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THE IMPACT OF MODAL ANALYSIS ON THE
ENGINEERING CURRICULUM

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H. METIN NUS RIZAI

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THE IMPACT OF MODAL ANALYSIS ON THE ENGINEERING CURRICULUM

By

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ABSTRACT

THE IMPACT OF MODAL ANALYSIS ON THE ENGINEERING CURRICULUM

By

H. Metin Nus Rizai

Modal analysis is a procedure for describing the motion of a structure by identifying its modes of vibration. A brief description of modal analysis techniques is presented as a prelude to a discussion of modal testing. This leads to a detailed discussion of modal testing technology and the potential impact of this technology on the Mechanical Engineering Curriculum at Michigan State University.

DEDICATION

To my major professor and my good friend Jim Bernard, my parents, Mr. and Mrs. Andrew TenEyck and my mother, who gave purpose to this work.

ACKNOWLEDGEMENTS

I particularly wish to express my gratitude to Dr. James E. Bernard, my major professor, for all his assistance and guidance as a teacher and a friend, on this thesis and throughout my M.S. program.

Many thanks to Bill Grafton from Gen-Rad Corporation, who helped me a great deal to do the experiments and also to Dr. R.C. Rosenberg and Dr. W.V. Nack for their valuable comments throughout this work.

Thanks also to all the friends who helped make my graduate studies an enjoyable experience.

Finally many, many thanks to my parents Mr. and Mrs. Andrew TenEyck who brought me to where I am today. Without their help I could not have made it. And thanks to my brothers and sisters who gave me a great support during my work.

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CHAPTER 1

INTRODUCTION

Modal analysis is a procedure for describing the motion of a structure by identifying its modes of vibration. The motion of the structure is assumed to be linear and a mode of vibration may be thought of as a property of a structure. Each mode has a specific resonant frequency, damping factor and mode shape which identifies the mode spatially over the entire structure. Once these properties are known, the response of the structure to any input force can be predicted and, if necessary, modified.

This thesis will discuss several different approaches to the determination of the motion of a structure. Chapter 2 gives a brief overview of analytical techniques, including dynamic sub-structuring and complications due to damping. Chapter 3 discusses the details of modal testing and Chapter 4 presents an example of testing techniques. Chapter 5 is the study of the potential impact of modal analysis on the Mechanical Engineering curriculum at Michigan State University. Chapter 6 presents a summary and recommendations.

CHAPTER 2

ANALYTICAL APPROACH

In recent years, computer based analytical techniques to aid in the understanding of dynamics of mechanical structures have become more sophisticated. These techniques may be purely analytical, starting with a mathematical model and proceeding through desired calculations. Or they can start from measured results, and perform calculations which cast these results into more useful form. Initially, I will discuss a common purely analytical approach, the finite element method.

2.1 The Finite Element Method

In order to get accurate results, one has to depend on an accurate mathematical model. Since analytical techniques work with mathematical models, modeling has become an important part of analysis.

Because of the requirement for a generalized method for modeling the dynamics of large, complex structures with nonhomogeneous physical properties, an analytical technique called the finite element method [1] has been developed and used as a modeling tool. The object of the finite element method is to subdivide a structure into many smaller elements such as plates, beams, etc. Then the equations describing structure are constructed from equations describing each of the individual elements plus all the boundary and loading conditions on the model.

When the finite element method is used for vibration problems, the model leads to a set of simultaneous second order linear differential equations which describe the elastic motion of a complex mechanical structure. These equations are often written as:

$$[M]\{\ddot{x}(t)\} + [K]\{x(t)\} = \{F(t)\} \qquad (2\text{-}1)$$

where:

$[M]$ = Mass matrix $\{\ddot{x}(t)\}$ = Acceleration vector

$[K]$ = Stiffness matrix $\{x(t)\}$ = Displacement vector

$\{F(t)\}$ = Applied force vector

If the system has n degrees of freedom, then the matrices are n by n, and vectors are n-dimensional. The matrices are real and symmetric.

2.2 Diagonalization

A common method to solve these equations is to diagonalize them [2]. This is done by transforming the equations of motion to a new coordinate system called generalized coordinates in diagonal or uncoupled form as shown in equation (2-2).

$$\begin{bmatrix} I \end{bmatrix} \begin{Bmatrix} \ddot{q}_1(t) \\ \ddot{q}_2(t) \\ \vdots \end{Bmatrix} + \begin{bmatrix} \lambda_1^2 & 0 & \cdot & \cdot \\ 0 & \lambda_2^2 & & \\ \vdots & & \ddots & \end{bmatrix} \begin{Bmatrix} q_1(t) \\ q_2(t) \\ \vdots \end{Bmatrix} = \begin{bmatrix} u_{11} & u_{12} & \cdot & \cdot \\ u_{21} & u_{22} & & \\ \vdots & & & \cdot \end{bmatrix}^T \begin{Bmatrix} f_1(t) \\ f_2(t) \\ \cdot \end{Bmatrix} \tag{2-2}$$

The transformation relating the generalized coordinates to the original coordinate system is a matrix, the columns of which are the eigenvectors of the system.

$$\begin{Bmatrix} x_1(t) \\ x_2(t) \\ \cdot \\ \cdot \end{Bmatrix} = \begin{bmatrix} u_{11} & u_{12} \cdots \\ u_{21} & u_{22} \\ \cdot & & \cdot \\ \cdot & & & \cdot \end{bmatrix} \begin{Bmatrix} q_1(t) \\ q_2(t) \\ \cdot \\ \cdot \end{Bmatrix} \tag{2-3}$$

Therefore diagonalization involves finding the eigenvalues, λ_i, and eigenvectors, u_{ij}. Once the equations of motion are in diagonal form it is much easier to understand them and to solve for the motion resulting from applied forces [2,3].

2.3 Modal Analysis

Modal analysis may be defined as the process of characterizing the dynamics of a structure in terms of its modes of vibration. These are the eigenvalues and eigenvectors of the mathematical model. That is, the eigenvalues of the equations of motion correspond to frequencies at which the structure tends to vibrate with a predominant well-defined deformation. The relative deformation is specified by the corresponding eigenvector. Therefore each mode of vibration is defined by an eigenvalue (resonant frequency) and corresponding eigenvector (mode shape).

A schematic diagram of the first two modes of a cantilever beam is shown in figure 2.1. Each of these modes corresponds to motion at a particular natural frequency.

The natural frequencies and modes of vibration of a structure are very useful information, for they tell the frequencies at which the structure can be excited easily and relate the excitation to the applied forces. This information in many cases is sufficient to indicate how to modify the structural design in order to deal with its noise and vibration effectively.

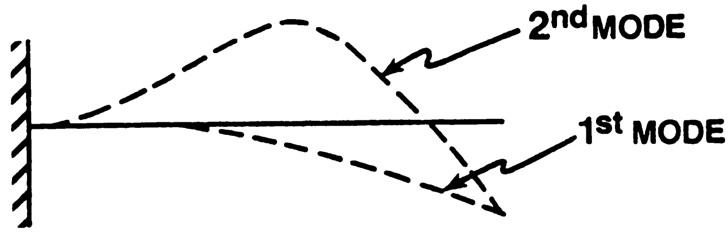

Figure 2.1 Modes of Vibration

2.4 Damping

Of course, damping is present in every structure. However, since the stiffness of an element is, in general, much easier to estimate than the damping, it is often hard to deduce a reasonable damping matrix for equation (2-1). A common technique is to add a diagonal [C] matrix to the left hand side of equation (2-2) such that:

$$\begin{bmatrix} & & \\ & I & \\ & & \end{bmatrix} \begin{Bmatrix} \ddot{q}_1(t) \\ \ddot{q}_2(t) \\ \vdots \end{Bmatrix} + \begin{bmatrix} c_1 & 0 & \cdots \\ 0 & c_2 & \\ \vdots & & \ddots \end{bmatrix} \begin{Bmatrix} \dot{q}_1(t) \\ \dot{q}_2(t) \\ \vdots \end{Bmatrix} + \begin{bmatrix} \lambda_1^2 & 0 & \cdots \\ 0 & \lambda_2^2 & \\ \vdots & & \ddots \end{bmatrix} \begin{Bmatrix} q_1(t) \\ q_2(t) \\ \vdots \end{Bmatrix} = \begin{bmatrix} u_{11} & u_{12} & \cdots \\ u_{21} & u_{22} & \\ \vdots & & \ddots \end{bmatrix}^T \begin{Bmatrix} f_1(t) \\ f_2(t) \\ \vdots \end{Bmatrix} \quad (2\text{-}4)$$

In this case, the C_{ii}'s are modal damping factors and are chosen from experience with the structure as a whole, rather than a particular knowledge of the elements that make it up. The diagonal [C] assumption in equation (2-4) assumes the modes do not interfere with each other through a viscous mechanism. This is usually quite reasonable, especially since, in structures of interest, the damping tends to be very small. (Complications are discussed in [2].) Another common technique is to include a [C] matrix in equation (2-1) which is proportional to mass and/or stiffness matrix [3]. Such a proportional damping matrix can be diagonalized by the eigenvectors from the undamped system. Therefore the governing equation of the structure can still be uncoupled [3].

2.5 Dynamic Substructuring

The dynamic analysis of structures by modal synthesis [4] approach is a very useful analytical method for obtaining the dynamic response of extremely large structures. The basic approach is to divide the structure into a number of smaller substructures each of which can be analyzed seperately. The total response of the system then obtained by appropriately coupling the dynamic characteristics for each of the component structures.

This technique is very useful in design of process of a large system since major substructures are analyzed by different engineering groups or at different times. Some groups can rely on experimental test results while other groups can use analytical investigations. Therefore, modifications on the substructures may proceed quite independently with later consideration given to the complete structure.

2.6 Some Final Remarks

Finite element methods can also be used for simulation of the dynamic response of the structure to external forces. Investigations of structural response to these forces can be done without building the prototype of the structure. But one should keep in mind that the utility of the results depends on the correspondence between the mathematical model and the structure of interest. In particular, the success of the model often depends on the ability of the analyst to procure and enter reasonable parametric data and to properly estimate load and boundary conditions.

There is another point which should be considered here, the numerical burden of the finite element method. The calculations are usually very lengthy and costly. These considerations might lead to testing, either as a replacement to or a verification for purely analytical work. The next chapter will discuss modal testing.

CHAPTER 3

MODAL TESTING

The objective of modal testing is to excite a mechanical structure by applied forces so that its natural frequencies and mode shapes may be identified. We have already mentioned the potential for verification of the analytical model using modal testing. Once the mathematical model is verified, one can depend on the analytical model and make simulations which usually take less time than testing. In addition, modal testing can be used for trouble shooting noise and vibration problems. These problems can occur because of less than ideal design, production error or as a result of wearout or failure in some of the components. Finally, modal testing can be used to construct a dynamic model for components of a structure which is too difficult to model analytically.

3.1 Transfer Function and Frequency Response Function

The equation of motion of an undamped dynamic structure is given by the equation (2-1). This equation, which is in the time domain, contains useful information about the system's response for arbitrary input forces. However, in many cases, the frequency domain information turns out to be even more useful. To aid in the discussion of the frequency domain, it will be useful to take a look at the transfer function representation of the set of differential equations in equation (2-1). Taking the Laplace transform of equation (2-1) and assuming all initial conditions are zero yields;

$$[B(s)]\ \{x(s)\} = \{F(s)\} \tag{3-1}$$

where:

$[B(s)] = [M]s^2 + [K]$

$\{x(s)\}$ = Laplace transform of displacement vector

$\{F(s)\}$ = Laplace transform of applied forces.

The $[B(s)]$ matrix is referred to as the system matrix. The matrix $[H(s)]$ is defined as the inverse of the system matrix $[B(s)]$, that is,

$$[H(s)] = [B(s)]^{-1} \tag{3-2}$$

Therefore $[H(s)]$ satisfies the following equation,

$$\{x(s)\} = [H(s)]\ \{F(s)\}. \tag{3-3}$$

Equations (3-2) and (3-3) indicate that $[H(s)]$, which is a function of the complex variable is the ratio of the output of the system to the input of the system in s domain. For an n-dimensional system $[H(s)]$ matrix is an nxn matrix. It is called a transfer matrix, and can be written as,

$$[H(s)] = \begin{bmatrix} h_{11}(s) \dots h_{1n}(s) \\ \vdots \quad \ddots \\ h_{n1}(s) \dots h_{nn}(s) \end{bmatrix} \tag{3-4}$$

The transfer matrix may be evaluated along the frequency axis of the complex Laplace plane using Fourier transforms. In this case, the transfer matrix [H(s)] becomes the frequency response matrix [H(jw)]. Each element of the matrix is a transfer function where h_{ij} is the transfer function which relates the response of the i^{th} point to an input at the j^{th} location.

There are several different forms of frequency response functions which are useful for modal testing. They all contain the same information and they are obtainable from each other. They are summarized in Table 3.1.

The frequency response matrix [H(s)] contains all the necessary information to characterize the modal parameters. If the roots of Det[B(s)] are distinct in equation (3-2), [H(s)] can be expanded into a partial fraction form [5] as follows,

$$[H(s)] = \sum_{k=1}^{n} \left(\frac{A_k}{s-p_k} + \frac{A_k^*}{s-p_k^*}\right) \tag{3-5}$$

where

p_k = k^{th} root of Det[B(s)]

p_k^* = Complex conjugate of p_k

$[A_k]$ = Residue matrix for the k^{th} root

$[A_k^*]$ = Complex conjugate of $[A_k]$.

The roots of Det [B(s)] can be written as:

$$p_k = -\sigma_k + iw_k \qquad p_k^* = -\sigma_k - iw_k \tag{3-6}$$

where

σ_k = modal damping $\qquad$ w_k = damped natural frequency

Equation (3-5) yields two of the three modal parameters, the resonant frequency and the damping. The modal vectors (eigenvectors) are also needed. They are the solution to the homogeneous equation:

$$[B(p_k)]\{U_k\} = 0 \tag{3-7}$$

The eigenvectors are proportional to the residue matrix in equation (3-5) [6], and the modal vectors represent a deformation pattern of the structure for a particular frequency. The deflected deformation of a structure which describes a natural mode of vibration is defined by known ratios of the amplitude of the motion at the various points on the structure.

TABLE 3.1

DIFFERENT FORMS OF TRANSFER FUNCTION FOR MECHANICAL STRUCTURES

Dynamic Compliance	= Disp/Force	Stiffness	= Force/Disp
Mobility	= Vel/Force	Mech. Impedance	= Force/Vel
Acceleration	= Acc/Force	Dynamic Mass	= Force/Acc

3.2 Digital Signal Processing

The main function of modal testing is to analyze the frequency response functions of mechanical structures. The general scheme for measuring frequency response functions consists of measuring simultaneously an input excitation and response signal in the time domain, Fourier transforming the signals and then forming the transfer functions by dividing the transformed response by the transformed input. This procedure is based upon the use of digital signal processing. The development, within the last decade, of both digital hardware and computer algorithms for the various transform techniques has made digital signal processing practical for the solution of structural dynamics problems.

The area of digital signal processing is very broad. Here the focus is limited to those topics which are useful for estimating the frequency response and modal properties. These topics are (a) inter-relationship between the time, frequency and s-domain [7], (b) Fourier transform, discrete Fourier transform [8,9], (c) signal sampling [8,10], (d) Correlation and Power spectrum [8,10], (e) Transfer Function and Coherence Function.

