WARRANT VALUATION IN RATIONAL SECURITY MARKETS

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This is to certify that the

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ABSTRACT

WARRANT VALUATION IN RATIONAL SECURITY MARKETS

By

Herbert Joel Weinraub

Problem

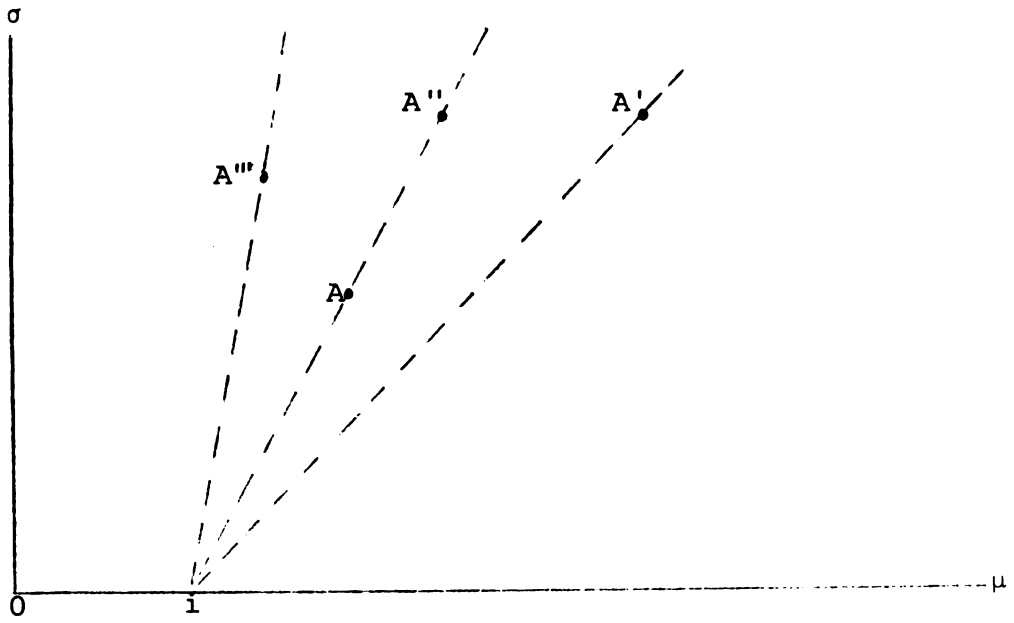
The valuation of stock purchase warrants has been studied based on three objectives: First, to develop a valuation model that generates a lower boundary value for a given warrant at a given time. Second, to describe and justify a new approach to the incorporation of warrants in the investment decision. Third, to apply the model in a market test to determine the frequency of warrant undervaluation, and the factors that influence the probability of acquiring undervalued warrants.

Model development

The analysis utilizes the two-parameter model, with the standard deviation and the arithmetic rate of return specified as the parameters. Additionally, the concept of a risk-free rate of interest at which an investors funds can be all or partially employed is incorporated in the model.

Future price expectations can be transformed into

a probability analysis that can be used to generate the two parameters for both a common stock (A) and its associated warrant (A'). Since the primary objective is the development of the warrant premium, it is initially assumed that the warrant sells for its base (non-premium) value. Under the base value assumption, a warrant will lie above and to the right of its associated stock in the two-parameter space.



The model to be developed is predicated on the assumption that an investment in a warrant should be combined with the risk-free rate of interest, and together should be considered as an alternative to an investment in the common stock. If the anticipated risk-return relationship for a stock is deemed acceptable, the alternative

warrant-i strategy should then be considered with the objective of proportioning funds in a manner that achieves a higher μ, at no increase in risk. This situation will be possible whenever the warrant occupies a point in the two-parameter space to the right of the ray iA''.

The valuation model developed is:

$$Pw^* = Ps - \frac{Po + D}{(1 + i)^n} \quad \text{with,}$$

Pw* = the price that places the warrant on the same ray from i as the associated common stock.

Ps = the current market price of the common stock.

Po = the current option price of the warrant.

D = the total dollar dividend per share, over the time horizon of the investment, on the common stock.

i = the risk-free rate of return.

n = the time horizon of the investment.

At Pw* an investor can proportion his funds between A' and i in a manner that equates the warrant strategy with the stock, and represents the minimum value a warrant can rationally have at a given time. With any market price less than Pw*, an investor can achieve a higher μ with the warrant-i combination, at no increase in risk, relative to the common stock.

If an investor's specified degree of acceptable

risk is σA, he will seek that investment on the σAL line furthest to the right. Whenever the market price of a warrant is less than its Pw*, a warrant-i combination can be purchased which is to the right of the stock on the σAL line, hence, rational investors will purchase the warrant in lieu of the stock, and this action will tend to force the price of the warrant back to a minimum of Pw*.

Market Test Methodology

A market test is developed to determine the practical significance of the valuation model. There are 35 randomly generated weeks between the dates of January 1, 1961 and June 1, 1971. Stock and warrant price observations are selected from the closing prices of the respective securities on the Friday of each week. The risk-free rate used is the prevailing maximum interest rate on savings accounts of commercial banks. The investment holding period is the remaining length of a warrant's life. For warrant's whose option price changed before maturity, the option price prevailing on the date examined is used and the warrant is assumed held until the date the terms changed. In addition, the last specified option price is used with the assumption that the warrant is held to expiration. A

warrant is considered undervalued if its market price is below Pw* with either option price, but counted only once if undervalued under each option price.

Test Results

It was found that 4.7 per cent of all stock price observations had associated undervalued warrants, and 11.3 per cent of all stock prices above their respective option prices had associated undervaluations. The probability of undervaluations increased when the remaining maturity of the warrant was less than two years, and particularly increased with less than one year to maturity. There is also an increase in the percentage of undervaluations when the associated stocks have a high Ps/Po ratio. Over two-thirds of all undervaluations occurred under the above maturity and Ps/Po conditions, although such conditions account for less than 13 per cent of total observations.

The general state of the market (bull or bear) has an effect on the frequency of undervaluations, with less occurring in bear, more in bull, than would be expected if the events were independent. Of all tested dates 63 per cent had at least one undervaluation. The conclusion reached is that warrant undervaluation occurs with sufficient frequency to merit investor attention.

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Of course, I, alone, am responsible for any errors or omissions that may appear on the following pages.

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CHAPTER I

PURPOSE, TERMINOLOGY, AND REVIEW OF LITERATURE

Purpose of Study

The primary objective of this study is to develop a model of warrant valuation that establishes a minimum price at which a given warrant at a given time can rationally trade. It will be demonstrated that a warrant market price below that determined by the model leads to an investment strategy that is superior to an investment in the warrant's associated common stock. A second objective is to describe and justify an investment strategy for warrants which represents a new approach to the incorporation of this vehicle in the investment decision. The final objective is an application of the developed model in a market test to determine if irrational pricing leads to substantial opportunities to acquire undervalued warrants, and under what circumstances, if any, are these opportunities most prevalent.

Review of Warrant Terminology

Stock Purchase Warrant: A stock warrant is a

security giving the bearer an option to purchase a specified number of common stock shares, at a specified price, on or before a specified date. Marketable warrants trade primarily in the over-the-counter market and on the American Stock Exchange. The New York Stock Exchange has recently allowed listing for a few select warrants. The principle methods by which warrants come into existence are: as part of the terms of a merger agreement; attached to an issue of long-term bonds; as part of the lending agreement for intermediate-term funds; as partial compensation for venture capitalists.

Associated Common Stock: The associated (related) common stock is that security which must be surrendered by the corporation when the option of a given warrant is exercised.

Option Price: The option (exercise) price, is the specified sum the warrant holder must pay to the corporation for each share of common stock received if the warrant is exercised. The option price may be a constant throughout the length of the warrant's life, or may change periodically at specified dates.

Expiration Date: The expiration date of a warrant is the date its terms expire. If a warrant remains unexercised at the close of trading on the expiration date

it becomes valueless. Warrant's at issuance can have original expiration dates ranging from a few months to perpetuity. Marketable warrants are normally intermediate-term securities with original life spans of three to ten years.

Dilution Protection: Dilution protection is a provision in a warrant's terms designed to protect a warrant holder from loss in the value of the option due to stock splits, stock dividends, or additional common stock financing. If a warrant is fully protected against dilution, the number of shares each warrant entitles its bearer to purchase is adjusted whenever there is a change in the amount of common stock outstanding. The majority of warrants are fully protected against dilution, some partially protected, and a few have no protection.

Theoretical Price: The theoretical or base price of a warrant is commonly defined as the current market price of the associated common stock minus the current option price of the warrant, multiplied by the number of shares the warrant entitles the holder to purchase.

Warrant Premium: The premium is the excess over the base (theoretical) price that investors pay for a warrant. Premiums arise due to the willingness of investors to acquire the potentially advantageous leverage

that warrants possess, relative to their related common stock. The establishment of the correct premium under a given set of circumstances has been the focus of this and past studies in this area.

Leverage: The concept of leverage can be variously defined. As defined here, leverage is the percentage movement in the market price of a warrant relative to a one per cent change in the market price of its associated common stock. The leverage is a function of the option price which is a fixed cost, as opposed to the variability of common stock prices.

Detachable: A detachable warrant can be removed from the security with which it was originally issued, and traded separately. If a warrant is nondetachable, a trading market for the warrant alone cannot be established.

Marketable Warrant: A marketable warrant is one which is detachable.

Arbitrage: For the purpose of this study, arbitrage is defined as the process of simultaneous purchase and sale of the same or equivalent securities to take advantage of a price discrepancy, when such action will result in immediate profit to the arbitrager.

Role of Warrants

The creation and distribution of warrants arises for a variety of purposes. The primary purpose of issuance has been as "sweeteners" added to long-term debt to make the issue more salable. For lessor known corporations, or in periods of relatively tight money, many firms can attract capital at reasonable prices only by including warrants as part of the security package. In addition, investors normally accept a lower bond yield when warrants are attached, due to the potentially advantageous equity participation offered by the security. The effect is to lower the cost of long-term debt to the corporation, relative to what it would have been with a straight bond issue.

Warrants also represent delayed equity financing. When the price of the stock rises above the option price, the corporation is assured that by the expiration date, at the latest, the warrants will be surrendered together with a specified sum of cash, and common stock issued in its stead. If a corporation forecasts a need for external equity funds in the future, the firm can arrange the option terms such that the expiration date coincides with the period when the cash inflow is required. In this manner, equity financing in a future period can be arranged as part of a present bond issue, lowering the cost of that

issue as well as part of the flotation costs of the new common stock. A major risk for the corporation is that the price of the stock may not rise or be maintained above the option price, and therefore, the warrant will not be exercised and additional funds not received. An additional risk is the stock price rising substantially above the option price necessitating the sale of the common well below then current market levels.

Warrants may also arise as part of the terms in a merger agreement. For the issuing corporation the use of warrants in this context represents a deferred payment to the other party. This was particularly true before the effect of warrants was required to be included in the reported earnings per share on a fully diluted basis.[1] Prior to 1969 the potential increase in common stock was not included when reporting the earnings of the issuing corporation.

Warrants have also been used as partial compensation to underwriters and other venture capitalists by a relatively new and/or unknown corporation when first going public.

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[1]Opinions of the Accounting Principles Board, Earnings Per Share, Number 15, May 1969.

With the recent issuance of warrants by AT&T and Tenneco, and with the inclusion of these securities on the New York Stock Exchange, it seems probable that the use of this security may increase in the future, and in particular, their use by the larger and less speculative corporations. From an investment perspective, the range and availability of warrants is likely to increase, and it seems an appropriate time to add to the knowledge of proper warrant valuation.

Background and Review of Literature

Giguere.[2] Giguere's basic hypothesis is that a mathematical relationship exists between the price of a stock and the price of its associated warrant. He attempts to discover this relationship and determine its predictive accuracy. "In fact, the value of a soundly based, reliable method of evaluating warrants would be to supply us with criteria against which actual market prices can be judged."[3]

Giguere states that certain warrant characteristics such as short life to expiration, a variable option

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[2]G. Giguere, "Warrants, A Mathematical Method of Evaluation," The Analysts Journal, No. 5, 17-25 (November 1958).

