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EXPECTED UTILITY MAXIMIZATION AND THE THEORY OF MULTI PRODUCT BANKING FIRM UNDER UNCERTAINTY

by

Elyas Elyasiani

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ABSTRACT

EXPECTED UTILITY MAXIMIZATION AND THE THEORY OF MULTI PRODUCT BANKING FIRM UNDER UNCERTAINTY

By

Elyas Elyasiani

An attempt is made in this dissertation to integrate some different aspects of bank behavior such as physical production, portfolio selection, costs, liquidity management, risk aversion, and institutional constraints into a unified banking model.

The existing banking models may be categorized as theories of costless intermediation. The banking literature has undersold the neoclassical theory of the firm for portfolio theory. Banking activity is summarized in portfolio management, operating costs are overlooked, and the role of banks as the administrators of the national payment mechanism is ignored.

In this study microeconomic theory is applied to banks as multiproduct firms. The banking firm is not a pure financial intermediary. In addition to loans and investments which it produces as an intermediary, it offers clearance services as a direct supplier. Banking outputs are joint and produced under uncertainty. Uncertainty is present due to randomness in deposits, interest rates, and output quantities. The bank maximizes the expected value of

an exponential utility function as the objective function.

The effects of changes in exogenous variables on bank behavior are investigated through comparative static results. The basic model indicates that portfolio composition and the price charged by the bank on clearance output are not independent of the rate paid on demand deposits. With an increase in the latter rate, liquidity declines and the percentage of the portfolio held in the risky asset increases. This means that regulation Q, when effective, serves the purpose of keeping banks liquid and safe. Regulation Q, however, is also found to lower the price paid for the clearance output. This artificially low price leads to overutilization of the product by consumers and a resource allocation which is not socially optimal. A trade-off may therefore be said to exist between bank liquidity and allocative optimality. It is desirable that the policy maker frequently chooses the optimal regulation Q ceiling on the trade-off possibility curve according to prevailing priorities. The existing zero ceiling is not necessarily appropriate over time.

Comparative static results on variances indicate that uncertainty increases liquidity, reduces the loan output, and lowers the certainty equivalent return on unencumbered funds. This feature should be considered in the making of monetary policy. The effect of uncertainty created by the policy should be taken into account, as well as the effect of the change in the policy instrument. The former effect may even overshadow the latter.

Asset reserve credit and payment of interest on excess reserves are investigated as potential monetary policy instruments. It is found that these instruments may be used to broaden the Fed's control over credit. The effects of an increase in the reserve requirement and the discount rate are found to be in general indeterminate. Only under specific conditions are these effects restrictive.

The basic model is extended to include more outputs and to incorporate liability management. The extended model indicates the possibility of both complementarity and substitution between outputs depending on the output mix. Results in the basic model do not all carry over to the extended model. In particular, fewer results have determinate signs.

Monthly data on the large weekly reporting banks in New York City are used to estimate the reduced form of the linearized version of the model. The sample covers the period June 1969-December 1978. To measure expectations on demand deposits and interest rates Box-Jenkins time series techniques are invoked to develop ARIMA models and make forecasts on these variables. The forecasts are used as data in estimation. The empirical results indicate that the hypotheses of risk aversion and dependence of bank decisions on operating costs can not be rejected.

To my Father,

and to the souls of

Mother, Moshe, and Avner

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INTRODUCTION

Commercial banks are one of the most vital institutions in today's economies. They constitute an important link in the monetary transmission process, play a key role in the capital market, and strongly influence the money supply, business activity, and the price level.

In spite of such prominence, a definitive model of bank behavior has not yet appeared. The existing models, greatly overshadowed by portfolio theory, treat banks as pure intermediaries that allocate a sum of funds among competing assets so as to optimize some objective function. These models are often unintegrated, lack a satisfactory theoretical framework, and overlook some of the relevant functions of banks. The marginalist micro theory has rarely been utilized to analyze the banking enterprise. The complex and diverse nature of commercial bank activities has made the establishment of a workable theory of production and cost prohibitively difficult.

In this study an attempt is made to develop an integrated model of bank behavior in a microeconomic firm theoretic context. The bank is considered as a multi product firm which transforms its inputs of capital, labor, and deposits into some outputs. The clearance activity which makes banks the administrators of the nation's payment mechanism is considered as a non-intermediary output. Loans and investments constitute intermediary outputs. The transformation of inputs into outputs is, like in any other firm, subject to a production function. It is argued that banking outputs are technically interdependent and therefore joint.

The study may be summarized as follows: The first chapter provides a brief review and a critique of the existing literature. In Chapter Two a theoretical framework is established and utilized to develop a model of multi product firm under uncertainty. In this chapter only two outputs are introduced, the clearance output and loans. The sources of funds are the exogenously determined demand deposits and the net worth. Uncertainty, is due to randomness in demand deposits, the loan rate, and the demand for the clearance output. The quantity of loans is deterministically set and achieved. The bank holds excess reserves to satisfy its liquidity needs and borrows at the discount window in case of a deficiency. The comparative static results are derived and interpreted. Randomness in loans is incorporated into the model in Chapter Three. This makes the quantity of loans and the loan rate simultaneously random. The comparative static results are found to remain essentially unchanged.

Commercial banks may not, in our time, be considered as passive intermediaries. The prevalence of liability management has transformed banks to aggressive enterprises. Chapter Four extends the model to introduce liability management. The number of outputs is also increased. With liability management the bank has access to new sources of funds which are under its control. It can sell CD's or buy funds in the federal funds market to increase the scale of its operation. The comparative static results for the extended model are derived, interpreted, and contrasted to previous models.

In Chapter Five the model is linearized, the adjustment mechanism between actual and desired values is hypothesized, and a system of reduced form equations is derived which is adaptable to estimation. To measure expectations on demand deposits and interest rates, which appear as regressors, in Chapter Six Box-Jenkins techniques are used to develop time series models and make forecasts on these variables.

In Chapter Seven the estimation techniques are discussed. The empirical results are displayed and contrasted to theoretical comparative static results.

Chapter Eight derives the policy implications of the results and provides some of the limiting features of the model. The mathematical derivations of moments of an incomplete distribution - discount window borrowing, - the second order conditions, total differentiation of the first order conditions, comparative static results, and the moments of products of random variables are provided in Appendices one through five respectively. Graphs of actual versus expected values of demand deposits and interest rates are given in Appendix Six.

CHAPTER ONE

A REVIEW OF THE BANK BEHAVIOR MODELS

Even though there exists an extensive literature on banking, a satisfactory model of bank behavior is still lacking. It is surprising that the neo-classical micro-economic theory did not find its way to banking literature until the 1960s, and even since then this apparatus has not been extensively used. The nature of the bank as a productive firm has not received adequate attention. In 1961 Porter rightly complained: "Over the course of the last century, the implications of the assumption of profit maximization for the behavior of the firm have been tracked down in ever greater detail, curiously however, this firm has almost always been a seller of non-financial goods; banking has been studiously exempted from the application of such theory. The exemption is curious because the commercial bank seems in many respectes more likely to fit the conditions of such static theory than the product manufacturer" [33 , p. 12].

To bridge the gap between the bank behavior models and the microtheory, Porter set up a model of banking firm which produces loans, securities, and cash under uncertainty. Porter recognized uncertainty as the crux of bank operation and explicitly introduced deposit variability into the asset selection process. This introduction came just about seventy years after Edgeworth had indicated the importance of such uncertainty in banking [9]. Porter applied an inventory-theoretic approach to the banking firm, considering cash and assets readily convertible to cash as inventories. The carrying costs of these inventories are the interest income foregone. Insufficient inventories are subject to penalty.

In Porter's model, the bank's sole concern is the deposit lowpoint. This is the lowest point to which deposits fall during the period and consequently, the point which necessitates the most radical adjustments in the bank's portfolio. In case of deficiency, at the deposit low point, the bank will sell securities at prevailing market prices, and when securities are exhausted, it will borrow from the discount window. Both competitive and imperfect loan markets are introduced and the firm is assumed to choose its portfolio so as to maximize the expected value of its profits.

Porter's model can explain the observed diversification behavior, the demand for excess reserves, and the dependence of the optimal asset-mix on the asset-returns, the discount rate, and the distribution of the deposit low.

In the Porter model, the bank's demand for reserves is not determined by the optimization process. It is set to be a fixed percentage of its deposits. The Porter banker always holds some interest-free excess reserves for no purpose, except to maintain the fixed desired excess reserve ratio. Porter's liquidity adjustment pattern is equally mechanical. In case of a deficiency, securities are completely exhausted before any other liquidity source is considered. This is a hierarchical pattern imposed on the bank and one that a rational decision making unit would not necessarily follow. Additionally, Porter's results are derived under specific simple distributions for the model's random variables and would not hold in general. Finally, Porter's model has not undergone any empirical testing; theory has not simply been made operational. To make the model testable, at least an expectation formation mechanism has to be introduced.

Another approach to bank behavior is the portfolio approach introduced by Markowitz [25]. In this approach returns on assets are taken to be random, and the investor minimizes the risk for any given expected return. The portfolio approach can explain the observed diversification phenomenon; when the returns are not perfectly correlated, diversification reduces the risk. This approach, however, ignores all the dimensions of the asset selection which are not translated into the two first moments of the return on portfolio. Specifically, if bank behavior can be divided into a portfolio decision and a liquidity decision, the portfolio approach certainly leaves the latter out.

Kane and Malkiel's (1965) model is based on the introduction of deposit variability into such a framework [20]. The model includes loans and securities as assets with equal maturities and random returns. No reserves are held and no borrowing is possible, so that deposit flows are supposed to be totally reflected in the securities.

Kane and Malkiel enrich this apparatus by introducing the idea of customer relationship and its implications on short and long-run earning and risk. They argue that there exists a class of loan requests L*, distinguished primarily by a continuing relationship between borrower and the bank, where the very failure to grant the loan itself would change the banker's opportunity set. It would reduce the strength of the relationship between the borrower and the bank, leading to a decline in the expected value of both short and long-run profits and an increase in aggregate risk [20 , p. 119]. In such a world, Kane-Malkiel claim that, in a tight money condition banks stretch the liquidity limits, sell their securities with capital losses, and ration credit on the basis of the strength of the customer relationship to satisfy the L* customers. This behavior, they conclude, is in contrast with the three basic points of the availability doctrine which ignores the bank's reaction to monetary policy.

One criticism of the model is that if the bank adjusts its portfolio of loans and securities with the possible arrival of the L* customers, the quantities of loans and securities may not be set and achieved. They must be stochastic. In addition, Kane and Malkiel do not provide an empirical test of their model.

Shull's (1963) banking model is based on the Clemens (1951) model of multi product price-discriminating firm [38,7]. The latter views the firm as a combination of Joan

Robinson's price-discriminating monopolist and Chamberlin's product-differentiating monopolistic competitor.

The firm uses its mobile resources to produce _m_ distinct outputs which are cost-wise homogenous. It sells each output _i_ to $\underline{n_i}$ separable groups of customers with different demand elasticities. Each group of customers demanding one output constitutes a market and is represented by a demand curve. The demand curves are assumed to be independent both across customer groups and across products.

The firm starts off producing for the market with the least elastic demand. It then diversifies into new markets with successively more elastic demand curves. It carries its production to the point at which marginal cost equals the marginal revenue in the least profitable (marginal) market. The quantities and prices are then adjusted in other markets so that all markets have an equal marginal revenue. This adjustment maximizes the firm's profits.

Such a firm will have a barely profitable marginal market which might have a quite elastic demand curve. It increases its prices and restricts its output in less elastic markets as it moves to new ones and tries to increase the number of its markets as much as possible, because this leads to greater profits. The firm, however, may end up with normal profits if the number of competitors is large.

Shull claims that such a model fits banking activity. He argues that: a) the bank's main resource (funds) is mobile, b) outputs can be considered cost-wise homogeneous since the main cost is the cost of funds, c) the bank customers can be divided into groups with different demand elasticities, d) banks do seem to diversify into new markets (new types of loans) instead of lowering the price in the old markets, and e) they do seem to discriminate in pricing by negotiating with each customer.

Shull's model may be criticized for its unrelated demand assumption. [See [[37]] and cost homogeneity. Besides, Shull fails to provide a mathematical formulation of the model.

Klein (1971) introduced a neoclassical microeconomic analysis of the banking firm [22]. In this analysis the bank is assumed to set the deposit rates in the deposit market and to use the funds to acquire loans, securities, and cash. The security market is assumed competitive and the loan market imperfect. Uncertainty is present due to randomness in deposits and asset returns. Cash holding represents a precautionary demand. The bank's objective is to maximize the rate of return on equity.

In this study Klein finds that: a) the optimal asset mix is independent of rates paid on deposits [22 , p. 215], so that the riskiness of the bank portfolio is not altered by variations in rates paid on deposits and the justification for regulation Q is unsound, b) loans are held up to the point at which their marginal return is the same as the exogenous expected return on riskless government securities E(g) [22 , p. 213], in other words, E(g) is the cut-off rate, c) the rate on each kind of deposit is set at a point which is below the portfolio yield by a factor which depends on the parameters of the deposit supply function (p. 214). Klein also estimates a production function for the bank's clearance output.

Pringle (1973) challenged the generality of these results [34]. He showed that result (a) "follows from his [Klein's] assumption that all controllable sources of funds have rising cost curves (all are endogenous) and that something other than deposit rates, namely the exogenous expected return on riskless government security E(g) constitutes the exogenous rate that pegs the system." In the same way Pringle showed that result (b) "follows from the assumption that E(g) alone is exogenous, and that investors are risk neutral" [34 , p. 992]. Pringle introduced a set of assumptions under which these results did not follow.

Hester and Pierce (1975) developed an econometric micro model of bank behavior in the framework of a profit maximizing firm subject to legal and institutional constraints, and deposit and loan uncertainty [17]. In this study, the exogenously determined bank liabilities--demand and time deposits--are assumed to be generated by auto-regressive stochastic processes. The processes are taken to be independent and uninfluenced by the portfolio composition [17 , p. 36]. Hester and Pierce banks consider the deposit inflows as largely transitory. They ultimately expect to retain only a small fraction of each inflow. Therefore, they try to lend only this small permanent component, holding the remaining in securities and cash, depending on the dates the deposits are expected to be withdrawn (p. 69). More specifically Hester and Pierce hypothesize a lagged portfolio adjustment mechanism by which deposit inflows are initially held in the form of cash, but are transferred gradually to less and less liquid assets. In this dynamic specification the asset structure is strongly dependent on the time pattern of changes in liability items. Hester and Pierce test their lagged adjustment hypothesis in an input-output version and an adaptive-expectation version. Their results for commercial banks are found to be satisfactory. The extent of their empirical work and the huge set of information which they use are impressive. Their empirical model and their empirical results, however, do not follow the theoretical framework they establish.

Scott (1977) developed a model of a multi product banking firm facing interrelated demand curves [37]. The idea is that prices charged to various customers for one output (e.g., loans) will influence their demand for other outputs (like deposits). Scott argues that with related demands "intuitively we might expect that the firm under certain conditions would set $P_1 < MC_1$ (marginal cost of good 1) to stimulate the demand for good 2. Good 1 would be a loss-leader in business terminology. The simple tactic of lowering prices on some goods to increase demand for others, becomes increasingly likely in a world of imperfect information where the seller wants to "get the customer's attention" and win his business [37 , p. 16].

In this model deposits are considered as both outputs and inputs or as outputs sold at ostensibly negative prices because they can support the sale the income-earning assets. "Formally the marginal revenue from an additional dollar of liabilities 'sold' includes a 'multiplier' from the relationship constraining assets given liabilities" (p. 22), so that if we incorporate the effect of this multiplier, the "effective marginal revenue" will be positive and equal to marginal cost of deposits production. Scott's objective is to examine the impact of concentration on prices, but his model can be used to examine the other aspects of bank behavior as well. He claims that his model includes Hodgman's model as a special case [19].

It is observed that banking models consider banks either as rational investors or productive firms. The rational investor models summarize banking activities to portfolio management and leave out the other dimensions. It may be noticed that the models developed by Porter, Kane and Malkiel, and Hester and Pierce are in this category. Among the models that consider banks as firms there is no general agreement on what constitute outputs, or what the technological and demand interdependence patterns between inputs or outputs are.

One output of the bank which is not often recognized is the clearance output. Generally, this output is overshadowed by portfolio management and taken as incidental to it. The clearance service is the distinctive feature between banks and other financial intermediaries; a service for which Pesek claims that portfolio management is maintained akin to the maintenance of cables for the phone services [31].

Another feature which is often ignored is the banker's porverbial conservatism. Models presented by Porter, Shull, Klein, and Hester and Pierce all display risk neutrality. Under uncertainty stochastic profits may deviate from their expected levels too much to be acceptable to the banker. The banker is therefore expected to consider the risk and to act accordingly. This point has been explained by Drhymes by using the Tchebychev inequality [8 , p. 245]. The inequality as applied to profit may be expressed as:

$$P_r\ [|\pi - E(\pi)| > K\sigma\pi] < \frac{1}{K^2} \qquad K > 0$$

where $E(\pi)$, $\sigma\pi$ are the mean and the standard deviation of profits (π), and K is a constant. In this inequality for large K the deviation $[\pi - E(\pi)]$ may be beyond the maximum level acceptable to the bank.

Exogeniety of deposits is another problem with some of the models. This feature might be adequate for pre 1960s, but no longer prevails. The banking world has undergone great changes over the last decade. Banks no longer leave the level and the composition of their liabilities to the public. Rather, they actively influence the size and the distribution of these liabilities by aggressively seeking CD's, federal funds, Eurodollars, etc.

The emergence of these new sources of funds discredits some of the conclusions which can be derived from the models based on exogenous liabilities. For example, in Hester and Pierce model satisfactory results were obtained for pre 1963 period on the assumption that demand and time deposits are independent. However, the existence of the CD market, which allows banks to optimize their CD sales in response to unexpected variations in deposits, falsifies such an assumption and restricts the cases in which the model can give satisfactory results.

In introduction to the next chapter, some other features of the existing models will be discussed.

CHAPTER TWO

A TWO PRODUCT BANKING FIRM UNDER UNCERTAINTY

2-1 Introduction

2-1-1 The Partial Nature of the Banking Models and the Necessity of a Theoretical Framework

A major problem with the banking models that have appeared in the literature is that they typically abstract from the interdependent nature of banking activities. Each function of the bank - portfolio selection, reserve management, or borrowing at the discount window - is analyzed in isolation and separated from other functions. [See for example [20], [26], [12]]. For unrelated functions such an approach is justified, but as soon as we realize that each function of the bank is an integral part of the whole and dependent upon others, such an isolation becomes equivalent to ignoring the linkages which relate these functions to one another to form an interdependent and unified complex. It is the conglomeration of all of the bank's functions which makes the bank a unique financial intermediary. Hence, the integration and unification of various dimensions of the bank behavior is essential to the better understanding of banking activity.

Another problem in the literature is that issues are often analyzed outside any explicitly introduced framework. As Klein [22] has pointed out, questions are often asked and answered about the effect on the bank behavior of changes in some exogenous factors - market structure, regulations, etc. - without bothering first to develop a theory which describes the bank behavior under any specific conditions. [See [22]]. Developing a model within which issues can be appropriately analyzed seems to be a prerequisite to a sensible analysis of those issues.

The model developed in this study integrates such different aspects of bank behavior as physical production, portfolio selection, liquidity, costs, risk attitude, and institutional constraints into a unified banking model. Then, on the basis of this model, the effect of variations in the exogenous factors on banking activities are analyzed.

2-1-2 The Nature of the Bank as a Pure Financial Intermediary

The banking literature treats the bank as a pure financial intermediary, which borrows short and lends long, expediting the transfer of funds from surplus spending units to deficit spending units. As a borrower, it issues deposit liabilities against itself. As a lender, it allocates the secured funds among competing assets so as to optimize some objective function. Such a view treats banks as rational investors or portfolio holders rather than productive firms. Consequently, banks have typically been analyzed in a portfolio-choice context, as opposed to microeconomic firm-theoretical context. [See [20] and [3]].

2-1-3 The Banking Firm, the Clearance Output, and the Intermediary-Direct-Supplier Theory

In spite of the traditional theory, it may be argued that the demand deposit holder is not a surplus spending unit who is trying to lend to the bank. Quite the contrary, it might be a deficit spending unit who has borrowed the deposited funds from the banker. A demand deposit holder does not necessarioy prefer to spend on current consumption and/or investment goods less than his current income as the definition of surplus spending unit requires [See [21], p. 49]. Instead, the depositor may plan to only use the bank's clearance service, one prerequisite of which is holding deposits.

The pure intermediary theory actually overlooks the non-intermediary portion of the banking activity. A distinct feature of banks as opposed to other financial intermediaries is that banks are the administrators of the nation's payment mechanism. As indicated by Klein [p. 206], such administration utilizes scarce resources and constitutes a service provided by the bank to the non-bank public; a service that may not be considered intermediation. Klein has estimated a linear homogenous Cobb-Douglas production function for the clearance output based on the Federal Reserve functional cost data. He imputes an implicit return of 1.6 percent on demand deposits through the clearance activity. The Klein model, however, isolates the clearance output from the rest of the banking activity and implies their independence.

Pesek [31] has also emphasized the non-intermediary nature of at least a portion of banking activity. He has proposed that "one part of our banking business does not consist of intermediation at all and does consist only of direct supply of a good (medium of exchange) and a service (accounting) to the demanders" (p. 876). Pesek offers an intermediary-direct-supplier view of banks. In this view a bank becomes, "a conglomerate of a pure bank....and a productive enterprise indistinguishable from say a C.P.A. firm or from H & R Block's tax advisory firm." (p. 875). It acts as an intermediary when it borrows from surplus spending units and lends to deficit spending units, but as a direct supplier when it produces the clearance output as a service for its depositors.

Following Klein [22], Pesek [31], and others, the present study views banks as multi product firms which are both a financial intermediary and a direct-supplier. In this view the bank becomes a technical unit which transforms some inputs into outputs subject to a production function. It employs capital and labor inputs to complement deposits in producing the clearance output (X_1) and the loan output (X_2). These outputs are claimed to be produced jointly.

2-1-4 The Technological Constraint and a Critique of Aggregation

The multiproduct banking firm has a technological constraint specified by its production function. Commonly two methods have been adopted in adapting single output production functions to multiple outputs. One method, adopted for example by Bell and Murphy [4], is to assume that outputs are independent so that a separate production function may be constructed for each individual product. This method ignores the interdependence bewteen different outputs. The other method is to construct a weighted index or an aggregate measure, which may be considered as a composite good. This method, which was adopted for example by Greenbaum [13], is further analyzed below.

2-1-4-1. A Critique of Aggregation

The most common procedure of aggregation is the weighted-average approach. In this approach output prices are generally used as weights to obtain total revenue as an aggregate measure. The problem with this approach is that it violates the neoclassical requirement that the production function should be single-valued [10 , p. 7]. It also leads to non-positive marginal products. The argument (due to Mundlak [27]) is as follows:

Consider the product transformation locus between outputs X_1 and X_2, on which α and $\alpha*$ are two optimal output combinations, selected under different price situations P_α and $P_{\alpha*}$, where each combination corresponds to a vector of outputs; e.g., $\alpha = (X_\alpha^{1}, X_\alpha^{2})$. Now if $\underline{X}$ is the aggregate output measure, the outputs at α and $\alpha*$ will be measured as:

$$X_\alpha = \sum_{i=1}^{2} P_\alpha^{\ i} X_\alpha^{\ i}$$

$$X_\alpha{}^* = \sum_{i=1}^{2} P_\alpha^{*i} X_\alpha^{*i}$$

The problem is that although all the output combinations on the product transformation curve, including α and α^* are by definition produced by the same bundle of inputs, they do not necessarily give rise to the same monetary value output measure (X). In fact, depending on the choice of prices, X_α can be larger than, equal to, or smaller than $X_\alpha{}^*$. $X_\alpha{}^* \gtrless X_\alpha$. This result indicates that the single output production function, with total revenue as the output measure, will not be single-valued [Figure 1].

Further, consider an increase in the bundle of inputs under the conditions that the relative output prices remain unchanged, i.e., assume the tangent isorevenue lines shift parallel to themselves so that we have:

$$\frac{P_\alpha^{\ 1}}{P_\alpha^{\ 2}} = \frac{P_\beta^{\ 1}}{P_\beta^{\ 2}} \quad , \quad \frac{P_\alpha{}^{*1}}{P_\alpha{}^{*2}} = \frac{P_\beta{}^{*1}}{P_\beta{}^{*2}} \quad .$$

Now consider that from α to β and from α^* to β^*, with outputs having increased and relative prices held constant, the aggregate output measure - total revenue - has increased, that is:

$$X_\beta > X_\alpha$$

$$X_\beta{}^* > X_\alpha{}^* \quad ,$$

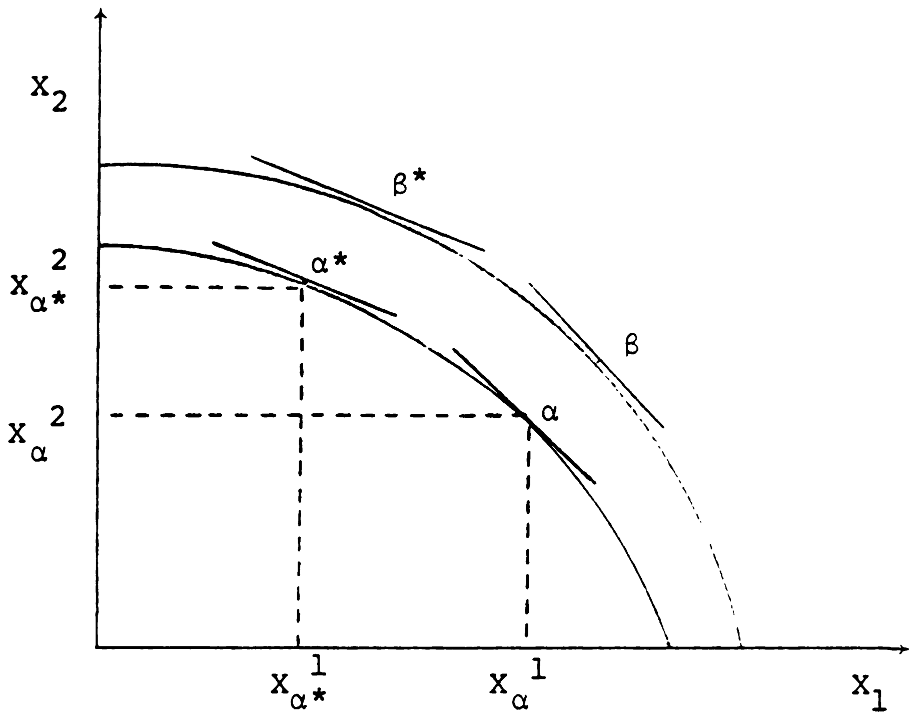


FIGURE 1

PRODUCT TRANSFORMATION ON AN

AGGREGATE PRODUCTION FUNCTION

the following were also shown to hold:

$$X_{\alpha}^{*} \gtreqless X_{\alpha}$$

$$X_{\beta}^{*} \gtreqless X_{\beta}$$

Putting these results together, it may be concluded that:

$$X_{\alpha}^{*} \gtreqless X_{\beta} \quad , \quad X_{\alpha} \gtreqless X_{\beta}^{*} \quad .$$

These relations indicate that an increase in the bundle of inputs may correspond to a larger, smaller, or equal value of aggregate output measure. In other words, these relations indicate that marginal products may be positive, negative, or zero [Figure 1].

