A MATRIX ALLOCATION PROCEDURE FOR UNIVERSITY COST ACCOUNTING

Thesis for the Degree of Ph. D.

MICHIGAN STATE UNIVERSITY
ALBERT A. EWALD

1968

This is to certify that the

thesis entitled

A MATRIX ALLOCATION PROCEDURE FOR UNIVERSITY COST ACCOUNTING

presented by

ALBERT A. EWALD

has been accepted towards fulfillment
of the requirements for

Ph.D. degree in Business Administration

Gardner M. Jones
Major professor

Date Nov. 18, 1968

O-169

ABSTRACT

A MATRIX ALLOCATION PROCEDURE FOR
UNIVERSITY COST ACCOUNTING

By Albert A. Ewald

This study uses matrix algebra and random sampling procedures to estimate the costs to the University of South Carolina (USC) of providing to the students that education upon which the awarding of degrees is based. These costs are estimated for degrees by academic level for the eight specific teaching (product) units and for the composite bachelor's and master's degree on a university-wide basis.

In using matrix algebra this study devises a partitioned matrix containing four matrices, A, B, C and D. A allocates the outputs of nonteaching (service) units to other service units. B allocates the service units' outputs to the product units. C and D are the actual expenditures of the product and service units for the fiscal year 1967. Allocations are based on floor space, expenditures, student-load and/or opinions elicited from interviews. Allocations are not 100 per cent of a service unit's output where services are provided to other than the eight product units under study.

The partitioned matrix generates all the financial data used in this study. First the A matrix is inverted by the Gauss-Jordan procedure to become A^{-1}. To check computational validity, A is multiplied by A^{-1} to obtain the identity matrix I. X, the product of $A^{-1}C$, is multiplied by B to determine the value of service units' outputs transferred to product units--matrix F. D is added to F to obtain T, the total cost matrix of the product departments' outputs.

Based on interviews, the percentage of each product unit's output devoted to teaching is obtained. This is multiplied by the total cost of each unit to obtain its teaching (net) cost. The credit hours delivered, by course and section number, are retrieved from the USC data bank. Based on interviews, each course of each college is weighted and a total number of equivalent units of output is obtained for each product unit.

The net cost is divided by the equivalent unit output to obtain the equivalent unit cost for each product department. Multiplication of this by the weighting factor yields the cost per semester hour for each course.

A random sample of 10 per cent, or five transcripts if larger, was drawn from the population of each specific degree type awarded during the fiscal year 1967. The semester hours completed for each graduate are classified according to semester hour cost. Accumulation multiplication of the number of hours by their cost equals the cost of each degree in the sample. From this, the estimate of the average cost of the specific degree and the estimate of the sampling error are calculated. Because of variations in sample size and sample means of degrees by type and level, stratified sampling techniques are applied to obtain an average cost of a bachelor's degree and of a master's degree.

Based on cost variations (from an input viewpoint) administrators should consider whether future tuition charges should be based upon current practice or if they should be based on university average costs for the degrees, or on an estimated market value of the degrees to the recipients. Administrators of state-supported institutions should consider whether education should be provided to nonresidents below cost and what reciprocity arrangements should be made with other states.

The inherent constraints of the university environment impose limits on this work. The two major limitations are: capital charges for land rental and building depreciation could not be obtained, therefore estimated degree costs are net of these charges; and, professorial time could not be isolated within the actual expenditures of the product units. As a result, all semester hours of credit produced by a college are charged with the same professorial cost although some variation exists with almost absolute certainty.

A MATRIX ALLOCATION PROCEDURE FOR UNIVERSITY COST ACCOUNTING

By

Albert A. Ewald

A THESIS

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Accounting and Financial Administration

1968

ACKNOWLEDGMENTS

Michigan State University through its Department of Accounting and Financial Administration provided me with an educational opportunity for which I am sincerely grateful. Dr. James Don Edwards, Department Chairman, provided encouragement and support which exceeded any rightful expectation.

The members of my dissertation committee, Doctors Gardner Jones (chairman), John Barson and Floyd Windal provided a professional environment rich with constructive criticism and illuminating insights and imbued with a sense of scholarship. Their guidance can not be over-valued.

The Department of Accounting and Financial Administration provided me with both financial support and the opportunity to work for Dr. Jones on the research project "A Procedural & Cost Analysis Study of Media in Instructional Systems Development," of which this dissertation is an outgrowth.

I am indebted to all these who supported me and especially to my immediate family and to the larger family of my professors and fellow students. I shall try, but this debt can not be paid in full.

TABLE OF CONTENTS

Chapter | Page

LIST OF TABLES

LIST OF APPENDICES

CHAPTER I

INTRODUCTION

Purpose of This Study

This dissertation describes an attempt to devise and apply a large cost allocation matrix in a university, as a means of systematically spreading indirect (service department) costs to producing (teaching) units whose primary output is degrees. The interdependency of dozens of service departments and the joint usage of common facilities make any more simple cost allocation procedure inappropriate.

When costs allocated to a college or department by the matrix allocation are added to costs incurred directly by such a teaching unit, it becomes possible to identify the appropriate average unit cost of a degree produced by the teaching unit. The end-product of the costing technique attempted in this dissertation is a statement of the comparative costs of producing degrees from each of the several colleges within the university.

The general purpose of this dissertation is to add empirical data to the information bank available to those who must solve the important problem of meeting the exploding demand for higher education with limited resources.

Need for This Study

The size and complexity of educational institutions have increased and continue to increase. The quantity and variety of degrees, research projects and publications--the products of these institutions--have also proliferated. This growth process involves elements both of horizontal and vertical expansion. Horizontal expansion is represented by the addition of a new college to a university. Vertical expansion is typified by the addition of a graduate degree program to an undergraduate degree program in the same academic field.

Background

A modern university is comprised of many colleges which in turn are jointly supported by many service centers.* Services provided to the colleges by the service centers do not account for the sum total of the costs of operating the service centers; some of their effort is used in serving each other. In fact, some service centers may exist to serve only service centers. In university accounting, service center costs directly allocated to the colleges--the "product divisions"--are generally valued at some percentage of the budget of the contributing service center, if indeed

---

*Service center is defined as a non-teaching unit whose services, nevertheless, are essential in the operation of a teaching institution. Examples are payroll, building maintenance, registrar.

they are allocated at all. A more realistic valuation of a service center's aggregate contribution to the teaching units is its budget total less services rendered to other service centers, plus services received from other service centers.

Three Basic Problems

Three basic problems evolve from this situation. First, and most basic, is the problem of determining the total cost of any individual service center. The major obstacle in this problem is the large number of service centers involved in mutually supporting each other as well as teaching departments. Were but a handful of service centers involved, the allocation of these cross-servicing costs would be mathematically easy and reasonably accurate. In actuality, however, there are in the order of 150 identifiable cost centers in large universities[1] of which several dozens may be service centers. Thus, an individual center may give and get services from each of the other dozens of service centers, and furnish services to the colleges. The traditional method of "successive allocation" in these kinds of situations is mathematically incorrect in that it ignores reciprocal services provided. The matrix allocation

---

[1] U.S. Department of Health, Education, and Welfare Office of Education, A Procedural and Cost Analysis Study of Media in Instructional Systems Development, Part B, U.S. Department of Health, Education, and Welfare, 1964, p. 104.

procedure by accommodating all the cross-services leads to a presumably more "accurate" allocation.[2] But even using a matrix procedure, the allocation of one center's services to up to 150 other centers and colleges requires exacting attention to causative relationships, which may, in fact, be quite elusive. Resulting allocations may be in error. By analogy, such an allocation is like cutting a pie into slices of one-hundredths of an inch with a knife that is thicker than the intended slice.

The second basic problem is the disposition of the service center's cost once it has been allocated to the colleges. Should the cost be allocated to undergraduate and graduate programs and to research? Should (can) graduate programs be divided into master's and doctoral levels for the purpose of charging them for allocated fixed costs? Should fixed costs be charged against research and publications? In short, can the costs of products of a university be meaningfully differentiated?

The third basic problem is that the expansion of programs and degrees has required increased and more varied services from the support departments ("service centers") of the institution. This increasing need has resulted in some service departments doing the bulk of their work for other service departments. Indeed, it is speculated that some

---

[2] Ibid.

support departments may provide no direct service to degree-producing divisions but expend all their efforts in supporting other service departments. As a result, many contemporary budgets for complex organizations require further analysis before their value as a planning and controlling device can be fully obtained. A budget for a department is in many respects an extrapolation of the expenditures of one or more prior periods modified for expected changes in activities, student loads, or prices. All too frequently, however, this modification does not take into full account the effect changes have on related support departments, especially when the decision for change is initiated in the product department. The product department cannot usually foresee the "ripple" effects its decisions will have on service departments, which support it indirectly by servicing other service departments. This study may provide some insights about the effects of such decisions and may focus attention more clearly on the total cost of such decisions.

Budgeting and Pricing

Managerial attention is frequently applied to the differences between budgeted and actual expenditures. "Management by exception" draws attention to unfavorable variances, whereas favorable variances are frequently "avoided" by the spending unit's expenditure of all budgeted funds to "protect" next year's budget. In some instances, unfavorable variances are avoided by having some effort

performed by support departments. This study will tend to gather all costs and allocate them to the product departments, thus drawing managerial attention toward total product department costs and their sources.

Any manager likes favorable and dislikes unfavorable variances. But in a conglomerate organization such as a university there may be product lines (degree programs) with <u>seemingly</u> favorable operating characteristics because the allocation procedure for costs favors some products to the disadvantage of others. Product lines (such as fields of study) are being added to such organizations because of their social worth or "image-building" value. What is needed is a more pragmatic and empirical basis upon which to budget or commit the usages of assets. This proposed research will make some contribution toward this end.

The main product line of a university is degrees composed of credit hours of instruction. Different degrees have different compositions of credit hours. Credit hours in some academic fields are expected to have different costs than hours in other fields. This study will attempt to generalize about the costs of various "kinds" of credit hours and to summarize the total cost of providing the instruction for different degrees. It is anticipated that, for example, a degree in an exotic and highly specialized science will cost more than a degree in a routine and highly stylized field. This study may provide a pragmatic and

empirical basis upon which university administrators can budget and commit assets to such usages.

It is most probable that the values of degrees, from a human capital point of view, vary widely. The value of one degree may have a much higher market value to its recipient than another degree to its recipient. This study may provide a point of departure for further studies into the pricing policy for various degrees and for more quantitative comparisons of the actual costs of programs with their social worth. Ultimately, help for answers to these interrelated questions may emerge; for example, should the price of a degree be based on its value to the recipient? Should the price be based on cost? Should the costs be borne by the larger community because of the social benefits derived from such programs?

An Outline of the Solution Method for the Cost Allocation Problem

The problem studied in this work will be stated in a general way and then in more detail. The specific methodology will be given in the next section.

In general, the problem solution is first to allocate the indirect costs of a university to its product-producing divisions. Second, these allocated costs are added to direct costs of the producing divisions to arrive at a total cost for all products. Third, these products are classified into categories to enable costs to be assigned to

them. Fourth, the cost of the product--be it a bachelor's, master's, or doctor's degree--can be compared to its price, and pricing and other implications can be drawn. As more and more such studies of degree costs are undertaken, results of this and future researches may indicate a measure of central tendency for the costs of each of the various products of a university.

