This is to certify that the

thesis entitled

INDUCED ELECTROMAGNETIC FIELDS IN CONDUCTING
BODIES IRRADIATED BY MAGNETIC FIELDS
AND MICROWAVES

presented by

Jen-Hwang Lee

has been accepted towards fulfillment
of the requirements for

Ph. D. degree in Elect. Engr. & Sys. Sci.

Major professor

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INDUCED ELECTROMAGNETIC FIELDS IN CONDUCTING BODIES IRRADIATED BY MAGNETIC FIELDS AND MICROWAVES

By

Jen-Hwang Lee

A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Electrical Engineering
and Systems Science

1979

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ABSTRACT

INDUCED ELECTROMAGNETIC FIELDS IN CONDUCTING BODIES IRRADIATED BY MAGNETIC FIELDS AND MICROWAVES

By

Jen-Hwang Lee

A new theoretical method for determining the eddy current induced by a uniform RF magnetic field or a beam of RF magnetic field in a biological body is developed in this research. This study was motivated by the fact that more biological research and medical applications utilize the irradiation of RF magnetic fields, and by the need for an efficient method of determining the magnetic mode of the induced electric field in a thick biological body irradiated by HF-VHF EM waves. The body of rotational symmetry is subdivided into a number of circular rings with various radii and cross-sectional areas. The eddy current induced by the impressed magnetic field can be considered, physically, as induced by a circulatory impressed electric field associated with the uniform impressed magnetic field. Induced eddy currents in all the rings maintain a scattered electric field which can be added to the impressed electric field to yield the total induced electric field in the body.

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At the same time, the eddy current is the product of the complex conductivity and the induced electric field. Using these relations and the point-matching method, a set of simultaneous equations for the induced electric fields can be obtained, and they are subsequently solved numerically. With the present method, the eddy currents induced by magnetic fields of 1 to 200 MHz in various biological bodies have been obtained. It was found that the conventionally used, quasi-static solution for the eddy current fails to yield accurate results for frequencies higher than 20 MHz, while the present method can be used to determine accurately and efficiently the induced eddy currents in biological bodies for frequencies higher than 20 MHz.

Also included in this thesis is a numerical method for quantifying the induced EM field on the surface of an irradiated biological body based on two coupled, surface integral equations. In the field of theoretical dosimetry, the quantification of the induced EM field inside an irradiated body has been performed mainly by volume integral equation methods. When a large biological body such as the human body is irradiated by an EM wave of microwave range, the body becomes electrically large and the induced EM field concentrates mainly in a thin layer of the body surface. A volume integral equation method then becomes inadequate or inefficient to handle this problem, and it is more efficient to quantify the induced EM field on the

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body surface based on a surface integral equation method for this case. After the tangential components of the induced electric and magnetic fields on the body surface are determined, the internal EM field can be calculated. To check the accuracy of this method, electrically small bodies are considered first, and results obtained from the surface integral equation method are compared with that obtained from the tensor integral equation method, a volume integral equation method. It was learned that, in some cases, the surface integral equation method showed advantages in accuracy and computational cost over the volume integral equation method.

To

To my mother, Kim-Chu Hong Lee,

my wife, Ai-Chih,

and my son, Yueh.

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I wish to express my sincere appreciation to my research advisor, Dr. K. M. Chen, for his constant support and guidance throughout the course of this research. I also wish to thank Dr. D. P. Nyquist for his helpful comments and encouragement about this study. Sincere appreciation is also extended to Dr. M. Siegel and Dr. D. Riska for their valuable help.

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Finally, I thank my wife and my son for their patience and love, which made completion of this work possible.

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CHAPTER I

INTRODUCTION

Since electromagnetic radiation and propagation were discovered more than a century ago, electromagnetic waves of various frequencies have been utilized in many ways to benefit the human society. Especially, due to the ingenious contributions of engineers and scientists, such incredible dreams as satellite communications, long-range radar detection, high power-rating microwave ovens, and many other products related to EM technology have been realized. Moreover, high-level beamed microwave power is proposed as a new scheme for transporting energy in the future, and the use of electromagnetically induced hyperthermia as an adjunct in cancer treatment is receiving increased attention from many medical researchers. Indeed, these highly advanced techniques of utilizing energy in EM form do benefit us in many respects; unfortunately, they also bring us certain unwanted potential hazards.

Over a decade ago, medical personnel and public health officials began to suspect that low-level, long-term ionizing radiation could be one of the major causes for several different cancers. Even more astonishingly, it is found

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impossible to completely avoid this low-level radiation since it exists everywhere in space, originating from either man-made or natural sources. Besides the above possible hazards due to low-level radiation, EM radiation in high intensities, under an uncontrolled condition, can be harmful to living systems. The thermal effects associated with such high-intensity radiation can produce burns, cataracts, and chemical changes, etc. All these effects have become the major concerns of the public, especially for those who are subject to possible long-term or high-intensity radiation exposure. As a result, there is strong demand for a more thorough understanding of the biological effects of EM radiation. There is therefore a demonstrated need to determine, for example, the safe power density for long-term human exposure, the correct applications methodology and techniques in utilizing high-intensity EM radiation for medical treatment, etc. However, based on the present state of knowledge in this area, many questions still remain unanswered. As a matter of fact, there exist strong arguments between involved groups about where realistic safety levels for microwave exposure should lie. It is surprising to observe that the maximum allowable safe power density for long-term human exposure varies from 10 mW/cm^2 in this country to as low as 0.01 mW/cm^2 in the USSR.

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To clarify these uncertainties, electromagnetic engineers should assume the responsibility for providing more complete, accurate, quantitative data on biological effects of EM radiation; they belong to the professional groups having adequate knowledge and ability to analyze this problem both theoretically and experimentally.

Under these circumstances, we complete the present research for the purpose of seeking for a better understanding of EM interactions with biological bodies. We study the eddy currents induced by RF magnetic fields inside biological bodies in Chapter II. The RF magnetic field has received increased attention due to its potential usefulness for local heating in cancer treatment. Based on our research, we obtain a better understanding about the mechanism exciting induced currents inside the bodies; furthermore, we are able to predict accurately the current distributions inside the bodies, which has not been accomplished by previous methods. In Chapter III, we study the microwave interactions with biological bodies through the solutions of two coupled, surface integral equations. This numerical technique can be applied to quantify the EM fields induced on the surfaces of arbitrarily shaped biological bodies. Through this study, we expect to obtain a more complete, accurate, quantitative evaluation of microwave interactions. Chapter IV describes

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in detail the computer programs used for the above two problems. Then, Chapter V summarizes this research.

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CHAPTER II

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In this chapter, we will present a new theoretical method for determining the electric field or the eddy current induced by a uniform RF magnetic field or a beam of RF magnetic field in a finite conducting body with rotational symmetry.

The body being considered is divided into a number of circular rings with various radii and cross-sectional areas. The induced electric field in each ring is then numerically determined based on the theory of vector potential and the moment method. Numerical examples are given and the results based on the present theory are found to deviate significantly from the often-used quasi-static solutions. An experiment was conducted to measure the electric fields induced by a UHF magnetic field in finite conducting phantom models. Experimental results were found to be in a good agreement with the theory. The accuracy of the present theory was also verified by the

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2.1. Introduction

Due to its increasing applications in the biomedical area, the radiation of RF magnetic field is becoming an important new technique which has drawn a large amount of attention from many investigators. Conventionally, the electric field or the eddy current induced by a magnetic field inside a finite conducting body is estimated by the quasi-static solution. Unfortunately, the quasi-static approximation can give accurate results only when the body is relatively small or when the frequency of the applied magnetic field is lower than about 20 MHz. If an electrically large body such as human body is exposed to a RF magnetic field with a frequency higher than 20 MHz, the quasi-static approximation becomes inadequate. It is desirable to solve this practical problem accurately and effectively.

Based on Mie theory, Lin et al. [1] have obtained a quasi-static solution for the magnetic mode of the electric field induced by a HF EM wave inside a spherical model of man. Their theory is valid up to 20 MHz and restricted to a spherical body. Spiegel [2] has used the same quasi-static solution to estimate the electric field induced by the magnetic field of an EHV power line inside a simplified

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model of man. His solution is valid only for the very low frequency range. A theory on the eddy current induced by an ac uniform magnetic field in a conducting sphere was presented by Van Bladel [3]. This theory imposes less approximations but only applies to a spherical geometry (see Section 2.5). There is a need for a new theoretical method which can be used to quantify the electric fields or the eddy currents induced by RF magnetic fields of up to VHF range in finite conducting bodies with geometries more complex than a sphere.

Another motivation of the present study is the need for improving the efficiency of a recent numerical method [4] developed by our group for quantifying the internal electric field induced by an EM wave in a human body. When a thick biological body is exposed to an EM wave of HF-VHF range, the induced electric field or current can be divided into the electric and magnetic modes. The electric mode being linear and relatively independent of the body shape is easily determined by the numerical method. However, the magnetic mode is circulatory and strongly dependent on the body shape. The determination of this magnetic mode with a numerical method often encounters the difficulty of numerical convergence. Thus, a new theoretical method which can efficiently determine the magnetic mode of the induced current in an irradiated biological body is desired.

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The theoretical method presented in this chapter is capable of predicting the electric field or the eddy current induced by a uniform RF magnetic field or a beam of RF magnetic field in a finite conducting body with rotational symmetry. The development of the theory is presented in Section 2.2. Some numerical examples are given in Section 2.3. The convergence of numerical results is discussed in Section 2.4. The comparison of the present solution with a closed form solution for a sphere is made in Section 2.5. The experimental setup is described in Section 2.6, and experimental results are compared with theoretical results in Section 2.7.

2.2. Theory

When a finite conducting body with rotational symmetry is exposed to a uniformly impressed, RF magnetic field $\vec{H}^i$, the induced electric field $\vec{E}$ inside the body is conventionally estimated by Maxwell equation of

$$\oint \vec{E} \cdot d\vec{\ell} = -j\omega\mu_o \int \vec{H}^i \cdot d\vec{s} . \tag{2.1}$$

The induced electric field $\vec{E}$ determined from the above equation is circulatory and its amplitude increases linearly with the radial distance from the central axis of the body and its phase is 90° out of that of impressed magnetic field $\vec{H}^i$. In this quasi-static solution, the interaction between the induced currents at different

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locations is completely ignored. However, to obtain accurate results, the scattered magnetic field produced by the induced current must be taken into consideration. The quasi-static approximation can predict good results only for frequencies lower than 20 MHz. A theoretical method which can be used to quantify the electric field in a finite conducting body with a rotational symmetry induced by a RF magnetic field with a frequency higher than 20 MHz is developed below.

We consider two different cases separately: (1) a finite conducting body immersed in an impressed, uniform RF magnetic field, and (2) the body irradiated by a cylindrical beam of RF magnetic field.

Physically, we may consider that the electric field in the body is induced by an impressed electric field which is associated with the impressed RF magnetic field.

For the case of an impressed, uniform RF magnetic field $\vec{H}^i$ as shown in Figure 2.1, the associated, impressed electric field $\vec{E}^i$ can be obtained from

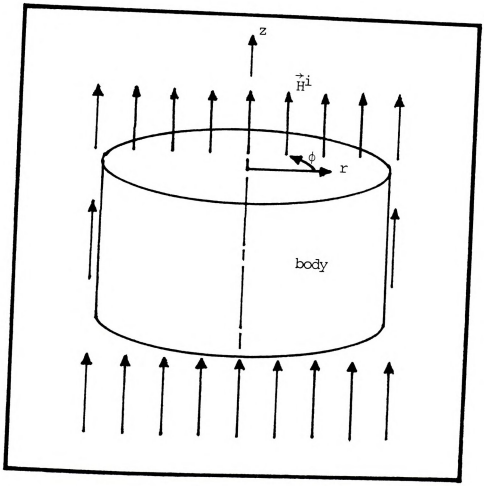
$$\oint \vec{E}^i \cdot d\vec{\ell} = -j\omega\mu_o \int \vec{H}^i \cdot d\vec{s}$$

as

$$\vec{E}^i = -\frac{j}{2}\omega\mu_o H^i r \hat{\phi} \qquad (2.2)$$

where r is the radial distance from the central axis of the body.

Figure 2.1



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For the case of a cylindrical beam of uniform magnetic field $\vec{H}^i$ with a beam radius of "b" as shown in Figure 2.2, the associated, impressed electric field can be found to be

$$\vec{E}^i = \begin{cases} -\frac{j}{2}\omega\mu_o H^i r\,\hat{\phi} & \text{for } 0 \le r < b \\ -\frac{j}{2}\omega\mu_o H^i \frac{b^2}{r}\,\hat{\phi} & \text{for } b \le r\,. \end{cases} \qquad (2.3)$$

These impressed electric fields $\vec{E}^i$ associated with the impressed magnetic fields $\vec{H}^i$ are in the azimuthal direction and they induce circulatory electric fields inside the finite conducting body.

If the rotational symmetry is assumed for the body geometry and the electrical properties of the body, the body can be subdivided into a number (N) of circular rings of various radii and cross-sectional areas as shown in Figure 2.3. We further assume that the induced electric field within each ring is uniform, but it can vary from ring to ring. Let's consider two sample rings shown in Figure 2.3; namely, the nth ring with radius r_n, cross-sectional area S_n and a reference point $(r_n, 0, z_n)$, and the mth ring with radius r_m, cross-sectional area S_m and a reference point $(r_m, 0, z_m)$. The geometries of these two rings are depicted in Figure 2.4, where the reference points of the rings, $\vec{r}_n$ and $\vec{r}_m$, the source point in the nth ring, $\vec{r}_n'$, and the distance between the source point

Figure 2.2

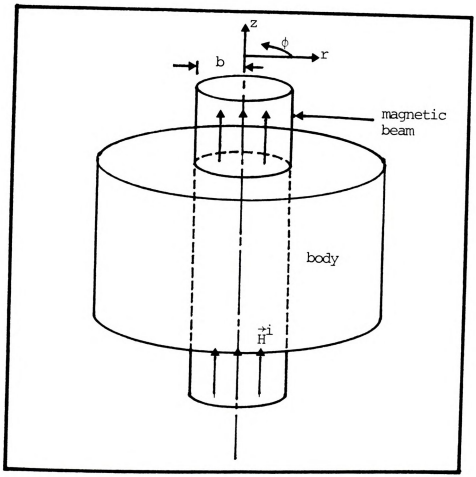

Figure 2.2. A biological body is irradiated by a beam of RF magnetic field.

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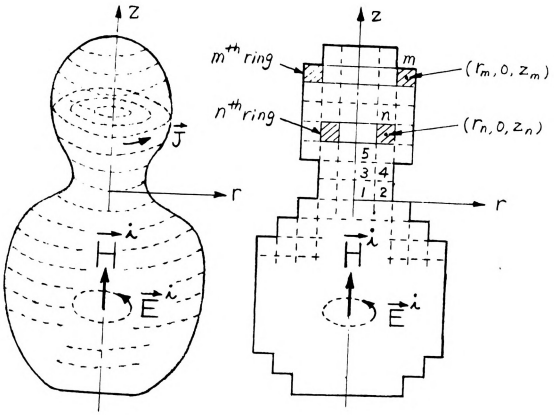

Figure 2.3. A biological body of rotational symmetry irradiated by a uniform RF magnetic field is subdivided into a number of rings of various radii.

$\vec{r}_n = (r_n, 0,$
$\vec{r}_m = (r_m, 0,$
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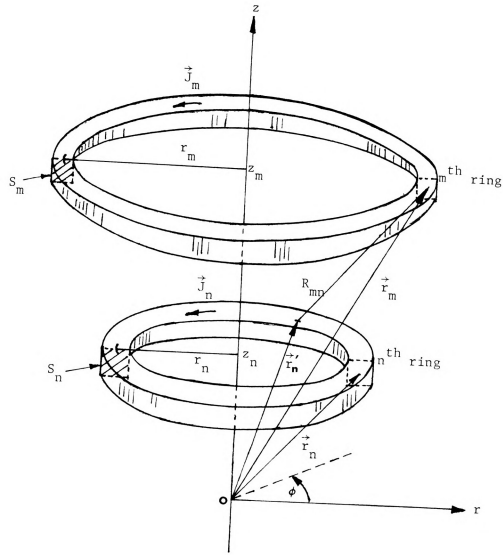

$\vec{r}_n = (r_n, 0, z_n)$ = reference point of the n^{th} ring

$\vec{r}_m = (r_m, 0, z_m)$ = reference point of the m^{th} ring

$\vec{r}_n' = (r_n, \phi, z_n)$ = source point in the n^{th} ring

$$R_{mn} = |\vec{r}_m - \vec{r}_n'| = [r_m^2 + r_n^2 - 2r_m r_n \cos\phi + (z_m - z_n)^2]^{\frac{1}{2}}$$

= distance between the source point in the n^{th} ring and the reference point of the m^{th} ring.

Figure 2.4. Geometries of two rings in the subdivided biological body.

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If the volume density of the induced current in the nth ring is denoted as $\vec{J}_n$, the vector potential at $\vec{r}_m$ of the mth ring maintained by $\vec{J}_n$ of the nth ring, $\vec{A}_{mn}$, can be determined as

$$\vec{A}_{mn} = \frac{\mu_o}{4\pi} \int_{\text{nth ring}} \vec{J}_n \frac{e^{-jk_o R_{mn}}}{R_{mn}} \, dv \tag{2.4}$$

where $R_{mn} = [r_m^2 + r_n^2 - 2\, r_m\, r_n \cos\phi + (z_m - z_n)^2]^{\frac{1}{2}}$

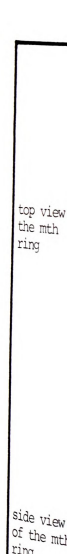
$k_o = \omega\sqrt{\mu_o \varepsilon_o}$.

Using $dv = S_n\, r_n\, d\phi$ and because of the rotational symmetry, eq. (2.4) can be evaluated as

$$\vec{A}_{mn} = \hat{\phi}\, \frac{\mu_o}{2\pi}\, J_n\, S_n\, r_n\, K_{mn} \tag{2.5}$$

where $$K_{mn} = \int_o^{\pi} d\phi \cos\phi \, \frac{e^{-jk_o R_{mn}}}{R_{mn}} \, . \tag{2.6}$$

To determine the vector potential $\vec{A}_{m_m}$ at $\vec{r}_m$ of the mth ring maintained by $\vec{J}_m$, the induced current in the mth ring, we consider the geometry of the mth ring shown in Figure 2.5. Note that we approximate the square ring cross-section by a circle with the same area. This leads to the following relationship:



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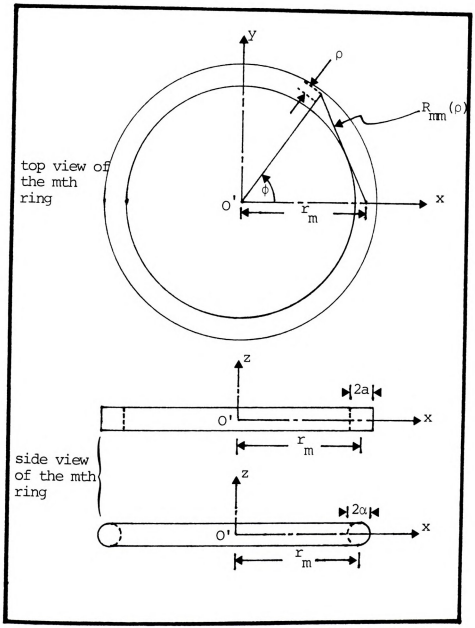

Figure 2.5. Geometries of a single ring inside the subdivided biological body.

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$$\alpha = \frac{2a}{\sqrt{\pi}}$$

where 2a is the dimension of the square cross-section, while α is the radius of the circular cross-section.

The distance between the source point inside the mth ring and the reference point $\vec{r}_m$, R_{mm}, can be expressed as

$$R_{mm}(\rho) \doteq r_m[2(1-\cos\phi) + \frac{\rho^2}{r_m^2}]^{\frac{1}{2}} .$$

The vector potential $\vec{A}_{mm}$ can be written in terms of R_{mm} as

$$\vec{A}_{mm} = \frac{\mu_o}{4\pi} \int_{\substack{\text{mth} \\ \text{ring}}} \vec{J}_m \frac{e^{-jk_o R_{mm}(\rho)}}{R_{mm}(\rho)} dv .$$

After substituting the expression of R_{mm} and some straightforward manipulations, we have the following form for the above equation:

$$\vec{A}_{mm} = \hat{\phi} \frac{\mu_o}{2\pi} J_m S_m r_m K_{mm} \qquad (2.7)$$

where $$K_{mm} = \frac{2\pi}{S_m}[R_{mm}(\alpha) - R_{mm}(0)] \int_0^{\pi} \cos\phi \, e^{-jk_o R_{mm}(0)} d\phi . \qquad (2.8)$$

The total vector potential at $\vec{r}_m$ maintained by the induced currents in all the rings, $\vec{A}_m$, can then be obtained as

$$\vec{A}_m = \sum_n \vec{A}_{mn} = \hat{\phi} \sum_n \frac{\mu_o}{2\pi} S_n r_n K_{mn} J_n . \qquad (2.9)$$

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At the same time, the induced current $\vec{J}_n$ in the nth ring can be related to the total induced electric field in the nth ring, $\vec{E}_n$, as

$$\vec{J}_n = [\sigma_n + j\,\omega\,(\varepsilon_n - \varepsilon_o)]\,\vec{E}_n = \tau_n\,\vec{E}_n. \qquad (2.10)$$

Equation (2.10) expresses $\vec{J}_n$ as the sum of the conduction current and the polarization current, and τ_n can be considered as the complex conductivity of the nth ring.

With eq. (2.10), $\vec{A}_m$ can be rewritten as

$$\vec{A}_m = \hat{\phi} \sum_n \frac{\mu_o}{2\pi} S_n\, r_n\, K_{mn}\, \tau_n\, E_n. \qquad (2.11)$$

This $\vec{A}_m$ is related to the scattered electric field $\vec{E}_m^s$ in the mth ring which is maintained by the induced currents in all the rings:

$$\vec{E}_m^s = -j\omega\vec{A}_m = -\hat{\phi} \sum_n jf\,\mu_o\, S_n\, r_n\, K_{mn}\, \tau_n\, E_n \qquad (2.12)$$

where f is the frequency. It should be emphasized that since the induced current in each ring is rotationally symmetrical, no electric charge is induced and, consequently, no scalar potential is maintained.

The total induced electric field $\vec{E}_m$ in the mth ring is the sum of the impressed electric field $\vec{E}_m^i$ at the mth ring and the scattered electric field $\vec{E}_m^s$ obtained in eq. (2.12):

$$\vec{E}_m = \vec{E}_m^i + \vec{E}_m^s \qquad (2.13)$$

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With eq. (2.12), eq. (2.13) can be rearranged as

$$E_m + \sum_n jf \mu_o S_n r_n K_{mn} \tau_n E_n = E_m^i. \qquad (2.14)$$

Equation (2.14) can be point-matched at N reference points of the N rings to yield N simultaneous equations in a matrix form as

$$\begin{bmatrix} M_{11}, M_{12}, \cdots\cdots\cdots, M_{1N} \\ M_{21}, M_{22}, \cdots\cdots\cdots, M_{2N} \\ \cdots\cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots\cdots \\ M_{N1}, M_{N2}, \cdots\cdots\cdots M_{NN} \end{bmatrix} \cdot \begin{bmatrix} E_1 \\ E_2 \\ . \\ . \\ E_N \end{bmatrix} = \begin{bmatrix} E_1^i \\ E_2^i \\ . \\ . \\ E_N^i \end{bmatrix} \qquad (2.15)$$

where

$$M_{nn} = 1 + jf \mu_o S_n r_n K_{nn} \tau_n \qquad (2.16)$$

$$M_{mn} = jf \mu_o S_n r_n K_{mn} \tau_n. \qquad (2.17)$$

The total induced electric fields, E_1 to E_n, in all the rings can be determined from eq. (2.15) by the matrix inversion technique. Note that if the induced eddy current is needed, it can be obtained readily from eq. (2.10).

A computer program has been developed based on this method to calculate induced electric fields in various finite conducting bodies. Some numerical results are discussed in the next section.

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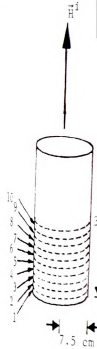
2.3. Numerical Results

The first numerical example deals with a cylindrical biological body with a diameter of 15 cm and a height of 30 cm being exposed to a uniform magnetic field of 100 MHz with an intensity of 1 Amp/m. The body is assumed to have a conductivity (σ) of 0.889 S/m and a dielectric constant (ε_r) of 71.7. In the numerical calculation, the body is subdivided into 100 rings of various radii and with a square cross-sectional area of 1.5 cm x 1.5 cm. The amplitude and phase angle of the induced electric field $\vec{E}$ in each ring were calculated, and the distributions of these quantities are plotted as functions of the radial distance from the cylindrical axis r for 10 different sections of the body as shown in Figure 2.6. It is observed in Figure 2.6 that the amplitude of the induced electric field $|\vec{E}|$ increases linearly with the radial distance; this result is close to the quasi-static solution as can be obtained from eq. (2.1). The phase angle of the induced electric field varies between around -110° and -150° (with respect to the impressed magnetic field $\vec{H}^i$) while the quasi-static solution predicts it to be -90° at any point in the body. For this cylindrical body, the often-used quasi-static solution can be used to estimate the amplitude of the induced electric field but not its phase angle. However, as the size of the cylindrical body is increased, the quasi-static solution is



H^i = 1 A/m
f = 100MHz
σ = 0.889 S/m
δ = 71.7

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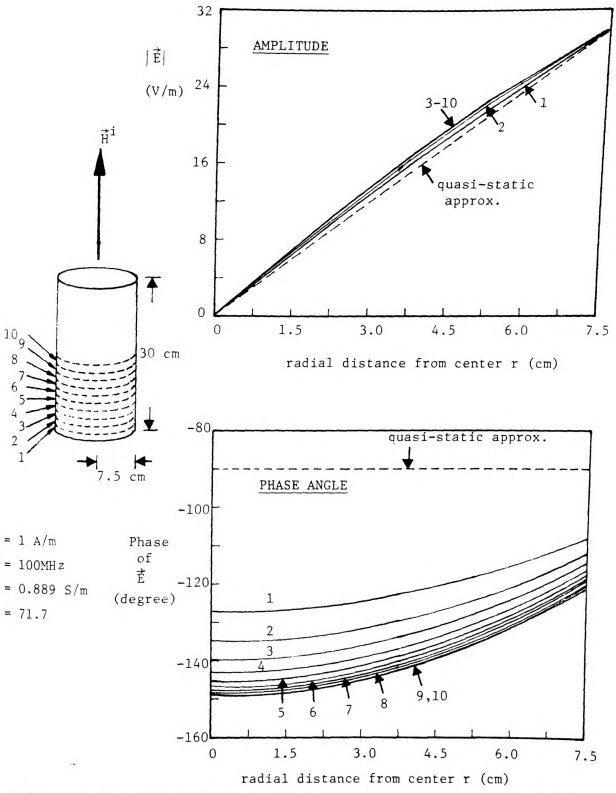

Figure 2.6. Distributions of amplitudes and phase angles of electric fields induced by a 100 MHz magnetic field of 1 A/m in a cylindrical biological body with a diameter of 15 cm and a height of 30 cm.

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In the second example, we consider a cylindrical biological body with the same electric properties (σ = 0.889 S/m and ε_r = 71.7) as the first example, but with a diameter of 36 cm and a height of 90 cm as shown in Figure 2.7. The same magnetic field of 100 MHz with an intensity of 1 Amp/m is impressed on the body. The distributions of the amplitudes and phase angles of the induced electric fields in Figure 2.7 show that the present results deviate greatly from the quasi-static solutions: the amplitude of the induced electric field is two or three times lower than the quasi-static solution, while the phase angles vary between -120° and -290° against the quasi-static solution of -90°. It is clear, from this example, that when a human body is exposed to a magnetic field of VHF range, the often-used, quasi-static solution becomes completely invalid.

The next example as shown in Figure 2.8 deals with the electric fields induced by magnetic fields of unit intensity (1 Amp/m) and of various frequencies (10, 40, 100 and 200 MHz) in a muscle disk with a diameter of 30 cm and a thickness of 1 cm. The electrical properties of the disk are assumed to have σ = 0.625 to 1.28 S/m and ε_r = 160 to 56.5 for the frequency range of 10 to 200 MHz. The amplitudes of the induced electric fields are found to

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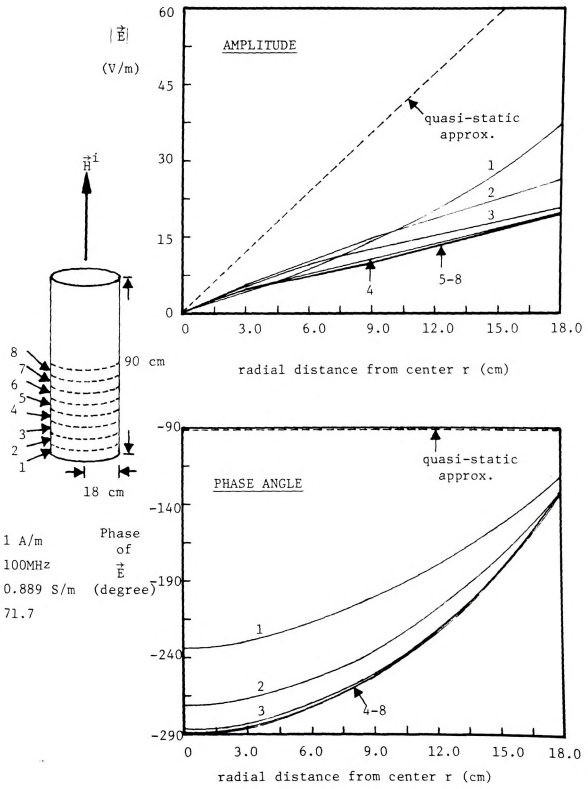

Figure 2.7. Distributions of amplitudes and phase angles of electric fields induced by a 100 MHz magnetic field of 1 A/m in a cylindrical biological body with a diameter of 36 cm and a height of 90 cm.

H^i = 1 A/m

σ = 0.625 to 1.28 S/m for 10-200 MHz

ε_r = 160 to 56.5 for 10-200 MHz

Figure 2.8.

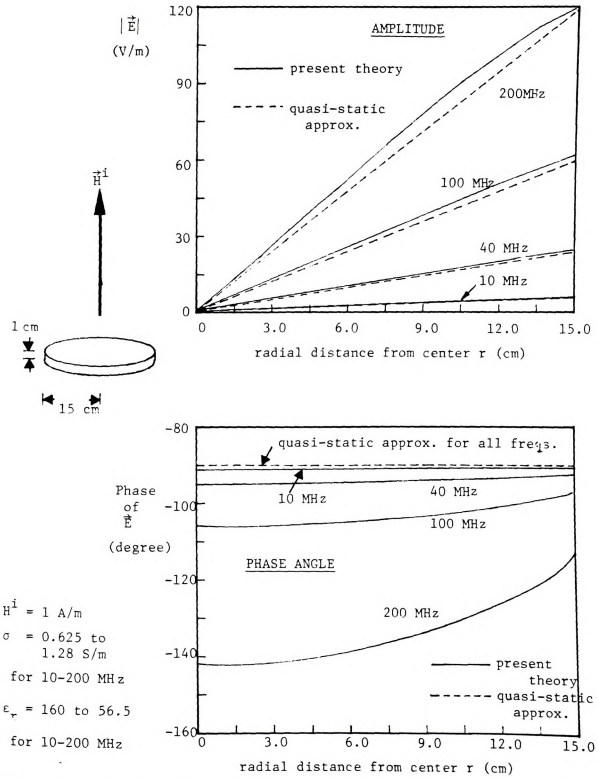

Figure 2.8. Distributions of amplitudes and phase angles of electric fields induced by magnetic fields of various frequencies in a muscle disk with a diameter of 30 cm and a thickness of 1 cm.

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increase linearly with the frequency and with the radial distance from the cylindrical axis; these results deviate slightly from the quasi-static solutions. The present results for the phase angles of the induced electric fields, however, deviate significantly from the quasi-static solutions, especially, for the high frequency cases.

In the example shown in Figure 2.9, we study the effect of the cylindrical height on the induced electric field at the central section of the cylinder. The cylindrical body has a diameter of 36 cm, but its height varies from 6 cm to 90 cm. The impressed magnetic field at 100 MHz has an intensity of 1 Amp/m. The electrical properties of the body at this frequency are assumed to have σ = 0.889 S/m and ε_r = 71.7. The induced electric fields at the central section of the cylinder are determined and the distributions of their amplitudes and phase angles are plotted for the cases of cylindrical heights; H = 6, 12, 18, 30 and 90 cm. It is observed in Figure 2.9 that as the cylindrical height is increased, the amplitude of the induced electric field at the central section of the cylinder decreases greatly, and its phase angle varies drastically from the quasi-static solution of -90°. This example indicates that the eddy current induced by a RF magnetic field in a cylindrical biological body is strongly dependent on the cylindrical height.

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H^i = 1 A/m

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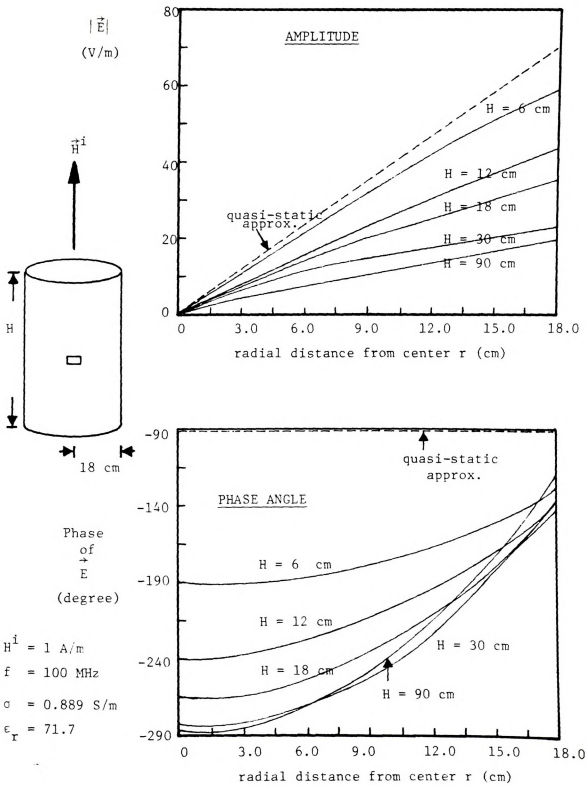

Figure 2.9. Distributions of amplitudes and phase angles of electric fields induced by a 100 MHz magnetic field in the central section of a cylindrical biological body of diameter 36 cm with various heights.

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The case of the eddy current induced by a beam of magnetic field in a biological body is studied next. Figure 2.10 shows the distributions of the amplitudes and phase angles of electric fields induced in a cylindrical body with a diameter of 32 cm and a height of 20 cm, by a beam of 100 MHz magnetic field with a beam diameter of 8 cm and an intensity of 1 Amp/m. The electrical properties of σ = 0.889 S/m and ε_r = 71.7 are assumed for the body. The impressed electric field for this case is calculated from eq. (2.3). The amplitudes and phase angles of the induced electric fields in five different sections of the cylinder are plotted in Figure 2.10. It is observed that the induced electric field or the induced eddy current is zero at the center of the magnetic beam and it increases linearly to a maximum value at the edge of the beam, and then decays down toward the edge of the body. The deviations between the present results and the quasi-static solutions are indicated in Figure 2.10. The application of a beam of RF magnetic field for the purpose of locally heating a biological body in a hyperthermia cancer therapy is feasible if the low heating at the beam center can be compensated by other means of EM heating.

The last numerical example is the quantification of the electric fields induced by a 100 MHz magnetic field with an intensity of 1 Amp/m in a cylindrical model of man with a height of 168 cm and a maximum diameter of


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H^i = 1 A/m
f = 100 MHz
σ = 0.889 S/m
ε_r = 71.7

Figure 2.10.

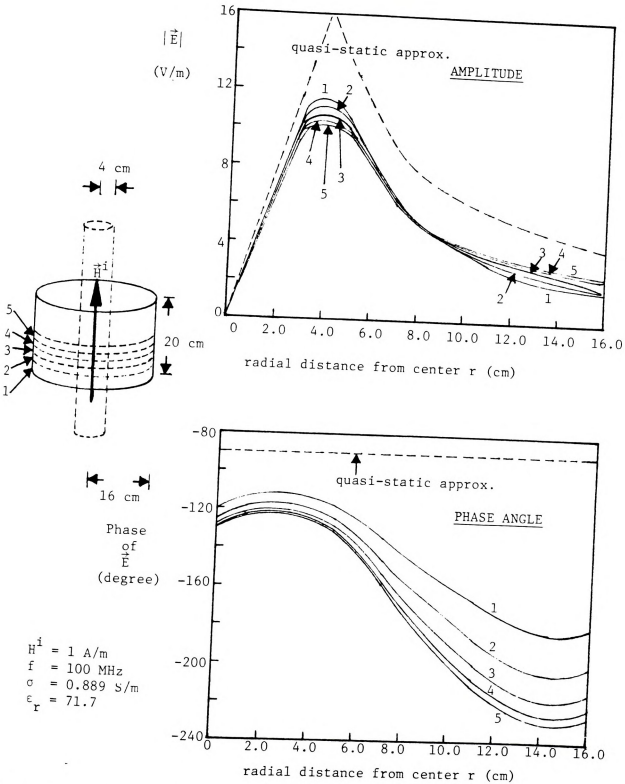

Figure 2.10. Distributions of amplitudes and phase angles of electric fields induced in a cylindrical biological body with a diameter of 32 cm and a height of 20 cm by a beam of 100 MHz magnetic field with a diameter of 8 cm and an intensity of 1 A/m.

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36 cm as shown in Figure 2.11. Again, σ = 0.889 S/m and ε_r = 71.7 are assumed for the body. In the numerical calculation, the body was subdivided into 66 rings of various radii and with a square cross-sectional area of 6 cm x 6 cm. The amplitudes and phase angles of the induced electric fields inside the body are indicated in Figure 2.11. It is noted that these results deviate greatly from the quasi-static solutions. For example, the induced electric field in the head can be greater than that in the middle section of the body even though the radius of the latter is greater than that of the former. The phase angles of the induced electric fields are very different from -90° and they vary widely throughout the body.

2.4. Convergence of Numerical Results

The accuracy and validity of numerical results presented in this chapter depend greatly on the convergence of numerical results as the number of subdivided rings is increased. Fortunately, the present theory gives an excellent convergence of numerical results. This fact is demonstrated in an example shown in Figure 2.12, in which a cylindrical body with a diameter of 36 cm and a height of 18 cm is exposed to a uniform magnetic field of 100 MHz with an intensity of 1 Amp/m. The electrical properties of σ = 0.889 S/m and ε_r = 71.7 are assumed for the body.

Figure 2.11

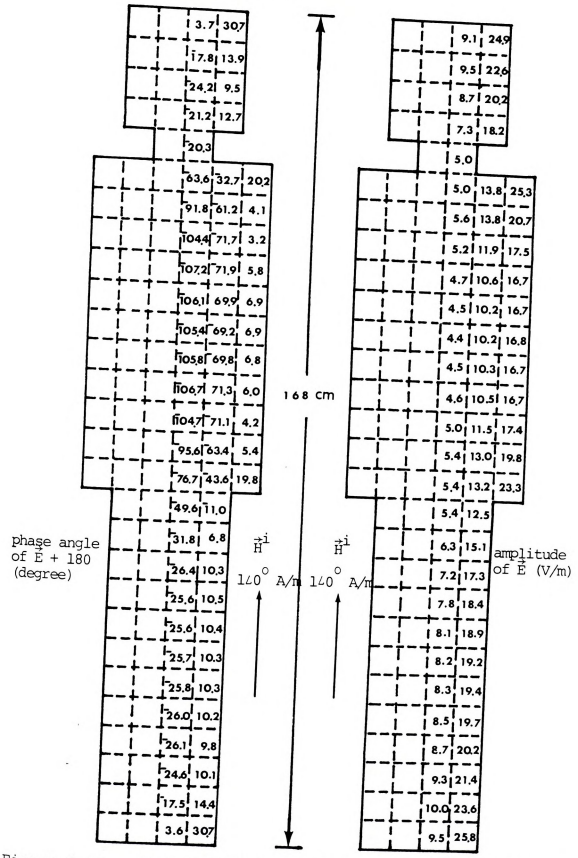

Figure 2.11. Distributions of amplitudes and phase angles of electric fields induced by a uniform magnetic field of 100 MHz in a model of man (σ = 0.889 S/m, ε_r = 71.7).

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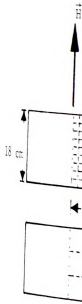
In the first numerical calculation, the body was subdivided into 9 rings of various radii and with a square cross-sectional area of 6 cm x 6 cm. In the second numerical calculation, the same body was subdivided into 81 rings of various radii and with a square cross-sectional area of 2 cm x 2 cm. The numerical results on the induced electric fields based on these two ring subdivisions are compared in Figure 2.12. It is observed that the agreement between these two sets of numerical results is excellent even though the number of rings in the second subdivision increases by a factor of 9 from that of the first subdivision. This excellent convergence of numerical results implies that it is possible to obtain accurate results on the electric fields or the eddy currents induced by a RF magnetic field in most biological bodies without subdividing the body into too many rings. This advantage provided by the present method can lead to a great saving in computer time and cost.

2.5. Closed Form Solution for Sphere and Comparison With Present Numerical Results

To compare our numerical results with the existing analytical solution [3], we consider the case of a conducting sphere with radius R which is immersed in a uniform time-harmonic magnetic field,

$$\vec{H}^i = \hat{z}\, H^i = \hat{r}\, H^i \cos\theta - \hat{\theta}\, H^i \sin\theta$$



H_0^i = 1 A/m
f = 100 MHz
σ = 0.889 S/m
ε_r = 71.7

Figure 2.12.

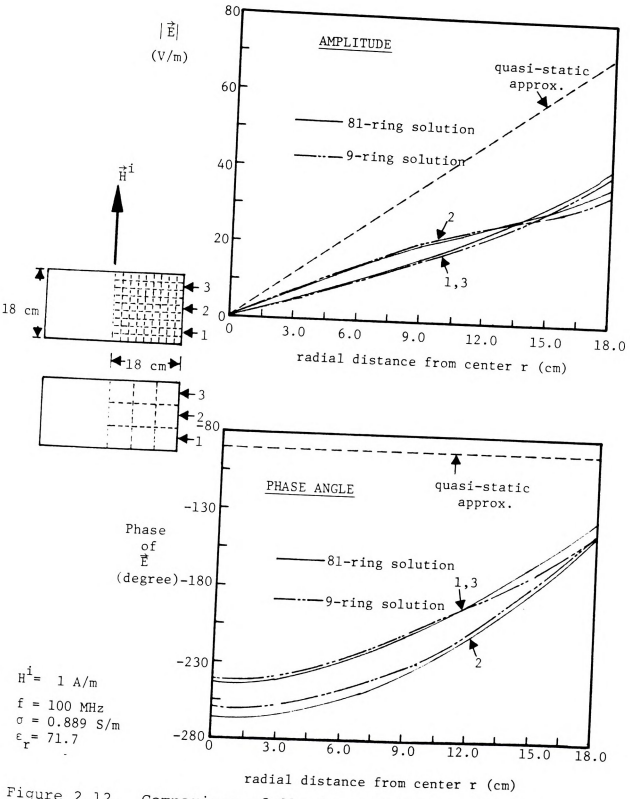


Figure 2.12. Comparison of 81-ring solutions and 9-ring solutions for the electric fields induced in a cylindrical biological body with a diameter of 36 cm and a height of 18 cm by a 100 MHz magnetic field with an intensity of 1 A/m.

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where $\hat{r}$ and $\hat{\theta}$ are unit vectors in spherical coordinates, as shown in Figure 2.13. The vector potentials maintained by the induced currents and the sources for the regions inside and outside the sphere can be written as

$$\vec{A}_{in} = (1/2)\ \mu_o\ H^i\ \sin\theta\ \frac{C_1}{\sqrt{r}}\ I_{3/2}\ [\ (j\omega\ \mu_o\ \sigma)^{\frac{1}{2}}\ r]\ \hat{\phi}$$

when $r < R$ (2.18)

$$\vec{A}_{out} = (1/2)\ \mu_o\ H^i\ \sin\theta\ (r + \frac{C_2}{r^2})\hat{\phi}\ \text{when}\ r > R \qquad (2.19)$$

where $I_{3/2}$ is the spherical Bessel function, and σ is the conductivity of the sphere. The constants C_1 and C_2 can be determined by use of the following boundary conditions at $r = R$,

1) $A_{in} = A_{out}$

2) $\frac{\partial}{\partial r}\ (r\ A_{out}) = \frac{\partial}{\partial r}\ (r\ A_{in})$.

After some manipulations and utilizing some identities of spherical Bessel functions we have

$$C_1 = \frac{3\ R^{3/2}}{w\ I_{1/2}(w)} \qquad (2.20)$$

$$C_2 = \frac{3\ w\ I_{-1/2}(w) - (3+w^2)\ I_{1/2}(w)}{w^2\ I_{1/2}(w)}\ R^3 \qquad (2.21)$$

where $w = \sqrt{2\ j}\ \frac{R}{\delta}$, and $\delta = (\frac{2}{\omega\ \mu_o\ \sigma})^{\frac{1}{2}}$ is the skin depth of the sphere. Combining eqs. (2.18) and (2.20), we obtain

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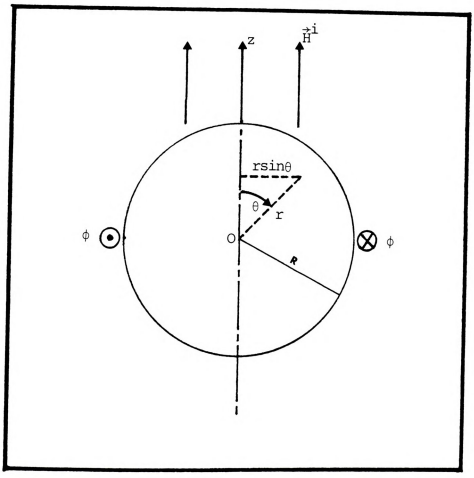

Figure 2.13. A finite conducting sphere with radius R is immersed in a uniform magnetic field polarized in the +z direction.

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the complete expression for $\vec{A}_{in}$,

$$\vec{A}_{in} = (3/2)\ \mu_o\ H^i\ \sin\theta\ \frac{\sqrt{Rr}}{\frac{wr}{R}}\ \frac{I_{3/2}(wr/R)}{I_{1/2}(w)}\ \hat{\phi}.$$

Then the electric field induced inside the sphere is given by

$$\vec{E} = -j\omega\vec{A}_{in} = -(3/2)\ j\omega\mu_o\ H^i\ \sin\theta\ \sqrt{Rr}\ \frac{R}{wr}\ \frac{I_{3/2}(wr/R)}{I_{1/2}(w)}\ \hat{\phi}. \qquad (2.22)$$

To compare eq. (2.22) with our numerical results, we assume that the sphere has a radius of 2 cm, a conductivity of 8.0 S/m, and a dielectric constant of 50, which is immersed in a uniform oscillating magnetic field with a frequency of 300 MHz. Under these assumptions the skin depth and the radius of the sphere are in the same order of magnitude. Now for the case of numerical evaluation, the sphere is simulated by a "ring sphere" which is constructed with 30 different rings, as shown in Figure 2.14. The numerically evaluated results of the induced electric fields in these circular rings are then compared with those values obtained from eq. (2.22) at corresponding points inside the sphere. The comparison is made in Table 2.1. Also listed in Table 2.1 are the results obtained by the quasi-static approximation.

It is apparent that the agreement between our numerical solution and the existing closed form solution is

Figure

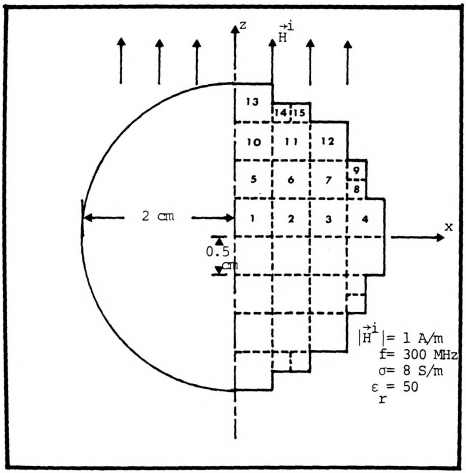

Figure 2.14. A finite conducting sphere with a radius of 2 cm is simulated by a "ring sphere" constructed with 30 circular rings of two different cross-sectional areas. The sphere is assumed to have a conductivity of 8.0 S/m and a dielectric constant of 50, and is immersed in a 300 MHz oscillating magnetic field with an intensity of 1 A/m.

Table 2.1. Comparison of the present solutions and a closed form solution for the electric fields induced in a conducting sphere with a conductivity of 8.0 S/m and a dielectric constant of 50 by a 300 MHz magnetic field with an intensity of 1 A/m. Also listed are the solutions obtained by the quasi-static approximation.

Table 2.1. Comparison of the present solutions and a closed form solution for the electric fields induced in a conducting sphere with a conductivity of 8.0 S/m and a dielectric constant of 50 by a 300 MHz magnetic field with an intensity of 1 A/m. Also listed are the solutions obtained by the quasi-static approximation.

Ring. No.	Quasi-Static Approximation		Closed Form Solution		Present Solution	
	Magnitude (10 V/m)	Phase (Degree)	Magnitude (10 V/m)	Phase (Degree)	Magnitude (10 V/m)	Phase (Degree)
1	0.296	-90	0.230	-25.6	0.259	-26.4
2	0.889	-90	0.689	-31.0	0.770	-31.7
3	1.480	-90	1.162	-41.8	1.269	-42.4
4	2.070	-90	1.681	-57.6	1.787	-58.3
5	0.296	-90	0.230	-30.9	0.256	-32.1
6	0.889	-90	0.692	-36.4	0.763	-37.3
7	1.480	-90	1.173	-47.2	1.264	-48.4
8	1.930	-90	1.557	-56.6	1.647	-58.2
9	1.930	-90	1.582	-60.7	1.658	-62.5
10	0.296	-90	0.232	-41.9	0.253	-43.4
11	0.889	-90	0.704	-47.1	0.759	-48.5
12	1.480	-90	1.206	-57.8	1.271	-59.0
13	0.296	-90	0.241	-57.6	0.255	-60.1
14	0.741	-90	0.600	-56.8	0.635	-59.0
15	1.040	-90	0.849	-60.6	0.894	-62.7

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excellent for both the magnitude and the phase distributions throughout the interior of the sphere. This excellent agreement confirms the accuracy of our numerical technique. Furthermore, the deviation of the quasi-static solution from the other two sets of solutions indicates that the conventional quasi-static solution is usually inadequate.

