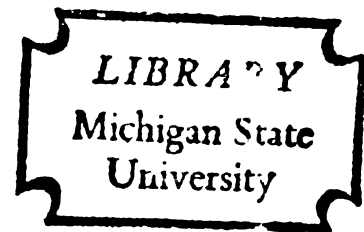


NONLEPTONIC DECAYS OF HYPERONS

Thesis for the Degree of Ph. D.
MICHIGAN STATE UNIVERSITY
YOSHIKAZO ERNESTO NAGAI
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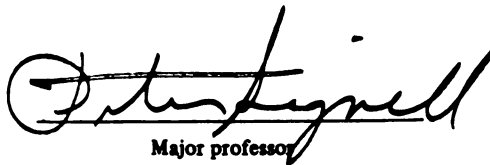
NONLEPTONIC DECAYS OF HYPERONS

presented by

Yoshikazo Ernesto Nagai

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Major professor

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ABSTRACT

NONLEPTONIC DECAYS OF HYPERONS

By

Yoshikazo Ernesto Nagai

Ever since the techniques of current algebra were first applied to nonleptonic decays of hyperons, each new effort to calculate the s- and p-wave amplitudes for these decays has led to more numerical puzzles than would seem reasonable for such apparently simple processes.

We present a new analysis of the problem by applying dispersion relation techniques to the scattering of a spurion from a hyperon. In a particular combination of amplitudes, the scattering process formally reduces to the weak decay process in the limit of vanishing four-momentum of the spurion. In this approach the Regge behavior of the scattering amplitude at high energies requires one subtraction in the dispersion relation, the subtraction point being chosen such that the calculable soft-pion amplitude gives the subtraction constant. Then the low-mass baryon pole contribution is separated from the dispersion integral with the remaining part of the integral coming from higher-mass resonances. Our method of evaluating this latter resonance contribution consists in assuming Regge behavior for the scattering amplitude at high energies and extrapolating this form of amplitude to the lower energy region. In this

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way the result of the higher-mass integration is just the real part of the Regge amplitude from the t-channel exchanges. This approach has its justification in the concept of local duality which states that the Regge amplitude, when extrapolated to lower energies, represents the true amplitude in an average sense.

The addition of the resonance contribution to the soft-pion-plus-pole amplitude increases the number of adjustable parameters to four. The numerical values of these parameters are then found by a χ^2 -fit to the experimental data.

As a further test of our analysis we make predictions for the amplitudes of the two-body radiative weak decays of hyperons. These decays are closely related to the nonradiative ones via low-energy theorems.

The main results can be summarized as follows:

(a) Our fit to the experimental pionic decay values is a considerable improvement over that of previous work. (b) The usual neglect of the higher-mass baryon resonances is found to be unwarranted for the s-wave. However, it is found to be fairly good for the p-wave. (c) A relatively large violation of unitary symmetry is present. (d) Octet dominance breaking is needed to explain the observed value $A(\Sigma_+^+) \approx 0$. (e) The available experimental information on radiative weak decays is in excellent agreement with our calculation.

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NONLEPTONIC DECAYS OF HYPERONS

By

Yoshikazo Ernesto Nagai

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Michigan State University

in partial fulfillment of the requirements

for the degree of

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I. INTRODUCTION

Since the historical paper of Lee and Yang on parity nonconservation,¹ weak interaction processes have been found to be a rich source of symmetry breaking. Apart from Lorentz invariance and the conservation of electric charge and baryon number, weak interactions appear to violate every symmetry that has been found to hold in strong interactions. Do these violations follow a definite pattern? Is it a large effect or merely a small correction?

The current×current theory^{2,3} of weak interactions provides a framework for the investigation of these questions. This theory emerged soon after parity nonconservation had been discovered; since then it has been modified to some extent, but its basic hypothesis remains intact. Namely, that all weak interactions, be it leptonic, semi-leptonic or nonleptonic, are generated by the interaction of a charged current with itself. Thus the general weak Hamiltonian in the current×current picture has the form

$$\mathcal{H}_w = \frac{G}{\sqrt{2}} \frac{1}{2} (J_\lambda J^{\dagger\lambda} + J_\lambda^\dagger J^\lambda) . \quad (1)$$

The weak current J_λ consists of a hadronic part, J_λ^h , and a leptonic one, J_λ^l . Both of these terms are equal admixtures

of vector and axial-vector components so that they have well-defined properties with respect to space-time transformations. In addition, the conserved vector current (CVC) hypothesis⁴ establishes a simple relationship between $\mathcal{F}_\lambda$, the vector component of J_λ^h , and the electromagnetic current J_λ^{em} ; this latter current is known to behave in a well-defined way with respect to internal symmetries, and so $\mathcal{F}_\lambda$ must do likewise. Similarly, the hypothesis of a partially conserved axial-vector current⁵ (PCAC) relates the divergence of $\mathcal{F}_\lambda^5$, the axial component of J_λ^h , to the meson field and thereby indicates specific symmetry properties for $\mathcal{F}_\lambda^5$.

A far reaching extension of these ideas is the identification of $\mathcal{F}_\lambda$ and $\mathcal{F}_\lambda^5$ with the elements of a Lie algebra.⁶ The success obtained by Adler and Weisberger⁷ in calculating the renormalized axial-vector coupling constant tends to confirm this identification, and it lead us to believe that symmetry is an essential ingredient for a dynamical theory of weak interactions.

Nonleptonic decay is an ideal application for such a theory. From the point of view of internal symmetries, most decay modes are different charge states of a few basic processes, and their amplitudes can be correlated by means of the transformation properties of the interaction Hamiltonian. If the hadronic current is of the Cabibbo type,⁸ then the nonleptonic interaction has specific transformation properties in $SU(3)$, and these properties

are sufficient to make a number of important predictions.

Among the nonleptonic processes, the two-body nonleptonic decays of hyperons have relatively accurate experimental information which makes the confrontation with the theory less ambiguous. The following decays will be considered:

$$\begin{aligned}
 \Lambda_c^0 &: \Lambda \rightarrow p + \pi^- \\
 \Sigma_c^- &: \Sigma^- \rightarrow n + \pi^- \\
 \Sigma_c^+ &: \Sigma^+ \rightarrow p + \pi^0 \\
 \Sigma_c^+ &: \Sigma^+ \rightarrow n + \pi^+ \\
 \Xi_c^- &: \Xi^- \rightarrow \Lambda + \pi^- .
 \end{aligned} \tag{2}$$

Because all baryons involved in these decays belong to an SU(3) octet, we can regard them as multiplet decays of a single one: $\alpha \rightarrow \beta + \pi$, with spin-parity assignment: $\frac{1}{2}^+ \rightarrow \frac{1}{2}^+ + 0^-$. Consider the decay of α at rest. Conservation of total angular momentum requires that the only partial waves allowed in the final state are $l=0$ and $l=1$, i.e., s- and p-wave only. The intrinsic parity of both α and β is +1 and that of the pion is -1. Then the parity of the final state is $-(-1)^l$ so that the s-wave decay amplitude is parity violating (pv) and the p-wave is parity conserving (pc).

According to the notion of the universality of the current $\times$ current form of weak Hamiltonian, the nonleptonic Hamiltonian $\mathcal{H}_{NL}$ is proportional to the strangeness-changing part of the symmetric product of the Cabibbo

current, J_λ , with itself, expression (1). Since J_λ and $J_\lambda^\dagger$ belong to the same octet of currents in the SU(3) group,⁹ the $\mathcal{H}_{\text{NL}}$ can only belong to the completely symmetric representation in the direct product decomposition,

$$8 \otimes 8 = 1 \oplus 8_s \oplus 8_a \oplus 10 \oplus \bar{10} \oplus 27 . \quad (3)$$

The symmetric representations are 1, 8_s , and 27. The first has only a strangeness preserving term, and since we are interested in strangeness-changing nonleptonic decays, $\mathcal{H}_{\text{NL}}$ can only transform as 8_s and 27. It is empirically known that the 27 part is small compared to the part coming from 8_s . This enhancement of the 8_s constitutes the so-called octet dominance,¹⁰ an extension of the isospin $\Delta I=1/2$ rule.¹¹

The first success along the line of making full use of symmetry properties was accomplished by Suzuki and by Sugawara in their current algebra study of the s-wave hyperon decays.¹² Using PCAC, the algebra of currents, and SU(3) symmetry, they derived the $\Delta I=1/2$ rule for Λ, Σ decays, the pseudo $\Delta I=1/2$ rule for Ξ decays, and the pseudo Lee-Sugawara relation.¹³ The extension of the same procedure to p-wave decays was considerably clarified by Brown and Sommerfield,¹⁴ and others;¹⁵ they confirmed the previous calculation of the s-wave amplitudes and, furthermore, found that the p-wave amplitudes can be computed in terms of the parameters appearing in the s-wave amplitudes. The

predicted values of the p-wave amplitudes, however, did not compare well with experiment. In general they were found to be two or three times smaller than the observed values. In view of this discrepancy for the p-wave, Kumar and Pati, and Itzykson and Jacob¹⁶ attempted to fix the p-wave while trying to maintain the apparent success of the s-wave calculations. They incorporated certain corrections, of the order of $\Delta M/2M$ compared to the Born terms, that were dropped by Brown and Sommerfield on the grounds that these were small. Also terms representing unitary symmetry breaking, previously neglected, were added. Quantitatively the agreement with experiment was found to be slightly better, but still not good enough. Several other refinements to the theory¹⁷⁻¹⁹ have been proposed from time to time, none of them, however, really convincing.

Meanwhile Okubo²⁰ attempted a new approach to the problem of the simultaneous description of both the s- and p-wave amplitudes by applying dispersion relation techniques to the scattering of a spurion from a hyperon. The scattering process formally reduces to the decay process in the limit of vanishing four-momentum of the spurion. In this approach Regge behavior²¹ of the scattering amplitude at high energies requires one subtraction to the dispersion relation in the energy, the subtraction point being chosen such that the calculable soft-pion amplitude gives the subtraction constant. Next the low-mass baryon pole contribu-



tion is separated from the dispersion integral, the remaining part of the integral coming from the higher-mass resonances. Difficulties in evaluating this latter resonance contribution with a minimum of free parameters has previously led to its neglect without justification.

We propose, within the scheme of current $\times$ current weak interaction and octet dominance, to implement Okubo's dispersion approach by making further use of the Regge theory. Our method of evaluating the resonance contribution consists in assuming Regge behavior for the scattering amplitude at high energies and extrapolating this form of amplitude to the lower energy region. In this way the result of the higher-mass integration is just the real part of the Regge amplitude from the t -channel exchanges, slightly modified due to the once-subtracted form of the dispersion relation. This approach has its qualitative justification in the concept of local duality²² which has been explored in the realm of high-energy phenomenology. Simply stated, local duality says that the Regge amplitude, when extrapolated to lower energies, reproduces the true amplitude in an average sense. The addition of the resonance contribution to the soft-pion-plus-pole amplitude increases the number of adjustable parameters to four. The numerical values of these parameters are found by fitting the experimentally determined amplitudes.

As a test of our approach we also consider the

two-body radiative decays of hyperons. These processes are closely related to the nonleptonic decays which will be referred as nonradiative decays of hyperons when dealing with the radiative processes. Insufficient experimental information on these radiative weak decays has prevented the selection of the best model among more than a half-dozen proposed since the first papers on the subject appeared more than a decade ago.^{23,24} All of these models predict decay rates and branching ratios that are in rough agreement with each other and with the available data. However, the first experimental determination²⁵ of the asymmetry parameter a for $\Sigma^+ \rightarrow p + \gamma$ gave the unexpected result $a = -1.03 \pm 0.52 - 0.42$, much larger than the theoretical predictions. The soft-pion-soft-photon approach, considered by Ahmed,²⁴ has previously led to a value for the asymmetry parameter consistent with the experimental value given above, but at the expense of an internal inconsistency in the calculation.²⁶ When the inconsistency is removed, Ahmed's calculation also yields a negligible asymmetry parameter like the other models. It is interesting, therefore, to test our results for nonradiative decays against the above experimental number, and also make predictions for the other radiative decays.

II. HAMILTONIAN AND INVARIANT DECAY AMPLITUDES

The starting point in most calculations of weak decay amplitudes is the specification of the effective Hamiltonian responsible for the decay; the choice is not unique. Here we consider the most popular nonleptonic weak interaction Hamiltonian, the current $\times$ current type, usually written as

$$\mathcal{H}_{NL} = \frac{G}{\sqrt{2}} J_{\lambda}^{\dagger} J^{\lambda} \quad , \quad (1)$$

where $G=1.0 \times 10^5 / M_p^2$ is the universal Fermi constant. The factor $1/\sqrt{2}$ appears for historical reasons, and J_{λ} is now the hadronic Cabibbo current⁸ assumed to transform like an octet of SU(3). This current is postulated to be made up of charged currents only (in contrast to charged plus neutral currents), both strangeness preserving and changing:

$$J^{\lambda} = (F_1^{\lambda} + i F_2^{\lambda} - F_1^{5\lambda} - i F_2^{5\lambda}) \cos \theta + (F_4^{\lambda} + i F_5^{\lambda} - F_4^{5\lambda} - i F_6^{5\lambda}) \sin \theta . \quad (2)$$

The superscript 5 stands for axial-vector current and the absence of it specifies the vector current; the subscripts denote the components of an octet; and θ is the Cabibbo angle assumed to be the same for vector and axial-vector currents.

Under the combined operation of charge conjugation and parity (CP), J_λ goes to $J_\lambda^\dagger$ so that we obtain explicit CP invariance if we choose the weak Hamiltonian in the symmetric form

$$\mathcal{H}_{NL} = \frac{G}{\sqrt{2}} \frac{1}{2} (J_\lambda^\dagger J^\lambda + J_\lambda J^\lambda{}^\dagger) \equiv \frac{G}{\sqrt{2}} [J_\lambda^\dagger J^\lambda]_s \quad (3)$$

instead of expression (1). Because J_λ and $J_\lambda^\dagger$ belong to an octet of currents in the SU(3) symmetric scheme, the symmetrized Hamiltonian (3) can only belong to the completely symmetric representation in the direct product decomposition of the two octets. Among the symmetric representations 1, 8_s and 27, the first has only a strangeness preserving term. Therefore in the case of strangeness changing nonleptonic decays, $\mathcal{H}_{NL}$ can only transform as 8_s and 27. Furthermore, we consider the octet dominance approximation.

In general an octet may be defined by its commutation relations with the vector octet charge:²⁷

$$F_k(x_0) = \int d^3x \mathcal{F}_k^0(x) . \quad (4)$$

The set of operators $O_l(x)$ form an octet if

$$[F_k(x_0), O_l(x)]_- = i f_{klm} O_m(x) . \quad (5)$$

Therefore for currents $\mathcal{F}_k^\lambda$ and $\mathcal{F}_k^{5\lambda}$ to be octets we require

$$[F_k(x_0), \mathcal{F}_l^\lambda(x)]_- = i f_{klm} \mathcal{F}_m^\lambda(x) , \quad (6a)$$

$$[F_k(x_0), \mathcal{F}_\ell^{5\lambda}(x)]_- = i f_{k\ell m} \mathcal{F}_m^{5\lambda}(x) . \quad (6b)$$

To preserve the complete symmetry between vector and axial-vector currents we postulate⁶ in addition that

$$[F_k^5(x_0), \mathcal{F}_\ell^\lambda(x)]_- = i f_{k\ell m} \mathcal{F}_m^{5\lambda}(x) , \quad (6c)$$

$$[F_k^5(x_0), \mathcal{F}_\ell^{5\lambda}]_- = i f_{k\ell m} \mathcal{F}_m^\lambda(x) . \quad (6d)$$

Hamiltonian (3) is a sum of products of octet vectors; it can be shown to be an octet tensor of second rank. To see more explicitly the tensor nature of the nonleptonic Hamiltonian, it is convenient to define the following quantities:

$$\begin{aligned} T_{kl}^{PC} &\equiv [\mathcal{F}_{k\lambda} \mathcal{F}_l^\lambda + \mathcal{F}_{k\lambda}^5 \mathcal{F}_l^{5\lambda}]_s , \\ T_{kl}^{PV} &\equiv [\mathcal{F}_{k\lambda} \mathcal{F}_l^{5\lambda} + \mathcal{F}_{k\lambda}^5 \mathcal{F}_l^\lambda]_s . \end{aligned} \quad (7)$$

In the product $J_\lambda^\dagger J^\lambda$, with J_λ given by (2), the term $\cos^2\theta$ is

$$\begin{aligned} T_{11}^{PC} + T_{22}^{PC} - T_{11}^{PV} - T_{22}^{PV} - i(\mathcal{F}_{1\lambda} \mathcal{F}_2^\lambda - \mathcal{F}_{2\lambda} \mathcal{F}_1^\lambda) + i(\mathcal{F}_{1\lambda} \mathcal{F}_2^{5\lambda} - \mathcal{F}_{2\lambda} \mathcal{F}_1^{5\lambda}) \\ - i(\mathcal{F}_{2\lambda} \mathcal{F}_1^{5\lambda} - \mathcal{F}_{1\lambda} \mathcal{F}_2^\lambda) - i(\mathcal{F}_{1\lambda}^5 \mathcal{F}_2^{5\lambda} - \mathcal{F}_{2\lambda}^5 \mathcal{F}_1^{5\lambda}) . \end{aligned} \quad (8)$$

The pieces in parenthesis give no contribution under symmetrization so that we are left with

$$(\cos^2\theta \text{ term of } [J_\lambda J^\lambda]_s) = 2(T_{11}^{PC} + T_{22}^{PC}) - 2(T_{11}^{PV} + T_{22}^{PV}) .$$

Similar results for the other terms are

$$(\sin^2\theta \text{ term of } [J_\lambda J^{\dagger\lambda}]_S) = 2(T_{44}^{PC} + T_{55}^{PC}) - 2(T_{44}^{PV} + T_{55}^{PV}) ,$$

$$(\sin\theta \cos\theta \text{ term of } [J_\lambda J^{\dagger\lambda}]_S) = 4(T_{14}^{PC} + T_{25}^{PC}) - 4(T_{14}^{PV} + T_{25}^{PV}) .$$

We can therefore write the nonleptonic Hamiltonian as

$$\mathcal{H}_{NL} = \mathcal{H}_{NL}^{PC} - \mathcal{H}_{NL}^{PV} \quad (9)$$

where

$$\mathcal{H}_{NL}^{PC} = \frac{G}{\sqrt{2}} \left[(T_{11}^{PC} + T_{22}^{PC}) \cos^2\theta + (T_{44}^{PC} + T_{55}^{PC}) \sin^2\theta + 2(T_{14}^{PC} + T_{25}^{PC}) \sin\theta \cos\theta \right] \quad (10)$$

and $\mathcal{H}_{NL}^{PV}$ is identical to $\mathcal{H}_{NL}^{PC}$ except for the replacement of the label PC by PV. Now, T_{kl}^{PC} and T_{kl}^{PV} can be shown to be tensors by proving that they satisfy the following relations:

$$\begin{aligned} [F_k(x_0), T_{lm}^{PC}(x)]_- &= i f_{kln} T_{nm}^{PC}(x) + i f_{kmn} T_{ln}^{PC}(x) , \\ [F_k^5(x_0), T_{lm}^{PC}(x)]_- &= i f_{kln} T_{nm}^{PV}(x) + i f_{kmn} T_{ln}^{PV}(x) , \\ [F_k(x_0), T_{lm}^{PV}(x)]_- &= i f_{kln} T_{nm}^{PV}(x) + i f_{kmn} T_{ln}^{PV}(x) , \\ [F_k^5(x_0), T_{lm}^{PV}(x)]_- &= i f_{kln} T_{nm}^{PC}(x) + i f_{kmn} T_{ln}^{PC}(x) . \end{aligned} \quad (11)$$

From the above tensor defining relations it follows that

$$[F_k^5, T_{lm}^{PC}]_- = [F_k, T_{lm}^{PV}]_- , \quad [F_k^5, T_{lm}^{PV}]_- = [F_k, T_{lm}^{PC}]_- . \quad (12)$$

Note that $\mathcal{H}_{NL}^{PC}$ and $\mathcal{H}_{NL}^{PV}$ are constructed from identical

combinations of T_{kl}^{PC} and T_{kl}^{PV} respectively. From this fact and Eq.(12) we get

$$[F_k^S, \mathcal{H}_{NL}^{PC}]_- = [F_k, \mathcal{H}_{NL}^{PV}]_- , \quad [F_k^S, \mathcal{H}_{NL}^{PV}]_- = [F_k, \mathcal{H}_{NL}^{PC}]_- \quad (13)$$

so that

$$[F_k^S, \mathcal{H}_{NL}]_- = - [F_k, \mathcal{H}_{NL}]_- . \quad (14)$$

Equation (14) is a basic one for the successful application of current algebra techniques to weak interactions.

We now proceed to consider the specific case of nonleptonic decays of hyperons. Let us simply write $\mathcal{H}$ for the weak Hamiltonian density responsible for the decay of a hyperon α into a baryon β and a pion π :

$$\alpha(p) \rightarrow \beta(p') + \pi^a(q') . \quad (15)$$

Here p , p' and q' denote the four-momentum of the respective particles, and a is the isospin index of the pion. We also denote the octet indices by α and β .

The S-matrix for process (15) to first order in weak interaction is

$$S \simeq 1 - i \int d^4x \mathcal{H}(x) \quad (16)$$

and its matrix element between initial and final states

$$\langle \beta(p') \pi^a(q') | S - 1 | \alpha(p) \rangle = -i \int d^4x \langle \beta(p') \pi^a(q') | \mathcal{H}(x) | \alpha(p) \rangle . \quad (17)$$

By writing $\mathcal{H}(x)$ in terms of the displacement (four-momentum) operator P ,

$$\mathcal{H}(x) = e^{iP \cdot x} \mathcal{H}(0) e^{-iP \cdot x}$$

we obtain

$$\langle \beta(p') \pi^a(q') | S-1 | \alpha(p) \rangle = -i(2\pi)^4 \delta^4(p' + q' - p) \langle \beta(p') \pi^a(q') | \mathcal{H}(0) | \alpha(p) \rangle. \quad (18)$$

The most general form for the matrix element (18) consistent with Lorentz invariance can be written as a linear combination of the five basic types of interactions: scalar, pseudoscalar, vector, axial-vector, and tensor. Because of the Dirac equation, the last three types reduce to either scalar or pseudoscalar. Hence the most general form for the matrix element of $\mathcal{H}(0)$ is

$$\langle \beta(p') \pi^a(q') | \mathcal{H}(0) | \alpha(p) \rangle = \frac{i}{(2\pi)^{q/2}} \frac{1}{(2q'_0)^{1/2}} \frac{1}{N_\alpha N_\beta} \bar{u}(p') (A_{\beta\alpha}^a - \gamma_5 B_{\beta\alpha}^a) u(p) \quad (19)$$

where $N_\alpha \equiv (E_\alpha/M_\alpha)^{1/2}$, $E_\alpha = (\mathbf{p}^2 + M_\alpha^2)^{1/2}$, and A, B are invariant functions of the Mandelstam variables:

$$\begin{aligned} s &= (p' + q')^2 = p^2, \\ t &= (p' - p)^2 = q'^2, \\ u &= (p - q')^2 = p'^2. \end{aligned} \quad (20)$$

If all three particles are on the mass-shell we have $s = M_\alpha^2$, $t = m_\pi^2$, $u = M_\beta^2$ with the implication that the invariant amplitudes A and B are constants. A complete theory of weak

interactions should allow a calculation of these amplitudes in terms of the masses involved. Note that A is the amplitude for the production of the pion in the s state (pv) and B is the one with the pion in the p state (pc).

