AN ANALYSIS OF THE EFFECTIVENESS OF THE WORKSHOP AS AN IN-SERVICE MEANS FOR IMPROVING MATHEMATICAL UNDERSTANDINGS OF ELEMENTARY SCHOOL TEACHERS

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ABSTRACT

AN ANALYSIS OF THE EFFECTIVENESS OF THE WORKSHOP AS AN IN-SERVICE MEANS FOR IMPROVING MATHEMATICAL UNDERSTANDINGS OF ELEMENTARY SCHOOL TEACHERS

by Mildred Jerline Dossett

Adviser: Calhoun C. Collier

The purpose of this study was to analyze the effectiveness of the workshop as a means of in-service education for elementary teachers for: (1) improving basic mathematical understandings, (2) changing attitudes toward mathematics, and (3) improving the classroom practices in the teaching of arithmetic. The two workshops used in this study were drawn from the 1963-64 series of workshops in mathematics for elementary school teachers, a part of an in-service education program being sponsored by the Missouri State Department of Education under Title III, National Defense Education Act.

The two state-sponsored workshops included sixty-seven participants, of whom forty-five were primary teachers and twenty-two intermediate teachers. These, along with the twenty-two teachers in the control group, were grouped, for purpose of analysis, as follows: (1) school system, (2) level of teaching assignment, (3) completion of in-service program, and (4) testing procedure.

The following hypotheses were advanced:

1. There will be a significant difference on a test of mathematical understandings between the post-test scores of a group of elementary teachers who participated in a mathematics workshop and their pre-test scores or the scores of a similar group who did not participate.
2. There will be a significant difference on an arithmetic attitude inventory between the post-test scores of a group of elementary teachers who participated in a mathematics workshop and their pre-test scores or the scores of a similar group who did not participate in the workshop.
3. There will be a significant difference between the class practices of a group of elementary school teachers who participated in the mathematics workshop and a similar group who did not participate.

In addition, two related questions were studied:

1. What effect, if any, does a pre-test have upon the post-test scores of a group of teachers who participated in the mathematics workshops?
2. What is the relationship of improvement in basic mathematical understandings or change in attitude toward arithmetic to teacher background factors?

The investigation proceeded as follows:

1. A form entitled "Teacher Background Information" was distributed to participants in the state-sponsored workshops and the control group. These data were transferred to a code sheet and used in the final analysis.

2. Two parallel forms of a "Test of Mathematical Understandings" were constructed, tried out in experimental form, revised, and administered to both the experimental and control groups at or near the first and last sessions of the two mathematics workshops.

3. The Dutton Arithmetic Attitude Inventory was administered to the experimental and control groups at the same time that the groups took the "Test of Mathematical Understandings" for the purpose of studying the effectiveness of the workshop in attitude changes toward arithmetic.

4. The _Texas Classroom Interview Question Schedule_ was used to elicit responses from a random selection of two pupils from eight classrooms of teachers in one of the experimental groups and an equal number of pupils from classrooms of teachers in the control group. Such a procedure provided data relative to the classroom practices of teachers in the teaching of arithmetic.

The analysis of covariance was utilized in the analysis of the data relative to the effectiveness of the workshop. Pearson Product-

Moment Correlation Coefficient was used in the relationship analyses reported in the study.

Conclusions which were an outgrowth of the findings of this study were:

1. Workshop participants, with the exception of one group of primary teachers, made statistically significant gains between pre- and post-test on a test of mathematical understandings and on an arithmetic attitude inventory.
2. When scores of a group of elementary teachers who participated in the mathematics workshop were compared with those of a group of teachers who did not participate in the workshop, it was found that, with initial differences allowed for, the workshop had contributed both to the development of mathematical understandings and to a change in attitude toward arithmetic.
3. Within the limitations of this study, and for the particular group used, a pre-test did not contribute significantly to the development of mathematical understandings or to a change in attitude toward arithmetic.
4. A significant relationship was found to exist between the pre- and post-test scores on the mathematical understandings test and semester hours credit in college mathematics; no significant relationship was found to exist between either the pre-test or post-test scores of an attitude inventory and background factors.

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By

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It is a pleasure to acknowledge the part each person had in bringing the study to a successful completion.

M. J. D.

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CHAPTER I

THE NATURE OF THE STUDY

"Education is being brought into the mainstream of national life."[1] The American people, operating on the premise that our welfare is dependent, among other factors, upon an enlightened citizenry, have focused a spotlight on public education. This spotlight has brought together groups of professional people representing different backgrounds of experience and training to study educational problems and to suggest directions. As a part of this nationwide attempt to revitalize and update the schools, changes have been and are being made in the mathematics curriculum of the elementary school.

The position that mathematics education needed revamping probably could have been taken any time in the last seventy-five years. In fact, David Maxey, writing for Look, said:

> Today's curriculum reform was forced on us. The Post-Sputnik wailing about our science gap is partly responsible. Computers dramatized the mathematical challenges of automation. Most of all, educators realized that the traditional curriculum was not answering the nation's need for scientists, teachers, businessmen--in fact, anybody who could deal with mathematics creatively.[2]

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[1] Jerome Bruner, The Process of Education (Cambridge: Harvard University Press, 1960), p. 18.

[2] David Maxey, "Why Father Can't Do Johnny's Math," Look, XXVII, No. 2 (November 5, 1963), p. 88.

This call for new programs in mathematics, programs which meet the changing needs of society, was also noted by John Kemeny.

> While Mathematics has always been recognized as one of the cornerstones of our educational system, we are entering an era in which an understanding of mathematics will be of even greater importance to all educated men. As our civilization grows in complexity and science plays a more vital role, the man ignorant of mathematics will be increasingly limited in his grasp of the main forces of civilization.[3]

Thus, as a result of recent developments in science and mathematics and also better use of our knowledge of how people learn, there is much activity and interest in the development of new elementary school mathematics programs that are different in both content and approach. In experimental programs, such as the School Mathematics Study Group (SMSG), the Greater Cleveland Mathematics Program (GCMP), and the University of Illinois Project, mathematicians and educators have worked together to set goals and develop new materials, materials which have stimulated new programs and changed existing mathematics curricula in the elementary school. These new programs can neither be ignored nor accepted blindly.

> One cannot predict with assurance either the mathematics needs of today's learner, or the mathematical ideas required for tomorrow's society, but we do know that the learner must be mathematically literate . . . children cannot be denied the

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[3] John Kemeny, "Teaching the New Mathematics," The Atlantic, CCX, No. 4 (October, 1962), p. 90.

> opportunity to learn the mathematics they need as a citizen or the mathematics that is a necessary stepping-stone to many careers.[4]

What do these changes imply for elementary school teachers? Since wise decisions about what to teach are dependent upon the teacher's understanding of curriculum reform, it is imperative that:

> . . . The elementary teacher should know the mathematics necessary to teach the content of the new curriculum.
>
> . . . .
>
> Teachers need a mathematics education which makes it possible for them to understand and appreciate the structure of the mathematics they teach, which makes it possible for them to help children develop a problem-solving technique, and which makes it possible for them to have confidence in their teaching of arithmetic.[5]

Dr. Bernard Gundlach, former Educational Consultant for the Greater Cleveland Mathematics Program, referred to the teacher as the key to progress. He declared that it was up to the teacher to prepare the next generation not only to step in and take over, but also to

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[4]Mary Folsom, "National Problems and Trends in Mathematical Training of Elementary School Teachers," Ten Conferences on the Training of Teachers of Elementary School Mathematics, A Seventh Report of the Committee on the Undergraduate Program in Mathematics to the Mathematical Association of America (Pontiac, Michigan: The Association, 1963), p. 4.

[5]Clarence Hardgrove, "National Problems and Trends in Mathematical Training of Elementary School Teachers," Ten Conferences on the Training of Teachers of Elementary School Mathematics, A Seventh Report of the Committee on the Undergraduate Program in Mathematics to the Mathematical Association of America (Pontiac, Michigan: The Association, 1963), pp. 30 and 34.

make its own contribution to the growing storehouse of human knowledge and development.[6]

Furthermore, he saw the elementary teacher as a builder. "Not only does she set the pace," he asserted, "but she has to build the very foundation upon which the mighty superstructure of human knowledge is to rest."[7]

Continuing to emphasize the importance of the elementary teacher, Gundlach noted that:

> . . . It is on the most elementary level that any curricular revision makes its strongest demands. . . . the elementary teacher cannot any longer fulfill his or her duties entirely by presenting choice bits of information; he or she must <u>interpret</u> and explain the language in detail for the benefit and future use of the young students.
>
> The essential condition for this to take place -- <u>understanding</u> and insight -- is that the teacher herself be thoroughly familiar with the language. . . .[8]

What mathematical understandings are needed by elementary teachers? What is the present status of elementary school teachers with respect to these important mathematical understandings? What means can be used to help teachers gain these needed mathematical understandings? These three questions are questions related to elementary school mathematics which have been pursued by many investigators.

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[6]Bernard Gundlach, Basic Mathematics for Elementary Teachers (Bowling Green, Ohio: Educational Research Council of Greater Cleveland, 1961, p. 3.

[7]Ibid., p. 4.

[8]Ibid.

The first question, that of mathematical understandings needed by elementary school teachers, has been the subject for several research studies. While there is a lack of agreement as to specific mathematical understandings, Newsom,[9] Stipanowich,[10] Schaaf,[11] and others have outlined what they believe should be included in a course dealing with the subject matter of arithmetic, more frequently referred to as elementary school mathematics. The material from the Greater Cleveland Mathematics Program (GCMP)[12] and the volumes on Number Systems[13] and Intuitive Geometry[14] prepared by the School Mathematics Study Group (SMSG) are also examples of materials available to help in planning programs for elementary school teachers.

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[9] C. V. Newsom, "Mathematical Background Needed by Teachers of Arithmetic," The Teaching of Arithmetic, Fiftieth Yearbook of the National Society for the Study of Education, Part II (Chicago: University of Chicago Press, 1951), p. 232.

[10] Joseph Stipanowich, "The Mathematical Training of Prospective Elementary Teachers," Arithmetic Teacher, IV (December, 1957), pp. 240-248.

[11] William L. Schaaf, "Arithmetic for Arithmetic Teachers," School Science and Mathematics, LIII (October, 1953), pp. 540-542.

[12] Robert E. Eicholtz and Martin Emerson, Topics for the Elementary Arithmetic Teacher (Cleveland, Ohio: Educational Research Council of Greater Cleveland, 1961).

[13] School Mathematics Study Group, Number Systems, VI: Studies in Mathematics (New Haven, Connecticut: Yale University Press, 1961).

[14] School Mathematics Study Group, Intuitive Geometry, VII: Studies in Mathematics (New Haven, Connecticut: Yale University Press, 1961).

Clarence Hardgrove, writing for the Committee on Undergraduate Programs in Mathematics, stressed that the emphasis should be the same for the teachers as it is for children. "The needed understandings, . . . are not only those understandings which give meaning to mathematics in the modern elementary school but also those which help the teacher tie the subject together as a related whole."[15]

Question two, the present status of elementary school teachers with respect to mathematical understandings, also has been the topic for several investigations. Robinson observed that "elementary teachers have at best only a mechanical knowledge of arithmetic."[16] Glennon gave further evidence of the arithmetic inadequacies of teachers. He noted that it is difficult for teachers to help children grow in understandings which they themselves do not possess.[17] Bean,[18] Orleans,[19]

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[15]Hardgrove, op. cit., p. 31.

[16]A. E. Robinson, The Professional Education of Elementary Teachers in the Field of Arithmetic ("Teachers College Contributions to Education," No. 672; New York: Bureau of Publications, Teachers College, Columbia University, 1936).

[17]Vincent J. Glennon, "A Study in Needed Redirection in the Preparation of Teachers of Arithmetic," Mathematics Teacher, XLII (1949), p. 395.

[18]John Bean, "Arithmetical Understandings of Elementary School Teachers," Elementary School Journal, LIX (May, 1959), pp. 47-50.

[19]Jacob S. Orleans, The Understandings of Arithmetic Processes and Concepts Possessed by Teachers of Arithmetic ("Office of Research and Evaluation, Division of Teacher Education," Publication No. 12; New York: College of the City of New York, 1952).

and a host of other investigators also have reported research studies which point to the deficiency of elementary school teachers with respect to mathematical understandings.

How can the mathematical understandings of elementary school teachers be improved? This question, a third question related to elementary school mathematics, has become, in the eyes of many educators, one of the most challenging and persistent problems faced by education. It is a question which calls for study of methods or procedures for remedying the situation rather than collecting and presenting additional evidence on the inadequacy of elementary school teachers. Numerous educators see this need for professional growth as a need far too pressing to be limited to either the pre-service or the in-service education of teachers.

Certainly, as suggested by Bean, Schaaf, Orleans, and Newsom, teacher trainees can be prepared through an up-to-date approach to mathematics. Yet, if an adequate program of mathematics could be inaugurated at once on a pre-service basis, only about five per cent of the 900,000 elementary school teachers would be directly affected each year.[20] At that rate, except for the spread from individual to individual, the schools would be at least twenty years incorporating present ideas.

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[20]Hardgrove, op. cit., p. 31.

Five years ago, just a few months after the appearance of the 1957 yearbook of the National Society for the Study of Education, which was devoted to in-service education, Ashby, in an editorial for Educational Leadership, wrote:

> The rapid acceleration of the phenomenon of change in modern society makes in-service education a more significant and challenging problem than ever before in the history of the teaching profession.
>
> . . . .
>
> Today's teacher must be alert to keep up with the out-of-school learnings of pupils. At the same time, the revolution in technology and production has provided new knowledges and tools to be used in teaching.
>
> . . . .
>
> As teaching becomes more of a profession and less of a procession, more teachers are removed from their preservice training than ever before. The training they received even a decade ago is inadequate today either as to substance or as to methodology. This underlines the demand for effective in-service education. . . . Without it, our schools cannot adequately prepare boys and girls for a dynamic society.[21]

Bean recommended the following two courses of action for the in-service program for improving arithmetic instruction:

1. A series of state, regional, and district workshops in arithmetic.
2. A series of regional extension courses under the direction of the colleges in each state.[22]

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[21]Lynn Ashby, "Today's Challenge to In-Service Education," Educational Leadership, XV, No. 5 (February, 1958), pp. 270-271.