In the first step in the solution of this problem, the recipients of the outputs of service departments are divided into two classes--service departments and product departments. Second, the output of each of the service departments is assigned to the recipients on the basis of the most closely identifiable usage relationship. In the third step the matrix of simultaneous equations is constructed that states the allocation pattern; this system is then solved for its inverse.

Once the inverse is obtained, the total costs of the product departments are obtained by a straightforward matrix multiplication. The products of the producing departments are identified and measured. They fall into the categories of delivered credit hours of graduate and undergraduate instruction, research, publications and other such scholarly endeavor. For each department, its total product costs are assigned to the several products, including the several degree categories. The differences between the costs of different degrees are calculated. The difference between

the costs of and prices paid for various degrees are examined, conclusions are drawn and recommendations are made.

Scope, Methodology and General Limitations

Scope

This study will be limited to a one-year (fiscal 1967) operating cycle of the University of South Carolina. Only those of its accounting records which are in the public domain will be used. Consideration of the university's products will be limited to the undergraduate and graduate degrees. These degrees will be classified and examined on the basis of the college which awards them and by their academic level, but no differentiation will be made among various degrees at the same level from the same college. The scope of this work, therefore, will be limited to those colleges which provide accredited instruction. Colleges which provide non-accredited training, if dealt with at all, will be treated in an incidental way for purposes such as clarity or possible future studies.

A study of one school for one year may seem to impose debilitating geographic, curriculum and time limitations. This is not the case. Professors, equipment, supplies and other factors used by a university for producing instruction are taken from regional, national and international markets. Curricula for similar degrees in different schools are acceptably similar in composition. In this

study, the matrix model is so constructed that the budget data for any year can be inserted to find current or future degree costs.

This study, because it determines one cost for one "composite" degree at each academic level from each college, will result in the same cost for very different degrees. This imposes a limitation. For example, the College of Arts and Science awards approximately twenty various degrees at the undergraduate level. Thus a degree in English or Chemistry or Social Science or Physics will have the same "composite" degree cost. The probability that all these degrees would in fact have the same cost is not high. Therefore, this study will generate cost data which are valid for "composite" degrees rather than cost data for specific degrees.

Methodology

The methodologies to be employed in this research fall into three clusters: the cost allocation techniques, the computer techniques and the statistical sampling techniques. Each of these techniques has some associated limitations.

The allocation of the outputs of the service departments will be based on interviews with the most knowledgeable personal available, the floor space occupied, the student load and/or the amount of the department's expenditures. The allocation of the efforts of the product departments will be based on interviews as will be the weighting of the teaching effort for various levels and types of instruction. Such allocations are tenuous and dependent on memory. They are, however, the best available.

The computer techniques to be used in this study are the Gauss-Jordan Matrix Inversion Procedure and several programs that multiply matrices and retrieve data from the central data bank. Computer usage introduces limitations. The computer truncates rather than rounds numbers thus introducing some minor errors. The matrix inversion and multiplication programs cannot be verified, practically, by hand. This limitation will be attenuated by solving a similar problem, whose answer is known, with the programs employed. The matrix inversion technique will be tested further for validity by a matrix multiplication that should produce the Identity Matrix. This should overcome the technical limitations of computer usage. However, there is no practical way to verify the tapes in the central data bank of the university and this study relies on them for the number of semester hours of credit produced by the colleges. They are assumed to be reliable.

This research will employ a stratified sampling technique as a basis for constructing the statistics relevant to the costs of the various degrees. The sample size for each "composite" degree will be 10 per cent or five samples, whichever is larger, without replacement of the

number of degrees, by academic level, granted by each college and school. Samples will be randomly selected. Since the stratified sample will not be exactly 10 per cent for each degree, the larger than 10 per cent samples will have disproportionately more weight. This will skew the cost slightly toward the less popular degrees.

General Limitations

This research requires identification of the products of a university and the costs which become embodied in these outputs. Such research also requires that the efforts which go into a particular output be sorted out and allocated to that product. If the product is a bachelor's degree, for example, it is nearly impossible to segregate specifically the degree yielding efforts from those yielding research results. Yet, some effort to do so is necessary regardless of the inherent and unavoidable inaccuracy of the results. The effort will at least provide a point of departure toward greater future accuracy.

The problem of sorting out costs requires the splitting of joint costs and measurement of intangible, invisible joint products. Further, the university is the type of organization where personnel and accounting structures are not geared to cost accounting and where almost every resource is used jointly. This environment is one in which accounting systems and procedures are designed for assuring budgetary compliance and in which cost accounting is quite foreign to

employees, especially faculty. To secure the information necessary for costing is of major importance to the problem because if the purpose is misunderstood the effort will be vigorously resisted. Top-level administrative support is essential.

The first step toward a definitive product-costing system for a university is the allocation of the support activities which the non-teaching organizational units (indirect costs) provide to the organizational teaching units. No really satisfactory basis of allocation exists but there are two reasonably satisfactory approaches. The first is the traditional service center allocation approach which is efficient when there are a few cost centers and the second is the matrix allocation procedure which is more suitable to large allocation problems. This work will use the matrix allocation procedure for indirect costs to determine a basis for recognizing cost differences in different subject matter areas. Some pricing implications should follow.

Organization of the Thesis

This research comprises four additional chapters each of which begins with a section that explains the purpose and nature of the chapter. The next chapter, the second, devises and defines the system of matrices and a matrix address system. In the third chapter, the matrix system is identified and the calculations required to

compute total costs of the product divisions of USC are performed and tested for accuracy. The fourth chapter will determine the average costs of each of the types of undergraduate and graduate level degrees awarded by USC and the average cost of the average or composite undergraduate and graduate degree. The last chapter, the fifth, contains the summary, conclusions and recommendations.

The computer programs employed in this study will be placed in the appendices in the order of their appearance.

CHAPTER II

THE UNIVERSITY MATRIX SYSTEM

Purpose of This Chapter

The purpose of this Chapter is to devise and define a system of matrices into which selected accounts of the University of South Carolina will be arranged for matrix calculations. This system will include an input set and an output set of matrices. The input set will be arranged into a partitioned matrix which will generate the set of output matrices.

A further purpose is to design the address system by which a matrix element can be efficiently and precisely located in the partitioned matrix space.

In this chapter some inference will be made to some of the allocation values and actual expenditures data which are given in Chapter III.

Background

Matrix algebra is covered comprehensively in many standard textbooks and is not, per se, within present scope. From those many standard references this work relies

primarily on four. The first by Kemeny et al.[1] provides good coverage of vectors and matrices. Chris Theodore[2] provides similar treatment. Howell and Teichroew[3] provide a straightforward and clear treatment of matrix algebra which is reproduced by Mattessich.[4] This study follows the conventions of matrix algebra outlined in these and the other standard textbooks dealing with the subject.

Definitions

The matrices used in this and the following chapters are verbally and, where relevant, mathematically defined as:

A - the allocation of the service departments' outputs to other service departments

A^{-1} - the inverse of A

B - the allocation of the service departments' outputs to the product departments

C - actual (or budgeted) service department expenditures

---

[1] John G. Kemeny, Arthur Schleifer, Jr., J. Laurie Snell, Gerald L. Thompson, Chapter V "Vectors and Matrices," Finite Mathematics with Business Applications (Englewood Cliffs, N.J.: Prentice Hall, Inc., 1964), pp. 229-311.

[2] Chris A. Theodore, Chapter 9 "The Algebra of Linear Programming," Applied Mathematics: An Introduction (Homewood, Illinois: Richard D. Irwin, Inc.), pp. 332-369.

[3] James E. Howell and Daniel Teichroew, Mathematical Analysis for Business Decisions (Homewood, Illinois: Richard D. Irwin, Inc., 1963), pp. 234-246.

[4] Richard Mattessich, Accounting and Analytical Methods (Homewood, Illinois: Richard D. Irwin, Inc., 1964), pp. 466-478.

D - actual (or budgeted) product department expenditures

I - the identity matrix

F - the value of the outputs transferred from the service departments to the product departments, BX = F

P - the partitioned matrix containing A, B, C, and D. The input set

T - the total cost of the product departments' outputs. T = D + F

X - the unknowns $CA^{-1} = X$*

In this work the matrix elements are identified by the lower case of that letter which identifies the matrix to which they belong. For example, a_{ij} is an element of A, b_{ij} is an element of B, etc.

In this work, actual expenditures are used. They can be replaced readily by budgeted expenditures when actual values are not available, or when administrators wish to use estimated data to determine their behavioral effect on costs.

The Matrix System Analogue

The Analogue

The large size of the matrix system of USC impairs clear discussion and manual algebraic operations. Its

---

*X - the unknowns is derived from:

AX = C, thus X = C/A. But the matrix equivalent to the reciprocal of A (the inverse) is A^{-1}, thus

$X = CA^{-1}$.

system, however, can be simulated by a small analogue whose matrices are consistent with the definitions and notations given above.

Assume a situation in which three service departments cross-service each other and provide services to two product departments. Each service department provides 20 per cent of its output to each of the other service and product departments. Every department has $100 of actual (or budgeted) expenditures.

The Matrix Arrangement

The first objective of the matrix arrangement is to gather into the A, X and C matrices those accounts having a reciprocal relationship. Reciprocity is the mutual giving and getting of support among service departments. These equations state this relationship in the analogue:

$$\begin{aligned} x_1 &= 100 + 0.25x_2 + 0.25x_3 \\ x_2 &= 0.25x_1 + 100 + 0.25x_3 \\ x_3 &= 0.25x_1 + 0.25x_2 + 100 \end{aligned} \qquad (2\text{-}1.0)$$

in which x_1, x_2 and x_3 are service department costs. Equation system (2-1.0) can be rewritten as:

$$\begin{aligned} 1.00x_1 - 0.25x_2 - 0.25x_3 &= 100 \\ -0.25x_1 + 1.00x_2 - 0.25x_3 &= 100 \\ -0.25x_1 - 0.25x_2 + 1.00x_3 &= 100 \end{aligned} \qquad (2\text{-}1.1)$$

This converts into the following matrices:

$$\begin{bmatrix} 1.00 & -0.25 & -0.25 \\ -0.25 & 1.00 & -0.25 \\ -0.25 & -0.25 & 1.00 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 100 \\ 100 \\ 100 \end{bmatrix} \qquad (2\text{-}1.2)$$

or in matrix notation:

$$AX = C \qquad (2\text{-}1.2M)$$

The second objective is to gather into the B, X and D matrices the accounts which have a non-reciprocal relationship. These accounts, the product departments, are non-reciprocal because they get support from but do not give support to the service departments. These equations state this relationship in the analogue:

$$\begin{aligned} t_1 &= 100 + 0.25x_1 + 0.25x_2 + 0.25x_3 \\ t_2 &= 100 + 0.25x_1 + 0.25x_2 + 0.25x_3 \end{aligned} \qquad (2\text{-}2.0)$$

in which t_1 and t_2 are the costs of the respective product departments' outputs. These equations can be manipulated to get:

$$\begin{aligned} t_1 - 100 &= f_1 = 0.25x_1 + 0.25x_2 + 0.25x_3 \\ t_x - 100 &= f_1 = 0.25x_1 + 0.25x_2 + 0.25x_3 \end{aligned} \qquad (2\text{-}2.1)$$

in which f_1 and f_2 are the values of the outputs transferred from the service departments to the product departments. Equation system (2-2.1) can be rewritten as:

$$\begin{bmatrix} t_1 \\ t_2 \end{bmatrix} - \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \quad \begin{bmatrix} 0.25 & 0.25 & 0.25 \\ 0.25 & 0.25 & 0.25 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad (2\text{-}2.2)$$

or in matrix notation as:

$$T - D = F = BX \qquad (2\text{-}2.2M)$$

This equation states that the total cost of all products less the actual (budgeted) expenditures of the product departments equals the value of the outputs transferred from the service departments.

