THE CONSTRUCTION OF THE FORMAL OPERATIONS OF IMPLICATION-REASONING AND PROPORTIONALITY IN CHILDREN AND ADOLESCENTS

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THESIS

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ABSTRACT

THE CONSTRUCTION OF THE FORMAL OPERATIONS OF IMPLICATION-REASONING AND PROPORTIONALITY IN CHILDREN AND ADOLESCENTS

By

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Two experiments are reported that are concerned with Piaget's theory of the development of formal-operational intelligence. For the most part, the experiments were designed as tests of specific predictions derived from Piaget's assertions concerning his two categories of formal operations ("propositional operations" and "formal-operational schemata"). The first experiment was focused on one of Piaget's "propositional operations" (implication), while the second experiment was focused on one of Piaget's "formal-operational schemata" (proportionality). Subjects from three age-groups were employed in both experiments: eight-nine year-olds; 11-12 year-olds; and 14-15 year-olds.

In experiment I, the "propositional operation" of implication was studied via its transitivity property (i.e., if A implies B and B implies C, then A implies C). A 2 x 3 x 2 design was employed with the factors being propositional syntax, age of subject, and propositional semantics. Two dependent measures were considered: correctness-incorrectness of implication-reasoning answers and the latencies of these same answers. The results indicated that: subjects' implication-reasoning abilities generally improved with age; the improvements with age in implication-reasoning tend to be linear only; propositional semantics strongly affected the correctness-incorrectness of implication-reasoning answers; propositional syntax affected answer latencies; and syntax affected younger

younger subjects more than older subjects.

In experiment II, three indices of Piaget's "proportionality schema" were investigated: density conservation; solid volume conservation; and liquid volume conservation. Both conservation answers and explanations were analyzed as dependent variables. The results of experiment II indicated that: subjects' abilities to conserve the three indices improved with age; the age increases in conservation ability were almost entirely linear; the conservations of solid and liquid volume tend to precede the conservation of density invariably; the three indices intercorrelate significantly; and older subjects learned density conservation more readily than younger subjects.

Since the same subjects were employed in both experiments, a prediction derived from Piagetian theory to the effect that the dependent variables of the two experiments should be related within subjects was tested but not confirmed. The results of the experiments were discussed in terms of general implications for the Genevan theory of cognitive development and Kagan's reflectivity-impulsivity dimension of cognitive style.

Approved Ellen A. Stromme

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By

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INTRODUCTION

Piaget has proposed a theory of the ontogeny of adult intelligence (e.g., Piaget, 1949, 1953; Inhelder & Piaget, 1958) which describes the 11-12 year-old child's acquisition of two hypothesized features of mature thought. The first of these (both ontogenetically and in terms of importance) is the acquisition of propositional logic as a dominant mode of thought. With the advent of these "propositional operations", Piaget holds that the emphasis of thought shifts from the <u>real</u> to the <u>possible</u> and that there is a consequent increase in the ability to reason by hypothesis. The second of the proposed features of mature thought involves the acquisition of several "formal-operational schemata" (Piaget, 1949) which owe their origins to the aforementioned coordination of propositional logic in the young adolescent. These operational schemata are said to correspond roughly to general equilibrium laws that characterize physical systems (e.g., mechanical equilibrium, 'action-reaction') and to be manifest in the child's increased ability to solve problems which involve such relations (Inhelder, 1953; Inhelder & Piaget, 1958).

The research to be reported herein was concerned with these two general features of the development of adult thought and their relation to each other. Two experiments were conducted to investigate the development of one of Piaget's "propositional operations" (implication) and one of his "formal-operational schemata" (proportionality). The first experiment was designed to examine the effects of age of subject, semantic content of propositions, and syntactic order of propositions on the propositional operation of implication. The second experiment concerned

developmental trends in three proposed (Inhelder & Piaget, 1958) indices of Piaget's "proportionality schema" (density conservation, solid volume conservation, liquid volume conservation) and the possible existence of an invariant acquisition sequence among the respective indices of the "proportionality schema." A final major objective of the present research was to determine the magnitude of the relation (if any) between the propositional operation and operational schema of interest. Consonant with this latter aim, the same children and adolescents served as subjects in both experiments.

Theoretical Background

To continue with Piaget's substantive notions, his theory of intellectual development is well known as a stage theory and it is difficult to justify predictions pertaining to the final stage of cognitive development (formal operations) without some consideration of the preceding stage (concrete operations). During the concrete-operational period, the child presumably acquires the ability to _classify_ objects simultaneously according to one or more criteria and to comprehend simple _relations_ among objects and events. (Hence, Piaget refers to the indigenous logic of this stage as the "logic of classes and relations"--Inhelder & Piaget, 1964) Another distinctive feature of this concrete-operational stage is that the child's thought operations are now held to be reversible (i.e., there exists the permanent possibility of any thought returning to its point of departure). As the present writer previously has noted (Brainerd, 1970; Brainerd & Allen, 1970), Piaget advocates two distinct forms of operational reversibility, viz. _inversion-negation_ (a _singularly_ operation analogous to negating a single affirmation) which applies exclusively

to the concrete logic of classes and reciprocity (a binary operation analogous to compensating changes in one affirmation with equal and opposite changes in a related affirmation) which applies exclusively to the concrete logic of relations.

Piaget argues (1949, 1953) that these classificatory, relational, and reversibility features of concrete-operational thought allow for the coordination of eight "elementary groupements" (four for classes and four for relations). Although these "elementary groupements" are limited in scope, they are said to facilitate some forms of intelligent behavior (e.g., seriation of asymmetrical transitive relations, conservation of simple quantitative invariants) and they may be described as follows:

1. Each "elementary groupement" is either a class or a relation.
2. Each "elementary groupement" is either symmetrical or asymmetrical.
3. Each "elementary groupement" is either multiplicative or additive.

Since each "elementary groupement" is classified in terms of three factors with two levels each, there are exactly eight possible "elementary groupements" (2 x 2 x 2 = 8).

It is out of these eight "elementary groupements" that the propositional operations ("the sixteen binary operations"), referred to earlier as characteristics of formal intelligence, are thought to be coordinated (Piaget, 1942, 1949, 1953). This coordination presumably produces the lattice structure of formal thought--the ultimate equilibrium. The principle reason why this coordination does not come about sooner in life is also the reason given for the limited generality of the eight "elementary groupements," viz. the two reversibilities of classes (inversion-negation) and relations (reciprocity) are themselves not coordinated and they only can be applied successively, not simultaneously.

The crucial feature of the consolidation of formal-operational intelligence, then, is said to be the coordination of the two forms of reversibility into a single schema--the INRC group--capable of simultaneous application to propositions. The two general advances of formal operations mentioned previously (propositional logic and dependent operational schemata) can be thought of as 'bonuses' entailed by the effect that this coordination has on the structures of concrete thought (the "elementary groupements").

The 'N' and 'R' of 'INRC' denote the two forms of reversibility (inversion-negation and reciprocity) which the group unites, while 'I' denotes an identity element and 'C' denotes a correlative element (the inverse of the reciprocal). As Parsons (1960) notes, Piaget's INRC group is isomorphic with the well-known four-group of mathematics. In the case of group INRC, the defining equalities are: CR = N; RN = C; NC = R; and I = NRC. Although a precise explication of the mathematics of the four-group is beyond the scope of the present report, it is important to consider some of the qualitative characteristics of Piaget's application of the four-group to the structure of adult cognition. Indeed, Piaget's <u>general</u> statements about the group INRC constituted the sole basis for a major hypothesis of the present research to the effect that the ontogenies of a particular propositional operation (implication) and a particular operational schema (proportionality) would be intertwined.

From the perspective of the present research, the most important characteristic of the group INRC is that it supposedly comes in two varieties: a 'logical' form and a 'physical' form. The logical form of the group INRC refers to the fact that the series of propositional operations which are acquired apropos formal thought ("the sixteen binary operations") are said to be structured according to the rules of the four-group (composition, associativity, identity, inversion). Similarly, the 'physical' form of the group INRC refers to the fact that special systems of physical transformations (such as those gathered under the rubric "operational schemata") also are said to be governed by the rules of the four-group. Hence, the group INRC is held to structure both the 'logical' operations of abstract thought and the interpolative cognitions about immediate physical experience.

Although these speculations about the nature and functions of the four-group may seem somewhat academic, the implications of these assertions--in terms of experimental predictions--are exceedingly concrete. In so far as the group INRC structures the transformations of both the internalized operations of propositional logic and our cognitions about systems of physical transformations, there comes to mind an obvious hypothesis to the effect that if behavioral manifestations of one of these two cognitive 'systems' are present in a given subject, then behavioral manifestations of the other also should be present. Hence, in addition to the predictions of specific developmental changes in the child's abilities to reason via propositional operations and to apply the dependent operational schemata, Piagetian theory also seems to authorize the conclusion that these two groups of skills should be related linearly within subjects.

Propositional Operations

Of the several principles of Piaget's theory of the development of formal intelligence, the most basic is the child's supposed acquisition of the system of "propositional operations" (Piaget, 1949, 1953, Piaget & Inhelder, 1969; Inhelder & Piaget, 1958). The coordination of these propositional operations is thought to precipitate the realization of a

generalized lattice-structure. The propositional operations which constitute the elements of this lattice are isomorphic to the 16 binary relations of formal porpositional logic. However, Piaget substitutes "operations" for 'relations'.

The difference between Piagetian "operations" and formal 'relations' is, in part, the difference between psychological activity and passivity. The metamathematical notion of 'relation' is completely abstract and may be defined only by resorting to certain 'formal characteristics' (symmetry, reflexivity, correlativity, transivity, etc.). Hence, by definition, the notion of relation is nonactive and only refers to the possible permutations and combinations observed when conjoining numbers or propositions. Conversely, Piaget's concept of "operations" derives from action--as does everything else in his system. These operations amount to internalized, reversible actions which have been organized according to principles that insure the equilibrium of the cognitive system. Although there apparently are no immediate behavioral consequences of this distinction--empirically the results of the "operation" of disjunction (p v q) appear to be the same as the results of the 'relation' of disjunction (also p v q)--Piaget probably is justified in drawing the distinction since it is difficult to conceive of any psychological isomorph of the metamathematical concept of 'relation'.

To return to the main thread of Piaget's argument, the formal-operational child is said to acquire the ability to think hypothetically and propositionally according to the rules of that species of generalized mathematical logic known as propositional logic. (It is important to note that there are infinitely many other formalized mathematical languages--some of which, as Parsons, 1960, has suggested, are more powerful than

the one employed by Piaget.) Formal discussions of propositional logic include the fact that if one considers only propositions <u>which admit of two truth values</u>, then there are precisely two <u>singularly</u> relations among such propositions (negation and affirmation) and 16 <u>binary</u> relations among such propositions. This latter fact serves as the source of the 16 elements of Piaget's lattice model of formal thought. The full compliment of 16 binary operations is tabled on p. 103-104 of Inhelder and Piaget (1958).

Experiment I is concerned with Piaget's logical operation of implication. An implication consists of an initial proposition (called an <u>antecedent</u>) and a second proposition (called a <u>consequent</u>). Depending on which of the two base propositions one wishes to consider as implying the other, the truth conditions for this operation may be found in either column three or column four of Table 1. The affirmation that this antecedent-consequent relation (implication) obtains between any two arbitrary propositions is equivalent to asserting 'if the antecedent is true, then the consequent must be true'. The truth-falsity of the consequent when the antecedent is false may vary.

The propositional operation of implication has been offered as the logical counterpart of what we commonly refer to as the cause-effect relation (Inhelder & Piaget, 1958)--i.e., the set of all possible cause-effect relations is a proper subset of all possible implications. Therefore, cause-effect is a <u>sufficient</u> but not <u>necessary</u> condition for implication. Alternatively, there is no conceivable cause-effect relation whose truth conditions are not specified adequately by the formal-implication model. Hence, in the initial experiment reported here, reasoning tasks were employed whose elements were propositions placed in

Table 1

Truth Conditions for the Propositional "Operation" of Implication

P	Q	$P \supset Q$	$Q \supset P$
True	True	True	True
True	False	False	True
False	True	True	False
False	False	True	True

a cause-effect relation to one another and whose solutions required that subjects of three chronological ages be able to reason on the basis of such relations. The effects of certain proposition-specific treatments (syntactic order of propositions and semantic content of propositions) on the same subjects' abilities to reach valid conclusions and their response latencies also were considered.