The non-single-valuedness, and the non-positive marginal product problems, will be removed only in the special case of fixed proportion production. This is because in this case no transformation is possible between the two outputs and the product transformation curve reduces to a single point. It may be concluded that the use of the monetary value measure is equivalent to ignoring the transformation possibilities [28].

2-1-4-2.The Multi Product Frontier

With the problems discussed above, it seems necessary that a multi product production frontier be used whenever multiple outputs are present. A multi product production frontier can be represented by an implicit function

$F(X,V) = \gamma$, where X and V denote vectors of n outputs and m inputs respectively and γ is a positive scalar.

$$X = (X_1 \ldots . X_n)$$

$$V = (V_1 \ldots . V_m)$$

The frontier can then be defined as a function which (i) for given values of all inputs and all but one output X_i, specifies the maximum amount of X_i producible, (ii) for given levels of all outputs and all but one input V_j, determines the minimum required V_j.

The production frontier is usually normalized so that its value varies directly with outputs and inversely with input levels. When the frontier is differentiable, normalization implies the following restrictions:

$$\frac{\partial F(X,V)}{\partial X_i} > 0 \qquad i = 1, \ldots . n \qquad \frac{\partial F(X,V)}{V_j} < 0 \qquad j = 1 \ldots . m$$

After the normalization, the positive scalar γ may be interpreted as the efficiency parameter, since for given inputs, larger output levels will correspond to larger γ values. Mathematically:

$$F(X,V) = \gamma$$

$$F(X^*,V) = \gamma^*$$

$$\gamma^* > \gamma$$

$$\text{imply } X^* > X \ .$$

Parameter γ can be absorbed in the frontier, so that the frontier may be written as $F(X,V) = 0$.

In this study the logarithmic version of the Mundlak trnascendental production frontier is adopted. This frontier is non-homogenous, has a variable elasticity of transformation, and is separable between outputs and inputs. The frontier can mathematically be represented as:

$$F(X,V) = X_1^{\alpha_1} X_2^{\alpha_2} e^{\beta_1 X_1 + \beta_2 X_2} - DD^{\alpha_D} L^{\alpha_L} K^{\alpha_K} e^{\gamma} = 0$$

or equivalently in logarithms as:

$$F(X,V) = \alpha_1 \log (X_1) + \alpha_2 \log (X_2) + \beta_1 X_1 + \beta_2 X_2$$
$$- \alpha_D \log (DD) - \alpha_L \log (L) - \alpha_K \log (K) - \gamma = 0$$

where X_i's are the clearance and loan outputs, K, L, and DD are capital, labor, and demand deposit inputs, and γ is the efficiency parameter. The following restrictions are required by normalization:

$$F_i > 0 \quad F_j < 0 \qquad \text{where } F_i = \partial F/\partial X_i \quad F_j = \partial F/\partial V_j$$

2-1-5 Jointness in Outputs and Separability in the Production Function

2-1-5-1. Theoretical Background

A multi product production process may correspond to a non-joint or a joint technology. If the technology is non-joint, multiple outputs are each produced under a separate process which is independent of other processes.

Hall [14] has defined non-jointness in the following manner.

> "A technology with transformation function $t(X,V)$ is non-joint if there exist functions $f^1(V^1)\ldots.f^n(V^n)$ (interpreted as individual production functions) with the properties: (i) There are no economies of jointness: if V can produce S, there is a factor allocation $V^1 + V^2 + V^n = V$ such that $f^i(V^i) > X^i$ $i = 1,\ldots.n$. (ii) There are no diseconomies of jointness: if $X^i = f^i(V^i)$, all i. Then $V = V^1 + \quad + V^n$ can produce X."

This definition implies that if there are any kind of interdependence between outputs, so that their co-production results either in some economies or diseconomies, the process is joint. In a joint technology all the outputs are produced through a single production process and are technically interdependent.

Separability is a feature related to jointness. The frontier $F(X,V) = 0$, is said to be separable between outputs and inputs if it can be written as $t(X) - g(V) = 0$. $t(X)$ is called the output or transformation function, and $g(V)$, the input or production function. Separability almost always implies jointness in outputs. In fact, Hall [14] has shown that if a multiple output technology which is separable is non-joint, it must be a fixed proportion process. In this case a single output production frontier may satisfactorily be used as explained before. The jointness implication of separability makes it practically equivalent to jointness. This equivalence forecloses the investigation of jointness hypothesis for any separable production frontier.

2-1-5 Jointness in Banking Outputs

Here it is claimed that the bank outputs are produced jointly. In financial intermediary-direct-supplier theory, the bank has at least two outputs of clearance (X_1) and intermediation (X_2). The production of X_1 provides a source of funds for the bank which it uses an an input to produce X_2. The two outputs are produced in a technically interdependent manner. It is possible, of course, for the two outputs to be produced separately, but in that case they will not be produced as efficiently. We can think of institutions which would accept deposits and clear checks for some fee without engaging in intermediation, where some others would borrow through time deposits and would lend without producing the clearance output. The aggregate cost of these two separate production processes would exceed their counterpart in the joint production case, because in the former case the demand deposits would not be as efficiently utilized.

It is noteworthy that these outputs have not always been produced jointly. Sixteenth century goldsmiths held gold coins and issued transferable liability notes which eased the clearance mechanism to a large extent, but for a long time did not engage in lending. However, as time progressed they recognized the profit opportunities inherent in the co-production of the two outputs and adopted a joint process.

The existence of profit opportunities in the co-production of different outputs indicates some economies of jointness. The interdependent nature of banking outputs alongside the presence of these economies in turn indicates that the production process has a joint nature. Bell and Murphy [4] have claimed that the bank's functions and services are not joint products, since they can be produced in varying proportions. There seems to be some misconception in such an argument. It is true that the bank outputs can be produced in varying proportions, but this does not rule out their jointness, since fixed proportion production is only a special case of joint production. Jointness exists whenever the quantities of outputs are technically interdependent [16]. The best way to resolve the controversy of course is to empirically test the jointness hypothesis. One such test is provided by Mundlak and Razin [28]. Another test is suggested by Samuelson [35] and adjusted to banking by Adar, Agmon, and Orgler [1]. These tests however, are based on functional cost data which are not available to the general researcher. To incorporate jointness, a separable production frontier is adopted in this study.

2-1-6 Banking Costs

It is customary in the banking literature, to ignore the bank's operating costs together with the production function constraint. These costs are considered to be irrelevant in the bank's decision making process. As a consequence of this tradition and as opposed to the neo-classical theory of the firm, the bank's output supply functions are not based on the production costs. The banking theory, as Pesek [31] has pointed out, is a theory of costless banking. Pesek [31] has argued that money supply theory should be based on the production costs, rather than the mechanics of money multiplier. In this study operating costs are incorporated into the decision making process. This will allow a test of significance of costs on output supply functions.

2-1-7 The Market Structure

In the clearance output market the proximity of the depositor to the bank is of prime importance. This market has therefore a local nature [22]. It seems unlikely that non-local competition can induce the depositors to transfer their transaction funds to non-local banks, or at least it is so in the presence of regulation Q which prohibits explicit interest payment on demand deposits.

Under this condition, the clearance output of different banks are spatially differentiated and the bank has a degree of monopoly power which enables it to set the price in the local clearance output market. The demand for clearance output is assumed to be represented by a linear function of the stock of demand deposits, the price charged by the bank, and a random term. The form of the demand function is expressed by equation (1) given in the next subsection. The loan market is assumed to be appropriately approximated by perfect competition, so that the demand function for loans is perfectly elastic. Capital and labor are also assumed to be hired competitively.

2-1-8 Sources of Uncertainty

One inherent characteristic of bank liabilities is deposit variability. Although this feature was recognized by Edgeworth (1888) [9], it was first incorporated into the banking models by Porter (1961) [33], and Mellon and Orr (1961) [29]. The stochastic nature of the deposits makes banks live under uncertainty created by random deposit flows and unexpected cash needs.

The demand for clearance output is assumed to be stochastic for two reasons besides deposit uncertainty. First, the transaction pattern of the demand deposit holders which is the source of their demand for clearance output is indeterminate. Second, technological changes in the payment mechanism may cause the clearance output to vary widely for given deposit levels and the clearance output prices.

A third source of uncertainty is randomness in the loan rate. To analyze the effect of default, assume the banker adds a premium to the contract rate to compensate for default [33]. With this additive default rate, the pure (net) loan rate P_2 may be written as the difference between the contract rate C_2 and the default rate d_2. It is assumed that both the contract rate and the default rate are random. Following these explanations we have:

(1) $X_1 = a_1 DD + b_1 P_1 + E_1 \quad a_1 > 0 \quad b_1 < 0$

(2) $DD \sim (\overline{DD}, V(D))$

(3) $E_1 \sim (0, V(E_1))$

(4) $C_2 \sim (\overline{C}_2, V(C_2))$

(5) $d_2 \sim (\overline{d}_2, V(d_2))$

(6) $P_2 \sim (\overline{P}_2, V(P_2))$

Where DD indicates demand deposits, E_1 is a disturbance term, and C_2, d_2, and P_2 were defined before.

The quantity of loans (X_2) is assumed to be set and achieved by the bank. Hence, any unexpected deposit flows will be reflected in reserve holdings. The distributions of the random variables are assumed known, and therefore bank behavior can be said to be under risk rather than under uncertainty.

2-1-9 The Precautionary Demand for Cash and the Recourse to the Discount Window

In the spirit of Mellon and Orr [29] and Klein [22] it is assumed that the bank holds a cash inventory as a precaution against the uncertainty in deposit flows. It plans to hold some optimum level of free reserves at the beginning of the period, but unexpected deposit shocks may make its actual reserve holdings deviate from the planned level. Unexpected deposit flows will lead to excess reserve holdings larger than the planned level and unexpected deposit drains may lead to insufficient reserves, in which case the bank will have to borrow at the discount window.

Since the level of free reserves is a random variable, its value can not be optimized. It is instead assumed that the bank chooses the mean value of this variable which will be its planned level of reserve holdings.

Excess reserves are assumed to have no explicit return while cash deficiencies are subject to a fixed penalty rate. It can be assumed that deficiencies will be eliminated by borrowing from the discount window or that they will occur at the end of the period and are covered by the sale of loans. The penalty rate in the first case will be the discount rate, and in the latter case the transaction cost of asset sale. The return on cash holding in this framework can be examined by evaluating its effect on the distribution of cash deficiencies and hence on the penalty costs incurred.

2-1-10 The Objective Function

The bank is assumed to be risk-averse and to maximize the expected value of an exponential utility function, where utility may be expressed as a function of the end of the period wealth (W).

$$U = a - be^{-2\alpha W} \qquad a \gtrless 0 \qquad \alpha, b > 0$$

where (W) is the sum of initial net worth (W_0) and the period's profit (π). α represents the risk-aversion coefficient.

The exponential utility function exhibits constant absolute and increasing relative risk-aversion. This may be illustrated by noting:

$$U' = 2\alpha be^{-2\alpha W} > 0$$

$$U'' = -4\alpha^2 be^{-2\alpha W} < 0$$

The degree of absolute risk-aversion as measured by $(\frac{-U''}{U'})$ has the constant value of 2α. The degree of relative risk-aversion as measured by $(\frac{-U''W}{U'})$ has the value of $2\alpha W$ which increases with wealth. The latter can be shown to be the wealth elasticity of marginal utility.

The choice of a utility function with constant absolute risk-aversion, though generally assumed to be inadequate for long-run and large decisions, seems reasonable for short-run and intermediate decisions. When decisions are revised in short intervals of time, no single decision can cause large changes in wealth [24].

To simplify matters, we assume that wealth or equivalently profits are distributed normally. If profits are normal it can be shown that maximization of the expected value of any utility function of wealth leads to maximization of a function of expected profits, variance of profits, and initial net worth. [See [41]]. With exponential utility and normally distributed profits, the expected utility function can be simplified as follows:

$$E(U) = a - bE\,(e^{-2\alpha W})$$

where $W \sim (\overline{W}, V(W))$. E indicates the expectation operator and V indicates the variance.

$$E(e^{-2\alpha W}) = \int_{-\infty}^{+\infty} \frac{e^{\frac{-(W-\overline{W})^2}{2v(W)}} * e^{-2\alpha W}}{(2\pi v(W))^{1/2}}\, dW$$

$$= \int_{-\infty}^{+\infty} \frac{e^{\frac{-1}{2v(W)}\left(W^2 + \overline{W}^2 - 2W\overline{W} + 4\alpha W V(W)\right)}}{(2\pi v(W))^{1/2}}\, dW$$

$$= \int_{-\infty}^{+\infty} \frac{e^{\frac{-1}{2v(W)}\left\{[W - (\overline{W} - 2\ v(W))]^2 - 4\alpha^2 (V(W))^2 + 4\alpha W V(W)\right\}}}{(2\pi v(W))^{1/2}}\, dW$$

$$= \int_{\infty}^{+\infty} e^{-2\alpha(\overline{W} - \alpha v(W))} \frac{* \, e^{\frac{-1}{2v(W)} \{W - [\overline{W} - 2\alpha v(W)]\}^2}}{(2\pi v(W))^{1/2}} dW$$

$$= \quad e^{-2\alpha \; (\overline{W} - \alpha v(W))}$$

Consequently, $\max E(U) \leftrightarrow \max - Ee^{-2\alpha W} \leftrightarrow$

$\max - e^{-2\alpha(W - \alpha v(W))} \leftrightarrow \max \overline{W} - \alpha v(W) \leftrightarrow$

$\max E(\pi) - \alpha v(\pi)$ as $W = W_0 + \pi$

To maximize expected utility, therefore, the bank has to maximize an operational objective function G where:

$$G = E(\pi) - \alpha v(\pi) \qquad (7)$$

If we are maximizing expected utility, it can be shown that a Taylor-series expansion of a logarithmic utility function of wealth, and also a quadratic utility function lead to a similar but not exactly the same objective function. The latter functions are quadratic in mean and linear in variance of profits. An affine transformation, however, would make these functions linear in both terms [8].

With adoption of the above objective function a few points should be noted. First, under risk-aversion, risk as measured by profit variability becomes a utility cost $\alpha v(\pi)$ which is subtracted from the expected profits. Second, with this measure, G as defined by equation (7), can be termed the certainty-equivalent profits. [See [24]].

Third, the indifference curves which are the loci of equal expected utility, can be equivalently defined as the loci of points on which G remains unchanged, so that on each indifference curve we have $dG = dE - \alpha dv(\pi) = 0$. For the measure of risk, we can take either the variance or the standard deviation of the profits. In the former case we can draw the indifference curves between the expected value and variance of profits. The trade-off between these two variables and the slope of the indifference curves are the constant α which merely depends on the degree of risk-aversion. In the latter case, however the indifference curves are drawn between the expected value and standard deviation of profits; the trade-off and the slope of the indifference curves are measured by $2\alpha(V(\pi))^{1/2}$ which is increasing with the level of risk as well as with the risk-aversion coefficient α. In the former case, the loci are straight lines, while in the latter case, each indifference curve is generated by the positive half of the parabola $E(\pi) = 2\alpha(V(\pi))^{1/2} + C$, where C is a constant. On each of the latter indifference curves, successive units of risk have to be compensated by larger and larger gains in expected profits [Figures 2 and 3].

2-1-11 The Constraints

The production function, the demand function for clearance output, the balance sheet identity, and reserve requirement equation constitute the constraints on the model.

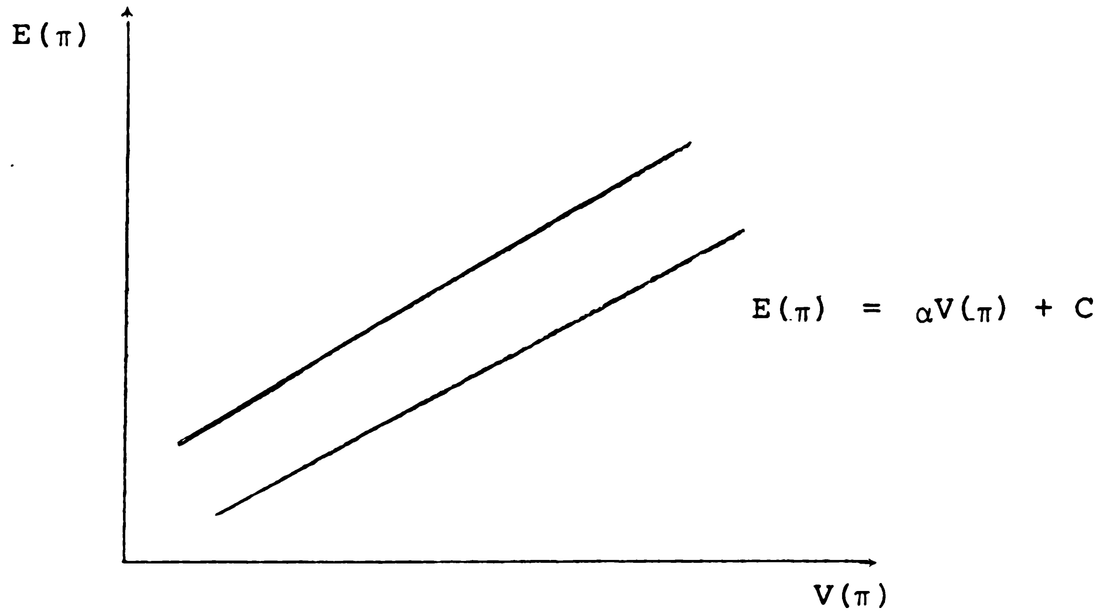


FIGURE 2 : Indifference Curves Between Mean and Variance

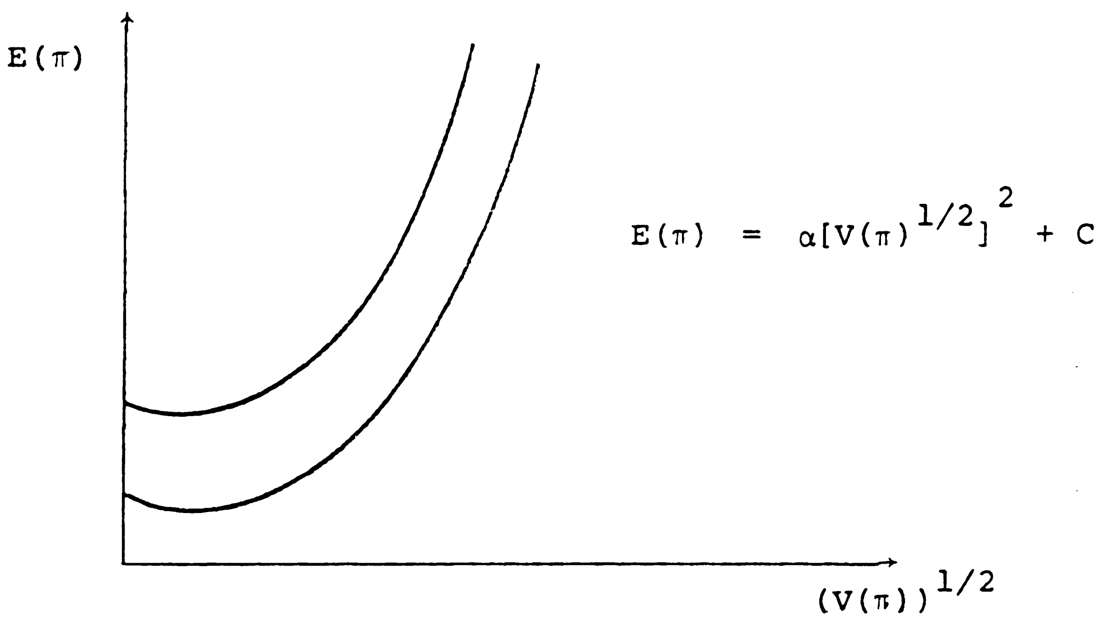


FIGURE 3 : Indifference Curves Between Mean and Standard Deviation

$$F = \alpha_1 \log (a_1\overline{DD} + b_1P_1) + \alpha_2 \log (X_2) + \beta_1(a_1\overline{DD} + b_1P_1) + \beta_2X_2 - \alpha_D \log (\overline{DD}) - \alpha_L \log (L) - \alpha_K \log (K) - \gamma = 0 \tag{8}$$

Now consider the other two constraints. We define free reserves as FR = ER - BR and substitute constraint (d) into (c) to get the substituted version of the balance sheet:

$$X_2(1 - s_2) + \overline{FR} = (1 - rd)\overline{DD} + W_0 \tag{9}$$

This form of the balance sheet implies that the sum of the planned values of assets and the sum of the expected liabilities must be the same. This formulation of the balance sheet is similar to the concept of "desired balance sheet" introduced by Tobin and Brainard [43] and has been adopted as the balance sheet constraint by Parkin, Gray and Barrett [30].

The equivalence of asset credit to compensating balances may be seen by considering the balance sheet and reserve requirement constraints. If r_2 percent of loans are to be held as compensating balances, these two constraints will be written as:

$$X_2 + \overline{R} = \overline{DD} + r_2X_2 + \overline{BR} + W_0$$

$$\overline{R} = rd(\overline{DD} + r_2X_2) + \overline{ER}$$

where r_2X_2 is the amount of deposits created through

compensating balances. Now substitution of the second constraint into the first will result in:

$$X_2\ [(1-r_2(1-rd)] + \overline{FR} = (1-rd)\overline{DD} + \overline{BR} + W_0$$

This relation is the same as the substituted version of the balance sheet given before except that S_2 is replaced by r_2 (1-rd).

2-1-12 Output and Input Measures

Outputs and inputs are measured as follows:

X_1, number of dollars debited per period

X_2, number of dollars of loans

DD, number of dollars of demand deposits held at bank (stock)

L, man-hours labor hired

K, units of machines rented

P_1, service charges per dollar of debits

P_2, pure loan rate, contract rate adjusted for default

R_D, rate paid on demand deposits

P_L, wage rate per man-hour

P_K, capital input rental price, dollars per machine

2-1 The Structure of the Model and the Optimality Conditions

The bank chooses its policy variables P_1, X_2, $\overline{FR}$, L, K, so as to maximize its objective function (7) subject to the constraints (8) and (9). The objective function (7) can be measured as follows:

$$\pi = P_1X_1 + (C_2 - d_2)X_2 - P_LL - P_KK - R_DDD - R_BBR$$

where X_1, DD, E_1, C_2, d_2 were given by (1), (2), (3), (4), (5). $C_2 - d_2 = P_2$, and BR is the borrowing from the discount window. We consequently have:

$$\pi = (a_1P_1 - R_D)\,DD + b_1P_1^{\,2} + P_1E_1 + (C_2 - d_2)X_2 - P_LL - P_KK - R_BBR$$

$$E(\pi) = (a_1P_1 - R_D)\,\overline{DD} + b_1P_1^{\,2} + (\bar{C}_2 - \bar{d}_2)X_2 - P_LL - P_KK - R_B\overline{BR}$$

$$\begin{aligned} V(\pi) = {} & (a_1P_1 - R_D)^2V(DD) + P_1^{\,2}\,V(E_1) + X_2^{\,2}\,V(C_2 - d_2) + R_B^{\,2}V(BR) \\ & + 2\,(a_1P_1^{\,2} - P_1R_D)\,C_{OV}(DD, E_1) + 2X_2(a_1P_1 - R_D) \\ & C_{OV}(DD, (C_2 - d_2)) - 2R_B\,(a_1P_1 - R_D)\,C_{OV}(DD, BR) \\ & - 2R_BX_2\,C_{OV}(BR, (C_2 - d_2)) + 2P_1\,X_2\,C_{OV}(E_1, (C_2 - d_2)) \\ & - 2P_1R_B\,C_{OV}(E_1, BR) \end{aligned}$$

It is assumed that the exogenous loan rate is uncorrelated with the individual bank's clearance output, and that borrowing is uncorrelated with the disturbance term E_1. It follows that:

$$C_{OV}((C_2 - d_2),\ DD) = 0$$

$$C_{OV}((C_2 - d_2),\ E_1) = 0$$

$$C_{OV}(BR,\ E_1) = 0 \quad .$$

These values are substituted in the objective function to obtain the Lagragian function G* to be maximized.

$$\begin{aligned} G^* = {} & (a_1P_1 - R_D)\ \overline{DD} + b_1P_1{}^2 + (\overline{C}_2 - \overline{d}_2)X_2 - P_LL - P_KK \\ & - R_B\overline{BR} - \alpha\{(a_1P_1 - R_D)^2V(DD) + P_1{}^2V(E_1) + X_2{}^2 \\ & V(C_2 - d_2) + R_B{}^2\ V(BR) + (2a_1P_1{}^2 - 2P_1R_D)\ C_{OV}(DD,\ E_1) \\ & - 2R_B\ (a_1P_1 - R_D)\ C_{OV}\ (DD,\ E_1) \\ & \qquad - 2X_2R_B\ C_{OV}((C_2 - d_2),\ BR)\} - \lambda F - \mu[X_2 \\ & (1 - S_2) + \overline{FR} - (1 - rd)\overline{DD} - W_0] \end{aligned}$$

where F is given by (8).