[3]Giguere, p. 17.

price, or a thin market, are factors that can distort the mathematical relationship between the securities. To avoid these problems he selects two perpetual warrants which have constant option prices. He then plots the prices of the stocks and their warrants and determines their relationship to be, $W = \frac{P^2}{4a}$, a parabola with its vertex at the origin. W = the warrant value, P = the price of the stock, a = the option price.

Giguere uses two "ideal" types to determine an equation of best fit, then applies this equation to other warrant types to see if they are under or over priced. There is no theory which explains the reasons for the warrant's behavior, and two highly specific and uncommon examples are used to formulate a general theory.

Morrison.[4] Morrison bases his valuation model on a break-even method of analysis. He attempts to determine if, at any given time, a commitment to a warrant is preferable to a commitment to its associated common stock.

The formulation derived is:

$$A = \frac{W}{1-X/Y} + Z \quad \text{with,}$$

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[4]R. J. Morrison, "The Warrants or the Stock", Analysts Journal, Vol. 13, No. 5 (November 1957), p. 52.

A = break-even point of stock.
B = warrant option price.
X = current market price of warrant.
Y = current market price of stock.
Z = total dividends anticipated per share during investor's time horizon.

As an example assume; W = \$3.00, Y = \$6.00, X = \$4.00, Z = annual dividend of 30 cents.

$$A = \frac{\$3.00}{1 - \$4.00/\$6.00} + \$1.20 = \$10.20$$

Thus, for warrants to be a superior vehicle for the employment of \$1200, the common must increase in price from \$6.00 to \$10.20. The formula can also be arranged to give the current worth of a warrant with any given estimate of the future price of its related stock.

Morrison recognizes that the risk relationship of the two securities is not incorporated in his model. In addition, only the mean value of the two alternative investments are equated, with other possible values ignored.

Shelton.[5] Shelton intuitively selects six factors he believes may influence the premium investors pay for warrants. By using regression analysis he finds three of the six materially affect the warrant-stock price relationship. In order of importance: the dividend yield of the

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[5]J. P. Shelton, "Relation of the Price of a Warrant to the Price of its Associated Stock," Financial Analysts Journal, J23:143-51 May; 88-99 July 1967.

common; its listing (on the American Exchange or over-the-counter); and the warrant's remaining life span. For example, the greater the foregone dividend the lower the warrant's premium, and the longer the life span the larger the premium. More than half a warrant's premium is unexplained by the tested variables, and Shelton attributes this to speculative emotions. His formula is designed to give appropriate weight to the factors he found significant. His model is:

$$Y = \left(\sqrt[4]{\frac{\text{longevity in months}}{72}} \right) (.47 - 4.25 \text{ yield} + .17, \text{ if listed})$$

Its interpretation is, initially place the warrant value 47 per cent of the distance from the top to the bottom of the two extremes that arbitrage defines, subtract percentage points if its associated stock pays a dividend, add 17 per cent if the warrant is listed, then multiply the resultant figure by the fourth root of the life span factor.

Sprenkle.[6] The Sprenkle study attempts to determine from warrant prices, what option investors'

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[6]C. M. Sprenkle, "Warrant Prices as Indicators of Expectations and Preferences", Yale Economic Essays, Vol. 1, No. 2 (1961), pp. 178-231.

expectations are toward the expected mean and standard deviation of a warrant's associated common stock. A second objective is to obtain quantitative measures of investors' risk preferences. He finds that two parameters can be estimated, expected variance and risk preference, and incorporates into his valuation model the assumption that future common stock prices are log-normally distributed.

His mathematically determined non-premium warrant value is based upon the assumptions that the expected price of the stock as well as the distribution of stock prices above the option price determines a warrant's basic value.

Warrant value equals:

$$\begin{array}{l} X - a \quad \text{if } X > a \\ 0 \text{ if } X < a \end{array} \qquad (1)$$

with a = option price, and X any expected stock price. If f(X), the distribution of possible stock prices on a target date, is continuous, then;

$$E(Pw) = \int_{a}^{\infty} (X - a)\ f(X)\ dX \qquad (2)$$

with E(Pw) = the expected warrant value. Equation (2) specifies that the expected price of the stock minus the option price is equal to the warrant's basic value.

With his assumption that future stock prices are

log-normally distributed, that is, f(X) is log-normal with variance LnX equal to σ^2, and mean LnX equal to μ, equation (2) takes the form of,

$$E(Pw) = \int_a^{\infty} \frac{(X - A)}{X \sigma \sqrt{2\pi}} \exp\left\{-1/2 \left[(LnX - \mu/\sigma\right]^2\right\} dX \qquad (3)$$

However, warrant's have a leverage factor compared to their Associated stock. Whatever the (σ,μ) of the stock, its warrant will have both greater expected return and greater risk. An investor will pay E(Pw) as determined by (3), only if he is neutral toward risk. The price an investor will normally pay for a warrant is equal to the expected value of that warrant plus a premium determined by what he is willing to pay for the warrant's leverage.

Sprenkle compares actual warrant prices to their E(Pw) to determine what the marginal option buyer's expectations are toward the future risk of a common stock, as well as his preferences for that risk. His warrant premium is derived not for the purpose of finding a rational price for the security, but for its usefulness in indicating option investors' risk expectations and preferences.

Samuelson.[7] Samuelson explicitly considers the

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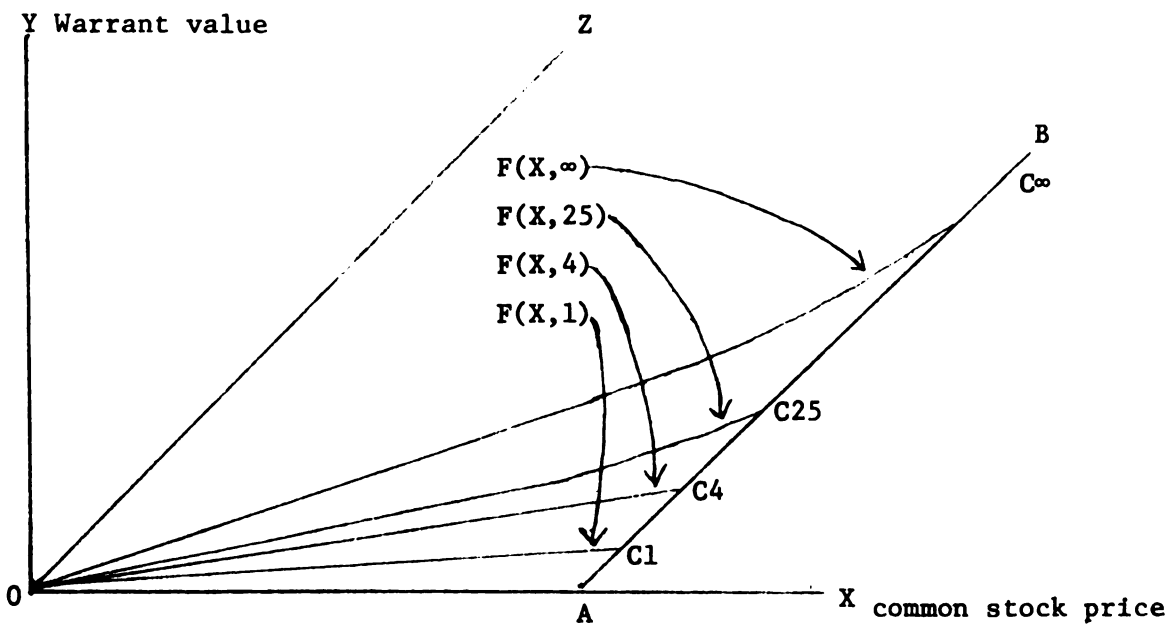
[7]P. A. Samuelson, "Rational Theory of Warrant Pricing", Industrial Management Review, Vol. 6, No. 2 (Spring, 1965), pp. 13-39.

additional worth of an American warrant due to the privilege of being able to convert it at any time on or before maturity. His final valuation formula is,

$$F(X) = \frac{(Y - 1)^{Y - 1}}{Y^{Y}} XY \quad , \quad Y = \frac{C}{C - 1}$$

With X the current price of the common stock, and C that point where it is no longer advantages to hold the warrant rather than the stock, and therefore, where it sells for its basic (non-premium) value. The formula applies only to the special case of a perpetual warrant whose associated stock is assumed to have a log-normal distribution of future prices.



For example, C_{25} is the point where a warrant with 25 years of remaining life will sell without a premium. He finds that at the maximum, when a common stock sells for four times its option price, the premium on its associated warrant will have disappeared. The rationale is that arbitrage will prevent a warrant from selling at a price lower than the locus of points represented by OAB. In addition, as a stock's price increases relative to its option price, the leverage inherent in the warrant diminishes, and by the time it sells for a multiple of four times its option, the leverage remaining in the warrant is too small to induce investors into paying a premium. The equation of the line between the origin and the C_T intercept is represented by Y, and can be empirically estimated by regressing the log warrant price against the log common price, Y being the regression coefficient.

Ayers.[8] Ayers attempts to illustrate how warrant prices reflect the risk preferences of investors. He assumes the average investor to be a risk averter.[9] His

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[8]H. F. Ayers, "Risk Aversion in the Warrant Markets," Industrial Management Review, Vol. 5, No. 1 (Fall, 1963), pp. 45-53.

[9]Risk aversion is defined as the unwillingness to accept additional risk without additional compensating return.

warrant valuation model is,

$$W = \sum_{S=E}^{\infty} D \left[S(t) - E(t) \right] \quad Pr\ (S(t)) \qquad (1)$$

where:

D = a discount factor less than one resulting from risk aversion.

W = current expected warrant price.

S(t) = stock price at time t from now.

E(t) = option price at time t from now.

Pr(S(t)) = the probability of S at t.

Ayers attempts to find the functional form of D. He assumes that future stock prices will have a log-normal distribution. The probability density function for the common stock is,

$$f\left(\text{Ln}\left[\frac{S(t)}{So}\right]\right) = \frac{1}{\sqrt{2\pi}\sigma t} \quad \exp -\left[\frac{\text{Ln}\left[\frac{S(t)}{So}\right] - \mu t^2}{2\sigma 2_t}\right] \qquad (2)$$

an unbounded random walk with trend μt.

His major departure from other studies is the incorporation of his assumption of risk averting behavior of investment company managers. He utilizes the Farrar objective function,[10] where,

$$U = R - A\sigma^2 \quad \text{with,} \qquad (3)$$

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[10] D. Farrar, "Investment Decision Under Uncertainty", Ford Foundation Prize Doctoral Dissertation Series, 1961.

U = expected utility.

R = expected return.

A = the coefficient of risk aversion.

σ^2 = expected variance.

Equation (3) is a postulated model of an investment manager's decision process. Ayers says that if S (stock price) in equation (2) represents a portfolio rather than a single stock, and if Farrar's model is accurate, investors expect an exponential rate of growth over time, and this rate increases with risk. Therefore, $D = e^{-rt}$ (4), with r = the expected return of the warrant. Combining equations (1), (2), and (4), he derives his final model,

$$W = \underset{t}{MAX} \left[e^{-rt}, We(t) \right] \qquad (5)$$

with We(t) equal to the expected value of the warrant at time (t).

The argument Ayers uses to explain the establishment of a warrant's market price is predicated on the assumption that investors do not regard any given warrant as an independent investment opportunity, but the warrant and its common stock together as a single composite opportunity. He assumes that investors will allocate their funds between the two securities in a manner that maximizes their utility. He claims that investors who are described

by the Farrar objective function dominate the market, so it is their preferences which determine a warrant's market price.

Chen.[11] As does Samuelson, Chen incorporates into his model the concept that an American warrant's value is increased due to its being convertible prior to maturity. All things equal, the longer the remaining life of a warrant, the greater its theoretical value.

In each period the investor must make the choice of either converting the warrant or holding it for one more period. A random walk in log-normal distribution is assumed for future stock prices, and a dynamic programming technique is used to develop his model.

