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TECHNOLOGICAL CHANGE, ECONOMIES OF SCALE, TRADED INTERMEDIATE PRODUCTS, AND SUBSTITUTION BETWEEN ENERGY AND NON-ENERGY INPUTS IN THE U.S. MANUFACTURING SECTOR

By

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ABSTRACT

TECHNOLOGICAL CHANGE, ECONOMIES OF SCALE, TRADED INTERMEDIATE PRODUCTS, AND SUBSTITUTION BETWEEN ENERGY AND NONENERGY INPUTS IN THE U.S. MANUFACTURING SECTOR

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In recent multi-input studies of energy demand in U.S. manufacturing, the most frequent model specification has consisted of employing a static profit-maximization framework defined over inputs of capital, labor, gross energy, gross materials, and gross output ("gross" meaning that these inputs include intra-industry, inter-firm shipments of traded intermediate products). In such models, the prices of the "energy" and "materials" aggregate inputs must be treated as endogenous rather than exogenous variables as has been commonly assumed. Thus, in such "gross" models, the application of Shephard's Lemma to obtain Hicksian industry factor demand functions is inappropriate as shown by Samuelson (1953).

This study has considered and estimated an alternative model in which cost and factor demand functions for U.S. manufacturing and nonenergy manufacturing sectors (for 1947-71 period) are conditional upon the level of output of these sectors delivered to final demand (i.e. "net" sector output).

This "net" model framework provides a proper context for energy policy discussion since we are usually concerned with the energy intensity of a given level of net output.

For purposes of estimation (via duality) a translog cost function is specified as a second order Taylor series approximation to the underlying production process. This study, then, presents estimates of two commonly used summary measures of price responsiveness for both sectors, namely, the factor price elasticities and the Allen partial elasticities of substitution among inputs.

Our "net" model framework shows considerably smaller values for factor price elasticities compared to the estimated values obtained in other studies. Regarding other issues, our empirical results indicate that homotheticity, homogeneity, constant returns to scale, and neutrality of technological change must all be rejected for the manufacturing sector, while for the nonenergy sector the homogeneous-Hicks-neutral specification is justifiable. The estimation of Hicks biases for the manufacturing sector reveals that over the 1947-71 period technological change has been labor-saving; capital, energy, and material using. This study has also examined returns to scale, and we conclude that in both sectors the source of growth has been primarily due to utilization of economies of scale. Finally, our data rejects value-added specification for both sectors.

DEDICATED TO

My wife, Fattaneh,

My parents,

and my daughter, Sheava.

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INTRODUCTION

The oil embargo of 1973 and subsequent enormous and continuous increases in the price of energy since then have accelerated extensive research on energy demand and possibilities for substitution among energy and non-energy inputs. Nowadays, the common belief that scarcity of energy will affect output, or at least retard its growth, looks so obvious as to require no argument. However, energy consumption in various sectors of the economy is affected differently by energy shortages. Differences in energy consumption are caused both by differences in total output and by differences in the energy intensiveness of production.

The manufacturing sector accounts for approximately one-fourth of the aggregate energy consumed for power and heat in the U.S. This sector, therefore, has been broadly classified as an important sector to examine if a wise energy policy is to be pursued. Pursuing such a policy, however, requires empirical estimates of energy demand functions and elasticities of substitution between energy and non-energy inputs, as the producing units' response to rising energy prices involves substitution.

Recently, a large number of econometric studies have focused on possibilities of factor substitution in U.S. manufacturing. Examples of such studies are: Berndt and Jorgenson (1973), Hudson and Jorgenson (1974), Berndt and Wood (1975, and 1979), Griffin and Gregory (1976), Berndt, Fuss and Waverman (1977), Pindyck (1978), Berndt and Khaled (1979), etc. The most frequent model specification in these studies has been consisted of utilizing a flexible functional form for the aggregate cost function defined over four aggregate inputs of capital services, labor services, energy and materials. Then, researchers have employed the well-known Shephard's Lemma of the theory of the firm to obtain the industry's conditional factor demand as first partial derivatives of the industry's aggregate cost function with respect to input prices. In the theory of the firm, application of Shephard's Lemma to obtain the firm's conditional factor demands is based on the critical assumption of exogenous input prices. Analogous application of Shephard's Lemma in the industry context violates this assumption, since the prices of energy and materials inputs are endogeneous to the manufacturing sector.

The aggregate output of the manufacturing sector and purchased materials and energy inputs have traditionally been measured as gross magnitudes. As such, these gross magnitudes contain the intra-industry, inter-firm shipments of intermediate products which move among firms. More specifically, a considerable portion of total materials and energy inputs purchased by the manufacturing sector is produced by other firms within the manufacturing sector; the balance are imported from other sectors of the economy (i.e. are "primary" to the manufacturing sector). While the prices of these primary energy and materials inputs can be considered exogeneous, the prices of the internally produced energy and materials are endogenous, since they respond to any change in the prices of primary factors of production. The subsequent application of Shephard's Lemma to obtain conditional factor demand functions, therefore, must be considered inappropriate in these models. This is precisely the specification error contained in these gross industry-level studies.

In this study we consider an alternative specification for the aggregate cost and conditional factor demand functions, and estimate the production structure of the U.S. manufacturing sector as well as the non-energy manufacturing sector for 1947-71 period separately. In particular, we specify our model, properly, over the level of deliveries of the aggregate manufacturing sector to the balance of the economy, i.e., the "net" output level of the sector. Energy policy discussions are usually, and properly, concerned with the energy intensity of net output. Consequently, we must seek estimates of the price elasticities of factor demand conditional upon the level of net output. In contrast, the "gross" model formulation does not provide an appropriate context for a meaningful energy policy discussion, since an industry's net output will fluctuate as primary factor prices change. The "net" model formulation also corresponds to the theory of consistent aggregation of factor demand functions of firms with neo-classical production functions (see Green (1964), chapter 9).

Once our model is properly specified, this study will present estimates of two commonly used summary measures of price responsiveness; namely, the price elasticity of demand for capital, labor, energy and materials, and the Allen partial elasticities of substitution between energy and non-energy inputs. These estimates, which are considerably different from their estimated values in other studies, constitute a challenge to the existing estimates of these elasticities.

Traditionally, a particular assumption frequently employed in econometric studies has been the absence of technical change. In connection with constant returns to scale this implies that all changes in input bundles result from price-induced substitution within a fixed technology. A slightly weaker maintained hypothesis would be that all technical change was of a "Hicks-neutral" character. Again, in such a specification input mix changes are due to factor price changes.

However, it is ideally desirable to estimate production structures under weaker assumptions. In particular, we relax both the assumption of "Hicks-neutral" technical change and constant returns to scale which have been maintained in previous studies of manufacturing. This will allow us to examine the effect of biased technical change, namely, input mix changes which occur independently of relative price changes over time. We also can examine the return-to-scale characteristic of the production process. This amounts, therefore, to testing the hypothesis of "Hicks neutrality" and constant returns to scale rather than to impose them a priori. Our empirical results for the manufacturing sector reveal that technical change has been decidedly non-neutral (labor-saving; and capital, energy, and materials using), and that scale economies are substantial.

Finally, this study examines various types of weak functional separability among inputs. The idea originated with Leontief (1947), and provides a criterion for input aggregation. According to Leontief, subsets of inputs which are weakly separable from others may be formed into consistent aggregates with the property that marginal changes in the level of other input outside the separable subset have no effect on the technical relation among inputs inside the subset. One important application of separability is in the derivation of a value-added function. A large number of empirical studies dealing with investment demand and degree of technical substitution between capital and labor have employed a value-added concept instead of output. In this study we put this hypothesis to test along with other weakly separable specifications.

To accomplish the objectives of this study we estimate the production relationship via the dual cost function because of its several econometric and theoretical advantages. Obviously it is desirable to employ a specific functional form which does not impose any a priori restriction on the Allen partial elasticity of substitution. The trancendental logarithmic (translog) function is such a candidate. Further, in the empirical implementation of the functional form some authors (e.g. Berndt and Wood (1975), Berndt and Christensen (1973a, 1973b) have taken the translog function as an exact representation of the true underlying production function. But Blackorby, Pirmont, and Russell (1977) and Denny and Fuss (1977) have shown that when the translog function is assumed to be an exact representation of the underlying technology, the separability conditions stated in these studies are too restrictive. This study departs from this restrictive assumption by assuming that the translog cost function is only an approximation to the underlying production technology.

CHAPTER I

APPLICATION OF DUALITY PRINCIPLE IN THE THEORY OF PRODUCTION AND COST - A REVIEW

1.0 Introduction

It is a tradition to start production theory with a set of physical, technological constraints or possibilities, usually called the production or transformation function or input requirement set, which describe the feasible production activity of the producing unit, also called the "firm". The theory develops, then, by formulating the decision of the firm which acts to achieve its objectives subject to the limitation of its technology and within a certain institutional context. This procedure results in constructed factor demands and output supplies being a function of the technical limitation and the economic environment surrounding the firm.

An interesting alternative is to approach the theory of production directly from observed economic data such as demands, supplies, prices, costs, profits, and revenues. This alternative method permits us to formulate the economic theory directly in terms of these functions. This approach is not only as fundamental as the traditional theory, but it is also more tractable. If the production function or input requirement set represents the firm's technological possibilities, the cost and profit functions are concerned with its economic behavior.

This procedure gets its power from the so called "duality theorem" between technology and cost functions or, more generally, profit function that establishes that the two approaches are equivalent and equally fundamental. There are two main practical advantages associated with the theory of production duality. First, it enables us to derive, painlessly, the system of demand and supply equations consistent with the optimization behavior of the firm just by direct differentiation of a cost, profit or revenue function, in contrast with solving explicitly a constrained optimization problem by the traditional (Lagrange multiplier) method, where optimization and obtaining the explicit solution involve messy algebraic operation even with objective functions of relatively simple form. Second, duality theory is attractive from the point of view that the "comparative static" results associated with optimizing behavior are very easily derived.

Duality theory has its roots in the work of Hotteling (1932), Roy (1942), Hicks (1946), and Samuelson (1947); but it was the pioneering work of Shephard (1953) which treated the subject comprehensively and provided the proof of basic duality between the technology and cost function. His work was extended and refined later on by a number of authors, among whom were McFadden (1962), Uzawa (1964), Shephard (1970), Diewert (1971), Lau (1978 and 1976) and others.

These works have built a framework and have paved the way for empirical research where use can be made of flexible functional forms such as the translog, and enabled researchers to use such complex functional forms rather easily, compared with the traditional methods. Empirical works such as Nerlove (1963), McFadden (1964), Diewert (1969a,b), Christensen, Jorgenson, and Lau (1971), Berndt and Christensen (1973a), Berndt and Wood (1975), Humphrey and Moroney (1975), Atkinson and Halversen (1976), and others are examples of works in which the dual cost and profit functions have been used as a basic tool in econometric production analysis.

In what follows we start with a description of production technology utilizing the concept of the input requirement set and then the production function (section 2). Then the duality between the input requirement set and the production function is explained. In section (4), the cost function and the duality between the cost and production function will be clarified; in particular we explain that, given fixed factor prices and a production function satisfying several properties, a total cost function may be derived under the assumption of cost-minimizing behavior, and conversely, given a cost function meeting some regularity conditions, a production function can be derived which in turn may be used to derive the original cost function. Finally, in the last section, we study the profit function and demonstrate the duality between the production function and profit function.

Now we will introduce the following notational conventions which will be utilized: x is an nx1 vector with elements $x_1, \ldots, x_n$; $x^1 \geq x^2$ (with x^1 and x^2 each being n dimensional vector) means that each element of x^1 is greater than or equal to the corresponding element of x^2; $x^1 > x^2$ means that each element of vector x^1 is greater than or equal to the corresponding element of x^2 and additionally at least one element of x^1 must be strictly greater than the corresponding element of x^2; $x^1 >> x^2$ means that each element of x^1 is strictly greater than that of x^2 correspondingly; x' represent the transpose of x; $\underline{0}$ is an nx1 vector whose elements are all zero; and $R^n{}_+ = (x : x \geq \underline{0})$ is the non-negative orthant in n dimentional Euclidian space.

1.1 Definition of Production Technology

The technology of a producing unit, utilizing the service of flows of several inputs to produce a single output, can be described in several ways. One convenient way is to use the notion of an "input requirement set".

Suppose that there are n inputs $x_i \geq 0$, i = 1, ..., n. Then the production structure of a producing unit can be characterized by

$$\Omega(y) \quad = \quad \{x' = (x_1, \ldots, x_n) \text{ can produce at least } y\} \tag{1.1}$$

where x is an nx1 vector of inputs, and $\Omega(y)$ specifies the set of all input combinations which result in a given level of output, y. The familiar isoquant might be seen as the boundary of $\Omega(y)$, i.e. the efficient set of the input requirement set which is associated with a specified level of output y:

$$I(y) = \{x' = (x_1, \ldots, x_n) \text{ can produce, exactly, } y\} \quad (1.2)$$

The input requirement set is assumed to have the following properties:

Assumption (1): $\Omega(y)$ is a nonempty subset of the non-negative orthant, $R^n{}_+$. It is possible that some inputs are not put to use; however, a zero level of output will result when no inputs at all are utilized. Then it must be true that $\Omega(0) = R^n{}_+$ and $y > 0$ implies that $0 \notin \Omega(y)$; or stating this differently: if $0 \varepsilon I(y) \Rightarrow y = 0$.

Assumption (2): $\Omega(y)$ is closed. Closure means that if a sequence of points x^n in $\Omega(y)$ converges, the limiting point, say x*, also belongs to $\Omega(y)$ and can produce y. Therefore $\Omega(y)$ contains all its limiting points. Considering the definition of the isoquant we see that the isoquant of $\Omega(y)$ belongs to $\Omega(y)$.

Assumption (3): $\Omega(y)$ is monotonic. If an input bundle x^1 can produce a given level of output, then this level of output can also be produced by a larger input bundle, i.e. if $x^1 \varepsilon \Omega(y)$ and $x^2 \geq x^1$, then $x^2 \varepsilon \Omega(y)$. Similarly, an input bundle capable of producing a given level of output can certainly produce a smaller output level, i.e. if $y \geq y'$, then $\Omega(y) \subseteq \Omega(y')$. This is what is termed "free disposal".

Assumption (4): $\Omega(y)$ is a convex set. Convexity implies that if x^1 and x^2 can both produce y, then any linear combination of x^1 and x^2 can also produce y, i.e. if x^1 and $x^2 \varepsilon \Omega(y)$, then $\theta x^1 + (1-\theta)x^2 \varepsilon \Omega(y)$, $0 \leq \theta \leq 1$. Convexity ensures that we have a well-behaved technology, i.e., the marginal rate of substitution between inputs is nonincreasing.

1.2 Production Function

An alternative way to describe the technology of a firm is to utilize the concept of a "production function". Samuelson (1947) refers to it as a "catelogue of possibilities" giving the maximum amount of output, y, which can be obtained from any given bundle of inputs $(x_1, \ldots, x_n)$. Using the concept of an input requirement set, the production function will be defined as:

$$f(x) = \max_y \{y \mid x \varepsilon \Omega(y)\} \tag{1.3}$$

This means that for any $x \varepsilon \Omega(y)$, f(x) is the largest output which can be produced. With the properties (1)-(4) assumed about $\Omega(y)$, f(x) has the following properties:

(i) Domain: f(x) is a real valued function of x defined for every $x \varepsilon R^n_+$ and it is finite if x is finite; and $f(\underline{0}) = 0$.

(ii) Monotonicity: the production function is nondecreasing in x; i.e.

$$x^2 > x^1 \Rightarrow f(x^2) \geq f(x^1).$$

iii) Continuity: The production function is continuous from above; i.e., if for every integer N, $x^N \geq 0$, $f(x^N) \geq y$, $\lim_{N\to\infty} x^N = x$, and $y = f(x)$, then we have $\lim_{N\to\infty} f(x^N) = y$. This is, of course, a weaker property than continuity, and is also consistent with certain discontinuous production processes.

(iv) Concavity: the production function is quasi-concave over R^n_+, because the set $\{x: f(x) \geq y,\ x \varepsilon R^n_+\}$ is convex for every $y \geq 0$. This property ensures that the marginal rates of substitution are now nonincreasing.

Proof: To show (i), $f(\underline{0}) = 0$, we must show that: $f(\underline{0}) \equiv A = \max_y \{y | 0 \varepsilon \Omega(y)\} = \{0\}$. That is, we must show that the only element of A is zero. By assumption, $y > 0$ implies that $\underline{0} \notin \Omega(y)$. Thus the elements of A <u>can not be positive</u>. Therefore the only elements of A are zero. Then $0 \varepsilon A$ and $f(0) = \max_y \{y | 0 \varepsilon \Omega(y)\} = \max_y \{0\} = \{0\}$. Q.E.D.

To see (ii), it must be shown that $x^2 \geq x^1$ implies that $f(x^2) \geq f(x^1)$. To prove this it is enough to show that

$$A = \{y : x^1 \varepsilon \Omega(y)\} \subseteq B = \{y : x^2 \varepsilon \Omega(y)\}.$$

Let $y_0 \varepsilon A$. This implies that $x^1 \varepsilon \Omega(y_0)$. But $x^2 \geq x^1$ and monotonicity of $\Omega(y)$ implies that $x^2 \varepsilon \Omega(y_0)$. Thus $y_0 \varepsilon B$. Therefore $A \subseteq B$.

$$\therefore f(x^2) = \max_y B \geq \max_y A = f(x^1). \quad \text{Q.E.D.}$$

1.3 Duality Between Input Requirement Set and Production Function

The production function is derivable from the input requirement set by definition (1.3). As shown by Diewert (1971) it is possible to assume a production function, $f(x)$, with the above properties and derive:

$$\Omega^*(y) \quad = \quad \{x : f(x) \geq y,\ x \varepsilon R^n_+\}. \qquad (1.4)$$

It can be shown that $\Omega^*(y)$ posses the four properties mentioned above about the input requirement set. On the other hand, using $\Omega^*(y)$ in (1.3) results in a production function, $f^*(x)$, which is identical to $f(x)$, i.e. $f^*(x) = f(x)$. In a similar way, starting with $\Omega(y)$ to derive $f(x)$, and then using $f(x)$ in (1.4), we obtain $\Omega^*(y)$. Then it is true again that $\Omega^*(y) = \Omega(y)$. This is what we refer to as full duality between the input requirement set and the production function.

1.4 Cost Function

One of the main behavioral assumptions, or rules of behavior, in micro analysis of a firm is that of profit maximization. Generally a firm will choose that input bundle that minimizes the cost of producing a given level of output, and then selects that output level, y, which maximizes profit. In what follows we first consider the problem of cost minimization and then that of profit maximization.

Suppose that a producing unit, whose technology is given by an n factor production function $y = f(x_1, \ldots, x_n)$, or equivalently by input requirement set, $\Omega(y)$, is facing a given vector of factor prices, $w' = (w_1, \ldots, w_n)$, $w_i > 0$, $i = 1, \ldots, n$, and wishes to produce a specified level of output y. Assuming that the producer will minimize the cost of production of that output level, the cost function is defined as

$$C(w,y) = \min_{x} \{w'x \mid x \varepsilon \Omega(y),\ x \geq \underline{0}\} \tag{1.5}$$

or equivalently

$$C(w,y) = \min_{x} \{w'x \mid f(x) \geq y,\ x \geq \underline{0}\} \tag{1.5'}$$

This simply says that the producing unit takes the factor prices as given, and attempts to minimize the total cost of a specified level of output. Then the total cost function, in general, depends upon the specified level of output, y, the given vector of factor prices, and f(x), the given production function.

Theorem (1): C(w,y) has the following properties:

(i) C(w,y) is a positive real valued function defined and finite for all finite y > 0.

$w' = (w_1, \ldots, w_n), w_i > 0$ and $C(w,0) = 0$

(ii) C(w,y) is differtiable in w and

$$\frac{\partial C(w,y)}{\partial w_i} = x_i(w,y), \quad i=1, \ldots, n.$$

where the $x_i(x,y)$ are the conditional factor demand functions, and depend upon the level of output produced and upon the input prices. This is known as Shephard's Lemma, and for a simple proof see Diewert (1971, pp. 495-496).

(iii) $C(w,y)$ is contineous in (w,y).

(iv) $C(w,y)$ is a nondecreasing left continuous function in y, and tends to plus infinity as y tends to plus infinity for every $w >> 0$.

(v) $C(w,y)$ is a nondecreasing function in w.

(vi) $C(w,y)$ is homogeneous of degree one in w.

(vii) $C(w,y)$ is concave in w.

(viii) $C(w,y)$ is strictly convex in y.

(ix) $x_i(w,y)$ is the cost minimizing bundle of input i needed to produce output $y > 0$ given factor prices $w >> 0$.

(x) $x_i(w,y)$ is continuous in (w,y).

(xi) $x_i(w,y)$ is homogeneous of degree zero in w.

1.5 Duality Between Cost Function and Input Requirement Set

We saw that by using a production function or input requirement set which satisfies certain regularity assumptions (i-iv) we are capable of deriving a cost function which satisfies some desirable properties (i-xi) via definition (1.5) or (1.5'). Now assume that we are given a cost function, $C(w,y)$, satisfying properties (i,iv,v,vi,vii). Then it can be shown that this cost function enables us to derive or generate a family of production possibility set or input requirement sets, $L(y)$, via the following:

$$L(y) = \{x\colon w'x \geq C(w,y),\ w >> 0 \text{ and } x \geq 0\} \qquad (1.6)$$

for $y > 0$ and

$$L(0) = R^n_+ = \{x\colon x \geq 0\} \qquad \text{for } y = 0.$$

The sets L(y) have properties (i-v) and is identical to $\Omega(y)$. In turn we can use L(y) in (1.5) to derive the cost function C*(w,y), which is identical to C(w,y), which has generated L(y). This is what we mean by the duality between the cost function and the input requirement set. But by virtue of the duality between the input requirement set and the production function, there also exists a duality between the cost and production function as

$$f(x) = \max_{y} \{y | w'x \geq C(w,y)\} \tag{1.7}$$

There are two important special cases which impose a certain functional form on the cost function, namely those of constant returns to scale (CRS) technology and homotheticity of the production function. As Varian (1978) puts it, "There is a convincing argument that all firms should exhibit at least constant return to scale. The reason is that the firm can always duplicate what it has been doing before...thus if the firm is currently producing y, (by doubling) all of its inputs it should be able to build another plant exactly like the first and produce exactly the same amount in each plant. Thus with twice the inputs, the firm can produce twice the output". However, he mentions several counter arguments against this replication argument as follows: First, doubling the inputs may give us twice the output, but this does not necessarily imply that utilizing half the inputs gives us half the output as is required for CRS. Second, the replication argument demands that all the inputs be increased. Considering that, in the short run, some inputs are fixed we would expect that short run technologies show decreasing rather than CRS, since it cannot be taken for granted that doubling only some of the inputs will result in a doubling of the output. However, even in the long run it might be impossible to increase all inputs. Finally, we might be faced with the possibility of increasing return to scale, since the replication argument for CRS says only that doubling this scale should give at least double the output. However, the long run CRS situation is considered to be a standard situation. The following proposition shows the form of cost function when we have CRS technology (see Diewert (1974)).

Proposition (1): If the production function, satisfying properties (i-iv), is, in addition, exhibiting CRS (or is homogeneous of degree one) the cost function defined by (1.5') may be written as:

$$C(w,y) = yC(w,1) = y\,\lambda(w).$$

where $C(w,1) = \lambda(w)$ is unit cost function.

Proof: This proposition may be proved directly from definition (1.5'). By definition (1.5'):

$$C(w,y) = \min_{x} \{w'x \mid f(x) \geq y,\ x \geq 0\} \qquad (1.5')$$

$$= \min_{x} \{w'x \mid y^{-1}f(x) \geq 1,\ x \geq 0\}.$$

Linear homogeneity of $f(x)$ implies that $tf(x) = f(tx)$. Thus by letting $t = y^{-1}$ the above can be written as

$$= \min_x \{w'x: \ f(y^{-1}x) \geq 1, \ x \geq \underline{0}\}$$

$$= \min_x \{y \ w'(y^{-1}x): \ f(y^{-1}x) \geq 1, \ y^{-1}x \geq \underline{0}\}$$

$$= y \min_u \{w'u: \ f(u) \geq 1, \ u \geq 0\} \ , \text{ where } u = y^{-1}x.$$

Therefore, $C(w,y) = y \ (w,1) = y\lambda(w)$ Q.E.D.

Another special case is that of homotheticity of the production function. Shephard (1953) introduced the concept of homotheticity and defined a homothetic function as follows:

Definition: A function, $F(x)$, is said to be homothetic if it can be written as $g(h(x))$, where g is a monotonic transformation of h and h is a homogeneous function of degree one; namely $\frac{\partial g}{\partial h} > 0$ and $h(tx) = th(x)$.

Proposition (2): If the production function, $F(x)$, is a homothetic one, then the cost function $C(w,y)$ can be factorized as $g^{-1}(y) \cdot \lambda(w)$, i.e. $C(w,y) = g^{-1}(y)\lambda(w)$, where $g^{-1}(\cdot)$ is the inverse function for g and $\lambda(w)$ is the unit cost function.

Shephard has proven this proposition by using the concept of a distance function, which is complex and lengthy. We can prove this proposition in a much simplier way directly from the definition.

Proof: By definition (5') we write:

$$C(w,y) = \min_x \{w'x: \ F(x) \geq y, \ \ x \geq \underline{0}\}$$

$$= \min_x \{w'x: \ g(h(x)) \geq y, \ \ x \geq \underline{0}\}$$

$$= \min_x \{w'x: h(x) \geq g^{-1}(y),\ x \geq \underline{0}\}$$, by monotonicity property.

$$= \min_x \{w'x: (g^{-1}(y))^{-1}\, h(x) \geq 1,\ x \geq \underline{0}\}$$

$$= \min_x \{g^{-1}(y) \cdot w'\ (g^{-1}(y))^{-1}x: h((g^{-1}(y))^{-1}x) \geq 1,\ (g^{-1}(y))^{-1} \cdot x \geq \underline{0}\}$$

$$= g^{-1}(y) \cdot \min_u \{w'u: h(u) \geq 1,\ u \geq \underline{0}\}$$, where $(g^{-1}(y))^{-1} \cdot x = u$.

$$= g^{-1}(y) \cdot c(w,1) = g^{-1}(y)\lambda(w).$$ Q.E.D.

This proposition will be utilized later on in testing homotheticity of production function. The reverse of this proposition can also be described and proved. It can easily be shown that a production function, $F(x)$, derived from a cost function of the form $g^{-1}(y) \cdot \lambda(w)$, is homothetic. Starting from the definition of production function, $F(x)$, we may write (assuming that the cost function is separable as above)

$$F(x) = \max \{y: w'x \geq g^{-1}(y)\lambda(w), \text{ for all } w >> 0 .$$

From above we obtain

$$h(x) = g^{-1}(F(x)) = \max \{g^{-1}(y): w'x \geq g^{-1}(y)\lambda(w),\ w >> 0\}.$$

But $h(x)$, as defined above, is clearly homogeneous of degree one and this proves that $F(x)$ is homothetic. Therefore the proposition may be stated in general as

Proposition: The production function, $F(x)$, is homothetic if and only if the associated cost function can be factorized as $C(w,y) = g^{-1}(y)\lambda(w)$.

1.6 Profit Function

A traditional method of obtaining a profit function is that of maximizing profit, defined as revenue less costs; i.e.

$$\text{Profit} = py - \Sigma w_i x_i \tag{1.8}$$

subject to technological constraints the producing unit is faced with, symbolized as production function, $f(x)$. Here we obtain the derived demand and supply, and then substitute back into the formula for profit given by (1.8) and obtain profit function as

$$\pi(p,w) = py^* - \Sigma w_i x_i^*$$

where y^* and x^* are the optimized values, each being a function of p and w. The difficulty with this procedure is that of tractibility; only those production functions of relatively simple form can be used to solve the profit maximization problem explicitly obtaining the derived demand and supply functions.

An alternative way to study the technology of a producing unit through the profit function is by invoking the duality theorem between the production and profit function. According to this theorem there exists a one-to-one relation between the production function and the profit function under some appropriate regularity conditions. Therefore for the purpose of theoretical or empirical analysis one may be better off to start by using an appropriate profit function.

Definition: Assuming that the assumptions (i-iv) are satisfied for the production technology, and assuming further that the production function is bounded[1] the profit function is defined as:

$$\pi(p,w) = \max_{x,y} \{py - w'x \mid f(x) \geq y\}.$$

Theorem 2: As in the case of cost function, the profit function possesses several properties as follows:

(i) $\pi(p,w)$ is nondecreasing in p and nonincreasing in w, i.e.; if $p^1 \geq p$ and $w^1 \leq w$, then $\pi(w^1,p^1) \geq \pi(w,p)$.

(ii) $\pi(p,w)$ is homogeneous of degree one in (w,p).

(iii) $\pi(p,w)$ is differentiable in p and w, and

$$\frac{\partial \pi(p,w)}{\partial p} = y(p,w) \; ,$$

$$\frac{\partial \pi(p,w)}{\partial w_i} = -x_i(w,p) \qquad i=1, \; \ldots, \; n.$$

(iv) $\pi(p,w)$ is a convex function in (p,w).

(v) $\pi(p,w)$ is a continuous function in (p,w); at least when $p > 0$ and $w_i > 0$ (i=1, ..., n).

(vi) x(p,w) and y(p,w) are continuous functions in (p.w).

(vii) $x(p,w)$ and $y(p,w)$ are homogeneous of degree zero in (p,w).

Proof: (i): for $p^1 \geq p$ we have $p^1 y \geq py$ (i-1)

for $w^1 \leq w$ we have $w^1 \cdot x \leq w \cdot x$ or

$-w^1 \cdot x \geq -w \cdot x$; (i-2)

adding (i-1) and (i-2) side by side we have:

$$p^1 y - w^1 \cdot x \geq py - w \cdot x;$$

then

$$\pi(p^1,w^1) = \max \{p^1 y - w^1 \cdot x \mid f(x) \geq y\}$$

$$\geq \max \{py - w \cdot x \mid f(x) \geq y\} = \pi(p,w)$$

$$\therefore \pi(p^1,w^1) \geq \pi(p,w)$$

From above it can be seen that for $p^1 = p$ but $0 \leq w^1 \leq w$ we have:

(a) $py - w^1 \cdot x \geq py - w \cdot y$

then $\pi(p,w^1) \geq \pi(p,w)$, i.e. the profit function is non-increasing in w.

(b) for $p^1 \geq p$ but $w^1 = w$, we have:

$$p^1 y - w \cdot x \geq py - w \cdot x$$

then $\pi(p^1,w) \geq \pi(p,w)$, i.e. the profit function is non-decreasing in p.

(ii): To show the linear homogeneity of $\pi(p,w)$ in (p,w) we multiply p and w by a scalar $\lambda > 0$; then we have:

$$\pi(\lambda p,\lambda w) = \max \{\lambda py - \lambda w' x \mid f(x) \geq y\}$$

$$= \lambda \max \{py - w' x \mid f(x) \geq y\}$$

$$= \lambda \pi(p,w) .$$

Q.E.D.

Using the above result we can show (vii) as follows:

$$\pi(\lambda p, \lambda w) = \lambda \pi(p,w)$$

which, in turn, can be written as

$$\lambda py \quad (\lambda p,\lambda w) - \lambda w' x(\lambda p,\lambda w) = \lambda py(p,w) - \lambda w' \ x(p,w).$$

Therefore it must be true (by the uniqueness property) that

$y(\lambda p,\lambda w) = y(w,p)$ and $x(\lambda p,\lambda w) = x(p,w) \therefore$ they are homogeneous of degree zero.

(iv) To prove the convexity of $\pi(p,w)$ we must show that:

$$\pi(p^*,w^*) \leq \alpha\pi(p^0,w^0) + (1-\alpha)\,\pi\,(p^1,w^1) \quad \text{for } 0 \leq \alpha \leq 1$$

where

$$p^* = \alpha p^0 + (1-\alpha)p^1 \tag{iv-1}$$

$$w^* = \alpha w^0 + (1-\alpha)w^1. \tag{iv-2}$$

By definition of profit function we have:

$$\pi(p^0 w^0) = p^0 y(p^0,w^0) - w^{0'} x(p^0,w^0)$$

$$\geq p^0 y(P^*,w^*) - w^{0'} x(p^*,w^*), \tag{iv-3}$$

where the inequality sign comes because $y(p^0,w^0)$ and $x(p^0,w^0)$ are the maximizing levels of output and inputs at prices (p^0,w^0). Similarly

$$\pi(p^1,w^1) = p^1 y(p^1,w^1) - w^{1'} x\,(p^1,w^1) \tag{iv-4}$$

$$\geq p^1\, y\,(p^*,w^*) - w^{1'} x(p^*,w^*).$$

Multiplying both sides of (iv-3) and (iv-4) by α and $(1-\alpha)$ respectively and adding them up:

$$\alpha\pi(p^0,w^0) + (1-\alpha)\pi(p^1,w^1) \geq (\alpha p^0+(1-\alpha)p^1)y(p^*,w^*)$$

$$- (\alpha w^{0\prime} +(1-\alpha)w^{1\prime})x(p^*,w^*) \qquad \text{(iv-5)}$$

Substituting (iv-1) and (iv-2) in (iv-5) we have:

$$\alpha\pi(p^0,w^0)+(1-\alpha)\pi(p^1,w^1) \geq p^*y(p^*,w^*)-w^{*\prime}x(p^*,w^*)$$

$$= \pi(p^*,w^*) \qquad \text{Q.E.D.}$$

(vi) To see the continuity of $\pi(p,w)$ in w,p, we make use of the differentiability of $\pi(p,w)$ in p and w. Assuming that $\pi(p,w)$ is differentiable at $w=\theta$, we can show that it is continuous at $w=\theta$ as follows:

Since $\pi(p.w)$ is differentiable at $w=\theta$ then by definition:

$$\lim_{w\to\theta} \frac{\pi(p,w)-\pi(p,\theta)}{w-\theta} = \pi'(p,\theta)$$

or

$$\frac{\pi(p,w)-\pi(p,\theta)}{w-\theta} = \pi'(p,\theta) + \varepsilon(w)$$

where

$\lim_{w\to\theta} \varepsilon(w)=0$. Then

$$\pi(p,w) = \pi(p,\theta) + (w-\theta)\ (\pi'(p,\theta) + \varepsilon(w)).$$

It can be seen from above that:

$$\lim_{w\to\theta} \pi(p,w) = \pi(p,\theta).$$

Therefore $\pi(p,w)$ is continuous in w. In a quite similar manner can can show that $\pi(p,w)$ is continuous in p too. Q.E.D.

An interesting approach (due to Lau (1978)) to obtain the profit function and to study its behavior, without actually constructing it, is through the classical Legendre's dual transformation (LT). LT changes a given function of a given set of variables into a new function of a new set of variables. The old and the new variables are related to each other by a point transformation.

To clarify this let us consider a given function of n variables $z_1, \ldots, z_n$,

$$F = F(z_1, \ldots, z_n).$$

A new set of variables may be introduced by the following transformation:

$$t_i = \frac{\partial F}{\partial z_i} \quad , \; i=1, \ldots, n \; . \tag{1.9}$$

Assuming that the determinant of the "Hessian" of F to be different from zero, which guarantees the independence of the n variables t_i, the equations (1.9) can be solved for z_i as functions of the t_i,

$$z_i = z_i\,(t_1, \ldots, t_n)\;,\; i=1, \ldots, n.$$

We define a new function G as follows:

$$G(t_1, \ldots, t_n) = \sum_{i=1}^{n} t_i z_i(t) - F(z_1(t), \ldots, z_n(t))$$

The function G is known as the Logendre dual transformation function of the primal function F. Consider the following partial differentiation of G with respect to t:

$$\frac{\partial G}{\partial t_i} = z_i(t) + \sum_{j=1}^{n} t_j \frac{\partial z_j(t)}{\partial t_i} - \sum_{j=1}^{n} \frac{\partial F}{\partial z_j(t)} \frac{\partial z_j(t)}{\partial t_i}$$

$$= z_i(t) + \sum_{j=1}^{n} \frac{\partial z_j}{\partial t_i} (t_i - \frac{\partial F}{\partial z_j}) \quad i=1, \ldots, n.$$

By substituting $\frac{\partial F}{\partial z_j} = t_j$ we obtain:

$$\frac{\partial G}{\partial t_i} = z_i, \quad i=1, \ldots, n.$$

This is a remarkable result which expresses the duality of LT. The following scheme summarizes this duality:

	Old System	New System
Variables:	$z_1, \ldots, z_n$	$t_1, \ldots, t_n$
Function:	$F=F(z_1, \ldots, z_n)$	$G=G(t_1, \ldots, t_n)$
	Transformation	
	$\frac{\partial F}{\partial z_i} = t_i$	$z_i = \frac{\partial G}{\partial t_i}$
	$G=\Sigma t_i z_i - F$	$F=\Sigma t_i z_i - G$
	$G = G(t_1, \ldots t_n)$	$F=F(z_1, \ldots, z_n)$

As this scheme reveals, the new variables are the partial derivatives of the old function with respect to the old variables, and the old variables are the partial derivatives of the new function with respect to the new variables.

The remarkable property of this transformation is that of its symmetry in both systems, i.e., the same transformation that leads from the old to the new system leads back from the new to the old system. This means that the LT of G, G*, is:

$$G^*(z_1, \ldots z_n) = \Sigma t_i(z)\, z_i - G(t_1(z), \ldots, t_n(z)) = F.$$

Therefore the two functions F and G are related to each other by the following set of dual relations:

$$F(z_1, \ldots, z_n) + G(t_1, \ldots, t_n) = \Sigma z_i t_i,$$

$$\frac{\partial F}{\partial z} = t \qquad\qquad \frac{\partial G}{\partial t} = z\ .$$

By using this transformation we are able to construct the profit function and study its behavior. Substituting the production function in the profit equation (1.8) results in

$$\text{Profit} = pf(x) - \Sigma w_i x_i$$

Assuming profit maximization as a behavioral assumption, a price-taker firm will maximize profit with respect to x, taking p and w as given. This results in the profit function π as a function of p and w, which gives the maximized level of profit for each set of values of p and w as

$$\pi(p,w) = pf(x^*) - \Sigma w_i x_i^*$$

Before going further it might be more convenient to work with normalized profit function given as

$$\pi^N(r) = f(x^*) - \Sigma r_i x_i^* ,$$

where $r_i = \frac{w_i}{p}$ is the normalized price of i^{th} input; while the one-to-one correspondence between $\pi(p,w)$ and $\pi^N(r)$ must be clear, because: $\pi(p,w) = \max_x \{pf(x) - \Sigma w_i x_i\} =$

$p \max_x \{f(x) - \Sigma r_i x_i\} \quad = \quad p \cdot \pi^N(r).$

In terms of our problem we are faced with the production function f(x) which may be identified as F(z), a normalized profit function $\pi^N(r)$ as G(t), and x as z. The partial derivative of f(x) with respect to x is set equal to t, the new variable according to LT:

$$\frac{\partial f}{\partial x} = t$$

But profit maximization implies that:

$$t = \frac{\partial f}{\partial x} = r \ .$$

Therefore r may be identified as t. The LT function can be constructed, by definition, and by recognizing that r=t, as

$$G(r) = \Sigma r_i x_i(r) - f(x_1(r), \ldots, x_n(r)) ,$$

which is precisely $-\pi^N(r)$, i.e., the negative normalized profit function. Moreover, from LT we obtain

$$\frac{\partial \pi^N(r)}{\partial r} = -x, \qquad (1.10)$$

which is the derived factor demand function. Furthermore, the relation between the "Hessian" matrices of the production function and the normalized profit function can be obtained by differentiating

$$\frac{\partial f}{\partial x} = r$$

with respect to r and by treating x as a function of r, which results in

$$\frac{\partial^2 f}{\partial r \partial r'} [\frac{\partial x}{\partial r}] = I \qquad (1.11)$$

where I is the matrix of unit.
But form (1.10)

$$\frac{\partial x}{\partial r} = -\frac{\partial^2 \pi^N}{\partial r \partial r'} = [-\pi^N_{ij}]. \qquad (1.12)$$

Therefore, by substituting (1.12) into (1.11) we obtain

$$[\pi^N_{ij}] = -[f_{ij}]^{-1}$$

where $[\pi^N_{ij}]$ and $[f_{ij}]$ are the "Hessian" matrices of the normalized profit function and production function respectively.

