VOLUME PREDICTION FROM STUMP DIAMETER AND STUMP HEIGHT FOR SELECTED SPECIES IN NORTHERN MINNESOTA

By

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ABSTRACT

VOLUME PREDICTION FROM STUMP DIAMETER AND

STUMP HEIGHT FOR SELECTED SPECIES

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Regression equations and volume tables are developed and presented for predicting tree volumes from measurements of stump diameter and stump height. Volumes are presented in cubic feet units. Pulpwood volume tables are presented for aspen (*Populus tremulodies*), paper birch (*Betula papyrifera*), red pine (*Pinus resinosa*), jack pine (*Pinus banksiana*), black spruce (*Picea mariana*), and balsam fir (*Abies balsamea*). Sawlog volume tables are presented for aspen, red pine, and balsam fir. Sample sizes ranged from 42 for balsam fir to 147 for jack pine. Data were collected from 41 logging sites. Coefficients of determination ranged from .733 for the aspen sawlog volume equation to .977 for the aspen pulpwood volume equation. Regression equations were evaluated by variable stump heights and by an independent test data set. Site index was not found to be a significant predictor variable. Volume tables and regression equations are applicable in northern Minnesota.

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INTRODUCTION

During my experience in a national forest district, there were occasions when loggers cut over a boundary on a logging contract, an action called "trespass cutting." Since the trees are usually removed from the site in such cases, managers can only approximate the volume of the tree. This volume is then used to estimate the value of the trees according to current timber prices so that restitution may be made. The objective of this study was to provide acceptable prediction equations for tree volumes using measurements of stump parameters and other site information.

Other possible uses of the results are:

1. Conducting growth and yield studies
2. Evaluating growth of previous stands
3. Check-cruising on marked timber sales (used for trees that were cut but not marked)
4. Estimating volumes from stump tallies

The criteria for selecting the best equations for such studies commonly includes small standard errors of estimate (SE), large coefficient of determination (R^2), examination of residuals of each equation, and use of the partial F-test on coefficients in the equation. Other selection criteria considered important in this study are a minimum number of variables in the equations, similar variables between species, and easy measurement of variables in the field.

There would be a need for volume predictions of pulpwood and sawtimber for many commercial species. The estimates should be sufficiently accurate to give reasonable predictions of volume.

Some variables that might be used to predict tree volume would include information on species, stump, and site quality. A clear definition of that part of the tree which is considered merchantable would also be important.

The trend in the U.S. Forest Service today is towards utilization of the whole tree. Cubic feet volumes are better representations of the solid wood of a tree than either cords or board feet. A problem with cords is that the volumes are different in different areas of the country. A problem with board feet is that there are different log rules which produce different volumes for the same tree. To alleviate these problems and to conform with the current trend, all volumes calculations in this study are expressed in cubic feet units.

Information collected as part of a wood utilization study in 1976 and 1977 served as the basic data for this study. The age and height parameters needed to determine site indices were obtained in 1978. The utilization study used U.S. Forest Service survey standards to classify whether a tree was sawtimber or pulpwood. A tree is classified as pulpwood if it is between 4.0" and 9.0" diameter at breast height (dbh) for softwood and between 4.0" and 11.0" dbh for hardwoods.[1] A tree is classified as sawtimber if it is greater than 9.0" dbh for softwoods and greater than 11.0" dbh for hardwoods.[2]

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[1] Forest Survey Handbook guidelines.

[2] Ibid.

Foresters would be the principal users of the volume tables and regression equations presented in this study. Extension agents and owners of timbered lands would also be possible users.

LITERATURE REVIEW

Available literature on prediction of tree volume from stump diameter was relatively sparse. The majority of the articles predicted diameter at breast height, rather than volume, from stump diameter. Generally, two phases of estimation were suggested. After dbh was predicted from stump diameter, estimated volumes were obtained by the use of local volume tables. The results from various studies have been presented in many different forms--charts, curves, graphs, tables, regression equations, "rule of thumb" equations, or combinations of the above. The earlier studies used charts whereas the latter studies used tables and regression equations.

Studies Utilizing Charts, Graphs, or Curves

Alignment charts were presented by Hough (1930) for beech in northwest Pennsylvania and by Ostrom and Taylor (1938) for beech, black cherry, sugar maple, and yellow poplar in Pennsylvania. Rapraeger (1941) presented charts for western white pine, ponderosa pine, western larch, Douglas fir, and Engelmann spruce in Idaho using stump height and stump diameter to predict dbh. Endicott (1959) presented a family of harmonized taper curves for eucalyptus species to provide estimation of dbh.

Studies Utilizing Tables and "Rule of Thumb" Equations

Some of the dbh predictions appear in tabular form. Cunningham *et al*. (1947) presented two sets of tables for 15 different trees species in Pennsylvania: one showing stump diameter when dbh is known; and the other showing dbh when stump diameter is known. McCormack (1953) presented tables of predicted dbh's for yellow pine and hardwoods in Georgia and North Carolina. He used stump diameters measured at stump heights of 6, 12, 18, and 30 inches as independent variables. Eie (1959) presented taper tables for five diameter classes of silver fir, spruce, Scotch pine, Austrian pine, beech, and oak. Stump measurements were taken at a height of one-third of the stump diameter at ground level. Decourt (1973) presented tables and graphs to predict tree volumes for eight softwoods in France. He claimed that it is impossible to obtain an unbiased and reasonably accurate estimate of volume (error less than 10%) removed in a thinning by subsequent measurement of the girth (diameter) of the stump unless thinning has taken place within the last five years. Almedag and Honer (1973, 1977) presented dbh--stump diameter relationships for eleven species in eastern and central Canada, in both English and metric units. Quigley (1954) presented a table for the average number of 16 foot logs per tree by dbh and a table of gross volume by dbh and numbers of 16 foot logs. He utilized a fixed stump height of one foot in his measurement of Central States hardwoods. Horn and Keller (1957) used a fixed stump height of 1.0 foot for sawtimber and a stump height of 0.5 foot for poletimber in his tables. They presented a "rule of thumb" equation of the form:

dbh = (diameter of stump) - ((diameter of stump/10) +1)

He also developed dbh:diameter of stump ratios for softwoods, hardwoods, and aspen in Minnesota.

Studies Utilizing Regression Equations

Studies using linear and multiple regression equations constitute the majority of the articles. These were divided into two categories: those that used stump height as an independent variable, and those that did not.

Equations Using Fixed Stump Height

Ostrom and Taylor (1938), in addition to alignment charts, presented regression equations using a fixed stump height of one foot for four species. Schaeffer (1953) presented equations:

$$dbh = D_1 + D_2$$

for oak, beech, hornbeam, and maple and

$$dbh = 1.2(D_1 + D_2)$$

for elm, poplar, Scotch pine, cherry, alder, robinia, and birch where D_1 and D_2 are the least and the greatest diameters of the stump respectively. Church (1953), working with Virginia pine in Maryland, presented a graph based on equations regressing stump diameter and stump diameter squared on dbh for fixed stump heights of 0.5 and 1.0 foot. Vimmerstedt (1957) used a stump height of 0.5 foot with a stump diameter measurements outside bark and inside bark when he developed seven regression equations and tables for yellow poplar, red maple, chestnut oak, black locust, yellow pine, and white pak. Bones (1960, 1961) presented dbh:stump diameter ratios for ponderosa pine, Douglas fir, white fir, western larch, lodgepole pine, subalpine fir, and Engelmann spruce in Washington and Oregon and for Sitka spruce and western hemlock in Alaska. Meyers (1963), working with ponderosa pine in the Southwest, gave two equations for predicting dbh; one for immature ponderosa pine (4-11 inches stump diameter) and one for old growth ponderosa pine (12-40 inches stump diameter) at a stump height of one foot. He gave a five step procedure to obtain volume from estimated dbh. Valiquette (1964) presented the relationship between dbh and stump diameter at different heights for Abies balsamea, Picea mariana, Picea glauca, Pinus banksiana, Populus tremuloides, and Betula papyrifera in Canada. Decourt (1964) gave equations and tables for the relationship between girth at breast height and butt girth for Pinus sylvestris, Pinus nigra var Austrica and corsicanna, Picea abies, Pseudotsuga taxifolia, Picea sitchensis, and Abies alba in France. Beck et al. (1966) presented regression equations for predicting dbh from stump diameter under (inside) bark and found that accurate results were obtained for Pinus ponderosa of greater than 33 inches dbh. Over- and under-estimation occurred with smaller Pinus ponderosa, Pinus lambertina, Abies concolor, Pseudotsuga taxifolia, and Lebocedrus decurrens in California. Kim and Yoo (1966) using linear relationships between dbh and stump diameter, found that dbh was approximately 86% of stump diameter for Pinus koraiensis, Pinus densiflora, Pinus rigida, Larix leptolepis, Abies holophylla and various hardwood species in Taiwan. Sukwong (1971) found a significant relationshp between dbh and stump diameter for teak (Tectona grandis) in Thailand. Lange (1973) presented tables and equations for Pinus ponderosa, Pinus contorta, Pseudotsuga taxifolia, and Larix occidentalis in Montana. Van Deusen (1975) using stump diameter measurements both inside bark and outside bark and fixed stump heights of 0.5 and 1.0 foot, presented equations and tables for ponderosa pine in South Dakota's Black Hills. Hann (1976) using stump heights of 1.0 and 1.2 feet presented tables and equations for ponderosa pine, Douglas fir, aspen, white fir, southwestern white pine, Engelmann spruce, and corkbark fir in Arizona and New Mexico.