Shelton and Giguere utilize the technique of regression analysis to develop their respective models. They attempt to formulate an equation, which describes the "typical" warrant-stock price relationship that has existed in the past. Each author assumes that if his model matches the warrant market prices that are observed, he has developed a theory of warrant valuation. At best their methodology leads to a theory on how investors' guide

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[11] A. H. Y. Chen, "A Model of Warrant Pricing in a Dynamic Market", The Journal of Finance, Vol. XXV, No. 5 (December, 1970), pp. 1041 - 1059.

their actions, not to how warrant's should be theoretically valued. Also, as Shelton admits, his model describes less than half of a warrant's premium fluctuation.

Samuelson, Ayers, Sprenkle, and Chen each requires knowledge of the future price action of common stocks. To determine the value of a warrant at time t, the probable price of its associated stock at t+n must be known, and each assumes that future stock prices have a positive trend and log-normal distribution.

CHAPTER II

DEVELOPMENT OF THE MODEL

Introduction

A warrant has value because it is substitutable for the stock, and changes in that value are dependent upon changes in the stock's future market price. The investment characteristics of the two securities are, however, quite different. Compared to its related stock, a warrant normally has a higher expected return in conjunction with a greater potential variance of return, or risk. The greater market price volatility inherent in a warrant is a function of the fixed option price required to purchase the associated stock. The pricing of a warrant on any given date is normally a function of marginal investors' expectations as to the future price performance of the related stock. A premium is established because of the desirability of obtaining the security's leverage characteristics, with the amount of the premium set so that the greater expected return of the warrant is sufficient to compensate marginal investors for the additional risk

assumed.

The warrant valuation model to be developed in this study differs substantially in concept, methodology, and conclusions from the papers outlined in Chapter One. This study does not attempt to derive a valuation for a given warrant based upon either past warrant-stock price relationships or future stock price predictions. The primary objective is to develop a model which generates a rational lower boundary valuation for any given warrant in relation to the current market price of its associated stock. The model is based upon the precept that a warrant and its associated stock should be considered as two alternative investment strategies, one consisting of the common stock alone, the other of the warrant in combination with a risk-free rate of interest. Considering these strategies as independent opportunities for investment, and utilizing the two-parameter model, it will be demonstrated that at any given time there is a unique warrant valuation that equates these strategies in terms of their respective standard deviations and expected rates of return. At this unique valuation the alternative strategies will occupy the same point in the two-parameter space, and can therefore be considered as identical assets. It will also be demonstrated that when a warrant's market price is below

that generated by the model, a higher return can be attained with the warrant-interest rate strategy than that possible with the associated stock, at no increase in risk. A rational investor will always seek to maximize return subject to a given level of risk, hence the warrant valuation derived will represent a lower boundary below which a market value cannot be rationally supported.

In developing the basic model the following assumptions are made:

(1) Rational investors.

(2) A two security world consisting of a common stock A, and its related warrant A'.

(3) The common stock pays no dividend.

(4) One hundred per cent of investable funds must be employed.

(5) A' does not initially sell at a premium.

The above assumptions are not critical. Their purpose is to simplify the analysis while developing the basic model, and each will be later removed, with the attending consequences discussed. It is additionally assumed that the warrant when purchased is held to maturity, and when a non-sustainable price discrepancy occurs the adjusting mechanism of arbitrage is a change in the warrant price, not in the stock price. The consequences of these latter

assumptions will also be discussed. The derived warrant value will be labeled Pw*.

Development of the Basic Model

The analysis utilizes the two-parameter model, with the standard deviation and the arithmetic rate of return specified as the parameters. Incorporated in the model is a risk-free rate of interest at which an investor's funds can be all or partially employed. When a range of potential values and their respective probabilities of occurrence is established for the common stock A, the expected return (μ) and the standard deviation (σ) of return can be calculated. Since it has been assumed that the warrant, if purchased, will be held to maturity, the time horizon for the estimates on the stock should coincide with the remaining length of its associated warrant's life. With μ and σ calculated the stock can be plotted in a two-parameter space (Figure 2-1). Assuming the warrant is currently priced at its base value, the option price can be subtracted from the values estimated for the stock, thereby generating the required information for calculating the μ and σ for the warrant.[1] When plotted, A' will lie above and to the right of A in the two-parameter space (Figure 2-1).

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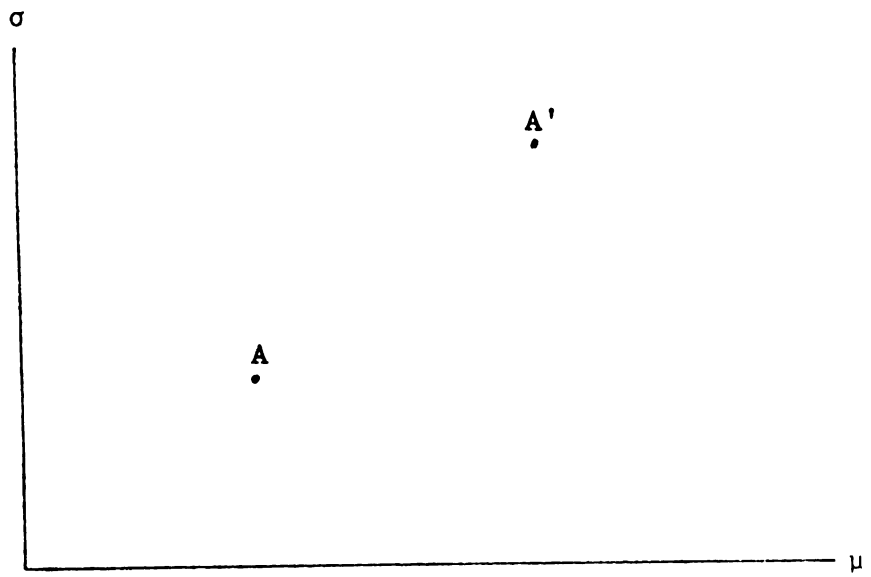
[1]Constrained by a lower limit of zero.

Figure 2-1

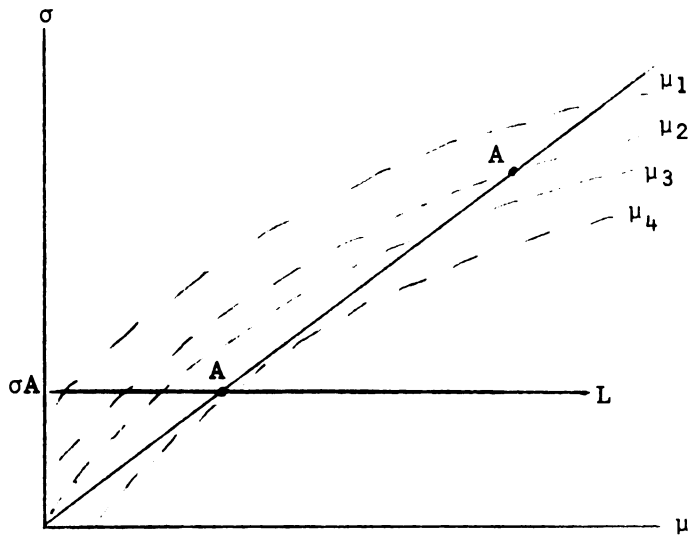
PLACEMENT OF THE STOCK AND WARRANT
IN THE TWO-PARAMETER SPACE



There is no indication which of the two securities a given investor will prefer. A' promises a greater expected return, but more risk must be assumed, as measured by σ, to be attained. The determination of which security an investor will favor requires the specification of his utility preference toward the tradeoff between risk and return. If his utility map is as depicted in Figure 2-2, the investor will chose to buy security A, since the combination of risk and return for that security places him on his highest attainable utility curve.

Figure 2-2

THE EFFECT OF UTILITY CURVES



An alternative approach is to specify the degree of risk willing to be assumed, then choosing that security which has the greatest μ for that risk level. If σA is specified, the security lying furthest to the right on the horizontal line σAL will be chosen. Since a two security world has been assumed, the common stock A would be chosen over A'.

A risk-free investment incorporated into the analysis modified the two-parameter model to that developed by Sharpe.[2] The rays extending out from the risk-free rate

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[2]W. F. Sharpe, Capital Asset Prices: A Theory of Market Equilibrium Under Conditions of Risk", Journal of Finance, Vol. XIX, No. 3 (September, 1964), pp. 425-442.

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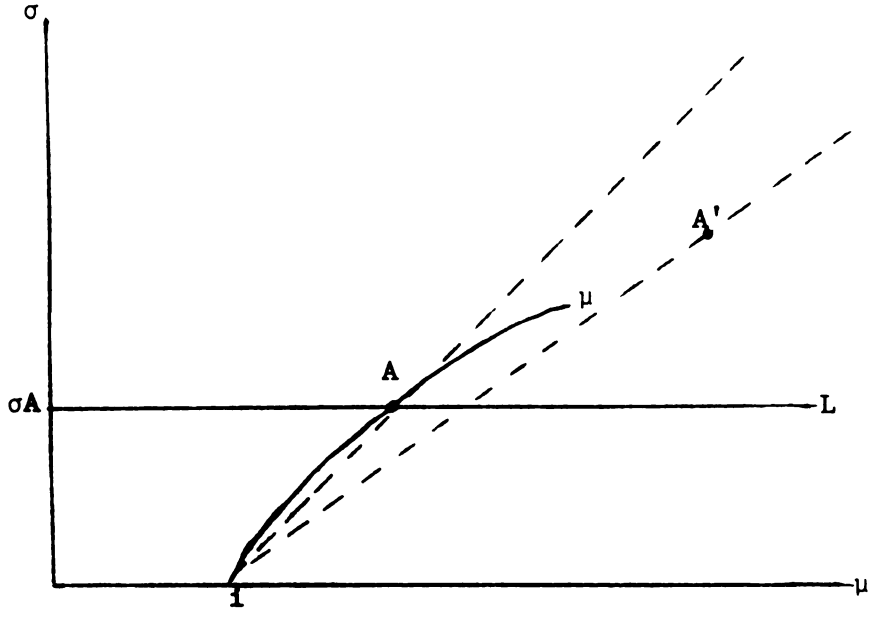
i are what Sharpe labeled capital market lines, and represent the division of an investor's funds between i and the respective securities illustrated (Figure 2-3). For example, if part of the investable funds are placed in i and part in A, the (μ, σ) combination the investor will receive falls somewhere on the ray between i and A. If funds are borrowed at rate i, for investment in A, the (μ, σ) combination will fall on the ray beyond point A. If σA is the specified degree of risk that an investor wishes to assume, he will place 100 per cent of his investable funds in the common stock, with none invested or borrowed at i.

Figure 2-3

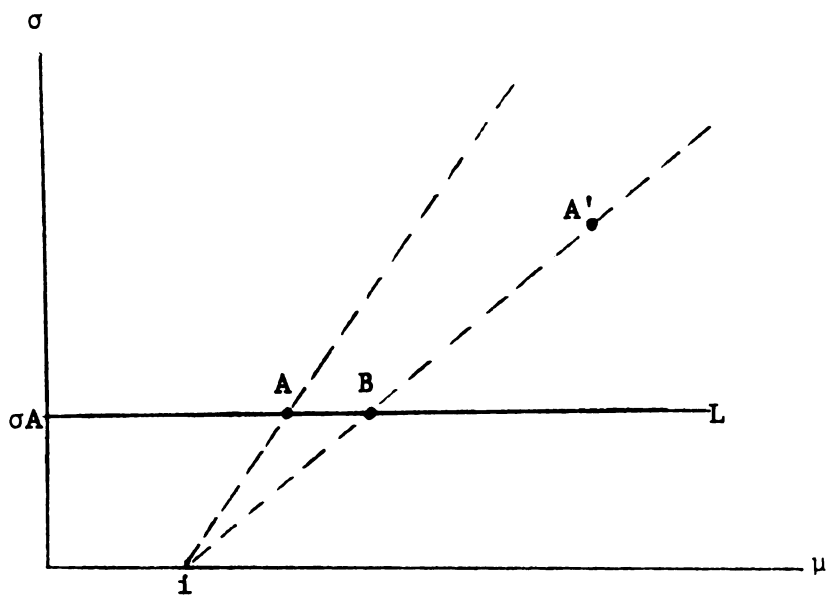
THE EFFECT OF THE RISK-FREE INTEREST



The same reasoning explains the ray i to A' and it is seen that a combination of i and A' exists that will place the investor at point B on the ray iA' (Figure 2-4).

