A COMPARISON OF THE ABILITIES OF LATE ELEMENTARY SCHOOL CHILDREN TO LEARN TASKS ON THE OPERATIONS OF SIGNED NUMBERS

Thesis for the Degree of Ph. D.

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ABSTRACT

A COMPARISON OF THE ABILITIES OF LATE ELEMENTARY SCHOOL CHILDREN TO LEARN TASKS ON THE OPERATIONS OF SIGNED NUMBERS

by

James F. Riley

Elementary school mathematics has undergone dramatic changes in both content and procedures within the past ten years. The "modern" mathematics revolution is continuing with recommendations for the inclusion of still newer ideas in the curriculum. One such topic is the study of the rules of operations on signed numbers. It was the purpose of this study to investigate the ability of children to learn and retain skills used in operations on signed numbers.

The numbers, in fact integers, were represented on the number line as bi-directional vectors. The number line was coordinated by indicating the direction and distance a point was located from zero. The operation of addition was defined as vector addition. The operation of subtraction was motivated by presenting that operation as the inverse operation of addition. The rules for multiplication were developed as a consequence of the distributive and additive inverse properties. The skills needed to effectively work with these operations were organized into seventeen objectively scored tasks. These tasks were further grouped into six lessons.

The subjects were members of twenty-one fourth, fifth, and sixth grade classes selected from school districts of southwestern Michigan. At each grade level the classes were assigned to one of two treatment groups (discovery or instructional) or to a control group. Each class was partitioned into four disjoint subclasses by sex and I.Q. A unit on signed numbers was taught to each class by the classroom teacher. The type of instruction or learning treatment they received was determined by the treatment group to which the class was assigned. An examination was given after each lesson, and a post test was given one month after the sixth lesson. The design permitted comparisons of a discovery type learning experience with a didactic type learning experience, boys with girls, high I.Q. with low I.Q., and one grade level with another. The post test gave the same comparisons on retention.

The hypotheses were statistically tested at $\alpha = .05$. This analysis of the data indicated that no significant difference existed between boys and girls in learning the 17 tasks. There was no difference in retention between the sexes at either the fourth or sixth grade level. However, at the fifth grade level girls did slightly better than boys. On 15 of the 17 tasks children with high I.Q. scored better than children with low I.Q., and on retention the high I.Q. subjects scored higher than the low I.Q. subjects at all grade levels. The subjects in the instruction group had higher scores in general than the subjects in the discovery group in 13 of the 17 tasks. Also, the subjects in the instruction classes retained more of what they learned than did the subjects in the discovery classes in the fourth and sixth grades. Only in the fifth grade was there no significant difference. The differences between the grades were more mixed. In 8 of the 17 tasks sixth graders scored higher than the fifth graders, and on only 3 of the 17 tasks did fifth graders score higher than fourth graders. However, on retention, the sixth graders retained more of what they learned than the fifth graders, who in turn retained more of what they learned than the fourth graders. Overall, a class was said to have reached a satisfactory level of achievement for a particular lesson if 50% or more of the class scored 50% or more on the test following that lesson. It was found that sixth graders could be expected to achieve at this level 90% of the time, whereas fourth and fifth graders could do so only 80% of the time.

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To my three girls: Netty, Lisalu, and Jennifer

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CHAPTER 1: THE PROBLEM

In 1963, a group of 25 mathematicians and scientists were brought together by Professor A.M. Gleason of Harvard and Professor W.T. Martin of the Massachusetts Institute of Technology for the purpose of conjecturing the content of the mathematics curriculum in the year 1990. The conclusions of the conference were published in a report[1] generally known as the Cambridge Conference report. Essentially the conference foresaw the mathematics content of the first sixteen years telescoped into a period of thirteen years. A number of topics normally introduced in the secondary school will necessarily be introduced in the elementary school. Operations on signed (positive and negative) numbers, a topic considered appropriate for seventh or eight grade by some present-day writers as Kingston[2] and Butler,[3] is proposed to be introduced at the third grade level. This curricular innovation is the motivation of this study to

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[1]Cambridge Conference on School Mathematics, Goals for School Mathematics (Boston: Published for Educational Services, Inc. by Houghton Mifflin, 1963).

[2]Kingston, J. Maurice, Mathematics for Teachers of the Middle Grades (New York: John Wiley & Sons, Inc., 1966), p. 59.

[3]Butler, Charles H. and Wren, Lynwood F., The Teaching of Secondary Mathematics (New York: McGraw-Hill Book Company, 1965), pp. 340-349.

compare the achievements of elementary school children in learning tasks involving signed numbers.

The Need

Since the ideas presented in the Cambridge Conference report reflect the thoughts of respected men active in the development of mathematics pedagogy, they need and deserve to be tested. Mathematics educators, like Irving Adler,[1] have strongly urged experimentation with topics found in the report. The reasons that justify this study then parallel those that motivate efforts as the Cambridge report.

A listing of writers that have outlined the causes and rationalizations of curricular changes in elementary school mathematics would be extensive. However, they all have common themes as cited in the following samples. Willoughby[2] states that the changes have been affected by acceleration in mathematics research, the reorganization and restructuring of mathematics, and new pedagogical methods. Folsom[3] and Butler[4] attribute the changes to the rapid development

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[1] Adler, Irving, "The Cambridge Conference Report: Blueprint or Fantasy?," The Arithmetic Teacher, Vol. 13 (March, 1966), pp. 179-187.

[2] Willoughby, Stephen S., Contemporary Teaching of Secondary School Mathematics (New York: John Wiley and Sons, Inc., 1967), pp. 29-35.

[3] Folsom, Mary, "Why the New Mathematics?," The Instructor, Vol. 73 (December, 1963), pp. 6-7.

[4] Butler, op. cit., pp. 4, 56-57.

of new mathematics and the changing needs of society for mathematics. The Cambridge Conference report[1] cites changing social needs, new developments in mathematics, and new teaching methods as reasons for change. In summary, four reasons are given as justification for mathematics curricular innovation, namely, (1) the increasing rate of the discovery of new mathematics, (2) the reorganization of mathematical structures, (3) the development of new educational methods, and (4) the changing need of society.

Consider the argument that the increasing volume of newly discovered mathematics justifies changes in the mathematics curriculum. Evenson[2] argues that since more mathematics is being created and used, there is a need for more mathematics to be learned. Frequently, the number of pages in the <u>Mathematical Review</u>[3] is cited as evidence of the expanding world of mathematical knowledge. However, a drive to learn more mathematics because there is more mathematics to learn, in some remote hope to close the gap, is indeed futile. Rather, the mathematics student must develop the skills of how to learn on his own the mathematics he will need in his lifetime.

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[1]Cambridge Conference on School Mathematics, <u>op. cit.</u>, pp. 7-12.

[2]Evenson, A.B., <u>Modern Mathematics</u> (Chicago: Scott, Foresman and Company, 1962), p. 8.

[3]Volume 35, 1968 contains 1,437 pages.

The reorganization of mathematical structures may be a more powerful force in the revamping of the mathematics curriculum. It is this reorganization that delineates "Modern" mathematics from "old" mathematics. Butler[1] describes the difference in this way:

> The origin of what might be called the modern point of view in mathematics can be traced to the pioneering efforts of Gauss, Bolyai, Lobachevski, and Riemann in the creation of non-Euclidean geometries. By daring to challenge that which for two millenniums had been accepted as absolute, they freed the intellect to reject the evidence of the senses for the sake of what the mind might produce . . . . This new method no longer recognizes postulates (axioms) as 'self evident truths,' but merely as 'acceptable assumptions.'

The "modern" mathematics growing out of this realization has resulted, according to Allendorfer,[2] in two trends. First, mathematical systems have been developed which exist only of and for themselves with no obligation to relate to the real world and, secondly, theories that may have grown from different models in nature are combined into a single abstract system that gives greater insight into the original systems as well as producing greater economy of thought. This structuring is in a sense the essence of mathematics, and, since an aim of mathematics education is to convey the nature of mathematics, it follows that this structuring should be a factor in determining the mathematics curriculum. Bruner[3]

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[1]Butler, op. cit., pp. 55-56.

[2]Allendorfer, Carl B., Mathematics for Parents (New York: The Macmillan Company, 1965), pp. 8-9.

[3]Bruner, J.S., The Process of Education (Cambridge, Massachusetts: Harvard University Press, 1962), p. 31.

states the case this way:

> . . . the curriculum of a subject should be determined by the most fundamental understanding that can be achieved of the underlying principles that give structure to the subject. Teaching specific topics or skills without making clear their context in the broader fundamental structure of a field of knowledge is uneconomical . . . .

Next, contributions in educational psychology by men like Skinner,[1] Bruner,[2] Piaget,[3] and Gagné,[4] have given rise to new theories of instruction. The new theories, while they do not suggest that revolutionary curricular changes as advocated in the Cambridge Conference report need to be undertaken, do indicate methods by which changes may be made. They give the curricular innovator a hope to succeed.

Finally, the changing ways in which we live have strong effects upon changes in the mathematics curriculum. People over thirty, remembering the neighborhood store, can probably recall a store clerk totaling the costs of groceries on a grocery list. Today, the supermarket check-out girl uses a very efficient machine, that not only totals the costs, but

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[1]Skinner, B.F., "Teaching Machines," Science, Vol. 128 (October, 1958), pp. 969-977.

[2]Bruner, J.S. Toward a Theory of Instruction (Cambridge, Massachusetts: The Belknap Press of Harvard University Press, 1966).

[3]Piaget, Jean, "How Children Form Mathematical Concepts," Scientific American, Vol. 189 (November, 1953), pp. 74-79.

[4]Gagné, Robert M., The Conditions of Learning (New York: Holt, Rinehart and Winston, 1965).

also calculates the change the customer is to receive. The world is in a computer revolution. Kemeny[1] has stated that one cent will buy about 2,000 arithmetical computations today and, therefore, no man can earn a living doing arithmetic. This is not to say that there is no longer a need for one to learn to compute. There is general need for numerous skills associated with the study of arithmetic, ranging from telling time to balancing a checkbook. However, the society no longer needs a large number of people, highly competent in arithmetic, to serve as accountants, bookkeepers, timekeepers, and stockmen. The computational aspects of their work is being increasingly handled by machines. Further, the growth of the use of computers is placing on our age a need for a new set of skills requiring more, not less, mathematics.

The introduction of signed numbers into the elementary school curriculum is justified on at least three of the four stated reasons. The rules of operations (addition and multiplication) on signed numbers provide an excellent illustration of the consequence of a mathematical structure. Also, an understanding of the operations of signed numbers is prerequisite to an understanding of the real number system. Knowledge of the real number system provides a foundation for a great deal of new mathematics. Finally, the real number system is

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[1] Kemeny, John, "The Impact of the Computer on Teaching," an address given at the Cleveland Meeting of the National Council of Teachers of Mathematics, Cleveland, Ohio, November 13, 1969.

probably the best model for application in the real world through disciplines as calculus and statistics. For these reasons the study is justified.

The Purpose

The purpose of this study is to investigate the abilities of elementary school children in learning tasks involving the operations of addition, subtraction, and multiplication of signed numbers. The effects of grade level, I.Q., sex, and different teaching methods upon the learning of tasks as measured by test scores are analyzed. An objective test for satisfactory achievement is defined and applied to the tasks.

The Assumptions

The crucial issue of the study is to consider the feasibility of introducing operations on signed numbers in the elementary school course of study. The assumptions used as the basis for the hypotheses are conservative.

(1) It is assumed that the general mathematical ability of boys and girls is the same. The results of research testing the mathematical abilities of boys and girls are mixed. Studies indicating that boys achieve better than girls in tasks dealing with mathematical concepts, where as girls achieve better on tasks involving computation are reported by

Jarvis[1] and Parsley[2].

(2) It is assumed that children, as they grow older and gain learning experience, can learn new tasks more readily and remember them longer.

(3) It is assumed that children with greater intellectual ability can learn new tasks more easily and remember them longer than children with less intellectual ability.

(4) It is assumed that teaching is an art. Theories of instruction may be constructed compatible with various theories of learning, but the success of the "average" teacher in the "average" classroom is due more to the personality of the teacher and her ability to adopt a teaching style that works for her.

(5) Bruner's[3] famous axiom that "any subject can be taught effectively in some intellectually honest form to any child at any stage of development" is accepted.

The Hypotheses

The following hypotheses, based upon the assumptions,

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[1] Jarvis, O.T., "Boy-Girl Ability Differences in Elementary School Arithmetic," <u>School Science and Mathematics</u>, Vol. 64 (November, 1964), pp. 657-659.

[2] Parsley, Kenneth M., "Further Investigation of Sex Differences in Achievement of Under-Average and Over-Average Achieving Students Within Five I.Q. Groups in Grades Four through Eight," <u>Journal of Educational Research</u>, Vol. 57 (January, 1964), pp. 268-270.

[3] Bruner, J,S., <u>The Process of Education</u>, p. 33.

are tested in the study:

(1) Hypothesis 1A: There will be no difference in the scores on the tasks between boys and girls.

Hypothesis 1B: There will be no difference in the retention of task skills between boys and girls

(2) Hypothesis 2A: The mean score of children on the tasks at any grade level will be higher than the mean score of children at a lower grade level on the same tasks.

Hypothesis 2B: The retention of task skills by children at any grade level will be greater than the retention of task skills by children at a lower grade level on the same tasks.

(3) Hypothesis 3A: The mean score on tasks of children with higher intellectual ability will be higher than the mean score on tasks of children with lower intellectual ability.

Hypothesis 3B: The retention of task skills by children with higher intellectual ability will be greater than the retention of task skills by children with lower intellectual ability.

(4) Hypothesis 4A: There will be no difference in the mean scores on the tasks between groups receiving different instructional methods.

Hypothesis 4B: There will be no difference in the retention of skills between groups receiving different instructional methods.

(5) Hypothesis 5: Fourth, fifth, and sixth grade classes will attain satisfactory achievement in learning tasks involving signed numbers.

Definitions

The following terms, unique in this study, are defined.

(1) Direction number: An integer represented on a number line as a vector.

(2) Discovery learning: A learning experience, as described on pages 35-36, where the responsibility for learning remains with the student.

(3) Instructional (Didactic) learning: A learning experience, as described on pages 35-36, where the responsibility for learning remains with the teacher.

The Overview

This chapter, the first, contains the statement of the problem and a justification for the study. In Chapter 2, the relevant literature is reviewed. The emphasis is placed on three areas; namely, the development of signed numbers in mathematics education, the use of mathematical structure in the elementary school, and the psychological foundations underlining the teaching methods used in the study. Those aspects of the study dealing with the design are found in Chapter 3. The selection of subjects, measures, and experimental design are reviewed, as well as the development of the curricular material. In Chapter 3 the hypotheses are restated in testable form and the statistical procedures for testing them are listed. Chapter 4 contains an analysis of the data and Chapter 5 ends the report with some conclusions and a summary.

CHAPTER 2: REVIEW OF LITERATURE

The Integers: Their Development and Pedagogy

The slow acceptance of the concept of negative numbers by mathematicians is remarkable. A survey of the development of integers by Gorza[1] states that not until 1637 were signed numbers firmly established as a number system through the work of Descartes, who referred to positive and negative numbers as true and false numbers. Prior to this, medieval mathematicians thought expressions as 2-5 to be "meaningless" and, even earlier, Diophantus (Ca. 275) called the equation $4x + 20 = 4$ absurd. However, the survey continues, not all mathematicians denied the existence of negative numbers. The Arabian al-Khowarizmi (Ca. 825) is known to have stated the rules of signed numbers, placing a "dot" over the numeral to indicate a negative number. At about the same time the Hindus denoted negative numbers by enclosing the numeral in a circle. But, according to Miller,[2] a refusal by some mathematicians to accept negatives persisted until the 19th century.

After the acceptance of signed numbers into the domain

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[1] Gorza, Vivian S., A Survey of Mathematics: Early Concepts and their Historical Development (New York: Holt, Rinehart and Winston, 1968), pp. 244-247.

[2] Miller, G.A., "Crusade Against the Use of Negative Numbers," School Science and Mathematics, Vol. 33 (December, 1933), pp. 959-964.

of mathematics by mathematicians, the study of operations on signed numbers became an integral part of the study of algebra. The teaching of signed numbers has evolved from a period when the study was developed by seemingly arbitrary rules of operation to the present attempt to show signed numbers as rational, necessary, functionaries in the structure of a number system. A survey of older algebra texts, as those by Wentworth,[1] Beman,[2] and Milne,[3] show the rules of operations on signed numbers to be based upon the "likeness" or "unlikeness" of the signs. In text books used today, as those by Beberman[4] and Price,[5] the rules are presented as the consequence of the algebraic structures of the number system.

The introduction of advanced mathematics topics into the elementary school curriculum brings with it problems

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[1]Wentworth, G.A., School Algebra (Boston: Ginn and Company, 1894), pp. 17-25.

[2]Beman, Woster W., Elements of Algebra (Boston: Ginn and Company, 1900), pp. 27-28.

[3]Milne, William J., High School Algebra (New York: American Book Company, 1892), pp. 20, 29, 43.

[4]Beberman, Max, and Vaughn, Herbert E., High School Mathematics (Boston: D.C. Heath and Company, 1966), pp. 20-29.

[5]Price, H.V., Peak, P., and Jones, P.S., Mathematics: An Integrated Series, Book One (New York: Harcourt, Brace and World, Inc., 1965), pp. 135-154.

not found in the secondary school. Romberg[1] has described this difficulty simply and adequately:

> The means of embodying advanced concepts in simple forms and the techniques of implementing such forms in successful instructional sequences remain to be found.

The literature provides some hints as to how this may be done in the case of signed numbers. Patterson[2] suggests using pictures on the number line to indicate positive and negative direction at the first grade level. For the fourth grade, Davis[3] suggests motivating the concepts of "plus" and "minus" numbers by "real life" credit and debit situations. The students then continue on to more abstract Problems involving frames as

$$^{+}5 + \square = {}^{+}3 \ .$$

Havenhill[4] proposes the use of arrows to indicate positive

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[1]Romberg, T.A. and DeVault, M.V., "Mathematics Curriculum: Needed Research," Journal of Research and Development in Education, Vol. 1 (Fall, 1967), pp. 95-110.

[2]Patterson, Katherine, "A picture line can be fun!," The Arithmetic Teacher, Vol. 16 (December, 1969), pp. 603-605.

[3]Davis, Robert B., The Madison Projects Approach to A Theory of Instruction, a report of the Madison Project, Webster College, St. Louis, Missouri, p. 12.

[4]Havenhill, Wallace P., "Though This Be Madness...," The Arithmetic Teacher, Vol. 16 (December, 1969) pp. 606-608.

and negative direction as well as magnitude. D'Augustine[1] recommends that the points on the number line be identified by numerals with arrows over them ($\overrightarrow{3}$). The arrows indicate the direction of the point from zero and the numeral indicates the distance of the point from zero. After a few exercises in addition using this representation, the arrows would be replaced by the traditional + and - signs. Further work would involve problems using frames similar to those previously attributed to Davis. Riedesel[2] and the School Mathematics Study Group[3] advocate using the thermometer for introducing signed numbers and then proceeding to addition by using arrows to find vector sums on the number line.

The suggestions offered thus far deal only with the representation, addition, and subtraction of signed numbers while ignoring the problems of multiplication. There is good reason for this. The teaching of the multiplication of signed numbers presents some imposing problems. The Cambridge report[4]

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[1]D'Augustine, C.H., Multiple Methods of Teaching Mathematics in the Elementary School (New York: Harper and Row, 1968), pp. 260-270.

[2]Riedesel, C. Alan, Guiding Discovery in Elementary School Mathematics (New York: Appleton-Century-Crofts, 1967), pp. 100-101.