Equations Using Variable Stump Heights

Stump height and stump diameter were used as variables in multiple regression equations for predicting dbh by several authors. Hampf presented regression equations and graphs for white pine (1954), sugar maple (1955a), American beech (1955b), yellow birch (1955c), northern red oak (1955d), yellow poplar (1955e), pitch pine (1957a), and white oak (1957b) in the northeast. Miller (1957) developed equations for dbh using stump diameter (inside and outside bark) and stump height on lowland and hill sites for slash pine in Georgia. McClure (1968) developed the following regression equation to predict dbh of 53 species in North Carolina, Virginia, and South Carolina:

$$dbh = D\,(b_0 + b_1\,(\log(H + 1.0) - \log(5.5)) + b_2\,(\log(H + 1.0) - \log(5.5))^2 + b_3(D(H - 4.5)))$$

where

dbh = diameter at breast height (inches)

D = stump diameter at point of measurement (inches)

H = stump height to the point of measurement in feet

Curtis and Arney (1977) presented three regression equations for Douglas fir in Washington and Oregon. He developed regression equations for study areas, individual heights, and all data combined. A weighted stepwise conditioned regression was fitted to the combined data. He also presented a nonlinear equation for stump diameters measured at variable heights:

$$dbh = .8522(H^{.1063})(dob)$$

where

dbh = diameter at breast height (inches)

H = stump height (feet)

dob = diameter of stump outside the bark (inches)

Nyland (1975) presented a sawlog volume table for northern hardwoods based on stump diameter inside bark and tree height. He used tree length tables prepared by the Applied Forest Research Institute to calculate the volume. Nyland (1977a), in his analysis of northern hardwoods, concluded that measuring stump heights improved the accuracy of predicting dbh. Raile (1978) used a regression model similar to McClure's (1968) in his analysis of over twenty species in Minnesota, Wisconsin, and Michigan.

Summary

There were a multitude of different methods to predict dbh from stump parameters. They ranged from simple ratio estimation of dbh: stump diameter to complex equations using weighted variables. Nyland (1975) was the only study that directly presented tree volume based upon stump measurements. Myers (1963) mentioned a procedure for obtaining volume after determining dbh. Raile's (1978) and Horn and Keller's (1959) study areas included the present study area in northern

Minnesota.

The literature review indicated that prediction of dbh from stump measurement is common. Predictions of volume are also possible but less common. Therefore, there exists a need for methods to predict volume directly from stump measurements. These methods for predicting volume of removed trees would be useful not only to foresters but also to the owner of private timber lands.

METHODS AND MATERIALS

The data for this study were obtained from a wood utilization study conducted in northern Minnesota during 1976-1977 by Mr. James E. Blyth of the North Central Forest Experiment Station. The data were collected by a survey crew of which I was a member.

Selection of Sites and Trees

The data came from logging operations during which the survey crew measured felled trees. Forty-one of a possible of sixty-five logging sites were used in this study. Aspen (*Populus tremuloides*), paper birch (*Betula papyrifera*) red pine (*Pinus resinosa*), jack pine (*Pinus banksiana*), white pine (*Pinus strobus*), balsam fir (*Abies balsamea*) and black spruce (*Picea mariana*) were sampled. The number of sites and range of site indices are presented in Table 1. The general location of logging sites for the seven species used in this study is shown in Figure 1.

Only a portion of the logged trees were sampled on each logging site. Sample trees were selected on the basis of convenience and safety. The first tree was arbitrarily selected from the felled trees on the logging site. The second tree and all successive trees were selected from those trees in the proximity of the first one. Selection of trees continued until an estimated total volume of 200 cubic feet for each species occurring on each logging operation was

Table 1. Species, Number of Sites, and Range of Site Indices Sampled

Species	Number of Sites	Site Index Range (ft.)[1]
Aspen	14	59-91
Paper Birch	6	49-73
Red Pine	9	55-69
Jack Pine	10	54-82
White Pine	5	48-59
Balsam Fir	5	42-73
Black Spruce	3	40-50
Total	41	

[1]Base age 50 utilizing regional site index curves.

obtained.

Table 2 presents the number of trees sampled for each species by stump diameter class. Table 3 presents information on stump diameter, stump height, tree dbh, merchantable sawlog height, and sawlog top diameter outside bark for each species sampled.

Data Collection

Information was collected on the following tree parameters: stump diameter outside bark (sdob); stump height (sh); diameters outside bark (dob) at heights of 0.5, 1.0, 1.5, 2.0, and 2.5 feet; dbh; dob at merchantable sawlog height; dob at merchantable pulpwood heights; dob at upper and lower end of variable bole segments; bark thickness at all diameter measurements; and length of each bole

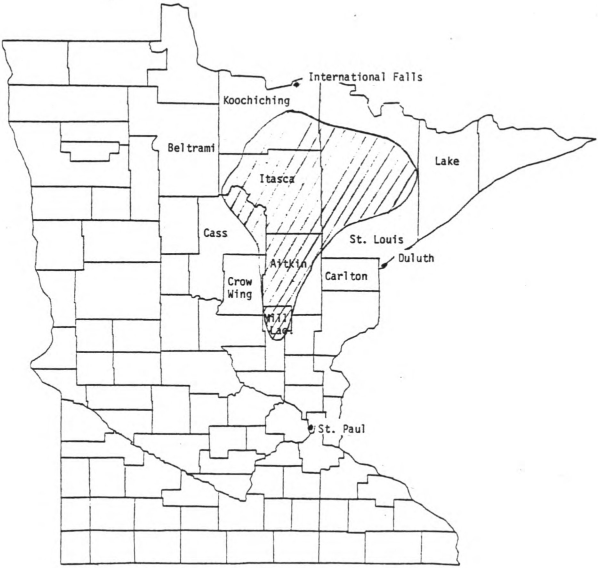

Figure 1. General Location of Logging Sites Sampled in Minnesota

Table 2. Number of Trees Sampled by Species and Stump Diameter Class

Stump Diameter (inches)	Balsam Fir	Jack Pine	Red Pine	Black Spruce	Aspen	Paper Birch	White Pine
4.5-5.5		2		1	1		
5.5-6.5		4	2	5	1	4	
6.5-7.5		10	3	8	2	12	
7.5-8.5	3	17	5	16	4	15	
8.5-9.5		23	4	9	11	26	
9.5-10.5	1	9	4	9	5	19	
10.5-11.5	7	20	5	4	14	13	1
11.5-12.5	6	13	7	1	13	11	1
12.5-13.5	9	8	5		18	8	
13.5-14.5	4	13	5		12	3	2
14.5-15.5	3	10	3		13	6	1
15.5-16.5	6	7	4		4	2	
16.5-17.5	1	2	2		10	1	
17.5-18.5	1	4	3		7		1
18.5-19.5	1	4	3		1		2
19.5-20.5			2				
20.5-21.5			1				
21.5-22.5		1	2		1		
> 22.5			2		1		1
Total	42	147	62	53	117	120	9

Table 3. Mean and Range of Selected Tree Parameters for Species Studied

Species	Stump Diameter (inches)		Stump Height (feet)		dbh(ob)[1] (inches)	Sawlog Merchantability Data: Top dob[2] (inches)	Sawlog Merchantability Data: Merchantable Height (feet)
	Mean	Range	Mean	Range	Range	Range	Range
Aspen	12.99	5.2-22.9	.24	0.1-1.2	5.5-17.9	9.0-11.5	15.0-51.0
Paper Birch	10.16	5.9-16.6	.42	0.0-2.1	5.0-12.5	9.0-9.9	16.0-38.0
Black Spruce	8.48	5.4-11.7	.48	0.2-0.9	5.0-10.1	7.0-7.3	20.0-39.0
Red Pine	13.34	6.3-23.3	.29	0.1-1.5	5.5-22.6	7.0-11.7	22.0-64.0
Jack Pine	11.37	5.1-21.8	.31	0.0-1.1	4.9-18.4	7.0-11.1	13.0-51.0
Balsam Fir	13.14	7.9-18.6	.55	0.1-1.1	5.7-13.4	7.0-7.0	14.0-46.0
White Pine	17.20	10.6-32.0	.98	0.4-2.0	9.4-27.4	7.0-11.8	22.0-70.0

[1] Diameter at breast height outside bark.

[2] Diameter outside bark at merchantable sawlog height.

segment. Merchantable sawlog heights were measured to a 7.0" minimum dob for softwoods and 9.0" minimum dob for hardwoods.[1] Merchantable pulpwood heights were measured to 4.0" minimum dob for all species.[2] Merchantable sawlogs were a minimum length of eight feet and merchantable pulpwood bolts were a minimum length of four feet. Diameters were measured with either diameter tape (D-tape) or tree calipers. Bark thickness was measured with a Swedish bark guage.

Site index (SI) parameters were collected during the summer of 1978. Site index parameters (age at dbh and total tree height) were measured on dominant trees within the logging sites. The site index was interpreted from regional site index curves.[3]

All diameters were measured and rounded down to the nearest tenth of an inch. In measuring stumps, the D-tape was located at the edge of the cut surface closest to the ground (Figures 2a, 2b, 2c, and 2d). All diameters were measured on a plane perpendicular to the centerline of the bole. Stump height was measured to the nearest tenth of a foot, from ground level to the point of measurement of stump diameter (Figures 2a, 2b, and 2c).

On a hill, measurement was on the uphill side of the tree (Figures 2d and 2e). On leaning trees, stump diameter was measured at the shortest length parallel to the stump (Figure 2f). When two stumps occurred on the same tree with a fork within one foot of ground level, the situation was treated as two separate trees. The

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[1] Forest Survey Handbook guidelines.

[2] Ibid.

[3] Ibid.

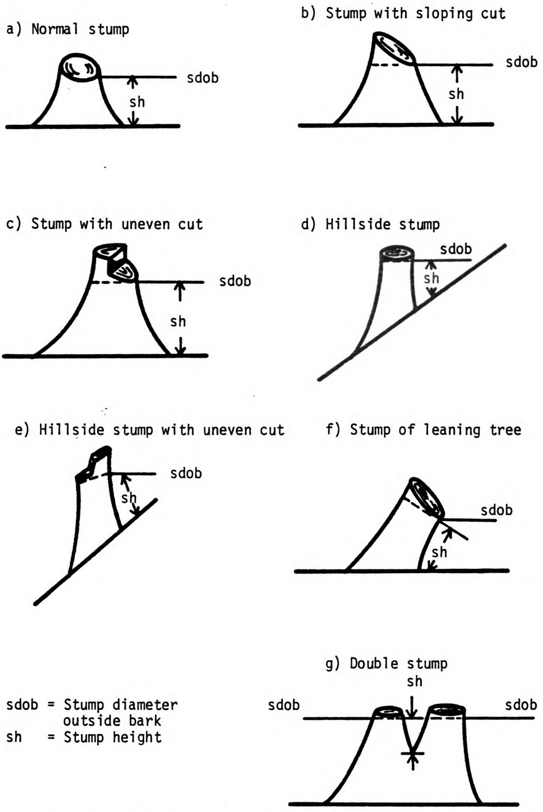

Figure 2. Schematic representation of stump variations and methods of field measurement

stump height of trees which forked between dbh and one foot was measured to the lowest point in the fork which produced the two separate trees (Figure 2g).

