THE PERFORMANCE OF A PHYSICAL DISTRIBUTION CHANNEL SYSTEM UNDER VARIOUS CONDITIONS OF LEAD TIME UNCERTAINTY: A SIMULATION EXPERIMENT

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ABSTRACT

THE PERFORMANCE OF A PHYSICAL DISTRIBUTION CHANNEL SYSTEM UNDER VARIOUS CONDITIONS OF LEAD TIME UNCERTAINTY: A SIMULATION EXPERIMENT

By

George D. Wagenheim

To achieve efficient and effective distribution of finished goods, it is necessary to understand the operation of a physical distribution channel system, the forces which impinge upon the system and the effects of these forces on successful channel operation. One such force is lead time uncertainty. Lead time is the elapsed time from placement to receipt of an order. Knowledge of how uncertain lead time affects the cost and the delivery capability of the system will result in partial understanding.

Lead time can be described by a probability distribution pattern, variance and average duration (level). Hypothetically, as pattern, variance and level change, the cost and service of the physical distribution channel system will change.

Using the Long Range Environmental Planning Simulator (LREPS) channel simulation model, 18 different combinations of pattern, variance and level representing uncertain lead time were tested for a 120 day operating period. Cost and service results were compared between uncertain systems and to the performance of a deterministic lead time

system (control). The following conclusions were drawn as a result of the analysis.

When comparing cost and service levels resulting from uncertain lead times to the deterministic control it was found that the pattern, variance and level unfavorably influenced physical channel system performance. In all cases, the cost resulting from uncertain lead times was higher and the service level was lower than the deterministic control case.

Comparison of cost and service levels between various uncertain systems revealed that all probability distribution patterns had similar effects on channel performance with the exception of the exponential distribution. The symmetrical distributions (normal and gamma) had the least effect on cost and service while the skewed distributions (exponential and erlang) had the greatest effect. Two coefficients of variation were used and compared. Those systems where the larger variation was used were consistently more costly and less effective than those systems with the smaller variation. Two average durations of lead time were also used and compared. The cost was consistently higher and the service level lower in those systems that employed the longer average lead time. Thus it was concluded that uncertainty adversely affected the cost and service of the physical distribution channel system. And different types of uncertainty had different effects which were guaranteed and judged to be statistically significant.

The cause of such behavior is the range of possible lead time durations. In the control situation, the fixed lead time guarantees that inventory replenishment will occur before stock is depleted. When lead time varies, durations above the average are possible which means inventory replenishment will occur after the stock is depleted. As the length of lead time durations above average increases (i.e., exponential), the number of days that a stocking location remains out of stock increases. Thus the service level of the system decreases. Even though inventory is not available at the retail level, it is still in the system. Thus average inventory increases at the wholesale and manufacturer level which increases the cost of the system.

In an effort to explain potential direction and possible results of future research, two simulation runs were conducted wherein both lead time and demand were allowed to simultaneously vary in accordance with a selected pattern, variance and level. The results, though tentative, indicate that lead time is the most critical of the two variables on physical channel performance. Demand somewhat neutralized the effect of lead time but the impact was insufficient to bring the system to the efficiency and effectiveness of the control system.

This research resulted in the following conclusions.

1. It is desirable to determine the type of lead time uncertainty that is confronted by the physical distribution channel system. If planning and operation is undertaken assuming an improper lead time distribution, the anticipated results will not be achieved.

2. With knowledge of the types of lead time distributions which will result in maximum efficiency and effectiveness, a systems planner can attempt to design the channel to enjoy the benefits of desirable distributions.

3. As conditions of lead time uncertainty change, the physical channel system must change in anticipation or in reaction. Without knowledge of the efficiency and effectiveness of particular distributions, such a change would seem unnecessary and if the need for change were known it would be unknown as to how to change.

4. The present research supports the systems concept. Decisions made by other members in the channel without knowledge of how it affects the overall system performance can lead to detrimental system performance. In addition, the importance of working together in order to optimize the system is emphasized.

5. Lastly, efforts to determine a system's lead time distribution appear worth the expense. It is important to point out that in all test cases, as alterations were made to reduce the cost, the same alterations resulted in increased service.

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A DISSERTATION

Submitted to
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To

my wife,

Charlotte
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With unselfish help from many individuals, this research became a reality. Academic, financial and moral support are all important, none are independent.

Johnson and Johnson Domestic Operating Company supported the original LREPS project. Their continued support of which I am a benefactor is greatly appreciated.

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CHAPTER I

INTRODUCTION

General Problem Statement

The physical distribution of goods represents a significant portion and an integral segment of the economy. The importance of physical distribution to the firm and to the economic sector at large cannot be denied. It has been variously reported that physical distribution costs account for 20% of the total sales dollar and in some cases may be as high as 50%.[1] In addition to aggregate cost, physical distribution is an integral part of overall distribution performance. Goods destined for consumption must be physically moved to the location of purchase or no transactions will result. Without physical distribution the economic sector would not function. To achieve efficiency and effectiveness in physical distribution, it is important to understand how the overall channel system operates, the forces which impinge upon the system and the effects of the forces on the successful operation of the system.

Only recently have serious attempts been made to understand these interrelationships. Although research has been conducted on all aspects of channel relationships, it has not been exhaustive nor have the conclusions been definitive. As a result, there is much research still to be done in the physical distribution of goods.[2]

Certain aspects of physical channel structure have been investigated. Decisions as to the overall structural design of the physical channel system have been effectively improved through the use of simulation models of such systems. Bowersox,[3] Shycon,[4] and Ballou[5] have made important contributions in the area of physical channel system simulation modeling. Behavioral dimensions of the channel are receiving more attention, with the works of Stern[6] and Bucklin[7] making significant impacts in this area. In addition, the location and inventory decisions have been exhaustively researched[8] and a number of effective models constructed.[9]

One aspect of physical distribution operations that has not been exhaustively researched is the impact of uncertainty upon system performance. Uncertainty influences physical distribution operations by introducing variable sales patterns and replenishment times. To the degree a better understanding of the impact of uncertainty is understood, it should lead to more effective planning and control of the system. If we were able to assess the impacts of uncertainty upon various aspects of the channel system, we would then be in a good position to account for these effects and take action to overcome them. The purpose of this research is to measure the impact of uncertainties (demand and lead time) on the performance (cost and service) of a physical distribution channel system.

An Overview of Physical Distribution

Physical distribution though variously defined will be used in this research to encompass the movement of finished goods from the manufacturing plant to the ultimate consumer.[10] The purpose of physical distribution is to move finished goods between these points in an efficient and effective manner. Performance is measured in terms of cost and service. Physical distribution is defined for this research to include transportation, warehousing, inventory, communication and handling.

The basic structure of a physical channel system is that of echeloned arrangement of institutions and/or functions. Echelon refers to a steplike formation. In the physical distribution context the echelon structure refers to the levels through which a product proceeds from production to a point of ultimate consumption. To measure the impact of uncertainty in this research an echelon structure is used. The echelon system rather than the direct system (one where there are no steps between the manufacturer of the product and the ultimate consumer) was selected for study for several reasons. Namely, it is a close replication of the real world, few products are directly distributed, the advent of the increasing number of products available both in kind and degree and the increase in scrambled merchandise necessitates the use of an echelon system for efficient distribution.[11] Furthermore, the effects of uncertainty on the system would seem to be magnified as additional levels are added to the system. Time delays, additional order cycles and the increased number of inventory points would account for these effects.

For this research each echelon has the following characteristics. They will hold inventory to facilitate the discrepancies between demand and production; they will be break bulk points, that is, they exist for the purpose of receiving larger volume shipments and dispersing these shipments to various customers and they will offer all the necessary facilitating activities to complete these operations such as handling and communication.

The operation of the physical channel system is defined as a system in which all the components interact to minimize the cost of the total system for a given level of service. System has been variously defined, but generally can be defined as, "a set or arrangement of things so related or connected as to form a unity or organic whole."[12] Bowersox defines the systems concept as, "one of total integrated effort toward the accomplishment of a predetermined objective."[13] The systems concept as cited by Alderson[14] can be viewed at any level of generalization. In terms of physical distribution the system can be seen as the components, i.e., the parts of the physical distribution system controlled by the firm such as transportation, handling, warehousing, inventory and communication.

Because the physical distribution segment of the overall economic sector is a system, these components or activity centers can be viewed as interrelated subsystems. Therefore, they behave not as entities but as interrelated parts of a whole. Trade-offs occur between and within these subsystems. The trade-offs can be arranged in such a way so as to influence total cost and service capability. The task of channel design is one of finding favorable trade-off relationships. The system can also be viewed at a higher level. That is, it would not only encompass the parts specific to an individual firm but could also include all the firms in a channel from manufacturer to ultimate consumption. It is in this context that system is defined for this research.

The argument for viewing the physical channel of distribution as a system rests upon the fact that all participants share in a unified goal. Thus, working in concert has the greatest potential for achieving desired results. That is, all the members of the channel have similar objectives. The objectives can be best reached through the systems approach which implies cooperation and concentration on a unified goal.

Attempts to improve unified operations across channel echelons can be witnessed by the increased moves to vertically integrate the channel in various ways.[15] Furthermore, the position has been presented by several authors that it is the channel that competes with other channels rather than firms competing against other firms.

For instance,

> Traditional economic and business analysis of strategic planning has tended to focus on the behavior of individual firms. More recent thinking suggests that the total channel systems might be the more appropriate unit of analysis. This view is taken below because, basically, economic systems are designed to satisfy customer needs and these needs are not completely satisfied until some

> package of goods and/or services has moved all along a channel of distribution to users or final consumers. The members of a channel system may not think of themselves as members of a system, but nevertheless their system will continue to exist only as long as their unique combination performs more effectively than competing channels.[16]

In this research, therefore, measures of performance relating to cost and service, are those associated with the channel system, rather than the individual channel members.

Uncertainty

A major force which affects the structure and operation of a physical distribution system is uncertainty. Uncertainty in the physical distribution context can be generally defined as not knowing what will occur or when it will occur. Although the sources of uncertainty are varied, it manifests itself in two general ways on the physical distribution system. First, there is demand uncertainty and, second, lead time uncertainty.

Demand can be defined as a request by the ultimate consumer made upon the system to deliver a product or service. Demand presents itself to the system in an uncertain fashion (i.e., it is a random variable). It is uncertain as to when demand will occur over time and when demand occurs it is uncertain as to how much will be demanded (i.e., level).

Lead time can be defined as the amount of time between placement of an order and receipt of that order. Specifically, it can be broken down into three components, order communication, order processing and transportation (see Figure 1-1). Each of these components represents a source of uncertainty. It is not known with certainty how long each one of these activities will take, thus taken together it is not known with certainty the overall time duration from placement to receipt of an order.

As pointed out previously, demand and lead time uncertainty affect the structure and operation of the physical distribution system. Uncertainty also affects the planning and control of the system. On planning and control, Lewis and Erickson say, "Management planning and control should concern itself with maximizing the efficiency and effectiveness of efforts used in attaining desired purposes."[17]

Thus, the significance of planning and control to the physical distribution system is established. Ideally, to plan and control effectively, we must know what will occur and when. However, the physical distribution system operates in a world of uncertainty, thus planning and control are adversely affected. Without effective planning and control efficiency and effectiveness are difficult to achieve.

Uncertainty is not new and it will always be with us as a simple fact of business. The majority of efforts in the past designed to cope with uncertainty have attempted to reduce its impact. For instance, more accurate sales forecasting, more accurate budgeting, etc. However, a potentially fruitful approach to solving the same problem is to first accept the fact that there will always be uncertainty and asking, can it be categorized and described, and if so, can one isolate how the various

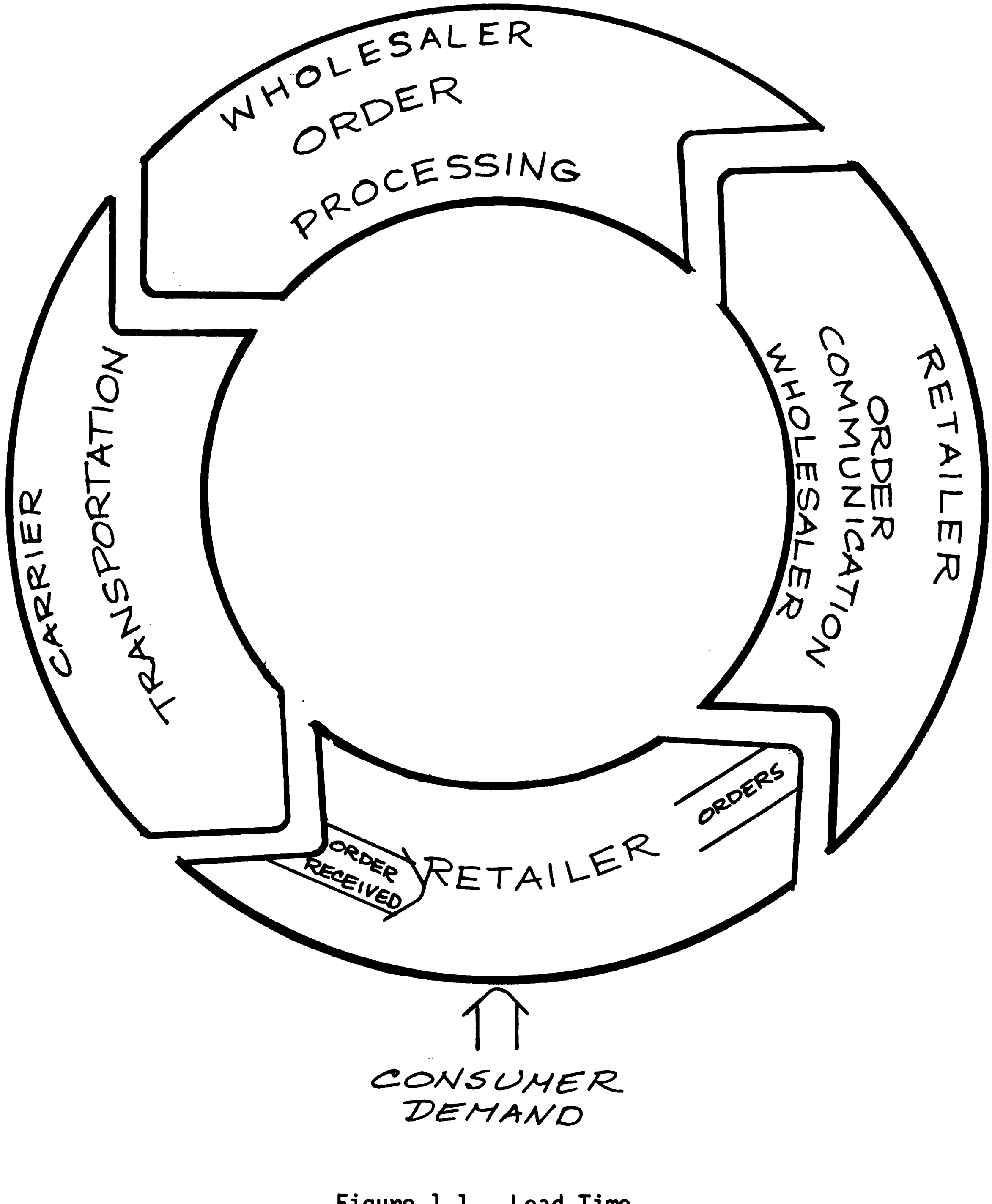

Figure 1-1. Lead Time.

types of uncertainty will affect a physical channel system. If one could isolate the impacts of uncertainty, which is the objective of this research, planning and control would be improved.

Research Procedure

The purpose of this research, as indicated earlier in this chapter, is to measure the impact of demand and lead time uncertainty on the cost and service capabilities of a physical channel of distribution. Demand and lead time uncertainty is evidenced in three material ways: (1) the level of demand and lead time, or average demand and lead time; (2) the variability or dispersion of demand and lead time about its average; and (3) the pattern or probability distribution of demand and lead time. Consequently, the research problem to be solved involves the development of a means by which the three material aspects of uncertainty may be impacted upon a physical channel system and the resultant cost and service levels measured.

Ideally, the solution to this problem could be obtained by performing a series of experiments on an existing channel of distribution. In this manner, the researcher could then observe how the system reacted to the changes in demand and lead time levels, variability and patterns. However, such a procedure is not feasible nor practical. It would not be possible to control all the relevant variables in the system in that cost and service measures could not be determined under "controlled" or identical conditions. Nor would it be possible to manipulate the level, variability and pattern of demand and lead time as is experienced by an ongoing physical channel of distribution. Therefore, the solution to the research problem lies not in actual experimentation, but with experimentation on a replication or model of a real world physical channel system.

A model is generally regarded as an abstraction or simplification of a system. A mathematical model describes the system, its components and their interactions in quantitative terms. The model thus allows one to abstract the essential characteristics of a system and thereby observe and eventually predict how that system will function. Models cannot replace actual experience; at best they reduce a complex system to manageable proportions or serve to crystallize our thinking or perceptions.[18] Once the analyst has achieved a parallelism between the actual situation and his model, it is usually easier to manipulate the model to study the characteristics in which he is interested than it is to try to work with the real world system.[19] The model of a system then provides the researcher with the means to experiment with variables both internal and external to the system model and thereby observe the reaction of the system to such variations.

Simulation is one form of modeling which has been successfully employed to replicate physical channel systems.[20] Simulation models mathematically represent a system, but when applied to problem solving do not necessarily lead to an optimal solution. Teichroew and Lubin provide insight into the nature of computer simulation:

> Computer simulation has come into increasingly widespread use to study the behavior of systems whose state changes over time. . . . Alternatives to the use of simulation are mathematical analysis, experimentation with either the actual system or a prototype of the actual system, or reliance upon experience and intuition. All, including simulation, have limitations. Mathematical analysis of complex systems is very often impossible; experimentation with actual or pilot systems is costly and time consuming, and relevant variables are not always subject to control. Intuition and experience are often the only alternatives to computer simulation available but can be very inadequate.
>
> Simulation problems are characterized by being mathematically intractable and having resisted solution by analytical methods. The problems usually involve many variables, many parameters, functions which are not well behaved mathematically, and random variables. Thus simulation is a technique of last resort. Yet, much effort is now devoted to "computer simulation" because it is a technique that gives answers in spite of its difficulties, costs and time required.[21]

Thus, simulation is a viable technique for modeling systems characterized by great complexity, probabilistic or stochastic processes and whose variables are difficult to analyze in precise mathematical terms. Simulation is also quite tractable for experimentation in that after a computer model of the system has been developed, the model may be sampled under different input conditions.[22] Therefore, a simulation model of a physical channel system has been selected as the means by which to measure the cost and service response of such a system to various types and levels of uncertainty.

The specific simulation model to be used in this research is the LREPS model.[23] The LREPS model has the following important characteristics:[24]

1. It provides a comprehensive model of physical distribution operations as an integrated system capable of total cost and customer service performance measurement.

2. The model incorporates a multiechelon structure.

3. The unifying dimension of the model is both spatial and temporal.

4. The model is dynamic, which permits physical distribution planning over time.

5. The model allows for both demand and lead time to be expressed in probabilistic terms. Thus, the model is capable of introducing simulated demand and lead time patterns based upon any one of a variety of probability distributions.

The design and operation of the LREPS model have been well documented in various works.[25]

The LREPS model provides the basic framework for the experimentation involving demand and lead time level, variability and pattern. The basic LREPS model was modified in accordance with the model description in Chapter II. Thus, one phase of the present research was to develop the necessary operating rules and cost functions to be employed in the modified model.

The effects of three material measures of uncertainty related to demand and lead time upon system cost and service are examined in the research. Each experimental run consists of impressing demand and lead time at a given level, with a given variability and a given probability distribution on the channel system model. In this manner, the impact of level, variability and pattern of uncertainty can be measured. The measures of system performance which serve as output of each experiment include:

1. Total system cost.

2. Individual activity center costs for the channel system.

3. System service level.

The probability distributions used in the experimental runs are computer generated. Each distribution reflects a particular probability function, mean and variance. The resulting distributions then serve as daily demand and lead time input for each experiment. The probability distributions selected for experimentation are those which have empirical justification and which have the potential to measurably affect channel performance.

Two "controlled" simulation runs were made for comparison purposes. The control or base system is completely deterministic in nature, that is, demand and lead time are given and fixed. As a result of this total certainty, no provision for safety stock is made. Thus, the experimental runs are also devoid of safety stock.

The experimental runs are short run in nature, i.e., the system's output is measured for a time span (simulated days) of less than one year. Because the system is evaluated over a short period of time, facility locations and numbers are not allowed to vary. Additionally, a time series of demand is not considered. In other words, the trend, seasonal and cyclical values of demand over the period are zero. All combination of patterns, levels and variances are imposed on a model that has no provision for backorders at the customer level. There are provisions for backorders within the system.

The method of experimentation in the simulation model is to make changes in the external and internal variables (demand and lead time) and then analyze the effects of these changes on the cost and service of the physical channel system. To study the results in some meaningful

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The method of experimentation in the simulation model is to make changes in the external and internal variables (demand and lead time) and then analyze the effects of these changes on the cost and service of the physical channel system. To study the results in some meaningful manner, a proper method of analysis, i.e., experimental design must be selected.

The experimental design employed in the research is a factorial design. A factorial experiment is one in which the effects of all the factors and factor combinations in the design are investigated simultaneously. In this case, three factors are to be analyzed: the probability distribution of demand and lead time, the average or level of demand and lead time; and the variance or dispersion of demand and lead time about the average. The factorial design is advantageous to the extent that effects of a particular factor are evaluated by averaging over a broad range of other experimental variables. For example, the factorial design will permit statements to be made as to the effect of a particular demand and lead time distribution, where the distribution is considered over a range of demand and lead time levels and variances.

The data is analyzed by standard analysis of variance techniques in addition to two multiple comparison techniques and standard t tests. Thus, the research develops comparisons, on the basis of cost and service, of the effects of probability distributions, levels and variances. Additionally, the cost and service performance of the system under each level of each factor is compared against the control system. Such a comparison is expected to provide a direct measure of the effect of the given type of uncertainty.

This research is basically a pilot inquiry into the effects of demand and lead time uncertainty on the performance of a physical channel system. To this extent, it is exploratory in nature, seeking to systematically analyze the sensitivity of physical distribution cost and service to uncertain conditions associated with demand and lead time. Thus, on the basis of research results, generalizations are expected on the impact of uncertainty. In addition, guidelines for further research will be established.

To be able to draw generalizations as to the effects of uncertainty on the system it is necessary to remove selected aspects of reality. As previously described, there are no safety stocks, no locational variations, no trends, etc. Inclusion of such factors (even though they would make the model more realistic) would only confuse and mask the effects of uncertainty. The intent is to systematically replace presently missing factors in future research. As factors are added and the model becomes more complete, the effects on the system of the newly introduced factors can be more accurately analyzed.

This research, which concentrates on demand and lead time uncertainties, should lead to the following results:

1. The testing of previously established hypotheses. Basic propositions as to how the channel system will react to various changes in key external and internal variables can be put to concrete test. Such hypotheses are formulated in Chapter IV.
2. Development of researchable hypotheses. The experiments conducted with this model should lead to a vast array of propositions as to effects upon the system when demand and lead time is varied in its material aspects. Hypotheses as to possible

changes in operating policies to mitigate the effects of demand and lead time variability should follow as a result of the experimental runs.

3. The results of this research should aid management of channel systems in formulating more satisfactory decision rules based upon the nature of the demand and lead time pattern faced by the channel, Different products experience different patterns of demand, and a knowledge of the effects of such patterns will assist management in the process of planning and controlling their systems to account for such patterns.

Division of the Problem

The research described in this chapter is completed in three aspects:

1. The effects of various levels, variability and patterns of demand on a physical channel system of distribution.
2. The effects of the same variations in lead time on the channel system.
3. To provide an indication as to possible areas for future research.

Therefore, three experimental runs are made which combine both lead time and demand uncertainty.

Each of the first two aspects are sufficiently broad and require an in depth evaluation of uncertainty consequences. The probability distributions assumed by demand and lead time are in some cases dissimilar and experience different ranges of level and variability. Additionally, each may be considered in isolation of the other without a great loss in empirical validity. Thus, two dissertations are undertaken using a common model. The research in one dissertation considers the effects of demand variations on the system cost and service, holding lead time constant. The other dissertation evaluates lead time variability, holding demand level, variability and pattern constant. Thus, with the exception of Appendix A (the physical distribution literature review) and Chapter VII (the analysis of the three experimental runs which combine demand and lead time uncertainty) the dissertations are separate and completed individually.

Specific Problem Statement

The purpose of this research is to observe and analyze the performance of a multiechelon, physical distribution system under various conditions of lead time uncertainty. Lead time is defined as the elapsed time from placement of an order to receipt of that order. If lead time were known with accuracy, an efficient and effective physical channel system could be designed and operated (all other things being equal). It is not known with accuracy the expected time span from placement to receipt of an order. As a result of this uncertainty in lead time, the planning and control of a physical channel system is adversely limited. System service level is difficult to predict and control and minimum system cost is difficult to predict and maintain.

Due to the sources which cause lead time uncertainty it is assumed that lead time uncertainty will never be eliminated. There is always some question as to how long it will take from placement of the order to receipt of the order. However, planning and control can be improved if the type of uncertainty and the effects of uncertainty are known. More specifically, planning and control would be more accurate if we knew how various types of uncertainty affected cost and performance of physical channel system activities and components. Likewise it would be helpful to know how various types of uncertainty affected total system cost and service.

Due to the multiplicity of causes, both known and unknown, which make lead times change, it can be treated as a random variable. Therefore, lead time can be represented by a probability distribution which describes its characteristics. Lead time can be described by its pattern, mean and variance. As these characteristics change, the effects upon a physical channel system are expected to change. The pattern or shape of the probability distribution describes the frequency of occurrence of lead time durations. The distributions show the probability of having an order delivered in a specific number of days. The mean of the probability distribution expresses the average time required to complete an order cycle. The variance indicates the extent to which the distribution is dispersed around the mean. For lead time the variance represents the inconsistency or variability.

Knowledge of how particular characteristics of lead time individually and collectively affect system performance would lead to a better understanding of physical channel systems. For instance, it is hypothesized that different patterns (i.e., negative exponential, gamma, etc.) will have different effects on the systems. Such effects have been shown in single point distribution problems, but have not been applied to a multiechelon channel system. In addition, it is hypothesized that lead time variability (i.e., the variance around the mean) affects total channel performance. As lead times become less variable, total system cost will decline for a given level of service. Lastly, the mean or average number of days required to complete an order cycle will affect the operation of a physical channel system.

Therefore, we have the problem of knowing what effects different patterns, variances and means of lead times have on the total cost and service level of a physical channel system. Conclusions regarding these effects are expected to enhance planning and control, thus increasing the efficiency and effectiveness of a physical channel system. Therefore, the purpose of this research is to observe and analyze the cost and service performance of a multiechelon, physical distribution system under various patterns, mean and variances of lead time.

Thesis Outline

This dissertation consists of seven chapters. After the introductory chapter, Chapter II describes the conceptualization of the channel system to be employed in this research. The model, its description, definition and relations are also developed in Chapter II. The modifications to the LREPS model, including decision rules, cost functions and output measures are detailed.

Chapter III describes the characteristics of lead time uncertainty. The nature of probability distributions and empirical justification of the existence of particular distributions are also reviewed. Criteria for the selection of a probability distribution as representative of lead time and final selection of those distributions are also considered.

Chapter IV details the research hypotheses to be tested. Additionally, the research methodology is presented. At this stage, the experimental design and measures of system output are specified in depth.

Chapter V details the findings of the experimental runs.

Chapter VI summarizes the findings and suggests generalizations to be drawn from the research. Areas of future research and the limitations of the present research are also outlined.

Chapter VII describes the procedures employed to make the experimental runs where lead time and demand are both random variables. The findings and conclusions relevant to these experiments are then presented. Finally, suggestions are developed as to the implications for future research.

Appendix A provides an overview of the more important simulation models specific to physical channel system modeling. Additionally, the more commonly applied inventory models are reviewed.

Appendix B details the statistical computations employed in the findings chapter.

CHAPTER I--FOOTNOTES

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CHAPTER II

RESEARCH MODEL

Introduction

To reach the stated objectives of this research it is necessary to employ a model that will simulate a physical channel system. It is the purpose of this chapter to develop and describe such a model. Before a specific model can be developed, however, the physical channel system must be conceptualized. It is here that the boundaries of the system are defined and the general purpose of the system is outlined. Conceptualization of the system is also necessary to force thinking about a channel of distribution in a non-traditional way. It is imperative that the physical distribution channel be seen as an integrated system of firms with a common goal and not as a group of separate, autonomous institutions with individual goals and objectives. Once the system is conceptualized, the LREPS model employed can be detailed. The model structure is outlined and its operation described.

Conceptualization of the System

As noted earlier, this research is concerned with that portion of the distribution system which begins at the end of the production line and ends with the ultimate consumer. This portion of the distribution system has several purposes among which are: communication, passage of title and physical movement. This research is specifically concerned with the physical movement purpose of this portion of the distribution system. Thus, the research interest is in the physical movement of goods from the time that the good assumes its final form (producer or manufacturer finished goods inventory) to the point that the good is in the physical possession of the ultimate consumer. This portion of the distribution system has been referred to as the "physical channel system" in this research. The purpose of the physical channel system is to service ultimate consumer demand by overcoming spatial and temporal gaps between the producer and the ultimate consumer.

The overall general criteria which dictates the structure and behavior of the physical channel system is that it (the physical channel system) performs its inherent function (servicing demand) in an efficient and effective manner within the environment in which it operates.

At this point, it is necessary to explain and/or define several of the above terms such as: efficiency, effectiveness, physical channel system structure and physical channel system behavior.

Efficient-Effective

Efficient can be defined as "producing the desired effect or result with a minimum of effort, expense or waste."[1] Or, "efficiency is a dollar measure of expenditure to get a specific job done."[2] To be efficient in a physical distribution sense is to perform a task at its minimum cost.

Effective can be defined as "producing a definite or desired result" or, "effectiveness is a measure of accomplishment in terms of objectives."[3] To be effective then in a physical distribution sense, is to meet the desired service level stated by objectives.

Therefore, a physical channel system could be efficient but not effective, i.e., operate at minimum cost but not reach the desired service level or it could be efficient but not effective, i.e., reach the desired service level, but not at a minimum cost. It is, however, the goal of the physical channel system to be both efficient and effective while performing its inherent function of servicing demand.

Structure

The inherent function of the physical channel system of servicing demand efficiently and effectively determines the structure of the system. The structure of such a system can be described with the aid of several principles, specifically, the principles of minimum possible engagements, maximum postponement in adjustment and minimum massed reserves.[4]

From the above, it can be seen that the system will have levels or echelons and each level will have the following characteristics. They will hold inventory to facilitate the discrepancies between demand and production; they will be break bulk points, that is, they exist for the purpose of receiving larger volume shipments and dispersing these shipments to various customers and they will offer all the necessary facilitating activities to complete these operations such as handling and communication.

To complete the structure there must be some means to physically move the goods between levels over space. In the physical channel system this is accomplished by the transportation component.

Behavior

The physical channel system as an entity has the purpose or objective of servicing demand efficiently and effectively. Thus, if we view the physical channel system as a system with components, it is clear that it is the goal of the system which determines the behavior of the system and thus its components. The components of the physical channel system could be viewed as the channel members and the activities of the physical channel system could be seen as inventory, warehousing, transportation, communication, and handling. A system has been defined as, "a set or arrangement of things so related or connected as to form a unity or organic whole."[5] In this case, our physical channel system can be seen as that "unity or organic whole."

Viewed as a whole, it can be seen that the physical channel system has an inherent function (service demand). Viewed from the perspective of the channel members it must be concluded that theirs too is to service demand. Thus, from the channel members' perspective we have a coincidence of function, thus, a unified function for the overall physical channel system.

Model Specifications

Although the overall purpose of this research is to generalize the effects of uncertainty on a physical channel system, such generalizations cannot be reached without the use of a specific model. In the previous section the model was bounded (manufacturer to ultimate consumer), its structure generalized (multiechelon) and measures of its operation specified (efficient and effective). In this section the details of the model being used are specified and described.

In construction of the specific model two criteria had to be balanced. On the one side, concluding generalizations are desired. To satisfy such a desire the model employed must be abstracted from a specific industry or product so that the conclusions would apply to all physical distribution systems. However, the logical extension of such thinking could be meaningless results. On the other side of the scale, conclusions are desired which will serve the advancement of the study of physical distribution and aid in the solution of present day physical distribution operational problems. To satisfy this criteria the model must, to a significant degree, be specific to an industry or product. Desiring neither useless generalizations nor conclusions that could only apply to one industry or product, a model was developed to balance these two criteria.

Because of the complexity of the system, computer simulation was chosen as the means of generating results. Specific numbers such as product weight and cube, costs for the system (the overall measure) and times (transit, packing, etc.) had to be employed. Thus, a true abstraction, even if desired, could not be achieved. The employment of such specific terms pulls the research in the direction of a specific system and reality. The actual numbers in absolute terms and the relationships between time, cost and product characteristics are important to the quality of generalizations generated. Thus a decision regarding the level and relationships of numbers to be used had to be made.

The criteria for selection was such that useful generalizations would result. The model developed is specific to the point that its structure and operation simulate real world conditions. However, the level of specificity has not been allowed to replicate a particular industry or product nor have the peculiarities of a particular physical channel system been allowed to enter. The result is a level of abstraction that permits useful generalization.

Structure

The model structure is shown in Figure 2-1. The model is multiechelon in structure, which is in keeping with the conceptualized physical channel system developed earlier. Additionally, it parallels the majority of finished goods physical channels. The three channel levels (institutions) have the general features of holding inventory, being break bulk points (with the exception of the primary stocking point) and have the necessary functional capabilities to carry out related activities (i.e., communication and handling).

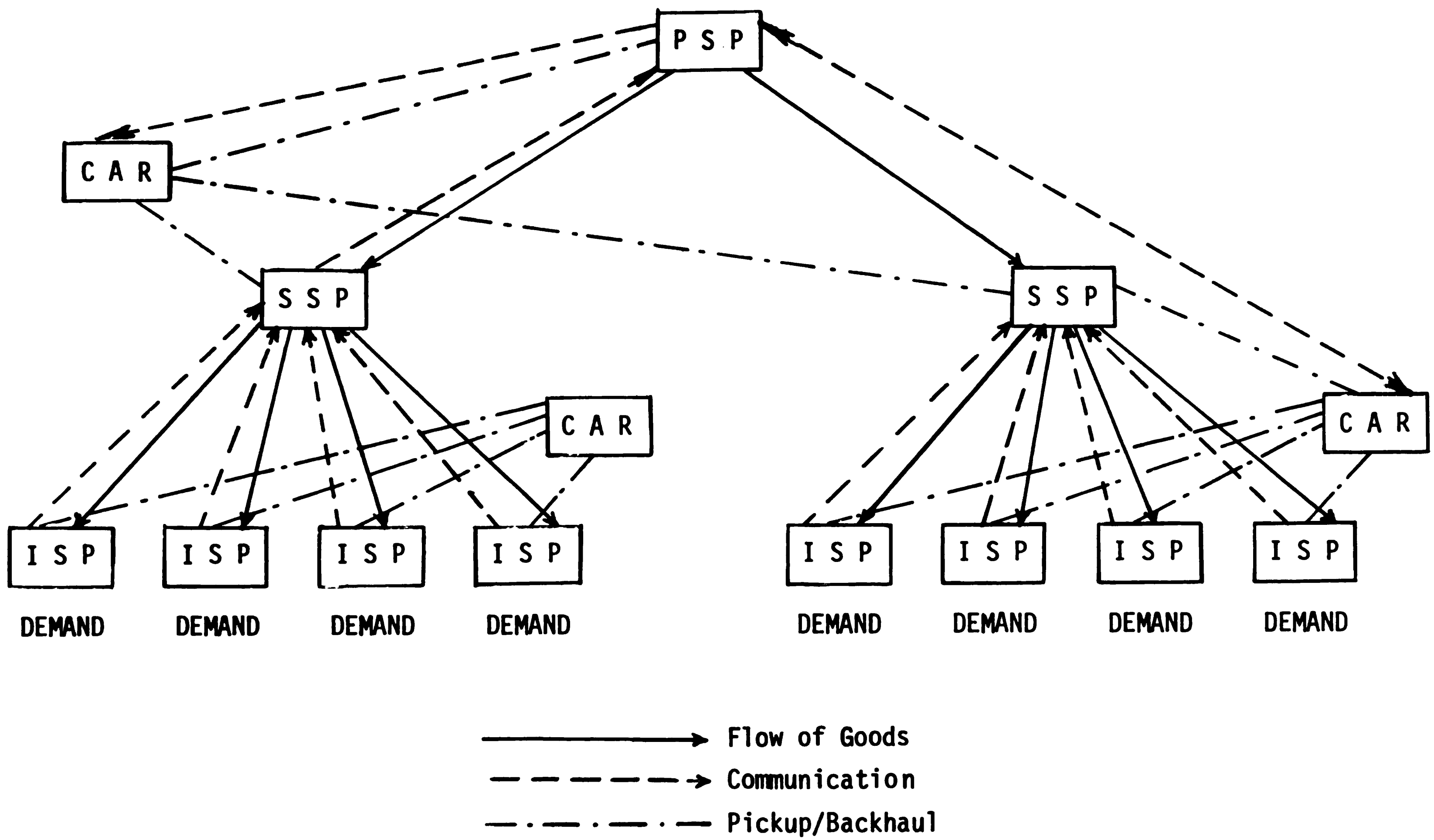


Figure 2-1. Physical Channel Structure

The modeled system handles one product. This abstraction was made for the sake of simplicity. The usefulness of the results will not be limited because only one product is employed. The product chosen is hypothetical in nature, This is in keeping with the general model specifications.

The Primary Stocking Point (PSP) is the first point in finished goods distribution. At this point the product is ready for distribution to the ultimate consumer. In an actual distribution system the PSP is comparable to a manufacturer's finished goods inventory. The source of this inventory is the production line. The PSP holds inventory and is capable of performing the handling function and prepares orders for delivery. The PSP also has a communications capability. The PSP communicates with the production line to request inventory and it receives communication from the Secondary Stocking Point (SSP) regarding the amount of products to be shipped (orders). The PSP can also communicate with the carrier to request service. As is shown in Figure 2-1, there is one PSP location. The addition of more than one PSP point to the model would add complexity but would offer no further information.

The primary stocking point deals with two SSP's, like a manufacturer would deal with two wholesalers. Each SSP is capable of preparing orders for shipment (handling) and each holds inventory. In addition, each has the following communications links. They can communicate with the PSP to place orders, they can communicate with the Interface Stocking Points (ISP) to receive orders, and they can call the carrier to have orders picked up and delivered. These are the only communication links possible. For instance, the SSP's cannot communicate with one another. The two SSP's deal with the same PSP and each SSP deals with four ISP's.

The ISP's are analogous to retail outlets. They sell to the ultimate consumer and buy from a wholesaler. Eight ISP's deal with two SSP's, each ISP deals with a specific SSP only and four ISP's deal with the same SSP. The ISP (as a retailer would) holds inventory and has a handling and communications capability. The ISP communicates with the ultimate consumer to receive orders and with the SSP to request shipment. The ISP does not communicate with the carriers.

The demand unit is analogous to the aggregate of ultimate consumers. The characteristics and level of this demand will be discussed in the operations section of this chapter.

In the physical channel system the carrier (CAR) is responsible for moving goods between physically separated inventory locations (PSP to SSP and SSP to ISP). The carrier has not been specifically defined, however, the rates used are motor rates. All carrier moves are independent of one another.

All inventory or nodal points are located equidistant from one another in terms of time and distance. The distance in time and miles from the PSP to all SSP's is the same. And, the distance in time and miles from the SSP's to the ISP's is the same. This assumption eliminates all spatial considerations from the model but allows the inclusion of freight rates and lead times. This assumption was made for several reasons. Allowing space, in the form of varied distances between nodal points would destroy the base of comparison between runs. Secondly, the purpose is to show the effects of uncertainty on the system and the exclusion of space makes the results more clearly attributed to uncertainty. In addition, the elimination of space from the model does not severely limit the conclusions reached.

Operation

The operation of the simulated physical channel system is described from the viewpoint of the activities performed by all the activity centers within the total system. In those situations where the particular activities would vary at any one of the three levels (ISP, SSP, PSP) these exceptions are noted and specifically defined and detailed.

Daily demand is the request made by the ultimate consumer for purchase of the product, and as such, it is the force which initiates the functioning of the channel system. The daily demand impressed at each ISP is held constant for the duration of the simulation run. Each ISP receives a demand of 75 units per day for a 120 day period. The demand at each ISP is independent. If demand cannot be satisfied at one ISP, then consumers would not travel to an alternate ISP to satisfy their demands.

Perpetual daily inventory, in contrast to a periodic inventory system, is maintained by all ISP's. In a periodic system, the inventory would be reviewed at specified time intervals, and orders placed for the quantity of goods necessary to bring inventory up to prescribed levels, However, with the perpetual daily inventory system, whenever inventory is reduced to a predetermined level or reorder point, an order is placed with the appropriate SSP. Upon receipt of an order by the SSP, the order is processed, filled and delivered by the transportation agent. The total order cycle or lead time (the time delay from placement of order to receipt of the order) is composed of three elements: order transmittal from ISP to SSP, order processing and handling at the SSP and transit time from the SSP to the ISP. Order transmittal from the ISP to the SSP and order processing and handling at the SSP is fixed and constant. Transit time varies in accordance with the probability distribution for a particular experiment and represents variability in lead times and, thus, become the source of uncertainty in the system. In a sense transit time can be viewed as the uncertainty proxy for the overall order cycle. The lead time for each order placed by an ISP is generated by the probability function which is under investigation. Thus, a set of lead times for each ISP is developed over the duration of the simulation run which represents the pattern, mean and variance of the distribution being used. The costs associated with the ordering process, inventory and transportation activities are noted and displayed at the conclusion of the simulation run.

The SSP's also follow a perpetual inventory policy, updating their inventory at the end of each operating day. Orders are placed with the PSP when the level of the SSP inventory reaches its predetermined reorder point. SSP orders are processed, filled and delivered from the location of the PSP. Total lead time between the SSP and the

PSP is fixed and constant at ten days. Inventory at the PSP is generated by daily production runs at the adjacent manufacturing facility. The production rate equals the average daily sales for all ISP's. Thus, the warehouse facility at the PSP receives daily inventory equal to the average daily demand at the ISP's.

The service level for the total channel system is measured at the ISP level, which means that service is defined in terms of stock availability. A channel system exists to satisfy the demands of the ultimate consumer in terms of place, time and possession utility. Consequently, the system should be organized and planned on the basis of making stock available at the consumer interface point. If the product is not there, the consumer is not satisfied nor assuaged by the fact that the average order cycle time is six days. The system service level is geared to the percentage of units out of stock at the consumer purchase point. Thus, a 90% service level implies that 90% of the units demanded over the length of the simulation would be available when the consumer demanded them.

The converse of service level, in terms of system performance, is that of the system costs necessary to meet that required service level. The costs generated for each simulation run (experiment) are of two types. First, activity center costs at each level in the channel are accumulated and reported at the end of the operating period. Total costs for each activity center within the system (inventory, transportation, etc.) are determined and reported. Thus, costs by activity center for the system are measured and analyzed. Secondly, the total cost for each experimental run is agglomerated and comparisons made between runs. Finally, total contribution margin for the system is calculated. These measures serve to indicate the combined effect of cost and service on the channel operation.

Behavioral considerations have been assumed away in the operation of the model. Although channel member relations and interrelations are critical to the smooth functioning of a channel system, the inclusion of such behavioral aspects would seem only to confuse the important cost and service relationships under consideration in this research.

Inventory.--The inventory policy followed at each level within the system is based on an economic order quantity (EOQ) which is ordered when a given reorder point is reached. In the initial system, the EOQ is determined by balancing carrying cost of the inventory against the costs of ordering. The reorder point is defined as that quantity in inventory which will just meet average demand over lead time. Finally, in the initial system, no safety stocks are carried at any level (nodal point) since demand per day and lead time are fixed. The specific values of all variables associated with inventory are presented in Tables 2-1 through 2-4.

An exception to the general EOQ formulation is made at the PSP level. The PSP receives daily inventory from the manufacturing facility. The daily production rate equals the total average demand for all ISP's. The inventory carrying charge is considered to be 25% of the value of an item in inventory as is the case at the ISP and SSP levels.

Communication.--Communication between levels consists of order generation and transmittal to a supplier and invoice preparation and order status from supplier to the demander. Thus, when the reorder point is reached at any level, the demander (ISP or SSP) processes the order through his purchasing and accounting department and transmits the order to the next level within the system (SSP or PSP). The channel member requesting replenishment of inventory directly bears this cost of order generation and transmittal. When the order is received by the supplier he processes the order, prepares a bill of lading, and performs all clerical functions. The demander is then notified that the order has been received and processed. An invoice is sent to the demander which contains a per unit charge for each item ordered and a separate charge for order processing and invoice preparation. These costs are considered part of the order processing cost to be borne by the channel member ordering inventory replenishment. Thus, such costs are an input into the generation of the EOQ values at the stocking points.

The final communication link is that between the SSP, PSP and the transportation agent. The cost for such communication is borne by the particular firm to which the shipment is made. Thus, the ISP is assessed a charge for the placement of an order by the SSP to a carrier. A similar situation occurs for the SSP when the PSP contracts a carrier to make a shipment to the SSP. All values for the variables associated with communication are contained in Tables 2-1 through 2-4.

Transportation.--The nature of the product, the quantity to be shipped and the locational points determine freight charges. The product in question has been arbitrarily determined to weigh 20 pounds and displace .75 cubic feet. The appropriate class rating is 65, and the rate for shipments over 10,000 pounds but less than a truckload is $2.82 per hundred weight. The minimum weight necessary for a truckload is 36,000 pounds, and the rate is $2.32 per hundred weight. The rates are based on a constant distance factor.

Shipments made between ISP's and the SSP are made in quantities of 520 units (unless on a backorder shipment--this situation will be explained in a subsequent section of this chapter) or 10,400 pounds. Thus, the applicable transportation charge is $2.82 per cwt. However, shipments between SSP and PSP are made in quantities of 1,162 units or 22,200 pounds. Since this item is assumed to be one of many moving between these channel members, the product in question moves in a mixed shipment and thus obtains a truckload rate of $2.32 per cwt for the movement.

In those situations where a backorder has been made, the products are shipped on the basis of the shipment size as shown in Table 2-1.

All shipments are made FOB destination to obtain the economies in shipment enjoyed by the greater shipping volume of the SSP and PSP level. Consequently, the cost of the product to the ISP includes transport charges as does the cost paid by the SSP. All values of the variables associated with transportation are found in Tables 2-1 through 2-4.

