data on them. The source used for elementary enrollment figures was the State Department of Education County Superintendent Summary Reports, State Financial and Enrollment Statistics, which records enrollment by grade.

2. Secondary Enrollment.

Secondary enrollment is defined as those full time students in non-special education classes attending grades 9 through 12. Non-public school children were not considered. The source used for figures on secondary enrollment was the State Department of Education County Superintendent Summary Reports, State Financial and Enrollment Statistics, which records enrollment by grade.

3. Births By City By Year.

Statistics on births by city by year were obtained from the Vital Statistics of the United States, Part II, Place of Residence. This source was considered the most accurate for purposes of this study since births are recorded by place of residence rather than merely by place of birth. Births were recorded for each district from 1938 to 1960, and extrapolated beyond 1960 to 1963 through a linear extrapolation.

4. Births By City By Period.

Statistics on births by city by period were computed for each district in the sample on the basis of data obtained from Vital Statistics of the United States, Part II, Place of Residence. For example, children that were in elementary school in 1955 were born between 1942 and 1950. Therefore, by adding up the number of births in a particular city between 1942 and 1950, the total number of children born in that city during the years that contribute to elementary enrollment in 1955 is obtained. In a like manner the number of children born between 1938 and 1942 should be related to secondary enrollment in 1955. Linear extrapolations beyond 1960 to 1963 were used as estimates of the number of births in the years 1961 through 1963.

5. County Population.

Population figures for Michigan Counties were obtained for the years 1950 and 1960 from reports of the United States Census Bureau. However, only estimates were available for the years 1955 through 1959, and 1965 and 1968. The county population estimates used were prepared by the University of Michigan Population Studies Center and are based on the assumptions outlined in Appendix A. These estimates were chosen because they are the most widely used estimates, as well as being the estimates utilized by the State of Michigan in projecting future population figures.

6. City Population.

Population figures for Michigan cities were obtained from the Center for Health Statistics, Michigan Department of Public Health. Statistics for the years 1950 and 1960 are the same as can be obtained from the U.S. Census Bureau. Estimates of city population between the years 1955 and 1960 were obtained through the use of a ratio correlation procedure which utilizes data on births, auto registration, sales tax, voter registration, and school enrollment, (See Appendix B.) Michigan Department of Health estimates were chosen because they are the most widely used estimates of city population.

Estimating the city population for 1965 and 1968 required a complicated process which involved the averaging of two separate estimates. The first estimate was obtained through a linear extrapolation of the ratios of city population to county population and the subsequent application of the extrapolated ratios to county population estimates for 1965 and 1968. The second estimate was obtained through averaging the ratios and applying the results to county population estimates. A subsequent averaging of the two estimates provided the needed projection of city population.[69]

7. City Elementary School-Age Population.

Data on city elementary school-age population was obtained for the years 1950 and 1960 from the Reports of the United States Census Bureau. Ratios of school-age to over-all population by city were established for 1950 and 1960. Linear interpolations and extrapolations were then completed in order to obtain estimates of the ratio of school-age population to total city population for each year. The resulting ratios were then applied to the estimates of total city population for relevant years in order to obtain estimates of city school-age population.

8. City Secondary School-Age Population.

Estimates of city secondary school-age population were obtained in the same manner as were the estimates of city elementary school-age population.

9. City Extrapolated Elementary School-Age Population.

This represents a special type of estimate for use in one of of the regression approaches. Data on city elementary school-age population was obtained for the years 1950 and 1960 from the Reports of the United States Census Bureau. Using these figures as a base, a straight line was applied to the data and extrapolated to 1965 and 1968.

10. City Extrapolated Secondary School-Age Population.

Same procedure as in "9" with data on secondary school-age population.

11. County Extrapolated Elementary School-Age Population.

Same procedure as in "9" with data at the county level.

12. County Extrapolated Secondary School-Age Population.

Same procedure as in "10" with data at the county level.

13. Occupied Housing Units.

Data on occupied housing units per city for the years 1950 and 1960, as well as the mean number of persons per occupied unit, was obtained from the U.S. Census Bureau. Data on occupied housing units was linearly interpolated and extrapolated to arrive at estimates for relevant dates. Another estimate was obtained through the application of the mean number of persons per occupied unit in a given city to the estimated number of people in that city. The two estimates were then averaged.

14. State Equalized Valuation.

Data on State Equalized Valuation was obtained from the Michigan State Department of Education State Aid Sheets. The data were linearly extrapolated to 1965 and 1968.

15. Annexations.

Although districts were chosen from a population of districts whose boundaries were largely coterminous with city boundaries, there were some exceptions. Data on annexations between 1950 and 1968 was therefore required. This information was obtained from the State Department of Education.

The following table presents a summary of the previous discussion.

TABLE 6

FACTORS SELECTED FOR STUDY AND THE SOURCES OF DATA DESCRIPTIVE OF EACH

Data	Year[a]	Source[b]
1. Elementary enrollment	1950,1955-1960, 1965,1968	Michigan State Department of Education.
2. Secondary enrollment	1950,1955-1960, 1965,1968	Michigan State Department of Education.
3. Births by City	1938-1960	Vital Statistics of the United States, Part II, Place of Residence.
	1961-1963	Linear extrapolation of pre-1960 data.
4. Births by City by Period	1938-1963	Births per year added over relevant periods. Utilize data on Births by City.
5. County Population	1950,1960	U.S. Census Bureau.
	1955-1959,1965, 1968	The University of Michigan Population Studies Center.

TABLE 6 (cont'd.)

Data	Year[a]	Source[b]
6. City Population	1950,1960	U.S. Census Bureau.
	1955-1959	Michigan Department of Public Health.
	1965,1968	Linear extrapolation of the ratios of city population to county population and the subsequent application of the resulting ratios to county population estimates for 1965 and 1968. In addition, the averaging of the same ratios and subsequent application to estimates of county population. Results are then averaged.
7. School-Age Population	1950,1960	U.S. Census Bureau.
	1955-1959	Linear interpolation of ratios of school-age population to total city population, and their subsequent application to estimates of city. population
	1965,1968	Linear extrapolation of ratios of school-age population to total city population and their subsequent application to estimates of city population.
8. Extrapolated City and County School-Age Population	1950,1960	U.S. Census Bureau.
	1955-1959	Linear interpolation of 1950 and 1960 school-age population figures.

TABLE 6 (cont'd.)

Data	Year[a]	Source[b]
	1965,1968	Linear extrapolation of 1950 and 1960 school-age population figures.
9. Occupied Housing Units	1950,1960	U.S. Census Bureau.
	1955-1959	Linear interpolation of 1950 and 1960 occupied housing figures and mean number of persons per occupied unit figures, application of the latter to population figures, and the subsequent averaging of the two estimates.
	1965,1968	Linear extrapolation of 1950 and 1960 occupied housing figures and mean number of persons per occupied unit figures, application of the latter to projected population figures, and the subsequent averaging of the two estimates.
10. State Equalized Valuation	1950,1955-1960	Michigan State Department of Education.
	1965,1968	Linear extrapolation of pre-1960 figures.
11. Annexations	1950-1968	Michigan State Department of Education.

[a]The symbol "-" is to be interpreted as the word "through". Thus 1955-1960 is to be read, "1955 through 1960".

[b]A detailed citing of each source appears in the part of the Bibliography that deals with data sources.

Regression and Ratio Methods Examined

This study examines the applicability of seventeen separate and distinct ratio and regression methodologies to the projection of future public school enrollment. The following criteria were used in choosing and developing projection methods:

1. The method should be uncomplicated enough so that it can be easily and quickly explained and simple enough so that computations can be made on a desk calculator or with standard electronic-computer programs.
2. There should be grounds for believing that the model will in fact approximate events.
3. The data necessary for making projections as well as for testing the model should be available.

As you recall, the distinction between ration and regression analysis methods is made on the basis of the measure of relationships used in the specific computation of projected enrollments. Ratio methods utilize the ratio or proportion of a given predictor to enrollment, while regression methods use coefficients of correlation and multiple correlation, statistics which purport to measure degree of association. Table 7 presents the seventeen projection methods that are considered in this study in terms of wehther each is a ratio or regression method, the predicor variable or variables used in each, and the relationship on which each projection is based. This Table should be read while keeping the previous discussions of assumptions and data limitations in mind.

TABLE 7

PROJECTION METHODOLOGIES EXAMINED IN THIS STUDY

Name	Type	Predictor (s)	Relationship
1. Cohort-Survival	Ratio	1. Births by City 2. Past District Enrollment	Past tendencies of students to advance from one grade to the next.
2. Grade-Retention	Ratio	Past District Enrollment	Past trends in public school enrollment.
3. Enrollment-Ratio Method I	Ratio	Total City Population	Past ratio of public school enrollment to total city population.
4. Enrollment-Ratio Method II	Ratio	Total County Population	Past ratio of public school enrollment to total county population.
5. Pupil-Home Unit	Ratio	Home Units (City)	Past ratio of public school enrollment to city occupied housing units.
6. Time Analysis I	Regression	Time	Past relationship between public school enrollment and time.
7. Time Analysis II	Regression	Time	Past relationship between the log of public school enrollment and time.
8. Transition Analysis	Regression	1. Births by City 2. Past District Enrollment	Past tendencies of students to advance from one grade to the next. (Also from birth to kindergarten)

TABLE 7 (cont'd.)

Name	Type	Predictor(s)	Relationship
9. Birth Analysis	Regression	Births by City	Past relationship between births and public school enrollment.
10. Enrollment Regression Analysis I	Regression	City School-Age Population	Past relationship between city school-age population and public school enrollment.
11. Enrollment Regression Analysis II	Regression	1. City School-Age Population 2. Total City Population	Past relationship between city school-age population, city total population, and public school enrollment.
12. Enrollment Regression Analysis III	Regression	1. City School-Age Population 2. Total County Population	Past relationship between city school-age population, county total population, and public school enrollment.
13. Enrollment Regression Analysis IV	Regression	1. Extrapolated City School-Age Population 2. Extrapolated County School-Age Population	Past relationship between extrapolated city school-age population, extrapolated county school-age population, and public school enrollment.
14. Housing Analysis	Regression	1. Housing 2. State Equalized Valuation	Past relationship between occupied housing, SEV, and public school enrollment.

TABLE 7 (cont'd.)

Name	Type	Predictor(s)	Relationship
15. Multi-Variable Analysis I	Regression	1. Births by City 2. City School-Age Population 3. Total City Population 4. Total County Population	Past relationship between the predictor variables and public school enrollments.
16. Multi-Variable Analysis II	Regression	Same variables as in 15	Past relationship between the predictor variables and the logs of public school enrollment.
17. Multi-Variable Analysis III	Regression	Logs of variables used in 15	Past relationship between the logs of the predictor variables and the logs of public school enrollment.

Discussion

It is one of the basic tenets of this study that most relationships can best be described by a linear approximation over a short period of time. This assumption is tested through the use of logarithmic transformations in three of the regression equations. These transformations provide varying approximations of positively accelerating curves. If these methods do in fact provide accurate projections of future enrollment, it will suggest that there should be greater experimentation with non-linear projection methodologies.

Design

The seventeen ratio and regression analysis projection methods presented in the preceding section were used to develop projections of elementary and secondary enrollment for the twenty-five school districts in the sample for the years 1965 and 1968. These projections were based on data which were available during or prior to the year 1960. In this manner it was possible to use actual enrollment in the years 1965 and 1968 as the criterion against which to judge the relative performance of the various projection methods.

As was previously mentioned, the sample school districts were divided into five strata on the basis of past community growth characteristics. Each of the seventeen projections methods was evaluated on the basis of its applicability to districts within each of the five stratum. The criterion of goodness of fit between estimated and actual district enrollment in 1965 and 1968 formed the basis for evaluation. Goodness of fit was determined in the following manner.

Each of the twenty-five districts represented in the sample has sixty-eight separate estimates of different aspects of its enrollment. Table 8 presents a breakdown of those estimates.

TABLE 8

NUMBER OF ENROLLMENT PROJECTIONS FOR EACH DISTRICT IN THE SAMPLE

	1965		1968	
Level	Ratio	Regression	Ratio	Regression
Elementary	5	12	5	12
Secondary	5	12	5	12

This means that for any given school district in any given stratum for any given method there are four separate estimates of enrollment; one for elementary enrollment in 1965, one for secondary enrollment in 1965, one for elementary enrollment in 1968, and one for secondary enrollment in 1968. Needless to say there are also four actual enrollment figures. Goodness of fit is determined by substracting the actual enrollment figure from the projected enrollment figure for each of the four aforementioned estimates, dividing each by its actual enrollment figure, and reporting each for its respective estimate. The resulting statistic, a measure of the percentage of error in each estimate, is called the Webster Coefficient and is labeled "W" whenever it appears. The smaller the Webster Coefficient, the better the estimate.

In other words, goodness of fit is determined by the Webster Coefficient (W). For any given school district within any given method there are four W Coefficients reported.

$$W_1 = \frac{(\text{Projected Elementary Enrollment}_{1965} - \text{Actual Elementary Enrollment}_{1965})}{\text{Actual Elementary Enrollment}_{1965}}$$

$$W_2 = \frac{(\text{Projected Secondary Enrollment}_{1965} - \text{Actual Secondary Enrollment}_{1965})}{\text{Actual Secondary Enrollment}_{1965}}$$

$$W_3 = \frac{(\text{Projected Elementary Enrollment}_{1968} - \text{Actual Elementary Enrollment}_{1968})}{\text{Actual Elementary Enrollment}_{1968}}$$

$$W_4 = \frac{(\text{Projected Secondary Enrollment}_{1968} - \text{Actual Secondary Enrollment}_{1968})}{\text{Actual Secondary Enrollment}_{1968}}$$

In addition, the Webster Coefficients are ranked within each district. As previously mentioned, each district has seventeen separate estimates of each of the four aforementioned enrollment figures. These seventeen separate estimates are ranked within each of the four sub-categories by district according to the size of the W Coefficient. The W Coefficients are then summed across the four sub-category estimates in order to obtain a composite W Coefficient for a given projection approach within a given district across the four estimates. The resulting composite W Coefficient is then ranked and is used as one index of the effectiveness of a given projection approach for a given district relative to other projection approaches for that district.

A second index of the effectiveness of a given projection approach is a frequency count by stratum of the number of projections yielded by each methodology that are within a given degree of accuracy of the actual enrollment figures. These observations are grouped by elementary and secondary projections, but collapsed over years. Although the observations certainly are not independent of one another, they nevertheless serve as indicants of the degree of success that a given methodology enjoys in a given stratum.

The two indicants discussed above are used, finally, to determine those projection formulae that are best suited for districts with growth characteristics similar to those described in each of the five strata that comprise the sample. Although at first glance it may appear as if the study utilizes a very small sample of districts, the reader is reminded that the entire population to which the study wishes to generalize consists of only some eighty Michigan school districts. Although there are many generalizations that may probably be extrapolated to other populations, the researcher would hesitate to recommend the unqualified application of the results of this study to states with growth patterns different from those of Michigan.

Footnotes for Chapter III

[67]Hauser and Kitagawa, op.cit., pp.69-70.

[68]Jack Rodgers, Criteria For the Establishment of Local Public Junior Colleges in Texas, unpublished Ph.D. Dissertation, University of Texas, June, 1956.

[69]This procedure was followed for every district in the sample except Livonia. In this one case it was obvious that the population estimates emanating from the method outlined in the text was completely unrealistic. Therefore, only the estimates obtained through a linear extrapolation of the ratios of city population to county population and the subsequent application of the extrapolated ratios to county population estimates for 1965 and 1968 were used.

CHAPTER IV

RESULTS

The results of the study are presented in seventeen separate sections, each section dealing with one of the seventeen projection methodologies examined in the study. In order to reorient the reader to the salient details of each of the methodologies, a short description of each is presented prior to the display of the results of the application of that methodology to the projection problem. The display of the results of each methodology is facilitated through the use of two tables. The major table presents the following data for each District.

1. Projected Elementary Enrollment, 1965. $\binom{EE}{1965}$

2. The W Coefficient for the 1965 elementary projection. (W)

3. The rank of the 1965 elementary projection relative to projections yielded by the other sixteen projection methodologies examined in the study. (R)

4. Projected Secondary Enrollment, 1965. $\binom{SE}{1965}$

5. The W Coefficient for the 1965 secondary projection. (W)

6. The rank of the 1965 secondary projection relative to projections yielded by the other sixteen projection methodologies examined in the study. (R)

7. Projected Elementary Enrollment, 1968. $\binom{EE}{1968}$

8. The W Coefficient for the 1968 elementary projection. (W)

9. The rank of the 1968 elementary projection relative to projections yielded by the other sixteen projection methodologies examined in the study.

10. Projected Secondary Enrollment, 1968. (SE) (1968)

11. The W Coefficient for the 1968 secondary projection. (W)

12. The rank of the 1968 secondary projection relative to projections yielded by the other sixteen projection methodologies examined in the study. (R)

13. A rank of the composite W Coefficients (R) The W Coefficients are summed within districts and projection methodologies across the four separate enrollment estimates. The resulting composite W Coefficient is then ranked across projection methodologies in order to provide one basis for assessing the relative performance of the seventeen separate projection methodologies. The composite ranks range from 1 to 17.

The Districts are organized by stratum in order to further facilitate comparison.

Another way of looking at the data is to look at the number of projections yielded by each methodology that are within a given degree of accuracy of the actual enrollment figure. The second table that displays information about each of the methodologies presents the number of estimates yielded by each methodology that were within a W Coefficient of .1000 of actual enrollment figures. Besides presenting this information by stratum, the second table also presents a composite picture of the data, that is, the percentage of the time by stratum that all seventeen approaches examined in this study yielded projections that were within a W Coefficient of .1000 of actual enrollment figures. The information supplied by this table thus provides another basis for an assessment of the performance of a given methodology relative to the performance of the other sixteen projection methodologies examined in this study. Projections that appear in this table are collapsed over years while maintaining the elementary and secondary breakdown.

Multiple correlation coefficients, regression coefficients, standard errors of regression coefficients, and F Statistics assessing the usefulness of the regression lines as predictors, are all available for each of the regression approaches. They are not reported because of a number of reasons:

1. The criterion by which the usefulness of a particular methodology is assessed is not the degree to which the regression line fits past data, but rather the accuracy of the projections yielded by that line as determined by the degree of discrepancy between those projections and actual data. That the regression line fit past data is a necessary but not sufficient requirement of the methodology.