Bark thickness was recorded to the nearest tenth of an inch. The length of each bole segment was measured to the nearest tenth of a foot. Total tree height was recorded to the nearest foot and age at dbh was recorded to the nearest year.

Many problems were encountered during data collection. Stump diameter was measured with a D-tape whenever possible, and by tree caliper otherwise. It was assumed that all stumps were perfectly round while, in fact, most stumps were either fluted, oblong, or irregular in shape. Measurements were taken on all stumps that were not split, regardless of their shape.

Stumps cut flush with the ground, beside being irregular in shape, were difficult to measure with either the D-tape or tree caliper. These trees resulted in a volume:stump diameter ratio larger than that which actually occurred. A later reference to this problem is made in this report. When all or part of the bark was missing from a stump (as was commonly caused by a feller buncher), a "best estimation" of stump diameter was made using the bark thickness and measured stump diameter.

Tree Volume Calculations

The volume of each tree was calculated by using Smalian's formula for each segment and adding all segment volumes to obtain the volume for pulpwood and for sawtimber. The formula is:

$$V = \Sigma\left(\frac{L \cdot \pi}{(2)\cdot(144)}\left(\left(\frac{dib1}{2}\right)^2 + \left(\frac{dib2}{2}\right)^2\right)\right)$$

where

dib_1 = Diameter inside bark at lower or larger end (inches)

dib_2 = Diameter inside bark at upper or smaller end (inches)

L = Length of tree segment (feet)

V = Volume ($feet^3$)

Σ = Summation symbol

π = Pi

Statistical and Computer Methods

Prediction equations were developed by regression analysis. Equations are of the form:

$$f(V) = b_0 + b_1 \cdot f(sdob) + b_2 \cdot sh + b_3 \cdot SI$$

where

f(V) = Volume, $volume^{-1}$, or $volume^2$ (cubic feet)

f(sdob) = stump diameter outside bark or (stump diameter outside $bark)^2$ (inches)

sh = Stump height (feet)

SI = Site index (based on age 50)

b_i = Regression coefficients; i = 0, 1, 2, 3

Regression equations were developed for each of the six species and for several species combinations. The methodology and theory of regression equations and analysis of variance are explained by Draper and Smith (1966), Cochran and Cox (1957) and Snedecor and Cochran (1971). Statistical Package for the Social Science (Nie, et al., 1975) was used for the analysis of the data using a CDC 6500 computer. FORTRAN programs were used to construct volume tables based on the regression equations.

Several variations of the above equations were examined and compared for each species. The specific equations that were developed for each species and combinations of species were as follows:

a) $V = b_0 + b_1 \cdot sdob + b_2 \cdot sh + b_3 \cdot SI$

b) $V = b_0 + b_1 \cdot sdob^2 + b_2 \cdot sh + b_3 \cdot SI$

c) $V = b_1 \cdot sdob^2 + b_2 \cdot sh + b_3 \cdot SI$

The following equations were developed for selected species to evaluate the potential for using them as alternative equation forms:

d) $V = b_1 \cdot sdob + b_2 \cdot sh + b_3 \cdot SI$

e) $V^2 = b_0 + b_1 \cdot sdob^2 + b_2 \cdot sh + b_3 \cdot SI$

f) $1/V = b_0 + b_1 \cdot sdob^2 + b_2 \cdot sh + b_3 \cdot SI$

Volume equations were developed for both pulpwood volumes (Vp) and for sawtimber volumes (Vs). The criteria used in developing and selecting the best equations were:

1. The significance of coefficients in the equations based upon the partial F-test with significance level of .05
2. Lack of trends in scatter plots of residuals
3. Minimum number of independent variables
4. Small standard error of estimate (SE)
5. Large coefficient of determination (R^2)
6. Similar equations variables between species
7. Ease of field measurement of variable in the equation

Draper and Smith (1966, p. 163) stated "there is no unique procedure for selecting the best regression equation and personal judgment will be a necessary part of any statistical method . . ."

The partial F-test on the regression coefficients was the first criteria used to eliminate nonsignificant variables. Differences (residuals) between the predicted volumes and the actual volumes were examined and related to the standard error of the equation. Residuals were also examined in a test procedure developed by Cady and Allen (1972). Personal judgment and experience, as well as the previously stated criteria, were used in the selection of the best equations.

Resultant pulpwood volume equations were verified by two methods. First, calculated individual tree volumes were compared to predicted volumes based on a range of stump heights and diameters. Second, volumes based on species specific equations were compared with tree volumes of an independent data set of the same species. These independent data sets were obtained from different logging sites.

It was desirable to have similar predictor variables between species. The user of these equations and volume tables would only need to measure the same variables for all species.

When species have equations of the same general form, it is desirable to compare equations to determine if they could be combined into one. Bartlett's chi-squared was used to test for homogeniety of variances. F-tests were then used to determine if two regression equations were the same. Comparison could not be made between regression equations having different variables or forms.

STATISTICAL ANALYSIS AND DISCUSSION

Equation Development and Selection

Regression equations were first developed using pulpwood volume as the dependent variable and different combinations of the three independent variables. These equations were developed with non-zero intercepts. Regression equations were then developed by forcing the equations through the origin using stump diameter squared and stump height as the independent variables. Analysis indicated that the variable stump diameter squared was better. The partial F-test of the regression coefficients showed that, in most cases, the site index variable wasn't significant and it was not included in any equation.

The equation selection, using the stated criteria, is discussed in detail for the aspen pulpwood volume equation. Selection for all other equations followed the same process. All equations developed for pulpwood volume are found in Appendix A, and all equations developed for sawlog volume are found in Appendix B.

The notations used for variables in the equations are:

sdob = Stump diameter outside bark (inches)

sh = Stump height (feet)

SI = Site index

Vp = Pulpwood volume (cubic feet)

Vs = Sawlog volume (cubic feet)

R^2 = Coefficient of determination

SE = Standard error of estimate

For aspen, based on a sample size of 117 trees, the equations developed were:

	R^2	SE
(1) $Vp = -38.8206 + 5.6200\ sdob + 17.7354\ sh$	.875	7.38
(2) $Vp = -35.2275 + 6.1032\ sdob$	.839	8.33
(3) $Vp = -3.7023 + 0.2110\ sdob^2 + 18.3884\ sh$	.874	7.41
(4) $Vp = 3.1512 + 0.2294\ sdob^2$	.837	8.39
(5) $Vp = 0.2031\ sdob^2 + 15.0507\ sh$	.977	7.48

Other equations were disregarded because the R^2 were less than .30, usually with a standard error of greater than 17 cubic feet.

The regression coefficients of the variables in all equations were tested for significance using partial F-test with an alpha significance level of 0.05. Site index coefficients were not significant and equations with site index variables were eliminated. Both stump height and stump diameter coefficients were significant. Equations 2 and 4 are simpler forms of the equations that might be selected for use if only stump diameter information were available. These two equations were not considered here because higher R^2 and lower SE are obtained by using equations 1, 3, or 5.

Criteria 2, 4, and 5 were used simultaneously to determine the best equation of equations 1, 3, and 5. Studying the scatter plots of the residuals, there was a slight indication that equations 3 and 5 had a better pattern (less scattering from the equation estimates and less numbers of residuals of $\pm$ two standard error) than equation 1. There was little difference in the scatter plots between equations 3 and 5 and little difference in standard error (SE) among the three equations. The coefficient of determination (R^2) was best for equation 5. Equation 5 was the equation selected for aspen pulpwood volume.

The same procedures were used to determine the best equations for all other species or combination of species. Table 4 presents the equations selected for pulpwood volume (cubic feet) and Table 5 presents the equations selected for sawlog volume (cubic feet). The site index variable was not selected for jack pine because of the desire for minimum number of independent varialbes and similar equations variables between species.

Equations with Transformation of Volume

Several transformations on the volume were developed. A regression was fitted to pulpwood volume squared for paper birch using a sample of 120 trees:

$$Vp^2 = -1079.1521 + 8.7424\ sdob^2 + 268.7942\ sh + 8.710\ SI;$$
$$R^2 = .705,\ SE = 322.17$$
$$Vp^2 = -544.5193 + 8.8550\ sdob^2 + 291.9912\ sh;$$
$$R^2 = .693,\ SE = 327.63$$
$$Vp^2 = -440.6160 + 9.0436\ sdob^2;$$
$$R^2 = .663,\ SE = 341.63$$

Another transformation involved reciprocal of the volume as the dependent variable in the regression equation. This was done for black spruce pulpwood volume using a sample of 53 trees:

$$1/Vp = .1080 - .0013\ sdob^2 - .0702\ sh + .0025\ SI;$$
$$R^2 = .646,\ SE = .03$$
$$1/Vp = .2440 - .0013\ sdob^2 - .1253\ sh;$$

Table 4. Pulpwood Volume (Cubic Feet) Equations[1] with Coefficients of Determination (R^2), Standard Errors (SE), and Sample Sizes

Species	b_0	b_1	b_2	R^2	SE	Sample Size
Aspen		.2031	15.0507	.977	7.48	117
Paper birch		.1689	5.9720	.955	5.02	120
Hardwood[2]		.2046	9.7315	.943	8.05	237
Red pine	-14.6711	.2761	21.1506	.968	7.23	62
Jack pine		.1857	6.8689	.974	5.38	147
Black spruce		.1353	9.0769	.941	3.78	53
Balsam fir		.1352	10.1287	.973	5.25	42
Conifer[3]	-13.4839	.2552	19.8723	.946	7.57	271

[1] $Vp = b_0 + b_1 \cdot (\text{stump diameter})^2 + b_2 \cdot (\text{stump height})$

[2] Applicable for aspen and paper birch.