The first order conditions (F.O.C.) for maximization are:

$$\begin{aligned} \partial G^*/\partial P_1 = {} & a_1\overline{DD} + 2b_1P_1 - \alpha\{2a_1(a_1P_1 - R_D)\ V(DD) \\ & + 2P_1V(E_1) + (4a_1P_1 - 2R_D)\ C_{OV}(DD,\ E_1) \\ & - 2a_1R_B\ C_{OV}(DD,\ BR)\} - \lambda F_{P_1} = 0 \end{aligned} \qquad (10)$$

$$\begin{aligned} \partial G^*/\partial x_2 = {} & \overline{C}_2 - \overline{d}_2 - \alpha\{2X_2V(C_2 - d_2) - 2R_B\ C_{OV} \\ & ((C_2 - d_2),\ BR)\} - \lambda F_2 - \mu(1 - S_2) = 0 \end{aligned} \qquad (11)$$

$$\partial G^*/\partial \overline{FR} = - R_B\,(\partial \overline{BR}/_{\partial \overline{FR}}) - \alpha R^2_B\,(\partial V(BR)/_{\partial \overline{FR}}) - \mu = 0 \tag{12}$$

$$\partial G^*/\partial L = - P_L - \lambda F_L = 0 \tag{13}$$

$$\partial G^*/\partial K = - P_K - \lambda F_K = 0 \tag{14}$$

$$\partial G^*/\partial \lambda = F = 0 \tag{15}$$

$$\partial G^*/\partial \mu = - [X_2(1 - S_2) + \overline{FR} - (1 - rd)\,\overline{DD} - W_0] = 0 \tag{16}$$

where F_1, $F_2 > 0$ F_L, $F_K < 0$ by normalization of the frontier,

$$\text{and } F_{P_1} = (\partial F/\partial \overline{X}_1)\,(\partial \overline{X}_1/\partial P_1) = b_1 F_1 < 0$$

If we consider γ and $(1 - rd)\,\overline{DD} + W_0$ as our constraint's constants, the Lagrangian multipliers satisfy the following relationships:

$$\partial G/\partial \gamma = \lambda,$$

$$\partial G/\partial\, [(1-rd)\overline{DD} - W_0] = \mu.$$

Since γ is the efficiency parameter, λ can be interpreted as the marginal effect of efficiency on the certainty-equivalent profit (G). It has also been termed the shadow price of efficiency by Hasenkamp [15]. In the same way μ can be interpreted as the equilibrium contribution of each dollar of unencumbered funds to the certainty-equivalent profit (G); in other words, μ is the opportunity

cost of each dollar of funds allocated in equilibrium.

It may be noted that the variable "borrowing" does not have a well-defined distribution. It represents the negative portion of the distribution of free reserves (FR), so that its moments can be considered as the incomplete moments of the latter variable. These moments and their partial derivatives with respect to $\overline{FR}$ are derived in Appendix 1. There it is shown that holding free reserves reduces both the mean and variance of the reserve deficiency or borrowing. Mathematically we have:

$$\partial\overline{BR}/\partial\overline{FR} = - P_r < 0$$

$$\partial V(BR)/\partial FR = - 2\overline{BR}\ (1 - P_r) < 0$$

where P_r represents the probability of reserve deficiency. Using these relations and the third of the first order conditions, the sign of μ is determined to be positive. The sign of λ is also positive due to the fifth of the first-order conditions.

The first order conditions implicitly define the asset demand functions for loans and free reserves and also the input demand functions for capital and labor. The optimal value of each endogenous variable is a function of all the exogenous variables in the model so that the decisions about production, portfolio selection, liquidity, etc. are simultaneous and interrelated.

The bank's demand for assets will generally depend on:

(i) the expected return on both assets

(ii) variances and covariances of returns

(iii) the distribution of deposits

(iv) the return on deposits (R_D)

(v) the risk attitude of the bank (α)

(vi) reserve requirement (rd), discount rate (R_B)

input prices (P_L, P_R), and net worth (W_0). Therefore one asset can be attractive to some banks and not to others, because of the differences in their distribution of deposits, their estimates of variances, and risk attitudes. The presence of the taste element (α) also rules out the separation feature in the portfolio.

The diversification can be explained by both risk-aversion and precautionary motive. Since cash holding reduces the mean and variance of borrowing, it will be held so long as its contribution exceeds its opportunity cost (μ), [see equation (12)]. One feature which makes the present model distinct from, for example, the one developed by Klein [22], is that in the latter, all the assets are pegged to a rate imposed on the model from outside. Each asset is held up to the point at which its marginal return, regardless of its riskiness, is the same as the expected rate of return on government securities. In such a model, riskiness of different assets has no effect on the asset mix. On the contrary, in the present model, assets are pegged to the equilibrium opportunity cost μ, which is endogenously determined by the system. Each asset is held up to the

point at which its return, adjusted for its riskiness, is the same as the common equilibrium contribution μ. In other words μ includes both risk and expected return elements.

One last point to consider is the dependence of the optimal values on the rate paid on deposits (R_D). This dependence has implications on the establishment of the regulation Q, which will be further explained in interpretations of the comparative static results.

The second order conditions for a maximum are derived in Appendix 2. The following restrictions are shown to be sufficient for these conditions to be satisfied.

$$F_{11},\ F_{22},\ F_{LL},\ F_{KK} \ > \ 0$$

$$\partial^2 V(BR)/\ \overline{FR}^2 \ \gtrsim \ 0$$

It is also shown that the first two conditions imply increasing marginal rate of product transformation between $\overline{X}_1$ and X_2, and the latter two imply decreasing marginal rate of substitution between K and L. The last term is shown to have an indeterminate sign in Appendix 1.

2-3 Displacement of Equilibria

Here the first order conditions (10) - (16) are totally differentiated to derive the comparative static results. One additional variable is also added to the model. To analyze the effect of improvement in the clearance technology on the system's variables, such improvement is

introduced as an increase in the productivity of inputs in producing output X_1. The marginal products of inputs for X_1 can be measured as follows:

$$MP_{V_j},\ \overline{X}_1 = \frac{\partial \overline{X}_1}{\partial V_j} = -\frac{F_{V_j}}{F_1} = \frac{\alpha_{j/V_j} + \beta_j}{\alpha_{1/\overline{X}_1} + \beta_1} \quad V_j = K,\ L,\ \overline{DD}.$$

For these marginal products to increase either α_1 and/or β_1 has to decline with technological progress. Here α_1 is taken to be a decreasing function of technology, so that $\partial\alpha_{1/\partial_t}$ is negative. This property will be used to determine the response directions of the endogenous variables to variations in technology.

Total differentiation of the first order conditions is given in Appendix 3, and summarized in Table 1. The cofactors H_{ij} of the hessian matrix H, are needed for comparative static results. The calculations are omitted here, their signs are displayed in Table 2. The comparative static results are derived in Appendix 4. The signs are displayed in Table 3.

The conditions under which some of the comparative static results are determined require some explanations. The terms $\frac{\partial \overline{BR}}{\partial V(FR)}$ and $\frac{\partial P_r}{\partial V(FR)}$ refer to the effect of V(FR) on the mean borrowing. These terms will vanish in the special case that $\overline{BR}$ is affected only by variations in $\overline{FR}$ and not those of V(FR). The condition $R_D < 2a_1P_1$ imposes a ceiling on R_D. The zero ceiling imposed by regulation Q is sufficient for this conditions to be satisfied. Since borrowing from the discount window results from unexpected

TABLE 1

THE DIFFERENTIATED FORM OF THE FIRST ORDER CONDITIONS

$$\begin{pmatrix} 0 & 0 & F_{P_1} & F_2 & 0 & F_L & F_K \\ 0 & 0 & 0 & 1-S_2 & 1 & 0 & 0 \\ F_{P_1} & 0 & A_{11} & 0 & 0 & 0 & 0 \\ F_2 & 1-S_2 & 0 & A_{22} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & A_{33} & 0 & 0 \\ F_L & 0 & 0 & 0 & 0 & -\lambda F_{LL} & 0 \\ F_K & 0 & 0 & 0 & 0 & 0 & -\lambda F_{KK} \end{pmatrix} \begin{pmatrix} -d\lambda \\ -d\mu \\ dP_1 \\ dX_2 \\ d\overline{FR} \\ dL \\ dK \end{pmatrix} =$$

TABLE 1 (cont'd)

$$-dDD[a_1F_1 - \alpha_D/DD] + d\gamma + \log(a_1DD + b_1P_1)\frac{\partial\alpha_1}{\partial_t}dt$$

$$(1-rd)d\overline{DD} - \overline{DD}drd + dW_O + X_2dS_2$$

$$(2\alpha a_1^2P_1 - 2\alpha a_1R_D)dV(DD) + 2\alpha P_1dV(E_1) + 2\alpha(2a_1P_1 - R_D)dC_{OV}$$
$$(DD,E_1) - 2\alpha a_1R_B\, dC_{OV}(DD,BR) - 2\alpha a_1C_{OV}(DD,BR)dR_B$$
$$- [2\alpha a_1V(DD) + 2\alpha C_{OV}(DD,E_1)]dR_D + [\lambda a_1b_1F_{11} - a_1]d\overline{DD}$$
$$+ [(2a_1^2P_1 - a_1R_D)V(DD) + 2P_1\, V(E_1) + (4a_1P_1 - 2R_D)C_{OV}$$
$$(DD,E_1) - 2a_1R_B\, C_{OV}(DD,BR)]\, d\alpha$$

$$-d\overline{C}_2 + d\overline{d}_2 - \mu dS_2 + 2\alpha X_2dV(C_2) + 2\alpha X_2dV(d_2)$$
$$- 4\alpha X_2dC_{OV}(C_2,d_2) - 2\alpha R_BdC_{OV}(C_2,BR) + 2\alpha R_BdC_{OV}(d_2,BR)$$
$$- 2\alpha C_{OV}(C_2-d_2,BR)dR_B + [2X_2V(C_2-d_2) - 2R_BC_{OV}(C_2,BR)$$
$$+ 2R_B\, C_{OV}(d_2,BR)]\, d\alpha$$

$$- \{[R_B - 2\alpha R_B^2\overline{BR}]\frac{\partial P_r}{\partial V(FR)}(1-rd)^2 + 2\alpha R_B^2(1-P_r)(1-rd)^2\frac{\partial\overline{BR}}{\partial V(FR)}\}$$
$$dV(DD) - (P_r + 2\alpha\overline{BR}\, R_B(1-P_r)]dR_B - [2R_B^2\, \overline{BR}(1-P_r)]$$
$$d\alpha + \{[R_B - 2\alpha R_B^2\, \overline{BR}]\,[2(1-rd)V(DD)\frac{\partial P_r}{\partial V(FR)}] + 2\alpha R_B^2$$
$$(1-P_r)(1-rd)V(DD)\frac{\partial\overline{BR}}{\partial V(FR)}\}\, drd$$

$$dP_L$$

$$dP_K$$

$$A_{11} = 2b_1 - 2\alpha V(X_1) - \lambda b_1^2 F_{11}$$

$$A_{22} = -2\alpha V(C_2 - d_2) - \lambda F_{22}$$

$$A_{33} = R_B \, \partial P_r/\partial \overline{FR} - 2\alpha R_B^2 \, [(1-P_r) \, P_r + 2\overline{BR} \, \partial P_r/\partial \overline{FR}]$$

$$A_{ii} < 0 \qquad i = 1,2,3.$$

TABLE 2

COFACTORS SIGNS

$H_{0,0}$ −	H_{01} +	H_{02} −	H_{03} +	H_{04} +	H_{05} +
$H_{0,00}$ +	H_{001} −	H_{002} −	H_{003} −	H_{004} −	H_{005} −
$H_{00,00}$ −	H_{11} +	H_{12} +	H_{13} −	H_{14} −	H_{15} −
		H_{22} +	H_{23} −	H_{24} +	H_{25} +
			H_{33} +	H_{34} −	H_{35} −
				H_{44} +	H_{45} −
					H_{55} +
					H −

TABLE 3

THE COMPARATIVE STATIC RESULTS

Exogenous Variables	Conditions Imposed	Endogenous Variables						
		P_1	X_2	$\overline{FR}$	L	K	μ	λ
$\overline{C}_2$		+	+	-	+	+	+	+
$\overline{d}_2$		-	-	+	-	-	-	-
S_2		+	+	?	+	+	?	+
R_B	$C_{OV}(DD,BR) < 0$, $C_{OV}(C_2-d_2,BR) = 0$	-	-	+	?	?	?	?
R_D	$C_{OV}(DD,X_1) > 0$	+	+	-	-	-	+	-
P_L		+	-	+	-	+	-	+
P_K		+	-	+	+	-	-	+
$V(C_2)$		-	-	+	-	-	-	-
$V(d_2)$		-	-	+	-	-	-	-
$V(DD)$	$\frac{\partial \overline{BR}}{\partial V(R_F)} = \frac{\partial P_r}{\partial V(FR)} = 0$, $R_D < 2a_1P_1$	-	-	+	+	+	-	+
$V(E_1)$		-	-	+	+	+	-	+
$C_{OV}(C_2,d_2)$		+	+	-	+	+	+	+
$C_{OV}(C_2,BR)$		+	+	-	+	+	+	+
$C_{OV}(d_2,BR)$		-	-	+	-	-	-	-

TABLE 3 (cont'd)

$C_{OV}(DD,BR)$		+	+	−	−	−	+	−
$C_{OV}(DD,E_1)$	$R_D < 2a_1P_1$ e.g. $R_D = 0$	−	−	+	+	+	−	+
r_d	$\frac{\partial P_r}{\partial V(FR)} = \frac{\partial \overline{BR}}{\partial V(FR)} = 0$	−	−	−	−	−	+	−
W_0		+	+	+	+	+	−	+
$\overline{DD}$		?	?	?	?	?	?	?
α	$R_D < 2a_1P_1$, $C_{OV}(DD,BR) < 0$, $C_{OV}(C_2-d_2,BR) = 0$	−	−	+	?	?	?	?
γ		−	+	−	−	−	+	−
t		−	?	?	?	?	?	?

deposit drains and occurs only in emergencies, it may be argued that BR varies indirectly with deposits and is uncorrelated with the loan rate. Following these arguments the conditions $C_{OV}(BR,DD) < 0$ $C_{OV}(C_2 - d_2, BR) = 0$ seem reasonable.

Some of the comparative static results are worth noting: The model shows that as $\overline{C}_2$ rises the bank switches from X_1 to X_2. It charges a higher P_1 to reduce X_1, and holds less reserves using the funds together with additional capital and labor inputs to produce more X_2. The bank achieves a larger certainty-equivalent return per dollar (μ). The negativity of $\partial\overline{FR}/\partial\overline{C}_2$ indicates that the bank's cash holding is interest elastic. (Row 1)

The effect of mean default rate $\overline{d}_2$ is in the opposite direction to mean contract rate $\overline{C}_2$. If expected mean default rate increases fewer loans will be planned, and more funds will be held as cash. The equilibrium contribution of each dollar of unencumbered funds (μ), will be smaller due to larger default. It may be of interest to consider $\overline{d}_2$ as a per unit tax (subsidy) on loans. The effect of such tax (subsidy) would be the same as (opposite to) that of default. The effect of collateral could also be analyzed in the same way. Collateral reduces the probability of default, and consequently the mean default rate $\overline{d}_2$. The effect of collateral therefore would be opposite to that of default.

The equivalence of asset reserve credit to compensating balances was shown before, both features may be considered

as additional compensation on loans, since they raise the effective loan rate as measured by $P_{2e} = \frac{P_2}{1-S_2}$, where S_2 is the percentage of loans credited to the bank's reserves. The comparative static effects of these variables are similar to those of the loan rate whenever the former have determinate signs.

The effect of asset reserve credit or compensating balances on the bank's liquidity-cash holding-is indeterminate. This is due to the fact that these variables - on one hand raise the effective loan rate, which increases the loans and reduces free reserves, and on the other hand provide a source of funds. The over all effect on cash can therefore be of either sign. The conditional supply of loanable funds, and demand for endogenous inputs have conventional signs, that is, the former is positively and the latter are negatively sloped (Columns 2, 4, 5).

Increased uncertainty represented by increases in the variances acts as an increase in the default rate; it lowers the clearance price P_1 and the loan output X_2, while it increases the mean cash holding $\overline{FR}$. These results on loans and free reserve holdings are consistent with Hicks [18]. The result about the loan output is also consistent with Sandmo [36]. All variances lower the equilibrium certainty-equivalent return μ. This result justifies Hester and Pierce's claim that realized profits under certainty fall short of their counterpart under perfect foresight [17].

The monetary policy effect of the discount window

operation is shown to be in the intended direction (Row 4). The monetary policy effect of the reserve requirement (rd) on the level of credit (X_2) is restrictive. It is also found that the reserve requirement and the mean free reserves move in opposite directions, so that in the absence of reserve requirement, a voluntary reserve holding would prevail (as $\partial\overline{FR}/\partial rd < 0$).

The portfolio composition and the risk undertaken by the bank are not independent of the rate paid on deposits (R_D). This result contradicts Benston [5] and Klein [22], but is consistent with Gambs [11]. The model indicates that the percentage of the portfolio held in the risky asset (X_2) does rise with R_D. Therefore regulation Q, when effective, does reduce the percentage of the risky asset in the portfolio, and does raise the liquidity as measured by $\overline{FR}/X_2$ or $\overline{FR}/\overline{DD}$. It can therefore be concluded that regulation Q serves its purpose.

Yet another conclusion can be drawn about regulation Q. The positive sign of $\partial P_1/\partial R_D$ gives a positive answer to the question posed by Klein [22] about whether a relationship exists between pricing policy of the clearance output and the rate of interest offered by the bank on the stock of demand deposits. By limiting the rate paid on demand deposits, regulation Q has led to a reduction of the service charges, so that the costs of the clearance output is at least partially offset by interest free demand deposits. In addition to this, the reduction in service charges leads to

over utilization of the clearance output by its demanders and hence to a resource allocation which is not socially optimal. These results about regulation Q, however, cease to hold true if the model is based on risk neutrality. This is one explanation why the Klein and the Benston models do not produce analogous results.

Inputs involved in a production process may be either substitutes or complements. A rise in an input price always lowers the demand for its complements, but it may increase or decrease demand for its substitutes depending on the magnitudes of substitution and output effects. Comparative static responses of capital and labor to input prices P_K, P_L, and R_D indicate that capital and labor are substitutes between themselves, but the relations between capital and labor on one hand and demand deposits on the other may be either one of substitution or complementarity.

An inferior input is defined as one an increase in whose price leads to an increase in the equilibrium output of the firm [10 , p.189]. In this model demand deposits, when considered as an input in producing loans, satisfy the inferiority condition [$\partial X_2/\partial R_D > 0$]. As R_D increases, the bank switches some of its cash funds to loans to increase its earning. The higher earning will offset the increment in costs due to the rise in R_D. Demand deposits are a normal (or superior) input for the clearance output. Capital and labor are normal (or superior) inputs for both outputs.

Increased risk aversion as measured by an increase

in the risk aversion coefficients α is shown to reduce the service charges P_1 and loans X_2. These features both lead to a higher X_1/X_2 ratio. Lower service charges lead to a larger clearance output level and increase the output ratio X_1/X_2, this increase is further reinforced by reduction in loans. Some of the loan funds are shown to be transferred to free reserves to increase liquidity as measured by the ratios $\overline{FR}/X_2$ and $\overline{FR}/\overline{DD}$. The comparative static effects of changes in demand deposits are indeterminate. An increase in wealth will not be totally allocated to loans as Porter claims [33]. It will be divided between loans and reserves.

The efficiency parameter γ may be considered as an indicator of technological progress. The model shows that as γ increases, the clearance output price P_1, the planned free reserves $\overline{FR}$, and capital and labor decline, while the loan output X_2 increases. The decline in the clearance output price P_1 leads to a higher demand for X_1. The bank will employ less of capital and labor inputs, will hold less reserves on the average, and will produce more of both clearance and loan outputs. The shadow price of efficiency λ (or the shadow price of technology) declines as efficiency increases. The equilibrium return or the opportunity cost of funds to the bank increases with the technology. [Row 2)]. The rise in the clearance output X_1 for given level of deposits indicates an increase in the velocity of deposits as defined by the ratio X_1/DD.

The variable t represents the technological

improvement in X_1 while the technology in production of X_2 is unchanged. From the comparative static results, it can only be said that such improvement reduces the service charge P_1. This result indicates that technological progress in the clearance mechanism - an electronic fund transfer system, VISA etc., - is beneficial to the consumer and increases the demand for clearance output.

It might be of interest to contrast these comparative static results to their counterparts when the model is adjusted to include only one of the two outputs. If the clearance output is excluded from the model, the results remain essentially unchanged - except that the rate paid on demand deposits (R_D) no longer appears in the first order conditions. The system is consequently independent of this rate and the establishment of regulation Q is found to have no theoretical basis. It may be argued that the results obtained on regulation Q by Klein [22], which contradict the results in the present study, may have followed his failure to include the clearance output.

A model without the loan output is equivalent to imposition of a 100 percent reserve requirement on demand deposits. Demand deposits receive no interest in this case, since they can not be used to make loans. In this model, few exogenous variables appear. The comparative static results indicate that the bank capital input rises with the expected demand deposit flow. The bank buys additional machinery to produce a larger clearance output. The remaining results are not surprising.

In this chapter an attempt was made to integrate such different aspects of bank behavior as physical production, portfolio selection, costs, liquidity, risk aversion, and institutional constraints into a unified banking model, but much more can be added. Randomness in outputs, simultaneous randomness in outputs and prices, and liability management will be incorporated in the following chapters.

CHAPTER THREE

THE BANKING FIRM UNDER PRICE AND OUTPUT UNCERTAINTY

3-1 Introduction

In the last chapter the bank was considered as a two product firm under deposit variability and price uncertainty, where the quantity of the loan output was deterministically set and achieved. In the real world, however, banks face quantity uncertainty as well as price uncertainty. If the bank's anticipations are not realized, the planned outputs will not necessarily prevail. When deposits unexpectedly run off, or the demand for loanable funds drops, the planned loans will be curtailed. On the other hand, when the bank confronts unexpected deposit inflows or loan demands which it is unwilling to turn down, it may end up lending beyond the planned level. Since decisions on loans have implications on the strength of customer relationship and long-run profits, loan uncertainty is critical to banks.

In this chapter output uncertainty is incorporated into the model. It is assumed that the bank plans the loan output, but the planned level is not necessarily achieved. The actual output is assumed to be stochastic and to deviate from the planned level by a random term with zero mean and a constant variance. The mean of the actual output will therefore be the planned output, and its variance the variance of the random term.

The incorporation of default and asset reserve credit instrument are abstracted in this chapter. The default rate d_2 is subtracted from the contract rate c_2, and the net loan rate P_2 will be used throughout. Loan rate uncertainty and perfect competition in the loan market are retained.

3-2 The Structure of the Model and the Optimality Conditions

3-2-1 The Objective Function

The bank chooses the service charges P_1, the planned loans $(\overline{X}_2)$, the planned level of free reserves $(\overline{FR})$, and capital and labor inputs so as to maximize its objective function. The objective function may be expressed as $G = E(\pi) - \alpha v(\pi)$, where the profit (π), and its mean and variance can be measured as follows:

$$\pi = P_1X_1 + P_2X_2 - P_LL - P_KK - R_BBR - R_DDD$$

Where: $$X_1 = a_1DD + b_1P_1 + E_1$$

$$\begin{pmatrix} DD \\ E_1 \\ X_2 \end{pmatrix} \quad \left[\begin{matrix} \overline{DD} \\ 0 \\ \overline{X}_2 \end{matrix} \quad \begin{pmatrix} V(DD) & C_{OV}(E_1, DD) & C_{OV}(DD, X_2) \\ C_{OV}(E_1, DD) & V(E_1) & C_{OV}(E_1, X_2) \\ C_{OV}(DD, X_2) & C_{OV}(E_1, X_2) & V(X_2) \end{pmatrix} \right]$$

$$\pi = (a_1P_1 - R_D)DD + b_1P_1^2 + P_1E_1 + P_2X_2 - P_LL - P_KK - R_BBR$$

$$E(\pi) = (a_1P_1 - R_D)\overline{DD} + b_1P_1^2 + E(P_2X_2) - P_LL - P_KK - R_B\overline{BR}$$

$$V(\pi) = (a_1P_1 - R_D)^2V(DD) + P_1^2V(E_1) + V(P_2X_2) + R^2_BV(BR)$$

$$+2\ P_1(a_1P_1 - R_D)\ C_{OV}(DD,E_1) + 2(a_1P_1 - R_D)\ C_{OV}(DD,P_2X_2)$$

$$-2(a_1P_1 - R_D)\ R_B\ C_{OV}(DD,BR) + 2\ P_1\ C_{OV}(E_1,P_2X_2)$$

$$-2P_1R_B\ C_{OV}\ (E_1,BR) - 2R_B\ C_{OV}(P_2X_2,\ BR)$$

The objective function includes moments and cross-moments of P_2X_2 which is the product of two random variables. To measure these moments some simplifications are necessary. Here each random price is written as $P = \overline{P} + U$ and each random quantity as $Z = \overline{Z} + V$ where $\overline{P}$ and $\overline{Z}$ represent means (expected price or planned output) and U and V are random terms. It is assumed that random terms in prices are independent from those in quantities and each term has a zero mean and a constant variance.

$$\begin{pmatrix} U \\ V \end{pmatrix} \left[\begin{matrix} 0 \\ 0 \end{matrix} \ , \ \begin{pmatrix} V\ (U) & 0 \\ 0 & V(U) \end{pmatrix} \right] \quad \text{where:} \quad \begin{matrix} V(U) = V(P) \\ V(V) = V(Z) \end{matrix}$$

In consequence to this treatment of the random variables, the following results are shown to hold in Appendix 5.

$$E\ (P_2X_2) = \overline{P}_2\overline{X}_2$$

$$V\ (P_2X_2) = V(P_2)V(X_2) + \overline{P}^2\ V(X_2) + \overline{X}^2\ V(P_2)$$

$$C_{OV}(BR,P_2X_2) = \overline{P}_2\ C_{OV}\ (BR,X_2)$$

$$C_{OV}(DD,P_2X_2) = \overline{P}_2C_{OV}(DD,X_2)$$

Besides, it is assumed that the random term in the demand function for clearance output (E_1) is uncorrelated with borrowing (BR) and total loan income P_2X_2 so that the following covariances vanish.

$$C_{OV}(E_1, BR) = 0$$

$$C_{OV}(E_1, X_2) = 0$$

3-2-2 The Relation Between Loans and the Loan Rate

It should be explained that the assumption of independence between the error terms in the quantity of loans and in the loan rate does not imply the independence of loans from the loan rate. The assumption indicates that deviations of loans from the planned level do not result from deviations of the loan rate from its expected value. For example, a situation is possible in which the loan rate is higher than expected and still the quantity of loans falls short of its planned value. The assumption can be justified on the ground that deviations in the loan output often result from unexpected deposit shocks and loan demand rather than price variations.

