SPAN OF CONTROL: A CRITICAL REVIEW AND SOME NEW CONSIDERATIONS

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ABSTRACT

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Statement of the Problem

Span of control is a concept in administration that may be defined as the greatest number of persons that can be adequately supervised by another person. The problem of this study was: (1) to identify the factors believed to determine span of control, (2) to establish the extent to which these factors have been precisely and operationally defined, (3) to report these findings in a context that is cohesive and useful for further research, and (4) to identify the areas within this structure that require additional research or development before the structure can be used as the basis for a more-nearly adequate restatement of the principle of span of control.

Methodology

This study was undertaken to provide a basis in theory and research for a more-nearly adequate restatement of the principle of span of control. It

was felt necessary first to establish that the present and prevailing conception of it was inadequate. If it were possible to dispose of this notion satisfactorily it would then be possible to substitute another for it which would, hopefully, explain the same phenomena with greater accuracy. In essence, this was the strategy of the study as set out in Chapter I.

In Chapter II, a perspective was provided for the critical review of span of control. The historical background of the development of the traditional concept was given and the Graicunas-Urwick version of span of control was identified as the object to be examined Their rationale for span of control was described in detail.

Chapter III had three objectives: (1) to show that the Graicunas-Urwick version of span of control was not a rigorous proof, (2) to show further that it failed to take into consideration some of the variables regarded to be the determiners of span of control, and (3) to show that, in spite of the dissatisfaction with the principle as stated, there is a large degree of agreement that something like span of control exists.

The objective of Chapter IV should have been to develop a refined version of span of control. It should be noted that the study was begun by making a number of assumptions which essentially declared that span of control was determined by Man's limited abilities to receive, process, and transmit information. To show the plausibility of these assumptions, two models were constructed, the first describing two-way interpersonal communication and the second shwoing Man to be a receiver, processer and transmitter of information. From the communication model, it was possible to conclude that there is a minimum amount of information that must be received and processed if a supervisor is to keep track of all of the aspects of a changing situation. However, knowledge about the mechanics of information handling has not yet advanced to the point where a satisfactory model of Man as an information processer may be constructed. When this became known, the objective of the chapter was modified to providing a basis for the eventual refinement of span of control. This was done by first showing how the factors already acknowledged to be among the determiners of span of control could be accounted for jointly by the communication and information processing models. The examples of successful supervisory situations having broad spans of control were then reviewed and

explained in terms of the models. The chapter was concluded by pointing out areas needing further study before the joint communication/information processing model could be satisfactorily established as a basis for a refined notion of span of control.

Summary and Conclusions

Summary of Contributions Made by This Study

This study makes three kinds of contributions. First, it defines communication and information theory as fields which can contribute to the advancement of the study of administration. Second, it reveals to some students of administration a need to increase their familiarity with topics in mathematics and physics so that they may examine more critically some of the ideas that have been developed in this study. Finally, it describes specifically how a basis for the re-definition of span of control can be constructed from some of these ideas that are new to administration.

Conclusions

The study shows that many areas still need to be developed before Man will adequately understand his own information handling capabilities and the concept of span of control can be adequately and precisely restated. More must be known about how the

brain handles simultaneously received messages. The input and transducing capacities of the several sensory receptors must be measured as well as the relative importance of the channels to which they are sensitive. Coding efficiency must be increased and the optimum rate of auditory reception should be established. An understanding of monitoring and the interruption of processing would shed light on what information the mind attends to and regards as worth processing. The various processing operations need to be analyzed so that the time it takes to perform each elementary operation, such as the decoding of a symbol or the addition of two one-digit numbers, can be distinguished. Knowledge about the structure and the operation of storage and retrieval as well as storage capacity would help administrators to avoid the design of situations that would be beyond a supervisor's span of control.

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CHAPTER I

INTRODUCTION

Span of control is a concept in administration that is defined as the number of persons being supervised by another person.[1] It is also defined as a limit; that is, as the greatest number of persons that can be adequately supervised by another person.[2] As a limit, span of control was chosen as the subject of this study for two reasons. First, if such a limit exists and can be accurately determined, either as a number or a range of numbers, its effects upon organizational planning and structure would be felt everywhere. However, it was observed that, although this limit had been declared to exist and had been stated as a range of numbers, it has been widely ignored by administrators and organizational planners and strongly criticized on both theoretical and practical grounds throughout its history. A desire to resolve this dissonance first prompted the selection of this topic for study. Second, although the notion of

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[1]Stephen J. Knezevich, Administration of Public Education (New York: Harper & Brothers, 1962), pp. 68-69.

[2]Rudyard K. Bent and Lloyd E. McCann, Administration of Secondary Schools (New York: McGraw-Hill Book Co., 1960), p. 57.

limit suggests that the key to span of control might be found in research on human limitations, the demonstration that has been offered as a "proof" of the principle is entirely mathematical and has not been related to any empirical findings.[3] Observing that thirty years of criticism have not yet totally discredited span of control--for it is still discussed in current books on administration--it was then anticipated that research in the intervening years had produced findings that would either support the present principle or provide the evidence needed to restate it with greater accuracy. Although this hope finally proved false, it was for some time the principal reason for the study. It was not until the study was in progress that its justification was shifted to providing a basis in research and theory for a future restatement of the principle.

A. The Importance of this Study

This study is important for three reasons: (1) it identifies fields outside of administration that have significant contributions to make to administration, (2) it stresses a need for an inter-disciplinary approach in the development of a science of administration, and (3) it describes specifically how developments in other fields might be used in administration.

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[3]Waino W. Suojanen, "The Span of Control--Fact or Fable?" Advanced Management (November, 1955), pp.5-13. Reprinted in Max D. Richards and William A. Nielander (eds.) Readings in Management (Cincinnati: Southwestern Publishing Company, 1958), p. 550.

During the study it was observed that mathematicians and scientists had produced many theoretical, scientific and technological advances in the decade from 1940 to 1950. These included the successful operation of the first stored-program digital computer, the quantification of the concept of information and the development of mathematical theories of communication (Shannon) and decision (Wald).[4] The word cybernetics was coined (Wiener) and with it came the realization that there were many analogies between control and communication mechanisms in automata and men.[5] Then, it was noted that, while some researchers, notably psychologists, have adopted the ideas inherent in these advances, writers in administration have not yet begun to reflect their acquaintance with them. If, in fact, they have not yet become acquainted with these ideas, unfortunate consequences follow: they can neither apply these ideas correctly to administration nor can they contribute to their further development. Thus, it was felt that, if span of control were investigated in terms of some of these new ideas, it might help administrators to identify new fields in which they would need to extend their acquaintance.

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[4] Robert E. Machol (ed.) Information and Decision Processes (New York: McGraw-Hill Book Co., 1960), p. vii.

[5] Ibid.

This study is also important for it underscores the need for the developers of ideas pertinent to administration to examine the areas between the field of administration and other fields such as information theory, communication, decision theory and psychology. For many years the separate developments of fields of study were assured by the tendency toward ever-increasing specialization of workers in those fields until, as Wiener wrote in 1948:

> Today there are few scholars who can call themselves mathematicians or physicists or biologists without restriction. A man may be a topologist or an acoustician or a coleopterist. He will be filled with the jargon of his field, and will know all its literature, and all its ramifications, but, more frequently than not, he will regard the next subject as something belonging to his colleague three doors down the corridor, and will consider any interest in it on his own part as an unwarrantable breach of privacy. 6

Yet, the no-man's land between the various established fields offers rich opportunities to the qualified researcher for it is out of this limbo that many of the advances of the 1940's came. At the same time, this area is the most refractory to the accepted techniques of mass attack and division of labor. For, if the difficulties of a problem in education are mathematical in essence, a corps of

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[6]Norbert Wiener, Cybernetics (New York: The Technology Press, John Wiley & Sons, Inc., 1948), p. 8.

educators ignorant of mathematics will get as far and no farther than one educator ignorant of mathematics. Similarly, if a mathematician who knows no education is teamed with an educator who knows no mathematics, the educator will not be able to state his problem in terms which the mathematician can manipulate nor will the mathematician be able to translate his answers so that the educator can understand them. However, the treasures that are being found in these areas by some are motivating others to acquire new skills in fields seemingly but remotely related to their primary interests and to develop new techniques of research which they hope will permit them to share in these discoveries. It is hoped that this study will stimulate a greater number of administrators to acquire the additional skills that they will need before they may adequately investigate the no-man's lands that surround their field.

Still, the greatest contribution that this study can make is to show how ideas from information theory and other advances in communication can be used in administration. This is done by constructing a model using components from those disciplines that can be used to explain span of control. Examples are given that show how the model may be applied. Those research findings are reviewed which either support the model or define quantitatively some limitations on human abilities to communicate or to handle information. That which emerges

is an incomplete but relatively cohesive body of theory and research which, it is asserted, could be used as a starting point for further research that could lead to an overall theory of administration and allow a more nearly adequate restatement of span of control in the process.

B. The Problem and Its Limiting Assumptions

The problem of this study which finally emerged was: (1) to identify the factors believed to determine span of control, (2) to establish the extent to which these factors have been precisely and operationally defined, (3) to report these findings in a context that is cohesive and useful for further research, and (4) to identify the areas within this structure that require additional research or development before the structure can be used as the basis for a more nearly adequate restatement of the principle of span of control.

Four major assumptions were made about this study. The first of these is that supervision and control were developed to meet Man's need to reduce the degree of uncertainty he perceives in his environment. This uncertainty may be of two kinds: (1) uncertainty about the present status of some aspect in that environment, and (2) uncertainty about what change may take place in an aspect. The first of these implies a need for knowledge about something whereas the second implies a need to predict that an event will occur or to cause it with a probability greater than chance.

The second assumption about supervision and control is that both are primarily sets of two-way communication and information-processing procedures. That is, to know what some aspect of the environment is like or is likely to do implies having received at least one information-bearing communication about it and being able to recall and perhaps otherwise process the received information. To be able to cause an event to occur implies having information and being able to transmit a communication back to the environment. It is recognized that Man has developed sophisticated manifestations of these procedures ranging from the manipulation of simple and predictable machines to the supervision and control of complex cooperative human endeavors such as our nation's space program. However, the assumption is only that, if each of these manifications were analyzed, the analysis would reveal a set of simple procedures for receiving, processing and transmitting information.

A third assumption recognized the fact that Man is not omnipotent. He has physical limitations imposed upon him by his very nature and cultural limitations that reflect the level of civilization of his society. These limitations place bounds on Man's actions as well as on his thinking. The assumption that was made postulates the existence of limitations on Man's ability to receive, process and transmit information and, therefore, on his ability to supervise and control subordinates. Examples

of physical limitations on information-processing ability might be the inability of the brain to receive two or more messages simultaneously, its limited span of immediate memory, or the maximum rate at which it can process a unit of information. An example of a corresponding cultural limitation is a clumsy coding system, such as Roman numerals, which does not lend itself to the process of multiplication. This forced the Romans to adopt the abacus as a device to speed up the process of multiplication which was needed in commerce and industry.

A fourth and final assumption was that a supervisor may spend a significant portion of his time doing non-supervisory tasks. There are two extremes of thought about supervision, one claiming that there are as many kinds of supervision as there are kinds of activities to be supervised, and the other asserting that supervision consists of but a single set of skills, those involved in getting people to work together effectively and efficiently. Quite probably, the truth lies somewhere between these two extremes. Hoffman[7] (1963) analyzed the dichotomy and concluded that no such thing as an all-purpose manager exists. He reasoned that there are at

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[7]F. O. Hoffman, "The All-Purpose Manager: Does He Exist?" Personnel, 1963, Vol. 40, No. 1, pp. 8-16.

least two aspects to a manager's job: (1) the specialized training and experience needed to build skills required to manage a particular field of work, and (2) the common denominator of all managerial jobs -- the responsibility for seeing to it that the organizational machinery runs at peak efficiency. This kind of a central position suggests that the supervisor must spend some time using his "common core" skills. However, any amount of time devoted to the use of specialized skills will reduce the amount of time left for supervision and it is reasonable to conclude that, other things being equal, it will reduce his span of control. This assumption serves to define the span of control as but a part of the supervisor's total capability for work.

C. Design and Procedures

Both the research design and the procedures of this study were simple. To achieve its objectives, it was felt necessary to establish three points: (1) that something on the order of span of control really exists, (2) that the Graicunas-Urwick explanation of it is inadequate, and (3) that new ideas adapted from fields outside of administration can provide the basis for an eventual restatement of span of control in terms that are more-nearly adequate. It had to be shown that authorities in administration still feel that something on the order of span of control exists and is operating in supervisory situations because of the possible contention that

span of control has been totally discredited. Also, such a demonstration supplies justification for proceeding with the investigation. In lieu of empirical evidence of span of control, which does not exist, the testimony of contemporary writers in administration was cited to provide at least circumstantial evidence in favor of span of control and to give this study a reason for being.

If the Graicunas-Urwick explanation for span of control were valid, this fact would also render such a study as this redundant. However, there are two good reasons for believing otherwise. First, the principle has been often ignored in practice with no apparent adverse effects. Second, neither Graicunas nor Urwick were able to relate their explanation of the phenomenon to observable reality.

In Chapter IV are to be found the beginnings of a rationale that, it is asserted, could lead to a more-nearly adequate restatement of span of control as well as to provide stimulation and direction for those interested in developing a research-based theory of administration. The third point to be established was to show how the new ideas that were incorporated into the rationale could lead to such a theory. Again, circumstantial evidence has been resorted to for it is impossible to predict the development of any theory with absolute accuracy. The theory and the research from which the rationale was

constructed were chosen with two criteria in mind: (1) the items used should relate to each other to a great degree so as to provide as large a measure of internal consistency as possible, and (2) the entire structure should be based upon the same few principles to lend credibility to that basis.

Procedures in the study included the following: (1) searching the literature to find probably relevant writings, (2) scanning the located materials quickly to determine their value to the study, (3) reading the valued material carefully and making notes from the articles and books read, (4) organizing these notes into chronological order (for Chapters II and III) and into logical order (for Chapter IV), (5) writing the rough draft of the study from the notes, and (6) refining the rough draft into its final form. It was frequently discovered at the times of taking notes, organizing, or rough-draft writing that pertinent points were not adequately covered; additional reading was done to make up these deficiencies where possible.

The procedures for searching the literature were organized into several phases. In addition to reviewing the Dissertation Abstracts,[8] the basic body of literature to be scanned was identified by reviewing

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[8] Dissertation Abstracts (Ann Arbor: University Microfilms, Inc.).

appropriate categories in the Psychological Abstracts.[9] A supplementary list of readings was obtained by noting the references in the articles that were carefully read. The actual scanning procedure started with the most recent writings. There were several reasons for scanning backward through time: (1) more recent writings in a given area are likely to be more sophisticated, (2) later writings supplied lists of relevant earlier studies, and (3) they frequently also provided evaluations of the earlier work of others. The list of readings for the historical sketch found in Chapter II was generated by starting with one source on span of control, Urwick's "The Manager's Span of Control,"[10] and tracing its references as systematically and thoroughly as possible.

In analyzing the present rationale for span of control, a logical flaw was discovered in it, that of the failure of Graicunas to consider all of the possibilities. In examining some of the overlooked cases and trying to count them, it was found to be helpful to develop Graicunas' patterns further using simple algebra. This analysis forms the Appendix.

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[9]Philip J. Siegmann (ed.) Psychological Abstracts (Washington: American Psychological Association, Inc.)

[10]Lyndall Urwick. "The Manager's Span of Control," Harvard Business Review, Vol. 34, No. 3 (May-June, 1956), pp. 39-47.

Readings were evaluated in terms of their relevance, validity and the reliability of their findings. A piece of research was considered to be relevant if it could be reasonably shown to pertain to one or more of the factors probably affecting span of control. It was considered to be valid if its research results could be abstractly expressed. This, in effect, is the definition of construct validity that was first enunciated by Cronbach and Meehl in 1955.[11] Construct validity is based upon the notion that all abstract concepts are devices which exist only in the minds of men and are, therefore, incapable of being measured directly. Experiments designed to substantiate such concepts can do so only through indirect measurements or indices. This means that there is no rigorous method by which an abstract concept pertaining to anything real can be firmly established in one experiment. Rather, a number of different indices in a succession of experiments must all suggest the validity of a concept before it is thought to be satisfactorily established. Similarly, a concept can be weakened but not demolished by one contradictory finding. Moreover, a contradictory result cannot be attributed to a faulty concept until it has been first determined that the contradiction is

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[11] Lee J. Cronbach, and Paul E. Meehl, "Construct Validity in Psychological Tests," Psychological Bulletin, 1955, Vol. 52, pp. 281-302.

not due to a flaw in the experimental design or method. The last consideration in the evaluation of research findings was that of reliability or freedom from the influences of transitory, outside factors. Such a consideration would not be necessary if it were known that validity were high but, in the case of construct validity, there can be no such simple determination. Consequently, in the review of research, only such research that was relevant to the immediate problem of the study was incorporated. In addition, findings had to appear to be valid and reliable.

In fabricating the rationale of Chapter IV, two problems had to be dealt with, that of consistency and that of the disposition of defects. For a rationale to be useful, it must be both externally and internally consistent, externally in the sense that none of its components can produce arguments with contradictory conclusions. That is, although no rationale needs to be complete, its points must correspond to points in the reality it purports to describe and no pair of its points may be mutually inconsistent. Furthermore, a rationale built from theory as well as research is likely to generate defects in its construction that are of two kinds. The first kind of defect is the area in which no theory or research is to be found. Where such defects occurred, a remedy suggesting a possible explanation for the phenomena encountered was offered or

an area was defined in which more research was needed. The second kind of defect arises from conflicts between theory and/or research findings. Conflicts between two theories were disposed of most easily, the disposition consisting of rejection of the position that was inconsistent with the rationale being fabricated or, if no inconsistency could be found, rejection of the position considered to be more complicated. Conflicts among research findings were judged on the basis of construct validity as mentioned above. In cases of theory/research conflicts, resolution depended on the weight of research findings, the greater the weight of respected research, the greater the probability that the theoretical position would be modified.

