A HIERARCHIC RANDOM UTILITY MODEL FOR WAGERS

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This is to certify that the

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A HIERARCHIC RANDOM UTILITY MODEL FOR WAGERS

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ABSTRACT

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By

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The 104 Ss made choices from 100 trinary decision problems. It was hypothesized that such choice would be a process of probabilistically choosing one of six decision rules. The dependent variable was a Bayesian estimate of the probability of choosing one of these rules, maximum expected value (EV). Independent variables were: sex (A), order of presentation of decision problems (B), whether or not S heard a lecture on EV (C), whether the EV choice was high risk or low risk (D), and whether the range of EV's for the wagers in a decision problem was large or small (E).

As significant main effects, the EV rule was used more by males ($p < .01$), by Ss who heard the EV lecture ($p < .05$, one-tailed), in problems for which the EV choice was low risk ($p < .001$), and for problems having large EV ranges ($p < .001$). The statistically significant interactions were A X B X C ($p < .05$), A X C X E ($p < .05$), D X E ($p < .001$), and C X D X E ($p < .001$). These results could not be attributed to arithmetically erroneous use of the EV rule.

It was concluded that the model helped to explicate the choice process but erroneously predicted no interaction between EV and risk characteristics of alternatives.

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By

Robert Gurney

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To my parents

and other victims

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INTRODUCTION

The juxtaposition of actual and optimal human behavior is widespread throughout psychology. The psychotherapist's questions of how a client behaves, how he ought to behave in order to satisfy criteria of efficiency, and how to change the client's behavior from the former to the latter are questions asked in all fields of human psychology. In this sense the layman who identifies psychology with clinical psychology is no more in error than the initiate who tells him he's wrong.

Marschak (1964) has noted that students of choice behavior also ask the psychotherapeutic questions. How do people choose? How would a rational man choose? And what are the conditions affecting the rationality of an actual choice? Hierarchic random utility models (HRUMs) describe contexts in which all three questions may be simultaneously considered.

HRUMs are random utility models, RUMs, (Block & Marschak, 1960; Becker, DeGroot, & Marschak, 1963a; Luce & Suppes, 1965) for which the set of alternatives is a set of decision rules. With respect to the decision problem, a decision maker using a HRUM must, in fact, make a choice from each of two sets of alternatives. One of these sets is the set of decision rules, denoted R_T. The other set is the set from which a choice is made by the decision maker using a decision rule. Denote this second set T. For example, consider someone buying a roast. The set T includes all the roasts available at the time of purchase. The various rules for selecting a roast specify R_T. R_T might include such alternatives as: "Buy a leg of lamb if it doesn't cost more than 99¢/lb; otherwise buy the best looking ham you can find;" or "Close your eyes and randomly pick a chuck roast." While not always easy to do, it is essential to maintain a distinction between T and R_T. In empirical applications it is typically the case that choice from R_T is a covert process while choice from T is overt. Choice from R_T is overt only to the extent that the decision maker expresses his rule for choice and the experimenter listens. Temporally, choice from R_T precedes choice from T.

Formally, let T be a set of alternatives, having subsets A, B, . . . and elements x, y, . . . Denote the number of elements in A as n(A). Defining a decision problem as the necessity of choosing one element from a set A, a "decision rule" is any set of instructions (*i.e.*, a program) that leads to a solution of a decision problem. More specifically, a decision rule at least implies a probability distribution specifying choice probabilities for any errorless user of the rule. That is,

(1) a "decision rule," $R_{i,A}$, implies $p_{i,A}$ where

(2) $p_{i,A}: A \rightarrow [0,1]$ such that $\sum_{x \in A} p_{i,A}(x) = 1$.

From the customary dichotomization of choice models into "algebraic" and "probabilistic" (Luce & Suppes, 1965), it follows that

$R_{i,A}$ is algebraic if it partitions A into k x's and n(A) - k y's such that $p_{i,A}(x) = 1/k$ and $p_{i,A}(y) = 0$. It is probabilistic if it satisfies (1) and (2), is not algebraic, and specifies a technique (such as using a random number table) for converting choice probabilities into a choice. This last requirement is true of algebraic rules only when $k > 1$. Finally, the set of all rules, $R_{i,A}$, is denoted R_A, has subsets S_A, and a probability distribution P, defined by (3).

(3) $P: R_A \rightarrow [0,1]$ such that $\sum_i P(R_{i,A}) = 1$.

$P(R_{i,A})$ is the probability with which the rule $R_{i,A}$ is chosen for making a choice from A.

As a RUM, a HRUM involves assuming that each rule is assigned a utility by a random variable, U_i. Then (3) is equivalent to (4).

(4) $P(R_{i,A}) = P\{U_i(R_{i,A}) \geq U_j(R_{j,A})$ for $R_{i,A}$ and all $R_{j,A}$ in $R_A\}$.

In other words, the probability of choosing a decision rule is specified by the probability with which that rule is most highly evaluated relative to the other rules. For formal reasons, it is assumed that no rule in the set R_A has zero probability of being chosen.

(5) $P(R_{i,A}) \neq 0$, for all i.

This assumption (5) could also be interpreted as part of the definition of R_A.

With assertions (1) through (5), the definition of HRUM readily follows. From (1) and (2), the probability of choosing an alternative is always conditional on the rule used. By (5) the conditional, $p(x|R_{i,A})$ exists. Since the $R_{i,A}$'s are simply elements of R_A, they partition R_A; hence, choice probabilities are given by (6), the definition of the HRUM.

(6) $p(x,A) = \sum_i p(x|R_{i,A})P(R_{i,A})$.

Verbally, a HRUM asserts that when a decision maker encounters a decision problem, he seeks a rule by which to make a choice. According to variable criteria, he evaluates the rules and picks one which seems to be optimal. He then makes a choice from A by applying the selected rule.

While HRUMs are relatively weak models, they are not without implications. In this section we shall explore some of these. In order to do so, however, we shall require the definition of a "trivial" choice set as a choice set having only one alternative.

Theorem 1 states the first result. While this implication is not very sensational, it does indicate one of the ways in which HRUMs are able to incorporate other models of choice. The theorem implies that for any model of choice, there is a HRUM having identical choice probabilities.

Theorem 1: For every distribution of choice probabilities, there is at least a trivial HRUM.

Proof: Let $p':A \rightarrow [0,1]$ such that $\sum_{x \varepsilon A} p'(x) = 1$.

For p' define a rule, $R_{\cdot,A}$, instructing the decision maker to choose from A according to p'.

Finally, let $R_A = \{R_{\cdot,A}\}$, a trivial choice set.

Then, by (4), $P(R_{\cdot,A}) = P\{U_{\cdot}(R_{\cdot,A}) \geq U_j(R_{j,A})\}$

$$= P\{U_{\cdot}(R_{\cdot,A}) \geq U_{\cdot}(R_{\cdot,A})\}$$

$$= 1.0,$$

and by (6), $p(x,A) = p(x|R_{\cdot,A})P(R_{\cdot,A}) = p(x|R_{\cdot,A}) = p'(x,A)$.

The value of Theorem 1 is primarily empirical. It indicates that HRUMs help us to abandon research designed to demonstrate the adequacy or inadequacy of a specific model. By equating various models with rules in R_A, it follows that a HRUM using decision maker may, in fact, behave according to several different rules in a sequence of choices. Since it is highly unlikely that any of our models are accurate for describing long sequences of decisions, HRUMs enable us to analyze such choice behavior as functions of many of our models. That some way in which to combine our models is an eventual necessity has been indicated by many studies. For example, behavior in the early trials of probability learning seems quite different from behavior after many hundreds of trials. The findings of Siegel, Siegel, and Andrews (1964) and Edwards (1961), for example, indicate that even if one insists that this behavior is governed by a subjective expected utility (SEU) strategy, the strategy changes over time. Similarly, Davidson, Suppes, and Siegel (1957) found that subjects were sometimes conservative in their choices and sometimes extravagant but never consistently one or the other. This points to the need for a probability distribution over rules specifying high risk and low risk behavior. Theorem 1 implies that HRUMs may be useful for coping with this kind of difficulty.

Theorem 2 is of theoretic interest. Perhaps, the most elegant probabilistic choice model available today is Amos Tversky's (1971, 1972a,b) "elimination by aspects" (EBA) model. Theorem 2 specifies conditions under which a nontrivial HRUM is an EBA. By noting that the theorem implies the EBA to be a special case of the HRUM and that both the Luce (1959) and Restle (1961) models are special cases of the EBA, it follows that the Luce and Restle models are also special cases of nontrivial HRUMs.

<u>Theorem 2</u>: If (a) all $U_j > 0$ and $P(R_{i,A}) = \dfrac{U_i(R_{i,A})}{\sum_j U_j(R_{j,A})}$ and

if (b) for any $R_{i,A}$ in R_A, $p_{i,A}(x) = 0$ for at least one $x \in A$, and

if (c) for any subset B of A such that $B = \{x \in A | p(x|R_{i,A}) \neq 0\} = \{x \in A | p(x|R_{j,A}) \neq 0\}$, $p_{i,A} = p_{j,A}$, then the HRUM satisfying (a), (b), and (c) is an EBA.

Proof: Let $\{B_k\}$ be the set of proper subsets of A such that for any B, there exists at least one $R_{i,A}$ in R_A for which $p(x \in B|R_{i,A}) \neq 0$ for all x in B and $p(y \notin B|R_{i,A}) = 0$ for all $y \notin B$.

Partition R_A into subsets $S_{k,A}$ so that if $R_{i,A} \in S_{k,A}$, then $R_{j,A} \in S_{k,A}$ if and only if $p(x|R_{i,A}) = p(x|R_{j,A})$ for all $x \in A$.

By the definition of B_k and the partitioning of R_A, each $S_{k,A}$ corresponds to one and only one B_k.

Let $\Psi(B_k) = \Sigma_{R_{i,a} \varepsilon S_{k,A}} U_i(R_{i,A})$. That is, the Ψ-value of a subset is the sum of the utilities of the rules that define the subset.

Then, since $S_{k,A}$ partitions R_A, $\Sigma_h \Psi(B_h) = \Sigma_j U_j(R_{j,A})$.

As a result of condition (a) and the definition of $S_{k,A}$, it follows that $\Sigma_{R_{i,A} \varepsilon S_{k,A}} P(R_{i,A}) = P(S_{k,A}) = \Psi(B_k)/ \Sigma_h \Psi(B_h)$.

Finally, since choosing a rule is equivalent to choosing a subset with a probability distribution defined over its elements,

$p(x|B_k) = p(x|S_{k,A}) = p(x|R_{i,A} \in S_{k,A})$.

Then by substitution into the definition of the HRUM,

$$p(x,A) = \Sigma_i p(x|R_{i,A})P(R_{i,A})$$

$$= \Sigma_k p(x|S_{k,A})P(S_{k,A})$$

$$= \frac{\Sigma_k p(x|B_k)\Psi(B_k)}{\Sigma_h \Psi(B_h)}$$

which is the EBA.

Corollary 2a: Every Luce model is a nontrivial HRUM.

Proof: See Tversky (1971).

Corollary 2b: A HRUM satisfying conditions (a), (b), and (c) of Theorem 2 and defined on a set A for which n(A) = 2 is a Restle model.

Proof: See Tversky (1971).

Corollary 2c: A HRUM satisfying conditions (a) and (b) of Theorem 2 and satisfying the condition that all $R_{i,A} \in R_A$ are algebraic, is an EBA.

Proof: This follows directly from the definition of algebraic rules (see page 3) which implies that if all rules are algebraic, condition (c) of Theorem 2 must be true.

Corollary 2c has many implications about the various algebraic models of choice. For example, suppose a decision maker consistently uses an SEU rule but the component utilities or probabilities or both are variable. In essence this would mean that he has a whole set of SEU decision rules from which to choose. If he chooses these rules (or equivalently if he chooses the utility and/or probability functions that specify the rules) according to a Luce model, his data will indicate that he is using an EBA.

It should be apparent from Theorem 2 and its corollaries that the HRUM provides an excellent context for comparative analysis of available models of choice. The HRUM is integrative to the extent that it employs notions common to much of decision theory. This is first reflected in the fact that RUM is the basis of the definition of the HRUM. The RUM is probably the most frequently encountered class of choice models. Second, the notion of "utility" while not defined on alternatives, is still an essential concept for the model.

Of considerable interest is the HRUM's postulation of a choice prior to a choice from $\underline{ACT}$. This idea, like so many others, seems to originate in The theory of games and economic behavior (von Neumann & Morgenstern, 1944; see also Luce & Raiffa, 1957). In that work the rational game player's main decision is not to make a choice at each move of the play but rather to select a strategy (or exhaustive decision rule) for specifying choices. In game theoretic terms, the decision problem is defined on the normal form of the game. In a sense the HRUM is a generalization of the game theoretic approach applied to individual choice behavior. It generalizes that theory by not insisting that R_A include all possible ways of making a choice. It also places fewer restrictions on $P(R_{i,A})$ than does game theory. On the other hand, nontrivial HRUMs imply, by assertion (5), that no element of R_A has zero probability. That is, if R_A is a strategy set for a game, a pure strategy solution would be impossible. This is not, of course, the case with a trivial HRUM in which R_A contains only the minimax rule.

Another class of models postulating a set of decision rules is "heuristic" decision theory (Newell, Shaw, & Simon, 1963; Simon, 1960). Choice from sets of heuristics or "rules of thumb" is prior to choice from offered alternatives. A similar emphasis on decision rules is to be found elsewhere (_e.g._, Yntema & Torgerson, 1961) in the literature on automated decision making.

In addition to analyses defining prior choice sets as sets of decision rules, some theorists and researchers (Becker _et al._, 1963a; Gurney, Phillips, Messé, & Lane, 1970; Tversky, 1971, 1972a,b) have defined the prior choice set as a set of aspects or attributes characterizing alternatives. With the exception of the Gurney _et al._ model, attribute analysts differ from rule theorists in placing less emphasis on specification of the elements of the prior choice set.

To conclude this theoretic discussion, reconsider the questions concerning actual _vs._ optimal choice. Since there seems to be no clear consensus on the meaning of "optimal choice" (Shelly & Bryan, 1964), it must be realized that to any one of the measures here suggested, there are objections.

Of the three measures to be offered, the one that follows most easily from a HRUM requires a definition of some rule, R_{OPT}, as somehow best. There may, of course, be several rules satisfying this requirement. Denoting the set of all such rules as S_{OPT}, then $OPT_1 = P(S_{OPT})$ measures the extent to which a sequence of choices coincides with those that might be made by a master or wizard of choice. For a specific decision maker, $1 - OPT_1$ measures the extent to which actual behavior deviates from optimal and, consequently, the extent to which "therapy" is needed. Moreover, changes in OPT_1 as a result of therapy or training measures the extent to which training has been beneficial or detrimental.

With respect to the HRUM, the other two measures of optimality, OPT_2 and OPT_3, are probably determinants of the utility functions, $U_i(R_{i,A})$, rather than consequences of these functions (as is the case with OPT_1). Both OPT_2 and OPT_3 require assigning to each alternative, $x \in A$, a number that reflects the excellence of the alternative. Denote this number $W(x)$, the worth of x. Then if a decision maker chooses $c \in A$, $W(c)$ reflects the value of his actual behavior, $W_{max}(x)$ represents the value of optimal behavior, and $W_{max}(x) - W(c)$ represents the extent to which training might prove beneficial. Moreover, changes in $W(c)$ as a result of training would indicate the worthwhileness of the training. It makes some sense, therefore, to let $OPT_2 = W(c)$.

While OPT_2 is a relatively conventional index of optimality, OPT_3 has been largely ignored by formal decision theory. It has its roots in the suggestion that the amount of effort involved in making a decision affects the value of a choice (Edwards, 1954; Simons, 1957). Specifically, assume that the value of a choice varies inversely with the amount of effort, E, and directly with the value, $W(c)$. (For a contrary viewpoint, see Lawrence & Festinger, 1962.) Then, let $OPT_3 = W(c)/ E$. While estimation of E could be an extremely complicated process, the problem can be simplified for empirical purposes by defining OPT'_3 as the ratio of the sum over a sequence of choices of W(c) divided by the time it takes to make the choices. Unfortunately, this measure is considerably more difficult to apply to a concrete case than are either of the other two. The reason for this difficulty is that OPT_3 varies not only with W(c) but also with E. Since E depends on the behavior of the decision maker, there is no obvious way in which to ascertain the maximum value of the index. If, for example, OPT_3 after training is the same as it was before training, this could indicate that the training procedure was worthless, that the individual prior to training was already functioning at optimum, or both. On the other hand, the measure is so compellingly reasonable that it should not be rejected out of hand.

Of the three measures, OPT_1 is probably the most generally applicable. OPT_2 and OPT_3, however, are easily obtained in decision contexts for which consequences are economic. OPT_2 corresponds to the notion of a salary or piecework wage while OPT'_3 corresponds to an hourly wage. In the formulation by exchange theorists (e.g., Thibaut & Kelley, 1959), OPT_2 is most closely related to the "rewards" concept, while OPT_3 combines "rewards" with "costs." OPT_1 is relatively peculiar to the HRUM formulation.

In order to explore the empirical applicability of HRUM theory, the model was focused on a group of subjects choosing from wager sets similar to those used by Becker *et al.* (1963b). OPT_1 was defined to be the probability with which a subject would use an expected value decision rule, R_1. R_1 instructs its user to choose a wager having the largest expected value or mean relative to the means of the other wagers offered. OPT_2 and OPT'_3 were also defined in order to serve as possible qualifications of any conclusions about optimality based on findings for OPT_1. OPT_2 was defined as the average of the expected values for choices made by a subject. OPT'_3 was OPT_2 divided by the time it took the subject to make his choices.