[22]Bean, op. cit., p. 450.

In the preceding introductory paragraphs it has been indicated that changes have been and are being made in the mathematics curriculum in the elementary school. Also, it has been pointed out that these changes have made it imperative for the elementary teachers "to understand and appreciate the structure of the mathematics they teach --a means which makes it possible for them to help children develop a problem-solving technique and a means which makes it possible for them to have confidence in their teaching of arithmetic."[23]

While various forms of in-service education have been utilized by school systems and state departments of education, few have been the reports which attempted to evaluate objectively the effectiveness of any of the techniques being employed in the in-service education program of teachers. The lack of such studies pointed toward the need for the present study.

Specifically, the present study attempted to evaluate the effectiveness of one series of state-sponsored workshops in arithmetic for elementary school teachers in: (1) improving basic mathematical understandings, (2) changing the attitudes of elementary school teachers toward mathematics, and (3) improving the classroom practices of teachers with respect to the teaching of arithmetic.

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[23] Hardgrove, op. cit., p. 34.

I. The Need for the Study

During recent years, there has been rather general acceptance among educators of the need for in-service education. Many educators see it as one of the outstanding needs in the field of education.

> Research on the effectiveness of various kinds of in-service programs is needed if efforts to improve understanding of teachers are to be efficient and effective in the improvement of educational opportunities for boys and girls.[24]

While research studies with respect to arithmetic have increased in number, there has been a paucity of studies in the area of in-service teacher education with respect to arithmetic. Since 1957, the annual bibliographies of research, prepared by J. Fred Weaver for the Arithmetic Teacher, list over three hundred research studies and dissertations on elementary school mathematics. However, less than twenty-five studies of which nine were dissertations had teacher education as the main area of emphasis. Only three of the dissertations, one by Rudd, another by Boyd, and a third by Houston, attempted to study specific in-service procedures. Several other studies, however, either surveyed present conditions or studied a particular aspect of the pre-service program.[25]

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[24]W. Robert Houston, Claude C. Boyd, M. Vere DeVault, "An In-Service Program for Intermediate Grade Teachers," Arithmetic Teacher, VIII (February, 1961), p. 65.

[25]For further information concerning these studies, see Chapter II, pp. 44-50.

Sparks, in an article which summarized recent research with respect to arithmetic understandings needed by elementary school teachers, concluded that the research to date has been "too limited in scope to offer substantial assistance in planning programs of teacher education."[26]

Barnett and Jansen, in a book of readings edited by DeVault, referred to the situation as a distinct challenge to those interested in teacher education. They suggested that there is a need today to discover ways of remedying the situation rather than to collect and to present additional evidence of the elementary school teacher's lack of mastery of mathematics.

They enumerated the following as possible areas to include in future studies:

1. Experimentation in search of effective ways to improve the teacher's mathematical knowledge and understanding.
2. Clarification of what the elementary school teacher should know.
3. Evaluation of the contribution of pre-service and in-service aspects of the teacher's preparation.
4. Utilization of various techniques to relate certain aspects of

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[26]Jack Sparks, "Arithmetic Understandings Needed by Elementary School Teachers," Arithmetic Teacher, VIII (December, 1961), p. 395.

preparation of improved teaching and learning mathematics in the classroom.[27]

Houston and DeVault, in describing a study carried out in connection with the elementary schools of Dallas, Texas, stated that:

> Studies are needed which investigate the changes brought about in the classroom as a result of in-service education programs. Case-studies, classroom observation, pupil interview technique, and experimental studies involving pre- and post-testing of both teachers and pupils are but some of the techniques which should be utilized. . . . Such studies must be forthcoming if continued effort devoted to in-service education is to make the greatest possible contribution to the continued growth of teachers.[28]

Thus the need for a study which would push further the "knowledge claim" that an in-service education program, with proper content emphasis, can raise the level of understandings of elementary school teachers with respect to basic mathematical understandings has made itself manifest.

II. The Problem

Statement of Problem

It was the purpose of this investigation to analyze the effectiveness of the "workshop" as a means of in-service teacher education for

[27]Glenn Barnett and Udo Jansen, "Planning for Teaching Competence in Mathematics Education," Improving Mathematics Program, ed. M. Vere DeVault, (Columbus: Charles E. Merrill Books, Inc., 1961), p. 466.

[28]Houston, DeVault, and Boyd, op. cit., p. 65.

developing needed mathematical understandings of elementary school teachers, for changing the attitudes of elementary school teachers toward mathematics, and for improving classroom procedures. The workshops used in the study were a part of the 1963-64 series of Science and Mathematics Workshops conducted by the Missouri State Department of Education under Title III, National Defense Education Act.

The effectiveness of the "workshop" was evaluated in terms of: (1) growth in mathematical understandings of elementary school teachers, (2) changes in attitudes of elementary school teachers toward mathematics, and (3) improvement in teacher classroom practices.

Hypotheses

The type of changes made by elementary school teachers and the effect of these changes upon classroom procedures, such as were examined as part of this study, were grouped into three categories:

A. Changes in mathematical understandings of elementary school teachers which were:

   1. Related to the basic structure of mathematics.
   2. Related to informational concepts.
   3. Related to logical abstractions or generalizations.

B. Changes in teacher attitudes toward mathematics which were:

   1. Related to over-all reaction to the subject.
   2. Related to its importance in the elementary school curriculum.

C. Changes in teacher performance in the classroom which were:

1. Related to use of materials, activities, and teaching aids.

2. Related to classroom organization and evaluation.

The following three hypotheses were formulated and tested in the present study.

Hypothesis Number One.--There will be a significant difference on a test of mathematical understandings between the post-test scores of a group of elementary school teachers who have participated in an in-service education program and their pre-test scores or the scores of a similar group who have not participated in the in-service teacher education program.

Two other questions directly related to this hypothesis were examined.

1. What effect, if any, does a pre-test have upon the post-test scores of teachers on a test of mathematical understandings?
2. What is the relationship of improvement in basic mathematical understandings to background factors?

Hypothesis Number Two.--There will be a significant difference on an arithmetic attitude inventory between the post-test scores of a group of elementary school teachers who have participated in an in-service education program and their pre-test scores or the scores of a similar group who have not participated in the in-service education program.

The two questions concerning test-retest effect and the relationship to background factors were noted here. The questions were stated as:

1. What effect, if any, does a pre-test have upon the post-test scores of teachers on an attitude inventory?
2. What is the relationship of improvement in arithmetic attitudes to teacher background factors?

Hypothesis Number Three.--There will be a significant difference between the classroom practices of a group of elementary school teachers who had participated in an in-service education program and the classroom practices of a similar group who had not participated in the in-service education program.

The Research Design

All participants in two of the five mathematics workshops for elementary school teachers sponsored by the Missouri State Department of Education during the second semester of 1963-64 under Title III, National Defense Education Act, constituted the sample for this study. A third group of teachers, who had indicated an interest in participating in a mathematics workshop but who had not begun the workshop experience, was used as a control group.

These workshops, specific in-service activities, a part of a state-wide in-service teacher education program, were conducted in the Southeast Missouri State College Service Area by the same mathematics consultant from the Missouri State Department of Education.

To test the three major hypotheses of the study and for the evaluation of the data relative to the background factors, a variation of the design referred to by Campbell and Stanley as an expanded "Recurrent Institution Cycle Design" was utilized.[29] Such a procedure allowed the researcher to utilize features of both the one-group "Pre-test, Post-test Design" and the "Static Group Comparison Design."

For this particular study, the following independent variables were used:

1. In-service presentation
2. Pre-testing
3. Post-testing
4. Grade level

Dependent variables were:

1. Growth in basic mathematical understandings of elementary school teachers.
2. Changes in attitudes of elementary school teachers toward mathematics.

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[29]Donald T. Campbell and Julian C. Stanley, "Experimental and Quasi-Experimental Designs for Research on Teaching," Handbook of Research on Teaching, ed. N. L. Gage (Chicago: Rand McNally and Company, 1963), p. 227.

3. Changes in teacher classroom procedures.[30]

The background information furnished by each teacher included: (1) sex, (2) years of teaching experience, (3) number of credits in high school mathematics, (4) number of semester hours credit in college mathematics, (5) number of semester hours credit in college mathematics teaching methods, and (6) highest degree attained.

Instruments selected or developed for use in the collection of data were:

1. Teacher Information Form and Problem Census Questionnaire.
2. A Test of Mathematical Understandings (two parallel forms).
3. Revised Form of Dutton Arithmetic Attitude Inventory.
4. Interview Question Schedule and Rating Scale.[31]

The analysis of covariance statistical technique was utilized in the analysis of the data relative to the effectiveness of the in-service presentation. Pearson Product-Moment Correlation Coefficient was used in the relationship analyses reported in the study.

III. The Limitations of the Study

It seems pertinent to point out at least three limitations in the present study. First, the length and number of sessions did not

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[30]For a more detailed explanation of the design, see Chapter III, pp. 72-112.

[31]See Appendices A, B, C, D, and E for samples of these instruments.

permit either treatment in depth of new concepts or extensive review of concepts teachers may have had in an arithmetic course for teachers. Second, since the data used in the study were obtained from one series of workshops, any generalizations concerning the results and conclusions drawn were limited by the extent to which the population represented regional and local groups. Finally, the extent to which the instruments used measured adequately the effects of the in-service program was a limitation. Certainly the instruments used in the study had the inherent limitations of paper-and-pencil tests and interview instruments. As pointed out by Glennon, far superior to the pencil-and-paper test would be "the study of the behavior of each person individually through conversing with him and keeping anecdotal records of his performances on the test items."[32]

IV. Definition of Terms

Attitude

Attitude was the term used to refer to the position assumed or reaction toward the subject of arithmetic that was or was not reflected by the participants' behavior pattern.

Classroom Procedures or Classroom Practices

The use of the term classroom procedures, or classroom

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[32] Glennon, op. cit., p. 395.

practices, in this study was used to refer to the teachers' performance in the classroom.

Educational Consultant

Educational consultant, as used in this study, referred to a qualified staff member from the state department of education, a person possessing special knowledge and experience, who had been asked by professional groups to work directly with them in providing assistance in connection with an educational problem.

Elementary School Teachers

The term elementary school teachers was used to identify persons teaching in grades kindergarten through six.

In-Service Education

The term in-service education, as used throughout this investigation, referred to those experiences, processes, procedures, and activities on the part of the employed teacher which were designed to contribute to professional growth.

Mathematical Understandings

Mathematical understandings, as used in this study, was the term used to refer to those generalizations about the structure of mathematics which give significance to computational skills, informational concepts, and logical abstractions.

Workshop

Workshop was the term used to refer to a particular form of in-service education designed to help teachers secure new or modified points of view and to acquire new knowledge, new understandings, and new techniques for classroom presentation. It involved planning with, active involvement of, and evaluation by the participants.

IV. Summary

This chapter's chief concern was to orient the reader to the study. The study was introduced through a discussion on the current significance of teachers understanding the mathematics which they teach. The problem was concisely stated and the nature and scope of the study described. In brief form the procedure for implementing the study was discussed. Chapter II will review the literature pertinent to the study. Chapter III will describe the study setting, the teacher population, and the procedures utilized in pursuing the study. Chapter IV will present, through tables, figures, and explanatory material, an analysis of data relative to the study. In Chapter V, a summary will be presented; conclusions and implications for further research also will be discussed.

CHAPTER II

REVIEW OF RELATED LITERATURE

Extensive research into the various aspects of elementary school mathematics has been reported not only in published and unpublished research but also in journals, monographs, periodicals, encyclopedias, professional books, and publications of the government and learned societies. Several authors have, at various times, presented exhaustive reviews of research in The Arithmetic Teacher, The Review of Educational Research, and School Science and Mathematics.[1] At least three annotated bibliographies also have been compiled.[2]

The review of literature pertinent to this study has been organized under four categories: (1) improving mathematical understandings of elementary school teachers, (2) utilizing in-service education procedures, (3) changing attitudes of elementary school teachers toward mathematics, and (4) improving classroom procedures of elementary school teachers with respect to teaching mathematics.

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[1]For further information see bibliographical entries under the names of: (1) J. Fred Weaver, (2) Robert L. Burch and Harold F. Moser, (3) Glenadine Gibb and Henry Van Engen, (4) Herbert F. Spitzer and Paul C. Burns, (5) Maurice L. Hartung, or (6) E. G. Summers.

[2]See bibliographical entries under Kenneth Brown, William L. Schaaf, or Guy T. Buswell.

I. Improving Mathematical Understandings of Elementary School Teachers

One of the main points of concern with respect to present-day education has been that arithmetic, a very important branch of mathematics, is failing to provide the solid basis of competence needed either for effective citizenship or to move up the path into higher mathematics. Many authorities believe that success in mathematics "rests on the foundation built in the first six grades, and an improvement in these grades will allow for a substantial strengthening of the program for the higher grades."[3]

Since the quality of mathematical instruction and hence the level of pupil achievement depend, in part, upon the mathematical competence of the teacher, a careful preparation of elementary school teachers in mathematics subject matter becomes an important aspect of an improved program of arithmetic in the elementary school. "A firm grasp of basic arithmetical concepts and processes is essential to teach arithmetic meaningfully. Teachers cannot teach understandings that they themselves do not have."[4]

Few research studies, however, can be found which closely relate to the problem of the mathematical competence of the elementary

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[3]J. Fred Weaver, "A Crucial Problem in the Preparation of Elementary School Teachers," Elementary School Journal (February, 1956), p. 436.

