METHODOLOGY FOR EVALUATING ECONOMIC THEORIES

Thesis for the Degree of Ph. D.

MICHIGAN STATE UNIVERSITY

Darrel Harvey Plaunt

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ABSTRACT

METHODOLOGY FOR EVALUATING ECONOMIC THEORIES

by Darrel Harvey Plaunt

The purpose of this study is to examine alternative procedures of theory evaluation with a view to identifying some of the more promising techniques and synthesizing these techniques into a more comprehensive framework of analysis.

The need for such a framework of analysis arises out of the attempts of researchers to explain and to predict the phenomena of reality. Efforts to improve the accuracy and reliability of explanation and prediction involve choosing among alternative theories and developing more powerful sets of hypotheses. As one distinguished philosopher has said: "All scientific activity amounts to the invention of and the choice among systems of hypotheses."

The development of this framework of analysis begins with an examination of the nature of theories and their role in scientific explanation and prediction. This examination provides a set of criteria for the logical and the empirical adequacy of theories. These criteria provide the broad outlines of the framework of analysis required for the examination of alternative techniques of theory evaluation.

The use of these techniques is illustrated by their application to the evaluation of the classical theory of

economic growth. This theory was chosen because it seemed sufficiently recalcitrant to evaluation to facilitate the illustration of many of the concepts involved.

The investigation of the logical adequacy of theories begins with the concept of an underlying structure or calculus, the concept of deduction relatedness and the concept of a model. This investigation is continued with the deduction of a number of apparently inconsistent theorems from the original formulation of classical axioms. Proof of such inconsistency would be extremely damaging to any putative theory.

Such proof is not easily forthcoming, however. An inconsistency may arise either from the nature of the axioms employed or from the vagueness and imprecision with which they are expressed. Since this vagueness and imprecision is characteristic of many social science theories, it is often necessary to more fully formalize the theory before unequivocal results can be obtained. This process of formalization is illustrated in two stages in this study. The first stage involves the interpretation of the classical axioms in terms of first differences. This interpretation provides a model of the classical axioms from which an inconsistency is deduced. This deduction would likewise have been extremely damaging if it had been possible to demonstrate that this formalization represents an adequate interpretation of the theory under examination. The second stage in this process

of formalization involved the specification of the axioms in general equation form.

The problems of assessing the logical adequacy of theories expressed in equation form require the use of more powerful analytic techniques. One such technique, applicable to linear systems, is the Test of Determinants. Another technique, applicable to both linear and non-linear systems is developed in detail in Chapter V. This technique is called the Method of Numerical Interpretation. It evolves out of the classification system developed in that chapter. It provides a direct means of proving the consistency of a set of axioms once a sufficient level of formalization has been attained. As such, it may usefully compliment the other analytic techniques more commonly employed in the evaluation of the logical adequacy of theories.

The investigation of the <u>empirical</u> as opposed to the <u>logical</u> adequacy of theories, on the other hand, comes to focus on the truth of the empirical propositions employed. Many of the problems of ascertaining the truth of these propositions involves questions of whether or not the variables can be measured and the accuracy and reliability of these measures. Other problems involve the accuracy and reliability with which the parameters can be estimated once the values of the variables are known. The concepts required for the investigation of the measurability of the variables are drawn from analytic philosophy. The concepts required for assessing the accuracy and reliability with which the

parameters could be estimated are drawn from mathematics and statistics.

These sets of concepts, together with the logical concepts employed in the examination of the consistency of theories, all come to focus within the general framework provided by the concepts of the logical and empirical adequacy of theories. Further progress in the development of this framework for the a priori analysis of theories is likely to go hand in hand with the refinement of the concepts of analytic philosophy and the development of more powerful mathematical and statistical techniques.

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By

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CHAPTER I

INTRODUCTION TO THE PROBLEMS OF THEORY EVALUATION

THE PROBLEM

The purpose of this study is to develop an approach to the problems of evaluating the explanatory and predictive potential of theories of economics. The need for such an approach arises out of the necessity for choosing among alternative theories, and for the development of more powerful sets of hypotheses. As Professor Goodman has put it, "All scientific activity amounts to the invention of and choice among systems of hypotheses."[1] It would seem, therefore, that the problems of theory evaluation are central to a major part of scientific inquiry.

The significance of this assertion becomes apparent upon examination of the nature of theories and their role in science. The nature and the role of theories will be presented in capsule form below and in more detail in Chapter II.

A theory is defined as "a systematically related set of statements, including some lawlike generalizations, which

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[1]Nelson Goodman, "The Test of Simplicity," Science, CXXVIII (1958), p. 1064.

is empirically testable."[2] The term "lawlike" here means, among other things, that the statement is of the universal conditional form. The statement "If investment increases then output will increase," for example, is of this form. It is universal in the sense that it means for all cases not just for some cases. It is conditional in the sense that it assumes the "if...then" form. A theory is systematically related in the sense that each statement in it is related to one or more other statements in the set in such a way that it functions either as an axiom or as a theorem within the set. Hence a theory is made up of two sets of statements--the axioms, and the theorems deduced therefrom.

A simple illustration of the nature of theories may be constructed out of a set of statements drawn from one of the more widely accepted theories of employment. This set might include the following types of statements. The first might say that investors will continue to adjust investment toward the level at which the marginal efficiency of capital equals the interest rate. The second might say that investment is a negative function of the interest rate. The third would say that income is a positive function of consumption plus the level of investment. Finally, the fourth would claim that employment is a positive function of income. If these

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[2]Richard S. Rudner, "On the Structure of Economic Theories," unpublished paper presented before the Joint Economics Agricultural Economics Seminar, Michigan State University, East Lansing, Michigan, May 26, 1958.

statements are treated as the axioms of a system, then one of the theorems that follows is that employment is a negative function of the interest rate.

One possible use of this theory might be to predict the level of employment to be expected in the future or to explain the level of employment attained in the past. The first of these tasks would require the conjunction of the relevant antecedent conditions, a decline in the interest rate, for example, with the theorems presented above, and with some assumption to the effect that other variables, not in the system, would remain constant. This latter assumption is required only if it is believed that there are other variables that may have an effect on the equilibrium of the system as a whole.

Given the antecedent conditions, and the theory, it should be possible to predict the consequent, an increase in employment, for example. Alternatively, given the consequent and the theory it should be possible to adduce the antecedent conditions, that interest rates fell, for example. Since the antecedents and the consequents are generally singular statements, the problem of ascertaining their truth is mainly one of observation and measurement. The problems associated with the theory, on the other hand, include these problems of observation and measurement and a large set of other problems as well.[3] It is this whole group of problems associated with

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[3]The types of problems included in this group will be developed in detail as the study progresses.

theories and hence central to the process of explanation and prediction that is the main concern of scientific endeavor.

The progress of science, then, depends, to a large extent, upon the ability of scientists to "choose among systems of hypotheses." This choice depends, upon their estimation of the explanatory and predictive potential of the alternative systems involved. This estimate, in turn, depends upon the group of problems mentioned above and the researcher's ability to understand and cope with them. If these problems were easily specified and solved then man's ability to explain, predict, and hence control his environment, would be much better than his record would indicate.

These problems are not easily solved, however. Professor Rudner has summarized the state of the arts with respect to man's ability to handle these problems of choice as follows: "Whatever may be the case for the serenity or unselfconsciousness with which practicing scientists go about the business of accepting or rejecting theories, it will surely not be denied that the problem of constructing an adequate philosophical rationale for such practice remains in its perennial state of crisis."[4]

Even though the rationale for accepting or rejecting theories, and hence for selecting among them may be in its "perennial state of crisis" this problem of developing an

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[4]Richard S. Rudner, "An Introduction to Simplicity," Philosophy of Science, Vol. 28, No. 2 (April, 1961), p. 109.

adequate rationale is attracting the attention of some of the ablest minds in economics. The 1962 Proceedings issue of the American Economic Review, for example, reports a session in which Machlup, Papandreou, Nagel, Krupp, Archibald, Simon and Samuelson, bend their efforts to the analyses of problems of the evaluation of theories.[5] Their main point of focus is Professor Milton Friedman's position that the adequacy of a theory must not be judged by the "realism of its assumptions" but by examining the concordance of the theory's logical consequences with the phenomena the theory is designed to explain.[6] Nor is this kind of interest limited to the field of general economics. At the meetings of the International Association of Agricultural Economics in Mexico in 1961 a substantial portion of the deliberations of the discussion groups were devoted to the process of theory evaluation, particularly as it applies to theories of economic growth. This interest is not limited to discussions at the annual meetings of professional associations, however. It is part and parcel of the everyday work of researchers in economics.

This evidence of both interest and involvement in the problems of evaluating theories merely points up the fact that these problems are both important and unsolved. The fact that

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[5]See the American Economic Review, "Problems of Methodology," (Proceedings Issue, May, 1963, Vol. LIII, No. 2), pp. 204-236.

[6]Milton Friedman, "The Methodology of Positive Economics," Essays in Positive Economics, (University of Chicago Press, 1953.)

they have hindered the development of economics ever since the inception of the science, and the fact that they have attracted the attention of some of its ablest minds over the preceding generations suggest that they are not amenable to quick and easy identification and solution. All that can be hoped in this study is that it will be possible to identify one or more of the connecting threads of thought that are relevant to theory evaluation, and to recognize and develop the tools required to bring that thesis to bear on the problems of choosing among these systems of hypotheses.

THE PROCEDURE

TRADITIONAL PROCEDURES

There are several approaches to the evaluation of theories that might be attempted here. One of these involves simply testing the accuracy with which the theory predicts. This generally involves quantifying the relationships employed, substituting in values for the exogenous variables, solving for the values of the endogenous variables, and comparing these results with past observations on the values of the variables to be predicted or explained. In this approach, the better its record of prediction, the better the theory is presumed to be. This procedure for the testing of theories is probably the most commonly used in economics. This is especially likely to be true in agricultural economics, where, if one permits the broad Rudnerian definition of theory to

include many of those systems commonly, and mistakenly, referred to as models, a relatively large proportion of research time is devoted to the development and testing of models (i.e. theories) in the manner described above.

This type of testing is an important method of procedure. It is particularly appropriate in circumstances in which the data is readily available or can be made available at a modest cost, and in which the statistical fitting procedures are not likely to be particularly expensive. For many theories this is not the case, however. In the first place, the tasks of observing, measuring, collecting and processing the data may become extremely expensive before sufficient reliability is attained to permit its useful application. In these cases it becomes important to obtain an understanding of its explanatory and predictive potential a priori. Such an understanding would enable the research group to avoid a substantial investment in quantification and testing of theories whose explanatory and predictive potential is particularly weak, and for which this conclusion could have been foreseen by adequate a priori analysis.

In addition, the sole reliance on testing, in contrast to other techniques of evaluation, is not likely to afford sufficient understanding of the causes of the adequacy or the inadequacy of the theory under examination. These causes may lie in either its logical structure or in its empirical

content. Testing alone does not necessarily help to identify either of these sets of causes. An understanding of both the logical structure and the empirical content is required both in the evaluation of extant theories and in the development of more powerful sets of hypotheses. Hence, a major portion of this study is devoted to a consideration of the logical and empirical adequacy of theories. These considerations apply to both theories in general and to economic theories in particular.

THE PROCEDURES EMPLOYED

Concepts Drawn from Analytic Philosophy

In view of the importance of developing an understanding of the logical and empirical conditions required for adequate explanation and prediction, the first phase of this study concentrates on a selection of concepts drawn from analytic philosophy. These concepts constitute the major ideas employed in the framework of analysis to be developed below.

An initial selection of these concepts is presented in Chapter II. These concepts include a group of notions about the logical structure of science, starting with the concept of a logical term and ending with a description of the structure of theories in general. They explain the manner in which the logical terms combine with descriptive terms in order to provide the empirical assertions of science. In

addition, they explain the manner in which these sentences are combined to form theories whose function it is to provide the lawlike statements used in explanation and prediction. This leads to a consideration of the role of these laws or lawlike statements in the structure of explanation and prediction. Examination of this structure provides an initial set of criteria for the logical and empirical adequacy of theories in general. These criteria focus attention on the concepts of deductive relatedness or deductive subsumption. This concept is explained in terms of the concepts of formalization. These concepts, in turn, provide the bases for a consideration of techniques of partial formalization and the use of models in theory construction and evaluation.

Out of this investigation of the logical structure of science evolves the thesis that guides the development of this study. This thesis asserts that the explanatory and predictive potential of any given theory depends on both the truth of its empirical propositions and the logical adequacy of its structure. Hence, the techniques designed to facilitate the evaluation of extant theories and the concepts required to guide the development of more powerful sets of hypotheses must perform a dual role. They must facilitate the recognition of the truth of the propositions employed. In addition, they must facilitate the recognition of the logical adequacy not only of individual propositions but also of sets

of propositions as they function in science.

This thesis, drawn from the field of analytic philosophy, provides the bases for an initial assault on the problems of theory evaluation. These problems are illustrated by examination of the classical theory of economic growth. Since there are a number of interpretations of this particular aspect of economic theory, it was decided to concentrate on one particular formulation and in so doing to avoid a number of questions more closely related to the history of economic thought than to the problems of choosing among alternative sets of hypotheses. The formulation chosen for examination is the Higgins summary of the classical theory.[7] This formulation was chosen because it has already attained a minimum level of formalization and because it seems to be sufficiently recalcitrant to evaluation to permit the illustration of most of the techniques developed below.

This summary of the classical theory is presented in Chapter III. It is followed by an examination of some of the logical consequences of the theory. This examination is carried to the point where evidence of inconsistency begins to arise, and where it becomes apparent that more rigorous techniques of analysis are required.

In the initial search for these techniques it was decided to limit the actual techniques employed to those with

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[7]Benjamin Higgins, Economic Development, Principles, Problems, and Policies, (New York: W.W. Norton and Co., Inc., 1959), pp. 85-106.

which researchers in economics could readily become familiar. In this stage a number of abortive attempts were made at ascertaining the logical consistency of the system using traditional and symbolic logic on the level of the calculus of propositions. Direct approaches to questions of consistency and inconsistency through the use of both economic and mathematical models were also attempted. None of these approaches produced unequivocal results. The reason for this may be largely attributable to the vagueness and imprecision with which the propositions of the theory were stated.

Since these problems of vagueness and imprecision are characteristics of a large numberof theories in economics, it was decided to allocate an important portion of this study to the examination and application of a variety of techniques of partial formalization. The first stage in this procedure will be the formalization of the classical axioms in terms of first differences so that the direction, if not extent, of each of these changes can be ascertained. This new formalization will then be analyzed in terms of its logical structure. If this analysis provides unequivocal results then it may be fruitful to proceed directly to a consideration of the application of selected techniques of mathematics and statistics. If the results are not unequivocal then it will become important to more thoroughly investigate both the techniques of formalization and the techniques of evaluation before the mathematical and statistical approaches are

attempted.

The development and application of the techniques of formalization and the techniques of analysis generally complement each other. The formalization of a theory generally permits the application of more rigorous techniques of analysis and the application of these techniques generally facilitates more adequate formalization.

The first step in this formalization would be to exhibit each of the classical axioms in terms of general equation forms, making certain that these forms adequately reflect the intent of these axioms. Once this is accomplished, attention can be returned to a consideration of analytic philosophy in order to develop a set of techniques which are more powerful than those available from the initial selection of concepts presented in Chapter II. In this initial selection, the unit of investigation was the concept of a theory as a whole. The difficulty in finding or developing a set of techniques of sufficient logical power and analytic precision to adequately evaluate these sets of statements suggests the more rigorous analysis of the individual statements that comprise the theory. This type of analysis may be termed the analysis of propositions and may provide the basis for the evaluation of the logical structure of theories and their role in explanation and prediction.

Concepts Drawn from Mathematics and Statistics

Up to this point attention has been focused on the

logical structure of theories and on a selection of concepts available from various areas of analytic philosophy. There is some question, however, whether this discipline can provide sufficiently precise tools for the analysis of the problems at hand. Most theories of economics require concepts of metricization. These concepts can not be handled adequately within the framework of formal logic without recourse at least to general quantification theory. The use of general quantification theory usually requires the skills of a professional logician. In order to avoid this arduous type of analysis and yet to employ a system of sufficient logical power, it would seem appropriate to turn to informal applications of mathematics and statistics.

After a sufficient level of formalization has been attained, it may be possible to bring certain mathematical techniques to bear on the problems of whether, for a given theory, it is possible to uniquely determine the values of the variables given the estimates of the parameters. In the same way, it may be possible to determine, a priori, whether it is possible to consistently estimate the values of the parameters, given the values of the variables involved. Finally, if the parameters can be estimated, and if the equations can be solved, it becomes important to assess the potential accuracy and reliability with which these parameters can be estimated and the predictions made.

These last three phases of the analysis, together with

the rigorous analysis of its logical structure, suggests that a comprehensive investigation of the techniques required to assess the explanatory and predictive potential of theories is likely to involve forays into a variety of areas in analytic philosophy, mathematics, and statistics. These forays will comprise the major investigations conducted below.

CHAPTER II

THE STRUCTURE OF SCIENTIFIC INQUIRY WITH SPECIAL EMPHASIS ON THE NATURE AND ROLE OF ECONOMIC THEORIES

INTRODUCTION

The purpose of this chapter is to examine some of the key concepts of analytic philosophy with a view to developing a thesis, or a set of concepts, which is powerful enough to guide the a priori analysis of theories of economics.[1] Since there is no well established set of criteria to guide the choice of these concepts, the initial selection depends, in large part, upon the insight and imagination that can be mobilized in the initial stages of such an inquiry. Once this initial selection is thoroughly examined, however, it is expected that a set of concepts and criteria can then be developed to the point where they can be used to guide and direct the investigation of the problems of theory evaluation.

At this stage of the inquiry it would seem appropriate to initiate an appraisal of the explanatory and predictive potential of theories with an examination of the language

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[1]The term a priori is used here to refer to the broad scope of scientific inquiry which is prior to systematic data collection and empirical analysis. The need for this type of analysis was established in Chapter I.

used to express the statements involved. This examination will concentrate on the terms and sentences employed in scientific discourse and set the stage for the examination of concepts, facts, laws, and theories. This will pave the way for a more detailed examination of laws or lawlike statements and their role in the structure of science. This examination comes to focus on the structure of explanation and prediction and provides an initial set of criteria for the evaluation of the role of theories in science.

This overall view of the structure of science provides the background for a more detailed study of the nature of theories. This study comes to focus on the concepts of deductive subsumption or systematic relatedness. These concepts can be explained in terms of the concept of a calculi. This concept of a calculi leads to a consideration of the broader field of formalization and eventually to a study of certain proposed techniques of partial formalization as a means of clarifying the nature and structure of theories, and rendering them amenable to logical analysis. This phase of the investigation culminates in a consideration of the method of models as a means of testing the explanatory and predictive potential of theories a priori.

In summary, then, this chapter progresses from an examination of individual terms to an examination of statements and finally to an examination of sets of statements. The examination of individual terms concentrates on logical

and descriptive terms as they function in declarative sentences. The consideration of individual statements focuses on the nature of lawlikeness. The study of sets of statements, on the other hand, comes to bear first on the structure of explanation and prediction and then on the nature of theories whose function it is to provide the lawlike statements used in this structure. This study of theories begins with a consideration of their underlying calculi, progresses to a consideration of the techniques of formalization, and finally culminates in an examination of models as a means of theory evaluation.

The concepts presented in this chapter are intended as an initial selection of the concepts useful in the process of theory evaluation. They constitute a very small sample of the concepts available from the broad scope of analytic philosophy. Other concepts will be drawn from analytic philosophy and from mathematics and statistics as the needs of the analysis arise. These needs continue to arise as these concepts are brought to bear on the problems under examination.[2]

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[2]This chapter represents, in some parts, a selection of some of the key points in Richard S. Rudner's paper entitled, "On the Structure of Economic Theories," presented to the Joint Economics - Agricultural Economics Seminar, May 26, 1958 at Michigan State University, East Lansing, Michigan, which presents ideas on theory construction which have become common in the literature. In addition, it includes drawing on: (1) Carl S. Hampel and Paul Oppenheim, "The Logic of Explanation," Readings in the Philosophy of Science, ed. Herbert Feigl, and May Brodbeck (New York: Appleton-Century-Crofts, Inc., 1953), pp. 319-331; and (2) May Brodbeck, "Models, Meaning

AN EXAMINATION OF THE LANGUAGE OF SCIENCE

Before any exposition of the underlying structure of science is attempted, it is desirable to explain some of the central terms and concepts employed. This section will be concerned with this explanation.

TERMS AND SENTENCES

The immediate product of any science is a set of declarative sentences--a set of linguistic entities. In thinking about science, it is useful to distinguish between the product and the process of science. The product itself contains only declarative sentences, yet the meta-language--the language used in discussions of science, may contain any or all of the sentence forms of a natural language.

These declarative sentences of science are made up of two types of terms: descriptive terms and logical terms. The descriptive terms or predicates may be usefully classified into several levels. The first level of predicates express properties, characteristics, or attributes of individual things or events. The second level of predicates denotes properties of these properties. The third level of predicates

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and Theories," Symposium on Sociological Theory, ed. Llewellyn Gross (New York: Row, Peterson & Co., 1959), pp. 373-401. These works present the standard treatments of some fairly well agreed upon concepts of concept formation, the structure of theories, and the logic of explanation and prediction. The problems of partial formalization have not been given extensive treatment in the literature. The considerations on partial formalization presented below are drawn from Rudner, op. cit.

express properties of properties of properties, and so on for higher level predicates. These descriptive terms are connected together to form statements of fact. These statements in turn may be jointed to each other to form compcund statements of fact. It is the logical terms mentioned above that do this connecting. These include: "and," "or," "if ... then," and "if and only if." They do not denote anything; they, and other logical terms, function only to give the language its form by connecting those terms that do denote something, and by expressing the connections which obtain among the facts expressed. If a sentence is stripped of its meaning by replacing its constituent statements with symbols, it still retains the form given to it by the logical terms employed.[3]

Thus far it has been pointed out that words can be classified into two types: logical words and descriptive words. The sentences of science can also be classified into two types (1) empirical or synthetic statements and (2) analytic

[3]It should be pointed out in passing that the family of logical terms alluded to above include more than just those connectives mentioned. They include: all (x), some ($\exists$x), is a member of (xεc), the terms of quantification and class membership and groups of other terms depending upon the formulation chosen. But in its most sophisticated forms, all of logic and mathematics can be constructed in terms of three primitives: (1) the Sheffer stroke, which is a connective in terms of which "or," "and," and "not" can all be defined; (2) the quantifier (x) or ($\exists$x); (3) the notion of class membership or identify; (4) variables; and (5) grouping indicators.

statements. These are distinguished from each other on the basis of whether or not their truth or falsity can be determined from their form and form alone. Statements of the form "If X then X" or "Either X or not X," for example, are judged to be true no matter what statement X stands for, as long as it always stands for the same statement. Similarly, statements of the form "X and not X" can be judged false without recourse to the meaning of the terms employed. Statements like these, whose truth or falsity does not depend upon the meaning of the descriptive terms employed, but depends on form and form alone, are called analytic statements. The other group of sentences, those whose truth or falsity depends upon their descriptive terms as well as their form, are called empirical, contingent, or synthetic statements. It will become apparent in the ensuing development that each of these broad classes of statements plays quite different roles in the process of empirical investigation.

CONCEPTS, FACTS, LAWS, AND THEORIES[4]

In addition to distinguishing between two different kinds of terms and two different kinds of statements, it is also useful to distinguish between concepts, statements of fact, laws, and theories.

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[4]Brodbeck, op. cit., pp. 373-401.

The word "concept" usually refers to a property or relation. A fact, on the other hand, is a particular event, that the general level of net new investment is such and such a value, for example. Part of the significance of factual statements in science arises from the way several such statements may lead to the formulation of generalizations or laws. A lawlike statement may be construed as a statement that says, given certain conditions, that wherever and whenever there is an instance of one kind of fact, there is also an instance of another kind of fact. It should be noted in passing that the occurrence of this second fact may or may not coincide in point of time with the occurrence of the first fact. It should be noted also that these laws are always empirical generalizations. They say, as Say's law does for example, that whenever aggregate supply expands, then aggregate demand also expands. This is an ordinally quantified law. A cardinally quantitative law, on the other hand, would say how much aggregate demand expanded when aggregate supply expanded by one unit. This type of law would likely be expressed in equation form. But it should be noted that even though it is in equation form, it is still an empirical or synthetic assertion whose truth or falsity depends upon its descriptive content; it is not analytic. It should also be pointed out, however, that these universal empirical assertions are not the only type of lawlike statement. As shall become apparent in a subsequent section, the

universal analytic assertions may also play an important role in science.

Thus far the language of science has been built up by combining terms to form statements, and statements by means of connectives to form compound statements, and finally, by attaching operators to sentence forms. Some of these statements are lawlike. Now some laws are so related to one another in such a way as to constitute a theory. A theory is a deductively related set of statements, at least one of which or perhaps all of which are lawlike. But the set of statements comprising a theory is also divided into two subsets: the axioms, and the theorems--the latter being deduced from the former. The axioms are generally empirical laws which are, for the moment at least, assumed to be true in order to determine what other statements, the theorems, would also be true if the axioms are in fact true. Axioms need not be self-evident or otherwise privileged. They are simply empirical laws that may at the same time function as axioms of one system, theorems of another, and hypotheses in a third. They are empirical lawlike statements which are the most general statements used--most general in the sense of being those from which all other statements of a theory, the theorems, are derived.

It should be noted in passing that some empirical laws are inductive generalizations or hypotheses. The term "hypothetico-deductive system" is often used to refer to such

empirical axiom systems or theories. The laws contained in these theories are usually expressed in the universal conditional form or as equations. But there are many equation forms that might be used, hence theories differ not only in their content (or descriptive terms) but also in their form. For example, laws in the social sciences may take the form of quantified linear equations expressing the correlation between the values of variables, while in the physical sciences they may take the form of the Cobb-Douglas, Carter-Halter, or Spillman function, etc.

LAWS IN THE STRUCTURE OF SCIENCE

Laws or lawlike statements are central to all scientific theorizing and to the structure of explanation and prediction. It is important, therefore, to examine the structure and meaning of laws in more detail than in the previous section. This examination will concentrate on two of the outstanding characteristics of laws. The first has to do with the truth of lawlike statements. The second concerns the characteristics of lawlikeness itself.

In order for a statement to be a law it must be a true statement. Hempel and Oppenheim argue that the requirement of a high confirmation instead of truth is not sufficient.[5] If a statement were considered to be a law

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[5]Carl G. Hempel and Paul Oppenheim, "The Logic of Explanation," Readings in the Philosophy of Science, ed. Herbert Feigl and May Brodbeck, (New York: Appleton-Century-Crofts, Inc., 1953), p. 322.

relative to one set of evidence and not a law relative to another set of evidence then some serious consequences would ensue. In the event that a lawlike statement was highly confirmed at an earlier stage in science and becomes highly disconfirmed as the result of more recent empirical evidence, this relativized concept of law would force the conclusion that it was a law in the earlier stages of the discipline but that it ceased to be a law as more evidence was acquired. This relativized concept of law does not accord with common usage of the term. The requirement that a statement must be true in order to be a law, on the other hand, would permit the more usual conclusion that the limited original evidence had given a high probability to the hypothesis that the statement was a law, but that more recent evidence had reduced the probability of it being true, and hence of it being a law. In order to avoid the problems associated with the relativized concept of law the requirement of truth will be taken as a necessary condition for a law throughout this study. Statements which have all the other necessary characteristics of scientific laws, with the possible exception of truth, are called lawlike.

The second outstanding characteristic of a law is the criterion of lawlikeness. The central notions in the concept of lawlikeness are the notions of the forms of statements and scope of the variables employed. The concept of lawlikeness was introduced in a previous section. It will be

examined in more detail below.

One of the major characteristics of lawlike statements is that they are of universal form. They may say, for example, that: "All firms tend to maximize profits," or "Every time supply decreases, with demand unchanged, the prices rise." In addition, lawlike statements are usually of the "if ... then ..." form. This is referred to as the conditional form. This combined universal conditional form is expressed symbolically as $(x)\ (C_x \supset E_x)$, which says that if any object is C it is also E. It should be noted in passing that these universal conditionals can be equivalently translated into other logical forms so that the requirement of lawlikeness is not that laws must be stated in the universal conditional form but they they must be at least translatable into the universal conditional form.

This concept of the universal conditional statement form takes on additional clarity when it is discussed in contrast with the existential conditional and singular forms. The existential conditional is expressed symbolically as $(\exists x)\ (C_x \cdot E_x)$. This says, for example, that there is at least one case in which if supply decreases, with demand unchanged, then prices rise. The comparable singular statement form would yield a statement to the effect that if supply decreased with demand unchanged, then prices fell. Both of these last two statements are quite different in their meaning from the universal conditional. Neither of these last two

types of statements are lawlike in that neither type has universal range although they are both of quantified conditional form.

There are a number of other types of statement forms whose predicates are of sufficient scope to make them universal conditionals. The two that should be distinghished here are the analytic universal conditionals and the empirical universal conditionals. This distinction is drawn at this point in order to expose the commonly held misconception that all laws are tautologies. These types of statement forms are analyzed in detail in Chapter V and the analysis extended to cover equation forms in Chapter VI. Hence only an intuitive notion of tautologies and empirical statement forms need be presented here.

It was pointed out in the foregoing section that the truth value of analytic statements (including tautologies) does not depend on the meanings of the descriptive terms employed. It depends upon their form and form alone. In the case of empirical statements, on the other hand, their truth values depend upon both their form and the descriptive terms employed. Laws may be expressed in either of these two types of statement forms. All the laws of mathematics, for example, are analytic statements. They contain no symbols referring to descriptive properties or relations. They say nothing in and of themselves about the real world.[6]

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[6]Brodbeck, op. cit., p. 377.

In the empirical assertions, on the other hand, the variable must be given meaning before its truth or falsity can be established.[7] For example, Galileo's law of falling bodies is stated $d = 16t^2$. This is an empirical law; it is not a tautology. The letters "d" and "t" must be given meaning as distance and time before its truth value can be ascertained. Its truth value can not be established (as in the case of tautologies) from examination of its form and form alone.[8]

The statements, "$Y = aX_1^{b_1} X_2^{b_2} X_3^{b_3}$" and "$D = a - bP$" drawn from production economics and from price theory respectively, can also be considered as empirical generalizations of universal conditional form whose truth values can not be ascertained without examination of both form and content. It will be demonstrated in Chapter V that it is these empirical, contingent or synthetic statements that perform the key role in expressing the empirical content of science. This does not deny a role to the tautological group of the analytic functions, however. In fact, it will also be demonstrated that tautological statements like "If A>B, and B>C, then A>C," and "$(x+y)^2 = x^2 + 2xy + y^2$", for example, play a key role in the logical structure of science.[9] The

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[7]Brodbeck, op. cit., p. 378.

[8]Brodbeck, loc. cit.

[9]Tautology is used here very broadly as a synonym for analyticity, i.e. truth in virtue of meaning alone.

intuitive distinction between tautological and empirical assertions presented above together with their more rigorous analysis in Chapter V should be sufficient to dispel the commonly held misconception that all laws in science are tautologies.

Having briefly examined the forms of lawlike statements and concluded that they must be universal conditionals, or translatable into universal conditionals, and either empirical or tautological in form, attention will now be turned to an examination of the scope of the statements employed.