<u>The Address System</u>

The partitioned matrix P used in this dissertation, when adapted to the analogue is:

$$\left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & c_{14} \\ a_{21} & a_{22} & a_{23} & c_{24} \\ a_{31} & a_{32} & a_{33} & c_{34} \\ \hline b_{41} & b_{42} & b_{43} & d_{44} \\ b_{51} & b_{52} & b_{53} & d_{54} \end{array}\right]$$

In this format, the section containing the a's constitutes the A matrix, the sections of the b's, c's and d's constitute the B, C and D matrices. The subscripts identify the row and column positions (the addresses) of the a, b, c and d elements in the partitioned matrix.

In equations (2-1.2) and (2-2.2) A, C, B and D are the input matrices of the partitioned matrix P. In the A matrix the conventional matrix addressing system is a_{ij}, (i = 1,2,3; j = 1,2,3) where the i value locates the row and the j value locates the column position of the specific matrix element. In this work this convention will be followed for the A matrix. The C matrix normally would have the conventional address system c_{ij}, (i = 1,2,3; j = 1). In this work its address system is modified so that c_{ij}, (i = 1,2,3; j = 4). Thus, the j value of this one column vector is one number more than the highest j value in the A matrix.

In the B matrix the conventional address system is b_{ij} (i = 1,2; j = 1,2,3). In this work the address system is b_{ij} (i = 4,5; j = 1,2,3). Thus, the i row of the B matrix begins with a number one higher than the highest i value in the A matrix. Similarly, the D address system becomes d_{ij} (i = 4,5; j = 4). Thus, any D matrix begins with an i and j value one higher than the highest i and j, respectively, in the A matrix. This system locates the articulated set of matrices and their elements in the partitioned matrix P. The articulation of the matrices becomes clearer when the matrix space is duscussed.

The Partitioned Matrix Space

In the analogue the **A** matrix is square and comprised of three row and three column vectors. Had the analogue consisted of n service departments the **A** matrix would have been square and comprised of n row and n column vectors. It is necessary to maintain squareness to invert the matrix. Squareness is maintained by assigning a coefficient of 0 to service departments from whom or to whom cross-servicing is not provided. For example, in the analogue, if department a_1 received no support from department a_2 but instead department a_3 received that incremental 25 per cent, the **A** matrix of equation (2-1.2) would have been:

$$A = \begin{matrix} 1.00 & 0 & -0.25 \\ -0.25 & 1.00 & -0.25 \\ -0.25 & -0.50 & 1.00 \end{matrix} \qquad \text{(a-1.2a)}$$

The matrices of the partitioned matrix articulate with each other in several ways. Each column in **P**, except the last, represents the total allocation of a specific service department. In each allocating column, the net value of the matrix elements in the **A** partition equals the value of that service department's output which is available to the product departments. The allocation of this output is expressed by the elements in the same column of the B partition. If the system is exhaustive, all this output available

in a column is allocated to the receiving product departments in the B matrix partition of that column.

The A, B, C and D matrices are articulated in this section identically as are the much larger matrices of the USC problem. The arrangement of the analogue matrices is:

A a_{ij}, $i = 1, 2, 3$ $j = 1, 2, 3$ equation (2-1.2)	**C** c_{ij}, $i = 1, 2, 3$ $j = 4$ equation (2-1.2)
B b_{ij}, $i = 4, 5$ $j = 1, 2, 3$ equation (2-2.2)	**D** d_{ij}, $i = 4, 5$ $j = 4$ equation (2-2.2)

$$= \begin{bmatrix} \mathbf{P} \\ P_{ij},\ i = 1, \ldots, 5 \\ j = 1, \ldots, 4 \end{bmatrix}$$

Substitution of the values from the matrices of the analogue into their address in the partitioned matrix yields:

i \ j =	1	2	3	4
1	1.00	-0.25	-0.25	100
2	-0.25	1.00	-0.25	100
i = 3	-0.25	-0.25	1.00	100
4	0.25	0.25	0.25	100
5	0.25	0.25	0.25	100

The USC Matrix System

USC is divided, for the purpose of this work, into two subsets. The producing subset consists of 10 elements which are located on campus and 23 elements which are widely dispersed throughout the state. The remainder of the University's activities comprise the subset of service departments. The 10 on-campus elements include 8 colleges and schools which produce college-level instruction, the College of General Studies which produces noncollege level instruction and the Department of Advanced Studies and Research which does not produce instruction. The 23 off-campus are 11 regional centers, 9 hospital affiliates and 3 field activities. Of these 33 elements, only the 8 colleges and schools which produce college-level instruction are relevant to this study because only they award degrees. The 25 remaining elements will be combined in one category for the purpose of assigning to them the service departments' outputs which they absorb. This will be accomplished by the allocation process of Chapter III in which only services absorbed by the 8 relevant elements will be carried forward.

The A Matrix Schema

There are 42 activities in the service department subset. Each service department has a number which summarizes subsidiary departmental accounts, in the chart of accounts.

These 42 activities, mutually supporting each other, constitute the A matrix that is 42 x 42 in dimension. The account numbers and titles of these activities will be identified by their matrix location number in Chapter III. The schema of the matrix is such that for any a_{ij}, i = j. Thus, for example, a service department, say Personnel Services, with an address which is $a_{ij} = p_{ij}$, (i = j = 5), is the fifth row vector of the matrix for allocation of services it receives and the fifth column vector for allocation of services it gives. This ordered arrangement ensures the required squareness of the A matrix and permits easier manipulation with an identity matrix.

The B Matrix Schema

There are 8 members of the product division subset that receive support from the service subset. These product divisions are arranged in an 8 x 42 matrix, the B matrix, and will be identified in Chapter III in the same way as the product departments. The schema of this matrix is such that for a $b_{ij} = p_{ij}$ (i = 43, 44.....50; j = 1,2,.....42). The address i locates the row vector which receives and j locates the column vector which provides services. Thus $b_{ij} = p_{ij}$ (i = 44, j = 5) locates the percentage of Personnel Services' output received by some product department, say the Business College.

The C Matrix

The C matrix is a single column vector of 42 rows. Each of the 42 elements is the actual expenditure of one of the service departments. The values were taken directly from the accounts of USC. The format of this matrix is $c_{ij} = p_{ij}$ and i = 1,.....42 and j = 43. Thus $c_{5,43} = p_{5,43}$ locates the amount actually spent by the Personnel Services Department as will be shown in Chapter III.

The D Matrix

The D matrix is, like the C matrix, a single column vector but it has 8 row elements. Each row element is the actual expenditure of a college or school as taken from the USC accounts. The format is $d_{ij} = p_{ij}$ and i = 43,.....50 and j = 43. Thus d44,43 = p44,43 locates the amount actually spent by the Business College as will be shown in Chapter III.

The Partitioned Matrix Space

The row dimensions of the P matrix is equal to the sum of the product and service departments or 50. This is equal to the highest i value in the system. The column dimension is equal to one more than the number of service departments or 43. This equals the highest j value in the system.

The general schema of the P matrix composed of the matrices--A,B,C and D--of the main allocation problem is:

	j = 1, 2, 3 42	43
i = 1, 2, 3, ... 42	A = $a_{ij} = b_{ij}$ i = 1,........42 j = 1,........42	C = $c_{ij} = p_{ij}$ i = 1,.....42 j = 43
43 50	B = $b_{ij} = p_{ij}$ i = 43,.........50 j = 1,..........42	D = $d_{ij} = p_{ij}$ i = 43,....50 j = 43

The matrix will differ from the partitioned matrix of the analogue in one way. The allocations made in the analogue are exhaustive. As a result, the algebraic value of the portion of any column falling within the A section of that partition will equal the algebraic value of that same column falling within the B section of that partition. In the USC matrix the allocations are not exhaustive. The services absorbed by the 25 "other" product departments, although carefully calculated are not carried forward because they are not relevant to this work. For these departments their combined proportional absorption of services causes and is equal to the algebraic difference between the two parts of any such column in the USC matrix.

CHAPTER III

THE MATRIX SYSTEM CALCULATIONS

Purpose of This Chapter

The overall purpose of this chapter is to compute the total costs of the product divisions of USC. This will be accomplished in five steps. First this chapter will identify the account titles, numbers, and actual expenditures of the product and service departments of USC with their address in the partitioned Matrix P. Next, the allocation bases for the service departments' outputs to the service departments and product departments will be discussed. This second step will include allocation of service departments' outputs to the recipient departments. This will complete the P-Matrix. The third step will be to determine A^{-1}, the inverse of the A partition of the USC Matrix. This will be accomplished by using the Gauss-Jordan Procedure. Also in this step the C partition will be multiplied by the inverse to determine X, the Matrix of unknowns needed for calculating F, the matrix of fixed cost. Fourth, a test of these calculations will be made in which A will be multiplied by A^{-1} to get the Identity Matrix, thereby to prove the quantitative integrity of the computer calculations.

The fifth and final step will be to add the F and D matrices to obtain the Total-Cost Matrix, T.

Background

The work in this and subsequent chapters will use the symbols as defined on pages 16 and 17 of Chapter II above with no further definition. New symbols and definitions will be added as needed.

The Addresses of Account Titles and Numbers

The P Matrix has 50 row locations of which 42 are in the A partition. This square partition, A, is arranged, as explained in Chapter II, so that the row and column locations of service departments are identical. Thus, the 42 service departments of USC, which comprise the A partition, are so arranged that each row or i location of an account is the same as its columnar or j location. As a result, the address $i = j$ obtains for each service department.

The remaining 8 row locations of the matrix contain the product departments. The product departments are identified by row location only. As a result, the address i for each product department has no counterpart j address. The 8 x 42 quadrant containing the product departments is the B partition. This and the A partition completely define the 50 row locations or addresses.

Of the 43 column addresses or locations, 42 have been defined as elements of the A partition. The 43rd column, a 50 x 1 vector, makes up the C and D partitions. This column vector articulates with the row identities in the following manner. Column 43 contains the actual expenditures in its appropriate ith row for the ith account number and title. Thus the address $p_{1,43}$ locates the first row account title and number and column 43 of that row states the amount of actual expenditure of that account.

In this work, the addresses, account titles, numbers, and amounts of actual expenditures are given in Table 1.

Basis for Allocating Service Departments' Outputs

The allocation of each of the outputs of the 42 service departments of USC to the other 41 service and to the 8 product departments involves 42 x 49 or 2,058 decisions. To discuss succinctly such an enormous allocation process requires a systematic treatment.