This is not the first time that span of control has been studied. However, it is an approach to it that would relate a formative theory of administration to observed reality and, in so doing, may give theoreticians as well as practicing administrators some valuable guidelines to consider.

CHAPTER II

THE HISTORY OF SPAN OF CONTROL

The objective of this chapter is to provide a perspective for the critical review of span of control. More particularly, it is hoped that the reader will observe that the principle of span of control was first suggested after World War I by the military who tend to adhere to it, at least at the higher levels of command and during wartime. It should also be noted that, while it found its way into the literature of management during the two decades following the Great War, there is no evidence to suggest that either military or non-military groups have subjected the principle to empirical testing.

A. Earliest References

As a discipline, administration or management appeared upon the scene rather lately. There are many reasons for this delay, among which might be included the many centuries during which business was held in low regard, the pre-occupation of business people with profit-making and technology, the tendency to compartmentalize the inter-related disciplines that comprise the humanities, and the widely-held belief that administration is incapable of reduction to a science.

Nevertheless, interest in administration is as old as civilization itself. In addition to the rich contents of Plato's Republic and Aristotle's Politics, concern over administrative problems and attempts at their solutions can be found in Egyptian papyrii dating back to 1169, B.C.,[1] in the writings of Confucius and his contemporaries (500,B.C.),[2] and in the works of Cicero.[3] Roman records leave much less information about their administrative skills than they do of their military history. However, Lepawsky has this to say about Roman managerial ability:

> There is some evidence to assume that even under the supposedly less efficient Republic the Romans knew much about how to administer their own affairs and those of their subject colonies. As for the Roman Empire, until external forces more weighty than administrative deficiencies alone brought about its destruction, it demonstrated a facility for management on a vast scale that has caused the envy of medieval as well as modern men.4

An examination of these writings has revealed that the origins of many of the concepts now employed in administration are, indeed, very old. Most of these early

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[1]James H. Breasted, Ancient Records of Egypt (Chicago: University of Chicago Press, 1906), Vol. 4.

[2]Confucius, Analects, Book XIII, World's Great Classics, Oriental Literature, Vol. 4, "The Literature of China--The Analects of Confucius," Translated by William Jennings (New York: The Colonial Press, 1899.)

[3]Cicero, De Officiis.

[4]Albert Lepawsky, Administration (New York: Alfred Knopf, Inc., 1949), pp. 91-92.

sources deal with the more basic ideas such as organization and division of labor, however, and rarely allude to the more sophisticated concepts such as span of control.

Two early sources clearly hint at span of control. The former of these is the Old Testament where, in the book of Exodus, Jethro, Moses' father-in-law, observed that Moses was spending too much time sitting in judgment of the Israelites and made the following comments:

> The thing that thous doest is not good. Thou wilt surely wear away, both thou, and this people that is with thee: for this thing is too heavy for thee; thou art not able to perform it thyself alone. Hearken now unto my voice, I will give thee counsel, and God shall be with thee: Be thou for the people to Godward, that thou mayest bring the causes unto God: And thou shalt teach them ordinances and laws, and shall show them the way wherein they must walk, and the work that they must do. Moreover thou shalt provide out of all the people able men, such as fear God, men of truth, hating covetousness; and place such over them, to be rulers of thousands, and rulers of hundreds, rulers of fifties, and rulers of tens: And let them judge the people at all seasons: and it shall be, that every great matter they shall bring unto thee, but every small matter they shall judge: so shall it be easier for thyself, and they shall bear the burden with thee.[5]

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[5]Ex. 18:17-22.

Evidently Moses did as he was told, for:

> . . . Moses chose able men out of all Israel, and made them heads over the people, rulers of thousands, rulers of hundreds, rulers of fifties, and rulers of tens. And they judged the people at all seasons: the hard causes they brought unto Moses, but every small matter they judged themselves.6

As national states developed during the Middle Ages, a corresponding development of administrative procedures followed in the fiscal field. The second source which may be said to refer to span of control is dated 812, A.D. and has to do with the manner in which Charlemagne had his estates administered:

> Let our mayors, foresters, stablemen, cellarers, deans, tollgatherers, and other officers to regular and fixed duties, and let them pay land taxes for their holdings; and for the manual work due to them let them perform their office well. And whatever mayor has a benefice let him find a substitute, so that the substitute may relieve him of his manual work and other services. No mayor shall have more land in his district than he can cover and administer in a day.7

It should be noted that neither of the above writers showed a conscious awareness of the concept or principle. Both were more concerned with the successful accomplishment of a task than they were with the number of persons requiring supervision that are necessary to the accomplishment of the task.

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[6]Ex. 18:25-26.

[7]Roy C. Cave and Herbert H. Coulson, "Capitulary of the Imperial Estates of Charlemagne," in A Source Book for Medieval Economic History (New York: Biblo and Tannen, 1936), p. 19.

B. Span of Control in the Twentieth Century

In 1922, following the experiences of the first world war, military observers began to focus their attention specifically on span of control. Among the first of these was Sir Ian Hamilton, a general of the British army, who wrote that:

> The average human brain finds its effective scope in handling from three to six other brains. If a man divides the whole of his work into two branches and delegates his responsibility, freely and properly, to two experienced heads of branches he will not have enough to do. The occasions when they would have to refer to him would be too few to keep him fully occupied. If he delegates to three heads he will be kept fairly busy, whilst six heads of branches will give most bosses a ten-hour day. Those data are the results of centuries of the experiences of soldiers, which are greater, where organization is in question, than those of politicians, business men or any other class of men . . . .
>
> Of all the ways of waste there is none so vicious as that of your clever politician trying to run a business concern without having any notion of self-organization. One of them who took over Munitions for a time had so little idea of organizing his own energy that he nearly died of overwork <u>through holding up the work of others</u>, i.e., by delegating responsibility coupled with <u>direct access to himself</u> to seventeen sub-chiefs. Now it will be understood why a Battalion has four companies (and not seventeen): why a Brigade has three or four Battalions (and not seventeen).
>
> Organizations are run by rule then; a rule whereby from three to six "hands" are shepherded by one "head," each "head" in turn being member of a superior group of from three to six who are being wheeled into line one by one. . . .

> As to whether the groups are three, four, five or six it is useful to bear in mind a by-law: The smaller the responsibility of the group member, the larger may be the number of the group and vice-versa. That is to say, one N.C.O. in charge of three private soldiers would be too idle; one lieutenant general in charge of six divisional generals would be too busy. The nearer we approach the supreme head of the whole organization, the more we ought to work toward groups of three, the closer we get to the foot of the whole organization (the Infantry of the Line), the more we work towards groups of six. 8

The principle also made its first, though rather informal, appearance in management circles in the year 1922 when H. P. Kendall of Boston addressed a meeting of the Taylor Society in the following terms:

> At a dinner the other evening, I heard the President of the General Electric Company asked how many people should report directly to the President of a large industrial company. He said that eight or nine were reporting at present, but that it was too many, and he was reorganizing his functions so that only four or five would report directly to himself, and I imagine that four or five is enough. Not that a chief executive should not have contact with others; but that is about as many general functions as should regularly and directly lead up to him.9

These two writers succeeded in focusing the attention of people in circles of management upon the notion of span of control and in fixing the limit rather firmly in the interval from three to six. However, the

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[8]Sir Ian Hamilton, The Soul and Body of an Army (London: Edward Arnold & Company, 1921), pp. 229-230.

[9]H. P. Kendall, "The Problem of the Chief Executive," Bulletin of the Taylor Society, Vol. 7, No. 2, April, 1922.

question was regarded as an open one until, mainly through the efforts of two other contributors, a rationale was built for it and the notion was advanced to its present status. The first of these is Lyndall Urwick, the prominent British management consultant, who has become the chief proponent of the concept and principle of span of control as it is presently stated. Urwick discovered General Hamilton's principle of organization shortly after its appearance and he mentioned it to some of his friends who were interested in problems of management. He said that he had found, from experience and from reading, "the ideal number of subordinates for all superior authorities . . . to be four," and "at the lowest level of organization, where what is delegated is responsibility for the performance of specific tasks and not for the supervision of others, the number may be eight or twelve."[10] Some time later, he adopted the term "span of control" and applied it to a formal statement of his conception, the wording of which now stands as "No supervisor can supervise directly the work of more than five or, at the most, six subordinates, whose work interlocks."[11] He has

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[10]Lyndall Urwick, "Axioms of Organization," Public Administration Magazine (London) October, 1935, pp. 348-349.

[11]Lyndall Urwick, Scientific Principles and Organization (New York: American Management Association) Institute of Management Series, No. 19, 1938, p.8.

restated this principle a number of times in his writing since its first introduction, and it is referred to, both directly and indirectly, in many textbooks and articles on administration that have appeared since 1938.[12] Urwick has taken it upon himself to answer the criticisms that have been leveled at the principle since its introduction into management literature and he regards the concept with paternal fondness.[13]

One of the persons to whom Urwick mentioned the writings of General Sir Ian Hamilton was V. A. Graicunas, a French management consultant, who became interested in it and began to build a theoretical foundation for it in the early 1930's. First published in the Bulletin of the International Management Institute, it was reprinted in 1937 in the Papers on the Science of Administration, edited by Luther Gulick and Lyndall Urwick, and has become the main support for the existing concept.[14] Graicunas' rationale is mathematical in nature; it is based on the assumption that effective supervision depends on the ability of the supervisor to keep in mind simultaneously all of the elements of three

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[13]Lyndall Urwick, "The Span of Control--Some Facts About the Fables," Advanced Management, Vol. 22, No. 11 (November, 1956), pp. 5-15.

[14]V. A. Graicunas, "Relationship in Organization," Bulletin of the International Management Institute. (Geneva: International Labour Office, 1933) as reprinted in Luther Gulick and Lyndall Urwick (eds.) Papers on the Science of Administration (New York: Institute of Public Administration, 1937), pp. 181-187.

distinct sets of human inter-relationships: (1) the direct relationships between the supervisor and each of his subordinates in the absence of any of the other subordinates, (2) the direct relationships between the supervisor and each of his subordinates in the presence of all possible combinations of other subordinates, and (3) the indirect cross-relationships between all possible ordered pairs of subordinates.[15] Graicunas noted that, computed on a maximum basis, a supervisor over n subordinates must keep in mind n relationships of the first set, n(n - 1) relationships of the third set, and $n(2^n/2 - 1)$ relationships of the second set. For each subordinate added thereafter, the number of relationships in the first set increases by just <u>one</u> while that of the third set is increased by ?n. However, the number of relationships in the second set is increased <u>exponentially</u> and at a much more rapid rate, being at least doubled for each additional subordinate. He felt that, as the number of such subordinates is increased, the corresponding increase in the number of relationships in the second set would soon exceed the capacity of the human mind to keep track of them all simultaneously.[16] The sum of the numbers of all of

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[15] <u>Ibid</u>., p. 184.

[16] <u>Ibid</u>., p. 184.

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[15] Ibid., p. 184.

[16] Ibid., p. 184.

these relationships, taken together, is:

$$N = n(2^n/2 + n - 1). \qquad (1)$$

The following table shows dramatically how the number N of relationships increases as the number n of subordinates increases:

TABLE 1

VARIATIONS OF NUMBERS OF RELATIONSHIPS WITH THE NUMBER OF SUBORDINATES

n	1st Set	2nd Set	3rd Set	N
1	1	0	0	1
2	2	2	2	6
3	3	9	6	18
4	4	28	12	44
5	5	75	20	100
6	6	186	30	222
7	7	441	42	490
8	8	1016	56	1080
9	9	2295	72	2376
10	10	5110	90	5210
11	11	11253	110	11374
12	12	24564	132	24708

These data were given by Graicunas as being the maximum numbers of relationships in each set for a given number n of subordinates.[17] They could as well have been derived from Equation (1) and the three expressions given on page 24 of this study. Equation (1) can also be

[17] Ibid., p. 186.

used to obtain the total number of relationships N for any n larger than twelve.

The thinking of two other men needs to be mentioned to round out the present picture of span of control. The first of these is C. I. Barnard who, in The Functions of the Executive, had the following to say about it:

> Under most ordinary conditions, even with simple purposes, not many men can see what each is doing or the whole situation, nor can many communicate essential information regarding or governing specific action without a central channel or leader. But a leader likewise is limited in time (and capacity) in communicating with many persons contemporaneously, especially if they are widely separated so that he must move about. In practice a limit of usually less than fifteen persons obtains, and for many types of cooperation five or six persons is the practicable limit . . . .
>
> . . . The complexity of the relationships in any group increases with great rapidity as the number of persons in the group increases. If the simplest possible relationship between two persons is that of "knowing" each other as accomplished by a mutual introduction, then the relational complexity at the very least increases as follows:

Number in Group	Number of Relationships	Increase in Relationships With Each Addition to Group
2	1	.
3	3	2
4	6	3
5	10	4
6	15	5
7	21	6
8	28	7
9	36	8
10	45	9
15	105	.
20	190	.
50	1225	.

> . . . A person has relationships not only with others individually and with groups, but groups are related to groups. As the number of possible groups increases, the complexity of group relationship increases in greater ratio.18

There is no question that Barnard's thinking was influenced by that of Graicunas for he makes reference to Graicunas' "Relationship in Organization."[19] Too, it does not seem likely that the number of relationships listed by Barnard coincidentally just happened to be half the number of relationships in the third set found by Graicunas and given in Table 1.

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[18] Chester I. Barnard, The Functions of the Executive (Cambridge, Massachusetts: Harvard University Press, 1960), pp. 106-109.

[19] Ibid., p. 109.

The final contributor to the development of the present principle of span of control is Ralph C. Davis who furtber amplified the Graicunas hypothesis by distinguishing two qualitative levels of supervision. The first level is the unit of supervision which applies to the first-line supervisor and wherein the activities are mostly physical and mainly concerned with the control of current operations. He then defines a span of executive control which he applies to all executives above the level of the first-line supervisor. According to Davis, executives are more concerned with the planning and organizing of activities which he feels are more exacting types of work since they require working with intangibles and abstractions. Davis claims that, as a result, the executive cannot effectively supervise more than three to seven first-line supervisors or subordinate executives. He indicates that the optimum span of executive control is, in most cases, limited to not more than five people.[20]

The foregoing sketch essentially summarizes the theoretical development of the span of control principle as it is known today. It is suspected that there is no empirical evidence to support it for both its protagonists and its adversaries are in agreement on this point. As Urwick stated:

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[20]Ralph C. Davis, "The Influence of the Unit of Supervision and the Span of Executive Control on the Economy of Line Organization Structure," Bureau of Research, Research Monograph No. 26 (Columbus: The Ohio State University, 1941).

> No-one has yet developed a satisfactory laboratory technique to prove that A can conduct conversations ("communicate with") five persons simultaneously with a reasonable hope of mutual comprehension, but that if the number is raised to eight or ten the prospect of success is less assured. Some practical men might even feel that such tests, even if they were practicable, were labouring the obvious. In any event they are not, within the limits of our present knowledge, practicable.21

Of the practicability of determining more nearly precisely how many conversations an individual can conduct simultaneously, of pragmatic approaches that have been made toward developing "proper" spans of control, and of the present status of the principle this study will have many comments to make in the chapters which follow.

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[21]Lyndall Urwick, "The Span of Control--Some Facts About the Fables," Advanced Management, Vol. 22, No. 11 (November, 1956), p. 7.

CHAPTER III

A CRITICAL ANALYSIS OF SPAN OF CONTROL

This chapter has three objectives. The first is to show that the Graicunas demonstration of span of control is not a rigorous proof. This is done by identifying and discussing some of its shortcomings. The second objective is to show why the Graicunas-Urwick version of span of control is unacceptable to both students and practitioners of administration. This is done by pointing to factors which the present principle overlooks and upon which some writers feel it depends, and by identifying and discussing other arguments, both theoretical and practical, that have been mustered against it. The third objective is to show that, in spite of their dissatisfaction with the principle, students of administration generally concede that something similar to it does actually affect supervisory situations. This is done by describing in detail the approaches that have been taken by large organizations to determine "proper" spans of control for their administrators.

A. Evaluation of Graicunas' Work

From his article it can be seen that Graicunas intended to support the principle of span of control by revealing theoretical evidence of its validity and clarifying some matters concerning relationships among individuals and groups which he felt were not obvious.[1] Whether or not he intended his work as a proof cannot be stated with certainty. However, as a proof it has a number of shortcomings of which this study has discovered three: (1) There is no evidence to show that a supervisor's ability is limited by his capacity for remembering simultaneously all of the relationships defined by Graicunas, (2) He failed to consider (or to explain away) all of the logically possible types of relationships that can be postulated to exist between a supervisor and/or his subordinates or groups of subordinates, and (3) He failed to show that, because an individual's memory span is generally limited to the immediate recall of about six digits, his capacity for supervising subordinates whose work "interlocks" is also limited to the same number. Examining the first of these shortcomings, Graicunas implied that there are two kinds of evidence, "theoretical" and practical.[2]

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[1] V. A. Graicunas, "Relationship in Organization," Bulletin of the International Management Institute. (Geneva: International Labour Office, 1933), as reprinted in Luther Gulick and Lyndall Urwick (eds.) Papers on the Science of Administration (New York: Institute of Public Administration, 1937), p. 184.

[2] Ibid.

By "theoretical" evidence he meant the abstract model of relationships which he defined and developed to help describe the supervisor's "burden of responsibility."[3] However, such models exist only in the mind and do not necessarily bear any relationship to observable reality. For this reason, a theoretical model standing alone cannot be said to constitute evidence of any kind. To be sure,any model may be analogous to reality and it is true that, the closer and more nearly complete the analogy, the more nearly accurately it may be used to help describe, explain, and predict about the reality that it resembles. It is in this consideration that the seriousness of Graicunas' mistake lies for he built a model to explain span of control in terms of certain kinds of relationships without ever establishing that these relationships really matter!