It will be convenient to define the amplitude

$$M = -i(2\pi)^{9/2} (2q'_0)^{1/2} N_\alpha N_\beta \langle \beta(p') \pi^a(q') | \mathcal{H}(0) | \alpha(p) \rangle \quad (21)$$

so that

$$M = \bar{u}(p') (A - \gamma_5 B) u(p) . \quad (22)$$

Then the decay rate is given by

$$\Gamma = \frac{1}{2} \frac{1}{(2\pi)^6} \int \frac{M_\beta}{E_\beta} d^3p' \frac{d^3q'}{2q'_0} (2\pi)^4 \delta^4(p' + q' - p) |M|^2 . \quad (23)$$

Performing the trace calculations and integrating over the phase space we obtain (Appendix C)

$$\Gamma = C_1 (|A|^2 + C_2 |B|^2) \quad (24)$$

where

$$C_1 \equiv \frac{1}{8\pi} \frac{(M_\alpha + M_\beta)^2 - m_\pi^2}{M_\alpha^2} |p'| , \quad C_2 \equiv \frac{(M_\alpha - M_\beta)^2 - m_\pi^2}{(M_\alpha + M_\beta)^2 - m_\pi^2} . \quad (25)$$

III. SOFT-PION THEOREM

The separation of the decay amplitude into two invariant amplitudes A and B has been accomplished on the grounds of Lorentz invariance alone. We can also write it in terms of field operators according to the LSZ reduction formula:²⁸

$$\langle \beta(p') \pi^a(q') | \mathcal{H}(0) | \alpha(p) \rangle = i \int d^4x f_{q'}^*(x) (\Box^2 + m_\pi^2) \langle \beta(p') | T(\phi_a(x) \mathcal{H}(0)) | \alpha(p) \rangle . \quad (1)$$

Here $f_{q'}(x)$ is the pion plane wave function,

$$f_{q'}(x) \equiv \frac{e^{-iq' \cdot x}}{(2\pi)^{3/2} (2q'_0)^{1/2}} \quad (2)$$

and $\phi_a(x)$ is the pion field operator. In terms of the amplitude M defined in (II.21), LSZ formula (1) becomes

$$M(q') = (2\pi)^3 N_\alpha N_\beta \int d^4x e^{iq' \cdot x} (\Box^2 + m_\pi^2) \langle \beta(p') | T(\phi_a(x) \mathcal{H}(0)) | \alpha(p) \rangle . \quad (3)$$

The soft-pion technique^{12,14} consists in letting q' go off the mass shell in an appropriate manner as follows. Integration by parts twice in (3) yields

$$M(q') = (2\pi)^3 N_\alpha N_\beta \int d^4x e^{iq' \cdot x} (m_\pi^2 - q'^2) \langle \beta(p') | T(\phi_a(x) \mathcal{H}(0)) | \alpha(p) \rangle . \quad (4)$$

The pion field operator may be replaced by the divergence

]

of the axial-vector current according to the PCAC relation

$$\partial_\lambda \mathcal{F}_a^{5\lambda}(x) = c \phi_a(x) \quad , \quad c \equiv \frac{0.95}{\sqrt{2}} m_\pi^3 . \quad (5)$$

Inserting $\phi_a(x)$ from (5) into (4) we obtain

$$M(q') = (2\pi)^3 N_\alpha N_\beta \frac{m_\pi^2 - q'^2}{c} \int d^4x e^{iq' \cdot x} \langle \beta(p') | T(\partial_\lambda \mathcal{F}_a^{5\lambda}(x) \mathcal{H}(0)) | \alpha(p) \rangle . \quad (6)$$

The next step is to consider the expression

$$T^\lambda(q') \equiv -i(2\pi)^3 N_\alpha N_\beta \int d^4x e^{iq' \cdot x} \langle \beta(p') | T(\mathcal{F}_a^{5\lambda}(x) \mathcal{H}(0)) | \alpha(p) \rangle . \quad (7)$$

If we multiply (7) by q'_λ and integrate by parts we get

$$q'_\lambda T^\lambda(q') = (2\pi)^3 N_\alpha N_\beta \int d^4x e^{iq' \cdot x} \langle \beta(p') | \partial_\lambda T(\mathcal{F}_a^{5\lambda}(x) \mathcal{H}(0)) | \alpha(p) \rangle . \quad (8)$$

Making use of the identity

$$\partial_\lambda T(\mathcal{F}_a^{5\lambda}(x) \mathcal{H}(0)) = T(\partial_\lambda \mathcal{F}_a^{5\lambda}(x) \mathcal{H}(0)) + \delta(x_0) [\mathcal{F}_a^{50}(x), \mathcal{H}(0)]_- \quad (9)$$

we can write (8) as

$$\begin{aligned} q'_\lambda T^\lambda(q') &= (2\pi)^3 N_\alpha N_\beta \int d^4x e^{iq' \cdot x} \langle \beta(p') | T(\partial_\lambda \mathcal{F}_a^{5\lambda}(x) \mathcal{H}(0)) | \alpha(p) \rangle \\ &\quad + (2\pi)^3 N_\alpha N_\beta \int d^4x e^{iq' \cdot x} \langle \beta(p') | [\mathcal{F}_a^{50}(x), \mathcal{H}(0)]_- | \alpha(p) \rangle \delta(x_0) . \end{aligned} \quad (10)$$

The first integral is related to M through Eq.(6), and in the soft-pion limit the second integral can be written as

$$\lim_{q' \rightarrow 0} \int d^4x e^{iq' \cdot x} \langle \beta(p') | [\mathcal{F}_a^{50}(x), \mathcal{H}(0)]_- | \alpha(p) \rangle \delta(x_0) = \langle \beta(p') | [\mathcal{F}_a^5, \mathcal{H}]_- | \alpha(p) \rangle . \quad (11)$$

Hence from relations (6), (10) and (11) we get

$$M(q'=0) = \lim_{q' \rightarrow 0} \frac{m_\pi^2}{c} q'_\lambda T^\lambda(q') - \frac{m_\pi^2}{c} (2\pi)^3 N_\alpha N_\beta \langle \beta(p') | [F_a^5, \mathcal{H}] | \alpha(p) \rangle. \quad (12)$$

It is customary to name the first term on the RHS the surface term, and the last the current algebra or equal-time-commutator (ETC) term. The extrapolated amplitude given by (12) expresses the content of the soft-pion theorem.

In their work Suzuki and Sugawara¹² neglected the possible contribution of the surface term and approximated the physical amplitude by $M(q'=0)$ given by (12). They considered only the s-wave amplitude A because in the limit of exact $SU(3)$ symmetry it has been shown²⁹ that

$$\langle \text{baryon} | \mathcal{H}^{PV} | \text{baryon} \rangle = 0. \quad (13)$$

In fact, using (II.13), expression (12) without the surface term gives

$$M^{PC}(q'=0) = - \frac{m_\pi^2}{c} (2\pi)^3 N_\alpha N_\beta \langle \beta(p') | [F_a, \mathcal{H}^{PV}] | \alpha(p) \rangle = 0$$

because of (13), whereas

$$M^{PV}(q'=0) = - \frac{m_\pi^2}{c} (2\pi)^3 N_\alpha N_\beta \langle \beta(p') | [F_a, \mathcal{H}^{PC}] | \alpha(p) \rangle$$

is not necessarily zero.

Brown and Sommerfield,¹⁴ and others,¹⁵ split the amplitude $M(q')$ in two parts, the Born (pole) term $M^B(q')$ and the nonBorn (remainder) $M^N(q')$:

$$M(q') = M^B(q') + M^N(q') . \quad (14)$$

Combining (12) and (14) in the soft-pion limit we have

$$\begin{aligned} M^N(q'=0) = \lim_{q' \rightarrow 0} \left(\frac{m_\pi^2}{c} q'_\lambda T^\lambda(q') - M^B(q') \right) \\ - \frac{m_\pi^2}{c} (2\pi)^3 N_\alpha N_\beta \langle \beta(p') | [F_a^5, \mathcal{H}] | \alpha(p) \rangle . \end{aligned} \quad (15)$$

When the intermediate states contributing to the pole and to the surface terms are degenerate in mass with either the initial α or final β states, the limits on the RHS of (15) are ambiguous³⁰ if taken separately. However, the ambiguity disappears when both limits are considered together. We will come to this point again in the next section where the surface term will be treated in detail. For our present discussion it is sufficient to know that the ambiguities cancel each other, leaving a well-defined limit.

Difficulties in evaluating the on-mass-shell remainder $M^N(q')$ led to the smoothness assumption expressed by the approximation

$$M^N(q') \simeq M^N(q'=0) \quad (16)$$

i.e., the value of the remainder M^N at a physical q' is approximately given by the value of M^N at the soft-pion point $q'=0$. This is a reasonable approximation in some cases, but it is not valid in general and should be checked for each particular case. The pragmatic attitude, when it comes to

applications of the soft-pion theorem, is to accept (16) as a working hypothesis, perform the necessary calculations and confront the results with the experiment. If the agreement is good one says that the smoothness assumption is valid for the particular case; if not, one tries to pinpoint the trouble, in general without much success.

Assuming (16) we can combine (14) with (15) to obtain

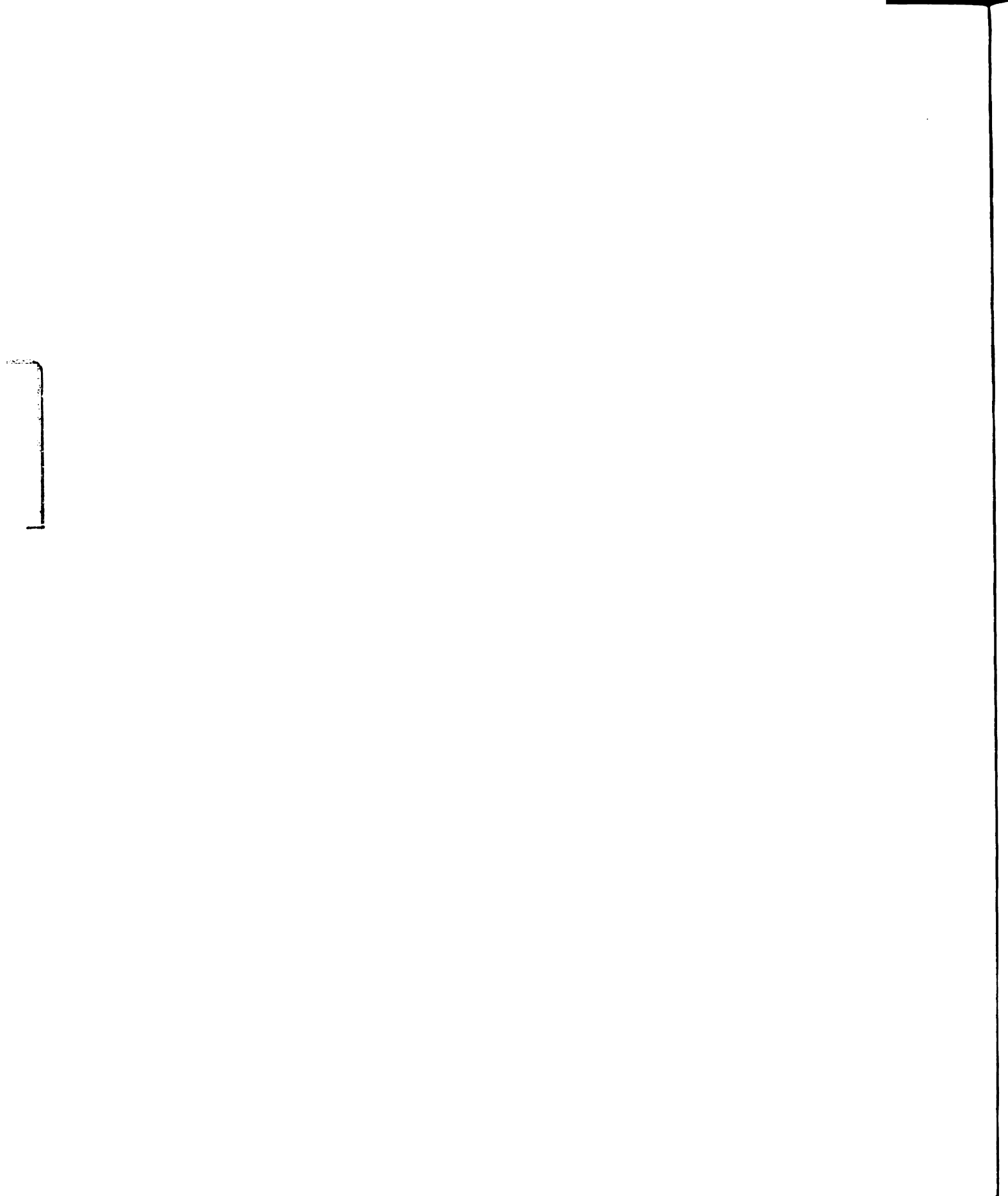
$$M(q') = M^B(q') + \lim_{q' \rightarrow 0} \left[\frac{m_\pi^2}{c} q'_\lambda T^\lambda(q') - M^B(q') \right] - \frac{m_\pi^2}{c} (2\pi)^3 N_\alpha N_\beta \langle \beta(p') | [F_3^5, \mathcal{H}] | \alpha(p) \rangle . \quad (17)$$

The surface-Born term is $\Delta M/2M$ of the on-mass-shell Born term, ΔM being a typical difference in baryon mass. Because the mass shift among the octet baryons is relatively small, Brown and Sommerfield neglected the surface-Born term. Under exact unitary symmetry the s-wave Born term is zero because of (13) so that the s-wave amplitude is given solely by the ETC term, thus reproducing the earlier result of Suzuki and Sugawara. On the other hand, the p-wave amplitude receives contribution from the Born term only because the ETC term involves the matrix element (13) which vanishes in the SU(3) limit.

The failure of Brown and Sommerfield, and others, to fit the data may have its origin in one or more of the following assumptions which they took for granted: (a) the

smoothness assumption defined by the approximation (16). According to expression (17) this assumption implies that the higher-mass contributions are negligible; and/or (b) the exact validity of unitary symmetry, implying the vanishing of the matrix element of $\mathcal{H}^{PV}$ between baryon states of a same octet.

Kumar and Pati, and others,¹⁶ introduced the effect of symmetry breaking terms by allowing them to vary through an overall adjustable parameter in order to fit the data. The agreement with observation improved somewhat, but was still far from being satisfactory. Their conclusion was that the symmetry breaking terms are not significant, being at most 15% of the observed amplitudes, which has been used by several workers as evidence in order to disregard symmetry breaking effects in nonleptonic decays and related processes such as radiative weak decays of hyperons. We argue that the fit of Kumar and Pati is not good enough to draw conclusions about the magnitude of these symmetry breaking effects.



IV. THE SURFACE-BORN TERM AND THE SOFT-PION AMPLITUDE

The evaluation of the surface-Born term,

$$\lim_{q' \rightarrow 0} \left[\frac{m_\pi^2}{c} q'_\lambda T^\lambda(q') - M^B(q') \right] \quad (1)$$

and discussion of the ambiguity inherent in it will be considered in detail; also the ETC part of the amplitude will be given explicitly.

The starting point is the definition (III.7):

$$T^\lambda(q') \equiv -i(2\pi)^3 N_\alpha N_\beta \int d^4x e^{iq' \cdot x} \langle \beta(p') | T(F_a^{5\lambda}(x) \mathcal{H}(0)) | \alpha(p) \rangle . \quad (2)$$

Insertion of the time-ordering operator,

$$T(F_a^{5\lambda}(x) \mathcal{H}(0)) \equiv \theta(x_0) F_a^{5\lambda}(x) \mathcal{H}(0) + \theta(-x_0) \mathcal{H}(0) F_a^{5\lambda}(x) \quad (3)$$

in (2) gives two additive pieces for T^λ :

$$T^{1\lambda}(q') \equiv -i(2\pi)^3 N_\alpha N_\beta \int d^4x e^{iq' \cdot x} \theta(x_0) \langle \beta(p') | F_a^{5\lambda}(x) \mathcal{H}(0) | \alpha(p) \rangle , \quad (4)$$

$$T^{2\lambda}(q') \equiv -i(2\pi)^3 N_\alpha N_\beta \int d^4x e^{iq' \cdot x} \theta(-x_0) \langle \beta(p') | \mathcal{H}(0) F_a^{5\lambda}(x) | \alpha(p) \rangle . \quad (5)$$

A complete set of intermediate states put between $F_a^{5\lambda}(x)$ and $\mathcal{H}(0)$:

$$T^{1\lambda}(q') = -i(2\pi)^3 N_\alpha N_\beta \int d^4x e^{iq' \cdot x} \theta(x_0) \sum_n \langle \beta(p') | F_a^{5\lambda}(x) | n \rangle \langle n | \mathcal{H}(0) | \alpha(p) \rangle \quad (6)$$

allow us to use the displacement operator (four-momentum) P to write

$$\mathcal{F}_a^{5\lambda}(x) = e^{iP \cdot x} \mathcal{F}_a^{5\lambda}(0) e^{-iP \cdot x}$$

and integrate by parts to obtain

$$T^{1\lambda}(q') = (2\pi)^3 N_\alpha N_\beta \sum_n \int d^4x \frac{e^{i(q'+p'-p_n) \cdot x}}{q'_0 - p'_0 - p_{n0}} \delta(x_0) \langle \beta(p') | \mathcal{F}_a^{5\lambda}(0) | n \rangle \langle n | \mathcal{H}(0) | \alpha(p) \rangle. \quad (7)$$

Integration with respect to x yields

$$T^{1\lambda}(q') = (2\pi)^6 N_\alpha N_\beta \sum_n \frac{\delta^3(q' + p' - p_n)}{q'_0 + p'_0 - p_{n0}} \langle \beta(p') | \mathcal{F}_a^{5\lambda} | n \rangle \langle n | \mathcal{H} | \alpha(p) \rangle. \quad (8)$$

The summation in n involves sum over intermediate states as well as sum over spins and three-momentum for each intermediate state. Thus, for a particular single particle intermediate state δ we have

$$\sum_n \equiv \sum_{\delta \text{ spin}} \int d^3p_n. \quad (9)$$

Performing the 3-momentum integration in (8) we obtain

$$T^{1\lambda}(q') = (2\pi)^6 N_\alpha N_\beta \sum_{\delta \text{ spin}} \frac{\langle \beta(p') | \mathcal{F}_a^{5\lambda} | \delta(p'+q') \rangle \langle \delta(p'+q') | \mathcal{H} | \alpha(p) \rangle}{q'_0 + E_\beta(p') - E_\delta(p'+q')}. \quad (10)$$

Now the weak vertex can be written as

$$\langle \delta(p_n) | \mathcal{H}(0) | \alpha(p) \rangle = \frac{1}{(2\pi)^3} \frac{1}{N_\alpha N_\beta} \bar{u}(p_n) (C_{\delta\alpha} - \gamma_5 V_{\delta\alpha}) u(p). \quad (11)$$

The matrix element of the axial-vector current $\mathcal{F}_a^{5\lambda}$ between

baryon states can be obtained as follows. The PCAC relation

$$\partial_\lambda \mathcal{F}_a^{5\lambda}(x) = c \phi_a(x) \quad (12)$$

combined with the displacement operator gives

$$\partial_\lambda e^{i(p'-p_n) \cdot x} \langle \beta(p') | \mathcal{F}_a^{5\lambda}(0) | \delta(p_n) \rangle = c \langle \beta(p') | \phi_a(x) | \delta(p_n) \rangle$$

so that

$$i(p'-p_n)_\lambda \langle \beta(p') | \mathcal{F}_a^{5\lambda}(0) | \delta(p_n) \rangle = c \langle \beta(p') | \phi_a(0) | \delta(p_n) \rangle. \quad (13)$$

Similarly the wave equation

$$(\square^2 + m_\pi^2) \phi_a(x) = j_a(x) \quad (14)$$

yields

$$(m_\pi^2 - (p' - p_n)^2) \langle \beta(p') | \phi_a(0) | \delta(p_n) \rangle = \langle \beta(p') | j_a(0) | \delta(p_n) \rangle. \quad (15)$$

The strong vertex can be written as

$$\langle \beta(p') | j_a(0) | \delta(p_n) \rangle = \frac{1}{(2\pi)^3} \frac{1}{N_p N_\delta} \bar{u}(p') i \gamma_5 u(p_n) K_{\beta\delta}^a \quad (16)$$

where

$$K_{\beta\delta}^a \equiv 2 g_{\pi NN} (\alpha F^a + (1-\alpha) D^a)_{\beta\delta}. \quad (17)$$

Here α is the experimentally known F,D mixing parameter

and $F_{\beta\delta}^a = -i f_{a\beta\delta}$, $D_{\beta\delta}^a = d_{a\beta\delta}$ (Appendix B).

From (13), (15) and (16) we get

$$(p'-p_n)_\lambda \langle \beta(p') | \mathcal{F}_a^{5\lambda}(0) | \delta(p_n) \rangle = \frac{1}{(2\pi)^3} \frac{1}{N_p N_\delta} \frac{c K_{\beta\delta}^a}{m_\pi^2 - (p'-p_n)^2} \bar{u}(p') i \gamma_5 u(p_n). \quad (18)$$

Dirac equation allow us to make the replacements

$$\bar{u}(p') = \frac{1}{M_\beta} \bar{u}(p') \not{p}' \quad , \quad u(p_n) = \frac{1}{M_\delta} \not{p}_n u(p_n)$$

on the RHS of (18) leading to

$$M_\beta (p'-p_n)_\lambda \langle \beta(p') | \mathcal{F}_a^{s\lambda} | \delta(p_n) \rangle = \frac{1}{(2\pi)^3} \frac{1}{N_\beta N_\delta} \frac{c K_{\beta\delta}^a}{m_\pi^2 - (p'-p_n)^2} \bar{u}(p') \not{p}' \gamma_5 u(p_n) \quad , \quad (19)$$

$$M_\delta (p'-p_n)_\lambda \langle \beta(p') | \mathcal{F}_a^{s\lambda} | \delta(p_n) \rangle = \frac{1}{(2\pi)^3} \frac{1}{N_\beta N_\delta} \frac{c K_{\beta\delta}^a}{m_\pi^2 - (p'-p_n)^2} \bar{u}(p') \gamma_5 \not{p}_n u(p_n) \quad .$$

Addition of these two expressions gives the desired matrix element for the axial-vector current:

$$\langle \beta(p') | \mathcal{F}_a^{s\lambda} | \delta(p_n) \rangle = \frac{c}{m_\pi^2} \frac{1}{M_\beta + M_\delta} \frac{1}{(2\pi)^3} \frac{1}{N_\beta N_\delta} \bar{u}(p') \gamma^\lambda \gamma_5 u(p_n) K_{\beta\delta}^a \quad (20)$$

where we have made the approximation $(p'-p_n)^2=0$ to be consistent with the value of the form factor $K_{\beta\delta}^a$ evaluated at zero momentum-transfer squared.