2.6. Experimental Setup

A series of experiments has been conducted to measure the electric fields induced by UHF magnetic fields inside some experimental models constructed with plexiglass and filled with phantom biological materials for the purpose of substantiating the accuracy of theoretical results presented in this chapter.

The experimental setup is depicted schematically in Figure 2.15. The experiment was conducted inside a large microwave anechoic chamber in which a standing EM wave was created by radiating an EM wave upon a metallic reflector. The electric field of the wave was polarized horizontally and the magnetic field was vertically polarized. The experiment was conducted in the frequency range of 500 to 750 MHz. The phantom body was placed at the location of a maximum magnetic field or a minimum electric field. If the body dimension in the direction of wave propagation is small compared with the wavelength, the

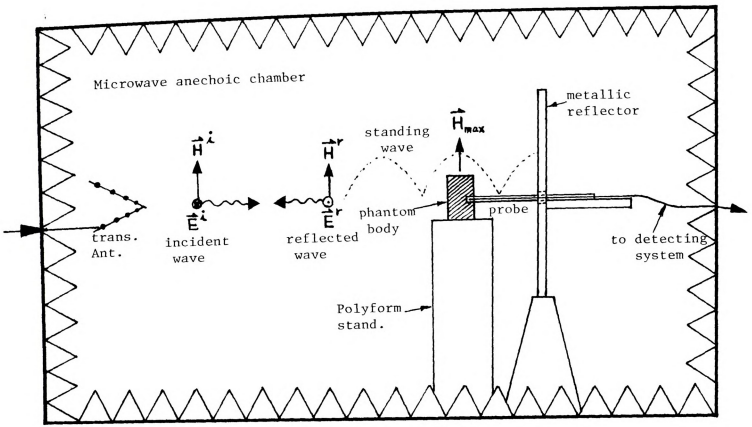

Figure 2.15. Experimental setup for measuring the electric fields inside the phantom biological bodies induced by RF magnetic fields.

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impressed magnetic field at any point inside the phantom body can be assumed to be uniform. The induced electric field inside the phantom body was measured by an implantable electric field probe with an interference-free lead-wire system. This probe was described elsewhere [5]. The output of the probe was connected to a detecting system outside the anechoic chamber.

2.7. Experimental Results and Comparison With Theoretical Results

The first experiment was conducted on a cylindrical phantom body with a diameter of 4 cm and a variable height. The electric properties of the phantom body were estimated to be approximately σ = 5.0 S/m and ε_r = 50. The experiment was conducted at 750 MHz and the cylinder was placed at the location of a maximum magnetic field, one wavelength (40 cm) in front of the metallic reflector. The direction of the magnetic field was in parallel with the cylindrical axis. The induced electric field in the central section of the cylinder was probed for three cases of cylindrical heights; H = 2, 4 and 8 cm. The measured amplitudes of the induced electric fields are plotted in Figure 2.16 with dashed lines, in comparison with corresponding theoretical results which are indicated with solid lines. It is noted that with an implantable electric field probe loaded with a microwave diode, only the amplitude of the induced electric field could be measured. In

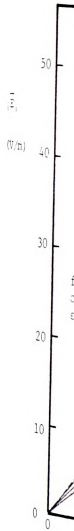

Figure 2.16.

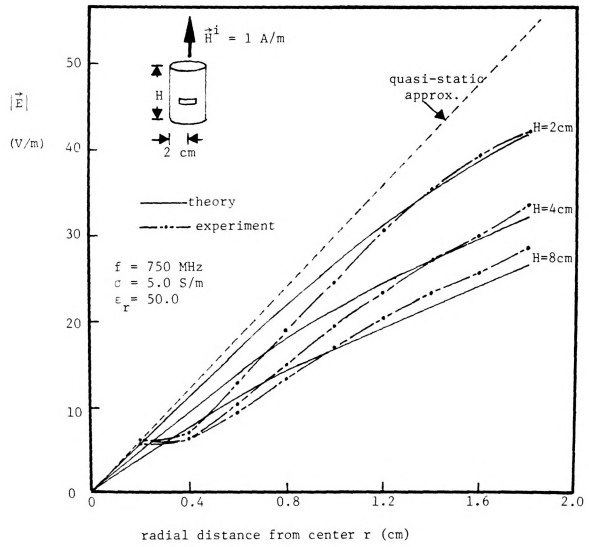

Figure 2.16. Comparison of theoretical and experimental results for the amplitudes of electric fields induced in a phantom biological cylinder of 4 cm diameter with various heights by a 750 MHz magnetic field.

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Figure 2.16, the agreement between experimental and theoretical results is very good except in the region near the cylindrical axis where the perturbation of the probe (about 1 cm in size) on the induced current becomes significant. Another cause of discrepancy between the theory and the experiment can be attributed to the image effect of the metallic reflector; this effect is not severe if the distance between the body and the reflector is about one wavelength or greater. The important finding in Figure 2.16 is that it was observed experimentally and predicted theoretically that as the cylindrical height was increased, the induced electric field in the central section of the cylinder decreased. These results were also found to be significantly different from the quasi-static solutions.

The second experiment was conducted on a phantom cylinder with a diameter of 3.8 cm and a height of 3.8 cm to study the effect of body conductivity on the induced electric field. Three kinds of phantom materials with conductivities of σ = 2.2, 4.5 and 6.0 S/m were used. The experiment was conducted at 750 MHz, and the cylinder was placed at the location of a maximum magnetic field as in the previous experiment. The induced electric fields at the central section of the cylinder were probed, and the distributions of their amplitudes are plotted in Figure 2.17 in comparison with corresponding theoretical

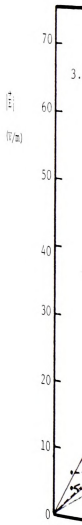

Figure 2.17.

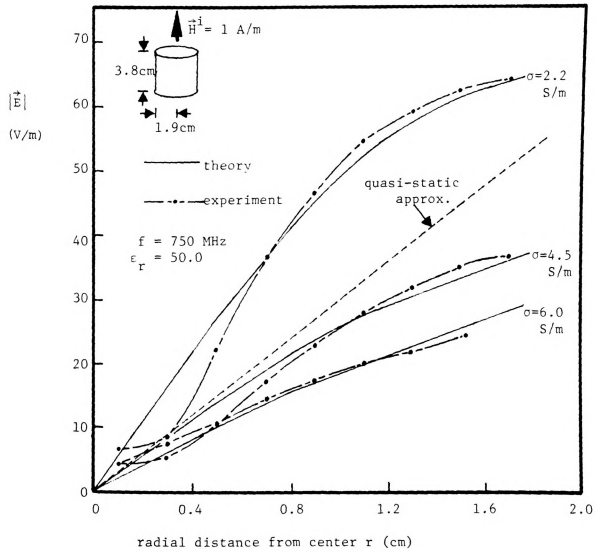

Figure 2.17. Comparison of theoretical and experimental results for the amplitudes of electric fields induced in a phantom biological cylinder with a diameter of 3.8 cm, a height of 3.8 cm and various conductivities by a 750 MHz magnetic field.

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results. It was observed that as the body conductivity is increased, the induced electric field decreases. This effect was accurately predicted by theory and confirmed by experiment. It is also noted that both theoretical and experimental results deviate significantly from the quasi-static solutions. The agreement between theory and experiment as shown in Figure 2.17 is very good.

The third experiment was conducted to measure the induced electric fields inside a phantom model of man. The phantom model of man was constructed with plexiglass and filled with phantom material, and has a height of 22.2 cm and a maximum diameter of 3.8 cm as shown in Figure 2.18. The experiment was conducted at 750 MHz, and electrical properties of the model at this frequency are approximately σ = 5.0 S/m and ε_r = 50. The model was placed upright at the location of a maximum magnetic field with the impressed magnetic field in parallel with the long dimension of the body. The induced electric fields at 32 locations of the body were probed. The measured amplitudes of the induced electric fields are indicated in Figure 2.18 in comparison with the corresponding theoretical results. The agreement between theory and experiment is considered to be very good in this rather complicated body. It is to be noted that experimental and theoretical results shown in Figure 2.18 are very different from the quasi-static solutions.

Figure 2.18

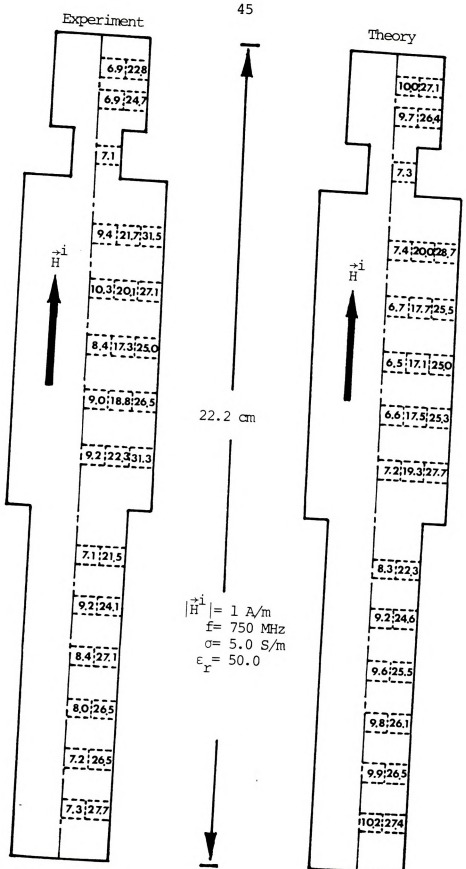

Figure 2.18. Comparison of theoretical and experimental results for the amplitudes of electric fields induced in a phantom model of man (height = 22.2 cm, max. diameter = 3.8 cm) by a 750 MHz magnetic field.

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A very good agreement between theory and experiment as demonstrated in Figures 2.16, 2.17 and 2.18 confirms the validity and accuracy of the theoretical method presented in this chapter.

2.8. Discussion

In the experiment, the body was placed at the location of a maximum magnetic field in a standing EM wave. Since the body dimension in the direction of wave propagation was small compared with the wavelength, the impressed magnetic field inside the body was assumed to be uniform. Under this approximation, the impressed electric field associated with the impressed magnetic field was

$$\vec{E}^i = -\frac{j}{2}\omega\mu_o H^i r\hat{\phi} \tag{2.2}$$

as has been given before. This $\vec{E}^i$ was used in the numerical calculation of the induced electric field inside the body.

From a different point of view, the body in the experiment was located at a minimum electric field of a standing EM wave, and the actual impressed electric field was

$$\vec{E}^i = \hat{x}\, j\sqrt{\frac{\mu_o}{\varepsilon_o}}\, H^i \sin ky \doteq \hat{x}\, j\omega\mu_o H^i y \tag{2.23}$$

where $y = 0$ corresponds to the center of the body.

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$\vec{E}^i$ given in eq. (2.2) is circulatory and that given in eq. (2.23) is linear and antisymmetrical. Also the amplitude of the latter is twice that of the former along the y axis.

It is important to ask whether these two different $\vec{E}^i$'s can yield the same theoretical values for the induced electric fields in the body? We have numerically proved that, indeed, these two $\vec{E}^i$'s give very similar numerical results on the induced electric fields in the same body. This phenomenon resulted in a very good agreement between experimental and theoretical results reported in this chapter.

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CHAPTER III

SURFACE INTEGRAL EQUATION METHOD FOR INTERACTION OF MICROWAVE WITH BIOLOGICAL BODY

Only in recent years has beamed microwave power been proposed as a new technique in transporting high-level energy between two remote transmitting-receiving stations, and has microwave heating been considered as an effective method in hyperthermia cancer therapy. Although EM transmission and scattering phenomena have been major and classical topics in electrodynamics since the end of the last century, uncertainties about the potential hazards of these high-power microwave radiations are still major concerns relating to the acceptability of these newly developed technologies. In order to fully understand the biological effects of microwave radiation, it is necessary to quantify the electric fields induced in the biological bodies when they are irradiated by the incident EM waves.

Existing methods of quantifying the electric fields induced by EM waves are mostly based on the solutions of volume integral equations. Unfortunately, when the body is large compared with the wavelength of the incident

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wave, the volume integral equation method becomes inefficient. Since for this case, the induced electric field is mainly concentrated in the region near the body surface, it is believed that the surface integral equation technique is potentially more efficient.

In this chapter, we develop a new technique based on two coupled surface integral equations. A good agreement between our solutions and the existing results obtained by volume integral equation method has been verified.

3.1. Introduction

High-level, beamed-power microwave transmission between satellites and earth has been proposed as an alternate scheme for providing energy in the future. However, the potential hazards which exist in the interface between the beamed-power microwave and personnel should be carefully considered by the microwave systems design engineers. Also, the thermal-heating effects of intense microwaves are becoming well known, and have been demonstrated effective in hyperthermia cancer therapy. But the allowable field intensity and the permissible level of absorbed power for the human body still remain unknown. It is desirable to obtain some quantitative information about microwave interactions with human or other biological bodies.

Johnson and Durney [6] have investigated the plane wave irradiation of a prolate-spheroid model of man based

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on the perturbation theory described by Van Bladel [7]. Their theory is valid only when the wavelength is long compared to the dimensions of the spheroid. The same approach has been adopted by Massoudi [8] to solve for the internal electric field induced by a plane wave inside an ellipsoidal model of man. Livesay and Chen [9] have developed a theoretical method based on a volume integral equation, the so called Tensor Integral Equation, which can be used to quantify the electric field induced inside an arbitrarily shaped biological body by an incident EM wave. This volume integral equation method removes the restrictions that the irradiated bodies have simple shapes and small electrical dimensions. However, when the body is electrically large, the EM waves cannot penetrate deeply inside the body, and will concentrate near the body surface. Consequently, the volume integral equation method becomes inefficient. For this case, the surface integral equation method seems to be a reasonable alternative approach. A surface integral equation method has been employed by Wu et al. [10] to solve for the EM fields induced inside arbitrary cylinders of biological tissue; only infinitely long cylinders were considered. Another surface integral formulation has been developed by Poggio and Miller [11]; however, in their formulation the derivatives of the surface unknowns are involved in the integrands of the surface integrals. This makes the numerical

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solutions difficult, if not impossible, to implement. Wang [12] has determined the scattering characteristics of conducting bodies by use of the moment-method, which divides the surface of the body into trilateral surface cells, but only results for sphere have been reported. It is desirable to obtain more complete information about the fields induced by EM waves, based on the surface integral equations, for finite conducting bodies with geometries more complicated than that of the sphere.

In this chapter, we will develop a new surface integral equation technique to quantify the induced EM field on the surface of an arbitrarily shaped, finite conducting body. It should be emphasized that the internal field can be readily determined in terms of the surface field, once it is found.

Before the surface integral equations are derived in Section 3.4, two preliminary theorems and a general solution of Maxwell's equations are described in Sections 3.2 and 3.3, respectively. The application of the coupled, surface integral equations to the special case of an infinite interface is considered in Section 3.5. The moment-method of solution of these integral equations is briefly reviewed in Section 3.6. The development of the numerical technique for the solutions of the surface integral equations is presented in Section 3.7. Finally, some numerical examples are given in Section 3.8.

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3.2. The Preliminary Theorems

Before the coupled, surface integral equations are derived, we must introduce two preliminary theorems which consist of several vector integral identities. These theorems are necessary for the derivation, and will be described without proof. The related proof can be found elsewhere [13].

Theorem 1.

Let $\vec{v}$, $\nabla_o \cdot \vec{v}$ be continuous on the closed surface F, and U be a differentiable function. Then

$$\int_F U \nabla_o \cdot \vec{v} \, dF' = -\int_F \vec{v} \cdot \nabla_o U \, dF'$$

with the surface divergence $\nabla_o \cdot \vec{v}$ defined as

$$\nabla_o \cdot \vec{v} \equiv \lim_{|S| \to 0} \frac{1}{|S|} \int_C (\hat{n}_o \cdot \vec{v}) \, dl$$

where $|S|$ is the area of a small surface element S enclosed by contour C, and $\hat{n}_o$ is a unit normal vector on C directed exterior to S. It can be shown that $\nabla_o = \nabla - \hat{n} \frac{\partial}{\partial n}$, where ∇ represents the conventional three-dimensional differential del-operator.

Theorem 2.

If the surface field $\vec{j}(\vec{r}')$ is continuous on F, then

$$\hat{n}(\vec{\xi}) \times \int_{F_i} [\vec{j}(\vec{r}') \times \nabla' \Phi(\vec{r}, \vec{r}')] \, dF' = -2\pi \vec{j}(\vec{\xi}) +$$

$$\hat{n}(\vec{\xi}) \times \int_F [\vec{j}(\vec{r}') \times \nabla' \Phi(\vec{\xi}, \vec{r}')] \, dF'.$$

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The sign $\int_{\vec{r}_{Fi}}$ implies that $\vec{r}$ is the field point in the integration, and lying in the interior side of the surface F. Also

$$\hat{n}(\vec{\xi}) \times \int_{\vec{r}_{Fe}} [\vec{J}(\vec{r}') \times \nabla' \Phi(\vec{r},\vec{r}')] \, dF' = 2\pi \vec{J}(\vec{\xi}) +$$

$$\hat{n}(\vec{\xi}) \times \int_{F} [\vec{J}(\vec{r}') \times \nabla' \Phi(\vec{\xi},\vec{r}')] \, dF'$$

The sign $\int_{\vec{r}_{Fe}}$ implies that $\vec{r}$ is the field point in the integration, and lying in the exterior side of the surface F.

The eventual field point $\vec{\xi}$ and the source point $\vec{r}'$ are both on the surface F. We denote with $\int_{\vec{r}_{Fi}}$ and $\int_{\vec{r}_{Fe}}$ the limits which we obtain if $\vec{r}$ approaches the point $\vec{\xi}$ of the surface F from the interior or exterior, respectively.

3.3. The General Solution of Maxwell's Equations in an Unbounded Homogeneous Space

Let us consider the problem where a finite source region G_s, consisting of electric and magnetic sources with volume current densities of $\vec{J}_e$ and $\vec{J}_m$, respectively, radiates into a lossless, unbounded, homogeneous, linear, and isotropic medium with permeability μ and permittivity ε, as shown in Figure 3.1 where we assume that the source region G_s is enclosed by surface F_s.

Based on the continuity equation, the volume charge densities are given as

$$Q_e = \frac{j}{\omega} \nabla \cdot \vec{J}_e \qquad Q_m = \frac{j}{\omega} \nabla \cdot \vec{J}_m$$

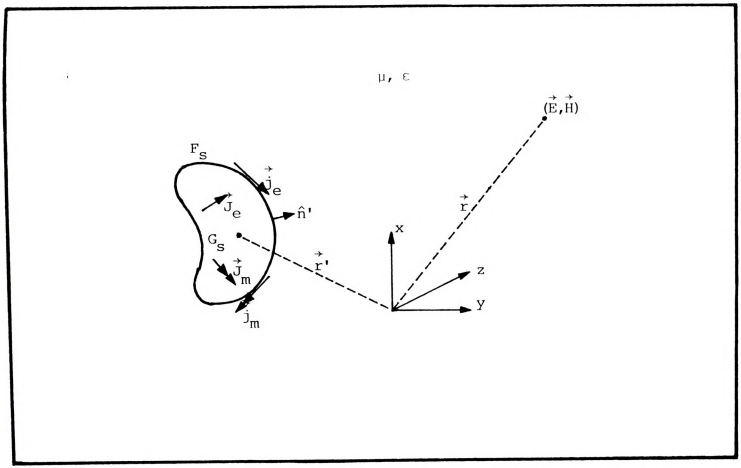

Figure 3.1. A source region G_s enclosed by surface F_s is characterized by volume current densities ($\vec{J}_e$, $\vec{J}_m$) and equivalent surface current densities ($\vec{J}_e$, $\vec{J}_m$). These current sources radiate into a homogeneous space with permeability μ and permittivity ε, and maintain an EM field ($\vec{E}$, $\vec{H}$) at point $\vec{r}$.

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where a time variation of $e^{j\omega t}$ has been assumed, and ω is the angular frequency of the sources.

From the general solution of Maxwell's equations [14], the electromagnetic field at any point $\vec{r}$ maintained by the above sources can be expressed in terms of $\vec{J}_e$, $\vec{J}_m$, Q_e, and Q_m as

$$\vec{E}(\vec{r}) = \frac{1}{4\pi} \int_{G_s} [-\vec{J}_m \times \nabla'\Phi + \frac{Q_e}{\varepsilon}\nabla'\Phi - j\omega\mu\vec{J}_e\Phi] \, dG' \qquad (3.1)$$

$$\vec{H}(\vec{r}) = \frac{1}{4\pi} \int_{G_s} [\vec{J}_e \times \nabla'\Phi + \frac{Q_m}{\mu}\nabla'\Phi - j\omega\varepsilon\vec{J}_m\Phi] \, dG' \qquad (3.2)$$

with $\Phi = \frac{e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}$, where $\vec{r}'$ is the position vector locating the source point inside the source region G_s, and $k = \omega\sqrt{\mu\varepsilon}$ is the wavenumber in unbounded space.

Now let's define the following source densities of the equivalent surface currents and charges on the closed surface F_s

$$\vec{J}_e \equiv \hat{n}' \times \vec{H}$$

$$\vec{J}_m \equiv -\hat{n}' \times \vec{E}/\eta_o$$

$$q_e = \frac{j}{\omega} \nabla_o \cdot \vec{J}_e$$

$$q_m = \frac{j}{\omega} \nabla_o \cdot \vec{J}_m$$

where $\hat{n}'$ is the outward unit normal vector on F_s, and $\eta_o = \sqrt{\mu_o/\varepsilon_o}$ is the intrinsic wave impedance in free space.

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The above electromagnetic field ($\vec{E}$, $\vec{H}$) at position $\vec{r}$ can also be expressed in terms of $\vec{J}_e$, $\vec{J}_m$, q_e, and q_m as [14]

$$\vec{E}(\vec{r}) = \frac{1}{4\pi} \int_{F_s} [-\eta_o \vec{J}_m \times \nabla'\Phi + \frac{q_e}{\varepsilon}\nabla'\Phi - j\omega\mu\vec{J}_e\Phi] \, dF' \quad (3.3)$$

$$\vec{H}(\vec{r}) = \frac{1}{4\pi} \int_{F_s} [\vec{J}_e \times \nabla'\Phi + \frac{q_m}{\mu}\nabla'\Phi - j\omega\varepsilon\eta_o\vec{J}_m\Phi] \, dF' \quad (3.4)$$

where Φ has the same form as that of eqs. (3.1) and (3.2). The last two equations are very important, and will serve as the basic tool in the following derivation.

3.4. Derivation of the Coupled Surface Integral Equations

To derive the coupled, surface integral equations, we first consider the geometry shown in Figure 3.2. We assume that a finite conducting body G enclosed by surface F is irradiated by the electromagnetic wave from the source region G_s. The medium in the region external to F is assumed to be free space with permeability μ_o and permittivity ε_o. The finite conducting body is constructed with a homogeneous material characterized by permeability μ_o and complex permittivity ε_d. The complex permittivity ε_d is defined by

$$\varepsilon_d = \varepsilon_o(\varepsilon_r - j\frac{\sigma}{\omega\varepsilon_o})$$

where ε_r is the dielectric constant, and σ is the conductivity of the body. The source region G_s is

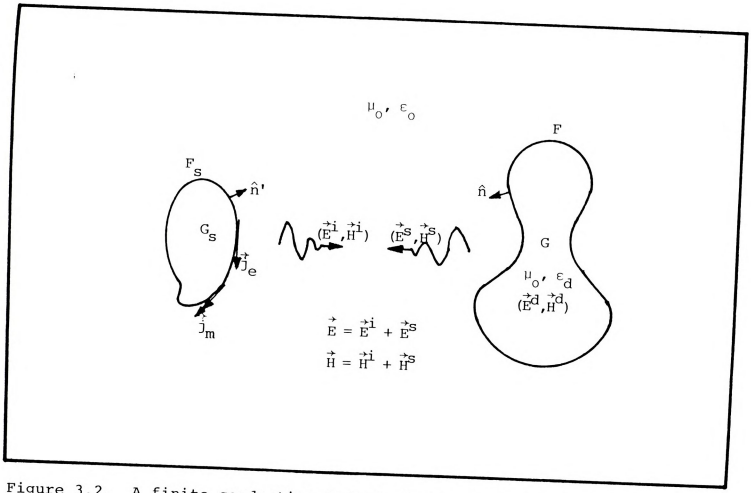

Figure 3.2. A finite conducting body G, immersed in free space and enclosed by surface F, with a permeability μ_o and a complex permittivity ε_d is irradiated by the incident field $(\vec{E}^i, \vec{H}^i)$ radiated from the source region G_s where the equivalent surface current densities $(\vec{J}_e, \vec{J}_m)$ are defined on the surface F_s. The EM fields scattered from and induced in the body are denoted as $(\vec{E}^s, \vec{H}^s)$ and $(\vec{E}^d, \vec{H}^d)$, respectively.

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characterized by the equivalent surface current densities $(\vec{J}_e, \vec{J}_m)$ on surface F_s.

Let us denote the incident field which is radiated by the sources in the absence of the finite conducting body by $(\vec{E}^i, \vec{H}^i)$. With the body present, we denote the fields scattered from and induced in it as $(\vec{E}^s, \vec{H}^s)$ and $(\vec{E}^d, \vec{H}^d)$, respectively. Now we are in a position to consider these three sets of EM fields separately.

I) The Scattered Field $(\vec{E}^s, \vec{H}^s)$:

To begin with, let's consider Maxwell's equations in the region internal to F. They are

$$\nabla \times \vec{H}^d = j\omega\varepsilon_o \vec{E}^d + j\omega(\varepsilon_d - \varepsilon_o)\vec{E}^d \tag{3.5}$$

$$\nabla \times \vec{E}^d = -j\omega\mu_o \vec{H}^d. \tag{3.6}$$

Defining the last term of eq. (3.5) as an equivalent volume current density, then eq. (3.5) becomes

$$\nabla \times \vec{H}^d = j\omega\varepsilon_o \vec{E}^d + \vec{J}_e^{eq} \tag{3.5'}$$

where

$$\vec{J}_e^{eq} \equiv j\omega(\varepsilon_d - \varepsilon_o)\vec{E}^d. \tag{3.7}$$

After substituting the expression for ε_d into eq. (3.7), we may see that $\vec{J}_e^{eq}$ consists of two components: the conduction current and the polarization current. We may think of $\vec{J}_e^{eq}$ as a current existing in free space as shown in

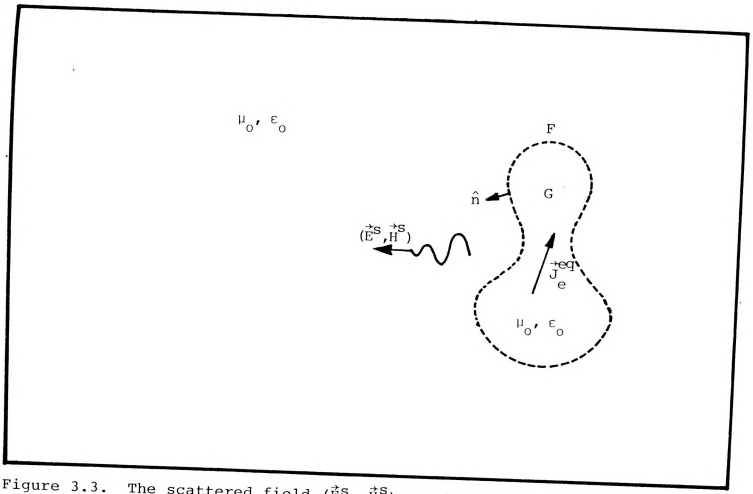

Figure 3.3. The scattered field $(\vec{E}^s, \vec{H}^s)$ can be considered as originated from an equivalent current source $\vec{J}_e^{eq}$ existing inside the body region which has been replaced by free space. This equivalent current source consists of two components, the conduction current and the polarization current.

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Figure 3.3, since only μ_o and ε_o appear in eqs. (3.5') and (3.6).

It is to be noted that the source region G_s is not included in Figure 3.3, because $\vec{J}_e^{eq}$ is the only source of the scattered field ($\vec{E}^s$, $\vec{H}^s$). Furthermore, the equivalent current $\vec{J}_e^{eq}$ radiates into an unbounded, homogeneous space. Hence, based on eqs. (3.1) and (3.2), $\vec{E}^s$ and $\vec{H}^s$ can be written as

$$\vec{E}^s = \frac{1}{4\pi} \int_G [\frac{Q_e^{eq}}{\varepsilon_o} \nabla' \phi_f - j\omega\mu_o \vec{J}_e^{eq} \phi_f] \, dG'$$

$$\vec{H}^s = \frac{1}{4\pi} \int_G [\vec{J}_e^{eq} \times \nabla' \phi_f] \, dG'$$

where $Q_e^{eq} = \frac{j}{\omega} \nabla \cdot \vec{J}_e^{eq}$ is the equivalent volume charge density,

$\phi_f = \frac{e^{-jk_o|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}$ is the Green's function for free space,

and $k_o = \omega\sqrt{\mu_o \varepsilon_o}$ is the wavenumber in free space.

Now let's define the equivalent surface current densities in terms of the scattered field ($\vec{E}^s$, $\vec{H}^s$) on the surface F as

$$\vec{J}_e^s \equiv \hat{n} \times \vec{H}^s$$

$$\vec{J}_m^s \equiv -\hat{n} \times \vec{E}^s / \eta_o$$

$$q_e^s = \frac{j}{\omega} \nabla_o \cdot \vec{J}_e^s$$

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where $\hat{n}$ is the outward unit normal vector on F.

From the equivalence theorem [15], [16], we know that the equivalent surface currents ($\vec{j}_e^s$, $\vec{j}_m^s$) on F can support ($\vec{E}^s$, $\vec{H}^s$) external to F and zero field internal to F as shown in Figure 3.4. Since the surface currents ($\vec{j}_e^s$, $\vec{j}_m^s$) radiate in an unbounded homogeneous space (free space), we can evaluate the EM fields supported by them with eqs. (3.3) and (3.4). From the uniqueness theorem, we know that the EM fields so calculated will be those postulated by the equivalence theorem. Hence we have

$$\left.\begin{aligned} \vec{E}^s &= \frac{1}{4\pi} \int_F [-\eta_o \vec{j}_m^s \times \nabla' \Phi_f + \frac{q_e^s}{\varepsilon_o} \nabla' \Phi_f - j\omega\mu_o \vec{j}_e^s \Phi_f] \, dF' \qquad (3.8) \\ \vec{H}^s &= \frac{1}{4\pi} \int_F [\vec{j}_e^s \times \nabla' \Phi_f + \frac{q_m^s}{\mu_o} \nabla' \Phi_f - j\omega\varepsilon_o \eta_o \vec{j}_m^s \Phi_f] \, dF' \qquad (3.9) \end{aligned}\right\} \text{ external to F}$$

as the scattered field at any point in the region external to F. In the region internal to F, the above two integrals vanish, i.e.,

$$\left.\begin{aligned} 0 &= \frac{1}{4\pi} \int_F [-\eta_o \vec{j}_m^s \times \nabla' \Phi_f + \frac{q_e^s}{\varepsilon_o} \nabla' \Phi_f - j\omega\mu_o \vec{j}_e^s \Phi_f] \, dF' \qquad (3.10) \\ 0 &= \frac{1}{4\pi} \int_F [\vec{j}_e^s \times \nabla' \Phi_f + \frac{q_m^s}{\mu_o} \nabla' \Phi_f - j\omega\varepsilon_o \eta_o \vec{j}_m^s \Phi_f] \, dF' \qquad (3.11) \end{aligned}\right\} \text{ internal to F}$$

because there is a null field internal to F.

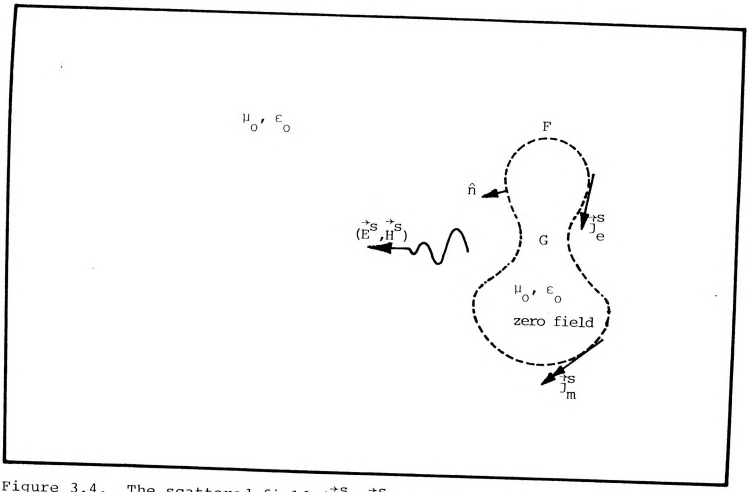

Figure 3.4. The scattered field $(\vec{E}^s, \vec{H}^s)$ in the exterior of the body and a zero field in the interior of the body are maintained by the equivalent surface currents $(\vec{J}^s_e, \vec{J}^s_m)$, which are defined in terms of the scattered field on surface F. The body region G has been replaced by free space.

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II) The Induced Field $(\vec{E}^d, \vec{H}^d)$:

To investigate the induced field $(\vec{E}^d, \vec{H}^d)$, let us consider Figure 3.2 again. This time we treat the interior region of G as source free with the induced field $(\vec{E}^d, \vec{H}^d)$ maintained by the sources existing in the exterior region of G. As for the case of scattered field, we may define a set of equivalent surface current densities $(\vec{J}_e^d, \vec{J}_m^d)$ in terms of the surface value of $(\vec{E}^d, \vec{H}^d)$:

$$\vec{J}_e^d \equiv \hat{n} \times \vec{H}^d$$

$$\vec{J}_m^d \equiv -\hat{n} \times \vec{E}^d/\eta_o$$

$$q_e^d = \frac{j}{\omega} \nabla_o \cdot \vec{J}_e^d$$

$$q_m^d = \frac{j}{\omega} \nabla_o \cdot \vec{J}_m^d$$

Again, from the equivalence theorem [15], [16], we know that, with the exterior region of G being replaced by a homogeneous source free region with permeability μ_o and complex permittivity ε_d, the negative of the equivalent surface currents $(\vec{J}_e^d, \vec{J}_m^d)$ will support the total induced field $(\vec{E}^d, \vec{H}^d)$ in the region internal to F and a null field in the region external to F, as shown in Figure 3.5. Equations (3.3) and (3.4) can be applied to determine these fields; a sign change is necessary since we define $(\vec{J}_e^d, \vec{J}_m^d)$

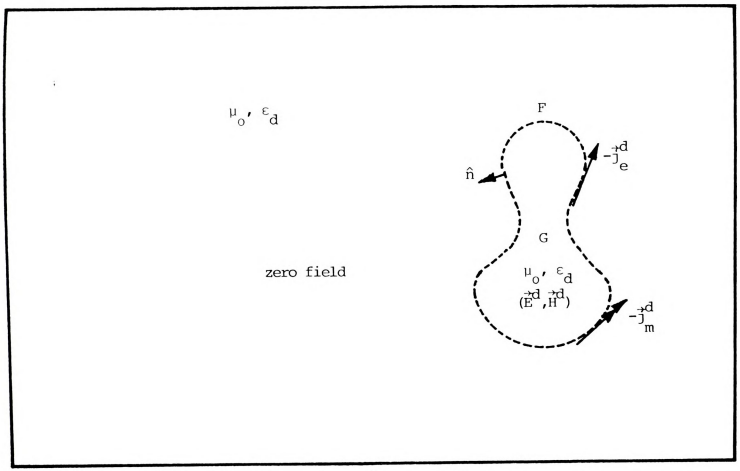

Figure 3.5. The induced field $(\vec{E}^d, \vec{H}^d)$ in the interior of the body and a zero field in the exterior of the body are maintained by the negative of the equivalent surface currents $(\vec{J}_e^d, \vec{J}_m^d)$, which are defined in terms of the induced field on surface F. The region external to F is replaced by a medium with permeability μ_o and complex permittivity ε_d.

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in terms of the unit normal vector $\hat{n}$ which is pointing into the source region. Hence we have

$$\vec{E}^d = \frac{1}{4\pi}\int_F [\eta_o \vec{J}_m^d \times \nabla'\Phi_d - \frac{q_e^d}{\varepsilon_d}\nabla'\Phi_d + j\omega\mu_o \vec{J}_e^d \Phi_d]\, dF' \quad (3.12)$$

$$\vec{H}^d = \frac{1}{4\pi}\int_F [-\vec{J}_e^d \times \nabla'\Phi_d - \frac{q_m^d}{\mu_o}\nabla'\Phi_d + j\omega\varepsilon_d \eta_o \vec{J}_m^d \Phi_d]\, dF' \quad (3.13)$$

} internal to F

as the induced field at any point in the region internal to the surface F, where $\Phi_d = \frac{e^{-jk_d|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}$ is the Green's function for the conducting body region, and $k_d = \omega\sqrt{\mu_o \varepsilon_d}$ is the complex wavenumber in the body. In the region external to F, the above two integrals vanish, i.e.,

$$0 = \frac{1}{4\pi}\int_F [\eta_o \vec{J}_m^d \times \nabla'\Phi_d - \frac{q_e^d}{\varepsilon_d}\nabla'\Phi_d + j\omega\mu_o \vec{J}_e^d \Phi_d]\, dF' \quad (3.14)$$

$$0 = \frac{1}{4\pi}\int_F [-\vec{J}_e^d \times \nabla'\Phi_d - \frac{q_m^d}{\mu_o}\nabla'\Phi_d + j\omega\varepsilon_d \eta_o \vec{J}_m^d \Phi_d]\, dF' \quad (3.15)$$

} external to F

because there is a null field external to F.

III) The Incident Field $(\vec{E}^i, \vec{H}^i)$:

To investigate the incident field $(\vec{E}^i, \vec{H}^i)$, let's consider the geometry shown in Figure 3.2 again. Based on $(\vec{E}^i, \vec{H}^i)$, we define the following equivalent surface currents on the body surface F:

$$\vec{J}_e^i \equiv \hat{n} \times \vec{H}^i$$

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$$\vec{J}_m^i \equiv -\hat{n} \times \vec{E}^i/\eta_o$$

$$q_e^i = \frac{j}{\omega} \nabla_o \cdot \vec{J}_e^i$$

$$q_m^i = \frac{j}{\omega} \nabla_o \cdot \vec{J}_m^i .$$

From the induction theorem [15], [16], we know that the negative of these equivalent surface currents, radiating in the presence of the conducting body, will support the total induced field ($\vec{E}^d$, $\vec{H}^d$) in the region internal to surface F, and the scattered field ($\vec{E}^s$, $\vec{H}^s$) external to F. This is illustrated in Figure 3.6.

Suppose that we are considering an extreme case with the complex permittivity of the body ε_d being replaced by the free space permittivity ε_o, then the negative of the surface currents ($\vec{J}_e^i$, $\vec{J}_m^i$) will support the incident field ($\vec{E}^i$, $\vec{H}^i$) in the region internal to F, and a null field in the region external to F. Because the entire space becomes homogeneous (free space), there exists no real scattering obstacle—the conducting body, so there is no scattered field. This situation is shown in Figure 3.7.

Again, eqs. (3.3) and (3.4) can be used to express the EM fields shown in Figure 3.7. We have

$$\vec{E}^i = \frac{1}{4\pi} \int_F \left[\eta_o \vec{J}_m^i \times \nabla'\Phi_f - \frac{q_e^i}{\varepsilon_o}\nabla'\Phi_f + j\omega\mu_o \vec{J}_e^i \Phi_f\right] dF' \Bigg\} \begin{matrix} \text{internal} \\ \text{to F} \end{matrix} \qquad (3.16)$$

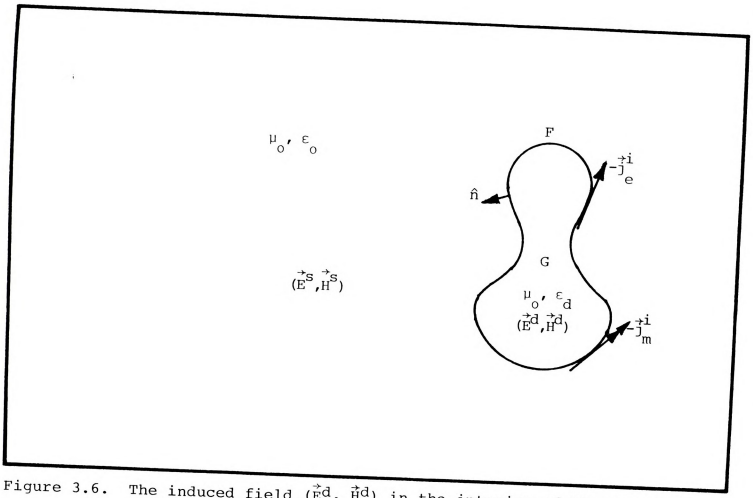

Figure 3.6. The induced field $(\vec{E}^d, \vec{H}^d)$ in the interior of the body and the scattered field $(\vec{E}^s, \vec{H}^s)$ in the exterior of the body are maintained by the negative of the equivalent surface currents $(\vec{J}_e^i, \vec{J}_m^i)$, which are defined in terms of the incident field on surface F. The body G with permeability μ_o and complex permittivity ε_d is immersed in free space.

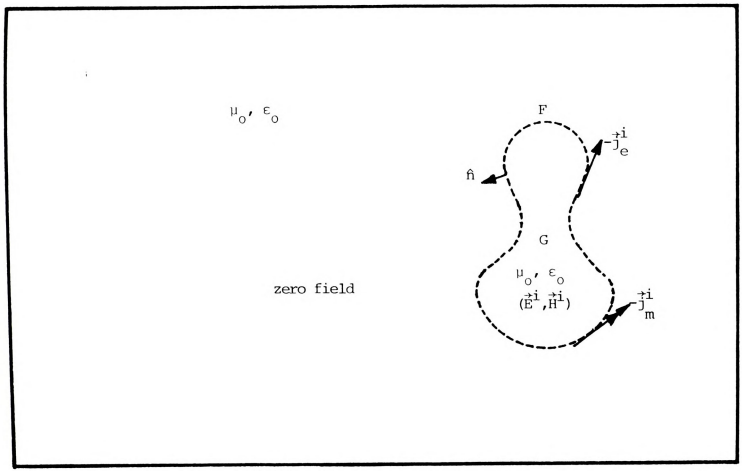

Figure 3.7. With the body region G replaced by free space, the incident field $(\vec{E}^i, \vec{H}^i)$ in the interior of the body and a zero field in the exterior of the body are maintained by the negative of the equivalent surface currents $(\vec{J}_e^i, \vec{J}_m^i)$, which are defined in terms of the incident field on surface F.

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as the incident field at any point in the region internal to F. However, the above two integrals vanish in the exterior region of the body, i.e.,

$$0 = \frac{1}{4\pi} \int_F [\eta_o\vec{j}^i_m \times \nabla'\phi_f - \frac{q^i_e}{\varepsilon_o}\nabla'\phi_f + j\omega\mu_o\vec{j}^i_e\phi_f]\, dF' \quad (3.18)$$

$$0 = \frac{1}{4\pi} \int_F [-\vec{j}^i_e \times \nabla'\phi_f - \frac{q^i_m}{\mu_o}\nabla'\phi_f + j\omega\varepsilon_o\eta_o\vec{j}^i_m\phi_f]\, dF'. \quad (3.19)$$

external to F

Up to this point, we have considered the incident field ($\vec{E}^i$, $\vec{H}^i$), the induced field ($\vec{E}^d$, $\vec{H}^d$), and the scattered field ($\vec{E}^s$, $\vec{H}^s$) individually. Now we are ready to derive the coupled surface integral equations based on the results we just obtained.