[4]Bean, op. cit., p. 447.

school teacher. The first known study was an investigation made by E. H. Taylor in 1938. Taylor administered a test on meanings in arithmetic to three hundred and thirty-three freshmen at Eastern Illinois State Teachers College. He found that the group tested was deficient in both the mechanics and the understandings of arithmetic.[5]

Vincent J. Glennon, in his pioneer study on the growth and mastery of certain basic mathematical understandings, brought to light serious deficiencies in the mathematical background of teachers and also began a needed redirection in the preparation of teachers of arithmetic. He had difficulty locating a suitable instrument for evaluating the mathematical understanding of teachers and subsequently developed, as part of his study, an eighty-item test for measuring basic mathematical understandings. Since that time, this multiple-choice test, referred to as a "Test of Basic Mathematical Understandings," has been used by numerous other investigators.

For his study, which attempted to obtain an index of prevailing conditions within the groups being studied, Glennon used the results from administering the test to persons at seven different educational levels (pupils in grades seven, eight, nine and twelve, freshmen, college seniors, and teachers-in-service). He found that there was no

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[5]E. H. Taylor, "Mathematics for a Four-Year Course for Teachers in the Elementary School," School Science and Mathematics, XXXVIII (May, 1938), pp. 409-503.

significant difference in achievement of basic mathematical understandings between teachers' college freshmen and teachers' college seniors. He also found that the teachers-in-service understood about fifty-five per cent, or slightly more than one-half, of the total items. From his data, he concluded "that significant growth was not being accomplished by persons at any step of the educational ladder."[6]

Glennon's findings supported two other hypotheses:

1. There was no significant difference in achievement of basic mathematical understandings between a teachers' college senior who had taken a course in the Psychology and Teaching of Arithmetic and a teachers' college senior who had not taken such a course.
2. There was no significant difference in achievement of basic mathematical understandings between teachers-in-service who had done graduate work in the Psychology and Teaching of Arithmetic and those who had not taken such graduate work.[7]

In his summary, Glennon suggested that one aspect of a needed redirection in the training of teachers seemed to lie in the professional training offered in teachers colleges and schools of education.

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[6]Glennon, _op. cit._, pp. 392-393.

[7]_Ibid._

> This training, as it is usually set up at the present time, consists of a single course in the methodology of teaching arithmetic as a "tool" subject. Little emphasis is placed upon the professional study of arithmetic as a science of number, as a system of related ideas, or as a series of number relationships.[8]

His findings also seem to suggest several aspects of needed redirection in the program of in-service education of teachers of arithmetic.

> Curriculum revision of the professional courses must be concerned with emphasizing the subject matter as well as with the principles of teaching the subject-matter.[9]

Many investigations have substantiated Glennon's major findings and conclusions. For example, J. Fred Weaver administered the Glennon test as a pre-test to four groups of upper-classmen prior to their taking the course entitled "Methods of Teaching Arithmetic." The average score made by the students was forty-four per cent, a score which did not vary greatly from the average score as noted by Glennon when he administered the same test.[10]

For one of the four groups taking the methods course, Weaver used the test both as a pre-test and as a post-test. When the average level of student understanding was raised from slightly more than forty-five per cent to seventy per cent, he noted the gain as significant and concluded that:

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[8] Ibid.

[9] Ibid.

[10] Weaver, op. cit., p. 261.

> A methods course with proper content emphasis . . . can result in significant improvement in the students' level of mathematical understanding.[11]

Bean, working at Stanford University, used the Glennon test for a comparative study of the mathematical understandings of four hundred and fifty Utah teachers. The mean score for the Utah teachers was 52.46, or 65.58 per cent of all items, answered correctly. However, when questioned, 64.57 per cent of the teachers did not think the test adequately measured the extent of their understandings while 35.43 per cent thought it did.[12]

Rodney saw the improvement of pre-service preparation of teachers as one approach to the improvement of mathematics in the elementary school. In 1951, he carried out an investigation which attempted to evaluate the pre-service preparation for teaching mathematics in the elementary schools as offered by the State College for Teachers at Buffalo, New York.

He developed a forty-item test of mathematical understandings and administered the test to over three hundred students at the State College for Teachers at Buffalo. Then, using the normative survey methods of research, he attempted to: (1) analyze the content of the teacher preparation program at that institution, (2) compile a list of

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[11] Ibid.

[12] John Bean, "The Arithmetic Understandings of Elementary School Teachers," (unpublished Ph. D. dissertation, Stanford University, 1958).

major objectives for the teaching of elementary school mathematics, and (3) study the conditions existing in three groups (freshmen, seniors, and classroom teachers) with respect to mathematical understandings.

His findings, which did not vary significantly among the three groups, supported previous investigations.[13] In summarizing his study, Rodney saw the need for organizing a program for the professional preparation of teachers which would "alleviate, to the greatest extent possible, apparent deficiencies."[14]

He also endorsed a program of in-service education when he said:

> An in-service program for the study of the meanings of the content of the elementary school mathematics should contribute to an improvement of teaching elementary school mathematics.[15]

That the present condition of teacher understanding in arithmetic leaves much to be desired also was noted by Orleans in a study published by the Office of Research and Evaluation, College of the City of New York. He made "a systematic effort to ascertain the extent to which teachers and prospective teachers of arithmetic understand the arithmetic processes and concepts represented by the short cuts they teach."[16]

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[13]Cecil T. Rodney, "An Evaluation of Pre-Service Preparation for Teaching the Mathematics of the Elementary School," (unpublished Ph. D. dissertation, University of Buffalo, Buffalo), 1951, p. 105.

[14]*Ibid.*, p. 108.

[15]*Ibid.*, p. 107.

[16]Orleans, *op. cit.*, p. 1.

For his first test, Orleans administered a free-answer type test to seven hundred twenty-two persons from five different levels: (1) undergraduates from four colleges, (2) upper classmen who were doing their student teaching, (3) students in graduate courses in education, (4) classroom teachers, and (5) other persons.

Then, using many of the responses given, he constructed an eighteen-item test which was of the multiple-choice type. This test, designed to measure teachers' understanding of the processes they teach, included items concerning long division and multiplication, the meaning of dividing by a fraction, reducing and raising fractions, remainders, subtraction, and per cent.

After administering the test to three hundred twenty-two teachers (fifty-three primary teachers, seventy-six upper grade teachers, sixty-seven junior and senior high mathematics teachers, and one hundred twenty-six other teachers), he noted that there were "few processes, concepts, or relationships which were understood by a large percentage of teachers."[17]

While the test was a relatively short one and no attempt was made to establish validity, Orleans concluded that "it was difficult for this group of people to verbalize their thoughts when they attempted to explain arithmetic concepts and processes."[18] He contributed this lack of

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[17] Ibid., pp. 1-59.

[18] Ibid., p. 37.

understanding of arithmetic to the rote process which has been so prevalent in the past.[19]

Other individuals have carried out studies directed to the pre-service level. Of particular interest were Fulkerson's findings with respect to teaching experience and college classification. Since students with teaching experience did significantly better than those without experience and performance became increasingly better as the level of college classification increased, findings which somewhat contradicted previous research, Fulkerson suggested further study of this phase of teacher training.[20]

Carroll's study attempted to deal directly with the problem of teachers' background in mathematics. An evaluation instrument, which was designed to assess the mathematical background of a group of prospective teachers, was the primary research tool.

The initial part of the study sought to establish a list of mathematical understandings which elementary teachers should know. To devise such a list, five series of elementary arithmetic textbooks, published between 1946 and 1952, were analyzed. A panel of sixteen authorities in the field of mathematics education evaluated the list

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[19] Ibid.

[20] Elbert Fulkerson, "How Well Do 158 Prospective Elementary Teachers Know Arithmetic?" The Arithmetic Teacher, VII (March, 1960), pp. 141-146.

of understandings and the questions which were subsequently developed.[21]

Using questions on which there was almost unanimous agreement, Carroll assembled two eighty-one item tests which she administered to teacher-training classes at Wayne State University. After the resulting data were statistically analyzed, Carroll selected items for a test which she "hoped would aid in the problem of diagnosing and remedying inadequate teacher background in mathematics."[22]

A study of university students' comprehension of arithmetical concepts was carried out by Dutton in 1961. His study was designed to measure students' understanding of arithmetical concepts as they progressed through courses designed to teach these processes. Two classes, composed of fifty-five prospective teachers, were tested at the beginning of the semester and again at the end with the University of California Achievement Test for Sixth Grade, a test covering basic arithmetical concepts that students are expected to know at the completion of grade six.[23]

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[21]Emma Carroll, "A Study of the Mathematical Understandings Possessed by Undergraduate Students Majoring in Elementary Education," (unpublished Ed. D. dissertation, Wayne State University), 1961.

[22]Ibid.

[23]The appendix of Evaluating Pupils' Understanding of Arithmetic, pp. 143-148, contains a copy of this test. For full bibliographic reference, see entry under Wilbur H. Dutton in the bibliography.

The conclusions drawn were that prospective elementary school teachers adhered to many arithmetical concepts and procedures learned in elementary and junior high school. While there were many basic arithmetical concepts understood by these prospective teachers, the students often adhered to traditional methods and mechanical procedures when attempting to explain the following concepts: (1) partial products in multiplication, (2) placement of quotient figures in long division, (3) placement of the decimal point in problems involving decimal fractions, and (4) understanding of and use of denominate numerals.[24]

Augustine P. Cheney, a graduate student at the University of California, constructed a fifty-two item test using thirty items from the instrument prepared by Dutton to measure understanding of basic mathematical concepts. After administering the test to one hundred twenty teachers in eight selected elementary schools in Ventura and Los Angeles Counties, he reported that these teachers understood some aspects of arithmetic but had difficulty with other phases such as place value, remainders in division, fractions involving multiplication and division, comparison of decimal fractions, and use of denominate numbers.

He also compared the mean scores of the primary teachers with those of the intermediate grade teachers. With a mean score of 29.91

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[24]Wilbur H. Dutton, Evaluating Pupils' Understanding of Arithmetic, Englewood Cliffs, N. J.: Prentice-Hall, 1964, pp. 54-55.

for the primary group and 38.74 for the intermediate grade teachers, he reported the difference as significant well beyond the one per cent level.[25]

Dutton, in a recent publication, called the study of Cheney important for the following reasons:

> . . . (1) His careful review of research studies dealing with the evaluation of teachers' understanding of arithmetic points out the paucity of studies and valid instruments in this area. (2) He accentuates the important fact that teachers have difficulties with certain aspects of arithmetic and that wide differences exist among teachers. (3) Primary-grade teachers seem to have an understanding of the simple concepts taught to young children and have neglected more advanced concepts taught in intermediate grades.[26]

Thus, in study after study, the need for teachers to have a better understanding of the basic concepts of mathematics has been stressed. Elementary teachers have "neither the facility in the computation processes which they are expected to teach nor a firm grasp of the basic mathematical concepts underlying the processes."[27]

Weaver, in reviewing what he termed a "crucial problem" in education, noted that:

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[25]Augustine B. Cheney, "Evaluation of Elementary School Teachers' Understanding of Basic Arithmetic Concepts," (unpublished M. A. thesis, University of California, Los Angeles, 1961).

[26]Dutton, _op. cit._, pp. 55-56.

[27]Doyal Nelson and Walter Worth, "Mathematical Competence of Prospective Elementary Teachers in Canada and United States, _The Arithmetic Teacher_, VIII (April, 1961), p. 147.

> Two things stand out clearly from the conditions discussed thus far: (1) the general level of arithmetic on the part of the undergraduate in representative teacher-training institutions is inexcusably low; (2) all too few teacher-training programs provide appropriate work in background mathematics that could be definitely helpful in raising the undergraduates' level of arithmetic scholarship.[28]

II. Utilizing In-service Education Procedures

Increasing standards of pre-service education have not lessened the need for continued in-service education. New chapters in the history of education have been written. Experimental programs have been tried out in all parts of the country. Some of these experiments have been pronounced worthless and discarded; others were deemed good and, therefore, widely adopted, thus influencing aims, methods, curricula, and course content. As a result, teachers, whether "neophytes" or "old-timers," found themselves needing to acquire new knowledges, new points of view, and new methods.

> . . . Our rapidly changing culture and its implication for curriculum change, the continuing increase in pupil enrollments and numbers of teachers, the need for improved school leadership, and the continuous additions to our knowledge about children and youth and the learning process . . . mean that professional school people need to work continuously to keep abreast of what they must know and must be able to do.[29]

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[28] Weaver, _op. cit._, p. 258.

[29] Stephen M. Corey, "Introduction," Chapter I, _Fifty-sixth Yearbook of the National Society for the Study of Education, Part I_ (Chicago, Ill.: University of Chicago Press, 1957), p. 1.

That there is a need for programs of in-service education is rarely contested. Baker, in commenting on the urgency, not only for adequate and up-to-date preparation of teacher trainees, but also for the development of means for today's teachers to be cognizant of the new content and new tools which the technological revolution has provided for education, said:

> Whether a teacher is new to a school system or is an "old-timer," whether he is a beginning teacher or one of long experience, there is need for an effective in-service education program. . . . The teacher who does not have the opportunities afforded by an in-service program, in too many cases, soon becomes antiquated.[30]

Actually, in-service education, defined as those experiences engaged in by the employed teacher during her service which are designed to contribute to professional growth, does not represent a new idea. "Such activities have been part and parcel of American education for more than a century."[31]

Programs, designed for contributing to the in-service stimulation and growth of the teacher, have included: (1) the teachers' institute, (2) the reading circle, (3) the summer school, (4) university extension, (5) supervision, (6) the teachers' meeting, (7) voluntary associations, and (8) the workshop.

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[30]T. P. Baker, "What Is An Effective In-Service Education Program?" Proceedings of The Thirty-Eighth Annual Convention, National Association of Secondary School Principals (Washington: The Association, March, 1951), p. 46.

[31]Corey, op. cit., p. 2.

