

THE EFFECT OF PORTFOLIO SIZE
ON PORTFOLIO PERFORMANCE:
AN EMPIRICAL ANALYSIS

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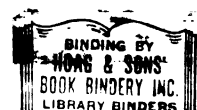
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ABSTRACT

THE EFFECT OF PORTFOLIO SIZE ON PORTFOLIO PERFORMANCE: AN EMPIRICAL ANALYSIS

By

Thomas A. Ulrich

Two factors determine the risk-return performance of a portfolio. They are the individual securities held and the diversification strategy of the portfolio. Diversification strategy can be divided into two parts, the number of securities held and the proportion of funds invested in each. This research focuses on the effect of portfolio size on portfolio performance where size is measured by the number of different securities held. Portfolio performance is measured in two dimensions, return and risk. The objective of this research is to strengthen the existing empirical knowledge regarding the relationship between portfolio size and portfolio return and between portfolio size and portfolio risk. In addition, the intent of this research is to determine an optimal portfolio size for common stock portfolios.

A number of research studies have attempted to determine the effect of portfolio size on portfolio performance. To date, the basic empirical research has been done with random portfolios. Random selection of securities has two limitations. First, random

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selection of securities for investment portfolios is practiced only by academicians, not practitioners. Second, while the random selection procedure provides the effect of portfolio size on portfolio risk, it precludes the determination of the effect of portfolio size on portfolio return. The return on a portfolio of common stocks selected randomly from a feasible set of common stocks is an unbiased and consistent estimator of the mean return on that feasible set. Hence, the effect of portfolio size on portfolio return is lost when random selection is employed. Consequently, only one dimension, risk, has been considered in determining the effect of portfolio size on portfolio performance. This research represents an initial attempt to employ nonrandom portfolios in determining the effect of portfolio size on portfolio performance. Due to the employment of nonrandom portfolios, both dimensions of portfolio performance, return and risk, are investigated to determine the effect of portfolio size.

The year-end portfolios (1967-1970) of eight randomly selected growth-income mutual funds provided thirty-two nonrandom portfolios. In order to measure the effect of portfolio size on portfolio return, portfolio risk and overall portfolio performance, it was necessary to simulate the portfolio building process. This was accomplished by ranking the securities within each mutual fund portfolio in order of portfolio inclusion, where the market value of the mutual fund's investment in each security was used as a proxy for the portfolio manager's ranking for portfolio inclusion. Furthermore, to permit the measurement of the effect of portfolio size on the various portfolio parameters, this research abstracted from the funds

allocation decision and assumed the equal allocation of investment funds.

The simulation of the nonrandom portfolios of increasing size led to statistical dependence among the parameter measurements on the simulated portfolios. While statistical dependence among the simulated portfolios precludes the statistical testing of regression parameters individually, they can be pooled and tested for statistical significance as a group. Hence, analysis of variance models were utilized to determine the effect of portfolio size on portfolio return, portfolio risk and overall portfolio performance.

There is no optimal portfolio size for these common stock mutual funds. It was found that the effect of portfolio size on the various portfolio parameters was dependent on the holding period. Hence, what may have been optimal one period may be less than optimal the next. Moreover, when the longer four year period was considered, the null hypotheses that portfolio return and overall portfolio performance were independent of portfolio size could not be rejected. The way in which securities are selected for portfolio inclusion (randomly vs. nonrandomly) affects the relationship between portfolio size and portfolio risk. While the benefits of diversification dropped off rapidly, the degree of this relationship was found to be significantly less than for random selection. The large portfolio sizes of the common stock mutual funds do not represent constrained optima. Some have argued that due to the large dollar investment of mutual funds, they must hold large portfolios so that their trading does not affect security prices. It is these large portfolio sizes,

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it is argued, that constrain mutual funds from above average performance. However, the lack of evidence of an unconstrained optimal size in this research leads to the conclusion that the large portfolio sizes are not optima either constrained or unconstrained.

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CHAPTER I

INTRODUCTION

Purpose of Research

Two factors determine the risk-return performance of a portfolio. They are the individual securities held and the diversification strategy of the portfolio. Diversification strategy can be divided into two parts, the number of securities held and the proportion of funds invested in each. This research focuses on the effect of portfolio size on portfolio performance where size is measured by the number of different securities held.¹ Portfolio performance is measured in two dimensions, return and risk. The objective of this research is to strengthen the existing inadequate empirical knowledge regarding the relationship between portfolio size and portfolio return and between portfolio size and portfolio risk. In addition, the intent of this research is to determine an optimal portfolio size for common stock portfolios and to rationalize the observed portfolio sizes of common stock mutual funds.

¹Unless otherwise specified, portfolio size will be measured by the number of different securities held.

A number of research studies have attempted to determine the optimal portfolio size for common stock portfolios.² To date, the basic empirical research has been done with random portfolios.³ Random selection of securities has two limitations in determining the optimal portfolio size. First, random selection of securities for investment portfolios is practiced only by academicians, not practitioners. Second, while the random selection procedure provides the effect of portfolio size on portfolio risk, it precludes the determination of the effect of portfolio size on portfolio return. Consequently, only one dimension, risk, is considered in determining the optimal portfolio size. The return on a portfolio of common stocks which are randomly selected from a feasible set of common stocks is an unbiased and consistent estimator of the mean return on that feasible set.⁴ That is, a portfolio consisting

²John L. Evans and Stephen H. Archer, "Diversification and the Reduction of Dispersion: An Empirical Analysis," Journal of Finance, XXII (December, 1968), 761-768; Lawrence Fisher and James H. Lorie, "Some Studies of Variability of Returns on Investment in Common Stocks," Journal of Business, XLIII (April, 1970), 99-134; Jack E. Gaumnitz, "Maximal Gains from Diversification and Implications for Portfolio Management," Mississippi Valley Journal of Business and Economics, VI (Spring, 1971), 1-14; Henry A. Latane and William E. Young, "Test of Portfolio Building Rules," Journal of Finance, XXIV (September, 1969), 595-612; Per B. Møkkelbost, "Unsystematic Risk Over Time," Journal of Financial and Quantitative Analysis, VI (March, 1971), 785-795.

³Unless otherwise specified "random portfolios" will refer to portfolios where securities are chosen for portfolio inclusion at random and "nonrandom portfolios" will refer to portfolios where securities are chosen for portfolio inclusion by some nonrandom means.

⁴An estimator is unbiased if its expected value is identical with the population parameter being estimated. An estimator is consistent if the probability of it approaching the parameter being estimated is one as the sample size approaches infinity.

of randomly selected common stocks from the New York Stock Exchange has a return which is an unbiased and consistent estimator of the return on a portfolio consisting of all common stocks on the New York Stock Exchange. Hence, the effect of portfolio size on portfolio return cannot be determined if random selection is employed. This research represents an initial attempt to employ nonrandom portfolios in determining the effect of portfolio size on portfolio performance.

Due to the employment of nonrandom portfolios, both dimensions of portfolio performance, return and risk, can be investigated for the effect of portfolio size. Moreover, both return and risk can be considered in determining the optimal portfolio size. Common stock portfolios of growth-income mutual funds are the nonrandom portfolios employed in this research.⁵

In addition to focusing on the portfolio return function, this research is employing two risk surrogates which have not been employed previously in determining the effect of portfolio size on portfolio risk. These surrogates are the modified quadratic mean⁶ and the index of unfavorable variation.⁷ Both risk surrogates

⁵For classification, see Wiesenberger Services, Inc., Investment Companies (New York: Wiesenberger Services, Inc., 1968-1971).

⁶Robert A. Levy, "Measurement of Investment Performance," Journal of Financial and Quantitative Analysis, III (March, 1968), 45-46.

⁷Richard S. Bower and Ronald F. Wipperfurth, "Risk-Return Measurement in Portfolio Selection and Performance Appraisal Models: Progress Report," Journal of Financial and Quantitative Analysis, IV (December, 1969), 423-427.

measure only downside variability, and therefore, intuitively appear to be more meaningful risk proxies than other risk surrogates which measure total variability.

Background and Existing Research

At this point it is essential that an objective measure of portfolio performance be established. Equally essential is the review of earlier research dealing with the effect of portfolio size on portfolio performance. This section first establishes the portfolio performance measure to be employed in this research. Then, since both portfolio risk and portfolio return are considered in measuring portfolio performance, the effect of portfolio size on each is examined. Finally, the conclusions drawn from these inquiries will be combined to discuss the effect of portfolio size on portfolio performance.

Portfolio Performance

What are the objectives of portfolio management? Traditional texts in portfolio management imply that investment portfolios are individual in nature and that there is no one set of objectives.⁸ However, recent advances in portfolio theory allow the establishment of some general portfolio management objectives. But in setting up these objectives one should keep in mind the measurement of portfolio performance. For, if performance cannot be measured in some

⁸See for example, Harry Sauvain, Investment Management (3rd ed.; Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1967), pp. 113-252. Here the portfolio manager is a kind of financial interior decorator, designing portfolios to reflect the owner's individual personality.

consistent way, there is little use in setting objectives. Without a measure of performance one will never know whether the objectives are being met, and one will certainly have very little idea of whether or not any changes in the portfolio, e.g., portfolio size, are improving performance.

Modern portfolio theory is based on Markowitz's mean return-risk criteria.⁹ The criteria describe a superior portfolio as one which has the highest mean return for a given level of risk or the smallest risk for a given level of mean return. While the mean return-risk criteria allow the ranking of portfolios in order of performance from best to worst for a given level of return or a given level of risk, the ranking of portfolios is not permitted if one of the dimensions of performance, return or risk is not on the same level. This can best be illustrated with an example.

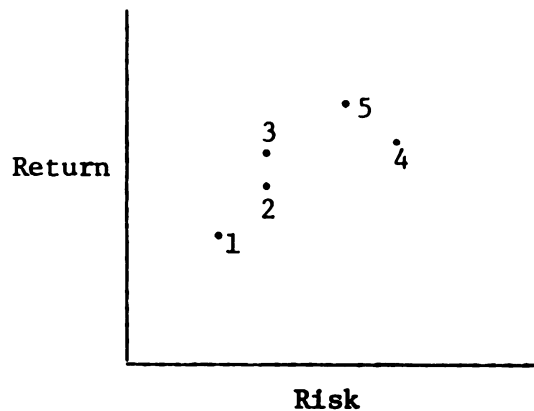


Figure 1

Risk-Return Space

⁹Harry M. Markowitz, "Portfolio Selection," Journal of Finance, VII (March, 1952), 77-91. Markowitz used variance of return as his risk measure.

Which portfolio of the five plotted in risk-return space in Figure 1 has the best performance? Based on the mean return-risk criteria portfolio 3 is superior to both 2 and 4. In the first case, 3 has achieved a higher level of return with the risk being the same as that of portfolio 2. In the second case, 3 has achieved a lower risk level with the mean return being the same as 4. But, what about portfolios 1 and 5? Are they better or worse than 3? Which of the three is best?

A criterion of best for portfolio ranking was developed independently by Sharpe¹⁰ and Lintner.¹¹ For a mathematical development and proof of the criterion the reader should consult these two sources directly.¹² For the reader an intuitive justification is given here.¹³ By jointly evaluating return and risk along with the risk free rate of interest, a means is provided for the ranking of portfolio performance. For each portfolio a ratio denoted by θ is computed. θ is equal to the portfolio's observed excess return divided by the portfolio's observed risk, S , where excess portfolio

¹⁰William F. Sharpe, "Capital Asset Pricing: A Theory of Market Equilibrium Under Conditions of Risk," Journal of Finance, XIX (September, 1964), 425-442.

¹¹John Lintner, "Security Prices, Risk and Maximal Gains from Diversification," Journal of Finance, XX (December, 1965), 587-615.

¹²Eugene F. Fama, "Risk, Return and Equilibrium: Some Clarifying Comments," Journal of Finance, XXIII (March, 1968), 29-40. Fama shows that the differences between the Sharpe and Lintner models are easily reconciled and that Sharpe's model is actually a special case of Lintner's more general model.

¹³The following discussion is adapted from Bower and Wipperfurth, op. cit., pp. 418-421.

return is equal to the difference between the mean portfolio return, R , and the risk free rate of interest, r_f .

$$\theta = \frac{R - r_f}{S} \quad (1)$$

The portfolio with the highest θ value is the best portfolio; it is best because it equals any other portfolio of the set of ranked portfolios in one dimension, either return or risk, while it does better in the other dimension. The matching is accomplished by leveraging the portfolio or by offsetting it with riskless investments. This is shown graphically in Figure 2, which is the same as Figure 1 except that the risk free rate of return, r_f , and a straight line drawn through the two points, r_f and portfolio 3, have been added.

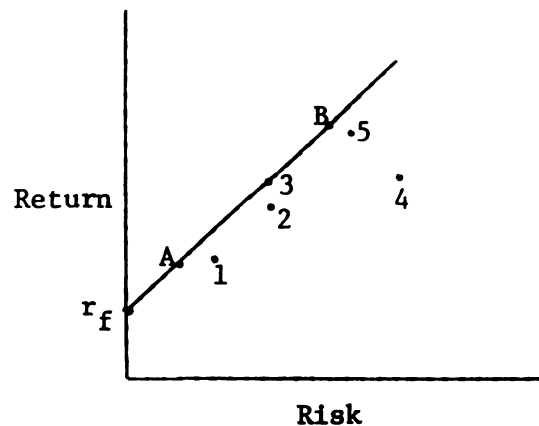


Figure 2

Performance Ranking

For any portfolio plotted in Figure 2 there is a straight line that can be drawn through it and r_f which will yield all the risk-return combinations that may be achieved by leveraging or offsetting the portfolio. Points, A and B, along the line through portfolio 3

indicate portfolios which match portfolios 1 and 5, respectively, on return but have less risk. Portfolio A is a combination of portfolio 3 and a riskless investment yielding rate r_f . Portfolio B is a combination of portfolio 3 and borrowed funds at rate r_f which are invested in portfolio 3. θ reports the slope of the straight line through the portfolio and the risk free rate of interest.

The portfolio that has the highest slope, and therefore the largest θ value, is the best portfolio in the sense that it can give the investor more return at any given risk level than any other portfolio in the set.¹⁴ Consequently, a generalized portfolio objective for portfolio management is to maximize θ . It is this θ measure that will be used to measure portfolio performance in this study.

Effect of Portfolio Size on Portfolio Risk

To date, the existing research on the effect of portfolio size has dealt almost exclusively with risk, and risk reduction has been the sole benefit of diversification. Until the advent of Markowitz's 1952 article,¹⁵ diversification was typically the "not all your eggs in one basket" approach. Risk was either assumed away or treated qualitatively. An example of a sophisticated approach to handling risk qualitatively is Professor Sauvain's textbook.¹⁶ Here the stated

¹⁴The ranking of the portfolios in Figure 2 from best to worst using θ is 3, 5, 2, 4, and 1.

¹⁵Markowitz, loc. cit.

¹⁶Sauvain, loc. cit.

objective of diversification is to minimize losses. Sauvin, however, ignores the problem of portfolio size.

Employing the variance of portfolio return as the measure of risk, Markowitz diversification combines securities that are less than perfectly positively correlated to reduce the variance of portfolio return without sacrificing portfolio return. From a feasible set of securities Markowitz diversification allows the investor to obtain a set of optimum or efficient portfolios according to the mean return-variance criteria. From this optimum set the investor selects that portfolio which best satisfies his risk-return preferences. The optimum number of securities to hold, therefore, is a function of the desired risk-return preference and the feasible set of securities. In practice, unless constrained, the portfolios on the Markowitz efficient frontier typically contain six to ten securities.¹⁷ Implementation of Markowitz's model, however, is very costly and time consuming because of the enormous data requirements.