3.3 Excitation Methods

There are various types of input excitation methods including random (pure, pseudo, periodic), sinusoidal, transient (impact, step relaxation). They each have their advantages and disadvantages. Reference [11] discusses each method in some detail. In this thesis, the emphasis will be put on impact testing.

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Impact testing is fast, easy to perform and requires less time for the setup than the shakers which are used in other methods. The steps used in impact testing are shown in Figure 3.1. The figure illustrates a hand-held hammer with a load cell mounted to it to impact the structure. The load cell measures the input force and an accelerometer mounted on the structure measures the response. The frequency content and duration of the input force can be altered by using a softer or harder hammer tip. In general the longer the duration of the force impulse the lower the frequency range of the excitation. Therefore a hammer with a hard tip can be used to emphasize higher frequency excitation whereas a softer tip can be used to emphasize lower frequency excitation. Figures 3-2 and 3-3 present the force impulse of different hammer tips in the time and frequency domain.

Figure 3.1 also illustrates the accelerometer which measured the instantaneous acceleration of a vibrating structure. Reference 12 gives a good explanation of the use of accelerometers in these measurements.

The process of measuring a set of responses (i.e. transfer functions) may be either mounting a stationary accelerometer on the structure and moving the input force from point to point, or exciting the structure at one location and moving the accelerometer from point to point. In the former case, a row of the transfer matrix is being measured, whereas in the latter case, column of the transfer matrix is being measured. Either a row or a column contains enough information to construct the rest of the transfer matrix [5].

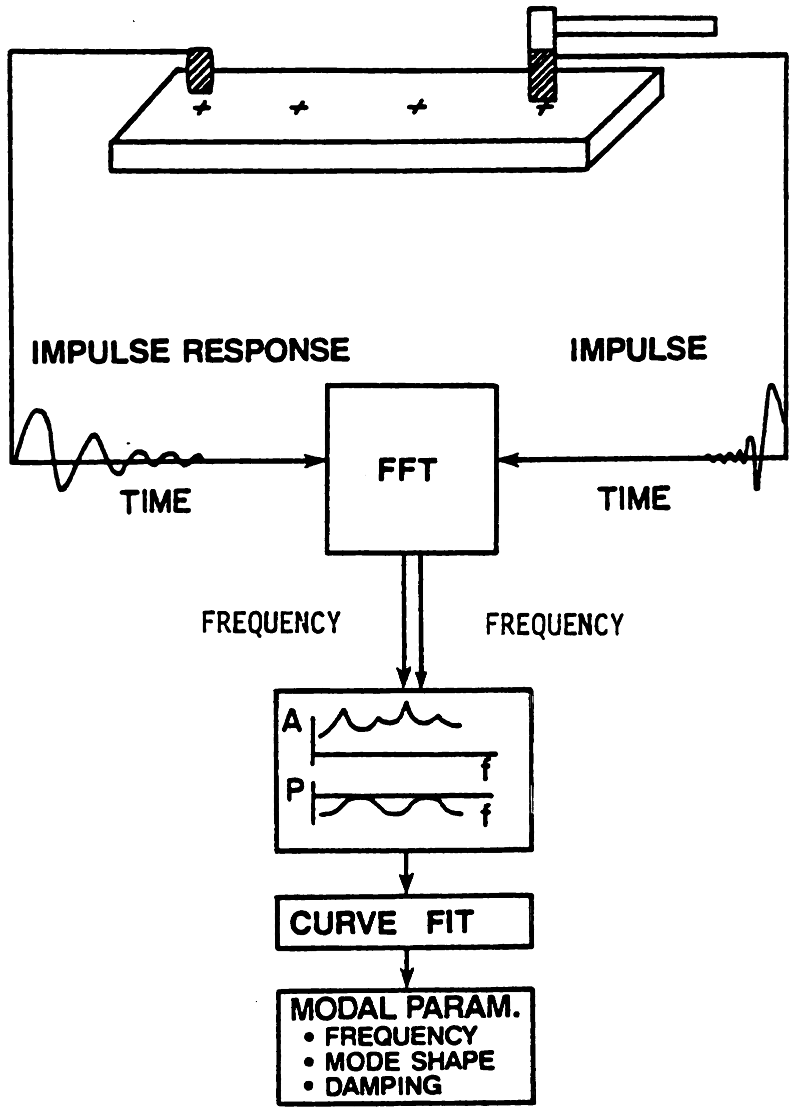


Figure 3-1 Impact Testing Method.

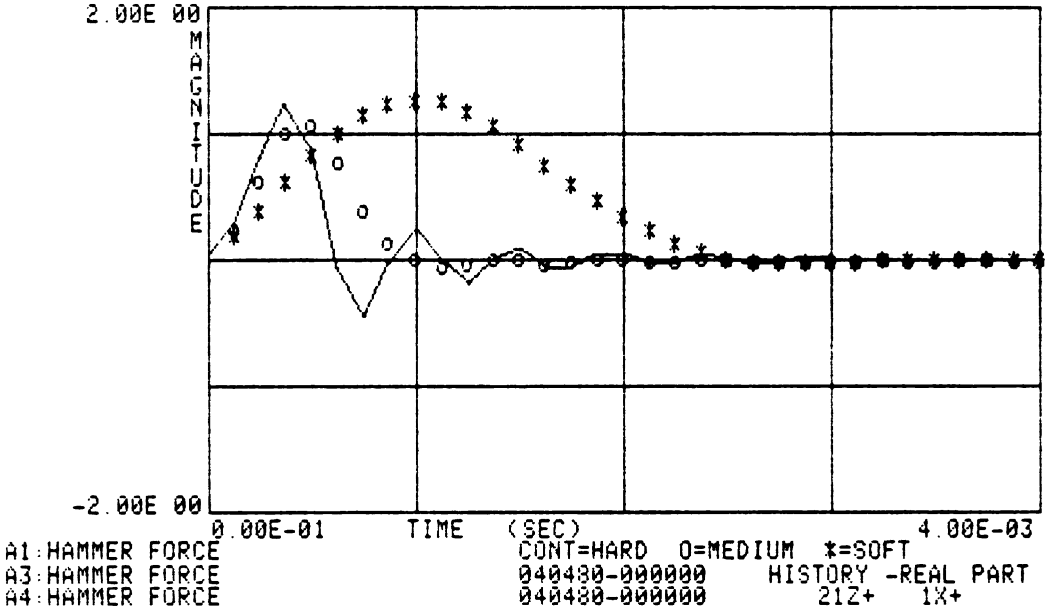


Figure 3-2 Time History of Impact Force Using Three Different Hammer Tips.

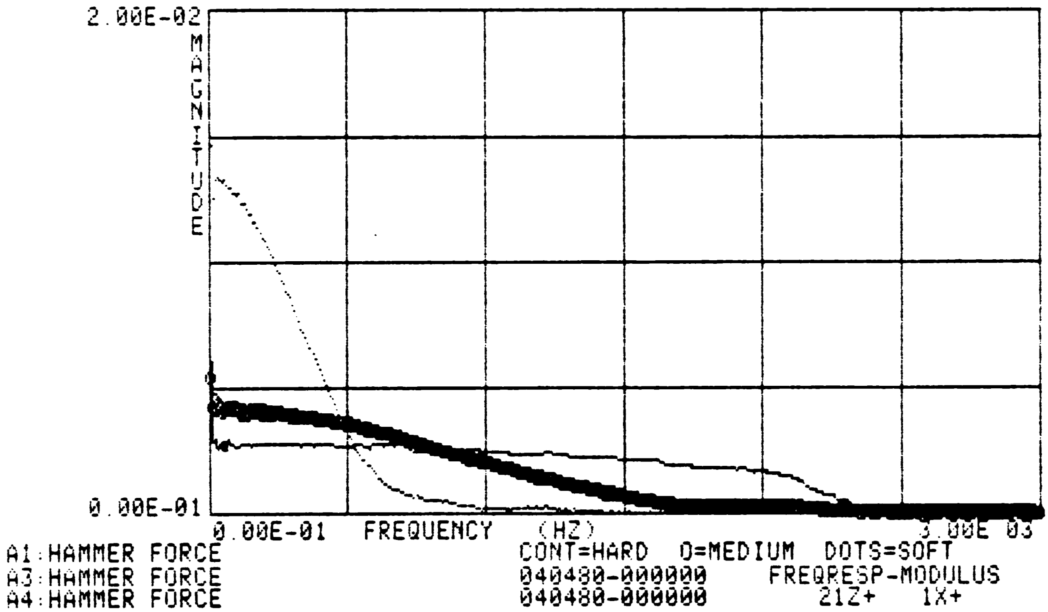


Figure 3-3 Frequency Domain of Impact Force Using Three Different Hammer Tips.

The impact testing has some advantages. It is easy to use and fast. It gives good accuracy and it can have very good frequency resolution. But it also has some drawbacks. The amplitude of input force is not easily controlled. It's energy density may not be high enough to excite the entire structure. More energy can be provided by hitting the structure harder but damage may result.

Despite these disadvantages, impact testing provides fast solution for trouble shooting vibration problems. For large variety of mechanical structures this method gives satisfactory results.

3.4 Modal Data Identification

When a structure is excited by a broadband input force, many of its modes are excited simultaneously. Since the structure is assumed to behave linearly, its transfer functions are the sum of the resonance curves for each of its modes as shown in Figure 3.4. Therefore at any given frequency the transfer function represents the sum of motion of all the modes which have been excited. The response in a certain frequency range can be approximately described in terms of the "Inertia Restraint" of the lower modes of vibration, the modes of vibration which are resonant in that frequency range, and the "Residual Flexibility" of the higher frequency modes (see Figure 3.5). The effects of lower and higher modes can be represented in additional stiffness and mass matrices [13].

The amount of overlap from one mode to another depends on (a) frequency separation, (b) damping of the structure and (c) nonlinear effects. Figure 3.6 shows transfer functions for the difference between light and heavy damping. In cases where modal overlap is light the transfer function data can be treated in the vicinity of each peak (resonance) as if it were a single degree of freedom system. In other words it is assumed that the contribution of the tails of adjacent modes near each modal resonance is negligibly small. In these cases, single degree of freedom curve fitting algorithms may be used to identify the characteristics of each resonance.

Figure 3.7 presents alternative forms of single degree of freedom curve fitting. The first method is the so-called co-quad method where the modal frequency can be obtained by simply taking the frequency of the peak of the imaginary part of the transfer function or the frequency where the real part of the transfer function is zero. And the residue can be estimated by using the peak value of the imaginary part of the transfer function.

The second method is magnitude phase technique, where the modal frequency is the frequency of the peak of the transfer function magnitude or it is the frequency where the phase angle is 90 degrees.

The third method, so called "circle fitting", gives the most accurate results. It is a way of estimating the modal parameters by least squared fitting of the parametric form of a circle to

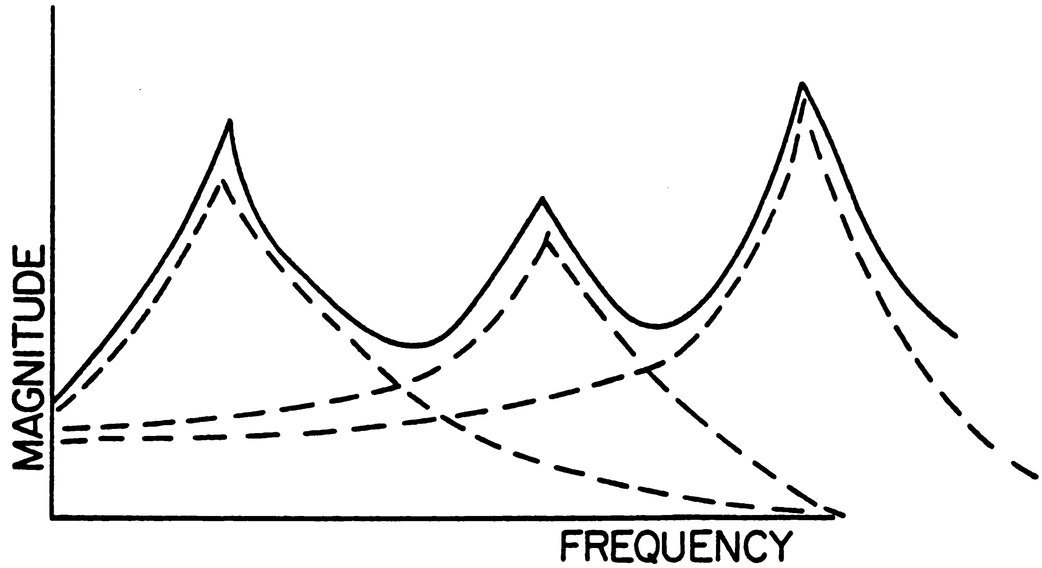


Figure 3-4 Frequency Response of a Multi-Degree-of-Freedom System.