Historians are in general agreement that the teachers' institute, wholly American in origin, was the earliest form of in-service education. Originating at a time when there was a shortage of well-trained teachers, the institute was said to have been designed to provide intensive additional training for the inadequately prepared teacher.[32]

The nature and purpose of the early institute as described by Barnard, Page, Sweet, and others was summarized by Horace Mann when, in 1845, he wrote:

> It is the design of a Teacher's Institute to bring together those who are actually engaged in teaching Common Schools, or who propose to become so, in order that they may be formed into classes, and that these classes, under able instructors, may be exercised, questioned and drilled, in the same manner that the classes of a good Common School are exercised, questioned, and drilled.[33]

Thus the institute, teaching content which the prospective teachers would later teach, served to provide, when well conducted, instruction in the approved methods of teaching.

Then, shortly after 1900, the institute began to disappear. Its value as an agency of in-service education, however, had been recognized. Today it is acknowledged as a forerunner of other improved practices.

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[32]Willard S. Elsbree, The American Teacher (New York: American Book Company, 1939), p. 135.

[33]Teacher Institutes or Temporary Normal Schools, pp. 45-46, cited by Herman Richey, In-Service Education (Chicago: University of Chicago Press, 1957), p. 39.

As normal schools and the evolving teachers' colleges came to require high school graduation for admission to their programs and as many of the college graduates moved into teaching, reading circle courses, conducted by various related state organizations, came to be recognized as a source from which lists of books on the new movements in education and on various subjects relating to the profession could be obtained. Even today, in some states, reading-circle services continue to be maintained and used widely by teachers.

Other forms of in-service teacher education--the summer school, university extension, supervision, teachers' meetings, and voluntary associations--have all aided in the upgrading of the teaching staff.[34] It was, however, the Eight-Year Study of the Commission on the Relation of School and College in the Progressive Education Association which brought significance to the workshop movement and gradually changed the concept of in-service education from that of "upgrading" to "keeping abreast of a changing world."[35]

> At one time the main emphasis in the in-service education of teachers was on bringing up to "standard" persons who had been employed with what was deemed to be inadequate preparation. There was a certain implication that, once this level was reached, and suitably recognized either by a degree, a higher

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[34]Elsbree, op. cit., pp. 135-137.

[35]C. Glen Hass, "In-Service Education Today," Ch. II. In-Service Education, Fifty-sixth Yearbook of the National Society for the Study of Education, Part I (Chicago, Ill.: University of Chicago Press, 1957), p. 16.

type of certificate, or an automatic increase in salary, the education of the teacher would be complete. . . . The newer emphasis is on every teacher continuing to give a certain amount of time to experiences calculated to lead to personal and professional growth.[36]

The Workshop Idea

Since its inception in 1936, when the first workshop for teachers was organized at the University of Ohio under the leadership of Ralph W. Tyler, Chairman of the Department of Education, University of Chicago, the workshop idea has not only gained acceptance but has been widely used as one means for the in-service education of teachers.[37] While often differing in type of organization, kind of facilities, length of operation, frequency of meetings, source of leadership, and sponsoring agencies, the workshop is thought normally to consist of opportunities for teachers, under expert guidance, to work on individual problems while acquiring new knowledge, new understandings, and new techniques for classroom presentation.[38] This type of in-service education has appeared in the form of courses offered for credit, seminars for studying particular problems, clinics, field trips, testing programs, discussion groups, and curriculum studies.

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[36]Commission on Teacher Education, Teachers for Our Times (Washington: American Council on Education, 1944), p. 19.

[37]Mary A. O'Rourke and William Burton. Workshop for Teachers (New York: Appleton-Century-Crofts, Inc., 1957), p. 1.

[38]Ibid., pp. 3-4.

Earl C. Kelley, in his book, The Workshop Way of Learning, referred to the workshop as an educational method which placed the responsibility for learning upon the student--a possible means for "putting into practice the truths that have become known about how people learn."[39] He also called the workshop an adventure "on the growing edge of the learning process."[40]

Mitchell, in a report for the North Central Association of Colleges and Secondary Schools which attempted to identify the characteristics of the workshop, contended that the workshop idea was a widespread and generally accepted form of in-service education.[41] However, he found only nine dissertations which were devoted to some phase of the workshop experience. Three of these studies investigated the workshop in general, two others considered the contribution of the workshop experience to classroom practices, another evaluated an organized program of workshops conducted by a state department of education, and the remainder described workshops conducted in specific areas of education or in selected situations.[42]

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[39] Earl C. Kelley, The Workshop Way of Learning, (New York: Harper and Brothers, 1951), p. ix.

[40] Ibid., p. 1.

[41] James Mitchell, "The Workshop as an In-service Education Procedure," Report of the Sub-committee on In-service Education of Teachers of the North Central Association of Colleges and Secondary Schools, North Central Quarterly, XXVIII (April, 1954), pp. 423-457.

[42] Ibid., pp. 424-425.

An early study concerned with the contribution of the workshop experience to classroom practices was based on observation of participants upon their return to their classrooms. Heaton stated that:

> As near as sincere professional appraisals could tell by using observation guides, there was valid evidence that the participants return to their classrooms with: (1) recognizable and notable changes in attitude toward their work and (2) a strong drive toward doing something about their curricular and instructional problems.[43]

Improvement in use of materials and in instructional practices also was observed and attributed to workshop presentation.[44]

Forest Mitchell investigated the effect of participation in a workshop upon classroom practices. The purpose of the study was to identify changes that were made in selected classroom practices and to discover any contributions that participation in a summer workshop might make.[45]

Another of the early studies devoted to the workshop idea was the investigation carried out by Mode Lee Stone at George Peabody College for Teachers. Often referred to as the first comprehensive study on workshops, this research presented an analysis of the total faculty

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[43]Kenneth L. Heaton et al, Professional Education for Experienced Teachers (Chicago: University of Chicago Press, 1940).

[44]Ibid.

[45]For a full report on this study, see page 60.

workshop technique as it related to the Florida Program of Curriculum Improvement.

The researcher sought to explore the consistency with which the procedures and products of the total faculty workshop technique: (1) met the criteria of a good learning experience, (2) contributed to the in-service education of teachers, and (3) resulted in better opportunities for learning on the part of children.

First, from professional books, current bulletins, dissertations, and periodicals, Stone assembled a list of sixteen techniques used in in-service education. Then state departments of education were asked to respond to a questionnaire on the value of each technique.

In addition, data from two school systems which had participated in the in-service program for more than one year were also examined. From this, Stone concluded that there was direct evidence demonstrating the benefits of this type of workshop.[46]

Another intensive study on workshop participation, according to many sources, is the one prepared by Mary O'Rourke. In this research, O'Rourke, after reviewing more than 200 references on the in-service workshop, traced the history of the workshop movement. The historical account was supplemented with a body of original data gathered from a questionnaire sent to 261 administrators in 43 states.

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[46]Mode Lee Stone, "An Analysis of the Total Workshop Technique," (unpublished Ed. D. dissertation, George Peabody College for Teachers, 1941).

In addition, a group of fifty-one elementary teachers in New England was interviewed and observed in the classroom.[47]

Seeing the in-service education workshop as: (1) an effective vehicle for cooperatively attacking a problem, (2) a valuable means of inducting new teachers, and (3) an opportunity for administrators and teachers to evaluate and retrain their skills, O'Rourke concluded that the "in-service workshop, one current type of professional education, was a positive force in raising the level of teacher education as testified by teachers and administrator-participants."[48]

While it has been nearly twenty years since Henderson reported a study on the Evaluation of the workshop program directed by the Ohio State Department of Education, the investigation continues to hold the unique position of being one of the few known investigations relative to a workshop program conducted by a state department of education.

Collecting the data over a period of three years (1944-47), Henderson attempted to evaluate the effectiveness of the workshop program for elementary school teachers in Ohio by determining the degree and extent to which the workshop program contributed to the development in teachers of democratic attitudes toward teaching.

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[47] Mary O'Rourke, "The In-service Workshop in Elementary Education: Its Effect Upon Participants," (unpublished dissertation, Cambridge, Massachusetts: Harvard Graduate School of Education, 1954).

[48] Ibid., p. 9.

Approximately 1600 teachers, 155 principals, and 26 county, village, and city superintendents participated in the program. Also, nearly two hundred consultants, who had served the workshops, were asked to evaluate the effectiveness of the workshops. Questionnaires, attitude inventories, and letters of inquiry were used.

The following were included in the findings:

1. Superintendents, in most cases, had initiated the workshops in their school systems; few teachers had participated in planning and evaluating the workshops.
2. Inadequate pre-planning of each project constituted a major weakness.
3. Most consultants and teachers indicated that the length of time devoted to each workshop was inadequate.
4. In most instances, the workshop group was too large for effective work.[49]

Hempel and Engle conducted investigations pertaining to the attitudes of teachers toward in-service programs. Hempel reported the relationship between the attitude of a selected group of teachers and their knowledge of the agreement of educational psychologists toward

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[49]Clara Henderson, "An Evaluation of the Workshop Program for the In-service Teacher Education directed by the Ohio State Department of Education, 1944-1947," (unpublished Ph. D. dissertation, Ohio State University, 1948).

the learning process.[50] Engle's study noted the relationship between teachers who were identified as "more open" and their response to new ideas in educationally significant ways.[51]

Robert Anderson reported a three-year cooperative staff study in the LaGrange (Illinois) Public Schools. The study was concerned with the influence of the in-service program on teacher test behavior and classroom procedure.

From a representative sample of the total staff participating in the workshop program, the investigator selected thirty-three classroom teachers and measured some of the influences of the in-service program upon the group through a series of tests and questionnaires given to the teachers and their pupils. Data were also gathered through personal observations in the classrooms and through evaluation questionnaires completed by the participating teachers.

As a basis for comparison of the differences noted, a control group of thirty teachers was selected from neighboring school systems in comparable communities.

Anderson summarized his findings as follows:

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[50]Carl Hempel, "Attitude of a Selected Group of Elementary School Teachers Toward In-service Education," (unpublished Ph. D. dissertation, University of Connecticut, 1960).

[51]Harry Engle, "A Study of Attitudinal Change in Teachers and Administration during a Summer Workshop," (unpublished Ed. D. dissertation, Auburn University, 1960).

1. The experimental group of teachers made numerically higher but not particularly significant scores on all instruments in the post-test administration.
2. Two of the seven measures of teacher progress indicated significant gains for the experimental group.
3. Teachers who had participated in several years of child study made slightly higher gains over other LaGrange teachers on the Purdue Test.

Anderson suggested that teacher attitude and practices can be improved through cooperative and continuous in-service procedures.[52]

Now, after nearly thirty years of use, the workshop continues to be one of the most used forms of in-service education. Its history has been fast moving; its variety has been intriguing. While not a panacea for all the ills of teacher education, investigators continue to view the workshop as a useful agency for the in-service education of the professional staff.

In-service Education for Teachers of Mathematics

Rare indeed were the research studies which the writer was able to locate with respect to in-service education programs for teachers of elementary school mathematics. A study by Rudd which included an in-service aspect, a cooperative in-service study in arithmetic by the

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[52]Anderson, op. cit., pp. 205-215.

New York Council, an investigation by Procunier on the impact of Title III, National Defense Education Act, a report of research operations at the University of Texas by DeVault, Houston, and Boyd, and an investigation by Ruddell and Brown on the effect of three different programs were the only investigations of this nature which the writer was able to locate.

Rudd's study had to do with the effectiveness of in-service procedures. For measuring teacher growth in mathematical understandings, Rudd utilized: (1) the Glennon test, (2) teacher conferences, (3) teacher questionnaire, (4) teacher summary, (5) classroom visitation, and (6) teacher opinion check list.

After administering the Glennon test to fourteen groups of teachers, he asked one group to participate in an eight-session course devoted to the development of seventy-two arithmetical understandings. A group who did not participate in the course served as a control group. Rudd concluded that, while teacher growth may be produced by means of such practices, the modest gains made by teachers in mathematical understandings were not large enough to produce a wide margin of improvement on test scores. . . ."[53]

He also stated that:

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[53]Lonnie Edgar Rudd, "The Growth of Elementary School Teachers in Arithmetical Understandings Through In-service Procedures," (unpublished Ph. D. dissertation, Ohio State University, Columbus, 1960), p. 109.

Perhaps the most conclusive evidence of teacher growth was to be found in the results of teacher summaries.[54]

A cooperative in-service study in arithmetic was conducted by the ✓ ?

Central New York School Study Council during the school year of 1949-50. The objectives of the study, as stated in the report, were:

> . . . (1) to explore the place of mathematical meanings in the teaching and learning of arithmetic and (2) to develop some teaching techniques through which pupils could be led to gain essential mathematical understandings and some test items through which pupil mastery of these understandings could be evaluated.[55]

The study involved teachers, principals, supervisors, and consultants in a program which utilized committees at both local and district level. Meetings were scheduled during school hours, thus helping teachers feel that the work of the council was a part of their school work and not an extra burden.

In describing the results of the study, Norem stated:

> Teachers and supervisors are understanding arithmetical processes for the first time and consequently are more interested in teaching them.[56]

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[54] Ibid., p. 109.

[55] Richard C. Lonsdale, "Preface," Developing Meaningful Practices in Arithmetic, A Third Report to the Central New York School Study Council prepared by the Committee on Flexibility (Syracuse, N. Y.: Bureau of School Services, School of Education, Syracuse University, 1951), pp. viii-ix.

[56] Evelyn Norem, "Some Results of Our Working Together to Improve Learning in Arithmetic," Developing Meaningful Practices in Arithmetic. A Third Report of the Committee on Flexibility, Central New York School Study Council (Syracuse, N. Y.: Bureau of School Service, School of Education, Syracuse University, 1951), p. 106.

DeVault, Houston, and Boyd investigated the relative effectiveness of television, television supplemented by classroom consultant services, face-to-face lecture-discussion, and face-to-face lecture discussion supplemented by consultant services as methods of in-service education for elementary school teachers. These four methods of in-service education were evaluated in terms of change in teacher achievement, change in teachers' classroom practices, and change in pupil achievement and interest.[57]

Eighty-nine teachers of elementary school mathematics in grades four, five, and six in one school system volunteered to participate in the study. The content for the in-service education program was prepared by the research team from the University of Texas. The same professor of mathematics served as the instructor for all groups.