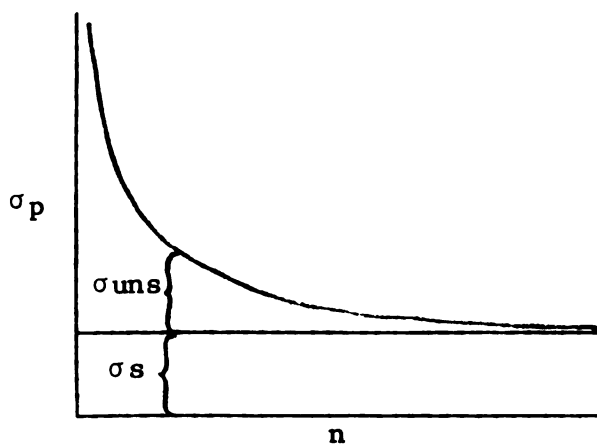
The dichotomization of risk into systematic and unsystematic risk by Sharpe¹⁸ has elucidated the effect of portfolio size on portfolio risk.¹⁹ Systematic risk is that portion of the variance of return explained by the market, i.e., it is the covariance of

¹⁷Peter O. Dietz, "Review of an Empirical Analysis of Some Aspects of Common Stock Diversification," by Edward H. Jennings, Journal of Financial and Quantitative Analysis, VI (March, 1971), 851.

¹⁸Sharpe, loc. cit.

¹⁹Apart from negatively correlated securities, all risk reduction results from averaging over the independent components of risk of the individual securities. It is in this area that the size factor has its effect.

the return of a security with the market. Unsystematic risk is that portion of the variance of return that remains unexplained by the market and is unique to that individual security. The implication is that the systematic risk of securities is that portion of risk that remains when a security is combined with others to form an efficient portfolio; all unsystematic risk is diversified away.



σ_p = the total variation in portfolio rate of return
 σ_s = the systematic risk component (the asymptotic value)
 σ_{uns} = the unsystematic risk component
 n = the number of securities in the portfolio

Figure 3

The Effect of Portfolio Size on Portfolio Risk

That is, increasing the number of different securities held reduces the unsystematic component of return variation. As the investor increases the number of securities in his portfolio, the unsystematic component of the portfolio's standard deviation asymptotically approaches its lower limit of zero leaving only the systematic component.

Several studies²⁰ have attempted, with the use of random portfolios, to measure the effect of portfolio size on portfolio risk in order to determine an optimal portfolio size. These empirical studies conclude that the common stock investor can virtually eliminate unsystematic risk with a portfolio that contains a small number of securities. Evans and Archer found that a typical random portfolio with equal dollar amounts in five securities will have only fourteen percent more risk than the most highly diversified portfolio. A typical random portfolio of ten securities will have only seven percent more than the minimum possible, while a typical random portfolio of twenty will have only three percent more than the minimum. Latane and Young found that for random portfolios with equal dollar amounts in each security, a four-stock portfolio has only fifteen percent more risk than the minimum, and eight-stock portfolio has only eight and a half percent more, and a sixteen-stock portfolio only five percent more. Fisher and Lorie also found that the opportunity to reduce dispersion by increasing the number of stocks in the random portfolio is rapidly exhausted. In addition to using the standard deviation of portfolio return as the risk measure as in the two studies above, Lorie and Fisher also employed the mean absolute deviation, Gini's mean difference, the coefficient of variation, the relative mean absolute deviation, and Gini's coefficient of concentration.²¹ The latter three are

²⁰ Evans and Archer, loc. cit.; Fisher and Lorie, loc. cit.; Latane and Young, loc. cit.

²¹ Gini's statistics are discussed in Fisher and Lorie, op. cit., pp. 102-104.

relative measures of dispersion. The results of Fisher and Lorie's study are summarized in Table 1 below.

TABLE 1
DISPERSION OF RETURNS ON N-STOCK PORTFOLIOS AS A
PERCENTAGE OF DISPERSION OF MARKET PORTFOLIOS*

	Number of Stocks in Portfolio						Entire Market
	1	2	8	16	32	128	
Standard Deviation	180	146	113	107	103	101	100
Mean Absolute Deviation	167	140	113	107	104	101	100
Gini's Mean Difference	168	141	113	107	104	101	100
Coefficient of Variation	176	143	112	106	103	101	100
Relative Mean Absolute Deviation	142	124	107	104	102	101	100
Gini's Coefficient of Concentration	148	128	109	105	103	101	100

*These results are based on 40 one-year holding periods of portfolios of stocks from NYSE 1926-1965.

The conclusion drawn from these studies using random portfolios is that the effect of portfolio size on portfolio risk is to decrease risk as size increases and, moreover, that the risk reduction effect drops off rapidly as portfolio size increases.

Because of the explicit relationship determined by Evans and Archer²² between the reduction of unsystematic risk and portfolio size, the following unwarranted implication is sometimes made. The reduction of unsystematic risk by increasing portfolio size has the same average effect no matter what stocks are held and no matter what

²²Evans and Archer, loc. cit.

the point in time. If this implication were true, then given the investor's risk-return preference function, he will always hold approximately the same number of securities in his portfolio. Mokkelbost,²³ working with random portfolios from different time periods, checked to see if the relationship between units of risk reduction and increments of portfolio size are stable or not. Mokkelbost's conclusion is that there is not a numerically stable and predictable relationship between the number of different securities held in a portfolio and the reduction of the unsystematic portion of the variability of the portfolio's rate of return. The only relationship that does hold is the general one that most of the unsystematic variation is reduced when a relatively few different securities are included in the portfolio.

Whitmore²⁴ gives an exact mathematical expression of the relationships between the number of securities in a portfolio, n , and the reduction in portfolio variance using the properties of simple random sampling and the definition of portfolio variance:

²³Mokkelbost, loc. cit.

²⁴G. A. Whitmore, "Diversification and the Reduction of Dispersion: A Note," Journal of Financial and Quantitative Analysis, V (June, 1970), 263-264.

$$\begin{aligned}
 V(n) &= \frac{\sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N \sigma_{ij}}{N(N-1)} + \frac{\frac{\sum_{i=1}^N \sigma_{ii}}{N} - \frac{\sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N \sigma_{ij}}{N(N-1)}}{n} \\
 &= c + \frac{(v-c)}{n}
 \end{aligned} \tag{2}$$

where

$V(n)$ = expected portfolio variance for a portfolio of size n

$v = (\sum \sigma_{ii}/N)$ = the average variance

N = number of securities in feasible set

$c = (\sum \sum \sigma_{ij})/N(N-1)$ = the average covariance

σ_{ij} = the covariance between securities i and j

σ_{ii} = the variance of security i

Equation 2 shows that as the portfolio size, n , increases, portfolio variance decreases, and that for large portfolios it is the covariance of a security that is most important in determining the incremental risk that a security adds to the portfolio rather than its variance. This relationship between portfolio variance and portfolio size however, is valid only for randomly selected portfolios and does not hold generally for portfolios obtained by other selection criteria. Consequently, it is necessary to investigate the effect of portfolio size on portfolio risk for nonrandom portfolios.

There is an additional benefit derived from increasing portfolio size. It deals with the systematic component of risk which is measured by the portfolio's beta factor. The beta factor is an index of systematic risk. While no economic variable including the beta

factors is constant over time. Blume²⁵ has shown empirically that beta factors for portfolios of increasing size show increasing intertemporal stability. Thus, increasing portfolio size allows one to better design the level of systematic risk for the portfolio.

Effect of Portfolio Size on Portfolio Return

The empirical research on the effect of portfolio size on portfolio return is almost nonexistent. As was pointed out above, random selection of securities for portfolios precludes the determination of the effect of portfolio size on portfolio return. Latane and Young²⁶ attempted to show the effect of portfolio size on portfolio return but in fact were really measuring the reduction in variability. For random portfolios with equal investment in each security, their study showed that as portfolio size increased from one to 224, the portfolio geometric mean return increased from 12.6 percent to 15.1 percent. Again, the benefits from diversification were rapidly exhausted. A four-stock portfolio achieved 67 percent of the maximum potential gain from diversification; 84 percent was achieved with an eight-stock portfolio. This gain in the portfolio geometric mean return, however, reflects the reduction in variability of return in as much as the geometric mean can be approximated as:²⁷

²⁵ Marshall E. Blume, "On the Assessment of Risk," Journal of Finance, XXVI (March, 1971), 1-10.

²⁶ Latane and Young, loc. cit.

²⁷ William E. Young and Robert H. Trent, "Geometric Mean Approximations of Individual Security and Portfolio Performance," Journal of Financial and Quantitative Analysis, IV (June, 1969), 181-182.

$$G^2 \approx A^2 - (SD)^2 \quad (3)$$

where

G = geometric mean

A = arithmetic mean

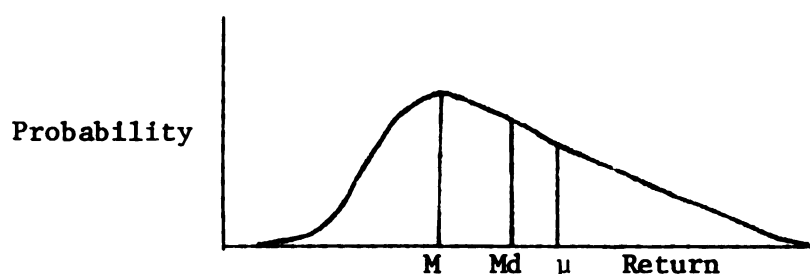
SD = standard deviation

The geometric mean return for random portfolios increases as size increases because while the arithmetic mean return remains constant the standard deviation decreases. While the concept should not be ignored, this research also focuses on the effect of portfolio size on the first term on the right hand side of equation 3. In order to do so, nonrandom portfolios are required.

While the results of empirical studies with random portfolios imply that anyone can achieve the same magnitude of risk reduction by increasing portfolio size, nothing can be implied about the effect on portfolio return. The size effect on portfolio return is dependent on one's ability to rank securities in order of portfolio inclusion. To be sure, an investor with perfect foresight who is constrained to invest his funds equally among n securities will have a portfolio return function that is monotonically decreasing with size. Few investors, however, if any, seem to possess such foresight. Historically, the benefit of diversification has been risk reduction. With the recent evidence that the minimum possible risk level can almost be achieved with a relatively few securities, the optimal portfolio size has been set at or less than sixteen securities.²⁸ A sixteen-stock portfolio has only five percent more

²⁸ Henry A. Latane and Donal L. Tuttle, Security Analysis and Portfolio Management (New York: The Ronald Press, 1970), p. 576.

risk than the most highly diversified portfolio, and the additional risk reduction as portfolio size increases is slight. Implicit in this optimal size is the assumption of a portfolio return function that is not increasing with increasing portfolio size. That is, since portfolio return is not increasing with increasing portfolio size and since the marginal reduction in risk with increasing portfolio size is slight once sixteen securities are held, there is no need to hold more than sixteen securities.



M = mode
Md = median
 μ = mean

Figure 4

Positively Skewed Probability Distribution

While it may not be rational for one who can predict security values to hold more than a few securities, it may be very rational for one who cannot predict security values to hold a large number of securities. In a positively skewed distribution, like that for security return in Figure 4, the mean lies above the mode and median. Therefore, as one increases his portfolio size the most likely portfolio return will increase, from the mode to the mean. That is, if one return is randomly selected from the probability distribution given by Figure 4, the most likely selected return would be the mode,

because that return has the greatest chance of being selected. But, as additional returns are randomly selected and the average computed, the most likely average or portfolio return approaches the mean of the distribution due to the property of consistency.²⁹

What does the introduction of capital market theory add to the discussion of the effect of portfolio size on portfolio return? More specifically, what is the contention as to the ability of investors to predict security values, since the necessary prerequisite of a monotonically decreasing portfolio return function is the ability to make such predictions. Two main conclusions of capital market theory are that the capital markets are efficient and that an investor only receives compensation for the systematic risk that he bears since the remainder, the unsystematic portion, can be diversified away. Without going into detail,³⁰ capital market efficiency means that the current market price of a security is the best estimate of its true value, and the expected return on that security can be best estimated from the security market line. That is, the expected return from a security, $E(R)$, depends on the risk free rate of interest, r_f , the expected return on the market, $E(R_m)$, and the security's beta factor, B .

²⁹See footnote 4.

³⁰For excellent reviews of both the theory and the empirical research dealing with capital markets the reader is directed to Michael C. Jensen, "Capital Markets: Theory and Evidence," Bell Journal of Economics and Management Science, III (Autumn, 1972), 357-398 and Eugene F. Fama, "Efficient Capital Markets: A Review of Theory and Empirical Work," Journal of Finance, XXV (May, 1970); 383-417.

$$E(R) = r_f + B(E(R_m) - r_f) \quad (4)$$

The implication here is that undervalued securities are rare and returns in excess of that justified by the level of systematic risk are random events. Hence, with the assumption that a portfolio manager is constrained to selecting securities from a given risk class,³¹ i.e., a given level of systematic risk, capital market theory predicts no effect of portfolio size on portfolio return. Furthermore, given that an investor does possess a talent for predicting security values, the rarity of undervalued securities implies a portfolio return function which would decline rapidly with size asymptotically approaching the level of return justified by its risk class.³²

Effect of portfolio Size on Portfolio Performance

Based on the inquiries into the effect of portfolio size on portfolio risk and portfolio return, what can be said about the effect of portfolio size on overall performance as measured by θ ? No relationship, either theoretical or empirical, has been established between portfolio size and portfolio risk for other than the situation where securities are selected randomly for portfolio

³¹If the portfolio beta factor increased or decreased as size increased, then, of course, the portfolio return function would increase or decrease, respectively.

³²Such a return function could be represented mathematically as $R_n = b_0 + b_1(1/n)$ where R_n = return on a portfolio of size n , b_0 = expected return on a portfolio of that particular risk class, b_1 = excess return on the rare undervalued securities, and n = portfolio size.

inclusion. Assuming risk behaves in a similar fashion in nonrandom portfolios as it does in random portfolios, the effect of size on portfolio risk is given by equation 5:

$$S_n = a_0 + a_1 (1/n) \quad (5)$$

where

- S_n = portfolio risk for a portfolio size n
- a_0 = systematic portfolio risk
- a_1 = unsystematic portfolio risk
- n = portfolio size

The portfolio return functions based on capital market theory are given by equations 6 and 7. Equation 6 represents the situation where the investor is able to predict security values but, due to efficient capital markets, undervalued securities and therefore returns in excess of those justified by the level of systematic risk are rare.

$$R_n = b_0 + b_1 (1/n) \quad (6)$$

In this case, the return on a portfolio of size n , R_n is equal to the return level justified by the portfolio's level of systematic risk, b_0 , plus the excess return from the limited number of undervalued securities, b_1 , averaged over n . Equation 7 represents the situation where the investor is unable to predict security values. Therefore, portfolio return, R_n , is independent of portfolio size and equal to the justified return based on the portfolio's level of systematic risk, b_0 .

$$R_n = b_0 + (0) n \quad (7)$$

Substituting equations 5 and 6 into equation 1, the effect of portfolio size on portfolio performance can be determined for the situation where the investor is able to predict security values in an efficient capital market.

$$\theta_n = \frac{R_n - r_f}{S_n} = \frac{b_o + b_1(1/n) - r_f}{a_o + a_1(1/n)} \quad (8)$$

The marginal effect of size on θ is best illustrated by the derivative of θ_n with respect to n , which provides the rate of change of θ_n with change in n . The derivative, therefore, provides a basis for a decision policy with respect to the optimal portfolio size. If $d\theta_n/dn$ is greater than zero, the policy of increasing portfolio size increases θ and therefore performance. If $d\theta_n/dn$ is less than zero, the policy of decreasing portfolio size increases θ . And, if $d\theta_n/dn$ is equal to zero and $d^2\theta_n/dn^2$ is less than zero,³³ θ_n is at its maximum point, and the optimum portfolio size is located at this point. The differentiation of equation 8 is presented below.

$$\begin{aligned} d\theta_n/dn &= \frac{(a_o + a_1 n^{-1}) (-b_1 n^{-2}) - (b_o + b_1 n^{-1} - r_f) (-a_1 n^{-2})}{a_o^2 + 2a_o a_1 n^{-1} + a_1^2 n^{-2}} \quad (9) \\ &= \frac{-a_o b_1 n^{-2} - a_1 b_1 n^{-3} + a_1 b_o n^{-2} + a_1 b_1 n^{-3} - r_f a_1 n^{-2}}{a_o^2 + 2a_o a_1 n^{-1} + a_1^2 n^{-2}} \\ &= \frac{(a_1 b_o - a_o b_1 - r_f a_1) n^{-2}}{a_o^2 + 2a_o a_1 n^{-1} + a_1^2 n^{-2}} \end{aligned}$$

³³The necessary and sufficient conditions for a maximum of a function are that the first derivative of the function be equal to zero and that the second derivative be less than zero with respect to the independent variable.