Table 2-1. Partial Shipment Rate Schedule

Weight (lbs.)	Units	Rate/Cost ($)
Under 500	25	4.53
500-999	50	4,34
1,000-2,000	100	3,84
2,000-5,000	250	3,56
5,000-10,000	251-499	3,25

Handling.--The product is loaded and shipped on pallets containing 130 units, A charge is made for handling the pallets both coming into and out of all inventory points. The charge is $1.00 per pallet for handling into the inventory point and the same charge for taking it from these stocking points.

Orders received at the SSP and PSP levels are handled in a first come first serve basis. If an entire order cannot be filled, a partial shipment of all remaining stock is made. The backorder is then placed for the remainder. The backorder quantity is processed and shipped as soon as the goods arrive at the stocking point. Thus, all backorders are filled immediately upon the availability of stock. All values associated with handling are found in Tables 2-1 through 2-4.

Warehousing (facility).--Each level within the system maintains a warehousing facility for the purpose of holding inventory. The costs for such facilities is stated on a square foot per year basis. The cost per square foot is identical at all stocking points and this cost is included as one input to inventory carrying cost. The effective space necessary to store one unit of product is assumed to be .25 square feet. The storage charge for all stocking points is $1.50 per square foot per year. All values related to storage are included in Tables 2-1 through 2-4.

Backorders.--Backorders are demands which cannot be filled immediately due to a stockout, but are eventually filled when stock is available. Stockouts can occur at any level, therefore a backorder could occur at the ISP, SSP, or PSP. However, in this research, all experimental runs, except one, are made with no backorders at the ISP. In all experimental runs backorders can occur at the SSP and the PSP.

When there is no provision to backorder and demand is made at the ISP by the ultimate consumer and the ISP is out of stock, the demand is recorded as a lost sale. The analogous situation is the ultimate consumer demanding a good at the retail level. When the good is not available the customer will do without, go to a competitor or find an acceptable substitute. There are no provisions to attempt to save the sale and the sale is lost. There is no additional charge associated with a backorder under these conditions with the exception of the cost of a stockout. There is a backorder capability at the ISP for one simulation run. Consumer demand is backordered when variable demand and variable lead time are combined. This condition is considered separately and is detailed in Chapter VII.

Table 2-2. Product Specifications

1. Physical

Weight: 20 pounds
Cubic feet: .75
Square feet: .25

2. Cost Related

Cost at PSP: $2.40[a]
Cost at SSP: $3.20[a]
Cost at ISP: $4.00[a]
Consumer price: $5.00

3. Transport Related

Class: 65
Rate basis: Average between rate basis Numbers 421 to 600
Rate: 10,000 pounds but less than truckload: $2.82
Rate: Truckload: $2.32
Mode: Motor truck
Tariff authority: Eastern Central

4. Handling

Units per pallet: 130
Weight per pallet: 2,600 pounds

[a]Transportation FOB destination. Transportation included in purchase cost.

Table 2-3. Inventory Decisions

	Stocking Level	
	ISP	SSP
Average demand per day (units)	75	300
Number of days	360	360
Demand per year	27,000	108,000
Order cost (fixed per order)	$5.00	$5.00
Carrying cost (%)	25%	25%
Cost of product (per unit)	$4.00	$3.20
Lead time (days)	7	10
Economic order quantity[a]	520	1,162
Reorder point (units)[b]	525	3,000
ROP (average daily sales)	7	10
Number of orders per year	52	97
Order interval (days)	7	4
Average inventory	260	581

[a]No stockouts or backorders considered.

[b]No safety stock included.

Table 2-4. Cost Computation/Cost Basis

Activity	ISP	SSP	PSP
Inventory[a]			
Interest	A.I x $4.00 x .10	A.I x $3.20 x .10	A.I x $2.40 x .10
Insurance and taxes	A.I x $4.00 x .025	A.I x $3.20 x .017	A.I x $2.40 x .024
Obsolescence	A.I x $4.00 x .0313	A.I x $3.20 x .015	A.I x $2.40 x .031
Total carrying	A.I x .25	same	same
Facility	A.I x .25 sq ft x $1.50	same	same
Communications per order			
To supplier			
Order generation	$ .47	$ .25	
Order transmittal	.50	.30	
From supplier			
Invoice preparation	2.28	2.60	
Order processing	1.45	1.65	
To carrier	.30	.20	
Total	$5.00	$5.00	
Transportation			
To demander	0	$2.82/cwt	$2.32/cwt
From supplier	0	0	0
Backordered orders[b]			
Storage	..[c]	..[c]	..[c]
Handling[d]			
Into stock	$1.00 per pallet	$1.00 per pallet	$1.00 per pallet
Out of stock	$1.00 per pallet	$1.00 per pallet	$1.00 per pallet
Value of item in inventory[e]	$4.00	$3.20	$2.40

[a]No quantity discounts.

[b]See Table 2-1.

[c]Included in facility cost.

[d]No economies included in handling.

[e]Value of an item in inventory is equal to its landed cost.

At all times backorders occur at the SSP level. If the model is running with or without backorders at the ISP, there is a backorder capability at the SSP. This capability must be available because the ISP is facing continuous daily demand which must be satisfied. The ISP cannot easily and quickly change suppliers and the alternate procedure of the ISP repeatedly placing orders for the duration of the SSP stock-out is unrealistic and inefficient. When a backorder occurs at the SSP two conditions can be present: (1) there can be inventory on hand, but it is insufficient to fill the entire demand; and (2) there is no inventory on hand. We will look at each case separately.

When there is partial inventory available, a partial shipment is made (which exhausts the stock at the SSP). The ISP is notified that a backorder has been placed for the difference between the quantity ordered and the quantity shipped and the balance of the order is shipped when the stock becomes available. There are additional costs associated with this procedure which can be directly allocated to backorders. When a partial shipment is made, the freight rate per cwt will increase (see Table 2-1). Therefore, the difference between the normal freight rate which would be paid to ship a full order and the new rate to ship a partial order can be allocated to backordering. There is no additional cost associated with notifying the customer of a backorder because this can be handled on the confirmation of order and the packing slip. However, when stock is available and the balance of the order is shipped, there are several additional charges. There is an additional order processing and invoice preparation that would not have been necessary if the full order could have been satisfied. Therefore, these charges can be allocated to backordering. Secondly, there is the difference between the normal freight rate and the partial shipment freight rate as was the case with the first partial shipment. All additional charges which were created due to a partial shipment are the responsibility of the SSP.

The second possible condition when a backorder occurs at the SSP is less complex. When an order arrives at the SSP and no stock is available, the customer is notified of the backorder and shipment is made when stock is available. The additional costs under these conditions which are directly allocated to backorders is an additional order processing and invoice preparation which is necessary to notify the customer and hold the order for future shipment. Backorders at the PSP are handled with the same procedure outlined for the SSP.

The model detailed in this chapter meets the criteria set forth. It is sufficiently broad to allow concluding generalizations, sufficiently specific to meet the demands of simulation and structurally and operationally simple to allow the effects of lead time to be seen and measured. Now that the model is set, it is necessary to describe how lead time uncertainty is imposed on the system and to generate and select the distributions that are used in the experiments.

CHAPTER II--FOOTNOTES

[1]Webster's New World Dictionary.

[2]Donald J. Bowersox, Edward W. Smykay and Bernard J. LaLonde, Physical Distribution Management (New York: The Macmillan Company, 1961), p. 360.

[3]*Ibid.*, p. 360.

[4]*Ibid.*, pp. 54-55.

[5]Webster, *op. cit.*

CHAPTER III

EXPERIMENTAL FACTORS--PROBABILITY DISTRIBUTIONS

Introduction

The focus of this research is to measure impacts of lead time uncertainty upon physical channel performance. As mentioned in Chapter I, lead time uncertainty is evidenced by the probability distribution, level and variance of lead time. It is the purpose of the present chapter to explore the nature of probability distributions in general and to discuss those particularly relevant for representing lead time. Criteria for including a particular distribution in this study are presented. Finally, those distributions to be used as experimental factors in the research are considered and their selection justified.

Nature of Lead Time

Lead time is the variable and the source of uncertainty that is introduced into the research model. Lead time is defined as the elapsed time from placement of an order to receipt of the order. As shown in Figures 3-1 and 3-2, lead time begins with recognition that an order should be placed and ends with the merchandise on the shelf and ready for sale. It is composed of several activities and generally viewed as having three elements: order communication, order processing and order shipment.

Activity	Elements	
Recognition that an order should be placed. Order generation and internal processing. Communication of order.	Order Communication	Lead Time
Receipt of order at supplier and processing. Merchandise picking, packing and readying for shipment.	Order Processing	
Communication to carrier, Movement to pick up goods. Loading Line haul Receiving Stocking shelves	Order Shipment	

Figure 3-1. Lead Time Elements.

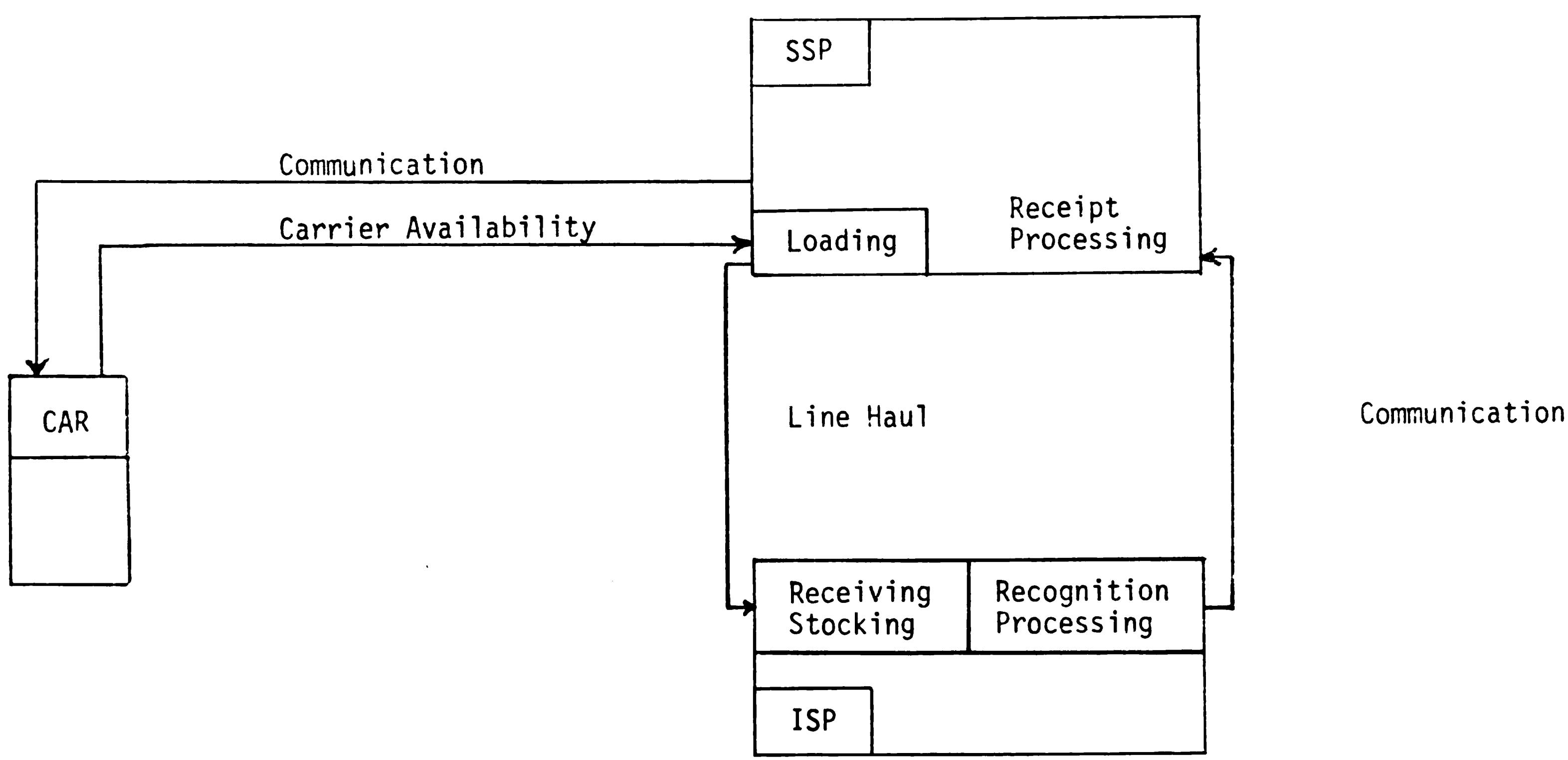


Figure 3-2. Lead Time Linkages

The one common characteristic of all lead time activities is that each requires some time to be performed, In a deterministic model the time lapse from the beginning to the end of each activity and thus total lead time is treated as a constant. Each and every time merchandise is ordered the elapsed time to receive the order is the same, Such behavior is not normally the case in business operation. In reality, individual activity time and total lead time varies from one order to the next. The possible reasons for time differences between successive lead time performances are many, It is impossible to consider all factors which have the capability of influencing the duration of lead time. Therefore, variation cannot be forecasted with accuracy. The elapsed time from placement of an individual order to receipt of that order can be expected to vary in a random fashion.

Given the nature and characteristics of lead time, it can be viewed probabilistically. "Probability enters into the process by playing the role of a substitute for complete knowledge."[1] In this research we are not only interested in the probability of a particular event, but we are also interested in the whole range of possible outcomes. Therefore, our attention must also be on probability distributions. It is, therefore, necessary that we look more closely at probabilities and their distributions.

Probability and Probability Distributions

In this research lead time is represented by several probability distributions. It has been established that lead time is a random variable in that all the relevant factors which create it are unknown. To show that probability distributions are appropriate theoretical models to represent lead time, an overview of probability distributions is necessary.[2]

Although individual events of a chance process cannot be predicted with accuracy, something can be said about the occurrence of particular events if the process is repeated. As a process is repeated which meet the following criteria: (1) that it can be repeated physically or conceptually; (2) that the set consisting of all its possible outcomes can be specified in advance; and (3) its various repetitions do not always yield the same outcome; the occurrence of particular events begin to stabilize. It is a characteristic of random data that if the experiment is repeated an indefinite number of times that any particular outcome that is observed will become more and more nearly a constant as the number of repetitions of the experiment is increased.[3]

Through observations and repetitions of the experiment, it is possible to determine the relative frequency of an occurrence. Relative frequency is the ratio of the number of times the outcome takes place to the total number of times the experiment is performed. If all the observations are grouped or classified a frequency distribution is created.

It is only a short conceptual step from a frequency distribution to a probability distribution. A probability distribution is a

> theoretical model of the relative frequencies of a finite number of observations of a variable. It is a systematic arrangement of the probabilities associated with the mutually exclusive and collectively exhaustive elementary events of an experiment.[4]

Thus, the probability distribution shows the probability of an event and the distribution of probabilities over a whole range of possible outcomes.

The probability distribution is an appropriate theoretical model to represent lead time. Lead time is uncertain and prediction with accuracy is impossible. Probability theory represents uncertainty and the probability distribution describes the whole range of possible outcomes which is necessary for the simulation.

To formulate the experiment it is desirable to look closely at the characteristics of probability distributions, i.e., the type of phenomena they describe and their characteristics.

Discrete/Continuous

The random variable under consideration may be either discrete or continuous.

> A discrete random variable can take on only a finite number of values. Also its distribution function, F(x), is one which increases only in finite jumps and which is constant between jumps.[5]

A continuous random variable takes on uncountably infinite values, such as time and weight whose counting is only limited by the measuring instruments.

> The probability that a continuous random variable assumes any single particular value is zero, since there are infinite numbers of real numbers within the intervals over which x (the random variable) is defined.[6]

To overcome this problem, the continuous random variable is viewed as intervals and the interval can take on values and probabilities as the finite numbers do in the discrete case.

Probability Function

A probability function assigns a chance of selection to each of the elementary events of an experiment. A probability function is distinguished from a probability distribution in that the function is a rule for assigning selection chances to the elementary events of an experiment, while a probability distribution is a systematic presentation or arrangement of probabilities. The probability function of a random variable is a description of its mathematical behavior, that is, the range of its possible values together with their respective probabilities.

The probability function can describe a specific point on the range or it can describe the range between points. A distinction must be made between the functions which describe points (discrete random variables) and the functions which describe discrete ranges between points (continuous random variables). The probability mass function assigns the probability of a point in both the discrete and continuous case. It is applicable to the discrete case because each point has a value. However, in the continuous case the probability of any given point must be zero, because of the nature of the variable.

> The probability that a continuous random variable assumes any particular value is zero, since there are infinite numbers of real numbers within the intervals over which x is defined. Consequently, a continuous random variable cannot be described by the probability function for discrete random variables.[7]

The probability mass function is used in this research for the discrete random variables.

As described above, the continuous random variable must be described in terms of subintervals or ranges between points. The probability function which describes the values and the probabilities associated with each is the probability density function or simply referred to as a density function. The discrete random variable can also be described over a range by its distribution function. However, the distribution function is not used in this research. The probability mass function is used to describe the discrete cases and the density function is used to describe the continuous case.

Measures

Given a probability distribution it must be described to operationalize the research. Statisticians have developed measures to describe distributions. Of the many ways a distribution can be measured, the expected value or central tendency and the variance are employed in the research.

The expected value is a measure of magnitude which considers the range of values of the random variable and their probabilities of occurrence. The term is synonymous with mathematical expectation, central tendency and mean. The expected value of a random variable measures the center mass of the probability function. It provides a quick picture of the long-run average result when the experiment is repeated an extremely large number of times.

The expected value in the discrete and continuous cases is defined as:

$$E(X) = \sum_{i=1}^{n} x\, f(x_i)$$

for the discrete random variable case. If x is a continuous random variable with probability density function f(x), the expected value of x is defined as:

$$E(X) = \int_{-\infty}^{\infty} x\, f(x)dx$$

The expected value does not adequately describe the random variable. "The expected value of a random variable indicates little or nothing about the range of values that the variable can assume, nor does it give any indication of the dispersion of the values of the variable."[8] The measure which overcomes this problem is variance.

"Variance represents the spread or scatter of the values of a random variable around its expected value."[9] If most of the area under the curve lies near the mean, the variance is small, while if the curve is spread out over a considerable range, the variance is large. In statistical terms, the variance represents the sum of the squared deviations around the mean divided by the number of observations. Thus,

$$\text{variance} = \frac{\Sigma(x-\bar{x})^2}{n}$$

A more common measure of variability is the standard deviation, which is the square root of the variance. The standard deviation allows the measurement of dispersion in the same units as the original values of the random variable X.

Parameters

In general, a parameter is defined as any descriptive measure of the characteristics of a population. "It is a single value derived by statistical methods in order to describe in summary fashion the pertinent characteristics about a population."[10] More specifically, it is some constant which describes a probability density function. For example, the mean is called the location parameter because it describes the position of the distribution on the x axis and the standard deviation is called the shape parameter because it alters the shape of the density with respect to a fixed scale. Each distribution is described by one or more parameters which singly or collectively affect the location and shape of the curve. The specific parameters for each of the special distributions discussed is covered in the following section.

Distribution Patterns

Although each random process can generate a different probability distribution, it has been found that certain "types" of distributions are generated over and over again. Thus, a group of special distributions have been catalogued. These are discussed in the following section.

Theoretical Probability Distributions

A broad range of theoretical probability distributions are available by which to represent random variables. "Probability distributions arise most naturally in terms of families of distributions that share selected common characteristics."[11] Each distribution family may be catalogued or characterized by a variety of factors, including its density or mass function, parameters, distributional shape, inherent generating process, assumptions and the kinds of experiments in which they commonly arise. It is the purpose of this section to briefly review the more commonly encountered distribution families and to systematically describe common distribution families according to their most relevant characteristics. From this review the distribution families applicable for representing lead time are selected. The distribution families so selected become the focus of this research in the experimental phase.

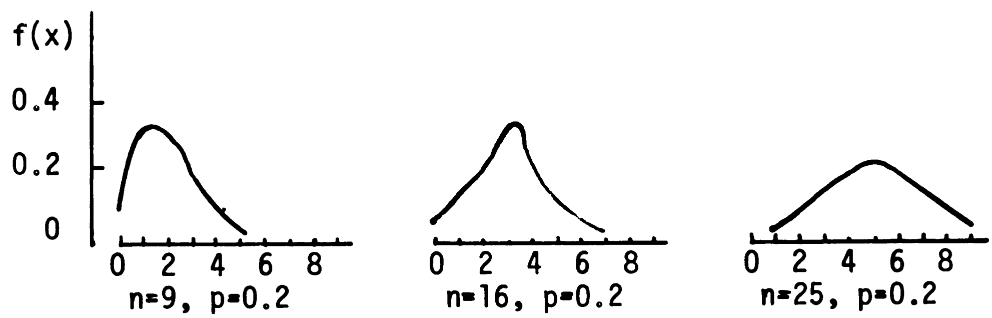
Discrete Distribution Families

The binomial family.--A random variable x is said to have a binomial distribution if its probability mass function is given by:

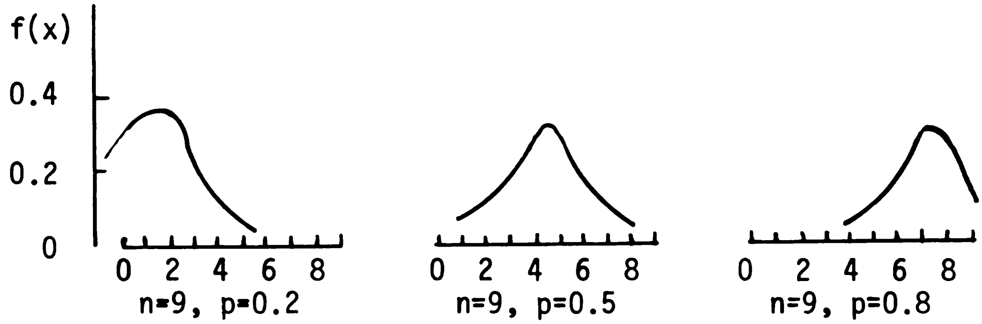
$$f(x;n,p) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x}, & x = 0,1,2, \ldots n \\ & 0 < p < 1 \\ 0, \text{ elsewhere} & \end{cases}$$

The parameters of the family are (n) and (p), where (n) represents the number of trials of an experiment and (p) represents the probability of success on a given trial. [q, the probability of failure, is equal to (1-p)]. The probability function thus describes a whole family of distributions of the binomial random variable (x), one for each possible combination of the values (n) and (p).[12] The random variable, (x), is defined as the number of successes.

The binomial distribution will assume different distributional shapes depending on the values assumed by (n) and (p). The distribution is symmetrical in situations where p = .5 and skewed when (p) takes on any value other than .5. However, as n approaches infinity, the distribution approaches symmetry and zero kurtosis.[13] For values of (p) less than .5, the binomial distribution is skewed to the right (long tail to the right of the mode) and skewed to the left for (p) greater than .5. These effects are somewhat mitigated when (n) is large. Finally, the binomial probability distribution may be approximated by the normal distribution and thus becomes almost continuous when (n) is very large. The distribution is represented by the following shapes with the values of (n) and (p) as specified in each illustration.



Binomial distributions with fixed p.



Binomial distributions with fixed n.

The mean or expected value of the binomial random variable is:

$$E(X) = np$$

The variance and standard deviation respectively are:

$$V(X) = npq$$

$$\sqrt{V(X)} = \sqrt{npq}$$

Theoretically, the binomial family of distribution is generated when we can assume that the following assumptions are met:[14]

If we consider a series of events or experiments:

1. The result of each experiment can be classified into one of two categories, such as success-failure, heads-tails, yes-no, and so on.

2. The probability (p) of a success (head, yes, etc.) is the same for each trial of the experiment.

3. Each experiment is independent of all others.

4. The trials of the experiments are performed a fixed number of times, say (n).

Thus, the binomial family describes random variables which are generated from populations having only two possible values. The probability mass function may be said to answer the question: "What is the probability of obtaining exactly (x) successes in (n) trials of an experiment, given the probability of success on any one trial is (p)?" The random variable of interest is thus the number of times in which the experiment results in a success.

The binomial family of distributions is usefully applied in many situations where its assumptions are at least approximated. Consequently, it is and has been successfully applied to quality control problems, where (p) represents the probability of obtaining a non-defective product or part. Additional situations, such as consumer surveys, where (p) refers to the proportion of favorable responses to a given question, have also been analyzed by use of the binomial distribution. In summary, the binomial distribution may obtain in a host of experimental situations where a constant (p), large (n) and independent trials are at least approximated.

The negative binomial family.--A random variable x is said to have a negative binomial function if its probability is given by:

$$f(x;r,p) = \begin{cases} \binom{x-1}{r-1} p^r q^{x-r}, & x = 1,2,3, \ldots\ldots \\ 0, \text{ elsewhere} \end{cases}$$

The parameters of the family are (r) and (p), where (r) represents the number of successes achieved in a given number of trials of an experiment and (p) represents the probability of success on a given trial.

The probability function describes a whole family of distributions of the negative binomial random variable, (x), one for each possible combination of the values (r) and (p). The random variable (x) is defined as the number of repetitions of the experiment that are required in order to achieve r successes.

The negative binomial distribution will assume various shapes depending on the values assumed by (r) and (p). The distributional shapes should vary in similar fashion as does the binomial.

The mean or expected value of the negative binomial random variable is:

$$E(X) = \frac{rq}{p}$$

The variance and standard deviation respectively are:

$$V(X) = \frac{rq}{p^2}$$

$$\sqrt{V(X)} = \sqrt{\frac{rq}{p}}$$

The negative binomial is said to be generated when the following assumptions are satisfied:[15]

1. The result of an experiment can be classified into one of two categories.
2. The probability of a success, (p), is constant.
3. Each trial of the experiment is independent.
4. The series of experiments is performed a variable number of times until a fixed number of successes is achieved.

The probability mass function of the negative binomial random variable is employed to determine the probability that the r^{th} success occurs on the x^{th} trial of a binomial experiment which meets the above four assumptions. Thus, the function describes the probability that (x) repetitions of the experiment are required in order to achieve (r) successes.[16] The number of successes, (r), is fixed and the number of trials (x) is the random variable.

Having two parameters, the negative binomial family provides a large class of distributions that serve as an assumption for an integer valued random variable.[17] It may serve as a model for a large number of real world applications, when the possible events are dichotomized and we wish to examine the probability of achieving a given number of successes in a fixed number of trials. Thus, potential applications exist in quality control, inspection sampling, sample surveys and the like. It has also been shown to be applicable in inventory studies for representing the total number of units demanded.

<u>The geometric family</u>.--A random variable, x, is said to have a geometric distribution if its probability mass function is given by:

$$f(x;p) = \begin{cases} pq^{x-1}, & x = 1, 2 \ldots \\ 0, & \text{elsewhere} \end{cases}$$

This family has only one parameter, (p), which is the probability of success on a given trial.

The mean or expected value of the geometric random variable is:

$$E(X) = \frac{1}{p}$$

which may be considered as the expected number of successes until a failure occurs. The variance and standard deviation respectively are:

$$V(X) = \frac{q}{p^2}$$

$$\sqrt{V(X)} = \sqrt{\frac{q}{p^2}}$$

The assumptions necessary to generate the geometric distribution are similar to those necessary to generate the binomial distribution.

The geometric family of probability distributions describes the probability distributions of the random variable (x), which is the number of trials necessary to achieve a success. Thus, the distribution refers to the number of trials, (x), needed for the first occurrence of a success.

The geometric distribution has very similar applications to those of the negative binomial, especially assembly line problems and those related to mechanical failure. Thus, the geometric may be applied to evaluate the reliability of various types of operating equipment by assessing the probability of a given number of cycles of a machine until it fails.[18]

<u>The multinomial</u>.--A group of random variables are said to have a multinomial distribution if their probability mass function is given by:

$$f(x_1, x_2, x_3, \ldots x_k; p_1, p_2, p_3, \ldots p_k, n =$$

$$\begin{cases} \frac{n}{X_1\ X_2\ X_3\ X_k}\, P_1^{x_1} \cdot P_2^{x_2} \cdot P_3^{x_3} \cdot P_k^{x_k}, \quad X = 0,1,2, \ldots n & 0 < P_i < 1 \\ & i = 1,2,3, \ldots k \\ & 0, \text{ elsewhere} \end{cases}$$

The multinomial is merely an extension of the binomial distribution. Whereas the binomial pertains to two alternative events, success, and failure of an experiment, the multinomial distribution applies to experimental trials for which more than two outcomes are possible. Thus, the likelihood that a specified number of each of multiple outcomes is obtained in n trials, for which the probability of the outcome of each is constant from trial to trial, is called a multinomial probability.[19]

The remaining characteristics, parameters, and assumptions of the multinomial are similar to the binomial distribution, but are of course different to the extent that more than two outcomes of an experiment are permitted. Thus, the multinomial can be applied to situations in which one desires to answer the question, "What is the probability of in (n) independent trials of an experiment, with x_1, x_2, ... x_k outcomes of each trial, with p_1, p_2, ... p_k probabilities, of getting exactly x_1, x_2, ... x_k of each possible outcome?"

<u>The hypergeometric family</u>.--A random variable x is said to have a hypergeometric distribution if its probability mass function is given by:

$$f(x;N,n,k) = \frac{\binom{N-k}{n-x}\binom{k}{n}}{\binom{N}{n}}$$

The parameters of the hypergeometric distribution include (N), the total number of objects in the population, (n), the number of objects in the sample or number of trials and (k), the total number of successes in the population or number of successful trials. The hypergeometric describes a whole family of distributions of the random variable (x), one for each combination of its parameters. The random variable x represents the number of successes.

The hypergeometric distributional shapes are quite similar to those assumed by the binomial, specifically as N becomes very large.

The mean or expected value of the hypergeometric random variable is:

$$E(X) = \frac{nk}{N}$$

The variance and standard deviation respectively are:

$$V(X) = \frac{nk(N-k)(N-n)}{N^2\ (N-1)}$$

$$\sqrt{V(X)} = \sqrt{\frac{nk(N-k)(N-n)}{N^2\ (N-1)}}$$

The hypergeometric distribution is generated when the following conditions are assumed:[20]

1. The result of each experiment can be classified into one of two categories, such as success or failure.
2. The probability of success changes on each trial.
3. Successive trials are dependent.
4. The trials are repeated a fixed number of times.

Thus, the hypergeometric distribution applies to processes similar to those for which the binomial obtains, except that the probability of success changes on each trial. The probability changes because trials (or draws) are made from a finite population, and thus the probability of success changes on each trial as the fraction $\frac{k}{N}$ changes. The process would be analogous to drawing spades from a deck of cards without replacement. The probability of drawing a spade on any draw is conditional upon previous draws, as the sample space is reduced for each card drawn.

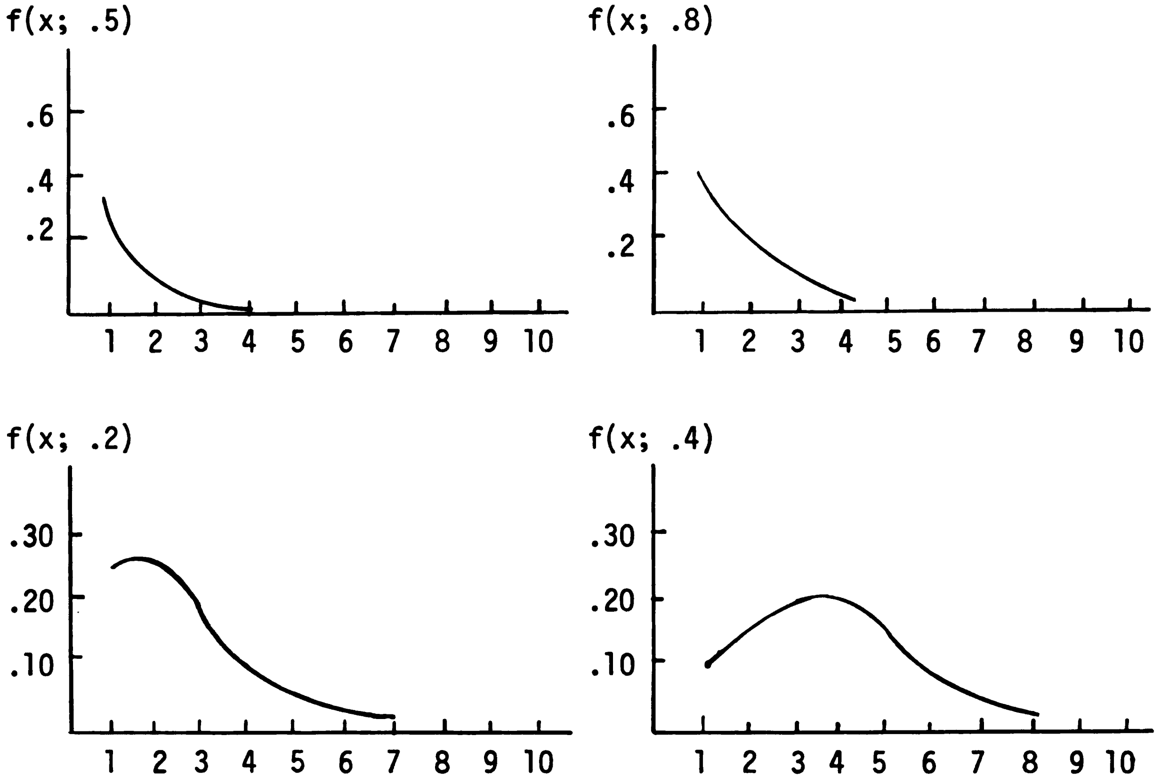
The most important application of the hypergeometric distributions are to those experiments or studies which are conducted with a finite population.

The poisson family.--A random variable x is said to have a poisson distribution if its probability mass function is given by:

$$f(x;\lambda) = \{ \; e^{-\lambda} \frac{\lambda}{x}! \qquad x = 0,1,2, \ldots$$
$$\lambda < 0$$
$$0, \text{ elsewhere}$$

The parameter of the poisson family is λ, the mean number of the occurrences of an event per unit time over a given number of trials. A whole family of distributions are then obtained based on the value of λ. The random variable x may thus be described as the number of occurrences of an event over some time or over space.

The poisson distribution will assume different distributional shapes depending on the value of λ. Thus, the distribution is highly skewed to the right when $\lambda \leq 1$, but becomes symmetrical as λ increases. The following are representative of the shapes taken by the poisson.



the poisson distribution

The expected value of the poisson random variable is:

$$E(X) = \lambda$$

The variance and standard deviation respectively are:

$$V(X) = \lambda$$

$$\sqrt{V(X)} = \sqrt{\lambda}$$

The following assumptions are necessary in order to generate the poisson distribution:[21]

1. Events that occur in one time (space) interval are independent of those occurring in any other non-overlapping time interval.

2. For a small time (space) interval the probability that one event occurs is proportional to the length of the time (space) interval.

3. The probability that two or more events occur in a very small time (space) interval is so small that it can be neglected.

There is no theoretical way of judging whether or not the basic assumptions are satisfied.[22] Thus, the assumptions are just that. Usually, the independence assumption is judged as satisfied unless there is overwhelming evidence to the contrary. The assumption that the event occurs only once in the interval can be circumvented by making the time interval extremely small.

The poisson distribution family therefore describes a situation where one counts the number of times an event occurs over some time interval. The events seem to occur random in time (space) and may thus be represented along a time (space) axis. The poisson thereby indicates the distribution of the probabilities of the numbers of rare events (whose probability is small in the interval) which occur in numerous trials. The probability mass function may be said to answer the questions: "What is the probability that an event A will occur exactly x times when a large number of trials are made in each of which the probability of the event A is very small?"[23]

According to Zehna, "the poisson family of probability distribution is used in many experimental situations in which integer-valued random variables are called for."[24] This is true in studies where a count is made of the number of times an event occurs, events being the number of misprints on a page, the number of calls received per minute on a telephone exchange, the number of accidents per hour on a highway, or the number of demands per day received by an inventory system. Bryan and Wadsworth suggest that many random phenomena of interest in sicence and industry yield a discrete variate x having a finite number of possible integral values, 0, 1, 2, 3--and satisfying conditions which lead to the poisson distribution.[25] Thus, additional applications of the poisson would include insurance problems, where the variable of interest is the number of deaths per time period, or a supermarket problem involving the formation of waiting lines at service facilities, and the counting of the number of defects in a manufactured item in a quality control situation.

Continuous Distribution Families

Uniform family.--A random variable x is said to have a uniform distribution if the probability density function is given by:

$$f(x;\ a,b) = \begin{cases} \frac{1}{b-a}, & \text{if } a \leq x \leq b \\ 0, & \text{elsewhere} \end{cases}$$

The parameters of this two parameter family are (a) and (b), the end points of the interval. Thus, the probability of x occurring is proportional to the length of the interval, and hence, intervals of the same length have the same probability.

The distributional shape is simply the representation of a horizontal line. The density is symmetrical about the center of the interval (a + b / 2) and thus this value is both the mean and median of the distribution. The expected value of the uniform distribution is:

$$E(X) = \frac{a+b}{2}$$

which simply represents the average of the end points. The variance and standard deviation, respectively, are:

$$V(X) = \frac{(a+b)}{12}$$

$$\sqrt{V(X)} = \sqrt{\frac{(a+b)}{12}}$$

Theoretically, the uniform distribution applies in situations when one can assume each event of a random process to be equally likely of occurring.

Zehna points out that, "as a model for random experiments, the uniform family is, first of all, suitable for bounded random variables whose essential range coincides with the interval (a, b)."[26]

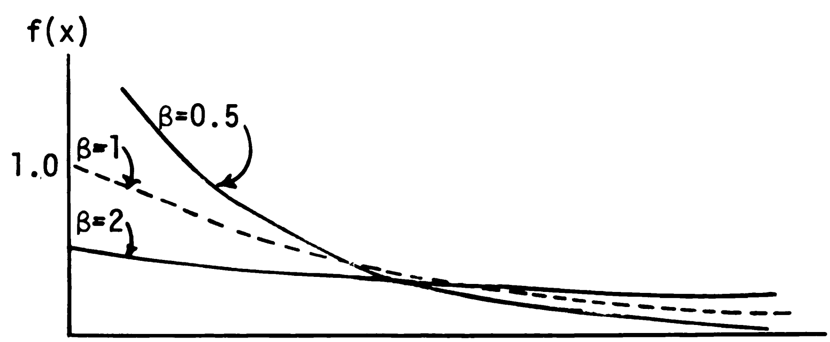
The uniform distribution also applies in situations where all events are equally likely or when numbers are to be generated by a purely chance process. Thus, tables of random numbers are generated from uniform distributions.

The exponential distribution family.--A random variable x is said to have an exponential distribution if its probability density function is given by:

$$f(x\ ;\ \beta) = \begin{cases} \frac{1}{\beta} e^{\frac{-x}{\beta}}, & \text{if } x > 0 \\ 0, & \text{elsewhere} \end{cases}$$

The parameter of the exponential distribution is β, which is generated from a poisson distribution. Thus, the exponential distribution is generated by a poisson process, and its parameter, β is defined as the reciprocal of the average number of successes per interval, i.e., $\beta = \frac{1}{\lambda}$. Thus, $\frac{1}{\beta}$ refers to the average length of the interval between two occurrences of the event. The random variable, x is defined as the width of the interval to the first occurrence of the event.

The exponential is a decaying type of probability function whose rate of decay depends upon the parameter β. It generally takes the following shapes:



Exponential distribution for various selections of β.

The mean of the exponential random variable is: $E(X) = \beta = \frac{1}{\lambda}$, and thus, the mean of the exponential is the reciprocal of the mean of the poisson. This result is to be expected since the exponential variable refers to time between successive poisson occurrences. Hence, the mean of the exponential is considered as the average time interval between poisson occurrences, or the expected time until the first occurrence of the event.[27]

The variance and standard deviation respectively are:

$$V(X) = \beta^2 = \frac{1}{\lambda^2}$$

$$\sqrt{V(X)} = \sqrt{\beta^2} = \sqrt{\frac{1}{\lambda^2}}$$

The most essential assumption necessary in order to generate an exponential distribution is that the random event occurs in time according to a poisson process. Additionally, the density function applies only to non-negative random variables.

The exponential family thus describes the probability distribution of the time between occurrences of an event that is developed from a poisson process. The exponential answers the question: "Through how long an interval must one wait in order to observe the first occurrence of an event if one is observing a sequence of events occurring in accordance with the poisson probability function?"[28] The random variable of interest is the length of the interval between occurrence of the desired event.

The exponential is found to be useful for representing a number of random variables which cannot assume negative values. For example, the time to failure of a machine is well represented by an exponential probability function. Such variables as waiting times for service, life of an electron tube, time intervals between successive breakdowns of an electrical system and the time intervals between accidents also are exponentially distributed. Important applications in business include the distribution of the length of time between successive arrivals at a service counter and the distribution of time wise variable demand that occurs in numerous situations.

The gamma probability family.--A random variable x is said to have a gamma probability distribution if its probability density function is given by:

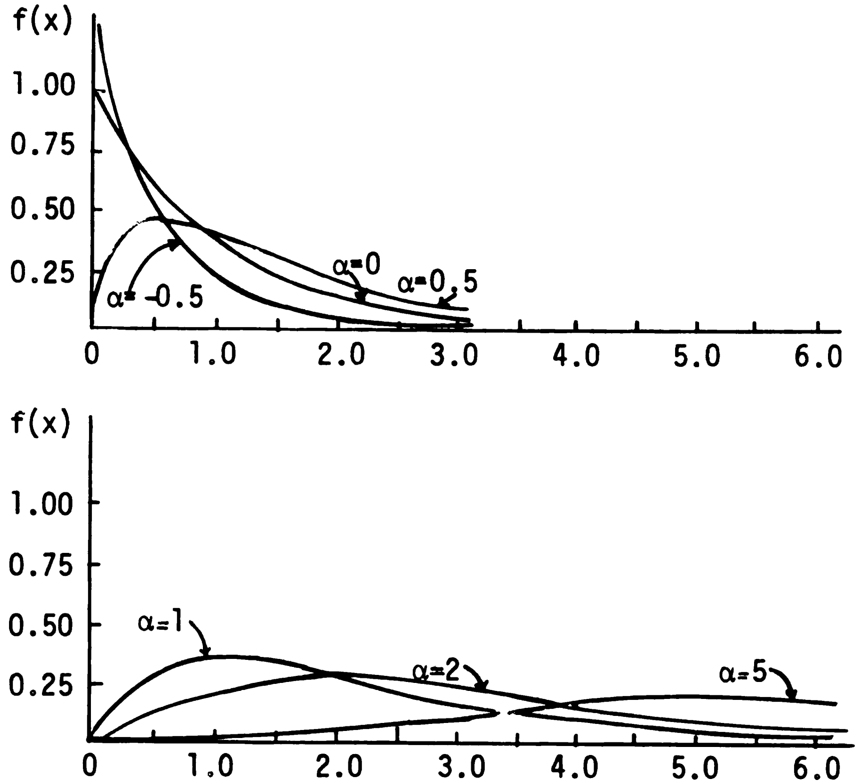
$$f(x;\ \alpha, \beta) = \begin{cases} \dfrac{X^{\alpha-1}e^{-(x/\beta)}}{\beta^{\alpha}\Gamma(\alpha)} & \text{for } x > 0 \\ 0, & \text{elsewhere} \end{cases}$$

The parameters of the gamma distribution are (α) and (β), where α refers to the number of successes per interval or unit space and (β) represents the reciprocal of the average number of successes per interval ($\frac{1}{\lambda}$). The gamma is thus related to both the poisson and the exponential distributions, and the exponential is a special case of the gamma for which $\alpha = 1$.

The gamma probability function describes a whole family of distributions of the gamma random variable (x), one for each possible combination of the values (α) and (β). The random variable x may be considered as the number of units of length (intervals) between one success and the α^{th} succeeding success.

The parameters α and β determine the shape of the density function, which is skewed to the right for all values of α and β.

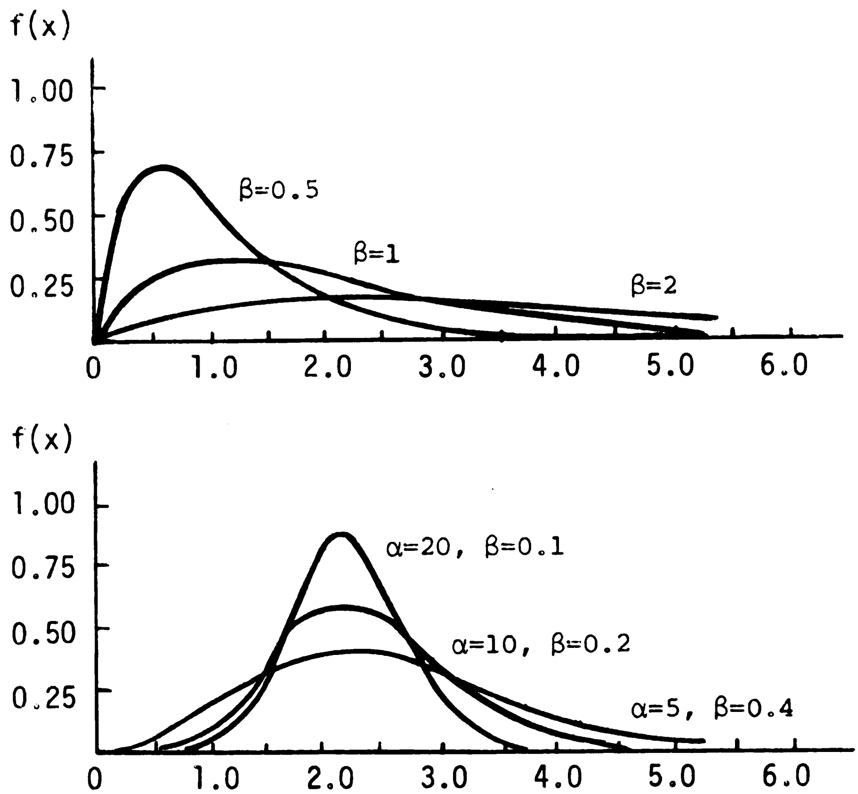
The skewness will decrease as α increases, as previously noted, when α = 1, the gamma is an exponential distribution, and therefore assumes the shape of a decay function as seen below.



Gamma distributions with unit β but different values of α.

if α is a positive interger, then the gamma becomes and Erlang distribution.

