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RESOURCE FIXITY, CREDIT AVAILABILITY AND
AGRICULTURAL ORGANIZATION

By
CLARK EDWARDS

AN ABSTRACT

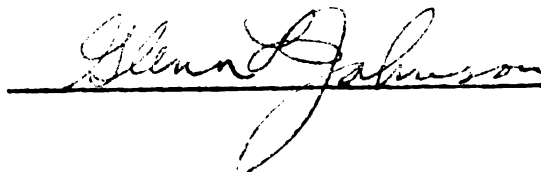
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DOCTOR OF PHILOSOPHY

Department of Agricultural Economics

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Approved

A handwritten signature, likely "Glenn Johnson", is written over a horizontal line.

AN ABSTRACT

This study helps to relate resource fixity and credit availability to the organization of agriculture. Factors affecting fixity and credit help to define the alternatives from which an optimal organization is chosen for an individual farm. The interest rate for borrowed funds on a farm depends on the total quantity of investment and working capital used and the fixity of a stock of productive services depends on on-farm relative to off-farm opportunities for using the services.

Credit and fixity are analyzed in the framework of the static theory of the firm. The amount of credit used depends on the farm demand for capital funds relative to an upward sloping supply function. Fixity of services depends on the value of services in production relative to off-farm opportunities for acquisition and salvage. The cost of acquiring additional services is greater than the salvage value of services in use for stocks of services which are subject to fixity.

The procedure of this study results in endogenous determination of the best list of fixed productive services. Services are regarded as not fixed to the farm if the quantity in use is worth changing. A necessary, but not sufficient, condition that the quantity of a productive service is fixed to the farm at the quantity initially used is that on-farm

opportunity costs for the service are bounded by off-farm opportunities for acquisition and salvage. Composing the best list of fixed services implies optimizing investments in stocks which are regarded as fixed to the farm.

The best reorganization of a farm business is a function of the existing organization. The best size of farm may depend as much on capital restrictions as it does on physical production relationships.

Supply response for products from individual farms is non-reversible with respect to price reversals when the best list of fixed services is determined endogenously rather than given among the fixed conditions of the problem. Non-reversibility means that reversal of the economic environment to a former state need not be accompanied by a complete reversal of output to its former level. Availability of additional capital funds through improved equity, capital accumulation and/or credit offers is apt to change the list of fixed services on individual farms and shift product supply functions so that additional supplies are forthcoming for stated prices.

The optimal farm organization exhibits maximum flow of returns to the equity of the farmer in his business. Improvements in either the rate of returns to equity or the size of the equity improve farmer welfare. Important among such conditions are factor prices, technology, offers of credit, and capital gains.

The results of this study indicate the nature of the relation of resource fixity and credit availability to the organization of agriculture. Propositions used to derive these results have an empirical as well as a theoretical origin. Rules for optimizing the use of durable stocks, given the conditions of resource fixity and credit availability, appear to conform to established principles of farm management as well as to observed behavior of farm output and prices.

It is hoped that the results will prove useful in future research in helping to quantify relationships and solve important farm problems involving resource fixity, credit availability and the organization of agriculture.

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A THESIS

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To

My wife, Rose
And my three children,
Suzan, Jane and Don

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TABLE OF CONTENTS

	PAGE
Acknowledgement	iii
CHAPTER 1: THE PROBLEM AND THE ASSUMPTIONS	1
Introduction	1
Farmers make choices subject to restrictions .	1
Purpose	3
Procedure and organization	4
Flow models	7
The flow concept	7
Stock and investment identification	7
Available flows are not necessarily used . . .	8
Investment and working capital funds	9
Supply and demand for funds	9
The marginal rate of interest	11
Alternative abstractions	12
The fixed asset structure	13
Composing lists of fixed assets	13
An operational definition of fixed flows . . .	14
Fixity of flows related to flow prices	16
Application of the definition	18
Fixed assets in the literature	20
Other assumptions on which the analysis rests . .	26
CHAPTER 2: THE MODELS	32
Introduction	32
Glossary	33

	PAGE
Model 1: A standard version	35
Model 2: The capital restriction	44
Model 3: Shifting asset structure	55
Model 4: The capital restriction and the shifting asset structure	64
CHAPTER 3: APPLICABILITY OF THE MODELS	72
Introduction	72
Farm organization	73
Opportunity costs	73
Determining the best organization	76
New resource combinations	79
Farm size and credit supply functions	80
Supply and demand for capital funds	80
Investment and working capital	83
Farm demand for funds and non-reversible changes in farm size	84
Supply response	87
Product supply functions for individual farms	87
Non-reversibilities	87
Supply shifters	93
Supply responses and the business cycle	97
Welfare and policy	105
Returns to owned assets	105
Changing the returns to owned assets	108
Summary and conclusions	111
BIBLIOGRAPHY	116

CHAPTER 1

THE PROBLEM AND THE ASSUMPTIONS

Introduction

Farmers in charge of going businesses have restrictions on their capacity to change their present use of productive services and their present stocks of assets. These restrictions constitute fixed conditions in the economic environment of farmers. As individuals in the farm economy, they have no control over these conditions. The conditions or restrictions limit the possibilities for alternative farm organization.

An alternative to the present farm organization is preferred by profit motivated farmers if adjustment from the present organization produces a positive gain. Gain is the change in the flow of net revenue produced by an adjustment in the farm organization. Economizing farmers seek the farm organization which maximizes gain subject to the fixed conditions.

In economic analyses of farmers problems, the results depend in part on statements of the fixed conditions. Alternative statements about supply schedules for productive services, for example, lead to alternative indications about the relation of conditions in the factor market to optimal farm organization and size, supply response, farmer welfare

and national policy recommendations. The importance of supply and demand conditions in the factor markets is recognized in the descriptive literature of agricultural economics. This importance has not been adequately reflected in many quantitative analyses because simplifying assumptions which becloud the fundamental relationships are frequently made.

Conditions in the factor market must be studied and reasonably accounted for in economic analyses. Important among these are conditions which (1) determine the slope of service supply functions and (2) cause discontinuities in the functions. Two restrictions are studied in this thesis, one in each of the two classes listed above. Other restrictions in these classes which are not studied in this thesis may be closely associated and the procedures for analysis may be similar, but examination of such restrictions is not undertaken herein.

One restriction studied in this thesis is the supply schedule to an individual farm for investment and working capital funds. This schedule restricts the quantity of funds the farmer is able to use to gain control of productive services. The other restriction describes the acquisition and disposal functions for factor services and/or stocks. These functions restrict (1) the possibilities of resource control with a given quantity of capital funds, and (2) exchanges of inputs in use for either capital funds or for other inputs.

In combination, the restrictions on capital funds and on acquisition and salvage possibilities for inputs serve to define relevant supply functions for factor services. While these restrictions receive special study in this thesis, other important restrictions used to define feasible farm organizations are drawn upon as needed with little ado. The other restrictions are often used in farm business analysis and include demand schedules for farm products, technical relationships among factors and products, and institutional arrangements.

Purpose.

Two specific properties of supply functions for farm factor services are examined in this thesis. The properties are (1) upward sloping supply functions as a consequence of imperfect competition in the farmer's money market and (2) discontinuous supply functions for services whose acquisition costs are greater than salvage values.

Simpler assumptions about fixed conditions in the factor market are often used which imply perfectly elastic supply functions for investment and working capital funds and for factor services which are regarded as variable to the farm. They imply perfectly inelastic supply functions for services regarded as fixed to the farm.

The assumptions about fixed conditions in the factor market which receive special attention in this thesis are more flexible and more realistic than the simpler assumptions

mentioned above. Comparison of the consequences of using either the simpler or the more flexible statements of fixed conditions permits an evaluation of the potential contribution of the more flexible and more realistic statements.

The purpose of this thesis is to make the above mentioned comparison and to demonstrate some important formal relationships among credit availability, resource fixity and farm production. The results should serve as a basis for future accumulation of necessary empirical information to quantify these relationships. The procedures used should prove useful in examining (1) other areas of imperfect competition such as the labor market and (2) other sources of discontinuities such as contract pricing or discontinuous stocks. The conceptualization should sharpen understanding of optimal organization and size of farms, supply response, farmer welfare and the requirements of national farm policies.

Procedure and organization.

In this study, some alternative statements of fixed conditions in the factor and money markets are incorporated in mathematical statements of the theory of the firm. Propositions about farm income, net worth, efficiency and supply response are deduced from the alternative models. Comparisons are made of the alternative deductions. Speculations are put forward as to the usefulness of the results. Measurements of the relationships displayed, as well as conceptualizations which are dynamic and macro-economic in nature, are relegated to future study.

The ends of this thesis are sought within the flow model of economic theory known as the static theory of the firm. The theory is a useful and readily accessible framework in which to conduct the analysis. This is not to say that it is the only useful framework. For example, activity analysis might have been adopted. The framework for the analysis is not the important part of this thesis. The important part involves the logical implications of changing lists of fixed factor services and of changing supplies of capital funds on farm production. These general implications could have been produced from widely divergent frameworks for analysis. Similar assumptions about fixed conditions in the market for factor services should lead to similar conclusions.

The remainder of this chapter is devoted to the main assumptions on which the analysis rests. The use of flow models, which necessitates certain abstractions from important intertemporal relationships, is discussed. A conceptualization of the supply of investment and working capital funds to the farm is presented. Relationships of prices in the factor market and of present use of productive services to optimal farm organization and to the fixed asset structure on farms are discussed. Some requirements of a procedure for endogenous determination of a list of fixed factor services are outlined. Finally, some of the essential assumptions found in usual versions of the static theory of the firm are briefly reviewed.

Chapter 2 presents four models. Model 1 is a usual version of the static theory of the firm in a competitive industry. Common assumptions of perfectly elastic supply curves for productive services are reflected in this model. Model 2 adopts an upward sloping supply function for investment and working capital funds and assumes price discrimination in the money market. Imperfections in the farmer's money market are well recognized in the descriptive literature of agricultural economics. The imperfections are frequently neglected in applications of static flow models to farm problems. Model 3 adopts supply functions for some factor services which reflect a divergence of acquisition costs from salvage values. A predetermined list of fixed factor services is not included in the set of fixed conditions for this model, the list is determined endogenously. Model 4 combines the features of the other models.

Chapter 3 interprets the models of chapter 2 and discusses some implications. Some effects of imperfect competition in the money market and of two prices in the factor market are discussed as they relate to farm income, net worth and efficiency. Commodity supply functions are discussed for individual farms. Alternative conceptualizations of supply relationships are derived from each of the models and compared. Speculations are put forward as to the usefulness of the results with respect to explanation of farmer response to changes in his economic environment; formulation of

agricultural policy; and, orientation and methodology in agricultural economics research.

Flow Models

Employment of durable stocks involves commitments for which responsibility is borne over several time periods. To lessen the difficulties of analysing the use of stocks which do not have identical lengths of useful life, it is customary to construct flow models. Such models require that all elements of the system are defined relative to the time interval over which production is measured.

The flow model chosen for the exposition of this thesis is the static theory of the firm. The use of durable stocks is introduced into this model through measures of flows of service from the stocks. Services are measured as units of flow per production period. Measures of sub-time intervals within the production period, such as hours per year, are not separated from the production period for examination in this study.

Stock and investment identification.

Given a quantity of stock, specification of the available flow of services per production period from the stock is assured by the physical relationship between stock and flow. The inverse identification of stock from flow may or may not be evident. For example, it may be possible to associate a given flow of services with either of two stocks.

Should such an ambiguity occur in a particular problem, it may be handled by including instructions for identifying the intended stock. One such instruction might be to choose the stocks which provide the required flow for the minimum investment.

The flow models used in this thesis presume that a one to one correspondence between flows and stocks can be established. Similarly, a correspondence of flow prices and investments is presumed. This means that a farm organization comprising a given set of service flows implies a unique, identifiable set of stocks. Associated with use of the stocks and services is a unique requirement for investment and working capital funds.

Available flows are not necessarily used.

Services available from existing stocks need not be fully utilized. If the available flow from a stock is greater than the utilized flow, the distinction is important in farm business analysis because production is a function of utilization while expenditures depend on availability as well. If available flows equal utilized flows, on the other hand, the distinction need not be maintained.

Available flows are equal to utilized flows in the optimal organization on profit maximizing farms if farmers have the opportunity of salvaging unused services by selling stocks in such a way that expenditure is reduced without reducing revenue. One set of conditions that can assure

this result includes (1) prices which are independent of quantity and (2) stocks which are perfectly divisible. These conditions are assumed in the models of chapter 2 as a means of simplifying the mathematics.

The assumption of perfect divisibility of stocks limits the range of application of the models. At first glance this restriction may seem severe because most stocks used on farms are not divisible into indefinitely small pieces. However, the restriction is not as damaging as it appears because the failure to include fixities as a consequence of imperfect divisibilities does not destroy the conclusions in this thesis about fixities as a consequence of other things. On the contrary, while the relation to production of discontinuities in the cost functions for services which result from imperfect divisibilities of stock is more complex than for the discontinuity in the cost function examined in this thesis, the results have been examined by the author and found to be analogous.

Investment and Working Capital Funds

The quantity of capital used by an individual farmer depends upon the supply of investment and working capital funds relative to the farm demand for funds. Farm demand is derived from the productivities of inputs controlled with the funds.

The supply of capital funds available to a farm reflects offers of credit from various sources in addition to equity or net worth of the farmer. These offers may be arrayed in order of increasing cost to form upward sloping supply functions. It is reasonable to assume that farmers employ capital they own plus capital borrowed at low rates of interest before considering offers at higher rates. In this way, farmers practice price discrimination on the money market.

The total interest payment made per production period is an important determinant of farm profits. The total payment is the sum of the payments to each source of funds. If price discrimination is practiced on upward sloping supply curves, the marginal rate of interest is greater than the average rate. For continuous supply functions, the measure of interest payment is the integral over the function from the farmers equity in the business to the total quantity of funds used. The integral taken from zero to the total quantity of funds used provides the same answer as when taken from farmer equity if the supply function is written in a manner which indicates no interest payments by the farmer to himself for the use of his equity.

Equity is an important determinant of the shape and position of the investment and working capital supply function for an individual farm. Other determinants include the character and personality of the farmer, uses to which the

funds are put, risks involved in the enterprise and insurance taken against risk. Some of these determinants are dynamic in nature and are not subjects for analysis in the static models of chapter 2. Others are not amenable to measurement for quantitative analysis.

The offers of lenders depend a great deal on equity and on factors correlated with equity. In abstracting from the many determinants of the supply of funds to an individual farmer, it is convenient to regard equity as the principal determinant. Given the farmer's equity in his business, his supply of available funds is reasonably stable. Improvements in equity shift the supply function to the right and make it more elastic. Such improvements can result from increases in the value of owned assets or from accumulated savings.

The marginal rate of interest.

If there is imperfect competition in the money market, then the marginal rate rather than the average rate of interest is important in decisions to change the quantity of funds used. For example, if the marginal capital cost is less than the marginal value product of funds used on the farm, additional investment and working capital funds can be employed profitably.

The marginal rate of interest to a farm is an opportunity cost that affects the farm supply functions for services. The opportunity cost is an interest charge against the additional investment and working capital associated with a unit of flow

of the service, which is added to flow price in constructing the relevant supply function. Inasmuch as the opportunity cost increases as additional funds are borrowed, supply functions for factor services become upward sloping as a consequence of conditions in the money market.

Alternative abstractions.

Flow model analyses frequently abstract from conditions determining the supply of capital funds and simply state an average rate of interest which is independent of the quantity of funds borrowed. Using this formulation implies that farmers may borrow indefinitely large quantities of funds at the stated rate of interest.