The concept of the profit function may easily be extended to a case when some inputs are fixed. If u represents the vector of fixed factors of production, then the production function may be written as

$$y = f(x,u)$$

Then the profit function may be defined as

$$\pi(p,w,u) = \max_{y,x} \{py-\Sigma w_i x_i | f(x,u) \geq y\}.$$

This may be called the "variable profit function" due to Samuelson (1953-1954, p. 20). The dual corresponding to this is obtained as follows:

$$f(x,u) \equiv \{(x,u)|\pi(p,w,u) \geq py-\Sigma w_i x_i \text{ for } p>0,\ w >>0,\ u \geq 0\}.$$

Alternatively, Legendre's transformation may be used to obtain the restricted or short run normalized profit function as $\pi^N(t,u)$ with the following dual transformation relations:

(i) $f(x,u) - \pi^N(r,u) = \Sigma r_i x_i$

(ii) $\frac{\partial f(\cdot)}{\partial x} = r \qquad \frac{\partial \pi^N(r,u)}{\partial r} = -x$

(iii) $x = -\frac{\partial \pi^N(\cdot)}{\partial r} \qquad r = \frac{\partial f(\cdot)}{\partial x}$

(iv) $\frac{\partial f(\cdot)}{\partial u} = \frac{\partial \pi^N(\cdot)}{\partial u} \qquad \frac{\partial \pi^N(\cdot)}{\partial u} = \frac{\partial f(\cdot)}{\partial u}$

(v) $f(\cdot) = \pi^N(\cdot) - \Sigma r_i \frac{\partial \pi^N}{\partial r_i} \qquad \partial \pi^N(\cdot) = \partial f(\cdot) - \Sigma x_i \frac{\partial f}{\partial x_i}$

(vi) $u \qquad u$

From this LT relationship, which represents a system of partial differential equations, one may either construct the normalized profit function or study its behavior, given the production function and the first order necessary condition for max (and vice versa).

Moreover, as is shown by the fact that the production function and the normalized profit function are LT of each other, a partial differential equation for f(x,u) in x and u transforms to a partial differential equation for $\pi^N(r,u)$ in r and u; therefore the equivalent properties of the production function and the normalized profit function may be deduced immediately. This technique has been extensively used by Lau (1978) to study the consequences of assuming several differential properties regarding the production function on the normalized profit function and vice versa.

Furthermore, the LT has the advantage of being quite useful in deriving the solution of certain partial differential equations, being intractible otherwise.

1.7 Separability: Definition and Two Related Theorems

The concept of separability is of quite an importance in some areas of economic theory. Sono (1945) and Leontief (1947) are pioneers of the subject, the former, on the theory of consumer behavior, and, the latter, on the theory of production. Sono's work on the theory of consumer behavior was published in Japanese in 1945 and remained not very well-known outside Japan until it was translated into English in 1961. Leontief's paper, developed completely independently of Sono's, appeared in 1946-1947, and was primarily concerned with the theory of production.

Since the publication of these papers, the concept of separability has been widely used in these two areas.

To have a clear concept, consider the production function $y = f(x_1, \ldots, x_n)$ where y is a product, say, steel, which has been produced by using many different kinds of inputs, such as various kinds of heterogeneous labor, machinery, and material. This function, in which a great number of inputs have been involved, may be broken down into several simpler relationships, each having fewer variables, that is, one might think of this production function as a combination of some separate intermediate relationships, each containing not only the original variable but also some additional, intermediate variables. Then the production function, f(x), may be written, say, as

$$f(x) = G(g^1(x_1,\ldots,x_r),\ g^2(x_{r+1},\ldots,x_k),$$
$$g^3(x_{k+1},\ldots,x_{n-1}),\ x_n)$$

Here one might think of g^i (i=1,2,3) as the intermediate output of sector i (iron mining industry, coal industry,...) which later is combined with x_n, say labor, to produce f(x), steel. In fact g^i acts as a new factor which is combined with the original input, x_n, to produce steel. One might ask, as Leontief (1947b) puts it, "given a quantitative description of the overall relationship such as f(x), can one without any additional outside information, i.e., solely through examination of the mathematical properties of the function, f(x), establish the possible existence of such subsidiary groups of variables (such as $(x, \ldots, x_r)$, $(x_{r+1}, \ldots x_k) \ldots$) and describe the properties of the corresponding intermediate functions $(g^1(\ ), g^2(\ ) \ldots)$?" Different theorems on separability will provide a general answer to this question.

The concept of separability has been studied under two conditions which have been named weak and strong separability, coined by Strotz (1957) and (1959).

Let $y = f(x)$ represent the production function, where $x = (x_1, \ldots, x_n)$ is a vector of inputs. Partitioning this vector into s mutually exclusive and exhaustive subsets $N^* = (N_1, \ldots, N_s)$, the weak separability of f(x) may be defined as follow:

Definition: The production function, f(x), is weakly separable with respect to the above position, if the marginal rate of substitution (MRS) between any two inputs i and j from any subset $N_m, m=1, \ldots, s$ is independent of the inputs outside N_m, namely,

$$\frac{\partial}{\partial x_k}\left(\frac{f_i}{f_j}\right) = 0, \qquad \forall\ i,j\ \ \varepsilon N_m \text{ and } k \notin N_m$$

Definition: Letting $s > 2$, the production function f(x) is said to be strongly separable if the following condition is fulfilled:

$$\frac{\partial}{\partial x_k}\left(\frac{f_i}{f_j}\right) = 0, \qquad \forall\ i \varepsilon N_m,\ j \varepsilon N_h,\ k \notin N_m \cup N_h \quad h \neq m$$

This means that MRS between any two inputs from subsets N_m and N_h is independent of the quantities of third inputs which are not in N_m or N_h. These conditions may also be written as

$$f_j f_{ik} - f_i f_{jk} = 0 \quad \forall\ i,j \in N_m \text{ and } k \notin N_m \text{ for weak}$$

separability and $\forall\ i \in N_m$, $j \in N_h$ and $k \notin N_m \cup N_h$ for strong separability, where

$f_i = \frac{f(\ )}{x_i}$, partial derivative of the production function with respect to i^{th} input and $f_{ij} = \frac{\partial^2 f}{\partial x_i \partial x_j}$.

Note that if s=2, then $i \in N_1$ implies that $k \in N_2$ and hence $j \in N_1$; then the condition for strong separability reduces to the condition for weak separability. The following two theorems are of fundamental importance regarding functional separability:

Theorem (1): The production function f(x) is weakly separable with respect to the partition $N^* = (N, \ldots, N_s)$ if and only if f(x) is of the form $G(g^1(x^1), \ldots, g^s(x^s))$, where $g^m(x^m)$ is a function of the subvector x^m alone which are the elements of N_m only.

Theorem (2): The production function f(x) is strongly separable with respect to the partition $N^* = (N_1, \ldots, N_s)$ if and only if f(x) is of the form $G(g^1(x^1) + \ldots + g^s(x^s))$ where $g^m(x^m)$ and x^m are defined as before.

These theorems have been proved by Goldman and Uzawa (1964).

1.7A Separability and Elasticity of Substitution

To understand the nature of the relation between functional separability and the elasticity of substitution it is necessary to interpret functional separability in economic language. Keeping the argument simple, let us assume a production function with three inputs, x_1, x_2, and x_3; and let inputs x_1 and x_2 be functionally separable from the third input, x_3. This means, by the definition of weak separability, that the marginal rate of substitution between inputs 1 and 2 is independent of the level of input 3 used. In other words, if the usage of x_1 and x_2 is held constant and the usage of x_3 increases, the increased flow of x_3 makes x_1 and x_2 more effective at the margin, and raises their effectiveness by exactly the same relative amount. This, as we saw above, was stated as $\frac{d}{dx_3}(\frac{f_1}{f_2}) = 0$. Here we see that the augmented usage of x_3 shifts the marginal products of x_1 and x_2 by the same proportion (which is observationally the same as the familiar Hick's-neutral technological change); therefore x_3 must have an equally close substitution or complementary relationship to both inputs, namely, the partial elasticity of substitution between x_1 and x_3 is the same as the partial elasticity of substitution between x_2 and x_3, one appropriate measure of the elasticity of substitution being the Allen partial elasticity of substitution in cases where more than two inputs are involved.

This inequality of the partial elasticities of substitution, in the case of functional separability, can be demonstrated mathematically. Berndt and Christensen (1973b) have shown this equality in the case of the homothetic production function, but in what follows it can be shown that the homotheticity of the production function is not required.

Before showing the relation between the separability of the production function and the equality of the elasticities of substitution, we shall first transform the Allen partial elasticity of substitution into a definition in terms of the cost function. This transformation has been shown by Uzawa (1962) for a production function which is homogeneous of degree one and subject to a diminishing marginal rate of substitution, and in terms of the unit cost function; while this may be shown for a twice differentiable, strictly quasi-concave production function f(x) with strictly positive marginal products.

Let $y = f(x_1,...,x_n)$ be a twice differentiable, strictly quasi-concave production function with a finite number, n, of inputs, each having a strictly positive marginal product. Also assume that the vector $x = (x_1,...,x_n)$ is partitioned into s mutually exclusive and exhaustive subsets $N^* = (N_1,...,N_s)$. The Allen partial elasticity of substitution, σ_{ij}, between two factors i and j ($i \neq j$) is defined

$$\sigma_{ij} = \frac{\sum_{h=1}^{n} x_h f_h |F_{ij}|}{x_i x_j |F|}$$

where

$$f_h = \frac{\partial f}{\partial x_h}\ ,\ f_{hg} = \frac{\partial^2 f}{\partial x_h \partial x_g}\ ;\ h,g=1,\ \ldots,\ n;$$

$$|F| = \begin{vmatrix} 0 & f_1 \ldots . f_n \\ f_1 & f_{11} \ldots . f_{1n} \\ \vdots & \vdots \\ f_n \ldots . & f_{n1} \ldots . f_{nn} \end{vmatrix}$$

and $|F_{ij}|$ is the determinant of $(ij)^{th}$ cofactor in F. Using this formula for two particular inputs, i and k, such that $i \varepsilon N_m$ and $k \notin N_m$ $(m=1, \ldots, s)$, the partial elasticity of substitution, σ_{ik}, between them can be written by:

$$\sigma_{ik} = \frac{\sum_{h=1}^{n} x_h f_h\ |F_{ik}|}{x_i x_k\ |F|} \quad \text{for } i \varepsilon N_m \text{ and } k \notin N_m\ .$$

It is a well-known fact that the firm, in order to minimize the cost of producing of a specific level of output, must adjust the factor inputs such that the ratio of price to marginal product will be the same for each factor or

$f_h = \frac{w_h}{\lambda}$; where λ is interpreted as being the marginal cost of output.

Also the rate of change of the independent variables $(x_1, \ldots, x_n)$, with respect to changes in factor input prices, is obtained (see Samuelson (1947), pp. 63-69) as

$$\frac{\partial x_i}{\partial w_k} = \frac{|F_{ik}|}{\lambda |F|}$$

Substituting these relations in σ_{ik} we obtain

$$\sigma_{ik} = \frac{\sum_{h=1}^{n} w_h x_h \ \partial x_i/\partial w_k}{x_i x_k}$$

On the other hand by Shephard's Lemma we have:

$$x_i = \frac{\partial C(w,y)}{\partial w_i} = C_i \text{ and}$$

$$\frac{\partial x_i}{\partial w_k} = \frac{\partial^2 C(w,y)}{\partial w_i \partial w_k} = C_{ik} .$$

By substituting these in σ_{ik} and utilizing the fact that $\Sigma w_h x_h = C(w,y)$ we get:

$$\sigma_{ik} = \frac{CC_{ik}}{C_i C_k} , \qquad i \varepsilon N_m,\ k \not\varepsilon N_m \qquad (1.13)$$

Similarly

$$\sigma_{jk} = \frac{CC_{jk}}{C_j C_k} . \qquad j \varepsilon N_m,\ k \not\varepsilon N_m$$

Now setting σ_{ik} and σ_{jk} equal to each other we get:

$$\frac{CC_{ik}}{C_i C_k} = \frac{CC_{jk}}{C_j C_k} , \qquad i,j \varepsilon N_m,\ k \varepsilon N_m .$$

Or

$$C_i C_{ij} - C_j C_{ik} = 0, \qquad i,j \varepsilon N_m,\ k \varepsilon N_m \qquad (1.14)$$

This means that $\sigma_{ik} = \sigma_{jk}$ $(i,j \varepsilon N_m,\ k \not\varepsilon N_m)$ if and only if (1.14) holds. But (1.13) is nothing but the condition for weak separability of the cost function C(w,y). On the other hand these conditions, under a suitable assumption such as homotheticity of the production function or homotheticity of the micro production functions $g^m(x^m)$, are also the conditions for weak separability of $f(x)$. Thus, it was shown how weak separability of the production function with respect to the partition N* implied that $\sigma_{ik} = \sigma_{jk}$ $(i,j \epsilon N_m,\ k \notin N_m)$. But still we need to know under what conditions weak separability of the production function implies weak separability of the cost function in the same partition. To see this, let $y = f(x) = G(g^1(x^1), \ldots, g^s(x^s))$ where $g^m(x^m)$ is a micro production function and x^m is a subvector with its elements being that of N_m only. The cost function associated with G( ) may be defined by:

$$C(w^1, \ldots, w^s, y) = \min_{x^1,\ldots,x^s} \{ \sum_{m=1}^{s} w'^m x^m | G(g^1(x^1), \ldots, g^s(x^s)) \geq y \}$$

$$= \min_{x^1,\ldots,x^s; y^1,\ldots, y^s} \{ \sum_{m=1}^{s} w'^m x^m | G(y^1, \ldots, y^s) \geq y; \quad y^m = g^m(x^m) \}.$$

The above is a two step cost minimization; in the first step we minimize $\Sigma w'^m x^m$ with respect to x^m and subject to $y^m = g^m(x^m)$ $(m=1, \ldots, s)$. Now if we assume that $y^m = g^m(x^m)$ are homothetic functions, the result of this minimization will be a cost function of the form

$\phi^m(y^m)\cdot\lambda^m(w^m)$ (see section 5) where $\lambda^m(w^m) = \min_{x^m} \{w'^m x^m \mid f^{\bar{m}}(x^m) \geq 1\}$.

In the second step we must minimize $\sum_{m=1}^{s} \phi^m(y^m)\cdot\lambda^m(w^m)$ with respect to y^m and subject to $G(y^1, \ldots, y^s)$, i.e.,

$$C(w^1, \ldots, w^s;y) = \min_{y^1,\ldots,y^s} \{\sum_{m=1}^{s} \phi^m(y^m)\cdot\lambda^m(w^m): G(y^1, \ldots, y^s) \geq y\}$$

$$= C(y; \lambda^1(w^1), \ldots, \lambda^s(w^s)). \quad \text{Q.E.D.}$$

In this proposition it should be noted that G is a homothetically separable function.[2]

Next we might consider the case in which the G function itself is homothetic in $x^1, \ldots, x^s$. In this case it can easily be shown that a weakly separable and homothetic function implies the homotheticity of each subfunction,[3] and so the problem becomes like the one we had before.

Another case is the one in which the G function and the subfunctions $g^m(x^m)$ exhibit constant return to scale, i.e., they are homogeneous of degree one. The cost function in this case is obtained as:

$$C(w^1, \ldots, w^s;y) = \min_{x^1,\ldots,x^s} \{\sum_{m=1}^{s} w'^m x^m : G(g^1(x^1), \ldots, g^s(x^s)) \geq y\}$$

$$= \min_{x^1,\ldots x^s; y^1,\ldots y^s} \{\Sigma w'^m x^m : G(y^1,\ldots y^s) \geq y; \; y^m = g^m(x^m)\} .$$

Since each $g^m(x^m)$ is homogeneous of degree one, it has a dual cost function of the form $y^m \lambda^m(w^m)$, where

$$\lambda^m(w^m) = \min_{x^m}\{w'^m x^m : g^m(x^m) \geq 1\};\ m=1,\ldots,s.$$

Therefore we have:

$$C(\cdot) = \min_{y^1,\ldots,y^s} \{\Sigma y^m \lambda^m(w^m) : G(y^1,\ldots,y^s) \geq y\}$$

$$= y \cdot c(\lambda^1(w^1),\ldots,\ \lambda^s(w^s))$$ since G is homogeneous of degree one.

As Diewert (1974) has pointed out, the elasticity of substitution will have a special feature in this case as will be described below. Evaluating the elasticity of substitution between the i^{th} input from group one, x_i^1, and the j^{th} input from group two, x_j^2, we have, by definition (1.13)

$$\sigma_{ij}^{12} = \frac{C \dfrac{\partial^2 C}{\partial w_i^1 \partial w_j^2}}{\dfrac{\partial C}{\partial w_i^1} \dfrac{\partial C}{\partial w_j^2}}$$

$$= \frac{(y \cdot c)\ y \cdot c_{12} \dfrac{\partial \lambda^1}{\partial w_i^1} \dfrac{\partial \lambda^2}{\partial w_j^2}}{y \cdot c_1 \dfrac{\partial \lambda^1}{\partial w_i^1} \quad y \cdot c_2 \dfrac{\partial \lambda^2}{\partial w_j^2}} = \frac{c\, c12}{c_1 c_2} = \sigma^{12}$$

where c_i is the first order partial derivative of c with respect to its i^{th} argument, and c_{12} is the second order partial derivative of c with respect to its first and second arguments.

The special feature about this elasticity is that σ_{ij}^{12} does not depend on the subgroup indices i and j or generally this means that $\sigma_{ij}^{mn} = \sigma^{mn}$. Therefore, the elasticity of substitution between the two primary inputs i and j from subgroups (intermediate inputs) m and n is the same as the elasticity of substitution between intermediate inputs m and n.

Interestingly, the relation between a weakly separable production function and the profit function may be established. Let $\pi(p,w^1, \ldots w^s)$ be the profit function associated with $G(g^1(x^1),\ldots,g^s(x^s))$; then we can show that the profit function is also weakly separable in input price aggregate as follows.

$$\pi(p,w^1,\ldots,w^s) = \max_{y;x^1,\ldots,x^s} \{py - \sum_{m=1}^{s} w'^m x^m | G(g^1(x^1), \ldots, g^s(x^s)) \geq y\}$$

$$= \max_{y;y^1,\ldots,y^s;x^1,\ldots,x^s} \{py - \sum_{m=1}^{s} w'^m x^m | G(y^1,\ldots,y^s) \geq y, g^m(x^m) = y^m\}$$

This consists of two steps: in the first step, we maximize $py - \sum_{m=1}^{s} w'^{m}x^{m}$ subject to $g^{m}(x^{m}) = y^{m}$ and with respect to $x^{1},\ldots,x^{s}$, which results in a cost function of the form $y^{m}\lambda^{m}(w^{m})$, provided that $g^{m}(x^{m})$ is homogeneous of degree one. In the second step, we maximize $py - \Sigma y^{m}\lambda^{m}(w^{m})$ subject to $G(y^{1},\ldots,y^{s}) = y$ and with respect to y and $y^{1},\ldots,y^{s}$, i.e.,

$$\max_{y;y^{1},\ldots,y^{s}} \{py - \Sigma y^{m}\lambda^{m}(w^{m}) : G(y^{1},\ldots,y^{s}) \geq y\},$$

which results in $\pi(p;\lambda^{1}(w^{1}),\ldots, \lambda^{s}(w^{s}))$, i.e., if the production function is weakly separable and the subfunctions are homogeneous, then the profit function will be weakly separable.

1.8 Functional Forms: Choice Criteria

In studying a producing unit, it is a common procedure to assume an objective function and then a behavioral assumption which enables the producing unit to optimize its objective subject to a side relation, given the conditions surrounding the problem such as market conditions which determines the prices of inputs and outputs, etc. However, to answer questions such as the elasticity of demand for fuel, elasticity or ease of substitution between capital and labor, and economies of scale, which are the most concrete applications of economic analysis to policy questions, it is necessary to know the specific parameters describing the producing unit's behavior.

The answers to such questions which might have serious policy implications can be found only by examining the data and estimating the parameters of the agent's objective function, using some statistical method. In doing so, it is necessary to choose a functional form for the objective function, say, the production function, and try to estimate its unknown parameters.

The task of choosing a functional form is not an easy one, since the researcher must compromise between different and often unreconcilable properties a functional form has. The main two desirable properties a function must have are: first, the capability of representing a wide range of technologies, in order to minimize the prior assumptions imposed on estimating the equations, and second, the tractability, i.e., the ease of computation, estimation, and interpretation. Flexible functions such as the translog (TL), the generalized Leontief (G.L.), the generalized Cobb-Douglas (GCD), etc., are the ones with the first property. These functions may be considered as a second order approximation to any arbitrary function; their elasticities of substitution between different pairs of factors are variable, they permit the existence of uneconomic regions, and, as far as estimation is concerned, they are linear in parameters. However, these functions have a complex form and often involve too many parameters, which makes estimation not an easy job. On the other hand, there are simple functional forms such as the Cobb-Douglas (CD), the constant elasticity of substitution (CES), and the

Leontief functions which are tractable, while they have restrictions on the elasticity of substitution, form of separability, constancy of factor shares (in CD case); still the choice between these forms needs careful consideration and depends upon the use to which they are to be put.

It is a well known fact that the CD and the Leontief production functions have an Allen elasticity of substitution (AES) of unity and zero for all input pairs respectively, although there is nothing in theory which suggests that such a restriction will be universally met. The degree of factor substitution is of great importance, since it has a number of repercussions. A change in an input price ratio will affect the cost of production, product prices, and income distribution according to factor ownership, and consequently consumption and savings patterns will be affected. In this sequence of events, a proper measure of the degree of factor substitution plays an important role.

This shortcoming decreases to some extent in the case of the CES function, since it allows the AES to deviate from unity, although it is doomed to be constant by construction. Although in the case of the two factor production function it might be justified, on the ground that the elasticity of substitution is constant, in the case of a multi-factor technology it necessitates that all inputs be equally substitutable, which is not realistic unless it is tested empirically. In fact, in the multi-factor case the CES model stands in sharp contradiction to economic common sense, as well as to the very purpose of such studies. This theoretically and empirically unjustifiable restriction has led to several studies which have indicated that the elasticity of substitution between capital and labor varies, usually inversely and sometimes significantly, as capital deepening occurs. This evidence casts doubt on the empirical usefulness of any CES function, and has led several authors[4] to develop functions with the property of making the common elasticities of substitution a function of some variables such as output level of factor ratio, etc.

However, the CD function has been widely used in empirical work, regardless of its shortcomings apparently because, first, the direct estimation of the CD function, using aggregate time series data, is not inappropriate, due to the fact that substitution effects are not well identified by highly colinear data, and second, as Fisher (1969) argues, the constancy of the factor shares of labor and capital in aggregate data fits the CD hypothesis.

With this general background in mind, we will now review several functional forms, and will study their properties in brief.

1.8A Functional Forms Summarized

There is an interesting way, due to Mundlack (1973), to describe several kinds of functional forms by using a quadratic form and by imposing some appropriate constraints on that. The form is

$$y_0 = (1, \underline{y}') \begin{pmatrix} \alpha_0 & \frac{1}{2}\underline{\alpha}' \\ \frac{1}{2}\underline{\alpha} & B \end{pmatrix} \begin{pmatrix} 1 \\ \underline{y} \end{pmatrix} = \alpha_0 + \underline{y}'\underline{\alpha} + \underline{y}'B\underline{y} \qquad (1.15)$$

To derive various functions, the following transformations will be utilized:

T_1: $y_i = x_i^{\rho i}$; $\rho_i \neq 0$

T_2: $y_i = \ln x_i$; $\rho_i = 0$, $i=0,1,\ldots,n$;

where x_0 is output and $x_1, \ldots, x_n$ are inputs.

(i) Cobb-Douglas (CD)

The CD function is derivable from (1.15) by imposing: $B \equiv 0$, and by obtaining variables by applying the logarithmic transformation T_2. The following notation is used to state this as

$(1.15) \cap (B \equiv 0) \cap T_2$

The result is

$$\ln x_0 = \alpha_0 + \sum_{i=1}^{n} \alpha_i \ln x_i$$

(ii) CES-like

This function may be obtained by

$(1.15) \cap (\alpha_0=0) \cap (B \equiv 0) \cap T_1 \cap \{\rho_i=\rho\}$ for $i=0,1,\ldots,n$;

which results in

$$x_0 = \left(\sum_{i=1}^{n} \alpha_i x_i^{\rho}\right)^{\frac{1}{\rho}}$$

(iii) CRES (Constant Ratio of Elasticity of Substitution)

The CRES function is developed by Mukerji (1963) and Forman (1965), and can be derived from (1.15) by imposing:

(1.15) $\cap(\alpha_0=0) \cap (B \equiv 0) \cap T_1$

which results in

$$x_0 = (\sum_{i=1}^{n} \alpha_i x_i^{\rho_i})^{\frac{1}{\rho_0}}$$

Unlike the CES, this function does not have the identical and constant partial elasticities of substitution. However, the ratios of the elasticities of substitution are constant for this function, and are not necessarily the same. The CRES is a homogeneous function only when the ρ's are all equal, and it obviously reduces to the CES-like in this case.

(iv) CRESH (Homogeneous or Homothetic CRES) (Hanoch 1971). The CRES functions are not homogeneous or homothetic, and this causes the Allen-Uzawa elasticities of substitution to vary with output, as well as with the factor combination. This makes the expansion path (for given factor prices) curved in a predetermined and often undesirable way, dictated by the form of the function.

In the CRESH, this problem is removed due to homogeneity and homotheticity of the function, so that the elasticities of substitution vary along the isoquants and differ between pairs of factors, although the elasticities of substitution stand in fixed ratios everywhere.

The CES and its limiting forms (the CD(σ=1), the Leontief (σ=0), and linear (σ=∞) functions) are special cases of CRESH. Its functional form is

$$\sum_{i=1}^{n} \alpha_i \left(\frac{x_i}{x_0}\right)^{\rho_i} - 1 \equiv 0$$

The parameters of this function are estimatable from a system of log-linear equations, given data on factor prices, quantities, and output, and assuming cost minimization.

(v) Generalized Leontief (Diewert 1971)

This function is obtained from (1.15) as:

$$(1.15) \cap (\alpha_0 \equiv 0) \cap (\alpha \equiv 0) \cap T_1 \cap \{\rho_0 = 1 \text{ and } \rho_j = \tfrac{1}{2}\} \quad j=1, \ldots, n .$$

The functional form for this function is written as:

$$x_0 = \sum_i \sum_j \beta_{ij} \, x_i^{\frac{1}{2}} x_j^{\frac{1}{2}} .$$

(vi) Generalized Diewert (Generalization of the Generalized Leontief)

This function is derived as (v) by setting the parameters as

$$\rho_0 = \rho \quad \text{and}$$

$$\rho_j = \tfrac{1}{2}\rho \quad j=1, \ldots, n .$$

The resulting functional form is also known as the quadratic mean of order ρ, and is written as

$$x_0 = \left(\sum_i \sum_j \beta_{ij} \, x_i^{\rho/2} x_j^{\rho/2}\right)^{1/\rho}$$

(vii) Translog Function (Christensen, Jorgenson and Lau (1971) (1973)). The translog (TL) function developed by Christensen, Jorgensen and Lau may be obtained from (1.15) by imposing:

(1.15) $\cap T_2$

which results:

$$\ln x_0 = \alpha_0 + \sum_{t=1}^{n} \alpha_i \ln x_i + \sum_i \sum_j \beta_{ij} \ln x_i \ln x_j$$

(viii) Quadratic form:

This form is obtained by imposing

(1.15) $\cap T_1 \cap \{\rho_j = 1\}$ $j=0,1,\ldots,n$.

which results in

$$x_0 = \alpha_0 + \Sigma\alpha_i x_i + \Sigma\Sigma\beta_{ij} x_i x_j \ .$$

1.9 <u>Translog Production Function and Its Properties</u>

This function is defined in Christensen et al (1970) as:

$$y = \alpha_0 \left(\prod_{i=1}^{n} x_i^{\alpha_i}\right)\left(\prod_{i=1}^{n} x_i^{\frac{1}{2}\left(\sum_{j=1}^{n} \gamma_{ij} \ln x_j\right)}\right) .$$

Taking logarithm of both sides we obtain:

$$\ln y = \ln \alpha_0 + \sum_{i=1}^{n} \alpha_i \ln x_i + \tfrac{1}{2} \sum_i \sum_j \gamma_{ij} \ln x_i \ln x_j \ , \tag{1.16}$$

where y is the quantity of output and the x's are inputs; α_0 is a parameter which represents the state of technical knowledge; α_i and γ_{ij} are technologically determined parameters Equality of γ_{ij} and γ_{ji} $(i \neq j)$ is necessary for the applicability of Young's theorem to integrable functions, while this equality could in principle be tested. In this function all input quantities must be strictly positive, because $\ln x \to -\infty$ as $x \to 0$ and output would be ill-defined.

The translog function has several interesting properties in both its theoretical and empirical application, especially if all the log quadratic terms are omitted, when it reduces to a Cobb-Douglas function; and as Christensen et al (1970) have shown, most of the CES-like functions may be derived from it as special cases when appropriate restrictions are imposed.

The interesting feature of the translog function, which has entitled this function, is its flexibility. It can be considered a second order approximation to any functional form for values of factors near unit. To see this, let us start, in general, with the following function:

$$y = \phi(x_1, \ldots, x_n)$$

where y denotes output, and the x's are services of inputs. This function may be rewritten as:

$$\ln y = \ln \phi(e^{\ln x_1}, \ldots, e^{\ln x_n}) ,$$

or

$$\ln y = f(\ln x_1, \ldots, \ln x_n) . \qquad (1.17)$$

Applying Taylor's method (see Allen (1938)), we expand (117) around point $\underline{x} = (\underline{1})$ or $\ln \underline{x} = (\underline{0})$, (where $\underline{x}$ is a n dimentional vector of input), obtaining:

$$\ln y = f(\ln 1, \ldots, \ln 1) + \sum_{i=1}^{n} \left.\frac{\partial f}{\partial \ln x_i}\right|_{\ln \underline{x}=\underline{0}} \ln x_i$$

$$+ \tfrac{1}{2} \sum_{i} \sum_{j} \left.\frac{\partial^2 f}{\partial \ln x_i \, \partial \ln x_j}\right|_{\ln \underline{x}=\underline{0}} \ln x_i \ln x_j + R \qquad (1.18)$$

where R represents the higher order terms. Note that $\left.\frac{\partial f}{\partial \ln x_i}\right|_{\ln \underline{x}=\underline{0}}$ and $\left.\frac{\partial^2 f}{\partial \ln x_i \partial \ln x_j}\right|_{\ln \underline{x}=\underline{0}}$ are constant for $i,j=1, \ldots, n$. Therefore we let the following:

$$\begin{cases} \left.\frac{\partial f}{\partial \ln x_i}\right|_{\ln \underline{x}=0} \equiv \alpha_i \quad i=1, \ldots, n. \\ \gamma_{ij} \equiv \left.\frac{\partial^2 f}{\partial \ln x_i \partial \ln x_j}\right|_{\ln \underline{x}=\underline{0}} = \left.\frac{\partial^2 f}{\partial \ln x_j \partial \ln x_i}\right|_{\ln \underline{x}=\underline{0}} \\ \equiv \gamma_{ji}, \quad i,j=1, \ldots, n. \end{cases} \qquad (1.19)$$

$$f(\ln 1, \ldots, \ln 1) = \ln \alpha_0 .$$

Therefore, by substituting (1.19) in (1.18) and omitting the higher order terms, R, we get (1.16) which is the translog function. Q.E.D.

1.9A Monotonicity and Convexity Properties

An important neoclassical assumption on production function is that of increasing marginal productivity of all factors, namely:

$$f_i = \frac{\partial y}{\partial x_i} \geq 0 , \quad i=1, \ldots, n .$$

This implies that

$$\frac{\partial \ln y}{\partial \ln x_i} = \frac{x_i}{y} \frac{\partial y}{\partial x_i} \geq 0 , \quad i=1, \ldots, n .$$

Because x_i and y are positive therefore

$$\frac{\partial \ln y}{\partial \ln x_i} = \alpha_i + \sum_{j=1}^{n} \gamma_{ij} \ln x_j \geq 0 \quad i=1, \ldots, n . \tag{1.20}$$

It is obvious that in general these monotonicity conditions cannot be globally satisfied for all quantity configurations. Notice that the translog function possesses an uneconomic region over ranges of input space if

(i) $x_j \to 0$ ($j=1, \ldots, n$) and $\gamma_{ij} > 0$,

or

(ii) $x_j \to \infty$ ($j=1, \ldots, n$) and $\gamma_{ij} < 0$.

Both cases imply negativity of f_i, the marginal product of the i^{th} input. This indicates that the translog function exhibits much more flexibility than either the Cobb-Douglas or the C.E.S. function which do not allow the existence of an uneconomic region.

Should this case (negativity of marginal product) happen, a profit maximizing or cost minimizing producer will not operate in that region as long as there exists a non-negative price associated with that input. In general, however, a local satisfaction of the monotonicity conditions, especially at the point of approximation, $\underline{x} = (\underline{1})$, is expected. This implies that

$$\alpha_i \geq 0 \quad i=1, \ldots, n .$$

Having $\alpha_i \geq 0$, $i=1, \ldots, n$, and an arbitrary set of γ_{ij}'s, it is always possible to find $\ln x_j$'s, not all zeros, in order to have (1.20) satisfied.

Another property that a neoclassical production function must also have, in addition to the monotonicity property, is that of the concavity, i.e., it exhibits decreasing returns to scale. To satisfy this property, $[f_{ij}]$ must be negative semidefinite. Hence a necessary condition is that $f_{ii} \leq 0$, or

$$f_{ii} = \frac{\partial^2 y}{\partial x_i^2} = \frac{\partial}{\partial x_i}\left(\frac{\partial y}{\partial x_i}\right) = \frac{\partial}{\partial x_i}\left(\frac{y}{x_i}\frac{\partial \ln y}{\partial \ln x_i}\right)$$

$$= \frac{\partial \ln y}{\partial \ln x_i}\left(\frac{x_i \frac{\partial y}{\partial x_i} - y}{x_i^2}\right) + \frac{y}{x_i}\frac{\partial}{\partial x_i}\left(\frac{\partial \ln y}{\partial \ln x_i}\right)$$

$$= \frac{y}{x_i^2}\left(\left(\frac{\partial \ln y}{\partial \ln x_i} - 1\right)\frac{\partial \ln y}{\partial \ln x_i} + \gamma_{ii}\right) \leq 0$$

This inequality must be satisfied in particular at the point of approximation. Therefore

$$\left.\frac{\partial^2 y}{\partial x_i^2}\right|_{\ln \underline{x}=\underline{0}} = y\left((\alpha_i - 1)\,\alpha_i + \gamma_{ii}\right) \leq 0 \tag{1.21}$$

To have (1.21) satisfied, given monotonicity, i.e. $\alpha_i \geq 0$, the following must be true:

$$0 \leq \alpha_i \leq 1$$

$$\gamma_{ii} \leq 0 \,.$$

Moreover, we compute cross partial derivatives as

$$\frac{\partial^2 y}{\partial x_i \partial x_j} = \frac{y}{x_i x_j} \left(\frac{\partial \ln y}{\partial \ln x_i} \frac{\partial \ln y}{\partial \ln x_j} + \gamma_{ij} \right) .$$

Evaluating this at the point of approximation we obtain

$$\left. \frac{\partial^2 y}{\partial x_i \partial x_j} \right|_{\ln \underline{x}=\underline{0}} \quad y \, (\alpha_i \alpha_j + \gamma_{ij}) \; .$$

Concavity of production function at ln $\underline{x}=\underline{0}$ implies, by a continuity argument, that $f(\cdot)$ is locally concave in the neighborhood of ln $\underline{x}=\underline{0}$; and this is all we need, because, as has been shown in standard microeconomic texts, the economic region is the concave region, even though the function might have uneconomic or convex regions which are locally convex and might exhibit increasing return to scale in certain ranges of the inputs. Therefore a necessary and sufficient conditions for $f(\cdot)$ to be locally concave at ln $\underline{x}=\underline{0}$ is that the matrix

$$[f_{ij}]_{\ln \underline{x}=0} = \begin{bmatrix} (\alpha_1 - 1)\alpha_1 + \gamma_{11} & \alpha_1\alpha_2 + \gamma_{12} & \alpha_1\alpha_n + \gamma_{1n} \\ \vdots & & \vdots \\ \alpha_n\alpha_1 + \gamma_{n1} & \cdots\cdots\cdots & (\alpha_n - 1)\alpha_n + \gamma_{nn} \end{bmatrix}$$

be negative semidefinite which in turn requires that all the principal minors be negative semidefinite. Therefore to check concavity we must compute these principal minors to see whether the concavity conditions are met.

It might seem desirable to actually test for the concavity assumption either in general or at some predetermined level of inputs. In this regard, the translog function is appropriate and ideal since global concavity is not assumed a priori.

In addition to the above test, it is very convenient to test statistically the homogeneity of the production function. If the production function exhibits constant returns to scale (CRS), first degree of homogeneity, it must be true that:

$$\ln f(\lambda x_1, \ldots, \lambda x_n) = \ln \lambda \cdot f(x_1, \ldots, x_n)$$

$$= \ln f(x_1, \ldots, x_n) + \ln \lambda$$

This implies the following set of linear parametric restrictions:

$$\Sigma\alpha_i = 1$$

$$\sum_i \gamma_{ij} = \sum_j \gamma_{ji} = \Sigma\Sigma\gamma_{ij} = 0 \qquad \forall \; i,j=1, \ldots, n .$$

which can be tested easily, given the required data. Thus we notice that how flexible the translog production function is, since it allows increasing, decreasing, and CRS, as well as completely variable returns to scale over the range of inputs.

The translog function will also provide a suitable framework for empirical work since the function and its corresponding marginal productivity conditions are linear in their parameters and therefore may be estimated, taking into account the linear equality restrictions across equations, by applying multiple regression techniques.

1.9B Translog Function and Separability

When we were discussing separability we saw that different aggregate indices of heterogeneous capital, material, and labor inputs could be used in the production function, based on the assumption that the production function was separable in those aggregate indices. Separability allows us to use aggregate data when disaggregated data do not exist or have poor quality. Another advantage of separability is that of its consistency with decentralization in decision making or equivalent optimization by stages (multistage). In some cases the appropriate disaggregated data exist, but even in these cases only multistage optimization and estimation is feasible, due to the large number of inputs involved. On the other hand, the separability specification will cause severe restrictions on the structure of technology, and hence on the possible form of the production function. In spite of this shortcoming, separability is a pivotal concept in production function estimation; although in most of the production function studies separability and the existence of aggregate inputs have been assumed *a priori*. However, there are two recent studies, done by Berndt and Christensen (1973 and 1974), in which empirical tests of separability and the possible existence of consistent aggregates of labor and capital have been furnished for the first time, using a translog function. Instead of using the translog function as a second order approximation to some unknown arbitrary production function, they have implicitly assumed the translog function as a <u>true</u> representation of the underlying technology. This procedure results in a different and more restrictive test for separability, and is not accepted as a general test of the separability hypothesis. In fact, the real problem with tests of separability, based on an exact interpretation of the translog function, is that they are not only tests of the null hypothesis of separability. Instead they result in tests of the joint null hypothesis of separability and a particular inflexible functional form for either the aggregate functions or the production function as a function of the aggregate inputs. The following proposition is due to Denny and Fuss (1977) and summarizes the whole argument.