[3] Applicable for red pine, jack pine, white pine, and black spruce.

Table 5. Sawlog Volume (Cubic Feet) Equations[1] with Coefficients of Determination (R^2), Standard Errors (SE), and Sample Sizes

Species	b_0	b_1	b_2	R^2	SE	Sample Size
Aspen	-13.2597	.2075	20.7462	.733	9.74	61
Hardwood[2]	-16.8581	.2098	22.9846	.700	10.22	72
Red pine	-20.4576	.2833	20.6706	.954	8.40	45
Jack pine		.1651	8.3650	.967	6.99	82
Balsam fir		.1206	6.8955	.961	5.92	35
Conifer[3]	-22.5873	.2669	21.5344	.931	9.69	152

[1] $Vs = b_0 + b_1 \cdot (\text{stump diameter})^2 + b_2 \cdot (\text{stump height})$

[2] Applicable for aspen and paper birch.

[3] Applicable for red pine, jack pine, white pine, and black spruce.

$R^2 = .615$, SE = .03

$1/Vp = .1788 - .0012\ sdob^2$;

$R^2 = .418$, SE = .04

Based upon R^2, the above transformations generally did not improve the accuracy of the selected equations (Table 4) for paper birch and for black spruce. Further transformations of the dependent variable were not tried.

Summary of Equation Selection and Justification

Stump diameter squared, instead of stump diameter, was used as an independent variable in the equations selected because it gave a better volume predictiability. Diameter squared and volume are often highly correlated. This fact was used in the judgment decision whether stump diameter squared or stump diameter was better in the selected equations.

Height of the stump was used as an independent variable because it contributed significantly to all equations. This agreed with the results of Nyland (1975).

Reviewing all equations that had site index as an independent variable, the partial F-test on the coefficients (criterion 1) indicated that the site index variable was not significant enough to justify its presence in the selected equations. This trend was evident in all species equations for both pulpwood and sawlog volumes. The range of the site indices was limited due to the fact that the logging sites were usually on higher site index lands.

Sawlog volume prediction equations for paper birch, white pine, and black spruce were not developed because of the sample sizes

(11, 9, and 6) were too small to make statistical inference with any degree of accuracy. Pulpwood volume prediction equation for white pine was not developed because of insufficient sample size (n = 9).

The scatter plots of the combined conifer residuals indicated that balsam fir volumes were always overestimated. Therefore, combined conifer pulpwood and sawlog equations were developed with the exclusion of balsam fir data. The conifer equations were developed from data that included the nine white pine trees. Prediction of white pine volume can therefore be made from the conifer equations in Tables 4 and 5.

Anomalies in the Statistical Analysis

During the analysis of the data, certain trends emerged. The coefficients of determination (R^2) were usually larger for all equations that were forced through the origin. The difference in R^2 ranged from .173 for paper birch to .007 for red pine in the pulpwood equations. In almost every case, the standard error was also larger for equations with non-zero intercept.

For those equations with non-zero intercept, the residuals (observed volume minus estimated volume) increased in a linear trend as the stump diameter increased. The equations generally overestimated the volumes of smaller trees (cira 4.0" - 7.0" stump diameters). The residuals of the equations in predicting volume of larger trees (cira 20.0" stump diameter and larger) indicated that an underestimation occurred.

The red pine diameter distribution differed from that of the other species. Table 2 shows there was little variation in the number of trees in each diameter class. The fact that the standard errors differed between equations (see Appendix A) with stump diameter and those with stump diameter squared may be attributed to this distribution.

In most cases where R^2 was best for equations with zero intercept and simultaneously SE was better for equations with non-zero intercept, the scatter plots of residuals were better for the equation with non-zero intercept.

Residuals tended to increase linearly as the stump diameter increases because larger trees have more variability in volume than smaller trees. Hann (1976) reported that the squared residuals increase as a linear function of stump diameter and he used the reciprocal of stump diameter in weighted least squares regression equations. Another technique that could be used to correct this trend would be to divide the data into two parts: one with smaller trees and the other with larger trees. This would create two separate equations for each species and for both pulpwood and sawlog volumes. The user would then have four equations (two for pulpwood and two for sawlog) for each species and their use would be more complex. These techniques were not utilized in this study but warrant future investigation.

After the volume tables were prepared using the selected regression equations, negative volumes resulted in the smaller diameter classes. These entries were eliminated because they were extrapolations outside the distribution of the species stump diameter and stump height ranges.

Commonalities of Selected Equations

Tests were made to determine if regression equations for any of the species were the same. Only those selected equations that had the same form and the same variables were tested. Bartlett's chi-squared (Snedecor and Cochran, 1971, p. 296) and the two tail F-test of homogeneity of variances were used to compare residual variances. The two tests produced the same results for comparison of two equations.

The results of conifer species comparison with each other and with the group conifer data (without balsam fir) showed that variances were heterogeneous for all except the jack pine - balsam fir, and red pine - conifer combinations. Since the variances of the two combinations noted above were homogeneous, the next step was to compare the slopes of the two individual equations. The slopes were different using the F-test with a significance level of 0.05. The variances between aspen and paper birch were heterogeneous. The variances between aspen and hardwood were homogeneous but the slopes were significant at the 0.05 level.

Therefore, no two equations or combination of equations can be combined into one equation.

Equation Verification

The equations were verified using two different techniques. The first compared the volume obtained from the regression equations, using stump heights from 0.5 feet to 2.5 feet and their respective diameters at these heights, with the actual measured tree volume. The second technique used independent data from trees in areas other than those used for equation development. Each species specific equation was compared with the same species in the other areas.

Equation Verification Using Varying Stump Heights

The selected equations (Tables 4 and 5) were verified using the volume data and the diameter at stump heights of 0.5, 1.0, 1.5, 2.0, and 2.5 feet. Residuals scatter graphs were used to determine how well the calculated volumes fit the data for each test height. All pulpwood volume equations using fixed heights of greater than 1.5 feet had residuals that were large indicating that the prediction equations used with stump heights of greater than 1.5 feet were very inaccurate. Stump heights greater than 1.5 feet were not within the range of sampled heights as shown in Table 3. Therefore, these equations are valid only for stump heights of less than 1.5 feet and the best volume prediction occurs at the mean stump height of each species.

The estimation of volume was good for fixed stump heights of 0.5, 1.0 and 1.5 feet. Good estimations of pulpwood volume for aspen were obtained with stump heights at 0.5 and 1.0 feet. Paper birch volume estimation was best at stump height of 1.0 foot and good at stump heights of 0.5 and 1.5 feet. Balsam fir volume estimation was equally good at stump heights of 0.5 and 1.0 feet. Black spruce, red pine, and jack pine volume estimations were best at a stump height of 0.5 foot.

Aspen and balsam fir sawlog volume prediction equations were good at stump heights of 0.5, 1.0, and 1.5 feet. Jack pine sawlog volume equation was good at stump heights of 0.5 and 1.0 feet. The best sawlog volume prediction equation for red pine was at a stump height of 0.5 feet where sawlog volumes were overestimated by approximately 5 to 15 cubic feet.

Equation Verification Using Data in Different Areas

Regression equations for pulpwood and sawlog volumes (Tables 4 and 5) were tested with data of their respective species that were from other areas. Residuals were examined and the quantity within $\pm$ one standard error and withing $\pm$ two standard errors were noted.

Cady and Allen's (1972) predictions sum of squares test was also used. The test used the following equation:

$$T^2 = \frac{\Sigma(V_i - \hat{V}_i)^2}{\Sigma V_i - \frac{(\Sigma V_i)^2}{n}}$$

where

V_i = Volume from the independent data set

$\hat{V}_i$ = Volume from the prediction equation

n = Numbers of V_i's

The regression equation being tested has a better fit as T^2 approches zero. Table 6 presents the results of this prediction sum of squares test of the selected equations with their respective independent test sets. Balsam fir volume equations had the best T^2 values. Black spruce pulpwood volume equation had the largest T^2 value of all pulpwood volume equations tested. This value is not conclusive as to whether or not the equation is a "good fit" because of the small sample. Although balsam sawlog volume equation has the best T^2 value, it also was not conclusive due to small test sample size. Paper birch, jack pine, and aspen pulpwood

Table 6. Results of Prediction Sum of Squares Test, Sample Size, and Stump Diameter Ranges of Independent Test Data

Species	T^2	Sample Size	Stump Diameter Range (inches)
Pulpwood equations[1]			
Aspen	.233	41	5.0-18.7
Paper birch	.183	18	6.0-15.7
Black spruce	.305	7	8.2-13.7
Balsam fir	.158	19	5.6-16.3
Jack pine	.224	23	9.0-23.4
Sawlog equations[1]			
Aspen	.479	25	10.8-18.7
Jack pine	.428	21	10.5-23.4
Balsam fir	.067	4	10.8-16.3

[1]Equations from Tables 4 and 5.

volume equations, in respect to their T^2 values, were concluded to have good fits. The T^2 values for jack pine and aspen sawlog volume equations, .43 and .48 respectively, indicate that the residuals were large for the above equations and that these equations did not fit their respective independent data sets as well as the other equations.

Another similar test involved the examination of the distribution of the residuals. A good prediction equation would have the residual values near zero. As the residual's values deviate from zero, the accuracy of the predictability of the equation decreased. A good prediction equation would have all residuals within $\pm$ one standard error.

All of the following species were tested with their respective pulpwood volume equation. Balsam fir, with 19 test trees, had all 19 residuals (100%) within $\pm$ one standard error. Paper birch, with 18 test trees, had 9 residuals (50%) within $\pm$ one standard error and 18 residuals (100%) within $\pm$ two standard errors. Jack pine, with 23 test trees, had 9 residuals (39%) within $\pm$ one standard error and 17 residuals (74%) within $\pm$ two standard errors. Black spruce, with 7 test trees, had 4 residuals (57%) within $\pm$ one standard error and 5 residuals (71%) within $\pm$ two standard errors. Aspen, with 286 test trees, had 185 residuals (65%) within $\pm$ one standard error and 271 residuals (95%) within $\pm$ two standard errors.

Table 7 presents the number of residuals and percentages within $\pm$ one standard error and within $\pm$ two standard error for test data of pulpwood and sawlog of selected species.