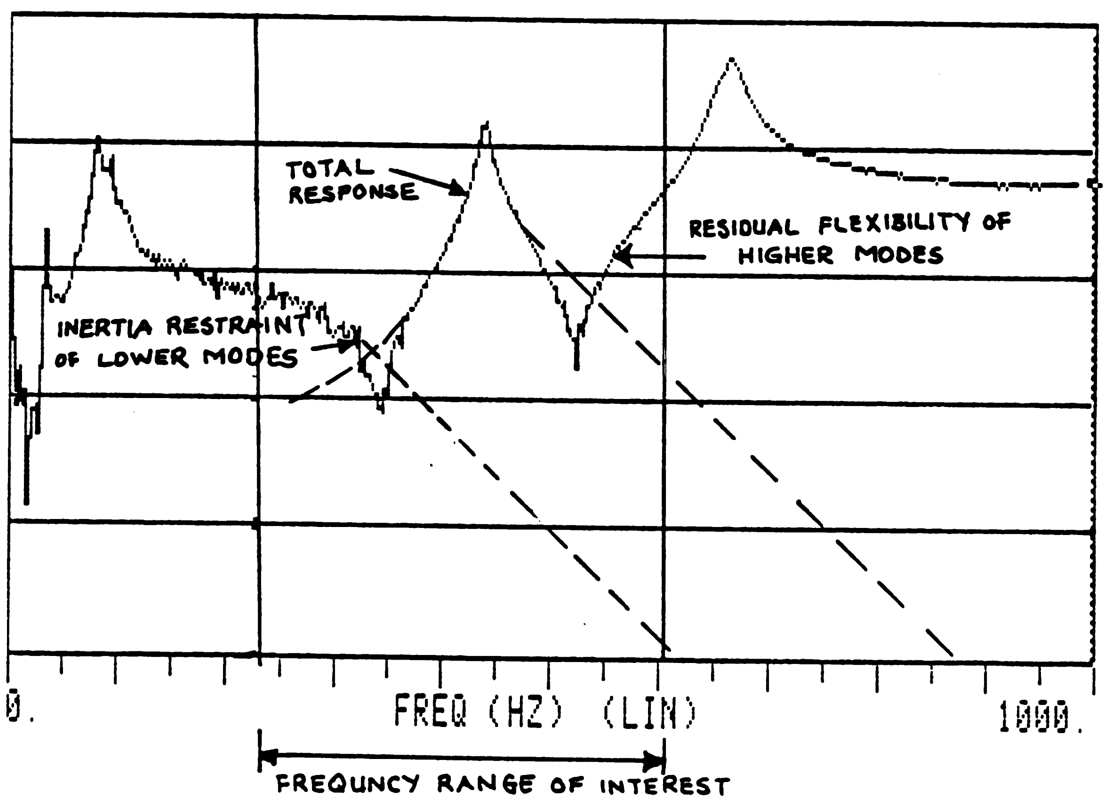
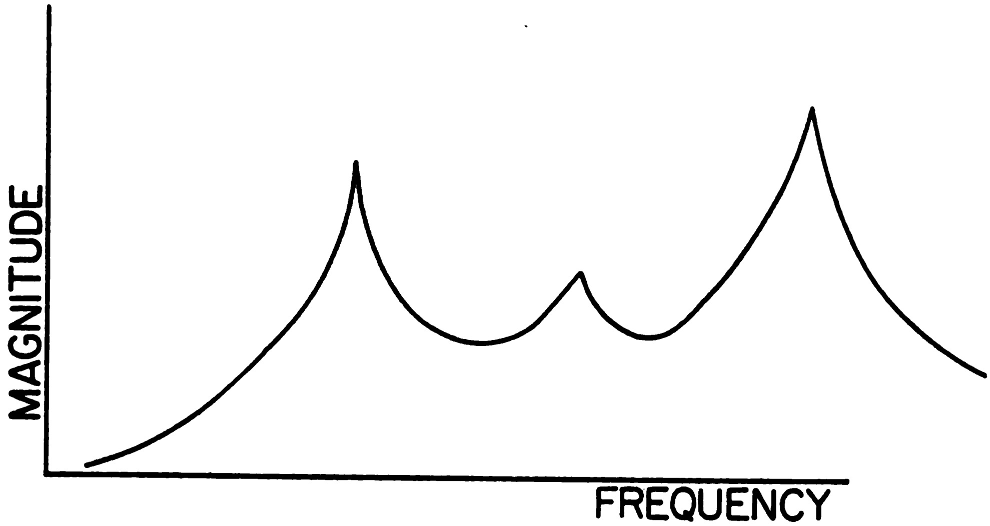
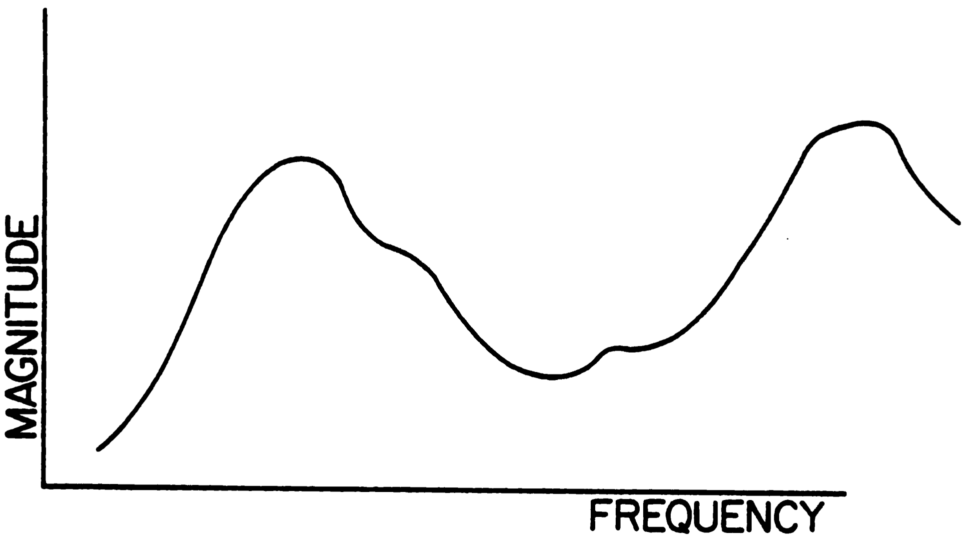


Figure 3-5 The effects of Lower and Higher Modes.

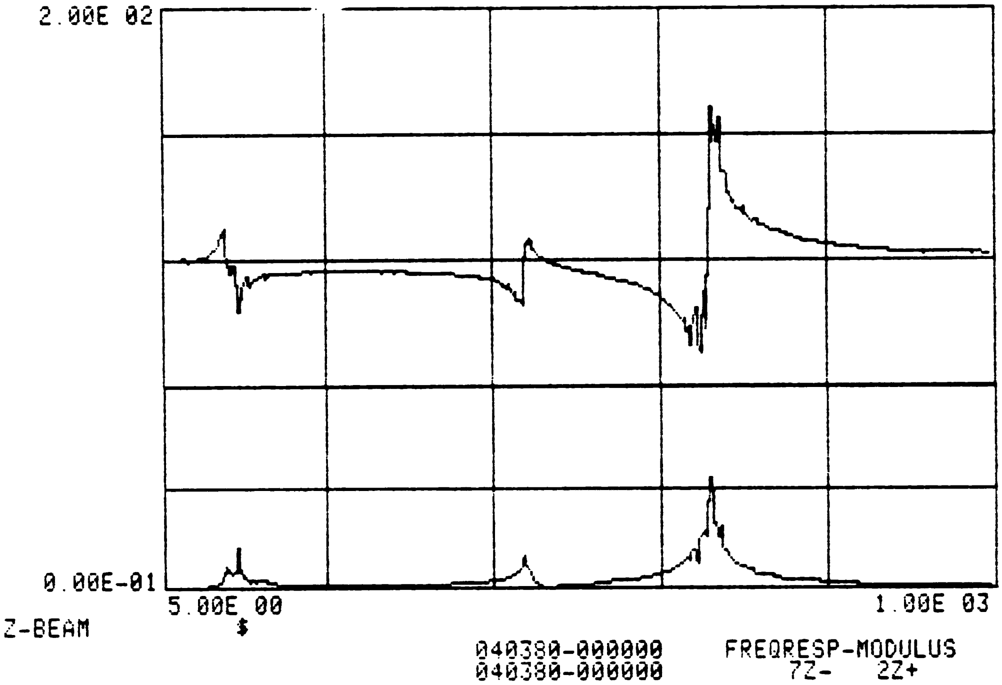


a) Light Modal Overlap.

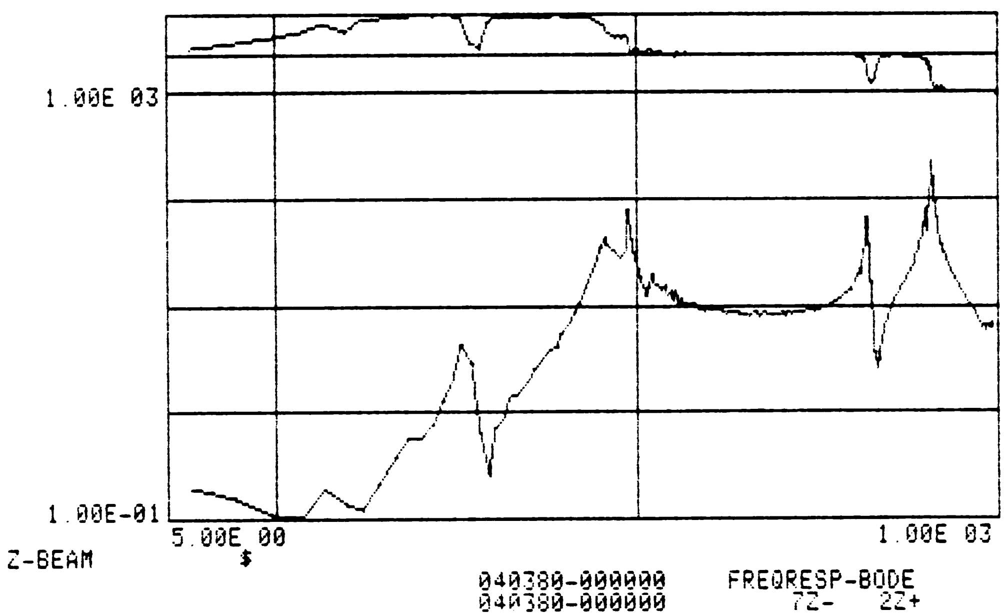


b) Heavy Modal Overlap.

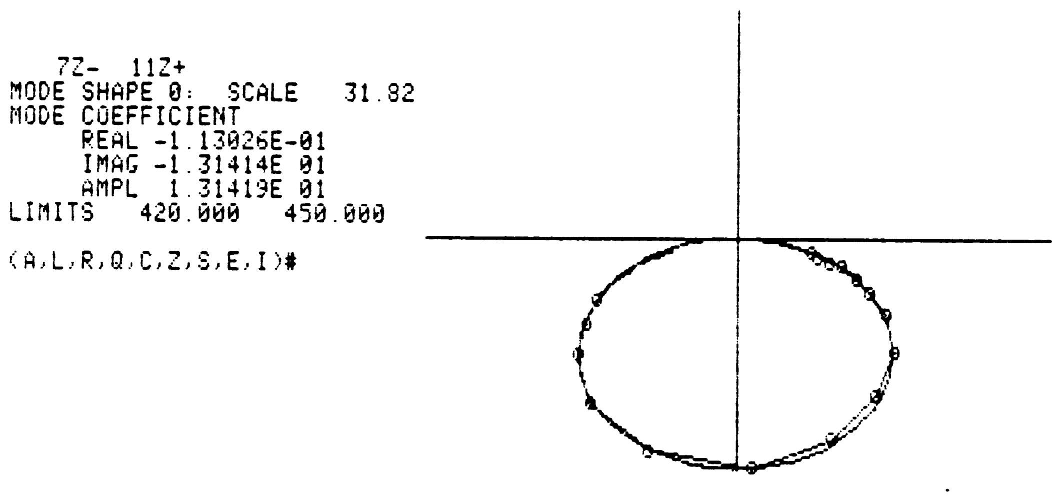
Figure 3-6 The Difference Between Light Modal Overlap and Heavy Modal Overlap.



a) Real and Imaginary Plots of Frequency Response Function (Co-Quad method).



b) Bode Plot and Phase Plot of Frequency Response Function.



c) Nyquist Plot (Circle Fit)

Figure 3-7 Alternative Forms of Frequency Response and Single-Degree-of-Freedom Curve Fitting.

the measurement data in Nyquist form. These methods are explained detail in references [15] and [16].

If the modes are closely spaced, then multi-degree of freedom techniques give much more accurate results [17]. These techniques involve curve fitting a multiple mode form of the transfer function to a frequency interval of measurement data containing several modal resonance peaks. In the process, all the modal parameters for each mode in a given frequency range are simultaneously identified. Figure 3.8 shows the polynomial form of the transfer function which can be used for curve fitting and an illustration of it on a transfer function data. The coefficients of the polynomials in the numerator and denominator are identified by curve fitting, and roots of the polynomials which contain modal parameters are found by a root finding routine. This and other multi degree of freedom methods are explained in detail in references [16], [17], [18].

Many times, modal coupling or noise on the measurement may make it difficult to identify the number of modes and and their parameters from any single measurement. In these cases a curve fitting procedure that identifies modal parameter from the multiple set of measurement should be used. In other words, multiple row or column of transfer matrix should be measured by mounting more than one accelerometer to get more accurate results. This technique is discussed in more detail in Reference [19].

MULTIPLE MODE METHODS

Polynomial Form:

$$H(s) = \left. \frac{A_o + A_1 s + A_2 s^2 + \ldots + A_m s^m}{B_o + B_1 s + B_2 s^2 + \ldots + B_n s^n} \right|_{s = jw}$$

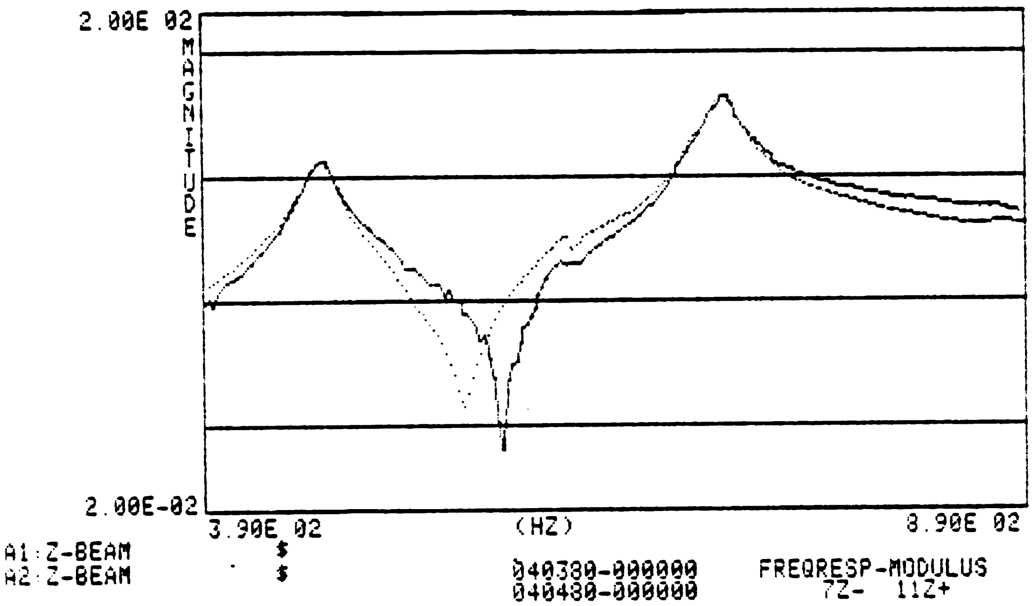
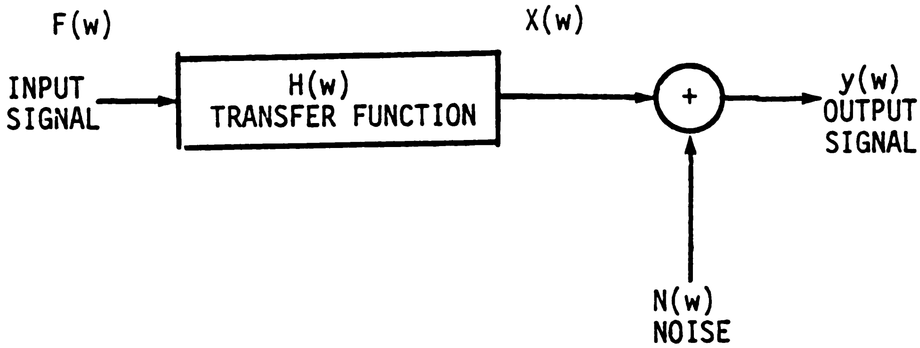


Figure 3-8 Multi-Degree-of-Freedom Curve Fit. Dotted Line. The Curve Fit.

3.5 Noise and Distortion

Another important matter in modal testing is the extraneous noise which is included in the measurement along with the desired signal. Since we are interested in identifying modal parameters from measured input and output, the reliability of the parameter estimates is reduced in proportion to the amount of noise in the measurements. In general, we measure input and output signals and obtain an estimate of transfer function. However, since there is always noise to be considered, the transfer function is obtained in more accurate fashion as shown in Figure 3.9 [16]. The effect of noise is reduced as the number of averages grows (the noise term in the Figure 3.9 gets smaller) and the ratio of output to input more accurately estimates the time transfer function. This effect can be quantified in the coherence function.

The coherence function is the ratio of response power caused by applied input to measured response power. As the number of averaging goes up, the coherence function becomes much smoother (see Figure 3.10). Whenever transfer functions are measured on a digital Fourier analyzer, the coherence function can also be calculated in terms of averaged input and output autopower and crosspower spectrums [8,9]. The coherence function indicates whether the response is being caused by the input. Values of coherence function less than 1 indicate that an amount of extraneous noise is being measured with the signal. Coherence is used to determine how much averaging is necessary to effectively remove the effects of noise from the measurement.