Let $b'_0 = b_0 - r_f$, then

$$d\theta_n/dn = \frac{(a_1 b'_0 - a_0 b_1) n^{-2}}{a_0^2 + 2a_0 a_1 n^{-1} + a_1^2 n^{-2}} \quad (10)$$

Since the denominator represents the square of the risk function and since the risk level asymptotically approaches the level of market risk as n approaches infinity, the denominator is finite and positive. Therefore, the decision policy with respect to portfolio size depends on the numerator. Table 2 presents the decision policy with respect to portfolio size when security values can be predicted in a relatively efficient capital market.

TABLE 2
DECISION POLICY WITH RESPECT TO PORTFOLIO SIZE
WHEN SECURITY VALUES CAN BE PREDICTED IN
A RELATIVELY EFFICIENT CAPITAL MARKET

Relationship	Policy
$a_1 b'_0 > a_0 b_1$	Increase portfolio size
$a_1 b'_0 < a_0 b_1$	Decrease portfolio size
$a_1 b'_0 = a_0 b_1$ and $d^2\theta_n/dn^2 < 0$	Optimum portfolio size, θ maximized

Similarly, equations 5 and 7 can be substituted into equation 1 to determine the effect of portfolio size on portfolio performance for the situation where the investor is unable to predict security values in an efficient capital market. Again, the decision policy with respect to portfolio size depends on the numerator of the first

derivative of θ_n with respect to n . The mathematics are presented below.

$$\theta_n = \frac{R_n - r_f}{S_n} = \frac{b_o - r_f}{a_o + a_1(1/n)} \quad (11)$$

$$\frac{d\theta_n}{dn} = \frac{(a_o + a_1 n^{-1})(0) - (b_o - r_f)(-a_1 n^{-2})}{a_o^2 + 2a_o a_1 n^{-1} + a_1^2 n^{-2}} \quad (12)$$

Let $b'_o = b_o - r_f$, then

$$\frac{d\theta_n}{dn} = \frac{(b'_o a_1) n^{-2}}{a_o^2 + 2a_o a_1 n^{-1} + a_1^2 n^{-2}} \quad (13)$$

Table 3 presents the decision policy with respect to portfolio size when security values cannot be predicted in an efficient capital market.

TABLE 3

DECISION POLICY WITH RESPECT TO PORTFOLIO SIZE
WHEN SECURITY VALUES CANNOT BE PREDICTED
IN AN EFFICIENT CAPITAL MARKET

Relationship	Policy
$b'_o a_1 > 0$	Increase portfolio size
$b'_o a_1 < 0$	Decrease portfolio size
$b'_o a_1 = 0$ and $a_1 = 0$	θ_n is independent of size
$b'_o a_1 = 0$ and $b'_o = 0$	θ maximized by holding only riskless securities yielding r_f

Equations 10 and 13 make one point clear. With the risk and return functions used, the marginal effect of portfolio size on θ

diminishes rapidly. Both equations have the n^{-2} factor in their numerators. For a portfolio containing twenty securities n^{-2} would equal 0.0025, while n^{-2} would equal 0.04 for a portfolio containing five securities. In other words, the marginal effect of increasing portfolio size would be 93.5% lower at the twenty security level than at the five security level.

What if, however, the investor is able to predict security values and the capital markets are not efficient? In this situation the portfolio return function could be approximated as:

$$R_n = b_o - b_1 n \quad (14)$$

where

R_n = return on a portfolio of size n

b_o = return on the most undervalued security

$-b_1$ = the marginal effect of size

Substituting equations 14 and 5 into 1, the effect of portfolio size on performance can be determined for this situation.

$$\theta_n = \frac{R_n - r_f}{S_n} = \frac{b_o - b_1 n - r_f}{a_o + a_1 (1/n)} \quad (15)$$

$$\frac{d\theta_n}{dn} = \frac{(a_o + a_1 n^{-1}) (-b_1) - (b_o - b_1 n - r_f) (-a_1 n^{-2})}{a_o^2 + 2a_o a_1 n^{-1} + a_1^2 n^{-2}} \quad (16)$$

Let $b'_o = b_o - r_f$, then

$$\begin{aligned} \frac{d\theta_n}{dn} &= \frac{-a_o b_1 - a_1 b_1 n^{-1} + a_1 b'_o n^{-2} - a_1 b_1 n^{-1}}{a_o^2 + 2a_o a_1 n^{-1} + a_1^2 n^{-2}} \\ &= \frac{-a_o b_1 - 2a_1 b_1 n^{-1} + a_1 b'_o n^{-2}}{a_o^2 + 2a_o a_1 n^{-1} + a_1^2 n^{-2}} \end{aligned} \quad (17)$$

Again, the numerator dictates the portfolio size policy depending on whether it is positive, negative or zero. But, note that in this situation the numerator is not dominated by the n^{-2} factor. The first term of the numerator is completely independent of size, and the negative sign favors the policy of decreasing portfolio size to increase performance.

Summary

From the investigation of the existing knowledge concerning the effect of portfolio size on portfolio performance presented in Chapter I, the necessity to expand the empirical evidence to include nonrandom portfolios is apparent. First, because nonrandom portfolios permit the measurement of the effect of portfolio size on portfolio return, and therefore allow portfolio performance to be measured in two dimensions, return as well as risk. Second, because investors in practice select the securities in their portfolios by other means than at random.

This research employs the nonrandom portfolios of growth-income mutual funds to investigate the effect of portfolio size on portfolio performance. Chapter II discusses the research hypotheses, sample selection and data collection. The presentation of the statistical models employed and the research results is the subject of Chapter III. Chapter IV contains the conclusions and recommendations for further research resulting from this study along with a summary of the research.

CHAPTER II

RESEARCH DESIGN

In this chapter, attention is first directed toward mutual funds, the nonrandom portfolios employed in this research. Initial discussion focuses on the implications of previous empirical research. The hypotheses investigated in this research are then classified into three categories, and each category is discussed individually. Subsequently, the hypotheses are operationalized by presenting the sampling and measurement procedures employed.

Mutual Funds

No set of portfolios has undergone more investigation than those of the mutual funds, due mainly to the ready availability of data. This study employs mutual fund portfolios as the nonrandom portfolios to be used in the investigation of the effect of portfolio size on portfolio performance.

The purported economic functions of mutual funds are to provide diversification and professional management. With the empirical evidence obtained from random portfolios indicating that a small portfolio can virtually eliminate the diversifiable risk, the implication follows that providing professional management is the major function. Being in competition with other saving and investment institutions, mutual funds, in order to attract and retain clientele,

have tried to convey a favorable image of their professional management expertise. Professional management connotes more than the selecting of securities from a desired risk class; it also implies achieving a higher than average return from the desired risk class. Moreover, professional management implies a portfolio return function that is monotonically decreasing with size.

The professional management image has been tarnished somewhat with the empirical research findings of Friend,³⁴ Sharpe,³⁵ Jensen,³⁶ and Williamson³⁷ which show that on the average mutual fund portfolios do not do significantly better or worse than portfolios of equal riskiness selected from the market randomly. As a result, mutual funds have been attacked for overdiversification. The Investment Company Act of 1940 and Subchapter M of the Internal Revenue Code require that a mutual fund hold no more than five percent of its assets in any given security if it is to obtain favorable tax treatment. While the five percent rule for these laws only applies to seventy-five percent of the mutual fund's total assets, state laws in Wisconsin and Ohio, among others, apply the five percent rule to 100 percent of the mutual fund's assets.

³⁴Irwin Friend, Marshall Blume, and Jean Crockett, Mutual Funds and Other Institutional Investors (New York: McGraw-Hill Book Company, 1970), pp. 52-59.

³⁵William F. Sharpe, "Mutual Fund Performance," Journal of Business, XXXIX (January, 1966), 119-138.

³⁶Michael C. Jensen, "The Performance of Mutual Funds in the Period 1945-64," Journal of Finance, XXIII (May, 1968), 389-416.

³⁷J. Peter Williamson, "Measuring Mutual Fund Performance," Financial Analysts Journal, XXVIII (November-December, 1972), 78-84.

Thus a mutual fund is constrained to hold at least twenty securities in its portfolio.

The previously cited research findings are purported to be the result of mutual funds holding considerably more securities than is required by law, i.e., overdiversification. It is argued that holding more securities than the legal minimum is a disservice to the shareholders because large portfolios deprive shareholders of the benefits of professional management, i.e., above average returns for a given risk class.

Yet the fact remains that mutual funds hold large portfolios. It is argued that mutual funds must hold a large number of different securities so that their buying and selling does not affect the price of the securities traded.³⁸ However, recent evidence presented by Scholes³⁹ questions this argument. For secondary offerings, Scholes found that the price of a common stock declines between one and two percent on the average. Moreover, Scholes does not attribute this to selling pressure, since he found no relationship between the size of the block either absolutely or as a percentage of the outstanding stock and the magnitude of the decline. Radcliffe's⁴⁰ research findings support Scholes as to the size of the relative price concession for large block trades, but Radcliffe found a

³⁸ Irwin Friend and others, A Study of Mutual Funds (Washington, D.C.: U.S. Government Printing Office, 1962), p. 361.

³⁹ Myron S. Scholes, "The Market for Securities: Substitutes Versus Price Pressure and the Effects of Information on Share Prices," Journal of Business, XLV (April, 1972), 179-211.

⁴⁰ Robert C. Radcliffe, "Liquidity Costs and Block Trading," Financial Analysts Journal, XXIX (July-August, 1973), 73-80.

significant positive relationship between the absolute size of the trade and the relative price concession. Whether mutual fund performance can be improved by decreasing portfolio size is the subject of this research.

Research Design

Hypotheses

The hypotheses delineated and tested in this study are classified into three categories. Each category has a number of specific hypotheses which deal with a particular area of concern. The first category deals with the effect of portfolio size on portfolio return. The second is concerned with the effect of portfolio size on portfolio risk. The effect of portfolio size on portfolio performance is the concern of the third. Each of these three categories will now be discussed in turn.

Since the previous research on the effects of portfolio size on portfolio performance dealt with random portfolios, any measurement of the effect of portfolio size on portfolio return was precluded. Due to the employment of the nonrandom portfolios of mutual funds in this research, the effect of portfolio size on portfolio return can be measured. The main hypothesis of the first category is that portfolio return is independent of portfolio size. Rejection of this hypothesis disputes the existence of an efficient capital market. An efficient capital market compensates the investor only for the systematic risk which he bears. Its existence would yield a portfolio return function independent of portfolio size, assuming that the portfolio manager is constrained to selecting

securities from a given risk class, i.e., the level of systematic risk does not increase or decrease with size. Furthermore, rejection of the hypothesis due to the existence of an inverse relationship between portfolio return and portfolio size would not only dispute the existence of an efficient capital market, but would also indicate the possibility of increasing portfolio return by decreasing portfolio size.

The effect of portfolio size on portfolio risk has been established for random portfolios. Moreover, it has been shown that apart from negatively correlated stocks, all the reduction in risk is the result of averaging over the independent components of the risks of individual securities. However, the validity of the conclusions derived from the research with random portfolios may not extend to the nonrandom portfolios of actual investors. Therefore the second category's main hypothesis is that the effect of portfolio size on portfolio risk is dependent upon whether the securities selected for portfolio inclusion are chosen in a random or nonrandom manner.

Preclusion of measuring portfolio return in previous research meant measuring portfolio performance in only one dimension. The employment of nonrandom portfolios allows the measurement of portfolio performance which simultaneously considers both return and risk. The main hypothesis of the third category is that portfolio performance is independent of portfolio size. Rejection of this hypothesis would indicate that portfolio managers can affect portfolio performance with the portfolio size decision. Moreover, rejection due to the existence of an inverse relationship between

portfolio performance and portfolio size would support the conclusion that the large portfolios of mutual funds represent constrained optima.

Sample

The randomly selected sample of mutual fund portfolios is limited to one class of mutual funds so that the selected portfolios have the same goals. Work by Farrar⁴¹ has shown that mutual funds of the same classification tend to have similar performance goals. The selected classification is the growth-income fund because these funds are common stock funds and because of the homogeneity of the goals of the mutual funds within this classification. From a group of thirty-two mutual funds classified as growth-income funds for the years 1968, 1969, 1970 and 1971 in Arthur Weisenberger's Investment Companies, eight mutual funds were randomly selected which had December 31 as their quarterly closing date.⁴² They are Dodge and Cox Stock Fund; Eaton and Howard Stock Fund; Fidelity Fund; Massachusetts Investors Trust; One William Street Fund; Scudder, Stevens and Clark Common Stock Fund; Varied Industry Plan; and Wisconsin Fund. The reason for the same quarterly closing date restriction is so that all the mutual funds have the same holding periods, and in particular the December 31 quarterly closing date is selected because the majority of the mutual funds have this as

⁴¹Donald E. Farrar, The Investment Decision Under Uncertainty (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1962).

⁴²Nineteen of the thirty-two growth-income funds had December 31 as their quarterly closing date.

their quarterly closing date. The December 31 portfolios were obtained from Moody's Bank and Finance Manual.

Portfolio performance is measured for one year. While the average holding period for the growth-income funds selected appears to be somewhat longer than one year, the implicit assumption of no portfolio revision precludes extension of the holding period beyond one year. The use of annual holding periods 1968, 1969, 1970 and 1971 provide thirty-two nonrandom portfolios.

Since our attention is directed towards the effect of portfolio size in common stock portfolios, only the common stock portion of the mutual fund portfolios is employed.⁴³ Moreover, the fixed income securities are viewed as the portfolio manager combining his risky portfolio with the riskless asset to move down along the capital market line. That is, the presence of fixed income securities causes the portfolio to be looked upon as a lending portfolio. The actual portfolio sizes of the selected growth-income mutual funds are given in Table 4.

⁴³The common stock portion of the mutual fund's total investments averaged for the selected mutual funds 95.83%, 94.71%, 90.46% and 90.80%, respectively for the annual holding periods 1968, 1969, 1970 and 1971.

TABLE 4

GROWTH-INCOME MUTUAL FUND PORTFOLIO SIZE

	1968	1969	1970	1971	Average
Dodge & Cox Stock Fund	43	45	40	39	41.75
Eaton & Howard Stock Fund	95	68	71	111	86.25
Fidelity Fund	98	94	94	106	98
Massachusetts Investors Trust	109	108	113	108	109.5
One William Street	64	70	71	72	69.5
Scudder, Stevens, & Clark Common Stock Fund	44	46	51	53	48.5
Varied Industry Plan	43	40	46	47	44
Wisconsin Fund	59	37	43	57	49

Ranking of Securities

It has been shown above that in order to investigate the effect of portfolio size on portfolio return, nonrandom portfolios are needed. To accomplish this goal, the individual securities in the selected growth-income mutual fund portfolios must be ranked. In this research, the ranking criterion employed is the market value of the individual securities in the mutual fund portfolio at the beginning of the holding period. The market value of the mutual fund's holdings of an individual security services as a proxy for the portfolio manager's ex ante return and risk evaluations for that individual security. Since the portfolio manager makes the buy and sell decisions, it is implicit that he places the greater proportion of his investment funds in those securities which have the best ex ante return-risk outlook. Therefore, ranking the securities by

the market value criterion is a proxy of the portfolio manager's ranking for portfolio inclusion.

Mutual fund management compensation is generally based on a percentage of the total assets managed. Total assets increase with the increasing value of the securities held in the portfolio and with the selling of mutual fund shares. Hence, management gives consideration to the sales appeal aspect of the portfolio as well as the investment aspect. Also, mutual funds may be constrained from holding larger amounts of the securities of certain firms because their buying and selling may affect the prices of these securities. To the extent that the ranking procedure employed abstracts from the marketability and sales appeal aspects of the portfolio, the ranking procedure is deficient.

