VISIBILITY THRESHOLDS FOR FLASHING LIGHTS

Thesis for the Degree of Ph. D.

MICHIGAN STATE UNIVERSITY

DOUGLAS HALL WILLIAMS

1971

This is to certify that the

thesis entitled

VISIBILITY THRESHOLDS
FOR
FLASHING LIGHTS

presented by

Douglas Hall Williams

has been accepted towards fulfillment
of the requirements for

Ph.D. degree in Psychology

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Date Aug 11, 1971

O-7639

ABSTRACT

VISIBILITY THRESHOLDS FOR FLASHING LIGHTS

By

Douglas Hall Williams

Absolute foveal thresholds were determined for lights flashing with pulse lengths of .01, .025, .05, .07, .1, .14, .2, .4, and 1.6 seconds, and null periods of .025, .05, .1, .4, and 3.2 seconds. The method of limits was used with three human subjects. It was concluded that neither the Blondel-Rey equation nor subsequent modifications of it adequately described the threshold of lights flashing with a null period of less than approximately one second. A modification of the Blondel-Rey equation was offered which described the data. It adds only one new term, the null period of the flashing light. The new equation is

$$\frac{E}{E_o} = \frac{\frac{.2n}{.2 + n} + t}{t}$$

where

E = Threshold illumination for the flashing light
E_o = Threshold illumination for a steady light
n = Null period
t = Pulse length

This equation can be used for any pulse length or null period, whether the light appears to flash or not.

VISIBILITY THRESHOLDS FOR FLASHING LIGHTS

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Douglas Hall Williams

A THESIS

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Psychology

1971

ACKNOWLEDGMENTS

To my long-suffering committee, and especially my Chairman, T. M. Allen, thanks are due.

All my friends, professors, and even the faceless multitudes of undergraduates had some part in the creation of this research, even though it may not show on the surface. My only hope is that we are all immortalized in some small way by the results of it.

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I. INTRODUCTION

Flashing lights have been studied by many investigators over the years. From these studies have come several useful laws and rules of thumb concerning the response of the human visual system to intermittent stimulation. As an upper bound of this area, Talbot's Law is well accepted as a description of the behavior of lights above CFF. For flashes with a duration less than .1 seconds, the Bunsen-Roscoe Law (also called Bloch's Law when the visual system is being considered as a whole) is applied. For flashes of longer duration, the Blondel-Rey equation is commonly accepted. Sometimes Hampton's modification of Blondel and Rey's equation is used. For longer flashes, (durations of more than a second or so) the steady threshold is thought to apply.

Exactly where one of these laws fades into another is not always clear. Often any differences in predictions between different equations are small and of no practical significance, and any given practical question can often be answered by a quick experiment. But it is of great importance to engineers designing visual warning systems to know what the response of the human element of the system will be. It would be much easier to specify with a single relation than with a collection of laws and equations, each with an ill-defined range of applicability.

The purpose of this paper will be to review the literature concerned with the derivation and practical application of these several laws. Other experiments which tend to show the inadequacy of all these laws in certain areas will be reviewed. Finally, data will be collected over the whole range and a model will be offered which should be of some use to engineers designing flashing light systems.

II. LITERATURE

The primary orientation of this paper will be to derive a relation which would be useful in an applied setting. However, in order to do this most efficiently, it is necessary to examine the theoretical foundations of the laws which are currently used.

Bloch's Law applies to flashes with a duration less than .1 seconds. For such flashes, it has been found that $I \times t = \text{Constant}$. For longer flashes, the law necessarily breaks down, since a long flash has the same visibility as a steady light. One of the most commonly used formulations is that of Blondel and Rey. In 1912, it was of importance to know something about the human eye so that an optimum flash duration and intensity could be selected for lighthouse flashes. Blondel and Rey's approach was to determine the brightness of a flashing light which would appear as bright as a steady light of known intensity. Subjects adjusted the single test flash to apparent equality with a long control flash of known candle power. In this way, the relative visibility of the stimuli could be determined. Three seconds were left between presentations of test flashes, as Blondel and Rey assumed that this would allow any effects of one flash to die out before the next was presented. They found that their data were described by an equation which they wrote:

$$\frac{E}{E_o} = \frac{a + t}{t}$$

where

E = Threshold illumination for a flashing light
E_o = Threshold illumination for a steady light
t = Length of the flash, in seconds
a = an empirical constant, .21

Several studies have been done since 1912 which have usually more or less supported Blondel and Rey's findings. In 1938, Neeland, Laufer and Schaub set up airway beacons on buildings 8.3 and 2.9 miles from observers. The observers matched the apparent intensity of the flashing beacons with an adjustable steady light from a remote projector. Despite several methodological difficulties, they obtained fair agreement with the Blondel-Rey equation.

In lighthouse use, interest is in comparison of signals somewhat above threshold, when the signal has "adequate" conspicuity. Toulmin-Smith and Green (1933) used a device similar to Blondel and Rey's to derive an equation for luminances greater than those used by them. Single pulses of different lengths (.05 to .5 seconds) with one second between successive flashes were used. The data were described by the equation

$$\frac{I_e}{I} = \frac{1.1\ t}{.15 + t}$$

where

t = duration of the flash (seconds)
I = Intensity of a steady light for threshold
I_e = Intensity of a flashing light for threshold ("apparent intensity")

This equation gives a ratio of the intensity of a flashing light to that of a steady light of equal visibility. Such an equation is in inverse form to that of Blondel and Rey, who gave the ratio of thresholds. This form is slightly more convenient to use.

Hampton, in 1934, objected to Toulmin-Smith and Green's formulation on the grounds that their equation led to the prediction that for very long flashes, the flashing light would appear brighter than the steady light. Using their data, he derived an equation which was of the same form as Blondel and Rey's, but with an additional parameter, E_c. This was defined as the "minimum illumination required for adequate conspicuity." His equation fit their data as well as their formula had.

$$\frac{I_o}{I} = \frac{t}{\left(\frac{.0255}{E_c}\right)^{.81} + t}$$

where

I_o = Apparent intensity
I = Intensity for threshold of a steady light
t = time of flash, in seconds
E_c = Conspicuity

Note that except for the inversion due to statement in terms of equivalent intensities, his equation is that of Blondel and Rey with their constant "a" replaced by $\left(\frac{.0255}{E_c}\right)^{.81}$. E_c could be set at any value of conspicuity the user felt was necessary for his application. Using a value for "useful" brightness of $E_c = .425$, his equation reduces to

$$\frac{I_o}{I} = \frac{t}{.1 + t}$$

Hampton's paper seems to be nearly the last word for this kind of study. One can find papers and practical applications using these equations and the value of .1 to .2 for the constant "a" right up until the present day. (Projector, 1957; Douglas, 1957).

With a very different application in mind, A series of widely quoted studies was performed by Gerathewohl (1951, 1952, 1953, 1954). He was interested in the conspicuity of supra-threshold flashing lights, similar to what would be encountered on a radarscope or control panel. His method consisted of placing a subject in front of a lighted screen on which were flashed signals of different luminances, colors, flash rates, and positions. The subject also had an auditory task. He responded to both tasks using footpedals and levers. The dependent variable in all studies was reaction time. He found that the response time decreased with increasing contrast, increasing flash frequency, and longer flash duration. His recommendation was that the most conspicuous signal would be three flashes per second when the signal was at least twice as bright as the background (1954).

However, in two similar studies, Dean (1962) was unable to duplicate these results with similar reaction time data. He hypothesized that flash rate might not be a determinant of signal conspicuity when apparent brightness had been adjusted by the Blondel-Rey formulas.

Gerathewohl's method suffered from problems reviewed in detail in Williams (1966). These problems included use of a cumulative clock, so that missed stimuli resulted in 15 seconds being added to the total reaction time, and failure to correct the stimulus brightnesses by using the Blondel-Rey equation. In general, the studies may provide data of some use in specific situations similar to those of the study, when reaction time is of importance; but the failure of Dean (1966) to replicate Gerathewohl's results make even this somewhat doubtful. Where reaction time can be definitely correlated with some variable of interest, methods similar to Gerathewohl's could be used; but even then, only the rankings of the various stimuli would be dependable, since it is almost certain that in a situation where more stimuli were available, or more distractions entered in, the reaction times themselves would change radically. Also, Gerathewohl was using supra-threshold stimuli, which made sense given the applications (radarscopes and panels) to which he was putting the data. One should, however, exercise caution in applying these results to the threshold situation.

In 1959, Clark and Blackwell studied thresholds of single and double pulses in the fovea. They used a temporal forced-choice procedure, with seven observers. Data on contrast thresholds were plotted on log-log axes, as shown in Figure 1.

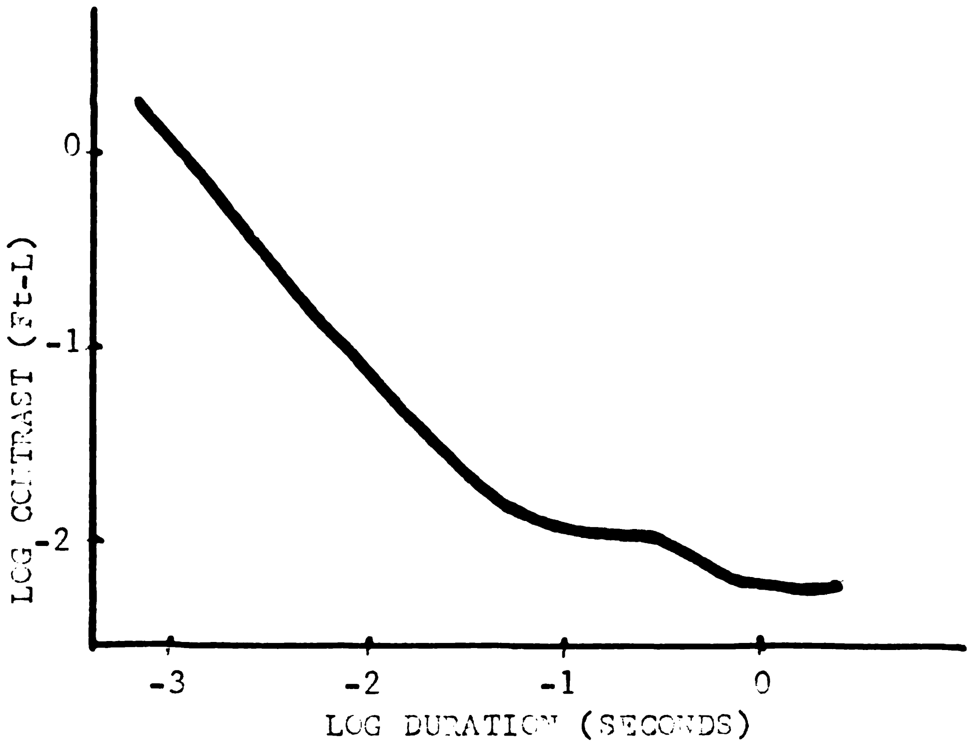

Figure 1. Contrast thresholds, from Clark and Blackwell, (1959).

Curves were fit by eye, and no mathematical relation was derived to describe them; but an interesting result is clear; between about .2 and about .075 seconds, a short plateau appears. This is a consistent finding for all background luminances. The break on this kind of plot from a downward-sloping line to a horizontal one is well-known and accepted and in fact a pulse length of .1 seconds is known as being of "critical duration" due to this effect; (Long, 1951) but the "bump" had not been found before.

In explanation for the failure of other investigators to find such a discontinuity, Clark and Blackwell point out that few other investigators took as many points in the region in question, nor as reliable ones (250-1000 observations per point). So there is little question that the plateau is a real effect. Unfortunately it is not easily explained. Clark and Blackwell do not consider it too important, explaining that

> "The interval of approximately .1 second appears to be a critical one for the visual system; it corresponds to the frequency of the alpha waves over the visual cortex and to the light frequency which produces a number of unusual visual effects . . . " (Clark and Blackwell, p. 2)

And it is, in fact, a very small effect, amounting to a fraction of a log contrast unit. For the purposes of practical application, we would be happy with predictions as accurate as a smooth curve ignoring this "bump."

Schmidt-Clausen (1967) exhaustively investigated the effects of five variables on the foveal threshold of short photic pulses. The variables were color, visual angle, background luminance, pulse shape, and exposure time. A total of more than 30,000 individual measurements were made on two subjects, and the data were extensively plotted and analyzed.