Even if Graicunas had shown that his "relationships" could be used as a valid index for a supervisor's span of control, it may be shown that he either failed to consider all of the possible types of relationships that could be defined or at least to dispose of them satisfactorily. That he overlooked some types of relationships can be shown with a hypothetical situation. Let supervisor A be in charge of three co-equal

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[3] Ibid.

subordinates, B, C and D. Following Graicunas, A must keep in mind all of the elements of three sets of relationships. Those in the first set he defined as the "direct single relationships between himself (the supervisor) and (each of) those he supervises."[4] These can be counted and described symbolically, as follows:

A * B (read "the relationship of A to B when B is alone with A"),
A * C, and
A * D.

The second set of relationships was defined by Graicunas as "direct group relationships" but was actually thought of by him as direct single relationships between a supervisor and each of his subordinates in the presence of combinations of the other subordinates.[5] These are:

A * B/C (read "the relationship of A to B in the presence of C"),
A * B/D,
A * C/B,
A * C/D,
A * D/B,
A * D/C,
A * B/CD (read "the relationship of A to B in the presence of C and D"),
A * C/BD, and
A * D/BC.

Finally, he recognized a third set of "cross relationships" which he took to mean what each subordinate thought of each other subordinate.[6] Again, these may be counted and described as follows:

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[4] Ibid.

[5] Ibid.

[6] Ibid.

B -- C (read "what B thinks of C"),
C -- B,
B -- D,
D -- B,
C -- D, and
D -- C.

That these relationships have been described and counted correctly can be verified either by checking Table 1 for n = 3, or by calculating N in Equation (1) for n = 3.

The first oversight that suggests itself is Graicunas' failure to consider the relationships between the supervisor and groups of his subordinates. Thus, in his own words, "If Tom supervises two persons, Dick and Harry, he can speak to each of them individually or he can speak to them as a pair" (underlining mine).[7] This opens the door to an entire new set of relationships that may be set out in the following manner:

A * (BC) (read "the relationship of A to the group composed of B and C when B and C are alone with A"),
A * (BD),
A * (CD),and
A * (BCD).

This set of relationships is analogous to the first of the three original sets defined by Graicunas. There is also an analogue to the second set:

A * (BC)/D (read "the relationship of A to the group composed of B and C in the presence of D"),
A * (BD)/C, and
A * (CD)/B.

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[7] Ibid.

This thinking can even be extended to Graicunas' third set of relationships, to include:

B -- (CD) (read "what B thinks of the group composed of C and D"),
C -- (BD), and
D -- (BC).

The total number of relationships has now risen to twenty-eight, an increase of ten over those defined by Graicunas. It should be noted that the last six of these newly-found relationships could not have been discovered with thinking confined to only two subordinates. One wonders whether span of control would have been different had not Graicunas chosen to do his thinking with Tom, Dick and Harry.

A second and more interesting oversight becomes apparent when it is noted that, in going from direct to cross relationships, Graicunas shifted his attention from relationships to perceptions. That this is not merely a problem in semantics can be simply shown. To begin with, a relationship between two things is often expressed in two different sets of terms. Thus, in describing one of the relationships that exists between the numbers two and three, one may say either that three is greater than two or that two is less than three. Similarly, if A is superior to B, then B is subordinate to A and either statement will describe the one relationship equally well. On the other hand, as Graicunas himself observed, that which A thinks of B needs not be

the same as what B thinks of A since these two thoughts are distinct perceptions existing in two separate minds. Graicunas apparently switched from relationships to perceptions without noticing it. In defining direct single relationships, he spoke of the relationship of Tom to Dick and Tom to Harry but not of Dick to Tom nor of Harry to Tom. Yet, he did define a pair of indirect cross "relationships" of Harry with Dick and of Dick with Harry.[8] Had he been consistent throughout his discussion, his set of cross relationships would have included only half the number of elements that he found, that is, three, in the case of the hypothetical situation established on page 31 of this study, as follows:

B * C,
B * D, and
C * D.

But now, having opened Pandora's Box of perception-relationships, at least two more sets each of relationships and perceptions can be defined. Of the relationships, the following indirect individual relationships may be defined:

B * C/D,
B * D/C, and
C * D/B.

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[8] Ibid.

These are the additional direct group relationships:

B * (CD),
C * (BD), and
D * (BC).

The two additional sets of perceptions include:

A -- B and B -- A,
A -- C and C -- A, and
A -- D and D -- A. Also:

A -- (BC),
A -- (CD),
A -- (BD), and, finally,
A -- (BCD).

The total number of relationships and perceptions, including those defined by Graicunas, has been increased from eighteen to forty-seven. Table 2 shows how this new number N' of relationships and perceptions of all sets increases as the number n of subordinates increases:

TABLE 2

VARIATIONS OF NUMBERS OF RELATIONSHIPS
WITH THE NUMBER OF SUBORDINATES

n	N_1	N_2	N_3	N'
1	3	0	0	3
2	8	2	3	13
3	17	12	18	47
4	34	50	80	164
5	67	180	320	567
6	132	602	1,212	1,946
7	261	1,932	4,424	6,617
8	518	6,050	15,696	22,264
9	1,031	18,660	54,432	74,123
10	2,056	57,002	185,300	244,358
11	4,105	173,052	621,368	798,525
12	8,202	523,250	2,058,168	2,589,620

There is no simple equation which would be similar to Equation (1) as defined by Graicunas. However, the following are true:

$$N' = N_1 + N_2 + N_3 \text{ where} \qquad (2)$$

$$N_1 = 2(2^n - 1) + n \qquad (3)$$

$$N_2 = {}_nC_1(2^{n-1}-1) + {}_nC_2\,(2^{n-2} - 1) + {}_nC_3\,(2^{n-3} - 1) + \ldots + {}_nC_{n-1}, \text{ and} \qquad (4)$$

$$N_3 = {}_3C_2 + 2n\,[2^{n-1} - n] + {}_nC_2[2^{n-2} - 1] + n[{}_{n-1}C_2(2^{n-3} - 1) + {}_{n-1}C_3(2^{n-4} - 1) + \ldots + {}_{n-1}C_{n-2}]. \qquad (5)$$

In Equations (4) and (5) ${}_nC_r = [n(n - 1)(n - 2)(n - 3) \ldots (n - r + 1)] / [r!]$ which is the standard definition for enumerating the number of possible combinations of n things taken r at a time. The derivations of Equations (2) through (5) are the subject of the Appendix, however, their accuracy may be verified by calculating N' for n = 3 and comparing the result with the total established by actual count. Also, the possible relationships and perceptions may be set out symbolically as was done on pages 33 through 37, for any n and their number may be compared to that derived for that n from Equations (2) through (5). These equations were used to construct Table 2. By comparing N' in Table 2 with N in Table 1 for any n, it can be seen that N' is always greater than N. Also, it increases at a

much more rapid rate as n increases. It is suggested that, had Graicunas found N' instead of N, span of control might be defined in a different way today. For example, had he noted the more rapid rate at which N' increases, he might have set the limit of supervision (span of control) at three instead of four and Urwick might then have stated the span of control principle using four or five and not five or six subordinates. As another possible alternative, Graicunas might then have observed the rapid rate of increase of N' with an increase in n and have concluded that, perhaps, keeping track of all of those relationships and perceptions is not the crucial factor limiting span of control that he first thought it to be. In any case, Graicunas' oversight or failure to explain away some possibilities that could be derived from a logical extension of his thinking damages his rationale. It is recognized in this study that these additional relationships and perceptions can have no validity until empirical evidence in support of them is created.

The last shortcoming of Graicunas' rationale to be discussed in this study was his failure to show that, because an individual's immediate memory span for digits is six, his capacity for the supervision of subordinates is limited to four. That is, he should have cited

evidence showing that, whereas an administrator's ability to supervise is judged to be adequate when there are no more than four subordinates with "interlocking" work, it rapidly becomes inadequate as the number of such subordinates is increased beyond four. Although Graicunas did state that the "span of attention" for digits is six,[9] he did not show that the capacity for remembering relationships is related to immediate memory span for digits and he did not show why he could increase the capacity for remembering relationships to forty-four (the maximum number of relationships he associated with four subordinates). Taken together, these shortcomings of Graicunas' demonstration suggest that it cannot stand as a proof of span of control.

However, his work is valuable for a number of reasons. First, it probably channeled and encouraged Lyndall Urwick's thinking on the subject, without which it could easily have lapsed into obscurity. Then, it has served as the object of study, stimulating students of administration to theoretical and practical refinements of organization design. Last, it has dramatized the complexity of the supervisory situation. As Koontz and O'Donnell so aptly put it:

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[9] Ibid.

> The Graicunas theory does serve to emphasize the dramatic complexity created by a manager's having more than a few subordinates, seeming to approve of the often accepted standard of three or four subordinates. But, most importantly, the formula serves as the key to the problem of span of management. For any managerial action that will reduce the number and frequency of relationships requiring the manager's attention will increase his span of management and thereby reduce the costs and inefficiencies of an undue number of departments. 10

B. Additional Factors Affecting Span of Control

Students of administration are in disagreement over the validity of the span of control principle, on both theoretical and practical grounds. One reason for its unacceptability to some is its failure to recognize the operation of factors other than "interlocking work" that would reasonably be expected to affect it. As Barnard points out:

> The size of the unit of supervision, therefore, usually is determined by the limitations of effective leadership. These limitations depend upon (a) the complexity of purpose and technological conditions; (b) the difficulty of the communication process; (c) the extent to which communication is necessary; (d) the complexity of the personal relationships involved, that is, of the social conditions.11

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[10] Harold Koontz and Cyril O'Donnel, Principles of Management (New York: McGraw-Hill Book Company, Inc., 1959), p. 74.

[11] Chester I. Barnard, The Functions of the Executive (Cambridge, Massachusetts: Harvard University Press, 1960), p. 107.

Griffiths feels that "structure should be related to the decision-making process, not to the number of human relationships which a mathematician believes an administrator can remember."[12] Knezevich, citing Newman, feels that the factors which influence the numbers involved in an effective span of supervision are:

1. The time available for such supervision. It follows the longer period the executive is willing and able to devote to his job, the larger his span of supervision;

2. The mental capacity and the personal adaptability of the executive responsible for supervision. These differences help to explain in part why the effective span of supervision will vary;

3. The complexity of the situation being supervised. A single situation with a number of workers doing fundamentally the same things will enable an executive to supervise a larger number of personnel than otherwise. On the other hand, if the executive is responsible for many unrelated activities, he will be able to supervise effectively a far smaller span of positions;

4. The other duties of the executive. If the executive is to be responsible for general planning and control of the institution as well as supervision of a number of other administrators, his effective span will be far smaller than that for an individual of similar ability and experience who does not have these added responsibilities.

5. The stability of operations. The greater the turnover among the teaching or administrative staff, the more difficult it becomes to supervise effectively a large span of operations.

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12 Daniel E. Griffiths, Administrative Theory (New York: Appleton-Century-Crofts, 1959), pp. 78-79.

6. The capability and experience of subordinates. The more capable, the better prepared professionally, and the greater the experience of principal and teacher the relatively easier it becomes to supervise larger numbers. This would imply that as administrators and teachers become more capable through greater professional preparation and experience, the larger would be the effective span of supervision. 13

"It follows that there is no magic number such as 3,7, or 11 that represents the effective span of supervision for any and all executives."[14] In a pragmatic approach to a determination of a "proper" span of control the Lockheed Missiles & Space Company defined the following as the critical factors affecting the span:

1. Similarity of function: the degree to which functions performed by the various components are alike or different.

2. Geographic contiguity: the physical location of the components and personnel reporting to a principal.

3. Complexity of functions: the nature of the duties being performed by the organization components or personnel. Takes into account the skills necessary to perform satisfactorily.

4. Direction and control: the nature of the personnel reporting directly to a principal. Includes the degree of the principal's attention which they require for proper supervision of their actions.

5. Coordination: the extent to which the principal must exert time and effort in keeping actions properly correlated and in keeping his activity keyed in with other activities of the company.

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[13] William A. Newman, Administrative Action (Englewood Cliffs, New Jersey: Prentice-Hall, 1950), p. 161.

[14] Stephen J. Knezevich, Administration of Public Education (New York: Harper & Brothers, 1962), pp. 67-68.

6. Planning: the importance, complexity, and time required to review and establish future programs and objectives.

7. Organizational assistance: the help received by the principal from direct-line assistants, staff activities, and assistants-to. (In the case of first-line supervision, lead men would be included.) 15

It can be noted that, while some factors, such as complexity of function, are widely regarded to have a bearing on span of control, there is no general agreement on all of them. Regardless, the apparent over-simplification of Urwick's principle has rendered it unacceptable to many.

The span of control principle has been challenged on both theoretical and practical grounds. Simon has questioned the validity of the principle in terms of its theoretical soundness. He writes:

> . . . A contradictory proverb of administration can be stated which, though it is not so familiar as the principle of the span of control, can be supported by arguments of equal plausibility. The proverb in question is the following:
>
> 'Administrative efficiency is enhanced by keeping at a minimum the number of organizational levels through which a matter must pass before it is acted on.'
>
> In many situations the results to which this principle leads are in direct contradiction to the requirements of the span of control. 16

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[15]Harold Stieglitz, "Optimizing Span of Control," Management Record, Vol. 24, No. 9, pp. 121-129. Reprinted in Harold Koontz and Cyril O'Donnell (eds.) Management: A Book of Readings (New York: McGraw-Hill Book Company, 1964), p. 165.

[16]Herbert A. Simon, Administrative Behavior (New York: The Macmillan Co., 1947), pp. 26-28.

Simon's point is that, since no respectable theory of administration can be allowed to reflect internal inconsistency through the inclusion of two mutually contradictory propositions, then either span of control or the principle of fewest possible organizational levels must be rejected.

The second criticism of span of control voiced by theoreticians is that it violates our democratic philosophy by encouraging centralization of authority and, thus, discouraging widespread participation in decision-making. The feeling is that such widespread participation is directly related to worker efficiency and morale. According to Gardner:

> There is good reason to believe that the gain in over-specialization and its twin brother, over-centralization of authority, has been lost in the debilitating and enervating effects they have had on employee morale and willingness to cooperate.17

Dale summarized the reasons for these challenges and the tendency toward a broader actual span of control:

1. The desire of executives to have access as high up as possible, as a means of advancement and a sign of status.

2. The need for keeping the chain of command as short as possible. The shorter the span of control, the more layers of supervision there will be and the longer the lines of communication, with corresponding disadvantages.

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[17] William H. Whyte, Jr. and the Editors of Fortune, Is Anybody Listening? (New York: Simon and Schuster, 1952), p. 129.

3. A natural tendency on the part of executives to take a personal interest in as many aspects of their job as possible, the lack of trust in the ability of subordinates, the fear of possible rivals, and the desire for power (as shown by the number of people reporting).

4. The political argument that as many interests as possible should be represented.

5. The danger of overly-close supervision which may discourage initiative and self-reliance.[18]

In real-life situations, span of control is generally ignored. One needs only to look at the common classroom situation to find one teacher supervising the work of thirty or more pupils. Still in the educational setting, one building principal is often placed over fifteen or more teachers. In the Roman Catholic Church, some 750 line bishops and 1200 other persons report directly to the Pope.[19] In 1955, although the President of the General Motors Corporation had only six line Group Executives reporting to him directly, two of these Group Executives had eight General Managers reporting to him and one had thirteen![20] Many other industrial concerns have, with interesting results, rejected the span of control concept of Graicunas-Urwick. For example, in a twelve-year study of morale and efficiency conducted Sears, Roebuck and Company, it was concluded that:

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[18]Ernest Dale, Planning and Developing the Company Organization Structure (New York: American Management Association, 1955), pp. 52-53.

[19]Harold Koontz and Cyril O'Donnell, Principles of Management (New York: McGraw-Hill Book Co., 1964),p.229.

[20]Harlow H. Curtice, "General Motors Organization Philosophy and Structure." "The Development and Growth

> Detailed studies on the span of control at Sears, Roebuck and Company very definitely showed the superiority in operating efficiency of a large span of control, provided subordinates are of high competence and self-reliance. Sears' regional vice-presidents now have full authority over everything in their territories, except purchasing, of course. These vice-presidents report to the president. As a result, Sears' president has now 13 executives directly under his supervision. These territorial vice presidents, in turn, have even more people reporting directly to them. In addition, other executives down the line have direct access to the president. 21

Military organizations, which first developed the notion of span of control and which are generally inclined to adhere to it closely in times of war, also modify existing spans of control by allowing large numbers of unsupervised subordinates to have the right of access to their chief. As Dale relates:

> General Eisenhower told me in an interview that in World War II he had at one time 150 battalion commanders reporting to him. This, he believed, resulted in cleaner understanding up and down the line, an opportunity for personal inspiration, and a chance to voice complaints. Now it would be physically quite difficult even to receive reports from 150 people or, to express it in organizational language, 'effectively supervise' so many people. What the General had in mind is that accessibility to the chief executive can make important contributions. The number of people he supervised was small, while the number who had access to him was large. 22

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of General Motors," a Statement before the Subcommittee on Anti-Trust and Monopoly of the United States Senate Committee on the Judiciary, Dec. 2, 1955, pp. 5-12. Reprinted in Harold Koontz and Cyril O'Donnell (eds.) Management: A Book of Readings (New York: McGraw-Hill Book Company, 1964), p. 196.

[21]Dale, op. cit., p. 53.

[22]Ernest Dale, "Dynamics and Mechanics of Organization," in Organization Planning and Management Development (New York: Am. Management Association, 1951), pp. 7-8.

Although Urwick argues quite adroitly that the nominal right of access to a chief executive is quite different from the frequent use of that right,[23] it goes without saying that even relatively infrequent use of such a right by each of a large number of people would significantly reduce a chief executive's amount of time for direct supervision and, with it, his effective span of control.

A research study has made available information on the number of executives reporting to the president in one hundred companies having over 5,000 employees each. One of the major conclusions of this study was that the theoretical limits of the executive span of control are in practice more often violated than they are observed, as indicated in the following table.

Table 3 indicates that the median number of executives reporting to the presidents of the organizations surveyed was between eight and nine. It is interesting to compare the column headed "Number of Executives" of Table 3 with the column headed "N" (the number of Graicunas-derived relationships) of Table 1. It is also interesting to note that, by Equation (1) that company president who has twenty-four executives

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[23]Lyndall Urwick, "The Span of Control--Some Facts About the Fables," Advanced Management, Vol. 22, No. 11 (November, 1956), p. 7.

reporting to him had 201,327,144 such relationships to keep track of! It should also be noted that the presidents of twenty-four firms had thirteen or more subordinates reporting to them directly.