The following represent some typical gamma density functions.



gamma distribution

The expected value of the gamma random variable is:

$$E(X) = \alpha\beta = \frac{\alpha}{\lambda}$$

The variance and standard deviation respectively are:

$$V(X) = \alpha\beta^2$$

$$\sqrt{V(X)} = \sqrt{\alpha\beta^2}$$

The gamma obtains in situations where the underlying process is a poisson, and thus the assumptions relevant to the poisson are applicable. Additionally, the gamma applies only to non-negative random variables. The tie between the gamma, poisson and exponential is close. The poisson resulted from an effort to determine the probability of (n) successes per unit lengths, given a mean of (λ) successes per unit of length. The exponential results from an effort to determine the probability of (x) units of length from one success to the next in a poisson process. The gamma distribution results from an effort to determine the probability of (x) units of length between one success and the (α^{th}) succeeding success.[29]

There is no direct answer to when the gamma is applicable, one must construct a histogram of the actual data.[30] The family is so extensive in shapes of densities available that it is a fairly safe assumption to make as a model for an experiment described by almost any non-negative random variable.[31] Parzen concludes that

> the gamma is of great importance in applied probability theory. In addition to describing lengths of waiting times, it also describes such numerical valued random phenomena as life of an electron tube, time intervals between successive breakdowns of an electrical system and time intervals between accidents.[32]

Basic found the gamma to provide an excellent description of the probability distribution of demands for a product.[33] Additionally, Bryan describes the gamma as applicable "when conditions of the problem exclude values of x smaller than some arbitrary minimum."[34]

<u>The erlang distribution</u>.--The erlang distribution is a special case of the gamma probability family. When $\alpha = 1$, the gamma is an exponential distribution which is a decay type function. When α becomes a positive interger above 1 the distribution is an erlang. As α goes from 1 to n, the shape of the distribution changes from a decay type

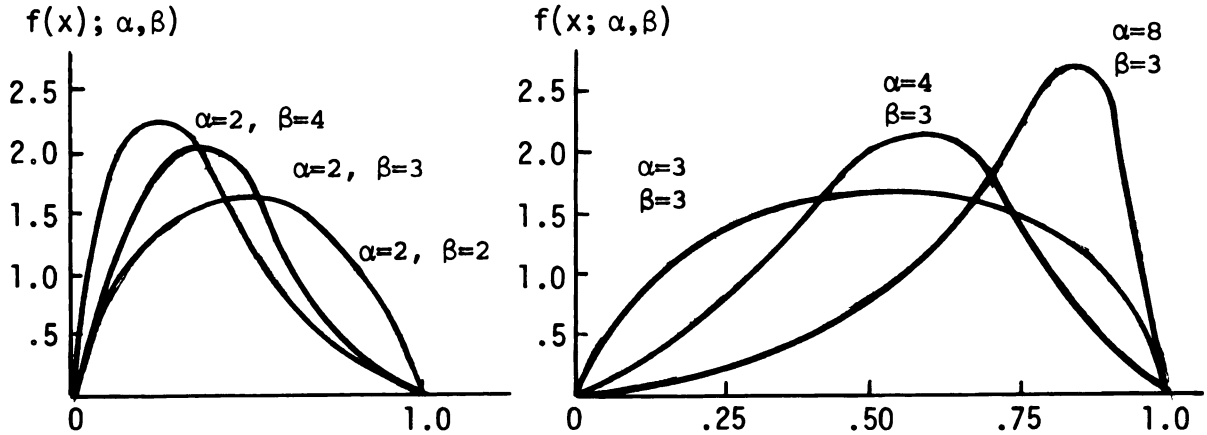
function through a series of shapes and eventually approximates the normal.

The primary application of erlang is a series of service times. A single service time can be viewed exponentially. As a second service time is added in series (i.e., a manufacturing process where two service type operations are performed consecutively) the process can be viewed as two independent exponentials. However, if the two service operations are to be viewed as one operation it can no longer be seen as an exponential distribution. A series of service type operations can be represented with an erlang distribution with the value of α equal to the number of stages. Thus, if there is a process which contains three exponential type service times, the entire operation can be represented by an erlang distribution with α equal to three. The forms of the density function, expected value, variance and standard deviation of the erlang are the same as the gamma.

Beta distribution family.--A random variable x is said to have a beta distribution if its probability density function is given by:

$$f\ (x;\alpha,\beta) = \begin{cases} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\ X^{\alpha-1}(1-x)^{\beta} & \text{for } 0 < x < 1 \\ & \alpha,\beta > 0 \\ 0, & \text{elsewhere} \end{cases}$$

Like the gamma, the parameters of the beta distribution include (α) and (β). There exists a broad family of distributions based upon the values of α and β. In the case where $\alpha = \beta$, the curve is symmetrical, otherwise it will be skewed. The variety of shapes is indicated below:



the beta density

The expected value of the beta random variable is:

$$E(X) = \frac{\alpha}{\alpha+\beta}$$

The variance and standard deviation respectively, are:

$$V(X) = \frac{\alpha\,\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

$$\sqrt{V(X)} = \sqrt{\frac{\alpha\,\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}}$$

The beta distribution applies when the admissible values of a random variable lie between 0 and 1. If both parameters, α and β, are equal to zero, then the distribution reduces to a rectangular or uniform distribution. The distribution is often a good representation for the random behavior of percentages. Additionally, the distribution is well suited for situations where values closer to zero have a greater probability than do those near unity.

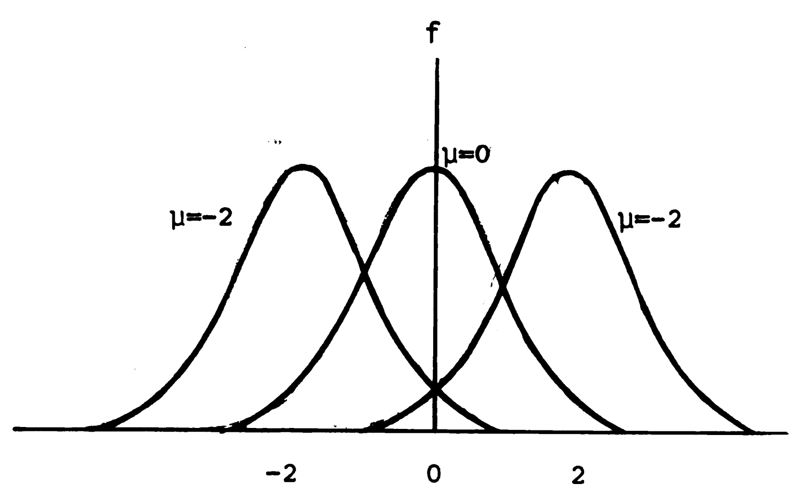
The normal distribution family.--A random variable x is said to have a normal distribution if its probability density function is given by:

$$f(x;\mu,\sigma^2) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{\frac{1}{2\sigma^2}(x-\mu)^2}$$

This two parameter family has, μ, the weighted average $E(X)$ and σ^2. The sum of the squared deviations, $E(V)$, and its parameters. As with most distributions, the probability function describes a whole family of distributions of the normal random variable (x), one for ecah combination of the values, μ and σ^2. The random variable (x) is simply the value of whatever variable is under consideration. The shapes assumed by the normal distribution are indicated below.

A shift in μ displaces the curve as a whole, whereas a change in σ^2 alters its relative proportions with reference to a fixed scale. The curve is always symmetrical about μ.

Additionally, the normal distribution is an excellent approximation of a number of continuous and discrete distributions.



The expected value of the normal distribution is:

$$E(X) = \mu$$

The variance and standard deviation respectively are:

$$V(X) = \sigma^2$$

$$\sqrt{V(X)} = \sigma$$

The normal distribution has become the most important probability model in statistical analysis.[35] Many continuous random variables, such as height, weight, I.Q., diameters of various manufactured items, tensile strength and the like are normally distributed. This is so because of the inherent attributes of measurements themselves. Errors in measurement seem to result from a vast collection of factors operative at a particular time. Each one of the factors has only a small effect on the magnitude and deviation of the error. Additionally, these errors work independently and with a force which is equal in both directions, therefore canceling in the long run. Thus, we think of errors of measurement as reflections of chance variations which are normally distributed with zero expectation.[36] Thus, other processes which possess this type of chance variation can often meet the assumptions of a normal distribution.

The important properties of the normal distribution include:

1. Symmetrical distribution.
2. Area under the density curve fully defined by μ and a specified value of σ.

3. Large deviations from μ less likely than small deviations due to the $[-(x-\mu)^2/2\sigma^2]$ exponent of the normal function.

4. Mean, median and mode are equal.

5. An infinite range to the distribution.

6. The average of n observations taken at random from almost any population tend to become normally distributed as n increases.

Many business processes may be represented by the normal distribution because of the frequent occurrence of variables in the analysis of business problems, which are the sums of independent random variables with very similar, if not identical probability distributions. The normal has thereby been applied to a wide variety of business problems, and even if the random variable so considered is not exactly normally distributed, the normal is such a good approximation to many distributions that the results are generally not impaired. Thus, the great value of the normal distribution is its ability to approximate many other distributions which are less tractable. The normal is considered a good approximation to the binomial, poisson and gamma distributions.

<u>The log-normal distribution</u>.--A random variable x is said to have a log-normal distribution if its probability density function is given by:

> If x is a random variable and y = log x and y is a normal random variable, then x is said to have a log normal distribution.[37]

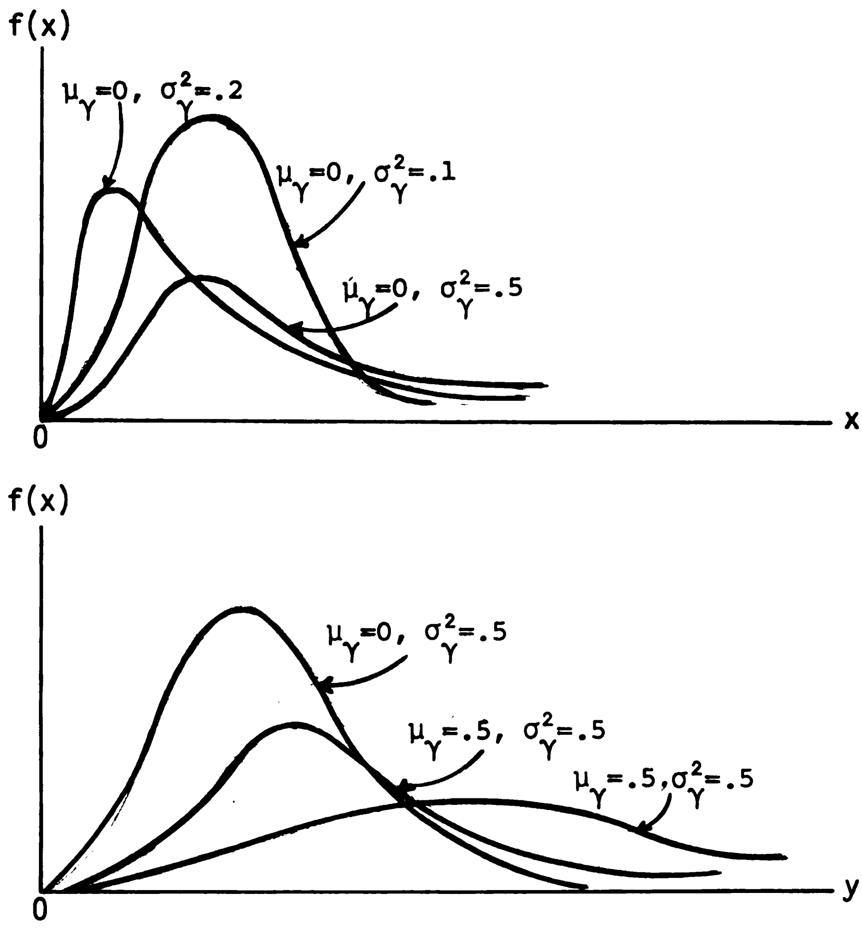
$$F\ (x;\ \mu_y, \sigma^2{}_y) = \left\{ \frac{1}{X\ \sigma_y \sqrt{\pi 2}}\, e^{\{\frac{1}{2\sigma_y^2}\,(\ln x - \mu_y)^2\}} \right.$$

The parameters of the log normal include μ_y and σ_y, where

$$\mu_y = \ln \sqrt{\frac{\mu_x^4}{\mu_x^2 + \sigma_x^2}} \qquad \sigma_y = \sqrt{\frac{\mu_x^2 + \sigma_x^2}{\mu_x^2}}$$

Thus, the log normal is nothing more than the probability distribution of a random variable whose logarithm obeys the normal probability density function.

The log normal is encountered in a variety of applications such as income studies and classroom sizes.[38] Additionally, it has been employed successfully to represent demand.



the log normal

Criteria for Selection

Of the probability distributions reviewed, not all are representative of lead time. It is necessary to select distributions which fit the nature of lead time and omit those not representative of the lead time process. To guide selection criteria for acceptance or rejection is necessary. These criteria are empirical validation, ability to produce differences in system operation and limitations due to resource constraints.

Empirical validation implies previous usage of the distribution to represent lead time. Most distributions have some practical foundation. They were discovered from observed phenomena. In addition to deriving a distribution from observation, particular distributions have been tested for fit with lead time and similar events. If a distribution has been derived from the observation of lead time or it has been shown to represent lead time, it will be selected for this research. The actual instances of the above two conditions are outlined in selection and justification.

One objective of this research is to examine physical channel reaction to different types of lead time uncertainty. Therefore, distributions are selected that are hypothesized to produce different system results. By looking at the type of process which generates the distribution, the distribution function and its parameters along with some empirical foundation, the possible reaction by the system can be hypothesized.

If it were practical it would be instructive to run all distributions that could possibly represent lead time. Realistically, however, there were time and resource constraints which made this impossible, therefore, selected distributions were eliminated.

By combining the above three criteria in consideration of the lead time process, particular distributions were selected and justified for use in the research.

Selection and Justification

Normal Distribution

The normal distribution was selected because of its widespread application and the ease with which it can be manipulated.

> In addition to portraying the distribution of many types of physical phenomena--it also serves as a convenient approximation to many other distributions which are less tractable.[39]

> The importance of the function arises from the fact that probabilities concerning random phenomena obeying a normal probability law with parameters μ and σ are easily computed, since they may be expressed in terms of the tabulated function.[40]

It has also been shown that the normal distribution approximates the limiting cases of many distributions. Thus, its universality and many applications make it the most important distribution discovered. One possible conclusion of this research is that the type of distribution employed to represent lead time is inconsequential in the decision process. That is, regardless of the lead time distribution form the effects on the system are the same. If the results of this research prove this hypothesis or lead toward this conclusion, the normal distribution would be the one to approximate all others. Thus, the normal distribution is one selected to represent lead time.

Exponential Distribution

Lead time as seen from the vantage point of the person placing the order can be seen as a time duration, i.e., five days from placement to receipt of order. This situation is exactly analogous with service time, i.e., how long to unload a ship, how long to wash a car, how long to service a machine. The exponential distribution describes these types of random phenomena. In the literature on queuing theory, the service times are exponentially distributed. "A surprisingly large number of service operations exhibit distribution functions which are equal to the exponential curve."[41] In addition, to empirical justification the exponential distribution can be deductively justified.

> Less is known (empirically) about lead-time distribution (than demand distribution), but on a priori grounds, one would expect skewed distributions with long tails on the right; lead times that are much shorter than average are less likely than lead times that are much larger than average.[42]

This seems reasonable considering the nature of lead time.

In addition, the exponential is quite different from the normal. Significant differences in system performances are hypothesized, especially for relatively inconsistent lead times. Thus, the exponential distribution will be used.

Poisson Distribution

The poisson distribution was chosen because of its large number of applications and its relationship to the phenomena of lead time.

The evidence is growing that many experimental situations fit the poisson assumption.

In addition to its wide range of applications, it has been shown that the poisson distribution is related to lead time and the exponential distribution.

> Irregular arrivals may be described in terms of probabilities in a manner quite analogous to service times. One measures the time between successive arrivals, and from these constraints a curve of probability that the next arrival comes later than time t after the previous arrival.[43]

> This is the distribution of arrivals called the poisson distribution, though to be consistent in our descriptions, we might better call it the exponential arrival case.[44]

Thus, through both empirical validation and association with service time phenomena, the poisson distribution is justified.

Gamma Distribution

The gamma distribution, similar to the poisson, was selected because of its many applications, and its relationship to the exponential or service time distribution.

> The gamma is so extensive in shapes of densities available that it is a fairly safe assumption to make as a model for an experiment described by almost any non-negative, random variable.[45]

The gamma distribution is also closely related to the exponential distribution and the service time phenomena. The standard form of the gamma when $\beta = 1$ is given by:

$$P_x(x) = \frac{x^{\alpha-1}e^{-x}}{\Gamma(\alpha)}, \qquad x > 0$$

If $\alpha = 1$, this is an exponential distribution and if α is a positive integer, it is an erlang distribution. Thus, the gamma, because of its wide range of applications and relationship to the service time phenomena, was used.

Erlang Distribution

Lead time as described earlier is composed of several "service type" operations. For instance, order communication, order processing and order shipment can be viewed individually as service operations. Thus as shown by Morse, lead time can be described as a series of exponential distributions.

> Therefore this facility (which has more than one station that can be described exponentially) viewed as a single facility, has a service-time distribution which is not exponential, though each of its phases is exponential.[46]

The resultant distribution as the number of stations is increased is called an erlang distribution.

> Erlang distributions provide a family of service--time distributions which range all the way from the "pure random" exponential type to the completely regular, constant service time situation. They will not fit all possible service time distributions, but they will fit many (and perhaps most) of the ones encountered in practice.[47]

Thus, the Erlang, because of its relationship with the gamma shown above, its relation to the exponential and its representation of the lead time phenomena, became useful in this research.

Log Normal Distribution

The log normal distribution was used because it describes service time phenomena and deductively it should apply because of its skewness to the right. The log normal has been shown to fit response time of humans to simple physical stimuli (the so-called Weber-Fechner Law), tool crib and library service time by Hawath, and maintenance down times.[48] Each of these phenomena are analogous to service time. In addition, this distribution clusters to the left and has a long tail to the right, which deductively seems to be the behavior of lead time. Holt has also indicated that assuming "lead times are log normally distributed may well provide a reasonable approximation."[49]

With few exceptions, there has been little empirical work done on lead time distributions. One possible explanation may be a lack of information. "It is rare indeed when sufficient data are available to yield a detailed histogram for the lead time distribution."[50] Therefore, the above selection and justification relied on distribution applications and reasoning on the relationship between the distributions and the nature of lead time.

These six probability distributions, along with selected variance and average levels, form the basis for the input to be made to the simulated channel system. The generation of these lead time distributions, the development of hypotheses relative to expected system results and the methods for measuring and analyzing the effects on the channel system are presented in the next chapter.

CHAPTER III--FOOTNOTES

[1]Charles T. Clark and Lawrence L. Schkade, Statistical Methods for Business Decisions (Cincinnati, Ohio: Southwestern Publishing Co., 1969), p. 181.

[2]Due to the scope and purpose of this research, only an overview of probability distributions is given. For more information in this area, see any one of the introductory statistics books listed in the bibliography.

[3]Ann Hughes and Dennis Grawaig, Statistics: A Foundation for Analysis (Reading, Mass.: Addison-Wesley Publishing Co., 1971), p. 3.

[4]Clark and Schkade, op. cit., p. 181.

[5]Ya-lun Chou, Statistical Analysis with Business and Economic Applications (New York: Holt, Rinehart and Winston, Inc., 1969), pp. 181-182.

[6]Ibid., p. 182.

[7]Ibid., p. 182.

[8]Richard C. Clelland et al., Basic Statistics with Business Applications (New York: John Wiley & Sons, Inc., 1966), p. 59.

[9]Ibid., p. 59.

[10]Clark and Schkade, op. cit., p. 3.

[11]Peter A. Zehna, Probability Distributions and Statistics (Boston: Allyn and Bacon, Inc., 1970), p. 122.

[12]Hughes and Grawaig, op. cit., p. 88.

[13]Kurtosis refers to the peakedness of the distribution, and is measured with reference to the peakedness of the normal distribution, which is of "intermediate peakedness." Thus, the normal distribution has "zero kurtosis."

[14]William C. Guenther, Concepts of Probability (New York: McGraw-Hill Publishing Co., 1968), p. 89.

[15]Ibid., p. 99.

[16]Hughes and Grawaig, op. cit., p. 99.

[17] Zehna, op. cit., p. 132.

[18] Ibid., p. 130.

[19] Clark and Schkade, op. cit., p. 214.

[20] Guenther, op. cit., p. 113.

[21] Ibid., p. 121.

[22] Zehna, op. cit., p. 135.

[23] V. E. Gmurman, Fundamentals of Probability Theory and Mathematical Statistics (London: Iliffe Books Ltd., 1968), p. 64.

[24] Zehna, op. cit., p. 134.

[25] George P. Wadsworth and Joseph P. Bryan, Introduction to Probability and Random Variables (New York: McGraw-Hill Book Co., Inc., 1960), p. 67.

[26] Zehna, op. cit., p. 141.

[27] Chou, op. cit., p. 216.

[28] Hughes and Grawaig, op. cit., p. 114.

[29] Claude McMillan and Richard F. Gonzalez, Systems Analysis: A Computer Approach to Decision Models (Homewood, Ill.: Richard D. Irwin, 1965), p. 159.

[30] Chris P. Tsokos, Probability Distribution: An Introduction to Probability Theory with Applications (Belmont, Calif.: Duxbury Press, 1972), p. 128.

[31] Zehna, op. cit., p. 148.

[32] Emanuel Parzen, Stochastic Processes (San Francisco: Holden-Day, 1962), p. 162.

[33] E. Martin Basic, "Development and Application of a Gamma Based Inventory Management Theory" (unpublished Ph.D. dissertation, East Lansing, Michigan, 1965), p. 8.

[34] Wadsworth and Bryan, op. cit., p. 91.

[35] Chou, op. cit., p. 221.

[36] Ibid., p. 222.

[37] Alexander Mood and Frank A. Graybill, Introduction to the Theory of Statistics (New York: McGraw-Hill Publishing Co., 1963), p. 132.

[38] Zehna, op. cit., p. 160.

[39] Hughes and Grawaig, op. cit., p. 117.

[40] Zehna, op. cit., p. 152.

[41] Phillip M. Morse, Queues, Inventories and Maintenance (New York: John Wiley and Sons, Inc., 1967), p. 12.

[42] Charles C. Holt et al., Planning Production Inventories and Work Force (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1960), p. 298.

[43] Morse, op. cit., p. 12.

[44] Ibid., p. 13.

[45] Zehna, op. cit., p. 148.

[46] Morse, op. cit., p. 40.

[47] Ibid., p. 41.

[48] R. L. Bovaird and H. I. Zagor, Naval Logistics Review Quarterly, 8 (1961), 348-349.

[49] Holt et al., op. cit., p. 298.

[50] Ibid., p. 419.

CHAPTER IV

HYPOTHESES AND RESEARCH METHODOLOGY

Introduction

The objective of this research is to measure the change in efficiency and effectiveness of a simulated physical distribution channel as a result of lead time uncertainties which are represented by probability distributions, variability and level. The statement of hypotheses and the research methodology required to test these hypotheses are delineated in this chapter.

The research methodology includes a justification for employing simulation experimentation, a description of the simulation model (LREPS) and the experimental design. The experimental design section considers the type of design to be used, description of experimental runs, the factors and their levels, the variables to be measured and the method of data analysis. Additionally, procedures for generating the distributions and their validation are discussed.

Hypotheses

The general hypothesis of this research is that lead time uncertainty has an effect on the efficiency and effectiveness of a multiechelon physical distribution system. Lead time uncertainty is represented by a probability distribution and is composed of three factors of uncertainty, pattern, variance and level (duration of lead time). The response variables which measure efficiency and effectiveness are total cost and stockouts (unsatisfied demand).

The general hypothesis can be segmented into two hypotheses. One concerns a comparison of the relationship between total cost and stockouts of a deterministic system with total cost and stockouts of an uncertain system. This first hypothesis can then be segmented into six subhypotheses which relate each of the three factors of uncertainty with the two response variables (total cost and stockouts). The second hypothesis concerns the relationship of two levels within a specific factor of uncertainty. For example, several distributions are used and the question asked is, "Will all the distributions have similar effects on the response variables (total cost and stockouts) or will the effects differ?" As with the first hypothesis, six subhypotheses which relate the three factors of uncertainty with the two response variables are developed for the second hypothesis. All the subhypotheses are stated below in the order in which they are presented in the conclusions.

1. Stockouts which result when the lead times of a physical channel system are particular types of probability distribution patterns will be different than those which result when the lead times experienced are constant at a fixed number of days.
2. Total cost which results when the lead times of a physical channel system are particular types of probability distribution

patterns will be different than those which result when the lead times experienced are constant at a fixed number of days.

3. Stockouts which result when the lead times of a physical channel system vary in duration around a mean will be different than those which result when the lead times experienced are constant at a fixed number of days.
4. Total cost which results when the lead times of a physical channel system vary in duration around a mean will be different than those which result when the lead times experienced are constant at a fixed number of days.
5. Stockouts which result when the lead times of a physical channel system vary in duration will be different than those which result when the lead times experienced are constant at a fixed number of days.
6. Total cost which results when the lead times of a physical channel system vary in duration will be different than those which result when the lead times experienced are constant at a fixed number of days.
7. Different types of lead time probability distributions produce different service levels (stockouts).
8. Different types of lead time probability distributions produce different total costs.
9. Different lead time coefficients of variation will produce different service levels (stockouts).
10. Different lead time coefficients of variation will produce different total costs.

11. Different lead time durations will produce different service levels (stockouts).
12. Different lead time durations will produce different total costs.

Hypotheses regarding the behavior of particular activity centers (i.e., transportation, inventory, facility and thruput) are not formally stated due to redundancy and the vast numbers of combinations possible. However, each channel activity center is measured in the simulation and the results are reported to the degree they clarify the behavior of stockouts and total cost.

Simulation Experimentation

Simulation is a technique for replicating the performance of an actual system or operation. The model or simulation, as a result of replicating performance, can serve as a base for experimental analysis. Martin Shubek succinctly describes the nature of simulation:

> A simulation of a system or organism is the operation of a model or simulator which is a representation of the system or organism. The model is amenable to manipulation which would be impossible, too expensive or impractical to perform on the entity it portrays. The operation of the model can be studied and, from it, properties concerning the behavior of the actual system or subsystem can be inferred.[1]

Thus, an important attribute of simulation experimentation is the capability to observe the performance of the system under a variety of conditions that would be otherwise impossible to achieve.

Naylor et al. provide an exhaustive set of rationale to justify the use of simulation experimentation as an alternative to actual observation and experimentation.[2] Their rationale include:

1. It may be impossible or extremely costly to observe certain processes in the real world. In these cases simulation can be used as an effective means of generating numerical data describing processes that otherwise would yield such information only at a very high cost, if at all.

2. Through simulation one can study the effects of certain environmental changes on the operation of a system by making alterations in the model of the system and observing the effects of these alterations on the system's behavior.

3. Simulation enables one to study and experiment with the complex internal interactions of a given system whether it be firm, an industry, an economy or some subsystem of one of these.

4. Simulation enables one to study dynamic systems in either real time, compressed time or expanded time.

Simulation experimentation appears well suited to the research objectives as presented in this thesis. The objectives of this research involve measuring the impact of environmental factors (lead time) on the performance of a complex system (physical channel system) over a time horizon. An attempt to experiment with uncertainty involved in various lead time distributions, levels and variances in actual practice would be almost impossible. The physical channel system would have to be isolated, and various segments of its operation held constant for each experiment. Consequently, controlled experimental conditions would be difficult, if not impossible to achieve. Problems would arise in being able to measure cost and service at all levels within the channel. Finally, the experimental factors, lead time, could hardly be controlled by the experimenter. In summary, a simulation model of a physical channel system and the performance of a structured set of experiments which vary the lead time distribution, level and variance offers a research opportunity not otherwise available.

Simulation Model--LREPS

To perform the specified research a valid simulation model is required. A number of excellent channel simulation models exist. These are reviewed in Appendix A. The simulation model employed in this research is known as LREPS. A brief description of the model is presented below, however, a more detailed description of the model is available.[3]

The LREPS model was developed by a Michigan State University research team under sponsorship of Johnson and Johnson Domestic Operating Company. The objective of the project was to design a planning model of a physical distribution system using dynamic simulation to evaluate the cost and service of alternative physical distribution system designs. The objectives have been realized; the model has been validated and successfully applied to numerous situations.[4]

In terms of the conceptual aspects of the model, an extensive variety of conditions can be simulated. LREPS replicates the logistics system of a manufacturer with national sales, on a multiproduct basis. The number of echelons may vary from 1 to 99, with either middlemen or company owned facilities at each nodal point. Product flow is not limited to a particular scheme, but may take numerous assignment paths or linkages depending on the system. Demand on the channel system may be evidenced individually by customer or aggregated into ZIP sectional centers. The system is capable of tracking up to 99 products with sales to as many as 10,000 customers.

The five logistical components, transportation, warehousing, inventory, communication and handling may be structured in a variety of ways. LREPS effectively handles all modes and legal forms of transportation, the reorder point, replenishment or combination inventory control system, all forms of communication, automated or manual materials handling and a variety of warehouse arrangements.

The experimental factors relevant to the LREPS model include target, controllable and uncontrollable variables. Target variables represent the performance of the system. Sales by echelon, weight, cases, items and lines; service levels in terms of stockouts and lead times; cost by activity center and echelon are the basic output measures of system performance. The controllable variables are those subject to managerial discretion and which become part of company strategy, or those dependent upon a given market situation. Order characteristics, product mix, and customer mix as well as facility network, inventory policy and transport modes, are the basic controllable variables. The model may then be deployed to test the sensitivity of various strategies to changes in these factors. Finally, uncontrollable variables include such factors as lead time determinants, competitive reactions and acts of God. The system's response to changes in these factors may also be assessed. The experimental factors are summarized in Figure 4-1.

TARGET VARIABLES (OUTPUT)

- Sales distribution
- Customer service
- Physical distribution system costs
- Physical distribution system flexibility

CONTROLLABLE VARIABLES (INPUT)

- Order characteristics
- Product mix
- New products
- Customer mix
- Facility network
- Inventory policy
- Transportation
- Communications
- Unitization

UNCONTROLLABLE VARIABLES (INPUT)

- Marketing environment
- Technology
- Acts of nature

Figure 4-1. Summary of Experimental Factor Categories.

The computer model is made up of three subsystems. The supporting data system loads all exogeneous variables, which include input variables such as cost factors, transport modes, decision rules and the like. The operating system simulates the actual operation of the logistical system. A demand and environmental system creates orders; the operations subsystem processes orders through the system; cost, sales and service measures are calculated through the measurement subsystem; the monitor and control subsystem compares actual cost and service to that desired and activates changes in the system. The third system, report generator, converts the raw data into useful management information. The conceptual scheme of the LREPS model is shown in Figure 4-2.

The LREPS model is highly flexible and dynamic. Its flexibility has already been alluded to in earlier paragraphs. It is dynamic in the sense that the model provides for time interval dependencies, i.e., deficiencies in one period are linked to future periods; feedback is provided to allow for the adjustment of controlled variables on the basis of system performance; and a variable time planning horizon is allowed. An additional significant feature of the model is the ability of the logistics system to be integrated on a temporal (total lead time) and spatial basis (location and transport modes). Further the model is set up on a sequential decision mode so that future decisions are influenced by past decisions. The system simulated using the LREPS model for this research was exhaustingly presented in Chapter II and will not be detailed in this section.

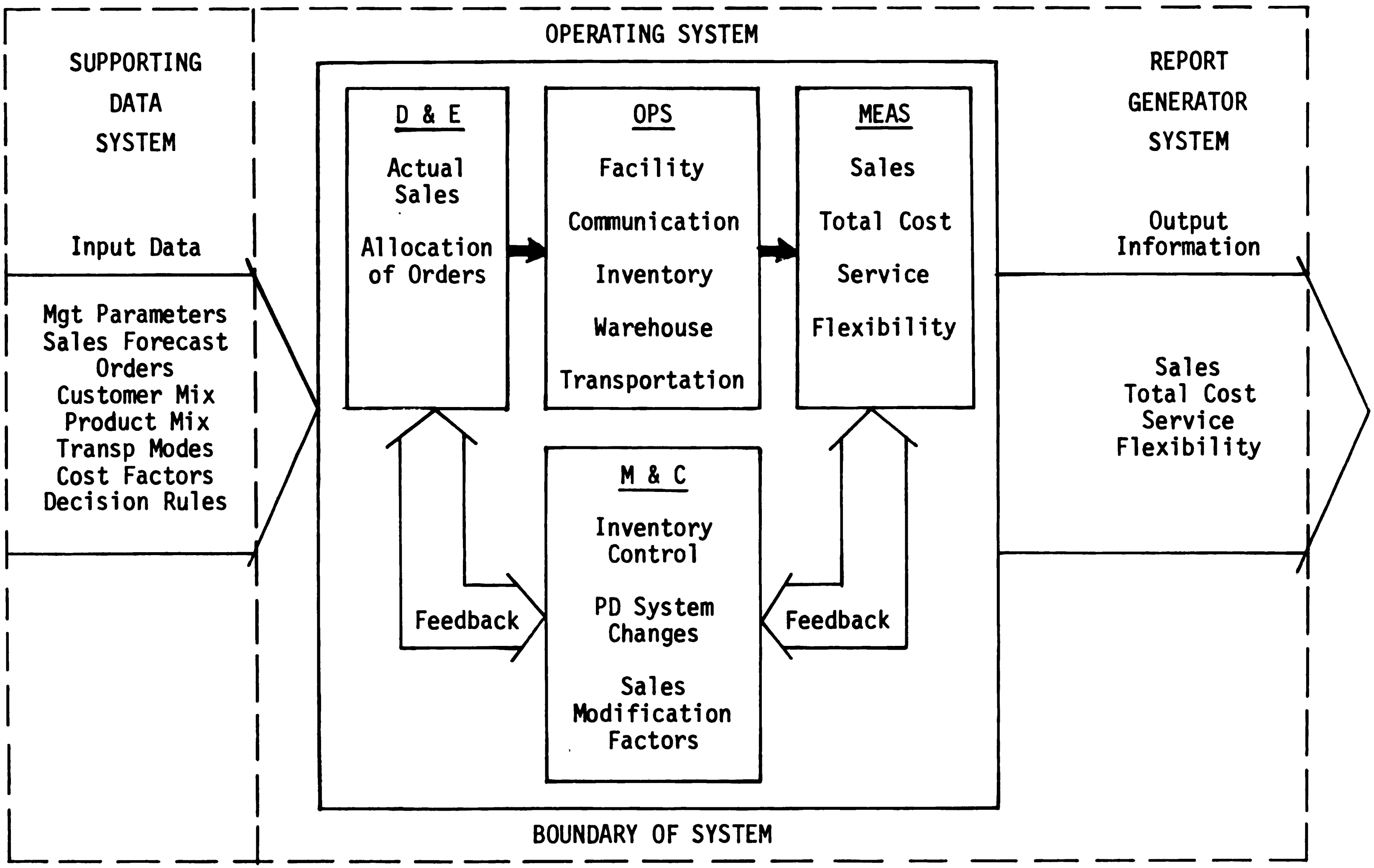


Figure 4-2. LREPS System Model Concept. (Source: See Helferich, footnote 25, page 23.)

Lead Time Generation

The first phase in the experimentation procedure was to generate the lead time used within the simulated physical channel system. The experimental factors or variables to be studied for each experimental run include the probability distribution of lead time, the average lead time and the variability or standard deviation of lead time. Thus, each experimental run involves a specific probability distribution, average or level, and standard deviation of lead time. Therefore, it was necessary to generate a set of lead time values which have the characteristics desired for the experimental run in question. For example, to evaluate the impact of the gamma distribution, with a given mean and standard deviation, it was necessary to create a set of lead times which follow a gamma distribution with a given mean and standard deviation.

To generate the appropriate lead time distributions which will serve as the order cycle time between the ISP and SSP for each experimental run, a set of computer programs presented by Pritsker and Kiviat[5] and Naylor et al.[6] were used. The normal, log normal, exponential, poisson and erlang were generated with Pritsker's GASP routine, the gamma program was developed by Naylor. Each distribution of lead time was generated from the same random number table, with the random number seed constant in every case.

To assure that the proper mean and standard deviation were generated, a t-test was employed to test the generated mean against the desired mean. In all cases the hypothesis of no difference was accepted at the .05 level.

To be sure that the assumed probability distribution (normal, poisson, etc.) had in fact been generated, it was necessary to compare the generated frequencies of the values of lead time to the theoretical frequencies that would occur if a given distribution applied. The Chi-square test and the Kolmogorov-Smirnov test are the most commonly applied statistical tests for comparing actual and theoretical frequencies.[7]

The Kolmogorov-Smirnov test (hereafter referred to as the K-S test) for goodness of fit was selected over the Chi-square test for the following reasons. The K-S test is more powerful than the Chi-square test, and thus provides better information. Secondly, the K-S test avoids the cell bias problem that is common to the Chi-square test. The K-S test treats individual observations separately and requires no grouping into cells or class intervals as does the Chi-square test. Additionally, the cell size requirements of the Chi-square tests are completely avoided. The Chi-square test is somewhat sensitive to nonnormality.[8]

The K-S test is concerned with the degree of agreement between a set of sampled values and some specified theoretical distribution.[9] It determines whether the frequencies in the sample can reasonably be thought to have come from a population having the theoretical distribution.[10] The procedure for the test is to compare the cumulative frequency of simulated lead time with the cumulative frequency distribution assumed. A "D" statistic is then computed which is the largest difference between actual and theoretical cumulative frequencies. The calculated "D" statistic is then compared with a critical "D" to determine whether the difference is significant. An example of the K-S test for goodness of fit as applied in this research is contained in Table 4-1. In each test the null hypothesis was accepted, i.e., that the desired distribution pattern was in fact generated.

Table 4-1. K-S test for Goodness of Fit--Log Normal Distribution

Random Variable	Observed Cumulative Frequency	Theoretical Cumulative Frequency	Difference
1	.010	.010	0
2	.135	.136	-.001
3	.405	.420	-.015
4	.690	.698	-.008
5	.825	.855	-.030
6	.935	.937	-.002
7	.975	.972	.003
8	.990	.990	0
9	.995	.996	-.001
10	1.000	.998	.002

D = .030 Critical D @ $\alpha = .05 = \frac{1.36}{\sqrt{200}} = .096$.

.030 < .096 Accept Ho that the distribution is log normal.

Design Considerations

In experimentation, three problems must be solved: (1) factor selection; (2) selection of experimental design; and (3) measuring results. These problems are solved in terms of the purpose and objectives of this research. Lead time uncertainties are defined for this research as the pattern or probability distribution, level and variability. Performance is defined in terms of cost and service. Thus, the goal is to measure the sensitivity of cost and service levels in a physical channel system to probability distributions, levels, and standard deviations of lead time. Having the objectives clearly in mind thus facilitates the decision to be made as to factor selection, experimental design and measurement.

The factors to be studied in this research include probability distributions, levels and variability of lead time. These so called "factors" might better be termed "conditions." In an actual situation, a channel system is faced with a given distribution of lead time and cannot readily change the distribution. Thus, this research proposes to investigate this condition, and its impact. The condition is not easily varied as are experimental factors in most research. However, changes in level and variability of lead time may be affected and thus these variables more readily assume the nature of experimental factors.

The factor or condition, lead time distribution is evaluated at six "levels." In other words, six types of probability distributions of lead time are investigated. The level, or average value of lead time is investigated at two levels, "high" and "low." Two levels of this variable were selected for a number of reasons. It is hypothesized that the level of lead time should affect costs and service in the system due to capacity constraints (truck load requirements, and the like). Secondly, it has been generally hypothesized by many authors that consistency rather than speed is the most important factor in efficient and effective channel operation. Thus two levels combined with two variances may help to confirm or disprove this thesis. The levels selected include an average lead time of four days and seven days. The specific values of these variables were selected arbitrarily, but the magnitude of the difference between them is felt to be great enough to show differences in system performance if these differences actually exist.

The third factor, variability of lead time must be clearly defined. The standard deviation of a variable is the most commonly accepted measure of variability. However, the standard deviation is an absolute measure of variability. Two sets of observations might be viewed considerably different in terms of variability if their standard deviations were the same but one of them had a mean three times as large as the other. Hamburg states, "For comparative purposes a relative measure of dispersion is required."[11] Measures of relative dispersion show some measure of scatter as a percent of the average about which they are computed.[12] Thus, variability in this research will focus upon relative variation. The measure of relative variability to be used is the coefficient of variation. The coefficient of variation, C.V., is the ratio of the standard deviation to the mean.[13] Hence, C.V. = α/μ.

Two levels of coefficient of variation are investigated in this research. The levels are defined as "low," and "high," and respectively correspond to a coefficient of variation of .18 and .375. The specific levels of the coefficient of variation were selected arbitrarily, but were set so that differences that might exist due to variability in lead time could be measured.

Experimental Design

The method of experimentation, as has been recounted before, is to make changes in lead time conditions and then to analyze the effects of these changes upon the behavior of the physical channel system. In order to effectively study the results in some systematic fashion, a proper method for analysis, i.e., an experimental design, must be selected.

The purpose of an experimental design is to provide a method for measurement of changes made in the factors and not other random fluctuations which might occur during the experimental run. Additionally, the experimental design should be effective, i.e., should yield the desired information at least possible cost.

Naylor and Hunter point out that a variety of experimental designs may be employed in simulation experiments when the objective is to explore the reaction of a system to changes in factors affecting the system.[14] Those designs considered to be particularly relevant include the full factorial, fractional factorial and response surface designs. The full factorial has been selected for use in this research.

A factorial experiment is one in which the effects of all the factors and factor combinations in the design are investigated simultaneously.[15] Each combination of factor levels is used the same number of times. In this research, the factors refer to lead time probability distribution, level and variability (coefficient of variation). A treatment, in the factorial sense, consists of some combination of all factors in the model. In this research, a treatment is made up of a probability distribution, with a given average lead time and a given coefficient of variation. A layout of the design is given in Figure 4-3.

The advantages of the factorial design, as opposed to randomized designs or one at a time approaches, are well summarized by Cox:

> To sum up, factorial experiments have, compared with the one factor at a time approach, the advantages of giving greater precision for estimating overall factor effects, of enabling the interactions between different factors to be explored, and of allowing the range of validity of the conclusions to be extended by the insertion of additional factors.[16]

It must be pointed out that interactions are not an important aspect of the investigation in this research. Interactions refer to the effect of combinations of experimental variables on the response variable that is above and beyond that which can be predicted from the variables considered singly. However, the nature of interactions seems to lose its meaning in the context of the present research problem. A channel system experiences a given pattern of lead time, with a given average level and variance. The system is not in a position to easily change one of these variables, i.e., combine it with another level of

Level	C.V.	DISTRIBUTION Normal	Log Normal	Gamma
1	.18			
	.375			
2	.18			
	.375			

Level	Poisson	Exponential	Erlang
1			
2			

Figure 4-3. Experimental Runs.

the other two variables and then commence operations. The levels of all three variables are fixed, and control over them somewhat limited. Thus, the nature of the experimental variables precludes a meaningful interpretation of the interaction effects.

The lack of attention to interaction effects does not diminish the applicability of the factorial design. The factorial design permits one to make statements as to the effect of each experimental variable which are based on observing that variable over a broad spectrum of conditions. Winer states:

> Apart from the information about interactions, the estimates of the effects of the individual variable is, in a sense, a more practically useful one; these estimates are obtained by averaging over a relatively broad range of other relevant experimental variables. By contrast, in a single-factor experiment some relevant experimental variables may be held constant, while others may be randomized. In the case of a factorial experiment, the population to which inferences can be made is more inclusive than the corresponding population for a single-factor experiment.[17]

Bonini concurs with this assessment, claiming that the factorial design provides for relatively wide generality of results.[18] Thus, the factorial design will allow statements to be made as to the effect of a particular lead time distribution, where the distribution is considered over a range of lead time levels and variances. In conclusion, the factorial design appears well suited to the objectives of this research.

There will be a deviation from the general factorial approach. Figure 4-3 indicates that the poisson, exponential and erlang distributions are not included in the layout matrix of the experimental design. The nature of the poisson, exponential and erlang distributions does not permit a fit into such a rigid pattern. All three distributions are one parameter distributions, and hence, cannot assume the total range of level (average lead time) and variance that the other distributions admit. Thus, these distributions untidy the analysis somewhat, but this problem is unavoidable due to the nature of their functional form.

Response Variables

In accordance with the objectives and hypothesis of the research, response variables are desired which most accurately and succinctly describe the effectiveness and efficiency of the system. In addition, information is desired on the behavior of key variables as a result of the imposition of uncertainty. Thus, measures of revenue, cost and its components, margin or profitability and service level are necessary.

To measure the effectiveness of the system the percentage of demand stocked out is used. This is the ratio of the unsatisfied demand (stockouts in dollars) to the total demand (in dollars) placed on the system. This measure is more useful than a simple revenue comparison, i.e., total sales or an unsatisfied demand comparison. By combining the two, a measure of the factor(s) effect on the system's ability to generate revenue and the system's service level is given. Thus this ratio describes the system's effectiveness (i.e., the ability to satisfy demand).

In addition to revenue and service, cost and its components are desired gauges of a system's performance. Total cost of the system is broken into transportation, thruput, facility, and inventory. It is necessary to look at total cost and its components because total cost could remain constant between two situations but its composition could be completely different. From the viewpoint of the manager or systems designer, cost components reveal more accurately the behavior of the system and may lead to defining system's interaction. From an experimental view, the effects of uncertainty on the components of cost are necessary for a more complete and useful analysis.

Thus, the response variables of interest in this research are percentage of stockouts at ISP, transportation costs, facility costs, inventory costs, thruput costs, and total costs.

Experimental Runs

The initial conditions and the experimental procedure of the data collection are discussed in this section. The system as described in Chapter II was modeled and simulated for 180 days to create the initial conditions. Then each of the factors of uncertainty and the control system were run from the initial conditions for 120 days. The output at the end of these runs is the data used in the analyses.

The initial system conditions which were employed as the starting point for all runs including the control runs were created first. Using the parameters of the system as described in Chapter II, a lead time of seven days was imposed between each ISP and SSP for the duration of the simulation.

Preparatory to day one, all the relationships in the system were set and inventory placed in the system. The level of inventory placed at each ISP and SSP in the system was randomly selected between ROP and ROP plus EOQ. This inventory level was selected because at any given point in time a stocking location would not have on hand and on order less than ROP or more than ROP plus EOQ. Thus the boundaries of the inventory are known. However, the actual amount is not known nor is the possibility that each location would have the same amount very great. Therefore the amount between these boundaries was randomly selected. The system was then simulated for a period of 180 days. This initial simulation period was chosen so that the effects of lead time would be seen at the highest level in the system (PSP) and to allow the system to stabilize. In effect, the system was "hot" after 180 days. A procedure identical to the one just described was carried out for a lead time of four days. The responses obtained after 180 days of simulation were used as the starting point for all experimental runs including the control runs.

The control system was then simulated. In the control system, as stated in experimental design, everything in the system is certain. Thus, lead time remains constant at four or seven days for the duration of the simulation. Employing the initial conditions obtained as described above, the system was simulated for 120 days and the results obtained after 120 days of simulation represented the control system responses which were used in the data analysis.

Every condition of uncertainty as described in the experimental design was simulated in the same manner as the control system. The initial conditions always remained the same and the simulation ran for 120 days. The simulation duration of 120 days was chosen for several reasons.