One of the basic requirements of lawlikeness is that of unlimited scope. For example, the statement, "Every time supply increases with demand unchanged, prices fall," would be considered lawlike. On the other hand, the statement, "Every time supply increased during the past fiscal year, with demand unchanged,prices fell," would not be lawlike. In the first instance the scope of the sentence is unlimited, in the second case it is limited to the finite number of changes occurring within a given year and this limitation on scope is apparent from the meaning of the terms employed. The second may therefore be simply an accidental generalization even though it may be fully confirmed by observation of all cases. This is perhaps the most important distinction between lawlike and non-lawlike statements. The first is accepted as true while many cases of it

remain to be determined, the further unexamined cases being predicted to conform with it.[10] The second is accepted as a description of certain facts after the determination of all cases has been accomplished. No prediction of any of its instances is based on it. A lawlike statement, if it is true, is therefore something that can be used in predicting other cases. It is lawlike only if it is acceptable prior to the determination of all its instances. Finally, in order for a sentence to be lawlike, its acceptance must not depend upon the determination of any given instance, that is, it must contain no essential occurrences of designations of particular objects.

This final requirement of lawlikeness poses some serious problems. Defined terms which appear to have no essential occurrences of references to particular things often turn out, upon examination, to be defined in terms referring to specific things. Hence certain restrictions have to be placed on the predicates that occur in fundamental lawlike sentences. The required restriction is that the predicates employed be purely qualitative. That is, that a statement of its meaning does not require reference to any one particular thing or spatio-temporal location. For example, the terms "elasticity of production," "consumption function," "marginal efficiency of capital" are acceptable

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[10] See Nelson Goodman, Fact, Fiction and Forecast, Cambridge University Press, p. 26.

terms or phrases within lawlike statements, while "rates of growth in excess of those experienced in the developed areas," "the rates of capital accumulation in the underdeveloped countries," or, "prices higher than in the 1930's," are not acceptable in lawlike statements. If this requirement of purely qualitative predicates is fulfilled within the fundamental laws then it is likewise fulfilled within the derivative laws that follow from them. At the same time, if this requirement is generally fulfilled, the requirements of non-limited scope is also fulfilled because sentences which violate the condition of non-limited scope generally do so by making explicit references to specific things.

A more rigorous analysis of lawlikeness and hence of laws is available in the literature of analytic philosophy.[11] Sufficient discussion has been presented, however, to provide an intuitive concept of lawlikeness and hence of laws. In summary, then, laws are true statements. They are usually stated in universal conditional form or translatable into universal conditional form.[12] Finally, the statements must be non-limited in scope or be derivable from more fundamental laws which are non-limited in scope. It is these lawlike

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[11]See Hempel, op. cit., and the footnotes and bibliography contained therein.

[12]It should be reemphasized, at this point, that while laws are generally universal conditionals or translatable into universal conditionals, it by no means follows that all universal conditionals are laws. In fact, Craig's Theorem, suggests the universal conditionals can be translated into single statements of equal predictive potential but no explanatory significance.

statements that are required in the structure of explanation and prediction.

THE ROLE OF LAWS IN THE STRUCTURE OF EXPLANATION AND PREDICTION

EXPLANATION

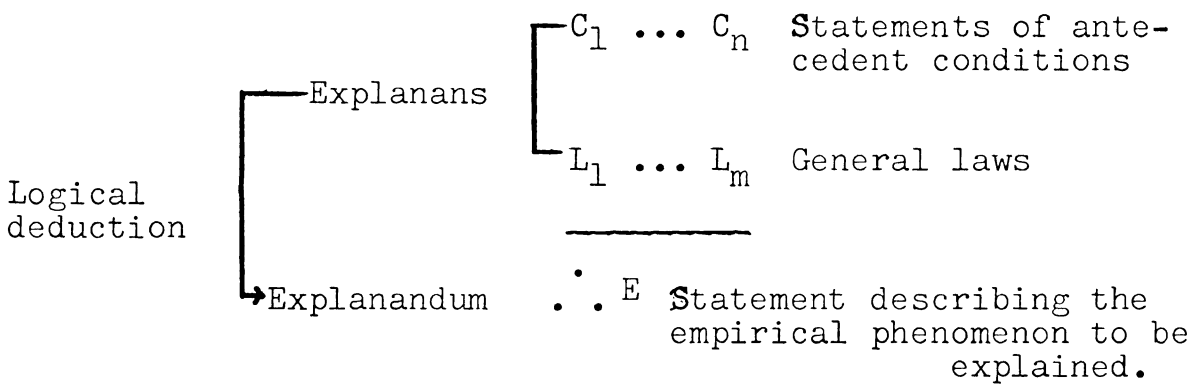
The processes of explanation and prediction employ the laws or lawlike statements discussed above, together with one or more singular statements, in the manner described below. Given the set of relevant lawlike statements $L_1 \ldots L_n$ and adducing the appropriate singular antecedent statements $A_1 \ldots A_n$ (statements which indicate certain conditions which occurred prior to, or at the same time as, or even later than, the phenomenon in question) one can explain the event E, by deducing E from the A's and L's. Hence, an event is explained by subsuming it under general laws, that is, by demonstrating that it occurred in accordance with those general laws, as a result of the occurrence of those particular antecedent conditions.

To explain a phenomenon, then, is to answer the question "Why does the phenomenon occur" and this is the same as answering the question "according to what general laws, and following what specific antecedent conditions does the phenomenon occur?" It should be noted that "why" is used here in the sense of "how come" not in the sense of "for what motive."

The foregoing structure is used in the explanation of

particular events. The explanation of general laws (statements of commonly observed regularities) is also accomplished by the same kind of procedure. For example, "Why does marginal cost eventually rise as output continues to expand?" i.e. "Why do costs conform to the law of eventually increasing costs?" The answer is that this law is a consequence of the law of diminishing returns. Hence, the explanation of a general regularity involves subsuming it under another, more comprehensive, regularity, i.e., under a more general law. Events are explained by subsuming them under laws, laws are explained by subsuming them under more general laws.

After the foregoing introduction to the structure of explanation, it becomes appropriate to examine this structure in more detail. Its outlines can be summarized as follows:



An explanation is made up of two sets of statements, an explanans, and an explanandum. The explanandum is usually a singular statement describing the phenomenon to be explained. The explanans, on the other hand, is made up of those two sets of statements which are adduced to account for (i.e., to

explain or imply) the explanandum. The first set is made up of statements of specific antecedent conditions, the second of statements of general laws.

In addition to having the foregoing structure, the following conditions of logical and empirical adequacy must be fulfilled in order for an explanation to be adequate.

LOGICAL ADEQUACY:

1) The explanans must imply the explanandum.

2) The explanans must contain at least one lawlike statement. It may be made up entirely of lawlike statements. One or more of these lawlike statements must be required in the deduction of the explanandum.

3) The explanans must have empirical content. It must be amenable to test by observation or experiment. Otherwise it could not imply E which is a description of the phenomenon in question.

EMPIRICAL ADEQUACY:

1) The sentences asserting the explanans must be true. The reasons for this are two-fold. The first is that to permit high confirmation instead of truth would lead, as in the case of laws, to a relativized concept explanation which is not in accordance with informed usage. The second is that adequate deduction assures the truth of the conclusion provided the premises are true. It does not, however, assure the truth of the conclusions if the premises should happen to be false.

PREDICTION

Both the structure and the necessary conditions of logical and empirical adequacy are the same for prediction as they are for explanation. Hence, if a structure can explain it can predict and if it can predict it can explain. The difference is only one of temporal orientation. If the event described by E is known to have occurred, and if the statements of the antecedents and the general laws are adduced after the event then the event E has been explained. If, on the other hand, the statements of the antecedent conditions and the general laws are given, and E is deduced from these prior to the occurrence of the phenomenon it describes, then E has been predicted.

Hence, an explanation is not fully adequate unless the explanans which has been adduced to explain the occurrence of the phenomenon E could have been used, with time taken properly into account, to predict the phenomenon in question.

This potentially predictive force is, after all, the reason why scientists are interested in explanation. Scientists explain so that they can predict, and hence provide a basis for control of the environment. More specifically, economists are interested in explaining the process of economic growth so that they can more accurately predict the values of the variables involved so that society can control the values of some of these variables in order to be able to increase the value of another variable--the real per capita

income of the economy, for example.

THEORIES IN THE STRUCTURE OF SCIENCE

Up to this point the structure of explanation and prediction and the central role of laws in this structure have been discussed, and the nature of laws investigated. The next task, then, is to show how these laws are combined to form theories, and how these theories function in providing the laws required to explain events, and to explain the lower level laws themselves.

THE NATURE OF THEORIES

It would be unwise to enter a discussion of the nature and function of theories without pausing for a moment to gain perspective. A theory is a deductively related set of laws. Laws are true hypotheses. Hypotheses are general statements about regularities. They are propositions asserting a universal connection between properties or relations. They state that every thing of a certain kind either has a certain property or stands in a certain relation to other things or events that have certain properties--all of which is to say that hypotheses are of the general form "everything which has the property A has the property B", where A and B are sufficiently complex properties or relations.

To assert that a theory is a deductively related set of laws is to assert that from some of the hypotheses taken as premises (axioms) all the other hypotheses (the theorems) logically follow. An intuitive grasp of deductive relatedness

can be obtained by considering the order in which a deductively related set of statements is usually presented in written form. Deductive systems are usually written so that the spatial relationships among the statements on the page correspond to the logical relationships among these statements. The logical relationship is one of deductive subsumption. The highest level hypotheses (the premises) are recorded first. These are followed by the intermediate level hypotheses which are deduced directly from the higher level hypotheses. These are followed by the lowest level hypotheses (the conclusions of the system) which are deduced from the premises, from the intermediate hypotheses, or from some combination of the two. The highest level hypotheses or premises constitute the axiom set. The intermediate level hypotheses function as conclusions from the highest level hypotheses and premises for the lowest level hypotheses. The lowest level hypotheses are conclusions derived from either of the previous levels. These intermediate and lowest level hypotheses constitute the theorems of the system.

THE NATURE OF CALCULI

One of the most useful concepts in understanding the nature of theories and of deductive relatedness is the concept of an underlying calculus or structure. This concept plays a key role in the process of theory evaluation.

During the last century it was discovered that it is possible to choose a set of symbols to substitute for the

propositions of a system, to codify a set of rules for the manipulation of these symbols, to manipulate them and translate them back into the natural language in order to obtain a set of statements which are the deductive consequences of the original propositions. This representation of a deductive system so that there is a rule of symbolic manipulation for each principle of deduction is called a calculus.

The practical advantages that derive from the use of a calculus include the facts that it enables deductions to be made by the manipulation of symbols alone and the further fact that the correctness of these deductions can be checked by inspection of the relationships between these symbols. In addition, once a deduction is proved in a calculus it is simultaneously proved for every interpretation of that calculus.

This concept of a calculus or an underlying structure is basic to many of those aspects of formalization concerned with the structure, as averse to the content, of the individual statements and the sets of statements of which theories are constructed.

THE NATURE OF DEDUCTIVE RELATEDNESS AND FORMALIZATION

The definition of theory as a deductively related set of laws can now be reformulated in order to point up some of the essential concepts involved: "A theory is a systematically related set of statements, at least one of which is lawlike and which is empirically testable." Of the three

pivotal terms in this definition, the term 'lawlike' has already been discussed. The discussion of the term 'empirically testable' will be deferred until the discussion of quantification. The concept requiring further elaboration, at this point, is the notion of 'systematic relatedness.'

'Systematically related,' as used here means deductively related or more precisely, the relationship of deductive subsumption. This is the relationship embodied in the concept of a calculus. The process of fully articulating the deductive system that constitutes a theory (which involves exhibiting its underlying calculus) is called the process of formalization. This process of formalization can be carried out to a whole series of different levels, from the lowest level where just one deduction has been made to the highest level where the entire framework has been articulated. Actually, a very few theories have ever been fully formalized, and for good reason. In many areas of economics, for example, knowledge is tentative. Time spent on formalization of these theories might often be better spent on clearing up the meaning and centrality of the concepts employed. Only a very few areas, where substantial agreement exists and where most of the significant assertions about the subject have already been made or where a purely formal system which constitutes the underlying calculus of the theory already exists, are ripe for easy full formalization.

FULL FORMALIZATION

The structure and import of full formalization becomes readily apparent when its relationship to the calculus of propositions is considered. An understanding of the concept of formalization can also be acquired by studying the procedure by which a language might be generated. As will soon become apparent, a language usually contains a deductive system and hence provides an appropriate place to begin an exposition of deductive systems in general.

In broad outline, the steps that would be followed in the generation of a language, English for example, are as follows: First, all the symbols of the language would be listed. Then, all the grammatical rules of the language (the entire syntax of English), for example, would be codified. Finally, all the sentences of the language would be generated by applying the rules to the symbols. The fact that a natural language is continually in a state of flux and hence does not have a finite set of terms or rules need not detract from the central point made here. The scientist usually works with an artificial, not a natural, language, and hence has control over the terms and rules that he admits into the system. This does not necessarily limit the number of terms and rules in an artificial language but it does mean that, conceptually at least, these large numbers do not create a problem.

Each of these three elements, the terms, the rules,

and the sentences are divided into sub-parts. The class of terms is divided into a set of primitive or undefined terms and a set of defined terms. The latter are defined in terms of the former, but do not enter in an essential (i.e. unelim-inable) fashion because they could always be replaced in any statement, by the primitives in terms of which they are defined. The fact that there must be a class of terms that function as primitives is apparent from the further fact that if all terms were to be defined they would have to be defined in terms of each other. This would involve vicious circul-arity of definition. The need for a set of defined terms, on the other hand, is apparent from the need to introduce new terms into the language of a theory, and the added notational convenience that definition provides.

The rules are also divided into two sub-classes--rules of formation and rules of transformation. From these elements of the system a group of all possible combinations of terms could be generated by some procedure designed to form all possible combinations and permutations of these terms. The formation rules are applied to this set of all possible combinations and permutations in order to select the permissable (or gramatically correct) statements from the group. The deductive system within the language will not contain all of these permissable statements. It will con-tain only a subset of statements which will become the axioms or underived statements of the deductive system. The rules

of transformation are then applied to this subset containing the axioms in order to generate all possible statements that can be deduced from these axioms. Hence, the rules of formation correspond, in the case of the generation of the English language, to the syntax of English. The rules of transformation, on the other hand, are comprised of the rules of deductive logic. The class of sentences on which attention is focused in the process of formalization is also divided into two sub-classes -- the axioms and the theorems. These two sets of sentences together constitute the deductive system.

In summary, then, a deductive system, or a deductively related set of statements, is generated by the application of two sets of rules on two sets of terms. In the foregoing example, the generation of the English language, terms are combined to form statements, and statements are combined to form more complex statements by the application of the rules of formation. The simple and complex statements are derived from other statements by the application of the rules of transformation, i.e. the rules of deductive logic.

The same procedure applies in the generation of an artificial system or a theory. The first task is to identify the relevant variables (terms) of the system and to generate a set of statements connecting these variables in such a way as to describe the phenomenon to be explained or

predicted. These statements, or a selection from them, become the axioms of the system. Then the theorems or the implications of these axioms are deduced with the aid of the transformation rules -- the rules of deductive logic. It is these axioms and theorems together that constitute the deductive formulation of a theory -- the part of the language system with which science is so vitally concerned. It is the codification of these axioms and theorems that constitutes the formalization of a theory.

Some of these axioms or theorems are then used as inputs into the structure of explanation or prediction and in this way are brought to bear on the problems with which science is concerned.

The power of formalization rests in part on the researcher's ability to elaborate the deductive consequences of the axioms chosen. These deductive consequences as well as the axioms themselves become the source of hypotheses for empirical investigation, and hence inputs into the structure of explanation and prediction. The more completely the axioms and the theorems can be codified, the more knowledge is obtained about the deductive consequences of these axioms and hence about the axioms themselves. The examination of such a system is usually both logical and empirical. The deduction of contradictory theorems supplies logical evidence of inconsistency. Empirical evidence for or against any one of the theorems is also evidence for or

against the axioms employed.

Interpretation of Fully Formalized Systems

Such a formalized system may be interpreted or it may be purely formal, with no recourse to meanings at all. If it is purely formal, then in order to make an interpretation of it, i.e., in order to make it refer to something or to take on some meaning, it is necessary to formulate a series of rules of interpretation. These rules of interpretation or semantical rules are also divided into two subsets -- the rules of designation and the rules of truth. Each one of these sets of rules applies to only one of the two sub-groups of terms that exist within the class of primitives. The rules of designation specify the meaning to be ascribed to each of the descriptive or empirically referential terms. The rules of truth specify the interpretations of the logical terms of the system, i.e., they determine a class of truth conditions for each type of logical connective employed.

The rules of designation apply to two subsets within the set of descriptive terms. The first comprises names which designate individuals or things. The second comprises a set of predicates which designate properties of individuals or things. It is the role of the rules of designation to specify for each of these names or predicates some thing or property which it is taken to designate.

The rules of truth on the other hand, specify that,

for example, an expression of the form "A is B" is true if and only if the individual or thing designated by the name in position 'A' actually has the property designated by the predicate in position 'B'. Other rules of truth specify the truth conditions of the other logical terms. These rules of truth of the formalized system known as the calculus of propositions constitute the key concepts involved in Chapter V. In that chapter they are employed in the development of a classification system which provides a means of recognizing certain methodologically important characteristics of the axioms and theorems of the theory under examination. Recognition of these characteristics becomes extremely important in the a priori evaluation of theories in general.

A complete codification of these rules of designation and of truth constitutes an interpretation of a formal system. A deductive system which has been thus formalized and interpreted and which contains among its axioms some empirically testable statements is what is meant by a fully formalized empirical theory.

Some appreciation of the power of such formal systems can be gained from a realization that when a theorem is proved in a formal system, like the lower functional calculus or some branch of mathematics,all of its alternative interpretations in formalized empirical systems are simultaneously proved, provided these interpretations satisfy the

axioms of this calculi. Some appreciation of the applicability of such formal systems or calculi to the problems of theory evaluation will be attained from consideration of the concept of a model discussed in this chapter and applied in Chapter VI.

PARTIAL FORMALIZATION

The foregoing examination of the nature of full formalization indicates that many theorists will likely have to settle for something short of full formalization for some time to come. But awareness of the power of full formalization in theory construction suggests that partial formalization may also be a fruitful method of inquiry. The next task, therefore, is to examine possible methods of partial formalization.

Systematic Presupposition in Partially Formalized Theories

It is commonly observed that partially formalized theories often presuppose large segments of theories from other disciplines. Price theory often presupposes segments of mathematics, for example. Theories of economic growth often presuppose concepts embodied in putative sociological theory.

The terms referring to those concepts borrowed from other disciplines are generally not indigenous to the theory in question. These terms, like "first derivative" as it occurs in economic theories, remain unexplained in most of

these theories. But unlike the groups of unexplained terms that are indigenous to the theory and function as relative primitives, this group of unexplained terms does not ordinarily have the same set of functions. The importance of recognizing this difference rests on the fact that this recognition is often sufficient to lead the researcher to an awareness that some statements from some other disciplines are being presupposed as premises in the theory in question or as transformation rules.

Current theorizing is sometimes carried out without making these deductions explicit, and without codification of the non-indigenous concepts which are actually being presupposed. The process of making these deductions and presuppositions explicit is part of the process of partial formalization. This aspect of formalization is just as important when the presupposed concepts come from the researchers intuition or common sense as when they come from other scientific disciplines. Systematic presupposition is an important aspect of partial formalization in that it helps to clarify the meaning of the concepts presupposed, and in so doing helps to render the otherwise unrecognized presuppositions amenable to logical and empirical examination and test.

Quasi Deduction in Partially Formalized Theories

Another important technique of partial formalization is that of quasi deduction. Quasi deduction is defined as

inference which fails to meet the rigorous requirements of deduction by failure to make all the premises and rules and steps explicit. This includes the indigenous as well as the non-indigenous premises referred to above. Here again explicit codification figures importantly in the process of partial formalization in that it subjects new theorems to the weight of empirical evidence. Evidence for or against these theorems constitute evidences for or against the theory as a whole.

Concept Formation in Partially Formalized Theories

There are three general techniques for introducing concepts into partially formalized theories.

The first is the technique of explicit definition. This technique is the same in partial formalization as it is in full formalization except that in the former there is no unique set of primitives in favor of which all the defined terms may be eliminated.

The second technique is that of specifying a sufficient condition for the use of these concepts. Consider the statement, "if a market is characterized by such a large number of small buyers and sellers that no single buyer or seller can influence the price, then that market is perfectly competitive." This statement specifies a sufficient condition for the occurrence of the property of being perfectly competitive, and affords a sufficient condition for the application of the term 'perfect competition', providing the

statement itself is a true statement. It should be noted, however, that this is not a definition, for the same theory may yield a number of different sufficient conditions none of which is a necessary condition for the term's use. These sufficient conditions may provide an entreé for the process of evaluation, however, in that they help clarify the concepts employed. In addition, if these concepts are couched in observation terms, they permit the researcher to submit these axioms and theorems to the weight of empirical evidence.

Finally, partially formalized theories often contain terms which cannot be introduced in either of the foregoing ways. These remaining terms generally function as relative primitives in the system. They are relative (or tentative) in the sense that terms once of primary importance in a theory may play only minor roles when the subject matter is more thoroughly understood. Hence terms may be primitive in one context and assume a non-primitive role in a more fully elaborated context. Another of the techniques of partial formalization, then, involves the identification of what is and what is not primitive in the system and in this way affords a clearer concept of the meaning of the terms employed. This differs from the technique of explicit definition in that the former makes the definition explicitly while the latter simply recognizes which concepts can not be explicitly defined and hence must be treated as a relative primitive.

All three of these techniques for introducing concepts into partially formalized theories suggest ways of recognizing the concepts employed in any theory under examination. Recognition is a prerequisite to ascertaining their testability. Assessment of their testability is in turn a useful first step in ascertaining the truth values of some of the axioms and theorems in the theory under examination. The truth values of these statements are of primary importance in the evaluation of theories in economics.[13]

MODELS IN THE STRUCTURE OF SCIENCE

This examination of the specially selected set of logical concepts required for the a priori evaluation of economic theories will conclude with a consideration of the nature and role of models in science. The concept of a model is an outgrowth of the concepts of calculi and formalization developed above and is basic to the particular application of the method of models presented in Chapter VI. As May Broadbeck has pointed out, not only has the term "model" appeared with increasing frequency in recent social science literature, it has taken on a decided halo effect.[14] According to current fashion, models are regarded as good

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[13]Geoffrey, P.E. Clarkson, The Theory of Consumer Demand: A Critical Appraisal (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1963), p. 11.

[14]May Broadbeck, "Models, Meaning and Theory", Symposium on Sociological Theory, Llewellyn Gross, ed., (Row, Peterson and Company, Evanston, Illinois,) p. 373.

things and needless to say, "mathematical models" are regarded as even better.

In non-technical use, the term 'model' usually refers to a norm or a replica. The non-technical use of 'model' to refer to a replica provides a point of departure from which to explain the technical meaning of model. A model train is a model of the real thing by virtue of the isomorphism, or sameness of structure, that exists between the two. The iso-morphism requires two conditions: (1) that there be a one-to-one correspondence between the elements of the model and the elements of the original and (2) that certain relations, like scale in this example, be preserved.

The foregoing notions can be extended to theories. For any two theories, then, if the laws of one have the same form as the laws of the other, they are isomorphic with each other, and one is a model of the other. This implies, of course, that there exists a one-to-one correspondence between statements in the two theories, and that certain relations are preserved upon translation.[15]

The next problem is to determine whether or not one theory is in fact a model of another. The following steps will illustrate the process. First, the underlying structure of the original theory may be developed by replacing each of

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[15]For a thorough treatment of the concept of isomorphism see Gerald J. Massey, "The Philosophy of Space and Time", (unpublished Ph.D. dissertation, Department of Philosophy, Princeton University, 1963), p. 123.

its descriptive terms by letter variables and the logical operators by the appropriate symbols. This leaves only the variables and the logical operators, as they would appear, for example, in the sentential calculus. Then, the model is generated by systematic substitution of the terms of the new theory for the variables in the underlying calculus. This is what is meant when it is claimed that a model is an alternative interpretation of an underlying calculus. In each of the laws, the concepts of the original have been replaced by the concepts of the new discipline. If empirical test shows that the resulting laws or hypotheses are true, then the laws of both the new and the old have the same form. This means that the two areas are isomorphic.

An excellent example of isomorphism obtains between the theories of production economics and consumption economics. If the underlying structure of production economics were developed by uniform substitution of letter variables for the descriptive terms, and symbols for the logical operators, and if uniform substitution of the concepts of consumption economics were effected, and if the laws of both production and consumption economics are true then isomorphism exists between these two areas, and production economics would provide a model of consumption economics.[16]

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[16]John R. Hicks established this isomorphism between production economics and the new consumption economics which permitted only cardinal utility measurement by showing that everything in the new consumption economics was also in production economics.

There is nothing absolute about the relationship between a theory and a model, however. Consumption economics could just as well be thought of as a model of production economics as vice versa. Usually, however, the better known area is taken as a model for the area about which less is known, the better known area providing the form of the laws for the area under investigation.

In current writings on theories of economic growth there are many putative models in vogue. Economic growth may be likened to the growth of a plant or an animal, for example, with life cycle and all. The ability to determine whether one such theory is a model of the other would be extremely useful in assessing the explanatory and predictive potential of certain recent theories of economic growth. Two ideas drawn from the above discussion are likely to be particularly useful in this regard. First, it must be possible to state what it is that is in one-to-one correspondence with what? Second, it must be possible to determine what formal similarities exist? Not only must the terms correspond, at least some of the laws connecting the concepts must have the same form in order for a putative model to be of any use in the explanation or prediction of the phenomena in question.

These conditions of: (1) one-to-one correspondence and (2) the preservation of the relationships that must obtain in order for a putative model to work on the same principles

as the theory in the new discipline suggests ways of analyzing putative models.

One of these ways is applied in Chapter VI. This particular approach differs from the usual use of models in science. The general use of models is in theory development rather than in theory evaluation. It is generally hoped that it will be possible to obtain new insights about the area under investigation like consumption economics, for example, by finding similarities between the new area and some more fully developed discipline like production economics, for example.

The use of the concept of a model as a diagnostic device is quite different from the approach specified above. One such use is in testing for the consistency of a theory or set of axioms. The general approach to this examination is as follows. The first step is to exhibit the underlying calculus of which the theory in question constitutes one interpretation. The second step is to give this calculus an interpretation which is different from the one represented by the classical axioms and which is analytic. Such interpretations include a wide variety of mathematical interpretations as well as those of Boolean Algebra, the higher functional calculi and other systems of formal logic.

This analytic interpretation, then, constitutes a model of the original axiom set. Further, it constitutes a model in which every statement is demonstrably true. As

pointed out above, when the statements in the model are all true the isomorphism is complete and the laws of the theory and the model have the same form. The fact that the statements in the model are all true has another extremely important consequence, that is, that the statements in the theory are mutually consistent. The fact that a true model of a theory implies that the theory is consistent follows as a corollary of the fact that if the theory were inconsistent then no true interpretation of the underlying calculus would be possible. Hence, if a set of statements which are an interpretation of the calculus are demonstrably true then the theory is proved to be consistent.[17]

The reason for translating the underlying calculus into an analytic interpretation rests on the fact that every statement in an analytic system is demonstrably true, while statements in the empirical theory are at best only highly confirmed.[18] If the underlying calculus has at least one true interpretation then that calculus is consistent. If the underlying calculus is consistent then, providing all the axioms of the theory are true, the original theory itself and every other interpretation of it, insofar as it is an instance of this calculus, is also consistent. The fact that the calculus is consistent simply means that it is

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[17]This conclusion is rigorously demonstrated in Chapter V.

[18]In light of Godel's incompleteness theorem this statement must be qualified, but it is considered to be sufficiently precise for the purposes at hand.

not self-denying. It may be true or it may be false depending upon the empirical interpretations employed but it is not self-denying or inconsistent.

The general use of the concepts of a model as a means of theory evaluation in economics is fraught with a number of problems. In the first place, only a very small percentage of extant economic theories have attained sufficient formalization to permit the identification of their underlying calculus. In the second place, it is often difficult to find an adequate interpretation which is also analytically true. Besides being difficult, any number of failures to find such an interpretation does not prove inconsistency, it only leaves the problem of consistency open to questions.

In view of the difficulties mentioned above, the problem at hand is to develop a procedure which will permit the researcher to avoid some of these difficulties and still obtain an unequivocal answer to the question of consistency of the theory under evaluation. This is accomplished in part by the combined use of the concepts of model construction and the concepts derived from the analysis of propositions developed in Chapter V. The concepts will then be applied, through the Method of Numerical Interpretation, to the evaluation of the classical theory conducted in Chapter VI.

SUMMARY

Up to this point the nature and structure of science has been examined. This examination started with a consideration

of terms and sentences used in science. Then it progressed to a study of laws and lawlike statements. This study was continued with an examination of the role of laws, or lawlike statements, in scientific explanation and prediction. This set the stage for an examination of sets of laws or theories as they function in providing the laws used in science.

The study of theories concentrated first on the nature of calculi or the logical structures of theories. This led to a consideration of deductive relatedness and formalization of theories. This consideration of formalization was divided into an examination of the concept of full formalization and a consideration of the techniques of partial formalization as a means of theory evaluation. These concepts of formalization and formal systems finally led to a consideration of the role of models in theory construction and evaluation.

Out of this initial selection of concepts from the field of analytic philosophy the thesis of this study evolves. This thesis asserts that the value of a theory depends upon its ability to provide the lawlike statements required for the explanation and prediction of the phenomenon in question. This ability depends upon the logical adequacy of its structure and the truth of the propositions employed.

Having examined this initial selection of concepts, the next step is to investigate their application to the

problems of theory evaluation. The theory chosen for investigation is the classical theory of economic growth. The outline of this theory is presented in the following chapter. This theory will be employed, throughout the remainder of this investigation, to illustrate the application of the techniques of theory evaluation which are developed as this study progresses.

CHAPTER III

EXAMINATION OF THE CLASSICAL THEORY OF ECONOMIC GROWTH WITH SPECIAL EMPHASIS ON THE TECHNIQUES OF LOGICAL ANALYSIS

A SUMMARY OF THE CLASSICAL THEORY OF ECONOMIC GROWTH

Higgins has summarized what he takes to be the main thesis of the classical theory of growth.[1] This summary no doubt does some injustice to most of the members of the classical school. It is a summary of their major points of agreement, and ignores, to a large extent, the differences and the polemics of that period's economics. It concentrates on the works of Smith, Mill, and Malthus and abstracts what he believes to be the bare bones of their theory of growth, translating them into modern terminology and stating them as functional relationships, taking care to close up the system in such a way that it appears, at least on the surface, to be determinate.[2] His presentation will be summarized below.[3]

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[1]Benjamin Higgins, Economic Development Principles, Problems and Policies, (New York: W.W. Norton and Company, Inc., 1959), pp. 85-106.

[2]The concept of determinateness will be defined and discussed in a later section.

[3]In this context the term theory of economic growth usually refers to both economic growth and decline. This type of theory is often applied to both the developed and the underdeveloped economies and is to be distinguished from theories of development. Development theories generally apply only to underdeveloped economies. They may include aspects of social and political change as well as economic change and development.

Then the theory will be examined and an attempt made at evaluation of its explanatory and predictive potential.