An eclectic approach was taken in allocating the outputs of the service departments. At the outset interviews with various administrators were used to determine if some empirically based method of allocation existed. The empirical methods for allocation were based on occupied floor space, actual direct departmental expenditures and number of credit hours of instruction completed. In some cases there was no sensible empirical basis for allocating either a part

Table 1. Addresses, account titles, numbers, and amounts of actual expenditures

Address i	Address j	Account Number	Account Title	Actual Expenditures
1	1	01000	Board of Trustees	55842.00
2	2	01100	President	63180.00
3	3	01150	Senior Vice-President	32831.00
4	4	01200	Vice Pres. for Bus. Affairs	70982.00
5	5	01201	Personnel Services	30920.00
6	6	01202	Accounting Services	133905.00
7	7	01205	Purchasing & Central Supply	104136.00
8	8	01207	Auxiliary Services (Admin.)	6000.00
9	9	02001	Vice Pres. for Student Affairs	102624.00
10	10	02002	Student Union	58303.00
11	11	02003	Counseling Bureau	5204.00
12	12	02004	Student Aid	20307.00
13	13	02006	Admissions & Registration	240571.00
14	14	02100	Vice-Pres. for Development	29541.00
15	15	02101	Alumni Association	9171.00
16	16	02102	Placement	14657.00
17	17	02150	Public Rel. & Information	36532.00
18	18	02201	Mail & Messenger Service	40067.00
19	19	02202	Printing	113306.00
20	20	02203	Telephone Exchange	3988.00
21	21	02204	Data Processing Service (Non-Acad.)	156860.00
22	22	02209	General (Association Dues)	4313.00
23	23	03000	General Instruction	84345.00
24	24	03020	Associate Vice-President	30955.00
25	25	03030	E.T.V. Center	28064.00
26	26	03030	Computer Science Center	210769.00
27	27	03040	University Press	68717.00
28	28	03100	Vice-Pres. for Academic Affairs	82944.00
29	29	07000	Administration & Supervision	68068.00
30	30	07001	Maintenance of Buildings	730865.00
31	31	07002	Maintenance of Grounds	144158.00
32	32	07003	Custodial Services	218910.00
33	33	07004	Police	96277.00
34	34	07006	Heating Plant	103966.00
35	35	07007	Utilities	110103.00
36	36	07009	Motor Pool	-9835.00*
37	37	07012	Insurance (Admin. & Acad. Bldgs.)	20079.00
38	38	07020	Campus Planning	54507.00
39	39	08400	Health Service	103205.00
40	40	06000	Libraries	668582.00
41	41	08100	Director of Housing	1654252.00
42	42	08500	Cafeterias	68016.00
43	43	03200	College of Arts & Science	3967256.50
44	43	03300	College of Business Admin.	751477.19
45	43	03400	School of Education	611504.46
46	43	03500	College of Engineering	561063.29
47	43	03700	School of Journalism	122593.38
48	43	03750	Law School	197989.12
49	43	03800	School of Nursing	225861.11
50	43	03850	School of Pharmacy	127543.81

*The Motor Pool collects charges from (i.e., transfers its costs to) users of its services. The rates set for its services during the fiscal year 1967 were too high. As a result, the motor pool account accrued credits in excess of its expenditures, or in other words, it transferred more than its actual costs to users of its services.

or the entire output of some departments. In these cases, the administrators were asked to allocate such departmental outputs based on their best judgment.

Some further distinction needs to be made between the general process of interviewing and interviewing as defined here is a method for allocation. In this dissertation, the interview method of allocating means that the actual allocations are based on the opinions of the interviewees. It is not an empirically based method and is therefore subject to more chance of error; there are no tangible, measurable, arithmetic bases beyond the proportions indicated by opinion.

Interview Method

All interviews had one underlying guideline. Interviews were sought with administrators at a high enough level for them to be knowledgeable of the subject matter but not at a level so high that the subject was trivial to them. The highest administrative officer interviewed was the vice-president for business affairs; the highest academic officer was the vice-president for academic affairs.

As a result of these interviews certain allocations were made on the basis of the interview method. Such allocations were made for either all or a portion of the output of those departments whose partitioned matrix column addresses are j = 1, 2, 3, 4, 5, 7, 8, 10, 11, 12, 13, 14, 17, 18, 20, 21, 24, 25, 26, 28, 29, 38 and 39.

If a department had its output allocated wholly on the basis of the interview method its column address will appear only in the preceding sentence. If its output is allocated partially on the interview method its column address will reappear in one or more of the empirical listings given below. A department partially based on the interview method occurs when some portion of its output is devoted to another department--in the opinion of the interviewee--and the balance of its output is allocable on some empirical basis.

<u>Floor Space Occupied</u>

The interviews with the most knowledgeable administrators revealed that some service departments should have their output allocated on the basis of floor space. For this allocation, the latest available blueprints of the USC buildings show that 2,730,140 square feet of space were being used. The floor space occupied by each department was converted to a percentage of the total USC space. However, during the year much construction was in progress and in the future some departments will have their locations changed both in locus and size. The allocation is reasonably valid for the period of this study but future work should incorporate new construction into the floor space base and reassess the space occupied by the departments.

Use of floor space occupied as a basis for allocation carries with it the associated problem of allocating maintenance, repair and utilities expenses. Newer buildings usually will have lesser maintenance expended on them than older buildings and will use lesser amounts of utilities because of more efficient insulation, lighting, building design, etc. The capital costs of newer buildings, however, will offset the occupancy costs because the unamortized investment in the new usually will be larger than the residual investments in the old buildings.[1] Specific identification of all these costs of the individual buildings is of importance to administrators who must decide on the design of new structures. For the purpose of this work, however, the "accident" of a department's location is irrelevant to the main purpose, computing the cost of delivering instruction. The results are more generally valid to present purpose if all floor space is treated as if it were homogeneous both in terms of capital investment costs and occupancy costs. Those departments with newer facilities have their lower occupancy costs offset by higher capital investment costs: the higher occupancy costs of old facilities are offset by their lower capital investment costs. In this manner, the "accident" of physical location and the effects this has on

---

[1]In other situations capital costs might be based on amortized costs of depreciable assets.

a department's cost tends to be mitigated and the overall results become more generally valid.

Departments whose outputs are allocable, in whole or in part, on the basis of floor space have the column addresses j = 30, 31, 32, 33, 34, 35 and 37 in the P matrix.

Credit Hours

Some departments provide services which are proportionate, in whole or in part, to the number of hours of instruction delivered. For a diversified university, such as USC, hours of instruction fall into three categories. First, some outputs are proportional to the total instruction delivered throughout the university system. This includes the instruction delivered in the centers, regions, and on campus. The second category comprises those outputs proportionate to the number of hours of instruction delivered on campus whether for regular college credit or not. The third category comprises on-campus instruction which is accredited college work. This third category is the actual product for which this work strives to determine a cost.

Examples of an output allocable on the basis of total hours taught in the university system are the Counseling Service and the President's office. Purchasing is proportionate to semester hours on campus; only the regions and centers are self supporting in this function. The Alumni Association is an example of a service proportionate only to on-campus credit hours of instruction. Allocations based on

total hours of instruction, in whole or in part, are applicable for departments whose columnar addresses in the P matrix are j = 2, 11, 13 and 17.

Allocations based, in whole or in part, on total hours of instruction on campus are made for addresses j = 5, 7, 9, 10, 12, 16, 18, 20, 22, 23, 27, 28, 39, 40, 41 and 42.

The total hours of on campus college credit instruction was the basis for allocating, in whole or in part, the output of only the Alumni Association whose address is j = 15.

Actual Expenditures or Budget

Some activities of the service departments of USC are proportionate to the amount of expenditures made by the recipient of these services. For example, the USC controller reported that his department rendered varying amounts of its output to differing groups of recipients depending on the time of year, the specific year in question (and construction activities currently being carried on). His considered opinion is that for general applicability and long range validity his services should be charged on the basis of the amount of money spent by the departments being serviced. Departments whose services are allocable on the basis of expenditures have the addresses i = j = 6, 19 and 36.

Cross-Allocation of the Service Department's Outputs

Thus far this chapter has established the address system for the partitioned USC matrix and identified the 42 service departments by account number and title. The amounts of departmental expenditures have also been given in Table 1. The various methods for allocation have been given in a general way. As a result, all that remains to cross allocate service departments' outputs is to complete the A partition of the USC matrix.

The actual allocations in the A partition entail 42 x 41 or 1,722 decisions. Each decision becomes one element in the 42 x 42 partition. These allocations are not listed in this dissertation because they are not generally applicable. They are relevant only to USC for the time period under study. The allocation values are analogous to the values listed in the A partition of the matrix of the analogue on page 23 of Chapter II above.

The equations upon which the allocation values are based are linear. There are 42 of these equations and 42 unknowns. These equations are of the general form

$$\begin{aligned} X_1 &= C_{1,1} + a_{1,2}x_2 + \cdots\cdots + a_{1,42}\, x_{42} \\ &\vdots \\ X_{42} &= a_{42,1} + a_{42,2}\, x_2 + \cdots\cdots + C_{42,1} \end{aligned} \qquad (3\text{-}1.0)$$

In this system, C_{ij} represents the actual departmental expenditures of the ith service department in the one column C partition of the USC matrix; a_{ij} represents the $a_{ij} = p_{ij}$ coefficient in the 42 x 42 A partition of the USC matrix;[2] and, X_i represents the ith unknown in the one column X matrix of unknowns as defined in Chapter II. Following the same algebraic steps as used in the Analogue, these 42 equations become, in matrix notation:

$$A \ X = C \qquad (2\text{-}1.2M)$$

The system of equations in 3-1.0 does not exhaust the entire output of the service departments. The remainder is allocated among the product departments in Chapter V below.

Calculation of the Inverse

This dissertation capitalizes on the work of Williams and Griffin,[3] particularly on their application of the matrix inversion procedure for cost allocation. Their matrix is 5 x 5, which is much smaller than the 42 x 42 matrix in the A partition of the USC Matrix. Their smaller matrix was used in test runs of the computer programs used in this work.

---

[2] Because of algebraic operations, however, these coefficients will be of different sign. For example, see equation 2-1.2a.

[3] Thomas H. Williams and Charles H. Griffin, "Matrix Theory and Cost Allocations," The Accounting Review, XXXIX, No. 3 (July, 1964), 671-678.

The results obtained were consistent with those found by Williams and Griffin.

Inversion of a matrix, when a computer is used, is procedurally routine. After the allocation values have been determined, as described above, the balance of the work is voluminous but straightforward. It consists of key punching data cards, writing the computer program, running the data through the computer and proofreading the printout of the input data.

The Gauss-Jordan procedure, a standard computer library program, is used to invert the A Matrix. This inversion program is given in Appendix 1. The inverse, A^{-1}, is an intermediate result. Because its main purpose is to serve as a matrix multiplier of the C partition, the A^{-1} Matrix is not reproduced in this work. The result of the A^{-1} multiplication of the C partition becomes the X matrix, dealt with below.

C Partition Multiplication

The multiplication of the C partition by the inverse is a straightforward calculation, but a very large one. The multiplication by the 42 x 42 inverse matrix of the 42 x 1 C matrix entails 1,764 multiplications and the addition of 42 columns of figures, each containing 42 values.[4] Part of

---

[4]This multiplication in matrix notation is equivalent to $A^{-1}\,C = X$.

the value of computer applications is to make it unnecessary to do such laborious work manually. The program listed in Appendix I includes instructions for the computer to perform these operations.

The result of these calculations is the X Matrix, which is multiplied by the B partition to determine the "actual allocation" of the service departments' outputs to each of the product departments.