First, it was imperative that the effects of lead time were seen throughout the system and that a sufficiently long run was made so that the PSP or highest echelon in the system would feel the effects of lead time. With a lead time of 10 days between the PSP and the SSP and seven days between the SSP and the ISP coupled with the fact that inventory turned approximately 30 and 40 times at the ISP and PSP, respectively, 120 days was seen as sufficient.

Secondly, the simulation should be long enough to allow the system to stabilize. One simulation was allowed to run for 720 days with reports every 30 days. The system stabilized rapidly and the results at day 120 as compared to 150, 180, etc. indicated that 120 days was sufficient.

Third, the run should be long enough to generate the desired distribution. Thus, there should be a sufficient number of points or observations to create the chosen probability distribution. Each time an ISP placed an order one point on the distribution was obtained, and over 120 days approximately 20 to 30 points were obtained. Considering the range of numbers available (in most cases approximately ten values), 120 days were seen as sufficient.

Lastly, the simulation cannot run forever and there is a real limitation of cost associated with length of run. One hundred and twenty days satiated all the previous conditions and the gain that would be made to run past 120 days would not be worth the cost. Thus, 120 days became the duration of all simulation runs.

Data Analysis

The final consideration in the design of experiments is the methods used to analyze the data generated in the experiments. A very broad range of data analysis techniques exist, and selection of techniques is dependent upon the objective of the research and the inherent assumptions of the techniques employed. The basic question to be answered is: "Does the pattern (probability distribution), average level and variability of lead time make a significant difference as to the system's performance?" Three forms of the analysis of variance technique plus the standard t test have been selected for data analysis in this research. These techniques appear to meet the objective of measuring differences in system performance caused by lead time uncertainty. Additionally, the necessary assumptions of the techniques do not seem to be violated.

The three analyses of variance techniques are the F-test, Tukey's test of multiple comparisons and Dunnett's method of multiple comparisons. These three forms of analysis of variance are particularly well suited for comparing outputs of computer models.[19] The F-test is appropriate to testing the hypothesis that the average response (cost, service level) for each of the distribution types, levels or variances are equal. Thus, the test assesses whether these alternatives differ in terms of their effect on system performance. Tukey's multiple comparison technique may then be applied to the question of how they differ. Finally, Dunnett's method provides the necessary analysis of how one specific mean, a control mean, compares with all other output means.[20]

The application of analysis of variance techniques rests upon meeting three key assumptions. These assumptions include: (1) the independence of statistical errors; (2) equality of variance; and (3) normality.[21] The independence assumption is met if the observations are uncorrelated in time. Since the experiments set forth relate to one time period, the correlation of observations over a time frame does not appear to be a problem. As for the second and third assumptions, the experimenter rarely, if ever, knows whether these assumptions are satisfied.[22] However, minor deviation from assumptions two and three will not greatly affect the results. The procedures employed are said to be "robust," that is, quite insensitive to departures from assumptions.[23] This is particularly true of the F-test as argued by Scheffe, especially when the cell sizes are equal as is true in the present case.[24] As for Tukey's and Dunnett's multiple comparison techniques, reference is made to Naylor:

> Unfortunately, the robustness properties of multiple comparisons . . . are not as well known as the ones of the simple F-test. One can safely conclude that departure from the assumptions of common variance and normality are small enough to not seriously matter.[25]

The F-test tests the hypothesis that the average response for each of the distribution types, levels or variances are equal.[26]

$$Ho: \; D_1 = D_2 = D_3 \ldots D_n$$

The decision rule for accepting or rejecting Ho is: If

$$F \geq F_{\alpha}, \; m-1, \; n\,(n-1) \text{ reject Ho}$$

Otherwise accept Ho

where:

F = appropriate percentile of the F distribution.

α = significance level.

m = number of distributions or variances or levels.

n = number of replicates per factor level.

If the hypothesis Ho is accepted, it is implied that the differences between distributions, levels or variances were caused by random fluctuation rather than actual differences in the factors. If the hypothesis is rejected, it is concluded that variations in the response variable are caused by the factor. In either case additional analysis is required. In this research the additional analysis will be multiple comparisons.

Given the research objective previously stated, it is also desirable to make individual mean comparisons among the alternative probability distributions, levels and variances of lead time. Multiple comparison techniques are tools relevant to meeting this query, since they have been designed specifically to attack questions of how the means of many populations differ.[27]

Multiple comparison procedures employ confidence intervals rather than strict hypothesis tests. Confidence intervals are constructed for the difference $(U_i - U_j)$ and the actual difference in the sample means $(\overline{X}_i - \overline{X}_j)$ are compared with the confidence interval so constructed. If the difference $(\overline{X}_i - \overline{X}_j)$ falls within the interval, it is concluded that the population means do not differ.

It would be tempting to employ the t-statistic to calculate the confidence intervals necessary for multiple comparisons. If a number of confidence intervals are calculated for a given experiment with a given value of (α), all the intervals will not be simultaneously true at the α level selected.[28] If an experimenter conducts K independent t-tests, each with the same (α), the probability of falsely rejecting at least one of the K hypotheses, assuming all are true, is $1 - p$ (not rejecting all K tests) or $\{1 - (1 - \alpha)^K\}$.[29] For a very large K, the value for all tests becomes quite small. Thus, the risk of a type 1 error is considerable using repeated t-tests.

To avoid the problems stated above, two methods of multiple comparisons, that produce confidence intervals which are all simultaneously true at a given (α) have been selected for use. The methods to be employed are Tukey's method and Dunnett's method. Both of these methods require that treatment means be uncorrelated and have equal variances.[30]

Tukey's method produces simultaneous confidence intervals for the comparison of any or all pairs of treatment means. Tukey's confidence intervals are calculated using the following:

$$(\overline{X}_i - \overline{X}_j) \pm q(p,v)\sqrt{\frac{MSe}{n}}$$

where p equals the number of treatments and v equals the degrees of freedom associated with MSe. q(p,v) is tabulated as "Percentage Points of the Studentized Range." To test the difference between treatment means, the difference $(\overline{X}_i - \overline{X}_j)$ is calculated and compared to $q(p,v)\sqrt{\frac{MSe}{n}}$.

An important aspect of this research is to compare the system performance (cost and service) associated with a probability distribution, level and variance of lead time with the performance of the system under "certain conditions," i.e., where lead time is fixed. What is desired then, is a test of the hypothesis of no difference between a base or control run (fixed lead time) and all other runs. Dunnett's method is well suited for such comparisons.[31]

Dunnett's method of multiple comparisons compares each treatment mean with a control condition. The confidence intervals constructed are calculated using the following:

$$(\overline{X}_i - \overline{X}_j) \pm t_1 - (\alpha/2)\sqrt{\frac{2MSe}{n}}$$

where $t_1 - (\alpha/2)$ is a tabled value from Dunnett's tables.

The hypotheses stated in this chapter will be tested using the techniques described above. The results of the simulation runs and the statistical tests of the hypotheses are presented in the next chapter.

CHAPTER IV--FOOTNOTES

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[27] Ibid., p. 203.

[28] Mendenhall, op. cit., p. 175.

[29] Ibid., p. 175.

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[31] C. W. Dunnett, "A Multiple Comparison Procedure for Comparing Several Treatments with a Control," Journal of the American Statistical Association, 50 (1955), 1097-1121.

CHAPTER V

EXPERIMENTAL RESULTS

Introduction

The general hypothesis of this research is that the presence of lead time uncertainty in the form of a probability distribution (pattern, variance and level) has a significant effect on the behavior of a physical channel system. The primary response variables which represent the behavior of the system are service level and cost. The findings, which are the values of the response variables as a result of the imposition of various types of uncertainty, are given in this chapter.

The overall results of the simulation runs are summarized in Table 5-1. The average response across all treatments is shown for each of the factors (distribution, variance and level) and for each response variable (demand stocked out in percent, the ratio of total cost to total revenue in percent, and the following cost figures in cents per unit: total cost, transport cost, facility cost, thruput cost, and inventory cost). As an example, the average demand stocked out as a result of the presence of a normal distribution is 6.98%. This figure was obtained by taking all the runs in which the normal distribution was used. Specifically, these runs are the normal distribution at two different coefficients of variation and two different

Table 5-1. Average Responses Over All Experimental Runs

		Distribution						Level (Days)		Coefficient of Variation (%)	
	Control	Normal	Log Normal	Gamma	Exponential	Erlang	Poisson	4	7	.18	.375
Demand stocked out (%)	0.0	6.98	7.43	8.05	17.66	13.24	10.43	6.20	8.77	5.52	9.46
Cost/revenue ratio (%)	24.17	24.99	25.06	25.15	27.31	26.30	25.67	24.73	25.40	24.67	25.46
Total cost (¢/unit)	120.87	124.94	125.29	125.76	136.56	131.48	128.33	123.70	127.00	123.35	127.32
Transportation (¢/unit)	104.59	103.36	103.36	103.58	102.35	102.34	103.02	103.38	103.49	103.73	103.14
Facility (¢/unit)	9.93	14.48	14.77	14.99	25.21	20.96	17.67	13.37	16.13	12.79	16.71
Thruput (¢/unit)	4.61	4.73	4.76	4.76	5.02	4.88	4.80	4.73	4.77	4.72	4.78
Inventory (¢/unit)	1.73	2.37	2.40	2.43	3.97	3.30	2.82	2.91	2.60	2.11	2.69

levels (durations). The average results for each factor (distribution pattern, variance and level) and for each response variable (demand stocked out, etc.) are obtained in this fashion. The values in the control section of Table 5-1 are the result of the two simulation runs in which lead time was held constant (four days in one case and seven days in the other).

Table 5-1 shows in a general sense that the simulation runs under uncertainty are generally more costly than the control runs and that the service level drops (to a maximum of 17.66% of all demand not filled in the exponential) across all conditions. It can also be seen that as the average level of lead time duration increases from four to seven days and the coefficient of variation increases from .18 to .375, the channel system becomes more costly and less effective.

More detailed findings are presented in the balance of this chapter and are organized around the two main research questions. The initial question regards the behavior of a system with lead time uncertainty vs. a control or deterministic system. These results are shown in the first section of the chapter and are organized by lead time pattern effects, lead time variance effects and lead time level (duration) effects. Within this section, comparisons are made between average responses for a factor (i.e., normal distribution) vs. control using Dunnett's multiple comparison technique and, in those cases where Dunnett is not applicable, a t-test is employed. Individual cell comparisons are also made for which no statistical inferences are implied.

The second section of the chapter focuses on the comparison of factors within themselves. First, the F Test is employed to discover if particular types of uncertainty have a significant effect collectively. For instance, the question of whether the effects due to the normal, log normal and gamma distributions are different is answered. Then Tukey's method of multiple comparison is used to discover if particular factors were significantly different. In those cases where the Tukey method is not applicable a t-test is employed. Lastly, individual cell comparisons are made on a nonstatistical basis. As with section I, this section is organized by distribution pattern, variance and level.

Figure 5-1 represents the general procedure for analyzing the results of the simulation experiments. Portions of this figure will be reproduced at the beginning of each subsection of this chapter to indicate the nature of the analysis presented.

Comparison of Factor and Control Responses: Average Response Comparisons

Probability Distribution

Figure 5-2 indicates that the demand stocked out and costs which result from the pattern of lead time will be compared to demand stocked out and costs which result when lead time is constant (the control simulation run).

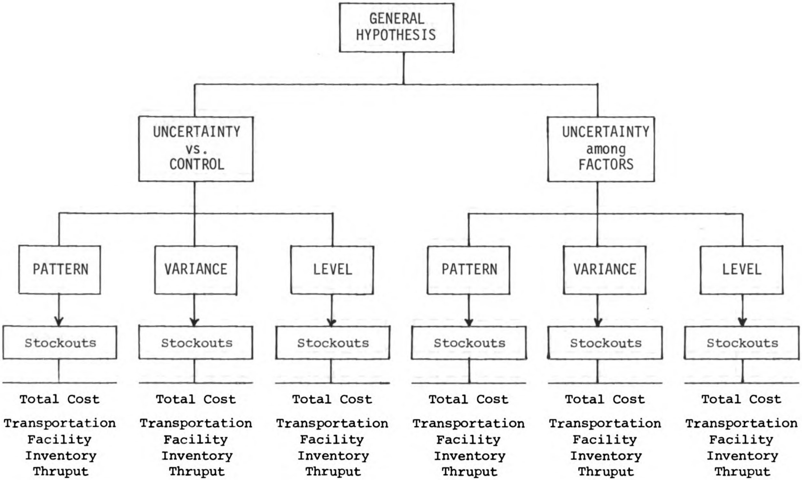

Figure 5-1. Research Analysis Organization.

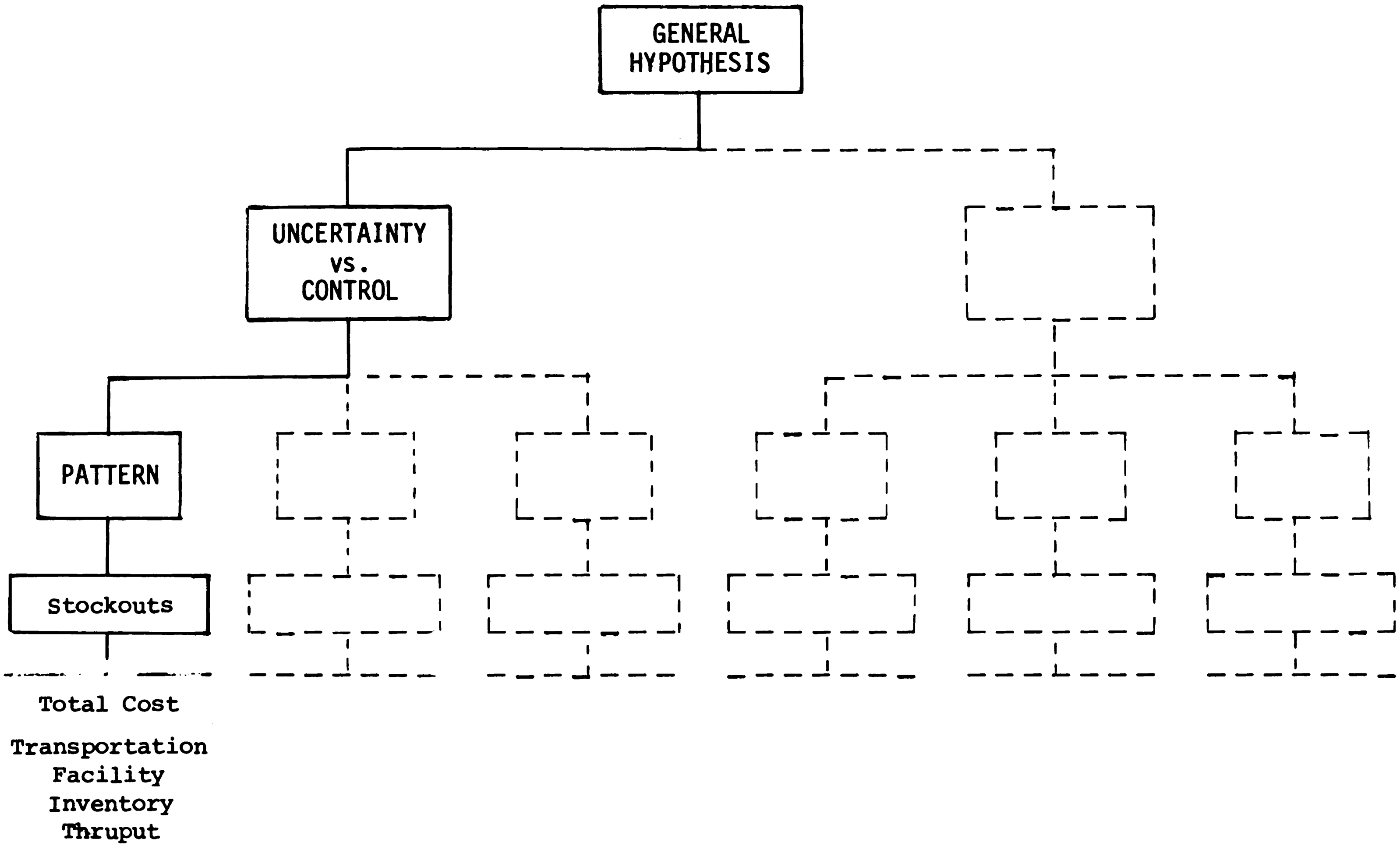

Figure 5-2. Research Analysis Organization: Control/Pattern.

Demand stocked out.--Figure 5-3 shows the average demand stocked out in percent due to the imposition of the lead time probability distribution patterns. Comparing each against the control, Dunnett's* comparison shows that the normal, log normal and gamma are significant. The critical value at the .05 level is 1.94 and the respective values are 6.98, 7.43, and 8.05. Due to the nature of the distributions, Dunnett's test could not be employed for the other distributions. Thus, a t-test is employed matching each against the control. The t-test indicates that the poisson was significant (critical value equals 12.47; poisson equals 25.00) while the exponential and erlang are not significant. A note of caution is necessary here. Due to the number of simulation runs involved for each distribution (two), the degrees of freedom are very small (one degree of freedom). In addition, large variances also compound the problem.

Cost/revenue ratio.--Figure 5-4 presents the ratio of the cost to revenue. The control average is 24.17% and this is set equal to 100%. The distribution pattern averages are then shown as a percentage of the control. For instance, the normal distribution is 103.4% of the control. The balance of the response variables in this section will be shown in this manner. Dunnett's critical value is .423. The normal is .815, the log normal is .885, and the gamma is .978, indicating these three are significant. The t-test shows no significant difference between the control and the exponential, poisson and erlang.

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*See Appendix B for sample calculations.

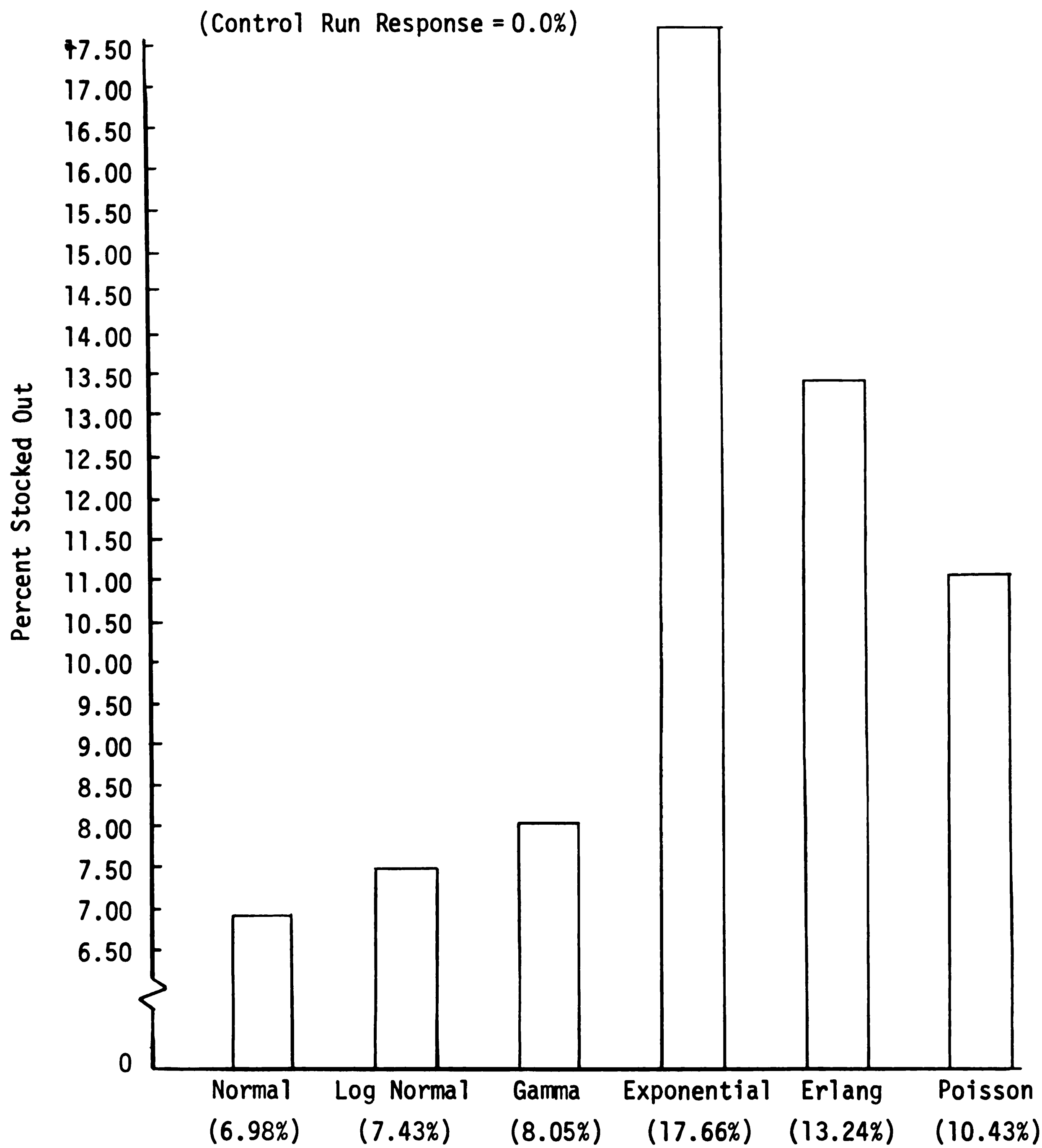

Figure 5-3. Probability Distribution Response Compared to Control Run Response: Percent of Demand Stocked Out.

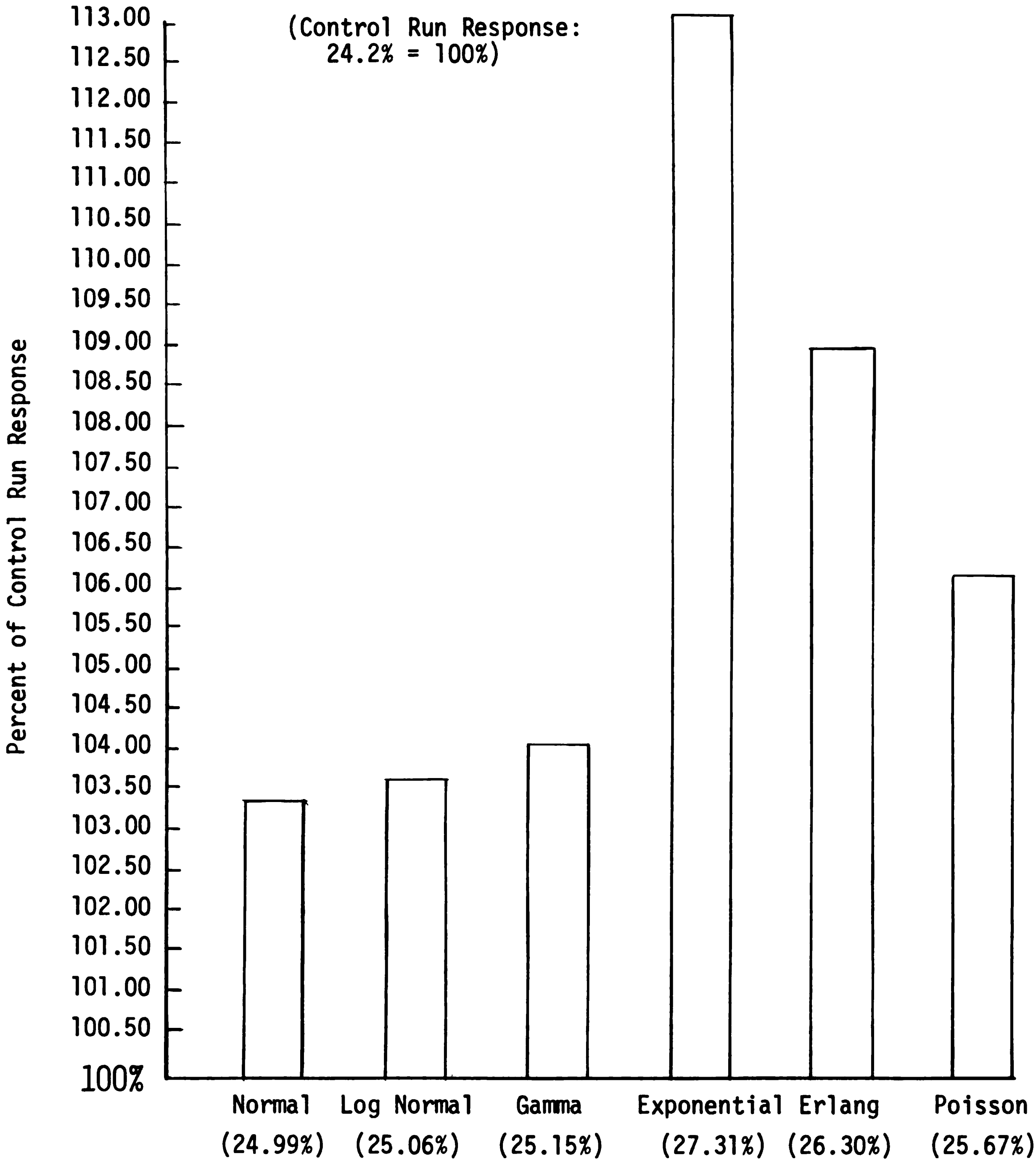

Figure 5-4. Ratio of Probability Distribution Response to the Control Run Response: Cost/Revenue Ratio.

Total cost.--Figure 5-5 shows the total cost, due to distribution, as a percent of control. Dunnett's critical value is 1.78. The value for normal is 4.07; log normal, 4.42 and gamma, 4.89. Thus, all are significant. The t-test used to compare exponential, erlang and poisson against control indicates that none are significant.

Transportation cost.--Figure 5-6 shows the first of the activity center costs. The activity center costs are composed of transportation, facility, thruput and inventory and collectively equal total cost. Figure 5-6 shows the transportation cost as a percent of the control transportation cost. All probability distributions cost less than the control. Dunnett's critical value equals .108. The gamma distribution is not significant (1.01), while the log normal (.123) and the normal (.123) are significant. The t-test reveals that none of the other distributions are significant.

Facility cost.--Figure 5-7 shows the facility costs as a percentage of the control. Dunnett's critical value is 1.69. The normal value equals 4.54, the log normal equals 4.84 and the gamma equals 5.06. Thus, all are significantly different. The t-test is not significant for exponential, erlang and poisson. However, the poisson value (9.3) is close to the critical value (12.47).

Thruput cost.--Figure 5-8 shows the thruput costs as a percentage of the control. Dunnett's critical value is .137. The normal distribution is not significant, while the log normal (.146) and the gamma (.150) are significant. The exponential, erlang and poisson are not significant using the t-test. However, the poisson (10.65) is close to the critical value (12.47).

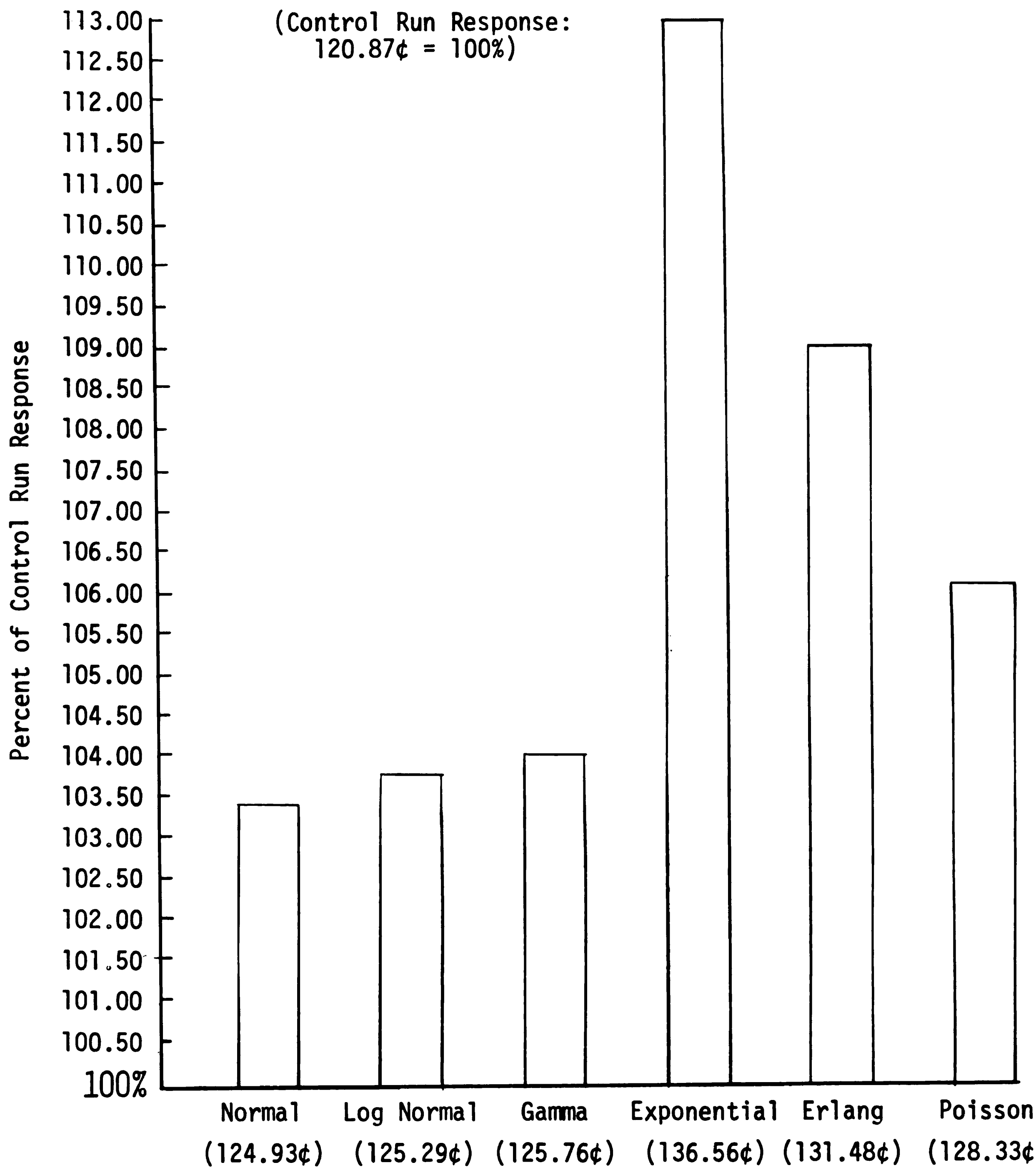

Figure 5-5. Ratio of Probability Distribution Response to the Control Run Response: Total Cost.

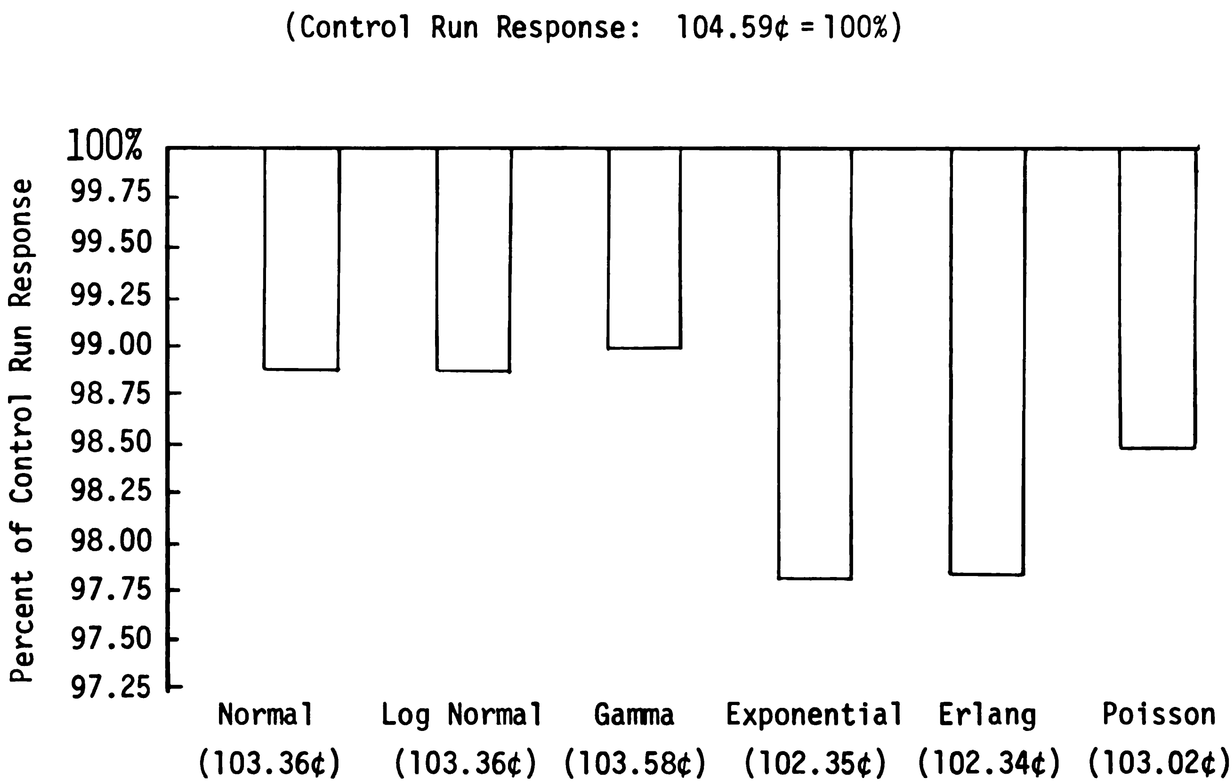

Figure 5-6. Ratio of Probability Distribution Response to the Control Run Response: Transportation Cost.

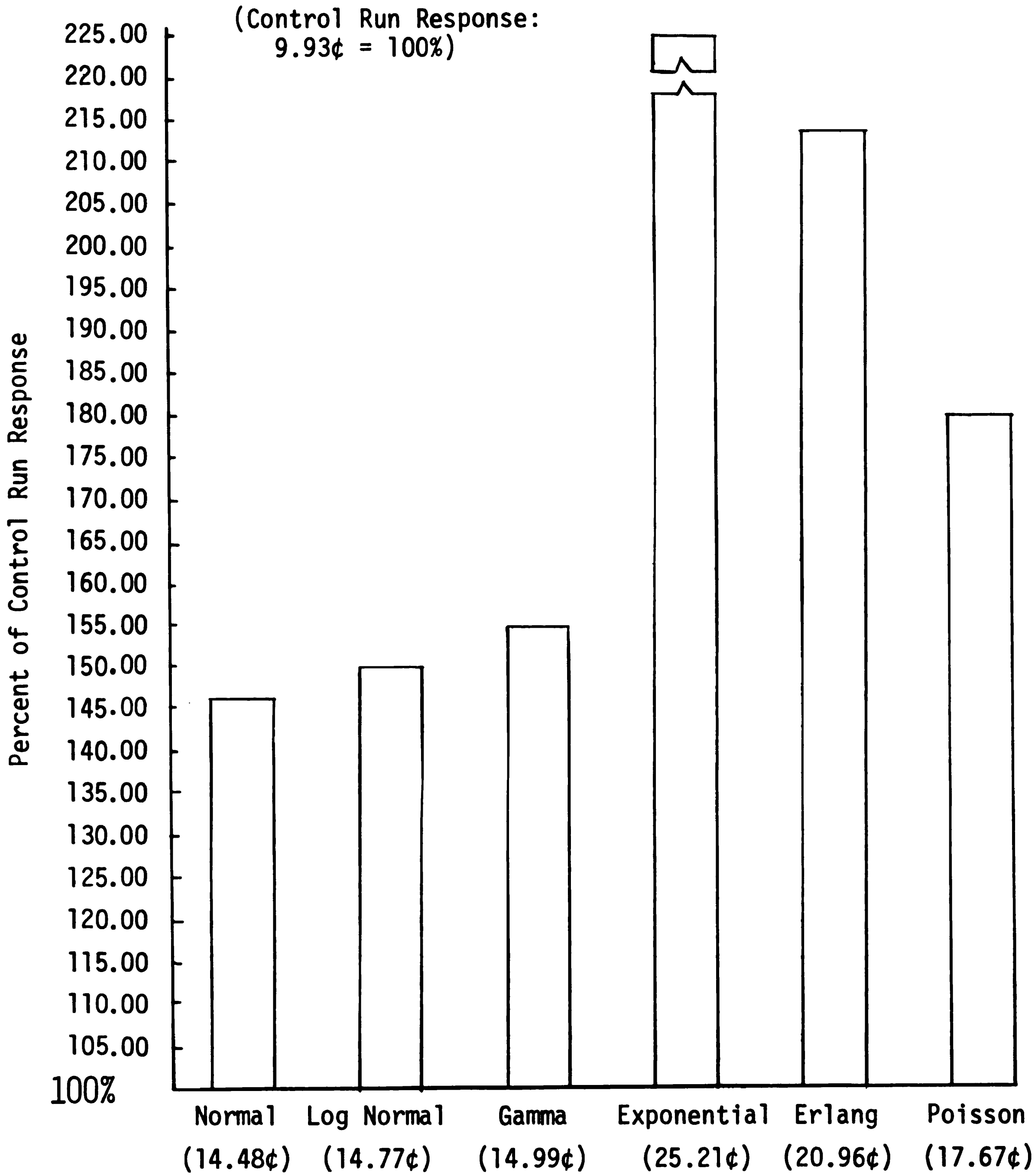

Figure 5-7. Ratio of Probability Distribution Response to the Control Run Response: Facility Cost.

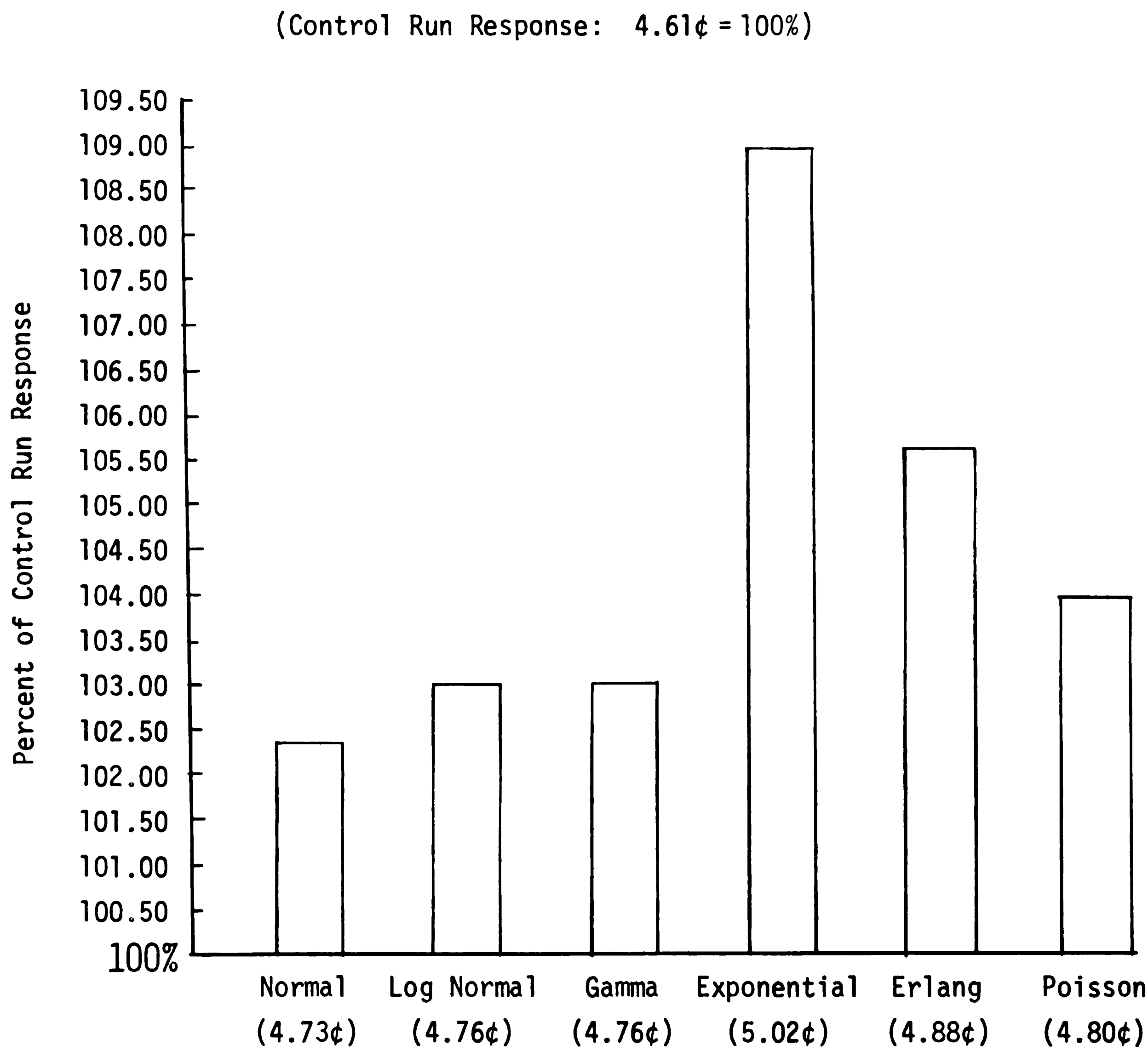

Figure 5-8. Ratio of Probability Distribution Response to the Control Run Response: Thruput Cost.

Inventory cost.--Figure 5-9 shows inventory costs as a percent of control. Dunnett's critical value equals .235. The normal distribution equals .631, the log normal is .665 and the gamma is .695. Thus, all are significant. The t-test indicates that the exponential, poisson and erlang are not significant.

Conclusion.--Regarding the findings for the probability distribution response vs. the control, the following general observations should be noted. In all cases, with the exception of transportation costs, the uncertain lead times are more expensive. The demand stocked out in all cases is more than the control. The relative difference between particular distributions across the response variables remains basically the same. For instance, the exponential is always the most costly (or the least effective), while the normal is the least costly or most effective. The relative effectiveness of the other distributions remain basically the same with few exceptions. Even the transportation costs (although lower than control) remain relatively the same between particular distributions and the control.

Variance

Figure 5-10 indicates that the demand stocked out and costs which result from the variance of lead time will be compared to the same measures which result when lead time is constant (the control simulation run).

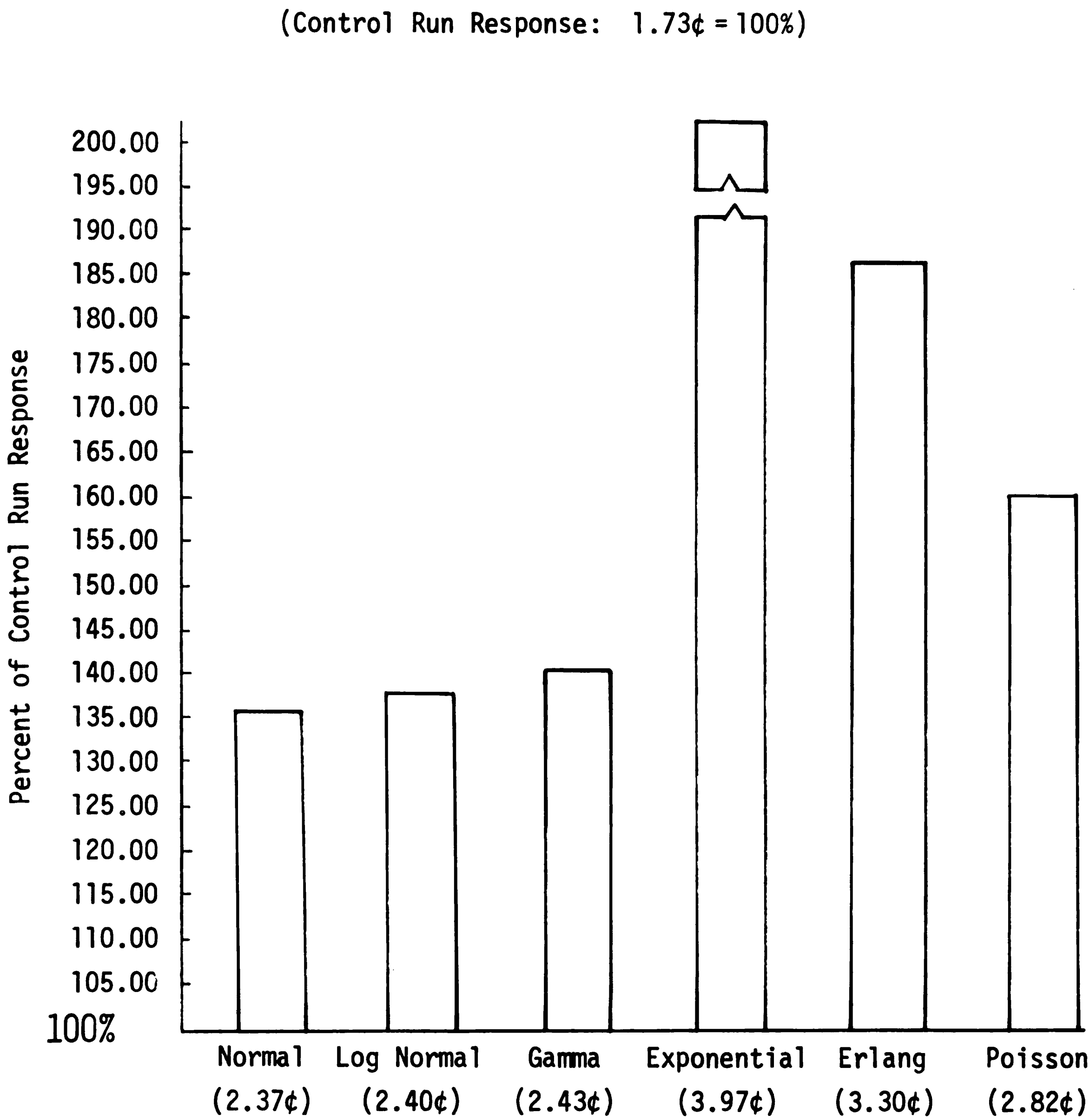

Figure 5-9. Ratio of Probability Distribution Response to the Control Run Response: Inventory Cost.