Proposition 1: The Production Function

The classical concept of the aggregate production function has prevailed in many theories of growth down to modern times. It claimed that output, O, was a function of the size of the labor force, L, the supply of known resources, K, the stock of capital, Q, the level of technique, T, and the way in which these factors are combined.

$$O = f_1 (L, K, Q, T) \qquad (1)$$

They thought that this function would probably exhibit constant returns to scale, but that, since the supply of known resources (principally land in this context) was fixed, the most important function was that subfunction which treated the level of land as a constant and traced the relationship between output and population at the existing level of technology. Both increases in the supply of known resources and of technology would shift this subfunction upwards. It is also fairly clear that they had in mind the traditional three stage production function and that they thought that Europe was, in their time, well into stage two, with APP falling but MPP still positive.

Proposition 2: Technological Progress Depends on Capital Accumulation

Another proposition that has stood the test of time

is that technological progress, T, is a function of investment, I.[4]

$$T = f_2 (I) \qquad (2)$$

They assumed that there was always a plentiful supply of better techniques and new products to introduce (except in agriculture), but that the rate of introduction was limited by the rate of capital formation. Hence, they stressed the need for savings and capital accumulation rather than for technological advance.

Proposition 3: Investment is a Function of Profits

A third proposition that has played a central role in the classical theory, and ever since, is that investment, I, is a function of profits, R.

$$I = f_3 (R) = dQ \qquad (3)$$

Here investment refers to net new investment or the net addition to the capital stock, dQ, and R means profits in the sense of returns to the fixed factors, or returns above variable costs.

Proposition 4: Profits Depend on the Labor Supply and the Level of Technique.

Unlike some of the more recent theories, the classical theory held that profits were a function of the labor supply

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[4]The author has taken the liberty at this point, of replacing the variety of symbols used by Higgins for "is a function of" with the more common "f_n" nomenclature.

and the level of technique.

$$R = f_4 (L, T) \qquad (4)$$

The classicists looked upon the whole process of growth as a race between technical progress and population growth. It was through the rate of profits and hence of capital accumulation that the outcome of this race was to have its effect. Improvement in technique tended to increase profits. But the growth of the labor force was associated with the employment of more workers on the same land, lowering APP, raising AVC, and thus lowering profits for any given index of product prices. This ignored the response of product prices to increased aggregate output on one hand and increased consumer damand due to both increased output and increased population on the other. The effect of increased population on profits is an empirical question, depending for its answer on the shape of the aggregate production function and the shifts in aggregate demand. The classicists seemed to ignore these effects, but realized that the final outcome of what they assumed to be the depressing effect of population growth on profits and the presumed increasing effects of technology on profits could not be answered a priori. Mill, for example, thought that technology was winning in the short run at least. This will be discussed more fully later in one of the succeeding sections.

Proposition 5: The Size of the Labor Force Depends on the Size of the Wage Bill.

Another part of their theory was the 'wages fund

doctrine' i.e., that the rate of population growth depends on how much money is available to pay wages. The labor force was thought to vary in proportion to the growth in population,

$$L = f_5 (W) \qquad (5)$$

where L is the total labor force and W represents the wage bill.

Their contention was that there were no checks on the size of labor-class families except the wages available to them and the number of children that could subsist on a given level of real wages. Hence, as wages rose, population rose and the real wage per capita was driven back down to the subsistence level. This assumption was probably quite appropriate to eighteenth century Europe and remains so for many underdeveloped countries today.

Proposition 6: The Wage Bill Depends on the Level of Investment.

The classicists thought of capital as consisting partly of a "wages fund", W, which was made up of savings and put to work through investment. Hence,

$$W = f_6 (I) \qquad (6)$$

The foregoing six equations constitute the basic functional relationships involved in the classical theory of growth. But this results, according to Higgins in seven endogenous variables, and only six equations. Hence, to

close the system, i.e., to make it determinate, he adds equation seven, which says that total output is equal to profits plus wages.

$$O = R + W \qquad (7)$$

This means, in effect, that the national income is the total cost of all goods and services produced, and that this amount is divided between workers and others.

Then Higgins added an eighth variable, w, to represent the minimum wage rate, and an eighth equation to indicate the long run equilibrium condition,

$$W = wL \qquad (8)$$

Summary of the Classical System

In summary, then, the "classical system", as interpreted, contains the following equations:

(1) $O = f_1 (L, K, Q, T)$ L_1

(2) $T = f_2 (I)$ L_2

(3) $I = f_3 (R) = dQ$ L_3

(4) $R = f_4 (T, L)$ L_4

(5) $L = f_5 (W)$ L_5

(6) $W = f_6 (I)$ L_6

(7) $O = R + W$

and in equilibrium in the long run

(8) $W = wL$

While Higgins discusses these relationships in a general way in terms of one variable as a function of one or more other variables, it is apparent from the text that he means that one variable is a positive monotonic function of the others. This is true for equations one through six except for number four. Equation four, on the other hand, says that profits are a positive function of the rate of application of new technology and a negative function of the size of the labor force.

EVALUATION OF THE CLASSICAL THEORY OF ECONOMIC GROWTH

IMPLICATIONS OF THE CLASSICAL THEORY

One of the first steps in the evaluation of this particular theory is to see what it implies with respect to economic growth in general.

VICIOUS CIRCLES AND GROWTH SPIRALS

It would appear that the circularity of causal connections is among the most important phenomena that the classicists had in mind. This idea will be discussed briefly below.

The writer has previously defined the concept of vicious circles and growth spirals as follows: "When certain conditions or processes exist, they tend to set up a chain of causes and effects which act and react upon each other so as to maintain the state or direction of those conditions or

processes."[5]

It should be pointed out that this concept is quite general. It may be useful in explaining why poor countries remain poor. Nelson's "low-level equilibrium trap,"[6] why poor countries sometimes become poorer, how poor countries can be made to grow, and how rich countries can become richer, remain at a high level of production or decline. Implicit in this idea is the concept of the circularity of the causal relations, that is, that an exogenous change in the magnitude and direction of one of the variables will set off a chain of actions and reactions which will bring about further changes in the same direction, while a change in the opposite direction will cause a circular chain reaction in that direction. It should be noted in passing that this adaptive or feedback concept of circularity bears no relation to the concepts of circularity of definition or of proof as discussed in the previous chapter. The present concept involves a purely empirical as opposed to a logical concept of circularity.

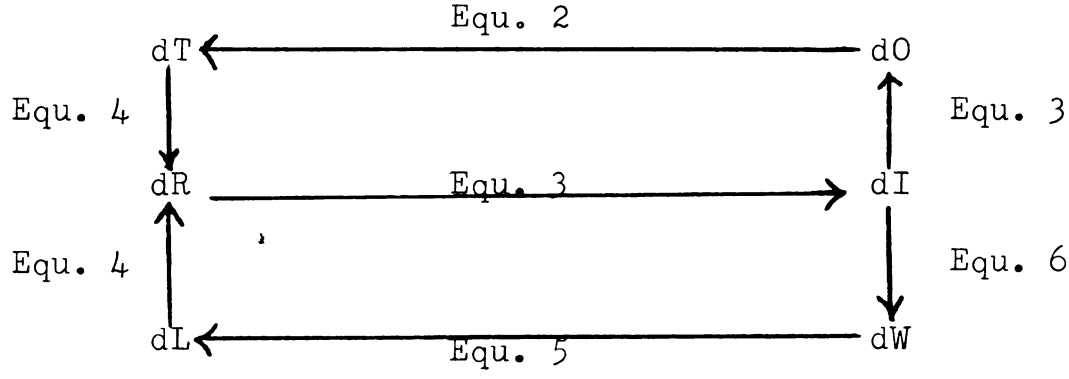
These adaptive ideas of circularity appear to be central to the classical theory. The classicists were looking at the possibility of sustained high level production rather

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[5]Darrel Plaunt and Lawrence Witt, "Recent Theories of Economic Development," unpublished paper prepared for discussion at the Interregional Marketing Committee meeting in Lexington, Kentucky, October, 1959.

[6]R.R. Nelson, "A Theory of the Low-Level Equilibrium Trap," American Economic Review, December, 1959, pp. 894-908.

than low level stagnation, but the general circularity of causes and effects is similar. If the theory were examined at equation three, for example, the following type of circularity may be seen:



If some exogenous force changes (increases) profits (the opening of a new export market for example) then investment will increase, equation three; this will lead to an increase in the stock of capital, also equation three, and hence to an increase in technology, equation two, which in turn causes profits to rise still further, equation four. If nothing happens to stop this chain reaction the increase in profits will cause further increases in investment, the stock of capital, and the level of technology and hence further increases in profits. If other things remain constant, the continually increasing stock of capital and level of technique

may cause a steady rise in aggregate output, which now becomes cumulative over time.

But other things do not remain constant. It is inherent in this very growth spiral, according to the classical theory, that this increase in profits and hence in investment causes an increase in the wage fund, equation six, which in turn causes an increase in the labor force, equation five, and this increased labor force applied to a fixed stock of land and capital encounters diminishing returns to labor causing labor costs to rise and profits to fall, equation four. If this vicious circle were to continue, investment, the wage fund, and the labor force would continue to rise in a cumulative fashion and to force profits downward, finally choking off the very process of growth itself. This means of choking off the growth process was implicit in their thesis of stagnation at a high level of output. It also helps explain what they meant by saying that the continuity of economic growth depended on the outcome of the race between technology and population growth. They assumed that the development of technology was steady and that there was always a plentiful supply of new products and processes to be exploited--their application being largely controlled by the rate of new investment. They also thought that population growth, on the other hand, would soon bring about a sharp rise in wage costs as a result of sharply decreasing returns in agriculture. The outcome for them depended upon

the relative strengths of these two opposing forces.

Perhaps these postulates are not too far from the situation of certain underdeveloped countries today. In areas of extreme population pressure when the man-land ratio is very high and where the rate of creation and application of new technology is quite slow (for sociological, religious or whatever reasons) it can still be expected that the rising prices of wage goods will make severe inroads into the general level of profits. Here, again, it might be argued that it is only to the extent that investment and the application of new technology proceeds at a rapid enough rate that profits can be maintained at high enough levels to induce the private sector to institute or to maintain a sustained and cumulative rate of economic growth. Indeed, this is the presupposition behind W.A. Lewis' arguments about the nature of economic growth with unlimited supplies of labor, and the difficulty of maintaining growth when the costs of labor inevitably rise due to whatever the cause may be, whether it is trade union activity, the application of social legislation from developed economies to underdeveloped economies, diminishing returns in the agricultural sector, or any one of the several other possible sets of causes.[7]

OTHER IMPLICATIONS OF THE CLASSICAL THEORY

All of this argument appears to have a high level of

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[7]W.A. Lewis, "Economic Development with Unlimited Supplies of Labor," Manchester School, May, 1955.

prima facie plausibility. Furthermore, it appears to be consistent with both a larger body of growth theories and with some of the evidence available from the underdeveloped areas. But some curious consequences ensue. These consequences become apparent upon further examination of the nature of the equations presented.

Each of these equations, except equation number four, is assumed to express a positive monotonic relationship between the variables involved. Equation four, on the other hand, says two things. It says that an increase in technology causes an increase in profits, and at the same time, an increase in the labor force causes a decrease in profits. On the other hand, the general level of technology and the size of the labor force are both considered to be positive monotonic functions of the rate of investment.

The next step in the evaluation of this theory might be to determine what these equations imply with respect to the direction of change in each of the variables in the system and hence the direction of change in output. It was argued in the previous section, for example, that an increase in profits, acting through equations three, two, and four, causes a further increase in profits. It was also argued, at the same time, that this increase in profits, acting through equations three, six, five, and four, causes a decrease in profits. In short, these axioms imply that an increase in profits causes both an increase and a decrease in profits.

After further analysis, it was found that the system also implies the same kind of conclusion for any of its other endogenous variables except output. An increase in the variable causes a decrease in the same variable. In addition, an increase in any one of the endogenous variables, except output, leads to both an increase and a decrease in any other endogenous variable. Hence it may lead to both an increase and a decrease in output. As the theory is currently formulated, it is extremely difficult to predict the direction of the change in any one of the endogenous variables given the change in any of the other variables. This would suggest that this particular formulation of the classical theory of economic growth is not particularly useful for explanatory or predictive purposes.

It would appear that there may be two reasons for the inadequacy of the present formulation. The first may be that this formulation does not provide enough information on the nature of the physical and economic relationships involved to render it useful for explanation and prediction. The restatement of the vague relationships discussed by Higgins in terms of almost completely unspecified functional forms does not add sufficient information to render the system capable of prediction or to render it readily amenable to further analysis. The second reason for the inadequacy of the present formulation may lie in its logical structure. The foregoing analysis suggests that the system would imply a simultaneous

increase and decrease in any of the endogenous variables. This suggests that the system may be inconsistent.

Since the question of consistency of the axiom set is altogether crucial to the evaluation of the explanatory and predictive potential of any given formulation of a theory, this question will be examined in some detail below. This examination is limited by the imprecision of the current formulation of the theory, on the one hand, and the applicability of the logical apparatus examined thus far, on the other. Both the theory and the logic are developed with additional precision as the study progresses, but certain methodologically important considerations in the evaluation of theories can be developed with the information already available at this stage of the inquiry.

EVALUATION IN TERMS OF THE LOGICAL STRUCTURE

The logical structure of a theory includes the axioms and the theorems deduced therefrom. The axioms are, for the present at least, assumed to be true in order to determine what theorems are true if the axioms are in fact true. In order for the axioms to be true they must, among other things, be consistent. Hence, one of the criteria for the adequacy of the logical structure of a theory is that the axioms be consistent.

One of the functions of a theory is to provide true theorems which function as the laws or lawlike statements required for the explanation or prediction of the events in

question. The discussion of the Hempel and Oppenheim paradigm of explanation and prediction, presented in Chapter II, listed four criteria for the logical and empirical adequacy of the structure of explanation and prediction. It will be recalled that these criteria include the logical requirements that the explanans imply the explanandum and that the explanans contain at least one lawlike statement and have empirical content. They also include the empirical requirement that the statements in the explanans be true. Since one of the functions of a theory is to provide the lawlike statements required for an adequate structure of explanation or prediction these criteria also function as criteria for the evaluation of theories in general.

A large proportion of the remainder of this study is devoted to an examination, refinement and application of these criteria. This examination proceeds in several stages. Some of the analytical concepts have already been examined. The classical theory, used to illustrate their application, has already been presented. Conclusive analysis requires both the refinement of the tools of analysis and the clarification and specification of the theory under examination. The immediately succeeding analysis employs the information and the tools already available. Subsequent analyses require the refinements of the theory that will be developed in Chapter IV and the refinements in the analytical technique to be examined in Chapter V. The application of these refinements

is made in Chapter VI and Chapter VII of this study.

In the initial approach to the evaluation of the classical axioms, it has already been demonstrated that the requirement that the explanans imply the explanandum has not been adequately fulfilled by this particular formulation of the theory. The other criteria of logical adequacy, that the explanans must contain at least one lawlike statement, and that it must have empirical content would appear to be fulfilled by this theory. Hence, a more general discussion of these criteria can be deferred until a later chapter. This clears the way, then, for an initial assault on the question of consistency in general and the question of the consistency of the classical axioms in particular.

THE CRITERION OF CONSISTENCY

The requirement of consistency is perhaps the most important requirement of the logical structure of a theory and is thought to be one of the most difficult to prove. Its importance is such that the existence of an inconsistency completely destroys the explanatory and predictive potential of that particular formulation of the theory. It will be rigorously demonstrated in Chapter V that the presence of an inconsistency permits the theory to imply anything at all about anything. Hence, it permits it to imply many things which are false as well as many things which are true, and in so doing, destroys the truth preserving characteristics so important to adequate explanation or prediction.

A set of axioms is said to be inconsistent if the negatation of any member of the set can be deduced from this set. Alternatively, a set of statements is inconsistent if a contravalid statement, that is one that denies itself, for example, "p.~p", can be deduced from this set. The logical structure of inconsistency will be further clarified and rigorously defined in Chapter V. Techniques for testing for inconsistencies will be developed in Chapter VI. It will be sufficient for the study of the present formulation of the classical axioms, however, to point out two general approaches to testing for consistency and to apply the one appropriate to the level of formalization currently available.

METHODS OF TESTING FOR CONSISTENCY

The method selected for testing for the consistency of a set of axioms depends in part upon the type of theory in question and the degree of specification or formalization attained. The methods discussed below represent two extremes in their requirements of specification and formalization.

The Method of Deduction

The method of deduction is examined in detail in Chapter VI. Only its broad outlines will be described here. It involves deducing the negation of one of the axioms or a contravalid statement from the axiom set. Success in making such a deduction proves inconsistency. But any number of failures to make such a deduction does not prove consistency. It is always possible that these failures may result from the

inability of the researcher to make a difficult deduction when such a deduction is in fact possible.

Direct application of the method of deduction may be made at any level of formalization from an almost completely unspecified system to one that is completely formalized. The major difference is in the level of formal logic appropriate to the analysis. Most economic theories that have attained a relatively high degree of formalization employ concepts of metricization. Adequate translation of these concepts into a logical calculus usually requires the use of one of the higher functional calculi of formal logic. The use of such calculi generally requiresthe skills of a professional logician. Fortunately, as will be demonstrated in Chapter VI, there is a way of avoiding this kind of arduous analysis by employing a direct proof of consistency. This procedure is based on a classification system developed out of the analysis of propositions and the method of models. The analysis of propositions is developed in Chapter V and the method of models elaborated in Chapter VI. This latter method was introduced in Chapter II. Its direct applicability to this initial formulation of the classical axioms will be considered below.

The method of deduction, discussed above, provides a proof of <u>inconsistency</u>. The method of models, on the other hand, provides a proof of <u>consistency</u>. As in the case of the method of deduction, just discussed, the converse is not true.

The failure to derive such a proof of consistency does not prove inconsistency.

As explained in Chapter II, the first step in the method of models is to adequately exhibit an underlying calculus of the axiom set. The axioms of the classical system constitute only one of many possible interpreations of their underlying calculus. The second step is to give this underlying calculus an interpretation which is different from its current interpretation and which is analytic. An analytic interpretation is one whose truth values can be ascertained by examination of its form and form alone. Examples of such interpreations include a wide variety of mathematical interpretations as well as those, for example, of Boolean Algebra and other analytic systems. It was argued in Chapter II and it will be rigorously demonstrated in Chapter V that success in finding such an analytically true interpretation of an underlying calculus proves the consistency of that calculus.

The problem with the application of this approach to the current formulation of the classical axioms is that it would be extremely difficult to adequately exhibit the underlying calculus of such a system. This difficulty stems from the imprecision with which the axioms are expressed. The application of the method of models will have to await the further formalization of these axioms. This formalization is conducted in Chapter IV.

REFORMULATION OF THE CLASSICAL AXIOMS

Up to this point in the analysis of the classical theory it has been concluded that the current formulation of the theory renders it inadequate for explanatory and predictive purposes. It has also become apparent that it is doubtful whether the current formulation is amenable to fruitful analysis by either the method of deduction or the method of models. The next step, then, is to reformulate these axioms in such a way as to render them amenable to such analysis. The investigation required to specify the axioms with sufficient precision to render them amenable to analysis by the method of models is beyond the scope of this chapter. This investigation will be deferred until Chapter IV. The axioms can be reformulated, however, on the basis of the information already available in the Higgins summary of the theory.[8] This reformulation can be effected with sufficient precision to permit the direct application of the method of deduction. This affords an initial test for the consistency of the system.

Perhaps the most effective way to formulate the information presented by Higgins is in terms of first differences. Let t_1 refer to the base period and t_2 refer to any subsequent period with the restriction that t_1 and t_2 maintain the same time interval for all variables in the system.

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[8]Higgins, loc. cit., pp. 85-99.

The direction of change of each of the variables can then be indicated by the appropriate "greater than" or "less than" signs between subscripted variables. The expression $[I_{(t_2)} > I_{(t_1)}]$, for example, can be used to represent the statement that investment in t_2 is greater than investment in the base period. If investment can be measured in each of these time periods then its correspondence with reality can be tested and its truth value confirmed. These statements can then be combined to form compound statements by the use of the statement connective "⊃" to mean "if....then....", the symbol "." to mean "and", and the symbol "V" to mean "or" as these symbols are employed in the calculus of propositions.[9] Equation two of the axiom set can then be stated as follows:

$$[I_{(t_2)} > I_{(t_1)}] \supset [T_{(t_2)} > T_{(t_1)}]$$

This says that if investment increases between t_1 and t_2 then the rate of application of new technology increases between t_1 and t_2.

If the propositions of the theory were clearly and precisely expressed then there would be little difficulty expressing it in terms of first differences. The problems arise in trying to ascertain the meaning of the propositions from the vague and imprecise manner in which they are stated.

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[9] These terms were introduced in Chapter II and will be rigorously defined in Chapter V.

All that the researcher can do under these circumstances is endeavor to insure that his reformulation is consistent with the spirit and intent of the author.

In the discussion of equation four, for example, it becomes apparent that, for the classicists, the development of capitalist economies was a race between technological progress and population growth.[10] As population grew, diminishing returns would be encountered in agriculture, raising labor costs and reducing profits. But offsetting this tendency was historically increasing returns, especially in industry, through improvement in technique.[11] Higgins' expression for these phenomena is R = R(T, L) which says only that "the level of profits depend on the level of technique and the size of the labor force."[12] This expression is so general that it appears to be always true. By itself it says little more than that profits either rise, fall, or stay constant as a result of changes in the level of technology and the size of the labor force. The statement in this form is a tautology and, hence, is not amenable to empirical test. It is thought that what Higgins is really claiming, however, is that the classicists asserted that profits are a positive function of technology and at the same

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[10]Higgins, op. cit., p. 87.

[11]Higgins, op. cit., p. 91.

[12]Higgins, op. cit., p. 91.

time profits are a negative function of the labor force. Hence, expression four can be written in two statements as follows:

$$4(a) \quad \left[T_{(t_2)} > T_{(t_1)}\right] \supset \left[R_{(t_2)} > R_{(t_1)}\right]$$

$$4(b) \quad \left[L_{(t_2)} > L_{(t_1)}\right] \supset \left[R_{(t_2)} < R_{(t_1)}\right]$$

If these formulations adequately express the meaning of the classical axioms then the classical system can be summarized as follows:

1. $\left[L_{(t_2)} > L_{(t_1)}\right] \cdot \left[K_{(t_2)} > K_{(t_1)}\right] \cdot \left[Q_{(t_2)} > Q_{(t_1)}\right] \cdot \left[T_{(t_2)} > T_{(t_1)}\right] \supset \left[O_{(t_2)} > O_{(t_1)}\right]$

2. $\left[I_{(t_2)} > I_{(t_1)}\right] \supset \left[T_{(t_2)} > T_{(t_1)}\right]$

3. $\left[R_{(t_2)} > R_{(t_1)}\right] \supset \left[I_{(t_2)} > I_{(t_1)}\right]$

4(a) $\left|T_{(t_2)} > T_{(t_1)}\right| \supset \left|R_{(t_2)} > R_{(t_1)}\right|$

4(b) $\left[L_{(t_2)} > L_{(t_1)}\right] \supset \left[R_{(t_2)} < R_{(t_1)}\right]$

5. $\left[W_{(t_2)} > W_{(t_1)}\right] \supset \left[L_{(t_2)} > L_{(t_1)}\right]$

6. $\left[I_{(t_2)} > I_{(t_1)}\right] \supset \left[W_{(t_2)} > W_{(t_1)}\right]$

7. $O = R + W$

This summary should now be amenable to analysis with the use of the calculus of propositions.

PROOF OF INCONSISTENCY

The method of deduction discussed above indicates that a set of axioms is inconsistent if a statement and its negation can be deduced from the same set of axioms. In order to deduce a single proposition from a set of conditionals, it is often useful to assert an antecedent condition. If one were to assert that investment increased from t_1 to t_2 then it follows from this antecedent condition and statements 2 and 4(a) that profits will increase.

2. $\left[I_{(t_2)} > I_{(t_1)}\right] \supset \left[T_{(t_2)} > T_{(t_1)}\right]$

A_1 $\left[I_{(t_2)} > I_{(t_1)}\right]$

$\therefore C_1$ $\left[T_{(t_2)} > T_{(t_1)}\right]$

and;

4(a) $\left[T_{(t_2)} > T_{(t_1)}\right] \supset \left[R_{(t_2)} > R_{(t_1)}\right]$

C_1 $\left[T_{(t_2)} > T_{(t_1)}\right]$

$\therefore C_2$ $\left[R_{(t_2)} > R_{(t_1)}\right]$

At the same time, an increase in investment in conjunction with statements 6, 5, and 4(b) implies a decrease in profits.

$$6. \quad \left[\overline{I_{(t_2)} > I_{(t_1)}}\right] \supset \left[\overline{W_{(t_2)} > W_{(t_1)}}\right]$$

$$A_1 \quad \left[\overline{I_{(t_2)} > I_{(t_1)}}\right]$$

$$\therefore C_3 \quad \left[\overline{W_{(t_2)} > W_{(t_1)}}\right]$$

and,

$$5. \quad \left[\overline{W_{(t_2)} > W_{(t_1)}}\right] \supset \left[\overline{L_{(t_2)} > L_{(t_1)}}\right]$$

$$C_3 \quad \left[\overline{W_{(t_2)} > W_{(t_1)}}\right]$$

$$\therefore C_4 \quad \left[\overline{L_{(t_2)} > L_{(t_1)}}\right]$$

and, finally

$$4(b) \quad \left[\overline{L_{(t_2)} > L_{(t_1)}}\right] \supset \left[\overline{R_{(t_2)} < R_{(t_1)}}\right]$$

$$C_4 \quad \left[\overline{L_{(t_2)} > L_{(t_1)}}\right]$$

$$\therefore C_5 \quad \left[\overline{R_{(t_2)} < R_{(t_1)}}\right]$$

In the event of an increase in investment, the classical axioms imply both an increase in profits, C_2, and a decrease in profits, C_5, over the same time period, t_1 to t_2. Hence, this interpretation of Higgins' statement of the theory would appear to be inconsistent.

CONCLUSIONS

The analysis conducted thus far reveals that this particular formulation of the classical axioms seems to be inconsistent. This renders it useless, in its present form, for explanation or prediction. Finding an inconsistency, however, does not mean that the whole set of axioms must be discarded. Identification and removal, or amendation of one of offending statements might be all that is required to render the theory useful for explanation and prediction. Alternatively, in the case that the inconsistency arises from a contravalid statement the problem could be solved by its removal. Finally, the inconsistency might be removed by changing some of the functions or by adding restrictions which would change the offending relationships in such a way as to make the whole set mutually consistent.

At least two more steps must be taken before much progress can be made in identifying the offending statements and in taking the appropriate remedial action. The first is to specify the relationships in such a way as to determine the nature of each of the equations involved. This specification involves further a priori analysis in terms of economic theory and in terms of empirical evidence. This process is closely allied to the types of things econometricians do in choosing the forms of the functions for statistical analysis. It employs some of the tools of partial formalization examined in Chapter II. This inquiry will be pursued

in the subsequent chapter. The second step is to develop the analytic apparatus required to facilitate the recognition of those characteristics of statements and arguments which render them either acceptable or unacceptable for purposes of explanation and prediction. This inquiry will be pursued in Chapter V. Both of these steps involve the development and use of a number of concepts having widespread significance for research methodology in economics.

CHAPTER IV

TECHNIQUES OF PARTIAL FORMALIZATION AND THE NATURE OF THE CLASSICAL AXIOMS

TECHNIQUES OF PARTIAL FORMALIZATION

The purpose of this chapter is to introduce certain aspects of the application of the techniques of partial formalization with a view to illustrating ways of clarifying a theory in order to render it amenable to a priori analysis. This inquiry is introduced at this juncture to clarify the nature of the functional relationships contained in classical theory.

In Chapter III it was concluded that it would be necessary to clarify the nature of the relationships involved before further attempts could be made to evaluate the theory in question. In Chapter II it was pointed out that this clarity is generally acquired as a theory develops toward full formalization. Formalization involves the explicit development of a theory as a completely elaborated deductive system. As Rudner points out, this goal is seldom attained.[1]

In Chapter III it was also concluded that there was simply not enough information available on the nature of the relationships employed, to render the classical theory

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[1]Rudner, op. cit., p. 5.

amenable to conclusive analysis. Little more can be accomplished without additional knowledge about the nature of the equations employed. One approach to the clarification and specification of these equations, and of similar statements in the social sciences in general, has been proposed by Professor Rudner in the work cited in Chapter II. He proposed the formalization of theories in the manner accomplished for Classical Mechanics. He points out the often observed notion that the ideal of science is not to heap together disconnected bits of information about the universe but to synthesize these into generalizations about reality. The next step is to fit these statements together in the relation of subsumption in order to construct the theories employed in the explanation or prediction of the phenomenon in question. The relationship among the statements of the theory will then approximate, to varying degrees, a complete deduction system. The full formalization of such a theory involves its complete articulation as a deductive system.

Such a system has the advantages that all the axioms and the theorems are made explicit and the rules of logic specified. The explicit statement of the axioms and theorems renders these statements amenable to logical analysis and empirical test. Evidence for or against any of these statements constitutes evidence for or against the theory as a whole. Making the theorems and the axioms of the theory explicit is an extremely important step in the corroboration

of a theory. It should be pointed out, however, that the overwhelming majority of extant scientific theories and especially theories in the social sciences are not at present susceptible to easy or fruitful full formalization.

Nevertheless, there may be substantial gains from the partial formalization of social science theories. The classical theory of economic growth is a case in point. The Higgin's summary already represents such a partial, though extremely elementary, formalization of the classical system. Such formalizations, it should be noted, range all the way from the extreme of almost negligible formalization to an opposite extreme of almost complete elaboration as a deductive system. This range of formalization provides substantial leeway in the extent to which it may be fruitful to formalize.

The extent to which it may be fruitful to formalize depends upon the cost in terms of research time and effort, and the benefits to be derived. At this particular juncture in this study it is fruitful to formalize the classical system to a point where it becomes more readily amenable to the application of the logical tests to be developed in Chapter V and applied in Chapter VI. After these tests are applied it will probably become fruitful to again carry the process of formalization to a more advanced level in order to make some assessment of the empirical significance of the classical theory.

Some of the techniques of partial formalization were introduced in Chapter II. These include the techniques of: (1) systematic presupposition, (2) quasi deduction and, (3) concept formation. The key concepts in each of these techniques will be presented below.

The key concepts in the technique of systematic presupposition includes the notion that many social science theories presuppose large segments of other scientific disciplines or other prescientific areas of "common knowledge" or common sense lore. The presupposition of any such concept or set of concepts assumes that these concepts are true and that therefore deductions based on them are likewise true. It is only after these presuppositions are made explicit that the weight of evidence or of logical analysis can be brought to bear on their evaluation. Evidence for or against these presuppositions becomes evidence for or against the theory itself.

It is a relatively short step from Rudner's approach of systematic presupposition to the slightly more general procedure of adducing explanations for the propositions of the theory in question. The lawlike statements and the antecedents that explain the propositions may themselves presuppose major bodies of concepts. The investigation of these presuppositions may be carried back to any other level of investigation required in order to obtain the evidence necessary for the corroboration of the presuppositions of

the theory at hand.

In the technique of quasi deduction, on the other hand, the central idea is that it may be possible to deduce revealing conclusions from the theory even though these deductions fail to make explicit all of the statements requisite as premises and all the rules of logic employed. The function of these deductions is again to facilitate corroboration of the theory. Several such deductions have already been made in the discussion of the classical system presented in Chapter III.

A third approach to partial formalization involves an examination of the concepts employed. As pointed out in Chapter II, concepts are generally introduced into a theory in any one of three ways: (1) through explicit definition, (2) through specification of a sufficient condition for their use, or finally, (3) as relative primitives. Careful examination of the concepts of a theory may enable the researcher to clarify these concepts, to eliminate unnecessary concepts and to ascertain the measurability of the variables and the empirical significance of the relationships involved. This examination of the measurability of these concepts will be deferred until the later part of Chapter VII when these questions, as they relate to the classical axioms, will be studied in some detail. The techniques of systematic presupposition and quasi deduction will be applied, in an informal way, to the formalization conducted below.