Inserting the weak (11) and the strong (20) vertices in the expression (10) for $T^{1\lambda}$ we obtain

$$T^{1\lambda}(q') = \frac{c}{m_\pi^2} \frac{1}{M_\beta + M_\delta} \frac{M_\delta}{E_\delta} \sum_{\text{spin}} \frac{\bar{u}(p') \gamma^\lambda \gamma_5 u(p_n) \bar{u}(p_n) (Kc - \gamma_5 K v) u(p)}{q'_0 + E_\beta(p') - E_\delta(p' + q')} \quad (21)$$

where we have simply written Kc for $K_{\beta\delta}^a c_{\delta\alpha}$, etc. Summation over spin states

$$\sum_{\text{spin}} u(p_n) \bar{u}(p_n) = \frac{\not{p}_n + M_\delta}{2M_\delta}$$

yields

$$q'_\lambda T^{1\lambda}(q') = \frac{c}{m_\pi^2} \frac{1}{M_\beta + M_\delta} \frac{\bar{u}(p') \not{A}' \gamma_5 (\not{p}_n + M_\delta) (Kc - \gamma_5 K v) u(p)}{2 E_\delta(p' + q') [q'_0 + E_\beta(p') - E_\delta(p' + q')]} . \quad (22)$$

Following step by step the preceding calculation we also get

$$q'_\lambda T^{2\lambda}(q') = -\frac{c}{m_\pi^2} \frac{1}{M_\beta + M_\delta} \frac{\bar{u}(p') (cK - \gamma_5 v K) (\not{p}_m + M_\gamma) \not{A}' \gamma_5 u(p)}{2 E_\gamma(p - q') [q'_0 + E_\gamma(p - q') - \Xi_\alpha(p)]} . \quad (23)$$

If $M_\delta \neq M_\beta$ and $M_\gamma \neq M_\alpha$, the soft-pion limit of both terms $q'_\lambda T^{1\lambda}$ and $q'_\lambda T^{2\lambda}$ vanishes. There is an ambiguity, however, when at least one of the equalities $M_\delta = M_\beta$ or $M_\gamma = M_\alpha$ holds. In this case expressions (22) and (23) can be simplified to yield

$$\begin{aligned} \frac{m_\pi^2}{c} q'_\lambda T^\lambda(q') = & \bar{u}(p') \left[\left(\frac{\not{A}'}{2 p' \cdot q'} - \frac{1}{2 M_\beta} \right) \gamma_5 (Kc - \gamma_5 K v) \right. \\ & \left. - (cK - \gamma_5 v K) \left(\frac{\not{A}'}{2 p \cdot q'} + \frac{1}{2 M_\alpha} \right) \gamma_5 \right] u(p) . \end{aligned} \quad (24)$$

It turns out that there is also an ambiguity of the same nature in the Born term which cancels the ambiguity in the surface term. In fact, Feynman rules applied to the diagrams of Figure 1 give

$$M^B(q') = -\bar{u}(p') \left[\gamma_5 \frac{1}{\not{p}' + \not{A}' - M_\delta} (Kc - \gamma_5 K v) + (cK - \gamma_5 v K) \frac{1}{\not{p} - \not{A}' - M_\gamma} \gamma_5 \right] u(p) \quad (25)$$

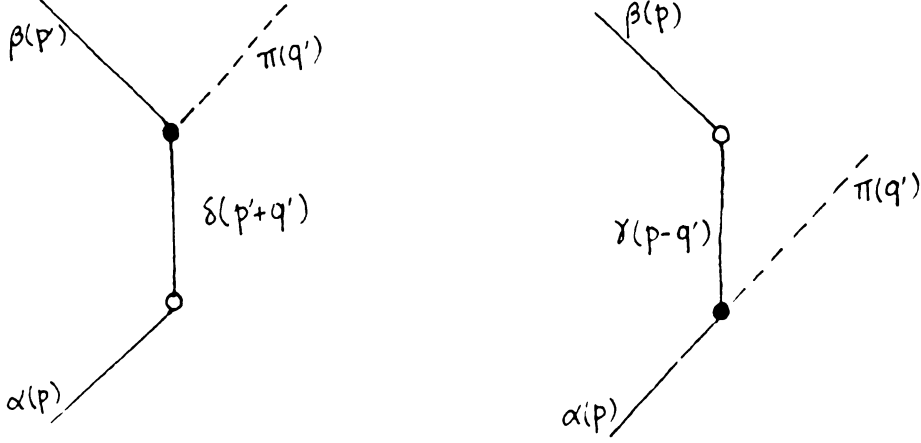


Figure 1. Born diagrams for the s and u channels.

which, for $M_\delta = M_\beta$ and $M_\gamma = M_\alpha$, reduces to

$$M^B(q') = \bar{u}(p') \left[\frac{\not{A}'}{2 p' \cdot q'} \gamma_5 (K \not{C} - \gamma_5 K \not{V}) - (C \not{\Lambda} - \gamma_5 \not{V} K) \frac{\not{A}'}{2 p \cdot q'} \gamma_5 \right] u(p) . \quad (26)$$

From (24) and (26) we see that the surface-Born term (1) is well-defined and of the order $\Delta M/2M$ relative to the on-mass-shell Born term. Note that for all practical purposes we may take $M_\delta \neq M_\beta$ and $M_\gamma \neq M_\alpha$ throughout the calculation and only at the end set $M_\delta = M_\beta$ and/or $M_\gamma = M_\alpha$ at the appropriate places. The final expression for the Born term is the same as that one would get by taking care of the ambiguities.

In what follows we write the nonBorn soft-pion amplitude as simply given by the ETC part:

$$M^N(q'=0) = \frac{m_\pi^2}{C} (2\pi)^3 N_\alpha N_\beta \langle \beta(p') | [F_a, \mathcal{H}] | \alpha(p) \rangle \quad (27)$$

with the understanding that the surface-Born term has been added to the on-mass-shell Born term.

We now consider a more explicit form for the ETC term. Any cartesian component F_a of the isospin operator can be written in terms of the raising, lowering, and third component isospin operators. The action of these operators on the baryon states are known so that for each particular decay the ETC can be easily evaluated. The result may be put in a closed form¹⁴

$$M^N(q'=0) = \frac{m_\pi^2}{c} \left([F_a, c]_{\beta\alpha}^- - \gamma_5 [F_a, v]_{\beta\alpha}^- \right) \quad . \quad (28)$$

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V. DISPERSION RELATION FORMALISM

The energy-momentum conservation for the decay

$$\alpha(p) \rightarrow \beta(p') + \pi^a(q') \quad (1)$$

requires $p=p'+q'$. Then the invariant s and p -wave amplitudes are constant and we are not able to write dispersion relations directly for the process (1). This difficulty can be avoided by considering the associated scattering process of a spurion κ from the hyperon α ,

$$\kappa(q) + \alpha(p) \rightarrow \beta(p') + \pi^a(q') \quad (2)$$

and properly taking the limit $q \rightarrow 0$ in the scattering amplitudes so as to recover the amplitudes for the decay (1).

The most general transition amplitude for (2) can be written as

$$\begin{aligned} M(s,t,u) = \bar{u}(p') \{ & F_1(s,t,u) - \gamma_5 F_2(s,t,u) + \frac{1}{2}(\not{q} + \not{q}') \\ & \times [G_1(s,t,u) - \gamma_5 G_2(s,t,u)] \} u(p) \end{aligned} \quad (3)$$

where the invariant amplitudes F_i and G_i are functions of the Mandelstam variables defined by

$$\begin{aligned}
s &= (p+q)^2 = (p'+q')^2 \quad , \\
t &= (p-p')^2 = (q'-q)^2 \quad , \\
u &= (p-q')^2 = (p'-q)^2 \quad , \\
s+t+u &= M_\alpha^2 + M_\beta^2 + m_\pi^2 + q^2 .
\end{aligned} \tag{4}$$

It is convenient to take s, t, q^2 as independent variables. Also, using the Dirac equation, we can express amplitude (3) in a more suitable form for taking the limit $q \rightarrow 0$. If we multiply (3) by M_β and add to it the same amplitude (3) multiplied by M_α , we obtain

$$\begin{aligned}
(M_\beta + M_\alpha)M &= \bar{u}(p') \left[(F_1 - \gamma_5 F_2)(M_\beta + M_\alpha) + (\not{p}'(\not{q}'+\not{q}) + (\not{q}'+\not{q})\not{p}) \frac{G_1}{2} \right. \\
&\quad \left. - \frac{\not{q}'+\not{q}}{2} (M_\beta + M_\alpha) \gamma_5 G_2 \right] u(p) .
\end{aligned} \tag{5}$$

If instead of adding the results of the multiplications by M_β and M_α , we subtract them we get

$$\begin{aligned}
(M_\beta - M_\alpha)M &= \bar{u}(p') \left[(F_1 - \gamma_5 F_2)(M_\beta - M_\alpha) + \frac{\not{q}'+\not{q}}{2} (M_\beta - M_\alpha) G_1 \right. \\
&\quad \left. - (\not{p}'(\not{q}'+\not{q}) + (\not{q}'+\not{q})\not{p}) \gamma_5 \frac{G_2}{2} \right] u(p) .
\end{aligned} \tag{6}$$

Now a simple, but tedious, algebraic manipulation shows

$$\not{p}'(\not{q}'+\not{q}) + (\not{q}'+\not{q})\not{p} = [\not{q}, \not{q}']_- + s - u . \tag{7}$$

Inserting this identity into (5) and (6) we get

$$M = \bar{u}(p') \left[(F_1 - \gamma_5 F_2) + \frac{[\not{A}, \not{A}']_- + s - u}{M_\beta + M_\alpha} \frac{G_1}{2} - \frac{\not{A}' + \not{A}}{2} \gamma_5 \frac{G_2}{2} \right] u(p) ,$$

$$M = \bar{u}(p') \left[(F_1 - \gamma_5 F_2) + \frac{\not{A}' + \not{A}}{2} \frac{G_1}{2} - \frac{[\not{A}, \not{A}']_- + s - u}{M_\beta - M_\alpha} \gamma_5 \frac{G_2}{2} \right] u(p) .$$

Adding again gives the result

$$M = \bar{u}(p') \left[(F_1 - \gamma_5 F_2) + \frac{1}{2} ([\not{A}, \not{A}']_- + s - u) \left(\frac{G_1}{M_\beta + M_\alpha} - \gamma_5 \frac{G_2}{M_\beta - M_\alpha} \right) \right] u(p) . \quad (8)$$

In terms of new invariant amplitudes defined by

$$\begin{aligned} H_1 &\equiv F_1 + \frac{s - u}{2(M_\alpha + M_\beta)} G_1 , & H_2 &\equiv F_2 - \frac{s - u}{2(M_\alpha - M_\beta)} G_2 , \\ J_1 &\equiv \frac{G_1}{2(M_\alpha + M_\beta)} , & J_2 &\equiv - \frac{G_2}{2(M_\alpha - M_\beta)} . \end{aligned} \quad (9)$$

the scattering amplitude M becomes

$$\begin{aligned} M(s, t, q^2) = \bar{u}(p') \{ & [H_1(s, t, q^2) - \gamma_5 H_2(s, t, q^2)] \\ & + [\not{A}, \not{A}']_- [J_1(s, t, q^2) - \gamma_5 J_2(s, t, q^2)] \} u(p) . \end{aligned} \quad (10)$$

In the limit $q \rightarrow 0$ we see that $s = M_\alpha^2$, $t = q'^2 = m_\pi^2 = 0$, and $q^2 = 0$ so that the transition amplitude M reduces to

$$M(M_\alpha^2, 0, 0) = \bar{u}(p') [H_1(M_\alpha^2, 0, 0) - \gamma_5 H_2(M_\alpha^2, 0, 0)] u(p) . \quad (11)$$

We formally identify $H_1(M_\alpha^2, 0, 0)$ and $H_2(M_\alpha^2, 0, 0)$ with the s - and p -wave decay amplitudes respectively,

$$A = H_1(M_\alpha^2, 0, 0) \quad \text{and} \quad B = H_2(M_\alpha^2, 0, 0) . \quad (12)$$

Before we write down dispersion relations in s and fixed t, q^2 for the amplitude $H_i(s, t, q^2)$, we have to settle the question of subtractions. Assuming Regge behavior for $H_i(s, t, q^2)$ at high energies, the need for subtraction depends on the trajectories exchanged in the t -channel. For the s -wave amplitude, $H_1(s, t, q^2)$, the exchanged trajectory in the t -channel is that of K^* , whereas for the p -wave amplitude, $H_2(s, t, q^2)$, we have the trajectories of K and K_A mesons exchanged. Although the trajectory parameters for these particles are not firmly established at present, under the reasonable assumption of a linear trajectory with a universal slope of about 1 GeV^{-2} , we obtain the following intercepts for K^* , K , and K_A respectively:

$$\alpha_V \simeq 0.25 \quad , \quad \alpha_P \simeq -0.25 \quad , \quad \alpha_A \simeq -0.75 \quad . \quad (13)$$

An unsubtracted dispersion relation involves an integral of the form

$$\int_{-\infty}^{+\infty} ds' \frac{\text{Im } H_i(s', t, q^2)}{s' - s - i\epsilon}$$

which for large s' would give contributions

$$\int^{\infty} ds' s'^{\alpha_i - 1} = \frac{s^{\alpha_i}}{\alpha_i} \Big|_{-\infty}^{\infty} = \begin{cases} \infty & \text{for } i = V \\ 0 & \text{for } i = A, P \end{cases} .$$

Therefore intercepts (13) imply one subtraction for H_1 and none for H_2 . However, we make one subtraction to both

amplitudes and write the dispersion relation

$$H_i(s, t, q^2) = H_i(s_0, t, q^2) + \frac{1}{\pi} (s - s_0) \int_{-\infty}^{+\infty} ds' \frac{\text{Im } H_i(s', t, q^2)}{(s' - s_0)(s' - s - i\epsilon)} \quad (14)$$

where the subtraction point s_0 should be chosen in such a way that we are able to compute the subtraction constant $H_i(s_0, t, q^2)$. For this purpose recall that in the soft-pion calculation we let $q' \rightarrow 0$ and find the extrapolated amplitude in terms of an equal-time-commutator. In this limit $q' \rightarrow 0$ we have $s = p'^2 = M_\beta^2$ and $t = q^2$. Therefore, if we choose $s_0 = M_\beta^2$ and let $q \rightarrow 0$ in order to recover the desired decay amplitudes, the subtraction constant $H_i(M_\beta^2, 0, 0)$ will be given by the soft-pion amplitude.

There are three reasons why we wish to separate the pole contribution from the dispersion integral in (14). First, we need the pole term to remove the well-known ambiguity associated with the soft-pion extrapolation. Second, the soft-pion term (subtraction constant) plus the pole contribution reproduces the old results so that the magnitude and form of the remaining integral provides a possible explanation for the numerical puzzle associated with the problem. Third, the removal of the pole makes the idea of extrapolating the Regge amplitude down to low energies more plausible, as discussed below. Then, denoting H_i^P for the pole contribution to the dispersion integral, we rewrite (14) as

$$H_i(s) = H_i(s_0) + H_i^P(s) + \frac{1}{\pi} (s-s_0) \int_{-\infty}^{+\infty} ds' \frac{\text{Im } H_i'(s')}{(s'-s_0)(s'-s-i\epsilon)} \quad (15)$$

where H_i' stands for the remaining part of H_i after the removal of the one-baryon intermediate states. Also the arguments of the H 's have been simplified for ease of notation. Note that the point $s=M_\alpha^2$, to which we wish to extrapolate in order to obtain the decay amplitude, lies at the lower end of the resonance region, and since we already extracted the large pole contribution from this region, the remaining amplitude $H_i'(s)$ can be assumed to be represented approximately by the extrapolated Regge amplitude according to the concept of local duality.²² In what follows we elaborate on this qualitative idea and evaluate the remaining integral explicitly.

With the help of the formal identity

$$\frac{1}{s'-s-i\epsilon} = P \frac{1}{s'-s} + i\pi \delta(s'-s)$$

the integral in (15) can be written as

$$\int_{-\infty}^{+\infty} ds' \frac{\text{Im } H_i'(s')}{(s'-s_0)(s'-s-i\epsilon)} = \frac{i\pi}{s-s_0} \text{Im } H_i'(s) + P \int_{-\infty}^{+\infty} ds' \frac{\text{Im } H_i'(s')}{(s'-s_0)(s'-s)} \quad (16)$$

Now, the assumption of local duality implies,

$$P \int_{-\infty}^{+\infty} ds' \frac{\text{Im } H_i'(s')}{(s'-s_0)(s'-s)} = P \int_{-\infty}^{+\infty} ds' \frac{\text{Im } H_i^R(s')}{(s'-s_0)(s'-s)} \quad (17)$$

where the Regge amplitude $H_i^R(s)$ has the form

$$H_i^R(s) \equiv \gamma_i \frac{e^{-i\pi\alpha_i}}{\sin \pi\alpha_i} s^{\alpha_i} . \quad (18)$$

Here the minus sign refers to K^* and K_A trajectories, and the plus sign to that of K . From this form for the Regge amplitude we see that

$$\begin{aligned} H_i^R(s)^* &= -H_i^R(-s) \quad \text{for } K^* \text{ and } K_A , \\ H_i^R(s)^* &= +H_i^R(-s) \quad \text{for } K . \end{aligned} \quad (19)$$

These identities imply that the imaginary part of H_i^R is even for K^* and K_A , whereas it is odd for K :

$$\begin{aligned} \text{Im } H_i^R(s) &= \text{Im } H_i^R(-s) \quad \text{for } K^*, K_A , \\ \text{Im } H_i^R(s) &= -\text{Im } H_i^R(-s) \quad \text{for } K . \end{aligned} \quad (20)$$

Hence we can write

$$\int_{-\infty}^{+\infty} ds' \frac{\text{Im } H_i^R(s')}{(s'-s_0)(s'-s)} = \pm \int_0^{\infty} ds' \frac{\text{Im } H_i^R(s')}{(s'+s_0)(s'+s)} + \int_0^{\infty} ds' \frac{\text{Im } H_i^R(s')}{(s'-s_0)(s'-s)} . \quad (21)$$

Inserting the explicit Regge form (18) into (21) we get

$$\begin{aligned}
P \int_{-\infty}^{+\infty} ds' \frac{\text{Im } H_i^R(s')}{(s'-s_0)(s'-s)} &= \pi \gamma_i \tan \frac{\pi}{2} \alpha_i \frac{s^{\alpha_i} - s_0^{\alpha_i}}{s-s_0} \quad \text{for } K^*, K_A, \\
&= -\pi \gamma_i \cot \frac{\pi}{2} \alpha_i \frac{s^{\alpha_i} - s_0^{\alpha_i}}{s-s_0} \quad \text{for } K.
\end{aligned} \tag{22}$$

With these values for the integral, expression (16) becomes

$$\begin{aligned}
\int_{-\infty}^{+\infty} ds' \frac{\text{Im } H_1'(s')}{(s'-s_0)(s'-s-i\epsilon)} &= \frac{i\pi}{s-s_0} \text{Im } H_1'(s) + \frac{\pi \gamma_V}{s-s_0} (s^{\alpha_V} - s_0^{\alpha_V}) \tan \frac{\pi}{2} \alpha_V, \\
\int_{-\infty}^{+\infty} ds' \frac{\text{Im } H_2'(s')}{(s'-s_0)(s'-s-i\epsilon)} &= \frac{i\pi}{s-s_0} \text{Im } H_2'(s) + \frac{\pi \gamma_A}{s-s_0} (s^{\alpha_A} - s_0^{\alpha_A}) \tan \frac{\pi}{2} \alpha_A \\
&\quad - \frac{\pi \gamma_P}{s-s_0} (s^{\alpha_P} - s_0^{\alpha_P}) \cot \frac{\pi}{2} \alpha_P.
\end{aligned} \tag{23}$$

Note that the equality (17) is less restrictive than local duality itself. By taking the values of the integrals obtained in (23) into the expression (15) we get

$$\begin{aligned}
A &= K_1 + \text{Re } H_1^P(M_\alpha^2) + \gamma_V (M_\alpha^{2\alpha_V} - M_\beta^{2\alpha_V}) \tan \frac{\pi}{2} \alpha_V, \\
B &= K_2 + \text{Re } H_2^P(M_\alpha^2) + \gamma_A (M_\alpha^{2\alpha_A} - M_\beta^{2\alpha_A}) \tan \frac{\pi}{2} \alpha_A \\
&\quad - \gamma_P (M_\alpha^{2\alpha_P} - M_\beta^{2\alpha_P}) \cot \frac{\pi}{2} \alpha_P.
\end{aligned} \tag{24}$$

where we have written K_i for the ETC term and have eliminated the soft-pion ambiguity between the surface and the pole terms.

With the intercepts given in (13) and the small mass splitting among the octet baryons we see that

$M_{\alpha}^{2\alpha_P} - M_{\beta}^{2\alpha_P}$ is proportional to $M_{\alpha}^{2\alpha_A} - M_{\beta}^{2\alpha_A}$. Also recent studies³¹ suggest the same D/F ratio for the strong $\overline{\text{B}}\text{BK}$ and $\overline{\text{B}}\text{BK}_A$ vertices. Therefore we can replace the two resonance contributions to the p-wave amplitude B by an effective term and write

$$\begin{aligned} A &= K_1 + \text{Re } H_1^P(M_{\alpha}^2) + \gamma_V (M_{\alpha}^{2\alpha_V} - M_{\beta}^{2\alpha_V}) \tan \frac{\pi}{2} \alpha_V , \\ B &= K_2 + \text{Re } H_2^P(M_{\alpha}^2) + \gamma' (M_{\alpha}^{2\alpha_A} - M_{\beta}^{2\alpha_A}) \tan \frac{\pi}{2} \alpha_A . \end{aligned} \quad (25)$$

The form of the resonance terms is reminiscent of the t-channel pole models.^{31a}

VI. POLE TERMS

For the evaluation of the pole term

$$\operatorname{Re} H_i^p(s) = \frac{s-s_0}{\pi} P \int_{-\infty}^{+\infty} ds' \frac{\operatorname{Im} H_i^p(s')}{(s'-s_0)(s'-s)} \quad (1)$$

we need to know the pole contribution to the imaginary part of $H_i(s)$.

The scattering amplitude $M(s, t, q^2)$ reduces to the decay amplitude $M(M_\alpha^2, 0, 0)$ in the limit $q \rightarrow 0$. Before taking the limit, however, we consider a convenient form for M in order to obtain the imaginary part of the invariant amplitudes H_1 and H_2 . The connection between M and the H 's is

$$M_{\beta\alpha} = \bar{u}(p') (H_1 - \gamma_5 H_2) u(p) \quad (2)$$

where we have dropped the term with the factor $[\not{q}, \not{q}']_-$ for this term will vanish in the limit $q \rightarrow 0$.