The relationship between loans and the loan rate may be explained better in a hypothetical one output model. As the output decisions occur prior to production period, the planned output level depends on the expected rather than actual price. For a given distribution of the loan rate the hypothetical marginal cost curve intersects the expected price $\overline{P}_2$ at point A which determines the planned output $\overline{X}_2$. The actual output will have a distribution whose mean is the

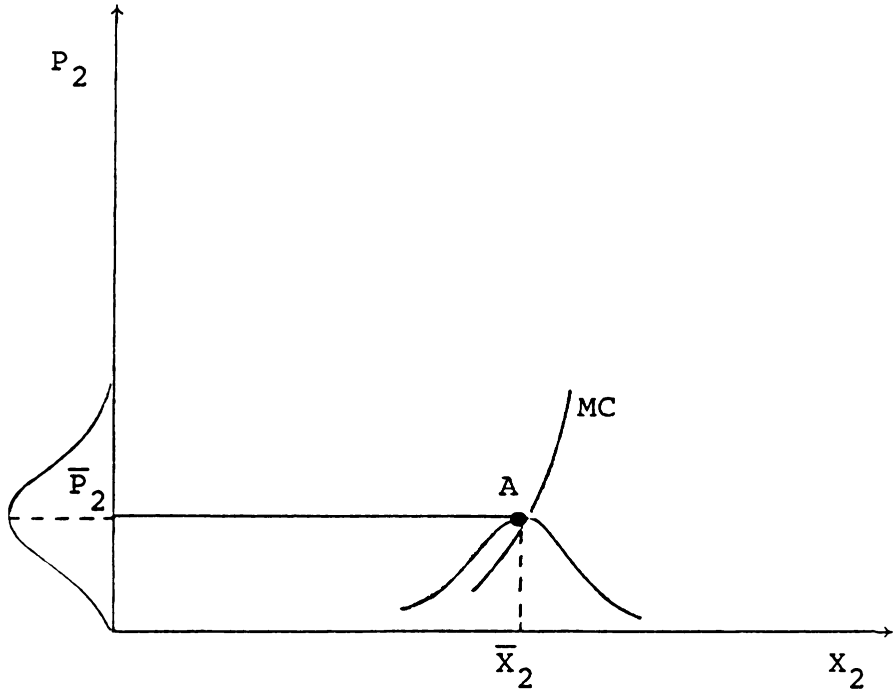


FIGURE 4

DEPENDENCE OF THE PLANNED OUTPUT ON THE

EXPECTED PRICE

planned output $\overline{X}_2$. It may take an infinite number of values, however, when the production occurs only one of these values will materialize and will become the actual output level. The fact that when $\overline{P}_2$ shifts up, a larger output will be planned explains the dependence of the planned output on the expected price, in other words it shows how the location of the distribution of the loan rate determines the location of the loan distribution. However, since loans and the loan rate can, regardless of each other, take on any of the values described by respective distributions, their deviations from respective means may be considered independent [Figure 4].

3-2-3 The Constraints

As in the preceding chapter, the production function, the balance sheet, the reserve requirement equation and the demand function for clearance output must be imposed on the model. For simplicity the demand function for clearance is substituted in the production function, and the reserve requirement constraint is substituted in the balance sheet. The substituted version of the constraints may be expressed as:

$$F = \alpha_1 \log (a_1\overline{DD} + b_1P_1) + \alpha_2 \log (\overline{X}_2) + \beta_1 (a_1\overline{DD} + b_1P_1)$$

$$+ \beta_2\overline{X}_2 - \alpha_D \log (\overline{DD}) - \alpha_L \log (L) - \alpha_k \log (K) - \gamma = 0$$

$$\overline{X}_2 + \overline{FR} - (1\text{-}rd)\ \overline{DD} - W_0 = 0$$

3-2-4 The Lagrangian Function

After substitution for moments of products the Lagrangian function may be written as G*:

$$\max G^* = (a_1P_1 - R_D)\overline{DD} + b_1P_1^2 + \overline{P}_2\overline{X}_2 - P_LL - P_KK$$

$$- R_B\overline{BR} - \alpha\{(a_1P_1 - R_D)^2 \, V(DD) + P_1^2V(E_1) + V(P_2)$$

$$V(X_2) + \overline{P}_2^2 \, V(X_2) + \overline{X}_2^2 \, V(P_2) + R_B^2 \, V(BR) + 2P_1$$

$$(a_1P_1 - R_D) \, C_{OV}(DD,E_1) + 2(a_1P_1 - R_D) \, \overline{P}_2 \, C_{OV}(DD,$$

$$X_2) - 2(a_1P_1 - R_D) \, R_B \, C_{OV}(DD,BR) - 2R_B \, \overline{P}_2$$

$$C_{OV}(BR,X_2)\} - \lambda F - \mu[\overline{X}_2 + \overline{FR} - (1\text{-rd}) \, \overline{DD} - W_0]$$

3-2-5 The First Order Conditions (F.O.C.)

The F.O.C. require that the partial derivatives of the Lagrangian function with respect to the endogenous variables be set to zero.

$$\partial G^*/\partial P_1 = a_1\overline{DD} + 2b_1P_1 - \alpha\{2a_1(a_1P_1 - R_D) \, V(DD) +$$

$$2P_1 \, V(E_1) + (4a_1P_1 - 2R_D) \, C_{OV}(DD,E_1) +$$

$$2a_1\overline{P}_2 \, C_{OV}(DD,X_2) - 2a_1R_B \, C_{OV}(DD,BR)\} - \lambda F_{P_1}$$

$$= 0$$

$$\partial G^*/\partial \overline{X}_2 = \overline{P}_2 - 2\alpha\overline{X}_2 \, V(P_2) - \lambda F_2 - \mu = 0$$

$$\partial G^*/\partial \overline{FR} = - R_B \, \partial\overline{BR}/\partial\overline{FR} - \alpha R_B^2 \, \partial V(BR)/\partial\overline{FR} = 0$$

$$\partial G^*/\partial L = -P_L - \lambda F_L = 0$$

$$\partial G^*/\partial K = -P_K - \lambda F_K = 0$$

$$\partial G^*/\partial \lambda = -F = 0$$

$$\partial G^*/\partial \mu = -[\overline{X}_2 + \overline{FR} - (1-rd)\overline{DD} - W_0] = 0$$

$\overline{BR}$ and V (BR) are the incomplete moments of the distribution of FR. The derivation of their partial derivatives with respect to $\overline{FR}$ follows the same lines as in the preceding chapter. Although double integrals should be used due to randomness in loans, the partial derivatives are the same as in the last chapter.

Notice the first order conditions are essentially the same as in the last chapter, where now $\overline{X}_2$ appears instead of X_2. The same interpretation of the optimality conditions is still applicable.

3-2-6 The Second Order Conditions (S.O.C.)

The Hessian matrix and the S.O.C. are similar to their counterparts in the last chapter, where now $\overline{X}_2$ is substituted for X_2 and S_2 is set to zero. The S.O.C. lead to the following restrictions:

$$F_{11}, F_{22}, F_{LL}, F_{KK} > 0$$

$$\frac{\partial^2 V(BR)}{\partial \overline{FR}^2} \gtrsim 0$$

3-3 Displacement of Equilibria

The first order conditions are totally differentiated to derive the comparative static results. The differentiated form of the first order conditions appears in Table 4. The cofactors Hij of the Hessian matrix have similar signs to their counterparts in the last chapter. These signs were displayed in Table 2. The comparative static results are presented in Table 5. Notice most results are the same as in the last chapter. The covariation between loans and deposits has led to indeterminacy of some previously determinate results (Row 1). Due to randomness in loans two new variables, $V(X_2)$ and $C_{OV}(DD,X_2)$, are now added to the set of the model's exogenous variables, but they do not have a determinate effect. Only under the special condition, that variability in free reserves does not affect the probability of borrowing, can we say that with uncertainty in loans the bank prudently plans smaller loans and holds more free reserves.

TABLE 4

THE DIFFERENTIATED FORM OF THE FIRST ORDER CONDITIONS

$$\begin{bmatrix} 0 & 0 & F_{P_1} & F_2 & 0 & F_L & F_K \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ F_{P_1} & 0 & A_{11} & 0 & 0 & 0 & 0 \\ F_2 & 1 & 0 & A_{22} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & A_{33} & 0 & 0 \\ F_L & 0 & 0 & 0 & 0 & -\lambda F_{LL} & 0 \\ F_K & 0 & 0 & 0 & 0 & 0 & -\lambda F_{KK} \end{bmatrix} \begin{bmatrix} -d\lambda \\ -d\mu \\ dP_1 \\ d\overline{X}_2 \\ d\overline{FR} \\ d_L \\ d_K \end{bmatrix} =$$

TABLE 4 (cont'd)

$$\begin{pmatrix}
-(a_1F_1 - \alpha_D/\overline{DD})\, d\overline{DD} + d\gamma \\[1ex]
(1-rd)\, d\overline{DD} - \overline{DD}\, drd + dW_O \\[1ex]
2\alpha a_1\, C_{OV}(DD,X_2)\, d\overline{P}_2 + 2\alpha a_1(a_1P_1 - R_D)\, dV(DD) + 2\alpha P_1 dV(E_1) \\
\quad + (4\alpha a_1P_1 - 2\alpha R_D)\, dC_{OV}\,(DD,E_1) + 2\alpha a_1\overline{P}_2\, dC_{OV}\,(DD,X_2) \\
\quad - 2\alpha a_1R_B\, dC_{OV}(DD,BR) - 2\alpha[a_1V(DD) + C_{OV}(DD,E_1)]\, dR_D \\
\quad - 2\alpha a_1\, C_{OV}(DD,BR)\, dR_B - (a_1 - \lambda a_1b_1F_{11})\, d\overline{DD} \\
\quad + [2a_1(a_1P_1 - R_D)V(DD) + 2P_1V(E_1) + (4a_1P_1 - 2R_D)\, C_{OV} \\
\quad (DD,E_1) + 2a_1\overline{P}_2\, C_{OV}(DD,X_2) - 2a_1R_B\, C_{OV}(DD,BR)]\, d\alpha \\[1ex]
d\overline{P}_2 + 2\alpha\overline{X}_2\, dV(P_2) + 2\overline{X}_2\, V(P_2)\, d\alpha \\[1ex]
-Q\,(1-rd)^2\, dV(DD) - QdV(X_2) - 2Q[C_{OV}(DD,X_2) - (1-rd)V(DD)]drd \\
\quad + 2Q(1-rd)dC_{OV}(DD,X_2) - dR_B[P_r + 4\alpha R_B\overline{BR}(1-P_r)] - 2R_B^2\overline{BR} \\
\quad (1-P_r)d\alpha \\[1ex]
dP_L \\[1ex]
dP_K
\end{pmatrix}$$

$$A_{11} = 2b_1 - 2\alpha V(X_1) - \lambda b_1^2F_{11}$$

$$A_{22} = -2\alpha V(P_2) - \lambda F_{22}$$

$$A_{33} = R_B\ \partial P_r/\partial_{\overline{FR}} - 2\alpha R_B^2\ [(1-P_r)P_r + 2\overline{BR}\ \partial P_r/\partial_{\overline{FR}}]$$

TABLE 4 (cont'd)

$A_{ii} < 0 \qquad i = 1,2,3$

$Q = R_B \; \partial P_r/\partial_V(FR) + 2\alpha R_B^2(1-P_r) \; \partial \overline{BR}/\partial_V(FR) - 2\alpha R_B^2 \overline{BR} \; \partial P_r/\partial_V(FR)$

TABLE 5

THE COMPARATIVE STATIC RESULTS

Exogenous Variables	Conditions Imposed	Endogenous Variables						
		P_1	$\overline{X}_2$	$\overline{FR}$	L	K	μ	λ
$\overline{P}_2$		?	?	?	+	+	?	?
R_B		-	-	+	?	?	?	?
R_D		+	+	-	-	-	+	-
P_L		+	-	+	-	+	-	+
P_K		+	-	+	+	-	-	+
$V(P_2)$		-	-	+	-	-	-	-
$V(DD)$	$\partial\overline{BR}/\partial V(FR) = \partial P_r/\partial V(FR) = 0$, $R_D < 2a_1P_1$	-	-	+	+	+	-	+
$V(E_1)$		-	-	+	+	+	-	+
$V(X_2)$	$\partial\overline{BR}/\partial V(FR) > 0$, $\partial P_r/\partial V(FR) = 0$	-	-	+	-	-	+	-
$C_{OV}(DD,X_2)$	$\partial P_r/\partial V(FR) = 0$	?	?	?	+	+	-	+
$C_{OV}(DD,E_1)$		-	-	+	+	+	-	+
$C_{OV}(DD,BR)$		+	+	-	-	-	+	-

TABLE 5 (cont'd)

r_d	$\partial\overline{BR}/\partial V(FR) = \partial P_r/\partial V(FR) = 0$	-	-	-	-	-	+	-
W_0		+	+	+	+	+	-	+
$\overline{DD}$		?	?	?	?	?	?	?
α	$C_{OV}(DD,E_1)$, $C_{OV}(DD,X_2) > 0$; $C_{OV}(DD,BR) < 0$; $R_D < 2a_1P_1$	-	-	+	?	?	?	?
γ		-	+	-	-	-	+	-

CHAPTER FOUR

THE MULTIPRODUCT BANK AND LIABILITY MANAGEMENT

4-1 Introduction

In this chapter we introduce a third output and liability management into the model. The bank now has a new source of funds which is under its control. It can choose the level of time deposit liabilities. This can be done through the sale of large negotiable certificates of deposits, CD's. It is also assumed that the bank has access to the federal funds market. It sells its excess reserves to and borrows its reserve deficiencies from other banks through this market. The funds secured through sale of CD's are used alongside funds acquired through clearance activity to produce three outputs X_1, X_2 and X_3. X_1 is the clearance output. X_2 and X_3 may be considered as two categories of loans, or as loans and investments respectively.

4-1-1 Sources of Uncertainty

The bank faces uncertainty in both quantities and prices. In regard to quantity uncertainty, the quantity of demand deposits, DD, time deposits, CD, federal funds, FF, and the three outputs X_1, X_2 and X_3 are assumed to be random. In regard to price uncertainty, the CD rate, R_{CD}, the federal funds rate R_F, the loan rate, P_2, and the security rate, P_3, are assumed to be stochastic.

Since X_1, X_2, X_3, CD and FF are random variables, their values cannot be optimized. It is therefore assumed that the bank optimizes their mean values $\overline{X}_1$, $\overline{X}_2$, $\overline{X}_3$, $\overline{CD}$, and $\overline{FF}$. In the decision period the bank can be considered to be facing a series of distributions for each variable with different means. By making arrangements, the bank chooses the distribution with the optimal mean. When the mean is chosen, it will constitute the planned or the desired quantity of the corresponding variable (loans for example). This quantity will not necessarily prevail however. When the production period arrives, the variable will take on one of the possible values on the corresponding distribution. This actual value will deviate from the planned or desired level by a random term.

In the same way, demand deposits, the loan rate, the security rate, the CD rate and the federal funds rate are random variables whose distributions are exogeneous to the bank. It is assumed that the bank can forecast and somehow measure the first two moments of these distributions. The forecasted mean will be considered as the expected deposit or expected price by the bank. The actual deposits or actual price will deviate from their expected levels by a random factor. Mathematically, this argument may be introduced by writing each variable Z as the sum of the corresponding mean $\overline{Z}$ and an error term E_z. $Z = \overline{Z} + E_z$. This formulation will later be used to measure and simplify the objective function.

4-1-2 Market Structure

The market structure for the clearance output is assumed imperfect, while the market for demand deposits, CD's, federal funds, loans, securities and capital and labor inputs are competitive. The demand for clearance output is consequently less than perfectly elastic; the bank sets the clearance output price or equivalently the mean output level; while CD rate, loan rate, security rate and federal funds rate are exogenous to the individual bank.

4-2 The Structure of the Model and the Optimality Conditions

4-2-1 The Objective Function

The bank's objective is to maximize the certainty-equivalent profit G where $G = E(\pi) - v(\pi)$. Here the profit equation π is used to derive its two first moments in order to measure the objective function.

$$\pi = P_1X_1 + P_2X_2 + P_3X_3 + R_F FF - R_D DD - R_C CD - P_L L - P_K K$$

where FF is the net federal funds sold.

$$E(\pi) = P_1\overline{X}_1 + E(P_2X_2) + E(P_3X_3) + E(R_F FF) - R_D\overline{DD}$$
$$- E(R_{CD}CD) - P_L L - P_K K$$

$$V(\pi) = P_1{}^2V(X_1) + V(P_2X_2) + V(P_3X_3) + V(R_F FF) + R_D{}^2V(DD)$$

$$+ V(R_{CD}CD) + 2P_1[C_{OV}(X_1,P_2X_2) + C_{OV}(X_1,P_3X_3)$$

$$+ C_{OV}(X_1,R_FFF) - R_DC_{OV}(X_1,DD) - C_{OV}(X_1,R_{CD}CD)]$$

$$+ 2C_{OV}(P_2X_2,P_3X_3) + 2C_{OV}(P_2X_2,R_FFF) - 2R_DC_{OV}$$

$$(P_2X_2,DD) - 2C_{OV}(P_2X_2,R_{CD}CD) + 2C_{OV}(P_3X_3,R_FFF)$$

$$- 2R_DC_{OV}(P_3X_3,DD) - 2C_{OV}(P_3X_3,R_{CD}CD) - 2R_DC_{OV}$$

$$(R_FFF,DD) - 2C_{OV}(R_FFF,R_{CD}CD) + 2R_DC_{OV}(DD,R_{CD}CD)$$

As explained before, some simplifications are required to derive the desired product moments. Random prices and quantities are written as the sum of the respective means and random terms which represent the deviations of the actual values from the corresponding expected or planned levels, $P = \overline{P} + U$, $Z = \overline{Z} + V$. Then, it is assumed that the random terms in prices (U) are independent from those in quantities (V). Following these assumptions it is shown in Appendix 5 that the desired product moments may be written as follows:

$$E(P_iX_i) = \overline{P}_i\overline{X}_i \qquad i = 2,3$$

$$V(P_iX_i) = \overline{X}_i{}^2V(P_i) + \overline{P}_i{}^2V(X_i) + V(P_i)V(X_i) \qquad i = 2,3$$

$$V(R_{CD}CD) = \overline{R}_{CD}{}^2V(CD) + \overline{CD}^2V(R_{CD}) + V(CD)V(R_{CD})$$

$$V(R_FFF) = \overline{R}_F{}^2V(FF) + \overline{FF}^2V(R_F) + V(R_F)V(FF)$$

$$C_{OV}(P_iX_i,DD) = \overline{P}_iC_{OV}(X_i,DD) \qquad i = 2,3$$

$$C_{OV}(P_iX_i,X_1) = \overline{P}_iC_{OV}(X_i,X_1) \qquad i = 2,3$$

$$C_{OV}(R_FFF,X_1) = \overline{R}_FC_{OV}(X_1,FF)$$

$$C_{OV}(R_FFF,DD) = \overline{R}_FC_{OV}(DD,FF)$$

$$C_{OV}(R_{CD}CD,DD) = \overline{R}_{CD}C_{OV}(DD,CD)$$

$$C_{OV}(P_2X_2,P_3X_3) = C_{OV}(P_2,P_3)C_{OV}(X_2,X_3) + \overline{X}_2\overline{X}_3C_{OV}(P_2,P_3) + \overline{P}_2\overline{P}_3C_{OV}(X_2,X_3)$$

$$C_{OV}(P_iX_i,R_FFF) = C_{OV}(P_i,R_F)C_{OV}(X_i,FF) + \overline{X}_i\overline{FF}\ C_{OV}(P_i,R_F) + \overline{R_F}\ \overline{P}_iC_{OV}(X_i,FF) \qquad i = 2,3$$

$$C_{OV}(P_iX_i,R_{CD}CD) = C_{OV}(P_i,R_{CD})C_{OV}(X_i,FF) + \overline{X}_i\overline{CD}\ C_{OV}(P_i, R_{CD}) + \overline{P}_i\ \overline{R}_{CD}\ C_{OV}(X_i,CD) \qquad i = 2,3$$

$$C_{OV}(R_FFF,R_{CD}CD) = C_{OV}(R_F,R_{CD})\ C_{OV}(FF,CD) + \overline{R}_{CD}\overline{R}_F\ C_{OV}(FF,CD) + \overline{FF}\ \overline{CD}\ C_{OV}(R_F.R_{CD})$$

These relations will be used to set up the Lagrangian function.

4-1-3 The Constraints

The following Constraints will be imposed on the model

a - Production Function

$$F = \alpha_1\log(\overline{X}_1) + \alpha_2\log(\overline{X}_2) + \alpha_3\log(\overline{X}_3) + \beta_1\overline{X}_1 + \beta_2\overline{X}_2$$

$$+ \beta_3\overline{X}_3 - \alpha_{DD}\log(\overline{DD}) - \alpha_{CD}\log(\overline{CD}) - \alpha_L\log(L) - \alpha_L\log(K)$$

$$- \gamma = 0$$

b - Balance sheet identity

$$X_2 + X_3 + FF = DD + CD + W_O$$

c - Demand function for clearance output

$$X_1 = a_1DD + b_1p_1 + E_1$$

d - Reserve requirement

$$RR = rd * DD + r_c * CD$$

where rd and r_c are reserve requirement ratios for demand deposits and CD's respectively.

Due to the accessibility of the federal funds market, the bank is assumed not to hold any interest-free excess reserves. Since the optimization occurs prior to the production period, the Constraints will be imposed in their expected value form. To simplify, we substitute the last constraint into the balance sheet constraint. We also use the mean demand function for the clearance output to derive P_1 in terms of X_1 and substitute that in the objective function.

$$\overline{X}_1 = a_1\overline{DD} + b_1p_1$$

or

$$P_1 = \frac{\overline{X}_1 - a_1\overline{DD}}{b_1}$$

Consequently there are only two constraints which must be imposed on the objective function.

4-1-4 The Langrangian Function

If the mean demand function for clearance output is used to substitute for P_1, and the simplifications given in Appendix 5 are used to substitute for the means, variances, and covariances in the objective function, the Lagrangian function can then be written as follows:

$$
\begin{aligned}
G^* \quad = \quad & \left(\frac{\overline{X}_1 - a_1\overline{DD}}{b_1}\right) \overline{X}_1 + \overline{P}_2\overline{X}_2 + \overline{P}_3\overline{X}_3 + \overline{R}_F\overline{FF} - R_D\overline{DD} \\
& - \overline{R}_{CD}\overline{CD} - P_L L - P_K K - \alpha\{\left(\frac{\overline{X}_1 - a_1\overline{DD}}{b_1}\right)^2 \; V(X_1) + V(P_2) \\
& V(X_2) + \overline{X}_2^{\,2} \; V(P_2) + \overline{P}_2^{\,2} \; V(X_2) + V(P_3) \; V(X_3) + \overline{X}_3^{\,2} \\
& V(P_3) + \overline{P}_3^{\,2} \; V(X_3) + V(R_F) \; V(FF) + \overline{FF}^2 \; V(R_F) + \overline{R}_F^{\,2} \\
& V(FF) + R_D^{\,2} \; V(DD) + V(R_{CD}) \; V(CD) + \overline{R}_{CD}^{\,2} \; V(CD) \\
& + \overline{CD}^2 \; V(R_{CD}) + 2 \left(\frac{\overline{X}_1 - a_1\overline{DD}}{b_1}\right) [\overline{P}_2 \; C_{OV}(X_1,X_2) + \overline{P}_3 \\
& C_{OV}(X_1,X_3) + \overline{R}_F \; C_{OV}(X_1,FF) - R_D \; C_{OV}(DD,X_1) - \overline{R}_{CD} \\
& C_{OV}(X_1,CD)] + 2C_{OV}(P_2,P_3) \; C_{OV}(X_2,X_3) + 2\overline{P}_2\overline{P}_3 \\
& C_{OV}(X_2,X_3) + 2\overline{X}_2\overline{X}_3 \; C_{OV}(P_2,P_3) + 2C_{OV}(P_2,R_F) \; C_{OV} \\
& (X_2,FF) + 2\overline{R}_F\overline{P}_2 \; C_{OV} \; (X_2,FF) + 2\overline{FF} \; \overline{X}_2 \; C_{OV}(P_2,R_F) \\
& - 2R_D\overline{P}_2 \; C_{OV}(X_2,DD) - 2C_{OV}(P_2,R_{CD}) \; C_{OV}(X_2,CD) \\
& - 2\overline{P}_2 \; \overline{R}_{CD} \; C_{OV}(X_2,CD) - 2\overline{X}_2\overline{CD} \; C_{OV} \; (P_2, R_{CD}) +
\end{aligned}
$$

$$2C_{OV}(P_3, R_F)\ C_{OV}(X_3, FF) + 2\ \bar{P}_3\ \bar{R}_F\ C_{OV}(X_3, FF) + 2\bar{X}_3$$

$$\overline{FF}\ C_{OV}\ (P_3, R_F) - 2R_D\bar{P}_3\ C_{OV}(DD, X_3) - 2R_D\bar{R}_F\ C_{OV}(DD, FF)$$

$$- 2C_{OV}(P_3, R_{CD})\ C_{OV}(X_3, DD) - 2\bar{P}_3\ \bar{R}_{CD}\ C_{OV}(X_3, CD)$$

$$- 2\bar{X}_3\ \overline{CD}\ C_{OV}\ (P_3, R_{CD}) - 2C_{OV}\ (R_F, R_{CD})\ C_{OV}(FF, CD)$$

$$- 2\bar{R}_F\ \bar{R}_{CD}\ C_{OV}\ (FF, CD) - 2\overline{FF}\ \overline{CD}\ C_{OV}(R_F, R_{CD}) + 2R_D$$

$$\bar{R}_{CD}\ C_{OV}(DD, CD)\} - \lambda F - \mu[\bar{X}_2 + \bar{X}_3 + \overline{FF} - (1-rd)\overline{DD}$$

$$- (1-r_c)\ \overline{CD} - W_0]$$

4-1-5 First Order Conditions For Optimality (F.O.C.)