Individual farmers do not face perfectly elastic supply functions for capital funds and using such functions in an analysis neglects money market determinants of farm size, and of fixed asset structure. It weakens the flow model as a means of relating equity, capital gains and inflations to farm production and farmer welfare.

This usual formulation of perfectly elastic supply functions for investment and working capital funds is used in models 1 and 3 of chapter 2. The other models use the alternative abstraction of upward sloping supply functions and price discrimination. Results of these alternative conceptualizations of conditions in the money market, as they affect the rules for the optimal use of farm resources, are compared in chapter 3.

The Fixed Asset Structure

The fixed asset structure of a farm is made up of the durable stocks whose quantities are not worth changing. An asset is regarded as fixed if the existing quantity is optimal and remains optimal subsequent to at least an indefinitely small change in the environment. The definition precludes classification of an asset as fixed if small changes in prices, for example, would always lead to reclassification of the asset as not fixed.

Decisions to invest in durable stocks are based on answers to questions of intertemporal relationships. Answers to such questions are assumed in the static framework of this thesis. Intertemporal problems are reduced in importance in static economic analyses because of the perfect knowledge assumption. Relaxation of this assumption opens many interesting and important questions which must remain for analysts employing dynamic models.

The important role played by uncertainty in the fixity of assets is therefore abstracted away from in this study. That is, we do not examine cases for which the stock of an asset on the farm may be worth changing under present conditions where the farmer elects not to make the change because of his imperfect knowledge about his own future demands for the asset.

Composition of a list of fixed assets involves inter-temporal relationships whether the list is given among the fixed conditions or is endogenously determined. In static analyses, the list of fixed assets is frequently regarded as equivalent to a statement of the relevant length of run. This thesis is concerned with rules for composing appropriate lists.

Inasmuch as the exposition of this thesis is in the context of flow model analysis, rules for composing lists of fixed services are developed. A list of fixed services implies a set of stocks from which the services flow. A change in the flow of services implies a change in stock. Identification of stocks from flows was discussed on page 7.

An operational definition of fixed flows.

The rules for deciding that an existing flow is not worth changing should relate fixity to the economic environment. Resource fixity is determined largely by technical, institutional and economic factors. Technical factors include discontinuous and immobile stocks of resources. Institutional factors include social custom, personal preference, habit, government policy and market structure. Economic factors include debt commitments, capital gains taxes, transportation costs, transfer fees, depreciation, physical productivities and product and factor prices.

Factors affecting fixity are reflected in the costs of acquiring additional services, in the salvage values of

services in use, and in the value of services in production. The relations among these are abstracted from to form an operational definition of a fixed factor service.

The operational definition for fixed flows states that if no feasible change in the existing quantity of a service generates a positive gain in net revenue then the flow is fixed. The outcome of applying this definition depends upon the levels of other services used on the farm because changes in the use of one service can change the optimal use of another. Failure to generate positive gains for each service singly does not preclude the possibility of gainful change in the use of two or more services jointly. A general definition must consider the gains generated by all feasible changes in farm organization. If maximizing gain does not require a change in the existing quantity of a service which is subject to fixity, then the service is not worth changing and is regarded as fixed.

A general application of the definition can be made by examining gains on a finite number of the indefinitely many feasible organizations defined by the fixed conditions of the farm environment. This is because for each service there are only three possibilities: The optimal quantity may be greater than, less than or equal to the initial quantity. Thus, for n factor services used on a farm there are 3^n ways to compose lists which regard n services as fixed, variable upward or variable downward. The best use of

services for each of the 3^n listings can be determined by existing procedures for determining optimal use of variable services subject to fixed conditions. From the 3^n candidates, the listing of services which generates the greatest gain in net revenue is the best list.

Fixity of flows is related to flow prices.

Some services which were initially optimal in an application may not remain optimal subject to indefinitely small changes in the economic environment. There is little advantage in regarding services as fixed that can become variable so easily. It is useful to regard services as subject to fixity only if they can remain fixed for small environmental changes.

An economically important condition for service fixity is a condition affecting pricing in the market for durable stocks. The acquisition cost of additional services from additions to durable stocks is frequently greater than the salvage value of services from comparable stocks existing on the farm. This divergence of acquisition costs from salvage values results in a discontinuity in the supply function and makes the service subject to fixity. The severity of the discontinuity depends upon the extent of the divergence of acquisition cost from salvage value. The location of the point depends on the initial quantity of the service on hand.

Other reasons for discontinuities, such as discontinuous inputs, and contract or volume discount pricing, do not uniquely locate the point at the initial use of the service. While these other causes may induce similar effects in cases where corner or border optima exist, they do so for different reasons. In such instances, optimal use of these services is independent of initial use. Most cost functions are discontinuous according to French et al¹ who state that the discontinuities require restatement of the profit maximizing conditions of conventional theory in which continuous functions are assumed.

Other students may wish to examine border optima when supply functions are discontinuous for these other reasons. The general statement of the operational definition given above includes such cases. In this thesis, effort is concentrated on discontinuities arising from differences between acquisition costs and salvage values. Discontinuities from this important source relate resource fixity to the existing use of services on farms.

When acquisition costs and salvage values are both describable by continuous functions, the gain function for the farm is easily defined if it includes the rule that the acquisition function is used to examine increased use of

¹French, Sammet and Bressler, "Economic Efficiency in Plant Operations," Hilgardia, Vol. 24, No. 19, July, 1956. p. 708.

the service whereas the salvage function is used to examine diminished use. The procedure for applying this rule is developed in chapter 2.

An interesting property of the definition is that a necessary, but not sufficient, condition that a flow of service is fixed is that the marginal value product of a service is bounded by acquisition cost and salvage value for the service. This property is useful in seeking optimal use of a service subject to fixity when it is appropriate to examine one service at a time. It is also useful for examining the gainfulness of changes in aggregates of services where an aggregation is defined according to least cost combinations.

Application of the definition.

In flow model analyses, the list of fixed service flows is usually stated among the fixed conditions. Reasons for regarding services as fixed are not necessarily given. Changes in the list as a result of changes in the economic environment are not examined by that procedure. However, one may compare organizations that result from using alternative lists. Results of the usual procedure are conditional in the sense that if that is the relevant list of fixed services then this is the optimal farm organization. For static analysis, a list of fixed services is equivalent to a statement of length of run; alternatively, it identifies which sub-production function out of a more general function is under consideration.

The usual procedure for including a list of fixed services among the fixed conditions is used in models 1 and 2. The other models apply the operational definition of a fixed service and compose endogenously the list of services fixed for the farm. Results of the alternative procedures are compared in chapter 3.

In this thesis, the operational definition of a fixed service is applied to static flow models. An alternative formulation which relates the capitalized value of stock to acquisition and salvage values for the stock might be useful in some contexts. However, the definition in the flow context is shown below to be equivalent to the definition in the stock context.

The acquisition and salvage values for flow are functions of acquisition and salvage values for stock. Alternatively, the capitalized value of the stock is a function of the marginal value product of a unit of flow. The value is net of periodic expenses such as personal property taxes, license fees, and utilization costs, and net of the junk value of the depreciated durable. If the rules for going from flows to stocks and from flow prices to investments are known, translation of variables from one time dimension to another is straight-forward.

To the extent that the definitions in either time dimension are equivalent, the definition in the flow dimension is preferable in the context of this thesis. The

flow definition is easily integrated into the flow models of the following chapter and clears away complications of examining investments on durables with different lengths of useful life without destroying the usefulness of the results.

Fixed assets in the literature.

The relation of fixed assets to production efficiency, supply response and farm income has long been considered important in economic analysis. However, most principles used in explaining use of fixed assets contain a degree of vagueness and lack a systematic approach to determining whether an asset is fixed. Makeshift arrangements for fixed asset analysis are frequently incorporated which, while not untrue, gloss over some important elements of asset fixity. Little attention has been paid, until a few short years ago, to the economic implications of the divergence of acquisition costs from salvage values in the factor market.

Descriptions of the law of diminishing returns depend on fixed inputs as well as fixed conditions. The law describes in great detail the character of returns to variable inputs. It does not describe returns to the fixed factor for such is not the purpose of the law. It does not provide reasons why the fixed factor remains fixed.

One concept of length of run involves the relevant list of fixed productive factors. For example, in the ultimate short run all factors are fixed and in the ultimate

long run all are variable. This concept does not provide rules for composing the relevant list of fixed assets.

The concepts of opportunity costs and comparative advantage have evolved as principles to explain allocation of quantities of fixed services among alternative uses. These concepts provide internal pricing mechanisms to aid in resource allocation when no applicable market prices exist to serve as guides.

The concepts of rent, quasi-rent, surplus and excess profits include returns to fixed assets. In short or intermediate lengths of run, such returns (either positive or negative) generally exist. The surplus returns are eliminated, however, in the ultimate long run when all assets are listed as variable and external pricing adjustments occur. It is seldom recognized that internal pricing adjustments which assign different rates of payment for the use of fixed assets can also eliminate the surplus returns and can do so in any length of run.

Procedures for organizing a farm business around fixed supplies of land and/or labor reflect the importance of a fixed asset approach to resource allocation problems. One example is the land use approach to farm planning used by the Soil Conservation Service. Such approaches are useful when the factors listed as fixed are appropriately chosen.

To compose a list of fixed assets, one needs a definition of what a fixed asset is. Such definitions are sparse in the literature. Adam Smith¹ stated that capitals employed without changing masters may very properly be called fixed capitals. Other classical writers said that fixed capital exists in durable shape in contrast to capital which fulfills the whole of its office by a single use.² The operational definition of a fixed asset as a durable stock whose initial use is optimal is close to the classical conceptualization.

References to the relation of price to resource fixity are mostly in very recent literature. An exception is in the literature of farm management. G. F. Warren³ was on the verge of a fixed asset analysis when he wrote "If hay is worth \$15 a ton at the railroad station, it is usually not worth more than \$12.50 on the farm, because the cost of baling and hauling to the station must be deducted. Live-stock need only return \$12.50 for hay to make it pay to feed rather than to sell." Warren omitted the part of the fixed asset definition which asserts: Livestock need return at least \$15 for hay to make it pay to buy more from the railroad

¹Smith, Adam, The Wealth of Nations, edited by Cannan, The Modern Library, New York, 1937. p. 263.

²Marshall, Alfred, Principles of Economics, Macmillan Co., New York, Eighth edition. p. 75.

³Warren, G. F., Farm Management, Macmillan Co., New York, 1913. p. 208.

station to supplement the supply of hay produced on the farm. In Warren's case, if livestock returns more than \$12.50 for hay but less than \$15, then hay is fixed to the livestock enterprise.

Weintraub¹ provides a more recent example of an incomplete definition. He states the part that Warren omitted. Weintraub states that "so long as a firm estimates that a further unit of an agent would not be profitable, then the factor is fixed", but he fails to note that the firm must also determine that disposal of a unit on hand would not be profitable before the necessary condition is met that the factor is fixed.

Institutional economists have recognized the capital losses incurred when owners sell assets for less than acquisition costs. Capital losses of this sort are included in the concept of supercession costs.² When the supercession cost exceeds the additional returns obtained by changing from one use to another, an asset remains fixed in its present use.

Dorfman³ asked which types of resources should be acquired and which disposed of. He then replied, "The question can

¹Weintraub, Sidney, An Approach to the Theory of Income Distribution, Chilton Company, Philadelphia, 1958. p. 170.

²Ely and Wehrwein, Land Economics, Macmillan Co., New York, 1940. p. 149.

³Dorfman, Robert, Application of Linear Programming to the Theory of the Firm, University of California Press, Berkeley and Los Angeles, 1951. p. 46.

be answered only by comparing the value of the contribution of each resource to net revenue with its acquisition cost or disposal price."

The divergence of acquisition cost from salvage value was related to asset fixity by Joseph W. Willet at the University of Kentucky when he noted that if the marginal value product of a flow is bounded by acquisition and salvage flow prices for a service used at diminishing returns then the quantity of flow is not worth changing.

Bradford and Johnson¹ used the definition to display the relevant length of run in a single product model. They failed to note the problem of applying the definition proposed by Willet to services used at increasing returns. They examined some important relations of fixed asset structure to farm management problems.

Johnson and Hardin² used the operational definition of a fixed asset as an aid to estimating the value of forage on farms. This work corresponds directly with the principle in farm appraisal that if market prices are not relevant for establishing the value of an asset, then the capitalized value of the flow in production is useful.

¹Bradford and Johnson, Farm Management Analysis, Wiley and Sons, Inc., New York, 1953. p. 133 and see reference to "Assets, fixed" in the index.

²Johnson and Hardin, Economics of Forage Valuation, Station Bulletin 623, Agricultural Extension Service, Purdue University, April, 1955.

Johnson¹ has proposed that incorporation of the operational definition of fixed asset into economic analysis can contribute to an explanation of individual farmer behavior and of aggregate supply response. He asserts that the most neglected aspect of current aggregative supply analysis for agriculture is the theory of fixed assets. He proposed and tested, in a rough way, some aggregate hypotheses about asset fixity, resource use and general levels of employment and business activity.

Hathaway² has tested some hypotheses about agriculture and the business cycle. His findings give further empirical support to Johnson's above-mentioned hypotheses. Hathaway's discussion is drawn upon in chapter 3 of this thesis in discussing the relation of shifting asset structures to supply response.

Smith³ applied the operational definition to some inputs in an individual farm business analysis. He used divergent acquisition costs and salvage values for farm

¹Johnson, Glenn L., "Supply Functions--Some Facts and Notions," Agricultural Adjustment Problems in a Growing Economy, edited by Heady, Diesslin, Jensen, and Johnson, Iowa State College Press, Ames, Iowa, 1958. pp. 74-93.

²Hathaway, D. E., "Agriculture and the Business Cycle," Policy for Commercial Agriculture, Joint Economic Committee, November 33, 1957. pp. 51-76.

³Smith, Victor E., "Perfect vs. Discontinuous Input Markets; A Linear Programming Analysis," Journal of Farm Economics, Vol. 37, August, 1955. p. 538.

produced hay and corn in a linear programming problem. Some other inputs were regarded as fixed *a priori*. He demonstrated that recognition of the two prices in the factor market can have a significant effect on the organization of farm activities and on the flow of profits.

Other Assumptions on Which the Analysis Rests

The main assumptions about the motivation of farmers and about their economic environment are adopted for this thesis directly from the static theory of the firm. Among numerous contributors to this topic are Hicks, Samuelson, Carlson and Allen.¹

The two modifications in the usual assumptions which are important to the conclusions of this thesis were discussed above. They relate to supply functions for investment and working capital funds as well as for assets subject to fixity. Also discussed above were assumptions required to make inferences about stocks from flow models. Other main assumptions are quite usual and are discussed briefly below.

¹Hicks, J. R., Value and Capital, Oxford University Press, London, Second edition, 1953. Chapter 6 and Appendix to Chapter 6.

Samuelson, Paul A., Foundations of Economic Analysis, Harvard University Press, Cambridge, 1948. Chapter 4.

Carlson, Sune, Pure Theory of Production, King and Son, London, 1939. Chapter 3.

Allen, R. G. D., Mathematical Economics, Macmillan and Co., Ltd., London, 1956. Chapter 18.

The motivation of farmers is to maximize gain subject to fixed conditions or restrictions in their economic environment. Gain is the change in profits with respect to a change from the existing to an alternative farm organization. Gain and profit maximization are equivalent under the assumptions of this thesis. The gain equation is convenient in the fixed asset model of chapter 2 because the values of services whose initial levels are optimal are not important in the gain equation.