2. Due to the nature of the projection problem and the rather unorthodox use made of the regression model in this study, the statistics mentioned above do not supply a great deal of useful information. This point will be discussed in greater detail shortly.

3. There are some eight-hundred and fifty separate and distinct regression equations used in this study, requiring some eight-hundred and fifty pages of data to describe them all.

The above-mentioned data is of only limited usefulness because of the following considerations. As one might suspect, due to the nature of the data and the small number of data points, less than five percent of the correlation and multiple correlation coefficients computed for the eight-hundred and fifty regression equations used in this study are less than .9000. It is therefore rather obvious that most of the F Statistics assessing the usefulness of the various regression lines as predictors are significant at $p \geq .05$. This is not suprising since it is one of the basic tenets of this study that, over a relatively short period of time or small number of data points, any function of the type examined here can best be approximated by a straight line.

The predictor variables, as was suspected, are all highly intercorrelated. This fact of course throws some doubt on the usefulness of multiple linear regression as opposed to simple linear regression as a projection technique. As the number of predictor variables in the regression equations are increased from one to four, the multiple correlations increase to the point where, with four predictor variables, there is not a single multiple correlation coefficient below .9800. However, the fact that the high multiple correlation coefficients are artifacts of the computational process in many instances is attested to by greatly increasing standard errors of regression coefficients as the number of predictor variables increase. There are cases when, with four predictor variables, the standard errors of the regression coefficients are larger than the regression coefficients themselves, suggesting the presence of a great deal of error. From an examination of the standard errors of the regression coefficients, one must conclude that, given the nature of the projection problem as outlined in this study, one or two carefully chosen predictor variables are sufficient. This of course may not be the case with an increased number of data points. However, it is relevant at this point to remind the reader of the difficulty of procuring data. At the local level in Michigan, ten years is about the most one can expect to obtain data for.

Table 9 presents the actual enrollment figures for each of the twenty-five districts that comprise the sample used in the study. These are the figures that provide the criterion against which the accuracy of the various methods is judged. Notice that the districts are grouped by stratum, as they will appear throughout the remainder of this manuscript.

TABLE 9

ACTUAL ENROLLMENT OF SAMPLE DISTRICTS

District	EE 1965	SE 1965	EE 1968	SE 1968
Allen Park	4629	2135	4677	2301
Garden City	10223	2981	10841	3124
Inkster	3652	1140	3687	1231
Livonia	22868	7019	25151	9936
Oak Park	5120	1956	4987	2110
Ann Arbor	11947	3881	14039	4746
Birmingham	11462	4557	11882	5633
Lincoln Park	9192	3965	9507	4063
Midland	8340	2970	8721	3396
Royal Oak	13883	5680	13305	6026
Alpena	5548	1882	5961	2256
Dearborn	13374	8395	13290	8998
Kalamazoo	13018	4750	13226	5084
Pontiac	16865	5529	17617	5798
Wyandotte	5797	2655	5722	2654
Bay City	8817	5202	9063	5452
Grand Rapids	23887	7814	24250	8774
Ferndale	5585	2463	5531	2528
Monroe	3688	2123	6065	2458
Saginaw	15526	6148	16079	6368
Battle Creek	8361	2958	8111	2952
Hamtramck	2375	1271	2207	1004
Highland Park	4258	1827	4140	2071
Ironwood	1930	748	1916	780
Muskegon	6397	2694	6125	2684

Cohort-Survival Ratio Method

The Cohort-Survival Ratio approach was used in the form presented in the Review of the Literature on pages 28 through 30. This means that nine separate ratio equations are used to obtain each of the elementary projections and that an additional four ratio equations are used to obtain each of the projections of secondary enrollment. The mean number of births per year for the relevant period of time is applied to the ratio equations to obtain the projection. Table 10 presents the projections yielded by the Cohort-Survival Ratio Approach. The key to the symbols in this table and the other sixteen tables like it that appear in this chapter is on pages 73 and 74.

Table 11 displays the number of projections yielded by the Cohort-Survival Ratio Method, within and across strata, that are within a W Coefficient of .1000 of the actual enrollment figure. Please observe the footnotes to this table that explain what the headings stand for, since there are sixteen more tables in this Chapter that are set up exactly like this one.

Appendix B-1 outlines in detail the procedures used to obtain projections from the Cohort-Survival Ratio method. In a like manner, procedures used in obtaining projections from each of the sixteen other methodologies examined in this study are outlined in detail in Appendices B—2 through B-16.

TABLE 10

PROJECTIONS BY THE COHORT-SURVIVAL RATIO METHOD

District	EE 1965	W	R	SE 1965	W	R	EE 1968	W	R	SE 1968	W	R	R
Allen Park	4836	+ .0447	5	2462	+ .1531	15	4917	+ .0513	6	2580	+ .1212	13	7
Garden City	11108	+ .0865	7	2800	- .0607	5	11796	+ .0880	5	3317	+ .0617	3	2
Inkster	3684	+ .0083	3	1086	- .0473	1	3697	+ .0027	2	1190	- .0333	1	1
Livonia	24122	+ .0543	5	6225	- .1131	3	28126	+ .1182	8	8047	- .1901	5	2
Oak Park	7861	+ .5353	10	1970	+ .0071	3	8023	+ .6087	4	2067	- .0203	1	5
Ann Arbor	13808	+ .1557	12	5702	+ .4692	17	16508	+ .1758	10	6086	+ .2823	15	13
Birmingham	12168	+ .0615	9	4360	- .0432	3	11484	- .0334	6	5348	- .0505	1	1
Lincoln Park	10242	+ .1142	10	4218	+ .0638	7	10751	+ .1308	9	4503	+ .1082	11	5
Midland	9662	+ .1585	16	3605	+ .2138	16	10444	+ .1975	15	4115	+ .2117	16	16
Royal Oak	14746	+ .0621	5	4728	- .1676	14	13390	+ .0063	1	6411	+ .0638	6	4
Alpena	5995	+ .0805	7	1894	+ .0063	1	6519	+ .0936	6½	2204	- .0230	1	1
Dearborn	15022	+ .1232	15	7149	- .1484	14	13728	+ .0329	3	8439	- .0621	6	10
Kalamazoo	16771	+ .2882	14	4758	+ .0016	1	15768	+ .1921	8	5196	+ .0220	1	6
Pontiac	16569	- .0175	1	5449	- .0144	1	16692	- .0525	1	5896	+ .0169	3	1
Wyandotte	6225	+ .0738	7	2283	- .1401	13	5628	- .0164	4	2998	+ .1296	10	6
Bay City	8117	- .0793	10	4688	- .0988	9	8840	- .0246	1	4820	- .1159	9	7
Grand Rapids	23460	- .0178	4	8780	+ .1236	13	22779	- .0606	9	9806	+ .1176	9	11
Ferndale	5307	- .0502	9	2119	- .1396	13	5257	- .0495	9	2256	- .1075	10	11
Monroe	3447	- .0653	6	2392	+ .1267	13	3376	- .4433	13	2339	- .0484	2	9
Saginaw	15463	- .0040	3	6103	- .0073	2	14918	- .0722	7	7290	- .1447	12	7
Battle Creek	7685	- .0808	6	3166	+ .0703	13	7777	- .0411	4	3192	+ .0813	13	5
Hamtramck	2375	.0000	1	969	- .2376	14	2085	- .0552	3	926	- .0776	12	5
Highland Park	4150	- .0253	8	2006	+ .0979	16	3718	- .1019	9	2079	+ .0038	1	4
Ironwood	1778	- .0787	12	760	+ .0160	2	1666	- .1304	11	756	- .0307	2	5
Muskegon	6069	- .0512	4	1890	- .2984	12	5615	- .0832	3	2829	+ .0540	5	7

TABLE 11

ACCURACY OF PROJECTIONS YIELDED BY THE COHORT-SURVIVAL RATIO METHOD

	EE[a]				SE[b]			
Stratum	H[c]	M[d]	P[e]	Ṗ[f]	H[g]	M[h]	P[i]	P̊[j]
A	7	3	.70	.38	6	4	.60	.29
B	4	6	.40	.50	4	6	.40	.49
C	7	3	.70	.52	7	3	.70	.39
D	10	0	1.00	.61	3	7	.30	.52
E	8	2	.80	.49	8	2	.80	.52
Total[k]	36	14	.72	.50	28	22	.56	.44

[a]Elementary Enrollment.

[b]Secondary Enrollment.

[c]Hits, that is, instances in which the W Coefficient is within .1000 of the actual enrollment figure. The frequency count is taken over both 1965 and 1968. This column records elementary hits.

[d]Misses, that is, instances in which the W Coefficient is not within .1000 of the actual enrollment figure. The frequency count is taken over both 1965 and 1968. This column records elementary misses.

[e]Percentage of hits for a given stratum for one particular projection approach. (elementary)

[f]Percentage of hits for a given stratum for all projection approaches. (elementary)

[g]Same as "c" but for secondary enrollment.

[h]Same as "d" but for secondary enrollment.

[i]Same as "e" but for secondary enrollment.

[j]Same as "f" but for secondary enrollment.

[k]Presents totals across strata of all of the above information.

The results suggest that the Cohort-Survival Ratio Method provides generally adequate projections of future enrollment in Districts with growth characteristics similar to those classified in Strata A, C, and E.[70] Projections yielded by this approach are, as has been suggested in the Literature, the most accurate of projections yielded by ratio approaches.

Grade-Retention Ratio Method

The Grade-Retention Ratio Method was used in the form presented in the Review of the Literature on page 31. Table 12 presents the projections yielded by the Grade-Retention Ratio Approach.[71]

Table 13 displays the number of projections yielded by the Grade-Retention Ratio Method, within and across strata, that are within a W Coefficient of .1000 of the actual enrollment figures.

TABLE 12

PROJECTIONS BY THE GRADE-RETENTION RATIO METHOD

District	EE 1965	W	R	SE 1965	W	R	EE 1968	W	R	SE 1968	W	R	R̄
Allen Park	5403	+ .1672	13	1809	- .1526	14	5696	+ .2178	12	1941	- .1564	14	15
Garden City	13653	+ .3335	9	2300	- .2284	9	18878	+ .7413	15	2989	- .0432	1	13
Inkster	3756	+ .0284	7	905	- .2061	7	3900	+ .0577	8	878	- .2867	12	9
Livonia	31732	+ .3876	15	6651	- .0524	2	50898	+1.0236	15	10449	+ .0516	2	13
Oak Park	11760	+1.2968	17	2173	+ .1109	8	17979	+2.6051	17	3116	+ .4767	14	16
Ann Arbor	13448	+ .1256	10	3540	- .0878	11	16866	+ .2013	11	4166	- .1222	6	10
Birmingham	10878	- .0509	8	4189	- .0807	7	11892	+ .0008	1	5115	- .0919	3	2
Lincoln Park	10864	+ .1623	12	3746	- .0552	5	11391	+ .1981	12	4473	+ .1009	10	9
Midland	8867	+ .0631	10	3032	+ .0208	2	10310	+ .1822	13	3240	- .0459	9	10
Royal Oak	13850	- .0023	1	5529	- .0265	3	15099	+ .1348	6	6768	+ .1231	10	3
Alpena	6082	+ .0962	10	1622	- .1381	12	7155	+ .2003	11	1873	- .1697	9	11
Dearborn	12467	- .0678	5	7052	- .1599	15	11817	- .1108	7	6960	- .2264	14	13
Kalamazoo	13428	+ .0314	1	3949	- .1686	13	13726	+ .0378	3	4106	- .1923	13	3
Pontiac	18115	+ .0741	12	3861	- .3061	15	19784	+ .1230	13	3909	- .3258	15	15
Wyandotte	5748	- .0084	1	2291	- .1370	11	5675	- .0082	2	2325	- .1239	9	3
Bay City	9279	+ .0523	9	4519	- .1312	12	10564	+ .1656	14	4883	- .1043	6	11
Grand Rapids	24239	+ .0147	3	7119	- .0889	10	26860	+ .1076	13	7871	- .1029	7	10
Ferndale	5721	+ .0238	5	2208	- .1035	10	5639	+ .0195	5	2366	- .0640	7	5
Monroe	3629	- .0159	4	2085	- .0178	3	3792	- .3747	8	2183	- .1118	4	3
Saginaw	16367	+ .0541	8	5861	- .0466	8	17682	+ .1014	11	5772	- .0935	10	11
Battle Creek	9591	+ .1471	12	2893	- .0219	6	10326	+ .2730	13	2923	- .0098	3	10
Hamtramck	2692	+ .1334	12	916	- .2793	16	2644	+ .1980	12	850	- .1533	15	15
Highland Park	4498	+ .0563	10	1679	- .0810	15	4601	+ .1113	11	1651	- .2028	16	12
Ironwood	2047	+ .0606	11	635	- .1510	15	2096	+ .0939	10	596	- .2358	12	11
Muskegon	7054	+ .1027	10	2932	+ .0823	6	7204	+ .1761	10	3083	+ .1486	9	8

TABLE 13

ACCURACY OF PROJECTIONS YIELDED BY THE GRADE RETENTION RATIO METHOD[a]

	EE				SE			
Stratum	H	M	P	$\dot{P}$	H	M	P	$\dot{P}$
A	2	8	.20	.38	3	7	.30	.29
B	4	6	.40	.50	7	3	.70	.49
C	7	3	.70	.52	0	10	.00	.39
D	6	4	.60	.61	5	5	.50	.52
E	3	7	.30	.49	4	6	.40	.52
Total	22	28	.44	.50	19	31	.38	.44

[a]For an explanation of the table headings, see the footnotes to Table 11, page 80.

The results suggest that the Grade-Retention Ratio Method provides generally inadequate estimates of public school enrollment. While it occasionally provides a reasonably accurate projection, this does not occur often enough to warrant any undue faith in the method. The results also suggest that the Grade-Retention Ratio Method does not yield estimates that resemble those yielded by the Cohort-Survival Ratio Method, as was suggested in the Review of the Literature

Enrollment-Ratio Method I

This method is described on pages 32 and 33 of the Review of the Literature. The total population figure to which the ratios were applied was total city population. This approach, like all approaches that depend on some form of population that must be estimated as the predictor variable, is limited by the accuracy of the projected population figures to which the various ratios are applied. Table 14

TABLE 14

PROJECTIONS BY ENROLLMENT-RATIO METHOD I

District	EE 1965	W	R	SE 1965	W	R	EE 1968	W	R	SE 1968	W	R	• R
Allen Park	5076	+ .0965	9	1729	- .1901	16	5620	+ .2016	10	1882	- .1820	15	14
Garden City	10900	+ .0662	7	2368	- .2056	7	12231	+ .1282	6	2596	- .1690	6	7
Inkster	4550	+ .2458	13	1051	- .0780	3	5055	+ .3710	14	1168	- .0511	2	14
Livonia	20747	- .0927	8	4480	- .3617	11	25416	+ .0105	2	4848	- .5120	13	8
Oak Park	5727	+ .1185	2	1388	- .2906	13	6825	+ .3685	2	1577	- .2526	5	3
Ann Arbor	11352	- .0498	7	3787	- .0242	5	13306	- .0522	5	4482	- .0556	3	1
Birmingham	11969	+ .0442	6	3198	- .2982	16	13611	- .1455	11	3542	- .3712	15	13
Lincoln Park	9603	+ .0447	6	3016	- .2393	15	10183	+ .0711	6	3168	- .2202	14	11
Midland	8728	+ .0465	7	3764	+ .2673	17	10213	+ .1710	12	4422	+ .3021	17	17
Royal Oak	15395	+ .1089	8	4888	- .1394	13	17328	+ .3023	12	5528	- .0826	7	11
Alpena	5063	- .0874	8	1488	- .2093	16	5403	- .0936	6½	1645	- .2708	16	12
Dearborn	14345	+ .0726	6	7957	- .0521	10	15057	+ .1329	11	8701	- .0330	4	6
Kalamazoo	19087	+ .4662	16	4688	- .0130	3	21974	+ .6614	16	5212	+ .0251	2	14
Pontiac	19920	+ .1811	16	4652	- .1748	7	20724	+ .1763	16	4609	- .2050	10	12
Wyandotte	6474	+ .1167	14	2420	- .0885	7	7031	+ .2287	14	2618	- .0135	1	7
Bay City	7903	- .1037	12	4069	- .2178	14	8283	- .0860	9	4167	- .2356	13	13
Grand Rapids	22684	+ .0040	8	6886	- .1187	12	23149	- .0454	6	7668	- .1260	11	7
Ferndale	6989	+ .2507	15	2253	- .0852	7	7271	+ .3145	16	2379	- .0589	5	13
Monroe	3669	- .0051	2	2100	- .0108	2	3973	- .3449	6	2170	- .1171	5	2
Saginaw	16393	+ .0558	9	6203	+ .0089	3	17437	+ .0844	8	6343	- .0039	1	3
Battle Creek	9332	+ .1161	9	3104	+ .0493	8	9721	+ .1984	9	3099	+ .0497	9	9
Hamtramck	2948	+ .2412	16	1181	- .0708	5	2937	+ .3307	16	1194	+ .1882	17	16
Highland Park	4890	+ .1484	14	1835	+ .0043	1	5092	+ .2299	14	1871	- .0965	5	13
Ironwood	1922	- .0041	3	647	- .1350	14	1906	- .0052	1	617	- .2089	10	6
Muskegon	7221	+ .1288	13	2648	- .0170	1	7539	+ .2308	13	2697	+ .0048	1	5

presents the projections yielded by Enrollment-Ratio I.[72]

Table 15 displays the number of projections yielded by Enrollment-Ratio Method I, within and across strata, that are within a W Coefficient of .1000 of the actual enrollment figures.

TABLE 15

ACCURACY OF PROJECTIONS YIELDED BY ENROLLMENT-RATIO METHOD I[a]

	EE				SE			
Stratum	H	M	P	$\dot{P}$	H	M	P	$\dot{P}$
A	4	6	.40	.38	2	8	.20	.29
B	6	4	.60	.50	3	7	.30	.49
C	4	6	.40	.52	0	10	.00	.39
D	5	5	.50	.61	8	2	.80	.52
E	4	6	.40	.49	3	7	.30	.52
Total	23	27	.46	.50	16	34	.32	.44

[a]For an explanation of the table headings, see the footnotes to Table 11, page 80.

