ESSAYS ON INFORMATION AND STRATEGY

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ABSTRACT

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Chapter 1: Reverse Price Discrimination and Information Discrimination with Bayesian Buyers: The Case of Monopoly

The paper studies price discrimination under the situation where buyers' prior valuations are initially observable to a seller but the buyers receive further information about a product or service which remains private thereafter. The buyers interpret new information via Bayes' rule. We show that, in this environment, prices are not monotone in buyers' prior valuations. Interestingly, this results in the possibility that a seller intentionally offers a higher price to a low valuation buyer rather than a high valuation buyer (Reverse Price Discrimination), which sharply contrasts with the standard result of price discrimination. We further explore the seller's incentives to discriminate in provision of information.

Chapter 2: Reverse Price Discrimination with Competition: a Hotelling Duopoly

We extend the previous analysis of Reverse Price Discrimination to a duopoly market using a standard Hotelling model. We show that the equilibrium prices can be non-monotone in the buyer's prior valuation even with competition. The buyers with relatively "weak" prior beliefs are charged higher prices, since they are more likely to view the products of two sellers vertically differentiated when receiving different signals from

them, which mitigates sellers’ price competition.

Chapter 3: Price Discrimination via Information Provision: Online vs. Offline Shoppers

We study a new type of second-degree price discrimination where a different price is offered as a bundle with a different level of information about a product. We consider the situations where buyers are uncertain about the value of the product, and may update their expected valuation in a Bayesian way after observing a signal coming from the product while shopping at a store. When an online store provides less information on the product than an offline (brick-and-mortar) store, the seller's price discrimination induces high valuation buyers to shop at the online store and low valuation buyers at the offline store. Furthermore, the buyers' incentive compatibility constraints ensure that the price should be lower at online stores than at offline stores when both stores incur the same transportation (shopping) costs. The conditions under which it is more profitable for the seller to sell at both online and offline stores than to sell only at either of the stores are examined. We then explore the case in which buyers are sophisticated enough that they may "milk" the information from one store and use it at the other store (Information Arbitrage). It is also discussed that, in an ex post sense, the low valuation buyer's purchase is socially optimal, whereas the high valuation buyer's purchase is suboptimal due to excessive consumption.

To my beloved wife, Suzie,

and children, Ikhan and Amy

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1.1 Introduction

When you open the door to a car dealership, a salesperson will welcome you and ask how you are. At this moment, the salesperson may try to figure out your willingness to pay by looking at your appearance such as your gender, race, and age, and further by asking some related questions from which he could infer your income and so forth. Then the salesperson may offer you a personalized price based on this observation. This looks like a typical third-degree price discrimination: a seller is able to observe a buyer's willingness to pay and charges the optimal price using that information.

However, the seller's pricing can be different from the one in a standard (third-degree) price discrimination if buyers' valuations can change depending on private information they additionally gather. For example, car buyers can do test drives and home buyers can visit houses before their purchase. These advance activities allow consumers to learn more about the products in which they are interested. In this situation, although buyers' prior willingness to pay might be observable to a seller, their posterior valuations formed by the new information they acquire may remain private. A key feature is that buyers update their valuations in a Bayesian way in the sense that buyers' posterior valuations are systematically dependent on their prior beliefs through Bayes' rule.

The analysis of the optimal pricing in this situation is of particular interest because it begins by departing from the standard classification of price discrimination. As is well known, third-degree price discrimination is charging different prices based on some observable characteristics, whereas second-degree price discrimination considers the case where the buyers' types are not observable. Our model considers the intermediate case in the following sense. Although a buyer's prior valuation is perfectly observable to the seller, the buyer receives new private information and updates his valuation before purchasing. As a consequence, the seller has to choose the optimal price based only on the buyer's prior valuation, but not on the posterior valuation because it is not observable. Then, what would be the optimal pricing rule for the seller who faces this environment?

We address this question in a monopoly setup, and we show that prices can be non-monotone in buyers' *prior* valuation. In other words, interestingly, there is a situation where the seller offers a higher price to low valuation buyers than to high valuation buyers. Our result seems contradictory to the general conjecture that it would be optimal to charge a higher (lower) price if buyers are with high (low) valuation. Here is the intuition underlying our result.

When the seller makes a decision on the prices, she faces a traditional trade-off between getting a higher margin and getting a greater market share. This trade-off depends crucially on the price elasticity of demand, which is essentially determined by the interaction between a buyer's prior valuations and the precision of information available to the buyer. When information precision is not extreme (i.e., neither too accurate nor too vague), we find that the demand is elastic (inelastic) when the buyer's prior valuation is high (low). Thus, if the buyer's prior valuation is high, the seller chooses to offer a low price because the buyer's updated valuation does not decline significantly even when he receives unfavorable private information. On the other hand, if his prior valuation is low, it is more profitable for the seller to charge a high price and target only the buyer who receives favorable private information because the price under which the buyer with unfavorable information is covered is too low to be profitable. This leads to the result that a higher price can be offered to buyers with lower willingness to pay and vice versa.

Our main results suggest that the seller can charge a higher price to buyers with lower willingness to pay as a profitable strategy. We refer to this outcome as *reverse price discrimination*. In particular, the process we suggest as a cause of reverse price discrimination consists of the following three simple steps: i) consumers have prior valuation, ii) consumers update their belief using Bayes rule after observing a private signal, and iii) consumers have private posterior valuation. We believe that this is a quite general process that we experience frequently in our everyday life. Our model shows how this common process provides a sufficient condition for which reverse price discrimination can arise. In other words, our model shows how the posterior valuations which yield reverse price discrimination are *endogenously* formed in terms of information quality.

Our approach enables us to answer the following interesting questions: i) what product markets are more likely to give rise to reverse price discrimination? or ii) what would be the characteristics of products for which we would expect reverse price discrimination more often? We will highlight that information should be neither too precise nor too imprecise for reverse price discrimination to arise. Thus, reverse price discrimination can be prevalent in a market for professional services such as the ones provided by lawyers, car mechanics, and home improvement contractors. Consumers in these markets are definitely uncertain of the quality of services, so they try to gather additional private information. Of course, the sellers in these markets often offer personalized prices to different individuals.

The result derived in our model may provide an alternative explanation to the discrimination in car sales observed by Ayres (1995) and Ayres and Siegelman (1995).[1] They tested how new-car dealerships quoted differently across customers' races. They found that dealers offered significantly lower prices to white buyers than to non-white buyers, even though white people are believed to have a higher willingness to pay than non-white people, which is why price discrimination appears to be the racial discrimination.[2] However, our model suggests the possibility that a salesperson may offer a higher price to the group which is more likely to have a lower willingness to pay because it is profitable to target only those who have favorable posterior impressions.

Our result can also explain reward programs for frequent customers. When a seller can identify repeated customers, she regards them as consumers with a high willingness to pay. The recent literature on purchase-based price discrimination has studied this issue extensively.[3] A common finding in this literature is the so called "ratchet effect," which describes price discrimination against loyal customers by charging a higher price. However, it is also not hard to find numerous opposite examples where sellers often use price discrimination favorable to loyal customers. For example, airline companies provide discounts and free upgrades to frequent customers. Automobile companies offer loyalty rebates to customers who currently own the same brand car. In our model, a buyer's prior valuation can be thought of as a brand loyalty which can be assessed by a seller through a customer's purchase history. Our model then provides one reasoning about why a seller may offer discount prices to the loyal customers.

The topic of third-degree price discrimination, which considers the case where the char-

[1] See Yinger (1998) for a good survey and summary of the discrimination literature in consumer markets.

[2] White people are believed to have a higher willingness to pay because of their greater ability to pay from the greater average income and the higher opportunity costs of search time.

[3] See Armstrong (2006) and Fudenberg and Villas-Boas (2005) for the review of the literature on behavior-based price discrimination.

acteristics of buyers (who have no private information) are observable, by a monopolist has been covered extensively in literature after the seminal work of Robinson (1933).[4] In particular, Borenstein (1985), Holmes (1989), Corts (1998), and Chen (1999) are the recent papers which extend the analysis of third-degree price discrimination to the setting of oligopoly. Price discrimination with incomplete information has been studied in the environment of screening or self-selection mechanism followed by Mussa and Rosen (1978) and Maskin and Riley (1984). In particular, Lewis and Sappington (1994) and Courty and Li (2000) are closely related to ours because they also study the environment where consumers are initially uncertain of their valuations that are in part revealed afterwards. Both papers study second-degree price discrimination in which firms are assumed not to be able to observe consumers' expected valuations. On the other hand, in our paper, the seller is able to observe consumers' expected valuations and price discriminate based on them. Most significantly, to the best of our knowledge, our paper is the first study of reverse price discrimination.

The rest of this article proceeds as follows. Section 1.2 describes the structure and assumptions of the basic model. Section 1.3 studies the monopolist's price discrimination. We first consider the case where a buyer has a unit demand and receives a binary signal. We then relax these two key assumptions and show the robustness of our main result even after incorporating a non-unit demand function and general information structure. Section 1.4 concludes.

1.2 The Model

In this section, we explain the basic setup of the model which will be used throughout the paper. The additional assumptions or setups needed in each scenario will be introduced as needed in each section.

Players. The buyers have a unit demand for the good, which is solely supplied by seller(s).

[4] Armstrong (2006), Stole (2007) and, Varian (1989) are the excellent survey papers about price discrimination.

The true value of the good depends on the match between a buyer's preference and the features of the good, which is denoted by $v \in \{H, L\}$. If $v = H$, the good is a good match with a buyer and if $v = L$, it is a bad match. A buyer's prior belief for $v = H$ is denoted as $\theta \in [0, 1]$. We normalize the buyer's valuation for the good to be 1 for $v = H$ and 0 for $v = L$. Accordingly, θ can be thought of as the buyer's *prior* valuation for the good and therefore as the willingness to pay. The buyer's type θ is public information and observable to the seller(s). For simplicity, the reservation value of seller(s) for the good is assumed to be 0.

Information. The buyers receive private information about the good while they are inspecting the good before purchase. Although the buyers are ex ante homogeneous, they may draw different binary signals, $s \in \{s_H, s_L\}$, on their match value. The realization of a signal is privately observed by the buyers, so it is private information. As is standard, the signals partially reveal the true match value of the good in the sense of Blackwell,

$$\begin{aligned}
\Pr(s_H|v &= H) = \Pr(s_L|v = L) = \alpha \\
\Pr(s_L|v &= H) = \Pr(s_H|v = L) = 1 - \alpha
\end{aligned}$$

where $\alpha \in \left(\frac{1}{2}, 1\right)$ without loss of generality. α is often interpreted as the precision or the quality of a signal and is common knowledge.

In addition, one interesting interpretation of α is related to the products classification. Nelson (1970) classifies products into two categories: search goods and experience goods. He defines a search good as one whose qualities can be easily evaluated by the consumer before purchase. Similarly, he defines an experience good as one whose qualities are difficult to observe before purchase. According to this definition, $\alpha = 1$ is comparable to a search good because the buyers are able to observe the quality of a good completely before their purchase. By contrast, $\alpha = 1/2$ represents an experience good because the buyers are not

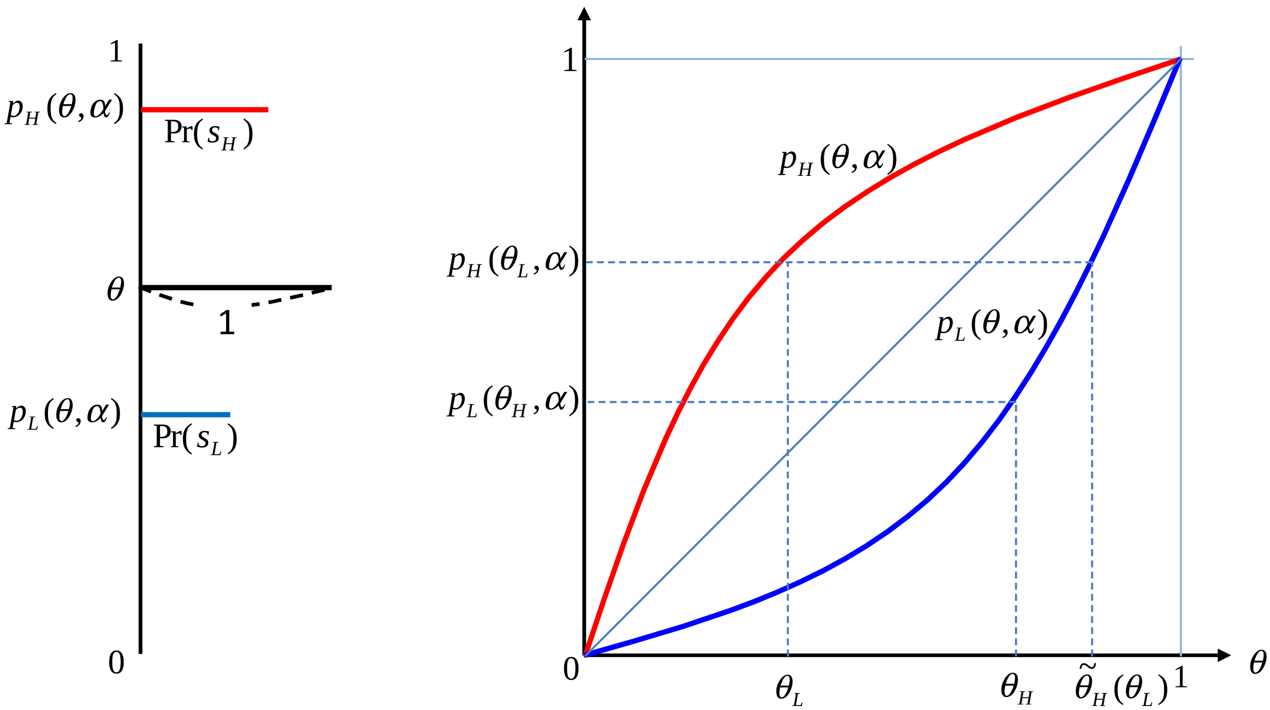

Figure 1.1: Demand (For interpretation of the references to color in this and all other figures, the reader is referred to the electronic version of this dissertation.)

able to observe the quality in advance.[5] Thus, if $\alpha \in \left(\frac{1}{2}, 1\right)$, the good can be thought of as one which is neither perfectly a search good nor perfectly an experience good. In reality, there would be many goods and services which fall somewhere between those two extreme types of goods, and those are our main interest in this paper.

As the buyer receives a signal $s \in \{s_H, s_L\}$, he updates his beliefs on the match value for the good. Let us refer to $p_H(\theta, \alpha)$ and $p_L(\theta, \alpha)$ as the buyer's posterior valuation when the buyer with a prior $\theta \in (0, 1)$ draws a signal s_H and s_L, respectively. Then, Bayes' rule leads to

$$p_H(\theta, \alpha) = \frac{\alpha\theta}{\alpha\theta + (1-\alpha)(1-\theta)} \text{ and} \tag{1.1}$$

$$p_L(\theta, \alpha) = \frac{(1-\alpha)\theta}{\alpha(1-\theta) + (1-\alpha)\theta} \tag{1.2}$$

[5]If $\alpha = \frac{1}{2}$, it means that the private signal s does not provide any information about the true value.

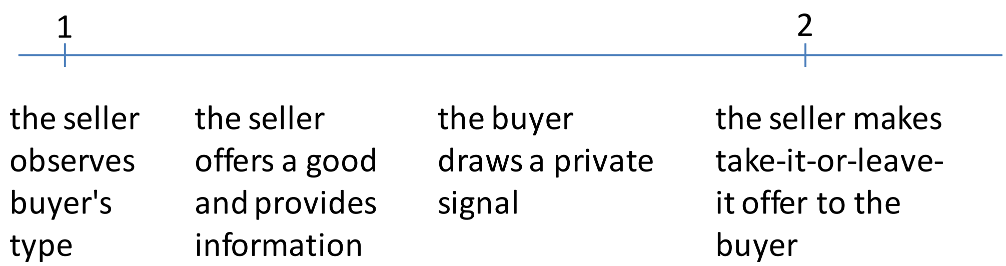

Figure 1.2: Timing

because we normalize the buyer's valuation for the good to be 1 for $v = H$ and 0 for $v = L$.

The buyer's posterior valuation is a mean-preserving spread of his prior belief. A signal disperses the prior belief to two-point distribution: $p_H(\theta, \alpha)$ with the probability $\Pr(s_H)$ and $p_L(\theta, \alpha)$ with the probability $\Pr(s_L)$. The probabilities that the buyer receives a good signal and bad signal are given by

$$\Pr(s_H) = \sum_{v \in \{L,H\}} \Pr(s_H|v) \Pr(v) \quad \text{and} \quad \Pr(s_L) = \sum_{v \in \{L,H\}} \Pr(s_L|v) \Pr(v),$$

respectively. In this information structure, the following properties are well-known. First, the posterior valuations $p_H(\theta, \alpha)$ and $p_L(\theta, \alpha)$ are increasing in θ at a decreasing rate and at an increasing rate respectively:

$$\frac{\partial p_H(\theta, \alpha)}{\partial \theta} > 0, \ \frac{\partial p_L(\theta, \alpha)}{\partial \theta} > 0, \ \frac{\partial^2 p_H(\theta, \alpha)}{\partial \theta} < 0, \text{ and } \frac{\partial^2 p_L(\theta, \alpha)}{\partial \theta} > 0 \tag{1.3}$$

as represented in Figure 1.1. Second, $p_H(\theta, \alpha)$ is an increasing function and $p_L(\theta, \alpha)$ is a decreasing function in α:

$$\frac{\partial p_H(\theta, \alpha)}{\partial \alpha} > 0 \text{ and } \frac{\partial p_L(\theta, \alpha)}{\partial \alpha} < 0 \tag{1.4}$$

Timing. The timing of the game is summarized in Figure 1.2. In the first stage, the seller can observe the buyer's prior valuation θ perfectly. At the same time, the buyer draws a private signal about the match value of the good. In the second stage, the seller quotes price P to the buyer and makes a take-it-or-leave-it offer. We assume that the buyer prefers to purchase the good when he is indifferent between buying and not buying the good.