Production functions relate flows of used factor services to flows of output over the production period. The relation assumes given technological principles for combining services. Substitution among services may or may not require substitution among the known technological principles. For every factor combination, it is assumed that the technological principles used maximize output with respect to available principles.

Interdependence among production functions for different enterprises may occur on farms. Some of these enterprise relationships are covered by the models of chapter 2. Examples which are covered include (1) an output of one enterprise serving as an input to another, (2) joint products from a single production process, (3) pricing interrelationships such as on-farm opportunity costs, and (4) accounting procedures which cause apparent interdependencies among production functions.

A fifth kind of interdependence is not covered by the models. This is the physical interdependence for which a change in resource use in one enterprise results in a change in the flow of output from another enterprise.

Using physically independent production functions in the models of chapter 2 has the advantage of making explicit the allocation of services among enterprises. The advantage is gained at the expense of developing models which are not applicable to situations for which the output of one enterprise is subject to change with respect to change in the use of a service in another enterprise. In addition, the procedure implies that it is both possible and meaningful to display an allocation of a given factor service among all the enterprises in which the service is used.

Factor services and flows of output are assumed perfectly divisible. Hence production functions are continuous functions. In addition, stocks of assets from which services flow are assumed perfectly divisible in the models of chapter 2. It was stated above that one consequence of perfectly divisible stocks is that used flows of factor services are always equal to available flows at the optimal organization for profit maximizing farms. Analysis of discontinuous stocks requires models which distinguish between used and available flows.

Supply functions for factor services differ among the models of chapter 2. It is the differences in these functions that distinguish the models from one another.

Perfectly elastic, continuous supply functions are used for variable services in model 1. Numerous imperfections in the factor and money market exist in the farm economy and perfectly elastic supply functions for inputs are far less realistic than perfectly elastic demand functions for products. The perfectly elastic functions are frequently used in analyses of the farm economy and they are included here for comparison with the results of the other models.

The supply functions assumed for the remaining model are not the usual ones assumed in the theory of the firm. Propositions about service supply functions in models 2, 3, and 4 are drawn from previous discussions in this chapter about the supply of capital funds and the fixed asset structure. By way of recapitulation, upward sloping, continuous supply functions are used for variable services in model 2. The slope is determined entirely by the marginal rate of interest. The constant market flow price determines the price axis intercept of the function.

A point of discontinuity exists in supply functions used for services subject to fixity in model 3. Functions for acquisition cost and for salvage value are linear and of zero slope. The supply function is formed by the acquisition function for quantities of services greater than the initial quantity existing on the farm and by the salvage function for quantities less than initial. The point of discontinuity is determined by the initial use of the service.

Model 4 combines the features of models 2 and 3. Supply curves are upward sloping as a consequence of imperfect competition in the capital market and have a point of discontinuity as a consequence of a divergence of acquisition costs from salvage values.

From the point of view of the mathematics involved, the interpretations made in chapter 3 may be more restrictive than the models necessitate. For example, supply curves may have the properties assumed in models 2, 3 and 4 from other causes besides imperfect competition in the money market and divergence of acquisition costs from salvage values. It is therefore possible to use the models of chapter 2 to examine the consequences of other causes for upward sloping supply functions with a lone point of discontinuity. Such other causes are not of concern in this thesis.

Demand functions for products produced on the farm are taken as continuous and perfectly elastic in each model of chapter 2. The use of perfectly elastic demand functions conforms reasonably to the observation that most of the product sales from America's five million farms are on competitive markets where the transactions of an individual farmer are not of sufficient magnitude to affect price.

Perfect knowledge of both present and future production relationships and of present and future supply and demand schedules for farm factors and products is assumed to be held by farm operators. The knowledge provides certainty of

the consequences of decisions made in the current period for which responsibility is borne over several periods.

Institutional arrangements in the economic environment of the farmer are stable. This includes governmental and marketing arrangements.

CHAPTER 2

THE MODELS

This chapter develops four models based on alternative assumptions discussed in the previous chapter. The models use the static theory of the firm to display the consequences of imperfect competition in the money market and of two prices in the factor market on farm organization.

The purpose of this chapter is to present the models and briefly point out differences among them. Discussion of the important differences is relegated to chapter 3. To assist the reader in following the notation used in this chapter, a glossary is presented at this point. Do not read it now. Definitions of symbols are not always made as the symbols are presented in the models, and the glossary is placed here to facilitate easy reference as symbols are encountered.

Glossary

G - - The gain, or the change in profits with respect to a change in farm organization. G^* is an equivalent expression. L is a Lagrangian function of gain. If π is profits and π^0 is the flow of profits from the initial farm organization, then $G = \pi - \pi^0$.

Y_j, Y_j^0 - - The flow of output from the j^{th} enterprise for $1 \leq j \leq m$. The superscript $(^0)$ means the flow from the initial farm organization which is given as a fixed condition. An analogous meaning is implied for subsequent uses of the superscript and the definition shall not be repeated there.

f_j, f_j^0 - - the flow of output measured as a function of inputs. Equivalent to Y_j and Y_j^0 .

X_i, X_i^0 - - The flow of the i^{th} service used on the farm for $1 \leq i \leq n$.

X_{ij}, X_{ij}^0 - - The flow of the i^{th} service used in the j^{th} enterprise.

P_{y_j} - - The market price of the j^{th} product.

P_{x_i}, A_i, S_i, a_i - - Various measures of flow prices for services. If the flow price of a service variable to the farm is assumed to have only one value, as in models 1 and 2, then the four measures are equivalent and the term P_{x_i} is used to represent the flow price of the i^{th} service. If acquisition cost is greater than

salvage value for a service, as in models 3 and 4, then the term α_i is used to represent the flow price with the understanding that $\alpha_i = A_i$ for acquisition of additional service and $\alpha_i = S_i$ for salvage of existing service where $A_i \geq S_i$. Flow prices include interest charges in models 1 and 3. They are net of interest charges in models 2 and 4.

$q_i, q_i^a, q_i^s, \beta_i$ - - Various measures of investment and working capital requirements per unit of flow. Analogously to the above distinctions among measures of flow prices, q_i is used in model 2 and β_i is used in model 4. $\beta_i = q_i^a$ for acquisition of additional services and $\beta_i = q_i^s$ for salvage where $q_i^a \geq q_i^s$.

K, K^0 - - The total quantity of investment and working capital funds used on the farm.

$g(u), g(K)$ - - $g(u)$ is the supply function for capital funds and $g(K)$ is the interest rate for the K^{th} dollar.

Model 1

A Standard Version

The first model is a rather standard version of the static theory of the firm in an industry of perfect competition which illustrates the consequences of regarding (1) the supply function for investment and working capital funds to the farm as perfectly elastic and (2) the list of fixed services as predetermined. Salvage values for variable services are assumed equal to acquisition costs.

Model 1 is sterile insofar as it relates availability of capital funds to optimal resource allocation. The predetermined rate of interest is the only reflection of conditions in the money market evident in this model. An increase in the rate usually produces an output reduction. Other changes in money market conditions which are not reflected in this given rate of interest are not related to resource allocation in this model. Because of this sterility, there is no need to display the interest payment explicitly. Interest charges are implicit in flow prices for services.

Model 1 does not explain how an appropriate list of fixed services is composed. However, some questions concerning allocation among enterprises of services fixed to the farm are appended to model 1. Explaining this allocation is part of the task of fixed asset theory. Allocation of

services among enterprises is according to an internal pricing mechanism called the principle of opportunity costs. The procedure is makeshift in model 1 because it does not relate on-farm opportunity costs to off-farm alternatives.

A multiple product situation is used for the exposition of this and subsequent models. Multiple product models are useful because some of the important consequences of imperfect capital funds markets and of shifting asset structures on farm production are from interproduct relationships.

Profit maximizing farmers prefer an alternative to the initial or existing farm organization if a positive gain is produced by the change. Using the notation of the glossary on page 33, the gain function is

$$(1) \quad G = \sum_{j=1}^m P_{y_j} (Y_j - Y_j^0) - \sum_{i=1}^n P_{x_i} (X_i - X_i^0)$$

The measure of gain is independent of prices of services regarded as fixed to the farm. This is a convenience because it eliminates the need for placing values on fixed flows of services. Evaluation of fixed services is required in the measure of profits. It is not required in the measure of gain or in determining the optimal farm organization. While market prices of fixed services are not of consequence in models 1 and 2, they will be shown to be important in models 3 and 4.

The optimal or most efficient use of resources is one which maximizes gain subject to restrictions limiting the

possibilities for reorganizing the farm. One important restriction is the production function which states technical relationships among input and output flows.

A general statement of technical restrictions is summarized in equation (2) which allows for interdependence

$$(2) \quad \theta(Y_1, Y_2, \dots, Y_m, X_1, X_2, \dots, X_n) = 0$$

among enterprises. If the output of the j^{th} enterprise is independent of a change in an input to the k^{th} enterprise ($1 \leq j, k \leq m$) then allocation of services among enterprises can be displayed in the following m independent equations.

$$(3) \quad \theta_j(Y_j, X_{1j}, X_{2j}, \dots, X_{ij}, \dots, X_{nj}) = 0 \quad j = 1, 2, \dots, m$$

Under reasonable assumptions about the form of equations (3) they can be solved for

$$(4) \quad Y_j = f_j(X_{1j}, X_{2j}, \dots, X_{ij}, \dots, X_{nj}) \quad j = 1, 2, \dots, m$$

In this and subsequent models, it is assumed that production functions can be displayed as in equations (4). The X_{ij} appearing in the production functions reflect the quantity of the i^{th} service used in the j^{th} enterprise. Some of the X_{ij} may be zero. A zero value simply means that the i^{th} service is not used in the j^{th} enterprise.

Some of the zero X_{ij} mean that the i^{th} service is not useful as a milking machine is not useful in the corn enterprise or as a corn picker is not useful in the dairy enterprise. For these we assume that productivities are not positive for all admissible X_{ij} . Other zero values occur when the i^{th} service

is not used as when wheat is not fed to hogs. These occur as consequences of economic rather than technical considerations. Answers to questions about economic reasons for zero input levels are provided by the solution for the gain maximizing farm organization. It is necessary to impose the restriction that the X_{ij} are not negative if meaningful results are to be assured.

Another fixed condition for model 1 is the list of quantities of services fixed to the farm. Let all X_i for which $d < i \leq n$ be on this list. Then all X_i for which $1 \leq i \leq d$ are variable to the farm. The list of quantities of fixed services may be written as

$$(5) \quad \sum_{j=1}^m X_{ij} = X_i^0 \quad i = d+1, \dots, n$$

The problem is to maximize gain equation (1) subject to production functions (4) and the list of fixed services (5) with the understanding that all service uses are non-negative. It is convenient in the exposition of this model to substitute production functions (4) into gain equation (1). This eliminates m dependent unknown quantities from the gain equation. The quantities, which measure the flows of output from each enterprise, may be computed by equations (4) once the optimal uses of factor services are determined.

Making these substitutions, and remembering that elements in the expenditure term of the gain equation vanish for $d < i \leq n$ because there can be no change in expenditure for

these fixed inputs, the problem is transformed to one of maximizing

$$(6) \quad G^* = \sum_{j=1}^m P_j (f_j - f_j^0) - \sum_{i=1}^d P_i \left(\sum_{j=1}^m X_{ij} - X_i^0 \right)$$

subject to constraints imposed by the list of fixed services in equations (5) and with the understanding that the X_{ij} are non-negative.

Kuhn and Tucker¹ have shown that the problem can be transformed into an equivalent saddle value (minimax) problem by an adaptation of the calculus method customarily applied to constraining equations. In conformance with the Kuhn and Tucker formulation, form the Lagrangian function

$$(7) \quad L \equiv G^* + \sum_{i=d+1}^n \lambda_i (X_i^0 - \sum_{j=1}^m X_{ij})$$

Then a particular set of X_{ij} maximizes G^* subject to the list of fixed services if and only if there is a set of λ_i (for $i = d + 1, \dots, m$) such that equation (7) is maximized with respect to the X_{ij} and minimized with respect to the λ_i for all non-negative X_{ij} . Kuhn and Tucker point out that such a saddle point provides a solution for a related zero sum two person game.

There are $(mn + n - d)$ unknowns in addition to the unknown gain in equation (7). These unknowns are the X_{ij}

¹Kuhn, H. W., and Tucker, A. W., "Non-linear Programming," Second Berkeley Symposium on Mathematical Statistics and Probability, Neyman, J., editor, University of California Press, Berkeley and Los Angeles, 1951. pp. 481-92.

and the λ_i . For a solution to the saddle value problem it is necessary, according to Kuhn and Tucker, that the unknowns satisfy the following $(mn + n - d)$ conditions:

$$(8) \quad \frac{\delta L}{\delta X_{ij}} \leq 0 \quad \frac{\delta L}{\delta X_{ij}} X_{ij} = 0 \quad X_{ij} \geq 0 \quad \begin{matrix} 1 \leq i \leq n \\ 1 \leq j \leq m \end{matrix}$$

$$(9) \quad X_i^0 - \sum_{j=1}^m X_{ij} = 0 \quad d < i \leq n$$

If a derivative in (8) is negative then it follows that the X_{ij} is equal to zero in the optimal organization if the three conditions in (8) are to be satisfied. On the other hand, if an X_{ij} is positive, then it follows from (8) that the derivative vanishes for that X_{ij} .

To understand the necessary conditions stated in equations (8) and (9) it is helpful to compare the properties of the solution for variable services and for fixed services. If the i^{th} service is variable to the farm and is used in the j^{th} enterprise then

$$(10) \quad P_{y_j} \frac{\delta Y_j}{\delta X_{ij}} - P_{x_i} = 0 \quad 1 \leq i \leq d$$

which means that the marginal value products are equal to each other and equal to flow price for each enterprise in which the service is used. This is a well known and frequently described result of the static theory of the firm. If the marginal value product of a service is less than flow price in some enterprise, it is implied that the service is not used in that enterprise.

For services fixed to the farm, a property of the necessary conditions is that if the i^{th} service is used in the j^{th} enterprise then

$$(11) \quad P_{y_j} \frac{\partial Y_j}{\partial X_{ij}} - \lambda_i = 0 \quad d < i \leq n$$

which means that the marginal value products are equal to each other and equal to the Lagrangian multiplier λ_i for each enterprise in which the service is used. A marginal value product less than λ_i implies that the service is not used in that enterprise.

The multiplier λ_i is a measure of on-farm opportunity cost. Using an additional unit of the i^{th} service in any enterprise in the optimal farm organization means withdrawal of a unit from an enterprise for which the marginal value product is equal to λ_i . Opportunity costs rather than market prices are used to allocate fixed services among enterprises.

One shortcoming of this model is that opportunity costs are not defined relative to off-farm opportunities. For example, there is no assurance in this model that the on-farm opportunity cost is not greater than the market price for which additional units of the service may be acquired. This shortcoming is amended in models 3 and 4 where the list of fixed services is determined endogenously rather than given as a fixed condition.

The relation of flow prices to market prices for durable stocks and to interest charges against borrowed funds are not satisfactorily handled in this model. Inasmuch as flow prices are used to allocate variable services, these relationships are rather important in determining the optimal use of services from durables when the stocks are regarded as variable to the farm. Efforts to amend this shortcoming are made in models 2 and 4.

Sufficient conditions for a solution to the saddle value problem as stated in model 1 are satisfied if production surfaces defined by equations (4) are concave with respect to services regarded as variable to the farm. This means that diminishing physical returns must occur for these services if they are used in the optimal organization. Rates of return from services regarded as fixed to the farm are not subject to this restriction. It is necessary but not sufficient that the production surfaces are not convex which permits constant but not increasing returns from services regarded as variable to the farm.