Figure 2-4

THE EFFECT OF PROPORTIONING FUNDS

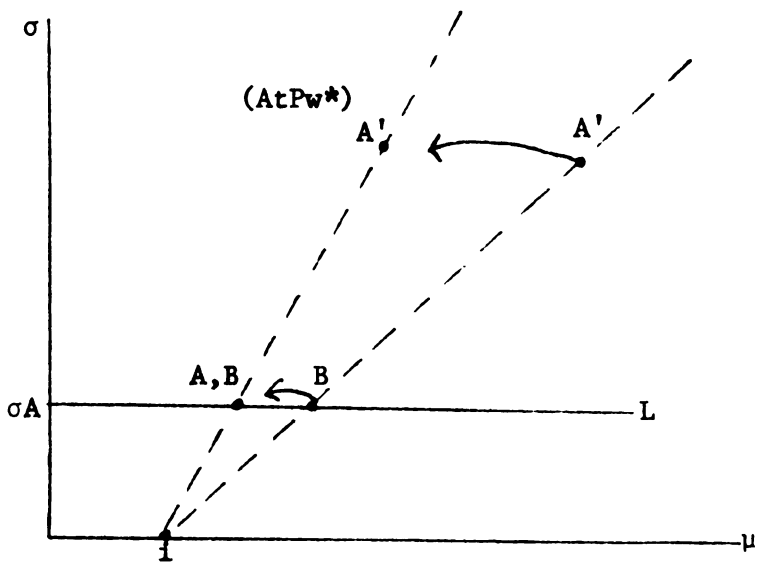


If the correct iA' investment combination is undertaken, the expected return the investor will achieve is greater than what can be achieved by investing in the common stock, and with no attending increase in risk. A rational investor will seek that strategy furthest to the right on the σAL line, thus increasing the demand for portfolio B relative to the demand for the common stock A. Since A' is part of B, the demand for the warrant will increase relative to the demand for its related common stock.

If Ps is held constant, Pw will increase due to the higher relative demand. The increase in the price of the warrant will move it to the left in the two-parameter space, with the process continuing till the price of the warrant is such that both it and the stock lie on the same ray from i (Figure 2-5). At this point a combination of A' and i can be purchased which equates that investment strategy with the common stock and will result in indifference between investing in the common stock or warrant strategy.

Figure 2-5

THE EFFECT OF A CHANGING WARRANT PRICE



To aid in illustrating the concepts involved, and in developing the Pw* model, a hypothetical distribution of common stock prices and other relevant information is

set forth below.[3] The method of probability analysis is used, by which an investor at time period t estimates the values a security may have in period t+n, together with their attendant probabilities. The expected rate of return (μ) and the standard deviation (σ) of the return can then be computed. If Po equals $20 the non-premium Pwt will be

TABLE 2-1

PROBABILITY TABLE FOR STOCK

Assume Pst = $30

Pst+n	Gain Per Share	RoR(%)	Prob.	Expected Gain
$45	$15	50.0	.10	1.50
40	10	33.3	.20	2.00
35	5	16.7	.40	2.00
30	0	0.0	.20	0
25	(5)	-16.7	.10	(.50)

Pst = The current price of the stock.
RoR = The rate of return.
Po = The warrant's option price.
Put = The current price of the warrant.

$\mu A = \Sigma(PiRoRi) = 16.7\%$

$\sigma A = \sqrt{\Sigma Pi(RoRi-\mu A)^2} = 18.26$

$10, assuming each warrant entitles the holder to purchase one share of stock. The probability distribution for the warrant is shown in Table 2-2.

[3]This is one of two major distribution classifications that affect the model. The effects of the other distribution will be discussed in a later section.

TABLE 2-2

PROBABILITY TABLE FOR WARRANT

Pwt+n	Gain Per Warrant	RoR(%)	Prob.	Expected Gain
$25	$15	150.0	.10	1.50
20	10	100.0	.20	2.00
15	5	50.0	.40	2.00
10	0	0.0	.20	0
5	(5)	-50.0	.10	(.50)

$\mu A' = 50\%$
$\sigma A' = 54.77$

The objective is to find a Pw* that equalizes the two alternative investment strategies to the extent that they occupy an identical point in the two-parameter space. That is, to find a warrant price such that the μ and σ of the warrant-i combination is identical to the μ and σ of the common stock. With the type of probability distributions hypothesized above, this objective can be satisfied by equating any two of the possible outcomes.

$$X(LP - Pw^*) + [F - (X \cdot Pw^*)] (i) = T \qquad (1)$$

$$X(Ep - Pw^*) + [F - (X \cdot Pw^*)] (i) = G \qquad (2)$$

X = the number of warrants to be purchased.
Lp = the lowest possible price the warrants can attain.
Ep = the expected price at time t+n per warrant.
F = funds to be invested
Pw* = the current price of the warrant, consisting of its base price plus premium, that will place the warrant on the same ray from i as the stock.
i = the risk-free rate.
T = possible loss in dollars with the stock.
G = expected gain in dollars with the stock.

Using the information in Tables 2-1 and 2-2, and assuming i = 5 per cent, and F = $3000:

$$X(5 - Pw^*) + [3000 - X \cdot Pw^*)] (.05) = -500 \quad (1)$$

$$X(15 - Pw^*) + [3000 - (X \cdot Pw^*)] (.05) = 500 \quad (2)$$

Solving,

$X = 100$

$Pw^* = \$10.95$

The premium = $Pw^* - Pw = \$10.95 - \$10.00 = \$0.95$

The distribution for the rate of return on the common stock remains unchanged (Table 2-3). However, the distribution of the rate of return for the warrant in Table 2-4 was based upon a beginning value of $10 (no premium), and the investment at that time did not involve the (A', i) combination. The recalculated distributions for each strategy are shown below.

TABLE 2-3

PROBABILITY TABLE FOR STOCK (STRATEGY A)

Pst+n	RoR_A	Prob.	$ profit A	on $3,000
$45	50.0	.10	$1,500	
40	33.3	.20	1,000	
35	16.7	.40	500	
30	0.0	.20	0	$\mu A = 16.7\%$
25	-16.7	.10	-500	$\sigma A = 18.26$

TABLE 2-4

PROBABILITY TABLE FOR WARRANT -i COMBINATION (STRATEGY B)

Pwt+n	$RoR_{A',i}$	Prob.	$ profit B	on $3,000
$25	50.0	.10	$1,500	
20	33.3	.20	1,000	
15	16.7	.40	500	
10	0.0	.20	0	$\mu A'i = 16.7\%$
5	-16.7	.10	-500	$\sigma A' = 18.26$

It is apparent that when the warrant sells for a premium of 95 cents, the two alternative investment strategies, strategy A consisting of 100 shares of the common stock, and strategy B, consisting of 100 warrants plus the balance of F in i, are identical and will occupy the same point in the two-parameter space. Each strategy has identical rates of return and standard deviations.

Below is a sample calculation used to determine the probability distribution for strategy B.

X = 100

Pw* = $10.95

Total investment in Warrants = $1095

Investment in i = F - $1095 = $1905

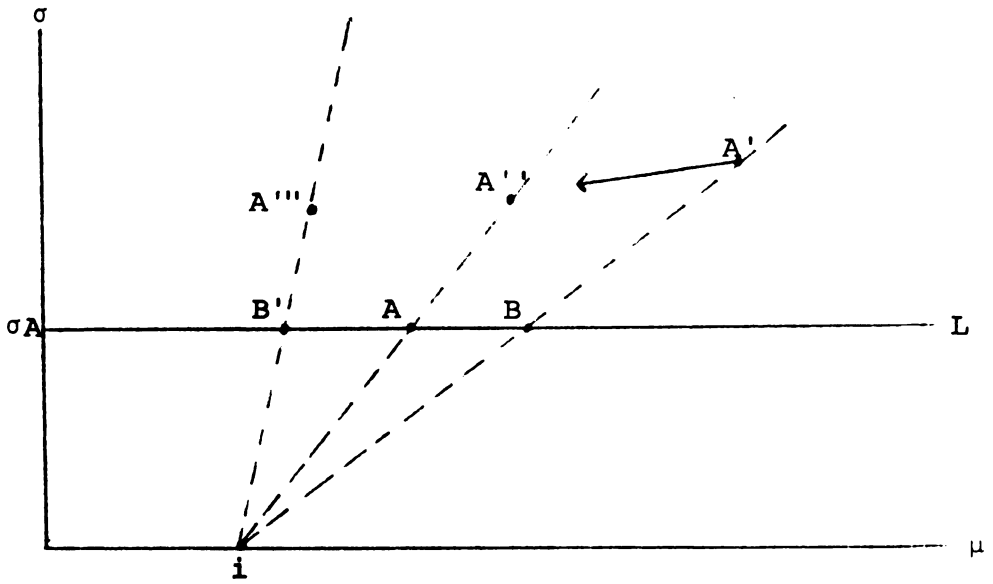
If Pw increases to $25, a profit of $25 - $10.95 = $14.05 on each warrant will be realized, or a total warrant profit of $1405. In addition, the profit from i will be $1905(.05)

= $95. Therefore, the total (A',i) profit will be $1500.

As the above demonstrates, when the warrant sells for a 95 cent premium, and is purchased in proper combination with i, the two alternative investment strategies facing an investor will be identical in terms of their respective expected profitability and risk. As a result, since the cost of strategy B is a function pf Pw*, and since a Pw* of $10.95 is the unique price that equates the two alternatives, a market price greater than $10.95 makes the common stock investment relatively more attractive, and a market price less than $10.95 makes the warrant investment, in combination with i, relatively more attractive.

Figure 2-6

COMPARATIVE RETURNS AT VARYING WARRANT PRICES



At any Pw* less than \$10.95 a combination of (A'i) can be purchased, such as portfolio B, that will have a greater μ than the common, at no increase in risk (Figure 2-6). At any Pw* greater than \$10.95, the (A',i) combination, such as B' will have less μ for a given risk than the common. One strategy will always be favored compared to the other except when the warrant sells for Pw*. The action of arbitragers will insure a shifting of funds in and out of the warrant strategy depending upon whether the market price of the warrant is over or under Pw*. This action will tend to maintain the warrant's price at Pw*.

Generalization of the Model

Previously, two simultaneous equations were used to solve for Pw*. Generalizing the equations,

$$XLP - XPw^* + iF - iXPw^* = T \qquad (1)$$

$$XEP - XPw^* + iF - IXPw^* = G \qquad (2)$$

Solving for X,

$$XLP - XEP = T - G$$

$$X(LP - EP) = T - G$$

$$X = \frac{T - G}{LP - EP} \qquad (3)$$

Using (1) to solve for Pw*,

$$Pw^*(-X - iX) = T - iF - XLP$$

$$Pw^* = \frac{T - iF - XLP}{-X(1 + i)} \qquad (4)$$

Substituting for X

$$Pw^* = \frac{T - iF - \frac{(T - G)LP}{LP - EP}}{\frac{- T + G}{LP - EP}\ (1 + i)} \qquad (5)$$

$$Pw^* = \frac{EP(- T + iF) + LP(G - iF)}{(G - T\ \ (1 + i)} \qquad (6)$$

T and G have been expressed in terms of the common stock, representing the dollar loss and gain. EP and LP are expressed in terms of the warrant. Converting EP and LP into common stock equivalent terms, and restating T and G,

$EPW = EPs - Po$

$LPw = LPs - Po$

$T = (Ps - Po)X$, $X = \frac{F}{Ps}$, therefore $T = F(-1 + \frac{LPs}{Ps})$

$G = (EPs - Ps)X$, therefore $G = F(1 - \frac{LP}{Ps})$

Substituting these terms into quation (6),

$$Pw^* = \frac{(-1 + \frac{LP}{Ps})\ (-EP + Po) + (\frac{EP}{Ps} - 1)(Lp - Po) + i(EP - LP)}{\frac{(EP - LP)}{Ps}\ (1 + i)} \qquad (7)$$

Equation (7) reduces to,

$$Pw^* = \frac{Ps + iPs - Po}{1 + i} \qquad (8)$$

Recalculating the numerical example using (8):

$$Pw^* = \frac{\$30 + .05(\$30) - \$20}{1 + .05} = \frac{\$11.50}{1.50} = \$10.95$$

Implications of the Basic Model

(1) Pw* represents the unique warrant price which equates the strategy of investing in the warrant-i combination, with that of investing in the common stock. The model is developed from the perspective of the common stock investor who first determines whether a given stock at a given market price is an acceptable vehicle for the employment of his funds. If it is, he should then compare the market price of the associated warrant (if one exists) with its calculated Pw*, and purchase the warrant-i combination if Pw* is greater than the market price.