The hypotheses are simple and straightforward. Hypothesis 1 asserts that instruction in the use of R_1 increases the likelihood with which it is, in fact, used. In accord with the preceding discussion, this is the therapeutic hypothesis.

Hypothesis 1: Instruction in the use of R_{OPT} increases OPT_1.

The second hypothesis asserts that when the expected values of wagers in a choice set differ by relatively large amounts, the probability of using R_1 is larger than when the expected values do not differ by much. That is, the probability of using the optimal rule increases as the wisdom of so doing increases.

Hypothesis 2: OPT_1 is larger when the expected values of alternatives have a large range than when the range is small.

Hypothesis 3 states that OPT_1 should differ as a function of time. No directionality is asserted since there seem to be good reasons for believing the difference to be in either direction. On one hand, experience with decision problems could be expected to lead to an increase in optimal behavior. On the other hand, use of R_1 is probably quite fatiguing. This fatigue might reduce OPT_1.

Hypothesis 3: OPT_1 in the first half of a sequence of decisions differs from OPT_1 in the second half of the sequence.

Specification of the HRUM for the situation being investigated involved specifying R_A and P. The definition of R_A was accomplished partially by guessing and partially by interviewing subjects. The resultant R_A contains six rules: R_1 = the expected value (EV) rule; R_2 = the "hunch" rule, a generalization of "event matching" (Simon, 1959); R_3 = low risk; R_4 = moderate risk; R_5 = high risk; and R_6 = random selection.

The EV rule, R_1, was historically the first seriously considered decision rule (Savage, 1954). Today the model has largely been replaced by the "Expected Utility" (EU), "Subjective Expected Utility" (SEU), "Subjective Expected Value" (SEV), "Nonadditive Subjective Expected Utility" (NSEU), and "Nonadditive Subjective Expected Value" (NSEV) models (Savage, 1954; Edwards, 1968; Lindman, 1966).

Formally, let

W_i = a wager, $[(o_{i1}, P(e_1)), (o_{i2}, P(e_2)), \ldots]$ where

E = a set of [probabilistic] events $e_1, e_2, e_3, \ldots,$

$P(e_j)$ = the probability with which e_j occurs,

$\Sigma P(e_j) = 1.0$, and

o_{ij} = the outcome if W_i were chosen and e_j were to occur.

Then, the EV rule instructs its user to choose that W_i for which

$\Sigma_j o_{ij} P(e_j)$ is maximal. This rule is typically included in analyses only as a baseline from which to estimate the extent to which other models improve predictability.

There are cases, however, in which the corresponding EV model provides a respectable fit to data. For three of eleven prisoners at Jackson State Prison in Michigan, the EV yielded almost perfect predictions of their choices in a relatively complex set of gambles (Tversky, 1967). Also, contrary to the Suppes and Walsh (1959) conclusion that their own model had "clear predictive superiority" over EV, DeGroot (1963) has argued that the Suppes and Walsh data indicate that for some subjects the EV model is better. In particular, by considering only predictions from the Suppes and Walsh model and from EV that disagree with one another, EV makes more accurate predictions for four out of eight subjects. A similar conclusion pertains for data from the Royden, Suppes, and Walsh (1959) study. In that case, the authors' model was a better predictor for only six of thirteen (or of sixteen if one includes the three subjects omitted from the analysis) subjects than was EV. In short, while EV is no longer fashionable, evidence that it does not play an important role in decision making is lacking. Rather, some of the evidence indicates that EV is still a potentially useful concept for describing actual behavior as well as optimal.

The "hunch" rule, R_2, has received little explicit attention except under the guise of "event matching" (Simon, 1959) or "probability matching" (Siegel *et al.*, 1964). This decision rule is of theoretic interest because it contradicts two common ideas in decision theory. These are the "lack of illusion" principle (Pfanzagl, 1968; also, Becker *et al.*. 1963a) and the assumed necessity of an interval scale for risky choice (Luce & Suppes, 1965).

The underlying notion of hunch rules is that the decision maker converts risky decision problems to riskless problems by guessing which of the set of probabilistic events will occur. For some $e_j \in E$, he assumes $P(e_j) = 1$. He then chooses that wager for which o_{ij} is maximal. Common examples of this rule are even or probability matching and the gambler's fallacy (Lindman & Edwards, 1961) or the Monte Carlo fallacy (Cohen, 1964). While there is debate about the appropriate use of the phrase "gambler's fallacy" (Cohen, 1964), the various definitions agree that the gambler sometimes changes P(e) values as a function of previously occurring events. For example, if a gambler playing roulette has bet on red (or *rouge*) for a large number of times in sequence and has lost every time, the fallacy leads him to believe that P(red) for the next bet is virtually 1.0.

To see how the hunch rule, R_2, contradicts the lack of illusion principle, consider the two games in Figures 1 and 2. The lack of illusion principle asserts that the arrangements of payoffs in a wager does not affect the decision process. Suppose that subjects are playing in a Davidson, Suppes, and Siegel (1957) casino. A die is rolled on

	W_1	W_2
ZEJ	10	3
ZOJ	1	0

Figure 1.

Game 1.

	W_1	W_2
ZEJ	1	3
ZOJ	10	0

Figure 2.

Game 2.

three sides of which is the nonsense syllable ZEJ. The other three sides have the syllable ZOJ. According to the casino operators, the subjective probability of ZEJ occurring is equal to the subjective probability of ZOJ occurring. Now suppose the decision maker decides that P(ZEJ) is 1.0 (perhaps, due to a Monte Carlo fallacy). Then, in game 1, he will choose W_1 while in game 2, he will choose W_2. In other words, for an R_2 user it does make a difference how the payoffs are arranged. While this consequence is probably of most importance in sequential gambling with feedback after each bet, it can also be formulated within the context of sequential independence. Suppose, for example, that it is, in fact, true on every trial that the decision maker believes the probability of ZEJ to be equal to the probability of ZOJ. He can still eliminate the riskiness of the problem by probability matching the events ZEJ and ZOJ. He simply flips an unbiased coin. If it comes up heads, he lets P(ZEJ) = 1. If it comes up tails, he lets P(ZOJ) = 1. Then, fully aware of the riskiness of the problem and of his technique, he behaves as if the problem has no risk by making his decision on the basis of the coin toss.

The second principle of decision theory violated by R_2 is stated by Luce and Suppes (1965).

> Once uncertainty in the consequences is admitted, no ordinal theory of choice can be satisfactory. The simplest sorts of example suffice to make this fact clear. (Pp. 281-282)

They then cite as examples a standard gambling experiment and decisions about insurance policies. Consider the insurance problem.

Typically what one is insuring against, at the time of buying the policy, is an unlikely event. Insurance buyers behave as if the catastrophe will, in fact, happen. Once they make this assumption, they require only an ordinal scale of preference (e.g., I'd rather have Allstate foot the bill than me). Similarly, the non-buyers need only an ordinal scale (e.g., I'd rather spend the money myself than give the Citizens' Man a fling). In short, while an all-encompassing theory of choice probably requires more than ordinality, the hunch rule can stand as an ordinal model of risky choice. The examples cited by Luce and Suppes do not present problems to an R_2 user.

The three "risk" rules, R_3, R_4, and R_5, may be considered as a unit. They direct the decision maker to attend to the same general aspect of the alternatives. This aspect is the spread (e.g., range or variance) of the outcomes, o_{ij}. R_3 directs the decision maker to choose the wager having smallest spread; R_4 implies choosing medium spread; and R_5 implies choosing maximum spread. Moreover, the rules are defined relative to one another. The fact that there are three rules is an artifact of the triadic decision problems presented to the subjects. Had the decision problems been binary, there would be only two rules. Had they involved twenty alternatives, there may well have been twenty risk taking rules. The reason for this situational specification of the number of risk taking rules is related to "unfolding theory" (Coombs, 1964; Coombs & Pruitt, 1961). Theories of choice are based on the assumption that a decision maker attempts to maximize something (Edwards, 1954). For rule R_1 and rule R_2, the "somethings" to be maximized are invariant across decision makers. The R_1 "something" is the EV of a wager. For the R_2 this "something" is the value associated with a wager when some specific event occurs. R_3 through R_5, on the other hand, have a "something" that depends on the decision maker. In terms of unfolding theory, the "something" is proximity of actual to the "ideal" variance of the payoffs within the wager. Each decision maker selects the wager whose variance is closest to his "ideal." That this "ideal" point varies from subject to subject no doubt is one of the reasons for the popular tendency to use risk taking as a personality variable (e.g., Kogan & Wallach, 1964). Contrary to this tendency, there is evidence that subjects are not consistently high or low risk takers (Davidson et al., 1957). Rather they vary from one extreme to the other, including the points in between.

The R_3 through R_5 rules are in a somewhat different spirit than unfolding theory. The notion underlying these rules is that the ideal point is largely determined by the decision problem itself. Suppose there are two decision problems. In the first problem, the wagers have risk values (e.g., range or variance) of 5, 10, and 15. In the second problem, the values are 15, 20, and 25. Instead of postulating an ideal point, the present formulation postulates an ideal relative position. An R_3 user would choose the wager whose value is smallest relative to the values of the other two wagers. An R_4 user would choose the wager having the middle value; and an R_5 user would choose the wager having the largest value. The absolute values associated with the wagers are irrelevant. For example, in the first problem the wager with value 15 would be high risk while in the second problem, the wager with value 15 is low risk.

The last rule in R_A, the random selection rule R_6, is typically not discussed. Yet, interviews with subjects often reveal that subjects do use this decision rule. It is also easy to show that for some decision problems the measure OPT_3 indicates that R_6 is an optimal decision rule. If one makes the reasonable assumption that R_6 is the easiest rule to use, then E_6 will be minimal. Since OPT_3 = $W(c)/ E$, it follows that if $W(c)$ is the same for all possible choices (as would be true, for example, if $W(c)$ were the EV of the alternative and the decision problem were a game of roulette), and $W(c) > 0$, then OPT_3 would be maximal when E is minimal. That is, OPT_3 would be largest for an individual choosing randomly. Depending on the nature of the $W(c)$ and E distributions for various sets of rules and decision problems, R_6 would often yield the largest OPT_3 value.

In addition to this specification of R_A, the application of the

HRUM requires specification of the $P(R_{i,A})$ values. The most closely related attempt to estimate the extent to which various decision rules are used is that of Kogan and Wallach (1964). They presented subjects with 66 binary choice problems, each alternative of which was a wager. Their set R_A contained five risk taking rules. Rather than estimating probabilities, their measures were the number of times subjects chose alternatives specified by the rules. Suppose, for example, that for a specific problem with alternatives x and y, an R_1, R_2, or R_3 user would choose x while an R_4 or R_5 user would choose y. If a subject chose x, he would get one point for each of the first three rules and no points for the other two. These points were then summed and the resulting totals used as indices of the extent to which each rule was used. In spite of the generally high quality of this study, Kogan and Wallach's technique for measuring these variables is quite unsatisfactory. The amount of overlap or dependence present in their measures makes it impossible to know what is, in fact, represented by any specific score.

The technique used in the present study suffers from the same difficulty as the Kogan and Wallach technique but considerably less so. The technique employs a Bayesian formula defined on each choice made by a subject. If there were no two rules in R_A having the same probability distributions for a decision problem, it would make sense to use (6).

$$(6)\quad P(R_{i,A}|\text{choice}) = \frac{P(\text{choice}|R_{i,A})P(R_{i,A})}{\sum_j P(\text{choice}|R_{j,A})P(R_{j,A})}$$

However, because of the way in which Becker _et al._ (1963b) decision problems are constructed, it is always the case that either R_1 and R_3 or R_1 and R_5 specify the same choice with probability 1.0. This implies that using (6) would lead to probable distortion of $P(R_{1,A}|\text{choice})$ because of subjects using R_3 or R_5. In order to reduce this distortion, the present technique first estimates the probability with which a choice indicates that either R_1 or R_3 is being used or, in the second case, that either R_1 or R_5 is being used. Then, by making a convenient assumption about the use of R_3 and R_5, the probability of using R_1 and one of the other two rules is separated into two components--one being the probability of R_1 conditional on the data and the other being the conditional probability of either R_3 or R_5. The result for R_1 constitutes the dependent variable, an estimate of OPT_1.

The decision problems fall into eight partitioning categories, denoted by subscripts f, g, and h. Let h = 1 when R_1 and R_3 specify the same choice, and h = 2 when R_1 and R_5 specify the same choice. Assume that the prior probabilities, $P(R_{i,A})$ are all identical, as in (7).

(7) $P(R_{1,A}) = P(R_{2,A}) = P(R_{3,A}) = P(R_{4,A}) = P(R_{5,A}) = P(R_{6,A}) = 1/6.$

The effect of assuming equality of the prior probabilities in (7) is to minimize the variance of resultant posterior probabilities. A more realistic technique would involve beginning with assumption (7) but then iteratively equating the priors in (7) with posteriors obtained by the estimation procedure. This iterative procedure, however, would have the undesirable property of giving too little weight to decision rules R_2 and R_6. Then, for any decision problem in the present study, the posterior probabilities are given by (8) and (9).

(8) $P_{fg1}(R_1 \cup R_3|\text{choice}) =$

$$\frac{P(\text{choice}|R_1 \cup R_3)}{P(\text{choice}|R_1 \cup R_3) + 1/2 \sum_{i=2,4,5,6} P(\text{choice}|R_i)}$$

(9) $P_{fg2}(R_1 \cup R_5|\text{choice}) =$

$$\frac{P(\text{choice}|R_1 \cup R_5)}{P(\text{choice } R_1|\cup R_5) + 1/2 \sum_{i=2,3,4,6} P(\text{choice}|R_i)}$$

In order to separate the rules, assume that on the average the probability of either R_3 or R_5 is independent of h, as in (10) where $\overline{P}$ indicates a mean.

(10) $\overline{P}_{fg1}(R_3|\text{choice}) = \overline{P}_{fg2}(R_3|\text{choice})$ and

$\overline{P}_{fg1}(R_5|\text{choice}) = \overline{P}_{fg2}(R_5|\text{choice}).$

That is, a subject using a risk rule for a choice is using only a risk rule. He is not using risk in combination with an EV rule. If there were a [used] rule that combined EV with the spread of wagers, R_A would require redefinition.

Since the decision problems in a group fg1 are matched with the problems in a group fg2 only as an aggregate, it is necessary to work with expected values. Again using $\overline{P}$ to indicate mean probabilities, the formulae used for estimating $P(R_{1,A})$ are given by (11) and (12).

(11) $\overline{P}_{fg1}(R_1|\text{choice}) = \overline{P}_{fg1}(R_1 \cup R_3|\text{choice}) - \overline{P}_{fg2}(R_3|\text{choice})$.

(12) $\overline{P}_{fg2}(R_1|\text{choice}) = \overline{P}_{fg2}(R_1 \cup R_5|\text{choice}) - \overline{P}_{fg1}(R_5|\text{choice})$.

The other values needed are the various $P(\text{choice}|R_i)$'s. For all rules except R_2 and R_6, these values are given by the definitions of the rules. For R_2 it is assumed that a subject selects any $e_i \in E$ with an equal probability. For R_6 it is assumed that each choice from A is equally likely (i.e., for the three choices possible in a Becker et al. decision problem, each has probability 1/3 for an R_6 user).

PROCEDURE

Subjects

Subjects (Ss) were 104 undergraduates at Michigan State University. They were members of a subject pool recruited from a newspaper advertisement by the Cooperation/Conflict Research Group. There were 52 females and 52 males. Thirteen Ss of each sex were randomly assigned to each of four conditions.

Apparatus

A PDP8/I computer controlled presentation of stimuli and recording of responses. Peripheral apparatus included a tape recorder and headphones, a slide projector and screen, a three choice response box, and a numerical response keyboard. The 8 inch by 11 inch projection screen was on a table and approximately three feet in front of S. The two response boxes were also on the table, between the screen and S.

Method

The experimental design was a six factor experiment with repeated measures on three of the factors. The three between Ss variables were sex, order of presentation, and hearing or not hearing a lecture on the EV technique. The three within variables were EV Risk, EV Range, and Problem Sequence Number, all of which are explained below.

Experimental Ss began their session by listening to the following lecture on using an EV decision rule.

> This lecture introduces you to one of many techniques for making decisions while gambling. Some psychologists believe that you will use this technique. Other

psychologists believe that you will not use this technique. We are interested in determining which of these psychologists is correct. It is important to us only that you understand what the technique is. Whether or not you actually use it is up to you.

This method is called the "Average Winnings" technique. It has this title because its application involves calculating arithmetic averages. An average is the sum of a set of numbers divided by the number of numbers added up. For example, to find the average of the numbers 3 and 7, you add 3 to 7 to get 10 and then divide by 2. Since 10 divided by 2 is 5, the average of 3 and 7 is 5. As a slightly more difficult example, what is the average of 3, 7, and 12? The sum of these numbers would be 3 + 7 + 12, which is 22. Since 22 divided by 3 is 7 1/3, the average of 3, 7, and 12 must be 7 1/3. So an average is simply the sum of a set of numbers divided by the number of numbers in the set.

The Average Winnings technique provides an answer to a common question. Which betting scheme or wager should be chosen if there are several wagers available? The answer involves calculating an average for each wager. Then you find the wager that has the largest average. This wager with the largest average is the wager that should be chosen.