The F.O.C. require that partial derivatives of the lagrangian function with respect to endogenous variables be set to zero, that is:

$$\partial G^*/\partial\bar{X}_1 = \frac{2\bar{X}_1 - a_1\overline{DD}}{b_1} - 2\alpha\ \{(\frac{\bar{X}_1 - a_1\overline{DD}}{b_1})\ V(X_1) + \frac{1}{b_1}$$

$$[\bar{P}_2\ C_{OV}(X_1, X_2) + \bar{P}_3\ C_{OV}(X_1, X_3) + \bar{R}_F\ C_{OV}(X_1, FF)$$

$$- R_D\ C_{OV}(X_1, DD) - \bar{R}_{CD}\ C_{OV}(X_1, CD)]\} - \lambda F_1 = 0$$

$$\partial G^*/\partial\bar{X}_2 = \bar{P}_2 - 2\alpha\ \{\bar{X}_2\ V(P_2) + \bar{X}_3\ C_{OV}\ (P_2, P_3) + \overline{FF}$$

$$C_{OV}\ (P_2, R_F) - \overline{CD}\ C_{OV}\ (P_2, R_{CD})\} - \lambda F_2 - \mu = 0$$

$$\partial G^*/\partial\bar{X}_3 = \bar{P}_3 - 2\alpha\ \{\bar{X}_3\ V(P_3) + \bar{X}_2\ C_{OV}(P_2, P_3) + \overline{FF}$$

$$C_{OV}(P_3, R_F) - \overline{CD}\ C_{OV}(P_3, R_{CD})\} - \lambda F_3 - \mu = 0$$

$$\partial G^*/\partial \overline{FF} = \overline{R}_F - 2\alpha \{\overline{FF}\ V(R_F) + \overline{X}_2\ C_{OV}\ (P_2, R_F)$$

$$+ X_3\ C_{OV}\ (P_3, R_F) - \overline{CD}\ (R_F, R_{CD})\}$$

$$- \mu = 0$$

$$\partial G^*/\partial \overline{CD} = - \overline{R}_{CD} - 2\alpha \{\overline{CD}\ V(R_{CD}) - \overline{X}_2\ C_{OV}\ (P_2, R_{CD}) - \overline{FF}$$

$$C_{OV}\ (R_F, R_{CD}) - \overline{X}_3\ C_{OV}\ (P_3, R_{CD})\} - \lambda F_{CD}$$

$$+ \mu(1-r_c) = 0$$

$$\partial G^*/\partial L = -P_L - \lambda F_L = 0$$

$$\partial G^*/\partial K = -P_K - \lambda F_K = 0$$

$$\partial G^*/\partial \lambda = -F = 0$$

$$\partial G^*/\partial \mu = -[\overline{X}_2 + \overline{X}_3 + \overline{FF} - (1-rd)\overline{DD} - (1-r_c)\overline{CD} - W_O] = 0$$

where $F_z = \partial F/\partial_z$

The interpretation of the F.O.C. follows the same lines as in the previous chapters. These conditions define the implicit demand function for CD's, Capital, and labor inputs. They also define the implicit supply functions for loanable funds, investment funds, and federal funds. The model explains the observed diversification behavior and the dependence of the loan output on the rate paid on demand deposits.

4-1-6 Second Order Conditions (S.O.C.)

The S.O.C. require the Hessian matrix H to be negative definite. This, in turn, implies that the principal minors alternate in sign with the first being positive. In the present form of the model, it can be shown that the S.O.C. become very complicated and will not necessarily be satisfied. The comparative static results will also be indeterminate in large part. However, if we assume that the relationships between different interest rates involved are highly nonlinear, so that the degree of their linear relationship as measured by their correlation coefficient is zero, then the covariances in the Hessian matrix disappear, the S.O.C. can be satisfied by certain requirements and some comparative static results will become determinate at least under certain conditions. Having made these assumptions, the S.O.C. require:

$$F_{ii} > 0 \qquad i = 1,2,3 \qquad \text{where } F_{ii} = \frac{\partial^2 F}{\partial \bar{X}_i^{\,2}}$$

$$F_{jj} > 0 \qquad j = L,K \qquad F_{jj} = \frac{\partial^2 F}{\partial_j^{\,2}}$$

4-3 <u>Displacement of Equilibria</u>

To get the comparative static results, the first order conditions are totally differentiated. The differentiated form provides a simultaneous equation system. The matrix of the coefficients in the system is the Hessian matrix. If a consistent solution exists, the system may be solved for the endogenous variables by using Cramer's rule.

The total differentiation and the cofactors of the Hessian matrix H will not be given here. The differentiated form of the first order conditions is given in Table 6. The signs of the cofactors H_{ij} are given in Table 7. The comparative static results are given in Table 8. The cofactors and the comparative static results have the given signs under the conditions specified in respective tables.

The cofactors which are marked by questions marks (?) have indeterminate signs. The conditions given beneath Table 7 can be translated into meaningful concepts. The conditions 1 and 2 can be written as $F_2/F_3 \gtrless 1$, where, $F_2/F_3 = -\dfrac{d\bar{\bar{X}}_3}{d\bar{X}_2}$ represents the marginal rate of product transformation between outputs X_2 and X_3. Conditions 3 and 4 can be written as $\dfrac{F_{CD}}{r_c-1} / F_2 \gtrless 1$. $\dfrac{F_{CD}}{(r_c-1)} / F_2$ represents the marginal product of the unencumbered CD input in producing output X_2. Analogously, conditions 5 and 6 have implications on the level of marginal product of unencumbered CD's in producing output X_3. Some of the comparative static results are especially noteworthy. In the profit maximizing competitive multiproduct firm, the conditional supply and demand functions for outputs and inputs are upward and downward sloping respectively [16, p. 98]. Such a determinacy is not present in our model. The conditional supply functions of loanable and investment funds are upward

TABLE 6

THE DIFFERENTIATED FORM OF F.O.C.

$$\begin{pmatrix} 0 & 0 & F_1 & F_2 & F_3 & 0 & F_{CD} & F_L & F_K \\ 0 & 0 & 0 & 1 & 1 & 1 & r_{c-1} & 0 & 0 \\ F_1 & 0 & A_{11} & 0 & 0 & 0 & 0 & 0 & 0 \\ F_2 & 1 & 0 & A_{22} & 0 & 0 & 0 & 0 & 0 \\ F_3 & 1 & 0 & 0 & A_{33} & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & A_{44} & 0 & 0 & 0 \\ F_{CD} & r_{c-1} & 0 & 0 & 0 & 0 & A_{55} & 0 & 0 \\ F_L & 0 & 0 & 0 & 0 & 0 & 0 & -\lambda F_{LL} & 0 \\ F_K & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\lambda F_{KK} \end{pmatrix} \begin{pmatrix} -d\lambda \\ -d\mu \\ d\bar{X}_1 \\ d\bar{X}_2 \\ d\bar{X}_3 \\ d\overline{FF} \\ d\overline{CD} \\ dL \\ dK \end{pmatrix} =$$

TABLE 6 (cont'd)

$$\begin{cases}
-F_D \, d\,\overline{DD} \\[2ex]
(1-rd)\, d\overline{DD} - \overline{DD}\, drd - \overline{CD}\, dr_c + dW_O \\[2ex]
\frac{a_1}{b_1}\, d\overline{DD} - \frac{2\alpha a_1 V(X_1)}{b_1{}^2}\, d\overline{DD} + 2\alpha\{\frac{\overline{X}_1 - a_1\overline{DD}}{b_1{}^2}\, dV(X_1) + \frac{1}{b_1}\, C_{OV}(X_1,X_2) \\
\qquad d\overline{P}_2 + \frac{1}{b_1}\, \overline{P}_2\, dC_{OV}(X_1,X_2) + \frac{1}{b_1} C_{OV}(X_1,X_3)\, d\overline{P}_3 + \frac{1}{b_1}\, \overline{P}_3 dC_{OV} \\
\qquad (X_1,X_3) + \frac{1}{b_1}\, \overline{R}_F dC_{OV}(X_1,FF) + \frac{1}{b_1}\, C_{OV}(X_1,FF)\, d\overline{R}_F - \frac{R_D}{b_1}\, dC_{OV} \\
\qquad (DD,X_1) - \frac{C_{OV}(DD,X_1)}{b_1} - \frac{\overline{R}_{CD}}{b_1}\, dC_{OV}(X_1,CD) - \frac{C_{OV}}{b_1}\, (X_1,CD) \\
\qquad d\overline{R}_{CD}\} + 2d\alpha\{(\frac{\overline{X}_1 - a_1\overline{DD}}{b_1{}^2})V(X_1), + \frac{1}{b_1}\, [\overline{P}_2 C_{OV}(X_1,X_2) + \overline{P}_3 \\
\qquad C_{OV}(X_1,X_3) + \overline{R_F} C_{OV}(X_1,FF) - R_D C_{OV}(DD,X_1) - \overline{R}_{CD} C_{OV}(X_1,CD)] \\[2ex]
-d\overline{P}_2 + 2\alpha\overline{X}_2\, dV(P_2) + 2\overline{X}_2\, V(P_2)\, d\alpha \\[2ex]
-d\overline{P}_3 + 2\alpha\overline{X}_3\, dV(P_3) + 2\overline{X}_3\, V(P_3)\, d\alpha \\[2ex]
-d\overline{R}_F + 2\alpha\overline{FF}\, dV(R_F) + 2\overline{FF}\, V(R_F)\, d\alpha \\[2ex]
+d\overline{R}_{CD} + 2\alpha\overline{CD}\, dV(R_{CD}) + 2\overline{CD}\, V(R_{CD})\, d\alpha + \mu dr_c \\[2ex]
dP_L \\[2ex]
dP_K
\end{cases}$$

where $A_{11} = \frac{2}{b_1} - \frac{2\alpha V(X_1)}{b_1{}^2} - \lambda F_{11}$ $\qquad A_{44} = -2\alpha V(R_F)$

$A_{22} = -2\alpha V(P_2) - \lambda F_{22}$ $\qquad A_{55} = -2\alpha V(R_{CD}) - \lambda F_{\overline{CD}}$

$A_{33} = -2\alpha V(P_3) - \lambda F_{33}$

TABLE 7

SIGNS OF THE COFACTORS

H	−														
$H_{0,0}$	−														
$H_{0,00}$	+	$H_{00,00}$	−												
H_{01}	+	H_{001}	−	H_{11}	+										
H_{02}	?	H_{002}	−	H_{12}	1,4 + 2,5	H_{22}	+								
H_{03}	?	H_{003}	2,3 −	H_{13}	+	H_{23}	−	H_{33}	+						
H_{04}	+	H_{004}	1,6 −	H_{14}	−	H_{24}	2,3 − 2,5	H_{34}	2,5 − 1,4	H_{44}	+				
H_{05}	4,5 −	H_{005}	?	H_{15}	3,6 −	H_{25}	+	H_{35}	+ 1,4	H_{45}	4,5 −	H_{55}	+		
H_{06}	+	H_{006}	−	H_{16}	−	H_{26}	1,4 +	H_{36}	+ 2,6	H_{46}	−	H_{56}	?	H_{66}	+
H_{07}	+	H_{007}	−	H_{17}	−	H_{27}	1,4 +	H_{37}	+	H_{47}	−	H_{57}	?	H_{67}	−

H_{77}	+

Condition 1: $F_2 > F_3$

Condition 2: $F_2 < F_3$

Condition 3: $F_2(r_c-1) > F_{CD}$

Condition 4: $F_2(r_c-1) < F_{CD}$

Condition 5: $F_3(r_c-1) > F_{CD}$

Condition 6: $F_3(r_c-1) < F_{CD}$

TABLE 8

THE COMPARATIVE STATIC RESULTS

ROW		CONDITION IMPOSED	$\overline{X}_1$	$\overline{X}_2$	$\overline{X}_3$	$\overline{FF}$	$\overline{CD}$	L	K	λ	μ
1	$\overline{P}_2$	$C_{OV}(X_1,X_2) > 0$	+ 1,4	+ 1,4	?	− 2,3	?	?	?	?	+ 2,3
2		$C_{OV}(X_1,X_2) = 0$	+ 1,4	+ 1,4	−	− 2,3	+ 2.5 or 1,6	+ 1,4	+ 1,4	?	+ 2,3
3	$\overline{P}_3$	$C_{OV}(X_1,X_3) > 0$	+ 2,5	?	+ 2,5	− 2,5	?	?	?	?	+ 1,6
4		$C_{OV}(X_1,X_3) = 0$	+ 2,5	−	+ 2,5	− 2,5	+ 1,4 or 2,3	+ 2,5	+ 2,6	?	+ 1,6
5	$\overline{R}_F$	$C_{OV}(X_1,FF) = 0$	−	− 2,3	− 2,5	+	− 4,5	−	−	−	+
6	$\overline{R}_{CD}$	$C_{OV}(X_1+CD) > 0$	?	− 1,4,6	− 2,3,5	+ 4,5	?	?	?	?	?
7		$C_{OV}(X_1,CD) = 0$	+ 3,6	− 2,5 or 1,6	− 1,4 or 2,3	+ 4,5	−	?	?	+ 4,5	?
8	R_D		−	− 1,4	− 2,5	+	+ 3,6	+	+	+	−
9	P_L		+	− 1,4	− 2,5	+	?	−	+	+	−
10	P_K		+	− 1,4	− 2,6	+	?	+	−	+	−
11	$V(P_2)$		− 1,4	−	+	+ 2,3	− 2,5 or 1,6	− 1,4	− 1,4	?	− 2,3
12	$V(P_3)$		− 2,5	+	−	+ 2,5	− 1,4 or 2,3	− 2,5	− 2,6	?	− 1,6
13	$V(R_F)$	$\overline{FF} > 0$	+	+ 2,3	+ 2,5	−	+ 4,5	+	+	−	−
14		$\overline{FF} < 0$	−	− 2,3	− 2,5	+	− 4,5	−	−	+	+

TABLE 8 (cont'd)

ROW		CONDITION IMPOSED	X_1	X_2	X_3	FF	CD	L	K	λ	μ
15	$V(R_{CD})$		+ 3,6	- 2,5 or 1,6	- 1,4 or 2,3	+ 4,5	-	?	?	+ 4,5	?
16	$V(X_1)$		-	- 1,4	- 2,5	+	+ 3,6	+	+	+	-
17	$C_{OV}(X_1,X_2)$		+	+ 1,4	+ 2,5	-	- 3,6	-	-	-	+
18	$C_{OV}(X_1,X_3)$		+	+ 1,4	+ 2,5	-	- 3,6	-	-	-	+
19	$C_{OV}(X_1,DD)$		-	- 1,4	- 2,5	+	+ 3,6	+	+	+	-
20	$C_{OV}(X_1,FF)$		+	+ 1,4	+ 2,5	-	- 3,6	-	-	-	+
21	$C_{OV}(X_1,CD)$		-	- 1,4	- 2,5	+	+ 3,6	+	+	+	-
22	W_O		+	+ 2,3	+ 1,6	+	?	+	+	-	+
23	rd		-	- 2,3	- 1,6	-	?	-	-	+	-
24	r_c		?	- 2,3,5	- 1,4,6	?	?	?	?	- 4,5	?
25	$\overline{DD}$		?	?	?	?	?	?	?	?	?
26	α		?	?	?	?	?	?	?	?	?

Conditions 1-6 were defined in Table 7.

(?) Indicates indeterminate

sloping only if these outputs each has a positive correlation with clearance output. (Rows 1 and 3). the conditional supply of federal funds is upward sloping and the conditional demand for CD's is downward sloping only if each one is uncorrelated with clearance output. (Rows 5 and 7). Only the conditional demand for capital and labor are negatively sloped in general (Rows 9 and 10).

In the profit maximizing model with a general form production function, the comparative static effects of a change in price of a given output or input on other outputs and inputs are indeterminate [16 ,p. 98]. With our model, however, it was shown in the two product case (Chapter Two) that when the loan rate rises, the bank will charge a higher price on the clearance output to reduce its quantity demanded. In other words it switches from X_1 to X_2. This result does not carry over to the present three output case. Here $\overline{X}_1$ rises with $\overline{X}_2$ (or $\overline{X}_3$) as the latter's price is expected to increase (Rows $\underline{1}$ and $\underline{3}$). It is not even clear whether the bank will switch between X_2 and X_3 when one of their prices change (Rows $\underline{1}$ and $\underline{3}$). It is possible for X_2 (X_3) to rise with $\overline{P}_3$ ($\overline{P}_2$) at a given output mix and to fall at another. This allows the possibility of both complementarity and substitution between the two outputs.

The relation between demand-deposits and CD's is shown to be one of substitutability. The substitution effect, which results from the increase in the rate paid on demand deposits, outweighs the output effect so that CD's rise with an increase in the rate paid on demand deposits. (Row 8). The same argument holds about the relationship between capital and labor inputs (Rows 9 and 10). The nature of the relationship between capital and labor in one hand and CD's on the other is indeterminate (Rows 9 and 10).

The comparative static effects of input prices on outputs are in general indeterminate [16 , p. 98]. In our model a rise in expected input prices $\overline{R}_{CD}$, R_D, P_L and P_K reduces the planned outputs of X_2 and X_3 (Rows 6 through 10). These inputs are therefore normal (or superior) in production of these two outputs.

Demand deposits are normal (or superior) inputs for X_1. The larger the rate paid on demand deposits, the lower will be the planned clearance output (Row 8). This feature has implications on regulation Q. Since the clearance output $\overline{X}_1$ and its price (service charges) move in the opposite directions, the above result indicates that service charges and the rate paid on demand deposits have a positive relationship, i.e., $\partial P_1/\partial R_D > 0$. As explained in Chapter Two, this means that regulation Q, which lowers the rate paid on demand deposits to zero, artificially reduces the service charges, leads to over utilization of the clearance output by consumers, and an allocation of resources which is not socially optimal. Capital and labor are shown to be inferior inputs of X_1 (Rows 9 through 10).

The federal funds market in the model can be considered as a discount window at which the bank can borrow and lend at the same rate. The federal funds rate plays the role of the discount rate, or equivalently, the interest rate paid on cash reserves. Such a mechanism has actually been suggested by Tobin [42]. Tobin suggests that, this mechanism can be a strong monetary policy instrument for the Fed. An increase in the rate paid on reserves would be a restrictive monetary policy and a decrease would be expansionary. Our model confirms Tobin's view. With an increase in the federal funds rate the bank lowers its scale of operation. It plans to buy less CD's, to lend less, and to invest less funds. It would rechannel the funds to interest bearing reserves (federal funds) (Row 5).

The effect of uncertainty can be evaluated through the comparative static results of variances and covariances. Increased uncertainty in price of each output X_2 or X_3, as measured by price variabilities, makes the bank switch to the other output (Rows 11 and 12). The bank will plan to buy less CD's and hire fewer units of capital and labor (Rows 11 and 12). In brief, uncertainty reduces the bank's scale of operation.

The effect of federal funds rate variability depends on whether the bank is a seller or a buyer in the federal funds market. We define a seller (buyer) bank as one whose average federal funds holding is positive (negative), i.e. although it may both buy and sell funds its sales exceed (fall short of) its purchases. The model indicates that when federal funds rate becomes more uncertain, a seller plans to sell less federal funds and allocate more funds to production of its outputs (Row 13). A buyer on the other hand will plan to buy less federal funds and to reduce its outputs (Row 14). In other words, uncertainty in the federal funds rate redistributes outputs from buyer to seller banks. [Note: the model was developed in terms of a seller bank, so that $\overline{FF}$ is the average federal funds sold. Then for a buyer bank $\partial\overline{FF}/\partial V(RF) > 0$ indicates that a larger sale (and therefore a smaller purchase) will correspond to greater variability in the federal funds rate].

Variability in the CD rate reduces the bank's demand for CD's. The bank will also restrict its loans and investments (Row 15). Variability in the clearance output $V(X_1)$ includes both variations in demand deposits and their turn-over rate. This is evidenced by the demand function for the clearance output $X_1 = a_1DD + b_1p_1 + E_1$. More uncertainty in either of the two components lowers all the three outputs. When the availability of funds grows more uncertain, that is variance of demand deposits rises, the banker becomes cautious in order to avoid fund insufficiencies. The bank plans to buy more CD's, produce less, and hold more of its funds in federal funds market (Row 16).

As explained in Chapter Two, μ is the equilibrium contribution of each dollar of unencumbered funds to certainty equivalent profit. The model indicates that this contribution declines as uncertainty grows. (Rows 11 through 14, and 16).

Increased covariation between the clearance output in one hand and loans and investments on the other induces the bank to plan larger quantities of all outputs. As the clearance output X_1 is related to demand deposits through the demand function $X_1 = a_1 DD + b_1 p_1 + E_1$, the covariances between X_1 and X_i $(i = 2,3)$ can be written as $C_{OV}(X_1, X_i) = a_1 C_{OV}(DD, X_i) + C_{OV}(E_1, X_i)$ $i = 2,3$ so that $C_{OV}(X_1, X_i)$ mainly represents $C_{OV}(DD, X_i)$. This relation can be used to explain the above comparative static result. As covariation between demand deposits and loans increases, the bank becomes more confident that deviations of loans above their planned level can be covered by simultaneous covariations in demand deposits, therefore it plans to buy less CD's and make more loans in the same time. The same argument holds about investments.

The reserve requirement instrument of monetary policy is shown to work in the intended direction. Larger reserve requirements on demand deposits or CD's will lower the planned outputs of loans and investments. This effect is similar to that of a decline in the net worth. The effect of changes in reserve requirements on CD transactions is not clear. The comparative static results for expected demand deposits and risk aversion coefficient are indeterminate.

Contrasting these results with those of the preceding chapters, it may be concluded that as the model extends to include more inputs and outputs, the results become less determinate. This is because when more alternatives are available the choice of each alternative is less likely. Notice in the extended model results lose their generality and hold only over certain ranges of variations as specified in Table 7. This indicates that it is quite possible for two variables to have a certain relationship over one range of variations and a different relationship over another.

In the next chapter, the model will be adapted to estimation and the sources of data will be described.

CHAPTER FIVE

THE ESTIMATION PROCEDURES

5-1 System of Reduced Form Equations

To empirically test the model, either the first order conditions or the system of equations displayed in Table 6 may be used. One problem with estimating the model is that the data on all the variables are not available. The best that can be done is to proxy some of these variables by other variables or, if they are endogenous, eliminate the corresponding equations.

If the first order conditions are to be estimated, a linearization of the system is necessary to allow the use of the ordinary linear techniques. Here, the first order conditions are approximated by a Taylor series up to the first component. The resulting linear system may now be used to obtain a reduced form in terms of the level of the variables. The reduced form will in general represent each of the endogenous variables as a function of all the exogenous variables plus an error term. The error term in each equation is a linear function of all the remainder terms in the Taylor series approximation. Since the Lagrangian multipliers cannot be measured and the data on capital and labor inputs are not available, the equations describing these variables are eliminated from the system.

The system of equations displayed in Table 6 may also be used to derive a set of reduced form equations in

terms of first differences. The system may be written as $Adz = b$, where dz is the vector of first differences of endogenous variables, A, when evaluated around some initial condition, is the matrix of the coefficients, and b, when evaluated around some initial condition, is a vector of constants. The rows in b are linear functions of the first differences of the exogenous variables. To get the reduced form, the system is alternatively written as $dz = A^{-1}b = C$ and partitioned to two parts:

$$\begin{pmatrix} dz_1 \\ dz_2 \end{pmatrix} = \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$$

dz_1 includes the endogeneous variables on which the data is available and dz_2 includes the remaining variables. $dz_1 = C_1$ is a system of linear equations which can be adapted to estimation.

5-1 Planned and Expected Values

The dependent variables in the reduced form equations are the planned or desired values of the endogenous variables. These values cannot be directly measured. To get the system in terms of actual values, one way is to assume that the planned values ($\bar{z}$) differ from the actual values (z) by a random term (E). $z = \bar{z} + E$. In this case the random term in each equation will be added to the disturbance term. A second way is to assume that the actual

values adjust to the desired values by a partial adjustment mechanism [23 , p. 476].

$$Z_t - Z_{t-1} = \gamma(\overline{Z}_t - Z_{t-1}) + U \qquad 0 < \gamma \leq 1$$

or

$$\overline{Z}_t = \frac{1}{\gamma} Z_t + \frac{\gamma-1}{\gamma} Z_{t-1} - \frac{1}{\gamma} U$$

In this case, after substitution in each equation for the desired value in terms of the actual value, the lagged value of the dependent variable will be added to the set of regressors and the error term U will be added to the disturbance term. The presence of the own lagged value allows the own adjustment coefficient (the coefficient of the lagged value) to be estimated. The partial adjustment hypothesis requires this coefficient to be between zero and unity. $0 < 1 - \gamma < 1$.

The set of regressors in the system includes the expecged values of interest rates and demand deposits. Here it is assumed that the bank uses Box-Jenkins time series models to forecast or measure these expected values. In the next chapter, such models are developed for demand deposits, the prime loan rate, the treasury bill rate, the CD rate, and the federal funds rate. These models are used to generate data on the corresponding variables over the sample period.

5-3 The Disturbance Term Properties

Since all the non-linearities in the model (remainder terms in the Taylor series approximation), and also the random terms in adjustment of actual to desired values, are relegated to the disturbance terms, there is a high probability that the effect of the omitted variables (variables relegated to the disturbance term) will carry over into subsequent periods and lead to serial correlation. The use of monthly time series data increases this probability. The disturbance terms therefore may not be spherical.

Here, it is assumed that the disturbance term for each equation in the reduced form system has the following properties:

(a) it is normally distributed

(b) it has a zero mean

(c) it is homoskedastic $U_{it} \sim N(0, \sigma_i^2)$

(d) if the data show that serial correlation is present, it is assumed that the disturbance term follows a first order autoregressive scheme.

$$U_{it} = \rho_i U_{it-1} + V_{it} \qquad |\rho| < 1$$

where V_{it} is spherical, that is:

$$V_{it} \sim N(0, \sigma_i^2 V)$$

$$E(V_{it}, V_{it-1}) = 0$$

Besides, after removing the serial correlation the disturbance terms might be correlated across the equations so that $E(V_{it}, V_{jt}) = \sigma_{ij}$ is non-zero. If this condition prevails, we have a case of "seemingly uncorrelated" regressions.

It is well known [23 , p. 520] that in this case the generalized least squares approach, which takes into account the correlations across equations, is more efficient than the ordinary least squares. The gain in efficiency, however, is zero for the present system since the regressors of all the equations are the same [40 , p. 309].

5-4 The Data

A combination of cross-section and time series observations on a group of homogenous individual banks may be considered as an ideal data set for the model. Since the model includes four simultaneous interest rates as regressors, a reasonably large number of observations might be necessary to reduce the degree of multicollinearity. Measurement of expectations, variances, and covariances, as conceived by the bankers, require formulation of moment generating mechanisms which generate these moments over time. Developing such mechanisms is an art rather than a science.

The data used in this study were drawn from a survey of weekly reporting large commercial banks in New York City. The sample period for the estimation starts at June, 1969. For loans, investments, federal funds, and CD's the sample ends at December, 1976. Since some of the items were available only in monthly figures, the weekly figures during each month were averaged to construct monthly series for the rest of the items.