The Hermitian conjugate of (2) is

$$M_{\beta\alpha}^\dagger = \bar{u}(p) (H_1^* + \gamma_5 H_2^*) u(p') \quad (3)$$

Under the interchange $\alpha(p) \leftrightarrow \beta(p')$ the Mandelstam variables in H_i do not change, but

$$\begin{aligned}\bar{u}(p) u(p') &\longrightarrow + \bar{u}(p') u(p) , \\ \bar{u}(p) \gamma_5 u(p') &\longrightarrow - \bar{u}(p') u(p) .\end{aligned}\tag{4}$$

Therefore, performing the interchange $\alpha(p) \leftrightarrow \beta(p')$ in (3)

$$M_{\alpha\beta}^{\dagger} = \bar{u}(p') (H_1^* - \gamma_5 H_2^*) u(p) .\tag{5}$$

Subtraction of (5) from (2) yields

$$M_{\beta\alpha} - M_{\alpha\beta}^{\dagger} = 2i \bar{u}(p') (\text{Im } H_1 - \gamma_5 \text{Im } H_2) u(p) .\tag{6}$$

With this combination of amplitude M in mind we work on the field theoretical form for the same amplitude:

$$M_{\beta\alpha} = (2\pi)^3 N_{\alpha} N_{\beta} \int d^4x e^{iq \cdot x} (\Box^2 + m_{\pi}^2) \langle \beta(p') | T(\phi_a(x) \mathcal{H}(0)) | \alpha(p) \rangle .\tag{7}$$

An alternate form³² of (7) more convenient for our purpose is

$$M_{\beta\alpha} = (2\pi)^3 N_{\alpha} N_{\beta} \int d^4x e^{iq \cdot x} (\Box^2 + m_{\pi}^2) \langle \beta(p') | \theta(x_0) [\phi_a(x) \mathcal{H}(0)]_- | \alpha(p) \rangle .\tag{8}$$

Taking the Hermitian conjugate of (8) and performing the interchange $\alpha(p) \leftrightarrow \beta(p')$ we get

$$M_{\alpha\beta}^{\dagger} = (2\pi)^3 N_{\alpha} N_{\beta} \int d^4x e^{iq \cdot x} (\Box^2 + m_{\pi}^2) \langle \beta(p') | \theta(-x_0) [\mathcal{H}(0), \phi_a(x)]_- | \alpha(p) \rangle .\tag{9}$$

Subtraction of (9) from (8) gives

$$M_{\beta\alpha} - M_{\alpha\beta}^{\dagger} = (2\pi)^3 N_{\alpha} N_{\beta} \int d^4x e^{iq \cdot x} \langle \beta(p') | [\phi_a(x), \mathcal{H}(0)]_- | \alpha(p) \rangle\tag{10}$$

where we have introduced the pion current

$$j_a(x) = (\square^2 + m_\pi^2) \phi_a(x) \quad .$$

The pole contribution to (10) arises from the $J^P = \frac{1}{2}^+$ baryon intermediate states in s and u channels. Let δ be a typical s-channel intermediate baryon state and γ an u-channel intermediate state, Figure 2. Then the pole

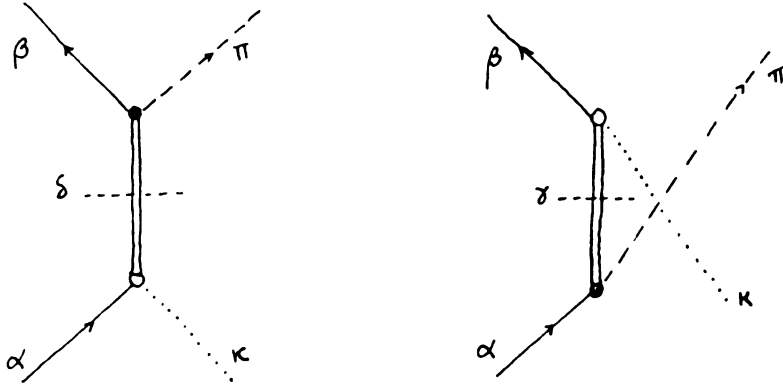


Figure 2. Baryon intermediate states in s and u channels.

contribution to (10) can be written

$$M_{\beta\alpha} - M_{\alpha\beta}^\dagger = (2\pi)^3 N_\alpha N_\beta \int d^4x e^{iq \cdot x} \sum_n \left(\langle \beta(p') | j_a(x) | \delta(p_n) \rangle \right. \\ \left. \times \langle \delta(p_n) | \mathcal{H}(0) | \alpha(p) \rangle - \langle \beta(p') | \mathcal{H}(0) | \gamma(p_n) \rangle \langle \gamma(p_n) | j_a(x) | \alpha(p) \rangle \right) . \quad (11)$$

Using the 4-momentum operator to translate $j_a(x)$ to $j_a(0)$ we obtain

$$M_{\beta\alpha} - M_{\alpha\beta}^\dagger = (2\pi)^3 N_\alpha N_\beta \sum_n \left\{ (2\pi)^4 \delta^4(q' + p' - p_n) \langle \beta(p') | j_a | \delta(p_n) \rangle \langle \delta(p_n) | \mathcal{H} | \alpha(p) \rangle \right.$$

$$-(2\pi)^4 \delta^4(q' + p_n - p) \langle \beta(p') | \mathcal{H} | \gamma(p_n) \rangle \langle \gamma(p_n) | j_a | \alpha(p) \rangle . \quad (12)$$

The strong and weak vertices have been introduced before:

$$\begin{aligned} \langle \beta(p') | j_a(0) | \gamma(p_n) \rangle &= \frac{1}{(2\pi)^3} \frac{1}{N_\beta N_\gamma} \bar{u}(p') i \gamma_5 u(p_n) K_{\beta\gamma}^a , \\ \langle \gamma(p_n) | \mathcal{H}(0) | \alpha(p) \rangle &= \frac{1}{(2\pi)^3} \frac{1}{N_\gamma N_\alpha} \bar{u}(p_n) (C_{\gamma\alpha} - \gamma_5 V_{\gamma\alpha}) u(p) . \end{aligned}$$

Insertion of these matrix elements in (12) and summation over spin states of the intermediate states give

$$\begin{aligned} M_{\beta\alpha} - M_{\alpha\beta}^\dagger &= i \sum_n (2\pi) \delta^4(p' + q' - p_n) \bar{u}(p') \gamma_5 \frac{\not{p}' + M_\gamma}{2E_\gamma} (K_C - \gamma_5 K_V) u(p) \\ &\quad - i \sum_n (2\pi) \delta^4(q' + p_n - p) \bar{u}(p') (C_K - \gamma_5 V_K) \frac{\not{p}' + M_\gamma}{2E_\gamma} \gamma_5 u(p) . \quad (13) \end{aligned}$$

The identity

$$\int \frac{d^3 p_n}{2E_n} (\quad) = \int d^4 p_n \delta(p_n^2 - M_n^2) (\quad)$$

can be used to perform the integration implicit in the summation sign. Thus we get

$$\begin{aligned} M_{\beta\alpha} - M_{\alpha\beta}^\dagger &= i 2\pi \delta((p' + q')^2 - M_\gamma^2) \bar{u}(p') \gamma_5 (\not{p}' + \not{q}' + M_\gamma) (K_C - \gamma_5 K_V) u(p) \\ &\quad - i 2\pi \delta((p - q')^2 - M_\gamma^2) \bar{u}(p') (C_K - \gamma_5 V_K) (\not{p} - \not{q}' + M_\gamma) \gamma_5 u(p) . \quad (14) \end{aligned}$$

We separate (14) in smaller pieces and write (14) as

$$M_{\beta\alpha} - M_{\alpha\beta}^{\dagger} = 2\pi i \left[\delta(s - M_{\delta}^2) M_{1s} K c - \delta(u - M_{\gamma}^2) M_{1u} c K \right. \\ \left. - \delta(s - M_{\delta}^2) M_{2s} K v + \delta(u - M_{\gamma}^2) M_{2u} v K \right] \quad (15)$$

where

$$M_{1s} \equiv \bar{u}(p') \gamma_5 (\not{p}' + \not{q}' + M_{\delta}) u(p) \quad , \quad M_{2s} \equiv \bar{u}(p') \gamma_5 (\not{p}' + \not{q}' + M_{\delta}) \gamma_5 u(p) , \\ M_{1u} \equiv \bar{u}(p') (\not{p} - \not{q}' + M_{\gamma}) \gamma_5 u(p) \quad , \quad M_{2u} \equiv \bar{u}(p') \gamma_5 (\not{p} - \not{q}' + M_{\gamma}) \gamma_5 u(p) . \quad (16)$$

At this stage of the calculation it is important not to replace $p' + q'$ by p or $p - q'$ by p' as we would in case of decay for which $p = p' + q'$. Dropping of some terms because of $q \rightarrow 0$ should be made carefully.

Dirac equation is applied to rewrite (16) in a more convenient form as

$$M_{1s} (M_{\alpha} - M_{\beta}) = \bar{u}(p') \gamma_5 (\not{q}' \not{p} + \not{p}' \not{q}' + M_{\beta}^2 - M_{\alpha} M_{\beta} + M_{\delta} M_{\alpha} - M_{\delta} M_{\beta}) u(p) , \\ M_{1u} (M_{\alpha} - M_{\beta}) = \bar{u}(p') (\not{q}' \not{p} + \not{p}' \not{q}' - M_{\alpha}^2 + M_{\alpha} M_{\beta} + M_{\gamma} M_{\alpha} - M_{\gamma} M_{\beta}) \gamma_5 u(p) , \\ M_{2s} (M_{\alpha} + M_{\beta}) = \bar{u}(p') (-\not{q}' \not{p} - \not{p}' \not{q}' - M_{\beta}^2 - M_{\alpha} M_{\beta} + M_{\delta} M_{\alpha} + M_{\delta} M_{\beta}) u(p) , \\ M_{2u} (M_{\alpha} + M_{\beta}) = \bar{u}(p') (\not{q}' \not{p} + \not{p}' \not{q}' - M_{\alpha}^2 - M_{\alpha} M_{\beta} + M_{\gamma} M_{\alpha} + M_{\gamma} M_{\beta}) u(p) . \quad (17)$$

We can repeat these manipulations on the M 's by replacing $p' + q'$ by $p + q$ and $p - q'$ by $p' - q$ in the expression (16). We get the same value for the M 's because of the energy-momentum conservation. Thus

$$\begin{aligned}
M_{1s}(M_\alpha - M_\beta) &= \bar{u}(p') \gamma_5 (\not{q} \not{p} + \not{p}' \not{q} + M_\alpha^2 - M_\alpha M_\beta + M_\delta M_\alpha - M_\delta M_\beta) u(p) , \\
M_{1u}(M_\alpha - M_\beta) &= \bar{u}(p') (\not{q} \not{p} + \not{p}' \not{q} - M_\beta^2 + M_\alpha M_\beta + M_\delta M_\alpha - M_\delta M_\beta) \gamma_5 u(p) , \\
M_{2s}(M_\alpha + M_\beta) &= \bar{u}(p') (-\not{q} \not{p} - \not{p}' \not{q} - M_\alpha^2 - M_\alpha M_\beta + M_\delta M_\alpha + M_\delta M_\beta) u(p) , \\
M_{2u}(M_\alpha + M_\beta) &= \bar{u}(p') (\not{q} \not{p} + \not{p}' \not{q} - M_\beta^2 - M_\alpha M_\beta + M_\delta M_\alpha + M_\delta M_\beta) u(p) .
\end{aligned} \tag{18}$$

In section V we have used the identity

$$\not{p}'(\not{q} + \not{q}') + (\not{q} + \not{q}')\not{p} = s - u + [\not{q}, \not{q}']_- .$$

At this point we drop the commutator in the q 's because its contribution becomes irrelevant in the limit $q \rightarrow 0$. Also $s - u = 2s - M_\alpha^2 - M_\beta^2$ at the point of interest, $t = 0$. Hence the above identity reduces to

$$\not{p}'(\not{q} + \not{q}') + (\not{q} + \not{q}')\not{p} = 2s - M_\alpha^2 - M_\beta^2 . \tag{19}$$

Addition of (17) and (18), and the use of (19) yields

$$\begin{aligned}
M_{1s}(M_\alpha - M_\beta) &= \bar{u}(p') \gamma_5 (s - M_\alpha M_\beta + M_\delta M_\alpha - M_\delta M_\beta) u(p) , \\
M_{1u}(M_\alpha - M_\beta) &= \bar{u}(p') (s - M_\alpha^2 - M_\beta^2 + M_\alpha M_\beta + M_\delta M_\alpha - M_\delta M_\beta) \gamma_5 u(p) , \\
M_{2s}(M_\alpha + M_\beta) &= \bar{u}(p') (-s - M_\alpha M_\beta + M_\delta M_\alpha + M_\delta M_\beta) u(p) , \\
M_{2u}(M_\alpha + M_\beta) &= \bar{u}(p') (s - M_\alpha^2 - M_\beta^2 - M_\alpha M_\beta + M_\delta M_\alpha + M_\delta M_\beta) u(p) .
\end{aligned} \tag{20}$$

The delta functions in (15) allow further simplification of the expressions (20) to give

$$\begin{aligned}
M_{1s} &= \bar{u}(p') \gamma_5 \frac{(M_\delta + M_\alpha)(M_\delta - M_\beta)}{M_\alpha - M_\beta} u(p) , \\
M_{1u} &= \bar{u}(p') \gamma_5 \frac{(M_\gamma + M_\beta)(M_\alpha - M_\gamma)}{M_\alpha - M_\beta} u(p) , \\
M_{2s} &= \bar{u}(p') \frac{(M_\alpha - M_\delta)(M_\delta - M_\beta)}{M_\alpha + M_\beta} u(p) , \\
M_{2u} &= \bar{u}(p') \frac{(M_\alpha - M_\gamma)(M_\gamma - M_\beta)}{M_\alpha + M_\beta} u(p) .
\end{aligned} \tag{21}$$

With these values for the M 's, expression (15) becomes

$$\begin{aligned}
M_{\rho\alpha} - M_{\alpha\beta}^\dagger &= 2\pi i \left[KC \frac{(M_\alpha + M_\delta)(M_\delta - M_\beta)}{M_\alpha - M_\beta} \delta(s - M_\delta^2) - cK \right. \\
&\times \frac{(M_\alpha - M_\gamma)(M_\gamma + M_\beta)}{M_\alpha - M_\beta} \delta(u - M_\gamma^2) \left. \right] \gamma_5 + 2\pi i \left[-KV \frac{(M_\alpha - M_\delta)(M_\delta - M_\beta)}{M_\alpha + M_\beta} \right. \\
&\times \delta(s - M_\delta^2) + vK \frac{(M_\alpha - M_\gamma)(M_\gamma - M_\beta)}{M_\alpha + M_\beta} \delta(u - M_\gamma^2) \left. \right] .
\end{aligned} \tag{22}$$

Comparing (6) and (22) we obtain

$$\begin{aligned}
\text{Im} H_1^P &= -\pi KV \frac{(M_\alpha - M_\delta)(M_\delta - M_\beta)}{M_\alpha + M_\beta} \delta(s - M_\delta^2) + \pi vK \frac{(M_\alpha - M_\gamma)(M_\gamma - M_\beta)}{M_\alpha + M_\beta} \delta(u - M_\gamma^2) , \\
\text{Im} H_2^P &= -\pi KC \frac{(M_\alpha + M_\delta)(M_\delta - M_\beta)}{M_\alpha - M_\beta} \delta(s - M_\delta^2) + \pi cK \frac{(M_\alpha - M_\gamma)(M_\gamma + M_\beta)}{M_\alpha - M_\beta} \delta(u - M_\gamma^2) .
\end{aligned} \tag{23}$$

Inserting these values for the $\text{Im} H_i^P$ in the dispersion integral (1) we finally obtain the desired pole terms:

$$\begin{aligned}
\text{Re} H_1^P(M_\alpha^2) &= \frac{M_\alpha - M_\beta}{(M_\alpha + M_\delta)(M_\delta + M_\beta)} KV - \frac{M_\alpha - M_\beta}{(M_\alpha + M_\gamma)(M_\gamma + M_\beta)} vK , \\
\text{Re} H_2^P(M_\alpha^2) &= \frac{M_\alpha + M_\beta}{(M_\alpha - M_\delta)(M_\delta + M_\beta)} KC - \frac{M_\alpha + M_\beta}{(M_\alpha + M_\gamma)(M_\gamma - M_\beta)} cK .
\end{aligned} \tag{24}$$

VII. SYMMETRY BREAKING IN THE TADPOLE MODEL

The weak vertex has been written as

$$\langle \beta(p') | \mathcal{H} | \delta(p_n) \rangle = \frac{1}{(2\pi)^3} \frac{1}{N_\beta N_\delta} \bar{u}(p') (c_{\beta\delta} - \gamma_5 v_{\beta\delta}) u(p) . \quad (1)$$

The form factor $v_{\beta\delta}$ is zero in the limit of exact SU(3) symmetry because it has been shown that the matrix element of the pv Hamiltonian between baryon states belonging to a same octet vanishes if exact SU(3) symmetry is assumed.

In a broken symmetry scheme such as the present one, $v_{\beta\delta}$ does not necessarily vanish. In fact, a tadpole model³³ has been proposed to evaluate $v_{\beta\delta}$. In this model the matrix element

$$\langle \beta(p') | \mathcal{H}^{pv} | \delta(p_n) \rangle = \frac{1}{(2\pi)^3} \frac{1}{N_\beta N_\delta} \bar{u}(p') \gamma_5 u(p) v_{\beta\delta} \quad (2)$$

is given by the tadpole Feynman diagram, Figure 3. Here the meson K_1^0 is a spin zero uncharged particle with CP=+1 which can communicate with the vacuum in the presence of the weak interaction. The tadpole diagram yields

$$\langle \beta(p') | \mathcal{H}^{pv} | \delta(p_n) \rangle = \frac{1}{(2\pi)^3} \frac{1}{N_\beta N_\delta} \bar{u}(p') i \gamma_5 K_{\beta\delta}^{(\kappa_1^0)} u(p) \frac{A(\kappa_1^0 \rightarrow 0)}{m_\kappa^2 - (p' - p_n)^2} \quad (3)$$

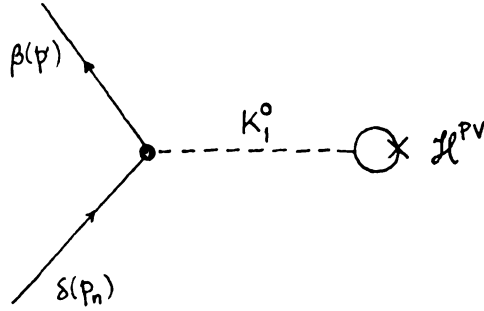


Figure 3. Tadpole diagram

where $A(K_1^0 \rightarrow 0)$ is the transition amplitude of the K_1^0 into the vacuum.

Comparing (2) and (3) at zero momentum transfer we obtain

$$V_{\beta\delta} = i K_{\beta\delta}^{(K_1^0)} \frac{A(K_1^0 \rightarrow 0)}{m_K^2} . \quad (4)$$

Since we are working with the cartesian octet states³⁴ we need the upper index in $K_{\beta\delta}^{(K_1^0)}$ in this system. In our phase convention

$$|K_1^0\rangle = \frac{i}{\sqrt{2}} (|K^0\rangle - |\bar{K}^0\rangle) = \frac{i}{\sqrt{2}} \left[\frac{1}{\sqrt{2}} (P_6 - iP_7) - \frac{1}{\sqrt{2}} (P_6 + iP_7) \right]$$

so that $|K_1^0\rangle = P_7$, where $P_1, \dots, P_8$ are the cartesian pseudo-scalar meson octet states.

The strong coupling constant in (4) is, therefore, given by

$$K_{\beta\delta}^7 = 2 g_{\pi NN} (\alpha F^7 + (1-\alpha) D^7)_{\beta\delta} . \quad (5)$$

We rewrite (4) as

$$V_{\beta\delta} = 2iG(\alpha F_{\beta\delta}^7 + (1-\alpha) D_{\beta\delta}^7) \quad (6)$$

where

$$G = g_{\pi NN} \frac{A(K_1^0 \rightarrow 0)}{m_K^2} . \quad (7)$$

Soft-pion techniques can be applied to evaluate the amplitude $A(K_1^0 \rightarrow 0)$ in terms of the amplitude $A(K_1^0 \rightarrow 2\pi^0)$ for the decay $K_1^0 \rightarrow 2\pi^0$. This latter amplitude can be defined more precisely according to

$$\langle \pi^0(q) \pi^0(q') | \mathcal{H}^{PV} | K_1^0(k) \rangle = \frac{1}{(2\pi)^{9/2}} \frac{1}{(2q_0 2q'_0 2k_0)^{1/2}} A(K_1^0 \rightarrow 2\pi^0) . \quad (8)$$

On the other hand the LSZ reduction formalism allow us to write

$$\langle \pi(q) \pi(q') | \mathcal{H}^{PV} | K(k) \rangle = \frac{i}{(2\pi)^{3/2} (2q'_0)^{1/2}} \int d^4x e^{iq' \cdot x} (\Box^2 + m_\pi^2) \langle \pi(q) | T(\phi_3(x) \mathcal{H}^{PV}(0)) | K(k) \rangle . \quad (9)$$

We replace the pion field operator by the divergence of the axial-vector current according to the PCAC relation, let q' go off the mass shell, and integrate by parts twice. The result is

$$\langle \pi(q) \pi(q') | \mathcal{H}^{PV} | K(k) \rangle = \frac{i(m_\pi^2 - q'^2)}{c(2\pi)^{3/2} (2q'_0)^{1/2}} \int d^4x e^{iq' \cdot x} \langle \pi(q) | T(\partial_\lambda F_3^{5\lambda}(x) \mathcal{H}^{PV}(0)) | K(k) \rangle . \quad (10)$$

From (8) and (10) we get

$$A(K_1^0 \rightarrow 2\pi^0) = i(2\pi)^3 (2q_0)^{1/2} (2k_0)^{1/2} \frac{m_\pi^2 - q'^2}{c} \int d^4x e^{iq' \cdot x} \\ \times \langle \pi(q) | T(\partial_\lambda F_3^{5\lambda}(x) \mathcal{H}^{PV}(0)) | K(k) \rangle . \quad (11)$$

ext, the usual surface term is introduced,

$$T^\lambda \equiv -i(2\pi)^3 (2q_0)^{1/2} (2k_0)^{1/2} \int d^4x e^{iq' \cdot x} \langle \pi(q) | T(F_3^{5\lambda}(x) \mathcal{H}^{PV}(0)) | K(k) \rangle . \quad (12)$$

Multiplying (12) by q'_λ and integrating by parts we get

$$q'_\lambda T^\lambda = (2\pi)^3 (2q_0 2k_0)^{1/2} \int d^4x e^{iq' \cdot x} \langle \pi(q) | \partial_\lambda T(F_3^{5\lambda}(x) \mathcal{H}^{PV}(0)) | K(k) \rangle . \quad (13)$$

The identity

$$\partial_\lambda T(F_3^{5\lambda}(x) \mathcal{H}(0)) = T(\partial_\lambda F_3^{5\lambda}(x) \mathcal{H}(0)) + \delta(x_0) [F_3^{50}(x), \mathcal{H}^{PV}(0)]_- \quad (14)$$

allow us to rewrite (13) as

$$q'_\lambda T^\lambda = (2\pi)^3 (2q_0 2k_0)^{1/2} \int d^4x e^{iq' \cdot x} \langle \pi(q) | T(\partial_\lambda F_3^{5\lambda}(x) \mathcal{H}^{PV}(0)) | K(k) \rangle \\ + (2\pi)^3 (2q_0 2k_0)^{1/2} \int d^4x e^{iq' \cdot x} \delta(x_0) \langle \pi(q) | [F_3^{50}(x), \mathcal{H}^{PV}(0)]_- | K(k) \rangle . \quad (15)$$

Taking the limit $q' \rightarrow 0$ we get

$$A(K_1^0 \rightarrow 2\pi^0) = -i \frac{m_\pi^2}{c} (2\pi)^3 (2q_0 2k_0)^{1/2} \langle \pi(q) | [F_3^5, \mathcal{H}^{PV}]_- | K(k) \rangle \quad (16)$$

where we have dropped the surface term.