Testing sawlog volume equations using their respective data

Table 7. Number of Volume Residuals Within ± One and ± Two Standard Errors and Sample Size of Independent Test Data

Species	Sample Size	Number (Percent) of Residuals Within ± 1 SE[1]	± 2 SE
Pulpwood			
Balsam fir	19	19(100%)	19(100%)
Paper birch	18	9(50%)	18(100%)
Jack pine	23	9(39%)	17(74%)
Black spruce	7	4(57%)	5(71%)
Aspen	286	185(65%)	271(95%)
Sawlog			
Balsam fir	4	4(100%)	4(100%)
Jack pine	21	10(48%)	17(81%)
Aspen	80	56(70%)	71(89%)

[1]Standard Error

sets, the following residuals distributions were obtained. Balsam fir, with 4 test trees, had all 4 residuals (100%) within $\pm$ one standard error. Jack pine, with 21 test trees, had 10 residuals (48%) within $\pm$ one standard error and 17 residuals (81%) within $\pm$ two standard errors. Aspen, with 80 test trees, had 56 residuals (70%) within $\pm$ one standard error and 71 residuals (89%) within $\pm$ two standard errors.

Balsam fir had the best residuals distribution. Aspen and paper birch had good residuals distributions. Jack pine and black spruce residuals distributions were not as good as the others.

The residuals distributions were an indication of the "goodness of fit" or accuracy of the prediction equations. The conclusion was that balsam fir, aspen, and paper birch prediction equations had good predictability of volume. Jack pine and black spruce predictability of volume equations were not as accurate as other species prediction equations. It is noted that this comparison of residuals distribution results were analogous to the T^2 test results.

Based upon the T^2 test and the residual distributions comparison test, several equations were considered to be accurate for the prediction of volume. Pulpwood volume equations were acceptable for aspen, paper birch, balsam fir, and black spruce. Sawlog volume equations were acceptable for aspen, jack pine, and balsam fir.

Red pine did not have an independent test sample. Its equations were accepted on the comparison of various stump heights procedures.

TABULAR RESULTS AND DISCUSSION

The volume tables in this section were calculated directly from the selected regression equations found in Tables 4 and 5. The range of stump heights (Table 3) of each species were usually between 0.0 and 1.5 feet with the majority were between 0.0 and 1.0 feet. The mean stump height was approximately 0.4 foot. The predictions using stump heights of greater than 1.5 feet were very inaccurate. Therefore, the stump heights in the volume tables ranged from 0.0 to 1.2 feet due to the above reasons. Trees with flaring characteristics and those that were measured at stump heights less than 0.2 feet often have estimated volumes larger than the actual volume. If a stump is measured within the flaring part of the stump, than the volume tables should be used with caution.

The volume tables and regression equations can be used with confidence in northern Minnesota. The validity of the equations should be tested in any other area by using a sample of trees of the appropriate species within that area.

These tables are valid only for the prediction of volumes of aspen, paper birch, red pine, jack pine, balsam fir, and black spruce. Volume predictions for white pine can be obtained from the conifer volume tables. The hardwood volume tables are only valid for the prediction of aspen and paper birch. The conifer tables are valid for the prediction of volume of red pine, jack pine, white pine, and black spruce, but not balsam fir.

To use the volume tables, stump diameter outside bark and stump height are required. Stump diameter is measured to the nearest inch and stump height is measured to the nearest tenth of a foot. The intersection of the stump height and stump diameter is the tree's predicted volume (cubic feet).

If either the stump height or stump diameter are not presented in the volume table, than interpolation may be used. The regression equation can be used to calculate the volume as an alternative method. Caution is necessary if one extrapolates equations beyond the ranges given in the tables.

Pulpwood Volume Tables and Their Use

Pulpwood volume tables are presented for aspen (Table 8), paper birch (Table 9), hardwood (Table 10), red pine (Table 11), jack pine (Table 12), balsam fir (Table 13), black spruce (Table 14), and conifer (Table 15).

This cubic feet volume could be converted into cords. A cord is defined as a stack of logs 4 feet by 4 feet by 8 feet of wood, bark, and air. The range of solid wood in a cord is 60 to 95 cubic feet. Some typical values used for a cord are 79 cubic feet in the Lake States, 72 cubic feet for southern pines, and 79 cubic feet for pulping hardwoods (Avery, 1967).

For example, an aspen stump having a stump diameter of 14.0 inches outside bark and a stump height of 0.5 feet will have a volume of 47 cubic feet of pulpwood material.

Table 8. Aspen Pulpwood Volume (Cubic Feet) Predictions from Stump Diameter and Stump Height

Stump Diameter (inches)	Stump Height (Feet)												
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
4	3	5	6	8	9	11	12	14	15	17	18	20	21
5	5	7	8	10	11	13	14	16	17	19	20	22	23
6	7	9	10	12	13	15	16	18	19	21	22	24	25
7	10	11	13	14	16	17	19	20	22	23	25	27	28
8	13	15	16	18	19	21	22	24	25	27	28	30	31
9	16	18	19	21	22	24	25	27	28	30	31	33	35
10	20	22	23	25	26	28	29	31	32	34	35	37	38
11	25	26	28	29	31	32	34	35	37	38	40	41	43
12	29	31	32	34	35	37	38	40	41	43	44	46	47
13	34	36	37	39	40	42	43	45	46	48	49	51	52
14	40	41	43	44	46	47	49	50	52	53	55	56	58
15	46	47	49	50	52	53	55	56	58	59	61	62	64
16	52	53	55	57	58	60	61	63	64	66	67	69	70
17	59	60	62	63	65	66	68	69	71	72	74	75	77
18	66	67	69	70	72	73	75	76	78	79	81	82	84
19	73	75	76	78	79	81	82	84	85	87	88	90	91
20	81	83	84	86	87	89	90	92	93	95	96	98	99
21	90	91	93	94	96	97	99	100	102	103	105	106	108
22	98	100	101	103	104	106	107	109	110	112	113	115	116
23	107	109	110	112	113	115	116	118	119	121	122	124	125
24	117	118	120	121	123	124	126	128	129	131	132	134	135
25	127	128	130	131	133	134	136	137	139	140	142	143	145
26	137	139	140	142	143	145	146	148	149	151	152	154	155
27	148	150	151	153	154	156	157	159	160	162	163	165	166
28	159	161	162	164	165	167	168	170	171	173	174	176	177
29	171	172	174	175	177	178	180	181	183	184	186	187	189

Volume = .2031 (Stump Diameter)^2 + 15.0507 (Stump Height)
Coefficient of Determination (R^2) = .977
Standard Error = 7.48

Table 9. Paper Birch Pulpwood Volume (Cubic Feet) Predictions from Stump Diameter and Stump Height

Stump Diameter (inches)	Stump Height (Feet) 0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
4	3	3	4	4	5	6	6	7	7	8	9	9	10
5	4	5	5	6	7	7	8	8	9	10	10	11	11
6	6	7	7	8	8	9	10	10	11	11	12	13	13
7	8	9	9	10	11	11	12	12	13	14	14	15	15
8	11	11	12	13	13	14	14	15	16	16	17	17	18
9	14	14	15	15	16	17	17	18	18	19	20	20	21
10	17	17	18	19	19	20	20	21	22	22	23	23	24
11	20	21	22	22	23	23	24	25	25	26	26	27	28
12	24	25	26	26	27	27	28	29	29	30	30	31	31
13	29	29	30	30	31	32	32	33	33	34	35	35	36
14	33	34	34	35	35	36	37	37	38	38	39	40	40
15	38	39	39	40	40	41	42	42	43	43	44	45	45
16	43	44	44	45	46	46	47	47	48	49	49	50	50
17	49	49	50	51	51	52	52	53	54	54	55	55	56
18	55	55	56	57	57	58	58	59	60	60	61	61	62
19	61	62	62	63	63	64	65	65	66	66	67	68	68
20	68	68	69	69	70	71	71	72	72	73	74	74	75
21	74	75	76	76	77	77	78	79	79	80	80	81	82
22	82	82	83	84	84	85	85	86	87	87	88	88	89
23	89	90	91	91	92	92	93	94	94	95	95	96	97
24	97	98	98	99	100	100	101	101	102	103	103	104	104
25	106	106	107	107	108	109	109	110	110	111	112	112	113

Volume = .1689 (Stump Diameter)^2 + 5.9720 (Stump Height)
Coefficient of Determination (R^2) = .955
Standard Error = 5.02

Table 10. Hardwood[1] Pulpwood Volume (Cubic Feet) Predictions from Stump Diameter and Stump Height

Stump Diameter (inches)	Stump Height (Feet)												
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
4	3	4	5	6	7	8	9	10	11	12	13	14	15
5	5	6	7	8	9	10	11	12	13	14	15	16	17
6	7	8	9	10	11	12	13	14	15	16	17	18	19
7	10	11	12	13	14	15	16	17	18	19	20	21	22
8	13	14	15	16	17	18	19	20	21	22	23	24	25
9	17	18	19	19	20	21	22	23	24	25	26	27	28
10	20	21	22	23	24	25	26	27	28	29	30	31	32
11	25	26	27	28	29	30	31	32	33	34	34	35	36
12	29	30	31	32	33	34	35	36	37	38	39	40	41
13	35	36	37	37	38	39	40	41	42	43	44	45	46
14	40	41	42	43	44	45	46	47	48	49	50	51	52
15	46	47	48	49	50	51	52	53	54	55	56	57	58
16	52	53	54	55	56	57	58	59	60	61	62	63	64
17	59	60	61	62	63	64	65	66	67	68	69	70	71
18	66	67	68	69	70	71	72	73	74	75	76	77	78
19	74	75	76	77	78	79	80	81	82	83	84	85	86
20	82	83	84	85	86	87	88	89	90	91	92	93	94
21	90	91	92	93	94	95	96	97	98	99	100	101	102
22	99	100	101	102	103	104	105	106	107	108	109	110	111
23	108	109	110	111	112	113	114	115	116	117	118	119	120
24	118	119	120	121	122	123	124	125	126	127	128	129	130
25	128	129	130	131	132	133	134	135	136	137	138	139	140
26	138	139	140	141	142	143	144	145	146	147	148	149	150
27	149	150	151	152	153	154	155	156	157	158	159	160	161
28	160	161	162	163	164	165	166	167	168	169	170	171	172
29	172	173	174	175	176	177	178	179	180	181	182	183	184