The variables in the model were measured as follow:

demand deposits (DD): total demand deposits adjusted to exclude U.S. government demand deposits, domestic commercial banks demand deposits, and cash items in process of collection.

loans (X_2): total gross loans, adjusted to exclude loans and federal funds transactions with domestic commercial banks. The adjective "gross" indicates that reserves for loans losses have not been deducted from total loans.

investments (X_3): total security investments

CD's: large negotiable time certificate of deposits. These are the certificates of deposits issues at demoninations of $100,000 or more.

federal funds (FF): net federal funds sold = federal funds sold - federal funds purchased

debits (X_1): data on debits are not available for the banks under consideration, but can be measured as the product of demand deposits and their turn over rate. The turn over rate for the banks were proxied by monthly turn over rate of adjusted demand deposits in New York City.

Net Worth (WO): total capital account.

reserve requirement ratio: the ratio of the reserves held by the banks over total time and demand deposits.

The above items were published in:

The Federal Reserve Bulletin.

loan rate (P_2): the prime loan rate.

security rate (P_3): monthly rate on three months treasury bill, rate on new issues.

CD rate (R_{CD}): monthly rate on three months CD's in the secondary market. The rate in the primary market was not available for the whole period.

federal funds rate (R_F): monthly effective rate on federal funds.

rate on demand deposits (R_D): R_D was set to zero

The above items were published in:

Banking Monetary Statistics, 1941-1970 [Tables 12.5, 12.6, 12.7].

Annual Statical Digest, 1971-1975 [Tables 24,26].

Annual Statical Digest, 1972-1976 [Tables 22,24].

Federal Reserve Bulletin [Tables A20-24, A29].

wage rate (P_L): gross-earning of production or non-supervisory workers in banking (average hourly earnings)

Published in:

Employment and Earning, U.S. Department of Labor, Bureau of Labor Statistics.

price of capital (P_K): the wholesale price for producers' durable goods.

Published by:

Survey of Current Business, (p. S.8).

Data on some other variables do not exist and have to be generated. Expected values, variances, and covariances are of this category. Data on expected values are generated by the time series models developed in the following chapter. Each variance is measured as the sum of the squared deviations of the variable from its mean over the preceding four months. Each covariance is in the same way measured as the sum of products of deviations of the two variables from the corresponding means over the preceding four months.

CHAPTER SIX

TIME SERIES FORECASTING

6-1 Introduction

The purpose for this chapter is twofold. First to develop time series models to forecast demand deposits, the prime loan rate, the 90-day treasury bill rate, the federal funds rate, and the CD rate. Second, to use these models to generate data on expected values of these variables, which can be used in the next chapter for empirical estimation.

A time series is a set of observations generated sequentially over time. Each time series is a realization of a specific stochastic process. A stochastic process is said to be stationary if its stochastic features are invariant with respect to displacements in time. Most of the time series encountered in economics are nonstationary; however stationarity can usually be induced, by suitable differencing of these series [32].

A time series model, in its most general form, can be written as an integrated autoregressive moving average process ARIMA (P, d, q):

$$\phi(B)(1-B)^{d}Z_t = \theta(B)E_t$$

where $\phi(B)$ and $\theta(B)$ are autoregressive (AR) and moving average (MA) operators respectively, B is the backward shift

operator, i.e. $B^k Z_t = Z_{t-k}$, d is the order of homogeneity, and E_t is a white noise residual. An AR operator of order P, and a MA operator of order q, can be expressed as follows:

$$\psi(B) = 1 - \psi_1 B - \psi_2 B^2 \; . \; . \; . \; . \; -\psi_m B^m$$

$$\psi = \phi, \theta$$

$$m = p, q$$

The general ARIMA process subsumes the pure AR scheme when d = 0 and $\theta(B) = 1$, and the pure MA scheme when d = 0 and $\phi(B) = 1$. The mixed autoregressive moving average process ARMA is subsumed when the condition d = 0 is satisfied. If this condition is violated, the pure schemes will be replaced by integrated ones.

6-2 Time Series Model Building

Time series modelling is an iterative process. A model is tentatively specified and estimated. The estimated model is then subjected to diagnostic checks. If the model is found to fit the data adequately, it can be used for forecasting purposes, otherwise the process is repeated, until a satisfactory model is found.

6-2-1 Identification

In the identification or specification stage, the purpose is to determine the orders P, d, and q in the general

ARIMA (P, d, q) process. The main tools in this stage are the autocorrelation and partial autocorrelation functions. A feature of stationary series is that their autocorrelation functions die out rapidly, while those of non-stationary series remain high. It is also known that autocorrelation and partial autocorrelation of each class of time series have unique patterns. For an AR(P) model, the autocorrelation function tails off, while the partial autocorrelation function cuts at lag P. On the other hand, for a MA(q) model, the autocorrelation function cuts at lag q, while the partial autocorrelation function tails off. For mixed processes, both functions tail off and dampen geometrically [6 , p. 79]. These features can be used to determine the parameters P, d, and q.

The time series models here, are constructed using monthly data on the prime loan rate, the treasury bill rate (rate on new issues of three months bills), the federal funds rate (effective rate), the CD rate, and demand deposits, for large weekly reporting banks in New York City. The sources of data were described in the last chapter. Non-seasonally-adjusted data are used for all the models to avoid the effect of such adjustments on the structural form of the models.

The data on the prime loan rate are available since 1949, at which time the sample period for this variable starts. For the treasury bill rate, however, the starting point is chosen to be 1955 to avoid the possible change in

behavioral structure of the rate due to the Fed-Treasury accord. The sample chosen for the federal funds rate starts from 1954. It should be remembered that federal fund activity increased substantially in 1960s and the behavioral structure of this rate might not have remained unchanged during the sample period. The data on the CD rate are available beginning in 1964 and the sample starts at that time. The data for demand deposits start at April 1961. The classification of banks as "large weekly reporting banks of New York City" has been revised different times. The last revisions occurred in April 1961 and May 1969. Consequently, the sample period was chosen to begin with the first revision and end with the second.

6-2-1-1.Seasonality

The autocorrelation function for the demand deposit series exhibits a substantial amount of seasonality. This does not seem to be the case for other series. This seasonality effect on demand deposits is eliminated by annual differencing.

6-2-1-2.The Order of Homogeneity

The fact that the autocorrelation functions for the series do not quickly die suggests that series are non-stationary. The autocorrelation function for first differenced series, however, do typically drop off fairly rapidly, indicating that first differencing the data does indeed generate a stationary time series.

6-2-1-3. Autoregressive and Moving Average Orders

One common feature among the series under consideration is that the autocorrelation of these series lack qualities that would lead to a definite clear-cut specification. These autocorrelations have in fact few clear-cut patterns. This makes the specification difficult and imprecise. To determine the orders P and q for a series, it is necessary to know whether the autocorrelation function cuts off at a certain lag. This information can be obtained by testing for the significance of the estimated autocorrelation. On the hypothesis that theoretical autocorrelation is zero, the estimated autocorrelation coefficient, when divided by its standard error, is approximately distributed as a standard normal variate [6 , p. 178]. Therefore, if this ratio has a value of two or larger, the theoretical autocorrelation coefficient has a probability of 95% to be non-zero. This approach provides a guide in finding the significant autocorrelation and consequently the AR and MA orders of the process. The orders adopted for our series, by using this approach, are given in Table 9.

As shown in the table, the models corresponding to the loan rate, P_2, the federal funds rate, R_F, and the CD rate, R_{CD}, are represented by integrated AR processes. After differencing, each model has only one AR factor. The loan rate model includes two terms of order 1 and 3. This indicates that, the change in this rate can be expressed, by

a weighted average of its past values during last month and three months ago. The AR processes expressing the changes in the CD rate and the federal funds rate have single terms of order 1 and 3, respectively. The treasury bill model follows an ARIMA process. This process makes the first difference in the bill rate a function of its past value during three months ago and the shock which occurred six months ago. The demand deposit series, after annual differencing for seasonal adjustment, follows a MA scheme which makes it a function of shocks occurring last month and last year. Parsimonious as they may be, each model passes the Q test for white noise. This test is discussed in detail below.

6-2-2 Estimation

Time series models can be estimated by a conditional maximum likelihood approach. If the disturbance terms are independent normal, have a zero mean, and a constant variance, the estimates will be the same as conditional least squares estimates. The identification process provides a set of preliminary estimates for the parameters of the models. These estimates are used as initial values in the iteration process. For our models, in all cases the estimates seemed to converge rapidly following one or two iterations. With the final estimates, the models can be written as in Table 9.

TABLE 9

BOX-JENKINS MODELS

Series	n		Fitted Form	Q	D·F	S·E
P_2	245	ARI (3,1)	$(1-.34B-.2B^3)(1-B)P_{2t} = E_t$	32.3	28	.0977
P_3	197	ARIMA (1,1,6)	$(1-.35B)(1-B)P_{3t} = (1-.11B^6)E_t$	37.8	28	.2035
R_F	173	ARI (3,1)	$(1-.36B^3)(1-B)R_{Ft} = E_t$	24.0	29	.2932
R_{CD}	65	ARI (1,1)	$(1-.52B)(1-B)\ R_{CDt} = E_t$	29.3	29	.1481
DD	96	IMA (1,12)	$(1-B^{12})\ (1-B)\ DD_t = (1-.28B-.71B^{12})E_t$	17.8	28	259.63

n = number of observations

$Q = n \sum_{k=1}^{k} r_K^2$ $\quad r_K$ = residual's correlation of lag k

D·F = degrees of freedom

S·E = standard error of residuals

6-2-3 Diagnostic Checking

Diagnostic checks are used to determine the adequacy of the models; they are tests of "goodness of fit." One way to investigate the model adequacy is to study the residuals of estimation. Since we have assumed that the disturbance terms are distributed normally and independently, then, if the model is specified correctly, we should expect the residuals to be nearly uncorrelated with each other. If this is true, the sample autocorrelations of these residuals are expected to be zero or insignificant. This argument is the basis of the portmanteau lack of fit test. In this approach the significance of the first k autocorrelations of the estimated residuals is tested. Under the null hypothesis of independence between residuals, it can be shown [32, p. 491] that the estimated autocorrelations of lag k of the residuals (K = 1,2....K) are independent normal, have a zero mean, and a constant variance 1/n. Following this, if the autocorrelation of lag K is represented by r_k, the variate $Q = \sum_{k=1}^{K} r_k^2$ will be approximately distributed as χ^2_{k-p-q} where n is the number of observations on the stationary series and p and q are autoregressive and moving average orders respectively. On the other hand if the residuals are correlated, r_k's are large, Q exceeds the value of χ^2_{k-p-q}, and the model is inappropriate. For our models Q values are given in Table 9. These values indicate that the models do pass the portmanteau test of fit.

6-2-4 Forecasting

The models that have passed the diagnostic checks may be used to make forecasts. The models developed in the last section were used to make monthly forecasts over the period June 1969-December 1978. To make forecasts for each successive period, a new actual observation was fed into the model, the forecasts, therefore, have an adaptive nature. The forecasting performance of the models is difficult to evaluate, except other models are available to which these models can be contrasted. The sample periods for the forecasts is one of extraordinary fluctuations in interest rates. The forecasting performances of the corresponding models should therefore be judged taking this point into account.

One point should be emphasized: the forecastability of the model can be properly judged only if it is used to forecast beyond the estimation data period. If the same set of data is used to estimate the model, and to make the forecast, the performance of the forecasts does not denote the adequacy of the model. Such an approach has been used by Hester and Pierce [17 , Chapter 8].

6-3 Accuracy Analysis of Postsample Forecasting

A perfect forecast series is one that is identical with the actual series. Performance of forecasting models are therefore compared to such an ideal model. Means and

standard deviations of forecast and actual (perfect forecast) series are contrasted. The correlation coefficient between the two is used as the basis of comparison, or some error measures are adopted to indicate the degree of forecasting imperfection.

For the models in Table 9, these measures are displayed in Table 10. The means and standard deviations of forecast series deviate from their actual counterparts by very narrow margins. The correlation coefficient between actual and forecast series, and the regression coefficient of actual on forecast series are generally larger than .94. The error measures are meaningful, when compared with a particular realization, or the mean of the corresponding series. The mean square error (MSE) may be decomposed into three components as displayed in Table 10 [39].

The results reported in Table 10 indicate that the smallest mean square error, mean absolute error, or mean error do not necessarily correspond to series with least variations. Some series may have larger variations, and still be forecasted with more precision than others. Variability should be distinguished from predictability. The former should not be emphasized to the exclusion of the latter.

In addition to these measures, Theil [39] has introduced an inequality coefficient (U) between actual and forecast series which is the root mean square error, adjusted to lie between zero and unity.

TABLE 10

ACCURACY MEASURES FOR FORECASTING MODELS

MEASURES SERIES	DD	P_2	P_3	RF	R_{CD}
Mean (Actual Series)	20706.9	7.64	5.96	6.78	6.92
Mean (Forecast Series)	20684.7	7.61	5.95	6.78	6.90
Standard Deviation (Actual Series)	3456.59	1.76	1.41	2.19	2.01
Standard Deviation (Forecast Series)	3453.40	1.77	1.41	2.26	2.06
Correlation Coefficient Between Actual and Forecast Series	.9818	.9854	.9529	.9742	.9662
Regression Coefficient of Actual on Forecast Series	.9827	.9775	.9551	.9461	.9431
Mean Error	22.22	.0259	.0098	.0020	.0166
Mean Absolute Error	530.9	.2123	.3337	.3630	.3933
Root Mean Square Error	658.9	.3032	.4342	.5110	.5332
Mean Square Error	434149	.0919	.1885	.2611	.2843
UM - $(\overline{F} - \overline{A})^2$	492.84	.0009	.0001	.0000	.0004
US = $(S_F - S_A)^2$	10.17	.0001	.0000	.0049	.0025
UC = $2(1-r) S_F S_A$	434506	.0909	.1872	.2553	.2799

Where $\overline{F}$, $\overline{A}$, S_F, S_A are means and standard deviations of forecasts and actual, and r is their correlation coefficient.

$$U = \frac{\sqrt{1/n \; \Sigma \; (F_i - A_i)^2}}{\sqrt{1/n \; \Sigma \; F_i^2} + \sqrt{1/n \; \Sigma \; A_i^2}}$$

where F_i and A_i represent forecast and actual values. Three measures U^m, U^s, and U^c are developed, using this coefficient, to represent the fractions of error due to unequal central tendency, to unequal variation, and to imperfect correlation between actual and forecast series. These measures may be expressed as:

$$U^m = (\overline{F} - \overline{A})^2 / MSE$$

$$U^s = (SF - SA)^2 / MSE$$

$$U^c = 2(1-r) \; SF \; SA/MSE$$

where $\overline{F}$, $\overline{A}$, SF, and SA are means and standard deviations of forecast and actual series, and <u>r</u> is their correlation coefficient. The size of the inequality coefficient and the fraction of error due to bias, to different variation, and to different covariation for our models are displayed in Table 11. The inequality coefficient has its lowest value for the demand deposits model, but it is small for all models. With the minute size of the fractions of error due to bias, it can be concluded that the models produce unbiased forecasts. Since the fraction of error due to different variations is also small, it can be said that the distribution of actual and forecast series are quite similar at least up to the second moment. The fraction of error due to

TABLE 11

THEIL'S ACCURACY MEASURES

MEASURES/SERIES	DD	P_2	P_3	R_F	R_{CD}
Theil's Inequality Coefficient	.0157	.0193	.0354	.0357	.0369
Fraction of Error Due to Bias (U^m)	.0011	.0073	.0005	.00001	.0009
Fraction of Error Due to Different Variation (U^s)	.00002	.0022	.00005	.0163	.0085
Fraction of Error Due to Different Covariation (U^c)	.9988	.9904	.9994	.9837	.9905
Alternative Decomposition:					
Fraction of Error Due to Difference of Regression Coefficient from Unity	.00818	.0174	.0213	.0569	.0485
Fraction of Error Due to Residual Variance	.9907	.9753	.9782	.9430	.9505

different covariations is large as desired. These values should be compared with the ideal case of $U^m = U^s = 0$, $U^c = 1$, which represents perfect forecasting.

In an alternative decomposition, the error is divided to two fractions due to difference of regression coefficient of actual on forecast from unity, and the other due to our residual variance. The size of these fractions for our models indicate a satisfactory performance.

Plots of actual versus forecast series, may be used for visual inspection of the forecasting performance. Plots of actual and forecast series for demand deposits, the Prime loan rate, the treasury bill rate, the CD rate, and the federal funds rate, are given in Appendix 6. These plots show that time series forecasts do capture the actual trends, however, they do not always predict the sharp turns.

The plots for mid 1973-mid 1975 are especially interesting. All the interest rates show erratic behavior, and wild month-to-month gyrations. An inquiry into the data reveals, that within 1973-1975 period the treasury bill rate rose more than 60 percent, while the loan rate, the federal funds rate, and the CD rate, more than doubled. Month to month variations in the interest rates go as high as two full percentages (federal funds rate June to July 1973), and a full one percent rise or fall is frequent. It should be remembered that this wild gyration is just mirroring the money supply during its most erratic course since the creation of the Federal Reserve System. Money supply data show that during 1973, money supply (M_1) sometimes grew at an annual rate of 15 percent (May and June, 1973), and sometimes even declined in absolute value (August and September, 1973). Whether we consider these conditions an exception to the course of conducting monetary policy or not, a time series forecasting model should not, by its nature, be expected to perform too well under these circumstances.

In general the performance of the models indicate that demand deposits and interest rates are satisfactorily predictable, although loan rate and treasury bill rate are less volatile and more precisely predictable than the CD rate and the federal funds rate. The forecast precision of all the models could most likely be improved if time series and structural models were combined, and/or multivariate time series techniques were utilized. In the next chapter the forecast values generated by these models, are used as data for the estimation of the bank behavior model.

CHAPTER SEVEN

THE EMPIRICAL RESULTS

In previous chapters a model of bank behavior was developed and some comparative static results were derived. In this chapter the magnitudes of these results will be examined. In Chapter Five the model was linearized, the adjustment mechanisms between actual and desired values were hypothesized, and the properties of the disturbance terms were specified. In the last chapter time series models were developed to generate data on expected values of demand deposits and interest rates to be used in empirical estimation reported in this chapter.

After eliminating the equations describint λ, μ, K, and L, on which data are not available, there remains a system of five equations. The dependent variables are the loans, the investment securities, the CD's, the net federal funds sold, and the debits. The regressors for all the equations are the same except for the lagged values of the dependent variables which appear due to the partial adjustment hypothesis. The remaining regressors include expected demand deposits ($\overline{DD}$), the expected prime loan rate, the expected treasury bill rate, the expected CD rate, the variance of demand deposits, the variance of foretold interest rates, the covariances between demand deposits in one hand and loans, securities, net federal funds sold, and CD's on the other, the reserve requirement ratio, the

initial net worth, and the prices of capital and labor inputs. Besides, eleven monthly dummy variables are added to the set of regressors to take account of seasonality.

To find the appropriate estimation technique, ordinary least squares was used to run a preliminary set of regressions on the levels of the variables. The results indicated that substantial serial correlation was present. Therefore a first order autoregressive scheme was hypothesized and the standard Cochrane-Orcut technique was implemented. The results of this set of regressions revealed that the serial correlation paramater ρ was close to unity. Hence, the estimation of the first difference version of the system seemed to be appropriate. The system was transformed into first differences (except for seasonal dummies), and each equation was estimated by ordinary least squares. The estimates of the behavioral coefficients, and the corresponding t ratios are displayed in Table 12. Besides tests of the significance of regressors, these results shed light on the actual, as opposed to predicted, behavior of banks.

Clearly the empirical signs displayed in Table 12 are not all the same as those suggested by comparative static results represented in Table 8. But it should be remembered that the comparative static signs were determinate only under the conditions specified for each result in the latter table, so that they could have different signs under other conditions. Therefore, although the empirical and

TABLE 12

THE EMPIRICAL RESULTS

Regressors	Regressands DX_2	DX_3	DCD	DFF	DX_1
C	1194.79 (3.02)****	671.15 (2.77)****	-85.94 (-.24)	238.80 (.46)	78728.6 (4.10)****
DZLAG	.2785 (2.42)****	-.1142 (-1.01)	.3620 (3.13)****	-.39 (-4.01)****	-.5505 (-3.93)****
$D\overline{DD}$	-.1521 (-.96)	-.1294 (-1.40)*	.1384 (1.03)	-.27 (-1.38)*	3.34 (.44)
$D\overline{P}_2$	170.40 (.52)	30.28 (.15)	-362.94 (-1.23)	-420.09 (-.95)	-7646.99 (-.51)
$D\overline{P}_3$	319.76 (1.44)*	-56.33 (-.41)	-42.52 (-.21)	55.90 (.19)	7667.23 (.76)
$D\overline{R}_F$	295.41 (1.25)	-8.33 (-.05)	334.43 (1.64)**	-39.88 (-.13)	2666.45 (.25)
$D\overline{R}_C$	-234.50 (-1.31)*	-111.08 (-1.02)	220.75 (1.42)*	769.38 (3.32)****	4879.63 (.57)
DV(DD)	-.0005 (-1.34)*	-.0004 (-1.84)**	-.0001 (-.48)	.0003 (.70)	-.1302 (-.06)
$DV(P_2)$	-179.57 (-.25)	-505.95 (-1.19)	433.18 (.68)	634.72 (.68)	-4328.12 (-.13)

TABLE 12 (cont'd)

Regressors	Regressands DX_2	DX_3	DCD	DFF	DX_1
$DV(P_3)$	-652.65 (-.54)	105.22 (-.14)	1570.16 (1.50)	2370.22 (1.52)*	33850.1 (.61)
$DV(R_F)$	691.54 (1.22)	-267.86 (-.79)	362.27 (.73)	-554.73 (-.75)	-12450.7 (-.48)
$DV(R_C)$	-248.30 (-.41)	100.48 (.27)	-516.55 (-.98)	-100.89 (-.12)	-4583.11 (-.15)
$DC_{OV}(DD,X_2)$	.0005 (1.36)*	.0006 (2.83)****	.00007 (.24)	-.0011 (-2.45)****	.0012 (.06)
$DC_{OV}(DD,X_3)$	.0006 (.76)	.0010 (2.18)***	.0001 (.21)	-.0017 (-1.70)**	-.0043 (-.09)
$DC_{OV}(DD,FF)$	.0008 (1.75)**	.0007 (2.49)****	.000001 (.003)	-.0014 (-2.24)***	-.0297 (-.99)
$DC_{OV}(DD,CD)$	-.0008 (-2.12)***	-.0005 (-2.28)***	.00002 (.07)	.0006 (1.34)*	.0147 (.71)
DRR	-21268.1 (-1.48)*	-1019.75 (-.11)	-7652.30 (.59)	-16655 (-.88)	-292179 (.30)
DW_O	.4008 (.65)	-.1665 (-.47)	-.3071 (-.55)	-.2894 (-.36)	43.53 (1.64)**

TABLE 12

Regressors	Regressands DX_2	DX_3	DCD	DFF	DX_1
DP_L	-6448.07 (-1.28)*	4235.62 (1.29)*	-534.99 (-.12)	-9238.66 (-1.41)*	-610580 (-2.25)***
DP_K	284.14 (2.57)****	-66.69 (-1.02)	176.47 (1.85)**	102.19 (.73)	10115.4 (2.05)***
JAN	-2092.37 (-3.28)****	-1045.72 (-2.79)****	-114.62 (-.21)	1016.22 (1.24)	-53536.4 (-1.70)**
FEB	-1507.83 (-2.51)****	-1360.76 (-3.73)****	169.68 (.31)	-16.25 (-.02)	-126746 (-4.31)****
MAR	-672.29 (-1.50)*	-485.71 (-1.65)**	127.94 (.32)	-433.72 (-.73)	-38924 (-1.83)**
APR	-1562.48 (-3.46)****	-363.34 (-1.34)*	-207.59 (-.56)	-609.72 (-1.04)	-49765.4 (-2.02)***
MAY	-1593.68 (-2.60)****	-794.02 (-2.15)***	441.94 (.79)	-091.59 (-1.12)	-111525 (-3.91)****
JUNE	-907.61 (-1.95)**	-312.43 (-1.10)	-260.52 (-.64)	-10.50 (-.01)	-72924 (-3.38)****
JULY	-694.96 (-1.38)*	-627.79 (-2.07)***	450.37 (.97)	-330.46 (-.50)	-78083 (-3.19)****

TABLE 12

	Regressands				
Regressors	DX_2	DX_3	DCD	DFF	DX_1
AUG	1676.9 (-3.37)****	-485.05 (-1.60)*	9.53 (.021)	-780.49 (-1.21)	-101068 (-4.15)****
SEP	-735.19 (-1.58)*	-329.54 (-1.17)	-11.26 (-.12)	-558.79 (-.92)	-61605 (-2.66)****
OCT	-1465.24 (-2.76)****	-787.80 (-2.49)****	246.93 (.51)	-431.05 (-.63)	-53047 (-2.08)***
NOV	-783.74 (-1.57)*	-181.68 (-.60)	368.63 (.85)	-906.47 (-1.42)*	-98341 (-4.05)****
R^2	.56	.49	.41	.48	.76
D·W	2.15	1.91	2.05	2.09	2.19
F	3.40	2.58	1.88	2.47	6.14

* significant at 10% level, one-tailed test

** significant at 5% level, one-tailed test

*** significant at 2.5% test, one-tailed test

**** significant at 1% level, one-tailed test

the displayed comparative static signs are not always identical, this does not refute the model.

Some of the empirical results are worthy of interpretation. The significance of the lagged values of the endogeneous variables indicates the possibility of a lock-in effect. The bank may find it difficult to alter the magnitude of the loans, CD's, federal funds, and clearance output as much as it desires by one period. This does not seem to be the case however (as might be expected) for government securities. The assumption of partial adjustment behavior seems therefore appropriate, at least for the former variables. Surprisingly, expected demand deposits are not a "significant" determinant of loans. The comparative static results of changes in demand deposits were indeterminate, but the empirical result seems to be in accordance with the Hester and Pierce result that demand deposits are allocated to loans only after a lag [17 , Chapter 9].

The variance of demand deposits, which may be taken to represent deposit uncertainty, is shown to reduce the loan output as expected, while the effect of variations in interest rates are insignificant. The effects of covariances, whenever significant, are in the directions predicted by the model. The significance of variance of demand deposits and significance, at least in some regressions, of covariances between demand deposits in one hand and loans, securities, CD's, and federal funds on the other lend support to the risk-aversion hypothesis as opposed to risk-neutrality

implicit in the models which maximize expected profits. The latter models may therefore be considered misleading.

The reserve requirement instrument of monetary policy is shown to have the predicted restrictive impact on loans. The initial net worth has a significant effect only on the clearance output.