TABLE 3

NUMBER OF EXECUTIVES REPORTING TO PRESIDENT IN 100 LARGE COMPANIES [24]

Number of Executives	Number of Companies
1	6
2	0
3	1
4	3
5	7
6	9
7	11
8	8
9	8
10	6
11	7
12	10
13	8
14	4
15	1
16	5
17	0
18	1
19	0
20	1
21	1
22	0
23	2
24	1

[24] Ernest Dale, Planning and Developing the Company Organization Structure (New York: American Management Association, 1952, Research Report No. 20), p. 57.

C. Present Status of Span of Control

Despite the reception that span of control has been accorded by administrators, there are at least two kinds of indicators of general agreement by them that something like it does affect supervisory situations. The first of these is the observation that, even though it is often not accepted, the Graicunas-Urwick conception is repeatedly found in new texts on administration.[25] The other indication can be inferred from noting the repeated attempts that have been made to study it or to establish it for practical situations. The Sears, Roebuck and Company study that has already been mentioned is one of these. Another is the study made by the Lockheed Missiles and Space Company.[26] In the latter, wherein a "proper" span of control was pragmatically defined as one which was not improper, Lockheed first isolated seven factors (see pp. 43-44 of this study) deemed to be the most significant to the span of management and then weighted six of them in order of their importance. The seventh factor, Organizational Assistance, was treated differently inasmuch as it was designed to lighten rather than increase the supervisory burden. So, instead of giving this factor a straight point value, Lockheed assigned percentage values to various types of

[25] The reader is requested to review the footnotes of this study.

[26] Harold Stieglitz, op.cit., pp. 164-169.

assistants. These point values were then set up as a supervisory index to "suggested spans." The higher the point value the greater the supervisory burden and the lower the suggested span. Relating of actual spans to point values was done by analyzing different supervisory situations and fitting a trend line to the results. From the analysis, it became apparent that two scales would be required--one for middle managers; another for first-line supervisors. In the latter case, the same index numbers provided for approximately twice the span. So far, Lockheed has used this program in only a few units of the company. In each case, the average span of control was extended from about a half person to slightly more than a person and the number of levels was reduced by about one. It was stated that reductions in managerial personnel and supervisory payroll were "substantial." It is to be quickly pointed out that, while Lockheed's organization analyst felt that their approach could be generally used throughout all other companies, its mere application did not bring about the reductions mentioned above. They felt that their approach was but a useful guide to be used along with and not as a substitute for astute judgments.

In conclusion, it seems fair to say that span of control has evolved amid speculation, theorizing,

and practical experimentation, with few if any attempts to relate it empirically to observed reality. It is to this problem that the attention of this study is now focused.

CHAPTER IV

SOME NEW CONSIDERATIONS

The objective of this chapter is to establish a basis in theory and research for a more-nearly adequate restatement of the principle of span of control. Consistent with the assumption that both supervision and control can be analyzed into communication and information-processing procedures, it was felt that the theoretical basis should describe the actual mechanisms of inter-personal communication and human information processing. To do so,each point in the theoretical model or models developed should correspond to a point in reality as evidenced by supporting research. In this chapter the two mechanisms are modeled separately. Of the two the communication model is given first. It is the less difficult to develop because, while there are many such models to consider, these have been categorized and are capable of separate discussion and application. The model chosen as the basis for this study was taken from information theory because it was designed to accommodate a measure of the amount of information in a communication. Such a measure is necessary if span of control is to be redefined

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in terms of Man's information receiving, processing and transmitting capabilities. On the other hand, if knowledge may be equated with consensus among researchers, then relatively little is known about human capacities for information processing. As a result the information processing model which is given second is the less comprehensive and the more tentative of the two. Nevertheless, some studies have been made in this field that are valuable both for their findings as well as their methods. From such studies a tentative description of some parameters on human information processing has been modeled.

Because relatively little is known about human information-processing mechanisms, it has been necessary to assume that there exist limitations on this capability that determine span of control. However, it is desirable to show that this assumption is both reasonable and useful so that researchers might be stimulated to efforts which will yield the findings needed before the principle can be established with satisfactory rigor. Since it is possible in this instance to demonstrate both reasonableness and utility, the third part of this chapter is devoted to this task. Reasonableness is established by showing how the factors that are held by administrators to be the

determiners of span of control may be translated into the terms of communication and information theory and treated in one cohesive theoretical structure. Utility is established by showing how the joint model may be used to explain the satisfactory operation of some particular supervisory situations having "broad" spans of control which apparently contradict the present concept. It is recognized that the utility of the concept would be enhanced by its ability to predict whether or not a proposed supervisory situation is within the intended supervisor's span of control. However, it is at this point that much patient study is needed to develop the concepts and to define the constants that will make the following body of ideas into an instrument that can be used successfully in organizational design.

A. A Communication Model

The communication model about to be developed is based upon that of Claude Shannon, the father of information theory.[1] In Shannon's model, a source

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[1]Claude E. Shannon and Warren Weaver, The Mathematical Theory of Communication (Urbana, Illinois: The University of Illinois Press, 1964), p. 7.

generates a message that is sent by a transmitter as a coded signal through a noisy channel to a receiver that decodes the signal and sends the message along to a destination. Figure 1, below, shows the essential components of this model.

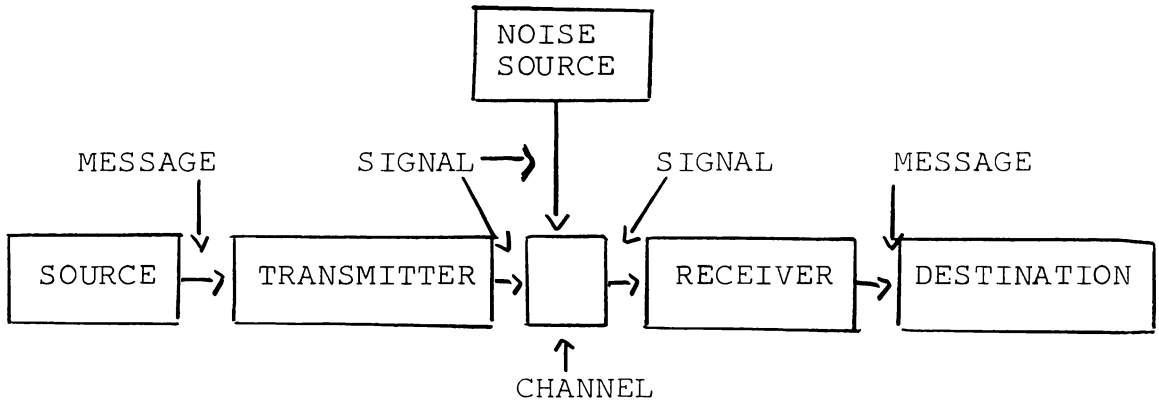

FIGURE 1

A DIAGRAM OF SHANNON'S COMMUNICATION MODEL

A discussion of this model begins with a definition of terms. It is felt that knowledge is stored as potential energy. When it is desired to convey knowledge, the device in which the knowledge is stored, called the source,[2] formulates a message which consists

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[2]Definitions on pp.56 and 57 are taken from Ibid. pp. 7, 8, and 33, 34; also from Thomas H. Crowley, et. al. Modern Communications (New York: Columbia University Press, 1962) Chapter I.

of a sequence of symbols. These symbols are then transformed into an amount of kinetic energy called a signal, the forms of the symbols in the signal being called its code. For this discussion, the transformation of potential energy into kinetic energy is called encoding, the transformation of kinetic energy from one form into another is called re-coding and the transformation of kinetic energy into potential energy is called decoding. These three processes are carried out by arbitrary devices called transducers. A transducer is a transmitter if it emits output further from its source than it receives input and a receiver if it emits output nearer to its destination than it receives input.

A pair of transducers is connected by a channel or band which is the physical medium for propagating the signal energy. The channel is defined more nearly precisely by the kind of signal energy it carries, such as visual or auditory, while its ability to accept variations in code defines its band width. Within a given band may exist sub-bands. That is, within the visual band there are the literal and pictorial sub-bands and within the auditory band there are the corresponding verbal and non-verbal (sound effects and music) sub-bands. Each of these sub-bands may carry signal energy in many different codes.

As a signal is carried along a channel, it is attenuated or weakened by resistance which is a characteristic of every channel. Every real channel also carries unwanted signal energy or noise which serves to confound the intended signal. The destination is the device for which the knowledge is intended and at which the signal is decoded and the message reproduced. Because of attenuation and noise, it is not possible to reproduce a message faithfully. That is, every received message carries with it a degree of uncertainty or equivocation. One goal of communication engineering is to build communication systems capable of reproducing messages as nearly faithfully as possible. Such systems are commonly known as having high fidelity.

The system illustrated in Figure 1 falls far short of describing typical two-way inter-personal communication. To increase its usefulness for this purpose it needs further development. First of all, let it be noted that, in such communication, source and destination are frequently interchanged as messages are sent back and forth. Also, more than one channel segment is usually found between a source and a destination. That is, there may be any number of channel segments and transducers connected in series if some qualifications may be specified: (1) the

transducer must be able to accept the signal from one segment and recode it so that it can be accepted by the other segment, (2) attenuation must not reduce the signal strength below that needed for satisfactory reception, and (3) the amount of noise introduced must not garble the message beyond recognition. Finally, it is to be observed that inter-personal communication is frequently multi-modal; that is, a "message" is frequently if not usually derived from the synthesis of several messages, simultaneously received along two or more modalities. For example, in a face-to-face conversation between two individuals, an objective message received in the auditory verbal modality is modified by the speaker's tone of voice (auditory non-verbal modality), by gestures he makes (visual pictorial modality) and, perhaps, by others as well. By means of these accompanying modalities the affective dimension of personal interaction may be achieved. Taken together, such "messages" must be conveyed by sets of channel segments and transducers that are connected in parallel between two source-destination combinations. The system illustrated in Figure 2 incorporates these additional features.

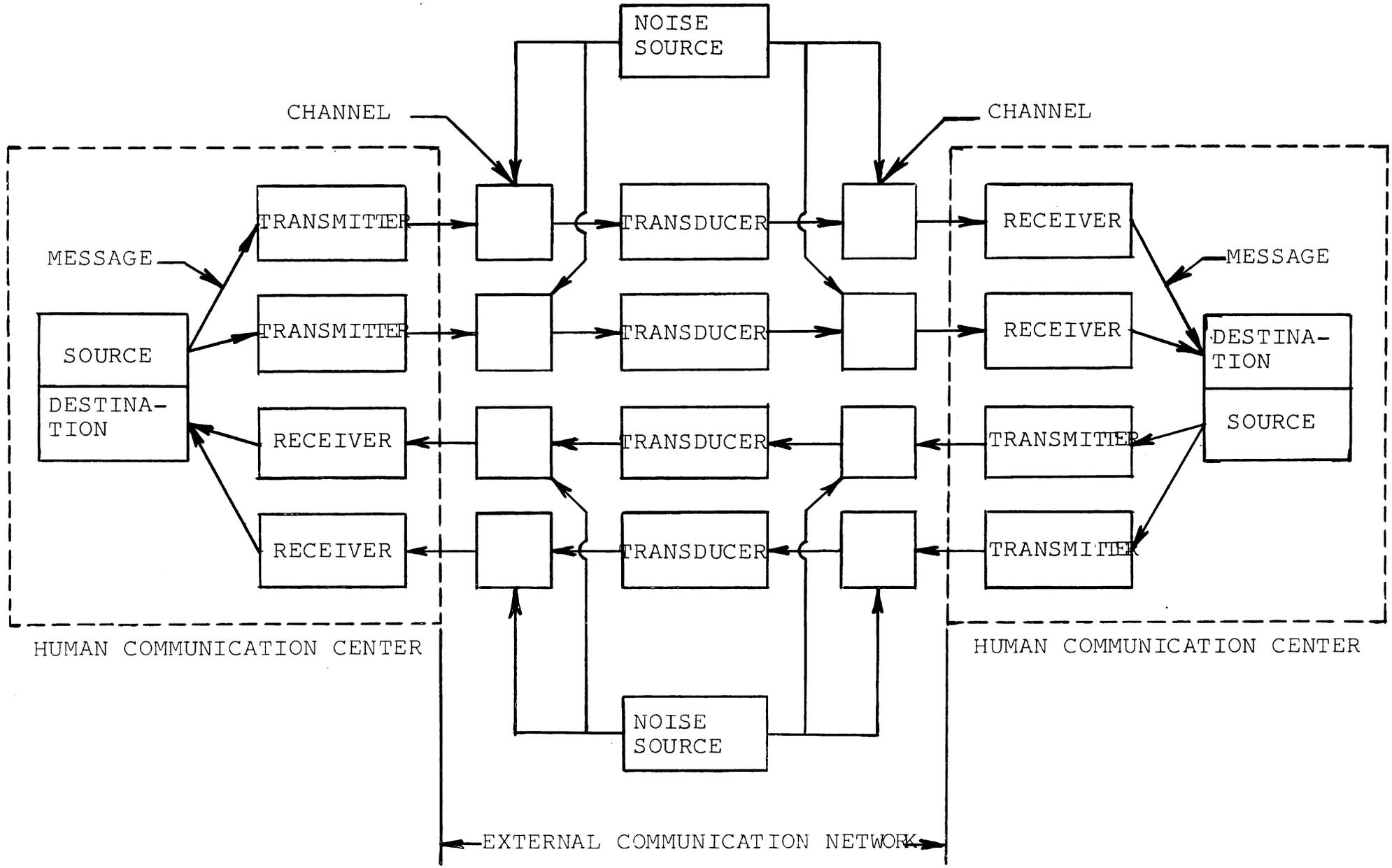

FIGURE 2

A MODEL OF TWO-WAY INTER-PERSONAL COMMUNICATION

Figure 2 dramatizes the complexity of typical two-way inter-personal communication. It shows that an individual, labeled "HUMAN COMMUNICATION CENTER" in the figure, frequently transmits signals simultaneously along two or more modalities (spoken words and visual gestures, for example) to another indiviudal who may then have to decode them simultaneously. At the same time as he is transmitting, signals may be arriving at the first individual along a number of modalities. Perhaps these may be placed into sequence so that they may then be received and processed one at a time. However, if this is not convenient or possible, they must either be dealt with as they arrive or be lost. The figure also shows that noise is introduced into every channel as unwanted signal energy. The effect of noise is to render each transmitting individual uncertain as to whether his message reached its intended destination and it renders each receiving individual uncertain as to what message was sent. The figure belies the apparent simplicity of communication channels that are shown in diagrams of formal organizations by single lines and it emphasizes the need for the student of span of control to determine what limitations exist on an individual's ability to receive, process and transmit information and how these limits might be shifted.

However, before an investigation of such human limitations can be conducted, it is necessary to introduce and to define three concepts of information theory: (1) a measure of the amount of information in a communication, (2) the capacity of a communication channel, and (3) the average output or entropy associated with a source.

1. The Amount of Information in a Communication. Since the beginnings of recorded time, Man has been aggressively engaged in trying to understand and manipulate the elements of his world (including the people in it). The continuing interest he has shown and the successes he has had makes it self-evident that successive generations of men have learned how to acquire, process, preserve and transmit, both to one another and to succeeding generations, ever-increasing amounts of knowledge about their worlds. It is an assumption of this study that knowledge of one's environment is only possible if one receives information-bearing communications about it. Man derives knowledge only by processing the information from these communications which are received by his sensory receptors. Man has a set of sensory receptors to correspond to each of the recognized senses of sight, hearing,touch, smell, taste, temperature, and the manipulational-situational (the ability of Man to

derive communications from his world by physically handling some of its elements). Each of these sensory receptors responds to certain kinds of stimuli by sending messages to the brain. The brain processes these messages and, sometimes, is able to derive additional knowledge from them. If a message causes additional knowledge to be derived, it is said to contain information. It is seen that the presence of information in a message is determined by at least two factors: (1) its novelty with regard to the receiver, and (2) the ability of the receiver to process the message correctly. That is, if the receiver has already received, suitably processed and placed into retrievable storage a message bearing the same potential information, then he can derive no increase in knowledge from it and, hence, it contains no information for him. It is important to note that messages describe either states of being or changes to states of being. As stated above, messages describing states of being may or may not bear information to a given individual. However, messages describing changes to states of being, referred to here as events, always bear information as long as the events that occur are only probable. An investigation of this notion of probable events leads to a definition of the amount of information in a message that is widely accepted today.

Intuitively it is agreed that information is received whenever a person is informed of the actual occurrence of an event that was previously not certain.[3] Furthermore, it seems reasonable that, within limits, the more probable the occurrence of an event is, the less information is transmitted by a message asserting its actual occurrence.[4] The effect of information, therefore, is to increase the probability of the occurrence of an event as far as the receiver is concerned. In general, then, it would appear that the amount of information in a message about the occurrence of a probable event should be measured by the extent to which it changes this probability for the receiver. Such a measure has been defined and is in use in physics, biology and psychology as well as in communication engineering. Briefly, the amount of information, I, in a message is defined as:

$$I = \log_b(p_2/p_1) \qquad (6)$$

where b is the base of logarithms, p_2 is the probability at the receiver <u>after</u> the message was received that the event occurred and p_1 is the probability at the receiver <u>before</u> the message was received that the event occurred.[5]

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[3]Amiel Feinstein, <u>Foundations of Information Theory</u> (New York: McGraw-Hill Book Company, Inc., 1958), p. 2.

[4]<u>Ibid</u>.

[5]Norbert Wiener, <u>Cybernetics</u> (Cambridge, Mass.: The Massachusetts Inst. of Technology Press, 1961), 2nd ed. pp. 61-62.

