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A Preliminary Investigation
Of Narrow-Strip Sampling
As Applied To Forestry

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James Edward Kearis

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Victor J. Rudolph
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A PRELIMINARY INVESTIGATION
OF NARROW-STRIP SAMPLING
AS APPLIED TO FORESTRY

By

James Edward Kearis

A DISSERTATION

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ABSTRACT

A PRELIMINARY INVESTIGATION
OF NARROW-STRIP SAMPLING
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By

James Edward Kearis

Narrow-strip sampling consists of sampling a forest stand by means of strips so narrow that the trees may be linearly ordered along the strips. The term 'narrow-strip' has been used in two basically different senses in previous research. On the one hand the 'narrowness' meant that a strip of a width much narrower than usually used was run through the stand. On the other hand, as used in this paper, 'narrowness' means that the trees which are sampled by a narrow-strip are ordered, one after the other, along the narow-strip. Pielou (1962) used narrow-strip sampling in the second sense but her theoretical analyses are totally different from those developed in this paper. In this study, a theoretical basis is given for narrow-strip sampling, a major componet of which is the derivation of the expected distance between adjacent stems along a narrow-strip given the density of stems per unit area, the average diameter breast high of the stems, and the width of the narrow-strip.

Monte-Carlo techniques are used to verify theoretical statements and to examine biases and precisions of

narrow-strip estimators. A range of real and computer-generated forest stands is used in the study and consists of combinations of random, regular, and clustered stem patterns with a realistic range of densities and diameter distributions generated from an empirical distribution function. Density and diameter considerations follow from field data gathered in stands located in the New Jersey Pine Barrens.

Distance sampling is an unpractical method of sampling for density but has been studied because of its theoretical appeal. Three supposedly robust methods of estimating density using distance sampling are compared with a density estimator using narrow-strip sampling in a Monte-Carlo study of 34 real and computer-generated stem maps. It is found that narrow-strip sampling gives highly robust estimates of density outperforming the best of the distance estimators.

Narrow-strip sampling, circular-plot sampling, and point sampling are used to sample the 34 stem maps for basal area, species proportion, diameter, and density using both random and systematic location of the three types of sampling elements which define clusters of stems. Strip width and length, plot radius, and basal area factor are chosen such that all three methods have an equal expected sample size. Methods are compared on the bases of bias and precision. On these bases, narrow-strip sampling generally performs as well as the other two methods.

When data gathered in the field in one oak-dominated and in one pine-dominated stand are analyzed, it is found that narrow-strip sampling performs very well. Narrow-strip sampling takes about one-half the time of plot sampling to estimate basal area, density, diameter, species proportions, and stand table entries. These estimates are close to the plot sample estimates. One low-intensity narrow-strip sample takes half the time that a point sample does to estimate basal area but, in addition, yields good density and diameter estimates. Diameters were not taken in the point sample in order to avoid tree-to-tree travel time, but, since there is no travel time involved in measuring diameters of trees which are narrow-strip sampled, diameters were taken in the low-intensity narrow-strip sample.

It seems likely that many areas of application lie ahead for narrow-strip sampling, and a few are described briefly. However, additional research will be required to verify results of this study under different conditions. Narrow-strip sampling is an attractive and practical alternative to circular-plot sampling, point sampling, and distance sampling for estimation of basal area, species proportions, diameter, density, and stand table entries in forest stand situations included in this study. Theoretically there is no reason why narrow-strip sampling for these characteristics cannot be applied to any forest situation.

ACKNOWLEDGMENTS

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Thanks is given to the Doctoral Committee members who gave useful comments and criticisms as the study progressed. The committee members are:

Dr. Daniel Chappelle, Department of Resource Development

Dr. Dennis Gilliland, Department of Statistics and Probability

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Dr. Victor Rudolph, Department of Forestry (chairman as of 8/78)

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LIST OF SYMBOLS

$p(t)$	=	conditional probability of a stem being narrow-strip sampled given that it is a distance t from a stem which has been narrow-strip sampled
D	=	random variable of the distance between two successive stems along a narrow-strip
$E(D)$	=	expected value of D
DN	=	density of a stand in stems per unit area
e(DN)	=	the narrow-strip estimator of density derived from the infinite series expansion of E(D)
S_i	=	the number of stems in a stand of species i
S	=	number of stems in a stand
R	=	random variable of stem radius breast-high
$E(2R)$	=	expected value of diameter breast-high
BA	=	basal area of a stand in square units per unit area
s_*	=	number of stems narrow-strip sampled
t_j	=	random variable of the j^{th} nearest neighbor distance
PPS	=	Probability Proportional to Size
$\hat{\theta}$	=	Cox's distance sampling estimator of density
γ^*	=	Diggle's distance sampling estimator of density
e*(DN)	=	Monte-Carlo version of e(DN) derived by sampling truncated t_j distributions from stem maps
w	=	narrow-strip width
$Pr(E)$	=	probability of the event E

$A \cap B$ = intersection of the two sets A and B

ϕ = the empty set

$A=\{\cdot\}$ = notation for the set A='the set of all x such that (:) x has property P' ($A=\{x: x \text{ has } P\}$)

$Pr(A|B)$ = conditional probability of the event A occurring given that the event B has occurred

$X \sim U(a,b)$ = the random variable X has the uniform distribution on the interval (a,b)

dbh = diameter breast high

$\overset{\frown}{AB}$ = arclength from point A to point B

BUF = blow-up factor is the reciprocal of the sampling intensity

BAF = Basal area factor used in point sampling

PRF = plot radius factor used in point sampling

$$PRF = \left\{ \frac{\frac{43560}{BAF} - 1}{576} \right\}^{1/2}$$

CHAPTER I

INTRODUCTION

To describe a forest stand, a decision must be made to employ a sampling technique which will provide desired information on selected stand characteristics. After the stand characteristics have been selected, the choice of a sampling technique is affected by available resources (funds, manpower, time, and equipment) and by the desired accuracy and precision of the required estimates. All else equal, the most cost-efficient sampling technique is preferred.

A new method of sampling called <u>narrow-strip sampling</u> is investigated in this paper. Narrow-strip sampling consists of sampling a forest stand by means of strips so narrow that the trees may be linearly ordered along the strips. No matter how a narrow-strip is placed in the forest, no two trees can lie side-by-side along the narrow-strip (see trees A and B in Figure 1); i.e., the trees are linearly ordered along the narrow-strip. This means that for any pair of trees sampled by the narrow-strip, one tree must precede the other. Because of the narrowness of the strip, inter-tree distances between neighboring stems along the narrow-strip can be taken and this allows an

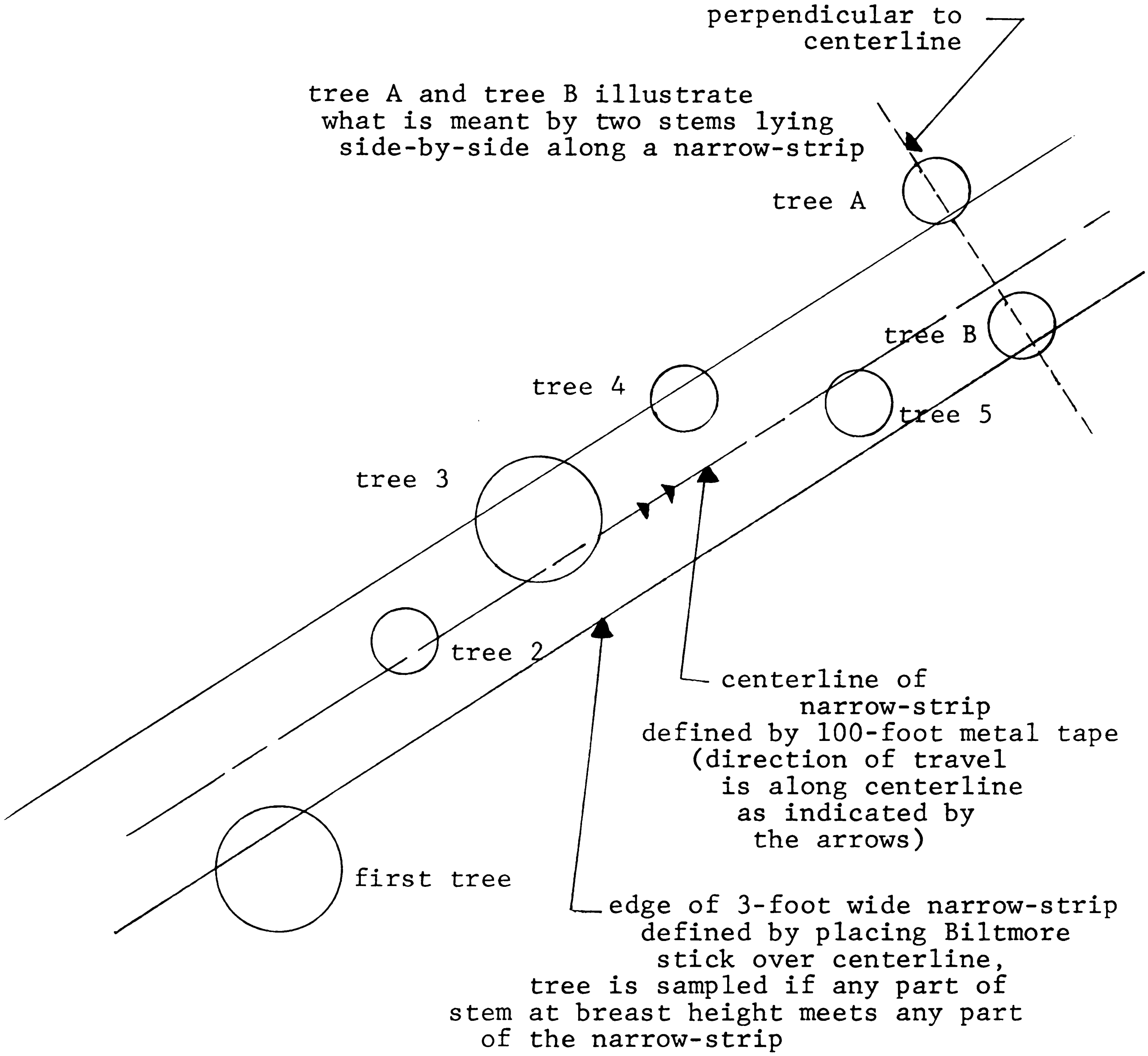


Figure 1. Trees ordered along a narrow-strip.

estimate to be made of density (the number of stems per unit area). Though trees can grow with stems in contact at breast height, this is rare and can be adjusted for in practice.

Besides density, volume and biomass are also important characteristics of a forest stand. Estimation of these characteristics using narrow-strip sampling could not be covered in this paper because of the preliminary scope of the study. However, diameters are measured as a part of the narrow-strip samples, and since heights can be measured as in any other sampling technique, volumes can also be estimated. In fact, narrow strip sampling provides a convenient frame from which to sample for any characteristic desired.

This paper demonstrates that narrow-strip sampling can provide cost-efficient estimates of density, diameter, basal area, and species proportions while achieving an accuracy and precision which compare quite favorably to accuracies and precisions experienced with circular-plot and point sampling. A major conclusion is that narrow-strip sampling may be an attractive and practical alternative to circular-plot and point sampling.

CHAPTER II

LITERATURE REVIEW

Narrow-strip sampling has not been used before as used in this study. Warren (1971, 1972) mentioned the possibility of some fruitful research in narrow-strip sampling of forest stands. He pointed out that Pielou (1962, 1963) used narrow-belt transects (another name for narrow-strips) to study runs of one species with respect to another and to study runs of healthy with respect to diseased trees along the strips but he adds:

> '... we have only one type of individual, and we record the distance between successive individuals as projected on the long axis of the plot.'

Warren also points out some practical advantages of narrow-strip sampling:

1. Searching for boundaries, a difficult task in heavy understory, is eliminated.

2. It is a simple matter to tell whether or not a tree is in the sample.

3. The crew is not as prone to feel it is wasting time in travelling from sampling point to sampling point since the crew is involved in cruising continuously along the strip.

Narrow-strips can be used to estimate density. Because of its effect on density estimation, spatial pattern is an important characteristic of a plant community. Pattern is also difficult to assess. Three basic patterns are generally recognized (Figure 2). The three areas have the same density but possess widely divergent spatial patterns. (The use of the word 'pattern' instead of the word 'distribution' will be consistent throughout this paper to avoid the connotation of an underlying statistical distribution which the word 'distribution' carries with it.) The fact that most plant populations have a clustered pattern is generally accepted throughout plant ecology. Most of the studies of pattern occur in forestry because trees are generally very easily distinguishable as individuals.

Two measures of the presence of a species in an area of interest are density, the number of plants per unit area, and cover, the proportion of the total area covered by the vertical projection of aerial shoots of the species (Greig-Smith, 1964). Another measure of presence important in forestry is basal area, the number of square units of stem per unit area occupied by trees using the diameter of the stem at breast height, 4.5 feet above the ground. There are a variety of ways to sample for these indicators of the degree of presence of a species.

In quadrat sampling for density (Cowlin, 1932; Greig-Smith, 1952; Thompson, 1958) the number of individuals contained in a rectangle of fixed size is examined. Circular-plots are also used for this purpose. The rectangles are located randomly, systematically or contiguously. The size of the rectangle is important and can cause problems with the bias of the density estimate (Pielou, 1977).

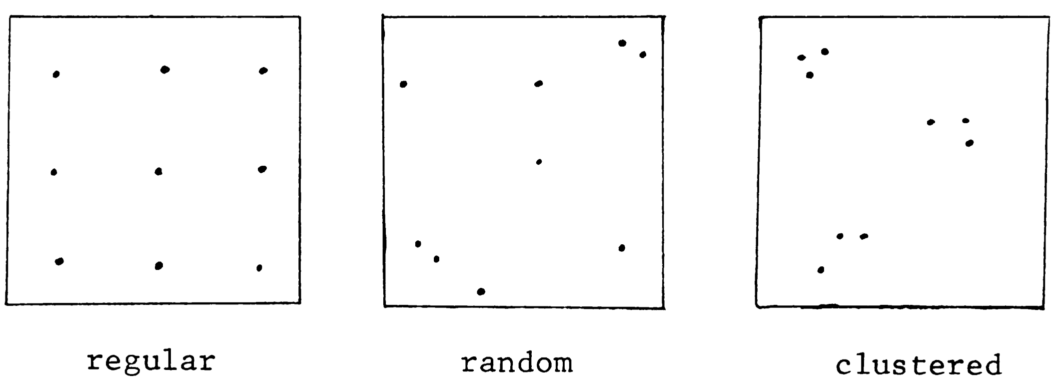


Figure 2. The three basic stem patterns.

Cover can be estimated by using the line-intercept method of sampling. The proportion of the length of lines that the species intercepts is used to estimate cover. The lines are generally placed systematically (Johnston, 1957). The line-intercept method assumes that aerial portions of plants do not overlap or intermingle; i.e., individual plant boundaries are well defined. That this is not the case can cause a serious bias in the estimate.

The most popular method of estimating basal area in forestry is the point sampling method (Grosenbaugh, 1952; Dilworth and Bell, 1968). A fixed angle is projected (Figure 3) and depending on the distance of the tree from the sampling point and on the tree's diameter breast high, the tree appears larger than, smaller than or the same size as the projected angle. If the tree is a true 'borderline' tree (one which is exactly the same size as the projected angle), it is counted as a sample tree. Each sample tree represents the same amount of basal area. The basal areas sampled are averaged over all sampling points to obtain the estimate.

Density can also can be measured by distance sampling. There are two types of distances measured in distance sampling: point-to-plant and plant-to-plant. In point-to-plant sampling, a point is chosen at random in the forest and the distance from the point to the nearest stem is measured. Plant-to-plant sampling assumes a stem is chosen at random and the distance to the nearest neighboring stem is measured. Distances can also be measured from a point or a plant to the k^{th} nearest

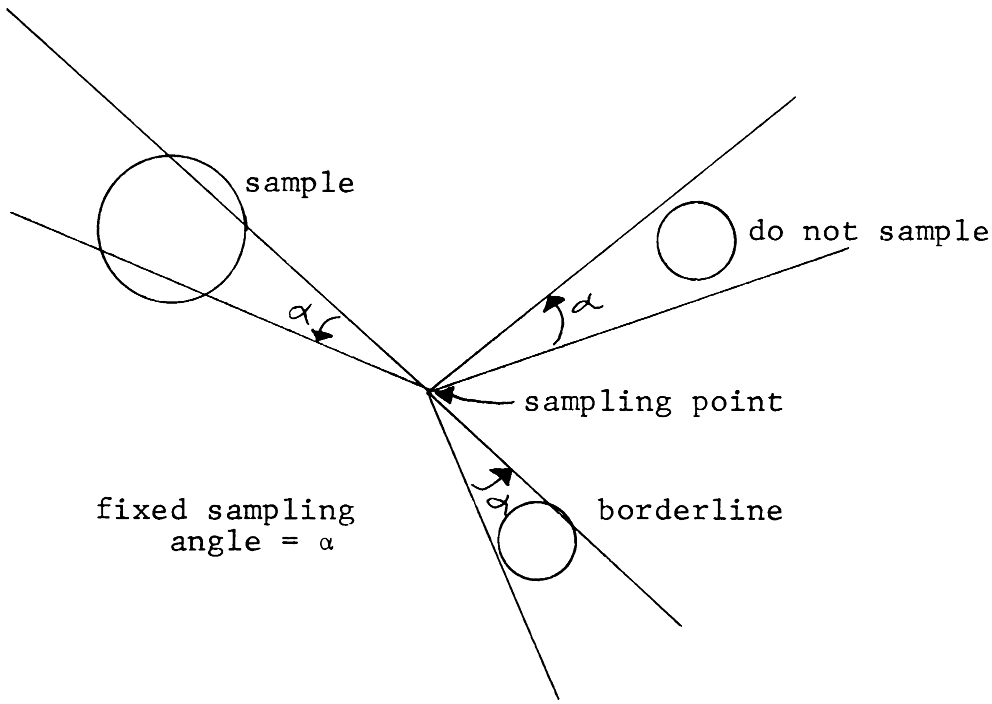


Figure 3. Point sampling.

plant where k>1. Various estimators of density have been derived using one or the other or some combination of the two methods (for reviews see Mueller-Dombois (1974), Persson (1971), and Pielou (1977)). All derivations assume a random pattern of stems. Distance sampling is laborious, contributes nothing to estimating stand characteristics other than density and pattern, and must be carried out independently of the sampling for these other characteristics. As a result, distance sampling is rarely done except for research purposes. A good study using distance methods to estimate density is Persson (1964).

The introduction of bias into density estimates due to non-random patterning of stems is covered by Persson (1971). One way density can be estimated is to estimate the expected value of t_1^2 where t_1 is the distance between a stem and its nearest neighbor. If $e(t_1^2)$ is such an estimate, then $\pi e(t_1^2)$ is the estimated average area occupied by a stem and the density estimate (in stems per acre if t_1 is in feet) is $43560/(\pi e(t_1^2))$. A small bias in $e(t_1^2)$ can cause a very large bias in the density estimate. Robust techniques of density estimation are currently being tested (Cox, 1976). Robust techniques are those which are not seriously affected by non-random patterns of stems.

The term 'narrow-strip' denotes strips narrower than the usual 1/2-chain or 1-chain widths which have been used in the past to cruise a certain proportion of the stand area --- the same as circular-plot sampling does today. 'Narrow' means 16.5 feet to Meyer (1942) and 3.3 feet to Cottam and Curtis (1949). At the other extreme, line-intersect sampling uses lines to sample logs lying askew on the forest floor (Bailey, 1969, 1970; DeVries, 1973a, 1973b, 1974; Warren and Olsen, 1964). An interesting summary of similar methods used in wildlife studies is given in Eberhardt (1978). Of particular interest in Eberhardt's paper is a denisty estimator using strips which are not necessarily narrow. Eberhardt's density estimator will be discussed in detail in Chapter VIII.

Studies using narrow-strip sampling have been conducted by Pielou (1962, 1963, 1965). Pielou places narrow-strips systematically and records the occurrence of one or the other type of tree (diseased/healthy or first-species/second-species) along the strips. When studying the incidence of disease, she assumes that healthy trees can either be placed in disease-free 'gaps' or infected 'patches'. She can conclude from her analysis whether or not the two types of trees are randomly mingled along the strips. She obtains probabilities of runs of length k=1,...,5 (see Vitayasai, 1971); i.e., she finds, for example, the probability of three healthy trees occurring in succession.

Most recently Birth (1977) stated:

> 'Rock outcrops, minor slope changes, and other site factors often result in micro-stands that are substantially different from the major stand encompassing them. Sampling with continuous, narrow, fixed-area strips that traverse the entire stand and are oriented across terrain features is an appropriate way of including the variation.'

CHAPTER III

DESCRIPTION OF DATA AND METHODS OF ANALYSIS

A description of two real stands which are used in field applications of narrow-strip sampling along with a description of 34 real and computer-generated stem maps used in Monte-Carlo analyses are given in Sections A and B. General discussions of the methods of analysis which are used in Chapters IV-VII are given in Section C. Chapter IV is the theoretical basis for this study. Chapters V and VI are Monte-Carlo studies of 34 stem maps, and any statement about bias or precision of an estimator in these chapters refers only to these stem maps unless otherwise stated. Chapter VII gives results of two field applications in the stands mentioned in Section A. Full details of the methods of analysis precede the disucssions of results in the appropriate chapters.

A. Description of stands used in field studies.

The two stands which form the empirical basis for this paper are located on lands managed by the New Jersey Division of Fish, Game, and Shellfisheries. Stand 1 is 62.5 acres and Stand 2 is 43.0 acres. Stand 1 is dominated by white oak (*Quercus alba* L.) and red oak (*Quercus velutina*

L.). Pitch pine (Pinus rigida Mill.) is also present. Stand 1 is contained in the Peaslee Wildlife Management Area approximately six miles northwest of Tuckahoe. Stand 2 is contained in the Bevans Wildlife Management Area located approximately six miles south of Millville. Stand 2 is dominated by pitch pine but contains some white and red oak. Distribution of stems by species and dbh (diameter breast high) is given in Table 1. These distributions were obtained from stem counts in twenty-five 0.2-acre circular-plots located systematically in each stand. These distributions are also used to generate species and diameters for computer-generated stem maps.

At each circular-plot center a 2-chain and 4-chain rectangular plot was laid out (Figure 4). A point sample was taken from the plot center and three narrow-strips were run parallel to the 4-chain sides of the rectangular plot and equidistant from each other. The strips were three feet wide.

Both stands are in an area of very little topographic relief. Elevation in the area is 80 to 120 feet above sea level. The area is part of a broad sand, silt and gravel plain sloping gently southwestward into Delaware Bay. The area has a mild climate with high humidity. Drought is not usually a problem. Since the area is well drained and the soil is acidic and coarse textured, fires can be a problem. Stand 2 was burned in the past ten years as evidenced by scars on the pitch pine stems.

Table 1. Stand tables

Stand 1

dbh (inches)	red oak	white oak	pitch pine	species proportions	
	(stems per acre)				
4	9.87	37.62	0.60	red oak	0.49
5	15.90	33.46	1.39	white oak	.49
6	13.50	26.28	2.98	pitch pine	.11
7	15.10	10.95	2.98		1.00
8	14.50	6.37	4.17		
9	13.90	3.60	4.77		
10	7.05	--	2.58		
11	4.23	--	2.98		
1/ 12	2.82	--	4.18		
	96.87	118.68	26.23	density = 242.18	

Stand 2

dbh (inches)	oaks	pitch pine	species proportions	
	(stems per acre)			
4	22.35	19.42	oaks	0.25
5	12.44	33.24	pitch pine	.75
6	9.00	33.62		1.00
7	7.35	27.22		
8	5.69	23.41		
9	2.55	19.82		
10	4.07	12.01		
11	--	8.60		
12	--	5.20		
13	--	7.82		
	63.45	190.36	density = 253.18	

1/ 12 inches (13 inches) and up counted as one dbh class.