To get the desired amplitude $A(K_1^0 \rightarrow 0)$ we have to

contract the remaining pion. Let

$$\mathcal{H}' \equiv [F_3^5, \mathcal{H}^{PV}]_- . \quad (17)$$

By repeating the previous procedure in obtaining (16) to the matrix element of $\mathcal{H}'$ we are led to

$$\langle \pi(q) | \mathcal{H}' | K(k) \rangle = \frac{i m_\pi^2}{C} \frac{1}{(2\pi)^{3/2}} \frac{1}{(2q_0)^{1/2}} \langle 0 | [F_3^5, \mathcal{H}']_- | K(k) \rangle . \quad (18)$$

Inserting (18) in (16) we obtain

$$A(K_1^0 \rightarrow 2\pi^0) = \frac{m_\pi^4}{C^2} (2\pi)^{3/2} (2k_0)^{1/2} \langle 0 | [F_3^5, [F_3^5, \mathcal{H}^{PV}]_-]_- | K(k) \rangle . \quad (19)$$

Application of the property (II.13) of the weak Hamiltonian yields

$$[F_3^5, [F_3^5, \mathcal{H}^{PV}]_-]_- = F_3 [F_3, \mathcal{H}^{PV}]_- - [F_3, \mathcal{H}^{PV}] F_3 . \quad (20)$$

The matrix element of this commutator is

$$\langle 0 | [F_3^5, [F_3^5, \mathcal{H}^{PV}]_-]_- | K(k) \rangle = \langle 0 | \mathcal{H}^{PV} F_3 F_3 | K(k) \rangle . \quad (21)$$

But

$$F_3 F_3 | K_1^0 \rangle = \frac{1}{4} | K_1^0 \rangle$$

so that

$$\langle 0 | [F_3^5, [F_3^5, \mathcal{H}^{PV}]_-]_- | K_1^0 \rangle = \frac{1}{4} \langle 0 | \mathcal{H}^{PV} | K_1^0 \rangle . \quad (22)$$

From the definition for the transition amplitude $A(K_1^0 \rightarrow 0)$,

$$\langle 0 | \mathcal{H}^{PV} | K_1^0 \rangle = \frac{1}{(2\pi)^{3/2}} \frac{1}{(2k_0)^{1/2}} A(K_1^0 \rightarrow 0) \quad (23)$$

and equations (22) and (19) we obtain

$$A(K_1^0 \rightarrow 0) = \frac{4c^2}{m_\pi^4} A(K_1^0 \rightarrow 2\pi^0) . \quad (24)$$

With the numerical value for the constant c given by

$$c = \frac{0.95}{\sqrt{2}} m_\pi^3$$

we finally have the desired relationship:

$$A(K_1^0 \rightarrow 0) = 2 F_\pi^2 A(K_1^0 \rightarrow 2\pi^0) \quad (25)$$

where $F_\pi = 0.95 m_\pi$.

Note that the amplitude on the RHS of (25) is the soft-pion amplitude for the decay $K_1^0 \rightarrow 2\pi^0$. In a hard-pion study³⁵ of the decays $K \rightarrow 2\pi$ it has been found that the physical amplitude differs by only about 10% from the one given by the soft-pion technique so that the use of the experimental value of $A(K_1^0 \rightarrow 2\pi^0)$ in (25) cannot be a large source of error.

So far we have assumed that the coupling constants of the vertices $\bar{B}BK$ for the various decay modes are given by the SU(3) values, in agreement with a dispersion theoretical evaluation³⁶ of some of these coupling constants. However, bootstrap calculations³⁷ have suggested

considerable reduction from the $SU(3)$ values. For the numerical analysis in the next section we consider a middle point between these two extremes by properly inserting a suppression factor.

VIII. NUMERICAL ANALYSIS AND RESULTS

Previous sections have considered the derivation of the general expressions for the three pieces which make up the s- and p-wave amplitudes for the nonleptonic decay $\alpha \rightarrow \beta + \pi^a$. In this section those pieces are put together and the final form of the A,B amplitudes are written in terms of the adjustable parameters which are determined by means of a χ^2 -fit to the experimental values of the amplitudes.

There are a few details about coupling constants that must be clarified before the final form for the amplitudes can be written. Recall that the $\bar{B}B\kappa$ weak vertex has the SU(3) symmetric coupling constants:

$$\begin{aligned} C_{\beta\delta} &= 2 \left(f F_{\beta\delta}^6 + d D_{\beta\delta}^6 \right) , \\ V_{\beta\delta} &= 2iG \left(\alpha F_{\beta\delta}^7 + (1-\alpha) D_{\beta\delta}^7 \right) . \end{aligned} \quad (1)$$

In the tadpole model G takes the form

$$G = \xi g_{\pi NN} \frac{A(K_1^0 \rightarrow 0)}{m_K^2} , \quad A(K_1^0 \rightarrow 0) = 2 F_\pi^2 A(K_1^0 \rightarrow 2\pi^0) . \quad (2)$$

This value of G differs from the one defined in section VI by the factor ξ , which is a corrective suppression factor put by hand in order to account for the coupling constant

shift from the SU(3) values in the $\bar{B}BK$ vertex as well as for the 10% uncertainty involved in the use of the physical value of $A(K_1^0 \rightarrow 2\pi^0)$ instead of the unknown soft-pion limiting value. Also ξ takes care of the extrapolation to zero momentum transfer squared in the tadpole model. For reasons given previously and assuming relatively small coupling constant shift from SU(3) values, we expect ξ to be nearly unity.

As we have seen the resonance terms come from the exchange of mesons in the t-channel, i.e., from t-channel Regge poles, Figure 4. Regarding the SU(3) structure of the

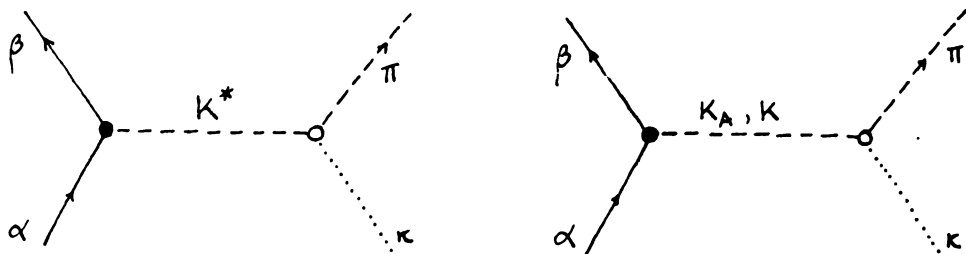


Figure 4. t-channel Regge poles.

strong and weak vertices in these resonance terms, we assume the coupling of the K^* to the baryons to be of the F type only,³⁸ whereas the $\bar{B}BK_A$ and $\bar{B}BK$ vertices have both F and D type couplings with the same mixing parameter α for both vertices. As for the weak vertex, we take it to be proportional to D^6 for both the s- and p-wave amplitudes.²⁹

Now we have all the information needed to write

the s- and p-wave amplitudes:

$$A = \frac{\sqrt{2}}{F_\pi} [F^a, C]_{\beta\alpha} + \frac{(M_\alpha - M_\rho) K_{\rho\delta}^a v_{\delta\alpha}}{(M_\alpha + M_\delta)(M_\delta + M_\rho)} - \frac{(M_\alpha - M_\rho) v_{\rho\gamma} K_{\gamma\alpha}^a}{(M_\alpha + M_\gamma)(M_\gamma + M_\rho)} \\ + \gamma_v K_{\rho\alpha}^b d_{6ab} (M_\alpha^{2\alpha_v} - M_\rho^{2\alpha_v}) \tan \frac{\pi}{2} \alpha_v, \quad (3)$$

$$B = \frac{\sqrt{2}}{F_\pi} [F^a, v]_{\beta\alpha} + \frac{(M_\alpha + M_\rho) K_{\rho\delta}^a c_{\delta\alpha}}{(M_\alpha - M_\delta)(M_\delta + M_\rho)} - \frac{(M_\alpha + M_\rho) c_{\rho\gamma} K_{\gamma\alpha}^a}{(M_\alpha + M_\gamma)(M_\gamma - M_\rho)} \\ + \gamma' K_{\rho\alpha}^b d_{6ab} (M_\alpha^{2\alpha_A} - M_\rho^{2\alpha_A}) \tan \frac{\pi}{2} \alpha_A, \quad (4)$$

where we have extracted the SU(3) dependence from γ_v and γ' defined in (V.27). For convenience we rewrite the above expressions in terms of equal-time-commutator (E), pole (P), and resonance (R) parts in the following form:

$$A = \frac{1}{F_\pi} A^E + R \frac{G}{m_\pi} A^P + A^R, \\ B = \frac{G}{F_\pi} B^E + \frac{R}{m_\pi} B^P + B^R, \quad (5) \\ R \equiv \frac{g_{\pi NN} m_\pi}{\sqrt{2} M_N}.$$

The expressions for A^E , A^P , etc. for the various decays are displayed in Tables 1, 2, 3, and 4. There the parameters γ_v and γ' have been redefined to absorb overall factors.

For the numerical evaluation the masses in the resonance terms are specified in GeV and the values of the

fixed constants are

$$\frac{1-\alpha}{\alpha} \simeq \sqrt{3} \quad , \quad \frac{g_{\pi NN}^2}{4\pi} = 14.6 \quad , \quad A(K_1^0 \rightarrow 2\pi^0) = 2.58 \times 10^{-4} \text{ MeV}. \quad (6)$$

A priori the coupling constant shift should be about 25%, corresponding to the mass splitting within the baryon octet. In fact, we have found that the χ^2 increases rapidly for $\xi \leq 0.7$ and increases gently for $\xi \geq 0.7$. This is consistent with the expectation of ξ being nearly unity as discussed before. With $\xi=0.7$ we obtain $G=11.25 m_\pi$ in units of $10^5 m_\pi^{-\frac{1}{2}} \text{ sec}^{-\frac{1}{2}}$. This leaves four adjustable parameters: f , d , γ_V , γ' . The best fit to the experimental amplitudes gave the following values for these parameters:

$$\frac{d}{f} = -1.09 \quad ,$$

$$f = 1.513 m_\pi \quad , \quad \gamma_V = -20.52 \quad , \quad \gamma' = -3.25 \quad , \quad (7)$$

in units of $10^5 m_\pi^{-\frac{1}{2}} \text{ sec}^{-\frac{1}{2}}$. The corresponding theoretical amplitudes are displayed in Table 5 with the contribution of each part given explicitly. Also shown are the experimental values.³⁹

Table 5 shows a very good overall fit, being sufficiently accurate to discriminate between the two experimental values for the decay Σ_0^+ . The only appreciable deviation occurs in the amplitude $A(\Sigma_+^+)$ in which none of the four adjustable parameters appears. Then, in the framework of the current×current interaction and octet dominance, the observed value $A(\Sigma_+^+) \approx 0$ requires the symmetry

breaking terms to be negligible. However, deviations from octet dominance and from unitary symmetry have been found to occur in many other decays, indicating that the interpretation of the observed value $A(\Sigma_+^+) \approx 0$ as evidence for the simultaneous validity of octet dominance and unitary symmetry to a high degree of accuracy does not seem plausible. Instead, we argue that these two types of deviations tend to cancel each other in the amplitude $A(\Sigma_+^+)$ so as to reproduce the observed value. On the other hand, the vanishing p-wave amplitude, $B(\Sigma_-) \approx 0$, has all three parts combined to reproduce the observed value. Thus, having several adjustable parameters, the addition of the 27 representation can be made without spoiling the present good fit for this amplitude. Similar arguments hold for the other amplitudes. In the approximation of octet dominance, therefore, we do not see the nonvanishing result for $A(\Sigma_+^+)$ in Table 5 as unreasonable, although the validity of our arguments remains to be seen by actual evaluation of the 27 contribution and re-search of the parameters.

Another important result displayed in Table 5 concerns the relative magnitude of the resonance contribution to the s-wave amplitude. Note that a suitable choice of the d/f ratio (≈ -0.3) makes the current algebra term for the s-wave proportional to the observed amplitude. This has previously led to the belief that the current algebra term was the dominant one, so that the assumption of

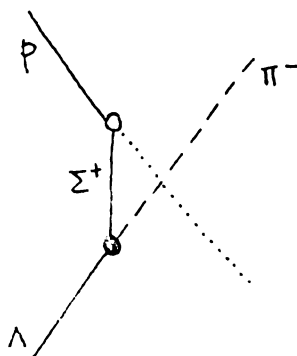
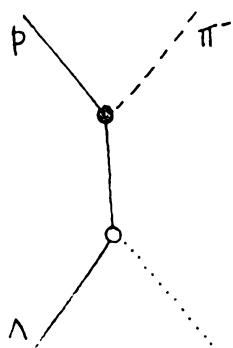
negligible higher-mass baryon contribution to the s-wave was considered to be on a good footing. Table 5 does not support this assumption. In contrast, the assumption works well for the p-wave, explaining the rough fit of the old p-wave pole model results.

Finally, the magnitude of the current algebra contribution to the p-wave amplitude indicates a relatively large violation of unitary symmetry, a result quite different from that of any previous work.

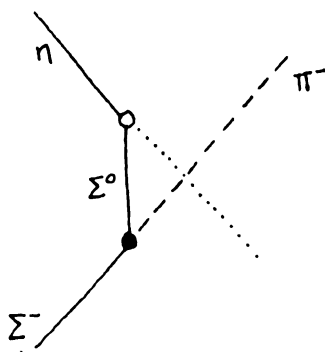
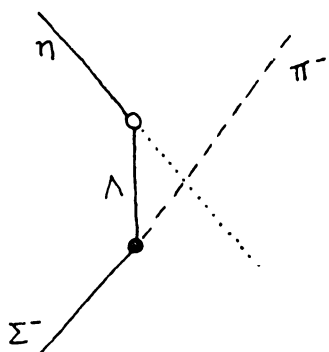
In summary, the main results of our analysis are:

- (a) Our fit to the experimental values is a considerable improvement over that of previous work.
- (b) The usual neglect of the higher-mass baryon resonances is found to be unwarranted for the s-wave. However, it is found to be fairly good for the p-wave.
- (c) A relatively large violation of unitary symmetry is present.
- (d) Deviation from octet dominance is needed to explain the observed value of $A(\Sigma_+^+) \approx 0$.
- (e) No simple dynamical explanation seems to emerge for the value of $B(\Sigma_-^-) \approx 0$.

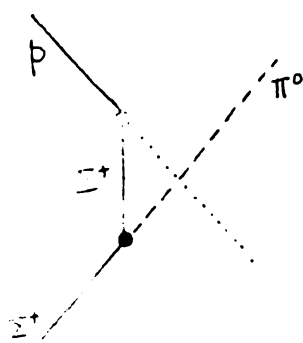
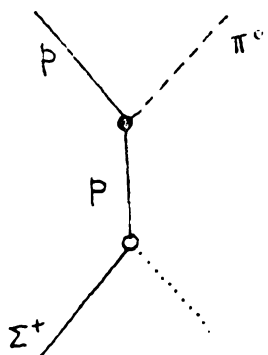
$$\Lambda \rightarrow p + \pi^-$$



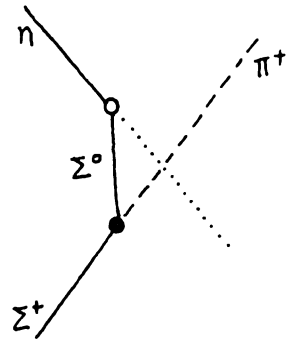
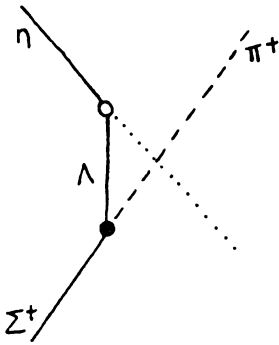
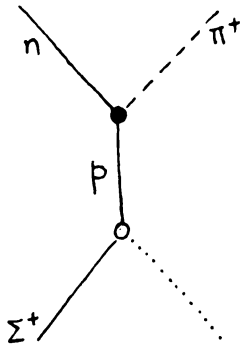
$$\Sigma^- \rightarrow n + \pi^-$$



$$\Sigma^+ \rightarrow p + \pi^0$$



$$\Sigma^+ \rightarrow n + \pi^+$$



$$\Xi^- \rightarrow \Lambda + \pi^-$$

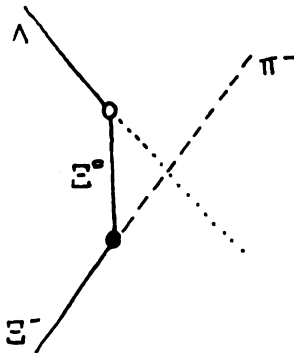
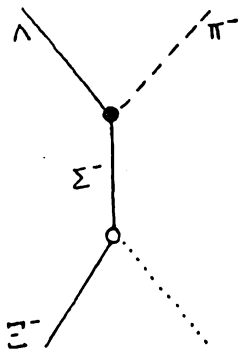


Figure 5. Pole diagrams for the various decay modes.



Table 1. Equal-time-commutator and resonance parts of the s-wave amplitudes.

Decay	A^E	A^R
$\Lambda \rightarrow p + \pi^-$	$\frac{1}{\sqrt{6}} (3f + d)$	$\gamma_v \sqrt{\frac{3}{2}} (M_\Lambda^{2\alpha_v} - M_N^{2\alpha_v}) \tan \frac{\pi}{2} \alpha_v$
$\Sigma^- \rightarrow n + \pi^-$	$f - d$	$\gamma_v (M_\Sigma^{2\alpha_v} - M_N^{2\alpha_v}) \tan \frac{\pi}{2} \alpha_v$
$\Sigma^+ \rightarrow p + \pi^0$	$\frac{1}{\sqrt{2}} (f - d)$	$\gamma_v \frac{1}{\sqrt{2}} (M_\Sigma^{2\alpha_v} - M_N^{2\alpha_v}) \tan \frac{\pi}{2} \alpha_v$
$\Sigma^+ \rightarrow n + \pi^+$	0	0
$\Xi^- \rightarrow \Lambda + \pi^-$	$\frac{1}{\sqrt{6}} (3f - d)$	$\gamma_v \sqrt{\frac{3}{2}} (M_\Xi^{2\alpha_v} - M_\Lambda^{2\alpha_v}) \tan \frac{\pi}{2} \alpha_v$

Table 2. Equal-time-commutator and resonance parts of the p-wave amplitudes.

Decay	B^E	B^R
$\Lambda \rightarrow p + \pi^-$	$\frac{1}{\sqrt{6}} (1 + 2\alpha)$	$\gamma' \frac{1}{\sqrt{6}} (2\alpha + 1) (M_\Lambda^{2\alpha_A} - M_N^{2\alpha_A}) \tan \frac{\pi}{2} \alpha_A$
$\Sigma^- \rightarrow n + \pi^-$	$(2\alpha - 1)$	$\gamma' (2\alpha - 1) (M_\Sigma^{2\alpha_A} - M_N^{2\alpha_A}) \tan \frac{\pi}{2} \alpha_A$
$\Sigma^+ \rightarrow p + \pi^0$	$\frac{1}{\sqrt{2}} (2\alpha - 1)$	$\gamma' \frac{1}{\sqrt{2}} (2\alpha - 1) (M_\Sigma^{2\alpha_A} - M_N^{2\alpha_A}) \tan \frac{\pi}{2} \alpha_A$
$\Sigma^+ \rightarrow n + \pi^+$	0	0
$\Xi^- \rightarrow \Lambda + \pi^-$	$\frac{1}{\sqrt{6}} (4\alpha - 1)$	$\gamma' \frac{1}{\sqrt{6}} (4\alpha - 1) (M_\Xi^{2\alpha_A} - M_\Lambda^{2\alpha_A}) \tan \frac{\pi}{2} \alpha_A$

Table 3. Pole part of the s-wave amplitudes.

Decay	A^P
$\Lambda \rightarrow p + \pi^-$	$\frac{1}{\sqrt{6}} (1+2\alpha) \frac{M_\Lambda - M_N}{M_N + M_N} \frac{2M_N}{M_\Lambda + M_N} + \frac{2}{\sqrt{6}} (1-\alpha)(1-2\alpha) \frac{M_\Lambda - M_N}{M_\Lambda + M_\Sigma} \frac{2M_N}{M_\Sigma + M_N}$
$\Sigma^- \rightarrow n + \pi^-$	$-\frac{1}{3}(1-\alpha)(1+2\alpha) \frac{M_\Sigma - M_N}{M_\Sigma + M_\Lambda} \frac{2M_N}{M_\Lambda + M_N} + \alpha(1-2\alpha) \frac{M_\Sigma - M_N}{M_\Sigma + M_\Sigma} \frac{2M_N}{M_\Sigma + M_N}$
$\Sigma^+ \rightarrow p + \pi^0$	$\frac{1}{\sqrt{2}} (1-2\alpha) \frac{M_\Sigma - M_N}{M_N + M_N} \frac{2M_N}{M_\Sigma + M_N} - \frac{2}{\sqrt{2}} \alpha(1-2\alpha) \frac{M_\Sigma - M_N}{M_\Sigma + M_\Sigma} \frac{2M_N}{M_\Sigma + M_N}$
$\Sigma^+ \rightarrow n + \pi^+$	$-(1-2\alpha) \frac{M_\Sigma - M_N}{M_N + M_N} \frac{2M_N}{M_\Sigma + M_N} - \frac{1}{3}(1-\alpha)(1+2\alpha) \frac{M_\Sigma - M_N}{M_\Sigma + M_\Lambda} \frac{2M_N}{M_\Lambda + M_N}$
	$\alpha(1-2\alpha) \frac{M_\Sigma - M_N}{M_\Sigma + M_\Sigma} \frac{2M_N}{M_\Sigma + M_N}$
$\Xi^- \rightarrow \Lambda + \pi^-$	$\frac{2}{\sqrt{6}} (1-\alpha) \frac{M_\Xi - M_\Lambda}{M_\Sigma + M_\Lambda} \frac{2M_N}{M_\Xi + M_\Sigma} - \frac{1}{\sqrt{6}} (1-2\alpha)(4\alpha-1) \frac{M_\Xi - M_\Lambda}{M_\Xi + M_\Xi} \frac{2M_N}{M_\Xi + M_\Lambda}$

Table 4. Pole part of the p-wave amplitudes.