Conclusions drawn from the results of the study were:

1. Television was as effective as face-to-face lecture discussion in changing the mathematics and methods understanding of teachers, in the reaction of teachers to the in-service education program, in changing all but one of the nine components of the classroom practices of teachers, and in changing the

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[57]M. Vere DeVault, Robert Houston, and Claude Boyd, Television and Consultant Services as Methods of In-Service Education for Elementary School Teachers of Mathematics (Bureau of Laboratory Schools Publication No. 15. Austin, Texas: University of Texas, 1962), pp. 1-102.

mathematics achievement and interest of pupils in classes of the participating teachers.

2. Consultant services as a supplement to television and face-to-face lecture-discussion made a significant contribution in some situations.[58]

Tests and conferences were the means used to obtain this information. Both Houston and Boyd completed dissertations related to aspects of the study. Houston's study had to do with pupil achievement;[59] Boyd studied teacher achievement and reaction.[60]

In their suggestions for further research, these researchers not only suggested educational television as a promising means of in-service education, but also advised school systems to study the effect of written materials and test-retest effect upon teacher change in achievement and in classroom practices.[61] Among the questions suggested for further research were:

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[58]The Bureau of Laboratory Schools, University of Texas, published the findings of the major study, _Television and Consultant Services as Methods of In-service Education for Elementary School Teachers of Mathematics_.

[59]W. R. Houston, "Selected Methods of In-service Education and the Mathematical Achievement and Interest of Elementary School Pupils," (unpublished Ph. D. dissertation, University of Texas, Austin, 1962).

[60]Claude C. Boyd, "A Study of Relative Effectiveness of Selected Methods of In-service Education for Elementary School Teachers," (unpublished Ph. D. thesis, University of Texas, Austin, 1962).

[61]DeVault, Houston, and Boyd, _op. cit._, pp. 101-102.

1. Would written materials and a testing program . . . be as effective as television or face-to-face lecture-discussion series in bringing about hypothesized outcomes?
2. Is in-service education by television without testing likely to be as effective in bringing about specific changes as when testing is included?
3. What anxieties are produced as a result of the testing program and to what extent are the outcomes of an in-service problem restricted by the nature of the test instruments?[62]

Procunier, working under the direction of Maurice Stapley, carried out an investigation which had to do with the impact of Title III, National Defense Education Act of 1958, upon the arithmetic programs in the public schools of Illinois.

This researcher, using interviews and an opinion questionnaire, found that the state supervisory and consultant service, developed under Title III, had become an integral function of the State Superintendent's office. The consultant staff, so he reported, had been of value to participating school districts through demonstrating worthwhile experiences and assisting in the in-service training of local staffs. He did, however, note that, even though participants and supervisory staff found the Title III workshop to be extremely valuable, there was an over-all lack of utilization.[63]

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[62] Ibid., p. 100.

[63] Robert Wilford Procunier, "The Impact of Title III, NDEA Programs in the Public Schools of Illinois," (unpublished Ed. D. dissertation, University of Illinois, 1962).

A recent investigation by Ruddell and Brown studied the effect that three different programs of in-service education had on teachers and pupils. Mathematics consultants were used in different ways in an attempt to improve instruction in elementary school mathematics. One group met with the consultant for one six-hour session during orientation week. Two other groups were involved in long range programs. One group participated in ten meetings scheduled at spaced intervals throughout the year. The third group also had ten meetings but were served by an intermediary.

Both a pre-test and post-test were used with teachers in all three groups. None of the groups, so the investigators said, made impressive gains although the second group made significantly greater gains than did the third group.[64]

III. Changing the Attitudes of Elementary School Teachers Toward Arithmetic

It has been said often that one of the factors which may limit the effectiveness of any teacher is his attitude toward the subject he is attempting to teach. Indeed, there are those who say that fear, dislike, and frustration toward a subject build up because of insufficient challenge, too difficult work, or poor presentation of the subject.

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[64] Arden Ruddell and Kenneth Brown, "In-service Education in Arithmetic: Three Approaches," Elementary School Journal (April, 1964), pp. 377-382.

> . . . The teacher's attitude is a more important factor than his formal preparation in his effectiveness. . . . If he is flexible, willing to try new things, and exercises critical judgment, he is likely to do well. If he is rigid, resistant to change, or uncritically for or against, he is likely to do poorly.[65]

Could not an elementary teacher with an inadequate understanding of mathematics and a genuine dislike of the subject infest a large number of boys and girls with an enduring fear and hatred to mathematics? While much of the current literature concerning arithmetic in the elementary school has indicated that arithmetic is a much disliked subject, there is not complete agreement on the effect of the teacher's attitude.

Dyer, Lakin, and Lord reported a study in which nearly three-fourths of the elementary teachers interviewed expressed a long standing hatred of arithmetic. They suggested that:

> . . . Elementary school teachers pass through the elementary school learning to detest mathematics. They drop it in high school as early as possible. They avoid it in college because it is not required. They return to the elementary school to teach a new generation to detest it.[66]

Poffenberger and Norton also placed strong emphasis upon the attitude of the teacher toward the subject. They stressed that one

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[65]Paul C. Rosenbloom, "Mathematics K-14," Educational Leadership, IX, No. 6 (March, 1962), p. 361.

[66]Henry Dyer, Robert Lakin, and Frederick M. Lord, "The Teacher," Problems in Mathematical Education (Princeton, N. J.: Educational Testing Service, 1956), pp. 7-12.

teacher's dislike of the subject could destroy favorable attitudes toward arithmetic and mathematics.[67]

> Any teacher who fears mathematics, who teaches the subject in a climate of distrust--or who believes that elementary arithmetic is a set of arbitrary mystical rules to be swallowed like a distasteful medicine, is bound to transmit some of these attitudes to the student.[68]

McDermott, for his doctoral dissertation, used structured interviews and the case-study approach to obtain data relative to factors that cause fear and dislike of mathematics. After interviewing forty-one students (seven proficient in mathematics and thirty-four who had been recommended for remedial help by their teacher), he concluded that "most students who have a fear and dislike of mathematics met with some frustration in the elementary grades."[69]

As early as 1951, Wilbur H. Dutton, after observing the overt behavior of university students as he taught methods courses in arithmetic, secured data pertinent to attitudes toward arithmetic. These data were obtained through written statements from two hundred and eleven students. He examined the statements and separated them into

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[67] Thomas Poffenberger and Donald Norton, "Factors Determining Attitudes toward Arithmetic and Mathematics," _The Arithmetic Teacher_, III (April, 1956), p. 114.

[68] _Ibid._

[69] Leon McDermott, "A Study of Factors that Cause Fear and Dislike of Mathematics," (unpublished Ed. D. dissertation, Michigan State University, 1958).

one of two groups: (1) factors responsible for favorable attitudes or (2) factors causing unfavorable attitudes.

After studying the findings, four conclusions were drawn:

1. There was a tremendous outpouring of unfavorable attitudes toward arithmetic.
2. There was a clustering of unfavorable responses around: (a) lack of understanding, (b) teaching unrelated to life, (c) too many pages of word problems, (d) boring drill, (e) poor teaching, (f) lack of interest, and (g) fear of making mistakes.
3. University students often came to methods classes with antagonistic attitudes toward arithmetic.
4. Student reaction often was so charged emotionally that learning could be affected.[70]

Dutton has continued to study the attitudes of prospective teachers toward arithmetic. In 1954, he reported a second investigation for which he constructed and evaluated an instrument designed to measure the attitude of prospective teachers toward arithmetic. The responses which nearly six hundred students had written over a five-year period were grouped around forty-five statements. Using a technique developed by Thurstone and Chave, students were asked to sort the statements using a scale of one to eleven (extreme like to extreme dislike). Twenty-two statements then were selected for incorporation into the evaluation instrument.

The instrument was used to test two hundred and eighty-nine students. No attempt was made to develop a total score or an average

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[70]Wilbur H. Dutton, "Attitude of Prospective Teachers Toward Arithmetic," Elementary School Journal, LVI (October, 1951), pp. 85-87.

score for each student. The data, however, were organized into tables in such a way that positive liking and pronounced unfavorable feeling could be noted.

Obtaining a reliability of .94 through a re-test procedure, Dutton concluded that attitudes can be measured objectively and significant data can be obtained which will be helpful in the education of prospective elementary school teachers.[71]

A later study by Dutton, recently reported in The Arithmetic Teacher, included a revision of the earlier instrument. The original scale was reduced to fifteen items and five new sections were added. A reliability of .84 was obtained for this revision by test-retest procedure.

Dutton administered the revised attitude scale to one hundred twenty-seven prospective teachers enrolled in classes at the University of California (Los Angeles). Both favorable and unfavorable attitudes were expressed by the students. Further study of the findings revealed that:

> . . . Liking or disliking arithmetic is an individual affair. Diagnosing students' feelings about arithmetic and planning corrective measures must be directed toward individual pupils. . . .[72]

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[71]Wilbur H. Dutton, "Measuring Attitudes Toward Arithmetic," Elementary School Journal, LV (September, 1954), pp. 24-31.

[72]Wilbur H. Dutton, "Attitude Changes of Prospective Elementary School Teachers Toward Arithmetic," The Arithmetic Teacher, IX (December, 1962), p. 424.

Also, while he noted that attitudes toward arithmetic of students responding to the revised attitude inventory were ambivalent, he concluded that they were, when compared with those of the 1954 sample, almost identical.[73]

Perhaps the most outstanding facet of the Dutton studies is the conclusion that attitudes toward arithmetic, once developed, are tenaciously held. "Continued efforts to redirect the negative attitudes of students into constructive channels have not been very effective. . . . The best antidote is probably improved teaching in each elementary school grade. . . ."[74]

O'Donnell made a study of attitudes of one hundred and nine elementary education seniors in Pennsylvania. His findings, however, were not in total agreement with those of Dutton.

Using the Remmer's "Attitude Toward Any Subject Scale," he obtained scores which were inconsistent with either the students' expressed like or dislike of arithmetic or achievement scores.[75]

Stright also conducted an attitudinal study which involved 1,023 students and 29 teachers. The Dutton Attitude Scale was revised for

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[73]Ibid.

[74]Ibid.

[75]J. R. O'Donnell, "Levels of Arithmetical Achievement, Attitudes Toward Arithmetic and Problem Solving Behavior Shown by Prospective Elementary School Teachers," (unpublished Ed. D. thesis, Pennsylvania State, 1958).

use in this study. This revised form, including twenty-five items for students and thirty-five items for teachers, asked the subjects of the study to either "agree" or "disagree" with the items presented. From the data relative to the replies of the teachers, the following conclusions were cited as significant:

1. Ninety per cent of all teachers said that no matter what happens, they fit arithmetic into their schedule each day.
2. Ninety-three per cent stated that they really enjoy teaching arithmetic, while 97 per cent indicated that they thoroughly enjoy teaching arithmetic. (Several questions were repeated in different form as a check; this was one which varied slightly.)
3. Ninety per cent of the teachers felt that a good teacher should keep up with modern methods, but twenty-one per cent felt that they teach arithmetic well without reading periodicals and methods books.
4. Seventeen per cent felt that methods of teaching arithmetic had not changed in the past thirty years.
5. All of the teachers agreed that arithmetic is a great value.[76]

Thus the assertion that arithmetic, when taught by teachers who dislike the subject, produces some undesirable attitudes has been open to dispute. The investigations of Dutton and McDermott supported the assertion while the data from studies by O'Donnell and Stright tended to disagree. Perhaps a statement in the preface to the material prepared by the School Mathematics Study Group summarizes rather well the present thinking with respect to the effect of attitudes.

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[76]Virginia Stright, "A Study of the Attitudes Toward Arithmetic of Students and Teachers in the Third, Fourth, and Sixth Grades," <u>The Arithmetic Teacher</u>, (October, 1960), pp. 280-286.

> If mathematics is taught by people who do not like, and do not understand the subject, it is highly probable that pupils will not like and will not understand it as well.[77]

Certainly it is hoped that the vicious circle can be broken. Certainly it is hoped that a new generation is not being taught to detest mathematics. Has it not been said that the teacher who enjoys his subject has the best start in the world?

> . . . A good teacher, a challenging experience and numerous practical application of arithmetic are highly significant factors in the development of favorable attitudes toward the subject.[78]

IV. Improving the Classroom Procedures of Elementary School Teachers in Teaching Arithmetic

Stephen M. Corey said that the acquiring of new understandings and new attitudes is but a means to an end. "Improvement of professional behavior is the main objective."[79] He stressed this objective when he wrote that one of the real values of any in-service education is the possibility of helping people to change and grow.

> . . . Reading something in a book, discussing it intelligently, or even memorizing it is completely inadequate. The test of in-service education is whether it results in better

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[77]School Mathematics Study Group, Number Systems, VI: Studies in Mathematics (New Haven, Connecticut: Yale University Press, 1961), p. i.

[78]Dutton, op. cit., p. 104.

[79]Stephen M. Corey, "Introduction," Ch. I, Fifty-sixth Yearbook of the National Society for the Study of Education, Part I (Chicago, Ill.: University of Chicago Press, 1957), p. 1.

learning and in better living experiences for boys and girls.[80]

Yes, the true test of an in-service program is how it affects the behavior of the participants. Did the teachers who participated change in their classroom procedures? Did these teachers relate some of the new ideas learned to actual classroom instructional procedures? Were they able to present new content in a way that could be easily understood?

These basic questions--to what extent did teachers' increase in understanding of the subject and a more favorable attitude toward the subject result in a change in classroom procedure--are questions open for investigation.

In the comprehensive study, Television and Consultant Services, by DeVault, Houston, and Boyd, an attempt was made to evaluate the classroom practices of eighty-seven teachers through the use of pupil interview technique. Six pupils from one class of each of forty-five teachers were randomly selected. An interview instrument, a twenty-seven item questionnaire which was patterned after one used by Shannon and Wishard for an Elementary School Science Program, was used to elicit pupil response on the following components: (1) materials, (2) classroom activities, (3) organization, (4) teaching aids, and (5) evaluation.