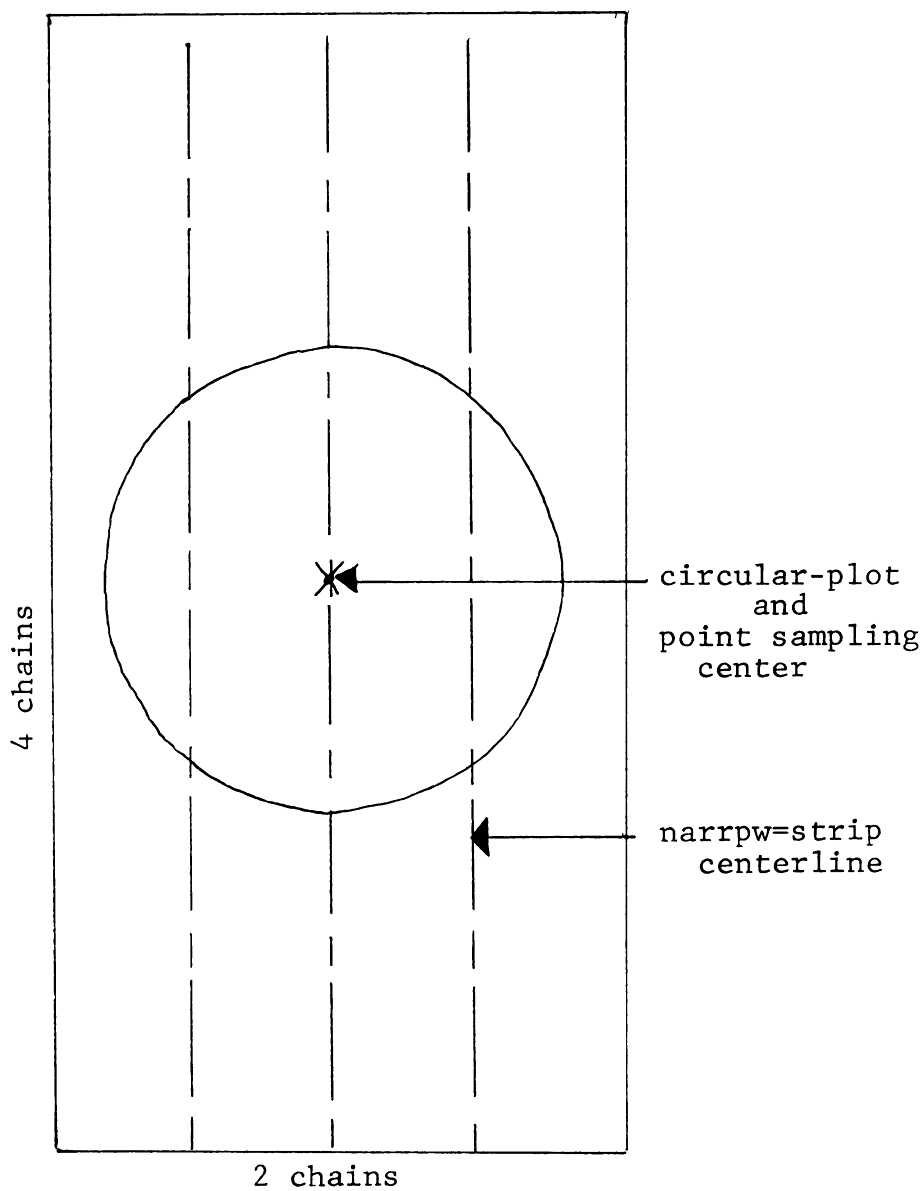


Figure 4. Sample plot layout. Orientation of rectangular plots to stands shown in Figure 19.

B. Description of stem maps sampled by computer.

There are 34 stem maps analyzed by Monte-Carlo techniques. The maps are numbered F = 1,...,34. Important characteristics of the maps are listed in Table 2. F = 1,...,30 are computer-generated and F = 31,...34 are actual stem maps. The patterns used in the generation of F = 1,...,30 are:

i. random (F = 1,...,5). stem center = (X,Y). $X \sim U(0,173')$ and $Y \sim U(0,346')$ where U denotes the uniform probability density function.

ii. regular (Figure 5a.)

  a. square (F = 6,...,10)

  b. rectangular (F = 11,...,15)

  c. equilateral-triangular (F = 16,...,20)

iii. clustered (Figures 5b and 5c.)

  a. square (F = 21,...,25)

  b. equilateral-triangular (F = 26,...,30)

Clusters consist of stems equally spaced around the circumference of a circle of radius 10 feet with a stem at the center. Cluster centers are then located either on a square or equilateral-triangular pattern. The number of stems per cluster is constant for a given stem map since only a certain number of clusters could fit within a 173 foot by 346 foot map. The number of stems per cluster varies from 7 to 12 depending on the desired density of the map as discussed below.

Species and diameter generation are accomplished using probability density functions empirically determined from an eight percent systematic sample by 0.2-acre circular-plots located

Table 2. Characteristics of the stem maps.

Stem map	Density (stems/ acre)	Stems[1/] per map	Red oak density	White oak density	Pitch pine density	Basal area (sq.ft./ acre)	Average dbh (inches) [2/]	Red oak dbh	White oak dbh	Pitch pine dbh
1	194	267	78	93	23	51.0	6.5	7.0	5.5	9.3
2	219	301	82	111	27	56.0	6.5	7.1	5.3	9.2
3	243	334	91	130	22	60.5	6.4	7.1	5.4	9.1
4	267	368	102	139	26	68.6	6.5	7.3	5.4	9.1
5	291	400	111	138	42	75.0	6.5	7.1	5.2	8.7
6	192	264	77	94	21	49.0	6.5	7.2	5.4	8.6
7	218	300	93	101	24	57.6	6.6	7.0	5.5	9.5
8	246	338	100	111	35	66.9	6.6	7.1	5.4	9.3
9	264	364	111	123	30	69.3	6.5	7.2	5.3	9.0
10	285	392	103	149	33	68.0	6.3	7.0	5.3	8.7
11	192	264	76	97	19	46.4	6.3	7.0	5.3	8.9
12	222	306	77	116	29	59.4	6.6	7.3	5.4	9.4
13	242	333	92	124	26	59.8	6.4	7.1	5.4	8.5
14	269	370	116	123	30	70.3	6.7	7.2	5.5	8.7
15	291	400	117	144	30	73.1	6.5	7.1	5.5	8.7
16	190	262	80	93	17	46.0	6.3	7.3	5.1	8.3
17	225	310	89	112	24	55.2	6.4	7.2	5.3	8.4
18	245	337	89	122	34	64.1	6.5	7.2	5.4	9.1
19	263	362	96	138	29	67.5	6.5	7.2	5.4	9.4
20	186	391	112	145	27	68.1	6.3	7.1	5.4	7.9
21	284	256	83	84	19	51.7	6.7	7.5	5.2	10.0
22	209	288	69	110	30	55.8	6.6	7.2	5.4	9.1
23	232	320	82	120	30	60.4	6.5	7.3	5.3	9.0
24	256	252	98	136	22	61.9	6.3	7.0	5.3	8.9
25	279	384	101	149	29	70.2	6.5	7.0	5.5	9.0
26	203	280	76	107	20	49.9	6.3	7.0	5.2	9.3

Table 2. continued

Stem map	Density (stems/ acre)	Stems[1/] per map	Red oak density	White oak density	Pitch pine density	Basal area (sq.ft./ acre)	Average[2/] dbh Inches)	Red oak dbh	White oak dbh	Pitch oak dbh
27	224	308	96	101	27	62.2	6.8	7.5	5.4	9.2
28	244	336	92	122	30	63.4	6.5	7.2	5.5	8.9
29	264	364	111	125	28	71.0	6.7	7.4	5.5	8.8
30	285	392	117	138	30	73.5	6.5	7.4	5.3	8.7
31	205	172	1	143	61	67.9	7.3	6.2	6.0	10.4
32	309	251	163	109	37	80.3	6.6	7.1	5.1	8.5
33	230	183	34	77	119	80.5	7.6	7.5	6.6	8.2
34	223	176	--	--	223	111.3	9.2	--	--	9.2

1/ Computer-generated maps (1,...,30) are 173 feet by 346 feet (1.37 acres) and actual stem maps (31,...,34) are 132 feet by 264 feet (0.8 acres).

2/ All species combined.

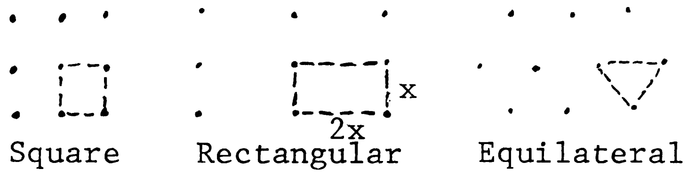


Figure 5a. Regular patterns.

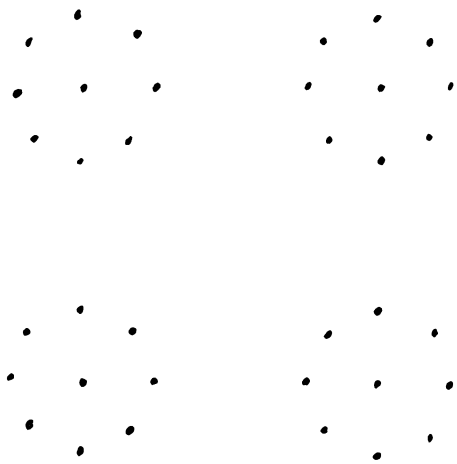


Figure 5b. Square clustered.

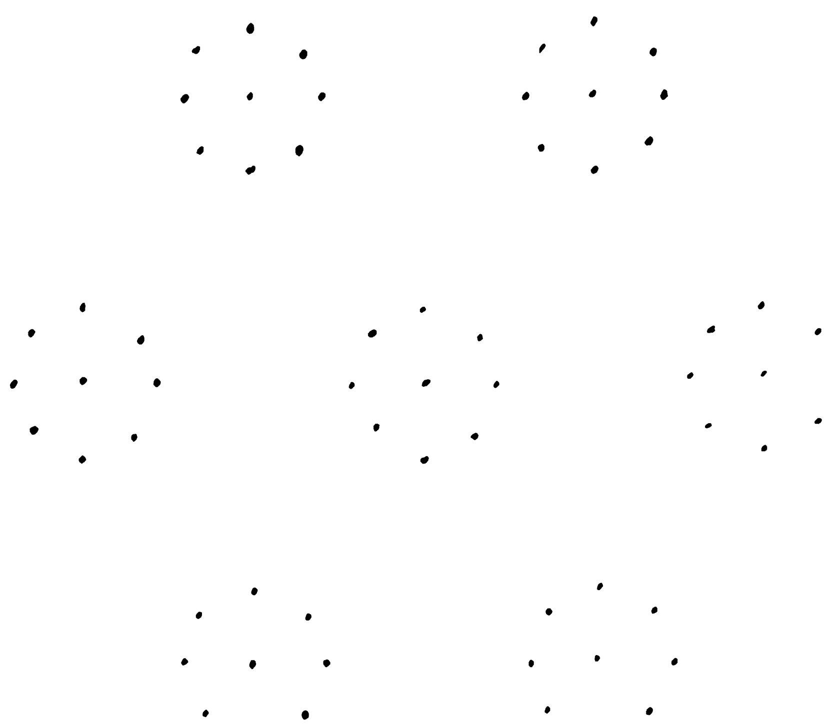


Figure 5c. Equilateral clustered.

Figure 5. Patterns for computer-generated stem maps.

in Stand 1. The method of generating a specific stem map is as follows:

1. choose a pattern.
2. choose a density.
3. generate (X,Y).
4. for each (X,Y):
    a. generate a species.
    b. generate a diameter.

F = 31,...,34 are from field maps using a Suunto compass and a 100 foot metal tape. F = 31 and F = 32 were made in Stand 1 and F = 33 and F = 34 were made in Stand 2.

All values in Table 2 are population values for the 34 stem maps. Density in Stand 1, averaged over twenty-five 0.2-acre circular-plots, is 242.2 stems per acre. This average density is denoted by $\overline{DN}$. Densities in the stem maps cycle from 'low' (0.8$\overline{DN}$) through 'medium-low' (0.9$\overline{DN}$) through 'average' ($\overline{DN}$) through 'medium-high' (1.1$\overline{DN}$) through 'high' (1.2$\overline{DN}$) as the F cycle through 5j+1, 5j+2, 5j+3, 5j+4, 5j+5 for j = 0,1,....,5. Densities, diameters, and species proportions for F = 31,...,34 are exactly as mapped. For F = 1,...,30 the area is 173 by 346 feet or 1.37 acres. For F = 31,...,34 the area is 132 by 264 feet or 0.8 acres. Patterns for F = 1,...,30 are as previously discussed. Patterns for F = 31 and F = 33 were found to be clustered and patterns for F = 32 and F = 34 were found to be random by use of Pielou's index of non-randomness (1959).

C. Methods of analysis

1. Theoretical basis.

Narrow-strip sampling is a cluster sampling technique

(Cochran, 1963) and a PPS (Probability Proportional to Size) technique. Chapter IV includes the theoretical framework of narrow-strip sampling upon which Monte-Carlo studies of Chapters V and VI and field applications of Chapter VII are based. Narrow-strip estimators of density, diameter, basal area, species proportions, and stand table entries are defined. Since the density estimator for narrow-strip sampling establishes the narrow-strip technique as a viable and practical sampling method, this estimator is the most important theoretical concept of this paper. The effect upon the density estimator of narrow-strip width, stem radius, and probability density functions of j^{th} nearest neighbor distances is contained in the derivation of the expected distance between two successive stems along a narrow-strip. General considerations about narrow-strip sample size, the ability of narrow strips to sample variation continuously, effects of wrongly including or excluding trees from the sample, and a mathematical definition of 'narrowness' are also given in Chapter IV.

2. Comparison with distance sampling.

Monte-Carlo studies are discussed in detail in Chapters V and VI. Such studies are designed to model particular situations. A general reference dealing with the components and design of such studies is Schmidt and Taylor (1970). Some papers dealing with simulation studies in forestry are Mawson (1968), Newnham (1966, 1968), Newnham and Maloley (1970), O'Regan and Palley (1965), and Payandeh (1970a).

A paper by Mohn and Stavem (1974) disucsses randomly located, non-intersecting discs in the x-y plane. An article on simulation of distance sampling is Diggle, Besag, and Gleaves (1976).

Basic to the study of all systems containing stochastic components is the generation of random numbers. Generators are available which generate psuedo-random numbers. The generator chosen for this paper is a modified version of IBM's RANDU and it was found to be sufficiently random over the range of values generated and over subsets of that range according to the purposes for which the numbers were used. No degenerate tendencies were exhibited and the cycle length was at least ten times that of the number of psuedo-random numbers generated.

In Chapter V distance sampling estimators of density are compared with the narrow-strip density estimator. Three supposedly robust estimators of density (Diggle, 1975; Lewis, 1976; Cox, 1976) are chosen to compare with the narrow-strip density estimator. Dependence of the accuracy of density estimation on spatial pattern prompted selection of all robust estimators. The three estimators chosen appeared to be successful.

3. Comparison with plot and point sampling.

Use of 'small' stem maps was necessitated by the enormous number of calculations required for each of the 100 Monte-Carlo realizations run for each sampling method over every stem map. Since narrow-strip, plot, and point sampling were being studied as statistical methods, sizes of stem maps were large enough so that results could be extended to larger stands. Similarly, the rectangular shape of stem maps offers no real restriction to generaliztion of results.

Systematic location of sampling units was also employed to study the three sampling methods. Behaviour of the three methods was analyzed with respect to estimation of the following characteristics: density, average diameter (total and by species), basal area, and species proportions. Behaviour was analyzed for random and systematic location of sampling units over all 34 stem maps.

Methods are compared on the basis of absolute value of bias (accuracy), sign of the bias, and the coefficient of variation of estimates averaged over 100 trials (precision). The range of densities and patterns, combined with empirical distribution functions for species and diameter-given-species, allowed for a comprehensive study of how well the three sampling methods estimated the nine characteristics.

4. Field applications.

In Chapter VII the design of a narrow-strip sample is discussed by means of two field applications. One application is in Stand 1 and the other is in Stand 2. The design includes specification of narrow-strip width to be used in the cruise, choice of sampling intensity, location of narrow-strips, equipment, methods, and cost-efficiency. Results of the narrow-strip technique are compared with results of circular-plot and point samples taken in both stands.

CHAPTER IV

THEORETICAL BASIS OF NARROW-STRIP SAMPLING

This chapter discusses narrow-strip sampling theoretically to explain how the method works and to provide a basis for the Monte-Carlo studies in Chapters V and VI. Accuracy and precision of the estimators will be considered in Chapters V and VI because they provide the basis upon which the judgements about the efficiency of narrow-strip sampling will be made.

Section A is the derivation of E(D) = the expected distance between two successive stems along a narrow-strip. Derivation of E(D) results in a relationship which is used to define the narrow-strip density estimator, e(DN). Narrow-strip density estimation is accomplished in three parts: (1) derivation of p(t) = the probability that a stem of radius r which is a distance t from its neighbor (a neighboring stem along the narrow-strip) is sampled by a narrow-strip of width w given that its neighbor has already been sampled; (2) derivation of E(D) resulting in an infinite series whose terms involve integrals of p(t) and their products; (3) definition of e(DN). Because of the apparently highly robust nature of e(DN), the expression for E(D) eliminates the necessity of either field sampling or Monte-

Carlo sampling stem maps to find numerical values of e(DN) corresponding to different values of w, r, DN and E(D). Numerical integration programs (see IBM, 1974) can be used to generate tables needed to look up e(DN) once the parameters are specified and an estimate of E(D) is made. The Appendix includes a description of a procedure used to obtain numerical values which define e(DN) and a listing of the Fortran IV program NRSTRP along with sample input and output for that program. NRSTRP generates the table of values needed to define e(DN) numerically for a given situation.

In Section B estimators used in Chapter VI are discussed. Narrow-strip sampling is more of a PPS technique than plot sampling, but less of a PPS technique than point sampling; however, simple counts, instead of weighted counts, of stems are used to define estimators for species proportions. Later on (Chapter VIII) use of weighted counts is suggested for this purpose. Diameter and density estimates are used to estimate basal area. The estimated stand table entries, like the basal area estimate, depend on diameter and density estimates but the entries also depend on the accuracy with which species proportions, as functions of species and diameter class, can be estimated.

Section C includes a discussion of narrow-strip sample size. Narrow-strip sampling nearest neighbor distances is considered as a PPS technique. This section also includes remarks about the way in which narrow-strips sample variation continuously as they are run through a stand and about the effect that wrongly including or excluding stems in a narrow-strip sample has on estimates. A mathematical definition of 'narrowness' is also given.

A. Estimation of density.

1. Derivation of the probability that a stem is narrow-strip sampled given that a neighboring stem has been narrow-strip sampled.

Let C_0 be a stem of radius r_0 and center x_0. Let C_j be a j^{th} nearest neighbor to C_0 where $j \leq S-1$ and S = the number of stems in a population. C_j has radius r_j and center x_j. Let t_j = the distance between x_0 and x_j so that t_j is the random variable denoting the j^{th} nearest neighbor distance in the population. Define the j^{th} nearest neighbor circle to be that circle centered at x_0 of radius t_j (Figure 6). Select one edge, e, of the narrow-strip and fix this edge for the following derivation. Consider a line, e', parallel to e and a distance r_0 from e. For notational purposes let 'NS' denote 'narrow-strip'. Then e' lies outside the narrow-strip. $C_0 \cap NS \neq \phi$ if, and only if, x_0 lies within a strip of width $w+2r_0$ with centerline CL. Let w = width of NS and CL be located randomly. Let Q = the distance from x_0 to e' measured perpendicular to CL. Q is a random variable. Figures 6a and 6b show how sampling of C_0 and C_j is done and how Q is measured. C_j is sampled only if it touches the NS and this implies that x_j falls within a strip of width $w+2r_j$ centered on CL.

Let D = the random variable of the distance between

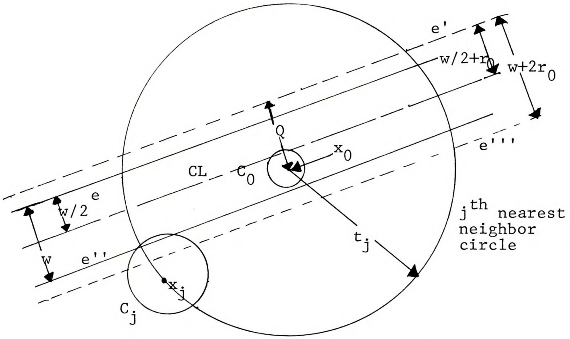


Figure 6a. Both stems sampled.

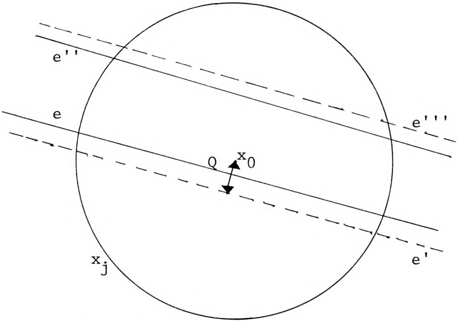


Figure 6b. One stem sampled.