(2) If a warrant can be purchased in the market for less than its Pw*, an investor is guaranteed of a rate of return on the warrant-i combination which will at all times be greater than the rate of return on the common stock, regardless of the future performance of that stock. If the stock increases in value over time, the warrant strategy will increase to a greater extent. If the stock declines in value, the warrant strategy will decline to a lessor extent. The greater the difference between the market price of the warrant and its Pw*, the greater the difference in the performance of the two strategies.

(3) Pw* is an equilibrium price. In a rational market a movement away from Pw* will set up an arbitrage

process that will tend to reestablish the price.

(4) It is evident that the model for Pw* (formula 8) is independent of the prospective probability distribution of rate of return for the common stock. Probability distributions were used to develop the model, but once the formula for Pw* was generalized, probability distributions became incidental for establishing a warrant's price.

The last point does not belie the importance of estimating the future price performance of common stocks. The efficient utilization of the model is dependent upon these estimates. The underlying position of this paper is that a warrant should be considered as an alternative to its related common stock, after that stock has been decided upon as an acceptable vehicle for investment. However, it should be reiterated that if a warrant can be purchased for less than its indicated Pw*, then regardless of the future price performance of the common stock, the (A',i) combination will have superior performance.

Model Modification Due to Dividend

It has been assumed that the common stock pays no dividend. In fact, the dividend which the stock is expected to pay is an important consideration when valuing its related warrant. Warrants do not convey the rights of

ownership, they merely represent options to buy such rights. As such, dividends never accrue to warrant holders. From the perspective of the warrant investor, each dividend paid by the corporation represents lost income relative to the common stock investor.

If the common in the example paid a dividend, in addition to having the hypothesized appreciation, the two strategies would no longer be equal at Pw*. The dollar profit for the warrant strategy would be less than that for the stock strategy by the total amount of the dividends paid over the time horizon, on the number of shares held.

Putting the dividend consideration into the generalized model,

$$Pw^* = \frac{Ps + iPs - Po - D}{1 + i} \qquad (9)$$

with,

D = the total dollar dividend to be paid per share, over the time horizon.

Model Modification Due to Multiperiod Analysis

The time horizon for the investment has been implicitly assumed a single unit period regardless of the remaining length of the warrant's life. In correcting for this simplification it must be recognized that a warrant can have n years of life remaining, and the i in equation

(9) must be adjusted to so reflect. Equation (9) becomes,

$$Pw^* = \frac{Ps + [(1+i)^n - 1]\, Ps - Po - D}{(1 + i)^n} \qquad (10)$$

reducing,

$$Pw^* = \frac{Ps\ (1 + i)^n - Po - D}{(1 + i)^n} \qquad (11)$$

and,

$$Pw^* = Ps - \frac{Po + D}{(1 + i)^n} \qquad (12)$$

Equation (12) represents the completed model. Po is the future sum that must be paid upon exercise of the warrant. Relative to the owner of the common, it represents a negative cash flow to be incurred in the future if the stock is acquired. D represents the total dollar dividend which the stock will pay over the period, and is also a future negative cash flow to the warrant holder, relative to the stockholder. Equation (12) stipulates that the current value of the warrant is equal to the current value of the common, minus the future negative cash flows the warrant holder will incur, discounted to present value.

The development of Pw* as an arbitrage supported valuation has been based upon the supposition that a warrant investment is combined with an investment in the risk-free rate of interest. The objective was to find the Pw* that would equate the warrant strategy with an investment

already deemed acceptable. If, through market imperfections, the market price of the warrant is less than Pw*, the warrant strategy will be purchased in lieu of its associated stock, thereby assuring greater relative market performance than the common can provide.

It must be recognized that many investors may have a risk-return preference that precludes them from considering the employment of any portion of their funds in the relatively low yielding i. These investors will consider the warrant alone (rather than in combination with i) as a possible investment vehicle. They may be willing to accept the higher risk inherent in a warrant, in order to attain the higher anticipated return. Since investors vary as to their individual risk-return preferences, it is possible for a market price higher than Pw* to be rationally supported. Given a favorable attitude toward the future price performance of a given stock, the lower the demand for an increased return per incremental increase in risk, the higher the price an investor will be willing to pay for the warrant.[4]

With the above consideration, Pw* can no longer be considered as <u>the</u> valuation for a given warrant at a given

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[4]Subject to an upper boundary, the price of the stock.

time. It is a lower boundary valuation. If the market price of the warrant falls below Pw* the demand for the warrant strategy must increase if the situation is recognized by rational investors. Demand must increase at this undervalued price because superior performance of the warrant strategy is guaranteed. Therefore, rational investors in the market for the common, will switch their purchase to the warrant strategy, forcing the warrant price back to Pw*.

In constructing the model it was also assumed that if the warrant is purchased it will be held to its expiration date. As a warrant approaches maturity its premium declines. Immediately before expiration a premium will not be paid for a warrant and it will trade at base value. When developing the model the expected values for the warrant strategy incorporated this assumption. The warrant value for t+n did not include an allowance for a premium. The assumption is valid if the warrant is held to expiration. However, allowance must be made for the possibility of selling the warrant prior to its maturity date. If this occurred the subsequent purchaser of the security will pay a premium, making the rate of return on the warrant strategy greater than under the base value assumption. Hence a purchaser of the warrant strategy, at any given time, will be willing to pay more than Pw* for the warrant

if he contemplates its sale before expiration. How much more than Pw* the warrant is worth to a current investor is a function of the remaining lifespan of the warrant at the time of its subsequent sale, as well as the number of subsequent sales the warrant will be subject to before expiring. Therefore, based upon the above, a market price greater than Pw* can be rationally supported. However Pw* still represents a lower boundary valuation for the security.

In the numerical illustration presented earlier, hypothetical probability distributions for the future price action of a common stock and its related warrant were set forth.[5] This was one of two types of distributions that could have been hypothesized. The delineation of the distribution classifications is a function of the number of zero warrant values that are assumed possible. When one or less zero warrant values are forecast, use of the model generates a Pw* which equates all characteristics of the two alternative strategies. As indicated in the dollar profit columns, Pw* equates more than the μ and σ of the strategies, it results in an equality of all possible outcomes. It makes the strategies identical not only in their occupation of a point in the two-parameter space, but

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[5]Tables 2-1 and 2-2.

identical in every respect.

When the range of the probability distribution is such that more than one zero warrant value is possible, the meaning of Pw* is modified. To illustrate, additional values, below that which hypothesized, are added to the example.[6]

TABLE 2-5

STRATEGY A, THE STOCK

Pst+n	$ Profit
$45	$1,500
40	1,000
35	500
30	0
25	(500)
20	(1,000)
15	(1,500)
10	(2,000)
5	(2,500)

TABLE 2-6

STRATEGY B, THE WARRANT WITH i

Pwt+n	$ Profit
$25	$1,500
20	1,000
15	500
10	0
5	(500)
0	(1,000)
0	(1,000)
0	(1,000)
0	(1,000)

The upper part of the profit distribution for the warrant strategy is identical to that of the stock strategy, but the part below the first possible zero value for the warrant is muted compared to the possible performance of the stock. A warrant's value can never fall below zero,

[6]Probabilities are omitted as they are not important to the illustration of the concept involved.

and this will occur when the stock still has a positive value and the possibility of further decline. As a result, the warrant strategy has a built in limited loss relative to the stock strategy.

To the extent that the limited loss aspect of the warrant strategy gives additional value to an investor, a market price higher than Pw* can be rationally supported. The extent of the higher price will be a function of the relative market power of the investors who, at any given time, are forecasting multiple zero distributions, and the value they place on the limited loss aspect of the strategy. The Pw* remains the price that investors who do not forecast such a distribution will be willing to pay, and again represents a lower boundary valuation.

Summary of Valuation Significance

In the basic analysis the hypothetical distribution of future stock prices was assumed to be of the type that led to a maximum of one possible zero value for the related warrant. Under these circumstances the valuation model generates a Pw* which results in a warrant-i strategy identical in every respect to an investment in the associated stock. They are, in fact, identical assets, and as such, a cost differential in their purchase cannot exist

in a rational market. However, other factors may be present that lead to a modification of the interpretation of Pw* from the only valuation to a lower boundary valuation. Individual risk-return preferences, the opportunity of selling a warrant prior to its expiration, and the possibility of muted distributions, all combine to rationally support a higher warrant price than its Pw*. How much higher is dependent on the force of these factors at any given time, but no factor can rationally support a warrant market price less than Pw*.

CHAPTER III

RESEARCH DESIGN AND ANALYSIS OF RESULTS

As developed in Chapter II, the Pw* model generates a warrant price which is a lower boundary valuation for a given warrant at a given time. Factors not explicitly incorporated in the model may be dominant, which justify a rational investor paying a higher price for a warrant than its indicated Pw*. But if the assumptions upon which the model is based are accepted, a market price lower than Pw* always guarantees better investment performance with the warrant strategy than that possible with the warrant's associated stock.

Market Test Methodology

A market test is developed to determine the practical significance of the valuation model. The objective of the test is twofold: to determine the frequency of undervalued warrant's; and to determine the factors, if any, that provide an environment that leads to a warrant being undervalued.

The test period is from January 1, 1961 to June 1,

1971, with 35 randomly generated weeks within that period. The specific dates selected are the Friday of each randomly generated week. Each warrant that was outstanding and traded on the American or New York Stock Exchange, on each selected date, is included in the sample.[1] The information required for utilization of the model consists of the market price of the common stock, the market price of the stock's associated warrant, the option price of the warrant, the dividend of the common stock, the length of the warrant's remaining life, and the risk-free rate of interest.

The market prices of each stock and warrant are taken from the closing price of each security on the Friday of each week selected. The option price of each warrant is the prevailing option price on the respective dates. If the option price on the date examined to the maturity of the warrant is a constant, it is assumed that the warrant will be held to maturity. If the contract terms stipulate an increase in the option price before the warrant's expiration, the prevailing option price is used with the assumption that the warrant will be held to

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[1]Warrants whose terms associated value with a multiplicity of securities, and warrants with perpetual lives were omitted from the sample.

the date the option price increases, and in addition, the final option price stipulated is used with the assumption that the warrant will be held to maturity. The total dollar dividend the holder of each stock will receive during the holding period is computed by assuming that the prevailing dividend will remain constant during the time span of the warrant's life. The rate selected as a risk-free rate of interest is the maximum allowable interest rate payable on savings deposits of 12 months or more by commercial banks which are members of the Federal Reserve System.

Test Results

During the period covered by the study, there were a total of 816 price observations for warrants, and an equal number of associated stock price observations. During the test period there were 38 instances of undervaluation, as defined by the model, or 4.7 per cent of all observations. A warrant whose option price changed over time was counted as undervalued if its calculated Pw* was greater than its market price under any of its multiple of option prices. If Pw* was greater than the current market price under more than one option price, it was considered as only one undervaluation. For example, on June 23, 1967 Realty Equities warrant had a current option price of $5.71

which in time increased to $6.34. It was determined as undervalued regardless of which option price was used in the model, but was counted as one undervaluation. It is felt that counting this and similar securities as multiple undervaluations would overstate the number of opportunities for investing in warrants of this nature.