Models 2 and 4 are not as restrictive on the production surfaces. Increasing returns from services which are variable to the farm may occur in the optimal farm organization according to those models. Such results lead to interesting implications about important determinants of farm size. For model 1, sufficient conditions for a solution to the saddle value problem are satisfied if in addition to equations (8)

and (9), L is a concave function of the X_{ij} given the optimal values for the λ_i ; and if L is a convex function of the λ_i given the optimal values for the X_{ij} .

Model 2

The Capital Funds Restriction

Versions of the static theory of the firm which are rooted in assumptions similar to model 1 are scattered widely throughout the literature. Many of these are for single rather than multiple product analysis and therefore omit rules for optimal allocation of fixed services among enterprises. The abstraction is very useful for solving some important problems, but it leaves several things to be desired. Two of its major weaknesses include its failure to recognize investments required in durable assets that are variable to the farm, and its makeshift arrangement for answering limited questions about fixed assets.

Model 2 helps amend one of these weaknesses by showing in mathematical terms some relationships of the farmer's money market to optimal farm organization. The results of model 2 are recognized in the descriptive literature of agricultural economics, but are frequently omitted from mathematical formulations of farm problems. Mathematical statements of these relationships can facilitate research on the empirical relationships of availability of investment capital and farm organization.

Conditions in the farmers money market are related to farm organization in model 2 through an upward sloping supply

function for investment and working capital funds. This replaces the assumption of a constant rate of interest given as a fixed condition in model 1 with its implication that a farmer may borrow as much money as he cares to at the stated rate.

Interest payments are separated from other cost components in model 2. This procedure clearly displays the relation of borrowed funds to gainful farm reorganization.

Interest payments affect supply curves for services regarded as variable to the farm. These supply curves are defined by marginal factor costs, or rates of change in expenditure with respect to changes in service use. The supply functions reflect constant market flow prices for services, as in model 1, but in addition include a non-constant interest charge. The interest charge in marginal factor costs is based on the marginal rate of interest to the farm. Inasmuch as the endogenously determined marginal rate of interest is presumed to increase with increased use of capital funds, factor supply curves to the farm are upward sloping. Individual service supply functions are interdependent because (1) physical productivities are interdependent and (2) the interest charge for using one service depends on the funds required to use all services.

With interest charges separated from other cost components, the gain function to maximize in model 2 is

$$(1) \quad G = \sum_{j=1}^m P_{y_j} (Y_j - Y_j^0) - \sum_{i=1}^n P_{x_i} (X_i - X_i^0) - \int_{K^0}^K g(u) du$$

where K is the investment and working capital funds used on the farm, and $g(u)$ is the supply function for these funds. The marginal rate of interest is $g(K)$.

Both the interest payment and the measure of gain are regarded as independent of the value of fixed services. To assure that this is so, it is understood in model 2 that the funds supply function g describes interest payments relative to K^0 independently of arbitrary changes in the measure of K^0 imposed, for example, by arbitrary revaluation of durable stocks which are regarded as fixed to the farm. This precaution is handled more systematically in model 4 where the list of fixed services is endogenously determined. The relevant interest payment as far as gains are concerned is the payment associated with changes in the use of funds where the change is measured by $(K - K^0)$.

While evaluation of fixed services is not required in model 2 to determine the gain maximizing farm organization, an evaluation is required in measures of profits and of equity. These topics are of interest in model 4 where appraisal of fixed services based on marginal value productivities is discussed.

The problem of model 2 is to maximize gain subject to restrictions on capital funds, production functions and the list of fixed services.

The total quantity of capital funds used on the farm is the sum of the investment capital and working capital required for each of the n services used. Funds required per unit of the i^{th} service are denoted by q_i which reflects the investment in stock associated with one unit of flow plus the working capital required to maintain the utilized flow. That is, q_i is the price of stock plus flow price less any double counting such as the depreciation component of flow price.

Capital funds requirements measured in this way assign to each unit of utilized flow a share of the expenditure for using the stock. The share covers (1) acquisition costs of stock, transfer fees, sales taxes, and other costs of making the services available for use, (2) annual, or periodic expenditures for license fees, property taxes and other costs of keeping the stock of services available for use once the stock has been acquired, and (3) wear and tear charges and other costs of utilization which vary with the actual use of the available services. Because of the assumption of perfect divisibility of stock, discussed on page 28, it is implied that one can acquire exactly enough additional stock to make one additional unit of flow of the service available however small the unit may be. It is further implied that available flow is equal to utilized flow on optimally organized

farms. Thus, q_i represents the funds required to incorporate one more unit of utilized flow in the production process except for interest charges. The quantity q_i may be paid either from cash funds held by the farmer, or from credit. Interest charges are handled separately.

For all funds used in all enterprises on the farm, the q_i must be summed for each of the X_{ij} . Therefore, the total quantity of investment and working capital funds is

$$(2) \quad K = \sum_{i=1}^n q_i \sum_{j=1}^m X_{ij}$$

Equation 2 is one of the fixed conditions imposed on the gain equation. It constitutes the major departure from model 1. The other two fixed conditions are exactly as in model 1. First there is a set of m independent production functions

$$(3) \quad Y_j = f_j(X_{1j}, X_{2j}, \dots, X_{nj}) \quad j = 1, 2, \dots, m$$

which are exactly as in equation (4) of model 1. Similarly, there is a list of fixed factor services

$$(4) \quad \sum_{j=1}^m X_{ij} = X_i^0 \quad i = d+1, \dots, n$$

which is exactly as equation (5) of model 1.

It is convenient in the exposition to eliminate some of the dependent unknowns from the gain equation as was done in model 1. Substitution of the capital funds restriction (2) and production functions (3) eliminates K and the Y_j .

Gain is then expressed as a function of the X_{ij} . When the optimal quantities of input uses (X_{ij}) are determined, the quantity of funds and the rates of output flows may be calculated. Making these substitutions, and remembering that the change in expenditure for services from fixed stocks is zero, the problem is reduced to one of maximizing

$$(5) \quad G^* = \sum_{j=1}^m P_{y_j} (f_j - f_j^0) - \sum_{i=1}^d P_{x_i} \left(\sum_{j=1}^m X_{ij} - X_i^0 \right) - \int_{K^0}^{\sum_{i=1}^n q_i \sum_{j=1}^m X_{ij}} g(u) du$$

with respect to the X_{ij} subject to constraints imposed by the list of fixed services in equations (4) and with the understanding that the X_{ij} are non-negative.

Using the Kuhn and Tucker procedure for transforming the problem into an equivalent saddle value (minimax) problem as described for model 1, form the Lagrangian function

$$(6) \quad L \equiv G^* + \sum_{i=d+1}^n \lambda_i (X_i^0 - \sum_{j=1}^m X_{ij})$$

which is to be maximized with respect to the X_{ij} and minimized with respect to the λ_i for all non-negative X_{ij} .

Necessary conditions for a solution to the saddle value problem, according to Kuhn and Tucker, are

$$(7) \quad \frac{\partial L}{\partial X_{ij}} \leq 0 \quad \frac{\partial L}{\partial X_{ij}} X_{ij} = 0 \quad X_{ij} \geq 0 \quad \begin{matrix} 1 \leq i \leq n \\ 1 \leq j \leq m \end{matrix}$$

$$(8) \quad X_i^0 - \sum_{j=1}^m X_{ij} = 0 \quad d < i \leq n$$

which have the same appearance as equations (8) and (9) in model 1.

Interpretation of the results of model 2 are different in some important respects from interpretations of the previous model. The differences follow from statements about fixed conditions in the money market, and are reflected through equations (7) in the explicit form taken by derivatives of the Lagrangian with respect to services regarded as variable to the farm.

On the other hand, rules for allocating fixed services among enterprises according to opportunity costs are identical in the two models. Changing conditions in the money market do not change the rules for determining the best use of fixed services. Hence the rules shall not be repeated here.

For services regarded as variable to the farm, if the i^{th} service is used in the j^{th} enterprise then

$$(9) \quad P_{y_j} \frac{\delta Y_j}{\delta X_{ij}} = P_{x_i} + q_i [g(K)] \quad 1 \leq i \leq d$$

which differs from the analogous equation (10) in model 1 in several respects.

The marginal value products of the i^{th} service are equal to each other and equal to marginal factor costs for each enterprise in which a service variable to the farm is used. This statement about the relation of costs to productivities for optimally used services is a well known result.

The special claim of model 2 is in its measure of marginal factor costs which, by the way, defines the supply function of the i^{th} service to the farm. The marginal factor cost in equation (9) is flow price plus an interest charge. The flow price (P_{x_i}) is regarded as a constant in this model. The price includes a depreciation reserve designed to recover the investment in the durable. The interest charge is shown separately from the flow price. It is an increasing function of the total quantity of capital used which means that marginal factor costs are increasing.

The interest charge is computed relative to the total quantity of investment and working capital funds required (q_i) and not relative to flow price. In the special case of non-durables, where total funds required is equivalent to flow price, the marginal factor cost for the i^{th} service simplifies to $p_{x_i} [1 + g(K)]$ in equation (9).

Furthermore, the interest charge is computed relative to the marginal rate of interest $g(K)$ and not relative to the average rate. The interest charge is an opportunity cost which does not necessarily reflect a rate actually paid in connection with the purchase of the marginal unit of service. It does reflect the payment that would be contracted by increasing borrowings sufficiently to place another unit of utilized service on the farm. Alternatively, it reflects the extent to which debt commitments may be reduced, and a reduction in the total interest payment effected, if a unit

of the i^{th} service were salvaged. The measure is the same in either direction in model 2. It need not be the same in model 4.

The average rate of interest paid on borrowed funds is less than the marginal rate when price discrimination is practiced on upward sloping supply curves for funds. Consequences of this are (1) restricted output and (2) a surplus as a component of profits. Both consequences are well known results of theories about firms where there is an element of imperfect competition or of monopoly in the market structure. In the case of model 2, imperfect competition is introduced through the money market.

The capital funds supply function g may vary widely among farmers. Consequently, the marginal rate of interest and hence marginal factor cost need not be the same for different farmers who purchase inputs in a single, competitive market. This helps explain how farmers with similar technical possibilities for production but with different facilities for gaining funds may have considerably different farm organizations. A farmer with limited capital would need to get a greater return from an additional expenditure on his barn, for example, than a wealthier neighbor not because the materials were purchased at different prices but because the only two sources he has of obtaining funds with which to acquire the additional materials are (1) substitution of funds from another use where the productivity is high or

(2) from the money market at a higher rate of interest than his more fortunate neighbor.

Non-increasing physical returns were required of services variable to the farm in model 1. Model 2 is more flexible in this regard because sufficiency conditions that a solution to the saddle value problem is optimal include the possibility of increasing returns to variable services. The restriction imposed is simply that the rate of change in additional returns is less than the rate of change in additional cost. For example, if the Lagrangian function is concave with respect to the X_{ij} , given the optimal values for the λ_i , it is implied that if the i^{th} service variable to the farm is used in the j^{th} enterprise then

$$(10) \quad P_y \frac{\delta^2 Y_j}{\delta X_{ij}^2} < q_{id}^2 \frac{dg}{dK} \quad 1 \leq i \leq d$$

where the right hand side is not negative under the assumptions of model 2. This leads to interesting speculations on conditions related to optimal use of resources and on the relative importance of various determinants of farm size which are not recognized in model 1.

Statements in the last few paragraphs stress some important possibilities for answering questions about farm organization and supply response which are related to investments in durables. They indicate areas in which advantages may be gained from application of abstractions similar to

model 2 compared to model 1. An important weakness of model 1 remains in model 2, however. The weakness is a failure to relate on-farm opportunity costs of services regarded as fixed to the farm to off-farm possibilities for either acquisition or salvage of some of these services. An effort to amend this weakness is made in model 3. The results of models 2 and 3 are then blended in model 4. Evaluation of the relationships among supply functions for investment and working capital funds, lists of fixed services and optimal farm organization remains for chapter 3.

Model 3

The Shifting Asset Structure

An asset is defined as fixed in chapter 1 if it is not worth varying. This model examines whether the flow of services from durable and non-durable stocks is worth varying. The procedure does not use a predetermined list of fixed services such as is used in the two previous models. The best list of fixed services is endogenously determined in model 3 simultaneously with the determination of optimal use of services which are variable to the farm. The best list of fixed services depends on the economic environment of the farm. Changes in the environment can change the list.

More precisely, services are fixed to the farm if the initial quantities on hand are optimal and remain so subsequent to at least small changes in the economic environment. Changes may occur, for example, in flow prices, product prices and/or physical productivities of services.

Services were shown in chapter 1 to be subject to fixity when acquisition costs (A_i) of additional services are greater than salvage values (S_i) of existing services on the farm. Services variable upward are priced according to acquisition costs, services variable downward, according to salvage values. Services remaining fixed to the farm may be valued between these limits.

The problem of model 3 is to maximize the gain equation defined by

$$(1) \quad G = \sum_{j=1}^m P_{x_j} (Y_j - Y_j^0) - \sum_{i=1}^n a_i \left(\sum_{j=1}^m X_{ij} - X_i^0 \right)$$

subject to fixed conditions defined by production functions and by price functions for services where the price functions reflect divergences of acquisition costs from salvage values for productive services. (see glossary on page 33)

The price functions for inputs are defined by

$$\sum_{j=1}^m X_{ij} > X_i^0 \Rightarrow a_i = A_i$$

$$(2) \quad \sum_{j=1}^m X_{ij} < X_i^0 \Rightarrow a_i = S_i \quad 1 \leq i \leq n$$

$$\sum_{j=1}^m X_{ij} = X_i^0 \Rightarrow A_i \geq a_i \geq S_i$$

which state that acquisition costs apply if the use of services is greater than the initial quantity used and that salvage values apply if the use is less than initial.

The problem of model 3 is difficult to solve when stated in the above form because without prior knowledge about whether the i^{th} service is variable upwards, variable downwards, or fixed to the farm, one does not know the appropriate solution to equations (2). The problem is easier to solve if it is restated in another form which is amenable to the Kuhn and Tucker procedure previously cited for maximization subject to inequalities.

To restate the problem, let γ_i be the quantity of X_i that is sold on the salvage market, and let ω_i be the quantity that is acquired in addition to the existing quantity X_i^0 . Obviously, γ_i and ω_i cannot be positive simultaneously because one cannot use both more and less at the same time. The total quantity of the i^{th} service used in a farm organization is defined by

$$(3) \quad \sum_{j=1}^m X_{ij} = X_i^0 - \gamma_i + \omega_i \quad 1 \leq i \leq n$$

where either γ_i or ω_i must vanish. If both should vanish, the initial quantity is used. If γ_i is positive, less than the initial quantity of the i^{th} service is used, and if ω_i is positive, more than initial is used. We require that both γ_i and ω_i are non-negative. A negative quantity of γ_i , for example, would mean that some of the i^{th} service is acquired at its salvage value which is presumed to be impossible. An additional restriction required on γ_i is

$$(4) \quad X_i^0 - \gamma_i \geq 0 \quad 1 \leq i \leq n$$

to assure that one does not sell more of the i^{th} service on the salvage market than initially exists on the farm.

With these definitions, and eliminating the production functions from equation (1), the restated gain equation is

$$(5) \quad G^* = \sum_{j=1}^m P_j y_j (f_j - f_j^0) + \sum_{i=1}^n S_i \gamma_i - \sum_{i=1}^n A_i \omega_i$$

which is to be maximized subject to equations (3) and (4) for non-negative X_{ij} , γ_i and ω_i .