The purpose of the remainder of this chapter is to carry the formalization of the classical system to the point where the probable nature of the functions employed become sufficiently clear that they can, at least for illustrative purposes, be subjected to the additional tests of consistency developed in the ensuing chapter.

THE NATURE OF THE CLASSICAL AXIOMS

In order to clarify the nature of the classical axioms, each of the six basic functional relationships will be discussed below. These relationships are presented in Chapter III. They are intended to represent the Higgin's summary of the classical system. The Higgin's summary is taken as given for the purposes of this discussion with little attention being paid to questions of whether it adequately or accurately represents the hard core of classical thought. Interesting as these questions may be, their consideration at this point would detract from the main stream of inquiry.

The questions at hand concern both the nature of these particular relationships and the general techniques available for ascertaining the nature of the relationships presented in any theory in which the level of formalization is less than desired.

Higgins makes it clear in his discussion of the aggregate production function that he believes that the classicists thought of this function as exhibiting constant

returns to scale and diminishing returns to each of the variable inputs. He also makes it clear that, according to the classicists, the relevant range of this function is that stage in which both the marginal physical product and the average physical product are falling but still positive. Perhaps the simplest type of function to represent these characteristics is a Cobb-Douglas production function in which the sum of the elasticities is equal to one. Hence, the equation $O = a_1 L^{b_1} K^{b_2} Q^{b_3} T^{b_4}$ will be used to represent the aggregate production function of the classical system.

The Higgins' summary is not as clear, however, on the nature of each of the other axioms employed. The second proposition says that the rate of application of new technology is a positive function of the rate of new investment. It is not clear, however, just what type of function this might be. Two of the presuppositions of this relationship are that new technology is capital absorbing and that reinvestment of depreciation reserves is not enough to take full advantage of the steady flow of new technology. This implies that they thought that the y-intercept of this relationship would likely be small or even zero. Considering the slope of the function, there is very little evidence to suggest that they thought that the rate of application of new technology would increase either at an increasing or a decreasing rate as investment increased. In addition, it was assumed that there was always a plentiful supply of new

techniques to be introduced with each new increase in investment. It might therefore be thought that the function would be linear. The general equation for this function would then be $T = a_1 + b_5 I$.

The third basic relationship in the classical system is that investment is a positive function of profits. Among its many presuppositions this relationship assumes that capitalists make investments because they expect to earn profits. Here profit is defined as return to the fixed factors or returns above variable costs. This implies that if they expected profits to be zero, they would refrain from investing. At the same time, it is to be expected that depreciation would continue. This implies that the combined effects of depreciation together with the absence of new investments could result in a net disinvestment when profits were zero. This suggests that the y-intercept may be negative. At the same time, there is little suggestion that the classicists thought that investment increased either at an increasing or a decreasing rate as profits increased. It may be convenient to conclude, therefore, that the relationship could be expressed in general form as follows: $I = a_3 + b_6 R$.

The fourth basic relationship in the classical scheme is that profits are a positive function of the rate of application of new technology and a negative function of the labour force. This relationship was discussed and treated

as two separate equations in Chapter III. If a_4 is permitted to take on any value then this relationship may well be summarized in the following equation: $R = a_4 + b_7T - b_8L$.

The fifth axiom in the classical system said that the labour force was a function of the wage fund. This presupposes the concept of an economic limit on family size. It assumes that there are no checks on the size of working-class families except the amount of wages available to them and the number of children that can subsist on those wages. If the wage fund were very low for a given economy, it would be expected that the labour force would be small even though a sizeable population may be supported in the subsistence sector. If the subsistence wage is assumed to be constant, then the equation $L = a_5 + b_9W$ may be used to represent this iron law of wages.

The final axiom in the classical scheme held that the wage fund was a positive function of the level of investment. This assumes that the wage fund is some fraction of the amounts of money being invested. Since there is little evidence that this fraction increased or decreased as investment increased, it would appear reasonable to assume that an equation of the form $W = a_6 + b_{10}I$ might adequately express the wage fund doctrine.

These six equations, together with equation seven, represent a higher level of formalization than the original Higgins' presentation. This formalization will form the

basis of the inquiry to be developed in Chapter VI. It should be noted, however, that this summary represents not only a relatively elementary level of formalization of the system, but also one that is highly tentative. There is simply not enough information presented in the Higgins' summary of the classical system to render it amenable to definitive formalization even at the level presented above. The same charge can be made against a wide variety of putative theories of economic growth, and against a wide variety of social science theories in general. Nevertheless, some additional clarification of these concepts can often be obtained by adducing some of the premises from which these axioms may have been deduced, and deducing some of the theorems that these axioms imply. Evidence for and against these premises and theorems provide additional knowledge of the shapes of the functions employed. This knowledge, tentative as it may be, provides an example to which the analytic techniques developed in Chapter V can be applied. Further efforts at partial formalization -- effort drawing more heavily on the ideas of concept formation, and the measurability of the concepts employed, will be conducted in Chapter VII.

CHAPTER V

METHODS OF ANALYSIS OF PROPOSITIONAL FUNCTIONS AS A MEANS OF EVALUATING THE EXPLANATORY AND PREDICTIVE POTENTIAL OF THEORIES

INTRODUCTION

The purpose of this chapter is to formulate a systematic approach to the evaluation of theories. This approach is a direct outgrowth of the concepts presented in Chapter II. Some of these concepts, the method of deduction and the procedures of model construction, were further elaborated in Chapter III. The method of deduction was then applied to one particular formalization of the classical theory of economic growth. It became apparent from this application that unequivocal results were not likely to be obtained without further refinement of both the theory itself and the logical framework employed. Chapter IV was devoted to certain aspects of the further formalization of the theory. The current chapter will be devoted to the further development of the analytical framework required.

The additional framework of analysis developed in the current chapter together with the more fully formalized version of the theory presented in the previous chapter provides the basis for the analysis conducted in Chapter VI. In that chapter the additional framework of analysis is

applied to the more completely specified theory in order to illustrate the way these techniques may be employed in the evaluation of the classical axioms. This illustration provides some basis for consideration of the analytical potential of these techniques in the a priori evaluation of theories in general.

In Chapter II theories were defined as systematically related sets of statements, at least one of which was law-like and which was empirically testable. The concept of systematic relatedness was defined as deductive relatedness, or the relationship of deductive subsumption. Examination of the relationship of deductive subsumption suggested the use of models as a means of theory evaluation. This method was limited in its application to the examination of the classical theory because of the relatively low level of formalization attained and because of the difficulty of finding another theory exhibiting sufficient similarity of structure. Since these difficulties are likely to arise for a very large proportion of extant theories, it becomes important to go beyond the examination of models or sets of statements to a more detailed examination of the individual statements themselves. This examination will be referred to as the analysis of propositions. This is the analysis that will be developed in the current chapter and applied in Chapter VI.

PROCEDURE

The procedure for the development of this new approach to the evaluation of theories is as follows. A distinction is drawn between the object-language of science and the meta-language in which science is discussed. Then the statements comprising the object language are analyzed with respect to form and content. The statement forms which are thus derived are then subjected to truth tabular analysis and classified according to the manner in which their truth values are ascertained. This provides a mutually exclusive and exhaustive classification of statement forms, and, hence, with certain reservations, of statements themselves. In addition, provided these statements can be adequately translated into a formalized language, it affords a procedure for ascertaining the classification to which any statement belongs. The effect of each of these types of statements on the logical structure of theories, and on the empirical content of theories can then be investigated. This investigation provides insights into some of the conditions that must be fulfilled by an empirical theory in order for it to be useful for explanation and prediction. It also provides a means of testing for the presence or absence of these characteristics in theories in general.

THE STRUCTURE OF THE PROPOSITIONS OF SCIENCE

As was pointed out in Chapter II, the product of

science is a set of linguistic entities called declarative sentences. This set can be referred to as the object-language of science. The object-language is to be distinguished from the meta-language--the language in which methodologists discuss the language of science. In the meta-language, scientists employ any kind of sentence, but the object-language is limited to the use of declarative sentences alone.

An examination of the propositions of science, then, comes to focus on the nature of the declarative sentences employed. This examination will concentrate first on the nature of single statements taken by themselves. In a later section these statements will be examined as members of groups of statements constituting axiom sets or theories. Restricting this initial investigation to <u>single</u> statements does not limit its applicability to <u>singular</u> statements, however. Indeed, attention is finally focused on compound statements or sets of statements of any degree of complexity. This specification of the scope of the investigation limits the application of the term "any statement", in the ensuing paragraphs to simple and complex declarative sentences as they function in science.

If the investigation is limited to what is sometimes called the 'molecular' considerations, that is, those which a statement has by virtue of its sentence connectives, then one of the first steps in an examination of the statements of

science is to distinguish between their form and their content. The next step is to abstract out from all the contexts in which they ordinarily occur, the forms of the statements under examination. This is accomplished by replacing the propositional constituents in the statement with letter variables like "p", "q", or "r", and by replacing the sentence connectives by appropriate symbols like "$\sim$", ".", "v", or "$\supset$" which are also called sentence connectives and which read "not", "and", "or", and "if ... then ..." respectively.

The scope of this investigation will be limited to the use of statements as substitution instances for variables and to the use of the four logical operators listed above. These restrictions are added in order to keep the logical apparatus at a minimum but nonetheless adequate for the development of those concepts that later become useful in the evaluation of theories. These restrictions will gradually be removed in order to include the examination of mathematical functions as the investigation proceeds.

Statement forms are constructed by combining variables, sentence connectives and parentheses into certain allowable patterns.[1] The variables "p" and "q" and the connective "$\supset$" can be combined to form the allowable

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[1]An intuitive understanding of the term "allowable patterns" is sufficient for present purposes. A more precise notion of this concept is available in the description of the role of the formation rules and well formed formulae in the section on the formalization of theories which was presented in Chapter II.

pattern "p⊃q". This is a statement form. It becomes a statement when "p" and "q" are replaced by statements. For example, if "p" were replaced with the statement "Investment increases", and "q" were replaced with "Output increases," then this statement form would yield the statement "If investment increases then output increases." This process of synthesizing a statement out of an allowable combination of logical operators and variables illustrates the construction of one type of sentence in science. What is more important for the purpose at hand, however, is that it is also illustrative of the manner in which a statement may be analyzed, and its form abstracted out of the context in which it occurs. The key to this process is the recognition of the logical words employed. The other words of the statement determine the content. If one can recognize the logical words, it is a relatively easy matter to abstract the form from the content. It must however, be recognized that any given statement may have many forms and that the major problem in this type of analysis is to provide an adequate translation into an appropriate calculus.[2]

At least one of the logical connectives "and", "or",

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[2]The concept of an 'adequate' translation or analysis refers to the selection the relevant structure, or, a relevant structure, from the many possible forms into which a statement may be analyzed. The relevant class of forms will be determined in part by the problem at hand. If the problem is to ascertain the truth functional characteristics of a statement, for example, then the analysis would be concerned, as a minimum, with selecting a statement form from one or the other of the three mutually exclusive classes of statement forms developed below.

"not", "if ... then ..." is involved in every compound statement. If this list were complete at least one of the words in the list would be involved not only in compound statements but in every statement. A form of any given statement can be exhibited by replacing the logical words by the appropriate symbols and by uniformly replacing the smallest part of the statement which is a sentence by letter variables. This permits the methodologist to examine the form of a given proposition without being hindered by the special subject matter of the unlimited number of propositions which exemplify that form. For example, one form of the statement "Investment rose or the labour force declined" can be exhibited by replacing the logical operator "or" with "v", the proposition "Investment rose" with "p" and "The labour force declined" with "q" to obtain the statement form "pvq". This is a way of abstracting the form from any number of statements which may be substitution instances of that form. This is, however, only one of the forms that such an assertion may have. Any assertion may exhibit a number of different forms.

It should be noted that some restrictions have to be placed on the kinds of substitutions permitted in given statement forms. These restrictions may be imposed intuitively by limiting the range of values or field of application to those things the names of which can be significantly substituted. That is, it can be limited to those

things which yield something meaningful upon substitution. This sense of meaningfulness is usually conditioned by the context in which the statement is understood.

Sufficient logical material has now been presented to permit the definition of certain methodologically fundamental terms. A propositional function is a linguistic entity which involves at least one variable and for which, when all variables are replaced by the names of appropriate values, a proposition results. Thus, "$p \supset q$", "$2x = y^2$", and "$p.q$", are all propositional functions. They all have at least one variable and yield a proposition when the variables are specified. They differ only in the restriction placed upon the variables involved. They have no truth value in and of themselves. They become true or false propositions on the basis of the variables they take on as substitution instances.

A formal function can now be defined as a propositional function which has been abstracted to the extent that it is constructed out of only formal concepts. It follows, then, that the form of a proposition or a statement form is a formal function obtained when all the non-formal concepts are replaced by appropriate variables. That is, a statement form is the form that the particular statement under examination exemplifies.

THE NATURE OF THE LOGICAL TERMS, AND THEIR ROLE IN THE DETERMINATION OF TRUTH VALUES

The variables in the statement forms mentioned above, can take on a wide range of values, and any number of interpretations, within the ranges specified. The sentence connectives take on one and only one interpretation, however. It is the meanings assigned to these symbols that now come under investigation.

Paramount among the concepts of interest to the methodologist is the concept of truth. Truth, as used here, is limited in its extension as a predicate of propositions which are the abstract entities indicated by sentences.[3] Statements are substitution instances of statement forms. It is the logical operators within these statement forms together with knowledge of the truth values of the constituent propositions that determines the truth values of a large class of statements--statements which are substitution instances of one particular class of statement forms. These classes of statement forms will be discussed in detail in a later section. The current discussion of logical operators and their particular role in determining truth values of statements forms is a prerequisite to much of the following investigation.

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[3]This study is concerned with truth as a predicate of statements. A statement is true or false accordingly as the proposition it expresses is true or false.

Each of the four types of logical operators presented above will be discussed below. The negation sign "~" is usually read "it is not the case that ..." or, "it is false that ...". Hence, "~ p" would read "p is false". If "p" were to take the statement "output increased" as a substitution instance, the whole statement would read "It is false that output increased". Now, if it were concluded upon investigation that output did in fact increase then the statement "~ p" is false. If it were ascertained that it is false that output increased then the statement is true. Hence, a contradictory statement form takes on truth values opposite to the truth values of the constituent proposition. This is illustrated in the first table presented in Appendix A.

It should be noted in passing that the contradictory statement form is so named because it contradicts another statement. It should not be confused with the self-contradictory form "p.~ p" which, as its name implies, contradicts itself. It should also be noted in passing that while it is only statements which are substitution instances of given statement forms that can properly be called true or false, it is often more convenient to talk about the truth value of the statement forms themselves. This convention will be followed hereafter whenever it is convenient.

The conjunction sign "." is usually read "and". It asserts the joint truth of both of the constituent propositions,

regardless of how complicated, in the statement form "p.q". For this kind of statement to be true, both of the constituent propositions must be true. Otherwise, the statement is false.

The disjunction sign "v" is usually read "or". It asserts that at least one of the constituent propositions in the statement form "pvq" is true. Hence, if both are false the statement is false. Otherwise, the statement is true.

The conditional sign or horseshoe "⊃" is usually read "If ... then ...". It asserts that it is true that if the first proposition in the statement form "p⊃q" is true then the second proposition of that statement form is also true. The statement is true for all possible combinations of truth values except when the first proposition is true and the second is false.

A more detailed analysis of each of these logical terms or statement connectives is presented in Appendix A. This analysis provides an introduction to truth table procedures. These procedures are then discussed in relation to other procedures and selected as the technique to be employed in ascertaining the truth value characteristics of alternative types of statement forms. The analysis presented in Appendix A provides a means of unequivocally determining the truth value characteristics of alternative types of statement forms. This in turn provides an unequivocal means of classifying statement forms and hence statements

themselves on the basis of their truth value characteristics. (See Appendix A).

On the basis of the analysis conducted in Appendix A it becomes apparent that all statements which can be adequately translated into the calculus of propositions can be classified as tautological, contingent or inconsistent. It is not yet clear, however, whether this classification is exhaustive. This point becomes clearer with the development of a new classification of statement forms--a classification which is also based on the truth value characteristics of statement forms and made unequivocal by the use of truth table procedures. This new system of classification and identification provides a broader basis for the analysis of the role and the effect of each of these three types of statement forms in the structure of science. It becomes part of the framework for bridging the gap between the analysis of individual statement forms and the development of criteria and techniques for the evaluation of the explanatory and predictive potential of theories in general.

A GENERAL CLASSIFICATION OF STATEMENT FORMS BASED ON THEIR TRUTH VALUE CHARACTERISTICS

This classification is built around two pairs of characteristics of statement forms, both of which are dichotomies. Every statement form is either <u>valid</u> or <u>non-valid</u>, but not both. Every statement form is either <u>consistent</u> or <u>inconsistent</u>, but not both. Further, every

statement form, and hence every statement has exactly two of these characteristics.

It should be emphasized at this juncture that the term "valid" is used in two entirely different senses in this study. In the present context it is used as a predicate of statement forms and might be more adequately expressed as truth functionally valid. In a later section it is used as a predicate of forms of inference. In both cases, the sense in which it is used can readily be ascertained from the context in which it is employed. Failure to make this distinction would inevitably lead to confusion about the role of valid statement forms in science.

The concepts valid, non-valid, consistent, and inconsistent are defined and illustrated below. Two other distinctions are required before proceeding, however. The first has to do with the formal notion that the truth values of statements can be ascertained on the basis of an examination of their form and form alone. This notion comes to bear at two points below. The first is in the classification of statement forms. This was accomplished in Appendix A. It was based on an examination of form alone. The next is in the establishment of the truth values of individual statements and arguments. The truth values of some kinds of statements can be ascertained on the basis of their form alone. For others, it cannot be accomplished without a knowledge of both form and content. It is in the first of

these two senses that this concept is applied to the taxonomical processes developed below.

The other distinction has to do with the kinds of substitution instanceswhich are permissable. In a previous section it was pointed out that the scope of the variables in a statement form is limited to those that "make sense", or those which can "meaningfully" be substituted. It is readily apparent that if the substitution instances of a contingent statement form are not meaningful, then the characteristics of truth or falsity could never be ascribed to these components, and the truth value of the statement could not be ascertained. In the case of tautologies and inconsistencies, on the other hand, truth value can be ascertained on the basis of examination of the form alone. This does not imply, however, that the substitution of nonsensical statements for the variables in a tautology would generate a true statement, or that the same procedure applied to an inconsistency would generate a false statement. In these cases, as in the case of contingent statements, the predicates truth and falsity can only be applied to statements which are meaningful. Hence it becomes important, throughout the remainder of this investigation, to limit the scope of substitutions to those which take on meaning within the context in which they are employed.

CHARACTERISTICS OF STATEMENT FORMS

The concepts of validity, non-validity, consistency

and inconsistency can now be defined and developed below:

1. A statement form is valid if every substitution instance which has that form is true. For example, every substitution instance of the form "$pv\sim p$", as demonstrated in Appendix A, yields statements which are true. These statements are true regardless of the truth values of their constituent statements and hence, regardless of their empirical content. There are infinitely many of these valid statement forms, among the most common of which are: "$pv\sim p$", "$\sim(p.\sim p)$", "$p\supset p$", "$(p.q)\supset p$". These and other valid statement forms will be examined in Appendix B.

2. A statement form is non-valid if at least one substitution instance which has that form is false. For example, the form $p\supset q$ has as a substitution instance, "If employment increases then output will fall." This statement may be true or it may be false. This becomes apparent from examination of the truth tabular analysis of its form alone. Whether a given statement which is a substitution instance of that form is true or whether it is false, however, depends not only on its truth tabular analysis, but also on knowledge which may require empirical investigation of the truth values of the component statements "p" and "q".

3. A statement form is consistent if at least one substitution instance which has that form is true. For example, the form "$p\supset q$" has as a substitution instance, "If

employment increases then output will increase." This statement may also be either true or false. This becomes apparent from examination of the truth tabular analysis of its form alone. Whether a given statement which is a substitution instance of this form is true or whether it is false, however, may depend not only on the truth tabular analysis but also on empirical knowledge of the constituent statements employed.

4. A statement form is inconsistent if every substitution instance which has that form is false. For example, the statement form "p.~ p" has as a substitution instance, "An increase in investment causes an increase in output, and it is not the case that an increase in investment causes an increase in output." This is always false. This becomes apparent from examination of the truth table analysis of its form alone. Whether a given statement which is a substitution instance of this form is true or false is therefore also apparent from examination of the form alone, with no need for empirical investigation.

CLASSIFICATION OF STATEMENT FORMS

It was stated at the beginning of this section that every statement was either consistent or inconsistent but not both and that every statement was likewise either valid or non-valid but not both, and further, that every statement had exactly two of these characteristics. The foregoing definitions in terms of truth characteristics permit the

classification of statement forms into the following categories:

1. A statement form may be consistent and valid. To be consistent means to be capable of being true or to be sometimes true. To be valid means to be always true. Hence, a statement form which is consistent and valid has at least one instance which is true and further, all instances are true. In this case, the weaker of the two conditions falls within the scope of the stronger. It follows that the truth characteristics of statement forms which are valid and consistent are ascertainable by examination of the form and form alone and that they are always true and hence tautologous.

2. A statement form may be consistent and non-valid. As above, consistent means to be capable of being true. Non-valid, on the other hand, means to be capable of being false. Hence, a statement which is consistent and non-valid is capable of being true and capable of being false. The truth values of statements which are substitution instances of these statement forms are contingent upon the truth values of the constituent propositions. The truth value of the substitution instance cannot be ascertained without recourse to empirical investigation. Hence, a statment form having this pair of characteristics is synthetic, logically indeterminate, or contingent.

3. A statement form may be inconsistent and non-valid.

As above, non-valid means to be capable of being false. Inconsistent, on the other hand, means to be always false. Hence, a statement form which is non-valid and inconsistent has at least one substitution instance which is false; further, all substitution instances are false. Here again the weaker condition falls within the scope of the stronger and the truth value turns out, upon examination, to be ascertainable on the basis of form and form alone and also to be always false and hence inconsistent.

The two pairs of characteristics, validity and non-validity and consistency and inconsistency, can be considered in only four ways, the three discussed above and finally, the combination valid and inconsistent. The first characteristic of this pair means always true and the second means always false. Since any given statement must be either true or false but not both, there are no statement forms having both of these characteristics simultaneously.

It follows that since every statement form is either valid or non-valid and consistent or inconsistent and since each statement has exactly two of these characteristics, there are exactly four pairs of these characteristics. One of these pairs is incompatible. Hence, the three pairs of characteristics listed above are exhaustive.

Their truth characteristics permit one to ascertain whether or not they are also mutually exclusive. According to the truth tabular analysis, statement forms which are

valid and consistent are always true; those that are non-valid and consistent are sometimes true and sometimes false, while those that are non-valid and inconsistent are always false. It is not possible for any given statement form to have any more than one of these three possible sets of truth values. Hence, each of the sets within this classification is mutually exclusive.

This mutually exclusive and exhaustive classification of statement forms provides a mutually exclusive and exhaustive classification of statements provided the statements are adequately analyzed into their appropriate statement forms. The characteristics of being valid and of being inconsistent are dominant characteristics. Hence, a statement which is a substitution instance of a valid statement form is valid. Similarly a statement which is a substitution instance of an inconsistent statement form is inconsistent. The characteristics of being non-valid and of being consistent, on the other hand, are recessive to their stronger counterparts. Hence, a statement is said to be consistent only if every statement form of which it is a substitution instance is consistent. Similarly, a statement is said to be non-valid only if every statement form of which it is an instance is non-valid.[4]

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[4]It should be remembered, however, that there are other classes of statement forms in addition to the truth-functional statement forms and that this analysis is limited to those statement forms which are truth functional.

This classification of the statements of science, based on the analysis of their underlying statement forms, is designed to facilitate the analysis of the role of each of these kinds of statements in science. This analysis will be conducted below. This analysis, in turn, is designed to facilitate the evaluation of the explanatory and predictive potential of economic theories a priori.

THE IMPORT OF THE STRUCTURE OF PROPOSITIONS IN THE STRUCTURE OF SCIENCE

INTRODUCTION

Thus far the truth functional properties of the statements of science have been analyzed and examined. The next step is to examine the role of each of the three types of statement forms in the structure of theories. This offers a systematic approach to the evaluation of the explanatory and predictive potential of theories a priori. It enables the specification of some of the characteristics requisite of theories. It provides a method for testing the extent to which some of these characteristics are fulfilled in any given theory.

Prior to proceeding with this investigation, it is important to distinguish between two elements of empirical science: (1) the axioms and theorems in which the empirical assertions of the science are expressed, and (2) the rules of logic, or the transformation rules, by which the theorems are derived from the axioms. It has been traditionally argued

that the axioms summarize, at least in part, the results of the inductive processes of science. The application of the rules of logic on the other hand, constitute, at least in part, the deductive processes involved in science. The first aspect will be referred to as the empirical assertions of science and the second as the logical structure of science.

The need for this distinction arises out of the need for one type of statement form in the inductive processes and a different type to provide the rules employed in the deductive processes. As will become apparent in the ensuing paragraphs, one particular type of statement form fulfills part of the all important role of providing the valid forms of inference which comprise the logical framework of science. The presence of these same statement forms to the exclusion of other types of statement forms in the axiom set, on the other hand, is enough to render the axioms vacuous. Failure to make the distinction between the inductive and the deductive aspects of science leads to confusion about whether some of the assertions being made about the nature of statement forms are limited to statement forms, or whether they can be exteneded to cover forms of inference as well. Once this distinction is made, and as long as the use of the term valid as a predicate of statement forms is distinguished from its use as a predicate of forms of inference, then much of the possible confusion arising over discussion of the roles

of valid statement forms tends to disappear.

THE ROLE OF VALID STATEMENT FORMS, OR TAUTOLOGIES, IN THE LOGICAL STRUCTURE OF SCIENCE

It has previously been established, with the aid of the truth table procedures, that the statements of science can be divided into three mutually exclusive and jointly exhaustive categories on the basis of their truth value characteristics. Statement forms which always yield false statements regardless of the truth values of the statements substituted for the variables are generally called inconsistencies and more accurately described as forms which are non-valid and inconsistent. Those which sometimes yield a true statement and sometimes yield a false statement, depending on the substitution instances employed, are generally called contingent but more accurately described as non-valid and consistent. Those which yield true statements regardless of the substitution instances employed are generally called tautologies and are more accurately described as valid and consistent statement forms. In the first and last classes of statement forms, the weaker of the two characteristics falls within the scope of the stronger so that the first may be referred to simply as inconsistent and the last as valid, or tautological.

It is this latter group of statement forms which is of major concern in a study of the logical structure of theories. For, as Ambrose puts it, it is "Out of these

latter the deductive system is constructed".[5] It is these forms that provide a major portion of the laws of logic. It is these laws of logic which correlate the pieces of empirical information expressed in the contingent statement forms. They permit the researcher to gain new knowledge about the real world by permitting him to deduce what else will be true about reality if the contingent statements in the axiom set are in fact true.

The details of the role of valid statement forms in the logical structure of science are developed in Appendix B. It need only be pointed out at this juncture that the valid statement forms together with a number of other logical concepts fulfill the indispensable role of providing valid forms of inference.

This indispensable role of the valid statement forms in the logical structure of science is to be clearly distinguished from the vacuous effects of the exclusive use of these statement forms in attempting to express the empirical assertions of science.

THE EFFECT OF VALID STATEMENT FORMS IN THE EMPIRICAL ASSERTIONS OF SCIENCE

In order to more clearly delimit the role of the valid statement forms in science, their effect on the empirical aspects of science will be examined below.

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[5]Alice Ambrose and Morris Lazerowitz, Fundamentals of Symbolic Logic, (New York: Holt, Reinhart and Winston, Inc., 1962), p. 18.

It should be remembered, from Chapter II, that the major role of theories in science is to provide the law-like empirical statements used in explanation and prediction. These statements, together with statements of the antecedent conditions, permit the explanation and prediction of events in reality. These empirical assertions are substitution instances of contingent statement forms.

Consider the contingent statement form "p⊃q", for example. In a theory of economic growth, this statement form might take as a substitution instance the statement, "If investment increases, then population will expand." If this is a true statement and if, in addition, investment does increase, then it is possible to deduce that output will expand. This deduction provides new knowledge about reality without having to observe it directly. It provides evidence that would not be available, except by observation, unless it was implied by one of the statements in the theory itself.

The effect of the counterpart valid statement form, "p⊃p", is quite different. Suppose, for example, that the statement "Output increases" were substituted for "p". Then the statement of the form "p⊃p" would say, "If output increases, then output increases". If this statement were to be used in order to deduce anything about reality, it would be necessary to assert the antecedent condition "p" which says that output increased. The conjunction of the assertion "Output increases", together with the assertion "If output

increases then output increases" would imply that output increases. The problem is that this provides no new information about reality. In order to deduce the conclusion it is necessary to know the truth of the antecedent in the first place. Since the antecedent of a valid conditional form always implies its consequent it cannot provide any new information about reality.

Further examples of pairs of contingent and valid statement forms and individual valid forms are examined in Appendix C. This examination leads to the conclusion that when the valid form is employed in an attempt to make an empirical assertion, the results are trivial. Even though both the proposition and the antecedent are known to be true, and the reasoning is demonstrably valid, the conclusion about reality necessarily turns out to be trivial. It only concludes what had to be asserted in the first place, in order to make the deduction from the valid statement form. These statements are obviously true but pointless and trivial in terms of the information they convey. The whole point is put into its proper perspective by Ambrose. She points out that they are trivial only from the point of view of what they <u>say</u>, however, not from the point of view of what they <u>imply</u>, that is, not from the point of view of the rules of deductive logic they provide.

The thesis of the present section has been that these statement forms which are valid and consistent are

trivial in terms of what they say, that is, in terms of their empirical content. The thesis of the foregoing section was that these same valid statement forms were anything but trivial in terms of what they imply. In fact, they provide a major proportion of the laws of logic which underly the structure of scientific inquiry.

It would seem to follow, therefore, that one of the problems in theory evaluation is to be able to determine whether or not a given statement form is a tautology. If it is, then the next problem is to determine its role in the theory. If it is used as a rule of inference in a theory, and if it is used in conjunction with one or more contingent statement forms, then it may perform a very useful function in explanation and prediction. On the other hand, if the statement of the theory which is a substitution instance of this particular statement form is intended to convey some emperical information and if all other statements in the theory are also substitution instances of tautologous statement forms, then that theory is vacuous.

THE EFFECT OF INCONSISTENT STATEMENT FORMS IN THE LOGICAL STRUCTURE OF SCIENCE

The foregoing sections have established a small part of the groundwork required for the examination of the formal structure of theories. They divide the statements of science into three mutually exclusive and exhaustive categories (tautologies, inconsistencies and contingent statements) on

the basis of the truth characteristics of their statement forms. They examined the indispensable role of tautologies in the logical structure of theories and their vacuous effect when employed, to the exclusion of other statement forms, in the axiom set. The next task is to examine the effects of inconsistencies in the logical structure of theories and as members of the axiom set.