Proposition (3): The separable form of a translog function, interpreted as an exact production function, must be either a Cobb-Douglas function of translog subaggregates or a translog function of Cobb-Douglas subaggregates.

To clarify this proposition, assume that the three input translog function (1.16), interpreted as being exact, is weakly separable as

$$\ln y = f(\ln g\ (\ln x_1,\ \ln x_2),\ \ln x_3)\ ,$$

where g is an input aggregator function. Since f is weakly separable then it must be true that:

$$f_i f_{jk} - f_j f_{ik} = 0 \qquad i,j=1,2; \text{ and } k=3$$

Substituting for first and cross partial derivatives in the above we obtain:

$$S_i \gamma_{jk} - S_j \gamma_{ik} = 0$$

$$(\alpha_i + \sum_j \gamma_{ij} \ln x_j)\gamma_{jk} - (\alpha_j + \sum_i \gamma_{ji} \ln x_i)\gamma_{ik} = 0$$

or

$$(\alpha_i \gamma_{jk} - \alpha_j \gamma_{ik}) + \sum_{m=1}^{3} (\gamma_{im}\gamma_{jk} - \gamma_{ik}\gamma_{jm}) \ln x_m = 0 \qquad (1.22)$$

To have (1.22) equal to zero we need both parantheses equal to zero. However, a sufficient condition for (1.22) to hold is:

$$\gamma_{ik} = \gamma_{jk} = 0 \ .$$

which are termed "linear" separability constraints. For nonzero γ_{ik} and γ_{jk} a necessary and sufficient condition for (1.22) to hold are

$$\alpha_i \gamma_{jk} - \alpha_j \gamma_{ik} = 0$$

$$\gamma_{im}\gamma_{jk} - \gamma_{jm}\gamma_{ik} = 0 \ ,$$

which may be written all together as

$$\frac{\alpha_i}{\alpha_j} = \frac{\gamma_{ik}}{\gamma_{jk}} = \frac{\gamma_{im}}{\gamma_{jm}} = \delta . \qquad m=1, \ldots, 3$$

Now there are two possibilities:

(a) Substituting the linear separability constraints ($\gamma_{13} = \gamma_{23} = 0$ in 3 inputs case) in translog production function (interpreted as exact) results in a production of the form (see Denny and Fuss (1977))

$$\ln y = \ln \alpha_0 + \delta_g \ln g + \delta_h \ln h \tag{1.23}$$

where g is a translog function of x_1 and x and h is a translog function of x_3, while δ_g and δ_h are the corresponding parameters. Obviously (1.23) is a Cobb-Douglas function of translog input aggregates. Here separability implies a unitary elasticity of substitution between aggregate inputs g and h along the y isoquant as well as $\sigma_{23} = \sigma_{13}$.

(b) Now if we substitute the non-linear constraints $(\frac{\alpha_1}{\alpha_2} = \frac{\gamma_{13}}{\gamma_{23}} = \frac{\gamma_{11}}{\gamma_{12}} = \frac{\gamma_{12}}{\gamma_{22}} = \delta)$ in translog function, again interpreted as exact, we obtain a production function of the form[5]:

$$\ln y = \ln \alpha_0 + \beta_g \ln g + \beta_h \ln h + 1/2 \sum_{i,j=g,h}\sum \beta_{ij} \ln g \ln h, \tag{1.24}$$

where g here represents a Cobb-Douglas function of x_1 and x_2 and α_0, β_g, and β_h are parameters. The function (1.24) is a translog function of Cobb-Douglas input aggregates. In this case, separability implies a unitary elasticity of substitution between x_1 and x_2 (i.e. along a g isoquant) as well as

$\sigma_{13} = \sigma_{23}$, $_{ij}$ being the Allen partial elasticity of substitution between factor i and j along a y isoquant. Then a rejection of the maintained (separability) hypothesis might be due to rejecting a unitary elasticity of substitution for the subaggregate function instead of the correct maintained hypothesis $\sigma_{13} = \sigma_{23}$. To avoid this problem one must consider the translog function not as a true function but rather as a second order approximation.

The following proposition about the approximate weak separability, which has been proven by Denny and Fuss (1977), will illustrate the point.

Proposition (4)[6]: The translog function (16) is a quadratic approximation to an arbitrary weakly separable production function $\ln y = f(\ln g(\ln x_1, \ln x_2), \ln x_3)$ if

$$\frac{\alpha_1}{\alpha_2} = \frac{\gamma_{13}}{\gamma_{23}} \qquad (1.25)$$

As we see the constraint (1.25) in this case is identical to the first set of constraints needed for separability in the exact case $(\frac{\alpha_1}{\alpha_2} = \frac{\gamma_{13}}{\gamma_{23}} = \frac{\gamma_{11}}{\gamma_{21}} = \frac{\gamma_{12}}{\gamma_{22}})$. For inputs 1 and 2 being separable from 3, it is shown by Berndt and Christensen (1973a) that the remainder of the constraints reduce to one independent constraint of the form $\gamma_{11}\gamma_{22} = (\gamma_{12})^2$. But this constraint is the one which forces $\ln g(\ln x_1, \ln x_2)$ to become a Cobb-Douglas sub-function, as can be seen from the proof of the proposition (3). Therefore, approximate weak separability involves imposing the constraint

$\alpha_1\gamma_{23} - \alpha_2\gamma_{13} = 0$ while for testing exact weak separability the imposition of the additional constraint $\gamma_{11}\gamma_{22} - (\gamma_{12})^2 = 0$ is needed.

1.10 Translog Cost Function

We saw that a producing unit was capable of producing alternative rates of output according to a cost function $C = C(y; w_1, \ldots, w_n)$. A translog cost function may be represented as:

$$\ln C = \alpha_0 + \theta_1 \ln y + \tfrac{1}{2} \theta_2 (\ln y)^2 + \Sigma\alpha_i \ln w_i + \tfrac{1}{2} \Sigma\Sigma\gamma_{ij} \ln w_i \ln w_j + \Sigma\beta_i \ln w_i \ln y . \quad (1.26)$$

One of the properties of the cost function for the cost minimizing firm is that of linear homogeneity in factor prices (see section 4). For this property to hold, it must obey the following restrictions on the parameters of (1.26):

$$\Sigma \alpha_i = 1$$

$$\sum_j \gamma_{ij} = \sum_i \gamma_{ji} = \sum_i \sum_j \gamma_{ij} = 0$$

$$\Sigma\beta_i = 0.$$

One may wish to test the validity of these restrictions as a test of the cost minimization hypothesis; or estimate the cost with these restrictions imposed *a priori*.

As was shown (section 4), invoking the well known Shephard's Lemma, the derived factor demand function can be obtained by partial differentiation of the cost function with respect to the factor prices; namely,

$$x_i = \frac{\partial C}{\partial w_i} \qquad i=1, \ldots, n;$$

which can appropriately be written in logarithmic form for the translog cost function as

$$\frac{\partial \ln C}{\partial \ln w_i} = \frac{\partial C}{\partial w_i} \frac{w_i}{C} = \frac{x_i w_i}{C} = S_i \qquad i=1, \ldots, n.$$

where S_i represents the relative share of the i^{th} input in total cost, which is computed from translog as

$$S_i = \alpha_i + \beta_i \ln y + \sum_j \gamma_{ij} \ln w_j \qquad i=1, \ldots, n .$$

By the monotonicity property the cost function must be an increasing function of input prices, i.e., $S_i \geq 0$. Unlike the derived factor demand function, x_i, these shares are linear in parameters and may conveniently be estimated. Note that since these shares add up to one, i.e., $\Sigma S_i = 1$, only (n-1) of these equations are independent; therefore, the (n-1) equations may be estimated in conjunction with the cost function itself. Later, the estimation procedure will be discussed in detail.

Another property of the cost function is that of concavity in input prices, which implies that the matrix $(\frac{\partial^2 C}{\partial w_i \partial w_j})$ must be negative semidefinite for a specified range of input prices.

There are other economically interesting hypotheses which may be suitably be tested within the framework of the translog cost function. These are homogeneity and homotheticity of the production function in inputs. Recall that homogeneity of the production function implies that

$$C(y,w) = y \cdot \lambda(w) \ .$$

To have this satisfied the following parametric restrictions must hold for (1.26).

$$\theta_2 = 0$$

$$\beta_i = 0 \qquad\qquad i=1, \ldots, n \ .$$

For a homothetic production function the cost function is factorized as

$$C(y,w) = h(y) \cdot \lambda(w) \ ,$$

which implies the following restriction on (1.26)

$$\beta_i = 0 \qquad\qquad i=1, \ldots, n \ .$$

As was shown, the Allen partial elasticity of substitution between two inputs i and j could be calculated from the cost function as

$$\sigma_{ij} = \frac{CC_{ij}}{C_i C_j}$$

where the indices on C represent partial differentiation of the cost function with respect to factor prices. Computing this for the translog cost function we obtain,[7]

$$\sigma_{ij} = \frac{\gamma_{ij}}{S_i S_j} + 1 \qquad i \neq j$$

$$\sigma_{ii} = \frac{\gamma_{ii} + S_i\ (S_i - 1)}{S_i^2}$$

Similarly the price elasticity of demand for a factor of production $E_{ij} = \frac{\partial \ln x_i}{\partial \ln w_j}$ is computed as

$$E_{ij} = \sigma_{ij} S_j$$

and the own-price elasticity of demand for i^{th} input is (see Allen (1938, p. 519, problem number 12)),

$$E_{ii} = \sigma_{ii} S_i \ .$$

Finally economies of scale are widely defined as the relative increase in output resulting from a proportional increase in all inputs along a ray through the origin. However, Hanoch (1975) has discussed a more relevant concept for microeconomic analysis as "the increase in output relative to costs for variations along the expansion path where input prices are constant and costs are minimized at every output." Thus the extent of scale economies can be expressed as the elasticity of total cost with respect to output, i.e., $\partial \ln C / \partial \ln y$. An alternative way to arrive at this elasticity is the following argument: positive economies of scale are associated with decreasing average cost (AC), i.e., $\frac{d}{dy}\ (\frac{C}{y}) < 0$ or $yC' - C < 0$, where $C' = \frac{dC}{dy}$ is the marginal cost (MC). This implies that $\frac{MC}{AC} < 1$, which

in turn can be written as $\frac{MC}{AC} = (\frac{dC}{dy})/(\frac{C}{y}) = \frac{d \ln C}{d \ln y} < 1$.
Thus we can measure economies of scale by d ln C/d ln y; and in order to make positive scale economies associated with positive numbers and negative scale economies (scale diseconomies), we subtract it from unity, i.e., scale economies $= 1 - \partial \ln C / \partial \ln y$.

1.11 Translog Profit Function

The translog profit function is given as

$$\ln \pi (p, w_1, \ldots w_n) = \alpha_0 + \theta_1 \ln p + \tfrac{1}{2} \theta_2 (\ln p)^2$$
$$+ \Sigma \alpha_i \ln w_i + \tfrac{1}{2} \Sigma\Sigma \gamma_{ij} \ln w_i \ln w_j + \Sigma \beta_i \ln w_i \ln p .$$

where p and w_i are the money prices of the output and the i^{th} factor of production respectively. It was shown that a valid profit function was characterized by certain conditions such as linear homogeneity in p and w, monotonicity, and concavity. Linear homogeneity in p and w implies following restrictions on the parameters:

$$\theta_1 + \sum_{i=1}^{n} \alpha_i = 1$$

$$\sum_{i=1}^{n} \gamma_{ji} + \sum_i \beta_i = 0 \qquad j=1,\ldots, n .$$

$$\theta_2 + \sum_{i=1}^{n} \beta_i = 0.$$

Monotonicity requires that $\pi(p,w)$ be an increasing function in p and a decreasing one in w. Thus the following must be met:

$$\frac{\partial \pi}{\partial p} = \frac{\pi}{p} \frac{\partial \ln \pi}{\partial \ln p} = \frac{\pi}{p} (\theta_1 + \theta_2 \ln p + \sum_i \beta_i \ln w_i) \geq 0$$

$$\frac{\partial \pi}{\partial w_i} = \frac{\pi}{w_i} \frac{\partial \ln \pi}{\partial \ln w_i} = \frac{\pi}{w_i} (\alpha_i + \sum_j \gamma_{ij} \ln w_j + \beta_i \ln p) \leq 0 \tag{1.27}$$

It is obvious that these conditions can not be globally met for all price combinations. But, as long as there exists the actual observed ranges of price for which the monotonicity conditions are met, we should not be concerned so much about global monotonicity. These conditions must be particularly satisfied at the point of approximation, $\ln p = 0$, $\ln \underline{w} = \underline{0}$. This implies, since π, w, and p are positive, that:

$$\theta_1 \geq 0$$

$$\alpha_i \leq 0 \qquad i=1, \ldots, n .$$

These restrictions are easily testable.

For the convexity condition it is required that the matrix (π_{ij}) be positive semidefinite - a necessary condition is that $\pi_{ii} \geq 0$, or

$$\frac{\partial^2}{\partial p^2} = \frac{\pi}{p^2} (\frac{\partial \ln \pi}{\partial \ln p} (\frac{\partial \ln \pi}{\partial \ln p} - 1) + \theta_2) \geq 0$$

$$\frac{\partial^2 \pi}{\partial w_i^2} = \frac{\pi}{w_i^2} (\frac{\partial \ln \pi}{\partial \ln w_i} (\frac{\partial \ln \pi}{\partial \ln w_i} - 1) + \gamma_{ii}) \geq 0 .$$

Evaluating these at the point of approximation, we obtain:

$$\left.\frac{\partial^2 \pi}{\partial p^2}\right|_{\ln p=0,\ \ln \underline{w}=\underline{0}} = \pi(\theta_1(\theta_1-1) + \theta_2) \geq 0$$

$$\left.\frac{\partial^2 \pi}{\partial w_i^2}\right|_{\ln p=0,\ \ln \underline{w}=\underline{0}} = \pi(\alpha_i(\alpha_i-1) + \gamma_{ii}) \geq 0,\ i=1,\ldots,n$$

From these conditions, given monotonicity, we derive a sufficient set of conditions as

$$\theta_1 \geq 1, \qquad \theta_2 \geq 0, \qquad \gamma_{ii} \geq 0 .$$

In order to form the matrix (π_{ij}) we compute the cross partial derivatives as:

$$\frac{\partial^2 \pi}{\partial p \partial w_i} = \frac{\pi}{p w_i} \left(\frac{\partial \ln \pi}{\partial \ln p} \frac{\partial \ln \pi}{\partial \ln w_i} + \beta_i\right)$$

$$\frac{\partial^2 \pi}{\partial w_i w_j} = \frac{\pi}{w_i w_j} \left(\frac{\partial \ln \pi}{\partial \ln w_i} \frac{\partial \ln \pi}{\partial \ln w_j} + \gamma_{ij}\right) .$$

At the point of approximation these are equal to:

$$\left.\frac{\partial^2 \pi}{\partial p \partial w_i}\right|_{\ln p=0,\ \ln \underline{w}=\underline{0}} = \pi(\theta_1 \alpha_i + \beta_i)$$

$$\left.\frac{\partial^2 \pi}{\partial w_i \partial w_j}\right|_{\ln p=0,\ \ln \underline{w}=\underline{0}} = \pi(\alpha_i \alpha_j + \gamma_{ij}) .$$

Hence the matrix $(\pi_{ij})_{\ln \underline{w}=\underline{0},\ \ln p=0}$ becomes:

$$\begin{bmatrix} \theta_1(\theta_1-1)+\theta_2 & \theta_1\alpha_1+\beta_1 & \theta_1\alpha_2+\beta_2 & \cdots & \theta_1\alpha_n+\beta_n \\ \theta_1\alpha_1+\beta_1 & \alpha_1(\alpha_1-1)+\gamma_{11} & \alpha_1\alpha_2+\gamma_{12} & \cdots & \alpha_1\alpha_n+\gamma_{1n} \\ \vdots & & & & \\ \theta_1\alpha_n+\beta_n & \alpha_n\alpha_1+\gamma_{n1} & \alpha_n\alpha_2+\gamma_{n2} & \cdots & \alpha_n(\alpha_n-1)+\gamma_{nn} \end{bmatrix}$$

To see that convexity conditions are satisfied locally we must have all the principal minors positive semidefinite.

One of the main advantages of using the profit function is that, by invoking the duality theorem as we saw, we are able to derive the supply and demand functions conveniently by differentiating the profit function with respect to the prices of output and inputs respectively. For the translog profit function the supply function of output and the demand functions for factors of production are given by the linear forms (1.27). Note that only n of these equations are independent and that linearity of the equations is an additional advantage for econometric estimation.

The own and cross-price elasticities of demand for the factors of production are derivable from the demand functions, and are defined as:

$$E_{ii} = \frac{\partial x_i^*}{\partial w_i} \frac{w_i}{x_i^*} \quad \text{and}$$

$$E_{ij} = \frac{\partial x_i^*}{\partial w_j} \frac{w_j}{x_i^*} .$$

Utilizing:

$$- \frac{\partial \pi}{\partial w_i} = x_i^* \quad \text{and}$$

$$\frac{\partial x_i^*}{\partial w_i} = - \frac{\partial^2 \pi}{\partial w_i^2}$$

E_{ii} can be written as

$$E_{ii} = \frac{w_i \frac{\partial^2 \pi}{\partial w_i^2}}{\frac{\partial \pi}{\partial w_i}} = \frac{- S_i^2 - S_i - \gamma_{ii}}{S_i} \tag{1.28}$$

where S_i, as before, is the ratio of expenditure on i^{th} input to profit, i.e.,

$$S_i = - \frac{\partial \ln \pi}{\partial \ln w_i} = - \frac{\partial \pi}{\partial w_i} \frac{w_i}{\pi} = \frac{w_i x_i^*}{\pi} \tag{1.29}$$

Similarly

$$E_{ij} = \frac{w_j \frac{\partial^2 \pi}{\partial w_i \partial w_j}}{\frac{\partial \pi}{\partial w_i}} = \frac{- S_i S_j - \gamma_{ij}}{S_i} \tag{1.30}$$

Partial elasticities of substitution are obtained by the following definitions:

$$\sigma_{ii} = \frac{E_{ii}}{S_i} = \frac{-(S_i^2 + S_i + \gamma_{ii})}{S_i^2}$$

$$\sigma_{ij} = \frac{E_{ij}}{S_j} = \frac{-(S_i S_j + \gamma_{ij})}{S_i S_j}$$

An interesting formula for the partial elasticities of substitution is derived by substituting for S_i and S_j, E_{ii} and E_{ij} from (1.28) - (1.30) which results in

$$\sigma_{ii} = \frac{\frac{w_i}{x_i^*}\ \frac{\partial x_i^*}{\partial w_i}}{-\frac{\partial \pi}{\partial w_i}\ \frac{w_i}{\pi}} = \frac{\pi\ \frac{\partial^2 \pi}{\partial w_i^2}}{\left(\frac{\partial \pi}{\partial w_i}\right)^2} = \frac{\pi \pi_{ii}}{(\pi_i)^2} \qquad (1.31)$$

and similarly

$$\sigma_{ij} = \frac{\pi \pi_{ij}}{\pi_i\ \pi_j}$$

This formula, (1.31), for the elasticity of substitution is similar to Uzawa's formulation of the elasticity of substitution which was in terms of the cost function and its first and second partial derivatives. Here we have shown that it is possible to formulate the Allen partial elasticity of substitution in terms of the profit function and its first and second partial derivatives with respect to factor prices. This may be considered a generalization of Uzawa's.

To summarize, the purpose of this chapter is to provide a description of the theoretical issues which form the basis of empirical research. We have reviewed the principle practical application of duality theory between production and cost function, which establishes that the two approaches are equivalent and equally fundamental. In particular, we have studied the two main advantages of the duality approach to the theory of production. First, the advantage that enables us to derive, painlessly, the system of demand and supply equations consistent with the optimization behavior of the firm, just by direct differentiation of cost or profit functions. Second, the advantage that the "comperative static" results (elasticity of substitution, etc.) associated with optimizing behavior are very easily derived. We have then studied the concept of weak separability among inputs and have seen that the assumption of weak separability on the cost function leads to severe restrictions on the partial elasticities of substitution. In this chapter we also have surveyed various functional forms; in particular, the many features of the translog functions have been described. More specifically, we have seen that (i) the translog function provides a second order Taylor's series approximation to any twice differentiable function, and therefore, there are no *a priori* assumptions on functional form to be estimated in empirical investigations; (ii) many economically meaningful hypotheses appear as linear restrictions on the parameters of the translog function and thus may be readily tested; (iii) the translog function and the marginal conditions are linear in parameters and hence may be easily estimated by standard linear regression methods; and (iv) the translog cost or production function, unlike the CES, or CD functions, imposes no *a priori* constraints on the partial elasticities of substitution and, therefore, is a powerful vehicle for the testing of specific functional forms.

CHAPTER I

FOOTNOTES

[1]The production function f(x) is said to be bounded if

$$\lim_{\lambda \to \infty} \frac{f(\lambda x)}{\lambda} = 0$$

This ensures that an attainable solution exists for the normalized profit maximization problem.

[2]A function is defined as being "homothetically separable" if it is weakly separable and each subfunction is homothetic; however, note that this does not necessarily imply the homotheticity of the function itself in $x^1, \ldots, x^s$.

[3]For proof, see Lau (1969) p. 385.

[4]One obvious approach is to make the common AES, σ, a function of some variable such as the level of output or the factor ratio or factor share, etc. Such generalizations have been called Variable Elasticity of Substitution (VES) functions, and have been discussed by Ravankar (1971), Lu and Fletcher (1968), Sato and Beckmann (1968), and Lovell (1973).

[5]For the details, see Denny and Fuss (1977).

[6]Instead of repeating their proof of this proposition a simpler proof is given as follows. Since the translog of the form (1.16) with symmetry imposed ($\gamma_{ij}=\gamma_{ji}$) is a quadratic approximation (around the expansion point $\underline{x} = (1, \ldots, 1)$) to an arbitrary production function of the form $\ln y = f(\ln x_1, \ldots, \ln x_n)$, by evaluating the Leontief conditions for weak separability of the translog function (i.e., equation (1.22)) at the point of approximation (by substituting $\ln \underline{x}=\underline{0}$ in equation (1.22) one obtains $\alpha_i\gamma_{jk} - \alpha_j\gamma_{ik} = 0$. For the three input case, where the third input is weakly separated from the other two, this condition can be written as

$$\frac{\alpha_1}{\alpha_2} = \frac{\gamma_{13}}{\gamma_{23}}$$ which is equation (1.25) Q.E.D.

[7]To see this we substitute in σ_{ij} for C_i, C_j, and C_{ij} in terms of S_i, S_j, and the parameters of the translog cost function. We obtain C_i and C_{ij} as follows:

$S_i = \frac{\partial \ln C}{\partial \ln w_i} = \frac{\partial C_i}{\partial w_i} = \frac{w_i}{C}$; therefore $C_i = \frac{S_i C}{w_i}$, and similarly $C_j = \frac{S_j C}{w_j}$. Then $C_{ij} = \frac{\partial C_i}{\partial w_j} = \frac{\partial}{\partial w_j} (\frac{S_i C}{w_i})$

$= \frac{C}{w_i} \frac{\partial S_i}{\partial w_j} + \frac{S_i}{w_i} \frac{\partial C}{\partial w_j}$. But $\frac{\partial S_i}{\partial w_j} = \frac{\gamma_{ij}}{\partial w_j}$. Thus $C_{ij} = \gamma_{ij} \frac{C}{w_i w_j}$

$+ S_i S_j \frac{C}{w_i w_j}$. Substituting for C_i, C_j, and C_{ij} in σ_{ij}

we obtain the above formulas.

CHAPTER II

SPECIFICATION AND ESTIMATION OF INDUSTRIAL FACTOR DEMAND FUNCTIONS WITH EXPLICIT ACCOUNT OF INTERNALLY PRODUCED ENERGY AND MATERIALS INPUTS

2.0 Introduction

Tremendous increases in energy prices and interruption in energy supplies associated with the oil crises of 1973 have led to a rising interest in extensive research on the characteristics of energy demand and factor substitution. The fact that the manufacturing sector accounts for more than one quarter of annual energy consumption in the United States has made this sector a potential source of reduction in energy demand.

The growing number of econometric studies, which have appeared in the literature in recent years with the objective of examining the possibilities of dealing with continued increases in the real cost of energy, is an indication. Examples of such studies are: Berndt and Jorgenson (1973), Hudson and Jorgenson (1974, 1976, 1978a, 1978b), Berndt and Wood (1975, 1979), Griffin and Gregory (1976), Atkinson and Halvorsen (1976), Berndt, Fuss and Waverman (1977), Brock and Nesbit (1977), Fuss (1977), Halvorsen (1977), Pindyck (1978), Berndt and Khaled (1979), Berndt and White (1977), and Magnus (1979).

Since energy inputs along with other inputs (capital, labor, and material) enter a producing unit's production process, a cost minimizing producing unit responds to a continued energy price increase obviously through substitution. These studies have sought to deal with the degree of energy substitution with other inputs, which becomes a crucial factor in driving policy implications of increasingly scarce and higher priced energy inputs. If one finds that energy and labor are substitutable, then ceteris paribus, an increase in the price of energy leads to an increase in the demand for labor, and therefore employment rises. On the other hand, if it is found that energy and labor are complements, then ceteris paribus, higher priced energy will restrain the demand for energy, and the demand for labor and therefore employment will fall. As another example, one may consider the relation between capital and energy inputs. If energy and capital inputs complement each other in the production process, then any restraint on energy prices (e.g. energy price control) will increase the demand for capital goods, and therefore favor capital formation, while an increase in energy cost reduces the demand for new plant and equipment and consequently discourages capital formation. On the other hand, if energy and capital inputs turn out to be substitutable in the production process, then rising energy prices facilitate capital formation by increasing the demand for capital goods.

Aside from its attractiveness, the knowledge of technical substitutability of energy and non-energy inputs is very important in the choosing of an appropriate policy. If nonreplenishable, or slowly growing energy such as oil and natural gas must be technically employed in fixed or almost fixed proportion with other inputs such as labor or capital, then the growth of real national product in resource-scarce economies may become severely restricted in the near future. Accordingly then, some knowledge of technical substitution is essential for rational planning of private and government policy concerning foreign trade and allocation of revenues for energy resource development.

In general, it can be argued that the limited degree of substitution between energy and non-energy inputs makes the adjustment process by an industry to higher priced energy somewhat difficult, and might cause a substantial rise in the unit cost. The industry in question might even shift the composition of the product away from an existing energy intensive to a non energy-intensive production process.

The estimation of Hicks Allen elasticity of substitution between energy and non-energy inputs in manufacturing have, therefore, been the center of attention in these recent studies. Review of the literature indicates that these estimates have not always been consistent. In fact, we have witnessed apparently contradictory results, the most interesting being the substitution possibilities between energy and capital, which may be summarized briefly in two main categories, according to energy-capital complementarity and energy-capital substitutability.

Among these studies Berndt and Wood (1975), Berndt and White (1979), Berndt and Khaled (1979), Berndt and

Jorgenson (1973), using time series data on capital (K), labor (L), energy (E), and materials (M) for the United States manufacturing, have found energy and capital inputs as complements. Similarly Fuss (1977) and Magnus (1979) employing the above four (KLEM) time series data for Canadian manufacturing pooled by different regions and KLE time series data for Dutch manufacturing respectively found the same K-E complementarity result.

In contrast to the above results, in studies by Griffin and Gregory (1976), as well as Pindyck (1979), based on KLE time series data pooled by organization for Economic Cooperation and Development (OECD) countries, both found K-E substitutability. Also Wills (1979), based on KLEM time series data on the United States primary metal industry, found capital and energy as being substitutable. Similar results obtained in a study by Halvorsen and Ford (1970), where employing cross section data on capital, two types of labor and three types of energy by state for eight two-digit SIC manufacturing industries, found either significant (E-K) substitutability or insignificant (E-K) complementarity.

In contrast to the conflicting evidence from econometric studies regarding E-K substitution possibilities, the substitution possibilities regarding (K-L), (E-L), (E-M), and (L-M) have been relatively consistent throughout these studies with few exceptions. It is not, however, the purpose of this study to reconcile these apparent contradictions regarding factor substitution possibilities, since this has been done elsewhere.[1] This study rather deals with another subtle issue raised by Anderson (1980) in his estimation of the United States non-energy manufacturing sector.

In the large number of the empirical studies on the possibilities of energy reduction in the manufacturing sector which has been conducted in the recent years - which were mentioned above - the most frequent model specification has been consisted of choosing a flexible aggregate cost function defined over four aggregate inputs of capital services (K), labor services (L), energy (E), and materials (M). Then, utilizing the well-known Shephard Lemma of the theory of the firm, researchers have, similarly, obtained the industry's conditional factor demand functions, as first partial derivatives of the industry's aggregate cost function with respect to factor prices. In the theory of the firm the application of Shephard's Lemma for obtaining the firm's conditional factor demand, as first partial derivatives of the firm's cost function with respect to factor prices, is based on the crucial assumption of exogenous input prices. The analogous application of Shephard's Lemma in the context of industry violates this crucial assumption, and leads to an erroneous result as we will see below.

The source of this error rests in the measure of "output" in the industry-level studies due to the lack of sufficiently detailed data. Aggregate "output" and purchased "material" and "energy" have traditionally been measured as "gross" magnitudes. As such, these gross magnitudes contain the inter-industry shipment of intermediate products which move between firms. Therefore, this method measures industry-level "output" as firm's total value-of-shipments; and similarly "energy" and "materials" as total amount of "energy" and "materials" purchased by all the firms which comprise an industry, regardless of source. Thus, given these measures of gross output, energy, and materials inputs, the application of the well-known Shephard Lemma of the theory of the <u>firm</u> to obtain the conditional factor demand function for industry fails. The utilization of Shephard's Lemma, as was discussed above, is based on the assumption that the prices of inputs are exogeneous, while at industry level studies this assumption obviously does not hold.

A glance at table (2.1)[2] reveals that of the total "materials purchased by all firms in the non-energy manufacturing sector a considerable portion have been produced by the firms within the <u>same</u> non-energy manufacturing sector. These "intra-industry inter-firm shipments" of traded intermediate products (materials) constitute slightly more than 60 percent of total "materials" purchased by firms in the non-energy United States manufacturing sector. Similarly, in the manufacturing sector (energy and non-energy sectors combined) the intra-industry inter-firm shipment of traded intermediate products ("materials" and "energy")

TABLE 2.1

TOTAL COST OF INPUTS AND SOURCE OF ENERGY AND MATERIALS INPUTS FOR MANUFACTURING; NON-ENERGY MANUFACTURING, AND ENERGY-PRODUCING SECTORS OF THE U.S. ECONOMY - SELECTED YEARS, (Millions of Current $'s)

Year	MANUFACTURING Total Cost*	Materials+ Total	Materials+ Own-Sector	%	Energy+ Total	Energy+ Own-Sector	%	NON-ENERGY MANUFACTURING Total Cost*	Materials+ Total	Materials+ Own-Sector	%	Energy	ENERGY-PRODUCING MANUFACTURING Total Cost*	Energy+ Total	Energy+ Own-Sector	%	Material
1947	196205	120207	69687	58	7944	1725	22	187693	118229	69251	59	3492	8512	4452	618	14	1978
1950	236745	145461	88213	61	10447	1940	19	225870	143045	87493	61	4367	10875	6080	597	10	2416
1953	314988	186548	126028	68	13025	2484	19	300391	183442	125044	68	5057	14597	7968	807	10	3106
1956	354834	209907	134497	64	15389	3305	21	336928	205430	133480	65	6048	17906	9341	1182	13	4477
1959	383492	222044	138425	62	16707	3438	21	363609	216362	137390	64	6679	19883	10028	1244	12	5682
1962	422329	240702	147683	61	18084	3721	21	400199	233923	146764	63	7459	22130	10625	1311	12	6779
1965	518896	296491	184896	62	19959	4250	21	494056	289302	184033	64	8801	24840	11158	1341	12	7189
1968	634554	359397	221766	62	23731	5223	22	604506	350623	220545	63	10692	30048	13039	1730	13	8774
1971	705569	407712	227048	56	30042	6527	22	668710	394712	225946	57	14351	36859	15691	2122	14	13000

* Total cost of all firms' purchases of all inputs.
+ Energy and materials purchased from firms within the sector ("own-sector") and from firms outside the sector.

Source: Calculated from tables in Faucett Associates (1973).

constitute 60 percent and 20 percent of total purchase of "materials" and "energy" respectively (see table (2.1)). The balance of "energy" and "materials" then are imported from outside of the manufacturing sector, and therefore, these "imported" portions are primarily to the manufacturing sector.

While the price of these "primary" energy and material inputs are exogeneous in a partial-equilibrium competitive industry model, the prices of the internally (within manufacturing) produced energy and materials are not exogeneous. These products are produced by some firms in the industry. A portion of these products are sold to other firms within the same industry to be used as inputs and the rest are sold to firms, households, government, etc., outside the manufacturing sector. This latter portion which is delivered outside of the sector is what we call "net output". Obviously, the prices of these internally produced products, considered as the output of some of the firms in the sector, will respond to any change in the price of primary inputs and hence these prices must not be considered as exogenous but endogenous variables in the competitive industry general equilibrium model. The application of Shephard's Lemma to obtain conditional factor demands is therefore incorrect in these models, and this is exactly the specification error contained in all of the econometric studies mentioned above which have included "materials", "energy", or both inputs.

Having recognized the source of error in industry-level studies as the employment of concepts, such as gross industry output, aggregate "energy" and "materials" inputs, which include intra-industry shipments of intermediate products moving between firms, we consider an alternative specification for non-energy manufacturing and manufacturing sectors (non-energy as well as energy sectors combined). In this alternative specification, we consider the aggregate cost and factor demand functions not conditional upon the "gross" output, but properly upon the level of the output _delivered_ by the manufacturing sector to the rest of the economy, i.e. upon "net output level of the manufacturing sector.

Nearly all energy policy discussions are, correctly, concerned with the energy intensity of the products delivered by the manufacturing sector to the balance of the economy, i.e., we are interested with estimates of the potential reduction in energy consumption for each level of the sector's "net" output (i.e., net output held constant). Therefore, it is proper to estimate the price elasticities of factor demand and elasticities of factor substitution conditional upon the level of net output. In contrast, in the "gross" model formulation of the manufacturing sector, the researcher is forced to include internally produced and traded intermediate products as an input. Holding each firm upon its initial isoquant (i.e., gross output held constant), an increase in the industry's price for a primary factor (e.g., energy) will urge each firm to change its input mix away from energy and toward other factors. This, however, will increase every firm's unit cost, and therefore the price of its output. Consequently firms will attempt to substitute away from energy and away from the products produced by the other firms within the industry. Such an adjustment will alter the net output of each firm delivered to the rest of the economy (since each firm was assumed to be upon its initial isoquant). Thus, the "gross" model formulation does not provide an appropriate context for a meaningful energy policy discussion since the industry's net output oscillates as primary factor prices change.

In this study we estimate the U.S. non-energy manufacturing sector as well as the manufacturing sector (non-energy and energy producing combined) for the 1947-71 period, separately. In our model specification for the <u>non-energy manufacturing</u> sector, like Anderson (1980), we emphasize the role of "materials" inputs, having netted out "gross" materials of intra-industry shipments of intermediate products which move between firms. We have then employed in our model the resulting primary materials input along with the other primary inputs of capital, labor, and energy. Our data, however, is different from that of Anderson's (1980). In his study of non-energy manufacturing, Anderson has utilized capital, labor, and energy data developed and utilized by Berndt and Wood (1975) in their study of the

United States manufacturing. He then has constructed the "primary" materials data from Faucett Associates (1973). But one must realize that the capital, labor, and energy data used by Berndt and Wood (1975) are not appropriate for non-energy manufacturing studies, since they have modeled the manufacturing sector which combines the energy as well as non-energy sector. To study the non-energy sector, therefore, a different and more appropriate data set must be utilized. Thus, for the purpose of estimating the non-energy manufacturing sector, we employ the capital, labor, and energy data corresponding to the non-energy sector developed and used by Hudson and Jorgenson (1974) in their study of the business sector of United States economy - which has been subdivided into nine industrial groups, non-energy manufacturing being one of them - while the required data on materials, for the non-energy sector, has been constructed from Faucett Associates (1973), with consideration of inter-firms flow of materials input.

However, to analyze more completely the structure of technology in United States manufacturing - which is, traditionally, the non-energy and energy sectors combined - we employ the capital and labor data developed and used by Berndt and Wood (1975), while the energy and materials data have been constructed from tables in Faucett Associates (1973), again with careful consideration of inter-firms flow of traded intermediate products (energy as well as materials inputs[3]).

We conclude our discussion by noting that in nearly all of the industry-level studies the failure of researchers to consider the role of inter-firms flow of intermediate products (energy and materials inputs), which have been included in their "gross" measure of aggregate "output", "energy", and "materials", has introduced a serious specification error and internal inconsistency, due to improper consideration of these aggregate energy and aggregate materials prices as exogenous and subsequent application of Shephard's Lemma, when in fact these prices are endogenous and the application of Shephard's Lemma must be considered as improper.

Considering these issues, this study will provide technological substitution possibilities (the Allen elasticities of substitution) between energy and non-energy inputs, which challenge the existing estimates of these substitution possibilities. One must note, however, that these Allen elasticities of substitution have been measured not along an aggregate isoquant for firm's total value-of-shipment but along an isoquant corresponding to the total deliveries (total sales) of the manufacturing sector to the balance of the economy (outside of the manufacturing sector), due to considerating and purging the aggregate energy and materials input of inter-firms flow of traded intermediate product, i.e., internally produced energy and materials.

In Section 1 we discuss a model of an industry's conditional factor demand functions derived from the industry's dual unit cost function and show the consequence of the application of Shephard's Lemma, when some factor input prices have been assumed, improperly, as exogenous. In Section 2 we briefly describe the translog cost model and estimation procedure, and then report the results of estimation.

2.1 A Simple Industrial Factor Demand Model

Consider a competitive industry composed of n firms, each producing a single product according to a positive, nondecreasing, continuously twice differentiable, and strictly concave aggregate production function. Each firm purchases, as its inputs, r primary products - primary in the sense of being produced outside of the industry in question - and (n-1) products produced by the other firms which comprise the industry. We also assume that no firm buys any product from itself in an observable market transaction; and that the industry purchases the r primary inputs at constant prices.

We may recall from chapter one that, assuming perfect competition, each firm's dual unit cost function can be obtained as a result of minimizing the cost equation subject to its production function, i.e.,

$$c^i(w_1,\ldots,w_r,w_{r+1},\ldots,w_{r+n}) = \min_x \{\sum_{k=1}^{r} w_k x_{ik} + \sum_{j=r+1}^{r+n} w_j x_{ij} \mid f^i(x_{i1},\ldots,x_{ir},\ldots,x_{i,r+n}) \geq 1\}$$

$$i=1,\ldots,n; \tag{2.1}$$

where $w_k(k=1,\ldots,r)$ are the prices, and $x_{ik}(k=1,\ldots,r)$ are the quantities of primary factor employed by ith firm in the industry, while $w_j(j=r+1,\ldots,r+n)$ are the market prices, and $x_{ij}(j=r+1,\ldots,r+n)$ are the quantities of n products produced and sold to the ith firm by the other firms which comprise the industry. $c^i(\cdot)$ and $f^i(\cdot)$ are the ith firm's unit cost and production function respectively.

The unit cost function for the ith firm is obtained from (2.1) as:

$$c^i(w_1,\ldots,w_r,w_{r+1},\ldots,w_{r+n}) = \sum_{k=1}^{r} w_k x^*_{ik}(w_1,\ldots,w_r,w_{r+1},\ldots w_{r+n})$$

$$\sum_{j=r+1}^{r+n} w_j x^*_{ij}(w_1,\ldots,w_r,\ w_{r+1},\ldots,\ w_{r+n}) \tag{2.2}$$

where x^*_{ik} and x^*_{ij} are the cost minimizing levels of the k^{th} primary and the j^{th} non-primary input employed by the ith firm, and $c^i(\cdot)$ is its dual unit cost function. By applying the well-known Shephard's Lemma to the cost function of a competitive firm, equation (2.2), the conditional factor demand function can be derived, assuming that the firm is facing exogenous input prices.