The restated problem can be put in the Lagrangian form

$$(6) \quad L \equiv G^* + \sum_{i=1}^n \lambda_i (X_i^0 - \gamma_i + \omega_i - \sum_{j=1}^m X_{ij}) + \sum_{i=1}^n \mu_i (X_i^0 - \gamma_i)$$

which is to be maximized with respect to the X_{ij} , γ_i and ω_i and minimized with respect to the λ_i and μ_i for all non-negative X_{ij} , γ_i , ω_i and μ_i .

The necessary conditions for a solution to the saddle value problem, adapted from the previously cited Kuhn and Tucker procedures, are

$$(7) \quad \frac{\partial L}{\partial X_{ij}} = p_{y_j} \frac{\partial Y_j}{\partial X_{ij}} - \lambda_i \leq 0 \quad \frac{\partial L}{\partial X_{ij}} X_{ij} = 0 \quad X_{ij} \geq 0$$

$$(8) \quad \frac{\partial L}{\partial \omega_i} = \lambda_i - A_i \leq 0 \quad \frac{\partial L}{\partial \omega_i} \omega_i = 0 \quad \omega_i \geq 0$$

$$(9) \quad \frac{\partial L}{\partial \gamma_i} = S_i - \mu_i - \lambda_i \leq 0 \quad \frac{\partial L}{\partial \gamma_i} \gamma_i = 0 \quad \gamma_i \geq 0$$

$$(10) \quad \frac{\partial L}{\partial \mu_i} = X_i^0 - \gamma_i \geq 0 \quad \frac{\partial L}{\partial \mu_i} \mu_i = 0 \quad \mu_i \geq 0$$

$$(11) \quad \frac{\partial L}{\partial \lambda_i} = X_i^0 - \gamma_i + \omega_i - \sum_{j=1}^m X_{ij} = 0$$

for all i and j .

Necessary and sufficient conditions for a solution to the saddle value problem are satisfied if, in addition to equations (7) to (11), L is a concave function of the X_{ij} , γ_i and ω_i given the optimal values of the Lagrangian multipliers and if L is a convex function of the Lagrangian multipliers given the optimal values for the X_{ij} , γ_i and ω_i .

To understand the necessary conditions and see how they differ from the results of previous models, it is helpful to examine some of the properties. If the i^{th} service is used in the j^{th} enterprise, it follows from equation (7) that

$$(12) \quad P_{y_j} \frac{\delta Y_j}{\delta X_{ij}} = \lambda_i$$

which means that the marginal value products are equal to each other and equal to λ_i for each enterprise in which the service is used. The multiplier λ_i is the on-farm opportunity cost of using the i^{th} service. A marginal value product less than the on-farm opportunity cost implies that the service is not used in that enterprise.

On-farm opportunity costs are bounded by off-farm opportunities for acquisition and salvage on optimally organized farms. This follows from relations (8), (9) and (10) which imply that if the i^{th} service is used on the farm, then

$$(13) \quad A_i \geq \lambda_i \geq S_i$$

If the optimal farm organization uses more than X_i^0 units of the i^{th} service, it is implied by equation (8) that on-farm opportunity cost equals acquisition cost for the service. If the optimal organization uses less than X_i^0 it is implied by equation (9) that on-farm opportunity cost equals salvage value. If the initial quantity of the service remains optimal, then on-farm opportunity cost is bounded by the acquisition and salvage values but need not equal either bound. An on-farm opportunity cost which is less than salvage value implies

by equations (9) and (10) that X_i^0 is worth more on the salvage market than it is worth in production and that consequently the i^{th} service is not used on the farm.

This completes the development of model 3 in which services, whose initial quantities remain optimal and whose acquisition cost is greater than salvage value, are listed as fixed services. A solution to the saddle value problem is also a solution to the problem initially stated for model 3 where the α_i in equations (1) and (2) are set equal to the λ_i , or on-farm opportunity costs.

The λ_i are useful in appraising the value of services used on farms. For services which are exchanged on the market, it is customary to appraise according to off-farm value. For such services, off- and on-farm values are equal to one another in the optimal organization. For services fixed to the farm, and to which market values are not assigned, it is useful to appraise according to value in production. In either event, λ_i is useful in valuing a unit of the i^{th} service. This value may be capitalized into an appraisal value for the stock from which the service flows.

Profits and returns to owned assets. According to Euler's Theorem¹, profits for the optimal farm organization are zero if (1) production functions are homogenous of degree

¹Brand, Louis, Advanced Calculus, John Wiley and Sons, Inc., New York, 1955. p. 161.

one and (2) fixed services are paid according to their on-farm opportunity costs.

This well known result has an interesting interpretation in model 3 where the application is made for all services regardless of the relevant length of run or fixity of assets. The application does not require a length of run long enough for all economic forces to work themselves out. Nor does it require that homogeneity is only with respect to services which are variable to the farm. It does assert that for optimally organized farms, profits can vanish when farmers charge for the use of fixed assets according to the on-farm opportunity costs of the services.

While profits may be zero on gain maximizing farms, the rate of returns to fixed factor services is greater than for other organizations. Maximizing gains is equivalent to maximizing the value in production of fixed services. If services are paid according to their value in production, it is equivalent to maximizing the rate of returns to fixed services.

Some of the fixed services on the farm are owned by the farmer as part of his equity. The farmer's wellbeing, according to the shifting asset structure model, depends on the rate of returns he can pay himself for the use of owned assets as well as on the quantity he owns or the size of his equity. In model 4, it is shown that the supply of funds

which a farmer can borrow to supplement his equity in the business affects the rate of returns to owned assets.

Optimal use of services depends on initial use. The gain maximizing farm organization depends upon, or is a function of the initial farm organization. That is, for two farms with identical production functions and market prices but with different initial farm organizations the gain maximizing farm organizations need not be identical. Differences in the supply functions for productive services on these farms lead to the different solutions. Supply functions for farm services which are subject to fixity have a discontinuity which is unique to each farm and which is determined by the initial farm organization.

Applications of models 1 and 2 have the property that the solution depends upon the levels at which fixed factor services are assumed to be fixed. For model 3, the solution depends upon the initial quantities of all factor services which are subject to fixity.

Non-reversibilities. The gain equation used in model 3 is not reversible. In models 1 and 2 the gain equation is reversible in the sense that the gain in changing from one farm organization to another is the negative of the gain for reversing the direction of change. The gain is not reversible in model 3 because acquisition costs are used in the measure of gain in one direction and salvage values in the reverse direction.

Non-reversible (1) supply functions for farm products and (2) demand functions for capital funds result from the non-reversible property of the gain function. But these are topics of chapter 3; first there remains the task of integrating the findings of the capital restriction of model 2 into the shifting asset structure model. This task is done in model 4.

Model 4

The Capital Restriction and the
Shifting Asset Structure

This model combines the features of the previous models to display the interaction of shifting asset structures and changing capital requirements on farm production.

The problem of model 4 is to maximize the gain equation defined by

$$(1) \quad G = \sum_{j=1}^m P_{y_j} (Y_j - Y_j^0) - \sum_{i=1}^n \alpha_i \left(\sum_{j=1}^m X_{ij} - X_i^0 \right) - \int_{K^0}^K g(u) du$$

subject to fixed conditions defined by production functions, acquisition and salvage functions for services and investment and working capital requirements for services, and where $g(u)$ is the supply function for capital funds. (see glossary on page 33) As in model 3, the fixed conditions do not include a predetermined list of fixed services.

The total requirement for investment and working capital funds in a farm reorganization is equal to the initial quantity of funds used plus the change required in funds by the change from the initial organization to the reorganization. The total requirement is defined by

$$(2) \quad K = K^0 + \sum_{i=1}^n \beta_i \left(\sum_{j=1}^m X_{ij} - X_i^0 \right)$$

where the β_i are the investment and working capital requirements for units of each of the n services used on the farm. For services whose acquisition costs are greater than salvage values, the additional funds required to acquire stock and thus provide more services on the farm probably differ from the savings in funds effected by salvaging stock and reducing the quantity of services available. Consequently, the investment and working capital funds required per unit of service may depend on whether the service is variable upward, variable downward, or fixed to the farm. Values for the β_i are assigned according to the rules in equations (3) which also contain rules for assigning values to the flow prices (α_i) appearing in equation (1).

(3)

$$\sum_{j=1}^m X_{ij} > X_i^0 \Rightarrow \alpha_i = A_i \quad \text{and} \quad \beta_i = q_i^a$$

$$\sum_{j=1}^m X_{ij} < X_i^0 \Rightarrow \alpha_i = S_i \quad \text{and} \quad \beta_i = q_i^s$$

$$\sum_{j=1}^m X_{ij} = X_i^0 \Rightarrow A_i \geq \alpha_i \geq S_i \quad \text{and} \quad q_i^a \geq \beta_i \geq q_i^s$$

for all i .

where q_i^i represents the quantity of funds required per unit of the i^{th} service, and the superscripts (a) and (s) distinguish whether the requirement is computed relative to acquisition cost or salvage value of the stock from which the i^{th} service flows.

In model 2, it was mentioned that arbitrary changes in K^0 caused by changes in appraisal values of fixed assets must not be permitted to bias the conditions for optimal use of productive services. As a precaution against having the solution depend on arbitrary changes in the measure of K^0 , we define the capital supply function $g(u)$ relative to K^0 and then define

$$(4) \quad K^0 = \sum_{i=1}^n \beta_i X_i^0$$

For β_i such that $q_i^a > \beta_i > q_i^s$, β_i may be defined unambiguously as a function of the marginal value product of the i^{th} service valued at its optimal use. The definition (4) may be substituted into equation (2) to simplify the expression for the total requirement for investment and working capital funds to

$$(5) \quad K = \sum_{i=1}^n \beta_i \sum_{j=1}^m X_{ij}$$

As in model 3, the problem of model 4 is easier to solve if it is restated in another form. In the restatement, we again let γ_i be the quantity of the i^{th} service that is sold on the salvage market and let ω_i be the quantity acquired. Remember that γ_i and ω_i cannot be positive simultaneously. With these definitions, the quantity of the i^{th} service used on the farm is

$$(6) \quad \sum_{j=1}^m X_{ij} = X_i^0 - \gamma_i + \omega_i \quad 1 \leq i \leq n$$

where the γ_i and ω_i are non-negative and where

$$(7) \quad X_i^0 - \gamma_i \geq 0 \quad 1 \leq i \leq n$$

Now the requirement for funds can be restated as

$$(8) \quad K = K^0 - \sum_{i=1}^n q_i^s \gamma_i + \sum_{i=1}^n q_i^a \omega_i$$

With these definitions, and eliminating the production functions by substitution, the restated gain equation is

$$(9) \quad G^* = \sum_{j=1}^m P_{y_j} (f_j - f_j^0) + \sum_{i=1}^n S_i \gamma_i - \sum_{i=1}^n A_i \omega_i - \int_K^K g(u) du$$

which is to be maximized subject to equations (6), (7) and

(8) for non-negative X_{ij} , γ_i and ω_i .

The restated problem can be put in the Lagrangian form

$$(10) \quad L \equiv G^* + \sum_{i=1}^n \lambda_i (X_i^0 - \gamma_i + \omega_i - \sum_{j=1}^m X_{ij}) + \sum_{i=1}^n \mu_i (X_i^0 - \gamma_i) \\ + \phi (K - K^0 + \sum_{i=1}^n \gamma_i q_i^s - \sum_{i=1}^n \omega_i q_i^a)$$

which is to be maximized with respect to the X_{ij} , γ_i , ω_i and K and minimized with respect to the λ_i , μ_i and ϕ .

The necessary conditions for a solution to the saddle value problem are

$$(11) \quad \frac{\partial L}{\partial X_{ij}} = P_{y_j} \frac{\partial Y_j}{\partial X_{ij}} - \lambda_i \leq 0 \quad \frac{\partial L}{\partial X_{ij}} X_{ij} = 0 \quad X_{ij} \geq 0$$

$$(12) \quad \frac{\partial L}{\partial \omega_i} = \lambda_i - A_i - \phi q_i^a \leq 0 \quad \frac{\partial L}{\partial \omega_i} \omega_i = 0 \quad \omega_i \geq 0$$

$$(13) \quad \frac{\delta L}{\delta \gamma_i} = S_i + \phi q_i^s - \mu_i - \lambda_i \leq 0 \quad \frac{\delta L}{\delta \gamma_i} \gamma_i = 0 \quad \gamma_i \geq 0$$

$$(14) \quad \frac{\delta L}{\delta K} = \phi - g(K) \leq 0 \quad \frac{\delta L}{\delta K} K = 0 \quad K \geq 0$$

$$(15) \quad \frac{\delta L}{\delta \mu_i} = X_i^0 - \gamma_i \geq 0 \quad \frac{\delta L}{\delta \mu_i} \mu_i = 0 \quad \mu_i \geq 0$$

$$(16) \quad \frac{\delta L}{\delta \lambda_i} = X_i^0 - \gamma_i + \omega_i - \sum_{j=1}^m X_{ij} = 0$$

$$(17) \quad \frac{\delta L}{\delta \phi} = K - K^0 + \sum_{i=1}^n \gamma_i q_i^s - \sum_{i=1}^n \omega_i q_i^a = 0$$

for all i and j .

Necessary and sufficient conditions for a solution to the saddle value problem are satisfied if, in addition to equations (11) to (17), L is a concave function of the X_{ij} , γ_i , ω_i and K given the optimal values of the Lagrangian multipliers; and if L is a convex function of the Lagrangian multipliers given the optimal values for the X_{ij} , γ_i , ω_i and K .

To understand the necessary conditions and see how they compare with the results of the previous models, it is helpful to examine some of the properties. If the i^{th} service is used in the j^{th} enterprise, it follows from equation (11) that

$$(18) \quad P_{y_j} \frac{\delta Y_j}{\delta X_{ij}} = \lambda_i$$

where λ_i is the on-farm opportunity cost of using the i^{th} service. The marginal value products of the i^{th} service are equal to each other and equal to on-farm opportunity cost in

each enterprise in which the service is used. A marginal value product less than the opportunity cost in an enterprise implies that the service is not used in that enterprise.

If the i^{th} service is used on the farm, it follows from equations (12) to (15) that

$$(19) \quad A_i + q_i^a [g(K)] \geq \lambda_i \geq S_i + q_i^s [g(K)]$$

which means that the on-farm opportunity cost is bounded by off-farm opportunities for acquisition and salvage of services.

The off-farm opportunity costs include flow prices and interest charges. The interest charges are money market opportunity costs based on the marginal rate of interest on the farm $[g(K)]$ and on the change in investment and working capital funds requirements (either q_i^a or q_i^s) with respect to a change in the quantity of the i^{th} service used on the farm. The interest charge is greater for acquisition than for salvage just as flow prices and stock prices diverge for acquisition and salvage.

If more than the initial quantity of the i^{th} service is used on the farm, it follows from equation (12) that on-farm opportunity cost is equal to off-farm opportunity for acquisition. If less than initial is used, it follows from (13) to (15) that on-farm opportunity cost is equal to off-farm opportunity for salvage. If on-farm opportunity cost lies between these bounds but equals neither bound, it is implied by equations (12), (13), and (16) that the initial quantity

is optimal and the service is regarded as fixed to the farm. The i^{th} service is not used on the farm if on-farm opportunity cost is less than off-farm opportunity for salvage.