It will be recalled from the foregoing introduction to truth functional analysis of statement forms that a statement which is a substitution instance of an inconsistent statement form is always false regardless of the truth or falsity of its constituent components. Statements which are tautologous, on the other hand, are always true. It follows that inconsistencies are the negations of tautologies. Examination of the statement forms $\sim(p.q \supset p)$, $\sim((p \supset q).p \supset q)$, and $\sim((p \supset q).\sim q \supset \sim p)$, for example, will help to establish this point. Each of the foregoing statement forms is the negation of a tautology. Each of them becomes an inconsistency when the tautology is denied. This is true for any tautology.

This conclusion is also true for valid forms of inference. A tautologous chain of reasoning claims that if the premises are true the conclusion is true and this is found upon analysis to be true. Its self-inconsistent negation, on the other hand, claims that it is false that, for a tautologous function, if the premises are true then the conclusion is also true, and this is found upon analysis to

be false. Hence, just as a tautology is an instance of a correct form of reasoning so an inconsistency is an instance of an incorrect form of reasoning. It is, therefore, of major concern to the methodologist to be able to recognize inconsistencies in the logical structure of theories. A technique designed to facilitate the recognition of inconsistencies in partially formalized theories will be developed in Chapter VI.

THE EFFECT OF INCONSISTENT STATEMENT FORMS ON THE EMPIRICAL ASSERTIONS OF SCIENCE

Inconsistencies can exist within given statements in the axiom set, but even if none of the individual axioms are inconsistent, the conjunction of two or more of them in the axiom set may constitute an inconsistency. It is the consequences of finding an inconsistency in either of these ways that is of concern here.

The consequences of inconsistencies have been succinctly summarized by Tarski who said that as soon as we succeed in showing that an empirical theory contains (or implies) false sentences, it cannot be any longer considered acceptable ...A well known logical law shows that a theory which enables us to derive two contradictory sentences enables us also to derive every sentence; therefore, such a theory is trivial and deprived of any scientific interest. Ambrose makes this more specific by saying:

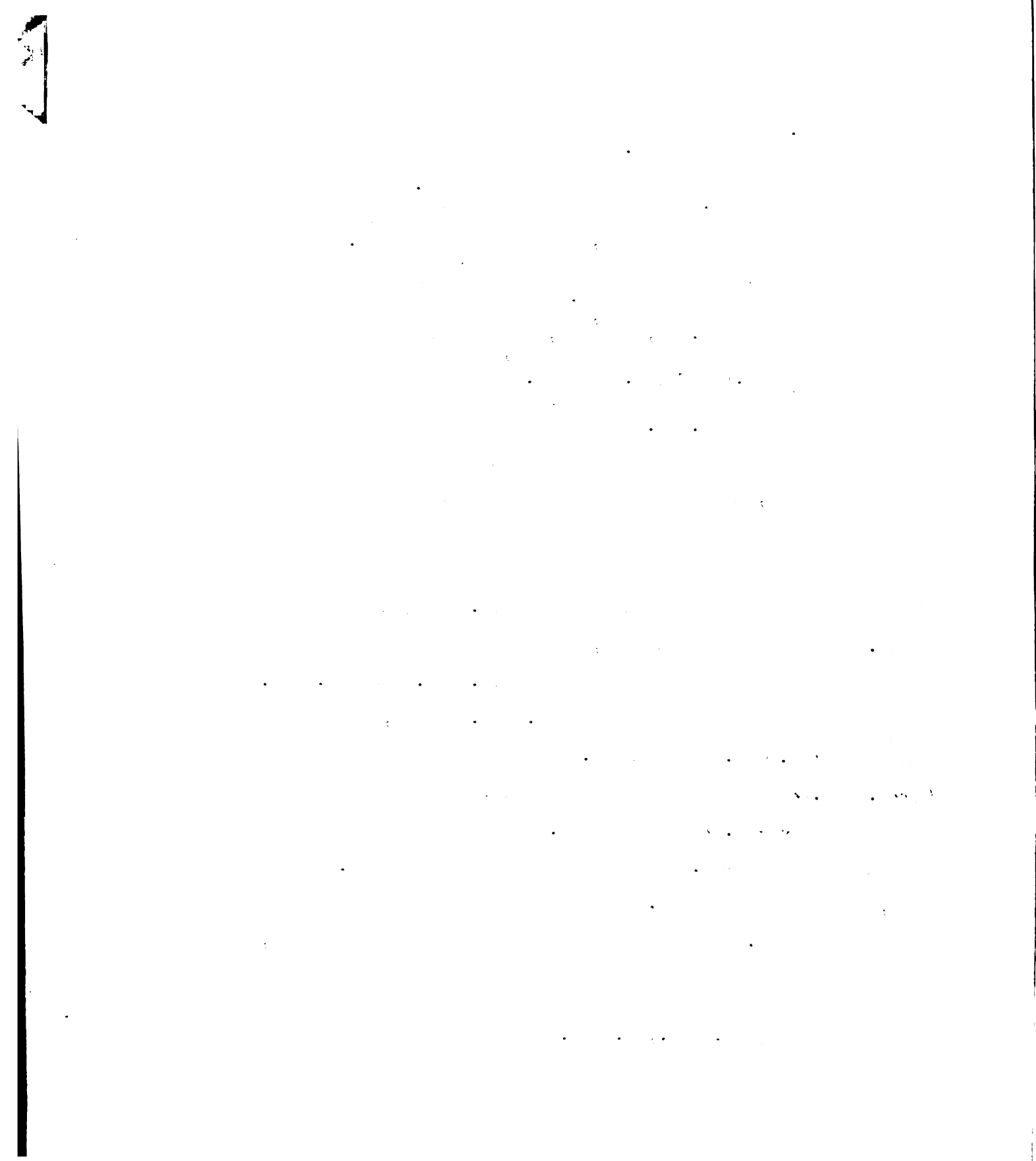
> "Every deductive system is such that there are properties which it is either necessary or desirable for a set of postulates to

> have. First and foremost is the requirement of consistency. It is an essential property of any set of postulates that from them no proposition of the form $p.\sim p$ be derivable. Unless it is impossible to prove both a proposition and its contradictory within the system, the set is worthless. This follows from the logical fact, already remarked, that a false proposition implies any proposition whatever. Given that the set contains both p and $\sim p$, and hence the false proposition $p.\sim p$, we have, as special instances of that logical fact,
>
> $$\sim(p.\sim p)\supset((p.\sim p)\supset q).$$
>
> Any proposition q is implied by the false proposition $p.\sim p$."[6]

The reader who is interested in actually proving this last proposition, and hence in seeing precisely why it is that the existence of an inconsistency in the axiom set permits it to imply anything at all might proceed as follows: Starting with two tautologies $\sim(p.\sim p)$ (tautology) and $(p.\sim q)\supset p$ (simplification), and following the rules of uniform substitution one can derive $\sim(p.\sim p).\sim q\supset\sim(p.\sim p)$. But the theory in question implies $(p.\sim p)$. Hence, by modus tollens $\sim(\sim(p.\sim p).\sim q)$ follows. But this implies $\sim(\sim q.\sim(p.\sim p))$ by the commutative laws, which in turn simplifies to $\sim q\supset(p.\sim p)$ by duality. This together with the familiar tautology $\sim(p.\sim p)$ implies q by modus tollens. Hence, the existence of $(p.\sim p)$ in a theory permits that theory to imply q. Since q can be any proposition whatsoever, the theory can then imply anything whatever about reality

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[6] Ambrose, op. cit., p. 148.



without any regard for the truth or falsity of the assertion made. Thus, a theory of economic growth, if it contained an inconsistency, might imply that Rome is a city in New South Wales or that G.N.P. in Ghana is a function of the number of kangaroo in Australia. In short, the presence of a statement form which is non-valid and inconsistent in the empirical context of a theory renders that theory entirely useless for explanatory and predictive purposes.

It follows from the foregoing discussion, that the consequences of an inconsistency in either the deductive or the inductive parts of a theory are extremely damaging. The major concern of this chapter is with the logical apparatus required in order to enable the researcher to recognize the presence of inconsistencies in partially formalized theories.

EFFECT OF CONTINGENT STATEMENT FORMS IN THE LOGICAL STRUCTURE OF SCIENCE

It will be recalled that the non-valid and consistent statement forms, that is the contingent statement forms, take on substitution instances which yield true statements for some combinations of truth values of the component statements, and false statements for others. These statement forms cannot function as rules of correct thinking. Nor are they examples of incorrect reasoning. In short, they do not function in the deductive structure of theories at all. Their role is to convey the empirical content of science.

EFFECT OF CONTINGENT STATEMENT FORMS ON THE EMPIRICAL ASSERTIONS OF SCIENCE

From the section on the effect of valid and consistent statement forms, that is, tautologies, on the empirical content of theories it becomes apparent that, on the basis of what they say, as averse to what they imply, their presence adds little to the empirical content of a theory. From the section on the effect of non-valid and inconsistent forms, that is, the inconsistencies, on the empirical content of theories, it becomes apparent that they say anything at all, true or untrue, relevant or irrelevant, and, as a result, render a theory useless for explanatory and predictive purposes. The problem with tautologies is that they say too little. The problem with inconsistencies that they say too much.

The only remaining alternative, then, is the category of statement forms which are non-valid and consistent, that is, the contingent statement forms. As indicated above, these are the group which are neither always true nor always false. The truth value of statements which are substitution instances of these forms are contingent upon states of affairs in reality. When sufficient evidence is amassed that their truth can be asserted, then their assertion conveys information about reality. One such assertion might say that output increases as investment increases. When sufficient evidence is amassed that their truth can be denied, then their denial also conveys information about reality. Such an assertion might say that output does not increase as investment increases. These are the types of assertions most useful in empirical theories. They are the

only kind that provide the relevant information about reality. This is why the criterion of empirical adequacy of explanation and prediction, discussed in Chapter II, requires that at least one of the statements be empirical, that is, contingent rather than tautological or self-inconsistent.

The criteria of logical adequacy, on the other hand, requires an additional restriction, however. Not only must at least one of the statements be a substitution instance of a non-valid and consistent statement form, at least one of these must contain empirically descriptive terms. Otherwise, it could not imply any empirical conclusions.

In addition, at least one of the statements must be lawlike. That is, it must be a general assertion about reality. It must say, for example, "Whenever investment rises, output rises" not "When investment rose in 1952 output also rose." The conjunction of a law-like statement with an empirical statement about an increase in investment implies something about reality. The conjunction of the same antecedent with a statement which is not lawlike, on the other hand, is not generally useful in explanation or prediction.

Further, substitution instances of the contingent statement form (or forms) employed in the axiom set must not only be required in the deduction of the conclusion, the conclusions or theorems must also follow tautologously from

these axioms. They must state what else is true about reality if the axioms are in fact true, or what else is likely to be the case if the axioms are highly confirmed. Otherwise, the effort spent on developing a theory would generally come to naught.

SUMMARY

The analysis of the propositions of the object language of science conducted in this chapter reveals certain concepts which are fundamental to an understanding of the requirements of science. Some of these concepts will be summarized below.

The object language of science is composed of declarative sentences. These sentences generally have both forms and empirical content. Their meaning is dependent upon both forms and content and the interrelationships between the two. The forms of these statements and their effect on the meaning of the statements is one of the major concerns of this investigation. These forms can be abstracted out from their empirical content by the uniform substitution of variables for constituent propositions and the uniform substitution of appropriate symbols for the logical operators employed. This manner of exhibiting the forms of these statements facilitates the analysis of the different kinds of statements used in the object language.

This analysis permitted the classification of statements into three mutually exclusive and exhaustive categories

on the bases of their truth tabular analysis.[7] The characteristics of each of these categories of statement forms were then analyzed with respect to their role and their effect in the logical structure of science and the empirical content of theories, particularly as they come to focus in the structure of explanation and prediction. Consideration was also given to the role of each category of statements as it functions in conjunction with other statements.

An understanding of the nature of these three categories of statement forms permits a clearer understanding of their role in the logical and empirical structure of science. It explains why at least one of the statements must be contingent and contain empirically observable terms, and why the conclusion must follow tautologously from these axioms. In addition, it provides both procedures and decision rules for testing for consistency. These procedures will be the major concern of the succeeding chapter.

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[7]This particular procedure was used because it permits examination of some of the concepts underlying the structure of theories with a minimum of logical apparatus.

CHAPTER VI

APPLICATION OF THE ANALYSIS OF PROPOSITIONS TO THE EVALUATION OF THE LOGICAL ADEQUACY OF THEORIES

INTRODUCTION

The general thesis of this study is that the value of a theory depends upon its usefulness in explaining and predicting the phenomena of reality, and that this ability to explain and predict depends upon the truth of its propositions and the logical adequacy of its structure.

In Chapter II some of the more widely held modern concepts of the nature of theories and the logical structure of science were examined. Some of these concepts were then applied to the evaluation of the classical theory of economic growth as presented in Chapter III. The results of this analysis were something less than unequovical. They focused attention on the need for further formalization of the theory itself and further development of the logical concepts required. Chapter IV was devoted to the further formalization of the classical theory. Chapter V was devoted to the development of the logical concepts required for the evaluation of theories in general.

These concepts were developed out of an analysis of the propositions of science. This analysis permitted the classification of statements into three mutually exclusive

and exhaustive categories on the basis of their truth value characteristics. The role and the effect of each of these types of statements on the logical structure and the empirical content of science were then analyzed.

This analysis provided a general means of recognizing tautologies, inconsistencies and contingent statements on the basis of their truth-value characteristics. It was argued that if it can be demonstrated, on the basis of an examination of form and form alone, that a given statement is always true, regardless of states of affairs in reality, then that statement is a tautology. If it can be demonstrated that a given statement is always false, regardless of states of affairs in the real world, then that statement is inconsistent. If the truth value of a statement cannot be ascertained by examination of its form and form alone, that is, if its truth value cannot be ascertained without knowledge of reality then it is a contingent statement.

If these general methods of identifying tautologies, contingencies and inconsistent statements are to be brought to bear on the evaluation of economic theories, the first step is to exhibit the forms of the individual statements employed in the theory. This task is not too difficult for those theories whose statements can be adequately translated into the calculus of propositions. There are few such theories in economics, however. Most economic theories involve the use of concepts of metricization. The logical

analysis of statements including such concepts generally requires the use of one of the higher functional calculi. The manipulation of these calculi is usually more formidable than the use of mathematical formulations having sufficient logical power. Hence, in order to avoid the use of these calculi it often becomes good strategy to translate the propositions of a theory into equation form rather than into one of the higher functional calculi. This renders the theory amenable to analysis by one of the more familiar types of formal logic employed in mathematics. The next step in the development of this process of evaluation is to extend the concepts developed in Chapter V to deal with the types of equation forms typically employed in economic theories.

This will be followed by an examination of the concepts of logical and empirical adequacy as they apply to the evaluation of theories expressed in equation form. The application of these concepts will then be illustrated with reference to the classical axioms.

This illustration leads to a consideration of the question of consistency. This question was considered in Chapter III and the method of formal deduction applied. It will now be reconsidered in light of the further formalization of the theory presented in Chapter IV, and in view of the additional framework of analysis developed in Chapter V. The question of consistency will first be considered in general terms and then with direct application to the classical axioms. This application is conducted by means of a method

which the author has chosen to call 'the method of numerical interpretation'. This method grew out of the concepts developed in Chapter V. It will ultimately be extended in more general mathematical terms by the use of 'the method of determinants', presented below. Examination of the method of determinants will lead directly to a consideration of the problems of estimation of the parameters involved and measurement of the values of the variables employed. These problems are examined in Chapter VII.

In the initial stages of this study attention was focused on the consideration of whole sets of statements. In the next stage attention was focused on the analysis of individual statements. Finally attention is focused on the consideration of individual parameters and variables within these statements. This sequence of the investigation, first of sets of propositions, then of individual propositions as they function within sets of propositions, and finally of individual variables as they function within propositions within sets of propositions, covers the broad scope of the investigation required in the analysis of the logical and empirical adequacy of theories. The next step in this sequence of investigation is to apply the concepts derived from the analysis of propositions to the examination of the logical adequacy of theories stated in equation form.

THE ANALYSIS OF PROPOSITIONS APPLIED TO EQUATION FORMS

In the analysis of propositions developed in Chapter V, the unit of investigation was the single proposition as it functions as a part of a compound proposition. The terms truth and falsity were used as predicates of these compound propositions. This is the lowest level of logical investigation and is referred to as the calculus of propositions. Higher levels of investigation include the higher functional calculi, Boolean Algebra, and etc. Investigations on the level of the calculus of propositions are sufficient for the purposes of developing the classification system employed in this study. This investigation provides a mutually exclusive and exhaustive classification of the statements of science on the bases of their truth value characteristics. This classification was based upon truth tabular analysis of statement forms. It could just as well have been established on the bases of Boolean expansion, for example, or any other technique for the analysis of truth values. The particular technique employed in establishing the truth value of the propositions is not of major concern. The major methodological consideration is that truth-functional tautologies, inconsistencies, and contingent statements can all be recognized on the bases of examination of their form and form alone.

If it is possible for the researcher to recognize the

truth value characteristics of individual statements and arguments then it is also possible to judge the explanatory and predictive potential of a theory on several crucial considerations. It is not until these considerations have been met that it becomes meaningful to begin to conduct that phase of the analysis designed to ascertain the potential accuracy and reliability with which the theory in question helps explain or predict the phenomena of reality.

TRUTH VALUE CHARACTERISTICS OF EQUATION FORMS

Throughout the remainder of this study attention will be focused on statements which have been formalized in equation form. It is important, therefore, to consider the methods by which the truth value characteristics of statements presented in equation form can be ascertained. Three examples should suffice to initiate investigation of these methods. In order to simplify this examination the investigation is limited to the use of whole numbers as substitution instances in equation forms. If an equation of the form $(X + Y)^2 = X^2 + 2XY + Y^2$, for example, were to come under consideration, it would become apparent that this statement is always true. There are no values which could be substituted for X and Y which would make it false. The statement is tautological. On the other hand, if an equation of the form $X = 1 + 2X^2$, for example, were to come under consideration, it would soon become apparent that this statement is always false. There are no values of X which could

make it come out true. This statement is inconsistent. Finally, if an equation of the form $Y = 100 + 6X_1$ were to come under consideration, it would become apparent that it is possible to choose a value for Y which would make this statement true for any given value of X. Similarly, it is possible to find a value for Y which would make it false for a given value of X. This statement is said to be contingent. Its truth value is contingent on the observed values of X and Y.

One further example drawn from the field of agriculture should suffice to illustrate the nature of contingent statements or equations. The equation $Y = A + bX_1$ might be used to express a simple production function in which Y is the yield of corn, A is the yield associated with the residual level of soil fertility, b is the increase in yield associated with a unit increase in X_1, where X_1 is fertilizer measured in hundredweights of some specified complete fertilizer. If A is 50 bushels and b is 10 bushels and a yield of 80 bushels is associated with three hundredweight of this particular fertilizer, then the statement $Y = 50 + 10X_1$ is true. If, however, it is found that a yield of 100 bushels is commonly associated with the application of three hundred pounds of fertilizer, on this particular soil type, then the statement $Y = 50 + 10X_1$ is false. Its truth or falsity depends on what happens in reality. It cannot be ascertained from examination of the form and form

alone. This is what is meant by a contingent or an empirical assertion; its truth value can only be ascertained after recourse to empirical investigation.

The foregoing examples illustrate the three mutually exclusive and exhaustive categories into which statements may be divided on the basis of their truth value characteristics. The preceeding chapter discussed the role and the effect of each of these types of statement forms in the structure of science. What is required now is a means of ascertaining these truth value characteristics for any given equation. This would enable the researcher to ferret out inconsistencies, to test implications, and to ascertain the empirical nature of the statements in a theory. This ability to identify the truth value characteristics of any statement in a theory would be a major step forward in the a priori evaluation of theories.

ASCERTAINING TRUTH VALUES OF EQUATION FORMS

Knowledge of the truth value characteristics of an equation can often be obtained by substituting values for its variables. Since there are only three types of statement forms in the classification under consideration, the substitution of values for the variables often permits the elimination of one of these equation forms. Tautologies, for example, are always true, regardless of the values substituted for the variables. Hence, if one succeeds in finding an assignment of values for the variables which make

the equation false, one can be sure that this equation is not a tautology. Inconsistencies, on the other hand, are always false. Hence, if one succeeds in finding an assignment of values which makes it true, one can be sure that it is not inconsistent. If one can demonstrate that it is true for at least one set of values and false for another, it is clearly contingent. The results of such investigation are not always this unequivocal, however. If one finds an assignment of values for which the equation is false then that equation is not tautological but it may be either contingent or inconsistent. Similarly, finding an assignment of values for which it is true proves that it is not inconsistent. It may still be either contingent or tautological. The continued failure to find an assignment of values which would make the equation come out true does not prove anything, however, unless all possible combinations of values have been tried. More will be said on this point later. It is sufficient to say, for the present, that the recognition of the assymetry in truth value characteristics of equations can be useful in the a priori evaluation of theories. This concept will be employed in the evaluation of the classical system illustrated below.

THE CRITERIA OF LOGICAL AND EMPIRICAL ADEQUACY

The criteria of logical and empirical adequacy of the structure of explanation and prediction were introduced in Chapter II. They were applied to the evaluation of the classical system in Chapter III. This evaluation was limited

by the inadequacy of the calculus of propositions as a diagnostic device for the theory at hand. Now that the analysis of propositions has been developed, however, the criteria of logical and empirical adequacy once again becomes a useful point of departure.

It will be recalled that the structure of explanation and prediction involves (a) one or more antecedent conditions, (b) one or more lawlike statements, and (c) a conclusion or a description of the phenomenon to be explained. These first two sets of statements constitute the explanans and the last the explanandum. The role of theory in this structure is in providing the lawlike statements required to predict the event given the antecedent conditions, or to explain the event in terms of the antecedent conditions. Hence, an evaluation of a theory is likely to depend very heavily on how well it performs these roles.

The criteria of logical and empirical adequacy constitute a minimum but not a sufficient set of criteria for the evaluation of the potential explanatory and predictive role of a theory. The criterion of empirical adequacy requires that the explanans be true. The criteria of logical adequacy require that: (1) the explanans must have empirical content, (2) the explanans must contain at least one lawlike statement, and (3) the explanans must imply the explanandum. In order to evaluate the extent to which each of these four criteria are fulfilled, and to lay the foundation for the

investigation of more precise criteria which will be employed later, it is useful to express the theory in the framework suggested by Hempel.[1] For this purpose it would be sufficient to express the seven basic propositions of the classical theory as L_1 ... L_7. These can be conjoined with one or more antecedent conditions A_1 to A_n. The antecedent A_1 might assert that investment increased, A_2 might assert that the general level of technology also increased, etc. These two sets of statements, the lawlike statements and the antecedents, make up the explanans. The explanandum is comprised of the singular statement that output increased, or decreased,as the case may be.

One such argument or chain of reasoning would say, for example, that since investment increased, A_1, and since technology is a positive function of investment, L_2. and since output is a positive function of land, labour, capital and technology, L_1, therefore output will increase, E. This is a series of propositions of the form, "since ... therefore ...."[2] In it A_1, L_1 and L_2 are asserted to be true. Since they are asserted to be true and since E follows logically, E is also asserted to be true. Since the A's and L's are asserted to be true, E can be asserted independently. This form of argument is very closely related to the Principle of Inference and as such is stronger than its tautologous

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[1]Hempel, op. cit., p. 322.

[2]Ambrose, op. cit., p. 156.

counterpart and hence implies that tautology.[3] The tautology would not say that since the A's and L's are true, therefore E is true. It would make the much weaker statement that if the A's and L's are true then E is true. This would be written $A_1.L_1.L_2 \supset E$. In this latter, much weaker form, it is subject to the tools available in this study and is consistent with the methodology presented thus far. The implicative form will be used, therefore, in preference to the "since ... therefore ..." form of inference, in subsequent examinations of the classical axioms.

The next step in the evaluation of theories is to begin to appraise the classical system in terms of the extent to which the criteria of logical and empirical adequacy are fulfilled. This appraisal will necessarily lead to considerations beyond the scope of analysis of propositions as presented thus far. It will lead into areas drawing more heavily on mathematics and statistics but still based on the analysis of propositions. It will lead to questions of whether the variables can be measured, whether the coefficients can be estimated, the accuracy and reliability of these measures and the significance of the predictions made.

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[3] The term 'Principle of Inference' is used by Ambrose to denote the rule "Given that A is a postulate or a proved theorem, and that $A \supset B$, we may infer B." Ambrose argues that without this rule no tautology which is a logical consequent in a deductive chain leading back to the postulates could ever be stated separately as a theorem. This concept is more completely discussed in Appendix B.

THE ANALYSIS OF PROPOSITIONS AND THE CRITERIA OF LOGICAL ADEQUACY

In this section an attempt will be made to point out some of the ways in which the analysis of propositions can facilitate the examination of the role of theories in the structure of explanation and prediction. Attention will be focused on the criteria of lawlikeness and of empirical content. Questions of whether or not the explanans implies the explanandum and whether or not the statements of the explanans are true will lead to an investigation which will occupy much of the remainder of this study.

LAWLIKENESS

In Chapter II it was pointed out that for a statement to be "lawlike" it must be of the universal conditional form or translatable into the universal conditional form.[4] Lawlike statements say in general, that everytime something happens, something else happens. Proposition L_2 of the system says, for example, "everytime investment increases the general level of technology increases." This is to be distinguished from the existential statement form which would have said, for example, "the general level of technology sometimes increases when investment increases." In addition, it is to be distinguished from singular statement forms which would have said, for example, "investment rose, and the

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[4]It should be noted in passing that while lawlike statements may be universal conditionals, this does not suggest that all universal conditionals are lawlike. Indeed, the literature of analytic philosophy suggests that the characteristics of lawlikeness require much more than universal conditional form and that these additional characteristics have not yet been adequately defined. See Goodman, op. cit., p. 31.

general level of technology increased." These three examples are illustrative of three types of quantification, and typical of a large proportion of scientific discourse. With reference to the particular theory at hand, they point up the fact that, while the Classicists did not generally use these terms, they did intend to express universal connections between events. An understanding of the nature of lawlikeness and the intent of the classical argument should constitute reasonably good evidence that the classical axioms do fulfill the requirement that at least one statement in the explanans be lawlike. The same kind of evidence, with respect to the generality of statements can usually be obtained with relatively little difficulty for many theories in economics.

EMPIRICAL CONTENT

Questions of the empirical content of theories can generally be divided into two parts, questions concerning the nature of the statement forms, and questions concerning the observability and measurability of the variables employed. Questions of observability and measurability of variables will be discussed in general and with reference to the classical axioms in a later section. Questions concerning the nature of the statement forms employed in the classical axioms are examined below.

The nature of statement forms was studied in the analysis of propositions in Chapter V and their relation to

equation forms elaborated in a previous section of Chapter VI. It was concluded that equation forms, just like statement forms, could be classified into three types: tautologies, inconsistencies and contingent statement forms. It became further apparent that each of these types of equation forms play a role and have an effect on science. These are comparable to the roles and the effects of their counterpart statement forms. It follows, therefore, that in order to be useful for explanation and prediction each of the statements must be consistent and the whole set taken together must be consistent. It also follows that the whole set of statements, expressed in the implicative form $A_1 \ldots A_n \bullet L_1 \ldots L_n \supset E$ must be tautologous. It likewise follows from the analysis of statement forms in Chapter V, that, even though most of the individual statements in the set may be tautologous, one of the necessary conditions for empirical explanation and prediction is that at least one of these statements must be non-tautologous. If at least one of these statements must be non-tautologous and none may be inconsistent then at least one equation must be contingent. In the section on the truth value characteristics of equation forms presented in this chapter it was demonstrated that if an equation form is contingent it is generally possible to demonstrate it rigorously. If the statement whose form is under examination can be assigned a set of values for the variables which make the statement true and

if it can likewise be assigned a different set of variables which makes it false, then that statement is proved to be contingent.

Examination of the individual axioms of the classical system reveals that each of the equations can be assigned values which would make it true and each can be assigned values which would make it false. Hence each equation is contingent, and the classical system fulfills the criterion that the explanans have empirical content. The same type of investigation could be applied to any theory provided that it is recognized that failure to find an interpretation which makes it true does not prove that a statement is inconsistent and failure to find an interpretation which makes it false does not prove that it is a tautology.

It might be hypothesized, in passing, especially in view of the role of definition in tautological constructs, that several widely discussed theories of economics may fail to meet this requirement of logical adequacy . The approach discussed above could be used as a general procedure for testing for the criterion of empirical content of economic theories before the question of implication is examined.

IMPLICATION

Having examined the questions of lawlikeness and empirical content of the explanans, what remains in the

investigation of the criterion of logical adequacy is to determine whether the explanans implies the explanandum. This criterion generally poses the most difficult set of problems in theory evaluation. Further investigation of these problems will be conducted below.

Most theories in economics require some degree of quantification before they can be brought to bear on economic problems. The deductive logic required for the study of partially quantified propositions involves levels of investigation in the higher functional calculi in which the deductive processes become extremely formidable. Hence the researcher may often find it is more fruitful to specify the relationship in equation form, and then apply the rules of mathematics. This may be easier for most researchers than to attempt a full scale a priori investigation of the theories deductive consequences by the methods of formal logic. The same general approach will be taken here.

If it is assumed, for example, that the production function is of the Cobb-Douglas form and that the sum of the b_i's is less than one, that K and Q are exogenously determined, that L and T are uniquely determined by the other equations in the system, and that their values fall within the range for which the production functions was fitted, then it would seem to follow that the conjunction of specified values for L and T would imply the value of O. It might perhaps be calimed, therefore, that the explanans does imply the explanandum

and that the final criterion of logical adequacy is fulfilled.

The evidence is far from conclusive, however. While it is true that it could be claimed that the system implies a value for O, it can not yet be claimed that it uniquely determines the value of O for any given set of values of the other variables involved. Nor can it be claimed that this is all that it implies. It was demonstrated in Chapter V that if a theory contains an inconsistency, it implies anything at all, and hence, a fortiori, it could imply the value of O. One of the requirements of a theory is that it uniquely determine the values of the endogenous variables, that is, that it be consistent and determinate. A major portion of the remainder of this chapter will be devoted to an investigation of consistency as a means of determining in general whether or not a particular explanans is capable of implying a particular explanandum.

TESTS FOR CONSISTENCY AND THE CRITERIA OF LOGICAL ADEQUACY

INTRODUCTION TO TESTS FOR CONSISTENCY

In Chapter V, in the section entitled "The Effect of Inconsistent Statement Forms on the Empirical Assertions of Science" it was pointed out that consistency is the first and foremost requirement of any theory. It was demonstrated that the presence of an inconsistency in a theory permits that theory to imply not only the truth about reality but anything at all about reality. This renders the theory useless for explanatory and predictive purposes. Hence, one of the

first requirements of an investigation of the methodology of economic research is that it bring together whatever tools can be brought to bear on the problem of identifying inconsistencies.

There are a number of concepts that should be brought into focus and clarified before the nature of these tools are investigated. The first is that the analysis of propositions provides one of the basic concepts which can be employed in this investigation. The analysis of propositions indicated that all declarative sentences employed in the object-language of science could be classified into three categories on the basis of their truth value characteristics. One of these categories is known as the class of inconsistencies. Their truth-values, like those of tautologies, can be ascertained from an examination of their form and form alone. Examples of such inconsistent forms include $p.\sim p$, $\sim(p \vee \sim p)$, $\sim(p \supset p)$, $\sim(p.q \supset q)$, etc. Statements which are substitution instances of these inconsistent statement forms are always false, regardless of the truth values of their constituent propositions. This concept, together with the fact that the characteristics of consistency and inconsistency are dichotomies, and hence that every statement is either one or the other, implies that if any statement can be shown to be true it is not inconsistent and is therefore consistent. This conclusion becomes of primary importance in investigations of the consistency of theories conducted below.