An advantage of dealing and starting with the cost function is that the set of factor demand functions is available directly from the cost function as the first derivative of the cost function without solving a set of simultaneous equations.

These results, however, can be extended to include explicit consideration of the internal flow of traded intermediate products in the industry, thereby showing how failure to account properly for the traded intermediate goods results in substantial errors in the estimated factor demand elasticities - errors which are implicit in most of the recent empirical studies of manufacturing factor demand functions.

Starting with the unit cost function of the ith firm, expressed in equation (2.2) above, and differentiating it with respect to the price of the kth primary input we obtain:

$$\frac{\partial c^i(\cdot)}{\partial w_k} = x^*_{ik}(w_1,\ldots,w_r,w_{r+1},\ldots w_{r+n})$$

$$+ \sum_{k=1}^{r} w_k \frac{\partial}{\partial w_k} x^*_{ik}(w_1,\ldots,w_r, w_{r+1},\ldots,w_{r+n})$$

$$+ \sum_{j=r+1}^{r+n} w_j \frac{\partial}{\partial w_k} x^*_{ij}(w_1,\ldots,w_r, w_{r+1},\ldots,w_{r+n})$$

$$+ \sum_{j=r+1}^{r+n} \frac{\partial w_j}{\partial w_k} x^*_{ij}(w_1,\ldots,w_r, w_{r+1},\ldots w_{r+n})$$

$$i = 1,\ldots,n. \qquad (2.3)$$

Now let us regard this single firm in <u>isolation</u> from the others. Since a competitive firm considered in isolation faces <u>exogenous</u> input prices (by definition), then it must be true that:

$$\frac{\partial w_j}{\partial w_k} = 0, \quad j=r+1,\ldots,r+n; \text{ and } k=1,\ldots,r . \tag{2.4}$$

Equation (2.4) means that any change in primary factor prices induce no effect on the prices of the inter-firm flow of traded intermediate products used as materials input. The other two terms on the right hand side of (2.3) may be written as:

$$\sum_{k=1}^{r} w_k \frac{\partial}{\partial w_k} x^*_{ik}(\cdot) + \sum_{j=r+1}^{r+n} w_j \frac{\partial}{\partial w_k} x^*_{ij}(\cdot) = \sum_{m=1}^{r+n} w_m \frac{\partial}{\partial w_m} x^*_{im}(\cdot) \tag{2.5}$$

Since $x^*_{im}(\cdot)$'s are the factor demand functions, and therefore are homogendous of degree zero in the set of all factor prices, equation (2.5) must be identically equal to zero by Euler's theorem.[4] Thus equation (2.3) reduces to:

$$\frac{\partial c^i(\cdot)}{\partial w_k} = x^*_{ik}(w_1,\ldots,w_r, w_{r+1},\ldots,w_{r+n}) , \tag{2.6}$$

which has the familiar interpretation. Therefore, in the case where we consider each competitive firm facing <u>exogenous</u> factor prices, in isolation from all other firms, the first derivative of the firm's unit cost function results in the set of factor demand functions, and that is what Shephard's Lemma is all about.

The model formulated in equation (2.1) to (2.6) is the one used empirical specification of factor demand studies in U.S. and Canadian manufacturing in recent years. In these studies the services of four aggregate inputs have been utilized: capital services (K), labor services (L), energy (E), and materials (M). The first two items are clearly primary to the manufacturing sector of the economy, and do not concern us here. The problematic issue is the treatment of the "materials" and "energy" aggregates, since they are composed of both "primary" material and energy, such as transportation, communication services, crude petroleum, and electricity, and industry-produced traded intermediate products such as plastic, steel, glass, kerosine, and jet fuel. While the exogeneity of prices for the former group may not be questionable, the prices of the latter group undoubtedly must be considered as <u>endogenous</u> variables in the study of an industry's demand for primary inputs such as capital, labor, and primary materials.

The case considered above, where we assumed that the competitive firm is isolated from the other firms may be termed the "gross output" model of industry factor demand, because the input demand functions of each firm were obtained conditional upon a <u>constant</u> level of output for each firm.

To study an industry, the interaction between the firms which comprise the industry must be taken into account. Hence, it is totally unrealistic to assume that the firms behave in isolation from each other. It is also unrealistic to assume that the prices of traded intermediate products - which are equal to the unit cost of their production in perfect competition - will not be affected by changes in the prices of the primary inputs facing the industry. An expected repercussion of a change in the price of a primary input, in the analysis of industry level factor demand functions, is that the gross output level of each firm may not remain constant. On the contrary, the firm moves off its initial isoquant, following a primary price change, due to the attempt of the other firms (within the industry) to substitute toward, or away from, the use of the firm's product as an intermediate material input. Then in this model (entitled a "net" model by Anderson (1980)) the inter-firm flows and gross outputs of the n traded intermediate products are supposed to respond and adjust as their market prices vary, due to a change in the price of a primary input, while maintaining a constant level of industry deliveries (net output) of these products outside of the manufacturing sector. Therefore, the case most expected is that of an increase in the supply price of a primary factor used by the industry that will disturb the existing equilibrium, and eventually increase the costs and thereby the price of the internally traded intermediate products. Even in the cases that a firm does not purchase a particular primary factor directly, in an indecomposable industry every firm necessarily purchases directly, or indirectly, some quantities of every primary factor.[5]

Ignoring the effect that a change in primary factor price could have on the price of internally traded intermediate products is precisely the specification error contained in the past studies.

The mathematical demonstration of this error, which has resulted from the unjustified assumption of exogenous "energy" and "materials" input prices, is as follow. Starting again from the cost function of the ith firm and taking its derivative with respect to the kth primary input we obtain equation (2.3) which we reproduce here:

$$\frac{\partial c^i(\cdot)}{\partial w_k} = x^*_{ik}(\cdot) + \sum_{k=1}^{r} w_k \frac{\partial x^*_{ik}(\cdot)}{\partial w_k} + \sum_{j=r+1}^{r+n} w_j \frac{\partial x^*_{ij}(\cdot)}{\partial w_k} + \sum_{j=r+1}^{r+n} \frac{\partial w_j}{\partial w_k} x^*_{ij}(\cdot), \quad i = 1,\dots,n. \tag{2.3}$$

Again according to Euler's Theorem, equation (2.5) holds, but we relax the assumption we made in equation (2.4), which was the result of considering the competitive firm as being isolated from all other firms in the industry and being faced with exogenous factor prices. Therefore we make the following more realistic assumption, namely,

$$\frac{\partial w_j}{\partial w_k} \neq 0 \qquad \text{for all or some of } j = r+1,\dots,r+n. \tag{2.7}$$

Considering (2.7) and (2.5), equation (2.3) reduces to:

$$\frac{\partial c^i(\cdot)}{\partial w_k} = w^*_{ik}(\cdot) + \sum_{j=r+1}^{r+n} \frac{\partial w_j}{\partial w_k} x^*_{ij}(\cdot), \qquad i=1,\dots,n; \tag{2.8}$$

which is the result of considering the p_j's as properly being <u>endogenous</u> to the industry model. From the above argument it is clear that, in order to apply Shephard's Lemma, the items labeled as "energy" and "materials" must refer solely to the primary materials and energy which have been purchased by the firms which comprise the industry, and must not include the firms' purchases of traded intermediate products; and traded intermediate products must be netted out. This is the case which we may term the "net" output model of industry factor demand, where the cost and the factor demand functions may be utilized conditional upon the level of deliveries (total sales) of an aggregate manufacturing sector to the balance of the economy (outside of manufacturing sector), i.e., upon the "net" output level of the manufacturing sector.

Next we may consider the response of conditional factor demands to a change in primary factor price by these "net" and "gross" models. Differentiating the conditional factor demand function (2.6) - for the "gross" model - and (2.8) for the "net" model - with respect to the prices of primary factor s = 1, ..., r, respectively, we obtain from (2.6):

$$\frac{\partial^2 c^i(\cdot)}{\partial w_k \partial w_s} = \frac{\partial}{\partial w_s} x^*_{ik}(\cdot), \quad i=1,\dots,n; \quad k,s=1,\dots,r \tag{2.9}$$

and from (2.8):

$$\frac{\partial^2 c^i(\cdot)}{\partial w_k\, \partial w_s} = \frac{\partial}{\partial w_s} x^*_{ik}(\cdot) + \sum_{j=r+1}^{r+n} \frac{\partial^2 w_j}{\partial w_k \partial w_s} x^*_{ij}(\cdot) + \sum_{j=r+1}^{r+n} \frac{\partial w_j}{\partial w_s} \frac{\partial}{\partial w_k} x^*_{ij}(\cdot) \tag{2.10}$$

$$i=1,\dots,n; \; k,s=1,\dots,r$$

Comparison of (2.9) and (2.10) obviously reveals the different responses of these two models to a change in a primary factor price. The first term on the right hand side of (2.10) is equivalent to (2.9), which according to the law of demand for factors is negative or equal to zero for $k = s = 1,\ldots,r$; i.e., the conditional demand curves for each factor are downward sloping. This follows from the fact that the cost function is concave in input prices, and that the second derivative of a concave function is non-positive. This term can be regarded as the direct effect of a change in primary factor prices. The other terms on the right hand side of (2.10), representing the terms relating to the effects on total unit cost of substitution among intermediate products within the industry, can be regarded as indirect substitution among primary factor inputs. The different responses of these two models may also imply different factor demand elasticities as can be seen from equations (2.9) and (2.10).

One must, however, note that under two special circumstances both the "net" and the "gross" model will be equivalent: first, if the firms comprising the industry employ a fixed coefficient Leontier technology with, obviously, no induced factor substitution among the traded intermediate products; second, if there are no traded intermediate products. In this case the traded intermediate products must have been netted out in terms of their primary factor content.[6] One also must realize from equations (2.9) and (2.10) that the "gross" model can be considered a special case of the more general "net" model.

In the next section we briefly describe the translog unit cost function and the system of share equations derived from it, then describing the estimation problems associated with the system of share equations, finally presenting the empirical results based upon the U.S. manufacturing and the U.S. non-energy manufacturing data, and comparing these results with other studies.

2.2 Empirical Specification and Estimation

As was discussed in the previous chapter, the fundamental result of the duality theorem states that, given certain regularity conditions, the specification of the production function implies a particular cost function and vice versa. Therefore, the structure of technology can be studied empirically employing either cost or production function - the choice being a statistical matter. Direct estimation of the production function is attractive when the output level is endogenous, while direct estimation of the cost function becomes more attractive whenever the level of output is exogenous.

Suppose the technology for the manufacturing sector can be represented by a positive, finite, continuously twice differentiable, strictly monotone increasing, and strictly concave aggregate production function as:

$$Q = F(K,L,E,M) \tag{2.11}$$

where (K,L,E,M) are the input quantities of capital services, labor services, primary energy, and primary materials. We further assume that any technological change affecting the aggregate inputs of K,L,E, and M is Hicks neutral. Now given the production function (2.11) and the vector of input prices, (W_K, W_L, W_E, W_M), the corresponding dual cost function of the manufacturing sector can be obtained as the solution to the following constrained minimization problem:

$$c(W_K,W_L,W_E,W_M;Q) = \min_{K,L,E,M} \{W_K K+W_L L+W_E E+W_M M \mid F(K,L,E,M) \geq Q\}. \tag{2.12}$$

Further, if the production function is assumed to be a first degree homogeneous function in input space, then the cost function (2.12) factors into the following expression:

$$c(W_K,W_L,W_E,W_M;Q) = Q\cdot c(W_K,W_L,W_E,W_M) \tag{2.13}$$

where $c(\cdot)$ is the manufacturing sector's unit cost function.

For the purpose of estimating the unit cost function $c(\cdot)$, a specific functional form must be employed. Until very recently the estimation of the parameters of the cost or production function has been based upon highly restrictive functional forms, the most popular being the Cobb-Douglas and C.E.S. The shortcomings of these functional forms - as we have seen - have led researchers to employ a relatively new generation of flexible functions such as the translog, generalized Leontief, etc. These functional forms permit the technology to exhibit an arbitrary set of partial elasticities of substitution between pairs of inputs at a given point in input price or quantity space. These functional forms, therefore, offer substantial gain of flexibility and create a great opportunity for the investigator to test important maintained hypotheses of previous works. With these points in mind the functional form we choose to work with is necessarily of this family. In particular we choose to work with the translog functional form proposed by Christensen, Jorgenson, and Lau (1971). This function provides a second order local approximation to an arbitrary underlying function about a point.

Therefore, we assume that the unit cost function can be approximated up to the second order by the following translog cost function:

$$\ln c(\underline{W}) = \alpha_0 + \sum_i \alpha_i \ln W_i + \tfrac{1}{2} \sum_i \sum_j \gamma_{ij} \ln W_i \ln W_j , \qquad i,j=K,L,E,M; \tag{2.14}$$

where (γ_{ij}) is a symmetric matrix. The equality of $\gamma_{ij} = \gamma_{ji}$ is a necessary for the applicability of Young's theorem to integrable functions, and as Denny and Fuss (1977) have stated in proposition (1) of their paper, the symmetry constraints must hold when one views the translog function as a quadratic approximation to an arbitrary cost function.

Recalling the Samuelson (1947) - Shephard (1953) duality theorem of chapter one, if we differentiate the unit cost function with respect to the price of an input we obtain the derived demand for that input. Therefore, assuming cost minimizing behavior and industry-level exogenous factor prices, we apply Shephard's Lemma which yields a set of estimable factor share equations linear in logarithm of factor prices,

$$S_i = \frac{\partial \ln c}{\partial \ln w_i} = \alpha_i + \sum_j \gamma_{ij} \ln W_j, \qquad i,j = K,L,E,M \tag{2.15}$$

where S_i is the cost share for ith factor. To correspond to a well-behaved production function, a cost function must be homogeneous of degree one in factor prices. This yields the following set of parametric restrictions:

$$\sum \alpha_i = 1$$

$$\sum_i \gamma_{ij} = \sum_j \gamma_{ij} = \sum_i \sum_j \gamma_{ij} = 0 \tag{2.16}$$

These restrictions will be imposed throughout the chapter.

Following Uzawa (1962) the Allen partial elasticity of substitution (AES) can be computed from the cost function by the following formula:

$$\sigma_{ij} = c(\underline{W},Q) \cdot c_{ij}(\underline{W},Q)/c_i(\underline{W},Q) \cdot c_j(\underline{W},Q) \tag{2.17}$$

and since $c(W,Q) = Q \cdot c(W)$, equation(2.17) can be reformulated in terms of the unit cost function:

$$\sigma_{ij} = c(\underline{W}) \cdot c_{ij}(\underline{W})/c_i(\underline{W})c_j(W), \qquad i,j = K,L,E,M \tag{2.18}$$

where $c_i = \frac{\partial c}{\partial W_i}$, and $c_{ij} = \frac{\partial^2 c}{\partial W_i \partial W_j}$. Clearly, as has been seen from the above formulas, AES's are variable. For the translog cost function, we earlier saw that the own and cross AES could be derived as:

$$\sigma_{ij} = 1 + \frac{\gamma_{ij}}{S_i S_j} , \qquad i \neq j$$

$$\sigma_{ii} = 1 + \frac{(\gamma_{ij} - S_i)}{S^2_i} \qquad (2.19)$$

The corresponding own and cross-price elasticities of demand for factors were obtained as:

$$\eta_{ij} = \sigma_{ij} S_j$$

$$\eta_{ii} = \sigma_{ii} S_i \qquad i,j = K,L,E,M; \qquad (2.20)$$

where η_{ij} (cross-price elasticity of demand) measures the percentage change in derived demand for input i for an exogeneous change in the price of input, given that all other input prices and output quantity remain constant. Note that while $\sigma_{ij} = \sigma_{ji}$ by definition $\eta_{ij} \neq \eta_{ji}$ in general.

To characterize the structure of technology of the manufacturing and non-energy manufacturing sectors, we estimate separately the parameters of the unit cost function, using the stochastic version of the cost share equations (2.15) as a multivariate regression system. To do this we specify an additive disturbance μ_i for each of the share equations on the assumption that entrepreneurs make random errors in adjusting to their exact cost-minimizing input levels.

The disturbance μ_i's are likely to be correlated, because random deviation from cost minimization should affect all of the market for the inputs, and hence the estimation procedure suggested by Zellner (1962) is expected to yield more efficient estimates. Because of adding-up constraints on the cost shares - which imply a zero sum of disturbances across the four equations at each observation - together with the symmetry restrictions across equations, one of the cost shares must be deleted to avoid a singular estimated disturbance var - cov matrix.

By deleting one of the share equations from the system the Zellner procedure can be made operational; however, the estimate so obtained will not be invariant to which equation is deleted. Barten (1969) has shown that the maximum likelihood estimates of the system of share equations with one equation deleted would be invariant to which equation is dropped.

Kmenta and Gilbert (1968) and Dhrymes (1970) have shown in a series of Monte Carlo experiments that the iteration of the Zellner (IZEF) estimation procedure until convergence results in estimates which are identical to those of maximum likelihood estimates. Accordingly the application of the iterative Zellner estimation procedure will yield, computationally, maximum likelihood and therefore consistent and asymptotically efficient estimates for the parameters of (2.15). This is the procedure we employ here.

In preliminary application of IZEF procedure to our data we have found a substantial degree of serial correlation in the postwar U.S. manufacturing and non-energy producing manufacturing sectors. To increase the efficiency for this case we assume that the additive disturbance term, μ_i, in each input share equation is both serially and contemporaneously correlated.

As was discussed above, the adding-up constraint on the dependent variables (2.15), together with the symmetry restrictions across equations imply the parametric restriction of (2.16), which in turn requires that one cost share equation be omitted. Therefore, imposing parametric restriction (2.16) on the system of equations (2.15), and adding an additive disturbance term μ_i to each of the equations we find:

$$S_{it} = \alpha_i + \sum_j \gamma_{ij} \ln (W_{jt}/W_{Mt}) + \mu_{it} \qquad i,j = K,L,E; \quad t = 1,\ldots,T \qquad (2.21)$$

Writing (2.21) in matrix notation we have:

$$\begin{bmatrix} S_{Kt} \\ S_{Lt} \\ \vdots \\ S_{Et} \end{bmatrix} = \begin{bmatrix} \alpha_K & \gamma_{KK} & \gamma_{KL} & \gamma_{KE} \\ \alpha_L & \gamma_{KL} & \gamma_{LL} & \gamma_{LE} \\ \alpha_E & \gamma_{KE} & \gamma_{LE} & \gamma_{EE} \end{bmatrix} \begin{bmatrix} 1 \\ \ln(W_{Kt}/W_{Mt}) \\ \ln(W_{Lt}/W_{Mt}) \\ \ln(W_{Et}/W_{Mt}) \end{bmatrix} + \begin{bmatrix} \mu_{Kt} \\ \mu_{Lt} \\ \mu_{Et} \end{bmatrix},$$

or more compactly:

$$\underline{S}_t = \Gamma \underline{W}_t + \underline{\mu}_t \,. \qquad (2.23)$$

Following Berndt and Savin (1975), we assume that the autoregressive process is first order, and that the vector of disturbances $\underline{\mu}_t$ satisfies the following specification:

$$\underline{\mu}_t = R\,\underline{\mu}_{t-1} + \underline{\varepsilon}_t \, , \qquad (2.24)$$

where $\underline{\varepsilon}_t$ is a 3x1 vector of random disturbance terms with the following property:

$$E(\varepsilon_{it}) = 0$$

$$E(\varepsilon_{it}, \varepsilon_{js}) = \begin{cases} \delta_{ij} & \text{for } t=s \\ 0 & \text{for } t \neq s \end{cases} \qquad i,j = K,L,E, \qquad (2.25)$$

and R is the following three by three matrix of the assumed first-order autocorrelation process:

$$R = \begin{pmatrix} \rho_{KK} & \rho_{KL} & \rho_{KE} \\ \rho_{KL} & \rho_{LL} & \rho_{LE} \\ \rho_{LE} & \rho_{LE} & \rho_{EE} \end{pmatrix}$$

Lagging and rearranging (2.23) and (2.24), we obtain the following difference equation system:

$$\underline{S}_t = \Gamma \underline{W}_t + R\underline{S}_{t-1} - R\Gamma \underline{W}_{t-1} + \underline{\varepsilon}_t \qquad t=2,\ldots,T \qquad (2.26)$$

As it might be clear, this difference equation system has non-linear terms due to combination of the paramters R and Γ. Further we assume that the vector of the random variable ε_t is normally distributed with mean zero and cov matrix Ω with elements δ_{ij} as in (2.25), where Ω is equal to $\Sigma \otimes I$ and

$$\sum = \begin{bmatrix} \delta_{KK} & \delta_{KL} & \delta_{KE} \\ \delta_{KL} & \delta_{LL} & \delta_{LE} \\ \delta_{KE} & \delta_{LE} & \delta_{EE} \end{bmatrix} .$$

One must note that the error specification (2.24), used by Hendry (1971), is in fact a generalization of the specifications utilized by Zellner (1962) and Parks (1967). If $\rho_{ij} = 0$ for all i and j, we have Zellner's specification of contemporaneous correlation but no autocorrelation; while if $\rho_{ij} = 0$ only for $i \neq j$, we have Parks' case of first-order autocorrelation in each equation,as well as contemporaneous correlation. Parks' diagonal specification, however, is not applicable for our model since the maximum likelihood estimates so obtained will not be invariant to which equation is deleted. This difficulty, however, can be overcome if we choose the R matrix as a diagonal matrix with equal elements (scalar matrix), i.e., $\rho_{ii} = \rho$ and $\rho_{ij} = 0$ for all i and j (see Berndt and Saving (1975)).

In this chapter,we specifically have estimated four specifications, namely, the unrestricted R matrix, symmetric R matrix, the scaler R matrix, and finally the null matrix (R=0). The results for manufacturing, as well as non-energy manufacturing sectors, are reported in table (2.2) and (2.3).

The hypothesis that R is the null matrix (R=0) is decidedly rejected for both sectors. The hypothesis that R is diagonal (with equal elements) is rejected for the manufacturing sector. For the non-energy manufacturing sector, this hypothesis is rejected at the five percent level, but accepted at the one percent level, since the value of the LR test statistic (19.24) falls between these two χ^2 critical values. Finally, the hypothesis that R is a symmetric matrix cannot be rejected for the non-energy manufacturing sector, while for the manufacturing sector this hypothesis is rejected at the five percent level but accepted at the one percent level.

We now proceed to our discussion of the empirical results in the next section.

2.3 Empirical Results

We have characterized the structure of technology in U.S. manufacturing as well as non-energy manufacturing over the 1947-1971 period by the estimation of a translog unit cost function (2.14) through the system of share equations (2.26) with linear homogeneity restrictions (2.16) imposed. These IZEF estimates shown in Tables (2.2) and (2.3) are based on the corresponding data for input prices and cost shares, where data construction and sources have been discussed in the appendix. Before proceeding to the price and substitution elasticities it is necessary to discuss and establish whether the estimated translog unit cost functions for both sectors are well-behaved.

A cost function is said to be well-behaved if it is: (i) concave in input prices and if (ii) its derived conditional input demand functions are strictly positive. Since the translog cost function does not meet these conditions globally, we must look for the fulfillment of these restrictions at each point of observation. The second condition, namely the positivity, is satisfied if the fitted cost shares are positive. Based on our IZEF parameter estimates, we have found that this condition is satisfied at each annual observation for both sectors. To check the concavity, we must look for a negative semi-definite Hessian matrix. Again, based on our IZEF estimate this condition is also satisfied at most of our annual observations, and we can conclude that our estimated translog unit cost functions are well-behaved for both sectors.

The empirical results presented in Tables (2.2) and (2.3) show that most of our IZEF parameter estimates are different from zero at a five percent level of significance for the manufacturing as well as the non-energy manufacturing sector. Each sector has been estimated with and without the autocorrelation correction. As is apparent from the Tables, the estimation of both sectors without autocorrelation results in low values of Durbin-Watson statistics, suggesting that our system of regression equations has disturbances which may be both serially and contemporaneously correlated. Correcting for such a correlation as was described, substantially improves our Durbin-Watson statistics for both sectors, as can be seen by a direct comparison between the two models. Improvement also occurs in the conventional R^2 (computed as one minus the ratio of the residual sum of squares to the total sum of squares in each equation), and in the logarithm of likelihood function after this correction, (see Tables).

The estimated Allen partial elasticity of substitution (AES), σ_{ij}, and the own and cross-price elasticities, η_{ij}, for capital, labor, energy, and materials inputs obtained from the cost function are reported for the manufacturing and non-energy manufacturing sectors in Tables (2.4) and (2.7). Elasticity estimates are the essence of this study. Since these estimates are fairly stable over the period of the study, we only present the results for three selected years. We will make some comments about them and compare them with the results of other studies. In particular we are interested in comparing our manufacturing results - as results obtained from our "net" model - with those of Berndt and Wood (1975), interpreted as the "gross" model. We then talk briefly about the non-energy manufacturing sector. Tables (2.8) and (2.9) presents a summary of some comparable estimates from other studies.

A direct comparison of our AES, σ_{ij}, and the own and cross-price elasticities, η_{ij}, with those of Berndt and Wood (1975) reveals that both sets of parameter estimates are in substantial agreement at least in terms of signs, with the σ_{ii} (K,L,E,M) and η_{ii} (K,L,E,M) all having the correct negative sign implied by satisfaction of the concavity condition. On the other hand, the magnitude of difference between our substitution and price elasticities and those of Berndt and Wood is quite sizable, and we should make a few comments about them.

Comparison of our energy-energy AES, σ_{EE}, with that of Berndt and Wood reveals that this elasticitity has changed substantially from -10.7 to -2.0, while the important own-price elasticity of energy, η_{EE}, has increased to about -.13 from -.47, namely, about one-third of the value obtained from the inconsistent gross output model of Berndt and Wood (1975), Berndt and Khaled (1979), and Berndt and White (1979). This means that the response of energy demand for changing energy prices (in the net model) is likely to be more inelastic than in the gross output model. Our inelastic result may indicate that, after an energy price increase, each firm finds itself facing higher priced produced inputs, including intermediate energy products as well as higher priced energy input; therefore the substitution possibilities away from energy would be smaller than if only energy increased in prices. An alternative estimate of own price elasticity is provided by Pindyck (1979) and also by Griffin and Gregory (1976). Using pooled international data, they argue that their estimates "provide a reasonable long run alternative to the time series literature" on energy, capital, and labor substitution elasticities. They report a "long-run" own price elasticity for energy of about -.08, whereas the estimates of the other time series studies, which probably elicits a short-run elasticity, have generally been reported to be around -.4 to -.5.

Comparing our estimated σ_{ii} (K,L,M) with those obtained by Berndt and Wood exposes the difference between these two studies. While our own AES for capital and labor increase to -2.12 and -.66 respectively form -8.75 and -1.66 estimated by Berndt and Wood, our own AES for materials fell to -.75 from their estimate of -.36. Our estimate of σ_{KK}, however,

TABLE 2.2

MAXIMUM LIKELIHOOD (IZEF) PARAMETER ESTIMATES OF TRANSLOG COST FUNCTION, NET ENERGY AND NET MATERIALS DATA, WITHOUT AND WITH FIRST-ORDER VECTOR AUTOCORRELATION - U.S. MANUFACTURING, 1947-1971

Parameters	Estimates	t-Statistics+	Estimates	t-Statistics+	Estimates	t-Statistics+	Estimates	t-Statistics
α_K	.097*	(63.1)	.099*	(84.3)	.100*	(92.5)	.099*	(65.0)
α_L	.461*	(130.0)	.451*	(47.4)	.453*	(76.7)	.452*	(86.3)
α_E	.062*	(127.8)	.062*	(232.3)	.062*	(224.0)	.061*	(72.5)
α_M	.380*	(88.8)	.388*	(42.0)	.385*	(65.6)	.387*	(74.8)
γ_{KK}	.053*	(8.4)	.070*	(16.2)	.070*	(16.6)	.065*	(14.2)
γ_{KL}	.002	(.4)	- .031*	(3.9)	- .022*	(2.8)	- .018*	(2.4)
γ_{KE}	- .016*	(7.9)	- .012*	(7.4)	- .014*	(8.5)	- .013*	(4.9)
γ_{KM}	- .040*	(4.4)	- .027*	(3.6)	- .035*	(4.9)	- .034*	(4.3)
γ_{LL}	.095*	(4.5)	.118*	(3.0)	.019	(.6)	.027	(.8)
γ_{LE}	- .010*	(4.8)	- .014*	(7.3)	- .012*	(6.6)	- .024*	(6.1)
γ_{LM}	- .089*	(3.8)	- .072*	(2.0)	.015	(.5)	.015	(.5)
γ_{EE}	.053*	(19.9)	.050*	(25.1)	.052*	(24.5)	.047*	(10.9)
γ_{EM}	- .028*	(7.5)	- .024*	(8.9)	- .026*	(9.3)	- .011*	(1.9)
γ_{MM}	.156*	(5.5)	.123*	(3.5)	.046	(1.4)	.030	(.9)
ρ_{KK}			.604*	(4.1)	.417*	(3.7)		
ρ_{KL}			.042	(1.1)	.046	(1.2)		
ρ_{KE}			.066	(.1)	- .069	(1.6)		
ρ_{LK}			- .568	(1.1)				
ρ_{LL}			.755*	(5.6)	.621*	(4.1)		
ρ_{LE}			- 5.003*	(3.0)	.013	(1.0)		
ρ_{EK}			- .021	(.4)				
ρ_{EL}			.006	(.5)				
ρ_{EE}			- .194	(1.2)	.212	(1.6)		
ρ							.554*	(6.7)
R^2:								
S_K	.671		.866		.863		.868	
S_L	.462		.683		.552		.557	
S_E	.957		.978		.981		.951	
Durbin-Watson:								
S_K	.813		1.464		1.292		1.722	
S_L	.702		1.971		1.745		1.665	
S_E	2.253		1.794		1.955		1.824	
Log-Likelihood Function:	314.039		337.580		332.725		319.933	

* Coefficient is significantly different from zero at .05 level of significance.
\+ Absolute value of t-ratios in parentheses.

TABLE 2.3

MAXIMUM LIKELIHOOD (IZEF) PARAMETER ESTIMATES OF TRANSLOG COST FUNCTION, NET MATERIALS DATA, WITHOUT AND WITH FIRST-ORDER VECTOR AUTOCORRELATION-U.S. NON-ENERGY MANUFACTURING SECTOR, 1947 - 71

Parameters	Estimates	t-Statistics+	Estimates	t-Statistics+	Estimates	t-Statistics+	Estimates	t-Statistics+
α_K	.170*	(91.8)	.170*	(55.9)	.172*	(32.6)	.170*	(80.2)
α_L	.458*	(131.9)	.461*	(41.4)	.454*	(35.7)	.452*	(66.6)
α_E	.030*	(87.3)	.029*	(32.6)	.029*	(30.8)	.030*	(42.9)
α_M	.342*	(77.5)	.340*	(28.5)	.345*	(31.9)	.348*	(55.2)
γ_{KK}	.088*	(9.9)	.084*	(17.3)	.084*	(16.9)	.082*	(16.7)
γ_{KL}	.009	(1.2)	-.022*	(2.4)	-.022*	(2.9)	-.016*	(2.6)
γ_{KE}	-.007*	(5.3)	-.005*	(4.0)	-.005*	(3.6)	-.005*	(2.7)
γ_{KM}	-.089*	(6.1)	-.056*	(6.8)	-.057*	(7.8)	-.062*	(8.7)
γ_{LL}	.101*	(6.4)	-.054	(1.6)	-.020	(.8)	.014	(.6)
γ_{LE}	-.002	(1.6)	.006*	(2.2)	.005*	(1.8)	-.002	(.8)
γ_{LM}	-.108*	(5.5)	.070*	(2.4)	.037	(1.5)	.003	(.1)
γ_{EE}	.010*	(4.0)	.002	(.8)	.002	(.6)	.004	(1.1)
γ_{EM}	.001	(.3)	-.003	(.7)	-.002	(.4)	.002	(.6)
γ_{MM}	.198*	(6.5)	-.011	(.4)	.022	(.9)	.056*	(2.1)
ρ_{KL}			.771*	(6.5)	.827*	(8.0)		
ρ_{KK}			.038	(.8)	-.009	(.3)		
ρ_{KE}			.541	(1.0)	-.074*	(2.3)		
ρ_{LK}			.836*	(2.1)				
ρ_{LL}			.784*	(4.5)	.820*	(7.3)		
ρ_{LE}			.715	(.4)	-.035*	(2.6)		
ρ_{EK}			-.090*	(2.7)				
ρ_{EL}			-.040*	(3.0)				
ρ_{EE}			.207	(1.5)	.339*	(2.9)		
ρ							.685*	(8.8)
R^2:								
S_K	.811		.961		.957		.958	
S_L	.629		.799		.772		.767	
S_E	.396		.732		.696		.460	
Durbin-Watson:								
S_K	.490		1.806		2.035		1.705	
S_L	.840		1.861		1.793		1.690	
S_E	1.343		2.199		2.261		2.204	
Log-Likelihood Function:	308.869		332.711		329.678		323.091	

+ Absolute value of t-ratios in parentheses.
* Coefficient is significantly different from zero at .05 level of significance.

TABLE 2.4

MAXIMUM LIKELIHOOD (IZEF) ESTIMATES OF THE ALLEN ELASTICITIES OF SUBSTITUTION (AES) UNDER FIRST ORDER AUTOCORRELATION SELECTED YEARS, U.S. MANUFACTURING

AES	1948	1959	1971
σ_{KK}	-1.29	-2.12	-1.10
σ_{LL}	- .69	- .66	- .61
σ_{EE}	-3.02	-2.04	- .60
σ_{MM}	- .70	- .75	.70
σ_{KL}	.17	.31	.30
σ_{KE}	- .98	- .90	-2.16
σ_{KM}	.27	.35	.08
σ_{LE}	.53	.48	.44
σ_{LM}	.59	.58	.62
σ_{EM}	.17	.02	- .05

TABLE 2.5

MAXIMUM LIKELIHOOD (IZEF) ESTIMATES OF
PRICE ELASTICITIES OF CONDITIONAL FACTOR DEMAND
UNDER FIRST ORDER AUTOCORRELATION (UNRESTRICTED R)
SELECTED YEARS, U.S. MANUFACTURING

η_{ij}	1948	1959	1971
η_{KK}	-.11	-.22	-.08
η_{KL}	.07	.14	.01
η_{KE}	-.07	-.06	-.12
η_{KM}	.11	.14	.03
η_{LK}	.01	.03	.002
η_{LL}	-.30	-.29	-.28
η_{LE}	.04	.03	.02
η_{LM}	.24	.23	.26
η_{EK}	-.09	-.09	-.15
η_{EL}	.23	.21	.21
η_{EE}	-.21	-.13	-.03
η_{EM}	.07	.01	-.02
η_{MK}	.02	.04	.01
η_{ML}	.26	.26	.29
η_{ME}	.01	.001	-.002
η_{MM}	-.29	-.29	-.29

TABLE 2.6

MAXIMUM LIKELIHOOD (IZEF) ESTIMATES OF THE ALLEN ELASTICITIES OF SUBSTITUTION (AES) UNDER FIRST ORDER AUTOCORRELATION (UNRESTRICTED R) SELECTED YEARS, U.S. NON-ENERGY MANUFACTURING

AES	1948	1959	1971
σ_{KK}	- 1.99	- 1.99	- 1.80
σ_{LL}	- 1.73	- 1.53	- 1.38
σ_{EE}	-28.41	-29.72	-28.88
σ_{MM}	- 1.63	- 1.85	- 1.80
σ_{KL}	.67	.70	.65
σ_{KE}	- .07	- .10	- .33
σ_{KM}	.12	.06	- .14
σ_{LE}	1.44	1.43	1.39
σ_{LM}	1.43	1.44	1.41
σ_{EM}	.78	.75	.76

TABLE 2.7

MAXIMUM LIKELIHOOD (IZEF) ESTIMATES OF
PRICE ELASTICITIES OF CONDITIONAL FACTOR DEMAND
UNDER FIRST ORDER AUTOCORRELATION (UNRESTRICTED R)
SELECTED YEARS, U.S. NON-ENERGY MANUFACTURING

η_{ij}	1948	1959	1971
η_{KK}	-.32	-.33	.24
η_{KL}	.28	.31	.30
η_{KE}	-.002	-.003	-.01
η_{KM}	.05	.02	-.05
η_{LK}	.11	.12	.09
η_{LL}	-.72	-.68	.65
η_{LE}	.04	.04	.04
η_{LM}	.56	.52	.52
η_{EK}	-.01	-.02	-.04
η_{EL}	.60	.63	.65
η_{EE}	-.89	-.89	-.89
η_{EM}	.31	.27	.28
η_{MK}	.02	.01	-.02
η_{ML}	.59	.64	.66
η_{ME}	.02	.02	.02
η_{MM}	-.64	-.67	-.66

TABLE 2.8

COMPARISON OF ESTIMATED ALLEN PARTIAL ELASTICITIES OF SUBSTITUTION (AES) U.S. MANUFACTURING

AES	R=0	Unrestricted R	Diagonal $R:\rho_{ii}=\rho$	Symmetric R	Berndt and Wood (1975)	Berndt and Khaled (1979) (1)	Berndt and White (1978)	Magnus (1979) (2)	Griffin and Gregory (1976) (3)	Pindyck (1979) 3	Hudson and Jorgenson (1974) 4	Wills (1979) 5
σ_{KK}	-3.66	-2.12	-2.57	-2.06	- 8.75	- 6.47 (-6.0 to -9.7)	- 5.84			-1.66 (-.5 to -1.8)		
σ_{LL}	- .72	- .66	-1.11	-1.16	- 1.66	- 1.61 (-.6 to -1.6)				-1.29 (-.3 to -1.3)		
σ_{EE}	-1.33	-2.04	-2.71	-1.51	-10.70	-10.23 (-10.2 to -15.7)	- 9.24			-27.21 (-10.9 to -27.2)		
σ_{MM}	- .55	- .75	-1.36	-1.25	- .36	- .35 (-.02 to -.4)	- .39					
σ_{KL}	1.05	.31	.60	.52	1.10	.78 (.78 to 2.1)		.60	.06 (.06 to .5)	1.41 (.6 to 1.4)	1.09	1.00 (2.54)
σ_{KE}	-1.58	- .90	-1.00	-1.21	-3.14	- 2.19 (1.4 to -3.0)	- 3.66	-.30	1.07 (1.02 to 1.07)	1.77 (-.4 to 1.8)	-1.39	.58 (1.32)
σ_{KM}	- .08	.35	.15	.13	.56	.40 (-.02 to .6)	.26					.48 (.88)
σ_{LE}	.66	.48	.13	.55	.64	.57 (.4 to 2.4)		.86	.87 (.7 to .9)	.05 (.05 to 1.2)	2.16	-1.17 (-3.5)
σ_{LM}	.50	.58	1.09	1.08	.59	.60 (-.03 to .6)						.01 (.41)
σ_{EM}	- .17	.02	.54	-.07	.75	.67 (.1 to 1.1)	.51					.92 (.94)

(1) The main entries of this column correspond to the constant returns to scale Hicks - neutral specification, while the values reported in parentheses obtained from other specifications as restrictions on Box-Cox form vary.

(2) Estimates are for the Netherlands 1950-76.

(3) Estimates for U.S. manufacturing; estimates for the manufacturing sector of eight industrialized countries in parentheses.

(4) Estimates taken from Griffin and Gregory (1976).

(5) The U.S. primary metals industry; estimates are given for the unrestricted model, and in parentheses, we have reported estimates for the (rejected) Hicks neutral specifications.