Since eq. (3.16) is true at any point $\vec{r}$ in the interior region of G, if we let the field point $\vec{r}$ approach a point $\vec{\xi}$ on the surface F along the interior normal, then the limiting value of eq. (3.16) will be the incident electric field at the point $\vec{\xi}$ on the surface F. Hence we may write

$$\vec{E}^i(\vec{\xi}) = \frac{1}{4\pi} \int_{F_i} [\eta_o\vec{j}^i_m \times \nabla'\phi_f - \frac{q^i_e}{\varepsilon_o}\nabla'\phi_f + j\omega\mu_o\vec{j}^i_e\phi_f]\, dF'$$

where $\int_{F_i}$ has the same meaning as that described in theorem 2. After taking the cross-product of the above

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equation with the unit normal vector $\hat{n}(\vec{\xi})$, we will arrive at the following equation

$$\hat{n} \times \frac{1}{4\pi} \int_{F_i} [\eta_o \vec{J}_m^i \times \nabla'\Phi_f - \frac{q_e^i}{\varepsilon_o}\nabla'\Phi_f + j\omega\mu_o \vec{J}_e^i \Phi_f] dF'$$

$$= -\eta_o \vec{J}_m^i \,. \tag{3.20}$$

Applying the same arguments to eq. (3.17), we have

$$-\hat{n} \times \frac{1}{4\pi} \int_{F_i} [-\vec{J}_e^i \times \nabla'\Phi_f - \frac{q_m^i}{\mu_o}\nabla'\Phi_f + j\omega\varepsilon_o\eta_o \vec{J}_m^i \Phi_f] \, dF'$$

$$= -\vec{J}_e^i \,. \tag{3.21}$$

Also, from eqs. (3.10) and (3.11), it is straightforward to obtain

$$\hat{n} \times \frac{1}{4\pi} \int_{F_i} [-\eta_o \vec{J}_m^s \times \nabla'\Phi_f + \frac{q_e^s}{\varepsilon_o}\nabla'\Phi_f - j\omega\mu_o \vec{J}_e^s \Phi_f] \, dF' = 0 \tag{3.22}$$

and

$$-\hat{n} \times \frac{1}{4\pi} \int_{F_i} [\vec{J}_e^s \times \nabla'\Phi_f + \frac{q_m^s}{\mu_o}\nabla'\Phi_f - j\omega\varepsilon_o\eta_o \vec{J}_m^s \Phi_f] \, dF' = 0. \tag{3.23}$$

Before going any further, we should introduce the boundary conditions which must be satisfied at the surface F. The boundary conditions require that the tangential components of the electric and magnetic field must be continuous across F, or equivalently we may write

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$$\vec{J}_m^s + \vec{J}_m^i = \vec{J}_m^d$$

Now, take the difference between eqs. (3.20) and (3.22), and apply the above boundary conditions. We have

$$\hat{n} \times \frac{1}{4\pi} \int_{F_i} [\eta_o \vec{J}_m^d \times \nabla'\Phi_f - \frac{q_e^d}{\varepsilon_o}\nabla'\Phi_f + j\omega\mu_o \vec{J}_e^d \Phi_f]\, dF'$$

$$= -\eta_o \vec{J}_m^i . \tag{3.24}$$

Similarly, subtracting eq. (3.23) from eq. (3.21), we obtain

$$-\hat{n} \times \frac{1}{4\pi} \int_{F_i} [-\vec{J}_e^d \times \nabla'\Phi_f - \frac{q_m^d}{\mu_o}\nabla'\Phi_f + j\omega\varepsilon_o\eta_o \vec{J}_m^d \Phi_f]\, dF'$$

$$= -\vec{J}_e^i . \tag{3.25}$$

On the other hand, we may derive the following two equations from eqs. (3.14) and (3.15):

$$\hat{n} \times \frac{1}{4\pi} \int_{F_e} [\eta_o \vec{J}_m^d \times \nabla'\Phi_d - \frac{q_e^d}{\varepsilon_d}\nabla'\Phi_d + j\omega\mu_o \vec{J}_e^d \Phi_d]\, dF' = 0 \tag{3.26}$$

$$-\hat{n} \times \frac{1}{4\pi} \int_{F_e} [-\vec{J}_e^d \times \nabla'\Phi_d - \frac{q_m^d}{\mu_o}\nabla'\Phi_d + j\omega\varepsilon_d\eta_o \vec{J}_m^d \Phi_d]\, dF' = 0 \tag{3.27}$$

It can be shown that equalities exist between the following limiting surface integrals and their corresponding regular surface integrals

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$$\hat{n} \times \int_{F_i} q_e^d \nabla'\Phi_f \, dF' = \hat{n} \times \int_F q_e^d \nabla'\Phi_f \, dF'$$

$$\hat{n} \times \int_{F_e} q_e^d \nabla'\Phi_d \, dF' = \hat{n} \times \int_F q_e^d \nabla'\Phi_d \, dF'.$$

Subtracting the second equation from the first equation, we find that

$$\hat{n} \times \int_{F_i} q_e^d \nabla'\Phi_f \, dF' - \hat{n} \times \int_{F_e} q_e^d \nabla'\Phi_d \, dF' =$$

$$\hat{n} \times \int_F q_e^d \nabla'(\Phi_f - \Phi_d) \, dF' = -\hat{n} \times \nabla \int_F q_e^d(\Phi_f - \Phi_d) \, dF'.$$

However, since $q_e^d = \frac{j}{\omega} \nabla_o' \cdot \vec{J}_e^d$, the integral on the right-hand side of the above equation can be rewritten as

$$\int_F q_e^d(\Phi_f - \Phi_d) \, dF' = \frac{j}{\omega} \int_F (\nabla_o' \cdot \vec{J}_e^d)(\Phi_f - \Phi_d) \, dF'.$$

Applying theorem 1 to the right-hand side of the above equation, we can rewrite it as

$$\int_F q_e^d(\Phi_f - \Phi_d) \, dF' = -\frac{j}{\omega} \int_F \vec{J}_e^d \cdot \nabla_o'(\Phi_f - \Phi_d) \, dF'$$

$$= -\frac{j}{\omega} \int_F \vec{J}_e^d \cdot \nabla'(\Phi_f - \Phi_d) \, dF'.$$

It should be mentioned that the last equality is true, because $\nabla_o' = \nabla' - \hat{n}' \frac{\partial}{\partial n'}$, and $\vec{J}_e^d$ has no normal component. Then finally, we have

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or

$$\hat{n} \times \int_{F_i} q_e^d \nabla' \Phi_f \, dF' - \hat{n} \times \int_{F_e} q_e^d \nabla' \Phi_d \, dF'$$

$$= -\hat{n} \times \nabla \left[-\frac{j}{\omega} \int_F \vec{J}_e^d \cdot \nabla' (\Phi_f - \Phi_d) \, dF' \right]$$

$$= \frac{j}{\omega} \hat{n} \times \int_F \nabla [\vec{J}_e^d \cdot \nabla' (\Phi_f - \Phi_d)] \, dF'.$$

Based on the following vector identity

$$\nabla (\vec{A} \cdot \vec{B}) = (\vec{A} \cdot \nabla) \vec{B} + (\vec{B} \cdot \nabla) \vec{A} + \vec{A} \times (\nabla \times \vec{B}) + \vec{B} \times (\nabla \times \vec{A}),$$

it is easy to show that

$$\hat{n} \times \int_{F_i} q_e^d \nabla' \Phi_f \, dF' - \hat{n} \times \int_{F_e} q_e^d \nabla' \Phi_d \, dF'$$

$$= -\frac{j}{\omega} \hat{n} \times \int_F [(\vec{J}_e^d \cdot \nabla') \nabla' (\Phi_f - \Phi_d)] \, dF'. \qquad (3.28)$$

Now, subtracting eq. (3.26) from eq. (3.24) with the latter multiplied by ε_o and the former multiplied by ε_d, the result is

$$\varepsilon_o \hat{n} \times \frac{1}{4\pi} \int_{F_i} \left[\eta_o \vec{J}_m^d \times \nabla' \Phi_f - \frac{q_e^d}{\varepsilon_o} \nabla' \Phi_f + j\omega \mu_o \vec{J}_e^d \Phi_f \right] dF' -$$

$$\varepsilon_d \hat{n} \times \frac{1}{4\pi} \int_{F_e} \left[\eta_o \vec{J}_m^d \times \nabla' \Phi_d - \frac{q_e^d}{\varepsilon_d} \nabla' \Phi_d + j\omega \mu_o \vec{J}_e^d \Phi_d \right] dF'$$

$$= -\varepsilon_o \eta_o \vec{J}_m^i$$

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$$\frac{1}{4\pi}\{[\hat{n}\times\int_{F_i}\varepsilon_o\eta_o\vec{j}_m^d\times\nabla'\Phi_f\,dF' - \hat{n}\times\int_{F_e}\varepsilon_d\eta_o\vec{j}_m^d\times\nabla'\Phi_d\,dF'] - [\hat{n}\times\int_{F_i} q_e^d\nabla'\Phi_f\,dF' - \hat{n}\times\int_{F_e} q_e^d\nabla'\Phi_d\,dF'] + [\hat{n}\times\int_{F_i} j\omega\mu_o\varepsilon_o\vec{j}_e^d\Phi_f\,dF' - \hat{n}\times\int_{F_e} j\omega\mu_o\varepsilon_d\vec{j}_e^d\Phi_d\,dF']\} = -\varepsilon_o\eta_o\vec{j}_m^i .$$

Based on eq. (3.28) and theorem 2, the above equation becomes

$$\vec{j}_m^d = \frac{2}{\kappa^2+1}\vec{j}_m^i - \frac{1}{2\pi(\kappa^2+1)}\hat{n}\times\int_F [jk_o\vec{j}_e^d(\kappa^2\Phi_d - \Phi_f) + \vec{j}_m^d\times\nabla'(\kappa^2\Phi_d - \Phi_f) + \frac{j}{k_o}(\vec{j}_e^d\cdot\nabla')\nabla'(\Phi_d - \Phi_f)]\,dF'.$$

Similarly, the other surface integral equation can be derived from eqs. (3.25) and (3.27),

$$\vec{j}_e^d = \vec{j}_e^i + \frac{1}{4\pi}\hat{n}\times\int_F [jk_o\vec{j}_m^d(\kappa^2\Phi_d - \Phi_f) - \vec{j}_e^d\times\nabla'(\Phi_d - \Phi_f) + \frac{j}{k_o}(\vec{j}_m^d\cdot\nabla')\nabla'(\Phi_d - \Phi_f)]\,dF'$$

where $\kappa = \sqrt{\varepsilon_d/\varepsilon_o}$ is the refractive index of the body. If we normalize all the quantities concerning the dimension of length by $1/k_o$, the above two coupled surface integral equations become (these two equations in similar forms have also been given by Müller [13] and Morita [19] through different derivations)

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$$\vec{J}_m^d = \frac{2}{\kappa^2+1}\vec{J}_m^i - \frac{1}{2\pi(\kappa^2+1)}\hat{n} \times \int_F [j(\kappa^2\Phi_d - \Phi_f)\vec{J}_e^d + \vec{J}_m^d \times \nabla'(\kappa^2\Phi_d - \Phi_f) + j(\vec{J}_e^d \cdot \nabla')\nabla'(\Phi_d - \Phi_f)]\, dF' \tag{3.29}$$

and

$$\vec{J}_e^d = \vec{J}_e^i + \frac{1}{4\pi}\hat{n} \times \int_F [j(\kappa^2\Phi_d - \Phi_f)\vec{J}_m^d - \vec{J}_e^d \times \nabla'(\Phi_d - \Phi_f) + j(\vec{J}_m^d \cdot \nabla')\nabla'(\Phi_d - \Phi_f)]\, dF' \,. \tag{3.30}$$

The induced equivalent surface currents ($\vec{J}_e^d$, $\vec{J}_m^d$) (or the induced surface field ($\vec{E}^d$, $\vec{H}^d$)) can be calculated based on the above two coupled, surface integral equations. In these two equations, due to normalization with respect to $1/k_o$, the Green's functions Φ_d and Φ_f have the forms of

$$\Phi_d = \frac{e^{-j\kappa|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} \quad \text{and} \quad \Phi_f = \frac{e^{-j|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} \,.$$

3.5. The Special Case of an Infinite Plane Interface

In this section, we apply the surface integral equations to the special case of an infinite plane interface between two half spaces, a free-space in the region $z < 0$ with permeability μ_o and permittivity ε_o and a finite conducting medium in the region $z > 0$ with permeability μ_o and complex permittivity ε_d, as shown in Figure 3.8. We assume that the incident plane wave, which propagates in

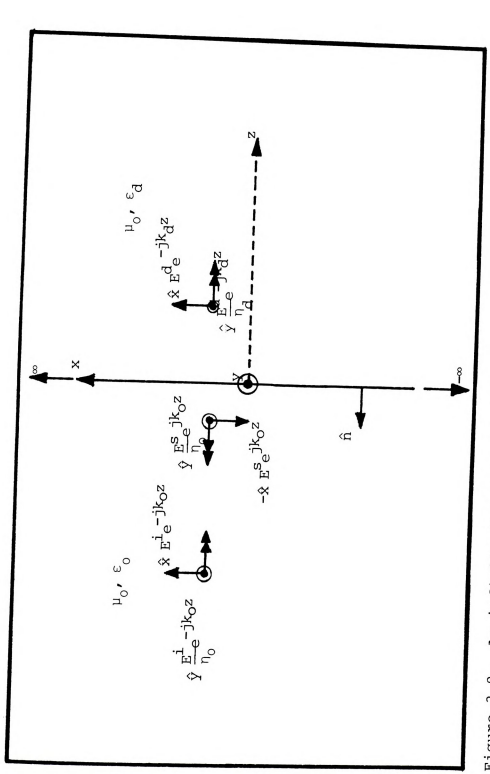

Figure 3.8. An infinite interface between free space and a finite conducting medium with permeability μ_o and complex permittivity ε_d is impinged by a plane wave propagating in the +z direction.

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the +z direction with its electric field $\vec{E}^i$ polarized in the +x direction and its magnetic field $\vec{H}^i$ polarized in the +y direction, is normally impinging on the interface, and produces a reflected field ($\vec{E}^s$, $\vec{H}^s$) and a transmitted field ($\vec{E}^d$, $\vec{H}^d$) in the free space region and the conducting medium region, respectively. Intuitively, these fields can be expressed as

$$\left.\begin{aligned} \vec{E}^i &= \hat{x}\, E^i\, e^{-jk_o z} \\ \vec{H}^i &= \hat{y}\, H^i\, e^{-jk_o z} = \hat{y}\, \frac{E^i}{\eta_o}\, e^{-jk_o z}\,, \\ \vec{E}^s &= -\hat{x}\, E^s\, e^{jk_o z} \\ \vec{H}^s &= \hat{y}\, \frac{E^s}{\eta_o}\, e^{jk_o z} \end{aligned}\right\} \text{in the region } z < 0$$

and

$$\left.\begin{aligned} \vec{E}^d &= \hat{x}\, E^d\, e^{-jk_d z} \\ \vec{H}^d &= \hat{y}\, H^d\, e^{-jk_d z} = \hat{y}\, \frac{E^d}{\eta_d}\, e^{-jk_d z} \end{aligned}\right\} \text{in the region } z > 0$$

where $\eta_d = \sqrt{\mu_o/\varepsilon_d}$ is the complex wave impedance in the conducting medium. By solving a simple boundary value problem, it is easy to show that, on the interface, the transmitted field (E^d, H^d) is related to the incident field (E^i, H^i) by

$$\begin{aligned} E^d &= \frac{2}{1+\kappa}\, E^i \\ H^d &= \frac{2\kappa}{1+\kappa}\, H^i. \end{aligned} \tag{3.31}$$

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It would be interesting to show that the results of eq. (3.31) can be deduced from the rather complicated surface integral equations which have been derived in the previous section. To do that, let's start from the following definitions of the various equivalent surface current densities.

$$\left.\begin{aligned} \vec{J}_e^{\,i} &\equiv \hat{n} \times \vec{H}^i = -\hat{z} \times \hat{y}\, H^i = \hat{x}\, \frac{E^i}{\eta_o} \\ \vec{J}_m^{\,i} &\equiv -\hat{n} \times \frac{\vec{E}^i}{\eta_o} = \hat{z} \times \hat{x}\, \frac{E^i}{\eta_o} = \hat{y}\, \frac{E^i}{\eta_o} \end{aligned}\right\} \text{ at } z = 0$$

also

$$\vec{J}_e^{\,d} \equiv \hat{n} \times \vec{H}^d = \hat{x}\, j_e^d$$

$$\vec{J}_m^{\,d} \equiv -\hat{n} \times \frac{\vec{E}^d}{\eta_o} = \hat{y}\, j_m^d$$

where $\hat{n}$ is the unit normal vector on the interface pointing in the -z direction.

Now, substituting the above expressions of surface current densities into eq. (3.29), we may show that the last two terms of the surface integration contribute nothing to the total surface integral. We should consider these two terms one by one as below.

I) Let us consider the term $\hat{n} \times \int_F \vec{J}_m^{\,d} \times \nabla'(\kappa^2 \Phi_d - \Phi_f)\, dF'$ first.

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It is easy to show that

$$\hat{n} \times \int_F \vec{j}_m^d \times \nabla'(\kappa^2\Phi_d - \Phi_f)\, dF' = (-\hat{z}) \times \int_F \hat{y}\, j_m^d \times \nabla'(\kappa^2\Phi_d - \Phi_f)\, dF' = 0 \tag{3.32}$$

is true, since the term $\nabla'(\kappa^2\Phi_d - \Phi_f)$ has only two components, x and y components but not z component (nothing is z-dependent in this problem), both of which vanish after cross-producting with unit vectors $\hat{y}$ and $\hat{z}$ consecutively.

II) Secondly, consider the term of $\hat{n} \times \int_F (\vec{j}_e^d \cdot \nabla')\, \nabla'(\Phi_d - \Phi_f)\, dF'$.

Since the surface current $\vec{j}_e^d$ is x-directed, we may express the first part of the integrand as

$$(\vec{j}_e^d \cdot \nabla') = j_{ex}^d \frac{\partial}{\partial x'} .$$

Because everything is z-independent, we may write the second part of the integrand as

$$\nabla'(\Phi_d - \Phi_f) = \hat{x} \frac{\partial}{\partial x'}(\Phi_d - \Phi_f) + \hat{y} \frac{\partial}{\partial y'}(\Phi_d - \Phi_f).$$

Combining the above two equations, we have the following expression for the integrand:

$$(\vec{j}_e^d \cdot \nabla')\, \nabla'(\Phi_d - \Phi_f) = \hat{x}\, j_{ex}^d \frac{\partial^2}{\partial x'^2}(\Phi_d - \Phi_f) + \hat{y}\, j_{ex}^d \frac{\partial^2}{\partial x' \partial y'}(\Phi_d - \Phi_f).$$

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Hence, the integral considered can be rewritten as

$$\hat{n} \times \int_F (\vec{j}_e^d \cdot \nabla') \nabla'(\Phi_d - \Phi_f)\, dF' = \hat{n} \times \int_F \hat{x}\, j_{ex}^d \frac{\partial^2}{\partial x'^2} (\Phi_d - \Phi_f)\, dF' + \hat{n} \times \int_F \hat{y}\, j_{ex}^d \frac{\partial^2}{\partial x' \partial y'} (\Phi_d - \Phi_f)\, dF'. \tag{3.33}$$

If we choose the field point to be at origin; i.e., $\vec{r} = 0$, we may write the Green's functions Φ_d and Φ_f as

$$\Phi_d = \frac{e^{-j\kappa r'}}{r'}, \quad \text{and} \quad \Phi_f = \frac{e^{-jr'}}{r'}$$

where $r' = |\vec{r}'| = \sqrt{x'^2+y'^2}$. With some straightforward algebraic manipulations, it can be shown that

$$\frac{\partial^2}{\partial x'^2}(\Phi_d - \Phi_f) = [j \frac{e^{-jr'-\kappa e^{-j\kappa r'}}}{r'^2} + \frac{e^{-jr'-e^{-j\kappa r'}}}{r'^3}] +$$

$$\cos^2\theta\, [\frac{e^{-jr'-\kappa^2 e^{-j\kappa r'}}}{r'} - 3j \frac{e^{-jr'-\kappa e^{-j\kappa r'}}}{r'^2}$$

$$-3 \frac{e^{-jr'-e^{-j\kappa r'}}}{r'^3}]$$

where θ is the angle between the source-point position vector $\vec{r}'$ and the x axis, as shown in Figure 3.9. Similarly, it can be shown that

$$\frac{\partial^2}{\partial x' \partial y'}(\Phi_d - \Phi_f) = \sin\theta \cos\theta\, [\frac{e^{-jr'-\kappa^2 e^{-j\kappa r'}}}{r'}$$

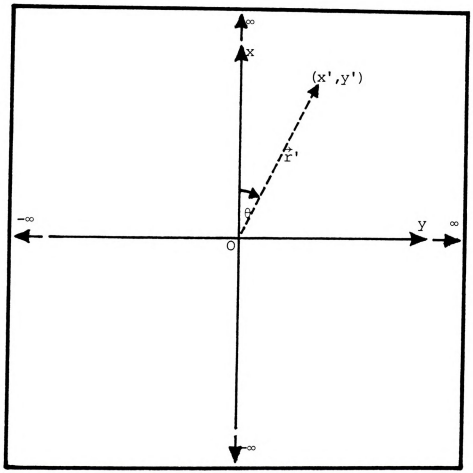

Figure 3.9. The coordinate system utilized in the infinite interface problem.

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$$- 3j \frac{e^{-jr'} - \kappa e^{-j\kappa r'}}{r'^2} - 3 \frac{e^{-jr'} - e^{-j\kappa r'}}{r'^3}] .$$

In the above derivation, the field point has been arbitrarily chosen at the origin. This assumption doesn't destroy the generality of this treatment, since the interface is infinitely large, and the field induced on it is uniformly distributed over it.

Now, we must show that both of the integrals on the right-hand side of eq. (3.33) vanish. Let us consider the last term of that equation first.

$$\int_F \frac{\partial^2}{\partial x' \partial y'} (\Phi_d - \Phi_f) \, dF' = \int_0^{2\pi} \sin\theta \cos\theta \, d\theta \cdot \int_0^{\infty} [e^{-jr'} -$$

$$\kappa^2 e^{-j\kappa r'} - 3j \frac{e^{-jr'} - \kappa e^{-j\kappa r'}}{r'} - 3 \frac{e^{-jr'} - e^{-j\kappa r'}}{r'^2}] \, dr'$$

It is clear that the first integral on the right-hand side of the above equation is equal to zero. Hence we have

$$\int_F \frac{\partial^2}{\partial x' \partial y'} (\Phi_d - \Phi_f) \, dF' = 0. \qquad (3.34)$$

For the first term on the right-hand side of eq. (3.33), we may write that

$$\int_F \frac{\partial^2}{\partial x'^2} (\Phi_d - \Phi_f) \, dF' = 2\pi \int_0^{\infty} [j \frac{e^{-jr'} - \kappa e^{-j\kappa r'}}{r'} +$$

$$\frac{e^{-jr'}-e^{-j\kappa r'}}{r'^2}]\,dr' + \int_0^{2\pi} \cos^2\theta\, d\theta \cdot \int_0^{\infty} [e^{-jr'} - \kappa^2\, e^{-j\kappa r'}$$

$$-3j\,\frac{e^{-jr'}-\kappa\, e^{-j\kappa r'}}{r'} - 3\,\frac{e^{-jr'}-e^{-j\kappa r'}}{r'^2}]\,dr'$$

$$= -j\pi\int_0^{\infty} \frac{e^{-jr'}-\kappa\, e^{-j\kappa r'}}{r'}\,dr' - \pi\int_0^{\infty} \frac{e^{-jr'}-e^{-j\kappa r'}}{r'^2}\,dr'$$

$$+ j\kappa\pi - j\pi = \pi\int_0^{\infty} \frac{d}{dr'}\left(\frac{e^{-jr'}-e^{-j\kappa r'}}{r'}\right) dr' + j\kappa\pi - j\pi$$

$$= \pi \lim_{r'\to 0} \left[\frac{e^{-j\kappa r'}-e^{-jr'}}{r'}\right] + j\kappa\pi - j\pi = 0. \qquad (3.35)$$

Note that for the last step in the above derivation, the radiation condition is applied.

After substitution of eqs. (3.34) and (3.35) into eq. (3.33), the result is

$$\hat{n} \times \int_F (\vec{j}_e^{\,d} \cdot \nabla')\, \nabla'(\Phi_d - \Phi_f)\, dF' = 0. \qquad (3.36)$$

Then, substituting eqs. (3.32) and (3.36) into eq. (3.29), we obtain

$$j_m^d = \left(\frac{2}{\kappa^2+1}\right) \frac{E^i}{\eta_o} + \frac{j}{2\pi(\kappa^2+1)}\, j_e^d \int_F (\kappa^2\, \Phi_d - \Phi_f)\, dF'. \qquad (3.37)$$

The integral term is easy to evaluate. We write down the result directly.

$$\int_F (\kappa^2\, \Phi_d - \Phi_f)\, dF' = j\, 2\pi(1 - \kappa)$$

Hence, eq. (3.37) becomes

$$j_m^d = \left(\frac{2}{\kappa^2+1}\right) \frac{E^i}{\eta_o} + \frac{\kappa-1}{\kappa^2+1}\, j_e^d. \tag{3.38}$$

Following a similar procedure, we can obtain another equation from eq. (3.30), which is

$$j_e^d = \frac{E^i}{\eta_o} + \frac{\kappa-1}{2}\, j_m^d. \tag{3.39}$$

Equations (3.29) and (3.30) have been simplified to become two linear equations with unknowns j_e^d and j_m^d. The solutions of these two unknowns are

$$j_e^d = \left(\frac{2\kappa}{\kappa+1}\right) \frac{E^i}{\eta_o}$$

and

$$j_m^d = \left(\frac{2}{\kappa+1}\right) \frac{E^i}{\eta_o},$$

which also can be written as

$$E^d = \left(\frac{2}{\kappa+1}\right) E^i$$

and

$$H^d = \left(\frac{2\kappa}{\kappa+1}\right) H^i$$

which are exactly the same as eq. (3.31).

We have shown that the coupled, surface integral equations can predict correct results for the simplified special case of an infinite two-dimensional interface between a finite conducting medium and free space. This

example analytically proves the validity of the surface integral equations. In the following sections, we will develop a numerical technique for solving these surface integral equations for the cases of arbitrarily shaped, finite conducting bodies; after that, some numerical examples will be presented to discuss the accuracy of this numerical technique.

3.6. Review of Moment-Method

Before actually solving the surface integral equations, we should briefly review the basic steps involved in numerical implementation of such solutions, or the so-called Method of Moments (MOM) [17]. The idea of moment-method solutions is that the linear operator equation

$$L(f) = g \tag{3.40}$$

where L is a linear operator, g is some known driving function, and f is to be determined, may be solved by first expanding f in a series of basis functions f_n with unknown weighting coefficients α_n,

$$f = \sum_n \alpha_n f_n \tag{3.41}$$

Next, a set of weighting or testing functions w_m is chosen and an inner product (denoted by symbol < >) taken in eq. (3.40). After substitution of eq. (3.41) into eq. (3.40), the result is

$$\sum_n \alpha_n < w_m, L f_n > = < w_m, g > \tag{3.42}$$

which may be written in matrix form as

$$[l_{mn}] [\alpha_n] = [g_m].$$

As illustrated in Figure 3.10, the idea when applied to the cubic box assumes that the box surface may be divided into square patches over each of which both components of surface current are assumed constant with unknown amplitudes (subdomain flat-top pulse-expansion functions). The testing functions chosen in this problem are δ functions at the patch centers (collocation or point matching). Thus, by successively enforcing the surface integral equations to be satisfied at these match points, a system of linear algebraic equations is generated. Matrix inversion then yields the solution for the unknown coefficients of the current samples.

3.7. The Numerical Technique

The validity of eqs. (3.29) and (3.30) has been checked in Section 3.5 through the consideration of a special case of a two-dimensional, infinite-plane interface between a finite conducting medium and free space. Since it is our goal to quantify the EM fields induced on the surfaces of arbitrarily shaped, finite conducting bodies, such as human bodies, it seems more realistic to consider a simulated human body with shape as shown in

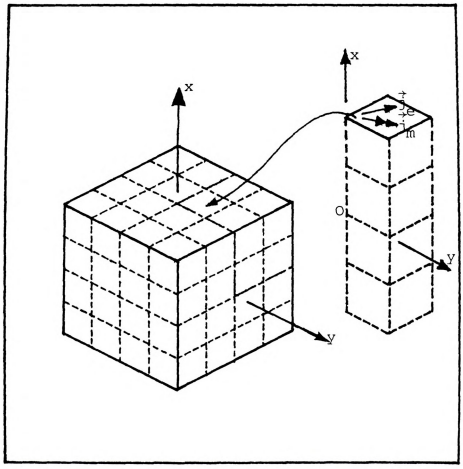

Figure 3.10. Division of the surface of a cubic body into square surface cells for moment-method solution using two-dimensional flat pulses as expansion functions and δ functions as testing functions.

Figure 3.11. We assume that the body region G, being characterized by permeability μ_o and complex permittivity ε_d and enclosed by surface F, is illuminated by an incident plane wave which is propagating in the +z direction with its electric field $\vec{E}^i$ polarized in the +x direction. The body is immersed in free space as shown in Figure 3.11.

It is easy to see from Figure 3.11 that, for a simulated body, the body surface F consists of a number of plane surface elements. If the body is described in a rectangular coordinate system, which is the one used in Figure 3.11, the problem can be simplified by constructing the body surface with surface elements having normal vectors pointing in one of the six possible along-axis directions, i.e., the $\pm\hat{x}$, the $\pm\hat{y}$, and the $\pm\hat{z}$ directions. For the purpose of clarity, we will name these surface elements according to the directions of their unit normal vectors. For example, the top surface element of the body head shown in Figure 3.11 will be considered as one of the +x surface elements, since its unit normal vector is pointing in the +x direction. It should be emphasized that a closed surface can consist of several surface elements with unit normal vectors pointing in the same direction, as can be seen from Figure 3.11.

Before going any further, we should mention that we choose a plane wave as the incident wave, because it is a realistic approximation when the body is distant from a

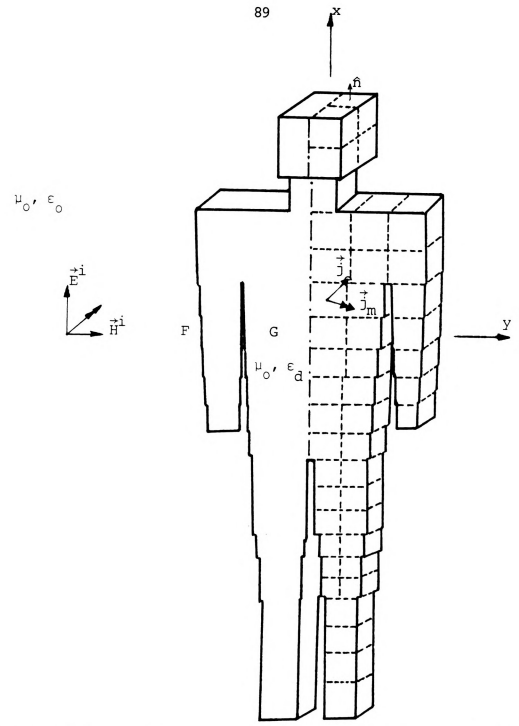

Figure 3.11. A simulated human body is irradiated by an incident plane wave propagating in the +z direction with a x-polarized electric field. The body is characterized by complex permittivity ε_d, and its surface is subdivided into a number of square surface cells. The induced equivalent surface currents ($\vec{J}_e$, $\vec{J}_m$) at different locations on the surface can be obtained by solving two coupled, surface integral equations.

radiating source, and it facilitates the analysis. Also, we choose the rectangular coordinate system since it is the most adaptable to arbitrarily shaped bodies. However, it is to be noted that the surface integral equations derived previously are so general that they can be applied to any form of incident field, as well as any orthogonal coordinate system.

Now, let us consider Figure 3.11 again, where each surface element of F has been subdivided into several square subareas named surface cells. Each surface cell can have any arbitrary dimension which depends on the accuracy desired. Furthermore, the center point of each surface cell is considered as a reference point which could be either a field point or a source point or both. The EM field induced inside each surface cell is assumed to be uniform. We will use the notation F_x to represent the combined surface of all the +x surface elements, and use the symbol N_x to denote the total number of surface cells included in F_x. Similar definitions will be applied to the notations, F_y, N_y; F_z, N_z; F_{-x}, N_{-x}; F_{-y}, N_{-y}; and F_{-z}, N_{-z}, etc. In other words, we may say that the body surface F has been divided into totally N_{cell} surface cells, where N_{cell} is the sum of N_x, N_y, N_z, N_{-x}, N_{-y}, and N_{-z}.

To solve the surface integral equations numerically by use of the moment-method, we must decompose the integral

equations first and then point-match them at the reference point of each surface cell. By doing this, a system of simultaneous, scalar, linear algebraic equations will be obtained. After transforming these linear equations into a matrix form, it can be solved by conventional matrix inversion techniques. To describe the procedures in more detail, let's start with the following definitions:

m: the numbering index of the surface cells in each subsurface (F_x, F_y, F_z, F_{-x}, F_{-y}, F_{-z}), when center points of the surface cells are considered as field points.

n: the numbering index of the surface cells in each subsurface, when center points of the surface cells are considered as source points.

A^n: the area of the nth surface cell.

$\vec{r}^m$: the position vector locating the mth field point, where $\vec{r}^m = (x^m, y^m, z^m)$.

$\vec{r}^n$: the position vector locating the nth source point, where $\vec{r}^n = (x^n, y^n, z^n)$.

Also,

$$r^{nm} \equiv |\vec{r}^n - \vec{r}^m|, \quad x^{nm} \equiv x^n - x^m, \quad y^{nm} \equiv y^n - y^m,$$

$$z^{nm} \equiv z^n - z^m.$$

and

$a^m = \sqrt{A^m/\pi}$: the equivalent radius of the mth surface cell.

We also like to introduce the following associated Green's functions; the reasons behind these definitions will be clear when the surface integral equations are decomposed:

$$G_1^{nm} = j\frac{e^{-jr^{nm}} - \kappa\, e^{-j\kappa r^{nm}}}{r^{nm^2}} + \frac{e^{-jr^{nm}} - e^{-j\kappa r^{nm}}}{r^{nm^3}}$$

$$G_2^{nm} = \frac{e^{-jr^{nm}} - \kappa^2 e^{-j\kappa r^{nm}}}{r^{nm^3}} - 3j\frac{e^{-jr^{nm}} - \kappa\, e^{-j\kappa r^{nm}}}{r^{nm^4}} - 3\frac{e^{-jr^{nm}} - e^{-j\kappa r^{nm}}}{r^{nm^5}}$$

$$G_3^{nm} = j\frac{e^{-jr^{nm}} - \kappa^3 e^{-j\kappa r^{nm}}}{r^{nm^2}} + \frac{e^{-jr^{nm}} - \kappa^2 e^{-j\kappa r^{nm}}}{r^{nm^3}}$$

$$G_4^{nm} = -\frac{e^{-jr^{nm}} - \kappa^2 e^{-j\kappa r^{nm}}}{r^{nm}} \qquad \Bigg\} \quad (3.43)$$

$$H_1^m = \pi\left(j\, e^{-ja^m} - j\kappa e^{-j\kappa a^m} + \frac{e^{-ja^m} - e^{-j\kappa a^m}}{a^m}\right)$$

and

$$H_2^m = 2\pi(j\kappa e^{-j\kappa a^m} - j e^{-ja^m} - j\kappa + j).$$

Now, we are in a position to decompose the coupled, surface integral equations, i.e., eqs. (3.29) and (3.30). For convenience, we repeat them here,

$$\vec{J}_m = \frac{2}{\kappa^2+1}\vec{J}_m^i - \frac{1}{2\pi(\kappa^2+1)}\hat{n} \times \int_F [j(\kappa^2\Phi_d - \Phi_f)\vec{J}_e + \vec{J}_m \times \nabla'(\kappa^2\Phi_d - \Phi_f) + j(\vec{J}_e \cdot \nabla')\nabla'(\Phi_d - \Phi_f)]\, dF' \qquad (3.29')$$

$$\vec{J}_e = \vec{J}_e^{\,i} + \frac{1}{4\pi}\,\hat{n} \times \int_F [j(\kappa^2 \Phi_d - \Phi_f)\,\vec{J}_m - \vec{J}_e \times \nabla'(\Phi_d - \Phi_f)$$

$$+ j(\vec{J}_m \cdot \nabla')\,\nabla'(\Phi_d - \Phi_f)]\,dF'. \qquad (3.30')$$

Note that, without any ambiguity, the superscript "d" for the induced equivalent surface current densities has been omitted in the above equations.

Let us consider eq. (3.29') first. To numerically implement the various surface integrals of this equation, we have to consider each possible case about the field point locations.

I) First of all, we consider the situation when the field point is located in one of the +x surface elements. Which means that the unit normal vector at field point, $\hat{n}$, is the same as the x-direction unit vector $\hat{x}$. For this case, we know that $1 \le m \le N_x$. Based on the previous definitions, it is easy to see that the last integral of eq. (3.29') can be written as

$$\hat{n} \times \int_F (\vec{J}_e \cdot \nabla')\,\nabla'(\Phi_d - \Phi_f)\,dF' = \hat{x} \times \{[\int_{F_x} + \int_{F_y} + \int_{F_z} +$$

$$\int_{F_{-x}} + \int_{F_{-y}} + \int_{F_{-z}}](\vec{J}_e \cdot \nabla')\,\nabla'(\Phi_d - \Phi_f)\,dF'\} \qquad (3.44)$$

where we have decomposed the total surface integral $\int_F$ into six different subintegrals, one for each subsurface. For example, $\int_{F_x}$ represents the integration of the integrand over all the surface elements having $\hat{x}$ as the unit normal

vector. The remaining five subintegrals in eq. (3.44) have similar meanings. Now, as an example, let's carry out the first subintegral of eq. (3.44).

$$\hat{x} \times \int_{F_x} (\vec{j}_e \cdot \nabla') \nabla' (\Phi_d - \Phi_f)\, dF' = \hat{x} \times \int_{F_x} [(j_{ey}\, \hat{y} + j_{ez}\, \hat{z}) \cdot (\hat{x} \frac{\partial}{\partial x'} + \hat{y} \frac{\partial}{\partial y'} + \hat{z} \frac{\partial}{\partial z'})] \; [(\hat{x} \frac{\partial}{\partial x'} + \hat{y} \frac{\partial}{\partial y'} + \hat{z} \frac{\partial}{\partial z'}) (\Phi_d - \Phi_f)]\, dF'$$

$$= \hat{x} \times \int_{F_x} [\hat{x}(j_{ey} \frac{\partial^2}{\partial x' \partial y'} + j_{ez} \frac{\partial^2}{\partial x' \partial z'}) + \hat{y}(j_{ey} \frac{\partial^2}{\partial y'^2} + j_{ez} \frac{\partial^2}{\partial y' \partial z'}) +$$

$$\hat{z}(j_{ey} \frac{\partial^2}{\partial y' \partial z'} + j_{ez} \frac{\partial^2}{\partial z'^2})] (\Phi_d - \Phi_f)\, dF'$$

$$= \hat{x} \times \int_{F_x} [\hat{y}(j_{ey} \frac{\partial^2}{\partial y'^2} + j_{ez} \frac{\partial^2}{\partial y' \partial z'}) + \hat{z}(j_{ey} \frac{\partial^2}{\partial y' \partial z'} + j_{ez} \frac{\partial^2}{\partial z'^2})]$$

$$(\Phi_d - \Phi_f)\, dF'$$

Since $\Phi_d = \frac{e^{-jkr}}{r}$ and $\Phi_f = \frac{e^{-jr}}{r}$ where $r = |\vec{r} - \vec{r}'|$, we may show that

$$\frac{\partial^2}{\partial y'^2}(\Phi_d - \Phi_f) = G_1 + (y' - y)^2 G_2$$

$$\frac{\partial^2}{\partial z'^2}(\Phi_d - \Phi_f) = G_1 + (z' - z)^2 G_2$$

and

$$\frac{\partial^2}{\partial y' \partial z'}(\Phi_d - \Phi_f) = (y' - y)(z' - z)\, G_2$$

where G_1 and G_2 have the same expressions as that of G_1^{nm} and G_2^{nm} defined in eq. (3.43) except r^{nm} being replaced by $\underline{r}$. Hence we have

$$\hat{x} \times \int_{F_x} (\vec{j}_e \cdot \nabla') \nabla' (\Phi_d - \Phi_f) \, dF' = -\hat{y} \int_{F_x} \{ j_{ey} (y' - y)(z' - z) \, G_2 +$$

$$j_{ez} [G_1 + (z' - z)^2 \, G_2] \} \, dF' + \hat{z} \int_{F_x} \{ j_{ey} [G_1 + (y' - y)^2 \, G_2] +$$

$$j_{ez} (y' - y)(z' - z) \, G_2 \} \, dF'.$$

The integrals on the right-hand side of the above equation can be numerically implemented by summing up the products of integrands and areas of the corresponding surface cells with a special consideration being paid to the singular terms which occur when the source point coincides with the field point. This process is completely based on the moment-method technique with two-dimensional pulse functions as the basis functions, and δ functions as the testing functions. So we may rewrite the above equation as

$$\hat{x} \times \int_{F_x} (\vec{j}_e \cdot \nabla') \nabla' (\Phi_d - \Phi_f) \, dF' = -\hat{y} \sum_{\substack{n=1 \\ n \neq m}}^{N_x} [j_{ey}^n A^n y^{nm} z^{nm} G_2^{nm} +$$

$$j_{ez}^n A^n (G_1^{nm} + z^{nm} \cdot z^{nm} \, G_2^{nm})] + \hat{z} \sum_{\substack{n=1 \\ n \neq m}}^{N_x} [j_{ey}^n A^n (G_1^{nm} + y^{nm} \cdot y^{nm} \, G_2^{nm}) +$$

$$j^n_{ez} A^n y^{nm} z^{nm} G_2^{nm}] - \hat{y}\int_{A^m}\{ j^m_{ey}(y' - y)(z' - z)G_2 + j^m_{ez}[G_1 +$$

$$(z' - z)^2 G_2]\} dF' + \hat{z}\int_{A^m}\{j^m_{ey}[G_1 + (y' - y)^2 G_2] +$$

$$j^m_{ez}(y' - y)(z' - z)G_2\} dF', \quad 1 \le m \le N_x \tag{3.45}$$

where j^n_{ey}, j^n_{ez}, j^m_{ey}, and j^m_{ez} are the unknown expansion coefficients of the equivalent surface current densities. Note that the above equation is satisfied at every field point location on the subsurface F_x. The notation $\sum_{\substack{n=1 \\ n\neq m}}^{N_x}$ implies a summation with respect to all the source points on F_x excluding the one coinciding with the field point where the singularities of Green's functions occur. The contributions of the singular terms are accounted for by the last two terms in eq. (3.45), where the notation $\int_{A^m}$ means the integration of integrand over a circle centered at the field point with area A^m. Now, let's investigate these singular terms more specifically. It can be shown that

$$\int_{A^m}(y' - y)(z' - z)G_2 \, dF' = \int_0^{a^m}\int_0^{2\pi} \underline{r}^2 \cos\theta \sin\theta \, G_2 \, d\theta \, \underline{r} \, d\underline{r}$$

$$= \underbrace{\int_0^{2\pi} \cos\theta \sin\theta \, d\theta}_{0} \int_0^{a^m} \underline{r}^3 G_2 \, d\underline{r} = 0, \tag{3.46}$$

also,

$$\int_{A^m}[G_1 + (y' - y)^2 G_2] \, dF' = \int_0^{2\pi}\int_0^{a^m} [G_1 + \underline{r}^2 \cos^2\theta \, G_2] \, \underline{r} \, d\underline{r} \, d\theta$$

and

$$\int_A^m [G_1 + (z' - z)^2 G_2]\, dF' = \int_o^{2\pi} \int_o^{a^m} [G_1 + \underline{r}^2 \sin^2\theta\, G_2]\, \underline{r}\, d\underline{r}\, d\theta.$$

After substituting the expressions of G_1 and G_2 into the above equations, we have

$$\int_A^m [G_1 + (y' - y)^2 G_2]\, dF' = \int_A^m [G_1 + (z' - z)^2 G_2]\, dF' = H_1^m, \tag{3.47}$$

where H_1^m is defined in eq. (3.43). Then, substituting eqs. (3.46) and (3.47) into eq. (3.45), finally we obtain

$$\hat{x} \times \int_{F_x} (\vec{J}_e \cdot \nabla')\, \nabla'(\phi_d - \phi_f)\, dF' = -\hat{y} \{ \sum_{\substack{n=1 \\ n\neq m}}^{N_x} A^n [j_{ey}^n\, y^{nm}\, z^{nm}\, G_2^{nm} +$$

$$j_{ez}^n (G_1^{nm} + z^{nm} \cdot z^{nm}\, G_2^{nm})] + j_{ez}^m\, H_1^m \} +$$

$$\hat{z} \{ \sum_{\substack{n=1 \\ n\neq m}}^{N_x} A^n [j_{ey}^n (G_1^{nm} + y^{nm} \cdot y^{nm}\, G_2^{nm}) +$$

$$j_{ez}^n\, y^{nm}\, z^{nm}\, G_2^{nm}] + j_{ey}^m\, H_1^m \}, \quad \text{where } 1 \leq m \leq N_x. \tag{3.48}$$

This concludes the evaluation of the first subintegral in eq. (3.44). The remaining five subintegrals in that equation can be carried out similarly. Combining these results with eq. (3.44), the latter will become

$$\hat{x} \times \int_F (\vec{J}_e \cdot \nabla')\, \nabla'(\phi_d - \phi_f)\, dF' = -\hat{y} \{ (\sum_{\substack{n=1 \\ n\neq m}}^{N_x} + \sum_{n=1}^{N-x}) A^n [j_{ey}^n\, y^{nm}\, z^{nm}\, G_2^{nm} +$$

$$j^n_{ez}(G^{nm}_1 + z^{nm}\cdot z^{nm} G^{nm}_2)] + (\sum_{n=1}^{N_y} + \sum_{n=1}^{N-y}) A^n[j^n_{ex} x^{nm} z^{nm} G^{nm}_2 +$$

$$j^n_{ey} y^{nm} z^{nm} G^{nm}_2] + j^m_{ez} H^m_1\} +$$

$$\hat{z}\ \{(\sum_{\substack{n=1\\ n\neq m}}^{N_x} + \sum_{n=1}^{N-x}) A^n[j^n_{ey}(G^{nm}_1 + y^{nm}\cdot y^{nm} G^{nm}_2) +$$

$$j^n_{ez} y^{nm} z^{nm} G^{nm}_2] + (\sum_{n=1}^{N_y} + \sum_{n=1}^{N-y}) A^n[j^n_{ex} x^{nm} y^{nm} G^{nm}_2 +$$

$$j^n_{ey} y^{nm} z^{nm} G^{nm}_2] + (\sum_{n=1}^{N_z} + \sum_{n=1}^{N-z}) A^n[j^n_{ex} x^{nm} y^{nm} G^{nm}_2 +$$

$$j^n_{ey}(G^{nm}_1 + y^{nm}\cdot y^{nm} G^{nm}_2)] + j^m_{ey} H^m_1\}, \text{ where } 1 \le m \le N_x. \tag{3.49}$$

This concludes the investigation of the last integral on the right-hand side of eq. (3.29').

If we repeat the whole process for the second surface integration of eq. (3.29'), we will find that it can be decomposed into the following form:

$$\hat{x}\ X \int_F \vec{j}_m\ X\ \nabla'(\kappa^2\ \phi_d - \phi_f)\ dF' = \hat{y}\ \{(\sum_{\substack{n=1\\ n\neq m}}^{N_x} + \sum_{n=1}^{N-x}) A^n\ x^{nm} G^{nm}_3\ j^n_{my} -$$

$$(\sum_{n=1}^{N_y} + \sum_{n=1}^{N-y}) A^n\ y^{nm} G^{nm}_3\ j^n_{mx} - (\sum_{n=1}^{N_z} + \sum_{n=1}^{N-z}) A^n[j^n_{mx}\ y^{nm} G^{nm}_3 -$$

$$J^n_{my}\, x^{nm}\, G_3^{nm}] \} + \hat{z}\, \{ (\sum_{\substack{n=1 \\ n\neq m}}^{N_x} + \sum_{n=1}^{N-x}) A^n\, x^{nm}\, G_3^{nm}\, J^n_{mz} +$$

$$(\sum_{n=1}^{N_y} + \sum_{n=1}^{N-y}) A^n [J^n_{mz}\, x^{nm}\, G_3^{nm} - J^n_{mx}\, z^{nm}\, G_3^{nm}] -$$

$$(\sum_{n=1}^{N_z} + \sum_{n=1}^{N-z}) A^n\, z^{nm}\, G_3^{nm}\, J^n_{mx} \}, \quad \text{where } 1 \leq m \leq N_x, \qquad (3.50)$$

and G_3^{nm} is defined in eq. (3.43). It is noted that the singular terms have no contribution to eq. (3.50).

Similarly, the first integral term on the right-hand side of eq. (3.29') can be transformed to become

$$\hat{x} \int_F \vec{J}_e (\kappa^2 \phi_d - \phi_f)\, dF' = -\hat{y}\, \{ (\sum_{\substack{n=1 \\ n\neq m}}^{N_x} + \sum_{n=1}^{N-x}) A^n\, G_4^{nm}\, J^n_{ez} +$$

$$(\sum_{n=1}^{N_y} + \sum_{n=1}^{N-y}) A^n\, G_4^{nm}\, J^n_{ez} + J^m_{ez}\, H_2^m \} + \hat{z}\, \{ (\sum_{\substack{n=1 \\ n\neq m}}^{N_x} + \sum_{n=1}^{N-x}) A^n\, G_4^{nm}\, J^n_{ey} +$$

$$(\sum_{n=1}^{N_z} + \sum_{n=1}^{N-z}) A^n\, G_4^{nm}\, J^n_{ey} + J^m_{ey}\, H_2^m \}, \quad \text{where } 1 \leq m \leq N_x, \qquad (3.51)$$

and G_4^{nm}, H_2^m are defined in eq. (3.43).

If we enforce satisfaction of eq. (3.29') at each reference point on the subsurface F_x, with the help of eqs. (3.49), (3.50), and (3.51), we can obtain totally N_x vector equations, i.e., one equation for each matching point. Furthermore, it is possible to decompose these N_x vector equations into $2N_x$ scalar, linear algebraic equations since

each surface current possesses two tangential components. Based on eqs. (3.29'), (3.49), (3.50), and (3.51), it can be shown that the following are those $2N_x$ linear equations:

$$\{ \Big(\sum_{\substack{n=1 \\ n\neq m}}^{N_x} + \sum_{n=1}^{N_x} \Big) \cdot [-j_{ey}^{n} \Psi_{1x}(n,m) - j_{ez}^{n} \Psi_{2z}(n,m) + j_{my}^{n} \Psi_{3x}(n,m)] -$$

$$j_{ez}^{m} H(m) + j_{my}^{m} C \} - \{ \Big(\sum_{n=1}^{N_y} + \sum_{n=1}^{N_y} \Big) \cdot [j_{ex}^{n} \Psi_{1y}(n,m) + j_{ez}^{n} \Psi_{2z}(n,m) +$$

$$j_{mx}^{n} \Psi_{3y}(n,m)] \} - \{ \Big(\sum_{n=1}^{N_z} + \sum_{n=1}^{N_z} \Big) \cdot [j_{ex}^{n} \Psi_{1y}(n,m) + j_{ey}^{n} \Psi_{1x}(n,m) +$$

$$j_{mx}^{n} \Psi_{3y}(n,m) - j_{my}^{n} \Psi_{3x}(n,m)] \} = 4\pi \, j_{my}^{m\,i} \qquad (3.52)$$

also,

$$\{ \Big(\sum_{\substack{n=1 \\ n\neq m}}^{N_x} + \sum_{n=1}^{N_x} \Big) \cdot [j_{ey}^{n} \Psi_{2y}(n,m) + j_{ez}^{n} \Psi_{1x}(n,m) + j_{mz}^{n} \Psi_{3x}(n,m)] +$$

$$j_{ey}^{m} H(m) + j_{mz}^{m} C \} + \{ \Big(\sum_{n=1}^{N_y} + \sum_{n=1}^{N_y} \Big) \cdot [j_{ex}^{n} \Psi_{1z}(n,m) + j_{ez}^{n} \Psi_{1x}(n,m) -$$

$$j_{mx}^{n} \Psi_{3z}(n,m) + j_{mz}^{n} \Psi_{3x}(n,m)] \} + \{ \Big(\sum_{n=1}^{N_z} + \sum_{n=1}^{N_z} \Big) \cdot [j_{ex}^{n} \Psi_{1z}(n,m) +$$

$$j_{ey}^{n} \Psi_{2y}(n,m) - j_{mx}^{n} \Psi_{3z}(n,m)] \} = 4\pi \, j_{mz}^{m\,i} \qquad (3.53)$$

where $1 \leq m \leq N_x$, and the various Ψ functions are defined as

$$\Psi_{1x}(n,m) = j\, A^n\, y^{nm}\, z^{nm}\, G_2^{nm}$$

$$\Psi_{1y}(n,m) = j\, A^n\, x^{nm}\, z^{nm}\, G_2^{nm}$$

$$\Psi_{1z}(n,m) = j\, A^n\, x^{nm}\, y^{nm}\, G_2^{nm}$$

$$\Psi_{2x}(n,m) = j\, A^n (G_1^{nm} + x^{nm} \cdot x^{nm}\, G_2^{nm} + G_4^{nm})$$

$$\Psi_{2y}(n,m) = j\, A^n (G_1^{nm} + y^{nm} \cdot y^{nm}\, G_2^{nm} + G_4^{nm})$$

$$\Psi_{2z}(n,m) = j\, A^n (G_1^{nm} + z^{nm} \cdot z^{nm}\, G_2^{nm} + G_4^{nm})$$

$$\Psi_{3x}(n,m) = A^n\, x^{nm}\, G_3^{nm}$$

$$\Psi_{3y}(n,m) = A^n\, y^{nm}\, G_3^{nm}$$

$$\Psi_{3z}(n,m) = A^n\, z^{nm}\, G_3^{nm}$$

$$H(m) = j(H_1^m + H_2^m)$$

$$C = 2\pi(\kappa^2 + 1).$$

This completes the whole analysis for the case of $\hat{n} = \hat{x}$.

II) If the field point is located in one of the +y-surface elements, i.e., when $\hat{n} = \hat{y}$, other $2N_y$ linear equations can be obtained by following a similar procedure as that described in case I). We will present these equations without repeating the details of the derivations:

$$\{\left(\sum_{n=1}^{N_x} + \sum_{n=1}^{N_{-x}}\right)\cdot[j_{ey}^{n}\,\Psi_{1x}(n,m) + j_{ez}^{n}\,\Psi_{2z}(n,m) - j_{my}^{n}\,\Psi_{3x}(n,m)]\} +$$

$$\{\left(\sum_{\substack{n=1\\ n\neq m}}^{N_y} + \sum_{n=1}^{N_{-y}}\right)\cdot[j_{ex}^{n}\,\Psi_{1y}(n,m) + j_{ez}^{n}\,\Psi_{2z}(n,m) + j_{mx}^{n}\,\Psi_{3y}(n,m)] +$$

$$j_{ez}^{m}\,H(m) + j_{mx}^{m}\,C\} + \{\left(\sum_{n=1}^{N_z} + \sum_{n=1}^{N_{-z}}\right)\cdot[j_{ex}^{n}\,\Psi_{1y}(n,m) + j_{ey}^{n}\,\Psi_{1x}(n,m) +$$

$$j_{mx}^{n}\,\Psi_{3y}(n,m) - j_{my}^{n}\,\Psi_{3x}(n,m)]\} = 4\pi\, j_{mx}^{m\,i} \qquad (3.54)$$

also,

$$\{\left(\sum_{n=1}^{N_x} + \sum_{n=1}^{N_{-x}}\right)\cdot[-j_{ey}^{n}\,\Psi_{1z}(n,m) - j_{ez}^{n}\,\Psi_{1y}(n,m) - j_{my}^{n}\,\Psi_{3z}(n,m) +$$

$$j_{mz}^{n}\,\Psi_{3y}(n,m)]\} - \{\left(\sum_{\substack{n=1\\ n\neq m}}^{N_y} + \sum_{n=1}^{N_{-y}}\right)\cdot[j_{ex}^{n}\,\Psi_{2x}(n,m) + j_{ez}^{n}\,\Psi_{1y}(n,m) -$$

$$j_{mz}^{n}\,\Psi_{3y}(n,m)] - j_{ex}^{m}\,H(m) + j_{mz}^{m}\,C\} - \{\left(\sum_{n=1}^{N_z} + \sum_{n=1}^{N_{-z}}\right)\cdot[j_{ex}^{n}\,\Psi_{2x}(n,m) +$$

$$j_{ey}^{n}\,\Psi_{1z}(n,m) + j_{my}^{n}\,\Psi_{3z}(n,m)]\} = 4\pi\, j_{mz}^{m\,i} \qquad (3.55)$$

where $1 \leq m \leq N_y$.