The second clarification involves the meta-language in which the object-language of science is discussed. The meta-language contains several uses of the terms "consistency" and "inconsistency." Statisticians use the term "consistent", for example, as a predicate of estimators-- a "consistent estimator" being one that tends to approach the true parameter as the number of observations approaches infinity. In order to avoid confusion with this and other concepts of consistency it would be possible to adopt the convention of using the term "non-self-contradictory" in place of "consistency" and "self-contradictory" in place of "inconsistency", but this would encumber the discussion unnecessarily. Throughout the remainder of this study the term "inconsistency" will continue to be used in the older and more customary way, as a predicate of statements that are self-denying. The term "consistency" will be used to mean not self-denying. These terms will also be used in the same way as predicates of equations, sets of statements, arguments and theories.

The third clarification involves a more intuitive approach to an understanding of inconsistency. A statement is inconsistent if it denies itself. A theory is inconsistent if it implies one statement "p" under one set of conditions and the statement "$\sim$p" under the same set of conditions. For example, if a theory of economic growth were to imply that, under a given set of antecedent conditions output would increase and the same theory using a different chain of

reasoning but the same set of antecedent conditions implied that there would be no increase in output, then that theory is self-denying or inconsistent. The statement that there will be no increase in output is the denial of the assertion that output will increase. It is not generally as easy as this example suggests, however, to recognize that one assertion is the denial of another. Two contradictory assertions are often couched in different terms so that the contradiction in meaning does not become apparent without careful and arduous analysis.

One further concept that is of some assistance in determining whether one concept is the contradictory of another is contained in the difference between the propositions "p.~ p" and "pvq". The first is like the example presented above. The second is like saying that the average total cost equals the price of the product at 100 units of output or at 200 units of output. The statement of the form "p.~ p" is inconsistent. The statement that the average total cost equals the price where output equals 100 units or output equals 200 units is perfectly consistent. This is the difference between claiming that one or the other of two different conditions is true at any given time and claiming that both of two opposite conditions are true at the same time.

ALTERNATIVE TESTS FOR CONSISTENCY

In view of the importance of being able to recognize

the presence of an inconsistency, a number of techniques for testing for consistency will be discussed below. This selection of techniques is not intended to be exhaustive but is intended to be illustrative of some of the types of approaches that may be fruitful. They are also illustrative of the differences in techniques applicable to the investigation of theories having attained different degrees of formalization. These techniques can be divided into three broad classes which will be called, for want of better names, the formal deductive techniques, traditional deductive techniques and mathematical techniques. In order to be consistent with the format of the study, the formal deductive techniques will be discussed first.

Formal Deductive Techniques

Among the formal deductive techniques, Truth Tabular Analysis, Boolean Expansion and the method of Universe Construction might hold some promise for researchers in economics.

The broad outlines of Truth Tabular Analysis were presented in Chapter V and Appendix A. This method can be used for sets of statements that can be adequately translated into the calculus of propositions. Once this is accomplished, unequivocal results can be obtained. The methods of Truth Tabular Analysis appear to provide relatively little potential for direct application in economic research, however. There are two reasons for this. The first is that

the number of alternative truth-value assignments increases in geometric progression with the number of variables involved so that the investigation becomes formidable for a relatively small number of variables, unless the process is set up for analysis on an electronic computer. The second is that most economic theories require some degree of quantification before they begin to take on much explanatory and predictive potential, and this generally requires the use of quantification theory to which truth tabular techniques are not as readily applicable.

The application of the methods of Boolean Algebra also provides a rountine procedure for testing a set of statements to determine whether or not an argument contains an inconsistency. In the case of Boolean Expansion, as in the case of the Truth Table Procedures, the technique is limited in its application to economic investigation by the use of a calculi which is not adequate for the analysis of the types of propositions commonly employed.

The methods of universe construction as applied to sets of sentences exhibiting forms of the Monadic Predicate Calculus appear to hold somewhat more promise for economic investigation than does Boolean Algebra or the Truth Tabular Analysis. It is designed specifically to deal with the nonvalidity of sets of quantificational statements. Evidence as to whether or not this development will prove to be substantially more useful than the other methods discussed

will have to await its fuller elaboration in the literature.

Traditional Deductive Techniques

The term 'traditional deductive techniques' is employed here to denote what is commonly referred to as formal and traditional logic in the investigation of the validity of non-validity of arguments. These are the techniques that have had by far the longest application in economic theorizing--their history dates back to the Mercantilist era when economics was the infant child of philosophy. These are the techniques economists intuitively employ in their search for consistent theories.

The procedure is generally to attempt to deduce the negation of one of the statements in the axiom set or one of the theorems of the theory. An alternative procedure is to try to deduce some easily recognizable inconsistent form. Success in deducing the negation of an axiom or theorem or in deducing an easily recognizable inconsistent form is enough to prove inconsistency and to destroy the theory. The procedure is then to examine the axioms or assumptions of the theory with a view to identifying the source of the inconsistency and then to modify the set accordingly.

The next step in the examination of these techniques is to consider those things that can be done to improve upon the effectiveness of the traditional deductive approach to inconsistency. Some students of economics have hoped to find the answer in formal logic. These answers are seldom

forthcoming, however; the discipline of formal logic deals only with part of the tools of analysis required for the elaboration of the deductive consequences of most theories. It is concerned with the formal elements of propositions not with their content. Economists are often as interested in the analysis of the latter as they are in the former. Other students of economics have hoped to obtain increased ability at ferreting out the deductive consequences of a set of axioms with the aid of improved skills in mathematics, with similar limitations on their success. Both approaches have some merit in and of themselves, but the merit of each is likely to be enhanced through an increased awareness of the power and the limitations of the other. The proper utilization of any or all of these formal tools is likely to increase the effectiveness of the intuitive approach that economists have employed, with varying degrees of success, for many generations. This use of mathematics together with certain concepts drawn from formal logic will be explored below.

In the traditional deductive approaches to the identification of an inconsistency, the deduction of the negation of one of the axioms or theorems, or the deduction of any easily recognizable inconsistent statement form, is sufficient to prove inconsistency. The failure to make such a deduction does not prove consistency. The researcher never knows, with this type of approach, that it would not be

possible eventually to find such a contradiction. It is possible to prove inconsistency with this approach, but it is not possible to prove consistency. There exists another approach, however, with which it is possible to prove consistency directly. It is an outgrowth of the analysis of propositions conducted in Chapter V, and it turns out upon analysis to be a special case of the Method of Models introduced in Chapter II. Throughout the remainder of this study, it will be called, for lack of a better name, the Method of Numerical Interpretation.

METHOD OF NUMERICAL INTERPRETATION

The basic concepts behind the Method of Numerical Interpretation are the concepts of consistency and inconsistency of equation forms. These concepts are reviewed below. In the first part of this chapter it was pointed out that equations were subject to the same kind of analysis as statement forms except that the substitution instances employed were values for the variables involved. As in the case of statements, an inconsistent equation form is one that always denies itself, that is, one for which every substitution instance is false. If permissible substitutions are limited to whole numbers, the equations $X = 1 + 10X$, $Y = 2 + 2Y^2$, $Y = 1 + 6Y^2 + X^2$, etc. are examples of inconsistent equations. Every substitution for the variables in an inconsistent equation produces a false statement. This provides a means of testing for consistency

in both single equations and sets of equations.

In order to determine whether a given equation is consistent or inconsistent it is useful to substitute numerical values for the variables. If a set of values can be found such that it satisfies the equation, that is, makes it true, then that equation is not inconsistent. If it were inconsistent all substitution instances would be false. If it is not inconsistent, then it is consistent. In short, any substitution of a set of values for variables which satisfies that equation is sufficient to prove that the equation does not deny itself. In this way, it proves that the equation is consistent.

The same procedure for testing for consistency can also be applied to sets of equations. A set of equations is said to be consistent if all of the equations in the set can be true simultaneously. A set of equations is inconsistent if one or more of the equations is made necessarily false by every set of values that satisfies the others. Briefly, the set is inconsistent if there is no set of values for the variables which permits all of the equations to be true simultaneously, that is, if they are always false when taken together. As in the case of single equations, it is possible to test the consistency of a set of equations by assigning numerical values to the variables. If a set of values is found for the variables which makes all equations true simultaneously, then that set of equations is not self-denying.

If it does not deny itself then it is consistent.

APPLICATION OF METHOD OF NUMERICAL INTERPRETATION

The next step in the development of the Method of Numerical Interpretation is to illustrate the procedure by applying it to the classical system and in so doing to learn some additional characteristics of consistency.

In the most general cases, where the propositions of the theory are almost completely unspecified there is very little that can be said about its consistency. It is only after some degree of specification that questions of consistency begin to arise. Even when the relationships expressed in the system are stated in general equation form there can be little evidence as to its consistency or inconsistency. It is usually not clear precisely what statements are being made about reality until the researcher is able to specify both the types of functions involved and the ranges in values the parameters are likely to assume. In fact, until such specification is made, the researcher has not actually made <u>any</u> statement about reality. Until such statements are made, there is little that can be said about the consistency or inconsistency of such possible sets of statements. Once these assertions have been clearly stated in equation form and at least one possible set of values for the parameters has been specified, then it becomes possible to test for consistency by the Method of Numerical Interpretation.

This method involves attempting to find a set of values for the variables which will satisfy all the equations simultaneously. If such a set of values is found, then every statement in the system becomes a true statement and the system does not deny itself. It is therefore consistent for the specific types of equations employed and the values of the parameters assumed. Failure to find such a set of values for the variables does not prove inconsistency, however. It simply leaves the problem of consistency open to question.

Additional understanding of this test can be obtained by examination of its application to one particular interpretation of the classical axioms. This is only one of a large number of possible sets of parameters for the classical system. The parameters assumed here were chosen for illustrative purposes only. They are not intended to be typical of the parameters to be expected in any particular economy.

The unspecified functions of the classical system are presented on the extreme left in the following example. The general functions are presented in the center. The specified functions to which the Method of Numerical Interpretation is to be applied is presented at the extreme right.

Once these values of the parameters were specified it was found that it is possible to obtain a set of values for

the variables which satisfies all seven equations simultaneously. When the exogenous variables K and Q are 312 and 5502 respectively, the values O = 397, L = 4.4, T = 752, I = 75, R = 380, and W = 17, satisfy all seven equations simultaneously.

	Unspecified Functions	General Functions	Specified Functions
(L_1)	$O = f_1(L,K,Q,T)$	$O = a_1 L^{b_1} K^{b_2} Q^{b_3} T^{b_4}$	$O = 10L^{.1} K^{.2} Q^{.2} T^{.1}$
(L_2)	$T = f_2(I)$	$T = a_2 + b_5 I$	$T = 2 + 10I$
(L_3)	$I = f_3(R)$	$I = -a_3 + b_6 R$	$I = -1 + .2R$
(L_4)	$R = f_4(T,L)$	$R = a_4 + b_7 T - b_8 L$	$R = 26 + .5T - 5L$
(L_5)	$L = f_5(W)$	$L = a_5 + b_9 W$	$L = 1 + .2W$
(L_6)	$W = f_6(R)$	$W = a_6 + b_{10} I$	$W = 2 + .2I$
(L_7)	$O = R + W$	$O = R + W$	$O = R + W$

This demonstrates that, for this particular selection of parameters, there is one set of values for the variables which satisfies all seven equations simultaneously. Each statement in the system is a true statement for this particular set of values. If this system denied itself there would be no combination of values which would make it true. The fact that there is at least one set of values for the variables which makes the set true proves that it is not always false. If the set is not always false it is not inconsistent and is therefore consistent.

The next task is to examine the significance of this conclusion for the theorist. With respect to the analysis of

the classical system, this test demonstrates that there is at least one set of values for the parameters such that the system does not deny itself. Each equation expressed above constitutes a statement form. This set of equations constitutes a set of statement forms which function as the axioms of the theory. Hence, to show that these equations are consistent is to show that the classical axioms are consistent. But what does it mean to say the classical axioms? It could well be argued that there is no unique set of axioms in favor of which all other sets of axioms may be eliminated. This follows from the fact that each of these functions may take on different sets of parameters at different times in different economies. If, for example, the production function expressed above is representative of economy A, and the function $O = 500L^{.3}K^{.1}Q^{.4}T^{.1}$ is representative of economy B, then these two functions express different statements about the same things. When one or more statements are different, then the sets of statements are different, and these differences constitute different theories about the same phenomena.

When it is recognized that there may be a large number of different theories about the same phenomena it becomes important to have some idea of how many of these theories might fulfill the requirements of consistency. This may be the only one, or there may be many others. Since there may be many others, and since the system tested involves

a set of structural equations which seem to contain the basic ideas of the classical system, the results of this test provide some hope for the useful application of this system once the parameters have been estimated statistically. Further application of the Method of Numerical Interpretation awaits this estimation.

The significance of the Method of Numerical Interpretation for the methodologist is somewhat more far reaching, however. It provides a test of consistency which may be applied on any one of several levels. One such level is likely to be that of the axiom set, or assumptions, of a theory before any functions are fitted, or deductions made. Finding an inconsistency at this stage renders all the remaining steps unnecessary. It can be also applied at almost every turn of the investigation as more evidence is acquired about the parameters of the equations employed. In this way it can be used to guide the selection of functions for fitting and hence guide the development of the theory itself.

Before leaving the development of the Method of Numerical Interpretation it would be well to consider some of the similarities and some of the differences between this method and the methods referred to as the 'traditional deductive techniques.' One of the major similarities between these two methods is that they both involve the application of the techniques of deductive logic. In the case of the traditional deductive techniques, the application

is made directly. In the case of the Method of Numerical Interpretation this application is made in terms of the concepts derived from the analysis of propositions.

One of the major differences between these techniques is that the deductive techniques can be applied to any one of a variety of different formulations of the theory. They can be applied to an almost completely unspecified set of propositions as long as they can be translated into one of the lower functional calculi. They can be applied to a theory formulated in terms of first difference as they were in Chapter III. Finally, they can be applied to completely specified functions by the use of one of the higher functional calculi. The Method of Numerical Interpretation, on the other hand, can only be applied to formulations which are almost completely specified.

Another major difference is that the traditional deductive techniques can only be used to prove inconsistency. At the same time, any number of failures to deduce an inconsistent or contravalid statement does not prove consistency. The Method of Numerical Interpretation, on the other hand, is capable of proving consistency directly. In contrast to the traditional deductive techniques, it is not capable of proving inconsistency.

In view of these two differences between these types of techniques, it may be useful to consider them as complementary rather than competitive in the evaluation of theories.

The deductive techniques are likely to be most useful in the initial stages of an inquiry before estimation of the parameters is attempted. The Method of Numerical Interpretation, on the other hand, is likely to become most useful as the process of quantification proceeds.

It should also be remembered that the Method of Numerical Interpretation is only one of the several methods of applying the analysis of propositions to the problems of consistency. The methods of truth tabular analysis, Boolean expansion and universe construction are all methods which facilitate the identification of truth value characteristics, and which can take advantage of the asymmetry of truth value characteristics in order to identify consistency. The choice of the most appropriate method depends, in large part, upon the types of statements employed and the levels of formalization obtained in the particular theory at hand.

TEST OF DETERMINANTS

In the previous section it was concluded that, while the Method of Numerical Interpretation enabled the researcher to demonstrate consistency for the values of the parameters assumed, it did not permit him to make any statements about whether there were likely to be other sets of values, that is, other theories, which would also be consistent. Nor did it specify the range over which these parameters could vary without making a theory inconsistent. This points up

the need for a set of techniques which permit the researcher to ascertain some of the restrictions which must be placed on the system in order for it to be consistent, and something of the probabilities of these conditions being fulfilled in reality. Examination of the expansion of the determinant of a set of linear equations holds some promise in this regard.

This concept can also be illustrated by application to the classical system, but this time the application will be to the general interpretation in which the value of the coefficients are not assumed a priori. Since this concept applies only to linear equations, the subset L_2 to L_7 will constitute the first unit of investigation. Then the analysis will be extended to include the production function.

The matrix of coefficients of equations L_2 to L_7 of the Classical System is:

$$\begin{bmatrix} 1 & -b_5 & 0 & 0 & 0 & 0 \\ 0 & 1 & -b_6 & 0 & 0 & 0 \\ -b_7 & 0 & 1 & +b_8 & 0 & 0 \\ 0 & 0 & 0 & 1 & -b_9 & 0 \\ 0 & -b_{10} & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & -1 & 1 \end{bmatrix}$$

This in turn has associated with it the determinant Δ. The expansion of this determinant is:

$$\Delta = 1 + b_6 b_8 b_9 b_{10} - b_5 b_6 b_7$$

In order for any system of linear equations to be consistent, the determinant must be non-zero. Hence for this linear subsystem of the classical axioms to be consistent the following condition must hold:

$$1 + b_6\ b_8\ b_9\ b_{10} - b_5\ b_6\ b_7 \neq 0.$$

In order to examine this condition, it is useful to examine the circumstances under which it does not hold, that is to examine the condition under which the expansion of the determinant is equal to zero. This expression can then be written:

$$1 + b_6\ b_8\ b_9\ b_{10} = b_5\ b_6\ b_7.$$

In the general case, each of the b's can take on any value, and all values are equally likely. Under these conditions, for any given value of the left hand member of this particular equation only one value of the right hand member is exactly equal to it. At the same time there are an infinite number of values for the right hand member which are larger than it and an infinite number of the values for the right hand member that are smaller than it. It follows that the b-values estimated for this subsystem of the classical axioms are not likely to satisfy this equation and hence that they are likely to satisfy the inequality specified above. This means that this subsystem of the classical axioms is likely to fulfill the requirements of consistency. This conclusion is consonant with the commonly held notions about the requirement that there be an equal number of

equations and endogenous variables in order for the system to be determinate.

The foregoing conclusion was obtained on the basis of an a priori examination of the coefficients employed. More conclusive evidence is not likely to be forthcoming until further knowledge is obtained about the values the parameters are likely to assume as a result of empirical investigation.

The next step in this approach is to examine the implications of the expansion of the determinant of the subset for the system as a whole. This can be accomplished by substituting expressions for the values of the variables in the production function. The values of the variables O, L, and T are given by:

$\frac{\Delta O}{\Delta}$, $\frac{\Delta L}{\Delta}$, and $\frac{\Delta T}{\Delta}$ respectively, where Δ is the determinant of the coefficients and ΔO, ΔL, and ΔT are formed by replacing the coefficients of O, L and T in the determinant by the constant terms. The production function then becomes:

$$\frac{\Delta O}{\Delta} = A_1 \left(\frac{\Delta L}{\Delta}\right)^{b_1} K^{b_2} Q^{b_3} \left(\frac{\Delta T}{\Delta}\right)^{b_4}.$$

A priori examination of this equation would suggest that this condition, for the consistency of the system as a whole, is particularly restrictive. The values of all the endogenous variables in the system are determined by the subsystem independent of the production function. It would therefore be only by chance that the values of the parameters would satisfy this relationship. It seems unlikely, therefore,

that the system as a whole would be consistent for any set of parameters that are likely to result from empirical investigation.

Further examination reveals that, even though the whole system is likely to be inconsistent once the parameters are obtained, there are at least two ways of making it consistent. The first is by forcing one of the b's to take on a value which would satisfy the equality

$$1 + b_6 b_8 b_9 b_{10} - b_5 b_6 b_7 = 0.$$

The second is by adding a restriction on the parameters so that they would satisfy the production function. Neither of these restrictions are likely to make any good economic sense a priori. They are likely to be imposed for purely mechanical or mathematical reasons. Perhaps they should be viewed as attempts to figuratively bend reality to suit the model rather than to rebuild the model to correspond with reality.

The relationship between the Method of Numerical Interpretation and the Test of Determinants now becomes clear. With respect to the classical system the first method revealed that there was at least one interpretation (one assignment of values for the parameters) such that the system was consistent. This supplied little evidence as to whether there were likely to be other values for which it would also be consistent, andin particular whether these values were of sufficiently wide range to accomodate the

values that the parameters are likely to acquire as a result of further empirical investigation. The Test of Determinants, on the other hand, revealed that there is likely to be relatively little chance that the parameters assumed by the Classical Axioms would fall within the range which would permit consistency. Conclusive evidence from either test will have to await further evidence on the actual value of the parameters.

The methods of numerical interpretation and of determinant expansion differ widely in their approach. One functions as a method for proving consistency. The other states a condition required for consistency. One asks the question of a precisely specified set of statements. The other asks the question of an unspecified set of statements. One applies to any system. The other is more generally applicable to linear systems or sub-systems and can be extended to cover some cases of non-linearity. It is thought that both approaches can play a significant role at various levels in the evaluation of the explanatory and predictive potential of theories.

Two additional notions should be presented before leaving the Test of Determinants. The first is that for linear systems with n equations and n endogenous variables the presence of a non-zero determinant establishes two other characteristics besides consistency. It establishes that each of the equations is linearly independent of the others

and that the system is determinate. Fulfilling these restrictions fulfills the requirements of the logical adequacy of a structure with respect to its ability to determine the values of the endogenous variables given the values of the parameters. The second is that, at least to the author's knowledge, there is no set of concepts comparable to those discussed in the Test of Determinants that is generally applicable to non-linear systems of equations. There are a number of equations, however, which become linear when transformed into logs. These include all the monotonic functions including the Cobb-Douglas function, for example, and some of the non-monotonic functions like the Carter-Halter-Hocking function. Hence an extension of the Test of Determinants is to use the logarithmic form of those functions which become linear in logs. This adaptation renders a substantially wider group of theories amenable to examination by these particular techniques.

SUMMARY

The foregoing investigation has extended the analysis of propositions to deal with equation forms and with sets of equations. This investigation revealed that equations, like statements, could be classified into three mutually exclusive and exhaustive categories on the basis of their truth value characteristics. It indicated that knowledge of the truth value characteristics of equation forms could be obtained by assigning numerical values to the

variables involved. When this is accomplished inconsistent equations always turn out false, tautologies are always true, and contingent equations are sometimes true and sometimes false depending upon the values of the variables employed. It follows as a corollary of the fact that inconsistent equations are always false, that if an equation turns out to be true then it cannot possibly be inconsistent. Therefore it must be consistent.

The same kind of reasoning also holds for sets of equations considered simultaneously. If one or more members in the set deny themselves or deny each other then every member of the set cannot be true simultaneously. If an assignment of values is found such that every equation of the group is true simultaneously then each member is consistent with each other and the whole set is consistent. Hence, what is required to prove consistency for a set of equations, just as in the case of single equations, is to demonstrate that there is at least one assignment of values for the variables such that every equation in the set is simultaneously true. This was demonstrated in one of the preceeding sections for one particular formulation of the classical axioms.

This only proves consistency for one set of parameters, however, that is, for one particular theory. It should be noted that a similar set of statements with different parameters constitutes a <u>different theory</u> even

though it is couched in the same terms and is about the same phenomena. Even if only one equation assumes a different set of parameters than it did in the previous example, then it is a different theory. Having demonstrated that the theory represented by one set of parameters is consistent, however, in no way suggests that similar theories, that is, those comprised of similar equations but different parameters, are also consistent.

While a theory, in these terms, may be expressed as a set of equations having one unique set of values for its parameters, it may at the same time have a wide variety of values for its variables. Having demonstrated that a theory is consistent means that it is not always false. Substitution of other values for its variables may make it true, or it may make it false, but it is not logically false or inconsistent. Having found one set of values of the variables for which a theory is true, is enough to demonstrate that it is not always false and hence not inconsistent. Since it must be either consistent or inconsistent, finding one set of values for its variables for which it is true is enough to prove that it is consistent. If it is proved to be consistent, it is always consistent regardless of the number of sets of valuesfor its variables which make it true and the number which makes it false. The properties of consistency and inconsistency are logical properties. They cannot change without a change in the logical structure. Hence when any

theory is demonstrated to be consistent, it is always consistent, for any number of substitution instances for the variables involved.[5]

Since there is no way of knowing, a priori, the values that the parameters are likely to assume as a result of empirical investigation, it becomes important to consider the range of values of the parameters for which similar theories are likely to be consistent. This can be accomplished, in part, by the use of the Method of Determinants. This procedure, although primarily useful for linear systems, can also be extended, with careful adaptation, to certain non-linear systems. It exhibits the conditions that must exist in order for a given theory to be consistent. This information, together with any knowledge that can be obtained about values that the parameters are likely to assume, can provide some evidence as to the possible consistency or inconsistency and hence the explanatory and predictive potential of the theory in question.

The Method of Numerical Interpretation is a means of testing the consistency of a given theory. The Method of Determinants, on the other hand, is a means of showing the conditions required for the consistency of a whole family of potential theories. These methods may become increasingly

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[5]This conclusion would not hold in the event that one or more of the substitution instances themselves contained an inconsistency, however.

useful in the evaluation of the logical adequacy of theories as these theories are developed toward higher levels of formalization. They may be used to complement each other and to complement the techniques of formal and traditional logic in the process of theory evaluation.

This whole set of tools for testing for consistency concentrates on the requirement that the explanans imply the explanandum. The analysis of propositions while playing the initial role in these considerations, also plays a key role in determining whether the criteria of lawlikeness and empirical content are also fulfilled. These criteria comprise the criteria of logical adequacy as they apply to the problems of theory evaluation.

Having considered the logical basis for the analysis of the criteria of logical adequacy as it applies to the classical theory and to theories in general, attention is now turned to the question of the empirical adequacy of these theories, with special reference to theories of economic growth.

CHAPTER VII

PROBLEMS OF THE EMPIRICAL ADEQUACY OF THEORIES

INTRODUCTION

Up to this point in the examination of the methodology for the a priori evaluation of theories, the main emphasis has been on ferreting out the implications of the analysis of propositions for the evaluation of the logical adequacy of theories. Attempts have been made to carry this analysis past the level permitted by the analysis of propositions with the aid of certain concepts drawn from linear algebra. The next step is to explore the implications of the analysis of propositions for the evaluation of the empirical adequacy of theories. In this case attempts will also be made to carry the analysis past the level permitted by the analysis of propositions. This will involve an examination of the variables employed.

This brings the development of the study to its third and final stage. The first stage involved the analysis of groups of statements or theories. The second involved the analysis of propositions. The final stage involves the examination of the variables employed. This examination will require the use of techniques drawn from mathematics and statistics. Their application is based on the concepts

derived from the analysis of propositions which provides the logical framework for their use.

The investigation of the logical adequacy of theories was concluded with an illustration involving the examination of the consistency of the classical axioms. This culminated in the recognition that further application of the same tools would require further empirical evidence as to the values the parameters might assume. This brings the investigation to the point where it becomes desirable to focus attention on the problems of estimating these parameters and measuring the variables.

The empirical aspects of the investigation of theories center largely on the extent to which they fulfill the criteria of empirical adequacy. The central idea of empirical adequacy is the requirement that the statements in the explanans be true. If the statements in the explanans are true and the theorems are correctly deduced, then these theorems, conclusions, or predictions must also be true. The truth of the explanans in this way assures the truth of the explanandum.

Any or all of the three types of statements identified in the analysis of propositions may appear, with different effects, in the explanandum. According to the analysis of propositions, both the tautologies and the inconsistencies may be identified by examination of their form and form alone with no recourse to empirical investigation. These

are not always easily identified, however. The researcher may have to turn to the Method of Numerical Interpretation which grew out of the analysis of propositions, or to some other technique like the Method of Determinants, in order to ferret out the inconsistencies and identify the tautologies.

The problem of establishing the truth value of the contingent statements is another matter, however. It requires knowledge of reality. An examination of the types of statements commonly involved in economic theories will serve to illustrate the problems involved. The classicists, for example, claimed that the labor force was some function of the wage fund. In order to employ this statement in a set of axioms designed to predict changes in output, it would be useful to specify the particular function more precisely. Stating the function in the form $L = a_5 + b_9W$ would not be very useful for prediction, but stating $L = 1 + .2W$, on the other hand, would be extremely useful, if it is in fact true. This statement says that the labor force is one plus one-fifth the size of the wage fund. The truth of the statement $L = 1 + .2W$ depends upon the observed values of L and W. The truth of the statement $L = a_5 + b_9W$, however, depends upon whether the values assumed by a_5 and b_9 make the statement true for a large enough number of values for L and W to satisfy the decision rule required to enable the researcher to treat it as true. In other words, getting a reasonable hypothesis about the relationship between

L and W depends on the researcher's ability to estimate the parameters involved.

Knowledge of the truth of the hypothesis depends not only upon the researcher's ability to estimate the parameters, but also upon his ability to observe and to measure the variables. If it is established that the variables are measurable and the parameters can be estimated then questions about the usefulness of the axiom hinge upon the reliability with which the coefficients can be estimated and the amount of change in one associated with a unit change in the other.

The next step, in this procedure, will be to examine some of the problems of estimating these parameters. These problems are sufficiently important that they will be dealt with in some detail below.

PROBLEMS OF ESTIMATING PARAMETERS

Paramount among the problems of estimating the parameters of a system are the problems of identification.

PROBLEMS OF IDENTIFICATION

The question raised at this point is not whether the coefficients can be estimated, because they generally can. The question is whether they can be estimated reliably. If they cannot be estimated by any of the modern statistical procedures then the question becomes one of how else they might be estimated. It is hoped that they can be estimated so that they can provide statements about reality which are reliable enough to be regarded as true. Then the researcher

will be able to ascertain what else will be true about reality if these statements are in fact true.

SIMULTANEOUS EQUATION METHODS

The classical theory of economic growth is usually thought to be a general system in which all of the relationships are taken to hold simultaneously and to act and react upon each other in such a way as to mutually determine their values. It would be appropriate, therefore, if these equations could be fitted by simultaneous equation procedures. The basic theory underlying the simultaneous equations approach was developed about 1943. The methods based on this theory have come into wide use over the past twenty years.[1] One of its advantages over traditional least squares methods of estimating the parameters of single equations is that it is designed to take the simultaneous nature of the relationships into account.

The simultaneous equations procedures were designed for use with systems of linear equations. Their use can readily be extended to systems which become linear in logs. The classical system, as interpreted in this study, is taken

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[1]This general approach to systems of equations can be found in such publications as U.S.D.A. Agricultural Handbook, No. 94, "Computational Methods for Handling Systems of Equations Simultaneously," by Joan Friedman and Richard J. Foote, and in Chapter 10 by Chernoff and Divinsky of "Studies in Econometric Method," Cowles Committee for Research in Economics, Monograph 14, p. 236.

to contain a production function of the Cobb-Douglas type, and six linear equations. Leaving aside the questions of the best fitting structure for the time being, the researcher may fruitfully ask if the system would be identifiable if it were treated as linear in logs.

Identifiability is a mathematical property of an equation that indicates whether the structural coefficients can be estimated by statistical means. The degree of identification refers to whether the equation is under-identified, just identified, or overidentified. A just identified equation has the mathematical property that permits unique determination of its coefficients by the usual statistical means. An overidentified equation, on the other hand, has the mathematical property that a number of alternative estimates of its structural coefficients can be obtained. In this case, the limited information or some other procedures must be employed in order to estimate its coefficients uniquely. An under-identified equation is one that has the mathematical property that its structural coefficients can not be estimated by statistical means. The mathematical properties mentioned above refer to the rank and order conditions of the set of equations of which each structural equation is a member. Rank and order both refer to the matrix of coefficients of such a system. The order refers to the number of columns in the matrix and the rank refers to the order of the minor

of highest order whose determinant is non-zero.