Figure 6. Given one stem is sampled, the other stem is or is not sampled. Narrow-strip lies between lines e and e''. C_0 touches the narrow-strip only if x_0 falls in the strip between e' and e'''.

successive stems along a narrow-strip. For the time being, the direction of travel along the narrow-strip is not specified so that D is measured in either direction. Since CL is randomly located, the stem pattern does not affect $p(t_j) = Pr(C_j \cap NS \neq \phi | C_0 \cap NS \neq \phi)$. Also, the random location of CL implies that $Q \sim U(0, w+2R)$, independently of stem pattern, where Q = the random variable of the distance between x_0 and e' and R = the random variable of stem radius in the population. Refer to Figure 7:

$$\begin{aligned} p(t_j) &= 2(\widehat{AB}+\widehat{BC})/(2\pi t_j) \\ &= 2(\theta_1+\theta_2)t_j/(2\pi t_j) \\ &= \tfrac{1}{\pi}[\mathrm{Sin}^{-1}((q-r_0+r_j)/t_j) + \mathrm{Sin}^{-1}((w+2r_0-(q+r_0) + \\ &\quad r_j)/t_j)] \\ &= \tfrac{1}{\pi}[\mathrm{Sin}^{-1}((q-r_0+r_j)/t_j) + \mathrm{Sin}^{-1}((w+r_0+r_j-q)/t_j)] \end{aligned}$$

See Figure 8. Assuming $r_0 = r_j = E(R)$ and denoting E(R) by r:

$$p(t_j) = \tfrac{1}{\pi}[\mathrm{Sin}^{-1}(q/t_j) + \mathrm{Sin}^{-1}((w+2r-q)/t_j)]$$

The assumption on the radii can be justifed if a uniform diameter distribution is assumed and this assumption simplifies later integrations and so it is made here. Further studies could investigate the effect which non-uniform diameter distributions might have on density estimation. Assuming $q = E(Q) = w/2+r$:

$$\begin{aligned} p(t_j) &= \tfrac{1}{\pi}[\mathrm{Sin}^{-1}((w/2+r)/t_j) + \mathrm{Sin}^{-1}((w+2r-w/2-r)/t_j)] \\ &= \tfrac{2}{\pi}\,\mathrm{Sin}^{-1}((w+2r)/(2t_j)) \end{aligned}$$

Finally:

$$p(t_j) = \begin{cases} 1 & \text{if } t_j \leq 1/2(w+2r) \\ \tfrac{2}{\pi}\,\mathrm{Sin}^{-1}((w+2r)/(2t_j)) & \text{otherwise} \end{cases} \qquad (1)$$

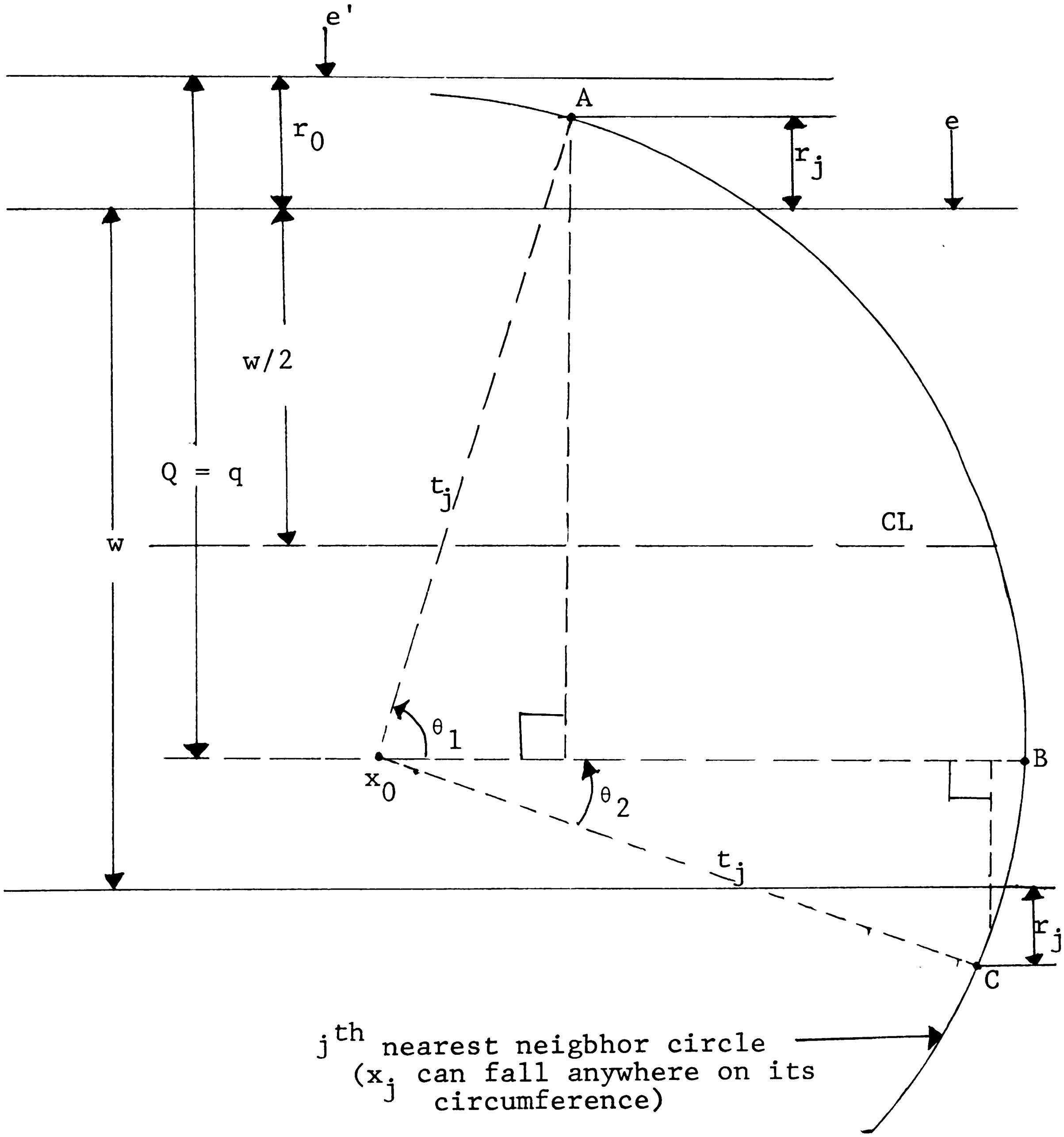


Figure 7. Derivation of the probability that a stem is narrow-strip sampled given that a neighboring stem has been narrow-strip sampled. x_k = center of stem C_k of radius r_k $(k = 0,j)$ and t_j = distance between x_0 and x_j. w = width of narrow-strip and e is fixed edge of that strip. e' = line parallel to and a distance r_0 away from e. Q (= q in the Figure) = distance from x_0 to e'.

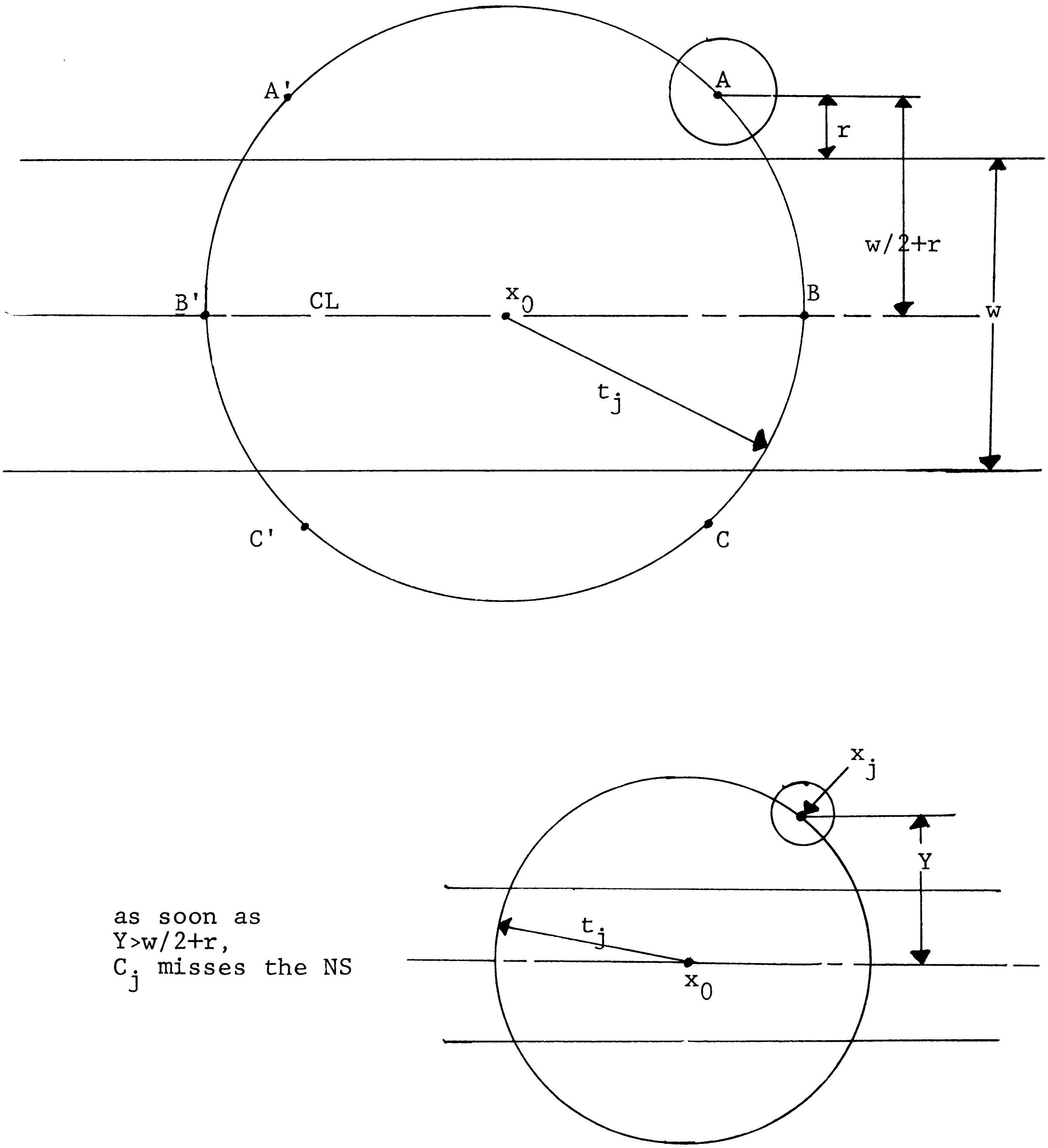


Figure 8. Assume the population stem radii to be constant and the center of the already sampled stem falls on the narrow-strip centerline.

2. Derivation of the expected distance between two adjacent stems along a narrow-strip.

Refer to Figure 9. The situation described in Section A1 now exists. The following derivation assumes a random pattern of stems. For $D = t_j$ to occur (i.e., the narrow-strip distance between two successive stems is in fact the j^{th} nearest neighbor distance in the population), it must be true that $C_j \cap NS \neq \phi$. $D = t_j$ does not occur just because $C_j \cap NS \neq \phi$. It must also be true that $C_k \cap NS = \phi$ for $k = 1,\ldots,j-1$ such that no C_k fall between (this is where 'narrowness' is essential) C_0 and C_j. These j-1 events still allow $C_k \cap NS \neq \phi$ to the left of C_0 (since C_j falls to the right of C_0 in Figure 9). These j-1 events occur independently of one another and independently of $C_j \cap NS \neq \phi$ with probabilities:

$$1 - (1/2)p(t_k) \text{ for } k = 1,\ldots,j-1$$

This implies:

$$\begin{aligned} Pr(D=t_j) &= E[(1/2)p(t_j) \prod_{k=1}^{j-1} (1-(1/2)p(t_k))] \\ &= (1/2)E(p(t_j)) \prod_{k=1}^{j-1} [1-(1/2)E(p(t_k))] \qquad (2) \end{aligned}$$

The $(1/2)p(t_j)$ is used instead of $p(t_j)$ since now the travel is in a fixed direction (it does not matter which direction is chosen) along the narrow-strip so that each inter-tree distance is counted only once. Let I_j be the indicator random variable for the event $D = t_j$:

$$I_j = \begin{cases} 1 \text{ if } D = t_j \\ 0 \text{ if } D \neq t_j \end{cases}$$

Finally:

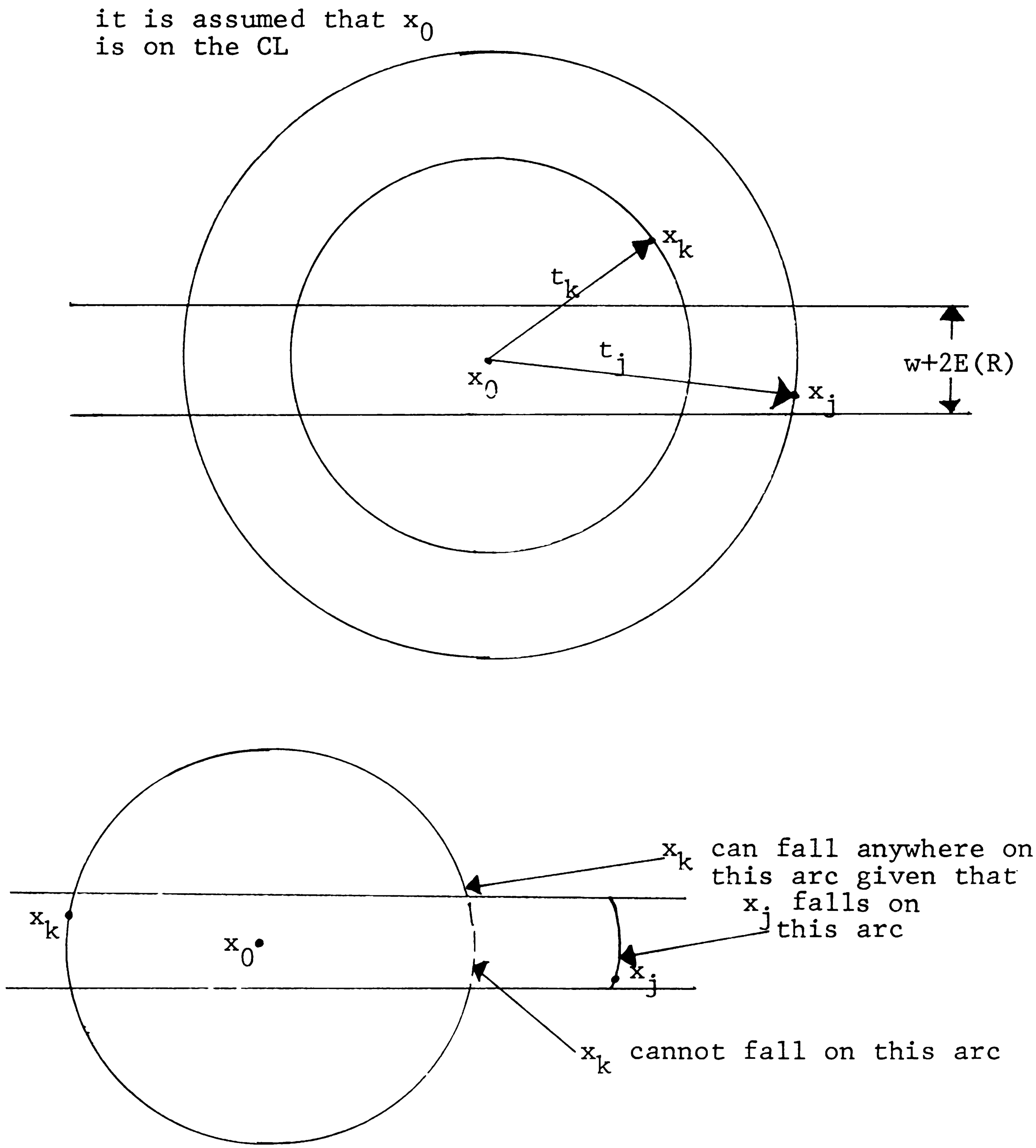


Figure 9. Expected distance between two adjacent stems along anarrow-strip. x_j falls on <u>either one</u> (right or left) of the two small arcs. <u>Given this event occurs</u>, the x_k must fall on the larger arcs like the ones indicated for $k = 1,\ldots,j-1$.

$$D = \sum_{j=1}^{\infty} t_j I_j$$

and:

$$E(D) = \sum_{j=1}^{\infty} E(t_j)E(I_j)$$

$$= \sum_{j=1}^{\infty} E(t_j)(1 \cdot Pr(D = t_j) + 0 \cdot Pr(D \neq t_j))$$

$$E(D) = \sum_{j=1}^{\infty} E(t_j)Pr(D = t_j) \tag{3}$$

where in (2):

$$\prod_{k=1}^{0}(1-(1/2)E_k(p(t))) = 1 \text{ (letting } E(p(t_k)) = E_k(p(t)) \text{ for notational purposes)}$$

$$E_i(X) = \int_0^{\infty} XK_i(t)dt$$

$$K_i(t) = 2(\pi m)^i t^{2i-1} e^{-\pi m t^2}/(i-1)! \quad (i = 1,2,\ldots)$$

The probability density functions, K_i, are for the t_i in a random population with density m (Thompson, 1956).

3. Definition of the narrow-strip density estimator.

Equation (3) gives E(d) in terms of w, E(R), and m for a random pattern. For English units m = DN/43560 stems per square foot and for Metric units m = DN/1000 stems per square meter. In practice w is known while E(D) and E(R) are estimated by e(D) and e(R) from a narrow-strip sample. What is then required is an estimate, e(DN), of density. A brief description of how to obtain e(DN) from a table of values generated by numerical integration will now be given. A full description is contained in the Appendix.

Suppose that in an area of interest it is known:

$7 \text{ inches} \leq E(R) \leq 10 \text{ inches}$

$200 \leq DN \leq 280$

Suppose also that it has been decided to sample with strips of width w = 3 feet (see Chapter VII for a determination of w). By numerically integrating (3) a table of the following form may be generated (refer to Figure 10). The dbh range from 5.0 to 12.0 inches and the densities range from 180 stems per acre to 300 stems per acre. These ranges have been extended from the previously mentioned 7.0 to 10.0 inches and 200 stems per acre to 280 stems per acre to faciliatate interpolation.

To generate the table in Figure 10, w is fixed at 3 feet while the dbh varies from 5.0 to 12.0 inches in steps of 1.0 inches. For each dbh the densities are allowed to vary from 180 stems per acre to 300 stems per acre by steps of 20. Each of the 56 tabular values is the result of one numerical intergration. Different forest situations will require different ranges and increments for the dbh and densities as well as a different choice of w. The integrations can be formed for any situation.

After a narrow-strip sample of width w = 3 feet is completed, we have estimates of the average dbh and E(D) of a a forest stand. All that is required to find the narrow-strip estimate of density, e(DN), is to search down the column corresponding to the dbh estimate until the estimate of E(D) is reached and then proceed to the left to read e(DN). Some interpolation will usually be required (see the Appendix). Chapter V uses this same method of interpolation

<u>w = 3 feet</u>

```
         dbh = 5.0   6.0   7.0   8.0   9.0   10.0   11.0   12.0
DN =     180                      *
         200                      *
         220                      *
         240*****************E(D)
         260
         280
         300
```

Figure 10. Form for a density estimation table using estimates from a narrow-strip sample. w = width of narrow strip (feet); dbh = diameter breast high (inches); DN = density (stems/acre); E(D) = narrow-strip nearest neighbor distance expected for the given w, dbh, and DN (feet).

applied to the 34 stem maps. Monte-Carlo values of E(D) are generated and differ significantly from the numerically integrated values in the Appendix. The reason for these differences is that the distributions of the t_j are truncated by the sizes of the stem maps.

B. Estimation of other characteristics

1. Estimation of species proportions.

Simple (unweighted) counts are used to estimate species proportions in this paper. Weighted counts will be mentioned in Chapter VIII. If:

S_i = number of stems in a stand of species i $(i = 1,\ldots,s)$

$$S = \sum_{i=1}^{s} S_i = \text{number of stems in stand}$$

s_i = number of stems of species i which are narrow-strip sampled

$$s_* = \sum_{i=1}^{s} s_i$$

then the narrow-strip estimator of the proportion of the i^{th} species in a stand is:

$$e(S_i/S) = s_i/s_*$$

2. Estimation of diameter.

If:

$R_{i,j}$ = radius of the j^{th} stem of species i $(j = 1,\ldots,S_i)$

$r_{i,j}$ = radius of the j^{th} stem of species i which is narrow-strip sampled $(j = 1,\ldots,s_i)$

then:

$$E(2R_i) = \frac{2}{S_i} \sum_{j=1}^{S_i} R_{i,j} = \text{average diameter of } i^{th} \text{ species}$$

$$E(2R) = \frac{2}{S} \sum_{i=1}^{s} \sum_{j=1}^{S_i} R_{i,j} = \text{average diameter of all stems}$$

and the narrow-strip estimators of these quantities are:

$$e(E(2R_i)) = \frac{2}{s_i} \sum_{j=1}^{s_i} r_{i,j}$$

$$e(E(2R)) = \frac{2}{s_*} \sum_{i=1}^{s} \sum_{j=1}^{s_i} r_{i,j}$$

3. <u>Estimation of basal area</u>.

If:

$$BA_i = \left(\frac{\pi}{S} \sum_{j=1}^{S_i} R_{i,j}^2\right) DN = \text{basal area in a stand for species } i$$

$$BA = \left(\frac{\pi}{S} \sum_{i=1}^{s} \sum_{j=1}^{S_i} R_{i,j}^2\right) DN = \text{basal area of stand}$$

then the narrow-strip estimators of these quantities are:

$$e(BA_i) = \left(\frac{\pi}{s_*} \sum_{j=1}^{s_i} r_{i,j}^2\right) e(DN)$$

$$e(BA) = \left(\frac{\pi}{s_*} \sum_{i=1}^{s} \sum_{j=1}^{s_i} r_{i,j}^2\right) e(DN)$$

4. <u>Estimation of stand table entries</u>.

If:

$S_{i,j}$ = number of stems in a stand of species i which are also in diameter class $j = 1,\ldots,v_i$

$s_{i,j}$ = number of stems of species i in class j which are narrow-strip sampled

then the $(i,j)^{th}$ entry in the stand table is:

$(S_{i,j}/S)DN$

and the narrow-strip estimator of this quantity is:

$e((S_{i,j}/S)DN) = (s_{i,j}/s_*)e(DN)$

C. General considerations.

1. Narrow-strip sample size.

Suppose that F_0 is a forest stand of area A. Let C be a stem of radius r such that C is located randomly in F_0. Let NS denote a narrow-strip of width w and length l such that the centerline is located randomly over F_0. Let PT denote a circle of radius (2r)(PRF) located randomly in F_0 where PRF = some plot radius factor used in point sampling. Finally PL denotes a circular-plot of radius Z located randomly in F_0. See Figure 11 from which the derivations of the following formulae follow:

$$Pr(C \cap NS \neq \phi) = \frac{(2(r/12) + w)l}{A} = \frac{(w^{+}r/6)l}{A}$$

$$Pr(C \cap PT \neq \phi) = \frac{\pi(2rPRF)^2}{A}$$

$$Pr(C \cap PL \neq \phi) = \frac{\pi(Z + r/12)^2}{A}$$

It is instructive to examine ratios of these three probabilities because this gives a comparison of how much more (or how much less) likely C is to be sampled according to one (or to another) of the three sampling techniques. To study this behavior, r

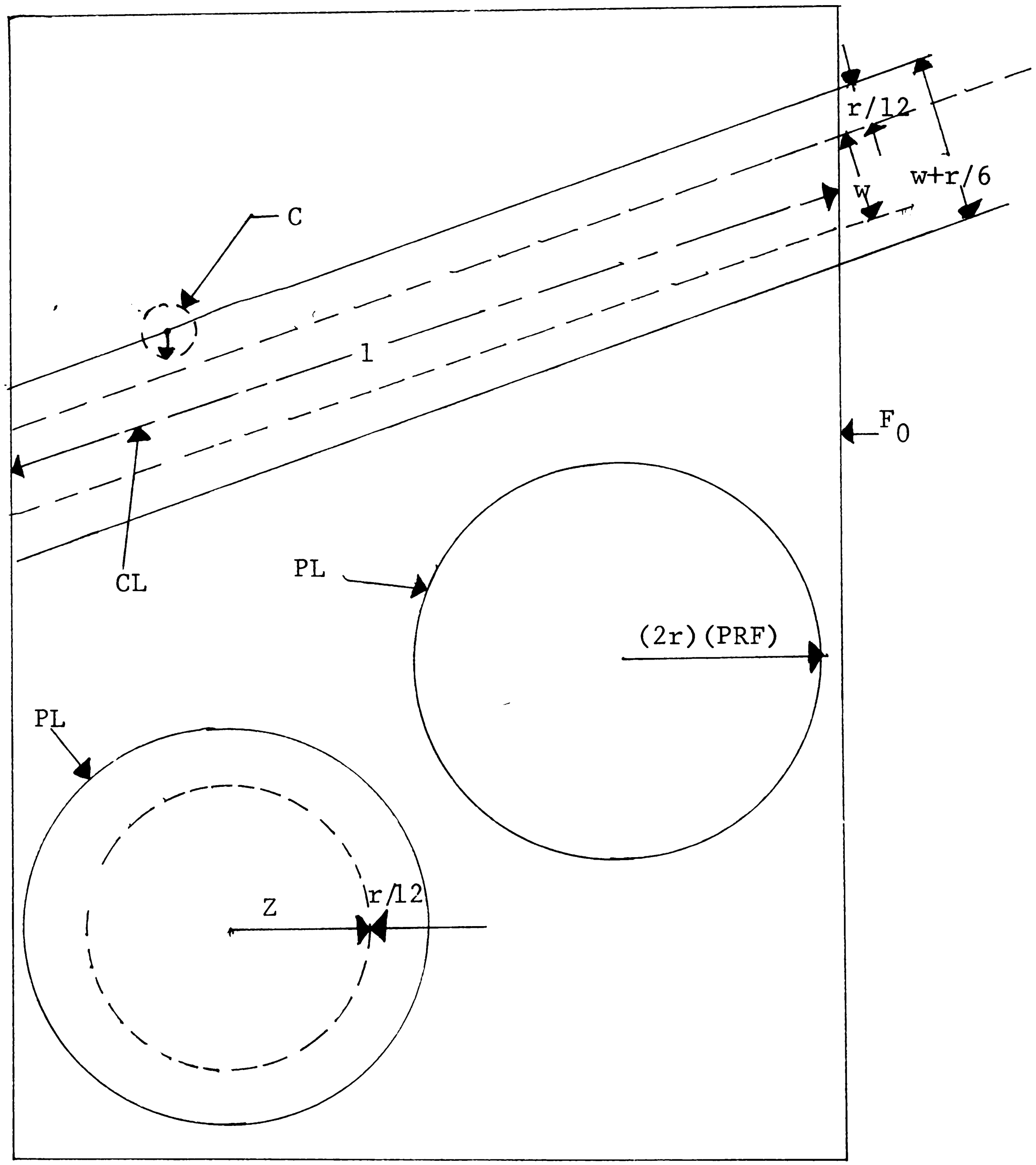


Figure 11. Geometry of stem sampled by the three techniques. F_0 = forest stand of area A; C = stem of radius r (inches); CL = centerline of narrow-strip of width w (feet) and length l (feet); PRF = plot radius factor of point sampling circle PR; PL = circular-plot of radius Z (feet).

is allowed to vary while w, l, Z, and PRF are given the values in Table 3. These values are representative of real situations.