For example, suppose that for some game there are only two possible wagers. One wager is characterized by the numbers 3 and 7 while the other is characterized by 3, 3, and 7. Which bet or wager should you choose? To use the Average Winnings technique, you first calculate the average for each wager. Since the first wager involves the numbers 3 and 7, its average must be its sum, 10 divided by 2. This means the average for the first wager is 5. The second wager involves three numbers, 3, 3, and 7. The average of these numbers must equal the sum, 3 + 3 + 7 = 13, divided by 3. The average for the second wager, therefore, is 4 1/3. Since the first wager had an average of 5, someone using the Average Winnings technique would choose the first wager because 5 is larger than 4 1/3.

Let's summarize what has been said. First, an average is the sum of a set of numbers divided by the number of numbers in the set. Second, the Average Winnings technique involves first calculating the average of each wager that could possibly be chosen. Then the gambler ought to choose the wager having the highest average.

> But now we are left with the question, what are the numbers associated with a wager? Once we have these numbers, we know what to do--calculate the average for each wager, then pick the wager having the largest average. But average of what? What specifically are the numbers associated with a wager?
>
> Without going into elaborate detail, these numbers are possible winnings. Each winning represents the amount of money a gambler wins if something happens. For example, suppose a gambler is flipping coins. If the coin comes up heads, he wins $5. If it comes up tails, he wins $3. Then the winnings associated with this wager are 5 and 3. To get the average winning for the wager, first add 5 and 3 which equals 8. Then divide this sum by 2 which results in the average winning being 4. If the gambler had a choice between this wager and another wager having an average of 2, the Average Winnings technique would instruct him to choose the wager whose average is 4. Four is larger than 2.
>
> Let's summarize the steps involved in using the Average Winnings technique. First, you list each and every winning associated with each wager. Second, you calculate the average winnings for each wager. To do this you just add up all the winnings listed for a specific wager, then divide by the number of winnings added up. You must do this for each wager. Finally, you choose the wager having the largest average winnings.
>
> Remember it is up to you whether or not you use this technique. It is important to us only that you understand what the technique is.
>
> If you understand the Average Winnings technique, press button "A" when the ready light goes on. If you do not understand, press button "B".

All <u>S</u>s then took a test of basic arithmetic and understanding of simple gambling problems (Appendix A). For Control <u>S</u>s this test began the session. This test served to familiarize <u>S</u> with the equipment in addition to providing information about <u>S</u>'s aptitude for coping with such problems.

After the test, <u>S</u> listened to taped instructions for responding to the Becker <u>et al</u>. decision problems. These instructions are in Appendix B. Briefly, the instructions told <u>S</u> that he would receive no feedback until the end of the experiment. At that time, one decision problem would be randomly selected and he would receive five cents for each point he won in that problem.

Each of the presented problems contained three wagers, W_1, W_2, and W_3, involving four payoffs, w, x, y, and z. See Appendix C for problems as presented. The payoffs were always such that $0 < w < x < y < z$. W_1 was a fifty fifty chance of winning w or winning z. For W_2 there was a .5 chance of winning x and a .5 chance of winning y. W_3 was a .25 probability of winning any one of the four payoffs. This problem structure implied that an R_1 choice would never be W_3, an R_3 choice would be W_2, an R_4 choice would be W_3, and an R_5 choice would be W_1. Except for reward sets corresponding to Becker _et al_.'s set 21, fifty of the one hundred reward sets in the present study were equivalent to those used by Becker _et al_. except that they were incremented by one point to eliminate payoffs of zero. The payoffs for set 21 were increased by 2 points in order to have a maximum payoff of 100 points. Each problem was presented in two arrangements. These arrangements were random except for the stipulation (not of immediate relevance to the present study) that for one of each pair of arrangements an R_2 user would never select W_2. The other 50 problems (25 pairs) were transformations of the Becker _et al_. problems. The payoffs w and z were the same in the transformed problems as they were in the original. Also, y - x was kept constant. The transformation involved increasing the discrepancy between EV(W_1) and

Table 1.

Transformed Reward Sets

Becker *et al*. (1963b) Set	w,z	Transformed x,y
1	4,90	44,78
2	1,73	9,43
3	2,46	18,42
4	11,63	25,59
5	61,89	78,84
6	21,57	32,56
7	3,45	7,29
8	3,81	10,56
9	23,79	29,57
10	4,88	17,45
11	30,62	45,59
12	5,55	25,49
13	9,51	14,30
14	17,85	18,72
15	7,91	14,66
16	15,89	70,74
17	24,56	43,53
18	22,94	25,71
19	5,63	15,27
20	7,77	15,75
21	4,100	37,91
22	4,72	19,23
23	17,43	27,41
24	3,33	13,31
25	1,37	4,22

$EV(W_2)$. If $EV(W_1)$ were the larger EV for a problem, x and y were decreased by one half the x - w interval. If $EV(W_2)$ were larger, then x and y were increased by one half the z - y interval. Table 1 presents the results of these transformations. The effect of this transforming the payoffs was to hold $EV(W_1)$ constant, hold both the variance of W_1 and the variance of W_2 constant, but move $EV(W_2)$ farther away from $EV(W_1)$. This defined the within variable, EV Range, for testing hypothesis 2.

The decision problem slides were presented in two orders, Order 2 being the reverse of Order 1. This defined the between variable Order. The two within variables in addition to EV Range were EV Risk (whether the R_1 choice was also the R_2 choice or was the R_5 choice), and Sequence Number (defined by dividing the problems in each Range-Risk category into two sets, one being roughly the problems having sequence numbers 1 through 50 and the other having numbers 51 through 100).

When _S_ completed the 100 choices, he was given a short break during which time he was interviewed about how he had made his decisions. After the interview, he calculated the arithmetic mean of all 150 distinct wagers he had been offered in the preceding part. For ethical reasons, he was instructed to work on this part only as long as feasible. Finally, _S_ received his payoff.

RESULTS

Results of this study fall under two headings. Under the first heading are findings centrally related to the experimental design. These include testing whether the dependent variable, OPT_1, satisfied the definition of probabilities and the results of an analysis of variance, including a test of the three formally stated hypotheses. The second set of results are peripheral to the experimental design but have important bearing on the interpretation of analysis of variance results. These peripheral results answer the questions of the extent to which OPT_1 values were distorted by computational errors, the relationship between OPT_1 and the other two optimality indices, OPT_2 and OPT'_3, and the degree of correspondence between parameter estimates and interview results.

Central Results

Estimated OPT_1 values for each of the eight conditions for each of the 104 Ss are in Appendix D. Since these values are interpreted to be probabilities, the fact that 42 of the 832 values are negative numbers suggests a difficulty with the dependent variable. In order to determine whether these negative values could be attributed to chance, t tests were performed. To perform these tests it was assumed that a negative OPT_1 indicated a zero probability of choosing R_1.

From assumptions (11) and (12), this was equivalent to stating the null hypothesis that $P(R_1 \cup R_2) = P(R_3)$ or $P(R_1 \cup R_5) = P(R_5)$, whichever pertained to the specific estimate being tested. Since these hypotheses were assumptions of the HRUM for wagers, the most powerful of conventional tests of proportion differences were used. First, the estimates of $P(R_1 \cup R_3)$, $P(R_3)$, $P(R_1 \cup R_5)$, and $P(R_5)$ were divided by their maximum possible values, .82, .69, .72, and .56, respectively. (That the maximum values were not 1.0 was a joint result of the parameter estimation procedure, _q.v._, pp. 21-25, and the decision problems, Appendix C). The effect of this correction was to increase the power of each test. The power was also increased by using the corrected estimates to estimate population variances rather than the conventional but conservative value of .25 as the estimate. In spite of these efforts, assumptions (11) and (12) could not be rejected. Out of 42 one tailed t tests, each with 23 degrees of freedom, only four were significant at the .10 level. None were significant beyond the .05 level. Since these results were approximately what would be expected if the null hypotheses were true, the negative OPT_1 values may be attributed to chance fluctuations in the parameter estimates.

Table 2 presents the results of an analysis of variance including all independent variables except Sequence Number. This variable was eliminated because, in the original analysis, it was statistically significant only as a main effect. Its conclusion in Table 2 would double the length of the table and concomitantly increase the difficulty of understanding the table's contents. The results of the original analysis, including Sequence Number, are in Appendix E.

Table 2.

Analysis of Variance Summary Table

Source	df	Mean Square	F
Sex (A)	1	1.140604	7.9857**
Order (B)	1	.200075	1.4008
EV Lecture (C)	1	.551958	3.8644
A X B	1	.287558	2.0133
A X C	1	.040849	.2860
B X C	1	.027506	.1926
A X B X C	1	.704694	4.9338*
Subjects (S), error	96	.142830	
EV Risk (D)	1	1.062732	199.0507***
A X D	1	.001582	.2963
B X D	1	.002764	.5177
C X D	1	.000637	.1193
A X B X D	1	.000004	.0007
A X C X D	1	.000357	.0669
B X C X D	1	.000064	.0120
A X B X C X D	1	.005508	1.0316
D X S, error	96	.005339	
EV Range (E)	1	3.501568	185.1016***
A X E	1	.059088	3.1236
B X E	1	.032270	1.7058
C X E	1	.033035	1.7463
A X B X E	1	.006980	.3690
A X C X E	1	.099801	5.2757*
B X C X E	1	.003968	.2098

Table 2 (cont'd.)

Source	df	Mean Square	F
A X B X C X E	1	.014580	.7707
E X S, error	96	.018917	
D X E	1	.066354	22.2703***
A X D X E	1	.001245	.4178
B X D X E	1	.000460	.1544
C X D X E	1	.036929	12.3944***
A X B X D X E	1	.007579	2.5437
A X C X D X E	1	.000083	.0279
B X C X D X E	1	.000477	.1601
A X B X C X D X E	1	.000057	.0191
D X E X S, error	96	.002980	

Note. *** signifies $p < .001$, ** signifies $p < .01$, and * signifies $p < .05$.

Simple effects analyses relevant to the statistically significant interactions of Table 2 may be found in Appendix F. It is to be noted that because of the large number of degrees of freedom for the error estimates of the analysis of variance, these estimates were also used for the simple effects analyses. This eliminated the necessity of making additional assumptions for pooling the estimates.

The first hypothesis of this study was that Ss who heard the experimental lecture on the EV rule would have a larger OPT_1 mean than would control Ss who did not hear the lecture. This implied a directional planned comparison. While EV Lecture was not statistically significant in the undirectional analysis of variance, the directional test was significant at the .05 level ($t = 1.966$, $df = 94$).

The second hypothesis also implied a directional test. This hypothesis asserted that the larger the differences between the EV's of the wagers in a decision problem, the larger would OPT_1 be. This variable corresponds to EV Range in Table 2. This hypothesis was strongly supported ($t = 13.605$, $df = 94$, $p < .001$).

The third hypothesis stated that there is a difference between OPT_1 in early decision trials and OPT_1 in later trials. Unlike the first two hypotheses, this was not directional. The independent variable in the original analysis of variance (see Appendix E) that corresponded to the independent variable of this hypothesis was Order X Sequence Number. The F-ratio for this interaction was only 1.58 which was not significant.

Before delving more deeply into effects of independent variables on OPT_1, there is a preliminary note. The OPT_1 values evidence large individual differences among Ss. This is reflected in the fact that over fifty percent of the variance of OPT_1 can be attributed to the one source, Subjects (S). Consequently, while individual differences may subsequently be ignored, they constitute the overriding fact of these data.

Of the three variables Sex, Order, and EV Risk for which no formal hypotheses were stated, EV Risk may be considered first--primarily because its significant main effect may have been a contrived result. It would have been surprising, in fact, if EV Risk had not had a significant effect. The reason for this would involve simultaneous consideration of the parameter estimation technique, the four decision rules, R_1, R_2, R_3 and R_5, and the specific matrices describing the decision problems. Bypassing this morass of complexity, the end result is the conclusion that when R_1 and R_5 specify the same choice, OPT_1 has a smaller maximum value (about .72) than when R_1 and R_3 specify the same choice (in which case the maximum is about .82). The difference of about .10 between the maximum values was reflected in the obtained OPT_1 means for the two levels of EV Risk. When R_1 and R_3 specified the same choice, the OPT_1 mean was equal to .3952. When R_1 and R_5 specified the same choice, the mean was equal to .2943. Yet, while coincidence of the obtained difference, .1011, with the difference of .10 between maxima seems initially to resolve the problem of why D had a significant main effect, there is a perspective from which the D effect was not simply an artifact. In order to be consistent with the previous evaluation of negative OPT_1 values, the appropriate correction for differences between maxima would be to correct the obtained OPT_1 means and look for a zero difference between the adjusted values. As before, this correction involved dividing each OPT_1 mean by its maximum possible value. For problems in which R_1 and R_3 specified the same choice, the adjusted OPT_1 mean = .4700. Where R_1 and R_5 specified the same choice, the new mean = .4088. For the analysis of variance, this correction reduced the mean square for D from 1.062732 to .389526. It concomitantly increased the error term from .005339 to about .009742. The resultant F-ratio, however, was still significant beyond the .001 level ($F = 39.9843$, $df = 1,94$).

The end result of analyzing the effect of EV Risk is serious doubt about the validity of the HRUM for wagers. If attention is restricted to only the two techniques used for adjusting OPT_1 means for the two levels of EV Risk, then it is impossible to reconcile the negative OPT_1 values discussed earlier with the present analysis of variance findings. Either finding alone could be explained by one technique or the other. The additive technique (i.e., taking the difference between maxima into account) resolved the analysis of variance finding but would have led to rejection of assumptions (11) and (12) if the analogous, but considerably less reasonable, correction had been applied to the negative OPT_1 values. On the other hand, the multiplicative technique (i.e., dividing obtained values by their

theoretic maxima) resolved the negative OPT_1 difficulties but did not eliminate the main effect of D. Consequently, having used either the additive or multiplicative correction technique, the two sets of results could not be simultaneously justified within the HRUM for wagers. Rather the evidence has indicated that sometimes EV and risk characteristics of a decision problem had a combinatorial influence on choice. The specific nature of this influence was dependent on which adjustment technique was used. If additive, either the presence of high EV had aversive properties or its absence had attractive properties. If multiplicative, high EV and low risk mutually increased an alternative's attractiveness more than high EV and high risk did. Since the conclusion from the additive correction is relatively unreasonable, it may be assumed that the multiplicative correction was the more reasonable of the two and that its implication is more accurate than that of the additive correction.

That OPT_1 scores were influenced by risk characteristics was also indicated by significant EV Risk X EV Range and EV Lecture X EV Risk X EV Range interactions (see Table 2). The simple effects analyses indicated that for the control group, there was an EV Risk X EV Range ($p < .001$) interaction not present in the experimental group and for large EV Range, there was an EV Lecture X EV Risk ($p < .05$) interaction not present for small EV Range. Since the simple EV Risk X EV Range interaction effect was the stronger of the two simple interactions, the control group EV Risk X EV Range matrix was compared with that of the experimental group. This comparison led to the conclusion that the control, large range, R_3 choice mean was larger than it would

Table 3.

OPT_1 Means for EV Lecture X EV Risk X EV Range

	R_3 Choice		R_5 Choice	
	Small Range	Large Range	Small Range	Large Range
Control	.237579	.482996	.178019	.335254
Experimental	.344530	.516613	.252434	.411663

be if there were no interaction and the control, small range, R_3 choice mean was smaller (see Table 3). By subtracting about .045 from the former and adding .045 to the latter, the EV Risk X EV Range and EV Lecture X EV Risk X EV Range interactions, together with the two simple interactions, would be virtually eliminated.

While the law of parsimony points toward focusing explanation of the EV Risk X EV Range and EV Lecture X EV Risk X EV Range interactions on the two deviant means for the control Ss when the R_1 choice is low risk, the explanation would be inappropriate to the data. Clarification of this assertion must await analysis of the Sex X EV Lecture X EV Range interaction. At that time, it will be clear that the means of the EV Lecture X EV Risk X EV Range interaction are most reasonably divided into three sets rather than one deviant and one nondeviant set. These three sets were the two control, low risk means, the two control, high risk means, and the four experimental means--the latter being itself broken down by the Sex X EV Lecture X EV Range interaction.

On the basis of the main and interaction effects involving EV Risk, it must be concluded that the HRUM for wagers needs revision. The data have indicated that the probability with which R_1 was used was affected by risk characteristics of the decision problems. Moreover, the effect of these risk characteristics was itself influenced by the experimental treatment (EV Lecture) and the size of difference between EV's of offered alternatives (EV Range). This stands in contradiction to the notion that Ss selected one and only one of the six rules in R_A. Empirically, either the set R_A contained additional rules that were combinations of EV and risk rules or the defined EV and risk rules were not defined with sufficient complexity to describe actual behavior.

In addition to EV Risk, the other variable for which no formal hypotheses were stated but which clearly played an important role in the analysis of variance results was Sex. As a main effect it was significant at the .01 level. In interaction with Order and EV Lecture and in interaction with EV Lecture and EV Range, it was significant at the .05 level. The overall mean of OPT_1 for males was .3972 while for females the mean was .2925.

The Sex X Order X EV Lecture interaction was the most complex of the significant interaction effects. Table 4 presents the mean OPT_1 values for each of the eight groups defined by these variables. The simple effects analyses indicated statistically significant effects for Sex for the Order 1 level of Order ($p < .01$), the experimental level of EV Lecture ($p < .05$), and the Order 1, experimental level of Order X EV Lecture ($p < .001$). Order was significant ($p < .05$) only for experimental females. EV Lecture was significant ($p < .05$) for Order 1 males and Order 2 females. Finally, Sex X Order was significant ($p < .05$) for experimental Ss. As already noted, the error term for these effects was extremely large. It was the mean square for Subjects in Table 2 and contains over half the OPT_1 variance. Consequently, the significant simple effects implied extremely large differences between the OPT_1 means (see Table 4).