Decay	B^P
$\Lambda \rightarrow p + \pi^-$	$\frac{1}{\sqrt{6}}(3f+d) \frac{M_\Lambda + M_N}{M_N + M_N} \frac{2M_N}{M_\Lambda - M_N} - \frac{2}{\sqrt{6}}(1-\alpha)(f-d) \frac{M_\Lambda + M_N}{M_\Lambda + M_\Sigma} \frac{2M_N}{M_\Sigma - M_N}$
$\Sigma^- \rightarrow n + \pi^-$	$\alpha(f-d) \frac{M_\Sigma + M_N}{M_\Sigma + M_\Sigma} \frac{2M_N}{M_\Sigma - M_N} - \frac{1}{3}(1-\alpha)(3f+d) \frac{M_\Sigma + M_N}{M_\Lambda + M_\Sigma} \frac{2M_N}{M_\Lambda - M_N}$
$\Sigma^+ \rightarrow p + \pi^0$	$-\frac{1}{\sqrt{2}}(f-d) \frac{M_\Sigma + M_N}{M_N + M_N} \frac{2M_N}{M_\Sigma - M_N} + \frac{2}{\sqrt{2}}\alpha(f-d) \frac{M_\Sigma + M_N}{M_\Sigma + M_\Sigma} \frac{2M_N}{M_\Sigma - M_N}$
$\Sigma^+ \rightarrow n + \pi^+$	$(f-d) \frac{M_\Sigma + M_N}{M_N + M_N} \frac{2M_N}{M_\Sigma - M_N} - \frac{1}{3}(1-\alpha)(3f+d) \frac{M_\Sigma + M_N}{M_\Sigma + M_\Lambda} \frac{2M_N}{M_\Lambda - M_N}$ $-\alpha(f-d) \frac{M_\Sigma + M_N}{M_\Sigma + M_\Sigma} \frac{2M_N}{M_\Sigma - M_N}$
$\Xi^- \rightarrow \Lambda + \pi^-$	$\frac{2}{\sqrt{6}}(1-\alpha)(f+d) \frac{M_\Xi + M_\Lambda}{M_\Sigma + M_\Lambda} \frac{2M_N}{M_\Xi - M_\Sigma} - \frac{1}{\sqrt{6}}(1-2\alpha)(3f-d) \frac{M_\Xi + M_\Lambda}{M_\Xi + M_\Xi} \frac{2M_N}{M_\Xi - M_\Lambda}$

Table 5. Best fit solution to both the s- and p-wave amplitudes, and experimental values, in units of $10^5 m_\pi^{-\frac{1}{2}} \text{ sec}^{-\frac{1}{2}}$.

Decay	ETC	Baryon Pole	Resonance	Total (Theory)	Experimental values
$A(\Lambda_-^0)$	1.241	1.113	-0.909	1.444	1.545 ± 0.024
$A(\Sigma_-^-)$	3.330	-0.437	-1.047	1.846	1.859 ± 0.017
$A(\Sigma_0^+)$	2.355	0.151	-0.740	1.766	1.568 ± 0.142 (1.155 ± 0.187)
$A(\Sigma_+^+)$	0	-0.942	0	-0.942	0.016 ± 0.034
$A(\Xi_-^-)$	2.660	0.489	-0.954	2.195	2.020 ± 0.029
$B(\Lambda_-^0)$	8.374	4.090	-1.390	11.074	10.644 ± 0.475
$B(\Sigma_-^-)$	-3.173	2.357	0.697	-0.120	-0.549 ± 0.386
$B(\Sigma_0^+)$	-2.244	-11.203	0.493	-12.954	-11.573 ± 1.880 (-15.713 ± 1.420)
$B(\Sigma_+^+)$	0	18.200	0	18.200	19.078 ± 0.347
$B(\Xi_-^-)$	2.244	-9.780	-0.278	-7.814	-6.831 ± 0.574

IX. RADIATIVE WEAK DECAYS

The remarkably good fit of our model for nonleptonic decays of hyperons has led to results quite different from those of previous work on the same problem. In particular, the relative importance of the symmetry breaking terms has afforded a new interpretation of the well-known experimental result $A(\Sigma_+^+) \approx 0$.

Apart from explaining nonleptonic decays, the analysis also yields further consequences in the form of predictions for the two-body radiative weak decays of hyperons. Lack of sufficient data on these processes does not allow a full test of the theory, but the few available experimental numbers are in good agreement with our calculations.

The following two-body radiative weak decays of hyperons will be considered in detail:

$$\begin{aligned}\Lambda &\rightarrow n + \gamma \\ \Sigma^+ &\rightarrow p + \gamma \\ \Sigma^0 &\rightarrow n + \gamma \\ \Xi^0 &\rightarrow \Lambda + \gamma \\ \Xi^0 &\rightarrow \Sigma^0 + \gamma \\ \Xi^- &\rightarrow \Sigma^- + \gamma\end{aligned}\tag{1}$$

There exists no experimental information on these, except for the second decay.

The above decays have the general form

$$\alpha(p) \rightarrow \beta(p') + \gamma(k) \quad (2)$$

where α, β also denote SU(3) indices for the respective baryons. The invariant amplitudes, A' and B' , for the radiative weak decay (2) are defined as

$$\langle \beta(p') \gamma(k) | \mathcal{H} | \alpha(p) \rangle = \frac{1}{(2\pi)^{9/2}} \frac{1}{(2k_0)^{1/2}} \frac{1}{N_\alpha N_\beta} e \bar{u}(p') (A' + \gamma_5 B') \not{k} u(p) \quad (3)$$

on general grounds of Lorentz invariance.⁴⁰

The method used here to compute A' and B' requires the consideration of the companion three-body radiative weak decay

$$\alpha(p) \rightarrow \beta(p') + \pi^0(q') + \gamma(k) \quad (4)$$

for which we define the transition amplitude R^μ as

$$\langle \beta(p') \pi(q') \gamma(k) | \mathcal{H} | \alpha(p) \rangle = \frac{i}{(2\pi)^6} \frac{1}{(2k_0)^{1/2}} \frac{1}{(2q_0')^{1/2}} \frac{1}{N_\alpha N_\beta} \epsilon_\mu R^\mu \quad (5)$$

where ϵ_μ is the polarization 4-vector of the photon. An outline of the procedure helps to introduce the basic ideas and to guide the derivation of the final expressions for the invariant amplitudes.

The soft-pion theorem applied to the three-body

radiative weak decay (4) yields the familiar formula

$$\epsilon.R^N(q'=0) = \lim_{q' \rightarrow 0} \left[\frac{m_\pi^2}{c} \epsilon_\mu q'_\lambda S^{\mu\lambda}(q') - \epsilon.R^B(q') \right] + \frac{m_\pi^2}{c} (2\pi)^{9/2} (2k_0)^{1/2} \\ \times N_\alpha N_\beta \langle \beta(p') \gamma(k) | [F_3, \mathcal{H}]_- | \alpha(p) \rangle. \quad (6)$$

In section X we show that the surface-Born term in (6) is zero in the soft-pion limit as long as we use the derivative coupling for the strong $\bar{B}B\pi$ vertex. Then the nonBorn soft-pion amplitude on the LHS of (6) is proportional to the ETC term which is essentially the amplitude for the desired two-body radiative weak decay.

In section XI the amplitude R^N for $\alpha \rightarrow \beta + \pi^0 + \gamma$ is related to M^N , the amplitude for the nonradiative weak decay $\alpha \rightarrow \beta + \pi^0$, in the soft-pion limit via Low's soft-photon theorem.⁴¹ Since the parameters for the amplitude M^N have already been determined, through this chain of low-energy theorems we are able to express the invariant amplitudes, A' and B' , in terms of known parameters and predict the values of several experimentally accessible quantities such as decay rates, branching ratios, and asymmetry parameters.

As usual the starting point in deriving the soft-pion theorem (6) is the LSZ reduction formula for the matrix element of the Hamiltonian:

$$\langle \beta(p') \pi(q') \gamma(k) | \mathcal{H}(0) | \alpha(p) \rangle = \frac{i}{(2\pi)^{3/2}} \frac{1}{(2q_0)^{1/2}} \int d^4x e^{iq'x} (\Box^2 + m_\pi^2) \\ \times \langle \beta(p') \gamma(k) | T(\phi_3(x) \mathcal{H}(0)) | \alpha(p) \rangle \quad (7)$$

which in terms of R^μ takes the form

$$\epsilon.R(q') = (2\pi)^{q/2} (2k_0)^{1/2} N_\alpha N_\beta \int d^4x e^{iq' \cdot x} (\Box^2 + m_\pi^2) \langle \beta(p') \gamma(k) | T(\phi_3(x) \mathcal{H}(0)) | \alpha(p) \rangle . \quad (8)$$

Allowing q' to go off the mass shell and integrating by parts twice we get

$$\epsilon.R(q') = (2\pi)^{q/2} (2k_0)^{1/2} N_\alpha N_\beta \frac{m_\pi^2 - q'^2}{c} \int d^4x e^{iq' \cdot x} \langle \beta(p') \gamma(k) | T(\partial_\lambda F_3^{5\lambda}(x) \mathcal{H}(0)) | \alpha(p) \rangle \quad (9)$$

where the PCAC relation has been used to replace the pion field operator by the divergence of the axial-vector current.

Multiplying by q' the surface term $S^{\mu\lambda}$ defined as

$$\epsilon_\mu S^{\mu\lambda}(q') = -i(2\pi)^{q/2} (2k_0)^{1/2} N_\alpha N_\beta \int d^4x e^{iq' \cdot x} \langle \beta(p') \gamma(k) | T(F_3^{5\lambda}(x) \mathcal{H}(0)) | \alpha(p) \rangle \quad (10)$$

and integrating by parts we obtain

$$\epsilon_\mu q'_\lambda S^{\mu\lambda}(q') = (2\pi)^{q/2} (2k_0)^{1/2} N_\alpha N_\beta \int d^4x e^{iq' \cdot x} \langle \beta(p') \gamma(k) | \partial_\lambda T(F_3^{5\lambda}(x) \mathcal{H}(0)) | \alpha(p) \rangle . \quad (11)$$

If the identity

$$\partial_\lambda T(F_3^{5\lambda}(x) \mathcal{H}(0)) = T(\partial_\lambda F_3^{5\lambda}(x) \mathcal{H}(0)) + \delta(x_0) [F_3^{50}(x), \mathcal{H}(0)]_- \quad (12)$$

is used, the RHS of (11) becomes split in two parts, one proportional to R^μ of (9) and the other proportional to an ETC. Thus, in the soft-pion limit, expression (11) gives

$$\epsilon.R^N(q'=0) = \lim_{q' \rightarrow 0} \left[\frac{m_\pi^2}{c} \epsilon_\mu q'_\lambda S^{\mu\lambda}(q') - \epsilon.R^B(q') \right] \\ + \frac{m_\pi^2}{c} (2\pi)^{3/2} (2k_0)^{1/2} N_\alpha N_\beta \langle \beta(p) \gamma(k) | [F_3, \mathcal{H}]_- | \alpha(p) \rangle \quad (13)$$

where the amplitude R^μ has been separated in a Born ($R^{B\mu}$) and a nonBorn ($R^{N\mu}$) parts in order to get rid of the ambiguities arising in the soft-pion limit.

From Eq. (13) we see that the on-mass-shell amplitude for $\alpha \rightarrow \beta + \gamma$ is given in terms of the off-mass-shell surface-Born terms and the current algebra term for $\alpha \rightarrow \beta + \pi^0$. These off-mass-shell terms take different values for derivative and nonderivative couplings of pseudoscalar mesons to baryons. In fact, working in the limit of exact unitary symmetry and assuming nonderivative coupling for the $\bar{B}B\pi$ vertex, Ahmed²⁴ obtained a relatively large pv amplitude for the decay $\Sigma^+ \rightarrow p + \gamma$ coming from these off-mass-shell surface-Born terms. On the other hand, assuming unitary symmetry, CP invariance, and the usual octet dominance, Hara has shown⁴² that the pv amplitude for $\Sigma^+ \rightarrow p + \gamma$ and $\Xi^- \rightarrow \Sigma^- + \gamma$ is zero in the current $\times$ current interaction picture. This apparent contradiction has its origin in the use of the nonderivative coupling for the $\bar{B}B\pi$ vertex as pointed out by Ram Mohan.²⁶ Therefore, within the framework of current algebra and current $\times$ current interaction, the derivative coupling should be used consistently throughout the calculation.

In light of the above arguments the derivative coupling will be used for the $\bar{B}B\pi$ vertex, in which case it will be shown in the next section that the surface-Born term of Eq.(13) vanishes for both the pc and pv amplitudes regardless of unitary symmetry arguments. By allowing symmetry breaking we restore a nonvanishing pv amplitude for $\alpha \rightarrow \beta + \gamma$ proportional to the symmetry breaking pc current algebra amplitude for $\alpha \rightarrow \beta + \pi^0$.

X. THE SURFACE-BORN TERM

In this section we show explicitly that the surface-Born term vanishes in the soft-pion limit, provided the strong $\bar{B}B\pi$ vertex is assumed to be of the derivative type.

The surface term defined in (IX.10) is written

$$\epsilon_\mu S^{\mu\lambda} = -i(2\pi)^{q/2} (2k_0)^{1/2} N_\alpha N_\beta \int d^4x e^{iq'x} \langle \beta(p') | T(\mathcal{F}_3^{5\lambda}(x) \mathcal{H}(0)) | \alpha(p) \rangle \quad (1)$$

without the photon in the final state because in what follows the line from which the photon is emitted can be the final leg as well as the initial one, and also from intermediate states.

To avoid carrying along the nonessential normalization factors in (1) we introduce the integrals

$$\begin{aligned} I^\lambda &\equiv \int d^4x e^{iq'x} \theta(x_0) \langle \beta(p') | \mathcal{F}_3^{5\lambda}(x) \mathcal{H}(0) | \alpha(p) \rangle, \\ J^\lambda &\equiv \int d^4x e^{iq'x} \theta(-x_0) \langle \beta(p') | \mathcal{H}(0) \mathcal{F}_3^{5\lambda}(x) | \alpha(p) \rangle \end{aligned} \quad (2)$$

corresponding to the definition of the time-ordering operator T . Of course

$$\epsilon_\mu S^{\mu\lambda} = -i(2\pi)^{9/2} (2k_0)^{1/2} N_\alpha N_\beta (I^\lambda + J^\lambda) . \quad (3)$$

In the soft-pion limit only the pole contributions to $S^{\mu\lambda}$ coming from one-baryon intermediate states have a chance to survive. Thus we insert a complete set of intermediate states between the operators $F_3^{5\lambda}$ and $\mathcal{H}$, but keep one-baryon states only:

$$I^\lambda = \sum_n \int d^4x e^{iq'x} \theta(x_0) \langle \beta(p') | F_3^{5\lambda}(x) | n \rangle \langle n | \mathcal{H}(0) | \alpha(p) \rangle . \quad (4)$$

Depending on the position of the photon line, the integral (4) can be split in two parts corresponding to the first two diagrams and the last two in Figure 6:

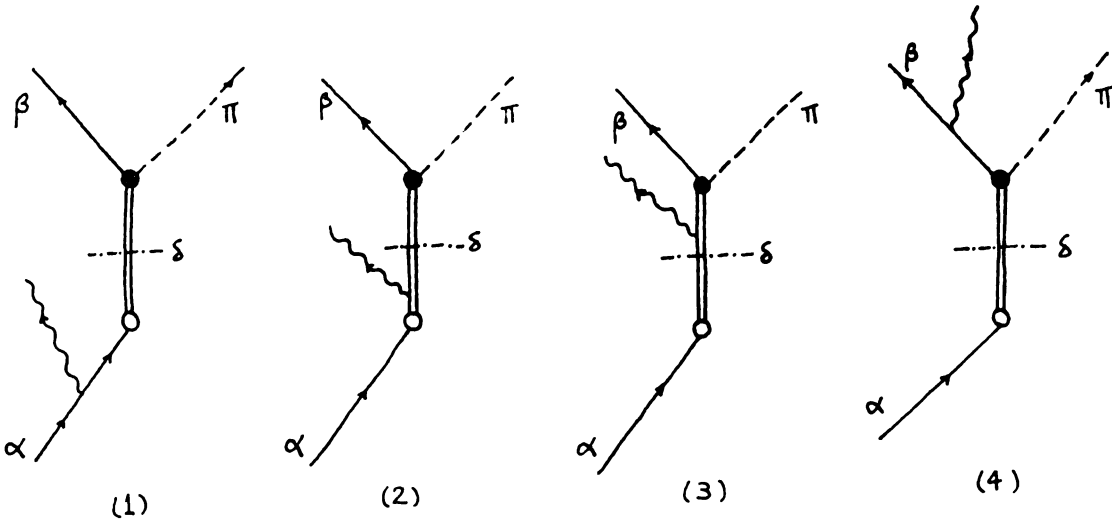


Figure 6. s-channel one-baryon intermediate state in radiative decays.

$$I_{12}^\lambda = \sum_n \int d^4x e^{i(q'+p'-p_n) \cdot x} \theta(x_0) \langle \beta(p') | \mathcal{F}_3^{5\lambda} | \delta(p_n) \rangle \langle \delta(p_n) | \mathcal{H} | \alpha(p) \rangle , \quad (5)$$

$$I_{34}^\lambda = \sum_n \int d^4x e^{i(q'+p'+k-p_n) \cdot x} \theta(x_0) \langle \beta(p') | \mathcal{F}_3^{5\lambda} | \delta(p_n) \rangle \langle \delta(p_n) | \mathcal{H} | \alpha(p) \rangle .$$

Integration by parts with respect to x_0 transforms $\theta(x_0)$ into a delta function, $\delta(x_0)$. Then the integration in x can be readily performed to give a delta function in 3-momenta. Integration with respect to p implicit in the summation $\sum_n$ can now be carried out to yield

$$I_{12}^\lambda = i(2\pi)^3 \sum_{\text{spin}} \frac{\langle \beta(p') | \mathcal{F}_3^{5\lambda} | \delta(p'+q') \rangle \langle \delta(p'+q') | \mathcal{H} | \alpha(p) \rangle}{q'_0 + E_\beta(p') - E_\delta(p'+q')} , \quad (6)$$

$$I_{34}^\lambda = i(2\pi)^3 \sum_{\text{spin}} \frac{\langle \beta(p') | \mathcal{F}_3^{5\lambda} | \delta(p'+q'+k) \rangle \langle \delta(p'+q'+k) | \mathcal{H} | \alpha(p) \rangle}{q'_0 + E_\beta(p') + k_0 - E_\delta(p'+q'+k)} .$$

In the soft-pion limit, $q' \rightarrow 0$, the denominator of I_{34}^λ does not vanish so that this term makes no contribution to the surface-Born term. Hence we have to deal with I_{12}^λ only.

The strong $\bar{B}B\pi$ vertex can be written as

$$\langle \beta(p') | \mathcal{F}_3^{5\lambda} | \delta(p_n) \rangle = \frac{1}{(2\pi)^3} \frac{1}{N_p N_s} \bar{u}(p') \gamma^\lambda \gamma_5 u(p_n) (K_A)_{\beta\delta}^3 \quad (7)$$

with

$$\frac{m_\pi^2}{C} (K_A)_{\beta\delta}^2 = K_{\beta\delta}^2 , \quad (8)$$

whereas the weak-electromagnetic vertex take the form

$$\langle s(p_n) | \mathcal{H} | \alpha(p) \rangle = \frac{1}{(2\pi)^{9/2}} \frac{1}{(2k_0)^{1/2}} \frac{1}{N_\alpha N_s} \bar{u}(p_n) [(C - \gamma_s v) \frac{1}{\not{p} - \not{k} - M_\alpha} \\ \times (\not{\epsilon} + \frac{\mu_\alpha}{2M_\alpha} \not{k} \not{\epsilon}) + (\not{\epsilon} + \frac{\mu_s}{2M_s} \not{k} \not{\epsilon}) \frac{1}{\not{p}_n + \not{k} - M_s} (C - \gamma_s v)] u(p) \quad (9)$$

corresponding to (1) and (2) in Figure 6. By inserting (7) and (9) into (6a) and summing over spins we get

$$\epsilon_\mu q'_\lambda S_I^{\mu\lambda} = \frac{\bar{u}(p') \not{\epsilon}' \gamma_s (\not{p}_n + M_s)}{2 E_s(p' + q') [q'_0 + E_s(p') - E_s(p' + q')]} [(K_A C - \gamma_s K_A v) \frac{1}{\not{p} - \not{k} - M_\alpha} \\ \times (\not{\epsilon} + \frac{\mu_\alpha}{2M_\alpha} \not{k} \not{\epsilon}) + (\not{\epsilon} + \frac{\mu_s}{2M_s} \not{k} \not{\epsilon}) \frac{1}{\not{p}_n + \not{k} - M_s} (K_A C - \gamma_s K_A v)] u(p) \quad (10)$$

where we have written $S_I^{\mu\lambda}$ for the part of $S^{\mu\lambda}$ corresponding to I^λ in (3).

The other part J^λ of $S^{\mu\lambda}$ has associated with it the graphs of the Figure 7. Of these graphs, the last two do not contribute to the surface term in the soft-pion limit just as corresponding graphs did not to I^λ . Repeating the procedure followed in the calculation of I^λ we obtain

$$\epsilon_\mu q'_\lambda S_J^{\mu\lambda} = -\bar{u}(p') [(\not{\epsilon} + \frac{\mu_\beta}{2M_\beta} \not{k} \not{\epsilon}) \frac{1}{\not{p} + \not{k} - M_\beta} (C K_A - \gamma_s v K_A) + (C K_A - \gamma_s v K_A) \\ \times \frac{1}{\not{p}_n - \not{k} - M_r} (\not{\epsilon} + \frac{\mu_r}{2M_r} \not{k} \not{\epsilon})] \frac{(\not{p}_n + M_r) \not{\epsilon}' \gamma_s u(p)}{2 E_r(p - q') [q'_0 + E_r(p - q') - E_\alpha(p)]} . \quad (11)$$

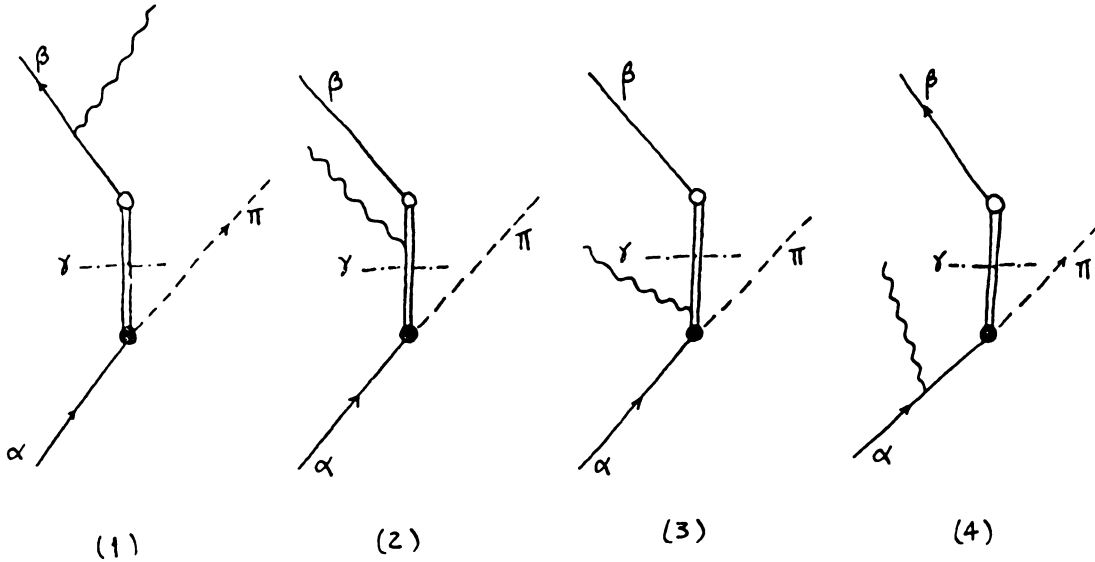


Figure 7. u-channel one-baryon intermediate state in radiative decays.