Piaget points out that there are cognitive skills present at the concrete level which are deceptively similar to formal-operational implication. These concrete skills constitute inherent sources of error in any investigation of the ontogeny of reasoning via implication. Without considering his assertions in depth, it suffices to say Piaget's premise is that the concrete-operational child can come close to solving <u>two-element</u> implications ($p \supset q$ or $q \supset r$) by employing simple concrete correspondences. Rather than study such two-element implications with their built-in error factor, it seemed more appropriate to study a <u>property</u> of the implication relation which can be arrived at only if the ability to handle the implication relation is well developed. Like most other asymmetric relations, the implication relation is <u>transitive</u> and it is precisely this transitivity property that was employed in experiment I. This approach to the study of implication via its transitivity property is roughly analogous to previous studies of the concrete concepts of 'less than' and 'more than' via their transitivity property (e.g., Piaget, Inhelder, & Szeminska, 1960; Inhelder & Piaget, 1964; Smedslund, 1963a; Murray & Youniss, 1968).

To say that a binary relation R is <u>transitive</u> on or among a set of elements (for present purposes, the set of elements is restricted to a series of verbal propositions conjoined by cause-effect relations) is

to assert that for any arbitrary elements x, y, and z, R relates x to z whenever R relates x to y and y to z. In the case of a simple relation such as 'less than', the transitive property is intuitively as well as logically evident (if $A > B$ and $B > C$, then trivially $A > C$). Although the transitivity property of implication is perhaps not so intuitively apparent, it is nonetheless logically evident and is specified clearly by Table 2.

The fact that implication is a transitive relation suggested that one might devise three-element implications similar to the three-element problems employed in the previously mentioned studies of concrete transivity. Thus if one has three base propositions (in lexicographical order: p, q, and r) and if one links the first with the second and the second with the third by the operation of implication, then it is valid to conclude (by Table 2) that the first and third also are linked by the same operation. Therefore, if it is the case that 'p implies q and q implies r', then it also must be the case that 'p implies r'--i.e., $(p \supset q) \cdot (q \supset r) \supset (p \supset r)$ is valid. On this point Piaget's theory of formal thought reduces to the hypothesis that if the formal-operational child is presented with the first two implications, he will infer that the third implication also holds. The general procedure for assessing children's abilities to reason in terms of implication thus becomes somewhat clearer, viz. to present subjects with the first two implications and to determine the relative presence of the remaining implication through appropriate interogation. Reasonably enough, the "appropriate interogation" corresponds to the fundamental truth conditions for implication set down in Table 1.

As previously mentioned, the truth conditions of the cause-effect relation are representable in terms of the implication relation. Therefore,

Table 2

The Transitivity of Truth-Functional Implication

p	q	r	$(p \supset q) \cdot (q \supset r) \supset (p \supset r)$
True	True	True	True
True	True	False	True
True	False	True	True
False	True	True	True
False	False	True	True
False	True	False	True
True	False	False	True
False	False	False	True

to study the cause-effect relation is to study a 'pure' example of an environmental isomorph of the logical operation of implication (Inhelder & Piaget, 1958). In conformity with the above model, these considerations suggested a series of problems that presents subjects with three base propositions of which the first and second, and the second and third are conjoined in terms of cause and effect. The problems were designed to discern the extent to which subjects inferred that the cause-effect relation also obtained between the first and third propositions. Thus the procedure for investigating reasoning by implication consisted of a series of eight problems for which the following example served as a prototype:

1. Propositions: p = Jack washes the family car; q = Jack's father is very pleased; r = Jack receives 50¢.

   a. $(p \supset q)$: If Jack washes the family car, then Jack's father will be very pleased.

   b. $(q \supset r)$: Whenever Jack's father is very pleased, Jack receives 50¢.

2. Evaluation: Subjects' inferences that the third implication (if Jack washes the car, then he will receive 50¢) held were assessed by four questions of the following form:

   a. $(p \cdot r)$: If Jack washes the family car, then what else will happen?

   b. $(\bar{p} \cdot r)$: If Jack doesn't wash the family car, is it possible that he still might receive 50¢?

   c. $(p \cdot \bar{r})$: If Jack washes the family car, is it possible that he won't receive 50¢?

   d. $(\bar{p} \cdot \bar{r})$: If Jack doesn't wash the family car, then what else will

happen?

Each of the preceding questions corresponds to one of the four cells of the fundamental truth table for implication (Table 1). It should be noted that this method did not yield an 'all-or-none' judgment vis-à-vis implication; rather, incremental evidence was adduced about the extent to which the implication relation is grasped in each problem. Obviously, this feature of experiment I facilitated parametric analyses of the data.

The first line of evidence deriving from experiment I concerns the ontogeny of implication-reasoning. Data indicating an increasing ability to reach those conclusions which are authorized by the premisses of the proposed problems would be supportive of Piaget's hypotheses about the development of propositional operations. The ontogenetic question aside, experiment I also was concerned with the effects of two further variables on subjects' abilities to reason in terms of implication and the rapidity of their implication responses, viz. the semantic content of the base propositions and the presentation order of the two initial implications--$(p \supset q) \cdot (q \supset r)$ vs. $(q \supset r) \cdot (p \supset q)$.

The work of De Vries (1969) suggests that the semantic content of simple Piagetian reasoning tasks has a pervasive effect on the conclusions of three to six year-olds. In experiment I an attempt was made to discover whether or not this effect extends to more complex reasoning tasks such as implication. The semantic manipulation was quite simple: the resultant effect (r) of each three-element implication problem was pleasant for (randomly) one-half the subjects and aversive for the remaining half of the subjects. The second influence to be investigated was the order in which the two initial implications were presented to subjects. Logically, the presentation order has absolutely no effect on the ultimate validity

of the third implication (thanks to the associativity property of the logic of propositions). Empirically, however, the order in which facts or statements are proposed has been shown to affect subjects' judgments (Inhelder & Piaget, 1958, 1964). The order manipulation consisted of reversing the order of the two initial implications for (randomly) one-half of the subjects. Hence, one-half of the subjects considered the formula $(p \supset q) \cdot (q \supset r)$ and the other half considered the formula $(q \supset r) \cdot (p \supset q)$. These two formuli are logically equivalent--the question remains whether or not they are psychologically equivalent.

If a syntactic (order) treatment were effective, one might expect a decrease in the influence of order reversal in favor of older subjects. This prediction derives from Piaget's numerous assertions that the concrete-operational child is influenced strongly by time-space correspondence phenomena such as serial-order. Conversely, the abstract operations of formal thought, with their relative independence from concrete phenomena, are said to approximate more closely the rules of 'pure' mathematical relations (such as, in this case, associativity).

<u>Formal-Operational Schemata</u>

Piaget argues (1949, 1953; Piaget & Inhelder, 1969; Inhelder & Piaget, 1958) that the coordination of the previously mentioned propositional operations produces an n-by-n combinatorial system (<u>structure d'ensemble</u>) whose structure is that of the lattice and whose laws of composition are those of the mathematical group. This formal structuring of thought in turn precipitates the acquisition of several "conceptual instrumentalities" (Flavell, 1963) which Piaget has called (e.g., 1949, 1953) "formal-operational schemata." If the propositional operations and their inherent structure are the most general features of formal thought and if immediate

sense impressions constitute the least general feature of formal thought, then these formal-operational schemata are at some intermediate level of generality. To continue the analogy, the formal-operational schemata are supposed to reconcile the most general features of formal intelligence with the least general features thereof.

Several formal-operational schemata have been referred to by Piaget and among these are: proportionality; mechanical equilibrium; "all-other-things-being-equal"; 'action-reaction'; and others. The development of these operational schemata is said to be necessitated by the occurence and reoccurence of certain general 'forms' or types of problems in every-day life.

Of the several operational schemata, the schema of proportionality probably has received the most extensive consideration in Piaget's theoretical expositions. The proportionality schema is of particular interest when considered in relation to experiment I, because Piaget offers the schema as a physical realization of the group INRC--the same group that is held to be influential in the coordination of propositional operations such as implication.

As Piaget defines the notion, the schema of proportionality refers to a qualitative structure which facilitates the understanding of complex physical systems that contain many factors or forces which compensate each other. The intended range of application of the proportionality schema may be illustrated for the case of the conservation of volume. As a spatial concept, volume conservation requires that subjects be able to place the three relevant spatial dimensions (length, width, depth) of two different containers (or objects) into a compensating proportionate relation to one another (i.e., given $XYZ = X'Y'Z'$, subjects must be capable of

inferring $XY/X'Y' = Z'/Z$) if they are to conserve volume. In addition, Inhelder and Piaget (1958) assert that it follows that the concept of <u>density</u> also requires application of a proportionality schema, since density is a second-order concept based on a weight per unit <u>volume</u> relation.

In view of their presumed representativeness as applications of the proportionality schema to the real world, the concepts of volume and density were examined in the second experiment reported here. As was the case for implication-reasoning, the age changes in children's understanding of the concepts of volume and density were assessed. Previous research (Elkind, 1961) tentatively suggests that the median age for acquisition of volume conservation is greater than 11 years. There is a complete absence of objective data relative to the ontogeny of density conservation. Further, the linearity of the acquisition functions also was of interest. Since three equally spaced age-groups were studied (third, sixth, and ninth graders), the possibilities of positively or negatively accelerated acquisition functions existed.

An additional aim of experiment II was to investigate a hypothesized (Inhelder & Piaget, 1958) invariant sequence in the acquisition of volume and density concepts. In so far as density is a concept that is dependent upon the notion of volume, Inhelder and Piaget assert that an adequate conception of volume must antedate an adequate conception of density.

To briefly characterize the relevant procedures of experiment II, the concept of volume was assessed by a typical conservation method which takes account of Piaget's (1968) warnings about "psuedo-conservation" and Rothenberg's (1969; Rothenberg & Courtney, 1969) warnings about "False positives." The general method for evaluating conservation of volume has been described correctly by Elkind (1961) and incorrectly described by

Trabasso (1968). The reality of whether or not one actually is investigating volume turns on whether or not one asks a spatial question as opposed to a quantity question (Trabasso's error) of the subjects. Obviously, the spatial question (e.g., "Which one takes up more space or room?") is the appropriate choice. Conservation of density was assessed via a new technique generated from the results of a broader investigation of density concepts conducted by Inhelder and Piaget (1958). The density technique employed in experiment II makes use of the fact that concrete nonconservers of density think of the concept in terms of absolute weight rather than weight-per-unit-volume.

Summary

Broadly speaking then, the aim of the two experiments reported here was to investigate some of the hypotheses offered by Piaget as part of his theory of the ontogeny of formal thought. Specifically, experiment I was concerned with the ability of children and adolescents to reason in terms of formal-logical implication. Experiment II was concerned with the articulation of the concepts of volume and density, in the same children and adolescents, as a means of adducing data pertinent to Piaget's proportionality schema.

It also was of interest to consider just what relation obtained between subjects' abilities to reason via implication and their abilities to apply the proportionality schema to concrete situations (i.e., to conserve density and volume). As previously mentioned, the prediction that these two cognitive skills should be related within subjects follows directly from Piaget's explication of the four-group INRC as a structuring agent of formal thought. In sum, if the rules and properties of the group

INRC organize the cognitive structuring of both the more general propositional operations (of which implication is one) and the less general operational schemata (of which proportionality is one), then the ontogenies of implication-reasoning and the proportionality schema should be similar within subjects.

EXPERIMENT I: IMPLICATION-REASONING

The initial experiment was focused on subjects' (Ss') implication-reasoning abilities. A 2 x 3 x 2 factorial design was employed with the factors being semantic content of component propositions, age of S (third grade, sixth grade, ninth grade), and syntactic order of component propositions. Two dependent variables were measured, viz. the answers to the implication-reasoning problems and the latencies of these same implication-reasoning answers. Five major questions were of interest in experiment I.