One interesting result is the significance of prices of capital and labor on both the clearance and loan outputs. Labor is found to be a normal input for both outputs since its coefficients have a negative signs in both regressions. Capital is an inferior input for both outputs. Besides, it behaves as a substitute for CD's. The coefficients of the capital input have positive signs in both loan and clearance output regressions. Positive signs indicate that with an increase in the price of capital, outputs will rise. This result about loans makes sense if it can be argued that at higher capital prices the banker transfers some of the cash assets to loans in order to increase its earning. The significance of the coefficients of capital and labor prices supports the Pesek point of view that operating costs do have an effect on a bank's decision making. The dummy coefficients indicate substantial seasonality in loans and clearance outputs. The seasonality in investment securities is small, and CD's and federal funds hardly exhibit any seasonality effects.

In general the model estimated for the large weekly reporting banks in New York City does not show a good

empirical performance. Several explanations may be provided. It should be recalled that estimation did not take place under appropriate conditions. The model was developed in terms of an individual bank, while the data refers to a group of banks which are not even homogenous. By using aggregate data, it is implicitly assumed that all the individual banks involved have the same objective functions, same constraints, identical behavioral parameters, and identical expectations about the future, while there is indeed some evidence to suggest the contrary [2 , p. 219].

Besides, the model is represented by a system of non-linear equations subject to some behavioral constraints. The estimated unconstrained linear version may simply be oversimplified and inaccurate. Multicollinearity between regressors may also lead to insignificance of the coefficients. To investigate the effect of multicollenearity between the first differences of interest rates included in the model, two of the interest rates and the corresponding variances were dropped from the set of regressors. The R^2 delete was close to total R^2 as in the presence of multicollinearity, but the regression results did not substantially change. It was concluded that the adverse effect of multicollinearity is slight. One final explanation may be that the static single period theory can not satisfactorily explain the multiperiod dynamic behavior of banks.

CHAPTER EIGHT

CONCLUSION

8-1 Results and Policy Implications

In this study a model of bank behavior under uncertainty was developed. The bank was considered as a multi product firm which produces its clearance output as a direct-supplier, and loans and securities as a financial intermediary. An expected utility maximization framework was adopted, and portfolio selection, liquidity and liability management, risk-aversion, jointness in outputs, and the bank's operating costs were incorporated into the model.

The comparative static results about regulation Q in a two asset model indicate the presence of such regulation reduces the percentage of the risky asset in the portfolio, leading to an increase in liquidity. It is also shown that this regulation, by allowing the cost of the clearance output to be partially offset by interest free demand deposits, leads to a reduction in the service charges. This artificially low price for the clearance output leads in turn to over utilization of this product by demanders and a resource allocation which is not socially optimal. A trade-off may therefore be said to exist between the share of the liquid asset in portfolio, and an optimal allocation of resources. It is desirable that the policy maker chooses the optimal point on the trade-off possibility curve by choice of the

ceiling for regulation Q. The existing zero ceiling is not necessarily appropriate over time. These results about regulation Q hold true only so long as risk-aversion prevails; models based on expected profit maximization cannot therefore be expected to produce analogous results.

The comparative static results also indicate the possibility of both complementarity and substitution between outputs. More specifically, the model shows that two outputs can be complements (substitutes) for a certain range of their variations, while possibly having a different relationship over another range or for another output mix. Uncertainty, as measured by variances, was found to lead to both a reduction in outputs, and a change in the output mix. This feature should be considered in the making of the monetary policy. Frequent and unusually large changes in monetary policy instruments might increase uncertainty and consequently strengthen or partially offset the policy's effect.

Reserve credit (reserve requirement) on assets, and payment of interest on excess reserves were investigated as potential monetary policy instruments. In a two asset model it was shown that asset reserve credit on one asset will lead the bank to increase that asset's share in its portfolio. This instrument can therefore be used to favor or disfavor a certain category of assets. With payment of interest by the Fed on excess reserves the discount window operation is complemented to work on excess reserves as well as reserve deficiencies. The Fed's control over credit is consequently

broadened.

Lack of appropriate data does not allow a proper test of the theoretical results. However, the empirical results with aggregate data indicate that banks do display risk-aversion, and do consider the operating costs in their decision makings. It may be concluded that depending on the rate of inflation, a smaller or larger quantity of loans will be made by the banking system and consequently a smaller or larger quantity of demand deposits and money supply will be created. This effect may distort the money supply from its targeted level by the Fed. The accuracy of money supply control may therefore improve if operating costs are considered in policy making.

8-2 Limitations

It should be recalled that the model's results are based on the specific underlying framework. Exponential utility, profit normality, and linear demand for the clearance output are some of the restricting features of the model. It is very desirable to determine the bank's response directions under a general utility function, but the problem is that determinacy is often lacking even for specific utility functions. The relative determinacy of the comparative static results in the present model is felt worth the slight loss of generality.

The static, single period nature of the model is another special feature. The empirical results support the

presence of a lock-in-effect in banks' assets and liabilities. This suggests that bank behavior may embody adjustment lags in response to stochastic variations in its portfolio. These adjustments which reflect a multi period optimization behavior are necessarily omitted in a single period context. In order to capture the dynamic nature of bank behavior, multi period models which consider the time path of variations in the variables seem to be the most desirable models to be examined. As another alternative a combination of time series and structural models may be utilized to explain bank behavior. In either case, the introduction of market imperfections and interrelations in output demands would enrich the model.

APPENDICES

APPENDIX ONE

The Moments of an Incomplete Distribution (Borrowing)

For a group of banks the term free reserves (FR) may be defined as the difference between excess reserves (ER) and borrowing from the discount window (BR): FR = ER - BR. Following this definition, for an individual bank which does not hold excess reserves and borrow at the same time, free reserves equal excess reserves when positive, and the negative value of borrowing when negative. The portion of the free reserves distribution to the right of point zero refers to excess reserve holdings, and the portion to the left, the negative value of borrowing.

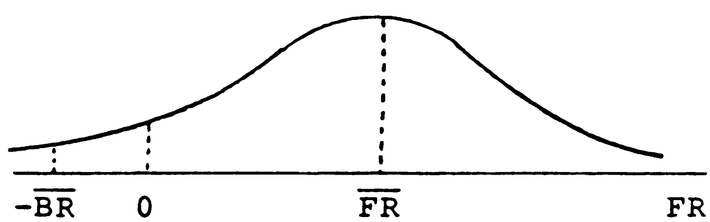


FIGURE 5
THE DISTRIBUTION OF FREE RESERVES

Since the variable free reserves is a linear function of demand deposits, its distribution may be derived from that of the latter. Utilizing the balance sheet identity, the value and the moments of free reserves may be obtained as follows:

$$FR = (1-rd)DD + W_0 - (1-S_2)X_2$$

$$\overline{FR} = (1\text{-}rd)\overline{DD} + W_O - (1\text{-}S_2)X_2$$

$$V(FR) = (1\text{-}rd)^2\, V(DD)$$

The borrowing variable does not have a well-defined distribution. Mean and variance of this variable can be considered as incomplete moments of the distribution of free reserves, where the values to the right of point zero are all treated as zero borrowing. To find the range of demand deposits which correspond to negative values of reserves (borrowing), free reserves may alternatively be written as:

$$FR = \overline{FR} + (1\text{-}rd)\ (DD - \overline{DD})$$

From this relation it can be seen that negative free reserves correspond to the values of demand deposits between zero and $\overline{DD} - \overline{FR}/1\text{-}rd$, that is:

$$FR < 0 \ \rightleftarrows\ DD < \overline{DD} - \frac{\overline{FR}}{1\text{-}rd}$$

The moments of borrowing may consequently be defined as:

$$\overline{BR} = \int_{-\infty}^{0} FR\,dF(FR) - \int_{0}^{\infty} 0\ dF(FR)$$

$$= \int_{0}^{\overline{DD} - \overline{FR}/1\text{-}rd} [\overline{FR} + (1\text{-}rd)\ (DD - \overline{DD}]\ g(DD)dDD$$

$$V(BR) = \int_{-\infty}^{0} (-FR-\overline{BR})^2 \, dF(FR) + \int_{0}^{\infty} (0-\overline{BR})^2 \, dF(FR)$$

$$= FR^2 \, dF(FR) + \overline{BR}^2 \int_{-\infty}^{0} dF(FR) + 2\overline{BR} \int_{-\infty}^{0} FR dF(FR)$$

$$+ \overline{BR}^2 \int_{0}^{\infty} dF(FR)$$

$$= EBR^2 + \overline{BR}^2 (Pr) + 2\overline{BR}(-\overline{BR}) + \overline{BR}^2 (1-Pr)$$

$$= EBR^2 - \overline{BR}^2$$

Where Pr is the probability of borrowing, g(DD) is the density function for demand deposits, and F(FR) is the cumulative distribution of free reserves.

2. Partial Derivatives

The variations in BR and V(BR) come about because of changes in the distribution of free reserves. To make matters simple it may be assumed that the latter changes can be properly represented by changes in mean and variance of free reserves. This simplification can be justified by assuming that free reserves have a two parameter distribution. The partial derivatives of the moments of borrowing may consequently be obtained as:

$$\frac{\partial \overline{BR}}{\partial \overline{FR}} = \frac{-\partial}{\partial \overline{FR}} \int_{0}^{\overline{DD} - \overline{FR}/1-rd} [\overline{FR} + (1-rd) \; (DD-\overline{DD}) \; g(DD) \; dDD$$

$$= \quad - \int_0^{\overline{DD}-\overline{FR}/1-rd} g(DD)\,d(DD) \quad = \quad -G(\overline{DD}-\overline{FR}/1-rd) \quad = \quad -P_r$$

where G is the cumulative distribution of DD.

$$\frac{\partial V(BR)}{\partial \overline{FR}} \quad = \quad \frac{\partial [EBR^2 - \overline{BR}^2]}{\partial \overline{FR}}$$

$$= \quad \frac{\partial}{\partial \overline{FR}} \int_0^{\overline{DD}\ -\ \overline{FR}/1-rd} [\overline{FR} + (1-rd)(DD-\overline{DD})]\ g(DD)dDD \ - \ \frac{\partial \overline{BR}^2}{\partial \overline{FR}}$$

$$= \quad 2 \int_0^{\overline{DD}\ -\ \overline{FR}/1-rd} [\overline{FR} + (1-rd)(DD-\overline{DD})]\ g(DD)\ dDD \ - \ 2\overline{BR}\,\frac{\partial BR}{\partial \overline{FR}}$$

$$= \quad -2\overline{BR} + 2\overline{BR}(P_r) \quad = \quad 2\overline{BR}(P_r-1) \quad < \quad 0$$

To derive the second partial derivatives, the relation $P_r = G(\overline{DD} - \overline{FR}/1-rd)$ may be utilized to conclude:

$$\frac{\partial P_r}{\partial \overline{FR}} \ < \ 0, \qquad \frac{\partial P_r}{\partial rd} \ < \ 0$$

and consequently:

$$\frac{\partial^2 \overline{BR}}{\partial \overline{FR}_2} \quad = \quad \frac{-\partial P_r}{\partial \overline{FR}} \quad > \quad 0$$

$$\frac{\partial^2 V(BR)}{\partial \overline{FR}^2} \quad = \quad \frac{\partial}{\partial \overline{FR}}\ [2\overline{BR}(P_r-1)] \quad = \quad 2\ [(P_r-1)(-P_r) \ + \ \overline{BR}\ \frac{\partial P_r}{\partial \overline{FR}}\ ],$$

has an indeterminate sign.

Since free reserves were assumed to have a two parameter distribution, the total derivative of P_r can be

measured as follows:

$$dP_r = \frac{\partial P_r}{\partial \overline{FR}} d(\overline{FR}) + \frac{\partial P_r}{\partial V(FR)} dV(FR)$$

where: $$V(FR) = (1-rd)^2 V(DD)$$

$$dV(FR) = -2\ (1-rd)\ V(DD)drd + (1-rd)^2\ dv(DD)$$

The sign of $\frac{\partial P_r}{\partial \overline{FR}}$ was shown to be negative. The sign of $\frac{\partial P_r}{\partial V(FR)}$ is indeterminate. Notice that from $d\overline{FR} = (1-rd)d\overline{DD} - \overline{DD}$ $drd - dX_2 - dW_O$, the effects of changes in X_2, rd, DD, W_O on the probability of borrowing (P_r) are reflected through changes in $\overline{FR}$.

APPENDIX TWO

The Second Order Conditions

For second order conditions the Hessian matrix H should be negative definite. This requires its bordered principal minors to alternate in sign with the first being positive.

$$H = \begin{pmatrix} 0 & 0 & F_{P_1} & F_2 & 0 & F_L & F_K \\ 0 & 0 & 0 & 1-S_2 & 1 & 0 & 0 \\ F_{P_1} & 0 & A_{11} & 0 & 0 & 0 & 0 \\ F_2 & 1-S_2 & 0 & A_{22} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & A_{33} & 0 & 0 \\ F_L & 0 & 0 & 0 & 0 & -\lambda F_{LL} & 0 \\ F_K & 0 & 0 & 0 & 0 & 0 & -\lambda F_{KK} \end{pmatrix}$$

$$A_{11} = 2b_1 - 2\alpha V(X_1) - \lambda b_1^2 F_{11}$$

$$A_{22} = -2\alpha V(P_2) - \lambda F_{22}$$

$$A_{33} = R_B\, \partial P_r/\partial \overline{FR} - 2\alpha R^2_B [P_r(1-P_r) + \overline{BR} \frac{\partial P_r}{\partial \overline{FR}}]$$

We need: $H_{55,44,33} > 0$ $\quad H_{55,44} < 0$ $\quad H_{55} > 0$ $\quad H < 0$

where $H_{ii,jj}$ indicates the minors of H obtained by deleting rows and columns i and j.

$$H_{55'44'33} = \begin{pmatrix} 0 & 0 & F_{P_1} & F_2 \\ 0 & 0 & 0 & 1 \\ F_{P_1} & 0 & A_{11} & 0 \\ F_2 & 1 & 0 & A_{22} \end{pmatrix} = F_{P_1}^{\;2} > 0$$

$$H_{55'44} = \begin{pmatrix} 0 & 0 & F_{P_1} & F_2 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ F_{P_1} & 0 & A_{11} & 0 & 0 \\ F_2 & 1 & 0 & A_{22} & 0 \\ 0 & 1 & 0 & 0 & A_{33} \end{pmatrix} = A_{33}F_{P_1}^{\;2} + A_{11}F_2^{\;2} < 0$$

$$H_{55} = \begin{pmatrix} 0 & 0 & F_{P_1} & F_2 & 0 & F_L \\ 0 & 0 & 0 & 1 & 1 & 0 \\ F_{P_1} & 0 & A_{11} & 0 & 0 & 0 \\ F_{P_2} & 1 & 0 & A_{22} & 0 & 0 \\ 0 & 1 & 0 & 0 & A_{33} & 0 \\ F_L & 0 & 0 & 0 & 0 & -\lambda F_{LL} \end{pmatrix} = \begin{matrix} F_{LL}(H_{55'44}) \\ + F_L^{\;2}A_{11}A_{33} > 0 \end{matrix}$$

$$H = \begin{vmatrix} 0 & 0 & F_{P_1} & F_2 & 0 & F_L & F_K \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ F_{P_1} & 0 & A_{11} & 0 & 0 & 0 & 0 \\ F_2 & 1 & 0 & A_{22} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & A_{33} & 0 & 0 \\ F_L & 0 & 0 & 0 & 0 & -\lambda F_{LL} & 0 \\ F_K & 0 & 0 & 0 & 0 & 0 & -\lambda F_{KK} \end{vmatrix} = -\lambda F_{KK}(H_{55}) - \lambda F_K{}^2 F_{LL} A_{11} (A_{22} + A_{33}) < 0$$

For these relations to hold the following conditions are sufficient.

$$A_{ii} < 0 \quad i = 1,2,3 \qquad F_{LL}, F_{KK} > 0$$

in turn for $A_{ii} < 0$ $i = 1,2,3$ it is sufficient to have:

$$F_{11}, F_{22} > 0 \qquad P_r(1-P_r) + \overline{BR}\,\frac{\partial P_r}{\partial \overline{FR}} < 0$$

the last condition will be satisfied if V(BR) declines at a constant or an increasing rate when $\overline{FR}$ increases. (See Appendix one).

Consider the marginal rate of product transformation (MRPT) and marginal rate of substitution (MRS).

$$MRPT = \frac{dX_2}{d\overline{X}_1} = \frac{F_1}{F_2} \text{ (by implicit function theorem).}$$

MRPT is increasing if:

$$\frac{d^2X_2}{d\overline{X}_1^{\,2}} > 0 \quad \text{or} \quad \frac{F_{11}F_2^{\,2} + F_1^{\,2}F_{22}}{F_2^{\,3}} > 0$$

$$MRS = \frac{-dK}{dL} = \frac{F_L}{F_K}$$

MRS is decreasing if:

$$\frac{d}{dL}\,(F_L/F_K) = \frac{F_{LL}F_K^{\,2} + F_{KK}F_L^{\,2}}{F_K^{\,3}} < 0$$

Since these conditions are satisfied by normalization and second order conditions, it can be concluded that MRPT is increasing and MRS decreasing.

APPENDIX THREE

Total Differentiation of The First Order Conditions

Condition 1:

$$a_1 d\overline{DD} + 2b_1 dP_1 - \{2a_1(a_1P_1 - R_D)V(DD) + 2P_1V(E_1)$$

$$+ (4a_1P_1 - 2R_D)\, C_{OV}(DD,E_1) - 2a_1R_B\, C_{OV}(DD,BR)\}d\alpha$$

$$- \alpha\{2a_1^2\, V(DD)dP_1 - 2a_1V(DD)\, dR_D + 2a_1(a_1P_1 - R_D)dV(DD)$$

$$+ 2V(E_1)\, dP_1 + 2P_1\, dV(E_1) + 4\, a_1\, C_{OV}(DD,E_1)\, dP_1$$

$$-2\, C_{OV}(DD,E_1)\, dR_D + (4a_1P_1 - 2R_D)\, d\, C_{OV}\, (DD,E_1)$$

$$-2\, a_1\, C_{OV}\, (DD,BR)\, dR_B - 2a_1R_B\, d\, C_{OV}(DD,BR)\} - F_{P_1}\, d\lambda$$

$$-\lambda F_{11}b_1(a_1\, d\overline{DD} + b_1dP_1) - \lambda b_1 \frac{\partial\alpha_1/\partial_t}{a_1\overline{DD} + b_1P_1}\, dt = 0$$

or:

$$-F_{P_1}d\lambda + dP_1[2b_1 - 2\alpha V(X_1) - \lambda b_1^2\, F_{11}] =$$

$$(2\alpha a_1^2P_1 - \alpha a_1R_D)dV(DD) + 2\alpha P_1dV(E_1) + (4\alpha a_1P_1 - 2\alpha R_D)$$

$$dC_{OV}(DD,E_1) - 2\alpha C_{OV}(DD,X_1)\, dR_D - 2\alpha a_1\, C_{OV}(DD,BR)dR_B$$

$$- 2\alpha a_1R_B\, dC_{OV}(DD,BR) + (\lambda a_1b_1F_{11} - a_1)\, d\overline{DD}$$

$$+ [2a_1(a_1P_1 - R_D)\, V(DD) + 2P_1\, V(E_1) + (4a_1P_1 - 2R_D)$$

$$C_{OV}(DD,E_1) - 2\, a_1R_B\, C_{OV}(DD,BR)]\, d\alpha$$

Condition 2:

$$d\overline{C}_2 - d\overline{d}_2 - [2X_2V(C_2 - d_2) - 2\ R_B\ C_{OV}\ (C_2 - d_2, BR)]\ d\alpha$$

$$- \alpha\{2X_2[dV(C_2) + dV(d_2) - 2\ dC_{OV}(C_2, d_2)] + 2V(C_2-d_2)dX_2$$

$$- 2R_B dC_{OV}(C_2, BR) + 2R_B dC_{OV}(D_2, BR) - 2C_{OV}(C_2-d_2, BR)dR_B\}$$

$$- F_2 d\lambda - \lambda F_{22} dX_2 - (1-S_2)\ d\mu + \mu dS_2 = 0$$

or:

$$- F_2 d\lambda - (1-S_2)\ d\mu - [2\alpha V(C_2-d_2) - \lambda F_{22}]\ dX_2 =$$

$$-d\overline{C}_2 + d\overline{d}_2 + 2\alpha X_2\ dV(C_2) + 2\alpha X_2\ dV\ (d_2) - 4\alpha X_2\ dC_{OV}$$

$$(C_2, d_2) - 2\alpha R_B\ d\ C_{OV}\ (C_2, BR) + 2\alpha R_B dC_{OV}(d_2, BR)$$

$$- 2\alpha C_{OV}\ (C_2-d_2, BR)\ dR_B - \mu dS_2 + [2X_2\ V(C_2-d_2) - 2R_B$$

$$C_{OV}(C_2-d_2, BR)]\ d\alpha$$

Condition 3:

$$R_B dP_r + P_r dR_B + 2R_B^2\ \overline{BR}\ (1-P_r)d\alpha + 2\alpha R_B^2\ (1-P_r)\ d\overline{BR}$$

$$- 2\alpha R_B^2\ \overline{BR}\ DP_r + 4\alpha\overline{BR}\ R_B(1-P_r)\ dR_B - d\mu = 0$$

where:

$$dP_r = \frac{\partial P_r}{\partial \overline{FR}}\ d\overline{FR} + \frac{\partial P_r}{\partial V(FR)}\ dV(FR)$$

$$d\overline{BR} = \frac{\partial \overline{BR}}{\partial \overline{FR}}\ d\overline{FR} + \frac{\partial \overline{BR}}{\partial V(FR)}\ dV(FR)$$

$$dV(R_F) = (1-rd)^2\ dV(DD) - 2(1-rd)\ V(DD)\ drd$$

or:

$$- d\mu + (R_B - 2\alpha R_B^2\overline{BR})\{\frac{\partial P_r}{\partial \overline{FR}} \; d\overline{FR} + \frac{\partial P_r}{\partial V(FR)} \; [(1-rd)^2 dV(DD)$$

$$- 2(1-rd) \; V(DD) \; drd]\} + [P_r + 4\alpha\overline{BR} \; R_B \; (1-P_r)] \; dR_B$$

$$+ 2 \; R_B^2 \; \overline{BR}(1-P_r)d\alpha - 2\alpha R_B^2(1-P_r) \; \{\frac{\partial \overline{BR}}{\partial \overline{FR}} \; d\overline{FR} + \frac{\partial \overline{BR}}{\partial V(FR)}$$

$$[(1-rd)^2 \; dV(DD) - 2 \; (1-rd)V(DD) \; drd]\} \; = \; 0$$

or:

$$-d\mu + \{[R_B - 2\alpha R_B^2 \; \overline{BR}] \; \frac{\partial P_r}{\partial \overline{FR}} - 2\alpha R_B^2 \; (1-P_r)P_r\} \; d\overline{FR} \; =$$

$$-dV(DD)\{[R_B - 2\alpha R_B^2 \; \overline{BR}](1-rd)^2 \; \frac{\partial P_r}{\partial V(FR)} + 2\alpha R_B^2(1-P_r)(1-rd)$$

$$\frac{\partial \overline{BR}}{\partial V(FR)}\} - dR_B \; [P_r + 4\alpha\overline{BR} \; R_B(1-P_r)] - 2R_B^2\overline{BR} \; (1-P_r) \; d\alpha$$

$$+ dr_d\{2[R_B - 2\alpha R_B^2 \; \overline{BR}](1-rd)V(DD)\frac{\partial P_r}{\partial V(FR)} + 4\alpha R_B^2(1-P_r)(1-rd)$$

$$V(DD)\frac{\partial \overline{BR}}{\partial V(FR)}$$

Condition 4:

$$- F_L d\lambda - \lambda F_{LL} dL \; = \; dP_L$$

Condition 5:

$$-F_K d\lambda - \lambda F_{KK} d_K \; = \; dP_K$$

Condition 6:

$$F_{P_1} \; dP_1 + F_2 \; dX_2 + F_L d_L + F_K dK \; = \; -(a_1 F_1 - \alpha D/\overline{DD})d\overline{DD}$$

$$+ d\gamma + \log \; (a_1\overline{DD} + b_1 P_1) \; \frac{\partial \alpha_1}{\partial_t} \; d_t$$

Condition 7:

$$(1-S_2)\ dX_2 + d\overline{FR} = (1-rd)\, d\overline{DD} - \overline{DD}drd + dW_0 + X_2 dS_2$$