The results suggest that Enrollment-Ratio Method I provides generally inadequate projections of public school enrollment for Districts across all Strata.

Enrollment-Ratio Method II

This method uses the same procedures as Enrollment-Ratio Method I, except that it uses total county population as the figure to which the ratios are applied. The methodology is described on pages 32 and 33 of the Review of the Literature. Table 16 presents the projections yielded by Enrollment-Ratio Method II.[73]

TABLE 16

PROJECTIONS BY ENROLLMENT-RATIO METHOD II

District	EE 1965	W	R	SE 1965	W	R	EE 1968	W	R	SE 1968	W	R	• R
Allen Park	4846	+ .0468	6	1672	- .2168	17	4894	+ .0463	5	1685	- .2677	16	12
Garden City	8887	- .1306	8	1868	- .3733	16	8954	- .1740	8	1879	- .3985	14	9
Inkster	3404	- .0679	10	893	- .2166	8	3438	- .0675	9	901	- .2680	10	12
Livonia	18201	- .2040	10	3902	- .4440	14	18381	- .2691	11	3920	- .6054	16	14
Oak Park	5362	+ .0472	1	1342	- .3139	14	5744	+ .1517	1	1409	- .3322	13	1
Ann Arbor	10261	- .1407	11	3384	- .2568	15	11162	- .2048	12	3665	- .2777	14	11
Birmingham	10749	- .0622	10	3071	- .3260	17	11516	- .0308	5	3290	- .4159	16	12
Lincoln Park	4141	- .0055	1	2877	- .2744	16	9232	- .0289	4	2900	- .2862	15	14
Midland	7658	- .0817	11	3231	+ .0878	10	8275	- .0511	4	3492	+ .0282	7	7
Royal Oak	14649	+ .0551	4	4122	- .2742	16	15838	+ .1903	7	4416	- .2671	12	13
Alpena	4845	- .1267	11	1508	- .1987	15	5031	- .1560	9	1659	- .2646	15	13
Dearborn	13855	+ .0359	3	6520	- .2233	17	13991	+ .0527	4	6584	- .2682	16	14
Kalamazoo	15041	+ .1554	9	4084	- .1402	9	15884	+ .2009	10	4296	- .1549	9	12
Pontiac	18669	+ .1069	14	4526	- .1814	8	20002	+ .1315	14	4849	- .1636	8	9
Wyandotte	5705	- .0158	2	1902	- .2836	17	5716	- .0010	1	1920	- .2765	14	12
Bay City	7400	- .1607	14	4029	- .2254	15	7589	- .1626	12	4132	- .2421	14	14
Grand Rapids	22167	- .0720	10	7176	- .0816	8	22975	- .0525	7	7931	- .0960	6	8
Ferndale	7516	- .3450	16	2263	- .0812	6	8053	+ .4559	17	2425	- .0407	2	14
Monroe	3679	- .0024	1	2138	+ .0070	1	3888	- .3589	7	2459	+ .0004	1	1
Saginaw	15544	+ .0011	2	6595	+ .0728	12	16045	- .0021	2	6807	+ .0689	7	2
Battle Creek	8618	- .0307	3	2423	- .0118	5	8690	+ .0713	5	2943	- .0030	1	2
Hamtramck	2445	+ .0294	3	815	- .3587	17	2469	+ .1187	5	823	- .1802	16	13
Highland Park	3803	- .1068	13	1558	- .1472	17	3841	- .0722	8	1572	- .2409	17	14
Ironwood	1664	- .1378	16	629	- .1590	17	1559	- .1863	16	589	- .2448	14	16
Muskegon	7427	+ .1610	14	2871	+ .0657	5	7658	+ .2502	14	2961	+ .1032	7	9

Table 17 presents the number of projections yielded by Enrollment-Ratio Method II, within and across strata, that are within a W Coefficient of .1000 of the actual enrollment figures.

TABLE 17

ACCURACY OF PROJECTIONS YIELDED BY ENROLLMENT-RATIO METHOD II[a]

	EE				SE			
Stratum	H	M	P	$\dot{P}$	H	M	P	$\dot{P}$
A	5	5	.50	.38	0	10	.00	.29
B	7	3	.70	.50	2	8	.20	.49
C	4	6	.40	.52	0	10	.00	.39
D	5	5	.50	.61	8	2	.80	.52
E	4	6	.40	.49	3	7	.30	.52
Total	25	25	.50	.50	13	37	.26	.44

[a]For an explanation of the table headings, see the footnotes to Table 11, page 80.

The results suggest that Enrollment Ratio Method II provides generally inadequate estimates of public school enrollment for districts across all Strata.

Pupil-Home Unit Ratio Method

The Pupil-Home Unit Ratio Method as used in this study bases its projections on the past ratio of elementary and secondary pupils to home units in each district. Table 18 presents the projections yielded by the Pupil-Home Unit Ratio Method.[74]

TABLE 18

PROJECTIONS BY PUPIL-HOME UNIT RATIO METHOD

District	EE 1965	W	R	SE 1965	W	R	EE 1968	W	R	SE 1968	W	R	• R
Allen Park	6005	+ .2972	17	1923	- .0992	12	6916	+ .4787	17	2173	- .0556	9	17
Garden City	9843	- .0371	3	2180	- .2687	11	11282	+ .0406	3	2425	- .2237	11	6
Inkster	4560	+ .2486	14	1057	- .0728	2	5011	+ .3590	13	1162	- .0560	4	13
Livonia	18843	- .1760	9	3644	- .4808	16	21351	- .1510	9	4091	- .5882	15	11
Oak Park	6889	+ .3455	3	1562	- .2014	11	8159	+ .6360	7	3516	+ .6663	15	9
Ann Arbor	11233	- .0597	8	3285	- .1525	13	13905	- .0095	2	3953	- .1670	10	8
Birmingham	10201	- .1100	12	3540	- .2231	14	10787	- .0921	9	3721	- .3394	14	11
Lincoln Park	11163	+ .2634	17	3470	- .1248	11	12622	+ .3276	17	3437	- .0802	8	16
Midland	8601	+ .0312	4	3413	+ .1491	15	9932	+ .1388	9	3940	+ .1601	15	13
Royal Oak	12173	- .1231	9	5103	- .1015	10	14120	+ .0612	3	5377	- .1076	9	5
Alpena	4518	- .1856	13	1475	- .2162	17	4505	- .2422	12	1571	- .3036	17	15
Dearborn	14475	+ .1025	13	6680	- .2042	16	15051	+ .1325	10	6819	- .2421	15	15
Kalamazoo	12547	- .0361	2	4024	- .1528	11	13366	+ .0105	1	4267	- .1607	10	2
Pontiac	16017	- .0502	6	4024	- .2722	13	15715	- .1079	10	3948	- .3190	14	13
Wyandotte	5291	- .0872	11	2174	- .1811	15	5401	- .0560	6	2219	- .1639	11	10
Bay City	7486	- .1509	13	4773	- .0824	8	7585	- .1630	13	4822	- .1155	8	12
Grand Rapids	21368	- .1054	14	6544	- .1625	14	21343	- .1198	14	7036	- .1980	14	14
Ferndale	5153	- .0778	12	2127	- .1201	12	4583	- .1713	12	2247	- .1111	11	12
Monroe	3198	- .1328	14	2028	- .0447	12	3253	- .4636	16	2060	- .1619	6	13
Saginaw	14374	- .0741	12	5915	- .0376	6	14448	- .1013	10	5945	- .0664	6	10
Battle Creek	8004	- .0426	4	2931	- .0091	2½	8018	- .0114	2	2935	- .0057	2	1
Hamtramck	2641	+ .1120	10	1008	- .2069	13	2511	+ .1377	6	958	- .0458	6	10
Highland Park	3868	- .0915	11	1911	+ .0459	11	3710	- .1038	10	1842	- .1105	6	9
Ironwood	1760	- .0880	13	658	- .1203	12½	1648	- .1398	12	616	- .2102	11	12
Muskegon	6855	+ .0715	6	2591	- .0382	4	6767	+ .1048	5	2557	- .0473	4	4

Table 19 presents the number of projections yielded by the Pupil-Home Unit Ratio Method, within and across strata, that are within a W Coefficient of .1000 of the actual enrollment figures.

TABLE 19

ACCURACY OF PROJECTIONS YIELDED BY THE PUPIL-HOME UNIT RATIO METHOD[a]

	EE				SE			
Stratum	H	M	P	$\dot{P}$	H	M	P	$\dot{P}$
A	2	8	.20	.38	4	6	.40	.29
B	5	5	.50	.50	1	9	.10	.49
C	5	5	.50	.52	0	10	.00	.39
D	2	8	.20	.61	4	6	.40	.52
E	5	5	.50	.49	6	4	.60	.52
Total	19	31	.38	.50	15	35	.30	.44

[a]For an explanation of the table headings, see the footnotes to Table 11, page 80.

The results suggest that the Pupil-Home Unit Ratio Method provides generally inadequate estimates of public school enrollment for Districts across all Strata.

Time Analysis I

Time Analysis I represents the first of the regression approaches examined in this study. The approach bases its projections on past relationships between enrollment and time. Table 20 presents the projections yielded by Time Analysis I.[75]

TABLE 20

PROJECTIONS BY TIME ANALYSIS I (REGRESSION)

District	EE 1965	W	R	SE 1965	W	R	EE 1968	W	R	SE 1968	W	R	R̄
Allen Park	5390	+ .1643	12	2184	+ .0229	3	5637	+ .2052	11	2480	+ .0777	11	9
Garden City	13072	+ .2786	12	2114	- .2908	12	15560	+ .4352	12	2492	- .2023	8	11
Inkster	3863	+ .0577	8	988	- .1333	6	4058	+ .1006	11	1053	- .1445	6	4
Livonia	22876	+ .0003	1	4991	- .2889	9	28032	+ .1145	7	6127	- .3833	7	4
Oak Park	8378	+ .6363	11	2026	+ .0357	6	10075	+1.0202	13	2504	+ .1867	4	10
Ann Arbor	13138	+ .0996	9	3560	- .0827	10	15365	+ .0944	9	4038	- .1491	9	9
Birmingham	11653	+ .0166	3	4264	- .0642	5	12912	+ .0866	8	4981	- .1157	5	4
Lincoln Park	10830	+ .1782	14	3515	- .1134	10	11506	+ .2102	13	3978	- .0209	3	10
Midland	8845	+ .0605	9	3174	+ .0686	7	9986	+ .1450	10	3474	+ .0229	5	9
Royal Oak	15899	+ .1452	11	5373	- .0540	4	17026	+ .2796	10	6205	+ .0297	3	7
Alpena	5870	+ .0580	5	1710	- .0913	8	6602	+ .1075	8	1970	- .1267	7	4
Dearborn	12154	- .0912	12	8620	+ .0268	4	11267	- .1522	13	9249	+ .0278	2	7
Kalamazoo	15314	+ .1763	10	4508	- .0509	6	16459	+ .2444	12	4932	- .0298	3	5
Pontiac	17617	+ .0445	5	4128	- .2533	12	18818	+ .0681	3	4331	- .2530	12	10
Wyandotte	6246	+ .0774	8	2894	+ .0900	8	6385	+ .1158	8	3161	+ .1910	13	8
Bay City	8639	- .0201	6	4959	- .0467	6	9402	+ .0374	3	5475	+ .0042	2	3
Grand Rapids	23705	- .0076	1	7469	- .0441	4	24280	+ .0012	1	8381	- .0447	3	1
Ferndale	5588	+ .0178	3	2423	- .0162	2	5599	+ .0122	3	2658	+ .0514	3	1
Monroe	4059	+ 1055	11	2043	- .0376	7½	4411	- .2727	3	2196	- .1065	3	4
Saginaw	15435	- .0058	4	6635	+ .0792	13	16083	+ .0002	1	6945	+ .0906	9	4
Battle Creek	9350	+ .1182	10	2721	- .0801	15	9806	+ .2089	11	2678	- .0928	15	12
Hamtramck	2610	+ .0989	8	1027	- .1919	11	2519	+ .1413	7	1004	.0000	1	6
Highland Park	4258	+ .0070	2	1737	- .0492	13	4292	+ .0367	5	1734	- .1627	13½	6
Ironwood	2027	+ .0502	9	683	- .0868	6½	2063	+ .0767	8	663	- .1500	7	8
Muskegon	7097	+ .1094	11	3497	+ .2980	11	7239	+ .1818	11	3848	+ .4336	12	12

Table 21 presents the number of projections yielded by Time Analysis I, within and across Strata, that are within a W Coefficient of .1000 of the actual enrollment figures.

TABLE 21

ACCURACY OF PROJECTIONS YIELDED BY TIME ANALYSIS I (REGRESSION)[a]

	EE				SE			
Stratum	H	M	P	$\dot{P}$	H	M	P	$\dot{P}$
A	2	8	.20	.38	3	7	.30	.29
B	5	5	.50	.50	7	3	.70	.49
C	5	5	.50	.52	7	3	.70	.39
D	8	2	.80	.61	9	1	.90	.52
E	5	5	.50	.49	5	5	.50	.52
Total	25	25	.50	.50	31	19	.62	.44

[a]For an explanation of the table headings, see the footnotes to Table 11, page 80.

The results suggest that Time Analysis I provides adequate enrollment projections for Districts with growth characteristics similar to those classified in Stratum D.

Time Analysis II

Time Analysis II bases its projections on the past relationship between time and the logarithms of public school enrollment. The logarithmic transformations provide higher estimates of future enrollment than those provided by the linear approximations yielded by Time Analysis I. Table 22 presents the projections yielded by Time Analysis II.[76]

TABLE 22

PROJECTIONS BY TIME ANALYSIS II (REGRESSION)

District	EE 1965	W	R	SE 1965	W	R	EE 1968	W	R	SE 1968	W	R	R
Allen Park	5429	+ .1728	15	2458	+ .1512	13	5716	+ .2221	13	3051	+ .3259	17	16
Garden City	17061	+ .6688	17	2655	- .1093	4	24798	+1.2874	17	3715	+ .1891	7	16
Inkster	3901	+ .0681	11	1012	- .1122	5	4134	+ .1212	12	1099	- .1072	5	3
Livonia	37010	+ .6184	16	8197	+ .1678	6	63669	+1.5314	16	14271	+ .4326	8	16
Oak Park	11359	+1.2185	16	4138	+1.1155	17	17205	+2.4499	16	8113	+2.8450	17	17
Ann Arbor	15961	+ .3359	13	3938	+ .0146	3	21683	+ .5444	15	4845	+ .0208	2	12
Birmingham	12373	+ .0794	11	5168	+ .1340	10	14397	+ .2116	12	6982	+ .2394	10	10
Lincoln Park	10996	+ .1963	16	3852	- .0284	3	11845	+ .2459	15	4701	+ .1570	13	15
Midland	9694	+ .1623	17	3274	+ .1023	13	11769	+ .3495	17	3636	+ .0706	10	15
Royal Oak	16625	+ .1686	14	6205	+ .0924	9	17694	+ .3298	14	8030	+ .3325	13	14
Alpena	6326	+ .1402	12	1793	- .0472	3	7531	+ .2633	13	2143	- .0500	3	9
Dearborn	12309	- .0796	9	8877	+ .0574	11	11572	- .1292	9	9747	+ .0832	9	9
Kalamazoo	15758	+ .2112	12	4758	- .0025	2	17330	+ .3102	14	5398	+ .0617	4	9
Pontiac	17927	+ .0629	10	4169	- .2459	11	19463	+ .1047	9	4413	- .2388	11	11
Wyandotte	6260	+ .0798	9	3020	+ .1374	12	6411	+ .1204	9	3417	+ .2874	15	13
Bay City	8904	+ .0098	2	5236	+ .0065	1	9968	+ .0998	11	6045	+ .1087	7	5
Grand Rapids	24103	+ .0090	2	7569	- .0313	3	25118	+ .0357	4	8583	- .0217	1	2
Ferndale	5691	+ .0184	4	2539	+ .0308	3	5607	+ .0137	4	2896	+ .1455	12	4
Monroe	4207	+ .1407	16	2041	- .0386	9	4710	- .2234	1	2040	- .1700	10½	6
Saginaw	15516	- .0006	1	6703	+ .0902	15	16259	+ .0111	3	7076	+ .1111	11	5
Battle Creek	9427	+ .1274	11	2723	- .0794	14	9965	+ .2285	12	2683	- .0911	14	13
Hamtramck	2619	+ .1027	9	1028	- .1911	10	2537	+ .1495	9	1007	+ .0029	2	8
Highland Park	4287	+ .0068	1	1737	- .0492	13	4291	+ .0364	4	1734	- .1627	13½	5
Ironwood	2029	+ .0512	10	684	- .0855	5	2066	+ .0782	9	665	- .1474	6	7
Muskegon	7103	+ .1103	12	3682	+ .3667	13	7254	+ .1843	12	4225	+ .5741	14	14

Table 23 presents the number of projections yielded by Time Analysis II, within and across Strata, that are within a W Coefficient of .1000 of the actual enrollment figures.

TABLE 23

ACCURACY OF PROJECTIONS YIELDED BY TIME ANALYSIS II (REGRESSION)[a]

	EE				SE			
Stratum	H	M	P	$\dot{P}$	H	M	P	$\dot{P}$
A	1	9	.10	.38	0	10	.00	.29
B	1	9	.10	.50	5	5	.50	.49
C	3	7	.30	.52	6	4	.60	.39
D	8	2	.80	.61	6	4	.60	.52
E	4	6	.40	.49	5	5	.50	.52
Total	17	33	.34	.50	22	28	.44	.44

[a]For an explanation of the table headings, see the footnotes to Table 11, page 80.

The results suggest that Time Analysis II could be used to project enrollment in Districts with growth characteristics similar to those classified in Stratum D. However, it is significant to note that Time Analysis I provides more accurate estimates of enrollment in every case than does Time Analysis II, and therefore should be used in preference to Time Analysis II. The results of a comparison of the projections yielded by these two approaches is significant in that they, in conjunction with the extremely high simple and multiple correlation coefficients that characterize all of the relationships examined in this study, provide further support for the hypothesis, implicit in the methodology used in this study, that enrollment growth over a relatively short period of time can best be approximated by a straight line.