1.3 Monopoly

1.3.1 Basic model

Now we consider the case where there is only one seller. In this section, we assume that there are two types of buyers and type-i buyer's prior valuation is $\theta_i \in \{\theta_L, \theta_H\}$ where $0 \le \theta_L < \frac{1}{2} < \theta_H \le 1$.[6] We define the buyer as a type-H if $\theta_i = \theta_H$ and a type-L if $\theta_i = \theta_L$.

Given the information structure, we find the optimal prices that the seller offers to each type of buyer. Note that when buyer i receives a good signal, he will purchase the good if and only if $p_H(\theta_i, \alpha) - P_i \ge 0$. Likewise, when he receives a bad signal, he will purchase if and only if $p_L(\theta_i, \alpha) - P_i \ge 0$. Hence, the expected demand is the equilibrium probability that buyer i accepts an offer for a given price P_i, which is given by

$$D(P_i) = \begin{cases} 1 & \text{if } P_i \le p_L(\theta_i, \alpha), \\ \Pr(s_H) = \alpha\theta_i + (1-\alpha)(1-\theta_i) & \text{if } p_L(\theta_i, \alpha) < P_i \le p_H(\theta_i, \alpha), \\ 0 & \text{if } P_i > p_H(\theta_i, \alpha). \end{cases}$$

Figure 1.1 illustrates the demand function depending on the realization of a private signal.

For each type of buyer, the seller has to choose between two alternative prices: (i) a high price at which only the buyer who receives a good signal $s = s_H$ can afford to buy, i.e., $P_i = p_H(\theta_i, \alpha)$ and (ii) a low price at which everyone including the buyer who receives a bad

[6] This can also be interpreted as follows: a seller regards the buyer with $\theta \in \left[0, \frac{1}{2}\right)$ as a type-L and the one with $\theta \in \left(\frac{1}{2}, 1\right]$ as a type-H. This assumption simplifies our analysis without a significant change in our main results.

signal $s = s_L$ can afford to buy, i.e., $P_i = p_L(\theta_i, \alpha)$. When a seller charges $p_H(\theta_i, \alpha)$ to a type-i buyer, she can sell the good with probability $\Pr(s_H)$ which is the probability that the buyer receives a signal $s = s_H$. On the other hand, she can sell the good with probability 1 if she charges $p_L(\theta_i, \alpha)$. Keep in mind that the seller can observe the buyer's type (θ_i, *prior* valuation) and therefore can offer different prices to two different types of buyers. As in any model of price discrimination, we rule out a resale possibility.

Let $\pi_H(\theta_i, \alpha)$ and $\pi_L(\theta_i, \alpha)$ be the seller's expected profit when she charges $p_H(\theta_i, \alpha)$ and $p_L(\theta_i, \alpha)$ to a type-i buyer. Then, we obtain from (1.1) and (1.2)

$$\pi_H(\theta_i, \alpha) = p_H(\theta_i, \alpha) \Pr(s_H) = \alpha\theta_i \quad \text{and} \tag{1.5}$$

$$\pi_L(\theta_i, \alpha) = p_L(\theta_i, \alpha) \cdot 1 = \frac{(1-\alpha)\theta_i}{\alpha(1-\theta_i) + (1-\alpha)\theta_i}. \tag{1.6}$$

Comparing $\pi_H(\theta_i, \alpha)$ and $\pi_L(\theta_i, \alpha)$ determines the optimal price the seller charges to type-i buyer.[7] The tension in determining the price is the trade-off between getting a higher margin by charging $p_H(\theta_i, \alpha)$ and getting a greater market share by charging $p_L(\theta_i, \alpha)$. The seller's pricing decision is determined by the price elasticity of demand. Now, let us show that the elasticity systematically depends on the interaction between a buyer's prior valuation and the precision of information. There are three possible cases. The following proposition summarizes the result.

[7]In fact, the comparison tells us whether the expected demand is inelastic, unit-elastic, or elastic. Since the expected demand is a two-point distribution, we shall use the midpoint method, $\varepsilon_p = -\frac{\Pr(s_H)-1}{p_H(\theta_i,\alpha)-p_L(\theta_i,\alpha)} \cdot \frac{\frac{p_H(\theta_i,\alpha)+p_L(\theta_i,\alpha)}{2}}{\frac{1+\Pr(s_H)}{2}}$, for calculating the price elasticity of demand. Then, we obtain $\pi_H(\theta_i, \alpha) \gtreqless \pi_L(\theta_i, \alpha)$ as $\varepsilon_p \lesseqgtr 1$. The seller charges a higher price, $p_H(\theta_i, \alpha)$, when the demand is inelastic and a lower price, $p_L(\theta_i, \alpha)$, when the demand is elastic.

Proposition 1 *Let* $\alpha_1 = \alpha^*(\theta_i = \theta_L)$ *and* $\alpha_2 = \alpha^*(\theta_i = \theta_H)$ *where*

$$\alpha^*(\theta_i) = \frac{\left((\theta_i + 1) - \sqrt{\theta_i^2 - 6\theta_i + 5}\right)}{2(2\theta_i - 1)}.$$

Then, given $\theta_H > 1/2 > \theta_L$, *there exist* $\alpha_1, \alpha_2 \in \left(\frac{1}{2}, 1\right)$ *such that the following results hold.*

(i) If $\alpha \in \left(\frac{1}{2}, \alpha_1\right)$, *a seller charges* $p_L(\theta_i, \alpha)$ *for each type* i.

(ii) If $\alpha \in [\alpha_1, \alpha_2]$, *a seller charges* $p_H(\theta_L, \alpha)$ *for a type-L buyer and* $p_L(\theta_H, \alpha)$ *for a type-H buyer.*

(iii) If $\alpha \in (\alpha_2, 1)$, *a seller charges* $p_H(\theta_i, \alpha)$ *for each type* i.

Proof of Proposition 1

In the appendix.

If information is sufficiently precise (case iii), the demand becomes inelastic. In this case, the seller prefers getting a higher margin to getting a greater market share because she has to lower the price too much in order to sell to the buyer with a bad signal. In other words, the price $p_H(\theta_i, \alpha)$ is high enough to compensate the loss in profit from losing the buyers with a bad signal. Hence the seller charges $p_H(\theta_i, \alpha)$ to each type of buyer. By contrast, if information is too vague (case i), the demand is elastic, so the seller prefers to have the large market share and serve all buyers by charging $p_L(\theta_i, \alpha)$. In these two cases, the elasticity is solely determined by the quality of information.

The most interesting case is the one where information quality is intermediate and the buyer's prior valuation determines the elasticity (case ii). When a buyer is type-H, it is more profitable to increase a market share because a type-H buyer's posterior valuation even with $s = s_L$ is relatively high enough to make the seller offer $p_L(\theta_L, \alpha)$ and serve all type-H buyers. On the other hand, when a buyer is type-L, the price for serving the buyers with $s = s_L$ should be significantly low. Hence, increasing a market share is not attractive, so the seller charges the price $p_H(\theta_L, \alpha)$ to type-L buyers. Now we compare the prices offered to each type of buyer.

Proposition 2 *If* $\alpha \in \left(\frac{1}{2}, \alpha_1\right)$ *or* $\alpha \in (\alpha_2, 1)$*, the price offered to a type-H buyer is higher than that offered to a type-L buyer.*

If information quality is sufficiently precise, i.e., $\alpha \in (\alpha_2, 1)$, the seller charges $p_H(\theta_i, \alpha)$ to each type i. Since $p_H(\theta_i, \alpha)$ is an increasing function in θ_i from (1.1), we obtain $p_H(\theta_H, \alpha) > p_H(\theta_L, \alpha)$. On the other hand, if information quality is sufficiently imprecise, i.e., $\alpha \in \left(\frac{1}{2}, \alpha_1\right)$, the seller charges $p_L(\theta_i, \alpha)$ for each type i. Again, we obtain $p_L(\theta_H, \alpha) > p_L(\theta_L, \alpha)$ because $p_L(\theta_i, \alpha)$ is also an increasing function in θ_i from (1.2). Hence, if information quality is either sufficiently precise or imprecise, the price offered to a type-H buyer should be higher than that for a type-L buyer. This implies that the standard result of price discrimination holds if the goods are close to search goods or experience goods.

Now, the striking result is that we find the possibility of *reverse price discrimination* for the case of in-between goods where $\alpha \in [\alpha_1, \alpha_2]$. The price offered to a type-L buyer can be higher than that offered to a type-H buyer. Since $p_H(\theta_i, \alpha)$ is concave in θ_i and $p_L(\theta_i, \alpha)$ is convex in θ_i, we can easily denote the case in which $p_H(\theta_L, \alpha) > p_L(\theta_H, \alpha)$ by looking at a certain θ_L and θ_H, as shown in Figure 1.1. We summarize this result more formally in the following proposition.

Proposition 3 *Suppose that* $\alpha \in [\alpha_1, \alpha_2]$*. Let us define* $\widetilde{\theta}_H(\theta_L)$ *such that* $p_L(\widetilde{\theta}_H(\theta_L), \alpha) = p_H(\theta_L, \alpha)$ *for given* θ_L *and* $\widetilde{\theta}_L(\theta_H)$ *such that* $p_H(\widetilde{\theta}_L(\theta_H), \alpha) = p_L(\theta_H, \alpha)$ *for given* θ_H*. We obtain the reverse price discrimination,*

$$p_H(\theta_L, \alpha) > p_L(\theta_H, \alpha),$$

if $\theta_H < \widetilde{\theta}_H(\theta_L) = \frac{\alpha^2\theta_L}{\alpha^2\theta_L+(1-\alpha)^2(1-\theta_L)}$*, or equivalently* $\widetilde{\theta}_L(\theta_H) = \frac{(\alpha-1)^2\theta_H}{\left(\theta_H-2\alpha\theta_H+\alpha^2\right)} < \theta_L$*.*

Proof of Proposition 3

In the appendix.

According to Proposition 3, for the reverse price discrimination to arise, the difference between two prior valuations θ_L and θ_H should be bounded above. Also note that

$$\frac{\partial\left(\widetilde{\theta}_H(\theta_L)\right)}{\partial\alpha} = \frac{2\left(\theta_L - 1\right)\left(\alpha - 1\right)\alpha\theta_L}{\left(2\alpha\theta_L - \theta_L - 2\alpha + \alpha^2 + 1\right)^2} > 0$$

and

$$\frac{\partial\left(\widetilde{\theta}_L(\theta_H)\right)}{\partial\alpha} = \frac{2\left(1 - \theta_H\right)\left(\alpha - 1\right)\alpha\theta_H}{\left(2\alpha\theta_H - \theta_H - \alpha^2\right)^2} < 0.$$

That is, for given θ_L, as α increases, the value of $\widetilde{\theta}_H(\theta_L)$ below which $p_H(\theta_L, \alpha) > p_L(\theta_H, \alpha)$ increases. Also for given θ_H, as α increases, the value of $\widetilde{\theta}_L(\theta_H)$ above which $p_H(\theta_L, \alpha) > p_L(\theta_H, \alpha)$ decreases. These imply that as information quality α increases (decreases), the parameter set of θ_L and θ_H for which the reverse price discrimination is derived becomes larger (smaller). Here is the reason. As information quality increases, the type-L buyer's posterior valuation, after observing a good signal, becomes relatively high enough to compensate the initial low prior belief. Also, although the type-H buyer's initial prior belief is high, his posterior belief after observing a bad signal becomes relatively low. These mean that the curvatures of both functions $p_H(\theta_L, \alpha)$ and $p_L(\theta_H, \alpha)$ become larger as α increases, as seen in Figure 1.1. Hence, even though the prior valuations differ much, as information quality increases, the reverse price discrimination is more likely to arise.

Our results say that the information quality should be intermediate, i.e., $\alpha \in [\alpha_1, \alpha_2]$ for the reverse price discrimination to arise. In other words, the signals buyers observe should be neither too precise nor too vague. We believe this is more applicable to many cases because a large number of goods are neither perfectly search goods nor perfectly experience goods. Consider a car sales market. Consumers are usually given some opportunities to inspect and test drive a car before the purchase is made, but it is still hard for them to completely figure out how well the car fits their taste. In this sense, the car sales market is a good example where reverse price discrimination may occur. In sum, our theory implies that while standard result of price discrimination applies to search goods and experience goods, the reverse price discrimination can arise for intermediate types of goods.

We believe that our result may provide an alternative explanation for Ayres and Siegelman (1995). According to their findings, the car dealers offered the $1,061 higher price to non-white buyers rather than to white buyers, although the non-white buyers are believed to have a lower willingness to pay than white buyers.[8] Our model suggests that their findings can be due to the dealer's strategy, which is to target only the buyers having a higher posterior valuation in the case of non-white buyers.

In general, the statistical survey shows that the average income of white American is higher than that of non-white American. If this statistical information induces the dealer to have a perception that the non-white buyers have relatively low initial willingness to pay, the dealer may think that he has to lower the price significantly to serve everyone including the ones with low posterior valuation.[9] Thus, it may yield a larger profit for the seller to target only the ones with high posterior valuation. On the other hand, if the dealer deals with the white buyers who are believed to have relatively high willingness to pay, the seller may want to offer a relatively low price in order to entice all the buyers regardless of the signals they receive, since even the ones getting a bad signal still have a relatively high posterior

[8] Ayres and Siegelman (1995) discuss the following three hypotheses: i) More people in minority groups are not aware of the fact that the sticker price is negotiable. ii) They are more likely to be averse to conducting negotiations. iii) Black Americans might have higher willingness to pay in terms of search costs. Especially, iii) is directly related to price discrimination based on the difference in consumers' willingness to pay. As to this, our hypothesis is that salespeople may charge a higher price to black Americans even though they are more likely to have lower willingness to pay.

[9] In our basic model, the buyer's initial willingness to pay θ is defined as the buyer's initial valuation of whether the given good would be a good match or not. However, it would be natural to conjecture that the income level is the other important factor which determines the buyer's willingness to pay.

valuation.[10]

Until now, the information structure α has been given exogenously. What happens if we relax this assumption so that the seller can choose the level of information to some extent? In particular, it would be interesting to ask whether the seller has incentives to engage in information discrimination. If we restrict our attention to the relevant case for reverse price discrimination, it can be easily understood that the seller has opposite incentives in providing information to different people. While $\pi_H(\theta_L, \alpha)$ is increasing in α, $\pi_L(\theta_H, \alpha)$ is decreasing in α. That is, the seller would provide as much (precise) information as possible to the L buyer, whereas she would provide as little (precise) information as possible to the H buyer. Since the seller's optimal strategy for L buyers is to charge a high price and serve only those who receive a good signal, she wants to allow them to update their beliefs more precisely so that they have a higher posterior valuation upon a good signal. On the contrary, for H buyers, since the seller charges a low price and targets all buyers including the ones with a bad signal, the more precise information would only make the buyers with a bad signal more disappointed.[11] This result may be able to explain one interesting observation in Ayres and Siegelman (1995). They tested whether salespeople might discriminate between white and non-white buyers simply because of their animus or hostility. However, they found that salespeople actually spent more time with non-white consumers while offering higher prices, which is consistent with our argument.[12]

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[10] We do not claim that our reasoning is the most proper explanation for the finding in Ayres and Siegelman (1995). We believe that price discrimination in car sales can stem from a complex interaction of all factors discussed in Ayres and Siegelman (1995) and our complementary reasoning.

[11] Technically, this argument is reminiscent of Johnson and Myatt (2006). They show that profits are "U-shaped" in the dispersion of demand. When the marginal consumer is above average, as in the case of the L buyer, profits increase as the demand curve is more dispersed. In contrast, when the marginal consumer is below average, as in the case of the H buyer, the less dispersion of the demand curve raises profits. In our model, the demand curve is dispersed by more precise information. A major difference in our model is that the degree of dispersion is systemically dependent on a buyer's prior valuation through Bayes' rule. Thus, we show that the seller provides the L buyer with more precise information to induce the dispersion of the demand curve, whereas she provides the H buyer with less precise information to reduce the dispersion of the demand curve.

[12] If buyers can decide whether to receive more information or not, there is no reason for

In the following two sections, we relax two key assumptions underlying our basic model, the unit demand and the binary signals, to check the robustness of the main results. In each case, we retain the other assumption to isolate the effect of each assumption.

1.3.2 General Information Structure

Now, we incorporate a general information structure. The buyer observes a signal $s \in [0,1]$. The signal is drawn from one of two distributions: $F_H(s)$ (with density $f_H(s)$) if $v = H$, or $F_L(s)$ (with density $f_L(s)$) if $v = L$. Both densities are bounded and twice differentiable and the likelihood ratio $M(s) \equiv f_L(s)/f_H(s)$ is decreasing in s. This monotone likelihood ratio property (MLRP) implies that higher signals are more associated with the high quality. The support of $f_L(s)$ is defined to be the set, $\{s : 0 \leq s \leq \underline{s}\}$, whereas the support of $f_H(s)$ is defined to be the set, $\{s : \overline{s} \leq s \leq 1\}$ with $\underline{s} > \overline{s}$. Accordingly, we must have

$$\lim_{s \to 0} M(s) = \infty \text{ and } \lim_{s \to 1} M(s) = 0.$$

This implies that the extreme signals are perfectly informative. When $s > \underline{s}$, buyers can be sure that the quality of the good is certainly H, while when $s < \overline{s}$, it is certainly L. In the case when $s \in [\overline{s}, \underline{s}]$, the quality of the good is unclear and the buyer's posterior valuation is

$$p(\theta, s) = \frac{\theta f_H(s)}{\theta f_H(s) + (1-\theta) f_L(s)} = 1 \Big/ \left[1 + \frac{(1-\theta)}{\theta} M(s)\right]. \tag{1.7}$$

It is important to note that the two density functions should overlap partially to have reverse price discrimination. What if two density functions do not overlap as in a case of $\underline{s} \leq \overline{s}$? In this case, the information structure is perfect because buyers can surely tell whether the good is H or L. This case corresponds to $\alpha = 1$ in our basic model. On the other hand, what if two density functions overlap substantially by having the same support

L buyers to refuse further information since they will end up with zero net utilities after all regardless of the level of information they obtain.

as in a case of $\underline{s} = 1$ and $\overline{s} = 0$? This case is now equivalent to $\alpha = 1/2$. That is, our modelling strategy again captures the notion that the good is neither perfectly search good nor perfectly experience good.