The Identity Matrix

The calculations performed by the computer to invert the A partition if done manually would require at least several man-years of effort. Because of this it is not feasible to check the computations manually, nor is it satisfying to accept blindly the computer's results. It is possible to authenticate the results by observing that a matrix, if multiplied by its inverse (its matrix-algebra reciprocal), yields the identity matrix.

As a brief proof of the matrix multiplication program of this work, this program was used to multiply the 5 x 5 matrix of Williams and Griffin by its inverse and it yielded the identity matrix.[5]

---

[5] T. W. Williams and C. H. Griffin, *op. cit.*, p. 676.

The A partition of the USC Matrix when similarly multiplied by its inverse does not yield the exact identity matrix. In several elements of the approximate identity matrix there are values that appear larger in the fifth place to the right of the decimal point than the exactly accurate 0.00000 element value. This acceptably small error results primarily from two causes. First, the allocation of the service departments' outputs were rounded to the fifth decimal place. Second, the computer truncates rather than rounds values.

The program used for multiplying the matrix by its inverse is given in Appendix I. The results of the multiplication are deemed to validate the calculations.

Thus far in this chapter the A, B, C and D matrices have been defined, and the A^{-1} and X matrices have been calculated. The work remaining to be accomplished is to compute the F Matrix and add it to the D matrix to obtain the T Matrix.

T Matrix Calculation

The F Matrix is obtained by multiplying the X Matrix by the B Matrix as outlined in Chapter II above, with specific reference to equations 2-2.2 and 2-2.2M. The actual calculation involves 42 x 8 = 336, multiplications and additions which are done by the computer to capitalize on its high speed. The program for this multiplication is, except for dimensions, similar to the program portion which multiplies A^{-1} by C. This program is given in Appendix II.

The F Matrix when added to the D Matrix yields the T Matrix. These values are given in Table 2 below.

Table 2. Values of D, F and T matrices

Product Department Address i	j	D	F	T
43	43	$3,967,256	$3,516,629	$7,483,885
44	43	751,477	831,124	1,582,601
45	43	611,504	390,101	1,001,605
46	43	561,063	239,914	800,977
47	43	122,593	52,154	174,747
48	43	197,989	163,030	361,019
49	43	225,861	85,152	311,018
50	43	127,543	94,175	221,718

The values of the T Matrix are the total costs of the product departments under the allocation scheme used herein. These costs will be used in the next chapter as a basis to determine the cost of the products of the schools and colleges of USC which produce accredited college-level education.

CHAPTER IV

COSTS OF DEGREES

Purpose

The purpose of this chapter is two-fold: first, the individual input costs of the several degrees awarded by USC will be determined; second, the average costs of the baccalaureate and master's level degrees will be calculated. The first purpose supports the second purpose; each purpose requires several preparatory steps.

There are five steps necessary to accomplish the first purpose. First, the outputs of the USC Product Departments will be determined in terms of course hours. Second, these outputs will be weighted to permit expression of them as equivalent units. Third, the cost per equivalent unit will be calculated and converted into a cost per hour. The fourth step will be to determine the number of credit hours by type that comprise the various degrees. Fifth, the costs of degrees will be calculated, and the reliability of the results will be determined.

The first five steps will determine the average cost of a bachelor's degree and the average cost of a graduate degree for each of the schools and colleges at USC. The

specific costs of the individual degrees which generate these average degree costs will enter into the computation of the costs of an average or composite undergraduate and an average graduate degree on a university-wide basis.

To calculate the costs of the university-wide composite or average degrees for both the graduate and undergraduate level requires the use of random stratified sampling techniques for two reasons. First, there is wide variability in the number of degrees awarded at the same level by different colleges and schools. Second, there is wide variability in the costs for degrees at the same level from the different colleges and schools. Use of a simple average (mean) cost of the specific degrees fails to take into account that there are more degrees awarded at one cost than another: use of a simple random sample fails to take into account that a disproportionate number of degrees may be drawn from the specific degree types.

There are three steps required to accomplish the second purpose because the statistical data developed in accomplishing the first purpose must be rearranged to provide the basis for application of the stratified sampling technique. First, the stratified sampling technique will be introduced. Second, this technique will be applied against the unit cost data developed by the first five steps. Third, the results will be stated.

Product Departments' Outputs

Background

The outputs of colleges and schools constitute a product line of almost infinite variety. Research, publications, public services of an administrative and advisory nature, plus student education are examples of this variety. This dissertation, however, concentrates on but one specific product--accredited semester hours of successful work--to the exclusion of all other outputs. This concentration requires a nonempirically based judgment of the percentage of each product department's effort that is devoted to teaching. This is a nonempirically based judgment because almost no objective detailed data are available relative to the use of professorial time.[1] The need for such objective data is well known and detailed in Doctor Jones' 1964 study for U.S. Office of Education.[2] Until research on this subject is completed, this and other studies must rely on the best nonempirical opinions available relative to the disposition of an educational institution's output.

---

[1]Gardner M. Jones, "A Procedural & Cost Analysis Study of Media in Instructional Systems Development," Part B. Office of Education, Department of Health, Education and Welfare, 1964, p. 103.

[2]*Ibid*.

Product Input Value

Colleges and schools produce such a wide variety of services that some procedure is required to assess the value of any one service. The procedure used in this work is to determine the input value of teaching to the exclusion of the other products. The input values of the total products of the colleges and schools of USC have been determined in the T Matrix of Table 2 and will be used in Table 5 below. However, this T Matrix does not include capital charges such as land rental, building depreciation and the like. These capital costs are subject more to local conditions than to general economic circumstances whereas other university costs are more subject to general economic conditions. This is because the specific location in an urban center as compared to the suburbs greatly affects the value of land and buildings both in exchange and as a tax basis. Other university costs are incurred to acquire educational and professional equipment, the services of educational and administrative staffs, etc. Such costs items are taken from regional and national markets and therefore will not vary as much from one school to another as will capital costs. As a result, the elimination of capital costs makes this work more generally applicable.

As previously stated the costs of each product department's output is contained in the T Matrix. The value of the teaching output is that percentage of a college's or

a school's output devoted to teaching. To determine this percentage, the deans, acting deans, assistant deans and other senior and knowledgeable people in each college and school were interviewed. In these interviews two things were sought: first, the percentage of effort going into teaching; second, the relative weights of the various courses. This second interview objective will be treated later.

The results of the interviews relative to the percentage of effort devoted to teaching are given in Table 3.

Table 3. Percentage of total effort devoted to teaching

College or School	Percentage
Arts and Science	90
Business	80
Education	90
Engineering	65
Journalism	75
Law	80
Nursing	85
Pharmacy	90

These data will be incorporated in calculations later in this chapter and are less objective than if there were empirical data available relative to the disposition of professorial time.

Equivalent Units of Output

Weighting of the various courses offered by the Product Departments permits all courses to be expressed in terms of an equivalent unit. To accomplish this, courses were divided into graduate and undergraduate levels and further subdivided into laboratory and nonlaboratory elements. This four-way weighting is relevant to the situation at USC although it is clearly recognized that other weighting systems may obtain for situations at other universities. For example, some universities may be such that courses should be divided into upper, lower, master and doctoral levels; then further subdivided into laboratory, nonlaboratory, large lecture, television, etc., elements. The results of the subdivision system employed in this work are given in Table 4.

Table 4. Course weights*

	Course Type			
	Undergraduate		Graduate	
College or School	Lab.	Non-Lab.	Lab.	Non-Lab.
Arts and Science	1	1	0	2
Business	0	1	0	2
Education	10	10	17	17
Engineering	1	1	0	2
Journalism	8	6	0	9
Law	0	0	0	1
Nursing	1	1	1	1
Pharmacy	1	1	0	0

*The value 0 indicates no such course available.

Table 4 expresses the weight ratio between courses of the same college but not among courses of the various colleges. For example, in the case of the School of Journalism the ratios of 6, 8 and 9 express the opinion of the Dean of the School of Journalism that graduate work in Journalism requires 150 per cent of the effort required for undergraduate work if the course does not involve a laboratory session. If the course involves a laboratory session the effort required is increased by one-third. These insights were then converted to ratios containing only the whole numbers 6, 8 and 9, to expedite calculations of unit cost of output. The numbers are relevant only to Journalism.

Cost per Equivalent Unit

Output of Product Departments

The semester hours of successfully completed college-accredited work at USC are recorded in the central data bank of USC. These data are so arranged that it is possible to identify semester hours by the college or school of their origin, their department and their specific class section but not by teacher. A computer program was written to retrieve this information for the two summer sessions of 1966, the fall semester of 1966, and the spring semester of

1967.[3] This time period coincides almost exactly with the fiscal period for which costs are relevant. In actuality the teaching calendar begins approximately two weeks before the fiscal calendar. Both the chief accountant and business manager of USC state that this negligible mismatch has no measurable or meaningful effect.

The computer program for retrieving these data generates the number of hours of work attempted, failed, and completed. Only successfully completed hours are treated as product. This is consistent with the normal practice of not considering rejected items as output. Unsuccessfully attempted semester hours are considered to be in the same category as unacceptable industrial product, that is, they are part of the cost of producing the useful products.

Calculation of Cost per Equivalent Unit

The successfully completed semester hours of each product division of USC are multiplied by the weighted values in Table 4. This multiplication converts the successfully completed hours of each course into equivalent units of output. These units of output are then totaled for each school or college to accumulate the total number of equivalent

---

[3]This computer program is relative only to the storage routines used by the USC data bank or to identical systems. Because this severely limits its general applicability this program is not reproduced in this work.

units delivered. The results are arranged in Column c of Table 5.

The Column (d) of Table 5 is determined in a straightforward manner. For example, 90 per cent of the $7,483,885 input value of the College of Arts and Sciences is devoted to producing the 169,783 equivalent units of output. This yields a cost per equivalent unit of:

$$\frac{\$7,483,885}{169,783} \times 0.9 = \$39.67 \qquad (4\text{-}1.0)$$

Table 5. Equivalent units

Column College or School	(a) T-Matrix Value	(b) Table 3 Value	(c) Table of Equivalent Units	(d) Cost per Equivalent Unit
Arts & Science	$7,483,885	0.9	169,783	$39.67
Business	1,582,601	0.8	41,358	30.62
Education	1,001,605	0.9	170,288	4.705
Engineering	800,977	0.65	7,922	65.72
Journalism	174,747	0.75	14,952	8.765
Law	361,019	0.80	6,775	42.63
Nursing	311,018	0.85	3,819	69.22
Pharmacy	221,718	0.90	3,857	51.74

Unit Cost Matrix

Table 5 lists the equivalent unit cost for each college or school and Table 4 lists the course weights. Multiplication of the equivalent unit cost by its course weight yields the input cost per semester hour of delivered

instruction. The results of this multiplication are given in Table 6. In this table only one listing is made for each nonzero course weight of each school. This table is designed to identify the 14 x 1 matrix which will be used later.

The unit cost of output having been determined and arranged into the matrix format desired, the next step is to determine the number and types of semester hours which comprise the delivered education upon which awards of various degrees are based.