Transition Analysis

Transition Analysis is the regression equivalent of the Cohort-Survival Ratio Method. It bases its projections on the past tendencies of students to advance from one grade to the next and from birth to kindergarten. Nine separate regression equations are used to obtain each of the elementary projections and an additional four are used to obtain each of the projections of secondary enrollment. The mean number of births per year for the relevant period of time is applied to the regression equations to obtain the projections. Table 24 presents the projections yielded by Transition Analysis.[77]

Table 25 presents the number of projections yielded by Transition Analysis, within and across Strata, that are within a W Coefficient of .1000 of the actual enrollment figures.

The results suggest that this is the best overall projection methodology. It seems particularly suitable for Districts with growth characteristics similar to those classified in Strata A, B, C, and E. and quite adequate for Stratum D districts.

TABLE 24

PROJECTIONS BY TRANSITION ANALYSIS (REGRESSION)

District	EE 1965	W	R	SE 1965	W	R	EE 1968	W	R	SE 1968	W	R	$\dot{R}$
Allen Park	4719	+ .0194	3	2237	+ .0477	6	4782	+ .0224	3	2346	+ .0195	1	1
Garden City	10567	+ .0336	2	2698	- .0949	2	10967	+ .0116	2	2976	- .0473	2	1
Inkster	3614	- .0114	6	1020	- .1052	4	3680	- .0018	1	1167	- .0519	3	2
Livonia	23933	+ .0465	4	7857	+ .1193	4	27851	+ .1073	6	10008	+ .0072	1	1
Oak Park	7619	+ .4880	8	1957	+ .0005	1	8117	+ .6276	6	2189	+ .0374	2	4
Ann Arbor	11626	- .0268	4	3624	- .0662	7	13348	- .0492	4	4085	- .1392	8	4
Birmingham	11576	+ .0099	2	4650	+ .0204	1	11533	- .0293	4	4684	- .1684	9	3
Lincoln Park	9869	+ .0737	7	3781	- .0464	4	10413	+ .0952	7	3942	- .0297	5	3
Midland	8266	- .0088	2	2659	- .1047	14	8538	- .0209	1	2897	- .1469	14	8
Royal Oak	13930	+ .0033	2	5635	- .0079	2	13639	+ .0251	2	5667	- .0595	5	1
Alpena	5304	- .0620	6	1729	- .0812	6	5652	- .0518	3	1863	- .1742	10	2
Dearborn	13681	+ .0229	2	8306	- .0105	1	13388	+ .0073	1	8489	- .0565	5	1
Kalamazoo	13886	+ .0666	4	4521	- .0482	5	13661	+ .0328	2	4528	- .1093	6	1
Pontiac	15690	- .0696	11	5685	+ .0282	3	15705	- .1079	11	5720	- .0134	1	2
Wyandotte	5986	+ .0326	4	2316	- .1276	10	5803	+ .0141	3	2517	- .0516	3	1
Bay City	8364	- .0513	8	5426	+ .0430	5	8540	- .0577	5	5390	- .0113	3	4
Grand Rapids	24356	+ .0196	5	8347	+ .0682	7	23207	- .0430	4	9766	+ .1130	8	5
Ferndale	5629	+ .0073	1	2092	- .1506	14	5535	+ .0007	1	2557	+ .0114	1	3
Monroe	3339	- .0946	9	2078	- .0211	4	3349	- .4511	14	2030	- .1741	13	11
Saginaw	14278	- .0803	14	6332	+ .0299	5	14261	- .1130	14	6313	- .0086	2	8
Battle Creek	8436	+ .0089	2	3344	+ .1304	17	8441	+ .0406	3	3349	+ .1344	17	6
Hamtramck	2525	+ .0631	6	1065	- .1620	8	2350	+ .0647	4	1055	+ .0507	8	4
Highland Park	4181	- .0180	5	1792	- .0191	4	4054	- .0207	3	1792	- .1347	8	2
Ironwood	1870	- .0310	4	742	- .0080	1	1832	- .0438	4	742	- .0487	4	1
Muskegon	6457	+ .0093	1	2762	+ .0252	2	6225	+ .0163	1	2732	+ .0178	2	1

TABLE 25

ACCURACY OF PROJECTIONS YIELDED BY TRANSITION ANALYSIS (REGRESSION)[a]

	EE				SE			
Stratum	H	M	P	$\dot{P}$	H	M	P	$\dot{P}$
A	7	3	.70	.38	8	2	.80	.29
B	10	0	1.00	.50	6	4	.60	.49
C	9	1	.90	.52	7	3	.70	.39
D	8	2	.80	.61	7	3	.70	.52
E	10	0	1.00	.49	6	4	.60	.52
Total	44	6	.88	.50	34	16	.68	.44

[a]For an explanation of the table headings, see the footnotes to Table 11, page 80.

Birth Analysis

Birth Analysis bases its projections on the past relationship between births and public school enrollment. Table 26 presents the projections yielded by this approach.[78]

Table 27 presents the number of projections yielded by Birth Analysis, within and across Strata, that are within a W Coefficient of .1000 of the actual enrollment figures.

The results suggest that while this approach appears to be quite adequate for projecting elementary enrollment in Strata C and D, it is quite inadequate for projecting secondary enrollment. This is probably true because it makes no provisions for migration, except indirectly through the number of births in the area. (Assuming the more people the more births.) All things considered, this approach appears to be quite inadequate and cannot be recommended for use in any Stratum, except for Stratum E.

TABLE 26

PROJECTIONS BY BIRTH ANALYSIS (REGRESSION)

District	EE 1965	W	R	SE 1965	W	R	EE 1968	W	R	SE 1968	W	R	Ṙ
Allen Park	3941	- .1486	11	2182	+ .0220	2	3981	- .1488	9	2373	+ .0312	5	8
Garden City	8537	- .1649	9	2667	- .1053	3	8996	- .1701	7	2809	- .1008	4	5
Inkster	3332	- .0876	12	858	- .2473	12	3859	+ .0466	7	950	- .2282	8	11
Livonia	17966	- .2143	11	3894	- .4452	15	20522	- .1840	10	4452	- .5519	14	12
Oak Park	7247	+ .4154	5	1986	+ .0153	4	7055	+ .4146	3	2028	- .0388	3	2
Ann Arbor	16186	+ .3548	14	3202	- .1749	14	20260	+ .4431	13	3386	- .2865	16	15
Birmingham	11188	- .0239	5	5058	+ .1099	9	11140	- .0624	7	6454	+ .1457	8	5
Lincoln Park	8801	- .0425	5	3924	- .0103	1	9167	- .0357	5	4011	- .0127	1	1
Midland	9145	+ .0965	13	3237	+ .0898	11	10196	+ .1691	11	3460	+ .0188	4	11
Royal Oak	14992	+ .0798	6	6322	+ .1130	11	14771	+ .1101	5	8771	+ .4555	15	12
Alpena	6600	+ .1896	14	1805	- .0409	2	7720	+ .2950	14	2191	- .0288	2	10
Dearborn	12398	- .0729	7	8635	+ .0285	5	12232	- .0796	5	9729	+ .0812	8	3
Kalamazoo	12510	- .0390	3	3547	- .2532	14	11922	0 .0985	5	3653	- .2814	14	13
Pontiac	16419	- .0264	2	4418	- .2009	10	16690	- .0526	2	4795	- .1729	9	3
Wyandotte	6067	+ .0465	5	3145	+ .1845	16	6007	+ .0498	5	3869	+ .4577	17	15
Bay City	8828	+ .0012	1	4602	- .1153	10	8649	- .0646	6	4599	- .1564	12	10
Grand Rapids	22018	- .0782	12	7851	+ .0047	1	21435	- .1160	12	8425	- .0397	2	4
Ferndale	5858	+ .0483	8	2643	+ .0730	4	5892	+ .0652	10	2783	+ .1008	9	7
Monroe	2874	- .2207	17	2037	- .0405	10	2953	- .5131	17	2038	- .1708	12	14
Saginaw	15085	- .0284	7	6847	+ .1136	16	15082	- .0620	6	7962	+ .2503	17	13
Battle Creek	7022	- .1601	13	2781	- .0598	12	6428	- .2074	10	2778	- .0589	12	11
Hamtramck	2662	+ .1208	11	1184	- .0684	4	2577	+ .1676	11	1092	+ .0876	13	7
Highland Park	4315	+ .0133	3	1751	- .0415	8½	4337	+ .0475	7	1749	- .1554	12	7
Ironwood	1925	- .0025	1½	813	+ .0868	6½	1889	- .0140	3	811	+ .0397	3	2
Muskegon	6646	+ .0389	2	2995	+ .1117	7	6613	+ .0796	2	2860	+ .0655	6	3

TABLE 27

ACCURACY OF PROJECTIONS YIELDED BY
BIRTH ANALYSIS (REGRESSION)[a]

	EE				SE			
Stratum	H	M	P	Ṗ	H	M	P	Ṗ
A	2	8	.20	.38	4	6	.40	.29
B	6	4	.60	.50	4	6	.40	.49
C	8	2	.80	.52	4	6	.40	.39
D	8	2	.80	.61	4	6	.40	.52
E	6	4	.60	.49	8	2	.80	.52
Total	30	20	.60	.50	24	26	.48	.44

[a]For an explanation of the table headings, see the footnotes to Table 11, page 80.

Enrollment Regression Analysis I

Enrollment Regression Analysis I bases its projections on the past relationship between city school-age population and public school enrollment. Table 28 presents the projections yielded by this approach.[79]

Table 29 presents the number of projections yielded by Enrollment Regression Analysis I, within and across Strata, that are within a W Coefficient of .1000 of the actual enrollment figures.

The results suggest that this approach, while it appears to be quite adequate for Districts having growth characteristics similar to those in Strata D and E, is probably not the best approach for either of those Strata. It should be noted that this approach also did a commendable job of projecting elementary enrollment in that most difficult of Strata, Stratum A.

TABLE 28

PROJECTIONS BY ENROLLMENT REGRESSION ANALYSIS I

District	EE 1965	W	R	SE 1965	W	R	EE 1968	W	R	SE 1968	W	R	R
Allen Park	4912	+ .0611	8	1964	- .0800	10	5157	+ .1026	8	2370	+ .0303	2	6
Garden City	10150	- .0071	1	2436	- .1828	5	11573	+ .0675	4	2404	- .2304	12	3
Inkster	3611	- .0112	5	849	- .2552	13	3738	+ .0138	3	875	- .2891	13	8
Livonia	23400	+ .0232	3	4425	- .3695	12	26144	+ .0394	4	5023	- .4944	11	7
Oak Park	7114	+ .3894	4	1947	- .0046	2	9317	+ .8682	9	2763	+ .3094	9	6
Ann Arbor	12253	+ .0256	3	3611	- .0695	8	14481	+ .0314	3	4126	- .1306	7	6
Birmingham	13603	+ .1867	14	5552	+ .2183	13	16009	+ .3473	14	6990	+ .2409	11	9
Lincoln Park	10101	+ .0989	9	3408	- .1404	12	10877	+ .1441	11	3921	- .0349	6	6
Midland	7554	- .0942	12	2921	- .0164	1	8433	- .0330	2	3143	- .0744	11	14
Royal Oak	15909	+ .1459	12	5331	- .0614	5	17062	+ .2823	11	6142	+ .0192	1	10
Alpena	5719	+ .0308	3	1615	- .1418	13	6288	+ .0548	4	1811	- .1972	12	6
Dearborn	14368	+ .0743	8	8218	- .0210	2	14428	+ .0856	6	9169	+ .0190	1	2
Kalamazoo	14856	+ .1411	8	4265	- .1021	7	15928	+ .2042	11	4615	- .0922	5	8
Pontiac	20468	+ .2136	17	5617	+ .0159	2	21354	+ .2121	17	5718	- .0137	2	8
Wyandotte	6216	+ .0722	6	2587	- .0256	4	6525	+ .1403	11	2948	+ .1107	8	4
Bay City	8703	- .0129	3	5158	- .0084	2	9339	+ .0304	2	5521	+ .0126	4	1
Grand Rapids	21888	- .0836	13	7463	- .0449	5	21948	- .0949	11	7980	- .0904	5	9
Ferndale	5731	+ .0255	6	2709	+ .0998	9	5683	+ .0274	6	2970	+ .1748	13	10
Monroe	3402	- .0775	7	2049	- .0348	5	3541	- .4161	10	2055	- .1639	7	10
Saginaw	14152	- .0884	15	6055	- .0151	4	14291	- .1112	13	6135	- .0365	5	9
Battle Creek	9064	+ .0840	7	2803	- .0524	10	9361	+ .1541	7	2806	- .0494	8	7
Hamtramck	2858	+ .2033	15	1139	- .1038	6	2856	+ .2940	15	1146	+ .1414	14	14
Highland Park	4145	- .0265	9	1765	- .0339	7	4102	- .0091	2	1773	- .1438	9	3
Ironwood	1935	+ .0025	1½	690	- .0775	4	1933	+ .0088	2	675	- .1346	5	3
Muskegon	6695	+ .0465	3	2624	- .0259	3	6662	+ .0876	4	2631	- .0197	3	2

TABLE 29

ACCURACY OF PROJECTIONS YIELDED BY
ENROLLMENT REGRESSION ANALYSIS I[a]

	EE				SE			
Stratum	H	M	P	$\dot{P}$	H	M	P	$\dot{P}$
A	7	3	.70	.38	3	7	.30	.29
B	5	5	.50	.50	6	4	.60	.49
C	5	5	.50	.52	6	4	.60	.39
D	8	2	.80	.61	8	2	.80	.52
E	7	3	.70	.49	6	4	.60	.52
Total	32	18	.64	.50	29	21	.58	.44

[a]For an explanation of the table headings, see the footnotes to Table 11, page 80.

Enrollment Regression Analysis II

This approach represents the first of the regression approaches that uses more than one predictor variable. The predictor variables utilized are city school-age population and total city population. Table 30 presents the projections yielded by this approach.[80]

Table 31 presents the number of projections yielded by Enrollment Regression Analysis II, within and across Strata, that are within a W Coefficient of .1000 of the actual enrollment figures.

The results suggest that Enrollment Regression Analysis II, while providing a number of accurate projections, cannot be recommended for use in any given Stratum.

TABLE 30

PROJECTIONS BY ENROLLMENT REGRESSION ANALYSIS II

District	EE 1965	W	R	SE 1965	W	R	EE 1968	W	R	SE 1968	W	R	• R
Allen Park	5302	+ .1453	10	2051	- .0393	5	5778	+ .2354	14	2569	+ .1164	12	11
Garden City	12495	+ .2222	10	2201	- .2616	10	15550	+ .4363	11	2442	- .2183	10	10
Inkster	3648	- .0010	1	832	- .2701	14	3793	+ .0287	5	859	- .3021	14	10
Livonia	28250	+ .2353	12	6012	- .1434	5	38698	+ .5386	12	8144	- .1803	4	9
Oak Park	9705	+ .8955	15	2032	+ .0388	7	13398	+1.6865	15	2644	+ .2527	6	14
Ann Arbor	12448	+ .0419	5	4165	+ .0731	9	14969	+ .0662	6	5271	+ .1106	4	6
Birmingham	13311	+ .1613	13	4972	+ .0910	8	15594	+ .3124	13	6169	+ .0951	4	9
Lincoln Park	10712	+ .1654	13	3889	- .0191	2	11572	+ .2172	14	3981	- .0201	2	7
Midland	9165	+ .0989	15	3251	+ .0946	12	10968	+ .2576	16	3658	+ .0771	12	14
Royal Oak	16101	+ .1597	13	5692	+ .0021	1	17419	+ .3092	13	6808	+ .1297	11	10
Alpena	6076	+ .0951	9	1757	- .0664	5	6892	+ .1561	10	2032	- .0992	5	5
Dearborn	12217	- .0865	11	8780	+ .0458	8	11304	- .1494	12	9776	+ .0864	10	11
Kalamazoo	13934	+ .0703	5	3979	- .1623	12	14453	+ .0927	4	4152	- .1833	11	7
Pontiac	17469	+ .0358	3	6557	+ .1859	9	18863	+ .0707	4	6548	+ .1293	6	7
Wyandotte	6358	+ .0967	13	3027	+ .1401	14	6705	+ .1717	12	3525	+ .3281	16	14
Bay City	8678	- .0157	5	5141	- .0117	3	9524	+ .0508	4	5454	+ .0003	1	2
Grand Rapids	23364	- .0218	6	7593	- .0282	2	24707	+ .0188	2	8068	- .0804	4	3
Ferndale	5665	+ .0137	2	2700	+ .0962	8	5563	+ .0057	2	3004	+ .1882	14	9
Monroe	4205	+ .1401	15	1842	- .1323	14	4670	- .2300	2	1762	- .2831	14	12
Saginaw	15247	- .0179	5	6951	+ .1306	17	15827	- .0156	4	7394	+ .1611	14	14
Battle Creek	9124	+ .0912	8	2806	- .0513	9	9444	+ .1643	8	2804	- .0501	10	8
Hamtramck	2591	+ .0909	7	1080	- .1502	7	2521	+ .1422	8	1067	+ .0627	11	9
Highland Park	4164	- .0220	7	1802	- .0136	3	4126	- .0033	1	1822	- .1202	7	1
Ironwood	2015	+ .0440	6	671	- .1029	8	2039	+ .0641	6	649	- .1679	8	9
Muskegon	7041	+ .1006	9	3719	+ .3804	14	7156	+ .1683	9	4163	+ .5510	13	13

TABLE 31

ACCURACY OF PROJECTIONS YIELDED BY
ENROLLMENT REGRESSION ANALYSIS II[a]

	EE				SE			
Stratum	H	M	P	$\dot{P}$	H	M	P	$\dot{P}$
A	2	8	.20	.38	2	8	.20	.29
B	3	7	.30	.30	8	2	.80	.49
C	7	3	.70	.52	4	6	.40	.39
D	8	2	.80	.61	5	5	.50	.52
E	6	4	.60	.49	4	6	.40	.52
Total	26	24	.52	.50	23	27	.46	.44

[a]For an explanation of the table headings, see the footnotes to Table 11, page 80.

Enrollment Regression Analysis III

This approach uses city school-age population and total county population as its predictor variables. Table 32 presents the projections yielded by Enrollment Regression Analysis III.[81]

Table 33 presents the number of projections yielded by Enrollment Regression Analysis III, within and across Strata, that are within a W Coefficient of .1000 of the actual enrollment figures.