The role played by Order in this interaction seems largely

Table 4.

OPT_1 Means for Sex X Order X EV Lecture

	Males		Females	
	Experimental	Control	Experimental	Control
Order 1	.480967	.322252	.221515	.267088
Order 2	.406208	.379567	.416550	.264941

attributable to the EV ranges of each order's initial decision problems. In spite of randomization procedures, these ranges differed considerably from one another (see Appendix C for decision problems). For Order 1 the differences between maximum and minimum EV values for the first five problems were 1, 3, 1, 6 and 4. For Order 2 the corresponding values were 5, 6, 20, 12, and 20. If experience with these initial problems had a lasting effect, there would be reason to expect the two orders to produce different OPT_1 means. This was significantly ($p < .05$) the case for experimental females. The difference between their OPT_1 means was .195. Only for control females was it clearly not the case. On the other hand, it could be argued that risk characteristics of initial problems for the two orders may have affected the OPT_1 values. For Order 1, the variances of the five initial problems were 9, 841, 289, 324, and 144. For Order 2, the corresponding variances were 289, 1156, 4, 729, and 4. If these variances were averaged or ranked and then averaged to control for the difference in variability of these variances, Order 2 EV choices had slightly larger variances than did Order 1. On this basis, then, using R_1 for Order 1 would imply encountering lower risk than using R_1 for Order 2. Following more directly from the HRUM, however, would be the question of whether or not the R_1 choices were also R_3 or were R_5. From this perspective, Order 1's initial problems were slightly higher risk. Two of the five R_1 choices were also R_5 while for Order 2 only one of the five was a high risk choice. It seems, however, that from either

perspective, the orders do not differ enough with respect to average variance or proportion of R_5 choices to account for the large differences found in Sex X Order X EV Lecture. The only risk characteristic that clearly differentiates between the orders is the variability of the variances--Order 2's being more variable than Order 1's. Arguing against explaining the Order effect with a risk interpretation was the additional fact that while EV Lecture did interact with EV Risk (see Table 3), Sex apparently did not. Whether or not the high EV choice was also low risk or was high risk did not differentially affect the two sexes (see Table 2). If anything, the data suggested that females were more low risk takers, more moderate risk takers, and more high risk takers than were the males (see Table 9). Only the difference for moderate risk, however, was significant (for R_3, low risk, $t = 1.0748$; for R_4, moderate risk, $t = 2.6280$, $p < .02$; for R_5, high risk, $t = 1.5082$).

For the other variables, Sex and EV Lecture, involved in the Sex X Order X EV Lecture interaction, the simple effects analyses and Table 4 indicate the extent to which it is impossible to make any simple statements about these variables. Both the largest and smallest sex differences were for experimental _S_s--the largest being for Order 1 (.26), the smallest for Order 2 (.01). For control _S_s, on the other hand, the larger difference was for Order 2 (.11) while the smaller was for Order 1 (.05). For EV Lecture, differences between control and experimental OPT_1 means divided the Sex X Order matrix into two groups. The larger differences for EV Lecture were found for

Table 5.

OPT_1 Means for Sex X EV Lecture X EV Range

	Males		Females	
	Small Range	Large Range	Small Range	Large Range
Control	.253814	.448005	.161784	.370245
Experimental	.333356	.553819	.263608	.374457

Order 1 males (.16) and Order 2 females (.15). The smaller differences groups were Order 2 males (.03) and Order 1 females (.05).

The Sex X EV Lecture X EV Range interaction may be attributed to the OPT_1 mean for experimental females responding to problems having a large EV range (see Table 5). If this mean, .3744, were increased by about .10, the original and simple interaction effects (EV Lecture X EV Range for females and Sex X EV Range for experimental Ss) would disappear. If the other groups (males and control females) were used as a standard, then, experimental females did not increase their probability of using R_1 for large EV range problems as much as was to be expected.

Table 6 presents means for the nonsignificant Sex X EV Lecture X EV Risk X EV Range interaction. It was previously asserted that the EV Lecture X EV Risk X EV Range effect (Table 3) could not be understood apart from the Sex X EV Lecture X EV Range effect. Considering only EV Lecture X EV Risk X EV Range seemed to indicate that control Ss responding to problems having a high EV but low risk alternative produced data deviant from the rest. Yet, the Sex X EV Lecture X EV Range interaction showed that combination of experimental males with experimental females distorted the data of Table 3. Similarly, the Sex X EV Lecture X EV Range interaction was itself distorted by combining data for problems for which R_1 and R_3 specified the same choice with data for R_1 and R_5 choice problems. Together the Sex X EV Lecture X EV Range and EV Lecture X EV Risk X EV Range effects pointed to the means of Table 6.

Table 6.

OPT_1 Means for Sex X EV Lecture X EV Risk X EV Range

	R_3 Choice		R_5 Choice	
	Small Range	Large Range	Small Range	Large Range
Control				
Males	.288691	.522396	.218937	.373641
Females	.186467	.443595	.137101	.296894
Experimental				
Males	.381669	.606214	.285044	.501424
Females	.307392	.427012	.219824	.321902

These means only moderately qualify the assertion that experimental females had a comparatively small difference between OPT_1 means for large and small EV ranges. The qualification is that experimental females did not differ very much from control Ss responding to problems for which R_1 and R_5 specified the same choice.

Table 6 also indicates that the EV Lecture X EV Risk X EV Range interaction could not be attributed solely to OPT_1 means for control Ss responding to R_3 choice problems. The difference between those means for large and small EV range was very close to the difference for experimental males. On the other hand, control Ss differed from experimental Ss in having smaller large range, small range differences for R_5 choice problems than they had for R_3 choice problems.

By combining the analysis of variance results, the first two hypotheses may be reevaluated. The third hypothesis is simply incorrect.

The first hypothesis, that instruction in the use of R_1 would increase OPT_1, was significant at the .05 level. This earlier conclusion must now be qualified. Converting the results of the simple effects analyses to one tailed tests, the hypothesis was tenable for only two of the four Sex X Order groups. These were Order 1 males ($t = 2.1414$, $df = 94$, $p < .025$) and Order 2 females ($t = 2.0455$, $df = 94$, $p < .025$).

With respect to the within variables, the Sex X EV Lecture X EV Range interaction ($p < .05$) indicated that for females the lecture on EV did not lead to an appreciable increase in OPT_1 for large EV range (mean for control = .370, for experimental = .374). It did, however, lead to the expected increase for small EV range (control mean = .162, experimental mean = .264). In other words, while experimental females used EV more than controls when it made little difference whether they used it nor not, they did not use it more frequently than controls when more frequent use would have made a difference. Similarly, analysis of the EV Lecture X EV Risk X EV Range interaction ($p < .001$) indicated that control Ss were less affected by EV range when R_1 and R_5 specified the same choice than they were when R_1 and R_3 choices were the same.

The second hypothesis, that large EV range leads to larger OPT_1 than small EV range, was strongly and consistently supported. Both as a main effect in the analysis of variance and as a simple effect in fourteen cases (see Appendix F), EV Range was significant beyond the .001 level. Moreover, for the analysis of variance in Table 3, EV Range , as a main effect, accounted for more (14.5 percent) of the variance in OPT_1 scores than did any other source of variance except Subjects. The point biserial coefficient between EV Range and OPT_1 means was .814. The corresponding biserial coefficient was 1.02--exceeding unity probably because the OPT_1 means were relatively bimodal (McNemar, 1962). This effect was qualified only by the finding that control Ss were less influenced by it when responding to R_5 problems than when responding to R_3 problems.

Peripheral Results

The magnitude of the EV Range effect was sufficiently large to suggest searching for an alternative to the hypothesis that people simply use R_1 more when the EV range is large than when it is small. This search was further motivated by the fact that in the present study, the difference between the large EV range and the small EV range was not very big. For the large EV range, the mean difference between maximum EV and minimum EV for each decision problem was 8.72 points and the standard deviation was 4.2569. According to the system for paying Ss, this corresponded to 43.6 cents. For the small EV range, the mean difference was 2.4 points (or 12 cents) with a standard deviation of 1.5232.

A reasonable alternative to the hypothesis that EV range determined the probability with which R_1 was used is the hypothesis that computational errors caused the significant EV Range effect. One would expect that the closer the EV's are to one another, the more likely it would be for an R_1 user to select the wrong wager as the one having the largest EV. In other words, it is not the probability of using R_1 that varies with EV range but rather it is the probability of erroneously using R_1 that varies with EV range.

Calculating ability would also be a likely explanation for the sex differences found in OPT_1 scores. Common stereotypes suggest that women using R_1 would do so with more errors than would men. Hence, their lower OPT_1 scores might indicate erroneous application of the

Table 7.

Summary Table for Test of Averaging Competence.

Source	df	Mean Square	F
Sex	1	.001073	.0391
Error	94	.027457	
EV Range	1	.170289	17.4548***
EV Range X Sex	1	.000009	.0009
Error	94	.009756	

*** implies $p < .001$.

rule rather than a genuine sex difference in the probability of using R_1.

In order to test these two hypotheses, an analysis of variance was performed on the data obtained by having Ss calculate averages for all fifty distinct decision problems used in the experiment. For both ethical and mechanical reasons, however, these data were not complete. Eight of the 104 Ss were eliminated from the analysis because they supplied information about only 15 or fewer of the 50 problems. For the remaining 96 Ss, the mean number of decision problems for which information was complete was 42.8125. For small EV range the mean was 21.375 and the standard deviation was 4.9671. For large EV range, the mean was 21.438 and the standard deviation was 4.9177.

The results of the analysis are in Table 7. The dependent variable was the proportion of times the wager having largest EV was not the wager for which S calculated the highest average. It is first clear from Table 8 that the hypothesized sex difference is untenable. The stereotype of greater arithmetic competence for males was simply not supported in these calculations. For small EV range the male mean was .1948. For females it was .1991. For large EV range the male mean was .1348 and the female mean was .1400.

On the other hand, there was a significant difference between error scores for large and small EV range. On the basis of the analysis of variance, therefore, the calculation hypothesis for EV Range remains tenable. In order to determine whether this effect would predict the difference for OPT_1 scores, two more hypotheses were stated. First, if M_S = the mean proportion of errors for small

Table 8.

Correlations Between Optimality Indices

Condition	Indices OPT_1, OPT_2	OPT_1, OPT'_3	OPT_2, OPT'_3
Male, Order 1, Control	.962	-.445	-.413
Male, Order 2, Control	.970	-.717	-.703
Male, Order 1, Experimental	.969	-.860	-.807
Male, Order 2, Experimental	.974	-.761	-.720
Female, Order 1, Control	.973	-.696	-.745
Female, Order 2, Control	.963	-.500	-.399
Female, Order 1, Experimental	.964	-.278	-.280
Female, Order 2, Experimental	.975	-.393	-.396

range, M_L = the mean proportion for large range, m_S = the mean of OPT_1 scores for small range, and m_L = the mean OPT_1 for large range, then $m_S/(1 - M_S) = m_L(1 - M_L)$. That is, adjusting the OPT_1 means for the amount of calculation error should remove the mean difference if the calculation hypothesis is correct. This correction for small EV range increased the OPT_1 mean from .2531 to .3152. The change for large EV range was from .4366 to .5063. The difference between the values, therefore, increased (from .1835 to .1911) rather than decreased.

A second hypothesis following from the calculation hypothesis is that there is a significant correlation between error differences and OPT_1 differences. That is, the larger the effect of EV range on calculation errors, the larger should be the difference between an _S_'s large EV range OPT_1 and small range OPT_1. For males this correlation was .147 which was in the appropriate direction but not statistically significant. The female correlation was .190. Since low reliability of error differences may have led to the low correlations, these coefficients were computed. For males the reliability estimate was .520. It was .768 for females. Adjusting the obtained correlations for these low reliabilities resulted in an increase of the male correlation from .147 to .204. With 46 degrees of freedom, this was still not statistically significant. For females the corrected correlation was .217 which also did not reach the .05 level of significance.

It can be concluded, therefore, that the calculation hypothesis does not explain the OPT_1 results either with respect to the sex difference or the EV range difference. Between men and women, no calculational difference was found. Between large and small EV range, a difference was found but it was not in statistically significant correspondence with the OPT_1 results.

The second peripheral result focuses on the question of whether or not OPT_1 can be equated with the general notion of optimality. Two other measures, OPT_2 and OPT'_3, were obtained for each S. OPT_2 was the average of the EV's of each S's choices. OPT'_3 was OPT_2 divided by the amount of time S spent making his decisions. These values, expressed in monetary units, are reported in Appendix G. The measures were then correlated with one another. The results are in Table 8. The general pattern of the relationships is clear. OPT_1 and OPT_2 measure much the same kind of optimality. OPT'_3, however, is negatively related to the other two measures.

Interview Data

Appendix G presents a comparison of interview results with means of estimated probabilities with which Ss used each of the rules in R_A (i.e., R_1 = EV, R_2 = hunch, R_3 = low risk, R_4 = moderate risk, R_5 = high risk, R_6 = random selection). For each of the six rules, interview responses were coded into one of four categories. If S reported using a rule frequently, it was coded as a major decision rule

Table 9.

Mean Parameter Estimates for Decision Rules and Interview Categories.

	Interview Category: Unused	Inferred	Minor	Major	Total
R_1, Males					
Mean	.2096	.4183	.2875	.4847	.3992
S. D.	.0946	.1308	.0865	.1784	.1968
n	14	3	2	33	52
R_1, Females					
Mean	.1890	.2550	.2270	.3746	.2945
S. D.	.0935	.0000	.0000	.2084	.1896
n	21	1	1	29	52
R_2, Males					
Mean	.1149	.1050	----	.1500	.1161
S. D.	.0237	.0000	----	.0040	.0239
n	49	1	0	2	52
R_2, Females					
Mean	.1246	.0880	----	.1135	.1235
S. D.	.0280	.0000	----	.0125	.0277
n	49	1	0	2	52

Table 9 (cont'd.)

	Interview Category				Total
	Unused	Inferred	Minor	Major	
R_3, Males					
Mean	.0902	.1615	.2533	.2932	.2094
S. D.	.0814	.1325	.1752	.1364	.1563
n	19	2	6	25	52
R_3, Females					
Mean	.1710	.3648	.1830	.3075	.2441
S. D.	.1380	.1766	.0800	.1675	.1695
n	24	4	2	22	52
R_4, Males					
Mean	.0641	.0850	.0640	.2730	.0766
S. D.	.0469	.0000	.0050	.1320	.0729
n	46	1	2	3	52
R_4, Females					
Mean	.0984	----	.1195	.2031	.1153
S. D.	.0576	----	.0445	.1010	.0758
n	42	0	2	8	52
R_5, Males					
Mean	.0340	.1290	.0467	.0771	.0442
S. D.	.0396	.0000	.0300	.0538	.0450
n	36	1	7	8	52

Table 9 (cont'd.)

	Interview Category				Total
	Unused	Inferred	Minor	Major	
R_5, Females					
Mean	.0507	.1510	.0643	.0780	.0594
S. D.	.0562	.0000	.0498	.0521	.0568
n	35	1	6	10	52
R_6, Males					
Mean	.1548	----	.1580	.1538	.1549
S. D.	.0147	----	.0057	.0143	.0142
n	44	0	3	5	52
R_6, Females					
Mean	.1637	.1640	.1600	.1596	.1628
S. D.	.0149	.0000	.0011	.0083	.0134
n	39	1	7	5	52

(indicated by +++ in the Appendix). If a rule was used occasionally or seldom, it was coded as a minor rule (indicated by ++). If the interviewer inferred that an inarticulate or confusing _S_ used a rule, the rule was coded as inferred (indicated by +). Finally, if _S_ gave no indication that he used a rule, it was coded as unused (indicated by no symbol). These data are summarized in Table 9.

The information obtained from the interviews has bearing on two questions. First, how well does the set R_A describe the variety of techniques _Ss_ reported using? Second, what is the correspondence between the numerical parameter estimates and the categories into which interview responses were coded? Relevant to the first question, there were two sets of _Ss_ who clearly did not choose their techniques exclusively from R_A. One of these sets included _Ss_ who used decision rules not at all included in R_A. The other group were _Ss_ who used rules similar but not equivalent to those in R_A.

In the first group (those using techniques markedly different from those in R_A), there were seven _Ss_ of whom five were female and all but one were in control groups. These _Ss_ (denoted by a three digit identification number, an M or F to indicate sex, a 1 or 2 to indicate order of presentation of decision problems, and a C or E to denote control or experimental group) were 109M1C, 108M1C, 130F2C, 128F1C, 156F1E, 141F1C, and 193F2C. While 109M1C was coded as using R_4 as a minor decision rule, he said he chose W_3 because of its skew rather than its moderate riskiness. He reported choosing W_3 if the payoffs were "lumped at the top" (i.e., negatively skewed).

The second and third Ss in this first list of deviants, 108M1C and 130F2C, reported using a very large number of minor decision rules. They could not describe these rules, however. The next two Ss, 128F1C and 156F1E, also claimed to use minor rules not contained in R_A. Unlike 108M1C and 130F2C, however, these Ss could describe their techniques. 128F1C would occasionally select a nonsense syllable having identical payoffs in two of the three cells corresponding to the syllable (see Appendix C, Problem 2 in which either ZEJ with payoffs 5, 5, and 25 or XIH with payoffs 63, 63, 37 exemplifies the kind of payoff matrix for which this technique is appropriate). She then used the EV rule R_1 on the two wagers containing these two payoffs. It made no difference whether the two identical payoffs were larger than or less than the third payoff. 156F1E was an experimental S who clearly did not realize the rationale behind the EV decision rule. She said she calculated the averages of the wagers but occasionally did not choose the wager with the largest EV. Rather she sometimes chose the wager with either minimal or intermediate EV.