III) Next, let's consider the case where the field point is located in one of the +z-surface elements. As in the previous two cases, by use of the point-matching technique, we may obtain the following $2N_z$ scalar, linear equations:

$$\{\left(\sum_{n=1}^{N_x} + \sum_{n=1}^{N_{-x}}\right)\cdot[-j_{ey}^{n}\,\Psi_{2y}(n,m) - j_{ez}^{n}\,\Psi_{1x}(n,m) - j_{mz}^{n}\,\Psi_{3x}(n,m)]\} -$$

$$\{\left(\sum_{n=1}^{N_y} + \sum_{n=1}^{N_{-y}}\right)\cdot[j_{ex}^{n}\,\Psi_{1z}(n,m) + j_{ez}^{n}\,\Psi_{1x}(n,m) - j_{mx}^{n}\,\Psi_{3z}(n,m) +$$

$$j_{mz}^{n}\,\Psi_{3x}(n,m)]\} - \{\left(\sum_{\substack{n=1\\ n\neq m}}^{N_z} + \sum_{n=1}^{N_{-z}}\right)\cdot[j_{ex}^{n}\,\Psi_{1z}(n,m) + j_{ey}^{n}\,\Psi_{2y}(n,m) -$$

$$j_{mx}^{n}\,\Psi_{3z}(n,m)] - j_{ey}^{m}\,H(m) + j_{mx}^{m}\,C\} = 4\pi\, j_{mx}^{m\,i} \tag{3.56}$$

also,

$$\{\left(\sum_{n=1}^{N_x} + \sum_{n=1}^{N_{-x}}\right)\cdot[j_{ey}^{n}\,\Psi_{1z}(n,m) + j_{ez}^{n}\,\Psi_{1y}(n,m) + j_{my}^{n}\,\Psi_{3z}(n,m) -$$

$$j_{mz}^{n}\,\Psi_{3y}(n,m)]\} + \{\left(\sum_{n=1}^{N_y} + \sum_{n=1}^{N_{-y}}\right)\cdot[j_{ex}^{n}\,\Psi_{2x}(n,m) + j_{ez}^{n}\,\Psi_{1y}(n,m) -$$

$$j_{mz}^{n}\,\Psi_{3y}(n,m)]\} + \{\left(\sum_{\substack{n=1\\ n\neq m}}^{N_z} + \sum_{n=1}^{N_{-z}}\right)\cdot[j_{ex}^{n}\,\Psi_{2x}(n,m) + j_{ey}^{n}\,\Psi_{1z}(n,m) +$$

$$j_{my}^{n}\,\Psi_{3z}(n,m)] + j_{ex}^{m}\,H(m) + j_{my}^{m}\,C\} = 4\pi\, j_{my}^{m\,i} \tag{3.57}$$

where $1 \leq m \leq N_z$.

Similarly, we can obtain $2N_{-x}$, $2N_{-y}$, and $2N_{-z}$ linear equations by enforcing eq. (3.29') at various reference points on the subsurfaces F_{-x}, F_{-y}, and F_{-z}, respectively. We list all these equations as follows:

IV) $\hat{n} = -\hat{x}$:

$$\{\Big(\sum_{n=1}^{N_x} + \sum_{\substack{n=1\\ n\neq m}}^{N_{-x}}\Big)\cdot[-j_{ey}^{n}\,\Psi_{1x}(n,m) - j_{ez}^{n}\,\Psi_{2z}(n,m) + j_{my}^{n}\,\Psi_{3x}(n,m)] - j_{ez}^{m}\,H(m) -$$

$$j_{my}^{m}\,C\Big\} - \{\Big(\sum_{n=1}^{N_y} + \sum_{n=1}^{N_{-y}}\Big)\cdot[j_{ex}^{n}\,\Psi_{1y}(n,m) + j_{ez}^{n}\,\Psi_{2z}(n,m) +$$

$$j_{mx}^{n}\,\Psi_{3y}(n,m)] \} - \{\Big(\sum_{n=1}^{N_z} + \sum_{n=1}^{N_{-z}}\Big)\cdot[j_{ex}^{n}\,\Psi_{1y}(n,m) + j_{ey}^{n}\,\Psi_{1x}(n,m) +$$

$$j_{mx}^{n}\,\Psi_{3y}(n,m) - j_{my}^{n}\,\Psi_{3x}(n,m)]\} = -4\pi\, j_{my}^{m\,i} \qquad (3.58)$$

also,

$$\{\Big(\sum_{n=1}^{N_x} + \sum_{\substack{n=1\\ n\neq m}}^{N_{-x}}\Big)\cdot[j_{ey}^{n}\,\Psi_{2y}(n,m) + j_{ez}^{n}\,\Psi_{1x}(n,m) + j_{mz}^{n}\,\Psi_{3x}(n,m)] + j_{ey}^{m}\,H(m) -$$

$$j_{mz}^{m}\,C\} + \{\Big(\sum_{n=1}^{N_y} + \sum_{n=1}^{N_{-y}}\Big)\cdot[j_{ex}^{n}\,\Psi_{1z}(n,m) + j_{ez}^{n}\,\Psi_{1x}(n,m) -$$

$$j_{mx}^{n}\,\Psi_{3z}(n,m) + j_{mz}^{n}\,\Psi_{3x}(n,m)]\} + \{\Big(\sum_{n=1}^{N_z} + \sum_{n=1}^{N_{-z}}\Big)\cdot[j_{ex}^{n}\,\Psi_{1z}(n,m) +$$

$$j_{ey}^{n}\,\Psi_{2y}(n,m) - j_{mx}^{n}\,\Psi_{3z}(n,m)]\} = -4\pi\, j_{mz}^{m\,i} \qquad (3.59)$$

where $1 \le m \le N_{-x}$.

V) $\hat{n} = -\hat{y}$:

$$\{\Big(\sum_{n=1}^{N_x} + \sum_{n=1}^{N_{-x}}\Big)\cdot[j_{ey}^{n}\,\Psi_{1x}(n,m) + j_{ez}^{n}\,\Psi_{2z}(n,m) - j_{my}^{n}\,\Psi_{3x}(n,m)]\} +$$

$$\{\left(\sum_{n=1}^{N_y} + \sum_{\substack{n=1\\ n\neq m}}^{N_{-y}}\right)\cdot[j_{ex}^{n}\,\Psi_{1y}(n,m) + j_{ez}^{n}\,\Psi_{2z}(n,m) + j_{mx}^{n}\,\Psi_{3y}(n,m)] +$$

$$j_{ez}^{m}\,H(m) - j_{mx}^{m}\,C\} + \{\left(\sum_{n=1}^{N_z} + \sum_{n=1}^{N_{-z}}\right)\cdot[j_{ex}^{n}\,\Psi_{1y}(n,m) + j_{ey}^{n}\,\Psi_{1x}(n,m) +$$

$$j_{mx}^{n}\,\Psi_{3y}(n,m) - j_{my}^{n}\,\Psi_{3x}(n,m)]\} = -4\pi\, j_{mx}^{m\,i} \qquad (3.60)$$

also,

$$\{\left(\sum_{n=1}^{N_x} + \sum_{n=1}^{N_{-x}}\right)\cdot[-j_{ey}^{n}\,\Psi_{1z}(n,m) - j_{ez}^{n}\,\Psi_{1y}(n,m) - j_{my}^{n}\,\Psi_{3z}(n,m) +$$

$$j_{mz}^{n}\,\Psi_{3y}(n,m)]\} - \{\left(\sum_{n=1}^{N_y} + \sum_{\substack{n=1\\ n\neq m}}^{N_{-y}}\right)\cdot[j_{ex}^{n}\,\Psi_{2x}(n,m) + j_{ez}^{n}\,\Psi_{1y}(n,m) -$$

$$j_{mz}^{n}\,\Psi_{3y}(n,m)] - j_{ex}^{m}\,H(m) - j_{mz}^{m}\,C\} - \{\left(\sum_{n=1}^{N_z} + \sum_{n=1}^{N_{-z}}\right)\cdot[j_{ex}^{n}\,\Psi_{2x}(n,m) +$$

$$j_{ey}^{n}\,\Psi_{1z}(n,m) + j_{my}^{n}\,\Psi_{3z}(n,m)]\} = -4\pi\, j_{mz}^{m\,i} \qquad (3.61)$$

where $1 \leq m \leq N_{-y}$.

VI) $\hat{n} = -\hat{z}$:

$$\{\left(\sum_{n=1}^{N_x} + \sum_{n=1}^{N_{-x}}\right)\cdot[-j_{ey}^{n}\,\Psi_{2y}(n,m) - j_{ez}^{n}\,\Psi_{1x}(n,m) - j_{mz}^{n}\,\Psi_{3x}(n,m)]\} -$$

$$\{\left(\sum_{n=1}^{N_y} + \sum_{n=1}^{N_{-y}}\right)\cdot[j_{ex}^{n}\,\Psi_{1z}(n,m) + j_{ez}^{n}\,\Psi_{1x}(n,m) - j_{mx}^{n}\,\Psi_{3z}(n,m) +$$

$$j_{mz}^{n}\,\Psi_{3x}(n,m)]\} - \{\left(\sum_{n=1}^{N_z} + \sum_{\substack{n=1\\ n\neq m}}^{N_{-z}}\right)\cdot[j_{ex}^{n}\,\Psi_{1z}(n,m) + j_{ey}^{n}\,\Psi_{2y}(n,m) -$$

$$j^n_{mx}\,\Psi_{3z}(n,m)\,] - j^m_{ey}\,H(m) - j^m_{mx}\,C\,\} = -4\pi\, j^{m\,i}_{mx} \qquad (3.62)$$

also,

$$\{\,(\sum_{n=1}^{N_x} + \sum_{n=1}^{N_{-x}})\cdot[\,j^n_{ey}\,\Psi_{1z}(n,m) + j^n_{ez}\,\Psi_{1y}(n,m) + j^n_{my}\,\Psi_{3z}(n,m) -$$

$$j^n_{mz}\,\Psi_{3y}(n,m)\,]\,\} + \{\,(\sum_{n=1}^{N_y} + \sum_{n=1}^{N_{-y}})\cdot[\,j^n_{ex}\,\Psi_{2x}(n,m) + j^n_{ez}\,\Psi_{1y}(n,m) -$$

$$j^n_{mz}\,\Psi_{3y}(n,m)\,]\,\} + \{\,(\sum_{\substack{n=1\\ n\neq m}}^{N_z} + \sum_{n=1}^{N_{-z}})\cdot[\,j^n_{ex}\,\Psi_{2x}(n,m) + j^n_{ey}\,\Psi_{1z}(n,m) +$$

$$j^n_{my}\,\Psi_{3z}(n,m)\,] + j^m_{ex}\,H(m) - j^m_{my}\,C\,\} = -4\pi\, j^{m\,i}_{my} \qquad (3.63)$$

where $1 \le m \le N_{-z}$.

Up to this point, we have obtained totally $2N_{cell}$ ($N_{cell} = N_x + N_y + N_z + N_{-x} + N_{-y} + N_{-z}$) scalar, linear algebraic equations based on eq. (3.29'). Repeating the same procedure for eq. (3.30'), we can obtain another $2N_{cell}$ linear equations. We therefore have totally $4N_{cell}$ equations, which are adequate to solve for the surface unknowns. For the purpose of saving space, we will not list the other $2N_{cell}$ equations; however, since eqs. (3.29') and (3.30') have very similar forms, they can be carefully deduced from eqs. (3.52)-(3.63). The combination of these $4N_{cell}$ linear equations can be transformed into a matrix form as shown in eq. (3.64), which then can be solved by a conventional

$$
\begin{array}{c}
\begin{matrix} & 4N_x & 4N_y & 4N_z & 4N_{-x} & 4N_{-y} & 4N_{-z} \end{matrix} \\
\begin{matrix} 4N_x \\ 4N_y \\ 4N_z \\ 4N_{-x} \\ 4N_{-y} \\ 4N_{-z} \end{matrix}
\left\{ \begin{array}{cccccc}
[M_{x,x}] & [M_{x,y}] & [M_{x,z}] & [M_{x,-x}] & [M_{x,-y}] & [M_{x,-z}] \\
4N_x \times 4N_x & 4N_x \times 4N_y & 4N_x \times 4N_z & & & \\
[M_{y,x}] & [M_{y,y}] & [M_{y,z}] & [M_{y,-x}] & [M_{y,-y}] & [M_{y,-z}] \\
4N_y \times 4N_x & 4N_y \times 4N_y & 4N_y \times 4N_z & & & \\
[M_{z,x}] & [M_{z,y}] & [M_{z,z}] & [M_{z,-x}] & [M_{z,-y}] & [M_{z,-z}] \\
4N_z \times 4N_x & 4N_z \times 4N_y & 4N_z \times 4N_z & & & \\
[M_{-x,x}] & [M_{-x,y}] & [M_{-x,z}] & [M_{-x,-x}] & [M_{-x,-y}] & [M_{-x,-z}] \\
[M_{-y,x}] & [M_{-y,y}] & [M_{-y,z}] & [M_{-y,-x}] & [M_{-y,-y}] & [M_{-y,-z}] \\
[M_{-z,x}] & [M_{-z,y}] & [M_{-z,z}] & [M_{-z,-x}] & [M_{-z,-y}] & [M_{-z,-z}]
\end{array} \right\}
\left\{ \begin{array}{c}
j^n_{cy} \\ j^n_{cz} \\ j^n_{my} \\ j^n_{mz} \\ \hline
j^n_{cx} \\ j^n_{cz} \\ j^n_{mx} \\ j^n_{mz} \\ \hline
j^n_{cx} \\ j^n_{cy} \\ j^n_{mx} \\ j^n_{my} \\ \hline
-j^n_{cy} \\ j^n_{cz} \\ j^n_{my} \\ j^n_{mz} \\ \hline
j^n_{cx} \\ j^n_{cz} \\ j^n_{mx} \\ j^n_{mz} \\ \hline
-j^n_{cx} \\ j^n_{cy} \\ j^n_{mx} \\ j^n_{my}
\end{array} \right\}
= 4\pi
\left\{ \begin{array}{c}
j^{m\,i}_{cy} \\ j^{m\,i}_{cz} \\ j^{m\,i}_{my} \\ j^{m\,i}_{mz} \\ \hline
j^{m\,i}_{cx} \\ j^{m\,i}_{cz} \\ j^{m\,i}_{mx} \\ j^{m\,i}_{mz} \\ \hline
j^{m\,i}_{cx} \\ j^{m\,i}_{cy} \\ j^{m\,i}_{mx} \\ j^{m\,i}_{my} \\ \hline
-j^{m\,i}_{cy} \\ -j^{m\,i}_{cz} \\ -j^{m\,i}_{my} \\ -j^{m\,i}_{mz} \\ \hline
-j^{m\,i}_{cx} \\ -j^{m\,i}_{cz} \\ -j^{m\,i}_{mx} \\ -j^{m\,i}_{mz} \\ \hline
-j^{m\,i}_{cx} \\ -j^{m\,i}_{cy} \\ -j^{m\,i}_{mx} \\ -j^{m\,i}_{my}
\end{array} \right\}
\begin{matrix} \\ 4N_x \\ 4N_y \\ 4N_z \\ 4N_{-x} \\ 4N_{-y} \\ 4N_{-z} \end{matrix}
\end{array}
\tag{3.64}
$$

matrix inversion technique. The matrix elements in eq. (3.64) may be obtained from the above linear equations.

It should be mentioned that the matrix [M] shown in eq. (3.64) possesses a four-fold symmetry, which is easy to see from the coefficients of the linear equations. These symmetry properties of the equations have been found very useful for the computer implementation of the numerical method. The initial version of the computer program is implemented directly based on eq. (3.64). Unfortunately, this program has only limited usefulness due to the prohibitive requirement of the matrix size needed to adequately sample current variations. However, if the incident plane wave is decomposed into four basic modes (cosine and sine variations of E and H fields), an eight-fold symmetrical property can be found for each type of surface current. These symmetrical properties are shown in Figure 3.12. A final version of the computer program, with the above symmetry conditions imposed, is then developed. By use of this computer program, it is possible to reduce the matrix size by a factor of 8 when an eight-fold symmetrical body is considered since it is sufficient to solve for the surface currents induced in only 1/8 of the biological body. The induced surface currents for the rest of the body can be readily obtained by intuition. Of course, since the original incident plane wave has been

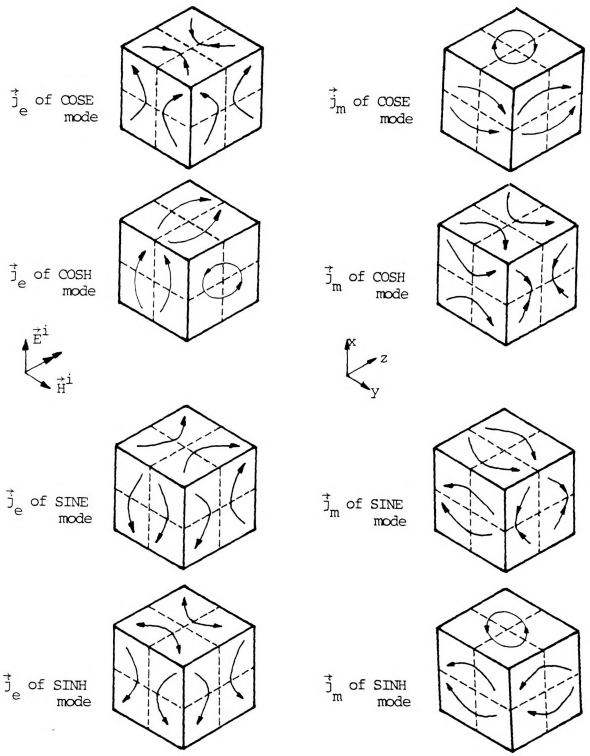

Figure 3.12. The symmetrical properties of various modes of the induced equivalent surface currents ($\vec{J}_e$, $\vec{J}_m$) on the surface of a cubic body.

separated into four different modes, the final solutions must be the combinations of results obtained for each basic mode. More details about the computer programs will be discussed in Chapter IV.

3.8. Numerical Results

The numerical technique developed in the previous section has been applied to solve for the EM fields induced on the surfaces of several finite conducting bodies. These results of induced surface fields obtained by the surface integral equation method (SIEM), accompanied with the results of internal fields induced in the same bodies obtained by the volume integral equation method (VIEM) [18], will be presented in this section. It should be mentioned that although SIEM gives solutions for both the induced electric field and induced magnetic field, for the purpose of simplifying the presentation we will only emphasize the induced electric fields, especially the component which predominates.

Figure 3.13 shows the vertical component of the induced electric field, E_x, in a muscle layer of 6cm x 6cm x 1 cm irradiated by an EM wave of 100 MH_z with a vertical incident electric field of 1 V/m at side-on incidence. The conductivity and dielectric constant of the body are assumed to be 0.889 S/m and 71.7. The top figure of Figure 3.13 shows the results for E_x (magnitude

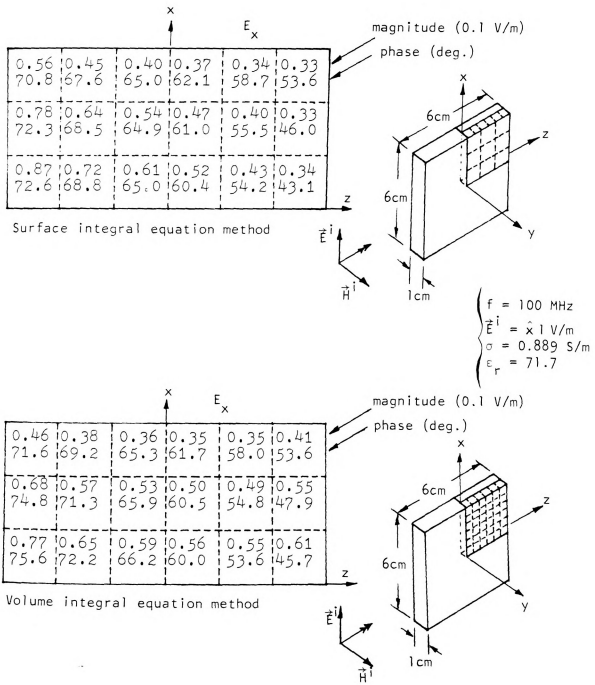

Figure 3.13. The x-components of the induced electric fields determined based on the surface integral equation method and the volume integral equation method.

and phase angle) on the surface of the body obtained with the surface integral equation method. In this numerical calculation, 1/8 of the body surface is divided into 21 subareas of two different sizes. The bottom figure of Figure 3.13 shows the results for E_x (magnitude and phase angle) at the centers of the first layer cells, or at y = 0.25 cm plane, obtained with the tensor integral equation method. In this numerical calculation, 1/8 of the body is divided into 36 volume cells. Comparison of these two sets of results, obtained with a surface integral equation method and a volume integral equation method, shows a qualitatively good agreement. This comparison is possible because the body is electrically thin in the y-direction, and the induced EM field should remain quite uniform in that direction. The disagreement between the two sets of results occurs mainly over the vertical edges of the body where the circulatory magnetic mode of the induced electric field has a significant contribution; the volume integral equation method usually gives poor results for that circulatory magnetic mode of the induced electric field.

Figure 3.14 shows the horizontal component of the induced electric field, E_z, in the same body under the same irradiation as the case of Figure 3.13. E_z near the central portion of the body, or the region near the x-axis, can be considered as mainly consisting of the

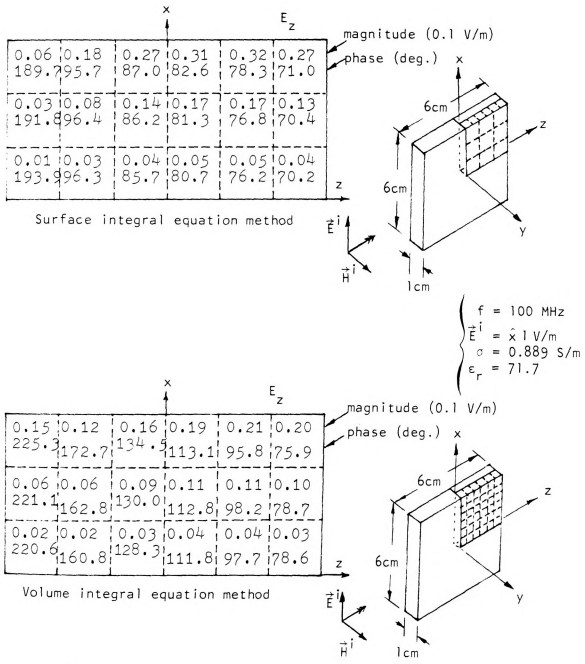

Figure 3.14. The z-components of the induced electric fields determined based on the surface integral equation method and the volume integral equation method.

circulatory magnetic mode of the induced electric field. For this component of induced electric field, only a fair agreement was obtained between the two sets of results obtained with the surface integral equation method and the volume integral equation method. The main reason for discrepancy is the inefficiency of a volume integral equation method in quantifying the circulatory magnetic mode of the induced electric field. The results obtained with the surface integral equation method appear to be more accurate for this case, because this set of results agrees with the results calculated by a method of quantifying the eddy current induced by a RF magnetic field as discussed in Chapter II.

As the second numerical example, we consider a rectangular body with a height of 180 cm, a width of 30 cm, and a thickness of 7.5 cm illuminated by a plane wave as shown in Figure 3.15. The incident wave is assumed to be x-polarized with a frequency of 10 MH_z and an electric field intensity of 1 V/m, and the body is assumed to have a conductivity (σ) of 0.625 S/m and a dielectric constant (ε_r) of 160. For the purpose of comparison, we plot the amplitude and phase distributions of the x-component of induced electric field in the top and bottom figures of Figure 3.15, where the results of both SIEM and VIEM are included. In the calculation by SIEM, 1/8 of the body surface is divided into 52 surface cells of two different

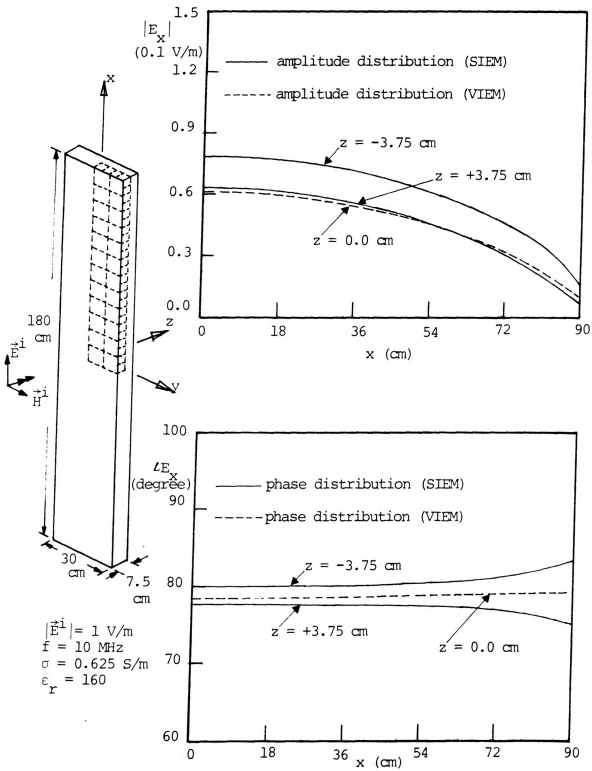

Figure 3.15. Amplitude and phase distributions of the x-component of induced electric field along the x direction in various layers at y = 3.75 cm.

sizes. The EM fields induced at center points of the surface cells are numerically evaluated. In the top figure of Figure 3.15, we plot, in solid lines, the amplitude distributions of the x-component of electric field, E_x, induced at y = 3.75 cm on the front surface and the back surface as functions of the x-coordinate. The corresponding phase distributions are plotted in the bottom figure of Figure 3.15. For the numerical calculation by VIEM, 1/4 of the body is divided into 24 volume cells. The internal electric fields induced at center points of the volume cells are then numerically calculated. In Figure 3.15, we plot, in dashed lines, the amplitude and phase distributions of the x-component of internal electric field induced at y = 3.75 cm and z = 0.0 cm as functions of the x-coordinate. The comparison between these two sets of results, the results from SIEM and that from VIEM, shows a reasonably good agreement. However, we observe that the amplitude of the surface field induced on the front surface is almost 25% larger than that induced on the back surface, while the latter is very close to the results of VIEM as shown in Figure 3.15. Based on this observation, we would tend to conclude that the SIEM doesn't predict accurate results for the field induced on the front surface since we expect that the induced field is quite uniform along the z-direction. But, if we consider an extreme case with the assumption that the body

is extended to infinity in both transverse directions, we find that the surface field induced on the front surface of this infinite plane slab is about 40% larger than that induced on the back surface (based on solution of plane-wave reflection and transmission in multiple-layer region). This special case gives some qualitative explanations regarding the difference between the surface fields of the front surface and the back surface. Finally, we should emphasize that the comparison of the phase distributions obtained by SIEM and VIEM shows a very good agreement, especially in the central portion of the body.

Figure 3.16 shows the amplitude and phase distributions of the x-component of induced electric field along the x-direction in various layers at y = 11.25 cm for the same body illuminated by the same incident wave. The results shown in Figure 3.16 are very similar to those of Figure 3.15. This is understandable since for this case, the x-component of the induced electric field should be rather insensitive to the location along the y-direction. In Figure 3.16, the results obtained by SIEM are plotted in solid lines, while those obtained by VIEM are plotted in dashed lines. As we can see from the top figure of Figure 3.16, the amplitude distributions of the x-component of induced electric field decrease monotonically along the x-direction. On the other hand, we observe from the bottom figure of Figure 3.16 that the phase distributions are

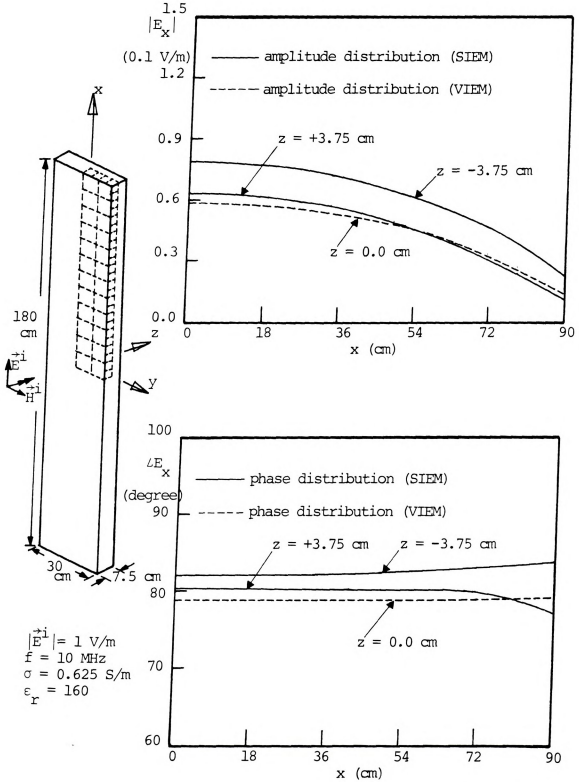

Figure 3.16. Amplitude and phase distributions of the x-component of induced electric field along the x direction in various layers at y = 11.25 cm.

nearly constant along the x-direction. Again, as in the case of Figure 3.15, the comparison between SIEM and VIEM shows reasonably good agreement, especially for the phase distributions.

The amplitude and phase distributions of the y-component of induced electric field along the x-direction in various layers at y = 11.25 cm and y = 3.75 cm are plotted in Figure 3.17. It is shown in Figure 3.17 that the field distributions at y = 11.25 cm (top figure) are very similar to those at y = 3.75 cm (bottom figure), except that the amplitude distributions of the former are three times as large as that of the latter. This is reasonable since the y-component of induced electric field must depend strongly on the locations along the y-direction. Figure 3.17 shows that the amplitude distributions of the y-component of induced electric field increase monotonically along the x-direction, while the phase distributions are nearly constant along the x-direction. It is to be noted that the y-component of induced electric field on the front surface and that on the back surface are exactly the same. This is quite different from what was observed in the previous two figures. Finally, it is clear from Figure 3.17 that the comparison between SIEM and VIEM demonstrates very good agreement for this case.

As the last example, we consider a more complicated geometry as shown in Figures 3.18 and 3.19. The body

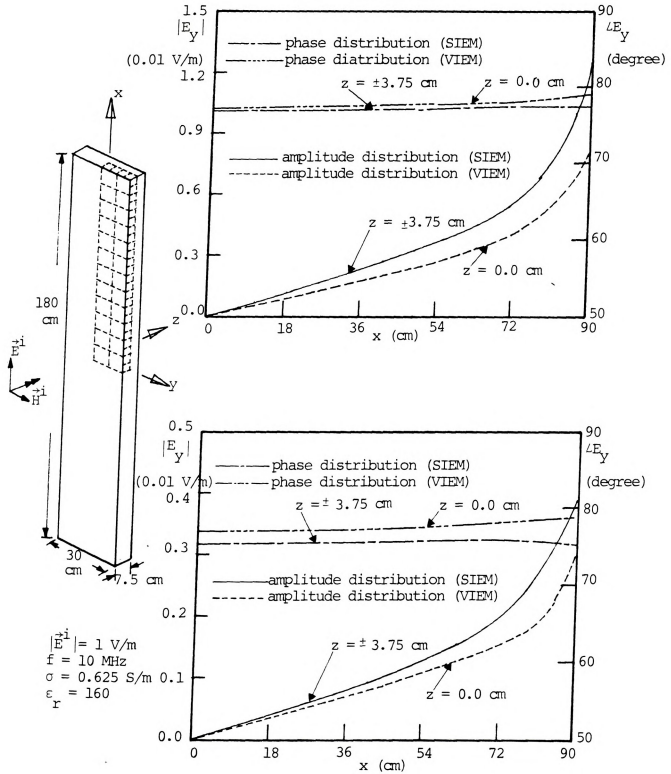

Figure 3.17. Amplitude and phase distributions of the y-component of induced electric field along the x direction in various layers at y = 11.25 cm (top figure) and y = 3.75 cm (bottom figure).

considered has a shape similar to that of a human body with the following dimensions: 70 cm in height, 20 cm in maximum width, and 10 cm in thickness. This situation can be considered as a simulation to the case of a two-year-old child innocently exposed to a high-intensity EM wave. For the purpose of reducing the number of unknowns to keep the computational cost low, we still assume that the frequency is as low as 10 MH_z. The electrical properties of the body are assumed to be $\sigma = 0.625$ S/m and $\varepsilon_r = 160$. In the calculation by SIEM, 1/4 of the body surface is divided into 29 surface cells of two different sizes. The amplitude distribution of the x-component of induced electric field is shown in Figure 3.18 for both the front surface and the back surface of the body. It is observed from Figure 3.18 that the electric field induced on the front surface is larger than that induced on the back surface. In Figure 3.19, the phase distribution of the x-component of induced electric field is shown. Unlike the amplitude distribution, the phase distribution is quite uniform over the entire body surface.

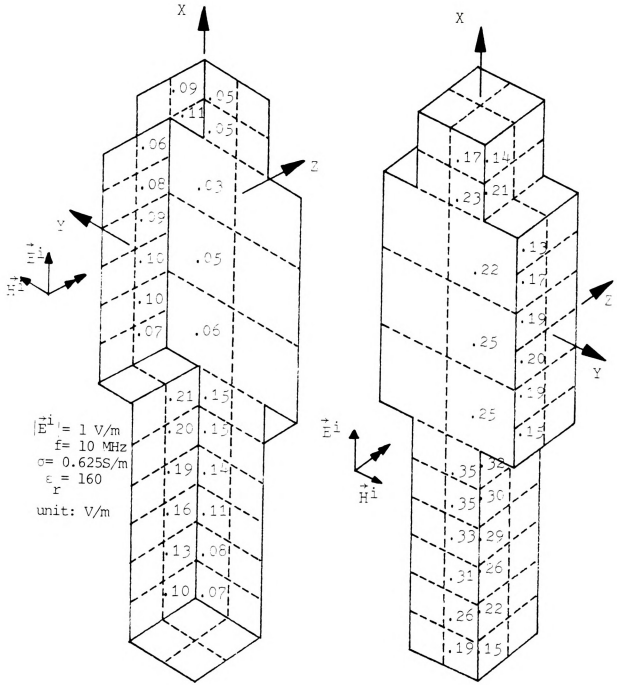

Figure 3.18. Amplitude distribution of the x-component of induced electric field on the surface of a simulated human body.

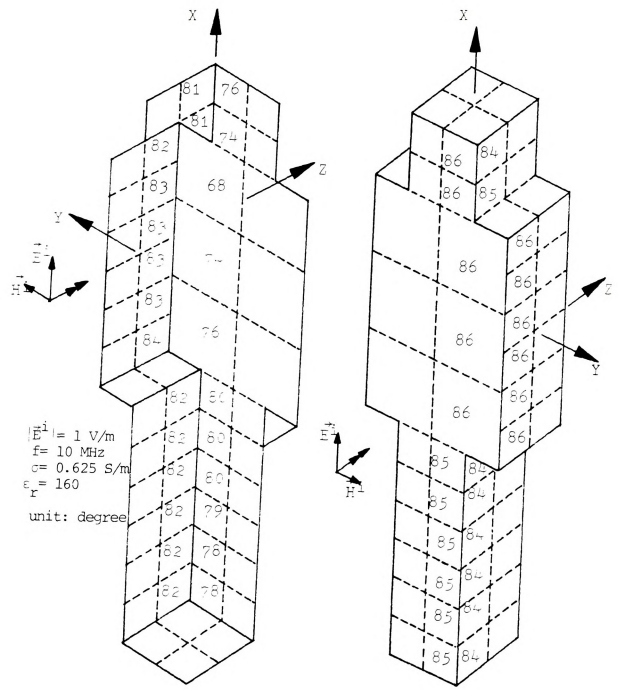

Figure 3.19. Phase distribution of the x-component of induced electric field on the surface of a simulated human body.

CHAPTER IV

PART 1

A USER'S GUIDE TO COMPUTER PROGRAM FOR INDUCED EDDY CURRENT INSIDE A FINITE CONDUCTING BODY WITH ROTATIONAL SYMMETRY

The first part of Chapter IV explains the computer program used to evaluate the eddy current induced at various locations within a finite conducting body with rotational symmetry. The theory behind this numerical technique has been presented in Chapter II. In addition to the listing of the program, an example will be worked out to help readers understand the sequential order of data files and the sample print out.

4.1. Formulation of the Problem

The program "EDDY" can handle two different cases: (1) the case of a body immersed in a uniform magnetic field and (2) the case of a body irradiated by a uniform magnetic beam. The first step in utilizing this program is to divide a body with rotational symmetry into N circular rings with various dimensions and radii. Inside each circular ring the induced electric field (induced electric field and induced eddy current are interchangeable, since

they are only related by the conductivity) and the electrical properties are assumed to be uniformly distributed; however, they may be changed from ring to ring.

For the first case considered, the impressed magnetic field can be written as

$$\vec{H}^i(r) = \hat{z}H^i.$$

Similarly, for the second case, it can be expressed as

$$\vec{H}^i(r) = \hat{z}H^i \qquad \text{for } 0 \leq r < b$$
$$= 0 \qquad \text{for } b \leq r.$$

Here the time-varying factor $\exp(j\omega t)$ has been suppressed. Usually, we assume that the impressed magnetic field has unit intensity, $H^i = 1$ A/m. The body considered has a symmetrical axis in the +z direction, as shown in Figure 4.1. For convenience, we also assume that the cross-section of each ring is a sqaure which is then approximated by a circle in the integration. Since the induced electric fields have only one component (in the $+\phi$ direction), we have N unknowns in total.

The second step in the numerical formulation of the problem is the specification of the location of each ring, its physical dimensions and its electrical parameters. As mentioned earlier, this program has been written for the case of uniformly impressed magnetic field or for the case of uniform magnetic beam. However, other types of

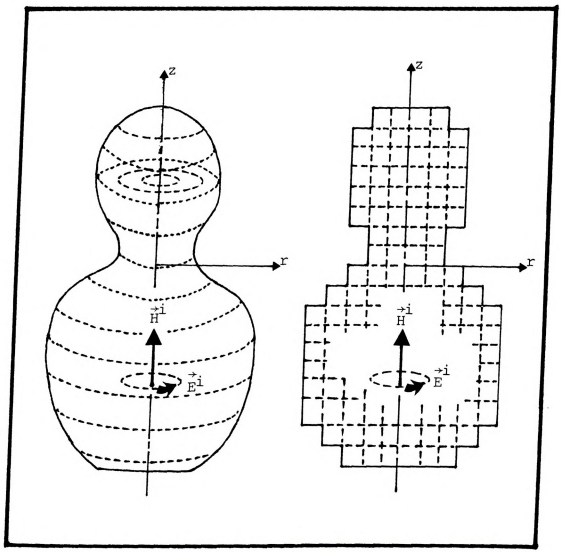

Figure 4.1. A rotationally symmetrical body with the symmetrical axis in the +z direction is irradiated by a magnetic field with unit intensity. The body is subdivided into a number of circular rings of various radii.

impressed magnetic fields may be used with few changes in the main program.

4.2. Description of Computer Program

The program "EDDY" is coded in FORTRAN. It used the following complex functions and subroutines:

"LEEMAT"--is a subprogram which calculates the elements of [M] matrix based on eqs. (2.16) and (2.17).

"FMM C," "FMMS," "FMNC," "FMNS"--are function subprograms which determine the integrands of integrals Kmn, and Kmm as expressed in eqs. (2.6) and (2.8). Note that we decompose each integrand into real and imaginary parts. Since these four functions are used as calling arguments for function subprogram "DCADRE," they must be declared external in subroutine "LEEMAT" which calls subprogram "DCADRE" for numerical integration.

"DCADRE"--is an IMSL routine which integrates a function by using technique of Cautious Adaptive Romberg Extrapolation.

"CMATP"--is a subprogram which calculates the induced electric field in each ring by solving N x N matrix. It is actually a Gauss-Seidel

method for solving a system of N equations in N unknowns.

"MAGPHA"--this subprogram, as the name suggests, determines magnitude and phase of the induced electric field in each ring.

"MCURVE," "PCURVE"--these two subroutines plot the magnitude and phase distributions of the induced electric fields as functions of radial distance.

A listing of the program "EDDY" is given at the end of this chapter.

4.3. Data Structure and Input Variables

Figure 4.2 shows a typical example of a body with rotational symmetry we refer to from time to time. This body is divided into six circular rings. The origin of the coordinate system can be arbitrarily chosen at any point along the central axis of the body; then the location of each ring is determined with respect to this origin. In this example, it is assumed that all the rings have the same cross-sectional area and electrical parameters.

The sequential structure of the input data files, the format specifications and the symbolic names of the variables appearing on each file are outlined in Table 4.1. There are totally three input files; each of them consists

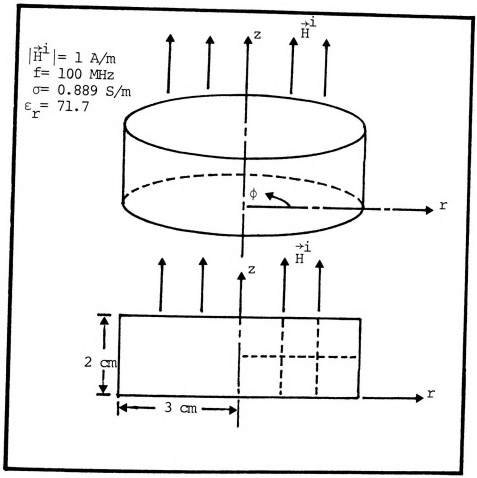

Figure 4.2. A cylindrical biological body with a diameter of 6 cm and a height of 2 cm is subdivided into 6 circular rings with a cross-section of 1 cm x 1 cm for each ring. The body is assumed to have a conductivity of 0.889 S/m and a dielectric constant of 71.7, and is immersed in a 100 MHz uniform magnetic field with an intensity of 1 A/m.

Table 4.1. The symbolic names of input variables and corresponding specifications for the data files used in data structure for the program "EDDY."

File No.	Card No.	Symbolic Name	Columns	Format
1	1	N	1-5	I5
		FREINM	6-15	F10.5
		NX	16-20	I5
		NXB	21-25	I5
2	1	NPAR	1-5	I5
		AERR	6-15	F10.5
		RERR	16-25	F10.5
3	1-N	XEND	1-12	F12.5
		ZEND	13-24	F12.5
		XAA	25-36	F12.5
		ZBB	37-48	F12.5
		REP	49-60	F12.5
		SIG	61-73	E13.6

of at least one data card. Specifically, the first two data files consist of only one data card each, while the third file consists of N data cards. The information on each data file is explained as below:

First Data File

This data file defines four variables with symbolic names N, FREINM, NX, and NXB. Where N is the total number of circular rings inside the body. NX is the number of rings per layer, and NXB is the number of rings per layer within the range of the magnetic beam. Finally FREINM is simply the frequency of the impressed magnetic field in MHz.

Second Data File

This file consists of only one data card and determines the following variables:

"NPAR"--for the integrals Kmn and Kmm, the integration limits 0° and 180° will be divided into "NPAR" equal-size subintervals. The purpose of this partitioning is to save the computational cost.

"AERR"--desired minimum absolute error for the numerical integration.

"RERR"--desired minimum relative error for the numerical integration.

Third Data File

This data file consists of N data cards, one for each circular ring. This set of data cards helps simulate the finite conducting body being considered. Each data card contains the following information:

"XEND," and "ZEND"--these codes correspond to the maximum boundaries of a ring cross-section in the x-, and the z-directions with reference to the origin. This information is supplied by the user in centimeters.

"XAA," and "ZBB"--are the symbolic names for the dimensions of the ring cross-section in the x-, and the z-directions. Since each ring cross-section is a square, the column for "XAA," and "ZBB" will contain the same information.

"REP," and "SIG"--are the codes for dielectric constant and conductivity (S/m) of each circular ring.

This completes the structure of the input data files needed to specify all the necessary information for the quantification of induced electric fields inside a finite conducting body with rotational symmetry. An example is worked out in the next section.

4.4. An Example to Use the Program

As an example, let us try to determine the electric field induced inside a circular cylinder, as shown in Figure 4.2. We assume that the body has radius 3 cm and

height 2 cm, and is immersed in a uniform magnetic field with an intensity of 1 A/m. Let us further assume that the frequency of the impressed magnetic field is 100 MHz, and the electrical parameters of the conducting body are ε_r = 71.7, σ = 0.889 S/m, and with a ring cross-section of 1 cm x 1 cm. With the aids of Section 4.3 and Table 4.1, the sequential order of the input data files is as follows:

File No.	Information on the File					
1	6	100.0		3	3	
2	18	0.0		0.01		
3.1	1.0	1.0	1.0	1.0	71.7	0.88900E+00
3.2	2.0	1.0	1.0	1.0	71.7	0.88900E+00
3.3	3.0	1.0	1.0	1.0	71.7	0.88900E+00
3.4	1.0	2.0	1.0	1.0	71.7	0.88900E+00
3.5	2.0	2.0	1.0	1.0	71.7	0.88900E+00
3.6	3.0	2.0	1.0	1.0	71.7	0.88900E+00

Now suppose that the program "EDDY" and the input data files are both stored on the magnetic disc under the permanent file names of "EDDYCURRENTEW" and "EDDYCURRENTDATA," respectively. In this case, the list of the commands needed for the execution of the program when submitted through a terminal interactively is as follows:

Statement No.	Its Purpose	Information in the Statement
1.	Job request	*JOBCARD*,CM170000, RG1,T600,JC1000.
2.	Dispose to engineering terminal	DISPOSE,**,V.
3.	Identification name	HAL,BANNER,LEE, EDDY,CURRENT.
4.	Calling subroutine DCADRE	HAL,LGO=DCADRE, UERTST.
5.	Calling the PF	ATTACH,EWFILE, EDDYCURRENTEW.
6.	Calling the EDITOR	EDITOR.
7.	Compile the program	FTN,I=Z,R,T.
8.	Return the EWFILE	RETURN,EWFILE.
9.	Calling the data	ATTACH,EWFILE, EDDYCURRENTDATA.
10.	Calling the EDITOR	EDITOR.
11.	Execute the program	LGO,W.
12.	End of Record	*EOR
13.	Change EWFILE to scope coded format	SAVE,Z.
14.	End of Record	*EOR
15.	Change EWFILE to scope coded format	SAVE,W,NS.
16.	End of file	*EOF

4.5. Printed Output

Now we would like to explain briefly the various sections of the output data files.

First, the coordinates of the maximum boundary limits on the x-z plane for each ring cross-section are listed, accompanied with the dimension and the cross-sectional area for each ring.

Then, the second output file contains the internally calculated coordinates for the central location of each ring cross-section, as well as the dielectric constant and the conductivity.

The third output file includes the information of the associated impressed electric field due to the impressed magnetic field of unit intensity. Also printed out is type of magnetic field applied.

The fourth output file has the information of the real and imaginary parts of the induced electric field inside each ring. Also included are the parameters used for numerical integrations, and the frequency of the impressed magnetic field.

The last output file is the induced electric field in magnitude and phase form, where the phase angles are in degrees.

An example print out of this program is shown in Table 4.2.

4.6. Listing of the Program

A FORTRAN listing of the program "EDDY" begins on page 175. Generally speaking, when compiled, the program "EDDY" requires about 46500B storage units of core memory.

Table 4.2. Printed output of program "EDDY."

THE EXTEND+DIMENSION+AS WELL AS CRO-SEC AREA OF EACH RING ARE GIVEN BELOW IN METERS

N	XEND	ZEND	X-DIMENSION	Z-DIMENSION	CRO-SEC AREA
1	+1[illegible]E-01	+1[illegible]E-01	+1[illegible]E-01	+1[illegible]E-01	+1[illegible]E-03
2	+2[illegible]E-01	+1[illegible]E-01	+1[illegible]E-01	+1[illegible]E-01	+1[illegible]E-03
3	+3[illegible]E-01	+1[illegible]E-01	+1[illegible]E-01	+1[illegible]E-01	+1[illegible]E-03
4	+1[illegible]E-01	+2[illegible]E-01	+1[illegible]E-01	+1[illegible]E-01	+1[illegible]E-03
5	+2[illegible]E-01	+2[illegible]E-01	+1[illegible]E-01	+1[illegible]E-01	+1[illegible]E-03
6	+3[illegible]E-01	+2[illegible]E-01	+1[illegible]E-01	+1[illegible]E-01	+1[illegible]E-03

Table 4.2. (Cont'd.)

THE CENTRAL POINT LOCATION AS WELL AS ELECTRICAL PROPERTIES OF EACH RING ARE GIVEN BELOW

N	XCEN(METERS)	ZCEN (METERS)	DIELEC.CONST.	COUDUC.(MHO/M)
1	.500000E-02	.500000E-02	.717600E+02	.889048E+00
2	.150000E-01	.500620E-02	.717200E+02	.889038E+00
3	.250000E- 1	.500000E-02	.717000E+02	.889000E+00
4	.500000E-02	.150000E-01	.717000E+02	.889000E+00
5	.150000E- 1	.150000E-01	.717000E+02	.889000E+00
6	.250000E- 1	.150000E-01	.717000E+02	.889000E+00

Table 4.2. (Cont'd.)