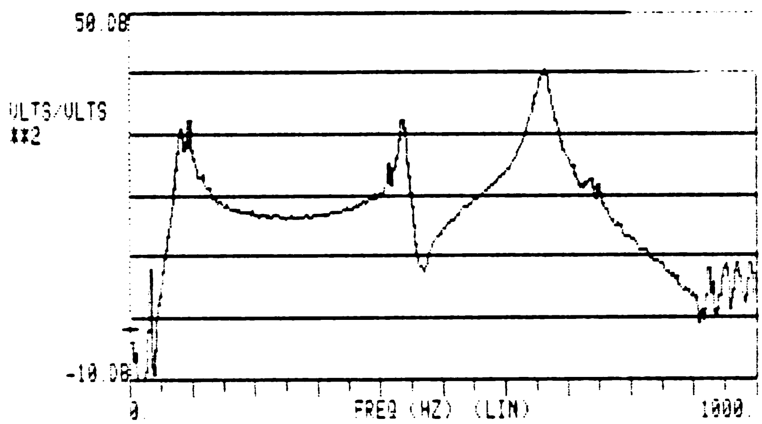
$$H = \frac{\overline{G}_{yf}}{\overline{G}_{ff}} - \frac{\overline{G}_{nf}}{\overline{G}_{ff}} \quad \text{where:}$$

$\overline{G}_{ff}$ = Averaged Auto Spectrum

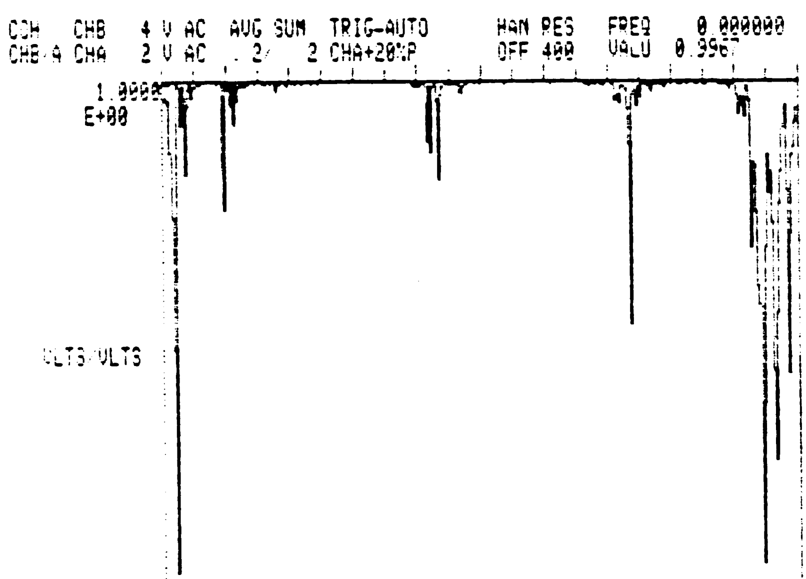
$\overline{G}_{yf}$ = Averaged Cross Spectrum

$$\text{COHERENCE FUNCTION } (\gamma^2) = \frac{|\overline{G}_{yf}|^2}{\overline{G}_{ff}\,\overline{G}_{yy}}$$

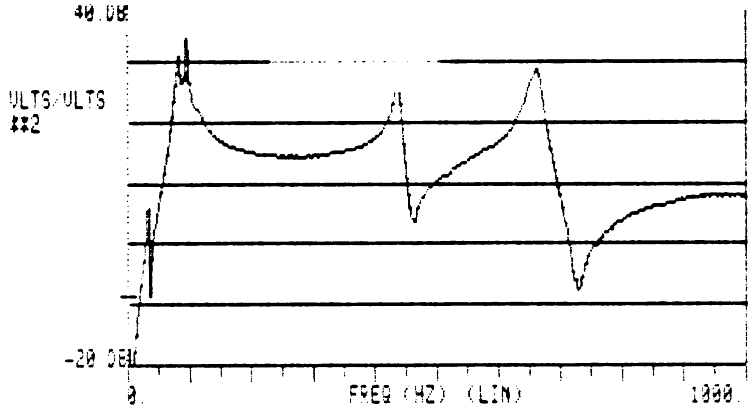
Figure 3-9 Measurement of Signal Plus Noise.



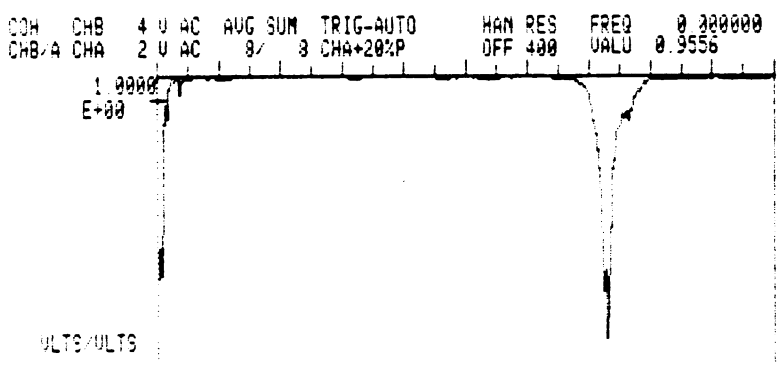
a) Frequency Response Data After 2 Averages.



b) Coherence Function of the Same Data After 2 Averages.



c) Frequency Response Data After 8 Averages.



d) Coherence Function of the Same Data After 8 Averages.

Figure 3-10 The Use of the Coherence Function and the Effects of Averaging.

Distortion or nonlinear motion is another important subject to consider in vibration measurements. Since modal analysis techniques are based on assumed linearity of the dynamic model, the measurements should not reflect any nonlinear motion. Power spectrum averaging does help to reduce this kind of an effect [16]. In addition, different types of excitation techniques to use for testing in order to reduce nonlinear effects [11].

3.6 Measurement Resolution

Since the accuracy of modal parameters depends on the accuracy of the transfer function measurements, frequency resolution is extremely important. In addition, curve fitting algorithms are heavily dependent on adequate resolution.

In the past, many Fourier analyzers have been limited to Base Band Fourier Analysis (BBFA), i.e., the Fourier transform is computed in a frequency range from zero to some maximum frequency F_{max}. This digital Fourier transform is spread over a fixed number of frequency lines which limits the frequency resolution between lines. Therefore BBFA provides uniform frequency resolution from 0 to F_{max} and the frequency resolution can be expressed as $\Delta f = F_{max}/(N/2)$, where N is the number of sampling points. From a practical point of view, in many structures modal coupling is so strong that increased frequency resolution is a necessity for achieving reliable results. In BBFA, the only way to obtain better resolution over this bandwith is to use a larger memory, collect more data and compute the spectral functions using more points which would increase the processing time.

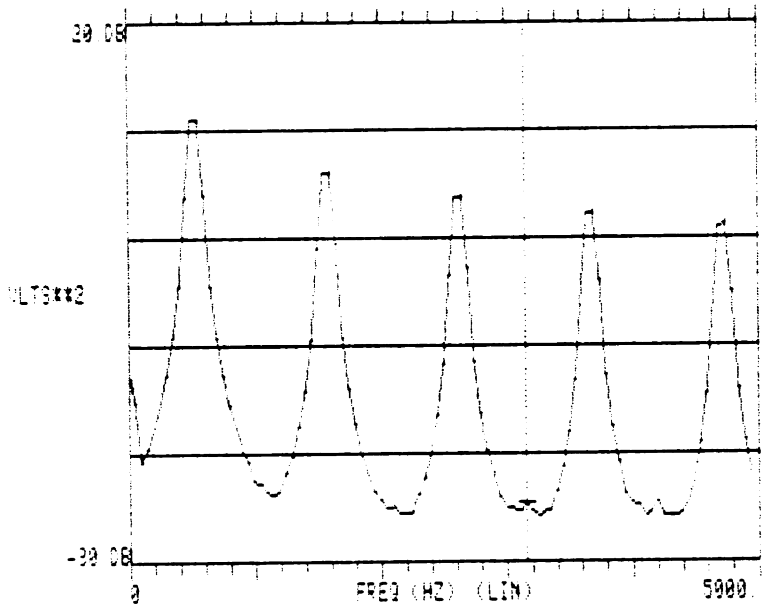
More recently the implementation of Band Selectable Fourier Analysis (BSFA), the so-called "zoom" transform, has made it possible to perform Fourier analysis over a frequency band whose upper and lower frequency limits are independently selectable. The resolution obtained in the frequency band of interest is approximately $\Delta f = BW/(N/2)$ where bandwith is the frequency region of interest. Therefore a narrow region of interest would increase the frequency resolution without increasing the number of spectral lines in the computer. However, the processing time gets longer as the bandwith gets narrower. Figure 3.11 shows the comparison between BBFA and BSFA. Reference 20 gives a good discussion about both BBFA and BSFA.

3.7 Some Other Considerations in Modal Testing

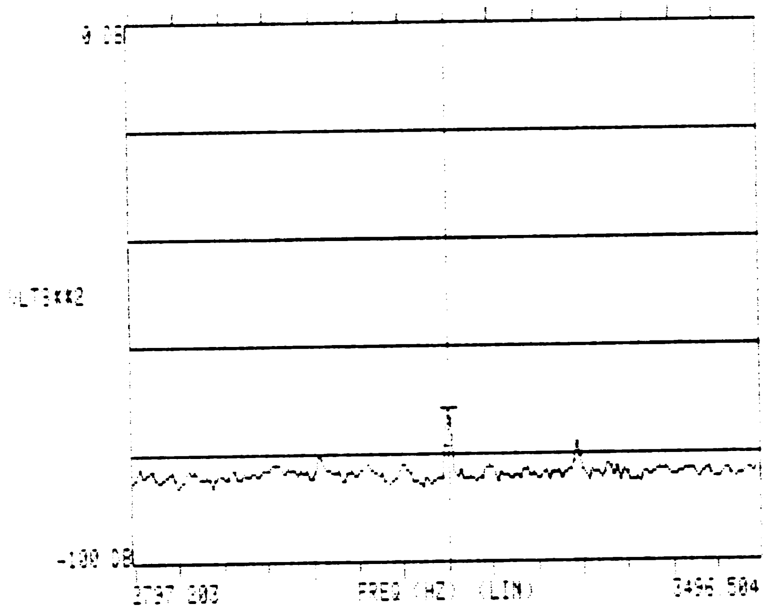
There are several factors that contribute to the quality of actual measured transfer and coherence function estimates. I have already discussed several important ones such as the excitation method, noise, distortion and frequency resolution. There are a few more considerations to be mentioned in modal testing. They are:

3.7.1 Aliasing

Sampling a signal at discrete times introduces a form of amplitude distortion called aliasing that converts high frequency energy to lower frequencies. If the sampling rate for an incoming signal is not greater than twice the highest frequency of any component in the signal, then some of the high frequency



a) Base Band Transfer Function.



b) BSFA Transfer Function (Zoom).

Figure 3-11 Base Band vs. Zoom Transform.

components of the signal will be effectively translated down to be less than one half of the sampling rate [8]. This translation may cause serious problems with interference between high and low frequency components.

In Figure 3-12, cosine waves with two different frequencies are shown in the frequency domain. Since sampling frequency was not greater than twice the highest frequency, it caused that frequency to appear in the lower frequency region. To avoid these interference effects, the signals can be sampled at a sufficiently high rate and/or a low pass filter can be put to reduce the amplitude of the higher frequency component so they are not longer large enough to be troublesome. References [8], [9] and [10] give more detail about aliasing and filtering.

3.7.2 Leakage

When a signal of finite length is sampled and Fourier transformed, the resulting transform is representative of a periodic signal for which the sampled signal is one period. If the original signal before sampling was not periodic, it will cause a smearing of data in the vicinity of peaks in the spectrum, which introduces another type of distortion called leakage. A simple example of what can happen is shown in Figure 3.13, where a sinewave has been sampled. The discontinuity at the ends leads to set of components in the analysis that may interfere with the components of interest. Several techniques have been developed to reduce the effects of "leakage", one of which is Hanning window. Reference [10] gives a good discussion of leakage.

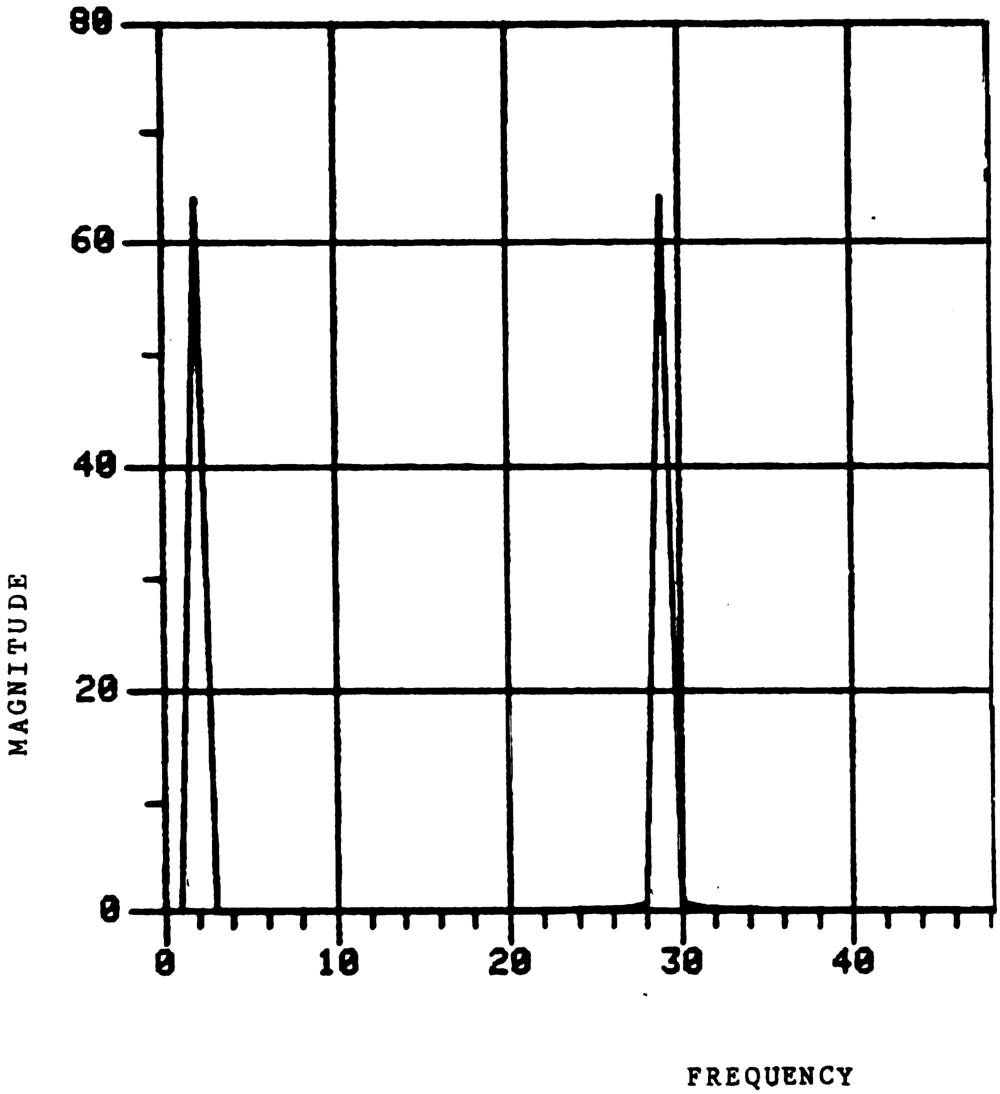


Figure 3-12 Illustration of Aliasing. Cosine Waves at 2 Hz and 99 Hz. Sampling frequency less than 2 x 99.