Measurement of Portfolio Return

Before proceeding further, it is necessary that the concepts of portfolio return, portfolio risk and overall portfolio performance become operational. Portfolio return is the primary dimension on which portfolio performance is evaluated. However, before the portfolio return can be computed, the return on the individual securities must be computed. The market rate of return for the individual securities, whose computation is given by equation 18, is equal to the capital gains (or losses) plus dividends for the period divided by the initial price:

$$r_{i,t} = \frac{P_{i,t+1} - P_{i,t} + D_{i,t}}{P_{i,t}} \quad (18)$$

where

$r_{i,t}$ = rate of return on security i during time period i

$P_{i,t}$ = price of security i at the start of period t

$P_{i,t+1}$ = price of security i at the end of period t

$D_{i,t}$ = dividends received during time period t

Price data for the securities held in the selected mutual fund portfolios and the Standard and Poor's 500 Index were obtained from ISL Daily Stock Price books and the Bank and Quotation Record. Dividend data for the individual securities were obtained from Moody's Dividend Annual. Dividend data for the Standard and Poor's 500 Index were obtained from Standard and Poor's Trade and Securities Statistics.

In order to be meaningful, return must be defined over some period of time. Because the study of portfolio performance is an ex post process, it is necessary to divide the annual holding period into subperiods in order to measure ex post variability. Hence twelve monthly returns are computed for each individual security.

Several phenomena occurred during the annual holding period that required adjustment. They were stock dividends, stock splits, stock rights, spin-offs and mergers. In the case of stock dividends and stock splits, the month-end prices and dividends after the stock dividend or stock split were adjusted to reflect the increase in the number of shares outstanding. Stock rights were not exercised, but were sold and recorded as a cash inflow in the month in which the stock went ex rights. The value of the rights was obtained from Moody's Dividend Annual. In the case of spin-offs, the month-end price and dividend data used thereafter reflect both the price changes and dividends of the parent company and the shares of the

company spun-off. Likewise, in the case of mergers, the month-end price and dividend data used after the merger pertain to the securities received for one share of the company's stock at the time of the merger.

As equation 19 shows, portfolio return is computed as the weighted average of the individual returns of the securities contained in the portfolio where the proportion of funds invested in each security is the same. This abstraction from the allocation of funds decision permits the measurement of the effect of portfolio size on portfolio return.

$$R_t = \frac{\sum_{i=1}^n r_{i,t}}{n} \quad (19)$$

where

R_t = rate of return on the portfolio during
time period t

$r_{i,t}$ = rate of return on security i during time
period t

n = number of securities held in the portfolio

The portfolio return measures employed are the arithmetic and geometric mean portfolio returns of the distribution of the twelve monthly portfolio returns computed by equation 19. The arithmetic mean portfolio return, A , is simply the sum of the monthly portfolio returns divided by the number of monthly portfolio return observations. This computation is given by equation 20.

$$A = \frac{\sum_{t=1}^{12} R_t}{12} \quad (20)$$

To compute the geometric mean portfolio return, multiplication and root taking are substituted for addition and division. However,

since it is possible that the multiplication of returns may end up with a negative product which would complicate the taking of the root, portfolio value relatives rather than portfolio returns are used in the computation of the geometric mean portfolio return. The portfolio value relative, VR_t , is simply equal to the portfolio return R_t plus 1.

$$VR_t = R_t + 1 \quad (21)$$

The computation of the geometric mean portfolio return is given by equation 22.

$$G = \left(\prod_{t=1}^{12} VR_t \right)^{1/12} - 1 \quad (22)$$

Both means are widely used in the investment field, with the arithmetic mean being the more familiar. The advantage of the geometric mean is that it measures the true rate of return over the holding period. More specifically, if initial dollar investment V_0 is compounded each subperiod over the holding period at the geometric rate of return G , its value at the end of the holding period V_{12} would equal the actual portfolio value observed. That is,

$$V_0 (1 + G)^{12} = V_{12} \quad (23)$$

The arithmetic mean portfolio return for the same twelve monthly periods is larger or equal to the geometric mean monthly portfolio return. That is, the following inequality holds.

$$A \geq G \quad (24)$$

The equality will hold only if there is no variability in the monthly portfolio rates of return. Any variability in the monthly portfolio

returns causes the inequality to hold due to the upward bias of the arithmetic mean. This upward bias can be aptly illustrated with an example. Suppose \$100 is invested, and increases 50 percent the first year and loses 50 percent the second. The arithmetic mean return is zero, but certainly the investment did not break even. The initial investment of \$100 increased to \$150 and then decreased to \$75, a \$25 loss over the two year period. The geometric mean return reflects this, and its value is -13.5 percent.

The advantage of the arithmetic mean monthly portfolio return is that it is the most likely or expected return in a single month. That is, if all that is known about the portfolio is rate of return each month over a series of months, then the best guess as to what the return on the portfolio will be in any one month of the series is the arithmetic mean. Both mean portfolio returns are used in this study as portfolio return measures.

Measurement of Portfolio Risk

While the measurement of portfolio return is fairly straight forward, its performance mate, portfolio risk, is not so easily quantified. Since the rate of return on a portfolio is a critical characteristic of the portfolio, risk in this research is defined as the uncertainty of the rate of return. This definition is deceptively simple and presents a problem in measurement. The degree of uncertainty emanates from the individual, and therefore, is not solely dependent upon the portfolio in question. Individuals with superior information would be more certain of the return on their portfolio than those who selected the same portfolio at random.

Also, since the utility of money is a function of wealth and since individuals are not homogeneous in wealth, the degree of risk aversion or risk acceptance is not homogeneous across individuals. What is needed is a means of measuring risk that is objective and independent of the individual doing the measuring so that portfolios can be compared with one another. Variability of return is such a measure of risk. Moreover, there is empirical evidence that substantiates the uses of variability of return as a measure of risk.⁴⁴ This evidence shows that, on the average, the higher the variability of return (risk) on a security or portfolio, the higher is the observed return. This follows one's intuitive feeling about risk, i.e., as risk increases, one would require a higher return to bear the higher risk. Therefore, variability of portfolio return is employed as an operational definition of portfolio risk in this research.

Many statistical measures of variability exist. Four measures are selected for use in this research. Two, the standard deviation of portfolio returns and the standard deviation of the natural logarithms of the portfolio value relatives, are selected because they have been employed previously and will therefore provide a point of reference. Unlike these two measures of variability, which consider all variability as risk, the other two treat only unfavorable

⁴⁴Shannon P. Pratt, "Relationship Between Variability of Past Returns and Levels of Future Returns for Common Stocks, 1926-1960," Frontiers of Investment Analysis, ed. E. Bruce Fredrikson (Scranton, Pennsylvania: Intext Educational Publishers, 1971), pp. 338-352; Jensen, loc. cit.; William F. Sharpe, "Risk Aversion in the Stock Market," Journal of Finance, XX (September, 1965), 416-422.

variation as risk. These latter two measures are the modified quadratic mean of the natural logarithms of the portfolio value relatives⁴⁵ and the index of unfavorable variation of portfolio return.⁴⁶

The standard deviation of the portfolio returns is computed as follows:

$$SD = \left[\frac{\sum_{t=1}^{12} (R_t - A)^2}{11} \right]^{1/2} \quad (25)$$

where

SD = standard deviation of the monthly portfolio returns

R_t = portfolio return in month t

A = arithmetic mean of the monthly portfolio returns

The standard deviation of the natural logarithms of the portfolio value relatives is computed as follows:

$$SDVR = \left[\frac{\sum_{t=1}^{12} (\ln VR_t - \ln (G+1))^2}{11} \right]^{1/2} \quad (26)$$

where

SDVR = standard deviation of the monthly portfolio value relatives

VR_t = portfolio value relative in month t

G = geometric mean of the monthly portfolio returns

The SD is more widely used than the SDVR, but the SDVR has several advantages over the SD. Rather than measuring variability about the arithmetic mean, the SDVR measures variability about the

⁴⁵Levy, loc. cit.

⁴⁶Bower and Wipperfurth, loc. cit.

geometric mean, the true measure of return over the entire holding period. Also, the SDVR is a relative measure of dispersion, whereas the SD is an absolute measure of dispersion.

As mentioned above, the modified quadratic mean of the logarithms of the portfolio value relatives and the index of unfavorable variation of portfolio return do not measure all variability, but treat only unfavorable variation as risk. Consequently they would not consider a portfolio growing at a variable rate to be more risky than a portfolio decreasing at a constant rate. There is, however, a definitional problem present in that unfavorable variation is not homogeneous across investors and indeed not across investments either. As a solution unfavorable return is defined as loss in computing the modified quadratic mean of the logarithms of the portfolio value relatives, since all losses are unfavorable. For the index of unfavorable variation, the definition of unfavorable variation is defined as those occasions on which the portfolio rate of return falls more or increases less than the return on the Standard and Poor's 500 Index.

The computation of the modified quadratic mean of the logarithms of the monthly portfolio value relatives is given by equation 27:

$$Q = \left[\frac{\sum_{t=1}^{12} (\ln VR_t)^2}{12} \right]^{1/2} \quad (27)$$

where if $\ln VR_t \geq 0$, then set $\ln VR_t = 0$ and where

Q = modified quadratic mean of the logarithms of the monthly portfolio value relatives

VR_t = portfolio value relative in month t

The computation of the index of unfavorable variation of portfolio return is given by equation 28:

$$U = \left[\frac{\sum_{t=1}^{12} d_t^2}{12} \right]^{1/2} \quad (28)$$

where if $d_t \geq 0$, then set $d_t = 0$ and where

U = index of unfavorable variation of monthly portfolio return

$$d_t = (R_t - R_{t-1}) - (I_t - I_{t-1})$$

R_t = portfolio return in month t

I_t = return on Standard and Poor's 500 Index in month t

Note that this latter measure allows only comparisons of riskiness among investments in common stocks due to the relationship with a stock market index.

Measurement of Portfolio Performance

The general portfolio performance measure employed in this research is explained in Chapter I. By jointly evaluating return and risk along with the risk free rate of interest, a means is provided for the ranking of portfolio performance. For each portfolio, a ratio denoted by θ is computed. θ is equal to the portfolio's observed excess return divided by the portfolio's observed risk, S , where excess portfolio return is equal to the difference between the mean portfolio return, R , and the risk free rate of interest, r_f .

$$\theta = \frac{R - r_f}{S} \quad (1)$$

The portfolio with the highest θ value is the best portfolio. Best, because it can be made to match any other portfolio of the set of

ranked portfolio in one dimension, either risk or return, while it is better in the other dimension. Several different operational definitions of portfolio performance will be used in this study employing the previously defined measures of portfolio return and portfolio risk.

The risk free rate of return for each of the four annual holding periods is approximated from the reported yields on U.S. Government securities obtained from Solomon Brothers' An Analytical Record of Yields and Yield Spreads. These rates are presented below in Table 5.

TABLE 5

RISK FREE RATE OF RETURN

Year	Treasury Bill Rates
1968	5.68%
1969	6.32%
1970	8.24%
1971	5.15%

Sharpe's reward to variability ratio is obtained by substituting the arithmetic mean monthly portfolio return, A , and the standard deviation of the monthly portfolio returns about the arithmetic mean, SD , for the mean portfolio return, R , and portfolio risk, S , in equation 1, respectively.

$$\theta_1 = \frac{A - r_f}{SD} \quad (29)$$

Recognizing the arguments that the geometric mean return is the more favorable measure of return because it reflects the true rate

of return over the holding period and that the SDVR is preferred to the SD because the former is measured about the geometric mean and is a measure of relative variability, the second operationally defined measure of portfolio performance is obtained.

$$\theta_2 = \frac{\ln (G + 1) - \ln (r_f + 1)}{\text{SDVR}} \quad (30)$$

The SDVR is used as the measure of portfolio risk, while the numerator becomes the mean of the natural logarithms of the monthly portfolio value relatives, $\ln (G + 1)$, minus the natural logarithm of the risk free value relative, $\ln (r_f + 1)$. The numerator is equal to the excess return given in terms compatible with the denominator.

Two additional operational definitions of portfolio performance are obtained by employing the previously defined risk measures which consider only unfavorable variation as risk. Levy⁴⁷ has developed the reward to vulnerability ratio. Like θ_2 , the reward to vulnerability ratio has its excess portfolio return given by the mean of the natural logarithms of the monthly portfolio value relatives minus the natural logarithm of the risk free value relative. The modified quadratic mean of the logarithms of the monthly portfolio value relatives, which is defined above is used as the risk measure. Thus we have:

$$\theta_3 = \frac{\ln (G + 1) - \ln (r_f + 1)}{Q} \quad (31)$$

The fourth operationally defined performance measure is obtained by modifying Sharpe's reward to variability ratio, such that the risk

⁴⁷Levy, loc. cit.

measure used considers only unfavorable variation as risk. The index of unfavorable variation, U , is substituted for SD in equation 29 to give:

$$\theta_4 = \frac{A - r_f}{U} \quad (32)$$

These then are the four operationally defined performance measures that are employed in this research to investigate the effect of portfolio size on portfolio performance.

CHAPTER III

STATISTICAL MODELS AND RESULTS

In order to measure the effect of portfolio size on the various portfolio parameters, it is necessary to simulate the portfolio building process from the first security to the addition of the last. This is accomplished by ranking the securities within the mutual fund portfolio where the market value of the mutual fund's investment in each security is used as a proxy for the portfolio manager's ranking for portfolio inclusion. In this manner securities are added one at a time to simulate portfolios of size one up to the actual size of the mutual fund portfolio. This simulation yields the effects of portfolio size on portfolio return, portfolio risk and portfolio performance. Unfortunately, it also leads to statistical dependence. Consequently, the selection of the statistical models for this research is constrained by the existence of statistical dependence. This chapter discusses the models employed along with the results of the statistical tests.

Portfolio Return

Statistical Model

Since the portfolio return measurements on the portfolios of increasing size are statistically dependent, regression of these measurements on the corresponding portfolio size does not provide

a means of testing the hypothesis that portfolio return is independent of portfolio size. While regression coefficients can be determined, the existence of statistical dependence precludes their being tested for statistical significance; hence, the preclusion of hypothesis testing.

Fund Size	Mutual Fund	Annual Holding Period			
		1968	1969	1970	1971
Large	Eaton & Howard Stock Fund				
	Fidelity Fund				
	Massachusetts Investors Trust				
	One William Street Fund				
Small	Dodge & Cox Stock Fund				
	Scudder, Stevens & Clark Common Stock Fund				
	Varied Industry Plan				
	Wisconsin Fund				

Figure 5

Three-Way Mixed Effects Analysis of Variance

The statistical model employed to test the effect of portfolio size on portfolio return is a three-way mixed effects analysis of variance model with nesting of one of the independent variables. The model is presented schematically in Figure 5. Two of the three independent variables, annual holding period and fund size, are fixed, while the third, mutual fund, is random and is nested within fund size. The classification of mutual funds with respect to fund

size, large or small, is carried out by ranking the mutual funds on the actual number of different securities held.

The mathematical presentation of the analysis of variance model is given by equation 33:

$$X_{ijk} = \mu + \alpha_i + \beta_j + C_{k(j)} + \alpha\beta_{ij} + \alpha C_{ik(j)} + \epsilon_{ijk} \quad (33)$$

where

- X_{ijk} = the dependent variable measurement for mutual fund k nested in mutual fund size class j in holding period i
- μ = the grand mean
- α_i = the main effect of holding period i
- β_j = the main effect of mutual fund size class j
- $C_{k(j)}$ = the main effect of mutual fund k, which is nested in mutual fund size class j
- $\alpha\beta_{ij}$ = the interaction effect between holding period i and mutual fund size class j
- $\alpha C_{ik(j)}$ = the interaction effect of holding period i and mutual fund k nested in mutual fund size class j
- ϵ_{ijk} = the experimental error term, which is normally and independently distributed with a mean of zero and a standard deviation of σ_ϵ

While individually the regression coefficients cannot be tested for statistical significance, they can be pooled and tested for statistical significance as a group. That is the purpose of this statistical model. The dependent variable is the linear trend coefficient or slope obtained from the linear regression of portfolio return on portfolio size for the simulated portfolios. Both the arithmetic mean portfolio return and the geometric mean portfolio return are determined for the simulated portfolios. Consequently, two different dependent variables are used in this statistical model. The specific hypotheses dealing with the effect of portfolio size on portfolio return presented below reflect this use of two dependent variables.