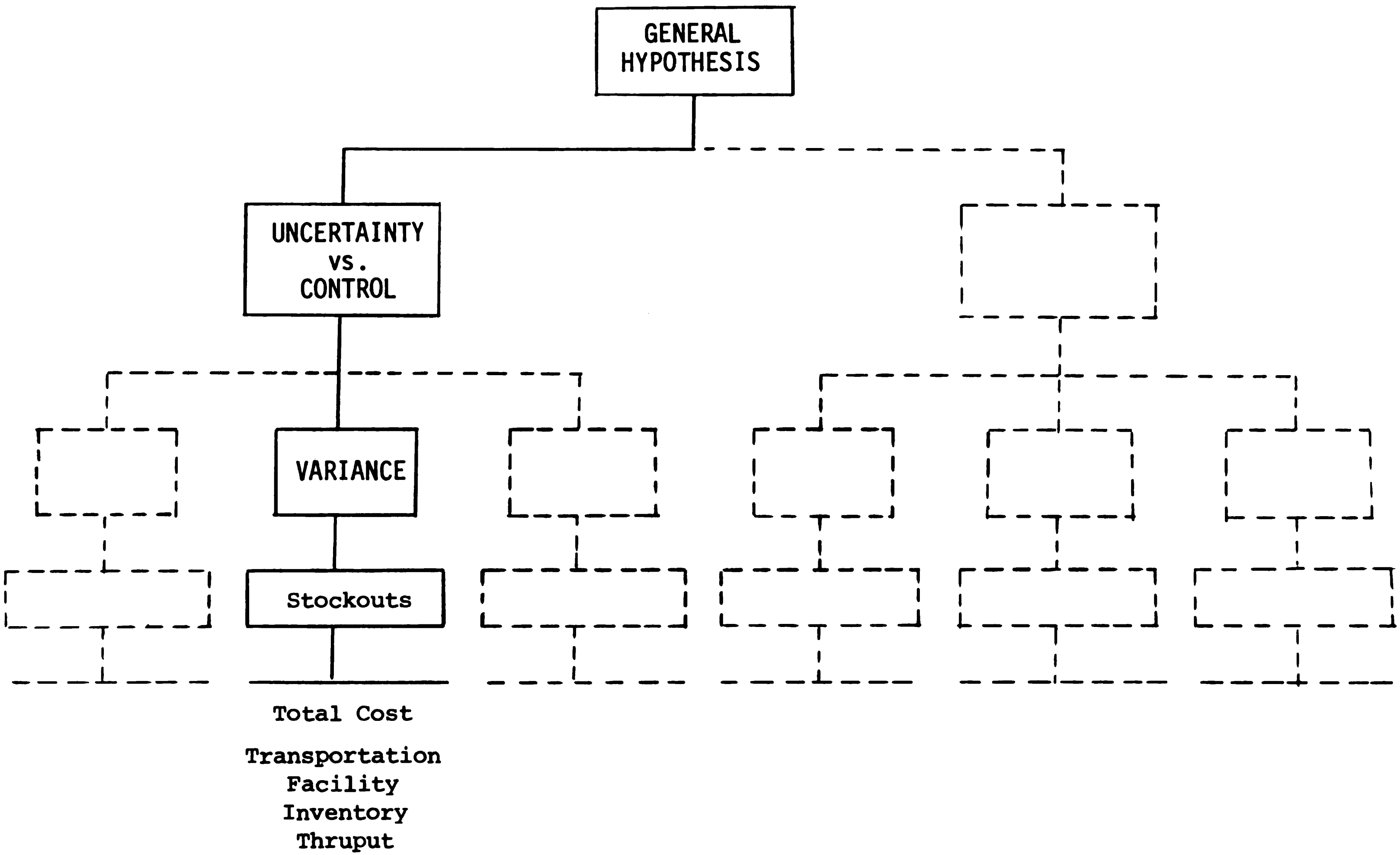

Figure 5-10. Research Analysis Organization: Control/Variance.

The responses of those treatments which included the coefficient of variation of .18 are averaged and those for .375 are averaged. These averages are then compared to the control average for each response variable using Dunnett's method. Due to the nature of the probability distributions, the variance is averaged over two levels and three distributions (normal, log normal and gamma).

The results are shown in Figures 5-11 through 5-17 for each individual response variable. Figure 5-11 shows the demand stocked out in percent (control equals 0.0%, .18 equals 5.55% and .375 equals 9.45%). In Figures 5-12 through 5-17, variance 1 (.18) and variance 2 (.375) are shown as a percentage of the control run response. For instance, in Figure 5-12 the cost to revenue ratio is 24.17% for the control run and is set equal to 100%. Thus, variance 1 is 103.1% of control and variance 2 is 105.35% of control.

The results of Dunnett's test for each response variable are as shown in Table 5-2. Each response variable for both levels of the coefficient of variation are significant, as shown in Table 5-2. When viewing Figures 5-11 through 5-17, note that across all response variables, with the exception of transportation, the uncertain runs cost more (stock out more) than the control runs. In addition, the larger coefficient of variation .375 has greater costs (greater stockouts) than the smaller coefficient of variation.

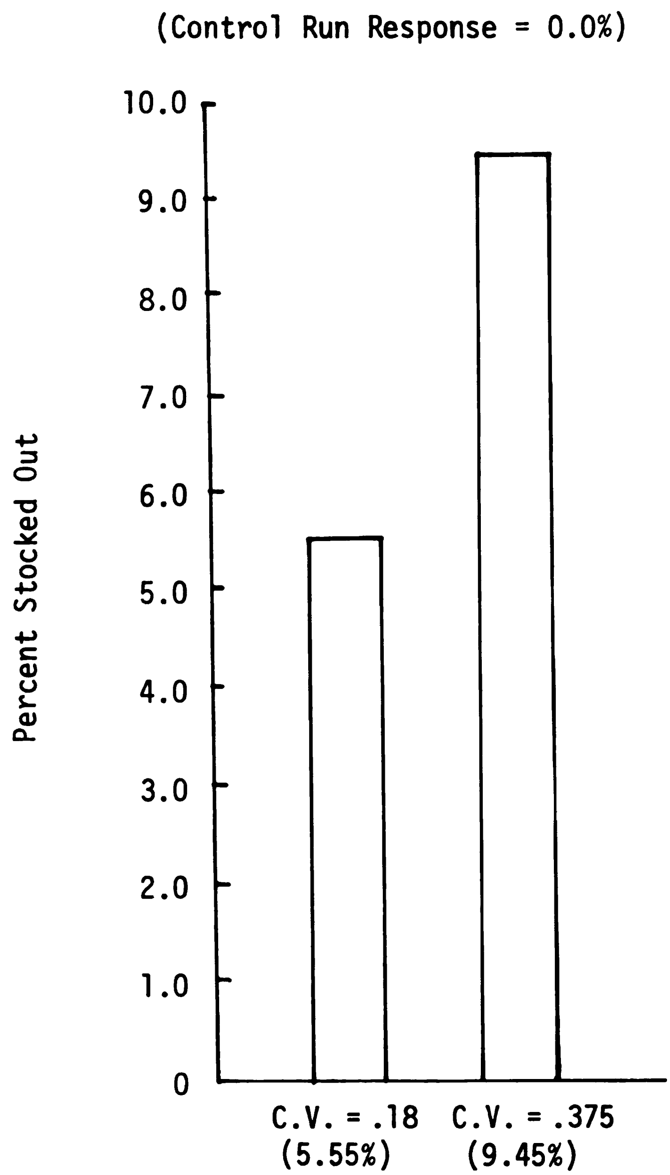

Figure 5-11. Coefficient of Variation Response Compared to Control Run Response: Percent of Demand Stocked Out.

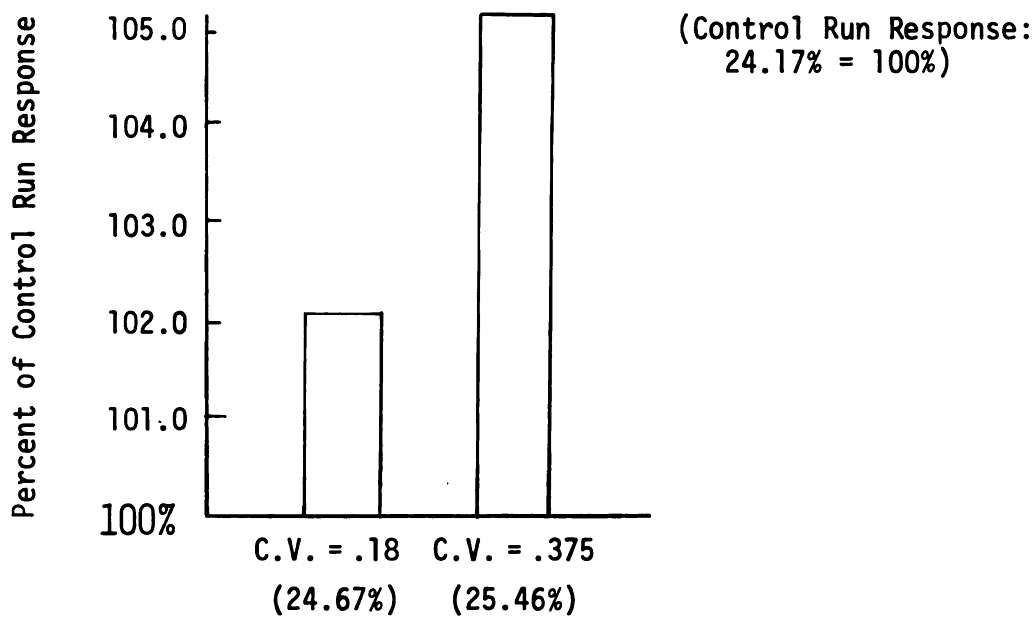

Figure 5-12. Ratio of the Coefficient of Variation Response to the Control Run Response: Cost/Revenue Ratio.

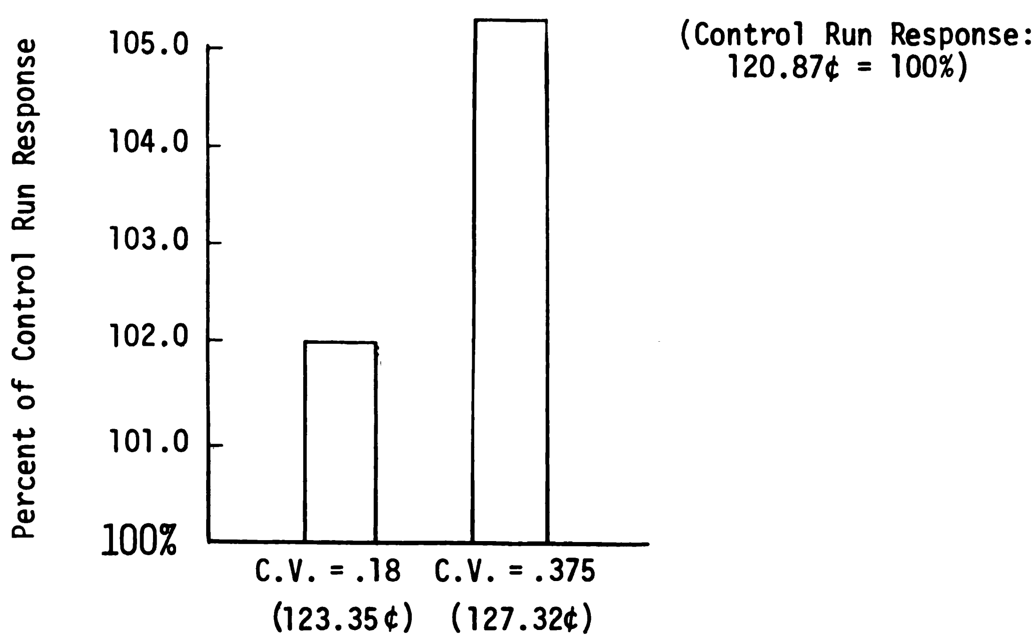

Figure 5-13. Ratio of the Coefficient of Variation Response to the Control Run Response: Total Cost.

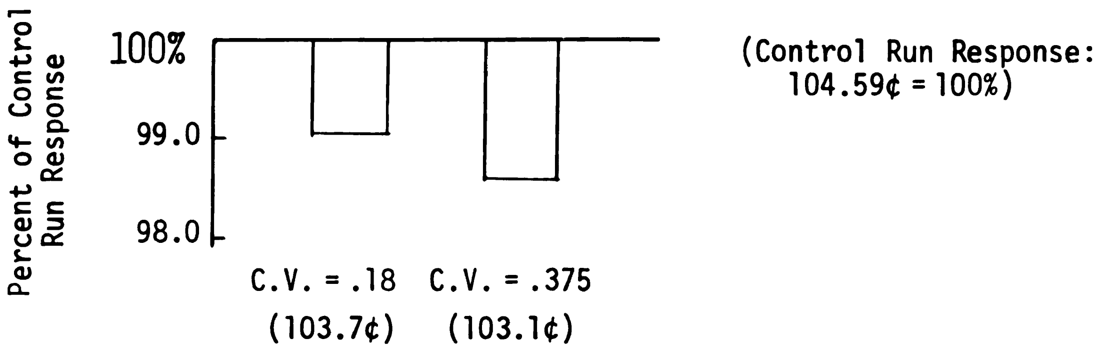

Figure 5-14. Ratio of the Coefficient of Variation Response to the Control Run Response: Transportation Cost.

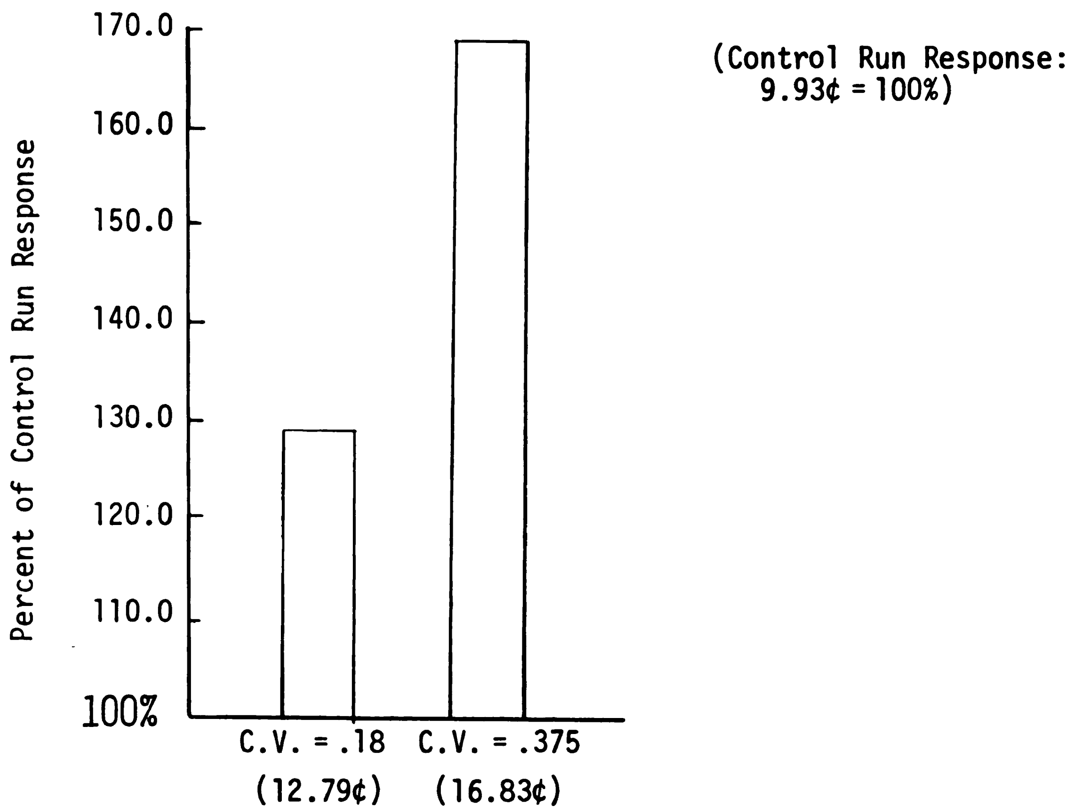

Figure 5-15. Ratio of the Coefficient of Variation Response to the Control Run Response: Facility Cost.

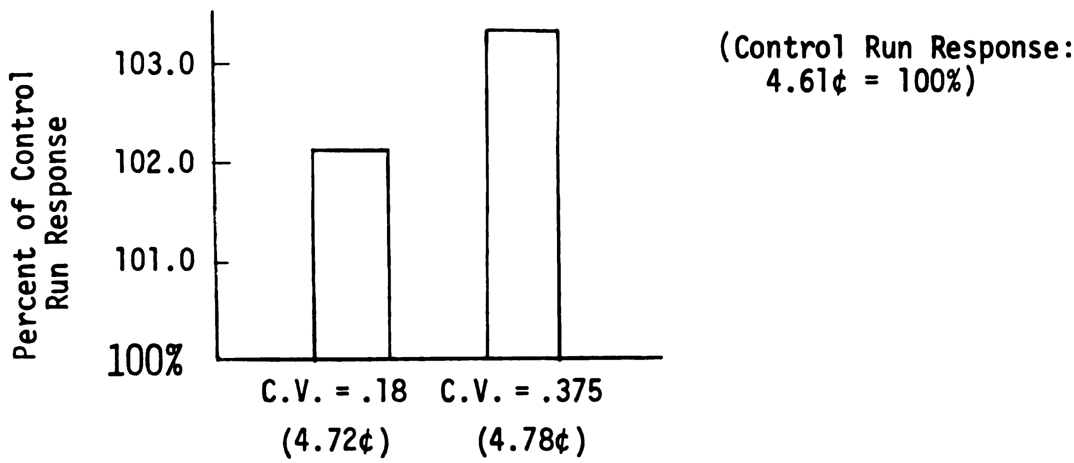

Figure 5-16. Ratio of the Coefficient of Variation Response to the Control Run Response: Thruput Cost.

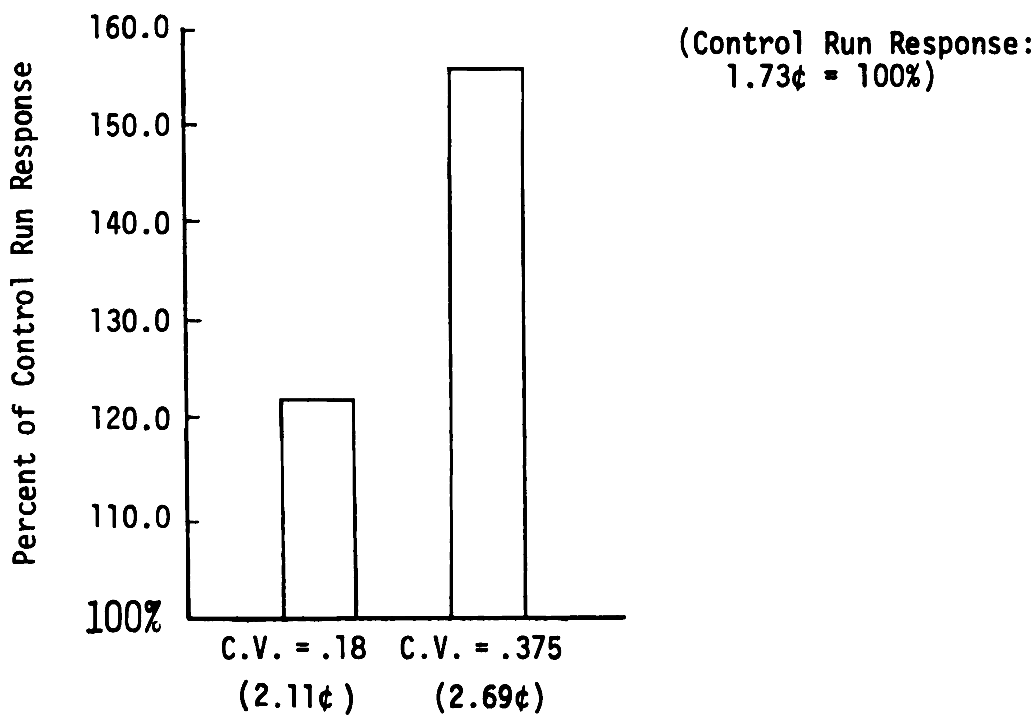

Figure 5-17. Ratio of the Coefficient of Variation Response to the Control Run Response: Inventory Cost.

Table 5-2. Results of Dunnett's Test (Variance vs. Control)

Figure	Response Variable	Critical Value	Actual Value .18	Actual Value .375
5-11	Demand stocked out (percent)	1.43	5.52	8.46
5-12	Cost/revenue ratio (percent)	0.312	0.493	1.29
5-13	Total cost (¢/unit)	1.315	2.478	6.446
5-14	Transportation (¢/unit)	0.798	0.86	1.45
5-15	Facility (¢/unit)	1.25	2.86	6.78
5-16	Thruput (¢/unit)	0.033	0.108	0.169
5-17	Inventory (¢/unit)	0.174	0.371	0.96

Level of Lead Time (Average Duration)

Figure 5-18 indicates that the demand stocked out and costs which result from the level of lead time will be compared to the same measures which result when lead time is constant (the control simulation runs). Two different lead times (levels) are used in this experiment. One-half of the simulation runs are made with a four day lead time, the balance with a seven day lead time. An average is drawn over all runs which have a four day lead time and an average is drawn across all runs which have a seven day lead time. These averages are compared against the control average using Dunnett's test. Due to the nature of the distributions, the levels are averaged over the two coefficients of variation and three distribution patterns (normal, log normal, and gamma).

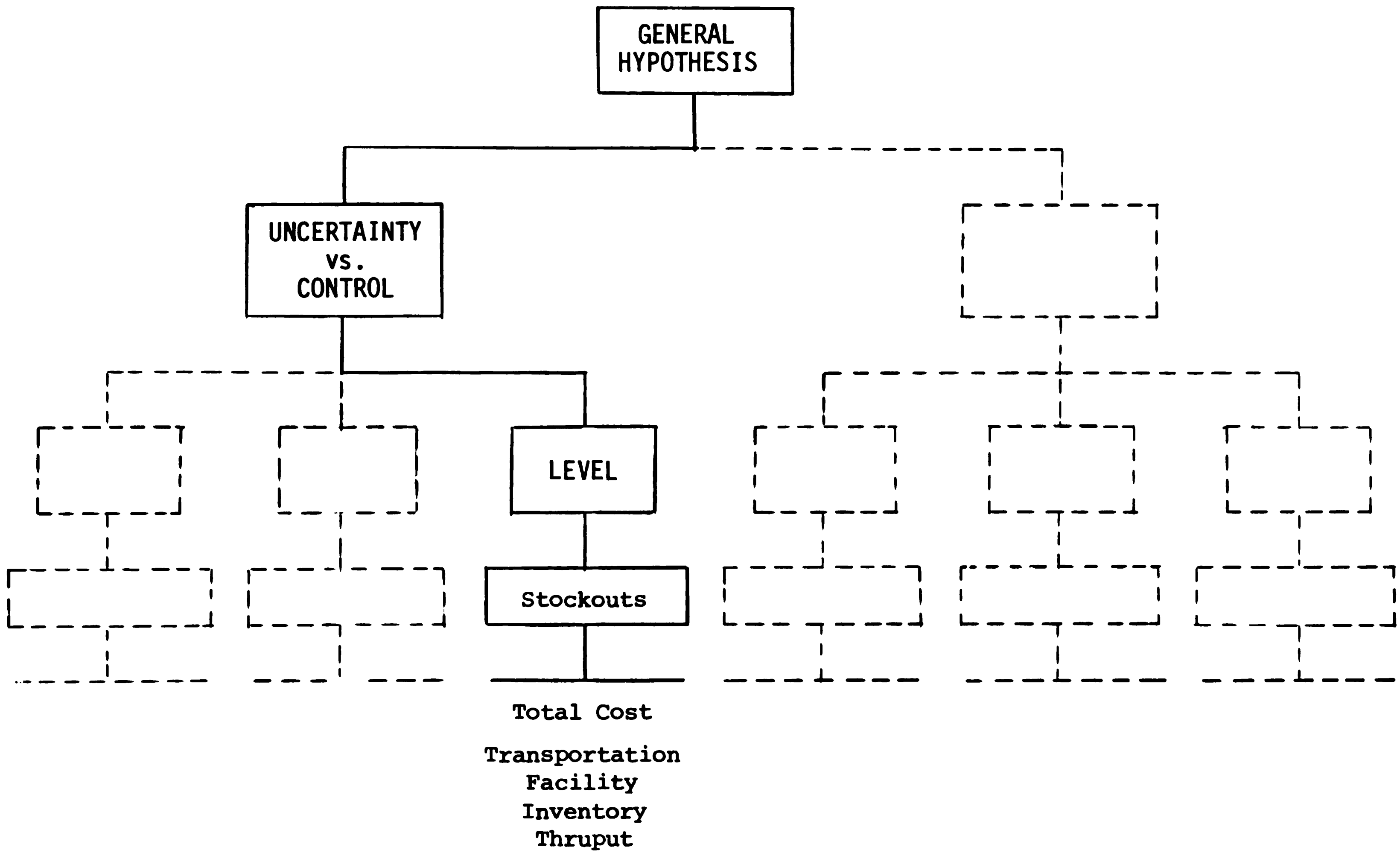

Figure 5-18. Research Analysis Organization: Control/Level.

The level averages vs. the control average are shown in Figures 5-19 through 5-25. Figure 5-19 shows the demand stocked out in percent (control equals 0.0%; 4 days equals 6.20%; 7 days equal 8.77%). Figures 5-20 through 5-25 are given as a percentage of the control. For instance, in Figure 5-20 the control cost/revenue ratio is 24.17% and this is set equal to 100%. Then the four day average and the seven day average is expressed as a percentage of the control (4 day equals 102.32% and 7 day equals 105.09%).

The results of Dunnett's test are shown in Table 5-3.

Table 5-3. Results of Dunnett's Test (Level vs. Control)

Figure	Response Variable	Critical Value	Actual Value 4 Day	Actual Value 7 Day
5-19	Demand stocked out (percent)	1.43	6.20	8.77
5-20	Cost/revenue ratio (percent)	0.312	0.559	1.226
5-21	Total cost (¢/unit)	1.315	2.79	6.13
5-22	Transportation (¢/unit)	0.798	1.21	1.10
5-23	Facility (¢/unit)	1.25	3.43	6.20
5-24	Thruput (¢/unit)	0.033	0.114	0.162
5-25	Inventory (¢/unit)	0.174	0.46	0.87

As Table 5-3 shows, each response variable for both levels is significant. Looking at Figures 5-19 through 5-25 it is seen that in all cases, with the exception of transportation, the uncertain runs cost more (stock out more) than the control runs. In all cases, with the exception of transportation, the runs with the seven day lead time cost more (stock out more) than the four day runs.

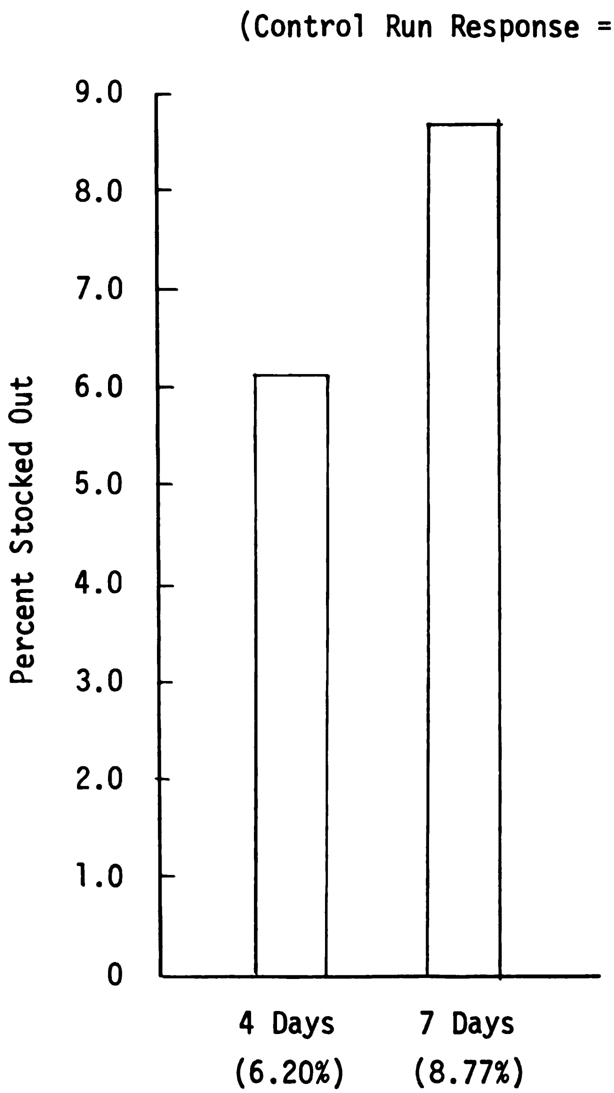

Figure 5-19. The Duration of Lead Time Response Compared to the Control Run Response: Percent Demand Stocked Out.

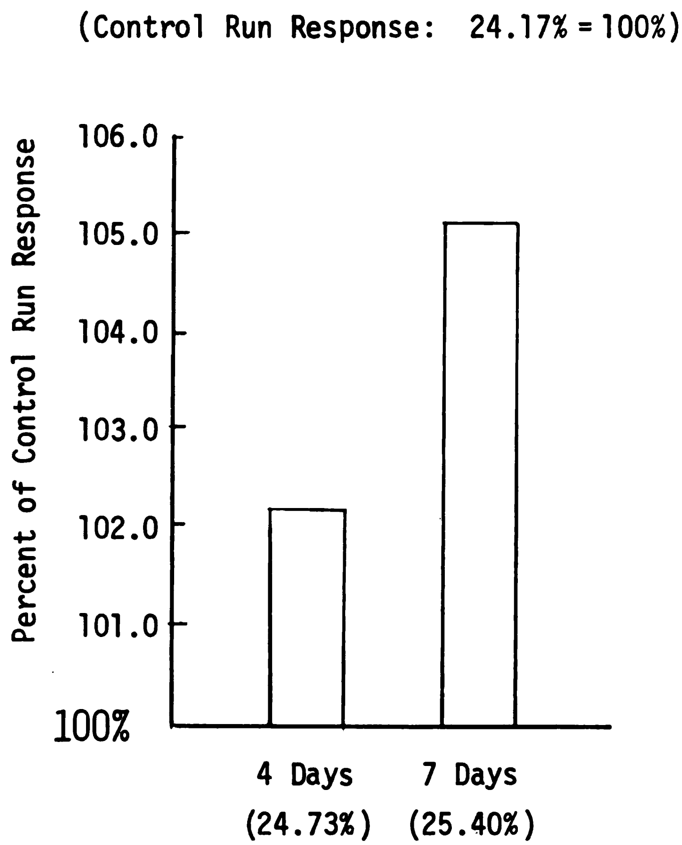

Figure 5-20. Ratio of the Duration of Lead Time Response to the Control Run Response: Cost/Revenue Ratio.

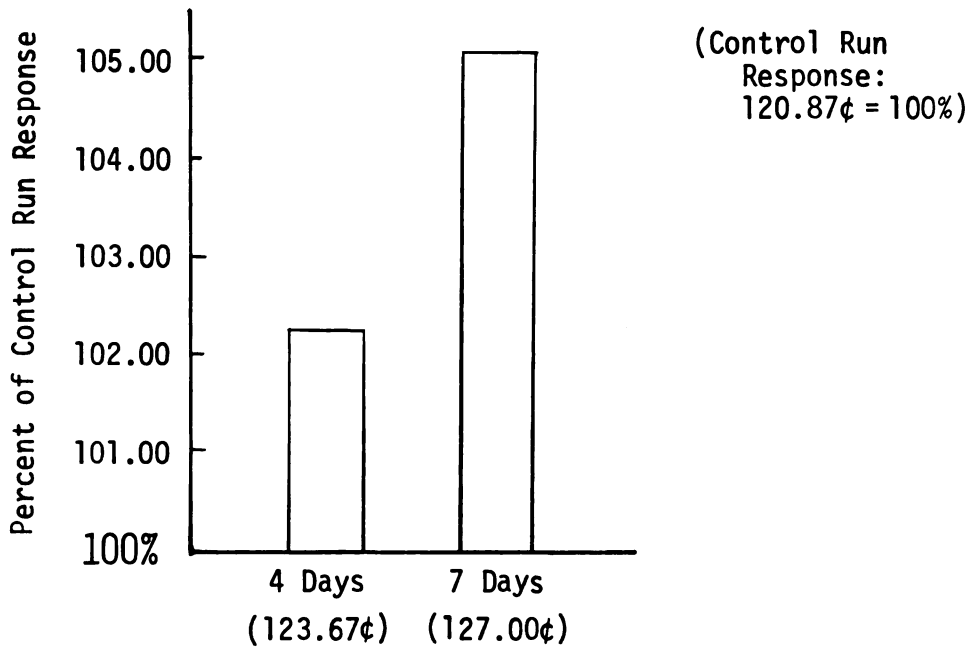

Figure 5-21. Ratio of the Duration of Lead Time Response to the Control Run Response: Total Cost.

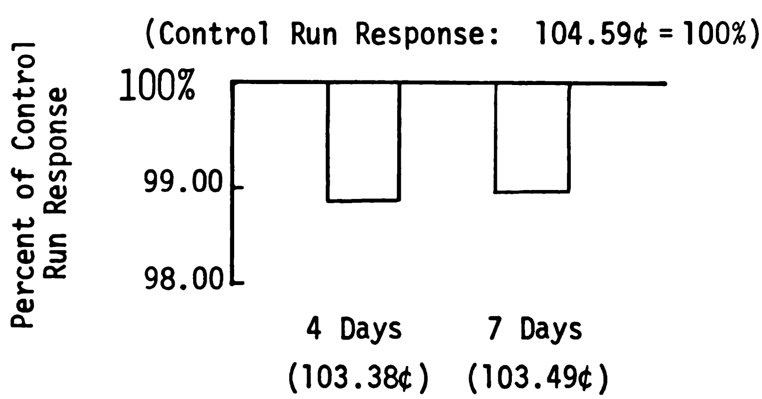

Figure 5-22. Ratio of the Duration of Lead Time Response to the Control Run Response: Transportation Cost.

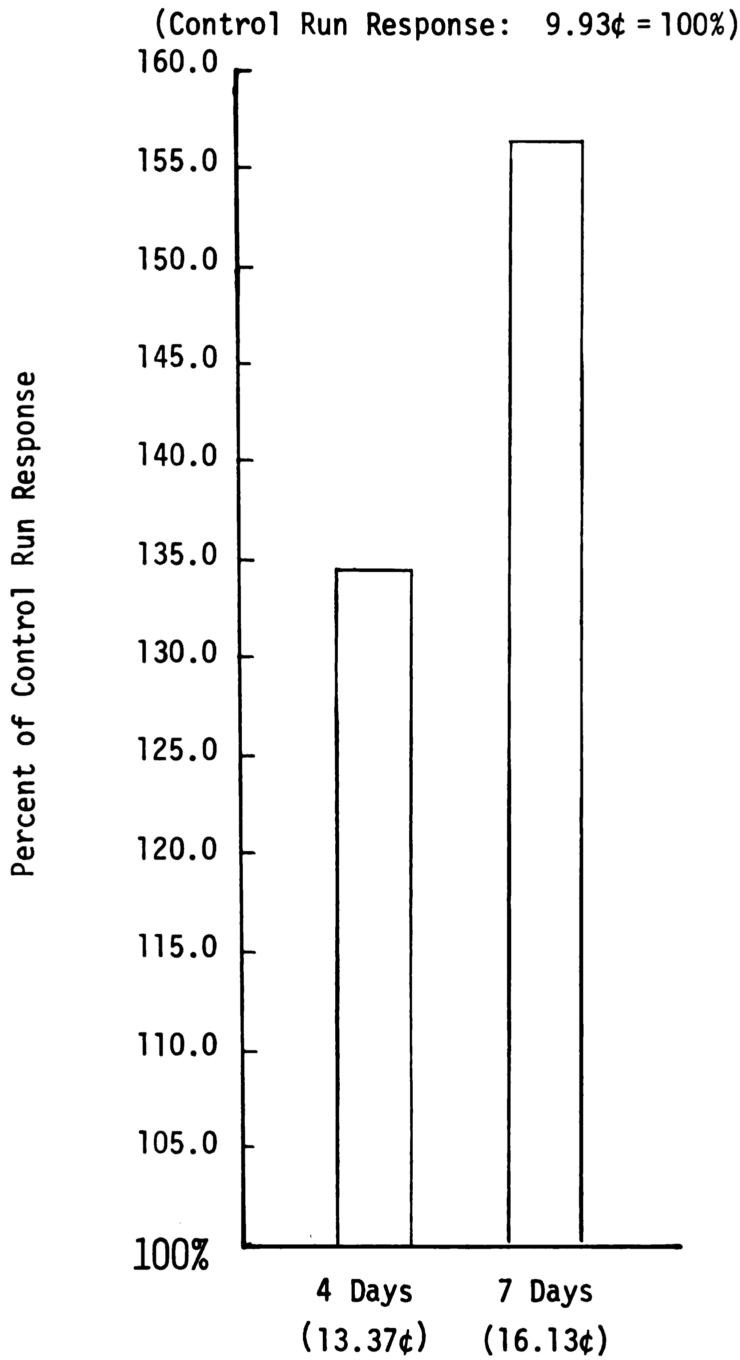

Figure 5-23. Ratio of the Duration of Lead Time Response to the Control Run Response: Facility Cost.

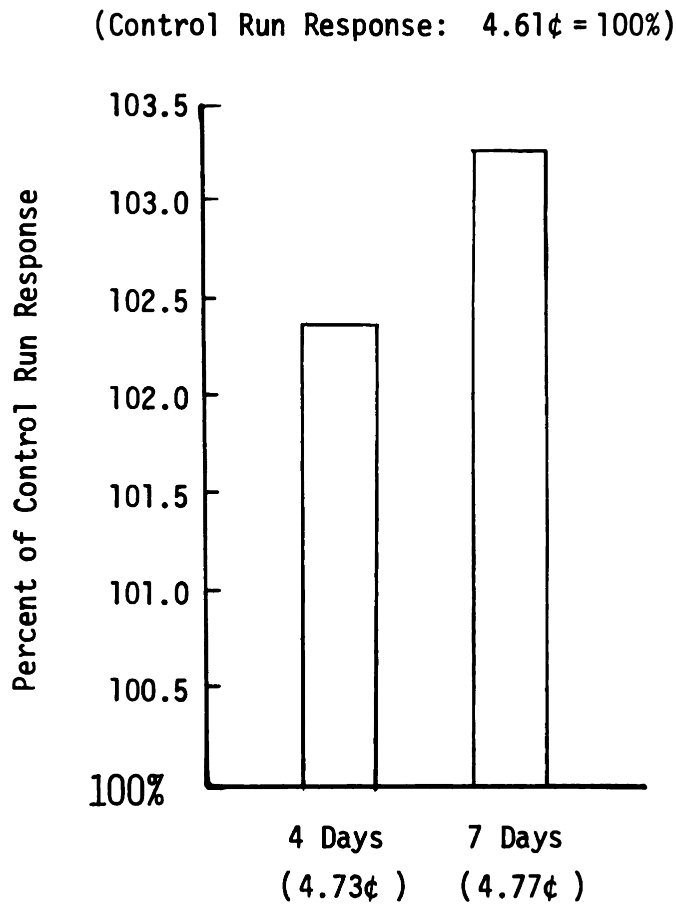

Figure 5-24. Ratio of the Duration of Lead Time Response to the Control Run Response: Thruput Cost.

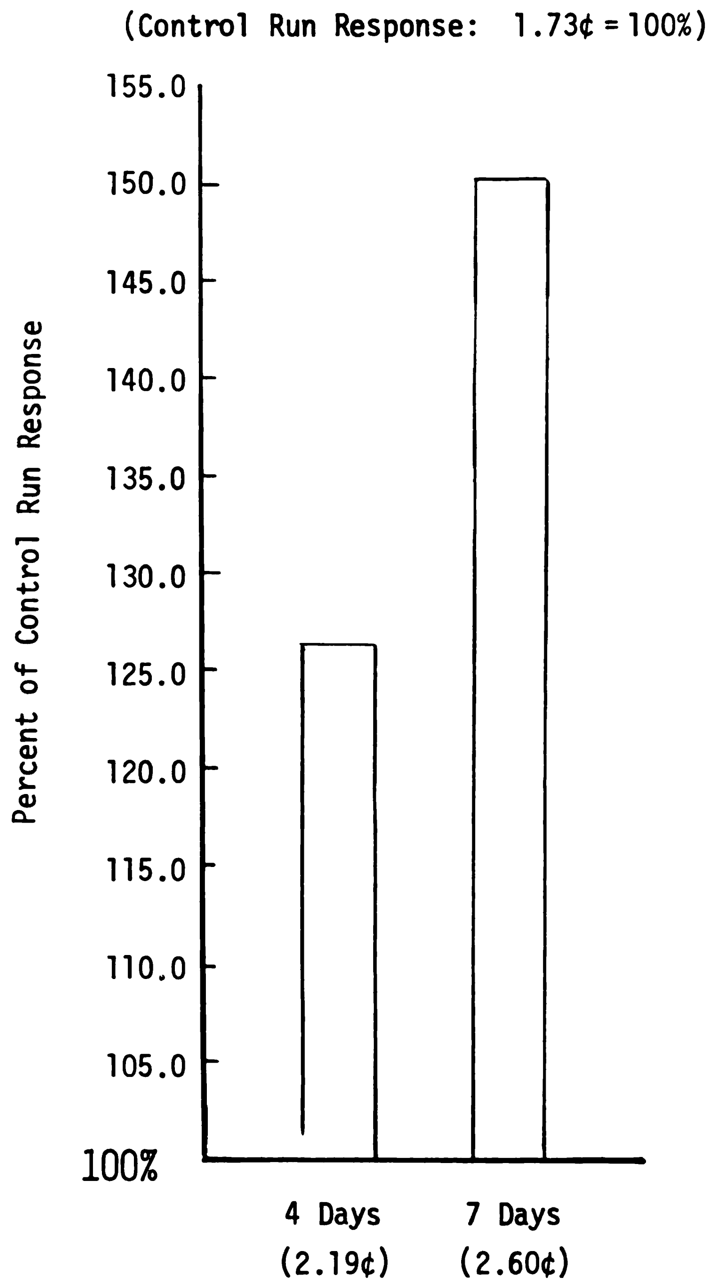

Figure 5-25. Ratio of the Duration of Lead Time Response to the Control Run Response: Inventory Cost.

Comparison of Factor and Control Responses: Individual Cell Comparisons

In the previous section the averages across factors (distribution pattern, variance and level) were compared through statistical tests to the average of the control system. Although such a procedure develops much information, more can be gleaned from the data. Therefore, data on a cell-to-cell basis, with the emphasis on nonstatistical analysis, is discussed in this section.

Individual cells represent the results of a particular simulation run for a particular response variable. For instance, Table 5-4 presents individual cell comparisons for demand stocked out. Each cell represents a particular combination of distribution pattern, coefficient of variation and level (four day or seven day lead time). The data in the cell is the demand stocked out as a result of this combination.

Cell to cell comparison by response variable reveals findings not possible when comparing averages. Trends may be identified and the behavior of particular cells revealed, which may or may not be shown when comparing averages. The emphasis in this section is to compare individual cells to control. A more extensive comparison among the cells representing uncertainty will be made in the next section of this chapter.

Demand Stocked Out

Table 5-4 shows individual cell comparisons for demand stocked out in percent. The control simulation runs (4 and 7 days) have no stockouts. All other runs incur some stockouts. The stockouts range from a low of 3.97% (normal distribution, coefficient of variation equals .18 and a 4 day lead time) to a high of 22.09% (exponential distributions with a 7 day lead time). In all cases the shorter lead time (4 days) has fewer stockouts than the longer lead time (7 days) and in those cases where two coefficients of variation are possible (.18 and .375), the smaller variation (.18) stocks out less than the larger variation (.375).

Cost/Revenue Ratio

Table 5-5 shows the individual cell comparisons for the cost/revenue ratio in percent. These figures are obtained by dividing the total revenue (sales) into the total cost for the system. In all cases the uncertain runs with a four day lead time have a higher ratio (more costly) than the four day control runs. The same holds true for the seven day level. In all cases the seven day runs (including the control) are more costly than the four day runs. In the four day runs the ratio closest to the control is normal at .18, while the largest difference is between control and the exponential. In the seven day runs the ratio closest to control is the log normal at .18 (24.80) followed closely by the normal at .18 (24.83). As with the four day runs the largest difference in the seven day is between control and exponential (24.51 vs. 28.20).

Table 5-4. Individual Cell Comparison: Percent Demand Stocked Out

Coefficient of Variation	Control[a]		Normal		Log Normal		Gamma		Exponential[b]		Erlang[c]		Poisson[d]	
	Level		Level		Level		Level		Level		Level		Level	
	4	7	4	7	4	7	4	7	4	7	4	7	4	7
.18	0.0	0.0	3.97	6.25	4.16	6.98	4.73	7.01	13.23	22.09	10.94	15.54	10.02	10.85
.375			8.70	8.95	7.76	10.83	7.84	12.62						

Table 5-5. Individual Cell Comparison: Cost/Revenue Ratio (Percent)

Coefficient of Variation	Control[a]		Normal		Log Normal		Gamma		Exponential[b]		Erlang[c]		Poisson[d]	
	Level		Level		Level		Level		Level		Level		Level	
	4	7	4	7	4	7	4	7	4	7	4	7	4	7
.18	23.84	24.51	24.27	24.83	24.42	24.80	24.51	25.20	26.43	28.20	25.75	26.84	25.51	25.82
.375			25.24	25.62	25.04	25.98	24.92	25.98						

[a]Not applicable. [b]$\sigma = \mu$. [c]$\sigma = .57$. [d]$\sigma = \sqrt{\mu}$.

Total Cost

Table 5-6 shows individual cell comparisons for total cost in cents per unit. In all cases the four day uncertain runs cost more than the four day control runs and in all cases, the seven day runs, including the control, cost more than the four day runs. The smallest increase above the cost of the control runs for the four day level is the normal distribution at a coefficient of variation of .18. The largest increase above control for the four day runs is the exponential distribution. This situation is almost duplicated at the seven day level. The smallest increase over control is shared by the log normal at .18 (123.99) and the normal at .18 (124.13) while the largest increase is the exponential.

Transportation Cost

Table 5-7 shows the individual cell comparisons for the cost of transportation in cents per unit. This is the only response variable where the uncertain runs cost less than the control runs. All cells on both levels are less than the control responses with the exception of the gamma distribution with a coefficient of variation of .18 and with a seven day lead time. Both control cells are equal on a per unit basis and the systematic progression of increase from one level to another or from one variance to another does not occur. In some of the cases, the seven day lead time responses are closer to the control response than the four day lead time. This is true for the normal distribution at both coefficients of variation, the log normal at .375 and the gamma at .18. In the balance of the cases, the four day runs are closer to

Table 5-6. Individual Cell Comparison: Total Cost (¢/Unit)

Coefficient of Variation	Control[a]		Normal		Log Normal		Gamma		Exponential[b]		Erlang[c]		Poisson[d]	
	Level		Level		Level		Level		Level		Level		Level	
	4	7	4	7	4	7	4	7	4	7	4	7	4	7
.18			121.34	124.13	122.08	123.99	122.57	125.98						
	119.20	122.54							132.13	140.99	128.77	134.20	127.56	129.09
.375			126.21	128.10	125.20	129.90	124.58	129.92						

Table 5-7. Individual Cell Comparison: Transportation Cost (¢/Unit)

Coefficient of Variation	Control[a]		Normal		Log Normal		Gamma		Exponential[b]		Erlang[c]		Poisson[d]	
	Level		Level		Level		Level		Level		Level		Level	
	4	7	4	7	4	7	4	7	4	7	4	7	4	7
.18			103.07	103.45	103.92	102.92	104.14	104.91						
	104.59	104.59							103.19	101.51	102.82	101.87	103.20	102.84
.375			103.32	103.63	103.05	103.57	102.79	102.48						

[a] Not applicable. [b] $\sigma = \mu$. [c] $\sigma = .57$. [d] $\sigma = \sqrt{\mu}$.

the control. This is true for the log normal at .18, the gamma at .375, the exponential, erlang and poisson. Considering the responses from one coefficient of variation to another, the results are just as randomly distributed. The smallest difference between the control and an uncertain cell is control vs. gamma, seven day at .18 (the only positive cell) and the largest difference is between control and the exponential at seven days. In the previous response variables, patterns and trends could be seen. In this response variable no evident pattern or trend is observed.