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[80]Robert Gilchrist, "Highway to Quality Teaching," NEA Journal, XL (May, 1959), pp. 18-19.

The investigators noted significant changes in the classroom practices of elementary school teachers involved in the program. In addition, they observed that teachers in the face-to-face lecture discussion group changed more in their manner of using materials than did teachers in the television group.[81]

Can teachers grow through in-service education? This question was asked by Ned Flanders in Educational Leadership as he reported on two recently completed in-service training projects which attempted to measure change in teacher behavior.

The purpose of the study carried out by Flanders and others at the University of Michigan under Title VII, NDEA, was to increase the flexibility of the teachers' influence and also to increase the use of those teacher behavior patterns which support pupil participation. Using tests to select teachers whom they thought would benefit from such an in-service education program, they hypothesized that a compatibility between preferred patterns of learning and in-service education procedures would affect the progress of teachers.

In their conclusions they noted that "very few in-service education programs are evaluated with enough care to tell whether or not the quality of classroom instruction has been affected."[82]

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[81]Houston, Boyd, and DeVault, "An In-Service Mathematics Education Program," p. 68.

[82]Ned Flanders, "Teacher Behavior and In-Service Programs," Educational Leadership, XXI, No. 1 (October, 1963), pp. 25-29.

In the same article, Flanders reported a project in human relations which was undertaken by Bowers and Soar. The purpose of the study was to help teachers achieve their own preferred degree of democratic classroom management by increased sensitivity to their own behavior, increased sensitivity to causes of pupil behavior, and greater self-direction by pupils working in groups. In their conclusions they indicated that "not all teachers can benefit from this kind of training."[83]

Two studies previously mentioned, one by Anderson and another by Mitchell, investigated the effect of participation in a workshop upon classroom practices. Anderson's study, a three-year cooperative staff study, was concerned with the influence of the in-service education program on teacher test behavior and classroom procedure.[84]

Forest C. Mitchell investigated the effect of participation in a workshop upon classroom practices. The purpose of the study was to identify changes that were made in selected classroom practices and to discover any contributions that participation in a summer workshop might make. His data consisted of case-study material obtained through

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[83]N. D. Bowers and R. S. Soar, *Studies in Human Relations in Teaching-Learning Process*, Final Report of Cooperative Research Project, No. 469, 1961, cited by Ned Flanders, "Teacher Behavior and In-Service Programs," *Educational Leadership*, XXI, No. 1 (October, 1963), p. 25.

[84]Anderson, *op. cit.*, pp. 205-215.

interviews, classroom observation, and daily logs collected on twenty-seven elementary school teachers. He concluded that the workshop was a rich experience with many opportunities for teachers to get the kind of help solicited. The greatest change he detected was in the use of materials.[85]

Other individuals, such as Turner,[86] who studied the problem-solving proficiency among elementary school teachers, and Mork,[87] who investigated the effects of an in-service education program on the science knowledge of fifth- and sixth-grade pupils, have reported closely related studies. However, the idea of a teacher's own learning being related to effective teaching is a question which must be answered by additional research.

> Studies are needed which investigate the changes brought about in the classroom as a result of in-service education programs. Case studies, classroom observation, pupil interview technique, and experimental studies involving pre- and post-testing of both teachers and pupils are but some of the techniques which should be utilized.[88]

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[85] Forest C. Mitchell, "The Effect of Participation in a Summer Workshop Upon Selected Classroom Procedures," (unpublished Ed. D. dissertation, University of California, 1951).

[86] Richard L. Turner, _Problem Solving Proficiency Among Elementary School Teachers_, Part II (Bloomington, Indiana: Institute of Educational Research, School of Education, Indiana University, 1960).

[87] Gordon M. A. Mork, "Effects of an In-service Teacher Training Program on Pupil Outcomes in Fifth and Sixth Grade Science," (unpublished Ph. D. dissertation, University of Minnesota, 1953).

[88] DeVault, Houston, and Boyd, _Television and Consultant Services_, pp. 99-100.

Have not good teachers always been recognized as those who were in the process of searching for new ideas, new understandings, and interesting ways to present their subject? DeVault, Houston, and Boyd said that, while the extent to which this "searching and experimenting" is related to increased understanding and a more favorable attitude yet remains to be determined, educators may continue to plan in-service education activities in the belief "that change does beget change"--that professional growth of teachers can be carried over into classroom procedures.[89]

V. Summary

Investigation after investigation has stressed the need not only for teachers "to grow on the job," but also the need for studies which investigate the effectiveness of various types of in-service education.

> . . . An in-service education program just makes it possible for the teacher to make better use of pre-service instruction and to keep up with the demands and problems of a changing society.[90]

There is a need for the critical examination and evaluation of in-service programs planned to meet the need of teachers if more

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[89]Ibid.

[90]DeVault, Houston, and Boyd, Television and Consultant Services, p. 102.

efficient in-service education is to contribute to the continued success of the American education effort.[91]

This chapter on related literature has sought to organize and summarize the major research development which was considered relevant to the present study. This review abstracted major studies which considered the following topics: (1) improving the mathematical understandings of teachers, (2) utilizing in-service education procedures, (3) changing the attitudes of elementary school teachers toward mathematics, and (4) improving the classroom procedures of teachers with respect to arithmetic.

Various sources indicated that an inexcusably large number of prospective and experienced elementary school teachers simply do not know as much arithmetic as they should in order to teach it effectively. They saw the teacher's attitude toward the subject as a factor which might limit his or her effectiveness. The various states, the many colleges and universities, and the local school districts were urged to continue to study and evaluate the many forms of in-service teacher education. "Only continuous study and growth will provide teachers sufficiently up-to-date to cope with the task at hand."[92]

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[91]C. W. Phillips, "What are the Characteristics of an Effective In-service Program?" Issues presented at Thirty-sixth Annual Conference, National Association of Secondary School Principals (February, 1952), pp. 360-361.

[92]A. S. Barr, "Teacher Personnel III, In-service Education," Encyclopedia of Educational Research. A project of the American Research Association (New York: Macmillan and Company, 1950), p. 1421.

The following chapter will describe the setting for the study, an in-service education program conducted by a mathematics consultant from the Missouri State Department of Education, and the procedures utilized in carrying out the study. Chapter IV will present through tables, graphs, and explanatory materials the findings from the study. In Chapter V, findings will be summarized and implications from the study discussed.

CHAPTER III

THE PROCEDURES OF THE STUDY

This chapter is concerned with the procedures which were followed in carrying out the present investigation. It has been divided into four sections. The first part of the chapter describes the state-sponsored workshop in mathematics for elementary school teachers, the in-service education program which was investigated in this study. The second section is a report on the population sample used in the study. It includes a description of the sample as well as an explanation of the procedures used for assignment of the sample to groups for purpose of analysis. The third part of the chapter is concerned with the collection of data. It includes a discussion of the instruments used, the administration of the instruments, and the preparation of the data for analysis. A brief summary concludes the chapter.

I. The State-Sponsored Workshop in Mathematics for Elementary School Teachers

Fred Weaver, in describing an in-service education program in mathematics for elementary school teachers, said:

> Rare indeed today is the school system that does not recognize a need for in-service education in mathematics for its

elementary teachers. Rare indeed, too, is the school system that is not seeking some means to satisfy this need effectively.[1]

The state of Missouri, through an in-service teacher education program financed by Title III, National Defense Education Act, has been able to assist nearly one hundred school systems in satisfying such a need. This study attempted to analyze the effectiveness of the 1963-64 series of state-sponsored workshops in mathematics for elementary school teachers. Specifically, this study attempted to evaluate the effectiveness of one series of state-sponsored workshops in mathematics for elementary school teachers in: (1) improving basic mathematics understandings, (2) changing the attitudes of elementary school teachers toward mathematics, and (3) improving the classroom practices of teachers with respect to the teaching of arithmetic.

Since the inception of the Title III, National Defense Education Act Program in Missouri in 1959, the state-sponsored workshops in mathematics for elementary school teachers have been a major aspect of Missouri's plan for administering this type of financial assistance. Viewed as one means for helping teachers to "modernize the content and upgrade their teaching of arithmetic,"[2] these state-sponsored workshops have been and continue to be important elements in the

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[1] J. Fred Weaver, "Focal Points," The Arithmetic Teacher, X, No. 6 (October, 1963), p. 359.

[2] "Elementary Mathematics (K-8) In-service Program," (Department of Education, State of Missouri, 1963), p. 2. (Mimeographed.)

in-service teacher education program. Through these mathematics workshops, teachers have had opportunities to learn of new developments, to examine new materials, and to observe and study new methods of presentation.

Objectives of Elementary School Mathematics (K-8) In-service Education Program

A bulletin issued by the State Department of Education sets forth the following as objectives for the state-sponsored workshops in mathematics for elementary school teachers:

1. To provide teachers with the opportunity to raise their level of understanding of the concepts of mathematics and to lay the foundation for further self-improvement.
2. To illustrate through demonstration teaching some of the new approaches to, and methods of, presenting mathematical ideas to elementary students.
3. To create an atmosphere of enthusiasm around mathematics so that teachers and students alike may enjoy the pursuit of its excellence.
4. To acquaint the teachers with new teaching aids, manipulative devices and the laboratory approach to the teaching of elementary school mathematics.
5. To provide experiences and materials whereby teachers may broaden their horizons relative to experimental programs, pertinent literature, and extracurricular activities in the field of mathematics.[3]

Description of the Workshops

These state-sponsored workshops, directed by competent and well-trained mathematics consultants from the state department, were

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[3] Ibid., pp. 3-4.

organized at the request of local school systems or adjoining school districts. They were open to all elementary school teachers. There was no fee and no college credit was given.

Each group met for a period of ten weeks, one session per week. The weekly sessions, approximately two hours in length, were scheduled either from 4:30 to 6:30 or from 7:00 to 9:00 in the evening.

The workshop sessions were, for the most part, built around the lecture-demonstration method of presentation. At least two sessions were devoted to the construction of and the demonstration of exploratory materials deemed vital to the presentation of a well-rounded mathematics program in the elementary school. In addition, each participant received printed materials which supplemented the oral presentations.

The mathematics consultant also visited the classrooms of teachers participating in the workshops and, upon request, taught demonstration classes using modern concepts and some of the newer approaches to the teaching of elementary school mathematics.

II. The Population Sample Used in the Study

Permission was obtained from the Missouri State Department of Education to evaluate the effectiveness of the 1963-64 series of state-sponsored workshops in mathematics for elementary school teachers,

the in-service teacher education program set up by the Missouri State Department of Education under Title III, National Defense Education Act.

In order to control as many variables as possible, the population from which a sample of two workshops was drawn consisted of only those state-sponsored workshops in mathematics for elementary school teachers conducted in the Southeast Missouri State College Service District by one and the same mathematics consultant during the 1963-64 school year.

<u>The Workshop Sites</u>

The sample for this study utilized workshops conducted in two school systems located in suburban areas of a metropolitan city.[4] One school system was the site for the first workshop; the other was the site for the second workshop, as well as being the school system from which a third group (used as a control group) was drawn.[5]

In this metropolitan area, often referred to as a large diversified industrial center, are located a number of industrial plants, some the

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[4]In accordance with an agreement between personnel from the Missouri State Department of Education and the superintendents in the participating school districts, no mention will be made of the name of any particular school system.

[5]When volunteers from one of the school systems exceeded the maximum number, it became necessary to divide the group and a third workshop planned for a later date. This third workshop group, since it also was made up of volunteers, was used as a control group.

largest of their kind in the world. Many employees in these industrial plants live in suburban areas surrounding the city. Both school systems utilized in this study were a part of a network of public schools which had been set up to care for an expanding population. The findings of Rogers with respect to a high degree of similarity in public education in Missouri, irrespective of differing geographical, cultural, and economic environment, was considered by the investigator as sufficient evidence that this sample did not differ significantly from normal.[6]

During the school year of 1963-64, the first school system, later to be referred to as School System X, had seven elementary schools with an enrollment of 3500 pupils (grades K-6) and an instructional staff of 148 persons assigned to work with children in the elementary grades. The second school system, referred to as School System Y, had nine elementary schools with an enrollment of 7300 students and 349 elementary school teachers.[7]

Description of Teacher Population

During the first semester of the 1963-64 school year, teachers in both school systems were given the opportunity to volunteer to

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[6]William R. Rogers, "Public Opinion Regarding Selected Public Education in Missouri," (unpublished Ed. D. dissertation, University of Missouri, 1949).

[7]Data obtained from Missouri School Directory, 1963-64. (Department of Education, State of Missouri, 1963), pp. 171-172; pp. 181-183. (Mimeographed.)

participate in a workshop in mathematics for elementary school teachers. One hundred twenty teachers, or approximately thirty per cent, of the nearly five hundred and fifty elementary school teachers in the two school systems indicated an interest in participating in the mathematics workshops. Of the teachers who originally indicated interest in participation, ten withdrew, four attended only the first session, six did not attend sufficient sessions to obtain in-service credit, five did not take the post-test, and six returned their papers with incomplete data. The remaining eighty-nine teachers constituted the population sample for this study. Henceforth, all descriptions and analyses will be made relative to this group.

Assignment Criteria Utilized

For purposes of analysis, the data concerning the participants were grouped according to the following four criteria: (1) school system, (2) level of teaching assignment (primary or intermediate), (3) completion of in-service education program, and (4) testing procedure.

School system. --The assignment of teachers to either Group A, B, or C was determined by the first assignment criterion, school system. All teachers in Group A were from School System X. The participants in Groups B and C were from School System Y.

Level of teaching assignment. --The second assignment criterion utilized had to do with the grade level at which the teachers were teaching. The workshop participants in Group A were divided into two groups, primary and intermediate. The primary teachers were identified as Group A_1; the intermediate teachers were designated as A_2. Group B was made up of teachers who were teaching at the primary level. A third group, Group C, included teachers whose assignment was at the intermediate level.