Given the buyer's prior belief and a signal about the quality, he will make a decision on whether to buy or not. The buyer decides to purchase the good if he receives a signal better than his standard. Since the buyer's expected net payoff from buying the good is $p(\theta, s) - P$, he purchases if and only if $p(\theta, s) \geq P$, i.e., $M(s) \leq \frac{\theta}{(1-\theta)} \frac{1-P}{P}$. The cutoff signal is defined as

$$\widehat{s}(\theta, P) \equiv \min \left\{ s \in [0,1] \,\middle|\, M(s) \leq \frac{\theta}{(1-\theta)} \frac{1-P}{P} \right\}. \tag{1.8}$$

If the buyer receives a signal better (worse) than the standard $\widehat{s}(\theta, P)$, he decides to buy (not buy) the good or service. $\widehat{s}(\theta, P)$ is decreasing in θ, i.e., the buyer with a more optimistic belief will set a lower standard because $p(\theta, s)$ is increasing in both θ and s. Obviously, the buyer sets a higher standard for a higher price.

We turn to the seller's problem, where she chooses the price P to maximize her profits. There are two possible cases. First, the seller can choose $P = 1$ only to target the buyer with $s > \underline{s}$. That is, when $P = 1$, the fraction of buyers $\theta(1 - F_H(\underline{s}))$ purchase the good. The corresponding profit is $\pi_H(\theta) = \theta(1 - F_H(\underline{s})) \cdot 1$. On the other hand, the seller can choose $P < 1$. In this case, the buyer purchases the good when he receives a signal $s \geq \widehat{s}$, the expected demand can be written by

$$D(\widehat{s}) = \theta[1 - F_H(\widehat{s})] + (1 - \theta)[1 - F_L(\widehat{s})].$$

The seller maximizes

$$\begin{aligned} \underset{P}{Max} \;\; \pi_L &= P\left\{\theta(1 - F_H(\widehat{s})) + (1 - \theta)(1 - F_L(\widehat{s}))\right\} \\ s.t. \;\; M(\widehat{s}) &= \frac{\theta}{(1-\theta)} \frac{1-P}{P}, \end{aligned}$$

and from the analysis, the following proposition summarizes the possibility of reverse price discrimination.

Proposition 4 *If* $-\frac{f_L(\underline{s})}{M'(\underline{s})} < (1 - F_H(\underline{s}))$, *there exists at least one cutoff value* $\widehat{\theta}$ *so that* $P^* = 1$ *if* $\theta \leq \widehat{\theta}$, *and* $P^* < 1$ *if* $\theta > \widehat{\theta}$.

Proof of Proposition 4

In the appendix.

The necessary condition for reverse price discrimination is intuitively appealing. First, $\underline{s}$ should not be too large because when $\underline{s}$ is close to 1, the right-hand side $1 - F_H(\underline{s})$ approaches to 0. Second, $\overline{s}$ should not be too small. As $\overline{s}$ decreases, $M'(\underline{s})$ in the left-hand side will be decreasing as well. Combining the two arguments, we can conclude that the two density functions should not overlap too much. That is to say, information should not be too imprecise. In addition, when we consider the two types of buyers fixed as in the baseline model, the cutoff $\widehat{\theta}$ must exist between θ_L and θ_H. Then, the type-L buyer is offered a higher price than the type-H buyer. Intuitively, for $\widehat{\theta}$ to be smaller than θ_H, $\underline{s}$ should not be too close to $\overline{s}$. It is because the seller's incentive to choose $P = 1$ is greater as the probability of drawing a signal $s \in (\underline{s}, 1]$ becomes larger. On the other hand, the seller's incentive to $P < 1$ is smaller because the probability of drawing a signal $s \in [\overline{s}, \underline{s}]$ becomes smaller. Thus, information should not be too precise either.

As an example, we can consider the case in which a bad signal follows the uniform distribution on $[0, \underline{s}]$, and a good signal follows the uniform distribution on $[\overline{s}, 1]$, where $\overline{s} < \underline{s}$. Note that when $s \in [\overline{s}, \underline{s}]$, the likelihood ratio $\frac{1-\overline{s}}{\underline{s}}$ is constant and buyers purchase the good as long as

$$\frac{\underline{s}\theta}{\underline{s}\theta + (1 - \overline{s})(1 - \theta)} \geq P.$$

When $P = 1$, the buyer purchases the good with probability $\frac{1-\underline{s}}{1-\overline{s}}\theta$. This is the probability that buyers draw a signal from $(\underline{s}, 1]$. On the other hand, when $P = \frac{\underline{s}\theta}{\underline{s}\theta+(1-\overline{s})(1-\theta)}$, the fraction of buyers $\theta + (1-\theta)\frac{\underline{s}-\overline{s}}{\underline{s}}$ purchase the good. This is the probability that buyers draw a signal from $[\overline{s}, 1]$. We can easily compute the seller's profit at each price as follows.

$$\begin{cases} \pi_H = \frac{1-\underline{s}}{1-\overline{s}}\theta & \text{at } P = 1, \text{ and} \\ \pi_L = \frac{\underline{s}\theta + (\underline{s}-\overline{s})(1-\theta)}{\underline{s}\theta + (1-\overline{s})(1-\theta)}\theta & \text{at } P = \frac{\underline{s}\theta}{\underline{s}\theta + (1-\overline{s})(1-\theta)}. \end{cases}$$

Both π_H and π_L are monotonically increasing in θ. We can find $\widehat{\theta} = \frac{(1-\overline{s})(1+\overline{s}-2\underline{s})}{\overline{s}(\underline{s}-\overline{s})}$ such that $\pi_H(\theta) \gtreqless \pi_L(\theta)$ as $\theta \lesseqgtr \widehat{\theta}$, where $\pi_H(\widehat{\theta}) = \pi_L(\widehat{\theta})$. For $\widehat{\theta}$ to exist between 0 and 1, we should have $1 + \overline{s} > 2\underline{s}$. Again, this condition implies that the two density functions should not overlap too much. Now, consider θ_H and θ_L fixed. Then, for $\widehat{\theta}$ to be smaller than θ_H, $\underline{s} - \overline{s}$ should be large enough. That is, the two density functions should not overlap too little.

The result is very similar to what has been shown in our basic model. When the buyer has a relatively low valuation, the seller charges the maximum price 1 and serves only the buyer who can be sure of the high quality. By contrast, when the buyer's valuation is greater than the threshold $\widehat{\theta}$, the seller offers a lower price to serve the buyers who are uncertain of the quality. The reverse price discrimination,

$$P(\theta_L) = 1 > \frac{\underline{s}\theta_H}{\underline{s}\theta_H + (1-\overline{s})(1-\theta_H)} = P(\theta_H),$$

arises if $\theta_L \in [0, \widehat{\theta}]$ and $\theta_H \in (\widehat{\theta}, 1]$. Of course, this is possible only when information is neither too precise nor too imprecise.

1.3.3 Non-unit Demand: Linear Demand Case

We now relax the buyer's unit demand to a linear demand function. A buyer's demand function is given by $D(P) = \theta - P$ where θ is the buyer's prior valuation about the good. Depending on what signal $s \in \{s_H, s_L\}$ the buyer draws, his posterior demand function is either $D(P) = p_H(\theta, \alpha) - P$ or $D(P) = p_L(\theta, \alpha) - P$. As $s \in \{s_H, s_L\}$ is the buyer's private information, a seller does not know the actual demand and therefore should calculate the

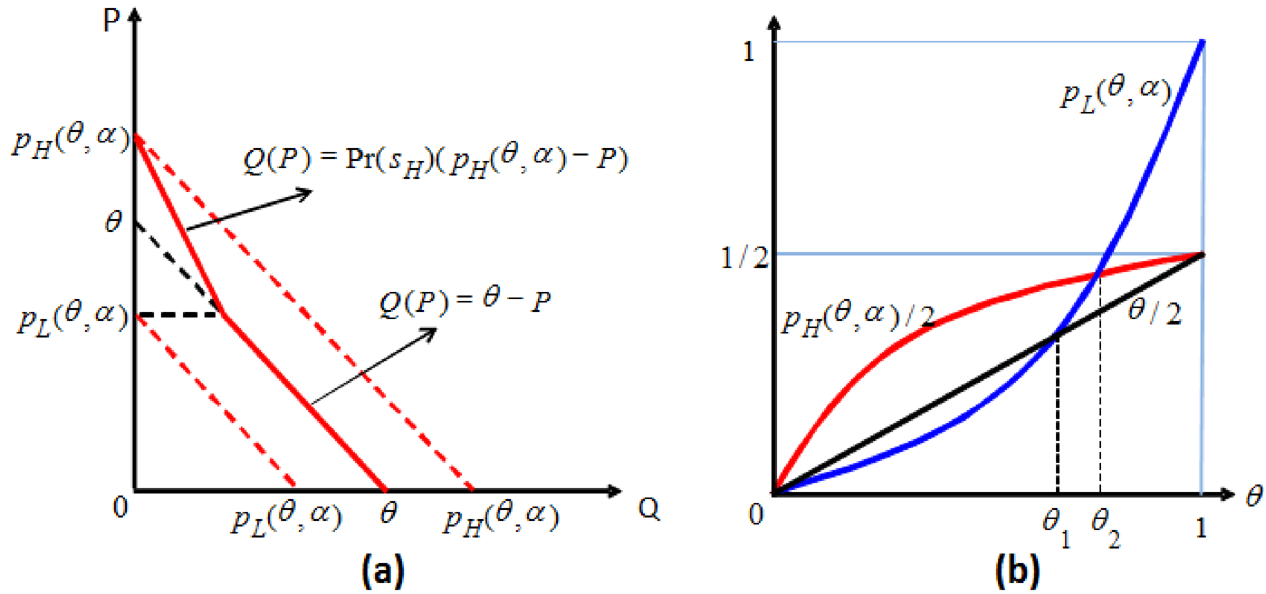

Figure 1.3: (a) Linear demand function (b) The optimal price

expected demand. If $P \leq p_L(\theta, \alpha)$, the expected demand is

$$D(P) = \Pr(s_H)\left(p_H(\theta,\alpha) - P\right) + \Pr(s_L)\left(p_L(\theta,\alpha) - P\right),$$

which turns out to be $D(P) = \theta - P$. On the other hand, if $P > p_L(\theta, \alpha)$, the buyer with a bad signal does not want to purchase and thus the seller faces $D(P) = p_H(\theta, \alpha) - P$ with probability $\Pr(s_H)$. Then, the seller's expected demand with the Bayesian buyer is inwardly kinked as shown in Figure 1.3 (a).

$$D(P) = \begin{cases} \theta - P & \text{if } P \leq p_L(\theta,\alpha), \\ \Pr(s_H)\left(p_H(\theta,\alpha) - P\right) & \text{if } P \in (p_L(\theta,\alpha), p_H(\theta,\alpha)], \\ 0 & \text{if } P > p_H(\theta,\alpha). \end{cases}$$

Since we assume zero production cost, the optimal price should be either $P^* = p_H(\theta, \alpha)/2$ or $\theta/2$. With a linear demand curve, the monopoly price is simply half of the price intercept. The analysis yields that the optimal price P^* depends on the value of α and θ as follows.

Lemma 1 *(1) Suppose that* $\alpha \in \left(\frac{1}{2}, \frac{\sqrt{5}-1}{2}\right)$*. Then,* $P^* = \theta/2$*. (2) Suppose that* $\alpha \in \left(\frac{\sqrt{5}-1}{2}, 1\right)$*. If* $\theta \in \left(0, \frac{\alpha^2+\alpha-1}{2\alpha-1}\right)$*,* $P^* = p_H(\theta, \alpha)/2$ *and if* $\theta \in \left(\frac{\alpha^2+\alpha-1}{2\alpha-1}, 1\right)$*,* $P^* = \frac{\theta}{2}$*.*

Proof of Lemma 1

In the appendix.

If the information quality is relatively low, i.e., $\alpha \in \left(\frac{1}{2}, \frac{\sqrt{5}-1}{2}\right)$, the optimal price is a monotone increasing function of θ. On the other hand, if the information quality is relatively high, i.e., $\alpha \in \left(\frac{\sqrt{5}-1}{2}, 1\right)$, there exist two optimal prices contingent on the prior valuation θ. The seller's decision is whether to charge a low price, $\theta/2$, for the sake of obtaining a high demand or charge a high price, $p_H(\theta, \alpha)/2$, for the sake of earning a high margin.

Keep in mind (1.4): $p_H(\theta, \alpha)$ is increasing in α and $p_L(\theta, \alpha)$ is decreasing in α. When α is relatively low, the demand with a bad (good) signal is not much lower (higher). In this case, charging the low price for serving a buyer with a bad signal is more profitable. On the other hand, when α is relatively high, the demand with a bad (good) signal becomes low (high) enough so that sometimes charging a high price can be more profitable. More precisely, charging a high price is optimal if θ is relatively low. Otherwise, with a high θ, the demand with a bad signal is not likely to be low enough for the high price to be optimal.[13]

Figure 1.3 (b) demonstrates the optimal price P^*, when $\alpha \in \left(\frac{\sqrt{5}-1}{2}, 1\right)$, as a function of θ. There is a downward jump in the optimal price at a certain threshold. As a result, the optimal price is not monotone with respect to θ. Then we can show the parameter set of θ for which the reverse price discrimination is obtained.[14]

Proposition 5 *Suppose that* $\alpha \in \left(\frac{\sqrt{5}-1}{2}, 1\right)$*. If* $\theta_L \in \left(\frac{\alpha+\alpha^2-1}{\alpha+2\alpha^2-1}, \frac{\alpha+\alpha^2-1}{2\alpha-1}\right)$ *and* $\theta_H \in \left(\frac{\alpha+\alpha^2-1}{2\alpha-1}, \dot{\theta}\right)$ *where* $\dot{\theta} = \frac{\alpha\theta_L}{(2\theta_L\alpha-\alpha-\theta_L+1)}$*, then* $P^*(\theta_L) > P^*(\theta_H)$ *for* $\theta_L < \theta_H$*.*

[13] If α is sufficiently high, we need very high prior valuation in order to sustain this low optimal price.

[14] In other words, here we find the necessary condition for reverse price discrimination, which says that information should not be too imprecise. When we consider θ_H and θ_L fixed, we can find the sufficient condition for α which should not be too large. This can be easily seen when α is cloase to 1, because $\theta_H \notin \left(\frac{\alpha+\alpha^2-1}{2\alpha-1}, \dot{\theta}\right)$ in this case.

1.4 Concluding Remarks

The model presented in this paper studies price discrimination under partially incomplete information in the sense that a buyer's prior valuation can be observed by the seller(s) but the buyer further draws a private signal which may give him or her additional information about a product sold by the seller(s). In this environment, we demonstrate the possibility that the buyer with a higher willingness to pay is offered a lower price and vice versa, since the seller only targets the buyers who draw a good signal by charging a high price when they are of a low type, whereas she wants to serve all of the buyers by offering a low price when they are of a high type.

A possible criticism to our main finding is that reverse price discrimination is not surprising because it is an existing result in the literature of statistical discrimination. According to the reasoning of statistical discrimination, the price reversion is mainly induced by the difference in the variance of posterior distributions.[15] But this is simply not true in our model because the optimal pricing is determined by the price elasticity of demand, not by the variance. Another criticism is that one can easily construct an example of two distributions where the optimal price is non-monotone in the mean valuation. That is, it can be argued that reverse price discrimination can arise without information inference. However, only providing a necessary condition is not the main point of our paper. We are not trying to find the arbitrary distributions, without economic motivation, for which reverse price discrimination can be derived. Our model shows how the posterior valuations generating reverse price discrimination are formed *endogenously* in terms of information quality. In particular, our paper answers what product markets and what characteristics of products

[15]This idea originated from Phepls (1972) that proposes the seminal model in which group average is used as a proxy for unobserved exogenous difference that are relevant to economic outcomes. See Fang and Moro (2011) for an excellent survey of the theories of statistical discrimination. In fact, Ayres and Siegelman (1995) borrow this idea to explain discrimination in car sales markets and argue "it may be profitable for dealers to offer higher prices to a group of consumers who have a lower average reservation price, if the variance of reservation prices within the group is suffciently large." Certainly, this is possible. But this is not the mechanism of our model. For example, there is a case where the optimal price is higher with a lower variance in our model.

give rise to reverse price discrimination.

APPENDIX

Appendix

Proof of Proposition 1

$\pi_H(\theta_i, \alpha) \gtreqless \pi_L(\theta_i, \alpha) \Longrightarrow \alpha\theta_i \gtrless \frac{(1-\alpha)\theta_i}{\alpha(1-\theta_i)+(1-\alpha)\theta_i} \Longrightarrow \theta_i\alpha^2 (1-2\theta_i) + \alpha\left(\theta_i + \theta_i^2\right) - \theta_i \gtrless 0$. Let $f(\alpha) = \theta_i\alpha^2 (1-2\theta_i) + \alpha\left(\theta_i + \theta_i^2\right) - \theta_i$.

Case (1) $\theta = \theta_H > \frac{1}{2}$

Then, (a) $f(\alpha)$ is a concave function, (b) $f(\alpha)$ attains max value at

$$\alpha = \frac{\theta_H + 1}{2(2\theta_H - 1)} > 1$$

and

$$f\left(\alpha = \frac{\theta_H + 1}{2(2\theta_H - 1)}\right) = \frac{(\theta_H - 5)(\theta_H - 1)\theta_H}{4(2\theta_H - 1)} > 0,$$

(c) $f\left(\alpha = \frac{1}{2}\right) = \left(-\frac{1}{4}\right)\theta_H < 0$, (d) $f(\alpha = 1) = -\theta_H(\theta_H - 1) > 0$. So $\exists\alpha_2$ such that if $\alpha \in \left(\frac{1}{2}, \alpha_2\right]$, $f(\alpha) \leq 0$ and if $\alpha \in (\alpha_2, 1)$, $f(\alpha) > 0$. This implies that if $\alpha \in \left(\frac{1}{2}, \alpha_2\right]$, $\pi_H(\theta_H, \alpha) \leq \pi_L(\theta_H, \alpha)$ and if $\alpha \in (\alpha_2, 1)$, $\pi_H(\theta_H, \alpha) > \pi_L(\theta_H, \alpha)$.