To be able to compare the techniques we fix $wl = \pi Z^2 =$ 0.2 acre. The three ratios of interest are:

$$k_1(r) = \frac{Pr(C \cap PT \neq \phi)}{Pr(C \cap PL \neq \phi)}$$

$$k_2(r) = \frac{Pr(C \cap PT \neq \phi)}{Pr(C \cap NS \neq \phi)}$$

$$k_3(r) = \frac{Pr(C \cap NS \neq \phi)}{Pr(C \cap PL \neq \phi)}$$

These ratios are graphed in Figure 12.

The k_3 curves in Figure 12 illustrate the phenomenon that plot shape does make a difference when sampling stems as circles as opposed to sampling stems as points. In each situation the narrow-strip has a higher probability of sampling stems of a given radius than does the circular-plot even though $wl = \pi Z^2$. This occurs partly because:

$2/w = 2l/wl$ = perimeter-to-area ratio for the narrow-strip

$2/Z = 2\pi Z/(\pi Z^2)$ = perimeter-to-area ratio for the circular-plot

so that:

$Z/w = (2/w)/(2/Z)$ = relative amount of perimeter in a narrow-strip as compared to a circular-plot

In the three situations given in Table 3 we have, respectively, $Z/w = 17.55$, 8.78, and 6.58.

Over the seven random stem maps ($F = 1,\ldots,5,32,34$):

Table 3. Example of stem being sampled by the three techniques.

DN	$E(t_1)$	w	l	Z	BAF	PRF	E(R)	range of r
250	6.60	3	2904	52.66	10	2.750	3.50	2-10
75	12.05	6	1452	52.66	30	1.588	14.00	10-20
35	17.64	8	1089	52.66	60	1.123	25.00	20-30

DN = density (stems/acre); $E(t_1)$ = expected value of 1^{st} nearest neighbor distance in random pattern with density = DN; w = width of narrow-strip; l = length of narrow-strop; Z = circular-plot radius; BAF = basal area factor; PRF = plot radius factor; E(R) = average stem radius in population; r = stem radius (inches).

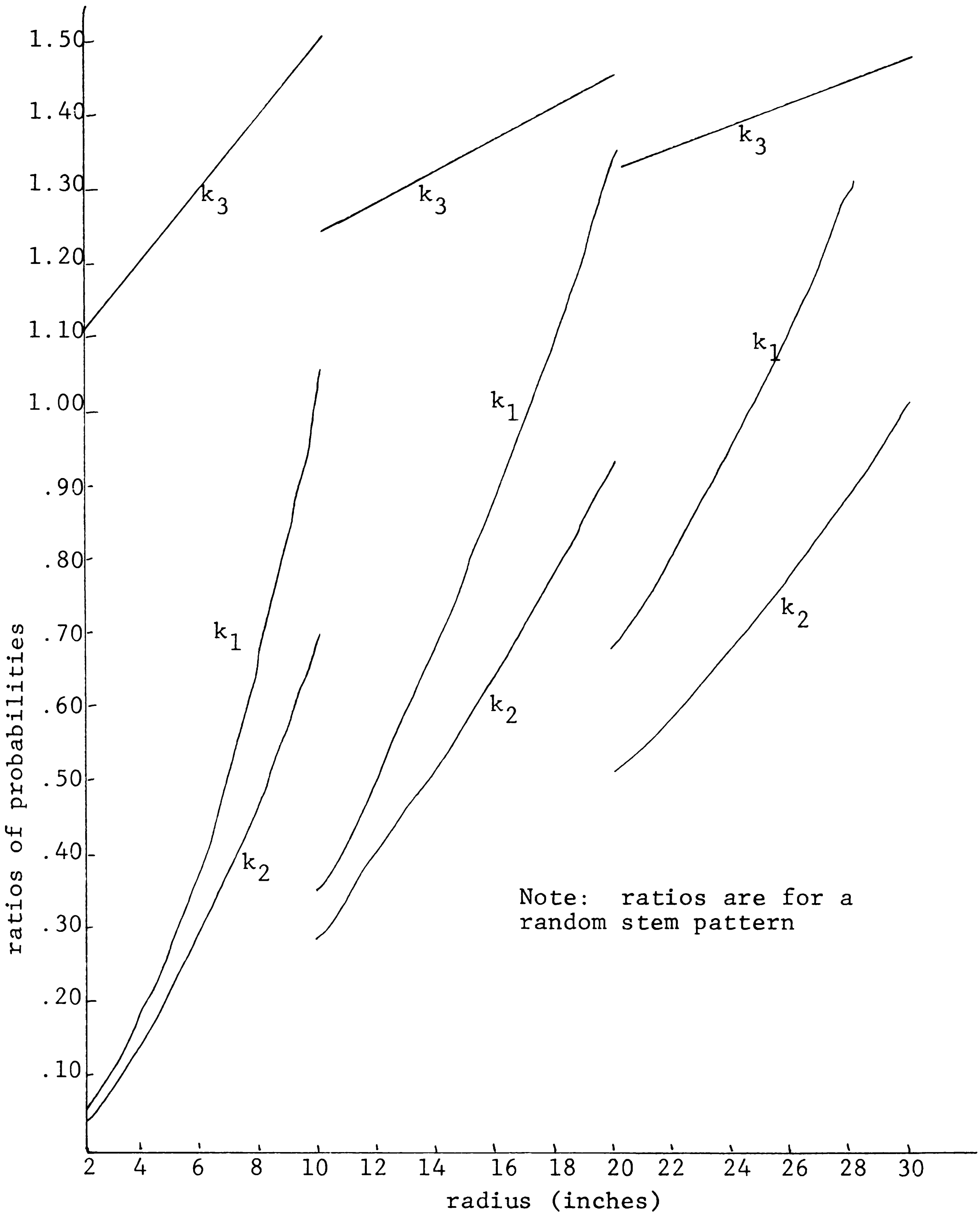


Figure 12. Comparisons of stem being sampled by three methods.

$s_* = (1+t)(a/A)S$

where a/A = sampling intensity

a = area covered by narrow-strips

A = area of F (1.37 or 0.80 acres)

S = number of stems in F

$1+t$ = proportion of stems narrow-strip sampled in 100 Monte-Carlo realizations as an excess of $(a/A)S$ = number of stems that the narrow-strips should have sampled on a strictly areal basis

For these seven stem maps $\bar{t} = .1218$ and $.0919 \le t \le .1770$. Over all 34 stem maps $\bar{t} = .1068$ and $.0584 \le t \le .1770$. This says that narrow-strips sampled, on the average, 10.68% more stems than would be implied purely by their area.

Since the k_2 curves fall below the line $y = 1$, the narrow-strip also has a higher probability of sampling a stem of a given radius than does the point sample. The k_1 curves show the point to circular-plot sampling relationship and are included for the sake of completeness.

2. Narrow-strip sampling of nearest neighbor distances as a PPS method.

In PPS sampling, the size of a population element is a measure of that element's importance in estimating some characteristic of a population. For example, in point sampling for basal area, if two stems, C_1 and C_2, are such that $r_2 = kr_1$, then $Pr(C_2 \cap PT \neq \phi) = k^2 Pr(C_1 \cap PT \neq \phi)$. When narrow-strip sampling D to estimate E(D), the smaller distances

are the more likely distances to be sampled. Consider Figure 13 in which a first and second nearest neighbor circles are pictured along with a narrow-strip. In the sense that trees which are closer together exert more of an influence on each other than trees which are further away, t_j is, in general, 'more important' than t_k when $j<k$. $t_1<t_2$ implies $p(t_1)>p(t_2)$ so that $k<1$ in $p(t_2) = kp(t_1)$ since Sin^{-1} is a strictly increasing function. This says that a stem which is a distance t_1 away from a narrow-strip sampled stem, C_0, has a higher probability of being sampled than does a stem which is a distance t_2 away from C_0.

3. Continuous sampling of variation.

Refer to Figure 14. This figure shows what is meant by the statement that narrow-strips sample variation continuously as they are run through the stand. The shaded portions represent area sampled by narrow-strips and not sampled by circular-plots.

It is assumed that significant variation in a forest will generally occur at right angles to the topography. The narrow-strips are oriented to pick up this variation. If the assumption is correct, then, generally, insignificant variation will occur inside the circular-plots but outside the narrow-strips. The notion of variation in this case is quite elusive analytically and deserves further study.

4. Wrongly sampled stems.

In Section C1 it was mentioned that:

$$\Pr(C \cap NS \neq \phi) = (w+r/6)1/A$$

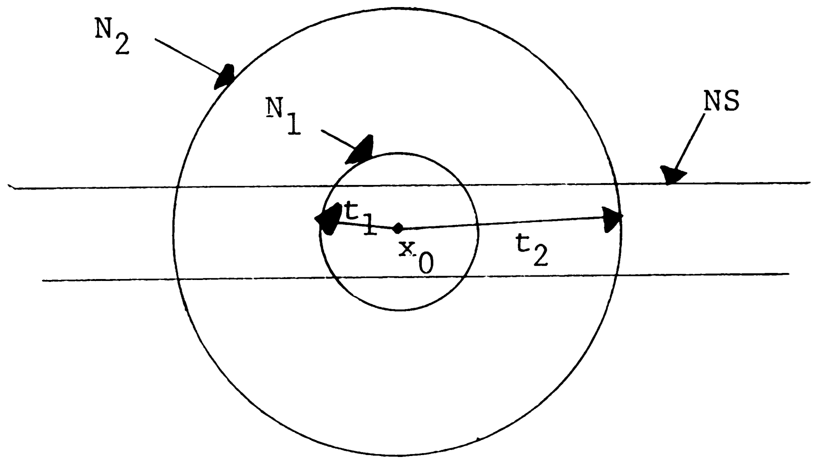


Figure 13. Narrow-strip sampling of nearest neighbor distances as a PPS method. x_0 = center of stem sampled by a narrow-strip; NS = narrow-strip; t_1 = first nearest neighbor distance to sampled stem; t_2 = second nearest neighbor distance to sampled stem; N_1,N_2 = nearest neighbor circles.

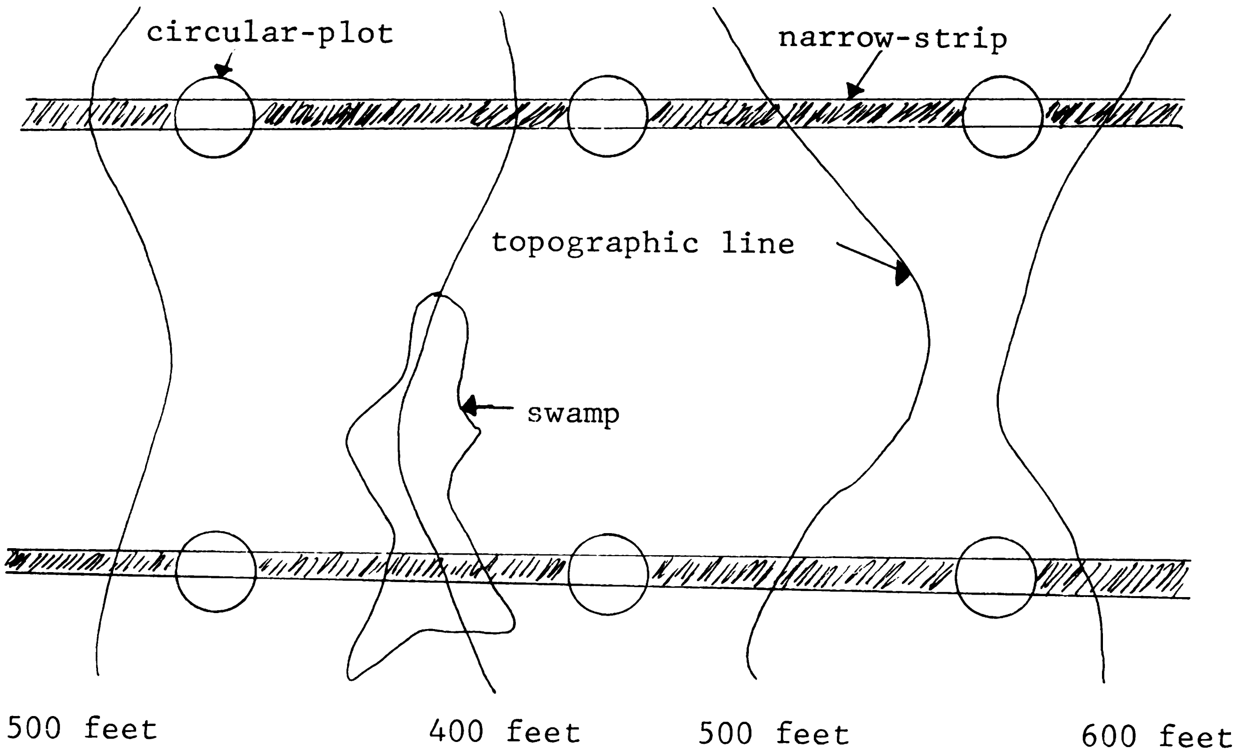


Figure 14. Continuous sampling of variation.

Suppose C_1 has radius r_1, C_2 has radius r_2, and $r_2 = kr_1$:

$$Pr(C_1 \cap NS \neq \phi) = (w^+ r_1/6)1/A$$

$$Pr(C_2 \cap NS \neq \phi) = (w^+ r_2/6)1/A$$

so that:

$$Pr(C_2 \cap NS \neq \phi) = vPr(C_1 \cap NS \neq \phi)$$

implies:

$$v = \frac{Pr(C_2 \cap NS \neq \phi)}{Pr(C_1 \cap NS \neq \phi)} = \frac{w^+ kr_1/6}{w^+ r_1/6}$$

For example, if w = 3 feet, r_1 = 6 inches and k = 2:

$$v = \frac{3+12/6}{3+6/6} = 1.25$$

A stem, C_2, twice as large in dbh as a stem, C_1, has an unconditional probability 1/4 as great as does C_1 of being narrow-strip sampled by strips located randomly. This suggests that stems which should be sampled and are not sampled tend to be of smaller dbh while stems that should not be sampled and are sampled tend to be of larger dbh. If this tendency were the sole factor in sampling stems, there would be a positive bias in dbh estimation; however, larger dbh stems tend to lie further apart and so, given that a larger stem is wrongly sampled, the probability of sampling another large stem is lessened. These two factors tend to compensate for each other it appears since, as seen in Chapters VI and VII, narrow-strip sampling gives accurate estimates of dbh.

5. Mathematical definition of 'narrowness'

Consider the forest to be a collection of n circles:

$$F = \{C_i : i = 1,\ldots,n\}$$

where $C_i = \{(x,y):(x-x_i)^2 + (y-y_i)^2 \leq r_i^2\}$

(x_i,y_i) = center of C_i

r_i = radius of C_i

It is assumed that none of the circles intersect. Denote the distance between (x_i,y_i) and (x_j,y_j) by D_{ij} $(1 \leq i<j \leq n)$. See Figure 15. Let $K_{ij} = D_{ij}-(r_i+r_j)$. Let w = width of the strip. Define $K = \min\{K_{ij}: 1 \leq i<j \leq n\}$. The strip is narrow if, and only if, $w<k$.

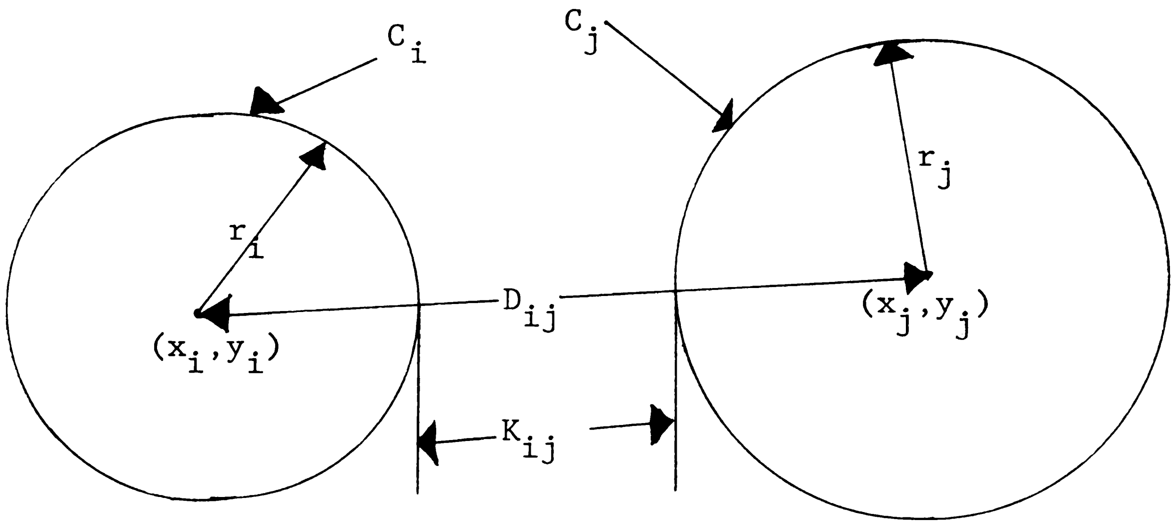


Figure 15. Distances used to define 'narrowness'.

CHAPTER V

COMPARISON OF NARROW-STRIP SAMPLING WITH DISTANCE SAMPLING FOR ESTIMATING DENSITY

Three supposedly robust density estimators using statistics derived from distance sampling are defined. Next a narrow-strip density estimator, e*(DN), is defined by means of Monte-Carlo sampling in contrast to the definition of e(DN) by means of numerical integration in Chapter IV. Finally the three distance estimators and e*(DN) are compared and conclusions are drawn.

A. Definition of distance estimators.

1. Cox's estimator.

Cox (1976) defines $\hat{\theta}$ by sampling N points at random. He defines N = n+m pairs of distances thusly:

X_i = distance from i^{th} point to nearest tree

Y_i = distance from tree nearest the i^{th} point to its nearest neighbor

$$A = \{(X_{1i}, Y_{1i}) : Y_{1i} \leq 2X_{1i};\ i = 1, \ldots, n\}$$

$$B = \{(X_{2i}, Y_{2i}) : Y_{2i} > 2X_{2i};\ i = 1, \ldots, m\}$$

Figure 16a is an example of an 'A-situation' and Figure 16b is an example of a 'B-situation'. A random variable W_{1i} is now defined:

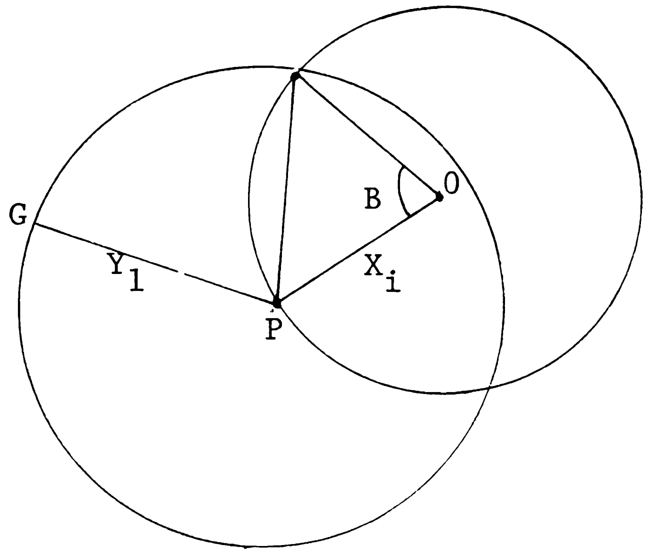


Figure 16a. 'A-situation'.

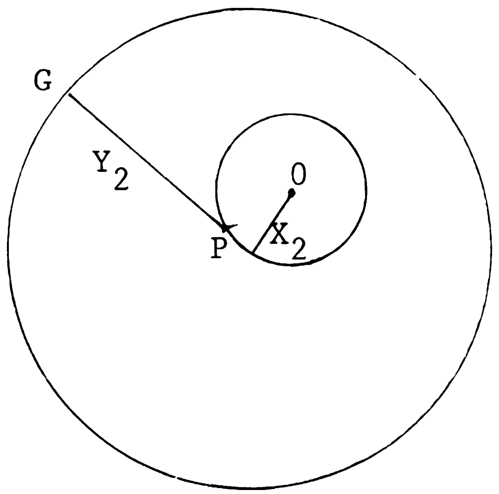


Figure 16b. 'B-situation'.