1. Does the implication-reasoning facility of Ss increase linearly from, say, age eight to age 15 as Piagetian theory predicts (Piaget, 1949, 1953; Inhelder & Piaget, 1958)?

2. Does the implication-reasoning facility of Ss vary with the semantic content of component propositions?

3. Does the implication-reasoning facility of Ss vary with the syntactic order of the antecedent propositions?

4. If semantic content and/or syntactic order affect implication-reasoning facility, does the effect or effects vary in intensity depending on the age of S?

5. If implication-reasoning facility does increase from age eight to age 15, is this increase entirely linear or are there some nonlinear features about it?

In the preceding five questions, the term 'facility' denotes both correctness-of-answers and response latencies. In addition to these major questions, some related though minor questions, such as the reliabilities of the dependent variables also were of interest.

Method

Subjects

The experimental Ss were drawn from three age-levels: eight-nine year-olds (third graders); 11-12 year-olds (sixth graders); and 14-15 year-olds (ninth graders). These age-ranges were chosen because they correspond to Piagetian levels of cognitive development that were particularly relevant to the present research. Piaget has reported (e.g., Piaget & Inhelder, , 1969) that the initial group of Ss (eight-nine year-olds) is well into the period of concrete operations, the second group of Ss (11-12 year-olds) is in a time of transition from concrete to formal thought, and the final group of Ss (14-15 year-olds) is well into the period of formal thought. In so far as one of the objectives of experiment I was to study the transition from concrete to formal thought via implication-reasoning (cf. questions 1, 4, and 5), these three age-levels were logical choices.

Twelve boys and 12 girls were selected from each level for a total of 72 Ss. All Ss were pupils of the Holt Public Schools, a moderately-sized semirural school system. For purposes of generality, it seemed more appropriate to study 'average' children than either 'bright' or 'dull' children. Therefore, only Ss within the 90-110 IQ range and with an academic grade-point in the 2.0-2.5 range (on a four-point scale) were included in the present experiment.

All Ss had to meet three other criteria to be included in experiment I: conservation of number (Piaget, 1952); conservation of length (Piaget, Inhelder, & Szeminska, 1960); and minimal reading ability. The classic

tests of number and length conservations that are so ubiquitous in the developmental literature were used as the first two criteria. To meet the third criterion, it was necessary for S to read six sentences that were similar in difficulty to those he would encounter in the form of implication-reasoning propositions. All Ss met the number criterion, only one S (a third grader) failed length conservation, and only one S (another third grader) failed to display minimal reading ability. These two Ss were replaced by two other third graders who met all criteria. In theory, the pretests of number and length conservation served as an assurance that all Ss--by Piagetian criteria--had attained at least the period of concrete operations.

One-half of the 24 Ss in each age-group participated in experiment I first. The remaining half participated in experiment II first.

Apparatus and Materials

A Sony 230 tape recorder was used throughout the experiment. Upon entering the experimental room, each S was fitted with Sony DR-6A headphones which were not removed until S left the room. Instructions were prerecorded on the left channel track. Ss' responses were recorded on the right channel track. Thus, all Ss heard E's instructions and comments through the left headphone, while their own responses simultaneously were being recorded on the right channel track. This particular record-playback-record technique is referred to by the audio industry as "sound-with-sound." Sony PR-150 recording tape was used exclusively and the recording speed was a constant $7\frac{1}{2}$ i.p.s.

The other important materials employed in the first experiment were white stimulus cards by which the implication-reasoning problems were presented to the Ss. All cards were three inches by five inches and the

component propositions of an implication-reasoning problem were typed in large red letters on each card.

Treatment Conditions

Age. As noted above, three age-levels were studied in the present experiment (eight-nine year-olds, 11-12 year-olds, 14-15 year-olds). Also as noted, these three age-levels presumably correspond to Piaget's periods of concrete operations, concrete-formal transition, and formal operations.

Syntax. The 48 propositions employed in the present experiment are enumerated in Table 3. The assessment procedure focused on the extent to which Ss concluded that the cause-effect relation held between the propositions of columns A and C of Table 3. Obviously, there are two possible orders in which the propositions of columns A, B, and C may be conjoined to necessitate the transitive inference 'A causes C', viz. 'A causes B and B causes C' or 'B causes C and A causes B'. For convenience, the former ordering is referred to as the forward syntactic treatment (FS) and the latter ordering is referred to as the reverse syntactic treatment (RS). One-half of the Ss received FS implication problems and the other half received RS implication problems.

Semantics. Examination of the propositions of Table 3 will reveal that each implication-reasoning problem centered on a single central character (either Jack or Jill depending on the sex of S). The semantic manipulation consisted of varying the reinforcement consequences of the implication problems for this central character. One-half of the Ss were given implication-reasoning problems in which the transitive inference (A causes C) involved some pleasant consequence for this central character. The other half of the Ss were given implication-reasoning problems in which the transitive inference involved some unpleasant consequence for this

Table 3

Implication-Reasoning Propositions

A	B	C
Jack (Jill) sweeps the kitchen floor	Mother is very pleased	Jack (Jill) gets all the dessert he (she) wants
Jack (Jill) washes the dishes	Father is very pleased	Jack (Jill) gets 50¢
Jack (Jill) gets all 'A's' on his (her) report card	Both of Jack's parents are very pleased	Jack (Jill) gets to stay up later as a reward
Jack (Jill) does well on an English test	Teacher is happy	Jack (Jill) gets less schoolwork to do
Jack (Jill) mows the lawn	Jack (Jill) works hard at something	Jack's (Jill's) father treats him (her) to an ice cream cone
Jack (Jill) has a birthday	All of Jack's (Jill's) relatives come to see him (her)	Jack (Jill) gets a lot of money
Jack (Jill) helps mother with the shopping	Mother does not have much work to do	Mother fixes Jack's (Jill's) favorite food for dinner
Jack (Jill) plays a part in a school play	Jack (Jill) does a very good job	Jack (Jill) gets an award from the school
Jack (Jill) throws a rock at a window	A window breaks	Mother sends Jack (Jill) to bed without supper
Jack (Jill) complains about how bad things are	Jack's (Jill's) friends get mad	Jack's (Jill's) friends won't talk to him (her)
Jack (Jill) plays with matches	Jack (Jill) starts a fire in his (her) House	Jack's (Jill's) father takes away his (her) allowance

Table 3 (cont'd.)

A	B	C
Jack (Jill) leaves his (her) bicycle in the driveway	Father hits Jack's (Jill's) bike with his car	Father takes away Jack's (Jill's) bike
Jack (Jill) forgets something he (she) is supposed to do	Father tells Jack (Jill) what to do	Jack (Jill) feels ashamed
Jack (Jill) wispers in class	Teacher gets mad	Jack (Jill) has to stay after school
Jack (Jill) breaks one of mother's favorite dishes	Mother is unhappy	Jack (Jill) cannot go outside and see his (her) friends
Jack (Jill) doesn't come home right after school	Jack's (Jill's) parents worry about him (her)	Jack (Jill) cannot watch television

central character. The pleasant outcome problems are referred to as the positive semantic treatment (+S) and the unpleasant outcome problems are referred to as the negative semantic treatment (-S). The eight +S problems appear in the top half of Table 3, while the eight -S problems appear in the bottom half of the same table.

Randomizations. Equal numbers of Ss from the three age-groups were assigned randomly to the two syntactic conditions and to the two semantic conditions with the single provision that the treatment levels be divided equally with respect to sex. From the 8! ways in which the eight implication problems might have been ordered, 24 problem orders were selected at random and randomly assigned to Ss within each age-group. Finally, the order in which the four assessment questions were asked was varied randomly for each randomly ordered problem.

Procedure

Each S was presented with either the eight implication-reasoning problems appearing in the top half of Table 3 or the eight implication-reasoning problems appearing in the bottom half of Table 3. The elements of these problems appearing in columns A, B, and C of Table 3 were three base assertions. As a means of controlling for possible age changes in short term memory, the problems were presented one-at-a-time on 3 x 5 cards and E read each one aloud. Ss were allowed to retain and reread each card during the interim during which E asked them questions concerning it.

In reading the problem E connected the column A proposition with the column B proposition and the column B proposition with the column C proposition by means of a cause-effect relation (i.e., 'A causes B and B causes C'). Following the reading of each 3 x 5 card, E assessed the extent to which Ss inferred that a cause-effect relation also obtained between the

column A and C propositions (i.e., 'A causes C') via four questions of the following general form:

a. E: "If A occurs, then what else will occur?" (correct answers = C or both B and C)

b. E: "If A occurs, then is it possible that C won't occur?" (correct answer = no)

c. E: "If C occurs sometime, is it possible that A didn't occur?" (correct answer = yes)

d. E: "If A doesn't occur, then what else could occur?" (correct answer = C may or may not occur or B and C may or may not occur)

The first and last questions obviously required greater extemporization on the part of S than did questions b and c. Whenever such a situation obtains, there is always the possibility that individual differences in motivation or anxiety may increase error variance. Hence, it seemed advisable to institute some precautionary measures to minimize the chances that failure to answer questions a and d might have been due to reticence on the part of S. These precautionary measures were a maximum of two promptings that were provided whenever S failed to answer (i.e., gave an, "I don't know") either the first or last question.

Prior to seeing the initial problem, Ss heard the following instructions: "I am going to show you some white cards one-at-a-time. On each card there is a short little story about a boy named Jack (girl named Jill). I shall read each story to you as you look at the card. I shall then ask you some questions about the story on the card. When you have answered the questions, we will go on to a new card and do the same thing again until we are finished."

To summarize the procedural details of experiment I, Ss from each

age-group were assigned randomly to either condition FS or condition RS and to either condition +S or condition -S. Next, they were read the above instructions. Finally, the eight cards with the implication-reasoning problems were presented and read aloud one-at-a-time; Ss were asked four questions pertaining to each card.

Dependent Variables

The dependent variables of interest were the content of Ss' answers to E's implication-reasoning questions (i.e., the correctness-incorrectness of Ss' responses) and the latencies of Ss' answers (i.e., the length of the intervals between E's questions and Ss' responses). It should be noted that Piagetian theory focuses exclusively on the former of these two dependent variables and does not include any consideration of response latencies as a reasoning parameter.

Implication-Reasoning Answers. The complete set of 32 implication reasoning questions that were posed to the 36 Ss in condition +S appears in Appendix I, and the alternative set of 32 questions posed to the 36 Ss in condition -S appears in Appendix II. The four assessment questions for each of the problems were scored in the following manner:

1. Question a ("If A occurs, then what else will occur?"): Ss received a score of '3' if they gave a correct answer on the first try; Ss received a score of '2' for a correct answer following a single prompting; Ss received a score of '1' for a correct answer following a second prompting.

2. Question b ("If A occurs, then is it possible that C won't occur?"): Ss received a score of '2' for a correct 'no' answer.

3. Question c ("If C occurs sometime, is it possible that A didn't occur?"): Ss received a score of '2' for a correct 'yes' answer.

4. Question d ("If A doesn't occur, then what else could occur?"):

Ss received a score of '3' if they gave a correct answer on the first try; Ss received a score of '2' for a correct answer following a single prompting; Ss received a score of '1' for a correct answer following a second prompting.

The four scores within individual problems were summed for each S and these totals were in turn summed across the eight problems to yield a single - 'implication-reasoning' score. This single score represented each S's status on the present dependent variable and it was used in all analyses of the variable save reliability estimates.

Response Latencies. The second dependent variable was the elapsed time between the last words of E's questions and the first words of the Ss' responses. All estimates of the second dependent variable were made from the tape recordings after Ss had participated in both experiment I and experiment II.

The latency of each of the Ss' 32 responses was estimated three times to the nearest 1/10 of a second. The average of the three estimates then was taken as the latency of a particular response. As was the case for the first dependent variable, latencies for individual Ss were summed within problems and then across problems to yield a single response latency value. This value represented a S's status on the second dependent variable and was used in all analyses of that variable save reliability estimates.

Results

Separate 2 x 3 x 2 analyses of variance for fixed-effects were performed on the data pertaining to the two dependent variables, the factors being syntax (A), age (B), and semantics (C). A summary of the analysis

of variance for the first dependent variable (correctness-incorrectness of implication-reasoning answers) appears in Table 4 and a summary of the analysis of variance for the second dependent variable (response latencies) appears in Table 5.