We can immediately see from (10) and (11) that the surface term vanishes in the soft-pion limit when $M_\delta \neq M_\beta$ and $M_\gamma \neq M_\alpha$. However, if at least one of these mass inequalities does not hold, an ambiguity arises which can be canceled by a corresponding ambiguity in the Born term. The net result is a well-defined limit whose value depends on the type of interaction (derivative or nonderivative) adopted for the strong vertex.

Consider $M_\delta = M_\beta$ and $M_\gamma = M_\alpha$, in which case expressions (10) and (11) can be put in a more convenient form. In fact, note that in the limit $q' \rightarrow 0$ we can make the fol-

lowing replacements:

$$p_n \xrightarrow{s} p' , \quad p_n \xrightarrow{v} p ,$$

$$2 E_s(p' + q') [q'_0 + E_p(p') - E_s(p' + q')] \rightarrow 2 p' \cdot q' ,$$

$$2 E_s(p - q') [q'_0 + E_p(p - q') - E_s(p)] \rightarrow 2 p \cdot q ,$$

so that in the limit $q' \rightarrow 0$, (10) and (11) reduce to

$$\begin{aligned} \epsilon_\mu q'_\lambda S_I^{\mu\lambda} = & \bar{u}(p) \frac{\not{q}' \gamma_5 (\not{p}' + M_p)}{2 p' \cdot q'} \left[(K_A C - \gamma_5 K_A v) \frac{1}{\not{p} - \not{k} - M_\alpha} \left(\not{\epsilon} + \frac{\mu_\alpha}{2 M_\alpha} \not{k} \not{\epsilon} \right) \right. \\ & \left. + \left(\not{\epsilon} + \frac{\mu_\alpha}{2 M_\alpha} \not{k} \not{\epsilon} \right) \frac{1}{\not{p}' + \not{k} - M_p} (K_A C - \gamma_5 K_A v) \right] u(p) , \quad (12a) \end{aligned}$$

$$\begin{aligned} \epsilon_\mu q'_\lambda S_J^{\mu\lambda} = & -\bar{u}(p) \left[\left(\not{\epsilon} + \frac{\mu_p}{2 M_p} \not{k} \not{\epsilon} \right) \frac{1}{\not{p}' + \not{k} - M_p} (C K_A - \gamma_5 v K_A) \right. \\ & \left. + (C K_A - \gamma_5 v K_A) \frac{1}{\not{p} - \not{k} - M_\alpha} \left(\not{\epsilon} + \frac{\mu_\alpha}{2 M_\alpha} \not{k} \not{\epsilon} \right) \right] \frac{(\not{p} + M_\alpha) \not{q}' \gamma_5}{2 p \cdot q'} u(p) . \quad (12b) \end{aligned}$$

Let us now turn our attention to the Born term. There are altogether six Feynman diagrams of which four present singularities of the nature discussed in the case of the surface term. These singular diagrams are almost identical to those contributing to the surface term, the difference being the appearance of off-mass-shell propagators in the Feynman diagrams instead of on-mass-shell intermediate states. The four diagrams of interest are given in Figure 8. The amplitudes corresponding to these diagrams are

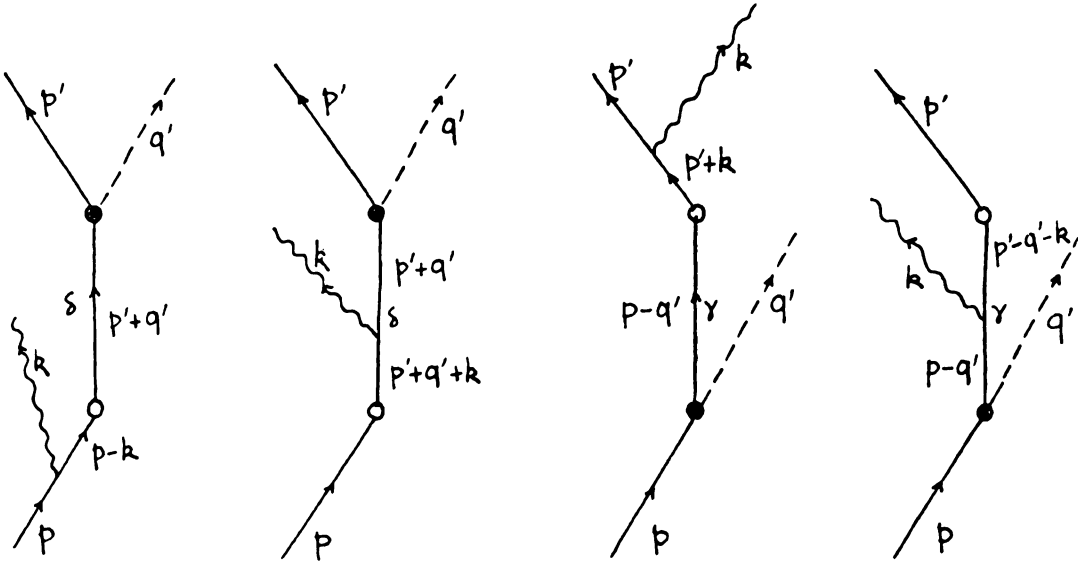


Figure 8. Born diagrams for the radiative decays.

$$\epsilon \cdot R_I^B = \bar{u}(p') \not{q}' \gamma_5 \frac{1}{\not{p}' + \not{q}' - M_s} [(K C - \gamma_5 K V) \frac{1}{\not{p} - \not{k} - M_\alpha} (\not{\epsilon} + \frac{M_\alpha}{2M_\alpha} K \not{\epsilon}) + (\not{\epsilon} + \frac{M_s}{2M_s} K \not{\epsilon}) \frac{1}{\not{p}' + \not{k} + \not{q}' - M_s} (K C - \gamma_5 K V)] u(p) , \quad (13a)$$

$$\epsilon \cdot R_J^B = \bar{u}(p') [(\not{\epsilon} + \frac{M_p}{2M_p} K \not{\epsilon}) \frac{1}{\not{p}' + \not{k} - M_p} (C K - \gamma_5 V K) + (C K - \gamma_5 V K) \frac{1}{\not{p} - \not{q}' - M_\gamma} (\not{\epsilon} + \frac{M_\gamma}{2M_\gamma} K \not{\epsilon})] \not{q}' \gamma_5 u(p) . \quad (13b)$$

In the soft-pion limit and for $M_\delta = M_\beta$ and $M_\gamma = M_\alpha$, the above

amplitudes can be put in the following form:

$$\begin{aligned} \epsilon.R_I^B = & \bar{u}(p') \frac{\not{A}' \gamma_5 (\not{p}' + M_\rho)}{2 p' \cdot q'} \left[(K C - \gamma_5 K V) \frac{1}{\not{p} - \not{k} - M_\alpha} \left(\not{\epsilon} + \frac{\mu_\alpha}{2 M_\alpha} \not{k} \not{\epsilon} \right) \right. \\ & \left. + \left(\not{\epsilon} + \frac{\mu_\rho}{2 M_\rho} \not{k} \not{\epsilon} \right) \frac{1}{\not{p}' + \not{k} - M_\rho} (K C - \gamma_5 K V) \right] u(p) , \quad (14a) \end{aligned}$$

$$\begin{aligned} \epsilon.R_J^B = & -\bar{u}(p') \left[\left(\not{\epsilon} + \frac{\mu_\rho}{2 M_\rho} \not{k} \not{\epsilon} \right) \frac{1}{\not{p}' + \not{k} - M_\rho} (C K - \gamma_5 V K) + (C K - \gamma_5 V K) \right. \\ & \left. \times \frac{1}{\not{p} - \not{k} - M_\alpha} \left(\not{\epsilon} + \frac{\mu_\alpha}{2 M_\alpha} \not{k} \not{\epsilon} \right) \right] \frac{(\not{p} + M_\alpha) \not{A}' \gamma_5}{2 p \cdot q'} u(p) . \quad (14b) \end{aligned}$$

Recalling the connection between K_A^a and K^a we see from (12) and (14) that the surface-Born term indeed vanishes in the soft-pion limit.

XI. SOFT-PHOTON THEOREM AND DECAY AMPLITUDES

The result of the previous section states that the surface-Born term vanishes in the soft-pion limit. Then the soft-pion formula of section IX connecting the two- and the three-body radiative weak decay amplitudes can be simply written as

$$\frac{m_\pi^2}{C} (2\pi)^{9/2} (2k_0)^{1/2} N_\alpha N_\beta \langle \beta(p') \gamma(k) | [F_3, \mathcal{H}] | \alpha(p) \rangle = \epsilon_\mu R^{\mu\alpha}(q'=0) . \quad (1)$$

The next step consists in applying Low's soft-photon theorem⁴¹ to relate R^μ and the amplitude M for the nonradiative decay $\alpha \rightarrow \beta + \pi^0$, by expanding R^μ in powers of k , the relation being exact to orders k^{-1} and k^0 .

The choice of the kinematical variables is the following:

$$\begin{aligned} s &= (p' + q')^2 , \\ t &= (p' - p)^2 , \\ \Delta_1 &= p^2 - M_\alpha^2 , \\ \Delta_2 &= p'^2 - M_\beta^2 . \end{aligned}$$

Of course the dependence of the nonradiative amplitude on the last two variables indicates off-mass-shell amplitudes

which will be written as $M(s, t, \Delta_1, \Delta_2)$. Note that Eq. (1) is a relationship between the physical amplitude for the two-body radiative weak decay of interest, $\alpha \rightarrow \beta + \gamma$, and the unphysical, soft-pion extrapolated nonBorn part of the three-body radiative weak decay amplitude $R^{N\mu}(q'=0)$. By "nonBorn" we mean what is left after the pole term of the nonradiative part have been removed, Figure 9.

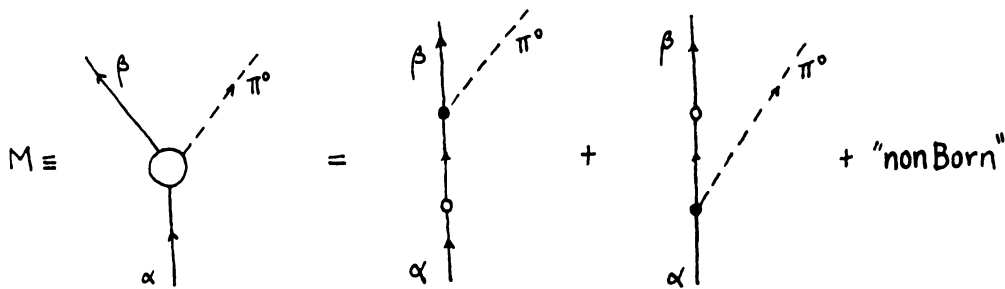


Figure 9. Definition of "nonBorn" amplitude.

The diagrams contributing to R^μ are given in Figure 10.

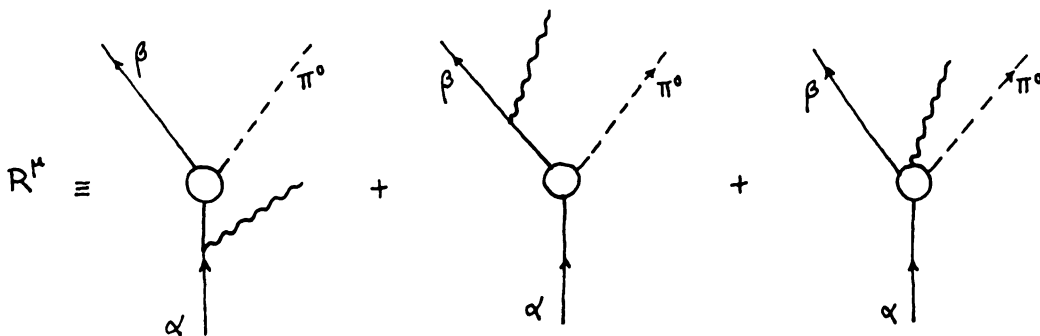


Figure 10. Diagrams for the three-body radiative decay.

Feynman rules applied to these diagrams give a relationship between $R^\mu(q')$ and $M(q')$. Then the removal of the pole contributions from $M(q')$ leads to the nonBorn part of $R^\mu(q')$:

$$\begin{aligned} \epsilon.R^N(q') = e \bar{u}(p') & \left[\left(\not{\epsilon} + \frac{\not{p}}{2M_\rho} \not{k} \right) \frac{1}{\not{p}' + \not{k} - M_\rho} \bar{M}^N(s + 2k.(p' + q'), t \right. \\ & + 2k.(p' - p), 0, 2p'.k) + \bar{M}^N(s, t + 2k.(p' - p), -2p.k, 0) \frac{1}{\not{p} - \not{k} - M_\alpha} \\ & \left. \times \left(\not{\epsilon} + \frac{\not{p}_\alpha}{2M_\alpha} \not{k} \right) \right] u(p) + [\epsilon.R^N(q')]^{int} \end{aligned} \quad (2)$$

where $\bar{M}^N$ stands for M^N without the spinors, and $[R^{N\mu}(q')]^{int}$ is the part of $R^{N\mu}(q')$ corresponding to the diagram in which the photon is emitted directly from the blob.

Low's soft-photon theorem states that, to orders k^{-1} and k^0 , Eq.(2) can be written in the following form

$$\begin{aligned} \epsilon.R^N(q') = e \bar{u}(p') & \left[\left(\not{\epsilon} + \frac{\not{p}}{2M_\rho} \not{k} \right) \frac{1}{\not{p}' + \not{k} - M_\rho} \bar{M}^N(s + 2k.(p' + q'), t \right. \\ & + 2k.(p' - p), 0, 0) + \bar{M}^N(s, t + 2k.(p' - p), 0, 0) \frac{1}{\not{p} - \not{k} - M_\alpha} \left(\not{\epsilon} + \frac{\not{p}_\alpha}{2M_\alpha} \not{k} \right) \\ & \left. - 2\epsilon.(p' + q') \frac{\partial}{\partial s} \bar{M}^N(s, t + 2k.(p' - p), 0, 0) \right] u(p). \end{aligned} \quad (3)$$

Note that now $\bar{M}^N$ is evaluated on the mass-shell: $\Delta_1=0$, $\Delta_2=0$. Because the physical amplitude $\bar{M}^N$ for the nonradiative decay, $\alpha \rightarrow \beta + \pi^0$, is independent of the kinematical variables, formula (3) can be simply written as

$$\begin{aligned} \epsilon.R^N(q'=0) = e \bar{u}(p') \left[\left(\not{\epsilon} + \frac{\mu_\beta}{2M_\beta} \not{k} \right) \frac{1}{\not{p}' + \not{k} - M_\beta} \bar{M}^N(q'=0) \right. \\ \left. + \bar{M}^N(q'=0) \frac{1}{\not{p} - \not{k} - M_\alpha} \left(\not{\epsilon} + \frac{\mu_\alpha}{2M_\alpha} \not{k} \right) \right] u(p) \end{aligned} \quad (4)$$

where we have taken the soft-pion limit of both sides.

We have derived previously the soft-pion amplitude

$$M^N(q'=0) = \frac{m_\pi^2}{c} (2\pi)^3 N_\alpha N_\beta \langle \beta(p') | [F_3, \mathcal{H}] | \alpha(p) \rangle \quad (5)$$

with the surface-Born term being set equal to zero in the derivative coupling for the $\bar{B}B\pi$ vertex. Expression (5) yields

$$\bar{M}^N(q'=0) = \frac{m_\pi^2}{c} (I_3^\beta - I_3^\alpha) (c - \gamma_5 v) \quad (6)$$

where I_3^β is the eigenvalue of the third component of the isospin operator for the baryon β . Insertion of (6) into (4) and some algebraic manipulations give

$$\begin{aligned} \epsilon.R^N(q'=0) = \frac{m_\pi^2}{c} (I_3^\beta - I_3^\alpha) \frac{e}{2p.k} \bar{u}(p') \left[(M_\alpha + M_\beta) \left(\frac{\mu_\beta}{2M_\beta} - \frac{\mu_\alpha}{2M_\alpha} \right) c \right. \\ \left. + \gamma_5 (M_\alpha - M_\beta) \left(\frac{\mu_\beta}{2M_\beta} + \frac{\mu_\alpha}{2M_\alpha} \right) v \right] \not{k} \not{\epsilon} u(p). \end{aligned} \quad (7)$$

From (1), (7), and (IX.3) we immediately obtain the expression for the invariant amplitudes,

$$\begin{aligned}
 A' &= \frac{1}{M_\alpha - M_\beta} \left(\frac{M_\beta}{2M_\beta} - \frac{M_\alpha}{2M_\alpha} \right) C, \\
 B' &= -\frac{1}{M_\alpha + M_\beta} \left(\frac{M_\beta}{2M_\beta} + \frac{M_\alpha}{2M_\alpha} \right) V.
 \end{aligned}
 \tag{8}$$

Expression (8) is good for the radiative weak decay of a charged particle. For the decay of neutral hyperons, however, (8) is not complete because diagrams in which the purely electromagnetic transition $\Sigma^0 \leftrightarrow \Lambda$ can occur also contribute. These diagrams are of the general form shown in Figure 11(a) with the electromagnetic vertex of the Figure 11(b).

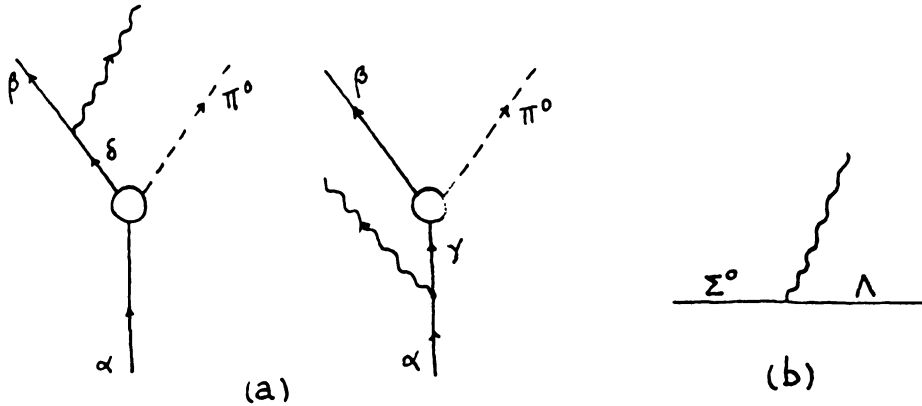


Figure 11. (a) Diagrams for the radiative decay of a neutral hyperon. (b) Electromagnetic $\Sigma^0 \Lambda$ vertex.

In case of neutral hyperons, therefore, in addition to (8) there are terms arising from interactions of the form

$$e \bar{\psi} i \frac{\mu_T}{2M_T} \sigma^{\mu\nu} k_\nu \psi \quad (9)$$

where $\mu_T \equiv \mu(\Sigma^0 \Lambda)$ is the transition anomalous magnetic moment, and the average $M_T \equiv (M_{\Sigma^0} + M_\Lambda)/2$ has been taken as the transition mass. If all four types of diagrams occur in a radiative weak decay the amplitudes will be given by

$$\begin{aligned} A'_{\beta\alpha} &= \frac{1}{M_\alpha - M_\beta} \left(\frac{\mu_\beta}{2M_\beta} - \frac{\mu_\alpha}{2M_\alpha} \right) C_{\beta\alpha} + \frac{1}{M_\alpha - M_\delta} \frac{\mu_T}{2M_T} C_{\delta\alpha} - \frac{1}{M_\gamma - M_\beta} \frac{\mu_T}{2M_T} C_{\beta\gamma}, \\ B'_{\beta\alpha} &= \frac{1}{M_\alpha + M_\beta} \left(\frac{\mu_\beta}{2M_\beta} + \frac{\mu_\alpha}{2M_\alpha} \right) V_{\beta\alpha} + \frac{1}{M_\alpha + M_\delta} \frac{\mu_T}{2M_T} V_{\delta\alpha} + \frac{1}{M_\gamma + M_\beta} \frac{\mu_T}{2M_T} V_{\beta\gamma}. \end{aligned} \quad (10)$$

Once the amplitudes are known the decay rate

$$\Gamma = \frac{1}{2} \frac{1}{(2\pi)^6} \int \frac{M_\beta}{E_\beta} d^3p' \frac{d^3k}{2k_0} (2\pi)^4 \delta^4(p' + k - p) |e \bar{u}(p') (A' + \gamma_5 B') \not{k} \not{\epsilon} u(p)|^2 \quad (11)$$

can be evaluated (Appendix C) to yield the result

$$\Gamma = \frac{e^2}{8\pi} \left(\frac{M_\alpha^2 - M_\beta^2}{M_\alpha} \right)^3 (|A'|^2 + |B'|^2). \quad (12)$$

Measurement of the decay rate alone does not always discriminate among several models, in the same sense that cross-section data sometimes accommodate conflicting models to describe the same scattering process. In such cases polarization measurements are needed to test the models. In a decay process the additional measurable quantity

is the asymmetry parameter defined as

$$a = \frac{2 \operatorname{Re}(A'^* B')}{|A'|^2 + |B'|^2} . \quad (13)$$

It is related to the right and left helicities of the photon.

With the expressions (10), (12), and (13) we are in a position to perform the numerical computation which is done in the next section.