Figure 16. Geometry of Cox's estimator. For both figures the random point is 0, the nearest stem to 0 is P and G is P's nearest neighbor.

$$W_{1i} = [2\pi+\sin B_i-(\pi+B_i)\cos B_i]^{-1}$$

where $\sin((1/2)B_i) = Y_{1i}/(2X_{1i})$

Finally:

$$\hat{\theta} = \begin{cases} \hat{\theta}_1 \text{ if } m \geq (1/4)N \\ \hat{\theta}_2 \text{ if } m < (1/4)N \end{cases}$$

where $\hat{\theta}_1 = (\pi/2)(1.17-.68m/N)H/N$

$$\hat{\theta}_2 = (\pi/2)(.20+3.20m/N)H/N$$

$$H = \sum_1^n Z_{1i}^2 + \sum_1^m Y_{2i}^2$$

$$Z_{1i}^2 = (1/\pi)X_{1i}^2 W_{1i}$$

2. Diggle's estimator.

Diggle (1975) claims his γ^* is 'moderately' robust. By robust he means that the mean standardized bias is 'small' in absolute value for a wide variety of patterns. Mean standardized bias is defined by:

$$E((\gamma^*-\gamma)/\gamma)$$

where γ is the density. γ^* is defined by:

$$\hat{\gamma}_X = (\pi/n)\sum_1^n X_i^2$$

$$\hat{\gamma}_Y = (\pi/n)\sum_1^n Y_i^2$$

where X_i = random point-to-plant distance

Y_i = random plant to nearest neighbor distance

$$\hat{\gamma} = (\hat{\gamma}_X\hat{\gamma}_Y)^{1/2}$$

$$\gamma^* = 43560\hat{\gamma}^{-1}$$

3. Lewis' estimator.

Lewis (1975) defines $R^{\dagger}$ as follows:

$R^{\dagger} = 43560/(\pi R^2)$

where $R = -(1/4)R_1+(3/2)R_2$

$$R_1 = (1/n)\sum_1^n R_{1i}$$

$$R_2 = (1/n)\sum_1^n R_{2i}$$

R_{1i} = random point to nearest plant distance

R_{2i} = random point to second nearest plant distance

B. Monte-Carlo narrow-strip density estimator.

The interpolation procedure in the Appendix which defines e(DN) is now used to define e*(DN). (See also IVA3). There is one important difference in the two parameters, a and b, which define e*(DN) and e(DN). That difference derives from the fact that in regressing to obtain a and b for e(DN), the 'actual' (or 'numerically integrated') values for E(D) were used. In the case of e*(DN) the values for E(D) which are used to determine a and b are derived from 1500 Monte-Carlo realizations over 34, 1.37-acre or 0.80-acre stem maps and so the resulting values for E(D) are substantially less than the numerically integrated values for E(D) used to define e(DN). In effect, the sizes of the stem maps truncate the distributions of the t_j. Nonetheless the regressions which yield the values of a and b for e*(DN) exhibit very good fits to the E(D) values.

Over populations with random, regular, and clustered patterns and 6.5 inches$\leq$E(2R)$\leq$7.5 inches and 190$\leq$DN$\leq$290, narrow-strips of width w, where 2 feet$\leq$w$\leq$9 feet, were randomly located. Fifteen hundred narrow-strips were used per stem map per width. For a given stem map and a given width the 1500 narrow-strips yielded approximately 10,500 values of D and these values were summed and averaged to obtain one Monte-Carlo value for E(D). The table of E(D) values in the Appendix is for a width of 3 feet. E(D) values used to define e*(DN) were obtained for widths of 2, 3,...,9 feet. Values of a and b were obtained for each of these widths. The lowest r^2 for any w was .9596 and the largest average absolute bias was .0637. Absolute bias is the absolute value of the mean standardized bias as defined by Diggle using e*(DN) in place of γ^*. The absolute biases are then averaged over the 34 stem maps for a given width.

Eight pairs of a and b values (one pair for each w) were regressed to obtain over-all values for a and b. The fit for a was linear and the fit for b was quadratic:

$$a = .004633665(w+E(2R))+.10961952$$

with $r^2 = .9999$

$$b = -5.9988485((w+E(2R))-4.8391573)^2+725.84898$$

with $r^2 = .9831$

Finally:

$$e^*(DN) = be^{-aE(D)}$$

where E(D) = Monte-Carlo average for D

C. Comparison of the estimators.

For the three distance estimators of DN, 25 and 50 random points and/or plants were chosen in each stem map to generate estimates. To evaluate e*(DN) 100 Monte-Carlo realizations

were performed for each stem map using a width of 5 feet. E(2R) was known. There were about 35 distances per realization. Each realization gave an estimate of E(D) and these estimates were averaged over all realizations to obtain an over-all estimate of E(D). The over-all estimate of E(D), the known value of E(2R), and the width of 5 feet, were then used to determine a and b and hence to evaluate e*(DN) for a given stem map. The average absolute bias, denoted by $|\overline{B}|$, was averaged over all 34 stem maps for each estimator to yield Table 4.

Of the three distance estimators, $R^{\dagger}$ performed uniformly best over all stem maps for each n. Though γ^* did not perform as well as $R^{\dagger}$, it generally outperformed $\hat{\theta}$ except for the two 'most regular' patterns which are the square regular and the equilateral-triangular regular. The narrow-strip estimator e*(DN) performed better than the distance estimators. $|\overline{B}|$ for e*(DN) ranged from .0000 to .1402. No correlation between pattern and $|\overline{B}|$ was evident and this suggests that e*(DN) is quite robust. Recalling the discussion in Chapter IVA1, it is possible that the robustness of e*(DN) derives from the fact that, since narrow-strips were randomly located, pattern did not affect the $p(t_j)$.

Table 4. Bias of density estimators.

	n = 25	n = 50
γ^*	.193	.161
$R^\dagger$	.151	.116
$\hat{\theta}$	.238	.213
e*(DN)	.026	

CHAPTER VI

COMPARISON OF NARROW-STRIP SAMPLING WITH CIRCULUAR-PLOT SAMPLING AND POINT SAMPLING

A description of how the three sampling methods were applied to the 34 stem maps is given along with necessary formulae used to calculate estimates. Next the results of a Monte-Carlo study which investigated the behaviour of the three techniques under random location of their sampling elements is presented. Finally a discussion of the behaviour of the three sampling methods using standard systematic location of their sampling elements is presented.

A. Sampling methods and formulae.

The three sampling techniques were applied to all 34 stem maps using random and systematic location of the sampling units. Both random and systematic samples had a sampling intensity of 10 percent for circular-plot sampling. Twelve plot centers were located. Plot radii were 12.60 feet for the computer-generated stem maps and 9.61 feet for the actual stem maps. The difference in radii occurs because the two sets of stem maps are of two different sizes. The systematic centers were located in the standard way (Figure 17a) and the random centers were located so that the entire plot was contained within

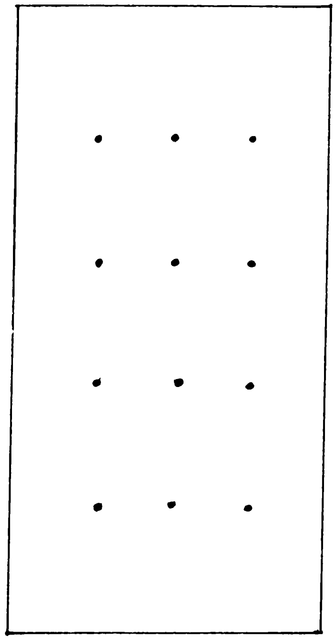


Figure 17a. Standard

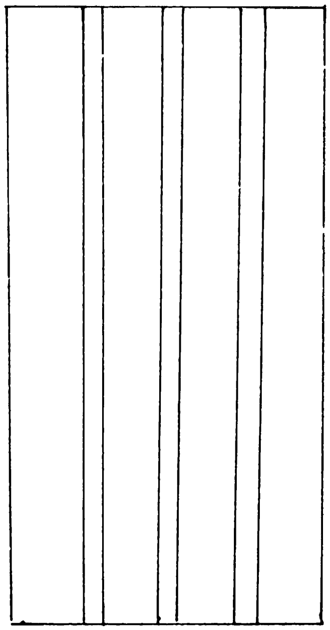


Figure 17b. 'Standard'

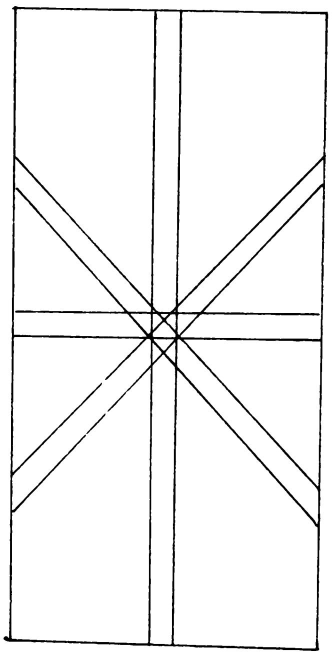


Figure 17c. 'Spoked'

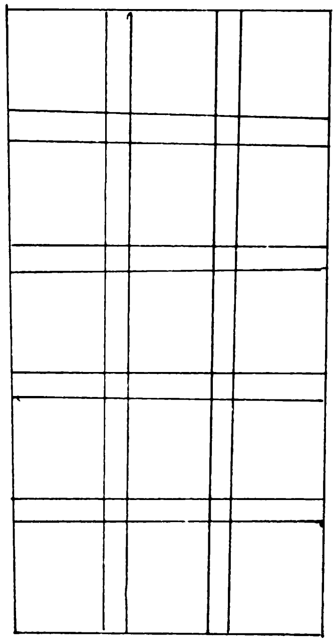


Figure 17d. 'Lattice'

Figure 17. Systematic layout for sampling elements.

the boundaries of a stem map. This is standard practice in real situations. Since there were twelve plots, each comprising 1/120 of the area of a stem map, it is seen that the expected number of stems per plot is S/120 where S = number of stems in a map. The circular-plot estimators of basal area and density are the usual ones and had a constant blow-up factor of ten.

Point sampling was done from the plot centers. Averaging S/120 stems per point was achieved by:

S/120 = BA/BAF

BAF = 120BA/S

where BA = basal area of stem map

BAF = basal area factor to be used

As a result, basal area factors varied from stem map to stem map depending on the number of stems per map and on the basal area of the map.

Narrow-strips of width w = 5 feet were passed through the stem maps until a sampling intensity of at least ten percent was reached in the random case. These intensities varied from 10.87 to 11.41 percent. In systematic sampling (Figure 17b) narrow-strips were located and w was fixed within each stem map so that the sampling intensity was 10 percent. The formula used to calculate basal area is:

$$e(BA) = ((\pi/s_*)\sum_{i=1}^{s}\sum_{j=1}^{s_i} r_{i,j}^2)e^*(DN)$$

This is the same formula as given on page 36 except e*(DN) is used in place of e(DN).

The point sampling estimators (Dilworth and Bell, 1968) used are:

$$e(BA) = (N/12)BAF$$

$$e(DN) = (1/12) \sum_{j=1}^{12} \sum_{i=1}^{27} n_{ij}(BAF/BA_i)$$

$$e(DT) = (1/N) \sum_{j=1}^{12} \sum_{i=1}^{27} d_i n_{ij}(BAF/BA_i)$$

$$e(PR) = (1/N) \sum_{j=1}^{12} \sum_{i=1}^{27} n_{ij}(R)(BAF/BA_i)$$

where N = total number of stems sampled $= \sum_{j=1}^{12} \sum_{i=1}^{27} n_{ij}$

n_{ij} = number of stems in i^{th} diameter class at j^{th} point

BA_i = basal area of stem in i^{th} diameter class

d_i = diameter of i^{th} diameter class (d_1 = 4 inches, d_2 = 5 inches,...,d_{27} = 30 inches)

$n_{ij}(R)$ = number of red oak stems in i^{th} diameter class at j^{th} point

PR = proportion of red oak stems in map

The above point sampling estimators were also used to estimate diameters within species classes (red oak, white oak, and pitch pine), basal areas within species classes, and species proportions for white oak and pitch pine. The formulae are similar and so are not listed.

B. Random Sampling

Bias is the average absolute bias defined on page 53. Precision is the Monte-Carlo average of the coefficients of variation over 100 realizations in each stem map for each technique.

Table 5 summarizes the bias and precision for nine stand characteristics for all three sampling methods under the random location of sampling elements. Biases are computed using the population means for each characteristic within each stem map. Entries in Table 5 are defined by:

$$\text{Bias} = (1/34) \sum_{i=1}^{34} B_i$$

$$\text{Precision} = (1/34) \sum_{i=1}^{34} P_i$$

where B_i = Monte-Carlo average of the absolute bias of a characteristic over 100 realizations in the i^{th} stem map

P_i = Monte-Carlo average of precision of a characteristic over 100 realizations in the i^{th} stem map

It is evident that plot sampling and narrow-strip sampling are quite accurate for dbh estimation while point sampling is not as accurate as the other two methods. According to a paper by Schreuder (1970), point sampling furnishes unbiased estimates of characteristics given certain circumstances. These results do not necessarily contradict Schreuder's paper because the stem maps were small due to: (1) computer storage requirements allowed a

Table 5. Random sampling comparisons.

	Bias			Precision		
	plot	point	narrow-strip	plot	point	narrow-strip
basal area (ft^2/acre)	.07	.03	.05	.20	.16	.18
density (stems/acre)	.05	.13	.03	.16	.20	.15
dbh (in.) (all species)	.01	.13	.01	.07	.07	.06
dbh (red oak)	.01	.14	.02	.10	.09	.09
dbh (white oak)	.01	.14	.01	.08	.11	.06
dbh (pitch pine)	.04	.14	.02	.21	.12	.18
proportion (r. oak)	.07	.13	.12	.28	.31	.24
proportion (w. oak)	.06	.16	.09	.26	.35	.23
proportion (p. pine)	.11	.10	.14	.50	.45	.46

maximum of 400 stems per map in order to contain information on 1500 narrow-strip clusters of stems when a particular stem map was being studied; (2) in order to compare the three methods on a common basis it was necessary to sample the same stem maps with points and plots as were sampled by narrow-strips. The resulting size of the stem maps (as mentioned in Chapter III, stem maps were either 132 by 264 feet or 173 by 346 feet) did not allow for an area within the maps in which no point centers could fall. It is believed that, since species and dbh were randomly generated, the sizes of the maps did not unduly affect the results.

Precisions for all three methods are quite good except for estimation of species proportion. Estimation of the frequency of occurrence of pitch pine was quite poor for each method. Pitch pine was by far the least frequent of the three species with a frequency of .11. This was true because the empirical distributions used to generate the stem maps were based on data obtained in Stand 1.

In Chapter V e*(DN) was obtained by averaging observed values of D which were measured between adjacent stems along narrow-strips. In practice a quicker method would be to calculate an estimate, e(D), of E(D) as follows:

$$e(D) = \sum_{i=1}^{n} d_i / \sum_{i=1}^{n} (m_i - 1) \qquad (4)$$

where d_i = distance between first and last stem on the i^{th} narrow-strip

m_i = number of stems on the i^{th} narrow-strip

n = number of narrow-strips

This method was used to caculate e(D) in this chapter. By using this method of estimating e(D), an insignificant bias is introduced but this method is much quicker than measuring every inter-tree distance along a narrow-strip. Refer to Figure 18. It is assumed that, on the average, one stem center will fall on the narrow-strip centerline and the other stem center will fall on one of the edges. This is equilavent to assuming that the y-coordinate of the centers differ by w/2. The average value of D_k' is what is being calculated in (4) where D_k' = biased inter-tree distance. The density and corresponding expected nearest neighbor distance will determine the choice of w (see Chapter VIIA1). In a situation similar to that in Stand 1, we might encounter E(D) = 40.0 while using a width of 3 feet. This implies that $E(D_k') = 39.972$ which differs very little from E(D). As the density decreases, the expected nearest neighbor distance and the width used for cruising increase, so that the magnitude of the above discrepancy between $E(D_k')$ and E(D) is typically small.

Plot sampling and narrow-strip sampling for density were the most accurate of the three methods and point sampling provided inaccurate density estimates. All three methods did well for estimating basal area with point sampling being the most accurate. Narrow-strip sampling

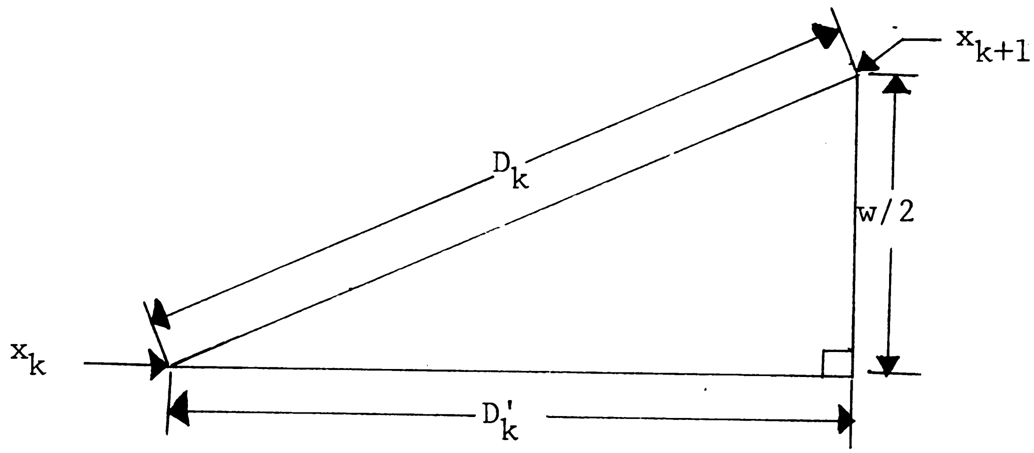


Figure 18. Bias in estimating inter-tree distance using the quick method of measurement. x_k, x_{k+1} are stem centers for neighboring stems along a narrow-strip; w = width of narrow-strip; D_k = inter-tree distance; D_k' = biased inter-tree distance as computed in Equation (4).

does not suffer from 'edge effect'; i.e., narrow-strip centerlines were located at random without restriction while the plot (and therefore the point) sampling centers could not fall within a border one plot radius (12.60 or 9.61 feet) wide around the map. Point sampling suffered more from this bias than did plot sampling.

For estimation of the proportions of red and white oak, plot sampling performed best while point and narrow-strip sampling did reasonably well. All three methods performed poorly in detecting pitch pine proportions because, though of larger dbh than the oaks, pitch pine occurred relatively infrequently.

C. Systematic sampling.

Refer to Table 6. Since the behaviour of systematic sampling with narrow-strips was quite erratic in the non-random stem maps, the three methods were compared over only random and real maps. The erratic behaviour of narrow-strips was due to the deterministic placement of stems in conjunction with the standard location (Figure 17b) of narrow-strips and the narrowness of the strips. Some strips missed stems completely in some cases while other strips sampled only a few stems in non-random maps. This behaviour caused very serious inaccuracies in the estimates. The nine stem maps in which the three methods were compared would seem to represent real situations much better than the stem maps which were not used in the analyses.

Over all 34 stem maps the 'spoked' and 'lattice' place-

Table 6. Systematic sampling comparisons

	Bias			Precision		
	plot	point	narrow-strip	plot	point	narrow-strip
basal area (ft^2/acre)	0.18	0.16	0.23	0.81	0.67	0.65
density (stems/ acre	.15	.20	.16	.66	.85	--1/
dbh (in.) (all species)	.08	.17	.04	.40	.44	.32
dbh (red oak)	.07	.13	.08	1.02	1.05	.29
dbh (white oak)	.06	.17	.04	1.04	1.38	.26
dbh (pitch pine)	.08	.16	.19	1.98	1.39	.23
proportion (r. oak)	.34	.46	.22	1.18	1.28	--1/
proportion (w. oak)	.26	.37	.20	1.12	1.43	--1/
proportion (p. pine)	.29	.37	.58	1.79	1.37	--1/

1/ Only two degrees of freedom, so coefficients of variation were not calculated (see page 67).

ments of narrow-strips (Figures 17c and 17d) were analyzed also. These two types of placements were unquestionably outperformed by the standard placement. In practice the standard placement is the one which will be used, therefore study of only the standard placement is justified.

For systematic sampling the methods are compared on the bases of absolute bias = |(estimated-actual)/actual| and coefficient of variation = (standard deviation of estimate)/estimate. All stems sampled were considered as one collection for the narrow-strip estimates of density and species proportions so there was no coefficient of variation calculated for narrow-strip sampling of these characteristics since, with three narrow-strips, there would have been only two degrees of freedom to calculate the variance of the estimates. The narrow-strip coefficients of variation were calculated from deviations of each stem from the estimated value. The plot and point sampling coefficients of variation were calculated over the twelve sampling centers in the usual manner.

All three methods performed poorly with respect to accuracy under systematic sampling except: (1) plot sampling for density and dbh; (2) point sampling for red oak dbh; (3) narrow-strip sampling for total dbh, red oak and white oak dbh. The poorest performances for all three methods were estimation of the species proportions. Narrow-strip sampling performed more poorly than plot and point sampling in basal area estimation but performed almost as well as plot sampling in estimating density.

Narrow-strip and point sampling had smaller coefficients of variation than did plot sampling for basal area. The narrow-strip sampling coefficient of variation for total dbh is lower than those for plot and point sampling. In estimating the diameters of the individual species the narrow-strip coefficients of variation are strikingly lower than those for the other two methods. This could be due to the two different ways in which variation was calculated, so it doesn't necessarily mean that variation along the narrow-strips is lower in these three cases. With larger maps, more strips would have been used and, as would occur in practice, each strip (just as each plot) would be considered a cluster and variation would be calculated in the usual manner.

D. Conclusions.

Under random and systematic sampling by means of narrow-strips, circular-plots and points over thirty computer-generated and four real stem maps, it was found that narrow-strip sampling was generally about as accurate and at least as precise (sometimes more precise) than the other two methods. This suggests that narrow-strip sampling might be used to advantage in real situations for the estimation of density, diameter, basal area, and species proportions. With this in mind, the application of narrow-strip sampling to the estimation of these characteristics is presented in Chapter VII for two real stands.

CHAPTER VII

PRACTICAL APPLICATION OF NARROW-STRIP SAMPLING

Design of a narrow-strip sample is explained by means of examples using data gathered in the New Jersey Pine Barrens in two stands: (1) oak-dominated (Stand 1); (2) pine-dominated (Stand 2). Topics covered first are choice of narrow-strip width, sampling intensity, location of narrow-strips, equipment, methodology, and cost-efficiencies. Next, narrow-strip sampling is compared with circular-plot sampling and point sampling using 'high-intensity' narrow-strip cruises. Finally narrow-strip sampling is compared with point sampling using a 'low-intensity' narrow-strip cruise in Stand 2. Figure 19 shows locations of sample plots and Figure 4 shows sample plot layout.

A. Design of a narrow-strip sample.

1. Choice of width.

In order to determine an appropriate width, w, before a narrow-strip cruise is made, a rough idea of density is required:

$$w = \begin{cases} 1 & \text{if } E(t_1) < 2 \\ [E(t_1)/2] & \text{if } 2 \leq E(t_1) < 18 \\ 8 & \text{if } 18 \leq E(t_1) \end{cases}$$

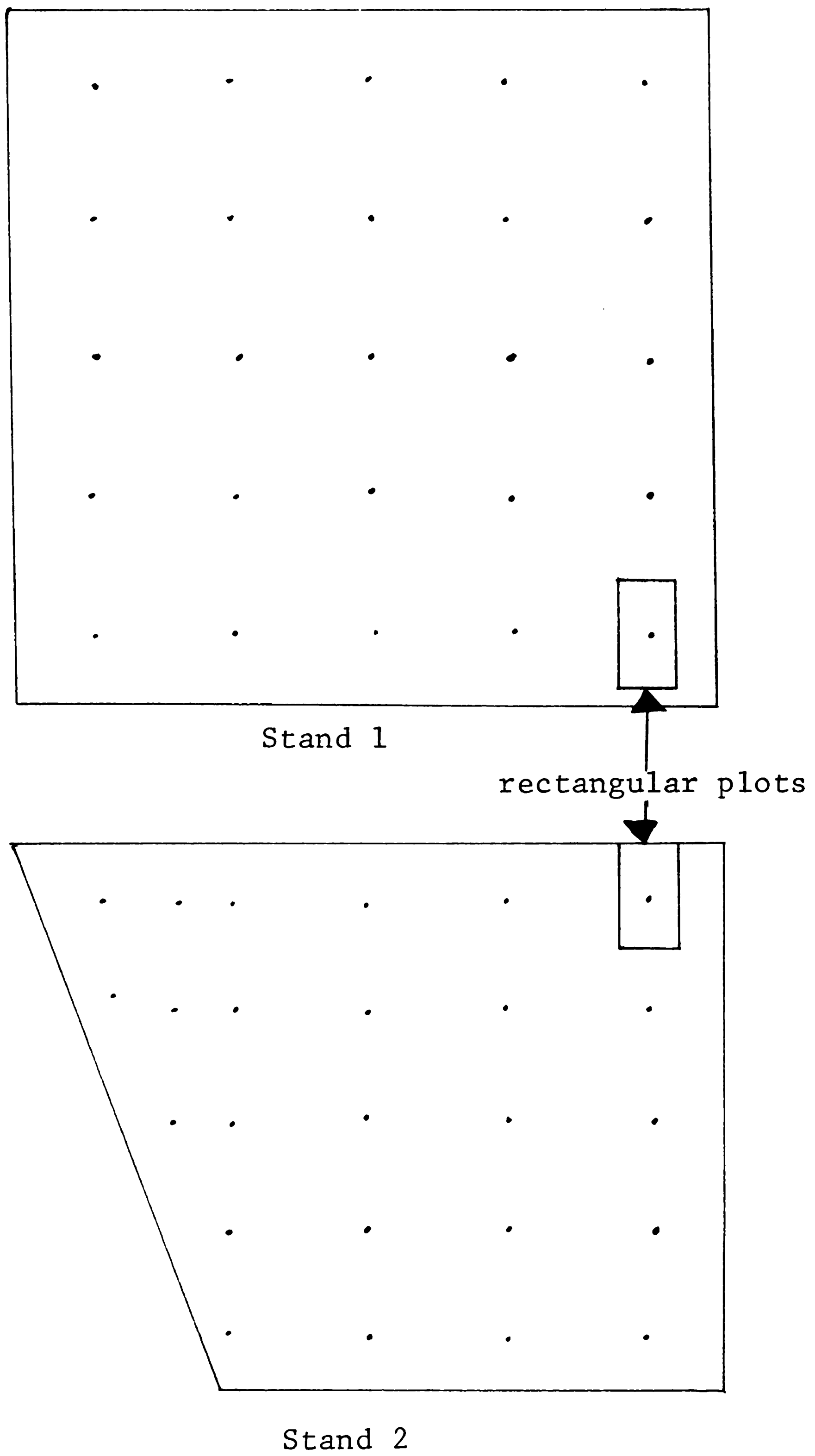


Figure 19. Locations of sample plot centers.