So as to appreciate more fully the theoretical and practical implications of Equation (6), the following discussion is given:

(a) The probability p_E of the occurrence of an event E is commonly defined as the ratio of the number of favorable ways that E can occur to the total number of ways E can occur. Thus, the probability of an event must lie between zero and one. If $p_E = 0$, E is an <u>impossible</u> event; if $p_E = 1$, E is a <u>certain</u> event. For all other values of p_E, E is a <u>probable</u> event.

(b) Information theory does not consider the possibility of the intentionally misleading message. That is, if E occurred, there would be no message sent stating that E did not occur. Thus, the only factor that could make p_2 be less than one is noise, not introduced by the transmitter, which has a garbling effect upon the received message. In the noisy case it is even possible for p_2 to be less than p_1. That is, a garbled message may be incorrectly processed. In such a case, I is negative. However, in the noiseless case which is only theoretically possible, this measure must be either

zero or positive. In order to describe the properties of Equation (6) more simply, the remainder of the discussion considers the noiseless case wherein $p_2 = 1$ and wherein Equation (6) becomes:

$$I = \log_b(1/p_1). \qquad (7)$$

(c) Consider now the two limiting cases, where $p_E = 0$ and $p_E = 1$. If $p_E = 0$, $1/p_1 = 1/p_E = 1/0$. But, because division by 0 is not defined, $I = \log_b(1/0)$ is meaningless. If $p_E = 1$, $1/p_1 = 1/p_E = 1/1 = 1$. This yields $I = \log_b 1$. But, since the logarithm of one to any base is zero, $I = 0$. That is, information that asserts the occurrence of an impossible event is without meaning and the amount of information in any message asserting the occurrence of a certain event is zero. These are consistent with the intuitive notions people have about information.

(d) The base of logarithms, b, may be any number greater than one. If $b = 2$, I is expressed in <u>binary digits</u> or <u>bits</u>.[6] Consider as an example the two possible events that can occur when an unbiased coin is

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[6] Shannon and Weaver, <u>op. cit.</u>, p. 9.

tossed that are called "heads" and "tails." These are the only two events that are possible, each is equally likely to occur, and the occurrence of one precludes the simultaneous occurrence of the other. Thus, the probability of "heads" occurring is $p_h = 1/2$ and, similarly, that of "tails" occurring is $p_t = 1/2$. The number of bits of information in a message transmitted in a noiseless channel asserting only that either "heads" or "tails" occurred is given by Equation (7) as:

$$I_h = \log_2(1/p_h) = \log_2 \frac{1}{1/2} = \log_2 2, \text{ and}$$

$$I_t = \log_2(1/p_t) = \log_2 \frac{1}{1/2} = \log_2 2.$$

But the logarithm of any number to its own base is 1. Therefore, $I_h = 1$ bit and $I_t = 1$ bit. This conclusion provides a useful index as to the magnitude of 1 bit of information: it is the amount of information needed to determine which of two equally likely and mutually exclusive events actually occurred.

(e) Table 4, below, yields values of I in bits of information as a function of p_E for ten different values of p_E. Note that, as the probability of the occurrence of an event

increases, the amount of information transmitted by a message of its actual occurrence diminishes in agreement, again, with intuitive notions.

TABLE 4

I AS A FUNCTION OF p_E

p_E	1/pE	$I = \log_2(1/p_E)$
1/64	64	6.000 bits
1/32	32	5.000
1/16	16	4.000
1/8	8	3.000
1/4	4	2.000
1/2	2	1.000
3/4	4/3	0.415
7/8	8/7	0.193
15/16	16/15	0.093
1	1	0.000

2. Channel Capacity. The notion of channel capacity stems from the recognition that communication channels have a maximum rate (expressed in bits per unit of time) at which they can convey information without error. It has been demonstrated that this

rate, called channel capacity, is a joint function of band width, W, signal level, P, and noise level, N, as follows:

$$C = W \log_2(1 + P/N). \qquad (8)^7$$

Equation (8) has some interesting implications. First, Shannon established mathematically that it is possible to transmit information through a channel at a rate of (C - e) bits per second where e may be made arbitrarily small.[8] Now, suppose that information is flowing through a given channel at very nearly its capacity with a very low error rate. That is, the given channel is being utilized very efficiently. Then, suppose that its noise level is increased by a small amount. Equation (8) reveals that an increase in noise level reduces channel capacity. As channel capacity is diminished, its error rate will increase rapidly. This is called the threshold effect.[9] The implication is that, if information is to be conveyed

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[7] Raisbeck, op. cit., p. 24.

[8] Crowley, op. cit., p. 294.

[9] Raisbeck, op. cit., p. 24.

accurately through a channel at nearly channel capacity, there must be some assurance that its noise level will not be increased. Finally, it suggests that, for optimum utilization of any communication channel, impulses passing through it should be uniformly distributed through time. That is, to take full advantage of any communication channel, it should be in continuous use at nearly its capacity.

3. Entropy of a Source. Entropy is a measure of the average amount of information produced by a source.[10] Such a measure is easily derived by multiplying the amount of information provided by the actual occurrence of each possible event by the probability of that event and adding the products. For the example of the tossed coin this becomes:

$$H = p_h \cdot I_h + p_t \cdot I_t = 1/2 \cdot 1 = 1 \text{ bit per event.}$$

Generally, if p_1, p_2, p_3, . . . ,p_n are probabilities of n different and mutually exclusive events produced by a source such that $p_1 + p_2 + p_3 + \ldots + p_n = 1$, then the entropy of that source is given by:

$$H = p_1 I_1 + p_2 I_2 + p_3 I_3 + \ldots + p_n I_n. \quad (9)$$

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[10] F. M. Reza, An Introduction to Information Theory (New York: McGraw-Hill Book Co., Inc., 1961), pp. 77.

But, since $I_n = \log_2 1/p_n$ (by definition), Equation (9) becomes:

$$H = p_1 \log_2 1/p_1 + p_2 \log_2 1/p_2 + p_3 \log_2 1/p_3 + \ldots + p_n \log_2 1/p_n.$$

However, since $\log 1/a = -\log a$, the above equation may be re-expressed as:

$$H = -(p_1 \log_2 p_1 + p_2 \log_2 p_2 + p_3 \log_2 p_3 + \ldots + p_n \log_2 p_n). \quad (10)$$

A more concise way of writing Equation (10) is:

$$H = -\sum_{k=1}^{n} p_k \log_2 p_k. \quad (11)$$

Equation (11) is given by both Shannon and Wiener as a measure of the uncertainty or entropy associated with a source.[11]

The three measures just described permit significant strides to be taken toward re-defining span of control in terms of Man's information receiving, processing and transmitting capabilities. The unit of information or the bit allows the amount of information in a communication to be expressed quantitatively. This unit is doubly valuable in that the measure attached to a given amount of information can be independent of any codes used to transmit it from place to place.

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[11] Ibid.

Thus, in the communication model just described, the recoding of a message by a string of transducers connected in series has no effect on the amount of information it contains except for attenuation due to transducer inefficiency. The concept of entropy allows a measure to be attached to the average output of a source. If the entropy of each source that is transmitting information to a given individual could be calculated, the total of all such entropies would represent the information-handling capability required of him if he were to appreciate all of the information transmitted to him. Since the concept of entropy used here is expressible in bits per event or in bits per unit of time, it is compatible with the information unit also used in this study. The third measure, that of channel capacity, is also compatible with the bit of information. It measures the capacity of a channel to convey information without error in bits per unit of time. Researchers in psychology have begun to consider the limited information-processing capability of the human brain as though it were a communication channel of limited capacity.

The two notions of entropy and channel capacity are of significant value to those who design supervisory situations. The entropy of each individual in the proposed situation might be determined to learn

how much information he is transmitting so that this could be mimimized and distributed more evenly over time. Such a measure might be obtained either by sampling actual communications or by calculating the minimum rate at which he would have to transmit. Actual entropies could then be minimized by removing the irrelevant and the redundant information. The remaining transmissions should then be arranged so that they would occur with relatively constant frequency during the life of the supervisory situation. The objective of both minimization and distribution is to reduce the information handling load on any destination by a given source so as to come within the channel capacity of that destination if possible. The channel capacity of each destination could then be calculated to determine whether or not it is great enough to handle the proposed entropies. To the extent that it is, the situation is tolerable from a communications point of view. If any channel capacity is exceeded by the entropies aimed at it, it may be said that that individual's span of control has been exceeded for he will not be able to process all of the necessary information destined for him without incurring a large proportion of errors. As an example, suppose that Supervisor A has two subordinates, B and C, as in the diagram below. The channel

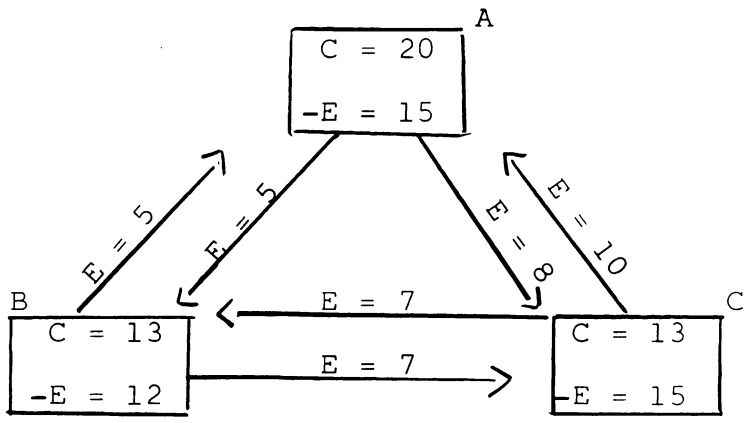

FIGURE 3

ENTROPY AND CHANNEL CAPACITY
OF A SUPERVISORY SITUATION

capacities, C_A, C_B and C_C, are as shown. Entropies emanating from each individual are given by labeled vectors. Entropies with minus signs shown in each box denote the average amount of information aimed at each individual. It can be seen from the diagram that A and B are both receiving information at rates inside their respective channel capacities. Error rates of information processing for both individuals should be low although somewhat greater for Individual B than Individual A because B is operating at a rate nearer to his channel capacity. However, the situation for Individual C is inherently untenable for he is receiving more information than he is able to process without a large number of errors. Other things remaining equal, two remedies suggest themselves: (1) the replacement of Individual C with another having a greater information-processing (channel) capacity,

or (2) the reconstruction of the situation so that Individual C needs to receive no more than thirteen bits of information per unit of time. The attention of this study is now turned to a review of the extent to which human information-handling capacity is presently assessable.

B. An Information-Processing Model

1. Input. Man reacts to but a fraction of the stimuli that bombard him. This may be due to the failure of the stimulus message to reach its destination which may be due to any combination of four causes. First, there may be no sensory receptor that can be tuned to receive a given stimulus signal. An example of this is the type of sound which Man cannot hear but to which a dog responds. Second, the channel capable of carrying a particular signal may be momentarily overloaded. If the human eye had greater capacity for transmitting information, the illusion of motion pictures would disappear and fluorescent lights would flutter. Third, the intensity of the signal at any transducer may not be strong enough to cause it to recode the signal and send it toward its destination. At the human level, both sensory receptors and synapses may be termed transducers for communication purposes.

A sensory receptor is a set of nerve fibers. A synapse is a point at which a number of incoming nerve fibers are connected to a single outgoing nerve fiber.[12] Wiener defines a threshold of action as the number of incoming fibers that must fire before an outgoing fiber fires.[13] "If a combination of incoming messages will not cause an outgoing fiber to fire, it is said to be below threshold; otherwise it is said to be above threshold."[14] Fourth, the signal-to-noise ratio of the channel may be such as to drown out the intended message. That is, if the intensity of noise in the channel exceeds the intensity of the signal, the signal will pass undetected.[15]

However, even if a signal reaches the brain, there is no assurance that it will cause a reaction. One reason is that it may arrive simulatensouly with one or more other signals and must then compete with them for processing priority. An early finding that agrees with everyday experience is that it is more difficult to understand two messages arriving simultaneously than

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[12] Norbert Wiener, The Human Use of Human Beings (Boston: Houghton-Mifflin Co., 1950), p. 34.

[13] Ibid.

[14] Ibid.

[15] Thomas H. Crowley, et. al, Modern Communications (New York: Columbia University Press, 1962), p. 5.

two messages arriving one after the other. Later studies have indicated that the degree of difficulty in dealing with two simultaneous spoken messages depends upon the number of other messages that might have arrived instead of the two that did. That is, when a listener knows to within a small number of alternatives what each message will be, he can comprehend two simultaneous messages.[16]

As the number of possible alternative messages increases, the brain apparently resorts to processing information sequentially. Emerging conceptions of researchers indicate that the human perceptual system generally functions as a single-channel system with information from only one source gaining access to it at any given time. These include Broadbent (1958),[17] Feigenbaum and Simon (1963)[18] and Colin Cherry (1953).[19]

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[16]Donald E. Broadbent, "Attention and the Perception of Speech," Scientific American, 1962, Vol. 206, No. 4,pp. 146.

[17]Donald E. Broadbent, Perception and Communication (New York: Pergamon Press, 1958).

[18]E. A. Feigenbaum and H. A. Simon, "Brief Notes on the EPAM Theory of Verbal Learning," in C. N. Cofer and Barbara S. Musgrave (eds.) Verbal Behavior and Learning (New York: McGraw-Hill Book Co., 1963), pp. 333-335.

[19]Colin E. Cherry, "Some Experiments on the Recognition of Speech With One and Two Ears," Journal of the Acoustical Society of America, Vol. 25 (September, 1953), pp. 975-979.

The first three are psychologists while the last is a communications engineer who has studied the problem of transmitting information to human receivers.

There is evidence to suggest that the brain employs one mechanism to screen out all but one of a number of simultaneously-arriving messages and another to monitor or scan messages for matters likely to require attention. It is known that an individual may "tune out" one ear and attend to the message arriving from the other ear.[20] That screening is not an either-or proposition, however, has been shown by Moray (1959)[21] who demonstrated that a man fully occupied in listening to speech in one ear will hear his own name in the other although he remains quite unresponsive to any other word in that ear. Treisman (1960)[22] found that speech entering the rejected ear could break through to the subject's attention if it consists of words that could appropriately follow words that have just been heard by the ear receiving attention.

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[20] Donald E. Broadbent, "Attention and the Perception of Speech," Scientific American, 1962, Vol. 206, No. 4, p. 151.

[21] N. Moray, "Attention in Dichotic Listening, Affective Cues and the Influence of Instructions," in Quarterly Journal Experimental Psychology, 1959, Vol. 11, pp. 50-60.

[22] Anne M. Treismen, "Contextual Cues and Selective Listening," in Quarterly Journal of Experimental Psychology, Vol. 12, No. 4 (Nov., 1960), pp. 242-248.

Considering the present state of research, there appears to be no way to predict which of two simultaneously-arriving messages will be processed first if they cannot be processed together. The matter may depend upon the maximum processing rate of the brain and upon the function of attention mechanisms. It is plausible to believe that there are at least two types of attention mechanisms, one selecting messages on the basis of physical characteristics and the other on the basis of content.[23] Regarding the mechanism that operates on physical characteristics of messages, it is now a generally-accepted principle of neurophysiology that messages traveling along a particular nerve channel can differ either by involving different nerve fibers or by producing a different number of impulses per unit of time in the fibers. It is suggested that the rate at which sounds are pulsed controls the rate at which fibers fire.[24] If this is so, the brain could pick out one voice from others by focusing on all nerve fibers that are firing at the same rate.[25] Even less is known about the attention

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[23]Donald E. Broadbent, "Attention and the Perception of Speech," Scientific American, 1962, Vol. 206, No. 4, p. 151.

[24]Ibid., p. 148.

[25]Ibid.

mechanism that operates on content. It is now believed that subjects pay attention in two types of situations, the unexpected and the imperfectly learned. The difference, however, is but a matter of degree. In both cases, attention can be considered directed as a result of an imperfect prediction.[26] It may, thus, be that the content-oriented attention mechanism monitors all incoming messages, switching one into phase for processing only when it deviates from that which had been expected.

Another reason why a signal reaching the brain may not cause a reaction is interference. Interference may take a number of novel forms. For example, while it has been shown that the brain might be able to distinguish between two sounds having different pulse rates, it has also been shown that two sounds having the same pulse rate are perceived by the listener as one sound.[27] Thus, there is a real possibility that parts of two messages may arrive simultaneously and the brain will have no way to unscramble them. Another kind of interference results from channel

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[26]Nicholas Brown, "Attention: A Theoretical Note," The Journal of General Psychology, 1960, Vol. 62, p. 109.

[27]Donald E. Broadbent, "Attention and the Perception of Speech," Scientific American, 1962, Vol. 206, No. 4, p. 147.

overloading. At high transmission speeds, where the amount of information from a single source is more than the system can handle, switching from source to source often takes place. This time loss is directly related to a loss in learning in that learning rate decreases in proportion to the number of times that switching takes place.[28] The time loss in such switching is estimated at about 200 milli-seconds.[29]

A third reason why a signal reaching the brain may not cause a reaction is that it may be incomprehensible. That is, either the recipient does not know how to decode the message or, if he is able to decode it satisfactorily, it has no meaning for him. Both may be regarded as functions of experience and are, thus, potentially capable of being learned. However, if either the language or content is not known, the receiver cannot respond intelligently to the message, indeed he may not respond at all, and the assertion is made that no communication took place.[30]

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[28] Robert M. W. Travers, "The Transmission of Information to Human Receivers," Audio-Visual Communication Review, 1964, Vol. 12, No. 4, p. 376.

[29] Ibid.

[30] S. S. Stevens, "Proceedings of the Speech Communication Conference at MIT; Introduction: A Definition of Communication," Journal of the Acoustic Society of America, 1950, Vol. 22, p. 689.

A final reason for non-reaction to a received signal is that the message may either be redundant or *appear* to be redundant to its recipient. It has already been pointed out (p. 63) that, if the receiver has already received and suitably processed a message bearing the same potential information, then he can derive no increase in knowledge from receiving the same message again and,hence, it contains no information for him. In such a case, a duplicate message may cause no reaction. However, it may be that, while the message is not *in fact* redundant, it may appear so to the receiver. Sebald (1962)[31] cites evidence that learned subjective realities persist and are resistant to change and that,to a great extent, an individual perceives only those meanings that reinforce prior images. We also observed that "selective distortion" took place to screen out dissonant features -- features apt to disturb pre-conceived images.[32] That is, selective perception and selective distortion may act subtly to make novel messages seem redundant.