Especially deviant were 141F1C and 193F2C. As her major rule, 141F1C chose that wager that most frequently had payoffs that were neither maximal nor minimal in the rows defined by the nonsense syllables (for example, in Problem 1 in Appendix C, this rule would lead to choosing A since for all syllables A's payoffs are in between those of B and C). The other particularly deviant S, 193F2C, used two major techniques not in R_A. One implied choosing wagers having the largest number of even integer payoffs because even numbers were her lucky numbers. She also used a distorted EV rule that involved discarding the smallest payoff in each wager, then using EV on the remaining payoffs.

In the second deviant group (S who used techniques similar but not identical to those in R_A), there were three Ss, all of whom were female. 123F2E added the digits in the tens column for each wager, then chose the wager having the maximum tens column sum. If this process did not lead to a choice, she used the standard EV. 189F2C used a similar technique. Unlike 123F2E, however, if the tens column sums had no maximum, she chose randomly. Finally, 176F1C rounded all payoffs to units of five, then used EV on the rounded payoffs.

Combining the two groups of deviants, the total number of these Ss was ten, of whom eight were female and eight were control Ss. The set R_A, therefore, corresponded to interview results for at most 90.4 percent of the Ss. A lower limit on this percent was obtained by assuming that all inferred rules were incorrectly inferred to be in R_A. This increased the number of deviant Ss to a total of twenty, of whom twelve were female and thirteen were control Ss. This lower limit was equal to 80.8 percent of the Ss.

The second question raised about the interview data was the amount of correspondence between numerical parameter estimates and verbal responses. To answer this question for the dependent variable, point biserial correlations were computed. The variables for these correlations were interview category (major *vs.* unused) and the mean of the eight OPT_1's estimated for each S. This included all but five males and two females for whom R_1 was coded as either a minor rule or inferred (see Table 2). For males, $r_{pb} = .623$ ($t = 5.34$, $df = 45$, $p < .001$); for females, $r_{pb} = .475$ ($t = 3.74$, $df = 48$, $p < .001$). The corresponding biserial coefficients were .820 for males and .600 for females.

For the remaining five rules, only R_3 and, to a less extent, R_4 show a reasonable correspondence between $\overline{P}(R_i)$, the mean estimated probability with which R_i was used, and interview categories. Again letting the major rule and unused rule categories define a dichotomy, $r_{pb} = .655$ ($t = 5.62$, $df = 42$, $p < .001$) for 44 of the 52 males. For 46 females, $r_{pb} = .407$ ($t = 2.96$, $df = 44$, $p < .01$). The biserial coefficients were .826 for males and .511 for females.

R_4 was unused by 46 males and 42 females. The resultant disproportionality placed such severe restrictions on the values of the point biserials (Nunnally, 1967) that they were not computed. A crude indication that $\overline{P}(R_4)$ is somewhat related to the interview categories is in Table 2. For males the mean of the three values for which R_4 was categorized a major rule was .273. The mean for the unused rule category was .064. For females the corresponding values were .203 and .098, respectively.

For the three rules R_2, R_5, and R_6, the numerical estimates and verbal response classifications do not agree with one another. The means in Table 2 clearly indicate this for R_2 and R_6. For R_5 there does seem to be a relationship but the absolute magnitude of $\overline{P}(R_5)$ for the major rule category was much smaller than its maximum possible value of .555. For this major rule category, the male mean of $\overline{P}(R_5)$ was only .077. For females it was .078.

There are at least two conclusions to be drawn from the preceding comparison of interviews with parameter estimates. First, with three qualifications, the evidence indicated that R_A described the decision rules of eighty to ninety percent of the _Ss_. The first qualification of this statement is that the validation procedure assumed interview results to be a valid index of the actual decision process. It is inconceivable, however, that this assumption was completely correct. Second, while ten to twenty percent of the _Ss_ used rules not in R_A, these same _Ss_ also used rules that were included in R_A. On this basis, the eighty to ninety percent range was a conservative index of R_A's adequacy. On the other hand, the evidence did not indicate that each _S_ used the entire set R_A (see Appendix G, showing 43 different patterns of interview codings). Many _Ss_ may have chosen their rules from proper subsets of R_A. On this basis, then, the eighty to ninety percent range may be a gross exaggeration.

Second, it may be concluded that there was a reasonably large correspondence between interview results and the mean numerical estimates of OPT_1. The obtained point biserial coefficients of .623 for males and .475 for females are more meaningful when the possible values of these coefficients are considered. They are not limited by -1.0 and +1.0. Rather the r_{pb} for males has .92 as an absolute upper limit (which would occur only if the OPT_1 means formed a dichotomy). If the mean OPT_1's were normally distributed, the r_{pb} would be limited to about .75. For females, the same limits would be .99 and a number between .75 and .80.

Finally, it is of some interest to note that the data in Appendix G suggest the possibility of stating a stronger HRUM for wagers than the one presented in this study. This stronger HRUM would have only two rules, R_1 and R_3, in R_A. Eliminating R_2, R_4, R_5, and R_6 would be justified both by the relatively small $\overline{P}(R_i)$ values found for these rules and also by considering the effect of the iterative procedure for parameter estimation that was suggested in the Introduction. $\overline{P}(R_1)$ was the largest $\overline{P}(R_i)$ for 61.5 S̲s̲ (the .5 being a tie $\overline{P}(R_1) = \overline{P}(R_3)$). $\overline{P}(R_3)$ was largest for 36.5 S̲s̲; $\overline{P}(R_4)$ was largest for 5 S̲s̲; and $\overline{P}(R_5)$ was largest for only one S̲. $\overline{P}(R_2)$ and $\overline{P}(R_6)$ were never maximal but this can be attributed to the parameter estimation procedure. In other words if one were to attend only to the $\overline{P}(R_i)$ values in Appendix G and one were to assume that only major decision rules were important, one would be tempted to define the elements of R_A as only R_1 and R_3. This would be further supported by using the iterative procedure. This would increase the values of $\overline{P}(R_1)$ and $\overline{P}(R_3)$ for all cases in which they presently exceed 1/6. This stronger model was not defined in the present experiment because $\overline{P}(R_2)$ and $\overline{P}(R_6)$ were artifactually small and the elements of R_A were based on interviews rather than postparameter estimation theorizing.

There is also an apparent tendency for the distribution of $\overline{P}(R_i)$ values for the Ss to fall into two categories. In one category there is a clearly maximal value--thereby resulting in a peaked distribution. The other category includes relatively flat distributions.

DISCUSSION

Two questions arise from the preceding presentation of model and data. First, what were the immediate effects of the experimental lecture on optimality of choice behavior? Second, how is the HRUM for wagers to be revised in order to be consistent with the findings of this study?

From obtained relationships between OPT_1, OPT_2, and OPT'_3, it is initially clear that the answer to the first question depends on which concepts of optimality are used. Empirically, two concepts were found. One, including OPT_1 and OPT_2, was interpreted either as the probability of using the EV decision rule or as the average EV characterizing the decision maker's choices. The other concept, OPT'_3 = OPT_2 divided by the time involved in the decision making, correlated negatively with the first concept. Consequently, unless one assumes that effort increases the value of a choice, any statement to the effect that a variable increased optimality of one kind is probably also a statement that the variable decreased optimality of the other kind. In a sense, therefore, to answer the question of how the experimental lecture affected optimality necessitates taking a stand on one side or the other of Alice's Looking Glass. In one world, OPT_1 corresponds to excellence while in the other, $1 - OPT_1$ measures excellence.

While this bifurcation of optimality is presently unavoidable, a study somewhat similar to the present may eliminate the difficulty. The future study would entail presenting the same sets of wagers but changing the nature of the decision problem. Instead of having _S_ choose a wager, he would be offered the set R_A of decision rules from which to choose. Once he chooses a rule, the computer would do the rest. If, for example, he were to choose R_1, the computer would calculate the EV's of the three wagers and pick the wager with the largest EV. Then a new Becker _et al_. problem would be presented, _S_ would again make a choice from R_A, and so on. The hypothesized result of this revision would be to eliminate the effort involved in applying a rule, thereby leading to an equivalence of OPT_2 and OPT_3. This study also has the advantage of converting _S_'s assumed covert choice from R_A into an overt process with concomitant illumination of the black box.

For the present, however, we are left with two antithetical kinds of optimality. Consequently, in restricting optimality to OPT_1 for answering the question of how the experimental lecture affected the excellence of choice behavior, we simultaneously reach the opposite conclusions for $1 - OPT_1$ which when averaged across the within variables, correlated positively with OPT'_3. Yet, even with this qualification, the therapy (_i.e._, experimental lecture) did not consistently improve the quality of choice behavior. At least four variables, Sex, Order of Presentation, EV Risk, and EV Range, interacted with the therapeutic variable, EV Lecture. For the between variables, EV Lecture was significant only for Order 1 males and Order 2 females. If the orders be interpreted as smaller (Order 1) or larger (Order 2) EV ranges for initial problems in the sequence, this effect indicated that for males, the experimental group had a larger mean than the control group when use or nonuse of R_1 made little difference in the EV of early choices. For females, on the other hand, the experimental group had a larger mean when use or nonuse of R_1 made a great deal of difference in early choice EV's. EV Lecture made little difference for Order 1 females and Order 2 males.

For the within variables EV Risk and EV Range, an EV Risk X EV Range interaction was found for control Ss but not for experimental. This arose because experimental Ss reacted to large and small EV range in the same manner for both levels of EV Risk. Control Ss, on the other hand, were less affected by EV range when the R_1 choice was high risk than when it was low risk. When it was low risk, their behavior was similar to that of experimental males. When it was high risk, their behavior was much closer to that of experimental females. As main effects, experimental females were significantly affected by EV range but less than were experimental males.

Apart from the effect of therapy, the probability of using the EV rule was influenced by personality variables or individual differences (including sex) and characteristics of the decision problems

themselves. More specifically, OPT_1 scores (a) showed large individual differences, (b) except for Order 2, experimental Ss, were larger for males than for females, (c) were consistently greater for large EV range than for small range, and (d) were consistently greater for problems having low risk R_1 choices than for problems having high risk R_1 choices.

Finally, using the Hippocratic principle that therapy is alright if it does no harm to the patient, the prescriptive generalization following from the present findings is that instruction in the use of EV was detrimental (but not significantly) only to females encountering, as their initial decision problems, cases for which the decision rule was not very useful (i.e., Order 1). If, on the other hand, optimality be defined as $1 - OPT_1$, then the conclusions must be reversed, with only Order 1 females being at all benefited by the lecture. Moreover, this beneficial effect for Order 1 females was not significantly large.

A second question arising from this study is how to reformulate the HRUM for wagers to be consistent with the findings. While it is primarily the effect of EV Risk that indicated the necessity for such a reformulation, it is somewhat intriguing to attempt to cope also with the other findings. EV Risk alone could be quickly resolved by adding rules to R_A or redefining the rules in R_A so that risk and EV would not be the unique foci of the corresponding rules.

To attempt to approximate more of the results, however, it is necessary to involve ourselves in more extensive repairwork than simply

patching the holes created by the EV Risk, EV Risk X EV Range and EV Lecture X EV Risk X EV Range effects. The first note to be sounded is that the data have indicated an interplay of decision rules and aspects of alternatives. For experimental Ss the probability of using R_1 was an additive function of EV Range and EV Risk. For control Ss, additivity was not found. It seems probable that a major difference between an experimental S and a control S is that the former begins the choice process with a clearly articulated decision rule while the latter probably does not. By guess, the effect of having such a rule is to drastically alter the decision maker's relationship with the first encountered decision problems. If he accepted the rule as a reasonable technique to consider using, he would first attend to the rule specified aspect of the alternatives. Decision makers without such an articulated rule, on the other hand, would perhaps probabilistically sample an aspect that would differentiate and permit comparative evaluation of the alternatives. For testing this conjecture, one could assume that the probabilistic aspect choosing would be according to a Luce model and that the choice process prior to rule articulation would be according to Tversky's EBA. Then the consequent hypothesis would be that while the EBA might be tenable for control data, it would not be tenable for Ss given a reasonable, relevant and articulated rule (by lecture or some other appropriate technique). These experimental Ss would first choose the aspect specified by the rule (except for rules like random choice in which case they would ignore all aspects).

Until the study is performed, assume that articulation of a rule focuses the decision maker on the relevant aspect of offered

alternatives. This does not imply, however, that choice will be according to the particular rule. Rather we may guess a certain amount of doubt on the S's part. Instead of saying, "That's it--that's the rule for me," he says, "I'll try it for awhile and if it makes sense, I'll use it but if it makes little sense, I won't use it." In other words, the decision maker tests the reasonability of the rule for some time period or number of choices. This need not imply that he actually selects the rule specified choice--only that he attends to the relevant aspect. Similar to Tversky's assertion that aspects characterizing all alternatives in the choice set do not affect the choice process, the decision maker testing an articulated rule probably has some threshold answer to the question of how much the rule separates its set of specified choices from the alternatives. If the separation exceeds threshold, he makes his choice by the rule. If it does not exceed threshold, he does something else. Finally, if the end result of this testing of the rule's usefulness is favorable to the rule, then he increases the probability of actually using it. In HRUM terms, the rule's utility is a function of the results of testing. If the rule is found useless, on the other hand, its utility decreases and perhaps concomitantly the utilities of aspects related to the rule also decrease.

It is also to be hypothesized that articulation of a rule results not only from instruction but also from decision making experience. In line with the Gurney et al. (1970) model of order effects in wage distribution booklets, it seems reasonable to assert that when attributes of alternatives are selected, they constitute the basis for defining less probabilistic decision rules. In line with more recent thought, it may be the case that as a decision maker gains experience in a type of decision problem, he moves from probabilistic "elimination by aspects" processes to more regulated or "programmed" (Simon, 1960) processes. By restricting his attention to a small set of techniques, he can ignore most aspects characterizing alternatives even though these aspects may be relatively highly evaluated. For example, many people may go through elaborate processes for selecting an automobile. Others no doubt automatically buy Fords -- not because the aspect "Ford," relative to "Chevrolet" or "Plymouth," has such high utility but because avoiding consideration of the alternatives has such high utility. Having gone through the "dissonance arousing" choice problem enough times to not want to go through it again, they settle on a few rules that let them bypass the problem of considering what they are buying. Perhaps they buy what they bought last time or they let the wife and kids decide. Experience may be as effective as instruction for articulating a decision rule.

On the basis of the preceding, the fairly complex Sex X Order X EV Lecture interaction (See Table 4) may be explained. It has been suggested that articulated rules are tested for some time period or number of choices. To explain the Sex X Order X EV Lecture effect, let us first assume that this trial period was shorter for females than for males. The women were less willing to continue calculating averages for small EV range problems at the beginning of the decision sequence. Let us also assume that males would be more likely to attend to EV's of wagers than would females. These findings that indicated males to be more likely to use R_1 seemed to indicate that EV is less alien to males than to females. The assumptions also rest on the stereotype that women find arithmetic less interesting than men do.

Noting that this explanation is largely conjecture, the means of Table 4 can be relatively well explained. There were five groups for whom R_1 was articulated early in the decision process. These are the four experimental groups and Order 2, control males who by encountering the very high EV ranges of early problems, defined the rule as a result of experience. There is very little difference between these Ss and the two Order 2, experimental groups. That their mean is at all smaller reflects a greater effect of the lecture for articulating R_1. Even with the large EV ranges of Order 2, some of the control males probably did not attend to this aspect. That the largest difference between any pair of groups was for Order 1, experimental Ss resulted from the sex difference in the trial period given R_1. The males continued attending to EV's for a long enough time to overcome the small EV ranges of the first few problems. The effect of their persistence was to encounter an upward trend in EV ranges. This made R_1 seem to become better and better through time. The Order 1, experimental females, on the other hand, gave R_1 a much shorter trial period. Deciding very early that it was a useless technique, they rejected it and decreased the probability of using R_1 below that of any other group. The other three groups, control females and Order 1, control males reflect the assumed sex difference and probably indicate the overall mean OPT_1's to be expected when R_1 is not articulated early in the choice process.

The results of the Sex X EV Lecture X EV Range and EV Lecture X EV Risk X EV Range interactions largely speak for themselves. But again somewhat loosely conjecturing, the fact that experimental females were less affected by EV Range may indicate less flexibility in using R_1. This would be reasonable if, as earlier hypothesized, R_1 was more alien to females than to males. Feeling less comfortable with the technique, they were less able to modify it to suit characteristics of the decision problems.

That neither experimental males nor experimental females showed an interactive effect of EV Risk X EV Range while control _S_s did suggests a property of the utility function of articulated rules. When a clearly defined rule is evaluated, its utility is some value V plus the sum of utilities of salient aspects characterizing the alternative specified by the rule. This conjecture certainly requires further testing with other aspects before it can be given much credence.

For the control _S_s, not having been told about R_1 either reduced the effect of EV range for high risk R_1 choices or increased the effect for low risk R_1 choices or perhaps both. The reason for this is certainly not apparent. It may have resulted from less attention being paid the actual EV values when the R_1 choice was characterized by the relatively aversive high risk property. Conversely, more attention may have been paid EV value for low risk R_1 choices but this seems unreasonable. It would suggest that when a choice is characterized by the relatively attractive aspect of low risk, the decision maker becomes more concerned about the extent to which it is an attractive R_1 choice.