THE IMPRESS ELECTRIC FIELD DUE TO INCIDENT MAGNETIC FIELD (WITH UNIT INCIDENT ELECTRIC FIELD) IS AS FOLLOW

N	IMPRESS ELECTRIC FIELD (VOLTS/METER)	
	REAL	IMAG
1	0.	-.523956E-02
2	.0	-.157187E-01
3	0.	-.261978E-01
4	0.	-.523956E-02
5	.0	-.157187E-01
6	0.	-.261978E-01

THE BODY IS IMMERSED IN AN UNIFORM MAGNETIC FIELD

Table 4.2. (Cont'd.)

THE INDUCED ELECTRIC FIELD IS IN AZIMUTHAL DIRECTION. THE ESTIMATED VALUES AT CENTRAL OF EACH SUBAREA ARE GIVEN BELOW

THE FREQUENCY BEING USED IS 1[illegible]MHZ.ZERO TO PI BEING DIVIDED INTO [illegible] SUBINTERVALS DURING NUMERICAL INTEGRATIONS

THE INTEGRATIONS INVOLVED RETAIN [illegible] ABSOLUTE ERROR AND/OR .01% RELATIVE ERROR

N	EINDUCED-REAL(VOLTS/METER)	EINDUCED-IMAG(VOLTS/METER)
1	-.[illegible]E-03	-.5[illegible]E-02
2	-.[illegible]E-02	-.1[illegible]E-01
3	-.1179[illegible]E-02	-.2[illegible]E-01
4	-.[illegible]E-03	-.5[illegible]E-02
5	-.[illegible]E-02	-.1[illegible]E-01
[illegible]	-.1179[illegible]E-02	-.2[illegible]E-01

Table 4.2. (Cont'd.)

THE MAGNITUDES AND PHASES OF INDUCED E FIELDS ARE GIVEN BELOW

N	EINDUCED-MAGN(VOLTS/METER)	EINDUCED-PHASE(IN DEGREES)
1	.530621E-02	-.940210E+02
2	.613461E-01	-.935704E+02
3	.266805E-01	-.935240E+02
4	.522628E-02	-.940219E+02
5	.101366E-01	-.935704E+02
6	.266805E-01	-.935240E+02

PART 2

A USER'S GUIDE TO COMPUTER PROGRAM FOR INDUCED EM FIELD ON THE SURFACE OF A FINITE CONDUCTING BODY WITH ARBITRARY SHAPE

The purpose of Part 2 is to explain the computer program used for quantifying the EM field induced on the surface of a finite conducting body when illuminated by an incident plane wave. The theoretical derivation and the development of this numerical technique have been discussed in Chapter III. Besides explaining the usage of this program, we will also present an example accompanied with a sample print out for the purpose of better understanding.

4.7. Formulation of the Problem

We are considering a finite conducting body illuminated by an incident wave which, for the purpose of simplicity, has been assumed to be a plane wave. Mathematically, the incident plane wave can be expressed in the following form:

$$\vec{E}^i(z) = \hat{x}e^{-jz} = \hat{x}\,[\cos(z)-j\,\sin(z)]$$
$$\vec{H}^i(z) = \hat{y}\frac{1}{\eta_o}e^{-jz} = \hat{y}\frac{1}{\eta_o}[\cos(z)-j\,\sin(z)] \qquad (4.1)$$

We like to mention a few points concerning the above expressions:

1) The wave is propagating in the +z direction, and has a time factor exp(jωt) which is suppressed.
2) All quantities with the dimension of length are normalized by $1/k_o$.
3) The incident electric field has unit intensity, 1 V/m.
4) η_o is the intrinsic wave impedance of free space.

The first step in utilizing this program is the numerical formulation of the problem; hence, the first thing to do is to visualize the shape, dimensions and the orientation of the conducting body with respect to the incident electromagnetic wave. The incident wave may illuminate the body either at normal incidence or at end-on incidence. To begin with, the body surface is divided into NCELL surface cells with NX, NY, NZ, NNX, NNY, and NNZ cells facing the positive-x, positive-y, positive-z, negative-x, negative-y, and negative-z directions, respectively. Each surface cell is a square with suitable dimension in order to obtain optimum results.

The maximum number of unknowns that can be handled is about 150. Since there are four unknowns for each surface cell, the maximum number of surface cells, without any simplifications, can not be greater than 40. This imposes a restraint on the physical size of the body. But, due to the symmetrical properties which exist in most of the bodies we are considering, it seems adequate to

apply these properties and reduce the number of unknowns by a factor of 4 or 8. It must be mentioned here that in order to apply symmetrical conditions the incident wave has to be decomposed into four different modes named COSE, COSH, SINE, and SINH, respectively, as can be understood from eq. (4.1). The above-mentioned symmetry conditions are justifiable since most of the bodies considered (either biological bodies or other finite conducting materials encountered in engineering) possess four or eight similar-looking segments and hereafter called quadrants, such that it is sufficient merely to calculate the induced electromagnetic field at each surface cell of the first quadrant and then convert them into the induced field for the rest of the body.

After determining the symmetry conditions, the next step in the numerical formulation of the problem is the specification of the location of each surface cell, its physical dimensions, and the electrical parameters of the body. The size of each cell can vary but the electrical properties are assumed to be uniform throughout the body. Note that the central location of each surface cell is predetermined by the user; then the incident field intensities are automatically evaluated for each cell.

4.8. Description of Computer Program

This program is also coded in standard FORTRAN and can be compiled on either FTN or MNF compilers. The program is symbolically named as "SURFLDS" with input and output formats on any undefined logic units in conjunction with four magnetic tapes, "TAPE1," TAPE2," "TAPE3," and "TAPE4." "TAPE1"-"TAPE4" are the names of the local files which are used to temporarily store the computed results due to COSE, COSH, SINE, and SINH components of incident field.

Program "SURFLDS" makes use of the following subprograms:

"MNSB"--is a subroutine mainly used to print out the echo information which includes all the messages just read in from the input data files by the main program.

"MATRI," "LEE"--are subroutines which generate the matrix of the linear equations set. The matrix elements are related to the associated Green's functions evaluated at various locations.

"ELEMX," "ELEMY," and "ELEMZ"--are subroutines which evaluate the matrix elements when source points are in the ±x-, the ±y-, and the ±z-directed surfaces, respectively. In these

subroutines, the symmetry conditions have been imposed.

"EXIM," "CEIM," "CHIM," "SEIM," and "SHIM"--are subroutines which calculate the incident electric and magnetic fields for EXPZ, COSE, COSH, SINE, and SINH modes, respectively. In these subroutines, the incident electric field is polarized in the +x direction and has unit intensity, 1 V/m.

"T"--is a subroutine which converts the equivalent surface currents to surface field quantities.

"PRNT"--this subroutine, as the name suggests, is a program used to generate the print out.

"COEFX," "COEFY," "COEFZ," "FCNXX"-"FCNZZ," and "CEFXX,"-"CEFZZ," etc.--are subprograms which evaluate the elements of the Tensor Green's function.

"CMATP"--is the program being used to solve a system of N equations in N unknowns. It is actually a Gauss-Seidel method of numerical technique.

A listing of the program "SURFLDS," including all the subprograms called, is given at the end of this chapter. In the next section, the structure of the input data files as well as the associated input variables are explained in detail.

4.9. Data Structure and Input Variables

Figure 4.3 shows a sample body with finite conductivity. With origin of the coordinate system being chosen at the center of the body, we divide it into eight different sections which are called quadrants. The numbering system used is also shown in Figure 4.3. Note that the symmetry conditions exist if the physical dimensions of each surface cell in the first quadrant are the same as their counterparts in other quadrants, which is the case for this sample body because we assume that all the surface cells have the same physical dimensions. Based on these existing symmetry conditions, we only intend to solve for the induced electromagnetic fields at various surface cells of the first quadrant and then convert them into the fields induced at the surface cells on the rest of the body. Now we like to introduce the input data files before we go any further in determining the induced surface field. First of all, the sequential structure of the data files, the format specifications and the symbolic names of the variables appearing on each file are outlined in Table 4.3.

There are totally five input data files. Only the fourth data file contains NCELL data cards, while the rest of the data files contain one data card each. The input variables associated with these input data files are discussed in detail as below:

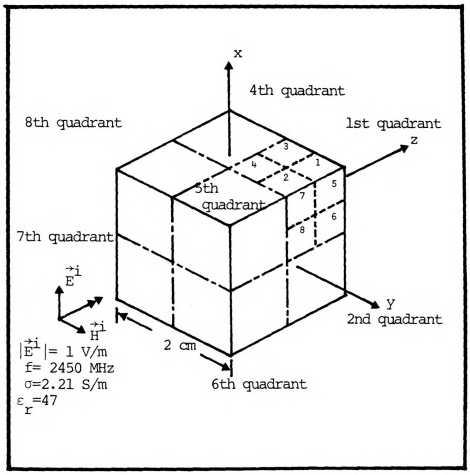

Figure 4.3. A cubic biological body (σ = 2.21 S/m, ε_r = 47) with dimensions of 2 cm x 2 cm x 2 cm is irradiated by an incident plane wave which has a frequency of 2.45 GHz and a unit intensity, x-polarized electric field. The body surface is subdivided into 95 surface cells with dimensions of 0.5 cm x 0.5 cm.

Table 4.3. The symbolic names of input variables and corresponding specifications for the data files used in data structure for the program "SURFLDS."

File No.	Card No.	Symbolic Name	Columns	Format
1	1	NDIV	1-2	I2
2	1	IQ	1-4	I4
3	1	NX	1-3	I3
		NY	4-6	I3
		NZ	7-9	I3
		NNX	10-12	I3
		NNY	13-15	I3
		NNZ	16-18	I3
4	1-NCELL	X	1-10	F10.5
		Y	11-20	F10.5
		Z	21-30	F10.5
		DCELL	31-40	F10.5
5	1	RLEP	1-13	E13.6
		SIG	14-26	E13.6
		FMEG	27-39	E13.6
		MODE	40-43	A4

First Data File

This file has only one data card with only one variable on it; namely, NDIV. NDIV is the number of subdivisions each side of the square surface cell will be divided into when the numerical integration over the surface cell is performed. The purpose of subdividing the surface cell is to improve the accuracy of the results. But the computational cost increases substantially when the value of NDIV goes up. Usually, we find that NDIV with value 3 gives excellent results and still keeps the required CPU time reasonably low. For the case of NDIV=3, each surface cell is subdivided into nine subsurface cells when numerical integration is carried out by the computer.

Second Data File

This data file has only one data card which defines a variable with symbolic name IQ. IQ is a four-digit code of the symmetrical property used. There are four different types of symmetrical property that can be handled by this program, as described in the following:

IQ = 0008 This is for the case of an eight-quadrants symmetrical body.

IQ = 1234 This is used when the body has symmetry with respect to the y-z plane and the x-z plane.

IQ = 1458 This is used when the body considered has symmetry with respect to the x-y plane and the x-z plane.

IQ = 1256 This is used when symmetry properties with respect to the x-y plane and the y-z plane exist.

Third Data File

This file has only one data card which defines six integer variables, NX, NY, NZ, NNX, NNY, and NNZ. These six integers are the numbers of surface cells on the +x, the +y, the +z, the -x, the -y, and the -z-directed surfaces of the first quadrant, respectively. The total number of surface cells of the first quadrant, NCELL, is obtained by summing up all these six integers.

Fourth Data File

This data file consists of NCELL data cards; i.e., one data card for each surface cell. On each data card, it defines four real variables, X, Y, Z, and DCELL. Where X, Y, Z are the coordinates of the central location for each corresponding surface cell with reference to the origin, while DCELL is the dimension of the corresponding surface cell. Note that these quantities are in centimeters.

Fifth Data File

This data file consists of one data card with four variables defined on it. The first three variables, RLEP, SIG, and FMEG, are specified under E13.6 format. RLEP is the symbolic name for dielectric constant, and SIG is that for conductivity in S/m, while FMEG stands for frequency in terms of MHz. Finally, the last variable of this last data file is symbolically named MODE, which is read in under format A4 and may have one of the following values:

"EXPZ"--is a four-letter code for plane wave which has complex exponential variation in z direction.

"COSE"--is the code for incident field which has only the x-component of E field with cosine variation in the z direction.

"COSH"--is the code for incident field which has only the y-component of H field with cosine variation in the z direction.

Similarly, "SINE" ("SINH")--is the code for incident field which has only the x-component (the y-component) of E (H) field with sine variation in the z direction.

Obviously, the effect of EXPZ is equivalent to the combined results due to COSE, COSH, SINE, and SINH. As we have mentioned, the reason of dividing a plane wave into

four different modes is for the purpose of applying symmetry conditions to a symmetrical body.

We have explained all the details about the input data files in this section. Now we are ready to use "SURFLDS" to solve for the EM field induced on the surface of an arbitrary body. An example is worked out in the next section.

4.10. An Example to Use the Program

As an example about the usage of the program "SURFLDS," let us again consider the case of a cube illuminated by a plane wave as shown in Figure 4.3. We assume that the cube has dimensions 2 cm x 2 cm x 2 cm, and is constructed with a material with the following electrical parameters at the frequency of 2.45 GHz: σ = 2.21 S/m, ε_r = 47. The cube has eight-fold symmetry, such that it can be divided into eight different quadrants. Although the same problem can be solved by dividing the body into four quadrants, for the purpose of demonstrating the logic of decomposing the plane wave into four separate modes, we apply eight-quadrant symmetry conditions to this body. We further divide each surface of the first quadrant into four surface cells, such that there are totally 12 surface cells (or totally 48 surface unknowns) to be considered, where each of these surface cells has dimensions of 0.5 cm x 0.5 cm.

As mentioned previously, since the incident plane wave is decomposed into COSE, COSH, SINE, and SINH modes, we have to run the program in four parts--one for each constituting mode of the incident wave. The results due to each incident mode will be buffered into a logical unit with one of the following logical names, "TAPE1," "TAPE2," "TAPE3," and "TAPE4." And then the results contained within these units will be appropriately combined in order to obtain the true EM fields induced on the surfaces of the first and the fifth quadrants.

With the help of Section 4.9 and Table 4.3, we can easily list the input data files for the case of COSE incident mode as follows:

File No.	Information on the File
1	03
2	0008
3	004 004 004 000 000 000
4.1	1.0 0.75 0.75 0.5
4.2	1.0 0.75 0.25 0.5
4.3	1.0 0.25 0.75 0.5
4.4	1.0 0.25 0.25 0.5
4.5	0.75 1.0 0.75 0.5
4.6	0.25 1.0 0.75 0.5
4.7	0.75 1.0 0.25 0.5
4.8	0.25 1.0 0.25 0.5

4.9	0.75	0.75	1.0	0.5
4.10	0.25	0.75	1.0	0.5
4.11	0.75	0.25	1.0	0.5
4.12	0.25	0.25	1.0	0.5

5 47.000000E+00 2.210000E+00 2.450000E+03 COSE

Now we assume that both the program "SURLDS" and the input data files are already stored in different permanent files with "SURFACEEW" and "SURFACEDATACOSE" as permanent file names, respectively. In this case, the commands needed in order to submit the job through the interactive terminals should be as follows:

Statement No.	Information in the Statement
1.	*JOBCARD*,RG1,JC1000,T600,CM170000.
2.	DISPOSE,**,V.
3.	HAL,BANNER,LEE,SURFLDS,8SYMM,COSE.
4.	ATTACH,EWFILE,SURFACEEW.
5.	EDITOR.
6.	ATTACH,A,SURFACEB.
7.	FTN,I=Z,R,T.
8.	COPYL,A,LGO,B.
9.	PURGE,A.
10.	CATALOG,B,SURFACEB,RP=999.
11.	RETURN,EWFILE.
12.	ATTACH,EWFILE,SURFACEDATACOSE.
13.	EDITOR.

```
14.                B,W.
15.                CATALOG,TAPE1,SURFACEDUETOCOSE.
16.                *EOR
17.                SAVE,Z,100-780.
18.                *EOR
19.                SAVE,W,NS.
20.                *EOF
```

There are several points we like to mention about the above job:

1) We assume that the program "SURFLDS" has been compiled, and the compiled binary codes are already stored in a file with permanent file name "SURFACEB."

2) We don't have to recompile the whole program each time we use it. If the array dimensions have been changed, the only section that needs to be recompiled is the main program.

3) The computed induced surface fields are buffered out to TAPE1, and then cataloged as a permanent file named "SURFACEDUETOCOSE."

So the results due to COSE mode of incident field have been obtained and stored into "SURFACEDUETOCOSE." To obtain the results due to the other three incident modes, the same program has to be rerun three more times with the only change in the last data file where the correct code

for the incident mode must be used. For example, when COSH mode is considered, the program can be executed without any recompilation by the use of the following commands:

Statement No.	Information in the Statement
1.	*JOBCARD*,RG1,JC1000,T600,CM170000.
2.	DIPOSE,**,V.
3.	HAL,BANNER,LEE,SURFLDS,8SYMM,COSH.
4.	ATTACH,A,SURFACEB.
5.	ATTACH,EWFILE,SURFACEDATACOSH.
6.	EDITOR.
7.	A,W.
8.	CATALOG,TAPE2,SURFACEDUETOCOSH.
9.	*EOR
10.	SAVE,W,NS.
11.	*EOF

After obtaining results for all four different modes and creating four different permanent files for them separately, we are now in a position to combine these results in such a way so as to yield the total induced surface fields on the body surfaces. Another program called "COMBINE" has been written for this purpose. Program "COMBINE" has output including the magnitude and phase information of the total surface fields in quadrants 1 and 5. The surface fields on the rest of the body, either

magnitude or phase, can be deduced from the information of the first or the fifth quadrant by intuition.

The data files for "COMBINE" are the same as those for program "SURFLDS," except that the variable MODE of the last data file for "SURFLDS" is redundant here. Hence the input data files for "SURFLDS" can be used for "COMBINE" without any modifications.

The commands needed in order to submit the program "COMBINE" interactively, assuming that both the program and the data files have been cataloged into permanent files, will be as follows:

Statement No.	Information in the Statement
1.	*JOBCARD*,RG1,JC500,T100.
2.	DISPOSE,**,V.
3.	HAL,BANNER,LEE,SURFLDS,8SYMM,COMBINE.
4.	ATTACH,EWFILE,COMBINEEW.
5.	EDITOR.
6.	ATTACH,A,COMBINEB.
7.	FTN,I=Z,R,T.
8.	COPYL,A,LGO,B.
9.	PURGE,A.
10.	CATALOG,B,COMBINEB,RP=999.
11.	RETURN,EWFILE.
12.	ATTACH,TAPE1,SURFACEDUETOCOSE.
13.	ATTACH,TAPE2,SURFACEDUETOCOSH.

```
14.    ATTACH,TAPE3,SURFACEDUETOSINE.
15.    ATTACH,TAPE4,SURFACEDUETOSINH.
16.    ATTACH,EWFILE,SURFACEDATACOSE.
17.    EDITOR.
18.    B,W.
19.    RETURN,EWFILE.
20.    PURGE,TAPE1.
21.    PURGE,TAPE2.
22.    PURGE,TAPE3.
23.    PURGE,TAPE4.
24.    *EOR
25.    SAVE,Z,100-1820.
26.    *EOR
27.    SAVE,W,NS.
28.    *EOF
```

Note that in the above job, we has assumed that program "COMBINE" has been cataloged as permanent file "COMBINEEW," and that the compiled program has been stored in permanent file named "COMBINEB." Furthermore, instead of creating a new input data file, the file "SURFACEDATACOSE" has been used.

4.11. Printed Output

The first output file of program "SURFLDS" mainly performs the echo checking of the input data. The location and dimension of each surface cell are listed first, then

the number of surface cells on surfaces of the first quadrant, the electrical parameters of the body, as well as the frequency and the type of mode of the incident field are all printed out as shown in Table 4.4.

The second output file is the listing of the real and imaginary parts of the incident equivalent surface currents which are internally determined for each surface cell. This file is shown in Table 4.5.

Then the computed induced surface currents at various surface locations are listed in the third output file where all the surface currents are expressed in terms of Amp/meter, as shown in Table 4.6.

The last output file consists of the information of the induced surface fields at various locations on the body surface as shown in Table 4.7. It is this file which will be buffered out to a magnetic tape with one of the following local file names, TAPE1, TAPE2, TAPE3, and TAPE4. Then this magnetic tape will be cataloged as a permanent file which has one of the following PFN's, "SURFACEDUETOCOSE," "SURFACEDUETOCOSH," "SURFACEDUETOSINE," and "SURFACEDUETOSINH."

Now we briefly discuss the output files of the program "COMBINE." The first file is the echo messages of the input variables. The next four files contain the values of the induced surface fields which are buffered in from TAPE1, TAPE2, TAPE3, TAPE4, respectively.

Table 4.4. First output file of program "SURFLDS."

```
THIS IS SOLUTION OF INDUCED FIELDS ON THE SURFACES OF AN   A DPS STRUCT BODY WITH ARBITRARY ELECT PROPERTIES
THE LOCATION AND DIMENSION OF EACH SURFACE-CELL ARE LISTED BELOW
 N        X (IN CM)          Y (IN CM)          Z (IN CM)          D (IN CM)
 1          1.00000            .75000             .75000             .50000
 2          1.20063            .75000             .25000             .50000
 3          1.00000            .25000             .75000             .50000
 4          1.00000            .25000             .25000             .50000
 5           .75000           1.00000             .75000             .50000
 6           .25000           1.00000             .75000             .50000
 7           .75000           1.00000             .25000             .50000
 8           .25000           1.00000             .25000             .50000
 9           .75000            .75000            1.00000             .50000
10           .25000            .75000            1.00000             .50000
11           .75000            .25000            1.00000             .50000
12           .25000            .25000            1.00000             .50000
THERE ARE NX=  4  *NY=  4  * NZ=  4  *NNX=  0  *NNY=  0  *AND NNZ=  0  CELLS ON SURFACES OF THE FIRST QUADRANT
EACH SURFACE CELL IS DIVIDED INTO  1 BY  1 SUB-SURFACE CELLS
DIELECTRIC CONST. =  .47000E+02
CONDUCTIVITY=  .22100E+01MHO/METER
FREQUENCY=  .24500E+10MEGA. HZ
TYPE OF IMPRESSED FIELD=0SF
```

Table 4.5. Second output file of program "SURFLDS."

THE INFAR CURT ARE LISTED AS FOLLOWS (AMPLIFIER FOR CURRENTS ON H FIELDS= VOLTMETER FOR E FIELDS)

N	REAL	IMAG	REAL	IMAG	REAL	IMAG	REAL	IMAG
	ON THE TOP SURFACE WE HAVE THE FOLLOWS							
	JFY OR -HZ		J Z OR HY		JHX OR -EZ/U0		JHZ OR -EY/U0	
1	(0.	0.)	(0.	0.)	(0.	0.)	(0.	0.)
2	(0.	0.)	(0.	0.)	(0.	0.)	(0.	0.)
3	(0.	0.)	(0.	0.)	(0.	0.)	(0.	0.)
4	(0.	0.)	(0.	0.)	(0.	0.)	(0.	0.)
	ON THE RIGH SURFACE WE HAVE THE FOLLOWS							
	JFX OR HZ		JFZ OR -HX		JHX OR -EZ/U0		JHZ OR EX/U0	
5	(0.	0.)	(0.	0.)	(0.	0.)	(.2958D8E-02	0.)
6	(0.	0.)	(0.	0.)	(0.	0.)	(.2958D8E-02	0.)
7	(0.	0.)	(0.	0.)	(0.	0.)	(.2638D6E-02	0.)
8	(0.	0.)	(0.	0.)	(0.	0.)	(.2638D6E-02	0.)
	ON THE REAR SURFACE WE HAVE THE FOLLOWS							
	JFX OR -HY		JFY OR HX		JHX OR EY/U0		JHY OR -EX/U0	
9	(0.	0.)	(0.	0.)	(0.	0.)	(-.231D8E-02	0.)
10	(0.	0.)	(0.	0.)	(0.	0.)	(-.231D8E-02	0.)
11	(0.	0.)	(0.	0.)	(0.	0.)	(-.231D8E-02	0.)
12	(0.	0.)	(0.	0.)	(0.	0.)	(-.231D8E-02	0.)

Table 4.6. Third output file of program "SURFLDS."

THE ENDU CHAT ARE LISTED FOLLOWS EXPRESSED FOR ERRORS ON H FIELDS. NUMBER OF FIELDS

N	REAL	IMAG	REAL	IMAG	REAL	IMAG	REAL	IMAG
	ON THE TOP SURFACE WE HAVE THE FOLLOWS							
	JEY OR -HZ		JEZ OR HY		JMX OR EZYMS		JMZ OR -EXYMS	
1	(-.1545414E-03	-.4986629E-04)	(-.1519131E- 3	-.5399982E- 4)	(-.5266499E-04	-.1508051E-03)	(.5362215E-04	.1547195E-03)
2	(-.5871499E-03	.1826611E-03)	(-.1141838E-03	.4081948E- 4)	(-.1076868E-03	-.6796461E-04)	(.1295965E-03	-.1756387E-04)
3	(-.1097212E-03	.5663750E-04)	(-.5114081E-04	.1410953E-03)	(-.1311922E-03	-.1671201E-03)	(.1117179E-03	.7382620E-04)
4	(-.1448908E-03	.1654802E-03)	(-.1951176E-03	.1580677E- 3)	(-.1601995E-03	-.6498815E- 4)	(.1434577E-03	.7595265E-04)
	ON THE HIGH SURFACE WE HAVE THE FOLLOWS							
	JEX OR HZ		JEZ OR -HX		JMX OR -EZYMS		JMZ OR EXYMS	
5	(.1015563E-02	.6680485E-03)	(.2955021E-03	-.2726727E-03)	(.6431969E-04	.1154481E-03)	(.3395465E-04	.1247495E-03)
6	(.1341153E-02	.8770680E-03)	(.3176937E-04	.1282137E-03)	(.3573809E-04	.2964656E-04)	(.5746256E-04	.2333955E-04)
7	(.1309259E-02	.9139665E-04)	(.7101145E-04	.2083942E-03)	(.7138584E-04	.4673352E-04)	(.2184645E-03	.5744936E-04)
8	(.1701814E-02	.4592543E-03)	(.1554118E-04	.1294445E-03)	(.2098853E-04	.1349726E-04)	(.3732609E-03	.3784205E-04)
	ON THE REAR SURFACE WE HAVE THE FOLLOWS							
	JEX OR -HY		JEY OR HX		JMX OR EYZMS		JMY OR -EXZMS	
9	(.9996501E-03	.6465366E-03)	(.2516477E- 3	-.2287293E-03)	(-.6463648E-04	-.1179999E-03)	(-.3258188E-03	-.1112 [illegible])
10	(.1311291E-02	.8761996E- 3)	(.4264774E- 4	-.1357148E-03)	(-.3598882E-04	-.1209934E-04)	(-.5736251E-03	-.2222498E-03)
11	(.1345492E-02	.1435477E-03)	(.8534761E- 4	-.2972434E-03)	(-.7266795E-04	-.4488827E-04)	(-.2333895E-03	-.4535878E-04)
12	(.1693838E-02	.6451 [illegible]E- 3)	(.2067277E- 3	-.1342884E- 3)	(-.2993792E- 4	-.1215347E-03)	(-.3550385E-03	-.7311547E-04)

Table 4.7. Fourth output file of program "SURFLDS."

```
      THE INDUCED FIELDS ARE LISTED AS FOLLOWS (PARAMETER FOR CURVES ON H FIELDS: VOLUMETRIC FLUX)

N          REAL              IMAG              REAL              IMAG              REAL              IMAG

                ON THE TOP SURFACE WE HAVE THE FOLLOWS

                  HZ                                HY                                EZ                                EY

 ************************** ********************************* ********************************* *********************************

1  (  .3545101-03  .4986291E-04) (  -.3519131- 4 -.5999982F-04) (  -.1985659F-01 -.9988795F-03) (  -.2021437E-01  -.452 435E-03)
2  (  .5071497-03 -.3826113E-03) (  -.1081384F-03  .4861134F-04) (  -.4066626F-01 -.3622599F-03) (  -.4084266E-01  -.6559078E-03)
3  (  .1077121F-03 -.9665759E-04) (  -.5114888F-03  .1810153E-03) (  -.6958046E-01 -.6370817F-03) (  -.6173731E-01  -.9277462E- 03)
4  (  .1440061-03 -.1658A2E- 03) (  -.1951776E- 3  .1956677E- 3) (  -.6629015E-01 -.9588175F- 03) (  -.6137423F-01  -.1255333E-02)

 ************************** ********************************* ********************************* *********************************

                ON THE HIGH SURFACE WE HAVE THE FOLLOWS

                  HZ                                HY                                EZ                                EX

 ************************** ********************************* ********************************* *********************************

5  (  .1815631-02  .5688A8E-03) (  -.2055F2E-03  .2727?2E-03) (  -.2424A9E-01 -.4352591F-03) (  .1246231+00  .4564 33E-03)
6  (  .1341131-02  .8790A0E-03) (  -.5179?7F-03  .1282136-03) (  -.1347327F-01 -.1317761-03) (  .2161192F+00  .9659?3E-03)
7  (  .13 939F-02 -.9139?5E-03) (  -.7191136F-03  .2883?2F-03) (  -.2688156F-01 -.1761471-03) (  .3242935- 0  .1214 69E-02)
8  (  .1700197-02 -.5929A3E-03) (  -.1554113F- 3  .1270A5F-03) (  -.1091116F-01 -.4437211F-03) (  .4 7?1E+00  .5 08F-03)

 ************************** ********************************* ********************************* *********************************

                ON THE REAR SURFACE WE HAVE THE FOLLOWS

                  HY                                HX                                EY                                EX

 ************************** ********************************* ********************************* *********************************

9  (  -.9996501-03 -.6653961-03) (  .2516471-03 -.2853293E-03) (  -.2435081E-01 -.4488?21- 1?) (  .1220?5E+00  .4 3?5E-03)
10 (  -.1311201-02 -.8763761-03) (  .6247?1E-03 -.1317?3F- 3) (  -.1356?5F-01 -.1142?8?- 1?) (  .2139?3E+00  .8 5?3E-03)
11 (  -.1305921-02  .1439?7F-03) (  .4?5631- 4 -.2?7243E-03) (  -.2739?2F-01 -.1842?81- 1?) (  .3?67?3E+00  .1 35?0E-02)
12 (  -.1691?31-02  .6531?1- 3) (  .2?72?1- 3 -.1392?8F- 3) (  -.1107?0F-01 -.4?5?761- 2?) (  .4?11?E+00  .2?3?6E-02)

 ************************** ********************************* ********************************* *********************************
```

The sixth and the eighth output files list the total surface fields induced by plane wave on the surfaces of the first quadrant in real-imaginary and magnitude-phase forms, respectively.

Finally, the seventh and the last output files list the total surface fields for the fifth quadrant in real-imaginary and magnitude-phase forms, respectively. The above output files of the program "COMBINE" are shown in Table 4.8.

4.12. Listing of the Program

For the purpose of reference, the programs "SURFLDS" and "COMBINE" are listed on pages 183 -229 . For about 150 surface unknowns, "SURFLDS" requires approximately 170000B words memory, while "COMBINE" requires about 30000B words. When the number of unknowns is more than 150, the memory required for "SURFLDS" exceeds that obtainable from CDC 6500 at MSU. To overcome this difficulty, the job may be submitted through Merit Network System for execution at other computing facilities. In this case, due to different operating systems involved, apparently different control commands must be employed. Nevertheless, since the program itself has been coded in standard FORTRAN, there is no modification needed except a subroutine named "BLOCK DATA" must be added into program "SURFLDS." The function

Table 4.8. Printed output of program "COMBINE."

```
      THIS IS THE TOTAL INDUCED FIELDS ON THE SURFACES OF AN    A 0'S SEMI-ELLIPTICAL BODY
THE RESULTS ARE OBTAINED BY COMBINING RESULTS FOR    IN FOUR DIFFERENT TYPES OF IMPRESSED FIELDS (WHICH ARE COSE, C SIN, SINE, AND SIN(
      THE LOCATION AND DIMENSION OF EACH SURFACE-CELL   ARE LISTED BELOW
        N        X (IN CM)          Y (IN CM)          Z (IN CM)          D (IN CM)
        1        1.00000            .75000             .75000             .50000
        2        1.00000            .75000             .25000             .50000
        3        1.00000            .25000             .75000             .50000
        4        1.00000            .25000             .25000             .50000
        5         .75000           1.00000             .75000             .50000
        6         .25000           1.00000             .75000             .50000
        7         .75000           1.00000             .25000             .50000
        8         .25000           1.00000             .25000             .50000
        9         .75000            .75000            1.00000             .50000
       10         .25000            .75000            1.00000             .50000
       11         .75000            .25000            1.00000             .50000
       12         .25000            .25000            1.00000             .50000
THERE ARE NX=  4  NY=  4  NZ=  4  NNX=  0  NNY=  0  AND NNZ=  0  CELLS ON SURFACES OF THE FIRST QUADRANT
EACH SURFACE CELL IS DIVIDED INTO  5 BY  5 SUB-SURFACE CELLS
DIELECTRIC CONST.=  .472000E+02
CONDUCTIVITY=  .221600E+01MHO/METER
FREQUENCY=  .240000E+09MEGA HZ
THE  TOTAL INCIDENT FIELD I: A PLANE WAVE
```

Table 4.8. (Cont'd.)

THE FIELDS INDUCED BY EACH TYPE OF IMPRESSED FIELD IN FIRS QUADRANT ARE LISTED BELOW EXPRESSED IN THE H FIELDS. VOLTAGE IS (EE-10)

N	REAL	IMAG	REAL	IMAG	REAL	IMAG
	ON THE TOP SURFACE WE HAVE THE FOLLOWS					
	HZ		HY		EZ	
1	(+.3545194E-03 +.4986429E-03)		(-.3519813E-03 -.5009982E-03)		(-.1985669E-01 -.5008755E-01)	(-.2021675E-01 -.452 355E-01)
2	(+.5717490E-03 -.1826118E-03)		(-.1008138E-03 +.8981198E-03)		(-.4966662E-01 -.2622598E-01)	(-.4984026E-01 -.6503337E-01)
3	(+.1997121E-03 -.9667590E-03)		(-.5114080E-03 +.1013554E-03)		(-.4350865E-01 -.6304815E-01)	(-.4173731E-01 -.2757749E-01)
4	(+.1448008E-03 +.1459809E-03)		(-.1951778E-03 +.5966778E-03)		(-.6839155E-01 -.2581756E-01)	(-.6117062E-01 -.6755531E-01)
	ON THE RIGH SURFACE WE HAVE THE FOLLOWS					
	HZ		HX		EY	EX
5	(+.1019463E-02 +.6684848E-03)		(-.2855802E-03 +.2720232E-03)		(-.2424885E-01 -.4352599E-01)	(+.1246235E+00 +.6667335E-01)
6	(+.1341153E-02 +.8790657E-03)		(-.5176977E-04 +.1208133E-03)		(-.1347352E- 1 -.1117567E-01)	(+.2161825E+00 +.8667339E-01)
7	(+.1599397E-02 +.9119967E-03)		(-.7191180E-04 +.2883429E-03)		(-.2668815E-01 -.1761477E-01)	(+.2928935E-01 +.5173435E-01)
8	(+.1708184E-02 +.9925439E-03)		(-.1559411E-04 +.1299845E-03)		(-.1891164E-01 -.4332771E-01)	(+.1407259E+00 +.2700665E-01)
	ON THE REAR SURFACE WE HAVE THE FOLLOWS					
	HY		HX		EY	EX
9	(-.9995608E-03 -.6653346E-03)		(+.2316437E- 4 +.2483299E-03)		(+.2436181E-01 -.4848021E- 1)	(+.1220859E+00 +.6833135E-01)
10	(-.1311201E-02 +.8761994E-03)		(+.6247731E-04 +.1357334E-03)		(+.1356757E-01 -.1114228E-01)	(+.2133935E+00 +.8363735E- 1)
11	(-.1359492E-02 +.1047571E-02)		(+.8343631E-04 +.2972243E-03)		(+.2739372E-01 -.1983981E- 1)	(+.2662761E-01 +.1057138E-01)
12	(-.1691853E-02 +.6551980E-03)		(+.2027279E-04 +.1382084E-03)		(+.1187762E-01 -.4554771E- 2)	(+.1331435E+00 +.3733655E-01)

Table 4.8. (Cont'd.)

THE FIELDS INDUCED BY EACH TYPE OF IMPRESSED FIELD IN FIRS QUADRANT ARE LISTED BELOW (NORMALIZED FOR 1 ETEUDS VOLTAGE FOR 1 METHOD

N	REAL	IMAG	REAL	IMAG	REAL	IMAG	REAL	IMAG
	ON THE TOP SURFACE WE HAVE THE FOLLOWS							
	HZ		HY		EZ		EY	
	**********	**********	**********	**********	**********	**********	**********	**********
1	(-.659494E-03	-.794146E-01)	(.247794E-02	.567746E-03)	(.782061E-01	.867161E-01)	(.162426E-01	.571165E-02)
2	(-.279252E-03	-.673870E-05)	(.266931E-02	.581152E-03)	(.147867E+00	.176476E-01)	(.193225E-01	.111455E-01)
3	(-.276864E-03	.715234E-03)	(.315396E-02	.277994E-03)	(.782886E-01	.431508E-01)	(.561175E-02	.427126E-02)
4	(-.122280E-03	.422479E-03)	(.353835E-02	.132191E-03)	(.130596E+00	.819197E-01)	(.119945E-02	.599815E-02)
	**********	**********	**********	**********	**********	**********	**********	**********
	ON THE RIGH SURFACE WE HAVE THE FOLLOWS							
	HZ		HY		EZ		EX	
	**********	**********	**********	**********	**********	**********	**********	**********
5	(-.539218E-03	-.692071E-01)	(-.795732E-03	-.813765E-03)	(.691313E-01	.417911E-01)	(-.577228E-01	-.431593E-01)
6	(-.154627E-02	-.893852E-01)	(-.612179E-03	-.244649E-03)	(.236376E-01	-.993010E-02)	(-.118182E+ 0	.203449E-02)
7	(-.508291E-03	-.150111E-03)	(-.169961E-02	-.117315E-02)	(.118072E+ 0	.121953E-01)	(-.193947E-01	.53 .5E-02)
8	(-.133294E-02	-.144551E-03)	(-.117782E-02	-.277116E-03)	(.391588E-01	-.368592E-01)	(-.157844E-01	.355392E-01)
	**********	**********	**********	**********	**********	**********	**********	**********
	ON THE REAR SUN ACE WE HAVE THE FOLLOWS							
	HY		HX		EY		EX	
	**********	**********	**********	**********	**********	**********	**********	**********
9	(.223492E-02	.382157E-03)	(-.515213E-03	-.835628E-04)	(.199313E-01	-.780296E-11)	(-.643E+0E-01	-.871439E-01)
10	(.216880E-02	.233020E-03)	(-.268890E- 3	-.171243E-04)	(.724997E- 2	-.174618E- 11)	(-.149917E+01	-.111218E-01)
11	(.282492E-02	.883559E-04)	(-.241953E- 3	.756128E-04)	(.215515E-02	-.134612E- 11)	(-.5224 3E-01	-.877119E-01)
12	(.284978E-02	-.522541E-03)	(-.114193E- 3	.814566E-04)	(.372104E-03	-.397720E-12)	(-.116120E+0	.791755E-01)
	**********	**********	**********	**********	**********	**********	**********	**********

Table 4.8. (Cont'd.)

THE FIELDS INDUCED BY SOME TYPE OF IMPRESSED FIELD IN FIRS QUADRANT ARE LISTED BELOW EXPECTED FOR THE SPECIFIED + MULTIPLICATION FIELDS

N	REAL	IMAG	REAL	IMAG	REAL	IMAG
	ON THE TOP SURFACE WE HAVE THE FOLLOWS					
	HZ		HY		EZ	EX
1	(.242949E-05 -.332084E-05)	(.853888E-06 -.397339E-06)	(-.559176E-02 -.717825E-02)	(-.135713E-01 -.137191E-01)		
2	(.622075E-05 -.117258E-04)	(.179194E-05 -.899147E-06)	(.102834E-01 -.106958E-01)	(-.655271E- 3 -.711397E-03)		
3	(-.151848E-04 .791664E-05)	(.112483E-05 -.208056E-06)	(-.687751E-02 -.834342E-02)	(-.917360E-02 -.277483E-02)		
4	(-.611918E-05 .193789E-05)	(.239059E- 5 -.173599E-05)	(.143422E-01 -.608858E-02)	(-.483869E-02 -.739422E-03)		
	ON THE RIGH SURFACE WE HAVE THE FOLLOWS					
	HZ		HY		EZ	EX
5	(.261773E-03 -.156206E-03)	(-.585761E-04 .719667E-04)	(-.623412E-02 -.410854E-02)	(.111837E-01 -.295154E-01)		
6	(.289698E-03 -.248873E-03)	(-.222508E-04 .234795E-04)	(-.295913E-02 -.761979E-03)	(.208036E-01 -.236859E-01)		
7	(.107161E-03 -.108036E-03)	(-.599213E-04 .207436E-04)	(.691261E-03 .686485E-02)	(.987806E-03 -.969172E-03)		
8	(.123425E-03 -.185552E-03)	(-.387886E-04 .220949E-04)	(.167401E-02 .677117E-03)	(.206402E-02 -.371111E-02)		
	ON THE REAR SURFACE WE HAVE THE FOLLOWS					
	HY		HX		EY	EX
9	(-.227301E-03 .124688E-03)	(.955711E- 5 -.499107E-04)	(-.122497E-01 -.124779E- 1)	(.112708E-01 -.735171E-01)		
10	(-.271018E-03 .214546E-03)	(-.760165E-05 .353564E-04)	(-.477588E-02 -.477881E-03)	(.206177E-01 -.241758E-01)		
11	(-.185883E-03 .268823E-03)	(-.712964E- 5 -.532182E-04)	(-.665179E-02 -.594714E- 2)	(.124461E- 2 -.211353E- 1)		
12	(-.216642E-03 .478558E- 3)	(-.612731E- 5 -.271081E-04)	(-.263277E-02 -.731851E-03)	(-.859815E- 3 -.178049E-01)		

Table 4.8. (Cont'd.)

THE FIELDS INDUCED BY SINK TYPE OF IMPRESSED FIELD IN THIS QUADRANT ARE LISTED BELOW (AMPLITUDE FOR A FIELDS WHEREAS FOR A FIELDS)

N	REAL	IMAG	REAL	IMAG	REAL	IMAG
	ON THE TOP SURFACE WE HAVE THE FOLLOWS					
	HZ		HY		FZ	FY
1	(-.159725E-03	+.256634E-03)	(.223598E-03	-.159788E-02)	(.130638E+00	-.762917E-01) (+.660184E-01 -.194511E-01)
2	(.366598E-03	+.789256E-03)	(-.269568E-03	-.591433E-03)	(.598803E-01	-.884677E-11) (+.666335E-11 -.770719E-01)
3	(-.888821E-05	+.218749E-03)	(.458105E-03	-.117988E-02)	(.126435E+00	-.128388E+00) (.228581E-01 -.435993E-01)
4	(.196434E-03	+.216882E-03)	(-.193592E-03	-.590202E-03)	(.589803E-01	-.125181E+01) (.179588E-01 -.703407E-01)
	ON THE RIGH SURFACE WE HAVE THE FOLLOWS					
	HZ		HX		FZ	FX
5	(-.669512E-03	+.183169E-02)	(-.980432E-03	-.935237E-03)	(.729819E-01	-.613956E-01) (-.339587E-01 +.170222E+00)
6	(-.119081E-02	+.181251E-02)	(-.782796E-03	-.359678E-04)	(.117069E-01	-.295334E-01) (-.665192E-01 +.219566E+00)
7	(.936953E-03	+.170019E-02)	(-.727044E-03	-.981936E-04)	(.340517E-01	-.568811E-01) (.585665E-01 +.431799E-01)
8	(.185849E-02	+.222875E-02)	(-.314770E-03	-.185347E-04)	(.589739E-02	-.215583E-11) (.539845E-01 +.323881E-01)
	ON THE REAR SURFACE WE HAVE THE FOLLOWS					
	HY		HX		FY	FX
9	(.533663E-03	+.278639E-02)	(.360777E-03	-.104722E-03)	(.951237E- 1	-.471767E-11) (-.799325E-01 +.184965E+00)
10	(.715778E-03	+.331561E-02)	(.536259E-04	-.359278E-04)	(.798013E-02	-.219358E- 1) (-.127862E-01 +.228338E+00)
11	(-.195471E-03	+.142779E- 2)	(.293883E-03	-.588253E-04)	(.129976E-01	-.362315E-11) (-.889528E-01 +.171653E+00)
12	(-.867928E-03	+.427164E-02)	(.119281E-03	-.440816E-04)	(.110739E-02	-.180120E-11) (-.889528E-01 +.277618E+00)

Table 4.8. (Cont'd.)

THE FIELDS INDUCED BY EACH TYPE OF IMPRESSED FIELD IN FIRS QUADRANT ARE LISTED BELOW EACH FIELD'S VOLUME INTEGRAL FIELDS

N	REAL	IMAG	REAL	IMAG	REAL	IMAG
	ON THE TOP SURFACE WE HAVE THE FOLLOWS					
	HZ		HY		EZ	EY
1	(-.3672276E-03 .5990407E-03)	(.2435761E-02 -.1128467E-02)	(.1833726E+00 -.5649477E-01)	(.3385237E-01 -.1784467E+00)		
2	(.4087161E-03 .5891815E-03)	(.2737180E-02 -.1816455E-02)	(.1764071E+00 -.6635756E-01)	(.1139081E-01 -.1534809E+00)		
3	(-.1704291E-01 .2328935E-03)	(.2889571E-02 -.1368998E-02)	(.1402296E+00 -.1901780E+00)	(-.7154945E-01 -.7433037E-01)		
4	(.2009356E-03 .9476568E-04)	(.3515117E-02 -.4818023E-03)	(.1294777E+00 -.2296127E+00)	(-.8464715E-01 -.9753035E-01)		
	ON THE RIGH SURFACE WE HAVE THE FOLLOWS					
	HZ		HX		EZ	EX
5	(-.5131798E-04 .1342220E-03)	(-.1960284E-02 -.8650937E-03)	(.1115958E+00 -.4721981E-01)	(.8569175E-01 .11 [illegible])		
6	(-.1106367E-02 .1629935E-03)	(-.1869908E-02 -.1240856E-03)	(.1891158E-01 -.6643147E-01)	(.5749137E-01 .3481935E+00)		
7	(.1845522E-02 .1550665E-03)	(-.2556670E-02 -.8631187E-03)	(.1326555E+00 -.6584357E-01)	(.1035997E+00 .69 [illegible])		
8	(.2644071E-02 .1278181E-03)	(-.1542967E-02 -.1035431E-03)	(.3581039E-01 -.4284458E-01)	(.1689958E+00 .1551187E+00)		
	ON THE REAR SURFACE WE HAVE THE FOLLOWS					
	HY		HX		EY	EX
9	(.1541531E-02 .2788609E-03)	(-.1172434E-03 -.2088489E-03)	(.2753022E-01 -.1221991E+00)	(.8620825E-02 .8020207E-01)		
10	(.1292156E-02 .1588894E-03)	(-.1694350E-03 -.1747677E-03)	(.3112480E-02 -.5265558E-01)	(.4126035E-01 .1781524E-01)		
11	(.1137731E-02 .1251770E-03)	(.1186762E-03 .7339999E-03)	(.1988942E-01 -.2073581E-01)	(.1557837E-01 .5623334E-01)		
12	(.9418201E-03 .1247538E-03)	(.1360865E-03 .1276618E-03)	(.1298478E-01 -.1023564E-01)	(.2152767E-01 .4133 [illegible])		
	ABOVE ARE TOTAL FIELDS IN THE FIRS QUADRANT					

Table 4.8. (Cont'd.)

THE FIELDS INDUCED BY THIS TYPE OF IMPRESSED FIELD IN FIFT QUADRANT ARE LISTED BELOW (ANALYTIC FOR A FIELDING WIRE

N	REAL	IMAG	REAL	IMAG	REAL	IMAG
	ON THE TOP SURFACE WE HAVE THE FOLLOWS					
	HZ		HY		EZ	EY
1	(.751894E-03 .751587E-03)	(.769364E-02 .218569E-02)	(-.381632E- 1 .919866E+01)	(.577467E-01 -.498693E-01)		
2	(.114678E-02 .124109E-03)	(.295886E- 2 .984291E- 3)	(.138138E+00 .163781E+01)	(.241217E-01 -.141335E+00)		
3	(.399677E-03 .714533E-04)	(.572772E-02 .186854E-02)	(-.135673E-01 .210901E+00)	(-.162174E-01 -.278371E-01)		
4	(.968833E-03 .819421E-04)	(.901765E-03 .398127E-03)	(.140379E+00 .758781E-01)	(-.437861E-01 -.181558E+00)		
	ON THE HIGH SURFACE WE HAVE THE FOLLOWS					
	HZ	HX	EZ	EX		
5	(.723555E-03 .780787E-02)	(.251632E-03 -.439073E-03)	(.142443E-01 .142599E+00)	(.139468E+00 .247736E+01)		
6	(.149680E-02 .473538E-02)	(-.199871E-03 -.304547E-03)	(.224455E-01 .249870E-01)	(.246294E+00 .345197E+00)		
7	(.264747E-02 .910669E-02)	(-.986570E-03 -.124041E-02)	(.118314E+00 .935550E-01)	(.138395E+01 .735719E-01)		
8	(.446374E-02 .195831E-02)	(-.882375E-03 -.391518E-03)	(.456193E-01 -.182792E- 1)	(.220895E+00 .139995E+00)		
	ON THE FRNT SURFACE WE HAVE THE FOLLOWS					
	HY	HX	EY	EX		
9	(.247351E-02 .395878E-02)	(-.912677E- 3 -.552472E-03)	(.139739E-01 -.111582E-01)	(.790713E-01 .387245E+00)		
10	(.250328E-02 .848739E-02)	(-.732648E-03 .496455E-03)	(-.886224E-02 -.127774E-01)	(.211530E+00 .496126E+00)		
11	(.814951E-02 .317066E-02)	(-.631707E-03 .204854E-03)	(-.999185E-02 -.380218E-01)	(.679415E-01 .281135E+00)		
12	(.515209E-02 .277442E-02)	(-.760329E-03 .157275E-03)	(-.769307E-02 -.659099E- 2)	(.159795E+00 .227139E+00)		
	ABOVE ARE TOTAL FIELDS ON THE FIFT QUADRANT					

Table 4.8. (Cont'd.)