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[31]Hans Sebald, "Limitations of Communication: Mechanisms of Image Maintenance in Form of Selective Perception, Selective Memory and Selective Distortion," *Journal of Communication*, 1962, Vol. 12, No. 3, pp. 142-149.

[32]*Ibid.*, p. 149.

From the available research it is difficult to arrive at meaningful figures for the input capacities of the several different kinds of sensory receptors, Attempts to measure input capacities in bits per unit of time are non-existent. There appears to be only the work of Jacobson (1951)[33] who estimated the information capacity of the ear at 10,000 bits per second. Before span of control can be set out in terms of information and communication theories, it will be necessary to measure precisely the transducing capacities of the sensory modalities so as to get a measure of the maximum rate at which information may impinge upon the brain. These measures of transducing capacity must be taken under a wide variety of conditions, to include such modifiers as fatigue, stress, and aging.

Before information input may be processed at conscious levels, it must be suitably decoded. At least two dimensions of decoding ability need to be examined which are the rates at which decoding in the several modalities can be done and the ability of sensory receptors to discriminate among various types of coded signals. Research to date is not able

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[33]H. Jacobson, "Information and the Human Ear," Journal of the Acoustic Society of America, 1951, Vol. 23, pp. 463-471.

to measure decoding (or encoding) rates as such because research designs do not allow these processes to be distinguished from other operations performed in sequence on the same information such as arithmetic or logical processing or reporting operations. As a result, it may be fairly said that the measures so derived apply only to the slowest of the sequential operations and that, if they cannot otherwise be distinguished, it cannot be said that the observations pertain to encoding or decoding ability. For example, if one tries to measure reading speed by having the subject read aloud, the resultant measure is smaller than if the person is asked to read silently. This indicates that, when the rate of reading aloud is tested, the measure may be depressed by the sequential encoding process which follows it. On the other hand, if the person is asked to read to himself and then to indicate the amount of material covered, the only way to determine the validity of the measure is to examine the subject on what he read. This also implies the further processing of information. The same qualifications apply to the discriminatory abilities of sensory receptors for, while some interesting research has been done in this area, it is difficult to determine whether the findings reported are due to decoding ability or comparing ability.

2. Storage and Retrieval. Man has evolved or consciously developed a number of storage and retrieval devices and procedures for the preservation, location and recall of information. Research is leading many psychologists to believe that there are at least two kinds of internal storage or memory. However, very little is presently known about the mechanics of either storage or retrieval. It is felt that there is a primary internal storage capability that is limited in capacity and in the ability to store information for longer periods of time. Complementing this is a much larger and more-nearly permanent secondary internal storage. In addition to these forms of internal storage, Man has learned to build and use several forms of external storage which may be either permenent or temporary and which are typified by the written record.

The inaccessibility to researchers of the internal storage device has prompted many different explanations for the physical processes of storage and retrieval. Nearly all agree that information is placed into storage by inducing a physical change in a definite part of the brain, that only a part of the information presented for storage can be later retrieved, that permanence of storage is facilitated by a re-examination or "rehearsal" of the information, and that re-organization

and modification of storage contents take place. One partial model of the storage system is given by Prywes (1961)[34] which assumes the existence of a hierarchy of memories. In the model, all incoming messages are recoded by an encoder for storage in the Immediate Memory. The Immediate Memory is assumed to be the point of storage for all information being processed or in transit. The Associative Memories are much larger and more complex, storing past experiences that can be retrieved by inter-connected associations. Transfers from the Immediate Memory to the Associative Memories is through the establishment of associations with past experiences or by generating new associations. Waugh and Norman (1965)[35] have developed a model for primary internal storage that makes the following points: (1) unrehearsed verbal stimuli tend to be quickly forgotten because they are interfered with and not because their "traces" decay, (2) rehearsal of information may cause it to be transferred from a very limited primary storage to a larger and more stable secondary storage, and (3) recently-perceived items may be stored in both

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[34]Noah S. Prywes, "Data Processing Aspects of Some Psychological Experiments," Perceptual and Motor Skills, 1961, Vol. 12, pp. 155-160.

[35]Nancy C. Waugh and Donald A. Norman, "Primary Memory," Psychological Review, 1965, Vol. 72, No. 4, pp. 89-104.

systems of storage simultaneously. There is no agreement as to what sort of a physical change takes place when information is placed into storage. Semon (1921)[36] described the process as an enduring if not permanent change somewhere in the nuclear material of the brain cell. Terms such as "imprinting" and "memory traces" suggest an engraving process. McCullough (1951)[37] thought of it as an electrical process with "reverberatory circuits." However, electrical resonance has yet to be demonstrated[38] and the present point of view is that information storage is achieved by a molecular change.[39]

It may be that each of the two internal storage-retrieval systems helps the supervisor keep track of his situation in its own way. It has been shown by Yntema and Mueser (1962)[40] that, given enough time, an individual can keep track of a highly-elaborate

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[36]R. Semon, The Mneme (New York: Allen and Urwin, 1921).

[37]W. S. McCullough, "Why the Mind is in the Head," in L.A. Jeffress, (ed.) Cerebral Mechanisms in Behavior; the Hickson Symposium (New York: Wiley and Sons, 1951).

[38]David Wechsler, "Engrams, Memory Storage, and Mnemonic Coding," American Psychologist, 1963, Vol. 18, No. 3, pp. 149-153, p. 150.

[39]Ibid.

[40]Douwe B. Yntema and Gayle E. Mueser, "Keeping Track of Variables that Have Few or Many States," Journal of Experimental Psychology, 1962, Vol. 63, No. 4, pp. 391-395.

situation; however, if the situation begins to change, the ability of an individual to keep track of its aspects falls off rapidly. The foregoing discussion seems to indicate that secondary storage can retain large amounts of information but that association time and the need for rehearsal impede the rate at which it can be placed into storage. If this is true, the stable aspects of a supervisory situation may be placed into permanent secondary storage. Such aspects may be stored in great detail because no empirical limit has been discovered on the capacity of internal secondary storage.

However, it may be argued that every supervisor's situation has some dynamic aspects. If it had none, it would soon become predictable and, according to information theory, require no supervision. It is these changing aspects of a situation that tax the supervisor's ability and one reason for this may be his limited capacity for primary internal storage or immediate memory. The nature of immediate memory has been of great concern to psychologists ever since the classical memory span experiments of Jacobs in 1887.[41] Because of this concern, some of the characteristics of immediate memory have emerged from the many studies

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[41] J. Jacobs, "Experiments in Prehension," Mind 1887, Vol. 12, pp. 75-79.

that have been focused upon it. These characteristics include capacities or memory spans for various kinds of information, the effects of decay, interference, interpolated material and the organization of storage. In one experiment to measure retention during steady-state conditions, Mackworth (1959)[42] made a detailed study to determine the number of stimuli that subjects could successfully recall in a simple stimulus-response task. He found that, to achieve a level of correct performance of 80%, one second per stimulus was required for each stimulus to be held in memory. In such a continually changing serial task, a subject's effective memory span was about three to four items of information. In a similar experiment which substituted recognition procedures for recall, Shepard and Teghtsoonian (1959)[43] found that subjects are capable of carrying along at least fifty bits of information as compared with the twenty-three bits they have been found capable of retaining from a single exposure to a sequence of decimal digits (Miller, 1956).[44]

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[42]J. F. Mackworth, "Paced Memorizing in a Continuous Task," Journal of Experimental Psychology, 1959, Vol. 58, pp. 206-211.

[43]Roger N. Shepard and Martha Teghtsoonian, "Retention of Information Under Conditions Approaching a Steady State," Journal of Experimental Psychology, 1961, Vol. 62, pp. 302-309.

[44]George A. Miller, "The Magical Number Seven, Plus or Minus Two: Some Limits on Our Capacity for Processing Information," Psychological Review, 1956, Vol. 62, pp. 302-309.

Posner (1963)[45] made a review of the research on immediate memory to which the interested reader is directed. As much of what is known is either qualitative or relevant for only highly specific forms of stimuli, much replication needs to be done in ways which will increase the generality of the findings. Two starting points for research on immediate memory as it applies to the supervisory situation are suggested: (1) the translation of existing findings into universal information units or bits so that a basis for this comparison might be obtained, and (2) the definition and measurement of change in specific supervisory situations to determine whether or not its rate exceeds the capacity of immediate memory.

3. Arithmetic and Logical Processing. There are some studies of human information processing capacity in which the results have been translated into bits or bits per unit of time. Perhaps the most illustrative and celebrated of these is the work of G. A. Miller (1956).[46] First discussing such concepts as bits and channel capacity, he then reviewed studies of absolute judgment of uni-dimensional stimuli such

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[45]Michael I. Posner, "Immediate Memory in Sequential Tasks," Psychological Bulletin, 1963, Vol. 60, No. 4,pp. 333-349.

[46]Goerge A. Miller, "The Magical Number Seven, Plus or Minus Two: Some Limits on Our Capacity for Processing Information," Psychological Review, 1956, Vol. 63, No. 2 (March, 1956), pp. 81-97.

discrimination of frequency of pure tones (Pollack, 1952)[47] and their loudness (Garner, 1953),[48] taste intensities (Beebe-Center, Rogers and O'Connell, 1955)[49] and visual position (Hake and Garner, 1951).[50] Finding that the human capacity for discriminating among such stimuli ranged from only 1.6 to 3.9 bits, he then made the following observations: (1) considering the wide variety of different variables that had been studied, the range of capacities was remarkably narrow, and (2) it appears to be safe to say that there is a limitation, either of learning or of the design of the nervous system, that limits human channel capacities.[51]

Since his findings were inconsistent with human abilities to recognize hundreds of faces and thousands of words, Miller then went on to examine studies of absolute judgment of two-dimensional stimuli such as the position of a dot in a square (Klemmer and Frick,

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[47] I. Pollack, "The Information of Elementary Auditory Displays," Journal of the Acoustic Society of America, 1952, Vol. 24, pp. 745-749.

[48] W. R. Garner, "An Informational Analysis of Absolute Judgments of Loudness," Journal of Experimental Psychology, 1953, Vol. 46, pp. 373-380.

[49] J. G. Beebe-Center, M. S. Rogers, and D. N. O'Connell, "Transmission of Information About Sucrose and Saline Solutions Through the Sense of Taste," Journal of Psychology, 1955, Vol. 39, pp. 157-160.

[50] H. W. Hake and W. R. Garner, "The Effect of Presenting Various Numbers of Discrete Steps on Scale Reading Accuracy," Journal of Experimental Psychology, 1951, Vol. 42, pp. 358-366.

[51] George A. Miller, op. cit., p. 86.

1953)[52] and combined loudness and pitch of pure tones (Pollack, 1953).[53] As he expected, abilities to judge increased but not to so great a value as would have been obtained by multiplying a corresponding uni-dimensional stimulus judgment value by two. He concluded that the addition of independently variable attributes to the stimulus increased the channel capacity but at a decreasing rate.[54] He then proposed to call this limit the span of absolute judgment.[55] Men have learned to circumvent this limit three ways: (1) by making relative rather than absolute judgments, (2) increasing the number of dimensions along which the stimuli can vary, and (3) arranging the task so that a sequence of several absolute judgments is made.[56]

Finally, Miller noted that his span of absolute judgment for uni-dimensional stimuli was about seven items, the same as the span of immediate memory and nearly the same as the span of attention which encompasses about six items. He warned that, although these three spans appeared to be but different aspects

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[52]E. T. Klemmer and F. C. Frick, "Assimilation of Information from Dot and Matrix Patterns," Journal of Experimental Psychology, 1953, Vol. 45, pp. 15-19.

[53]I. Pollack, "The Information of Elementary Auditory Displays," II. Journal of the Acoustic Society of America, 1953, Vol. 25, pp. 765-769.

[54]George A. Miller, op. cit., p. 88.

[55]Ibid., p. 90.

[56]Ibid.

of a single underlying process,the span of immediate memory was not limited to $\log_2 7 = 2.81$ bits of information but, because of the ability of the mind to reorganize information, it could handle seven "chunks" of information where the limit to the number of bits per "chunk" has not yet been defined.[57] The ability to reorganize information has been recognized by psychologists ever since Freeman showed an increase in the span of apprehension for well-grouped dots.[58] Allan (1961)[59] stated that research suggests that perceptual organization may be a basic activity of the human communication system, that it is a function of channel overloading and that it will not occur when the channel is operating below capacity but only under stress. This tends to confirm Miller's "chunk" theory.

Some other parameters on human information processing capacity have been tentatively determined. Some of these involve the concept of reaction time. As early as 1953, Human found that reaction time is a

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[57] Ibid., p. 93.

[58] R. S. Woodworth, Experimental Psychology, (London: Methuen, 1938), p. 693.

[59] Mary D. Allan, "Memorizing, Recoding and Perceptual Organization," British Journal of Psychology, 1961, Vol. 52, pp. 25-30.

monotonically increasing function of the amount of information in the stimulus series.[60] Behar (1963)[61] found that average reaction time was proportional to the $\log_2$ of the number of response alternative which is the amount of information in bits. Wiegand (1963)[62] did three studies to show that reaction time increased linearly as each of (1) uncertainty, (2) joint stimulus-response uncertainty, and (3) uncertainty in a complex information task increased. It is to be realized that reaction time is analogous to the data-processing concept of "turn-around time" in that it includes reception and transmission times as well as information processing time. More interesting but less defensible are estimates that have been made of actual information processing rates of humans. Jacobson (1951)[63] made an estimate of human ability to process auditory

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[60] R. Hyman, "Stimulus Information as a Determinant of Reaction Time," Journal of Experimental Psychology, 1953, Vol. 45, pp. 188-196.

[61] I. Behar, "On the Relations Between Response Uncertainty and Reaction Time in Category Judgments," Perceptual and Motor Skills, 1963, Vol. 16, No. 2, pp. 595-596.

[62] Karl L. Wiegand, "Information Theory and Human Behavior: Uncertainty as a Fundamental Variable in Information Processing Tasks," USAF AM RL TDR 63-89, 92 pp.

[63] H. Jacobson, op. cit.

input. For a person with a vocabulary of 150,000 words, speech "appreciation" was estimated at about ninety bits per second. Thomas (1963)[64] assumed that information translation tasks are strictly proportional to the amount of information to be handled and estimated that the human information handling rate is from 2.07 to 5.78 binary units per second where one binary unit may be the same as one bit. Raisbeck (1964)[65] made an interesting estimate of Man's ability to process visual-literal material. Observing that a good, average reading speed is 500 words per minute (without resorting to skipping), he then calculated that, at an average of five letters per word, a good reader averages forty-two letters per second. Using Shannon's estimate of the entropy of written English,[66] one bit per letter, he established that a man can process with his eyes and his mind about forty bits per second, paying at least some attention to all of it and making some use of it.

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[64] H. B. G. Thomas, "Communication Theory and the Constellation Hypothesis of Calculation," Quarterly Journal of Experimental Psychology, 1963, Vol. 15, No. 3, pp. 173-191.

[65] Gordon Raisbeck, Information Theory. An Introduction for Scientists and Engineers (Cambridge, Massachusetts: The Massachusetts Institute of Technology Press, 1964),pp. 47-49.

[66] Claude E. Shannon, "Prediction and Entropy of Printed English," Bell Systems Technical Journal, Vol. 30 (Jan., 1951), pp. 50-64.

C. The Mechanism of Control

Man is often described as being deliberate, conscious and purposeful. These are terms that imply that Man wishes on occasion to act independently of the stimuli that are impinging upon him. In this connection it is possible that the actions he contemplates require him to have knowledge which he does not possess and which he cannot derive from these impinging stimuli. To put it another way, uncertainty about the present state of one or more elements of a Man's environment or uncertainty about probable changes in their states may prevent him from doing what he wants to do. To increase the degree to which he may act freely, Man has developed a number of devices which reduce the degree of uncertainty he perceives in his environment to a satisfactorily low level. One such device consists of testing the environment by establishing a two-way communication system or loop between the individual and the elements of the environment wherein he wishes to reduce the level of uncertainty. First, the individual forms a mental hypothesis or ideal of what he hopes that element is like. Then, he does something to it (transmits a communication) and observes it (receives a communication) to determine the extent to which his concept is accurate. Each such reinforced concept causes an increase in knowledge and a reduction in uncertainty.

To the extent that doing something to the environment causes a change in the communication being received by the acting individual, that is, if the system's output causes an alteration in its input, it is said that feedback occurs.[67] A communication system having the feedback feature, called a feedback loop, has recently been recognized as a basic form of human behavior. Miller, Galanter and Pribram (1960)[68] have tried to evaluate the possible role of feedback as the basic element of behavior. They state that the reflex has been grossly over-rated as the basic unit of behavior for anything except highly-restricted experiments such as bar-pressing. They propose instead what is known as the Cybernetic Hypothesis: The fundamental building block of the nervous system is the feedback loop.[69] Simultaneously but independently, Powers, Clark and McFarland (1960)[70] have also developed a feedback

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[67] H. Von Foerster (ed.) Cybernetics: Circular Causal, and Feedback Mechanisms in Biological and Social Systems, Transactions of the Tenth Conference (New York: Josiah Macy, Jr., Foundation, 1953), p. 71.

[68] George A. Miller, Eugene Galanter and Karl H. Pribram, Plans and the Structure of Behavior (New York: Henry Holt and Co., 1960).

[69] Ibid.

[70] W. T. Powers, R. K. Clark and R. L. McFarland, "A General Feedback Theory of Human Behavior: Part I and Part II," Perceptual and Motor Skills, Monograph Supplements, I-VII, pp. 71-88, 3-VII, pp. 309-323, (1960).

explanation of human behavior in which they state that communication is a high-order, externally-fed, feedback system.