Schmidt-Clausen compares all his variables with the function Lu = f(t), where Lu = threshold luminance and t = length of flash. For example, the results for different pulse shapes are presented as a family of curves of Lu = f(t), one for each pulse shape. Often, the Blondel-Rey relation is plotted on the same graph for comparison. Often also, in the text, a calculation of the "constant" a in the Blondel-Rey equation is presented. It turns out not to be very constant; in fact it varies from .1 to 1.45 seconds as a function of background luminance, visual angle, pulse shape, and color.

Schmidt-Clausen's procedure was to present single short flashes foveally, apparently using the method of adjustment (though the exact procedure in regard to the responses is not stated). His flashes were a minimum of five seconds apart, (somewhat longer than the three seconds used by Blondel and Rey) and were apparently presented at the experimenters' convenience, without S knowing one was coming. There seems to be little or no effect due to the procedural differences, since in cases which are most similar to Blondel and Rey's experiment, nearly exact agreement is obtained with their equations. Schmidt-Clausen shows that the shape of the Blondel-Rey curves changes very little over all the variables investigated, except for color; he expressed his results in terms of a modification of the equation

$$\frac{Ls}{Ls\infty} = \frac{f(\alpha)+t}{t}$$

where

Ls = Threshold luminance

Ls∞ = Threshold luminance for a long pulse

f(α) for the pulse shapes, surround luminance, and visual angle were given in the report. Note that here again, the equation is of the same form as that of Blondel and Rey.

It should be pointed out that the graphs he presents do not seem to have been derived from a mathematical curve fit of the data, although he doesn't say. Rather it appears that the shape was derived from the Blondel Rey equation, and the constant "a" varied, or the curves raised and lowered to provide a fit.

The discontinuity which Clark and Blackwell (1959) found with similar presentations of single flashes does not show in any of Schmidt-Clausen's graphs. He was apparently not aware of the findings of Clark and Blackwell, and even though he plots several data points well within the region in which one would expect to find the irregularity, no such effects can be seen in the graphs which he presents. However, the graphs are a little difficult to read in the region in question, and it is possible that the very small effect could be obscured.

In general, Schmidt-Clausen's results seem to support, with some modifications, the results of Blondel and Rey, and Bloch. The effects of the wave-shape, color, background luminance, and visual angle can be calculated using formulas developed by Schmidt-Clausen. However, the caution which applies to use of the Blondel-Rey equation applies to his study also: it was based on single flashes, several seconds apart. It is clear that flashes, even if not perceived, can affect the threshold of subsequent flashes (Clark, 1959).

In common with the other studies reviewed so far, null period (or frequency) is not considered. One cannot expect one study to consider all possible variables; but it should be pointed out that all of Schmidt-Clausen's results might be changed if a train of pulses were studied, instead of the single pulses.

One other law deals with intermittent photic stimulation and needs to be discussed here. It does not, however, deal with discriminable flashes. Talbot's Law applies only to lights which are intermittent, but appear fused to a continuous light. Ideally, any mathematical relation which deals with intermittent photic pulses should reduce to Talbot's Law above fusion.

Graham (1965) states the Talbot Law as

$$L_m = \frac{1}{t} \int_o^t L dt$$

where

Lm = the luminance of a steady light which matches the time-varying luminance

L = the instantaneous luminance of the time-varying field

t = the period of a single cycle of the varying field

For the case of square pulses, which is what the Blondel-Rey and Bloch laws deal with, we may express the Talbot Law as

$$\frac{I_s}{I_f} = \frac{t_\ell}{t_\ell + t_d}$$

where

I_s = Equivalent intensity
I_f = Intensity of the pulse
t_ℓ = Pulse length
t_d = Null period

Notice that the null period of the pulse is a necessary factor in Talbot's Law. None of the previously mentioned laws have taken null period of the pulse into account. Hence, using any of these other laws, one would calculate that a train of .1 second pulses with .05 seconds of null period between them would be exactly as visible as a train of .1 second pulses of the same luminance with 1 second of null period between them. But if null period enters into determinations of the threshold above CFF, as Talbot's Law states it must, it is reasonable to think that it might enter in, to some degree, below CFF.

All the formulas reviewed, again excepting Talbot's Law, which would currently be applied to a continuous light with a null period of, say, .1 second duration every second would predict that it would be seen at exactly the same luminance as a continuous flash. As far as the author can determine, no studies have been done on the threshold of this sort of pulse. It is certainly an area worthy of investigation, if for no other reason than to validate any mathematical generalizations over the entire range of possible pulse-length--null period variations.

As was noted above, all the equations describing intermittent photic pulses, except Talbot's Law, fail to take into account the dark time of the pulse. Dark time is almost certainly an important variable which needs to be incorporated into an equation relating pulse characteristics and threshold. One of the major drawbacks seems to be lack of an adequate way of representing data taken at different frequencies. If one assumes frequency is an important variable, one must draw a separate graph for each frequency, and the result quickly becomes confusing (Erdmann, 1962). Hence, null period or frequency of the flashes tends to be ignored, and graphs are presented (Grahan, 1965; Schmidt-Clausen, 1967) showing threshold plotted against length of pulse, regardless of the time between pulses.

Distinction should be made here between several experimental procedures which are used. One can hold null period constant, and vary pulse length. This yields data for various pulse lengths of approximately the same frequency. (For exactly the same frequency, the null period would need to be varied so that pulse length + null period = constant.) Here, both pulse length and null period vary; but a pulse length-null period plot is typically not used. One can also study various dependent variables. The present study, like many others, will study absolute thresholds (no background luminance). It is possible to do a similar study using a comparison flash which is varied to appear the same as the test flash. Or, the luminances may all be held constant and the frequency necessary for fusion of the flashing light may be studied. Which of these is chosen depends upon the purpose for which the study is being done.

This paper will not be the last word in any of these areas. However, a way of looking at all the threshold studies simultaneously will be offered. When such laws as Bloch's, Talbot's, and the Blondel-Rey relation are all seen to be related and flow into one another, the necessity for more studies of for different studies can be quickly seen. Also, the border between the predictions of two equations can be seen, and often this is an area on which a great deal of effort should be spent, since the rates of change of the threshold function are higher here.

Figure 2 is a representation which is a partial solution to problems of plotting these equations. It is a plot with null period on a log scale on one axis, and log pulse length on the other axis. Note that for certain combinations of light and dark time (near the origin) Talbot's Law will apply, and threshold will be a function of $\frac{\text{light time}}{\text{dark time}}$. The exact point where Talbot's Law ceases being applicable is not easily discovered from the literature. The Ferry-Porter Law (Garham, 1965) can be used to estimate CFF but according to Graham, applying that law at threshold luminance is suspect.

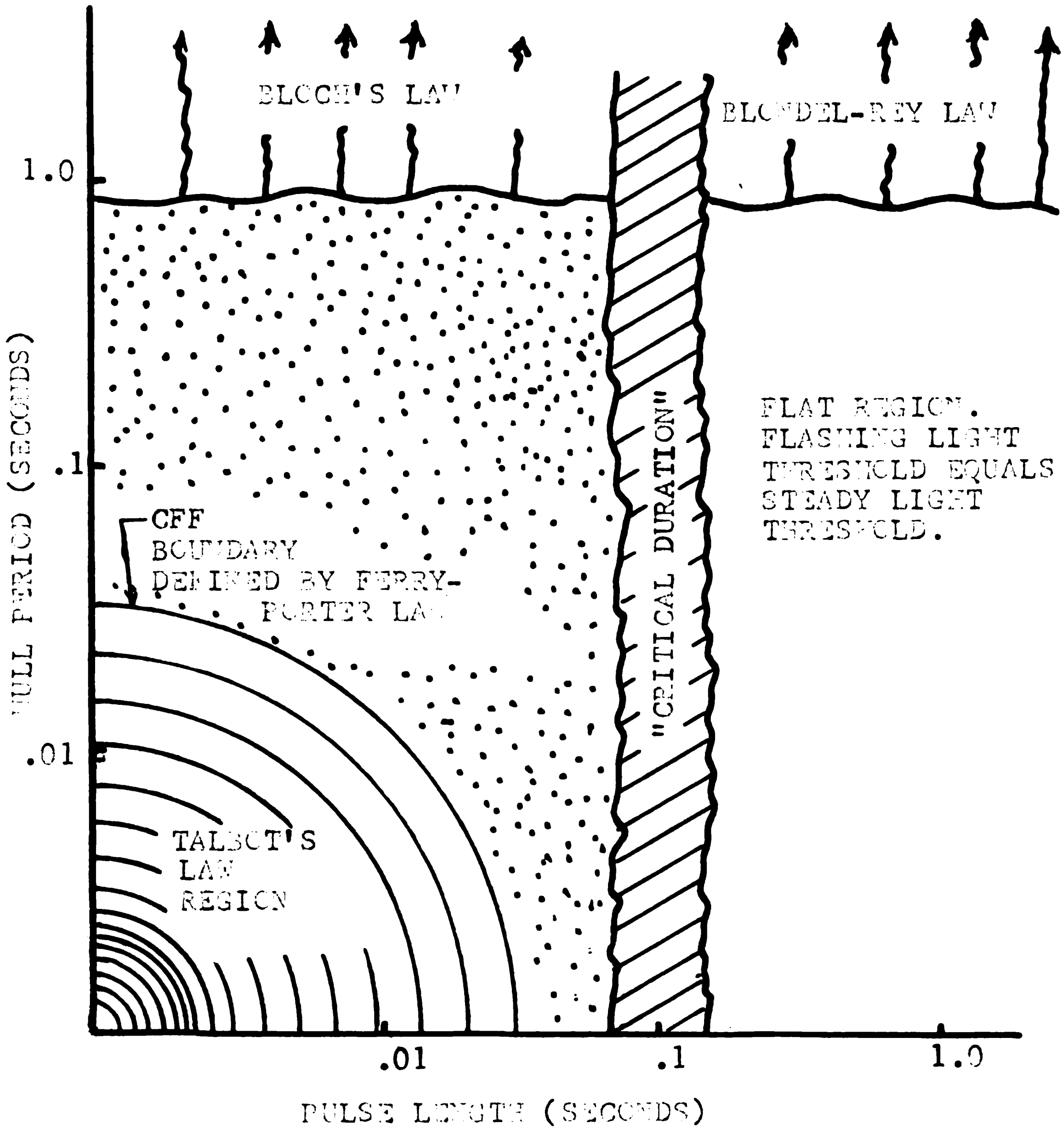

Figure 2. Overview of relevant laws. Stippled area indicates unknown regions.

As dark time increases so that the intermittency is below CFF, Bloch's Law is said to hold. This would imply that the entire surface below a pulse length of .1 seconds (critical duration) is a plane, sloping upward from a minimum threshold at .1 seconds pulse length. Blondel and Rey's equation would substitute a slight curve for the flashes near .1 second, and raise the level of the plane slightly, the greatest difference occurring for the slower flashes.

Both Blondel and Rey and Bloch neglect to mention null period. One could either assume that their derived relations apply for only the null periods they used in their experiments, or that they apply for any null period. The latter assumption is commonly made.

Above a certain length of flash (which is not too clearly defined), the steady threshold is said to apply to intermittent sources. Data here are rather sparse. "Flashing light" studies are usually motivated by some applied problem (such as lighthouses or roadway flashers) and usually no one is able to provide the large amounts of energy that such a long flash requires in a practical field flasher. Hence, almost no research is done on flashes of lengths greater than one second, and the author knows of no research whatsoever on the threshold of flashes with long pulse lengths and short null periods. A threshold value the same as for a steady source, or higher, would not be surprising. If it were lower, none of the present equations would handle it.

The purpose of the present study was to investigate the entire range of flash length and null periods between them. For simplicity, absolute foveal thresholds of white square-wave pulses were obtained in a completely dark room.

III. METHOD

A. FACILITY DESCRIPTION

The experiment was run in the MSU psychology research building in a room 40 feet long by 8 feet wide. The walls and floor were black, as was the ceiling, except for light fixtures inset into it and covered with white plastic. The room was totally light-tight, it being impossible for the experimenter to see any stray light after one hour dark adaptation.

At one end of the room was the stimulus board (described in detail later). Twenty feet from the stimulus board, the S sat in an ordinary chair. Ss were free to move around in the chair and shift their bodies but were asked to fixate the red fixation light during experimental trials. Any failure on their part to do so resulted in a rather drastic drop in thresholds, and any such trials were discarded.