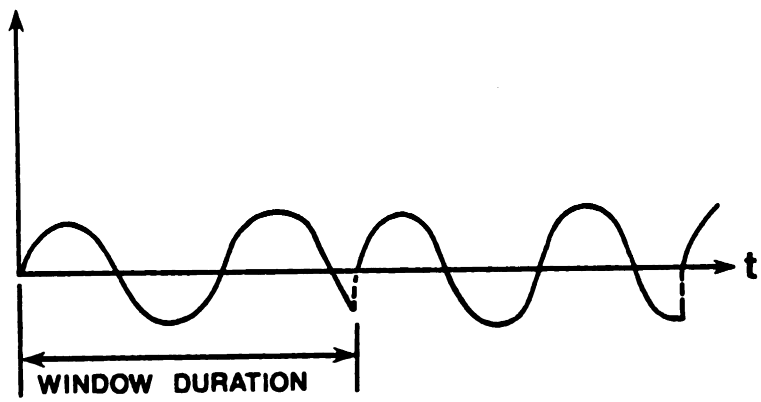


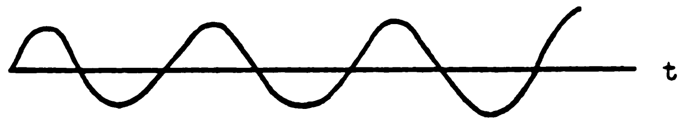
Figure 3-13 Illustration of Leakage.

3.7.3 Windowing

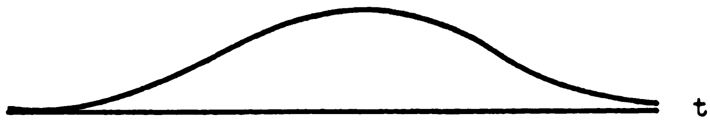
The purpose of windowing is to remove unwanted characteristics of the signals. The most commonly used windowing techniques are Hanning and exponential [9,10].

When Hanning is used, the data at the ends of the window are ignored since they are multiplied by a value near zero (Figure 3.14). It is important that the data window be wide enough to ensure that the important behavior is centered within the window.

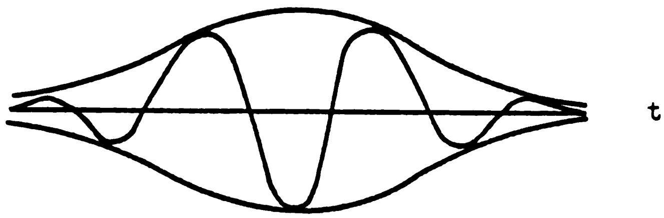
Another commonly used technique is exponential weighting, which multiplies both the input and output signals by an exponentially decaying envelope (see Figure 3.14). In the case of impulsive excitation, in which the signal/noise ratio is greater at smaller values of time, the weighting rejects most of the noise. This procedure leads to more consistent determination of resonant peak amplitudes. But it also makes the determination of closely spaced modes more difficult. Therefore "zoom" trans-



SINE WAVE

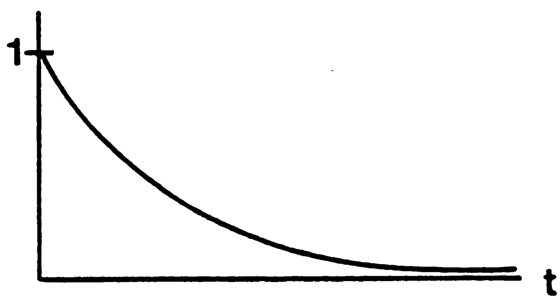


HANNING WINDOW



SINE WAVE MULTIPLIED BY THE HANNING WINDOW

a) Hanning Window.



b) Exponential Window.

Figure 3-14 Hanning Window and Exponential Window.

form analysis may be required in some cases to allow sufficient resolution of closely spaced modes.

3.8 Summary

Modal testing is based on frequency response information of the structure. The general scheme for measuring frequency response functions consists of measuring simultaneously an input excitation with a load cell mounted to a hammer in impact testing or a shaker in other methods of excitation, and the response signal with a transducer, preferably accelerometer, mounted to the structure. Digital signal processing techniques are applied to these signals. Then the modal parameters such as resonant frequency, damping and mode shapes can be obtained with using single degree-of-freedom or multi degree-of-freedom curve fitting algorithms.

There are several factors that should be considered to obtain accurate test results, including (a) the selection of input excitation method, (b) the selection of curve fitting algorithm, (c) noise and distortion, (d) Measurement resolution, and (e) aliasing, leakage and windowing.

CHAPTER 4

AN EXAMPLE OF MODAL TESTING

Modal testing was performed on a Z shaped aluminum beam (Z-Beam) using Gen-Rad 2508 Structural Analysis System [21] utilizing the SDRC MODAL PLUS software. The Case Center for CAD (Computer-Aided Design) at Michigan State University has a Gen-Rad 2507 [21] which is very similar to Gen-Rad 2508. SDRC MODAL PLUS is a joint software product of SDRC (Structural Dynamics and Research Corporation), Cincinnati, Ohio and Gen-Rad, Inc. AVA Div., Santa Clara, California. The Modal Plus software is also included in Gen-Rad 2507, Structural Analysis System.

This chapter uses the Z-Beam as an example to demonstrate the procedure of modal testing. The implementation of the methods in MODAL PLUS requires the following steps:

- definition of the geometry of a structure,
- excitation of the structure and data acquisition,
- computation of the frequency response function,
- estimation of modal parameters, and
- generation and display of mode shapes.

Each of these subjects is covered in this chapter.

4.1 Geometry Definition

The implementation of modal analysis depends on the geometric definition of the structure. That is, a coordinate system must be selected and several points on the structure must be defined.

Additional points yield better estimates of mode shapes, but more points require that more data must be collected.

The structure in this case was a "Z" shaped aluminum beam (Z-Beam) which was modeled with a 28 points. The base of the Z-Beam was clamped to the ground (see Figure (4-1).

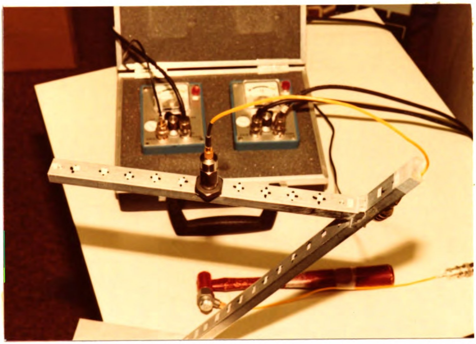


Figure 4-1 Z-Beam with the points that define its geometry.

4.2 Excitation Technique

In order to estimate frequency response function from measured data, one must supply an excitation function which is rich in energy at all frequencies of interest. In this case, the impact method was used with the hammer kit shown in Figure 4.2. The kit includes the hammer, a load cell attached to the hammer and an accelerometer.

In this experiment, the accelerometer remained at one location and the hammer was used to impact the 28 points (see Figure 4.3). A nylon tip was used for the hammer (medium tip). A time history and the corresponding frequency function of a hammer impact are shown in Figure 4.4.

Figure 4-2 Typical Hammer Kit for Modal Analysis.

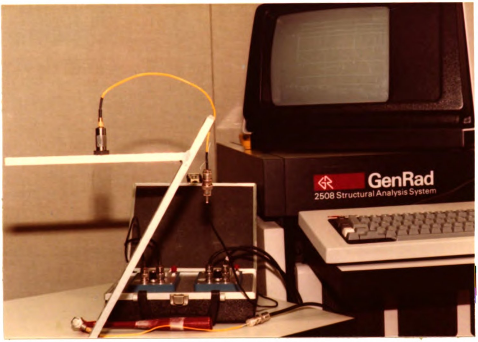
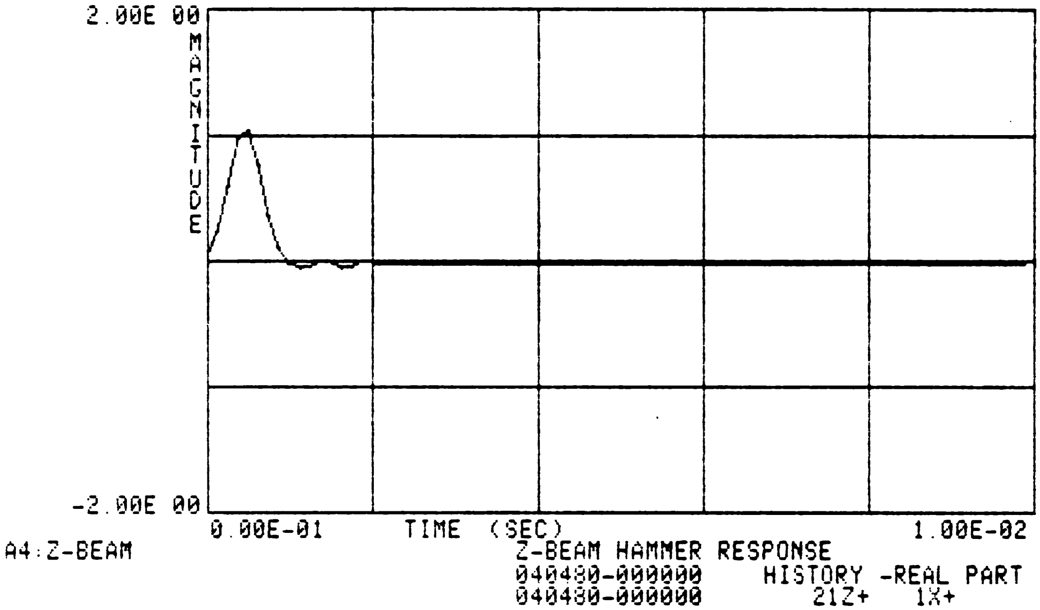
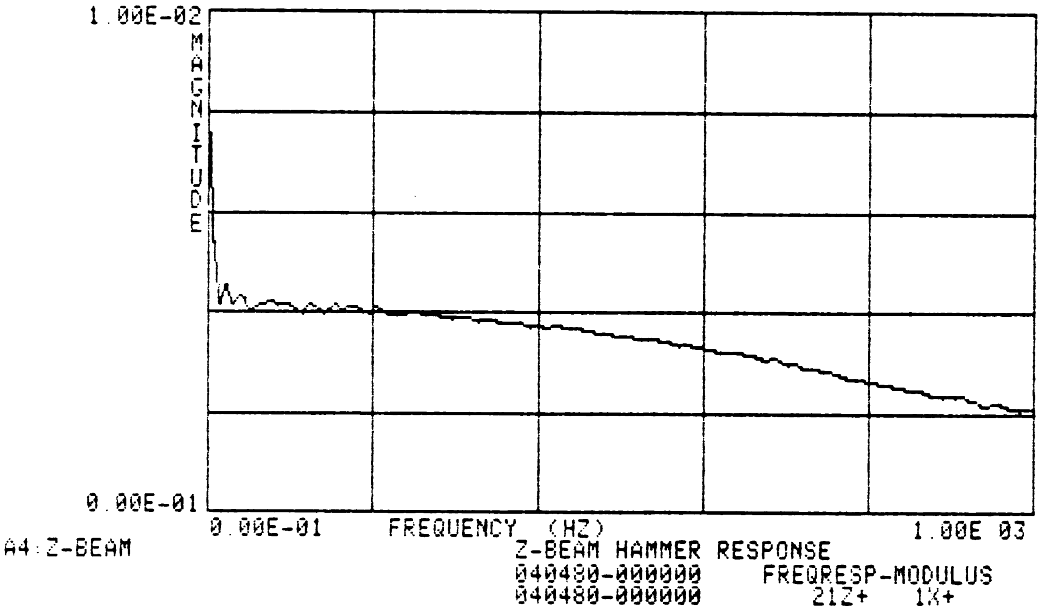


Figure 4-3 Z-Beam with the Instrumentation on it.



a) Time History



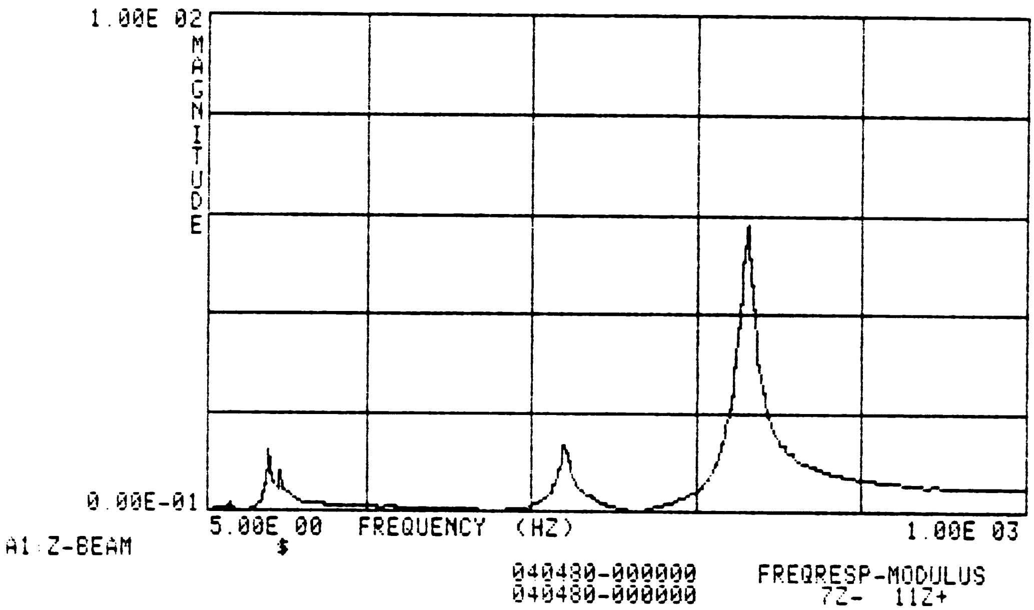
b) Frequency Domain

Figure 4-4 Time History and Frequency Domain of Impact Force Using a Nylon Tip.

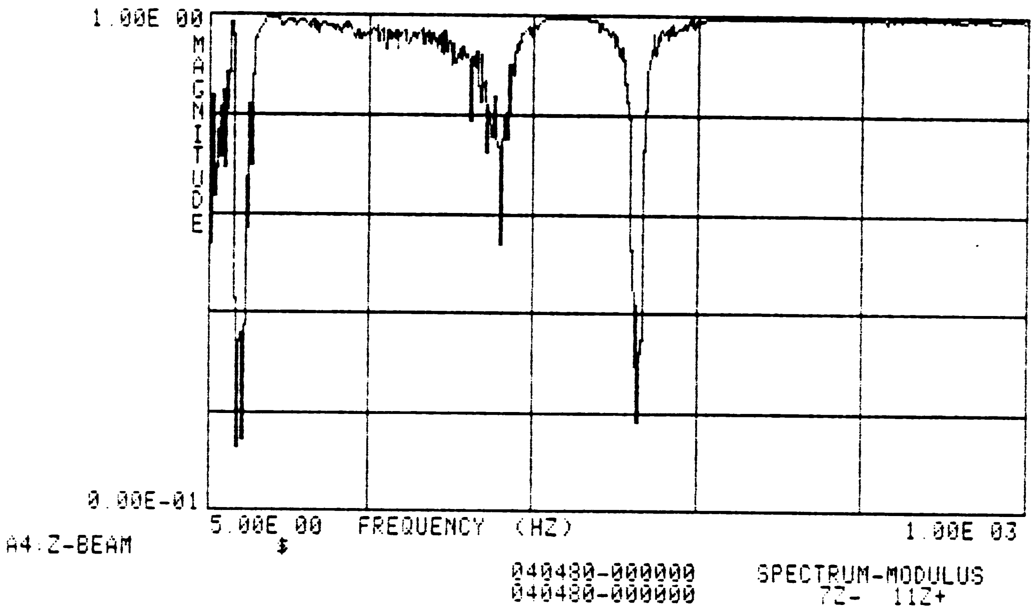
4.3 Frequency Response Data Analysis

After preliminary calculations it was decided that 0-1000 Hz frequency was adequate. Data was for accelerations in the vertical direction. Data was taken at each point with 5 averages and the coherence function was analyzed before accepting transfer function data. Figure 4-5 illustrates the difference between good and bad transfer function data with the aid of coherence function.