It is apparent from Table 4 that the adequacy of Ss' answers was influenced strongly by both their age and the semantic content of the problems to which they were exposed. Alternatively, the order in which Ss received the initial propositions ('A causes B and B causes C' vs. 'B causes C and A causes B') did not affect the adequacy of Ss' answers. Finally, the effects of the age and semantic factors were simple and additive with no interactive tendencies being noted.

On the other hand, it is apparent from Table 5 that the latency of Ss' answers was influenced by their age and the syntactic order of the component propositions of the implication-reasoning problems. The semantic content of the problems did not appear to affect the rapidity with which Ss answered the various questions. In addition, there was a significant tendency for the factors of age and syntax to interact. Post hoc analysis revealed that this was attributible to the fact that the syntactic manipulation was more effective with the two younger groups than with the older group. It is also apparent that the three significant F ratios of Table 5 account for a much smaller portion of the total variance than do the two significant F ratios of Table 4.

Since three age-groups were studied, the possibility existed that the age increases in the correctness and rapidity of implication answers were characterized by certain nonlinear features. For example, the difference between the third and sixth graders might have been proportionately larger than the difference between the sixth and ninth graders (or vice

Table 4

Summary of Analysis of Variance for Implication Answers

Source	SS	*df*	MS	*F*	*p*
Syntax (A)	66.12	1	66.12	<1	
Age (B)	3234.33	2	1617.17	19.98	$<.0005$
Semantics (C)	3029.01	1	3029.01	44.67	$<.0005$
A x B	316.34	2	158.17	1.95	
A x C	5.02	1	5.02	<1	
B x C	444.11	2	222.06	2.74	$<.10$
A x B x C	410.11	2	205.06	2.53	$<.10$
Error	4857.83	60	80.96		
Total	12362.87	71			

Table 5

Summary of Analysis of Variance for Implication Latencies

Source	SS	*df*	MS	*F*	*p*
Syntax (A)	7.93	1	7.93	8.35	$<.01$
Age (B)	14.02	2	7.01	7.38	$<.005$
Semantics (C)	1.48	1	1.48	1.56	
A x B	7.45	2	3.73	3.93	$= .025$
A x C	1.94	1	1.94	2.04	
B x C	5.32	2	2.66	2.80	$<.10$
A x B x C	2.75	2	1.38	1.45	
Error	56.84	60	.95		
Total	97.73	71			

versa). This nonlinear possibility was explored via an orthogonal polynomials analysis of both dependent variables. Summaries of these two analyses appear in Tables 6 and 7. Neither analysis provided any evidence of nonlinearity. While the linear component is large and highly significant in the case of both answers and latencies, the quadratic component is less than one for both dependent variables.

In so far as the analyses of variance demonstrated clear and consistent effects of the various factors, it is likely that the two dependent measures are characterized by adequate reliabilities. In addition, Cronbach's coefficient alpha was used to calculate exact reliability estimates for each dependent variable. These total reliability estimates, as well as the separate components contributed by each age-group, appear in Table 8. It can be seen that the rather short (eight problem) evaluation of implication-reasoning facility was quite reliable and that the reliability estimates do not vary notably with age of S. Hence even though eight-nine year-olds tend to answer implication-reasoning questions incorrectly, they nonetheless answer these same questions reliably. Since total reliability estimates for the two dependent variables are identical, the large difference in portion of variance accounted for in the separate analyses of variance (Tables 4 and 5) of the two variables cannot be attributed to differences in reliability.

The average number of points that Ss received for each problem are plotted against problem order for each of the age-groups in Figure 1. It can be seen that there was some improvement in the quality of the answers as Ss proceeded through the eight problems. As is typical of 'warm up' effects, the improvement was more pronounced for the first through the fourth problems than for the fifth through the eighth problems. The three

Table 6

Orthogonal Polynomials Analysis for Implication Answers

Source	SS	*df*	MS	*F*	*p*
Between	3234.33	2			
Linear	3117.09	1	3117.09	22.81	$<.0005$
Quadratic	117.24	1	117.24	<1	
Error	9128.54	69	132.30		
Total	12362.87	71			

Table 7

Orthogonal Polynomials Analysis for Implication Latencies

Source	SS	df	MS	F	p
Between	14.03	2			
Linear	14.00	1	14.00	11.57	$<.005$
Quadratic	.03	1	.03	<1	
Error	83.70	69	1.21		
Total	97.73	71			

Table 8

Reliabilities of Implication-Reasoning Dependent Variables

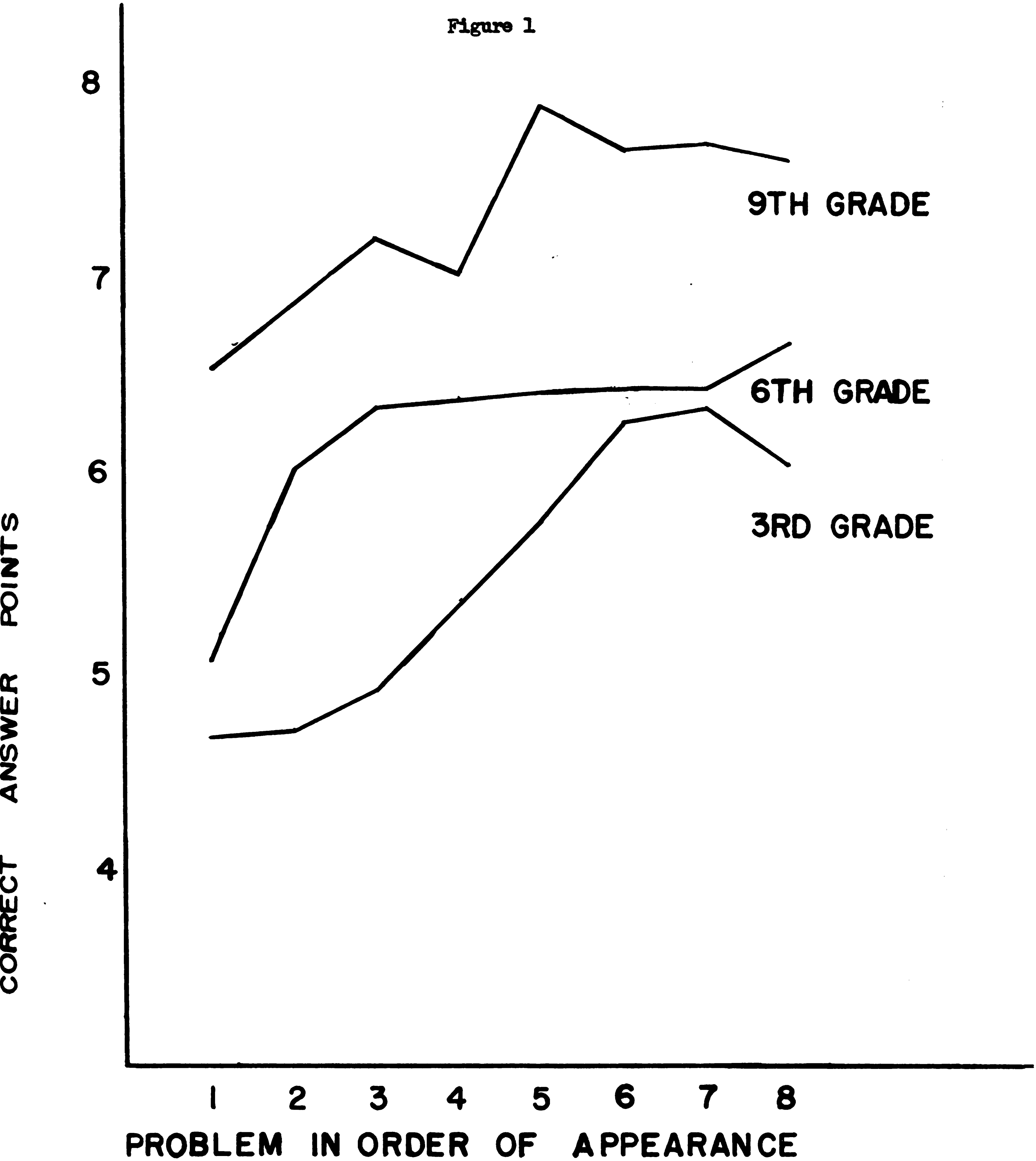
Variable	Age-Group			
	8-9 years[a]	11-12 years[a]	14-15 years[a]	all Ss[b]
Answers	.89*	.88*	.85*	.90*
Latencies	.88*	.85*	.94*	.90*

[a] N = 24 per cell

[b] N = 72 per cell

* $p < .0005$

Figure 1


8
7
6
5
4
9TH GRADE
6TH GRADE
3RD GRADE
CORRECT ANSWER POINTS
1 2 3 4 5 6 7 8
PROBLEM IN ORDER OF APPEARANCE

possible paired answer comparisons for each of the eight problems plotted in Figure 3 are given in Table 9. The generally large and highly significant t ratios of Table 9 authorize the conclusion that each of the three answer-by-problem curves of Figure 3 tends to be significantly different from the other two curves. In line with the significant linear component reported in Table 6, paired inspections of the three columns of Table 9 indicate that the differences between corresponding points on the ninth and third grade curves tend to be larger and more highly significant than the differences between corresponding points on either the third and sixth grade curves or the sixth and ninth grade curves.

The average total response latencies are plotted against problem order for each of the age-groups in Figure 4. As was the case for problem answers, there is a general improvement in the latencies across problems with the decrement being most pronounced for the first few problems. The three possible paired response comparisons for each of the eight problems plotted in Figure 4 are given in Table 10. Careful inspection of Figure 4 and Table 10 reveals the reason why the age main effect of Table 5 is so much smaller than the age main effect of Table 4. It is apparent that only the latter halves of the third and sixth grade response latency curves differ significantly from each other. It also is apparent that the sixth and ninth grade curves differ significantly only with respect to the first problem. In short, sixth grade latencies started out at a level not significantly different from the third grade latencies and 'warmed up' to a level not significantly different from ninth grade latencies.

Thus, the smaller age main effect evidenced in Table 5 as opposed to Table 4 is almost exclusively the result of the large and highly significant differences between the corresponding points on the third and

Table 9

t Tests for all Possible Pairs of Points in Figure 1

Problem Number	Age Pairings		
	Third Grade vs. Sixth Grade	Sixth Grade vs. Ninth Grade	Third Grade vs. Ninth Grade
First	.87	3.62****	5.29****
Second	4.36****	2.73***	7.99****
Third	4.96****	3.00***	8.04****
Fourth	3.57****	2.08**	5.38****
Fifth	2.33**	4.81****	6.42****
Sixth	1.00	4.69****	5.66****
Seventh	.30	3.71****	4.46****
Eighth	1.90*	2.82***	4.49****

*$p < .05$

**$p < .025$

***$p < .005$

****$p < .0005$

Figure 2

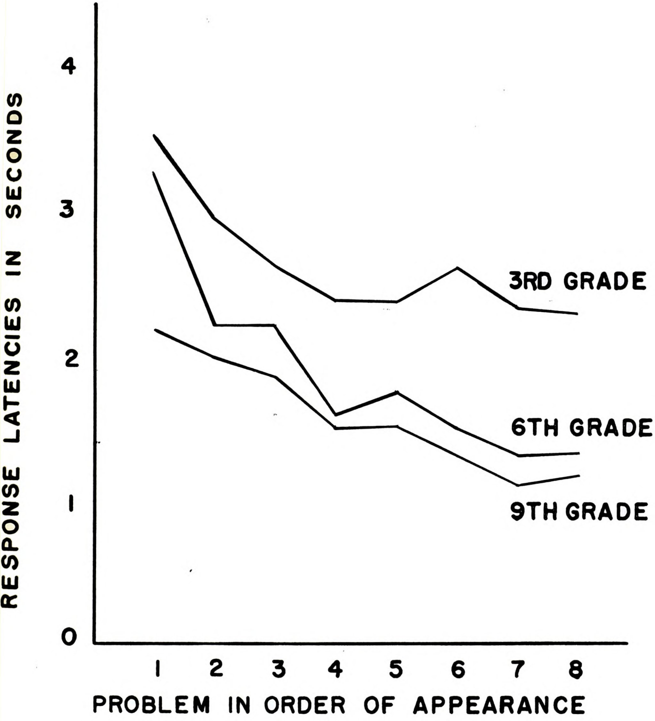

RESPONSE LATENCIES IN SECONDS
4
3
2
1
0
3RD GRADE
6TH GRADE
9TH GRADE
1 2 3 4 5 6 7 8
PROBLEM IN ORDER OF APPEARANCE

Table 10

$\underline{t}$ Tests for all Possible Pairs of Points in Figure 2

Problem Number	Age Pairings: Third Grade vs. Sixth Grade	Age Pairings: Sixth Grade vs. Ninth Grade	Age Pairings: Third Grade vs. Ninth Grade
First	.44	2.16**	3.69****
Second	1.69*	.81	2.96***
Third	.80	1.21	2.99***
Fourth	3.28***	.46	3.86****
Fifth	1.72*	.93	2.98***
Sixth	2.94***	.92	3.62****
Seventh	3.18***	.78	4.19****
Eighth	2.96***	.70	3.87****

*$\underline{p} < .05$

**$\underline{p} < .025$

***$\underline{p} < .005$

****$\underline{p} < .0005$

ninth grade curves. These data are also consistent with the significant linear component reported in Table 7. The fact that the linear term of Table 7 is roughly one-half the size of the linear term of Table 6 also is accounted for by the preceding considerations.