Table 6. Semester hour cost matrix

College or School	Course Type[a]	Equivalent Unit Weight	Equivalent Unit Cost	14 x 1 Matrix of Semester Hours Costs
Arts and Science	UGNL & L	1	$39.67	$39.67
	GNL	2	39.67	79.34
Business	UGNL	1	30.62	30.62
	GNL	2	30.62	61.24
Education	UGL & NL	10	4.705	47.05
	GL & NL	17	4.705	79.99
Engineering	UGL & NL	1	65.72	65.72
	GNL	2	65.72	131.44
Journalism	UGNL	6	8.765	52.59
	UGL	8	8.765	70.12
	GNL	9	8.765	78.88
Law	GNL	1	42.63	42.63
Nursing	ALL	1	69.22	69.22
Pharmacy	UGL & NL	1	51.74	51.74

[a]Code: UG = Undergraduate; G = Graduate; NL = Nonlaboratory; L = Laboratory.

Determination of Degree Composition

Background

Degrees are generally awarded for completing a specified number of courses and for attaining a grade point ratio above some minimum point. During any period a university delivers many courses with various numbers of hours of credit granted for those courses. In terms of degrees, a university's output could be measured in equivalent degree units. For example, if a school produced 120 semester hours of successful teaching during a period, and the degree requirements of that university were 120 semester hours, the equivalent unit of output would be one degree. Thus a portion of the product of a university can be measured in terms of degrees.

Within a university it is common for the requirements for different degrees to vary. For example, some degrees at USC require 120 semester hours whereas other degrees require 128 hours and still other require a different number of credit hours. This variable requirement of semester hour totals is not the only variable. Degree requirements also vary in the type and mix of courses required. As a result, one degree may differ very much from another in both the number of hours and in the type of hours required. This variability in the number and type of required courses necessitates that statistical sampling

techniques be used to identify the ingredients and therefore to calculate the costs of representative degrees.

Statistical Population

The population upon which the statistical measurements of this dissertation are based is taken from the USC roster of degrees awarded for the period under study. The number of these various degrees awarded is given in Table 7.

These degrees, which were awarded during the fiscal year 1967, are to a great extent based upon teaching delivered in prior years. This raises the question: Should not the costs of degrees be based upon the costs that pertained at the delivery date of each course comprising the basis for awarding the degree? The answer posed in this work is: The current going rate of cost is sought, not the historic cost. The purpose of the next section explains the statistical sampling technique used to determine degree costs.

Sampling Technique

The statistical portion of this dissertation relies upon the work of W. G. Cochran[4] and Taro Yamane.[5] The population from which the samples were drawn is taken from the

---

[4] W. G. Cochran, Sampling Techniques (2d ed.; New York: John Wiley & Sons, Inc., 1963), pp. 93-118.

[5] Taro Yamane, Elementary Sampling Theory (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1967), pp. 64-158.

Table 7. USC degrees awarded, fiscal year 1967

Degree Awarded	Number of Degrees (N_h)*	Sample Size (n_h)*
Bachelor's Level:		
Arts & Sciences	390	39
Business	352	36
Education	226	23
Engineering	79	8
Journalism	33	5
Nursing	40	5
Pharmacy	49	5
Totals (N)	1,169	121
Master's Level:		
Arts	74	9
Business	58	6
Education	31	5
Engineering	16	5
Sciences	59	6
Totals (N)	238	31
Graduate Level:		
Law	96	10

*For convenience, N_h, the population strata of Population N, and n_h, the sample size used later in this chapter are identified in Table 7.

rosters of the three USC graduations which took place during fiscal year 1967. The data in Table 7 were compiled from these rosters. The three rosters were reassembled in such a way that rosters of the recipients of similar degrees, say a Bachelor of Arts, were butted against each other to make one composite roster. Therefore, each such composite roster contained three alphabetized listings sequentially arranged.

These rosters were then subdivided into units containing 10 names each to ensure a minimum 10 per cent sample. A table of random numbers was then entered at a random point, and the list of random digits that followed was used to determine which name in each group of 10 names should be included in the sample. The first random digit would indicate which of the first ten names to use, the second random digit the name in the second decade, etc.

The transcripts of the selected graduates were then drawn from the record file of USC and the semester hours of work accomplished by those students were systematically tallied. Hours were categorized in a system compatible with the tabulation shown in Table 6. As a result, each transcript generated a 1 x 14 matrix element in which the row value was the number of hours completed in each of the 14 "course types" of Table 6. This arrangement generates an n x 14 matrix, where n is the sample size. This arrangement also facilitates additional computations using the 14 x 1 matrix of Table 6. These calculations determine the costs of degrees.

Degree Costs and Reliability

Degree Costs

The cost associated with the delivery of the instruction necessary for awarding a degree may be obtained by adding the products of multiplying the amount of each type of

instruction by its appropriate cost. The amount of instruction has been tallied from n transcripts for each of the 13 types of degrees shown in Table 7 above. This results in a series of 13 matrices that are n x 14 in dimension where n, it will be recalled, is the sample size. For example, 39 of the 390 transcripts of students who were awarded bachelor degrees in Arts and Science have been randomly selected. The first transcript selected generates the first row of the 39 x 14 matrix of bachelors' degrees in Arts and Science. Each of the 14 columns in this row indicates the number of credit hours taken in the matching "Course Type" column of Table 6 in the precise order shown in Table 6. For example, the first column of the 39 x 14 matrix is Arts and Science course type UGNL & L; the second column in Arts and Science Course type GNL; etc. Continuing this process for each randomly selected transcript from each type of degree of the 12 remaining results in a series of 13 matrices, each of which is n x 14 in dimension, where n is the sample size shown in Table 7.

Each of these matrices was then multiplied by the 14 x 1 matrix of Semester Hour Costs of Table 6. Because the sum of the n's was 162 and this results in a very burdensome series of 162 x 14 calculations, this operation and the statistical computations were accomplished on a computer. The results are given below in Table 8.

Table 8. Cost of degrees

Degree Awarded	Mean of Average Degree Cost	Standard Deviation	Standard Error	90 Per Cent Confidence Level
Bachelor's Level:	$\overline{X}$	$\sqrt{S_h^2}$		
Arts & Science	$4881	$211	$ 34	$4881 ± 56
Business	4508	681	115	4508 ± 189
Education	5317	350	75	5317 ± 123
Engineering	7687	657	248	7687 ± 408
Journalism	5780	577	288	5780 ± 474
Nursing	7254	396	20	7254 ± 33
Pharmacy	7687	908	454	7687 ± 747
Master's Level:				
Arts	2708	827	292	2708 ± 480
Business	1960	300	134	1960 ± 220
Education	3098	419	209	3098 ± 344
Engineering	4480	1421	710	4480 ±1168
Sciences	2776	460	206	2776 ± 339
Graduate Level:				
Law	3989	391	130	3989 ± 214

The computer program which performs the calculations which result in Table 8 is given in Appendix III. This program besides performing calculations which indicate the reliability of these results also calculates some data to be used in the stratified sampling calculations of a later portion of this chapter.

Reliability of the Results

The reliability of the statistical sampling is shown in Table 8. A 90 per cent confidence interval is given for each of the types and levels of degrees awarded. This was obtained by multiplying the Standard Error of the sample mean by the value of the standard Normal Deviate corresponding to 90 per cent of the area under the normal curve. The intervals for various degrees of confidence can readily be obtained by extracting the appropriate "Normal Deviate" given in any "Areas under the Normal Curve" table for the confidence desired.

The 90 per cent confidence interval in Table 8 may be taken to mean the following. If another sample of n size, say 39, were randomly drawn from the population, say the 390 graduates who were awarded a bachelor's degree in Arts and Science, there is a 90 per cent probability that the average cost of delivering the instruction necessary for that degree would be between $4825 and $4937. There is a 10 per cent probability that the average cost would be less than $4825 or greater than $4937.

The Stratified Costs of Degrees

The average cost of an undergraduate degree in each of the colleges--Arts and Science, Business, etc.--has been determined, as has been the average cost of each of the various graduate degrees. However, the costs of the average undergraduate degree at USC as a whole and of the average graduate degree have not been calculated. Determination of these costs requires further analysis.

The graduate degree in Law needs to be eliminated from further consideration for two reasons. First, this degree is currently (at the time of this study) being changed from a Bachelor of Laws to a Doctor of Laws degree. All other graduate degrees in this study are scheduled to continue at the master's degree level. Second, the education delivered to qualify students for a law degree are peculiar to the Law School and requires two years rather than one year of study.

After elimination of Law from further consideration one additional calculation is worthwhile. Thus far we have the mean cost and a confidence interval for each of the various degrees awarded by USC. We have no mean cost nor confidence interval for the average USC degree awarded at either the bachelor's or master's level. Further, the division into specific degree types of the populations of both master's and bachelor's level degrees has resulted in some very small subdivisions or strata. This division has

resulted in some very high and some very low costs for the specific degree types. A simple random sample of either population could lead to a less efficient estimate than other sampling routines because of this variation within each population. Stratified sampling mitigates these circumstances.

The Stratified Sampling Technique

Stratified sampling is a method which provides a more efficient estimate of the real mean of a population (than the simple random sampling) when the population can be stratified into homogeneous sub-populations. In essence the stratified sampling technique takes a sample from each of the various strata which composes the overall population.[6] The design of this work is to take from each stratum a sample whose size is proportionate to the size of its stratum. The overall population which encompasses the various strata is the roster of degrees awarded by USC during 1967.

The USC graduates are divided into one population of those receiving undergraduate degrees and another population of those receiving graduate degrees exclusive of Law. These two populations are next separately arranged into one stratum for each type of degree awarded. By previous design

---

[6]Stratified sampling is a well-defined technique covered in many standard texts. As previously stated, this dissertation relies upon the works of Cochran and Yamane cited above.

the sample size selected and used in the first sampling program was the greater of 10 per cent of each stratum (degree type) or 5 in order to avoid the chance that a smaller sample might not be as representative of the population. The computer program in Appendix III contains instructions for the appropriate data to be calculated. The statistical terms used in defining these values and the values given in Tables 7 and 8 are listed in Table 9.

Table 9. Table of statistical terms

N = Number of degrees in a population

N_h = Number of degrees in stratum h

n_h = Sample size for stratum h

$\bar{x}$ = Estimate of the mean of the population

$\bar{x}_h$ = Estimate of the mean of stratum h

s_h^2 = Estimate of the variance of stratum h

$\hat{V}$ = Estimate of the variance of x

$s = \sqrt{\hat{V}}$ = Estimate of the standard error.

Application of the Statistical Technique

Computation of the estimated average cost of an undergraduate and of a graduate degree is done by:[7]

[7]Cochran, op. cit., p. 89. The symbols in Cochran's equation (5.1) are modified to make it consistent with terminology used herein.

$$\overline{x} = \frac{\Sigma\, N_h \overline{x}_h}{N} \qquad (4\text{-}2.0)$$

Computation of the estimate of the variance of $\overline{x}$ is obtained by:[8]

$$V_{(\overline{x})} = \frac{1}{N^2}\, \Sigma\, N_h (N_h - n_h)\, \frac{s_h^{\ 2}}{n_h} \qquad (4\text{-}3.0)$$

The values for N_h and n_h are given in Table 7; the remaining values are given in Table 10. Solution of equations 4-2.0 and 4-3.0 yields:

$\overline{x}$ = \$5290 for undergraduate degrees

$\overline{x}$ = \$2925 for graduate degrees

$\hat{V}$ = \$1869 for undergraduate degrees

$\hat{V}$ = \$10,954 for graduate degrees

s = \$43 for undergraduate degrees

s = \$105 for graduate degrees.

Statement of Results

The results of the Stratified Sampling Technique as applied against the statistical data of this dissertation permit the following statement.