These rank and order criteria are variously stated in the literature in terms of rules of thumb usually called "counting rules." These counting rules are generally limited to a statement of a necessary but not a sufficient condition for identifiability in that the more exact rank criteria are often omitted. An equation is just identified if the number of variables in the system (endogenous and exogenous) minus the number of variables in the particular equation is just equal to the number of endogenous variables in the system minus one.[2] It is overidentified if the number of variables in the system minus the number in the equation is greater than the number of endogenous variables in the system minus one. It is underidentified if the number of variables in the system minus the number of variables in the equation is less than the number of endogenous variables in the system minus one. The same conditions are more succinctly summarized by Koopmans and Hood in the following way.[3] If K** represents the number of exogenous variables in the sytem but not in the equation in question and G^

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[2]A.M.S. Agricultural Handbook, No. 146, Analytical Tools for Studying Demand and Price Structures, (U.S.D.A., Washington, D.C.,) p. 62.

[3]Tjalling C. Koopmans or William C. Hood, "The Estimation of Simultaneous Linear Economic Relationships," Chapter VI, Studies in Econometric Method, Cowles Commission for Research in Economics, Mono. 14, (Chapman & Hall, Limited, London, 1953), p. 138.

represents the number of endogenous variables in the equation, then the condition for being just identified is as follows:

$$K^{**} = G^{\Delta} - 1.$$

If the equation is overidentified $K^{**} > G^{\Delta} - 1$, and if underidentified $K^{**} < G^{\Delta} - 1$.

If K and Q in the classical system are exogenous, the degree of identification of each equation, on the assumption that it can be treated linearly, is presented in the following table:

Test of Identifiability

Equation	K^{**}	G^{Δ}	$G^{\Delta} - 1$	Degree of Identification
L_1	0	3	2	Not identifiable
L_2	2	2	1	Over identified
L_3	2	2	1	Over identified
L_4	2	3	2	Just identified
L_5	2	2	1	Over identified
L_6	2	2	1	Over identified
L_7	2	3	2	Just identified

In the case in which the capital stock Q is treated as endogenous and the system again treated as linear in logs the results would also be something less than encouraging. In this case the production function would still be underidentified and all six of the other equations would be overidentified.

Other attempts to make it identifiable would include

treating two variables not in the production function as exogenous. This might be accomplished by using lagged variables for I, R and W but this is likely to involve some problems because of the pseudo-recursiveness of the system. All three of these variables both determine and are determined by other variables in the system.

Another attempt to make it identifiable would be to add extra variables. Durbin says in this connection, "It is easy to get formal identifications by adding extra variables to the model, but if the effect of these variables on the system is small, then for small samples the standard errors of the coefficients will be so large that the situation is almost as bad as if the model were not identified. Unfortunately, nothing can be done to remedy this."[4] In the case of the classical system it may be difficult to suggest another variable whose effect on the system is not relatively small. In addition, for growth problems for which annual data are likely to be the best available, the sample size is almost certain to be small. It is not likely, therefore, that this approach would be particularly rewarding.

Failure to establish the identifiability of the system suggests that the simultaneous equations techniques are not applicable to the system as a whole, though they may be applicable to sub-sets of equations within the system. This

[4]Foote, op. cit., p. 61.

does not necessarily mean that the parameters can not be estimated, it just means that it would likely be difficult to estimate them in such a way as to take account adequately of the simultaneous relationships involved.

Having reasonably good evidence that the classical system is not identifiable as presented and having failed to find a way of making it identifiable, perhaps the next step would be to investigate estimation procedures other than the methods of simultaneous equations.

REDUCED FORM METHODS

The next approach to estimation of the parameters of the classical axioms might be directly through least squares procedures. Foote sounds a warning on this point too, however. He says that "... we obtain estimates that are statistically biased if we use the least squares approach to estimate the coefficients in an equation that contains current values of two or more endogenous variables." Each of the equations in the classical system does contain current values of two or more endogenous variables. Hence, a direct least squares approach is not likely to be too promising.

A modification of this approach, and one that is likely to be more promising, is the method of reduced forms. This method can be summarized in capsule form as follows: The first step involves expressing each of the endogenous variables as a function of all the exogenous variables in the system. The next is to fit each of these equations by

least squares. The final step is to derive the structural coefficients from the parameters of these reduced form equations.

This procedure, while conceptually similar to the simultaneous equations techniques and closely related to the problem of identification, is sufficiently different to require different treatment at this point. An example of this procedure should help to clarify some of the concepts involved and in so doing to lay the foundation for the examination of the method as a possible approach to the estimation of the parameters of the classical system. The following example is presented for purely illustrative purposes with no claims being made about its application to reality.

Suppose a system were postulated in which output (O) was a function of investment (I) with the level of known resources (K) treated as an exogenous variable. Suppose in addition that investment is a function of output with the stock of capital available in the previous period treated as exogenous. These equations could be written as follows:

(L_1) $O = b_{11}I + b_{12}K$ (Production function)

(L_2) $I = b_{21}O + b_{22}Q$ (Investment function)

These equations constitute a system of two linear equations and two endogenous variables. Application of the counting rules reveals that $K^{**} = G^{\Delta} - 1$ for each equation so that the system is just identified.

These equations can be expressed in reduced form by substituting L_2 for I in equation L_1 and L_1 for O in L_2. The following reduced form equations are obtained:

$$(L'_1) \quad O = \frac{b_{11}b_{22}}{1 - b_{11}b_{21}} Q + \frac{b_{12}}{1 - b_{11}b_{21}} K$$

$$(L'_2) \quad I = \frac{b_{22}}{1 - b_{11}b_{21}} Q + \frac{b_{12}b_{21}}{1 - b_{11}b_{21}} K$$

If these equations were fitted by least squares procedures the estimates obtained for the parameters of the reduced forms, referred to as b_{11} and b_{12} in the production equation and b_{21} and b_{22} in the investment equation, should be both consistent and efficient. Consistent in this sense means unbiased, and efficient denotes an estimate having the smallest possible variance.

The next step is to estimate the values of the parameters of the structural equations from these parameters of the reduced form equations. Dividing γ_{11} by γ_{21} provides b_{11}. Dividing β_{22} by β_{12} provides b_{21}. Substituting b_{11} and b_{21} into γ_{21} provides b_{22} and dividing b_{11} and b_{21} into β_{12} yields b_{12}. These estimates of the structural coefficients will be statistically consistent if the predetermined variables are known without error and if the unexplained residual terms each have a probability districation whose averages is zero, and whose variance is independent of the independent variables.[5]

[5]Foote, op. cit., p. 58.

The foregoing example illustrates the use of the method of reduced forms for a system that is just identified. For such systems it is possible to uniquely determine the structural coefficients from the parameters of the reduced form equations. For overidentified equations, on the other hand, at least two values of the structural coefficients are obtained. Neither value is statistically consistent and there is no direct way of deciding which answer to use.

This problem can be illustrated briefly by assuming that L_1 now contains a second predetermined variable, say technology. The production function would still be just identified but the investment function would now be over-identified. The reduced form equations would now be:

$$(L''_1) \quad 0 = \frac{b_{11}b_{22}}{1 - b_{11}b_{21}} Q + \frac{b_{11}b_{23}}{1 - b_{11}b_{21}} T + \frac{b_{12}}{1 - b_{11}b_{21}} K$$

$$(L''_2) \quad I = \frac{b_{22}}{1 - b_{11}b_{21}} Q + \frac{b_{23}}{1 - b_{11}b_{21}} T + \frac{b_{12}b_{21}}{1 - b_{11}b_{21}} K$$

The structural coefficient b_{11} could now be obtained by dividing the coefficient of Q in (L''_1) by the coefficient of Q in (L''_2) or by dividing the coefficient of T in (L''_1) by the coefficient of T in (L''_2). This problem occasioned by the over identification of an equation could be handled directly by one of the maximum likelihood approaches: Its relevance at this point, however, is with respect to the applicability of reduced form procedures in estimating the

structural parameters of the classical system.

APPLICABILITY OF REDUCED FORM METHODS TO THE CLASSICAL SYSTEM

Two problems arise with respect to the applicability of reduced form procedures to the estimation of the structural coefficients of the classical system. The first has to do with the identifiability of the system; the second with the form of the functions.

Problems of Idnetifiability

It will be recalled that the application of the counting rules to the classical axioms, on the assumption that they could be treated linearly, revealed that one equation was under identified, two were just identified and four were over identified. The fact that one of these equations is under identified would make it extremely difficult to apply the method of reduced forms to this particular case.

If two more exogenous variables were added to make the system identifiable then the method of reduced forms might have some merit. Many of the problems in the application of this method would then be associated with taking care of the problems of over identification. These problems are often not insurmountable, however. The researcher can often bring other information to bear on the selection of the best of the several b-values to use--information that was not included when the system was specified. This information may have been overlooked or it may have been purposefully omitted because of the computational problems involved. More

likely, however, with the advent of larger computing facilities, the equations would be fitted simultaneously by one of the maximum liklihood procedures.

If the system were modified so that it became just identified the method of reduced forms holds considerable promise. If it is just identified it is possible to solve by simultaneous equations techniques, but, for limited computational facilities, it is often easier to solve by reduced forms and least squares.

Problems of the Forms of the Functions

The form of the production function in the classical system raises a number of problems with respect to the application of the reduced forms approach. These problems are quite apart from the problems of identifiability. In order to obtain an equation for any of the endogenous variables in terms of all the exogenous variables (K and Q) it is necessary to substitute equations for the desired endogenous variables into the production function. To solve for I, for example, in terms of K and Q, equation L_7 was substituted for O to obtain

$$R + W = a_1 L^{b_1} K^{b_2} Q^{b_3} T^{b_4} .$$

Expressions for R, W, L, and T were obtained in terms of I, and these equations were substituted into the production function above. The following expression was obtained.

$$\text{Log}\left\{\frac{a_3}{b} + \frac{a_6 - b_9 a_6}{1 - b_9} + \left(\frac{1}{b_6} + \frac{b_{10} - b_9 b_{10}}{1 - b_9}\right) I\right\} - b_1 \log\left\{a_5\right.$$

$$+ b_9 a_6 + b_9 b_{10} I \Big\} - b_4 \log \Big\{ a_2 + b_5 I \Big\}$$

$$= \log a_1 + b_2 \log K + b_3 \log Q.$$

Examination of this function reveals that it would be difficult to solve for I in such a way that it would be possible to determine the values of the structural coefficients from the values obtained from the parameters of the reduced form thus obtained.

After a number of attempts at solution, the same type of expression was obtained for each of the other five endogenous variables, and the same type of conclusion follows. It would be extremely difficult to estimate the values of the parameters of the structural equations of the classical system by the method of reduced form.

THE USE OF LAGGED VARIABLES

Before leaving the questions of whether the structural coefficients can be estimated by modern statistical methods, and which methods might be most appropriate for any given system of equations, it would be well to consider the use of lagged variables as a means of making a model identifiable. This consideration has been avoided up to this time, in the treatment of the classical axioms, in order that considerations with respect to the simultaneous equations and reduced forms procedures might be illustrated first.

The use of lagged variables, in the classical system, may be considered as one means of adding variables. It might

be argued, for example, in equation L_5, that the labor force is a function of the wage fund in some past period rather than the wage fund in the current year. If it is argued that population expands as a result of increased real incomes to working class families and hence increased ability to support larger families, then it follows that this increased population will not have its effect on the labor market until the children of this era have attained working age -- perhaps eighteen to twenty years after the increase in the wage fund. Similarly, it might be argued that entrepreneurs make their decisions to invest, not on the basis of profits in the current production period, but on the basis of some accustomed rate of profits. This accustomed rate might be a moving average of the past three years, for example. If it were thought that the use of these lagged variables were likely to account for a significant part of the unexplained residual, then R_1 could be used to replace R in equation L_3 and W_1 to replace W in L_5, and the system would then be:

L_1 $\quad 0 = a_1 L^{b_1} K^{b_2} Q^{b_3} T^{b_4}$

L_2 $\quad T = a_2 + b_5 I$

L_3 $\quad I = -a_3 + b_6 R_1$

L_4 $\quad R = a_4 + b_7 T - b_8 L$

L_5 $\quad L = a_5 + b_9 W_1$

L_6 $\quad W = a_6 + b_{10} I$

L_7 $\quad O = W + R$

Application of the counting rule now provides the following results:

Equation	K**	G^{Δ}	$G^{\Delta} - 1$	Degree of Identification
L_1	2	3	2	Just identified
L_2	4	2	1	Over identified
L_3	3	1	0	" "
L_4	4	3	2	" "
L_5	3	1	0	" "
L_6	4	2	1	" "
L_7	4	3	2	" "

The production function would now be just identified and the other equations over identified. On the assumption that the system can be treated linearly, the addition of these two lagged variables would permit estimation of the coefficients by simultaneous equations techniques.

If the addition of these two lagged variables permits the estimation of the coefficients then the next major question is whether these estimates are likely to be reliable enough to permit useful prediction and explanation. The answer to this question depends, in part, upon the accuracy and reliability with which the variables can be measured. Attention will be turned at this point to a brief consideration

of these problems.

PROBLEMS OF MEASURING THE VARIABLES[6]

It was pointed out at the first of this treatment of the question of empirical adequacy of a theory that the statements in the explanans must all be true. It was also indicated that both tautologies and contingencies were permitted in the explanans. In addition it was argued that at least one of the statements in the explanans must be contingent if the theory is to have any empirical import. It was pointed out in Chapter IV that these contingent statements are often expressed in equation form once a reasonably high level of formalization has been attained. The truth of these equation forms depends on both the accuracy and reliability with which the parameters are estimated, on the one hand, and the accuracy and reliability with which the variables are measured, on the other.

The accuracy and reliability with which the parameters are estimated determines, in part, the success of the statements in prediction. The closer the predicted values come to the observed values of the dependent variable, the more justification there is for considering them as the true (or highly confirmed) values of the parameters. A

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[6]The concept of a variable is used here in the usual manner in which it is applied in economics to refer to any statistic like population, land, capital, etc.; these statistics usually represent a characteristic of the phenomenon to be predicted or explained.

set of parameters could be taken as true parameters if they permit the researcher to make estimates of the dependent variable which come close enough to reality to be accepted as true.[7] It is in this sense that the problems of estimating the parameters of a given statement or theory affect the truth or falsity of the statements in the explanans and hence the explanatory and predictive potential of the theory itself. It is for this reason that any exhaustive investigation of a theory generally requires consideration of the problems of estimating the parameters.

Another major determinant of the truth of the statements in the explanans, and hence of the predictive potential of the theory, is the accuracy and reliability with which the variables can be measured. The problems of measuring the variables are essentially different from the problems of estimating the parameters. In many cases, however, the unexplained residual associated with the parameters may be due as much to errors in measurement of the variables as to the accuracy and reliability with which the parameters are estimated. Hence questions of the measurability of the variables, and of the accuracy and reliability with which they can be measured, are of primary importance in determining the truth of the propositions in question. The first requirement is that they must be measurable in principle; the

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[7]This acceptance as true is likely to be conditional on compliance with some kind of decision rule. This rule in turn is likely to be couched in terms of the extra costs of additional accuracy and the consequences of inaccuracy.

second is that they must be measurable with sufficient accuracy to permit reliable estimation of the parameters.

In any a priori analysis, then, one of the first questions one might ask is whether the variables are measurable. If the answer is negative for a particular variable then the statements containing it are not readily amenable to corroboration. These statements cannot be significantly denied and hence cannot be meaningfully asserted. It follows that to show that a given variable is not measurable is to show something particularly damaging about the current status of the theory's explanatory and predictive potential, provided that a lawlike statement containing this variable is required in the structure of explanation appropriate to the problem at hand. This is not to say anything particularly conclusive, however. Variables that are classified as not measurable at one point in an analysis may well become measurable at some later date. It is only after it is recognized that a certain variable is likely to be important that any research effort is devoted to the development of the techniques of measurement. It is likely to be more fruitful, therefore, to think of the values of the variables as being either measurable or not currently measurable, that is, non-measurable, rather than not measurable in any absolute sense.

The effect of non-measurability becomes more apparent when it is recalled that most economic theories that are

ready for empirical application are couched in terms of equation forms. Their explanatory and predictive performance requires that they determine the values or the direction of change in the endogenous variables given the values or the direction of change in the exogenous variables. If the values of these variables are not measurable at least in ordinal if not cardinal terms then this function cannot be performed and their empirical usefulness is seriously curtailed.

It would seem to follow that unless the variables in a system are currently or potentially measurable, that system can provide very little information about reality. It would be useful, therefore, if a set of criteria or techniques could be developed which would enable the researcher to ascertain whether a given variable is measurable. If it is measurable, then it becomes important to determine whether it can be measured with sufficient accuracy and reliability to permit useful explanation and prediction, and whether the tasks of measurement can be performed at less cost than the value of the information obtained.

MEASURABILITY AND THE EMPIRICIST THESIS

One possible approach to the question of measurability of a given variable would be to determine whether it is observable. If it is observable then it might be argued that even though the value of a given variable is not currently measurable, it may become measurable. On the other

hand, it might conceivably be thought that if it is not observable it is not likely to be measurable. The connection or lack of connection, between observability and measurability has led to a number of important questions in both economics and analytic philosophy.

Few economists would argue, for example, that utility is observable, but many have argued that it is measurable. Orthodox members of the cardinalist persuasion have argued that utility is in fact measurable. Their more modern colleagues have argued that it is at least measurable in principle. The ordinalists, on the other hand, have argued that it is not measurable either in fact or in principle. The existence of such polemics suggests that methodologically important questions of measurement are not likely to be amenable to quick and easy solution in terms of the observability of the variables.

Problems with respect to the relationship between observability and measurability have also had a long history in analytic philosophy. The earlier forms of positivism or empiricism held that: "Any term in the vocabulary of empirical science is definable by means of observation terms; i.e., it is possible to carry out a rational reconstruction of the language of science in such a way that all primitive terms are observation terms and all other terms are defined by means of them."[8] Hempel refers

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[8]Carl G. Hempel, "Fundamentals of Concept Formation in Empirical Science," International Encyclopedia of Unified Science, Vol. 2, No. 7, (Toronto: University of Toronto Press, 1952), p. 23.

to this notion as the "narrower thesis of empiricism."

The thesis is fraught with several difficulties, however, These center, according to Scheffler, around the use of theoretical terms and dispositional terms. Scheffler claims that the heart of more modern empiricism has been its doctrine of empirical meaning, with its sharp line between the verifiable and the univerifiable and its rejection of non-analytic, non-experiential statements as nonsense. He goes on to point out that this approach to ferreting out the nonsense propositions presents some difficulties. The problem is to ascertain whether a given statement about a particular variable expresses a genuine proposition about a matter of fact. The analytic question behind this problem is whether any of the current or proposed definitions of empirical significance or meaningfulness fulfill the criteria of meaningfulness in such a way as to permit the researcher to judge the significance of any candidate statement. While this battle rages and until its outcome is known, the problems of recognizing those variables which are and which are not meaningful, and which are or are not measurable, remains to plague the practitioner.

Hempel's analysis of this thesis comes to bear at two points: the concept of dispositional terms, and the concept of quantitative terms. Both of these concepts will be examined below.

A dispositional term designates, not a directly

observable characteristic but rather a disposition, on the part of some physical object or entity to display some specific reaction under certain specifiable circumstances. The 'marginal propensity to consume' is a case in point. It is not a directly observable characteristic but a tendency on the part of consumers to allocate their income in a given pattern under different given levels of income. The concepts of 'liquidity preference' and 'economic rationality' fall into the same category.

Hempel points out the problems associated with attempts to define dispositional predicates, like those mentioned above, by means of observation terms. These problems hinge on the use of the conditional statement form in the sense of material implication.[9] These problems direct attention to the pathbreaking work of Carnap in his introduction of the concept of 'reduction sentences' as a means of avoiding these difficulties.[10] The problems with the use of reduction sentences, however, is that they do not offer a complete definition. They only offer a partial, or conditional, determination of its meaning under specified test conditions. This indeterminacy of meaning can be reduced by the use of additional reduction sentences but in general, a set of reduction sentences only partially determines the empirical meaning of a dispositional predicate.

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[9]Hempel, op. cit., p. 25.

[10]Rudolf Carnap, "Testability and Meaning," Philosophy of Science, III (1936), pp. 419-471.

The other major problem area, associated with the idea of solving questions of measurability by defining these concepts with the exclusive use of observation terms, involves the treatment of the metrical terms. These terms are used to represent measurable qualities like length, mass, temperature, value, etc. These measurements may be expressed by any positive real numbers. Hempel argues that it is not possible to fully define these concepts purely in terms of observables. Here again Carnap's concept of a reduction sentence, or more properly the concept of an inductive chain of reduction sentences has been adduced to fill the gap.

In view of the problems associated with the use of dispositional terms and metrical terms it would appear that the 'narrow thesis of empiricism' may have to be abandoned in favour of what Hempel calls the 'liberalized thesis of empiricism.' This thesis holds that every term of empirical science can be introduced, on the bases of observation terms, by means of a suitable set of reduction sentences.[11]

The problem with this "liberalized" thesis is that it may perhaps be too 'liberal' to rigorously bridge the gap between the concepts employed in science and the requirement of the measurability of these concepts. The central point in this concern is that these sets of reduction sentences are not definitions. They provide both a necessary

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[11]Hempel, op. cit., p. 31.

and a sufficient condition for the use of terms, but these two conditions do not necessarily coincide as they do in the case of definition. It follows that the use of reduction sentences as a means of definition does not satisfy all the requirements of definition.

In view of the failure to adequately define dispositionals and metricals in observation terms or by use of reduction sentences, and in view of the vast body of literature dealing with these and related problems of concept formation in empirical science, it would appear reasonable to conclude that an adequate criteria for recognizing those terms which are empirically meaningful has not been conclusively established.[12] Since solution to these questions of meaningfulness and observability appears to be, in some senses at least, prior to the solution to questions of measurability, it would seem to follow that definitive solutions to questions of measurability will have to await the fruits of more comprehensive analyses of the problems the empiricists confront.

It would also appear, however, that there are a number of concepts of measurement which may permit a more direct approach to the problems of measurement than that provided by the traditional empiricist methods. Evidence for the need of such an approach is readily forthcoming from even

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[12]See Hempel, op. cit., bibliography.

the most cursory examination of the polemics associated with the measurability or non-measurability of 'utility', 'technological change', and 'management' as these variables are treated in agricultural economics. A selection of these concepts will be examined below.

CONCEPTS OF MEASUREMENT

Any science, like economics, in which mathematics is applied, involves both counting and measurement. Counting involves discrete objects and the establishment of a one-to-one correspondence between these objects or entities and numbers. The population of a country, for example, is counted, not measured. Measurements, on the other hand, are always concerned with a continuously variable property of objects of processes such as weight, length of duration, rate of return, etc.

The fact that measurement makes sense only with respect to variable properties can be seen by comparing one variable like 'income' with another like 'national image.' It does not make sense to say of one country that it has more national image than another. It does, however, make sense to say that one country has more income than another. That is, to say, properties like the size of the national income can be ordered by a comparative relation, whereas properties like national image cannot. It should be pointed out in passing that such things as the national image do have measurable dimensions such as the per capita income, the general

level of education, etc., but then it is these properties, not the concept of the national image itself, that can be arranged in order. For a property to be measurable, then, there must be some determinable characteristic which appears in determinate form and which can be ordered in terms of 'more' or 'less', like more weight or less weight, for example. The term 'quality', for example, generally refers to a determinable property of a determinate form but is not measurable because its determinate form cannot be ordered in terms of 'more than' or 'less than'.[13] Quantity, on the other hand, is usually a determinable property of a determinate form that is measurable in that different degrees of this determinate form can be ordered. Finally, certain properties like 'expectations' are not measurable in that there is no determinable property of which it might be said that one was more or less than another. In summary, in order for a property to be measurable, it must be determinable, it must be determinate, and it must be possible to put determinate forms in some kind of order.

These concepts of metricization are defined more rigorously by Hempel in his sections on 'Fundamental Measurement' and 'Derived Measurement'.[14] An intuitive notion of these concepts, starting with the concept of order, is

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[13]See Arthur Pap, An Introduction to the Philosophy of Science, (New York: The Free Press of Glencoe, 1962), pp. 125-135.

[14]Hempel, op. cit., pp. 62-74.

presented below.

A number of the standard treatments of measurement in analytic philosophy deal with two types of measurement, fundamental measurement and derived measurement. One of the basic concepts in fundamental measurement is the concept of quasi-serial ordering. This is an array that is serial except that several elements may occupy the same place in it. The basic notion of quasi-serial ordering of properties is that variables can be put in order of size, weight, or whatever characteristic is being measured. This ordering requires the specification of criteria of 'coincidence' and 'precedence.'[15] In operational terms, these criteria mean, in the case of mass, for example, that if for any two elements the first outweighs the second, then the second has precedence over the first. If, on the other hand, the first just balances the second, then the first is coincident with the second.

These two conditions satisfy the requirements of quasi-serial ordering, but they do not result in measurement. The next step is to metricize the quasi-serial order by assigning one real number to each element in the set in such a way that all elements of the same size will have the same number, and, for all pairs of elements, the larger element of each pair has the larger number. One of the crucial

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[15]See Pap, op. cit., p. 131.

phases in determining the values to be assigned in this manner is in selecting some specific way of combining any two elements in the set into a new element and stipulating that the number assigned to the new element is the arithmetic sum of the numbers assigned to its component elements.

Derived measurement, on the other hand, involves the determination of a metrical scale by means of criteria which presupposes at least one previous scale of measurement.[16] Derived measurement may be considered in two separate phases -- derived measurement by stipulation, and derived measurement by law. Derived measurement by stipulation, consists of defining some "new" quantity by means of others which are already available. For example, the capital-output ratio used in theories of economic development may be defined in terms of the capital stock of the nation and its gross national product. Derived measurement by law, on the other hand, generally introduces an alternative method of measuring some quantity that has already been introduced. This generally consists of discovering some law which represents this quantity as a mathemetical function of some other quantity for which methods of measurement are already known. Some of the best examples of this type of measurement are found among the physical sciences. The measurement of temperature by means of a thermocouple or

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[16]Hempel, op. cit., p. 70.

altitude by the use of a barometer should suffice to illustrate the concept.

The foregoing concepts of measurement together with a knowledge of the empiricist thesis provides some of the background required for an examination of questions of measurability. This background is supplemented with the considerations of partial formalization presented in Chapter II and with empirical knowledge of the variables to be employed. Incomplete as this information may be, it provides an illustration of some of the concepts required in this particular phase of a priori analysis.

MEASURABILITY OF THE CLASSICAL VARIABLES

Examination of the classical variables provides an illustration of several of the problems raised above. This brief examination will be considered in two phases -- the first will consider the problems of measuring technological change and the second will consider the other variables as a group.

Technological progress is generally thought to include such things as changes in organization and in ways of producing products and services. It often includes the introduction of new products and of new inputs. Sometimes it results in the production of more of a given product, or a different kind of product, from the same input, or from different combinations of the same or different inputs.

In view of the discussions of partial formalization

initiated in Chapter II and continued in Chapter IV it becomes apparent that this complex concept of technological advances is introduced into the classical theory without explicit definition and without a set of statements specifying a sufficient condition for its use. It would seem to follow, therefore, that the concept is introduced as a relative primitive in the classical system. This type of introduction into a system affords little evidence as to the measurability of the variable introduced.

One approach to ascertaining the measurability of such a complex variable is to ascertain which of its several characteristics are determinable, and determinate, and whether they can be put in order. If a quasi-serial order can not be established then it is not possible to measure the characteristics. In any attempt to establish a quasi-serial order it is necessary to have a technique for deciding, for any pair of determinate forms, whether they coincide or whether one takes precedence over the other. In the case of the general level of technology, it would seem apparent that it is clearly possible to recognize different levels of technology applied at the same time in different countries and at different times within the same country. To this extent it could be argued that, taken as a whole, the concept is determinable. The next question is whether there exists a determinate form of one or more of the characteristics of the general level of technology such that it

is possible to ascertain whether this characteristic coincides with or takes precedence over the others. The answer to this question again seems to be positive. It is to be expected that it could be possible to construct a labor-capital ratio, for example, to represent one aspect of differences in technology. In the same way, it would be possible to construct other ratios to facilitate comparison of other aspects of technological progress. The next question becomes one of whether it is possible to weight these indicators and to aggregate them in such a way as to adequately reflect differences in technology. If it is, then one would expect it would be possible to eventually carry out the other steps required for measurements.

There is some real question, however, whether these tasks of weighting and aggregating are likely to be accomplished in the forseeable future. Johnson, for example, attempts to avoid these problems by taking advantage of the notion that the so-called upward shift in the production function resulting from new technology is generally attributable to a new input or to the use of a new combination of old inputs. He argues that, in these cases at least, it becomes important to identify the new factor or new combination of factors and to treat it directly in the analysis.[17]

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[17]Glen L. Johnson, "A Note on Nonconventional Inputs and Conventional Production Functions," Agriculture in Economic Development, ed. Carl K. Eicher, Lawrence W. Witt, (New York: McGraw-Hill, 1964), pp. 121-122.

This assumes that there are characteristics of these new inputs or new combinations that are measurable. If there are, then this direct treatment of the inputs themselves would appear to be more promising than attempting to measure technological change, as such, and treating it as an explicit variable in the aggregate production function.

Technological change is only one of the variables in the classical system which is beset with problems of measurement. As in the case of technological change, all of these concepts were introduced as relative primitives in the system. Unlike technological change, on the other hand, they are all amenable to some form of measurement. The statistics on gross national product, for example, can be used as a measure of output. The labor statistics provide a measure of the labor force. In the same way, other sources of information provide indices for each of the other variables. For each of these variables, however, the questions of accuracy and reliability present a whole series of problems. Each of these variables is an aggregate of a complex set of inputs or products. Each is an index with all the problems of index numbers and time series data.

The problems that attend the observation collection and aggregation of these data are serious enough in the advanced countries; they are often more serious, however, in the underdeveloped areas in which this particular theory is to be applied. These problems include the difficulty of

obtaining and training skilled and reliable enumerators, processors and analysts. These problems of data collection and analysis together with the relatively short periods for which these data are available in most underdeveloped areas makes it extremely difficult to obtain reliable statistics. It also makes it extremely costly to improve the accuracy and reliability of the data already available. This extreme difficulty and high cost of obtaining reliable data places a premium on careful a priori analysis of putative theories, prior to commiting the resources required to obtain reliable measures.

CHAPTER VIII

SUMMARY

PURPOSE OF THE STUDY

The purpose of this study was to investigate ways and means of evaluating the explanatory and predictive potential of theories. Attention was focused on economic theories in general, with special reference to theories of economic growth. The main focus of the study was on the processes of evaluation, rather than on the techniques of testing commonly applied to theories. This process of evaluation includes the broad scope of inquiry that may be conducted prior to the collection of data and the empirical testing of hypotheses. It concentrates on the logical structure of theories, on the one hand, and on the empirical adequacy of sets of hypotheses, on the other.

It was recognized, at the beginning of this inquiry, that the process of a priori evaluation of theories has been carried out ever since the inception of science and that therefore it might be extremely difficult to say anything really new in this area. It was also recognized that the concepts that are likely to be most useful in the a priori evaluation of theories are drawn from a wide cross section of mathematics, statistics and analytic philosophy.

It was further apparent that few of these concepts have ever been drawn together into a unified approach to the problems of theory evaluation. It was hoped, at the same time, that each of these sets of concepts, drawn from different disciplines, could be clarified by their simultaneous application to these problems. Successful selection of these concepts, and their synthesis into a workable framework of analysis would constitute an important contribution to research methodology in economics. It was therefore hoped that, as a minimum, some of the more relevant questions of research methodology might be raised and some of the more promising approaches suggested.