B. CONTROL EQUIPMENT

Behind the S, and separated from him by a flat black plywood partition was the control and monitoring equipment. This consisted of a console from which the light time, dark time, and brightness of the light pulses could be controlled. Two power supplies (low and high voltage) were incorporated in the console, along with a dim red panel illumination and a writing surface. An oscilloscope provided a means of monitoring the outgoing pulses to the glow tube. The overhead illumination could be controlled from a wall switch near the console. (See Figure 3 for a plan of the room.)

C. STIMULUS GENERATION EQUIPMENT

Figure 4 is a system block diagram of the stimulus generation system. The duration of the pulses and null periods provided to the glow tubes could be regulated by means of a specially-constructed solid-state pulse generator. The two variables could be set independently on two 10-turn potentiometers, which had been calibrated using the calibrated sweep of a Tektronix oscilloscope. Null periods from less than .01 seconds to more than three seconds, and pulse lengths from less than .01 to more than three seconds could be obtained. A special setting resulted in a continuous pulse.

D. STIMULUS GENERATION

A control for the brightness of the stimulii was provided on the control console. The control was accomplished by decreasing the current fed to the glow-tubes, by changing the bias on a tube in the constant current circuit. The potentiometer shaft was geared to a mechanical counter so that a given position of the control could be read precisely. All data were recorded in terms of the arbitrary numbers, and converted to photometric units later. This brightness control was only used for the slight adjustments in brightness necessary to apply the limits method to determining the thresholds. Primary reduction of the high light output of the glow-tubes was provided by a set of neutral density filters, a diaphragm,

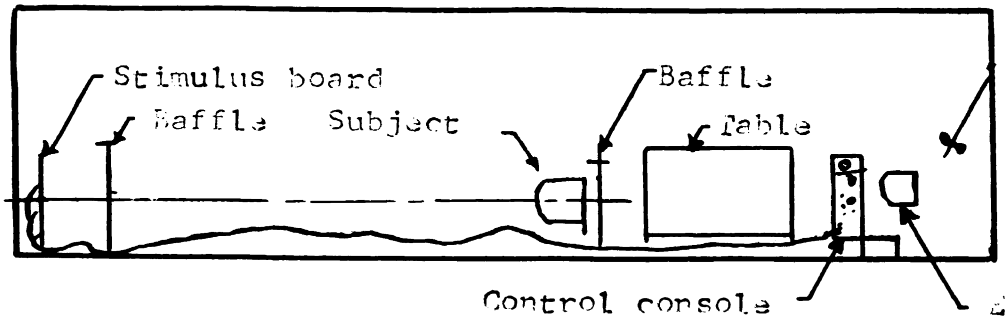

Figure 3. Plan of experimental room.

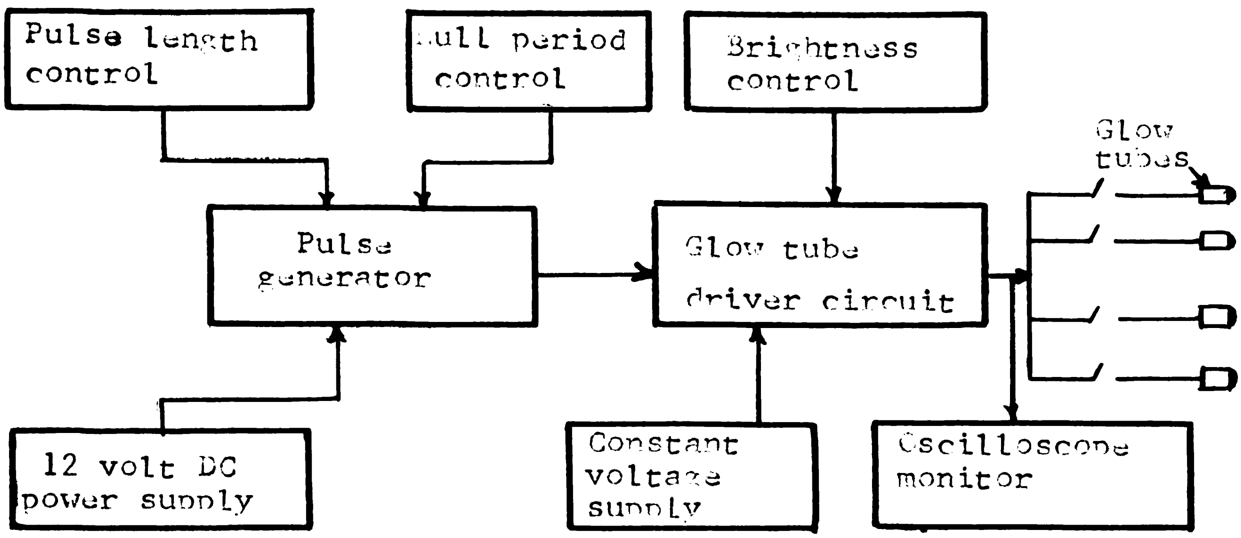

Figure 4. Equipment block diagram.

and an opaque plastic diffusion screen. (See Figure 5.) Glow tubes were Sylvania R-1131-C units, with a spectral distribution of output shown in Figure 6. Rise time and fall times of these units is negligible--in the fractions of a microsecond range.

Light output values for the stimuli were obtained using a Prichard spectra photometer. Each of the four aperatures were calibrated for the full range of the brightness control. Each of the aperatures differed in the brightness range it covered, so that a large range of brightnesses could be presented without disturbing the stimulus board to change filters. As near as was possible, the filters were set so that the dimmest panel control positions for the brightest aperature was nearly equal to the brightest panel control position for the second brightest aperature, etc. When pulse-length-null period combinations were found for which thresholds could not be determined with any of the four available aperatures, a note was made and when enough of these accumulated, a filter change was made to bring one of the stimuli to a point where a threshold could be taken.

E. STIMULUS PRESENTATION

The stimulus board and associated components had to accomplish several objectives:

1) presentation of the stimuli at constant locations

2) provision for some method of keeping the S's point of regard at a constant position when no stimulus was present.

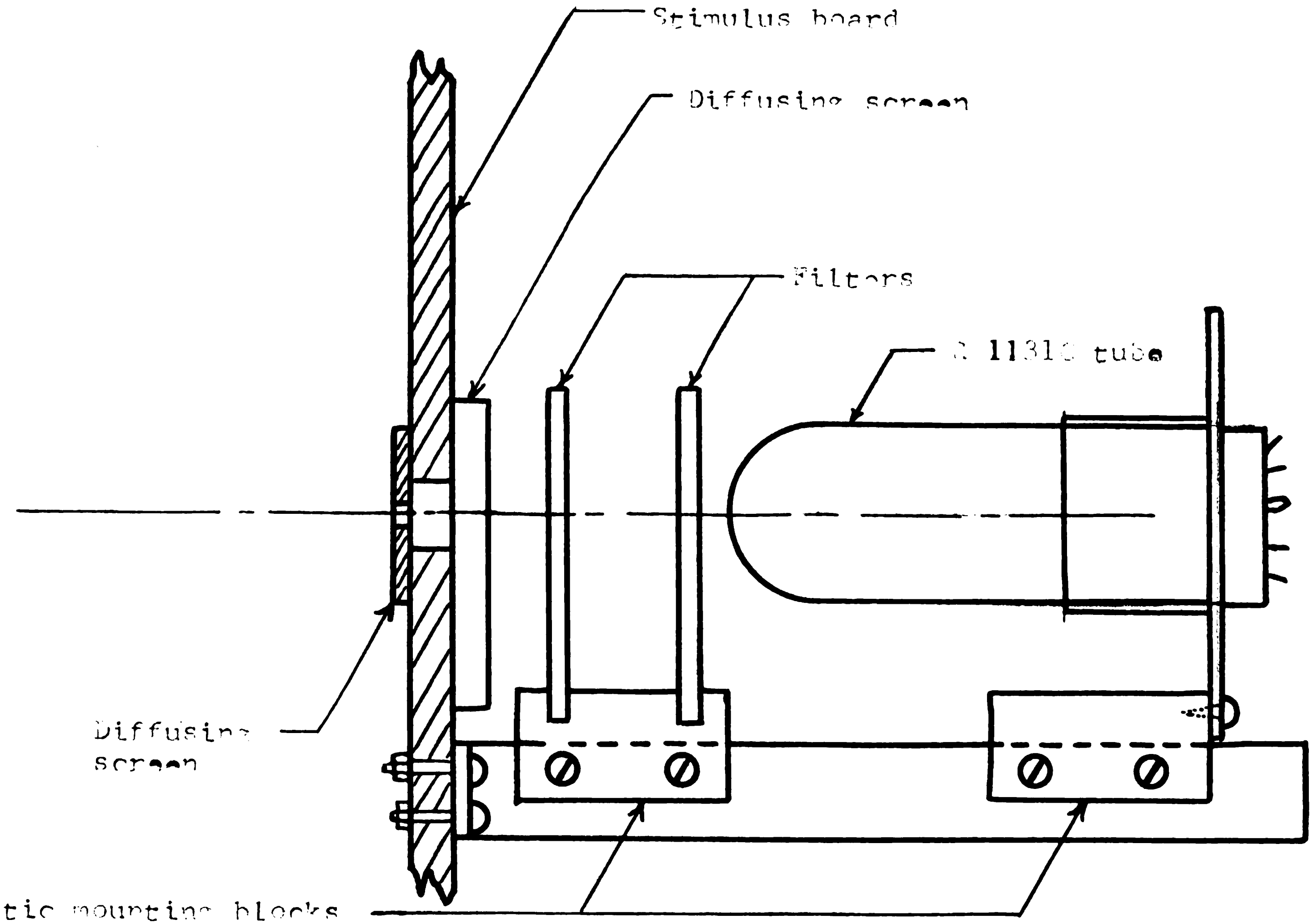

Figure 5. Detail of mounting of glow tubes and filters on stimulus board.

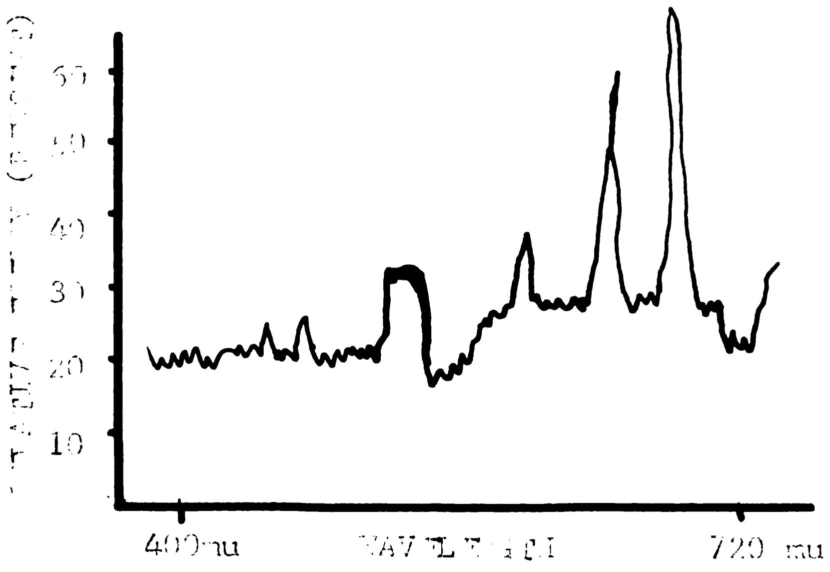

Figure 6. [illegible] vs. [illegible] for [illegible]