XII. PREDICTIONS FOR RADIATIVE WEAK DECAYS

The general expression (XI.10) for the radiative weak decay amplitudes derived in the previous section can be applied to each of the decays (IX.1). Below is a list of amplitudes obtained from (XI.10) with the help of the Appendix B on SU(3) invariant vertices. The respective diagrams are shown in Figure 12.

$$\Lambda \rightarrow n + \gamma$$

$$A' = - \frac{1}{M_\Lambda - M_n} \left(\frac{\mu_n}{2M_n} - \frac{\mu_\Lambda}{2M_\Lambda} \right) \frac{3f+d}{\sqrt{3}} - \frac{1}{M_{\Sigma^0} - M_n} \frac{\mu_\tau}{2M_\tau} (f-d)$$

$$B' = - \frac{1}{M_\Lambda + M_n} \left(\frac{\mu_n}{2M_n} + \frac{\mu_\Lambda}{2M_\Lambda} \right) \frac{1+2\alpha}{\sqrt{3}} G + \frac{1}{M_{\Sigma^0} + M_n} \frac{\mu_\tau}{2M_\tau} (2\alpha-1) G$$

$$\Sigma^+ \rightarrow p + \gamma$$

$$A' = \frac{1}{M_{\Sigma^+} - M_p} \left(\frac{\mu_p}{2M_p} - \frac{\mu_{\Sigma^+}}{2M_{\Sigma^+}} \right) \sqrt{2} (f-d)$$

$$B' = \frac{1}{M_{\Sigma^+} + M_p} \left(\frac{\mu_p}{2M_p} + \frac{\mu_{\Sigma^+}}{2M_{\Sigma^+}} \right) \sqrt{2} (2\alpha-1) G$$

$$\Sigma^0 \rightarrow n + \gamma$$

$$A' = \frac{1}{M_{\Sigma^0} - M_n} \left(\frac{\mu_n}{2M_n} - \frac{\mu_{\Sigma^0}}{2M_{\Sigma^0}} \right) (f-d) + \frac{1}{M_\Lambda - M_n} \frac{\mu_T}{2M_T} \frac{3f+d}{\sqrt{3}}$$

$$B' = \frac{1}{M_{\Sigma^0} + M_n} \left(\frac{\mu_n}{2M_n} + \frac{\mu_{\Sigma^0}}{2M_{\Sigma^0}} \right) (2\alpha-1) G - \frac{1}{M_\Lambda + M_n} \frac{\mu_T}{2M_T} \frac{2\alpha+1}{\sqrt{3}} G$$

$$\Xi^0 \rightarrow \Lambda + \gamma$$

$$A' = \frac{1}{M_{\Xi^0} - M_\Lambda} \left(\frac{\mu_\Lambda}{2M_\Lambda} - \frac{\mu_{\Xi^0}}{2M_{\Xi^0}} \right) \frac{3f-d}{\sqrt{3}} - \frac{1}{M_{\Xi^0} - M_{\Sigma^0}} \frac{\mu_T}{2M_T} (f+d)$$

$$B' = \frac{1}{M_{\Xi^0} + M_\Lambda} \left(\frac{\mu_\Lambda}{2M_\Lambda} + \frac{\mu_{\Xi^0}}{2M_{\Xi^0}} \right) \frac{4\alpha-1}{\sqrt{3}} G - \frac{1}{M_{\Xi^0} + M_{\Sigma^0}} \frac{\mu_T}{2M_T} G$$

$$\Xi^0 \rightarrow \Sigma^0 + \gamma$$

$$A' = -\frac{1}{M_{\Xi^0} - M_{\Sigma^0}} \left(\frac{\mu_{\Sigma^0}}{2M_{\Sigma^0}} - \frac{\mu_{\Xi^0}}{2M_{\Xi^0}} \right) (f+d) + \frac{1}{M_{\Xi^0} - M_\Lambda} \frac{\mu_T}{2M_T} \frac{3f-d}{\sqrt{3}}$$

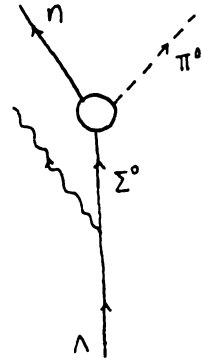
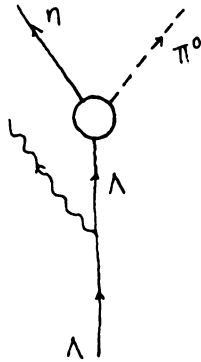
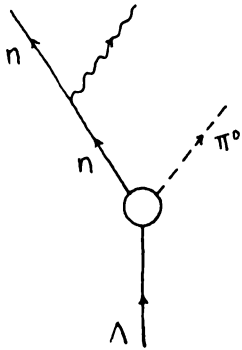
$$B' = -\frac{1}{M_{\Xi^0} + M_{\Sigma^0}} \left(\frac{\mu_{\Sigma^0}}{2M_{\Sigma^0}} + \frac{\mu_{\Xi^0}}{2M_{\Xi^0}} \right) G + \frac{1}{M_{\Xi^0} + M_\Lambda} \frac{\mu_T}{2M_T} \frac{4\alpha-1}{\sqrt{3}} G$$

$$\Xi^- \rightarrow \Sigma^- + \gamma$$

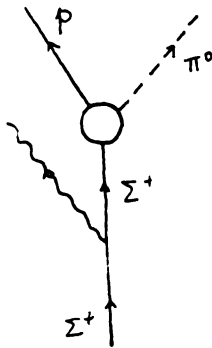
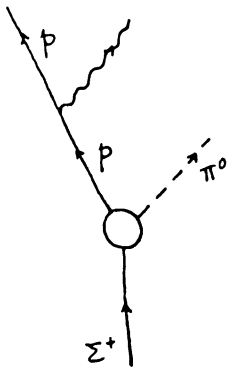
$$A' = -\frac{1}{M_{\Xi^-} - M_{\Sigma^-}} \left(\frac{\mu_{\Sigma^-}}{2M_{\Sigma^-}} - \frac{\mu_{\Xi^-}}{2M_{\Xi^-}} \right) \sqrt{2} (f+d)$$

$$B' = -\frac{1}{M_{\Xi^-} + M_{\Sigma^-}} \left(\frac{\mu_{\Sigma^-}}{2M_{\Sigma^-}} + \frac{\mu_{\Xi^-}}{2M_{\Xi^-}} \right) \sqrt{2} G$$

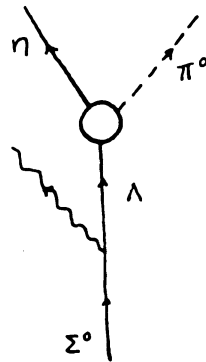
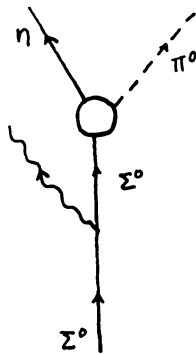
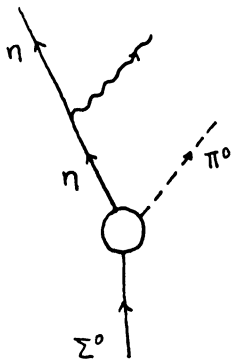
$$\Lambda \rightarrow n + \gamma$$



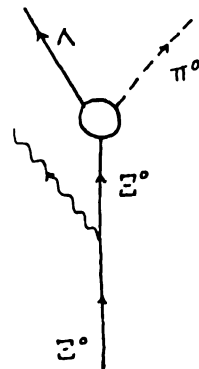
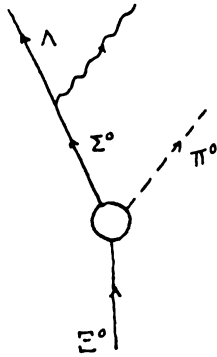
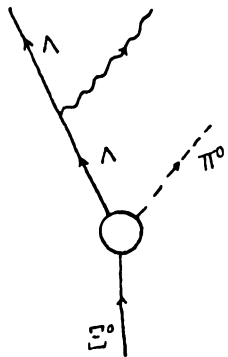
$$\Sigma^+ \rightarrow p + \gamma$$



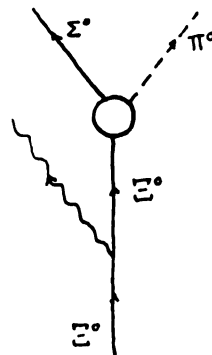
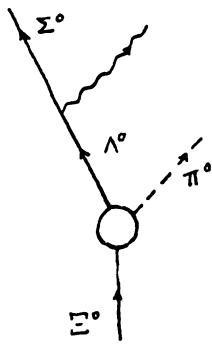
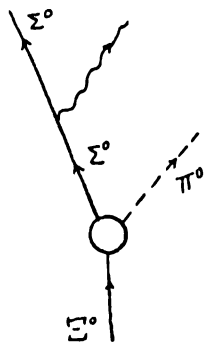
$$\Sigma^0 \rightarrow n + \gamma$$



$$\Xi^0 \rightarrow \Lambda + \gamma$$



$$\Xi^0 \rightarrow \Sigma^0 + \gamma$$



$$\Xi^- \rightarrow \Sigma^- + \gamma$$

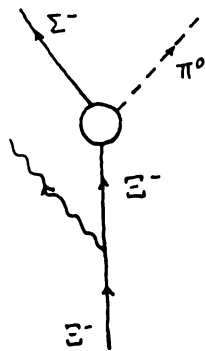
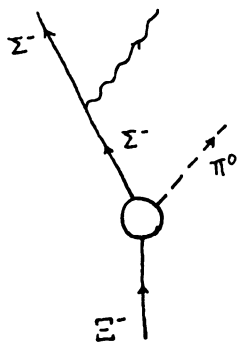


Figure 12. Diagrams for radiative weak decay modes.



For the numerical computation the experimentally determined (total) magnetic moments of the proton and of the neutron⁴³ are used:

$$\begin{aligned}\mu_p(\text{total}) &= 2.793 \mu_N, \\ \mu_n(\text{total}) &= -1.913 \mu_N,\end{aligned}\tag{1}$$

where μ_N is the nuclear magneton. The total magnetic moments of the other baryons are taken to be the SU(3) values⁴⁴ with appropriate "mass corrections":⁴⁵

$$\mu_\alpha = \mu_\alpha(\text{SU(3) value}) \times \frac{M_p}{M_\alpha}.\tag{2}$$

These "mass-corrected" values come closer to the existing experimental values for μ_Λ and μ_{Σ^+} .

The numerical results are displayed in Table 6. The only experimental numbers available at present²⁴ are those for the decay $\Sigma^+ \rightarrow p + \gamma$:

$$\frac{\Gamma(\Sigma^+ \rightarrow p + \gamma)}{\Gamma(\Sigma^+ \rightarrow p + \pi^0)} = (2.6 \pm 0.3) \times 10^{-3}, \quad a = -1.03^{+0.52}_{-0.42}.\tag{3}$$

The theoretical branching ratio and asymmetry parameter are 2.82×10^{-3} and -0.78 respectively. The agreement is excellent, although further reduction of the experimental errors is highly desired. Note that the relatively large violation of unitary symmetry found in nonradiative decays gives rise to the relatively large magnitude of B' which is the symmetry breaking amplitude for the radiative decay. Therefore

the measured asymmetry parameter of about -1.0 strongly supports breaking of unitary symmetry in both types of decays, which is a result quite different from that of previous models.

Table 6. Theoretical radiative weak decay amplitudes of hyperons, decay rates, and asymmetry parameters.

Decay	Amplitude ($10^5 \text{ m}_\pi^{-\frac{1}{2}} \text{ sec}^{-\frac{1}{2}} \text{ GeV}^{-1}$)		Decay rate (10^7 sec^{-1})	Asymmetry parameter
	A'	B'	Γ	a
$\Lambda \rightarrow n + \gamma$	-0.147	0.927	0.795	-0.309
$\Sigma^+ \rightarrow p + \gamma$	0.887	-0.429	2.328	-0.783
$\Sigma^0 \rightarrow n + \gamma$	-1.538	-0.304	6.009	+0.380
$\Xi^0 \rightarrow \Lambda + \gamma$	0.483	-0.512	0.653	-0.998
$\Xi^0 \rightarrow \Sigma^0 + \gamma$	1.574	0.226	0.846	+0.281
$\Xi^- \rightarrow \Sigma^- + \gamma$	0.0015	-0.062	0.0013	-0.049

In concluding this last section we briefly mention what remains to be done. We have seen in section VIII that our overall fit to the experimental data is very good, with the exception of a discrepancy in the s-wave $A(\Sigma_+^+)$. We argued that this discrepancy can be removed without disturbing the good fit by incorporating the contribution of

the 27 representation. This addition, however, implies an undesirable increase in the number of free parameters, unless a way is found to determine some of these parameters independently of the experimental decay amplitudes. For instance, the parameters γ_v and γ' are essentially residue functions evaluated at $t=0$ so that they might be determined from the high energy total cross-section data. Recall that the total cross-section is related to the imaginary part of the forward scattering amplitude through the optical theorem.

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LIST OF REFERENCES

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Note added. After this work has been completed, it came to our attention the existence of a slightly up-dated set of experimental nonleptonic decay amplitudes compiled by H. Filthuth, Proc. Topical Conf. on Weak Interactions, CERN, 1969. The new data, however, do not differ significantly from the old ones, so do not change appreciably the values of our searched parameters.
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APPENDICES

APPENDIX A. Notation and Conventions

We describe here the notation and conventions used in the text. The metric and gamma matrices are those defined by Bjorken and Drell.

The coordinates t, x, y, z are denoted by the contravariant four-vector x^μ ,

$$x^\mu = (x^0, x^1, x^2, x^3) = (t, x, y, z) = (t, \mathbf{x}). \quad (1)$$

The covariant four-vector x_μ is

$$x_\mu = (x_0, x_1, x_2, x_3) = (t, -x, -y, -z) = g_{\mu\nu} x^\nu \quad (2)$$

with the metric tensor $g_{\mu\nu} = g^{\mu\nu}$ given by

$$g = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}. \quad (3)$$

The four-momentum of a particle of mass m is defined by

$$p^\mu = (E, p_x, p_y, p_z) = ((p^2 + m^2)^{1/2}, \mathbf{p}) \quad (4)$$

so that (summing over repeated indices)

$$p^2 = p_\mu p^\mu = m^2. \quad (5)$$

The scalar product of two four-momenta is

$$p \cdot q = p^\mu q_\mu = p_0 q_0 - \mathbf{p} \cdot \mathbf{q} = E_p E_q - \mathbf{p} \cdot \mathbf{q} \quad . \quad (6)$$

Four-vector will always be set in lightface, three-vectors in double line.

For the four-dimensional gradient we use the abbreviations

$$\partial_\mu \equiv \frac{\partial}{\partial x^\mu} \quad , \quad \partial^\mu \equiv \frac{\partial}{\partial x_\mu} \quad . \quad (7)$$

For example, the four-divergence of $B^\mu = (B^0, \mathbf{B})$ is

$$\partial_\mu B^\mu = \frac{\partial B^0}{\partial t} + \frac{\partial \mathbf{B}}{\partial \mathbf{x}} \quad .$$

The four-dimensional Laplacian, denoted by $\square^2$, is defined by

$$\square^2 \equiv \partial_\mu \partial^\mu = \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial x_\mu} = \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial \mathbf{x}^2} \equiv \frac{\partial^2}{\partial t^2} - \nabla^2 \quad . \quad (8)$$

Let P^μ denote the four-momentum operator. If $F(x)$ is any field operator, its commutator with P^μ is

$$[P^\mu, F(x)] = -i \partial^\mu F(x) \quad . \quad (9)$$

This gives the translation formula

$$F(x) = e^{iP \cdot x} F(0) e^{-iP \cdot x} \quad . \quad (10)$$

The gamma matrices in the Dirac equation satisfy the anticommutation relations

$$[\gamma^\mu, \gamma^\nu]_+ \equiv \gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2 g^{\mu\nu} \quad . \quad (11)$$

Useful combinations are

$$\sigma^{\mu\nu} = \frac{i}{2} (\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu) = \frac{i}{2} [\gamma^\mu, \gamma^\nu] , \quad (12)$$

$$\gamma_5 \equiv i \gamma^0 \gamma^1 \gamma^2 \gamma^3 \equiv \gamma^5 , \quad (13)$$

$$\not{x} \equiv \gamma^\mu p_\mu = \gamma^0 p_0 + \gamma^a p_a = \gamma^0 p^0 - \boldsymbol{\gamma} \cdot \mathbf{p} , \quad (14)$$

$$\not{\partial} \equiv \gamma^\mu \partial_\mu = \gamma^\mu \frac{\partial}{\partial x^\mu} = \gamma^0 \frac{\partial}{\partial t} + \gamma^1 \frac{\partial}{\partial x} + \gamma^2 \frac{\partial}{\partial y} + \gamma^3 \frac{\partial}{\partial z} . \quad (15)$$

Let 1 and σ^a ($a=1,2,3$) denote the unit and the

Pauli spin matrices:

$$1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} , \quad \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} , \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} . \quad (16)$$

We adopt the representation

$$\gamma^a = \begin{pmatrix} 0 & \sigma^a \\ -\sigma^a & 0 \end{pmatrix} , \quad \gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} , \quad \gamma^5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} . \quad (17)$$

The positive-energy spinor $u(p,s)$ and its adjoint $\bar{u}(p,s) \equiv u^\dagger(p,s) \gamma^0$ normalized $\bar{u}(p,s)u(p,s)=1$, satisfy Dirac equation

$$\begin{aligned} (\not{x} - M)u(p,s) &= 0 , \\ \bar{u}(p,s)(\not{x} - M) &= 0 . \end{aligned} \quad (18)$$

Here s indicates spin variable. The positive-energy projection operator is

$$\sum_s u(p,s) \bar{u}(p,s) = \frac{\not{x} + M}{2M} . \quad (19)$$

APPENDIX B. SU(3) Invariant Vertices

SU(3) invariant interactions among various octets can be constructed with the help of the F_k and D_k matrices defined by

$$(F_k)_{lm} = -i f_{klm} \quad , \quad (D_k)_{lm} = d_{klm} \quad (1)$$

where the value of the structure constants are displayed below:

klm	f_{klm}	klm	d_{klm}
123	1	118	$1/\sqrt{3}$
147	1/2	146	1/2
156	-1/2	157	1/2
246	1/2	228	$1/\sqrt{3}$
257	1/2	247	-1/2
345	1/2	256	1/2
367	-1/2	338	$1/\sqrt{3}$
458	$\sqrt{3}/2$	344	1/2
678	$\sqrt{3}/2$	355	1/2
		366	-1/2
		377	-1/2
		448	$-1/(2\sqrt{3})$
		558	$-1/(2\sqrt{3})$
		668	$-1/(2\sqrt{3})$
		778	$-1/(2\sqrt{3})$
		888	$-1/(2\sqrt{3})$

f_{klm} is antisymmetric and d_{klm} is symmetric under interchange of any two indices.

The SU(3) invariant Yukawa coupling, for example, is written

$$\mathcal{H} = 2g_{\pi NN} \bar{B}_l i \gamma_5 [\alpha (F_k)_{lm} + (1-\alpha)(D_k)_{lm}] B_m P_k. \quad (2)$$

The relation between physical states and the octet operators B_k is

$$\begin{aligned} |p\rangle &= \frac{1}{\sqrt{2}} (B_4 + i B_5) |0\rangle \\ |n\rangle &= \frac{1}{\sqrt{2}} (B_6 + i B_7) |0\rangle \\ |\Sigma^+\rangle &= -\frac{1}{\sqrt{2}} (B_1 + i B_2) |0\rangle \\ |\Sigma^0\rangle &= B_3 |0\rangle \\ |\Sigma^-\rangle &= \frac{1}{\sqrt{2}} (B_1 - i B_2) |0\rangle \\ |\Lambda\rangle &= B_8 |0\rangle \\ |\Xi^0\rangle &= \frac{1}{\sqrt{2}} (B_6 - i B_7) |0\rangle \\ |\Xi^-\rangle &= -\frac{1}{\sqrt{2}} (B_4 - i B_5) |0\rangle \end{aligned} \quad (3)$$

Similar relations hold for the pseudoscalar meson states and the operators P_k . Using the structure constants given above we can determine $K_{\beta\delta}^a \equiv 2g_{\pi NN} [\alpha (F^a)_{\beta\delta} + (1-\alpha) (D^a)_{\beta\delta}]$ for any particular vertex.

APPENDIX C. Decay Rate

The decay rate of the radiative weak decay,
 $\alpha(p) \rightarrow \beta(p') + \gamma(k)$, is given by

$$\Gamma = \frac{1}{2} \frac{1}{(2\pi)^6} \int \frac{M_p}{E_p} d^3p' \frac{d^3k}{2k_0} (2\pi)^4 \delta^4(p' + k - p) \sum_{\text{spin}} |M|^2 \quad (1)$$

where

$$M = \bar{u}(p') (A' + \gamma_5 B') \not{k} u(p) . \quad (2)$$

The Hermitian conjugate of M is

$$M^\dagger = \bar{u}(p) \not{k} (A'^* - \gamma_5 B'^*) u(p) \quad (3)$$

so that

$$\sum_{\text{spin}} |M|^2 = \text{Tr} \left[\frac{\not{p}' + M_p}{2M_p} (A' + \gamma_5 B') \not{k} \frac{\not{p} + M_\alpha}{2M_\alpha} \not{k} (A'^* - \gamma_5 B'^*) \right] . \quad (4)$$

Inserting the identity

$$\not{k} \not{k} (\not{p} + M_\alpha) = (\not{p} + M_\alpha) \not{k} \not{k} - 2(k \cdot p) \not{k} \quad (5)$$

in (4) we obtain two pieces for the trace: one containing the factor $\not{k} \not{k} \not{k}$ and the other the factor $\not{k} \not{k} \not{k}$. Because

$$\not{\epsilon} \not{\epsilon} = \epsilon^2 = -1 \quad \text{and} \quad \not{k} \not{k} = k^2 = 0$$

the first piece is zero. Thus the trace (4) reduces to

$$\begin{aligned} \sum_{\text{spin}} |M|^2 &= \frac{2 \, k \cdot p}{4 M_\alpha M_\beta} \text{Tr} [(\not{p} + M_\beta)(A' + \gamma_5 B') \not{k} (A'^* - \gamma_5 B'^*)] \\ &= \frac{2 (k \cdot p')^2}{M_\alpha M_\beta} (|A'|^2 + |B'|^2) . \end{aligned} \quad (6)$$

Now, the energy-momentum conservation $p = p' + k$ gives

$$2 \, k \cdot p' = M_\alpha^2 - M_\beta^2 .$$

There is also a factor 2 coming from the two states of polarization of the photon. Then expression (6) becomes

$$\sum_{\text{spin}} |M|^2 = \frac{M_\alpha^2 - M_\beta^2}{M_\alpha M_\beta} (|A'|^2 + |B'|^2) . \quad (7)$$

Substituting (7) in the decay rate (1) we get

$$\Gamma = \frac{1}{2} \frac{1}{(2\pi)^2} \frac{(M_\alpha^2 - M_\beta^2)^2}{M_\alpha} (|A'|^2 + |B'|^2) \int \frac{d^3 p'}{E_\beta} \frac{d^3 k}{2k_0} \delta^4(p' + k - p) . \quad (8)$$

Integration in k yields

$$\Gamma = \frac{1}{2} \frac{1}{(2\pi)^2} \frac{(M_\alpha^2 - M_\beta^2)^2}{M_\alpha} (|A'|^2 + |B'|^2) \int \frac{d^3 p'}{E_\beta 2k_0} \delta(p'_0 + k_0 - p_0) . \quad (9)$$

Write $d^3 p' = \rho^2 d\rho d\Omega$ where $\rho = |\mathbf{p}'|$. Also let

$$f(\rho) \equiv p'_0 + k_0 - p_0 = (\rho^2 + M_\beta^2)^{1/2} + \rho - M_\alpha . \quad (10)$$

If ρ_0 is the value of ρ at which $f(\rho)$ is zero, then

$$\left. \frac{df}{d\rho} \right|_{\rho=\rho_0} = \frac{\rho_0}{(\rho_0^2 + M_\beta^2)^{1/2}} + 1 = \frac{M_\alpha}{(\rho_0^2 + M_\beta^2)^{1/2}} \quad (11)$$

The argument in the δ -function of (9) can be changed according to

$$\delta(\rho'_0 + k_0 - \rho_0) = \frac{\delta(\rho - \rho_0)}{\left. \frac{df}{d\rho} \right|_{\rho=\rho_0}} = \frac{(\rho_0^2 + M_\beta^2)^{1/2}}{M_\alpha} \delta(\rho - \rho_0) \quad (12)$$

so that

$$\begin{aligned} \int \frac{d^3 p'}{E_\beta 2k_0} \delta(\rho'_0 + k_0 - \rho_0) &= 4\pi \int \frac{\rho^2 d\rho}{(\rho^2 + M_\beta^2)^{1/2} 2\rho} \frac{(\rho_0^2 + M_\beta^2)^{1/2}}{M_\alpha} \delta(\rho - \rho_0) \\ &= 2\pi \frac{\rho_0}{M_\alpha} = \pi \frac{M_\alpha^2 - M_\beta^2}{M_\alpha^2} \end{aligned} \quad (13)$$

where we have replaced

$$\rho_0 = \frac{M_\alpha^2 - M_\beta^2}{2M_\alpha} \quad .$$

Therefore, inserting (13) into (9) we finally obtain

$$\Gamma = \frac{1}{8\pi} \left(\frac{M_\alpha^2 - M_\beta^2}{M_\alpha} \right)^3 (|A'|^2 + |B'|^2) \quad (14)$$

This is the expression for the decay rate used in the text.

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