To be effective as a reducer of uncertainty, a feedback loop must have certain characteristics. First, it must be power-amplifying; that is, it must have two inter-connected power sources such that the larger is controlled by the smaller. Second, it must be sensitive to error. Error is the difference between a system's desired output and its actual output. Error actuates the system by affecting its input. Third, it should be self-zeroing in that it causes convergence of actual and desired outputs. To be self-zeroing, its feedback must be negative. Feedback is formally defined to be negative if the ratio of its input to output, called gain, is less than one.[71] Zeroing can occur only if gain is less than one for, if a system's gain is greater than one, its error and oscillation are both increased and the system becomes unstable. A feedback loop oscillates due to the delay between the occurrence of error and the effect of that error

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[71] F. Johnson and George R. Klare, "Feedback: Principles and Analogies," Journal of Communication, 1962, Vol. 12, No. 3, p. 159.

upon input. If gain is equal to one, the system will oscillate at the same rate indefinitely. Fourth, the system should function in a pre-designed manner, and finally, it should adapt itself to unforeseen conditions.[72]

It is important to recognize the existence of one or more reliable feedback loops having the above characteristics in every successful supervisory situation. To begin with, each supervisor forms a concept of his situation which is to say that he has ideas about <u>what</u> he and his subordinates are doing and what they should be doing, both cooperatively and individually, as well as ideas about <u>how</u> they are and should be doing it. Until he makes observations of various aspects of his situation, he will be uncertain as to how these notions square with reality. As he makes observations he reduces his uncertainty about how things are and he is able to compare them with how he thinks things ought to be. To the extent that his perceptions agree with his observations, the supervisor does not disturb the situation but merely continues to observe until the general level of uncertainty is satisfactorily low. If his observations fail to provide him the information he needs to reduce uncertainty in a

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[72] <u>Ibid</u>.

particular area, he institutes procedures that are aimed at supplying that information.

All is well until reality and concepts of it begin to diverge. Then an open-minded supervisor might observe that (1) the concept should be modified to agree with reality, (2) adjustments should be made in the environment to bring the reality into agreement with the concept, or (3) both should be modified to reach mutual agreement. In each case, a feedback loop is set up to facilitate the change. If a modification of concept is desired, the loop supplies information that is compared against the concept as well as outoing communication aimed at testing the extent to which the modified concept agrees with reality. If it is desired to modify reality, the loop is set up to communicate adjustments, observe resultants, and communicate any needed further adjustments.

The establishment and maintenance of control of a situation by a supervisor,using uncertainty reduction as an end and the feedback mechanism as the means to an end, can be seen to depend upon a number of factors. First, it depends upon the nature of the situation itself. As has been reported earlier, the situation is determined by such factors as complexity of purpose, technological conditions, the number of

of persons being supervised, the other duties of the executive, the stability or predictability of operations and geographic contiguity.

Second, control is determined by the validity, the degree of detail and the amount of compatability of the set of concepts held by each individual in that supervisory situation. (By a model is meant a structure of symbols and operating rules which is supposed to match a set of relevant points in an existing structure or process.)[73] A model is valid to the extent to which its points actually correspond to analogous points in reality and its degree of detail indicates the number of points of it that correspond. Validity and detail are obviously useful characteristics as they determine to a great extent whether or not a person "knows what he is doing." Both can be outgrowths of education or experience and both are clearly susceptible to coloring by attitudes. In part, when a job calls for experience, it is asking for an individual who has valid and detailed notions about that job. The models of two people are compatable to the degree that they are consistent with each other. Compatability may not be a problem where a

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[73] K. W. Deutsch, "On Communication Models in the Social Sciences," Public Opinion Quarterly, Vol. 16 (Fall, 1952), p. 357.

task is highly concrete and specific. However, abstract concepts frequently mean different things to different people and it is obvious that incompatable abstract notions can cause misunderstandings requiring the attention and the time of members of the group. The set of concepts held by any individual was developed and is modified only through effective communication.

Third, the supervisor must set up and maintain a communication system by means of which he may receive reliable information about his actual situation. This information may be used to modify his concepts about the reality of the situation or it may be used to modify the reality to agree with his concepts. Information may be unreliable for a number of reasons. First, noise and ambiguity can confound a message so that it is misunderstood. Noise can be reduced by increasing the redundancy of coding but this causes a reduction in transmission rates. Codes may be generated that are free of ambiguities but these are likely to be imperfectly learned by their users. Second, messages may be unreliable if they are deliberately deceiving. Reducing the possibility of being misled by deceiving messages is done by consistency checking. A consistency check may be performed in three ways: (1) by comparing two or more messages recieved from the same channel, (2) by comparing

two or more messages received from two or more sources, and (3) by comparing two or more messages received from the same source but along different channels or modalities.

Fourth, the supervisor must establish procedures that will allow him to receive enough information about all significant aspects of his situation so that his uncertainties about them are reduced to and remain at satisfactorily low levels. These procedures may involve monitoring of or scanning all incoming messages (p. 80) switching them into channels for processing only when unexpected information is received. They may involve the employment of predictable (skilled and experienced) people and understandable machines and techniques. They may involve the deliberate establishment of procedures designed to supply needed information.

Fifth, the supervisor must be able to communicate information about his concepts and about the actual situation to his subordinates. This implies the understanding and use of mutually-understandable symbols and code words.

Sixth and finally, he must set up and maintain negative feedback loops to detect errors and communicate corrections such that the real situation will stabilize and that there will exist a high degree of agreement

between the reality and his concepts of it. It is often desirable to operate several different kinds of loops simultaneously between two individuals. One such loop might be used to bring the supervisor's notion of reality into agreement with that of his subordinate, another to increase the validity and detail of the supervisor's notions of reality, a third to check the reliability of communications being received, and a fourth to detect errors in the design of the loop itself. It is not suggested that each feedback loop be in continuous operation. Rather, each should be used only when the uncertainty it is designed to reduce rises above a satisfactory level. Knowing how, when, and how often to operate a given loop is the responsibility of the supervisor and it relates directly to his ability to store, retrieve and process information.

D. A Re-Statement of the Principle of Span of Control

It will be recalled that this study was undertaken with four assumptions: (1) that supervision and control were devised to reduce the uncertainty (increase the predictability) of situations, (2) that supervision and control consist of sequences of communication and information processing procedures, (3) that Man's ability to communicate and process information is limited, and (4) that a supervisor may perform non-supervisory tasks. Thus far, this chapter has been

devoted to developing and supporting a theoretical structure which will provide a framework in which these assumptions may be tested when it is more nearly complete. Some evidence has been given for the key assumption that Man's ability to communicate and process information is limited. It may also be deduced from the definition of an information measure that a positive amount of information must be transmitted to an individual if he is to know of the occurrence of a probable event. From this it is possible to generalize that an individual must receive and process information at a rate no less than n bits per unit of time if he is to know the current state of a situation in which probable events are producing information at a rate of n bits per unit of time. That is, there is a minimum rate at which information must be processed by an individual if he is to keep track of a continually changing situation. The remainder of this chapter is given to showing how the limitations on Man's communication and information-processing capabilities might reasonably be expected to determine span of control. Reasonableness is established by showing how the factors that are held by administrators to be the determiners of span of control may be translated into the terms of communication and information theory and treated in one cohesive theoretical structure.

At the same time, the usefulness of such a notion is demonstrated by showing how the joint communication-information processing model may be used to explain the satisfactory operation of supervisory situations having widely varying spans of control.

1. A New Look at the Determiners of Span of Control. In Chapter III (pp.41-45) were listed the factors held by some to the determiners of span of control. These were: (a) the complexity of purpose and technological conditions, (b) the difficulty of the communication process, (c) the extent to which communication is necessary, (d) the complexity of the personal relationships involved, (e) the time available for such supervision, (f) the mental capacity and the personal adaptability of the executive responsible for the supervision, (g) the complexity of the situation being supervised, (h) the other duties of the executive, (i) the stability of operations, (j) the capability and experience of subordinates, (k) similarity of functions, (l) geographic contiguity, (m) complexity of functions, (n) direction and control, (o) coordination, (p) planning, and (q) organizational assistance. To show how these factors may be related to interpersonal communication and human information-processing capabilities, they may be organized under two main headings: (a) factors affecting the extent

to which communication and information processing are necessary, and (b) factors affecting human communication and information processing capabilities. An outline showing such an organization is given below. Lower case letters in parentheses refer to factors (a) through (q) above. Some additional items were supplied for clarification or discussion purposes. The outline is not intended to be exhaustive but is only designed to show how these factors may be organized to reveal their relationship to communication and information theory. Broadly stated, a supervisor's span of control may be said to be a function of six major determiners: (1) the degree of uncertainty in the situation, (2) the prevailing concept of supervision, (3) the needs of all the individuals in the situation to reduce the degree of uncertainty, (4) the communication and information-processing capabilities of those individuals, (5) the difficulty of the communication process, and (6) the proportion of time available for supervision. These broad categories may be used to explain why the number of individuals being supervised may vary widely from situation to situation. Consider the six examples of broad spans of control cited in Chapter III (pp. 45-49).

TABLE 5

OUTLINE OF DETERMINERS OF SPAN OF CONTROL

I. Factors Affecting the Extent to Which Communication (c) and Information Processing are Necessary

- A. Factors Affecting the Uncertainties in the Supervisory Situation
  - 1. Complexity or simplicity of the situation (g)
    - a. complexity of purpose (a1) and functions (a)
    - b. technological conditions (a2)
    - c. stability of operations (i)
      - 1. personnel turnover rates
      - 2. ends and means to ends
    - d. similarity of functions (k)
  - 2. Abstractness-Concretness dimension
  - 3. Precision-Ambiguity/Vagueness dimension
- B. Prevailing Concept of Supervision
  - 1. Direction (m1)
  - 2. Control (m2)
  - 3. Planning (p)
  - 4. Coordination (o)
  - 5. Growth and development of personnel
- C. Needs of Individuals to Increase Certainty

II. Factors Affecting Human Communication and Information Processing Capabilities

- A. Mental Capability, Adaptability (Training, Experience and Intelligence)
  - 1. Mental capability and the personal adaptability of the executive responsible for supervision (f)
  - 2. The capability and experience of subordinates (j)
- B. The Difficulty of the Communication Process (b)
  - 1. Geographic contiguity (l)
  - 2. Complexity of human relationships involved (d)
  - 3. Coding
- C. The Time Available for [such] Supervision (e)
  - 1. The other duties of the executive (h)
  - 2. Organizational assistance (q)

2. Examples of Broad Spans of Control That Work.

The first example given of a broad span of control that works is that of the teacher in the traditional classroom situation. An examination of the above outline quickly reveals that such a situation may be easily managed without resorting to the establishment and maintenance of an elaborate communication system with feedback because the situation may be constructed to present a low degree of uncertainty to the teacher. The purpose of the situation may be to "cover" a textbook, a task which may be easily defined in terms of pages. Procedures can be routinized so that each day is like all of the others. Each child is given the same assignment to be completed at about the same pace and in the same way, and evaluated by one set of uniform and absolute standards. Only the exception must be spotted and dealt with on an individual basis. To the extent that the traditional classroom situation is manipulated by the teacher for ease of management, it is said to be a teacher-oriented classroom. Nothing is said here about the extent to which learning can occur in such a classroom. It is only suggested that the traditional classroom situation may have been developed for ease of management and not necessarily effective instruction as measured by learning outcomes.

Inexperienced teachers often strain the determining parameters of their situations -- and promptly lose control over them. They may simultaneously establish such a profusion of activities that they can neither keep track of them all nor find time to keep them stimulated. They may raise the level of uncertainty and anxiety among their students by defining ends and means in ways that are vague, contradictory or ambiguous. They may behave unpredictably with regard to items affecting the security of students such as marks and tests. Fortunately, experience can help inexperienced teachers in this respect. They may sense a need to reduce the uncertainties in their situations and simplify as well as routinize them to make them more manageable. However, as they gain experience, they also learn more about teaching. This internalizing of some of the parameters of education can also increase their ability to manage by reducing the amount of uncertainty they perceive in their classroom situations.

In the case of the building principal with fifteen or more teachers under him, a broad span of control can also be made to work successfully -- if the problem is properly approached. Perhaps the least satisfactory solution is to define supervision marginally; that is, in terms where the aspects to be supervised are both curtailed and capable of being easily monitored. Milk

money is accounted for and "order" in the classroom is assessed by the absence of noise, the presence of neat rows of quiet children, and of floors that are free of scrap papers. Teacher attendance and punctuality are observed and it is noted whether or not reports and report cards have been filled out correctly and on time. A better solution is for the principal to recognize the abilities of the faculty members being supervised, thereby reducing his need to know about everything that is going on at any instant. A most favorable solution for such a principal is for him to de-emphasize i-dotting and t-crossing, to define supervision in terms of educational parameters, and then to concentrate on those aspects of the supervisory situation where professional competency and experiences may be shared in the joint solutions of educational problems.

The stability of the operation is one factor which makes it possible for the Pope of the Roman Catholic Church to operate with a large span of control. The very low rate of change makes the operation of the church highly predictable to all. Also, there is much duplication of functions making it relatively easy for one to conceptualize its operation. Finally, the long period of training and its intense discipline for professionals guarantees a high degree of conformity or predictability.

Industrial concerns such as General Motors or Sears have such smaller spans of control though still exceeding the Graicunas-Urwick limit of six by a significant number. Such concerns generally have a simple purpose: to make money by manufacturing and marketing one or more products. They must be responsive to changes in a dynamic and still unpredictable economy. Thus, if a line of products fails to make money for its company,it is either withdrawn or modified. Such concerns generally have highly sophisticated and sensitive communication systems in the area of financial accounting to keep management at all levels informed as to the extent to which a given line is making money. A large span of control is feasible in the light of such a simple goal. Supervisors are chosen and promoted on their ability to turn a profit. As long as they do so, they are given a relatively free hand. If they fail, it is almost axiomatic that they will be replaced. In every case, the main feedback loop is in the financial accounting system.

The smallest spans of control are to be found in military organizations where the subject first became a matter for study. An army with an intelligent enemy faces a situation with a high degree of uncertainty. In addition, it has a high personnel turnover rate in combat and the typical line officer has a variety of functions to supervise. The life-and-death

nature of the task enhances the needs of all involved to increase the predictability of survival. Capabilities and adaptabilities of both officers and men vary widely for, while a cadre of professionals generally forms an army's core, it must rely on the judgment of individuals due to geographic dispersion and the need for security. To compensate for these disadvantages, the goal is simplified (to meet the enemy and defeat him) as is the concept of supervision ("Do what you're told and don't ask questions!"). The superior is given almost total power over his subordinates and, as the task grows more complex at higher levels, he is given considerable staff assistance so that he may devote nearly all of his time to the control of his subordinates.

CHAPTER V

CLOSING REMARKS

A. A Review of Developments

This study was undertaken to provide a basis in theory and research for a more-nearly adequate restatement of the principle of span of control. As a pre-requisite, it was felt necessary to show that the topic was in an area of legitimate concern. That is, it had to be shown that something like span of control exists and that the conception of it which prevailed was inadequate. Then, having satisfactorily disposed of the former notion, it would be possible to substitute another concept for it and to demonstrate its ability to explain the same phenomena with greater accuracy. In essence, this was the strategy of the study as set out in Chapter I.

In Chapter II a perspective was provided for the critical review of span of control. The historical background of the development of the traditional concept was given and the Gracicunas-Urwick version of span of control was identified as the object to be examined. Their rationale for span of control was described in detail.

Chapter III had three objectives: (1) to show that the Graicunas-Urwick version of span of control was not a rigorous proff, (2) to show further that it failed to take into consideration some of the variables regarded to be the determiners of span of control, and (3) to show that, in spite of the dissatisfaction with the principle as stated, there is a large degree of agreement that something like span of control exists.

The disposal of the Graicunas-Urwick version of span of control as a proof was relatively easy. First, it was shown that no evidence had been cited proving that Graicunas' "relationships" really mattered. Then it was demonstrated that he had made a logical error by failing to account for or to dispose of all possible types of such "relationships." Finally, it was shown that, while he tried to associate his rationale with the span of immediate memory, he failed to show any relationship between an average span of immediate memory of six digits and a supervisor's span of control of four individuals. To illustrate that the Graicunas-Urwick notion did not tell the entire story of span of control, a number of additional factors believed to affect it were listed. Also, several examples of supervisory situations were given in which broad spans of control had been seen to function satisfactorily.

Indications of the acceptance in principle of the notion of span of control were given in the concluding sections of the chapter to show that it may still be regarded as a valid concern.

The objective of Chapter IV should have been to develop a refined version of span of control. It should be noted that the study was begun by making a number of assumptions which essentially declared that span of control was determined by Man's limited ability to receive, process, and transmit information. To show the plausibility of these assumptions, two models were constructed, the first describing two-way interpersonal communication and the second showing Man to be a receiver, processer and transmitter of information. From the communication model it was possible to conclude that there is a minimum amount of information that must be received and processed if a supervisor is to keep track of all of the aspects of a changing situation. However, knowledge about the mechanics of information handling has not yet advanced to the point where a satisfactory model of Man as an information processer may be constructed. When this became known, the objective of the chapter was modified to providing a <u>basis</u> for the eventual refinement of span of control. This was accomplished by first showing how the factors already acknowledged to be among the determiners of span of control could be

accounted for jointly by the communication and information processing models. The examples of successful supervisory situations having broad spans of control were then reviewed and explained in terms of the models.

B. The Contributions Made by This Study

As was pointed out in Chapter I, this study can make three kinds of contributions. First, it identifies communication and information theory as fields which can contribute to the advancement of the study of administration. Second, it may reveal to some students of administration a desire to increase their familiarity with topics in mathematics and physics so that they might be able to examine more critically some of the ideas that have been developed in this study. Third, it describes specifically how a basis for the re-definition of span of control can be constructed from some of these ideas that are new to administrators, and, finally, it points out areas requiring further investigation before such a re-definition can be made.