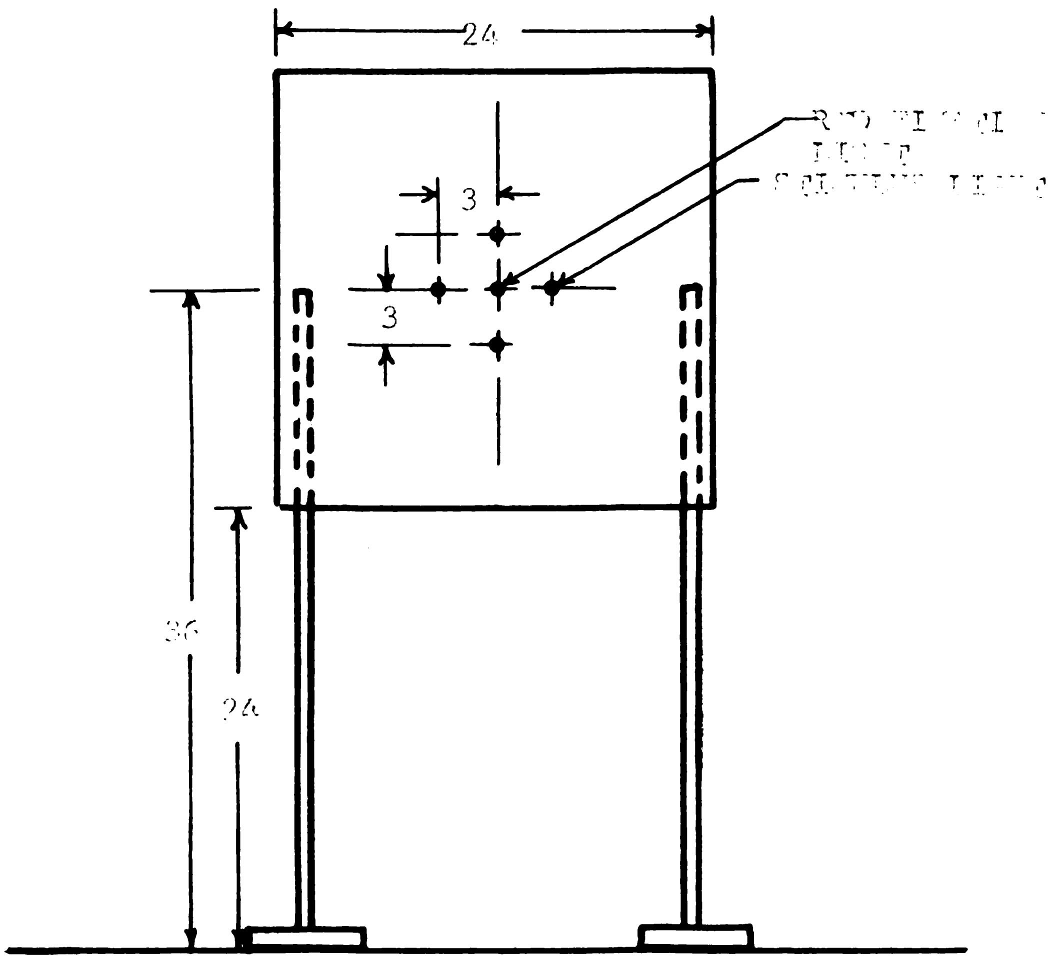

Figure 7. Dimensions of [illegible] board (inches).

This would also help prevent empty-field myopia.

3) elimination of all glare from shiny objects between the S and the stimulus.

4) prevention of all stray light from the back and sides of the glow tubes from reaching the S's eye.

In order to accomplish 1), four glow tubes were mounted on the stimulus board three inches from the central fixation light, as shown in Figure 7. Each glow tube and filter set was mounted on a rail and could be adjusted so as to line up the light beam from the tube with the aperture in the board. (See Figure 5.) The stimulus board was mounted two feet off the floor on two adjustable stands, painted flat black.

Condition 2) was provided for by a dim red fixation light. It was mounted in the center of the board. The light was an NE-2 neon bulb operated from the 110-volt supply thru a 100-Kohm resistor. The bulb was pushed into a 1/4-inch hole, and covered with a hemispherical red plastic filter.

3) was accomplished by removing all possible items which could reflect light, and thereby confuse the S. Some non-removable features of the room were found to reflect, and these were treated in various ways: metal fixtures in the floor were painted flat black; shelves along the side of the room were covered with black matte paper. A baffle 18 inches high was placed on the floor approximately five feet in front of the stimulus board to prevent reflections from the black vinyl floor.

4) was eliminated as a problem by enclosing the back and sides of the stimulus board in wood and covering the cracks with black tape; in addition, each glow tube was enclosed in a black cardboard tube. In this way, the light generated by each tube could be kept within the area directly behind the individual stimulus aperature.

F. DIMENSIONS AND VISUAL ANGLES

Table 1 gives the calculated visual angles at the Ss eye of various important features. The dimensions of interest are the size of the stimulus aperature and the distance of the stimulus from the fixation point.

TABLE 1

VISUAL ANGLES

Item	Calculated Visual Angle
Fixation light	4'
Fixation light to any stimulus light	45'
Stimulus aperatures:	
Top	2'
Bottom	2-1/2'
Left	3-1/2'
Right	3'

The other critical dimension is the distance between the fixation light and any stimulus light. Subjects would presumably have the fixation light centrally fixated during trials when the stimulus was dimmer than threshold; it is therefore necessary that the stimuli be on the fovea during this time. It is generally accepted that the fovea is 2° in extent, with the central fovea being half this size or less (Graham, 1965). In any case, the 45' used in the present study between the fixation light and any stimulus assures foveal thresholds. After a stimulus was defected on descenting trials, it would of course be centrally fixated by the S, as he was instructed to do so.

G. PULSE PARAMETERS

The number of thresholds needed in any area of the pulse length-null period threshold curve varied according to the expected rate-of-change of shape of the surface in that area. For instance, for very long flashes, there is little difference in thresholds for flashes of 1.0 seconds duration and flashes of 1.2 seconds duration. Hence, thresholds could be spaced widely apart and still reflect an accurate picture of the surface. For shorter flashes, say, durations of around .1 seconds, a much smaller change in the length of the pulse duration (or null period) has a great effect on the threshold. Here, thresholds were spaced as closely together as necessary until an accurate picture of the surface could be obtained. The pulse durations and null periods used are shown in Table 2.

TABLE 2

PULSE PARAMETERS

Pulse Length (Seconds)	Null Period (Seconds)
.01	.025
.025	.05
.05	.1
.07	.4
.1	3.2
.14	
.2	
.4	
1.6	

H. RATIONALE FOR SELECTED PULSE PARAMETERS

The values of the light pulse and null period duration were selected carefully to maximize information obtained from each combination. Each pulse length was paired with each null period, giving a total of 45 data points on the surface. The reasons for selecting each combination are given below.

1. Light pulses .01, .025 and .05; null period .025 and .05 seconds.

These combinations give a stimulus which is above CFF or is nearly so. In other words, the sensation is that of a continuous light. For these stimuli, Talbot's Law should predict the brightness necessary for threshold. They are included so that the position of the part of the surface representing Talbot's Law may be plotted. The threshold for these points should be higher than the steady threshold by an amount proportional to the PCF.

2. Light pulse of .07 thru .4 seconds, all null periods.

These were included as an investigation of the "bump" shown by Clark and Blackwell. Their study used "long" null periods, and the series of points at a null period of 3.2 seconds serves as a partial replication of their experiment. However, the shorter null periods were also run with these critical light pulse lengths to see what the behavior of the "bump" is over different frequencies.

3. Pulse length of 1.6 seconds, all null periods.

This series was included because a pulse of that length can be effectively a steady light. Most null periods have little or no effect on the threshold when the light pulse is this long, so these five points can all be considered as checks on each other. It is extremely important that the steady threshold be accurately determined since the calculation of the theoretical points in the Talbot's Law and Bloch's Law region depend on the determination of the steady threshold and since the actual thresholds are compared with these calculated values. Hence, it is felt that including five points with the expectation that the thresholds will be nearly the same for all of them is not excessive.

4. Pulse length of less than .1 seconds and null period of .1 or longer.

This group was included to verify and locate the Bloch's Law region. As with the Talbot's Law region, the theoretical points in this area can be calculated from the steady threshold. However, it is worthwhile checking Bloch's Law over several short null periods, since it was derived for long null periods only.

The spacing of the points on the surface was also chosen carefully. A previous study (Williams, 1966) used frequency and PCF as parameters of the flashes. On a logarithmic null period-pulse length plot this resulted in the data points being arranged in concentric arcs, and made the results difficult to interpret. For the present study, the points were purposely picked in terms of pulse length and null period. In addition, they were spaced in terms of a standard interval on the log scale. This interval is equal to the distance between .1 and .14 seconds. Starting with .025 seconds, the distance to .05 is 2 units; from .05 to .07 is one unit; from .1 to .14 is of course one unit; etc. This scheme allows several computational simplifications on the computer, and makes plotting results easier.

I. PHOTOMETRY

All brightness readings were taken with a Prichard Spectra-Photometer model 1970 Number 299. It was calibrated in terms of footcandles by use of an N.B.S. calibrated standard lamp. All readings were taken 10 feet from the stimulus board, in line with the subject's eye. The glow tubes were set for a long pulse length and the average of several flashes taken as the brightness of the stimulus.

J. EXPERIMENTAL DESIGN

The experiment was set up as a 9 by 5 factorial design, with two replications. Each of the nine pulse lengths was ordered randomly within each of the five null periods, and the null periods were ordered randomly within a replication. Randomization was accomplished by writing each of the five null periods on a card and each of the nine pulse periods on another set of cards. Each day, before the subject came in, one card was drawn (without replacement) from the "null period" deck. The number on the card drawn determined the null period for that day. The "pulse length" deck was then shuffled, and the order of the cards recorded on the data sheets. To guide the experimenter, and eliminate errors made in the dark, the dial settings for the pulse generator were also written on the data sheet at this time.

Each session consisted, then, of nine pulse lengths paired with one null period. Ten thresholds were taken for each pulse length-null period combination, for each replication. A total of three replications were performed, but the data of the first was not used, in order to allow practice and learning effects to subside. The data points, then, are based on twenty thresholds, ten taken on each of two occasions.

IV. PROCEDURE

Subjects were paid to participate. They were referred to the experimenter by a student work-study office, and their inclusion in the experiment was contingent on their passing the eye test and being willing to participate in the experiment for its full duration.

Each subject was checked for visual acuity with a projected acuity chart, to assure they had 20:20 vision, or had been corrected to that standard.

Subjects were given a practice session of one or two hours duration which included the eye test, administrative details, and a brief explanation of the purpose of the experiment. After a short period of dark adaptation, thresholds for two or three different points were taken. These thresholds gave the subject an idea of the nature of the experiment and allowed any practice effects to dissipate. After this session, the subject was asked if he was still interested in participating, and if so, an appointment was made for another session. The threshold data from the practice session was not used.

Sessions after the practice session were a minimum of two hours in duration. This included approximately 15 minutes for dark adaption. A session shorter than two hours was felt to be inefficient due to the necessity for dark adaptation; times longer than 2-1/2 - 3 hours were found in a pilot study to be fatiguing to the subject.

An attempt was made to schedule the majority of the sessions at night so as to minimize effects of walking into the experiment directly from bright sunlight. Occasionally daytime sessions had to be scheduled, and in these cases Ss were asked to wear dark glasses when outside, before the session.

Each session was sprinkled with rest periods at the discretion of the E or S. During a rest period a subject could stretch, move around, or close his eyes, as desired. He was not allowed to leave the room, smoke, or drink coffee or soda.

After a subject was dark adapted and had fixated the red light and stated his readiness to proceed, a stimulus was presented. Since the experimenter had little or no idea where the threshold would be for a particular combination of pulse length and null period, a guess was made as to which aperature should be used. Then the experimenter operated according to the flow chart shown in Figure 8.

A typical smooth-running sequence would consist of picking the correct aperature, then:

E: "Starting bright" (turns stimulus light on, and slowly starts decreasing its brightness).

S: "I don't see it."

E: (Writes number corresponding to position of brightness control, resets know to a low value, then says. . . .) "Starting dim."

S: "I see it."

E: (Writes number, resets brightness knob to some high value, then says. . . .) "Starting bright."