This completes the development of model 4 in which capital funds are acquired on an imperfect market and resources are subject to fixity. The results obtained are largely a combination of the results of the previous models. An important advantage of model 4 is in the analysis of interrelationships of investments in stock to fixity of flows from stock. Other major strengths of the model are also strengths of models 2 or 3.

The interesting features of model 4 that have already been encountered in previous models include the relation of on-farm opportunity costs to off-farm opportunities, the non-reversibilities of response to changes in the economic environment, and the relation of initial farm organization and supply of funds to optimal organization and farm size. These and other implications of model 4 are discussed and evaluated in the following chapter.

The theorems used to display the necessary conditions for optimal farm organization, given the fixed conditions, are useful for the conceptual purposes of this thesis. The theorems were formulated by Kuhn and Tucker (see previous citation) for maximizing non-linear functions subject to linear inequalities. They conveniently lead to statements of the necessary conditions, but do not contain rules for

computing solutions for specific applications to farm problems. Such computations would generally involve a series of successive approximations such as used in the usual computational procedures for activity analysis. In specific applications, such computations may become far more tedious than might appear as obvious to the casual reader of this chapter. There remains the question of whether or not procedures can be devised for computing solutions to some problems. For most of the usual applications in agricultural economics, the required computational procedures are, or can be devised.

CHAPTER 3

APPLICABILITY OF THE MODELS

This chapter explores some implications of the models developed in chapter 2 and discusses their comparative usefulness. The role of investment commitments in durable stocks, and factors affecting decisions to change existing investments, are of particular interest. Some hypotheses are projected which may be useful in explaining changes in farm organization and size resulting from changes in the economic environment of farmers. Resource fixities are associated with non-reversibilities in farmers adjustments.

Supply responses, at the individual farm level, are compared for each of the four models. Some conjectures are made about the effects of resource fixity and credit availability in aggregate supply response. Capital gains and losses are shown to affect levels of farm output. Macro-supply relationships are not conceptualized in a definitive way in this study.

Some remarks on the relation of farm equities and returns thereto on farm production and farmer welfare are included in this chapter. The indications are that farm policy recommendations could be improved by accounting for asset fixity and investment commitments on American farms.

The discussions emphasize the implications of alternative propositions about fixed conditions in the money and factor markets. The framework used to derive these results is regarded as one of several means to an end.

Some empirical evidence is presented which helps to confirm the validity of abstractions such as model 4 which account for resource fixity and credit availability. The evidence appears to contradict model 1, which fails to account realistically for the two factors. The special conditions are reasonable in model 4 that (1) acquisition costs are greater than salvage values for some factors used on farms and (2) supply functions for capital funds are upward sloping. Rules for optimal use of resources subject to these two conditions are discussed and shown to be consistent with observations of changes in farm organization, output and prices. Actual measurements, and the gathering of statistical confidence measures, are not undertaken in this study.

The value of the procedures developed in chapter 2 is suggested by the following discussions but will not be adequately determined until propositions derived from the models are proven either useful or not useful in helping to quantify relationships and to solve important farm problems.

Farm Organization

Farmers seeking gainful adjustments in their businesses compare the existing organization to alternative possibilities.

The set of alternatives is determined by the fixed conditions, and optimal choice is guided by economic considerations.

Market prices are frequently described as the primary economic guide to resource allocation although there is some dissention to this proposition in the literature. Other guides which supplement the role of market prices are developed in chapter 2. Among these are on-farm opportunity costs to guide the allocation among enterprises of services which are fixed to the farm; money market opportunity costs to help guide the allocation of services which are variable to the farm; and relative on-farm to off-farm opportunities to assist in deciding whether a given service is variable or fixed.

On-farm opportunity costs are the result of an internal pricing mechanism which guides resource allocation when market prices are not relevant. The mechanism is important in describing the best use of services which remain fixed to the farm. For example, Heady¹ found working capital funds allocated between crops and livestock production by on-farm opportunity costs on a sample of 160-acre, Iowa farms for which additional working capital was not worth acquiring.

Commitments in durable stocks are made in one period and the responsibility for these commitments may be extended over several future periods. If product prices diminish and

¹Heady, Earl O., Resource Productivity and Returns on 160-Acre Farms in North-Central Iowa, Research Bulletin 412, Agricultural Experiment Station, Iowa State College, Ames, Iowa, July, 1954. p. 1084.

a stock becomes less valuable in production than formerly, it may remain more valuable on the farm than on the salvage market. Services whose acquisition costs are equal to salvage values are not subject to fixity. Off-farm opportunities are a sufficient guide to the optimal use of such services because off- and on-farm opportunities are equal in the optimal farm organization.

A portion of the off-farm opportunities reflect conditions in the money market. Farmers with different access to funds have different interest charges to add to market prices. For example, Heady and Swanson¹ found that less than 32 percent of the farmers in a 1939 survey in Iowa would borrow additional funds if returns per dollar of additional investment and working capital were ten percent. An additional 39 percent of the farmers in the survey would borrow more funds if returns were 25 percent; and an additional 10 percent of the farmers if returns were 50 percent. This indicates that off-farm opportunity costs can differ among farmers acquiring factor services in a competitive market.

Farmers deliberating a change in investment commitments under the assumed conditions will always find changes profitable if on-farm opportunity costs are either (1) greater than off-farm opportunities for acquiring additional services by

¹Heady and Swanson, Resource Productivity in Iowa Farming, Research Bulletin 388, Agricultural Experiment Station, Iowa State College, Ames, Iowa, June, 1952. p. 771.

obtaining additional stock from which the services flow or (2) less than opportunities for salvaging existing services by selling some of the existing stock. If a change in the existing investment in stocks is not profitable, on-farm opportunity costs are the relevant guides to allocation of services among enterprises.

Determining the best reorganization.

Fixed services in the existing farm organization restrict the opportunities for farm reorganization. For example, Hasbargen and Pond¹, in proposing three alternative budgets for a 240-acre Minnesota farm, used a maximum of 33 litters of pigs per year because of the capacity of the present housing facilities. These facilities were regarded as fixed. Hasbargen and Pond discuss the importance of the present organization in planning farms for increased profits on pages 6-8 of their publication.

Services are not fixed absolutely, but are fixed relative to the environment. Important environmental conditions which determine the best list of fixed services according to models 3 and 4 include product prices, acquisition costs and salvage values of inputs, the supply function for funds, technical relationships among inputs, and the initial farm organization.

¹Hasbargen, Paul R., and Pond, George A., Planning Farms for Increased Profits, Station Bulletin 445, Agricultural Experiment Station, University of Minnesota, December, 1957.

The importance of the existing farm organization in determining the optimal is sometimes overlooked in farm business analysis. The influence is only partly accounted for when a predetermined list of fixed services is included among the fixed conditions. The initial use of variable services which are subject to fixity is neglected by this procedure. The initial use of all inputs for which acquisition cost is greater than salvage value influences the optimal use of services on the farm.

The observation that the optimal farm organization depends on the initial organization is associated with the farm management axiom that for every farm and for every farmer there is a unique best farm plan. Differences in farm organization among farmers of comparable ability and in similar economic environments can be explained by models 3 and 4 through differences in equities in the farm businesses and differences in ~~initial~~ farm organization.

The exposition of the procedure in chapter 2 was within the framework of marginal analysis. The same general results would follow from an analogous procedure for composing the best list of fixed services in an activity analysis framework. To illustrate the possibilities of application in linear programming, consider the marginal value productivity for April labor of \$13.90 displayed by Swanson¹ for a corn belt

¹Swanson, Earl R., "Application of Linear Programming Analysis to Corn Belt Farms," Journal of Farm Economics, Vol. 38, May, 1956. p. 418.

farm. If an additional hour of April labor, including capital inputs that are regarded as perfect complements of April labor and which are not existing unused on the farm, can be acquired for less than \$13.90 then April labor need not be regarded as a limiting resource.

When no factor services are included among the fixed conditions and production processes result in constant returns to scale, indefinitely large farm sizes might be indicated in an analysis. Finite farm size can be assured for these cases if the fixed conditions include (1) an indefinitely high acquisition cost for at least one limiting service and/or (2) a fixed quantity of investment and working capital funds, or (3) a rate of change in additional cost which is greater than the rate of change in additional revenue. More is said of farm size in the next section of this chapter.

Candler¹ presented a procedure for varying some "limiting inputs" continuously while regarding other inputs as fixed. He illustrated the procedure by using capital funds as a continuous variable while regarding land and labor inputs as fixed. Candler's procedure was not designed to decide whether the initial quantity of service is optimal, but rather to find what the optimal use is once the decision has been made to consider the profitability of alternative levels of use.

¹Candler, Wilfred, "A Modified Simplex Solution for Linear Programming with Variable Capital Restrictions," Journal of Farm Economics, Vol. 38, November, 1956. p. 240.

New resource combinations.

Some changes in the economic environment lead to substantially new resource combinations. Others lead to relatively minor recombinations. Most major farm reorganizations are associated, in the minds of observers, with advancing technology. However, some major changes in production processes may be adequately explained as adjustments within a stated technology. For example, in models 3 and 4 substantial farm reorganizations may occur subsequent to endogenous changes in the best list of fixed assets.

When changes in the list involve services used at levels of increasing returns, the optimal use of all services on the farm is likely to differ considerably from initial use because the optimal use of a variable service is either at zero or in the region of diminishing returns for services with elastic supply functions. For example, small dairy herds are sometimes kept on farms where the marginal value of a cow in production is greater than the average value. A change in the availability of capital funds might permit an increase in herd size to the region of diminishing returns and involve recombinations of other services used without involving changes in technology.

Other substantial changes in resource combinations occur within known technologies when (1) services used initially do not appear in the optimal organization and (2) services not used initially appear in the optimal organization.

Just as technological change leads to changing resource productivities and substantially new resource combinations, so may changes in the best list of fixed factor services bring about major farm reorganizations. Frequently a new technology and a change in the best list of fixed services both occur simultaneously and the two events are difficult to distinguish in the real world. Some major farm reorganizations can be fully explained by new resource combinations within a given technology. Others cannot.

Farm Size and Credit Supply Functions

One measure of farm size is the quantity of productive services used. The optimal quantity depends on the supply of and demand for capital funds with which to obtain services. Hence, the amount of investment and working capital funds used is a determinant of farm size.

Downward sloping farm demand curves for capital funds assure finite farm size in models 1 and 3 as they do in other abstractions which assume constant prices and for which fixed services imply diminishing physical productivities. Supply functions for capital funds are taken as perfectly elastic in these models which admit indefinitely large quantities of funds available to each farm at the stated rate of interest.

Upward sloping farm supply curves for capital funds help to assure finite farm sizes in models 2 and 4. A

finite quantity of funds is offered at each rate. Changes in offers induce changes in farm size. The upward sloping curves could be sufficient to assure finite farm size in abstractions which result in perfectly elastic demand functions for farm capital funds. This result is implied by the sufficiency conditions for an optimal solution in models 2 and 4.

Imperfections in the money market are, of course, but one source of upward sloping supply functions to farms. Imperfections in other markets may limit farm size as well. For example, Engene¹ states that the markets for labor, management, capital funds and insurance against risk may be important sources of increasing costs in American agriculture.

The importance of availability of capital funds in determining farm size is indicated by Schultz² who observes (1) that nearly all physical assets in agriculture are owned by individuals and (2) that when an individual does not own enough to organize a farm of optimum scale he rents or borrows additional inputs. In this way, equity and availability of credit are major determinants of farm size. Availability of credit is a function of equity which, in turn, depends on

¹Engene, S. A., "Concept of Limited Resources and Size of Farm," Resource Productivity, Returns to Scale, and Farm Size, Heady, Johnson and Hardin, editors, the Iowa State College Press, Ames, Iowa, 1956. p. 116.

²Schultz, T. W., The Economic Organization of Agriculture, McGraw-Hill Book Co., Inc., New York, 1953. pp. 302-3.

(1) the quantity of physical assets the individual owns and
(2) the value of each asset. Thus, capital gains or losses can generate changes in farm size by changing the value of owned assets.

One condition presented in chapter 2 for optimal size is that the rate of change in additional costs be not less than the rate of change in additional returns. Downward sloping demand curves for services make it easy to meet this condition. However, it can also be met with upward sloping supply curves and perfectly elastic demands for services resulting from constant returns. For this reason, simplified models using constant prices, linear production functions and a linear upward sloping supply function for investment and working capital funds will always indicate finite farm sizes whether or not a predetermined list of resources is included among the fixed conditions.

A linear programming procedure would be convenient for applying these concepts if the quantity of capital funds were given as a fixed condition. However, if a linear, upward sloping supply function for funds is used and price discrimination is assumed, an activity analysis with a quadratic interest payment term in the otherwise linear net revenue function would be appropriate. Activity analyses using these simplifying assumptions could compose lists of fixed factor services endogenously yet be assured of finite farm size.

Investment and working capital.

Both investment capital and working capital funds are important in determining the best use of available resources and optimal farm size. Criteria for optimizing the quantity of stocks through changes in investments are derived from the fixed conditions of models 2 and 4.

The relation of the investments to farm size remains implicit in models 1 and 3. Profitable changes in investment commitments for durable stocks can be traced in these models if flow prices are defined to include depreciation, repairs, taxes, and interest charges on both investment capital and working capital.

Investments are easily associated with fixed assets in models which include predetermined lists of such assets. If the investment function is implicit, and only the working capital is explicit, attention is focused on working capital alone. The result is that many applications of flow model analysis attribute all changes in farm size to changes in the working capital used, given the length of run implied in the predetermined list of fixed assets. This limited interpretation is valid only when the list of variable services is composed entirely of services from stocks which are fully exhausted in a single production period.

Optimizing the investment in durable stocks can be explicitly examined in models 3 and 4. Whether an existing investment is worth changing depends on the capitalized value

in production of the stock relative to off-farm opportunities for acquisition and salvage. Off-farm opportunities do not depend on market prices alone, but on money market opportunities as well. The decision to acquire additional stock depends partly on where the money is coming from and on how much the use of the money costs per year. Such opportunities vary with the net worth of the farmer, available credit and the existing use of funds. The decision to salvage stock depends partly on opportunities for using the salvage value either for other investments or for retiring existing debt commitments.

Farm demand for funds and non-reversible changes in farm size.

Farm demand for capital funds is derived from the productivities of services used. The demand schedule reflects the change in total revenue with respect to an additional dollar invested in additional services, where the services are added in least cost proportions.

Factor prices are reflected in the farm demand for funds. The divergence of acquisition costs from salvage values for some factors is reflected by a kink in the demand function. The kink is located at the initial quantity of funds used on the farm and helps explain why farmer response to improvements in loan offers is not necessarily the inverse of response to contractions. The demand function is more elastic for borrowing additional funds than for retiring existing debts. The segment of the curve associated with additional capital

reflects productivities of services added to the business at acquisition cost. In contrast, reductions in the quantity of funds used are effected by retiring debt commitments with funds raised through disposal of stocks on the salvage market at values often below acquisition prices.

Shifts in asset structure cause this kink in the demand function for funds, because the least cost combinations of services on the expansion path are not necessarily the same as on the contraction path.

Two illustrations are useful in explaining how the above mentioned kink is associated with non-reversible reorganization of farms in response to changes in supply and demand conditions for capital funds.

For one illustration, suppose that changes in the money market result in improved offers of funds to individual farmers. Each farmer would respond by using additional capital to increase the size of his business. With a downward sloping demand function the marginal rate of interest after the adjustment would be less than before. Now suppose that the capital funds supply function reverts to its former position. With increased interest charges, each farmer responds by reducing the size of his business in order to retire some of his debt commitments. He does this by salvaging stocks which he recently acquired but which are now worth less in production than the off-farm opportunity cost of using the stocks. Farm sizes after the readjustment would remain

larger than before the initial adjustment because the difference between acquisition costs and salvage values makes it ungainful to salvage all the durables acquired in the recent expansion.