[1] Applicable for aspen and paper birch.
Volume = .2046 (Stump Diameter)^2 + 9.7315 (Stump Height);
Coefficient of Determination (R^2) = .943; Standard Error = 8.05

Table 11. Red Pine Pulpwood Volume (Cubic Feet) Predictions from Stump Diameter and Stump Height

Stump Diameter (inches)	Stump Height (Feet) 0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
4	0	0	0	0	0	0	2	5	7	9	11	13	15
5	0	0	0	0	1	3	5	7	9	11	13	15	18
6	0	0	0	2	4	6	8	10	12	14	16	19	21
7	0	1	3	5	7	9	12	14	16	18	20	22	24
8	3	5	7	9	11	14	16	18	20	22	24	26	28
9	8	10	12	14	16	18	20	22	25	27	29	31	33
10	13	15	17	19	21	24	26	28	30	32	34	36	38
11	19	21	23	25	27	29	31	34	36	38	40	42	44
12	25	27	29	31	34	36	38	40	42	44	46	48	50
13	32	34	36	38	40	43	45	47	49	51	53	55	57
14	39	42	44	46	48	50	52	54	56	58	61	63	65
15	47	50	52	54	56	58	60	62	64	66	69	71	73
16	56	58	60	62	64	67	69	71	73	75	77	79	81
17	65	67	69	71	74	76	78	80	82	84	86	88	91
18	75	77	79	81	83	85	87	90	92	94	96	98	100
19	85	87	89	91	93	96	98	100	102	104	106	108	110
20	96	98	100	102	104	106	108	111	113	115	117	119	121
21	107	109	111	113	116	118	120	122	124	126	128	130	132
22	119	121	123	125	127	130	132	134	136	138	140	142	144
23	131	134	136	138	140	142	144	146	148	150	153	155	157
24	144	146	149	151	153	155	157	159	161	163	166	168	170
25	158	169	162	164	166	168	171	173	175	177	179	181	183
26	172	174	176	178	180	183	185	187	189	191	193	195	197
27	187	189	191	193	195	197	199	201	204	206	208	210	212
28	202	204	206	208	210	212	214	217	219	221	223	225	227
29	218	220	222	224	226	228	230	232	234	237	239	241	243

Volume = -14.6711 + .2761 (Stump Diameter)^2 + 21.1506 (Stump Height);
Coefficient of Determination (R^2) = .969; Standard Error = 7.23

Table 12. Jack Pine Pulpwood Volume (Cubic Feet) Predictions from Stump Diameter and Stump Height

Stump Diameter (inches)	Stump Height (Feet)												
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
4	3	4	4	5	6	6	7	8	8	9	10	11	11
5	5	5	6	7	7	8	9	9	10	11	12	12	13
6	7	7	8	9	9	10	11	11	12	13	14	14	15
7	9	10	10	11	12	13	13	14	15	15	16	17	17
8	12	13	13	14	15	15	16	17	17	18	19	19	20
9	15	16	16	17	18	18	19	20	21	21	22	23	23
10	19	19	20	21	21	22	23	23	24	25	25	26	27
11	22	23	24	25	25	26	27	27	28	29	29	30	31
12	27	27	28	29	29	30	31	32	32	33	34	34	35
13	31	32	33	33	34	35	36	36	37	38	38	39	40
14	36	37	38	38	39	40	41	41	42	43	42	44	45
15	42	42	43	44	45	45	46	47	47	48	49	49	50
16	48	48	49	50	50	51	52	52	53	54	54	55	56
17	54	54	55	56	56	57	58	58	59	60	61	61	62
18	60	61	62	62	63	64	64	65	66	66	67	68	68
19	67	68	68	69	70	70	71	72	73	73	74	75	75
20	74	75	76	76	77	78	78	79	80	80	81	82	83
21	82	83	84	84	85	85	86	87	87	88	89	89	90
22	90	91	91	92	93	93	94	95	95	96	97	97	98
23	98	99	100	100	101	102	102	103	104	104	105	106	106
24	107	108	108	109	110	110	111	112	112	113	114	115	115
25	116	117	117	118	119	119	120	121	122	122	123	124	124
26	126	126	127	128	128	129	130	130	131	132	132	133	134
27	135	136	137	137	138	139	139	140	141	142	142	143	144
28	146	146	147	148	148	149	150	150	151	152	152	153	154
29	156	157	158	158	159	160	160	161	162	162	163	164	164

Volume = .1857 (Stump Diameter)^2 + 8.8689 (Stump Height)
Coefficient of Determination (R^2) = .974; Standard Error = 5.38

Table 13. Balsam Fir Pulpwood Volume (Cubic Feet) Predictions from Stump Diameter and Stump Height

Stump Diameter (inches)	Stump Height (Feet) 0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
4	2	3	4	5	6	7	8	9	10	11	12	13	14
5	3	4	5	6	7	8	9	10	11	12	13	14	15
6	5	6	7	8	9	10	11	12	13	14	15	16	17
7	7	8	9	10	11	12	13	14	15	16	17	18	19
8	9	10	11	12	13	14	15	16	17	18	19	20	21
9	11	12	13	14	15	16	17	18	19	20	21	22	23
10	13	14	15	16	17	18	19	20	21	22	23	24	25
11	16	17	18	19	20	21	22	23	24	25	26	27	28
12	19	20	21	22	23	24	25	26	27	28	29	30	31
13	23	24	25	26	27	28	29	30	31	32	33	34	35
14	26	27	28	29	30	31	32	33	34	35	36	47	38
15	30	31	32	33	34	35	36	37	38	39	40	41	42
16	34	35	36	37	38	39	40	41	42	43	44	45	46
17	39	40	41	42	43	44	45	46	47	48	49	50	51
18	43	44	45	46	47	48	49	50	51	52	53	54	55
19	48	49	50	51	52	53	54	55	56	57	58	59	60
20	53	54	55	56	57	58	59	60	61	62	63	64	65
21	59	60	61	62	63	64	65	66	67	68	69	70	71
22	64	65	67	68	69	70	71	72	73	74	75	76	77
23	70	71	73	74	75	76	77	78	79	80	81	82	83
24	77	78	79	80	81	82	83	84	85	86	87	88	89
25	83	84	85	86	87	88	89	90	91	92	93	94	95

Volume = .1332 (Stump Diameter)^2 + 10.1287 (Stump Height)
Coefficient of Determination (R^2) = .973
Standard Error = 5.25

Table 14. Black Spruce Pulpwood Volume (Cubic Feet) Predictions from Stump Diameter and Stump Height

Stump Diameter (inches)	Stump Height (Feet) 0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
4	2	3	4	5	6	7	8	9	9	10	11	12	13
5	3	4	5	6	7	8	9	10	11	12	12	13	14
6	5	6	7	8	9	9	10	11	12	13	14	15	16
7	7	8	8	9	10	11	12	13	14	15	16	17	18
8	9	10	10	11	12	13	14	15	16	17	18	19	20
9	11	12	13	14	15	15	16	17	18	19	20	21	22
10	14	14	15	16	17	18	19	20	21	22	23	24	24
11	16	17	18	19	20	21	22	23	24	25	25	26	27
12	19	20	21	22	23	24	25	26	27	28	29	29	30
13	23	24	25	26	26	27	28	29	30	31	32	33	34
14	27	27	28	29	30	31	32	33	34	35	36	36	37
15	30	31	32	33	34	35	36	37	38	39	40	40	41
16	35	36	36	37	38	39	40	41	42	43	44	45	46
17	39	40	41	42	43	44	45	45	46	47	48	49	50
18	44	45	46	47	47	48	49	50	51	52	53	54	55
19	49	50	51	52	52	53	54	55	56	57	58	59	60
20	54	55	56	57	58	59	60	60	61	62	63	64	65

Volume = .1353 $(\text{Stump Diameter})^2$ + 9.0769 (Stump Height)
Coefficient of Determination (R^2) = .941
Standard Error = 3.78

Table 15. Conifer[1] Pulpwood Volume (Cubic Feet) Predictions from Stump Diameter and Stump Height

Stump Diameter (inches)	Stump Height (Feet)												
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
4	0	0	0	0	0	1	3	5	6	8	10	12	14
5	0	0	0	0	1	3	5	7	9	11	13	15	17
6	0	0	0	2	4	6	8	10	12	14	16	18	20
7	0	1	3	5	7	9	11	13	15	19	17	21	23
8	3	5	7	9	11	13	15	17	19	21	23	25	27
9	7	9	11	13	15	17	19	21	23	25	27	29	31
10	12	14	16	18	20	22	24	26	28	30	32	34	36
11	17	19	21	23	25	27	29	31	33	35	37	39	41
12	23	25	27	29	31	33	35	37	39	41	43	45	47
13	30	32	34	36	38	40	42	44	46	48	50	51	53
14	37	39	41	42	44	46	48	50	52	54	56	58	60
15	44	46	48	50	52	54	56	58	60	62	64	66	68
16	52	54	56	58	60	62	64	66	68	70	72	74	76
17	60	62	64	66	68	70	72	74	76	78	80	82	84
18	69	71	73	75	77	79	81	83	85	87	89	91	93
19	79	81	83	85	87	89	91	93	95	97	99	100	102
20	89	91	93	95	97	99	101	102	104	106	108	110	112
21	99	101	103	105	107	109	111	113	115	117	119	121	123
22	110	112	114	116	118	120	122	124	126	128	130	132	134
23	121	123	125	127	129	131	133	135	137	139	141	143	145
24	133	135	137	139	141	143	145	147	149	151	153	155	157
25	146	148	150	152	154	156	158	160	162	164	166	168	170
26	159	161	163	165	167	169	171	173	175	177	179	181	183
27	173	175	177	178	180	182	184	186	188	190	192	194	196
28	187	189	191	193	195	196	198	200	202	204	206	208	210
29	201	203	205	207	209	211	213	215	217	219	221	223	225

[1]Applicable for red pine, jack pine, white pine, and black spruce
Volume = -13.4839 + .2252 (Stump Diameter)^2 + 19.8723 (Stump Height)
Coefficient of Determination (R^2) = .946; Standard Error = 7.57

Sawlog Volume Tables and Their Use

Sawlog volume tables are presented for aspen (Table 16), hardwood (Table 17), red pine (Table 18), jack pine (Table 19), balsam fir (Table 20), and conifer (Table 21). The volume of paper birch sawlogs can be obtained from the hardwood sawlog volume table. The volume of white pine and black spruce sawlogs can be obtained from the conifer sawlog volume table. The volume for black spruce sawlogs should be used with caution in the conifer sawlog volume table because black spruce sawlog diameters were near the low diameter classes.