The exchange would continue until 10 thresholds (a threshold is one ascending and descending pair) had been done. Then

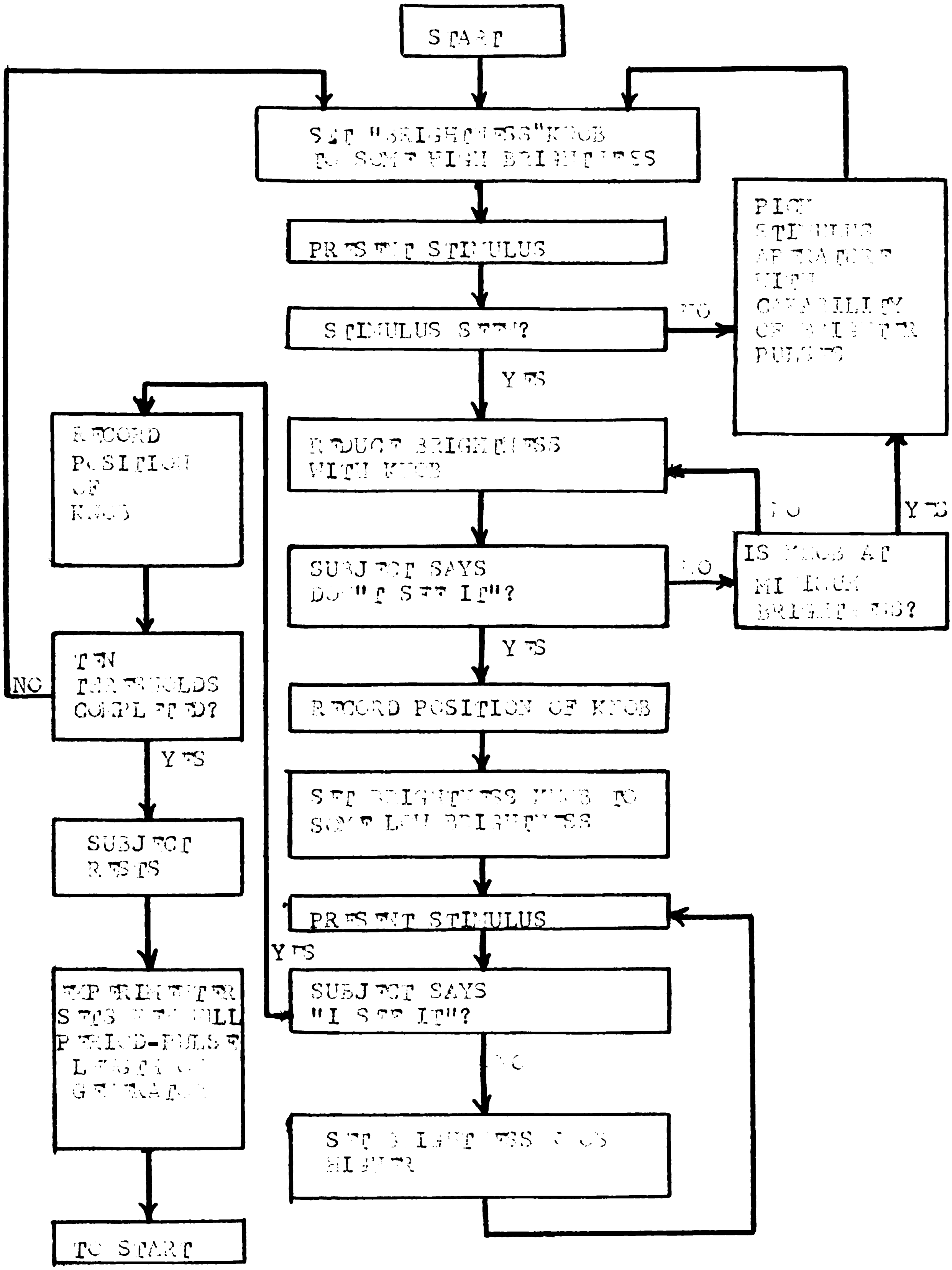

Figure 8. Experimenter actions

a rest period would be called while E set a new pulse-length--null period combination on the console.

Each of the three replications included 10 thresholds for each pulse length-null period combination. The first entire replication was completed but not used in the data analysis, in order to allow practice and learning effects to subside.

In case a brighter or dimmer aperature was needed than was available on the stimulus board, the threshold determination for that data point was postponed until a filter change could be made. Photometric readings were taken after each such change, and at bi-weekly intervals throughout the experiment.

Data sheets were specially prepared, listing the date, time, subject, and other pertinent data. Above each column of data was the pulse length and null period which was being used, along with the appropriate control settings.

When all thresholds for one combination for one subject had been completed, the data sheet was taken to the Olivetti-Underwood Programa desk computer and the means and standard deviations were calculated. Any data which had an unusually high variance were re-determined at the next session.

V. RESULTS

The major results are shown in Figures 9, 10, and 11. Each graph is the data for one subject. The two replications for each subject have been averaged. Figure 12 is a graph of the same results averaged over both replications and all three subjects.

In all of these graphs, the amount of illumination at the subject's eye is plotted on the vertical axis and the pulse length in seconds is plotted on the horizontal axis. The curves are identified by different kinds of lines with longer dashes for longer null periods. Each graph has five curves, one for each null period. Figures 9 - 11 are presented to show how the subjects differed; most of the discussion will center around the over-all average graph, Figure 12. In general, the perturbations and deviations from the expected curves which are found in the individual subject graphs disappear when they are averaged together. This is to be expected, for the laws which the curves are being compared to (Talbot's, Bloch's, etc.) are derived based on averages of large numbers of thresholds. No individual subject in only 20 thresholds could be expected to conform exactly to these laws.

In the Appendix, one graph for each subject will be found which shows how this threshold for the calibration standard flash varies from day to day. The dotted curves are for the

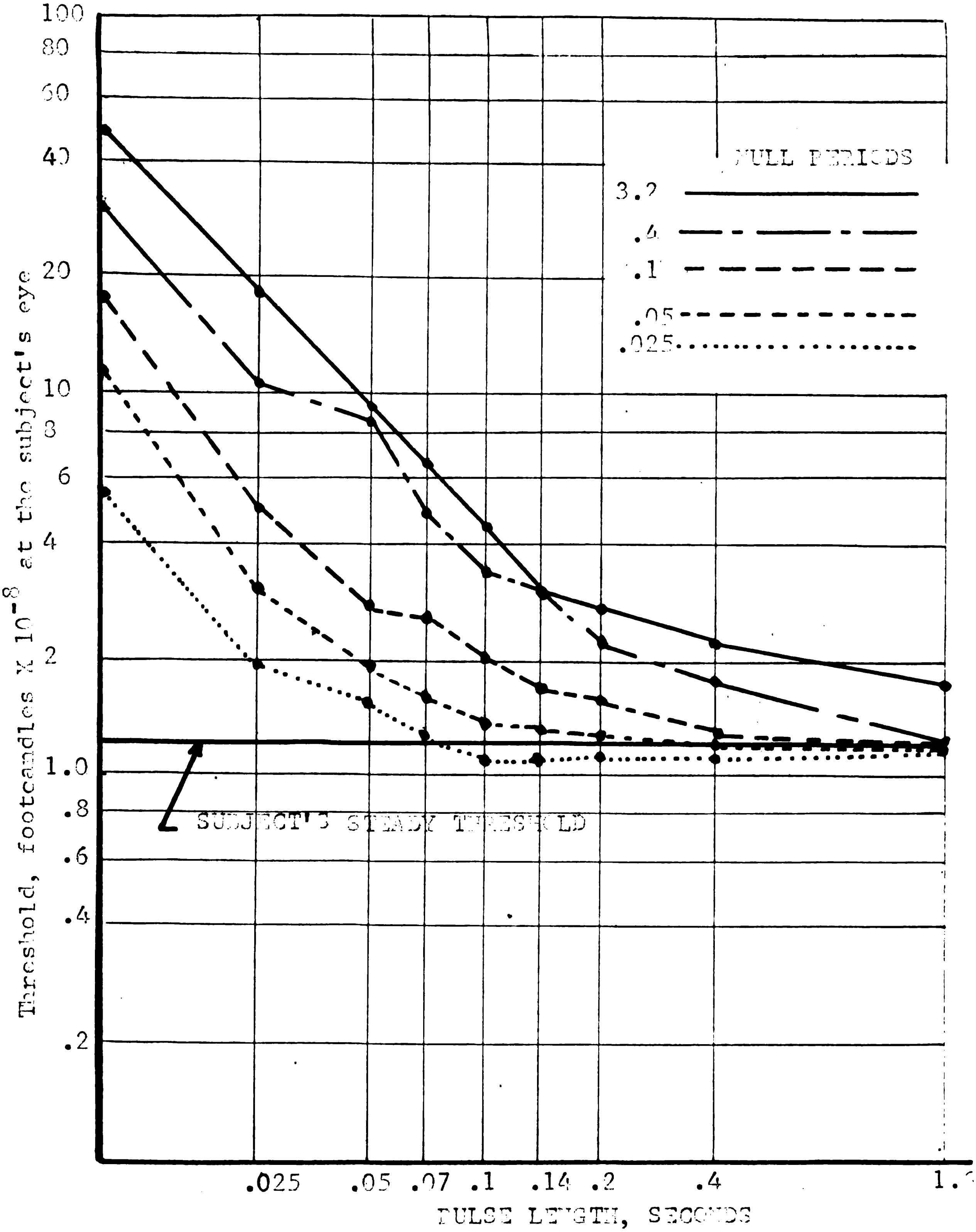

Figure 9. Results ofor subject 1. Average of two replications.

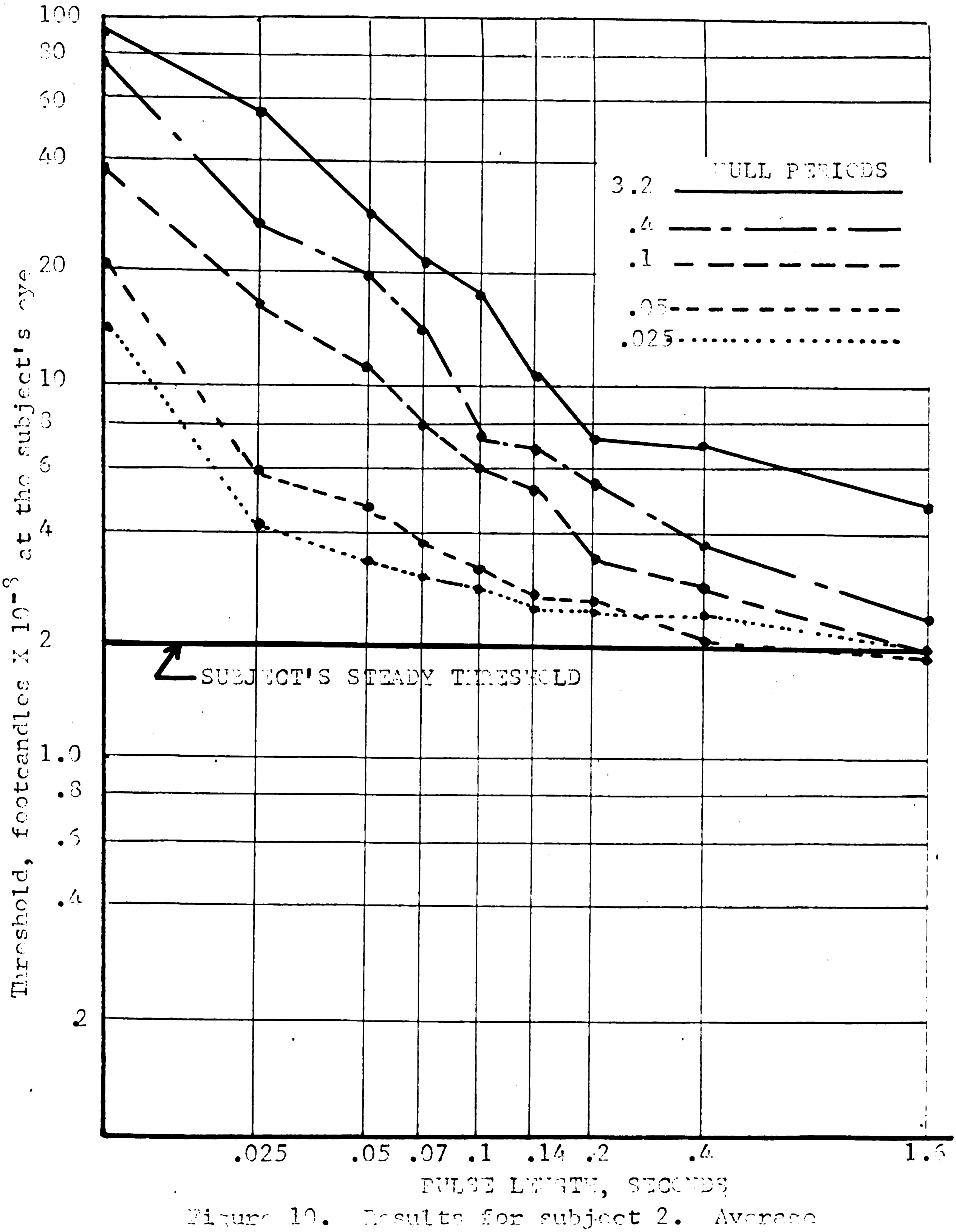

Figure 10. Results for subject 2. Average of two replications.