The results suggest that this method produces adequate enrollment projections for Districts having growth characteristics similar to those classified in Strata B and D, although it is probably not the best approach for use in either of those Strata. It should be noted that this approach also did a commendable job in projecting elementary enrollment for those Districts classified in Stratum A.

TABLE 32

PROJECTIONS BY ENROLLMENT REGRESSION ANALYSIS III

District	EE 1965	W	R	SE 1965	W	R	EE 1968	W	R	SE 1968	W	R	° R
Allen Park	4870	+ .0520	7	1933	- .0946	11	4987	+ .0662	7	2381	+ .0347	6	5
Garden City	9747	- .0465	4	2409	- .1918	6	10822	- .0017	1	2300	- .2637	13	4
Inkster	3615	- .0101	4	887	- .2219	10	3744	+ .0154	4	934	- .2412	9	5
Livonia	23270	+ .0175	2	4404	- .3725	13	26083	+ .0370	3	5021	- .4946	12	6
Oak Park	8691	+ .6974	13	2011	+ .0281	5	9928	+ .9907	11	2644	+ .2530	7	11
Ann Arbor	12043	+ .0080	1	3641	- .0618	6	14109	+ .0049	1	4189	- .1173	5	2
Birmingham	11242	- .0191	4	3812	- .1634	11	12210	+ .0276	3	4177	- .2691	12	7
Lincoln Park	9927	+ .0799	8	3322	- .1621	14	10480	+ .1023	8	3703	- .0886	9	8
Midland	8328	- .0014	1	3060	+ .0303	3	9323	+ .0690	5	3302	- .0276	6	1
Royal Oak	15706	+ .1313	10	5176	- .0887	8	16711	+ .2559	9	5876	- .0248	2	6
Alpena	5456	- .0165	2	1630	- .1339	11	5801	- .0268	1	1814	- .1959	11	3
Dearborn	14868	+ .1138	14	9170	+ .0923	12	15500	+ .1662	14	10592	+ .1771	13	12
Kalamazoo	14717	+ .1305	7	4194	- .1170	8	15810	+ .1953	9	4118	- .1900	12	11
Pontiac	18208	+ .0796	13	5113	- .0752	4	19017	+ .0794	6	5231	- .0977	4	5
Wyandotte	6347	+ .0948	12	2705	+ .0188	2	6739	+ .1777	13	3142	+ .1838	12	9
Bay City	8939	+ .0138	4	4558	- .1237	11	9680	+ .0680	7	4816	- .1166	10	8
Grand Rapids	22582	- .0546	9	7404	- .0524	6	23389	- .0355	3	7743	- .1175	10	6
Ferndale	5798	+ .0375	7	2492	+ .0117	1	5755	+ .0404	8	2694	+ .0656	8	2
Monroe	4024	+ .0911	8	2048	- .0353	6	4370	- .2744	4	2053	- .1647	8	5
Saginaw	14353	- .0755	13	6649	+ .0814	14	14389	- .1051	12	6897	+ .0830	8	15
Battle Creek	11537	+ .3798	17	2796	- .0547	11	13308	+ .6407	16	2797	- .0525	11	17
Hamtramck	2990	+ .2589	17	953	- .2501	15	2953	+ .3380	17	959	- .0448	5	17
Highland Park	4653	+ .0927	12	1751	- .0415	8½	4726	+ .1415	13	1758	- .1511	10	11
Ironwood	2023	+ .0481	7	660	- .1176	10½	2053	+ .0715	7	630	- .1923	9	10
Muskegon	6901	+ .0787	7	3330	+ .2360	10	6931	+ .1315	7	3576	+ .3323	11	11

TABLE 33

ACCURACY OF PROJECTIONS YIELDED BY ENROLLMENT REGRESSION ANALYSIS III[a]

	EE				SE			
Stratum	H	M	P	$\dot{P}$	H	M	P	$\dot{P}$
A	8	2	.80	.38	3	7	.30	.29
B	7	3	.70	.30	6	4	.60	.49
C	5	5	.50	.52	4	6	.40	.39
D	8	2	.80	.61	6	4	.60	.52
E	4	6	.40	.49	4	6	.40	.52
Total	32	18	.64	.50	23	27	.46	.44

[a]For an explanation of the table headings, see the footnotes to Table 11, page 80.

Enrollment Regression Analysis IV

This approach uses city and county extrapolated school-age population as the predictor variables. Table 34 presents the projections yielded by Enrollment Regression Analysis IV.[82] In the cases of those Districts that are starred, there was a perfect correlation between the two predictor variables thus necessitating the use of a single predictor.

Table 35 presents the number of projections yielded by Enrollment Regression Analysis IV, within and across Strata, that are within a W Coefficient of .1000 of the actual enrollment figures.

TABLE 34

PROJECTIONS BY ENROLLMENT REGRESSION ANALYSIS IV

District	EE 1965	W	R	SE 1965	W	R	EE 1968	W	R	SE 1968	W	R	• R
Allen Park	5410	+ .1687	14	2180	+ .0210	1	6474	+ .3842	16	2478	+ .0769	10	13
Garden City	13095	+ .2809	13	2078	- .3029	13	15501	+ .4298	10	2484	- .2048	9	12
Inkster	3878	+ .0618	9	891	- .2184	9	3962	+ .0745	10	1032	- .1616	7	6
*Livonia	21436	- .0626	6	5105	- .2726	8	27936	+ .0975	5	6295	- .3563	6	3
Oak Park	7604	+ .4851	6	2196	+ .1226	10	9418	+ .8885	10	2789	+ .3218	12	8
Ann Arbor	11425	- .0436	6	3841	- .0103	1	12722	- .0938	8	3935	- .1708	11	7
Birmingham	11370	- .0080	1	3599	- .2102	12	11863	- .0015	2	3956	- .2977	13	8
*Lincoln Park	10967	+ .1931	15	3516	- .1132	9	11882	+ .2498	16	3953	- .0270	4	13
Midland	8205	- .0161	3	3183	+ .0717	8	9011	+ .0332	3	3450	+ .0159	3	2
Royal Oak	14146	+ .0189	3	5180	- .0880	7	14765	+ .1097	4	5743	- .0469	4	2
Alpena	5342	- .0371	4	1576	- .1625	14	5760	- .0337	2	1765	- .2176	13	7
Dearborn	14526	+ .0861	10	8085	- .0369	7	14895	+ .1207	8	8411	- .0652	7	8
Kalamazoo	15410	+ .1837	11	4544	- .0433	4	16610	+ .2558	13	5680	+ .1172	7	10
Pontiac	17736	+ .0516	7	5086	- .0801	5	19026	+ .0799	7	5219	- .0998	5	4
Wyandotte	6302	+ .0871	10	2753	+ .0369	5	6476	+ .1317	10	2935	+ .1058	7	5
Bay City	8597	- .0249	7	4827	- .0720	7	9884	+ .0905	10	4723	- .1337	11	9
Grand Rapids	24870	+ .0411	7	7017	- .1019	11	22797	- .0599	8	7190	- .1805	13	12
Ferndale	5910	+ .0576	11	2273	- .0771	5	6177	+ .1167	11	2396	- .0522	4	8
Monroe	3809	+ .0328	5	2043	- .0376	7½	4020	- .3371	5	2044	- .1684	9	7
Saginaw	15214	- .0200	6	6131	- .0027	1	15740	- .0210	5	6159	- .0328	4	1
Battle Creek	8290	- .0084	1	2672	- .0966	16	8175	+ .0078	1	2598	- .1199	16	3
Hamtramck	2715	+ .1431	13	1022	- .1959	12	2678	+ .2134	13	1001	- .0029	3½	11
Highland Park	4347	+ .0209	6	1747	- .0437	10	4701	+ .1355	12	1750	- .1549	11	10
Ironwood	1995	+ .0336	5	862	+ .1524	16	2009	+ .0485	5	786	+ .0076	1	4
Muskegon	6960	+ .0880	8	3133	+ .1629	9	7023	+ .1466	8	3385	+ .2611	10	10

TABLE 35

ACCURACY OF PROJECTIONS YIELDED BY ENROLLMENT REGRESSION ANALYSIS IV[a]

	EE				SE			
Stratum	H	M	P	$\dot{P}$	H	M	P	$\dot{P}$
A	4	6	.40	.38	2	8	.20	.29
B	7	3	.70	.50	6	4	.60	.49
C	6	4	.60	.52	6	4	.60	.39
D	8	2	.80	.61	6	4	.60	.52
E	6	4	.60	.49	4	6	.40	.52
Total	31	19	.62	.50	24	26	.48	.44

[a]For an explanation of the table headings, see the footnotes to Table 11, page 80.

The results suggest that Enrollment Regression Analysis IV provides an adequate projection methodology for Districts having growth characteristics similar to those in Strata B and D, although the secondary projections leave something to be desired. It should be pointed out, however, that the projections yielded by this methodology, though adequate, are not the best for Strata B and D of any of the estimates examined in this study. (Also Stratum C).

Housing Analysis

The predictor variables used in this approach were occupied housing and state equalized valuation. Table 36 presents the projections yielded by Housing Analysis.[83]

Table 37 presents the number of projections yielded by Housing Analysis, within and across Strata, that are within a W Coefficient of .1000 of the actual enrollment figures.

TABLE 36

PROJECTIONS BY HOUSING ANALYSIS (REGRESSION)

District	EE 1965	W	R	SE 1965	W	R	EE 1968	W	R	SE 1968	W	R	° R
Allen Park	5446	+ .1764	16	2084	- .0230	4	5888	+ .2589	15	2231	- .0304	3	10
Garden City	11094	+ .0852	6	2304	- .2271	8	13040	+ .2028	9	2616	- .1626	5	8
Inkster	3682	+ .0082	2	881	- .2271	11	3803	+ .0314	6	894	- .2737	11	7
Livonia	21268	- .0699	7	4677	- .3336	10	25390	+ .0095	1	5601	- .4362	10	5
Oak Park	7606	+ .4855	7	2577	+ .3174	15	9297	+ .8642	8	2782	+ .3184	11	12
Ann Arbor	12054	+ .0089	2	3821	- .0154	4	12841	- .0853	7	3884	- .1816	13	5
Birmingham	12041	+ .0505	7	4213	- .0754	6	13525	+ .1382	10	4921	- .1263	7	6
Lincoln Park	10293	+ .1198	11	3689	- .0696	8	10814	+ .1374	10	3909	- .0379	7	4
Midland	8702	+ .0434	6	3131	+ .0542	5	9853	+ .1298	8	3400	+ .0011	1	5
Royal Oak	15238	+ .0976	7	4889	- .1392	12	15907	+ .1955	8	5385	- .1063	8	9
Alpena	5472	- .0136	1	1635	- .1312	10	5450	- .0857	5	1730	- .2331	14	8
Dearborn	13457	+ .0062	1	7435	- .1143	13	13397	+ .0080	2	7626	- .1524	12	5
Kalamazoo	14139	+ .0861	6	4070	- .1431	10	14758	+ .1158	6	4300	- .1542	8	4
Pontiac	15823	- .0617	9	3891	- .2962	14	16148	- .0833	8	3874	- .3318	16	14
Wyandotte	5968	+ .0294	3	2405	- .0941	9	6054	+ .0580	7	2504	- .0565	4	2
Bay City	7996	- .0931	11	5060	- .0272	4	8324	- .0815	8	5256	- .0359	5	6
Grand Rapids	22147	- .0728	11	7168	- .0826	9	22135	- .0872	10	7460	- .1497	12	13
Ferndale	5278	- .0554	10	2708	+ .1035	11	5361	- .0307	7	2372	- .0617	6	6
Monroe	3714	+ .0070	3	2036	- .0409	11	3763	- .3795	9	2040	- .1700	10½	8
Saginaw	14493	- .0665	10	6432	+ .0461	7	14490	- .0988	9	6463	+ .0149	3	6
Battle Creek	8875	+ .0614	5	3091	+ .0449	7	8888	+ .0957	6	3091	+ .0470	7	4
Hamtramck	2790	+ .1747	14	1044	- .1785	9	2759	+ .2501	14	1007	+ .0029	3½	12
Highland Park	4321	+ .0147	4	1737	- .0492	13	4319	+ .0432	6	1721	- .1690	15	8
Ironwood	2210	+ .1450	17	719	- .0387	3	2288	+ .1941	17	592	- .2410	13	13
Muskegon	6811	+ .0647	5	3077	+ .1421	8	6823	+ .1139	6	3023	+ .1263	8	6

TABLE 37

ACCURACY OF PROJECTIONS YIELDED BY HOUSING ANALYSIS (REGRESSION)[a]

	EE				SE			
Stratum	H	M	P	$\dot{P}$	H	M	P	$\dot{P}$
A	5	5	.50	.38	2	8	.20	.29
B	5	5	.50	.50	6	4	.60	.49
C	9	1	.90	.52	2	8	.20	.39
D	9	1	.90	.61	7	3	.70	.52
E	5	5	.50	.49	5	5	.50	.52
Total	33	17	.66	.50	22	28	.44	.44

[a]For an explanation of the table headings, see the footnotes to Table 11, page 80.

The results suggest that this approach is extremely useful for projecting elementary enrollment in Districts with growth characteristics similar to those in Strata C and D, although the approach appears to be relatively unsuccessful at projecting secondary enrollment, with the exception of Stratum D Districts.

Multi-Variable Analysis I (Regression)

This is the first of three methods that utilize four predictor variables; total city population, total county population, school-age population, and births by city. It was originally thought that, due to the nature of the projection problem, four predictor variables could profitably be used with only six observations. As has already been discussed earlier in this Chapter, this hypothesis was proven to be in error from a methodological standpoint. The projections yielded by the three methods tend to support what the standard errors of the regression coefficients suggested, that is, that there is a tremendous amount of error present in the projections made by these methods. There are, of course, some notable exceptions to this, but not in sufficient numbers to lead one to any conclusion other than that all three methods are generally inadequate for use with the small number of data points used in this study. Table 38 presents the projections yielded by Multi-Variable Analysis I.[84]

Table 39 presents the number of projections yielded by Multi-Variable Analysis I, within and across Strata, that are within a W Coefficient of .1000 of the actual enrollment figures.

TABLE 38

PROJECTIONS BY MULTI-VARIABLE ANALYSIS I (REGRESSION)

District	EE 1965	W	R	SE 1965	W	R	EE 1968	W	R	SE 1968	W	R	• R
Allen Park	4557	- .0112	1	1992	- .0669	9	4712	+ .0074	2	2230	- .0308	4	3
Garden City	6072	- .4060	16	4065	+ .3636	15	2086	- .8075	16	6144	+ .9667	16	15
Inkster	7850	+1.1495	15	559	- .5096	15	8089	+1.1939	15	478	- .6116	15	15
Livonia	30466	+ .3322	14	7209	+ .0207	1	43682	+ .7367	14	10532	+ .0599	3	10
Oak Park	7850	+ .5332	9	1473	- .2469	12	8089	+ .6220	5	1446	- .3146	10	7
Ann Arbor	16647	+ .3934	15	4325	+ .1144	12	21272	+ .5152	14	5583	+ .1763	12	14
Birmingham	13649	+ .1908	15	4306	- .0550	4	19101	+ .6075	15	6046	+ .0733	2	15
Lincoln Park	9088	- .0113	2	4546	+ .1465	13	9337	- .0178	2	5689	+ .4001	16	12
Midland	8678	+ .0405	5	3159	+ .0636	6	9544	+ .0955	6	3499	+ .0303	8	6
Royal Oak	4205	- .6971	17	7027	+ .2371	15	neg.	-	17	10265	+ .7034	16	16
Alpena	6797	+ .2251	15	1677	- .1089	9	7891	+ .3237	15	1917	- .1502	8	14
Dearborn	11022	- .1758	17	8642	+ .0294	6	5310	- .6044	17	10347	+ .1499	11	17
Kalamazoo	4161	- .6803	17	1872	- .6058	17	neg.	-	17	1135	- .7767	17	17
Pontiac	17604	+ .0438	4	6440	+ .1647	6	18932	+ .0746	5	6562	+ .1317	7	6
Wyandotte	6881	+ .1869	16	2591	- .0241	3	8302	+ .4508	16	2414	- .0904	6	16
Bay City	12098	+ .3721	15	3961	- .2385	16	14763	+ .6289	15	3805	- .3020	16	15
Grand Rapids	45627	+ .9101	15	5681	- .2729	17	66175	+1.7288	15	5381	- .3867	17	15
Ferndale	8698	+ .5565	17	3244	+ .3170	15	6979	+ .2617	13	3863	+ .5280	15	15
Monroe	3324	- .0986	10	1695	- .2016	16	3329	- .4511	15	1067	- .5659	17	17
Saginaw	13700	- .1176	17	6520	+ .0605	10	11212	- .3026	17	7458	+ .1711	15	17
Battle Creek	10824	+ .2945	14	2928	- .0101	4	12262	+ .5117	14	3031	+ .0267	5	14
Hamtramck	2406	+ .0130	2	1264	- .0055	3	2563	+ .1613	10	1056	+ .0517	9½	3
Highland Park	6441	+ .5126	15	1845	+ .0098	2	7243	+ .7495	15	1984	- .0420	4	15
Ironwood	1733	- .1020	15	658	- .1203	12½	1567	- .1821	15	572	- .3243	17	17
Muskegon	neg.	-	17	4007	+ .4873	15	neg.	-	17	4639	+ .7283	16	17

TABLE 39

ACCURACY OF PROJECTIONS YIELDED BY MULTI-VARIABLE ANALYSIS I (REGRESSION)[a]

	EE				SE			
Stratum	H	M	P	$\dot{P}$	H	M	P	$\dot{P}$
A	2	8	.20	.38	4	6	.40	.29
B	4	6	.40	.50	4	6	.40	.49
C	2	8	.20	.52	3	7	.30	.39
D	1	9	.10	.61	1	9	.10	.52
E	1	9	.10	.49	6	4	.60	.52
Total	10	40	.20	.50	18	32	.36	.44

[a]For an explanation of the table headings, see the footnotes to Table 11, page 80.

Multi-Variable Analysis II (Regression)

This method uses the same predictor variables a Multi-Variable Analysis I, but uses the logarithms of public school enrollment as the criterion variable. Table 40 presents the projections yielded by this approach.[85]

Table 41 presents the number of projections yielded by Multi-Variable Analysis II, within and across Strata, that are within a W Coefficient of .1000 of the actual enrollment figures.