Case (2) $\theta = \theta_L < \frac{1}{2}$

Then, (a) $f(\alpha)$ is a convex function, (b) $f(\alpha)$ attains min value at

$$\alpha = \frac{\theta_L + 1}{2(2\theta_L - 1)} < \frac{1}{2}$$

and

$$f\left(\alpha = \frac{\theta_L + 1}{2(2\theta_L - 1)}\right) = \frac{(\theta_L - 5)(\theta_L - 1)\theta_L}{4(2\theta_L - 1)} < 0,$$

(c) $f\left(\alpha = \frac{1}{2}\right) = \left(-\frac{1}{4}\right)\theta_L < 0$, (d) $f(\alpha = 1) = -\theta_L(\theta_L - 1) > 0$. So $\exists\alpha_1$ such that if $\alpha \in \left(\frac{1}{2}, \alpha_1\right)$, $f(\alpha) < 0$ and if $\alpha \in [\alpha_1, 1)$, $f(\alpha) \geq 0$. This implies that if $\alpha \in \left(\frac{1}{2}, \alpha_1\right)$, $\pi_H(\theta_L, \alpha) < \pi_L(\theta_L, \alpha)$ and if $\alpha \in [\alpha_1, 1)$, $\pi_H(\theta_L, \alpha) \geq \pi_L(\theta_L, \alpha)$.

Now let us define

$$\alpha^*(\theta_i) = \frac{1}{2\theta_i - 1}\left(\frac{1}{2}\theta_i - \frac{1}{2}\sqrt{-6\theta_i + \theta_i^2 + 5} + \frac{1}{2}\right).$$

The computation yields that $\alpha_2 = \alpha^*(\theta_i = \theta_H)$ and $\alpha_1 = \alpha^*(\theta_i = \theta_L)$. Also

$$\frac{\partial(\alpha^*(\theta_i))}{\partial\theta_i} = -\frac{\left(5\theta_i - 7 + 3\sqrt{\theta_i^2 - 6\theta_i + 5}\right)}{2(2\theta_i - 1)^2\left(\sqrt{\theta_i^2 - 6\theta_i + 5}\right)} > 0.$$

Here,

$$\left(3\sqrt{\theta^2 - 6\theta + 5}\right)^2 - (7 - 5\theta)^2 = (-4)(2\theta - 1)^2 < 0,$$

which implies $3\sqrt{\theta^2 - 6\theta + 5} < 7 - 5\theta$ because $3\sqrt{\theta^2 - 6\theta + 5} > 0$ and $7 - 5\theta > 0$ for $\theta \in [0, 1]$. As the numerator is negative, $\frac{\partial(\alpha^*(\theta_i))}{\partial\theta_i} > 0$. Then, this implies that $\alpha_2 > \alpha_1$ because $\theta_H > \frac{1}{2} > \theta_L$. Then, (i) If $\alpha \in \left(\frac{1}{2}, \alpha_1\right)$, $\pi_H(\theta_i, \alpha) < \pi_L(\theta_i, \alpha)$. (ii) If $\alpha \in [\alpha_1, \alpha_2]$, $\pi_H(\theta_L, \alpha) > \pi_L(\theta_L, \alpha)$ and $\pi_H(\theta_H, \alpha) < \pi_L(\theta_H, \alpha)$. (iii) If $\alpha \in (\alpha_2, 1)$, $\pi_H(\theta_i, \alpha) > \pi_L(\theta_i, \alpha)$. Note that $\pi_H(\theta_i, \alpha)$ ($\pi_L(\theta_i, \alpha)$) is the profit when a seller charges $p_H(\theta_i, \alpha)$ ($p_L(\theta_i, \alpha)$) for type i. Then this proves Proposition 1. ■

Proof of Proposition 3

Let us define $\widetilde{\theta}_H(\theta_L)$ such that $p_L(\widetilde{\theta}_H(\theta_L), \alpha) = p_H(\theta_L, \alpha)$ for given θ_L. Then, $p_L(\theta_H, \alpha) < p_H(\theta_L, \alpha)$ for $\theta_L < \theta_H < \widetilde{\theta}_H(\theta_L)$. By solving

$$\frac{\alpha\theta_L}{\alpha\theta_L + (1-\alpha)(1-\theta_L)} = \frac{(1-\alpha)\widetilde{\theta}_H(\theta_L)}{\alpha(1-\widetilde{\theta}_H(\theta_L)) + (1-\alpha)\widetilde{\theta}_H(\theta_L)}$$

in terms of $\widetilde{\theta}_H(\theta_L)$, we obtain

$$\widetilde{\theta}_H(\theta_L) = \frac{\alpha^2\theta_L}{\alpha^2\theta_L + (1-\alpha)^2(1-\theta_L)}.$$

Or if we define $\widetilde{\theta}_L(\theta_H)$ such that $p_H(\widetilde{\theta}_L(\theta_H), \alpha) = p_L(\theta_H, \alpha)$ for given θ_H, $p_L(\theta_H, \alpha) <$

$p_H(\theta_L, \alpha)$ for $\widetilde{\theta}_L(\theta_H) < \theta_L < \theta_H$. By solving

$$\frac{(1-\alpha)\theta_H}{\alpha(1-\theta_H)+(1-\alpha)\theta_H} = \frac{\alpha\theta_L}{\alpha\theta_L+(1-\alpha)(1-\theta_L)},$$

we obtain

$$\widetilde{\theta}_L(\theta_H) = \frac{(\alpha-1)^2\,\theta_H}{\left(\theta_H - 2\alpha\theta_H + \alpha^2\right)}.$$

■

Proof of Proposition 4

Note that we are considering the case where $s \in [\bar{s}, \underline{s}]$. When $P = 1$, $\pi_H(\theta) = \theta(1 - F_H(\underline{s}))$. When $P < 1$, the seller solves

$$\begin{aligned} \underset{P}{Max}\ \ \pi_L &= P\left\{\theta(1-F_H(\widehat{s})) + (1-\theta)(1-F_L(\widehat{s}))\right\} \\ s.t.\ \ M(\widehat{s}) &= \frac{\theta}{(1-\theta)}\frac{1-P}{P}. \end{aligned}$$

We can find the optimal price from the first-order condition,

$$D(\widehat{s}) - P\left[\theta f_H(\widehat{s}) + (1-\theta)f_L(\widehat{s})\right]\frac{\partial\widehat{s}}{\partial P} = 0. \tag{1.9}$$

Now, what we want to show is that there must exist a cutoff value $\widehat{\theta}$ such that $\pi_H(\theta) \gtreqless \pi_L(\theta)$ as $\theta \lesseqgtr \widehat{\theta}$. First, $\pi_H(\theta = 0) = \lim_{\theta\to 0}\pi_L(\theta) = 0$ and both $\pi_H(\theta)$ and $\pi_L(\theta)$ are non-decreasing in θ. It is easy to see that $\pi_H(\theta)$ is linearly increasing in θ. To investigate the slope of $\pi_L(\theta)$, we use the envelop theorem and obtain

$$\frac{\partial\pi_L}{\partial\theta} = P^*\left[\{F_L(\widehat{s}) - F_H(\widehat{s})\} - \{\theta f_H(\widehat{s}) + (1-\theta)f_L(\widehat{s})\}\frac{\partial\widehat{s}}{\partial\theta}\right] \geq 0. \tag{1.10}$$

This is always non-negative. The first term in the bracket, $F_L(\widehat{s}) - F_H(\widehat{s})$, represents the first-order stochastic dominance which always holds under the MLRP. The second term is negative because of $\frac{\partial \widehat{s}}{\partial \theta} \leq 0$.

Second, when θ is close to 1, we have $\pi_H(\theta) < \pi_L(\theta)$. Let us show that

$$\lim_{\theta \to 1} \pi_L > \pi_H(\theta = 1) = 1 - F_H(\underline{s}).$$

It can be shown that $\lim_{\theta \to 1} \pi_L = 1$. Intuitively, as θ is close to 1, the posterior belief (1.7) approaches to 1. In other words, when buyers believe almost surely that the good is of high quality, the seller's optimal price is 1 and buyers decide to purchase the good. Formally, let us show that

$$\lim_{\theta \to 1} \pi_L = \left(\lim_{\theta \to 1} P^* \right) \cdot \left(\lim_{\theta \to 1} D(\widehat{s}(P^*)) \right) = 1.$$

It is immediate to obtain $\lim_{\theta \to 1} D(\widehat{s}(P^*)) = 1 - F_H(\overline{s}) = 1$ because $\lim_{\theta \to 1} \widehat{s} = \overline{s}$. To prove $\lim_{\theta \to 1} P^* = 1$, let us show that the first-order condition is always positive as θ approaches to 1. Applying the implicit function theorem, the first-order condition (1.9) can be rewritten as

$$D(\widehat{s}) + \frac{\theta}{1 - \theta} \frac{1}{M'(\widehat{s})} \frac{1}{P} \left[\theta f_H(\widehat{s}) + (1 - \theta) f_L(\widehat{s}) \right] = 0.$$

Note that the second term is indeterminate in the limit because $\frac{\theta}{1-\theta}$ goes to infinity and $M'(\widehat{s})$ goes to ∞. Once we apply L'hopital's rule, it can be shown that the second term turns out to be 0. Then, as the first-order condition is always positive, the optimal pricing is to increase the price as much as possible. Hence, $P^* = 1$.

Next, let us find the condition for

$$\lim_{\theta \to 0} \frac{\partial \pi_L}{\partial \theta} < (1 - F_H(\underline{s})).$$

This implies that when θ is close to 0, we must have $\pi_H(\theta) > \pi_L(\theta)$. We find the limit value of (1.10) at $\theta \to 0$ as

$$\lim_{\theta \to 0} \frac{\partial \pi_L}{\partial \theta} = -\frac{f_L(\underline{s})}{M'(\underline{s})}.$$

This can be derived in the following steps. As θ approaches to 0, the posterior belief (1.7) becomes closer to 0 as well and so P^* must be 0. The formal proof is analogous to the one given for the case of $\theta \to 1$. If $\theta \to 0$, the first term becomes 0 because it goes to $P^*\{F_L(\widehat{s}) - F_H(\widehat{s})\}$. Now, let us investigate the second term, $-P^*\{\theta f_H(\widehat{s}) + (1-\theta) f_L(\widehat{s})\}\frac{\partial \widehat{s}}{\partial \theta}$. Using the implicit function theorem,

$$\frac{\partial \widehat{s}}{\partial \theta} = \frac{1}{(1-\theta)^2 M'(\widehat{s})} \frac{1-P^*}{P^*}.$$

The second term is rewritten as

$$-\{\theta f_H(\widehat{s}) + (1-\theta) f_L(\widehat{s})\} \frac{1}{(1-\theta)^2 M'(\widehat{s})} (1-P^*).$$

As θ approaches to 0, $\widehat{s}$ becomes $\underline{s}$. Consequently, this term turns out to be $-\frac{f_L(\underline{s})}{M'(\underline{s})}$. Note that the sign of this term is positive because the likelihood ratio, $M(s)$, is decreasing. As a result, if $-\frac{f_L(\underline{s})}{M'(\underline{s})}$ is smaller than $(1 - F_H(\underline{s}))$, at least there should exist one cutoff value $\widehat{\theta}$ such that $\pi_H \gtreqless \pi_L$ as $\theta \lesseqgtr \widehat{\theta}$. ■

Proof of lemma 1

Case (1) $P = p_H(\theta) - \frac{D}{\Pr(s_H)}$

In this case, the optimal price is

$$P^* = \frac{p_H(\theta, \alpha)}{2} = \frac{\alpha\theta}{2(2\theta\alpha - \alpha - \theta + 1)}$$

and it is binding only if $p_L(\theta, \alpha) < P^* < p_H(\theta, \alpha)$ is satisfied. It is always true that $P^* < p_H(\theta, \alpha)$ because

$$P^* - p_H(\theta, \alpha) = -\frac{\alpha\theta}{2(\theta(2\alpha - 1) + (1-\alpha))} < 0.$$

Also

$$P^{*}-p_{L}\left(\theta,\alpha\right)=-\frac{\left(2\theta+4\alpha-5\theta\alpha-\alpha^{2}+2\theta\alpha^{2}-2\right)\theta}{2\left(\theta\left(2\alpha-1\right)+\left(1-\alpha\right)\right)\left(2\theta\alpha-\alpha-\theta\right)}$$

where $2\theta\alpha-\alpha-\theta<0$ for all $\alpha\in\left(\frac{1}{2},1\right)$ and $\theta\in(0,1)$. The analysis of the numerator yields the following result.

Result 1. $p_{L}\left(\theta,\alpha\right)<P^{*}$ holds if $\alpha\in\left(2-\sqrt{2},1\right)$ and $\theta\in\left(0,\frac{\alpha^{2}-4\alpha+2}{2\alpha^{2}-5\alpha+2}\right)$.

Case (2) $P=\theta-D$

In this case, the optimal price is $P^{*}=\frac{\theta}{2}$ and it is binding only if

$$0<P^{*}<p_{L}\left(\theta,\alpha\right)=\frac{(1-\alpha)\theta}{\alpha(1-\theta)+(1-\alpha)\theta}$$

is satisfied. Here,

$$P^{*}-p_{L}\left(\theta,\alpha\right)=\frac{\left(2\theta\alpha-3\alpha-\theta+2\right)\theta}{2\left(2\theta\alpha-\alpha-\theta\right)},$$

where $2\theta\alpha-\alpha-\theta<0$ always. Then the analysis of the numerator yields the following result.

Result 2. $P^{*}<p_{L}\left(\theta,\alpha\right)$ holds in following two cases: (i) If $\alpha\in\left(\frac{2}{3},1\right)$ and $\theta\in\left(\frac{3\alpha-2}{2\alpha-1},1\right)$ or (ii) if $\alpha\in\left(\frac{1}{2},\frac{2}{3}\right)$ and $\theta\in\left(\frac{1}{2},1\right)$.

Then, from Results 1 and 2, the optimal price P^{*} can be described as follows.

Result 3. (1) If $\alpha\in\left(\frac{1}{2},2-\sqrt{2}\right)$, $P^{*}=\frac{\theta}{2}$. (2) Suppose that $\alpha\in\left(2-\sqrt{2},\frac{2}{3}\right)$. (i) If $\theta\in\left(0,\frac{\alpha^{2}-4\alpha+2}{2\alpha^{2}-5\alpha+2}\right)$, there exist two candidates $P^{*}=\frac{\alpha\theta}{2(2\theta\alpha-\alpha-\theta+1)}$ and $\frac{\theta}{2}$. (ii) If $\theta\in\left(\frac{\alpha^{2}-4\alpha+2}{2\alpha^{2}-5\alpha+2},1\right)$, $P^{*}=\frac{\theta}{2}$. (3) Suppose that $\alpha\in\left(\frac{2}{3},1\right)$. (i) If $\theta\in\left(0,\frac{3\alpha-2}{2\alpha-1}\right)$, $P^{*}=\frac{\alpha\theta}{2(2\theta\alpha-\alpha-\theta+1)}$, (ii) if $\theta\in\left(\frac{3\alpha-2}{2\alpha-1},\frac{\alpha^{2}-4\alpha+2}{2\alpha^{2}-5\alpha+2}\right)$, there exist two candidates: $P^{*}=\frac{\alpha\theta}{2(2\theta\alpha-\alpha-\theta+1)}$ and $\frac{\theta}{2}$, (iii) if $\theta\in\left(\frac{\alpha^{2}-4\alpha+2}{2\alpha^{2}-5\alpha+2},1\right)$, $P^{*}=\frac{\theta}{2}$.

Now, for given two candidates $P^{*}=\frac{\alpha\theta}{2(2\theta\alpha-\alpha-\theta+1)}$ and $\frac{\theta}{2}$, we check which price yields a greater profit to the seller. When $P^* = \frac{\alpha\theta}{2(2\theta\alpha-\alpha-\theta+1)}$, the profit is denoted by

$$\pi(P^* = \frac{p_H(\theta)}{2}) = \frac{\alpha^2\theta^2}{4(2\theta\alpha-\alpha-\theta+1)}$$

and when $P^* = \frac{\theta}{2}$, the profit is denoted by

$$\pi(P^* = \frac{\theta}{2}) = \frac{\theta^2}{4}.$$

The comparison of both profits yields the following result.

Result 4. (1) If $\alpha < \frac{\sqrt{5}-1}{2}$, $\pi(P^* = \frac{p_H(\theta,\alpha)}{2}) < \pi(P^* = \frac{\theta}{2}) \Longrightarrow P^* = \frac{\theta}{2}$. (2) When $\alpha > \frac{\sqrt{5}-1}{2}$, if $\theta \in \left(0, \frac{\alpha+\alpha^2-1}{2\alpha-1}\right)$, $\pi(P^* = \frac{p_H(\theta,\alpha)}{2}) > \pi(P^* = \frac{\theta}{2}) \Longrightarrow P^* = \frac{\alpha\theta}{2(2\theta\alpha-\alpha-\theta+1)}$ and if $\theta \in \left(\frac{\alpha+\alpha^2-1}{2\alpha-1}, 1\right)$, $\pi(P^* = \frac{p_H(\theta,\alpha)}{2}) < \pi(P^* = \frac{\theta}{2}) \Longrightarrow P^* = \frac{\theta}{2}$.

We apply Result 4 to 2-(i) and 3-(ii), Result 3 can be simplified as Lemma 3.

(1) Suppose that $\alpha \in \left(\frac{1}{2}, \frac{\sqrt{5}-1}{2}\right)$. Then $P^* = \frac{\theta}{2}$.