[3]School Mathematics Study Group, Mathematics for the Elementary School, Teacher Commentary, Part I (New Haven: Yale University Press, 1963), pp. 349-376.

[4]The Cambridge Conference on School Mathematics, op. cit., p. 37.

states:

> Perhaps no area of discussion brought more viewpoints than the question of how the multiplication of signed numbers should be introduced. The simple route via the distributive law was considered, but a closely related approach was more popular. One observes that the definition of multiplication is ours to make but only one definition will have the desireable properties. Others favored an experimental approach involving negative weights on balance boards, etc. Still others favored the "negative" debt approach. Even the immediate introduction of signed area was proposed . . . . The question is evidently not mathematical, it is purely pedagogic. The problem is to convey the "inner reasonableness of $(^{-}1) \times (^{-}1) = {}^{+}1$."

Havenhill[1] suggests that the rules for multiplication be developed in the following way:

> By utilizing the two interpretations of the + and - signs, the multiplication sentence, $a \times b = c$, may be interpreted as follows. The magnitude of the multiplicand (b) is the length of each arrow. Its sign points the arrows to the right (+) or left (-). The magnitude of the multiplier (a) tells how many arrows to lay end to end beginning at the origin. Its sign tells whether to reverse their direction (-) or not (+).

This may seem to be confusing. The fault is not Havenhill's. The pedagogical problem is real. Havenhill's procedure underlines the difficulty. The rules for multiplication can be justified many ways. But most, like the use of equivalence classes of ordered pairs of natural numbers as described by Banks,[2] the geometrical approach of using projections on the real line with the ratios of similar triangles suggested

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[1]Havenhill, loc. cit.

[2]Banks, J. Houston, Elements of Mathematics, Second Edition (Boston: Allyn and Bacon, Inc., 1960), pp. 136-148.

by Petro,[1] and the product line method, can be rejected, *a priori*, as unsuitable for the elementary school. The search for an adequate way to teach the multiplication of signed numbers continues.

Research specifically attending to the problems of developing the concepts of signed numbers is exceedingly rare. Parsons,[2] working in the Madison Project, reports trials with fourth grade children have been determined a "success" though a criteria for "success" is not reported. Carlton[3] reports that instruction in the elementary school on operations of positive and negative integers is under evaluation in the Soviet Union. No results are available at the present time.

A review of the current elementary texts used in the United States reveals that the study of signed numbers is being slowly introduced to sixth grade children. Most programs on this topic deal only with addition and, in some cases, with subtraction as exemplified in texts by Duncan,[4]

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[1]Interview with John Petro, Associate Professor of Mathematics, Western Michigan University, March 16, 1970.

[2]Parsons, Cynthia, "Algebra as Presented to Fourth Graders is Grasped with Enthusiasm," *Christian Science Monitor*, January 9, 1960, p. 11.

[3]Carlton, Virginia, "Mathematics Education in the Elementary Schools of the Soviet Union," *The Arithmetic Teacher*, Vol. 15 (February, 1968), pp. 108-114.

[4]Duncan, Ernest R., *Modern School Mathematics: Structure and Use* (Boston: Houghton Mifflin Company, 1970), pp. 332-339.

Fouch,[1] Hartung,[2] Keedy,[3] Glennon,[4] and Spitzer.[5] In every case the subtraction is considered, it is motivated as the inverse of addition. Only one author, Eicholz,[6] also includes the operation of multiplication. The justification of the rules of signed numbers is handled by the distributive law and the additive inverse property.

Teaching Mathematical Structure in the Elementary School

As previously stated, the teaching of signed numbers has evolved from a time when the study was developed from apparently arbitrary rules to the present procedure of developing the operations on the numbers as consequences

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[1]Fouch, Robert S., and Haas, Raymond, SRA Elementary Mathematics Program, Book 6 (Chicago: Science Research Associates, 1968), pp. 143-150.

[2]Hartung, Maurice L., et al., Seeing Through Arithmetic, 6 (Glenview, Illinois: Scott Foresman and Company, 1968), pp. 314-318.

[3]Keedy, Mervin J., et al., Exploring Elementary Mathematics, 6 (New York: Holt, Rinehart, and Winston, 1970), pp. 224-231, 234-235.

[4]Glennon, Vincent J., Short, Roy F., and Brownell, M.A., Mathematics We Need (Boston: Ginn and Company, 1966), pp. 312-313.

[5]Spitzer, Herbert F., et al., Elementary Mathematics (St. Louis, Missouri: McGraw-Hill Book Company, 1967), pp. 23-24, 30.

[6]Eicholz, Robert E. and O'Daffer, Phares C., Elementary School Mathematics, second edition, Book 6 (Menlo Park, California: Addison Wesley Publishing Company, 1968), pp. 291-293.

of the structure of the number system. This follows the generally accepted belief that mathematics that is learned through understanding is learned with greater retention and greater facility for transfer than mathematics learned by rote. Studies by Brownell,[1] Dawson,[2] Greathouse,[3] Krich,[4] Miller,[5] and Rappaport[6,7] confirm this belief. It is argued that meaning in arithmetic is attained through the laws that give the subject structure by mathematics educators as

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[1]Brownell, William A. and Moser, Harold E., "Meaningful vs. Mechanical Learning: A Study in Grade 3 Subtraction," Duke University Studies in Education, Vol. 8 (1949), pp. 1-207.

[2]Dawson, Dan T., "The Case for the Meaning Theory in Teaching Arithmetic," Elementary School Journal, Vol. 55 (March, 1955), pp. 393-399.

[3]Greathouse, Jimmie Joe, "An Experimental Investigation of Relative Effectiveness Among Three Different Arithmetic Teaching Methods," unpublished Ph.D. Thesis, The University of New Mexico, 1965.

[4]Krich, Percy, "Grade Placement and Meaningful Learning," School Science and Mathematics, Vol. 64 (February, 1964), pp. 131-137.

[5]Miller, G.H., "How Effective is the Meaning Method?," The Arithmetic Teacher, Vol. 4 (March, 1957), pp. 45-49.

[6]Rappaport, David, "Understanding Meanings in Arithmetic," The Arithmetic Teacher, Vol. 5 (March, 1958), pp. 96-99.

[7]__________, "The Meaning Approach in Teaching Arithmetic," Chicago School Journal, Vol. 44 (January, 1963), pp. 172-174.

Flournoy,[1] Gordon,[2] and Schraf.[3] They reason that, since computational algorithms are governed by algebraic structural laws, an understanding of these laws by students and the use of these laws by teachers in justifying the algorithms will result in more meaningful learning.

The research investigating the ability of elementary school children to learn and apply structural laws is fairly extensive. Studies by Schmidt[4] and Hall[5] indicate that children who have developed an understanding of the commutative and associative laws show an improvement in fundamental addition and multiplication skills. Research reports by Gray[6]

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[1]Flournoy, Frances, "Understanding Relationships: An Essential for Solving Equations," The Elementary School Journal, Vol. 64 (January, 1964), pp. 214-217.

[2]Gordon, David X., "Clarifying Arithmetic Through Algebra," School Science and Mathematics, Vol. 42 (March, 1942), pp. 288-289.

[3]Schraf, William L., "Arithmetic Taught as a Basis for Later Mathematics," School Science and Mathematics, Vol. 46 (May, 1946), pp. 413-423.

[4]Schmidt, Mary M., "Effects of Teaching the Commutative Laws, Associative Laws on Fundamental Skills of Fourth Grade Pupils," Dissertation Abstracts, Vol. 26 (February, 1966), p. 4510.

[5]Hall, Kenneth Dwight, "An Experimental Study of Two Methods of Instruction for Mastering Multiplication Facts at the Third Grade Level," unpublished Ph.D. Thesis, Duke University, 1967.

[6]Gray, Roland F., "An Experiment in the Teaching of Introductory Multiplication," The Arithmetic Teacher, Vol. 7 (March, 1965), pp. 199-203.

and Schell[1,2] show that children with an understanding of the distributive law develop a better understanding of multiplication than children motivated by "repeated addition" or "rectangular array" methods.

While knowledge of mathematical structure may help children learn arithmetical operations, the teaching of mathematical structure, itself, presents some problems. Baumann[3] found that the attainment of the concepts of commutativity, closure, and identity were quite difficult for second and fourth grade children. Flournoy[4] and Gray[5] have demonstrated that elementary school children could not apply the structural laws without specific instruction into the nature of the laws. The order of difficulty in learning the structural laws is reported by Crawford[6] to

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[1]Schell, Leo M., "Two Aspects of Introductory Multiplication: The Array and the Distributive Property," Dissertation Abstracts, Vol. 25 (April, 1965), p. 5161.

[2]Schell, Leo M., "Learning the Distributive Property by Third Graders," School Science and Mathematics, Vol. 68 (January, 1968), pp. 28-32.

[3]Baumann, Raemt R., "Childrens Understanding of Selected Mathematical Concepts in Grades Two and Four," Dissertation Abstracts, Vol. 26 (March, 1966), p. 5219.

[4]Flournoy, Frances, "Applying Basic Mathematical Ideas in Arithmetic," The Arithmetic Teacher, Vol. 11 (February, 1964), pp. 104-108.

[5]Gray, *op. cit.*

[6]Crawford, Douglas H., "An Inventory of Age-Grade Trends in Understanding the Field Axioms," Dissertation Abstracts, Vol. 25 (April, 1965), pp. 5728-5729.

be commutativity (easiest), inverse, closure, identity, associativity, and distributivity (most difficult). In at least one case, the structural development has proved less reliable than the traditional approach. Hervy,[1] comparing the equal additions approach with the use of cartesian products, reported that equal-additions multiplications problems were less difficult to solve and conceptualize, and that cartesian-product problems were more readily solved by high achievers than by low achievers.

Theories of Instruction in Mathematics

Developments in learning theory have lead to the establishment of theories of instruction in mathematics. A spectrum of ideas on teaching procedures range from rigidly guided learning experiences to those encouraging student experimentation and discovery. The two essential views that are being proposed have been summarized by Shulman[2] as follows:

> The controversy seems to center essentially about the question of how much and what kind of guidance ought to be provided to the students in the learning situation. Those favoring learning by discovery advocate the teaching of broad principles and problem-solving through minimal teacher guidance and

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[1]Hervey, Margaret A., "Childrens Responses to Two Types of Multiplication Problems," The Arithmetic Teacher, Vol. 13 (April, 1960), pp. 288-292.

[2]Shulman, Lee S., "Psychological Controversies in the Teaching of Science and Mathematics," The Science Teacher, Vol. 35 (September, 1968), pp. 34-37, 89-90.

maximal opportunity for exploration and trial-and-error on the part of the student. Those preferring guided learning emphasize the importance of carefully sequencing instructional experiences through maximum guidance and stress the importance of basic associations of facts in the service of the eventual mastering of principles and problem solving.

The learning objectives of the theories differ, and as such defy comparison. Bruner,[1] a strong proponent of discovery, describes the objectives of discovery as follows:

> . . . a theory of instruction seeks to take account of the fact that a curriculum reflects not only the nature of knowledge itself--the specific capabilities--but also the nature of the knower and of the knowledge getting process . . . To instruct someone in these disciplines is not a matter of getting him to commit the results to mind, rather it is to teach him to participate in the process that makes possible the establishment of knowledge.

Gagné,[2] who adamantly favors the guided learning approach, argues that to effectively solve problems the learner must have accumulated knowledge and that this is done best by leading students through guided learning experiences. Gaining knowledge is one objective of guided learning.

The reasons for choosing one set of objectives over another are epistemological. Bruner[3] declares:

> But I think we would all agree that, at the very least, an educated man should have a sense of what knowledge is like in some field of inquiry, to know it in its connectedness and with a feeling for how the knowledge is gained.

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[1]Bruner, Jerome S., Toward A Theory . . ., p. 72.

[2]Gagné, op. cit., p. 170.

[3]Bruner, Jerome S., "On Learning Mathematics," The Mathematics Teacher, Vol. 53 (December, 1960), pp. 610-619.

Ausubel[1] replies:

> This miracle of culture is made possible only because it is so much less time-consuming to communicate and explain an idea meaningfully to others than to require them to re-discover it by themselves.

In general, research studies in curricular development use diadactic teaching methods. Studies involving guided instruction, by the nature of the instruction, are easier to design, control, and the objectives can be described in terms of observable behavior. The researcher working with discovery methods is faced with some imposing problems. Wittock[2] characterizes these problems as:

(1) Conceptual Problems. Is discovery a way to learn subject matter or is it an end in its own right? Is it learning by discovery or learning to discover?

(2) Methological Problems. How does one control the rate and sequencing of stimuli in treatments? What are the dependent variables?

(3) Semantic Inconsistencies. How can operational definitions be developed? How can one avoid the naming of treatments in terms of responses, i.e., rote learning and discovery are responses, not stimuli.

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[1]Ausubel, David P., "Some Psychological and Educational Limitations of Learning by Discovery," The Mathematics Teacher, Vol. 57 (May, 1964), pp. 290-302.

[2]Wittock, M.C., "The Learning by Discovery Hypothesis," in Shulman, Lee (Editor), Learning by Discovery: A Critical Appraisal (Chicago: Rand, McNally and Company, 1966), pp. 42-48.

A closer look at one of these problems may bring the difficulty into sharper focus. Consider the conceptual problem of what does one mean by discovery teaching? For some, it means literally placing the child in a sea of stimuli and letting him sink or swim. For others, discovery teaching implies a highly structural system of dispensing stimuli leading the child in discoveries. Glaser[1] takes the first approach when he writes:

> . . . a learning by discovery sequence involves induction. This is the procedure of giving examples of a more general case which permits the student to induce the general propositions involved.

Johnson[2] takes the second point of view. He writes:

> What we really do is provide a setting where educational experiences are intelligible and understandable and we guide the mind of the child, as it were, along paths which cause him to see, not only the correctness of the manipulation, but also the rationale of the process.

Clearly, it is wise to heed Shulman's warning[3] that one man's discovery can easily be confused with another's guided learning.

The research dealing with discovery teaching centers largely around the relative effectiveness of discovery and

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[1]Glaser, Robert, "Variables in Discovery Learning," in Shulman, Lee (Editor), <u>Learning by Discovery: A Critical Appraisal</u> (Chicago: Rand, McNally and Company, 1966), p. 15

[2]Johnson, Harry C., "What Do We Mean by Discovery?," <u>The Arithmetic Teacher</u>, Vol. 11 (December, 1964), pp. 538-539.

[3]Shulman, <u>op</u>. <u>cit</u>., p. 34.

non-discovery teaching on the accumulation of knowledge, retention, and transfer as dependent variables. Studies by Bassler,[1] Fleckman,[2] Scandura,[3] Ter Keurst,[4] and Worthen[5] support the claims of the advocate of discovery in that didactic methods lead to better results in initial testing but that discovery methods result in better performance on retention tests. The results further indicate that the discovery groups transfer concepts more easily. A study by Wilson[6] shows that groups taught by discovery methods transfer discovery problem solving approaches to new situations.

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[1]Bassler, Otto C., "Intermediate Versus Maximal Guidance--A Pilot Study," The Arithmetic Teacher, Vol. 15 (April, 1968), pp. 357-362.

[2]Fleckman, Bessie, "Improvement of Learning Division Through Use of the Discovery Method," Dissertation Abstracts, Vol. 27A (April, 1967), pp. 3366-3367.

[3]Scandura, Joseph J., "An Analysis of Exposition and Discovery Modes of Problem Solving Instruction," Journal of Experimental Education, Vol. 33 (December, 1964), pp. 148-159.

[4]Ter Keurst, Arthur J., "Rote Versus Discovery Learning," School and Community, Vol. 55 (November, 1968), pp. 42-44.

[5]Worthen, Blaine R., "A Study of Discovery and Expository Presentation: Implications for Teaching," Journal of Teacher Education, Vol. 19 (Summer, 1968), pp. 223-242.

[6]Wilson, John H., "Differences Between the Inquiry Discovery and Traditional Approaches to Teaching Science in Elementary School," Research In Education, Vol. 4 (1969), p. 752.

Armstrong[1] reports that the inductive (discovery) approach fosters the learning of operations, while deductive (directed) methods result in greater learning of mathematical properties.

Kersh,[2,3] a critic of the discovery method, argues that research supports the claim that through discovery students (a) develop an interest in the task, and (b) understand what they learn and are better able to remember and to transfer what is learned. He denies that there is any evidence to support the conjecture that students learn strategies for discovering new generalizations. At this later date the criticism, in view of the studies cited, still has some validity.

Regardless which instructional strategy one may favor or what teaching procedures research may support, the problem of considering the effects of teaching procedures on curriculum development is with us. Any study that investigates the introduction of new curricular material should include the results obtained by differing modes of instruction.

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[1]Armstrong, Jenny Rose, "The Relative Effects of Two Forms of Spiral Curriculum Organization and Two Modes of presentation on Mathematical Learning," Dissertation Abstracts, Vol. 29 (July, 1968), p. 141.

[2]Kersh, Bert Y., "Learning by Discovery: What is Learned?," The Arithmetic Teacher, Vol. 11 (April, 1964), p. 226.

[3]__________, "Learning by Discovery: Instructional Strategies," The Arithmetic Teacher, Vol. 12 (October, 1965), pp. 414-417.

Summary

A survey of the literature indicates that mathematics educators recognize a need for introducing the algebra of signed numbers at the elementary school level. To some extent, this is being done at the sixth grade level in some programs. In these cases, the crucial problem of the multiplication of integers is ignored.

If the algebra of signed numbers is to be a part of the elementary school curriculum, the topic should be developed through an understanding of the structure of the mathematical system rather than through the assumption of seemingly arbitrary rules of operation. Research indicates that children who learn the "reasoning" behind mathematical concepts learn those concepts faster and retain them longer. Further, the "reasoning" is best learned through an understanding of the laws which give structure to the mathematics system. Studies show that the structural laws must be taught and that some of them, as the distributive law, are difficult for children to learn.

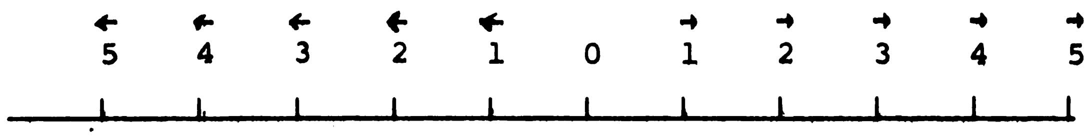
Finally, studies in learning theory have lead to the formation of theories of instruction in mathematics. Essentially, these theories follow one of two tracks: guided learning or discovery learning. The proponents of guided learning argue that their procedures provide for more efficient learning. Those who favor discovery learning maintain that one who learns through discovery will retain what he has learned for a longer period and will more easily transfer this knowledge. Research supports the claims of both groups. The time of investigating differing teaching strategies is here, and a study investigating the introduction of signed numbers in the elementary school should consider the effects of different instructional procedures.

CHAPTER 3: DESIGN OF THE STUDY

The Curriculum And Its Presentation

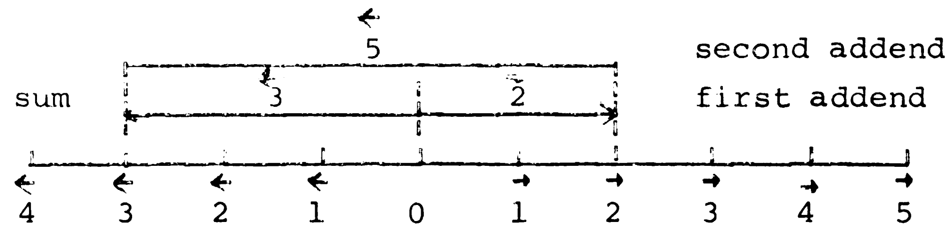
The purpose of the study is to investigate the ability of elementary school children to accomplish tasks related to operations on signed numbers. In the curricular material developed for the study signed numbers were represented on a number line as bi-directional vectors in the following way.