TABLE 2.9

COMPARISON OF ESTIMATED INPUT DEMAND ELASTICITIES - U.S. MANUFACTURING*

η_{ij}	R=0	Unrestricted R	Diagonal R: ii =	Symmetric R	Berndt & Wood (1975)	Berndt & Khaled (1979)	Berndt & White (1978)	Magnus (1979)	Griffin & Gregory (1976)	Pendyck (1979)	Hudson & Jorgenson (1974)	Wills (1979)
η_{KK}	-.35	-.22	-.27	-.21	-.49	-.37 (-.35 to -.54)	-.28	-.42	-.18 (-.18 to -.38)	-.71 (-.29 to -.78)	-.42	-.51 (-.95)
η_{KL}	.48	.14	.27	.23	.28	.22 (.21 to .59)			.05 (.05 to .29)		.29	
η_{KE}	-.10	-.06	-.06	-.07	-.14	-.10 (-.06 to -.14)	-.17		.13 (.08 to .17)		-.02	
η_{KM}	-.03	.14	.06	.05	.35	.25 (-.01 to .4)	.16				.15	
η_{LK}	.10	.03	.06	.05	.06	.05 (.05 to .12)			.01 (.01 to .19)		.14	
η_{LL}	-.33	-.29	-.49	-.51	-.46	-.44 (-.17 to -.46)		-.17	-.12 (-.12 to -.27)	-.65 (-.23 to -.66)	-.45	-.14 (-.69)
η_{LE}	.04	.03	.01	.03	.03	.03 (.02 to .11)			.11 (.05 to .15)		.04	
η_{LM}	.19	.23	.42	.43	.37	.37 (-.02 to .39)					.27	
η_{EK}	-.15	-.09	-.10	-.12	-.18	-.13 (-.08 to -.15)	-.18		.15 (-.15 to .4)		-.18	
η_{EL}	.31	.21	.06	.24	.18	.16 (.12 to .66)			.64 (.4 to .64)		.57	
η_{EE}	-.08	-.13	-.17	-.09	-.47	-.45 (-.45 to -.71)	-.42	-.57	-.79 (-.77 to -.8)	-.85 (-.83 to -.87)	.07	-.37 (-.29)
η_{EM}	-.07	.01	.21	-.03	.47	.42 (.09 to .66)	.31				-.46	
η_{MK}	-.01	.04	.02	.01	.03	.02 (-.01 to .04)	.01					
η_{ML}	.23	.26	.48	.48	.16	.16 (-.01 to .17)						
η_{ME}	-.01	.001	.03	-.004	.03	.03 (.01 to .05)	.02					
η_{MM}	-.21	-.29	-.53	-.49	-.22	-.22 (-.01 to .02)	-.24					-.13 (-.31)

*See footnotes of table (2.8).

is in the range estimated by Pindyck (1979), while our σ_{LL} estimate is close to the estimated Berndt and Khaled obtained from their more general specification.

Looking at our own-price elasticities in Table (2.9), and comparing these with those of Berndt and Wood, we realize that these estimates have the correct negative sign required for stability. However, our η_{ii} (K,L,E,M) for capital, labor, energy, and material, being -.22, -.29, -.13, and -.29 respectively, are more inelastic (except for η_{MM}) compared with these elasticities estimated to be, -.48, -.46, -.47, and -.22 respectively by Berndt and Wood.

Another interesting result is that the controversial energy-capital complementarity relationship persists in this study, though considerably weaker compared with estimates obtained in other studies, numerically our σ_{KE} = -.90 versus -3.14 of Berndt and Wood. The corresponding cross-price elasticities of η_{KE} = -.06 and η_{EK} - .09 show that capital (energy) is less responsive to a change in price of energy (capital) in this study than in Berndt and Wood's.

The labor-capital substitution elasticity of .31, which we have found, is well below that obtained in other studies. For example, Berndt and Wood found labor and capital quite substitutable, and have placed the estimate of σ_{KL} around 1.01, which is consistent with numerous traditional two input (capital-labor) studies. Our finding of lower σ_{KL} is, however, in line with the CES literature as surveyed by Nerlove (1967) and Nadiri (1970), which places the estimate of σ_{KL} between .3 and .7. It is also consistent with the translog study by Griffin and Gregory (1976), which places this elasticity between .39 for United Kingdom and Belgium and .52 for Denmark, while their estimate of σ_{KL} for U.S. is considerably lower at about .06. The corresponding η_{KL} and η_{LK} in our study are .14 and .03 respectively, which are, respectively, about one-half of the estimates reported by Berndt and Wood.

Energy and labor are slightly substitutable in our "net" model as well as in the "gross" model with an AES of .50. Substitution possibilities between energy and material were found to be very limited in this study, compared to what Berndt and Wood have found in their study. The cross price elasticity of materials with respect to the price of energy falls from .03 (estimated by Berndt and Wood) to almost zero (.001). The cross price elasticity of energy with respect to price of materials fall to .01 from .47 obtained by Berndt and Wood.

As for our manufacturing sector, we conclude that our findings on the AES and input price elasticities are in general agreement with past literature at least in terms of signs. But as far as the degree of these estimates is concerned, we observe that a respecification of our model over the net industry output and primary factors of production will result in a fall in the absolute value of these elasticities. We also observe that the technological possibilities for substitution between energy and non-energy inputs are present but to a very limited extent. Specifically, energy appears to be a complement with capital, while it is rather weak substitute for labor and materials. Our estimates of price elasticities suggest that higher priced energy - ceteris paribus - causes a slight decline in the quantity of capital and energy demanded, and at the same time, leads to an insignificant increase in demand for labor and material.

As for the non-energy manufacturing sector, the results on the AES and own and cross-price elasticities are presented in Tables (2.6), (2.7), (2.10), and (2.11). Looking at these elasticities, we find that these estimates are also in general agreement with other studies, at least in terms of signs. However, the magnitude of these estimates are quite different from those reported in other studies. For comparison purposes, Hudson and Jorgenson (1974) could be quite useful, as their study contains the non-energy manufacturing sector as one of the nine sectors into which they have subdivided the U.S. business sector; unfortunately, they have not reported estimated substitution or factor price elasticities. Here we make the following comments about our findings for the non-energy manufacturing sector.

(i) The estimate of the own price elasticities of factor demand, η_{ii} (i = K,L,E,M), indicate that labor, energy, and materials are quite responsive to a change in their own prices, η_{EE} is about -.89, while η_{LL} and η_{MM} are -.68 and -.67 respectively. Capital appears to be less responsive, as η_{KK} is about -.33.

(ii) again as expected, we find that capital and energy are complements, though substantially weak, even substantially weaker than what we found in the manufacturing sector. σ_{EK} is about -.10, and the corresponding cross-price elasticities, η_{KE}, η_{EK}, are about -.003 and 0.02.

(iii) Energy and labor appear to be quite substitutable with the estimated AES of about 1.40 versus the weak result obtained in our manufacturing model or in other studies. The estimated cross-price elasticities, η_{LE} and η_{EL}, are .04 and .63 respectively, more responsive than suggested by the estimates we found for our manufacturing sector.

(iv) Capital and labor are slightly substitutable with σ_{KL} of .70, although this substitutability appears to be stronger than the one we found in the manufacturing sector.

(v) Energy and materials remain weak substitutes with an AES of about .75, which is about the same value found by Berndt and Wood, and stronger than the result we obtained for the manufacturing sector. The cross-price elasticities, η_{EM} and η_{ME}, are .27 and .02 respectively.

(vi) Materials and labor are found to be close substitutes with σ_{LM} being about 1.40 in contrast with a weak substitutability of about .60 found in other studies. The cross price elasticities, η_{LM}, η_{ML}, are .52 and .64 respectively.

For the non-energy manufacturing sector, our findings

TABLE 2.10

ALTERNATIVE MAXIMUM LIKELIHOOD (IZEF) ESTIMATION OF ALLEN PARTIAL ELASTICITIES OF SUBSTITUTION (AES), UNDER VARIOUS SPECIFICATIONS OF AUTOREGRESSION: U.S. NON-ENERGY MANUFACTURING, 1959

AES	R=0	Unrestricted R	Diagonal R: $\rho_{ii}=\rho$	Symmetric R
σ_{KK}	- 1.85	- 1.99	- 2.04	- 1.99
σ_{LL}	- .70	- 1.53	- 1.15	- 1.34
σ_{EE}	-20.76	-29.72	-29.03	-30.38
σ_{MM}	- .23	- 1.85	- 1.38	- 1.64
σ_{KL}	1.11	.70	.79	.71
σ_{KE}	- .45	- .10	.02	- .03
σ_{KM}	- .54	.06	- .04	.05
σ_{LE}	.85	1.43	.87	1.35
σ_{LM}	.31	1.44	1.02	1.23
σ_{EM}	.90	.75	1.24	.83

TABLE 2.11

ALTERNATIVE MAXIMUM LIKELIHOOD (IZEF)
ESTIMATION OF DEMAND PRICE ELASTICITIES
UNDER VARIOUS SPECIFICATIONS OF AUTOREGRESSION
U.S. NON-ENERGY MANUFACTURING, 1959

η_{ij}	R=0	Unrestricted R	Diagonal R: $\rho_{ii}=\rho$	Symmetric R
η_{KK}	-.31	-.33	-.34	-.33
η_{KL}	.51	.31	.35	.32
η_{KE}	-.01	-.003	.001	-.001
η_{KM}	-.18	.02	-.01	.02
η_{LK}	.19	.12	.13	.12
η_{LL}	-.32	-.68	-.52	-.60
η_{LE}	.03	.04	.03	.04
η_{LM}	.11	.52	.36	.44
η_{EK}	-.08	-.02	.003	-.004
η_{EL}	.39	.63	.39	.61
η_{EE}	-.62	-.89	-.83	-.90
η_{EM}	.31	.27	.44	.30
η_{MK}	-.09	.01	-.01	.01
η_{ML}	.14	.64	.46	.55
η_{ME}	.03	.02	.04	.02
η_{MM}	-.08	-.67	-.49	-.58

show that the direction of these substitution and input price elasticities are consistent with results in other studies and those results obtained for our manufacturing study above. All of our own-Allen elasticities of substitution and own-price elasticities have the negative sign required for stability. These estimates, however, are somewhat different from the result obtained for manufacturing. Here, the important own-price elasticity of energy demand is estimated to be more elastic, compared with the value obtained in other studies and particularly compared with the value we found above for manufacturing. Our estimate of η_{EE} for the non-energy manufacturing sector is about -.89 and quite stable. This estimate is at the same level of the more elastic, long-run price elasticities of energy demand, estimated by Griffin and Gregory (1976) from pooled international data (four years: 1955, 1960, 1965, 1969) for the manufacturing sector of nine industrialized countries; while the own-price elasticity of manufacturing energy as surveyed by Waverman (1977), is found to be quite robust among studies, with a common value of around -.50. As we recall our estimate of η_{EE} for the manufacturing sector - around -.15 - about one-third of the value estimated in other studies.

We also observe that technological possibilities for substitution between energy and non-energy inputs are present in non-energy manufacturing sector. In particular, we find that: energy and labor are quite substitutable (σ_{LE} is about 1.40), energy and material are slightly substitutable with an AES of .75, and energy and capital are rather weak complements (σ_{KE} is about -.10). This suggests that, except for labor, the technological possibilities for substitution between energy and non-energy inputs are somewhat limited. For a given level of net output, an increase in the price of energy will increase the quantities of labor and materials, though not in a significant amount, while leading to a reduction in the quantity of energy demanded, and to a very small reductions in quantity of capital demanded.

CHAPTER II

FOOTNOTES

[1]See Berndt and Wood (1979) and Berndt (1976).

[2]This table has been calculated from data and tables in Faucett Associates (1973).

[3]Here one must realize that, like materials input, there are energy products which are primary to the manufacturing sector as well as the energy products which are produded by the firms within manufacturing sector. Necessary information and data for distinguishing these two types of energy products are obtained from Faucett Associates (1973).

[4]Euler's theorem states that: If a function $y = f(x_1,\ldots,x_n)$ is homogeneous of degree k, then:

$$\sum_{i=1}^{n} \frac{\partial f}{\partial x_i} x_i = ky.$$

[5]As an example we may consider that an increase in the price of a primary input such as labor or energy will increase the price of say textiles, leather, glass, rubber and plastic, steel,..., which are produced within manufacturing sector (the internally traded intermediate products). The firms producing furniture and automobiles would substitute away from labor, or energy, but are also faced with higher priced textiles, leather, glass, rubber and plastic, steel .... The industry's new optimum (cost minimizing) choice of primary input quantities is obviously through factor substitution involving internally-traded intermediate products.

[6]Green (1964, Chapter 9) has demonstrated that the second case is a necessary condition for consistent aggregation of the factor demand functions of the firms with neoclassical production function.

[7]For proof, see page 12.

CHAPTER III

TOWARD A MORE GENERAL SPECIFICATION AND ESTIMATION OF INDUSTRIAL FACTOR DEMAND FUNCTIONS

3.0 Introduction

The study of the production function entered a new era with the pioneering work of Cobb and Douglas (1928), titled "A Theory of Production Function". The new production function framework which they originated has stimulated a great deal of theoretical as well as empirical research, and has remained dominant in the field for more than thirty years.

Recognition of severe limitations of the traditional functional forms, such as the CES or Cobb-Douglas, has led researchers to an attractive class of flexible functional forms which started to emerge around 1970; these are the Generalized Leontief (Diewert (1971)), the translog (Christensen et al (1971, 1973)), the Generalized Cobb-Douglas, etc., which we reviewed in Chapter I. These functions have some desirable properties such as the capability of representing a wide variety of technologies. These functions can be considered as a second order Taylor series approximations to any arbitrary function. Unlike the CD and CES functions, these functional forms no longer assume a unitary or constant elasticity of substitution between different pairs of factors; and unlike the CD and CES they do not imply any stringent separability restrictions a priori. In fact the separability conditions which we are interested in can be tested rather than imposed as maintained hypothesis.

In recent years these flexible functional forms (especially the translog) have been utilized in different areas where a production, cost, or utility function has been required. The U.S. manufacturing sector has been such an area, where a growing amount of empirical analysis has emerged. Examples of such works have been cited above.

In the previous chapter we discussed a common problem with these industry-level studies, in which the researchers have employed static profit-maximization models defined over gross output, gross energy, and gross materials input (where these contain intra-industry shipments of traded intermediate products). In models of this kind the price of "energy" and "materials" aggregate inputs must be taken as endogenous, rather than exogenous variables as these authors have assumed. The subsequent utilization of Shephard's Lemma, which requires exogenous factor prices, is therefore inaccurate as a method of obtaining factor demand functions. In the previous chapter the exact nature of this error was examined and a proper alternative was proposed.

Another issue is that in the empirical implementation of the functional form some authors (e.g. Berndt and Wood (1975)) have assumed that the translog function is an exact representation of the true underlying production function. Blackorby, Primont and Russell (1977), and Denny and Fuss (1977) have shown that when the translog function is interpreted to be an exact representation of the technology the separability conditions outlined in these studies are too restrictive and cannot be accepted in performing statistical tests of the separability hypothesis. In this study we depart from this restrictive assumption by interpreting the translog cost function as an approximation to the underlying production structure.

The translog cost function does not oblige the structure of production to exhibit homotheticity, homogenity, or constant return to scale. Rather we can statistically test the validity of the parameter restrictions implied by these structures. If any of the restrictions are valid statistically, it is desirable to adopt the simplified model.

With these remarks in mind, in the present study of the U.S. manufacturing sector (1947-1971), we adopt a general, non-homothetic cost function considered to be a second-order Taylor series approximation to the underlying production structure. The technology involves four types of inputs: capital services (K), labor services (L), primary energy (E), and primary materials (M); and a technology index (T). By employing this general model we are able to measure the contribution of economies of scale and technological change as sources of growth in output after World War II. We can also study and measure the relative changes in various factor shares, Allen partial elasticities of substitution, and factor price elasticities of demand. This flexible model also permits us to perform several interesting tests about the structure of the production technology and examine the approximate weak separability conditions for the cost function.

3.1 Technical Change

One of the problems of considerable importance in applied economics is that of distinguishing movements along the production function from movements from one production function to another. Suppose we observe that point $P_1(X(t), Q(t))$ in input-output space shifts in time to point $P_2(X(t+1), Q(t+1))$ in the next period. Letting the level of output remain the same, one might ask whether the input movement from $X(t)$ to $X(t+1)$ has been along the isoquant of the assumed production function (substitution, or a movement from one production function to another (technical change). Similarly, it is of importance to identify whether, for $X(t+1) = \lambda X(t)$, $\lambda > 0$, the output has moved from one to another isoquant of the <u>same</u> production function (scale effect), or there has been a shift from one production technology to another (technical change). This suggests that it is useful to try to measure economies of scale and technical change, and separate the effect of one from the other. First we start with the definition of technical change.

Assume a neoclassical aggregate production function

$$Q = F(\underline{X},T), \qquad \frac{\partial F}{\partial \underline{X}} \geq 0, \qquad \frac{\partial F}{\partial T} \geq 0; \tag{3.1}$$

where Q and $\underline{X}$ represent output and a vector of inputs respectively while the variable T represents time to allow for technical change. Technological change by its very nature allows a producing unit to produce more output with the same level of input quantities. Or equivalently, the existing level of output, after technological advancement, can be produced with smaller quantities of at least one input while the quantities of other inputs remain the same. Therefore, it seems quite natural to measure the rate at which technology advances as

$$\frac{\dot{Q}}{Q} = \left.\frac{\partial \ln Q}{\partial T}\right|_{\underline{X}} \tag{3.2}$$

If factor input prices and output quantities are exogenously determined, the duality theory between cost and production implies that, assuming cost-minimizing behavior, the production technology given by (3.1) can be uniquely represented by the cost function

$$C = C(\underline{W},\ Q,\ T) \tag{3.3}$$

where C is total cost and $\underline{W}$ is the vector of factor input prices. Technological change, therefore, can be measured by utilizing the cost function. A producing unit at internal equilibrium, ceteris paribus, can produce a given level of output at a lower cost after technological advancement. Thus, the rate of technological change can directly be measured by

$$\frac{\dot{C}}{C} = \left.\frac{\partial \ln C}{\partial T}\right|_{\underline{W},\, Q} \tag{3.4}$$

on the cost side.

To undertake empirical work, we need to be more specific about the nature and character of technological change. Technological change may be biased with respect to one factor or the other, or it may be neutral with regard to all inputs involved. According to Hicks (1932, p. 121), technological changes are classified as labor-saving, neutral, or capital-saving respectively, "as their initial effects are to increase, leave unchanged, or diminish the ratio of the marginal product of capital to that of labor". If the producing unit is to attain a position of internal equilibrium, then one must examine the effect of a technological change along the firm's expansion path where the firm minimizes the cost of producing any given level of output. In this sense Hicks neutral technological change can be interpreted as "expansion path saving". Technological change is said to be Hicks neutral, in this sense, if marginal rates of technical substitution between each pair of factor inputs are independent of technological change. Therefore, representing technical change by the production function (3.1) the following can be defined (see Lau (1978)).

Definition: A production function exhibit Hicks neutrality, in the above sense, if it can be written in the form

$$F(\underline{X}, T) \equiv f(g(\underline{X}), T) \tag{3.5}$$

An alternative interpretation of Hicks' classification, which is widely accepted, is the one which seeks the effect of technical advancement along a ray from the origin where factor proportions remain constant at their pre-technical-change level. However, if one considers the effect of technological change along a ray, as suggested by this interpretation, then we are faced with a quite different notion of neutrality, i.e., implicit Hicks neutrality the term used by Blackorby et al (1976). Technological change, therefore, is defined to be implicitly Hicks neutral if marginal rates of technical substitution between each pair of factors, at constant factor ratios, are independent of technological change. Constructing the implicit representation of the production function (3.1) in the implicit form of $H(Q, \underline{X}, T) = 0$, Blackorby et al (1976) have proved that technical change is implicitly Hicks neutral if and only if $H(\cdot)$ can be written as

$$H(Q, \underline{X}, T) \equiv G(Q, T, g(Q, \underline{X})) \tag{3.6}$$

According to this interpretation, technical change is classified as labor-saving, neutral, or capital-saving depending on whether, at a constant capital-labor ratio, the marginal rate of technical substitution increases, stays unchanged, or decreases. This classification can immediately be interpreted in terms of factor input shares. If at a constant value of a factor ratio, the marginal rate of substitution (or the ratio of capital price to labor price) is rising, then the labor share is declining. Similarly, if the marginal ratio of substitution is declining, the capital share will decrease, and if technical change is classified as neutral, factor shares remains constant.[1]

Another type of technological change, different from the former two types, but often confused with them, is one which can be written in the following decomposable form:

$$F(\underline{X}, T) = A(T) \cdot f(\underline{X}) \qquad (3.7)$$

In this case the isoquant map remains unchanged, but the output number attached to each isoquant is multiplied by A(t). This type of neutrality is what Blackorby et al (1976) call "extended Hicks neutrality".

The practical implication of Hicks neutrality, as is evident from all three types of Hicks neutrality, is that the ratio of the marginal products of any two factors of production is independent of time.[2] However, it is clear that these three types of Hicksian neutrality are not equivalent in general. Besides the fact that extended Hicks neutrality (3.7) implies Hick neutrality (3.5), none of the three types of Hicks neutrality implies either of the other two, unless additional assumptions are imposed. One such assumption is that of <u>homotheticity</u> in inputs which serves as a necessary and sufficient condition for simultaneous Hicks and implicit Hicks neutrality. The reason is that the expansion path through any arbitrary point coincides with a ray through that point under input homotheticity. Therefore homotheticity is a necessary condition for the equivalence of all three types of Hicks neutrality. The second assumption is that of input homogeneity, which is a sufficient condition for the equivalence of all three types. (See Blackorby et al 1978).

Corresponding to Hicks neutrality in the production function F(·), we may define indirect Hicks neutrality for the dual cost function $C = C(\underline{W}, Q, T)$, as Lau (1978) did for the normalized profit function.

Definition: A cost function is said to be indirectly Hicks neutral if it can be written in the following form:

$$C = \tilde{C}(g(\underline{W},Q),\ Q,\ T). \qquad (3.8)$$

Practically, indirect Hicks neutrality implies that the ratio of the derived demands of each pair of inputs is independent of technical change.

In general Hicks neutrality does not imply indirect Hicks neutrality or vice-versa, unless additional assumptions are made. Under the homotheticity assumption, Hicks neutrality implies and is implied by indirect Hicks neutrality. Also, as Lau (1978) has shown in the context of the normalized profit function, "a technology is both directly and indirectly Hicksian neutral only if either it is homothetic or it is additive in T." (See Lau (1978))

An alternative classification of technical change, which has played a more central role in growth literature, is the Harrod classification of technical change. According to Harrod, technical change is defined to be neutral if at any constant value of the capital-output ratio the marginal product of capital remains unchanged. Here we compare points with constant capital-output ratios, while in Hicks' classification points with a constant capital-labor ratio are compared. The Harrod classification can also be stated in terms of the effect upon factor input shares as technological progress proceeds. Technological change is defined to be labor-saving (capital-saving) in Harrod's sense if, at any constant level of the capital-output ratio, the capital share is increasing (decreasing) relative to the labor share. Technical change is classified as Harrod neutral if, at any constant value of the capital-labor ratio, the capital share and labor share increase at the same rate.

The importance of Harrod neutrality stems from the fact that only technical change of this form can be consistent with balanced growth in the usual growth model. In particular, it has been demonstrated by Joan Robinson (1938) and Uzawa (1961) that Harrod neutral technical change is exactly equivalent to pure labor-augmenting technological progress, and can easily be incorporated in the usual growth model. Harrod neutral technical change, therefore, may be defined as labor-augmenting technical change as follows:

Definition: Production function (3.1) exhibits Harrod neutral technological change if one can write the production function in the following separable form:

$$F(\underline{X},T) \equiv f(h(L,T),\underline{X}) \qquad (3.9)$$

where L is labor, the primary factor of production.

The practical implication of Harrod neutrality, formulated in the weakly separable form above, is that the ratio of the marginal product of labor to the rate of technological change measured in terms of output, is independent of $\underline{X}$. For the dual side of the problem, and corresponding to the dual cost function (3.3), indirect Harrod neutrality can be defined as:

<u>Definition</u>: Cost function (3.3) exhibits indirect Harrod neutral technological change if it can be written in the following weakly separable form:

$$C = \tilde{C}(g(W_L,T),\ W_{\underline{X}},Q) \qquad (3.10)$$

The practical implication of indirect Harrod neutrality is that the ratio of the demand for labor to the rate of technical change, measured in terms of cost, is independent of W_X. Here again direct Harrod neutrality does not imply indirect Harrod neutrality or vice-versa. Only under the two following conditions is a production function both directly and indirectly Harrod neutral (See Lau (1978) for a proof in the case of the normalized profit function):

$$\begin{aligned} &1)\quad h(L,T) = \tilde{h}(A(T)L) \\ &2)\quad Q = F(\underline{X}) + h(L,T) \end{aligned} \qquad (3.11)$$

One interesting form of technical change is that of factor and/or output-augmenting technical change. Under general specification of factor and output-augmenting, the production function can be written as follows:

$$F(\underline{X},T) = \alpha(T)\ f(\alpha_1(T)X_1,\ \ldots,\ \alpha_n(T)X_n) \qquad (3.12)$$

where $\alpha(T)$ represents output-augmenting technical change and $\alpha_i(T)$ is "factor-augmenting" technical change corresponding to input i. Hicks and Harrod neutrality may be considered two special cases associated with (3.12). The former occurs when $\alpha_i(T) = \hat{\alpha}(T)$ for all i and f is a homothetic function. In this case it can be shown that:

$$Q = \alpha(T)\cdot g(\hat{\alpha}(T)^{-1}h(X_1,\ \ldots,\ X_n)) \qquad (3.13)$$

which clearly exhibits Hicks neutrality,[3] while Harrod neutrality occurs when the $\alpha(T)$ and $\alpha_i(T)$'s are all constant, except for the $\alpha_i(T)$ corresponding to labor, the primary factor of production.

3.2 Return to Scale

A common assumption in theoretical and empirical research is the assumption of linear homogeneity of the production function implying the existence of constant returns to scale. The assumption of constant returns to scale is of some importance, because the justification of this assumption at the industry level is that in the long run, a perfectly competitive industry with price taking firms and free entry and exit will be able to duplicate what it has been producing before. Thus, by doubling all of their inputs the firms should be able to build another plant identical to the first and produce twice the output. In this case, then, the average cost remains the same for all levels of output.

Scale effects, however, at the industry level may be present due to externalities or lack of freedom for entry and exit, or it may be the case that the firm's production functions are not homogenous of degree one. It might also be the case that the production function exhibits decreasing returns to scale, because the scale effects are genuinely decreasing or because some factor remains fixed in the long run. Similarly, we might be faced with increasing scale effects; for example one factor explaining the rapid post-World War II economic growth in the U.S. might be due to the presence of increasing scale effects. In these circumstances it may be desirable to allow for non-constant return to scale, and not to impose constant returns to scale a priori on the estimating model.

In the theory of production, we recognize two different concepts of returns to scale. The first concept, which has widely been used as a measure and definition of returns to scale, is the proportional change in output relative to the proportional change in the inputs for movements along a ray through the origin. This is, in other words, the elasticity of output with respect to an equiproportional variation of all inputs. This concept is known as the function coefficient, and using the production function (3.1), it may be given more precisely by

$$\varepsilon = \frac{dQ}{Q} \div \frac{dk}{k} = \frac{d \ln Q}{d \ln k} ,$$

where $k > 0$ is a scalar.

According to Hanoch (1975), the second concept, which is more relevant as a measure of scale economies, is obtained from the relationship between total cost and output along the expansion path where the producing unit must remain if it is to minimize the cost of producing any given level of output. For this purpose, elasticity of cost with respect to output appears to be a natural measure to express returns to scale. This elasticity measures the proportional increase in cost relative to the proportional increase in output for movement along the expansion path where input prices are constant and costs are minimized at every level of output. This can be written symbolically as:

$$E = \left. \frac{\partial \ln C}{\partial \ln Q} \right|_{W} ,$$

where $E \lesseqgtr 1$, depending on whether the cost function exhibits increasing, constant, or decreasing returns to scale.[4] By subtracting this elasticity from unity, one can associate positive numbers with economies of scale and negative numbers with scale diseconomies, i.e., we have $SE = 1 - E \gtreqless 0$ as there is increasing, constant, or decreasing returns to scale.

For purposes of empirical implementation, it is necessary to adopt an explicit function form for the cost function (3.3). Our choice is the translog function. In this flexible framework we are able to explicitly deal with these issues and further examine various other specifications as we proceed.

3.3 The General Empirical Model

We assume that the production technology in U.S. manufacturing can be presented by the production function

$$Q = F(K,L,E,M;T) \qquad (3.14)$$

where the flow of output Q is related to the service flow of four aggregate inputs; capital (K), labor (L), energy (E), and intermediate materials (M). T is an indicator of technical change and is measured in years. We further assume that the production function F is positive, finite, contineously twice differentiable, strictly monotone, strongly quasi concave, and nondecreasing in technical change, T. If factor prices and output levels are exogenously determined, and assuming cost-minimizing behavior, the production structure implied by (3.14) can uniquely be described by a cost function of the form:

$$C = C(W_K, W_L, W_E, W_M, Q; T) , \qquad (3.15)$$

where C represents total cost and the W_i, i = K,K,E,M, are factor prices.

For purposes of estimation, it is necessary to employ a specific functional form for the cost function C. The highly general functional form we have chosen for this study is the translog cost function proposed by Christensen, Jorgenson and Lau (1971, 1973). It places no apriori restrictions on the Allen partial elasticities of substitution (AES) among the factors of production. An important property of the translog function is that it can be interpreted as a second order approximation to an arbitrary cost function. It also allows scale economies to vary with the level of output. Also, technical change can be incorporated into translog cost function conveniently.

The production structure is represented by a non-homothetic translog cost function which can be written by the following approximation to an arbitrary cost function:

$$\ln C = \alpha_0 + \sum_i \alpha_i \ln w_i + \frac{1}{2} \sum_i \sum_j \gamma_{ij} \ln W_i \ln W_j$$

$$+ \sum_i \gamma_{iQ} \ln W_i \ln Q + \alpha_Q \ln Q + \frac{1}{2} \gamma_{QQ} (\ln Q)^2$$

$$+ \sum \gamma_{it} T \ln W_i + \alpha_t T + \frac{1}{2} \gamma_{tt} T^2 + \gamma_{tQ} T \ln Q,$$

$$i, j = K, L, E, M. \qquad (3.16)$$

The derived demand functions for each factor of production are conveniently obtained by partially differentiating the cost function with respect to factor prices, namely,

$$\frac{\partial C}{\partial W_i} = X_i$$

where X_i is the cost-minimizing quantity demanded of ith input. Using this result, known as Shephard's lemma (Shephard (1953, 1970)), one can express the input demand functions in terms of cost shares simply by logarithmic differentiation of the translog cost function as

$$\frac{\partial \ln C}{\partial \ln W_i} = \frac{W_i X_i}{C} = S_i$$

where S_i denotes the cost share of the ith factor input. Then the translog cost function produces cost share equations of the form

$$S_i = \alpha_i + \sum_j \gamma_{ij} \ln W_i + \gamma_{iQ} \ln Q + \gamma_{it} T, \qquad (3.17)$$

where $i,j = K,L,E,M$.

Since the systems of demand equations (3.17) must satisfy the adding-up restriction ($\sum S_i = 1$), the following parameter restrictions hold:

$$\begin{aligned} &\sum \alpha_i = 1 \\ &\sum_i \gamma_{ij} = \sum_j \gamma_{ij} = \sum_i \sum_j \gamma_{ij} = 0 \qquad (3.18) \\ &\sum_i \gamma_{iQ} = 0 \\ &\sum_i \gamma_{it} = 0 \qquad i,j = K,L,E,M \end{aligned}$$

In addition to these parameter restrictions (3.18) the following symmetry constraints, implied by equality of the cross partial derivatives, are required.

$$\gamma_{ij} = \gamma_{ji} \qquad i \neq j \tag{3.19}$$

These restrictions, (3.18) and (3.19), will be imposed throughout the study.

The translog cost function must satisfy the following conditions: (i) Linear homogeneity in prices; that is, for a given level of output, total cost must increase proportionally with a proportional increase in all factor prices. This implies the same parameter restrictions as (3.18). (ii) Monotonicity: that is, the cost function must be an increasing function of input prices. This implies that the cost shares be strictly positive. (iii) Concavity in input prices: this means that the Hessian matrix, $\frac{\partial^2 C}{\partial W_i W_j}$, must be negative semidefinite.

The Allen-Uzawa partial elasticity of substitution between input i and j, as we saw before, can be obtained from the cost function by the formula

$$\sigma_{ij} = \frac{CC_{ij}}{C_i C_j} \tag{3.20}$$

For the translog cost function these elasticities are:

$$\sigma_{ij} = 1 + \frac{\gamma_{ij}}{S_i S_j} \qquad i \neq j \tag{3.21}$$

$$\sigma_{ii} = 1 + \frac{\gamma_{ii} - S_i}{S_i^2} \tag{3.22}$$

Obviously these elasticities entail no apriori restrictions with respect to their value of constancy. The own and cross-price elasticities of demand for factors of production can be obtained as:

$$\eta_{ij} = \frac{\partial \ln X_i}{\partial \ln W_j} = \sigma_{ij} S_j \qquad (3.23)$$

$$\eta_{ii} = \frac{\partial \ln X_i}{\partial \ln W_i} = \sigma_{ii} S_i \quad i,j = K,L,E,M \qquad (3.24)$$

where the η_{ij} (demand price elasticity) measures the percentage change in demand for input i for an exogenous change in the price of input j, given all other input prices and output quantity remain constant. Note that while $\sigma_{ij} = \sigma_{ji}$ by definition, $\eta_{ij} \neq \eta_{ji}$ in general.

3.4 Technical Change and Bias

Inclusion of T, as an input, in the translog cost function (16) will facilitate the study of technical change biases. The Hicksian concept, which is the most common concept of the biases of technical change, can be handled conveniently in the cost function framework in terms of input cost shares. Letting S_i represent the share of the ith input of total cost, as in (3.17), a technical change is said to be i-using, i-saving, or neutral if the ith cost share (S_i), at constant input prices, increases, decreases, or remains the same. Therefore the γ_{it}'s are the estimates of factor using or factor saving Hicks biases of technological change. These parameters represent, in fact, the rate of change in the cost shares not attributable to prices, i.e.,

$$\frac{\partial S_i}{\partial T} = \gamma_{it} \qquad i = K,L,E,M. \qquad (3.25)$$

Here, a zero value for the γ_{it} (for all $i = K,L,E,M$) implies Hicks neutral technical change.

If the production structure is non-homothetic, technological change may be biased with respect to the returns to scale. A biased technological advancement of this kind will increase the scale level at which decreasing returns set in, and thus may change the output level at which minimum average cost could be attained. The scale bias (SB), therefore, can be defined as a time (technical change) derivative of the scale measure, i.e.,

$$SB = \frac{\partial}{\partial T}\left(\frac{\partial \ln C}{\partial \ln Q}\right) .$$

With respect to our translog cost function this, derivative is equal to γ_{tQ}, and thus we can measure the scale bias after estimation of the parameters of the translog cost function.

Other alternative models regarding technical change may be tested. For example a parametric restriction of $\gamma_{tt} = 0$, in addition to Hicks neutrality constraints ($\gamma_{it} = 0$), implies scale bias technical change along with a linear Hicks neutral technical change. While a further imposition of $\gamma_{tQ} = 0$, in addition to $\gamma_{it} = 0$ and $\gamma_{tt} = 0$, implies the usual type of exponential Hicks neutral technical change. Finally, we may have a production structure with no overall technological effect which implies the following parameter restrictions;

$$\alpha_t = 0, \quad \gamma_{it} = 0, \quad \gamma_{tt} = 0, \quad \gamma_{tQ} = 0 \qquad i = K,L,E,M \tag{3.26}$$

3.5 Homotheticity, Homogeneity, and Return to Scale

A common assumption in theoretical and empirical research is that the production function is linear homogenous, meaning that there are constant returns to scale. This is the kind of assumption which one may wish to test, rather than impose a priori. The cost function specified in (3.15) is assumed to be non-homothetic with the corresponding trans-log approximation represented by (3.16).

If the production technology exhibits homotheticity, the cost function (3.15) can be written as a separable function of output and factor prices; that is,

$$C = h(Q,T)\cdot C(W,T) \ , \tag{3.27}$$

where C(W,T) is the unit cost function corresponding to $F(\underline{X},T)$. To apply the translog approximation to (3.27) we take logarithms of both sides and obtain

$$\ln C = \ln h(Q,T) + \ln c(\underline{W},T) \tag{3.28}$$

The non-homothetic translog cost function (3.16) will correspond to a homothetic production technology, represented by the cost function (3.27), if we impose the restrictions:

$$\gamma_{iQ} = 0 \qquad i = K,L,E,M. \tag{3.29}$$

Two other versions of (27) which are also implied by the homotheticity of the production structure are

$$C = k(Q)\cdot c(W,T) \tag{3.30}$$

$$C = g(Q,T)\cdot c(\underline{W}). \tag{3.30}$$

For these cases the additional parameter restrictions are: $\gamma_{tQ} = 0$ for (3.30) and $\gamma_{it} = 0$ for (3.31) respectively. These two specifications will exhibit, in addition to homotheticity, scale unbiasedness ($\gamma_{tQ} = 0$), and Hicks neutrality ($\gamma_{it} = 0$) respectively.

A homothetic production technology is further restricted to be homogeneous (of degree $\frac{1}{k}$) if and only if the elasticities of cost with regard to output are constant. The cost function (3.15) can be written in the form

$$C = Q^k\cdot c(\underline{W},T) \tag{3.31}$$

The translog cost function (3.16) can serve as a second order approximation to (3.32) if we impose the following restrictions on (3.16).

$$\gamma_{QQ} = 0, \quad \gamma_{tQ} = 0, \quad \gamma_{iQ} = 0 \qquad i = K,L,E,M \tag{3.32}$$

Finally, if the underlying production function exhibits constant returns to scale (linear homogeneous), then the corresponding cost function can be written as

$$C = Q\cdot c(\underline{W},T) \tag{3.33}$$

This imposes an additional parameter restriction on the translog cost function (3.16), i.e., in addition to (3.33) we have[5]:

$$\alpha_Q = 1 .$$

Economies of scale are among the factors, such as technical change and relatively cheap material resource inputs, which have been mentioned as contributing phenomena in the growth of industrial output after WWII. Earlier, scale economies (SE) were defined in terms of the cost elasticities of output. With the translog cost function (3.16) this elasticity is obtained as:

$$\frac{\partial \ln C}{\partial \ln Q} = \alpha_Q + \gamma_{QQ} \ln Q + \sum \gamma_{iQ} \ln W_i + \gamma_{tQ} T \,. \quad (3.35)$$

Under a different specification and with different parameter restrictions imposed, this elasticity can be derived accordingly. For example, if the homotheticity restriction ($\gamma_{iQ} = 0$) is imposed, then, under this restriction, one can rewrite this formula as:

$$\frac{\partial \ln C}{\partial \ln Q} = \alpha_Q + \gamma_{QQ} \ln Q + \gamma_{tQ} T; \quad (3.36)$$

and so on.

3.6 Estimation and Hypothesis Testing

The parameters to be estimated are contained in the derived factor share equations (3.17) and the translog cost function itself, which form our estimable equations. It is of course feasible to estimate the parameters of the translog cost function alone, using ordinary least squares. This procedure, however, may result in a high degree of multicollinearity and therefore highly imprecise coefficient estimates, due to the large number of terms involved in the translog cost function. In addition, this method neglects the additional information contained in the factor share equations.

An alternative estimation method used by many (e.g. Berndt and Wood (1975)) is to estimate only the share equations as a multi-variate regression system. This method is satisfactory when the factor share equations contain all the parameters of the translog cost function (for example, in case we assume constant returns to scale - Hicks neutral technical change). Since we have adopted a nonhomothetic-nonneutral specification for the translog cost function, many parameters do not appear in the factor share equations. Consequently, the estimation of factor share equations is not an appropriate approach, and must be abandoned in this case.