(2) Suppose that $\alpha \in \left(\frac{\sqrt{5}-1}{2}, 1\right)$. If $\theta \in \left(0, \frac{\alpha+\alpha^2-1}{2\alpha-1}\right)$, $P^* = \frac{\alpha\theta}{2(2\theta\alpha-\alpha-\theta+1)}$ and if $\theta \in \left(\frac{\alpha+\alpha^2-1}{2\alpha-1}, 1\right)$, $P^* = \frac{\theta}{2}$. ■

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Chapter 2

Reverse Price Discrimination with Competition: A Hotelling Duopoly

2.1 Introduction

In the previous chapter, we show that prices can be non-monotone in buyers' *prior* valuation - the seller may offer a higher price to low valuation buyers than to high valuation buyers. Our result seems contradictory to the general conjecture that it would be optimal to charge a higher (lower) price if buyers are with high (low) valuation. The intuition underlying this result is as follows. When the seller makes a decision on the prices, she faces a traditional trade-off between getting a higher margin and getting a greater market share. This trade-off depends crucially on the price elasticity of demand, which is essentially determined by the interaction between a buyer's prior valuations and the precision of information available to the buyer. When information precision is not extreme (i.e., neither too accurate nor too vague), we find that the demand is elastic (inelastic) when the buyer's prior valuation is high (low). Thus, if the buyer's prior valuation is high, the seller chooses to offer a low price because the buyer's updated valuation does not decline significantly even when he receives unfavorable private information. On the other hand, if his prior valuation is low, it is more profitable for the seller to charge a high price and target only the buyer who receives favorable private information because the price under which the buyer with unfavorable information is covered is too low to be profitable. This leads to the result that a higher price can be offered to buyers with lower willingness to pay and vice versa.

We now want to explore a similar phenomenon in a duopoly market to see if reverse price discrimination still occurs even with competition between sellers. Consider that the competing sellers share buyers' prior valuations and the buyers gather further private information about the product or service provided by each seller. Again, we show that reverse price discrimination can arise in this case as well. In the duopoly case, new information plays a role of differentiating two products. Especially when the buyer gathers opposite information about each product (for example, the buyer receives a good signal from one seller while obtaining a bad signal from the other seller), he perceives that two products are more differentiated than before and thus the price competition between the sellers can be mitigated.

We find that the degree of differentiation becomes higher, and so does the equilibrium price, when the buyer has an intermediate prior valuation. The intuition is straightforward. The buyer with intermediate prior valuation is not certain whether the product would be a good match or a bad match for him, thus he is quite sensitive to the new information he gathers. Hence, when he receives opposite information from the two products he tends to be strongly biased toward the product from which he obtains relatively favorable information. This, in turn, gives the sellers a weaker incentive to offer a lower price to attract them, since the buyer with biased preferences is less likely to change his minds by price differential. For the opposite reason, the buyer with extreme (either high or low) prior valuation has the advantage of being offered a lower price, since he is likely to perceive that the two products are still similar even after updating his valuations. As a result, the equilibrium prices do not change monotonically with the buyer's prior valuation. This yields the possibility that the buyer with lower prior valuation is charged the higher price than the buyer with higher prior valuation.

The topic of third-degree price discrimination, which considers the case where the characteristics of buyers (who have no private information) are observable, by a monopolist has been covered extensively in literature after the seminal work of Robinson (1933).[1] In particular, Borenstein (1985), Holmes (1989), Corts (1998), and Chen (1999) are the recent papers which extend the analysis of third-degree price discrimination to the setting of oligopoly. Price discrimination with incomplete information has been studied in the environment of screening or self-selection mechanism followed by Mussa and Rosen (1978) and Maskin and Riley (1984). In particular, Lewis and Sappington (1994) and Courty and Li (2000) are closely related to ours because they also study the environment where consumers are initially uncertain of their valuations that are in part revealed afterwards. Both papers study second-degree price discrimination in which firms are assumed not to be able to observe consumers' expected valuations. On the other hand, in our paper, the sellesr are able to observe consumers' expected valuations and choose their prices based on them.

The rest of this article proceeds as follows. Section 2.2 describes the structure and assumptions of the basic model. Section 2.3 shows the possibilities of reverse price discrimination in duopoly case. Section 2.4 concludes.

2.2 The Model

In this section, we explain the basic setup of the model, which is consistent with the monopoly case in the previous chapter.

Players. The buyers have a unit demand for the good, which is supplied by two sellers. The true value of the good depends on the match between a buyer's preference and the features of the good, which is denoted by $v \in \{H, L\}$. If $v = H$, the good is a good match with a buyer and if $v = L$, it is a bad match. A buyer's prior belief for $v = H$ is denoted as $\theta \in [0, 1]$. We normalize the buyer's valuation for the good to be 1 for $v = H$ and 0 for $v = L$. Accordingly, θ can be thought of as the buyer's *prior* valuation for the good and therefore as the willingness to pay. The buyer's type θ is public information and observable

[1] Armstrong (2006), Stole (2007) and, Varian (1989) are the excellent survey papers about price discrimination.

to the sellers. For simplicity, the reservation value of sellers for the good is assumed to be 0.

Information. The buyers receive private information about the good while they are inspecting the good before purchase. Although the buyers are ex ante homogeneous, they may draw different binary signals, $s \in \{s_H, s_L\}$, on their match value. The realization of a signal is privately observed by the buyers, so it is private information. As is standard, the signals partially reveal the true match value of the good in the sense of Blackwell,

$$\begin{aligned} \Pr(s_H|v &= H) = \Pr(s_L|v = L) = \alpha \\ \Pr(s_L|v &= H) = \Pr(s_H|v = L) = 1 - \alpha \end{aligned}$$

where $\alpha \in \left(\frac{1}{2}, 1\right)$ without loss of generality. α is often interpreted as the precision or the quality of a signal and is common knowledge.

As the buyer receives a signal $s \in \{s_H, s_L\}$, he updates his beliefs on the match value for the good. Let us refer to $p_H(\theta, \alpha)$ and $p_L(\theta, \alpha)$ as the buyer's posterior valuation when the buyer with a prior $\theta \in (0, 1)$ draws a signal s_H and s_L, respectively. Then, Bayes' rule leads to

$$\begin{aligned} p_H(\theta, \alpha) &= \frac{\alpha\theta}{\alpha\theta + (1-\alpha)(1-\theta)} \text{ and} \\ p_L(\theta, \alpha) &= \frac{(1-\alpha)\theta}{\alpha(1-\theta) + (1-\alpha)\theta} \end{aligned}$$

because we normalize the buyer's valuation for the good to be 1 for $v = H$ and 0 for $v = L$.

The buyer's posterior valuation is a mean-preserving spread of his prior belief. A signal disperses the prior belief to two-point distribution: $p_H(\theta, \alpha)$ with the probability $\Pr(s_H)$ and $p_L(\theta, \alpha)$ with the probability $\Pr(s_L)$. The probabilities that the buyer receives a good signal and bad signal are given by

$$\Pr(s_H) = \sum_{v \in \{L,H\}} \Pr(s_H|v) \Pr(v) \quad \text{and} \quad \Pr(s_L) = \sum_{v \in \{L,H\}} \Pr(s_L|v) \Pr(v),$$

respectively.

Timing. The timing of the game is as follows. In the first stage, the sellers can observe the buyer's prior valuation θ perfectly. At the same time, the buyer draws a private signal about the match value of the good from each of two sellers. In the second stage, the sellers simutaneously choose their prices, and then the buyers make a purchasing decision given the prices.

2.3 Duopoly: Hotelling Model

We have shown in the previous chapter that reverse price discrimination can be derived in the monopoly case. Then, can it also be the case even when sellers compete with each other? In this chapter, we extend our analysis to a duopoly market and consider the case where two sellers are located at two end points on the Hotelling line of unit length. We also show that the equilibrium prices can be non-monotone in the buyer's prior valuation in the duopoly case.

Two sellers, A and B, supply differentiated products A and B respectively. As in a standard Hotelling model, buyers are uniformly distributed and indexed as $x \in [0, 1]$ which denotes each buyer's location or brand preference. Buyers purchase either one unit of a good from only one seller or nothing. Type (θ, x) buyer's value is $w + \theta - tx$ for good A and $w + \theta - t(1 - x)$ for good B. Note that $w + \theta$ is the intrinsic value of consuming the product. While w captures the product value with no uncertainty, θ represents the value with uncertainty. We assume that w is sufficiently large so that the market is fully covered. The parameter $t > 0$ reflects the degree of product differentiation. To keep consistency with the monopoly case, we assume that the buyers' prior valuations are public information and known to sellers. For analytical simplicity, we assume that the symmetric information structure is exogenously given to the sellers.

Now the buyers independently receive a private signal from each seller. They face

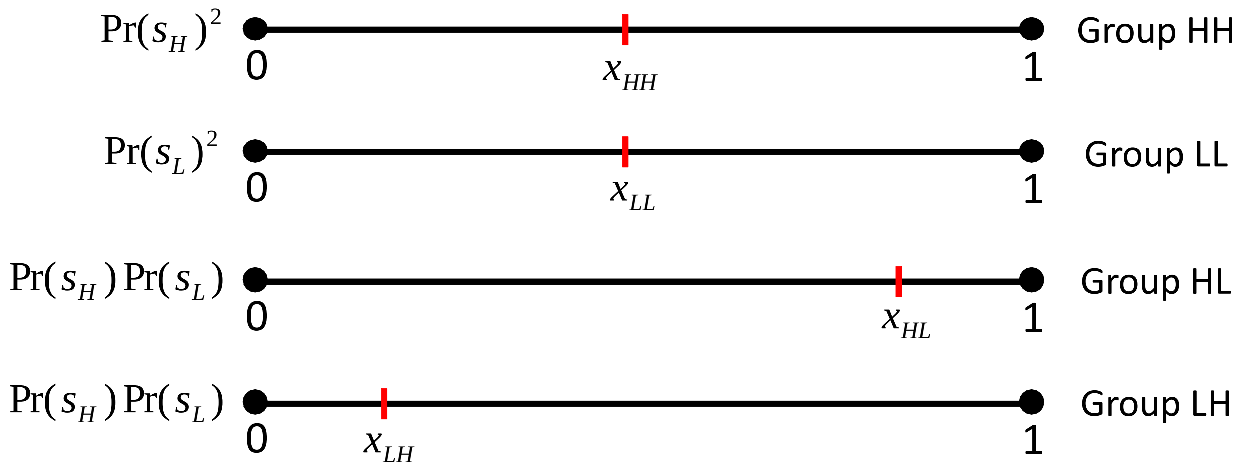

Figure 2.1: Demand in the Hotelling model

one of four possible cases according to the combinations of signal realizations $\left(s^A, s^B\right)$, where $s^A, s^B \in \{s_H, s_L\}$ denote the signals from sellers A and B. As both signals s^A and s^B are buyers' private information, each seller expects the possible outcome as follows: (i) $\left(s^A, s^B\right) = (s_H, s_H)$ with probability $\Pr(s_H)^2$, (ii) $\left(s^A, s^B\right) = (s_L, s_L)$ with probability $\Pr(s_L)^2$, (iii) $\left(s^A, s^B\right) = (s_H, s_L)$ with probability $\Pr(s_H)\Pr(s_L)$, and (iv) $\left(s^A, s^B\right) = (s_L, s_H)$ with probability $\Pr(s_H)\Pr(s_L)$.

Figure 2.1 illustrates the demand structure as a result of the realization of private signals. If the buyers receive the same signals from both sellers, i.e., $\left(s^A, s^B\right) = (s_H, s_H)$ or (s_L, s_L), the private signals do not provide additional information about which good would be more preferable than the other. As their relative preference between two goods does not change, there is no difference from the standard Hotelling model in this case. Hence the location of marginal buyers who are indifferent between the two goods is denoted by

$$x_{HH} = x_{LL} = \frac{1}{2} + \frac{P^B - P^A}{2t} \tag{2.1}$$

where the subscription for x denotes the group as illustrated in Figure 4, and P^A and P^B are the prices offered by sellers A and B, respectively.[2]

[2]We only consider the interior solution such as $0 < x_{HH}, x_{LL} < 1$. This implies that

On the other hand, if the different signals are drawn from two sellers, the marginal buyers for the case where $\left(s^A, s^B\right) = (s_H, s_L)$ and (s_L, s_H) are, respectively,

$$x_{HL} = \frac{1}{2} + \frac{P^B - P^A + \Delta(\theta,\alpha)}{2t} \text{ and } x_{LH} = \frac{1}{2} + \frac{P^B - P^A - \Delta(\theta,\alpha)}{2t} \tag{2.2}$$

where $\Delta(\theta,\alpha) \equiv p_H(\theta,\alpha) - p_L(\theta,\alpha)$. Here, $\Delta(\theta,\alpha)$ represents a buyer's bias after observing different signals. Note that depending on the size of the bias, the marginal buyers may not exist in the Hotelling line. That is, x_{HL} can be greater than 1 and/or x_{LH} can be smaller than 0.

Lemma 2 *(1) If $\Delta(\theta,\alpha) < t$, then $0 < x_{HL} < 1$ and $0 < x_{LH} < 1$. (2) If $\Delta(\theta,\alpha) \geq t$, then $x_{HL} \geq 1$ and $x_{LH} \leq 0$.*

Proof of Lemma 2

In the appendix.

When $\Delta(\theta,\alpha) < t$, seller A's demand function is written by

$$D_A = (\Pr(s_H)^2 + \Pr(s_L)^2)x_{HH} + \Pr(s_H)\Pr(s_L)(x_{HL} + x_{LH}).$$

In this case, since the bias is small, the marginal buyers exist and stand on the interior point in the Hotelling line. By contrast, when $\Delta(\theta,\alpha) \geq t$, seller A's demand function is given by

$$D_A = (\Pr(s_H)^2 + \Pr(s_L)^2)x_{HH} + \Pr(s_H)\Pr(s_L).$$

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the prices difference is less than the product differentiation. For example, if $P^A - P^B > t$, $\frac{1}{2} + \frac{P^B - P^A}{2t} < 0$ and if $P^B - P^A > t$, $\frac{1}{2} + \frac{P^B - P^A}{2t} > 1$. That is, we exclude the possibility that seller $j \in \{A, B\}$ dominates the market of group HH and LL only due to the price effects such that P^j is far lower than P^{-j}. This also implies that we focus on the symmetric equilibrium although there may exist the asymmetric cases where $0 < x_{HL} < 1$ and $x_{LH} < 0$ and $x_{HL} > 1$ and $0 < x_{LH} < 1$.

Now, the bias is large enough relative to the price difference, so the two sellers can avoid competition for the two groups of buyers, groups HL and LH. That is, all buyers in group HL purchase good A, while all buyers in group LH purchase good B. We can write down seller B's demand function in a similar way. Then, for $j \in \{A, B\}$, each seller j's demand function can be derived as follows.

$$D_j = \begin{cases} \frac{t+P^{-j}-P^j}{2t} & \text{if } \Delta(\theta,\alpha) < t, \\ \left(\Pr(s_H)^2 + \Pr(s_L)^2\right)\left(\frac{1}{2} + \frac{P^{-j}-P^j}{2t}\right) + \Pr(s_H)\Pr(s_L) & \text{if } \Delta(\theta,\alpha) \geq t. \end{cases}$$

In turn, solving the two sellers' maximization problems, the symmetric equilibrium prices can be readily shown as follows.

Lemma 3 *In the symmetric equilibrium, each seller's optimal price is as follows.*

$$P^*(\theta) = P^{A*} = P^{B*} = \begin{cases} t & \text{if } \Delta(\theta,\alpha) < t, \\ t\left(\frac{(\Pr(s_H)+\Pr(s_L))^2}{\left(\Pr(s_H)^2+\Pr(s_L)^2\right)}\right) & \text{if } \Delta(\theta,\alpha) \geq t; \end{cases}$$

where $\Pr(s_H) = \alpha\theta + (1-\alpha)(1-\theta)$ *and* $\Pr(s_L) = \alpha(1-\theta) + (1-\alpha)\theta$.

Proof of Lemma 3

In the appendix.

When the bias is less than the product differentiation parameter t, the equilibrium prices are t. However, when the bias is greater than t, the equilibrium prices become greater than t. Now, let us describe the condition, $\Delta(\theta,\alpha) \gtreqless t$, as a function of θ in detail in order to show the relationship between the buyer's prior valuation and the equilibrium prices.

Proposition 6 *(1) For* $t > 1$, $P^{A*} = P^{B*} = t$ *for all* $\theta \in (0,1)$. *(2) For* $0 < t < 1$, *if* $\alpha \in \left(\frac{1}{2}, \frac{t+1}{2}\right)$, $P^{A*} = P^{B*} = t$ *for all* $\theta \in (0,1)$. *On the other hand, if* $\alpha \in \left(\frac{t+1}{2}, 1\right)$, *there exist* $\underline{\theta} \in \left(0, \frac{1}{2}\right)$ *and* $\bar{\theta} \in \left(\frac{1}{2}, 1\right)$ *such that if* $\theta \in (0, \underline{\theta})$ *or* $\theta \in (\bar{\theta}, 1)$, $P^{A*} = P^{B*} = t$ *and if* $\theta \in [\underline{\theta}, \bar{\theta}]$, $P^{A*} = P^{B*} = \left(\frac{(\Pr(s_H)+\Pr(s_L))^2}{\Pr(s_H)^2+\Pr(s_L)^2}\right) t$.

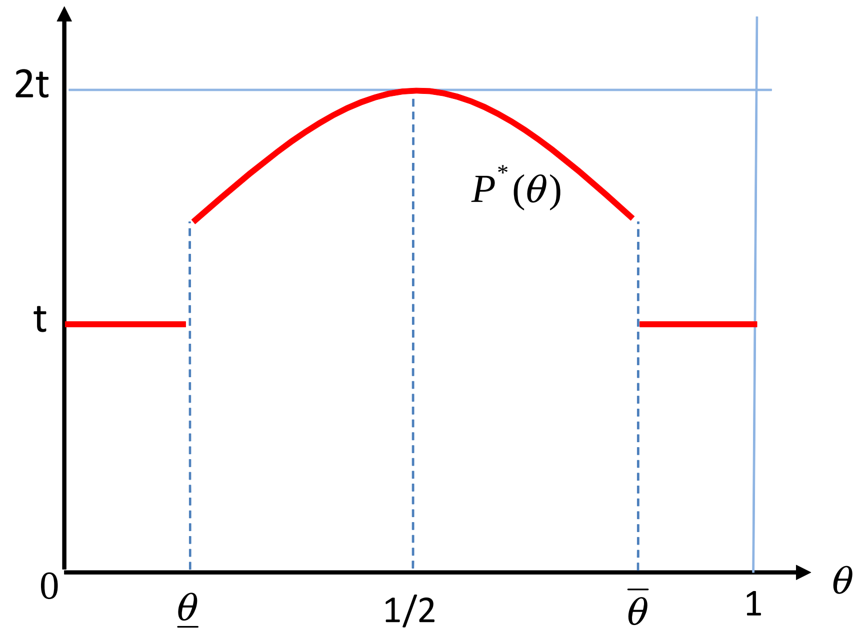

Figure 2.2: Equilibrium prices in duopoly

Proof of Proposition 6

In the appendix.