Signed numbers were called direction numbers in their presentation to the subjects. The number line was coordinated by indicating the direction and distance a point was located from zero.



The operation of addition was developed by placing the tail of the first addend vector at zero, placing the tail of the second addend at the head of the first addend, and naming the sum to be the vector extending from zero to the head of the second addend vector. The following example illustrates this operation.

Example: $\overset{\rightarrow}{2} + \overset{\leftarrow}{5} = \overset{\leftarrow}{3}$



The subtraction of direction numbers was motivated by presenting that operation as the inverse operation of addition.

Example: $\overset{\rightarrow}{3} - \overset{\leftarrow}{2} = \overset{\rightarrow}{5}$ since $\overset{\rightarrow}{5} + \overset{\leftarrow}{2} = \overset{\rightarrow}{3}$.

The multiplication of direction numbers of the form $\vec{a} \times \vec{b}$ was defined as $\vec{a} \times \vec{b} = \overrightarrow{a \times b}$. The rules for multiplication of direction numbers of the forms $\vec{a} \times \overset{\leftarrow}{b} = \overleftarrow{a \times b}$ and $\overset{\leftarrow}{a} \times \overset{\leftarrow}{b} = \overrightarrow{a \times b}$ were developed as consequences of the distributive and additive inverse properties. The division of signed numbers was not considered in the study.

It was assumed that the directional number approach would provide a better visual image that children need at this age level than would the use of "plus" and "minus" signs. The operations of subtraction and multiplication were developed using the structural approaches epistemologically proposed in the first chapter and somewhat empirically supported in the second chapter.

The material was organized into seventeen achievement tasks that could be objectively scored. The tasks were:

(1) Name[1] the points on the coordinated number line.

(2) Construct and name a direction number given its initial and terminating point.

(3) Name the terminating point of a direction number given the direction number and its initial point.

(4) Name the initial point of a direction number given the direction number and its terminating point.

(5) Construct and name the sum of direction numbers with the same direction.

(6) Construct and name the sum of direction numbers with different direction.

(7) Construct and name the additive inverse of a given direction number.

(8) Construct and name an unknown addend given the sum and the other addend.

(9) Demonstrate the ability to restate number sentences involving the operation of subtraction into sentences involving the operation of addition.

(10) Construct and name the solutions of subtraction problems.

(11) Name the product of direction numbers of the form $\vec{a} \times \vec{b}$.

(12) Name the product $\vec{a} \times 0$.

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[1]The underlined verbs in this list are operationally defined in AAAS Commission on Science Education, Science--A Process Approach: An Evaluation Model And Its Application, Second Report (The Association, 1968), pp. 7-9.

(13) Name the missing terms in equations illustrating the distributive law.

(14) Name the product of direction numbers of the form $\overrightarrow{a} \times \overleftarrow{b}$ using the distributive law.

(15) Name the product of direction numbers of the form $\overrightarrow{a} \times \overleftarrow{b}$ using the rule.

(16) Name the product of direction numbers of the form $\overleftarrow{a} \times \overleftarrow{b}$ using the distributive law.

(17) Name the product of direction numbers of the form $\overleftarrow{a} \times \overleftarrow{b}$ using the rule.

The seventeen tasks were organized into six lessons. Each lesson consisted of two sets of exercises. The first set, called the problem set for group work, was used by the teacher for instructional purposes. The second set, called the problem set for individual work, was used to test the subjects ability to solve direction number problems. The problem sets are found in Appendix A.

Sample

The 578 subjects in the study were children enrolled in twenty-one fourth, fifth, and sixth grade classes from various elementary schools in southwestern Michigan. The classes were from thirteen different schools in eleven different cities. The cities ranged in population from 5,000 through 500,000. The teachers that participated in the study were selected from volunteers enrolled in continuing education mathematics courses for elementary teachers offered by Western Michigan University at centers in Fremont, Grand Rapids, and Marshall, Michigan. The teachers, all of whom were certified and experienced, worked with their own classes in their own schools.

Measures

Eight measuring devices were used in the study. The problem sets for individual work, as previously mentioned, constituted six of the measures. A post test covering the curricular material in the problem sets (see Appendix A) and the Otis Quick-Scoring Mental Ability test[1] made up the remaining two. Four of the twenty-one classes in the study were selected at random and their test scores were used to compute reliability estimates. The equation employed for computing reliability was

$$r_{tt} = 1 - \frac{V_c}{V_t}$$

where V_c was the error variance and V_t was the individual variance of an analysis of variance upon the two classifications of subject and test item. The theory and computational procedures used to find the measures of reliability have been clearly explained by Kerlinger.[2] The reliability

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[1]Otis, Arthur S., Otis Quick-Scoring Mental Tests: New Edition, Beta Test Form Em (New York: Harcourt, Brace and World, Inc., 1954).

[2]Kerlinger, Fred N., Foundations of Behavioral Research (New York: Holt, Rinehart and Winston, Inc., 1964), pp. 429-443.

measures are summarized in Table 3.1. Further information on the Otis Test has been compiled by Buros.[1]

Table 3.1

RELIABILITY MEASURES

	Tests							
	1	2	3	4	5	6	Post	Otis
Measure	.475	.726	.824	.536	.956	.897	.547	.953

The Design of the study

The classes were divided as classes into two treatment groups (the pupil-discovery group and the teacher-instruction group) and one control group at each grade level. Each class was also partitioned into four disjoint subclasses by sex and high and low I.Q. The median raw score for the Otis Mental Abilities test was found for each class. Those subjects within the class with raw scores above this median were classified as high I.Q., and those with raw scores below this median were classified as low I.Q.

The seventeen tasks listed in the first section of this chapter were organized into six problem sets. The day after the subjects had a learning experience with a particular

[1]Buros, Oscar K., The Sixth Mental Measurements Yearbook (Highland Park, New Jersey: The Gryphon Press, 1963), p. 481.

problem set, by either the discovery or instructional treatment, they were given an examination on the tasks in the problem set. The control classes were given the examination without the learning treatments. All classes received a skill retention examination (post-test) one month after the sixth problem set examination.

The design permitted comparisons of a discovery type learning experience with a didactic type learning experience, boys with girls, high I.Q. with low I.Q., and one grade level with another on specific learning tasks involving signed numbers. The post test gave the same comparisons on retention. It was assumed that the learning due to maturation and test experience was uniform throughout all classes. The control classes were used to give some indication of the extent of this learning.

Treatment Procedures

A review of skills involving natural numbers on the number line was conducted by the classroom teacher for the purpose of defining the problems in the problem set under consideration. Classes in all three groups (instructional, discovery, and control) received this review. The pupil-discovery classes then organized themselves into pupil committees of about six mumbers each to cooperatively work for a period of 30 minutes toward the solutions of the task problems. The teachers in the discovery classes were permitted to answer questions concerning the correctness or incorrectness of the committee solutions to the problems. She could offer encouragement. She did not explain why a solution was incorrect nor suggest correct procedures. The classes in the teacher-instruction group were conducted by the teacher. She involved the students as much as possible in teaching the students to solve the task problems during a 30 minute period. Each teacher used her own instructional style. The classes in the control group received only the review of the skills in natural numbers, and then they worked individually on the task problems without any help whatsoever from the teacher.

The Hypotheses

The hypotheses of the study were grouped into three classifications: those dealing with learning, those dealing with retention, and one dealing with satisfactory achievement. The hypotheses related to learning were as follows.

(1) Hypothesis 1A: There will be no difference in the mean scores on tasks between boys and girls.

(2) Hypothesis 2A: The mean score on tasks at any grade will be higher than the mean score on the same tasks at a lower grade level.

(3) Hypothesis 3A: The mean score on tasks by children with higher intellectual ability as measured by the Otis Mental Abilities test will be higher than the mean score on tasks by children with lower intellectual ability.

(4) Hypothesis 3A: There will be no difference in

the mean scores on the tasks between groups receiving different instructional methods.

The hypotheses related to retention were as follows.

(1) Hypothesis 1B: There will be no difference in the mean scores on the retention of task skills between boys and girls.

(2) Hypothesis 2B: The mean scores on retention of task skills at any grade level will be greater than the mean scores on retention of task skills at a lower grade level.

(3) Hypothesis 3B: The mean scores on retention of task skills by children with higher intellectual ability as measured by the Otis Mental Abilities test will be greater than the mean scores on retention of task skills by children with lower intellectual ability.

(4) Hypothesis 4B: There will be no difference in the mean scores on the retention of task skills between groups receiving different instructional methods.

The following hypothesis was related to satisfactory class achievement.

(1) Hypothesis 5: The classes at all three grade levels will attain satisfactory levels of achievement on the learning of task skills.

Analysis

Just as the hypotheses were grouped into three different classifications, the analysis of these hypotheses require three different analytic procedures. An $\alpha = .05$ level of significance was used in each case to accept or reject a hypothesis.

At first glance an analysis of variance seemed to be an ideal vehicle for testing the hypotheses related to learning. However, this procedure must be rejected for good reason. The number of subjects in each cell would vary as a result of differing class size and mix. This leads to an unbalanced design and the assumptions of independence (or orthogonality) would not be valid. A five way unbalanced analysis of variance does not exist. An analysis of variance based upon a reduction of the number of factors, as the pooling of sex and class data, was possible. But this procedure would not have yielded full information on the interactions among the factors. Instead, the analysis used in testing the hypothesis on learning was the technique of planned comparisons as described by Hays.[1] This analysis can be used when a number of particular questions, formulated prior to data collection, are to be answered separately. In this procedure the means μ_i under comparison are expressed as a linear combination with weights c_j, not all equal to zero, in the form

$$\Psi_{H_i} = \sum_j c_j \mu_j \ .$$

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[1]Hays, William L., Statistics for Psychologists (New York: Holt, Rinehart and Winston, 1965), pp. 459-489.

The requirement is made that

$$\sum_j c_j = 0 \ .$$

If the c_j's are selected properly, the Ψ_{H_i}'s will be Orthogonal. In these cases the hypothesis generally tested is

$$H_0: \quad \Psi = 0$$

by the statistic

$$t = \frac{\Psi}{\sqrt{\text{est. var } (\Psi)}}$$

distributed as t with the degrees of freedom of the mean square error. Since the computational procedures were written specifically for the study they are shown in detail in Appendix B.

The testing of the hypotheses dealing with retention also presented their own peculiar problems. The differences in post-test scores could have easily been tested, but the question whether these differences were due to better retention or to better initial learning would remain. To avoid this difficulty, a multifactor analysis having repeated measures and unequal group size was used. The repeated measures used were the post-test scores and the sum of the task scores from the problem sets of those tasks that were identical to the ones found in the post-test. The analysis corrects for differences due to initial learning in the variance of the mean of the post-test scores by having each subject used as his own control. The computational procedures followed were found in Winer.[1]

Finally, to test the hypothesis concerned with satisfactory achievement, it was necessary to define satisfactory achievement. A class was said to have done satisfactory work on a problem set if 50% of the class correctly solved 50% or more of the problems on that set. This was clearly an arbitrary level. However, considering the extent of the material covered in the six lessons of only 30 minutes each, and considering that the curricular material was new to many of the teachers, the level of achievement was believed to be reasonable. The hypothesis was to be accepted if it could be expected that this level of accomplishment would be reached 90% of the time. The statistic used was

$$\chi^2 = \sum_{i=1}^{2} \frac{\left[f_i - F_i\right]^2}{F_i}$$

where f_i was the observed frequency of class having success or failure and F_i was the theoretical frequency of classes having success or failure. The two classifications of success or failure were represented, respectively, by $i = 1,2$. This statistic was assumed to have a χ^2

[1]Winer, B.J., Statistical Principles in Experimental Design (New York: McGraw-Hill Book Company, 1962), pp. 374-378.

distribution with one degree of freedom. The computational procedures recommended by Dixon[1] were followed.

Summary

In this study, seventeen tasks were selected as measures of the ability of elementary school children to perform and understand the operations of addition, subtraction and multiplication of signed numbers. These seventeen tasks were organized into six lessons. The subjects were members of twenty-one fourth, fifth, and sixth grade classes selected from school districts of southwestern Michigan. At each grade level the classes were assigned to one of two treatment groups (discovery or instructional) or to a control group. Each class was partitioned into four disjoint subclasses by sex and I.Q. A unit on signed numbers using the six lessons was taught to each class by the classroom teacher. The type of instruction or learning treatment they received was determined by the treatment group to which the class was assigned. An examination was given after each lesson, and a post-test was given one month after the sixth lesson. The design permitted comparisons of a discovery type learning experience with a didactic type learning experience, boys with girls, high I.Q. with low I.Q., and one grade level with another. The post-test gave the same

[1]Dixon, Wilfred J., and Massey Jr., Frank J., Introduction to Statistical Analysis (New York: McGraw-Hill Book Company, Inc., 1957), pp. 221-224.

comparisons on retention.

CHAPTER 4: ANALYSIS OF DATA

The purpose of this study was to investigate the ability of elementary school children to accomplish tasks related to operations on signed numbers. Twenty-one fourth, fifth, and sixth grade classrooms were divided into two treatment groups (a discovery group and an instructional group) and one control group. The classes were given six examinations covering seventeen tasks on the operations of signed numbers during a training period and a post-test one month after the training period. A summary of the mean scores on these tasks is found in Table 4.1.

A number of hypotheses on learning, retention, and achievement as measured by the test scores were tested. A planned comparisons test was developed for each of the seventeen tasks. For this test the classes within each treatment group were pooled even though the 102 analysis of variances comparing means between classes for each task indicated differences in 38 cases, no significant differences in 48 cases, and no analysis in 16 cases at $\alpha = .05$. This pooling was rationalized on the basis that the sample selections were classes and not the individual students within the classes. The analysis of variance tables using the planned comparisons computational procedures are found in Appendix C. The overall analysis of variance for each of the tasks indicated differences at

Table 4.1

SUMMARY OF MEAN SCORES FOR TASKS AND POST TEST

Test Number	Task	High Score	Fourth Grade		Fifth Grade		Sixth Grade	
			Disc.	Instr.	Disc.	Instr.	Disc.	Instr.
1	1	1	.958	.964	.972	.538	.628	1.000
	2	5	1.860	2.622	2.719	2.170	1.470	3.799
	3	3	1.040	.964	.573	1.446	.269	1.513
	4	3	.651	1.093	1.038	1.341	.499	1.471
2	5	2	1.753	1.801	1.283	1.784	1.511	1.828
	6	2	1.506	1.150	1.201	1.543	1.189	1.685
	7	3	2.419	2.266	2.026	2.594	1.987	2.732
	8	2	1.086	1.080	1.161	1.416	.809	1.742
3	9	4	2.337	2.673	2.138	2.581	.712	3.230
	10	4	1.293	1.266	.999	1.353	1.211	2.523
4	11	3	2.880	2.878	2.847	2.799	2.892	2.957
	12	2	1.969	1.976	1.969	1.987	1.976	2.000
	13	5	2.954	2.794	2.802	3.011	1.975	4.238
5	14	1	.614	.661	.314	.466	.528	.695
	15	4	1.830	2.359	1.328	1.612	2.140	3.202
6	16	1	.562	.575	.471	.609	.836	.825
	17	4	2.393	2.505	2.082	2.633	3.371	3.347
Post Test		11	9.137	9.656	8.535	8.741	9.963	13.333

$\alpha = .05$ for all tasks except eleven and twelve. The hypotheses on learning as measured by the test scores and tests of these hypotheses were as follows.

(1) Hypothesis 1A: There will be no difference in the mean scores on tasks between boys and girls.

Symbolically: $H_0 : \mu_{B_i} - \mu_{G_i} = 0$

Legend: μ_{B_i} = mean score of boys on task i, $i = 1,\ldots,17$,

μ_{G_i} = mean score of girls on task i, $i = 1,\ldots,17$.

Since the degrees of freedom exceeded 400 in each case, the distribution of t was considered normal. The null hypothesis was rejected if t was not in the interval $-1.960 < t < 1.960$ ($\alpha = .05$). The tests of the hypothesis for each of the tasks are listed in Table 4.2.

Table 4.2

THE TESTS OF HYPOTHESIS 1A ON THE COMPARISON OF LEARNING BETWEEN THE SEXES FOR 17 LEARNING TASKS

Task	Mean Difference	t-value	H_0
1	-.032	-1.431	ns*
2	-.348	-1.718	ns
3	-.023	-.213	ns
4	-.156	-1.652	ns
5	.055	1.046	ns
6	.020	.355	ns
7	-.065	-.755	ns
8	-.109	-1.855	ns

*not significant

Table 4.2

Continued

Task	Mean Difference	t-value	H_0
9	-.158	-1.107	ns
10	-.164	-.970	ns
11	.010	.294	ns
12	.005	-.331	ns
13	-.181	-.759	ns
14	-.052	1.433	ns
15	-.105	.585	ns
16	-.007	.212	ns
17	-.131	-.528	ns

(2) Hypothesis $2A_1$: The mean scores on tasks at the sixth grade level will be greater than the mean scores on tasks at the fifth grade level.

Symbolically: $H_0 : \mu_{6_i} - \mu_{5_i} = 0$

$H_A : \mu_{6_i} - \mu_{5_i} > 0$

Legend: μ_{6_i} = mean score of sixth graders on tasks i,

$i = 1,\ldots,17$.

μ_{5_i} = mean score of fifth graders on tasks i,

$i = 1,\ldots,17$.

Since the degrees of freedom exceeded 150 in each case, the distribution of t was considered normal. The null hypothesis was rejected if $t > 1.645$ ($\alpha = .05$). The tests of the hypothesis for each of the tasks are listed in Table 4.3.

Table 4.3

THE TESTS OF HYPOTHESIS 2A, ON THE COMPARISON OF LEARNING BETWEEN FIFTH AND SIXTH GRADERS FOR 17 LEARNING TASKS

Task	Mean Difference	t-value	H_0
1	.037	1.360	ns
2	.053	.213	ns
3	-.195	-1.473	ns
4	.257	2.211	rejected
5	.113	1.727	rejected
6	.036	.522	ns
7	.004	.040	ns
8	.060	.838	ns
9	-.581	-3.306	ns
10	.594	2.855	rejected
11	.100	2.237	rejected
12	.007	.397	ns
13	.095	.326	ns
14	.212	4.768	rejected
15	1.143	5.203	rejected
16	.286	6.974	rejected
17	1.049	3.450	rejected

(3) Hypothesis $2A_2$: The mean scores on tasks at the fifth grade level will be greater than the mean scores on the same tasks at the fourth grade level.

Symbolically: $H_0 : \mu_{5_i} - \mu_{4_i} = 0$

$$H_A : \mu_{5_i} - \mu_{4_i} > 0$$

Legend: μ_{5_i} = mean scores of fifth graders on task i,

$i = 1,\ldots,17.$

μ_{4_i} = mean scores of fourth graders on task i,

$i = 1,\ldots,17.$

Since the degrees of freedom exceeded 150 in each case, the distribution of t was assumed normal. The null hypothesis was rejected if $t > 1.645$ ($\alpha = .05$). The tests of the hypothesis for each of the tasks are listed in Table 4.4.