The average input cost of the education required for the awarding of a bachelor's degree is, with 90 per cent

---

[8] Cochran, op. cit., p. 93. Cochran's equation (5.11) modified for consistency with present terminology.

Table 10. Stratified costs of degrees

	$N_h - n_h$	s_h^{2*}	s_h^2/n_h	$N_h(N_h - n_h)$	$N_h(N_h - n_h)\, s_h^2/n_h$
Bachelor's Level:					
Arts & Science	351	44682	1145	136890	156739050
Business	316	463551	12876	111232	1432223232
Education	203	122795	5338	45878	244896764
Engineering	71	431915	53989	5609	302824301
Journalism	28	332648	66529	924	61472796
Nursing	35	1572	314	1400	439600
Pharmacy	44	825016	165003	2156	355746468
					2554342211
Master's Level:					
Arts	65	684125	76013	4810	365622530
Business	52	90008	15001	3016	45243016
Education	26	175368	35073	806	28268838
Engineering	11	2019018	403803	176	71069328
Sciences	53	211611	35268	3127	110283036
					620486748

*The square root of the values of S_h^2 are slightly different than the values listed in Table 8 because in Table 8 they have been rounded to the nearest dollar.

confidence, between \$5,219 and \$5,361. The average input cost for the education required for the awarding of a master's degree is, with 90 per cent confidence, between \$2,752 and \$3,098.

CHAPTER V

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Purpose

The purpose of this chapter is three-fold. First, the research methodology and results will be summarized. Second, conclusions will be drawn relative to other research methodology and results. Third, recommendations will be made with respect to possible use of the research results and future research of a similar or related nature.

Summary

This research divided the on-campus organizations of USC into product or service departments. The services rendered by the service departments, as measured by their input costs to USC, were allocated to the product departments. These allocations were based on the amount of space occupied, actual expenditures, student load and the opinions of the most knowledgeable personnel available when more empirically oriented grounds for allocation did not exist. These allocations were made by using the Gauss-Jordan matrix inversion technique as adapted by Williams and Griffin,[1] for cost

---

[1]T. H. Williams and C. H. Griffin, <u>op. cit</u>.

allocations. The computer program for this matrix inversion routine is shown in Appendix II-1.

After the allocations of the costs of the service departments to the product departments were determined, only those product departments which provided instruction for college accredited courses were given further consideration. These product departments--eight on-campus schools and colleges--are producers of research, publications, services and education; of these products only education was dealt with further. This was accomplished by dropping that portion of each product department's total cost not ascribable to education as determined by the judgments of the most knowledgeable personnel available. These steps produced a figure identified as the total cost, net of capital charges, of the education produced by the eight schools and colleges. It did not, however, measure, what education _per se_ had been produced.

Education output was measured, initially, in terms of successfully completed semester hours of instruction. This was accomplished by extracting from USC's data bank the total number of semester hours by course and section. Each course was then assigned a weight proportionate to the effort required to deliver that course. In making such weight assignments it was again necessary to rely on subjective evaluation made by the most knowledgeable persons in that area. Multiplying the weight so derived by the number of relative semester hours permitted expression of the entire output of each school and college in terms of equivalent units. Division of total costs by total output yielded the cost per equivalent unit of instruction: multiplication of the cost per equivalent unit by its relative course weight yielded the input cost per semester hour. Multiplication of the semester hours included on the transcript of a degree awardee, by the appropriate cost per semester hour yielded the input cost at USC for the education upon which award of degree was based.

Sampling techniques were selected to determine the cost of representative degrees by type and level. The statistical technique employed entailed a randomly selected sample, of either five cases from the population or 10 per cent of the population, whichever was larger, upon which to calculate the estimated mean and confidence intervals for each degree type and for each degree level. These calculations were accomplished by computer, the computer program for which is given in Appendix IV.

The estimated costs of degrees by type attained by use of these statistical techniques are shown in Table 10 and rearranged in a descending cost order by degree type in Table 11. The estimated average cost of degrees by level was found to be, with 90 per cent confidence, between $5,219 and $5,361 for undergraduate degrees and between $2,752 and $3,095 for master's level degrees.

Table 11. Cost of degrees

	Cost Rank	90 Per Cent Confidence Level
Bachelor's Level:		
Pharmacy	1	\$7687 ± 747
Engineering	1	7687 ± 408
Nursing	2	7254 ± 33
Journalism	3	5780 ± 474
Education	4	5317 ± 123
Arts & Science	5	4881 ± 56
Business	6	4508 ± 189
Master's Level:		
Engineering	1	\$4480 ± 1168
Education	2	3098 ± 344
Sciences	3	2776 ± 339
Arts	4	2708 ± 480
Business	5	1960 ± 220

Conclusions

Background

Conclusions based on the work accomplished in the dissertation need, at the outset, to be circumscribed by limitations imposed on the research. Afterward, tentative conclusions can be drawn from the research.

Research Limitations

The research in this dissertation has one basic limitation imposed upon it which is worthy of elaboration. This limitation is the lack of empirically based data upon which to make allocations.

Within this general limitation there are two specific areas which may be seriously affected by the lack of empirical data. In the first instance, some allocations of service departments' outputs had to be based upon the considered opinions of the most knowledgeable personnel available. In the second instance, the allocations of the producing departments were similarly based upon opinion.

In the later case, the proportion of professorial time devoted to teaching, as compared to time used in research, writing and providing other professional services, could not be precisely allocated.

Allocations by Nonempirical Judgments

Various allocations were made throughout this dissertation. In some instances allocations had to be based upon the judgments of the most knowledgeable personnel available and as such are the best available data. As previously noted, such allocations may be affected, among other things, more by recent experience and circumstances surrounding the individual person making the judgment than by the average or normal experiences and circumstances. Therefore, such judgmental allocations may be less objective than allocations based upon empirical data.

Professorial Time

The total teaching costs of the product departments of USC were spread over the total number of successful semester hours of delivered instruction by means of the equivalent unit. These hours of instruction were summed from the total number of hours delivered by each section of every course. Among these sections there existed variations in the number of students and the salaries of the teachers. Because no existing records were detailed enough for more penetrating analysis, this dissertation could not consider the cost differences that exist, from section to section and from course to course as section size and professorial salaries vary. For example, in this work, the hours of education produced by a small section taught by a highly paid faculty member would be costed at the same hourly rate as a large section taught by a low-paid graduate student, and vice-versa. Although there will be some averaging out of semester hour costs because so many courses, sections and faculty members are involved, clear probability exists that highly paid senior faculty members will tend to teach the most advanced courses and lower paid junior faculty members or graduate assistants will tend to teach the introductory courses.

The sections of the more advanced courses tend to be smaller than those of the introductory courses. As a result, a larger proportion of the costs of professorial salaries should be allocated to some sections and courses than to others. In this dissertation, these salary costs have been allocated as though all sections were homogeneously

constituted and taught. As a result, the costs of lower level courses may be overstated and those of higher level courses may be understated. This tendency will be offset in part by the heavier weight given to graduate courses as reflected in Table 4. It will be offset further because each degree requires a mixture of courses from various levels. Thus, the understated costs of the higher level courses will be offset by the overstated costs of the lower level courses. The computed degree costs should, therefore, serve as reasonable estimates of the real input costs of degrees.

Matrix Algebra Techniques

The matrix inversion technique and the matrix multiplications used in this dissertation were adaptable to computer usage. The inversion of the A Matrix--the cross-allocations of the service departments' outputs among service departments--when done by the computer required slightly over two minutes. This inversion required solution of 40 simultaneous equations in 40 unknowns. The approximate number of mathematical operations involved in this procedure on a matrix of this size can be estimated by taking into consideration the machine time and the machine speed. The computer performs an operation in 8 microseconds: it performs 120,000 operations per second. Use of machine time rounded to 120 seconds, multiplied by the rate of 125,000 operations per second yields 14,000,000 operations performed. If a manual operator could perform one operation per second, without human error, this calculation would require approximate 2.1 man years of 2,000 hours each. Thus matrix algebra techniques when performed on a computer and used to allocate overhead free enormous amounts of accounting time which can be devoted to refining the basis of the cost allocations or for other purposes.

Recommendations

The recommendations made in this work fall into three categories: first, a group of recommendations dealing with the pricing of education; second, a group of recommendations pertaining to the use of the techniques employed in this work; and third, a group of recommendations treating with the need for further research related to this work.

Pricing of Degrees

At USC and many other educational institutions the price of degrees--tuition--is invariant with respect to the type of degree sought by the student. This work demonstrates that the cost of producing all USC degrees is higher than the tuition charges of $500 per year for in-state and $1,000 per year for out-of-state students. For example, the bachelor's degree with the lowest input cost is in business for a total of $4,508, net of capital costs. The tuition charged for four years is $2,000 in-state and $4,000 out-of-state. The master's degree in business has the lowest input cost,

$1,960, net of capital costs, and the tuition charge for it is $750 in-state and $1,500 out-of-state.

The market value of degrees varies dependent upon earning opportunities and prestige associated with the fields of endeavor for which given degrees qualify their recipients. The value of degrees to their holders should be researched. Where the market value of a degree exceeds tuition consideration should be given to increasing tuition by the amount by which the input costs of the degree exceeds the tuition presently charged. This work does not recommend, per se, that there should be a different tuition rate for each of the degrees available. It recommends that since a cost difference exists, the value-to-holder difference may be a justifiable basis upon which to introduce a price difference to meet the need for additional funds or to establish some parity between value given and received.

Social Welfare Based Pricing

There are some degrees whose input costs may exceed their market value as well as the tuition charged for them. Some of these degrees may have a high social welfare value; for example, nurses provide a service that is essential to society in general. It may well be that irrespective of the market value of a nursing degree it is in the best interest of society to subsidize such training.

This work recommends that, where possible, some assessment be made of the social welfare value of the important services that degree holders provide to the general community. Administrators should, therefore, take into account the input cost, tuition price and value of degrees--both to the community and the degree holder and consider adjusting prices--tuition rates--to encourage students to undertake studies in areas which provide needed services to the larger community.

Out-of-State Tuition Rates

The average cost of either an undergraduate or a graduate USC degree exceeds the price paid for the education upon which it is based. For example, the out-of-state tuition at USC is $1,000 per two-semester academic year. The average undergraduate degree has an input cost of between $5,219 and $5,361, with 90 per cent confidence. An out-of-state student, therefore, receives a subsidy to the extent that the input cost of his degree exceeds the tuition charged him for his education. Administrators should consider whether out-of-state tuition rates should cover at least the input costs of delivering the education to the recipient. In particular, institutions like USC which are publicly supported, should take into consideration the number of out-of-state students it and other state institutions educate from each state and the number of students from its state that are being educated in each of those states. The relative differences between costs and prices of each such institution should be compared to USC's cost-price differential to evaluate whether quotas should be imposed against states who seem not to support "fairly" public education. To do otherwise could be interpreted as a subsidization of other states by USC or vice-versa.

Research Techniques

There are sizeable amounts of accounting and managerial time made available by the matrix inversion and matrix multiplication techniques used in--but not originated by--this dissertation. Further, these techniques are readily adaptable to other conglomerate type organizations where allocations of overheads to ultimate products is abstruse. This technique should be adopted in such cases. Part of the time saved could be spent in researching for better input data upon which to build a set of empirically based allocation decision rules.