Discussion

To return to the first of the five questions posed earlier in relation to experiment I, the data clearly support Piaget's contention that the implication-reasoning facility of children and adolescents increases linearly between the ages of eight to 15. This contention was supported by the age main effect of the analysis of variance for implication answers and by the age main effect of the analysis of variance for implication response latencies.

Concerning the second question, the very large F ratio of Table 4 corresponding to the semantic main effect suggests that Ss' abilities to reach appropriate transitive conclusions based on the implication relation is affected strongly by this variable. In the present experiment, those Ss receiving the +S treatment produced reliably better answers than those Ss receiving the -S treatment. Alternatively, the insignificant F ratio of Table 5 corresponding to the semantic main effect suggests that although semantic content clearly affects correctness-incorrectness of answers, this manipulation has a negligible impact on the rapidity of answer production.

The conclusions regarding the third question are essentially the inverse of those for the second question. The syntax main effects of Tables 4 and 5 suggest that the order in which implication-reasoning propositions are presented influences the rapidity with which implication answers are produced, but such order does not affect the correctness of implication-

reasoning productions. Those Ss receiving the FS treatment responded in reliably shorter periods of time than those Ss receiving the RS treatment. The absence of a reliable syntax by semantics interaction may imply that propositional sequence affects the production speed of incorrect and correct answers equally.

There was not a tendency for the semantic effect reported in Table 4 to be more effective for one age-group than for the others. Thus, the semantic content of the propositions did not affect the correctness of the implication-reasoning productions of the three age-groups differentially. Alternatively, the syntax by age term of Table 5 demonstrates that the order manipulation affected the latencies of the three age-groups differentially. Post hoc examination revealed that, as expected, propositional order produced the least latency variance in the oldest Ss (14-15 year-olds). Unexpectedly, however, propositional order produced the most latency variance in sixth grade Ss. In so far as Piaget argues that the 11-12 year-old child is in a period of transition from concrete to formal operations (specifically, substage IIIA of Inhelder & Piaget, 1958), this significant syntax by age interaction is certainly open to cognitive dissonance interpretations (e.g., Festinger, 1957; Smedslund, 1961b).

Regarding the fifth question, no quadratic (i.e., positively or negatively accelerated) tendencies were noted for either the age trend in correctness of implication answers or the age trend in the rapidity of answer production. Instead, the quadratic component was less than one for both dependent variables. This fact coupled with the high reliabilities of the dependent variables might be construed as a dim forcast indeed for those who would opt for quadratic trends within the specified age-ranges on such parameters.

Given the identical reliabilities of the two dependent variables, the discrepancies between the results of the analysis of variance for implication answers and the results of the analysis of variance for implication latencies imply something about the interchangeability of these two measures. Experiment I identified at least three differences between these two variables vis-à-vis implication-reasoning: semantic content of propositions affects correctness of answers but not answer latency; propositional order affects answer latency but not the correctness of answers; propositional order affects the three age-groups differentially. It is important to note these empirical discrepancies, because they indicate that the two dependent variables are in no wise equivalent measures of implication-reasoning in particular and perhaps of deductive reasoning in general. Since it typically is not a simple matter to record data pertaining to both of these dependent variables in a given experiment, experimenters frequently choose to study one at the expense of the other. The implication of the noted discrepancies is that any choice between response accuracy and response latency must result from carefully reasoned preference rather than simple expediency or caprice.

Finally, the fact that the main effect F ratios noted in relation to implication-reasoning answers were four to five times larger than those noted in relation to the latencies of these same answers suggests that the latter of the two variables probably is fixed earlier in life and remains relatively more resistant to manipulations such as those examined in the present experiment.

EXPERIMENT II: THE PROPORTIONALITY SCHEMA

The second experiment was designed to investigate ontogenetic changes in two cognitive skills that Inhelder and Piaget (1958) claim are indices of the ability to handle qualitative proportions, viz. volume conservation and density conservation. The Genevan school holds that children's ability to reason in terms of a "proportionality schema" increases during the ages studied in the present experiment. Experiment II was conducted in the hope that five major questions pertaining to Piaget's specualtions about the "proportionality schema" would be answered.

1. Does the frequency of the conservations of volume and density increase from, say, age eight to age 15 as Piagetian theory predicts?

2. To what extent is it true--as Piaget claims--that the indices of volume conservation (both solid and liquid) and the index of density conservation employed in the present experiment tap the <u>same</u> cognitive skill (the "proportionality schema")?

3. Is the Genevan school correct in its assumption that an adequate conception of volume must invariably precede an adequate conception of density?

4. Are there any nonlinear features about the age trends in answers and/or explanations given by <u>S</u>s in relation to volume conservation and density conservation?

5. Do older <u>S</u>s tend to 'learn' density conservation more readily than younger <u>S</u>s?

Minor questions concerning the relation between conservation answers vs. conservation explanations and possible differences in age trends for

these two estimates of conservation also were of interest.

Method

Subjects

The same 72 Ss employed in experiment I also participated in experiment II. One-half of the Ss (randomly) participated in experiment I first and then in experiment II. The remaining Ss did the reverse.

Apparatus

The same recorder, headphones, and microphones employed in experiment I were retained for experiment II. A large (5000 ml) water-filled beaker and three 50gm balls of rubber-base clay (blue, red, yellow) were used to assess density conservation. Each of the clay balls was more dense than water. The pieces of clay, a medium-sized rubber band, and an amber-colored glass 14.5cm high and 6.0cm in diameter were used to assess solid volume conservation. Two uncolored glasses 14.5cm high and 6.0cm in diameter, one uncolored glass 14.5cm high and 3.0cm in diameter, and one uncolored glass 14.5cm high and 9.0cm in diameter were used to assess liquid volume conservation.

Procedure

Three basic assessments were executed in experiment II: density conservation; solid volume conservation; and liquid volume conservation. Since Inhelder and Piaget (1958) do not separate liquid and solid volume conservations when making predictions, it was deemed advisable to measure both aspects of volume conservation.

Each of the six possible orders of presentation was assigned randomly to four Ss from each of the three age-groups. Because the assessment technique varied somewhat depending on the conservation concept, the

details of the three assessment procedures are given separately.

Density. S first was shown the 5000ml water-filled beaker and one of the 50gm clay balls was placed in S's hand. S was asked to predict whether or not the ball would fall to the bottom of the beaker if it was placed in the water and also to justify the prediction. The ball then was placed in the water and S observed that it sank to the bottom. Following this initial demonstration, the four steps of the assessment were executed:

Step 1: The ball was flattened into a "pancake" and S was asked whether or not the clay would float now that its shape had been altered. E required that S explain the basis for his prediction. The "pancake" then was placed in the water and S observed that it sank to the bottom.

Step 2: The "pancake" was removed from the water and dried. E then cut off approximately two-thirds of the clay and asked S if the remaining piece would float. Again, E required that S explain the basis for his prediction. The smaller piece of clay was placed in the water and S observed that it sank to the bottom.

Step 3: The reduced piece of clay was removed from the water and dried. From this E cut an even smaller piece that was approximately the size of a dime and S was asked whether or not this very small piece would float. Once again, S had to justify his prediction. The very small piece of clay then was placed in the water and S observed that it sank.

Step 4: After S had observed that the clay sank when it was in the four different sizes and shapes, E asked S, "Do you think we could ever get a piece of this clay small enough so that it would float?" S was not required to justify this last response.

Solid Volume. S first was shown an amber-colored glass which was

approximately two-thirds full of water and had a rubber band around it. S then was shown one (randomly selected) of the three balls of clay. E placed the ball in the glass and asked S to mark the new higher water level with the rubber band. After S had marked the new water level, E removed the ball, dried it, and rolled it into a "sausage." E then asked S the following (randomly ordered) three questions:

1. E: "If I place the sausage in the glass, will the water go above the rubber band now?" (S agreed or disagreed)

2. E: "If I place the sausage in the glass, will the water go below the rubber band now?" (S agreed or disagreed)

3. E: "If I place the sausage in the glass, will the water go right back to the rubber band?" (S agreed or disagreed)

Obviously, the correct sequence of answers involved disagreeing with the first two assertions and agreeing with the final assertion. S was given each question separately and subsequent questions were not posed until the previous question had been answered. Following the three questions, S was asked to explain the basis for his responses.

The method of requiring that Ss both agree and disagree with E's assertions during conservation assessment orginally was developed by Rothenberg (1969; Rothenberg & Courtney, 1969). Briefly, Rothenberg argues that the data of studies such as those of number conservation in very young children (Mehler & Bever, 1967) are confounded by the fact that Ss tend to agree with an E more frequently than they disagree. Since this state of affairs ushers in the possibility of "false positives," Rothenberg suggests that the ideal sequence of conservation assessment questions is structured so that S must both agree and disagree with E to be judged a conserver. The separate treatment of each of the three questions

enumerated above was precipitated by Rothenberg's contentions.

Liquid Volume. Ss first were shown two uncolored glasses 14.5cm high and 6.0cm in diameter. E then put one liter of water in each glass and E asked S if the water in the first glass took up the same amount of space or room as the water in the second glass and vice versa. If S agreed (and all Ss did), the water in one of the two identical glasses was poured into an uncolored glass 14.5cm high and 3.0cm in diameter. S then was asked two questions.

1. E: "Does the water in these two glasses take up the same amount of space or room?" (S agreed or disagreed)

2. E: "Does the water in one of the glasses take up more space or room than the water in the other glass?" (S agreed or disagreed)

S also was required to explain the rationale for his responses. As was indicated for solid volume, the procedure of using questions that involved both agreeing and disagreeing with E was modeled after Rothenberg's technique (Rothenberg, 1969; Rothenberg & Courtney, 1969).

The water in the 14.5cm x 3.0cm glass then was returned to the original 14.5 x 6.0 glass. S was asked whether or not the water in the two original glasses still took up equal amounts of space. If S agreed (and all Ss did), the water from one of the glasses was poured into a glass 14.5cm x 9.0cm and S was asked the same two questions as before. S again was required to justify his answers.

Dependent Variables

It has been noted elsewhere (Brainerd & Allen, 1970) that there exists some controversy concerning the appropriate dependent variables to measure as part of conservation assessment. Gruen (1966), Griffths, Shantz, and Sigel (1967), and Rothenberg (1969; Rothenberg & Courtney, 1969) have

contributed papers to this controversy. The most important psychometric feature of this debate concerns whether or not <u>S</u>'s explanations of his agreements-disagreements with <u>E</u>'s assertions should be <u>necessary</u> to the 'conserver-nonconserver' judgment. Some investigators (e.g., Bruner, 1964) have made the conserver-nonconserver judgment only on the basis of <u>Ss</u>' agreements-disagreements. Others (Smedslund, 1961a) also have required that <u>Ss</u> adequately explain their agreements-disagreements to be judged conservers.