There were a wide range of sawlog heights for each stump diameter class. The sawlog volume tables gives the volumes for the average sawlog height. The range of the sawlog heights are presented in Table 3.

The unit of measurement for sawlog volume is cubic feet. To convert volume into board feet, multiply it by twelve and compensate for saw kerf. This conversion would not produce the same results for all log rules (i.e. International 1/4 or Scribner rules).

If one would want the sawlog volume of a removed aspen tree which has a stump diameter of 20.0" and a stump height of 0.6 feet, Table 16 gives a volume of 82 cubic feet.

Table 16. Aspen Sawlog Volume (Cubic Feet) Predictions from Stump Diameter and Stump Height

Stump Diameter (inches)	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
11	12	14	16	18	20	22	24	26	28	31	33	35	37
12	17	19	21	23	25	27	29	31	33	35	37	39	42
13	22	24	26	28	30	32	34	36	38	40	43	45	47
14	27	29	32	34	36	38	40	42	44	46	48	50	52
15	33	36	38	40	42	44	46	48	50	52	54	56	68
16	40	42	44	46	48	50	52	54	56	59	61	63	65
17	47	49	51	53	55	57	59	61	63	65	67	70	72
18	54	56	58	60	62	64	66	68	71	73	75	77	79
19	62	64	66	68	70	72	74	76	78	80	82	84	87
20	70	72	74	76	78	80	82	84	86	88	90	93	95
21	78	80	82	84	87	89	91	93	95	97	99	101	103
22	87	89	91	93	95	98	100	102	104	106	108	110	112
23	97	99	101	103	105	107	109	111	113	115	117	119	121
24	106	108	110	112	115	117	119	121	123	125	127	129	131
25	116	119	121	123	125	127	129	131	133	135	137	139	141
26	127	129	131	133	135	137	139	142	144	146	148	150	152
27	138	140	142	144	146	148	150	153	155	157	159	161	163
28	149	152	154	156	158	160	162	164	166	168	170	172	174
29	161	163	165	167	170	172	174	176	178	180	182	184	186

Volume = -13.2597 + .2097 (Stump Diameter)^2 + 20.7462 (Stump Height)
Coefficient of Determination (R^2) = .733
Standard Error = 9.74

Table 17. Hardwood[1] Sawlog Volume (Cubic Feet) Predictions from Stump Diameter and Stump Height

Stump Diameter (inches)	Stump Height (Feet) 0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
11	9	11	13	15	18	20	22	25	27	29	32	34	36
12	13	16	18	20	23	25	27	29	32	34	36	39	41
13	19	21	23	25	28	30	32	35	37	39	42	46	46
14	24	27	29	31	33	36	38	40	43	45	47	50	52
15	30	33	35	37	40	42	44	46	49	51	53	56	58
16	37	39	41	44	46	48	51	53	55	58	60	62	64
17	44	46	48	51	53	55	58	60	62	64	67	69	71
18	51	53	56	58	60	63	65	67	70	72	74	76	79
19	59	61	63	66	68	70	73	75	77	80	82	84	86
20	67	69	72	74	76	79	81	83	85	88	90	92	95
21	76	78	80	83	85	87	89	92	94	96	99	101	103
22	85	87	89	92	94	96	98	101	103	105	108	110	112
23	94	96	99	101	103	106	108	110	113	115	117	119	122
24	104	106	109	111	113	115	118	120	122	125	127	129	132
25	114	117	119	121	123	126	128	130	133	135	137	140	142
26	125	127	130	132	134	136	139	141	143	146	148	150	153
27	136	138	141	143	145	148	150	152	154	157	159	161	164
28	148	150	152	155	157	159	161	164	166	168	171	173	175
29	160	162	164	166	169	171	173	176	178	180	183	185	187

[1]Applicable for aspen and paper birch.

Volume = -16.8581 + .2098 (Stump Diameter)^2 + 22.9846 (Stump Height)
Coefficient of Determination (R^2) = .700
Standard Error = 10.22

Table 18. Red Pine Sawlog Volume (Cubic Feet) Predictions from Stump Diameter and Stump Height

Stump Diameter (inches)	Stump Height (Feet)												
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
9	2	5	7	9	11	13	15	17	19	21	23	25	27
10	8	10	12	14	16	18	20	22	24	26	29	31	33
11	14	16	18	20	22	24	26	28	30	32	34	37	39
12	20	22	24	27	29	31	33	35	37	39	41	43	45
13	27	29	32	34	36	38	40	42	44	46	48	50	52
14	35	37	39	41	43	45	47	50	52	54	56	58	60
15	43	45	47	49	52	54	56	58	60	62	64	66	68
16	52	54	56	58	60	62	64	67	69	71	73	75	77
17	61	63	66	68	70	72	74	76	78	80	82	84	86
18	71	73	75	78	80	82	84	86	88	90	92	94	96
19	82	84	86	88	90	92	94	96	98	100	102	105	107
20	93	95	97	99	101	103	105	107	109	111	114	116	118
21	104	107	109	111	113	115	117	119	121	123	125	127	129
22	117	119	121	123	125	127	129	131	133	135	137	139	141
23	129	131	134	136	138	140	142	144	146	148	150	152	154
24	143	145	147	149	151	153	155	157	159	161	163	165	168
25	157	159	161	163	165	167	169	171	173	175	177	179	181
26	171	173	175	177	179	181	183	186	188	190	192	194	196
27	186	188	190	192	194	196	198	201	203	205	207	209	211
28	202	204	206	208	210	212	214	216	218	220	222	224	226
29	218	220	222	224	226	228	230	232	234	236	238	241	243

Volume = -20.4576 + .2833 (Stump Diameter)^2 + 20.6706 (Stump Height)
Coefficient of Determination (R^2) = .954
Standard Error = 8.40

Table 19. Jack Pine Sawlog Volume (Cubic Feet) Predictions from Stump Diameter and Stump Height

Stump Diameter (inches)	Stump Height (Feet)												
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
9	13	14	15	16	17	18	18	19	20	21	22	23	23
10	17	17	18	19	20	21	22	22	23	24	25	26	27
11	20	21	22	22	23	24	25	26	27	28	28	29	30
12	24	25	25	26	27	28	29	30	30	31	32	33	34
13	28	29	30	30	31	32	33	34	35	35	36	37	38
14	32	33	34	35	36	37	37	38	39	40	41	42	42
15	37	38	39	40	40	41	42	43	44	45	46	46	47
16	42	43	44	45	46	46	47	48	49	50	51	51	52
17	48	49	49	50	51	52	53	54	54	55	56	57	58
18	53	54	55	56	57	58	58	59	60	61	62	63	64
19	60	60	61	62	63	64	65	65	66	67	68	69	70
20	66	67	68	69	69	70	71	72	73	74	74	75	76
21	73	74	74	75	76	77	78	79	79	80	81	82	83
22	80	81	82	82	83	84	85	86	87	87	88	89	90
23	87	88	89	90	91	92	92	93	94	95	96	97	97
24	95	96	97	98	98	99	100	101	102	103	103	104	105
25	103	104	105	106	107	107	108	109	110	111	112	112	113
26	112	112	113	114	115	116	117	117	118	119	120	121	122
27	120	121	122	123	124	125	125	126	127	128	129	130	130
28	129	130	131	132	133	134	134	135	136	137	138	139	139
29	139	140	140	141	142	143	144	145	146	146	147	148	149

Volume = .1651 (Stump Diameter)^2 + 8.3649 (Stump Height)
Coefficient of Determination (R^2) = .967
Standard Error = 7.00

Table 20. Balsam Fir Sawlog Volume (Cubic Feet) Predictions from Stump Diameter and Stump Height

Stump Diameter (inches)	Stump Height (Feet)												
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
9	10	10	11	12	13	13	14	15	15	16	17	17	18
10	12	13	13	14	15	16	16	17	18	18	19	20	20
11	15	15	16	17	17	18	19	19	20	21	21	22	23
12	17	18	19	19	20	21	21	22	23	24	24	25	26
13	20	21	22	22	23	24	25	25	26	27	27	28	29
14	24	24	25	26	26	27	28	28	29	30	31	31	32
15	27	28	29	29	30	31	31	32	33	33	34	35	35
16	31	32	32	33	34	34	35	36	36	37	38	38	39
17	35	36	36	37	38	38	39	40	40	41	42	42	43
18	39	40	40	41	42	43	43	44	45	45	46	47	47
19	44	44	45	46	46	47	48	48	49	50	50	51	52
20	48	49	50	50	51	52	52	53	54	54	55	56	56
21	53	54	55	55	56	57	57	58	59	59	60	61	61
22	58	59	60	60	61	62	62	63	64	65	65	66	67
23	64	64	65	66	67	67	68	69	69	70	71	71	72
24	69	70	71	72	72	73	74	74	75	76	76	77	78
25	75	76	77	77	78	79	79	80	81	82	82	83	84

Volume = .1206 (Stump Diameter)^2 + 6.8955 (Stump Height)
Coefficient of Determination (R^2) = .967
Standard Error = 5.92

Table 21. Conifer[1] Sawlog Volume (Cubic Feet) Predictions from Stump Diameter and Stump Height

Stump Diameter (inches)	Stump Height (Feet)												
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
9	0	1	3	5	8	10	12	14	16	18	21	23	25
10	4	6	8	11	13	15	17	19	21	23	26	28	30
11	10	12	14	16	18	20	23	25	27	29	31	33	36
12	16	18	20	22	24	27	29	31	33	35	37	40	42
13	23	25	27	29	31	33	35	38	40	42	44	46	48
14	30	32	34	36	38	40	43	45	47	49	51	53	56
15	37	40	42	44	46	48	50	53	55	57	59	61	63
16	46	48	50	52	54	56	59	61	63	65	67	69	72
17	55	57	59	61	63	65	67	70	72	74	76	78	80
18	64	66	68	70	72	75	77	79	81	83	85	88	90
19	74	76	78	80	82	85	87	89	91	93	95	97	100
20	84	86	88	91	93	95	97	99	101	104	106	108	110
21	95	97	99	102	104	106	108	110	112	114	117	119	121
22	107	109	111	113	115	117	119	122	124	126	128	130	132
23	119	121	123	125	127	129	131	134	136	138	140	142	144
24	131	133	135	138	140	142	144	146	148	150	153	155	157
25	144	146	149	151	153	155	157	159	161	164	166	168	170
26	158	160	162	164	166	169	171	173	175	177	179	181	184
27	172	174	176	178	181	183	185	187	189	191	193	196	198
28	187	189	191	193	195	197	200	202	204	206	208	210	212
29	202	204	206	208	210	213	215	217	219	221	223	226	228

[1]Applicable for red pine, white pine, jack pine, and black spruce.