The 'greatest interger function' is defined by:

$[x] = j =$ greatest integer$\leq x$

For example:

$[3.1416] = 3$, $[1.999]=1$, $[j] = j$ for any integer j

The tabulation for w given density is shown in Table 7. Density for Stands 1 and 2 was though to be about 250 stems per acre. Since $170.16\leq 250\leq 302.50$, $w = 3$ was chosen for the cruises. Should the density estimate be too uncertain, choose the narrower of the two widths and adjust later if necessary.

2. Location of narrow-strips and sampling intensity.

Centerlines of narrow-strips should be located parallel and equidistant from each other so that travel along the narrow-strips is generally at a right angle to major gradients of variation in the stand. Orientation of the narrow-strip centerlines should be the same as orientation of lines upon which circular-plot centers would be located to include the maximum amount of variation in the sample. The spacing, u_1, between centerlines is governed by sampling intensity (s.i.) and is illustrated in Figure 20.

Refer to Figure 19. It is suggested as a 'rule-of thumb' that, for a 'high-intensity' narrow-strip sample, narrow-strip sampling intensity be about one-quareter that of the circular-plot sampling intensity which would normally be used. To ensure enough degrees of freedom with the 0.2-acre circular-plots, it was decided to use 25 plots. Stand 1 is 25 chains by 25 chains square. Plot spacing of

Table 7. Choice of width.

minimum[1/] density (stems/ acre)	maximum density	width (feet)	$E(t_1)$[2/] (feet)
680.63	--	1	4
302.50	608.63	2	6
170.16	302.50	3	8
108.90	170.16	4	10
75.63	108.90	5	12
55.56	75.63	6	14
42.54	55.46	7	16
--	42.54	8	18

1/ minimum density = $(43560/4E^2(t_1))$.

2/ $E(t_1)$ = expected value of 1^{st} nearest neighbor distance in random pattern.

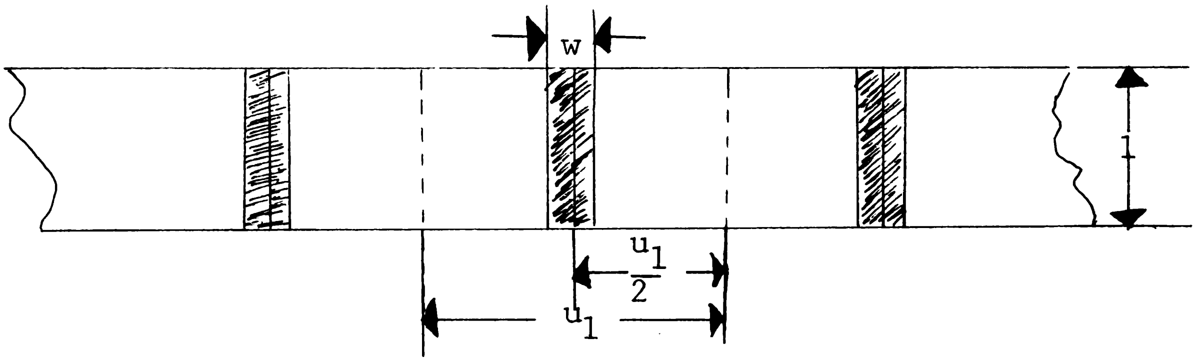


Figure 20. Spacing between narrow-strip center-lines.

5 chains by 5 chains in Stand 1 results in a circular-plot sampling intensity of eight percent. Running twelve narrow-strips of width 3 feet (a total length of 300 chains) through Stand 1 gives a narrow-strip sampling intensity of 2.18 percent and a centerline spacing of 137.5 feet.

Stand 2 is 25 chains along the top edge, 20 chains along the right edge, 18 chains along the bottom edge, and 21.2 chains along the left edge. Circular-plot sampling intensity for this stand was 12.5 percent. Running fifteen narrow-strips of width 3 feet beginning at the right edge of the stand gives a narrow-strip sampling intensity of 3.41 percent and a centerline spacing of 88.0 feet.

3. Equipment and methods.

Equipment used in a cruise depends on what is to be measured, how accurately measurements are to be taken, available equipment, available manpower, available funds, and personal preferences for such equipment. Equipment used on narrow-strip cruises of Stands 1 and 2 included a biltmore stick, 100-foot metal tape, Suunto compass, field book, and pencil.

The first narrow-strip centerline should be located randomly between 0 feet and u_1 feet from the stand boundary. The head chain-person is also the compass-person. The head chain-person walks 100 feet along the centerline trailing the metal tape then stops and records that 100 feet has elasped. The tape is the centerline. The rear chain-person calls out species and dbh of each tree sampled and the head chain-person records values in the field book.

If the rear chain-person encounters a borderline tree, that person places the biltmore stick over the centerline at breast height and determines whether any part of the trunk touches the stick. A tree is sampled only if its trunk touches the stick. A collapsable appendage could be attached to the stick such that the appendage opens up at a right angle to the stick, thus ensuring that the stick is at a right angle to the tape and providing a more accurate determination of whether or not a borderline tree should be sampled.

There are two distances which must be measured per narrow-strip. The distance along the centerline between the first stem and the boundary from which the narrow-strip was started, called the 'initial distance', and the distance between the last stem and the boundary at which the narrow-strip ends, called the 'terminal distance'. An example of a narrow-strip data sheet used for Stands 1 and 2 is given in Figure 21.

B. Results of narrow-strip cruises in two stands.

1. Comparison with plot and point sampling.

High-intensity narrow-strip samples are now compared with plot and point sampling in Stands 1 and 2. By 'high-intensity' it is meant that narrow-strips covered about one-quarter of the area that circular-plots covered. Point sampling is compared with the other two methods only with respect to basal area estimation since only stem counts were taken in the point sample. Plot sampling is compared

Stand___ Strip___ pg___of__

Initial Distance____ Terminal Distance____

stem	species	dbh	stem	species	dbh
1			26		
2			27		
.			.		
.			.		
.			.		
25			50		

Figure 21. Narrow-strip data sheet

to narrow-strip sampling with respect to estimation of species proportions, diameters, and density. The narrow-strip density estimate is derived from running twelve narrow-strips through Stand 1 and fifteen narrow-strips through Stand 2 as described in Section A. Since e*(DN) is used to estimate density, the narrow-strips were segmented by segments of length 4 1/6 chains in Stand 1 and of length 4 chains in Stand 2. Narrow-strip estimates of characteristics other than density and all plot and point sampling estimates are calculated using the formulae defined in Chapters IVB and VIA respectively. Variances were calculated using 24 degrees of freedom for the circular-plot and point samples (there were 25 plots and points in each stand) and using 417 and 434 degrees of freedom for the narrow-strip samples (there were 418 and 435 stems sampled by narrow-strips in Stands 1 and 2 respectively).

Estimates of the 0.2-acre circular-plots are used as standards of comparison. Tables 8 and 9 contain numeric results of the cruises. Figures 22 and 23 are graphs of Stand 1 and Stand 2 stand tables. Narrow-strip cruises, as described in Chapter IIIA, took about twice as long as did the point sample and about half as long as did the plot sample. Narrow-strip cruises took about eight hours in each stand. The difference in times between narrow-strip and point samples was due to the fact that species and dbh were recorded for each stem in the narrow-strip samples while only species were recorded in the point samples. The differences in time

Table 8. Basal area.

Stand 1

		all species	red oak	white oak	pitch pine
basal area (sq.ft./acre)	narrow-strip	73.98	39.69	21.94	12.35
	circular-plot	70.47	33.36	23.40	13.71
	point	63.20	28.80	16.80	17.60
coefficient of variation	narrow-strip	0.5287	0.5464	0.5020	0.5590
	circular-plot	.5514	.5703	.5384	.5370
	point	.4538	1.0432	1.0125	1.4041
bias with[1/] respect to plot sample	narrow-strip	.05	.19	-.06	-.10
	point	-.11	-.14	-.28	.28

Stand 2

		all species	red oak	white oak	pitch pine
basal area (sq.ft./acre)	narrow-strip	88.07	6.47	9.09	75.51
	circular-plot	82.93	5.49	8.83	68.61
	point	80.40	6.60	13.80	60.00
coefficient of variation	narrow-strip	.6807	.7995	.5013	.7032
	circular-plot	.6602	.7841	.5648	.6679
	point	.3953	1.4787	1.5346	.6124
bias with respect to plot sample	narrow-strip	.06	.18	.03	.06
	point	-.03	.25	.56	-.12

1/ bias = (other value - plot value)/plot value

Table 9. Species proportions, dbh and density.

species proportions

		Stand 1			Stand 2		
		red oak	white oak	pitch pine	red oak	white oak	pitch pine
mean	plot	0.40	0.49	0.11	.08	.17	.75
	strip	.46	.42	.12	.08	.18	.75
bias	strip	.15	-.14	.09	-.04	.07	.00
s.e.	plot	.26	.20	.09	.06	.13	.16
	strip	.29	.25	.14	.10	.16	.19
c.v.	plot	.65	.42	.85	.73	.78	.22
	strip	.62	.60	1.21	1.27	.94	.26

diameters

		Stand 1				Stand 2			
		sp.	oak	oak	pine	sp.	oak	oak	
mean	plot	6.98	7.71	5.77	9.09	7.39	6.45	5.75	7.81
	strip	7.04	7.59	5.87	8.42	7.53	6.73	7.76	7.95
bias	strip	.01	-.02	.03	-.07	.02	.04	.00	.02
s.e.	plot	.48	.87	.47	1.65	.78	1.43	.76	.87
	strip	.65	.84	.77	1.63	1.01	1.68	.84	1.20
c.v.	plot	.07	.11	.08	.18	.11	.22	.13	.11
	strip	.09	.11	.13	.19	.13	.25	.15	.15

density

Stand	$\sum_{i=1}^{n} d_i$	$\sum_{i=1}^{n} (m_i - 1)$	e(D)[1/]	a[2/]	b	e*(DN)	DN	bias
1	17709	484	36.59	.02758	716.44	261.15	242.80	.0782
2	17523	501	34.98	.02777	717.04	271.43	253.80	.0695

1/ d_i = distance along narrow-strip between 1^{st} and m_i^{th} stems

m_i = number of stems sampled on i^{th} narrow-strip

$$e(D) = \sum_{i=1}^{n} d_i \Big/ \sum_{i=1}^{n} (m_i - 1)$$

2/ see Chapter VB.

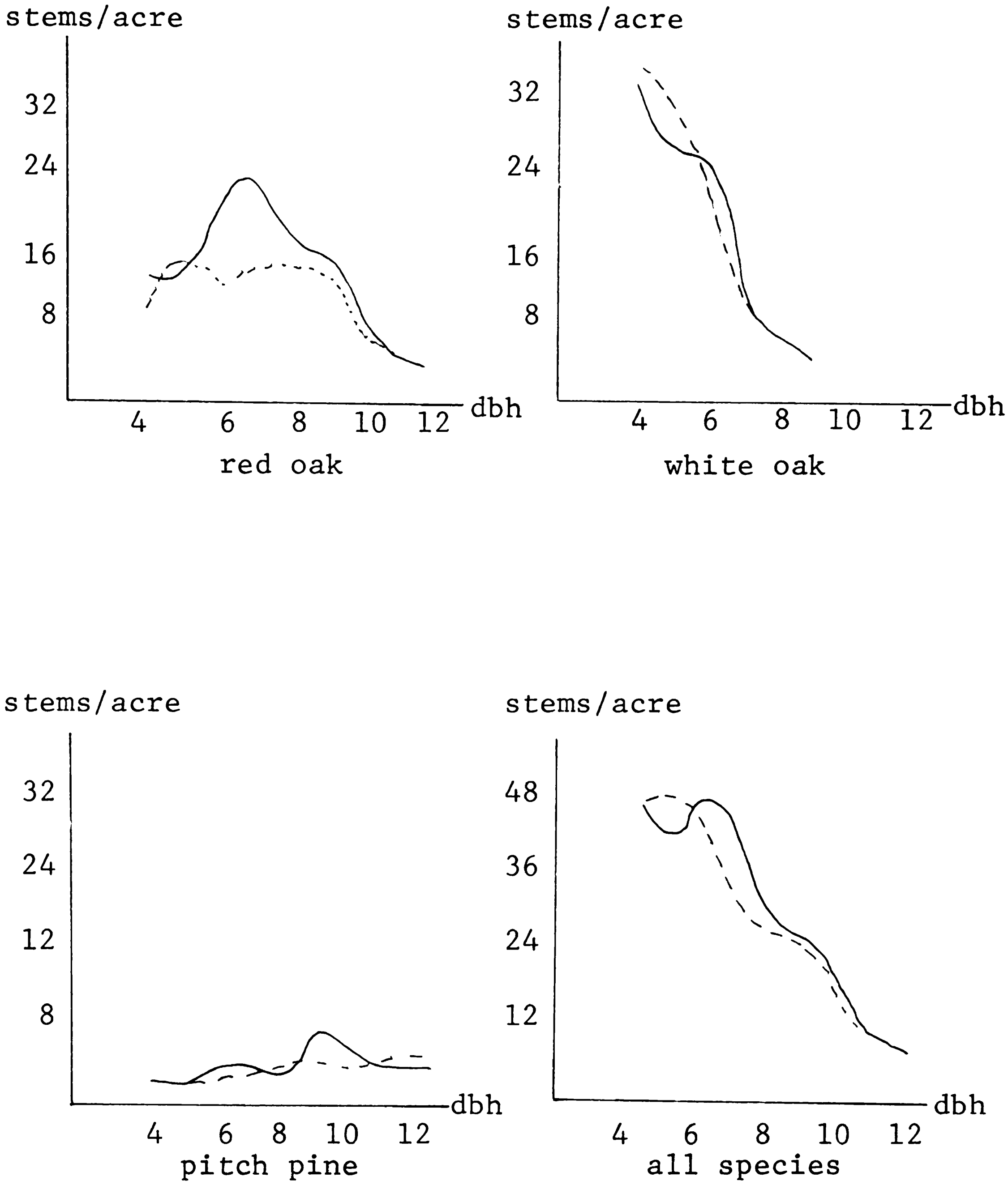


Figure 22. Stand 1 stand tables. Narrow-strip is solid line. Circular-plot is dashed line.

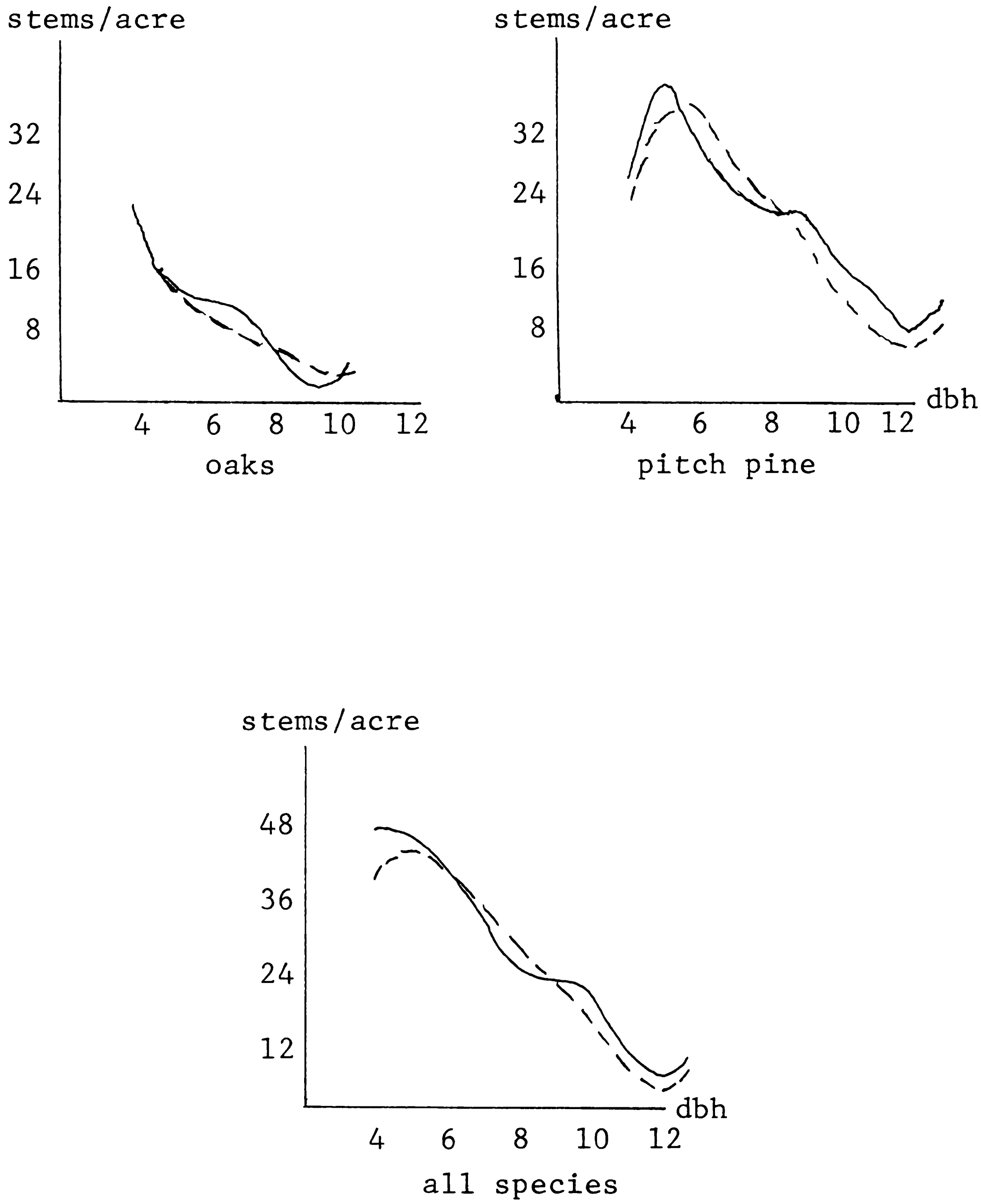


Figure. 23. Stand 2 stand tables. Narrow-strip is solid line. Circular-plot is doubled line.

between narrow-strip and plot samples was due to the facts that plot boundaries had to be searched and some extra walking was required to visit every stem in the plot.

Narrow-strip sampling gave good estimates of basal area, species proportions, diameters, density, and stand table entries. Narrow-strip sample size was about 0.37 times the sample size of the circular-plots in each stand. Density was calculated using e*(DN) as defined in Chapter VB and E(D) was estimated using the method of VIB.

2. Comparison with point sampling.

A low-intensity narrow-strip cruise is now compared with point sampling in Stand 2. By 'low-intensity' it is meant that narrow-strips covered about one-twelth the area covered by circular-plots. Four narrow strips were run through Stand 2 in two hours. This is about half the time the point sample took, but, in addition, narrow-strips gave very good estimates of average diameter and density while at the same time giving a very good estimate of basal area. e(DN) as defined in the Appendix was used to estimate density and the method of VIB was used to estimate E(D).

The narrow-strip estimate of basal area was 86.97 square feet per acre which has a bias of 0.05 from the circular-plot value given in Table 8. From Table 8 it is seen that the point sampling bias is -.03 so narrow-strip sampling did very well in estimating basal area. The estimated average dbh (all species combined) was 7.35 inches which is unbiased with respect to the plot sample estimate in Table 9. The desnity estimate is 275.16 which has a bias of 0.08. The estimator e(DN) is far more convenient to use than e*(DN) and so the use of e(DN) is suggested exclusively.

CHAPTER VIII

RECOMMENDATIONS

Further study is needed in order to establish narrow-strip sampling as a basic sampling technique for forestry. As has been demonstrated in this paper, narrow-strip sampling has much promise as a practical sampling technique. It is the intention of this chapter to list some possible applications of the method. It should also be mentioned that more theoretical development needs to be done, for example, the detection of the type and intensity of spatial pattern present in a population of stems. This chapter is not intended to be an exhaustive list of possible further areas of study of narrow-strip sampling and it is not intended to suggest that narrow-strip sampling can be used more effectively than existing methods of analysis which are currently being applied to these areas.

Pattern in a forest stand is studied by distance sampling (Pielou, 1960; Payandeh, 1975; Cox and Lewis, 1976). During the writing of this paper there were indications that spatial pattern might be detected by narrow-strip sampling. The most promising approach seemed to be:

1. Measure all values of t_1 occurring along a

narrow-strip where (see Chapter IVA1) values of D are measured in both directions.