THE FIELDS INDUCED BY EXEZ TYPE OF IMPRESSED FIELD IN FIRS QUADRANT ARE LISTED BELOW (EXPRESSED FOR A UNIFORM WIRE CURRENT)

N	ABSL	PHAS	ABSL	PHAS	ABSL	PHAS	ABSL	PHAS
	ON THE TOP SURFACE WE HAVE THE FOLLOWS							
	HZ		HY		EZ		EY	
	**********	**********	**********	**********	**********	**********	**********	**********
1	(.790380E-03	.121148E+ 3)	(.268569E-02	-.268569E+02)	(.191895E+00	-.171237E+02)	(.109835E+00	-.777458E+02)
2	(.841929E-03	.999444E+ 2)	(.273718E- 2	-.381195E-01)	(.188473E+00	-.225615E+02)	(.132898E+00	-.857036E+02)
3	(.300836E-03	.329272E+ 3)	(.311774E-02	-.260499E+02)	(.204809E+00	-.467334E+02)	(.778618E-01	-.195408E+03)
4	(.221347E-03	.253891E+ 2)	(.354787E-02	-.779220E+01)	(.260956E+00	-.602843E+02)	(.103754E+00	-.119699E+03)
	**********	**********	**********	**********	**********	**********	**********	**********
	ON THE HIGH SURFACE WE HAVE THE FOLLOWS							
	HZ		HX		EZ		EX	
	**********	**********	**********	**********	**********	**********	**********	**********
5	(.134259E-02	.913367E+ 2)	(.211925E-02	-.157664E+03)	(.130329E+00	-.431751E+02)	(.119117E+00	-.679455E+02)
6	(.196995E-02	.124164E+ 3)	(.187572E- 2	-.171592E+03)	(.501347E-01	-.478358E+02)	(.345806E+00	-.312660E+02)
7	(.241876E-02	.988423E+02)	(.269749E-02	-.161408E+03)	(.143772E+00	-.226709E+02)	(.123928E+00	-.557865E+02)
8	(.296611E-02	.961832E+02)	(.159693E- 2	-.176161E+03)	(.712434E- 1	-.685647E+02)	(.923554E+00	-.677499E+02)
	**********	**********	**********	**********	**********	**********	**********	**********
	ON THE REAR SURFACE WE HAVE THE FOLLOWS							
	HY		HX		EY		EX	
	**********	**********	**********	**********	**********	**********	**********	**********
9	(.432378E-02	-.427680E+02)	(.311325E-03	-.112121E+03)	(.125235E+00	-.773902E+ 2)	(.807445E-01	-.791798E+02)
10	(.581877E-02	-.471370E+02)	(.228497E-03	-.154013E+03)	(.527472E-01	-.933878E+02)	(.325539E+00	-.763903E+02)
11	(.281122 -02	-.465127E+02)	(.267737E-03	-.636997E+02)	(.732184E-01	-.104955E+ 3)	(.276279E-01	-.119142E+ 3)
12	(.394272E-02	-.492961E+02)	(.322256E-03	-.817287E+02)	(.420531E-01	-.117181E+03)	(.532178E+00	-.102704E+03)
	**********	**********	**********	**********	**********	**********	**********	**********

ABOVE ARE TOTAL FIELDS IN THE FIRS QUADRANT

Table 4.8. (Cont'd.)

THE FIELDS INDUCED BY EXYZ EXEC OF IMPOSSED FIELDS IN FIRST QUADRANT ARE LISTED BELOW (ONLY EVEN TERMS ARE WRITTEN)

N	ABSE	PHAS	ABSE	PHAS	ABSE	PHAS	ABSE	PHAS

ON THE TOP SURFACE WE HAVE THE FOLLOWS

	HZ		HY		EZ		EX	
1	(.106337E-02	.447890E+02)	(.966808E-02	.394491E+02)	(.219214E+00	.180826E+03)	(.197082E+01	-.217333E+03)
2	(.137541E-02	.785543E+02)	(.311827E-02	.180402E+02)	(.213602E+00	.497267E+02)	(.187908E+00	-.983498E+02)
3	(.430109E-03	.132873E+02)	(.817433E-02	.265930E+02)	(.219011E+00	.335458E+02)	(.779637E-01	-.117687E+03)
4	(.468915E-03	.190153E+02)	(.403234E-02	.566481E+01)	(.159772E+00	.283909E+02)	(.132275E+00	-.112997E+03)

ON THE RIGH SURFACE WE HAVE THE FOLLOWS

	HZ		HY		EZ		EX	
5	(.279892E-02	.157967E+02)	(.724196E-03	-.696677E+02)	(.143508E+00	.982956E+01)	(.283813E+00	.567098E+02)
6	(.399147E-02	.693626E+02)	(.364276E-03	-.122377E+03)	(.335879E-01	.881627E+02)	(.856872E- 0	.577159E+02)
7	(.335877E-02	.179800E+02)	(.156986E-02	-.127754E+03)	(.150834E+ 0	.383588E+02)	(.156735E+00	.271397E+02)
8	(.887441E-02	.236878E+02)	(.965337E-03	-.156074E+ 3)	(.477586E-01	-.124231E+02)	(.250477E+00	.269417E+02)

ON THE FRNT SURFACE WE HAVE THE FOLLOWS

	HY		HX		EY		EX	
9	(.966808E-02	.180826E+02)	(.983772E-03	-.176998E+03)	(.627341E-01	-.771134E+02)	(.321972E+00	.165.28E+02)
10	(.615204E-02	.497265E+02)	(.395728E-03	-.177798E+03)	(.151897E- 1	-.122251E+02)	(.498706E+ 0	-.693.72E+ 2)
11	(.523165E-02	.335458E+02)	(.678644E- 3	-.151.15E+ 3)	(.197758E- 1	-.108481E+ 3)	(.210655E+ 0	.731.88E+02)
12	(.985362E-02	.283128E+02)	(.364140E- 3	-.149862E+ 3)	(.111700E-01	-.177245E+03)	(.276973E+00	-.594.98E+ 2)

ABOVE ARE INTAL FIELDS ON THE FIRT QUADRANT

of "BLOCK DATA" can be found in most of the computer user's manuals.

```
C*************************************************************************
C*****PROGRAM EDDY DETERMINES THE CIRCULATORY ELECTRIC FIELDS INDUCED BY ********
C*****UNIFORM MAGNETIC FIELDS INSIDE FINITE CONDUCTING BODIES WITH **************
C*****ROTATIONAL SYMMETRY*************************************************
C*************************************************************************
      PROGRAM EDDY(INPUT,OUTPUT)
      DIMENSION XEND(12),ZEND(12),XAA(12),ZBB(12),B(12),PHA(12)
      COMPLEX LEE(12,13),EINC(12),DET,EIND(12)
      REAL KO,MUO,MAG(12)
      COMMON/CONS1/FREQ,MUO,EPO,PI,OMEG,KO
      COMMON/PARA1/XCEN(12),ZCEN(12),A(12)
      COMMON/PARA2/SIG(12),REP(12)
      COMMON/PARA6/AREA(12)
      PI=3.1415926535
      MUO=4.0*PI*1.0E-07
      EPO=8.854E-12
      READ 1, N,FREINM,NX,NXS
      READ 2, NPAR,AERR,RERR
      NP1=N+1
      FREQ=FREINM*1.0E+06
      OMEG=2.0*PI*FREQ
      RTMUEP=SQRT(MUO*EPO)
      KO=OMEG*RTMUEP
      DO 100 I=1,N
      READ 3, XEND(I),ZEND(I),XAA(I),ZBB(I),REP(I),SIG(I)
      XEND(I)=XEND(I)/100.
      ZEND(I)=ZEND(I)/100.
      XAA(I)=XAA(I)/100.
      ZBB(I)=ZBB(I)/100.
      AREA(I)=XAA(I)*ZBB(I)
      A(I)=XAA(I)/2.
      B(I)=ZBB(I)/2.
      XCEN(I)=XEND(I)-A(I)
      ZCEN(I)=ZEND(I)-B(I)
  100 CONTINUE
      PRINT 4
      DO 101 I=1,N
      PRINT 5, I,XEND(I),ZEND(I),XAA(I),ZBB(I),AREA(I)
  101 CONTINUE
      PRINT 6
      DO 102 I=1,N
      PRINT 7, I,XCEN(I),ZCEN(I),REP(I),SIG(I)
  102 CONTINUE
      CALL LEEMAT(N,NPAR,AERR,RERR,LEE)
      PRINT 8
      IF (NXS .LT. NX) GO TO 50
      DO 103 I=1,N
      EINC(I)=-1.0*CMPLX(0.0,1.0)*KO*XCEN(I)/2.0
```

```
      LEE(I,NP1)=EINC(I)
      PRINT 9,  I,EINC(I)
  103 CONTINUE
      PRINT 13
      GO TO 51
   50 DO 106 I=1,NXS
      EINC(I)=-1.0*CMPLX(0.0,1.0)*K0*XCEN(I)/2.0
  106 CONTINUE
      NXSP1=NXS+1
      DO 107 I=NXSP1,NX
      EINC(I)=-1.0*CMPLX(0.0,1.0)*XEND(NXS)*XEND(NXS)*K0/(2.0*XCEN(I))
  107 CONTINUE
      NZ=N/NX
      NZM1=NZ-1
      DO 108 J=1,NZM1
      DO 108 K=1,NX
      I=J*NX+K
      EINC(I)=EINC(K)
  108 CONTINUE
      DO 109 I=1,N
      LEE(I,NP1)=EINC(I)
      PRINT 9,I,EINC(I)
  109 CONTINUE
      PRINT 14
   51 CALL CMATP(LEE,N,1,DET,1.0E-50)
      PRINT 10,  FREINM,NPAR,AERR,RERR
      DO 104 I=1,N
      EIND(I)=LEE(I,NP1)
      PRINT 11,  I,LEE(I,NP1)
  104 CONTINUE
      CALL MAGPHA(EIND,N,MAG,PHA)
      PRINT 12
      DO 105 I=1,N
      PRINT 11,  I,MAG(I),PHA(I)
  105 CONTINUE

      CALL MCURVE(EINC,N,NX,XCEN,MAG)
      CALL PCURVE(N,NX,XCEN,PHA)
    1 FORMAT(I5,F10.5,2I5)
    2 FORMAT(I5,2F10.5)
    3 FORMAT(5F12.5,E13.6)
    4 FORMAT("1",10X,"THE EXTEND,DIMENSION,AS WELL AS CRO-SEC AREA OF EA
     2CH RING ARE GIVEN BELOW IN METERS"/"-",10X,"N",7X,"XEND",11X,
     3"ZEND",7X,"X-DIMENSION",4X,"Z-DIMENSION",4X,"CRO-SEC AREA")
    5 FORMAT("0",8X,I3,5(2X,E13.6))
    6 FORMAT("1",10X,"THE CENTRAL POINT LOCATION AS WELL AS ELECTRICAL P
     2ROPERTIES OF EACH RING ARE GIVEN BELOW"/"-",10X,"N",2X,"XCEN( METE
     3RS)",2X,"ZCEN (METERS)",2X,"DIELEC CONST.",2X,"CONDUC.(MHO/M)")
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    7 FORMAT("0",8X,I3,4(2X,E13.6))
    8 FORMAT("1",10X,"THE IMPRESS ELECTRIC FIELD DUE TO INCIDENT MAGNETI
     2C FIELD (WITH UNIT INCIDENT ELECTRIC FIELD) IS AS FOLLOW"/"-",10X,
     3"N",10X,"IMPRESS ELECTRIC FIELD (VOLTS/METER)"/"-",24X,"REAL",22X,
     4"IMAG")
    9 FORMAT("0",8X,I3,13X,E13.6,4X,E13.6)
   10 FORMAT("1","THE INDUCED ELECTRIC FIELD IS IN AZIMUTHAL DIRECTION,
     2THE ESTIMATED VALUES AT CENTRAL OF EACH SUBAREA ARE GIVEN BELOW"/
     3"-","THE FREQUENCY BEING USED IS",F10.5,"MHZ,ZERO TO PI BEING DIVI
     4DED INTO",I5,"SUBINTERVALS DURING NUMERICAL INTEGRATIONS"/"-",10X,
     5"THE INTEGRATIONS INVOLVED RETAIN",F10.5,"ABSOLUTE ERROR",2X,
     6"AND/OR",F10.5,"RELATIVE ERROR"/"-",10X,"N",10X,"EINDUCED-REAL(VOL
     7TS/METER)",10X,"EINDUCED-IMAG(VOLTS/METER)")
   11 FORMAT("0",8X,I3,16X,E13.6,23X,E13.6)
   12 FORMAT("1","THE MAGNITUDES AND PHASES OF INDUCED E FIELDS ARE GIVE
     2N BELOW"/"-",10X,"N",10X,"EINDUCED-MAGN(VOLTS/METER)",10X,
     3"EINDUCED-PHASE(IN DEGREES)")
   13 FORMAT("-",9X,"THE BODY IS IMMERSED IN AN UNIFORM MAGNETIC FIELD")
   14 FORMAT("-",9X,"THE BODY IS ILLUMINATED BY A MAGNETIC BEAM")
      STOP
      END
C*****SUBROUTINE LEEMAT*************************************************************
      SUBROUTINE LEEMAT(N,NUM,AERR,RERR,LEE)
      EXTERNAL FMMC,FMMS,FMNC,FMNS
      COMPLEX LEE(12,13),KINTEG(12,12),TAU(12)
      REAL KO,MUO
      COMMON/CONS1/FREQ,MUO,EPO,PI,OMEG,KO
      COMMON/PARA1/X(12),Z(12),A(12)
      COMMON/PARA2/SIG(12),EP(12)
      COMMON/PARA3/XM
      COMMON/PARA4/AM
      COMMON/PARA5/XN,ZN,ZM
      COMMON/PARA6/AREA(12)
      NP1=N+1
      DO 10 I=1,N
      DO 10 J=1,NP1
   10 LEE(I,J)=CMPLX(0.0,0.0)
      DELT=PI/NUM
      DO 20 I=1,N
      TAU(I)=SIG(I)+CMPLX(0.0,1.0)*OMEG*(EP(I)*EPO-EPO)
      XM=X(I)
      AM=A(I)
      SUMC=0.0
      SUMS=0.0
      ALPHA=0.0
      BETA=DELT
   15 SUMC=SUMC+DCADRE(FMMC,ALPHA,BETA,AERR,RERR,ERROR,IER)
      SUMS=SUMS+DCADRE(FMMS,ALPHA,BETA,AERR,RERR,ERROR,IER)
```

```
      ALPHA=BETA
      BETA=BETA+DELT
      IF(BETA.GT.PI+0.5*DELT) GO TO 20
      GO TO 15
   20 KINTEG(I,I)=(SUMC+CMPLX(0.0,1.0)*SUMS)*2.0*PI/AREA(I)
      DO 30 I=2,N
      IM1=I-1
      XM=X(I)
      ZM=Z(I)
      DO 30 J=1,IM1
      XN=X(J)
      ZN=Z(J)
      SUMC=0.0
      SUMS=0.0
      ALPHA=0.0
      BETA=DELT
   25 SUMC=SUMC+DCADRE(FMNC,ALPHA,BETA,AERR,RERR,ERROR,IER)
      SUMS=SUMS+DCADRE(FMNS,ALPHA,BETA,AERR,RERR,ERROR,IER)
      ALPHA=BETA
      BETA=BETA+DELT
      IF (BETA.GT.PI+0.5*DELT) GO TO 29
      GO TO 25
   29 KINTEG(I,J)=SUMC+CMPLX(0.0,1.0)*SUMS
   30 KINTEG(J,I)=KINTEG(I,J)

      DO 40 I=1,N
      DO 40 J=1,N
      IF (I.EQ.J) GO TO 35
      LEE(I,J)=CMPLX(0.0,1.0)*FREQ*MUO*AREA(J)*X(J)*TAU(J)*KINTEG(I,J)
      GO TO 40
   35 LEE(I,J)=1.0+CMPLX(0.0,1.0)*FREQ*MUO*AREA(J)*X(J)*TAU(J)*KINTEG(I,
     2J)
   40 CONTINUE
      RETURN
      END
C*****FUNCTION FMMC*****************************************************************
      FUNCTION FMMC(T)
      REAL KO,MUO
      COMMON/CONS1/FREQ,MUO,EPO,PI,OMEG,KO
      COMMON/PARA3/XM
      COMMON/PARA4/AM
      ALPHAM=2.0*AM/SQRT(PI)
      F1=2.0*XM*XM*(1.0-COS(T))
      F2=SQRT(F1)
      ROOT=SQRT(F1+ALPHAM*ALPHAM)-F2
      FMMC=COS(T)*COS(KO*F2)*ROOT
      RETURN
      END
```

```
C*****FUNCTION FMMS**********************************************************
      FUNCTION FMMS(T)
      REAL KO,MUO
      COMMON/CONS1/FREQ,MUO,EPO,PI,OMEG,KO
      COMMON/PARA3/XM
      COMMON/PARA4/AM
      ALPHAM=2.0*AM/SQRT(PI)
      F1=2.0*XM*XM*(1.0-COS(T))
      F2=SQRT(F1)
      ROOT=SQRT(F1+ALPHAM*ALPHAM)-F2
      FMMS=-COS(T)*SIN(KO*F2)*ROOT
      RETURN
      END
C*****FUNCTION FMNC**********************************************************
      FUNCTION FMNC(T)
      REAL KO,MUO
      COMMON/CONS1/FREQ,MUO,EPO,PI,OMEG,KO
      COMMON/PARA3/XM
      COMMON/PARA5/XN,ZN,ZM
      ROOT=SQRT(XM*XM+XN*XN-2.*XM*XN*COS(T)+(ZM-ZN)*(ZM-ZN))
      FMNC=COS(T)*COS(KO*ROOT)/ROOT
      RETURN
      END
C*****FUNCTION FMNS**********************************************************
      FUNCTION FMNS(T)
      REAL KO,MUO
      COMMON/CONS1/FREQ,MUO,EPO,PI,OMEG,KO
      COMMON/PARA3/XM
      COMMON/PARA5/XN,ZN,ZM
      ROOT=SQRT(XM*XM+XN*XN-2.*XM*XN*COS(T)+(ZM-ZN)*(ZM-ZN))
      FMNS=-COS(T)*SIN(KO*ROOT)/ROOT
      RETURN
      END
C*****SUBROUTINE CMATP*******************************************************
        SUBROUTINE CMATP(A,N,M,DET,EP)
C
        COMPLEX A,B,DET,CONST,S
      DIMENSION A(12,13)
C
        DET = CMPLX(1.0 , 0.0)
        NP1 = N + 1
        NPM = N + M
        NM1 = N - 1
 2      DO 4 J = 1,NM1
        C = CABS(A(J,J))
        JPJ = J + 1
        DO 5 I = JP1,N
        D = CABS(A(I,J))
```

```
          IF( C-D )6, 5, 6
 6        DET = DET
          DO 7 K = JP1,NPM
          B = A(I,K)
          A(I,K) = A(J,K)
 7        A(J,K) = B
          C = D
 5        CONTINUE
          IF(CABS(A(J,J))-EP)14, 15, 15
 15       DO 4 I = JP1,N
          CONST = A(I,J)/A(J,J)
          DO 4 K = JP1,NPM
 4        A(I,K) = A(I,K) - CONST*A(J,K)

          IF(CABS(A(N,N))-EP)14, 18, 18
 14       DET = CMPLX((0. , 0.)
          PRINT 30
          RETURN
 18       CONTINUE
          DO 12 I = 1,N
          K = N - I + 1
          KP1 = K + 1
          DO 12 L = NP1,NPM
          S = CMPLX(0. , 0.)
          IF( N - KP1 ) 12, 19, 19
 19       DO 13 J = KP1,N
 13       S = S + A(K,J) * A(J,L)
 12       A(K,L) = (A(K,L) - S)/A(K,K)
 30       FORMAT(1X,42HTHE DETERMINANT OF THE SYSTEM EQUALS ZERO./
     1            1X,36HTHE PROGRAM CANNOT HANDLE THIS CASE.//)
          RETURN
          END
C*****SUBROUTINE MAGPHA*************************************************************
      SUBROUTINE MAGPHA(EIND,N,MAG,PHA)
      COMPLEX EIND(12)
      REAL IM(12),MAG(12)
      DIMENSION RE(12),PHA(12)
      DO 10 I=1,N
      MAG(I)=CABS(EIND(I))
      RE(I)=REAL(EIND(I))
      IM(I)=AIMAG(EIND(I))
      PHA(I)=ATAN2(IM(I),RE(I))
      PHA(I)=PHA(I)*57.29577951
   10 CONTINUE
      RETURN
      END
```

```
C*****SUBROUTINE MCURVE**********************************************************
      SUBROUTINE MCURVE(EINC,N,NX,XCEN,MAG)
      COMPLEX EINC(30)
      REAL MAG(30),MEINC(17)
      DIMENSION XCEN(30),IBUF(385),X(17),TMAG(17,3)
      CALL PLOTS(IBUF,385,0)
      DO 10 I=1,NX
      MEINC(I)=CABS(EINC(I))
      X(I)=XCEN(I)
   10 CONTINUE
      NZ=N/NX
      JEN=NZ/2
      LEE=JEN*2
      IF(LEE.NE.NZ) JEN=JEN+1
      DO 30 J=1,JEN
      DO 20 K=1,NX
      L=(J-1)*NX+K
      TMAG(K,J)=MAG(L)
   20 CONTINUE
   30 CONTINUE
      CALL PLOT(0.0,0.5,-3)
      CALL SCALE(X,5.0,NX,1)
      DO 40 II=1,JEN
      CALL MCSCALE(TMAG(1,II),0,NX,1)
   40 CONTINUE
      CALL SCALE(MEINC,6.0,NX,1)
      DO 50 JJ=1,JEN
      TMAG(NX+1,JJ)=MEINC(NX+1)
      TMAG(NX+2,JJ)=MEINC(NX+2)
   50 CONTINUE
      CALL AXIS(0.0,0.0,11HX IN METERS,-11,5.0,0.0,X(NX+1),X(NX+2))
      CALL AXIS(0.0,0.0,23HMAG. OF E FIELDS IN V/M,23,6.0,90.0,
     2MEINC(NX+1),MEINC(NX+2))
      CALL LINE(X,MEINC,NX,1,1,3)
      DO 60 KK=1,JEN
      CALL LINE(X,TMAG(1,KK),NX,1,0,0)
   60 CONTINUE
      CALL PLOT(12.0,0.0,999)
      RETURN
      END
C*****SUBROUTINE PCURVE**********************************************************
      SUBROUTINE PCURVE(N,NX,XCEN,PHA)
      DIMENSION PHA(30),XCEN(30),IBUF(385),X(17),TPHA(17,3)
      CALL PLOTS(IBUF,385,0)
      DO 10 I=1,NX
      X(I)=XCEN(I)
   10 CONTINUE
```

```
      NZ=N/NX
      JEN=NZ/2
      LEE=JEN*2
      IF(LEE.NE.NZ) JEN=JEN+1
      JENM1=JEN-1
      DO 30 J=1,JEN
      DO 20 K=1,NX
      L=(J-1)*NX+K
      TPHA(K,J)=PHA(L)*90.0
   20 CONTINUE
   30 CONTINUE
      CALL PLOT(0.0,0.5,-3)
      CALL SCALE(X,5.0,NX,1)
      IF(JEN.EQ.1) GO TO 45
      DO 40 II=1,JENM1
      CALL MCSCALE(TPHA(1,II),0,NX,1)
   40 CONTINUE
   45 CALL SCALE(TPHA(1,JEN),6.0,NX,1)
      IF(JEN.EQ.1) GO TO 55
      DO 50 JJ=1,JENM1
      TPHA(NX+1,JJ)=TPHA(NX+1,JEN)
      TPHA(NX+2,JJ)=TPHA(NX+2,JEN)
   50 CONTINUE
   55 CALL AXIS(0.0,0.0,11HX IN METERS,-11,5.0,0.0,X(NX+1),X(NX+2))
      CALL AXIS(0.0,0.0,37HREL. PHASE (0 FOR INC. FIELD) IN DEG.,37,6.0,
     290.0,TPHA(NX+1,JEN),TPHA(NX+2,JEN))
      DO 60 KK=1,JEN
      CALL LINE(X,TPHA(1,KK),NX,1,0,0)
   60 CONTINUE
      CALL PLOT(12.0,0.0,999)
      RETURN
      END
```

```
C*************************************************************************
C*****PROGRAM SURFLDS QUANTIFIES THE EM FIELDS INDUCED BY INCIDENT PLANE WAVE***
C*****ON THE SURFACES OF FINITE CONDUCTING BODIES WITH ARBITRARY SHAPES*********
C*************************************************************************
      PROGRAM SURFLDS(INPUT,OUTPUT,PUNCH,TAPE1,TAPE2,TAPE3,
     +                TAPE4,TAPE5=INPUT,TAPE6=OUTPUT)
      REAL K0,MU0,MODE
      REAL INDU,IMPR                                                    120
      COMPLEX FK,L,DET,CJ,FHUN                                          130
      COMMON IQ                                                         140
      COMMON /NUMDV/NDIV                                                150
      COMMON /NXYZ/NX,NY,NZ,NNX,NNY,NNZ
      COMMON /CONS/PI,MU0,EPO                                           170
      COMMON /CPLX/CJ
      COMMON /PARA/FK,FHUN
      COMMON /MODS/MODE                                                 190
      COMMON /IMCR/IMPR,CRNT
      COMMON /CSEH/COSE,COSH,SINE,SINH,EXPZ
      DIMENSION X(12),Y(12),Z(12),DCELL(12),ARE(12),L(48,49)
      DATA INDU/4HINDU/,FLDS/4HFLDS/
      CALL NOBLANK
      PI=3.1415926535
      MU0=1.256637061E-06                                               250
      EPO=8.854E-12                                                     260
      CJ=CMPLX(0.0,1.0)                                                 270
      READ 11, NDIV                                                     280
   11 FORMAT(I2)
      READ 5, IQ
    5 FORMAT(I4)                                                        290
      READ 1,NX,NY,NZ,NNX,NNY,NNZ                                       300
    1 FORMAT(6I3)                                                       310
      NCELL=NX+NY+NZ+NNX+NNY+NNZ                                        320
      DO 2 I=1,NCELL                                                    330
      READ 3,X(I),Y(I),Z(I),DCELL(I)                                    340
    2 CONTINUE                                                          350
    3 FORMAT(4F10.5)                                                    360
      READ 4,RLEP,SIG,FMEG,MODE                                         370
    4 FORMAT(3E13.6,A4)                                                 380
      IF((IQ.EQ.0008).OR.(IQ.EQ.1234).OR.(IQ.EQ.1256)                   390
     +.OR.(IQ.EQ.1458)) GO TO 8                                         430
      PRINT 99,IQ                                                       440
      GO TO 9999                                                        450
   99 FORMAT(1H1,10X,22HILLEGAL SYMM CONDITION,                         460
     +1X,I4,1X,44HHAS BEEN APPLIED, PROBLEM CAN NOT BE HANDLED)         470
    8 IF ((MODE.EQ.EXPZ).OR.(MODE.EQ.COSE)                              480
     +.OR.(MODE.EQ.COSH).OR.(MODE.EQ.SINE)                              490
     +.OR.(MODE.EQ.SINH)) GO TO 9                                       500
                                                                        510
```

```
      PRINT 999, MODE
      GO TO 9999                                                        520
999   FORMAT(1HL, 10X, 22HILLEGAL INPRESSED MODE, 1X, A4, 1X,           530
     +44HHAS BEEN APPLIED.  PROBLEM CAN NOT BE HANDLED)                 540
9     CONTINUE                                                          550
      CALL MNSB(NCELL, X, Y, Z, DCELL, RLEP, SIG, FMEG, FREQ, OMEG,     560
     +KO, ARE, MAT, MATP)                                               400
      CALL MATRI(X, Y, Z, ARE, NCELL, L, MAT, MATP)                     410
      IF(MODE.EQ.EXPZ AND. IQ.EQ.1234) CALL EXIM(L, MAT, MATP, Z, NCELL) 420
      IF(MODE.EQ.COSE) CALL CEIM(L, MAT, MATP, Z, NCELL)
      IF(MODE.EQ.COSH) CALL CHIM(L, MAT, MATP, Z, NCELL)                430
      IF(MODE.EQ.SINE) CALL SEIM(L, MAT, MATP, Z, NCELL)                440
      IF(MODE.EQ.SINH) CALL SHIM(L, MAT, MATP, Z, NCELL)                450
      CALL CMATP(L, MAT, MATP, 1, DET, 1.0E-50)                         460
      CALL PRNT(INDU, CRNT, L, MAT, MATP)                               470
      CALL T(L, MAT, MATP)                                              480
      CALL PRNT(INDU, FLDS, L, MAT, MATP)                               490
      IF(MODE.EQ.EXPZ) GO TO 9999                                       500
      IF (MODE.EQ.COSE) MTAPE=1
      IF (MODE.EQ.COSH) MTAPE=2
      IF (MODE.EQ.SINE) MTAPE=3
      IF (MODE.EQ.SINH) MTAPE=4
      BUFFER OUT(MTAPE, 1) (L(1, MATP), L(MAT, MATP))
      CALL UNIT(MTAPE)
      DO 6 I=1, MAT
      PUNCH 7, L(I, MATP)
    6 CONTINUE
    7 FORMAT(2E13.6)
 9999 CONTINUE
      STOP
      END                                                               570
C*****BLOCK DATA*****************************************************  580
      BLOCK DATA
      REAL IMPR
      COMMON /IMCR/IMPR, CRNT
      COMMON /CSEH/COSE, COSH, SINE, SINH, EXPZ
      DATA IMPR/4HIMPR/, CRNT/4HCRNT/
      DATA COSE/4HCOSE/, COSH/4HCOSH/, SINE/4HSINE/, SINH/4HSINH/
      DATA EXPZ/4HEXPZ/
      END
C*****SUBROUTINE MNSB*************************************************
      SUBROUTINE MNSB(NCELL, X, Y, Z, DCELL, RLEP, SIG, FMEG, FREQ, OMEG,  590
     +KO, ARE, MAT, MATP)                                               600
      REAL KO, MUO, MODE, X(NCELL), Y(NCELL), Z(NCELL), DCELL(NCELL),   610
     +ARE(NCELL)                                                        620
      COMPLEX FK, CJ, FHUN                                              630
      COMMON IQ                                                         640
```

```
      COMMON /NUMDV/NDIV
      COMMON /NXYZ/NX, NY, NZ, NNX, NNY, NNZ
      COMMON /CONS/PI, MUO, EPO                                          650
      COMMON /CPLX/CJ
      COMMON /PARA/FK, FHUN
      COMMON /MODS/MODE                                                  670
      COMMON /CSEH/COSE, COSH, SINE, SINH, EXPZ
      PRINT 101, IQ
  101 FORMAT(1H1, 5X, 56HTHIS IS SOLUTION OF INDUCED FIELDS ON THE SURFACE  690
     2S OF AN, 1X, I4, 1X, 47HQ"S SYMMET BODY WITH ARBITRARY ELECT PROPERTIE
     3S)
      PRINT 102
  102 FORMAT(1H-, 5X, 64HTHE LOCATION AND DIMENSION OF EACH SURFACE-CELL A  730
     2RE LISTED BELOW)
      PRINT 103
  103 FORMAT(1H-, 5X, 3H N , 7X, 9HX (IN CM), 10X, 9HY (IN CM), 10X, 9HZ (IN CM)  760
     2, 10X, 9HD (IN CM))
      DO 104 I=1, NCELL
  104 PRINT 105, I, X(I), Y(I), Z(I), DCELL(I)                           790
  105 FORMAT(1H-, 5X, I3, 7X, F10.5, 10X, F10.5, 10X, F10.5, 10X, F10.5)  800
      PRINT 106, NX, NY, NZ, NNX, NNY, NNZ
  106 FORMAT(1H-, 5X, 13HTHERE ARE NX=, I3, 2X, 4H, NY=, I3, 2X, 5H,  NZ=, I3, 2X,  820
     25H, NNX=, I3, 2X, 5H, NNY=, I3, 2X, 9H, AND NNZ=, I3, 2X,
     339HCELLS ON SURFACES OF THE FIRST QUADRANT)
      PRINT 100, NDIV, NDIV
  100 FORMAT(1H-, 5X, 33HEACH SURFACE CELL IS DIVIDED INTO,
     21X, I2, 1X, 2HBY, 1X, I2, 1X, 17HSUB-SURFACE CELLS)
      PRINT 107, RLEP, SIG, FMEG, MODE
  107 FORMAT(1H-, 5X, 19HDIELECTRIC CONST  =, E13.6/1H-, 5X, 13HCONDUCTIVITY=  860
     2, E13.6, 9HMHO/METER/1H-, 5X, 10HFREQUENCY=, E13.6, 7HMEG.  HZ/1H-, 5X, 24H
     3TYPE OF IMPRESSED FIELD=, A4)
      FREQ=FMEG*1.0E+06                                                  900
      OMEG=2.0*PI*FREQ                                                   910
      KO=OMEG*SQRT(MUO*EPO)                                              920
      DO 5 I=1, NCELL                                                    930
      X(I)=(X(I)/100.0)*KO                                               940
      Y(I)=(Y(I)/100.0)*KO                                               950
      Z(I)=(Z(I)/100.0)*KO                                               960
      DCELL(I)=(DCELL(I)/100.0)*KO                                       970
      ARE(I)=DCELL(I)*DCELL(I)                                           980
    5 CONTINUE                                                           990
      FK=RLEP-CJ*SIG/(OMEG*EPO)                                          1000
      FK=CSQRT(FK)                                                       1010
      FHUN=2.0*PI*(FK*FK+1.0)                                            1020
      MAT=4*NCELL                                                        1030
      MATP=MAT+1                                                         1040
      RETURN                                                             1050
      END                                                                1060
```

```
C*****SUBROUTINE MATR(X,Y,Z,ARE,NCELL,L,MAT,MATP)*****************************************
      SUBROUTINE MATR(X,Y,Z,ARE,NCELL,L,MAT,MATP)                                    1070
      INTEGER NV(6)                                                                  1080
      REAL X(NCELL),Y(NCELL),Z(NCELL),ARE(NCELL),MUO,MODE                            1090
      COMPLEX L(MAT,MATP),E(16),CJ,FK,FHUN                                           1100
      COMMON IQ                                                                      1110
      COMMON /NXYZ/NX,NY,NZ,NNX,NNY,NNZ                                              1120
      COMMON /CONS/PI,MUO,EPO
      COMMON /CPLX/CJ
      COMMON /PARA/FK,FHUN
      COMMON /MODS/MODE                                                              1140
      COMMON /CSEH/COSE,COSH,SINE,SINH,EXPZ
      DO 1 I=1,MAT                                                                   1160
      DO 1 J=1,MATP                                                                  1170
      L(I,J)=0.0                                                                     1180
    1 CONTINUE                                                                       1190
      NV(1)=NX                                                                       1200
      NV(2)=NY                                                                       1210
      NV(3)=NZ                                                                       1220
      NV(4)=NNX                                                                      1230
      NV(5)=NNY                                                                      1240
      NV(6)=NNZ                                                                      1250
      DO 5 MDIR=1,6                                                                  1260
      MVV=NV(MDIR)                                                                   1270
      IF(MVV.EQ.0) GO TO 5                                                           1280
      DO 4 I=1,MVV                                                                   1290
      M=I
      IF(MDIR.EQ.1) MT=M                                                             1300
      IF(MDIR.EQ.2) MT=M+4*NV(1)                                                     1310
      IF(MDIR.EQ.3) MT=M+4*(NV(1)+NV(2))                                             1320
      IF(MDIR.EQ.4) MT=M+4*(NV(1)+NV(2)+NV(3))                                       1330
      IF(MDIR.EQ.5) MT=M+4*(NV(1)+NV(2)+NV(3)+NV(4))                                 1340
      IF(MDIR.EQ.6) MT=M+4*(NV(1)+NV(2)+NV(3)+NV(4)+NV(5))                           1350
      MP=MT+NV(MDIR)                                                                 1360
      MPP=MP+NV(MDIR)                                                                1370
      MPPP=MPP+NV(MDIR)                                                              1380
      DO 3 NDIR=1,6                                                                  1390
      NVV=NV(NDIR)                                                                   1400
      IF(NVV.EQ.0) GO TO 3                                                           1410
      DO 2 J=1,NVV                                                                   1420
      N=J                                                                            1430
      IF(NDIR.EQ.1) NT=N                                                             1440
      IF(NDIR.EQ.2) NT=N+4*NV(1)                                                     1450
      IF(NDIR.EQ.3) NT=N+4*(NV(1)+NV(2))                                             1460
      IF(NDIR.EQ.4) NT=N+4*(NV(1)+NV(2)+NV(3))                                       1470
      IF(NDIR.EQ.5) NT=N+4*(NV(1)+NV(2)+NV(3)+NV(4))                                 1480
      IF(NDIR.EQ.6) NT=N+4*(NV(1)+NV(2)+NV(3)+NV(4)+NV(5))                           1490
                                                                                     1500
```

```
      NP=NT+NV(NDIR)
      NPP=NP+NV(NDIR)                                                    1510
      NPPP=NPP+NV(NDIR)                                                  1520
      IF(NDIR.EQ.1.OR. NDIR .EQ.4) CALL ELEMX(MDIR,NDIR,M,N,X,Y,Z,ARE,NC1530
     +ELL,E)                                                             1540
      IF(NDIR.EQ.2.OR. NDIR .EQ.5) CALL ELEMY(MDIR,NDIR,M,N,X,Y,Z,ARE,NC1550
     +ELL,E)                                                             1560
      IF(NDIR.EQ.3.OR. NDIR .EQ.6) CALL ELEMZ(MDIR,NDIR,M,N,X,Y,Z,ARE,NC1570
     +ELL,E)                                                             1580
      CALL LEE(E,MT,MP,MPP,MPPP,NT,NP,NPP,NPPP,L,MAT,MATP)               1590
    2 CONTINUE                                                           1600
    3 CONTINUE                                                           1610
    4 CONTINUE                                                           1620
    5 CONTINUE                                                           1630
      RETURN                                                             1640
      END                                                                1650
C*****SUBROUTINE ELEMX*************************************************1660
      SUBROUTINE ELEMX(MDIR,NDIR,M,N,X,Y,Z,ARE,NCELL,E)                  1670
      REAL X(NCELL),Y(NCELL),Z(NCELL),ARE(NCELL),MODE,MUO                1680
      COMPLEX E(16),C(16),P12,CJ,P,FHUN,FK,FHUNM                         1690
      COMMON IQ                                                          1700
      COMMON /NXYZ/NX,NY,NZ,NNX,NNY,NNZ                                  1710
      COMMON /CONS/PI,MUO,EPO
      COMMON /CPLX/CJ
      COMMON /PARA/FK,FHUN
      COMMON /MODS/MODE                                                  1730
      COMMON /CSEH/COSE,COSH,SINE,SINH,EXPZ
      IF(MDIR.EQ.1) MM=M
      IF(MDIR.EQ.2) MM=M+NX                                              1750
      IF(MDIR.EQ.3) MM=M+NX+NY                                           1760
      IF(MDIR.EQ.4) MM=M+NX+NY+NZ                                        1770
      IF(MDIR.EQ.5) MM=M+NX+NY+NZ+NNX                                    1780
      IF(MDIR.EQ.6) MM=M+NX+NY+NZ+NNX+NNY                                1790
      IF(NDIR.EQ.1) NN=N                                                 1800
      IF(NDIR.EQ.4) NN=N+NX+NY+NZ                                        1810
      IF(MDIR.NE.NDIR) GO TO 201                                         1820
      IF(M.EQ.N) GO TO 3                                                 1830
  201 XM=X(MM)                                                           1840
      YM=Y(MM)                                                           1850
      ZM=Z(MM)                                                           1860
      XN=X(NN)                                                           1870
      YN=Y(NN)                                                           1880
      ZN=Z(NN)                                                           1890
      AREA=ARE(NN)                                                       1900
      CALL COEFX(MDIR,AREA,XM,YM,ZM,XN,YN,ZN,C)                          1910
      DO 2 I=1,16                                                        1920
      E(I)=C(I)                                                          1930
    2 CONTINUE                                                           1940
                                                                         1950
```

```
      GO TO 4                                                           1960
    3 CONTINUE                                                          1970
      XM=X(MM)                                                          1980
      YM=Y(MM)                                                          1990
      ZM=Z(MM)                                                          2000
      XN=XM                                                             2010
      YN=YM                                                             2020
      ZN=ZM                                                             2030
      AREA=ARE(MM)                                                      2040
      A=SQRT(AREA/PI)                                                   2050
      P12=CJ*(P(2,A)+PI*P(1,A))                                         2060
      PIM=PI                                                            2070
      FHUNM=FHUN                                                        2080
      IF(MDIR.EQ.4) PIM=-1.0*PI                                         2090
      IF(MDIR.EQ.4) FHUNM=-1.0*FHUN                                     2100
      E(1)=4.0*PIM                                                      2110
      E(2)=0.0                                                          2120
      E(3)=0.0                                                          2130
      E(4)=P12                                                          2140
      E(5)=0.0                                                          2150
      E(6)=E(1)                                                         2160
      E(7)=-1.0*P12                                                     2170
      E(8)=0.0                                                          2180
      E(9)=0.0                                                          2190
      E(10)=E(7)                                                        2200
      E(11)=FHUNM                                                       2210
      E(12)=0.0                                                         2220
      E(13)=P12                                                         2230
      E(14)=0.0                                                         2240
      E(15)=0.0                                                         2250
      E(16)=FHUNM                                                       2260
    4 CONTINUE                                                          2270
      IF(IQ.EQ.1234) YN=-1.0*YN                                         2280
      IF(IQ.EQ.1234) GO TO 102                                          2290
      ZN=-1.0*ZN                                                        2300
      CALL COEFX(MDIR,AREA,XM,YM,ZM,XN,YN,ZN,C)                         2310
      IF(MODE.EQ.COSH.OR.MODE.EQ.SINE) GO TO 9                          2320
      DO 6 I=1,4                                                        2330
      E(I)=E(I)+C(I)                                                    2340
    6 CONTINUE                                                          2350
      DO 7 I=5,12                                                       2360
      E(I)=E(I)-C(I)                                                    2370
    7 CONTINUE                                                          2380
      DO 8 I=13,16                                                      2390
      E(I)=E(I)+C(I)                                                    2400
    8 CONTINUE                                                          2410
      GO TO 21                                                          2420
```

```
  9 CONTINUE                                                            2430
    DO 10 I=1,4                                                         2440
    E(I)=E(I)-C(I)                                                      2450
 10 CONTINUE                                                            2460
    DO 11 I=5,12                                                        2470
    E(I)=E(I)+C(I)                                                      2480
 11 CONTINUE                                                            2490
    DO 12 I=13,16                                                       2500
    E(I)=E(I)-C(I)                                                      2510
 12 CONTINUE                                                            2520
 21 CONTINUE                                                            2530
    IF(IQ.EQ.1256) ZN=-1.0*ZN                                           2540
    IF(IQ.EQ.1256) XN=-1.0*XN                                           2550
    IF(IQ.EQ.1256) GO TO 101                                            2560
    YN=-1.0*YN                                                          2570
    CALL COEFX(MDIR,AREA,XM,YM,ZM,XN,YN,ZN,C)                           2580
    IF(MODE.EQ.COSH.OR.MODE.EQ.SINE) GO TO 24                           2590
    DO 23 I=1,16                                                        2600
    E(I)=E(I)-C(I)                                                      2610
 23 CONTINUE                                                            2620
    GO TO 30                                                            2630
 24 CONTINUE                                                            2640
    DO 25 I=1,16                                                        2650
    E(I)=E(I)+C(I)                                                      2660
 25 CONTINUE                                                            2670
 30 CONTINUE                                                            2680
    ZN=-1.0*ZN                                                          2690
102 CALL COEFX(MDIR,AREA,XM,YM,ZM,XN,YN,ZN,C)                           2700
    DO 32 I=1,4                                                         2710
    E(I)=E(I)-C(I)                                                      2720
 32 CONTINUE                                                            2730
    DO 33 I=5,12                                                        2740
    E(I)=E(I)+C(I)                                                      2750
 33 CONTINUE                                                            2760
    DO 34 I=13,16                                                       2770
    E(I)=E(I)-C(I)                                                      2780
 34 CONTINUE                                                            2790
    IF(IQ.EQ.1458) GO TO 100                                            2800
    XN=-1.0*XN                                                          2810
    YN=-1.0*YN                                                          2820
101 CALL COEFX(MDIR,AREA,XM,YM,ZM,XN,YN,ZN,C)                           2830
    DO 49 I=1,8                                                         2840
    E(I)=E(I)-C(I)                                                      2850
 49 CONTINUE                                                            2860
    DO 50 I=9,16                                                        2870
    E(I)=E(I)+C(I)                                                      2880
 50 CONTINUE                                                            2890
```

```
      IF(IQ.EQ.1234) YN=-1.0*YN                                          2900
      IF(IQ.EQ.1234) GO TO 103                                           2910
      ZN=-1.0*ZN                                                         2920
      CALL COEFX(MDIR,AREA,XM,YM,ZM,XN,YN,ZN,C)                          2930
      IF(MODE.EQ.COSH.OR.MODE.EQ.SINE) GO TO 66                          2940
      DO 62 I=1,4                                                        2950
      E(I)=E(I)-C(I)                                                     2960
   62 CONTINUE                                                           2970
      DO 63 I=5,8                                                        2980
      E(I)=E(I)+C(I)                                                     2990
   63 CONTINUE                                                           3000
      DO 64 I=9,12                                                       3010
      E(I)=E(I)-C(I)                                                     3020
   64 CONTINUE                                                           3030
      DO 65 I=13,16                                                      3040
      E(I)=E(I)+C(I)                                                     3050
   65 CONTINUE                                                           3060
      GO TO 81                                                           3070
   66 CONTINUE                                                           3080
      DO 67 I=1,4                                                        3090
      E(I)=E(I)+C(I)                                                     3100
   67 CONTINUE                                                           3110
      DO 68 I=5,8                                                        3120
      E(I)=E(I)-C(I)                                                     3130
   68 CONTINUE                                                           3140
      DO 69 I=9,12                                                       3150
      E(I)=E(I)+C(I)                                                     3160
   69 CONTINUE                                                           3170
      DO 70 I=13,16                                                      3180
      E(I)=E(I)-C(I)                                                     3190
   70 CONTINUE                                                           3200
   81 CONTINUE                                                           3210
      IF(IQ.EQ.1256) GO TO 100                                           3220
      YN=-1.0*YN                                                         3230
      CALL COEFX(MDIR,AREA,XM,YM,ZM,XN,YN,ZN,C)                          3240
      IF(MODE.EQ.COSH.OR.MODE.EQ.SINE) GO TO 85                          3250
      DO 83 I=1,8                                                        3260
      E(I)=E(I)+C(I)                                                     3270
   83 CONTINUE                                                           3280
      DO 84 I=9,16                                                       3290
      E(I)=E(I)-C(I)                                                     3300
   84 CONTINUE                                                           3310
      GO TO 94                                                           3320
   85 CONTINUE                                                           3330
      DO 86 I=1,8                                                        3340
      E(I)=E(I)-C(I)                                                     3350
   86 CONTINUE                                                           3360
```

```
      DO 87 I=9,16                                                      3370
      E(I)=E(I)+C(I)                                                    3380
   87 CONTINUE                                                          3390
   94 CONTINUE                                                          3400
      ZN=-1.0*ZN                                                        3410
  103 CALL COEFX(MDIR,AREA,XM,YM,ZM,XN,YN,ZN,C)                         3420
      DO 96 I=1,4                                                       3430
      E(I)=E(I)+C(I)                                                    3440
   96 CONTINUE                                                          3450
      DO 97 I=5,8                                                       3460
      E(I)=E(I)-C(I)                                                    3470
   97 CONTINUE                                                          3480
      DO 98 I=9,12                                                      3490
      E(I)=E(I)+C(I)                                                    3500
   98 CONTINUE                                                          3510
      DO 99 I=13,16                                                     3520
      E(I)=E(I)-C(I)                                                    3530
   99 CONTINUE                                                          3540
  100 CONTINUE                                                          3550
      RETURN                                                            3560
      END                                                               3570
C*****SUBROUTINE ELEMY*************************************************************
      SUBROUTINE ELEMY(MDIR,NDIR,M,N,X,Y,Z,ARE,NCELL,E)                 3580
      REAL X(NCELL),Y(NCELL),Z(NCELL),ARE(NCELL),MODE,MUO               3590
      COMPLEX E(16),C(16),P12,CJ,P,FHUN,FK,FHUNM                        3600
      COMMON IQ                                                         3610
      COMMON /NXYZ/NX,NY,NZ,NNX,NNY,NNZ                                 3620
      COMMON /CONS/PI,MUO,EPO
      COMMON /CPLX/CJ
      COMMON /PARA/FK,FHUN                                              3640
      COMMON /MODS/MODE
      COMMON /CSEH/COSE,COSH,SINE,SINH,EXPZ
      IF(MDIR.EQ.1) MM=M                                                3660
      IF(MDIR.EQ.2) MM=M+NX                                             3670
      IF(MDIR.EQ.3) MM=M+NX+NY                                          3680
      IF(MDIR.EQ.4) MM=M+NX+NY+NZ                                       3690
      IF(MDIR.EQ.5) MM=M+NX+NY+NZ+NNX                                   3700
      IF(MDIR.EQ.6) MM=M+NX+NY+NZ+NNX+NNY                               3710
      IF(NDIR.EQ.2) NN=N+NX                                             3720
      IF(NDIR.EQ.5) NN=N+NX+NY+NZ+NNX                                   3730
      IF(MDIR.NE.NDIR) GO TO 201                                        3740
      IF(M.EQ.N) GO TO 202                                              3750
  201 XM=X(MM)                                                          3760
      YM=Y(MM)                                                          3770
      ZM=Z(MM)                                                          3780
      XN=X(NN)                                                          3790
      YN=Y(NN)                                                          3800
```

```
      ZN=Z(NN)                                                          3810
      AREA=ARE(NN)                                                      3820
      CALL COEFY(MDIR,AREA,XM,YM,ZM,XN,YN,ZN,C)                         3830
      DO 2 I=1,16                                                       3840
      E(I)=C(I)                                                         3850
    2 CONTINUE                                                          3860
      GO TO 203                                                         3870
  202 CONTINUE                                                          3880
      XM=X(MM)                                                          3890
      YM=Y(MM)                                                          3900
      ZM=Z(MM)                                                          3910
      XN=XM                                                             3920
      YN=YM                                                             3930
      ZN=ZM                                                             3940
      AREA=ARE(MM)                                                      3950
      A=SQRT(AREA/PI)                                                   3960
      P12=CJ*(P(2,A)+PI*P(1,A))                                         3970
      PIM=PI                                                            3980
      FHUNM=FHUN                                                        3990
      IF(MDIR.EQ.5) PIM=-1.0*PI                                         4000
      IF(MDIR.EQ.5) FHUNM=-1.0*FHUN                                     4010
      E(1)=4.0*PIM                                                      4020
      E(2)=0.0                                                          4030
      E(3)=0.0                                                          4040
      E(4)=-1.0*P12                                                     4050
      E(5)=0.0                                                          4060
      E(6)=E(1)                                                         4070
      E(7)=P12                                                          4080
      E(8)=0.0                                                          4090
      E(9)=0.0                                                          4100
      E(10)=P12                                                         4110
      E(11)=FHUNM                                                       4120
      E(12)=0.0                                                         4130
      E(13)=E(4)                                                        4140
      E(14)=0.0                                                         4150
      E(15)=0.0                                                         4160
      E(16)=FHUNM                                                       4170
  203 CONTINUE                                                          4180
      IF(IQ.EQ.1458) ZN=-1.0*ZN                                         4190
      IF(IQ.EQ.1458) GO TO 300                                          4200
      XN=-1.0*XN                                                        4210
      CALL COEFY(MDIR,AREA,XM,YM,ZM,XN,YN,ZN,C)                         4220
      DO 4 I=1,4                                                        4230
      E(I)=E(I)+C(I)                                                    4240
    4 CONTINUE                                                          4250
      DO 5 I=5,12                                                       4260
      E(I)=E(I)-C(I)                                                    4270
    5 CONTINUE                                                          4280
```

```
      DO 6 I=13,16                                                      4290
      E(I)=E(I)+C(I)                                                    4300
    6 CONTINUE                                                          4310
      IF(IQ.EQ.1234) XN=-1.0*XN                                         4320
      IF(IQ.EQ.1234) YN=-1.0*YN                                         4330
      IF(IQ.EQ.1234) GO TO 302                                          4340
      ZN=-1.0*ZN                                                        4350
      CALL COEFY(MDIR,AREA,XM,YM,ZM,XN,YN,ZN,C)                         4360
      IF(MODE.EQ.COSH.OR.MODE.EQ.SINE) GO TO 22                         4370
      DO 21 I=1,16                                                      4380
      E(I)=E(I)+C(I)                                                    4390
   21 CONTINUE                                                          4400
      GO TO 28                                                          4410
   22 CONTINUE                                                          4420
      DO 23 I=1,16                                                      4430
      E(I)=E(I)-C(I)                                                    4440
   23 CONTINUE                                                          4450
   28 CONTINUE                                                          4460
      XN=-1.0*XN                                                        4470
  300 CALL COEFY(MDIR,AREA,XM,YM,ZM,XN,YN,ZN,C)                         4480
      IF(MODE.EQ.COSH.OR.MODE.EQ.SINE) GO TO 33                         4490
      DO 30 I=1,4                                                       4500
      E(I)=E(I)+C(I)                                                    4510
   30 CONTINUE                                                          4520
      DO 31 I=5,12                                                      4530
      E(I)=E(I)-C(I)                                                    4540
   31 CONTINUE                                                          4550
      DO 32 I=13,16                                                     4560
      E(I)=E(I)+C(I)                                                    4570
   32 CONTINUE                                                          4580
      GO TO 45                                                          4590

   33 CONTINUE                                                          4600
      DO 34 I=1,4                                                       4610
      E(I)=E(I)-C(I)                                                    4620
   34 CONTINUE                                                          4630
      DO 35 I=5,12                                                      4640
      E(I)=E(I)+C(I)                                                    4650
   35 CONTINUE                                                          4660
      DO 36 I=13,16                                                     4670
      E(I)=E(I)-C(I)                                                    4680
   36 CONTINUE                                                          4690
   45 CONTINUE                                                          4700
      IF(IQ.EQ.1256) GO TO 113                                          4710
      ZN=-1.0*ZN                                                        4720
      YN=-1.0*YN                                                        4730
  302 CALL COEFY(MDIR,AREA,XM,YM,ZM,XN,YN,ZN,C)                         4740
      DO 47 I=1,8                                                       4750
```

```
      E(I)=E(I)+C(I)
   47 CONTINUE
      DO 48 I=9,16
      E(I)=E(I)-C(I)
   48 CONTINUE
      IF(IQ.EQ.1458) ZN=-1.0*ZN
      IF(IQ.EQ.1458) GO TO 301
      XN=-1.0*XN
      CALL COEFY(MDIR,AREA,XM,YM,ZM,XN,YN,ZN,C)
      DO 60 I=1,4
      E(I)=E(I)+C(I)
   60 CONTINUE
      DO 61 I=5,8
      E(I)=E(I)-C(I)
   61 CONTINUE
      DO 62 I=9,12
      E(I)=E(I)+C(I)
   62 CONTINUE
      DO 63 I=13,16
      E(I)=E(I)-C(I)
   63 CONTINUE
      IF(IQ.EQ.1234) GO TO 113
      ZN=-1.0*ZN
      CALL COEFY(MDIR,AREA,XM,YM,ZM,XN,YN,ZN,C)
      IF(MODE.EQ.COSH.OR.MODE.EQ.SINE) GO TO 83
      DO 81 I=1,8
      E(I)=E(I)+C(I)
   81 CONTINUE
      DO 82 I=9,16
      E(I)=E(I)-C(I)
   82 CONTINUE
      GO TO 92
   83 CONTINUE
      DO 84 I=1,8
      E(I)=E(I)-C(I)
   84 CONTINUE
      DO 85 I=9,16
      E(I)=E(I)+C(I)
   85 CONTINUE
   92 CONTINUE
      XN=-1.0*XN
  301 CALL COEFY(MDIR,AREA,XM,YM,ZM,XN,YN,ZN,C)
      IF(MODE.EQ.COSH.OR.MODE.EQ.SINE) GO TO 98
      DO 94 I=1,4
      E(I)=E(I)+C(I)
   94 CONTINUE
      DO 95 I=5,8
      E(I)=E(I)-C(I)
```