Statistical Hypotheses

One of the implications of previous discussion is that mutual funds are doing a disservice to their shareholders by investing in more securities than the legal minimum. That is, portfolio return has an inverse relationship with portfolio size. Therefore, the following two hypotheses are tested.

Hypothesis 1:

H_0 : Arithmetic mean portfolio return of growth-income mutual funds is independent of portfolio size.

H_1 : Arithmetic mean portfolio return of growth-income mutual funds is dependent on portfolio size.

Hypothesis 2:

H_0 : Geometric mean portfolio return of growth-income mutual funds is independent of portfolio size.

H_1 : Geometric mean portfolio return of growth-income mutual funds is dependent on portfolio size.

Portfolio return is dependent on the holding period over which it is measured. Thus the holding period may have an effect on the relationship between portfolio return and portfolio size. This leads to the next two hypotheses.

Hypothesis 3:

H_0 : The effect of portfolio size on the arithmetic mean portfolio return of growth-income mutual funds is independent of the holding period.

H_1 : The effect of portfolio size on the arithmetic mean portfolio return of growth-income mutual funds is dependent on the holding period.

Hypothesis 4:

H_0 : The effect of portfolio size on the geometric mean portfolio return of growth-income mutual funds is independent of the holding period.

- H_1 : The effect of portfolio size on the geometric mean portfolio return of growth-income mutual funds is dependent on the holding period.

The next two hypotheses examine whether the effect of portfolio size on portfolio return is related to differences in actual mutual fund size as measured by the number of different securities held. That is, do the larger mutual funds possess a relationship between portfolio size and portfolio return that is different than that observed for the smaller mutual funds, and can this difference account for the size disparity?

Hypothesis 5:

- H_0 : No difference exists between the relationships of portfolio size and arithmetic mean portfolio return for large and small growth-income mutual funds.
- H_1 : A difference exists between the relationships of portfolio size and arithmetic mean portfolio return for large and small growth-income mutual funds.

Hypothesis 6:

- H_0 : No difference exists between the relationship of portfolio size and geometric mean portfolio return for large and small growth-income mutual funds.
- H_1 : A difference exists between the relationships of portfolio size and geometric mean portfolio return for large and small growth-income mutual funds.

The last two hypotheses of this section are concerned with the homogeneity of professional management across the growth-income mutual funds. That is, is there a difference among the individual mutual funds with respect to the effect of portfolio size on portfolio return.

Hypothesis 7:

- H_0 : No difference exists among the relationships of portfolio size and arithmetic mean portfolio return for the individual growth-income mutual funds.

H_1 : A difference exists among the relationships of portfolio size and arithmetic mean portfolio return for the individual growth-income mutual funds.

Hypothesis 8:

H_0 : No difference exists among the relationships of portfolio size and geometric mean portfolio return for the individual growth-income mutual funds.

H_1 : A difference exists among the relationships of portfolio size and geometric mean portfolio return for the individual growth-income mutual funds.

Test Statistics

A brief, intuitive explanation of the operation of the analysis variance model is now presented along with the development of the specific test statistics employed in the testing of the hypotheses.⁴⁸ The analysis of the variance table presented below facilitates this explanation.

Analysis of variance consists of partitioning the total variance observed into component parts, where each component part consists of variation due to possible experimental effects and chance phenomena. The analysis of variance table lists the possible sources of variation and the number of degrees of freedom available to estimate each source of variation. The expected mean squares in Table 6 show the composition of estimated variation for each source. Consider the first two hypotheses. The null hypothesis is that the mean portfolio return is independent of portfolio size, or equivalently, the mean linear trend coefficient is equal to zero. If the null hypothesis

⁴⁸ For a thorough explanation of the analysis of variance model see Roger E. Kirk, Experimental Design: Procedures for the Behavioral Sciences (Belmont, California: Wadsworth Publishing Company Inc., 1968), pp. 35-73.

is true, then the variance term σ_{μ}^2 will equal zero. That being the case, the expected mean square for the mean and the expected mean square for the mutual fund will be equal. Therefore, to test the first two hypotheses, two independent variance estimates, one for the mean and one for the mutual fund source, are tested for equality by means of an F test. If the F test statistic, shown in the fourth column of Table 6, is greater than the critical F value shown in the fifth column, the evidence is sufficiently great to reject the null hypothesis and accept the alternative hypothesis. This decision is made because if the null hypothesis were true, the probability of observing an F statistic greater than the critical value would be only .05. Such an event would be sufficiently rare that if the critical F value were surpassed it would be more likely due to $\sigma_{\mu}^2 > 0$, i.e., the mean linear trend coefficient not being equal to zero. If however the F statistic were less than the critical F value, there would not be sufficient evidence to reject the null hypothesis.

For hypotheses 3 and 4, if the null hypothesis is true, then $\sigma_{\alpha}^2 = 0$. Hence, the F test statistic is the ratio of the mean squares for holding period and holding period-mutual fund interaction. If the F test statistic exceeds the critical F value given in the fifth column, the null hypothesis is rejected and the alternative is accepted. Otherwise, the null hypothesis is not rejected. For hypotheses 5 and 6, $\sigma_{\beta}^2 = 0$ if the null hypothesis is true, and the F test statistic is the ratio of the mean squares for fund size and mutual fund. Again, the critical F value at the significance level .05 is given in the fifth column.

Since the statistical design model has only one replication

TABLE 6

THREE-WAY MIXED EFFECTS ANALYSIS OF VARIANCE TABLE

Source of Variation	Degrees of Freedom	Expected Mean Squares	F Statistics	Critical F Value (.05 level)
Mean, μ	1	$\sigma_{\epsilon}^2 + 32 \sigma_{\mu}^2 + 4 \sigma_C^2$	$F_{1,6} = \frac{E(MS_{\mu})}{E(MS_C)}$	5.99
Holding Period, α	3	$\sigma_{\epsilon}^2 + 8 \sigma_{\alpha}^2 + \sigma_{\alpha C}^2$	$F_{3,18} = \frac{E(MS_{\alpha})}{E(MS_{\alpha C})}$	3.16
Fund Size, β	1	$\alpha_{\epsilon}^2 + 16 \sigma_{\beta}^2 + 4 \sigma_C^2$	$F_{1,6} = \frac{E(MS_{\beta})}{E(MS_C)}$	5.99
Mutual Fund, C	6	$\sigma_{\epsilon}^2 + 4 \sigma_C^2$		
Holding Period-Fund Size Interaction, $\alpha\beta$	3	$\sigma_{\epsilon}^2 + 4 \sigma_{\alpha\beta}^2 + \sigma_{\alpha C}^2$	$F_{3,18} = \frac{E(MS_{\alpha\beta})}{E(MS_{\alpha C})}$	3.16
Holding Period-Mutual Fund Interaction, αC	18	$\sigma_{\epsilon}^2 + \sigma_{\alpha C}^2$		
Error, ϵ	<u>0</u>	σ_{ϵ}^2		
Total	32			

per cell, no degrees of freedom are available to estimate the error variance within cells. Consequently, there is no straight forward test of hypotheses 7 and 8. The F test statistic for these hypotheses is the ratio of mean squares for mutual fund and error, but the error variance cannot be estimated. Turning to Table 6 and the expected mean square of the holding period-mutual fund interaction, one can see that if there is no interaction effect, $\sigma_{\alpha C}^2$ would equal zero, leaving error as the only source of variation. Therefore, if it can be shown that $\sigma_{\alpha C}^2$ is equal to zero, the ratio of the mean squares for mutual fund and holding period-mutual fund interaction can be employed as an F test statistic in testing hypotheses 7 and 8. A means of testing the existence of this interaction was developed by Tukey.⁴⁹ If the employment of this test indicates the nonexistence of a holding period-mutual fund interaction, i.e., $\sigma_{\alpha C}^2 = 0$, hypotheses 7 and 8 can be tested by using the following F test statistic and critical F value respectively, $F = \frac{E(MS_C)}{E(MS_{\alpha C})}$ and $F_{.05;6,18} = 2.66$.

In applying Tukey's one degree of freedom for nonadditivity test on the holding period-mutual fund interaction, the level .25 of significance is adopted so that the chance of making a wrong decision will lean towards the rejection of $\sigma_{\alpha C}^2 = 0$. This liberalism guards against making a type II error which would result in a conservative test of the mutual fund main effect.

⁴⁹ John W. Tukey, "One Degree of Freedom for Nonadditivity Biometrics, V (September, 1949), 232-242.

Statistical Results

The results of the three-way mixed effects analysis of variance model are presented in Table 7. Hypotheses 3 and 4, dealing with the effect of the holding period on the relationship between portfolio size and the arithmetic and geometric mean portfolio returns respectively, were the only cases where the null hypothesis of no effect was rejected at the .05 level of significance. For all the other hypotheses the null hypothesis could not be rejected at the .05 level of significance.⁵⁰ The mean slopes for the 1968, 1969, 1970 and 1971 holding periods are .000148, -.000131, .000070, -.000063 and .000169, -.000123, .000068, -.000060, respectively for the arithmetic and geometric mean portfolio return situations. The overall means are .000006 and .000013 respectively.

The above mean slopes indicate the predicted increases or decreases in monthly portfolio return due to the addition or removal of the marginal security. The annual portfolio return corresponding to a monthly portfolio return is obtained from equation 34.

$$\text{Annual Rate} = (1 + \text{Monthly Rate})^{12} - 1 \quad (34)$$

⁵⁰It is noted at this point that the existence of heterogeneity in the variances and covariances in the statistical model employed causes the conventional F test of the hypotheses dealing with the holding period to be positively biased, i.e., an experimenter will err in the direction of rejecting the null hypothesis when it is true. To guard against such error, the Geisser-Greenhouse negatively biased F test was also employed when the null hypothesis was rejected by the conventional F test. This conservative test considers the maximum possible effect due to heterogeneity. In the case of the holding period main effect the conservative F critical value is $F_{.05;1,6} = 5.99$. Hence, the null hypotheses were still rejected.

In the case where the slope equals .000148, the addition of the marginal security would increase monthly portfolio return by .0148%, or on an annual basis the increase would amount to .18%. In the case where the slope equals .000006 the addition of the marginal security would increase the annual portfolio yield by less than one-hundredth of one percent.⁵¹ Of course the addition or deletion of more securities would have a larger impact on portfolio return.

TABLE 7
PORTFOLIO RETURN: ANALYSIS OF VARIANCE RESULTS I

Source of Variation*	Critical F Values (.05 level)	Arithmetic Mean Computed F Statistic	Geometric Mean Computed F Statistic
Mean	$F_{1,6} = 5.99$	(1)** 0.04	(2) 0.25
Holding Period	$F_{3,18} = 3.16$	(3) 9.65	(4) 9.49
Fund Size	$F_{1,6} = 5.99$	(5) 0.23	(6) 0.07
Mutual Fund	$F_{6,18} = 2.66$	(7) 1.97	(8) 1.54
Holding Period-Fund Size Interaction	$F_{3,18} = 3.16$	1.28	1.43

*Only sources of variation tested for significance are listed.

**Numbers in parentheses indicate the hypotheses corresponding to the computed F statistics.

⁵¹ These calculations assume that portfolio return was equal to zero before the addition of the marginal security. The impact on an annual basis would be somewhat larger for portfolio yields higher than zero prior to the addition of the marginal security due to the compounding effect.

The existence of the significant holding period main effect was analyzed further with Tukey's a posteriori multiple comparison test to determine which holding periods were different from each other at the .05 level of significance. Tukey's post-hoc test was employed because it is the most powerful multiple comparison test for pairwise comparisons.⁵² The results of the Tukey post-hoc test are presented in Table 8. Significant differences were found between holding periods 1968 and 1969, 1968 and 1971, and 1969 and 1970 for both the arithmetic and geometric mean portfolio return cases.

TABLE 8
PORTFOLIO RETURN: TUKEY POST HOC MULTIPLE
COMPARISONS OF HOLDING PERIODS

Holding Periods Compared	Critical Studentized Range Value (.05 level)	Computed Studentized Range Statistic	
		Arithmetic Mean	Geometric Mean
1968-1969	$q_{4,18} = 4.00$	6.87	6.89
1968-1970	$q_{4,18} = 4.00$	1.93	2.38
1968-1971	$q_{4,18} = 4.00$	5.19	5.40
1969-1970	$q_{4,18} = 4.00$	-4.94	-4.51
1969-1971	$q_{4,18} = 4.00$	-1.68	-1.49
1970-1971	$q_{4,18} = 4.00$	3.26	3.02

⁵² For a thorough explanation of the Tukey a posteriori multiple comparison test see Roger E. Kirk, loc. cit., pp. 88-90.

The conclusion drawn from these results is that the effect of portfolio size on the arithmetic and geometric mean portfolio return of growth-income mutual funds is dependent on the holding period. However, there is no indication that portfolio managers are taking advantage of this phenomenon as no difference in portfolio size was found between the holding periods.⁵³

Nonrejection of the .05 level at significance of the null hypotheses of hypotheses 1 and 2 fails to refute the existence of independence between portfolio return and portfolio size. However, the existence of the holding period main effect influences the interpretation of these results. The failure to refute the independence is for the longer run situation, more specifically the average effect of the four annual holding periods. The existence of the holding period main effect that shows a positive relationship between portfolio return and portfolio size one year and a negative relationship the next year can average out to an independent relationship over the two year period. Such is the situation in this experiment. However, as was noted above, it appears that no attempt was made on the part of the mutual funds to take advantage of such phenomenon.

The actual portfolio size differences for large and small portfolio, growth-income mutual funds cannot be explained by the

⁵³ The null hypothesis of no holding period main effect could not be rejected at the .05 level of significance in a two-way mixed effects analysis of variance experiment with holding period and mutual fund as the two independent variables and portfolio size as the dependent variable. The computed F statistic, $F = 2.60$, did not exceed the critical F value, $F_{.05;3,21} = 3.07$.

existence of different relationships between portfolio size and portfolio return. The null hypotheses of hypotheses 5 and 6 that no difference exists between the relationships of portfolio size and the arithmetic and geometric mean portfolio returns respectively for large and small growth-income mutual funds could not be rejected at the .05 level of significance. Moreover, they could not be rejected at the .25 level of significance. The mean slope values for the large and small mutual fund classifications are .000020, -.000008 and .000020, .000006 respectively for the arithmetic and geometric mean situations.

Homogeneity of professional management across the growth-income mutual funds is not rejected. The null hypotheses of hypotheses 7 and 8 that no difference exists among the relationships of portfolio size and the arithmetic and geometric mean portfolio returns respectively for the individual growth-income mutual funds could not be rejected at the .05 level of significance.⁵⁴

As portfolio size increases for the simulated portfolios the possibility exists that the slope obtained from the linear regression of portfolio return against portfolio size may be reduced in absolute value due to a decreasing marginal effect of portfolio size. Such a result may have biased the preceding statistical tests in

⁵⁴ In order to test these hypotheses it was first necessary to apply Tukey's one degree of freedom for nonadditivity test to determine whether a holding period-mutual fund interaction existed. The computed F statistics for the arithmetic and geometric mean portfolio return cases, .649 and .954 respectively, did not exceed the critical F value, $F_{.25;1,20} = 1.40$. Thus, the absence of the interaction effect allows the hypotheses to be tested employing the F statistic described in the previous section.

favor of nonrejection. The following two experiments investigate this possibility of measurement bias. The first experiment is exactly the same as the preceding except that the dependent variable is the slope obtained from the linear regression of portfolio return against portfolio size using only the first thirty-seven securities; the size of the smallest portfolio. As Table 9 shows, the results of the hypothesis testing are the same as in the preceding experiment. Only in the case of the holding period-fund size interaction is there any appreciable change in the computed F values.

TABLE 9
PORTFOLIO RETURN: ANALYSIS OF VARIANCE RESULTS II

Source of Variation*	Critical F Values (.05 level)	Arithmetic Mean Computed F Statistic	Geometric Mean Computed F Statistic
Mean	5.99	0.003	0.05
Holding Period	3.16	9.40	8.76
Fund Size	5.99	0.001	0.03
Mutual Fund	2.66	2.06	1.69
Holding Period- Fund Size Interaction	3.16	0.35	0.40

*Only sources of variation tested for significance are listed.