TABLE 40

PROJECTIONS BY MULTI-VARIABLE ANALYSIS II (REGRESSION)

District	EE 1965	W	R	SE 1965	W	R	EE 1968	W	R	SE 1968	W	R	• R
Allen Park	4563	- .0142	2	2012	- .0576	8	4696	+ .0040	1	2387	+ .0373	8	2
Garden City	8073	- .2106	10	4917	+ .6494	17	5167	- .5233	13	8026	+1.5691	17	17
Inkster	11826	+2.2382	17	82	- .9280	17	13627	+2.6959	17	238	- .8066	17	17
Livonia	62043	+1.7130	17	20148	+1.8704	17	199200	+6.9201	17	88410	+7.8979	17	17
Oak Park	9026	+ .7628	14	2869	+ .4667	16	10287	+1.0627	14	3926	+ .8606	16	15
Ann Arbor	24736	+1.0704	17	5015	+ .2921	16	44938	+2.2009	17	7922	+ .6691	17	17
Birmingham	16987	+ .4820	16	5621	+ .2334	15	37303	+2.1394	17	16289	+1.8917	17	17
Lincoln Park	9429	+ .0257	4	5162	+ .3018	17	9688	+ .0190	3	7718	+ .8995	17	17
Midland	9153	+ .0974	14	3225	+ .0858	9	10385	+ .1908	14	3661	+ .0780	13	12
Royal Oak	6086	- .5616	16	10585	+ .8635	17	1984	- .8508	16	31540	+4.2339	17	17
Alpena	8136	+ .4664	16	1780	- .0541	4	10598	+ .7778	17	2131	- .0554	4	17
Dearborn	11401	- .1475	16	8810	+ .0494	9	7732	- .4182	16	11426	+ .2698	17	16
Kalamazoo	7248	- .4432	15	2529	- .4675	16	4567	- .6546	15	1998	- .6070	16	16
Pontiac	17865	+ .0592	8	7927	+ .4337	17	19520	+ .1080	12	7934	+ .3684	17	17
Wyandotte	7003	+ .2080	17	2473	- .0685	6	8597	+ .5024	17	2416	- .0896	5	17
Bay City	14022	+ .5903	17	3893	- .2516	17	20122	+1.2202	17	3483	- .3611	17	17
Grand Rapids	84343	+2.5309	16	5891	- .2460	16	267501	+10.0309	16	5468	- .3767	16	16
Ferndale	6556	+ .1732	14	3955	+ .6057	17	7048	+ .2742	15	5506	+1.1780	17	17
Monroe	3283	- .1098	12	1726	- .1869	15	3499	- .4230	11	1285	- .4772	16	15
Saginaw	13733	- .1154	16	6567	+ .0681	11	11726	- .2707	16	7680	+ .2060	16	16
Battle Creek	11312	+ .3529	16	2934	- .0081	1	13454	+ .6587	17	3042	+ .0304	6	16
Hamtramck	2447	+ .0303	5	1277	+ .0047	2	2297	+ .0407	2	1056	+ .0517	9½	2
Highland Park	7073	+ .6611	17	1877	+ .0273	6	8526	+1.0594	17	2005	- .0318	2	17
Ironwood	1743	- .0968	14	661	- .1163	9	1599	- .1654	13	552	- .2923	15½	15
Muskegon	2200	- .6560	15	4720	+ .7520	17	1371	- .7761	15	6340	+1.3621	17	15

TABLE 41

ACCURACY OF PROJECTIONS YIELDED BY MULTI-VARIABLE ANALYSIS II (REGRESSION)[a]

	EE				SE			
Stratum	H	M	P	$\dot{P}$	H	M	P	$\dot{P}$
A	2	8	.20	.38	2	8	.20	.29
B	3	7	.30	.50	2	8	.20	.49
C	1	9	.10	.52	5	5	.50	.39
D	0	10	.00	.61	1	9	.10	.52
E	3	7	.30	.49	6	4	.60	.52
Total	9	41	.18	.50	16	34	.32	.44

[a]For an explanation of the table headings, see the footnotes to Table 11, page 80.

Multi-Variable Analysis III (Regression)

This method uses the logarithms of the predictor variables used in Multi-Variable Analysis I as the predictors, and the logarithms of public school enrollment as the criterion. Table 42 presents the projections yielded by this approach.[86]

Table 43 presents the number of projections yielded by Multi-Variable Analysis III, within and across Strata, that are within a W Coefficient of .1000 of the actual enrollment figures.

TABLE 42

PROJECTIONS BY MULTI-VARIABLE ANALYSIS III (REGRESSION)

District	EE 1965	W	R	SE 1965	W	R	EE 1968	W	R	SE 1968	W	R	° R
Allen Park	4441	- .0406	4	2028	- .0501	7	4505	- .0367	4	2383	+ .0356	7	4
Garden Park	7196	- .2960	14	3896	+ .3069	14	3512	- .6760	14	5219	+ .6706	15	14
Inkster	9026	+1.4715	16	417	- .6342	16	11112	+2.0138	16	393	- .6807	16	16
Livonia	29024	+ .2691	13	8490	+ .2095	7	41959	+ .6682	13	14270	+ .4361	9	15
Oak Park	8514	+ .6628	12	2195	+ .1221	9	10002	+1.0056	12	2682	+ .2710	8	13
Ann Arbor	20130	+ .6849	16	3833	- .0123	2	29387	+1.0932	16	4832	+ .0181	1	16
Birmingham	17935	+ .5647	17	4390	- .0366	2	31872	+1.6823	16	6302	+ .1187	6	16
Lincoln Park	9317	+ .0135	3	4217	+ .0635	6	9426	- .0085	1	4516	+ .1114	12	2
Midland	8763	+ .0507	8	3100	+ .0437	4	9738	+ .1166	7	3352	- .0129	2	4
Royal Oak	7581	- .4539	15	6161	+ .0846	6	3786	- .7154	15	8122	+ .3478	14	15
Alpena	8743	+ .5758	17	1716	- .0882	7	9165	+ .5374	16	1999	- .1139	6	16
Dearborn	12701	- .0503	4	8194	- .0239	3	11034	- .1697	15	9281	+ .0314	3	4
Kalamazoo	15769	+ .2113	13	2817	- .4069	15	15169	+ .1469	7	2571	- .4942	15	15
Pontiac	18879	+ .1194	15	7414	+ .3409	16	20178	+ .1453	15	7577	+ .3068	13	16
Wyandotte	6536	+ .1274	15	2623	- .0120	1	7662	+ .3390	15	2779	+ .0470	2	11
Bay City	13549	+ .5366	16	4090	- .2137	13	18597	+1.0519	16	4026	- .2615	15	16
Grand Rapids	119480	+4.0018	17	5905	- .2443	15	463900	+18.1298	17	5642	- .3569	15	17
Ferndale	6553	+ .1726	13	3755	+ .5245	16	6987	+ 2632	14	5020	+ .9857	16	16
Monroe	3258	- .1165	13	1633	- .2308	17	3468	- .4281	12	1362	- .4458	15	16
Saginaw	14405	- .0722	11	6453	+ .0496	9	13312	- .1720	15	7338	+ .1523	13	12
Battle Creek	10875	+ .3006	15	2931	- .0091	2½	12478	+ .5384	15	3028	+ .0257	4	15
Hamtramck	2434	+ .0248	4	1269	+ .0015	1	2271	+ .0289	1	1051	+ .0468	7	1
Highland Park	6928	+ .6270	17	1868	+ .0224	5	8351	+1.0171	16	1998	- .0352	3	16
Ironwood	1834	- .0497	8	660	- .1176	10½	1577	- .1769	14	522	- .2923	15½	14
Muskegon	2021	- .6840	16	4029	+ .4955	16	1177	- .8081	16	4625	+ .7231	15	16

TABLE 43

ACCURACY OF PROJECTIONS YIELDED BY MULTI-VARIABLE ANALYSIS III (REGRESSION)[a]

	EE				SE			
Stratum	H	M	P	$\dot{P}$	H	M	P	$\dot{P}$
A	2	8	.20		2	8	.20	
B	3	7	.30		7	3	.70	
C	1	9	.10		5	5	.50	
D	1	9	.10		1	9	.10	
E	3	7	.30		6	4	.60	
Total	10	40	.20		21	29	.42	

[a]For an explanation of the table headings, see the footnotes to Table 11, page 80.

Summary

Table 44 presents a breakdown by Strata of the effectiveness of the seventeen projection methodologies examined in this study. In order to be judged as adequate for a given Strata a projection methodology must meet the following criteria.

1. The methodology must receive a composite W Coefficient rank of 10 or better on four of the five Districts classified in that Strata.

2. The methodology must yield projections that are within a W Coefficient of .1000 of actual enrollment figures at least six out of ten observations on both elementary and secondary enrollment estimates.

Table 44 also identifies those formulae that appear to be best suited to the problems of projection relevant to Districts in each of the Strata identified in this study. Only one formula per Stratum is identified as best. Criteria are composite W Coefficient rank and number of projections yielded that are within a W Coefficient of .1000.

TABLE 44

EFFECTIVENESS OF PROJECTION METHODOLOGIES BY STRATA

	Stratum				
Projection Methodology	A	B	C	D	E
1. Cohort-Survival Ratio	X	0	X	0	X
2. Grade Retention Ratio	0	0	0	0	0
3. Enrollment Ratio Method I	0	0	0	0	0
4. Enrollment Ratio Method II	0	0	0	0	0
5. Pupil-Home Unit (Ratio)	0	0	0	0	0
6. Time Analysis I (Regression)	0	0	0	b	0
7. Time Analysis II (Regression)	0	0	0	X	0
8. Transition Analysis (Regression)	b	b	b	X	b
9. Birth Analysis (Regression)	0	0	0	0	X
10. Enrollment Regression Analysis I	0	0	0	X	X
11. Enrollment Regression Analysis II	0	0	0	0	0
12. Enrollment Regression Analysis III	0	X	0	X	0
13. Enrollment Regression Analysis IV	0	X	X	X	0
14. Housing Analysis (Regression)	0	0	0	X	0
15. Multi-Variable Analysis I	0	0	0	0	0
16. Multi-Variable Analysis II	0	0	0	0	0
17. Multi-Variable Analysis III	0	0	0	0	0
Total	2	3	3	7	4

X = projection methodology is adequate as determined by previously outlined criteria.

b = projection methodology yields best estimates for Districts in this Strata.

0 = projection methodology is not adequate as determined by previously outlined criteria.

Footnotes for Chapter IV

[70]In order to be judged as adequate for a given Strata a projection methodology must meet the following criteria.

a. It must receive a composite W Coefficient rank of 10 or better on four of the five Districts classified in that Strata.

b. It must yield projections that are within a W Coefficient of .1000 of actual enrollment figures at least six out of ten observations on both elementary and secondary enrollment estimates.

[71]The methodology is outlined in Appendix B-2.

[72]The methodology is outlined in Appendix B-3.

[73]The methodology is outlined in Appendix B-4.

[74]The methodology is outlined in Appendix B-5.

[75]The methodology is outlined in Appendix B-6.

[76]The methodology is outlined in Appendix B-7.

[77]The methodology is outlined in Appendix B-8.

[78]The methodology is outlined in Appendix B-9.

[79]The methodology is outlined in Appendix B-10.

[80]The methodology is outlined in Appendix B-11.

[81]The methodology is outlined in Appendix B-12.

[82]The methodology is outlined in Appendix B-13.

[83]The methodology is outlined in Appendix B-14.

[84]The methodology is outlined in Appendix B-15.

[85]The methodology is outlined in Appendix B-16.

[86]The methodology is outlined in Appendix B-16.

CHAPTER V

SUMMARY AND CONCLUSIONS

This study explores three major research questions.

1. Can educational attendance prediction problems be formulated in such a manner that the least squares regression model can be used in the analysis thereof? Ultimately we are asking if simple or multiple linear regression, or a transformation thereof, will yield more accurate projections of future school attendance than will the most popular educational projection methodologies currently in use at the district level.
2. Are different variables and diverse projection methodologies relevant to different types of school districts, or is there one projection methodology that provides the best estimates, ie. estimates with the least amount of error, across all school districts?
3. Is it worthwhile to make the necessary data transformations to estimate local population statistics for projection purposes, or will county population statistics, which are readily available from relatively reliable sources, provide more reliable estimates on which to base future school population projections?

The population identified in this study consists of those public school districts in the State of Michigan that are largely coterminous with cities of 10,000 population or more. It is divided into five strata on the basis of past community growth characteristics. Twenty-five school districts, five from each stratum, were randomly drawn from the population. Seventeen separate and distinct ratio and regression projection methodologies, using pre-1960 observations as base data, were used in order to obtain estimates of elementary and secondary enrollment for each of the twenty-five districts in the sample for the years 1965 and 1968. Actual 1965 and 1968 enrollment figures were then used as the criteria against which to judge the relative accuracy of the various ratio and regression approaches.

The ratio and regression analysis methods examined in this study all have one thing in common, they are all relatively simple to apply. Every one of the twelve regression approaches can be run on canned computer programs currently available at Michigan State University. This is their major strength, but also their major weakness. It is a strength in that, given the applicability of a given regression approach to enrollment projection, educators can apply that approach to their educational projection problems with relatively little difficulty. With some modifications of current computer programs, a canned program could be made available that, given the values of relevant predictor variables, would project elementary and secondary enrollment at given intervals into the future. It is a weakness in that, given districts with certain kinds of growth characteristics, particularly districts similar in growth patterns to those classified in Stratum A, the approaches examined in this study tend to oversimplify the estimation problem. It is probable that in districts that have unusual growth characteristics, a local survey that considers a large number of demographic variables is needed in order to obtain consistently adequate projections of future enrollment. It seems crucial that at least the approximate number of people that a given area can support be determined so that an upper limit can be established beyond which projected population values cannot increase.

Results and Discussion

A number of major conclusions that are pregnant with implications for education can be drawn from the results of this study. Of primary importance, the results suggest that educational attendance prediction problems can be formulated in such a manner that the least squares regression model can be used in the analysis thereof. One need only point to the relative accuracy of the projections produced by Transition Analysis, a regression approach, to support the contention that regression analysis offers a viable methodology that can be used to great advantage in the estimation of future public school enrollment. Adding further support is the fact that all of the regression methodologies examined in this study, with the exception of the abortive attempts to use four predictor variables with six observations, outperform all of the comparable ratio methods in terms of accuracy of projections yielded.

Also of major interest is the fact that it appears as if different projection methodologies perform differentially across districts that are located in communities with diverse past population growth characteristics. For example, Time Analysis I, a regression approach, appears to provide the most accurate projections for districts with past growth characteristics similar to those classified in Stratum D. Other projection methodologies also predict with differential success across strata. Adding additional support to this position is the differential ease with which all of the projection methodologies, taken as a group, predict across the five strata. For example, while it appears to be relatively difficult to obtain accurate estimates for those districts classified in Stratum A, there are numerous accurate projections of various aspects of enrollment of those districts classified in Statum D.

On the other hand, Transition Analysis, a regression approach, appears to supply adequate projections of future school enrollment across all districts, actually supplying the best estimates in four of the five strata. Therefore, it appears that one could not go too far wrong in using Transition Analysis, regardless of the past growth characteristics of the community in which the district of interest is located.

At another level of differentiation, many of the projection methodologies appear to have enjoyed differential success in the projection of elementary and secondary enrollment. In general, the various methodologies experienced more success in projecting elementary enrollment than in attempting to project secondary enrollment. This seems to be particularly true of Birth Analysis, Enrollment Regression Analysis III, Enrollment Regression Analysis IV, and Housing Analysis, all regression methodologies.

The question concerning whether it is more efficient to use population estimates derived from local or county data, appears to be an insignificant one. Births, used in conjunction with past tendencies of students to advance from one grade to the next, appear to form the most adequate basis from which to project future school enrollment. Both Transition Analysis, a regression approach, and Cohort-Survival Ratio Analysis, approaches that use the aforementioned predictors, far outperform all of the other methodologies examined in this study. Generally speaking, neither total city population estimates nor total county population estimates, when taken singularly, correlated exceptionally highly with enrollment across all districts. However, it is significant to note that correlations were increased by using school-age population, a variable that was computed in the manner outlined on page 62. This explains the relatively high multiple correlations that were observed in the cases of Enrollment Regression Analysis II and Enrollment Regression Analysis III, as well as casting serious doubt on the validity of using total city or total county population in conjunction with school-age population as predictors, since school-age population by itself appears to produce results that are at least comparable to the multi-variate approaches. All things considered, this researcher has grave doubts as to the viability of either city or county population estimates to the projection of future school enrollment.

Another interesting result, considered in the context of the question of the viability of using estimates of population variables as predictors of future school enrollment, is the performance of a relatively unique regression approach, Enrollment Regression Analysis IV. While the inter-censal population estimates produced by the University of Michigan Population Studies Center, and used as the basis for most of the population estimates used in this study, are based on such demographic variables as births, auto registration, school census count, voters, and sales tax, the inter-censal estimates used as predictor variables in Enrollment Regression Analysis IV were computed in strictly an algebraic manner. Census counts for the years 1950 and 1960 were obtained, and a straight line was simply drawn between those two points to obtain approximations of the base data on which the regression equations are formulated. The line was then extrapolated to obtain estimates of relevant population parameters for 1965 and 1968. Despite the relative simplicity of this method, as opposed to the complicated techniques used to obtain population estimates for all of the other methodologies examined in this study, Enrollment Regression Analysis IV outperformed the more complex methods in terms of accuracy of projections.

Finally, the results of the study suggest that the accuracy of projections is inversely related to the distance into the future that one wishes to make those projections. That is, the further into the future a projection is made, the more inaccurate it is likely to be. This is generally true of all projection methodologies across all strata.

Recommendations For Future Research

As the study progressed, an exponentially increasing number of unanswered questions made themselves apparent. These questions are incorporated into recommendations for future research.