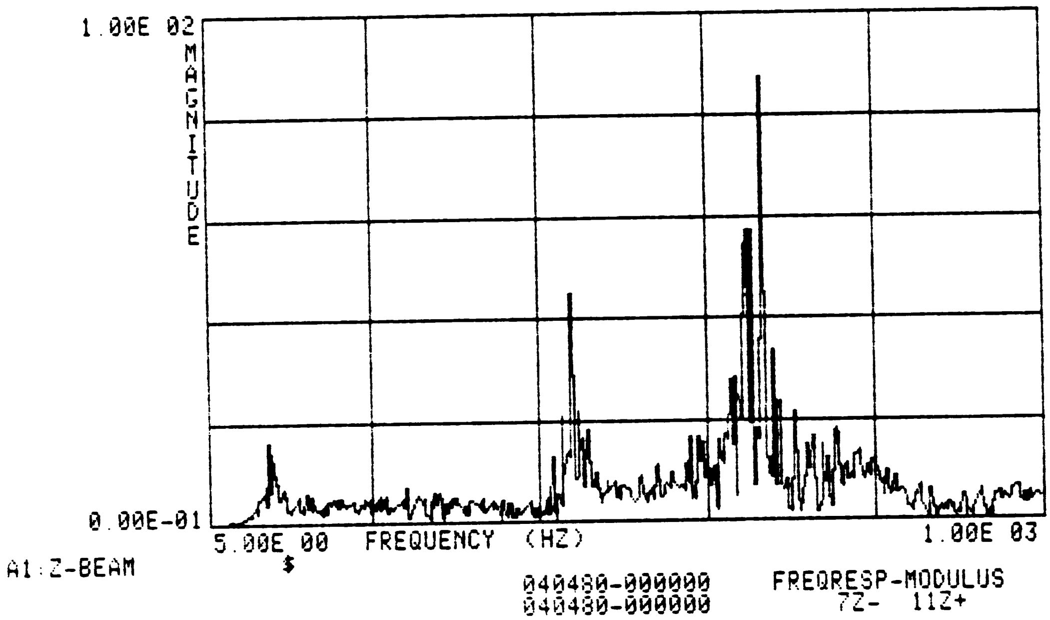
Frequency response functions such as Figure 4-6 were inspected for resonant peaks to determine at what frequencies modal estimates should be obtained. The numbers on the peaks indicate the use of a digital cursor to obtain the frequency and magnitude values listed on the left side of the plot. Inspection of this plot resulted in the selection of the frequency bands listed in Table 4.1 as the probable location of significant modes of vibration.



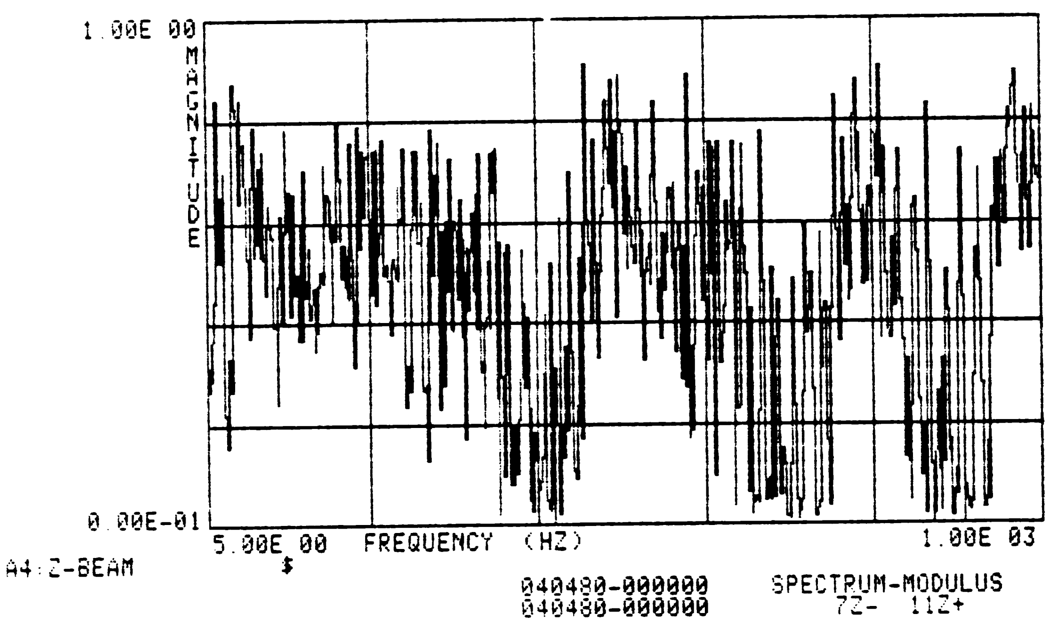
a) Good Transfer Function Data.



b) Good Coherence Function.



c) Bad Transfer Function Data



d) Bad Coherence Function

Figure 4-5 Difference Between Good and Bad Data.

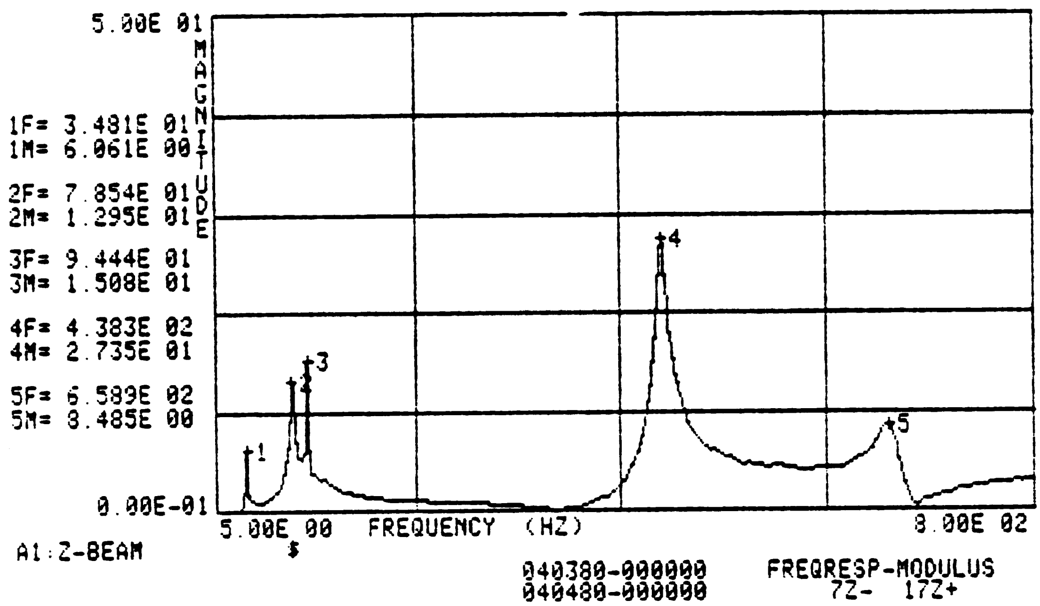


Figure 4-6 Example of Mode Frequency Selection.

TABLE 4.1

MODAL FREQUENCY BANDS

Mode Number	Frequency Band (Hz)
1	29 - 39.8
2	72 - 86
3	89 - 100
4	428 - 450
5	648 - 670

4.4 Modal Parameter Estimation

Two methods were employed in MODAL PLUS in extracting modal parameter; (1) Circle fit (2) Multiple degree of freedom curve fit.

Figure 4-7 and 4-8 are two examples of circles fitted to data. Figure 4.7 presents data in the frequency range from 360 to 500 Hz. The points are quite dense, indicating a well resolved spectral analysis. However, in Figure 4-8, which is in the frequency range of 600 to 700 Hz, there appears to be a second mode of smaller magnitude at the right hand side. Considering this point, circle fit was used in the frequency range of 550 to 650 Hz and 655 to 720 Hz, and two seperate modes were found between 550 and 700 Hz. (See Figure 4-9.)

Multi degree of freedom curve fitting method was employed in the frequency range of 500 to 1000 Hz and 10 to 1000 Hz (see Figure 4-10), the mode which was noticed in Figure 4-10 was also noticable in the 500 to 1000 Hz frequency range.

Table 4.2 shows the modal parameters for the modes of Z-Beam from Figure 4-6.

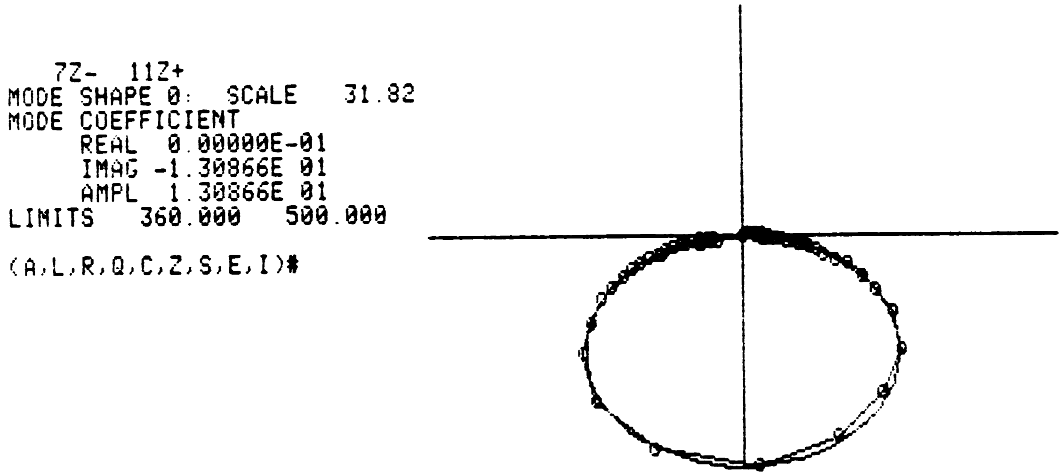


Figure 4-7 Circle Fit 360-500 Hz.

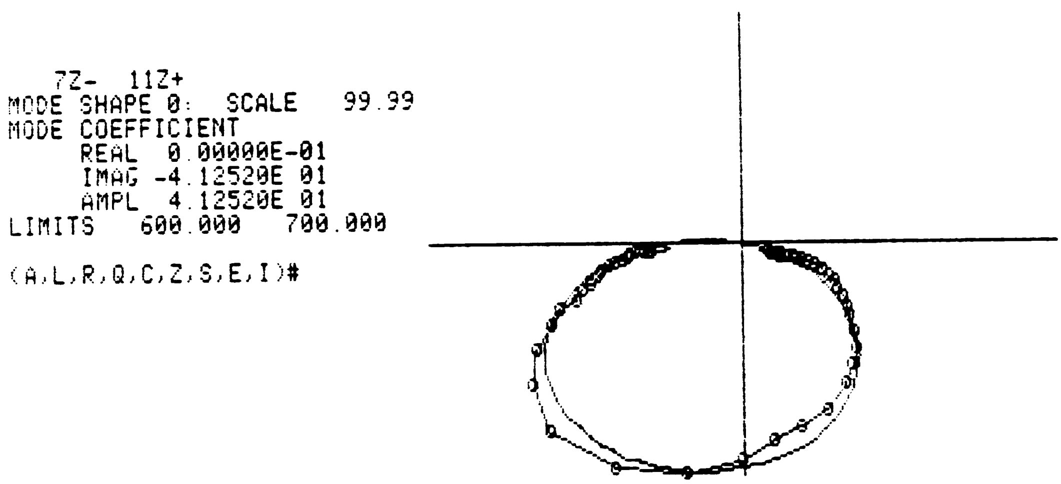
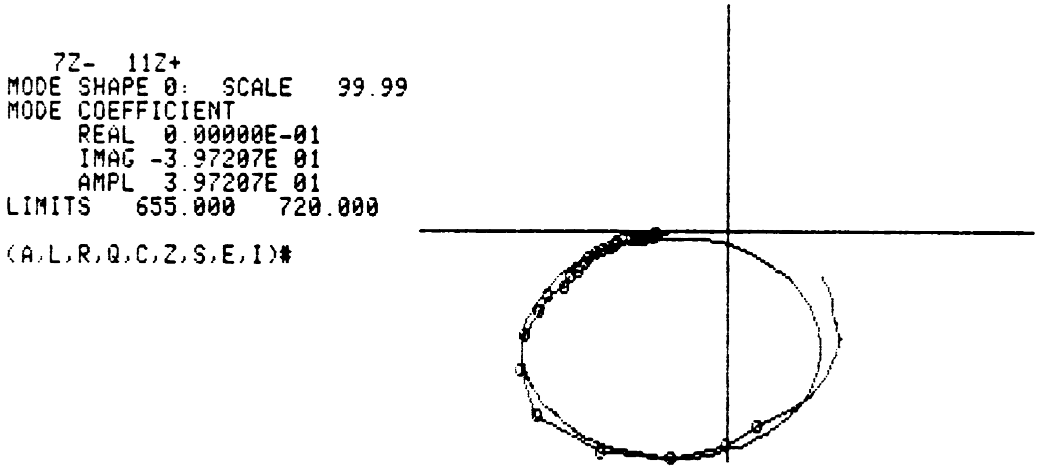
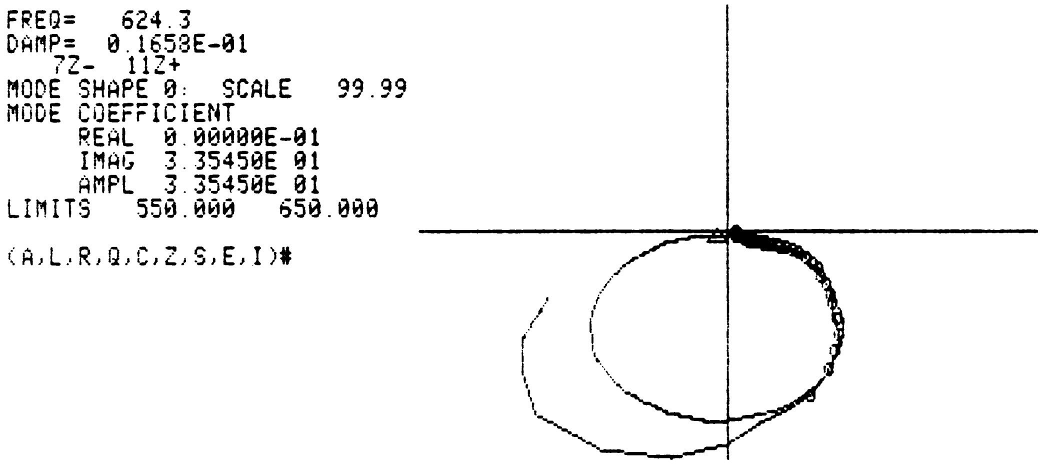


Figure 4-8 Circle Fit 600-700 Hz.