The second experiment is better able to measure a decreasing marginal effect of portfolio size. The statistical model employed is similar to the one used above except that another completely crossed independent variable, incremental step, is added to the

model to obtain a four-way analysis of variance model. In this experiment the linear regressions to obtain the dependent variable are performed on seven data points at a time in five incremental steps. A test for a significant incremental step main effect will provide a test for the existence of decreasing marginal effect of portfolio size. The results of the four-way analysis of variance are given in Table 10. The incremental step main effect was not significant at the .05 level. There was, however, a significant holding period-incremental step interaction effect which influences the interpretation of the nonsignificant incremental step main effect. From the raw data it appears that the nonsignificant incremental main effect was due to a positive relationship in some years and a negative relationship in others to yield a nonsignificant result overall. Using only the absolute values of the dependent variables the means of the five incremental steps respectively were .00187, .00073, .00036, .00027, and .00026 for the arithmetic mean case and .00193, .00073, .00036, .00030, and .00028 for the geometric mean case. These mean values indicate the existence of a decreasing absolute marginal effect of portfolio size.

These latter two experiments lead to the conclusion that there was a decreasing absolute marginal effect of portfolio size, but that it did not appreciably bias the measurement variable employed earlier in this research.

TABLE 10

PORTFOLIO RETURN: ANALYSIS OF VARIANCE RESULTS III

Source of Variation*	Critical F Value (.05 level)	Arithmetic Mean Computed F Statistic	Geometric Mean Computed F Statistic
Mean	$F_{1,6} = 5.99$	0.28	0.01
Holding Period	$F_{3,18} = 3.16$	10.38	8.65
Incremental Step	$F_{4,24} = 2.78$	0.31	0.20
Fund Size	$F_{1,6} = 5.99$	0.00	0.07
Holding Period- Incremental Step Interaction	$F_{12,72} = 1.89$	3.93	3.51
Holding Period- Fund Size Interaction	$F_{3,18} = 3.16$	0.67	0.72
Incremental Step- Fund Size Interaction	$F_{4,24} = 2.78$	0.38	0.36
Holding Period- Fund Size- Incremental Step	$F_{12,72} = 1.89$	1.11	1.11

*Only sources of variation tested for significance are listed.

Portfolio Risk

Statistical Model

For random portfolios, the effect of portfolio size on portfolio risk has been established by other researchers. The validity of these findings however, may not extend to the nonrandomly selected portfolios of actual investors. This section attempts to determine whether the security selection process affects the relationship between portfolio size and portfolio risk.

In the statistical models employed, the dependent variable is the correlation coefficient obtained from the regression of portfolio risk against the reciprocal of portfolio size for the simulated portfolios. The reason for this choice is that the above relationship between portfolio risk and portfolio size has been established empirically for random portfolios.⁵⁵ The correlation coefficient measures the degree of this relationship. As is true with portfolio return, the testing of individual correlation coefficients for statistical significance is precluded due to the existence of statistical dependence. The correlation coefficient also has the disadvantage in that its probability distribution is nonnormal, and normality of the dependent variable is an underlying assumption of the analysis of variance models employed. Fortunately, this does not pose a problem because a transformation exists which systematically alters the values of the correlation coefficient such that normality is achieved but order is unchanged. This transformation is the Fisher r to z transformation.⁵⁶ The transformed coefficient of correlation, then, is the dependent variable used in the statistical models.

The first model employed is a two-way mixed effects analysis of variance model with the fixed independent variable being the annual holding period and the random independent variable being the mutual fund. This model is used to determine whether a linear relationship exists between portfolio risk and the reciprocal of

⁵⁵ See Evans and Archer, loc. cit.

⁵⁶ William H. Beyer (ed.), Handbook of Tables for Probability and Statistics (2nd ed.; Cleveland, Ohio: The Chemical Rubber Company, 1968), p. 394.

portfolio size for the growth-income mutual funds, and whether that relationship is affected by the holding period. The model is presented in Figure 6.

Mutual Fund	Annual Holding Period			
	1968	1969	1970	1971
Dodge & Cox Stock Fund				
Eaton & Howard Stock Fund				
Fidelity Fund				
Massachusetts Investors Trust				
One William Street Fund				
Scudder, Stevens & Clark Common Stock Fund				
Varied Industry Plan				
Wisconsin Fund				

Figure 6

Two-Way Analysis of Variance

The mathematical representation of the analysis of variance model is given by equation 35:

$$X_{ij} = \mu + \alpha_i + B_j + \alpha B_{ij} + \epsilon_{ijk} \quad (35)$$

where

- X_{ij} = the dependent variable measurement for mutual fund j in holding period i
- μ = the grand mean
- α_i = the main effect of holding period i
- B_j = the main effect of mutual fund j
- αB_{ij} = the interaction effect between holding period i and mutual fund j
- ϵ_{ijk} = the experimental error term, which is normally and independently distributed with a mean of zero and a standard deviation of σ_ϵ

The test statistics employed to test the statistical hypotheses can be found in the analysis of variance table depicted in Table 11.

The model employed to determine whether the security selection process affects the relationship between portfolio size and portfolio risk is a three-way mixed effects analysis of variance model. The model is presented schematically in Figure 7. The two fixed independent variables are the annual holding period and the security selection process. Portfolio is the random independent variable. For each nonrandom, mutual fund portfolio, a corresponding random portfolio of equal size was obtained by randomly selecting securities for portfolio inclusion from the set of common stock securities held by the selected growth-income mutual funds.

The mathematical representation of this analysis of variance model is given by equation 36:

$$X_{ijk} = \mu + \alpha_i + \beta_j + C_k + \alpha\beta_{ij} + \alpha C_{ik} + \beta C_{jk} + \alpha\beta C_{ijk} + \epsilon_{ijk} \quad (36)$$

where

- X_{ijk} = the dependent variable measurement for portfolio k whose securities were selected by security selection process j for holding period i
- μ = the grand mean
- α_i = the main effect of holding period i
- β_j = the main effect of security selection process j
- C_k = the main effect of portfolio k
- $\alpha\beta_{ij}$ = the interaction effect between holding period i and security selection process j
- αC_{ik} = the interaction effect between holding period i and portfolio k
- βC_{jk} = the interaction effect between security selection process j and portfolio k
- $\alpha\beta C_{ijk}$ = the interaction effect between holding period i, security selection process j and portfolio k
- ϵ_{ijk} = the experimental error term, which is normally and independently distributed with a mean of zero and a standard deviation of σ_ϵ

TABLE 11
TWO-WAY ANALYSIS OF VARIANCE TABLE

Source of Variation	Degrees of Freedom	Expected Mean Squares	F Statistics	Critical F Value (.05 level)
Mean, μ	1	$\sigma_{\epsilon}^2 + 32 \sigma_{\mu}^2 + 4 \sigma_{\beta}^2$	$F_{1,7} = \frac{E(MS_{\mu})}{E(MS_{\beta})}$	5.59
Holding Period, α	3	$\sigma_{\epsilon}^2 + 8 \sigma_{\alpha}^2 + \sigma_{\alpha\beta}^2$	$F_{3,21} = \frac{E(MS_{\alpha})}{E(MS_{\alpha\beta})}$	3.07
Mutual Fund, β	7	$\sigma_{\epsilon}^2 + 4 \sigma_{\beta}^2$		
Holding Period-Mutual Fund Interaction, $\alpha\beta$	21	$\sigma_{\epsilon}^2 \sigma_{\alpha\beta}^2$		
Error, ϵ	<u>0</u>	σ_{ϵ}^2		
Total	32			

Security Selection Process	Portfolio	Annual Holding Period			
		1968	1969	1970	1971
Nonrandom	Dodge & Cox Stock Fund				
	Eaton & Howard Stock Fund				
	Fidelity Fund				
	Massachusetts Investors Trust				
	One William Street Fund				
	Scudder, Stevens & Clark Common Stock Fund				
	Varied Industry Plan				
	Wisconsin Fund				
Random					
	(Portfolios of randomly selected securities				
	corresponding to the above mutual funds in				
	size)				

Figure 7

Portfolio Risk: Three-Way Analysis of Variance

TABLE 12

PORTFOLIO RISK: THREE-WAY MIXED EFFECTS ANALYSIS OF VARIANCE TABLE

Source of Variation	Degrees of Freedom	Expected Mean Square	F Statistic	Critical F Value (.05 level)
Mean, μ	1	$\sigma^2_{\epsilon} + 64 \sigma^2_{\mu} + 8 \sigma^2_{\alpha C}$	$F_{1,7} = \frac{E(MS_{\mu})}{E(MS_C)}$	5.59
Holding Period, α	3	$\sigma^2_{\epsilon} + 16 \sigma^2_{\alpha} + 2 \sigma^2_{\alpha C}$	$F_{3,21} = \frac{E(MS_{\alpha})}{E(MS_{\alpha C})}$	3.07
Security Selection Process, β	1	$\sigma^2_{\epsilon} + 32 \sigma^2_{\beta} + 4 \sigma^2_{\beta C}$	$F_{1,7} = \frac{E(MS_{\beta})}{E(MS_{\beta C})}$	5.59
Portfolio, C	7	$\sigma^2_{\epsilon} + 8 \sigma^2_C$		
Holding Period-Security Selection Process Interaction, $\alpha\beta$	3	$\sigma^2_{\epsilon} + 8 \sigma^2_{\alpha\beta} + \sigma^2_{\alpha\beta C}$	$F_{3,21} = \frac{E(MS_{\alpha\beta})}{E(MS_{\alpha\beta C})}$	3.07
Holding Period-Portfolio Interaction, αC	21	$\sigma^2_{\epsilon} + 2 \sigma^2_{\alpha C}$		
Security Selection Process-Portfolio Interaction, βC	7	$\sigma^2_{\epsilon} + 4 \sigma^2_{\beta C}$		
Holding Period-Security Selection Process-Portfolio Interaction, $\alpha\beta C$	21	$\sigma^2_{\epsilon} + \sigma^2_{\alpha\beta C}$		
Error, ϵ	0	σ^2_{ϵ}		
Total	64			

The test statistics employed to test the statistical hypotheses can be found in the analysis of variance table depicted in Table 12.

Statistical Hypotheses

Since four different risk measurements are made on each of the simulated portfolios, there are four different dependent variables or one correlation coefficient for each risk measure. This is reflected in the specific hypotheses which are presented below.

The first four hypotheses of this section deal with whether a linear relationship exists between portfolio risk and the reciprocal of portfolio size.

Hypothesis 9:

- H_0 : There is no linear relationship between portfolio risk as measured by the standard deviation of portfolio return and the reciprocal of portfolio size for growth-income mutual funds.
- H_1 : There is a linear relationship between portfolio risk as measured by the standard deviation of portfolio return and the reciprocal of portfolio size for growth-income mutual funds.

Hypothesis 10:

- H_0 : There is no linear relationship between portfolio risk as measured by the standard deviation of the natural logarithms of the portfolio value relatives and the reciprocal of portfolio size for growth-income mutual funds.
- H_1 : There is a linear relationship between portfolio risk as measured by the standard deviation of the natural logarithms of the portfolio value relatives and the reciprocal of portfolio size for growth-income mutual funds.

Hypothesis 11:

- H_0 : There is no linear relationship between portfolio risk as measured by the modified quadratic mean of the natural logarithms of the portfolio value relatives

and the reciprocal of portfolio size for growth-income mutual funds.

- H_1 : There is a linear relationship between portfolio risk as measured by the modified quadratic mean of the natural logarithms of the portfolio value relatives and the reciprocal of portfolio size for growth-income mutual funds.

Hypothesis 12:

- H_0 : There is no linear relationship between portfolio risk as measured by the index of unfavorable variation of portfolio return and the reciprocal of portfolio size for growth-income mutual funds.
- H_1 : There is a linear relationship between portfolio risk as measured by the index of unfavorable variation of portfolio return and the reciprocal of portfolio size for growth-income mutual funds.

The return on a portfolio depends on the holding period, and portfolio risk is a measure of the variability of portfolio return. Therefore, the next four hypotheses focus on the effect of the holding period on the degree of the hypothesized relationship between portfolio risk and portfolio size.

Hypothesis 13:

- H_0 : The degree of the linear relationship between portfolio risk is measured by the standard deviation of portfolio return and the reciprocal of portfolio size is independent of the holding period.
- H_1 : The degree of the linear relationship between portfolio risk as measured by the standard deviation of portfolio return and the reciprocal of portfolio size is dependent on the holding period.

Hypothesis 14:

- H_0 : The degree of the linear relationship between portfolio risk as measured by the standard deviation of the natural logarithms of the portfolio value relatives and the reciprocal of portfolio size is independent of the holding period.

- H_1 : The degree of the linear relationship between portfolio risk as measured by the standard deviation of the natural logarithms of the portfolio value relatives and the reciprocal of portfolio size is dependent on the holding period.

Hypothesis 15:

- H_0 : The degree of the linear relationship between portfolio risk as measured by the modified quadratic mean of the natural logarithms of the portfolio value relatives and the reciprocal of portfolio size is independent of the holding period.
- H_1 : The degree of the linear relationship between portfolio risk as measured by the modified quadratic mean of the natural logarithms of the portfolio value relatives and the reciprocal of portfolio size is dependent on the holding period.

Hypothesis 16:

- H_0 : The degree of the linear relationship between portfolio risk as measured by the index of unfavorable variation of portfolio return and the reciprocal of portfolio size is independent of the holding period.
- H_1 : The degree of the linear relationship between portfolio risk as measured by the index of unfavorable variation of portfolio return and the reciprocal of portfolio size is dependent on the holding period.

The third group of hypotheses focuses on whether a difference in the degree of the linear relationship between portfolio risk and the reciprocal of portfolio size exists for random and nonrandom portfolios.

Hypothesis 17:

- H_0 : There is no difference in the degree of the linear relationship between portfolio risk as measured by the standard deviation of portfolio return and the reciprocal of portfolio size for random and non-random portfolios.
- H_1 : There is a difference in the degree of the linear relationship between portfolio risk as measured by the standard deviation of portfolio return and the reciprocal of portfolio size for random and non-random portfolios.

Hypothesis 18:

- H_0 : There is no difference in the degree of the linear relationship between portfolio risk as measured by the standard deviation of the natural logarithms of the portfolio value relatives and the reciprocal of portfolio size for random and nonrandom portfolios.
- H_1 : There is a difference in the degree of the linear relationship between portfolio risk as measured by the standard deviation of the natural logarithms of the portfolio value relatives and the reciprocal of portfolio size for random and nonrandom portfolios.

Hypothesis 19:

- H_0 : There is no difference in the degree of the linear relationship between portfolio risk as measured by the modified quadratic mean of the natural logarithms of the portfolio value relatives and the reciprocal of portfolio size for random and nonrandom portfolios.
- H_1 : There is a difference in the degree of the linear relationship between portfolio risk as measured by the modified quadratic mean of the natural logarithms of the portfolio value relatives and the reciprocal of portfolio size for random and nonrandom portfolios.

Hypothesis 20:

- H_0 : There is no difference in the degree of the linear relationship between portfolio risk as measured by the index of unfavorable variation of portfolio return and the reciprocal of portfolio size for random and nonrandom portfolios.
- H_1 : There is a difference in the degree of the linear relationship between portfolio risk as measured by the index of unfavorable variation of portfolio return and the reciprocal of portfolio size for random and nonrandom portfolios.

The F test statistics and the critical F values at the .05 significance level used for the testing of these hypotheses are presented in Table 11 and 12. The operation of the analysis of variance model and its testing procedure has been discussed previously.

Statistical Results

The results of the two-way and three-way analysis of variance models are presented in Table 13. The null hypotheses of hypotheses 9, 10, 11 and 12 were rejected at the .05 level of significance. Moreover, in every case except hypothesis 11, they were rejected at the .01 level of significance; the critical F value equals 12.2. This evidence supports the existence of a linear relationship between portfolio risk as measured by the standard deviation of portfolio returns (SD), the standard deviation of the natural logarithms of the portfolio value relatives (SDVR), the modified quadratic mean of the natural logarithm of the portfolio value relatives (Q), or the index of unfavorable variation of portfolio returns (U) and the reciprocal of portfolio size for growth-income mutual funds.