Facility Cost

Table 5-8 shows the individual cell comparisons for the facility costs in cents per unit. In all cases, the cost in the uncertain runs is greater than the cost in the control runs. In the four day control run the smallest difference vs. control is log normal at 11.55 vs. 8.51. However, both the normal at .18 (11.65) and the gamma at .18 (11.77) are very close. The greatest difference is again the exponential (control equals 8.51, exponential equals 20.77). In the seven day runs, which across all cells are more expensive than the four day runs, the results are slightly different. The smallest difference is between control (11.35) and normal at .18 (13.70). The log normal and the gamma are relatively close to the normal (14.3 and 14.04, respectively). The largest difference is again the exponential (29.65). In those cases where two coefficients of variation are used, the .375 variation is more costly than the .18 variation. Once again, a definite pattern can be seen in this response variable.

Table 5-8. Individual Cell Comparison: Facility Cost (¢/Unit)

Coefficient of Variation	Control[a]		Normal		Log Normal		Gamma		Exponential[b]		Erlang[c]		Poisson[d]	
	Level		Level		Level		Level		Level		Level		Level	
	4	7	4	7	4	7	4	7	4	7	4	7	4	7
.18			11.65	13.70	11.55	14.03	11.77	14.04						
	8.51	11.35							20.77	29.65	18.24	23.69	16.84	18.51
.375			15.64	16.95	14.96	18.56	14.65	19.50						

[a]Not applicable.

[b]$\sigma = \mu$.

[c]$\sigma = .57$.

[d]$\sigma = \sqrt{\mu}$.

Thruput Cost

Table 5-9 shows the individual cell comparisons for the thruput costs in cents per unit. In all cases the uncertain runs cost more than the control runs. The control costs for both the four day and seven day runs are equal. In all cases, with the exception of the control, the normal at .375 and the poisson costs increase as the lead time goes from four to seven days. In all cases where two coefficients of variation are used, the larger variation was more expensive. In both the four day and seven day runs, the uncertain run closest to control is normal at .18, and the furthest from control is the exponential distribution. The overall difference between control and all uncertain runs is very small (exponential 5.17,and control 4.61 is the largest difference).

Inventory Cost

Table 5-10 shows the individual cell comparisons for inventory costs in cents per unit. In all cases for the four day runs, the uncertain runs are more expensive than the control run. This is also true for the seven day runs where all uncertain runs are more than the control runs. In all cases, including the control, the seven day runs cost more than the four day runs. And in those runs where two coefficients of variation are used, the largest variation (.375) cost more (for both lead times) than the smaller variation (.18). In the four day runs the uncertain response closest to control is log normal at .18 (1.93) with normal and gamma at .18 very close (1.94 and 1.96, respectively). The largest difference in the four day runs is exponential (3.30). These same findings basically repeat themselves in

Table 5-9. Individual Cell Comparison: Thruput Cost (¢/Unit)

<table>
<tr><th rowspan="3">Coefficient of Variation</th><th colspan="2">Control[a]</th><th colspan="2">Normal</th><th colspan="2">Log Normal</th><th colspan="2">Gamma</th><th colspan="2">Exponential[b]</th><th colspan="2">Erlang[c]</th><th colspan="2">Poisson[d]</th></tr>
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<tr><td>.18</td><td rowspan="2">4.61</td><td rowspan="2">4.61</td><td>4.68</td><td>4.74</td><td>4.69</td><td>4.76</td><td>4.70</td><td>4.74</td><td rowspan="2">4.88</td><td rowspan="2">5.17</td><td rowspan="2">4.82</td><td rowspan="2">4.94</td><td rowspan="2">4.82</td><td rowspan="2">4.79</td></tr>
<tr><td>.375</td><td>4.76</td><td>4.75</td><td>4.77</td><td>4.81</td><td>4.76</td><td>4.84</td></tr>
</table>

Table 5-10. Individual Cell Comparison: Inventory Cost (¢/Unit)

<table>
<tr><th rowspan="3">Coefficient of Variation</th><th colspan="2">Control[a]</th><th colspan="2">Normal</th><th colspan="2">Log Normal</th><th colspan="2">Gamma</th><th colspan="2">Exponential[b]</th><th colspan="2">Erlang[c]</th><th colspan="2">Poisson[d]</th></tr>
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<tr><td>.18</td><td rowspan="2">1.49</td><td rowspan="2">1.98</td><td>1.94</td><td>2.24</td><td>1.93</td><td>2.29</td><td>1.96</td><td>2.28</td><td rowspan="2">3.30</td><td rowspan="2">4.65</td><td rowspan="2">2.90</td><td rowspan="2">3.71</td><td rowspan="2">2.69</td><td rowspan="2">2.96</td></tr>
<tr><td>.375</td><td>2.52</td><td>2.76</td><td>2.42</td><td>2.96</td><td>2.38</td><td>3.10</td></tr>
</table>

[a]Not applicable. [b]$\sigma = \mu$. [c]$\sigma = .57$. [d]$\sigma = \sqrt{\mu}$.

the seven day runs. The closest to control is normal at .18 (2.24) with log normal and gamma at .18 very close (2.29 and 2.28, respectively). The largest difference is the exponential distribution.

Conclusion

In the comparison of individual cell responses to control by response variable, several interesting findings evolve. With the exception of transportation, all uncertain runs cost more than the control runs. Generally, the seven day runs cost more than the four day runs and the largest variation (.375) cost more than the smaller variation (.18). In all cases where uncertain runs cost more than the control runs, the exponential distribution accounts for the largest deviation from control while the normal followed closely by the log normal, and gamma account for the smallest deviation from control.

Comparison Among Factor Responses: Introduction

In the previous section comparisons were made between factors (distribution pattern, coefficient of variation and lead time level) and the control or deterministic simulation runs. In comparing factors against control, the main question revolves around comparison of the behavior of the channel system when confronted with an uncertain lead time as opposed to a constant lead time. Basically, we are asking what are the effects of uncertainty. The next main question asks, is there a difference in how particular factors among themselves affect the system? For instance, does a normal distribution have the same or different effects on a system than a log normal or gamma distribution? It is this question that we address in this section of the experimental results. Thus, in this section the control responses are eliminated and only the responses from the uncertain runs are considered. Each experimental factor (distribution patterns, coefficient of variation and lead time levels) is analyzed to determine whether or not they create different effects on system performance.

The analysis of the factors among themselves is performed in three parts and the organization of this section follows this scheme. First, an analysis of variance is performed using the F test. The purpose of this procedure is to determine the relative impact of distribution patterns, variances and lead time levels on the response variables of the system. The F test is performed at the .05 level for all response variables. Because of the characteristics of the exponential, poisson and erlang distributions they are not included in this analysis.

Secondly, average response comparisons are made within each factor. For instance, the average response of the normal is compared against all other distributions. These comparisons are made for each response variable. Where possible (normal vs. log normal vs. gamma), Tukey's method of multiple comparisons is used. When the Tukey method does not apply (i.e., normal vs. exponential, erlang and poisson), a t-test is used to make the comparisons. Both the Tukey method and the t-test are performed at the .05 level. In the second part of this section, distribution patterns are presented first, then the coefficients of variation, and finally the lead time level. Within each factor, each response variable is analyzed.

The last part of this section analyzes the individual cell comparisons by response variable. The analysis in this section is nonstatistical in nature and is designed to reveal information obscured when working with averages.

Comparison Among Factor Responses: Analysis of Variance

Tables 5-11 through 5-17 present the analysis of variance results for each response variable (demand stocked out, cost/revenue ratio, total cost, transportation cost, facility cost, thruput cost and inventory cost, respectively).

For all response variables, with the exception of transportation costs, the F ratio for the duration of lead time (level, 4 or 7 days) and the coefficient of variation (.18 and .375) is significant. This indicates that both of these factors have an effect on all response variables (with the exception of transportation).

The analysis of variance results for transportation are presented in Table 5-14. Neither distribution patterns nor the level have significant F ratios, indicating that they do not have an effect on the transportation cost. However, the coefficient of variation (variance) has a significant F ratio, indicating that it has an effect on the transportation costs.

Table 5-11. Analysis of Variance: Demand Stocked Out (%)

Source	Sum of Squares	DF	Mean Square	F	Critical F $\alpha = .05$
Distributions	2.3	2	1.15	1.41	4.74
Levels	19.8	1	19.8	24.32*	5.59
Variances	46.7	1	46.7	57.35*	5.59
Error	5.7	7	0.814		

*Significant.

Table 5-12. Analysis of Variance: Cost/Revenue Ratio (%)

Source	Sum of Squares	DF	Mean Square	F	Critical F $\alpha = .05$
Distributions	0.05	2	0.025	0.65	4.74
Levels	1.33	1	1.33	34.46*	5.59
Variance	1.89	1	1.89	48.96*	5.59
Error	0.27	7	0.0386		

*Significant.

Table 5-13. Analysis of Variance: Total Cost (¢/Unit)

Source	Sum of Squares	DF	Mean Square	F	Critical F $\alpha = .05$
Distributions	1.4	2	0.7	1.02	4.74
Levels	34.4	1	34.4	50.15*	5.59
Variance	48.2	1	48.2	70.26*	5.59
Error	4.8	7	0.686		

*Significant.

Table 5-14. Analysis of Variance: Transportation Cost (¢/Unit)

Source	Sum of Squares	DF	Mean Square	F	Critical F $\alpha = .05$
Distributions	0.1256	2	0.0628	0.25	4.74
Levels	0.4494	1	0.4494	1.78	5.59
Variances	2.7135	1	2.7135	10.74*	5.59
Error	1.768	7	0.2526		

*Significant.

Table 5-15. Analysis of Variance: Facility Cost (¢/Unit)

Source	Sum of Squares	DF	Mean Square	F	Critical F $\alpha = .05$
Distributions	0.52	2	0.26	0.42	4.74
Levels	22.82	1	22.82	37.01*	5.59
Variances	45.88	1	45.88	74.52*	5.59
Error	4.31	7	0.616		

*Significant.

Table 5-16. Analysis of Variance: Thruput Cost (¢/Unit)

Source	Sum of Squares	DF	Mean Square	F	Critical F $\alpha = .05$
Distributions	0.002	2	0.001	2.33	4.74
Levels	0.007	1	0.007	16.33*	5.59
Variances	0.011	1	0.011	25.67*	5.59
Error	0.003	7	0.000429		

*Significant.

Table 5-17. Analysis of Variance: Inventory Cost (¢/Unit)

Source	Sum of Squares	DF	Mean Square	F	Critical F $\alpha = .05$
Distributions	0.008	2	0.004	0.57	4.74
Levels	0.502	1	0.502	7.17*	5.59
Variances	1.017	1	1.017	145.3*	5.59
Error	0.084	7	0.012		

*Significant.

Comparison Among Factor Responses: Average Response Comparisons

The analysis of variance technique is designed to show if various levels of an experimental factor (i.e., normal, log normal, gamma) as a whole have an effect on a response variable. Because it is a collective type of analysis, it does not indicate if particular levels within the factor (i.e., normal, log normal, gamma) have differing impacts on a particular response variable. Thus, even though the coefficient of variation of lead time had an effect on all the response variables as indicated by the analysis of variance, it cannot be said that the .18 variance had the same or a different effect than the .375 variance on the response variables. Therefore, individual comparisons between factor levels are made. Where applicable, Tukey's multiple comparison method is used. Where the properties of the factor levels prohibit this, a t-test is employed.

The average of the cells in which a particular factor level was used is compared to the average of the cells in which the comparison factor level was used. For instance, the normal distribution is run at two variances and two levels of lead time, thus four cells were averaged and the average is compared to the average across the cells in which the log normal was run. Thus, distribution patterns are compared, followed by variances and levels.

Probability Distributions

Figure 5-26 indicates that the demand stocked out and costs which result from each type of lead time pattern will be compared to one another.

Demand stocked out.--Figure 5-27 shows the amount of demand satisfied in percent due to the distribution pattern. Tukey's multiple comparison technique is used for normal, log normal and the gamma distribution comparisons. No significant difference exists among any possible combination. This, therefore, indicates that each has the same effect on the system. T-tests are then employed to individually compare the exponential, erlang and poisson against the normal, log normal and gamma (i.e., a t-test was used to compare poisson vs. normal, poisson vs. log normal and poisson vs. gamma, etc.). As a result of the nine possible combinations, two are significantly different and a third extremely close. The critical value equals 2.776. Normal vs. exponential has a value of 3.3, log normal vs. exponential has a value of 3.01 and normal vs. erlang has a value of 2.77. Thus, all distributions, with the exception of the exponential, have similar effects on the

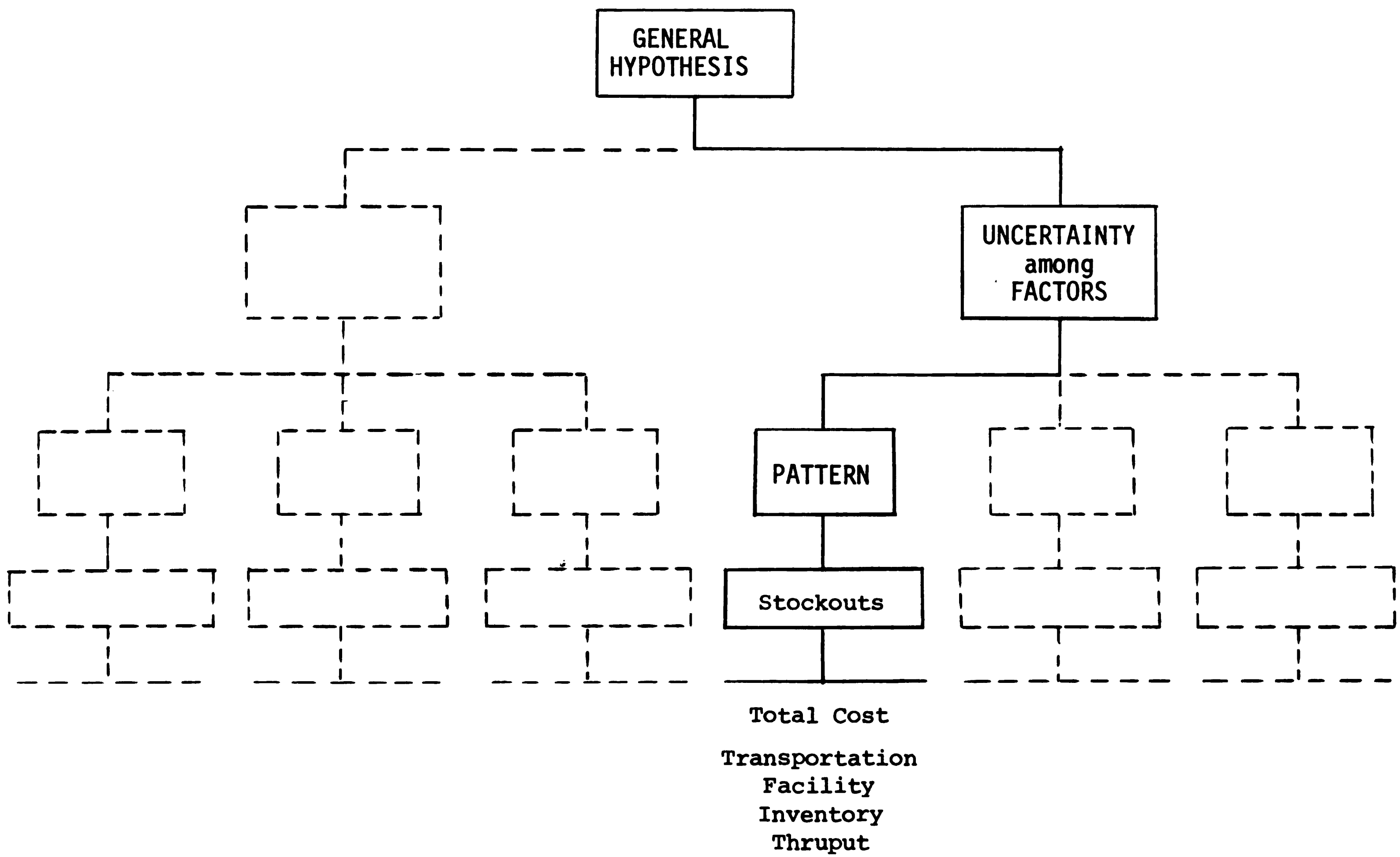


Figure 5-26. Research Analysis Organization: Among Factors/Pattern.

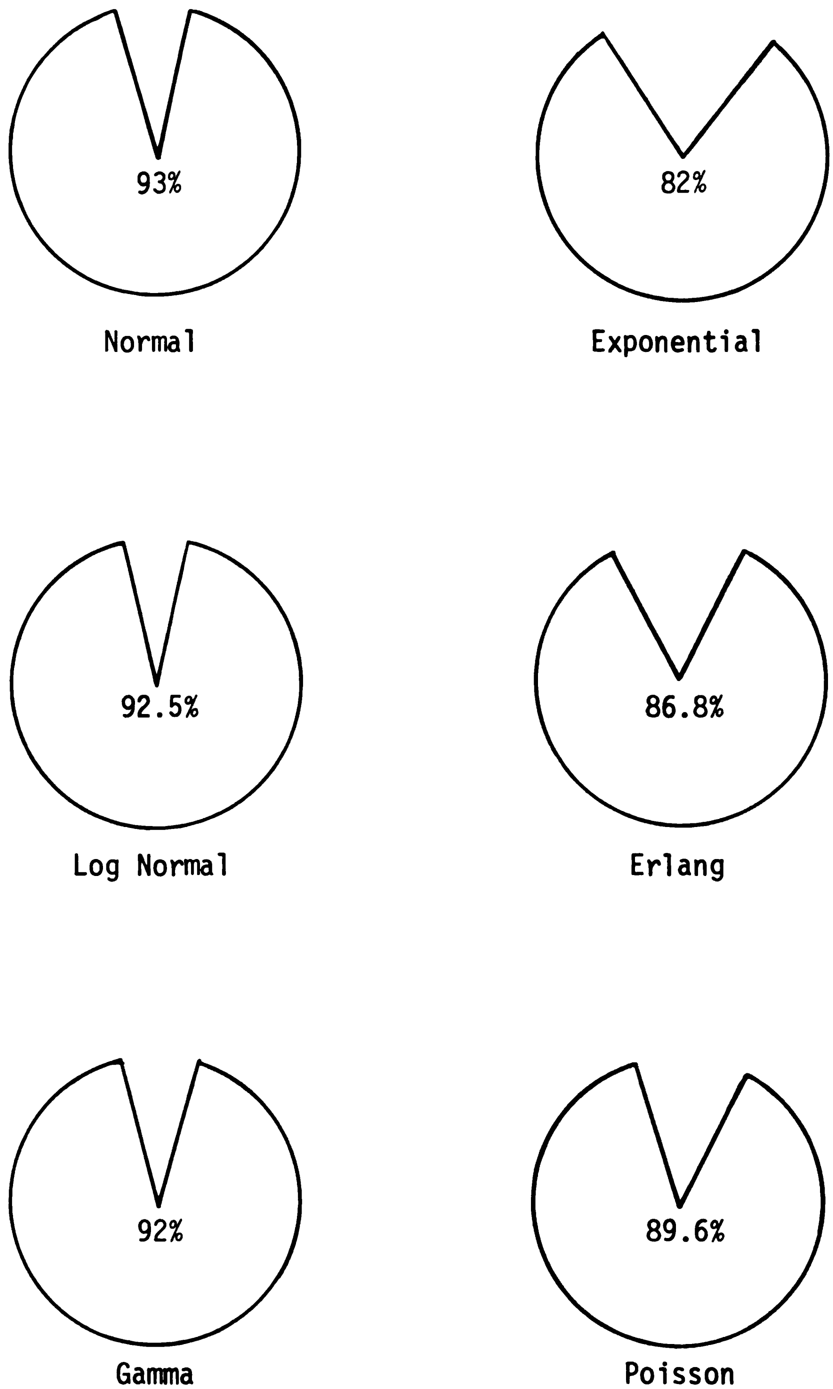


Figure 5-27. Probability Distribution Response: Demand Satisfied (%).

demand stocked out in the system. Due to one degree of freedom, a statistical analysis of the exponential vs. the erlang vs. poisson was not made. This condition is true for all response variables in this section.

Costs.--Figure 5-28 shows the average total cost due to distributions. Total cost is composed of four activity center costs (transportation, facility, thruput and inventory). All figures are shown in cents per unit. Individual tests were made on these five response variables.

The total cost of the normal, log normal and gamma distributions are compared using Tukey's technique. There are no significant differences between these factor levels. Thus, these distributions do not have different effects on total cost. The exponential, poisson and erlang distributions are compared against the normal, log normal and gamma using the t-test. Of the nine combinations, three are significant. The critical value is 2.776. The value for normal vs. exponential is 3.34, log normal vs. exponential is 3.06 and gamma vs. exponential is 3.02. Thus the exponential is the only distribution that has a different effect on the total cost of the system.

Transportation costs for distributions are shown as part of total cost in Figure 5-28. As with the previous response variables, the normal, log normal and gamma are compared using Tukey's technique. The poisson, exponential and erlang are compared to normal, log normal, and gamma using the t-test. There is one significant difference. The critical value is 2.776, and in the t-test comparing normal and erlang,

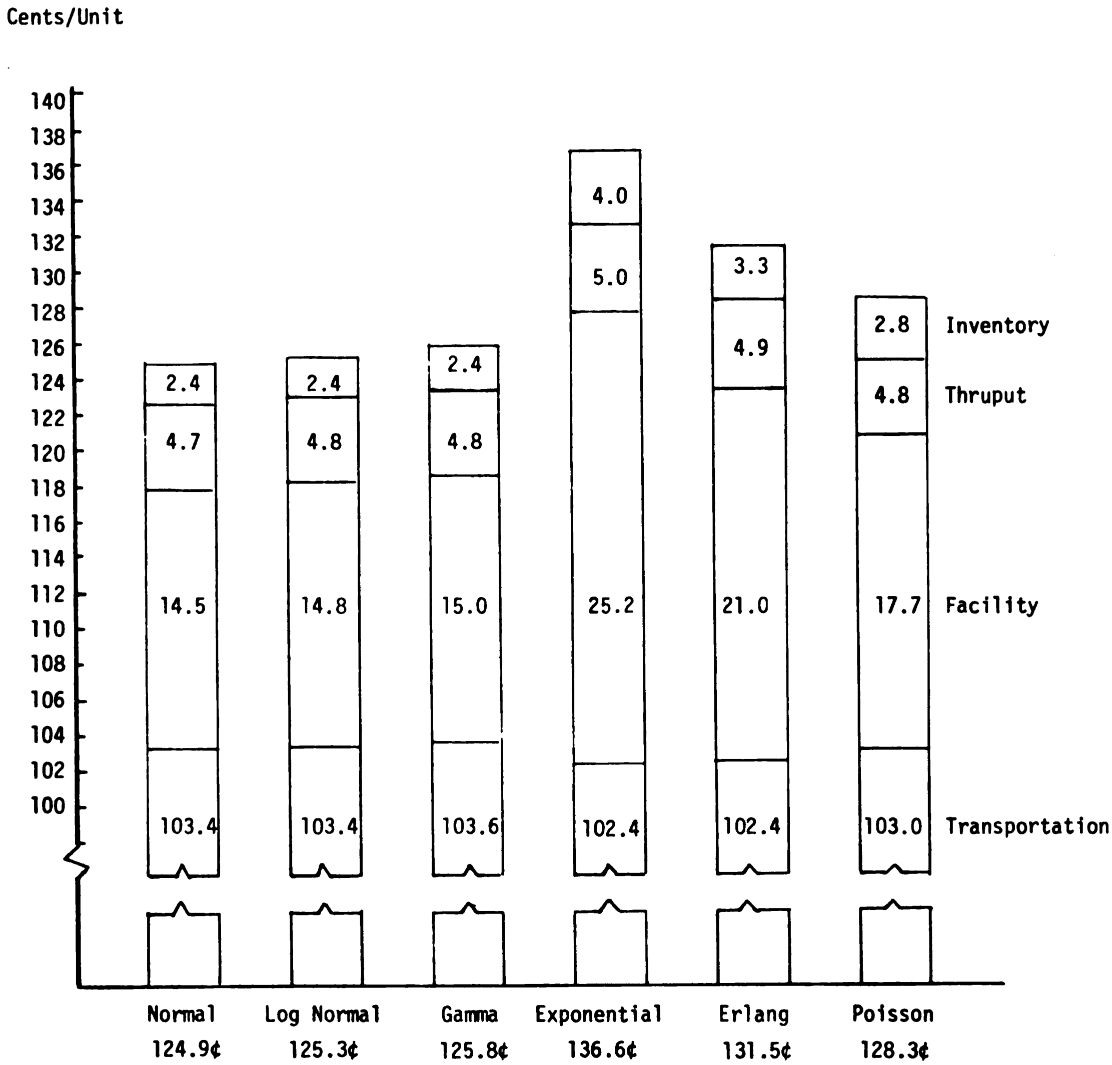


Figure 5-28. Probability Distribution Response: Total Cost (¢/Unit)

the t-value was 3.00. Thus all distributions have the same effect on transportation costs except this one situation.

Facility costs are shown as part of total costs in Figure 5-28. The same tests over the same distributions as previously described are used. The results show that three situations have significant t-values. The critical value is 2.776. Normal vs. exponential equals 3.33, log normal vs. exponential equals 2.99 and gamma vs. exponential equals 2.80. All other combinations do not show a significant difference.

Thruput costs are shown as part of total cost in Figure 5-28. Employing the same tests as in the other response variables, two combinations are significant. The critical value equals 2.776 and the value for normal vs. erlang is 3.27 and the value of normal vs. exponential is 3.11. No other combinations were found to be significant.

Inventory costs are shown as part of total cost in Figure 5-28. Using the same test as previously employed, three combinations are significant. The critical value is equal to 2.776. The t-value of normal vs. exponential is 3.26, log normal vs. exponential is 3.00 and gamma vs. exponential is 2.81. No other combinations are significant.

Conclusion.--Looking at the average distribution responses over all distributions over all response variables, several findings stand out. The comparison of the normal, gamma, and poisson for all combinations and all response variables, resulted in no significant differences. In the comparison of normal, log normal and gamma vs. erlang, only three significant differences are found: normal vs. erlang for transportation, normal vs. erlang for thruput costs and normal vs. erlang for demand stocked out. In the comparison between normal, log normal and gamma vs. exponential, eleven significant differences are found. Normal vs. exponential is significant for thruput, facility, inventory, total cost and demand stocked out. Log normal vs. exponential is significant for facility, inventory, total cost and demand stocked out. And, gamma vs. exponential is significant for facility, inventory and total cost.

Variance

Figure 5-29 indicates that the demand stocked out and costs which result from each level of lead time variance (C.V. of .18 and C.V. of .375) will be compared to one another.

Demand stocked out.--Figure 5-30 shows the demand satisfied in percent by the two coefficients of variation .18 and .375. A t-test is used to compare the two variations and they are significant. The critical value is 2.23 and the t value for the comparison is 4.10. Thus, the two variations have differing effects on the amount of demand stocked out (or satisfied).

Costs.--Figure 5-31 shows total cost and activity center costs (transportation, facility, thruput, inventory) in cents per unit for the two coefficients of variation.

In all cases, with the exception of inventory, the coefficient of variation of .18 has a different effect on the system than does the coefficient of variation at .375. The critical t value is 2.23 and the t values for the costs are: total cost, 3.41; transportation costs, 2.27; facility costs, 4.08, and thruput costs, 2.73.

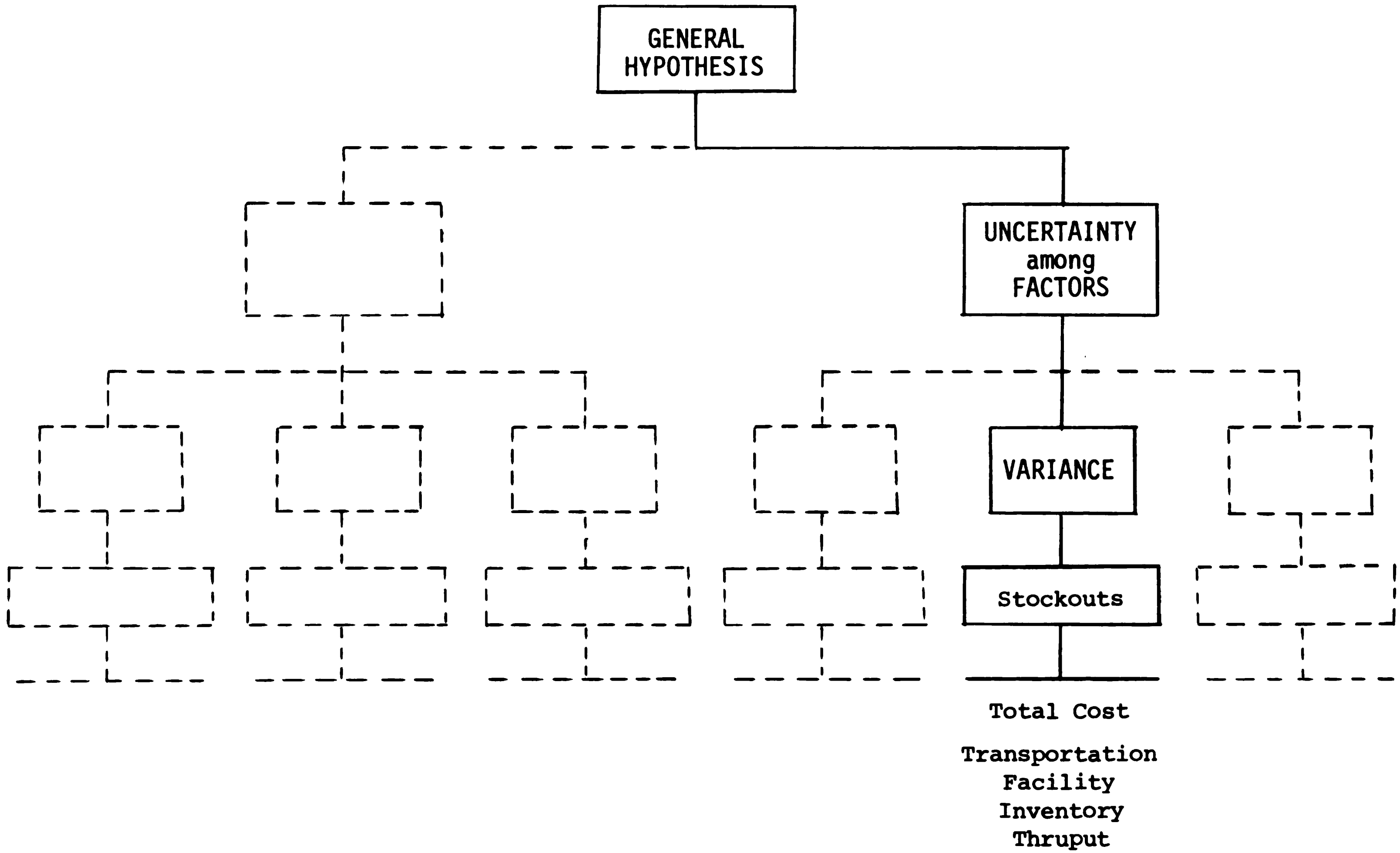
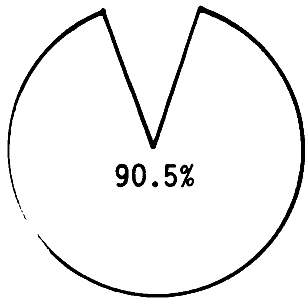
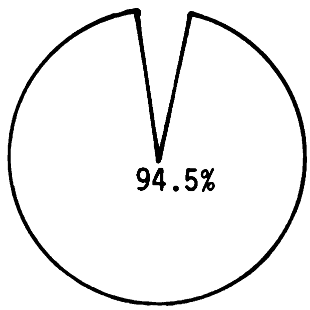


Figure 5-29. Research Analysis Organization: Among Factors/Variance.



C.V. = .375



C.V. = .18

Figure 5-30. Variance Response (Coefficient of Variation): Demand Satisfied (%).

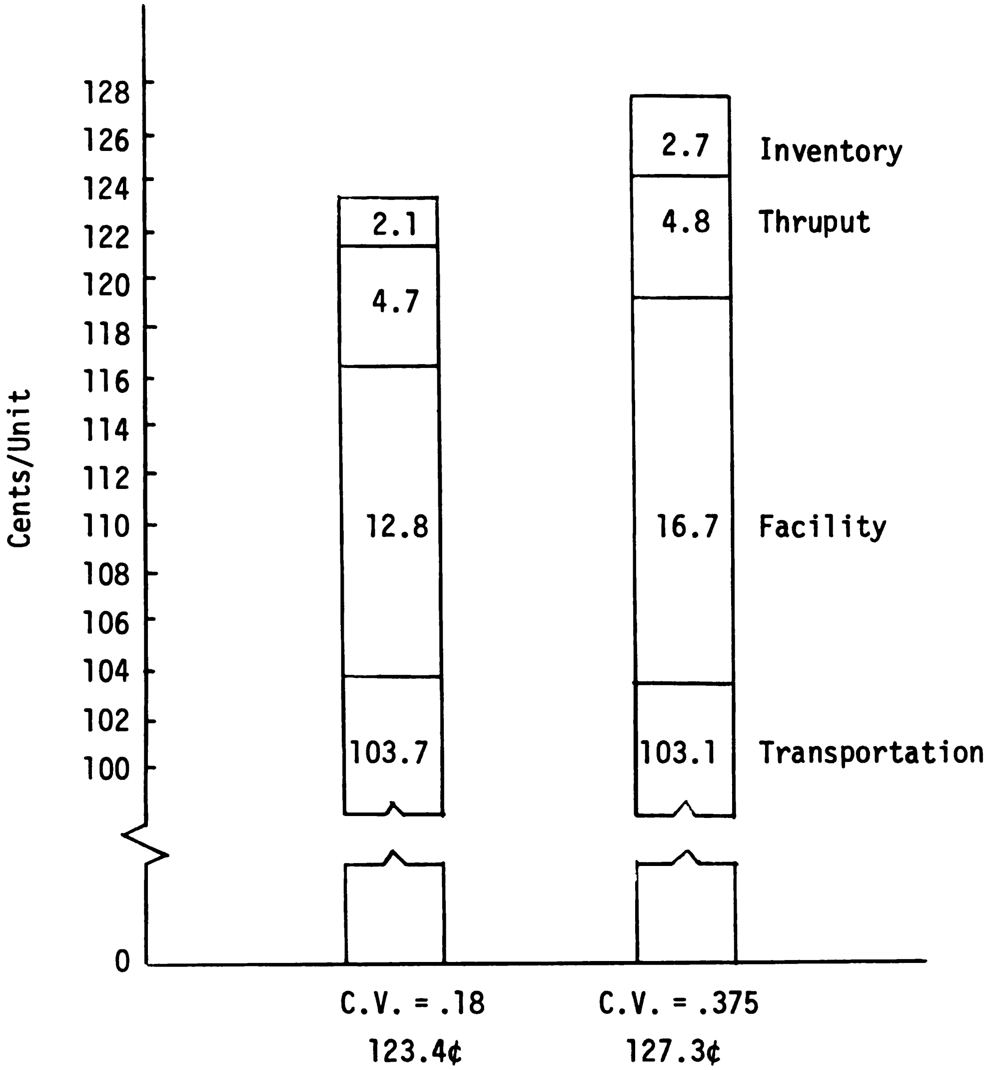


Figure 5-31. Variance Response (Coefficient of Variation): Total Cost (¢/Unit).

In summary then, the coefficient of variation of .18 and the coefficient of variation at .375 have different effects on all response variables except inventory. The figures used to obtain the average for each variance contained six observations, two levels (4 day and 7 day) and three distributions (normal, log normal and gamma).

Level of Lead Time (Average Duration)

Figure 5-32 indicates that the demand stocked out and costs which result from each average level of lead time (4 days and 7 days) will be compared to one another.

Demand stocked out.--Figure 5-33 shows the demand satisfied in percent for each average level of lead time. The results of the t-test to compare the two levels reveals that they are not different. Thus, both levels (4 days and 7 days) have the same effect on demand stocked out (demand satisfied).

Costs.--Figure 5-34 shows the total costs and the activity center costs (transportation, facility, thruput, inventory) in cents per unit for each average level of lead time. When using a t-test to compare levels for individual response variables, only total cost is significant. The critical t value is equal to 2.23 and the t value for level 1 vs. level 2 for total cost equals 2.48.

In summary, total cost is the only response variable for which the two levels have differing effects. The average figures for level are composed of six observations, two levels of the coefficient of variation (.18 and .375) and three distributions (normal, log normal and gamma).

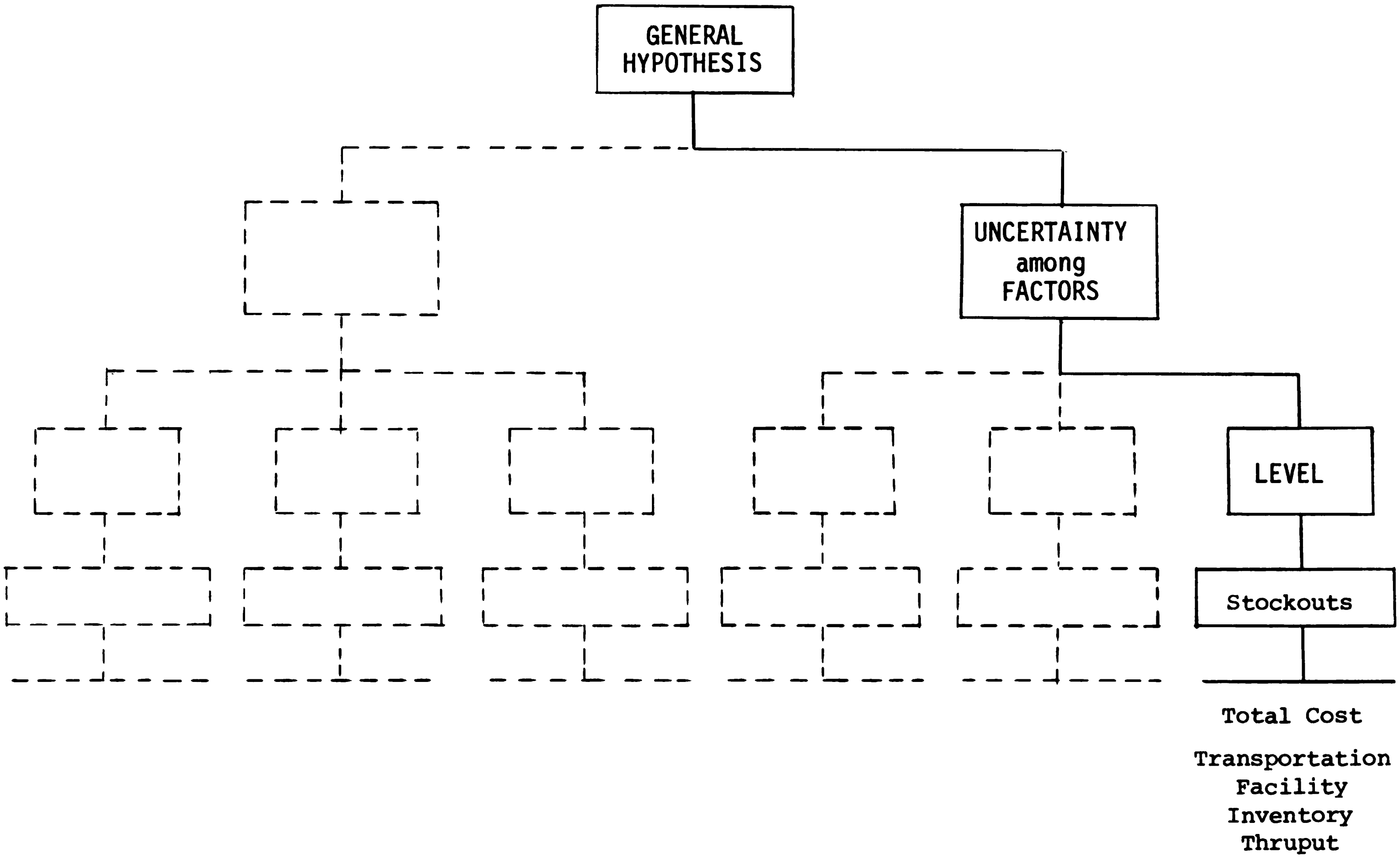
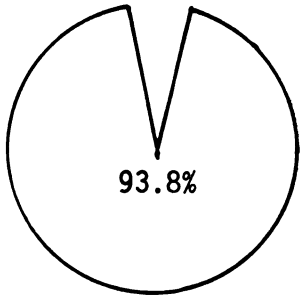
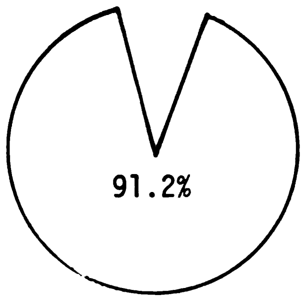


Figure 5-32. Research Analysis Organization: Among Factors/Level.



Level 1
4 Days



Level 2
7 Days

Figure 5-33. Lead Time Duration Response:
Demand Satisfied (%).

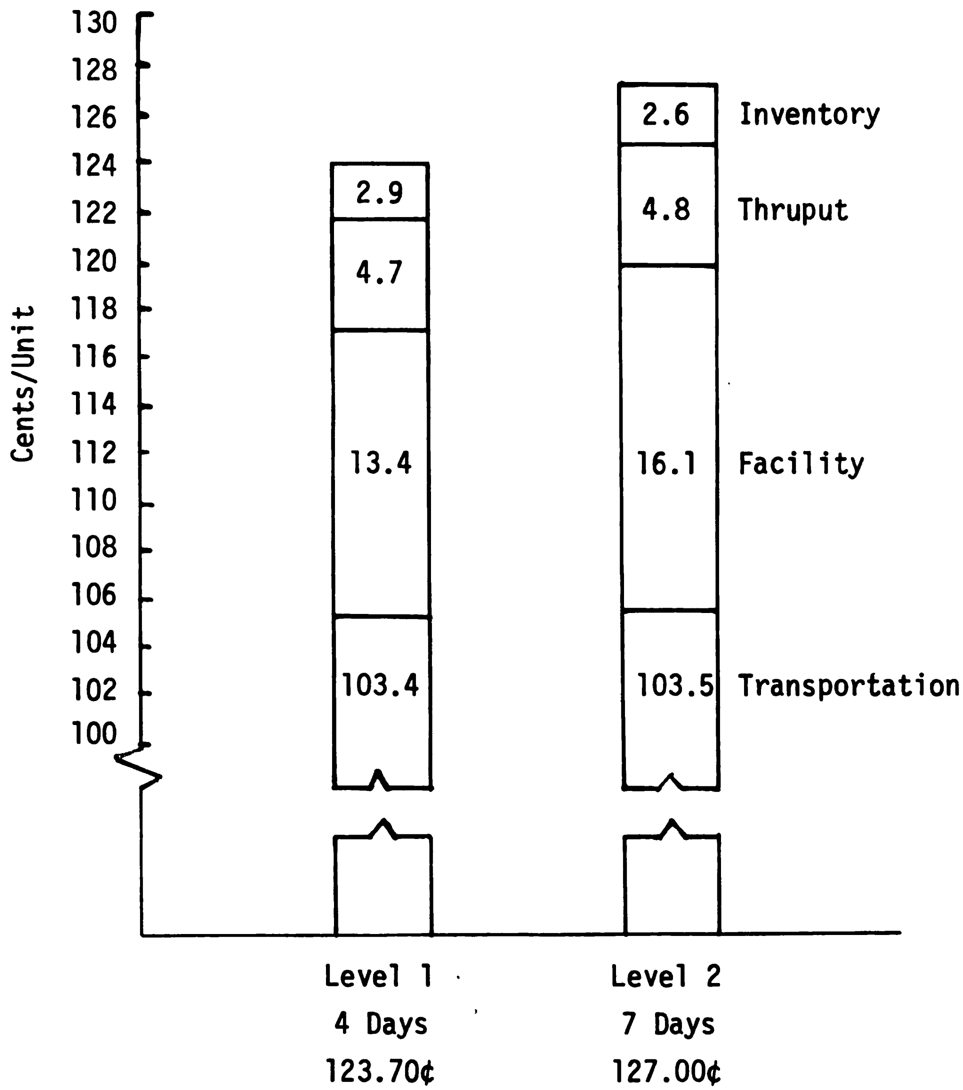


Figure 5-34. Lead Time Duration Response: Total Cost (¢/Unit).

Comparison Among Factor Responses: Individual Cell Comparisons

As in the case of factor responses vs. control in the previous section, it is necessary and informative to look at individual cell comparisons among factors. The analysis of variance using the F test and the comparison of levels of factors using the Tukey technique and the t-test both use average responses for the response variables. Such analysis may tend to obscure findings of importance. For this reason and for the fact that individual cell comparisons may reveal additional findings or tend to confirm present results, such an analysis is made.

The individual cell comparisons have no statistical foundation. The organization for this part of the section is by response variable.

Demand Stocked Out

Table 5-18 shows the individual cell comparisons for demand stocked out in percent. In all cases the seven day simulation runs stockout more than the four day runs. And, in all cases where two coefficients of variation are used, the larger variation stocks out more than the smaller variation. In the four day and seven day runs, the smallest stockout is normal at .18. The largest is the exponential distribution. From the smallest to largest stockout, the progression is normal at .18, log normal at .18, gamma at .18, log normal at .375, gamma at .375, normal at .375, poisson, erlang and exponential. The progression in the seven day runs is quite similar: normal at .18, log normal at .18, gamma at .18, normal at .375, log normal at .375, gamma at .375, poisson, erlang and exponential.

Table 5-18. Individual Cell Comparison: Percent Demand Stocked Out

Coefficient of Variation	Normal		Log Normal		Gamma		Exponential[a]		Erlang[b]		Poisson[c]	
	Level		Level		Level		Level		Level		Level	
	4	7	4	7	4	7	4	7	4	7	4	7
.18	3.97	6.25	4.16	6.98	4.73	7.01						
							13.23	22.09	10.94	15.54	10.02	10.85
.375	8.76	8.95	7.76	10.83	7.84	12.62						

[a] $\sigma = \mu$.