The result of the cycle is increased farm sizes and higher marginal rates of interest. Total revenues increase with increased farm sizes, but so do total expenditures. Each farmer may or may not be better off after the cycle than before. The increased farm sizes are explained by internal shifts in the fixed asset structures.

Another illustration of the non-reversible responses to changes in supply and demand conditions for capital funds leads to similar results. Suppose that farm demand for capital shifted through a cycle similar to the supply cycle described above. The cycle might involve a favorable change in relative product/factor prices followed by an unfavorable change. Results would be similar to the above. Shifts in the fixed asset structure would cause farm demand for funds to be greater after the cycle than before. Resultant farm sizes would be intermediate to the previous farm sizes and the marginal rates of interest would be intermediate to the previous rates. Again each farmer may or may not be better off after the completion of the cycle.

If both sorts of cycles described above operated jointly, the resultant farm size would be unequivocally greater than initial size. Whether the marginal rate of interest is

greater or less than the initial rate would depend on whether the supply effect overrides the demand effect.

Supply Response

Product supply functions (or marginal cost functions) for individual farms are implicit in the models of chapter 2. Some properties of these functions are discussed and compared in this section. Supply responses for aggregates of commodities and/or aggregates of farms are not derived. However, it is apparent that the models developed in this thesis can make substantial contributions in aggregate supply analysis. An effort to sketch the nature of these contributions is made below in conjunction with the discussions on individual farm supply response.

Non-reversibilities.

Non-reversible response to product price reversals are a consequence of changes in the best list of fixed factor services. The non-reversibilities are associated with kinks in product supply functions which are analogous to the previously discussed kinks in farm demand functions for capital funds. A kink is located at the quantity of that product produced. Changes in production relocate the kink. The segment of the supply function associated with product prices greater than the existing price is defined relative to acquisition costs of services. The segment for lower product prices is defined relative to salvage values.

The non-reversibilities discussed in this thesis are associated with changes in the best list of fixed factor services within a known technology. They are not the non-reversibilities which Cochrane¹ has rightly associated with advancing technology. He stated that because technological advance is incorporated into the response relation, the output response is not reversible. As explained on pages 79 and 80 of this thesis, changes in the list of fixed assets and advances in technology are closely associated with one another in the real world. However, for expositional convenience, the discussions below are confined to unchanging technologies.

The non-reversible character of supply response within a known technology comes only in part from the divergence of acquisition costs from salvage values in the real world. However, only non-reversibilities from this source are under discussion at the present time. Other potential sources of discontinuities in factor supply functions which can lead to non-reversible responses include institutional rigidities, resource immobilities and other imperfections in the factor markets.²

¹Cochrane, Willard W., "Conceptualizing the Supply Relation in Agriculture," Journal of Farm Economics, Vol. 37, December, 1955. p. 1172.

²Haver, Cecil B., "Institutional Rigidities and Other Imperfections in the Factor Markets," Agricultural Adjustment Problems in a Growing Economy, Edited by Heady, Diesslin, Jensen and Johnson, Iowa State College Press, Ames, Iowa, 1958. p. 130.

The non-reversible supply functions, because of their kink, do not have the same elasticity for product price increases as for decreases. Empirically, product supply functions may be more elastic for increases than for decreases. This is not a logical necessity, but is likely because of an empirical interdependence among product and factor prices. For example, when product prices decrease, both acquisition costs and salvage values are likely to decrease, with salvage values falling at a faster rate. This effect counteracts the tendency for services to become variable downward and makes the supply function less elastic with respect to product price decreases than to increases.

Non-reversibilities of supply response caused by changes in the best list of fixed factor services are accompanied by internally induced shifts in product supply functions. As an illustration, suppose that demand for a product shifts and the farm price increases. Farmers respond with output increases defined according to acquisition costs. Now if price reverts to its former level, a relatively inelastic adjustment follows which is defined according to salvage values of services in use. The resultant output from gain maximizing farms is greater than the output previous to the price rise. If product demand functions to farms were downward sloping, the equilibrium price after demand reverted to its former level would be lower than the price which was effective previous to the demand increase. The expected

consequence of a cycle consisting of a demand increase followed by a return to previous conditions is thus seen to be increased output and lower product prices. The lower prices may or may not be sufficient to reduce total revenue on individual farms. If it is, then the value in production of fixed assets is also reduced and capital losses are incurred. An alternative outcome, of course is that government policies are implemented to support prices. Such policies may sustain individual incomes and avert capital losses, but they may also create surpluses.

The reduced value in production of fixed assets was observed by D. Gale Johnson¹ who partially explained inelastic supply response to depression conditions by inelastic supply schedules for land, labor and some capital inputs. Output is maintained in depressions, according to Johnson, by paying lower rates of return without reducing employed quantities of such assets.

As another illustration of non-reversible supply response due to changes in the best list of fixed assets, suppose that the product supply function shifts to the right because of, for example, lower factor prices. The response is a relatively elastic output increase defined by acquisition costs. Now, if the supply shifter reverts to its former level,

¹ Johnson, D. Gale, "The Nature of the Supply Function for Agricultural Products," American Economic Review, Vol. 40, September, 1950, p. 539.

output is contracted along the relatively inelastic path described by salvage values. The resultant output from gain maximizing farms is greater than the output previous to the initial change in the economic environment. If product demand functions were downward sloping, resulting product prices would be less than either of the previous prices. Again the expected consequence is increased output and lower product prices. Lower incomes and capital losses are also likely to occur unless price supports are implemented by the government. It is through these effects that non-reversibilities of supply response can work against farmers during changes in general levels of employment and business activity.

Empirical substantiation that individual farm supply response is non-reversible is offered by Boyne and Johnson¹ who tested some hypotheses about farmer response to price increases relative to price decreases. The hypothesis that farmers are less responsive to input price increases than decreases was found significant at the 1.7 percent level. Additional substantiation is offered by Halvorson² who found evidence that farmers feeding grain to dairy cows may be more

¹Boyne, D. H., and Johnson, G. L., "A Partial Evaluation of Static Theory from Results of the Interstate Managerial Survey," Journal of Farm Economics, Vol. 40, May, 1958.

²Halvorson, Harlow W., "Supply Elasticity for Milk in the Short Run," Journal of Farm Economics, Vol. 37, December, 1955, pp. 1196-7.

responsive with respect to milk price increases than decreases. Aggregative evidence is discussed on pages 97 to 105.

The above illustrations display apparent rightward shifts in product supply functions as a direct consequence of shifting asset structures. Other shifters, including capital gains or losses and changes in availability of credit, are discussed in the next section. Supply responses involving changes in lists of fixed assets are frequently identified with responses in different lengths of run; that is, the non-reversibilities may be associated with changes in the relevant length of run.

Measuring supply response relative to endogenous changes in lists of fixed assets would provide more precision and less vagueness than most applications of the length of run concept in supply analysis. To the extent that a list of fixed assets and a statement of length of run are equivalent in static analysis, the list states precisely which length of run is being studied.

The tempered supply response to price reversals discussed above has a corollary. An augmented supply response follows large product price changes when lists of fixed asset change. The greater the magnitude of the price change the greater the number of services that become variable and the more elastic the supply response. This has the rather interesting implication that the greater the magnitude of the price change the longer the relevant length of run.

Supply Shifters.

Changes in the best list of fixed assets, discussed above, shift the price-quantity equations ordinarily used to describe supply functions. Such shifts were shown to be non-reversible. Three other supply shifters are discussed below. These supply shifters can be fully explained in the context of model 4 of chapter 2 which explicitly accounts for changes in credit availability and changes in the best list of fixed assets. These supply shifters are overlooked in contexts which do not account for resource fixity and credit availability. The three additional shifters are (1) the marginal rate of interest (2) prices of services listed as fixed to the farm and (3) quantities of services listed as variable to the farm.

The marginal rate of interest on an individual farm is a money market opportunity cost which is inversely related to farm output. Events which reduce the marginal rate of interest increase supplies of products by shifting supply schedules to the right. Among such events are capital gains or increases in the value of owned assets; internal capital accumulation from earnings, inflations and windfalls; improvements in loan offers; and lower factor prices. These shifters may depend on changes in absolute price levels, but they are independent of changes in relative prices. Their use in explaining the maintenance of output during periods of unfavorable relative prices for agriculture is illustrated

in the subsequent discussion of supply response and the business cycle which begins on page 97.

Improvements in loan offers frequently occur in conjunction with improved equities whether the latter is by capital gains or by accumulation. These shifters reduce marginal interest rates and increase farm output by making more funds available to individual farmers. Lower factor prices, on the other hand, can increase farm output by reducing the quantity of funds required to produce a given output.

The output increasing effects of improved capital positions of individual farmers may occur within a single production period or occur from one period to the next. The latter event links supply response to intertemporal changes in the availability of capital funds. These changes shift product supply functions to the right over time. Such intertemporal shifts have a non-reversible character of a different nature than the non-reversibilities associated with changes in the best lists of fixed assets. On the other hand withdrawal of loan offers, capital losses, reduced returns to owned assets, and inflated factor prices all serve to increase money market opportunity costs. Each reduce output independently of changes in relative product to factor prices.

The output reducing effect of higher prices is not well understood because inflations frequently include many other effects such as improved equities and better loan offers

which more than offset the contracting influence associated with rising costs of production. To illustrate, suppose an inflation occurs which increases all product and factor prices in proportion. To focus attention on money market opportunity costs, suppose further that no change occurs in the availability of capital funds. First note that with no change in relative prices, no change in output can be explained by model 1 of chapter 2. However, according to model 2, an output reduction will be induced by an increase in the marginal interest rate. The interest rate is increased by a movement along the capital supply function. The movement is an effort to obtain a sufficient quantity of additional capital to retain existing quantities of factor services. The effect is a worsening of off-farm opportunities for acquiring additional services, and an improvement in off-farm opportunities for salvaging existing services. Inasmuch as the value of services in production has not changed relative to factor prices, but only relative to the cost of money, the change in money market opportunities is sufficient to induce downward adjustments in the use of services with the consequence of a contraction in farm output.

The offsetting, output increasing effects of inflations ordinarily increase or at least maintain output, which lessens the importance of the output reducing effect of inflations discussed above. However, on farms where equities are small and where improvements in availability of capital funds are

not sufficient to offset increased costs of production, output reductions will occur. Conditions are discussed in the section on farm policy and welfare which begins on page 105 for which farmers might salvage their entire farms and seek employment elsewhere.

The supply function for investment and working capital funds could be such that farmers would not respond at all to changes in product prices. This would happen, for example, when supply functions for funds are perfectly inelastic. Farmers in this situation would experience changes in net revenue with respect to changes in product prices but would not have alternative organizations offering positive gains. Neither product prices nor interest payments have a direct influence on decisions to allocate available productive services among alternative enterprises when the quantity of funds used is not subject to change.

Prices of services listed as fixed to the farm are supply shifters. Variations in such prices may change the best list of fixed services and lead to adjustments in farm organization. Decreases in acquisition cost of fixed assets induce output increases. Increases in salvage values induce output decreases. Output is not responsive to increases in acquisition costs or decreases in salvage values. Increases in the marginal rate of interest have the same effect on the list of fixed services and on supply response as increases in salvage values. Decreases in interest are similar in effect to decreases in acquisition costs.

These output adjustments can be traced in models 1 and 2 only by seeking explanations in changes in the fixed conditions. These sources of change are explicit in supply response equations derived from models 3 and 4.

Quantities of services listed as variable to the farm affect supply response because they help locate the kinks in farm product supply curves, and farm capital demand curves as well as the points of discontinuity in factor service supply curves. The initial use of all services is important in supply response where shifting asset structures are concerned. Only the quantities of fixed services are important in other analyses where fixed asset structures are predetermined and where acquisition costs equal salvage values for variable services.

Changes in farm organizations are supply shifters. Supply analyses which account for initial farm organizations stress changes in product supplies from quantities being produced. When the quantity produced changes, the relevant supply function changes.

Supply response and the business cycle.

The product supply functions for individual farms discussed above may explain responses of each individual farmer to changes in price other things being equal, but they do not contain rules for aggregating the responses of all farmers. In this section, some hypotheses about farm credit and resource fixity are compared with aggregate supply response to (1)

changes in general price level and (2) changes in the index of prices received relative to prices paid by farmers. The discussion below is based on, but speculates beyond, the models of chapter 2. The comparisons indicate that the shifting asset structure and the restricted funds model is consistent with observed output responses. It further shows that since World War II abstractions which do not incorporate these features, such as model 1, are not consistent with output response.

The period 1910-56 covers ten business cycles according to indicators developed by the National Bureau of Economic Research. For each of these ten expansions and ten contractions, Hathaway¹ has recorded changes in farm output, prices received, prices paid, income and value of assets. (See accompanying table.) The observations can be used to test some hypotheses advanced by Johnson² on the basis of earlier versions of model 3 about resource use, aggregate output, and changes in the general level of employment and business activity. The hypotheses concern the relative fixity of

¹Hathaway, Dale E., "Agriculture and the Business Cycle," Policy for Commercial Agriculture, Joint Economic Committee, November 22, 1957, pp. 51-76.

²Johnson, Glenn L., "Supply Functions--Some Facts and Notions," Agricultural Adjustment Problems in a Growing Economy, Heady, Diesslin, Jensen and Johnson, editors, Iowa State College Press, Ames, Iowa, 1958. pp. 81-88.

Percentage changes in measures of selected indicators of conditions in the economic environment of farmers during periods of business expansion and contraction, 1910-56

Periods of business*	Index of net farm output (1)	Prices received by farmers (2)	Prices paid by farmers (3)	Net farm income (4)	Current value of real assets (5)
Expansion					
1911-13	1.7	8.5	2.0	9.6	6.7
1927-29	2.8	5.7	0	7.0	-0.1
1924-26	7.4	1.4	0	11.3	-3.1
1954-56	4.6	-5.6	2.1	-3.2	-9.5
1921-23	11.3	14.5	1.2	30.5	-15.7
1946-48	6.1	21.6	30.2	9.8	31.5
1949-53	6.9	3.2	13.9	1.4	12.7
1932-37	7.9	87.7	6.5	156.7	1.0
1914-19	0	114.9	94.1	157.4	39.6
1938-44	22.8	103.1	33.3	195.9	50.5
Contraction					
1910-11	-3.3	9.6	4.2	-8.7	--
1948-49	-2.9	-12.9	-2.4	-14.0	-2.4
1953-54	0	-3.5	1.1	-12.6	-2.6
1944-46	1.0	19.8	14.3	24.1	33.0
1923-24	-1.4	.7	0	4.5	-1.8
1926-27	-1.4	-3.4	-1.2	-0.8	-2.9
1913-14	10.0	-1.0	0	-6.5	1.6
1937-38	-3.7	-20.5	-4.5	-21.3	6.8
1920-21	-11.4	-41.2	-18.3	-45.0	-24.8
1929-32	2.7	-56.1	-24.8	-69.2	-43.3

Sources: Hathaway, Dale E., "Agriculture and the Business Cycle," Policy for Commercial Agriculture, Joint Economic Committee, November 22, 1957. pp. 51-76.

*Arrayed in terms of the magnitude of change in gross national product from the trough year to the peak year for expansions and the peak year to the trough year in contractions.

nine classes of factor services. Hathaway's data lend validity to Johnson's hypotheses and support the position that a fixed asset theory can be useful in explaining aggregate agricultural supply response. The hypotheses examined below, while less detailed than those mentioned above, include the investment capital dimension in addition to fixity of assets.