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4760
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4990
5000
5010
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5220
5230
```

```
 95 CONTINUE                                                            5240
    DO 96 I=9,12                                                        5250
    E(I)=E(I)+C(I)                                                      5260
 96 CONTINUE                                                            5270
    DO 97 I=13,16                                                       5280
    E(I)=E(I)-C(I)                                                      5290
 97 CONTINUE                                                            5300
    GO TO 113                                                           5310
 98 CONTINUE                                                            5320
    DO 99 I=1,4                                                         5330
    E(I)=E(I)-C(I)                                                      5340
 99 CONTINUE                                                            5350
    DO 100 I=5,8                                                        5360
    E(I)=E(I)+C(I)                                                      5370
100 CONTINUE                                                            5380
    DO 101 I=9,12                                                       5390
    E(I)=E(I)-C(I)                                                      5400
101 CONTINUE                                                            5410
    DO 102 I=13,16                                                      5420
    E(I)=E(I)+C(I)                                                      5430
102 CONTINUE                                                            5440
113 CONTINUE                                                            5450
    RETURN                                                              5460
    END                                                                 5470
C*****SUBROUTINE ELEMZ*************************************************************
      SUBROUTINE ELEMZ(MDIR,NDIR,M,N,X,Y,Z,ARE,NCELL,E)                 5480
      REAL X(NCELL),Y(NCELL),Z(NCELL),ARE(NCELL),MODE,MUO               5490
      COMPLEX E(16),C(16),P12,CJ,P,FHUN,FK,FHUNM                        5500
      COMMON IQ                                                         5510
      COMMON /NXYZ/NX,NY,NZ,NNX,NNY,NNZ                                 5520
      COMMON /CONS/PI,MUO,EPO
      COMMON /CPLX/CJ
      COMMON /PARA/FK,FHUN
      COMMON /MODS/MODE                                                 5540
      COMMON /CSEH/COSE,COSH,SINE,SINH,EXPZ
      IF(MDIR.EQ.1) MM=M
      IF(MDIR.EQ.2) MM=M+NX                                             5560
      IF(MDIR.EQ.3) MM=M+NX+NY                                          5570
      IF(MDIR.EQ.4) MM=M+NX+NY+NZ                                       5580
      IF(MDIR.EQ.5) MM=M+NX+NY+NZ+NNX                                   5590
      IF(MDIR.EQ.6) MM=M+NX+NY+NZ+NNX+NNY                               5600
      IF(NDIR.EQ.3) NN=N+NX+NY                                          5610
      IF(NDIR.EQ.6) NN=N+NX+NY+NZ+NNX+NNY                               5620
      IF(MDIR.NE.NDIR) GO TO 201                                        5630
      IF(M.EQ.N) GO TO 202                                              5640
201   XM=X(MM)                                                          5650
      YM=Y(MM)                                                          5660
                                                                        5670
```

```
      ZM=Z(MM)                                                          5680
      XN=X(NN)                                                          5690
      YN=Y(NN)                                                          5700
      ZN=Z(NN)                                                          5710
      AREA=ARE(NN)                                                      5720
      CALL  COEFZ(MDIR, AREA, XM, YM, ZM, XN, YN, ZN, C)                5730
      DO 2 I=1,16                                                       5740
      E(I)=C(I)                                                         5750
    2 CONTINUE                                                          5760
      GO TO 203                                                         5770
  202 CONTINUE                                                          5780
      XM=X(MM)                                                          5790
      YM=Y(MM)                                                          5800
      ZM=Z(MM)                                                          5810
      XN=XM                                                             5820
      YN=YM                                                             5830
      ZN=ZM                                                             5840
      AREA=ARE(MM)                                                      5850
      A=SQRT(AREA/PI)                                                   5860
      P12=CJ*(P(2,A)+PI*P(1,A))                                         5870
      PIM=PI                                                            5880
      FHUNM=FHUN                                                        5890
      IF(MDIR.EQ.6)  PIM=-1.0*PI                                        5900
      IF(MDIR.EQ.6)  FHUNM=-1.0*FHUN                                    5910
      E(1)=4.0*PIM                                                      5920
      E(2)=0.0                                                          5930
      E(3)=0.0                                                          5940
      E(4)=P12                                                          5950
      E(5)=0.0                                                          5960
      E(6)=E(1)                                                         5970
      E(7)=-1.0*P12                                                     5980
      E(8)=0.0                                                          5990
      E(9)=0.0                                                          6000
      E(10)=E(7)                                                        6010
      E(11)=FHUNM                                                       6020
      E(12)=0.0                                                         6030
      E(13)=P12                                                         6040
      E(14)=0.0                                                         6050
      E(15)=0.0                                                         6060
      E(16)=FHUNM                                                       6070
  203 CONTINUE                                                          6080
      IF(IQ.EQ.1458)  YN=-1.0*YN                                        6090
      IF(IQ.EQ.1458)  GO TO 300                                         6100
      XN=-1.0*XN                                                        6110
      CALL  COEFZ(MDIR, AREA, XM, YM, ZM, XN, YN, ZN, C)                6120
      DO 4 I=1,4                                                        6130
      E(I)=E(I)+C(I)                                                    6140
    4 CONTINUE                                                          6150
```

```
      DO 5 I=5,12                                                       6160
      E(I)=E(I)-C(I)                                                    6170
    5 CONTINUE                                                          6180
      DO 6 I=13,16                                                      6190
      E(I)=E(I)+C(I)                                                    6200
    6 CONTINUE                                                          6210
      IF(IQ.EQ.1256) XN=-1.0*XN                                         6220
      IF(IQ.EQ.1256) ZN=-1.0*ZN                                         6230
      IF(IQ.EQ.1256) GO TO 302                                          6240
      YN=-1.0*YN                                                        6250
      CALL COEFZ(MDIR,AREA,XM,YM,ZM,XN,YN,ZN,C)                         6260
      DO 21 I=1,16                                                      6270
      E(I)=E(I)+C(I)                                                    6280
   21 CONTINUE                                                          6290
      XN=-1.0*XN                                                        6300
  300 CALL COEFZ(MDIR,AREA,XM,YM,ZM,XN,YN,ZN,C)                         6310
      DO 30 I=1,4                                                       6320
      E(I)=E(I)+C(I)                                                    6330
   30 CONTINUE                                                          6340
      DO 31 I=5,12                                                      6350
      E(I)=E(I)-C(I)                                                    6360
   31 CONTINUE                                                          6370
      DO 32 I=13,16                                                     6380
      E(I)=E(I)+C(I)                                                    6390
   32 CONTINUE                                                          6400
      IF(IQ.EQ.1234) GO TO 113                                          6410
      ZN=-1.0*ZN                                                        6420
      YN=-1.0*YN                                                        6430
  302 CALL COEFZ(MDIR,AREA,XM,YM,ZM,XN,YN,ZN,C)                         6440
      IF(MODE.EQ.COSH.OR.MODE.EQ.SINE) GO TO 49                         6450
      DO 47 I=1,8                                                       6460
      E(I)=E(I)+C(I)                                                    6470
   47 CONTINUE                                                          6480
      DO 48 I=9,16                                                      6490
      E(I)=E(I)-C(I)                                                    6500
   48 CONTINUE                                                          6510
      GO TO 58                                                          6520
   49 CONTINUE                                                          6530
      DO 50 I=1,8                                                       6540
      E(I)=E(I)-C(I)                                                    6550
   50 CONTINUE                                                          6560
      DO 51 I=9,16                                                      6570
      E(I)=E(I)+C(I)                                                    6580
   51 CONTINUE                                                          6590
   58 CONTINUE                                                          6600
      IF(IQ.EQ.1458) YN=-1.0*YN                                         6610
      IF(IQ.EQ.1458) GO TO 301                                          6620
      XN=-1.0*XN                                                        6630
```

```
      CALL COEFZ(MDIR,AREA,XM,YM,ZM,XN,YN,ZN,C)                    6640
      IF(MODE.EQ.COSH.OR.MODE.EQ.SINE) GO TO 64                    6650
      DO 60 I=1,4                                                  6660
      E(I)=E(I)+C(I)                                               6670
   60 CONTINUE                                                     6680
      DO 61 I=5,8                                                  6690
      E(I)=E(I)-C(I)                                               6700
   61 CONTINUE                                                     6710
      DO 62 I=9,12                                                 6720
      E(I)=E(I)+C(I)                                               6730
   62 CONTINUE                                                     6740
      DO 63 I=13,16                                                6750
      E(I)=E(I)-C(I)                                               6760
   63 CONTINUE                                                     6770
      GO TO 79                                                     6780
   64 CONTINUE                                                     6790
      DO 65 I=1,4                                                  6800
      E(I)=E(I)-C(I)                                               6810
   65 CONTINUE                                                     6820
      DO 66 I=5,8                                                  6830
      E(I)=E(I)+C(I)                                               6840
   66 CONTINUE                                                     6850
      DO 67 I=9,12                                                 6860
      E(I)=E(I)-C(I)                                               6870
   67 CONTINUE                                                     6880
      DO 68 I=13,16                                                6890
      E(I)=E(I)+C(I)                                               6900
   68 CONTINUE                                                     6910
   79 CONTINUE                                                     6920
      IF(IQ.EQ.1256) GO TO 113                                     6930
      YN=-1.0*YN                                                   6940
      CALL COEFZ(MDIR,AREA,XM,YM,ZM,XN,YN,ZN,C)                    6950
      IF(MODE.EQ.COSH.OR.MODE.EQ.SINE) GO TO 83                    6960
      DO 81 I=1,8                                                  6970
      E(I)=E(I)+C(I)                                               6980
   81 CONTINUE                                                     6990
      DO 82 I=9,16                                                 7000
      E(I)=E(I)-C(I)                                               7010
   82 CONTINUE                                                     7020
      GO TO 92                                                     7030
   83 CONTINUE                                                     7040
      DO 84 I=1,8                                                  7050
      E(I)=E(I)-C(I)                                               7060
   84 CONTINUE                                                     7070
      DO 85 I=9,16                                                 7080
      E(I)=E(I)+C(I)                                               7090
   85 CONTINUE                                                     7100
   92 CONTINUE                                                     7110
```

```
      XN=-1.0*XN
  301 CALL COEFZ(MDIR,AREA,XM,YM,ZM,XN,YN,ZN,C)                          7120
      IF(MODE.EQ.COSH.OR.MODE.EQ.SINE) GO TO 98                          7130
      DO 94 I=1,4                                                        7140
      E(I)=E(I)+C(I)                                                     7150
   94 CONTINUE                                                           7160
      DO 95 I=5,8                                                        7170
      E(I)=E(I)-C(I)                                                     7180
   95 CONTINUE                                                           7190
      DO 96 I=9,12                                                       7200
      E(I)=E(I)+C(I)                                                     7210
   96 CONTINUE                                                           7220
      DO 97 I=13,16                                                      7230
      E(I)=E(I)-C(I)                                                     7240
   97 CONTINUE                                                           7250
      GO TO 113                                                          7260
   98 CONTINUE                                                           7270
      DO 99 I=1,4                                                        7280
      E(I)=E(I)-C(I)                                                     7290
   99 CONTINUE                                                           7300
      DO 100 I=5,8                                                       7310
      E(I)=E(I)+C(I)                                                     7320
  100 CONTINUE                                                           7330
      DO 101 I=9,12                                                      7340
      E(I)=E(I)-C(I)                                                     7350
  101 CONTINUE                                                           7360
      DO 102 I=13,16                                                     7370
      E(I)=E(I)+C(I)                                                     7380
  102 CONTINUE                                                           7390
  113 CONTINUE                                                           7400
      RETURN                                                             7410
      END                                                                7420
C*****SUBROUTINE EXIM*************************************************7430
      SUBROUTINE EXIM(L,MAT,MATP,Z,NCELL)                                7680
      REAL Z(NCELL),IMPR,MUO                                             7690
      COMPLEX L(MAT,MATP),CJ                                             7700
      COMMON /IMCR/IMPR,CRNT                                             7710
      COMMON /NXYZ/NX,NY,NZ,NNX,NNY,NNZ                                  7720
      COMMON /CONS/PI,MUO,EPO
      COMMON /CPLX/CJ
      DO 1 I=1,NX                                                        7740
      IP=I+NX                                                            7750
      L(IP,MATP)=CEXP(-1.0*CJ*Z(I))/377.0                                7760
    1 CONTINUE                                                           7770
      DO 11 I=1,NY                                                       7780
      II=I+NX                                                            7790
      IP=I+4*NX+3*NY                                                     7800
      L(IP,MATP)=CEXP(-1.0*CJ*Z(II))/377.0                               7810
```

```
11      CONTINUE                                                          7820
        DO 2 I=1,NZ                                                       7830
        II=I+NX+NY                                                        7840
        IP=I+4*(NX+NY)                                                    7850
        IPP=IP+3*NZ                                                       7860
        L(IP,MATP)=-1.0*CEXP(-1.0*CJ*Z(II))/377.0                         7870
        L(IPP,MATP)=L(IP,MATP)                                            7880
2       CONTINUE                                                          7890
        IF(NNX.EQ.0) GO TO 6                                              7900
        DO 4 I=1,NNX                                                      7910
        II=I+NX+NY+NZ                                                     7920
        IP=I+4*(NX+NY+NZ)+NNX                                             7930
        L(IP,MATP)=-1.0*CEXP(-1.0*CJ*Z(II))/377.0                         7940
4       CONTINUE                                                          7950
6       CONTINUE                                                          7960
        IF(NNY.EQ.0) GO TO 16                                             7970
        DO 14 I=1,NNY                                                     7980
        II=I+NX+NY+NZ+NNX                                                 7990
        IP=I+4*(NX+NY+NZ+NNX)+3*NNY                                       8000
        L(IP,MATP)=-1.0*CEXP(-1.0*CJ*Z(II))/377.0                         8010
14      CONTINUE                                                          8020
16      CONTINUE                                                          8030
        IF(NNZ.EQ.0) GO TO 7                                              8040
        DO 5 I=1,NNZ                                                      8050
        II=I+NX+NY+NZ+NNX+NNY                                             8060
        IP=I+4*(NX+NY+NZ+NNX+NNY)                                         8070
        IPP=IP+3*NNZ                                                      8080
        L(IP,MATP)=CEXP(-1.0*CJ*Z(II))/377.0                              8090
        L(IPP,MATP)=L(IP,MATP)                                            8100
5       CONTINUE                                                          8110
7       CONTINUE                                                          8120
        CALL PRNT(IMPR,CRNT,L,MAT,MATP)                                   8130
        DO 3 I=1,MAT                                                      8140
        L(I,MATP)=4.0*PI*L(I,MATP)                                        8150
3       CONTINUE                                                          8160
        IF ((NNX.EQ.0).AND.(NNY.EQ.0).AND.(NNZ.EQ.0)) GO TO 9             8170
        MATT=4*(NX+NY+NZ)                                                 8180
        MATTP=MATT+1                                                      8190
        DO 8 I=MATTP,MAT                                                  8200
        L(I,MATP)=-1.0*L(I,MATP)                                          8210
8       CONTINUE                                                          8220
9       RETURN                                                            8230
        END                                                               8240
C*****SUBROUTINE CEIM*************************************************************
        SUBROUTINE CEIM(L,MAT,MATP,Z,NCELL)                               7440
        REAL Z(NCELL),IMPR,MUO                                            7450
        COMPLEX L(MAT,MATP),CJ                                            7460
```

```
      COMMON /IMCR/IMPR,CRNT
      COMMON /NXYZ/NX,NY,NZ,NNX,NNY,NNZ                                 7470
      COMMON /CONS/PI,MUO,EPO                                           7480
      COMMON /CPLX/CJ
      DO 1 I=1,NY
      II=I+NX                                                           7500
      IP=I+4*NX+3*NY                                                    7510
      L(IP,MATP)=COS(Z(II))/377.0                                       7520
    1 CONTINUE                                                          7530
      DO 2 I=1,NZ                                                       7540
      II=I+NX+NY                                                        7550
      IP=I+4*(NX+NY)+3*NZ                                               7560
      L(IP,MATP)=(-1.0)*COS(Z(II))/377.0                                7570
    2 CONTINUE                                                          7580
      IF(NNY.EQ.0) GO TO 6                                              7590
      DO 4 I=1,NNY                                                      7600
      II=I+NX+NY+NZ+NNX                                                 7610
      IP=I+4*(NX+NY+NZ+NNX)+3*NNY                                       7620
      L(IP,MATP)=(-1.0)*COS(Z(II))/377.0                                7630
    4 CONTINUE                                                          7640
    6 CONTINUE                                                          7650
      IF(NNZ.EQ.0) GO TO 7                                              7660
      DO 5 I=1,NNZ                                                      7670
      II=I+NX+NY+NZ+NNX+NNY                                             7680
      IP=I+4*(NX+NY+NZ+NNX+NNY)+3*NNZ                                   7690
      L(IP,MATP)=COS(Z(II))/377.0                                       7700
    5 CONTINUE                                                          7710
    7 CONTINUE                                                          7720
      CALL PRNT(IMPR,CRNT,L,MAT,MATP)                                   7730
      DO 3 I=1,MAT                                                      7740
      L(I,MATP)=4.0*PI*L(I,MATP)                                        7770
    3 CONTINUE                                                          7780
      IF((NNX.EQ.0).AND.(NNY.EQ.0).AND.(NNZ.EQ.0)) GO TO 9              7790
      MATT=4*(NX+NY+NZ)
       MATTP=MATT+1
      DO 8 I=MATTP,MAT
      L(I,MATP)=-1.0*L(I,MATP)                                          7800
    8 CONTINUE                                                          7810
    9 RETURN                                                            7820
      END
C*****SUBROUTINE CHIM*************************************************7840
      SUBROUTINE CHIM(L,MAT,MATP,Z,NCELL)                               7850
      REAL Z(NCELL),IMPR,MUO                                            7860
      COMPLEX L(MAT,MATP),CJ                                            7870
      COMMON /IMCR/IMPR,CRNT                                            7880
      COMMON /NXYZ/NX,NY,NZ,NNX,NNY,NNZ                                 7890
      COMMON /CONS/PI,MUO,EPO
      COMMON /CPLX/CJ
```

```
      DO 1 I=1,NX                                                        7910
      IP=I+NX                                                            7920
      L(IP,MATP)=COS(Z(I))/377.0                                         7930
    1 CONTINUE                                                           7940
      DO 2 I=1,NZ                                                        7950
      II=I+NX+NY                                                         7960
      IP=I+4*(NX+NY)                                                     7970
      L(IP,MATP)=(-1.0)*COS(Z(II))/377.0                                 7980
    2 CONTINUE                                                           7990
      IF(NNX.EQ.0) GO TO 6                                               8000
      DO 4 I=1,NNX                                                       8010
      II=I+NX+NY+NZ                                                      8020
      IP=I+4*(NX+NY+NZ)+NNX                                              8030
      L(IP,MATP)=(-1.0)*COS(Z(II))/377.0                                 8040
    4 CONTINUE                                                           8050
    6 CONTINUE                                                           8060
      IF(NNZ.EQ.0) GO TO 7                                               8070
      DO 5 I=1,NNZ                                                       8080
      II=I+NX+NY+NZ+NNX+NNY                                              8090
      IP=I+4*(NX+NY+NZ+NNX+NNY)                                          8100
      L(IP,MATP)=COS(Z(II))/377.0                                        8110
    5 CONTINUE                                                           8120
    7 CONTINUE                                                           8130
      CALL PRNT(IMPR,CRNT,L,MAT,MATP)                                    8140
      DO 3 I=1,MAT                                                       8170
      L(I,MATP)=4.0*PI*L(I,MATP)                                         8180
    3 CONTINUE                                                           8190
      IF((NNX.EQ.0).AND.(NNY.EQ.0).AND.(NNZ.EQ.0)) GO TO 9
      MATT=4*(NX+NY+NZ)
      MATTP=MATT+1
      DO 8 I=MATTP,MAT
      L(I,MATP)=-1.0*L(I,MATP)                                           8200
    8 CONTINUE                                                           8210
    9 RETURN                                                             8220
      END
C*****SUBROUTINE SEIM*************************************************** 8240
      SUBROUTINE SEIM(L,MAT,MATP,Z,NCELL)                                ********
      REAL Z(NCELL),IMPR,MUO                                             8250
      COMPLEX L(MAT,MATP),CJ                                             8260
      COMMON /IMCR/IMPR,CRNT                                             8270
      COMMON /NXYZ/NX,NY,NZ,NNX,NNY,NNZ                                  8280
      COMMON /CONS/PI,MUO,EPO                                            8290
      COMMON /CPLX/CJ
      DO 1 I=1,NY                                                        8310
      II=I+NX                                                            8320
      IP=I+4*NX+3*NY                                                     8330
      L(IP,MATP)=(-1.0)*CJ*SIN(Z(II))/377.0                              8340
    1 CONTINUE                                                           8350
```

```
      DO 2 I=1,NZ                                                       8360
      II=I+NX+NY                                                        8370
      IP=I+4*(NX+NY)+3*NZ                                               8380
      L(IP,MATP)=CJ*SIN(Z(II))/377.0                                    8390
    2 CONTINUE                                                          8400
      IF(NNY.EQ.0) GO TO 6                                              8410
      DO 4 I=1,NNY                                                      8420
      II=I+NX+NY+NZ+NNX                                                 8430
      IP=I+4*(NX+NY+NZ+NNX)+3*NNY                                       8440
      L(IP,MATP)=CJ*SIN(Z(II))/377.0                                    8450
    4 CONTINUE                                                          8460
    6 CONTINUE                                                          8470
      IF(NNZ.EQ.0) GO TO 7                                              8480
      DO 5 I=1,NNZ                                                      8490
      II=I+NX+NY+NZ+NNX+NNY                                             8500
      IP=I+4*(NX+NY+NZ+NNX+NNY)+3*NNZ                                   8510
      L(IP,MATP)=(-1.0)*CJ*SIN(Z(II))/377.0                             8520
    5 CONTINUE                                                          8530
    7 CONTINUE                                                          8540
      CALL PRNT(IMPR,CRNT,L,MAT,MATP)                                   8550
      DO 3 I=1,MAT                                                      8580
      L(I,MATP)=4.0*PI*L(I,MATP)                                        8590
    3 CONTINUE                                                          8600
      IF((NNX.EQ.0).AND.(NNY.EQ.0).AND.(NNZ.EQ.0)) GO TO 9
      MATT=4*(NX+NY+NZ)
      MATTP=MATT+1
      DO 8 I=MATTP,MAT                                                  8610
      L(I,MATP)=-1.0*L(I,MATP)                                          8620
    8 CONTINUE                                                          8630
    9 RETURN
      END                                                               8650
C*****SUBROUTINE SHIM*************************************************************
      SUBROUTINE SHIM(L,MAT,MATP,Z,NCELL)                               8660
      REAL Z(NCELL),IMPR,MUO                                            8670
      COMPLEX L(MAT,MATP),CJ                                            8680
      COMMON /IMCR/IMPR,CRNT                                            8690
      COMMON /NXYZ/NX,NY,NZ,NNX,NNY,NNZ                                 8700
      COMMON /CONS/PI,MUO,EPO
      COMMON /CPLX/CJ
      DO 1 I=1,NX                                                       8720
      IP=I+NX                                                           8730
      L(IP,MATP)=(-1.0)*CJ*SIN(Z(I))/377.0                              8740
    1 CONTINUE                                                          8750
      DO 2 I=1,NZ                                                       8760
      II=I+NX+NY                                                        8770
      IP=I+4*(NX+NY)                                                    8780
      L(IP,MATP)=CJ*SIN(Z(II))/377.0                                    8790
    2 CONTINUE                                                          8800
```

```
      IF(NNX.EQ.0) GO TO 6                                              8810
      DO 4 I=1,NNX                                                      8820
      II=I+NX+NY+NZ                                                     8830
      IP=I+4*(NX+NY+NZ)+NNX                                             8840
      L(IP,MATP)=CJ*SIN(Z(II))/377.0                                    8850
    4 CONTINUE                                                          8860
    6 CONTINUE                                                          8870
      IF(NNZ.EQ.0) GO TO 7                                              8880
      DO 5 I=1,NNZ                                                      8890
      II=I+NX+NY+NZ+NNX+NNY                                             8900
      IP=I+4*(NX+NY+NZ+NNX+NNY)                                         8910
      L(IP,MATP)=(-1.0)*CJ*SIN(Z(II))/377.0                             8920
    5 CONTINUE                                                          8930
    7 CONTINUE                                                          8940
      CALL PRNT(IMPR,CRNT,L,MAT,MATP)                                   8950
      DO 3 I=1,MAT                                                      8980
      L(I,MATP)=4.0*PI*L(I,MATP)                                        8990
    3 CONTINUE                                                          9000
      IF((NNX.EQ.0).AND.(NNY.EQ.0).AND.(NNZ.EQ.0)) GO TO 9
      MATT=4*(NX+NY+NZ)
      MATTP=MATT+1
      DO 8 I=MATTP,MAT
      L(I,MATP)=-1.0*L(I,MATP)                                          9010
    8 CONTINUE                                                          9020
    9 RETURN                                                            9030
      END                                                               9050
C*****SUBROUTINE T*****************************************************************
      SUBROUTINE T(L,MAT,MATP)                                          9060
      COMPLEX L(MAT,MATP)                                               9070
      COMMON /NXYZ/NX,NY,NZ,NNX,NNY,NNZ                                 9080
      DO 130 I=1,NX                                                     9090
      L(I,MATP)=-1.0*L(I,MATP)                                          9100
      J=I+2*NX                                                          9110
      L(J,MATP)=L(J,MATP)*377.0                                         9120
      K=J+NX                                                            9130
      L(K,MATP)=-377.0*L(K,MATP)                                        9140
  130 CONTINUE                                                          9150
      DO 131 I=1,NY                                                     9160
      II=I+NY+4*NX                                                      9170
      L(II,MATP)=-1.0*L(II,MATP)                                        9180
      J=II+NY                                                           9190
      L(J,MATP)=-377.0*L(J,MATP)                                        9200
      K=J+NY                                                            9210
      L(K,MATP)=L(K,MATP)*377.0                                         9220
  131 CONTINUE                                                          9230
      DO 132 I=1,NZ                                                     9240
      II=I+4*(NX+NY)                                                    9250
      L(II,MATP)=-1.0*L(II,MATP)                                        9260
```

```
      J=II+2*NZ                                                          9270
      L(J,MATP)=L(J,MATP)*377.0                                          9280
      K=J+NZ                                                             9290
      L(K,MATP)=-377.0*L(K,MATP)                                         9300
  132 CONTINUE                                                           9310
      IF(NNX.EQ.0) GO TO 136                                             9320
      DO 133 I=1,NNX                                                     9330
      II=I+4*(NX+NY+NZ)+NNX                                              9340
      L(II,MATP)=-1.0*L(II,MATP)                                         9350
      J=II+NNX                                                           9360
      L(J,MATP)=-377.0*L(J,MATP)                                         9370
      K=J+NNX                                                            9380
      L(K,MATP)=377.0*L(K,MATP)                                          9390
  133 CONTINUE                                                           9400
  136 CONTINUE                                                           9410
      IF(NNY.EQ.0) GO TO 137                                             9420
      DO 134 I=1,NNY                                                     9430
      II=I+4*(NX+NY+NZ+NNX)                                              9440
      L(II,MATP)=-1.0*L(II,MATP)                                         9450
      J=II+2*NNY                                                         9460
      L(J,MATP)=377.0*L(J,MATP)                                          9470
      K=J+NNY                                                            9480
      L(K,MATP)=-377.0*L(K,MATP)                                         9490
  134 CONTINUE                                                           9500
  137 CONTINUE                                                           9510
      IF(NNZ.EQ.0) GO TO 138                                             9520
      DO 135 I=1,NNZ                                                     9530
      II=I+4*(NX+NY+NZ+NNX+NNY)+NNZ                                      9540
      L(II,MATP)=-1.0*L(II,MATP)                                         9550
      J=II+NNZ                                                           9560
      L(J,MATP)=-377.0*L(J,MATP)                                         9570
      K=J+NNZ                                                            9580
      L(K,MATP)=377.0*L(K,MATP)                                          9590
  135 CONTINUE                                                           9600
  138 CONTINUE                                                           9610
      RETURN                                                             9620
      END                                                                9630
C*****SUBROUTINE PRNT*************************************************************
      SUBROUTINE PRNT(IMIN,CRFL,L,MAT,MATP)                              9640
      REAL IMIN,LEFT                                                     9650
      COMPLEX L(MAT,MATP)                                                9660
      COMMON /NXYZ/NX,NY,NZ,NNX,NNY,NNZ                                  9670
      DATA TOP/4HTOP  /,RIGHT/4HRIGH/,REAR/4HREAR/,FLDS/4HFLDS/
      DATA BTTM/4HBTTM/,LEFT/4HLEFT/,FRNT/4HFRNT/
      PRINT 111,IMIN,CRFL                                                9700
  111 FORMAT(1H1,5X,4HTHE  ,A4,1X,A4,81H ARE LISTED AS FOLLOWS (AMP/METER
     2 FOR CRRNTS OR H FIELDS, VOL/METER FOR E FIELDS)/1H-,3H N ,6X,4HRE
```

```
     3AL, 11X, 4HIMAG, 11X, 4HREAL, 11X, 4HIMAG, 11X, 4HREAL, 11X, 4HIMAG, 11X, 4HRE
     4AL, 11X, 4HIMAG)
      PRINT 123, TOP
  123 FORMAT(1H-, 15X, 7HON THE , A4, 28H SURFACE WE HAVE THE FOLLOWS)          9750
      IF(CRFL.EQ.FLDS) GO TO 136
      PRINT 112                                                                9770
  112 FORMAT(1H-, 14X, 10HJEY OR -HZ, 20X, 9HJEZ OR HY, 20X, 12HJMY OR EZ/WO, 1 9780
     28X, 13HJMZ OR -EY/WO)
      GO TO 138
  136 PRINT 137                                                                9810
  137 FORMAT(1H-, 14X, 10H     HZ     , 20X, 9H     HY     , 20X, 12H     EZ      , 1 9820
     28X, 13H     EY      )
  138 PRINT 122
  122 FORMAT(1H-, 5X, 4(28H****************************, 2X))                    9850
      DO 12 I=1, NX
      II=I                                                                     9870
      J=I                                                                      9880
      IP=J+NX                                                                  9890
      IPP=IP+NX                                                                9900
      IPPP=IPP+NX                                                              9910
   12 PRINT 13, II, L(J, MATP), L(IP, MATP), L(IPP, MATP), L(IPPP, MATP)       9920
      PRINT 122                                                                9930
   13 FORMAT(1H-, I3, 1X, 4(1X, 1H(, 2E13.6, 1H), 1X))                         9940
      PRINT 123, RIGHT
      IF(CRFL.EQ.FLDS) GO TO 139                                               9960
      PRINT 113                                                                9970
  113 FORMAT(1H-, 14X, 9HJEX OR HZ, 20X, 10HJEZ OR -HX, 20X, 13HJMX OR -EZ/WO, 9980
     218X, 12HJMZ OR EX/WO)
      GO TO 141
  139 PRINT 140                                                                10010
  140 FORMAT(1H-, 14X, 9H     HZ     , 20X, 10H     HX     , 20X, 13H     EZ      , 10020
     218X, 12H     EX      )
  141 PRINT 122
      DO 14 I=1, NY                                                            10050
      II=I+NX                                                                  10060
      J=I+4*NX                                                                 10070
      IP=J+NY                                                                  10080
      IPP=IP+NY                                                                10090
      IPPP=IPP+NY                                                              10100
   14 PRINT 13, II, L(J, MATP), L(IP, MATP), L(IPP, MATP), L(IPPP, MATP)       10110
      PRINT 122                                                                10120
      PRINT 123, REAR                                                          10130
      IF(CRFL.EQ.FLDS) GO TO 142                                               10140
      PRINT 114                                                                10150
  114 FORMAT(1H-, 14X, 10HJEX OR -HY, 20X, 9HJEY OR HX, 20X, 12HJMX OR EY/WO, 1 10160
     28X, 13HJMY OR -EX/WO)
      GO TO 144
  142 PRINT 143                                                                10190
                                                                               10200
```

```
143 FORMAT(1H-,14X,10H     HY     ,20X,9H     HX     ,20X,12H     EY        ,1
   28X,13H     EX        )
144 PRINT 122                                                          10230
    DO 15 I=1,NZ                                                       10240
    II=I+NX+NY                                                         10250
    J=I+4*(NX+NY)                                                      10260
    IP=J+NZ                                                            10270
    IPP=IP+NZ                                                          10280
    IPPP=IPP+NZ                                                        10290
 15 PRINT 13,II,L(J,MATP),L(IP,MATP),L(IPP,MATP),L(IPPP,MATP)          10300
    PRINT 122                                                          10310
    IF(NNX.EQ.0) GO TO 19                                              10320
    PRINT 123,BTTM                                                     10330
    IF(CRFL.EQ.FLDS) GO TO 145                                         10340
    PRINT 146                                                          10350
146 FORMAT(1H-,14X,9HJEY OR HZ,20X,10HJEZ OR -HY,20X,
   213HJMY OR -EZ/WO,18X,12HJMZ OR EY/WO)
    GO TO 147                                                          10380
145 PRINT 137                                                          10390
147 PRINT 122                                                          10400
    DO 16 I=1,NNX                                                      10410
    II=I+NX+NY+NZ                                                      10420
    J=I+4*(NX+NY+NZ)                                                   10430
    IP=J+NNX                                                           10440
    IPP=IP+NNX                                                         10450
    IPPP=IPP+NNX                                                       10460
 16 PRINT 13,II,L(J,MATP),L(IP,MATP),L(IPP,MATP),L(IPPP,MATP)          10470
    PRINT 122                                                          10480
 19 CONTINUE                                                           10490
    IF(NNY.EQ.0) GO TO 20                                              10500
    PRINT 123,LEFT                                                     10510
    IF(CRFL.EQ.FLDS) GO TO 149                                         10520
    PRINT 150                                                          10530
150 FORMAT(1H-,14X,10HJEX OR -HZ,20X,9HJEZ OR HX,20X,
   212HJMX OR EZ/WO,18X,13HJMZ OR -EX/WO)
    GO TO 151                                                          10560
149 PRINT 140                                                          10570
151 PRINT 122                                                          10580
    DO 17 I=1,NNY                                                      10590
    II=I+NX+NY+NZ+NNX                                                  10600
    J=I+4*(NX+NY+NZ+NNX)                                               10610
    IP=J+NNY                                                           10620
    IPP=IP+NNY                                                         10630
    IPPP=IPP+NNY                                                       10640
 17 PRINT 13,II,L(J,MATP),L(IP,MATP),L(IPP,MATP),L(IPPP,MATP)          10650
    PRINT 122                                                          10660
 20 CONTINUE                                                           10670
```

```
      IF(NNZ.EQ.0) GO TO 21                                              10680
      PRINT 123,FRNT                                                     10690
      IF(CRFL.EQ.FLDS) GO TO 153                                         10700
      PRINT 154                                                          10710
  154 FORMAT(1H-,14X,9HJEX OR HY,20X,10HJEY OR -HX,20X,
     213HJMX OR -EY/W0,18X,12HJMY OR EX/W0)
      GO TO 155
  153 PRINT 143                                                          10740
  155 PRINT 122                                                          10750
      DO 18 I=1,NNZ                                                      10760
      II=I+NX+NY+NZ+NNX+NNY                                              10770
      J=I+4*(NX+NY+NZ+NNX+NNY)                                           10780
      IP=J+NNZ                                                           10790
      IPP=IP+NNZ                                                         10800
      IPPP=IPP+NNZ                                                       10810
   18 PRINT 13,II,L(J,MATP),L(IP,MATP),L(IPP,MATP),L(IPPP,MATP)          10820
      PRINT 122                                                          10830
   21 CONTINUE                                                           10840
      RETURN                                                             10850
      END                                                                10860
C*****SUBROUTINE COEFX*************************************************10870
      SUBROUTINE COEFX(MDIR,AREA,XM,YM,ZM,XN,YN,ZN,C)                    *******
      REAL D(3),R(5),MUO                                                 11730
      COMPLEX C(16),G(4),Q(4),F(10),CJ,FK,FHUN                           11740
      COMPLEX H(5),HT(5),FT(10)                                          11750
      COMMON /CONS/PI,MUO,EPO                                            11760
      COMMON /CPLX/CJ
      COMMON /PARA/FK,FHUN
      COMMON /NUMDV/NDIV                                                 11780
      NDIV2=NDIV*NDIV                                                    11790
      DLEN=SQRT(AREA)                                                    11800
      DL=DLEN/NDIV                                                       11810
      MDIRM=MDIR                                                         11820
      IF(MDIR.GE.4) MDIRM=MDIR-3                                         11830
      IF(MDIRM-2) 100,200,300                                            11840
100   DO 1 I=1,5                                                         11850
      HT(I)=0.0                                                          11860
1     CONTINUE                                                           11870
      YT=YN+(DLEN+DL)/2                                                  11880
      ZT=ZN+(DLEN+DL)/2                                                  11890
      DO 4 J=1,NDIV                                                      11900
      YT=YT-DL                                                           11910
      DO 3 K=1,NDIV                                                      11920
      ZT=ZT-DL                                                           11930
      CALL DRG(XM,YM,ZM,XN,YT,ZT,D,R,G)                                  11940
      CALL GQ(R,G,Q)                                                     11950
      CALL FCNXX(AREA,D,Q,H)                                             11960
      DO 2 I=1,5                                                         11970
      HT(I)=HT(I)+H(I)                                                   11980
2     CONTINUE                                                           11990
                                                                         12000
```

```
3       CONTINUE                                                        12010
        ZT=ZT+DLEN                                                      12020
4       CONTINUE                                                        12030
        DO 5 I=1,5                                                      12040
        HT(I)=HT(I)/NDIV2                                               12050
5       CONTINUE                                                        12060
        CALL CEFXX(HT,C)                                                12070
        RETURN                                                          12080
200     DO 101 I=1,10                                                   12090
        FT(I)=0.0                                                       12100
101     CONTINUE                                                        12110
        YT=YN+(DLEN+DL)/2                                               12120
        ZT=ZN+(DLEN+DL)/2                                               12130
        DO 104 J=1,NDIV                                                 12140
        YT=YT-DL                                                        12150
        DO 103 K=1,NDIV                                                 12160
        ZT=ZT-DL                                                        12170
        CALL DRG(XM,YM,ZM,XN,YT,ZT,D,R,G)                               12180
        CALL GG(R,G,Q)                                                  12190
        CALL FCNYX(AREA,D,Q,F)                                          12200
        DO 102 I=1,10                                                   12210
        FT(I)=FT(I)+F(I)                                                12220
102     CONTINUE                                                        12230
103     CONTINUE                                                        12240
        ZT=ZT+DLEN                                                      12250
104     CONTINUE                                                        12260
        DO 105 I=1,10                                                   12270
        FT(I)=FT(I)/NDIV2                                               12280
105     CONTINUE                                                        12290
        CALL CEFYX(FT,C)                                                12300
        RETURN                                                          12310
300     DO 201 I=1,10                                                   12320
        FT(I)=0.0                                                       12330
201     CONTINUE                                                        12340
        YT=YN+(DLEN+DL)/2                                               12350
        ZT=ZN+(DLEN+DL)/2                                               12360
        DO 204 J=1,NDIV                                                 12370
        YT=YT-DL                                                        12380
        DO 203 K=1,NDIV                                                 12390
        ZT=ZT-DL                                                        12400
        CALL DRG(XM,YM,ZM,XN,YT,ZT,D,R,G)                               12410
        CALL GG(R,G,Q)                                                  12420
        CALL FCNZX(AREA,D,Q,F)                                          12430
        DO 202 I=1,10                                                   12440
        FT(I)=FT(I)+F(I)                                                12450
202     CONTINUE                                                        12460
203     CONTINUE                                                        12470
```

```
      ZT=ZT+DLEN                                                        12480
204   CONTINUE                                                          12490
      DO 205 I=1,10                                                     12500
      FT(I)=FT(I)/NDIV2                                                 12510
205   CONTINUE                                                          12520
      CALL CEFXY(FT,C)                                                  12530
      RETURN                                                            12540
      END                                                               12550
C*****SUBROUTINE COEFY*************************************************
      SUBROUTINE COEFY(MDIR,AREA,XM,YM,ZM,XN,YN,ZN,C)                   12560
      REAL D(3),R(5),MUO                                                12570
      COMPLEX C(16),G(4),Q(4),F(10),CJ,FK,FHUN                          12580
      COMPLEX H(5),HT(5),FT(10)                                         12590
      COMMON /CONS/PI,MUO,EPO
      COMMON /CPLX/CJ
      COMMON /PARA/FK,FHUN
      COMMON /NUMDV/NDIV                                                12610
      NDIV2=NDIV*NDIV                                                   12620
      DLEN=SQRT(AREA)                                                   12630
      DL=DLEN/NDIV                                                      12640
      MDIRM=MDIR                                                        12650
      IF(MDIR.GE.4) MDIRM=MDIR-3                                        12660
      IF(MDIRM-2) 100,200,300                                           12670
100   DO 1 I=1,10                                                       12680
      FT(I)=0.0                                                         12690
1     CONTINUE                                                          12700
      ZT=ZN+(DLEN+DL)/2                                                 12710
      XT=XN+(DLEN+DL)/2                                                 12720
      DO 4 J=1,NDIV                                                     12730
      ZT=ZT-DL                                                          12740
      DO 3 K=1,NDIV                                                     12750
      XT=XT-DL                                                          12760
      CALL DRG(XM,YM,ZM,XT,YN,ZT,D,R,G)                                 12770
      CALL QQ(R,G,Q)                                                    12780
      CALL FCNXY(AREA,D,Q,F)                                            12790
      DO 2 I=1,10                                                       12800
      FT(I)=FT(I)+F(I)                                                  12810
2     CONTINUE                                                          12820
3     CONTINUE                                                          12830
      XT=XT+DLEN                                                        12840
4     CONTINUE                                                          12850
      DO 5 I=1,10                                                       12860
      FT(I)=FT(I)/NDIV2                                                 12870
5     CONTINUE                                                          12880
      CALL CEFXY(FT,C)                                                  12890
      RETURN                                                            12900
200   DO 101 I=1,5                                                      12910
      HT(I)=0.0                                                         12920
                                                                        12930
```

```
101   CONTINUE                                                          12940
      ZT=ZN+(DLEN+DL)/2                                                 12950
      XT=XN+(DLEN+DL)/2                                                 12960
      DO 104 J=1,NDIV                                                   12970
      ZT=ZT-DL                                                          12980
      DO 103 K=1,NDIV                                                   12990
      XT=XT-DL                                                          13000
      CALL DRG(XM,YM,ZM,XT,YN,ZT,D,R,G)                                 13010
      CALL GG(R,G,Q)                                                    13020
      CALL FCNYY(AREA,D,Q,H)                                            13030
      DO 102 I=1,5                                                      13040
      HT(I)=HT(I)+H(I)                                                  13050
102   CONTINUE                                                          13060
103   CONTINUE                                                          13070