Table 4.4

THE TESTS OF HYPOTHESIS $2A_2$ ON THE COMPARISON OF LEARNING BETWEEN FOURTH AND FIFTH GRADERS FOR 17 LEARNING TASKS

Task	Mean Difference	t-value	H_0
1	-.207	-7.462	ns
2	.169	.679	ns
3	.013	.103	ns
4	.307	2.655	rejected
5	-.238	-3.639	ns
6	.069	.992	ns
7	-.014	-.141	ns
8	.210	2.930	rejected
9	-.155	-.884	ns
10	-.093	-.453	ns
11	-.057	-1.274	ns
12	.006	.309	ns
13	.051	.174	ns
14	-.250	-5.603	ns
15	.479	1.888	rejected

Table 4.4

Continued

Task	Mean Difference	t-value	H_0
16	-.024	-.605	ns
17	.143	.472	ns

(4) Hypothesis 3A: The mean score on tasks by children with greater intellectual ability as measured by the Otis Mental Abilities test will be greater than the mean score on tasks by children with lower intellectual ability.

Symbolically: $H_0 = \mu_{H_i} - \mu_{L_i} = 0$

$$H_A = \mu_{H_i} - \mu_{L_i} > 0$$

Legend: μ_{H_i} = mean score of children with greater mental ability on task i, i = 1,...,17.

μ_{L_i} = mean score of children with lower mental ability on task i, i = 1,...,17.

Since the degrees of freedom exceeded 400 in each case, the distribution of t was considered normal. The null hypothesis was rejected if $t > 1.645$ ($\alpha = .05$). The tests of the hypothesis for each of the tasks are listed in Table 4.5.

Table 4.5

THE TESTS OF HYPOTHESIS 3A ON THE COMPARISON OF LEARNING BETWEEN HIGH AND LOW I.Q. GROUPS FOR 17 LEARNING TASKS

Task	Mean Difference	t-value	H_0
1	.015	.671	ns
2	1.140	5.619	rejected
3	.259	2.399	rejected
4	.641	6.789	rejected
5	.252	4.723	rejected
6	.411	7.198	rejected
7	.535	6.183	rejected
8	.540	9.225	rejected
9	.646	4.517	rejected
10	.656	3.874	rejected
11	.078	2.128	rejected
12	.012	.795	ns
13	.868	3.647	rejected
14	.180	4.973	rejected
15	.511	2.848	rejected
16	.120	3.598	rejected
17	.567	2.287	rejected

(5) Hypothesis 4A: There will be no difference in the mean scores on tasks between the discovery treatment group and the instructional treatment group.

Symbolically: $H_0 : \mu_{D_i} - \mu_{I_i} = 0$

Legend: μ_{D_i} = mean score of discovery treatment group on task i, $i = 1,\ldots,17$.

μ_{I_i} = mean score of instructional treatment group on task i, i = 1,...,17.

Since the degrees of freedom exceeded 400 in each case the distribution of t was considered normal. The null hypothesis was rejected if t was not in the interval $-1.960 < t < 1.960$ ($\alpha = .05$). The tests of the hypothesis for each of the tasks are listed in Table 4.6.

Table 4.6

THE TEST OF HYPOTHESIS 4A ON THE COMPARISON OF LEARNING BETWEEN DISCOVERY-INSTRUCTION GROUPS FOR 17 LEARNING TASKS

Task	Mean Difference	t-value	H_0
1	.003	.150	ns
2	-.843	-4.159	rejected
3	-.685	-6.353	rejected
4	-.563	-5.963	rejected
5	-.293	-5.487	rejected
6	-.151	-2.657	rejected
7	-.382	-4.417	rejected
8	-.382	-6.524	rejected
9	-1.147	-8.017	rejected
10	-.483	-2.855	rejected
11	.000	-.015	ns
12	-.005	-.331	ns
13	-.777	-3.263	rejected
14	-.124	-3.425	rejected
15	-.584	-3.257	rejected
16	-.023	-.701	ns
17	.090	-.364	ns

An analysis of variance for repeated measures using the unweighted means solution was used to test the hypotheses on retention. The repeated measures used were the post-test and the sum of the scores on the tasks that were the same as the tasks on the post-test. If a child had missed taking any one of the task tests or the post-test used in the repeated measure, he had to be removed from the analysis. This requirement resulted in the loss of about one-fourth of the subjects. Because of this loss, the data were pooled to form one factor designs. The analysis was computed for each of the factors of sex, I.Q., and instructional method for each grade level. One further analysis was computed for grade level. The results are summarized in Tables C.18 through C.27 included as part of Appendix C.

Each of the following hypotheses on retention was tested by the statistic

$$F = \frac{\left[\mu_1 - \mu_2\right]^2}{MS_{\text{subjects w groups}} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

where μ_1 and μ_2 were the post test means under comparison, n_1 and n_2 were the number of subjects in the mean groups, and $MS_{\text{subjects w groups}}$ was the subjects within groups mean square from the analysis of variance. Since the degrees of freedom of the denominator exceed 30 in each case, the function

$$\text{Log}_{10}\, F_{.05}\,(v_1 v_2) \approx \frac{1.4287}{\sqrt{\frac{2v_1 v_2}{v_1+v_2} - .95}} - (.681)\,\frac{v_2-v_1}{v_2 v_1}$$

in Dixon[1] was used as an approximation for the F distribution percentiles where v_1 and v_2 were the degrees of freedom of the numerator and denominator respectively.

(6) Hypothesis 1B: There will be no difference in the mean scores on the retention of task skills as measured by the post test between boys and girls at the same grade level. Symbolically: $H_0 : \mu_{B_i} = \mu_{G_i}$

Legend: μ_{B_i} = mean score on post test by boys in grade i, $i = 4,5,6$.

μ_{G_i} = mean score on post test by girls in grade i, $i = 4,5,6$.

The tests of the hypothesis for each of the grade levels are listed in Table 4.7.

Table 4.7

THE TESTS OF HYPOTHESIS 1B ON THE COMPARISON OF RETENTION BETWEEN THE SEXES IN GRADES FOUR THROUGH SIX

Grade	F	F at α = .05	H_0
4	.288	.738	ns

[1]Dixon, *Op. cit.*, p. 402.

Table 4.7

Continued

Grade	F	F at $\alpha = .05$	H_0
5	.861	.731	rejected
6	.079	.731	ns

(7) Hypothesis $2B_1$: The mean score on the retention of task skills as measured by the post test at the sixth grade levels will be greater than the same mean score at the fifth grade level.

Symbolically: $H_0 : \mu_6 - \mu_5 = 0$

$H_A : \mu_6 - \mu_5 > 0$

Legend: μ_6 = the mean score on the post test by sixth graders.

μ_5 = the mean score on the post test by fifth graders.

The region for rejection was $F \geq .720$ ($\alpha = .05$). F was computed to be 8.337. The null hypothesis was rejected.

(8) Hypothesis $2B_2$: The mean scores on the retention of task skills as measured by the post test at the fifth grade level will be greater than the same mean score at the fourth grade level.

Symbolically: $H_0 : \mu_5 - \mu_4 = 0$

$H_A : \mu_5 - \mu_4 > 0$

Legend: μ_5 = the mean score on the post test by fifth graders.

μ_5 = the mean score on the post test by fourth graders.

The region of rejection was $F \geq .720$ ($\alpha = .05$). F was computed to be 1.185. The null hypothesis was rejected.

(9) Hypothesis 3B: The mean score on retention of task skills as measured by the post test by children with higher mental ability as measured by the Otis Mental Abilities test will be greater than the same mean score by children with lower mental ability.

Symbolically: $H_0 : \mu_{H_i} - \mu_{L_i} = 0$

$H_A : \mu_{H_i} - \mu_{L_i} > 0$

Legend: μ_{H_i} = mean score on post test by children with higher mental ability in grade i, $i = 4,5,6$.

μ_{L_i} = mean score on post test by children with lower mental ability in grade i, $i = 4,5,6$.

The tests of the hypothesis for each of the grade levels are listed in Table 4.8.

Table 4.8

THE TESTS OF THE HYPOTHESIS 3B ON THE COMPARISON OF RETENTION BETWEEN HIGH AND LOW I.Q. GROUPS IN GRADES FOUR THROUGH SIX

Grade	F	F at $\alpha = .05$	H_0
4	28.686	.737	rejected
5	1.481	.731	rejected

Table 4.8

Continued

Grade	F	F at α = .05	H_0
6	2.770	.731	rejected

(10) Hypothesis 4B: There will be no differences in the mean scores on the retention of task skills as measured by the post test between groups at the same grade level receiving different instructional treatments.

Symbolically: $H_0 : \mu_{D_i} = \mu_{I_i}$

Legend: μ_{D_i} = mean score on post test by the discovery group in grade i, i = 4,5,6.

μ_{I_i} = mean score on post test by the instruction group in grade i, i = 4,5,6.

The tests of the hypothesis for each grade level are listed in Table 4.9.

Table 4.9

THE TESTS OF HYPOTHESIS 4B ON THE COMPARISON OF RETENTION BETWEEN DISCOVERY-INSTRUCTION GROUPS IN GRADES FOUR THROUGH SIX

Grade	F	F at α = .05	H_0
4	1.724	.731	rejected

Table 4.9

Continued

Grade	F	F at α = .05	H_0
5	.046	.731	ns
6	15.366	.731	rejected

A class was said to have achieved a satisfactory level of learning for a particular lesson if 50% of the class scored 50% or better on the test covering that lesson. The achievement levels of the classes in the study are summarized in Tables C.28 through C.30, found in Appendix C. Further, the classes were expected to achieve this level 90% of the time.

(5) Hypothesis 5: At each of the three grade levels, 50% of the students in a class will score 50% or higher on the six individual tests 90% of the time.

Using the computational procedures described in the analysis section of chapter three, page 42 the hypothesis was rejected if $\chi^2 > 3.84$ ($\alpha = .05$ with one degree of freedom).

At the fourth grade level, $\chi^2 = 16.89$, the hypothesis was <u>rejected</u>.

At the fifth grade level, $\chi^2 = 21.777$, the hypothesis was <u>rejected</u>.

At the sixth grade level, $\chi^2 = .604$, the hypothesis was <u>not rejected</u>.

Summary

In this chapter a number of hypotheses were tested on the ability of fourth, fifth, and sixth grade children to learn and retain skills involving the operations on signed numbers. A final hypothesis was tested on the level of achievement of the classes in lessons involving operations on signed numbers.

In tests investigating the ability of children to learn the skills, the mean scores of seventeen tasks were compared within classifications of sex, grade, I.Q. and method of instruction. Table 4.10 summarizes the findings of these comparisons.

Table 4.10

NUMBERS OF TESTS THAT SUPPORT THE HYPOTHESES COMPARING MEANS (μ) WITHIN THE CLASSIFICATIONS OF SEX, GRADE, I.Q. AND METHOD OF TEACHING

Comparison	Hypotheses	No. of tests supporting Hypotheses	Total No. of tests
Sex	$\mu_{Boy} = \mu_{Girl}$	17	17
Grade	$\mu_{G6} > \mu_{G5}$	8	17
	$\mu_{G5} > \mu_{G4}$	3	17
I.Q.	$\mu_{Hi\ IQ} > \mu_{Lo\ IQ}$	15	17
Method of Instruction	$\mu_{Disc} = \mu_{Instr}$	4	17

The mean scores on a post-test were used to compare the retention of subjects classified according to sex, I.Q., and method of instruction at each grade level and then between the grades themselves. Table 4.11 summarizes the findings of the comparison.

Table 4.11

SUMMARY OF TESTS OF HYPOTHESES COMPARING POST TEST SCORE MEANS (μ) WITHIN THE CLASSIFICATIONS OF SEX, I.Q., TEACHING METHOD, AND GRADE

Comparisons	Hypothesis	Result of Tests of Hypothesis Grade 4	Grade 5	Grade 6
Sex	$\mu_{Boy} = \mu_{Girl}$	supported	rejected	supported
I.Q.	$\mu_{Hi\ IQ} > \mu_{Lo\ IQ}$	supported	supported	supported
Teaching Method	$\mu_{Disc} = \mu_{Instr}$	rejected	supported	rejected
Grade	$\mu_{6G} > \mu_{5G}$: supported, $\mu_{5G} > \mu_{4G}$: supported			

Finally, the classes were said to have reached satisfactory achievement if 50% of students in the class scored 50% or higher on the tests given after each lesson. It was hypothesized that this level of achievement would be reached 90% of the time. The hypothesis was rejected at the fourth and fifth grade levels and supported at the sixth grade level.

CHAPTER 5: SUMMARY AND CONCLUSIONS

Summary

Elementary school mathematics has undergone dramatic changes in both content and procedures within the past ten years. The "modern" mathematics revolution is continuing with recommendations for the inclusion of still newer ideas in the curriculum. One such topic is the study of the rules of operations on signed numbers. It was the purpose of this study to investigate the ability of children to learn and retain skills used in operations on signed numbers.

The numbers, in fact integers, were represented on the number line as bi-directional vectors. The number line was coordinated by indicating the direction and distance a point was located from zero. The operation of addition was defined as vector addition. The operation of subtraction was motivated by presenting that operation as the inverse operation of addition. The rules for multiplication were developed as consequences of the distributive and additive inverse properties. The skills needed to effectively work with these operations were organized into seventeen objectively scored tasks. These tasks were further grouped into six lessons.

The subjects were members of twenty-one fourth, fifth, and sixth grade classes selected from school districts of southwestern Michigan. At each grade level the classes were assigned to one of two treatment groups (discovery or instructional) or to a control group. Each class was partitioned into four disjoint subclasses by sex and I.Q. A unit on signed numbers was taught to each class by the classroom teacher. The type of instruction or learning treatment they received was determined by the treatment group to which the class was assigned. An examination was given after each lesson, and a post test was given one month after the sixth lesson. The design permitted comparisons of a discovery type learning experience with a didactic type learning experience, boys with girls, high I.Q. with low I.Q., and one grade level with another. The post test gave the same comparisons on retention.

The hypotheses were statistically tested at $\alpha = .05$. This analysis of the data indicated that no significant difference existed between boys and girls in learning the 17 tasks. There was no difference in retention between the sexes at either the fourth or sixth grade level. However, at the fifth grade level, girls did slightly better than boys. On 15 of the 17 tasks children with high I.Q. scored better than children with low I.Q., and on retention the high I.Q. subjects scored higher than the low I.Q. subjects at all grade levels. The subjects in the instruction group had higher scores in general than the subjects in the discovery group in 13 of the 17 tasks. Also, the subjects in the instruction classes retained more of what they learned than did the subjects in the discovery classes in the fourth and sixth grades. Only in the fifth grade was there no significant difference. The differences between the grades were more mixed. In 8 of the 17 tasks sixth graders scored higher than the fifth graders, and on only 3 of the 17 tasks did fifth graders score higher than fourth graders. However, on retention, the sixth graders retained more of what they learned than the fifth graders who in turn retained more of what they learned than fourth graders. Overall, a class was said to have reached a satisfactory level of achievement for a particular lesson if 50% or more of the class scored 50% or more on the test following that lesson. It was found that sixth graders could be expected to achieve at this level 90% of the time, whereas fourth and fifth graders could do so only 80% of the time.

Conclusions

The following conclusions are stated as a result of the tests of the hypothesis.

(1) No difference existed between boys and girls in the fourth, fifth, and sixth grades in their ability to learn tasks involving the operations of signed numbers.

(2) No difference existed between boys and girls in the fourth, fifth, and sixth grade in their ability to retain skills learned involving the operations on signed numbers.

(3) Higher I.Q. children in the fourth, fifth, and sixth grades scored higher on tasks involving operations of signed

numbers than did lower I.Q. children.

(4) Higher I.Q. children in the fourth, fifth and sixth grades retained skills learned in the operations of signed numbers better than lower I.Q. children.

(5) Children in the sixth grade scored higher on tasks involving operations of signed numbers than did children in the fifth grade.

(6) Children in the sixth grade retained skills learned in the operations of signed numbers better than fifth grade children.

(7) No difference existed between fifth and fourth grade children in their ability to learn tasks involving the operations of signed numbers.

(8) Children in the fifth grade retained skills learned in the operation of signed numbers better than did fourth grade children.

(9) Children in the fourth, fifth, and sixth grades who had an instructional type learning experience scored higher on tasks on the operations of signed numbers than did fourth, fifth, and sixth grade children who had a discovery type learning experience.

(10) Children in the fourth, fifth, and sixth grades who had an instructional type learning experience retained skills learned on the operations of signed numbers better than did fourth, fifth, and sixth grade children who had a discovery type learning experience.

(11) Sixth grade classes attained satisfactory levels

of achievement on lessons on the operations of signed numbers.

Discussion

A number of the conclusions are easily justified by a cursory review of the analysis. Conclusion (1), that no difference existed between sexes in their ability to learn, was well supported in that all 17 tests were not significant (Table 4.2). Conclusion (3), that high I.Q. children were better able to learn than low I.Q. children was supported by 15 of the 17 tasks and the remaining two, while not significant, were positive (Table 4.5). Similarly, conclusion (4), that high I.Q. children were better able to retain what they had learned was well supported (Table 4.8). Conclusions (6) and (8) on the ability of higher grade children to better retain what was learned are supported by the statistical tests.

Conclusions (9) and (10), that children receiving instructional learning treatments learned better and retained more of what they learned, is contrary to what was conjectured in the hypotheses, namely, that no difference existed. The analysis, however, indicated that differences did exist in 12 of the 17 tasks in favor of instruction. The differences in four of the five non-significant cases were in the direction of the instructional procedure (Table 4.6). In defense of the discovery method it should be reported that this procedure was a new experience for both the subjects and their teachers. Even though the discovery groups did not learn as well, they did learn, as illustrated by the fact that the discovery classes reached satisfactory levels of achievement in 34 of 54 lessons. Also, it is interesting to note how these differences due to teaching method came about. A review of the mean scores (Table 4.1) revealed that in general, little difference existed between the methods at the fourth grade level, and that the differences at the sixth grade were greater. This occurred because the means of the instruction group increased with the grade level, and the means of the discovery group decreased with the grade level. The superior performance by the instructional groups was uniform over both the high and low I.Q. classification and the boy-girl classification.

Conclusion (7), that there was no difference in learning ability between fifth and fourth grade children, was also contrary to conjecture. Only 2 of the 17 tasks showed significant differences and the non-significant t's were positive in five cases and negative in six cases (Table 4.4). Conclusion (5), that children in the sixth grade learned better than children in the fifth grade, was not overwhelmingly supported. In only 8 of the 17 tasks was the null hypothesis rejected. However, the fact that the overall analysis of variance indicated that the data on two tasks contained no real differences and that 7 of the 9 non-significant tests favored the conclusion were considered indicative (Table 4.3).

Conclusion (2), that no difference existed between boys and girls in their ability to retain what they learned, was supported by the analysis at the fourth and sixth grade level, but not at the fifth grade level. When a large number of statistical tests are made at $\alpha = .05$, as was the case in this study, it is almost certain that type one errors will occur. There are three reasons why the significant difference at the fifth grade level was judged to be such an error. First, the F-ratio was not highly significant (Table 4.7). Second, all other comparisons based upon sex in the study were not significant. Third, and most importantly, a review of the raw data indicated that fifth grade girls had an unusually large share of high I.Q. subjects (54.9% compared with 48.8% in the fourth grade and 48.3% in the fifth grade).

The results of the study, in some cases, supported the findings of other researchers and, in other cases, questioned their findings. The problems in task 7, testing the understanding of the inverse property, were answered correctly 77.9% of the time, whereas the problems in task 15, testing the understanding of distributive property, were answered correctly only 52.1% of the time. This clearly supported Crawford[1] who reported that the inverse property was more easily learned than the distributive property.

No difference was found in the abilities of boys and girls in learning or retaining the material. Jarvis[2] and

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[1]Crawford, Op. cit., pp. 5728-5729.

[2]Jarvis, Op. cit., pp. 657-659.

Parsley[1] indicated that boys achieved better than girls on tasks dealing with mathematical concepts. This was not observed in this study, even though the learning of mathematical concepts was a large part of the learning.