Because the model cannot be used unless the market price of the stock is greater than the warrant's option price, this class of price observations is segregated from all others. Of the 816 observations, 336 (41.2 per cent) have a $Ps \geq Po$. The 38 undervaluations constitute 11.3 per cent of all warrants whose associated stock price is greater than the option price.

A total of 85 different warrants comprise the 816 observations. Of these, 48 separate warrants comprise the 336 observations when the price of the associated stock is greater than the option price, and 12 different warrants accounted for the 38 undervaluations. Hence, 14.1 per cent (12/85) of all warrants are undervalued at least once during the period, and 25 per cent (12/48) of all warrants whose associated stock price is greater than Po are undervalued at least once. The degree of undervaluation ranged from one cent to several dollars, (see Appendix A).

The presentation in Table 3-1 helps to illustrate

the meaning and consequences of undervaluation. On January 31, 1969 the closing price of AMK common stock was $45.13. The closing price of the associated warrant was $18.00. The valuation model generated a Pw* of $19.04, hence, by definition, the warrant was undervalued. In accordance with the concept of undervaluation used here, a combination of the warrant, risk-free rate of interest should have been purchased in lieu of the common stock. It can be reasonably assumed that the purchaser of AMK stock was optimistic about the future price performance of that security, but aware of the risk inherent with any investment. In any event, a greater return on investment (or a smaller loss) could have been guaranteed, regardless of any future price movements, had the stock been foregone and the warrant-i combination purchased instead.

Column A illustrates the dollar return on each strategy if the price of the stock is unchanged from the current price on the warrant's date of expiration. Column B shows the respective returns if the price of the stock is higher on the date of expiration, column C if the stock price is lower. Column D is a special case showing the effect if the stock price falls below the option price. In the first two cases the profit on the warrant-i strategy is greater than what can be realized on the stock. In the

TABLE 3-1

THE CONSEQUENCES OF UNDERVALUATION

	Stock Price	A Ps = $45.13	B Ps = $60.00	C Ps = $40.00	D Ps = $25.00
	COMMON STOCK				
(1)	Cost of stock (100 shares)	$4513	$4513	$4513	$4513
(2)	Amount received at sale	$4513	$6000	$4000	$2500
(3)	Profit (loss) (2) - (1)	$0	$1487	($513)	($2013)
(4)	Total dividends	$270	$270	$270	$270
(5)	Net profit (loss) (3) + (4)	$270	$1757	($243)	($1743)
	WARRANT				
(6)	Cost of warrant (100 warrants)	$1800	$1800	$1800	$1800
(7)	Amount in i ($4513 - $1800)	$2713	$2713	$2713	$2713
(8)	Price at expiration (Ps - Po)	$10.63	$25.50	$5.50	$0
(9)	Amount received at sale	$1063	$2550	$550	$0
(10)	Profit (loss) on warrants (9) - (6)	($737)	$750	($1250)	($1800)
(11)	Total interest received (7) x .426	$1155.74	$1155.74	$1155.74	$1155.74
(12)	Net profit (loss) (10) + (11)	$418.74	$1905.74	($94.26)	($644.26)

last two cases the loss on the warrant-i strategy is less than what may be realized with the stock. Important to note is the loss on the warrant strategy cannot exceed $644.26, whereas the stock purchaser can lose considerably more than $1743 with further stock price depreciation (column D).

The Risk-Free Rate of Interest

When analyzing the undervaluation of warrant's as defined by,

$$Pw^* = Ps - \frac{Po + D}{(1 + i)^n}$$

the key variable is the risk-free rate of interest. If the common pays no dividend, and the warrant has no option price, Pw* would equal Ps. The elimination of Po and D equates the two securities, making them identical and interchangeable at zero cost.[2] In fact, a Po must exist, representing a future sum to be paid if the warrant is transferred for the stock. Because of Po's existence, the warrant will always have a lower market price than the common.[3] To purchase the stock, 100 per cent of the cost for X number of shares must be committed to that investment. By

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[2]Brokerage fees and taxes not considered.

[3]Assuming a warrant to common stock transfer ratio of 1:1.

purchasing the warrant an investor can obtain an option for the same number of shares, plus, free a portion of his funds, which can then be placed in an additional investment (i). The greater the available yield on the additional investment the less desirable it is to have those funds committed to the stock. Ps and Po are known values at any given time, and if dividends are temporarily ignored, the only factor that can reduce the warrant value is a lower i. The lower the i used, the higher the present value of Po, hence, the smaller the difference between Ps and Po, and the lower the warrant value. Since Pw* is derived by using a currently available i, a market price lower than Pw* implies the use of an i lower than actually available. The time value of money is not being fully recognized because investors are implicitly assuming a lower rate of return on that portion of the funds invested in the risk-free rate of interest. The interest rate performs two functions simultaneously. The warrant enables the investor to participate in the price appreciation of the common, while i gives value to the funds that would otherwise be committed to that common. In addition, since i has zero correlation with the warrant, the portfolio risk of the warrant-i strategy is reduced to the same level of risk as the common.

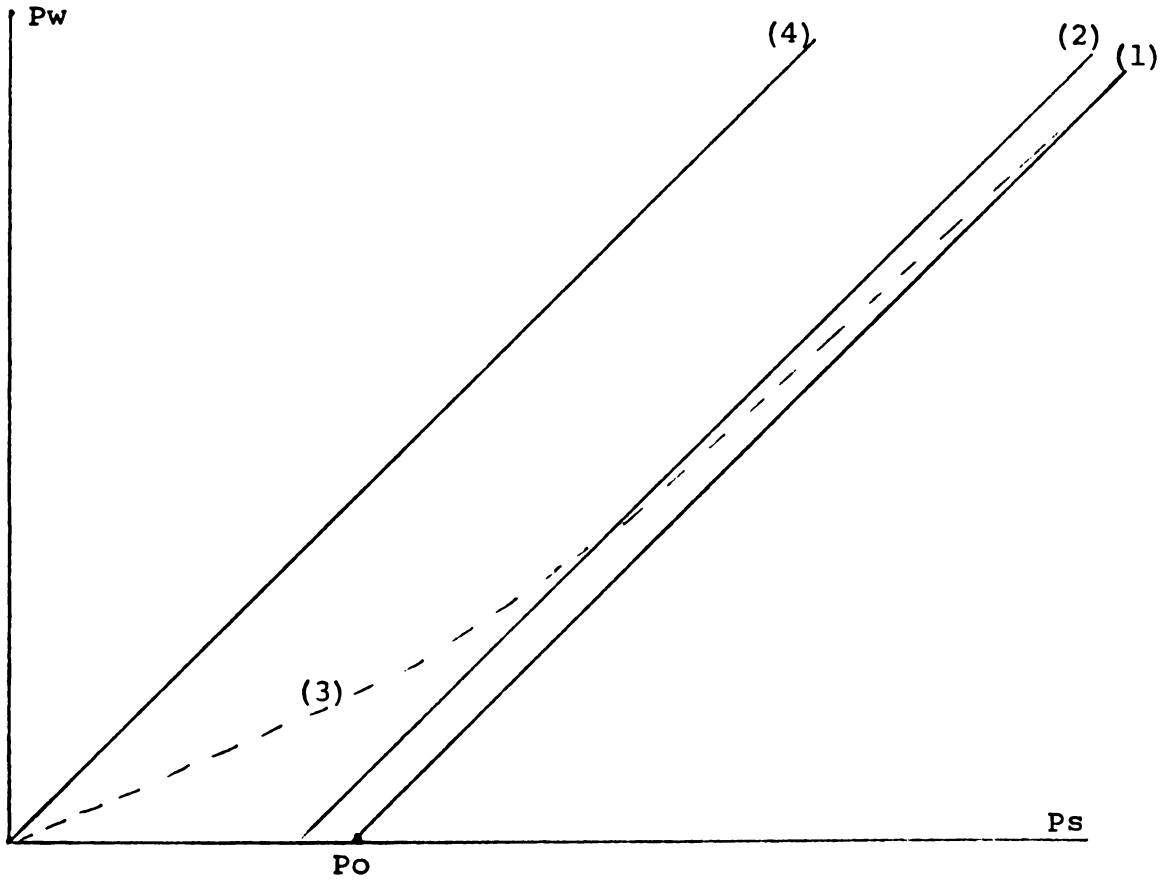
In sum, the warrant investor can purchase an option for X number of shares for a lower initial cost than a direct purchase, the difference reflecting the sum Po that must be paid upon conversion of the warrant. The lower cost temporarily frees funds that would be tied up by a direct purchase. When the market price of a warrant is below its Pw*, the productivity of those freed funds is greater with the warrant strategy then they would be with the direct purchase of stock. The introduction of dividends recognizes that the productivity of the freed funds must be great enough to compensate for the income lost by not owning the stock directly.

Relationship of Pw* to Other Valuation Concepts

Several warrant valuation concepts are depicted in Figure 3-1. Line (1) represents the traditional lower boundary valuation, and is derived by subtracting the current option price from the current stock price. The boundary is maintained by arbitrage. If a warrant's price placed it below this line, an immediate profit could be made by simultaneously purchasing the warrant and converting it into its associated stock. Arbitragers would enter the market, thereby driving up the warrant's price. Line (4) represents the upper boundary, and consists of the

Figure 3-1

RELATIONSHIP OF VALUATION THEORIES



common stock's price. The boundary is maintained since rational investors will not pay more for an option to buy a security than for the security itself. Together, the upper and lower boundary define the trading range of a warrant. Within that range a warrant will trade at a premium. The closer a warrant's price to the upper boundary the greater its premium, the closer to the lower

boundary, the smaller its premium. Line (3) represents a warrant's premium curve as developed in regression studies by Shelton, Samuelson, and others. Though the slope of the line and its intersection with the lower boundary differs among the studies, its general slope is as depicted, with the intersection at a point where the price of the stock is within three to five times greater than its related warrant's option price. The inherent reasoning is that warrant's will trade at their greatest premium when the stock's price is near the option price, since this is when the potential leverage effects of the warrant are greatest. As the stock's price increases the leverage effects of the warrant decline, and the security loses its speculative appeal. By the time a stock trades at approximately four times its option price, the potentially beneficial leverage will all but dissipate, hence, the warrant will no longer command a premium.

Line (2) depicts the lower boundary valuation of a given warrant at a given time, as generated by the Pw* model. This line will shift upward and to the left as the risk-free rate of interest increases. A higher i gives a greater time value to the funds which have been defined as freed by not investing directly in the common stock. The Pw* line shifts downward and to the right as the dividend

on the common increases.

Agreement on two warrant value characteristics is dominant in the literature. One, the premium on a warrant will tend to decline as the contract approaches its expiration date, and this decline accelerates during the last two years of a warrant's life. On this point the Pw* model is in partial agreement. As $n \to 0$ the value of D and Po increases, thereby decreasing Pw*. The pattern of decline is different however, as the loss in premium will not accelerate as the warrant approaches maturity. Two, the premium will decline as the price of the stock increases relative to the option price. Here the Pw* model disagrees. With all else constant, as the stock price increases the premium will decline as a percentage of the warrant's value, but the dollor amount of the premium remains constant. This is represented in Figure 3-1 by lines (1) and (2) being parallel.

In accordance with the contrasting opinion between the conclusions of other studies and the Pw* model, the probability of finding undervalued warrants should be greatest when the common is trading at a high Ps/Po ratio, or when the warrant has two or less years to maturity. To test the first proposition, the 336 stock price observations that were above their respective Po's, were divided

into five Ps/Po classifications (Table 3-2). Of the stock

TABLE 3-2

STOCKS ABOVE THEIR OPTION PRICE

Ps/Po	1.0-1.99	2.0-2.99	3.0-3.99	4.0-4.99	5.0
Number of warrants	254	55	18	7	2
Number of undervalued	16	16	3	1	2
Undervalued/warrants	6.3%	29.1%	16.7%	14.3%	100%

price observations that were close to the respective Po's, only 6.3 per cent of the associated warrant's were undervalued. Each of the higher Ps/Po ratios shows a substantial increase in the percentage of undervalued warrant prices, with 26.8 per cent of all observations with a Ps/Po $\geq$ 2 being undervalued. Stocks with a Ps/Po $\geq$ 2 comprise 25 per cent of the observations, but account for 58 per cent of all undervalued warrants.