An optimal approach, practiced by many authors, has consisted of joint estimation of the translog cost function and the factor share equations as a multivariate regression system. Therefore the model to be estimated consists of the translog function itself and the three share equations for capital, labor, and primary energy, after deleting one share equation - arbitrarily the materials factor share - to avoid singularity of the disturbance var-cov matrix. With constraints for linear homogeneity in factor prices and symmetry constraints imposed (i.e., restrictions (3.18) and (3.19)), the estimating equations are written as:

$$S_i = \alpha_i + \sum_j \alpha_{ij} \ln (W_j/W_M) + \alpha_{iQ} \ln Q + \gamma_{it} T + \mu_i$$

$$\ln(C/W_M) = \alpha_0 + \sum_i \alpha_i \ln(W_i/W_M)$$

$$+ \tfrac{1}{2} \sum_i \sum_j \gamma_{ij} \ln (W_i/W_m) \ln (W_j/W_M)$$

$$+ \sum_i \gamma_{iQ} \ln (W_i/W_M) \ln Q + \sum_i \gamma_{it} \ln (W_i/W_M) T$$

$$+ \alpha_Q \ln Q + \tfrac{1}{2} \gamma_{QQ}(\ln Q)^2 + \alpha_t T + \tfrac{1}{2} \gamma_{tt} T^2$$

$$+ \gamma_{tQ} T \ln Q + \mu_0; \qquad i,j = K,L,E.$$

where $\gamma_{ij} = \gamma_{ji}$, and μ_0 and μ_i are random disturbances. Assuming that the random error vector $\mu = (\mu_0, \mu_K, \mu_L, \mu_E)$ is independently and identically distributed as multivariate normal with mean vector zero and nonsingular covariance matrix, we have estimated the parameters of the model employing the "iterative Zellner" estimation method. This estimation procedure is well known to yield coefficient estimates identical to maximum-likelihood estimates (see Kmenta and Gilbert (1968)), and therefore the estimates are consistent and asymptotically efficient.

Since the parameter estimates so obtained are maximum-likelihood estimates, we can test the validity of various hypotheses (specifications) such as homotheticity, homogeneity, etc. Our statistical tests for various specifications are based on the likelihood ratio method. The likelihood ratio is $\lambda = L_{max}(\hat{\omega})/L_{max}(\hat{\Omega})$, where

$L_{max}(\hat{\omega})$ and $L_{max}(\hat{\Omega})$ stand for the maximum of the likelihood function under restricted and unrestricted models respectively. It is well known (Wilks, (1938)) that the test statistic $-2 \ln \lambda$ has an asymptotic distribution that is chi-square with degrees of freedom equal to the number of independently imposed restrictions.

3.7 Empirical Results

3.7.A Tests of Underlying Assumptions

3.7.A.1 Manufacturing

We now proceed to a discussion of our various specifications estimated by employing the Zellner iterative estimation method. The parameter estimates for these specifications are reported in Table (3.2) along with their corresponding sample log-likelihood values. The reported sample log-likelihood values indicate that casting Model 1 as the unconstrained model, the homothetic specification (Model 3, $\gamma_{iQ} = 0$) must be rejected. This can be seen by computing the likelihood ratio test statistic which is twice the difference of the sample log-likelihood values. The test statistic for the homothetic model (Model 3) is 19.854, while the .01 chi-square critical value with three (parameter restrictions) degrees of freedom (.01 χ^2_3) is 11.345. The two other versions of the homothetic production function, specified in equations (3.30) and (3.31), and presented as Model 4 and Model 5 in table (3.2), are also rejected. The likelihood ratio test statistics are 20.842 and 23.128 for Model 4 and Model 5 respectively, while the .01 χ^2_4 and .01 χ^2_6 critical values are 13.277 and 16.812 respectively. Model 4 assumes homotheticity but no scale bias ($\gamma_{tQ} = 0$), while Model 5 assumes homotheticity along with Hicks neutral technical change.

Since the homotheticity hypothesis is rejected, the technology is not homogeneous of any degree. As indicated in table (3.2) the homogenous and constant returns to scale (CRS) hypotheses are rejected, where test statistics for these models (Model 6 and 10) are 21.590 and 47.628, compared with the corresponding .01 χ^2 critical values of 11.070 and 12.592 respectively. However, if the (rejected) homothetic model is maintained, the homogeneith restrictions would not be rejected, while the CRS model would be rejected.

As for the effect of technological advancement, the model with "no technical change" (Model 12: $\alpha_t = \gamma_{tt} = \gamma_{it} = \gamma_{tQ} = 0$) has decisively been rejected against the unrestricted model (Model 1). For our nonhomothetic, unrestrained model (Model 1), "Hicks neutral technical change" (Model 13; $\gamma_{it} = 0$) is also rejected; the test statistics is 14.322 while the .01 χ^2_3 is equal to 11.345. However, once the (rejected) homothetic, homogeneous, or CRS specifications are maintained as out unconstrained models, the imposition of Hicks neutrality restrictions on each causes very little additional loss of fit respectively; and thus "Hicks neutrality" will not be rejected for these models (Model 5, 8, and 11). For example, if homotheticity is maintained, then the imposition of Hicks neutrality will result in a test statistics of 3.274, while the .01 χ^2_3 is 11.345, and hence "homothetic-Hicks neutral technical change" can not be rejected.

Model 11 (CRS-Hicks neutral technical change) is the same model chosen by Berndt and Wood (1975) in their study of the U.S. manufacturing sector. Berndt and Wood, however, did not estimate the rate of technical change, as they only

TABLE 3.1

ALTERNATIVE MODEL SPECIFICATIONS WITH
THEIR CORRESPONDING PARAMETER RESTRICTIONS

Model	Specification	Parameter Restrictions
1	Unrestricted:	None
2	No scale bias:	$\gamma_{tQ} = 0$
3	Homothetic:	$\gamma_{iQ} = 0$
4	Homothetic - no scale bias:	$\gamma_{iQ} = \gamma_{tQ} = 0$
5	Homothetic - Hicks neutral technical change	$\gamma_{iQ} = \gamma_{it} = 0$
6	Homogeneous:	$\gamma_{iQ} = \gamma_{QQ} = \gamma_{tQ} = 0$
7	Homogeneous/conditional on no scale bias:	$\gamma_{iQ} = \gamma_{QQ} = 0$
8	Homogeneous - Hicks neutral technical change:	$\gamma_{iQ} = \gamma_{QQ} = \gamma_{tQ} = \gamma_{it} = 0$
9	Homogeneous - Hicks neutral/conditional on no scale bias	$\gamma_{iQ} = \gamma_{QQ} = \gamma_{it} = 0$
10	Constant return to scale:	$\alpha_Q = 1, \gamma_{iQ} = \gamma_{QQ} = \gamma_{tQ} = 0$
11	Constant return to scale - Hick neutral technical change:	$\alpha_Q = 1, \gamma_{iQ} = \gamma_{QQ} = \gamma_{tQ} = \gamma_{it} = 0$
12	No technical change:	$\alpha_t = \gamma_{it} = \gamma_{tt} = \gamma_{tQ} = 0$
13	Hicks neutral technical change:	$\gamma_{it} = 0$
14	Hicks neutral technical change - no scale bias:	$\gamma_{it} = \gamma_{tQ} = 0$
15	Hicks neutral linear technical change:	$\gamma_{it} = \gamma_{tt} = \gamma_{tQ} = 0$
16	Hicks neutral linear technical change/conditional on no scale bias:	$\gamma_{it} = \gamma_{tt} = 0$
17	Unitary elasticity of substitution:	$\gamma_{ij} = 0$
18	Homogeneous - no technical change:	$\gamma_{iQ} = \gamma_{QQ} = \gamma_{tQ} = \alpha_t = \gamma_{it} = \gamma_{tt} = 0$
19	Homogeneous - Hicks neutral linear technical change:	$\gamma_{iQ} = \gamma_{QQ} = \gamma_{tQ} = \gamma_{it} = \gamma_{tt} = 0$

TABLE 3.2

MAXIMUM LIKELIHOOD (IZEF) PARAMETER ESTIMATES
OF TRANSLOG COST FUNCTION UNDER VARIOUS SPECIFICATIONS
U.S. MANUFACTURING, 1947-71

Parameters	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
α_0	.360	.367	.364	.369	.360	.356
α_K	.101	.101	.104	.104	.097	.104
α_L	.461	.462	.458	.460	.461	.459
α_E	.060	.059	.061	.060	.062	.060
α_M	.379	.378	.377	.376	.380	.376
α_Q	.607	.308	.596	.262	.640	.376
α_t	- .002*	.012	- .001*	.013	- .003*	.010
γ_{KK}	.058	.057	.048	.048	.053	.048
γ_{KL}	- .006*	- .006*	.027	.028	.002*	.028
γ_{KE}	- .014	- .013	- .016	- .016	- .016	- .016
γ_{KM}	- .038	- .039	- .060	- .060	- .039	- .060
γ_{LL}	.186	.190	.068*	.074*	.096	.072*
γ_{LE}	- .022	- .024	.014*	- .017*	- .009	- .016*
γ_{LM}	- .158	- .160	- .081*	- .085*	- .088	- .084*
γ_{EE}	.059	.060	.059	.060	.054	.059
γ_{EM}	- .024	- .024	- .028	- .028	- .028	- .028
γ_{MM}	.220	.222	.169	.173	.156	.172
γ_{KQ}	- .033	- .032				
γ_{LQ}	.177	.177				
γ_{EQ}	- .008*	- .008*				
γ_{MQ}	- .137	- .138				
γ_{Kt}	.001	.001	- .0005	- .0004		- .0005
γ_{Lt}	- .009	- .009	.0004*	.0003*		.0003*
γ_{Et}	.001*	.001*	.0001*	.0002*		.0001*
γ_{Mt}	.007	.007	- .00003*	.00004*		.00002*
γ_{QQ}	- 2.676*	.135*	- 3.083*	.202*	- 3.293*	
γ_{tt}	- .005*	.0*	- .006*	- .0001*	- .006*	.0002*
γ_{Qt}	.116*		.136*		.144*	
R^2						
LTC	.996	.996	.996	.996	.996	.996
SK	.708	.710	.721	.722	.670	.722
SL	.652	.651	.435	.435	.462	.435
SE	.959	.959	.960	.960	.958	.960
Log of Likelihood Function:	396.964	396.597	387.037	386.543	385.400	386.169
Restrictions:	None	1	3	4	6	5
Likelihood Ratio Statistic: $-2\frac{\ln L(\omega)}{\ln L(\Omega)}$		.734	19.854	20.842	23.128	21.590

* Coefficient is not significantly different from zero at .05 level of significance.

TABLE 3.2

(cont'd.)

	Model 7	Model 8	Model 9	Model 10	Model 11	Model 12
	.370	.354	.366	.256	.250	.323
	.104	.097	.097	.098	.097	.103
	.460	.461	.461	.478	.461	.454
	.060	.062	.062	.059	.062	.068
	.376	.380	.380	.365	.380	.374
	.276	.386	.299			.611
	.013	.009	.012	- .012	- .011	
	.048	.053	.053	.056	.056	.052
	.028	.002*	.002*	.002	- .00002*	.017*
	- .016	- .015	- .016	- .014	.015	- .016
	- .060	- .040	- .039	- .044	- .041	- .053
	.074*	.096	.096	.153	.103	.076
	- .017*	- .009	- .009	- .022*	- .010	.007*
	- .085*	- .089	- .089	- .133	- .093	- .100
	.060	.053	.054	.061	.052	.042
	- .028	- .028	.028	- .025	- .027	- .033
	.173	.157	.157	.201	.161	.186
						- .008*
						.014*
						- .009
						.003*
	- .0005			- .0001*		
	.0003*			- .001*		
	.0002*			.0003*		
	.00004*			.0008*		
						.113*
	- .0005*	.0002*	- .0005*	.00001*	- .0001*	
	.009*		.008*			
	.996	.996	.996	.721	.717	.993
	.721	.672	.671	.678	.670	.719
	.435	.462	.462	.454	.460	.471
	.960	.957	.957	.957	.955	.954
	386.603	384.493	384.888	373.150	372.463	383.000
	4	8	7	6	9	6
	20.722	24.942	24.152	47.628	49.002	27.928

TABLE 3.2

(cont'd.)

	Model 13	Model 14	Model 15	Model 16	Model 17
	.353	.361	.359	.360	.343
	.106	.106	.106	.106	.088
	.438	.439	.439	.439	.415
	.067	.067	.070	.067	.074
	.389	.388	.388	.388	.423
	.701	.308	.328	.313	.550*
	-.005*	.012	.012	.012	.004*
	.049	.049	.049	.049	
	.026	.027	.027	.027	
	-.016	-.015	-.015	-.015	
	-.060	-.060	-.060	-.060	
	.024*	.026*	.026*	.026*	
	.003*	.003*	.003*	.003*	
	-.052*	-.055*	-.055*	-.055*	
	.044	.044	.044	.044	
	-.031	-.031	-.031	-.031	
	.143	.147	.147	.147	
	-.012	-.012	-.012	-.012	-.002*
	.034	.033	.033	.033	.119
	-.007	-.007	-.007	-.007	-.040
	-.015*	-.014*	-.014	-.014*	-.076*
					.0002*
					-.003*
					.001
					.002*
	-3.641*	.140*	.102*	.150*	-1.561*
	-.006*	-.0001*			-.002*
	.157*			-.001*	.064*
	.996	.996	.996	.996	.996
	.734	.735	.735	.735	.009
	.479	.478	.478	.478	.509
	.956	.956	.956	.956	.755
	389.803	389.185	389.167	389.174	353.150
	3	4	5	4	10
	14.322	15.558	15.594	15.580	87.628

TABLE 3.3

VALUES OF $\chi^2_{\alpha,n}$

Degrees of freedom: n	1	2	3	4	5	6	7	8	9	10
α=.05	3.841	5.991	7.815	9.488	11.070	12.592	14.067	15.507	16.919	18.307
α=.025	5.024	7.378	9.348	11.143	12.832	14.449	16.013	17.535	19.023	20.483
α=.01	6.635	9.210	11.345	13.277	15.086	16.812	18.475	20.090	21.666	23.209
α=.005	7.879	10.597	12.838	14.860	16.750	18.548	20.278	21.955	23.589	25.188

estimated the three cost share equations which are independent of the rate of technical change. When the null hypothesis of "CRS-Hicks neutral technical change" is tested against the general unconstrained model, the null hypothesis is rejected (as was seen above) when estimating the system either with the cost function included or excluded.

The rejection of Hicks neutral technical change (Model 13) will make imposition and estimation of a further restriction of "no scale bias", (Model 14, $\gamma_{tQ}=0$), or $\gamma_{tt}=0$ (Model 16), somewhat pointless. These two models are rejected, as expected. Conditional on the "Hicks neutral technical change" we also have tested for the validity of the "Hicks neutral linear technical change" restrictions ($\gamma_{tt}=\gamma_{tQ}=0$; Model 15) against our unrestricted specification, Model 1. Based on the likelihood ratio test criterion, the test statistic for this hypothesis is 15.594, while the .01 (.005) χ^2_5 is 15.086 (16.750), which indicates rejection at .01 level of significance but "acceptance" at .005 level. However, if these models (i.e., models 14, 15, and 16) are tested against the (rejected) "Hicks neutral technical change," none of them will be rejected, while the null hypothesis of "no technical change" (Model 12) is rejected when tested against the same alternative.

We also have tested a model that assumes technological change but no scale bias ($\gamma_{tQ}=0$; Model 2). The sample log-likelihood value, as well as the estimates of the parameters for this model, are almost identical to those of the unrestricted model; therefore it has the same explanatory power as the unrestricted model. Accordingly, one may test for the validity of the "homothetic" specification with or without $\gamma_{tQ}=0$, as can be seen by comparing the sample log-likelihood values and the parameter estimates of Model 3 and Model 4. The same is true about the "homogeneous" specification of Model 7 and Model 6, the "Hicks neutral technical change" specification of Model 14 and Model 13, and the "homogeneous Hicks neutral" specification of Model 9 and Model 8 as indicated in table (3.2). All of these models, (i.e., Models 3, 4, 7, 14, 13, 9, and 8), however, are rejected when tested against the unrestricted specification (or the specification with no scale bias, Model 2). An exception occurs, however, in moving from Model 16 ($\gamma_{it}=\gamma_{tt}=0$) to Model 15 ($\gamma_{it}=\gamma_{tt}=\gamma_{tQ}=0$). Both Model 15 and 16 are rejected at the .01 level of significance when tested against the unrestricted specification (Model 1), while Model 15 (Hicks neutral linear technical change) can be accepted at the .005 level of significance. We may conclude that, in order to appropriately study the structure of production for the U.S. manufacturing sector, a flexible nonhomothetic cost function, which exhibits non-neutral technical change, should be employed.

The estimates of Hicks biases, γ_{it}, reported for the unrestricted model show that four estimates are significantly different from zero. These estimates indicate that biased technical change has been labor-saving; capital, energy, and intermediate material using. The labor-saving and other-factor-using biased technical change is consistent with factor price data, where the labor price had the fastest rate of growth. The labor-saving bias is somewhat strong. It amounts to an annual .86 of one percentage point change in the labor cost share not attributable to substitution within an unchanged technology. It implies that if the production technology had remained static over the period, the share of labor in "net" output would have been about 21.5% larger than it is. In the case of capital, energy, and material, their shares in "net" output would have been 3.25%, 1.5%, and 16.75% smaller than if the production technology had remained static over the period.

3.7.A.2 Non-energy Manufacturing

As for the non-energy manufacturing sector, we have tested the same seventeen specifications we tested for the manufacturing sector. The parameter estimates of these models are reported along with the maximum value of the log-likelihood functions in table (3.4). To our surprise, our non-energy manufacturing data can only reject three specifications at the .01 level of significance. These specifications are CRS, CRS-Hicks neutral technical change, and the unitary elasticity of substitution. All other models can be accepted even at .05 level of significance.

The "rejection" of CRS and "acceptance" of the homogeneous specification will obviously lead us to conclude

TABLE 3.4

MAXIMUM LIKELIHOOD (IZEF) PARAMETER ESTIMATES
OF THE TRANSLOG COST FUNCTION UNDER VARIOUS SPECIFICATION
U.S. NONENERGY MANUFACTURING, 1947-71

Parameters	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
α_0	.452	.449	.447	.447	.428	.442
α_K	.151	.152	.149	.149	.169	.149
α_L	.469	.469	.469	.469	.458	.470
α_E	.028	.028	.030	.030	.030	.030
α_M	.352	.352	.351	.351	.342	.351
α_Q	.074*	.259*	.309*	.339	.375	.413
α_t	.023	.015	.013*	.012	.012*	.009
γ_{KK}	.089	.089	.093	.092	.086	.093
γ_{KL}	- .021*	- .021*	- .031	- .031	.009*	- .031
γ_{KE}	- .003*	- .004	- .005	- .005	- .007	- .006
γ_{KM}	- .066	- .065	- .056	- .056	- .087	- .056
γ_{LL}	.120	.120	.118	.119	.100	.119
γ_{LE}	- .009*	- .007*	- .001*	- .001*	- .002*	- .001*
γ_{LM}	- .090*	- .092	- .086	- .087	- .107	- .087
γ_{EE}	.004*	.005*	.007*	.007*	.101	.007*
γ_{EM}	.008*	.006*	- .001*	- .001*	- .001*	- .001*
γ_{MM}	.147	.151	.143	.144	.195	.144
γ_{KQ}	.031*	.031*				
γ_{LQ}	.014*	.014*				
γ_{EQ}	- .011	- .099*				
γ_{MQ}	- .034*	- .036*				
γ_{Kt}	- .0002*	- .0002*	.001	.001		.001
γ_{Lt}	- .001*	- .001*	- .001*	- .001*		- .001*
γ_{Et}	.001*	.0004*	- .0004*	- .00004*		- .00004*
γ_{Mt}	.001*	.001*	- .0003*	- .0003*		- .0003*
γ_{QQ}	3.072*	.303*	.641*	.140*	.796*	
γ_{tt}	.004*	- .001*	.001*	- .0002*	.0004*	.00004*
γ_{Qt}	- .108*		- .020*		- .022*	
R^2						
LTC	.995	.995	.995	.995	.995	.995
SK	.857	.857	.854	.854	.811	.855
SL	.687	.686	.690	.690	.630	.691
SE	.515	.510	.452	.452	.402	.451
Log of Likelihood Function:	389.188	388.905	387.550	387.537	382.946	387.464
Restrictions:	None	1	3	4	6	5
Likelihood Ratio Statistic: $-2 \frac{\ln L(\omega)}{\ln L(\Omega)}$		.566	3.276	3.302	12.484	3.448

* Coefficient is not significantly different from zero at .05 level of significance.

TABLE 3.4

(cont'd.)

	Model 7	Model 8	Model 9	Model 10	Model 11	Model 12
	.446	.420	.427	.360	.338	.402
	.149	.169	.169	.143	.168	.156
	.470	.458	.458	.487	.458	.449
	.030	.030	.030	.030	.030	.031
	.351	.342	.342	.341	.341	.364
	.351	.527	.431			.739
	.011	.007	.010	.002*	- .001*	
	.092	.086	.086	.078	.070	.087
	- .031	.009*	.008*	- .048	.003*	- .130*
	- .005	- .007	- .007	- .004	- .006	- .006
	- .056	- .087	- .087	- .027	- .067	- .068
	.119	.100	.100	.154	.099	.075
	- .001*	- .002*	- .002*	- .003*	- .003	.001*
	- .087	- .107	- .107	- .104	- .100	- .062*
	.007*	.011	.010	.004*	.008	.005*
	- .001*	- .001*	- .001*	.002*	.001*	- .00002*
	.144	.195	.195	.128	.166	.131
						.019
						.009*
						- .003*
						- .025*
	.001			.001		
	- .001*			- .002*		
	- .00004*			- .00001*		
	- .0003*			.001*		
						- .144
	- .0004*	- .0001*	- .001*	- .001	- .001*	
	.005*		.008*			
	.995	.995	.995	.775	.814	.994
	.854	.811	.811	.826	.780	.852
	.690	.690	.630	.688	.644	.663
	.451	.391	.396	.438	.443	.442
	387.530	382.653	382.906	375.887	372.527	383.595
	4	8	7	6	9	6
	3.316	13.070	12.564	26.602	33.322	11.186

TABLE 3.4

(cont'd.)

	Model 13	Model 14	Model 15	Model 16	Model 17	Model 18	Model 19
	.448	.447	.444	.448	.414	.424	.431
	.151	.151	.151	.151	.163	.170	.170
	.459	.460	.459	.460	.421	.458	.458
	.031	.031	.031	.031	.032	.030	.030
	.358	.358	.359	.358	.384	.342	.342
	.223*	.285*	.378	.265*	.438	.650	.490
	.016	.014	.101	.015	.012*		.007
	.091	.090	.092	.090		.089	.089
	- .022	- .022	- .022	- .022		.012	.009*
	- .005	- .005	- .005	- .005		- .007	- .007
	- .064	- .064	- .644	- .064		- .094	- .091
	.097	.098	.095	.098		.092	.098
	- .001*	- .001*	- .001*	- .001*		- .002*	- .002*
	- .074	- .076	- .072	- .075		- .102	- .106
	.005*	.005*	.005*	.005*		.010	.011
	.001*	.001*	.001*	.001*		- .001*	- .001
	.138	.139	.136	.138		.197	.198
	.025	.025	.026	.025	.102		
	- .006*	- .007*	- .005*	- .007*	- .090*		
	- .002*	- .002*	- .002*	- .002*	- .008		
	- .017*	- .017*	- .019*	- .017*	- .004*		
					- .005		
					.006		
					.0002*		
					- .002*		
	1.263*	.229*	.040*	.488*	.162*		
	.001*	- .0004*			- .0001		
	- .041*			- .010*	- .006*		
	.995	.995	.995	.995	.996	.993	.995
	.857	.857	.858	.857	.225	.813	.811
	.681	.681	.681	.681	.625	.609	.627
	.466	.465	.464	.465	.356	.400	.395
	388.462	388.409	388.256	388.430	361.993	379.355	382.346
	3	4	5	4	10	10	9
	1.452	1.558	1.864	1.516	54.390	19.666	13.684

that, to portray the production structure of the non-energy manufacturing sector, it is very reasonable to assume a homogeneous production function. On the other hand, due to acceptance of the "no technical change" specification, and conditional on the homogeneous specification,we have tested for the validity of the "homogeneous-no technical change" specification (Model 18). So, maintaining the homogeneous specification (which could not be rejected), we impose "no technical change" restrictions ($\alpha_t=\gamma_{tt}=\gamma_{it}=\gamma_{tQ}=0$) which result in the likelihood ratio test statistic of 16.218. Consequently, this hypothesis (Model 18) is rejected at the .01 level of significance.[6] This suggests that it is not realistic to assume the absence of technological change.

Next, conditional on the homogeneous specification we test for the validity of the Hicks neutral technical change restrictions ($\gamma_{it}=0$). The resultant test statistic is 9.622, while the .025 (.01) χ^2 critical value with three degrees of freedom is 9.348 (11.345). This implies rejection of the null hypothesis at the .025 level of significance, but "acceptance" at the .01 level, since the test statistic falls between these two critical values. This rejection of Hicks neutral technical change at the .025 level may make estimation under the further restriction of "Hicks neutral linear technical change" ($\gamma_{it}=\gamma_{tt}=0$) somewhat pointless. For the record, however, when the null hypothesis of "homogeneous-Hicks neutral linear technical change" (Model 19) is tested against the alternative hypothesis of the homogeneous-Hicks neutral specification, the null hypothesis is rejected at the .05 level, but accepted at the .025 level of significance.

It must be noted, however, that neither of these two hypotheses (i.e. homogeneous-Hicks neutral and homogeneous-Hicks neutral linear technical change) was rejected when they were tested against the unrestricted model (Model 1).

On balance, we may conclude that, for the nonenergy manufacturing sector, characterization of the production structure by a function exhibiting homogeneity and Hicks neutrality is justifiable. Indeed, to examine the validity of various types of separability hypotheses for the non-energy-manufacturing sector, we have made such an assumption.

3.7.B Elasticity Estimates

3.7.B.1 Manufacturing

Estimates of the Allen partial elasticity of substitution (σ_{ij}) and of the factor input demand elasticities (η_{ij}), which have been derived under various specifications, are reported in table (3.5) and (3.6)[7]. The purpose of this is to observe the effect which these specifications have on these elasticities.

In the first column of table (3.5) we present the substitution elasticity estimates corresponding to our general nonhomothetic nonneutral model, while in other columns we report the estimates corresponding to the models specified at the top of each column. As is apparent from

TABLE 3.5

MAXIMUM LIKELIHOOD (IZEF) ESTIMATES OF ALLEN PARTIAL ELASTICITIES OF SUBSTITUTION (AES) UNDER VARIOUS TRANSLOG SPECIFICATIONS, U.S. MANUFACTURING
1958

AES	Model 1	Model 13	Model 15	Model 12	Model 3	Model 5	Model 6	Model 8	Model 10	Model 11
σ_{KK}	-3.03	-4.05	-4.05	-3.69	-4.14	-3.52	-4.17	-3.53	-3.16	-3.17
σ_{KL}	.86	1.65	1.67	1.41	1.69	1.04	1.69	1.05	1.05	1.0
σ_{KE}	-1.29	-1.73	-1.63	-1.73	-1.80	-1.78	-1.75	-1.75	-1.52	-1.66
σ_{KM}	- .05	- .69	- .70	- .51	- .71	- .13	- .72	- .14	- .25	- .18
σ_{LL}	- .29	-1.11	-1.09	- .84	- .88	- .75	- .86	- .74	- .47	- .71
σ_{LE}	.17	1.09	1.10	1.24	.50	.67	.46	.67	.24	.64
σ_{LM}	.10	.71	.69	.44	.55	.51	.53	.50	.25	.48
σ_{EE}	- .01	-3.88	-3.88	-4.43	- .34	-1.55	- .17	-1.59	.23	-1.93
σ_{EM}	.10	- .22	- .24	- .31	- .12	- .12	- .11	- .12	- .03	-.07
σ_{MM}	- .11	- .61	- .59	- .34	- .45	- .53	- .43	- .53	- .25	- .50

TABLE 3.6

MAXIMUM LIKELIHOOD (IZEF) ESTIMATES OF DEMAND PRICE ELASTICITIES UNDER VARIOUS TRANSLOG SPECIFICATIONS, U.S. MANUFACTURING 1958

η_{ij}	Model 1	Model 13	Model 15	Model 12	Model 3	Model 5	Model 6	Model 8	Model 10	Model 11
η_{KK}	-.27	-.36	-.36	-.33	-.37	-.31	-.37	.31	-.28	-.28
η_{KL}	.38	.74	.75	.64	.76	.47	.77	.47	.47	.45
η_{KE}	-.08	-.11	-.10	-.11	-.12	-.11	-.11	-.11	-.10	-.11
η_{KM}	-.02	-.27	-.28	-.20	-.28	-.05	-.28	-.05	-.10	-.09
η_{LK}	.08	.15	.15	.13	.15	.09	.15	.09	.09	.09
η_{LL}	-.13	-.50	-.49	-.38	-.40	-.34	-.39	-.34	-.21	-.32
η_{LE}	.01	.07	.07	.08	.03	.04	.03	.04	.02	.04
η_{LM}	.04	.28	.27	.17	.22	.20	.21	.20	.10	.19
η_{EK}	-.12	-.15	-.15	-.15	-.16	-.16	-.15	-.15	-.13	-.15
η_{EL}	.07	.49	.50	.56	.21	.30	.21	.30	.11	.29
η_{EE}	-.001	-.25	-.25	-.28	-.02	-.10	-.01	-.10	.01	-.12
η_{EM}	.04	-.09	-.10	-.12	-.05	-.05	-.04	-.05	-.01	-.03
η_{MK}	-.005	-.06	-.06	.05	-.06	-.01	-.06	-.01	-.02	-.02
η_{ML}	.04	.32	.31	.20	.25	.23	.24	.23	.11	.22
η_{ME}	.01	-.01	-.02	-.02	-.01	-.01	-.01	-.01	-.002	-.004
η_{MM}	-.05	-.24	-.23	-.13	-.18	-.21	-.17	-.21	-.10	-.20

table (3.5), the substitution elasticity estimates which we have obtained under the three specifications of "Hicks neutral technical change", "Hicks neutral linear technical change", and "no technical change" are generally of a larger magnitude than those estimates derived under the unrestricted specification. This is to be expected since some of what is classified as substitution response under one specification is reclassified as technical change under the other.

Comparing the estimated σ_{ij} for the homothetic specification with those of the homogeneous model, we observe that the estimates are very close. This closeness must be regarded as natural since, as we saw earlier, the homogeneous specification could not be rejected when homotheticity was maintained. The same conclusion holds when Hicks neutrality is imposed on the homothetic and homogeneous specifications. The resultant elasticity estimates of these two specifications (homothetic-Hicks neutral and homogeneous-Hicks neutral) are almost identical.

The estimated Allen elasticities of substitution (σ_{ij}) and price elasticities (η_{ij}), reported in tables (3.5) and (3.6), show that the own-Allen elasticities of substitution are negative for all specifications. But, here we obtain a smaller magnitude for these elasticities compared to the Berndt-Wood (1975), and Berndt-Khaled (1979) gross output models, excepting the σ_{MM} which is of a larger magnitude in our specifications. In particular, our η_{EE} (own-price elasticity of primary energy) is of a smaller magnitude compared to the robust estimate of about -.5 reported by Berndt-Wood (1975), Hudson-Jorgenson (1974), and Berndt-Khaled (1979). We have also found a stable and substantial primary energy-capital complementarity relationship under all specifications. Although it is not as strong as those reported by Berndt-Wood and Berndt-Khaled for their gross output model, the σ_{KE} is -1.3 for our unrestricted model and between -1.5 to -1.8 for other specifications, while the estimated cross-price elasticities (η_{KE} and η_{EK}) are about -.1 and -.15 respectively.

Capital and labor again appear to be substitutable as has been the case in numerous traditional two input (capital-labor) studies. The estimated σ_{KL} for our unrestricted model is about .85 (compared to 1.0 obtained by Berndt-Wood and .8 to 2.1 obtained by Berndt-Khaled), while for other specification (K-L) substitutability is quite stronger with an estimated σ_{KL} between 1.0 and 1.7. Our estimates for η_{KL} and η_{LK} are .38 and .08 for the unrestricted model compared to η_{KL}=.58 and η_{LK}=.12 obtained by Berndt-Khaled for their unrestricted specification. We find primary energy-labor and labor-primary materials substitution elasticities positive (substitutable) everywhere, with their magnitude oscillating mildly across the various specifications. Berndt-Khaled found these elasticities not quite stable; in fact the estimated σ_{LM} for their unrejected models turned out to be negative.

Finally, we have found a capital-primary material complementarity relationship (σ_{KM}<0), which persists for all specifications. Also, primary energy and primary materials are found to be complementary for all selected models except for our unrestricted one. Berndt-Wood (1975) found the material inputs as substitutable for both energy and capital with an estimated σ_{KM}=.55 and σ_{EM}=.75, while Berndt-Khaled (1979) found a (K-M) complementarity relationship (σ_{KM}<0) under their unrejected specifications and positive σ_{KM} under rejected ones. Their estimates of σ_{EM}, however, turn out to be positive for all specifications, but about .3 for the unrestricted model.

3.7.B.2 Non-Energy Manufacturing

Now we look at the estimated Allen partial elasticities of substitution, σ_{ij}, and the estimated price elasticities of demand for factors, η_{ij}, for the non-energy manufacturing sector, which are presented in tables (3.7) and (3.8). A glance at these tables reveals the equivalence of the estimated price and substitution elasticities we have obtained under the homothetic and homogeneous specifications. Estimated price and substitution elasticities derived under the homogeneous-Hick neutral technical change, the homogeneous-Hicks neutral linear technical change, and the homogeneous-no technical change specifications are also equivalent. As is evident from these tables, the price and substitution elasticities from these various models are reasonably close. This should not be surprising since none of these specifications (except the

TABLE 3.7

MAXIMUM LIKELIHOOD (IZEF) ESTIMATES OF ALLEN PARTIAL ELASTICITIES OF SUBSTITUTION (AES) UNDER VARIOUS TRANSLOG SPECIFICATIONS, U.S. NON-ENERGY MANUFACTURING, 1959

AES	Model 1	Model 12	Model 3	Model 6	Model 8	Model 19	Model 18	Model 11
σ_{KK}	- 1.77	- 1.87	- 1.66	- 1.65	- 1.90	- 1.81	- 1.81	- 2.48
σ_{KL}	.72	.83	.59	.59	1.11	1.18	1.16	1.04
σ_{KE}	.42	- .13	- .10	- .11	- .47	- .45	- .42	- .21
σ_{KM}	- .14	- .18	.02	.02	- .51	- .56	- .61	- .16
σ_{LL}	- .61	- .84	- .62	- .61	- .71	- .71	.75	- .71
σ_{LE}	.31	1.04	.93	.93	.85	.86	.88	.80
σ_{LM}	.43	.61	.46	.45	.32	.33	.35	.37
σ_{EE}	-28.78	-26.71	-24.49	-24.39	-20.55	-20.46	-21.09	-24.16
σ_{EM}	1.82	1.00	.94	.93	.89	.86	.88	1.12
σ_{MM}	- .66	- .79	- .69	- .68	- .26	- .23	- .24	- .51

TABLE 3.8

MAXIMUM LIKELIHOOD (IZEF) ESTIMATES OF DEMAND PRICE ELASTICITIES UNDER VARIOUS TRANSLOG SPECIFICATIONS, U.S. NON-ENERGY MANUFACTURING, 1959

η_{ij}	Model 1	Model 12	Model 3	Model 6	Model 8	Model 19	Model 18	Model 11
η_{KK}	-.29	-.31	-.27	-.27	-.32	-.31	-.31	-.42
η_{KL}	.33	.38	.27	.27	.51	.51	.53	.48
η_{KE}	.01	-.004	-.003	-.003	-.01	-.01	-.01	-.01
η_{KM}	-.05	-.06	.01	.01	-.17	-.19	-.21	-.06
η_{LK}	.12	.14	.10	.10	.19	.19	.20	.17
η_{LL}	-.28	-.38	-.29	-.28	-.32	-.33	-.34	-.33
η_{LE}	.01	.03	.03	.03	.03	.03	.03	.02
η_{LM}	.15	.21	.16	.16	.11	.11	.12	.13
η_{EK}	.07	-.02	-.02	-.02	-.08	-.08	-.07	-.04
η_{EL}	.14	.47	.42	.42	.39	.40	.40	.37
η_{EE}	-.85	-.80	-.73	-.73	-.61	-.61	-.63	-.72
η_{EM}	.64	.35	.33	.32	.30	.29	.30	.39
η_{MK}	-.02	-.03	.003	.003	-.09	-.10	-.10	-.03
η_{ML}	.20	.28	.21	.21	.15	.15	.16	.17
η_{ME}	.05	.03	.03	.03	.03	.03	.03	.03
η_{MM}	-.23	-.28	-.24	-.24	-.08	-.08	-.08	-.18

CRS-Hicks neutral) are rejected.

The calculated own price elasticities, η_{ij}, are all negative which indicates that the factor demand functions are all downward sloping and all are inelastic. For this sector, energy demand is more responsive to a change in its own price with own-price elasticities of about -.6 for the homogeneous model, and -.7 to -.8 for others. Again, the capital-energy complementarity relationship persists for all specifications, although it turned out to be much weaker than that reported for the manufacturing sector. The estimated σ_{KE} are between -.01 to -.47 for these selected specifications while the estimated cross-price elasticities, η_{KE} and η_{EK}, are about -.01 and -.08 respectively.

Capital and labor are found to be quite substitutable with the estimated substitution elasticity of 1.11 for the homogeneous-Hick neutral specification, while between .72 and 1.18 for other specifications. The estimated capital-labor cross price elasticities show a stronger responsiveness than those we found either for manufacturing, or those reported by Berndt and Wood (1975). The estimated η_{KL}, and η_{LK} for our homogeneous-Hicks neutral specification are 0.5 and 0.2 respectively. Labor and energy are inelasticly substitutable with σ_{LE}=0.85 and price elasticities of η_{LE}=0.03 and η_{EL}=0.4.

Capital and primary materials display complementarity with an estimated σ_{KM} of about -0.5 for our homogeneous-Hicks neutral specification. This complementarity, which is in agreement with findings we reported for the manufacturing sector, is not in accord with the weak substitutability found by Berndt and Wood. We also found energy and primary materials to be substitutable with an estimated $_{EM}=-.9$, which is in accordance with the Berndt and Wood findings, although a bit stronger than their results. The cross price elasticities, η_{EM} and η_{ME}, are 0.3 and .03 respectively.

3.7.C Economies of Scale and the Rate of Technological Advancement

Next, we may turn to the question of economies of scale. For this purpose we have calculated the index of scale economies, SE, suggested by Christensen and Green (1976) defined as one minus the cost elasticity along an output ray. For our translog cost function, this index is formulated as

$$SE = 1 - \partial \ln C/\partial \ln Q = 1-(\alpha_Q + {}_{QQ}\ln Q + \sum_i \gamma_{iQ}\ln W_i + \gamma_{tQ}T), \quad (3.37)$$

where SE is positive for scale economies and negative for scale diseconomies. This index has a natural interpretation (when multiplied by 100) as the percentage difference between total cost and total revenue, assuming that the output is priced at marginal cost, i.e., SE = (C-PQ)/C. The translog cost function allows scale economies to vary, in general, with the level of output, factor prices, and the state of technology, T (measured in years). If the production function is homothetic ($\gamma_{iQ}=0 \ \forall \ i$), and if there is no technological scale bias ($\gamma_{tQ}=0$), then $SE = 1-\alpha_Q-\gamma_{QQ}\ln Q$, which varies with output. If in addition, the production function exhibits homogeneity ($\gamma_{QQ}=0$) of degree $1/\alpha_Q$ in input quantities, then $SE = 1-\alpha_Q$, which is a constant.