      XT=XT+DLEN                                                        13080
104   CONTINUE                                                          13090
      DO 105 I=1,5                                                      13100
      HT(I)=HT(I)/NDIV2                                                 13110
105   CONTINUE                                                          13120
      CALL CEFYY(HT,C)                                                  13130
      RETURN                                                            13140
300   DO 201 I=1,10                                                     13150
      FT(I)=0.0                                                         13160
201   CONTINUE                                                          13170
      ZT=ZN+(DLEN+DL)/2                                                 13180
      XT=XN+(DLEN+DL)/2                                                 13190
      DO 204 J=1,NDIV                                                   13200
      ZT=ZT-DL                                                          13210
      DO 203 K=1,NDIV                                                   13220
      XT=XT-DL                                                          13230
      CALL DRG(XM,YM,ZM,XT,YN,ZT,D,R,G)                                 13240
      CALL GG(R,G,Q)                                                    13250
      CALL FCNZY(AREA,D,Q,F)                                            13260
      DO 202 I=1,10                                                     13270
      FT(I)=FT(I)+F(I)                                                  13280
202   CONTINUE                                                          13290
203   CONTINUE                                                          13300
      XT=XT+DLEN                                                        13310
204   CONTINUE                                                          13320
      DO 205 I=1,10                                                     13330
      FT(I)=FT(I)/NDIV2                                                 13340
205   CONTINUE                                                          13350
      CALL CEFZY(FT,C)                                                  13360
      RETURN                                                            13370
      END                                                               13380
```

```
C*****SUBROUTINE COEFZ*************************************************************
      SUBROUTINE COEFZ(MDIR,AREA,XM,YM,ZM,XN,YN,ZN,C)                    13390
      REAL D(3),R(5),MUO                                                 13400
      COMPLEX C(16),G(4),Q(4),F(10),CJ,FK,FHUN                           13410
      COMPLEX H(5),HT(5),FT(10)                                          13420
      COMMON /CONS/PI,MUO,EPO
      COMMON /CPLX/CJ
      COMMON /PARA/FK,FHUN                                               13440
      COMMON /NUMDV/NDIV                                                 13450
      NDIV2=NDIV*NDIV                                                    13460
      DLEN=SQRT(AREA)                                                    13470
      DL=DLEN/NDIV                                                       13480
      MDIRM=MDIR                                                         13490
      IF(MDIR.GE.4) MDIRM=MDIR-3                                         13500
      IF(MDIRM-2) 100,200,300                                            13510
100   DO 1 I=1,10                                                        13520
      FT(I)=0.0                                                          13530
1     CONTINUE                                                           13540
      YT=YN+(DLEN+DL)/2                                                  13550
      XT=XN+(DLEN+DL)/2                                                  13560
      DO 4 J=1,NDIV                                                      13570
      YT=YT-DL                                                           13580
      DO 3 K=1,NDIV                                                      13590
      XT=XT-DL                                                           13600
      CALL DRG(XM,YM,ZM,XT,YT,ZN,D,R,G)                                  13610
      CALL GQ(R,G,Q)                                                     13620
      CALL FCNXZ(AREA,D,Q,F)                                             13630
      DO 2 I=1,10                                                        13640
      FT(I)=FT(I)+F(I)                                                   13650
2     CONTINUE                                                           13660
3     CONTINUE                                                           13670
      XT=XT+DLEN                                                         13680
4     CONTINUE                                                           13690
      DO 5 I=1,10                                                        13700
      FT(I)=FT(I)/NDIV2                                                  13710
5     CONTINUE                                                           13720
      CALL CEFXZ(FT,C)                                                   13730
      RETURN                                                             13740
200   DO 101 I=1,10                                                      13750
      FT(I)=0.0                                                          13760
101   CONTINUE                                                           13770
      YT=YN+(DLEN+DL)/2                                                  13780
      XT=XN+(DLEN+DL)/2                                                  13790
      DO 104 J=1,NDIV                                                    13800
      YT=YT-DL                                                           13810
      DO 103 K=1,NDIV                                                    13820
      XT=XT-DL                                                           13830
```

```
      CALL DRG(XM,YM,ZM,XT,YT,ZN,D,R,G)                                 13840
      CALL GG(R,G,Q)                                                    13850
      CALL FCNYZ(AREA,D,Q,F)                                            13860
      DO 102 I=1,10                                                     13870
      FT(I)=FT(I)+F(I)                                                  13880
102   CONTINUE                                                          13890
103   CONTINUE                                                          13900
      XT=XT+DLEN                                                        13910
104   CONTINUE                                                          13920
      DO 105 I=1,10                                                     13930
      FT(I)=FT(I)/ND1V2                                                 13940
105   CONTINUE                                                          13950
      CALL CEFYZ(FT,C)                                                  13960
      RETURN                                                            13970
300   DO 201 I=1,5                                                      13980
      HT(I)=0.0                                                         13990
201   CONTINUE                                                          14000
      YT=YN+(DLEN+DL)/2                                                 14010
      XT=XN+(DLEN+DL)/2                                                 14020
      DO 204 J=1,NDIV                                                   14030
      YT=YT-DL                                                          14040
      DO 203 K=1,NDIV                                                   14050
      XT=XT-DL                                                          14060
      CALL DRG(XM,YM,ZM,XT,YT,ZN,D,R,G)                                 14070
      CALL GG(R,G,Q)                                                    14080
      CALL FCNZZ(AREA,D,Q,H)                                            14090
      DO 202 I=1,5                                                      14100
      HT(I)=HT(I)+H(I)                                                  14110
202   CONTINUE                                                          14120
203   CONTINUE                                                          14130
      XT=XT+DLEN                                                        14140
204   CONTINUE                                                          14150
      DO 205 I=1,5                                                      14160
      HT(I)=HT(I)/NDIV2                                                 14170
205   CONTINUE                                                          14180
      CALL CEFZZ(HT,C)                                                  14190
      RETURN                                                            14200
      END                                                               14210
C*****SUBROUTINE GG*****************************************************
      SUBROUTINE GG(R,G,Q)                                              11510
      REAL R(5),MU0                                                     11520
      COMPLEX G(4),Q(4),CJ                                              11530
      COMMON /CONS/PI,MU0,EPO
      COMMON /CPLX/CJ
      Q(1)=CJ*G(2)/R(2)+G(1)/R(3)                                       11550
      Q(2)=G(3)/R(3)-3.0*CJ*G(2)/R(4)-3.0*G(1)/R(5)                     11560
      Q(3)=CJ*G(4)/R(2)+G(3)/R(3)                                       11570
      Q(4)=-1.0*G(3)/R(1)                                               11580
      RETURN                                                            11590
      END                                                               11600
```

```
C*****COMPLEX FUNCTION P*************************************************************
      COMPLEX FUNCTION P(I,A)                                                    11610
      REAL MUO                                                                   11620
      COMPLEX CJ,FK,FHUN                                                         11630
      COMMON /CONS/PI,MUO,EPO
      COMMON /CPLX/CJ
      COMMON /PARA/FK,FHUN                                                       11650
      IF(I.EQ.2) GO TO 1                                                         11660
      P=CJ*CEXP(-1.0*CJ*A)-CJ*FK*CEXP(-1.0*CJ*FK*A)+(CEXP(-1.0*CJ*A)-CEX11670
     2P(-1.0*CJ*FK*A))/A                                                         11680
      RETURN                                                                     11690
    1 P=2.0*PI*CJ*((FK*CEXP(-1.0*CJ*FK*A)-CEXP(-1.0*CJ*A))-(FK-1.0))             11700
      RETURN                                                                     11710
      END                                                                        11720
C*****SUBROUTINE DRG*****************************************************************
      SUBROUTINE DRG(XM,YM,ZM,XN,YN,ZN,D,R,G)                                    11730
      REAL D(3),R(5),MUO                                                         11740
      COMPLEX G(4),EX(2),CJ,FK,FHUN                                              11750
      COMMON /CONS/PI,MUO,EPO
      COMMON /CPLX/CJ
      COMMON /PARA/FK,FHUN                                                       11770
      D(1)=XN-XM                                                                 11780
      D(2)=YN-YM                                                                 11790
      D(3)=ZN-ZM                                                                 11800
      DD=D(1)*D(1)+D(2)*D(2)+D(3)*D(3)                                           11810
      R(1)=SQRT(DD)                                                              11820
      R(2)=R(1)*R(1)                                                             11830
      R(3)=R(2)*R(1)                                                             11840
      R(4)=R(3)*R(1)                                                             11850
      R(5)=R(4)*R(1)                                                             11860
      EX(1)=CEXP(-1.0*CJ*R(1))                                                   11870
      EX(2)=CEXP(-1.0*CJ*FK*R(1))                                                11880
      G(1)=EX(1)-EX(2)                                                           11890
      G(2)=EX(1)-FK*EX(2)                                                        11900
      G(3)=EX(1)-FK*FK*EX(2)                                                     11910
      G(4)=EX(1)-FK*FK*FK*EX(2)                                                  11920
      RETURN                                                                     11930
      END                                                                        11940
C*****SUBROUTINE LEE*****************************************************************
      SUBROUTINE LEE(E,MT,MP,MPP,MPPP,NT,NP,NPP,NPPP,L,MAT,MATP)                 11950
      COMPLEX E(16),L(MAT,MATP)                                                  11960
      L(MT,NT)=E(1)                                                              11970
      L(MP,NT)=E(2)                                                              11980
      L(MPP,NT)=E(3)                                                             11990
      L(MPPP,NT)=E(4)                                                            12000
      L(MT,NP)=E(5)                                                              12010
      L(MP,NP)=E(6)                                                              12020
```

```
      L(MPP,NP)=E(7)                                                    12030
      L(MPPP,NP)=E(8)                                                   12040
      L(MT,NPP)=E(9)                                                    12050
      L(MP,NPP)=E(10)                                                   12060
      L(MPP,NPP)=E(11)                                                  12070
      L(MPPP,NPP)=E(12)                                                 12080
      L(MT,NPPP)=E(13)                                                  12090
      L(MP,NPPP)=E(14)                                                  12100
      L(MPP,NPPP)=E(15)                                                 12110
      L(MPPP,NPPP)=E(16)                                                12120
      RETURN                                                            12130
      END                                                               12140
C*****SUBROUTINE FCNXX*************************************************************
      SUBROUTINE FCNXX(AREA,D,Q,F)                                      12150
      REAL D(3),MUO                                                     12160
      COMPLEX Q(4),F(5),CJ                                              12170
      COMMON /CONS/PI,MUO,EPO
      COMMON /CPLX/CJ
      F(1)=AREA*D(1)*Q(1)
      F(2)=CJ*AREA*D(2)*D(3)*Q(2)                                       12190
      F(3)=CJ*AREA*(Q(1)+D(3)*D(3)*Q(2)+Q(4))                           12200
      F(4)=CJ*AREA*(Q(1)+D(2)*D(2)*Q(2)+Q(4))                           12210
      F(5)=AREA*D(1)*Q(3)                                               12220
      RETURN                                                            12230
      END                                                               12240
C*****SUBROUTINE FCNXY*************************************************************
      SUBROUTINE FCNXY(AREA,D,Q,F)                                      12250
      REAL D(3),MUO                                                     12260
      COMPLEX Q(4),F(10),CJ                                             12270
      COMMON /CONS/PI,MUO,EPO                                           12280
      COMMON /CPLX/CJ
      F(1)=AREA*D(2)*Q(1)
      F(2)=CJ*AREA*D(1)*D(3)*Q(2)                                       12300
      F(3)=CJ*AREA*(Q(1)+D(3)*D(3)*Q(2)+Q(4))                           12310
      F(4)=AREA*D(3)*Q(1)                                               12320
      F(5)=AREA*D(1)*Q(1)                                               12330
      F(6)=CJ*AREA*D(2)*D(1)*Q(2)                                       12340
      F(7)=CJ*AREA*D(2)*D(3)*Q(2)                                       12350
      F(8)=AREA*D(2)*Q(3)                                               12360
      F(9)=AREA*D(3)*Q(3)                                               12370
      F(10)=AREA*D(1)*Q(3)                                              12380
      RETURN                                                            12390
      END                                                               12400
C*****SUBROUTINE FCNXZ*************************************************************
      SUBROUTINE FCNXZ(AREA,D,Q,F)                                      12410
      REAL D(3),MUO                                                     12420
      COMPLEX Q(4),F(10),CJ                                             12430
      COMMON /CONS/PI,MUO,EPO                                           12440
      COMMON /CPLX/CJ
```

```
      F(1)=AREA*D(2)*Q(1)
      F(2)=AREA*D(1)*Q(1)                                                    12460
      F(3)=CJ*AREA*D(1)*D(3)*Q(2)                                            12470
      F(4)=CJ*AREA*D(2)*D(3)*Q(2)                                            12480
      F(5)=AREA*D(3)*Q(1)                                                    12490
      F(6)=CJ*AREA*D(2)*D(1)*Q(2)                                            12500
      F(7)=CJ*AREA*(Q(1)+D(2)*D(2)*Q(2)+Q(4))                                12510
      F(8)=AREA*D(2)*Q(3)                                                    12520
      F(9)=AREA*D(1)*Q(3)                                                    12530
      F(10)=AREA*D(3)*Q(3)                                                   12540
      RETURN                                                                 12550
      END                                                                    12560
C*****SUBROUTINE FCNYX*************************************************    12570
      SUBROUTINE FCNYX(AREA,D,Q,F)                                           12580
      REAL D(3),MUO                                                          12590
      COMPLEX Q(4),F(10),CJ                                                  12600
      COMMON /CONS/PI,MUO,EPO
      COMMON /CPLX/CJ
      F(1)=AREA*D(1)*Q(1)                                                    12620
      F(2)=CJ*AREA*D(2)*D(3)*Q(2)                                            12630
      F(3)=CJ*AREA*(Q(1)+D(3)*D(3)*Q(2)+Q(4))                                12640
      F(4)=AREA*D(3)*Q(1)                                                    12650
      F(5)=AREA*D(2)*Q(1)                                                    12660
      F(6)=CJ*AREA*D(1)*D(2)*Q(2)                                            12670
      F(7)=CJ*AREA*D(1)*D(3)*Q(2)                                            12680
      F(8)=AREA*D(1)*Q(3)                                                    12690
      F(9)=AREA*D(3)*Q(3)                                                    12700
      F(10)=AREA*D(2)*Q(3)                                                   12710
      RETURN                                                                 12720
      END                                                                    12730
C*****SUBROUTINE FCNYY*************************************************    12740
      SUBROUTINE FCNYY(AREA,D,Q,F)                                           12750
      REAL D(3),MUO                                                          12760
      COMPLEX Q(4),F(5),CJ
      COMMON /CONS/PI,MUO,EPO
      COMMON /CPLX/CJ
      F(1)=AREA*D(2)*Q(1)                                                    12780
      F(2)=CJ*AREA*D(1)*D(3)*Q(2)                                            12790
      F(3)=CJ*AREA*(Q(1)+D(3)*D(3)*Q(2)+Q(4))                                12800
      F(4)=CJ*AREA*(Q(1)+D(1)*D(1)*Q(2)+Q(4))                                12810
      F(5)=AREA*D(2)*Q(3)                                                    12820
      RETURN                                                                 12830
      END                                                                    12840
C*****SUBROUTINE FCNYZ*************************************************    12850
      SUBROUTINE FCNYZ(AREA,D,Q,F)                                           12860
      REAL D(3),MUO                                                          12870
      COMPLEX Q(4),F(10),CJ
      COMMON /CONS/PI,MUO,EPO
      COMMON /CPLX/CJ
```

```
      F(1)=AREA*D(2)*Q(1)
      F(2)=AREA*D(1)*Q(1)                                                12890
      F(3)=CJ*AREA*D(1)*D(3)*Q(2)                                        12900
      F(4)=CJ*AREA*D(2)*D(3)*Q(2)                                        12910
      F(5)=AREA*D(3)*Q(1)                                                12920
      F(6)=CJ*AREA*(Q(1)+D(1)*D(1)*Q(2)+Q(4))                            12930
      F(7)=CJ*AREA*D(1)*D(2)*Q(2)                                        12940
      F(8)=AREA*D(2)*Q(3)                                                12950
      F(9)=AREA*D(1)*Q(3)                                                12960
      F(10)=AREA*D(3)*Q(3)                                               12970
      RETURN                                                             12980
      END                                                                12990
C*****SUBROUTINE FCNZX*************************************************13000
      SUBROUTINE FCNZX(AREA,D,Q,F)                                       13010
      REAL D(3),MUO                                                      13020
      COMPLEX Q(4),F(10),CJ                                              13030
      COMMON /CONS/PI,MUO,EPO
      COMMON /CPLX/CJ
      F(1)=AREA*D(1)*Q(1)
      F(2)=CJ*AREA*(Q(1)+D(2)*D(2)*Q(2)+Q(4))                            13050
      F(3)=CJ*AREA*D(2)*D(3)*Q(2)                                        13060
      F(4)=AREA*D(3)*Q(1)                                                13070
      F(5)=AREA*D(2)*Q(1)                                                13080
      F(6)=CJ*AREA*D(1)*D(2)*Q(2)                                        13090
      F(7)=CJ*AREA*D(1)*D(3)*Q(2)                                        13100
      F(8)=AREA*D(1)*Q(3)                                                13110
      F(9)=AREA*D(3)*Q(3)                                                13120
      F(10)=AREA*D(2)*Q(3)                                               13130
      RETURN                                                             13140
      END                                                                13150
C*****SUBROUTINE FCNZY*************************************************13160
      SUBROUTINE FCNZY(AREA,D,Q,F)                                       13170
      REAL D(3),MUO                                                      13180
      COMPLEX Q(4),F(10),CJ                                              13190
      COMMON /CONS/PI,MUO,EPO
      COMMON /CPLX/CJ
      F(1)=AREA*D(3)*Q(1)
      F(2)=AREA*D(1)*Q(1)                                                13210
      F(3)=CJ*AREA*D(1)*D(2)*Q(2)                                        13220
      F(4)=CJ*AREA*D(2)*D(3)*Q(2)                                        13230
      F(5)=AREA*D(2)*Q(1)                                                13240
      F(6)=CJ*AREA*(Q(1)+D(1)*D(1)*Q(2)+Q(4))                            13250
      F(7)=CJ*AREA*D(1)*D(3)*Q(2)                                        13260
      F(8)=AREA*D(3)*Q(3)                                                13270
      F(9)=AREA*D(1)*Q(3)                                                13280
      F(10)=AREA*D(2)*Q(3)                                               13290
      RETURN                                                             13300
      END                                                                13310
                                                                         13320
```

```
C*****SUBROUTINE FCNZZ*************************************************************
      SUBROUTINE FCNZZ(AREA,D,Q,F)                                          13330
      REAL D(3),MUO                                                         13340
      COMPLEX Q(4),F(5),CJ                                                  13350
      COMMON /CONS/PI,MUO,EPO
      COMMON /CPLX/CJ
      F(1)=AREA*D(3)*Q(1)
      F(2)=CJ*AREA*D(1)*D(2)*Q(2)                                           13370
      F(3)=CJ*AREA*(Q(1)+D(2)*D(2)*Q(2)+Q(4))                               13380
      F(4)=CJ*AREA*(Q(1)+D(1)*D(1)*Q(2)+Q(4))                               13390
      F(5)=AREA*D(3)*Q(3)                                                   13400
      RETURN                                                                13410
      END                                                                   13420
C*****SUBROUTINE CEFXX*************************************************************
      SUBROUTINE CEFXX(F,C)                                                 13430
      COMPLEX F(5),C(16)                                                    13440
      C(1)=F(1)                                                             13450
      C(2)=0.0                                                              13460
      C(3)=-1.0*F(2)                                                        13470
      C(4)=F(4)                                                             13480
      C(5)=0.0                                                              13490
      C(6)=F(1)                                                             13500
      C(7)=-1.0*F(3)                                                        13510
      C(8)=F(2)                                                             13520
      C(9)=F(2)                                                             13530
      C(10)=-1.0*F(4)                                                       13540
      C(11)=F(5)                                                            13550
      C(12)=0.0                                                             13560
      C(13)=F(3)                                                            13570
      C(14)=-1.0*F(2)                                                       13580
      C(15)=0.0                                                             13590
      C(16)=F(5)                                                            13600
      RETURN                                                                13610
      END                                                                   13620
C*****SUBROUTINE CEFXY*************************************************************
      SUBROUTINE CEFXY(F,C)                                                 13630
      COMPLEX F(10),C(16)                                                   13640
      C(1)=-1.0*F(1)                                                        13650
      C(2)=-1.0*F(4)                                                        13660
      C(3)=-1.0*F(2)                                                        13670
      C(4)=F(6)                                                             13680
      C(5)=0.0                                                              13690
      C(6)=F(5)                                                             13700
      C(7)=-1.0*F(3)                                                        13710
      C(8)=F(7)                                                             13720
      C(9)=F(2)                                                             13730
      C(10)=-1.0*F(6)                                                       13740
      C(11)=-1.0*F(8)                                                       13750
                                                                            13760
```

```
      C(12)=-1.0*F(9)                                                   13770
      C(13)=F(3)                                                        13780
      C(14)=-1.0*F(7)                                                   13790
      C(15)=0.0                                                         13800
      C(16)=F(10)                                                       13810
      RETURN                                                            13820
      END                                                               13830
C*****SUBROUTINE CEFXZ*************************************************************
      SUBROUTINE CEFXZ(F,C)                                             13840
      COMPLEX F(10),C(16)                                               13850
      C(1)=-1.0*F(1)                                                    13860
      C(2)=-1.0*F(5)                                                    13870
      C(3)=-1.0*F(3)                                                    13880
      C(4)=F(6)                                                         13890
      C(5)=F(2)                                                         13900
      C(6)=0.0                                                          13910
      C(7)=-1.0*F(4)                                                    13920
      C(8)=F(7)                                                         13930
      C(9)=F(3)                                                         13940
      C(10)=-1.0*F(6)                                                   13950
      C(11)=-1.0*F(8)                                                   13960
      C(12)=-1.0*F(10)                                                  13970
      C(13)=F(4)                                                        13980
      C(14)=-1.0*F(7)                                                   13990
      C(15)=F(9)                                                        14000
      C(16)=0.0                                                         14010
      RETURN                                                            14020
      END                                                               14030
C*****SUBROUTINE CEFYX*************************************************************
      SUBROUTINE CEFYX(F,C)                                             14040
      COMPLEX F(10),C(16)                                               14050
      C(1)=-1.0*F(1)                                                    14060
      C(2)=-1.0*F(4)                                                    14070
      C(3)=F(2)                                                         14080
      C(4)=-1.0*F(6)                                                    14090
      C(5)=0.0                                                          14100
      C(6)=F(5)                                                         14110
      C(7)=F(3)                                                         14120
      C(8)=-1.0*F(7)                                                    14130
      C(9)=-1.0*F(2)                                                    14140
      C(10)=F(6)                                                        14150
      C(11)=-1.0*F(8)                                                   14160
      C(12)=-1.0*F(9)                                                   14170
      C(13)=-1.0*F(3)                                                   14180
      C(14)=F(7)                                                        14190
      C(15)=0.0                                                         14200
      C(16)=F(10)                                                       14210
      RETURN                                                            14220
      END                                                               14230
C*****SUBROUTINE CEFYY*************************************************************
      SUBROUTINE CEFYY(F,C)                                             14240
```

```
      COMPLEX F(5),C(16)                                                14250
      C(1)=F(1)                                                         14260
      C(2)=0.0                                                          14270
      C(3)=F(2)                                                         14280
      C(4)=-1.0*F(4)                                                    14290
      C(5)=0.0                                                          14300
      C(6)=F(1)                                                         14310
      C(7)=F(3)                                                         14320
      C(8)=-1.0*F(2)                                                    14330
      C(9)=-1.0*F(2)                                                    14340
      C(10)=F(4)                                                        14350
      C(11)=F(5)                                                        14360
      C(12)=0.0                                                         14370
      C(13)=-1.0*F(3)                                                   14380
      C(14)=F(2)                                                        14390
      C(15)=0.0                                                         14400
      C(16)=F(5)                                                        14410
      RETURN                                                            14420
      END                                                               14430
C*****SUBROUTINE CEFYZ*************************************************
      SUBROUTINE CEFYZ(F,C)                                             14440
      COMPLEX F(10),C(16)                                               14450
      C(1)=F(1)                                                         14460
      C(2)=0.0                                                          14470
      C(3)=F(3)                                                         14480
      C(4)=-1.0*F(6)                                                    14490
      C(5)=-1.0*F(2)                                                    14500
      C(6)=-1.0*F(5)                                                    14510
      C(7)=F(4)                                                         14520
      C(8)=-1.0*F(7)                                                    14530
      C(9)=-1.0*F(3)                                                    14540
      C(10)=F(6)                                                        14550
      C(11)=F(8)                                                        14560
      C(12)=0.0                                                         14570
      C(13)=-1.0*F(4)                                                   14580
      C(14)=F(7)                                                        14590
      C(15)=-1.0*F(9)                                                   14600
      C(16)=-1.0*F(10)                                                  14610
      RETURN                                                            14620
      END                                                               14630
C*****SUBROUTINE CEFZX*************************************************
      SUBROUTINE CEFZX(F,C)                                             14640
      COMPLEX F(10),C(16)                                               14650
      C(1)=0.0                                                          14660
      C(2)=F(4)                                                         14670
      C(3)=-1.0*F(2)                                                    14680
      C(4)=F(6)                                                         14690
      C(5)=-1.0*F(1)                                                    14700
```

```
      C(6)=-1.0*F(5)                                                    14710
      C(7)=-1.0*F(3)                                                    14720
      C(8)=F(7)                                                         14730
      C(9)=F(2)                                                         14740
      C(10)=-1.0*F(6)                                                   14750
      C(11)=0.0                                                         14760
      C(12)=F(9)                                                        14770
      C(13)=F(3)                                                        14780
      C(14)=-1.0*F(7)                                                   14790
      C(15)=-1.0*F(8)                                                   14800
      C(16)=-1.0*F(10)                                                  14810
      RETURN                                                            14820
      END                                                               14830
C*****SUBROUTINE CEFZY*************************************************
      SUBROUTINE CEFZY(F,C)
      COMPLEX F(10),C(16)                                               14840
      C(1)=F(1)                                                         14850
      C(2)=0.0                                                          14860
      C(3)=-1.0*F(3)                                                    14870
      C(4)=F(6)                                                         14880
      C(5)=-1.0*F(2)                                                    14890
      C(6)=-1.0*F(5)                                                    14900
      C(7)=-1.0*F(4)                                                    14910
      C(8)=F(7)                                                         14920
      C(9)=F(3)                                                         14930
      C(10)=-1.0*F(6)                                                   14940
      C(11)=F(8)                                                        14950
      C(12)=0.0                                                         14960
      C(13)=F(4)                                                        14970
      C(14)=-1.0*F(7)                                                   14980
      C(15)=-1.0*F(9)                                                   14990
      C(16)=-1.0*F(10)                                                  15000
      RETURN                                                            15010
      END                                                               15020
C*****SUBROUTINE CEFZZ*************************************************
      SUBROUTINE CEFZZ(F,C)                                             15040
      COMPLEX F(5),C(16)                                                15050
      C(1)=F(1)                                                         15060
      C(2)=0.0                                                          15070
      C(3)=-1.0*F(2)                                                    15080
      C(4)=F(4)                                                         15090
      C(5)=0.0                                                          15100
      C(6)=F(1)                                                         15110
      C(7)=-1.0*F(3)                                                    15120
      C(8)=F(2)                                                         15130
      C(9)=F(2)                                                         15140
      C(10)=-1.0*F(4)                                                   15150
      C(11)=F(5)                                                        15160
```

```
      C(12)=0.0                                                          15170
      C(13)=F(3)                                                         15180
      C(14)=-1.0*F(2)                                                    15190
      C(15)=0.0                                                          15200
      C(16)=F(5)                                                         15210
      RETURN                                                             15220
      END                                                                15230
C*****SUBROUTINE CMATP*************************************************************
      SUBROUTINE CMATP(A,N,NP,M,DET,EP)                                  15240
      COMPLEX A,B,DET,CONST,S                                            15250
      DIMENSION A(N,NP)                                                  15260
      DET=CMPLX(1.0,0.0)                                                 15270
      NP1=N+1                                                            15280
      NPM=N+M                                                            15290
      NM1=N-1                                                            15300
    2 DO 4 J=1,NM1                                                       15310
      C=CABS(A(J,J))                                                     15320
      JP1=J+1                                                            15330
      DO 5 I=JP1,N                                                       15340
      D=CABS(A(I,J))                                                     15350
      IF (C-D) 6,5,5                                                     15360
    6 DET=-DET                                                           15370
      DO 7 K=J,NPM                                                       15380
      B=A(I,K)                                                           15390
      A(I,K)=A(J,K)                                                      15400
    7 A(J,K)=B                                                           15410
      C=D                                                                15420
    5 CONTINUE                                                           15430
      IF (CABS(A(J,J))-EP) 14,15,15                                      15440
   15 DO 4 I=JP1,N                                                       15450
      CONST=A(I,J)/A(J,J)                                                15460
      DO 4 K=JP1,NPM                                                     15470
    4 A(I,K)=A(I,K)-CONST*A(J,K)                                         15480
      IF (CABS(A(N,N))-EP) 14,18,18                                      15490
   14 DET=CMPLX(0.0,0.0)                                                 15500
      PRINT 30                                                           15510
      RETURN                                                             15520
   18 CONTINUE                                                           15530
      DO 12 I=1,N                                                        15540
      K=N-I+1                                                            15550
      KP1=K+1                                                            15560
      DO 12 L=NP1,NPM                                                    15570
      S=CMPLX(0.0,0.0)                                                   15580
      IF (N-KP1) 12,19,19                                                15590
   19 DO 13 J=KP1,N                                                      15600
   13 S=S+A(K,J)*A(J,L)                                                  15610
   12 A(K,L)=(A(K,L)-S)/A(K,K)                                           15620
   30 FORMAT(1X,42HTHE DETERMINANT OF THE SYSTEM EQUALS ZERO./           15630
     11X,36HTHE PROGRAM CANNOT HANDLE THIS CASE.//)                      15640
      RETURN                                                             15650
      END                                                                15660
```

```
C*************************************************************************
C*****PROGRAM COMBINE IS TO COMBINE RESULTS FROM COSE, COSH, SINE, SINH MODES***
C*****AND TO OBTAIN THE TOTAL FIELDS ON THE SURFACES OF FIRST AND FIFTH QUADRANT
C*************************************************************************
      PROGRAM COMBINE(INPUT,OUTPUT,TAPE1,TAPE2,TAPE3,TAPE4)
      COMPLEX SUM1(208),SUM5(208),ECOS(208),HCOS(208)
      COMPLEX ESIN(208),HSIN(208)
      COMMON /NXYZ/NX,NY,NZ,NNX,NNY,NNZ                                 120
      DIMENSION X(52),Y(52),Z(52),DCELL(52)
      DIMENSION ABS1(208),ABS5(208),ANG1(208),ANG5(208)
      DIMENSION NV(6)
      DATA FIRS/"FIRS"/,CMPL/"CMPL"/,COSE/"COSE"/,COSH/"COSH"/,SINE/"SIN150
     +E"/,SINH/"SINH"/,EXPZ/"EXPZ"/,FIFT/"FIFT"/,ABSL/"ABSL"/           160
      CALL NOBLANK
      READ 500,NDIV
  500 FORMAT(I2)
      READ 5, IQ                                                        180
    5 FORMAT(I4)                                                        190
      PRINT 10,IQ                                                       200
      PRINT 11                                                          210
      PRINT 12                                                          220
      PRINT 13                                                          230
      READ 1,NX,NY,NZ,NNX,NNY,NNZ                                       240
      NCELL=NX+NY+NZ+NNX+NNY+NNZ                                        250
      DO 2 I=1,NCELL                                                    260
      READ 3,X(I),Y(I),Z(I),DCELL(I)                                    270
    2 CONTINUE                                                          280
      READ 4, RLEP,SIG,FMEG                                             290
      DO 14 I=1,NCELL                                                   300
   14 PRINT 15, I,X(I),Y(I),Z(I),DCELL(I)                               310
      PRINT 16,NX,NY,NZ,NNX,NNY,NNZ                                     320
      PRINT 101,NDIV,NDIV
  101 FORMAT(1H-,5X,33HEACH SURFACE CELL IS DIVIDED INTO,
     21X,I2,1X,2HBY,1X,I2,1X,17HSUB-SURFACE CELLS)
      PRINT 17,RLEP,SIG,FMEG                                            330
      MAT=4*NCELL                                                       340
      DO 18 J=1,MAT                                                     350
      SUM1(J)=CMPLX(0.,0.)                                              360
      SUM5(J)=CMPLX(0.,0.)                                              370
   18 CONTINUE                                                          380
      BUFFER IN (1,1) (ECOS(1),ECOS(MAT))
      CALL UNIT(1)
      BUFFER IN (2,1) (HCOS(1),HCOS(MAT))
      CALL UNIT(2)
      BUFFER IN (3,1) (ESIN(1),ESIN(MAT))
      CALL UNIT(3)
      BUFFER IN (4,1) (HSIN(1),HSIN(MAT))
      CALL UNIT(4)
```

```
      CALL PRFL(FIRS,CMPL,COSE,ECOS,MAT)                                470
      CALL PRFL(FIRS,CMPL,COSH,HCOS,MAT)                                480
      CALL PRFL(FIRS,CMPL,SINE,ESIN,MAT)                                490
      CALL PRFL(FIRS,CMPL,SINH,HSIN,MAT)                                500
      DO 19 I=1,MAT                                                     510
      SUM1(I)=ECOS(I)+HCOS(I)+ESIN(I)+HSIN(I)                           520
   19 CONTINUE                                                          530
      IF(IG.EQ.1234) GO TO 34                                           540
      NV(1)=NX                                                          550
      NV(2)=NY                                                          560
      NV(3)=NZ                                                          570
      NV(4)=NNX                                                         580
      NV(5)=NNY                                                         590
      NV(6)=NNZ                                                         600
      L=1                                                               610
      M=2                                                               620
      N=3                                                               630
      J=1                                                               640
      K=NV(L)                                                           650
   35 CONTINUE                                                          660
      DO 20 I=J,K                                                       670
      SUM5(I)=ECOS(I)-HCOS(I)-ESIN(I)+HSIN(I)                           680
   20 CONTINUE                                                          690
      J=J+NV(L)                                                         700
      K=K+NV(L)                                                         710
      DO 21 I=J,K                                                       720
      SUM5(I)=-ECOS(I)+HCOS(I)+ESIN(I)-HSIN(I)                          730
   21 CONTINUE                                                          740
      J=J+NV(L)                                                         750
      K=K+NV(L)                                                         760
      DO 22 I=J,K                                                       770
      SUM5(I)=-ECOS(I)+HCOS(I)+ESIN(I)-HSIN(I)                          780
   22 CONTINUE                                                          790
      J=J+NV(L)                                                         800
      K=K+NV(L)                                                         810
      DO 23 I=J,K                                                       820
      SUM5(I)=ECOS(I)-HCOS(I)-ESIN(I)+HSIN(I)                           830
   23 CONTINUE                                                          840
   36 CONTINUE                                                          850
      J=J+NV(L)                                                         860
      K=K+NV(M)                                                         870
      IF(NV(M).EQ.0) GO TO 37                                           880
      DO 24 I=J,K                                                       890
      SUM5(I)=ECOS(I)-HCOS(I)-ESIN(I)+HSIN(I)                           900
   24 CONTINUE                                                          910
      J=J+NV(M)                                                         920
      K=K+NV(M)                                                         930
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```
      DO 25 I=J,K                                                       940
      SUM5(I)=-ECOS(I)+HCOS(I)+ESIN(I)-HSIN(I)                          950
   25 CONTINUE                                                          960
      J=J+NV(M)                                                         970
      K=K+NV(M)                                                         980
      DO 26 I=J,K                                                       990
      SUM5(I)=-ECOS(I)+HCOS(I)+ESIN(I)-HSIN(I)                          1000
   26 CONTINUE                                                          1010
      J=J+NV(M)                                                         1020
      K=K+NV(M)                                                         1030
      DO 27 I=J,K                                                       1040
      SUM5(I)=ECOS(I)-HCOS(I)-ESIN(I)+HSIN(I)                           1050
   27 CONTINUE                                                          1060
   37 CONTINUE                                                          1070
      J=J+NV(M)                                                         1080
      K=K+NV(N)                                                         1090
      IF(NV(N).EQ.0) GO TO 38                                           1100
      DO 28 I=J,K                                                       1110
      SUM5(I)=-ECOS(I)+HCOS(I)+ESIN(I)-HSIN(I)                          1120
   28 CONTINUE                                                          1130
      J=J+NV(N)                                                         1140
      K=K+NV(N)                                                         1150
      DO 29 I=J,K                                                       1160
      SUM5(I)=-ECOS(I)+HCOS(I)+ESIN(I)-HSIN(I)                          1170
   29 CONTINUE                                                          1180
      J=J+NV(N)                                                         1190
      K=K+NV(N)                                                         1200
      DO 30 I=J,K                                                       1210
      SUM5(I)=ECOS(I)-HCOS(I)-ESIN(I)+HSIN(I)                           1220
   30 CONTINUE                                                          1230
      J=J+NV(N)                                                         1240
      K=K+NV(N)                                                         1250
      DO 31 I=J,K                                                       1260
      SUM5(I)=ECOS(I)-HCOS(I)-ESIN(I)+HSIN(I)                           1270
   31 CONTINUE                                                          1280
   38 CONTINUE                                                          1290
      IF(L.EQ.4) GO TO 34                                               1300
      L=4                                                               1310
      M=5                                                               1320
      N=6                                                               1330
      J=J+NV(3)                                                         1340
      K=K+NV(L)                                                         1350
      IF(NV(L).EQ.0) GO TO 36                                           1360
      GO TO 35                                                          1370
   34 CALL PRFL(FIRS,CMPL,EXPZ,SUM1,MAT)                                1380
      PRINT 32,FIRS                                                     1390
      CALL PRFL(FIFT,CMPL,EXPZ,SUM5,MAT)                                1400
      PRINT 32,FIFT                                                     1410
```

```
      DO 33 I=1,MAT                                                     1420
      ABS1(I)=CABS(SUM1(I))                                             1430
      ABS5(I)=CABS(SUM5(I))                                             1440
      CALL ANGLE(SUM1(I),ANG1(I))                                       1450
      CALL ANGLE(SUM5(I),ANG5(I))                                       1460
      SUM1(I)=CMPLX(ABS1(I),ANG1(I))                                    1470
      SUM5(I)=CMPLX(ABS5(I),ANG5(I))                                    1480
   33 CONTINUE                                                          1490
      CALL PRFL(FIRS,ABSL,EXPZ,SUM1,MAT)                                1500
      PRINT 32,FIRS                                                     1510
      CALL PRFL(FIFT,ABSL,EXPZ,SUM5,MAT)                                1520
      PRINT 32,FIFT                                                     1530
   10 FORMAT("1",10X,"THIS IS THE TOTAL INDUCED FIELDS ON  THE SURFACES 1540
     + OF AN",1X,I4,1X,"Q'S SYMMETRICAL BODY")                          1550
   11 FORMAT("-","THE RESULTS ARE OBTAINED BY COMBINING RESULTS DUE      1560
     +TO FOUR DIFFERENT TYPES OF IMPRESSED FIELDS WHICH ARE COSE, COSH,  1570
     +SINE, AND SINH")                                                  1580
   12 FORMAT("-",10X,"THE LOCATION AND DIMENSION OF EACH SURFACE-CELL    1590
     +ARE LISTED BELOW")                                                1600
   13 FORMAT("-",10X," N ",7X,"X (IN CM)",10X,"Y (IN CM)",10X,          1610
     +"Z (IN CM)",10X,"D (IN CM)")                                      1620
    1 FORMAT(6I3)                                                       1630
    3 FORMAT(4F10.5)                                                    1640
    4 FORMAT(3E13.6)                                                    1650
   15 FORMAT("-",5X,I3,7X,F10.5,10X,F10.5,10X,F10.5,10X,F10.5)          1660
   16 FORMAT("-",5X,"THERE ARE NX=",I3,2X,"NY=",I3,2X,",NZ=",I3,2X,     1670
     +",NNX=",I3,2X,",NNY=",I3,2X,",AND NNZ=",I3,2X,                    1680
     +"CELLS ON SURFACES OF THE FIRST QUADRANT")                        1690
   17 FORMAT("-",5X,"DIELECTRIC CONST.=",E13.6/"-",5X,"CONDUCTIVITY=",  1700
     +E13.6,"MHO/METER"/"-",5X,"FREQUENCY=",E13.6,"MEG HZ"/"-",5X,"THE  1710
     +TOTAL INCIDENT FIELD IS A PLANE WAVE")                            1720
   32 FORMAT("-",10X,"ABOVE ARE TOTAL FIELDS ON THE ",A4," QUADRANT")   1730
      STOP                                                              1740
      END                                                               1750
C*****SUBROUTINE PRFL*************************************************
      SUBROUTINE PRFL(QUAD,COAB,MODE,INFL,MAT)                          1760
      REAL MODE,LEFT                                                    1770
      COMPLEX INFL(MAT)                                                 1780
      COMMON /NXYZ/NX,NY,NZ,NNX,NNY,NNZ                                 1790
      DATA TOP,/"TOP "/,RIGHT/"RIGH"/,REAR/"REAR"/,CMPL/"CMPL"/,ABSL/"ABS1800
     +L"/,FIRS/"FIRS"/,FIFT/"FIFT"/,FRNT/"FRNT"/                        1810
      DATA BTTM/"BTTM"/,LEFT/"LEFT"/                                    1820
      PRINT 111,MODE,QUAD                                               1830
      IF (COAB.EQ.ABSL) PRINT 112                                       1840
      IF (COAB.EQ.CMPL) PRINT 113                                       1850
      PRINT 123, TOP                                                    1860
      PRINT 137                                                         1870
      PRINT 122                                                         1880
```

```
      DO 12 I=1,NX                                                      1890
      II=I                                                              1900
      J=I                                                               1910
      IP=J+NX                                                           1920
      IPP=IP+NX                                                         1930
      IPPP=IPP+NX                                                       1940
   12 PRINT 13,II,INFL(J),INFL(IP),INFL(IPP),INFL(IPPP)                 1950
      PRINT 122                                                         1960
      PRINT 123,RIGHT                                                   1970
      PRINT 140                                                         1980
      PRINT 122                                                         1990
      DO 14 I=1,NY                                                      2000
      II=I+NX                                                           2010
      J=I+4*NX                                                          2020
      IP=J+NY                                                           2030
      IPP=IP+NY                                                         2040
      IPPP=IPP+NY                                                       2050
   14 PRINT 13,II,INFL(J),INFL(IP),INFL(IPP),INFL(IPPP)                 2060
      PRINT 122                                                         2070
      IF(QUAD.EQ.FIRS) PRINT 123,REAR                                   2080
      IF(QUAD.EQ.FIFT) PRINT 123,FRNT                                   2090
      PRINT 143                                                         2100
      PRINT 122                                                         2110
      DO 15 I=1,NZ                                                      2120
      II=I+NX+NY                                                        2130
      J=I+4*(NX+NY)                                                     2140
      IP=J+NZ                                                           2150
      IPP=IP+NZ                                                         2160
      IPPP=IPP+NZ                                                       2170
   15 PRINT 13,II,INFL(J),INFL(IP),INFL(IPP),INFL(IPPP)                 2180
      PRINT 122                                                         2190
      IF(NNX.EQ.0) GO TO 19                                             2200
      PRINT 123,BTTM                                                    2210
      PRINT 137                                                         2220
      PRINT 122                                                         2230
      DO 16 I=1,NNX                                                     2240
      II=I+NX+NY+NZ                                                     2250
      J=I+4*(NX+NY+NZ)                                                  2260
      IP=J+NNX                                                          2270
      IPP=IP+NNX                                                        2280
      IPPP=IPP+NNX                                                      2290
   16 PRINT 13,II,INFL(J),INFL(IP),INFL(IPP),INFL(IPPP)                 2300
      PRINT 122                                                         2310
   19 CONTINUE                                                          2320
      IF(NNY.EQ.0) GO TO 20                                             2330
      PRINT 123,LEFT                                                    2340
      PRINT 140                                                         2350
      PRINT 122                                                         2360
```

```
      DO 17 I=1,NNY                                                     2370
      II=I+NX+NY+NZ+NNX                                                 2380
      J=I+4*(NX+NY+NZ+NNX)                                              2390
      IP=J+NNY                                                          2400
      IPP=IP+NNY                                                        2410
      IPPP=IPP+NNY                                                      2420
   17 PRINT 13,II,INFL(J),INFL(IP),INFL(IPP),INFL(IPPP)                 2430
      PRINT 122                                                         2440
   20 CONTINUE                                                          2450
      IF(NNZ.EQ.0) GO TO 21                                             2460
      IF(QUAD.EQ.FIRS) PRINT 123,FRNT                                   2470
      IF(QUAD.EQ.FIFT) PRINT 123,REAR                                   2480
      PRINT 143                                                         2490
      PRINT 122                                                         2500
      DO 18 I=1,NNZ                                                     2510
      II=I+NX+NY+NZ+NNX+NNY                                             2520
      J=I+4*(NX+NY+NZ+NNX+NNY)                                          2530
      IP=J+NNZ                                                          2540
      IPP=IP+NNZ                                                        2550
      IPPP=IPP+NNZ                                                      2560
   18 PRINT 13,II,INFL(J),INFL(IP),INFL(IPP),INFL(IPPP)                 2570
      PRINT 122                                                         2580
   21 CONTINUE                                                          2590
  111 FORMAT("1","THE FIELDS INDUCED BY ",A4," TYPE OF IMPRESSED FIELD",2600
     +1X,"IN ",A4," QUADRANT ARE LISTED BELOW (AMP/METER FOR H FIELDS,"2610
     +,1X,"VOL/METER FOR E FIELDS)")                                    2620
  112 FORMAT("-"," N ",6X,"ABSL",11X,"PHAS",11X,"ABSL",11X,"PHAS",      2630
     +11X,"ABSL",11X,"PHAS",11X,"ABSL",11X,"PHAS")                      2640
  113 FORMAT("-"," N ",6X,"REAL",11X,"IMAG",11X,"REAL",11X,"IMAG",      2650
     +11X,"REAL",11X,"IMAG",11X,"REAL",11X,"IMAG")                      2660
  123 FORMAT("-",15X,"ON THE ",A4," SURFACE WE HAVE THE FOLLOWS")       2670
  137 FORMAT("-",14X,"     HZ     ",20X,"     HY     ",20X,"     EZ        ",2680
     +18X,"     EY        ")                                           2690
  122 FORMAT("-",5X,4(28H**************************,2X))                 2700
   13 FORMAT("-",I3,1X,4(1X,"(",2E13.6,")",1X))                        2710
  140 FORMAT("-",14X,"     HZ     ",20X,"     HX     ",20X,"     EZ        ",2720
     +18X,"     EX        ")                                           2730
  143 FORMAT("-",14X,"     HY     ",20X,"     HX     ",20X,"     EY        ",2740
     +18X,"     EX        ")                                           2750
      RETURN                                                            2760
      END                                                               2770
C*****SUBROUTINE ANGLE*************************************************2780
      SUBROUTINE ANGLE(G,EF)                                            2790
      COMPLEX G                                                         2800
      F=57.2957795131                                                   2810
      IF (AIMAG(G)) 10,20,30                                            2820
   10 IF (REAL(G)) 40,50,40                                             2830
```

```
 40 EF=F*ATAN2(AIMAG(G),REAL(G))                                    2830
    GO TO 100                                                       2840
 50 EF=-90.0                                                        2850
    GO TO 100                                                       2860
 20 IF (REAL(G)) 60,65,65                                           2870
 60 EF=-180.0                                                       2880
    GO TO 100                                                       2890
 65 EF=0.0                                                          2900
    GO TO 100                                                       2910
 30 IF (REAL(G)) 40,75,40                                           2920
 75 EF=90.0                                                         2930
100 CONTINUE                                                        2940
    RETURN                                                          2950
    END                                                             2960
```

CHAPTER V

SUMMARY

A new theoretical method for determining the eddy current induced by an oscillating magnetic field in a finite conducting body with rotational symmetry was presented in Chapter II. This study was motivated mainly by two reasons: (1) More biological research and medical applications utilize the irradiation of RF magnetic fields. (2) When a thick biological body is irradiated by a HF-VHF EM wave, the induced current can be divided into the electric and magnetic modes. The magnetic mode of induced current, also called the eddy current, is predominant and is difficult to determine with a numerical method. The new theoretical method was derived by utilizing the theory of vector potential, and the numerical results were obtained by using the point-matching and matrix inversion techniques. Two different forms of magnetic fields were considered in Chapter II, namely, a uniformly impressed magnetic field and a beam of uniform magnetic field. For both cases, the results obtained by the new method were found to deviate significantly from the conventionally used, quasi-static solution. An experiment for measuring the magnitudes of the induced

electric fields inside rotationally symmetrical bodies was conducted, and some experimental results were presented. Also, a closed form solution for the induced electric field inside a finite conducting sphere was given in that chapter. The accuracy of the new method was verified by comparing the present solution with the experimental measurements as well as the closed form solution for a finite conducting sphere.

In the quantification of the magnetic mode of the induced current in an irradiated biological body, one often encounters the difficulty of numerical convergence. The new method presented in Chapter II shows an excellent numerical convergence and accuracy, and should help solve this problem. It is noted that the applicability of this new method is restricted to the cases of rotationally symmetrical bodies which are constructed with nonmagnetic materials. For the cases of magnetic materials without rotational symmetry, a different technique should be developed.

In Chapter III, the microwave interactions with finite conducting, biological bodies were studied. As the first step, we derived two coupled, surface integral equations which relate the induced fields to the incident fields. The validity of these equations was then verified by considering the special case of a two-dimensional, infinite interface between free space and a finite conducting medium.

These two surface integral equations were solved numerically by use of moment-method. The results so obtained agree quite well with the solutions obtained by a volume integral equation method--Tensor Integral Equation Method. For the purpose of reducing the number of unknowns, only the cases involved with electrically small bodies were demonstrated in Chapter III. It is believed that this surface integral equation method is potentially more efficient than the volume integral equation method since when an electrically large body is irradiated by EM waves, the fields induced inside the body mainly concentrate in the region near the body surface. Under this condition, solving the internal fields through the volume integral equation becomes impractical.

Moreover, with a finer subdivision of the body for the method of moment, the number of unknowns encountered in solving the surface integral equations is usually less than that encountered for the volume integral equation, if the number of unknowns exceeds a certain number as in an electrically large body. It is pointed out that the validity of the surface integral equation method is limited to the cases of homogeneous bodies due to the nature of deriving the equations.

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