The claim of Bassler,[2] Fleckman,[3] and others that didactic teaching methods lead to better results in initial testing was supported, but the claim that discovery methods result in better performance on retention tests was not verified. Also, the conjecture that a discovery approach aids in the learning of operations, while directed methods result in greater learning of mathematical properties as reported by Armstrong[4] was not substantiated.

A study of the data revealed some unexpected observations that were not directly related to the theories discussed in the study. Recall that in task 8 the subjects were required to find a missing addend given a sum, in task 9 the subjects were required to restate a subtraction problem as an addition problem, and in task 10 the subjects were required to combine tasks 8 and 9 to find a solution to a subtraction problem. One would think that a child who had mastered tasks 8 and 9 would find task 10 easy to solve. However,

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[1]Parsley, Op. cit., pp. 268-270.

[2]Bassler, Op. cit., pp. 357-362.

[3]Fleckman, Op. cit., pp. 3366-3367.

[4]Armstrong, Op. cit., p. 141.

this was not the case. Overall, the scores averaged 60.0% correct on task 8, 56.9% on task 9, and 36.6% on task 10. The subjects had difficulty in combining the two previously learned skills to form the new skill.

In another case the learning of tasks 14 and 16 was prerequisite to the learning of tasks 15 and 17, respectively. In tasks 14 and 16 the distributive property and the additive inverse property were used to justify the rules for finding the products of numbers with negative values. In tasks 15 and 17, the subjects were to apply the rules they learned in the previous task. Thus if a child had failed in task 14, one would expect him to fail in task 15. However, this was not always the case. Many subjects, after missing tasks 14 and 16, went on to correctly solve the problems in task 15 and 17.

Recommendations

The study of the operations of signed numbers is a topic that could well be taught within the sixth grade mathematics curriculum. This would include the operation of multiplication as well as the operations of addition and subtraction already included in some programs. The order of difficulty of the operations in the study were addition (easiest), multiplication, and subtraction (most difficult). Since the multiplication of signed numbers is apparently easier to learn than subtraction, there is no reason to exclude it.

The fears expressed in the Cambridge Report[1] about the pedagogical problems of teaching the multiplication of signed numbers seem to be exaggerated. The topics could be taught also in the fourth and fifth grades, but not with the degree of rigor used in the sixth grade. This is evident by the fact that while only the sixth graders achieved satisfactory levels of achievement 90% of the time, the fourth and fifth graders did so 80% of the time. The closeness of these percentages lends support to those who advocate non-graded schools.

Replications or further similar studies are needed before one can judge which conclusions can be generally accepted. The study contains a number of weaknesses that restrict such generalization. First, the classes used in the study were not selected at random, and little demographic information is available concerning the subjects. This makes it difficult to conjecture how other elementary school children would do in similar studies. Also, the relatively low reliability scores on some of the examinations were disappointing. In a replication of the study, where better instruments were developed, better results may be expected. Finally, further study using more instructional time might result in higher achievement. This is suggested in the mean scores (Table 4.1) for lessons five and six. The lessons were very similar in that they covered the multiplication of negative numbers.

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[1]The Cambridge Conference on School Mathematics, *op. cit.*, p. 37.

The mean scores on lesson six were higher than the mean scores on lesson five in every case. Since the lessons were similar the higher scores on lesson six may be attributed to the total experience, i.e. extra time.

Further investigations in the study of the operations on signed numbers might compare the "direction" number approach used in this study with the traditional plus and minus representation. The direction number approach was used because it was hoped that this would provide a better visual image that children need at this age. If it can be shown that the plus and minus symbols serve just as well, then they should be used since they are universally accepted and the students must adopt them sooner or later.

In further studies, the number system used should be extended to include rational numbers as well as integers. The number combinations used in this study were restricted purposefully to the easier combinations. The desire was to measure the ability of children to learn the concepts rather than to measure their arithmetic ability. The extension to include the rational numbers would permit an investigation of the operation of division, which was excluded from this study.

Finally, the positive results of this study should encourage similar investigations of other topics recommended in the Cambridge report[1] for the elementary school.

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[1]The Cambridge Conference on School Mathematics, op. cit.

BIBLIOGRAPHY

BIBLIOGRAPHY

A.A.A.S. Commission on Science Education, Science--A process Approach: An Evaluation Model and Its Application, Second Report (1968), The American Association for the Advancement of Science.

Adler, Irving, "The Cambridge Conference Report: Blueprint or Fantasy?," The Arithmetic Teacher, Vol. 13 (March, 1966), pp. 179-187.

Allendorfer, Carl B., Mathematics for Parents (New York: The Macmillan Company, 1965).

Armstrong, Jenny Rose, "The Relative Effects of Two Forms of Spiral Curriculum Organization and Two Modes of Presentation on Mathematical Learning," Dissertation Abstracts, Vol. 29 (July, 1968), p. 141.

Ausubel, David P., "Some Psychological and Educational Limitations of Learning by Discovery," The Mathematics Teacher, Vol. 57 (May, 1964), pp. 290-302.

Banks, J. Houston, Elements of Mathematics, Second Edition (Boston: Allyn and Bacon, Inc., 1961).

Bassler, Otto C., "Intermediate Versus Maximal Guidance - A Pilot Study," The Arithmetic Teacher, Vol. 15 (April, 1968), pp. 357-362.

Baumann, Raemt R., "Childrens Understanding of Selected Mathematical Concepts in Grades Two and Four," Dissertation Abstracts, Vol. 26 (March, 1966), p. 5219.

Beberman, Max, and Vaughn, Herbert E., High School Mathematics (Boston: D.C. Heath and Company, 1966).

Beman, Woster W., Elements of Algebra (Boston: Ginn and Company, 1894).

Brownell, William A., and Moser, Harold E., "Meaningful vs. Mechanical Learning: A Study in Grade 3 Subtraction," Duke University Studies in Education, Vol. 8 (1949), pp. 1-207.

Bruner, Jerome S., "On Learning Mathematics," The Mathematics Teacher, Vol. 53 (December, 1960), pp. 610-619.

Bruner, J.S., The Process of Education (Cambridge, Massachusetts: Harvard University Press, 1962).

Bruner, J.S., Toward A Theory of Instruction (Cambridge, Massachusetts: The Belknap Press of Harvard University Press, 1966).

Buros, Oscar K., The Sixth Mental Measurements Yearbook (Highland Park, New Jersey: The Gryphon Press, 1963), p. 481.

Butler, Charles H. and Wren, Lynwood F., The Teaching of Secondary Mathematics (New York: McGraw-Hill Book Company, 1965).

Cambridge Conference on School Mathematics, Goals for School Mathematics (Boston: published for Educational Services, Inc. by Houghton Mifflin, 1963).

Carlton, Virginia, "Mathematics Education in the Elementary Schools of the Soviet Union," The Arithmetic Teacher, Vol. 15 (February, 1968), pp. 108-114.

Crawford, Douglas H., "An Inventory of Age-Grade Trends in Understanding the Field Axioms," Dissertation Abstracts, Vol. 25 (April, 1965), pp. 5728-5729.

D'Augustine, C.H., Multiple Methods of Teaching Mathematics in the Elementary School (New York: Harper and Row, 1968).

Davis, Robert B., The Madison Projects Approach to a Theory of Instruction, a report of the Madison Project, Webster College, St. Louis, Missouri.

Dawson, Dan T., "The Case for the Meaning Theory in Teaching Arithmetic," Elementary School Journal, Vol. 55 (March, 1955), pp. 393-399.

Dixon, Wilfred J., and Massey Jr., Frank J., Introduction to Statistical Analysis (New York: McGraw-Hill Book Company, Inc., 1957).

Duncan, Ernest R., Modern School Mathematics: Structure and Use (Boston: Houghton Mifflin Company, 1970).

Eicholy, Robert E. and O'Daffer, Phares C., Elementary School Mathematics, Second Edition, Book 6 (Menlo Park, California: Addison Wesley Publishing Company, 1968).

Evenson, A.B., Modern Mathematics (Chicago: Scott, Foresman and Company, 1962).

Fleckman, Bessie, "Improvement of Learning Division Through Use of the Discovery Method," Dissertation Abstracts, Vol. 27A (April, 1967), pp. 3366-3367.

Flournoy, Frances, "Applying Basic Mathematical Ideas in Arithmetic," The Arithmetic Teacher, Vol. 11 (February, 1964), pp. 104-108.

Flournoy, Frances, "Understanding Relationships: An Essential for Solving Equations," The Elementary School Journal, Vol. 64 (January, 1964), pp. 214-217.

Folsom, Mary, "Why the New Mathematics?," The Instructor, Vol. 73 (December, 1963), pp. 6-7.

Fouch, Robert S., and Haas, Raymond, SRA Elementary Mathematics Program, Book 6 (Chicago: Science Research Associates, 1968).

Gagné, Robert M., The Conditions of Learning (New York: Holt, Rinehart and Winston, 1965).

Glaser, Robert, "Variables in Discovery Learning" in Shulman, Lee (editor), Learning by Discovery: A Critical Approach (Chicago: Rand, McNally and Company, 1966), p. 15.

Glennon, Vincent J., Short, Roy F., and Brownell, M.A., Mathematics We Need (Boston: Ginn and Company, 1966).

Gordon, David X., "Clarifying Arithmetic Through Algebra," School Science and Mathematics, Vol. 42 (March, 1942), pp. 288-289.

Gorza, Vivian S., A Survey of Mathematics: Early Concepts and Their Historical Development (New York: Holt, Rinehart and Winston, 1968).

Gray, Roland F., "An Experiment in the Teaching of Introductory Multiplication," The Arithmetic Teacher, Vol. 7 (March, 1965), pp. 199-203.

Greathouse, Jimmie Joe, "An Experimental Investigation of Relative Effectiveness Among Three Different Arithmetic Teaching Methods," unpublished Ph.D. Thesis, The University of New Mexico, 1965.

Hall, Kenneth Dwight, "An Experimental Study of Two Methods of Instruction for Mastering Multiplication Facts at the Third Grade Level," unpublished Ph.D. Thesis, Duke University, 1967.

Hartung, Maurice L., et. al., Seeing Through Arithmetic, 6 (Glenview, Illinois: Scott Foresman and Company, 1968).

Haverhill, Wallace P., "Though This Be Madness . . . ," The Arithmetic Teacher, Vol. 16 (December, 1969), pp. 606-608.

Hays, William L., Statistics for Psychologists (New York: Holt, Rinehart and Winston, 1965).

Hervey, Margaret A., "Childrens Responses to Two Types of Multiplication Problems," The Arithmetic Teacher, Vol. 13 (April, 1960), pp. 288-292.

Jarvis, O.T., "Boy-Girl Ability Differences in Elementary School Arithmetic," School Science and Mathematics, Vol. 64 (November, 1964), pp. 657-659.

Johnson, Harry C., "What Do We Mean by Discovery?," The Arithmetic Teacher, Vol. 11 (December, 1964, pp. 538-539.

Keedy, Mervin J., et. al., Exploring Elementary Mathematics, 6 (New York: Holt, Rinehart and Winston, 1970).

Kemeny, John, "The Impact of the Computer on Teaching," An address given at the Cleveland Meeting of the National Council of Teachers of Mathematics, Cleveland, Ohio, November, 13, 1969.

Kerlinger, Fred N., Foundations of Behavioral Research (New York: Holt, Rinehart and Winston, Inc., 1964), pp. 429-443.

Kersh, Bert Y., "Learning by Discovery: Instructional Strategies," The Arithmetic Teacher, Vol. 11 (April, 1964), p. 226.

Kingston, J. Maurice, Mathematics for Teachers of the Middle Grades (New York: John Wiley and Sons, Inc., 1966).

Krich, Percy, "Grade Placement and Meaningful Learning," School Science and Mathematics, Vol. 64 (February, 1964), pp. 131-137.

Miller, G.A., "Crusade Against the Use of Negative Numbers," School Science and Mathematics, Vol. 33 (December,

Miller, G.H., "How Effective is the Meaning Method?," The Arithmetic Teacher, Vol. 4 (March, 1957), pp. 45-49.

Milne, William J., High School Algebra (New York: American Book Company, 1892).

Otis, Arthur S., Otis Quick-Scoring Mental Tests: New Edition, Beta Test Form Em (New York: Harcourt, Brace and World, Inc., 1954).

Parsley, Kenneth M., "Further Investigation of Sex Differences in Achievement of Under-Average and Over-Average Achieving Students Within Five I.Q. Groups in Grades Four Through Eight," Journal of Educational Research, Vol. 57 (January, 1964), pp. 268-270.

Parsons, Cynthia, "Algebra as Presented to Fourth Graders is Grasped With Enthusiasm," Christian Science Monitor, January 9, 1960, p. 11.

Patterson, Katherine, "A Picture Line Can Be Fun!," The Arithmetic Teacher, Vol. 16 (December, 1969), pp. 603-605.

Piaget, Jean, "How Children Form Mathematical Concepts," Scientific American, Vol. 189 (November, 1953), pp. 74-79.

Price, H.V., Peak, P., and Jones, P.S., Mathematics: An Integrated Series, Book One (New York: Harcourt, Brace and World, Inc., 1965).

Rappaport, David, "The Meaning Approach in Teaching Arithmetic," Chicago School Journal, Vol. 44 (January, 1963), pp. 172-174.

Rappaport, David, "Understanding Meanings in Arithmetic," The Arithmetic Teacher, Vol. 5 (March, 1958), pp. 96-99.

Riedesel, C. Alan, Guiding Discovery in Elementary School Mathematics (New York: Appleton-Century-Crofts, 1967).

Romberg, T.A. and DeVault, M.V., "Mathematics Curriculum: Needed Research," Journal of Research and Development in Education, Vol. 1 (Fall, 1967), pp. 95-110.

Scandura, Joseph M., "An Analysis of Exposition and Discovery Modes of Problem Solving Instruction," Journal of Experimental Education, Vol. 33 (December, 1964), pp. 148-159.

Schell, Leo M., "Learning the Distributive Property by Third Graders," School Science and Mathematics (January, 1968), pp. 28-32.

Schell, Leo M., "Two Aspects of Introductory Multiplication: The Array and The Distributive Property," Dissertation Abstracts, Vol. 25 (April, 1965), p. 5161.

Schmidt, Mary M., "Effects of Teaching the Commutative Laws, Associative Laws on Fundamental Skills of Fourth Grade Pupils," Dissertation Abstracts, Vol. 26 (February, 1966), p. 4510.

School Mathematics Study Group, Mathematics for The Elementary School, Grade Six, Teacher Commentary, Part I (New Haven: Yale University Press, 1963).

Schraf, William L., "Arithmetic Taught As a Basic for Later Mathematics," School Science and Mathematics, Vol. 46 (May, 1946), pp. 413-423.

Shulman, Lee S., "Psychological Controversies in the Teaching of Science and Mathematics," The Science Teacher, Vol. 35 (September, 1968), pp. 34-37, 89-90.

Skinner, B.F., "Teaching Machines," Science, Vol. 128 (October, 1958), pp. 969-977.

Spitzer, Herbert F., et. al., Elementary Mathematics (St. Louis, Missouri: McGraw-Hill Book Company, 1967).

Ter Keurst, Arthur J., "Rote versus Discovery Learning," School and Community, Vol. 55 (November, 1968), pp. 42-44.

Wentworth, G.A., School Algebra (Boston: Ginn and Company, 1894).

Willoughby, Stephen S., Contemporary Teaching of Secondary Mathematics (New York: John Wiley and Sons, Inc., 1967).

Wilson, John H., "Differences Between the Inquiry Discovery and Traditional Approaches to Teaching Science in Elementary School," Research In Education, Vol. 4 (1969), p. 752.

Winer, B.J., Statistical Principles in Experimental Design (New York: McGraw-Hill Book Company, 1962).

Wittock, M.C., "The Learning by Discovery Hypothesis" in Shulman, Lee (editor), Learning by Discovery: A Critical Appraisal (Chicago: Rand, McNally and Company, 1966), pp. 42-48.

Worthen, Blaine R., "A Study of Discovery and Expository Presentation: Implications for Teaching," Journal of Teacher Education, Vol. 19 (Summer, 1968), pp. 223-242.

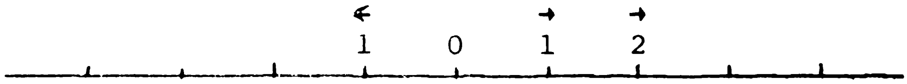
APPENDICES

APPENDIX A

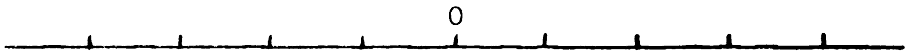
PROBLEM SETS AND POST-TEST

PROBLEM SET 1 FOR GROUP WORK

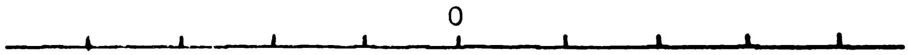
(1-1) Write the direction number at each point to tell the distance and direction the point is from zero.



(1-2) Choose any point you wish on the number line and name it 0.

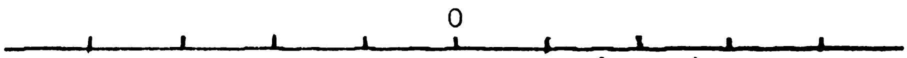


(2-1) Complete each number line below. Use the number line to answer the question following it.



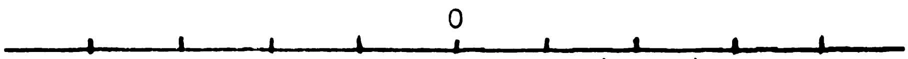
What direction number goes from $\overrightarrow{1}$ to $\overrightarrow{4}$? Ans.________

(2-2)



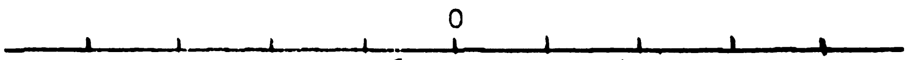
What direction number goes from $\overleftarrow{2}$ to $\overrightarrow{3}$? Ans.________

(2-3)



What direction number goes from $\overrightarrow{4}$ to $\overrightarrow{2}$? Ans.________

(3-1)

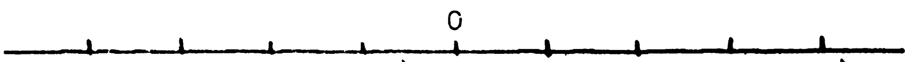


The direction number $\overleftarrow{4}$ goes from $\overrightarrow{1}$ to ________.

(3-2)

The direction number $\overrightarrow{3}$ goes from $\overrightarrow{2}$ to ________.

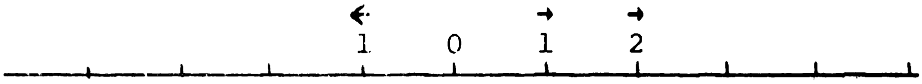
(4-1)



The direction number $\overrightarrow{5}$ goes from ________ to $\overrightarrow{2}$.

PROBLEM SET 1 FOR INDIVIDUAL WORK

(1-1) Write the direction number at each point to tell the distance and direction the point is placed from zero.



Complete each number line below. Use the number line to find the answer to the question following it.

(2-1)

What direction number goes from $\overrightarrow{1}$ to $\overrightarrow{3}$? Ans.________

(2-2)

What direction number goes from $\overrightarrow{5}$ to $\overrightarrow{1}$? Ans.________

(2-3)

What direction number goes from $\overrightarrow{2}$ to $\overleftarrow{1}$? Ans.________

(2-4)

What direction number goes from $\overleftarrow{4}$ to $\overrightarrow{2}$? Ans.________

(2-5)

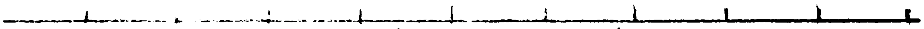
What direction number goes from $\overleftarrow{3}$ to $\overleftarrow{5}$? Ans.________

(3-1)

The direction number $\overrightarrow{3}$ goes from $\overrightarrow{2}$ to ________.