In testing the second proposition, there are 27 warrant price observations where the respective securities had less than two years to maturity, and 14.8 per cent were undervalued. Of the 14 observations when maturity was less than one year, 21.4 per cent were undervalued. The above compares with an average for the tested period of 11.3 per cent. Of the 38 undervaluations, 68.4 per cent

occurred when the stock had a Ps/Po ratio $\geq$ 2, or when the warrant had less than two years to maturity.

General Market Test

A series of tests are conducted to determine if general market movements, as measured by the Dow Jones Industrial Average, influence the probability of occurrence of undervalued warrants. The period studied includes five bull markets of 88 months total duration, and four bear markets of 37 months total duration. The general market tendency was therefore bullish 70.4 per cent of the time, and bearish 29.6 per cent. Of the 35 dates on which price observations are taken, 25 or 71.4 per cent occur during rising markets, 10 or 28.6 per cent during market declines. Of the 816 stock price observations, 71.9 per cent are during rising markets, 28.1 per cent during market declines.

During the period, 336 stock prices, or 41.2 per cent of the 816 observed, have Ps/Po ratios $\geq$ 1. This is unaffected by the general state of the market. During market declines 42.8 per cent of the observations have a Ps/Po $\geq$ 1, 40.5 per cent during bull markets. There are 8 and 30 warrant undervaluations during bear and bull markets respectively. Thus: (1) 21.1 per cent of the undervalued warrants occurred during market declines, while these

declines constituted 29.2 per cent of the tested period, and 28.6 per cent of the tested dates; and (2) 78.9 per cent occurred during bull markets, and these markets comprised 70.4 per cent of the tested period, and 71.4 per cent of the tested dates. In addition, 68 per cent of the dates tested during bull markets have one or more undervalued warrants, 50 per cent of the dates tested during bear markets have one or more undervalued. The percentage of undervalued warrants to total observations are 3.5 per cent and 5.1 per cent during bear and bull markets respectively, compared to an average of 4.7 per cent for the entire period. The percentage of undervalued warrants to those observations whose stock price has a $Ps/Po \geq 1$ is 8.2 per cent and 12.6 per cent during bear and bull markets respectively, compared to a period average of 11.3 per cent.

Each of the above tests indicates a definite but weak relationship between general market movements and undervalued warrants. In each test, undervaluations are less in bear, more in bull markets, than would be expected if general market movements and undervaluations were independent variables.

The Professional Investor Hypothesis

In examining the price action of common stocks before and after their associated warrants were undervalued, a tendency was observed for these stocks to have a substantial price decline after undervaluation. Since the model specifies warrant undervaluation relative to the current price of the common stock, and since the relationship between the model and other valuation theories implies that warrants will tend toward undervaluation when the Ps/Po ratio is relatively high, a substantial rise in stock price before undervaluation is a logical occurrence. Unexpected is that nine of the twelve stocks whose associated warrants were undervalued had substantial price depreciation within one year after the last date of undervaluation, and a tenth stock within two years. The concept of professional investors[4] as separate from all other investors has been discussed in stock market literature. It is the underlying concept in the odd-lot theory, and Cootner behaviorally explained his hypothesis of a bounded random walk stock price pattern on the assumption that professionals have a different pattern of investment than non-professionals.[5]

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[4]Professional investors are defined as those whose normal occupation involves the analysis of, and investment in the securities markets.

[5]Paul H. Cootner, "Random Vs. Systematic Changes", Industrial Management Review, Vol. 3, No. 2 (Spring, 1962), pp. 24-45.

Professionalism may also have applicability in warrant undervaluation. Most warrants are issued by firms whose stocks are classified as speculative. As these stocks become in favor with investors, their prices enjoy considerable appreciation. As the price advances toward the end of its speculative rise professionals may recognize overvaluation and cease to be in the market for these securities. If it is assumed that professionals constitute the bulk of demand for warrants, and if it is assumed that this class of investors recognizes the overvaluation of a warrant's related stock, it would be reasonable for them to forecast an impending price decline in said stock, and sell off the related warrant. If their concerted actions were strong enough the depressed warrant price could become undervalued. It must be emphasized that the above assumptions imply that the warrant is undervalued relative to an overvaluation of its associated stock. If the hypothesis has merit it would help explain why the funds freed by purchasing an option rather than the stock directly, are being undervalued. There is no value in purchasing an option to buy shares whose price decline is considered imminent.

Market Test Conclusions

The time period chosen for the market test contained several major price movements of both the bullish and bearish variety. The 35 randomly selected weeks, and the number of price observations on each selected date gave equal relative importance to the influence (measured in time) that each type of market movement had during the tested period. It was found that on any given date an average of 4.7 per cent of the outstanding warrants, and an average of 11.3 per cent of the warrants whose related stock had a Ps/Po ratio ≥ 1 were undervalued. Although some warrants tended to be undervalued on a multiple of dates, it was found that 14.1 per cent of all the outstanding warrants included in the study, and 25 per cent of those whose related stocks had a Ps/Po ≥ 1 were undervalued at least once during the period. It was found that 63 per cent of the tested dates contained at least one undervaluation. It is the conclusion of this study that the frequency of undervaluation is substantial, and merits the consideration of an investor contemplating the purchase of a common stock which has an associated warrant.

The general direction of market movements was found to have a direct but not overpowering effect on the probability of undervaluation occurrence. Market declines

constituted 29.6 per cent of the tested period, 28.6 per cent of the tested dates, and 28.1 per cent of the price observations. If undervaluations were independent of market movements, approximately 29 per cent of the undervaluations would have occurred during market declines. In fact, only 21.1 per cent of such observations occurred during bear markets. The corresponding figures for bull markets show a higher percentage of undervaluations than independence would indicate.

Many past studies in this area have attempted to define warrant valuation in terms of previous patterns of premium determination, as developed through regression analysis. The accuracy of a well constructed test of this nature, or of a mathematical model based upon the results of such a study is not in dispute. The disagreement is in what constitutes undervaluation. By graphically comparing valuation patterns (Figure 3-1) it would appear the probability of undervaluation is greatest when the price of the common is high relative to the option price, since undervaluation will occur when the warrant price is below line (2). This was borne out by the market test since 58 per cent of the undervaluations were associated with the 25 per cent of the stocks with a $Ps/Po > 2$. In addition, 26.8 per cent of all such stocks had an associated undervalued warrant, compared to 4.7 per cent for all observations.

CHAPTER IV

SUMMARY, IMPLICATIONS AND QUALIFICATIONS

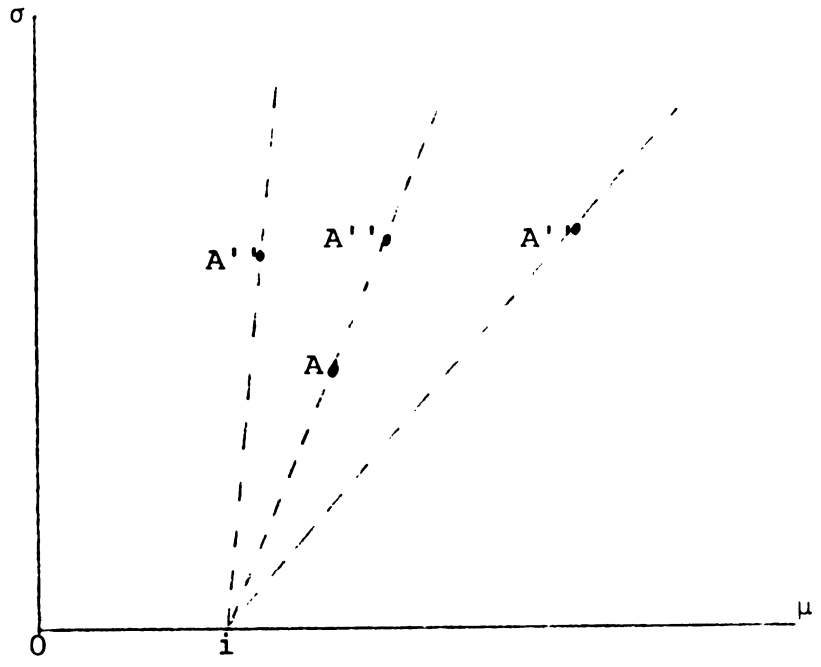
Model Development

Warrants are considered as alternative investments to their related common stocks. After a given stock has been analyzed and judged an acceptable vehicle for investment, its associated warrant (if one exists) would then be analyzed with the objective of determining if the market price of said warrant were undervalued. If found undervalued, the warrant, in combination with a risk-free investment, would guarantee a higher return on the funds that would have been committed with a direct purchase of the stock. Hence, warrant undervaluation affords the opportunity of improving upon a risk-return relationship which has already been found acceptable.

A warrant is defined as undervalued if its current market price places it on a ray from i to the right of a ray from i to its related stock (Figure 4-1). As illustrated, an investor can proportion funds between i and A' (the warrant) and achieve a higher μ with no increase in

Figure 4-1

THE RISK-RETURN RELATIONSHIP WITH VARYING WARRANT PRICES



risk relative to the associated stock.

In the valuation model,

$$Pw^* = Ps - \frac{Po + D}{(1 + i)^n}$$

Pw* represents the market price of a warrant that will place it on the same ray from i as its associated stock (point A''). At Pw* the proportioning of funds between A' and i will place the alternative investment strategies on an identical point in the two-parameter space. At this point the warrant strategy offers no relative advantage compared to a direct investment in the common stock. A

market price higher than Pw* moves the warrant to a ray left of iA (A'''). In sum, Pw* represents the unique value which equates an investment in the warrant-i strategy to an investment in the warrant's related stock. A market price less than Pw* allows the opportunity to proportioning funds so as to achieve a higher μ with no increase in risk.

Lower Boundary Valuation

A market price higher than Pw* can be rationally justified on several grounds. If an investor's risk-return preference is such that it precludes the allocation of funds in i, the warrant will not be looked upon as an alternative to its related stock, but as an independent opportunity. An investor will pay more than Pw* so long as the projected incremental return, relative to the stock, is great enough to compensate for the increased risk inherent in a warrant. In addition, the value of a warrant will decline to zero while its related stock still has a positive value. Since the return on the funds invested in i is independent of the stock price, the warrant-i strategy has limited loss potential relative to a direct investment in the stock. To the extent an investor places a premium on this limited loss, a higher price than Pw* can be justified. Lastly, the Pw* model implies the holding of a warrant to its date

of maturity, hence at its sale or conversion the premium will have dissipated. If a warrant is sold before maturity the subsequent buyer will pay a premium, generating additional cash inflow to the warrant holder not explicitly incorporated in the model. To the extent an investor foresees this possibility, he may be willing to pay more than Pw* for the security.

Though the above assumptions may justify a higher price than Pw*, a lower price will always allow the stock investor to improve his potential return, at no increase in risk, by purchasing the warrant-i combination in lieu of the common. Therefore, Pw* is a lower boundary valuation for the warrant, below which, no market price is rationally justifiable.

Market Test Methodology

A market test is developed to determine the practical significance of the valuation model. There are 35 randomly generated weeks between the dates of January 1, 1961, and June 1, 1971. Stock and warrant price observations are selected from the closing prices of the respective securities on the Friday of each week. The risk-free rate used is the prevailing maximum interest rate on savings accounts of commercial banks. The investment holding period

is the remaining length of a warrant's life. For warrant's whose option price changes before maturity, the option price prevailing on the date examined is used, and the warrant is assumed held until the date the terms change. In addition, the last specified option price is used with the assumption that the warrant is held to expiration. A warrant is considered undervalued if its market price is below Pw* with either option price, but counted only once if undervalued under each option price.