[b] $\sigma = .57$.

[c] $\sigma = \sqrt{\mu}$.

Cost/Revenue Ratio

Table 5-19 shows the individual cell comparisons for the ratio of cost to revenue in percent. In all cases the seven day runs cost more than the four day runs but the increases are quite small. In all cases where two coefficients of variation are used, the larger variation costs more than the smaller variation. In the four day runs, the least expensive treatment is normal at .18 and the most expensive is exponential. The progression from least expensive to most expensive in the four day runs is normal at .18, log normal at .18, gamma at .18, gamma at .375, log normal at .375, normal at .375, poisson, erlang and exponential. In the seven day runs the least expensive response is log normal at .18 and the most expensive is exponential. The progression from the least expensive to the most expensive in the seven day runs is log normal at .18, normal at .18, gamma at .18, normal at .375, poisson, log normal at .375 equal to gamma at .375, erlang and exponential.

Total Cost

Table 5-20 shows the individual cell comparisons for total cost in cents per unit. In all cases the seven day runs cost more than the four day runs, and in those cases where two coefficients of variation are used, the larger variation cost more than the smaller variation. In the four day runs the least costly response is normal at .18 and the most costly is exponential. The progression from the least expensive to the most expensive in the four day runs is: normal at .18, log normal at .18, gamma at .18, gamma at .375, log normal at .375, normal at .375, poisson, erlang and exponential. In the seven day runs the least

Table 5-19. Individual Cell Comparison: Cost/Revenue Ratio (Percent)

<table>
<tr><th rowspan="3">Coefficient of Variation</th><th colspan="2">Normal</th><th colspan="2">Log Normal</th><th colspan="2">Gamma</th><th colspan="2">Exponential[a]</th><th colspan="2">Erlang[b]</th><th colspan="2">Poisson[c]</th></tr>
<tr><th colspan="2">Level</th><th colspan="2">Level</th><th colspan="2">Level</th><th colspan="2">Level</th><th colspan="2">Level</th><th colspan="2">Level</th></tr>
<tr><th>4</th><th>7</th><th>4</th><th>7</th><th>4</th><th>7</th><th>4</th><th>7</th><th>4</th><th>7</th><th>4</th><th>7</th></tr>
<tr><td>.18</td><td>24.27</td><td>24.83</td><td>24.42</td><td>24.80</td><td>24.51</td><td>25.20</td><td rowspan="2">26.43</td><td rowspan="2">28.20</td><td rowspan="2">25.75</td><td rowspan="2">26.84</td><td rowspan="2">25.51</td><td rowspan="2">25.82</td></tr>
<tr><td>.375</td><td>25.24</td><td>25.62</td><td>25.04</td><td>25.98</td><td>24.92</td><td>25.98</td></tr>
</table>

Table 5-20. Individual Cell Comparison: Total Cost (¢/Unit)

<table>
<tr><th rowspan="3">Coefficient of Variation</th><th colspan="2">Normal</th><th colspan="2">Log Normal</th><th colspan="2">Gamma</th><th colspan="2">Exponential[a]</th><th colspan="2">Erlang[b]</th><th colspan="2">Poisson[c]</th></tr>
<tr><th colspan="2">Level</th><th colspan="2">Level</th><th colspan="2">Level</th><th colspan="2">Level</th><th colspan="2">Level</th><th colspan="2">Level</th></tr>
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<tr><td>.18</td><td>121.34</td><td>124.13</td><td>122.08</td><td>123.99</td><td>122.57</td><td>125.98</td><td rowspan="2">132.13</td><td rowspan="2">140.99</td><td rowspan="2">128.77</td><td rowspan="2">134.20</td><td rowspan="2">127.56</td><td rowspan="2">129.09</td></tr>
<tr><td>.375</td><td>126.21</td><td>128.10</td><td>125.20</td><td>129.90</td><td>124.58</td><td>129.92</td></tr>
</table>

[a] $\sigma = \mu$. [b] $\sigma = .57$. [c] $\sigma = \sqrt{\mu}$.

expensive response is log normal at .18 and the most expensive is exponential. The progression for the seven day runs from the least expensive to the most expensive is log normal at .18, normal at .18, gamma at .18, normal at .375, poisson, log normal at .375, gamma at .375, erlang and exponential.

Transportation Cost

Table 5-21 shows the individual cell comparisons for transportation costs in cents per unit. In the nine cases where costs can be compared from the four day to the seven day runs, the seven day runs are more expensive in four cases (normal at .18, normal at .375, log normal at .375, gamma at .18), and less expensive in five of the cases (log normal at .18, gamma at .375, exponential, erlang and poisson). In the six possible cases where the two coefficients of variation can be compared, the larger variation .375 is more expensive in three cases (normal, 4 day; normal, 7 day; and log normal, 7 day), and the smaller variation is more expensive in three cases (log normal, 4 day; gamma, 4 day; and gamma, 7 day). The least expensive response for the four day runs is gamma at .375 and the most expensive is gamma at .18. The progression from the least expensive to the most expensive in the four day runs is gamma at .375, erlang, log normal at .375, normal at .18, exponential, poisson, normal at .375, log normal at .18, gamma at .18. Note that the spread from the least expensive to the most expensive is very small. In the seven day runs the least expensive response is exponential and the most expensive is gamma at .18. The progression

Table 5-21. Individual Cell Comparison: Transportation Cost (¢/Unit)

<table>
<tr><th rowspan="3">Coefficient of Variation</th><th colspan="2">Normal</th><th colspan="2">Log Normal</th><th colspan="2">Gamma</th><th colspan="2">Exponential[a]</th><th colspan="2">Erlang[b]</th><th colspan="2">Poisson[c]</th></tr>
<tr><th colspan="2">Level</th><th colspan="2">Level</th><th colspan="2">Level</th><th colspan="2">Level</th><th colspan="2">Level</th><th colspan="2">Level</th></tr>
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<tr><td>.18</td><td>103.07</td><td>103.45</td><td>103.92</td><td>102.92</td><td>104.14</td><td>104.91</td><td rowspan="2">103.19</td><td rowspan="2">101.51</td><td rowspan="2">102.82</td><td rowspan="2">101.87</td><td rowspan="2">103.20</td><td rowspan="2">102.84</td></tr>
<tr><td>.375</td><td>103.32</td><td>103.63</td><td>103.05</td><td>103.57</td><td>102.79</td><td>102.48</td></tr>
</table>

[a] $\sigma = \mu$. [b] $\sigma = .57$. [c] $\sigma = \sqrt{\mu}$.

from the least costly to the most costly in the seven day runs is exponential, erlang, gamma at .375, poisson, log normal at .18, normal at .18, log normal at .375, normal at .375 and gamma at .18. Note again that the spread from the least costly to the most costly is very small.

Facility Cost

Table 5-22 shows the individual cell comparisons for facility costs in cents per unit. In all cases the seven day runs cost more than the four day runs and in all cases where two coefficients of variation are used, the larger variation is the most expensive. The least costly response in the four day runs is log normal at .18 and the most expensive is exponential. The progression from least expensive to most expensive in the four day runs is log normal at .18, normal at .18, gamma at .18, gamma at .375, log normal at .375, normal at .375, poisson, erlang, and exponential. The least costly response in the seven day runs is normal at .18, and the most costly, exponential. The progression from least expensive to most expensive in the seven day runs is: normal at .18, log normal at .18, gamma at .18, normal at .375, poisson, log normal at .375, gamma at .375, erlang and exponential.

Thruput Cost

Table 5-23 shows the individual cell comparison for thruput costs in cents per unit. In all cases with the exception of normal at .375 and poisson, the seven day runs are more costly. And in the cases where two coefficients of variation are used, the .375 variance is more

Table 5-22. Individual Cell Comparison: Facility Cost (¢/Unit)

<table>
<tr><th rowspan="3">Coefficient of Variation</th><th colspan="2">Normal</th><th colspan="2">Log Normal</th><th colspan="2">Gamma</th><th colspan="2">Exponential[a]</th><th colspan="2">Erlang[b]</th><th colspan="2">Poisson[c]</th></tr>
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<tr><td>.18</td><td>11.65</td><td>13.70</td><td>11.55</td><td>14.03</td><td>11.75</td><td>14.04</td><td rowspan="2">20.77</td><td rowspan="2">29.65</td><td rowspan="2">18.24</td><td rowspan="2">23.69</td><td rowspan="2">16.84</td><td rowspan="2">18.51</td></tr>
<tr><td>.375</td><td>15.61</td><td>16.95</td><td>14.96</td><td>18.56</td><td>14.65</td><td>19.50</td></tr>
</table>

Table 5-23. Individual Cell Comparison: Thruput Cost (¢/Unit)

<table>
<tr><th rowspan="3">Coefficient of Variation</th><th colspan="2">Normal</th><th colspan="2">Log Normal</th><th colspan="2">Gamma</th><th colspan="2">Exponential[a]</th><th colspan="2">Erlang[b]</th><th colspan="2">Poisson[c]</th></tr>
<tr><th colspan="2">Level</th><th colspan="2">Level</th><th colspan="2">Level</th><th colspan="2">Level</th><th colspan="2">Level</th><th colspan="2">Level</th></tr>
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<tr><td>.18</td><td>4.68</td><td>4.74</td><td>4.69</td><td>4.76</td><td>4.70</td><td>4.74</td><td rowspan="2">4.88</td><td rowspan="2">5.17</td><td rowspan="2">4.82</td><td rowspan="2">4.94</td><td rowspan="2">4.82</td><td rowspan="2">4.79</td></tr>
<tr><td>.375</td><td>4.76</td><td>4.75</td><td>4.77</td><td>4.81</td><td>4.76</td><td>4.84</td></tr>
</table>

[a] $\sigma = \mu$. [b] $\sigma = .57$. [c] $\sigma = \sqrt{\mu}$.

costly in all cases. In the four day runs normal at .18 is the least costly and exponential the most costly. The progression from least costly to most costly is normal at .18, log normal at .18, gamma at .18, normal at .375 tied with gamma at .375, log normal at .375, poisson tied with erlang and exponential. In the seven day runs the least costly response is normal at .18 and gamma at .18 and the most costly is the exponential. The progression from least costly to most costly is normal at .18 tied with gamma at .18, normal at .375, log normal at .18, poisson, log normal at .375, gamma at .375, erlang and exponential. Note that in the four day and seven day runs the spread from least costly to most costly is quite small.

Inventory Cost

Table 5-24 shows the individual cell comparison for inventory costs in cents per unit. In all cases the seven day runs are more costly than the four day runs and in all cases where two coefficients of variation are used, the larger variation (.375) is more costly than the smaller variation. The least costly and most costly responses in the four day runs are log normal at .18 and exponential. The progression from least costly to most costly in the four day runs is: log normal at .18, normal at .18, gamma at .18, gamma at .375, log normal at .375, normal at .375, poisson, erlang and exponential. In the seven day runs, the least costly and most costly responses are: normal at .18 and exponential, respectively. The progression from least costly to most costly is: normal at .18, gamma at .18, log normal at .18, normal at .375, log normal at .375, tied with poisson, gamma at .375, erlang and exponential.

Table 5-24. Individual Cell Comparison: Inventory Cost (¢/Unit)

Coefficient of Variation	Normal		Log Normal		Gamma		Exponential[a]		Erlang[b]		Poisson[c]	
	Level		Level		Level		Level		Level		Level	
	4	7	4	7	4	7	4	7	4	7	4	7
.18	1.94	2.24	1.93	2.29	1.96	2.28	3.30	4.65	2.90	3.71	2.69	2.96
.375	2.52	2.76	2.42	2.96	2.38	3.10	″	″	″	″	″	″

[a] $\sigma = \mu$.

[b] $\sigma = .57$.

[c] $\sigma = \sqrt{\mu}$.

Conclusion

In most cases, with the exception of transportation costs, the seven day runs are more expensive than the four day runs and the larger variance is more expensive than the smaller variance. Table 5-25 shows the progression from least expensive to most expensive responses by response variable.

Excluding transportation, the exponential is the most costly (most stockouts) in all cases and erlang is the second most costly in all cases. The least costly is, in most cases, normal at .18.

Summary of Findings

Considering all the experimental results, several findings stand out. All uncertain responses, with the exception of transportation costs, are higher than the control runs. The results of the uncertain responses are significantly different vs. the control runs for distributions, variances and levels. The seven day runs are generally more costly (more stockouts) than the four day runs and the runs with the larger coefficient of variation (.375) are more costly than the smaller coefficient of variation.

When looking at the factors within themselves, the variances and the lead time levels are significantly different for all response variables, thus indicating that the two variances have different effects on the system as did the two levels. Among the distributions, they generally have the same effects on the channel system. One

Table 5-25. Individual Cell Comparisons
(Least Costly, to Most Costly)

Demand Stocked Out	Total Cost/ Revenue Ratio	Total Cost	Transportation Cost	Facility Cost	Thruput Cost	Inventory Cost
4 Day						
Normal .18	Normal .18	Normal .18	Gamma .375	Log Normal .18	Normal .18	Log Normal .18
Log Normal .18	Log Normal .18	Log Normal .18	Erlang	Normal .18	Log Normal .18	Normal .18
Gamma .18	Gamma .18	Gamma .18	Log Normal .375	Gamma .18	Gamma .18	Gamma .18
Log Normal .375	Gamma .375	Gamma .375	Normal .18	Gamma .375	Normal .375	Gamma .375
Gamma .375	Log Normal .375	Log Normal .375	Exponential	Log Normal .375	Gamma .375	Log Normal .375
Normal .375	Normal .375	Normal .375	Poisson	Normal .375	Log Normal .375	Normal .375
Poisson	Poisson	Poisson	Normal .375	Poisson	Poisson	Poisson
Erlang	Erlang	Erlang	Log Normal .18	Erlang	Erlang	Erlang
Exponential	Exponential	Exponential	Gamma .18	Exponential	Exponential	Exponential
7 Day						
Normal .18	Log Normal .18	Log Normal .18	Exponential	Normal .18	Normal .18	Normal .18
Log Normal .18	Normal .18	Normal .18	Erlang	Log Normal .18	Gamma .18	Gamma .18
Gamma .18	Gamma .18	Gamma .18	Gamma .375	Gamma .18	Normal .375	Log Normal .18
Normal .375	Normal .375	Normal .375	Poisson	Normal .375	Log Normal .18	Normal .375
Log Normal .375	Poisson	Poisson	Log Normal .18	Poisson	Poisson	Log Normal .375
Gamma .375	Log Normal .375	Log Normal .375	Normal .18	Log Normal .375	Log Normal .375	Poisson
Poisson	Gamma .375	Gamma .375	Log Normal .375	Gamma .375	Gamma .375	Gamma .375
Erlang	Erlang	Erlang	Normal .375	Erlang	Erlang	Erlang
Exponential	Exponential	Exponential	Gamma .18	Exponential	Exponential	Exponential

exception is the exponential distribution which is significant in several cases. This observation is substantiated by the fact that in all cases, for both the four and seven day runs, exponential is the most costly.

CHAPTER VI

CONCLUSIONS

Introduction

This chapter brings together the hypotheses and the findings, relates conclusions to the present body of physical distribution knowledge and suggests areas for future research. In the first section, the hypotheses and findings are integrated and conclusions are reached regarding rejection or acceptance. As each hypothesis is rejected or accepted, the rationale for the behavior is given.

Next, implications regarding planning and operation of channel systems are discussed. Implications regarding the systems concept within the channel and modeling are also explored.

The last section looks into the limitations of the research and offers areas that should be researched in the future.

Integration of Findings and Hypotheses

Factors vs. Control

The first of two general hypotheses states that the presence of uncertainty will have a significant effect on the cost and service (demand stocked out) of a deterministic physical distribution system. The subhypothesis of the first general hypothesis concerns the effects of the factors of uncertainty (distributions, variances, and levels) on cost and service. In this first part of the integration of findings and hypotheses, each factor will be discussed in turn as to its effect on the total cost and demand stocked out in a deterministic system. As the effect of each factor on each response variable is discussed, the reasons for the observed behavior are given.

Distributions vs. control.--The first subhypothesis concerns the effect of distributions on stockouts. The hypothesis which states that stockouts for the uncertain systems when compared to the certain systems will be significantly different is affirmed. When stockouts for the normal, log normal and gamma distributions were compared to the control stockouts, they were found to be statistically significantly different. When exponential, erlang and poisson were compared to control, no statistically significant difference was found. However, there are reasons to believe that these are also significant.

The poisson, exponential and erlang all had stockouts that were above the normal, log normal and gamma. Because there were only two observations, thus one degree of freedom, the critical value was substantially inflated. In addition, lead time is basically the only stochastic variable in the simulation. For the above reasons it is felt that the results for the exponential, erlang and poisson could not have occurred by chance. Thus it is concluded that these distributions caused stockout responses that were different from the stockout response of the control system.

Stockouts due to distributions can be explained by the variability in lead time that was introduced. In the control system, lead time was constant. In the uncertain systems, lead time varied. Thus, the system could be confronted with a lead time over four or seven days (the lead time durations in the two control systems). When this occurs, the system stocks out. Even though a lead time above the mean can be cancelled by a lead time below the mean, the random presentation of times and the inventory ordering rule causes stockouts. The shorter lead times tend to increase inventory because demand over lead time is less than anticipated. The longer lead times create stockouts, which is in part caused by the economic order quantity reorder point system. If the ISP receives stock before anticipated, it simply waits longer before reordering. Thus it reorders on the premise that stock will be received in a designated period of time. When the lead time exceeds this duration the system stocks out.

The second subhypothesis concerns the effects that probability distribution patterns have on total cost. The hypothesis that total cost under various distribution patterns would be significantly different than the total cost in the deterministic system is affirmed. As shown in the findings, average total cost for the normal, log normal, and gamma distributions was statistically different in comparison to average total cost for the deterministic system. When the exponential, erlang and poisson distributions were compared to the control, the results were not statistically significant. However, based on the same reasoning as was discussed under stockouts in this section, it is concluded that the total cost due to these distributions was different from the stockouts in the control runs.

The uncertain systems' total cost was higher than control total cost for all distributions. All activity center costs were found to be higher for the uncertain systems with the exception of transportation which was found to be lower for all distributions. Thus, when every distribution was compared to control, it was found that total cost, thruput costs, facility costs and inventory costs were above their respective control responses, while transportation costs were found to be lower than control costs. Why this occurred is explained in this way. In the uncertain systems, inventory rose throughout the channel, particularly at the PSP and SSP levels. As a result, fewer partial shipments were made from the SSP to the ISP, thus transportation costs went down (see Table 6-1).

Table 6-1. Partial Shipments and Inventory for PSP, SSP, ISP

	Control		Exponential[a]	
	Partial Shipments (Thousands of Lbs.)	Inventory (Dollars)	Partial Shipments (Thousands of Lbs.)	Inventory (Dollars)
PSP	0	5,520	0	23,064
SSP	356	1,482	112	3,045
ISP	N/A	573	N/A	791

[a]The exponential was chosen for this example because it is the extreme case.

Inventory rose in the system under uncertain conditions for several reasons. When the lead time was constant, there were no stockouts, when lead time varied, stockouts occurred and only a portion of the demand made on the system was satisfied. Inventory rose at the PSP level because production was geared to meet average daily demand. Inventories rose at the SSP because demand over lead time was not as great as anticipated. The lead time from PSP to SSP was fixed at ten days. Under the economic order quantity system when the reorder point was reached at the SSP, an order was placed, designed to reach the SSP just before it ran out of stock. EOQ and ROP were computed on average daily demand (analogous to a certain condition). When the control system ran the anticipated conditions were realized. When the lead time was allowed to vary, there were some times when the demand coming from the ISP was below anticipated demand. Therefore, when orders were received from the PSP, SSP inventory went above the predetermined level and average inventory went up. Thus average inventory went up at the PSP and SSP levels primarily due to the stockout condition that was caused by the variable lead times.

Inventory rose at the ISP due to the skewed distributions where the frequency of occurrence of lead times at or below the mean are greater. This did not occur in all distributions, only in those which had long tails above the mean. Thus, the exponential which is a decay type function, increased inventory at the ISP the greatest.

It can therefore be concluded that the presence of patterns of distribution will increase stockouts and costs. The basic reason for this lies in the range of possible lead time durations.

Variance vs. control.--The third subhypothesis concerns the effects of the presence of variance on stockouts. The hypothesis which states that stockouts for the uncertain system due to the presence of variance will be different from the stockouts of the deterministic system is affirmed. It is difficult to conceive of variance without referring to a particular type of distribution. However, variance can be perceived as simply a deviation away from a point. In the control system there was no variance, thus all lead times were the same in duration. When variance is introduced, the possible number of lead time durations are increased. Instead of one lead time duration there are now several. Such a condition, if it can be seen devoid of a distribution, would cause stockouts to increase. As discussed under distributions, some of the lead times must be longer in duration than the fixed control system lead time. Thus, as soon as one lead time exceeds the control lead time, a stockout occurs.

The fourth subhypothesis concerns the effects of variance on total cost. The hypothesis which states that total cost for the uncertain systems due to the presence of variance will be different from the total cost of the certain system is affirmed. Viewing variance as we did in the previous subhypothesis, it can be seen that total cost goes up, facility, inventory and thruput costs go up, while decreasing transportation costs for the same reasons given for patterns of distribution. Thus it can be concluded that stockouts and total cost will go up as a result of the presence of variance. Again, the range of possible lead time durations is the base cause.

<u>Level of lead time (duration) vs. control</u>.--As described in the findings, an average was drawn over all experiments where uncertainty was present, which contained the same lead time. Therefore, two averages were obtained, one for the four day system and one for the seven day system. These averages were individually compared to the average of the control runs. By doing this, the fifth and sixth hypotheses, which concern the presence of fluctuating durations of lead time devoid of a pattern or variance, can be tested. The fifth hypothesis which states that the stockouts due to the presence of fluctuating, lead time duration will be different from the stockouts in the control system is affirmed. It was found that the stockouts due to lead time fluctuation were statistically significantly different from the control. This was true for both the four day and seven day lead times. The simple fluctuation, primarily above the mean, is the reason for this. As shown before, when a lead time duration, which is above the control lead time duration occurs, a stockout results.

The sixth subhypothesis which states that total cost as a result of lead time duration fluctuation will be different from the control total cost is affirmed. When the total costs were compared, it was found that the response due to the uncertain system was statistically significantly different from the four day and seven day cases. The total cost for the uncertain systems was above the control total cost. The rationale is the same for this case as it was for distributions and variances. As a result of stockouts, inventory goes up, transportation costs go down, but total cost goes up.

It can, therefore, be concluded that the presence of fluctuating lead times will cause significant changes in stockouts and total cost in comparison to the stockouts and total cost responses of a fixed lead time system. Again, the basic reason is the range of lead time durations.

Summary.--When comparing the factors (distribution patterns, variances and levels) to the control system it is concluded that the stockouts and total cost caused by the factors were statistically significantly different from the control system stockouts and total cost. Every factor had similar effects on stockouts and total cost. In every case, stockouts and total cost increased. The reason for this in-concert behavior is the range of lead times introduced into the system by the factors. In all cases the range of lead times caused stockouts, which in turn caused inventory and facility costs to increase. Therefore, transportation costs decreased but not enough to offset the increase in inventory and facility costs, therefore, total cost went up.

Comparison Among Factors

The second general hypothesis states that particular levels of a factor of uncertainty will have different effects on the response variables. For example, several distributions were used and the question asked is, "Will all the distributions have similar effects on the response variables or will the effects differ?" This question can be answered by comparing distributions among themselves. Within this section each type of uncertainty will be discussed, first distributions then variances and levels. As in the previous section, stockouts and total cost will be discussed separately and reasons for the behavior will be given.

Comparison of distributions.--The seventh subhypothesis concerns the relationship between particular pairs of distributions to determine if they had similar or different effects on the response variables. The general form of these hypotheses states that stockouts, as a result of one distribution, are different than the stockouts due to another distribution. Thus, several hypotheses are generated, one for each combination: normal vs. log normal; normal vs. gamma; normal vs. poisson; normal vs. exponential; normal vs. erlang; log normal vs. gamma; log normal vs. poisson; log normal vs. exponential; log normal vs. erlang; gamma vs. poisson; gamma vs. exponential; gamma vs. erlang; poisson vs. exponential; poisson vs. erlang; and erlang vs. exponential. Due to the possible number of specific hypotheses, conclusions regarding each combination will be brought together.

Statistically, the only distribution combinations that had differing effects on stockouts were exponential vs. normal, and exponential vs. log normal. However, if we were to stop here our conclusions would be incomplete. There is a direct relationship between the skewness, thus the range, of the distributions and the amount of stockouts. Thus, the normal distribution caused the fewest stockouts, and the exponential stocked out the most demand. As the range of the possible lead time durations increases, the number of days an ISP can be out of stock increases. For example, when the range of possible values goes from one to seven days, as in the case of the normal distribution at a mean of four and a coefficient of variation of .18, the maximum number of days a system can be out of stock is three days. (The ISP orders with four days of stock remaining.) However, as the range of values increases (i.e., up to fifteen days), the maximum number of days that an ISP can be out of stock increases (i.e., eleven days), thus stockouts go up. It can be concluded that as the distribution pattern changes the number of stockouts increases directly with the range of the distribution under consideration.

The eighth subhypothesis states that the total cost caused by one particular distribution will be different than the total cost caused by another distribution. As in the case of stockouts, there are many pairs of distributions, each of which forms a specific hypothesis. Again, amassing hypotheses and findings, it is concluded that generally these hypotheses must be statistically rejected. Of all the possible combinations, the exponential vs. the normal, and exponential vs. gamma, and exponential vs. the log normal were found to be significantly different. However, as in the case of stockouts, if we stopped here the analyses and conclusions would be incomplete.

As the symmetry and the skewness of the distribution patterns changed, their effects on the system changed, and the places in the system where the effects took place changed. In all cases, as the distribution patterns became more skewed, all costs went up in direct relationship with the exception of transportation costs which went down. Transportation costs are inversely related to the other costs. In addition, in those systems which cost the most, the transportation costs were the lowest.

Basically, the rationale for this behavior is the same as in the previous section. As stockouts go up, inventory goes up because demand over lead time is less than expected. Inventory goes up, partial shipments go down, thus lowering transportation costs. However, as the type of distribution changes, the point in the system where the effect takes place changes and the degree of effect is different.

For all response variables the normal distribution cost the least with the exception of transportation which cost the most and the exponential cost the most with the exception of transportation which cost the least. The normal, log normal and gamma distributions had similar effects. However, even among these distributions there were recognizable increases in cost. These distributions are relatively symmetrical and clustered about the mean, thus values above the mean are cancelled by values below the mean and the possible range of values is limited. Stockouts were relatively small, thus inventory did not get a chance to build. Thus, partial shipments could not go down drastically. The major effect was found at the PSP and SSP levels. In fact, inventory at the ISP levels for these distributions displayed no discernible differences, while inventory at the PSP and SSP levels built. On the other end of the continuum is the exponential and erlang distributions.

The exponential distribution is a decay function and the erlang is drastically skewed. These distributions had similar effects on the system, however, the exponential was consistently more deleterious. In addition to causing stockouts at the ISP level, thus causing the growth of stocks at the PSP and SSP levels and lower freight rates, the exponential and erlang distributions caused inventory at the ISP level to go up. Because the frequency of occurrences of lead time durations at and below the mean are greater than those above the mean, the ISP was experiencing a rapid lead time in some cases. Thus, anticipated demand over lead time was down and inventories rose. In addition, as the range of possible lead times increases (i.e., the exponential distribution with a mean of 4 had several lead times above 15), the probability of an extensive stockout condition exists, thus increasing inventory and decreasing transportation.

It is therefore concluded that different distributions have different effects on the stockouts and total cost. The basic reason is the skewness of the distributions. In addition, the system reacts differently as the skewness increases.

Comparison of variances.--The ninth subhypothesis concerns the two variances in comparison to one another. It states that the effect on stockouts due to one variance will be different than the effects due to the other variance. This hypothesis is affirmed. In those cases where two coefficients of variation were possible, stockouts due to the smaller variation (.18) were statistically significantly different from the stockouts due to the larger variation (.375). The stockouts increased as the coefficient of variation increased. The reason for this can be explained by looking at the range and probability of specific lead times. As variation increases, the range of lead time duration increases and as variation increases, the probability of getting a lead time further away from the mean increases. Thus, stockouts would increase.

The tenth subhypothesis concerns the effect of variance on total cost. It states that the total cost due to the coefficient of variation at .18 would be different than the total cost due to the coefficient of variation at .375. This hypothesis is affirmed. It was found that in those cases where two coefficients of variation were possible, that total cost due to the coefficient of variation at .18 was statistically significantly different than the total cost due to the coefficient of variation at .375. Total cost for the .375 coefficient of variation was greater than total cost for the .18 coefficient of variation. This condition was also true for all activity center costs with the exception of transportation where the larger variation had a lower cost. Such behavior can be explained. As the coefficient of variation increases, two things occur. First, the range of the distribution increases and, secondly, the probability of getting values further away from the mean increases. Thus, the probability that stockouts will increase goes up and the resultant behavior due to increased stockouts on total cost occurs. As stockouts go up, inventory goes up and partial shipments go down. Thus it can be concluded that different variances do cause different effects and as the variance goes up (i.e., from .18 to .375), the total cost goes up.

Once again, the range of lead time durations is the basic cause. Variance is a proxy for consistency and these conclusions tend to support the general belief that consistency in lead times is extremely important to the efficient and effective operation of a distribution system. The importance of variance in comparison to the other two uncertain factors (distributions and lead time durations) is more clearly seen when individual cell comparisons are made in the following section.

Comparison of levels (duration of lead time).--The eleventh subhypothesis concerns the effect of levels of lead time on stockouts. It states that stockouts as a result of a four day lead time will be different than the stockouts that result from a seven day lead time. This hypothesis is affirmed. It was found that stockouts due to the shorter lead time (4 days) were statistically significantly different from the stockouts due to the longer lead time. The seven day lead time stocked out more than the four day lead time. This conclusion seems reasonable because of the increased range of the possible lead time durations.

The twelfth and final subhypothesis concerns the effect of lead times on total cost. The hypothesis states that the total cost due to a lead time of four days would be different than the total cost due to a seven day lead time. This hypothesis is affirmed. It was found that the total cost that resulted from a four day lead time was statistically significantly different than the total cost generated by the seven day lead time. Total cost for the seven day lead time was above total cost for the four day lead time. Activity center costs, with the exception of transportation, were higher for the seven day lead time. The cost of transportation for the seven day lead time was lower than the cost of the four day lead time. These conclusions seem logical considering what we have found so far and relate directly to the range of the lead times. When comparing a four day lead time with a seven day lead time that has the same pattern and variance, the range of the possible lead time duration increases. Thus, if a four day average lead time has a maximum of seven days, the system can be out of stock for a period of three days. However, a seven day lead time would have a greater range, thus the number of days stocked out would be greater. As has been shown, there is a direct relationship between the number of stockouts and total cost. As stockouts go up, costs go up and as we go from a four to seven day lead time, stockouts go up.

Individual Cell Comparison

To complete the interpretation of findings and hypotheses it is useful to look at individual cell comparisons. Realistically, a channel of distribution is not confronted with just a distribution or just a level, it is confronted with uncertainty, that is a combination of distribution pattern, variance and duration of lead time.

When looking at the individual cells, it was found that as the distribution became more skewed, and the variance and average lead time increased, the stockouts and total cost increased. This observation is confirmed by our findings. As shown in Table 6-2, the four day lead time distributions with small variances consistently stocked out the

Table 6-2. Stockouts and Total Cost Ranked from Fewest Stockouts (Least Costly) to Most Stockouts (Most Costly)

	Stockouts			Comparison of Position			Total Cost		
Distribution	Variance	Lead Time	Percent Stocked Out	Stock-out	Total Cost	Distribution	Variance	Lead Time	Total Cost in ¢/Unit
Normal	.18	4	3.97	1	1	Normal	.18	4	121.34
Log normal	.18	4	4.16	2	2	Log normal	.18	4	122.08
Gamma	.18	4	4.73	3	3	Gamma	.18	4	122.54
Normal	.18	7	6.25	4	5	Log normal	.18	7	123.99
Log normal	.18	7	6.98	5	4	Normal	.18	7	124.13
Gamma	.18	7	7.01	6	8	Gamma	.375	4	125.20
Log normal	.375	4	7.76	7	7	Log normal	.375	4	125.58
Gamma	.375	4	7.84	8	6	Gamma	.18	7	125.98
Normal	.375	4	8.76	9	9	Normal	.375	4	126.21
Normal	.375	7	8.95	10	11	Poisson	.50	4	127.56
Poisson	.50	4	10.02	11	10	Normal	.375	7	128.10
Log normal	.375	7	10.83	12	14	Erlang	.57	4	128.77
Poisson	.375	7	10.85	13	13	Poisson	.375	7	129.09
Erlang	.57	4	10.94	14	12	Log normal	.375	7	129.90
Gamma	.375	7	12.62	15	15	Gamma	.375	7	129.92
Exponential	1.00	4	13.23	16	16	Exponential	1.00	4	132.13
Erlang	.57	7	15.54	17	17	Erlang	.57	7	134.20
Exponential	1.00	7	22.09	18	18	Exponential	1.00	7	140.99

least and were the least expensive. While the seven day exponential distribution is most expensive and has the most stockouts. The progression from low to high cost and low to high stockouts is consistent. Every seven day system costs more (stocks out more) than every comparable four day system; every system with a coefficient of variation of .375 cost more (stocks out more) than a comparable system with a variance of .18. And as the range and skewness of distributions increases, the costs (stockouts) go up.

The direct relationship between stockouts and total cost can also be seen in Table 6-2. With few exceptions as stockouts go up, total cost goes up.

We have firmly established that as the range and skewness of a distribution pattern increase, stockouts and total cost increase; as the variance increases, stockouts and total cost increase; and as the duration of lead time increases, stockouts and total cost increase.

Looking at individual cells, comparisons can be made between experiments that have different combinations of uncertainty. The question that is asked regards the neutralization of one type of uncertainty by another type of uncertainty. For instance, the four day exponential stocks out less and costs less than the seven day erlang. Thus, even though the exponential has a greater coefficient of variation the stockouts and costs can be reduced by speed. Looking at Table 6-2, there are only a few examples of this type of behavior.

For stockouts, all smaller variance systems (at both levels) stock out less than the larger variances. Thus, speed cannot overcome variance. Considering total cost, there are only two examples of where speed overcame variance. Gamma with a variance of .375 and a lead time of four days and log normal with a variance of .375 and a lead time of four days were less expensive than gamma with a variance of .18 and a lead time of seven days. However, this is an exception rather than the rule. As shown in Table 6-2, the first few least costly systems had variances of .18. Thus it can be concluded that variance has a stronger effect than does average lead time. This was confirmed by the F value in the analysis of variance. Thus, the contention that consistency is more important than speed is generally supported.

The factor which had the least power was the pattern of distribution. In the analysis of variance it was shown that effects on stockouts and total cost as a result of distributions was not significant. When individual comparisons were made, only the exponential distribution displayed significant differences. Thus, it can be concluded that of all the factors, distribution effects on response variables are the weakest.

Implications of the Research for Channel Planning, Operation and Control

The conclusions drawn in the previous section show that the presence of uncertainty does have an effect on the efficiency and effectiveness of a physical channel system. In addition, it was shown that different levels of a factor of uncertainty had different effects. The conclusions thus reached will have effects on the planning, operation and control of a physical distribution system. These implications are delineated in this section.

1. As the type of uncertainty changes, the efficiency and effectiveness of the system changes. Thus, it is imperative to determine empirically the type of uncertainty that is confronting the system. If a particular type of distribution, variance and level is assumed when planning a system and in actuality the uncertainty is different, the results will be incorrect. If various conditions of uncertainty had similar effects, empirical knowledge of the conditions would be unnecessary. For instance, the normal, log normal and gamma distributions had similar effects, thus one could be used in all cases. However, even in this example, one could not guess at the variation or the duration of lead time. Therefore, empirical validation would still be necessary. The task of discovering the pattern, variance and level of lead time is difficult at best. The task must be undertaken, however, if accurate planning and control is desired.

2. It has also been shown that certain types of uncertainty result in a more or less effective and efficient system. For instance, a normal distribution with a four day lead time and a coefficient of variation of .18 was much more efficient and effective than the exponential with a seven day lead time. With the knowledge of the relative efficiency and effectiveness of certain conditions, one is put in a position of increasing overall performance by moving from a particular type of uncertainty to another. An example will help explain the point. A system is confronted with an exponential lead time. Even if it

achieves the minimum cost conditions, it is not in the best possible position. By changing the lead time distribution from exponential to normal and maintaining an optimum system, efficiency and effectiveness increases. Two facts are necessary for such a conclusion. One, the present condition must be known and, two, the conditions which increase efficiency and effectiveness must be known.

3. It seems reasonable to assume that conditions of uncertainty may change over time. For instance, the duration may change, the dispersion around the mean may change or the pattern itself may change. With knowledge of how a system reacts to particular types of uncertainty it is possible to prepare for such contingencies. The sources of a change could be many and varied. As a result of the conclusions of this research, it was shown how the efficiency and effectiveness of a system changes as the lead time configuration changes. Thus, preparations could be made to change the system or possibly use a different system when conditions change.

4. As would be anticipated, the implications due to effects of uncertainty go beyond physical distribution. A firm, through its own actions, may disrupt and render inefficient its own distribution system. Actions taken by the firm such as changing carriers, decision rules, or order cycle procedures could change the conditions of uncertainty in lead time. As the conditions change, the responses change and the system could become less efficient and effective.

5. The point just discussed once more reinforces the systems concept. It is not enough to optimize one activity or one group in a

firm because the action in one may have adverse effects on another which result in a worse overall condition for the firm. Not only is the systems concept important at the firm level, but it is important at the channel level. It was shown that changes in inventory and stockouts occurred at different points in the system. Thus, to optimize the physical distribution of goods, cooperation among channel members is essential.

6. The research results also point to which type of uncertainty causes the greatest effect. Therefore, it indicates which type should be attacked first to increase efficiency and effectiveness. Variance which is representative of consistency should be the first uncertain factor considered. A reduction in the variance gives the greatest increase in efficiency and effectiveness. The least effective change comes through distribution pattern, as long as a highly skewed distribution is not present. The move from a normal to a log normal results in little change. However, moving away from an exponential or erlang type distribution would have significant impacts. As a move is made away from the exponential or erlang, both the pattern and the variance change. The skewness and coefficient of variation are reduced.

7. As shown in the conclusions, there is a direct relationship between stockouts and cost. Thus, as the stockouts decrease, cost decreases, therefore the system increases in effectiveness and efficiency simultaneously. Any effort put forth to seek a more efficient and effective system will be rewarded with decreases in stockouts and cost. Because of the double change in system performance, the returns gained will probably exceed the cost to create the change.

8. All previous implications assume that the uncertainty that is present in lead times can be altered. Very little work has been done to empirically determine the pattern, variance and level of lead time. The pattern and the variance of lead time could possibly be changed through carrier cooperation. Therefore, once again, the systems concept is reconfirmed. Levels can be changed by ordering decisions rules, however, total cost trade-offs throughout the system must be considered before such a move is made.

9. Lastly, the importance of the carrier in a channel of distribution is emphasized. Traditionally, the carrier has not been viewed as a channel member and little attention given to his role or impact. The results of this research conclude, without doubt, that the carrier has a significant impact on channel operations. Thus, not only must the channel members cooperate, they must view the carrier as an integral part of the channel whose impact can have significant effects on the system.

Limitations of the Research

Simulation studies are constrained to the extent that the simulation model accurately replicates the real world system. The present research is not free of that constraint. However, the LREPS model has been subjected to extensive testing and has been judged to be valid.

The model used in this research has been stripped of many important features. Demand is fixed and constant, there are no behavioral dimensions to channel interrelationships, location is given, there are no backorders at the ISP level, etc. To the extent that these features would change findings, the study is constrained.

The findings of the research are also limited by the lack of replication of experimental runs. In some cases, only one degree of freedom was available for difference between means tests. Hence, the possibility of falsely rejecting a true null hypothesis is quite real. Additionally, more replications would have allowed for better estimates of sampling error, and the ability to make cell by cell comparisons. Because of the limited experimental runs, nonstatistical comparisons of individual cells had to be made.

A further constraint upon the research is that various policies relative to channel operation could not be tested. Thus, an EOQ method of inventory control was employed, and to the extent the results would differ under different policies the research results are limited.

Future Research

Due to the nature of the present research, the course of future research has basically been charted. It was intended that this research be a foundation for work to come. Thus, variables which were purposefully omitted must be replaced in the model until the model is as close a replicate of the real world as possible. As this process of adding variables is carried out, questions that arise after each stage must be answered. Answering such questions represent research areas which will complement the adding process and more fully explain the distribution channel system. As was anticipated, the present research, which represents the lowest level of complexity, generated areas of research outside of the adding of variables to the model.

First, there is validation of the present conclusions. Additional replications of particular runs should be made to increase the validity of the present conclusions. In a sense, such a procedure could be seen as broadening the foundation and making it more secure.

It has been shown that operating policies affect the results that uncertainty has on the system. Thus, different inventory decisions could be tried to discover the resultant effects. Analogous to this recommendation is the entire question of system structure and functional relationships. There is nothing preordained about the traditional structure that is generally accepted today. It is felt that a combination of research done on shuffling functions and adding back variables may suggest a more efficient and effective functional relationship in the channel system.

Another path that must be followed in the future concerns the specificity of the model. This research attempted to make generalizations. Generalizations are seldom useful for operational decisions. Thus the peculiarities of specific industries and products must be considered. Such refinements will increase the probability of solving problems in specific instances.

Research into the empirical determination of lead time uncertainty must also be done. This is probably the single most important area that could result in increases in efficiency or effectiveness immediately. This research has shown that the effects of variance are disastrous. Thus, any knowledge of the variance of a specific lead time and any attempt to reduce such a variance would be fruitful.

Finally, the measurement of the joint effect of the principle uncertainties affecting a channel system--demand and lead time, would be the most logical next step in the continuation of research in this area. Additionally, the impact of a backorder system at the ISP level provides another fruitful area for investigation.

To provide a tentative indication as to the type of results that might occur under the situation of uncertain demand and lead times, with and without backorders, this research effort was coupled with that relating to demand and a number of joint experimental runs were made. It is hoped that the results of these runs will provide some indication as to the direction future research in this area should take. The results are reported in the following postscript chapter.

CHAPTER VII

A POSTSCRIPT: AN EXPLORATORY INVESTIGATION INTO THE EFFECTS OF VARIABLE LEAD TIME AND DEMAND

Introduction

Purpose

This chapter is designed to go beyond the scope of the present research and provide a more definitive statement on the course of future research. The chapter displays those elements of the model (omitted in the present research) that should be introduced in future research efforts. While this research was being completed, simultaneous research was being conducted where lead time was held constant and demand was allowed to vary.[1] The identical model was used with identical structure and operation and decision rules. The primary difference was in which type of uncertainty was held constant and which type was allowed to vary.

One of the purposes of the present research was to construct a foundation upon which future research could be conducted. Closely allied with that purpose was to suggest future research in which variables could be added to the model so that more complete information could be gained as these variables were systematically added. As indicated in the conclusions of this research, one of the most important variables to add would be variable demand.

It is realized that the addition of variable demand opens the door to research that cannot be accomplished with a few runs or a simple cursory look. It is not the purpose of this chapter to make a definite statement regarding the interaction of variable demand and lead time. Without at least one combination run, however, it would be very difficult to hypothesize the type of behavior that would result by combining demand and lead time; thus, it would be impossible to clearly indicate the most fruitful path for future research. With a few combination simulation runs the above question can be more easily answered.

Scope

To accomplish the above purpose, it was necessary to bring together in one simulation run variable lead time and variable demand. Decisions regarding the particular distributions used, modifications to the model, if any, and the number of simulation runs had to be reached. It is the purpose of this section to indicate and justify these decisions.

In the research with variable lead time, six different lead time distributions were run with two levels of variance and two lead time durations. In the research where demand was allowed to vary, six distributions were used, three levels of variance and two demand levels. Of all the possible combinations, the following combination runs were used.

Run 1. Gamma lead time with coefficient of variation = .375 and lead time = 7 days and

Gamma demand with coefficient of variation = .50 and daily demand = 75 units.

Run 2. Exponential lead time at 7 days and

Gamma demand with coefficient of variation = .50 and daily demand = 75 units.

Run 3. Exponential lead time at 7 days and

Normal demand with coefficient of variation = .50 and daily demand = 75 units.

More than one simulation was run to assure that the results were not atypical. Runs were chosen which had created differences when observed alone. Runs were also selected that represented two different points on a continuum from little effect to the greatest effect. Furthermore, runs were chosen which seemed to closely represent reality. The exponential distribution is well suited for lead time and the gamma and normal are well suited for demand.

No modifications were made to the simulation model when these runs were made. Thus, the same structure, operating procedure and decision rules applied. Any shift from the original conditions would have caused doubt as to the reasons for the behavior seen.

In addition to the three runs outlined above, one additional run was made with normal demand and exponential lead time. In the second run, stockouts at the ISP were filled which was not the case in any previous runs. The purpose of making such a run parallels the reason for making combination runs. One more piece of reality is added into the model, more specific future research areas can be offered and the addition of backorders exemplifies the procedure which should be used in future research. Adding a backorder provision at the ISP displays that lost sales can be captured at an additional cost. An analysis of the cost to capture these backorders was desired and an indication of the effects on the system in general, as a result of backorders at the ISP, was desired.

Thus, decisions regarding the backorders had to be made. With backordering at the ISP, the system would run the same as it did without backorders, except for the following modifications:

1. Demand that could not be satisfied from stock was recorded at the ISP and held for delivery when stock was available.

2. The order decision rule of EOQ was maintained and one modification made. When inventory on hand and on order drops below the reorder point, an additional order is placed. The demand recorded at the ISP depletes this total. Thus, if on a particular day an ISP has no stock on hand and receives an order for 75 units, these units are removed from the on hand-on order total. If as a result of that demand, the on hand-on order drops below the reorder point, an additional order would be made. Thus it would be possible to have more than one order in process at the same time.

3. To maintain a backorder system, additional costs are incurred. Additional costs were accounted for in the following ways:

    a. Order processing costs doubled from $5.00/order to $10.00/order.

    b. Handling at the SSP increased from $2.00/pallet to $4.00/pallet.

    c. All backorders moved at the same partial shipment rate of $4.53/cwt.

    d. A per unit charge of 10 cents was included to cover the cost at the ISP of recording the order and performing tasks generated by the backorder.

Although the backorder scheme presented above is only one of many that could be considered, it was felt that this scheme is representative.