Further Research

Cost allocation is usually based upon direct labor. In an educational institution professorial time is the direct labor element. Yet little if any empirical evidence exists relative to the way this time is devoted. Professorial time goes into research, administration, writing, teaching, and other related activities. What is needed is a definitive examination of professorial time utilization.

The work in the dissertation leads to the clear conclusion that detailed research should be done on statistically significant samples of professors at a large number of schools. Such research could provide the as yet missing link that is needed to definitively cost the output of the conglomerate institution which the university, or multiversity, has in fact become.

APPENDIX I

INVERSION PROGRAM

```
$JOB 157    04/25/68       USC CSC JOB 157
            $ID    50096,300523,010,050, MAIN PROGRAM, EWALD
            $IBJOB         WATFOR
            $IBFTC EWALD
                   DIMENSION A(42,42), U(42,42),C(42),X(42),B(42,42),AA(42,42)
                   READ ( 5, 100 ) N
               100 FORMAT ( 13 )
                   READ ( 5, 102 )  ( ( A(I,J), I = 1, N), J = 1, N )
               102 FORMAT (7F10.5)
                   WRITE (6, 1000)
              1000 FORMAT ( 1H1, /// 5X, 122H THIS IS THE INVERSION OF MATRIX A BY GAU
                  1SS-JORDAN METHOD AND THE SOLUTION OF SIMULTANEOUS EQUATIONS USING
                  2THE CONSTANTS C.  // 26X, 79H THE FIRST MATRIX PRINTED HERE IS MATR
                  3IX A AND THE SECOND IS THE INVERSION OF A.          )
                   WRITE(6,334)
                   DO 333 I=1,N
               333 WRITE(6,300) I, (A(I,J),J=1,10)
               300 FORMAT (15,10F12.5)
                   WRITE(6,334)
                   DO 356 I=1,N
               356 WRITE(6,300) I, (A(I,J),J=11,20)
                   WRITE(6,334)
                   DO 366 I-1,N
               366 WRITE(6,300) I, (A(I,J),J=21,30)
                   WRITE(6,334)
                   DO 376 I=1,N
               376 WRITE(6,300) I, (A(I,J),J=31,40)
                   WRITE(6,334)
                   DO 386 I=1,N
               386 WRITE(6,309) I, (A(I,J),J=41,42)
               309 FORMAT (15,2F12.5)
                   WRITE(6,334)
                   IFLAG = 0
                   DO 28 KKK = 1, 42
                   DO 28 LLL = 1, 42
                28 AA(KKK,LLL) = A(KKK,LLL)
```

Appendix I--Continued

```
304 FORMAT( 2F12.5)
334 FORMAT(1H1,//////////)
    DO 901 I = 1, N
    DO 901 J = 1, N
    U(I,J) = 0.0
    IF(I .EQ J) U (I,J) = 1.0
901 CONTINUE
    EPS = .1
    DO 915 I = 1,N
    K = 1
    IF( I - N ) 921, 907, 921
921 IF( A(I,I) - EPS )  905, 906, 907
905 IF( -A(I,I) - EPS ) 906, 906, 907
906 K = K + 1
    DO 923 J = 1, N
    U(I,J) = U(I,J) + U(K,J)
923 A(I,J) = A(I,J) + A(K,J)
    GO TO 921
907 DIV = A(I,I)
    DO 909 J = 1, N
    U(I,J) = U(I,J) / DIV
909 A(I,J) = A(I,J) / DIV
    DO 915 MM = 1, N
    EMS = 0.00001
    DELT = A(MM,I)
    IF(ABS(DELT) - EMS ) 915,915,916
916 IF(MM - I ) 910, 915, 910
910 DO 911 J = 1, N
    U(MM,J) = U(MM,J) - U(I,J) * DELT
911 A(MM,J) = A(MM,J) - A(I,J) * DELT
915 CONTINUE
    DO 933 I = 1, N
    DO 933 J = 1, N
933 A(I,J) = U(I,J)
 27 DO 335 I = 1,N
```

Appendix I--Continued

```
  335 WRITE(6,300) I, (A(I,J),J=1,10)
      WRITE(6,334)
      DO 345 I=1,N
  345 WRITE(6,300) I, (A(I,J),J=11,20)
      WRITE(6,334)
      DO 355 I=1,N
  355 WRITE(6,300) I, (A(I,J),J=21,30)
      WRITE(6,334)
      DO 365 (I=1,N
  365 WRITE(6,300) I, (A(I,J),J=31,40)
      WRITE(6,334)
      DO 375 I=1,N
  375 WRITE(6,309) I, (A(I,J),J=41,42)
      WRITE(6,334)
      IF(IFLAG .EQ. 1) GO TO 2
      DO 29 KK = 1,42
      DO 29 LL = 1,42
   29 B(KK,LL) = 0.0
      DO 30 IA = 1,42
      DO 30 IB = 1,42
      DO 30 IC = 1,42
   30 B(IA,IB) = B(IA,IB) + (A(IA,IC) * AA(IC,IB))
      DO 31 I = 1,42
      DO 31 J = 1,42
   31 A(I,J) = B(I,J)
      WRITE(6,334)
      IFLAG = 1
      GO TO 27
    1 READ ( 5, 120 )  ( C(I), I = 1, N )
  120 FORMAT (6F12.8)
      WRITE (6, 121 )
  121 FORMAT ( //// 15X, 102HWE NOW ARE GOING TO WRITE THE VALUES OF THE
     1CONSTANTS AND THEN BELOW THAT THE VALUES OF THE UNKNOWNS.      )
      WRITE(6,334)
```

Appendix I--<u>Continued</u>

```
        DO 880 IL = 1,N
    880 WRITE(6,202) IL, C(IL)
    202 FORMAT(   (15,F25.2))
        WRITE(6,334)
        DO 10 I = 1, N                                       79
     10 X(I) = 0.0                                           80
        DO 950 I = 1, N                                      81
        DO 950 J = 1, N                                     82
        X(I) = X(I) + A(I,J) * C(J)                            83
    950 CONTINUE                                           84
    109 DO 881 IL = 1,N
    881 WRITE(6,202)IL,  X(IL)
        WRITE(6,334)
        GO TO 1
      2 STOP                                                 86
        END                                                  87
   $ENTRY
----------------------- EXECUTION
```

APPENDIX II

MATRIX MULTIPLIER PROGRAM

```
$JOB 141   05/02/68       USC CSC JOB 141
           $ID    50096,300523,005,010, MATRIX MULTIPLY - EWALD
           $IBJOB
           $IBFTC EWALD
                  DIMENSION  A(10,42),  C(42),  X(10)
                 READ (5,101) A
             101 FORMAT (10F8.5)
                 READ (5,102)  C
             102 FORMAT  (F10.0)
                 DO 9 I = 1,10
               9 X(I) = 0.0
                 DO 7 J = 1,10
                 DO 7 K = 1,42
               7 X(J) = X(J)  + (A(J,K)*C(K))
             100 FORMAT(1H1)
                 WRITE(6,100)
                 DO 8 J = 1,42
               8 WRITE(6,104) (A(I,J),I=1,10)
             104 FORMAT(10X,10F11.5)
                 WRITE(6,100)
                 DO 6 I = 1,42
               6 WRITE(6,105) C(I)
             105 FORMAT(15X,F12.2)
                 WRITE(6,100)
                 WRITE(6,103) X
             103 FORMAT(10X,F20.4)
                 STOP
                 END
           $ENTRY
```

APPENDIX III

STATISTICAL PROGRAM

```
$JOB 156    04/25/68        USC CSC JOB 156
            $ID     50096,300523,003,050, CALCULATE Z       PROF. EWALD
            $IBJOB
            $IBFTC HONESS
                  DIMENSION IA(39,14),C(14),X(39),A(39,14),DIFFX(39),DIFXSQ(39)
                  READ(5,103) C
              103 FORMAT(7F10.2)
                  WRITE(6,106)
              106 FORMAT(10X,10H C MATRIX ,/)
                  WRITE(6,107) C
              107 FORMAT(10X,F10.2)
               10 READ(5,101) M,N
              101 FORMAT(213)
              102 FORMAT(1415)
                  DO 1 I = 1,M
                1 READ(5,102) (IA(I,J), J = 1,N)
                  WRITE(6,100)
                  WRITE(6,108)
              108 FORMAT(10X,3H M ,5X,3H N ,/)
                  WRITE(6,109) M,N
              109 FORMAT(10X,13,5X,13,///)
                  WRITE(6,110)
              110 FORMAT(10X,10H A MATRIX ,/)
                  DO 11 J = 1,M
               11 WRITE(6,111) (IA(J,L),L=1,N)
              111 FORMAT(5X,14(3X,15))
                  DO 2 J = 1,M
                2 X(J) = 0.0
                  DO 3 K = 1,M
                  DO 3 L = 1,N
                3 A(K,L) = IA(K,L)
                  DO 4 I = 1,M
                  DO 4 J = 1,N
                4 X(I) = X(I) + (A(I,J) * (C(J))
```

Appendix III--Continued

```
      WRITE(6,100)
  100 FORMAT(1H1)
      WRITE(6,104) (X(I),I=1,M)
  104 FORMAT(10X,F20.4)
      WRITE(6,100)
      XMEAN = 0.0
      DO 5 I = 1,M
    5 XMEAN = XMEAN + X(I)
      XM = M
      XMEAN = XMEAN / XM
      DO 6 I = 1,M
    6 DIFFX(I) = ABS(X(I) = XMEAN)
      DO 7 I = 1,M
    7 DIFXSQ(I) = DIFFX(I) * DIFFX(I)
      SUMDSQ = 0.0
      DO 8 I = 1,M
    8 SUMDSQ = SUMDSQ + DIFXSQ(I)
      MMONE = M - 1
      XMMONE = MMONE
      SSQ = SUMDSQ / XMMONE
      SRTSSQ = SQRT(SSQ)
      SRTXMM = SQRT(XMMONE)
      Z = SRTSSQ / SRTXMM
      WRITE(6,105) XMEAN
      WRITE(6,105) SUMDSQ
      WRITE(6,105) XMMONE
      WRITE(6,105) SSQ
      WRITE(6,105) SRTSSQ
      WRITE(6,105) SRIXMM
      WRITE(6,105) Z
  105 FORMAT(10X, E14.8)
      GO TO 10
    9 STOP
      END
$ENTRY
```

BIBLIOGRAPHY

Cochran, W. G. Sampling Techniques. 2d ed. New York: John Wiley & Sons, Inc., 1963.

Jones, Gardner M. "A Procedural & Cost Analysis Study of Media in Instructional Systems Development," Part B. Office of Education, Department of Health, Education and Welfare, 1964.

Howell, James B., and Daniel Teichroew. Mathematical Analysis for Business Decisions. Homewood, Illinois: Richard D. Irwin, Inc., 1963.

Kemeny, John G., Arthur Schleifer, Jr., J. Laurie Snell, and Gerald L. Thompson. Chapter V "Vectors and Matrices," Finite Mathematics with Business Applications. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1964.

Mattessich, Richard. Accounting and Analytical Methods. Homewood, Illinois: Richard D. Irwin, Inc., 1964.

Theodore, Chris A. Chapter IX "The Algebra of Linear Programming," Applied Mathematics: An Introduction. Homewood, Illinois: Richard D. Irwin, Inc.

Yamane, Taro. Elementary Sampling Theory. Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1967.