C. Conclusion

The foregoing overview shows that many areas still need to be developed before Man will adequately understand his own information handling capabilities

and the concept of span of control can be adequately and precisely restated. More must be known about how the brain handles simultaneously received messages. The input and transducing capacities of the several sensory receptors must be measured as well as the relative importance of the channels to which they are sensitive. Coding efficiency must be increased and the optimum rate of auditory reception should be established. An understanding of monitoring and the interruption of processing would shed light on what information the mind attends to and regards as worth processing The various processing operations need to be analyzed so that the time it takes to perform each elementary operation, much as the decoding of a symbol or the addition of two one-digit numbers, can be distinguished Knowledge about the structure and the operation of storage and retrieval as well as storage capacity would help administrators avoid the design of situations that would be beyond a supervisor's span of control.

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APPENDIX

APPENDIX

THE DERIVATION OF EQUATIONS (1) - (5)

According to Graicunas, there are three types of "relationships" between a supervisor and his subordinates and among his subordinates. The first of these is the set of all direct relationships between the supervisor and each of his subordinates in the absence of any of the other subordinates. If A represents a supervisor with four co-equal subordinates, B, C, D and E, these relationships are:

A * B, A * C, A * D, and A * E.

With four subordinates there are four such relationships; with n subordinates their number would be $N_{1\circ} = n$.

The second type of relationship is the set of all direct relationships between the supervisor and each of his subordinates in the presence of all possible combinations of other subordinates. In the above situation these are:

A * B/C, A * C/B, A * D/B, and A * E/B;

A * B/D, A * C/D, A * D/C, and A * E/C,

A * B/E, A * C/E, A * D/E, and A * E/D,

A * B/CD, A * C/BD, A * D/BC, and A * E/BC;

A * B/CE, A * C/BE, A * D/BE, and A * E/BD;

A * B/DE, A * C/DE, A * D/CE, and A * E/CD;

A * B/CDE, A * C/BDE, A * D/BCE, and A * E/BCD.

With four subordinates there are twenty-eight such relationships. With n subordinates there would be n columns of the remaining n - 1 subordinates combined one at a time, plus n columns of the remaining n - 1 subordinates combined two at a time, plus n columns of the remaining n - 1 subordinates combined three at a time , . . ., plus n columns of the remaining n - 1 subordinates combined n - 1 at a time. However, the number of combinations of n things combined r at a time is:

$$ {}_nC_r = [n(n-1)(n-2)(n-3)\ldots(n-r+1)]/r!. $$

Thus, with n subordinates the number of such relationships is given by:

$$ N_{II^\circ} = n \cdot {}_{n-1}C_1 + n \cdot {}_{n-1}C_2 + n \cdot {}_{n-1}C_3 + \ldots + n \cdot {}_{n-1}C_{n-1} $$

$$ = n[{}_{n-1}C_1 + {}_{n-1}C_2 + {}_{n-1}C_3 + \ldots + {}_{n-1}C_{n-1}] $$

Resorting temporarily to a numerical analysis, it may be seen that:

$$ {}_1C_1 = 1 = 2^1 - 1, $$

$$ {}_2C_1 + {}_2C_2 = 2 + 1 = 3 = 2^2 - 1, $$

$$ {}_3C_1 + {}_3C_2 + {}_3C_3 = 3 + 3 + 1 = 7 = 2^3 - 1, $$

. . . .

$$ {}_{n-1}C_1 + {}_{n-1}C_2 + {}_{n-1}C_3 + \ldots + {}_{n-1}C_{n-1} = 2^{n-1} - 1 $$

(*), and $ {}_nC_1 + {}_nC_2 + {}_nC_3 + \ldots + {}_nC_n = 2^n - 1 $ (**)

Substituting (*) obtains:

$$N_{II^\circ} = n\left[2^{n-1} - 1\right] = n\left[2^n/2 - 1\right] .$$

The third type of relationship is the set of all indirect cross-relationships between all possible ordered pairs of subordinates. With four subordinates these are:

B -- C, C -- B, C -- D and D -- C;

B -- D, D -- B, C -- E and E -- C;

B -- E, E -- B, D -- E and E -- D.

Of these there are twelve. With n subordinates there would be twice the number of n things combined two at a time since the ordering of the pairs is to be considered. This obtains:

$$N_{III''} = 2 \cdot {}_nC_2 = 2n(n - 1)/2 = n^2 - n.$$

N_{I°, N_{II°, and $N_{III''}$ define all of Graicunas' N relationships. Thus:

$$N = N_{I^\circ} + N_{II^\circ} + N_{III''}$$
$$= \not{n} + n(2^n/2 - 1) + n^2 - \not{n}$$
$$= n(2^n/2 + n - 1).$$ This is Equation (1).

For four subordinates this is 4 + 28 + 12 = 44 or

$$N = 4(2^4/2 + 4 - 1) = 4(16/2 + 4 - 1) = 4(11) = 44.$$

Refer, now, to Graicunas' statement that "If Tom supervises two persons, Dick and Harry, he can speak to each of them individually or he can speak to them <u>as a pair</u>" (underline added). His statement admits the set of all direct relationships between the supervisor

and each possible group (of two or more) subordinates in the absence of any of the remaining subordinates. With four subordinates these are:

A * BC, A * BD and A * BE;

A * CD, A * CE and A * DE;

A * BCD, A * BCE, A * BDE and A * CDE;

A * BCDE.

With four subordinates there are eleven such relationships that are analogous to Graicunas' first set. With n subordinates there would be:

$$N_{I'} = {}^{n}C_2 + {}^{n}C_3 + {}^{n}C_4 + \ldots + {}^{n}C_n$$
$$= {}^{n}C_1 + {}^{n}C_2 + {}^{n}C_3 + {}^{n}C_4 + \ldots + {}^{n}C_n - {}^{n}C_1.$$

Substituting from (**) obtains:

$$N_{I'} = 2^n - 1 - {}_{n}C_1 = 2^n - n - 1.$$

There is a similar analogue to Graicunas' second set, the set of all direct relationships between the supervisor and each possible group (of two or more) of subordinates in the presence of any possible combinations of remaining subordinates. These are:

A * BC/D, A * BD/C, A * BE/C, A * CD/B,

A * CE/B, A * DE/B, A * BC/E, A * BD/E,

A * BE/D, A * CD/E, A * CE/D, and A * DE/C;

A * BC/DE, A * BD/CE, A * BE/CD, A * CD/BE,

A * CE/BD, and A *BCE/D, A * BDE/C, and A * CDE/B.

Of these there are twenty-two. With n subordinates there would be:

$$
\begin{aligned}
N_{II'} = {} & {}_nC_2 \cdot {}_{n-2}C_1 + {}_nC_2 \cdot {}_{n-2}C_2 + {}_nC_2 \cdot {}_{n-2}C_3 + \cdots \\
& + {}_nC_2 \cdot {}_{n-2}C_{n-2} \\
& + {}_nC_3 \cdot {}_{n-3}C_1 + {}_nC_3 \cdot {}_{n-3}C_2 + {}_nC_3 \cdot {}_{n-3}C_3 + \cdots \\
& + {}_nC_3 \cdot {}_{n-3}C_{n-3} \\
& + {}_nC_4 \cdot {}_{n-4}C_1 + {}_nC_4 \cdot {}_{n-4}C_2 + \cdots + {}_nC_4 \cdot {}_{n-4}C_{n-4} \\
& + \cdots \\
& + {}_nC_{n-2} \cdot {}_2C_1 + {}_nC_{n-2} \cdot {}_2C_2 \\
& + {}_nC_{n-1} \cdot {}_1C_1, \\
= {} & {}_nC_2({}_{n-2}C_1 + {}_{n-2}C_2 + {}_{n-2}C_3 + {}_{n-2}C_4 + \cdots \\
& + {}_{n-2}C_{n-2}) \\
& + {}_nC_3({}_{n-3}C_1 + {}_{n-3}C_2 + {}_{n-3}C_c + \cdots + {}_{n-3}C_{n-3}) \\
& + {}_nC_4({}_{n-4}C_1 + {}_{n-4}C_2 + \cdots + {}_{n-4}C_{n-4}) \\
& + \cdots \\
& + {}_nC_{n-2}({}_2C_1 + {}_2C_2) \\
& + {}_nC_{n-1}({}_1C_1).
\end{aligned}
$$

Substituting from (*) and (**) obtains:

$$
N_{II'} = {}_nC_2(2^{n-2}-1) + {}_nC_3(2^{n-3}-1) + {}_nC_4(2^{n-4}-1) + \cdots + {}_nC_{n-1}.
$$

Extending this thinking to Gracicunas' third set defines the set of all indirect relationships between a subordinate and each possible group (of two or more) of other subordinates. These are:

B -- (CD), B -- (CE), B -- (DE) and B -- (CDE);

C -- (BD), C -- (BE), C -- (DE) and C -- (BDE);

D -- (BC), B -- (BE), D -- (CE) and D -- (BCE);

E -- (BC), E -- (BD), E -- (CD) and E -- (BCD).

Of these there are sixteen. For n subordinates there would be:

$$N_{III'''} = n({}_{n-1}C_2 + {}_{n-1}C_3 + {}_{n-1}C_4 + \ldots + {}_{n-1}C_{n-1})$$

$$= n({}_{n-1}C_1 + {}_{n-1}C_2 + {}_{n-1}C_3 + {}_{n-1}C_4 + \ldots + {}_{n-1}C_{n-1} - {}_{n-1}C_1)$$

$$= n(2^{n-1} - \chi - n + \chi)$$

$$= n(2^{n-1} - n).$$

Here it is observed that, in describing his "relationships," Graicunas changed his frame of reference so that what he defined as "indirect cross relationships" are not relationships but perceptions. That is, when he defined the direct single relationship A * B, he did not admit B * A as distinct from it. However, when he defined indirect cross relationships, he admitted both B -- C and C -- B. Since this change is unexplained, it creates two avenues of exploration. (1) a search for <u>true</u> indirect cross relationships consistent with those classes of relationships already defined by Graicunas, and (2) a search for all classes of perceptions that are similarly consistent. Of the true indirect individual relationships, defined as the set of all relationships between each possible

pair of subordinates in the absence of any of the other subordinates, there are:

B * C, B * D and B * E;

C * E and C * D;

D * E.

With four subordinates there are six such relationships; with n subordinates there would be $N_{III^\circ} = {}^{n}C_2\ n(N-1)/2$.

There are, as well, indirect individual relationships, defined as the set of all relationships between each possible pair of subordinates in the presence of all possible combinations of other subordinates. These are:

(B *C)/D, (B * C)/E and (B * C)/DE;

(B * D)/C, (B * D)/E and (B * C)/CE;

(B * E)/C, (B * E)/D and (B * E)/CD;

(C * D)/B, (C * D)/E and (C * D)/BE;

(C * E)/B, (C * E)/D and (C * E)/BD;

(D * E)/B, (D * E)/C and (D * E)/BC.

With four subordinates there are eighteen such relationships. With n subordinates there would be:

$$N_{\overline{III}^\circ} = {}_{n}C_2(2^{n-2}-1).$$

$N_{\overline{III}^\circ}$ is derived from N_{II° by treating the n subordinates as though each were one of n superiors relating to n - 1 remaining subordinates.

Analogous to Class I' there are indirect group relationships defined as the set of all relationships between each subordinate and each possible group (of two or more) of other subordinates. These are:

B * (CD), B * (CE), B * (DE) and B * (CDE);

C * (BD), C * (BE), C * (DE) and C * (BDE);

D * (BC), D * (BE), D * (CE) and D * (BCE);

E * (BC), E * (BD), E * (CD) and E * (BCD).

Of these there are sixteen. For n subordinates there would be n times the number of Class I' Relationships computed as for n - 1 subordinates, or:

$$N_{III'} = n\left[2^{n-1}-(n-\chi) - \chi\right] = n\left[2^{n-1}-n\right].$$

There is a final set of indirect group relationships analogous to Class II'. This is the set of all relationships between each subordinate and each possible group (of two or more) of other subordinates in the presence of all possible combinations of remaining subordinates. These are:

B * (CD)/E, B * *CE)/D and B * (DE)/C;

C * (BD)/E, C * (BE)/D and C * (DE)/B;

D * (BC)/E, D * (BE)/C and D * (CE)/B;

E * (BC)/D, E * (BD)/C and E * (CD)/B.

Of these there are twelve. For n subordinates there would be n times the number of Class II' Relationships computed for n - 1 subordinates, or:

$$N_{\overline{III}'} = n\left[{}_{n-1}C_2(2^{n-3}-1) + {}_{n-1}C_3(2^{n-4}-1) + \ldots + {}_{n-1}C_{n-2}\right]$$

Pursuing now the matter of consistent perceptions, two classes may be discovered. The first of these is the set of all perceptions held by the superior of each individual subordinate and of those held by each subordinate of his superior. These are:

A -- B, B -- A, A -- C and C -- A;

A -- D, D -- A, A -- E and E -- A.

Of these there are eight. For n subordinates there would be:

$N_{I'''} = 2n.$

The final class to be discussed is the set of all perceptions held by the superior of each possible group (of two or more) of subordinates. These are:

A -- (BC), A -- (BD), and A -- (BE);

A -- (CD), A -- (CE), and A -- (DE);

A -- (BCD), A -- (BCE), A -- (BDE), and A -- (CDE);

A -- (BCDE).

Of these there are eleven. For n subordinates there would be:

$N_{I''} = 2^n - n - 1.$

These twelve classes of relationships and/or perceptions are displayed below in Table 6.

TABLE 6

RELATIONSHIPS AND PERCEPTIONS OF GRAICUNAS AND LEHTO

	Relationships		Perceptions	
	Individual°	Group'	Individual"	Group"'
I	A * B	A * (BC)	A -- B	A -- (BC)
II	A * B/C	A * (BC)/D	--	--
III	B * C	B * (CD)	B -- C	B -- (CD)
$\overline{\text{III}}$	B * C/D	B * (CD)/E	--	--

The above matrix shows that not all logical possibilities have been considered. This was judged to be a weakness in Graicunas' rationale. However, it has not been the intention of this study to build a similar rationale but merely to show that Graicunas did, in fact, either overlook or fail to explain away some logically acceptable possibilities. This has been abundantly accomplished.

Combining these classes obtains:

$$N_1 = N_{I^\circ} + N_{I'} + N_{I''} + N_{I'''}.$$

For four subordinates this is:

$N_1 = 4 + 11 + 8 + 11 = 34$. This agrees with Table 2.

For n subordinates this is:

$$N_1 = N_{I^\circ} + N_{I'} + N_{I''} + N_{I'''}$$

$$= n + 2^n - \not{n} - 1 + \not{2n} + 2^n - \not{n} - 1$$

$= n + 2 \cdot 2^n - 2 = 2(2^{n-1}) + n$. This is Equation (3). For $n = 4$ it yields:

$$N_1 = 2(2^4 - 1) + 4 = 2(15) + 4 = 34.$$

Also:

$$N_2 = N_{II^\circ} + N_{II'}.$$

For four subordinates this is:

$N_2 = 28 + 22 = 50.$ This agrees with Table 2.

For n subordinates this is:

$$\begin{aligned} N_2 &= N_{II^\circ} + N_{II'} \\ &= n[2^n/2 - 1] + {}_nC_2(2^{n-2}-1) + {}_nC_3(2^{n-3}-1) \\ &\quad + \ldots + {}_nC_{n-1} \\ &= {}_nC_1(2^{n-1}-1) + {}_nC_2(2^{n-2}-1) + {}_nC_3(2^{n-4}-1) \\ &\quad + \ldots + {}_nC_{n-1}. \end{aligned}$$

This is Equation (4). For n = 4 it yields:

$$\begin{aligned} N_2 &= {}_4C_1(2^{4-1}-1) + {}_4C_2(2^{4-2}-1) + {}_4C_3 \\ &= 4(8 - 1) + 6(4 - 1) + 4 \\ &= 4(7) + 6(3) + 4 = 28 + 18 + 4 = 50. \end{aligned}$$

Finally:

$$N_3 = N_{III^\circ} + N_{III'} + N_{III''} + N_{III'''} + N_{\overline{III}^\circ} + N_{\overline{III}'}.$$

For four subordinates this is:

$N_3 = 6 + 16 + 12 + 16 + 18 + 12 = 80.$ This agrees with Table 2.

For n subordinates this is:

$$\begin{aligned} N_3 &= N_{III^\circ} + N_{III'} + N_{III''} + N_{III'''} + N_{\overline{III}^\circ} + N_{\overline{III}'} \\ &= n(n - 1)/2 + n(2^{n-1}-n) + n^2 - n + n(2^{n-1}-n) \\ &\quad + {}_nC_2(2^{n-2}-1) \\ &\quad + n[{}_{n-1}C_2(2^{n-3}-1) + {}_{n-1}C_3(2^{n-4} - 1) + \ldots \\ &\quad + {}_{n-1}C_{n-2}] \end{aligned}$$

$$= {}^{n}C_2 + 2n(2^{n-1}-n) + 2\cdot{}^{n}C_2 + {}^{n}C_2(2^{n-2}-1)$$

$$+ n\left[{}^{n-1}C_2(2^{n-3}-1) + {}^{n-1}C_3(2^{n-4}-1) + \cdots\right.$$

$$\left. + {}^{n-1}C_{n-2}\right]$$

$$= 3\,{}^{n}C_2 + 2n(2^{n-1}-n) + {}^{n}C_2(2^{n-2}-1) + n\left[{}^{n-1}C_2\right.$$

$$(2^{n-3}-1)$$

$$\left. + {}^{n-1}C_3\,(2^{n-4}-1) + \cdots + {}^{n-1}C_{n-2}\right]. \quad \text{This is}$$

Equation (5).

For n = 4 it yields:

$$N_3 = 3\cdot{}^{4}C_2 + 2\cdot 4(2^{4-1}-4) + {}^{4}C_2(2^{4-2}-1) + 4\left[{}^{4-1}C_2\right.$$

$$\left.\cdot(2^{4-3} - 1)\right]$$

$$= 3\cdot 6 + 8(2^3-4) + 6(2^2-1) + 4\left[3(2-1)\right]$$

$$= 18 + 8(4) + 6(3) + 4\,[3]$$

$$= 18 + 32 + 18 + 12$$

$$= 80.$$

Finally:

$N' = N_1 + N_2 + N_3$. This is Equation (2).

For four subordinates this is:

$N' = 34 + 50 + 80 = 164$. This agrees with Table 2.