2. Calculate $K = (n_1+n_2)/(2(m-1))$ where:

   n_1 = number of t_1 measured in one direction

   n_2 = number of t_1 measured in the other direction

   m = number of stems sampled by a narrow-strip (generally $2(m-1) > n_1+n_2$)

3. $E_1(p(t))$ is defined on page 32 :

$$\text{pattern} = \begin{cases} \text{random} & \text{if } K = E_1(p(t)) \\ \text{non-random} & \text{if } K \neq E_1(p(t)) \end{cases}$$

4. If pattern is non-random:

$$\text{pattern} = \begin{cases} \text{clustered} & \text{if } s^2(e(t_1)) \neq 0 \\ \text{regular} & \text{if } s^2(e(t_1)) = 0 \end{cases}$$

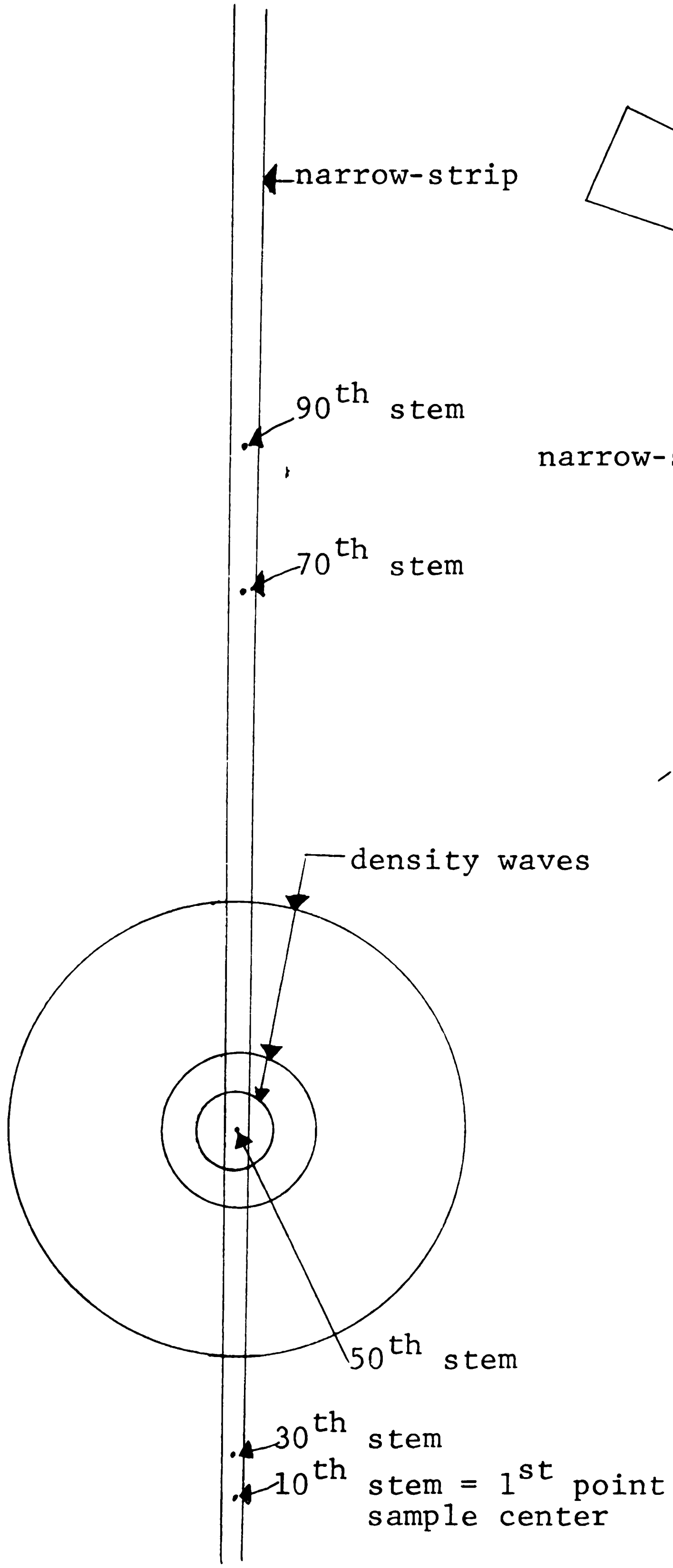
   where $s^2(e(t_1))$ = narrow-strip sample variance of the estimate of $E(t_1)$

Pattern estimates based on narrow-strip sampling of the 34 stem maps were not always accurate and no theoretical basis (only the intuitive basis of 1-4) could be found; however, it seems that, if pattern is a population characteristic important enough to study in some situation, there very well could be some quick method of estimating pattern as a part of a narrow-strip cruise for other characteristics.

Tree growth in an even-aged stand can be sampled from stems which are narrow-strip sampled and which are also first nearest neighbors. Nearest neighbors in such a stand should exhibit more significant interaction in terms of competition over their life-span than stems which are further apart. Thinning causes changes in nearest neighbors and growth responses could also be studied with respect to this aspect.

Considerations similar to those on growth studies also apply to yield studies in conjunction with narrow-strip sampling. For both growth and yield the narrow-strip method's ability to sample variation continuously might give new insight into the variability of these characteristics. In these cases, narrow-strip sampling offers a simple way (ocular estimation of successive stems along a narrow-strip) in which to choose nearest neighbors for measurement.

Narrow-strip sampling could also be combined with other methods of sampling. For example suppose a narrow-strip of length 1 is expected to encounter 100 stems. Also suppose that along a transect of length 1 some points are to be chosen for point sampling. At the tenth stem along a narrow-strip a point sample would be taken. If five points are to be chosen, the other four points would occur at every twentieth stem after the tenth one. Due to an effect called 'waves of density' (Zeide, 1972, 1975) a narrow-strip would determine sampling points which would allow point samples to occur in higher density regions thus reducing the variance of a point sampling estimate (Figure 24a).



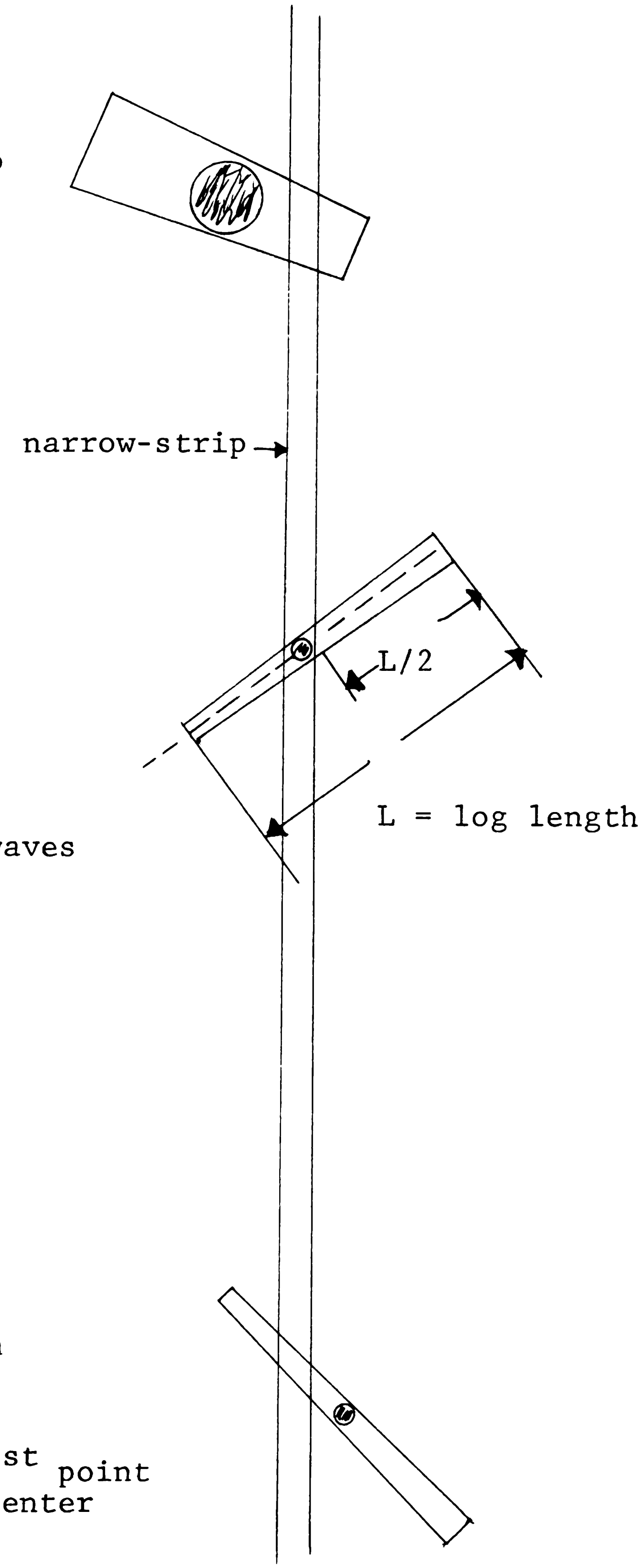


Figure 24a. Point sampling. Figure 24b. Fuel sampling

Figure 24. Two possible applications of narrow-strip sampling.

Similar considerations apply to 3P sampling but in this case every k^{th} stem ($k = 1,\ldots,50$) might be tested to see whether or not it fell into a 3P sample. Information on species, diameter, etc. need not be taken at all as a part of a narrow-strip sample in the 3P sampling and point sampling cases just mentioned, rather the narrow-strips would simply provide a sampling frame from which to sample stems using one of the other two methods.

Stem maps made from aerial photographs (Payandeh, 1970) could be used to gather pre-sampling information for a narrow strip cruise. The intersection of the major and minor diameters of a tree crown would be used as the coordinates of a stem center. A regression of crown area on the square of the dbh would be used to determine stem diameter.

Forest fuel sampling and sampling for logging residue have been done using the line-intersect method of sampling (Bailey, 1969, 1970; DeVries, 1973a, 1973b; VanWagner, 1968); Warren and Olsen, 1964). Narrow-strip sampling could be used in these types of situations by counting a piece of debris in the sample only if the shaded circles (Figure 24b) of diameter equal to the mid-diameter of the piece are met by a narrow strip. The number of pieces sampled should be much lower than in line-intersect sampling so that more intensive measurements of each piece could be accomplished in what should prove to be a shorter period of time. Also, because circles in a plane parallel to the forest floor are being sampled, orientation of the piece centerline with respect to the forest floor should not be a problem.

In a paper by Eberhardt (1978) discussion is given of strip sampling in which the strips are not assumed to be narrow. Density estimates using the method mentioned in that paper might be studied by running narrow-strips through a forest stand and comparing the density estimate given by Eberhardt, e'(DN), with the density estimate derived in this paper, e(DN). The derivation in Eberhardt's paper relies on the strip centerline being randomly located (see Figure 25) parallel to one edge of the area of interest.

Assume a narrow-strip has the x-coordinate of its centerline located such that $X \sim U(0,W)$. Let stems C_i of radius r_i be located independently of one another over a forest stand for $i = 1,...,S$. Suppose the forest stand is a W by L rectangle and the narrow-strip centerline is located parallel to the side of length L. Let k = the number of C_i sampled by this narrow-strip. Define:

$$X(C_i) = \begin{cases} 1 \text{ if } C_i \cap NS \neq \phi \\ \\ 0 \text{ if } C_i \cap NS = \phi \end{cases}$$

Now:

$$\begin{aligned} Pr(C_i \cap NS \neq \phi) &= (w+2wr_i)L/(WL) \\ &= (w+2r_i)/W \end{aligned}$$

where w = narrow-strip width

and so:

$$\begin{aligned} E(k) &= \sum_1^S [0.Pr(X(C_i) = 0)+1.Pr(X(C_i) = 1)] \\ &= \sum_1^S (w+2r_i)/W \end{aligned}$$

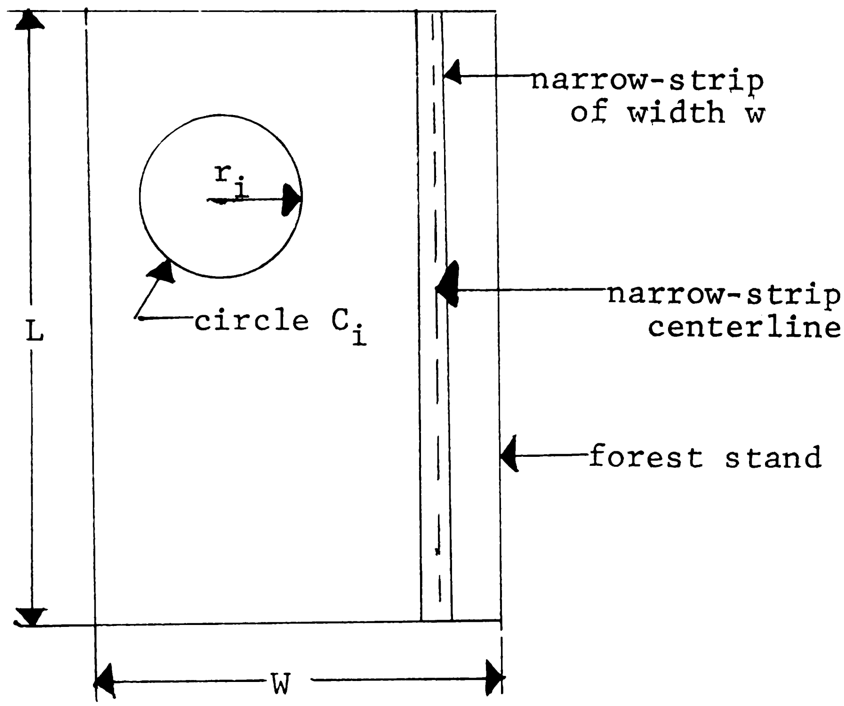


Figure 25. Another density estimator using narrow-strips.

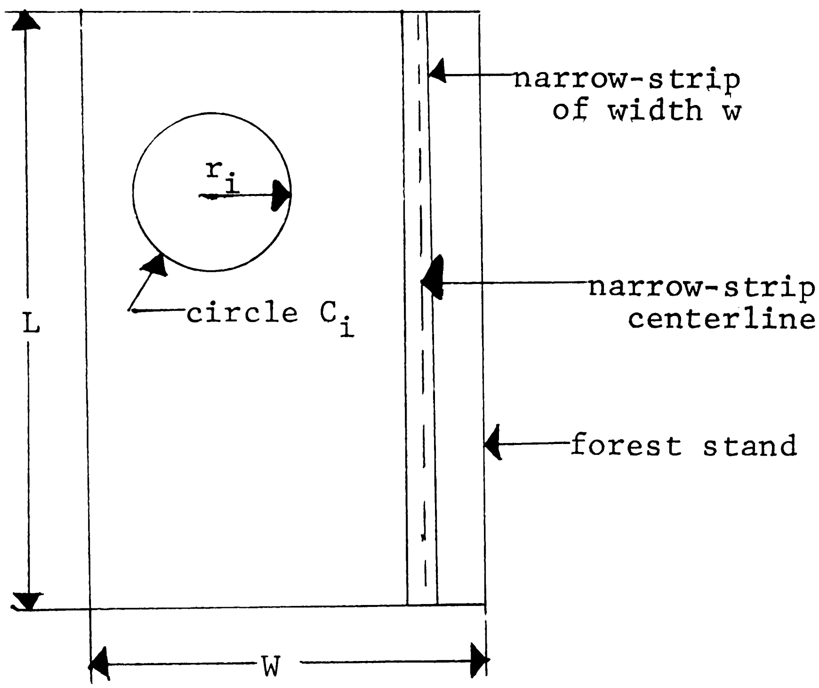


Figure 25. Another density estimator using narrow-strips.

$$= S(w+2\bar{r})/W$$

$$\text{where } \bar{r} = E(r) = \frac{1}{S} \sum_{1}^{S} r_i$$

This gives a density estimate via:

$$e(S) = e(k)W/(w+2e(r))$$

$$e'(DN) = e(S)/(WL)$$

$$= e(k)/((w+2e(r))L)$$

where $e(S)$ = estimate of total number of stems in the stand

$e(k)$ = estimate of $E(k)$

$e(r)$ = estimate of $E(r)$

As a closing remark it must be emphasized that more research is needed to compare narrow-strip sampling of varying intensities with currently used methods of sampling such as plot, point and 3P sampling. Weighted estimators, especially for dbh and species proportions, should be studied. The study of estimation by means of narrow-strips of important characteristics other than the ones studied in this paper is also needed to explore this promising sampling technique. Other characteristics which should be studied are volume, growth, and biomass. It is hoped that the few possible areas of study listed here will generate enough interest in narrow-strip sampling so that this method may begin to become established as a basic sampling technique in forestry.

CHAPTER IX

SUMMARY AND CONCLUSIONS

Narrow-strip sampling consists of sampling a forest stand by means of strips so narrow that the trees may be linearly ordered along the strips. Narrow-strip sampling is a form of cluster sampling in which the number of elements sampled is a random variable. Narrow-strip sampling is also a PPS technique.

A theoretical basis for narrow-strip sampling is given. Monte-Carlo techniques are used to verify theoretical statements and to examine the biases and precisions of narrow-strip estimators. A range of real and computer-generated forest stands are used in the study and they consist of combinations of random, regular, and clustered patterns with a realistic range of densities and diameter distributions which are generated from an empirical distribution function. Density and diameter considerations follow from field data gathered in two stands located in the New Jersey Pine Barrens.

Distance sampling is an unpractical method of sampling for density but has been studied because of its theoretical appeal. Three supposedly robust methods of density estimation using distance sampling are compared with a density estimator using narrow-strip sampling in a Monte-Carlo

study of 34 real and computer-generated stem maps. It is found that narrow-strip sampling gave highly robust estimates of density, outperforming the best of the distance estimators.

Narrow-strip sampling, circular-plot sampling and point sampling are used to sample the 34 stem maps for basal area, species proportions, diameters, and density using both random and systematic location of the three types of sampling elements which define clusters of stems. Strip length, plot radius, and basal area factor are chosen such that all three methods have an equal expected sample size. Methods are compared on the bases of bias and precision. On these bases, narrow-strip sampling generally performs as well as the other two methods.

When data gathered in the field (one oak-dominated 62.5-acre stand and one pine-dominated 43.0-acre stand) are analyzed, it is found that narrow-strip sampling performs very well. A high-intensity narrow-strip sample takes about one-half the time of a plot sample to give good estimates of the same characteristics which the plot sample estimates and, in about twice the time it takes to point sample only for basal area, the narrow-strip technique samples for basal area, density, diameters, species proportions, and stand table entries. A low-intensity narrow-strip sample is compared to a point sample in the pine-dominated stand. The low-intensity narrow-strip sample performs remarkably well since it takes half the time of the point sample but also gives very good estimates of average diameter (all species combined) and density.

It seems likely that many areas of application lie ahead for narrow-strip sampling. A few possible such areas are suggested in Chapter VIII. In the meantime many empirical studies will be required to further verify what this study has demonstrated: narrow-strip sampling is an attractive and practical alternative to circular-plot sampling, point sampling, and distance sampling for estimation of basal area, species proportions, diameters, stand table entries, and density in the types of situations included in this study. Theoretically there is no reason why narrow-strip sampling for these characteristics and other characteristics like growth, volume, and biomass cannot be applied to any forest situation.

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LITERATURE CITED

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APPENDIX

APPENDIX A

Generating a density equation

Table A1 is generated by means of a Fortran IV program called NRSTRP. NRSTRP approximates the sum of the infinite series for E(D) over a specified range of diameters and densities and for a given width. Table A2 gives the results of least-squares exponential curve fits for each graph in Figure A1. Figures A1 and A2 are graphs of Table A1 and A2. Table A3 gives input card layouts and input values for five test cases using NRSTRP. Table A4 gives the output of the test cases listed in Table A3. Table A5 is a listing of NRSTRP.

Since estimated diameters and inter-tree distances encountered in a narrow-strip sample will not in general match those values given in Table A1, interpolation will usually be required to estimate density. One method of interpolation is now discussed. This method results in:

$$e(DN) = be^{aD}$$

where $a = a(d)$

$b = b(d)$

d = diameter estimate

D = inter-tree distance estimate

e = 2.71828... = the base of the natural logarithms

By inspecting Figures A2 it appears that:

$$a(d) = rd+s$$

Table A1. Expected inter-tree distance.

density (stems/ acre)	dbh (inches) 5	6	7	8
180	66.989960	65.424316	63.927383	62.501740
200	60.349457	58.934082	57.588150	56.297546
220	54.906540	53.614029	52.393904	51.204926
240	50.356293	49.170822	48.043747	46.959978
260	46.504501	45.409073	44.359924	43.353516
280	43.200134	42.174591	41.187164	40.229218
300	40.319427	39.333603	38.433090	37.576920

Table A1. (continued)

density (stems/ acre)	dbh (inches) 9	10	11	12
180	61.134430	59.827271	58.566345	57.36.633
200	55.058044	53.878464	52.744034	51.656799
220	50.081451	49.005478	47.967697	46.963776
240	45.924438	44.924835	43.925690	43.034195
260	42.368347	41.449081	40.585861	39.756470
280	39.353806	38.519348	37.712692	36.937973
300	36.755264	35.966888	35.216080	34.489349

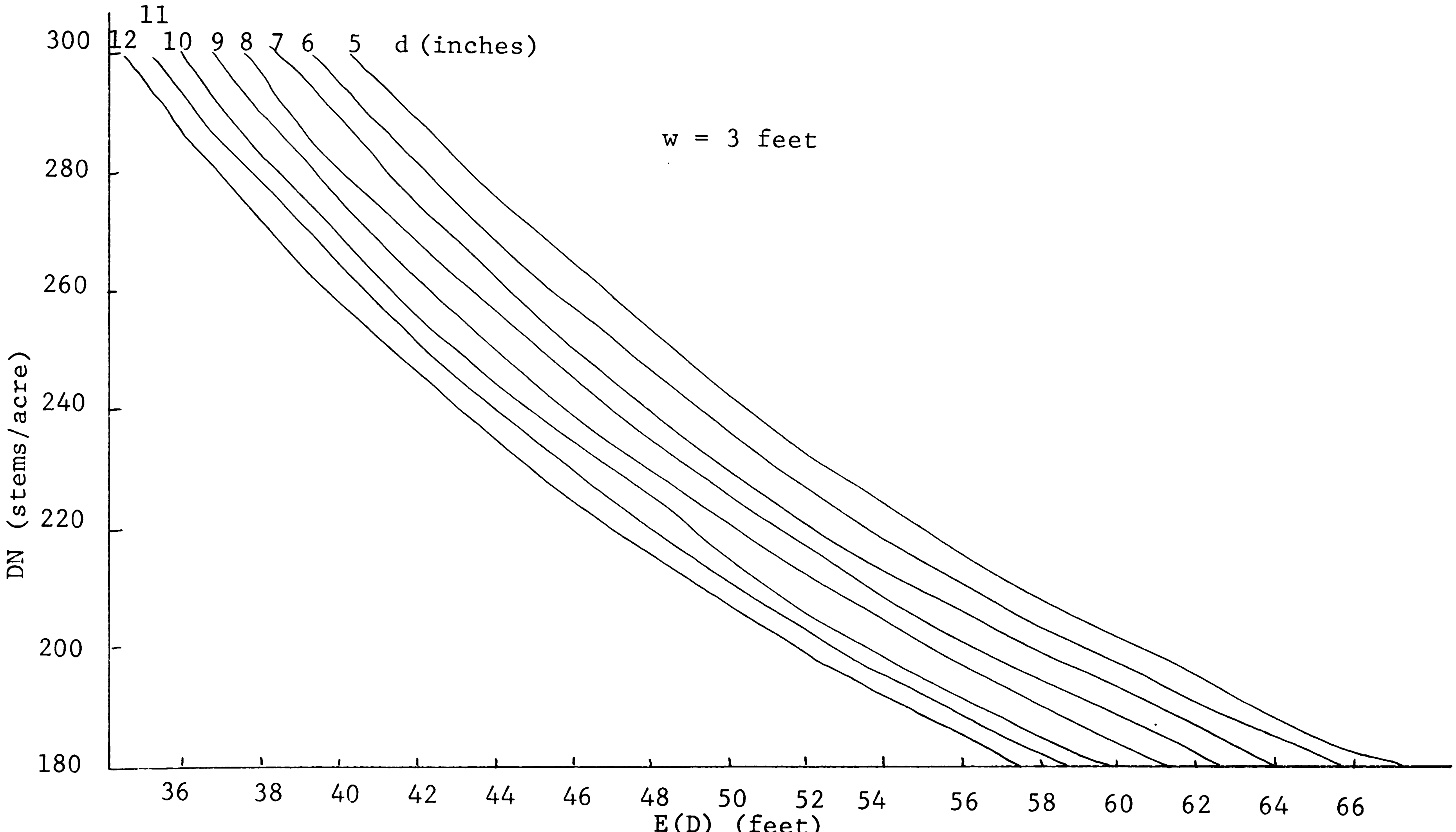

Figure A1. Density related to inter-tree distance.

$$b(d) = t(d-10)^2+u$$

Refer to Table A2. Least-squares regressions are performed on (d_i,a_i) and on (d_i,b_i) for $i = 1,\ldots,8$ yielding:

$$a(d) = -.00045145138d-.016857918$$
$$\text{with } r^2 = .9999$$
$$b(d) = .090081282(d-10)^2+635.18938$$
$$\text{with } r^2 = .9824$$

In Table A2:

$$B_i = (1/7)\sum_{j=1}^{7} |b(d_i)e^{a(d_i)E(D_{ji})}-DN_j| /DN_j$$

where $E(D_{ji}) = (j,i)^{th}$ entry in Table A1

and

j	DN_j
1	180
2	200
3	220
4	240
5	260
6	280
7	300

In view of the r^2 and B_i the interpolation equation:

$$e(DN) = b(d)e^{a(d)D}$$

is a very good one.