The results of testing also show that the holding period affects the degree of this relationship. The null hypotheses of hypotheses 13, 14, 15, and 16 were all rejected at the .05 level of significance.⁵⁷ This significant main effect was followed by Tukey's a posteriori multiple comparison test to locate the significant differences. The results are presented in Table 14. The lone significant difference found was between 1970 and 1971 for the

⁵⁷ The holding period main effect is not as strong as it was with portfolio return. In the latter case, the holding period main effect was significant employing the more conservative Geisser-Greenhouse F test (See footnote 50). In this case the holding period main effect is not significant employing the conservative Geisser-Greenhouse F test. Therefore, the possibility of liberal bias must be considered in the interpretation of these results.

TABLE 13

PORTFOLIO RISK: ANALYSIS OF VARIANCE RESULTS

Source of Variation*	Critical F Value (.05 level)	SD Computed F Statistic	SDVR Computed F Statistic	Q Computed F Statistic	U Computed F Statistic
(Two-Way Analysis of Variance)					
Mean	$F_{1,7} = 5.59$	(9)** 13.69	(10) 15.03	(11) 11.60	(12) 26.52
Holding Period	$F_{3,21} = 3.07$	(13) 4.64	(14) 4.49	(15) 4.66	(16) 3.39
(Three-Way Analysis of Variance)					
Security Selection Process	$F_{1,7} = 5.59$	(17) 6.33	(18) 4.99	(19) 0.18	(20) 4.60
Holding Period-Security Selection Process Interaction	$F_{3,21} = 3.07$	2.02	2.05	3.00	0.32

*Only sources of variation tested for significance are listed.

**Numbers in parentheses indicate the hypotheses corresponding to the computed F statistics.

TABLE 14

PORTFOLIO RISK: TUKEY POST HOC MULTIPLE
COMPARISONS OF HOLDING PERIODS

Holding Periods Compared	Critical Studentized Range Value (.05 level)	Computed Studentized Range Statistic			
		SD	SDVR	Q	U
1968-1969	3.95	2.79	3.09	4.07	-0.26
1968-1970	3.95	3.42	3.40	1.35	3.16
1968-1971	3.95	-1.05	-0.76	-0.87	-1.01
1969-1970	3.95	0.63	0.31	-2.71	3.42
1969-1971	3.95	-3.84	-3.85	-4.94	-0.75
1970-1971	3.95	-4.47	-4.15	-2.23	-4.16

situations using SD, SDVR and U as the risk measure. Where Q was used as the risk measure significant differences were found between 1968 and 1969, and 1969 and 1971. The overall and holding period mean correlation coefficients are given in Table 15. Note the relatively close correspondence between the overall means of the four risk measures.

When it comes to determining whether the security selection process affects the relationship between portfolio risk and the reciprocal of portfolio size, the results were less certain. Only in the case where risk is measured by the standard deviation of portfolio returns was the null hypothesis of no security selection process main effect rejected at the .05 level. Moreover, the non-rejection in the other situations could not be explained by a

TABLE 15

PORTFOLIO RISK: MEAN CORRELATION COEFFICIENTS

Mean	Nonrandom			
	SD	SDVR	Q	U
Overall	0.3978	0.4083	0.3684	0.5041
1968	0.5332	0.5669	0.5461	0.5438
1969	0.2158	0.1889	-0.1941	0.5648
1970	0.1337	0.1454	0.3306	0.2391
1971	0.6265	0.6373	0.6564	0.6208
Overall	Random			
	SD	SDVR	Q	U
Overall	0.6178	0.6062	0.4213	0.6763

significant holding period-security selection process interaction effect. However, when the significance was relaxed to the .10 level a security selection process main effect was observed for the standard deviation of the logarithms of the portfolio value relatives and the index of unfavorable variation situations. In the case of the modified quadratic mean the holding period-security selection process interaction effect was significant at the .10 level.

These less certain results are attributed to the fact that the randomly selected portfolios were not truly random in that the feasible set was constrained to the securities held in the growth-income mutual fund portfolios. In spite of this restriction a significant difference was observed between the two security selection processes at the .10 level. The random portfolios showed a stronger degree of linear relationship between portfolio risk and

the reciprocal of portfolio size than the nonrandom portfolios.

Portfolio Performance

Statistical Model

The hypotheses dealing with portfolio performance are very similar to those dealing with portfolio return, and the same three-way mixed effects analysis of variance model is used to test these hypotheses. Therefore, the reader is directed to Figure 5, equation 33 and Table 6 for the schematic representation, the mathematical representation, and the analysis of variance table of the three-way mixed effects analysis of variance model, respectively. The same F test statistics and critical F values that appear in Table 5 apply to the testing of the hypotheses which are presented below. Likewise, the Tukey one degree of freedom for nonadditivity test, the F test statistic for the mutual fund main effect, and Tukey's post hoc multiple comparison test apply.

The dependent variable used in this statistical model is the linear trend coefficient obtained from the linear regression of portfolio performance on portfolio size for the simulated portfolios. Since four different portfolio performance measures were determined for the simulated portfolios, four different dependent variables are employed in the statistical model.

The first of the four performance measures is Sharpe's reward to variability ratio, and it is represented by equation 29.

$$\Theta_1 = \frac{A - r_f}{SD} \quad (29)$$

The second portfolio performance measure is represented by equation 30. Here, the geometric mean and the SDVR are used for the measure of the mean portfolio return and portfolio risk, respectively.

$$\theta_2 = \frac{\ln (G+1) - \ln (r_f+1)}{\text{SDVR}} \quad (30)$$

Levy's reward to vulnerability ratio is the third portfolio performance measure. It is represented by equation 31.

$$\theta_3 = \frac{\ln (G+1) - \ln (r_f+1)}{Q} \quad (31)$$

The fourth portfolio performance measure substitutes the index of unfavorable variation, U, for SD in equation 29 to give:

$$\theta_4 = \frac{A-r_f}{U} \quad (32)$$

Statistical Hypotheses

As mentioned previously, nonrandom portfolios allow for the measurement of portfolio performance that simultaneously considers both portfolio return and portfolio risk. Therefore, the impact of portfolio size on portfolio performance can be tested. Based on the results of previous empirical research with random portfolios in which only risk was measured, it was concluded that relatively small portfolios, eight to sixteen securities, were optimal and that larger portfolios did not improve portfolio performance. The first four hypotheses of this section deal with the existence of a relationship between portfolio size and portfolio performance.

Hypothesis 21:

H_0 : Portfolio performance as measured by θ_1 is independent of portfolio size for growth-income mutual funds.

H_1 : Portfolio performance as measured by θ_1 is dependent on portfolio size for growth-income mutual funds.

Hypothesis 22:

H_0 : Portfolio performance as measured by θ_2 is independent of portfolio size for growth-income mutual funds.

H_1 : Portfolio performance as measured by θ_2 is dependent on portfolio size for growth-income mutual funds.

Hypothesis 23:

H_0 : Portfolio performance as measured by θ_3 is independent of portfolio size for growth-income mutual funds.

H_1 : Portfolio performance as measured by θ_3 is dependent on portfolio size for growth-income mutual funds.

Hypothesis 24:

H_0 : Portfolio performance as measured by θ_4 is independent of portfolio size for growth-income mutual funds.

H_1 : Portfolio performance as measured by θ_4 is dependent on portfolio size for growth-income mutual funds.

The next four hypotheses focus on the effect of the holding period on the relationship between portfolio performance and portfolio size.

Hypothesis 25:

H_0 : The effect of portfolio size on portfolio performance measure θ_1 is independent of the holding period for growth-income mutual funds.

H_1 : The effect of portfolio size on portfolio performance measure θ_1 is dependent on the holding period for growth-income mutual funds.

Hypothesis 26:

H_0 : The effect of portfolio size on portfolio performance measure θ_2 is independent of the holding period for growth-income mutual funds.

H_1 : The effect of portfolio size on portfolio performance measure θ_2 is dependent on the holding period for growth-income mutual funds.

Hypothesis 27:

- H_0 : The effect of portfolio size on portfolio performance measure θ_3 is independent of the holding period for growth-income mutual funds.
- H_1 : The effect of portfolio size on portfolio performance measure θ_3 is dependent on the holding period for growth-income mutual funds.

Hypothesis 28:

- H_0 : The effect of portfolio size on portfolio performance measure θ_4 is independent of the holding period for growth-income mutual funds.
- H_1 : The effect of portfolio size on portfolio performance measure θ_4 is dependent on the holding period for growth-income mutual funds.

The next set of four hypotheses examines whether a difference in the relationship between portfolio performance and portfolio size is associated with fund size. That is, do the larger mutual funds possess a relationship between portfolio performance and portfolio size that is different than that observed for the smaller mutual funds?

Hypothesis 29:

- H_0 : No difference exists between the relationships of portfolio size and portfolio performance measure θ_1 for large and small growth-income mutual funds.
- H_1 : A difference exists between the relationship of portfolio size and portfolio performance measure θ_1 for large and small growth-income mutual funds.

Hypothesis 30:

- H_0 : No difference exists between the relationships of portfolio size and portfolio performance measure θ_2 for large and small growth-income mutual funds.
- H_1 : A difference exists between the relationships of portfolio size and portfolio performance measure θ_2 for large and small growth-income mutual funds.

Hypothesis 31:

- H_0 : No difference exists between the relationships of portfolio size and portfolio performance measure θ_3 for large and small growth-income mutual funds.
- H_1 : A difference exists between the relationships of portfolio size and portfolio performance measure θ_3 for large and small growth-income mutual funds.

Hypothesis 32:

- H_0 : No difference exists between the relationships of portfolio size and portfolio performance measure θ_4 for large and small growth-income mutual funds.
- H_1 : A difference exists between the relationships of portfolio size and portfolio performance measure θ_4 for large and small growth-income mutual funds.

The final set of hypotheses pertains to whether there is homogeneity among the mutual funds with respect to the relationship between portfolio performance and portfolio size.

Hypothesis 33:

- H_0 : No difference exists among the relationships of portfolio size and portfolio performance measure θ_1 for the individual growth-income mutual funds.
- H_1 : A difference exists among the relationships of portfolio size and portfolio performance measure θ_1 for the individual growth-income mutual funds.

Hypothesis 34:

- H_0 : No difference exists among the relationships of portfolio size and portfolio performance measure θ_2 for the individual growth-income mutual funds.
- H_1 : A difference exists among the relationships of portfolio size and portfolio performance measure θ_2 for the individual growth-income mutual funds.

Hypothesis 35:

- H_0 : No difference exists among the relationships of portfolio size and portfolio performance measure θ_3 for the individual growth-income mutual funds.

H_1 : A difference exists among the relationships of portfolio size and portfolio performance measure θ_3 for the individual growth-income mutual funds.

Hypothesis 36:

H_0 : No difference exists among the relationships of portfolio size and portfolio performance measure θ_4 for the individual growth-income mutual funds.

H_1 : A difference exists among the relationships of portfolio size and portfolio performance measure θ_4 for the individual growth-income mutual funds.

Statistical Results

The results of the three-way mixed effects analysis of variance model are presented in Table 16. Hypotheses 25, 26, 27 and 28, dealing with the effect of the holding period on the relationship between portfolio size and the various portfolio performance measures, were the only cases where the null hypothesis of no effect was rejected at the .05 level of significance.⁵⁸ For all the other hypotheses the null hypothesis could not be rejected at the .05 level of significance. The mean slope values are given in Table 17.

The existence of the significant holding period main effect was analyzed further with Tukey's a posteriori multiple comparison test to determine which holding periods were different from each other at the .05 level of significance. The results of the Tukey post-hoc test are presented in Table 18. Significant differences were found between 1968 and 1969, and 1968 and 1971 for all four

⁵⁸Employing the conservative Geisser-Greenhouse F test, critical F value of 5.99, to guard against positive bias due to possible heterogeneity in the variances and covariances still led to rejection of the null hypothesis of no holding period main effect.

performance measures. In addition significant differences were found between 1968 and 1970 for θ_3 and θ_4 , between 1969 and 1970 for θ_1 , θ_2 , and θ_4 , and between 1969 and 1971 for θ_4 .

TABLE 16
PORTFOLIO PERFORMANCE: ANALYSIS OF VARIANCE RESULTS I

Source of Variation*	Critical F Values (.05 level)	Computed F Statistics			
		θ_1	θ_2	θ_3	θ_4
Mean	5.99	(21)** 0.29	(22) 0.48	(23) 0.92	(24) 0.12
Holding Period	3.16	(25) 15.76	(26) 15.88	(27) 11.53	(28) 20.75
Fund Size	5.99	(29) 0.14	(30) 0.07	(31) 0.28	(32) 0.18
Mutual Fund	2.66	(33) 1.47	(34) 1.30	(35) 0.20	(36) No test
Holding Period- Fund Size Interaction	3.16	1.90	1.92	1.80	1.91

*Only sources of variation tested for significance are listed.

**Numbers in parentheses indicate the hypotheses corresponding to the computed F statistics.

TABLE 17

PORTFOLIO PERFORMANCE: MEAN SLOPE VALUES

Mean	θ_1	θ_2	θ_3	θ_4
Overall	0.00023	0.00028	0.00062	0.00055
1968	0.00355	0.00378	0.00836	0.01264
1969	-0.00298	-0.00291	-0.00363	-0.01368
1970	0.00133	0.00126	0.00138	0.00188
1971	-0.00098	-0.00099	-0.00365	0.00138
Large Funds	0.00039	0.00039	0.00028	0.00123
Small Funds	0.00007	0.00018	0.00096	-0.00013

TABLE 18

PORTFOLIO PERFORMANCE: TUKEY POST HOC MULTIPLE
COMPARISONS OF HOLDING PERIODS

Holding Periods Compared	Critical Studentized Range Value (.05 level)	Computed Studentized Range Statistic			
		θ_1	θ_2	θ_3	θ_4
1968-1969	4.00	9.17	9.24	7.17	11.1
1968-1970	4.00	3.13	3.47	4.18	4.53
1968-1971	4.00	6.36	6.58	7.18	4.47
1969-1970	4.00	-6.04	-5.77	-2.99	-6.55
1969-1971	4.00	-2.81	-2.66	0.01	-6.34
1970-1971	4.00	3.23	3.11	3.00	0.21

The conclusion drawn from these results is that the effect of portfolio size on portfolio performance does depend on the holding period. But, as was pointed out previously, there is no indication that portfolio managers are taking advantage of this phenomenon as no difference in portfolio size was found between the holding periods.

The nonrejection of the null hypothesis for hypotheses 21, 22, 23, and 24 must be interpreted in light of the significant holding period main effect. The failure to refute the existence of independence between portfolio performance and portfolio size is for the longer run situation, the average effect over the four year period. Again, as was true for portfolio return, it is the positive effect one year and the negative effect the next that leads to the independent effect over the combined holding period. (Review the mean slope values in Table 17.)

No evidence was found to indicate a difference in the relationship between portfolio size and portfolio performance due to mutual fund size differences. The null hypotheses of hypotheses 29, 30, 31, and 32 that no difference exists between the relationships of portfolio size and portfolio performance for large and small growth-income mutual funds could not be rejected even at the .25 level of significance; the critical F value is 1.57.

No evidence was found to indicate heterogeneity of professional management across the growth-income mutual funds. The null hypotheses of hypotheses 33, 34, and 25 that no difference exists among the relationships of portfolio size and portfolio performance for the individual growth-income mutual funds could not be rejected at the

.05 level of significance. Moreover, except for hypothesis 33, these hypotheses could not be rejected at the .25 level of significance; the critical F value for that level is 1.45. In the case of hypothesis 36 no test for the mutual fund main effect existed. In order to test these hypotheses it was first necessary to apply Tukey's one degree of freedom for nonadditivity test to show that a holding period-mutual fund size interaction did not exist. Since an interaction effect was found in the case of hypothesis 36, the mean square for this interaction could not be used as the error term in the test for a mutual fund main effect. Hence, no test existed.⁵⁹

The possibility of measurement bias that existed for the tests involving portfolio return is also present here. Therefore, the same two experiments which were performed to determine the presence of this bias in the case of portfolio return were carried out with the substitution of portfolio performance for portfolio return. The first experiment is the same three-way analysis of variance model as employed above with the dependent variable based on only the first 37 securities. Its results are presented in Table 19. The results of the hypothesis testing are the same as those above. Only in the case of the holding period-fund size interaction was there any appreciable change in the computed F values.