Testing procedure.--A third assignment criterion was testing procedure. Groups A_1, A_2, B_1, and C were tested both before and after the second semester in-service education program; those in Group B_2, a randomized sample of primary teachers from School System Y, were tested only at the end of the mathematics workshop. The number of teachers initially assigned to each group is shown in Table 1, page 73.

Time of participation in the in-service education program.--A fourth assignment criterion was completion of the in-service education program during the time of this study. Group C, the control group, did not even begin the workshop experience even though they had volunteered to participate in a workshop in mathematics for elementary school teachers.

III. The Procedures Used in the Collection of Data

The specific procedures used in gathering the data for this study included: (1) the preparation and use of an information form, (2) the

TABLE 1

ASSIGNMENT FOR EIGHTY-NINE TEACHERS UTILIZED IN THE STUDY

School System[a]	Level of Teaching Assignment	In-service Education Group Pre-test		Control Group Pre-test		Total
		Yes	No	Yes	No	
X	Primary (Group A_1)	10				10
	Inter-mediate (Group A_2)	22				22
Sub-total		32				32
Y	Primary (Group B_1)	20				20
	Primary (Group B_2)		15			15
	Inter-mediate (Group C)			22		22
Sub-total		20	15	22		57
Total		52	15	22		89

[a]In accordance with an agreement between personnel from the Missouri State Department of Education and the superintendents in the participating school districts, no mention will be made of the name of any particular school system.

construction of, the administration of, and the scoring of a test of mathematical understandings, (3) the selection and use of an arithmetic attitude inventory, and (4) the procurement of and the utilization of a classroom interview instrument.

The Preparation and Use of an Information Form

An information form entitled "Teacher Background Information" was prepared for distribution at the first session of each workshop. This information form requested each teacher to furnish the following data relative to background information: (1) sex, (2) years of teaching experience, (3) grade level assignment, (4) highest degree attained, (5) number of semester high school credits in mathematics, (6) number of semester hours credit in college mathematics, and (7) number of semester hours credit in methods of teaching arithmetic.[8]

After the teacher had completed this information form, the data for each teacher were transferred directly from the information sheet completed by the teacher to a code sheet and subsequently punched into IBM cards and used in the final analysis.

Years of teaching experience.--The eighty-nine teachers utilized in the study varied widely in terms of teaching experience. The years of teaching experience for all teachers ranged from none to thirty-five

[8] For examination of this registration form, see Appendix I, p. 228.

years. For the total group, the mean was 11.11 and the standard deviation was 8.7178. Table 2, page 76, presents a summary of the population with respect to years of teaching experience.

Mathematics background.--The data concerning the mathematics background of the eighty-nine teachers has been organized under three headings: (1) high school credit in mathematics, (2) college credit in mathematics, and (3) college credit in methods of teaching arithmetic. Mean, standard deviation, and range were obtained on each item for each group. Table 3, page 77, shows this information.

Highest degree attained.--The professional training of the eighty-nine elementary school teachers was described in terms of one of the following five classifications: (1) less than a Bachelor's degree, (2) a Bachelor's degree, (3) more than a Bachelor's degree but less than a Master's degree, and (5) a Master's degree or more. Table 4, page 78, shows the number of teachers by group in each of the classifications.

The Construction of, the Administration of, and the Scoring of a Test of Mathematical Understandings

Even though most educators agree that paper-and-pencil tests are only one part of a well-rounded program of evaluation in mathematics, such tests always have been and probably will continue to be one of the most valuable means of obtaining evidence of achievement. "They [the paper-and-pencil tests] contribute much useful data to the total program

TABLE 2

COMPOSITION OF POPULATION WITH RESPECT TO YEARS OF TEACHING EXPERIENCE[a]

Years of Teaching Experience	School System X		School System Y			Total
	Group		Group			
	A_1	A_2	B_1	B_2	C	
32 - 35	0	0	1	0	1	2
28 - 31	0	0	2	0	1	3
24 - 27	0	0	3	1	0	4
20 - 23	1	3	1	1	2	8
16 - 19	0	1	0	5	3	9
12 - 15	0	2	3	2	4	11
8 - 11	3	6	0	1	5	15
4 - 7	3	5	2	3	3	16
0 - 3	3	5	8	2	3	21
N =	10	22	20	15	22	89
Mean =	8.2	8.68	12.50	12.60	12.54	11.1
Stand. Dev. =	5.20	8.15	11.00	8.14	8.31	8.7

[a]The decision to use nine intervals was based on a statement by Noll in which he said that the interval used for frequency distribution should be of "such a size as to give a distribution containing not less than eight nor more than sixteen intervals." Since the years of teaching experience ranged from none to thirty-five years, an interval of four established nine categories. For further information, see Victor Noll, Educational Measurement (Boston: Houghton Mifflin Company, 1957), pp. 393-402.

TABLE 3

CENTRAL TENDENCY AND VARIABILITY
OF MATHEMATICS BACKGROUND
FOR POPULATION SAMPLE

Background Factor	Mean	Standard Deviation	Range
High School Credit in Mathematics			
Group A_1	2.8	4.40	2 - 3
Group A_2	2.8	1.09	1 - 4
Group B_1	1.7	2.61	1 - 4
Group B_2	2.5	6.60	1 - 4
Group C	3.0	1.07	1 - 4
College Credit in Mathematics[a]			
Group A_1	2.1	1.1	2 - 3
Group A_2	6.0	1.30	0 -14
Group B_1	3.4	3.50	0 - 6
Group B_2	2.4	1.3	0 - 6
Group C	3.0	1.09	0 - 6
College Credit in Methods of Teaching Arithmetic[a]			
Group A_1	2.9	1.8	2 - 3
Group A_2	3.1	1.46	0 - 6
Group B_1	2.8	3.94	0 - 6
Group B_2	2.4	.71	0 - 6
Group C	2.13	3.10	0 - 6

[a]The mathematics preparation in college is expressed in semester hour credits.

TABLE 4

COMPOSITION OF POPULATION BY HIGHEST DEGREE HELD

Degree Held	School System X		School System Y			Total
	Group		Group			
	A_1	A_2	B_1	B_2	C	
More than Master's degree	0	2	1	0	5	8
Master's degree	0	1	0	0	0	1
More than Bachelor's degree but less than Master's degree	9	5	18	10	4	46
Bachelor's degree	1	13	1	5	12	32
Less than Bachelor's degree	0	1	0	0	1	2
Total	10	22	20	15	22	89

of evaluating mathematical learnings."[9] With these ideas in mind, the writer, for the initial part of this investigation, sought to find a paper-and-pencil instrument for evaluating the mathematical understandings needed by elementary school teachers.

For guidance, the investigator turned to the major reference work on tests and test reviews, the series of Mental Measurements Yearbooks, and a publication of the National Council of Teachers of Mathematics, Mathematics Tests Available in the United States.[10]

The first reference, one of the most important sources of information with regard to tests, is, up to now, a five volume publication. Each yearbook includes reviews of tests published during a specified period, thereby supplementing rather than supplanting earlier yearbooks.[11]

An examination of the arithmetic tests cited in any of the previously mentioned references revealed that no test commonly used at the present time contained more than a few items designed to measure mathematical understandings.

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[9]Noll, op. cit., p. 3.

[10]Sheldon S. Myers, Mathematics Tests Available in the United States. A bulletin to the National Council of Teachers of Mathematics, Prepared by the Secondary School Curriculum Committee (Washington: The Association, 1959).

[11]For further information concerning these yearbooks, see the bibliographical entries under Oscar Buros.

This was not surprising since, in recent years, many writers have stressed the lack of adequate testing instruments for evaluating mathematical understandings. As early as 1932, Butler wrote:

> It is certain that, in the domain of published tests, the specific testing for master of mathematical concepts [understandings] has received scant attention.[12]

Brownell, an outstanding authority in the field of elementary school mathematics, noted that:

> . . . Exceedingly little has been done either informally or systematically to find practical and valid procedures for evaluating the outcomes under the heading above [mathematical understandings].[13]

Collier, in his doctoral dissertation at Ohio State University, stated that:

> Evaluation of important mathematical outcomes has not kept pace with instruction in arithmetic. Standardized tests commonly used measure mainly computational skill. . . . There is a definite need for more systematic and comprehensive research relative to evaluating the newer outcomes of arithmetic instruction.[14]

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[12]C. H. Butler, "Mastery of Certain Mathematical Concepts at the Junior High School," Mathematics Teacher, (March, 1932), pp. 117-172.

[13]William A. Brownell, "The Evaluation of Learning in Arithmetic," Arithmetic in General Education. A Report of the National Council Committee on Arithmetic, Sixteenth Yearbook of the National Council of Teachers of Mathematics (New York: Bureau of Publications, Teachers College, Columbia University, 1941), p. 247.

[14]Calhoun C. Collier, "The Development of and the Evaluation of a Non-Computational Mathematics Test for Grades 5 and 6," (unpublished Ph. D. dissertation, Ohio State University), p. 43.

Van Engen contended that much attention has been given to teaching arithmetic meaningfully, but not nearly enough research has been carried out in the area of testing for meaning.[15]

Glennon, while conducting one of the most significant studies in the field of mathematics education, found it necessary to develop his evaluation instrument.[16] Since that time a number of researchers have used the Glennon test when conducting investigations pertaining to mathematical understandings. However, when the writer sought information and permission to use the test, Glennon wrote:

> If I were building the test today, it would be a somewhat modified test and, I think, a somewhat improved test. You may wish to use some of the items and replace others with items of your own to obtain a better coverage of the present-day elementary school mathematics program.[17]

When the review of literature and testing instruments indicated that, at the present time, there were no published tests available for measuring many of the important mathematical concepts or understandings, it became necessary to develop a pencil-and-paper test which could be used for evaluating the mathematical understandings of teachers both at the pre-service and in-service levels.

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[15]Henry Van Engen, "A Summary of Research and Investigation and Their Implications for the Organization and Learning of Arithmetic," Mathematics Teacher, XLI (October, 1948), pp. 260-265.

[16]Glennon, op. cit., p. 7.

[17]Letter from Vincent J. Glennon, Director, Arithmetic Studies Center, Syracuse University, Syracuse, N. Y., October 8, 1963.

It was Lindquist who wrote that preparation of an educational achievement test includes five major steps: (1) planning the test, (2) writing the test items, (3) trying out the test in preliminary form and assembling the finished test after tryout, (4) preparing the directions for administering and scoring the test, and (5) reproducing the test.[18] Other writers, even though their ideas were somewhat similar, have organized these steps in various ways. Noll proposed four steps: (1) planning the test, (2) constructing the test , (3) using the test, and (4) evaluating the testing instrument.[19] The procedure, however, followed in the construction of this paper-and-pencil test was a modification of the procedure as proposed by Lindquist.

In this particular study, planning involved not only the preparation of an outline or blueprint specifying the content to be covered by the test, but it also involved careful attention to types of items, to provision for review, to arrangements for tryout, to item difficulty, to problems of test reproduction, and to interpretation and use of test results. Determining test objectives and preparing an outline of content, however, were the first steps to receive consideration.

Determining test objectives.--Preliminary to actual test construction, the person attempting to develop an evaluation instrument must decide upon the purpose of objectives of the test.

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[18] E. F. Lindquist (ed.), Educational Measurement, (Washington, D. C.: American Council on Education, 1951), p. 19.

[19] Noll, op. cit., p. 3.

> . . . No single test . . . can measure all objectives; he, the testmaker, must choose from an adequate list of objectives those which he will attempt to measure . . . and then formulate his teaching and measurement program on the basis of the objectives selected.[20]

Often a statement of objectives has been worked out by committees or groups and the resulting formulation generally represents the best and most forward-looking ideas that the group can produce at that time.

Support for using such a list to serve as a basis upon which to build a test was discussed by Collier. He quoted a statement from Greene, Jorgenson, and Gerberich which read as follows: "Reports of national committees and the writings of subject and test specialists often serve as good guides to content in educational test construction."[21]

One list which the writer found most helpful was a list of understandings in elementary school mathematics prepared by Brownell.[22]

Though prepared several years ago, this list continues to be widely quoted by authorities. And, while changes have been and are

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[20]Noll, _op. cit._, p. 96.

[21]Calhoun C. Collier, "The Development and Evaluation of a Non-Computational Mathematics Test for Grades 5 and 6," (unpublished Ph. D. dissertation, Ohio State University, 1956), p. 47, quoting Harry A. Green, Albert Jorgensen, and J. Raymond Gerberich, _Measurement and Evaluation in the Elementary School_, (New York: Longmans Green and Company, Inc., 1953), p. 68.

[22]William A. Brownell, "The Evaluation of Learning in Arithmetic," _Arithmetic in General Education_, The Sixteenth Yearbook of the National Council of Teachers of Mathematics (New York: Teachers College, Columbia University, 1941), pp. 231-232.

being made in the mathematics program in the elementary school, there is almost unanimous agreement that today's program is made up of more mathematics new to the elementary school than new mathematics.

The objectives which were grouped under the heading of mathematical understandings were:

1. Meaningful conceptions of quantity, of the number system, of whole numbers, of common fractions, of decimals, of per cents, of measures, etc.
2. A meaningful vocabulary of the useful technical terms of arithmetic which designate quantitative ideas and the relationships between them.
3. Grasp of important arithmetical generalizations.
4. Understanding of the meanings and mathematical functions of the fundamental operations.
5. Understanding of the meanings of measures and of measurements as a process.
6. Understanding of important arithmetical relationships, such as those of which function in reasonably sound estimations and approximations, in accurate checking, and ingenious and resourceful solutions.
7. Some understanding of the principles which govern number relations and computational procedures.[23]

Determining test content.--The second step in planning the test was the outlining of the content to be covered by the test. Actually, the problem of selecting the content to be covered in the test was chiefly one of determining the arithmetical concepts or mathematical understandings most commonly taught in the elementary school. This was accomplished through an examination of recently published arithmetic

[23] Ibid., p. 231.

textbooks and experimental materials.[24] Detailed study of articles or books recently published by Schaaf,[25] Newsom,[26] Mueller,[27] and Brumfiel also proved helpful.[28]

After developing the list of important mathematical concepts, it became necessary to organize these understandings under appropriate headings. The writer found a summary which Grossnickle had adapted from a section of Growth of Mathematical Ideas, the Twenty-fourth Yearbook of the National Council of Teachers of Mathematics, very helpful for developing a framework around which to organize the list of mathematical understandings.[29]

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[24] A list of the materials used for this examination is included in Appendix H, page 226.