One principle that is suggested by the results of the study is tempered, at this point, by far too much uncertainty. Of major concern for future research is the apparent inapplicability of regression equations with more than two predictor variables to projection problems of the type outlined in this study. Unless predictor variables can be located that are characterized by low inter-correlations, yet maintain a high correlation with the criterion variable, it is extremely doubtful that more than two such variables are useful. However, before any conclusions can be drawn as to the validity of this hypothesis, a great deal of future research must be perpetrated. Of primary importance is a specification of the optimum length of past time series observations to be used in estimating the constants in the regression model, that is, providing such an optimum length exists. If, in fact, a greater number of observations provide more accurate projections, assuming that it is possible to obtain the data, can more predictor variables efficiently be added? As soon as an increase in the number of observations is contemplated, a new research question arises, that being a determination of the length of time, or number of data points, that are required before the linear model no longer furnishes an adequate projection methodology. This question calls for experimentation with transformed scales and higher order curves as non-linear projection techniques, in the context of varying the number of data points. The small amount of experimentation done with transformed scales in this study suggests that, for a relatively small number of data points, the linear model provides the most adequate projection technique. An empirical determination of the point at which this is no longer the case would be extremely valuable.

The reader will recall some earlier concern with the implications and effects of autocorrelated error terms on the regression model. The effects of autocorrelated error terms lie entirely in the computation of standard errors and confidence intervals for the regression coefficients. While the estimates are admittedly unbiased, their sampling variances may be unduly large. The problem arises when the usual least-squares formulae are applied to estimate the sampling variances of the regression coefficients. Serious underestimates of these variances are likely to result. Although this state of affairs did not effect the outcome of this study, it would be helpful at some time in the future to be able to compute realistic confidence intervals for population projections. This of course would also involve the development of methods designed to combine the error in the projection of the predictor variable with the error in the projection of the criterion variable into a meaningful statement of the amount of error inherent in the projection. A theoretical study of the applicability of the regression model to time series analysis, in the context of its use in this study, would provide invaluable assistance to the science of demography.

Also of major interest is a replication of the results of this study on districts other than those included in the sample and the population. Transition Analysis appears to offer a viable new approach to the projection of future public school enrollment. It would be extremely interesting and valuable to determine if these results hold across different populations and samples.

Finally, experimentation should be conducted with a greater diversity of projection methodologies utilizing a number of different demographic variables. The models investigated in this study might be improved in a number of ways. First, greater experimentation with methods of projecting various demographic variables, such as future city and county population, should be encouraged. If more adequate estimates of relevant demographic variables were available, it is probable that more reliable estimates of future school enrollment could be obtained. Second, non-public school data, if it were made available, could be used as additional variables in the projection equations. Lastly, a study of the feasibility of using various indicants of economic status should be encouraged, especially if it becomes possible to use several decades of data in projecting future public school enrollment.

This study only serves as a starting point in the investigation of the applicability of least-squares regression to the projection of future school enrollment. Educators have relied on largely inadequate techniques for far too long. The time for the application of some of the techniques of the behavioral sciences to the projection of future enrollment has long since passed. It is hoped that the results of this study, in conjunction with the recommendations for future research, will encourage such a course of action.

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APPENDICES

APPENDIX A

ASSUMPTIONS MADE BY THE UNIVERSITY OF MICHIGAN POPULATION STUDIES CENTER IN PROJECTING FUTURE STATE POPULATION

APPENDIX A

ASSUMPTIONS MADE BY THE UNIVERSITY OF MICHIGAN POPULATION STUDIES CENTER IN PROJECTING FUTURE STATE POPULATION

The state population was projected by using a component procedure. That is, births, deaths, and migration were projected individually. The assumptions about these components are described below.

Mortality

A life table was constructed based on mortality in the Michigan population for the period 1959-1961. It was assumed that age specific rates of mortality would remain constant throughout the projection period. Although this assumption is unrealistic it does not seriously affect the population projections. Gains in longevity are likely to be small and the impact on population growth is trivial by comparison to a 10 or 15 per cent change in fertility. The present population is "aged forward" by using survival rates for that age level constructed from the life table. The procedure is straight forward. If there are say, 1000 people at a certain age level in a given year, we take that 1000 times the survival rates for that age level in order to obtain a projection of the number of survivors. When this process is completed for all age groups, we obtain a projection of population by age for all groups 5-9 or older. Two elements are lacking - net migrants by age and the population 0-4. These components are handled by the assumptions on migration and fertility.

Migration

Census data from 1960 indicate that the state lost approximately 180.000 net migrants for the period 1955-1960. Estimates of net migration by age for the period 1960-65 were prepared by obtaining estimates of the state population by age in 1965 and noting the discrepancy between expected survivors of the 1960 population and estimated 1965 population:

Est. Pop. (1965) - Expected pop (1965) = Estimated Net Migrants

These estimates imply an out migration from the state about equal in magnitude to the out migration during the preceding five years. Two sources give us confidence in the migration estimates:

1. The school census shows a consistent year to year out migration of the population age 7-16 for the period 1956- 1964.

2. A recent release of the Census Bureau using different procedures obtains similar results.

For the period 1965-1980, we have assumed that net migration would gradually diminish toward zero by 1980, but with the age pattern of migration being fairly similar from period to period. If migration outward were to continue at the same pace experienced by the state in 1955-60 or 1960-65, the state total population would be about 350-375 thousand less than the figures shown in the projections for 1980.

Fertility

We have made fertility projections by using cohort procedures. This method of projecting fertility focuses on the size of completed family and the spacing of births, rather than on age specific birth rates (the Census Bureau has recently shifted over to this method in their national projections).

Several national studies conducted at the University of Michigan indicate that married women now in the childbearing ages are likely to complete their families with three children. Given the fact that about 95 per cent of all women eventually marry, this means that each woman will have approximately 2.85 children. The projections used in this report are based on the results of these national surveys and are consistent with them.

Projections of births are obtained as follows:

If there are say 1000 women age 15-19 in 1960 and an expected 950 women age 20-24 in 1965 (based on the mortality and migration assumptions) then based on the birth assumptions it is relatively simple to determine their prediction.

Calculations are carried out for all women in the childbearing ages This procedure generated a prediction of births for the State of Michigan for the period 1960-1965 that was less than one per cent difference from the actual number.

Once births have been projected for each five year period, they are "survived" to age 0-4, migrants added or subtracted and we are ready to repeat the entire operation for the next five year period.

APPENDIX B

PROCEDURES USED IN OBTAINING PROJECTIONS

APPENDIX B-1

PROCEDURES USED IN OBTAINING PROJECTIONS FROM THE COHORT-SURVIVAL RATIO METHOD

1. Obtain the actual number of births by place of residence by year for the period 1950 through 1955.

2. Obtain actual enrollment figures by grade for kindergarten through twelfth grade for the years 1955 through 1960.

3. Compute the average percentage of survivorship between the number of births five years earlier and kindergarten enrollment thusly:

$$^{A}B\text{-}K = \frac{\left(\frac{K_{1955}}{B_{1950}} + \frac{K_{1956}}{B_{1951}} + \frac{K_{1957}}{B_{1952}} + \frac{K_{1958}}{B_{1953}} + \frac{K_{1959}}{B_{1954}} + \frac{K_{1960}}{B_{1955}}\right)}{N}$$

where:

$^{A}B\text{-}K$ = Average percentage of survivorship from birth to kindergarten.

K = Kindergarten enrollment in a given year.

B = Births in a given year.

N = 6

4. Compute the average percentage of survivorship between successive grades. The computation of the average percentage of survivorship from kindergarten to grade one is illustrated below. All other needed average percentages of survivorship are computed in a similar manner, except obviously using different input variables.

$$^{A}K\text{-}1 = \frac{\frac{F_{1956}}{K_{1955}} + \frac{F_{1957}}{K_{1956}} + \frac{F_{1958}}{K_{1957}} + \frac{F_{1959}}{K_{1958}} + \frac{F_{1960}}{K_{1959}}}{N}$$

where:

$^{A}K\text{-}1$ = Average percentage of survivorship from kindergarten to grade one.

F = First grade enrollment in a given year.

K = Kindergarten enrollment in a given year.

N = 5

5. The procedures outlined in steps 3 and 4 yield thirteen separate ratios.

A_{B-K} = Average percentage of survivorship from birth to kindergarten.

A_{K-1} = " " kindergarten to first grade

A_{1-2} = " " first to second grade

A_{2-3} = " " second to third grade

A_{3-4} = " " third to fourth grade

A_{4-5} = " " fourth to fifth grade

A_{5-6} = " " fifth to sixth grade

A_{6-7} = " " sixth to seventh grade

A_{7-8} = " " seventh to eighth grade

A_{8-9} = " " eighth to ninth grade

A_{9-10} = " " ninth to tenth grade

A_{10-11} = " " tenth to eleventh grade

A_{11-12} = " " eleventh to twelfth grade

6. Determine the mean number of births by place of residence for the years 1952 through 1960, 1948 through 1951, 1955 through 1963, and 1951 through 1954. Those dates correspond to the birth dates of the children in elementary school in 1965, in secondary school in 1965, in elementary school in 1968, and in secondary school in 1968, respectively.

Symbolically:

$\overline{EB}_{1965}$ = Mean number of births by place of residence for the years 1952 through 1960.

$\overline{SB}_{1965}$ = Mean number of births by place of residence for the years 1948 through 1951.

$\overline{EB}_{1968}$ = Mean number of births by place of residence for the years 1955 through 1963.

$\overline{SB}_{1968}$ = Mean number of births by place of residence for the years 1951 through 1954.

7. An example of the procedures used in projecting elementary and

secondary enrollment to 1965 follows.

a. $\overline{EB}_{1965} \times A_{B-K} = K_{1965}$ K = kindergarten

b. $K_{1965} \times A_{K-1} = F_{1965}$ F = first grade

c. $F_{1965} \times A_{1-2} = S_{1965}$ S = second grade

d. $S_{1965} \times A_{2-3} = T_{1965}$ T = third grade

e. $T_{1965} \times A_{3-4} = Fo_{1965}$ Fo = fourth grade

f. $Fo_{1965} \times A_{4-5} = Fi_{1965}$ Fi = fifth grade

g. $Fi_{1965} \times A_{5-6} = Si_{1965}$ Si - sixth grade

h. $Si_{1965} \times A_{6-7} = Se_{1965}$ Se = seventh grade

i. $Se_{1965} \times A_{7-8} = E_{1965}$ E = eighth grade

j. Sum the nine separate estimates to obtain a projection of total elementary enrollment for 1965.

k. <u>In</u> order to obtain a projection of secondary enrollment, take $\overline{SB}_{1965}$ and recurse through the identical process outlined in steps (a) through (i). Once an estimate of eighth grade enrollment has been obtained in this manner, use that estimate as a basis for obtaining estimates of the various components of secondary enrollment thusly: (E_{1965} = eighth grade enrollment)

1. $E_{1965} \times A_{8-9} = N_{1965}$ N = ninth grade

2. $N_{1965} \times A_{9-10} = Te_{1965}$ Te = tenth grade

3. $Te_{1965} \times A_{10-11} = El_{1965}$ El = eleventh grade

4. $El_{1965} \times A_{11-12} = Tw_{1965}$ Tw = twelfth grade

5. Sum the four separate estimates to obtain a projection of total secondary enrollment in 1965.

8. Follow the same procedure, but input $\overline{EB}_{1968}$ and $\overline{SB}_{1968}$ respectively, to obtain projections of elementary and secondary enrollment for 1968.

APPENDIX B-2

PROCEDURES USED IN OBTAINING PROJECTIONS FROM THE GRADE-RETENTION RATIO METHOD

1. Obtain the actual enrollment figures by grade for kindergarten through twelfth grade for the years 1955 through 1960.

2. Combine the enrollment figures into a sum of kindergarten through grade eight and grade nine through grade twelve figures.

3. Compute the average percent increase in elementary school enrollment.

$$API_E = \frac{\left(\frac{EE_{1955}}{EE_{1956}} + \frac{EE_{1956}}{EE_{1957}} + \frac{EE_{1957}}{EE_{1958}} + \frac{EE_{1958}}{EE_{1959}} + \frac{EE_{1959}}{EE_{1960}}\right)}{N}$$

where:

API_E = Average percent of increase in elementary enrollment by year for the years 1955 through 1960.

EE = Elementary enrollment in a given year.

N = 5

4. Compute the average percent increase in secondary school enrollment in a like manner.

5. a. $PEE_{1965} = (5 \times API_E) \times EE_{1960}$

   b. $PSE_{1965} = (5 \times API_S) \times SE_{1960}$

   c. $PEE_{1968} = (8 \times API_E) \times EE_{1960}$

   d. $PSE_{1968} = (8 \times API_S) \times SE_{1960}$

   where:

   EE = Elementary enrollment in a given year.

   API_E = Average percent increase in elementary enrollment by year for the years 1955 through 1960.

   SE = Secondary enrollment in a given year.

   API_S = Average percent increase in secondary enrollment by year for the years 1955 through 1960.

P = When a P appears in front of EE or SE, it signifies projected enrollment.

APPENDIX B-3

PROCEDURES USED IN OBTAINING PROJECTIONS FROM ENROLLMENT-RATIO METHOD I.

1. Obtain population figures for total city population for the years 1955 through 1960.

2. Obtain actual elementary and secondary enrollment figures for the years 1955 through 1960.

3. Compute the average of the percents which elementary and secondary enrollment are of total city population.

4. Apply those average percentages to the future estimated total city population for the years 1965 and 1968 to obtain projections of elementary and secondary enrollment for 1965 and 1968.

5. An example of the formula, projecting elementary enrollment for 1965, looks as follows:

$$PEE_{1965} = \frac{\left(\frac{EE_{1955}}{TP_{1955}} + \frac{EE_{1956}}{TP_{1956}} + \frac{EE_{1957}}{TP_{1957}} + \frac{EE_{1958}}{TP_{1958}} + \frac{EE_{1959}}{TP_{1959}} + \frac{EE_{1960}}{TP_{1960}}\right)}{N}$$

$$\times \; ETP_{1965}$$

Where:

EE = Elementary enrollment in a given year.

TP = Total city population in a given year.

N = 6

ETP = Estimated total city pop8lation in a given year (1965).

P = When a P appears in front of EE or SE, it signifies projected enrollment.

APPENDIX B-4

PROCEDURES USED IN OBTAINING PROJECTIONS FROM ENROLLMENT-RATIO METHOD II

1. Obtain population figures for total county population for the years 1955 through 1960.

2. Obtain actual elementary and secondary enrollment figures for the years 1955 through 1960.

3. Compute the average of the percents which elementary and secondary enrollment are of total county population.

4. Apply those average percentages to the future estimated total county population for the years 1965 and 1968 to obtain projections of elementary and secondary enrollment for 1965 and 1968.

5. An example of the formula, projecting secondary enrollment for 1968, looks as follows:

$$PSE_{1968} = \frac{\left(\frac{SE_{1955}}{TP_{1955}} + \frac{SE_{1956}}{TP_{1956}} + \frac{SE_{1957}}{TP_{1957}} + \frac{SE_{1958}}{TP_{1958}} + \frac{SE_{1959}}{TP_{1959}} + \frac{SE_{1960}}{TP_{1960}}\right)}{N}$$

$$\times\ ETP_{1968}$$

where:

SE = Secondary enrollment in a given year.

TP = Total county population in a given year.

N = 6

ETP = Estimated total county population in a given year. (1968)

P = When a P appears in front of EE or SE, it signifies projected enrollment.

APPENDIX B-5

PROCEDURES USED IN OBTAINING
PROJECTIONS FROM THE PUPIL-HOME
UNIT RATIO METHOD

1. Obtain figures on the number of home units by city for the years 1955 through 1960.

2. Obtain actual elementary and secondary enrollment figures for the years 1955 through 1960.

3. Compute the average of the percents which elementary and secondary enrollment are of the number of home units.

4. Apply those average percentages to the future estimated total number of home units for the years 1965 and 1968 to obtain projections of elementary and secondary enrollment for 1965 and 1968.

5. An example of the formula, projecting secondary enrollment for 1965, looks as follows:

$$PSE_{1965} = \frac{\left(\frac{SE_{1955}}{HU_{1955}} + \frac{SE_{1956}}{HU_{1956}} + \frac{SE_{1957}}{HU_{1957}} + \frac{SE_{1958}}{HU_{1958}} + \frac{SE_{1959}}{HU_{1959}} + \frac{SE_{1960}}{HU_{1960}}\right)}{N}$$

$$\times\ EHU_{1965}$$

where:

PSE = Projected secondary enrollment in a given year. (1965)

SE = Secondary enrollment in a given year.

HU = Home units in a give year.

N = 6

EHU = Estimated housing units in a given year. (1965)

APPENDIX B-6

PROCEDURES USED IN OBTAINING PROJECTIONS FROM TIME ANALYSIS I

1. Obtain actual elementary and secondary enrollment figures for the years 1955 through 1960.

2. Utilize the values 1955 through 1960 as values of the predictor variable in establishing each of the two regression equations required. Use the values corresponding to elementary enrollment in the years 1955 through 1960 as the values of the criterion variable in one of the equations, and the values corresponding to secondary enrollment in those years as values of the criterion variable in the other. Each year on the x axis is paired with the corresponding value of elementary or secondary enrollment on the y axis in defining the needed parameters of the regression equations.

3. Utilizing the least-squares criterion, two equations of the form $y = a + bx$ are obtained. (One equation to be used in projecting future elementary enrollment, and one to be used in projecting future secondary enrollment.) For example, in projecting elementary enrollment for the year 1965, the least-square equation would be of the following form:

   $PEE_{1965} = a + b\,(1965)$

   where:

   PEE = Projected elementary enrollment in a given year. (1965)

   a = The y intercept of the line defined by the relationship between time and elementary enrollment for the years 1955 through 1960.

   b = The slope of the line defined by the relationship between time and elementary enrollment for the years 1955 through 1960.

   1965 = The value of the predictor variable as defined by the year to which it is desired to project enrollment.

APPENDIX B-7

PROCEDURES USED IN OBTAINING PROJECTIONS FROM TIME ANALYSIS II

1. Obtain actual elementary and secondary enrollment figures for the years 1955 through 1960. Transform those figures to logarithms to the base ten.

2. Utilize the values 1955 through 1960 as the values of the predictor variable in establishing each of the two regression equations required. Use the values corresponding to the logarithms of elementary enrollment in the years 1955 through 1960 as the values of the criterion variable in one of the equations, and the values corresponding to the logarithms of secondary enrollment in those years as the value of the criterion variable in the other. Each year on the x axis is paired with the corresponding value of the logarithm of elementary or secondary enrollment on the y axis in defining the needed parameters of the regression equations.