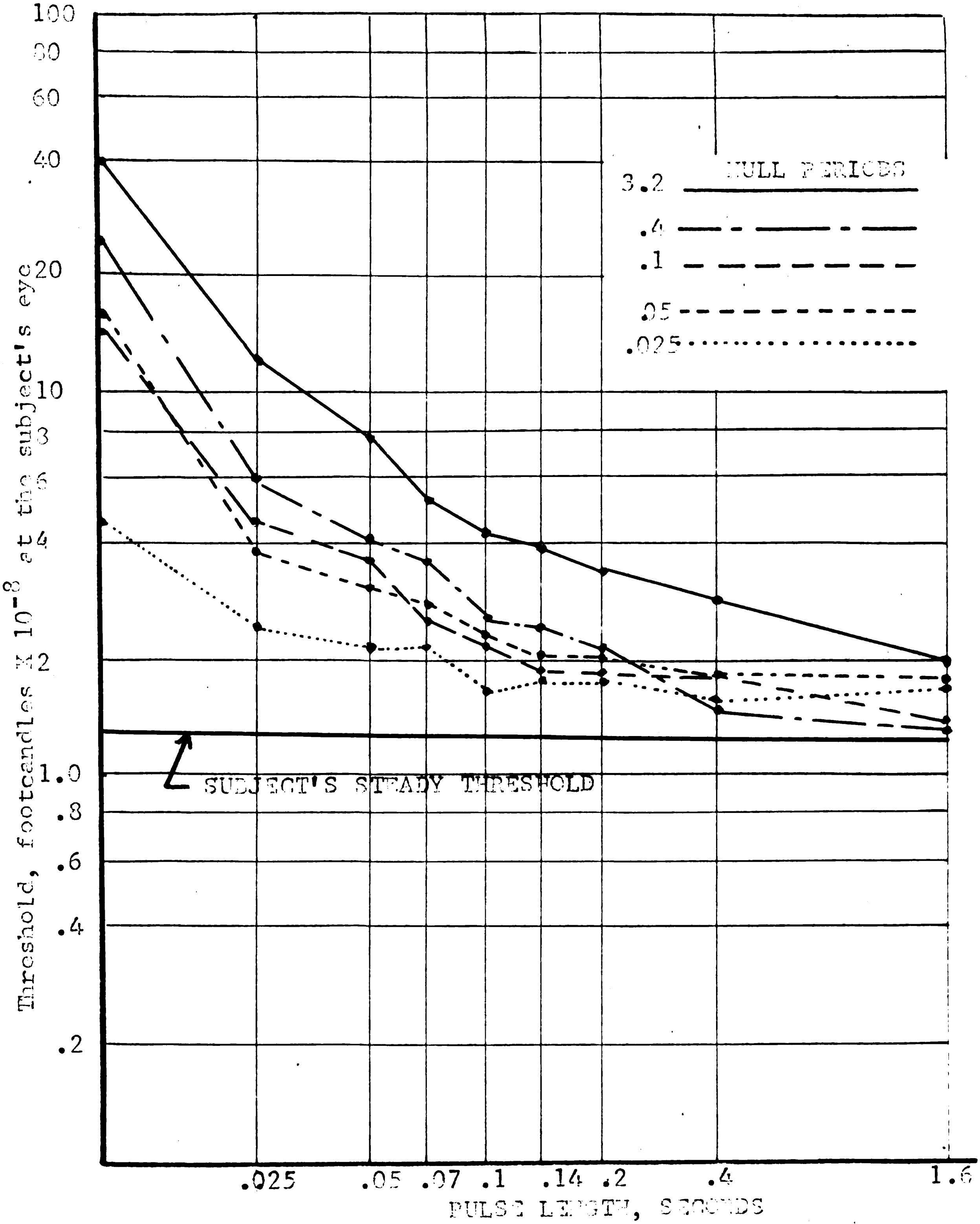

Figure 11. Results for subject 3. Average of two replications.

first calibration flash threshold of the day (at the start of the session), the solid curves for the last one of the session. Various small bumps and valleys appear in the average of these curves, and in general, these were minor and of no consequence. Where major changes in the threshold appeared, the cause was investigated and usually found to be due to subject fatigue, forgetting to wear sunglasses on a bright day, etc. Figure 12 is the key result graph of the experiment. For this reason, the comparison curves (Blondel-Rey, Bloch's Law, etc.) have been plotted here for quick comparison.

The straight lines in the left side of the graph, slanting downward at a 45° angle represent the predictions of Bloch's Law, Ixt = constant. One line has been drawn for each null period curve so that one may see how each compares with the reciprocity prediction.

The curves for .025 and .05 seconds null periods seem to break from the Bloch's Law line at shorter pulse lengths and slope toward the steady light threshold more quickly. Subjects were asked what these lights looked like, and almost unanimous agreement was shown that for these two null periods, with pulse lengths longer than .07 or .1 seconds the stimuli appeared fused. This varied some from subject to subject; but the reason for the shape of the curves is clear; when the stimuli became fused, the threshold is governed by Talbot's Law, and hence their shape will differ from the other curves, where the stimuli weren't fused.

The Blondel-Rey curve has been plotted near to the 3.2 second null period curve, to compare the fit of the data to that law. Up to about .1 seconds, the fit is excellent. As expected, between .1 and about .3 seconds, the data depart from the prediction of the law and rejoin it at about .4 seconds, for a good fit out to the longest pulse length used in this experiment.

All of the curves (except the .025 and .05 null period) show irregularities in the vicinity of .1-.2 seconds. This is slight for the 3.2 null period, extending only from .1 to about .3 seconds. For the .4 null period it is more extensive, starting at about .05 pulse length and extending to about .2. For the .1 null period it also starts around .05 pulse length and extends to about .2 seconds. This irregularity suggests that the "bump" Clark and Blackwell found does indeed exist for different null periods than they used; and is in fact more pronounced than the curves they showed.

The clearest finding of the study is seen by examining the null period curves of Figure 12. If Blondel and Rey's relation were completely correct, all the curves would fall on top of the 3.2 null period one: Obviously, they haven't and in fact, they have sorted themselves out in order of decreasing null period, with no overlaps except in the area of 1.0 seconds for the three most similar null periods.

The general shape of the curves, however, is similar to that predicted by Blondel and Rey, except for the "bump" region noted above. But in general this is to be expected,

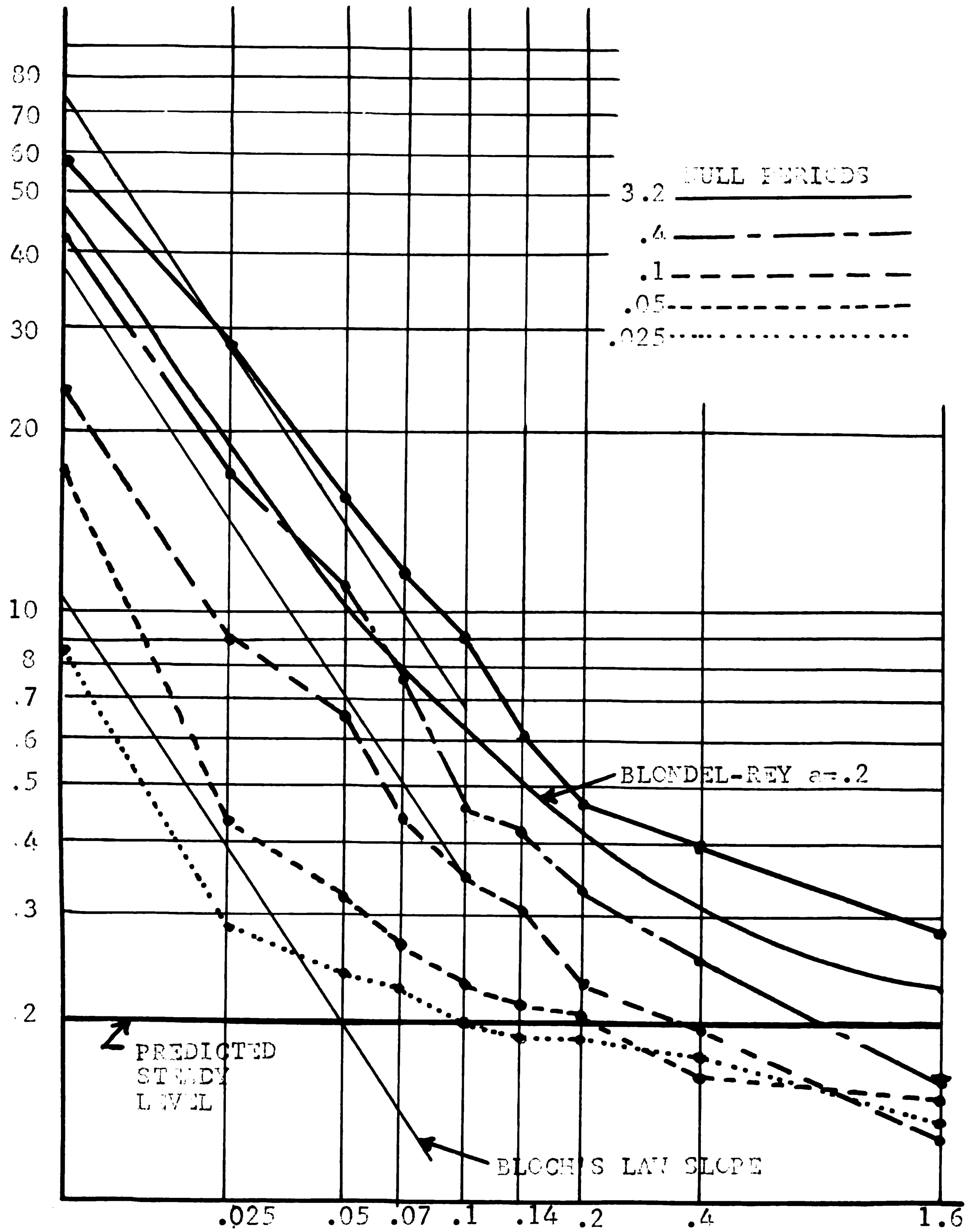

Figure 12. Grand Average over all three subjects, and two replications. Various theoretical curves also shown.

since the curves should match Bloch's Law (asymptotic to Blondel and Rey's for pulse lengths less than .1) and must at some point level off, implying that lengthening the pulse will not lower the threshold (steady level).

The raw data were punched onto cards, and an analysis of variance was carried out on the CDC 3600 computer. The results are shown in Table 3.

Note that the replications effect is not significant, indicating that practice was not a significant factor; for each subject, the first replication was not different from the second. All other main effects are significant. The "remaining error" consists of the sum of interactions between replications and all other effects.

The graphs and AOV table will be discussed and related to each other in detail in the discussion section, which follows.

TABLE 3

ANALYSIS OF VARIANCE

Source of Variance	Sum of Squares	Degrees of Freedom	Mean Square	F Statistic	Approximate Significance Probability of F Statistic
Replications	24.53	1	24.53	3.62	0.059
Subjects	4132.09	2	2066.05	304.89	<0.0005
Null Periods	6085.74	4	1521.44	224.51	<0.0005
Subjects X Null Periods	2252.39	8	281.55	41.55	<0.0005
Pulse Lengths	19447.73	8	2430.97	358.74	<0.0005
Subjects X Pulse Lengths	3813.98	16	238.37	35.18	<0.0005
Null Period X Pulse Lengths	8374.37	32	261.70	38.62	<0.0005
Subjects X Pulse Lengths X Null Periods	2068.70	64	32.32	4.77	<0.0005
Remaining Error	908.00	134	6.78		
Total	47107.57	269			

VI. DISCUSSION

A. ANALYSIS OF VARIANCE RESULTS

The analysis of variance Table, Table 3, gives an overview of the general outcome of the experiment. Each significant factor will be discussed in turn.

1. Subjects

This significant F merely indicates that subjects differ in their average threshold, in the over-all sensitivity of their eyes. Subject 1 seems to be the most sensitive, subject 2 the least.