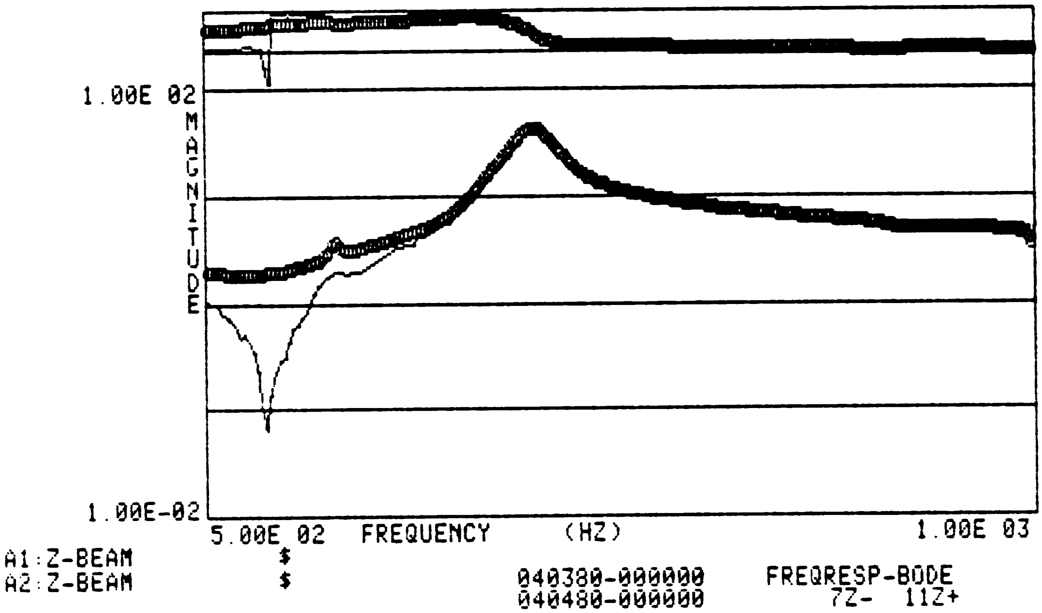


a) Circle Fit 655-720 Hz.

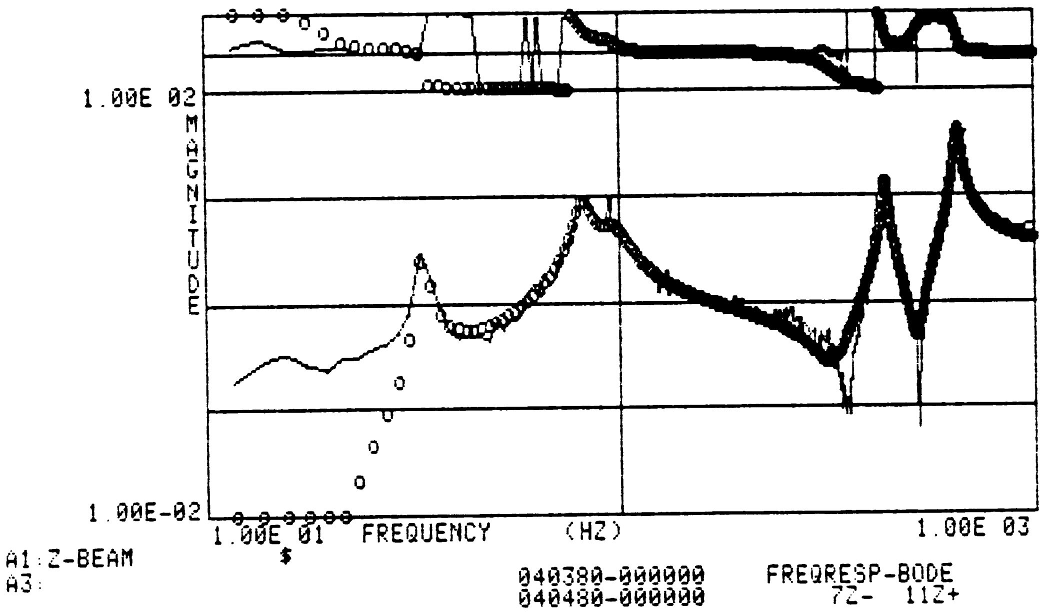


b) Circle Fit 550-650 Hz.

Figure 4-9 Determination of Modes Between 550-720 Hz via Circle Fit.



a) Multi Degree of Freedom Curve Fit 500-1000 Hz.



b) Multi Degree of Freedom Curve Fit 10-1000 Hz.

Figure 4-10 Multi Degree of Freedom Curve Fit. Solid Line-Data Circles Fit.

TABLE 4.2

MODE PARAMETERS

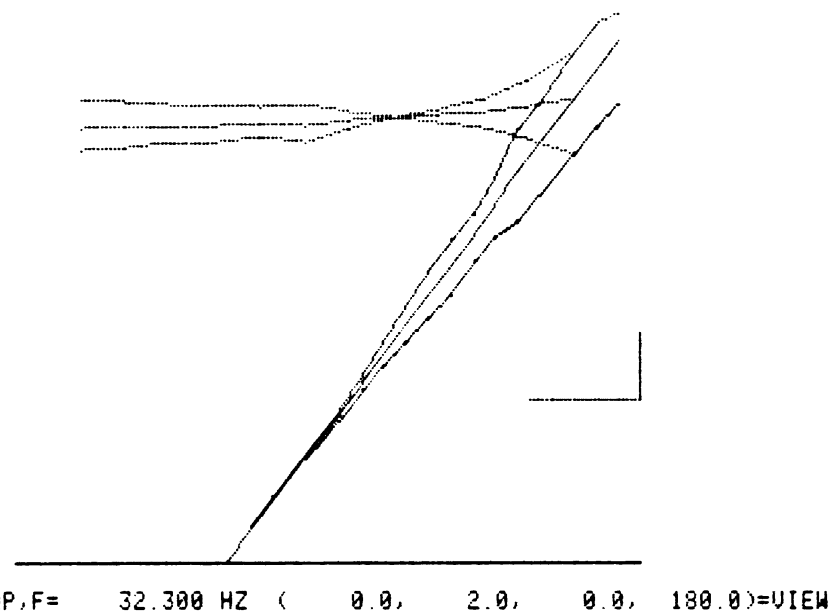
MODE	FREQUENCY	DAMPING	AMPLITUDE	PHASE
1	33.831	0.007358	10.33	1.7396
2	82.246	0.043136	179.7	-1.1411
3	98.664	0.74359	134.3	-2.6723
4	439.221	0.013872	507.0	-1.4423
5	658.719	0.015534	2542.0	-1.8938

4.5 Mode Shape Display and Interpretation of Results

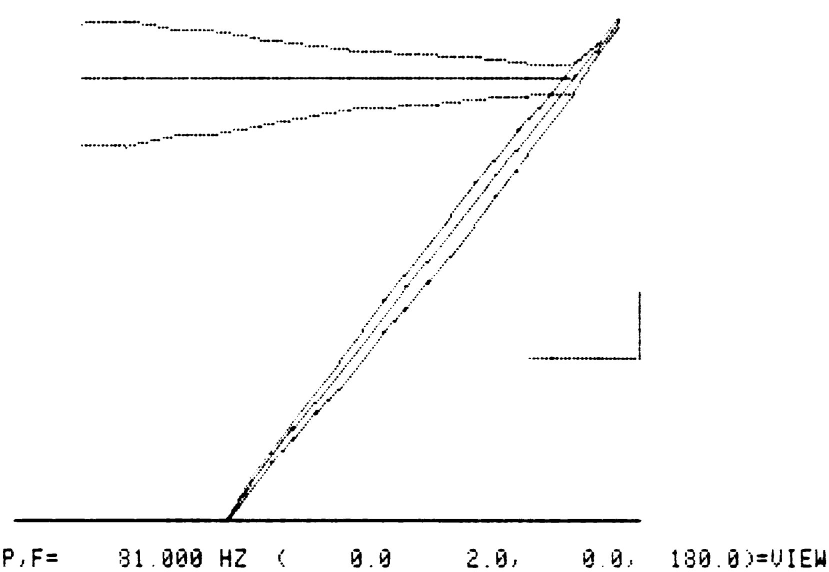
Once the modal parameters are estimated for all points on the geometry of a structure, then these parameters may be associated with the structural geometry. This facilitates the display of animated mode shapes and global visualization of the structural vibration.

For the Z-Beam, the mode shape display task software in MODAL PLUS was employed. Figure 4-11 shows four mode shapes of the Z-Beam.

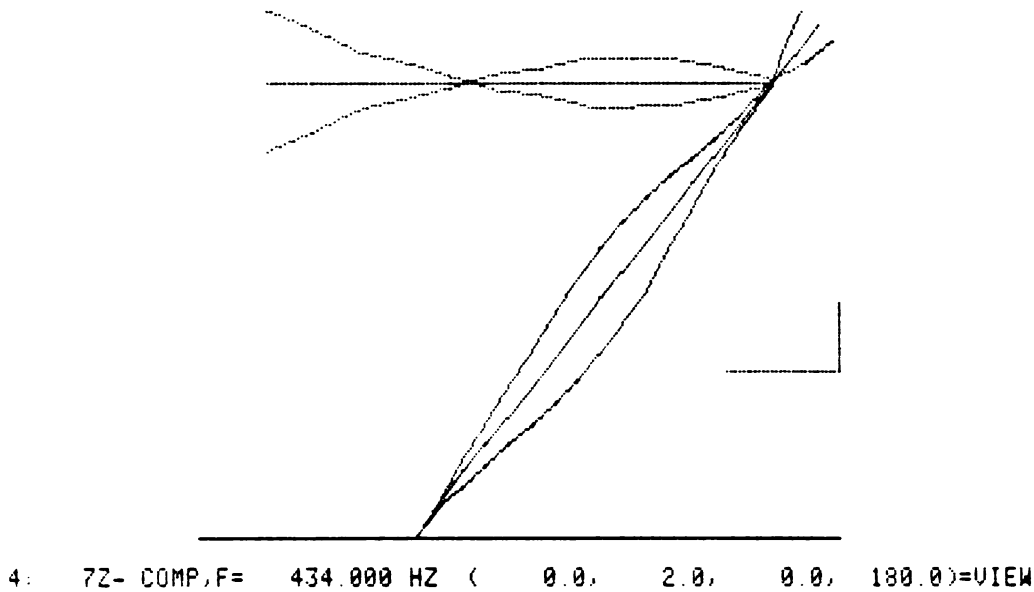
Figure 4-11a presents three frames of a mode at 32.3 Hz. This mode has a modal node very close to the position where the accelerometer was attached which causes it to almost disappear from the "driving point" frequency response data. This is illustrated in Figure 4-12.



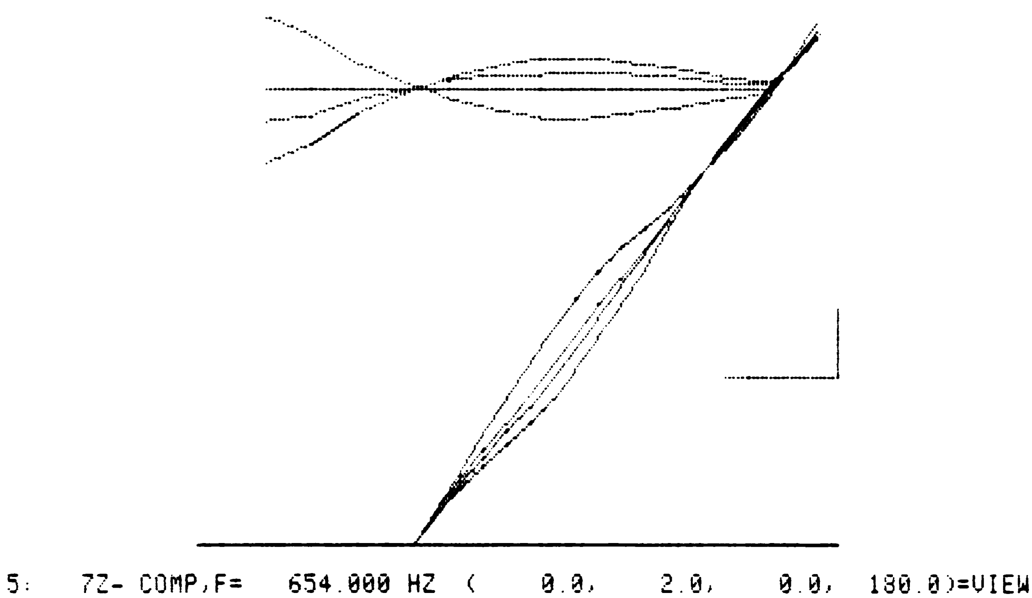
a) Mode at 32.3 Hz.



b) Mode at 81.00 Hz.

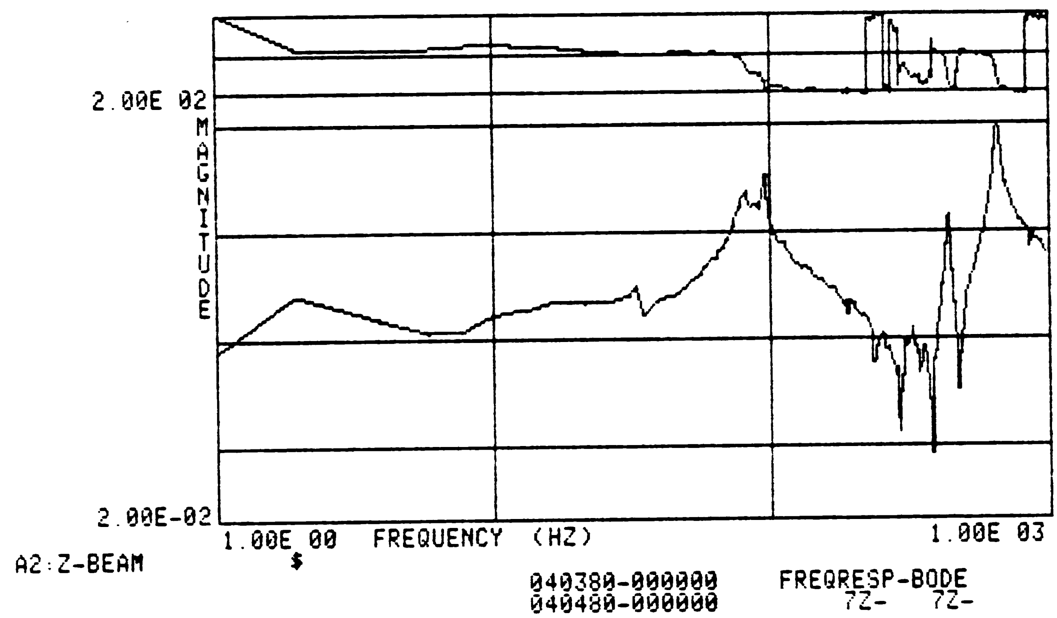


c) Mode at 434 Hz.