The post war decade from 1946 to 1956 covers three expansions and two contractions in general business activity according to the table on page 99. For each of these periods, the ratio of prices received by farmers to prices paid was less at the end of the period than at the beginning. That is, there was a continual tightening of the cost price squeeze. Usual hypotheses from abstractions similar to model 1, which do not allow for shifting asset structures or for changes in capital position, would predict output reductions in each of these periods because of the unfavorable changes in relative prices. Hathaway's observations, reported in the table, show output increases in three of the five post war periods and no change in output for a fourth period. The contraction of 1948-49 is the only post war period to show a decrease in farm output. These observations since World War II are not consistent with model 1.

To explain these events, economists properly seek answers in change in technology, institutional arrangements, expectations, education, specialization and aggregate employment.

On the other hand, shifts in fixed asset structures and changes in farmers' capital positions may explain some of these events. In the following paragraphs, it is demonstrated that resource fixity and credit availability can be sufficient to explain maintenance of farm output during periods of unfavorable prices. This is not to say that the two elements provide a complete explanation of the observed supply responses. It does not even prove they are necessary for an explanation. It does strongly indicate, however, that an account of changes in resource fixity and credit availability may be necessary, in conjunction with technology, education, non-monetary motivations and other factors, in a complete explanation of supply response.

The expansions of 1946-48 and 1949-53 measure major improvements in general business activity. Thus, while farmers experienced a cost price squeeze as measured by relative prices, both prices received and prices paid increased in absolute value. Farm output increased about six or seven percent in each of these periods. The output increases cannot be explained by changes in relative prices. They may be partially explained by changes in the best lists of fixed assets on individual farms and by improvements in farmers capital position, as sketched below.

Improved demand for farm products and higher product prices during these two expansions increased the value in production of services initially fixed to farms and the

services became variable upward. This made supply response more elastic than it might have been in more moderate periods of expansion. Internal capital accumulations accelerated during these years through (1) high rates of returns on owned assets, and (2) capital gains associated with the inflation which followed the removal of price ceilings in 1946. Apparently, increasing returns to owned assets in conjunction with capital gains resulted in output increasing improvements in loan offers by optimistic lenders. The value of real assets increased by nearly one-third during the 1946-48 expansion which may have contributed more to farmer welfare during the period than the ten percent increase in income. To the extent that salvage market prices were bolstered by a prosperous economy, the demand for used durables was strong relative to supply and farmers had good opportunities to reorganize businesses without undue sacrifice of existing investments.

Thus it can be hypothesized that output increasing adjustments occurred during the cost-price squeezes partly because increases in general business activity changed the fixed asset structures on American farms and increased the quantity of funds used in production of farm products.

In two other post war periods, a slightly different story is told. In the mild contraction of 1953-54 and in the moderate expansion of 1954-56, the index of prices received by farmers increased first by 3.5 percent and then by

5.6 percent. (see table on page 99) This is an opposite situation from the product price increases in the two periods discussed above. However, as in the other two periods, relative prices became more unfavorable as the index of prices paid increased slightly. Again, usual hypotheses would predict output reductions. And again, abstractions which account for shifting asset structures and changing capital positions help explain why farm output was not reduced during these two periods of economic activity. Note in the table that output increased nearly five percent in the 1954-56 expansion but remained unchanged in the 1953-54 contraction.

Falling product prices during these periods decreased the value in production of existing farm inputs. This reduced the demand for new inputs and increased supplies of used durables on the salvage markets. While some buyers shifted from markets for new inputs to the used markets, demand in the salvage markets was reduced in the aggregate and salvage values for many durable stocks plummeted. Stocks were worth less in production than formerly, but, with low salvage values, the stocks remained more valuable in production than on the salvage markets. This caused a severely inelastic output response to falling product prices and prevented the output reduction usually expected during cost price squeezes.

Two forces were favorable to output increases during these periods that can be traced in models similar to model 4 of chapter 2. One force is internal capital accumulation

with its output increasing effect on the quantity of available funds. Capital accumulation was moderate during these years because low product prices produced low rates of returns to owned assets and capital losses were incurred. The value of real assets fell nearly ten percent during the 1954-56 expansion. The other output increasing force was in the high salvage values for farm labor and real estate. Both these values were bolstered during the periods while salvage values for other inputs fell. To go beyond model 4, this bolstering probably facilitated the movement of some farmers out of agriculture which, in turn, implemented the long run trend toward fewer, larger and more productive farms.

Thus the usually expected output decreasing adjustments did not occur during the cost price squeezes because of improvements in supplies of capital funds and because of poor opportunities for disposal of durable stocks in use on farms.

Other periods of economic activity, from 1910 to 1946, are not inconsistent with model 1 according to observations recorded in the table on page 99. Directions of change during the fifteen periods up to and including World War II might be explained as well by one of the models as by another. This leads to the speculation that current trends toward larger, commercial, specialized farms makes shifting asset structures and changing capital requirements more important in the supply relationship in recent years than formerly.

The above explorations of increasing farm output in periods of unfavorable prices is in the context of the models of chapter 2. These abstractions, by their restrictive assumptions, omit the important effects in supply response of technology, education, specialization, risk and uncertainty, and non-monetary motivations of farmers. Thus, the above does not pretend to be a complete explanation of farm product supply response since World War II. Rather, it claims to offer a useful and important supplement to existing explanations. The task of integrating the effects of resource fixity and credit availability with other factors to form a thorough explanation is not undertaken in this study.

Welfare and Policy

To this juncture, statements about farm organization and supply response were related to the consequences of applying rules for gain maximization. No conclusions were intended as to whether or not farmers are better off after adjusting to one situation than after adjusting to another. No intimations were implied that society might implement policies which change the conditions farmers regard as fixed as a means of improving farmer welfare. However, welfare and policy considerations are prominent in many current discussions of farm problems and it is worth noting potentialities of fixed asset and investment theories for throwing some light on such discussions.

The returns to factors owned by a farmer indicate welfare according to model 4 of chapter 2. This includes returns to the management and labor services he supplies in addition to physical stocks comprising his equity. Increases in the quantity of owned assets and/or in the rate of returns are regarded as improvements in farmer welfare on the Pareto-better criterion that more is better than less.

The optimal farm organization exhibits maximum returns to owned assets subject to fixed conditions. Changes in fixed conditions which increase returns to owned assets on optimally organized farms may be regarded as welfare increasing changes. Profits, on the other hand, are not as useful an indicator of wellbeing when used as a measure of total revenue less total expenditure. This is because profits measured in this way are net of returns to owned assets and may be negligible or vanish on optimally organized farms.

In this regard, it is worth noting that measures of farm income which are net of returns to owned assets are notoriously low relative to non-farm incomes. The unfavorable comparison does not extend to measures of equities. The recent census reported a median income of farm managers about one-third the median income of all citizens. The net worth or equity of farmers, on the other hand, is high relative to the net worth of persons not in agriculture. The census reported a median net worth of farm operators about three times greater than the median net worth of all United States spending units.

Perhaps the measure of farmer equity in conjunction with the rate of returns on the equity is a more useful indicator of the wellbeing than usual measures of farm income, receipts per dollar of expenses, or receipts per dollar of investment.

Returns to owned assets are derived from time commitments or investments in durable stocks. In this respect, the returns are analogous to the rent concept of the classical economists. One interpretation is to regard the excess of the capitalized rate of return to fixed assets above the salvage value for the asset as a surplus above the return required to maintain the asset in production. Another interpretation is to accept the definition of Weintraub¹ that economic rent is the imputed earning of fixed factors.

Imputation of earnings is a useful means for evaluating fixed assets. To do so on optimally organized farms assures that such assets are not valued for more than the acquisition costs of additional units nor less than salvage value. Change in the value of fixed assets in production is reflected in the imputed value of the asset and implies a change in the measure of net worth or equity. In this way, changes in fixed conditions affect farmer welfare by (1) changing the returns to owned assets and (2) changing the measure of equity. Meanings of the latter change remain to be examined.

¹Weintraub, Sidney, An Approach to the Theory of Income Distribution, Chilton Company, Philadelphia, 1958. p. 169.

Changing the returns to owned assets.

Changes in fixed conditions which generate changes in farmer welfare may be consequences of social, political, economic or natural forces. Among these are changes which result from implementation of agricultural policy as, for example, supporting farm prices to improve farmer welfare.

High price supports might be defended as a good means for improving farmer welfare if an abstraction similar to model 1 is used for the defense. For example, one defense is that higher prices increase the value productivities of all services on the farm without increasing costs. However, model 1 neglects changes in the money market and shifts in fixed asset structure which might be associated with supported prices. It was through neglect of these that model 1 was shown to be contradicted by observations on supply response since World War II. Policy recommendations drawn from abstractions which neglect money markets and asset structures need not be as applicable currently as they were previous to World War II.

Policy recommendations based on abstractions such as model 4 might concentrate on other elements of the farmers environment in addition to product prices. For examples, (1) supported salvage markets might make contraction in supply more elastic, (2) new technologies which increase physical productivities of stocks farmers own rather than stocks which they buy might increase the value of farmer

equities as well as the rates of return to owned assets, and (3) curbs on credit can make capital funds supply functions less elastic and inhibit increases in output without necessarily affecting total returns to owned assets. Attention to these and other elements in the farmers environment associated with capital restrictions and shifting asset structures might offset the welfare reducing effect on farms of a reduction in farm prices.

Policies to support salvage markets for some services have been tried in American agriculture as, for example, salvaging land from production by putting it in a conservation reserve. Improving salvage markets for other key inputs on farms such as the labor input have been proposed but not implemented. Policies which help farmers to find good salvage values for entire farms might assist the exodus of farmers out of agriculture.¹ To leave farming, the farmer must salvage his entire business, find alternative employment for his own skill and alternative uses for the equity he salvaged from the business. The better the salvage markets, the greater this convertible equity and the stronger the likelihood that a move out of agriculture can be a gainful one. The gain maximizing person leaves the farm when his

¹Schultz, T. W., "Homesteads in Reverse," Farm Policy Forum, Iowa State College Press, Ames, Iowa, Summer, 1956. p. 12.

expected non-farm wage plus returns on his equity when transferred to non-farm uses are greater than his expected returns to owned assets on the farm, other things being equal.

The welfare implications discussed above are meaningful for each farmer individually. However, when farm policies are implemented which affect the entire economy, it usually happens that the welfare of some farmers is increased while that of others decreases. The policies affect the welfare of some members of the non-farm economy as well. No effort to provide interpersonally valid welfare comparisons is attempted in this study.

If the goal of agricultural policy includes improving farmer welfare without surplus inducing price supports and without counteracting the current trend toward a more efficient agricultural industry manned by fewer farmers, then successful farm policy must not neglect restrictions of capital funds and the shifting asset structures of individual farms.

Summary and Conclusions

The preceding pages present an attempt to combine some facts about the economic environment of farmers with some existing mathematical procedures for maximizing quantities subject to restrictions. Rules were derived for optimizing the investments of farmers in stocks of productive factors. Results of the attempt conform reasonably to expectations. The individual pieces of this study are not new but the synthesis of the pieces may be regarded as novel.

The major points of departure from other writings on associated subjects are (1) the formalization of investment and working capital supply functions for use in conjunction with static flow models, and (2) the use of on-farm opportunity costs, relative to off-farm opportunities for acquisition and salvage of factors, to determine the best list of fixed factor services. These departures assume answers to intertemporal questions and produce specific rules for optimal adjustments by farmers whose lists of fixed factors change and whose credit facilities are limited.

Beginning with results from the latter pages of this thesis and working backwards, the analysis explains net revenue as a return to fixed assets. This reminds one of the rent theories of the classical economists. Each farmer is regarded as owning an equity in his business. He supplements the equity with borrowed funds for additional investment

and working capital and uses the funds to control productive services in a fashion which maximizes the flow of net revenue to the equity.

Improvements in either the rate of returns to equity or in the size of the equity improve farmer welfare. Changes in environmental factors which farmers do not control, such as product and factor prices, technology and offers of credit, change the wellbeing of farmers by changing both the rate of returns and the measure of equity.

Adjustments to changes in environment, or fixed conditions, may result in product supply responses on individual farms which are not reversible. Non-reversibility of supply response means that reversal of fixed conditions to a former state need not be accompanied by a complete reversal of output to its former level. The expected consequence of an increase in demand for farm products followed by a reversal to the previous demand, for example, is increased farm output and lower prices relative to prices and quantities in the initial situation. Lower farm incomes and capital losses are probable but not necessary results of such a cycle. Surpluses are another possible result if prices are supported during the declining phase of the cycle. The non-reversibilities are traced through shifts in the fixed asset structures of farms and are characterized by kinked product supply curves. A kink is located by the quantity of product forthcoming. Output response is generally more elastic for increases in supply than for contractions.

A potentially important product supply shifter which is easily neglected is the availability of capital funds. As additional funds become available through improved equity and capital accumulation and/or through improved offers of credit, product supplies are apt to increase.

Restrictions on the availability of capital funds may be as helpful in explaining limited farm size as the concept of diminishing returns. In fact, the existence of many small farms may be more satisfactorily explained by small equities and restricted credit facilities than by diminishing returns.

The interrelation of credit offers and fixed asset structure probably contribute a more meaningful explanation of optimal farm size than either element can offer alone. For example, the supply of funds helps determine the fixed asset structure, and the fixity helps determine the rate of returns to services which are variable to the farm.

The optimal organization of resources on farms depends on opportunities for profitable adjustments in the existing organization. Whether the quantity of a service should be varied from its existing use depends on the on-farm opportunity cost of the service relative to off-farm opportunities for (1) acquisition of more of the resource, and (2) salvage of some of the existing quantity.

A necessary, but not sufficient condition that a resource remain at its initial level during reorganization by

economizing farmers is that on-farm opportunity costs for services are bounded by off-farm opportunities for acquisition and salvage.

Such off-farm opportunities are characterized by the condition that acquisition costs are frequently greater than salvage values. When they are greater, the supply function for services from a stock has a discontinuity located at the existing quantity of services, and the services are subject to fixity.

Such are the consequences of assuming (1) acquisition costs greater than salvage values and (2) upward sloping supply functions for investment and working capital funds. The framework used to derive the above results was the static theory of the firm. However, an important part of the mathematical apparatus was developed by Kuhn and Tucker for non-linear programming where maximization is subject to inequalities. The similarities of marginal analysis and activity analysis as vehicles for examining fixed asset structures and capital restrictions appear more striking than the differences. Analogous procedures to those applied in the marginal analysis of this thesis would produce similar results in an activity analysis framework.

Several limitations of this thesis depend on restrictive assumptions in the models. Most important among these, perhaps, is that static-micro models were used. The role played by risk in fixing assets and influencing decisions

to invest is thus neglected. Aggregate responses are not adequately examined. These and other restrictive assumptions mentioned in chapter 1 limit the range of application of the models developed in chapter 2. The effect of these limitations on the usefulness of the results was not fully determined.

The results of this study are supported in two ways. First, the two fundamental restrictions on resource fixity and credit availability have empirical as well as theoretical origins. Secondly, the rules for optimal allocation of resources subject to these two restrictions were shown to be consistent with (1) observed changes in farm prices and quantities and (2) established principles of farm management. There remains the task of making objective, statistically valid tests of confidence in the results.

It is hoped that the foregoing discussions of the role of resource fixity and credit availability in the organization of American farms will prove useful in future research by helping in the task of quantifying relationships and solving important farm problems concerning farm organization, supply response and credit.

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