In conclusion, the reformulation of the HRUM for wagers has largely involved distinguishing articulated from inarticulated rules. The latter type may very well be described by a model such as the EBA. When rules have been articulated, however, it has been hypothesized that they radically alter choice behavior. For example, selection of an aspect may be disconnected from direct implications for choice and, instead, be used to test the worthwhileness of the rule.

As a first approximation, the following very simple model may be stated. It is based on an earlier model of order effects (Gurney _et al._, 1970) and on the EBA. To date it is restricted to sets of alternatives for which any aspect characterizes only one alternative and each rule specifies only one alternative so that rules and aspects as alternatives to be chosen are formally equivalent. In this case, the EBA is equivalent to a Luce model defined on aspects or alternatives or rules. The present model assumes that in at least one case choice deviates from the EBA. This occurs when, for an articulated rule R_i chosen at time t, the utility of the aspect specified by R_i, $u(X_i)$, increases from t to t + 1 while utilities of aspects specified by other rules do not increase. In this case, the probability of choosing R_1 at t + 1 is equal to 1.0 - 1 (where k is infinitesimal). In all other cases, the EBA holds.

To test this _Ss_ will have R_1, R_3, R_4, and R_5 defined for them.

They will then choose one of these four rules during each trial of a study similar to that presented earlier for controlling for the influence of effort on optimality. By varying the decision problems at t + 1 in a manner determined by the S's choice at t, the predicted algebraic rule selections can be tested against transitions hypothesized to be probabilistic.

APPENDICES

APPENDIX A

Arithmetic and Gambling Test

1. What is 13 + 6?
2. What is the average of 7 and 3?
3. What is 24 divided by 3?
4. What is 12 + 11 + 7?
5. What is 7 + 5 + 14 + 10?
6. If a flipped coin lands with its "heads" side up, John wins. If its "tails" side comes up, he loses. The coin is tossed 100 times. How many times do you expect him to win?
7. When the "heads" side of a flipped coin comes up, John wins five dollars. When its "tails" side comes up, John wins one dollar. The coin is tossed 100 times. At the end of these 100 tosses, how many dollars do you expect John to have won?
8. What is the average of 14, 11, and 11?
9. What is 45 divided by 5?
10. When the "heads" side of a flipped coin comes up, John wins five dollars. When its "tails" side comes up, he wins one dollar. What is John's average winning (in dollars) for each time the coin is flipped?
11. What is the average of 7, 4, 5, and 4?
12. If a rolled six sided cube comes up on its 1st or 2nd side, John wins $6. If sides 3 or 4 come up, he wins $5. If sides 5 or 6 come

up, he wins \$4. On the average, how many dollars will John win each time he plays?

13. What is the average of 2, 2, 11, 15, and 15?

14. What is 5 + 9 + 7 + 8 + 7?

15. What is 6 + 3 + 7 + 15 + 12 + 3?

16. What is the average of 11, 5, 4, 11, 13, and 10?

17. What is 63 divided by 7?

18. What is 132 divided by 11?

19. What is 182 divided by 13?

20. When one of the numbers 1, 2, 3, or 4 is picked at random, John wins \$2 when 1 is picked. He wins \$4 if 2 is picked, \$6 if 3 is picked, and \$8 if 4 is picked. How much (in dollars) will John's average winnings be each time he plays?

APPENDIX B

Instructions for Decision Problems

In this part of the experiment, you will gamble with the computer. Each slide projected on the screen will offer you three wagers or betting schemes. They are called A, B, and C. You must select one of these wagers. Indicate your choice by pressing the A, the B, or the C button on the three button response box.

After you have made your choice, the computer will randomly select one of four nonsense syllables. These syllables are ZEJ, ZOJ, XEH, and XIH. You then win the number of points indicated by both your choice of A, B, or C and the computer's choice of ZEJ, ZOJ, XEH, or XIH.

An example (see below) is on the screen right now. If you were to select wager B, you could win either 8 points, 4 points, 6 points, or 8 points. If the computer selected ZEJ, you would win 8 points because this is the contents of the intersection of wager B and row ZEJ. If the computer picked ZOJ, you would win 4 points. If it selected XEH, you would win 6 points. Finally, if it picked XIH, you would win 8 points.

Unlike most gambling games, you will not be told how much you win until the end of the experiment. At that time, the computer will randomly select one game that you have played. It will then tell you how many points you won for that specific game. For this specific game, you will receive 5¢ for each point you win. This will be your payment for helping us in this study. Since you can win anywhere from 1 to 100 points, you can win anywhere from 5¢ (if you win only 1 point) to $5.00 (if you win 100 points).

Let's summarize these instructions.

First, you must select one of three wagers, A, B, or C. These wagers correspond to columns of numbers.

Second, indicate your choice by pressing either the A, the B, or the C button on the three button box.

Third, without telling you the result, the computer will randomly select a nonsense syllable. These syllables correspond to rows of numbers.

Fourth, you win the number of points contained in both the column (or wager) you have chosen and the row (or syllable) chosen by the computer.

Fifth, you must remember two facts.

Fact 1: One of your winnings will determine the amount of money you receive for participating in this experiment. Since neither you nor the experimenter know which winning will be selected, you must realize that any one of the games you play may be the one selected. Since the amount of money can be as much as $5 or as little as 5¢, it is reasonable to take every game seriously.

Fact 2: The computer is unbiased. This machine is equally likely to select any one of the nonsense syllables. It has no preferences. Also, it is equally likely to select any game as the payoff game. It is indifferent to how much money you win. Finally, just as you are free to pick A in one game and B in another and C in another or to pick nothing but A's for all games or, indeed, to pick any one of the three wagers for any specific game, so the computer pays no attention to its previously selected nonsense syllables. For each game, any syllable is as likely to be selected as any other.

To indicate that you understand these instructions, choose the wager (in the example being shown) with which you could never win 6 points.

	A	B	C
ZEJ	4	8	9
ZOJ	1	4	2
XEH	7	6	6
XIH	1	8	6

Figure 3.

Example Decision Problem

APPENDIX C

Decision Problems[1]

1. XIH,XEH,ZEJ,ZOJ; 73,79,73,79; 79,61,89,73; 61,89,61,89.

2. ZEJ,XEH,XIH,ZOJ; 5,37,63,25; 5,5,63,63; 25,25,37,37.

3. XIH,XEH,ZEJ,ZOJ; 63,11,63,11; 21,63,11,55; 55,21,55,21.

4. ZEJ,ZOJ,XEH,XIH; 4,22,2,22; 1,37,37,1; 1,37,22,4.

5. ZOJ,XIH,XEH,ZEJ; 21,57,21,57; 55,21,57,31; 31,31,55,55.

6. XEH,ZOJ,ZEJ,XIH; 56,10,81,3; 3,81,3,81; 10,10,56,56.

7. ZOJ,ZEJ,XIH,XEH; 89,58,54,15; 89,15,15,89; 58,58,54,54.

8. ZEJ,XIH,XEH,ZOJ; 71,25,71,25; 71,94,22,25; 22,94,94,22.

9. XEH,XIH,ZEJ,ZOJ; 29,7,29,7; 45,3,7,29; 45,45,3,3.

10. ZOJ,XEH,ZEJ,XIH; 49,25,49,25; 49,5,55,25; 55,55,5,5.

11. ZOJ,XEH,ZEJ,XIH; 13,31,3,33; 33,3,33,3; 31,13,31,13.

12. XEH,ZOJ,XIH,ZEJ; 55,31,31,55; 57,55,31,21; 21,57,57,21.

13. ZEJ,ZOJ,XEH,XIH; 43,53,43,53; 43,24,53,56; 24,56,24,56.

14. ZEJ,XEH,XIH,ZOJ; 63,5,5,63; 5,27,63,15; 15,15,27,27.

15. XEH,ZEJ,XIH,ZOJ; 3,3,81,81; 10,56,10,56; 56,3,81,10.

16. ZEJ,XEH,XIH,ZOJ; 66,66,14,14; 14,7,66,91; 91,91,7,7.

17. ZOJ,XEH,ZEJ,XIH; 25,49,49,25; 55,5,25,49; 5,55,55,5.

18. XIH,XEH,ZEJ,ZOJ; 17,45,17,45; 45,4,17,88; 4,88,4,88.

19. ZEJ,XIH,XEH,ZOJ; 73,73,79,79; 61,89,61,89; 61,79,73,89.

20. ZEJ,ZOJ,XIH,XEH; 4,100,100,4; 91,37,37,91; 91,37,4,100.

21. ZOJ,XEH,ZEJ,XIH; 34,38,34,38; 72,34,38,4; 4,72,4,72.

22. XEH,ZOJ,XIH,ZEJ; 56,32,32,56; 57,56,21,32; 57,21,21,57.

23. XIH,XEH,ZEJ,ZOJ; 3,81,81,3; 3,81,18,64; 64,64,18,18.

24. ZOJ,XIH,XEH,ZEJ; 72,72,4,4; 23,23,19,19; 19,4,72,23.

25. ZOJ,XIH,XEH,ZEJ; 28,82,28,82; 82,28,100,4; 4,100,4,100.

26. XIH,ZOJ,XEH,ZEJ; 5,19,43,55; 43,19,43,19; 55,5,55,5.

27. ZEJ,XIH,XEH,ZOJ; 73,73,1,1; 18,52,18,52; 18,1,52,73.

28. XEH,ZEJ,ZOJ,XIH; 4,22,4,22; 22,4,37,1; 1,37,1,37.

29. XIH,ZEJ,ZOJ,XEH; 74,28,28,74; 28,74,22,94; 94,22,22,94.

30. ZOJ,XEH,XIH,ZEJ; 78,61,89,84; 84,78,78,84; 89,89,61,61.

31. ZEJ,ZOJ,XIH,XEH; 43,17,17,43; 39,25,25,39; 25,43,39,17.

32. ZOJ,ZEJ,XIH,XEH; 58,54,58,54; 89,15,89,15; 54,58,15,89.

33. XIH,ZOJ,ZEJ,XEH; 88,4,58,30; 30,30,58,58; 88,88,4,4.

34. XEH,XIH,ZEJ,ZOJ; 39,39,25,25; 43,17,43,17; 39,25,43,17.

35. XEH,XIH,ZEJ,ZOJ; 24,56,56,24; 24,40,50,56; 40,50,40,50.

36. ZEJ,XIH,ZOJ,XEH; 91,91,7,7; 91,7,14,66; 14,66,14,66.

37. ZOJ,ZEJ,XIH,XEH; 36,36,64,64; 23,23,79,79; 64,79,36,23.

38. ZEJ,XEH,ZOJ,XIH; 9,43,73,1; 43,9,43,9; 73,1,1,73.

39. XEH,XIH,ZOJ,ZEJ; 11,29,11,29; 3,33,3,33; 33,11,29,3.

40. XEH,ZOJ,XIH,ZEJ; 32,66,66,32; 4,4,90,90; 4,66,90,32.

41. ZOJ,ZEJ,XEH,XIH; 38,46,14,2; 14,14,38,38; 2,2,46,46.

42. ZOJ,XIH,XEH,ZEJ; 20,20,36,36; 51,9,9,51; 9,36,51,20.

43. ZOJ,XIH,XEH,ZEJ; 11,63,63,11; 63,59,11,25; 25,59,59,25.

44. XEH,ZEJ,ZOJ,XIH; 78,90,44,4; 4,4,90,90; 44,78,44,78.

45. XIH,ZOJ,XEH,ZEJ; 23,23,79,79; 29,57,29,57; 57,79,23,29.

46. ZOJ,XIH,XEH,ZEJ; 75,15,15,75; 7,7,77,77; 75,7,15,77.

47. ZOJ,XIH,XEH,ZEJ; 8,26,37,1; 37,37,1,1; 26,26,8,8.

48. ZEJ,XEH,XIH,ZOJ; 18,42,42,18; 46,18,2,42; 2,2,46,46.

49. ZOJ,XEH,ZEJ,XIH; 78,84,78,84; 84,78,61,89; 61,89,89,61.

50. ZEJ,ZOJ,XIH,XEH; 29,11,29,11; 33,3,3,33; 29,11,33,3.

51. XEH,ZEJ,XIH,ZOJ; 27,41,27,41; 43,27,41,17; 17,43,17,43.

52. ZOJ,ZEJ,XEH,XIH; 14,14,38,38; 2,46,14,38; 46,46,2,2.

53. ZEJ,ZOJ,XEH,XIH; 64,64,36,36; 23,23,79,79; 36,64,79,23.

54. ZOJ,ZEJ,XIH,XEH; 46,46,2,2; 42,18,42,18; 2,18,46,42.

55. XEH,ZEJ,XIH,ZOJ; 71,71,25,25; 22,25,94,71; 94,94,22,22.

56. ZOJ,XEH,XIH,ZEJ; 17,41,27,43; 43,43,17,17; 41,41,27,27.

57. ZOJ,ZEJ,XEH,XIH; 51,9,14,30; 51,9,51,9; 30,14,30,14.

58. XIH,XEH,ZEJ,ZOJ; 32,66,90,4; 90,4,4,90; 66,32,66,32.

59. ZOJ,XIH,ZEJ,XEH; 11,11,63,63; 21,55,21,55; 63,21,11,55.

60. ZOJ,XIH,ZEJ,XEH; 30,58,88,4; 30,30,58,58; 88,4,4,88.

61. XIH,ZOJ,XEH,ZEJ; 15,15,75,75; 15,75,7,77; 77,7,77,7.

62. XIH,ZOJ,XEH,ZEJ; 31,31,13,13; 3,33,3,33; 33,31,13,3.

63. XEH,XIH,ZOJ,ZEJ; 3,3,45,45; 7,7,29,29; 45,29,7,3.

64. ZOJ,ZEJ,XIH,XEH; 8,26,26,8; 26,37,1,8; 1,1,37,37.

65. ZOJ,XIH,ZEJ,XEH; 22,74,22,74; 22,74,7,91; 7,91,7,91.

66. ZOJ,ZEJ,XEH,XIH; 34,34,38,38; 72,72,4,4; 4,38,72,34.

67. XIH,ZEJ,ZOJ,XEH; 56,32,56,32; 21,56,32,57; 57,21,57,21.

68. XEH,ZOJ,ZEJ,XIH; 30,62,62,30; 59,30,45,62; 45,45,59,59.

69. XIH,XEH,ZOJ,ZEJ; 44,78,90,4; 44,78,78,44; 4,4,90,90.

70. XIH,ZEJ,ZOJ,XEH; 19,43,43,19; 55,55,5,5; 19,5,55,43.

71. XIH,XEH,ZEJ,ZOJ; 24,56,43,53; 56,24,56,24; 53,43,53,43.

72. XIH,ZEJ,ZOJ.XEH, 36,20,36,20; 9,51,51,9; 9,20,36,51.

73. ZOJ,XIH,ZEJ,XEH; 14,30,14,30; 30,9,51,14; 9,51,9,51.

74. XEH,ZOJ,ZEJ,XIH; 42,56,42,56; 42,30,56,62; 62,30,30,62.

75. XIH,ZOJ,XEH,ZEJ; 18,18,72,72; 17,85,85,17; 85,17,72,18.

76. ZEJ,XIH,ZOJ,XEH; 19,23,23,19; 72,4,23,19; 72,4,72,4.

77. XIH,ZOJ,XEH,ZEJ; 9,1,43,73; 43,9,9,43; 1,1,73,73.

78. XIH,ZOJ,XEH,ZEJ; 12,3,45,34; 45,45,3,3; 34,12,34,12.

79. ZOJ,XEH,XIH,ZEJ; 13,7,77,73; 7,77,7,77; 13,73,73,13.

80. XEH,ZOJ,XIH,ZEJ; 100,100,4,4; 82,4,100,28; 82,82,28,28.

81. XIH,ZEJ,ZOJ,XEH; 45,45,17,17; 88,88,4,4; 4,17,88,45.

82. ZOJ,XEH,ZEJ,XIH; 79,29,57,23; 29,57,57,29; 79,23,23,79.

83. XIH,XEH,ZEJ,ZOJ; 56,56,24,24; 50,50,40,40; 40,24,50,56.

84. XEH,XIH,ZOJ,ZEJ; 73,1,52,18; 73,73,1,1; 18,52,18,52.

85. XIH,XEH,ZEJ,ZOJ; 12,12,34,34; 45,3,3,45; 12,3,45,34.

86. ZEJ,XEH,ZOJ,XIH; 74,22,28,94; 28,74,74,28; 22,94,94,22.

87. ZOJ,XEH,ZEJ,XIH; 7,74,91,22; 22,22,74,74; 91,7,7,91.

88. ZOJ,XEH,ZEJ,XIH; 25,37,5,63; 37,25,37,25; 63,5,63,5.

89. ZEJ,ZOJ,XEH,XIH; 77,77,7,7; 13,7,73,77; 73,73,13,13.

90. XEH,ZOJ,ZEJ,XIH; 17,85,85,17; 73,19,17,85; 19,73,73,19.

91. ZOJ,XEH,ZEJ,XIH; 63,5,63,5; 27,27,15,15; 5,27,15,63.

92. ZEJ,ZOJ,XIH,XEH; 64,18,64,18; 18,64,3,81; 81,3,81,3.

93. XEH,XIH,ZOJ,ZEJ; 45,30,62,59; 45,59,45,59; 30,30,62,62.

94. XEH,XIH,ZOJ,ZEJ; 30,56,42,62; 62,30,62,30; 56,42,56,42.

95. XEH,ZEJ,XIH,ZOJ; 85,19,73,17; 73,73,19,19; 85,85,17,17.

96. ZEJ,ZOJ,XIH,XEH; 70,70,74,74; 15,15,89,89; 89,74,15,70.

97. ZOJ,XEH,ZEJ,XIH; 91,37,91,37; 100,4,4,100; 4,91,100,37.

98. XIH,ZOJ,XEH,ZEJ; 74,70,15,89; 89,15,15,89; 74,70,74,70.

99. XEH,ZOJ,ZEJ,XIH; 72,17,18,85; 17,85,85,17; 18,72,18,72.

100. XEH,ZOJ,XIH,ZEJ; 59,63,25,11; 11,11,63,63; 25,25,59,59.

1. Note. These problems are according to Order 1 of presentation. Order 2 is the reverse (i.e., problem k in Order 1 was presented as the

101 - kth problem in Order 2). For a problem presented in this Appendix as the slide shown the S in the form of Figure 4 (see next page).

$$CVC_1, CVC_2, CVC_3, CVC_4; a_1, a_2, a_3, a_4; b_1, b_2, b_3, b_4; c_1, c_2, c_3, c_4,$$

	A	B	C
CVC_1	a_1	b_1	c_1
CVC_2	a_2	b_2	c_2
CVC_3	a_3	b_3	c_3
CVC_4	a_4	b_4	c_4

Figure 4.