Regardless of which of the preceding criteria is chosen, one is left with an 'all-or-none' model of conservation. Rather than choose either alternative, the data of both answers and explanations were combined and analyzed in the present experiment. Most generalizations are based on <u>F</u> ratios deriving from the <u>combined</u> answer and explanation data; however, the answer and explanation data also were analyzed separately to provide psychometric justification for employing a 'combined' criterion. The scoring procedure for both answers and explanations is given below.

<u>Answers</u>. In all three conservation assessments, appropriate agreements and disagreements were assigned a '1', while inappropriate agreements and disagreements were assigned a '0'. This method yielded a five-point range (0-4) for density and liquid volume and a four-point range (0-3) for solid volume.

<u>Explanations</u>. The rationales given by the <u>Ss</u> were sorted into five categories which are summarized below. Explanations were sorted by two judges and an overall interjudge agreement of .93 was noted. The first of the five categories below (inversion reversibility) was not deemed relevant to density conservation and was used only in relation to volume conservation explanations.

1. Inversion reversibility: S made reference to the fact that perceptual deformations performed by E can always be reversed. (e.g., "You could always pour it back.")

2. Reciprocity reversibility: S made reference to the fact that changes in certain dimensions of an object are compensated by changes in related dimensions (e.g., "This one is now taller but it is also skinnier"). Identity explanations (e.g., "It's the same") also fall in this category.

3. Conceptual irrelevant: These are explanations that, like categories 1 and 2, are not based on simple perceptual features. Nonetheless, they are irrelevant to why conservation actually obtains. For example, to say, "It's the weight that makes it so," is not relevant to either volume or density.

4. Perceptual Irrelevant: These are the explanations that typically follow incorrect conservation answers. Reference is made to deceptive perceptual features of the stimuli (e.g., "It's skinnier so it must take up more space").

5. Don't know: This category is self-explanatory.

Explanations falling in the first and second categories were assigned a '1' and explanations falling in the remaining three categories were assigned a '0'. Ss' total scores for explanations were added to their total answer scores for purposes of the preliminary analyses.

Results

General Age Trends

One way analyses of variance were performed on the dependent variable data. The F ratios for each of the three proportionality schema estimates appear in Table 11. Separate F ratios were calculated for each of the

Table 11

Age Trend F Ratios for Proportionality One Way Analyses of Variance

Concept	Criterion		
	Combined	Answers	Explanations
Density Conservation[a]	19.90**	10.71*	16.31**
Solid Volume Conservation[a]	16.99**	15.52**	10.50*
Liquid Volume Conservation[a]	8.89*	11.56*	2.40

[a] $\underline{df}$ = 1/69 for all cells

*p < .005

**p < .0005

three possible 'conservation criteria' (combined, answers only, explanations only). In general and by all criteria, the age differences in density and volume conservations were large and highly significant. For both density and solid volume, the _F_ ratio was larger for the combined criterion than were the separate _F_'s for answers and explanations; this suggests superior reliability of the combined criterion. This generalization does not hold for liquid volume, because explanations failed to discriminate the age-groups.

The age trends for density and volume conservations reported in Table 11 also were subjected to orthogonal polynomials analysis to determine whether or not there were any nonlinear features about these trends. The results of these analyses appear in Tables 12, 13, and 14 for density, solid volume, and liquid volume respectively. As was the case in experiment I, it was the linear terms of the orthogonal polynomials analyses that were the most highly significant and accounted for most of the variance. In addition, small but significant quadratic trends were noted for both density and solid volume conservations. In both instances, however, the quadratic trend was limited to _explanations_ and failed to show up for answers. For liquid volume conservation, complete linearity was the rule with all quadratic components being smaller than one.

The proportion of conservation explanations falling in each of the previously mentioned categories is reported by age-group and by conservation in Table 15. In line with the significant _F_ ratios of Table 11, it is apparent that _Ss_' conceptual justifications for density and solid volume conservations improved steadily with age. Alternatively, it also is apparent that the quality of _Ss_' rationales for liquid volume conservation do not vary appreciably between age-groups studied in the present experiment.

Table 12

Orthogonal Polynomials Analyses for Density Conservation

Source	SS	df	MS	F	p
Combined Criterion					
Between	133.86	2			
Linear	117.18	1	117.18	37.84	$<.0005$
Quadratic	16.68	1	16.68	4.96	$<.05$
Error	232.09	69	3.36		
Total	365.95	71			
Answers					
Between	38.68	2			
Linear	35.01	1	35.00	19.38	$<.0005$
Quadratic	3.67	1	3.67	2.03	
Error	124.64	69	1.81		
Total	163.32	71			
Explanations					
Between	28.78	2			
Linear	24.10	1	24.10	27.34	$<.0005$
Quadratic	4.68	1	4.68	5.19	$<.05$
Error	60.88	69	.88		
Total	89.66	71			

Table 13

Orthogonal Polynomials Analyses for Solid Volume Conservation

Source	SS	df	MS	F	p
Combined Criterion					
Between	35.58	2			
Linear	33.35	1	33.35	32.92	<.0005
Quadratic	2.23	1	2.23	2.21	
Error	69.92	69	1.01		
Total	105.50	71			
Answers					
Between	20.86	2			
Linear	20.00	1	20.00	29.77	<.0005
Quadratic	.86	1	.86	1.27	
Error	46.42	69	.67		
Total	67.28	71			
Explanations					
Between	3.36	2			
Linear	2.72	1	2.72	17.00	<.0005
Quadratic	.64	1	.64	4.01	<.05
Error	11.08	69	.16		
Total	14.44	71			

Table 14

Orthogonal Polynomials Analyses for Liquid Volume Conservation

Source	SS	*df*	MS	*F*	*p*
Combined Criterion					
Between	88.11	2			
Linear	85.33	1	85.33	17.95	<.0005
Quadratic	2.78	1	2.78	<1	
Error	341.84	69	4.95		
Total	429.95	71			
Answers					
Between	52.78	2			
Linear	52.12	1	52.12	22.83	<.0005
Quadratic	.66	1	.66	<1	
Error	157.54	69	2.28		
Total	210.32	71			
Explanations					
Between	4.08	2			
Linear	3.53	1	3.53	4.14	<.05
Quadratic	.55	1	.55	<1	
Error	58.79	69	.85		
Total	62.87	71			

Table 15

Percent of Explanations Falling in each Category

Grade and Concept		Explanation Category 1	2	3	4	5
Third Graders	Density	...	46.6	57.4	12.0	26.0
	Solid	0.0	13.8	51.7	20.7	13.8
	Liquid	24.1	35.6	4.4	35.9	0.0
Sixth Graders	Density	...	7.8	44.4	29.6	18.2
	Solid	10.0	13.3	46.7	16.7	13.3
	Liquid	23.1	42.3	9.6	25.0	0.0
Ninth Graders	Density	...	33.7	55.8	6.8	4.2
	Solid	28.0	28.0	35.7	8.3	0.0
	Liquid	16.7	50.3	22.9	2.1	0.0

The simple and multiple coefficients of correlation between the three dependent variables of the present experiment are given in Table 16. All multiple coefficients are significant and moderate to moderately high portions of the variance are accounted for by these values. Table 16 suggests that it is true--as Inhelder and Piaget (1958) contend--that the conservations of density and volume tap the same or related cognitive skills.

<u>Invariant Sequences</u>

For purposes of evaluating Inhelder and Piaget's (1958) contention that volume conservation invariably precedes density conservation, each of the 72 <u>S</u>s was classified as either a 'conserver' or a 'nonconserver' of density, solid volume, and liquid volume. This judgment was based entirely on <u>S</u>s' <u>answers</u> to conservation questions. The criterion for inclusion in each 'conserver' group was a perfect sequence of answers (agreements-disagreements) made in relation to the particular conservation under consideration. Thus, liquid volume conservers had given five consequtive correct agreements-disagreements, density conservers had given four consequtive correct agreements-disagreements, and solid volume conservers had given three consequtive correct agreements-disagreements.

These data were placed in four-fold contingency tables for significance testing (Tables 17 and 18). As McManis (1969) has pointed out, the upper right-hand and lower left-hand cells of such four-fold tables are the crucial cells pertaining to the question of invariant sequence. For the assumption of invariant sequence to be supported, the number of <u>S</u>s appearing in the lower left-hand cell should be significantly larger than the number of <u>S</u>s appearing in the upper right-hand cell. Theoretically, <u>no</u> <u>S</u>s should appear in the upper right-hand cell if the invariant sequence assumption is veridical. However, such a result would be expected only

Table 16

Simple and Multiple Correlations Between the Three Dependent Variables

Grade and Concept		Conservation Concept: Density	Solid	Liquid	Multiple
Third Graders	Density	...	.402*	.118	.403*
	Solid	.402*	...	.410**	.542****
	Liquid	.118	.410**	...	.412**
Sixth Graders	Density	...	.481***	.353*	.596****
	Solid	.481***	...	.004	.513***
	Liquid	.353*	.004	...	.401*
Ninth Graders	Density	...	.352*	.534****	.596****
	Solid	.352*	...	.375*	.513***
	Liquid	.534****	.375*		.570****

*$p < .05$

**$p < .025$

***$p < .01$

****$p < .005$

Table 17

Relationship Between the Conservations of Density and Solid Volume

Density	Solid Volume	
	Present	Absent
Present	25	2*
Absent	16*	29

*$p = 4.87 \times 10^{-4}$

Table 18

Relationship Between the Conservations of Density and Liquid Volume

Density	Liquid Volume	
	Present	Absent
Present	26	2*
Absent	26*	18

*$p = 1.41 \times 10^{-6}$

in the event that both measures were perfectly or near-perfectly reliable. Given the typically average to moderate reliabilities of conservation problems (Almy, Chittenden, & Miller, 1966), such a 'textbook' result was not anticipated.

Siegel's (1956) binomial test was used to test the significance of the differences between the upper right-hand and lower left-hand cells of Tables 17 and 18. The significance level was quite high for both density vs. solid volume and density vs. liquid volume. In light of the previously mentioned fact that the reliabilities of conservation problems frequently are marginal, Tables 17 and 18 would seem to provide strong support indeed for a volume-density invariant sequence.

<u>Age Trends in Conservation Learning</u>

Here we shall consider only the density conservation data of the third and sixth grade <u>S</u>s. The question that is of interest is whether or not there was a difference in the extent to which <u>S</u>s of different ages 'learned' the density conservation concept in the course of the assessment procedure. A ceiling effect for ninth grade <u>S</u>s (i.e., almost perfect agreement-disagreement performance) restricted the investigation of this question to third and sixth grade <u>S</u>s.

It will be remembered that in the final step of the density conservation assessment <u>E</u> posed the following question to <u>S</u>, "Do you think we could ever get a piece of this clay small enough so that it would float?" Since <u>E</u> had shown <u>S</u> that the clay did not float in the preceding three steps of the assessment, it was thought that the extent to which <u>S</u>s answered 'no' to this final question represented some estimate of how much they had been influenced by the preceding steps. Hence, third and sixth grade <u>S</u>s were classified according to two criteria. First, they were

divided into those Ss who made at least one error during the first three steps and those who had performed perfectly during the first three steps. Second, the former or critical group was divided into those Ss who had answered 'no' to the final question and those who had answered 'yes'.

These data appear in Table 19. It is apparent from the significant chi-square value that sixth graders who made one or more errors during the first three steps were more likely to 'learn' by their experiences than the third graders who also made one or more errors during the first three steps.

Discussion

Returning to the first of the five questions posed as a prelude to experiment II, the data indicate that Ss' performances in relation to the three indices of the proportionality schema do improve linearly between the ages of eight to 15. This prediction of Piagetian theory is supported by the large and highly significant F ratios of Table 11 for both conservation answers and explanations.