When $t > 1$, the bias is always less than the product differentiation due to

$$\Delta(\theta, \alpha) = p_H(\theta, \alpha) - p_L(\theta, \alpha) < 1.$$

Thus, the equilibrium price is t as in the standard Hotelling model. The interesting case is the one where $0 < t < 1$, in which the bias can be either greater or less than t. In this case, the information quality α matters in determining the equilibrium. If the information quality is relatively low, i.e., $\alpha \in \left(\frac{1}{2}, \frac{t+1}{2}\right)$, its effect is insignificant and the equilibrium price is still $P^{A*} = P^{B*} = t$. On the other hand, if the information quality is relatively high, i.e., $\alpha \in \left(\frac{t+1}{2}, 1\right)$, the private signal may have a dramatic effect on the buyer's posterior valuation, especially when the prior valuation is intermediate, thereby leading to changes in the equilibrium prices.

In the case that buyers have extreme θ (i.e., $\theta \in (0, \underline{\theta})$ or $\theta \in (\bar{\theta}, 1)$), they are stubborn about their prior valuation no matter what signals they draw. Thus, they hardly become inclined toward one seller over the other even if they receive different signals from the sellers. Because of this insensitivity to the signals, the equilibrium prices are still determined to be t, as in the standard Hotelling model. On the other hand, in the case that buyers have intermediate θ (i.e., $\theta \in [\underline{\theta}, \bar{\theta}]$), new information starts to play a role in buyer's relative preferences. Note that these buyers can be regarded as not so obstinate about their prior beliefs on the match value of the product: the probability that their match value turns out to be either $v = H$ or $v = L$ is similar. Therefore, they are relatively sensitive to the new information, and are easily swayed by the signals they receive.

In particular, we find that the equilibrium prices are increasing in $\theta \in \left(\underline{\theta}, \frac{1}{2}\right)$, decreasing in $\theta \in \left(\frac{1}{2}, \bar{\theta}\right)$, and maximized at $\theta = \frac{1}{2}$. The intuition to understand this is simple. Recall that the bias $\Delta(\theta, \alpha) = p_H(\theta, \alpha) - p_L(\theta, \alpha)$ is increasing in $\theta \in \left(\underline{\theta}, \frac{1}{2}\right)$ and decreasing in $\theta \in \left(\frac{1}{2}, \bar{\theta}\right)$. This implies that buyers become signal-sensitive as the prior valuation θ is closer to 1/2. As the buyers are more (less) signal-sensitive, they become less (more) sensitive to the price differential, which, after all, mitigates (intensifies) price competition between the sellers. Hence the equilibrium prices are higher as θ is closer to 1/2. To put it simply, the easier consumers are swayed by the signals, the more they are exploited by the firms.

As a result, the equilibrium prices are not monotone with respect to θ as in Figure 2.2. This non-monotonicity suggests that, even in the duopoly market, there is a possibility that sellers offer a higher price to the buyer with lower willingness to pay than to the one with higher willingness to pay. With slight abuse of notation, we reuse the notations θ_H and θ_L to show the reverse price discrimination explicitly.

Proposition 7 *Suppose that* $0 < t < 1$ *and* $\alpha \in \left(\frac{t+1}{2}, 1\right)$. *Then we obtain the reverse price discrimination,*

$$P^*(\theta_L) > P^*(\theta_H) \text{ for } \theta_L < \theta_H,$$

if (1) $\theta_L \in \left(\underline{\theta}, \frac{1}{2}\right)$ *and* $\theta_H \in (1 - \theta_L, 1)$ *or (2)* $\theta_L \in \left(\frac{1}{2}, \bar{\theta}\right)$ *and* $\theta_H \in (\theta_L, 1)$.

Our paper may provide complementary but sharply contrary results to Armstrong (2006). He shows that it has no effect on the firms' prices and profits in the Hotelling model even though the firms can observe a consumer's valuation and target a personalized price to the consumers. In our setting with private information, however, the sellers may offer different personalized prices based on buyers' prior valuations.[3]

2.4 Concluding Remarks

The paper studies price discrimination of two competing sellers under partially incomplete information where a buyer's prior valuation can be observed by the sellers but the buyer further draws private signals which may give him additional information about a product sold by the sellers. We show that the buyer with a higher willingness to pay may be offered a lower price than the one with a lower willingness to pay, even when there is price competition between sellers.

In the monopoly market, the reverse price discrimination occurs due to the elasticity of posterior demand, which was determined by the prior beliefs of buyers and the precision of signals. To illustrate, the seller only targets the buyers who draw a good signal by charging a high price when they are of a low type, whereas she wants to serve all of the buyers by offering a low price when they are of a high type. In the duopoly market, however, the sellers charge a higher price to the buyers with intermediate prior valuations than to the buyers with extreme prior valuations, since the formers are likely to perceive that two competing products are more differentiated when they receive different signals from the sellers.

[3]Damiano and Li (2007) study very similar issues such as price competition for privately informed buyers. A crucial difference is that they focus on the case in which the prior valuation is fixed as $\theta = 1/2$. In other words, price discrimination is not an issue in their paper. On the other hand, our focus is to find what prices two sellers offer to buyers based on θ. In addition, it is worthwhile to explain the difference of the modeling strategy between the two papers. In their model, the two goods are *ex ante* identical. Thus, there is no pure-strategy Nash equilibrium, and it is hard to characterize and compare the equilibrium prices with our general prior θ. This is the reason why we model the duopoly market by using the Hotelling model of product differentiation.

APPENDIX

Appendix

Proof of lemma 2

(1) $x_{HL} = \frac{1}{2} + \frac{P^B - P^A + \Delta(\theta,\alpha)}{2t}$.

(i) $x_{HL} > 0 \Longrightarrow P^B - P^A > -t - \Delta(\theta,\alpha)$, which is always true under our assumption.

(ii) $x_{HL} < 1 \Longrightarrow P^B - P^A < t - \Delta(\theta,\alpha)$. $x_{LH} = \frac{1}{2} + \frac{P^B - P^A - \Delta(\theta,\alpha)}{2t}$.

(iii) $x_{LH} > 0 \Longrightarrow P^B - P^A > -t + \Delta(\theta,\alpha)$.

(iv) $x_{LH} < 1 \Longrightarrow P^B - P^A < t + \Delta(\theta,\alpha)$, which is always true.

Then, from (ii) and (iii), $-t + \Delta(\theta,\alpha) < P^B - P^A < t - \Delta(\theta,\alpha)$. So, $\left|P^B - P^A\right| < t - \Delta(\theta,\alpha) \Longrightarrow \Delta(\theta,\alpha) < t - \left|P^B - P^A\right|$. It also should be that $t > \Delta(\theta,\alpha)$ from $-t + \Delta(\theta,\alpha) < t - \Delta(\theta,\alpha)$. However, if $\Delta(\theta,\alpha) < t - \left|P^B - P^A\right|$, $\Delta(\theta,\alpha) < t$ is always true. Moreover, as we consider the symmetric equilibrium, both sellers' optimal prices should be same. Then, $\Delta(\theta,\alpha) < t - \left|P^B - P^A\right| \Longrightarrow \Delta(\theta,\alpha) < t$.

(2) From (1), $x_{HL} \geq 1 \Longrightarrow P^B - P^A \geq t - \Delta(\theta,\alpha)$ and $x_{LH} \leq 0 \Longrightarrow P^B - P^A \leq -t + \Delta(\theta,\alpha)$. So $t - \Delta(\theta,\alpha) \leq P^B - P^A \leq -t + \Delta(\theta,\alpha)$. Then, $\left|P^B - P^A\right| \leq -t + \Delta(\theta,\alpha) \Longrightarrow \Delta(\theta,\alpha) \geq t + \left|P^B - P^A\right|$. It also should be that $t \leq \Delta(\theta,\alpha)$ from $t - \Delta(\theta,\alpha) \leq -t + \Delta(\theta,\alpha)$. However, if $\Delta(\theta,\alpha) \geq t + \left|P^B - P^A\right|$, $\Delta(\theta,\alpha) \geq t$ is always true. Also as we consider the symmetric equilibrium, $\Delta(\theta,\alpha) \geq t + \left|P^B - P^A\right| \Longrightarrow \Delta(\theta,\alpha) \geq t$. ■

Proof of lemma 3

Case (1) $\Delta(\theta,\alpha) \leq t$

Seller j's problem is

$$\underset{P_i}{Max}\ \pi^j = \left(\frac{t + P^{-j} - P^j}{2t}\right) P^j.$$

The first-order condition is

$$\frac{\partial \pi^j}{\partial P^j} = \frac{1}{2t}\left(t - 2P^j + P^{-j}\right) = 0.$$

Solving the two first-order conditions, we obtain $P^{A*} = P^{B*} = t$.

Case (2) $\Delta(\theta,\alpha) > t$

Seller j's problem is

$$\underset{P^j}{Max}\ \pi^j = \left(\left(\Pr(s_H)^2 + \Pr(s_L)^2\right)\left(\frac{1}{2} + \frac{P^{-j} - P^j}{2t}\right) + \Pr(s_H)\Pr(s_L)\right) P^j.$$

Then, the first-order condition is

$$\frac{\partial \pi^j}{\partial P^j} = \left(\Pr(s_H)^2 + \Pr(s_L)^2\right)\frac{1}{2t}\left(t - 2P^j + P^{-j}\right) + \Pr(s_H)\Pr(s_L) = 0.$$

Again, solving the two first-order conditions, we obtain

$$P^{A*} = P^{B*} = \left(\frac{(\Pr(s_H) + \Pr(s_L))^2}{\left(\Pr(s_H)^2 + \Pr(s_L)^2\right)}\right) t.$$

■

Proof of Proposition 6

Let us check the condition $\Delta(\theta,\alpha) \gtreqless t$. Here,

$$\Delta(\theta,\alpha) \equiv p_H(\theta,\alpha) - p_L(\theta,\alpha) = \frac{(\theta-1)(2\alpha-1)\theta}{(2\alpha\theta - \theta - \alpha + 1)(2\alpha\theta - \theta - \alpha)},$$

then we obtain

$$\frac{\partial \Delta(\theta,\alpha)}{\partial \theta} = \frac{(2\theta-1)(\alpha-1)(2\alpha-1)\alpha}{(2\theta\alpha - \alpha - \theta + 1)^2(2\theta\alpha - \alpha - \theta)^2}.$$

Case (1) $\theta > \frac{1}{2}$

(i) $\frac{\partial \Delta(\theta,\alpha)}{\partial \theta} < 0$, so $\Delta(\theta,\alpha)$ is a decreasing function

(ii) $\Delta\left(\theta = \frac{1}{2}, \alpha\right) = (2\alpha - 1) > 0$

(iii) $\Delta(\theta = 1, \alpha) = 0$. So if $(2\alpha - 1) > t$, $\exists \bar{\theta}$ such that if $\theta \in (\frac{1}{2}, \bar{\theta}]$, $\Delta(\theta,\alpha) > t$ and if $\theta \in [\bar{\theta}, 1)$, $\Delta(\theta,\alpha) < t$.

On the other hand, if $(2\alpha - 1) < t$, $\Delta(\theta,\alpha) < t$ for all $\theta \in \left(\frac{1}{2}, 1\right)$.

Case (2) $\theta < \frac{1}{2}$

(i) $\frac{\partial \Delta(\theta,\alpha)}{\partial \theta} > 0$, so $\Delta(\theta,\alpha)$ is an increasing function

(ii) $\Delta\left(\theta = 0\right) = 0$

(iii) $\Delta\left(\theta = \frac{1}{2}, \alpha\right) = (2\alpha - 1)$. So if $(2\alpha - 1) > t$, $\exists \underline{\theta}$ s.t. if $\theta \in (0, \underline{\theta}]$, $\Delta\left(\theta,\alpha\right) < t$ and if $\theta \in [\underline{\theta}, \frac{1}{2})$, $\Delta\left(\theta,\alpha\right) > t$.

On the other hand, if $(2\alpha - 1) < t$, $\Delta\left(\theta,\alpha\right) < t$ for all $\theta \in \left(0, \frac{1}{2}\right)$. In both cases, the condition that $(2\alpha - 1) \gtreqless t \Longrightarrow \alpha \gtreqless \frac{t+1}{2}$ matters. Note that

$$\frac{t+1}{2} - \frac{1}{2} = \frac{1}{2}t > 0$$

and

$$\frac{t+1}{2} - 1 = \frac{1}{2}\left(t - 1\right).$$

Then we have the following result. *(1) if $t > 1$, always for all $\theta \in (0,1)$, $\Delta\left(\theta,\alpha\right) < t$. (2) if $t < 1$, $\exists \alpha^{*} = \frac{t+1}{2}$ such that if $\alpha \in \left(\frac{1}{2}, \frac{t+1}{2}\right)$, for all $\theta \in (0,1)$, $\Delta\left(\theta,\alpha\right) < t$. On the other hand, if $\alpha \in \left(\frac{t+1}{2}, 1\right)$, there exist $\underline{\theta}$ and $\bar{\theta}$ such that if $\theta \in (0, \underline{\theta})$ or $\theta \in (\bar{\theta}, 1)$, $\Delta\left(\theta,\alpha\right) < t$ and if $\theta \in (\underline{\theta}, \bar{\theta})$, $\Delta\left(\theta,\alpha\right) > t$.* We already know that if $\Delta\left(\theta,\alpha\right) < t$, the optimal price is $P^{A*} = P^{B*} = t$ and if $\Delta\left(\theta,\alpha\right) \geq t$, the optimal price is

$$P^{A*} = P^{B*} = t\left(\frac{(\Pr(s_H) + \Pr(s_L))^2}{(\Pr(s_H)^2 + \Pr(s_L)^2)}\right).$$

■

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Chapter 3

Price Discrimination via Information Provision: Online vs. Offline Shoppers

3.1 Introduction

Second-degree price discrimination (monopolistic screening) is the optimal pricing scheme when buyers' individual valuations are not observable (Mussa and Rosen (1978); Maskin and Riley (1984)). A seller offers a menu of bundles that induces each type of buyers to select the menu designed for the type. In the literature, a popular form of second-degree price discrimination is offering different prices together with different quantities or different quality of products. In this paper, we study a new type of second-degree price discrimination in that a seller offers different prices with different level of information as a bundle, which can induce the self-selection of buyers through information discrimination.

In the real world, we can observe various types of information-differentiated price discrimination. One example is that many traditional merchants operate brick-and-mortar (offline) stores as well as online malls so that people can shop either at the offline or online stores. A crucial difference between online shopping and offline shopping is the accessibility to the information about products. In the offline store, people can learn better about the product, for example, by reading a part of novels, by trying on clothes, and so forth. On the other hand, when they shop online, it may be more difficult for consumers to decide whether the products really match their preferences.[1] Another example is travel agency companies such as priceline.com or hotwire.com. It usually offers two options to buyers: buyers can choose hotels and flights with detailed information, or they can do so without knowing the brand and location of hotels or the brand and schedule of flights. Namely, they are offered transparent travel services and opaque travel services at the same time.[2]

To capture these scenarios, we consider the environment where buyers have two kinds of private information. One is their *ex ante* valuation for a good or service. In addition, they receive a signal about how well the product fits their tastes when visiting a store, and may update their valuations in a Bayesian way according to the signal, which is another private information. That is, buyers' *ex ante* valuations and *ex post* valuations are privately known. We further consider that the precision of the signal depends on a marketplace: an online or an offline store. Thus, the seller may control the level of information, to some extent, by choosing the marketplace.

The purpose of the paper is to provide a new explanation about information-differentiated products in terms of second-degree price discrimination. In particular, we show that the self-selection is incentive compatible only when high valuation buyers purchase the product with less information and low valuation buyers purchase the one with more information. The intuition is as follows. When a buyer purchases a good or service without information, he has to take some risks of ending up being mismatched with it. The buyer who is sufficiently optimistic, thereby having a high *ex ante* expected valuation, will face relatively less risks,

[1]Without loss of generality, we shall focus on the direct (first-hand) information which potential buyers can obtain when inspecting products by themselves at a store. In some cases, however, one may argue that online stores endow better information than offline stores especially when it comes to indirect (second-hand) information – consumer's review, for example. In such cases where the indirect information is more relevant, we still can apply the results once appropriate redefinition is made. Among others, see Ghose (2009) for empirical studies on product-level uncertainty in online markets.

[2]Shapiro and Shi (2008) study the role of opaque travel agencies and suggest that service providers may be able to increase their profits by selling through opaque travel agencies since it helps distinguish the types of consumers thereby weakening the competition for high valuation consumers.

and so he may decide to buy the product even without further information. Knowing this, a seller is able to separate high valuation buyers from low valuation buyers by offering a cheaper price together with less information. As a result, the high valuation buyers may choose to shop at online stores, from which they hardly receive further information on the product, paying lower price than the low valuation buyers.

Information-driven price discrimination shows significantly different welfare implications from quantity or quality-driven price discrimination. In our settings, aside from the transportation costs, the low valuation buyer's purchase is socially optimal, whereas the high valuation buyer's purchase is suboptimal due to excessive consumption.[3,4] Among low valuation buyers, only the ones who find the product matching well with their preferences will buy it, i.e., they will buy only when they indeed want it. On the other hand, since high valuation buyers purchase a product without knowing whether it actually fits their tastes, their purchase may not be socially desirable - i.e., they happen to buy even when they do not want it in retrospect. This is a distinctive result because it is well-known from previous literature that second-degree price discrimination leads high valuation buyers to have the first-best optimal consumption and low valuations buyers to have suboptimal consumption.