METHOD OF ANALYSIS

The general method of analysis, employed in this study, involved forays into analytic philosophy, mathematics, and statistics. These forays were directed at selecting those concepts and tools likely to be most useful in the evaluation of the explanatory and predictive potential of theories. The use of these analytical tools and concepts was then illustrated by application to the classical theory of economic growth.

The main body of this study was initiated with an investigation of the logical structure of science. This was built up from a consideration of logical terms and descriptive terms and the manner in which these terms are combined to provide the empirical assertions of science. This lead to a

consideration of the logical structure of explanation and prediction and to the specification of a set of criteria for the logical and empirical adequacy of such structures. This lead to a consideration of empirical laws or lawlike statements as they function in science, and to an examination of theories. It is the function of these theories to provide those empirical laws employed in explanation and prediction. This analysis of theories involved an examination of formal structures and the relationship of deductive subsumption. This developed into a consideration of the formalization of theories. Recognizing that most economic theories are not currently amenable to easy and fruitful full-formalization, attention was turned to the concepts of partial formalization. This investigation culminated in an examination of the concept of a model and its use in theory construction and in theory evaluation.

The next phase of this study involved an illustration of the application of these concepts to the problems of theory evaluation. The theory chosen for examination was the classical theory of economic growth. This theory was chosen because it seemed to be sufficiently recalcitrant to facilitate the illustration of a number of aspects of the framework of analysis being developed. The basic concepts of the theory were presented in Chapter III. This presentation was followed by a brief examination of some of the logical consequences of this statement of the theory. This

examination provided a set of deductions which seemed to agree with the empirical evidence available from some of the underdeveloped areas and with a larger body of theories of economic growth. Further investigation suggested, however, that the theory contained an inconsistency. It revealed, for example, that one set of statements implied a decrease in output and, at the same time, an increase in output. Further investigation also revealed that the same types of implications hold for each of the variables in the system.

This discovery of an apparent inconsistency lead to an investigation and attempted application of a number of techniques for demonstrating inconsistency. These included an attempt at rigorous deduction of an inconsistency through the use of the calculus of propositions. It also included an attempted application of the method of models. None of these initial attempts at evaluation were definitive. They did, however, point up the necessity of clarifying the statements of the theory before any more conclusive analysis could be accomplished.

The first phase of this process of clarification was conducted in the latter part of Chapter III. It involved the restatement of the classical axioms in terms of first differences. This more rigorous statement of the classical axioms permitted the direct application of the tools of formal logic presented in the calculus of propositions. This

illustrated the point that, in the process of theory evaluation, the clarification and refinement of the theory itself goes hand in hand with the development and application of more powerful techniques of analysis.

In addition, it demonstrated that this particular formulation of the theory is inconsistent. This result is not entirely conclusive, however. Whenever a theory is stated with the vagueness and imprecision which characterizes the Higgins version of the classical theory of economic growth, it becomes extremely difficult to ascertain whether any given reformulation of the theory adequately expresses the meaning intended by the author. If it does not, then that restatement involves the specification of a different theory. It is different, even though it is about the same set of phenomena. This uncertainty about the adequacy of this particular reformulation of the classical axioms leads to the conclusion that unequivocal results could not be obtained until the theory had been reformulated. This reformulation had to be in terms that are more precise than those obtained by specification in terms of first differences. It is only after such additional precision is accomplished that more powerful sets of analytic techniques can be brought to bear on the problems of theory evaluation. The remainder of this study is therefore devoted to the development and application of the techniques of partial formalization, on the one hand, and of a more powerful framework of analysis,

on the other.

A number of the techniques of partial formalization introduced in Chapter II are employed in the clarification and specification of the classical theory of economic growth conducted in Chapter IV. The main focus of this chapter is on the nature of the functional relationships involved. Starting with the information available in the Higgins version of the theory, supplementing this with information drawn from other sources, and employing the techniques of systematic presupposition and quasi deduction, an attempt was made at specifying the shape of each of the functional relationships employed. In view of the paucity of information available, this reformulation is at best only highly tentative. It does, however, provide an illustration of some of the approaches to partial formalization. In addition, it provides an example which can be used to illustrate the application of the framework of analysis to be developed in Chapter V.

Unsuccessful attempts at the direct application of the method of models, to the formulation of the theory presented in Chapter III, suggested that the individual proposition itself rather than the whole set of propositions that constitute a theory might be a more appropriate unit of investigation. Attention was therefore turned, in Chapter V, to a set of techniques which the author has called 'the analysis of propositions'. This analysis concentrates on

the individual proposition as the unit of investigation, and on the calculus of propositions as the technique of analysis. There are a number of more powerful calculi which might have been used for this purpose, but the calculus of propositions was chosen in order to keep the logical apparatus at a minimum and at the same time to provide the classification system developed in Chapter V.

The key concepts developed in Chapter V are built around two pairs of dichotomies. These dichotomies are based on the truth value characteristics of statement forms. Every statement form yields a statement which is either valid or non-valid, but not both. Every statement form is either consistent or inconsistent, but not both. Furthermore, every statement form, has exactly two of these four characteristics. It follows that every statement form is one or the other of the following three types: A statement form may be valid and consistent in which case every substitution instance yields a true statement. These types of statement forms are called tautologies. Alternatively, a statement form may be non-valid and consistent in which case some substitution instances may be true and some substitution instances may be false. These types of statement forms are called logically indeterminate, or contingent. Finally, a statement form may be non-valid and inconsistent in which case every substitution instance yields a false statement. These types of statement forms are called inconsistencies. The final combination,

valid and inconsistent, would mean both always true and always false. This combination is logically incompatible. Hence, the three pairs of characteristics listed above provide a mutually exclusive and exhaustive set of characteristics of statement forms.

Each of these three types of statement forms plays a different role and has a unique effect on the logical structure and on the empirical assertions of science. These roles and effects are discussed in Chapter V. They are explained in more detail in Appendices A, B and C. These discussions are summarized below. It was demonstrated in Chapter V that the presence of an inconsistency permitted the theory to imply anything at all, true or false, sensible or nonsensical about the phenomenon in question. Hence the presence of a single inconsistency renders the theory useless for explanatory and predictive purposes. The role of the valid statement forms, on the other hand, is mainly in the provision of the rules of deductive logic. The third type of statement form, the contingency, may yield statements that are true and statements that are false. Substitution instances of this latter type of statement form comprise the empirical assertions of science.

In view of the roles and the effects of each of these types of statement forms in science, it would be useful to be able to recognize any given type of statement form. The criteria of logical and empirical adequacy require, among

other things, that the explanans imply the explanandum. It also requires that the explanans must have empirical content. This means that at least one of the statements in the explanans must be contingent or synthetic. That is, they cannot all be analytic even though the relationship between the explanans and the explanandum must be that of logical implication. Finally, each of the statements, taken individually, must be consistent, and the whole group, taken as a unit must be consistent.

The assymmetry of the truth value characteristics of statement forms described above provides the key to the development of methods of recognizing each of these types of statement forms and hence of assessing the explanatory and predictive potential of a theory. It will be recalled from this classification that all tautologies are true statements. Hence, if one finds a substitution instance which is not true, then that statement form is not tautologous. All that is required to prove that it is not tautologous is to find one substitution instance which is not true. Any number of true substitution instances, on the other hand, will not prove it tautologous. In the case of inconsistencies, every substitution instance is false. If one succeeds in finding one substitution instance which is true, then that statement form is not inconsistent. Since it must be either consistent or inconsistent, it therefore follows that it is consistent. Hence, finding a single substitution instance

which is true is enough to prove consistency. Finally, in the case of contingent statement forms, if one can find one substitution instance which makes it true, and one substitution instance which makes it false, then it is contingent.

In the case of economic theories, however, it is often difficult to analyze the statements in such a way as to adequately exhibit the statement forms involved. As a result, it is difficult to ascertain the truth value characteristics of the individual statements employed. These difficulties are compounded when attention is turned to whole sets of statements as they function in theories. The concepts summarized above provide a basis for the types of techniques developed for use in the evaluation of such theories.

Most theories of economics require the use of metric concepts. The logical analysis of statements employing these concepts usually requires the use of quantification theory. This requires the skill of a professional logician. The same kind of results can sometimes be obtained, however, without this arduous type of analysis. They may be obtained by the use of the concepts presented above together with a minimum set of concepts drawn from mathematics.

It was argued above, for example, that any statement or set of statements is either consistent or inconsistent. In addition, any statement or set of statements which is inconsistent is always false. In terms of equations, this

means that every equation or set of equations which is inconsistent is false for every possible substitution of values for its free variables. If, in examining the consistency or inconsistency of any equation or set of equations, one finds a set of values for the free variable such that all equations in the set are true simultaneously, then that equation or set of equations is not inconsistent. If it is not inconsistent, it must necessarily be consistent.

This is the basis for the approach employed in the Method of Numerical Interpretation as presented in Chapter VI. The statements of the classical axioms were presented in equation form with values of the parameters assumed. Then an attempt was made to find a set of values for the variables such that all equations would be true simultaneously. Success in finding such an assignment of values is enough to prove the consistency of this particular formulation of the theory. Failure to find such an assignment of values simply leaves the problem of consistency open to question.

It should be remembered at this juncture that each new set of values for the parameters of a set of equations constitutes a new set of statements about the phenomenon in question. Each new set of statements constitutes a new theory, even though it is about the same set of phenomena. Hence the proof of consistency for one set of values for the parameters does not necessarily prove that other sets of parameters, that is, other theories, would also be consistent.

It would, therefore, be extremely useful to be able to ascertain the range of values for the parameters which would permit a theory to be consistent. If it is not possible to ascertain the range of values, then it is to be hoped that it would at least be possible to obtain some assessment of the likelihood of a set of general equations taking on a set of values for the parameters which would provide a consistent theory. The Method of Numerical Interpretation is of limited use in this regard. It assumes a set of values for the parameters a priori, and is used to assess the consistency of the equations involving this particular set of parameters. What is required is a set of procedures which can be applied to a set of general equations in order to obtain some a priori assessment of the likelihood of their taking on a set of consistent parameters. One approach which appears particularly promising for linear systems may be called the Test of Determinants.

This test employs the notion that the expansion of the determinant of a set of linear equations must be non-zero in order for that set of equations to be consistent. The first step in applying this concept to the problem of assessing the consistency of a set of general functions is to solve for the determinant of the system. The next step is to write an inequality which specifies that the expansion of the determinant is not equal to zero. The final step is to examine this expression to determine the likelihood of the

parameters taking on a set of values which would satisfy this inequality.

This technique is illustrated by its application to the formulation of the classical axioms presented in Chapter VII. It is applied first to the linear sub-system comprised of equations two through seven. Examination of this sub-system revealed that there are a very large number of possible values and combinations of values for which the determinant would be non-zero, that is for which these equations would be consistent.

The next step in the examination of the classical axioms was to assess the implications of the expansion of the determinant of the sub-set for the classical system as a whole. This was accomplished by substituting functions of the parameters for the endogenous variables in the aggregate production function. In view of the fact that the values of all the endogenous variables in the system are determined by the sub-system independent of the production function, it would be only by chance that the values of the parameters would satisfy the production function. It seemed unlikely, therefore, that the classical system would be consistent.

This application of the Method of Determinants to the classical system illustrates its possible role in the process of theory evaluation in general. One of the primary requirements of the logical adequacy of a system is that it be consistent. The traditional methods, employed in

economics, for treating consistency concentrated on efforts to deduce a statement and its negation or to deduce a contravalid statement. The Method of Numerical Interpretation, on the other hand, concentrates on giving the theory an interpretation which is analytic, in fact, one which is analytically true. The fact that such a theory is consistent follows as a corollary of the concept that, if it were inconsistent, no true interpretation would be possible. This method has limited a priori value, however. Its application requires knowledge of, or at least the assumption of, given values for the parameters. The Method of Determinants, by contrast, states a necessary condition for consistency. This condition is stated in terms of an inequality comprising a general or unspecified set of parameters. Examination of this equation gives some concept of the likelihood of these parameters taking on a set of values that will satisfy this inequality and hence render the system consistent. This method, while most generally applicable to linear systems, can be extended to cover monotonic functions as well. This extension increases the applicability of the Method of Determinants to a substantial number of extant theories in economics.

The final phase of this study is developed in Chapter VII. It is concerned with the elaboration and application of techniques which can assist the researcher in assessing the empirical, in contrast to the logical,

adequacy of theories. These techniques centre around the problems of estimating the parameters of a theory, and of measuring the variables involved.

Consideration of the problems of estimating the parameters, given the values of the variables, comes to focus on the problems of identification, and on tests for recognizing the identifiability of sets of equations. Failure to meet the rigorous requirements of identifiability turns attention toward methods of modifying the structure in order to obtain identifiability. Failure to make satisfactory adjustments in the system ultimately directs attention to alternative types of estimation procedures. As in the case of the method of determinants applied to the problems of logical adequacy, these techniques have never been fully developed for systems of non-linear equations, except for the monotonic functions.

The questions of the measurability of the variables, on the other hand, are likewise fraught with a variety of problems both logical and empirical. These problems seem to centre around the empiricist criteria of meaningfulness and the problems of establishing quasi-serial orders and appropriate scales of measurement.

These problems of measurement, together with the problems of estimation of the parameters, constitute two of the major areas of research methodology in need of intensive future development. This kind of development is of primary

importance in assessing the empirical adequacy of extant theories and in the development of more powerful sets of hypotheses.

In summary, there are a number of things that can be accomplished in the a priori evaluation of theories. These things center around the thesis that the logical and the empirical adequacy of a theory depends upon the truth of the individual propositions employed and the truth preserving characteristics of the logical relationships that obtain among these propositions.

The logical adequacy of the theory depends, in part, upon the consistency of the structure and the implicative relationships between the axioms and the theorems employed. This study demonstrates that, contrary to the commonly held misconception that it is not possible to prove consistency, it is in fact possible to prove consistency directly. The Method of Numerical Interpretation was developed for this purpose. It is based on the concepts of formalization elaborated in Chapter II and the analysis of propositions developed in Chapter V. This Method of Numerical Interpretation is flexible enough to be applicable to a wide variety of function, including many non-linear functions, as long as they are virtually completely specified.

A second method of examining the consistency of a system involves the use of the Method of Determinants. This method is applicable to sets of statements which are stated

in general form and is therefore more applicable than the Method of Numerical Interpretation in the early phases of analysis. It is currently applicable to linear functions and to certain monotonic functions. Its extension to other systems of non-linear equations will have to await the development of this phase of mathematics.

The question of empirical adequacy, on the other hand, depends in large part, upon whether or not it is possible to measure the variables. The question of whether it is possible to estimate the parameters, like the question of consitency, is a matter of logical structure. The rank and order conditions required for the identifiability of a set of equations is another aspect of the problems of consistency. The determinant of the system must be non-zero in both cases. While this can be determined for linear systems, and for certain monotonic functions, neither the criteria, nor the techniques of analysis, are available to handle these questions for systems of non-linear, non-monotonic, equations.

Progress in the a priori evaluation of theories is likely to go hand in hand with the development of both analytic philosophy and mathematics -- particularly those branches of mathematics required to handle the complicated systems of non-linear, non-monotonic, functions required in the development of more powerful theories.

APPENDIX A

SENTENCE CONNECTIVES, TRUTH TABLE TECHNIQUES, AND A CLASSIFICATION OF STATEMENT FORMS

SENTENCE CONNECTIVES

The discussion in Chapter V defining and explaining the meaning of this group of sentence connectives is summarized in the following table. The left hand section of the table presents all the different possible combinations of truth values of the constituent propositions. These are referred to as the truth conditions. The right hand portion presents the truth value characteristics of each of the different statement forms under each of the different sets of truth conditions. The whole table is referred to as a truth table.

p	q	~p	p.q	pvq	p⊃q
T	T	F	T	T	T
F	T	T	F	T	T
T	F	F	F	T	F
F	F	T	F	F	T

The foregoing truth table was presented in order to exhibit the role that the sentence connectives play in determining the truth values of each of these four kinds of statement forms. The next step is to extend the use of the

truth table technique to the point where it becomes a method for establishing the truth value characteristics of statement forms in general.

THE TRUTH TABLE TECHNIQUE IN ASCERTAINING TRUTH VALUE CHARACTERISTICS

The truth table shown above is used as a method of exhibiting the truth value characteristics of sentence connectives. This same tool can be used as a method of testing or determining the truth value characteristics of statement forms using these connectives. It is not the only method used for this purpose. The method of negation of Boolean expansion, for example, has come into use since the development of the Boolean algebra. This latter procedure has some marked advantages over the Truth Table procedures for statement forms having a larger number of variables than can easily be handled by the truth table methods. The truth table concepts are generally simpler, however. Hence, in order to keep the logical apparatus at a minimum, the truth table procedure will be employed here. The essential concepts of the truth table procedures are presented below.

Limiting the scope of the discussion to the statement forms of relatively simple compound statements containing only two variables, it becomes apparent that the first component statement might turn out, on investigation, to be true, and the second might also be true. Alternatively, the

first might be false and the second true. Similarly, the first might be true and the second false, or the first might be false and the second also false. This exhausts all the combinations of truth values that two individual component statements may acquire. When there are three distinct variables in a given statement form, there are eight possible combinations of truth values. In general, there are 2^n possible combinations, where n is the number of distinct variables in the statement form. All these combinations, called systems of values of the variables, are generally recorded under the list of variables presented at the left hand side of a truth table.

The truth value of an individual statement depends on the truth values of its component statements and the particular sentence connectives employed. Hence, the definitions of the sentence connectives employed uniquely determine the truth value of the statement under examination for any given combinations of truth values of its individual components. In the same way, the truth value characteristics of a given statement form is uniquely determined for every possible combination of truth values. Each of the truth value characteristics which a given statement form is uniquely determined to have, is represented by a T or an F under the logical operator which has the widest scope in that particular statement form. In this way the truth value characteristics of the statement form can be exhibited.

Hence, any statement which can be adequately represented by one particular statement form in the calculus of propositions, may be analyzed in the foregoing manner. Its truth value can then be unequivocally ascertained.

A CLASSIFICATION OF STATEMENT FORMS

The next step is to apply the truth table techniques to the analysis of alternative types of statment forms. The relatively simple forms "pvq", "p.q", "(p.q)v~(p.q)", and "(p.q).~(p.q)" will be sufficient for illustrative purposes. Their truth tabular analysis is presented below:

p	q	(pvq)	(p.q)	(p.q)v~(p.q)	(p.q).~(p.q)
T	T	T	T	T T F	T F F
F	T	T	F	F T T	F F T
T	F	T	F	F T T	F F T
F	F	F	F	F T T	F F T

Examination of the truth value characteristics of the first statement form reveals that any given statement which is a substitution instance of this particular statement form will be a true statement if both of its component statements are in fact true or if at least one of them is true. It will be false if both are false. Whether it is true or false depends not only on the nature of the statement form but also on the truth values of the constituent components. For example, the statement, "Investment rose or the labour force declined" is true if either or both of its

constituent statements is true and false if both are false. The second statement form might take as a substitution instance, "Investment rose and output increased." This statement is only true if both of the constituent propositions are true and false otherwise. Sentences which are substitution instances of either of these statement forms may be sometimes true and they may be sometimes false; that is, they may take on either truth values but not both at the same time. Their truth values are contingent upon the truth and falsity of the constituent propositions. These types of statement forms are called <u>contingent</u>, synthetic or logically indeterminate.

The truth tabular analysis also indicates that statements which are substitution instances of the third statement form presented above are always true. Not only is it true when one or all of the constituent statements is true, it is still a true statement when all (in this case two) of its constituent statements are false. For example, the statement, "Investment rose and output increased" is a true statement regardless of states of affairs in the real world. Its truth can be ascertained without any empirical investigation. It can be ascertained on the basis of an examination of the form and the form alone. This type of statement form is called <u>tautologous</u>.

The fourth statement form analyzed in the foregoing table yields statements which are always false. Statements

which are substitution instances of this type of statement form are not only false when one or more of its constituent statements is false; they are false even when all the substitution instances (two in this case) are true. This is intuitively clear for substitution instances like: "Investment rose and output increased, and it is not the case that investment rose and output increased" which is false for this substitution instance and is likewise false for any other substitution instance of the same statement form. Substitution instances of this statement form are false regardless of states of affairs in the real world and their falsity can be ascertained on the basis of form and form alone. The terms 'self-inconsistent' or 'inconsistent' and the terms 'self-contradictory' or 'contradictory' are commonly used to describe this type of statement form.

Further application of the truth table procedures would suggest that any statement, which can be adequately translated into the calculus of propositions, can be analyzed in such a way as to determine its truth value characteristics. If the number of constituent statements in any compound statement exceeds the number that can be readily analyzed with truth table procedures, then other techniques like the method of Boolean Expansion can provide the decision rule. Hence it would seem to be possible to determine whether any such statement form is tautologous, contingent or inconsistent. This provides the beginning of

a classification system which is central to the analysis of propositions developed in Chapter V.

APPENDIX B

THE ROLE OF VALID STATEMENT FORMS IN THE LOGICAL STRUCTURE OF SCIENCE

This section on the role of the valid statement forms is devoted to explaining how these valid statement forms provide rules of inference employed in the deductive processes of science. This explanation can be initiated by examination of the following very simple tautologies: "$p \supset p$", "$p \vee \sim p$", "$\sim(p . \sim p)$". The form "$p \supset p$" might take as a substitution instance, "If investment rises then investment rises" which is a true statement regardless of whether or not investment actually rises. Any other substitution instance of this form like, "If investment falls then investment falls" is likewise true regardless of the case in reality. A little further thought reveals that all the above statement forms yield statements which are true regardless of what happens in reality. Truth, it will be recalled, is a predicate of statements, and to say that the statement itself is true or to say that it is false is not necessarily to say anything at all about reality.[1] An examination of the following truth tabular analyses of these statement forms reveals that they cannot possibly take on

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[1]Alice Ambrose, op. cit., p. 18.

substitution instances which yield statements which are false.

p	p⊃p	pv~p	~(p.~p)
T	TTT	TT F	T TF F
F	FTF	FT T	T FF T

It can be seen from the above truth table, if the constituent statement "p" is true, the resulting statement is true. If, on the other hand, the constituent proposition is false, the resulting statement is still true. No matter whether the statements substituted in these statement forms are true or false they yield statements which cannot be anything but true. To reason otherwise would be to reason fallaciously. If one were to reason, for example, as if the statement "Investment rose or investment did not rise" were false then one would not be reasoning logically. Similarly, if one were to reason as if the sentence, "It is false that output increased and output did not increase" were false then one would not be reasoning logically. Further illustration would reveal a large number of substitution instances of valid statement forms, all of which would yield true statements. The truth table analysis reveals that it could not possibly be otherwise. Hence, one can take it as a rule of logic that statements which are substitution instances of valid statement forms are true--their truth is conditioned by the forms of the

statements alone regardless of states of affairs in reality.

Other more complicated statements whose tautologous truth characteristics are not so readily apparent but which are nevertheless instances of valid statement forms are likewise laws of logic. Consider those statements analyzed in the following truth table, for example.

p	q	(p.q)⊃p	(p⊃q)⊃(∼q⊃∼p)	(p.q)⊃∼(∼p∨∼q)
T	T	T TT	T T T	T T T F
F	T	F TF	T T T	F T F T
T	F	F TT	F T F	F T F T
F	F	F TF	T T T	F T F T

Every statement which is a substitution instance of one of these forms is true. It is a true statement regardless of the truth or falisty of its components and hence, regardless of the case in reality. Reasoning that ran counter to these forms would not be reasoning logically.

Thus far attention has been focused on the nature and the role of individual statements in science. The next step is to consider groups of statements, particularly those groups that make up theories, and to use the same logical apparatus used to study single statements to bridge the gap to the study of groups of statements.

Chains of reasoning are made up of groups of statements which are usually expressed in the "since...therefore ..." framework. They assert that since the premises are

true, therefore the conclusion is true. That is, they assert not only that the premises are true but also that the conclusion is true and that the premises imply it. This framework implicitly employs the Principle of Inference.[2] The principle of inference applies only to cases where antecedents are asserted to be true, not just taken as hypotheses. It permits one to assert the conclusion by itself and independent of its antecedents and hence to use the conclusion by itself as a premise in establishing further theorems. In comparison, the antecedents in the valid statement form "$p.(p \supset q) \supset q$" are taken as hypotheses and not asserted as true. If they are true then q is true, but they are not asserted to be true. The difference is that the Principle of Inference says that since the premises are true, therefore, the conclusion is true. The valid statement forms say only that if the premises are true then the conclusion is true. It should be clear, therefore, that the Principle of Inference is not a tautology but a rule of inference used along with the tautologies to form other rules of logic.

These chains of reasoning employing the Principle of Inference are always open to questions, however, -- questions of whether or not the antecedents actually do imply the

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[2]This is the term used by Ambrose, op. cit., p. 156, to denote the rule: "Given that A is a postulate or a proved theorem, and that $A \supset B$, we may infer B." Ambrose argues that without this rule no tautology which is a logical consequent in a deductive chain leading back to the postulates could ever be stated separately as a theorem. It is for this reason that she calls it the Principle of Inference.

conclusions. These questions can be tested by the "if... then..." form of implication. The antecedents can be treated as a single conjunction. Then this conjunction is taken as a substitution instance for "p" and the conclusion as a substitution instance for "q" in the simple statement form "p ⊃ q". The argument or chain of reasoning is now expressed as a single statement--a substitution instance of a single statement form in which both the antecedent and the conclusion can be statements of any degree of complexity. This statement form can then be tested by any one of several procedures designed for testing tautologies.[3] In order to remain within the framework of analysis already employed, the truth table procedure will be used below.

If, upon analysis, the implicative function turns out to be true for all possible combinations of truth values of its constituent propositions, then the function is an instance of a valid statement form. As such it is tautologous and can be employed as a valid form of inference or a rule of logic. It is in this way that the valid statement forms provide a large number of the rules of deductive logic.

It should be noted, however, that the implicative function is significantly weaker than the Principle of Inference as employed in chains of reasoning, and that as

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[3]See Ambrose, op. cit., pp. 96-97, pp. 155-156. The reason for restricting the examination to tautological implication, at this juncture, is that this section is only concerned with the role of tautologies in the logical structure of science.

Ambrose puts it, "It is important to see clearly that the formal validity of an argument is entirely unaffected by the actual truth-values of the premises and conclusion."[4] Hence, because one or more of the premises may be false, it is entirely possible to deduce a conclusion which is false, even though the reasoning is correct, that is, even though it is an instance of a valid form of inference. This established the necessity of the criteria of empirical adequacy of explanation and prediction. The statements in the explanations must be true; otherwise, regardless of the logical rigor of the deduction, the conclusion may be false.

The final step in the development of the role of the valid statement forms in provinding valid forms of inference is to apply the procedures specified above to the evaluation of several chains of reasoning in order to ascertain which of the forms is, in fact, a valid form of inference. Consider the following simple chains of reasoning:

$$\begin{array}{cccc} p \supset q & p \supset q & p \supset q & p \supset q \\ \hline \therefore \sim p & \therefore p & \therefore \sim q & \therefore q \end{array}$$

The truth characteristics of these are analyzed in the table below.

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[4] Ambrose, op. cit., p. 122.

p	q	(p⊃q).~q⊃~p	(p⊃q).q⊃p	(p⊃q).~p⊃~q	(p⊃q).p⊃q
T	T	T F FT F	T T T TT	T F	FT F	T T	TTT
F	T	T F FT T	T T T FF	T T	TF F	T F	FTT
T	F	F F TT F	F F F TT	F F	FT T	F F	TTF
F	F	T T TT T	T F F TF	T T	TT T	T F	FTF

According to this analysis, the first and the last sets of statements analyzed are true for all possible combinations of truth values of their component statements. These two statement forms can, therefore, be employed in conjunction with the Principle of Inference as valid forms of inference or rules of logic. This is not true for the other two chains of reasoning, however. They are not tautologous, and their use as rules of inference would, sometimes at least, lead to errors in reasoning. Any chain of reasoning that would sometimes lead to faulty conclusions even though the antecedents are true could never function as a rule of inference.

The truth table, within its scope of application, provides unequivocal means of ascertaining whether or not a given statement form is valid and hence whether or not it can be used as a rule of inference. These tools apply to a wide variety of statement forms, but not to all statement forms. There is, in fact, no general test for implication. There are other sources of rules of inference, including definitions and equivalences but these will not be developed here.

Sufficient knowledge of the logical apparatus of science has been presented to demonstrate the indispensable role of the valid statement forms or tautologies in the logical structure of science. The next task is to show their vacuous effects in the empirical content of science.

APPENDIX C

THE EFFECT OF VALID STATEMENT FORMS IN THE EMPIRICAL ASSERTIONS OF SCIENCE

Further understanding of valid statement forms can be obtained by examination of the following pair of statement forms: "$p \vee \sim q$" and "$p \vee \sim p$". The first of these is a contingent statement form and the second is a valid statement form. A substitution instance of the first statement form might say in explaining an increase in the general level of profits that, "Technology improved or the labour force declined." Then, if one were to find that the labour force had not declined, one could conclude that the general level of technology had improved. This is new knowledge. One could ascertain that the general level of technology had improved without ever having observed it directly. All that is required is knowledge of truth of the first two propositions. The counterpart valid form, on the other hand, could not be used to make this kind of assertion. All it could imply is that "technology did not improve." That is, if one knew that it was not the case that technology improved, i.e., the denial of the antecedent, then one could deduce the consequent, that technology did not improve. This can be written as follows: "$(p \vee \sim p) . \sim p \supset \sim p$", and tested with truth table procedures, like any other

implication within the calculus of propositions. The important point here is that when the valid form is employed in an attempt to make an empirical assertion, the results are trivial. Even though the proposition is known to be true, and the truth value of its antecedent or consequent can be asserted independently, and the reasoning is demonstrably valid, the conclusion about reality is necessarily trivial. It merely asserts what had to be asserted in the first place in order to make any deduction from the valid statement form that was employed. Hence, the use of valid statement forms as vehicles for the communication of empirical information contributes nothing at all to the empirical progress of science.

A brief examination of some of the more common valid statement forms will help to further substantiate this point. Consider the form "$\sim(p.\sim p)$". This might say of investment that it is not true that investment rises and investment does not rise at any point in time. Or consider "$(p.q) \supset p$" which might take as a substitution instance, "If technology increases and the labour force expands then technology increases." The Principle of Tautology asserts "$(pvp) \supset p$" which might say, "If output increases or output increases then output increases." The Principle of Association "$pv(qvr) \supset qv(pvr)$" is equally illuminating. It might say, "If investment increases or the labour force or technology increases, then the labour force

increases or investment or technology increases." None of the statements which are substitution instances of these statement forms convey any empirical information which is not already contained in the antecedent with which it is employed.

All of these statement forms are tautologies. They are all true statements--true by virtue of their form and form alone. They are true regardless of the truth values of their constituents. The truth values of their constituents have no effect on the truth values of a tautologous proposition as a whole. Hence, these propositions could not be significantly denied. It would provide no new information to say, for example that it is not true that, "If there is an increase in output then there is an increase in output." This is patently true as is every other tautology. By the same token, the same proposition could not be significantly asserted. In fact, there is an adage whose intellectual roots reach far back into the philosophy of science that statements which cannot be meaningfully denied cannot be meaningfully asserted. This is the case in any attempted use of valid statement forms in the empirical content of a theory. If, for example, all of the six axioms expressing the empirical content of the classical theory of economic growth turned out, upon examination, to be tautologous, then the theory would clearly not be useful for explanation and prediction.

It might contain only one empirical non-valid and consistent statement form, however, and even if the rest were all valid forms it might still have valuable explanatory and predictive potential.

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