An estimation of scale economies can be derived for each specification by evaluating the formula (3.37) for the corresponding specification at the observed level of output and factor prices. For comparison with scale economies estimated from the unrestricted translog cost function, we have also presented scale economies estimates for other restricted specifications in tables (3.9) and (3.10) for manufacturing and nonenergy manufacturing respectively. The estimates of scale economies for both sectors indicate substantial positive scale economies for all selected specifications. These estimates (except for Model 5) all are statistically significant as judged by the corresponding standard error given in parentheses.

The estimates of scale economies for manufacturing (reported in table (3.39)) seem somewhat large for all specifications (except for CRS which is zero by definition). For our unrestricted nonhomothetic-nonneutral net output model the estimates of scale economies is .345. Berndt and Khaled (1979), however, obtained an estimate of .199 for their unrestricted gross output model, which they considered to be rather large when compared with the estimated obtained in other studies of returns to scale involving value-added[8].

As is indicated in table (3.9), the estimates of scale

TABLE 3.9

ALTERNATIVE ESTIMATE OF SCALE ECONOMIES AND RATE OF TECHNOLOGICAL CHANGE, U.S. MANUFACTURING, 1971
(ESTIMATED STANDARD ERROR IN PARENTHESES)

	Model 1	Model 13	Model 15	Model 12	Model 3	Model 5	Model 6	Model 8	Model 10	Model 11
$SE=1-\frac{\partial \ln C}{\partial \ln Q}$	.345	.300	.550	.260	.330	.320	.624	.614	0	0
	(.092)*	(.059)*	(.065)*	(.014)*	(.196)	(.218)	(.064)	(.064)	-	-
$\frac{\partial \ln C}{\partial T}$	.0035	.0021	.0117	0	.0029	.0026	.0147	.0145	-.0120	-.0132
	(.0220)*	(.0359)	(.0026)	-	(.0220)*	(.0393)	(.0006)*	(.0030)	(.0009)*	(.0026)

* Estimated standard errors of SE and $\partial \ln/\partial T$ for these models for the year 1971 are extremely burdensome to compute. Therefore, as an approximation, these standard errors have been computed in terms of the price and quantity indexes corresponding to mid-point of our sample (year 1959).

TABLE 3.10

ALTERNATIVE ESTIMATE OF SCALE ECONOMIES AND RATE OF TECHNOLOGICAL CHANGE, U.S. NON-ENERGY MANUFACTURING, 1971 (ESTIMATED STANDARD ERROR IN PARENTHESES)

	Model 1	Model 13	Model 12	Model 5	Model 3	Model 6	Model 8	Model 15	Model 18	Model 11
$SE=1-\frac{\partial \ln C}{\partial \ln Q}$	.405	.481	.420	.336	.514	.587	.473	.510	.350	0
	(.136)*	(.122)*	(.015)*	(.115)	(.216)	(.074)	(.062)	(.056)*	(.013)	-
$\frac{\partial \ln C}{\partial T}$	.0006	.0046	0	-.002	.006	.0094	.0037	.0066	0	-.024
	(.0017)*	(.108)	-	(.0062)	(.0056)*	(.0029)*	(.0038)	(.0023)	-	(.0025)

* See footnote of table (3.9).

economies are quite sensitive to the function specification. We obtain quite a large estimate of .624 for our (rejected) homogeneous specification (Model 6), while our estimate from Model 12 (nonhomothetic zero technical change) is the smallest, where SE = .260. This latter estimate is remarkably close to the estimate of .240 obtained by Nadiri and Schankerman (1981) for U.S. manufacturing sector.

Imposition of Hicks neutrality on the non-homothetic-nonneutral specification (Model 1) reduces the implied estimate of economies of scale to .300 for Model 13 (the Hicks neutral specification). The same results holds for the homothetic and homogeneous specification when we compare Model 6 (homogeneous) with Model 8 (homogeneous-Hicks neutral) and Model 3 (homothetic) with Model 5 (homothetic-Hicks neutral). The figures in table (3.9) in general indicate that the estimation of scale economies enlarges as we move from the unrestricted to the homogeneous specification. Berndt and Khaled (1979) found quite the contrary as their estimate of scale economies got smaller under the homogeneous-Hicks neutral specification.

As for the nonenergy manufacturing sector, the figures in table (3.10) once again display the sensitivity of the scale economies estimates to various specifications. For the homogeneous specification (Model 6) we obtain the high estimate of .587 for scale economies, while the homothetic-Hicks neutral model (Model 5) renders a relatively smaller estimate of .336. Furthermore, when the Hicks neutrality restriction ($\gamma_{it}=0 \ \forall \ i$) is imposed on the nonhomothetic-nonneutral model, the resulting specification (Model 13) produces a higher estimate of scale economies (i.e., .481). However, quite the opposite is true about the homothetic and homogeneous specifications, since the imposition of the Hicks neutrality restrictions yield relatively lower estimates of scale economies.

In tables (3.9) and (3.10) we also have reported the corresponding estimates of the rate of technological advancement for each specification measured by $\partial \ln C/\partial T$ (output quantity and input prices fixed). The negative of this measure is defined as "the dual rate of total cost diminution" by Ohta (1974) and utilized by Berndt and Khaled (1979). As is evident from tables(3.9) and (3.10), the estimated rate technical change are affected by various functional specifications. Our unrestricted model for manufacturing produces a positive and statistically insignificant estimate of the rate of technological change, while our (rejected) CRS-Hicks neutral model yields a negative and statistically significant estimate of 1.32% per annum. The sensitivity of the rate of technical change to functional specification was also realized by Berndt and Khaled as they got a statistically significant negative estimate of .72% per annum for their (rejected) CRS-Hicks neutral specification, and a positive and statistically insignificant estimate for their unrestricted model.

For the nonenergy manufacturing sector once again we obtain a negative and statistically significant estimate of the rate of technological change for our (rejected) CRS-Hicks neutral model, while other specifications imply positive estimates, some of which are statistically significant and some not. Earlier we concluded that the production structure for the nonenergy manufacturing sector may satisfactorily be characterized by a homogeneous-Hicks neutral specification. For this specification, however, we obtain a positive and statistically insignificant estimate for $\partial \ln C/\partial T$.

These results indicate that for the U.S. nonenergy manufacturing sector, like the manufacturing sector, the null hypothesis of a zero rate of technological change would not be rejected, while the null hypothesis of no scale economies (SE=0) would be rejected. Consequently we are persuaded that in U.S. manufacturing and nonenergy manufacturing (1947-71) the source of growth has been due mainly to the utilization of economies of scale. The same conclusion is reached by Berndt and Khaled (1979) although our estimates of scale economies are somewhat stronger.

Following Berndt and Khaled (1979), we may now examine the effect of changes in relative factor prices on the average productivity of various inputs. Average productivity, which is typically measured as Q/X_i (e.g., output per man hour), is obviously affected by (among other things) changes in relative input prices. A natural way to express this responsiveness is the elasticity of the average productivity of the ith input with respect to jth input price, i.e., $\partial \ln (Q/X_i)/\partial \ln W_j$ where output quantity and other input prices are held fixed. But $\partial \ln (Q/X_i)/\partial \ln W_j = -\partial \ln X_i/\partial \ln W_j$, which is the negative of the well known price elasticity of demand for the factor of production, namely,

$$\partial \ln (Q/X_i)/\partial \ln W_j = -\eta_{ij}, \quad i,j = K,L,E,M \tag{3.38}$$

We may also compute the elasticity of average productivity with respect to output and time by the following two formulas, i.e.,

$$\partial \ln (Q/X_i)/\partial \ln Q = 1-\partial \ln C/\partial \ln Q - \gamma_{iQ}/S_i, \text{ input prices fixed} \tag{3.39}$$

and

$$\partial \ln (Q/X_i)/\partial T = -\partial \ln C/\partial T - \gamma_{it}/S_i, \text{ output quantity and input prices fixed} \tag{3.40}$$

Table (3.11) summarizes estimates of these elasticities for U.S. manufacturing in 1971, based on our unrestricted, non-homothetic-nonneutral specification. The average productivity of various inputs, increases with growth in their own prices.[9] Average labor productivity increases as the price of the labor input rises, and decreases with growth in the prices of K, E, and M, since labor is substitutable with K, E, and M. Energy average productivity increases with growth in the price of the capital input, due to the complementarity relationship between energy and capital inputs. The response of average energy productivity to growth in the prices of K, L, and M (shown in Table (3.11)) are considerably weaker (in magnitude) in our net output model, compared to the estimates Berndt and Khaled (1979) obtained in their gross output model.[10]

TABLE 3.11

ESTIMATED ELASTICITIES OF AVERAGE PRODUCTIVITY OF INPUTS WITH RESPECT TO FACTOR PRICES, OUTPUT, AND TIME; NONHOMOTHETIC-NONNEUTRALMODEL, MODEL, U.S. MANUFACTURING, 1971

$-\eta_{ij}$	i=K	L	E	M
j=K	.15	.38	.12	.10
L	-.06	.14	-.01	-.06
E	.16	.07	-.11	.01
M	.02	.07	.002	.05
$\frac{\partial \ln (Q/X_i)}{\partial \ln W_j}$	.780	-.039	.479	.682
$\frac{\partial \ln (Q/X_i)}{\partial T}$	-.022	.015	-.013	-.020

TABLE 3.12

ESTIMATED ELASTICITIES OF AVERAGE PRODUCTIVITY OF INPUTS WITH RESPECT TO FACTOR PRICES, OUTPUT, AND TIME; HOMOGENEOUS HICKS-NEUTRAL MODEL, U.S. NON-ENERGY MANUFACTURING, 1971

$-\eta_{ij}$	i=K	L	E	M
j=K	.23	-.53	.02	.27
L	-.15	.32	-.03	-.14
E	.10	-.40	.62	-.33
M	.10	-.18	-.03	.10

For the average factor productivity elasticities with respect to output, formula (3.40) indicates that the value of these elasticities is determined by two terms, i.e., by the estimate of scale economies ($1-\partial \ln C/\partial \ln Q$) and by the value of $-\gamma_{iQ}/S_i$. But

$$\partial(SE)/\partial \ln W_i = -\gamma_{iQ} \qquad i=K,L,E,M$$

where $\gamma_{iQ}>0$ indicates that higher factor prices have a depressing effect on scale economies and cause Q/X_i to decline, while $\gamma_{iQ}<0$ indicates the opposite. Therefore, as long as economies of scale are present ($SE>0$), a negative value of γ_{iQ} implies a positive value for the elasticity of average factor product with respect to output. For the manufacturing sector (unrestricted model-1971) these elasticities are positive for K,E, and M since $\gamma_{iQ}<0$ ($i = K,E,M$).

For the labor input, however, γ_{LQ} turns out to be positive, which implies a depressing effect on scale economies. Therefore the average labor productivity elasticity is determined by the value of SE and the negative value of $-\gamma_{LQ}/S_L$. For the year 1971, we obtain a negative value (-.039) for this elasticity. Although Berndt and Khaled (1979) obtained a positive value for this elasticity for all input, their estimate of the average material productivity elasticity was the smallest since γ_{MQ} turned out to be positive in thier gross output model.

Finally, we turn to our estimates of the average factor productivity elasticity with respect to T. These estimates are quite consistent with the pattern of technical change biases we obtained for our general specification. Since technical change has been found to be labor-saving, the input-output ratio for L decreases over time, implying an increase in average labor productivity indicated by the positive value of this elasticity.

For the nonenergy manufacturing sector, we have reported in Table (3.12) the estimates of the average factor productivity elasticities for 1971, based on the homogeneous Hicks neutral specification.[11] The average productivity of all inputs

For the nonenergy manufacturing sector, we have reported in Table (3.12) the estimates of the average factor productivity elasticities for 1971, based on the homogeneous Hicks neutral specification.[11] The average productivity of all inputs increases with growth in their own price. For energy, this elasticity is the largest. Average labor productivity rises with increases in the price of the labor input and decreases with growth in the prices of capital, energy, and materials, since these inputs appear to be substitutable with labor. Finally, average energy productivity increases with growth in the prices of capital and energy, while it declines with growth in the price of labor and materials.

CHAPTER III

FOOTNOTES

[1]A point to be clarified about the relation between technical change and factor shares is that a labor-saving technical change, for example, which increases the ratio of the marginal product of capital to that of labor need not actually decrease the marginal product of labor and consequently labor's absolute share. At any rate, however, a labor-saving technical change will decrease the relative share of labor. Exactly a similar relation is true for a capital-saving technical change.

[2]Mathematically this can be stated as:

$$\frac{\partial}{\partial t}\left(\frac{F_i}{F_j}\right) = 0 \qquad \text{or} \qquad F_i F_{jt} - F_j F_{it} = 0$$

[3]If $\alpha_i(T) = \hat{\alpha}(T)$ for all i, then (12) may be written as $Q = \alpha(T)\ f(\hat{\alpha}(T)x_1, \ldots, \hat{\alpha}(T)x_n)$. By the homotheticity of $f(\cdot)$ we further have: $Q = \alpha(T)\ g\ (h(\hat{\alpha}(T)x_1, \ldots, \hat{\alpha}(T)x_n))$, where $h(\cdot)$ is homogeneous of degree one, i.e., $h(\lambda \underline{x}) = \lambda h(\underline{x})$, where λ is a positive scaler. Letting $\lambda = 1/\hat{\alpha}(T)$ we obtain: $Q = \alpha(T)\cdot g(\hat{\alpha}(T)^{-1}\ h(x_1, \ldots, x_n))$; this completes the proof.

[4]We can justify this formula as a measure of return to scale in the following way. It is well-known that increasing, constant, or decreasing returns to scale prevail as long run average cost decreases, stays constant, or increases respectively. Mathematically this can be stated as:

$$\frac{d(C/Q)}{dQ} \lesseqgtr 0, \text{ or } (Q\cdot\frac{dC}{dQ} - C)/Q^2 \lesseqgtr 0.$$

To have this satisfied we must have: $(Q\cdot\frac{dC}{dQ} - C) \lesseqgtr 0$, which implies $\frac{dC}{dQ} \lesseqgtr \frac{C}{Q}$. Or further,

$$\frac{dc}{dQ} \div \frac{c}{Q} = \frac{d\ \ln\ C}{d\ \ln\ Q} \lesseqgtr 1 .$$

[5]The parameter restrictions in this case and the homogeneous case involve the restriction $\gamma_{tQ} = 0$, due to the requirement that the elasticity of cost with respect to output must be constant or equal to one, depending on whether the production structure is homogeneous or linear homogeneous respectively. The imposition of this restriction implies, at the same time, the absence of scale bias.

[6]This model has been rejected at the .05 level of significance when tested against the unrestricted model, but accepted at the .025 level.

[7]The concavity condition is violated, though not severely at some point for models with no restriction on the γ_{it}(i=K,L,E,M) coefficient. The noncavities disappear for these models when neutrality (γ_{it}=0) is imposed. This violation of regularity conditions, as Wales (1977) explains, does not necessarily imply the absence of the underlying cost-minimization process, rather it may indicate the inability of the flexible functional form to approximate the true cost function over the range of the data. Here, in Tables (3.5) and (3.6), we have reported the estimated substitution and price elasticities for the manufacturing sector for the year 1958 (which is very close to the mid-point of the sample year, 1959), since the concavity condition is satisfied for all specifications at that point.

[8]Berndt and Khaled computed the dual rate of returns to scale as one over the elasticity of cost with respect to output, i.e., $(\partial \ln C/\partial \ln Q)^{-1}$. To compare with our estimates, we have inverted their estimate and subtracted it from one.

[9]Although the point estimate of η_{EE} is positive ($-\eta_{EE} < 0$) for the year 1971, this curvature violation does not seem to be statistically significant when judged by its standard error. The standard error for η_{EE}, under the assumption that the share S_E is constant and equal to the mean of its estimated value, turned out to be about .15, which is quite large. See also footnote 7 above.

[10]The elasticity estimates reported by Berndt and Khaled are: $-\eta_{Ej}$ = .293, -.434, .546, and -.405 for j = K,L,E,M respectively.

[11] The reasons for choosing the homogeneous Hicks neutral specification are twofold: (1) this specification appeared to be justifiable for representing the non-energy manufacturing sector, and (2) for the unrestricted model the estimates of γ_{iQ} and γ_{it} turned out to be insignificant. Since for the homogeneous Hicks neutral specification $\gamma_{iQ} = \gamma_{it} = 0$ holds, we only have reported $-\eta_{ij}$ in Table (3.12).

CHAPTER IV

TESTS FOR THE EXISTENCE OF REAL VALUE-ADDED AND OTHER TYPES OF INPUT AGGREGATION IN U.S. MANUFACTURING

4.0 Introduction

The estimation of the production relation and factor demand functions requires some measure of the output or activity level. Real value-added has been used as such a measure in virtually all empirical studies of labor-capital substitubility and investment demand, where in the existence of real value-added has been assumed *a priori*. It has been shown by some authors (e.g. Sims (1969), Gordon (1969), and Arrow (1972)) that when material inputs are used in the production process, as they are in the manufacturing sector, then the existence of real value-added rests upon the existence of weak separability between the primary input and material inputs. This allows us to write the production function, F, as $G(g(K,L),E,M)$ where $g(\cdot)$ has been identified as real value-added. In this chapter we will test the value-added specification and other types of separability among inputs; but first we summarize the discussion of weak separability in Chapter I.

4.1 Weak Separability Defined

The notion of weak separability, which we are mainly concerned with, is defined as follows: If we partition the n-tuple vector of the input $x = \{x_1, x_2, \ldots, x_n\}$ into r mutually exclusive and exhaustive subsets as $N^* = \{N_1, \ldots, N_r\}$, a function, $f(x)$, is said to be weakly separable with respect to the partition N* if the marginal rate of substitution between a pair of inputs i, $j \varepsilon N_m$ (m=1, ..., r) is independent of changes in the level of inputs outside N_m, i.e.,

$$\frac{\partial}{\partial x_k}\left(\frac{f_i}{f_j}\right) = 0$$

or

$$f_j f_{ik} - f_i f_{jk} = 0 \qquad \forall\ i,j \varepsilon N_m, \text{ and } k \notin N_m \tag{4.1}$$

Goldman and Uzawa (1964) have proved that f(x) is weakly separable with respect to the partition N* (r > 2) if and only if it is of the form

$$f(x) = G(g^1(x^1), \ldots, g^r(x^r)) \tag{4.2}$$

where $g^m(x^m)$ is a function of the subvector $x^m \varepsilon N_m$ alone.

Berndt and Christensen (1973a) have shown that if f(x) is homothetically separable, then the dual cost function C(Q,W) is also separable and therefore we must have:

$$C_j C_{ik} - C_i C_{jk} = 0 \tag{4.3}$$

Berndt and Christensen (1973a) have also proved, in the context of production theory, that a strictly quasi-concave homothetic production function, f(x), is locally weakly separable with respect to the partition N* if and only if $\sigma_{ik} = \sigma_{jk}$, i.e., the Allen partial elasticity of substitution between input i and k is the same as that between input j and k.

4.2 Various Specifications Discussed

In several recent studies of manufacturing a restrictive assumption about the structure of production has been the weak separability of capital, labor, and energy inputs, as a group from the materials input. Examples are Humphrey and Moroney (1975), Griffin and Gregory (1976), Pindyck (1979), and Magnus (1979). This separability assumption has been necessarily adopted in these studies due to unavailability of reliable data from which to construct price or quantity indices of the materials input. However, one might ideally wish to test this hypothesis rather to impose it apriori. But it is feasible to do so only when price and quantity indices for the materials input are available. In this section we attempt to test this and other similar restrictions on the production structure.

Specifically we assume that the general non-homothetic production function (3.14) is weakly separable as:

$$F(K,L,E,M;T) = f(g(K,L,E,T),M,T) \tag{4.4}$$

where $g(\cdot)$ is an input aggregator function or microfunction. If $g(\cdot)$ is homothetic in primary inputs, then the dual non-homothetic cost function (3.15) will be weakly separable as:

$$C(W_K,W_L,W_E,W_M,Q,T) = H(h(W_K,W_L,W_E,T),W_M,Q,T) \tag{4.5}$$

where $h(\cdot)$ is aggregate input price.

The translog cost function (3.16) will be approximately[1] weakly separable as in (4.5) if the following parameter restrictions hold:

$$\alpha_i \gamma_{jM} - \alpha_j \gamma_{iM} = 0$$

and

$$\alpha_i \gamma_{jQ} - \alpha_j \gamma_{iQ} = 0 \qquad i,j = K,L,E \qquad (4.6)$$

These constraints follow directly by applying condition (4.3) to the translog function (3.16).

The price aggregator function h(·) is homothetic, and thus independent of the level of output. Since homotheticity of the aggregator function is a necessary and sufficient condition for the validity of the two-stage optimization, one may estimate equation (4.5) in stages. Excellent examples of this two-stage optimization procedure are Fuss (1977), and Pindyck (1979). In the first stage, by choosing an appropriate price aggregator function for energy, they have optimized the mix of fuels that make up the energy input; and then they have optimally chosen quantities of capital, labor, and energy. While the price aggregator functions chosen by Fuss (1977) and Pindyck (1970) are linear homogeneous and homothetic translog functions respectively, we may observe a less restrictive alternative.

The less restrictive alternative we may consider is the following formulation which may be quite useful in many practical situations. More specifically, we assume that the non-homothetic cost function (3.15) is weakly separable as:

$$C = H(h(W_K, W_L, W_E, Q, T), W_M, Q, T) \tag{4.7}$$

where the microfunction h(·) is non-homothetic in terms of the primary inputs, and includes the output as one of its arguments.

The translog cost function (3.16) will be approximately weakly separable as in equation (4.7) if the parameter restrictions

$$\alpha_i \gamma_{jM} - \alpha_j \gamma_{iM} = 0 \qquad i,j = K,L,E \tag{4.8}$$

hold.

This model specification may be proposed when data for output can be constructed, although the information on the complete list of inputs may not be available. This being the case, the assumption of homotheticity or homogeneity of the aggregator function is unnecessarily restrictive. A multi-stage procedure may also be adopted even in cases when a complete set of data is available. For example, when a large number of disaggregated inputs are involved, or when the data are of low quality, a researcher may be forced to consider a two-stage procedure in order to overcome these limitations to some extent. And hence, this less restrictive version of separability may be adopted instead as alternative. An example of this procedure is the work by Denny and Fuss (1980) on labor.

Another form of separability is that of weak homotheticity. As was defined by Shepard (1970), a function is said to be homothetic if it is a monotonic transformation of a linear homogeneous function. Homotheticity of the production function implies the separability of the dual cost function in the form given in equation (3.27). As Denney and May (1978) discuss, there are two special economic properties associated with this definition as follow:

(i) Taking partial derivatives of (3.27) with respect to the price of factor inputs, one realizes that the ratio between any two input demand equations does not depend upon the level of output, i.e., $\frac{x_i}{x_j} = \frac{\partial c/\partial W_i}{\partial c/\partial W_j}$ $i,j = K,L,E,M$;

and

(ii) The elasticity of cost with respect to output is a function of output only, i.e., $\frac{\partial \ln C}{\partial \ln Q} = g(Q)$.

In the light of this argument a production function is defined to be <u>weakly homothetic</u> if it satisfies the first property. The dual cost function corresponding to a weakly homothetic production function may take the form

$$C = H(h(\underline{W},T),Q,T) \tag{4.9}$$

where the factor price vector, $\underline{W}$, is weakly separable from output Q. As it is obvious from (4.9), only the first property holds, where the factor demand equations are derived as

$$x_i = \frac{\partial H}{\partial h} \cdot \frac{\partial h}{\partial W_i} \qquad i = K,L,E,M.$$

Therefore, the dual production function is no longer a positive monotonic transformation of a linear homogeneous production function (see McFadden (1978, Ch. 1)).

To have a weakly homothetic production structure, the translog cost function (3.16) must satisfy the following parameter restrictions:

$$\alpha_i \gamma_{jQ} - \alpha_j \gamma_{iQ} = 0 \qquad i = K,L,E,M.$$

Adoption of weak homotheticity is sufficient for properties such as linear expansion paths, and therefore the stronger assumption of homotheticity may be too restrictive to assume.

However, an unfortunate limitation of weak homotheticity is that the imposition of parameter restrictions for weak homotheticity in all inputs, along with the adding-up constraints ($\sum S_i = 1$), immediately implies homotheticity, i.e., $\gamma_{iQ} = 0$. To see this let us assume that there are only two inputs. The condition for weak homotheticity is $\alpha_1 \gamma_{2Q} - \alpha_2 \gamma_{1Q} = 0$. But imposing this condition along with the adding-up constraints of $\alpha_1 + \alpha_2 = 1$ and $\gamma_{1Q} + \gamma_{2Q} = 0$ implies that the weak homotheticity condition cannot hold unless $\gamma_{1Q} = \gamma_{2Q} = 0$. Therefore, we only consider weak homotheticity in primary inputs and energy. The cost function, thus, may be written

$$C = H(h(W_K, W_L, W_E, W_M, T), W_M, Q, T), \tag{4.9a}$$

and the parameter restrictions for this case are

$$\alpha_i \gamma_{jQ} - \alpha_j \gamma_{iQ} = 0 \qquad i,j = K,L,E. \tag{4.10a}$$

One important application of separability is in the derivation of the value-added functions. National income accountants have measured the output of an industry or a sector in terms of value-added by allocating the origins of national income to the services of labor and capital. The vast majority of theoretical and empirical studies of production by economists deal with capital and labor and the degree of substitutability between them only and refer to value-added rather than to output. Materials or "non-factor" inputs are eliminated from the production function and subtracted from its results.

The omission of particular inputs imposes certain separability restrictions on the production structure and therefore affects the possible functional form of the production function. Again, consider production function (1) which relates the total output Q of a sector to the services of the four aggregate inputs of capital (K), labor (L), energy (E), and materials (M). In the context of production function (1), it is possible to use real value-added to measure the output if K and L are homothetically weakly separable from E and M. This condition holds if and only if we can write

$$F(K,L,E,M,T) = G(g(K,L,T),E,M,T) \tag{4.11}$$

where the subfunction $g(\cdot)$ is homothetic and represents real value-added. This separability assumption places severe restrictions on the Allen partial elasticity of substitution between pairs of inputs; namely it implies that $\sigma_{KE} = \sigma_{LE}$ and $\sigma_{KM} = \sigma_{KE}$.

Berndt and Wood (1975), using a four-input translog cost function, have tested the value-added assumption, and have concluded that the condition necessary for a value-added specification do not seem to be satisfied for the U.S. manufacturing sector during the period 1947-1971. In carrying out their test Berndt and Wood (1975) have implicitly assumed that the translog function is an exact representation of the true function, and therefore the separability conditions outlined in their study are too restrictive.[2] This study departs from Berndt and Wood (1975) by examining the approximate weak separability conditions for the value-added specification, as was the case in previous tests.

If the production function is weakly separable as in equation (4.11), then the dual cost function will be weakly separable as

$$C = H(h(W_K,W_L,T),W_E,W_M,Q,T) \tag{4.12}$$

where $h(W_K, W_L, T)$ is a real value-added aggregate price. The condition for the validity of the value-added specification can be checked statistically by testing whether the parameter restrictions imposed by (4.12) on the translog cost function (3.16) are satisfied with our (KLEM) data. The translog cost function (3.16) will be approximately weakly separable as in equation (4.12) if

$$\alpha_i \gamma_{jk} - \alpha_j \gamma_{ik} = 0$$

and (4.13)

$$\alpha_i \gamma_{jQ} - \alpha_j \gamma_{iQ} = 0 \qquad i,j = K,L, \text{ and } k = E,M .$$

We may consider a real-value added aggregate price which is more general than the one in (4.12) by assuming a non-homothetic real value-added aggregate price, as we did for the previous case in equation (4.7). If this is the case the cost function may be written as:

$$C = H(h(W_K, W_L, Q, T), W_E, W_M, Q, T) \qquad (4.14)$$

where the primary input prices are now separable from only the price of energy and materials and not from output. What are the parameter restrictions on the non-homothetic translog cost function (4.16) such that it will be approximately weakly separable as in equation (4.14)? The conditions are:

$$\alpha_i \gamma_{jk} - \alpha_j \gamma_{ik} = 0 \qquad i,j = K,L \text{ and } k = E,M. \qquad (4.15)$$

There might be other kinds of separability consistent with our data which are worth checking; especially the ((K,E), (L,M)) weak separability is an attractive candidate. Berndt and Wood (1975) have tested the linear weak separability restrictions for ((K,E), (L,M)) separability, and were not able to reject these restrictions with their data; consequently, they could not reject the consistent aggregation of K and E (an index of "utilized capital" as they called it) and of L and M. Here we test two versions of this hypothesis with our data, departing from Berndt and Wood (1975) again by examining the approximate weak separability conditions which are less restrictive.

The dual cost function corresponding to ((K,E), (L,M)) weak separability may be written as

$$C = H(h(W_K, W_E, T), g(W_L, W_M, Q, T), Q, T) \tag{4.16}$$

where $h(\cdot)$ and $g(\cdot)$ are homothetic in input prices and independent of the level of output. For our non-homothetic translog cost function, inputs K and E will be approximately weakly separable from L and M if the following parameter restrictions hold:

$$\alpha_i \gamma_{jk} - \alpha_j \gamma_{ik} = 0 \quad i,j = K,E, \text{ and } k = L,M,Q;$$

and (4.17)

$$\alpha_i \gamma_{jk} - \alpha_j \gamma_{ik} = 0 \quad i,j = L,M \text{ and } k = K,E,Q \ .$$

Another version of ((K,E),(L,M)) weak separability is

$$C = H(h(W_K, W_E, Q, T), g(W_L, W_M, Q, T)) \qquad (4.18)$$

where h(·) and g(·) are non-homothetic functions in input prices[3]. The non-homothetic translog cost function (3.15) is approximately weakly separable as specified in equation (4.18) if the parameter restrictions implied by (4.18) are satisfied, i.e.,

$$\alpha_i \gamma_{jk} - \alpha_j \gamma_{ik} = 0, \quad i,j = K,E, \text{ and } k = L,M; \text{ and}$$
$$\alpha_i \gamma_{jk} - \alpha_j \gamma_{ik} = 0, \quad i,j = L,M, \text{ and } k = K,E\ . \qquad (4.19)$$

There is of course no reason to restrict our analysis to various possibilities of weak separability among factor inputs. The separability specification also permits us to analyze technological change. We earlier saw that a production function is defined to exhibit Hicks neutrality if it can be written in the following form

$$C = H(h(\underline{W}, Q), Q, T) \qquad (4.20)$$

The parameter restrictions implied by (4.20) on our non-homothetic translog cost function (3.16) are

$$\alpha_i \gamma_{jT} - \alpha_j \gamma_{iT} = 0 \quad i,j = K,L,E,M\ . \qquad (4.21)$$

An unfortunate limitation of this specification is that the parameter restrictions (4.21) cannot be imposed simultaneously with adding up constraints ($\Sigma S_i = 1$) without imposing extended Hicks neutrality. For example, if there are only two inputs, the condition for indirect Hicks neutrality is $\alpha_1\gamma_{2T} - \alpha_2\gamma_{1T} = 0$. This constraint, however, cannot hold simultaneously with the adding-up constraints $\alpha_1 + \alpha_2 = 1$ and $\gamma_{1T} + \gamma_{2T} = 0$ unless $\gamma_{1T} = \gamma_{2T} = 0$, which only holds for extended Hicks neutrality. Therefore, we adopt and test indirect Hicks neutrality for a subset of the inputs; namely we write (4.20) as

$$C = H(h(W_K, W_L, W_E, W_M, Q), W_M, Q, T) \tag{4.20a}$$

and the implied parameter restrictions as

$$\alpha_i\gamma_{jT} - \alpha_j\gamma_{iT} = 0 \qquad i,j = K,L,E. \tag{4.21a}$$

Finally, we check the condition for the validity of Harrod neutral technical change with our data. We saw earlier that a cost function exhibits indirect Harrod neutral technological change if it can be written in the following weakly separable form:

$$C = H(h(W_K, W_E, W_M, Q), W_L, Q, T) \tag{4.22}$$

The non-homothetic translog cost function (3.16) will be approximately weakly separable if the following parameter restrictions are satisfied:

$$\alpha_i\gamma_{jL} - \alpha_j\gamma_{iL} = 0$$

$$\alpha_i\gamma_{jT} - \alpha_j\gamma_{iT} = 0 \qquad i,j = K,E,M. \tag{4.23}$$

4.3 Statistical Results

4.3.A Manufacturing

We have statistically checked the conditions for the validity of the imposed parametric restrictions implied by the various types of separability specifications, which were discussed above. To perform these statistical tests we have estimated all eleven specifications, and have computed the likelihood ratio test statistics as twice the difference of the sample log-likelihood values of the unrestricted and restricted models. As is generally well known, the asymptotic distribution of this test statistic is χ^2 with degrees of freedom equal to the number of restrictions.

Of the eleven specifications we have tested, five specifications could not be rejected as the likelihood ratio test statistic for these models turned out to be small compared to the .05 (.01) χ^2 critical values. These specifications are the ((K,L,E),M) separability specified in equation (4.7), the value-added specification of equation (4.18), and the two versions of the ((K,L,E),M) weak separability specifications (see footnote 3), and finally one of the ((K,E),(L,M)) weak separability specifications specified in equation (4.18), i.e., $H(h(W_K, W_E, Q, T),\ g(W_L, W_M, Q, T))$.

The likelihood ratio test statistics we have obtained for these five models (in above order) are 0.282, 3.732, 4.076, 2.818, and 7.346; while the 0.05 χ^2 critical values (with two and three degrees of freedom) are 5.991 and 7.815

TABLE 4.1

ALTERNATIVE WEAK SEPARABILITY TESTS
U.S. MANUFACTURING, 1947-71

Type of Separability	Independent Parameter Restrictions	Likelihood Ratio Statistics: $-2 \ln L(\omega)/L(\Omega)$	Test Result
$[(W_K, W_L, W_E, T), W_M, Q, T]$	4	26.290	*
$[(W_K, W_L, W_E, Q, T), W_M, Q, T]$	2	.282	
$[(W_K, W_L, W_E, W_M, T), W_M, Q, T]$	2	19.506	*
$[(W_K, W_L, T), W_E, W_M, Q, T]$	3	42.370	*
$[(W_K, W_L, Q, T), W_E, W_M, Q, T]$	2	3.732	
$[(W_K, W_E, T), (W_L, W_M, T), Q, T]$	5	15.986	*
$[(W_K, W_E, T), W_L, W_M, Q, T]$	3	4.076	
$[(W_K, W_E, Q, T), (W_L, W_M, Q, T)]$	3	7.346	
$[(W_K, W_E, Q, T), W_L, W_M, Q, T]$	2	2.818	
$[(\underline{W}, Q), W_M, Q, T]$	2	14.168	*
$[(W_K, W_E, W_M), W_L, Q, T]$	4	25.504	*
$[(W_K, W_L, W_E), W_M]$	2	12.536	*
$[(W_K, W_L), W_E, W_M]$	2	40.426	*
$[(W_K, W_E), W_L, W_M]$	2	14.748	*
$[(W_K, W_E), (W_L, W_M)]$	3	24.176	*

* Weak separability hypothesis is rejected at .01 level of significance.

TABLE 4.2

ALTERNATIVE WEAK SEPARABILITY TESTS
U.S. NON-ENERGY MANUFACTURING, 1947-71

Type of Separability	Independent Parameter Restrictions	Likelihood Ratio Statistics $-2\ln L(\omega)/L(\Omega)$	Test Results
$[(W_K, W_L, W_E, T), W_M, Q, T]$	4	7.644	
$[(W_K, W_L, W_E, Q, T), W_M, Q, T]$	2	6.522	
$[(W_K, W_L, W_E, W_M, T), W_M, Q, T]$	2	3.184	
$[(W_K, W_L, T), W_E, W_M, Q, T]$	3	1.310	
$[(W_K, W_L, Q, T), W_E, W_M, Q, T]$	2	.018	
$[(W_K, W_E, T), (W_L, W_M, T), Q, T]$	5	11.920	
$[(W_K, W_E, T), W_L, W_M, Q, T]$	3	10.498	
$[(W_K, W_E, Q, T), (W_L, W_M, Q, T)]$	3	10.744	
$[(W_K, W_E, Q, T), W_L, W_M, Q, T]$	2	10.498	*
$[(\underline{W}, Q), W_M, Q, T]$	2	3.616	
$[(W_K, W_E, W_M), W_L, Q, T]$	4	4.722	
$[(W_K, W_L, W_E), W_M]$	2	15.138	*
$[(W_K, W_L), W_E, W_M]$	2	15.772	*
$[(W_K, W_E), W_L, W_M]$	2	22.968	*
$[(W_K, W_E), (W_L, W_M)]$	3	41.432	*

*Weak separability hypothesis is rejected at .01 level of significance.

respectively. These five specifications, therefore, cannot be rejected based on these test statistics. However, we are inclined to reject the first four specifications on the grounds that none of them will satisfy the concavity condition required for well-behavedness of the cost function. The ((K,E),(L,M)) weak separability specification of equation (4.18) is the only specification that our net output data and model specification (nonhomothetic-nonneutral) cannot reject, while at the same time the concavity conditions are satisfied for this model. However, the other version of this separability specification, i.e., $H(h(W_K,W_E,T),\ g(W_L,W_M,T),\ Q,T)$ is rejected at .01 level of significance. It is interesting to note that the ((K,E),(L,M)) separability specification was the only specification that Berndt and Wood (1975) could not reject with their data and model specification (CRS - Hicks neutral technical change), and that the cost function was also well-behaved.[4]

One important result is the "rejection" of the value-added specifications. Many empirical studies of investment demand and capital-labor substitutability in the U.S. manufacturing sector have in fact assumed a priori the value-added specification. One, therefore, is inclined to view the results of such studies as unreliable due to the rejection of value-added specification.

4.3.B Non-manufacturing Energy

As for the non-energy manufacturing sector, we have repeated the same number of statistical tests to check whether any of the parametric separability conditions imposed on the translog cost function (16) are satisfied. Of the eleven separability specifications we have tested for the non-energy manufacturing sector, we could reject only one at the 0.01 level of significance, where the general "nonhomothetic-nonneutral" specification was chosen as the unrestricted model. The rejected model is the weak separability specification of $H(h(W_K, W_E, Q, T), W_L, W_M, Q, T)$[5].

This "nonrejection" situation here is quite similar to what we experienced in the previous chapter when we were estimating and testing for the validity of various model specifications for the nonenergy manufacturing sector. There we concluded that, for the nonenergy manufacturing sector, the characterization of the production structure by a nonhomothetic-nonneutral cost function was unnecessary, and that the adoption of a homogeneous-Hicks neutral specification was justifiable.

Maintaining the "homogeneous-Hicks neutral" specification as the unrestricted specification, we have tested for the validity of three well-known separability specifications; i.e., ((K,L,E),M) weak separability; the value-added specification, ((K,L),E,M); and ((K,E),(L,M)) weak separability. These specifications have frequently been utilized by many researchers in a number of empirical studies. For example, ((K,L,E),M) weak separability has been assumed by Griffin and Gregory (1976) and Magnus (1979) in their studies, due to lack of reliable data on the materials input price. The value-added specification has also been employed in many empirical studies of investment demand and capital-labor substitutability in U.S. manufacturing. And finally, the ((K,E),(L,M)) weak separability is the separability that Berndt and Wood (1975) were not able to reject in their study of U.S. manufacturing.

All these specifications, however, are rejected at the 0.01 level of significance with our data for the nonenergy manufacturing sector. The likelihood ratio test statistics for these specifications (in the same order mentioned above) are 15.138, 22.968, and 41.432, while the 0.01 χ^2 critical values with two and three degrees of freedom are 9.210 and 11.345 respectively.

CHAPTER IV

FOOTNOTES

[1]There is a distinction between the translog as an exact representation of a functional form and as an approximation to a functional form. Tests of the separability hypothesis based on an exact interpretation of the translog are restrictive. Specifically, the separability constraints imply that the separable form of a translog function must be either a Cobb-Douglas function of translog subaggragates or a translog function of Cobb-Douglas subaggragates. To avoid this restriction in carrying out our tests, we have chosen the more general interpretation of the translog as a second-order approximation to some unknown arbitrary cost function.