Test Results

It was found that 4.7 per cent of all stock price observations have associated undervalued warrants, and 11.3 per cent of all stock prices above their respective option prices have associated undervaluations. The probability of undervaluations increases when the remaining maturity of the warrant is less than two years, and particularly with less than one year to maturity. There is also an increase in the percentage of undervaluations when the associated stocks have a high Ps/Po ratio. Over two-thirds of all undervaluations occurred under the above maturity and Ps/Po conditions, although such conditions accounted for less than 13 per cent of total observations.

The general state of the market (bull or bear) has

an effect on the frequency of undervaluations, with less occurring in bear, more in bull, than would be expected if the events were independent. Of all tested dates, 63 per cent have at least one undervaluation. The conclusion reached is that warrant undervaluations occur with sufficient frequency to merit investor attention.

Qualifications

Undervalued warrants as defined and determined do not imply attractive investments. A warrant is only undervalued relative to the current market price of its associated stock. If such stock is overvalued and/or a price decline is imminent, a loss will likely be realized whether the stock is purchased directly or indirectly through the warrant strategy. If an investor is not optimistic concerning the future price action of the stock, the warrant strategy should not be undertaken even if the warrant is classified as undervalued. The valuation model is not designed for use independent of an investment decision on common stock. A decision on a given stock should first be developed, and if found acceptable, the model should then be applied to its associated warrant, with that warrant being purchased in combination with i, if found undervalued.

The model can also be used to scan all available

warrants. If any are classified as undervalued, commensurate analysis of the associated stocks can be undertaken to determine if one or more are acceptable. In this manner, analysis can be confined to situations presenting undervalued opportunities, but not all such opportunities present profitable investment possibilities.

Components of the Model

There are five variables required for utilization of the model. The Ps is the current market price of the stock, and requires no estimation. The Po and n are part of the warrant's contractual terms and also known with certainty at any given time.

The rate used for i is subject to estimation and individual determination. The rate on savings deposits in commercial banks is used for this paper. It is felt to most closely approximate the abstract concept of risk-free interest. Such deposits are insured against loss, and the rate is relatively stable. Dependent upon the time horizon of the investment, and the concept of what constitutes risk for an investor, other rates may reasonably be used for i. For example, if a warrant has less than one year to maturity, the rate on Treasury bills or the savings rate may be used, whichever is higher. If a warrant has ten

years to maturity, an adjustment to i may be desired to reflect the possibility of lower future interest rates.

Of the information required, D presents the greatest uncertainty, the greatest need for estimation, and the greatest degree of subjectivity. But again, the warrant should be considered, the model used, only after the stock has been analyzed. If fundamental (value) analysis is employed, prospective dividends will have already been estimated when determining the desirability of the stock, and since the warrant represents an alternative, the same estimates can be employed in the model. The models developed by Samuelson, Chen, Ayers, and Sprenkle, all require estimates of future stock prices. Dividends are far more stable than either stock prices or earnings,[1] hence their estimates should tend to be more accurate and dependable.

The Need for Additional Research

Warrants are only one type of option available in the security markets, and the model developed may have applicability to other option contracts. There are several areas that may provide fruitful research.

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[1] John Lintner, "Distribution of Incomes of Corporations Among Dividends, Retained Earnings, and Taxes", American Economic Review, XLIV (May, 1956), pp. 97-113.

1. A study of investors who frequently purchase stock options, to determine if the concept of professionalism has a bearing on valuation.

2. A study of put and call option contracts, utilizing the developed model. Call options, in particular, are identical to warrants with short maturities, and their shorter time horizon should reduce if not eliminate the uncertainty of dividend and interest rate estimations.

3. The examination of premium determination on convertible bonds and preferred stocks, incorporating the interest and dividend yield on the respective securities.

Contributions of the Study

This research should provide a new perspective to the interrelationships between the investment characteristics of warrants and their related common stocks. It presents an investment strategy, that of combining a warrant with a risk-free investment, that has not been previously explored, plus the justification of such a strategy, which together should result in more efficient allocation of an investor's funds.

Secondly, this paper should result in a supplementary definition of what constitutes a warrant's lower boundary valuation. This additional definition will lead

to a more rational pricing policy and a narrower trading range for warrants as a whole.

Third, the model developed in Chapter II has more practical applicability than those requiring a tenuous assumption of future stock price movements. From an investor's view, the benefit of a warrant valuation model should be its usefulness as an aid in the formulation of a rational investment decision. To the extent that investors consider estimates of future dividends more reliable than those of future stock prices, the model presented here will be a more accurate tool in the investment decision process.

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APPENDIX A

DEGREE AND TIME TO MATURITY OF UNDERVALUED WARRANTS

TABLE A-1

THE VARYING DEGREES OF UNDERVALUATION, LISTED CHRONOLOGICALLY

Date	Warrant	Warrant Market Price	Model Price	Market Price / Model Price (%)
11/15/63	TWA	12.88	13.79	93.4
1/29/65	TWA	33.00	36.56	90.3
4/16/65	TWA	35.38	38.60	91.7
	Realty Equities	2.00	2.29	87.3
8/13/65	TWA	29.13	31.07	93.8
	Realty Equities	1.88	2.14	87.9
11/19/65	TWA	35.88	40.07	89.5
	Realty Equities	5.63	7.18	78.4
3/17/67	TWA	66.25	66.31	99.9
	Realty Equities	5.63	6.44	87.4
	Braniff Airlines	41.50	48.09	86.3
	Universal American	1.50	1.66	90.4
4/7/67	TWA	56.75	56.81	99.9
	Realty Equities	6.13	7.55	81.2
	Braniff Airlines	35.38	39.66	89.2
5/5/67	Realty Equities	7.63	9.13	83.6
6/23/67	Realty Equities	6.50	9.13	71.2
1/26/68	Realty Equities	15.00	15.80	94.9
	Frontier Airlines	8.88	10.77	82.5
	Uris Buildings	23.50	23.82	98.7
3/15/68	Frontier Airlines	8.00	9.61	83.2
8/30/68	Realty Equities	27.38	30.40	90.1
	Martin Marietta	21.75	21.88	99.4
11/8/68	Realty Equities	31.00	31.66	97.9
1/31/69	AMK	18.00	19.04	94.5
	Frontier Airlines	8.75	16.37	53.5
11/21/69	Daylin	14.50	23.37	62.0
12/5/69	Daylin	14.25	22.36	63.7
1/1/71	Kaufman and Broad	22.62	23.93	94.5
1/22/71	" " "	26.00	28.02	92.8
2/5/71	" " "	25.37	27.40	92.6
2/26/71	" " "	28.50	30.37	93.8
5/7/71	Daylin	9.62	13.82	69.6
	Kaufman and Broad	40.00	41.32	96.8
	Williams	29.75	30.38	97.9
5/14/71	Daylin	9.25	12.94	71.5
	Kaufman and Broad	37.75	39.80	94.8
	Lerner Stores	31.12	31.58	98.5

TABLE A-2

TIME TO MATURITY IN DESCENDING ORDER FOR EACH UNDERVALUED WARRANT

Years, Days	Warrant
19,279	Braniff Airlines
19,238	Braniff "
19,120	Daylin
19,106	Daylin
19,34	Frontier Airlines
18,351	Frontier "
18,30	Frontier "
17,318	Daylin
17,311	Daylin
11,109	Lerner Stores
10,16	TWA
9,29	AMK
8,307	TWA
8,229	TWA
8,110	TWA
8,12	TWA
7,95	Uris Building
6,278	TWA
6,238	TWA
6,74	Realty Equities
4,300	Realty "
4,262	Realty "
4,239	Williams
4,223	Realty Equities
4,6	Realty "
3,155	Realty "
3,85	Realty "
3,59	Kaufman and Broad
3,38	Kaufman " "
3,24	Kaufman " "
3,3	Kaufman " "
2,298	Kaufman " "
2,291	Kaufman " "
2,156	Realty Equities
1,107	Realty "
0,353	Realty "
0,63	Martin Marietta
0,14	Universal American

APPENDIX B

UNDERVALUED WARRANTS RELATIVE TO CHARACTERISTICS OF THEIR ASSOCIATED COMMON STOCKS

TABLE B-1

UNDERVALUATIONS RELATIVE TO THE NUMBER OF LISTED WARRANTS

Date	Number of Eligible Warrants*	Undervaluations	Undervalued Eligible (%)
5/14/71	61	3	4.92
5/7/71	60	3	5.00
2/26/71	56	1	1.79
2/5/71	52	1	1.92
1/22/71	56	1	1.76
1/1/71	54	1	1.85
12/1/70	33	0	0
10/12/70	30	0	0
5/11/70	31	0	0
4/13/70	30	0	0
12/5/69	43	1	2.33
11/21/69	42	1	2.38
1/31/69	23	2	8.70
11/8/69	18	1	5.56
8/30/68	16	2	12.50
3/15/68	14	1	7.14
1/26/68	14	3	21.43
6/23/67	11	1	9.09
5/5/67	11	1	9.09
4/7/67	11	3	27.27
3/17/67	12	4	33.33
8/22/66	11	0	0
7/22/66	11	0	0
3/11/66	10	0	0
11/19/65	10	2	20.00
8/13/65	10	2	20.00
4/16/65	10	2	20.00
1/29/65	11	1	9.09
11/15/63	11	1	9.09
9/27/63	11	0	0
6/7/63	11	0	0
1/4/63	12	0	0
6/8/61	7	0	0
4/14/61	7	0	0
3/9/61	6	0	0
Total or Average	816	38	4.66%

*Listed on the American or New York Stock Exchange.

TABLE B-2

UNDERVALUATIONS RELATIVE TO THE NUMBER OF WARRANTS WHOSE ASSOCIATED STOCK PRICE WAS GREATER THAN THE WARRANT OPTION PRICE

Date	Number of Stocks With Ps > Po	Ps > Po Eligible (%)	Undervalued Ps > Po (%)
5/14/71	29	47.54	10.34
5/7/71	27	45.00	11.11
2/26/71	23	41.07	4.35
2/5/71	21	40.38	4.76
1/22/71	18	32.14	5.56
1/1/71	11	20.37	8.33
12/1/70	6	18.18	0
10/12/70	6	20.00	0
5/1/70	2	6.45	0
4/13/70	6	20.00	0
12/5.69	14	32.56	6.67
11/21/69	16	38.10	5.88
1/31/69	19	82.61	10.53
11/8/68	14	77.78	7.14
8/30/68	9	56.25	20.00
3/15/68	11	78.57	9.09
1/26/68	12	85.71	25.00
6/23/67	8	72.73	12.50
5/5/67	6	54.55	16.67
4/7/67	7	63.64	42.86
3/17/67	8	66.67	50.00
8/22/66	6	54.55	0
7/22/66	7	63.64	0
3/11/66	5	50.00	0
11/19/65	4	40.00	50.00
8/13/65	4	40.00	50.00
4/16/65	4	40.00	50.00
1/29/65	4	36.36	25.00
11/15/63	4	36.36	20.00
9/27/63	5	45.45	0
6/7/63	3	27.27	0
1/4/63	3	25.00	0
6/8/61	5	71.43	0
4/14/61	5	71.43	0
3/9/61	4	66.67	0
Total or Average	336	41.18%	11.31%

TABLE B-3

THE RELATIONSHIP OF THE STOCK PRICE TO THE WARRANT OPTION PRICE, PER YEAR, FOR EACH UNDERVALUATION

Date	Number of Undervaluations	Ps/Po
5/14/71	3	1.128, 2.746, 3.083
5/7/71	3	1.167, 2.815, 2.338
2/26/71	1	2.301
2/5/71	1	2.163
1/22/71	1	2.192
1/1/71	1	2.001
12/5/69	1	1.628
11/21/69	1	1.672
1/31/69	2	1.308, 1.172
11/8/68	1	5.191
8/30/68	2	1.479, 5.025
3/15/68	1	1.125
1/26/68	3	1.192, 3.095, 2.929
6/23/67	1	2.277
5/5/67	1	2.277
4/7/67	3	1.012, 2.058, 3.591
3/17/67	4	1.127, 1.905, 4.023, 1.119
11/19/65	2	2.065, 2.551
8/13/65	2	1.323, 2.136
4/16/65	2	1.345, 2.472
1/29/65	1	2.375
11/15/63	1	1.301