General Form of Decision Problem Slides

APPENDIX D

OPT_1 Estimates

Subject	Estimates*							
101M2C	.152174	.288632	-.023279	.089618	.294036	.209847	.083618	.072892
102M2E	.522882	.487337	.690055	.763298	.419910	.254875	.663348	.537557
103M1C	.134076	.274913	.271211	.502509	.316413	.335994	.195969	.529330
104M2C	.492009	.040476	.632912	.289037	.078404	.053312	.181900	.151378
105F1E	.180921	.306758	.130365	.496789	.000000	.179186	.000000	.235294
106F2C	.023658	.000000	.191097	.065934	-.089428	.000000	.366299	-.025092
107M1C	.745443	.816065	.818489	.825798	.677499	.714027	.722172	.716742
108M1C	.141361	.185942	.522384	.331756	.246293	.162377	.335994	.279669
109M1C	.234832	.181271	.224936	.497127	.120362	.134158	.181900	.251805
110F2E	.031413	.039453	.120010	.109575	.058824	.078404	.019580	.030661
111F1E	.180921	.306758	.155920	.383919	.000000	.058824	.000000	.238009
112F1E	.159378	.146896	.136122	.171197	-.056680	.092555	.151296	.067107

Subject	Estimates*							
113F1E	.084419	.086738	.516003	.267909	-.067406	.164819	.419910	.338492
114M2E	.128290	.316627	.182997	.442651	.000000	.058824	.061538	.235294
115F1E	.102060	-.006798	.223105	.051167	-.050895	-.070120	.173318	-.053255
116M1E	.275465	.227522	.417771	.780343	.181900	.137228	.302262	.655204
117F2C	-.118079	.336349	.449057	.631430	.007928	.212562	.217991	.319128
118M1E	.243421	.244177	.353707	.410375	.000000	.137228	.120362	.274455
119F2E	.121368	.303324	.235628	.453798	.061538	.120362	.123077	.210202
120F2C	.015051	-.212947	.195757	.329549	-.044755	-.039459	.383381	.125931
121F2E	.748877	.694012	.818489	.768994	.660633	.596380	.722172	.554067
122M1C	-.099687	.104483	.234292	.530018	.013796	.134158	.249363	.299548
123F2E	.516877	.693326	.518804	.693930	.551352	.596380	.484163	.716742
124M2E	.811377	.559197	.755908	.763217	.719457	.495599	.722172	.716742
125M1C	.346167	.395285	.818490	.825798	.299548	.299548	.722172	.716742
126F2E	.811377	.687549	.818489	.759864	.719457	.672069	.722172	.716742
127F1E	.173974	-.063603	.205601	.398917	.260304	-.0143670	.005512	.333279
128F1C	-.078588	.278510	-.045547	.348105	.033376	.179186	-.076344	.123216
129M1C	.159459	.193228	.689974	.763217	.200909	.249363	.663348	.674784

Subject	Estimates*							
130F2C	.149215	.348671	.686621	.574625	.173318	.181900	.618675	.492529
131M1C	-.012531	.072322	.179563	.207876	.058824	.000000	.061538	.000000
132F1E	-.017767	.117545	.314539	.248482	-.025448	-.011652	.097984	.109065
133M2C	-.169878	.268478	.439811	.417162	.011354	.154093	.257234	.196051
134M1C	-.044878	.476701	.427071	.384349	-.022378	.134513	.120362	.095270
135M1C	.183868	.128640	.232113	.314985	-.041958	.075689	-.083916	.238009
136F2C	.214245	.362255	.690055	.589409	.257234	.179186	.484163	.476018
137M1C	.379742	.750131	.761684	.759864	.319128	.593665	.604525	.596380
138M1E	.332448	.628565	.818489	.759864	.332924	.478733	.722172	.615961
139F2C	.211990	.348348	.573616	.763217	.288251	.251805	.593228	.557137
140F1E	.031775	.233404	-.013557	.483825	-.125874	.0364460	.047825	.268670
141F1C	.102964	.167552	.343583	.439985	-.109364	.120362	.243439	.238009
142F2C	.111553	.247205	.640857	.731563	.120362	.179186	.123077	.355656
143M1E	.268198	.296403	.761684	.777018	.265518	.212917	.722172	.596380
144M1E	.447996	.274390	.637342	.766245	.151023	.358371	.215632	.612891
145F1E	.106898	.355956	.345159	.408344	-.041958	.176471	.304977	.154093
146M1E	.691145	.750131	.689974	.768994	.674429	.714027	.638256	.716742

Subject	Estimates*							
147M1C	.117934	.240824	.178742	.368355	.000000	.137228	-.045028	.058824
148F1E	.613575	.750131	.693326	.825798	.459070	.714027	.680214	.716742
149M2C	.322915	.278845	.755908	.703060	.139942	.196050	.599095	.596380
150F1E	-.158811	.152859	.040195	.390125	.100699	.109065	.069467	.173674
151M2E	.358553	.355956	.636346	.707232	.238009	.196051	.542986	.655204
152M2C	.156900	.233821	.205118	.496789	.081119	.196051	.221061	.355656
153F1C	.206960	.027859	.561540	.467518	.061538	.092200	.383381	.355656
154F2E	.190790	.244258	.130365	.385508	.000000	.058824	.000000	.058824
155F2E	.027092	.244874	.247704	.254688	.047172	.212973	.271658	.441989
156F1E	.523745	.449273	.690055	.129607	.433705	.512464	.601810	.223915
157F2C	-.201005	.286452	.460976	.531524	.036446	.137228	.263019	.417195
158F1E	.330780	.088568	.442351	.314242	.335638	.008639	.414042	.509394
159M1C	.190710	.467154	.216237	.637811	.033376	.274455	.215276	.358371
160M1E	.371776	.152309	.336808	.275249	.156453	.145511	.338708	.212562
161F2E	.400079	.404612	.442757	.458254	.238009	.192981	.299548	.450571
162M2C	.451013	.634999	.818489	.825798	.484163	.476018	.722172	.716742
163M1E	.685890	.518641	.693326	.825798	.599095	.361086	.542986	.655204

Subject	Estimates*							
164F1C	.141513	.031347	.232194	.238951	.016126	-.008582	.120362	.330209
165M2E	.119755	.283505	.639293	.604743	-.022378	.120362	.425339	.417195
166M1E	.620443	.526584	.761346	.759864	.436775	.414480	.599095	.716742
167M1E	-.030725	.194574	.146479	.371789	.097629	.117647	.293680	.117647
168F2E	.379742	.325500	.818489	.825798	.179186	.196051	.542986	.716742
169M1E	.758745	.816065	.818489	.825798	.596380	.714027	.722172	.716742
170M2C	.574192	.458808	.686621	.759864	.436420	.302046	.621390	.615961
171F2E	.420149	.355874	.502474	.773166	.179186	.196051	.363801	.476018
172M2C	.048303	.306758	.624121	.606563	.033376	.120362	.400247	.294118
173M1E	.651816	.816065	.818489	.763217	.537557	.714027	.722172	.716742
174M2E	.358553	.401302	.703357	.653941	.179186	.257590	.601810	.495599
175M2E	.171970	.240742	.298128	.549420	.075334	.179186	.061538	.355656
176F1C	.120314	.270739	.086495	.445704	.294036	.198550	.184399	.450571
177M1E	.166380	.228556	.646134	.537195	.078404	.327494	.481448	.372522
178M2E	.027591	.190062	.633989	.448114	.184399	.151296	.540271	.512109
179M2C	.259591	.065318	.262569	.657267	.114850	.033376	.237927	.377952
180F2C	.172305	.197965	.564974	.585597	.137228	.246293	.123077	.327494

Subject	Estimates*							
181F2C	.109470	.247205	.290951	.425497	.000000	.058824	.061538	.117647
182F1C	.559276	.816065	.689974	.768994	.599095	.652489	.615605	.671714
183F1C	.154610	.035521	.322592	.316932	-.045028	.036446	.181900	.355656
184M2C	.408667	.380176	.758936	.759864	.434060	.383520	.680214	.674784
185F2E	.376444	.618263	.577788	.693930	.330481	.512464	.495599	.557137
186F2E	.324264	.414968	.274994	.493274	.123077	.296833	.120362	.196051
187M2E	.300019	.509999	.139865	.825798	.288168	.196051	.495599	.716742
188M2E	.287101	.408089	.818489	.645150	.299548	.201481	.663348	.554422
189F2C	.193657	.414968	.528591	.599278	.274100	.251805	.417195	.476018
190F1C	-.025416	.270821	.564974	.653443	.120362	.271385	.201481	.238009
191F1C	-.073751	.139556	.332298	.385871	.061538	.120362	.000000	.294118
192M2C	.699192	.759260	.755989	.825798	.657919	.534842	.663348	.716742
193F2C	.417202	.045834	.001850	.103055	.117647	.008284	.000000	.002772
194F2C	.095679	.308397	.465068	.268396	.075689	.296833	.302262	.374882
195F1C	.128290	.129127	.182997	.145376	.000000	.000000	.061538	.000000
196M2E	.745443	.410028	.818489	.825798	.599095	.476018	.722172	.537557
197M2C	.279762	.265098	.289256	.540548	.094914	.218485	.039161	.411765

Subject	Estimates*							
198F2E	.748877	.624644	.818489	.759864	.660633	.714027	.660633	.613246
199M2C	.233553	.359390	.636346	.825798	.000000	.179186	.363801	.417195
200F1C	.171970	.306758	.240140	.368355	.061538	.120362	.061538	.117647
201F1C	.075427	.433709	.693326	.700311	.108983	.167889	.439490	.436775
202F1C	.460882	.518641	.818489	.825798	.302262	.355656	.722172	.716742
203M2E	.204724	-.170764	-.160586	-.028639	-.045028	.000000	.000000	.000000
204M2E	.354702	.306758	.818489	.825798	.179186	.179186	.722172	.716742

* The first four estimates correspond to problems for which the R_1 choice is the same as the R_3 choice while the last four correspond to problems for which the R_1 choice is the same as the R_5 choice. Estimates 1, 2, 5, and 6 are from problems having a small EV range and 3, 4, 7, and 8 are from problems having a large EV range. Estimate 1 is based on problems 1, 3, 5, 7, 12, 19, 25, 26, 31, 32, 34, 35, and 39; Estimate 2 is based on 40, 41, 50, 52, 58, 59, 70, 74, 79, 80, 83, 89, 94; Estimate 3 is based on 10, 11, 13, 17, 20, 22, 30, 43, 44, 46, 48, 49, 51; Estimate 4 is based on 54, 56, 61, 62, 67, 68, 69, 71, 93, 96, 97, 98, and 100; Estimate 5 is based on 2, 21, 23, 27, 29, 33, 37, 42, 47, 53, 60, 64; Estimate 6 is based on 65, 66, 72, 78, 84, 85, 86, 87, 88, 90, 92, and 95; Estimate 7 is based on 4, 6, 8, 9, 14, 15, 16, 18,

24, 28, 36, and 38; and Estimate 8 is based on 45, 55, 57, 63, 73, 75, 76, 77, 81, 82, 91, and 99 (See Appendix C for decision problems).

APPENDIX E

Table 10

Analysis of Variance Summary Table

Source	df	Mean Square	F
Sex (A)	1	2.281209	7.9857**
Order (B)	1	.400150	1.4008
EV Lecture (C)	1	1.103915	3.8644
A X B	1	.575117	2.0133
A X C	1	.081698	.2860
B X C	1	.055011	.1926
A X B X C	1	1.409389	4.9338*
Subjects (S)	96	.285660	
EV Risk (D)	1	2.125463	199.0507***
A X D	1	.003165	.2963
B X D	1	.005527	.5177
C X D	1	.001274	.1193
A X B X D	1	.000008	.0007
A X C X D	1	.000714	.0669
B X C X D	1	.000128	.0120
A X B X C X D	1	.011015	1.0316
D X S, error	96	.010678	
EV Range (E)	1	7.003135	185.1016***
A X E	1	.118177	3.1236

Table 10 (cont'd.)

Source	df	Mean Square	F
B X E	1	.064539	1.7058
C X E	1	.066070	1.7463
A X B X E	1	.013959	.3690
A X C X E	1	.199602	5.2757*
B X C X E	1	.007935	.2098
A X B X C X E	1	.029160	.7707
E X S, error	96	.037834	
Sequence Number (F)	1	.644134	28.0131***
A X F	1	.008765	.3812
B X F	1	.036360	1.5813
C X F	1	.038642	1.6805
A X B X F	1	.011961	.5202
A X C X F	1	.008403	.3654
B X C X F	1	.021439	.9324
A X B X C X F	1	.001091	.0474
F X S, error	96	.022994	
D X E	1	.132709	22.2703***
A X D X E	1	.002490	.4178
B X D X E	1	.000919	.1544
C X D X E	1	.073858	12.3944***
A X B X D X E	1	.015158	2.5437
A X C X D X E	1	.000166	.0279
B X C X D X E	1	.000954	.1601
A X B X C X D X E	1	.000114	.0191
D X E X S, error	96	.005959	

Table 10 (cont'd.)

Source	df	Mean Square	F
D X F	1	.027481	3.7162
A X D X F	1	.000351	.0475
B X D X F	1	.002961	.4004
C X D X F	1	.010288	1.3912
A X B X D X F	1	.000005	.0007
A X C X D X F	1	.000124	.0168
B X C X D X F	1	.002016	.2726
A X B X C X D X F	1	.009682	1.3093
D X F X S, error	96	.007395	
E X F	1	.002177	.1269
A X E X F	1	.013531	.7887
B X E X F	1	.005915	.3448
C X E X F	1	.020157	1.1750
A X B X E X F	1	.013677	.7973
A X C X E X F	1	.000145	.0085
B X C X E X F	1	.007783	.4537
A X B X C X E X F	1	.009028	.5263
E X F X S, error	96	.017155	
D X E X F	1	.000283	.0549
A X D X E X F	1	.004139	.8029
B X D X E X F	1	.000329	.0638
C X D X E X F	1	.013031	2.5278
A X B X D X E X F	1	.000039	.0076
A X C X D X E X F	1	.000516	.1001
B X C X D X E X F	1	.001183	.2295

Table 10 (cont'd.)

Source	df	Mean Square	F
A X B X C X D X E X F	1	.000008	.0016
A X E X F X S, error	96	.005155	

APPENDIX F

Table 11.

Simple Effects Analysis for Appendix E[1].

Level	Source	df	Mean Square	F
A: Male	B	1	.007916	.0277
	C	1	.893318	3.1272
	E	1	4.470344	118.1568***
	B X C	1	.453532	1.5876
	C X E	1	.017946	.4743
A: Female	B	1	.967350	3.3864
	C	1	.292311	1.0233
	E	1	2.650898	70.0665***
	B X C	1	1.010901	3.5388
	C X E	1	.247732	6.5479*
B: Order 1	A	1	2.573630	9.0094**
	C	1	.332842	1.1652
	A X C	1	1.085074	3.7985
B: Order 2	A	1	.282755	.9898
	C	1	.826080	2.8918
	A X C	1	.406041	1.4214
C: Experimental	A	1	1.613477	5.6482*
	B	1	.376122	1.3167
	D	1	1.009524	94.5424***
	E	1	2.853960	75.4337***
	A X B	1	1.892509	6.6250*
	A X E	1	.312395	8.2570**

Table 11 (cont'd.)

Level	Source	df	Mean Square	F
	D X E	1	.004295	.7208
C: Control	A	1	.749561	2.6240
	B	1	.079132	.2770
	D	1	1.117305	104.6362***
	E	1	4.215345	111.4168***
	A X B	1	.091930	.3218
	A X E	1	.005294	.1399
	D X E	1	.202178	33.9282***
D: R_3 Choice	C	1	.513743	1.7984
	E	1	4.531962	119.7854***
	C X E	1	.139826	3.6958
D: R_5 Choice	C	1	.591415	2.0703
	E	1	2.603885	68.8240***
	C X E	1	.000104	.0027
E: Small Range	A	1	.680493	2.3822
	C	1	.855217	2.9938
	D	1	.597988	56.0018***
	A X C	1	.012908	.0452
	C X D	1	.027523	2.5775
E: Large Range	A	1	1.718933	6.0174*
	C	1	.314737	1.1018
	D	1	1.660185	155.4772***
	A X C	1	.268397	.9396
	C X D	1	.047611	4.4588*
A X B: Male, Order 1	C	1	1.309866	4.5855*

Table 11 (cont'd.)