[2]See footnote 1 above.

[3]Two less restrictive versions of (4.16) and (4.18) are $C = H(h(W_K, W_E, T), W_L W_M, Q, T)$ and $C = H(h(W_K, W_E, Q, T), W_L, W_M, Q, T)$ respectively. The parameter restrictions for the former specification are: $\alpha_i \gamma_{jk} - \alpha_j \gamma_{ik} = 0$, and $\alpha_i \gamma_{jQ} - \alpha_j \gamma_{iQ} = 0$; $i,j=K,E$, and $k=L,M$. While for the latter specification the following implied parametric restrictions must hold: $\alpha_i \gamma_{jk} - \alpha_j \gamma_{ik} = 0$; $i,j=K,E$, and $k=L,M$.

[4]For the record, however, we have also checked the validity of the separability conditions for three commonly utilized specifications using the same model specification (CRS-Hicks neutral technical change) employed by Berndt and Wood (1975). The specifications tested are: ((K,L,E),M) weak separability, the value-added specification, and ((K,E),(L,M)) weak separability. All these three specifications have been rejected at the .01 level of significance when each was tested against the maintained "CRS-Hicks neutral technical change" specification.

[5]In addition to this specification, there are four other separability specifications which our data can reject, although at a lower level of significance. Specifically, the specification numbers 2, 6, 7, and 8 in Table (4.2) are rejected (the two former ones at the .05, and the two latter ones at the .025 level of significance).

CHAPTER V

SUMMARY AND CONCLUSIONS

The oil crisis of 1973 and the subsequent continuing increases in the price of energy have led to increased interest in the characteristics of energy demand and substitution elasticities between energy and nonenergy inputs. The U.S. manufacturing industries that account for approximately one-fourth of aggregate energy consumption have attracted a great deal of attention as a potential sourse of reductions in energy demand.

In recent years, a growing number of econometric studies have focused on the estimation of the Hicks-Allen substitution elasticities among energy and nonenergy inputs in the manufacturing process. The information regarding these elasticities provided by empirical studies is very essential in deriving policy implication of increasingly scarce and higher priced energy inputs. A review of the literature, however, indicates that these estimates have not always been consistent. One interesting result is the contradictory evidence concerning substitution possibilities between capital and energy. It has not been, however, the purpose of this study to reconcile these differences concerning factor substitution. This study rather has dealt with another subtle issue ignored in past studies of manufacturing.

The primary objective of this study has been to examine past studies of manufacturing energy demand and to employ an alternative model for the estimation of industrial conditional factor demand functions. More specifically, in past econometric studies of manufacturing sectors the researchers have specified a flexible cost function over four aggregate inputs of capital services (K), labor services (L), energy (E), and materials (M). In these studies, output, materials and energy have been measured as "gross" magnitudes in the sense that they contained intra-industry shipments of traded intermediate products (energy and materials). But, as was revealed in table (2.1), a considerable portion of purchased materials and energy inputs have been produced within the same sector. The prices of these internally produced and traded materials and energy must be considered, properly, as endogenous variables in a competitive industry general equilibrium model, as shown by Samuelson (1953). In fact, the prices of these internally produced materials and energy respond to any change in the prices of primary inputs such as labor, capital, primary energy and materials. The application of Shephard's Lemma in these models to derive industry conditional factor demand functions is not proper, as exogeneity of input prices is a necessary condition for such an application. Therefore, the source of error in past studies has consisted of a model specification over "gross" industry output which has forced the researchers to include the internally produced products in their aggregate "energy" and "materials" inputs. The prices of these inputs are not exogeneous, and the application of Shephard's Lemma by these authors is inappropriate.

In this study, we have considered an alternative model for aggregate cost and conditional factor demand functions, and have estimated the production structure of the U.S. manufacturing sector as well as the non-energy manufacturing sector for the 1947-71 period separately. In particular, we have specified our model, properly, conditional upon the level of industry output net of internally produced materials and energy, (i.e., the level of industry deliveries to the balance of the economy). Such a model specification presents a proper context for energy policy discussions, since we are commonly interested in reducing the energy intensity of a given level of a sector's output delivered to final demand, i.e., the net output.

Having done that, this study has then addressed questions concerning factor demand, and possibilities for substitution among energy and non-energy inputs. To accomplish this we have estimated the production relationship for the U.S. manufacturing and non-energy manufacturing sectors, via the dual cost function, because of its several econometric and theoretical advantages. For purposes of estimation a translog cost function is specified as a quadratic approximation to the underlying production process. The relevant inputs are capital services (K), labor services (L), primary energy (E), and primary materials (M). Estimation of a translog cost function has made it possible to calculate the extent of factor substitution without imposing any a priori constraint on the values of AES. Moreover, the use of a flexible functional form (such as the translog) has made it possible to test a number of hypotheses concerning other characteristids of the production function, such as C.R.S., homogeneity, homotheticity, and the rate and bias of technological change. Here we briefly summerize our findings concerning the estimates of price and substitution elasticities which are the essence of this study.

Among the major findings of this study concerning the input price elasticities and factor substitution in the manufacturing sector are that: (i) the η_{ii} and σ_{ii} (i=K,L,E,M) (i = K,L,E,M) all have the correct negative sign indicating satisfaction of the concavity condition; (ii) the controversial energy-capital complementarity relationship has persisted in this study, but is substantially weaker than the estimates obtained by Berndt and Wood (1975); (iii) energy and labor are slightly substitutable; (iv) our net model has displayed a considerably smaller magnitude for factor price and substitution elasticities then the gross specification employed by Berndt and Wood and Berndt and Khaled (1979). In particular, the important own-price elasticity of energy, η_{EE} is found to be , substantially, of smaller magnitude (about -.13) compared to the robust estimate of about -.50 reported by Berndt and Wood, Hudson and Jorgenson (1974), and Berndt and Khaled. This means that the response of energy demand to a changing energy price is likely to be more inelastic in the net output model than in the gross output model. Our inelastic result may indicate that after an energy price increase, each firm finds itself facing higher priced produced inputs, including intermediate energy products as well as higher priced energy inputs. Therefore, the possibilities of substituting away from energy would be smaller than if only energy increased in price. For the manufacturing sector we conclude that our findings on the AES and input price elasticities indicate that respecification of our model over the net industry output and primary factors of production has resulted in a fall in the absolute value of these elasticities. We have also observed that the technological possibilities for substitution between energy and non-energy inputs are present, but are very limited. Specifically, energy appeared to be complement with capital, while it has been a rather weak substitute for labor and materials.

Relevant policy implications of our empirical results suggest that, since σ_{EE} and σ_{KE} are negative and σ_{LE} is positive, higher priced energy - ceteris paribus - dampens the demand for energy and new plant and equipment, while at the same time it will stimulate employment. This leads, therefore, to an increase in the labor intensiveness of the production process, and a decline in energy and capital intensiveness. Furthermore, investment tax credits, and accelerated depreciation allowances, which decrease the price of capital services, result in increased demand for capital and energy. This requires a careful handling of these policy instruments since a particular energy policy may contradict investment policy.

As for the non-energy manufacturing sector our estimates of the AES and η_{ij} indicate again that the technological possibilities for substitution among energy and non-energy inputs are present. It is unfortunate, however, that we cannot compare our results with similar studies of the non-energy manufacturing sector as we are not aware of any such studies. The estimated input price and substitution elasticities obtained for non-energy manufacturing reveals that (i) all of the own-Allen partial elasticities of substitution and own-price elasticities have the correct negative sign required for stability; (ii) labor, energy, and materials are quite responsive to a change in their own prices, while capital appears to be less responsive; (iii) capital and energy are again complementary but substantially less so than what we have found for the manufacturing sector; (iv) labor and energy are quite substitutable; (v) capital and labor are slightly substitutable, although this substitutability appears to be stronger than what we found in manufacturing sector; and (vi) the important own-price elasticity of energy demand is more elastic than the Berndt-Wood estimates and the estimate we obtained for the manufacturing sector. For the nonenergy manufacturing sector, therefore, we have found that, except for labor, the technological possibilities for substitution between energy and nonenergy inputs are somewhat limited. For a given level of net output, an increase in the price of energy will lead to small increases in the quantities of labor and materials demanded and to very small reductions in the quantity of capital demanded.

We also have tested a number of nypotheses concerning other characteristics of the production process for the manufacturing and nonenergy manufacturing sectors. For the manufacturing sector our empirical results imply that homotheticity, homogeneity, and C.R.S. must all be rejected. Neutrality of technical change has also been rejected. The estimation of Hicks biases obtained for our non-homothetic-nonneutral model indicates that over the 1947-71 period technical change has been labor-saving; capital, energy, and material-using. The labor-saving and other factor-using biased technical changes are consistent with input price data, since labor input price had the largest rate of growth over the 1947-71 period. Our result is in contrast with that of Berndt and Khaled (1979) since their "gross" output model displays technical change that is capital and energy-using, approximately labor neutral, and intermediate material-saving.

As for the nonenergy manufacturing sector, we have also done several tests concerning the nature of technical change and the characteristics of the production process for the 1947-71 period. Of the nineteen specifications tested, our nonenergy manufacturing data could only reject three of them at the .01 level of significance. These specifications are CRS, CRS-Hicks neutral technical change, and unitary elasticity of substitution. All other models could be accepted even at the .05 level of significance. On balance we have concluded that for the non-energy sector, the characterization of the production structure by a function exhibiting homogeneity and Hicks neutrality was justifiable.

This study has also addressed the question of economies of scale. For this purpose the index of scale economies suggested by Christensen and Green (1976) was calculated. This index was defined as one minus the cost elasticity along an output ray. The translog cost function allows scale economies to vary, in general, with the level of output, factor prices, and the state of technology. The estimates of scale economies for both sectors, reported in Tables (3.9) and (3.10), show substantial positive and statistically significant scale economies for all selected specifications. The estimates of scale economies are quite sensitive to the model specification. For the manufacturing sector we have obtained quite a large estimate of .624 for our (rejected) homogeneous specification (Model 6), while our estimate for Model 12 (nonhomothetic-no technical change) is .260. This latter estimate is remarkably close to the estimate of .240 obtained by Nadiri and Schankerman (1981) for the U.S. manufacturing sector. For our unrestricted net output model the estimate of scale economies is .345. Berndt and Khaled obtained an estimate of .199 for their unrestricted gross output model, which they judged to be somewhat large compared to the estimates obtained in other studies of returns to scale involving value added.

For the nonenergy manufacturing sector we have obtained the high estimate of .587 for scale economies for the homogeneous model (Model 6), while the homothetic Hicks neutral specification (Model 5) has rendered a relatively smaller estimate of .336. Moreover, the imposition of the Hicks neutrality restriction (γ_{it}=0) on the nonhomothetic-nonneutral model has produced a high estimate of scale economies, while for the homothetic and homogeneous models the imposition of γ_{it}=0 has rendered relatively low estimates.

We also have estimated the rate of technical advancement measured by $\frac{\partial \ln C}{\partial T}$ for both sectors over the 1947-71 period. The negative of this measure is defined as "the dual rate of total cost diminution" by Ohta (1974) and was utilized by Berndt and Khaled (1979). Our empirical results indicate that the null hypothesis of zero rate of technical advancement would not be rejected while the null hypothesis of no scale economies would be rejected. Consequently, we are persuaded to conclude that, in both sectors over the 1947-71 period, the source of growth has been primarily the utilization of economies of scale.

Finally, this study makes possible tests of the hypothesis of weak separability among inputs. Of the eleven specifications tested for the manufacturing sector, we could only accept the [(K,E),(L,M)] weak separability of equation (4.18), where the nonhomothetic nonneutral specification was maintained as the unrestricted model. It is interesting to note that the [(K,E),(L,M)] weak separability specification was the only one that Berndt and Wood were not able to reject with their data and model specification (CRS-Hicks neutral technical change). For the record, however, when the null hypothesis of [(K,E), (L,M)] weak separability was tested against the same model specification employed by Berndt and Wood as the unrestricted model, the null hypothesis was rejected.

For the nonenergy manufacturing sector we have maintained the "homogeneous-Hicks neutral" specification as our unrestricted specification, and we have tested for the validity of three well-inown weak separability specifications, i.e., the [(K,L,E,M),M)] weak separability, the value-added specification [(K,L),E,M], and the [(K,E),(L,M)] weak separability. These specifications have repeatedly been utilized by many researchers in a number of empirical studies. For example, the [(K,L,E),M] weak separability has been assumed by Griffin and Gregory (1976) and Magnus (1979) in their studies due to lack of reliable data on the materials input price; the value-added specification has also been employed in many empirical studies of investment demand and K-L substitutability in U.S. manufacturing; finally the [(K,E),(L,M)] weak separability is the form of separability found by Berndt and Wood (1975). All these specifications, however, were rejected.

One important result is the rejection of the value-added specification for both sectors. Many empirical studies of investment demand and capital-labor substitutability in the U.S. manufacturing sector have in fact assumed, a priori, the value-added specification. One, therefore, is inclined to view the results of such studies as unreliable due to rejection of the value-added specification.

One important requirement of the net model framework, utilized in this study, is the necessity of obtaining sufficiently detailed energy and materials data that enable us to partition these inputs into "internal" and "external" (primary) portions. This becomes a major deficiency of the "net" output model since recourse must be made to national input-output tables, which are available infrequently and with a time lag; however gross model estimation can be performed with readily available value-of-shipment data. But as this study has shown, estimation performed by the gross output data may lead to biased estimates of factor substitution for an industry.

This study has shed light on a number of issues such as factor demand and factor substitution elasticities, technical change, scale economies, characteristics of the production process, and weak separability among inputs. At the same time it suggests areas for further study. While this study has examined and estimated an alternative model conditional upon the industry's net output (wherein the rate of inter-firm flow of intermediate products is considered, a maintained hypothesis of the "net" model is that the primary input prices are exogenous to the industry as a whole. Since the use of Shephard's Lemma in this context is dependent upon the assumed exogenity of primary input prices, it is thus highly desirable to test the validity of this hypothesis (see Geweke (1978) for description of such a test). As far as our data is concerned, it unfortunately ends in 1971. It would of course seem desirable to find out whether our 1947-71 data yield results consistent with the post-1973 OPEC price increases. There is need for further work on reconciling substitution possibilities between energy and capital from time series analysis with those of cross-section studies, employing the "net" model instead of the gross model as the framework of study. For this purpose, the utilization of post-1971 data, and, perhaps more significantly, pooled cross-section and time series data would be useful and provide us with more accurate estimates of factor demand and substitution elasticities, returns to scale, and the rate and bias of technical change. Since the static profit- maximization model is an inadequate description of the actual decision process, a possible extension of this study would consist of the specification and estimation of a dynamic factor demand model, which would be more general than our instantaneous adjustment assumption. Another possible research project would consist of using the "net" model framework for specification and estimation of production relations for other sectors of the U.S. economy.

APPENDICES

APPENDIX A

DATA CONSTRUCTION AND SOURCES

Annual price and quantity indices for inputs of capital and labor services are obtained from Berndt and Wood (1975). The price and quantity indices for capital services have been constructed from nonresidential structures and producers' durable equipment, following the procedure outlined by Christensen and Jorgenson (1970). To construct a quantity index of capital input, it is necessary to start with the measurement of the capital stock corresponding to each type of capital service. For the purpose of such measurement a perpetual inventory method is chosen to estimate the level of capital stock corresponding to equipment and structures. In discrete time the perpetual inventory formula may be written in the form

$$K_{it} = I_{it} + (1 - \bar{d}_i) K_{i,t-1} \qquad i = e,s; \qquad (A.1)$$

where K_{it} denotes end-of-period capital stock, I_{it} the quantity of investment taking place in the period, $\bar{d}_i$ the average rate of replacement; and the subscripts e, s, and t refer to producer durable equipment, nonresidential structures, and the time period respectively. To implement this perpetual inventory method it suffices to have data on investment in constant dollars, a capital benchmark, and an average rate of replacement[1] for each type of capital stock. Then an aggregate quantity index of capital services is computed by Divisia[2] aggregation of the two capital services from nonresidential structures and producers' durable equipment by the following discrete approximation to the continuous Divisia formula,

$$\ln K_t - \ln K_{t-1} = \bar{S}_{it} (\ln K_{it} - \ln K_{i,t-1}), \quad i=e,s$$

$$\text{where } \bar{S}_{it} = \frac{P_{it}K_{it}}{P_{et}K_{et}+P_{st}K_{st}} . \qquad (A.2)$$

As for the capital service price, Berndt and Wood (1975), following Christensen and Jorgenson (1969), have constructed and used the rental price of capital services from nonresidential structures and producers' durable equipment. The imputation of rental prices for capital services is based on the correspondence between the acquisition price of an asset and the present value of the services it provides, where in competitive equilibrium these two prices are equal.

To make this correspondence explicit according to the perpetual inventory method, it is assumed that the quantity of service flow from an asset declines geometrically over time. The service price of an asset at time t is obtained as

$$P_t = q_{t-1} \cdot r_t + q_t \cdot d - (q_t - q_{t-1}) \qquad (A.3)$$

where q_t denotes the acquisition price of the asset at time t, r the nominal rate of return, and d the rate of replacement. In fact the service price is stated as the sum of the cost of capital, $q_{t-1} \cdot r_t$, the current cost of replacement,

$q_t \cdot d$, and the cost of capital gain or loss on the value of the asset, $(q_t - q_{t-1})$.

The service price may be formulated as the sum of the real cost of capital, $q_{t-1} \cdot r_t^*$, and the current cost of replacement $q_t \cdot d$; where $r_t^* = r_t - (q_t - q_{t-1})/q_{t-1}$ is the real rate of return. Since the U.S. manufacturing sector is overwhelmingly incorporated enterprises, Berndt and Wood have used the corporate service price formulas developed by Christensen and Jorgenson (1969).

The tax treatment of property income generated by each asset alters the service prices, and therefore the tax structure for property income generated by that asset must be incorporated in these service prices. For the corporate sector in U.S. manufacturing these service price formulas, derived by Christensen and Jorgenson (1970), are:

$$P_{et} = \left(\frac{1-\mu_t Z_{et} - k_t + y_t}{1-\mu_t}\right)\left(q_{e,t-1}\ r_t + q_{et}\bar{d}_e - (q_{et} - q_{e,t-1})\right) + q_{et}\ \theta_t \tag{A.4}$$

$$P_{st} = \left(\frac{1-\mu_t Z_{st}}{1-\mu_t}\right)\left(q_{s,t-1}\ r_t + q_{st}\bar{d}_s - (q_{st} - q_{s,t-1})\right) + q_{st}\ \theta_t \tag{A.5}$$

where r is the nominal after-tax rate of return on corporate property, $\bar{d}_e$ and $\bar{d}_s$ are the average rate of replacement for equipment and structures respectively, μ is the effective corporate profit tax rate, Z_{et} and Z_{st} refer to the present value of depreciation deductions for tax purposes on a dollar's investment in equipment or structures over the life time of investment, k is the investment tax credit, y is equal to $k\mu Z_e$ for 1962 and 1963 and zero for all other years, q_e and q_s are price indices of new equipment and structures, and θ is the tax rate on corporate property in U.S. manufacturing.

Having computed the service price measures of P_{et} and P_{st}, an aggregate service price of capital can be constructed by employing the discrete approximation to the Divisia index given by the following formula:

$$\ln P_{kt} - \ln P_{k,t-1} = \sum_i \bar{S}_{it}(\ln P_{it} - \ln P_{i,t-1}) \quad i=e,s \tag{A.6}$$

where $\bar{S}_{it} = \dfrac{P_{it}K_{it}}{P_{et}K_{et}+P_{st}K_{st}}$ $\quad$ i=e,s.

In practice, instead of constructing a Divisia price index for capital by (A.6), one may compute an <u>implicit</u> Divisia price index as $P_{kt} = (P_{et}K_{et}+P_{st}K_{st})/K_t$, where K_t is the Divisia quantity index obtained from (A.2). Or alternatively, given a Divisia price index P_{kt} by (A.6), one may construct an <u>implicit</u> Divisia quantity index as $K_t = (P_{et}K_{et}+P_{st}K_{st})/P_{st}$. The empirical results based on either alternative, however, remain basically unchanged.

The construction of a measure of a labor service price index, W_L, starts with constructing a measure of labor service, L. The quantity index for labor services has been constructed as a Divisia index of production and non-production[3] labor man-hours adjusted for quality changes using the educational attainment index of Christensen and Jorgenson (1970).

A man-hour index is traditionally measured as the product of the number of workers employed and the average hours per worker worked. Therefore, a man-hour series for production (non-production) workers may be constructed as the product of the number of production (non-production) workers and the average hour per worker. These man-hour series has been constructed for production and non-production workers based on data provided by the U.S. Bureau of Labor Statistics - the two series which reflect changes in the average hour worked by production and non-production labor as well as change in production and non-production persons engaged. These man-hours estimates, however, do not reflect changes in the quality of labor.

Quality adjustment may be achieved by incorporating an index of educational attainment constructed by Christensen and Jorgenson (1970). Multiplication of the production and non-production man-hours series by the Christensen and Jorgenson educational index results in a quantity series for production and non-production labor which takes account of changes in average hours worked, changes in workers engaged, and changes in the quality of labor as measured by changes in educational attainment.

Next, an aggregate quantity index of labor services may be constructed by Divisia aggregation of the two adjusted man-hours series. Total compensation of employees in U.S. manufacturing, which is the sum of wages, salaries and supplements to wages and salaries, is obtained from the U.S. Department of Commerce, Office of Business Economics. Total compensation is also adjusted for labor compensation to proprietors, which has been imputed on the assumption that proprietors, on the average, earn the same compensation as the average compensation of all labor types in U.S. manufacturing.

We then construct annual price and quantity indices for "primary" energy and intermediate materials in U.S. manufacturing, 1947-71. A rich source for this purpose is the annual interindustry flow tables described in Faucett (1973). These tables have been constructed on the basis of data from various sources such as the annual Bureau of Mines and Minerals Yearbook, the Census of Mineral Industries (1954, 1958, 1963, and 1967), the census of Manufacturers (1947, 1954, 1958, 1963, and 1967), the U.S. Department of Commerce Input Output Tables (1947, 1958, 1963), and the Annual Surveys of Manufacturers. The energy input-output matrices presented in Faucett measure flows of goods and services from 25 producing sectors of the economy (represented by the rows of the tables) to 15 consuming sectors (represented by the columns)[4] in current as well as constant dollars.

The producing sectors are: (1) Agriculture, non-fuel mining, and construction, (2) Manufacturing excluding petroleum products, (3) Transportation, (4) Communications, trade, and services, (5) Coal mining, (6) Crude petroleum, (7) Natural gas, (8) Motor gasoline, (9) Aviation gasoline, (10) Kerosine, (11) Distillat fuel oil, (13) Jet fuel, (14) Petroleum coke, (15) Still gas, (16) Liquified gases, (17) Other petroleum and coal fuels, (18) Other refined petroleum products, (19) Asphalt and related products, (20) Lubricating oil and grease, (21) Electric utilities, (22) Gas utilities, (23) Water and sanitary services, (24) Imports, and (25) Value added.

The consuming sectors consist of ten industrial categories and five categories of final demand. The ten industrial categories which we focus on in our data construction are enumerated below. The first five industrial categories of the consuming sector have the same title as the first five categories of the producing sector. The remaining five categories are: (6) Crude petroleum and natural gas, (7) Petroleum refining and related industries, (8) Electric utilities, (9) Gas utilities, and (10) Water and sanitary services.

There is harmony between the first 24 producing sectors and the ten industrial consuming sectors. Producing sector Nos. (6), Crude petroleum, and (7), Natural gas are a disaggregation of consuming sector No. (6), i.e., Crude petroleum and natural gas. Likewise, the detailed petroleum products of producing sectors (8) through (20) serve as a breakdown of the output of consuming sector No. (7), Petroleum refining and related industries. The U.S.

manufacturing sector, as has been conventionally defined, is the row sum of column (2), Nonenergy manufacturing, and column (7), Petroleum refining and related industries.

Looking at the input-output tables described in Faucett reveals that the manufacturing sector purchases "energy" and "materials" inputs from two sources. First, "energy" and "materials" inputs purchased from sectors other than the manufacturing (for example: transportation, communication services, crude petroleum, electricity). Second, those "energy" and "materials" inputs produced within manufacturing sector (for example: plastic, steel, glass, kerosin, jet fuel). This latter portion of "energy" and "materials" inputs is what constitutes our traded intermediate products such that their prices must be considered as endogenous variables; the former portion constitutes our primary "energy" and "materials" inputs and their prices can be taken as exogenous variables.

Based on tables described in Faucett we construct annual price indices of "primary" energy input as the Divisia price indices of coal, crude petroleum and natural gas, electricity, and gas purchased by firms in the U.S. manufacturing sector. Note that in constructing the Divisia price index for primary energy we only have taken into account those energy inputs (products) produced by sectors other than U.S. manufacturing; we have netted out the trade intermediate energy products produced within manufacturing sectors.[5]

Similarly we construct annual price indices of "primary" materials input purchased by establishment in the U.S. manufacturing as the Divisia price indices of non energy intermediate products (materials) produced by the following sectors: agriculture, non-fuel mining, and construction; transportation; communication, trade, and services; water and sanitary services; and imports. Note that we have constructed "primary" materials price indices having taken account of only those intermediate materials products (inputs) produced by sectors other than the U.S. manufacturing. The additional data needed for the manufacturing sector to implement the estimation are total input cost (in current dollars) and net aggregate output quantity series. These data series are constructed as total cost and gross output quantity, net of traded intermediate products, from input-output tables given in Faucett (1973).

As for the U.S. nonenergy manufacturing sector, i.e., sector (2), energy products (inputs) are entirely purchased from firms outside this sector, and thus, may entirely be considered as "primary" products (inputs) - primary in the sense of being produced outside of the sector in question - and therefore energy input prices may be taken as exogenous. While the nonenergy manufacturing sector purchases the materials input from two separate sources, it buys a portion of its materials input from firms outside of this sector, and the rest from firms within the sector. It is the former portion which constitutes "primary" materials input and its price may be considered exogenous.

The data series for prices of capital, labor, and energy have been obtained from a study done by Hudson and Jorgenson (1974), while data series for prices of materials, total input cost, and net aggregate output for nonenergy manufacturing is constructed from input-output tables given in Faucett (1973). Specifically, we construct annual price indices of "primary" materials input purchased by firms in the U.S. nonenergy manufacturing as the Divisia price indices of nonenergy intermediate products (materials input) produced by other sectors, namely: agriculture, non-fuel mining, and construction; transportation; communication, trade, and services; water and sanitary services; and imports.

TABLE A.1

TOTAL NET OUTPUT, TOTAL COST, AND COST SHARES OF CAPITAL, LABOR, NET ENERGY, AND NET MATERIALS INPUTS - U.S. MANUFACTURING, 1947-71

			Cost Shares			
Year	Total Net Output*	Total Input Cost**	Capital	Labor	Net Energy	Net Materials
1947	124.79300	111.14816	.83796160E-01	.40572306	.53952343E-01	.45452844
1948	121.85777	119.21338	.89373152E-01	.42583276	.70394784E-01	.41439980
1949	115.61305	113.70381	.75496576E-01	.42507427	.69047816E-01	.43038134
1950	139.82366	131.79132	.83962630E-01	.41710468	.64549014E-01	.43438368
1951	148.12859	148.40577	.86904091E-01	.43955637	.63467882E-01	.41007165
1952	151.35090	153.90912	.84547314E-01	.45912324	.62946237E-01	.39408321
1953	161.03758	162.95110	.84479607E-01	.47743250	.64688120E-01	.37139977
1954	155.74319	163.65639	.94500755E-01	.45559929	.64959274E-01	.38494068
1955	173.22500	178.91788	.90781213E-01	.45692750	.63476047E-01	.38881524
1956	176.49605	190.84256	.79197675E-01	.46238782	.63272049E-01	.39514246
1957	178.41709	199.77639	.85312376E-01	.46078515	.67085003E-01	.38681747
1958	171.83074	200.47443	.97007773E-01	.44001048	.64257572E-01	.39872417
1959	188.04831	216.92071	.10219958	.45114875	.61169816E-01	.38548186
1960	192.64856	223.59531	.94807927E-01	.45435077	.60779450E-01	.39006185
1961	194.81848	227.04338	.95200057E-01	.44897076	.60776931E-01	.39505225
1962	209.98934	239.65437	.90928702E-01	.46100102	.59932143E-01	.38813813
1963	218.13526	248.24839	.92998186E-01	.46437659	.59956079E-01	.38266915
1964	232.33839	263.31390	.89813078E-01	.46690610	.58094162E-01	.38518666
1965	250.02511	286.24288	.90715114E-01	.46454369	.54879968E-01	.38986123
1966	267.23054	313.64725	.90746812E-01	.47140144	.53037289E-01	.38481446
1967	272.27972	329.54338	.89346107E-01	.47022022	.53182680E-01	.38725099
1968	287.56365	358.94370	.93914557E-01	.47108964	.51562404E-01	.38343340
1969	297.58576	392.88928	.86811594E-01	.46594610	.50950741E-01	.39639157
1970	290.17252	404.19545	.81057663E-01	.45896684	.53286103E-01	.40668939
1971	294.65099	425.21431	.72369357E-01	.44745160	.55301525E-01	.42487751

* Billions of 1947 U.S. dollars.

** Billions of current U.S. dollars.

TABLE A.2

DIVISIA PRICE INDEXES* OF CAPITAL, LABOR, NET ENERGY, AND NET MATERIALS INPUTS - U.S. MANUFACTURING
1947-71

Year	W_K	W_L	W_E	W_M
1947	.76477156	.57660151	.71737357	.83078578
1948	.76683645	.66572681	.88222603	.85407191
1949	.56876826	.66645909	.81029988	.85748226
1950	.70739075	.71230468	.88439311	.91427722
1951	.80206947	.77140057	.89611520	.98509179
1952	.76281375	.79541602	.90402184	1.0067070
1953	.76976552	.82718100	.94692889	.99081923
1954	.83174261	.83815949	.95142633	.99265978
1955	.84365775	.87136020	.94823203	1.0251348
1956	.76175836	.91210287	.95203265	1.0535201
1957	.81311271	.94932249	1.0310870	1.0298797
1958	.88422123	.96516750	1.0206081	1.0535356
1959	1.0000000	1.0000000	1.0000000	1.0000000
1960	.95912276	1.0279652	.99985181	1.0633788
1961	.96612062	1.0492821	1.0087378	1.0819354
1962	.96762722	1.0870726	1.0159240	1.0894829
1963	1.0117469	1.1150262	1.0082460	1.0425924
1964	1.0156013	1.1589575	1.0084162	1.1285708
1965	1.0757200	1.1851410	1.0010161	1.1655857
1966	1.1096835	1.2307040	1.0069533	1.2226400
1967	1.0601034	1.2720752	1.0186071	1.2459966
1968	1.1464032	1.3484922	1.0350763	1.2609388
1969	1.1035899	1.4208153	1.0631259	1.4613036
1970	1.0130470	1.5022314	1.1170896	1.4829606
1971	.91907952	1.5915643	1.1769648	1.4732506

* Divisia price indexes have been normalized to equal unity at the midpoint of the sample so as to maximize the "goodness" of the translog quadratic approximation.

TABLE A.3

TOTAL NET OUTPUT, TOTAL COST, AND COST SHARES OF CAPITAL, LABOR, NET ENERGY, AND NET MATERIALS INPUTS
U.S. NONENERGY MANUFACTURING, 1947-71

Year	Total Net Output*	Total Input Cost**	Cost Shares			
			Capital	Labor	Net Energy	Net Materials
1947	118.44700	115.25480	.14688672	.39816930	.29689870E-01	.42495410
1948	116.22200	122.91040	.16216691	.41426275	.30959138E-01	.39261120
1949	115.66600	120.10510	.16561578	.40397868	.32504032E-01	.39790150
1950	127.36000	139.98160	.18046943	.39101711	.31661304E-01	.39685216
1951	136.55500	158.77500	.18695890	.41098725	.30622579E-01	.37143127
1952	141.34500	162.07550	.17167925	.43729620	.30772695E-01	.36025186
1953	149.98900	171.76440	.16837016	.46133425	.30306629E-01	.33998896
1954	144.76800	169.14160	.15951132	.45141704	.30307151E-01	.35876449
1955	160.18100	189.49200	.17716843	.44210837	.29257172E-01	.35146602
1956	162.03700	201.61020	.16199577	.45034825	.30779197E-01	.35687679
1957	162.85100	208.98400	.16039505	.45972985	.30475060E-01	.35040003
1958	156.89200	203.92570	.14679121	.45273499	.29358242E-01	.37111556
1959	171.57600	225.23820	.16313041	.45615144	.29802671E-01	.35061548
1960	176.00700	232.84450	.15101280	.46519307	.29579827E-01	.35391431
1961	178.10600	233.04160	.14959475	.45903753	.30844278E-01	.36152344
1962	192.22800	250.35460	.16101322	.46136490	.29415877E-01	.34820610
1963	199.03300	259.72820	.16542678	.46004400	.29934370E-01	.34459485
1964	214.94800	279.73940	.17097341	.45696530	.29531771E-01	.34252951
1965	232.13600	307.23820	.18069883	.44863171	.28039482E-01	.34262992
1966	249.26700	337.15220	.17863001	.45511819	.27959479E-01	.33829232
1967	255.37400	347.95030	.16659562	.45911046	.29306730E-01	.34599719
1968	272.26800	377.80410	.16698906	.46113492	.27576174E-01	.34429985
1969	283.91000	408.16440	.14955591	.46276373	.28702650E-01	.35947770
1970	277.69500	422.23690	.12922935	.47315584	.30293514E-01	.36672130
1971	287.02500	446.75800	.13539545	.45515917	.31688297E-01	.37775709

* Billions of 1947 U.S. dollars.

** Billions of current U.S. dollars.

TABLE A.4

DIVISIA PRICE INDEXES* OF CAPITAL, LABOR, NET ENERGY, AND NET MATERIALS INPUTS - U.S. NONENERGY MANUFACTURING, 1947-71

Year	W_K	W_L	W_E	W_M
1947	.83224756	.52335496	.84468938	.83306244
1948	.90390879	.57633598	.97895792	.85503462
1949	.84446254	.59828244	.89779559	.85829597
1950	1.0724756	.64694656	.93887776	.91362980
1951	1.1653094	.67843511	.97895792	.98588882
1952	.96661238	.70515267	.99799599	1.0067518
1953	.95195440	.72709924	.97394790	.98999181
1954	.85667752	.80152672	.95190381	.99189090
1955	1.0643322	.82729008	.98496994	1.0241544
1956	.97719870	.86545802	1.0110220	1.0513212
1957	.94218241	.91507634	.99098196	1.0293457
1958	.81433225	.95419847	1.0020040	1.0534361
1959	1.0000000	1.0000000	1.0000000	1.0000000
1960	.93811075	1.0085878	1.0170341	1.0618913
1961	.90879479	1.0419847	1.0160321	1.0811315
1962	1.0325733	1.0801527	1.0140281	1.0888739
1963	1.0602606	1.1040076	.98697395	1.0429792
1964	1.1392508	1.1727099	1.0200401	1.1281778
1965	1.2500000	1.1870229	1.0791583	1.1660756
1966	1.2622150	1.2547710	1.0360721	1.2237448
1967	1.1074919	1.3043893	1.0270541	1.2460704
1968	1.1433225	1.3883588	1.0681363	1.2618934
1969	1.0545603	1.4866412	1.0661323	1.4595950
1970	.90472313	1.6192748	1.0651303	1.4827018
1971	.97149837	1.7070611	1.2124248	1.4755299

* Divisia price indexes have been normalized to equal unity at the midpoint of the sample so as to maximize the "goodness" of the translog quadratic approximation.

APPENDIX A

FOOTNOTES

[1]These rates of replacement are .135 and .071 for equipment and structures respectively. These rates have been computed by Berndt and Christensen (1973) as "the arithmatic mean of the replacement rates implicit in the OBE capital stock and investment series".

[2]For a brief discussion of Divisia index see appendix (B).

[3]The concept of "nonproduction" and "production" workers have been defined in various studies. Berndt and Christensen (1974) have mentioned the BLS definitions of these terms as "office and related workers" for non-production workers, and "non office workers" for production workers; while the OBE definitions are "wage labor" for production labor and "salaried labor" for non-production labor; i.e., the distinction is mainly based on the degree of direct association with the physical production process. And, in that spirit one might use "blue collar labor" for production labor and "white collar labor" for non production labor.

[4]A sector's output distribution is described by its row and its input purchase by its column.

[5]These consist of energy products produced by sectors (8) through (20) (enumerated above) which constitute the energy producing section of the U.S. manufacturing sector, namely, Petroleum refining and related industries.

APPENDIX B

DIVISIA INDEX

Divisia price indices have been utilized in this study. A brief description of such indices and their construction may seem desirable in this section.

The Divisia index is defined in terms of a weighted sum of growth rates, the weights being the component's shares in total value. Specifically, if $x(t) = \{x_1(t),\ldots,x_n(t)\}$ is the set of observations on quantities of inputs and $p(t) = \{p_1(t),\ldots,p_n(t)\}$ the corresponding price observations which are to be indexed, the Divisia price index, in its continuous form, is defined as:

$$\exp \int_t \sum_{i=1}^{n} s_i \frac{\dot{p}_i(t)}{p_i(t)} , \tag{B.1}$$

where $s_i = p_i(t)x_i(t)/\Sigma p_j(t)x_j(t)$ is the relative share of the value of ith input in total value, and dots over variables denote derivatives with respect to time. A Divisia quantity index may also be constructed by invoking the above definition and replacing $\dot{p}/p$ by $\dot{x}/x$ in (B.1). The Divisia quantity index is dual to the price index. The rate of growth of these price and quantity indexes are

$$\begin{aligned} \dot{p}(t)/p(t) &= \sum s_i \dot{p}_i(t)/p_i(t), \\ \dot{x}(t)/x(t) &= \sum s_i \dot{x}_i(t)/x_i(t) \end{aligned} \tag{B.2}$$

respectively.

However, due to the nature of economic data, which take the form of observations at discrete points in time, the following discrete form of the Divisia index is used in practice:

$$\log p_t - \log p_{t-1} = \sum \tfrac{1}{2} (s_{i,t} + s_{i,t-1})(\log p_{i,t} - \log p_{i,t-1}) \tag{B.3}$$

where $s_{it} = p_{it}x_{it}/\Sigma\, p_{it}x_{it}$. The discrete Divisia index (B.3) is in fact the discrete approximation to the continuous-time Divisia index (B.1). This is Tornqvistt's discrete approximation to the continuous formula. It approaches the continuous form as $\Delta t \to 0$.

The Divisia index has many desirable properties such as an aggregation procedure discussed by Richter (1966), Theil (1967), Hulten (1973), and Diewert (1976). It is also a fact that this index suffers from one extremely serious problem. Since the index is a line integral, it is dependent, in general, upon the path on which the integral is taken. Hulten (1973), however, has shown that if the aggregate (x) exists, is homogeneous of degree one in its components (x_i), and there exists a corresponding price (p) normal at each point unique up to a scalar multiple, then the Divisia index is path independent, and retrieves the actual values of the aggregating function, subject to an arbitrary normalization in some base period. Therefore, the Divisia index preserves, up to the normalization, all the information in the problem; and it is at least as good as any other index. In other words, the Divisia index is the best choice among index numbers, given the above conditions.

The Tornqvist's discrete approximation (B.3) is proved by Diewert (1976) to belong to the class of "superlative" indexes. As such it is exact[1] for an aggregating function which is interpretable as a second order approximation to a first degree homogeneous function. Therefore, the discrete approximation to the Divisia index, (B.3), may itself serve as a second order approximation to an arbitrary (unknown) proper aggregation procedure.

APPENDIX B

FOOTNOTES

[1]An index is said to be exact for an aggregating function if it retrieves the actual value of the function.

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