(3-2)

The direction number $\overleftarrow{1}$ goes from $\overrightarrow{4}$ to ________.

(3-3) 

The direction number $\overleftarrow{2}$ goes from $\overleftarrow{3}$ to ________.

(4-1) 

The direction number $\overrightarrow{5}$ goes from ________ to $\overrightarrow{3}$.

(4-2) 

The direction number $\overleftarrow{2}$ goes from ________ to $\overleftarrow{1}$.

(4-3) 

The direction number $\overrightarrow{2}$ goes from ________ to $\overleftarrow{2}$.

PROBLEM SET 2 FOR GROUP WORK

Use the number line to find the missing number in **each number** sentence.

(5-1) $\overleftarrow{5}\ \overleftarrow{4}\ \overleftarrow{3}\ \overleftarrow{2}\ \overleftarrow{1}\ 0\ \overrightarrow{1}\ \overrightarrow{2}\ \overrightarrow{3}\ \overrightarrow{4}\ \overrightarrow{5}\ \overrightarrow{6}\ \overrightarrow{7}$

$$\overrightarrow{2} + \overrightarrow{3} = \square$$

(5-2) $\overleftarrow{6}\ \overleftarrow{5}\ \overleftarrow{4}\ \overleftarrow{3}\ \overleftarrow{2}\ \overleftarrow{1}\ 0\ \overrightarrow{1}\ \overrightarrow{2}\ \overrightarrow{3}\ \overrightarrow{4}\ \overrightarrow{5}\ \overrightarrow{6}$

$$\overleftarrow{3} + \overleftarrow{3} = \square$$

(6-1) $\overleftarrow{6}\ \overleftarrow{5}\ \overleftarrow{4}\ \overleftarrow{3}\ \overleftarrow{2}\ \overleftarrow{1}\ 0\ \overrightarrow{1}\ \overrightarrow{2}\ \overrightarrow{3}\ \overrightarrow{4}\ \overrightarrow{5}\ \overrightarrow{6}$

$$\overrightarrow{4} + \overleftarrow{1} = \square$$

(6-2) $\overleftarrow{6}\ \overleftarrow{5}\ \overleftarrow{4}\ \overleftarrow{3}\ \overleftarrow{2}\ \overleftarrow{1}\ 0\ \overrightarrow{1}\ \overrightarrow{2}\ \overrightarrow{3}\ \overrightarrow{4}\ \overrightarrow{5}\ \overrightarrow{6}$

$$\overleftarrow{3} + \overrightarrow{2} = \square$$

(7-1) $\overleftarrow{6}\ \overleftarrow{5}\ \overleftarrow{4}\ \overleftarrow{3}\ \overleftarrow{2}\ \overleftarrow{1}\ 0\ \overrightarrow{1}\ \overrightarrow{2}\ \overrightarrow{3}\ \overrightarrow{4}\ \overrightarrow{5}\ \overrightarrow{6}$

$$\overrightarrow{4} + \overleftarrow{4} = \square$$

(7-2) $\overleftarrow{6}\ \overleftarrow{5}\ \overleftarrow{4}\ \overleftarrow{3}\ \overleftarrow{2}\ \overleftarrow{1}\ 0\ \overrightarrow{1}\ \overrightarrow{2}\ \overrightarrow{3}\ \overrightarrow{4}\ \overrightarrow{5}\ \overrightarrow{6}$

$$\overrightarrow{2} + \square = 0$$

(7-3) $\overleftarrow{6}\quad \overleftarrow{5}\quad \overleftarrow{4}\quad \overleftarrow{3}\quad \overleftarrow{2}\quad \overleftarrow{1}\quad 0\quad \overrightarrow{1}\quad \overrightarrow{2}\quad \overrightarrow{3}\quad \overrightarrow{4}\quad \overrightarrow{5}\quad \overrightarrow{6}$

$$\boxed{} + \overleftarrow{3} = 0$$

(8-1) $\overleftarrow{6}\quad \overleftarrow{5}\quad \overleftarrow{4}\quad \overleftarrow{3}\quad \overleftarrow{2}\quad \overleftarrow{1}\quad 0\quad \overrightarrow{1}\quad \overrightarrow{2}\quad \overrightarrow{3}\quad \overrightarrow{4}\quad \overrightarrow{5}\quad \overrightarrow{6}$

$$\overrightarrow{1} + \boxed{} = \overrightarrow{4}$$

(8-2) $\overleftarrow{6}\quad \overleftarrow{5}\quad \overleftarrow{4}\quad \overleftarrow{3}\quad \overleftarrow{2}\quad \overleftarrow{1}\quad 0\quad \overrightarrow{1}\quad \overrightarrow{2}\quad \overrightarrow{3}\quad \overrightarrow{4}\quad \overrightarrow{5}\quad \overrightarrow{6}$

$$\boxed{} + \overleftarrow{2} = \overrightarrow{1}$$

PROBLEM SET 2 FOR INDIVIDUAL WORK

Use the number line to find the missing number in each number sentence.

(5-1) $\overleftarrow{5}\quad \overleftarrow{4}\quad \overleftarrow{3}\quad \overleftarrow{2}\quad \overleftarrow{1}\quad 0\quad \overrightarrow{1}\quad \overrightarrow{2}\quad \overrightarrow{3}\quad \overrightarrow{4}\quad \overrightarrow{5}\quad \overrightarrow{6}\quad \overrightarrow{7}$

$$\overrightarrow{3} + \overrightarrow{4} = \square$$

(5-2) $\overleftarrow{6}\quad \overleftarrow{5}\quad \overleftarrow{4}\quad \overleftarrow{3}\quad \overleftarrow{2}\quad \overleftarrow{1}\quad 0\quad \overrightarrow{1}\quad \overrightarrow{2}\quad \overrightarrow{3}\quad \overrightarrow{4}\quad \overrightarrow{5}\quad \overrightarrow{6}$

$$\overleftarrow{2} + \overleftarrow{4} = \square$$

(6-1) $\overleftarrow{6}\quad \overleftarrow{5}\quad \overleftarrow{4}\quad \overleftarrow{3}\quad \overleftarrow{2}\quad \overleftarrow{1}\quad 0\quad \overrightarrow{1}\quad \overrightarrow{2}\quad \overrightarrow{3}\quad \overrightarrow{4}\quad \overrightarrow{5}\quad \overrightarrow{6}$

$$\overrightarrow{3} + \overleftarrow{1} = \square$$

(6-2) $\overleftarrow{6}\quad \overleftarrow{5}\quad \overleftarrow{4}\quad \overleftarrow{3}\quad \overleftarrow{2}\quad \overleftarrow{1}\quad 0\quad \overrightarrow{1}\quad \overrightarrow{2}\quad \overrightarrow{3}\quad \overrightarrow{4}\quad \overrightarrow{5}\quad \overrightarrow{6}$

$$\overleftarrow{4} + \overrightarrow{2} = \square$$

(7-1) $\overleftarrow{6}\quad \overleftarrow{5}\quad \overleftarrow{4}\quad \overleftarrow{3}\quad \overleftarrow{2}\quad \overleftarrow{1}\quad 0\quad \overrightarrow{1}\quad \overrightarrow{2}\quad \overrightarrow{3}\quad \overrightarrow{4}\quad \overrightarrow{5}\quad \overrightarrow{6}$

$$\overrightarrow{3} + \overleftarrow{3} = \square$$

(7-2) $\overleftarrow{6}\quad \overleftarrow{5}\quad \overleftarrow{4}\quad \overleftarrow{3}\quad \overleftarrow{2}\quad \overleftarrow{1}\quad 0\quad \overrightarrow{1}\quad \overrightarrow{2}\quad \overrightarrow{3}\quad \overrightarrow{4}\quad \overrightarrow{5}\quad \overrightarrow{6}$

$$\overrightarrow{4} + \square = 0$$

(7-3) $\overleftarrow{6}\quad \overleftarrow{5}\quad \overleftarrow{4}\quad \overleftarrow{3}\quad \overleftarrow{2}\quad \overleftarrow{1}\quad 0\quad \overrightarrow{1}\quad \overrightarrow{2}\quad \overrightarrow{3}\quad \overrightarrow{4}\quad \overrightarrow{5}\quad \overrightarrow{6}$

$$\square + \overleftarrow{2} = 0$$

(8-1) $\overleftarrow{6}\quad \overleftarrow{5}\quad \overleftarrow{4}\quad \overleftarrow{3}\quad \overleftarrow{2}\quad \overleftarrow{1}\quad 0\quad \overrightarrow{1}\quad \overrightarrow{2}\quad \overrightarrow{3}\quad \overrightarrow{4}\quad \overrightarrow{5}\quad \overrightarrow{6}$

$$\overrightarrow{2} + \square = \overleftarrow{3}$$

(8-2) $\overleftarrow{6}\quad \overleftarrow{5}\quad \overleftarrow{4}\quad \overleftarrow{3}\quad \overleftarrow{2}\quad \overleftarrow{1}\quad 0\quad \overrightarrow{1}\quad \overrightarrow{2}\quad \overrightarrow{3}\quad \overrightarrow{4}\quad \overrightarrow{5}\quad \overrightarrow{6}$

$$\square + \overrightarrow{3} = \overleftarrow{1}$$

PROBLEM SET 3 FOR GROUP WORK

Rewrite each of the following subtraction number sentences as an addition number sentence.

(9-1) $\overset{\rightarrow}{5} - \overset{\rightarrow}{3} = \overset{\rightarrow}{2}$

(9-2) $\overset{\rightarrow}{2} - \overset{\leftarrow}{2} = \square$

(9-3) $\overset{\leftarrow}{2} - \square = \overset{\rightarrow}{3}$

(9-4) $\square - \overset{\leftarrow}{1} = \overset{\leftarrow}{4}$

Rewrite each of the following subtraction number sentences as an addition number sentence. Then use the number line to find the missing number.

(10-1) $\overset{\leftarrow}{6}\quad \overset{\leftarrow}{5}\quad \overset{\leftarrow}{4}\quad \overset{\leftarrow}{3}\quad \overset{\leftarrow}{2}\quad \overset{\leftarrow}{1}\quad 0\quad \overset{\rightarrow}{1}\quad \overset{\rightarrow}{2}\quad \overset{\rightarrow}{3}\quad \overset{\rightarrow}{4}\quad \overset{\rightarrow}{5}\quad \overset{\rightarrow}{6}$

$\overset{\leftarrow}{1} - \overset{\leftarrow}{4} = \square$

(10-2) $\overset{\leftarrow}{6}\quad \overset{\leftarrow}{5}\quad \overset{\leftarrow}{4}\quad \overset{\leftarrow}{3}\quad \overset{\leftarrow}{2}\quad \overset{\leftarrow}{1}\quad 0\quad \overset{\rightarrow}{1}\quad \overset{\rightarrow}{2}\quad \overset{\rightarrow}{3}\quad \overset{\rightarrow}{4}\quad \overset{\rightarrow}{5}\quad \overset{\rightarrow}{6}$

$\overset{\leftarrow}{1} - \overset{\rightarrow}{3} = \square$

(10-3) $\overset{\leftarrow}{6}\quad \overset{\leftarrow}{5}\quad \overset{\leftarrow}{4}\quad \overset{\leftarrow}{3}\quad \overset{\leftarrow}{2}\quad \overset{\leftarrow}{1}\quad 0\quad \overset{\rightarrow}{1}\quad \overset{\rightarrow}{2}\quad \overset{\rightarrow}{3}\quad \overset{\rightarrow}{4}\quad \overset{\rightarrow}{5}\quad \overset{\rightarrow}{6}$

$\overset{\leftarrow}{5} - \square = \overset{\leftarrow}{3}$

(10-4) $\overleftarrow{6} \quad \overleftarrow{5} \quad \overleftarrow{4} \quad \overleftarrow{3} \quad \overleftarrow{2} \quad \overleftarrow{1} \quad 0 \quad \overrightarrow{1} \quad \overrightarrow{2} \quad \overrightarrow{3} \quad \overrightarrow{4} \quad \overrightarrow{5} \quad \overrightarrow{6}$

$$\square - \overrightarrow{5} = \overrightarrow{1}$$

PROBLEM SET 3 FOR INDIVIDUAL WORK

Rewrite each of the following subtraction number sentences as an addition number sentence.

(9-1) $\overleftarrow{4} - \overleftarrow{3} = \overleftarrow{1}$

(9-2) $\overrightarrow{4} - \overleftarrow{2} = \square$

(9-3) $\overleftarrow{1} - \square = \overleftarrow{3}$

(9-4) $\square - \overrightarrow{4} = \overleftarrow{1}$

Rewrite each of the following subtraction number sentences as an addition number sentence. Then use the number line to find the missing number.

(10-1) $\overleftarrow{6}\quad \overleftarrow{5}\quad \overleftarrow{4}\quad \overleftarrow{3}\quad \overleftarrow{2}\quad \overleftarrow{1}\quad 0\quad \overrightarrow{1}\quad \overrightarrow{2}\quad \overrightarrow{3}\quad \overrightarrow{4}\quad \overrightarrow{5}\quad \overrightarrow{6}$

$\overrightarrow{3} - \overleftarrow{1} = \square$

(10-2) $\overleftarrow{6}\quad \overleftarrow{5}\quad \overleftarrow{4}\quad \overleftarrow{3}\quad \overleftarrow{2}\quad \overleftarrow{1}\quad 0\quad \overrightarrow{1}\quad \overrightarrow{2}\quad \overrightarrow{3}\quad \overrightarrow{4}\quad \overrightarrow{5}\quad \overrightarrow{6}$

$\overrightarrow{2} - \overleftarrow{2} = \square$

(10-3) $\overleftarrow{6}\quad \overleftarrow{5}\quad \overleftarrow{4}\quad \overleftarrow{3}\quad \overleftarrow{2}\quad \overleftarrow{1}\quad 0\quad \overrightarrow{1}\quad \overrightarrow{2}\quad \overrightarrow{3}\quad \overrightarrow{4}\quad \overrightarrow{5}\quad \overrightarrow{6}$

$\overrightarrow{4} - \square = \overleftarrow{2}$

(10-4) 

$$\boxed{} - \overleftarrow{3} = \overrightarrow{4}$$

PROBLEM SET 4 FOR GROUP WORK

Place a direction number in each of the following spaces to complete the number sentence.

(11-1) $\vec{2} \times \vec{4} = \square$

(11-2) $\vec{3} \times \square = \vec{6}$

(11-3) $\square \times \vec{2} = \vec{8}$

(12-1) $\vec{4} \times 0 = \square$

(12-2) $\vec{5} \times \square = 0$

(13-1) $\vec{3} \times (\vec{2} + \vec{4}) = \square \times \vec{2} + \square \times \vec{4}$

(13-2) $\vec{4} \times (\vec{5} + \vec{1}) = \vec{4} \times \square + \vec{4} \times \square$

(13-3) $\vec{2} \times (\square \times \vec{3}) = \vec{2} \times \vec{4} + \vec{2} \times \vec{3}$

(13-4) $\square \times (\vec{4} + \vec{2}) = \vec{3} \times \vec{4} + \vec{3} \times \vec{2}$

(13-5) $\square \times (\vec{3} + \square) = \vec{2} \times \square + \square \times \vec{5}$

PROBLEM SET 4 FOR INDIVIDUAL WORK

Place a direction number in each of the following spaces to complete the number sentences.

(11-1) $\vec{2} \times \vec{3} = \square$

(11-2) $\vec{2} \times \square = \vec{4}$

(11-3) $\square \times \vec{3} = \vec{9}$

(12-1) $\vec{5} \times 0 = \square$

(12-2) $\vec{3} \times \square = 0$

(13-1) $\vec{2} \times (\vec{3} + \vec{4}) = (\square \times \vec{3}) + (\square \times \vec{4})$

(13-2) $\vec{3} \times (\vec{2} + \vec{4}) = (\vec{3} \times \square) + (\vec{3} \times \square)$

(13-3) $\vec{4} \times (\square + \vec{3}) = (\vec{4} \times \vec{5}) + (\vec{4} \times \vec{3})$

(13-4) $\square \times (\vec{3} + \vec{5}) = (\vec{2} \times \vec{3}) + (\vec{2} \times \vec{5})$

(13-5) $\square \times (\vec{4} + \square) = (\vec{2} \times \square) + (\square \times \vec{3})$

PROBLEM SET 5 FOR GROUP WORK

Place a direction number in each of the following spaces to complete the number sentences.

(14-1) What is $\overrightarrow{2} \times \overleftarrow{3}$?

$$\overrightarrow{2} \times (\overrightarrow{3} + \overleftarrow{3}) = \overrightarrow{2} \times \overrightarrow{3} + \overrightarrow{2} \times \overleftarrow{3}$$

$$\overrightarrow{2} \times \square = \square + \overrightarrow{2} \times \overleftarrow{3}$$

$$\square = \square + \overrightarrow{2} \times \overleftarrow{3}$$

$$\square = \square + \square$$

(14-2) What is $\overrightarrow{4} \times \overleftarrow{2}$?

$$\overrightarrow{4} \times (\overrightarrow{2} + \overleftarrow{2}) = \overrightarrow{4} \times \overrightarrow{2} + \overrightarrow{4} \times \overleftarrow{2}$$

$$\overrightarrow{4} \times \square = \square + \overrightarrow{4} \times \overleftarrow{2}$$

$$\square = \square + \overrightarrow{4} \times \overleftarrow{2}$$

$$\square = \square + \square$$

Work each of the following problems using the rule you have discovered.

(15-1) $\overrightarrow{1} \times \overleftarrow{4} = \square$

(15-2) $\overrightarrow{2} \times \overleftarrow{2} = \square$

(15-3) $\overrightarrow{3} \times \overleftarrow{2} = \square$

PROBLEM SET 5 FOR INDIVIDUAL WORK

Place a direction number in each of the following spaces to complete the number sentence.

(14-1) $\overrightarrow{2} \times (\overrightarrow{1} + \overleftarrow{1}) = \overrightarrow{2} \times \overrightarrow{1} + \overrightarrow{2} \times \overleftarrow{1}$

$\overrightarrow{2} \times \square = \square + \overrightarrow{2} \times \overleftarrow{1}$

$\square = \square + \overrightarrow{2} \times \overleftarrow{1}$

$\square = \square + \square$

Work each of the following problems using the rule you have discovered.

(15-1) $\overrightarrow{3} \times \overleftarrow{3} = \square$

(15-2) $\overrightarrow{3} \times \overleftarrow{1} = \square$

(15-3) $\overrightarrow{3} \times \overleftarrow{4} = \square$

(15-4) $\overrightarrow{2} \times \overleftarrow{5} = \square$

PROBLEM SET 6 FOR GROUP WORK

Place a direction number in each of the following spaces to complete the number sentence.

(16-1) What is $\overleftarrow{3} \times \overleftarrow{2}$?

$$\overleftarrow{3} \times (\overrightarrow{2} + \overleftarrow{2}) = \overleftarrow{3} \times \overrightarrow{2} + \overleftarrow{3} \times \overleftarrow{2}$$

$$\overleftarrow{3} \times \square = \square + \overleftarrow{3} \times \overleftarrow{2}$$

$$\square = \square + \overleftarrow{3} \times \overleftarrow{2}$$

$$\square = \square + \square$$

(16-2)

$$\overleftarrow{2} \times (\overrightarrow{2} + \overleftarrow{2}) = \overleftarrow{2} \times \overrightarrow{2} + \overleftarrow{2} \times \overleftarrow{2}$$

$$\overleftarrow{2} \times \square = \square + \overleftarrow{2} \times \overleftarrow{2}$$

$$\square = \square + \overleftarrow{2} \times \overleftarrow{2}$$

$$\square = \square + \square$$

Work each of the following problems using the rule you have discovered.