Second, the simple and multiple correlation coefficients obtaining between density, solid volume, and liquid volume generally were significant and, therefore, suggest that these variables are, to some extent, all estimates of the same cognitive skill. This seems to be a significant result in that it represents the first clear empirical support for Inhelder and Piaget's (1958) assertion that density and volume both are conserved as a result of a single cognitive development, viz. the child's acquisition of the formal-operational "proportionality schema." Although Inhelder and Piaget present data from different groups of Ss which suggest that the ontogenies of density and volume concepts are roughly parallel,

Table 19

Age Differences in the Learning of Density Conservation

Age-Group	Ss Making some Density Conservation Errors	
	Ss Answering 'yes' to the Final Question	Ss Answering 'no' to the Final Question
Third Graders[a]	9	10
Sixth Graders[a]	15	0

[a] $\chi^2 = 8.79$; $p < .005$

such evidence is hardly an adequate substitute for the intercorrelational analyses that are necessary to establish that these measures are associated reliably <u>within</u> <u>Ss</u>. Thus, to a significant degree, the three dependent variables seem to measure the same underlying ability; whether or not this 'ability' is an incipient "proportionality schema" is, of course, another matter.

The argument that an adequate concept of volume invariably precedes an adquate concept of density also receives strong support from the data of the present experiment. This result is important from the general perspective of Piagetian theory in that it represents a point in favor of 'logical' methods of predicting concept acquisition. <u>Prima</u> <u>facie</u>, volume seems to be a more complex concept than density. While <u>Ss</u> <u>could</u> conserve density merely by thinking of it as a simple invariant characteristic of each substance, volume remains a property that is not observed or measured easily. The logic of physics, on the other hand, dictates that density is a more complex concept than volume, since density is defined as weight-per-unit-volume. In so far as the Genevan school typically employs such 'logical' approaches, the data of the present experiment might well be construed as support for a general method of making predictions as well as for a specific prediction.

There were some nonlinear features about the age trends for density and solid volume conservations. In general, however, the linear components were overwhelmingly larger than the quadratic components. In the two instances where quadratic components were found to be significant, the significance level was not great ($p < .05$) and the corresponding linear components were four to five times as large as the quadratic terms. In addition, the fact that significant quadratic terms were noted only in

relation to conservation explanations but not conservation answers suggests that these nonlinear results do not merit more general consideration.

The significance of the fact that older Ss tended to 'learn' density conservation more readily than did younger Ss also should be considered briefly. As previously noted, Piagetian theory predicts and developmental psychologists generally have believed that the extent to which conservation concepts are trainable is partly a function of the age of S. Given a particular conservation concept and a particular training technique, then, the prediction is that older Ss should be affected more by their experiences than younger Ss. Pinard and Larendeau (1969) argue that this prediction follows directly from the fact that among nonconservers of a particular concept, older Ss are more likely to possess already an operational basis for the concept and, therefore, are nearer to acquiring the conservation 'naturally' than are younger Ss (i.e., the closer S is to 'natural' acquisition, the more he or she presumably profits from the learning experience).

As Brainerd and Allen (1970) have pointed out, however, this prediction has not been supported as yet by studies of concrete conservation training. Some of the least successful conservation training experiments (e.g., Wohlwill & Lowe, 1962; Smedslund, 1963b) involved Ss that were older than the Ss used in some of the most successful of these experiments (e.g., Gelman, 1969; Wallach & Sprott, 1964). This result is not deemed to be particularly damaging to the preceding hypothesis by the present author, because of the relatively narrow age ranges used in previous conservation training experiments. Since preschoolers typically are more difficult to obtain than elementary schoolers and since a large number of second graders have acquired one or more of the concrete conservations,

kindergarten and first grade Ss have been employed almost exclusively. Given this constricted age-range (one year), perhaps it is not surprising that age has not been shown to be associated with amount of conservation learning.

Alternatively, the data of the present experiment clearly support the presumed association between age and conservation learning. This result is not seen as descrepant with the previously mentioned lack of support from earlier conservation training experiments. Rather, it is likely that this result can be attributed to the fact that the age difference between the groups of Table 19 is three times greater than has been the rule for similar experiments.

Finally, when taken together the data of Table 11, 12, 13, and 14 suggest that the 'controversy' concerning whether or not conservation explanations should be included as necessary criteria to be considered when judging Ss to be conservers or nonconservers may well be a pseudocontroversy. In so far as conservation answers and explanations manifest similar age trends, there seems to be no good reason for not combining these two criteria to yield a single criterion. This combined criterion is characterized by both higher reliability and susceptibility to parametric analyses. Such a combined criterion eliminates all-or-none conservation judgments with their large built-in random error.

GENERAL DISCUSSION

In this section it remains to consider the within Ss relations between the results of the two experiments. As indicated earlier, the same Ss were employed in both experiments so that implication-reasoning and the three indices of the "proportionality schema" could be compared within Ss. It will be remembered that on the basis of Piagetian theory one would predict that implication-reasoning ability should be associated positively with each of the three indices of the "proportionality schema." To reiterate, Piaget argues (e.g., 1949, 1953; Inhelder & Piaget, 1958) that the "sixteen propositional operations" (of which implication is one) and the "formal-operational schemata" (of which proportionality is one) are alternative manifestations of the same underlying cognitive structure, the group INRC. If two measurable cognitive skills are manifestations of the same incipient cognitive structure, it certainly seems legitimate to expect that estimates of two such cognitive skills should be related significantly related within Ss.

To ascertain the strength of the relation between implication-reasoning ability and the proportionality schema indices, the scores of individual Ss on these variables were correlated within each age-group. Because the semantic treatment of experiment I affected implication-reasoning ability significantly, the total scores of all the Ss in the -S condition were corrected so that their scores did not differ significantly from the other 12Ss in their particular age-group. To accomplish this, each -S score was transformed linearly by adding to it the difference between the means of conditions +S and -S for the particular age-group

that it was from. These corrected implication-reasoning values were correlated separately with density, solid volume, and liquid volume scores and also with a summed proportionality schema estimate based on Ss' total scores on all three variables of experiment II. The 12 coefficients appear in Table 20.

It is obvious from Table 20 that the predicted intrasubject association between the dependent variables of experiments I and II did not materialize. While the predicted correlations would be positive and significant, the obtained values range from negative and insignificant through zero to positive and insignificant. Clearly, if Piaget's assertions are correct, we must wait for future data to bear them out.

Of course, the inferences that can be made based on Table 20 are limited. Aside from the obvious problem of arguing for the null hypothesis, there is always the limitation of reliability to consider. As Almi et al. (1966) have pointed out, the respective reliabilities of Piaget's conservation problems are not overpowering. If the 'true' relation between implication-reasoning and the proportionality schema estimates is a small to moderate one, the reliability factor may be responsible for the absence of positive and significant validity correlations. However, the relatively large F ratios obtained in relation to the age trends for density and volume conservations, as well as the generally significant correlations of these variables with each other, lead one to suspect that the reliabilities of the proportionality schema estimates are quite adequate. Since the implication-reasoning reliability estimates are high (Table 8), reliability should be a prohibitive factor only in the event that the relation between implication-reasoning and the proportionality schema variables is indeed a precise one.

Table 20

Correlations of Implication Answers with Proportionality Variables

Age-Group[a]	Proportionality Schema Variables: Density	Solid	Liquid	Combined
Third Graders	-.230	.124	-.102	-.149
Sixth Graders	-.044	-.037	-.115	-.124
Ninth Graders	-.166	-.302	-.035	-.139

[a]N = 24 for all cells

Even if it is true--as the data suggest--that there is no significant relation between the dependent variables of experiments I and II, this fact does only limited damage to the Piagetian assumption that the "propositional operations" and the "formal-operational schemata" are manifestations of the same structural reality, the group INRC. The present experiments were focused on only one propositional operation and one formal-operational schema. Fifteen propositional operations and an indeterminant number of formal-operational schemata remain to be studied and interrelated. Unless similar data are forthcoming from studies of these other variables, the importance of the present data vis-à-vis the assumption of a single underlying determinant of the two categories of formal operations will remain limited.

Conclusions and Implications

In general, it can be said that the predictions pertaining to each experiment were well supported. In experiment I, age of S, semantic content of propositions, and syntactic order of propositions each was shown to have some influence on the dependent variables. In addition, a first order interaction between age and syntax suggested that--as Piagetian theory predicts--an effective order manipulation may have a greater influence on some age-groups than on others. The data of experiment II confirmed the major age trend and invariant sequence predictions offered by the Genevans in relation to the proportionality schema variables.

On the other hand, the interexperiment prediction concerned with the presumed relation between the dependent variables of the two experiments clearly was not supported. This result highlights one of the dangers of the clinical method employed by Piaget and his collaborators. Conclusions based on probabilistic inference frequently are called for when such a method is employed. For example, members of the Genevan school typically consider empirical evidence indicating that two or more aspects of cognitive development are characterized by similar age trends to be a sufficient basis for inferring that the cognitive variables are manifestations of the same underlying structure. This technique has been used by Inhelder and Piaget (1958) to bridge the gap between the parallel developmental trends they identified for the "propositional operations" and the "formal-operational schemata." However, the data of Table 20 do not support their inference.

This conflict is particularly unfortunate because it probably is unnecessary. When one wishes to establish an unequivocal relation between variables displaying similar developmental trends, there is but a single additional step involved in appropriately quantifying the variables for correlation. If Inhelder and Piaget had completed the analysis of their data in this matter, the status of the results in Table 20 would be clearer. The implications of these considerations, of course, are that probabilistic inference and statistical probability are two different (though related) things, and the latter is always preferrable to the former where choice is possible.

Some final mention should be made of the significance of the age trends in implication-reasoning response latencies noted in experiment I. It will be remembered that the analysis of variance revealed that response latencies <u>decreased</u> linearly between the ages of eight to 15. The surprising feature of this result is that it is clearly at odds with the developmental trends identified by Kagan and his associates (e.g., Kagan, Moss, & Sigel, 1963) in relation to their reflectivity-impulsivity dimension of cognitive style. Kagan's reflectivity-impulsivity results, like those appearing in Tables 5 and 7 of experiment I, are based on response latency measures. Contrary to the results of experiment I, however, Kagan finds that response latencies increase as a function of age. This trend is thought to be a function of a generalized tendency to "reflect" on all matters as one ages (a little generation gap here). Fortunately, there are at least two differences between Kagan's studies and experiment I of the present report that may be responsible for these inverse age trends.

First and perhaps least important, Kagan and his associates typically have studied Ss that were much younger than those employed in the present research. While Ss from up to and including the first year of high school were investigated in experiment I, the reflectivity-impulsivity age trends tend to be based on early elementary school Ss. Although much older Ss were used to study implication-reasoning, the present author finds it difficult to believe that a single cognitive variable characterized by a clear and consistent trend within one age-range would be characterized by the inverse trend within a slightly older age-range.

Second and seemingly more important, there are marked differences between the cognitive requirements of the implication-reasoning problems and the cognitive requirements of the problems employed in studies of reflectivity-impulsivity. The emphasis of the implication-reasoning problems clearly was on deductive species of reasoning. Alternatively, both Siegelman (1969) and Ward (1968) suggest that the emphasis of reflectivity-impulsivity studies is on perceptual-inductive reasoning. Reflectivity-impulsivity studies usually involve some variety of the matching-to-sample technique (e.g., the Matching Familiar Figures Test).

It does not seem unreasonable to conclude that latency intervals characteristic of matching-to-sample tasks are largely 'search times'--i.e., intervals during which S overtly scans a perceptually present array. On the other hand, no additional external information is required as part of the implication-reasoning problems and the latencies characteristic of such problems should be largely 'think times'--i.e., intervals during which S covertly combines and recombines given propositions. In so far as these requirements involve different or even reciprocal cognitive processes, it perhaps is not farfetched to expect nonequivalent developmental trends. At the very least, the latency data of experiment I suggest that the reflectivity-impulsivity dimension probably is less general and more task-dependent than previously has been acknowledged (e.g., Kagan, Rosman, Day, Albert, & Phillips, 1964).