Suppose a narrow-strip sample has been conducted and the estimates $d = 7.4$ inches and $D = 42$ feet have been obtained. This implies:

$$e(DN) = b(7.4)e^{a(7.4)(42)}$$
$$= 635.79833e^{(-.020198658)(42)}$$
$$= 272.20019$$

It is a straightforward matter to determine Table A1, Table A2, a(d) and b(d) once the d_i and DN_j have been chosen. The cost of generating Table A1 with NRSTRP was less than $9.00

Table A2. Exponential regression coefficients.

i	dbh (in.)	distance multiplier	leading constant	goodness of fit (r^2)	1/ bias
1	5	-.019130974	637.43707	0.9948	0.0112
2	6	-.019567004	636.57339	.9949	.0109
3	7	-.020014943	636.18527	.9949	.0108
4	8	-.020456716	636.57180	.9948	.0108
5	9	-.020909785	636.34289	.9947	.0110
6	10	-.021364480	635.21352	.9946	.0111
7	11	-.021827428	635.23293	.9944	.0114
8	12	-.102290705	635.36301	.9944	.0115

1/ see page 103.

Tabe A3. NRSTRP sample input.

1/W	D	DN	TOL1	TOL2	JMAX1	JMAX2	ISR	IUNITS
2/10	20	30	40	56	55	60	65	70
3.0	4.0	180.	.01	.01	25	1500	50	1
3.0	8.0	240	.01	.01	25	1500	50	1
3.0	12.0	300	.01	.01	25	1500	50	1
8.0	72.0	30	.01	.01	25	1500	50	1
1.0	2.0	2000	.01	.01	25	1500	50	1

1/ Fortran names for values (see page 107).

2/ values are right-justified in these card columns.

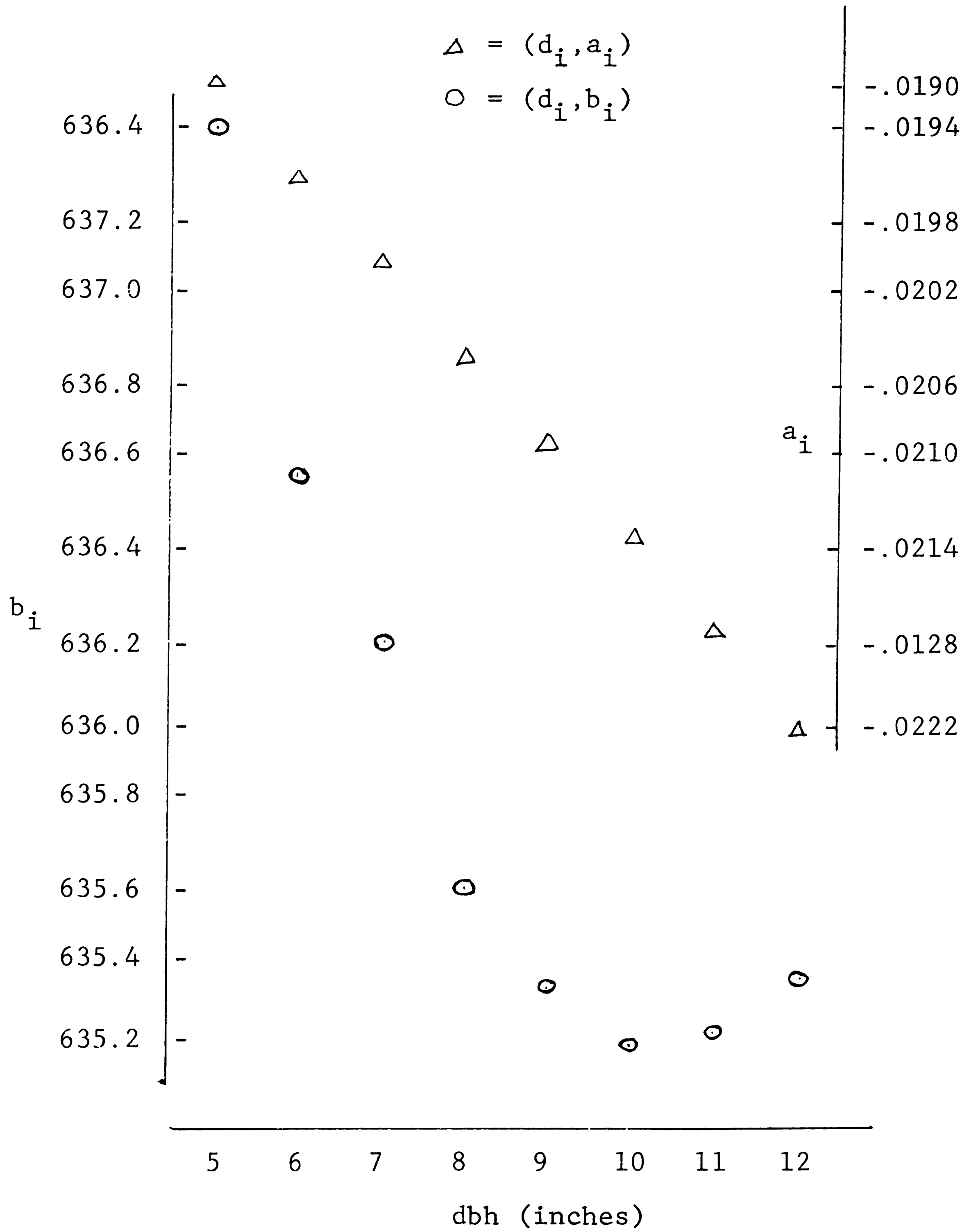

Figure A2. Regression constants related to diameter.

on an IBM 370/168 (OS/VS). Inputs to NRSTRP (width, diameter, density) can be in English (feet, inches, stems/acre) or in Metric (meters, centimeters, stems/hectare) units.

There is one input card per run. The order of the variables on the input card and their Fortran IV names are: W,D, DN, TOL1, TOL2, JAMX1, JAMX2, ISR, IUNITS. The meanings of these variables are:

W = narrow-strip width

D = average stem diameter to be encountered

DN = density to be encountered

TOL1 = tolerance on the integral of $K_j(t)$ using Simpson's rule. If S_j is the value of this integral, then the program checks to see whether or not $|S_j - 1.0| \leq$ TOL1.

TOL2 = tolerance on the sum of all probabilites of a narrow-strip distance between two successive stems, T, taking on the value of a random variable t_j. If k is the last term calculated, the program checks to see whether or not $\left| \sum_{j=1}^{k} \Pr(T=t_j) - 1.0 \right| \leq$ TOL2.

JMAX1 = maximum number of terms to be calculated using Simpson's rule.

JMAX2 = maximum number of terms to be allowed for the convergence of the infinite series for E(T).

ISR = the number of subdivisions to be used in Simpson's rule. Must be an even integer.

IUNITS = 1 (or 2) if English (or Metric) units are used.

NRSTRP is comprised of two sections. The first section uses Simpson's rule to integrate $p(t)K_j(t)$ for $j = 1,\ldots,l$ in order to estimate $E_j(p)$. $l \leq JMAX1$. Should numerical values become too large, the first section switches to calculations using natural logarithms. Should these values become too large or should any integral of $K_j(t)$ not meet the requirements of TOL1, control passes to the second section. In this way it is possible for $l < JMAX1$ to occur.

The second section of NRSTRP uses Stirling's approximation (see Thompson, 1956) to estimate the integrals of $p(t)K_j(t)$ in order to estimate $E_j(p)$. In the output there are three entries under Stirling's Approximation. The entry under 'Started' is equal to $l+1$. The entry under 'Finished' is the integer for which the program converged and is less than or equal to JMAX2. The third entry is the sum of all the $Pr(T = t_j)$ for $j = 1,\ldots,k$ = the integer for which the program converged. If the third entry is not within TOL2 of 1.0, then JMAX2 should be increased until convergence is achieved. Under 'Distance Statistics' are listed E(D) = the expected value of T, V(D) = the variance of T and CV(D) = the coefficient of variation of T.

The IBM subroutine OVERFL is used to test whether or not internal floating point capacity of the machine has been exceeded. If this condition exists, calculations proceed using natural logarithms and values are then converted using the exponential function.

Table A4. NRSTRP output.

```
* EXPECTED DISTANCE BETWEEN STEMS IN A NARROW-STRIP SAMPLE*

                         *** INPUT ***

            WIDTH                DBH               DENSITY
   0.30000000E 01     0.40000000E 01      0.18000000E 03

                       * SIMPSONS RULE *
        TOLERANCE               JMAX   NO. SUBDIVISIONS
   0.99999979E-02     0.25000000E 02      0.50000000E 02

                 * STIRLINGS APPROXIMATION *
        TOLERANCE               JMAX
   0.99999979E-02     0.15000000E 04

                        *** OUTPUT ***

                 * STIRLINGS APPROXIMATION *
          STARTED             FINISHED   PR(D=T(J):J<1448)
   0.26000000E 02     0.14470000E 04     0.99000084E 00

                  * DISTANCE STATISTICS *
             E(D)                 V(D)               CV(D)
   0.68766113E 02     0.40533047E  04    0.92582771E 00

*EXPECTED DISTANCE BETWEEN STEIMS IN A NARROW-STRIP SAMPLE *

                         *** INPUT ***

          WIDTH                  DBH               DENSITY
   0.300000E 01       0.80000000E 01      0.24000000# 03

                       * SIMPSONS RULE *
        TOLERANCE               JMAX   NO. SUBDIVISIONS
   0.99999979E-02     0.25000000E 02      0.50000000E 02

                 * STIRLINGS APPROXIMATION *
        TOLERANCE               JAMX
   0.99999979E-02     0.15000000E 04

                        *** OUTPUT ***

                 * STIRLINGS APPROXIMATION *
          STARTED             FINISHED   PR(D=T(J):J<894)
   0.26000000E 02     0.89300000E 03     0.099000067E 00

                  * DISTANCE STATISTICS *
             E(D)                 V(D)               CV(D)
   0.46958878E 02     0.18630012E 04     0.91915507E 00
```

Table A4. (continued).

* EXPECTED DISTANCE BETWEEN STEMS IN A NARROW-STRIP SAMPLE*

*** INPUT ***

WIDTH	DBH	DENSITY
0.30000000E 01	0.12000000E 02	0.30000000E 03

* SIMPSONS RULE *

TOLERANCE	JMAX	NO. SUBDIVISIONS
0.99999979E-02	0.25000000E 02	0.50000000E 02

* STIRLINGS APPROXIMATION *

TOLERANCE	JMAX
0.99999979E-02	0.15000000E 04

*** OUTPUT ***

* STIRLINGS APPROXIMATION *

STARTED	FINISHED	PR(D=T(J):J<599)
0.26000000E 02	0.59800000E 03	0.99003357E 00

* DISTANCE STATISTICS *

E(D)	V(D)	CV(D)
0.34489349E 02	0.98807300E 04	0.91140091E 00

EXPECTED DISTANCE BETWEEN STEMS IN A NARROW-STRIP SAMPLE

*** INPUT ***

WIDTH	DBH	DENSITY
0.80000000E 01	0.72000000E 02	0.30000000E 02

* SIMPSONS RULE *

TOLERANCE	JMAX	NO. SUBDIVISIONS
0.99999979E-02	0.25000000E 02	0.50000000E 02

* STIRLINGS APPROXIMATION *

TOLERANCE	JMAX
0.99999979E-02	0.15000000E 04

*** OUTPUT ***

* STIRLINGS APPROXIMATION *

STARTED	FINISHED	PR(D=T(J):J<486)
0.26000000E 02	0.48500000E 03	0.99000442E 00

E(D)	V(D)	CV(D)
0.98294693E 02	0.79728594E 04	0.90839979E 00

TABLE A4. (continued)

```
*EXPECTED DISTANCE BETWEEN STEMS IN A NARROW-STRIP SAMPLE*

                         *** INPUT ***

           WIDTH                  DBH             DENSITY
  0.10000000E 01     0.20000000E 01      0.20000000E 04

                      * SIMPSONS RULE *
       TOLERANCE                 JMAX    NO. SUBDIVISIONS
  0.99999979E-02     0.25000000E 02      0.50000000E 02

                  * STIRLINGS APPROXIMATION *
       TOLERANCE                 JMAX
  0.99999979E-02     0.15000000E 03

                        *** OUTPUT ***

                  * STIRLINGS APPROXIMATION *
         STARTED             FINISHED   PR.(D=T(J):J<1062)
  0.26000000E 02     0.10610000E 04      0.99002075E 00

                    * DISTANCE STATISTICS *
  0.17692978E 02     0.26627417E 03      0.92232902E 00
```

TABLE A5. Program NRSTRP.

```
C
C
C     CALCULATE E(D)=EXPECTED DISTANCE BETWEEN STEMS ALONG A
C        NARROW-STRIP GIVEN WIDTH OF STRIP, AVERAGE D.B.H. OF
C        STAND AND STAND DENSITY.
C
C
C        JAMES E. KEARIS                    7/22/78
C        REFERENCE: PH.D. DISSERTATION, MICHIGAN STATE
C        UNIVERSITY: 'A PRELIMINARY INVESTIGATION OF NARROW-
C        STRIP SAMPLING AS APPLIED TO FOREST SAMPLING'.
C
C
      DIMENSION GX(57), T(102)
C
C     GENERATE FACTORIAL TABLE: GX(J)=(J-1) FACTORIAL
C
      GX(1)=1.
      DO 1 I=2,57
      X=I-1
    1 GX(I)=X*GX(I-1)
C
C     READ AND WRITE INPUT
C
  998 READ(5,20,END=999) W,D,DN,TOL1,TOL2,JMAX1,JMAX2,ISR,
     1IUNITS
      WRITE(6,21) W,D,DN,TOL1,TOL2, JAMX1,JMAX2,ISR,IUNITS
   20 FORMAT(5F10.4,4I5)
   21 FORMAT('W=',F10.4,'  D=',F10.4,'  DN=',F10.4,'  TOL1='
     1,F10.4,'  TOL2=',F10.4,/,'JMAX1=',I5,'  JMAX2=',I5,
     2'  ISR=',I5,'  IUNITS=',I5)
C
C     DEFINE CONSTANTS
C
      XK=2
      PI=3.1415927
      GO TO (2,3), IUNITS
    2 XP=43560.
      WP=12.
      GO TO 4
    3 XP=10000.
      WP=100.
    4 XM=DN/XP
      XM1R=SQRT(XP/DN)
      PIXM=PI*XM
      PIXM1=XP/DN/PI
      PIXM1R=SQRT(PIXM1)
      XISR=ISR
      WPRIME=(W+D/WP)/2.
      BJ=1.
      SPDETJ=0.
      SED=0.
      SED2=0.
```

TABLE A5. (continued).

```
      ICNTRL=1
C
C
C     BEGIN SUM USING SIMPSON'S RULE
C
C
      DO 100 J=1,JMAX1
      IJ=1
      XJ=J
      GO TO (190,102), ICNTRL
C
C     IS FACTORIAL ARGUMENT WITHIN BOUNDS
C
  190 IF(J-28) 101,101,109
  109 ICNTRL=2
      GO TO 102
C
C     FIND J-1, J, 2*J FACTORIALS
C
  101 X1=GX(J)
      X0=GX(J+1)
      X2=GX(2*J+1)
C
C     ETJ=EXPECTED VALUE OF J-TH NEAREST NEIGHBOR DISTANCE
C
      ETJ=XM1R*X2*XJ/(2.**J*X0)**2
      CALL OVERFL(IOVFL)
      GO TO (110,111,110), IOVFL
  110 ICNTRL=2
C
C     CALCULATE ETJ USING LN FUNCTION
C
  102 J2=2*J
      J1=J+1
      S2=0.
      DO 201 I=J1,J2
      Y2=I
  201 S2=S2+ALOG(Y2)
      S1=0.
      IJ=2
      IF(J-2) 165,165,164
  164 J2=J-1
      DO 202 I=2,J2
      Y2=I
  202 S1=S1+ALOG(Y2)
  165 ETJ=S2-2.*XJ*ALOG(XK)-S1
      ETJ=XM1R* EXP(ETJ)
C
C     STJ=STANDARD DEVIATION OF J-TH NEAREST NEIGHBOR DISTANCE
C
  111 STJ= SQRT(XJ*PIXM1-ETJ**2)
```

TABLE A5. (continued)

```
C
C     A=LOWER LIMIT OF INTEGRATION
C     B=UPPER LIMIT OF INTEGRATION
C     H=WIDTH OF SUBDIVISION
C
      A=ETJ-4.*STJ
      B=ETJ+5.*STJ
      IF(A-.01) 171,170,170
  171 A=.01
  170 H=(B-A)/XISR
C
C     TEST FOR OVERFLOW AT T(I)=B FOR THIS J
C
      GO TO (140,141), ICNTRL
  140 XKJTI=2.*PIXM**J*B**(2*J-1)* EXP(-PIXM*B**2)X/1
      CALL OVERFL(IOVFL)
      GO TO (120,121,120), IOVFL
  120 ICNTRL=2
  141 GO TO (148,149), IJ
  148 S1=0
      IF(J-2) 149,149,147
  147 J2=J-;
      DO 146 I=2,J2
      Y2=1
  146 S1=S1+ALOG(Y2)
  149 (XKJTI=XJ*ALOG(PIXM)+(2.*XJ-1.)*ALOG(B)-PIXM*B**2-S1
      XKHTI=2.* EXP(XKJTI)
      CALL OVERFL(IOVFL)
  122 JJ=J
      GO TO 301
  121 T(1)=A-H
      ISS=ISR+2
      SKJTI=0.
      SPTHLF=0.
      DO 150 I=2,ISS
      T(I)=T(I-1)+H
C
C     Z=COEFFICIENT OF FIRST, LAST, ODD OR EVEN TERM IN
C     SIMPSON'S RULE
C
      IF(I-2) 151,151,155
  155 IF(I-ISS) 156,151,151
  156 GO TO (152,153), IZ
  151 IZ=1
      Z=1.
      GO TO (154,157), ICNTRL
  152 IZ=2
      Z=4
      GO TO (154,157), ICNTRL
```

TABLE A5. (continued)

```
  153 IZ=1
      Z=2.
      GO TO (154,157), ICNTRL
C
C     CALCULATE K(J) AT T(I)
C
  154 XKJTI=2.*PIXM**J*T(I)**(2(J-1) EXP(-PIXM*T(I)**2)/X1
      GO TO 158
  157 XKJTI=XJ*ALOG(PIXM)+(2.*XJ-1.)*ALOG(T(I))-
      1PIXM*T(I)**2-S1
      XKJTI=2.*EXP(XKJTI)
  158 XKJTI=Z*XKJTI
      SKJTI=SKJTI+KJTI
      ARG=WPRIME/T(I)
      IF(ARG=1.) 161,160,160
  160 PTHALF=.5
      GO TO 162
  161 PTHALF=ARSIN(ARG)/PI
  162 SPTHLF=SPTHFL+PTHALF*XKJTI
  150 CONTINUE
      SKJTI=SKJTI*H/3.
      SPTHLF=SPTHLF*H/3.
C
C     IS INTEGRAL OF K(J) BETWEEN A AND B WITHIN TOLERANCE
C
      IF( ABS(SKJTI-1.)-TOL1) 180,180,181
  181 WRITE(6,800) J, SKJTI
  800 FORMAT('INTEGRAL OF K('I4,')=',E14.8)
  180 PRDETJ=SPTHLF*BJ
      BJ=BJ*(1.-SPTHLF)
      SPDETJ=SPDETJ+PRDETJ
      SED=SED+ETJ*PRDETJ
      SED2=SED2+XK*PIXM1*PRDETJ
      IF( ABS(SPDETJ-1.)-TOL2) 801,801,100
  801 JJ=XJ+.05
      XJJ=XJ
      GO TO 450
  100 CONTINUE
      JJ=JMAX1+1
C
C
C     END SUM USING STIRLING'S APPROXIMATION
C
C
  301 XJJ=JJ
      D) 400 J=JJ,JMAX2
      XJ=J
      ETJ=PIXM1R* SQRT(CJ)
      PTHALF= ARSIN(WPRIME/ETJ)/PI
      PRDETJ=PTHALF*BJ
```

TABLE A5. (continued)

```
      SPDETJ=SPDETJ+PRDETJ
      SED=SED+ETJ*PRDETJ
      SED2=SED2+XJ*PIXM1*PRDETJ
      IF( ABS(SPDETJ-1.)-TOL2) 450,450,400
  400 CONTINUE
  450 VD=SED2-SED**2
      CVD= SQRT(VD)/SED
C
C     WRITE OUTPUT
C
      WRITE(6,22)
   22 FORMAT('1')
      WRITE(6,23)
   23 FORMAT(////)
      WRITE(6,24)
   24 FORMAT(5X,'* EXPECTED DISTANCE BETWEEN STEMS IN A
     1NARROW-STRIP SAMPLE*')
      WRITE(6,23)
      WRITE(6,25)
   25 FORMAT(28X,'*** INPUT ***',//)
      WRITE(6,26)
   26 FORMAT(20X,'WIDTH',19X,'DBH',15X,'DENSITY')
   27 FORMAT(11X,E14.8,8X,E14.8,8X,E14.8,//)
      WRITE(6,27) W,D,DN
      WRITE(6,28)
   28 FORMAT(26X,'* SIMPSONS RULE *')
      WRITE(6,29)
   29 FORMAT(16X,'TOLERANCE',18X,'JMAX',6X,'NO.SUBDIVISIONS')
      X1=JMAX1
      X2=ISR
      WRITE(6,27) TOL1, X1, X2
      WRITE(6,32)
   32 FORMAT(21X,'* STIRLINGS APPROXIMATION *')
      WRITE(6,30)
   30 FORMAT(16X,'TOLERANCE',18X,'JMAX')
      X1=JMAX2
      WRITE(6,27)TOL2,X1
      WRITE(6,23)
      WRITE(6,31)
   31 FORMAT(28X,'*** OUTPUT ***',//)
      J=XJ+1.05
      WRITE(6,32)
      WRITE(6,33)
   33 FORMAT(18X,'STARTED',14X'FINISHED',5X,'PR(D=T(J):J<',
     1I4,')')
      WRITE(6,27) XJJ, XJ, SPDETJ
      WRITE(6,35)
   35 FORMAT(23X,'* DISTANCE STATISTICS *')
      WRITE(6,36)
   36 FORMAT(21X,'E(D)',18X,'V(D)',17X,'CV(D)')
```

TABLE A5. (continued).

```
      WRITE(6,27) SED, VD, CVD
      WRITE(6,22)
      GO TO 998
  999 CONTINUE
      STOP
      END
```