(17-1) $\overleftarrow{2} \times \overleftarrow{4} = \square$

(14-2) $\overleftarrow{3} \times \overleftarrow{1} = \square$

(17-3) $\overleftarrow{2} \times \overleftarrow{5} = \boxed{}$

PROBLEM SET 6 FOR INDIVIDUAL WORK

Place a direction number in each of the following spaces to complete the number sentences.

(16-1) What is $\overleftarrow{2} \times \overleftarrow{4}$?

$$\overleftarrow{2} \times (\overrightarrow{4} + \overleftarrow{4}) = \overleftarrow{2} \times \overrightarrow{4} + \overleftarrow{2} \times \overleftarrow{4}$$

$$\overleftarrow{2} \times \boxed{} = \boxed{} + \overleftarrow{2} \times \overleftarrow{4}$$

$$\boxed{} = \boxed{} + \overleftarrow{2} \times \overleftarrow{4}$$

$$\boxed{} = \boxed{} + \boxed{}$$

Work each of the following problems using the rule you have discovered.

(17-1) $\overleftarrow{3} \times \overleftarrow{3} = \boxed{}$

(17-2) $\overleftarrow{2} \times \overleftarrow{3} = \boxed{}$

(17-3) $\overleftarrow{4} \times \overleftarrow{2} = \boxed{}$

(17-4) $\overleftarrow{1} \times \overleftarrow{4} = \boxed{}$

POST TRAINING WORK

Use the number line to find the missing direction number in each number sentence.

(5-1) $\overleftarrow{6}\ \overleftarrow{5}\ \overleftarrow{4}\ \overleftarrow{3}\ \overleftarrow{2}\ \overleftarrow{1}\ 0\ \overrightarrow{1}\ \overrightarrow{2}\ \overrightarrow{3}\ \overrightarrow{4}\ \overrightarrow{5}\ \overrightarrow{6}$

$$\overrightarrow{3} + \overrightarrow{4} = \square$$

(6-1) $\overleftarrow{6}\ \overleftarrow{5}\ \overleftarrow{4}\ \overleftarrow{3}\ \overleftarrow{2}\ \overleftarrow{1}\ 0\ \overrightarrow{1}\ \overrightarrow{2}\ \overrightarrow{3}\ \overrightarrow{4}\ \overrightarrow{5}\ \overrightarrow{6}$

$$\overrightarrow{3} + \overleftarrow{1} = \square$$

(7-1) $\overleftarrow{6}\ \overleftarrow{5}\ \overleftarrow{4}\ \overleftarrow{3}\ \overleftarrow{2}\ \overleftarrow{1}\ 0\ \overrightarrow{1}\ \overrightarrow{2}\ \overrightarrow{3}\ \overrightarrow{4}\ \overrightarrow{5}\ \overrightarrow{6}$

$$\overrightarrow{3} + \overleftarrow{3} = \square$$

(8-1) $\overleftarrow{6}\ \overleftarrow{5}\ \overleftarrow{4}\ \overleftarrow{3}\ \overleftarrow{2}\ \overleftarrow{1}\ 0\ \overrightarrow{1}\ \overrightarrow{2}\ \overrightarrow{3}\ \overrightarrow{4}\ \overrightarrow{5}\ \overrightarrow{6}$

$$\overrightarrow{2} + \square = \overleftarrow{3}$$

Rewrite each of the following subtraction number sentences as an addition sentence. Then use the number line to find the missing number.

(10-1) $\overleftarrow{6}\ \overleftarrow{5}\ \overleftarrow{4}\ \overleftarrow{3}\ \overleftarrow{2}\ \overleftarrow{1}\ 0\ \overrightarrow{1}\ \overrightarrow{2}\ \overrightarrow{3}\ \overrightarrow{4}\ \overrightarrow{5}\ \overrightarrow{6}$

$$\overrightarrow{3} - \overleftarrow{2} = \square$$

(10-2) $\overleftarrow{6}\ \overleftarrow{5}\ \overleftarrow{4}\ \overleftarrow{3}\ \overleftarrow{2}\ \overleftarrow{1}\ 0\ \overrightarrow{1}\ \overrightarrow{2}\ \overrightarrow{3}\ \overrightarrow{4}\ \overrightarrow{5}\ \overrightarrow{6}$

$$\overrightarrow{4} - \square = \overleftarrow{2}$$

Write the missing direction number in each of the following spaces to complete the number sentence.

(11-1) $\overrightarrow{2} \times \overrightarrow{3} = \square$

(11-2) $\overrightarrow{2} \times \square = \overrightarrow{4}$

(12-1) $\overrightarrow{5} \times 0 = \square$

(15-3) $\overleftarrow{3} \times \overrightarrow{2} = \square$

(17-1) $\overleftarrow{2} \times \overleftarrow{4} = \square$

APPENDIX B

PLANNED COMPARISONS COMPUTATIONAL PROCEDURES

The data, for each task, were pooled into 24 classifications by grade, teaching method, sex, and I.Q. for the planned comparison analysis. Each group was identified by a four space code. The first space identified the grade (4: fourth, 5: fifth, and 6: sixth), the second space identified the teaching method (D: discovery and I: instructional), the third space identified the sex (M: male and F: female), and the fourth space identified I.Q. level (H: high I.Q. and L: low I.Q.). Thus the code 5DML would identify those subjects who were fifth graders, taught by the discovery method, boys, and with low I.Q. measures. In the computational procedures described in this section, the groups are identified by subscripts, i, $i = 1,\ldots,24$. The identification codes and their corresponding subscripts are found in Table B.1.

Table B.1

SUBSCRIPT CODING FOR IDENTIFICATION OF CLASSIFICATION GROUPS USED IN PLANNED COMPARISON COMPUTATIONAL PROCEDURES

i	Code	i	Code	i	Code
1	4DMH	9	5DMH	17	6DMH
2	4DML	10	5DML	18	6DML
3	4DFH	11	5DFH	19	6DFH
4	4DFL	12	5DFL	20	6DFL
5	4IMH	13	5IMH	21	6IMH
6	4IML	14	5IML	22	6IML
7	4IFH	15	5IFH	23	6IFH
8	4IFL	16	5IFL	24	6IFL

For each group i, the mean (m_i), variance (S_i) and group size (N_i) was calculated. These statistics were then used to compute the following values used in the analysis of variance tables.

<u>The degrees of freedom for within mean squares</u>

(1) $$DF = \left[\sum_{i=1}^{24} N_i \right] - 24$$

<u>The overall mean</u>

(2) $$M = \sum_{i=1}^{24} N_i \; m_i$$

<u>The between sum of squares</u>

(3) $$SB = \sum_{i=1}^{24} N_i (m_i - M)^2$$

<u>The between mean square</u>

(4) $$MB = \frac{SB}{23}$$

<u>The within sum of squares</u>

(5) $$SW = \sum_{i=1}^{24} (N_i - 1) \; S_i^{\,2}$$

The within mean square

(6) $$MW = \frac{SW}{DF}$$

The F-ratio of mean square

(7) $$F = \frac{MB}{MW}$$

Sample size and overall mean for grade four

(8) $$N1 = \sum_{i=1}^{8} N_i$$

(9) $$M1 = \sum_{i=1}^{8} \frac{N_i m_i}{N1}$$

Sample size and overall mean for grade five

(10) $$N2 = \sum_{i=9}^{16} N_i$$

(11) $$M2 = \sum_{i=9}^{16} \frac{N_i m_i}{N2}$$

Sample size and overall mean for grade six

(12) $$N3 = \sum_{i=17}^{24} N_i$$

(13) $$M3 = \sum_{i=17}^{24} \frac{N_i m_i}{N3}$$

Sample size and overall mean for discovery group

(14) $$N4 = \sum_{i} N_i$$

(15) $$M4 = \sum_{i} \frac{N_i m_i}{N4}$$

$i = 1,2,3,4,9,10,11,12,17,18,19,20$

Sample size and overall mean for instruction group

(16) $$N5 = \sum_{i} N_i$$

(17) $$M5 = \sum_{i} \frac{N_i m_i}{N5}$$

$i = 5,6,7,8,13,14,15,16,21,22,23,24$

Sample size and overall mean for boys

(18) $$N6 = \sum_{i} N_i$$

(19) $$M6 = \sum_{i} \frac{N_i m_i}{N6}$$

$i = 1,2,5,6,9,10,13,14,17,18,21,22$

Sample size and overall mean for girls

(20) $$N7 = \sum_{i} N_i$$

(21) $$M7 = \sum_{i} \frac{N_i m_i}{N7}$$

$$i = 3,4,7,8,11,12,15,16,19,20,23,24$$

Sample size and overall mean for high I.Q. subjects

(22) $$N8 = \sum_{i=1}^{12} N_{2i-1}$$

(23) $$M8 = \sum_{i=1}^{12} \frac{N_{2i-1} m_{2i-1}}{N8}$$

Sample size and overall mean for low I.Q.

(24) $$N9 = \sum_{i=1}^{12} N_{2i}$$

(25) $$M9 = \sum_{i=1}^{12} \frac{N_{2i} m_{2i}}{N9}$$

Difference of means and t-value for grade 4 - 5 comparison

(26) $$M45 = M1 - M2$$

(27) $$T45 = \frac{M45}{\sqrt{MW \left(\frac{1}{N1} + \frac{1}{N2}\right)}}$$

Difference of means and t-value for grade 5 - 6 comparison

(28) $$M56 = M2 - M3$$

(29) $$T56 = \frac{M56}{\sqrt{MW \left(\frac{1}{N2} + \frac{1}{N3}\right)}}$$

Difference of means and t-value for discovery-instructional comparisons

(30) $$MDI = N4 - N5$$

(31) $$TDI = \frac{MDI}{\sqrt{MW \left(\frac{1}{N4} + \frac{1}{N5}\right)}}$$

Difference of means and t-value for boy-girl comparisons

(32) $$MMF = M6 - M7$$

(33) $$TMF = \frac{MMF}{\sqrt{MW \left(\frac{1}{N6} + \frac{1}{N7}\right)}}$$

Difference of means and t-value for high-low I.Q. comparisons

(34) $$MHL = M8 - M9$$

(35) $$THL = \frac{MHL}{\sqrt{MW \left(\frac{1}{N8} + \frac{1}{N9}\right)}}$$

<u>Difference of means and t-value for instructional method and boy-girl interactions</u>

(36) $$MGS = MDI - MMF$$

(37) $$TGS = \frac{MGS}{\sqrt{MW \left(\frac{1}{N4} + \frac{1}{N5} + \frac{1}{N6} + \frac{1}{N7}\right)}}$$

<u>Difference of means and t-value for I.Q. and boy-girl interactions</u>

(38) $$MSIQ = MMF - MHL$$

(39) $$TSIQ = \frac{MSIQ}{\sqrt{MW \left(\frac{1}{N6} + \frac{1}{N7} + \frac{1}{N8} + \frac{1}{N9}\right)}}$$

<u>Difference of means and t-value for instructional method and I.Q. interactions</u>

(40) $$MGIQ = MDI - MHL$$

(41) $$TGIQ = \frac{MGIQ}{\sqrt{MW \left(\frac{1}{N4} + \frac{1}{N5} + \frac{1}{N8} + \frac{1}{N9}\right)}}$$

<u>Difference of means and t-value for grade 4 - 5 and instructional method interactions</u>

(42) $$M45G = M45 - MDF$$

(43) $$T45G = \frac{M45G}{\sqrt{MW \left(\frac{1}{N1} + \frac{1}{N2} + \frac{1}{N4} + \frac{1}{N5}\right)}}$$

<u>Difference of means and t-value for grade 5 - 6 and instructional method interactions</u>

(44) $$M56G = M56 - MDI$$

(45) $$T56G = \frac{M56G}{\sqrt{MW \left(\frac{1}{N2} + \frac{1}{N3} + \frac{1}{N4} + \frac{1}{N5}\right)}}$$

<u>Difference of means and t-value for grade 4 - 5 and boy-girl interactions</u>

(46) $$M45S = M45 - MMF$$

(47) $$T45S = \frac{M45S}{\sqrt{MW \left(\frac{1}{N1} + \frac{1}{N2} + \frac{1}{N6} + \frac{1}{N7}\right)}}$$

<u>Difference of means and t-value for grade 5 - 6 and boy-girl interactions</u>

(48) $$M56S = M56 - MMF$$

(49) $$T56S = \frac{M56S}{\sqrt{MW \left(\frac{1}{N2} + \frac{1}{N3} + \frac{1}{N6} + \frac{1}{N7}\right)}}$$

<u>Difference of means and t-value for grade 4 - 5 and I.Q. interactions</u>

(50) $$M45IQ = M45 - MHL$$

(51) $$T45IQ = \frac{M45IQ}{\sqrt{MW \left(\frac{1}{N1} + \frac{1}{N2} + \frac{1}{N8} + \frac{1}{N9}\right)}}$$

Difference of means and t-value for grade 5 - 6 and I.Q. interactions

(52) $$M56IQ = M56 - MHL$$

(53) $$T56IQ = \frac{M56IQ}{\sqrt{MW \left(\frac{1}{N2} + \frac{1}{N3} + \frac{1}{N8} + \frac{1}{N9}\right)}}$$

APPENDIX C

TABLES

Table C.1

PLANNED COMPARISONS ANALYSIS OF VARIANCE FOR TASK 1

Comparison	Mean Difference	t-value	Significant
Grade 4-5	.207	7.462	*
Grade 5-6	-.037	-1.360	
Treatment	.003	.150	
Sex	-.032	-1.431	
I.Q.	.015	.671	
Interactions			
Group-Sex	.035	1.119	
I.Q.-Sex	-.047	-1.486	
Group-I.Q.	-.011	-.367	
Grade(4,5)-Treatment	.203	5.698	*
Grade(5,6)-Treatment	-.041	-1.149	
Grade(4,5)-Sex	.239	6.695	*
Grade(5,6)-Sex	-.005	-.150	
Grade(4,5)-I.Q.	.192	5.370	*
Grade(5,6)-I.Q.	-.052	-1.478	

Table C.2

PLANNED COMPARISONS ANALYSIS OF VARIANCE FOR TASK 2

Comparison	Mean Difference	t-value	Significant
Grade 4-5	-.169	- .679	
Grade 5-6	-.053	- .213	
Treatment	-.843	-4.159	*
Sex	-.348	-1.718	
I.Q.	1.140	5.619	*
Interactions			
Group-Sex	-.495	-1.725	
I.Q.-Sex	-1.488	-5.188	*
Group-I.Q.	-1.984	-6.914	*
Grade(4,5)-Treatment	.674	2.094	*
Grade(5,6)-Treatment	.790	2.461	*
Grade(4,5)-Sex	.178	.555	
Grade(5,6)-Sex	.295	.919	
Grade(4,5)-I.Q.	-1.309	-4.069	*
Grade(5,6)-I.Q.	-1.193	-3.714	*

Table C.3

PLANNED COMPARISONS ANALYSIS OF VARIANCE FOR TASK 3

Comparison	Mean Difference	t-value	Significant
Grade 4-5	-.013	-.103	
Grade 5-6	.195	1.473	
Treatment	-.685	-6.353	*
Sex	-.023	-.213	
I.Q.	.259	2.399	*
Interactions			
Group-Sex	-.662	-4.341	*
I.Q.-Sex	-.282	-1.847	
Group-I.Q.	-.944	-6.189	*
Grade(4,5)-Treatment	.672	3.927	*
Grade(5,6)-Treatment	.881	5.151	*
Grade(4,5)-Sex	.009	.054	
Grade(5,6)-Sex	.218	1.278	
Grade(4,5)-I.Q.	-.272	-1.593	
Grade(5,6)-I.Q.	-.063	-.369	

Table C.4

PLANNED COMPARISONS ANALYSIS OF VARIANCE FOR TASK 4

Comparison	Mean Difference	t-value	Significant
Grade 4-5	-.307	-2.655	*
Grade 5-6	.257	2.211	*
Treatment	-.563	-5.962	*
Sex	-.156	-1.652	
I.Q.	.641	6.789	*
Interactions			
Group-Sex	-.407	-3.047	*
I.Q.-Sex	-.797	-5.969	*
Group-I.Q.	-1.204	-9.016	*
Grade(4,5)-Treatment	.255	1.713	
Grade(5,6)-Treatment	.820	5.475	*
Grade(4,5)-Sex	-.151	-1.012	
Grade(5,6)-Sex	.413	2.758	*
Grade(4,5)-I.Q.	-.948	-6.351	*
Grade(5,6)-I.Q.	-.384	-2.565	*

Table C.5

PLANNED COMPARISONS ANALYSIS OF VARIANCE FOR TASK 5

Comparison	Mean Difference	t-value	Significant
Grade 4-5	.238	3.539	*
Grade 5-6	-.113	-1.727	
Treatment	-.293	-5.487	*
Sex	.055	1.046	
I.Q.	.252	4.723	*
Interactions			
Group-Sex	-.349	-4.619	*
I.Q.-Sex	-.196	-2.598	*
Group-I.Q.	-.545	-7.220	*
Grade (4,5)-Treatment	.531	6.289	*
Grade (5,6)-Treatment	.180	2.128	*
Grade (4,5)-Sex	.182	2.155	*
Grade (5,6)-Sex	-.169	-2.000	*
Grade (4,5)-I.Q.	-.014	-.168	
Grade (5,6)-I.Q.	-.365	-4.324	*

Table C.6

PLANNED COMPARISONS ANALYSIS OF VARIANCE FOR TASK 6

Comparison	Mean Difference	t-value	Significant
Grade 4-5	-.069	-.992	
Grade 5-6	-.036	-.522	
Treatment	-.151	-2.657	*
Sex	.020	.355	
I.Q.	.411	7.198	
Interactions			
Group-Sex	-.172	-2.129	*
I.Q.-Sex	-.390	-4.837	*
Group-I.Q.	-.563	-6.969	*
Grade (4,5)-Treatment	.082	.912	
Grade (5,6)-Treatment	.115	1.273	
Grade (4,5)-Sex	-.089	-.993	
Grade (5,6)-Sex	-.056	-.629	
Grade (4,5)-I.Q.	-.480	-5.321	*
Grade (5,6)-I.Q.	-.447	-4.953	*

Table C.7

PLANNED COMPARISONS ANALYSIS OF VARIANCE FOR TASK 7

Comparison	Mean Difference	t-value	Significant
Grade 4-5	.014	.141	
Grade 5-6	-.004	-.040	
Treatment	-.382	-4.417	*
Sex	-.065	-.755	
I.Q.	.535	6.183	*
Interactions			
Group-Sex	-.317	-2.588	*
I.Q.-Sex	-.600	-4.905	*
Group-I.Q.	-.918	-7.496	*
Grade(4,5)-Treatment	.397	2.903	*
Grade(5,6)-Treatment	.378	2.759	*
Grade(4,5)-Sex	.080	.587	
Grade(5,6)-Sex	.061	.445	
Grade(4,5)-I.Q.	-.520	-3.801	*
Grade(5,6)-I.Q.	-.539	-3.938	*