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APPENDICES

APPENDIX I

ASSESSMENT QUESTIONS FOR THE POSITIVE SEMANTICS CONDITION

1. If Jack (Jill) sweeps the kitchen floor, then Jack's (Jill's) mother will be very pleased. Whenever Jack's (Jill's) mother is very pleased, he (she) gets all the dessert he (she) wants.

a. If Jack (Jill) sweeps the kitchen floor, then what else will occur? Anything else? Could it be that Jack (Jill) won't get all the dessert that he (she) wants?

b. If Jack (Jill) sweeps the kitchen floor, then could it be that Jack (Jill) <u>won't</u> get all the dessert that he (she) wants?

c. If Jack (Jill) gets all the dessert he (she) wants sometime, could it be that Jack (Jill) <u>didn't</u> sweep the kitchen floor?

d. If Jack (Jill) <u>doesn't</u> sweep the kitchen floor, then what else will occur? Anything else? Could it be that Jack (Jill) could still get all the dessert that he (she) wants?

2. If Jack (Jill) washes the dishes, then Jack's (Jill's) father will be very pleased. Whenever Jack's (Jill's) father is very pleased, Jack (Jill) gets 50¢.

a. If Jack (Jill) washes the dishes, then what else will occur? Anything else? Could it be that Jack (Jill) won't get 50¢.

b. If Jack (Jill) washes the dishes, then could it be that Jack (Jill) <u>won't</u> get 50¢.

c. If Jack (Jill) gets 50¢ sometime, could it be that Jack (Jill) <u>didn't</u>

wash the dishes?

d. If Jack (Jill) <u>doesn't</u> wash the dishes, then what else will occur? Anything else? Could it be that Jack (Jill) could still get 50¢.

3. If Jack (Jill) gets all 'A's' on his (her) report card, then both of Jack's (Jill's) parents will be very pleased. Whenever both of Jack's (Jill's) parents are very pleased, Jack (Jill) gets to stay up later as a reward.

a. If Jack (Jill) gets all 'A's' on his (her) report card, then what else will occur? Anything else? Could it be that Jack (Jill) won't get to stay up later?

b. If Jack (Jill) gets all 'A's' on his (her) report card, then could it be that Jack (Jill) <u>won't</u> get to stay up later as a reward?

c. If Jack (Jill) gets to stay up later as a reward sometime, could it be that Jack (Jill) <u>didn't</u> get all 'A's' on his (her) report card?

d. If Jack (Jill) <u>doesn't</u> get all 'A's' on his (her) report card, then what else will occur? Anything else? Could it be that Jack (Jill) could still stay up later?

4. If Jack (Jill) does well on an English test, then Jack's (Jill's) teacher will be very pleased. Whenever Jack's (Jill's) teacher is very pleased, Jack (Jill) gets less schoolwork to do.

a. If Jack (Jill) does well on an English test, then what else will occur? Anything else? Could it be that Jack (Jill) won't get less schoolwork?

b. If Jack (Jill) does well on an English test, then could it be that Jack (Jill) <u>won't</u> get less schoolwork to do?

c. If Jack (Jill) gets less schoolwork to do sometime, could it be that Jack (Jill) <u>didn't</u> do well on an English test?

d. If Jack (Jill) <u>doesn't</u> do well on an English test, then what else will

occur? Anything else? Could it be that Jack (Jill) could still get less schoolwork to do?

5. If Jack (Jill) mows the lawn, then Jack (Jill) works very hard at it. Whenever Jack (Jill) works very hard at something, Jack's (Jill's) father treats him (her) to an icecream cone.

a. If Jack (Jill) mows the lawn, then what else will occur? Anything else? Could it be that Jack (Jill) won't get an ice cream cone?

b. If Jack (Jill) mows the lawn, then could it be that Jack's (Jill's) father won't treat him (her) to an icecream cone?

c. If Jack's (Jill's) father treats him (her) to an icecream cone sometime, could it be that Jack (Jill) didn't mow the lawn?

d. If Jack (Jill) doesn't mow the lawn, then what else will occur? Anything else? Could it be that Jack (Jill) will still get an icecream cone?

6. If Jack (Jill) has a birthday, then all of Jack's (Jill's) relatives come to see him (her). Whenever all of Jack's (Jill's) relatives come to see him (her), Jack (Jill) gets a lot of money.

a. If Jack (Jill) has a birthday, then what else will occur? Anything else? Could it be that Jack (Jill) won't get a lot of money?

b. If Jack (Jill) has a birthday, then could it be that Jack (Jill) won't get a lot of money?

c. If Jack (Jill) gets a lot of money sometime, could it be that Jack (Jill) didn't have a birthday?

d. If Jack (Jill) doesn't have a birthday, then what else will occur? Anything else? Could it be that he (she) could still get a lot of money?

7. If Jack (Jill) helps his (her) mother with the shopping, then Jack's (Jill's) mother does not have much work to do. Whenever Jack's (Jill's) mother does not have much work to do, she fixes Jack's (Jill's) favorite

food for dinner.

a. If Jack (Jill) helps his (her) mother with the shopping, then what else will occur? Anything else? Could it be that Jack (Jill) won't get his (her) favorite food?

b. If Jack (Jill) helps his (her) mother with the shopping, then could it be that Jack's (Jill's) mother won't fix Jack's (Jill's) favorite food?

c. If Jack's (Jill's) mother fixes his (her) favorite food for dinner sometime, could it be that Jack (Jill) didn't help mother with the shopping?

d. If Jack (Jill) doesn't help mother with the shopping, then what else will occur? Anything else? Could it be that Jack (Jill) could still get his (her) favorite food for dinner?

8. If Jack (Jill) plays a part in a school play, then he (she) will do a very good job. Whenever Jack (Jill) does a very good job at something, Jack (Jill) gets an award from the school.

a. If Jack (Jill) plays a part in a school play, then what else will occur? Anything else? Could it be that Jack (Jill) won't get an award?

b. If Jack (Jill) plays a part in a school play, then could it be that Jack (Jill) won't get an award from the school?

c. If Jack (Jill) gets an award from the school sometime, could it be that Jack (Jill) didn't play a part in a school play?

d. If Jack (Jill) doesn't play a part in a school play, then what else will occur? Anything else? Could it be that Jack (Jill) could still get an award from the school?

APPENDIX II

ASSESSMENT QUESTIONS FOR THE NEGATIVE SEMANTICS CONDITION

1. If Jack (Jill) throws a rock at a window, then the window will break. Whenever Jack (Jill) breaks a window, Jack's (Jill's) mother sends him (her) to bed without supper.

a. If Jack (Jill) throws a rock at a window, then what else will occur? Anything else? Could it be that Jack (Jill) won't be sent to bed without supper?

b. If Jack (Jill) throws a rock at a window, then could it be that Jack's (Jill's) mother <u>won't</u> send him (her) to bed without supper?

c. If Jack's (Jill's) mother sends him (her) to bed without supper sometime, could it be that Jack (Jill) <u>didn't</u> throw a rock at a window?

d. If Jack (Jill) <u>doesn't</u> throw a rock at a window, then what else will occur? Anything else? Could it be that Jack (Jill) could still be sent to bed without supper?

2. If Jack (Jill) complains about how bad things are, then Jack's (Jill's) friends get mad. Whenever Jack's (Jill's) friends get mad, they won't talk to Jack (Jill) anymore.

a. If Jack (Jill) complains about how bad things are, then what else will occur? Anything else? Could it be that Jack (Jill) will still be spoken to by his (her) friends?

b. If Jack (Jill) complains about how bad things are, then could it be that Jack's (Jill's) friends will still talk to him (her)?

c. If Jack's (Jill's) friends won't talk to him (her) sometime, could it be that Jack (Jill) didn't complain about how bad things are?

d. If Jack (Jill) doesn't complain about how bad things are, then what else will occur? Anything else? Could it be that Jack's (Jill's) friends could still not talk to him (her)?

3. If Jack (Jill) plays with matches, then he (she) will start a fire in his (her) house. Whenever Jack (Jill) starts a fire in his (her) house, Jack's (Jill's) father takes away his (her) allowance.

a. If Jack (Jill) plays with matches, then what else will occur? Anything else? Could it be that Jack (Jill) could still have his (her) allowance?

b. If Jack (Jill) plays with matches, then could it be that Jack's (Jill's) father won't take away Jack's (Jill's) allowance?

c. If Jack's (Jill's) father takes away his (her) allowance sometime, could it be that Jack (Jill) didn't play with matche

d. If Jack (Jill) doesn't play with matches, then what else will occur? Anything else? Could it be that Jack's (Jill's) father could take away Jack's (Jill's) allowance anyway?

4. If Jack (Jill) leaves his (her) bicycle in the driveway, then Jack's (Jill's) father will hit the bike with his car. Whenever Jack's (Jill's) father hits Jack's (Jill's) bike with his car, he takes the bike away from Jack (Jill).

a. If Jack (Jill) leaves his (her) bicycle in the driveway, then what else will occur? Anything else? Could it be that Jack (Jill) could still have his (her) bike?

b. If Jack (Jill) leaves his (her) bike in the driveway, then could it be that Jack's (Jill's) father won't take the bike away from Jack (Jill)?

c. If Jack's (Jill's) father takes away Jack's (Jill's) bike sometime, could it be that Jack (Jill) didn't leave the bike in the driveway?

d. If Jack (Jill) doesn't leave his (her) bicycle in the driveway, then what else will occur? Anything else? Could it be that Jack's (Jill's) father could still take the bike away?

5. If Jack (Jill) forgets something he (she) is supposed to do, then Jack's (Jill's) father tells him (her) what to do. Whenever Jack's (Jill's) father tells him (her) what to do, Jack (Jill) feels ashamed.

a. If Jack (Jill) forgets something he (she) is supposed to do, then what else will occur? Anything else? Could it be that Jack (Jill) won't feel ashamed?

b. If Jack (Jill) forgets something something he (she) is supposed to do, then could it be the Jack (Jill) won't feel ashamed?

c. If Jack (Jill) feels ashamed sometime, could it be that Jack (Jill) didn't forget something he (she) was supposed to do?

d. If Jack (Jill) doesn't forget something he (she) is supposed to do, then what else will occur? Anything else? Could it be that Jack (Jill) could feel ashamed anyway?

6. If Jack (Jill) wispers in class, then Jack's (Jill's) teacher will get mad. Whenever Jack's (Jill's) teacher gets mad, Jack (Jill) has to stay after school.

a. If Jack (Jill) wispers in class, then what else will occur? Anything else? Could it be that Jack (Jill) won't have to stay after school?

b. If Jack (Jill) wispers in class, then could it be that Jack (Jill) won't have to stay after school?

c. If Jack (Jill) has to stay after school sometime, could it be that Jack (Jill) didn't wisper in class?

d. If Jack (Jill) doesn't wisper in class, then what else will occur?

Anything else? Could it be that Jack (Jill) could still stay after school?

7. If Jack (Jill) breaks one of his (her) mother's favorite dishes, then Jack's (Jill's) mother will be unhappy. Whenever Jack's (Jill's) mother is unhappy, Jack (Jill) cannot go outside and see his (her) friends.

a. If Jack (Jill) breaks one of his (her) mother's favorite dishes, then what else will occur? Anything else? Could it be that Jack (Jill) could still go outside and see his (her) friends?

b. If Jack (Jill) breaks one of his (her) mother's favorite dishes, then could it be that Jack (Jill) could still go outside and see his (her) friends?

c. If Jack (Jill) cannot go outside and see his (her) friends sometime, could it be that Jack (Jill) <u>didn't</u> break one of mother's favorite dishes?

d. If Jack (Jill) <u>doesn't</u> break one of mother's favorite dishes, then what else will occur? Anything else? Could it be that Jack (Jill) still could not go outside and see his (her) friends?

8. If Jack (Jill) doesn't come home right after school, then Jack's (Jill's) parents worry about him (her). Whenever Jack's (Jill's) parents worry about him (her), Jack (Jill) cannot watch television.

a. If Jack (Jill) doesn't come home right after school, then what else will occur? Anything else? Could it be that Jack (Jill) could still watch television?

b. If Jack (Jill) doesn't come home right after school, then could it be that Jack (Jill) could still watch television?

c. If Jack (Jill) cannot watch television sometime, could it be that Jack (Jill) <u>did</u> come home right after school?

d. If Jack (Jill) <u>does</u> come home right after school, then what else will occur? Anything else? Could it be that Jack (Jill) could not watch television still?