2. Null Periods

This significant factor is a primary finding of the experiment, and is in fact the main phenomenon which was being investigated. Other investigators have implicitly assumed that null periods other than those which they used in their experiments would give the same results. Blondel and Rey, and Toulmin-Smith and Green used a long null period in the original studies, but their equations have no stated limiting values on null period. Reviewers who have applied the formulas to general cases have made no statement that the formulas should be used with caution. (Projector, 1957; Graham, 1964). Douglas (1957) applies his integral form of the Blondel-Rey equation to flashes with null periods of .1 and .01 seconds. He also states:

> "When the dark period is between .01 and .1 second, the effective intensity will lie between that of a single flash and that of the group. The behavior during the transition is not known." (Douglas, p. 644)

and:

> "If the (dark) periods . . . are of the order of .1 second or more, it is believed that the individual flashes will be seen. Therefore, the effective intensity should be computed on the basis of a single flash." (Ibid.)

The significant effect of the null period shows on all the subjects' graphs, Figures 9 - 11. It is most clearly shown on Figure 12, the over-all averages. The null periods themselves out in order, with the longest (3.2 seconds between flashes) being the least visible, and the shortest (.025 seconds between flashes) being the most visible. At very long pulse lengths (1.6 seconds) null period makes little difference in the threshold.

If the difference in threshold for differing null periods was significant but very small, one would tend to dismiss it as experimental error, or perhaps a real but not useful effect. But the difference shown in thresholds in Figure 12 amounts to as much as one full log unit. We will examine these differences in detail.

The 3.2 second null period can be considered a baseline, since this is closest to the long null periods used by Blondel and Rey, and others. The .4 null period curve has a slightly lower threshold, with the .1 null period below that. About half a log unit has been covered at this point. The flashes are still distinctly separate (the fastest is still only 10 per second, not above CFF). The longer flashes are like steady lights for all null periods, which was expected. The shape of the three curves is very similar.

The .05 and .025 second null periods fall below the .1, and account for the rest of the threshold differences between the longest and shortest null periods. The shapes of the curves for these are slightly different from the others. Subject reports provide the answer as to why: for pulse lengths of 0.1 sec. or longer, these flashes generally appeared to be either a steady light or a light with a slight rippling or fluttering effect superimposed on it, especially for the .025 second null period. Hence, we would not expect these null periods to have effects much different from a steady light. At shorter pulse lengths, the lights again appear to flash, and hence the threshold begins to rise. Since the length of these flashes is shorter than the critical duration (.1 sec.), the slope of the rise is determined by Bloch's Law, $I^{xt=C}$.

3. Pulse Length

It was anticipated that this factor would be significant; every study ever performed in this area has shown a significant effect of pulse length. The irregularities which appeared between .07 and .2 seconds will be discussed later.

4. Subject x Null Period Interaction

This significant interaction indicates that subjects responded differently to some null periods. The reason for it is clear in examining the individual subjects' graphs, Figures 9 - 11. Subject 2's curves separate in order according to null period, subject 1's do not except for the 3.2 and .4 null periods, and subject 3's curves run close together except for the 3.2 null period. Without a large number of subjects with which to explore these different responses more fully, no conclusions can be made about this interaction.

5. Subjects x Pulse Length Interaction

This interaction would not seem to be particularly meaningful. It merely states that some subjects reacted differently to some pulse lengths. Given the different over-all levels of sensitivity of the different Ss eyes, and the slight differences in the nature of the "bump" in the area of .07 to .2 second pulse lengths, it is not surprising to see a significant interaction here.

6. Null Period x Pulse Length Interaction

This significant interaction indicates that the null period curves are not parallel. Obviously they cannot be if they are all to converge at long pulse lengths.

7. Subjects x Pulse Length x Null Period Interaction

This second-order interaction means subjects are differentially affected by combinations of pulse length and null periods. Figures 9 - 11 show this effect; but the effect is small; the general shape of the curves is the same for all subjects.

B. GRAPHICAL RESULTS

Each subject's data is presented in Figures 9 - 11. Since the two replications were not significantly different from each other, they have been averaged together for these three graphs.

Subject 1, whose results are shown in Figure 9, shows a very low threshold over-all, but no abnormalities in the shapes of the curves. The 3.2 second null period line starts by following the Bloch's Law slope at the shortest pulse length used and continued to do so until the critical duration is reached. At this point a break occurs, similar to that shown by Clark and Blackwell. The curve then levels off near the steady threshold level. The .4 second null period performs similarly, except for a slightly high threshold at .05 seconds. This is most likely caused by some random fluctuation in the subject's sensitivity, as none of the other subjects show this particular abnormality. The .4 second null period curve also shows the expected "bump" near .14 seconds.

The rest of the curves are similar to the one for the 3.2 second null period, though below it (lower threshold). All of them (except .025 null period) show some irregularity between .05 and .2 seconds pulse length, before leveling off at the steady light level. The .025 second null period curve does not show this irregularity, but this is probably because the flash appears to the subject to be steady, with this combination of null periods and pulse lengths. Subject 2 has a higher over-all threshold than the other two, but in all other respects his curves are similar to those of subject 1.

Subject 3 is slightly different. The 3.2 second null period curve resembles that of the other subjects, but the curves for the .4, .1 and .05 second null periods are closer together than for the other subjects. At pulse lengths longer than the critical duration, all the curves (except the 3.2 second null period) clump together with a slope near zero. The shapes of the curves, however, are very similar to those of the other Ss. It appears that this subject reacts differently than the other Ss to changes in null period. However, the general trend is the same; the longer null periods are more difficult to see, the shorter ones easier. And again, the irregularities in the area of .07-.2 seconds pulse length appear.

Figure 12 is a composite graph, consisting of the average of the data which went to make up Figures 9 - 11. Hence individual subject idiosyncrasies have (hopefully) been averaged out. Since this is the easiest figure to work with, Bloch's Law, the Blondel-Rey relation, and a predicted steady level have been plotted here for convenience. The predicted steady level was derived by taking the average threshold obtained for each subject each day on the calibration flashes at the start and end of the session. This level (since the calibration flash was above CFF, and had a 50% PCF) was twice what the threshold would have been for a steady light. This over-all average value, for three subjects, was divided by 2. The value obtained should be a good estimate of the threshold which would have been obtained for these subjects if a steady light had actually been used. (A steady stimulus was not used in order not to overheat the glow tubes.) Note that the steady line value is quite close to the points for a 1.6 second pulse for all null periods except the 3.2 second null period. These two independent determinations of a steady light threshold (calibration flash and 1.6 second flash) should agree rather closely.

The 3.2 second null period curve is probably too high, that is, the subjects didn't see these lights as easily as expected. This is probably an artifact caused by the difficulty in seeing a very brief flash in an uncertain location a long time (3.2 seconds) after the previous flash.

The Blondel-Rey Law was derived based on flashes separated by several seconds. It is reasonable, then, that the Blondel-Rey curve would plot near the 3.2 second null period line. This is the line labeled "Blondel-Rey" in Figure 12. Note that the experimental curve follows it very closely up to the critical curation, 0.1 sec. At this point, the experimental data tumble down irregularly, down to a pulse length of about .4 seconds, where it again closely follows the Blondel-Rey curve out to the longest flash used in this experiment. The curves for the null periods 0.4 and 0.1 seconds have a very similar shape, but at a lower threshold. The only departures from the smooth curve are in the area between .07 and .2 seconds pulse length, the same area where Clark and Blackwell found a similar bump.

So, one could characterize the data as agreeing with the established laws where they are applicable (below critical duration and with long null periods). But the findings of this study also would have to cause revision of part of the established laws. In the case of a flash with a null period of less than .5 seconds or so, and a flash of duration less than .4 seconds or so, none of the old laws (Bloch's, Blondel-Rey, or Talbot's) will apply with complete accuracy.

As far as can be seen from the data of this experiment, the irregularities in the curves between .07 and .2 seconds pulse length have no simple mathematical description. Further definition of the area by a very fine-grained study of those pulse lengths might determine a more accurate shape for the curve, and from this, perhaps a simple formula could be derived. It will not be attempted in this paper.

However, the variation due to the change in null period seems amenable to a mathematical treatment. Various curves were plotted in an attempt to fit the data of the experiment. It was found that when "a" is decreased the fit becomes better for the curves obtained with the shorter null periods. The modification of the Blondel-Rey Law becomes obvious, then; for null periods shorter than about 1.0 seconds, the constant "a" must be changed for a good prediction of threshold to be made.

Dr. T. M. Allen, working with the author on the basis of these data, has derived an equation which not only fits the

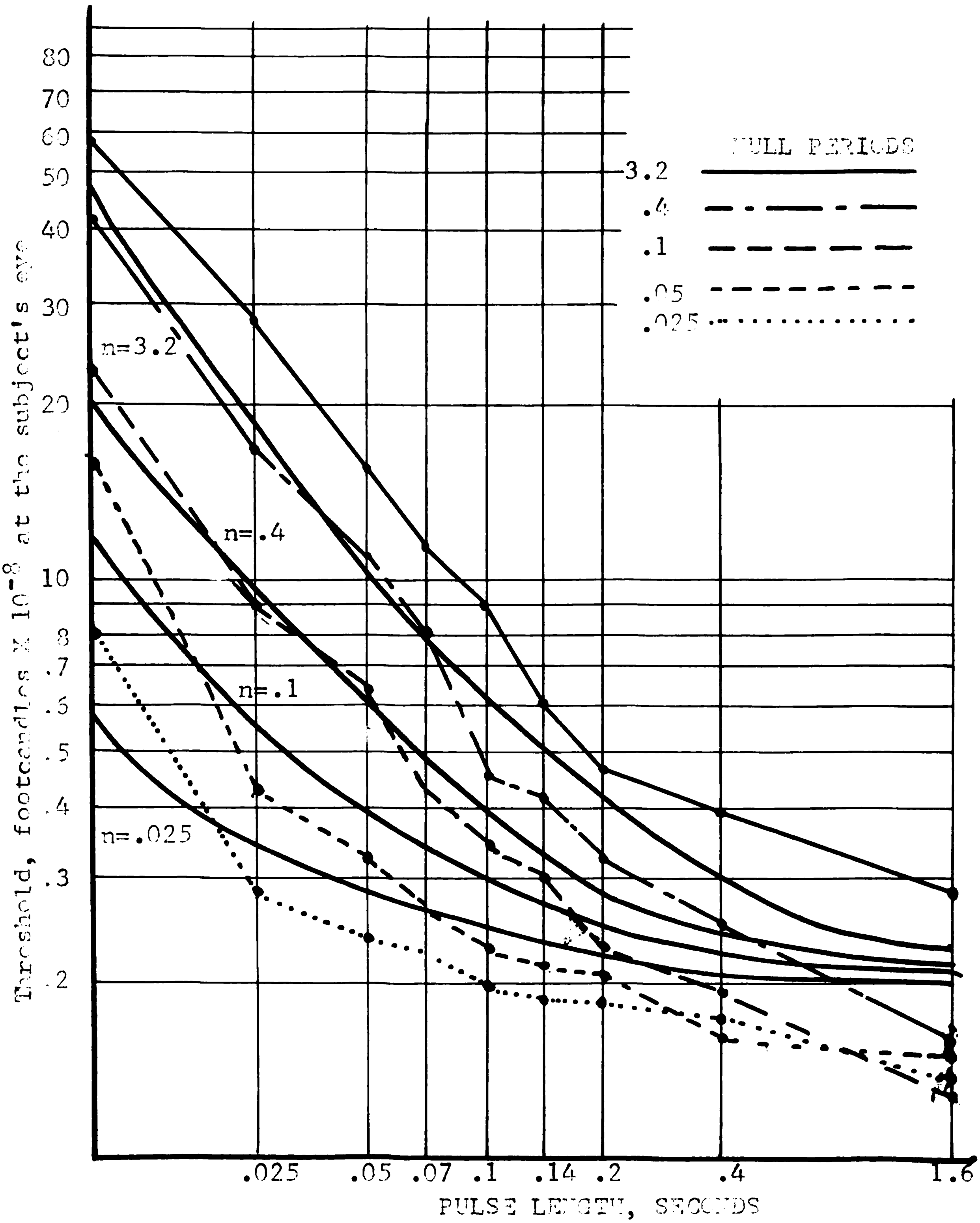

Figure 13. The curves of the data from Figure 12, with curves produced by the Williams-Allen formula described in the text.

data of this study, but also fits the data of other studies, such as those of Schuil, and accurately describes the behavior of the threshold for flashes from above CFF down to steady lights. Essentially, it incorporates Bloch's Law, Talbot's Law, and the Blondel-Rey equation; and in addition it adequately describes areas which none of these laws handle.