Table C.8

PLANNED COMPARISONS ANALYSIS OF VARIANCE FOR TASK 8

Comparison	Mean Difference	t-value	Significant
Grade 4-5	-.210	-2.930	*
Grade 5-6	.060	.838	
Treatment	-.382	-6.524	*
Sex	-.109	-1.862	
I.Q.	.540	9.225	*
Interactions			
Group-Sex	-.273	-3.295	*
I.Q.-Sex	-.649	-7.839	*
Group-I.Q.	-.922	-11.136	*
Grade(4,5)-Treatment	.172	1.857	
Grade(5,6)-Treatment	.442	4.772	*
Grade(4,5)-Sex	-.101	-1.091	
Grade(5,6)-Sex	.169	1.826	
Grade(4,5)-I.Q.	-.750	-8.104	*
Grade(5,6)-I.Q.	-.480	-5.179	*

Table C.9

PLANNED COMPARISONS ANALYSIS OF VARIANCE FOR TASK 9

Comparison	Mean Difference	t-value	Significant
Grade 4-5	.155	.884	
Grade 5-6	.581	3.306	*
Treatment	-1.147	-8.017	*
Sex	-.158	-1.107	
I.Q.	.646	4.517	*
Interactions			
Group-Sex	-.989	-4.882	*
I.Q.-Sex	-.805	-3.976	*
Group-I.Q.	-1.794	-8.863	*
Grade (4,5)-Treatment	1.302	5.757	*
Grade (5,6)-Treatment	1.729	7.626	*
Grade (4,5)-Sex	.313	1.386	
Grade (5,6)-Sex	.740	3.262	*
Grade (4,5)-I.Q.	-.491	-2.172	*
Grade (5,6)-I.Q.	-.065	-.288	

Table C.10

PLANNED COMPARISONS ANALYSIS OF VARIANCE FOR TASK 10

Comparison	Mean Difference	t-value	Significant
Grade 4-5	.093	.453	
Grade 5-6	-.594	-2.855	*
Treatment	-.483	-2.855	*
Sex	-.164	-.970	
I.Q.	.656	3.874	*
Interactions			
Group-Sex	-.318	-1.331	
I.Q.-Sex	-.820	-3.424	*
Group-I.Q.	-1.139	-4.758	
Grade (4,5)-Treatment	.577	2.159	*
Grade (5,6)-Treatment	-.111	-.414	
Grade (4,5)-Sex	.258	.965	
Grade (5,6)-Sex	-.430	-1.601	
Grade (4,5)-I.Q.	-.562	-2.103	*
Grade (5,6)-I.Q.	-1.250	-4.660	*

Table C.11

PLANNED COMPARISONS ANALYSIS OF VARIANCE FOR TASK 11

Comparison	Mean Difference	t-value	Significant
Grade 4-5	.057	1.274	
Grade 5-6	-.100	-2.237	*
Treatment	0.000	-.015	
Sex	.010	.294	
I.Q.	.078	2.128	*
Interactions			
Group-Sex	-.011	-.219	
I.Q.-Sex	-.067	-1.296	
Group-I.Q.	-.078	-1.515	
Grade(4,5)-Treatment	.058	.999	
Grade(5,6)-Treatment	-.100	-1.722	
Grade(4,5)-Sex	.046	.804	
Grade(5,6)-Sex	-.111	-1.918	
Grade(4,5)-I.Q.	-.020	-.349	
Grade(5,6)-I.Q.	-.178	-3.078	

Table C.12

PLANNED COMPARISONS ANALYSIS OF VARIANCE FOR TASK 12

Comparison	Mean Difference	t-value	Significant
Grade 4-5	-.006	-.309	
Grade 5-6	-.007	-.397	
Treatment	-.015	-.934	
Sex	-.005	-.331	
I.Q.	.012	.795	
Interactions			
Group-Sex	-.009	-.426	
I.Q.-Sex	-.018	-.796	
Group-I.Q.	-.027	-1.222	
Grade(4,5)-Treatment	.008	.348	
Grade(5,6)-Treatment	.007	.282	
Grade(4,5)-Sex	0.000	-.031	
Grade(5,6)-Sex	-.002	-.098	
Grade(4,5)-I.Q.	-.019	-.740	
Grade(5,6)-I.Q.	-.020	-.810	

Table C.13

PLANNED COMPARISONS ANALYSIS OF VARIANCE FOR TASK 13

Comparison	Mean Difference	t-value	Significant
Grade 4-5	-.051	-.174	
Grade 5-6	-.095	-.326	
Treatment	-.777	-3.263	*
Sex	-.181	-.759	
I.Q.	.868	3.647	*
Interactions			
Group-Sex	-.596	-1.770	
I.Q.-Sex	-1.049	-3.116	*
Group-I.Q.	-1.646	-4.886	*
Grade(4,5)-Treatment	.726	1.919	
Grade(5,6)-Treatment	.682	1.812	
Grade(4,5)-Sex	.129	.342	
Grade(5,6)-Sex	.085	.228	
Grade(4,5)-I.Q.	-.919	-2.431	*
Grade(5,6)-I.Q.	-.963	-2.559	*

Table C.14

PLANNED COMPARISONS ANALYSIS OF VARIANCE FOR TASK 14

Comparison	Mean Difference	t-value	Significant
Grade 4-5	.250	5.603	*
Grade 5-6	-.212	-4.768	*
Treatment	-.124	-3.425	*
Sex	-.052	-1.433	
I.Q.	.180	4.973	*
Interactions			
Group-Sex	-.072	-1.407	
I.Q.-Sex	-.232	-4.530	*
Group-I.Q.	-.304	-5.939	*
Grade(4,5)-Treatment	.374	6.509	*
Grade(5,6)-Treatment	-.087	-1.527	
Grade(4,5)-Sex	.302	5.252	*
Grade(5,6)-Sex	-.159	-2.786	*
Grade(4,5)-I.Q.	.069	1.212	
Grade(5,6)-I.Q.	-.392	-6.839	*

Table C.15

PLANNED COMPARISONS ANALYSIS OF VARIANCE FOR TASK 15

Comparison	Mean Difference	t-value	Significant
Grade 4-5	.659	2.984	*
Grade 5-6	-1.143	-5.203	*
Treatment	-.584	-3.257	*
Sex	-.105	-.585	
I.Q.	.511	2.848	*
Interactions			
Group-Sex	-.479	-1.888	
I.Q.-Sex	-.616	-2.427	*
Group-I.Q.	-1.095	-4.317	*
Grade(4,5)-Treatment	1.243	4.370	*
Grade(5,6)-Treatment	-.559	-1.970	*
Grade(4,5)-Sex	.764	2.685	*
Grade(5,6)-Sex	-1.038	-3.659	*
Grade(4,5)-I.Q.	.148	.520	
Grade(5,6)-I.Q.	-1.655	-5.831	*

Table C.16

PLANNED COMPARISONS ANALYSIS OF VARIANCE FOR TASK 16

Comparison	Mean Difference	t-value	Significant
Grade 4-5	.024	.605	
Grade 5-6	-.286	-6.974	*
Treatment	-.023	-.701	
Sex	-.007	-.212	
I.Q.	.120	3.598	*
Interactions			
Group-Sex	-.016	-.345	
I.Q.-Sex	-.127	-2.694	*
Group-I.Q.	-.144	-3.040	*
Grade(4,5)-Treatment	.048	.912	
Grade(5,6)-Treatment	-.263	-4.962	*
Grade(4,5)-Sex	.031	.603	
Grade(5,6)-Sex	-.279	-5.270	*
Grade(4,5)-I.Q.	-.095	-1.807	
Grade(5,6)-I.Q.	-.407	-7.680	*

Table C.17

PLANNED COMPARISONS ANALYSIS OF VARIANCE FOR TASK 17

Comparison	Mean Difference	t-value	Significant
Grade 4-5	.143	.472	
Grade 5-6	-1.049	-3.450	*
Treatment	-.090	-.364	
Sex	-.131	-.528	
I.Q.	.567	2.287	*
Interactions			
Group-Sex	.040	.116	
I.Q.-Sex	-.698	-1.991	*
Group-I.Q.	-.657	-1.875	*
Grade(4,5)-Treatment	.233	.596	
Grade(5,6)-Treatment	-.959	-2.444	*
Grade(4,5)-Sex	.274	.700	
Grade(5,6)-Sex	-.918	-2.339	*
Grade(4,5)-I.Q.	-.423	-1.081	
Grade(5,6)-I.Q.	-1.616	-4.120	*

Table C.18

ANALYSIS OF VARIANCE FOR REPEATED MEASURES ON THE FACTOR OF SEX IN THE FOURTH GRADE

Source	SS	df	MS	F
Between Subjects		117		
Sex	1.737	1	1.737	
Subjects w groups	4,735.512	116	40.823	
Within Subjects		118		
Tests	927.334	1	927.334	85.816*
Sex x Tests	11.698	1	11.698	1.082
Tests x Subjects w groups	1,253.598	116	10.806	

*Significant

Table C.19

ANALYSIS OF VARIANCE FOR REPEATED MEASURES ON THE FACTOR OF SEX IN THE FIFTH GRADE

Source	SS	df	MS	F
Between Subjects		113		
Sex	5.652	1	5.652	
Subjects w groups	2,719.864	112	24.284	
Within Subjects		114		
Tests	1,294.603	1	1,294.603	63.585*
Sex x Tests	15.639	1	15.639	.768
Tests x Subjects w groups	2,280.332	112	20.360	

*Significant

Table C.20

ANALYSIS OF VARIANCE FOR REPEATED MEASURES ON THE FACTOR OF SEX IN THE SIXTH GRADE

Source	SS	df	MS	F
Between Subjects		108		
Sex	8.918	1	8.918	
Subjects w groups	3,892.789	107	36.381	
Within Subjects		109		
Tests	282.319	1	282.319	28.046*
Sex x Tests	.429	1	.429	.042
Tests x Subjects w groups	1,077.122	107	10.066	

*Significant

Table C.21

ANALYSIS OF VARIANCE FOR REPEATED MEASURES ON THE FACTOR OF I.Q. IN THE FOURTH GRADE

Source	SS	df	MS	F
Between Subjects		117		
I.Q.	3.005	1	3.005	
Subjects w groups	3,125.745	116	26.946	
Within Subjects		118		
Tests	12,933.281	1	12,933.281	1,187.520*
I.Q. x Tests	1,608.911	1	1,608.911	147.728*
Tests x Subjects w groups	1,263.445	116	10.891	

*Significant

Table C.22

ANALYSIS OF VARIANCE FOR REPEATED MEASURES OF THE FACTOR OF I.Q. IN THE FIFTH GRADE

Source	SS	df	MS	F
Between Subjects		111		
I.Q.	238.588	1	238.588	
Subjects w groups	2,428.495	110	2,428.495	
Within Subjects		112		
Tests	1,466.264	1	1,466.264	77.645*
I.Q. x Tests	55.424	1	55.424	2.934
Tests x Subjects w groups	2,077.276	110	18.884	

*Significant

Table C.23

ANALYSIS OF VARIANCE FOR REPEATED MEASURES ON THE FACTOR OF I.Q. IN THE SIXTH GRADE

Source	SS	df	MS	F
Between Subjects		106		
I.Q.	50.730	1	50.730	
Subjects w groups	319.130	105	3.039	
Within Subjects		107		
Tests	305.079	1	305.079	30.615*
I.Q. x Tests	10.691	1	10.691	1.072
Tests x Subjects w groups	1,041.370	105	9.965	

*Significant

Table C.24

ANALYSIS OF VARIANCE FOR REPEATED MEASURES ON THE FACTOR OF TEACHING METHOD IN THE FOURTH GRADE

Source	SS	df	MS	F
Between Subjects		117		
Method	.579	1	.579	
Subjects w groups	549.229	116	4.734	
Within Subjects		118		
Tests	924.670	1	924.670	81.146*
Method x Tests	10.019	1	10.019	.879
Tests x Subjects w groups	1,322.271	116	11.395	

*Significant

Table C.25

ANALYSIS OF VARIANCE FOR REPEATED MEASURES ON THE FACTOR OF TEACHING METHOD IN THE FIFTH GRADE

Source	SS	df	MS	F
Between Subjects		113		
Method	142.056	1	142.056	
Subjects w groups	2,952.893	112	26.365	
Within Subjects		114		
Tests	1,274.801	1	1,274.801	8.108*
Method x Tests	107.468	1	107.468	.686
Tests x Subjects w groups	17,453.118	112	155.831	

*Significant

Table C.26

ANALYSIS OF VARIANCE FOR REPEATED MEASURES ON THE FACTOR OF TEACHING METHOD IN THE SIXTH GRADE

Source	SS	df	MS	F
Between Subjects		108		
Method	1,411.583	1	1,411.583	
Subjects w groups	2,170.239	107	20.282	
Within Subjects				
Tests	331.275	1	331.275	49.748*
Method x Tests	161.141	1	161.141	24.198*
Tests x Subjects w groups	712.516	107	6.659	

*Significant

Table C.27

ANALYSIS OF VARIANCE FOR REPEATED MEASURE ON THE FACTOR OF GRADE

Source	SS	df	MS	F
Between Subjects		340		
Grade	428.407	2	214.203	
Subjects w groups	11,178.281	338	33.071	
Within Subjects		341		
Tests	2,351.697	1	2,351.697	18.875*
Tests x Grade	155.226	2	77.613	.622
Tests x Subjects w groups	4,221.340	338	124.595	

*Significant

Table C.28

CUMULATIVE PERCENTAGE OF STUDENTS WITHIN FOURTH GRADE CLASSES TO REACH ACHIEVEMENT LEVELS

Test	Test Score as % Correct	Percentage of Class Reaching Achievement Levels						
		Discovery Group			Instruction Group			Control
		Class 1	Class 2	Class 3	Class 1	Class 2	Class 3	Class 1
1	75%	04.7%	11.5%	16.6%	16.6%	10.3%	00.0%	14.8%
	50%	09.5%	34.6%	* 66.6%	* 83.3%	48.2%	26.9%	25.9%
	25%	38.0%	61.5%	96.6%	100.0%	82.7%	92.3%	74.0%
2	75%	58.8%	37.0%	68.0%	39.9%	60.0%	70.3%	11.5%
	50%	*100.0%	* 74.0%	* 92.0%	* 65.5%	* 83.3%	* 85.1%	34.6%
	25%	100.0%	92.5%	96.0%	89.6%	90.0%	96.2%	80.7%
3	75%	29.4%	07.0%	34.7%	20.6%	03.3%	56.0%	3.5%
	50%	76.4%	17.8%	* 78.2%	* 65.5%	26.6%	* 73.0%	21.4%
	25%	94.1%	35.7%	91.3%	93.1%	83.3%	96.1%	67.8%
4	75%	50.0%	50.0%	82.6%	62.0%	25.8%	85.7%	00.0%
	50%	* 95.0%	* 70.8%	* 95.6%	*100.0%	*100.0%	*100.0%	64.2%
	25%	100.0%	95.8%	95.6%	100.0%	100.0%	100.0%	89.2%
5	75%	10.5%	16.6%	61.9%	51.7%	25.6%	84.0%	03.5%
	50%	42.1%	37.5%	* 76.1%	* 62.0%	40.0%	* 84.0%	14.2%
	25%	52.6%	45.8%	80.9%	72.4%	40.3%	88.0%	25.0%
6	75%	55.0%	40.7%	83.3%	70.0%	60.0%	45.8%	03.5%
	50%	* 55.0%	44.4%	* 87.5%	* 70.0%	* 66.7%	45.8%	14.2%
	25%	55.0%	44.4%	97.6%	70.0%	66.7%	50.0%	25.0%

* indicates satisfactory achievement

Table C.29

CUMULATIVE PERCENTAGE OF STUDENTS WITHIN FIFTH GRADE CLASSES TO REACH ACHIEVEMENT LEVELS

Test	Test Score as % Correct	Percentage of Class Reaching Achievement Levels						
		Discovery Group			Instruction Group			Control
		Class 1	Class 2	Class 3	Class 1	Class 2	Class 3	Class 1
1	75%	00.0%	08.3%	28.5%	21.4%	00.0%	36.0%	00.0%
	50%	25.0%	33.3%	* 60.7%	* 67.8%	08.3%	* 64.0%	00.0%
	25%	79.1%	66.7%	85.7%	89.2%	91.6%	80.0%	26.9%
2	75%	65.3%	52.3%	39.2%	74.1%	80.0%	65.2%	00.0%
	50%	* 80.7%	* 61.9%	* 50.0%	* 87.0%	* 96.0%	* 73.9%	63.7%
	25%	92.3%	76.1%	71.4%	96.0%	96.0%	91.3%	11.1%
3	75%	26.9%	10.0%	14.2%	23.3%	26.9%	39.1%	00.0%
	50%	* 50.0%	30.0%	25.0%	46.6%	42.3%	* 52.1%	03.7%
	25%	88.4%	80.0%	75.0%	86.6%	84.6%	78.2%	62.9%
4	75%	79.1%	33.3%	56.0%	77.4%	50.0%	52.1%	22.2%
	50%	*100.0%	* 88.8%	*100.0%	* 93.5%	* 96.1%	* 95.6%	* 66.6%
	25%	100.0%	100.0%	100.0%	100.0%	100.0%	95.6%	81.4%
5	75%	26.9%	00.0%	07.1%	65.3%	08.3%	08.6%	26.9%
	50%	* 76.9%	00.0%	14.2%	* 65.3%	08.3%	26.0%	34.6%
	25%	92.3%	13.0%	35.7%	73.0%	29.1%	65.2%	46.1%
6	75%	88.0%	6.6%	30.7%	96.7%	11.1%	91.6%	00.0%
	50%	* 92.0%	* 53.3%	34.6%	* 96.7%	11.1%	* 95.8%	03.8%
	25%	92.0%	53.3%	34.6%	100.0%	11.1%	95.8%	04.6%

Table C.30

CUMULATIVE PERCENTAGE OF STUDENTS WITHIN SIXTH GRADE CLASSES TO REACH ACHIEVEMENT LEVELS

Test	Test Score as % Correct	Percentage of Class Reaching Achievement Levels						
		Discovery Group			Instruction Group			Control
		Class 1	Class 2	Class 3	Class 1	Class 2	Class 3	Class 1
1	75%	00.0%	00.0%	06.8%	28.0%	43.4%	62.5%	00.0%
	50%	00.0%	46.4%	24.1%	* 52.0%	* 95.6%	* 91.6%	03.3%
	25%	08.3%	85.7%	44.8%	84.0%	100.0%	100.0%	13.3%
2	75%	34.7%	44.0%	35.7%	77.8%	91.3%	85.7%	23.3%
	50%	* 56.5%	* 60.0%	* 67.8%	* 92.6%	* 91.3%	*100.0%	30.0%
	25%	91.3%	96.0%	96.4%	100.0%	100.0%	100.0%	40.0%
3	75%	00.0%	29.1%	00.0%	45.5%	91.6%	70.0%	00.0%
	50%	00.0%	* 62.5%	10.7%	* 63.6%	* 91.6%	* 90.0%	03.7%
	25%	07.6%	87.5%	39.2%	81.8%	91.6%	100.0%	03.7%
4	75%	03.8%	53.8%	26.9%	84.6%	91.6%	95.4%	24.0%
	50%	* 92.3%	*100.0%	* 88.4%	*100.0%	*100.0%	*100.0%	* 84.0%
	25%	96.1%	100.0%	100.0%	100.0%	100.0%	100.0%	88.0%
5	75%	80.7%	26.9%	03.1%	88.0%	82.6%	59.0%	03.8%
	50%	* 84.6%	* 57.6%	15.6%	* 92.0%	* 91.3%	* 63.6%	11.5%
	25%	96.1%	84.6%	31.2%	92.0%	95.6%	77.2%	19.2%
6	75%	81.4%	92.5%	81.4%	76.9%	100.0%	75.0%	00.0%
	50%	* 81.4%	* 96.2%	* 85.1%	* 80.7%	*100.0%	* 80.0%	00.0%
	25%	81.4%	96.2%	85.1%	80.7%	100.0%	80.0%	00.0%

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