Another way of understanding this difference is to think of the level of information as a part of the product. Then, concealing important characteristics of the product can be regarded as providing a suboptimal quality or a damaged good. Now, interestingly, the incentive compatible screening requires the high valuation buyer to choose the damaged product. Moreover, some high valuation buyers will be suffering from negative net surplus more severely than the low valuation buyers.

There are several papers which study price discrimination in the environment that buyers learn their preferences over time. Miravete (1996) compares *ex ante* two-part tariffs and *ex post* two-part tariffs in telecommunication industry. Courty and Li (2000) study sequential screening through refund policy. Grubb (2010) also studies a similar issue but focuses on

[3]The high valuation buyer's consumption is said to be "excessive" in an *ex post* sense.
[4]See also Creane (2008) for insightful welfare implications in a similar environment.

the case that consumers are overconfident in that they overestimate the precision of their demand forecasts. In these papers, contracts are signed when consumers have partial private information, that is, before they learn their valuations. Compared to these papers, our paper differs in the sense that the provision of different levels of information is the screening mechanism itself. In other words, buyers' self selection arises by their purchase of different goods with different levels of information.[5] In this sense, the closest paper to ours is Nocke, Peitz and Rosar (2011) which studies how advance-purchase discount can serve to price discriminate. In an intertemporal setting where consumers' uncertainty is resolved over time, advance-purchasing can be thought of as purchasing with less information.

The rest of this paper proceeds as follows. Section 3.2 introduces the basic model. Section 3.3 analyzes the benchmark case where either transparent or opaque products are available. In Section 3.4, we allow the seller to price discriminate by offering two types of products together, and derive the optimal price discrimination. We then compare the profits of three cases – selling at online, offline, or both stores – to find out which is most profitable to the seller, in Section 3.5. Section 3.6 extends the model to incorporate buyers' rational behavior against price discrimination. Then Section 3.7 concludes.

3.2 The Model

Seller. There is a monopolistic seller with a single product, which can be sold at offline and/or online stores. We abstract from any costs that the seller might incur in producing the product or in operating the stores.

Buyers. There is a continuum of buyers with a unit demand. Each buyer's match value, v, for the product is either v_H with probability λ or v_L with probability $(1-\lambda)$, where $\lambda \in [0, 1]$.

[5] Relatedly, Lewis and Sappington (1994) study how information provision affects second-degree price discrimination of offering menus of different prices and quantities. Contrary to this paper, we study price discrimination of offering menus of different prices and different information. Also, Bar-Isaac et al. (2010) explore information provision and gathering issues in a similar environment, but do not allow prices to differ.

We normalize buyers' valuation as $v_H = 1$ and $v_L = 0$, then the buyers' *ex ante* expected valuation for the product is simply λ. We consider two types of buyers: type-i buyers have $\lambda = \lambda_i$, where $i \in \{L, H\}$ and $\lambda_L < \lambda_H$. In words, type-H buyers are more optimistic about their match values, v, than type-L buyers. The buyers' type is private information, and thus it is not observable to the seller. For simplicity, we assume that the mass of each type is one.[6] The buyers have to incur transportation costs to get to the store: t_N for online stores and t_F for offline stores. We assume $0 \leq t_N \leq t_F$. We further assume $t_F < \lambda_L$ so that we are assured to include all the possible cases in the analysis.

Information. Once a buyer arrives at a store, he may observe a binary private signal on his match value: a good signal s_H or a bad signal s_L. The signal provides information about the good's match value in the sense of Blackwell. The probability of the signal being s_k follows the conditional probability distributions $Pr(s_k|v_l) = \alpha$ for $l = k$ and $Pr(s_k|v_l) = 1 - \alpha$ for $l \neq k$, where $l, k \in \{L, H\}$. Note that α represents the precision or informativeness of the signal.[7] Given the prior belief λ_i, the probability of receiving signal s_k is

$$\Pr(s_k; \lambda_i) = \lambda_i \Pr(s_k|v_H) + (1 - \lambda_i) \Pr(s_k|v_L).$$

Let $\phi_k(\lambda_i, \alpha) \equiv \Pr(v_H|s_k)$ be the buyer's posterior belief about $v = v_H$, after receiving the signal s_k. Then, Bayes' rule leads to

$$\begin{aligned} \phi_H(\lambda_i, \alpha) &\equiv \Pr(v_H|s_H) = \frac{\alpha\lambda_i}{\alpha\lambda_i + (1-\alpha)(1-\lambda_i)}, \text{ and} \\ \phi_L(\lambda_i, \alpha) &\equiv \Pr(v_H|s_L) = \frac{(1-\alpha)\lambda_i}{\alpha(1-\lambda_i) + (1-\alpha)\lambda_i}. \end{aligned}$$

The type-i buyer's *ex post* valuation for the good is a mean-preserving spread for a degenerate distribution to two-point distribution: it will be $\phi_H(\lambda_i, \alpha)$ with the probability $Pr(s_H)$ and

[6] Even if the mass is different from type to type, our qualitative results remain the same.

[7] Without loss of generality, we consider $\alpha \in [\frac{1}{2}, 1]$. If α is less than $\frac{1}{2}$, then the buyers would regard $(1 - \alpha)$ as an effective signal, which provides the same level of information.

$\phi_L(\lambda_i, \alpha)$ with the probability $Pr(s_L)$.

It seems worth comparing the information structure in this model with that in the search literature.[8] In the search literature, consumers can learn prices only by visiting stores, whereas consumers are perfectly informed of prices beforehand but may learn about their match value when visiting stores.

Online vs. Offline. Now, a significant difference between shopping at online and at offline stores is the informativeness of the signal a buyer may receive. The buyer is able to examine a product face-to-face in offline stores, so he can precisely find out how well the product fits his tastes. In online stores, however, he only can inspect the product by browsing the pictures on website, which might prevent him from figuring out the product's exact characteristics.[9] To capture this phenomenon, we assume that the signal that the buyer receives from the offline stores is more informative than from the online stores, i.e., $\alpha_N < a_F$, where α_N and a_F are the informativeness of the signal from the online and the offline stores, respectively. Furthermore, we shall normalize these to $\alpha_N = 1/2$ and $\alpha_F = 1$ in the following analysis.[10]

If the buyer shops at the online store, his *ex post* valuation after visiting the store is still the same as the prior belief λ_i regardless of the signal he receives, since the signal conveys no additional information ($\alpha_N = 1/2$). If he shops at the offline store, however, his *ex post* valuation is 1 when receiving a good signal or 0 when a bad signal because the signal is perfectly informative ($\alpha_F = 1$). Therefore, in this case, he will purchase the product only when he receives a good signal. Given that the prices offered at online and at offline stores are p_N and p_F, respectively, type-i buyer's expected net utility from the purchase is

$$\begin{cases} EU_i = \lambda_i - p_N - t_N, & \text{if buys at online store,} \\ EU_i = \lambda_i(1 - p_F) - t_F, & \text{if buys at offline store.} \end{cases}$$

[8]See, among many others, Wilde and Schwartz (1979) and Stahl (1989)

[9]We assume that it is impossible for the buyers to return the product once they purchase it.

[10]That is, we consider that browsing the pictures on website was initially available to every potential buyer with no extra efforts, and that he can be perfectly informed of the match value of the product when inspecting the product face-to-face.

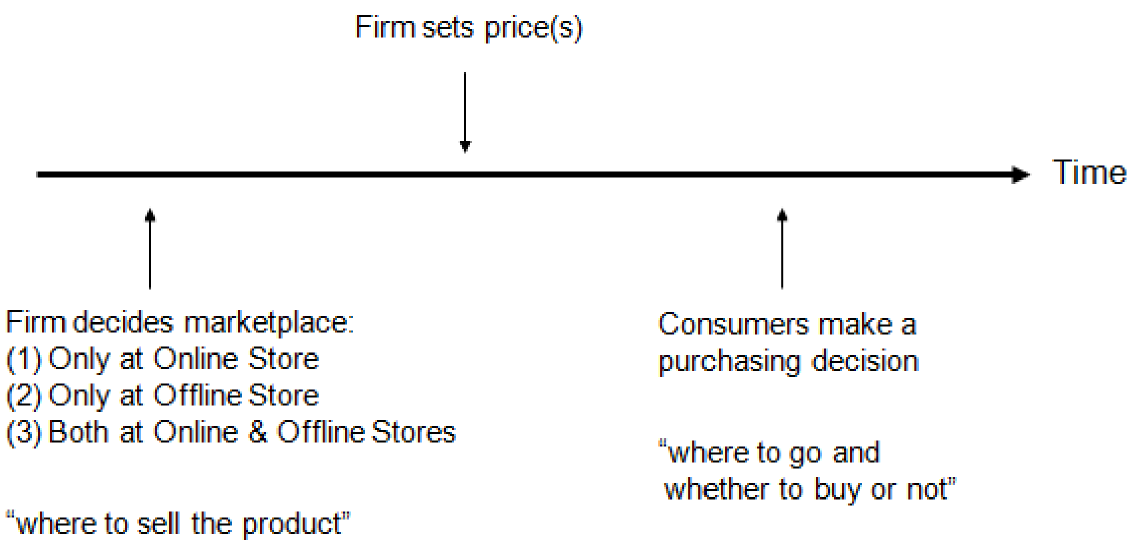

Figure 3.1: Timeline

Timeline. The timing of the game is as follows. First, buyers form their prior beliefs about their match value of the product. Second, the monopolist seller determines marketplace(s) to sell her product: selling only at online, only at offline, or both at online and offline stores. Third, the seller quotes price(s) which immediately become observable to all potential buyers. Last, the buyers make a purchase decision knowing the price(s): where to go shopping and whether to buy or not.

3.3 Online or Offline

To begin with, we study the benchmark case where the product is available only at either online or offline stores. In either regime, the seller's price choice is to decide whether to serve only type-H buyers or both types of buyers.

Only online store. The type-i buyer goes to the online store and buys if $\lambda_i - p_N - t_N \geq 0$. If the seller charges $\lambda_L - t_N < p_N \leq \lambda_H - t_N$, then only type-H buyers will purchase. Whereas, if he charges $p_N \leq \lambda_L - t_N$, then both types of buyers will buy. The optimal price and corresponding profits are

$$\overline{p}_N^* = \lambda_H - t_N \text{ and } \overline{\pi}_N^* = \lambda_H - t_N, \qquad \text{if } \lambda_H > 2\lambda_L - t_N, \tag{Case 1}$$

$$\underline{p}_N^* = \lambda_L - t_N \text{ and } \underline{\pi}_N^* = 2(\lambda_L - t_N), \quad \text{if } \lambda_H \leq 2\lambda_L - t_N. \tag{Case 2}$$

It is noteworthy that, since buyers' valuation does not change when they visit the online store, the buyers who decide to shop at the online store always end up buying the product – otherwise they would not have come at the first place. Thus, the seller can simply charge the expected gross utility of the targeted type of buyers, λ_i, but the compensation for the transportation cost t_N, as is reflected on the optimal prices above.

Only offline store. Now, the type-i buyer goes to the offline for shopping if $\lambda_i(1-p_F)-t_F \geq 0$. Only type-H buyers will go shopping at the offline store if the price is

$$1 - t_F/\lambda_L < p_F \leq 1 - t_F/\lambda_H,$$

but both types of buyers will do so if

$$p_F \leq 1 - t_F/\lambda_L.$$

Thus, the optimal price and the corresponding profits are now

$$\overline{p}_F^* = 1 - t_F/\lambda_H \text{ and } \overline{\pi}_F^* = \lambda_H - t_F, \qquad \text{if } \lambda_H > \lambda_L^2/t_F, \tag{Case 3}$$

$$\underline{p}_F^* = 1 - t_F/\lambda_L \text{ and } \underline{\pi}_F^* = (\lambda_H + \lambda_L)(1 - t_F/\lambda_L), \quad \text{if } \lambda_H \leq \lambda_L^2/t_F. \tag{Case 4}$$

Similarly to the previous cases, the optimal price chosen by the seller is essentially comprised of the buyer's gross utility and the compensation for the transportation cost. When the seller operates an offline store, however, the only ones receiving a good signal will decide to buy the product at last, but the others receiving a bad signal will walk away empty-handed. Since the compensation for the transportation cost can only be granted via the discount of price, those who leave empty-handed have no way to receive the compensation. Considering this risk from a buyer's perspective, the seller should compensate more than the transportation cost itself in order to entice all the potential buyers to visit the offline store. Therefore, the optimal price would incorporate the gross utility of the *actual buyers* and the compensation for the transportation costs of *all visitors* including even "window shoppers."

3.4 Online and Offline: Second-Degree Price Discrimination

Let us consider the case that the seller runs both online and offline stores and quotes prices at each store. There are possibly two self-selective ways in terms of buyers' choices of stores.[11] First, the self-selection may occur in the way that type-L buyers shop at the online store and type-H buyers at the offline store. However, this case never arises in the equilibrium.

Proposition 8 *There exists no pair of prices, (p_N, p_F), which induce type-L buyers to shop at online and type-H buyers at offline stores.*

Proof. Let us assume that there exists a pair of prices, $(\dot{p}_N, \dot{p}_F)$, where type-L buyers shop at online and type-H buyers at offline stores. Then, it must be true for $\dot{p}_N$ and $\dot{p}_F$ that

$$\lambda_L - \dot{p}_N - t_N \geq \lambda_L(1 - \dot{p}_F) - t_F$$

and

$$\lambda_H(1 - \dot{p}_F) - t_F \geq \lambda_H - \dot{p}_N - t_N.$$

[11] We shall focus on the cases in which each type of buyers is to choose different store. Otherwise, the analysis becomes exactly the same as only online (or offline) store is available.

Note these conditions ensure that type-L prefers online to offline stores but type-H prefers offline to online stores. Combining two conditions, we get

$$\dot{p}_N - \lambda_L \dot{p}_F \leq t_F - t_N \leq \dot{p}_N - \lambda_H \dot{p}_F,$$

which is not feasible for $\lambda_L < \lambda_H$. ■

The intuition to understand this result is quite simple. Type-H buyers are more optimistic about the match value of the product, and thus they are more likely to prefer shopping at the online store with no further information to shopping at the offline store than type-L buyers are. Hence, whenever type-L buyers find the price of online store a better deal than that of offline store, so do type-H buyers.

The other possible scenario is that type-L buyers shop at the offline store but type-H buyers at the online store. The seller maximizes the profit function with the following four constraints.

$$\underset{p_F, p_N}{Max} \quad p_F \lambda_L + p_N \tag{3.1}$$

subject to

$$\begin{array}{lll}
[IR_L] \;\; \lambda_L(1-p_F) - t_F \geq 0 & \Longleftrightarrow & p_F \leq 1 - t_F/\lambda_L \\
[IR_H] \;\; \lambda_H - p_N - t_N \geq 0 & \Longleftrightarrow & p_N \leq \lambda_H - t_N \\
[IC_L] \;\; \lambda_L(1-p_F) - t_F \geq \lambda_L - p_N - t_N & \Longleftrightarrow & t_F - t_N \leq p_N - \lambda_L p_F \\
[IC_H] \;\; \lambda_H - p_N - t_N \geq \lambda_H(1-p_F) - t_F & \Longleftrightarrow & t_F - t_N \geq p_N - \lambda_H p_F
\end{array}$$

Recall that type-L buyers, who shop offline, purchase the good with probability λ_L, i.e., only when they receive a good signal after examining the product at the store. On the other hand, type-H buyers make the purchase with probability 1, as long as they decide to visit the online store given the price. Thus, the profit function to maximize is $p_F \lambda_L + p_N$. The first two individual-rationality $[IR]$ constraints ensure that the buyers' expected net

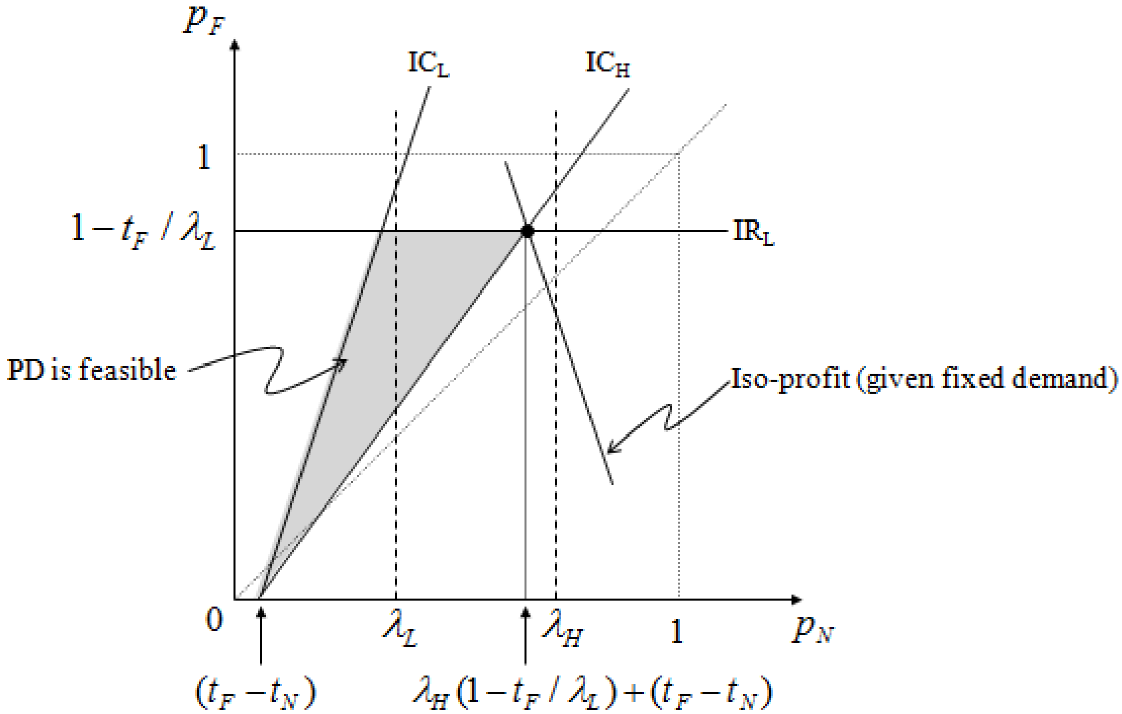

Figure 3.2: ICs and IRs

utility should be non-negative. The other two constraints are the incentive-compatible $[IC]$ constraints which require type-L buyers to choose the offline over the online store, and vice versa.