⁵⁹ The computed F statistics for θ_1 , θ_2 , θ_3 , and θ_4 were .02, 1.04, 1.22 and 3.99 respectively in determining whether a holding period-fund size interaction was present. Only for θ_1 , θ_2 , and θ_3 was the critical F value, $F_{.25;1,20} = 1.40$, not exceeded indicating the absence of the interaction effect and therefore allowing a test for the mutual fund main effect.

The second experiment is a four-way analysis of variance model with the additional independent variable being the incremental step. The results of this experiment are given in Table 20. There was no evidence of an incremental step main effect.

TABLE 19

PORTFOLIO PERFORMANCE: ANALYSIS OF VARIANCE RESULTS II

Source of Variation*	Critical F Values (.05 level)	Computed F Statistics			
		θ_1	θ_2	θ_3	θ_4
Mean	5.99	0.02	0.06	0.20	0.33
Holding Period	3.16	16.36	16.20	11.32	35.16
Fund Size	5.99	0.01	0.00	0.04	0.19
Mutual Fund	2.66	0.66	0.62	0.43	No test
Holding Period- Fund Size Interaction	3.16	0.44	0.52	0.76	0.42

*Only sources of variation test for significance are listed.

In each case the null hypothesis of no incremental main effect could not be rejected at the .05 level of significance. However, the holding period-incremental step interaction effect was significant at the .05 level in each case. From the raw data it appears that the nonsignificant incremental main effect was due to a positive relationship in some years and a negative relationship in others to yield a nonsignificant result overall. Employing the absolute values of the dependent variables in the same model yielded a significant incremental step main effect at the .05 level in each case.

The means of the five incremental steps respectively using the absolute values were .00031, .00014, .00007, .00006, and .00006 for θ_1 , .00071, .00047, .00029, .00026, and .00024 for θ_2 , .00032, .00014, .00007, .00006, and .00006 for θ_3 , and .00056, .00022, .00010, .00012, and .00010 for θ_4 . Hence, a decreasing absolute marginal effect of portfolio size exists.

TABLE 20
PORTFOLIO PERFORMANCE: ANALYSIS OF VARIANCE RESULTS III

Source of Variation*	Critical F Value (.05 level)	Computed F Statistics			
		θ_1	θ_2	θ_3	θ_4
Mean	5.99	0.61	0.31	0.15	0.03
Holding Period	3.16	19.31	19.29	8.39	37.91
Incremental Step	2.78	0.81	0.60	0.47	0.92
Fund Size	5.99	0.01	0.00	0.01	0.22
Holding Period- Incremental Step Interaction	1.89	6.38	6.42	5.36	3.29
Holding Period- Fund Size Interaction	3.16	1.04	1.02	1.49	1.15
Incremental Step- Fund Size Interaction	2.78	0.51	0.54	0.38	1.55
Holding Period- Fund Size- Incremental Step Interaction	1.89	1.16	1.08	1.63	1.11

*Only sources of variation tested for significance are listed.

The conclusion drawn from these two experiments is that there was a decreasing absolute marginal effect of portfolio size on portfolio performance, but that it did not appreciably bias the measurement variable employed.

CHAPTER IV

SUMMARY AND CONCLUSIONS

This final chapter begins with a summary of the research. The limitations are then discussed prior to the presentation of the conclusions. Finally, recommendations for further research are presented.

Summary

Objectives

The objectives of this research were to strengthen the existing inadequate empirical knowledge regarding the relationship between portfolio size and portfolio return, between portfolio size and portfolio risk, and between portfolio size and portfolio performance. In addition, the intent of this research was to determine an optimal portfolio size for common stock portfolios and to rationalize the observed portfolio sizes of common stock mutual funds.

To date, the basic empirical research has been done with random portfolios. The use of random portfolios has two limitations. First, random selection of securities for investment portfolios is practiced only by academicians, not practitioners. Second, while the random selection procedure provides the effect of portfolio size on portfolio risk, it precludes the determination of the effect of portfolio size on portfolio return. Since both portfolio risk and

portfolio return are necessary for the computation of portfolio performance, the effect of portfolio size on portfolio performance is also precluded when random portfolios are used. This research represents an initial attempt to employ nonrandom portfolios in determining the effect of portfolio size on the portfolio parameters of return, risk, and performance.

Sample

Growth-income mutual fund portfolios were the nonrandom portfolios selected for use in this study. Random selection of the portfolios was limited to this one classification so that the selected portfolios had the same stated goals. The growth-income classification was used because these mutual funds are common stock funds and because of the homogeneity of the goals among the mutual funds within this classification. The selected mutual funds were Dodge and Cox Stock Fund; Eaton and Howard Stock Fund; Fidelity Fund; Massachusetts Investors Trust; One William Street Fund; Scudder, Stevens and Clark Common Stock Fund; Varied Industry Plan; and Wisconsin Fund. From these eight mutual funds thirty-two nonrandom common stock portfolios were obtained by using their year-end portfolios for 1967, 1968, 1969 and 1970. Portfolio measurements were taken for these four annual holding periods. While the average holding period appeared to be somewhat longer for the selected mutual funds, the implicit assumption of no portfolio revision precluded extension of the holding period beyond one year.

Measurements

In order to measure the effect of portfolio size on the various portfolio parameters, it was necessary to simulate the portfolio building process from the first security to the addition of the last. This was accomplished by ranking the securities within each mutual fund portfolio where the market value of the mutual fund's investment in each security was used as a proxy for the portfolio manager's ranking for portfolio inclusion. To permit the measurement of the effect of portfolio size on the portfolio parameters, this research abstracted from the allocation of funds decision and assumed the investment of equal dollar amounts in each security.

Because the study of portfolio performance is an ex post process, it was necessary to divide the annual holding period into subperiods in order to measure ex post variability. Hence, twelve monthly returns were computed for each individual security. Two portfolio return measurements were used in this research, the arithmetic and geometric means of the twelve monthly portfolio returns. Four measures of portfolio variability were used. The standard deviation of portfolio returns and the standard deviation of the natural logarithms of the portfolio value relatives were selected because they have been employed previously and therefore, provide a point of reference. The other two measures were the modified quadratic mean of the natural logarithms of the portfolio value relatives and the index of unfavorable variation of portfolio return. These latter two measures treat only unfavorable variation as risk. Four performance measures which incorporate the above

risk and return measures were used. They were Sharpe's reward to variability ratio (θ_1), a modification of Sharpe's reward to variability ratio where the index of unfavorable variation replaces the standard deviation (θ_4), Levy's reward to vulnerability ratio (θ_3), and a modification of Levy's reward to vulnerability ratio where the standard deviation of the natural logarithms of the portfolio value relatives replaces the modified quadratic mean of the natural logarithms of the portfolio value relatives (θ_2).

Statistical Models

The simulation necessary to measure the effect of portfolio size on the various portfolio parameters led to statistical dependence. Consequently, the selection of the statistical models for this research was constrained by this existence of statistical dependence. Although statistical dependence precluded the testing of the individual regression coefficients (obtained by regression of a portfolio parameter against portfolio size) for statistical significance, the regression coefficients could be pooled and tested for statistical significance as a group.

In the case of portfolio return and portfolio performance, the slopes obtained from the linear regression of portfolio return and portfolio performance against portfolio size respectively, were pooled and tested for significance using a three-way mixed effects analysis of variance models. The three independent variables were the annual holding period, mutual fund size, and mutual fund nested within mutual fund size. In this way the two portfolio return measures and the four portfolio performance measures were tested to

see if they were independent of portfolio size, if the holding period affected the linear relationship between the portfolio parameter and portfolio size, if the size of the mutual fund affected this relationship, and if the mutual funds themselves differed among each other in this relationship.

Two analysis of variance models were required in the case of portfolio risk. A two-way analysis of variance model was utilized to determine whether there was a linear relationship between portfolio risk and the reciprocal of portfolio size, and also whether the holding period affected this relationship. The two independent variables were the holding period and the mutual fund. The dependent variable was the transformed correlation coefficient obtained from the linear regression of portfolio risk against the reciprocal of portfolio size. This test, like the one to follow, was performed on each of the four risk measures. In order to determine whether the security selection process (random vs. nonrandom) affected the relationship between portfolio risk and the reciprocal of portfolio size, a three-way analysis of variance model was used. The three independent variables were the annual holding period, the security selection process, and the mutual fund. This model required that the thirty-two nonrandom portfolios be balanced by thirty-two random portfolios. The latter were obtained by randomly selecting securities for portfolio inclusion from the set of securities held by the selected growth-income mutual funds. Again, the dependent variable was the transformed correlation coefficient.

Statistical Results

The effect of portfolio size on portfolio return was found to be dependent on the holding period. In some years the effect was positive and in others negative. The longer run effect over the four annual periods however, averaged out, and over this longer period portfolio return was found to be independent of portfolio size. There was no evidence to indicate that the effect of portfolio size on portfolio return differed between the large and the small growth-income mutual funds. Moreover, there was no evidence to indicate that this effect differed among any of the mutual funds.

A linear relationship between portfolio risk and the reciprocal of portfolio size was found to exist for all four of the risk measures employed. Also, it was found that the holding period affected these relationships. The results were less certain when it came to determining whether the security selection process (random vs. nonrandom) affected the relationship between portfolio risk and the reciprocal of portfolio size. Only the standard deviation of portfolio return showed a difference between the two selection processes. However, the standard deviation of the natural logarithms of the portfolio value relatives and the index of unfavorable variation gave a significant difference at the .10 level. In the case of the modified quadratic mean of the natural logarithms of the portfolio value relatives, the nonsignificance at the .10 level was attributed to the significant holding period-security selection process interaction. These less than certain results might be attributable to the fact that the randomly selected portfolios were not truly random in that the feasible set of securities

was constrained to the securities held in the growth-income mutual fund portfolios.

The effect of portfolio size on portfolio performance was found to be dependent on the holding period. Like portfolio return, in some years the effect was positive and in others negative. However, over the four annual periods portfolio performance was found to be independent of portfolio size. There was no evidence to indicate that the effect of portfolio size on portfolio performance differed between the large and small growth-income mutual funds, and there was no evidence to indicate that this effect differed among any of the mutual funds.

Limitations

The purpose of this section is to discuss the limitations in this research. Attention is first focused on the internal validity of the research design. Internal validity is concerned with the validity of the results obtained from the statistical model. Was there any measurement bias in the dependent variable or other biases present that would invalidate the results? Were the required assumptions of the statistical models met?

The following steps were taken to insure internal validity. Random sampling was employed to prevent biased comparisons. Mutual fund portfolios were blocked on actual portfolio size and annual holding period to control any bias due to these variables. A balanced experimental design was employed because under these conditions analysis of variance models are robust to the assumptions of normality and homoscedasticity. The possibility of measurement

bias was checked in the cases of portfolio return and portfolio performance by using the same number of observations in the regression and by employing incremental regressions. The findings indicated no bias. Finally, when the possibility of bias due to heterogeneity among the covariances was present, the conservative Geisser-Greenhouse test was employed which compensated for the maximum possible bias.

In interpreting the statistical results it should be pointed out that where the null hypothesis cannot be rejected, it should not automatically be accepted. There is always the possibility that the nonrejection is due to the smallness of the sample size. Acceptance may lead to a type II error, accepting the null hypothesis when it is false. Before any decision is made on the acceptance of the null hypothesis, the probability of a type II error is required. Unfortunately this cannot be determined without a knowledge of the true value of the parameter in question. That is, the probability of a type II error varies with the difference between the true value of the parameter and the hypothesized value. Hence, nonrejection is not the same as acceptance.

While the possibility of a type I error, rejecting the null hypothesis when it is true, can be controlled by prescribing the significance level, carrying out more than one separate test causes the probability of the type I error to increase. In this research the .05 level of significance was used. However, due to the carrying out of two, four, and four separate tests for portfolio return, portfolio risk, and portfolio performance respectively, the probabilities of making a type I error increased to

.10, .20 and .20 respectively.⁶⁰

Attention is now directed toward the external validity of the statistical results. That is, to what population can these results be generalized? Since the mutual fund portfolios were chosen randomly, the results can be generalized to the feasible set of growth-income mutual funds. A more liberal point of view is to generalize the results to common stock portfolios that have the same characteristics as the tested portfolios.⁶¹

Finally, mention is made again of the following procedural limitations. The measured monthly portfolio return was a gross return; no expenses were netted out. The market value of the mutual fund's investment in each security was used as a proxy for the portfolio manager's ranking for portfolio inclusion. To permit the measurement of portfolio size on the various portfolio parameters, this research abstracted from the funds allocation decision and assumed equal allocation of investment funds.

Conclusions

The empirical investigation of the nonrandom portfolios obtained from growth-income mutual funds has led to the conclusion that there is no optimal portfolio size for these common stock mutual funds. It was found that the effect of portfolio size

⁶⁰The probability of a type I error when several tests are made is computed by taking the product of k and the level of significance, where k equals the number of tests.

⁶¹See Jerome Cornfield and John W. Tukey, "Average Values of Mean Squares in Factorials," Annals of Mathematical Statistics, XXVII (December, 1956), 907-949.

on the various portfolio parameters of return, risk, and performance is dependent on the holding period. Hence, what may be an optimal size for one period may not be optimal for the next. Moreover, when the longer run situation was considered, no evidence was found that either portfolio return or portfolio performance were dependent on portfolio size. This lack of evidence of portfolio size affecting portfolio return or portfolio performance in the longer run situation tends to support the efficient market hypothesis, which predicts that portfolio return and performance are independent of portfolio size.

With regard to the effect of portfolio size on portfolio risk, it is concluded that the way in which the securities are selected for portfolio inclusion, i.e., randomly versus nonrandomly, does affect the relationship between portfolio risk and portfolio size. While the effect of portfolio size on portfolio risk for nonrandom portfolios can be represented by the same mathematical relationship as that observed for random portfolios, the degree of the relationship is not as great. That is, while like random selection the benefits of diversification drop off quickly, the degree of this relationship is significantly less for the nonrandom portfolios.

What can be concluded about the actual observed portfolio sizes of common stock mutual funds? From the results of this study it is concluded that these large portfolio sizes do not represent constrained optima. In order to be constrained optima, the large dollar investment in the mutual funds must prevent the portfolios from reaching their unconstrained optimal size. The evidence is somewhat mixed on whether the size of the transactions necessary

to partake of the portfolio manager's ability to predict security values affects security prices significantly so as to lose any advantage of prediction. However, the lack of evidence of an optimal size in this research leads to the conclusion that the large portfolio sizes are not optima either constrained or unconstrained. That is, since portfolio performance is independent of portfolio size in the long run situation, mutual fund portfolio performance can not be improved by decreasing portfolio size.

Recommendations for Further Research

This study represents an initial attempt to determine empirically the effect of portfolio size on portfolio performance through the employment of nonrandom portfolios. Hopefully, the results of this study will stimulate additional empirical research with nonrandom portfolios. In this vein, recommendations for further research are presented.

Future research should be oriented toward overcoming the limitations discussed earlier in this chapter. The inherent problem of statistical dependency resulting from the simulation of portfolios of increasing size might be handled by some statistical technique other than by the pooling of regression coefficients. Also, the statistical dependency problem could be eliminated if a sample of nonrandom portfolios of increasing size were available, and thus obviated the need for simulation. An attempt should be made to incorporate the funds allocation decision which this study abstracted from by assuming the investment of equal dollar amounts in all securities. An attempt should also be made to allow for portfolio

revision and thereby extension of the holding period under study rather than holding the portfolios fixed for one year as in this study. An attempt should also be made to incorporate additional information affecting the portfolio size decision which was not available for this study. This additional information includes transactions costs, research costs, clerical costs, and the actual ranking for portfolio inclusion held by the portfolio manager.

In addition, expansion of this work to include portfolios with different goals, e.g., the capital gains oriented mutual fund portfolios, would allow the determination of the generality of this study's conclusions. That is, are the conclusions of this study specific for growth-income mutual fund portfolios or more general and applicable to other types of portfolios?

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