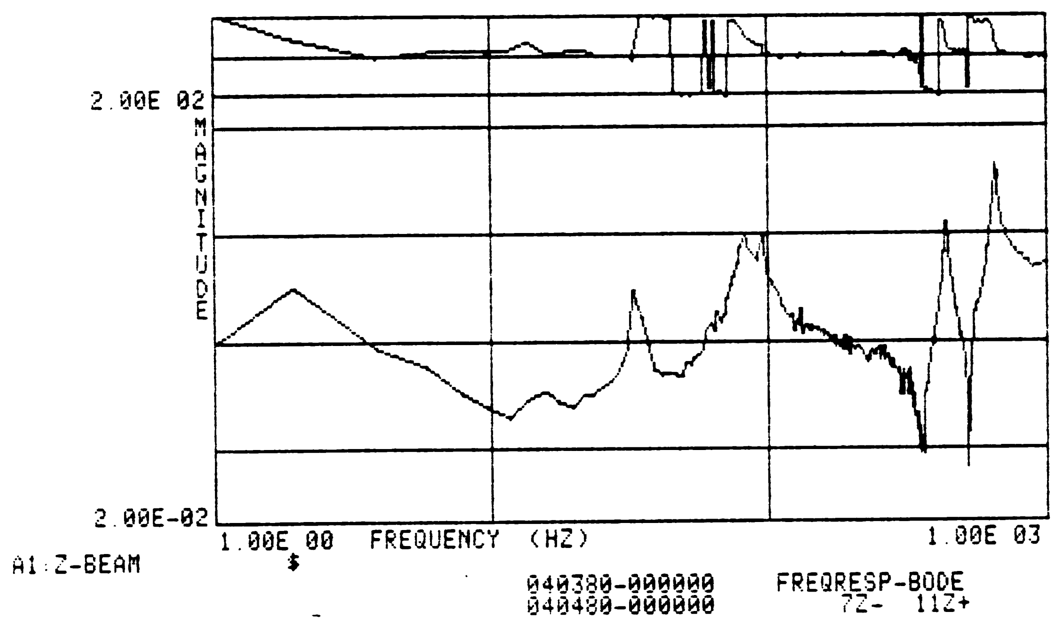


d) Mode at 654 Hz

Figure 4-11 Modes of Z-Beam; 3 Frames.



a) Transfer Function at the Driving Point.



b) Transfer Function Input at Point 11 response from point 7.

Figure 4-12 The Illustration of Modal Response at a Node.

4.6 The Validity of the Modal Data

The validity of the modal data can be assessed by synthesizing various response functions, and then comparing them with measured data. If the modal data is accurate, the fit from it should be fairly close to any frequency response function of the structure.

In this case, the modal data was obtained by analyzing 5 modes in frequency band of 0-1000 Hz. i.e., the Z-Beam frequency response representation was the sum of the 5 modes. Figure 4-13 shows the frequency data and the fit from the modal data, between points 7 and 19 where the accelerometer is at point 7. The fit indicates that the modal data matches fairly well with the modes for this frequency response data.

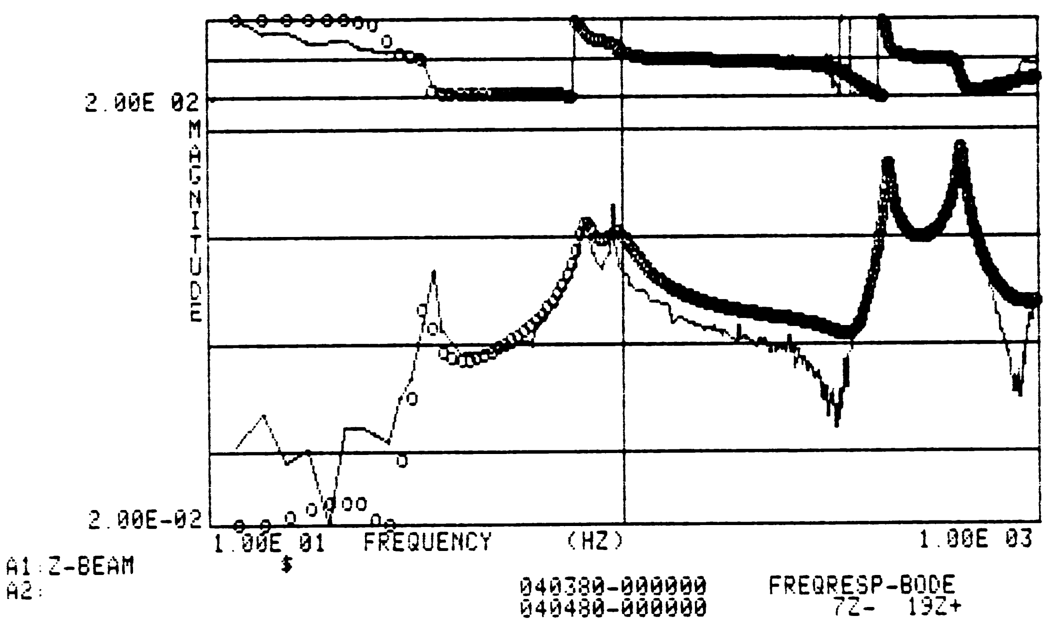


Figure 4-13 The Modal Fit on an Arbitrary Transfer Function Data. Circles-Fit. Solid Line-Data.

4.7 Summary

A typical modal test was performed on a Z shaped beam using GenRad 2508 Structural Analysis System with the SDRC MODAL PLUS software. The procedure was to define the geometry, choose an excitation techniques, and identify the characteristic of the dynamics of a system by analyzing its frequency response data and using curve fitting algorithms. The last step was to display the mode shapes, interpret the results and verify the modal data.

CHAPTER 5

MODAL ANALYSIS AT MICHIGAN STATE UNIVERSITY

The use of analytical and experimental modal analysis has become popular and widely used during the 1970's primarily because of the advances in the computer technology. Today, modal analysis has an important role in the design and development of mechanical systems. New developments and better techniques are being researched in industry [4,22] and in the university environment [4, 23].

Various subjects, including vibration theory, matrix algebra, Laplace and Fourier transforms, transfer functions, and electronic instrumentation are involved in applications of modal analysis. Most of these topics are covered, at least in part, as part of the required undergraduate curriculum in mechanical engineering at Michigan State University [24]. But in order to gain a satisfactory knowledge of modal analysis, the student should be able to combine these topics. This chapter presents a scenario for the coordinated introduction of modal analysis into mechanical engineering undergraduate and graduate curricula.

5.1 The Undergraduate Curriculum

The mechanical engineering undergraduate curriculum requires 180 credits, of which 128 credits are related to mathematics, science and engineering. I want to consider only those courses which relate modal analysis theory and its applications.

These courses will give students the opportunity to blend various topics on this subject.

5.1.1 ME 346: Instrumentation Laboratory (3 credits)

The prerequisite for this course is ME 351 which presents an overview of several mathematical methods which are essential to the understanding of the topics covered in modal analysis theory. The object of the course ME 346 is to present to students the principles and applications of instrumentation in Mechanical Engineering. Two weeks of the course are devoted to modal testing including equipment calibration, set up of an experiment, modal testing, and the display of animated mode shapes. Since the time is limited, a very simple modal experiment should be presented which emphasizes the impact of the instrumentation on modal testing. Thus, ME 346 will introduce modal testing techniques.

5.1.2 ME 464: Computer Assisted Design II

The objectives of this course are to introduce the modal analysis theory along with the digital signal processing and to help students to understand modal testing techniques by presenting several experiments. Table 5.1 presents the outline of the course. In addition, if time permits, a comparison between a modal testing of a sample structure and a computer simulation of the same structure should be demonstrated.

The present prerequisites are ME 455 (Mechanical Vibrations), which presents material covering up to two degrees of freedom in time domain, and ME 463 (Computer Assisted Design I) which ensures that the students understand interactive graphics. I feel that, in addition, the students also need ME 458 (Control Theory) as a prerequisite. ME 458 gives an overview of Laplace domain, time domain and frequency domain relationships along with the applications of transfer function analysis. Another course which is strongly recommended prior to ME 464 is MTH 334 (Theory and Applications of Matrices).

The structure to be tested should be small in size because of space limitations and have simple geometry since any elastic structure would be sufficient for the purpose of learning testing techniques. These considerations will simplify the set up of the experiments and make it easier to feed test point coordinates to the computer for the animation of mode shapes. It would also make it easier to construct a mathematical modal or a simple structure for computer simulation purposes.

5.1.3 Other Undergraduate Courses

Table 5.2 summarizes the courses which are related to Modal Analysis in the mechanical engineering undergraduate curriculum at Michigan State University. In my view, it would be appropriate for any of these courses to use the modal equipment on a demonstration basis, and perhaps, for ME 499, for hands on experiments.

TABLE 5.1

OUTLINE FOR ME 464

1. Analytical Approach to Modal Analysis
   i) Analysis of the mathematical model
   ii) Eigenvalues, eigenvectors and their applications
   iii) Normal mode shapes, complex mode shapes.

2. Fourier Analysis and Digital Signal Processing
   i) Time domain, Laplace domain and frequency domain.
   ii) Fourier transform
   iii) Signal sampling, power spectrum analysis
   iv) Aliasing, leakage, windowing.

3. Modal Testing
   i) Transfer function, frequency response function
   ii) Modes of vibration
   iii) Excitation methods
   iv) Modal data identification (i.e., curve fitting algorithms)
   v) Reduction of measurement noise and coherence function
   vi) Measurement resolution

4. Presentation of Several Laboratory Experiments and Class Participation in Interpreting the Results.

TABLE 5.2

UNDERGRADUATE COURSES RELATED TO MODAL ANALYSIS

COURSE		TOPICS
ME 346:	Instrumentation Laboratory	Instrumentation for mechanical engineering applications including demonstration of modal analysis.
ME 351:	Mechanical Engineering Analysis	Essential mathematical methods to the solution of engineering problems.
ME 422:	Mechanical Design Projects	Application of design concept. Team design project.
ME 455:	Mechanical Vibrations	Basic theory of mechanical vibration and it's applications. One degree and multiple degree of systems, time varying systems.
ME 458:	Control Theory	Control systems, application of transfer function analysis, time, Laplace, frequency, domains and stability.
ME 463:	Computer Assisted Design I	Basics of interactive graphics, line fitting and surface development, finite element analysis.
ME 464:	Computer Assisted Design II	Modal Analysis techniques.
ME 499:	Independent Study	Individual Projects.

5.2 The Graduate Curriculum

The purpose of the graduate program in modal analysis is to take a number of qualified students and lead them to a deeper level of understanding of modal analysis techniques. The graduate program will include more mathematical sophistication and an enhanced ability to exploit the laboratory facilities in complex applications. The present Mechanical Engineering graduate curriculum is given in the Appendix.

5.2.1 A Graduate Course in Modal Analysis (e.g., ME 824: Modal Analysis - Theory and Measurement Techniques)

The objectives of this course will be to present the analytical and experimental approach to Modal Analysis by building a mathematical foundation from the analytical and experimental point of view and to participate in the "hands on" experiments on various structures using the Modal Analysis testing facilities and Prime 750.

The students should understand the material presented in ME 823 (Theory of Vibration I) and ME 464 (Computer Assisted Design II) prior to this course. It is also recommended to take MTH 831 (Matrix Theory) to gain an in depth facility with linear algebra.

Table 5.3 presents a course outline.

5.2.2 Other Graduate Courses

Table 5.4 shows the graduate courses that are related to Modal Analysis theory in the Engineering College of Michigan State University.

By participating in such courses in the graduate curriculum, an engineering student will gain in depth knowledge of a type of structural testing that is becoming increasingly useful to industry. Therefore experience gained in studying modal analysis is often viewed as an asset by employers. This subject also carries exceptional academic utility, since it promotes understanding of multi-degree of freedom vibrations, and demonstrates the related applicability of several topics such as Laplace and Fourier Transforms and linear algebra.

TABLE 5.3

OUTLINE FOR ME 824

1. Modal Testing - Its Techniques and Applications

    i) Fourier analysis

    ii) Signal sampling and its considerations

    iii) Frequency response analysis

    iv) Modes of vibration

    v) Excitation methods

    vi) Curve fitting algorithms

    vii) Reduction of measurement noise, coherence function

2. Analytical Approach to Modal Analysis

    i) Eigenvalues, eigenvectors and their applications

    ii) Effects of various types of damping

    iii) Dynamic Substructuring

    iv) Modal Perturbation Analysis, i.e., modifications on the mathematical model with "what if" type of questions, e.g., what if we make changes in M, C or K matrices? How would the modified structure behave dynamically?

3. Group Projects

    Involving modal testing and computer simulation of a structure, and correlating the results.

TABLE 5.4

GRADUATE COURSES RELATED TO MODAL ANALYSIS

COURSE	TOPICS
ME 823: Theory of Vibrations I	Discrete and continuous systems with linear and nonlinear characteristics.
ME 824: Modal Analysis and its Applications	Analytical and experimental approach to Modal Analysis, modal analysis of complex structures.
ME 890: Special Topics	Special problems, projects in modal analysis.
ME 899: Research	Research projects in Modal Analysis.
MMM 805: Strain and Motion Measurements	Laboratory training in strain and motion measurement. Study of strain gages and accelerometers.
MMM 809: Finite Element Method	Theory and application of the finite element method.
SYS 826: Linear Concepts in System Science	State space and frequency domain models of interconnected systems, solution of continuous and discrete time linear systems.

CHAPTER 6

SUMMARY AND RECOMMENDATIONS

6.1 Summary

This thesis is a study of the theory and an approach to experimental modal analysis of structural dynamics. Initially, a brief overview of analytical techniques was given to show the relationship between purely analytical and purely experimental approaches. Then the experimental approach known as modal analysis was studied in some detail.

Various excitation methods were presented and the impact testing method was discussed in detail. Curve fitting methods to identify modal data were presented. Several factors that contribute to the quality of testing were discussed and an example was given to illustrate the technique.

Since modal analysis has wide application in industry and since it is topic worthy of academic study from several points of view, the impact of modal analysis on the Mechanical Engineering curriculum at Michigan State University was discussed, and several courses at different academic levels (i.e., junior, senior, graduate) were proposed.

6.2 Recommendations

Modal analysis offers the opportunity to establish an academic focal point for the department of Mechanical Engineering at Michigan State University. I recommend the establishment of several courses devoted to this area. This would bring to the department the

the following benefits

1) The students would gain an overview of several important tools which, though they are inherently worthwhile, are not now brought together in a cohesive unit.

2) A sophisticated overview of modal analysis is very much in demand throughout the industrial community. Currently, only the University of Cincinnati has a strong reputation in this area. MSU's department is now ideally suited through faculty, students, and equipment to become a major contributor in the field.

APPENDIX

APPENDIX

MASTER'S DEGREE PROGRAM IN MECHANICAL ENGINEERING*
(Beginning with Fall Term 1979)
(Revised - Effective Fall Term 1980)

1. Credit requirements

Total credits:	45 (minimum)
Plan A (thesis):	30 (minimum) at 800 level or above 8-12 thesis (These credits may count as part of the 30 credit minimum above. The booklet "The Graduate School Guide to the Preparation of Master's Theses and Doctoral Dissertations" is available from the Graduate School.)
Plan B (report):	27 (minimum) at 800 level or above. Six credits of ME 890 are to be taken and applied toward a research/design project and a report.

(In each plan, a minimum of 9 credits must be taken outside the Mechanical Engineering Department. Thesis and Report Form details are available at the Mechanical Engineering Headquarters 200 EB.)

2. Area requirements

Each student must take at least one regularly scheduled 800 level course in each of four areas, chosen from the list below:

a.	Design:	M.E. 826, 827, 828
b.	Fluid Mechanics:	M.E. 840, 841, 842, 843
c.	Heat Transfer:	M.E. 810, 813, 814, 817, 832
d.	Systems and Control:	M.E. 851, 860, 862
e.	Thermodynamics:	M.E. 815
f.	Vibrations:	M.E. 823

*The requirements listed here are in addition to those indicated in the college requirements document "MASTER'S DEGREE PROGRAM REGULATIONS COLLEGE OF ENGINEERING, MICHIGAN STATE UNIVERSITY."

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