As usual, $[IR_H]$ is always satisfied: $[IC_H]$ and $[IR_L]$ together imply

$$p_N \leq \lambda_H(1 - t_F/\lambda_L) + t_F - t_N,$$

then $p_N \leq \lambda_H - t_N$ holds given that $\lambda_L < \lambda_H$. In addition, one can easily show that $[IC_L]$ is not binding whenever both $[IC_H]$ and $[IR_L]$ hold with equality. Then the maximization problem reduces to maximize (3.1) subject to $[IR_L]$ and $[IC_H]$. Solving it leads to the following proposition.

Proposition 9 *Suppose the seller operates both online and offline stores and employs effective price discrimination. Then there exists an optimal pair of prices,* $\widehat{p}_F = 1 - t_F/\lambda_L$ *and*

$\widehat{p}_N = \lambda_H(1 - t_F/\lambda_L) + (t_F - t_N)$, *for which type-H buyers shop at the online and type-L buyers at the offline stores. The corresponding profits are*

$$\widehat{\pi}_{NF} = (t_F - t_N) + (\lambda_H + \lambda_L)(1 - t_F/\lambda_L).$$

The intuitive explanation on the optimal prices above is as follows. The type-H buyers will purchase at the offline store paying $\widehat{p}_F$, whenever they find the product matching well with their tastes. Since this probability is λ_H for type-H buyers, the expected price they are paying at the offline store is $\lambda_H \widehat{p}_F$. Knowing this, the seller can entice type-H buyers to choose the online store instead of offline store, if she sets the price of the online store as low as their expected price of the offline store. In addition, she can charge their savings in transportation costs, $(t_F - t_N)$, on top of that.

From the proposition above, we find some interesting observations on the optimal prices and welfare implications, which are summarized in the following two corollaries.

Corollary 1 *Suppose the seller operates both online and offline stores and employs effective price discrimination. Then the optimal price offered at the online store is lower than that at the offline store, i.e.,* $\widehat{p}_N < \widehat{p}_F$, *if* $(t_F - t_N) < (1 - \lambda_H)(1 - t_F/\lambda_L)$

Corollary 2 *Suppose the seller operates both online and offline stores, and sets the pair of prices,* $\widehat{p}_F = 1 - t_F/\lambda_L$ *and* $\widehat{p}_N = \lambda_H(1 - t_F/\lambda_L) + (t_F - t_N)$. *The type-H buyers with low match value will suffer from a severer negative ex post net surplus than the type-L buyers with low match value.*

Price discrimination through information provision shows several interesting differences from that through quality or quantity. Note first that, without informational discrepancies between online and offline stores, the price at the online store is supposed to be higher than at the offline store due to a lower transportation costs associated with the online store. When shopping at the online store, however, buyers have to take risks of buying an undesirable product with zero match value, and the risks are larger to type-L buyers than type-H buyers. Thus, the seller can screen out type-H buyers from type-L buyers by lowering the price at the online store where no further information on products is available. Such strategic effects to separate out two types of buyers may push down the price at the online store so dramatically that type-H buyers may pay a lower price than type-L buyers after all.[12,13]

Second, type-H buyer may end up having a larger negative net surplus than type-L buyers, in an *ex post* sense. In a traditional second-degree price discrimination model, low valuation consumers have no net surplus, whereas high valuation consumers always enjoy a positive net surplus. This is because a seller must leave some rents to high valuation consumers in order to prevent them from buying the bundle designed for low valuation consumers. In our settings, however, type-H buyers will suffer from a negative net surplus with probability $(1 - \lambda_H)$.[14]

Last but not the least, if there is a slight production cost, type-L buyer's purchase is socially optimal while type-H buyer's purchase is suboptimal, which is contrary to the common results in the literature on second-degree price discrimination. Since type-L buyers are perfectly informed of the characteristics of the product at the offline store prior to purchase decisions, only people who find a good match will buy the product. However, type-H buyers purchase the product without knowing how well it matches their preferences, their purchase may not be socially desirable with probability $(1 - \lambda_H)$ if a seller has to incur positive production costs.[15]

[12] If there is no difference in the transportation costs of online and offline stores, i.e., $t_F = t_N$, then only the strategic price discriminating effects will prevail. Therefore, in this case, the price is always lower at online stores than at offline stores.

[13] Brynjolfsson and Smith (2000) find that prices on the Internet are 9-16% lower than prices in conventional outlets. See also Carlton and Chevalier (2001), Goolsbee (2001), Brown and Goolsbee (2002), and Chevalier and Goolsbee (2003) for empirical studies on online prices.

[14] Of course, type-H buyers should enjoy strictly positive *ex ante* net surplus on average.

[15] More precisely, the social desirability would be determined by the relative size of production and transportation costs. Note that the statement is true given the assumption

3.5 Optimal Choice of Marketplace

Comparing the optimal profits in three cases – selling at the online, at the offline, or both stores, we obtain the following results.

Proposition 10 *(i) If* $\frac{\lambda_L - t_N}{1 - t_F/\lambda_L} \leq \lambda_H \leq 2\lambda_L - t_N$ *or* $2\lambda_L - t_N < \lambda_H \leq \lambda_L^2/t_F$*, then it is optimal for the seller to operate both online and offline stores and to employ price discrimination. (ii) If* $\lambda_H > \max\{2\lambda_L - t_N, \lambda_L^2/t_F\}$*, then it is optimal to operate only an online store and to sell only to type-H buyers. (iii) If* $\lambda_H < \min\{2\lambda_L - t_N, \frac{\lambda_L - t_N}{1 - t_F/\lambda_L}\}$*, then it is optimal to operate only an online store and to sell to both types of buyers.*

Proof. (i) If $\lambda_H \leq 2\lambda_L - t_N$, then $\max\{\overline{\pi}_N, \underline{\pi}_N\} = \underline{\pi}_N$. Since $\overline{\pi}_F < \overline{\pi}_N$ and $\underline{\pi}_F < \widehat{\pi}_{NF}$, now we should compare $\underline{\pi}_N$ and $\widehat{\pi}_{NF}$. It can be easily shown that $\underline{\pi}_N \leq \widehat{\pi}_{NF}$ when $\frac{\lambda_L - t_N}{1 - t_F/\lambda_L} \leq \lambda_H$. If $2\lambda_L - t_N < \lambda_H$, then $\max\{\overline{\pi}_N, \underline{\pi}_N\} = \overline{\pi}_N$. Similarly, we compare $\overline{\pi}_N$ and $\widehat{\pi}_{NF}$, then we get $\overline{\pi}_N \leq \widehat{\pi}_{NF}$ when $\lambda_H \leq \lambda_L^2/t_F$.

(ii) If $\lambda_H > \max\{2\lambda_L - t_N, \lambda_L^2/t_F\}$, then $\max\{\overline{\pi}_N, \underline{\pi}_N\} = \overline{\pi}_N$ and $\max\{\overline{\pi}_F, \underline{\pi}_F\} = \overline{\pi}_F$. Since $\overline{\pi}_F < \overline{\pi}_N$, we compare $\overline{\pi}_N$ and $\widehat{\pi}_{NF}$. It is straightforward to see $\widehat{\pi}_{NF} < \overline{\pi}_N$ for $\lambda_H > \lambda_L^2/t_F$.

(iii) If $\lambda_H \leq 2\lambda_L - t_N$, then $\max\{\overline{\pi}_N, \underline{\pi}_N\} = \underline{\pi}_N$. Since $\overline{\pi}_F < \overline{\pi}_N$ and $\underline{\pi}_F < \widehat{\pi}_{NF}$, we compare $\underline{\pi}_N$ and $\widehat{\pi}_{NF}$, then we get $\widehat{\pi}_{NF} < \underline{\pi}_N$ for $\lambda_H < \frac{\lambda_L - t_N}{1 - t_F/\lambda_L}$. ■

When the difference between λ_H and λ_L is sufficiently large (case ii), the seller is better off by targeting only type-H buyers. Due to the advantage of transportation costs of online stores, he will want to operate only online stores and sell only to type-H buyers. On the other hand, when the difference is small (case i and iii), the seller is drawn to sell to both types of buyer. Note that it becomes more difficult (or costly) for the seller to separate out two types as λ_H and λ_L come closer, i.e., their incentives become similar. Therefore, the seller will be better off running both stores and adopting price discrimination if the difference

$t_N \leq t_F$.

is intermediate (case i), but he will rather opt to run only an online store if it is too small (case iii).

3.6 Extensions: Sophisticated Buyers and Milking Strategy

Until now, we implicitly assumed that buyers are somewhat naive that they only can purchase at the store where they receive the signal. If buyers are sophisticated enough, however, the buyers might consider visiting one store to get information about the product and then purchasing from the other store (*milking strategy*). The buyers will be tempted to do so especially when they find out that the other store is offering a lower price than the store from which they gather the product information. In order to prevent buyers' such milking strategies, the seller's choice of prices must satisfy the following "arbitrage(milking)-proof" conditions.

$$
\begin{array}{lll}
[AP_L] \;\; \lambda_L(1-p_F)-t_F \geq \lambda_L(1-p_N-t_N)-t_F & \iff & p_N \geq p_F - t_N \\
[AP_H] \;\; \lambda_H - p_N - t_N \geq \lambda_H(1-p_N-t_N)-t_F & \iff & p_N \leq t_F/(1-\lambda_H) - t_N
\end{array}
$$

$[AP_L]$ implies that, for type-L buyers, buying at the offline store is better than switching to the online store after examining the product at the offline store. This constraint boils down to

$$t_N \geq p_F - p_N, \tag{3.2}$$

which is intuitively appealing because the type-L buyers would marginally compare (i) the extra transportation costs incurred from an additional visit to the online store with (ii) the benefits from paying a lower price at the online store.[16] Similarly, $[AP_H]$ assures that type-H buyers do not visit the offline store for the purpose of collecting information before making a

[16] If the price is higher at the online than the offline store, then it is senseless for buyers to switch to the online store after collecting the information from the offline store.

purchase at the online store. When solving the profit maximization problem, we can ignore the latter condition $[AP_H]$ since it is already implied by $[IC_H]$ and $[AP_L]$ together. The intuition is simple. $[AP_L]$ tells us that even if the buyer arrives at the offline store and finds the product perfectly matching his tastes, the price advantage of the online over the offline store is not prominent enough to provoke him an another trip to the online store. That is, buyers have no incentives to switch to the online store no matter what signal they receive from the offline store. Even for type-H buyer, the additional information from signals would not change his purchase decision on "where to buy," and therefore he is only concerned about "where to go shopping," online or offline stores, which is determined by $[IC_H]$ constraint.

Considering the two additional incentive compatibility constraints, $[AP_L]$ and $[AP_H]$, we obtain the following condition for which the optimal pair of prices derived in the previous section still can be an effective price discrimination.

Proposition 11 *Suppose that the seller operates both online and offline stores. When consumers are sophisticated enough to use milking strategy, the optimal pair of prices,* $\widehat{p}_F = 1 - t_F/\lambda_L$ *and* $\widehat{p}_N = \lambda_H(1 - t_F/\lambda_L) + (t_F - t_N)$,*are sustainable if* $t_F \geq \frac{\lambda_L(1-\lambda_H)}{\lambda_L+(1-\lambda_H)}$.

Seemingly, the milking strategy can be prevented when t_N is large – say, if t_N is negligible, whenever type-L buyer receives a good signal from the offline store, he may want to switch to the online store to buy it despite the additional transportation costs t_N. However, the "milking-proof" condition above is only a function of $(t_F, \lambda_L, \lambda_H)$, but not t_N.[17] The reason is straightforward from the price offered at online stores $(\widehat{p}_N)$. When t_N becomes smaller, the seller can raise $\widehat{p}_N$ as much without increasing the deviation incentives of type-H buyers, while keeping $\widehat{p}_F$ as it is. Therefore, the changes in t_N do not affect type-L buyer's incentives for milking strategy as the both sides of (3.2) would equally change. By contrast, when t_F gets smaller, the seller is tempted to raise the price for the offline store $(\widehat{p}_F)$, but not so

[17]For this reason, we may freely normalize as $t_N = 0$ without affecting any qualitative results.

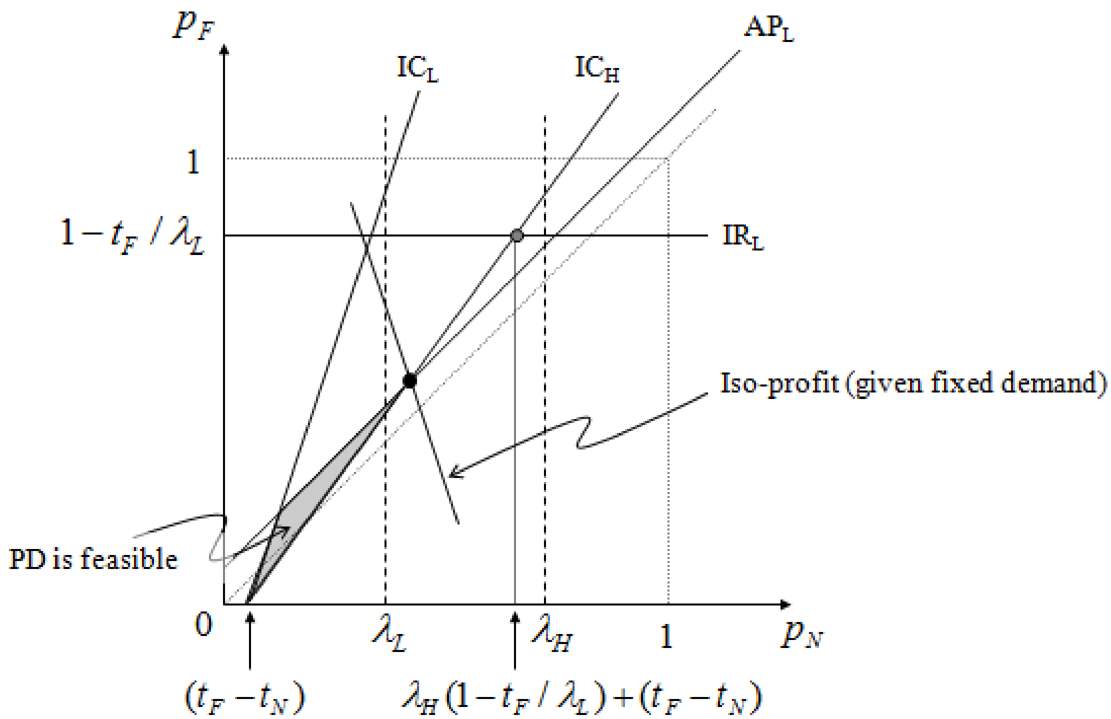

Figure 3.3: PD with Sophisticated Buyers

much for the online store since the advantage in transportation costs of online over offline store, $t_F - t_N$, becomes less. This, in turn, makes the online store more attractive for type-L buyers, i.e., the only right-hand side of (3.2) increases. As a result, type-L buyers are more likely to deviate from the offline store, when t_F is smaller.

Proposition 12 *Suppose that the seller operates both online and offline stores, and that consumers are sophisticated enough to use milking strategy. If* $t_F < \frac{\lambda_L(1-\lambda_H)}{\lambda_L+(1-\lambda_H)}$, *then the optimal pair of prices which can separate types of buyers are* $\widehat{p}_F = \frac{t_F}{1-\lambda_H}$ *and* $\widehat{p}_N = \frac{\lambda_H t_N+(t_F-t_N)}{1-\lambda_H}$, *and the corresponding profits are* $\widehat{\pi}_{NF} = \left(\frac{1+\lambda_L}{1-\lambda_H}\right) t_F - t_N$.

3.7 Concluding Remarks

In this paper, we study a new type of second-degree price discrimination where a different price is offered as a bundle with a different level of information about a product. When buyers are uncertain about their match value of the product, they may update their expected valuation in a Bayesian way after observing a signal coming from the product. The precision of the signal, or equivalently the amount of information buyers can collect from offline stores may be superior to that from online stores since the buyers can inspect and examine the product's characteristics more thoroughly in the former than the latter. In this environment, the seller may raise profits by using price discrimination in the way that high valuation buyers choose to shop at the online stores but low valuation buyers at the offline stores. This may result in the distortion of welfare: the low valuation buyer's purchase is socially optimal, but the high valuation buyer's is suboptimal, since the high valuation buyers make a purchase decision without perfectly knowing their match value, i.e., their consumption may be excessive in retrospect. In turn, the high valuation buyers would suffer from a severer negative *ex post* net surplus when they turn out to have a low match value than the low valuation buyers would.

We further study buyers' reaction against seller's such price discrimination. If buyers are sophisticated enough, they may "milk" the information from offline stores and make a purchase at online stores, when the online stores offer a better price deal. We find that, facing buyers' milking strategy, the seller's price discrimination can be successful only when the transportation costs for offline stores are sufficiently high. If the costs are low, the seller will have to lower the prices to effectively separate out the buyers, which makes the price discrimination no longer profitable at some point.

There can be some ways for the seller to prevent buyers from using milking strategy. One immediate way is to manipulate offline transportation costs either by relocating offline stores to remote sites or by reducing the number of offline stores.[18] An alternative way can be that

[18]Forman et al. (2009) empirically examine the trade-off between the benefits of online and offline shopping and show that offline transportation costs matter.

she differentiates products sold in online and offline stores. If the seller provides a slightly different version in online from offline, the product information buyers gather from offline stores will not perfectly show the match value of the product being sold in online stores. This would give an interesting explanation for marketplace-specific product versioning by which some products are only made available for either online or offline purchase.

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