3. Utilizing the least-squares criterion, two equations of the form $Y = a+bx$ are obtained. (One equation to be used in projecting future elementary enrollment, and one to be used in projecting future secondary enrollment.) For example, in projecting secondary enrollment for the year 1965, the least-squares equation would be of the following form:

   $$PSE_{1965} = a + b\ (1965)$$

   where:

   PSE = The logarithm of projected secondary enrollment in a given year. (1965)

   a = The y intercept of the line defined by the relationship between time and the logarithms of secondary enrollment for the years 1955 through 1960.

   b = The slope of the line defined by the relationship between time and the logarithms of secondary enrollment for the years 1955 through 1960.

   1965 = The value of the predictor variable as defined by the year to which it is desired to project enrollment.

4. Convert the logarithm of projected enrollment for a given year into the projected enrollment figure for that year.

APPENDIX B-8

PROCEDURES USED IN OBTAINING PROJECTIONS FROM TRANSITION ANALYSIS

1. Obtain the actual number of births by place of residence by year for the period 1950 through 1955.

2. Obtain actual enrollment figures by grade for kindergarten through twelfth grade for the years 1955 through 1960.

3. Utilizing the least-squares criterion, compute the following thirteen regression equations with predictor and criterion variables as specified.

	Predictor (enrollment)	Criterion (enrollment)
a.	Births, 1950-1955	Kindergarten, 1955-1960
b.	Kindergarten, 1955-1959	First Grade, 1956-1960
c.	First Grade, 1955-1959	Second Grade, 1956-1960
d.	Second Grade, 1955-1959	Third Grade, 1956-1960
e.	Third Graed, 1955-1959	Fourth Grade, 1956-1960
f.	Fourth Grade, 1955-1959	Fifth Grade, 1956-1960
g.	Fifth Grade, 1955-1959	Sixth Grade, 1956-1960
h.	Sixth Grade, 1955-1959	Seventh Grade, 1956-1960
i.	Seventh Grade, 1955-1959	Eighth Grade, 1956-1960
j.	Eighth Grade, 1955-1959	Ninth Grade, 1956-1960
k.	Ninth Grade, 1955-1959	Tenth Grade, 1956-1960
l.	Tenth Grade, 1955-1959	Eleventh Grade, 1956-1960
m.	Eleventh Grade, 1955-1959	Twelfth Grade, 1956-1960

4. The procedures outlined in step 3 yield thirteen separate regression equations of the form $Y = a + bx$. For example, transition from birth to kindergarten is defined as:

$$Y = a + bx$$

where:

Y = Projected enrollment in kindergarten in a given year.

a = The Y intercept of the line defined by the relationship between births for the years 1950-1955 and kindergarten enrollment for the years 1955-1960.

b = The slope of the line defined by the relationship between births for the years 1950-1955 and kindergarten enrollment for the years 1955-1960.

x = The value of the predictor variable, in this case births during a given period. (See step 5.)

5. Determine the mean number of births by place of residence for the years 1952 through 1960, 1948 through 1951, 1955 through 1963, and 1951 through 1954. Those dates correspond to the birth dates of children in elementary school in 1965, in secondary school in 1965, in elementary school in 1968, and in secondary school in 1968, respectively.

   Symbolically:

   $\overline{EB}_{1965}$ = Mean number of births by place of residence for the years 1952 through 1960.

   $\overline{SB}_{1965}$ = Mean number of births by place of residence for the years 1948 through 1951.

   $\overline{EB}_{1968}$ = Mean number of births by place of residence for the years 1955 through 1963.

   $\overline{SB}_{1968}$ = Mean number of births by place of residence for the years 1951 through 1954.

6. Define T = a + b (the regression line) for each of the thirteen cases, projected elementary and secondary enrollment for the year 1965 would be computed in the following manner.

   a. $T_{B-K}(\overline{EB}_{1965}) = K_{1965}$ K = kindergarten
   b. $T_{K-1}(K_{1965}) = F_{1965}$ F = first grade
   c. $T_{1-2}(F_{1965}) = S_{1965}$ S = second grade
   d. $T_{2-3}(S_{1965}) = T_{1965}$ T = third grade
   e. $T_{3-4}(T_{1965}) = Fo_{1965}$ Fo = fourth grade
   f. $T_{4-5}(Fo_{1965}) = Fi_{1965}$ Fi - fifth grade
   g. $T_{5-6}(Fi_{1965}- = Si_{1965}$ Si = sixth grade
   h. $T_{6-7}(Se_{1965}) = Se_{1965}$ Se = seventh grade
   i. $T_{7-8}(Se_{1965}) - E_{1965}$ E = eighth grade

Note: Every one of the above equations is of the form Y = a + bx. For example:

$T_{K-1}(K_{1965}) = F_{1965}$

where:

$T_{K-1} = a + b$

where:

a = The Y intercept of the line defined by the relationship between kindergarten enrollment in the years 1955-1959 and first grade enrollment in the years 1956-1960.

b = The slope of the line defined by the relationship between kindergarten enrollment in the years 1955-1959 and first grade enrollment in the years 1956-1960.

K_{1965} = Projected kindergarten enrollment in 1965.

F_{1956} = Projected first grade enrollment in 1965.

j. Sum the nine separate estimates to obtain a projection of total elementary enrollment for 1965.

k. In order to obtain a projection of secondary enrollment, take $\overline{SB}_{1965}$ and recurse through the identical steps outlined in steps (a) through (i). Once an estimate of eighth grade enrollment has been obtained in this manner, use that estimate as a basis for obtaining estimates of the various components of secondary enrollment thusly:

1. $T_{8-9}(E_{1965}) = N_{1965}$ N = ninth grade
2. $T_{9-10}(N_{1965}) = Te_{1965}$ Te = tenth grade
3. $T_{10-11}(Te_{1965}) = El_{1965}$ El = eleventh grade
4. $T_{11-12}(El_{1965}) = Tw_{1965}$ Tw = twelfth grade
5. Sum the four separate estimates to obtain a projection of secondary enrollment for 1965.

7. Follow the same procedure, but input $\overline{EB}_{1968}$ and $\overline{SB}_{1968}$ respectively, to obtain projections of elementary and secondary enrollment for 1968.

APPENDIX B-9

PROCEDURES USED IN OBTAINING PROJECTIONS FROM BIRTH ANALYSIS

1. Obtain actual elementary and secondary enrollment figures for the years 1955 through 1960.

2. Obtain the number of births by place of residence for the years 1938 through 1960. Determine the total number of births during each of the six nine year periods that correspond to the period during which children who were enrolled in elementary school in each of the six years, 1955 through 1960, would have been born. Do likewise for the total number of births during each of the six four year periods that correspond to the period during which children who were enrolled in secondary school in each of the six years, 1955 through 1960 would have been born.

3. Utilize the summated births that correspond to the years 1955 through 1960 as the predictor variables, one set for projecting elementary enrollment and the other for projecting secondary enrollment. Use elementary and secondary entollment, respectively, as the criterion variables.

4. Using the least-squares criterion, two equations of the form $Y = a + bx$ are obtained. (One equation to be used in projecting future elementary enrollment, and one to be used in projecting future secondary enrollment.) For example, in projecting elementary enrollment for the year 1965, the following procedure would be followed:

   a. Define the predictor and criterion variables.

	Predictor		Criterion
1.	births 1942 through 1950	1.	EE_{1955}
2.	births 1943 through 1951	2.	EE_{1956}
3.	births 1944 through 1952	3.	EE_{1957}
4.	births 1945 through 1953	4.	EE_{1958}
5.	births 1946 through 1954	5.	EE_{1959}
6.	births 1947 through 1955	6.	EE_{1960}

   b. $PEE_{1965} = a + b \quad EB_{1952-1960})$

   where:

   PEE = Projected elementary enrollment in a given year. (1965)

   a = The Y intercept of the line defined by the relationship between births and enrollment for the years 1955 through 1960. (as specified above)

b = The slope of the line defined by the relationship between births and enrollment for the years 1955 through 1960. (as specified above)

$EB_{1952-1960}$ = The sum of the number of births, 1952 through 1960.

APPENDIX B-10

PROCEDURES USED IN OBTAINING PROJECTIONS FROM ENROLLMENT REGRESSION ANALYSIS I

1. Obtain actual elementary and secondary enrollment figures for the years 1955 through 1960.

2. Obtain city school-age population figures for elementary and secondary school-age population for the years 1955 through 1960.

3. Utilize the values corresponding to elementary or secondary school-age population in the years 1955 through 1960 as the values of the predictor variable, and the values corresponding to elementary and secondary enrollment, respectively, in the years 1955 through 1960, as the values of the criterion variable in formulating the regression equations.

4. Utilizing the least-squares criterion, two equations of the form $Y = a + bx$ are obtained. (One equation to be used in projecting future elementary enrollment, and one to be used in projecting future secondary enrollment.) For example, in projecting secondary enrollment for the year 1965, the least-squares equation would be of the following form:

   $$PSE_{1965} = a + b\ (ESA_{1965})$$

   where:

   PSE = Projected secondary enrollment in a given year. (1965)

   a = The Y intercept of the line defined by the relationship between city elementary school-age population and elementary enrollment for the years 1955 through 1960.

   b = The slope of the line defined by the relationship between city elementary school-age population and elementary enrollment for the years 1955 through 1960.

   ESA = Estimated city elementary school-age population in a given year. (1965)

APPENDIX B-11

PROCEDURES USED IN OBTAINING PROJECTIONS FROM ENROLLMENT REGRESSION ANALYSIS II

1. Obtain the actual enrollment figures for elementary and secondary enrollment for the years 1955 through 1960. Elementary enrollment is the criterion variable in one of the regression equations while secondary enrollment is the criterion variable in the other.

2. Obtain city school-age population figures for elementary and secondary school-age population for the years 1955 through 1960. Elementary city school-age population represents one of the predictor variables in one of the regression equations while secondary city school-age population represents one of the predictor variables in the other.

3. Obtain total city population figures for the years 1955 through 1960. Total city population represents a second predictor variable used in both regression equations.

4. Utilizing the least-squares criterion, two equations of the form $Y = a + b_1 x_1 + b_2 x_2$ are obtained. (One equation to be used in projecting future elementary enrollment, and one to be used in projecting future secondary enrollment.) For example, in projecting elementary enrollment for the year 1968, the least-squares equation would be of the following form:

$$PEE_{1968} = a + b_1 (ESA_{1968}) + b_2 (TCP_{1968})$$

where:

PEE = Projected elementary enrollment in a given year. (1968)

a = The Y intercept of the line defined by the relationship between elementary enrollment, and city elementary school-age population and total city population for the years 1955 through 1960.

b_1 = The weight that city elementary school-age population contributes to the definition of the slope of the regression line.

ESA = Estimated elementary school-age population in a given year. (1968)

b_2 = The weight that total city population contributes to the definition of the slope of the regression line.

TCP = Estimated total city population in a given year. (1968)

APPENDIX B-12

PROCEDURES USED IN OBTAINING PROJECTIONS FROM ENROLLMENT REGRESSION ANALYSIS III

1. Obtain the actual enrollment figures for elementary and secondary enrollment for the years 1955 through 1960. Elementary enrollment is the criterion variable in one of the regression equations while secondary enrollment is the criterion variable in the other.

2. Obtain city school-age population figures for elementary and secondary school-age population for the years 1955 through 1960. Elementary city school-age population represents one of the predictor variables in one of the regression equations while secondary city school-age population represents one of the predictor variables in the other.

3. Obtain total county population figures for the years 1955 through 1960. Total county population represents a second predictor variable used in both regression equations.

4. Utilizing the least-squares criterion, two equations of the form $Y = a + b_1 x_1 + b_2 x_2$ are obtained. (One equation to be used in projecting future elementary enrollment, and one to be used in projecting future secondary enrollment.) For example, in projecting secondary enrollment for the year 1968, the least-squares equation would be of the following form:

$$PSE_{1968} = a + b_1 (SSA_{1968}) + b_2 (TP_{1968})$$

where:

PSE = Projected secondary enrollment in a given year. (1968)

a = The Y intercept of the line defined by the relationship between secondary enrollment, and city secondary school-age population and total county population for the years 1955 through 1960.

b_1 = The weight that city secondary school-age population contributes to the definition of the slope of the regression line.

SSA = Estimated secondary school-age population in a given year. (1968)

b_2 = The weight that total county population contributes to the definition of the slope of the regression line.

TP = Estimated total county population in a given year. (1968)

APPENDIX B-13

PROCEDURES USED IN OBTAINING PROJECTIONS FROM ENROLLMENT REGRESSION ANALYSIS IV

1. Obtain the actual elementary and secondary enrollment figures for the years 1955 through 1960. Elementary enrollment is the criterion variable in one of the regression equations while secondary enrollment is the criterion ovariable in the other.

2. Obtain extrapolated city school-age population figures for elementary and secondary school-age population for the years 1955 through 1960. City elementary school-age population represents one of the predictor variables in one of the regression equations while city extrapolated secondary school-age population represents one of the predictor variables in the other.

3. Obtain extrapolated county school-age population figures for elementary and secondary school-age population for the years 1955 through 1960. County extrapolated elementary elementary school-age population represents one of the predictor variables in one of the regression equations while county extrapolated secondary school-age population represents one of the predictor variables in the other.

4. Utilizing the least-squares criterion, two equations of the form $Y = a + b_1 x_1 + b_2 x_2$ are obtained. (One equation to be used in projecting future elementary enrollment, and one to be used in projecting future secondary enrollment.) For example, in projecting elementary enrollment for the year 1965, the least squares equation would be of the following form:

$$PEE_{1965} = a + b_1 (EECP_{1965}) + b_2 (EETP_{1965})$$

where:

PEE = Projected elementary enrollment in a given year. (1965)

a = The Y intercept of the line defined by the relationship between elementary enrollment, and city and county extrapolated elementary school-age population for the years 1955 through 1960.

b_1 = The weight that city extrapolated elementary school-age population contributes to the definition of the slope of the regression line.

EECP = City extrapolated elementary school-age population in a given year. (1965)

b_2 = The weight that county extrapolated elementary school-age population contributes to the definition of the slope of the regression line.

EETP = County extrapolated elementary school-age population in a given year. (1965)

APPENDIX B-14

PROCEDURES USED IN OBTAINING PROJECTIONS FROM HOUSING ANALYSIS

1. Obtain the actual elementary and secondary enrollment figures for the years 1955 through 1960. Elementary enrollment is the criterion variable in one of the regression equations while secondary enrollment is the criterion variable in the other.

2. Obtain figures on the number of home units by city for the years 1955 through 1960. Home units represents one of the predictor variables in both regression equations.

3. Obtain figures on State Equalized Valuation by district for the years 1955 through 1960. State Equalized Valuation represents the second predictor variable in both regression equations.

4. Utilizing the least-squares criterion, two equations of the form $Y = a + b_1 x_1 + b_2 x_2$ are obtained. (One equation to be used in projecting future elementary enrollment, and one to be used in projecting future secondary enrollment.) For example, in projecting secondary enrollment for the year 1965, the least-squares equation would be of the following form:

$$PSE_{1965} = a + b_1 (HU_{1965}) + b_2 (SEV_{1965})$$

where:

PSE = Projected secondary enrollment in a given year. (1965)

a = The Y intercept of the line defined by the relationship between secondary enrollment, and occupied housing and State Equalized Valuation for the years 1955 through 1960.

b_1 = The weight that occupied housing contributes to the definition of the slope of the regression line.

HV = The number of estimated occupied housing units in a given year. (1965)

b_2 = The weight that State Equalized Valuation contributes to the definition of the slope of the regression line.

SEV = Estimated State Equalized Valuation in a given year. (1965)

APPENDIX B-15

PROCEDURES USED IN OBTAINING PROJECTIONS
FROM MULTI-VARIABLE ANALYSIS I

1. Obtain the actual elementary and secondary enrollment figures for the years 1955 through 1960. Elementary enrollment is the criterion variable in one of the regression equations while secondary enrollment is the criterion in the other.

2. Obtain figures on total city population for the years 1955 through 1960. Total city population is one of the predictor variables used in both regression equations.

3. Obtain figures on total county population for the years 1955 through 1960. Total county population is the second predictor variable used in both regression equations.

4. Obtain city school-age population figures for elementary and secondary school-age population for the years 1955 through 1960. Elementary school-age city population represents a third predictor variable in one of the regression equations while secondary city school-age population represents a third predictor variable in the other.

5. Obtain birth statistics for elementary and secondary school-age population for the years 1955 through 1960. (For procedures, see pp. 156-157 of Procedures Used in Obtaining Projections from Birth Analysis.) Elementary births represents a fourth predictor variable in one of the regression equations while secondary births represents a fourth predictor variable in the other.

6. Utilizing the least-squares criterion, two equations of the form $Y = a + b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4$ are obtained. (One equation to be used in projecting future elementary enrollment, and one to be used in projecting future secondary enrollment.) For example, in projecting elementary enrollment for the year 1968, the least-squares equation would be of the following form:

$$PEE_{1968} = a + b_1(TCP_{1968}) + b_2(TP_{1968}) + b_3(ESA_{1968}) + b_4(EB_{1968})$$

where:

PEE = Projected elementary enrollment in a given year. (1968)

a = The Y intercept of the line defined by the relationship between elementary enrollment, and total city population, total county population, city elementary school-age population, and city elementary birth statistics for the years 1955 through 1960.

b_1 = The weight that total city population contributes to the slope of the regression line.

TCP = Estimated total city population in a given year. (1968)

b_2 = The weight that total county population contributes to the slope of the regression line.

TP = Estimated total county population in a given year. (1968)

b_3 = The weight that city elementary school-age population contributes to the slope of the regression line.

ESA = Estimated elementary school-age population in a given year. (1968)

b_4 = The weight that city elementary births contributes to the slope of the regression line.

EB = Total number of births during the years that contributed students to elementary school in a given year. (1968)

APPENDIX B-16

PROCEDURES USED IN OBTAINING PROJECTIONS FROM MULTI-VARIABLE ANALYSIS II AND MULTI-VARIABLE ANALYSIS III

1. Multi-Variable Analysis II uses the same procedures as Multi-Variable Analysis II, except that the criterion variables, elementary and secondary enrollment, are transformed into logarithms to the base ten. This of course means that the projections are in logarithmic form and must be transformed into raw numbers.

2. Multi-Variable Analysis III uses the same procedures as Multi-Variable Analysis I, except that all of the criterion and predictor variables are transformed into logarithms to the base ten. Once again the projections are in logarithmic form and must be transformed into raw numbers.