APPENDIX FOUR

Comparative Static Results

$$Z_i = P_1,\ X_2,\ \overline{FF},\ L,\ K,\ -\lambda,\ -\mu$$

$$i = 1,\ 2,\ 3,\ 4,\ 5,\ 0,\ 00$$

$$\frac{\partial Z_i}{\partial \bar{C}_2} = -\frac{H_{i2}}{H}$$

$$\frac{\partial Z_i}{\partial \bar{d}_2} = \frac{H_{i2}}{H}$$

$$\frac{\partial Z_i}{\partial S_2} = X_2 \frac{H_{i00}}{H} - \mu \frac{H_{i2}}{H}$$

$$\frac{\partial Z_i}{\partial R_B} = -2\alpha a_1 C_{OV}(DD,BR) \frac{H_{i1}}{H} - 2\alpha C_{OV}(C_2-d_2,BR) \frac{H_{i2}}{H}$$

$$- [P_r + 4\alpha \overline{BR}\ R_B\ (1-P_r)] \frac{H_{i3}}{H}$$

$$\frac{\partial Z_i}{\partial R_D} = -2\alpha C_{OV}(DD,X_1) \frac{H_{i1}}{H}$$

$$\frac{\partial Z_i}{\partial P_L} = \frac{H_{i4}}{H}$$

$$\frac{\partial Z_i}{\partial P_K} = \frac{H_{i5}}{H}$$

$$\frac{\partial Z_i}{\partial V(C_2)} = 2\alpha X_2 \frac{H_{i2}}{H}$$

$$\frac{\partial Z_1}{\partial V(d_2)} = 2\alpha X_2 \frac{H_{i2}}{H}$$

$$\frac{\partial Z_i}{\partial V(DD)} = 2\alpha a_1(a_1P_1 - R_D) \frac{H_{i1}}{H} - \{[R_B - 2\alpha R_B^2\overline{BR}](1-rd)^2 \frac{\partial P_r}{\partial V(FR)}$$

$$+ 2\alpha R_B^2 (1-P_r)(1-rd)^2 \frac{\partial \overline{BR}}{\partial V(FR)}\}\frac{H_{i3}}{H}$$

$$\frac{\partial Z_i}{\partial V(E_1)} = 2\alpha P_1 \frac{H_{i1}}{H}$$

$$\frac{\partial Z_i}{\partial C_{OV}(C_2,d_2)} = -4\alpha X_2 \frac{H_{i2}}{H}$$

$$\frac{\partial Z_i}{\partial C_{OV}(C_2,BR)} = -2\alpha R_B \frac{H_{i2}}{H}$$

$$\frac{\partial Z_i}{\partial C_{OV}(d_2,BR)} = 2\alpha R_B \frac{H_{i2}}{H}$$

$$\frac{\partial Z_i}{\partial C_{OV}(DD,BR)} = -2\alpha a_1 R_B \frac{H_{i1}}{H}$$

$$\frac{\partial Z_i}{\partial C_{OV}(DD,E_1)} = 2\alpha(2a_1P_1 - R_D) \frac{H_{i1}}{H}$$

$$\frac{\partial Z_i}{\partial rd} = -\overline{DD} \frac{H_{i00}}{H} + \{[R_B - 2\alpha R_B^2\overline{BR}][2(1-rd)V(DD)\frac{\partial P_r}{\partial V(FR)}]$$

$$+ 4\alpha R_B^2 (1-P_r)(1-rd)V(DD) \frac{\partial \overline{BR}}{\partial V(FR)}\} \frac{H_{i3}}{H}$$

$$\frac{\partial Z_i}{\partial W_O} = \frac{H_{i00}}{H}$$

$$\frac{\partial Z_i}{\partial \overline{DD}} = -\left(a_1F_1 - \alpha_D/\overline{DD}\right)\frac{H_{i0}}{H} + \left(\lambda a_1 b_1 F_{11} - a_1\right)\frac{H_{i1}}{H}$$

$$\frac{\partial Z_i}{\partial \alpha} = [(2a_1^2P_1 - a_1R_D)V(DD) + 2P_1V(E_1) + (4a_1P_1 - 2R_D)\, C_{OV}$$

$$(DD,E_1) - 2\, a_1R_B\, C_{OV}(DD,BR)]\,\frac{H_{i1}}{H} + [2X_2V(C_2 - d_2)$$

$$- 2R_B\, C_{OV}(C_2 - d_2,BR)]\,\frac{H_{i2}}{H} - 2R_B^2\, \overline{BR}\,(1-P_r)\,\frac{H_{i3}}{H}$$

$$\frac{\partial Z_i}{\partial \gamma} = H_{io}/H$$

$$\partial Z_i/\partial t = -\log\,(a_1\overline{DD} + b_1P_1)\,\frac{\partial \alpha_1}{\partial t}\,\frac{H_{i0}}{H} + \lambda b_1\left(\frac{\partial \alpha_1/\partial_t}{a_1\overline{DD} + b_1P_1}\right)$$

$$\frac{H_{i1}}{H}$$

APPENDIX FIVE

Moments of a Product of Random Variables

Consider two random variables P and Z which can be written as:

$$P = \overline{P} + U$$

$$Z = \overline{Z} + V$$

where $\overline{P}$ and $\overline{Z}$ represent mean values of P and Z respectively, and U and V are independent random terms with zero mean and constant variances.

$$\begin{pmatrix} U \\ V \end{pmatrix} \sim \left(\begin{matrix} 0 \\ 0 \end{matrix} , \begin{pmatrix} V(U) & 0 \\ 0 & V(V) \end{pmatrix} \right) \quad \begin{matrix} \text{where:} \\ V(U) = V(P) \\ V(V) = V(Z) \end{matrix}$$

The moments of the product PZ can be measured as follows:

$$PZ = (\overline{P}+U)(\overline{Z}+V) = \overline{P}\overline{Z} + \overline{P}\overline{V} + \overline{Z}\overline{U} + UV$$

$$(PZ)^2 = (\overline{P}\overline{Z})^2 + (\overline{P}\overline{V})^2 + (\overline{Z}\overline{U})^2 + (UV)^2 + 2\overline{P}^2\overline{Z}V + 2\overline{P}\overline{Z}^2U$$

$$+ 4\overline{P}\overline{Z}UV + 2\overline{P}UV^2 + 2\overline{Z}U^2V.$$

$$E(PZ) = \overline{P}\overline{Z}$$

$$E(PZ)^2 = (\overline{P}\overline{Z})^2 + \overline{P}^2V(Z) + \overline{Z}^2V(P) + V(P)V(Z)$$

$$V(PZ) = E(PZ)^2 - (E(PZ))^2 = \overline{P}^2V(Z) + \overline{Z}^2V(P) + V(P)$$
$$V(Z)$$

$$C_{OV}(P_iZ_i,P_jZ_j) = EP_iZ_iP_jZ_j - E(P_iZ_i)E(P_jZ_j)$$

$$= E\{(\overline{P}_i + U_i)(\overline{Z}_i + V_i)(\overline{P}_j + U_j)(\overline{Z}_j + V_j)\} -$$

$$\overline{P}_i\overline{Z}_i\overline{P}_j\overline{Z}_j$$

$$= E\ \{(\overline{P}_i\overline{Z}_i + \overline{P}_iV_i + \overline{Z}_iU_i + U_iV_i)(\overline{P}_j\overline{Z}_j + \overline{P}_jV_j$$

$$+ \overline{Z}_jU_j + U_jV_j)\} - \overline{P}_i\overline{Z}_i\overline{P}_j\overline{Z}_j$$

$$= E\{\overline{P}_i\overline{Z}_i\overline{P}_j\overline{Z}_j + \overline{P}_i\overline{P}_j\overline{Z}_iV_j + \overline{P}_i\overline{Z}_i\overline{Z}_jU_j + \overline{P}_i\overline{Z}_iU_iV_j$$

$$+ \overline{P}_i\overline{P}_j\overline{Z}_jV_i + \overline{P}_i\overline{P}_jV_iV_j + \overline{P}_i\overline{Z}_jV_iU_j + \overline{P}_iV_iU_jV_j$$

$$+ \overline{P}_j\overline{Z}_j\overline{Z}_iU_i + \overline{P}_j\overline{Z}_iU_iV_j + \overline{Z}_i\overline{Z}_jU_iU_j + \overline{Z}_iU_iU_jV_j$$

$$+ \overline{P}_j\overline{Z}_jU_iV_i + \overline{P}_jU_iV_iV_j + \overline{Z}_jU_iV_iU_j + U_iV_iU_jV_j\}$$

$$- \overline{P}_i\overline{Z}_i\overline{P}_j\overline{Z}_j$$

$$= \overline{P}_i\overline{P}_j\overline{Z}_iE(V_j) + \overline{P}_i\overline{Z}_i\overline{Z}_jE(U_j) + \overline{P}_iZ_iC_{OV}(U_i,V_j)$$

$$+ \overline{P}_i\overline{P}_j\overline{Z}_jE(V_i) + \overline{P}_i\overline{P}_jC_{OV}(V_i,V_j) + \overline{P}_i\overline{Z}_jC_{OV}$$

$$(V_i,U_j) + \overline{P}_iE(U_j)C_{OV}(V_i,V_j) + P_j\overline{Z}_j\overline{Z}_iE(U_i)$$

$$+ \overline{P}_j\overline{Z}_jC_{OV}(U_i,V_i) + \overline{Z}_i\overline{Z}_jC_{OV}(U_i,U_j) + \overline{Z}_iC_{OV}$$

$$(U_i,U_j)E(V_j) + \overline{P}_j\overline{Z}_jC_{OV}(U_i,V_i) + \overline{P}_jE(U_i)C_{OV}$$

$$(V_i,V_j) + \overline{Z}_jE(V_i)C_{OV}(U_i,U_j) + C_{OV}(U_i,U_j)C_{OV}$$

$$(V_i,V_j)$$

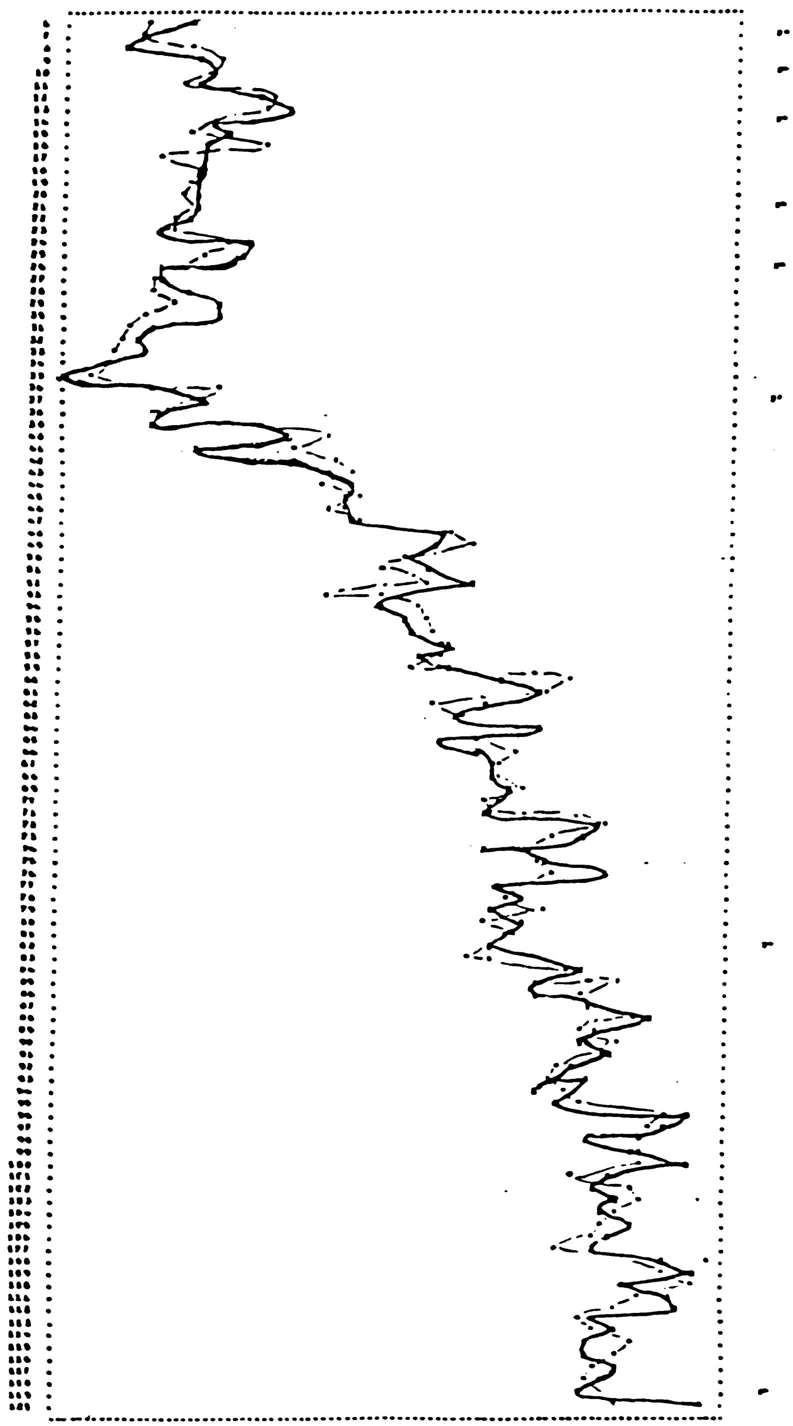
Since: $E(U_i) = E(U_j) = E(V_i) = E(V_j) = 0$

$$: \quad C_{OV}(V_i, V_j) = C_{OV}(Z_i, Z_j)$$

$$: \quad C_{OV}(U_i, U_j) = C_{OV}(P_i, P_j)$$

$$C_{OV}(P_i Z_i, P_j Z_j) = \overline{P}_i \overline{P}_j C_{OV}(Z_i, Z_j)$$

$$+ \overline{Z}_i \overline{Z}_j C_{OV}(P_i, P_j) + C_{OV}(Z_i, Z_j) C_{OV}(P_i, P_j)$$



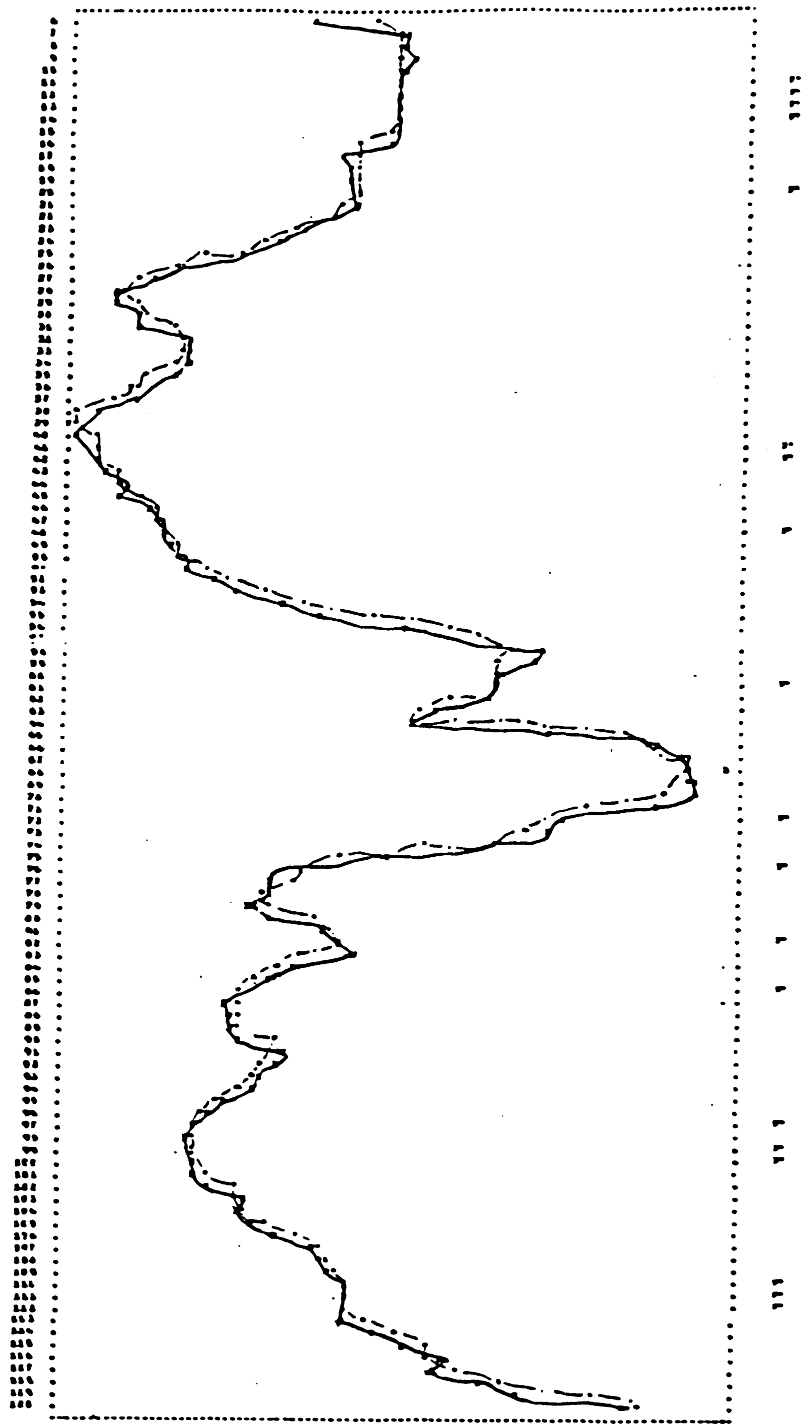
Actual

Forecasted

FIGURE 6:

GRAPH OF ACTUAL VERSUS FORECASTED

DEMAND DEPOSITS



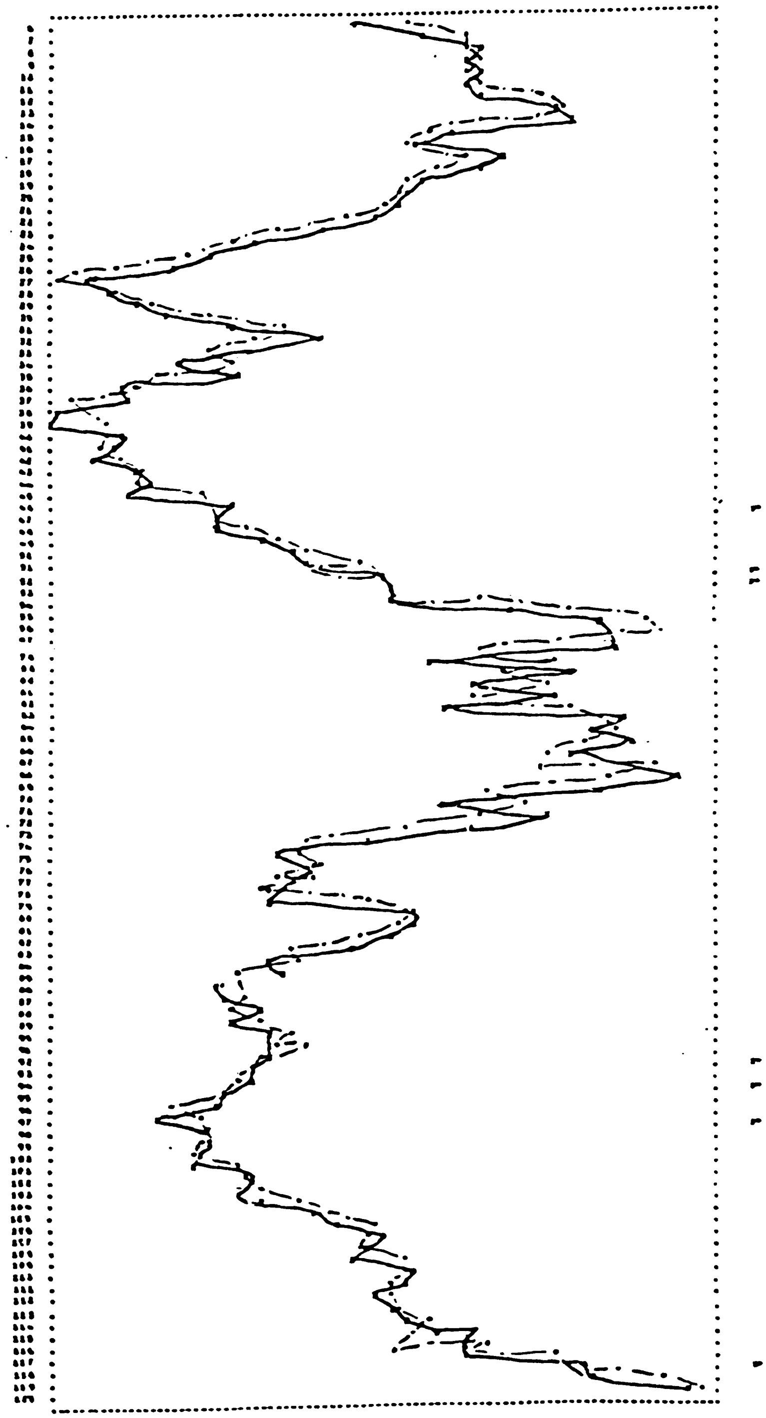
Actual

Forecasted

FIGURE 7:

GRAPH OF ACTUAL VERSUS FORECASTED PRIME

LOAN RATE



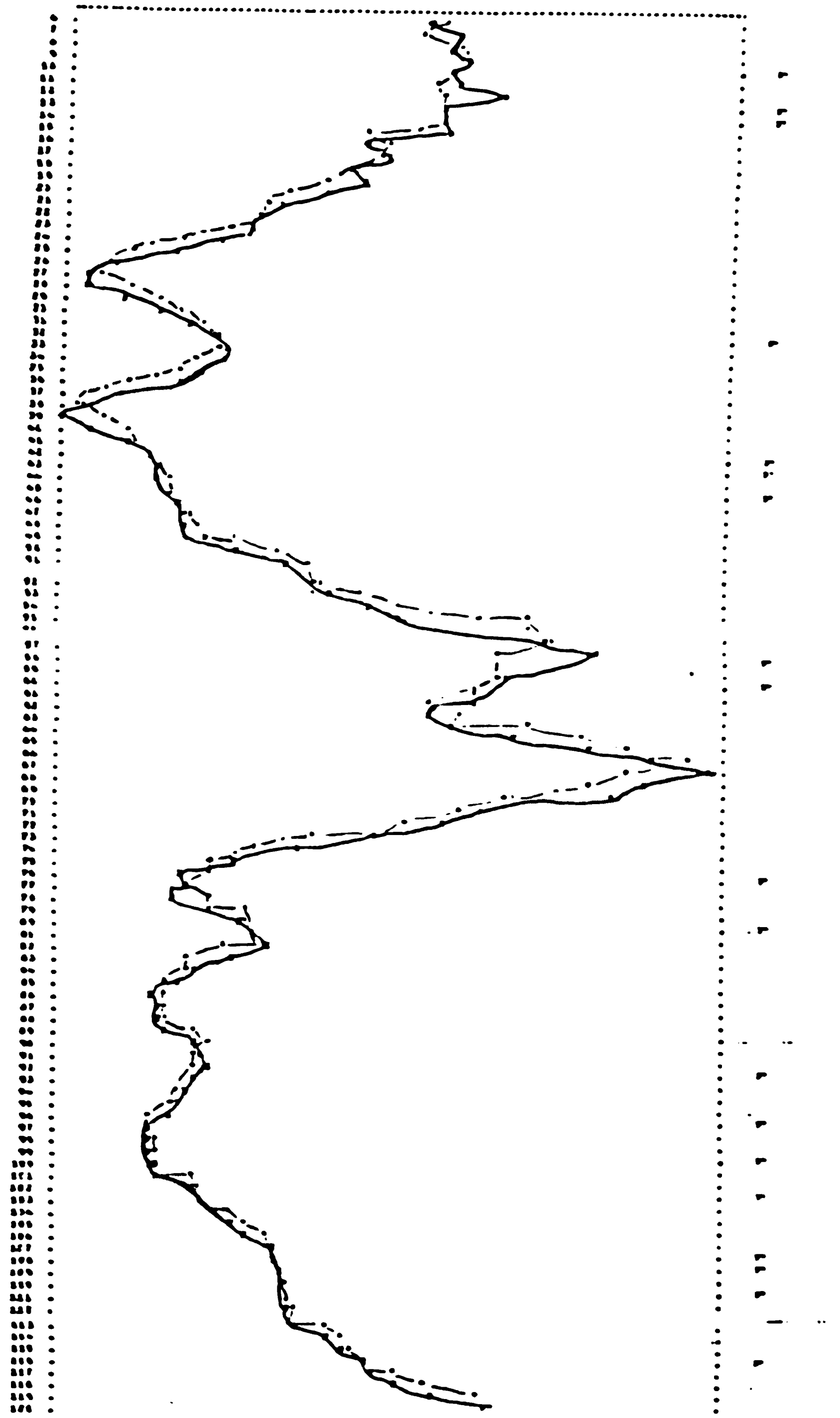
Actual

Forecasted

FIGURE 8:

GRAPH OF ACTUAL VERSUS FORECASTED

TREASURY BILL RATE

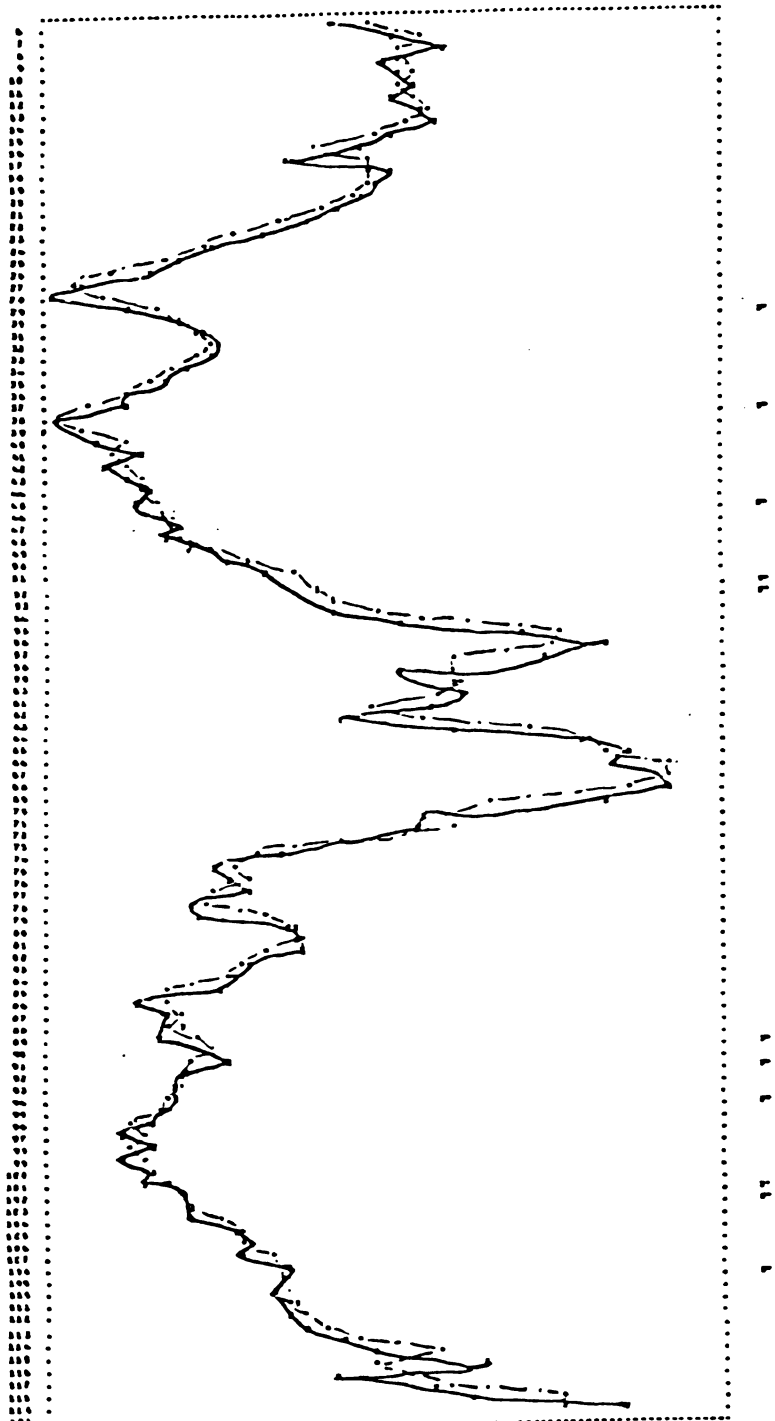


Actual

Forecasted

FIGURE 9:

GRAPH OF ACTUAL VERSUS FORECASTED FEDERAL FUNDS RATE



Actual

Forecasted

FIGURE 10;

GRAPH OF ACTUAL VERSUS FORECASTED CD RATE

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