Level	Source	df	Mean Square	F
A X B: Male, Order 2	C	1	.036903	.1292
A X B: Female, Order 1	C	1	.107994	.3780
A X B: Female, Order 2	C	1	1.195220	4.1841*
A X C: Male, Experimental	B	1	.290616	1.0173
	E	1	2.527426	66.8030***
A X C: Male, Control	B	1	.170826	.5980
	E	1	1.960947	51.8303***
A X C: Female, Experimental	B	1	1.977991	6.9243*
	E	1	.638938	16.8879***
A X C: Female, Control	B	1	.000239	.0008
	E	1	2.259689	59.7264***
A X E: Male, Small	C	1	.328550	1.1517
A X E: Male, Large	C	1	.382223	2.0382
A X E: Female, Small	C	1	.539142	1.8874
A X E: Female, Large	C	1	.000924	.0032
B X C: Order 1, Experimental	A	1	3.500399	12.2537***
B X C: Order 1, Control	A	1	.158240	.5539
B X C: Order 2, Experimental	A	1	.005562	.0195
B X C: Order 2, Control	A	1	.683234	2.3918
C X D: Experimental, R_3	E	1	1.539872	40.7007***
C X D: Experimental, R_5	E	1	1.318385	34.8466***
C X D: Control, R_3	E	1	3.131908	82.7803***
C X D: Control, R_5	E	1	1.285604	33.9801***

Table 11 (cont'd.)

Level	Source	df	Mean Square	F
C X E: Experimental, Large	A	1	1.672878	5.8562*
	D	1	.572755	53.6388***
C X E: Experimental, Small	A	1	.252970	.8856
	D	1	.441047	41.3043***
C X E: Control, Large	A	1	.314423	1.1007
	D	1	1.135039	106.2970***
C X E: Control, Small	A	1	.440415	1.5418
	D	1	.184465	17.2752***
D X E: R_3, Small	C	1	.594793	2.0822
D X E: R_3, Large	C	1	.058762	.2057
D X E: R_5, Small	C	1	.287947	1.0080
D X E: R_5, Large	C	1	.303586	1.0628

[1] Note. Error terms and corresponding degrees of freedom for testing the effect of a source are the same as those given in Appendix G.

APPENDIX G

Table 12.

Summary Data for Male, Order 1, Control Ss[1]

S	OPT_2	OPT'_3	$\overline{P}(R_1)$	$\overline{P}(R_2)$	$\overline{P}(R_3)$	$\overline{P}(R_4)$	$\overline{P}(R_5)$	$\overline{P}(R_6)$
103	$2.07	$20.73/hr.	.319	.162	.095	.162	.105+++	.157
107	$2.15	$3.80/hr.	.756+++	.104	.000	.000	.010	.129
108	$2.05	$6.85/hr.	.276+++	.164	.095	.116	.192+++	.157
109	$2.05	$5.85/hr.	.230+++	.092	.427+++	.059++	.023	.169
122	$2.06	$12.36/hr.	.184+++	.130	.339	.114	.064	.169
125	$2.14	$3.78/hr.	.555+++	.105	.142	.053	.000+++	.145
129	$2.12	$4.72/hr.	.449+	.133	.100	.063	.107	.148
131	$2.01	$8.05/hr.	.073+++	.082	.612	.052	.000	.180
134	$2.05	$8.22/hr.	.201++	.156	.204	.218+++	.052	.170
135	$2.02	$6.73/hr.	.134	.092	.510+++	.047	.042	.174
137	$2.12	$2.89/hr.	.598+++	.118	.044	.086	.010	.142
147	$2.02	$10.12/hr.	.136	.095	.497+++	.074	.022	.176
159	$2.06	$11.23/hr.	.302+++	.130	.202+++	.138	.064++	.163+++

Table 12 (cont'd.)

S	OPT_2	OPT'_3	$\overline{P}(R_1)$	$\overline{P}(R_2)$	$\overline{P}(R_3)$	$\overline{P}(R_4)$	$\overline{P}(R_5)$	$\overline{P}(R_6)$
Mean	$2.07	$8.10/hr.	.324	.120	.251	.091	.053	.160
S. D.	.045	44.606	.198	.027	.194	.055	.053	.014

Summary Data for Male, Order 2, Control Ss[1]

S	OPT_2	OPT'_3	$\overline{P}(R_1)$	$\overline{P}(R_2)$	$\overline{P}(R_3)$	$\overline{P}(R_4)$	$\overline{P}(R_5)$	$\overline{P}(R_6)$
101	$2.03	$20.32/hr.	.145	.162	.193++	.182	.149	.170+++
104	$2.06	$4.58/hr.	.245+	.129	.232+++	.163	.063++	.168
133	$2.05	$12.32/hr.	.198	.124	.294+	.085+	.129+	.170
149	$2.12	$6.70/hr.	.452+++	.129	.100++	.145	.021++	.154
152	$2.05	$7.69/hr.	.245	.102	.401+++	.046	.042	.165
162	$2.15	$3.30/hr.	.643+++	.100	.089++	.030	.000	.139
170	$2.12	$7.08/hr.	.559+++	.116	.073+++	.034	.076	.142+++
172	$2.08	$8.33/hr.	.308	.106	.312+++	.069	.043+++	.162++
179	$2.07	$15.55/hr.	.254	.119	.287++	.081	.095+++	.164
184	$2.13	$6.10/hr.	.561+	.121	.070	.024	.085+++	.140
192	$2.14	$2.42/hr.	.704+++	.104	.029	.029	.000	.134

Table 12 (cont'd.)

S	OPT_2	OPT'_3	$\overline{P}(R_1)$	$\overline{P}(R_2)$	$\overline{P}(R_3)$	$\overline{P}(R_4)$	$\overline{P}(R_5)$	$\overline{P}(R_6)$
197	$2.05	$11.19/hr.	.270	.123	.270+++	.069++	.106++	.162++
199	$2.09	$3.22/hr.	.382+++	.089	.304+++	.064	.000	.160
Mean	$2.09	$8.37/hr.	.382	.117	.204	.078	.062	.156
S. D.	.039	5.038	.176	.017	.114	.051	.048	.012

Summary Data for Male, Order 1, Experimental Ss[1]

S	OPT_2	OPT'_3	$\overline{P}(R_1)$	$\overline{P}(R_2)$	$\overline{P}(R_3)$	$\overline{P}(R_4)$	$\overline{P}(R_5)$	$\overline{P}(R_6)$
116	$2.10	$9.00/hr.	.374++	.097	.327+++	.034	.010	.157
118	$2.05	$8.77/hr.	.227+++	.093	.442+++	.040	.031++	.168
138	$2.14	$4.00/hr.	.588+++	.101	.120+++	.017	.032	.142
143	$2.13	$5.10/hr.	.489+++	.107	.171+++	.012	.075	.147
144	$2.10	$4.33/hr.	.437+++	.132	.099	.126	.054	.153
146	$2.14	$1.61/hr.	.706+++	.113	.000	.017	.032	.132+++
160	$2.06	$9.50/hr.	.250	.154+++	.170	.146+++	.118	.162+++
163	$2.13	$6.39/hr.	.613+++	.117	.029+	.099	.000	.142
166	$2.13	$3.55/hr.	.607+++	.111	.071+++	.059	.010	.141

Table 12 (cont'd.)

S	OPT_2	OPT'_3	$\overline{P}(R_1)$	$\overline{P}(R_2)$	$\overline{P}(R_3)$	$\overline{P}(R_4)$	$\overline{P}(R_5)$	$\overline{P}(R_6)$
167	$2.04	$7.21/hr.	.164	.105	.443+++	.063	.054+++	.171
169	$2.15	$2.48/hr.	.748+++	.103	.013	.006	.000	.130
173	$2.15	$2.74/hr.	.719+++	.104	.026	.018	.000	.132
177	$2.08	$6.56/hr.	.356	.129	.191+++	.123	.043+++	.158
Mean	$2.11	$5.48/hr.	.483	.113	.162	.058	.035	.149
S. D.	.038	2.530	.190	.016	.148	.047	.033	.013
Summary Data for Male, Order 2, Experimental Ss[1]								
102	$2.13	$3.65/hr.	.545+++	.105+	.154+++	.040	.010	.145
114	$2.03	$7.63/hr.	.182	.080	.550+++	.016	.000	.172
124	$2.15	$1.82/hr.	.694+++	.107	.031	.023	.010	.134
151	$2.11	$3.96/hr.	.463+++	.094	.270+++	.011	.010	.150
165	$2.08	$5.94/hr.	.327+++	.103	.313+++	.075	.021	.162
174	$2.10	$2.68/hr.	.459+++	.102	.239++	.028	.021++	.150++
175	$2.05	$2.46/hr.	.244+++	.091	.441+++	.045	.011	.167
178	$2.08	$1.98/hr.	.336+++	.146	.144	.102	.118	.155

Table 12 (cont'd.)

S	OPT_2	OPT'_3	$\overline{P}(R_1)$	$\overline{P}(R_2)$	$\overline{P}(R_3)$	$\overline{P}(R_4)$	$\overline{P}(R_5)$	$\overline{P}(R_6)$
187	$2.11	$2.34/hr.	.434+++	.146+++	.071	.114	.085	.150
188	$2.12	$3.35/hr.	.487+++	.104	.200+++	.040	.021	.149
196	$2.14	$2.85/hr.	.644+++	.112	.041	.064	.000	.139
203	$2.01	$10.03/hr.	-.026	.201	.167+++	.455+++	.011	.191
204	$2.14	$3.66/hr.	.515+++	.089	.242+++	.006	.000	.148
Mean	$2.10	$4.03/hr.	.408	.114	.220	.078	.024	.155
S. D.	.043	2.337	.188	.031	.145	.113	.034	.014
			Summary Data for Female, Order 1, Control Ss[1]					
128	$2.02	$8.68/hr.	.096+++	.140	.315	.144	.130	.175++
141	$2.04	$4.08/hr.	.196	.088+	.447+	.054	.043	.172
153	$2.07	$5.92/hr.	.271	.127	.234+++	.168+++	.032	.167
164	$2.03	$9.36/hr.	.139	.118	.376+++	.128	.065	.175
176	$2.07	$5.64/hr.	.255+	.172	.071+	.189	.151+	.161
182	$2.14	$3.57/hr.	.673+++	.116	.000	.052	.022	.135

Table 12 (cont'd.)

S	OPT_2	OPT_3	$\overline{P}(R_1)$	$\overline{P}(R_2)$	$\overline{P}(R_3)$	$\overline{P}(R_4)$	$\overline{P}(R_5)$	$\overline{P}(R_6)$
183	$2.05	$4.39/hr.	.171	.142	.263++	.218+++	.032++	.174
190	$2.08	$7.33/hr.	.290	.127	.216+++	.169	.032++	.166
191	$2.05	$7.24/hr.	.159	.149	.202+++	.310+++	.000	.180
195	$2.01	$12.07/hr.	.084	.065	.671+++	.000	.000	.180
200	$2.04	$10.18/hr.	.185	.084	.500+++	.059	.000++	.174
201	$2.10	$5.72/hr.	.386+++	.144	.072+++	.127	.117	.154
202	$2.14	$2.86/hr.	.593+++	.105	.086+++	.072	.000	.144
Mean	$2.06	$6.70/hr.	.269	.121	.266	.130	.048	.166
S. D.	.040	2.661	.175	.029	.187	.080	.050	.013

Summary Data for Female, Order 2, Control Ss[1]

S	OPT_2	OPT_3	$\overline{P}(R_1)$	$\overline{P}(R_2)$	$\overline{P}(R_3)$	$\overline{P}(R_4)$	$\overline{P}(R_5)$	$\overline{P}(R_6)$
106	$2.03	$7.17/hr.	.067+++	.205	.042	.388+++	.117	.181
117	$2.08	$6.56/hr.	.260+++	.157	.089	.230+++	.097	.167
120	$2.04	$15.30/hr.	.094	.167	.215	.202	.149+++	.174
130	$2.11	$7.03/hr.	.405+++	.119	.202	.067	.054	.153++

Table 12 (cont'd.)

S	OPT_2	OPT'_3	$\overline{P}(R_1)$	$\overline{P}(R_2)$	$\overline{P}(R_3)$	$\overline{P}(R_4)$	$\overline{P}(R_5)$	$\overline{P}(R_6)$
136	\$2.10	\$5.74/hr.	.409+++	.109	.229	.086	.011+++	.156
139	\$2.12	\$6.05/hr.	.449	.129	.124+++	.063	.086+++	.148++
142	\$2.08	\$4.45/hr.	.318	.102	.337+++	.080	.000	.163
157	\$2.06	\$7.73/hr.	.243+++	.122	.312+++	.115	.042	.166
180	\$2.07	\$5.92/hr.	.298	.132	.172+++	.156+++	.075++	.164
181	\$2.04	\$3.30/hr.	.168	.084	.488+++	.083	.000	.176
189	\$2.10	\$6.63/hr.	.396+++	.114	.228	.074	.033	.155+++
193	\$2.01	\$10.95/hr.	.089+++	.115	.453	.052	.117	.174+++
194	\$2.06	\$4.76/hr.	.274+++	.126+++	.269	.144	.022	.165
Mean	\$2.07	\$7.05/hr.	.267	.129	.243	.134	.062	.165
S. D.	.032	2.964	.125	.030	.125	.091	.047	.009

Summary Data for Female, Order 1, Experimental Ss[1]

S	OPT_2	OPT'_3	$\overline{P}(R_1)$	$\overline{P}(R_2)$	$\overline{P}(R_3)$	$\overline{P}(R_4)$	$\overline{P}(R_5)$	$\overline{P}(R_6)$
105	\$2.03	\$5.31/hr.	.195	.077	.539+	.018	.000	.171
111	\$2.03	\$8.71/hr.	.169	.087	.499+++	.070+++	.000	.175++

Table 12 (cont'd.)

$\underline{S}$	OPT_2	OPT'_3	$\overline{P}(R_1)$	$\overline{P}(R_2)$	$\overline{P}(R_3)$	$\overline{P}(R_4)$	$\overline{P}(R_5)$	$\overline{P}(R_6)$
112	$2.04	$1.77/hr.	.110+++	.169	.166	.220	.160	.173
113	$2.06	$17.70/hr.	.227++	.166	.123	.190	.130+++	.164+
115	$2.02	$12.10/hr.	.048	.126	.413	.085+++	.151++	.176
127	$2.05	$4.11/hr.	.162+++	.147	.244+++	.179	.096+++	.171
132	$2.03	$4.06/hr.	.107+++	.129	.342+++	.139	.108	.176
140	$2.02	$7.56/hr.	.111	.114	.402+	.081	.117+++	.174
145	$2.05	$5.35/hr.	.229+++	.101+++	.359+++	.109	.031	.171
148	$2.14	$2.47/hr.	.683+++	.119	.000	.033	.031	.133
150	$2.04	$4.89/hr.	.110+++	.151	.257	.179	.128+++	.174
156	$2.10	$3.93/hr.	.446+++	.131	.103++	.075++	.096++	.149+++
158	$2.08	$5.68/hr.	.305+++	.147	.145	.126	.118	.159+++
Mean	$2.05	$6.43/hr.	.223	.128	.276	.116	.090	.167
S. D.	.034	4.166	.166	.027	.158	.060	.053	.012

Table 12 (cont'd.)

Summary Data for Female, Order 2, Experimental Ss[1]

S	OPT_2	OPT'_3	$\overline{P}(R_1)$	$\overline{P}(R_2)$	$\overline{P}(R_3)$	$\overline{P}(R_4)$	$\overline{P}(R_5)$	$\overline{P}(R_6)$
110	$2.00	$4.29/hr.	.062	.120	.456	.139	.043+++	.180
119	$2.05	$3.41/hr.	.207	.084	.475+++	.042	.021	.171++
121	$2.14	$2.52/hr.	.698+++	.101	.031	.024	.011	.135
123	$2.12	$4.23/hr.	.597+++	.115	.072	.063	.011	.141
126	$2.15	$5.38/hr.	.740+++	.109	.000	.011	.010	.130
154	$2.02	$6.38/hr.	.138	.078	.580+++	.029	.000	.176
155	$2.05	$11.21/hr.	.218	.172	.089+++	.127	.235	.160
161	$2.09	$3.80/hr.	.363+++	.131	.130+++	.181	.033	.161+++
168	$2.13	$2.84/hr.	.502+++	.116	.118	.104	.010+++	.150
171	$2.10	$3.83/hr.	.413+++	.110	.226	.086	.010	.156++
185	$2.11	$1.89/hr.	.522+++	.137	.039	.064	.094	.142++
186	$2.06	$1.97/hr.	.284+++	.117	.256+++	.164++	.010+++	.168
198	$2.14	$1.57/hr.	.702+++	.110	.016	.029	.010	.133
Mean	$2.09	$4.10/hr.	.419	.115	.191	.082	.038	.154
S. D.	.047	2.449	.219	.023	.188	.054	.061	.016

Table 12 (cont'd.)

[1] Note. The $\overline{P}(R_i)$'s are means of estimated probabilities with which the rules were used. The +'s indicate codings of S's verbal descriptions of his decision rules: +++ = major decision rule, ++ = minor decision rule, and + = decision rule inferred by interviewer from ambiguous S statement.

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