Obviously, many changes could be made predicated on many different objectives. The problem of such a decision is representative of the type of problems that will confront future researchers in this area.

Research Questions

These simulation runs are exploratory in nature for they were made to enable a more definitive statement on future research and display the type of factor adding that should be done. The limited number of runs allows no statistical inferences as to the actual behavior of the systems running together. Therefore, a statement of hypotheses would be improper. More realistically, it can only be indicated as to the type of behavior that is anticipated.

For those runs where there are no backorders at the ISP, it was anticipated that the introduction of variable demand would simply compound the results with only variable lead time. Thus, total cost would go up and stockouts would increase.

Where backorders are allowed at the ISP, two conditions were anticipated. The costs would increase and demand satisfied would increase because of the model design. However, the amount of increased cost was basically unknown and the effects on the system simply due to the presence of a backorder rule at the ISP were unknown.

Experimental Findings

Table 7-1 presents the major output responses for the three experimental runs described earlier in this chapter. Additionally, the output responses for the situation where demand is fixed (and lead time

Table 7-1. Experimental Results with Variable Demand and Variable Lead Time

Response Variables	Variable[a] Demand and Lead Time Demand: Gamma (75, .5) Lead Time: Gamma (7, .375) No Backorder at ISP	Variable Demand Fixed Lead Time Demand: Gamma (75, .5)	Variable Lead Time Fixed Demand Lead Time: Gamma (7, .375)	Variable Demand and Lead Time Demand: Normal (75, .5) Lead Time: Exponential (7) No Backorder at ISP	Variable Demand and Lead Time Demand: Normal (75, .5) Lead Time: Exponential (7) Backorder at ISP	Variable Demand Fixed Lead Time Demand: Normal (75, .5)	Variable Lead Time Fixed Demand Lead Time: Exponential (7)
Percent of demand stocked out	13.66	.98	12.62	22.35	21.00[b]	5.64	22.10
Total cost (¢/unit)	129.00	124.90	130.00	137.00	127.13	124.60	141.00
Transportation (¢/unit)	103.00	108.00	102.50	104.50	106.00	111.00	101.50
Facility (¢/unit)	18.33	10.48	19.50	23.88	13.20	7.53	29.65
Thruput (¢/unit)	4.84	4.60	4.84	4.99	4.88	4.60	5.17
Inventory (¢/unit)	2.93	2.02	3.09	3.81	2.30	1.51	4.65
Special handling (¢/unit)	0.00	0.00	0.00	0.00	0.33	0.00	0.00
Special expediting (¢/unit)	0.00	0.00	0.00	0.00	0.42	0.00	0.00
Stockouts per day--ISP	84.00	12.00	75.70	142.00	127.00[b]	36.30	133.00
Stockouts per day--SSP	48.00	68.00	72.40	50.00	118.50[b]	225.90	19.00
Stockouts per day--PSP	0.00	140.00	0.00	0.00	0.00	426.60	0.00
Average inventory turnover--ISP	30.30	21.44	30.10	25.09	31.40	24.63	21.80
Average inventory turnover--SSP	33.60	33.24	9.78	27.60	54.90	55.07	23.50
Average inventory turnover--PSP	9.84	64.00	30.78	7.52	27.12	134.26	5.61

[a]The mean and coefficient of variation are given in parentheses after the distribution name.

[b]Refers to the amount of stockouts which occurred and were backordered.

is represented by the exponential and gamma distributions) and where lead time is fixed (and demand is represented by the gamma and normal distributions), are presented for comparison purposes. Because only three experimental runs were made, no statistical tests were made on the results. Thus, the findings are presented from a nonstatistical basis and no statistical inferences to the relevant populations can be made. The discussion of the findings will be presented on a general level.

Combination Experimental Runs (No Backorders)--Demand Stocked Out

In both combination runs, the percent of demand stocked out was greater than it was under either the variable lead time--fixed demand or variable demand--fixed lead time case. However, the percent stocked out did not increase greatly as compared to the lead time situation (13.66% vs. 12.62% and 22.35% vs. 22.10%). The result is rather unexpected as it might be hypothesized that the stockout rate would be tremendously magnified as a result of combining the two types of uncertainty.

It does appear that lead time has a much stronger impact upon stockouts than does demand variability. The stockout rate associated with variable demand and fixed lead times was 1.98% for the gamma and 5.64% for the normal. Thus, the stockout percentage, when both demand and lead time were variable, is not pulled in the direction of that which occurs under variable demand, but in the direction of, and in excess of, the results which occur under variable lead time.

The effects of the combination of variable lead time and demand appear to be felt at the ISP level. Stockouts per day increased at the ISP level as compared to the runs where one of the uncertainties was fixed. However, in both cases, stockouts at the SSP and PSP decline significantly from those occurring when only demand is variable. As compared to the variable lead time--fixed demand situation, SSP stockouts decline in one case (72 vs. 48) and increase in the other (19 vs. 50).

In summary, a variable demand--variable lead time condition appears to produce higher stockout rates than occur when only one factor is variable. The effect seems to be slight, and certainly not as great as might be expected. In fact, the stockout rates in the combination runs did not even exceed the sum of the stockout rates from the one variable factor situations. The effects of the combination runs are seen at the ISP level, thus allowing larger inventory buildup within the channel. (The SSP has a stock turnover rate of only 10 times in the combination run.) The relative increase of inventory within the channel creates a buffer for extremely large demands emanating from the ISP's and the possibility of longer than average lead times. Thus, the two types of uncertainty seem to create a type of "cancellation effect" whereby their combination does not produce stockouts greatly in excess of that produced by one of them. If specific short lead times (below average) were matched with a series of extreme demands, the chance of a stockout is substantially reduced in that instance. Such occurrences would seem to explain the relatively low stockout rates associated with the combined runs.

Combination Experimental Runs (No Backorders)--System per Unit Cost

For both combination runs, the total cost per unit incurred by the channel system fell between the total cost levels associated with those runs where one of the experimental variables was held constant. As with the percent of demand stocked out, the effect of lead time has a much greater impact than does demand variability. The total per unit cost is drawn more closely to those associated with lead time uncertainty. Again, it is somewhat unusual that the total cost is not in excess of the uncertain lead time cost. The "cancellation effects" appear to be operative for costs also.

Facility and inventory costs for the combination runs are very much above those obtaining as a result of variable demand, and very close, but below those incurred with the variable lead time run. In opposition to the general trend are transportation costs, which, for the combination runs are substantially below the variable demand situations and above those associated with variable lead time.

These results may be explained by reference to the inventory situation in the channel. In general, variable demand creates serious impacts within the channel because of the relatively constant demand that the ISP's put upon SSP inventory. This impact tends to create stockouts in the channel, which reduce average inventory at the SSP and PSP, increase inventory turnover and thereby increase the number of partial shipments experienced within the channel. The variable lead time tends to impact more directly at the ISP level, thereby reducing pressures on the SSP and PSP inventories. These average inventories build, and stock turn declines. However, the number of partial shipments are reduced, and thus transport rates are lower.

When the two situations are combined, the effects of both are felt, with the lead time variability predominating. The overriding impact of lead time is explained by the fact that a single extreme demand on any given day does not have the impact that a single extreme lead time might have. In other words, demand over lead time is the relevant consideration when looking at demand variability. Thus, an extremely large demand will more than likely be offset by an extremely small demand over a constant lead time of seven days. It is the occurrence of a number of extremely large demands over lead time that produce stockouts. Therefore, demand variability tends to average, and only in those cases where a stream of large demands are evidenced do stockouts occur.

However, when lead time experiences extreme variation in the form of a very large number of days between order transmittal and order receipt, demand at a constant amount per day will be lost for most of those days by which the average lead time is exceeded. Thus, one extreme lead time deviation can measurably increase the stockout rate. Generally, one extreme demand cannot. Therefore, the lead time uncertainty, when coupled with demand uncertainty, tends to have the greatest overall impact on channel performance. However, the effects do appear to cancel to some degree since the costs are not above those associated with the fixed demand--variable lead time situation.

Combination Experimental Runs (Backorders)--Demand Stocked Out

When the ISP was allowed to backorder, all demands presented were eventually satisfied. However, in the backorder case the number of stockouts which occurred (and were eventually filled) is less than the situation where no backorders were made. The explanation lies in the nature of the backorder process. Anytime the on hand and on order inventory dipped below the reorder point, an order was placed for the EOQ. Thus, in the case of backorders, a given day's demand may trigger an order, but an order for the entire economic order quantity. In the no backorder case, such an order would not be placed on the same day because the demand was lost. Thus, in the backorder situation, you may in fact order when you have more on hand and on order than is necessary to cover demand over lead time because the stockouts are recorded and backordered. Therefore over a period of time, there may be more inventory on hand at the ISP level than in the no backorder case, and thus fewer stockouts.

The backorder procedure did not appear to produce severe strains throughout the channel. Compared to the no backorder case, inventory turnover increased at all levels within the channel. Thus, ISP stockouts did not continue to build an abundance of inventory in the upper levels of the channel. In fact, the whole system appeared to function much more smoothly than it did under the no backorder case. There are additional stockouts at the SSP, as might be expected when unsatisfied demand at the ISP is backordered. The effect on overall system performance is not great.

Combination Experimental Runs
(Backorders)--System Cost per Unit

Total system cost per unit is very close but higher than those occurring when only demand is variable and materially lower than those with variable lead time. Additionally, total cost per unit are almost 10 cents per unit lower than those with the combination run and no backorders. The greatest difference in cost in comparing the backorder case to the no backorder case appears in inventory and facility cost. The backorder system is much lower for both costs (23.88¢ vs. 13.20¢ for facility and 3.81¢ vs. 2.30¢ for inventory). This result reiterates the findings suggested above, that the system functions more efficiently in the backorder case because inventories did not build within the system. Thus inventory turnover increases, and facility and inventory costs decline as average inventories are held in check. Transport costs are higher with the backorder situation due to the impact of premium transportation rates for the backordered merchandise.

The additional costs associated with the backordered goods, when allocated across the total units sold, does not have a great effect on total per unit cost (2.3¢ increase, a good portion of which was transportation). However, Table 7-2 reveals the impact of the additional costs on only those units backordered. Thus, 56 cents per unit is associated with backorder costs, which would increase the cost/revenue ratio to .36 for these items. (The ratio is .26 in the no backorder case.) It would therefore be necessary to compare these additional costs to the channel wide margin to assess whether it would be worthwhile to backorder the goods stocked out. However, it should be realized that backordering did produce positive system-wide effects to the extent that inventory and facility costs declined as compared to the no backorder situation. Also, the effect on future demand of eventually fulfilling present demand must also be evaluated in considering the backorder costs.

Table 7-2. Cost per Unit Associated with the Units Backordered under Variable Demand and Lead Time

	Backordered Units: Additional Cost per Unit (¢)
Transportation	32.00
Ordering	8.00
Special handling	6.00
Special expediting	10.00
	56.00

Total units backordered: 15,287

Total cost of backordering: $8,565.16

Cost/revenue ratio of units backordered: $\frac{1.81}{5.00} = .36$.

Implications for Future Research

The primary purpose for running combination variable lead time and demand simulations was to focus on some of the immediately useful areas of future research. The combination runs were also designed to display the viability of the research approach of creating a foundation and then adding back elements of reality, thus increasing the complexity of the model. These two goals have been met with these few combination runs.

Specific recommendations for future research lie in the particular combination of runs which should be made. Given efficiency and effectiveness as the goal, lead times with small variances and demand with symmetrical distributions and low variances should be combined, thus revealing if those combinations will, in fact, reduce costs and stockouts. Furthermore, combinations which appear as though they will neutralize one another's effects should be tested. For instance, lead time was found to be the dominant factor, but its effects could be dampened somewhat with a particular demand. There must be some types of demand that will dampen the effect of lead time more than others.

Decision rules regarding the specific activity centers should also be considered. It was found that transportation costs, inventory costs and facility costs reacted in predictable ways to certain stimuli. Therefore it is known how these costs will react and it now becomes necessary to create decision rules and/or system structures and functions which would enable these costs to move in the direction determined by the planner or modeler. In conjunction with looking at specific activity centers, the ordering procedure should be investigated.

The decision rules employed in the system regarding ordering procedure seem to have a significant impact. With an economic order quantity rule the stockouts primarily caused by variable lead time seem to be allowed to persist. Under these conditions an ISP waits until stock reaches a prescribed level before ordering. This prescribed level is primarily dependent upon average demand and average lead time. Thus, as soon as a lead time beyond the average is realized, a stockout occurs. If the decision rules were variable quantity based on fixed time, the problems would probably remain because both are predicated upon estimation of average demand. There is a definite need to employ an order decision rule which more accurately accounts for variable lead time.

To overcome the stifling effects of variable lead time we can go one of several ways: (1) control lead time, (2) be able to better predict its variability, or (3) be better prepared for the unexpected. Each alternative presented has its pitfalls and each has its associated costs. However, that is not the question at hand. More importantly, the significance of lead time variability has been established, and a prime area for research, regardless of the path, has been established.

The combination runs and the previous discussion on decision rules reemphasizes again two major areas of concern in distribution: (1) the systems concept at the channel level, and (2) the behavioral problems created by autonomous ownership of institutions in the channel. Although the combination runs did not compound the effects as anticipated, they did not make efficiency any better. More interestingly, the points in the system which feel the pinch seem to shift. With demand it was the PSP and SSP, with lead time it was the ISP, with demand and lead time together, it was primarily the SSP. It seems realistic that if the channel worked in concert, pressures and profits could be spread around in such a fashion as to make the entire system more efficient. The consumer is uninterested in how a product arrived or the status of the channel members; the consumer will patronize that channel which delivers the goods. Thus research, into a unified channel (one that doesn't optimize the individuals but rather optimizes the efficiency and effectiveness of the channel) is required.

If viewing the channel as a system is paramount, then investigation into the behavioral aspects must be of parallel importance. It has been shown that channel member cooperation could significantly increase the efficiency and effectiveness of a channel. The "I'm an island" mentality must be abolished and "Its my team against yours" must be adopted. This is a tall order, but one that must be explored if distribution efficiency and effectiveness is to be reached.

Conclusions

As a result of the combination runs presented in this chapter, specific areas of future research and those areas which need immediate attention have been indicated. In addition, even though the results are not conclusive, the procedure of adding back variables is workable. As a result of the combination runs, logical cause-effect relationships could be followed from one model variation to another.

CHAPTER VII--FOOTNOTE

[1]Thomas W. Speh, "The Performance of a Physical Distribution Channel System Under Various Conditions of Demand Uncertainty: A Simulation Experiment" (unpublished Ph.D. dissertation, East Lansing, Michigan, 1974).

APPENDIX A

LITERATURE REVIEW

APPENDIX A

LITERATURE REVIEW

The primary objective of this appendix is to present a description of the considerations given demand and lead time uncertainty in research concerned with physical distribution. Additionally, multiechelon physical channel simulation models capable of experimentation with uncertain demand and lead time will be investigated. Prior investigations into demand and lead time uncertainty in physical distribution are concentrated in the area of inventory control. Hence, this literature will be examined in the first section of this appendix. The second section reviews the relevant simulation models.

Demand and Lead Time Uncertainties--Inventory

Introduction

The purpose of this section of the literature review is to provide a perspective on the efforts made to examine the impacts of demand and lead time uncertainty on the inventory component of the physical channel system. In fact, most efforts to define and measure the effects of demand and lead time uncertainties in physical distribution have been made by those concerned with developing optimal inventory policies. Uncertainty is part and parcel of the inventory problem because the decision of when to order stock and how much to order is directly dependent upon the level and variability of demand over a variable lead time horizon.

The body of literature relevant to inventory management is extremely broad in terms of the specific problems examined and vast in terms of the number of expositions on the subject. The past two decades have witnessed the growth of a large array of articles, monographs and books concerned with a more or less mathematical treatment of inventory problems. Many contributors to these publications use as their point of departure a mathematical model, and then proceed to derive mathematical solutions and study their properties in great detail.[1] Thus, the emphasis is on determining optimal solutions as to when and how much to order under a copious number of conditions. The literature contains the presentation of optimal decision rules for recoverable items, seasonal goods, spare parts, "one-shot demand" items, slow moving goods, high demand per time period items and the like. The inventory problems associated with single station supply points, multi-facility supply points (many inventory points in the same echelon) and multiechelon supply points are extensively analyzed. Many combinations of certain and uncertain lead time are found within the recent literature. Variations on the basic economic order quantity formulation are abundant. However, in most inventory treatments reviewed, one fact remains: uncertainty in demand and lead time are important elements in the analyses and resulting decision rules. It is fair to say that generally, the focus in the inventory literature is to assume that demand and/or lead time uncertainty have certain characteristics, and proceed to develop the optimal rules. In most cases, uncertainty is not the critical issue, but rather it is noted and the analysis resumes toward its main objective. There are instances in which expositions are given as to the optimal inventory policy to follow when demand and lead time assume given probability distributions. In the main, the inventory literature generally does not contain any broad, systematic analysis of the impacts of demand and lead time pattern, level and variance on a multiechelon inventory system, let alone the physical channel system. There are, of course, exceptions to this statement, and the studies which approach such systematic analysis will be discussed.

The remaining sections of this appendix will be organized as follows. A brief review of a number of the more important inventory textbooks will be presented. The emphasis will be on the objectives of these texts and how uncertainty is considered. Next, a representative sample of the periodical literature relative to single station inventory analysis is reviewed. Again, the consideration of uncertainty will be the focal point. Thirdly, the literature relevant to multiechelon inventory control with demand and lead time uncertainties is discussed. Lastly, the more generalized systematic attempts to categorize and catalog the overall impact of demand uncertainty on the inventory function are reviewed.

It must be pointed out that the literature reviewed in this section is by no means a collectively exhaustive consideration of all the literature relevant to inventory control and uncertainty. As previously indicated, hundreds of articles exist which explore every facet of inventory control. This review thereby intends to provide a highly representative sample of the types of consideration given to inventory and uncertainty. Furthermore, the review is concentrated on the periodical literature since the mid-1960's. Extensive bibliographies exist for the relevant material appearing before this time.[2]

Inventory Texts

The early 1960's witnessed a great expansion in the publication of inventory textbooks. The great bulk of the most important texts in the field of inventory management were published between 1958 and 1965. Interest in and development of operations research techniques, realization of the importance and cost of inventory and the beginnings of widespread application of computer technology most likely account for development of texts at that time. A brief discussion of some of these texts is presented below.

Robert G. Brown's initial work in forecasting for inventory control appeared in 1959.[3] The objective of the text was to

> show how uncertainty can be kept to an irreducible minimum and how that minimum can be measured and accounted for in a well-designed inventory control system.[4]

Thus, Brown's efforts were focused upon estimating the demand which could occur during a lead time so that the decision of when to order and how much to order could be more optimally made. Emphasis was given to evaluating the characteristics of demand, including the overall average value of demand, trends in average, cycles, noise (random fluctuations) and autocorrelation.[5] Brown concludes,

> Any total demand pattern can be made up by the combination of these components in different proportions. The forecasting problem is to look at the aggregate and to identify and measure each of the components. The method of making these measurements should lead to economical, practical guides for routine decisions as to when and how much to order for replenishment of inventories.[6]

Demand distributions are given extensive consideration, but not in their actual form. The relevant distribution, according to Brown, is the distribution of forecast errors, i.e., the errors represent deviations of demand away from its forecast value. This measure is then the uncertainty associated with demand, and is the relevant variable in setting safety stocks. Additionally, Brown's Appendix A describes methods for generating demand from any given population distribution. In this light, the exponential, hypergeometric, poisson and normal distributions are considered. However, Brown felt the normal distribution to be a good enough approximation for any distribution of forecast errors.[7]

Magee[8] includes a chapter on uncertainty considerations for inventory control. However, he does not elaborate on the nature of the relevant probability distributions. Magee's emphasis is on the basis for scientific methods in inventory control and also on the necessary methodology for practical application. A later text by Magee,[9] although not an inventory text, discusses the concept of a probability distribution of demand, concluding that the pattern of individual customer orders is log normally distributed.[10] Holt et al.[11] devote a significant portion of their book to inventory problems. Chapter 15 describes empirical work done on determining demand distributions. The log normal, gamma and poisson distributions are described.

Lead time distributions, specifically the log normal distribution, are considered separately and in combination with demand distributions. The authors examine the necessary steps to determine the joint probability distribution of demand over lead time. This estimate of the demand over lead time distribution is then applied to determining safety stocks.

The purpose of Fetter and Dalleck's inventory text is to "provide a guide for use in the study of inventory problems which will lead to the development of ordering rules for effective inventory control."[12] They examine the variability of both demand and lead time, and demonstrate methods for dealing with variability. The models developed are primarily for single stations, but include multi-item problems. Probability distributions of demand and lead time are examined, but the normal distribution is generally assumed for lead time and the poisson and exponential for demand. The authors also note the importance of predicting future variability of both variables, and indicate that it is necessary to find a statistical distribution that is capable of generating the data desired for forecasting. However, empirical distributions may suffice if their pattern is not expected to change.

Hansmann[13] looks at inventory problems as static or dynamic, one or many items and single or multiechelon. He thus develops operating rules necessary for each situation. Hansmann indicates the need for forecasting demand distributions, and also includes probabilistic demand within his models, but spends little time discussing the various types of demand and lead time distributions and their effects.

Starr and Miller,[14] in presenting optimal inventory rules, also consider dynamic and static models. However, they make a distinction between the degree of uncertainty facing the decision maker. Thus, all inventory problems are analyzed under each of three conditions--certainty, where demand is known exactly; risk, where the probability distribution of demand is known; and uncertainty, where the distribution is unknown. The normal distribution of demand is used in most examples because of its tractability. However, the authors indicate that solution of the inventory problems under risk will not be diminished because the normal was used. Without assuming the normal the analysis would simply be more difficult. Additionally, the models are also analyzed under constant lead time and probabilistic lead time, and the difference in operating rules noted.

A strong mathematical orientation is the focus of Wagner's text.[15] However, broad coverage is afforded probability distributions of demand and their effects on operating decisions. A large section of the text is addressed to determining the relevant demand distributions for both single and multi-item systems. Optimal inventory policies are developed for the case of gamma, normal, poisson, geometric, negative binomial and uniform demand distributions. Additionally, lead time is seen as a "delivery lag" whose duration may be variable.

Hadley and Whitin[16] present the techniques for constructing and analyzing mathematical models of inventory systems for a single stocking point. Rather extensive treatment of demand and lead time uncertainties are developed throughout the text. Various distributions for demand and lead time are studied (including poisson, gamma, exponential, normal and negative binomial) and optimal policies thereby developed. They state that the normal distribution can be used for approximating the others, but it is really not known how rapidly each approaches the normal or how much error there is when the normal is used as an approximation. The convolution of demand and lead time distributions is also examined, and the resulting demand over lead time distributions developed.

The problems involved with securing information on demand and lead time distributions (and with the case where demand changes over time) are considered. The authors suggest the use of empirical data or theoretical distributions. However, a great deal of empirical data is required so that enough information can be gained as to the tail of the distribution. They also conclude that lead time information is much more difficult to secure.

Prichard and Eagle[17] take a somewhat less rigorous mathematical approach to inventory control than do other texts. However, they do have an excellent chapter dealing with uncertainty and probability. The nature of the demand distribution is presented in terms of the normal, poisson and negative binomial distributions. The conditions where each apply are discussed and supporting empirical evidence provided. Additionally, demand over lead time, with lead time and demand both variable, is developed and the impact upon safety stock shown.

Brown's[18] text of 1967 is basically an update of his earlier book. The primary emphasis again being the forecasting of all demand components--level, trend, seasonal and random. The distribution of demand over fixed lead time is investigated.

In summary, the objective of most inventory texts is to develop optimal policies of when to order and how much to order. The impacts of uncertainty are evaluated to the extent that different types of uncertainty lead to different decision rules. The texts vary in terms of the total consideration given to demand and lead time uncertainty. However, most texts assume a given distribution and then proceed to mathematically determine optimal policies.

Single Station Inventory Control

The recent periodical literature in inventory control is focussed upon very specific topics, i.e., management of seasonal goods inventory, control policies when demand is gamma distributed, order policies when lead time is dependent on demand and the like. This section will briefly review the recent literature in terms of the specific problem under investigation and the way in which lead time and/or demand uncertainty is handled.

Kaplan[19] considered the development of optimal policies with variable lead times. His purpose was to "characterize optimal policies for a dynamic inventory problem when the time lag in delivery of an item was a discrete random variable with a known probability distribution."[20] An interesting conclusion of his analysis is that the inventory policies which resulted were very much like those which obtain when lead times are deterministic.

Lead time, expressed as a stochastic variable with a given distribution was considered by a number of authors interested in optimal policies for an (S-1,S) inventory model. The (S-1,S) inventory policy means that whenever demand occurs for a given number of units, a reorder is placed for that number of units regardless of whether there is a stock of units on hand. Gross and Harris[21] studied the model for the case when lead times are dependent on the number of backorders. In their model, the service time contribution to lead time is an exponential distribution. Demand variability was also considered, and policies developed on the basis that demand is a compound poisson distribution.

A number of variations on the theme (S-1,S inventory policies) were considered. Galliher, Morse and Simond[22] looked at a number of possible situations. They considered an arbitrary demand distribution and constant lead time plus the poisson demand and exponential lead time distribution. Rose[23] evaluated the expected number of backorders and resupply times for the (S-1,S) policy when demand is arbitrary and lead times are constant. Hadley and Whitin[24] consider the case of poisson demand and arbitrary lead times. Their model includes both the stockout case and the backorder situation.

Particular types of demand distributions were also considered, and the appropriate inventory policy formulated. Sivazlian[25] studied the (s,S) inventory model and developed the optimal values of (s,S) for the case of demand which is gamma distributed. Burgin[26] concentrated on determining safety stock and potential lost sales for the situation in which demand is normally distributed and lead time assumes a gamma distribution. Burgin compares the results achieved from approximating demand over lead time to those achieved when the distribution is directly calculated. The approximation appeared to be adequate.[27]

Hausman and Thomas[28] also considered probabilistic demand, but their point of departure was somewhat different. They considered the type of policy to follow for equipment when there were two types of demands, those for original equipment (deterministic) and those for spare parts (probabilistic). Spare parts demand was considered to be a normal distribution and lead times were fixed. The continuous review policy was judged to work best when the demand for original equipment is small relative to total demand.

Control and management of seasonal or style goods also received some attention in the recent literature. Ravindron[29] evaluated an inventory model where the demand pattern was dependent. Thus, "contagious demand" related to the influence of past demands on future demands. The poisson function was the basic probability function associated with demand. However, a contagious demand rate $\alpha(t)$ was added to the basic function to account for the influence of past demand (i.e., friends' recommendations, word of mouth).[30] The contagious distribution reduces to a negative binomial distribution given certain parameter values. Ravindron proceeds to develop optimal ordering policies and he determines how long the inventory should be carried. Chang and Fyffe[31] attack the same problem concentrating on methods for reestimating sales of seasonal goods during their period of sale.

In summary, the literature relevant to single station inventory models is broad in its coverage of specific problems and conditions. In addition, the formulation of stochastic demand and lead time varied from constant rates of demand and fixed lead times to the consideration of demand with a "contagious demand rate." Again, the objective in viewing demand and lead time as random variables was to formulate optimal inventory policies under the given conditions.

Multiechelon Inventory Control

The literature relevant to multiechelon inventory control is not as abundant as that relating to single station inventory control. As Hadley and Whitin point out, it is very difficult to study analytically multiechelon inventory systems.[32] A brief review of the literature relevant to optimal multiechelon inventory policies indicates how recent its history is. Clark and Scarf[33] were one of the first to formulate the nature of the optimal policy involving uncertain demands in 1960. Fuhuda[34] extended the work of Clark and Scarf. Zangwill,[35] in 1966, studied optimal policies in multiechelon systems where demands are known with certainty. Bessler and Veinott[36] considered a multiechelon inventory system with random demands. They determined optimal policies for redistributing stock from facilities with excesses to those with shortages. The variance of demand experienced by each facility was shown to have an effect on the optimal policy. If the demand variance is less at one facility than another, then the optimal base stock at the first facility may be different than that at the second facility. Additionally, Sherbrooke,[37] in

1968, extended the work done on multiechelon inventory problems. He considered the optimal model for recoverable items.

More recently, Simon[38] studied a two echelon inventory model for low demand consumables or reparable parts. In this work, transportation times were assumed to be deterministic and the failure process generating demands was a poisson process. According to Simon, the results obtained are useful in a number of applications. If costs were imposed, optimal values for s and S could be derived, and if many products were involved, then optimal inventory investments in each product could be derived.[39]

Hockstaedeter[40] builds on the original work of Scarf and Clark. The objective was to determine an approximation to the cost function (upper and lower bounds) for a multiechelon inventory system. In the model both demand and lead time are variable. Demand was considered a random variable, whose particular value was independent from period to period. Lead time was viewed in terms of delivery lags, with the lag being a multiple of the review period.

In summary, the literature relevant to multiechelon inventory analysis concentrates upon devising optimal policies for specific circumstances. Various conditions of demand and/or lead time uncertainty are assumed for a particular model or problem.

Systematic Demand and/or Lead Time Analysis

The last section of the review of inventory literature relates to the efforts made to systematically evaluate the impact of a wide variety of demand and lead time uncertainty conditions. The work reviewed in this section is different from that considered in the previous inventory literature in that the focus is more towards systematically evaluating demand and/or lead time uncertainties on a specific inventory system or physical distribution system, rather than assuming a given form of uncertainty and designing optimal policies for inventory control. Thus, the research reviewed here primarily involves simulation, and more closely approximates the research problem studied in the present dissertation. Three research studies will be considered. However, the work done by Ballou and Camp will be considered in a later section dealing with simulation and will not be reviewed here.

Gross and Soriano[41] simulated an inventory system and studied the impact of various distributions and variances of demand and lead time on the base and safety stock requirements of the system. The major thrust of the research was directed toward evaluating the effects of reducing the average duration of lead time on base and safety inventory levels. More specifically, the authors desired to estimate achievable on-shelf inventory savings for a military overseas resupply system when resupply is performed by air rather than sea.[42] As a by-product of the research, estimates of the impacts of various parameters, such as average demand, variance of demand and lead time, distribution of demand and lead time, inventory review period and order quantity were studied.

The simulated system was a military resupply system (single echelon) with an s,S inventory policy, and periodic review. Demands were withdrawn from inventory in a "lump sum" at the end of a time period. The output measures included average on-shelf inventory and percent of units demanded but not filled from existing inventory. No costs were included in system performance measurement.

Simulation runs were 2,000 weeks in length and were replicated 15 times. Twenty-two cases were investigated, where demand assumed a poisson distribution in seven cases and a normal distribution in fifteen cases. Lead time was either normal, exponential, uniform or constant. Additionally, the variance and average level of each demand and lead time distribution assumed two levels.

The general results of the simulation indicated that reductions in the average lead time (from thirteen to two weeks) led to large reductions in inventory. Lead time variability also affected average inventory, in that lower variation led to lower levels of inventory for a fixed service level (on-shelf inventory availability). The sensitivity of the system to changes in demand variation appeared to be a great deal weaker than the sensitivity to lead time variations. The same is true concerning sensitivity to lead time and demand distributional shapes.[43] The order quantity size has little effect on inventory availability, nor is the effect of changing order quantities sensitive to average lead time. Finally, the length of the inventory review period led to differing performance.

In summary, the literature specific to inventory control indicates the extent to which demand and lead time uncertainties have received attention in the study of inventory. The types of analyses are extensive and varied, with the general objective of achieving optimal inventory policies under assumed uncertain conditions. No systematic analyses of the impacts of uncertainty on a multiechelon channel system were discovered, although the Gross and Soriano work was relevant to a single echelon inventory system. The remaining sections of this appendix will be addressed to evaluating the simulation models which are available for replicating a multiechelon physical channel system.

Model Selection and Criteria

The second section of the literature review concerns a search for a tool through which the objectives of this research can be met. As previously indicated, real world experimentation has been eliminated as a valid option. Thus, a model of some type and specifications must be employed. Creation of a model is not within the scope of the present research. There are many physical distribution system models in existence,[44] and experimentation with a system to increase the understanding of physical distribution is the goal rather than to refine system models or add to the number of models available. Thus, a model must be selected from those presently available.

The first step in model selection is to specify a set of criteria the model must meet. The criteria for the model derive from the objectives of the research. The objectives of this research are restated in the first section of this review, and the criteria are delineated and discussed.

Given a set of criteria, the existing models can be reviewed and one selected. The selection procedure is to pick a particular criteria which will eliminate a family or group of models. This procedure is repeated until the desired model is found. The review and elimination of families of models and individual models are discussed in the second section

Criteria

Criteria derive directly from the objectives of the research. Thus an explicit statement of objectives is necessary.

The objectives of the present research are:

1. To measure the effects of uncertainty on a multiechelon physical distribution system.
2. Construct a foundation that will facilitate future research and simulate a system which is an accurate, complete and valid representation of present operating conditions.
3. Meet the above criteria within given time and monetary resource constraints.

To meet the first objective, the model must have the following characteristics: It must be multiechelon and multifacility, encompass all the physical distribution components and be capable of employing stochastic lead times and demand.

To be multiechelon and multifacility, a model must be capable of replicating more than one stage of a physical distribution system and more than one stocking point or facility at each step. In a physical distribution system a step is at least a break bulk point and dispersion point and traditionally holds inventory, i.e., manufacturer, wholesaler, retailer. For this research it is necessary to be able to simulate at least these three steps. Provision for the increase of the number of steps is also desirable. As products pass through the steps of a physical distribution system the geographic dispersion increases thus the number of facilities on each step increase. Thus, the model must be capable of simulating multiple facilities on each step, i.e., two manufacturers, four warehouses, sixteen retailers. The absolute number of facilities available at each level is important for this research and the capability to expand the number of facilities is desired.

The model must be capable of simulating all the physical distribution system components. These components are: transportation, warehousing, inventory, handling and communication. The transportation component concerns the movement of finished goods between stocking points from manufacturer to consumer. It includes pick up, line haul, delivery and back haul. Warehousing concerns the stocking points in the system which hold and handle finished goods. It also includes the networks of facilities, their location, addition and deletion. Inventory refers to the amount of finished goods in the system necessary to overcome the discrepancies between production and consumption. Handling concerns those operations necessary at a stocking point to physically prepare an order for shipping, i.e., picking, packing and movement of the goods within the stocking point. Communication refers to all those activities which verbally link the system together: order communication, order processing, request for a carrier, etc.

Together the above five components completely describe the physical distribution system. If a model did not have all of them it would not completely represent the system. For this research it is desired to have all five components.

To complete the requirements of the first objective, the model must be capable of employing stochastic demand and lead time. The model must be able to function under lead times which have various durations. In addition, the model must be capable of simulating separately the three elements of lead time: order communication, order processing, and order transportation. To accept variable demand the model must be able to continue accurate and valid simulation while the demand fluctuates. Because demand is the initiator of the system, its effects are felt throughout, and the model must be capable of adjusting to variations in demand.

To meet the second objective of constructing a foundation for future research and employing a model which is an accurate, complete and valid representation, the following criteria are necessary. The model must be flexible; it must be capable of simulating an extended time horizon; it must be dynamic, allow for change, be unified on a spatial and temporal basis and be valid.

To be flexible, the model must be capable of operating under various conditions, for instance, one product or many products, different channel structures, order times, different backordering procedures, etc. This is a necessary criteria, for in this initial research the model is stripped of many complicating factors. As future research is attempted, selected elements of the model will be replaced until such time that a replication as close as possible to the real world is achieved. Thus, a single model which can be initially simple and in steps become increasingly complex is needed.

Coupled closely with the above criteria is the specification that the model be capable of long range planning. Closely related to long range planning is the model's ability to be dynamic and its ability to allow for changes in exogeneous and status variables.[45] To be dynamic the model must use the output of one time period as input to the next period. If periods are treated independently, then a series of simulated time periods are treated in isolation. In actuality, future time periods are dependent upon previous time periods. Thus, it is desired to have a model which has this capability. Another fact of life is change. Change occurs both internal and external to the system. A model should be capable of accounting for these changes. Thus once the simulation is in progress, it is necessary to have the capability of changing these variables and having the model account for them.

The previous discussion on variable change and dynamic operation directly affect the time horizon. Any model can be run for an infinite number of days, but if the end result is actually the simulation of a single time period where change is not accounted for, the results are not actually long run in nature. Thus, a model capable of simulating a long run time frame while being dynamic and allowing for change is required.

The model must also be unified on a spatial and temporal basis. "The unifying dimension of a model is classified as spatial if the cost and/or service are developed on location or transit time. If the model uses order cycle time as the measure of physical distribution performance, the model is classified as temporal or time oriented."[46] It is desired to have a model which is unified on both dimensions. The model should be structured "to cope with inventory planning and facility location on a simultaneous basis thereby integrating the temporal and spatial aspects of system design."[47]

Lastly, to meet the second objective the model should be valid. It would be desirable to have a model which was validated both experimentally and under actual conditions.

Model Selection

Given the previous criteria model selection is now possible. Due to the criteria of multiechelon, dynamic and the inclusion of all physical distribution components, many models can be eliminated, namely those that are single station, static and allow only a portion of the physical distribution components. With the review of inventory in the previous section, the discussion of possible models can be limited to those presented below.

Ballou.--Ballou's model[48] is basically a multiechelon, dynamic simulation model. However, it does not meet the present criteria for the following reasons: (1) it is basically an inventory model with the capability to simulate transportation and communications; however, it does not have the capability to consider the location problem and cannot simulate handling operations at the stocking location; (2) it is a short time horizon model; and (3) the unifying dimension is time without space consideration.

Camp.--In his dissertation, Camp[49] analyzes the effect of carrier service on the location of warehouses. He employs the measure of mean delivery time and standard deviation for carrier service. The model's unifying dimension is space and time; it is heuristic and will allow stochastic lead time. However, it does not meet the present criteria for the following reasons. It is not multiechelon. "The methodology selected to measure results was a heuristic computer simulation of a typical single echelon distribution system."[50] It is not dynamic and is basically designed for a short time horizon.

Distribution system simulation.--The distribution system simulation is a soft ware system designed by Michael M. Connors and others[51] for use on the IBM 360/370 computer. It is unique in the sense that the user does not need to know computer programming. As a result of the answers to a series of questions on a physical distribution system, the system can be modeled and results given. The authors claim the system is extremely flexible. "A large number of different distribution system models--over 10^{12} feasible models can be generated . . . these are all functionally different models not merely parametrically different."[52] The simulation is multiechelon and multifacility in nature. And, apparently will allow stochastic lead time and demand. As stated by the authors it is "clear that DSS views inventory and product movement as being the key elements in structuring a distribution system."[53] Thus a question arises as to the comprehensiveness of the model. It appears as though the simulation is not dynamic in the sense previously defined. In addition, Sumer Aggarwal points out that the simulation does not directly include facility evaluation nor does it permit inclusion of production subsystems.[54] "It (DSS) assumes that the plant maintains an infinite inventory that can satisfy any demands."[55] The DSS is an extremely complete simulation and closely approximates the "total distribution system." However, a lack of comprehensiveness and dynamic operation eliminates it from consideration.

Markland.--Markland has created a "comprehensive simulation modeling approach to the problem of locating warehouse facilities."[56] The model is multiechelon, multiservice, multidestination and multiproduct. It accepts stochastic lead time and demand and includes transportation costs, warehousing costs, inventory and handling. Apparently, it does not have the communication function and does not break down lead time into the components of order transmittal, order processing and order transportation. It is flexible in the sense previously defined and is capable of simulating several time periods. It is not dynamic in the sense that the output from T_1 is used as the input to T_2. It appears that the simulated time periods are independent.

The Markland model is very extensive and seems to accurately simulate a physical distribution system. However, an incomplete array of physical distribution components and the fact that the model is not dynamic eliminates it from consideration.

Forrester.--In an attempt to overcome the problem of matching production rates with consumption rates Forrester developed an industrial dynamic simulation.[57] It is multiechelon and comprehensive. The components that are included are: transportation, inventory, communication, handling and a fixed set of locations. Because locations are fixed, the multiwholesalers and retailers are aggregated to a single point. The unifying dimension is time and the time horizon is not stated.

Although this model contains the majority of desired attributes, and Forrester's pioneering effort has contributed immensely to simulation modeling, a more satisfactory model exists.

LREPS.--The Long Range Environmental Planning Simulator (LREPS) will be used in this research. It contains all the desired characteristics and meets the stated criteria. Details of the model are available in Chapter IV.

APPENDIX A--FOOTNOTES

[1]Frederick Hansmann, Operations Research in Production and Inventory Control (New York: John Wiley & Sons, Inc., 1962), p. 3.

[2]See any number of inventory texts, specifically: Frederick Hansmann, Operations Research in Production and Inventory Control (New York: John Wiley & Sons, Inc., 1962); G. Hadley and T. M. Whitin, Analyses of Inventory Systems (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1963); and Bibliography on Physical Distribution Management published by NCPDM.

[3]Robert G. Brown, Statistical Forecasting for Inventory Control (New York: McGraw-Hill Book Co., Inc., 1960).

[4]Ibid., p. 1.

[5]Ibid., pp. 21, 22.

[6]Ibid., p. 22.

[7]Ibid., p. 167.

[8]John F. Magee, Production Planning and Inventory Control (New York: McGraw-Hill Book Co., Inc., 1958).

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[12] Robert Fetter and Winston C. Dollech. Decision Models for Inventory Management (Homewood, Ill.: Richard D. Irwin, 1961), p. v.

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[23] Marshall Rose, "The (S-1,S) Inventory Model with Arbitrary Backordered Demand and Constant Delivery Times," Operations Research, 20 (September 1972), 1020-1032.

[24] Hadley and Whitin, op. cit., pp. 204-212.

[25] B. D. Sivazlian, "Dimensional and Computational Analysis in Stationary (s,S) Inventory Problems with Gamma Distributed Demand," Management Science, 17, No. 6 (February 1971), 307-311.

[26] T. A. Burgin, "Inventory Control with Normal Demand and Gamma Lead Times," Operations Research, 23 (May 1972), 73-78.

[27] Ibid., p. 76.

[28] Warren H. Hausman and L. Joseph Thomas, "Inventory Control with Probabilistic Demand and Periodic Withdrawals," Management Science, 18, No. 5 (January, Part 1, 1972), 265-275.

[29] Arunachalam Ravindron, "Management of Seasonal Style--Goods Inventories," Operations Research, 20, No. 1 (March 1972), 265-275.

[30] Ibid., p. 265.

[31] Sang Hoon Chang and David E. Fyffe, "Estimation of Forecast Errors for Seasonal-Style-Goods Sales," Management Science, 18, No. 2 (October 1971), B89-B96.

[32] Hadley and Whitin, op. cit., p. 6.

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APPENDIX B

EXAMPLES OF STATISTICAL TESTS USED FOR TESTING THE RESEARCH HYPOTHESES: DUNNETT'S TEST, TUKEY'S TEST AND STANDARD t-TEST

APPENDIX B

EXHIBIT I

EXAMPLE OF THE CALCULATIONS TO COMPARE FACTOR AND CONTROL RESPONSES USING DUNNETT'S METHOD

Dunnett's method of multiple comparisons compares the control mean with all other factor means. The formula for the comparisons of control and factor means is:

$$(\overline{X}_j - \overline{X}_c) \pm d \cdot \sqrt{2MS_e/n},$$

where $(\overline{X}_j - \overline{X}_c)$ is the difference between the control mean and the factor mean and $d \cdot \sqrt{2MS_e/n}$ is the confidence allowance against which $(\overline{X}_j - \overline{X}_c)$ is compared. If $(\overline{X}_j - \overline{X}_c)$ exceeds $d \cdot \sqrt{2MS_e/n}$ the difference between the factor mean and control mean is significant. The value of "d" is based on the level of significance and is found from Dunnett's tables. The value of MS_e is derived from the AOV tables in Chapter V.

Comparison of mean transportation costs: control versus distributions:

$$d \cdot \sqrt{2MS_e/n} = 3.04^* \cdot \sqrt{2\ (2.753/4)} = 1.08$$

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*The value of "d" from Dunnett's tables for an .05 level of significance.

Distribution	Mean Transport Cost	Mean Control Cost	$(\overline{X}_j - \overline{X}_c)$	Confidence Allowance	Significant
Normal	103.36	104.59	-1.23	1.08	Yes
Log normal	103.36	104.59	-1.23	1.08	Yes
Gamma	103.58	104.59	-1.01	1.08	No

EXHIBIT II

EXAMPLE OF THE CALCULATIONS TO COMPARE THE DIFFERENCES AMONG FACTOR RESPONSES USING TUKEY'S METHOD

Tukey's method of multiple comparisons compares the mean response associated with each level of a factor to the mean response for all other levels of the factor. The formula for the comparisons among factor level is:

$$(\overline{X}_j - \overline{X}_J) \pm q_{m,v} \cdot \sqrt{MS_e/n}$$

where $(\overline{X}_j - \overline{X}_J)$ is the difference between the average response for pairs of response means and $q_{m,v} \cdot \sqrt{MS_e/n}$ is the confidence allowance against which $(\overline{X}_j - \overline{X}_J)$ is compared. If $(\overline{X}_j - \overline{X}_J)$ exceeds the value of $q_{m,v} \cdot \sqrt{MS_e/n}$, the difference between the two means is significant. The value of q is based upon the number of sample means compared, the degrees of freedom, and the level of significance. Its value is found from Tukey's tables.

Comparison of mean stockout percentages: among distributions

$$q_{m,v} \cdot \sqrt{MS_e/n} = 4.16 \cdot \sqrt{.8143/4} = 1.877$$

Differences Between All Pairs of Sample Means

j \ J	Log Normal (7.43%)	Gamma (8.05%)
Normal (6.98%)	-0.45	-1.07
Log Normal (7.43%)	----	-0.62

EXHIBIT III

EXAMPLE OF THE CALCULATIONS TO COMPARE RESPONSE MEANS USING STANDARD t-TESTS

Standard t-tests were used for comparisons between factor responses and control responses and among factor responses when Tukey or Dunnett's methods were not applicable. The differences between the mean responses associated with factors, or between factors and control were calculated in terms of standard errors and compared to the critical t-value. The decision rules are:

If $\dfrac{\bar{X}_1 - \bar{X}_2}{\hat{\sigma}_{\bar{X}_1 - \bar{X}_2}} < |t|$ Accept the null hypothesis.

If $\dfrac{\bar{X}_1 - \bar{X}_2}{\hat{\sigma}_{\bar{X}_1 - \bar{X}_2}} > |t|$ Reject the null hypothesis.

Comparison of exponential vs. gamma: percentage of demand stocked out:

Critical t = 2.77 at .05 level of significance.

$\hat{\sigma}_{\bar{X}_e - \bar{X}_g} = 3.68$ $\bar{X}_e - \bar{X}_g = 9.61$

$t = \dfrac{9.61}{3.68} = 12.61$ 12.61 > 2.77. Reject that $\mu_e = \mu_g$.

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