The equation simply replaces "a" in the original Blondel-Rey equation with the quantity

$$a = \frac{na_{max}}{n+a_{max}}$$

where

n = null period

a_{max} = the Blondel-Rey constant, 0.2 sec.

substituting, we have

$$\frac{E}{E_o} = \frac{\frac{.2n}{.2 + n} + t}{t}$$

with n and t short, this reduces to approximately Talbot's Law,

$$\frac{E}{E_o} = \frac{n + t}{t} .$$

With n long and t more than .1 seconds, we have approximately

$$\frac{E}{E_o} = \frac{.2 + t}{t}$$

which is the Blondel-Rey equation. With n long and $t<.1$ seconds, we have approximately Bloch's Law.

In Figure 13, the curves for this equation are plotted along with the data of the present experiment. The fit, while not perfect, is certainly close enough to be useful, and is far better than any of the other equations discussed. Further research with more subjects would be necessary to refine the equation or reject it in favor of a better one.

VII. CONCLUSIONS

It can be concluded that the Blondel-Rey equation and subsequent modifications of it are inadequate to handle predictions of the threshold of lights flashing with a null period of less than approximately one second. However, the Blondel-Rey equation can be modified simply by changing "a" depending on the null period. The new equation is

$$\frac{E}{E_o} = \frac{\frac{.2n}{.2 + n} + t}{t}$$

where

E = threshold illumination for the flashing light
E_o = threshold illumination for a steady light
n = null period
t = pulse length

This equation can be used for any pulse length or null period, whether the light appears to flash or not. Although further data would be needed for a precise test of this equation, it is likely to be useful in practical settings due to its simplicity.

APPENDIX

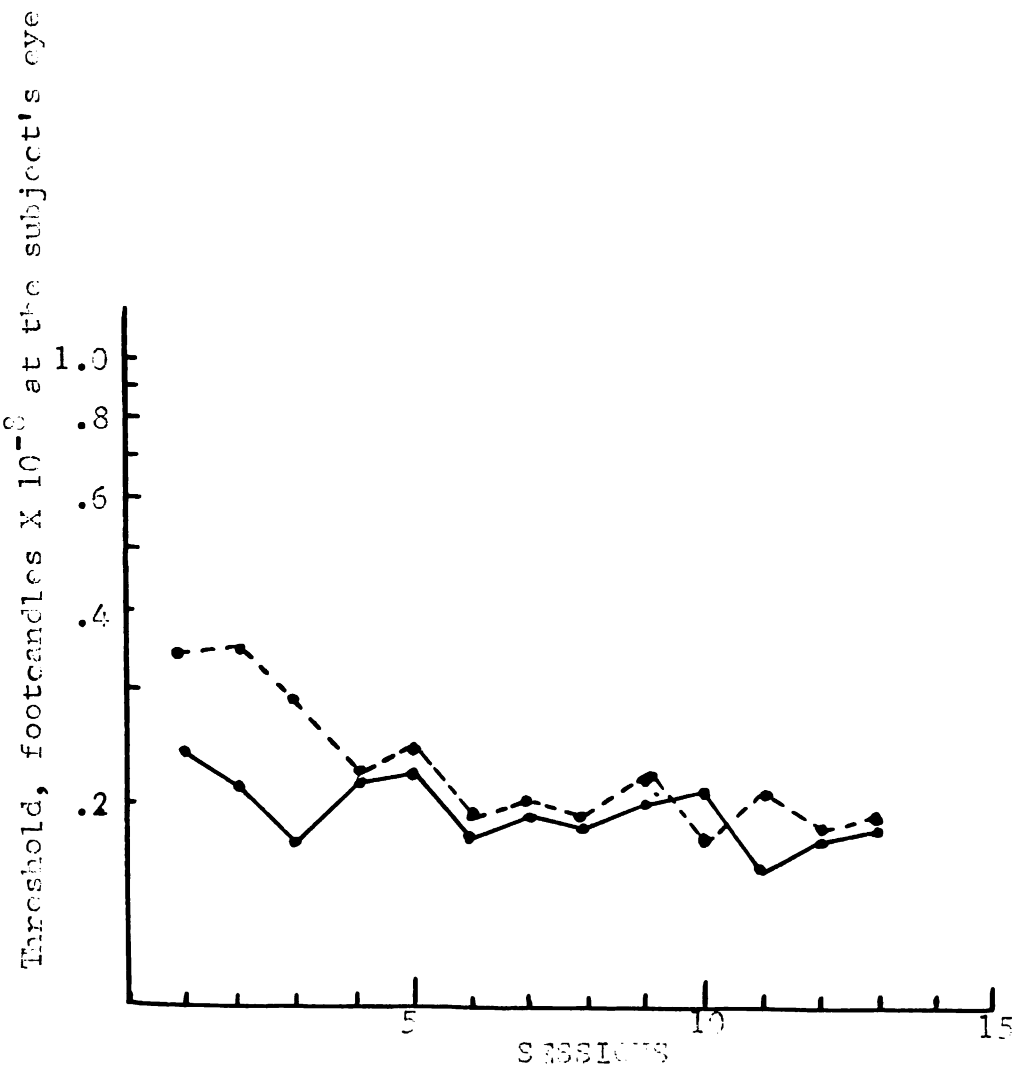

Figure A-1. Calibration flash data for subject 1.

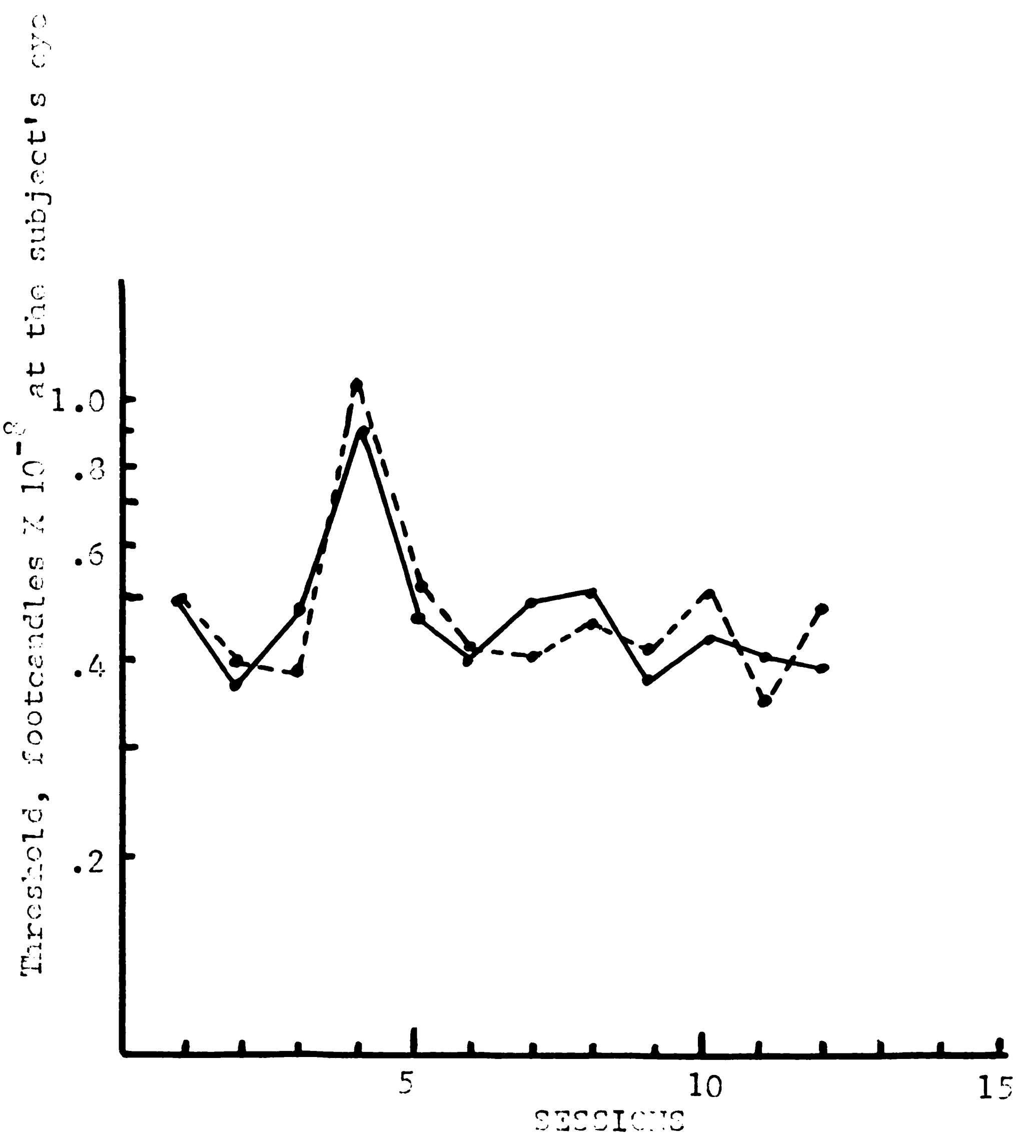

Figure A-2. Calibration flash data for subject 2.

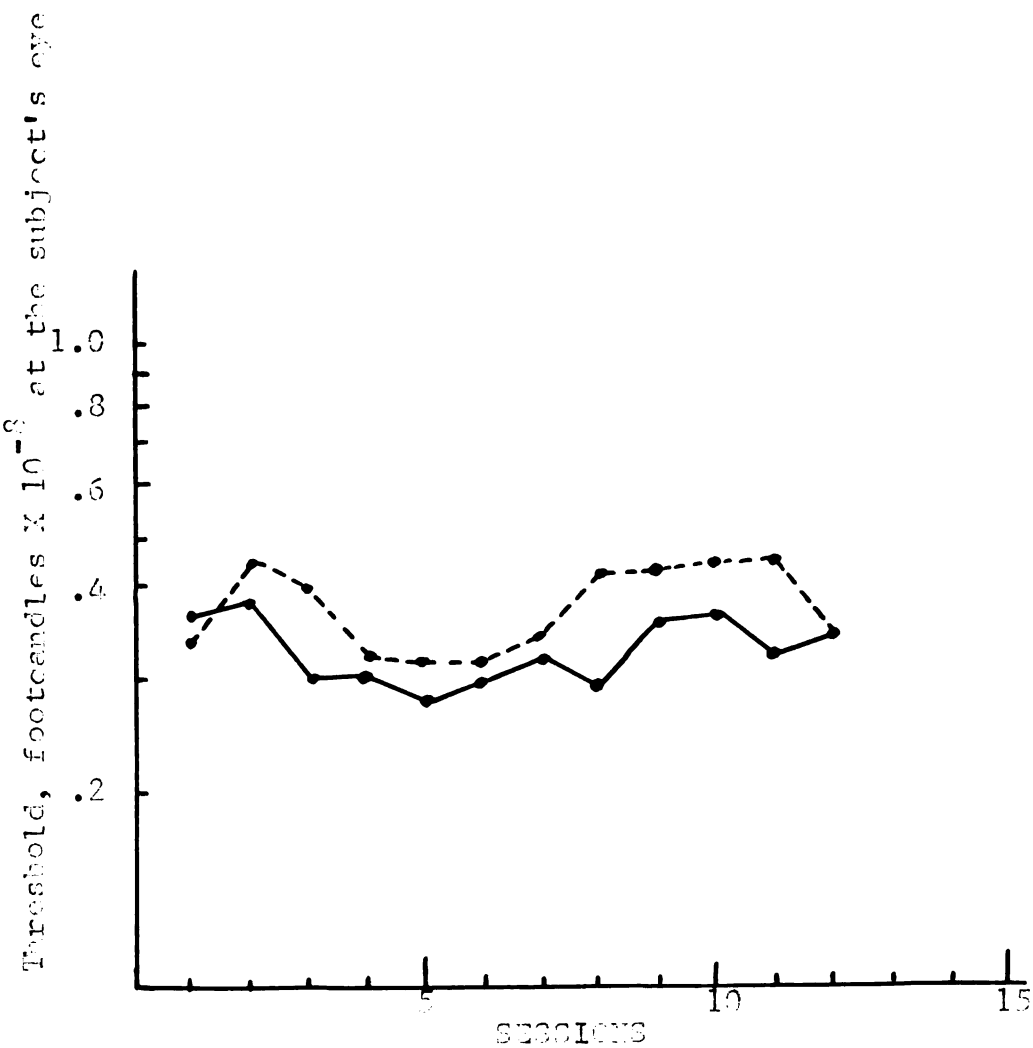

Figure A-3. Calibration flash data for subject 3.

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