Volume = -22.5873 + .2669 (Stump Diameter)^2 + 21.5344 (Stump Height)
Coefficient of Determination (R^2) = .931
Standard Error = 9.69

CONCLUSIONS

The regression equations and volume tables developed and presented can be used to predict volumes of missing trees. Only stump diameter outside bark and stump height are needed to use these tables. The regression equations can also be used to predict volumes. To use the tables with confidence in areas other than northern Minnesota, sample trees must be used to verify the accuracy of the equations.

The equations and volume tables tend to underestimate the volume of smaller trees (less than 8.0" sdob) and overestimated the volume of larger trees (greater than 20.0" sdob). These errors are not serious if the stump height of the smaller trees are at least 0.4 feet. Estimation of the volume of a tree between 8.0" and 20.0" sdob one can be confident that 68% of the time the actual volume will be within $\pm$ one standard deviation and that 95% of the time the actual volume will be within $\pm$ two standard deviations.

The sawlog volume tables and regression equations predicted volumes with less accuracy than the pulpwood equations due to the wide range of sawlog heights and the range of the diameters at the top height (Table 3).

Volume equations for white pine were not developed due to an insufficient sample size. White pine sample trees were incorporated in the development of the conifer volume equations and tables. Paper birch sawlog volumes can be predicted from the hardwood sawlog volume equation and table. Black spruce sawlogs were incorporated in the development of the conifer sawlog volume equation. This equation could be used to predict black spruce sawlog volumes although the predicted volume would not be very accurate because the range of black spruce was near the low end of the stump diameter class.

Other areas of investigation are suggested by this study. Curtis and Arney (1977) used weights in the prediction of dbh from stump diameter. The trends of linearly increasing residuals with stump diameters indicates that better equations may be developed by using weighted regression equations. Another approach to improve the regression equations would be to separate the sample into two diameter ranges and develop separate equations for each range. These areas warrant future studies.

APPENDICES

APPENDIX A

EQUATIONS DEVELOPED FOR PULPWOOD VOLUME

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EQUATIONS DEVELOPED FOR PULPWOOD VOLUME

Species: Aspen Sample size: 117

	R^2	SE
$Vp = -38.8206 + 5.6200\ sdob + 17.7354\ sh$	.875	7.38
$Vp = -35.2275 + 6.1032\ sdob$	.839	8.33
$Vp = -3.7023 + .2110\ sdob^2 + 18.3884\ sh$	.875	7.41
$Vp = 3.1512 + .2294\ sdob^2$	.837	8.39
$Vp = .2031\ sdob^2 + 15.0507\ sh$	.977	7.48

Species: Paper birch Sample size: 120

$Vp = -20.0576 + 3.7777\ sdob + 6.0434\ sh$	.792	4.93
$Vp = -18.3278 + 3.8583\ sdob$	.754	5.33
$Vp = -.3460 + .1709\ sdob^2 + 6.1380\ sh$	.782	5.04
$Vp = -1.8488 + .1746\ sdob^2$	.744	5.44
$Vp = .1689\ sdob^2 + 5.9720\ sh$	.955	5.02

Species: Aspen and paper birch combined Sample size: 237
(Hardwood)

$Vp = -5.4581 + .2258\ sdob^2 + 11.2335\ sh$	.861	7.52
$Vp = -1.7741 + .2377\ sdob^2$	.834	8.19
$Vp = .2046\ sdob^2 + 9.7315\ sh$	.943	8.05

Species: Red pine Sample size: 62

	R^2	SE
$Vp = -64.5973 + 7.7598\ sdob + 23.5302\ sh$	.939	10.15
$Vp = -63.3709 + 8.5187\ sdob$	.918	11.69
$Vp = -14.6711 + .2761\ sdob^2 + 21.1506\ sh$	.968	7.23
$Vp = -9.1861 + .3000\ sdob^2$	.952	8.93
$Vp = .2531\ sdob^2 + 7.0679\ sh$	.976	10.09

Species: Jack pine Sample size: 147

$Vp = -46.4575 + 4.6604\ sdob + 7.6991\ sh + .2991\ SI$	.921	5.06
$Vp = -31.6157 + 4.8563\ sdob + 12.9914\ sh$	.911	5.34
$Vp = -28.7475 + 4.9592\ sdob$	.893	5.86
$Vp = -19.9108 + .1884\ sdob^2 + 6.2481\ sh + .3052\ SI$	.927	4.87
$Vp = -3.6049 + .1965\ sdob^2 + 11.5601\ sh$	.917	5.18
$Vp = -.5739 + .2005\ sdob^2$	.902	5.60
$Vp = .1857\ sdob^2 + 6.8689\ sh$	.974	5.38

Species: Black spruce Sample size: 53

$Vp = -23.0927 + 3.3282\ sdob + 18.0544\ sh$	.769	3.15
$Vp = -13.2445 + 3.2032\ sdob$	.542	4.39
$Vp = -9.4545 + .1953\ sdob^2 + 18.3115\ sh$	.781	3.07
$Vp = .0778 + .1869\ sdob^2$	.548	4.36
$Vp = .1353\ sdob^2 + 9.0969\ sh$	.941	3.78

Species: Balsam fir Sample size: 42

$Vp = -34.8265 + 4.4332\ sdob + 14.2794\ sh$	.856	4.92

	R^2	SE
$Vp = -29.5703 + 4.332\ sdob$	.783	5.97
$Vp = -8.5492 + .1590\ sdob^2 + 16.0590\ sh$	.870	4.67
$Vp = .8002 + .1648\ sdob^2$	.777	6.05
$Vp = .1332\ sdob^2 + 10.1287\ sh$	.973	5.25

Species: All conifers combined Sample size: 313

$Vp = -13.1434 + .2456\ sdob^2 + 16.8927\ sh$	.914	8.99
$Vp = -8.1981 + .2635\ sdob^2$	.895	9.88

Species: Conifer without balsam fir Sample size: 271

$Vp = -13.4839 + .2552\ sdob^2 + 19.8723\ sh$	.946	7.57

APPENDIX B

EQUATIONS DEVELOPED FOR SAWLOG VOLUME

APPENDIX B

EQUATIONS DEVELOPED FOR SAWLOG VOLUME

Species: Aspen Sample size: 61

	R^2	SE
$Vs = -66.6867 + 6.7185\ sdob + 21.1427\ sh$	.748	9.47
$Vs = -13.2597 + .2075\ sdob^2 + 20.7462\ sh$	.733	9.74
$Vs = 2.8395\ sdob + 10.5340\ sh$	.937	13.20
$Vs = .1762\ sdob^2 + 12.6348\ sh$	.963	10.16

Species: Aspen and paper birch combined (Hardwood) Sample size: 72

	R^2	SE
$Vs = -70.7616 + 6.7022\ sdob + 25.1618\ sh$	.710	10.10
$Vs = -16.8581 + .2098\ sdob^2 + 22.9846\ sh$	.670	10.22
$Vs = 2.6372\ sdob + 12.3716\ sh$	.926	13.66

Species: Red pine Sample size: 45

	R^2	SE
$Vs = -91.2989 + 9.1989\ sdob + 21.0971\ sh$	.944	9.34
$Vs = -20.4576 + .2833\ sdob^2 + 20.6706\ sh$	.954	8.40
$Vs = 3.5844\ sdob + 20.2372\ sh$	.896	23.66
$Vs = .2391\ sdob^2 + 9.2084\ sh$	.974	11.80

Species: Jack pine Sample size: 82

	R^2	SE
$Vs = -48.7921 + 5.6979\ sdob + 14.0967\ sh$	.854	6.41
$Vs = -8.4512 + .1929\ sdob^2 + 14.6583\ sh$	.847	6.49

	R^2	SE
$Vs = 2.484\ sdob + 4.8065\ sh$	.920	10.87
$Vs = .1651\ sdob^2 + 8.3650\ sh$	.967	6.99

Species: Balsam fir Sample size: 35

$Vs = -47.0082 + 4.6526\ sdob + 16.5611\ sh$	.789	5.26
$Vs = -14.1411 + .1619\ sdob^2 + 16.2356\ sh$	.801	5.10
$Vs = 1.8755\ sdob + 3.0220\ sh$	.926	8.12
$Vs = .1206\ sdob^2 + 6.8955\ sh$	.961	5.92

Species: All conifers combined Sample size: 177

$Vs = -91.5122 + 8.5750\ sdob + 22.9465\ sh$	.829	14.17
$Vs = -24.2758 + .2686\ sdob^2 + 18.0678\ sh$	.893	11.11
$Vs = 2.5297\ sdob + 20.8128\ sh$	.789	24.71
$Vs = .2079\ sdob^2 + 3.6418\ sh$	.921	15.11

Species: Conifer without balsam fir Sample size: 142

$Vs = -22.5873 + .2669\ sdob^2 + 21.5344\ sh$	.931	9.69

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