[25] Jack N. Sparks, "Arithmetic Understandings Needed by Elementary-School Teachers," The Arithmetic Teacher, VIII (December, 1961), pp. 395-400, quoting W. L. Schaaf, "Arithmetic for Arithmetic Teachers," School Science and Mathematics, III (October, 1953), pp. 537-555.

[26] C. V. Newsom, "Mathematical Background Needed by Teachers of Arithmetic," The Teaching of Arithmetic, Fiftieth Yearbook of the National Society for the Study of Education, Part II.

[27] Francis J. Mueller, "Arithmetic and Teacher Preparation," The Mathematics Teacher, LII (November, 1959), pp. 572-573.

[28] Charles F. Brumfiel, Robert E. Eicholtz, and Merrill Shanks, Fundamental Concepts of Elementary Mathematics (Reading, Massachusetts: Addison-Wesley Publishing Company, 1962).

[29] A list of the important mathematical understandings may be found in Appendix J, page 228.

The following are key ideas or important strands in mathematics included in the summary by Grossnickle.[30]

1. Numerals and numeration systems
2. Principles underlying numbers
3. Relationships and generalizations
4. Measurement and approximation
5. Estimation and proof
6. Symbolism
7. Rational numbers
8. Geometry

The next step was the formulation of a blueprint or outline of the specific content to be covered by the test. Many authorities believe that content outline and statements of objectives represent two dimensions into which a test plan or blueprint must be fitted. "These two dimensions needed to be fitted together in order to give a complete framework and to decide upon the relative importance given to the several content areas and objectives."[31]

The distribution of items for the tests of mathematical understandings, both according to content and test form, is shown in Table 5.

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[30] Foster E. Grossnickle and Leo J. Brueckner, Discovering Meanings in Elementary School Mathematics (Philadelphia: Holt, Rinehart and Winston, Inc., 1963), pp. 10-14.

[31] Noll, op. cit., p. 98.

TABLE 5

DISTRIBUTION OF ITEMS ON TEST OF MATHEMATICAL UNDERSTANDINGS ACCORDING TO TEST FORM AND TEST-CONTENT OUTLINE

Test-Content Outline	Number of Items	
	Form A	Form B
1. Numerals and Numeration System	10	10
2. Symbolism (including introduction to sets)	2	2
3. Concept of Sets (including mathematics sentences)	6	6
4. Principles Underlying Number Operations	6	6
5. Relationships and Generalizations	5	5
6. Operations with Whole Numbers	11	11
7. Estimation and Proof	1	1
8. Measurement and Approximation	1	1
9. Geometry	4	4
10. Rational Numbers	9	9
Total	55	55

Developing the test instrument. --A review of the literature was made in order to gain information concerning the construction of good tests. It was found that various writers in the field of tests and measurements have issued statements or developed criteria of a general nature for the construction of tests. The suggestions for item writing which Lindquist included in Educational Measurement proved most helpful as the writer began to consider the construction of objective test items. The following were included in a list of suggestions made by Lindquist.

1. Express the item as clearly as possible.
2. Choose words that have precise meaning wherever possible.
3. Avoid complex or awkward word arrangements.
4. Include all qualifications needed to provide a reasonable basis for response selection.
5. Avoid unessential specificity in the stem or the responses.
6. Avoid irrelevant inaccuracies in any part of the item.
7. Adapt the level of difficulty of the item to the group and purpose for which it is intended.
8. Avoid irrelevant clues to the correct responses.
9. In order to defeat the rote-learner, avoid stereotyped phraseology in the stem or the correct response.
10. Avoid irrelevant sources of difficulty.[32]

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[32]Lindquist, op. cit., pp. 213-227.

Even though there are many forms of test items in general use--essay, true-false, short answer, matching, and multiple-choice--most authorities indicate that the multiple-choice item, consisting of a stem, which may be an incomplete statement or a direct question, followed by two or more alternatives or possible answers, is the "most valuable and the most generally applicable of all types of test exercises."[33]

Since the purpose of this particular test was to determine the extent of understanding in certain areas of elementary school mathematics and these understandings called for inferences, judgments, generalizations, and noting relationships, it was felt that these understandings fitted into categories which Hawkes said were tested by the multiple-choice items.

> The multiple-choice type of test can be made particularly effective in requiring inferential reasoning, reasoned understanding, sound judgment and discrimination on the part of the student; it is definitely superior to other types for these purposes.[34]

Several sources suggested specific principles for test construction with respect to multiple-choice items. Some of these principles were:

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[33]Noll, _op. cit._, p. 130.

[34]E. F. Lindquist, _Educational Measurement_ (Washington: The American Council on Education, 1951), p. 249, citing _The Construction and Use of Achievement Examinations_, p. 138.

1. The stem of a multiple-choice item should clearly formulate the problem.

2. All the options should be possible and plausible answers.

3. The stem should not be loaded down with irrelevant material.

4. One choice should be a best answer, but the others should appear plausible to the uninformed.

5. The writer should beware of clues from the length of the option.

6. Irrelevant grammatical cues should be avoided.

7. The best or correct answer should be placed equally often in each possible position.

8. Choices should be in parallel form whenever possible.

9. The choice of an item should come at or near the end of the sentence.

10. The number of choices in multiple-choice items should be at least four.[35]

Writing the test items.--Using the list of understandings, test items of the multiple-choice type, items which might be used in evaluating the mathematical understanding of elementary school teachers, were constructed. As these items were constructed, they were recorded on cards and filed under one of the eight categories around which the list of understandings was organized. Since permission had been obtained to use any or all items in the Glennon test, some twenty-two

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[35]Noll, op. cit., pp. 129-134.

of his items were included in the pool of items which was placed on cards. The objective here was to construct a pool of test items from which one hundred and ten of the best questions could be selected and assembled into two experimental tryout forms of fifty-five items each.

Selecting the best items.--When it was felt that a sufficient number of items had been formulated, attention was turned to the problem of selecting the best items which could be assembled into the two equivalent forms.[36] Each test was to contain one or more items for each of the ten categories in the Test Content Outline.[37] Form A and Form B of the test, each form consisting of fifty-five items, were developed. Except for the first few items in each test, no effort was made to arrange the items in order of difficulty. A set of test directions using simple, clear language was also prepared.

After items for the two comparable forms of the test were assembled, several sets were reproduced for review by persons considered to be competent judges.[38] One person was asked to review the items from an editorial standpoint, two test technicians were asked to review the items from the technical point of view, and five persons

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[36]The procedure used in test for equivalence of form is explained on page 96.

[37]The distribution of items both according to test form and test outline is shown in Table 5, page 87.

[38]The names and addresses of the judges are listed in Appendix I.

were asked to review the subject-matter. Comments and suggestions of the judges were utilized in revising many of the items in the try-out forms.

Administering the tryout forms.--After the test items had been written, reviewed by a team of experts, and revised on the basis of their suggestions, it became necessary for the test items to be tried out experimentally on a sample of examinees "as nearly like the population with whom the final form of the test was to be used as reasonably possible."[39]

As a pre-tryout, the preliminary form of the test was administered to approximately fifty elementary school teachers enrolled in a graduate course in "Problems in Teaching Arithmetic." These teachers, whose experience ranged from none to over thirty years, worked, two or three to a group, and informally discussed the various items as they took the test. Their questions and comments concerning each item were recorded by the test constructor. Ambiguous and cumbersome items were noted. Some respondents crossed out words and rewrote the questions in what they believed to be clearer and more concise language.

Next, on the basis of the data obtained in this pre-tryout, items were revised, deleted, or left unchanged. Two parallel forms of the

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[39] Lindquist, op. cit., p. 250.

test were prepared. Lindquist says that if, "in developing equivalent forms of a test, each form is built to conform to a detailed set of specifications, while at the same time care is taken to avoid identity or overlapping of content, the two resulting test forms should be truly equivalent."[40] A further check of equivalency was made by examining item correlations within and between the two test forms.[41]

For the trial administration, each form of the test was given to a group of fifty students who were majoring in elementary education and enrolled in a college course, Technique of Teaching Arithmetic, at Southeast Missouri State College, Cape Girardeau, Missouri. Form A was given during one class period and Form B on the day following. This trial administration, as Lindquist states, served to "indicate how the test would function in actual use."[42]

Since the test constructor administered the test, it was not necessary to prepare detailed directions for the test examiner. On the first page of each test, however, were directions for taking the test and a sample test item.[43]

There was no time limit to the test; however, each student was asked to record the time he completed the test on his answer sheet.

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[40] Lindquist, op. cit., p. 575.

[41] For details concerning this test for equivalency, see page 96.

[42] Lindquist, op. cit., p. 250.

[43] See Appendix B and C, pp. 183-203.

This information was helpful in determining appropriate time for the finished test.

In the scoring of the test, one point was allowed for each correct answer. If the student marked more than one choice, even though one of the choices was the right answer, the item was considered to be answered incorrectly. All work was checked by a person other than the original scorer in order to minimize errors.

Correction was made for "guessing" or chance success through the use of the following formula:

$$S = R - \frac{W}{n - 1},$$

where S = the score,

R = the number of correct responses,

W = the number of incorrect responses (excluding omitted items),

n = the number of choices.[44]

> . . . Because correction for chance success often makes test scores easier to interpret, the writer [Lindquist] is inclined to urge its use for almost all educational purposes. . . .[45]

The data obtained through the administration of the tryout forms of the test were recorded on IBM work sheets and punched into IBM cards for later use in: (1) computing the coefficient of reliability, (2) determining equivalency of test forms, (3) studying difficulty index, and (4) determining the discriminating power of the individual items.

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[44]Lindquist, _op. cit._, p. 365. [45]_Ibid._

An arithmetic mean and standard deviation were computed for both forms.

Studying the tryout tests.--It has been said that all good measuring instruments have certain qualities in common. While such characteristics as objectivity, ease of administration, ease of scoring, and ease of interpretation are important, the two primary qualities most agreed upon are reliability and validity.

> A test which lacks a known and satisfactory degree of these principal qualities is not a measuring instrument in any true sense, and little or no dependence can be placed upon results obtained by its use.[46]

Reliability.--Often, it is customary to think of validity as the most important quality of a test and to discuss it first. It was Noll, however, who said that "since reliability is essential to validity and the opposite is not so, there is something to be said for placing reliability at the head of the list."[47]

Reliability of measuring instruments is usually determined by one of three methods: (1) self-correlation, (2) correlation of equivalent forms, and (3) split-halves correlation. Since two equivalent forms of the test were available, the reliability for the two measuring instruments developed for use in connection with the present study was determined by correlating the scores made by the

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[46]Noll, op. cit., p. 66.

[47]Ibid., p. 67.

fifty college students on the two equivalent forms of the test. For purpose of analysis, Form A was referred to as X and Form B referred to as Y. Pearson Product-Moment Correlation Coefficient was used. The correlation coefficient between the two forms of the test was found to be .87. This correlation coefficient, indicative of a high positive relationship between the two sets of scores, also was significant at the five per cent (.05) level.[48] Such a degree of reliability is considered adequate by most test specialists.

> Desirable reliabilities differ according to purpose. Where a test is intended only for use in studying groups, a lower reliability and coefficient (around .75) may be sufficient to make fairly accurate comparisons. Where individual difference is the goal, reliability of .95 or higher is very desirable.[49]

Equivalency of forms.--The problem of preparing truly equivalent forms was, according to Lindquist, "a problem in the logic and practice of test construction."[50] He stated that the best guarantee for equivalency of two test forms is the preparation of a complete and detailed set of specifications for the test in advance of any final test construction.[51]

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[48]When a table of the "Values of the Correlation Coefficient for Different Levels of Significance" (df = n-2) was consulted, it was noted that, with fifty cases, any correlation of .280 or higher was significant at the .05 level and any correlation of .363 or higher was significant at the .01 level. For further information, see Allen L. Edwards, Statistical Methods for the Behavioral Sciences, (New York: Holt, Rinehart and Winston, 1961), p. 147.

[49]Noll, op. cit., p. 73.

[50]Lindquist, op. cit., p. 575. [51]Ibid., p. 575.

> If each test form is built to conform to the outline while at the same time care is taken to avoid identity or detailed overlapping of content, the two resulting forms will be truly equivalent.[52]

A further check upon the degree of equivalency was made by examining item correlation within and between the two test forms. Lindquist says that "on the average, these should be equal."[53]

Both Form A and Form B of the tests used in connection with this study were sub-divided into two halves (odd-numbered items and even-numbered items); alternate halves were used to obtain correlations among the resulting six scores.

McNemar suggested that the following t-test, which takes into account the relationship between the pairs of r's, be used:

$$t = (r_{12} - r_{13}) \sqrt{\frac{(N - 3)\ (1 + r_{23})}{2\ (1 - r_{12}^{2} - r_{13}^{2} - r_{23}^{2} + 2 r_{12} r_{13} r_{23})}}$$

where r_{12} = odd and even-numbered items, Form A, with odd and even-numbered items, Form B,

r_{13} = even-numbered items of both forms and odd-numbered items of both forms, and

r_{23} = even-numbered items, Form A, and odd-numbered items, Form B, with odd-numbered items, Form A, and even-numbered items, Form B.[54]

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[52] Ibid., p. 575.

[53] Ibid., p. 576.

[54] N. M. Downie and R. W. Heath, Basic Statistical Methods. New York: Harpers and Brothers, 1959, pp. 145-146, quoting Q. McNemar, Psychological Statistics, New York: John Wiley, 2nd ed., 1954.

The resulting t (1.02), with 47 (N-3) degrees of freedom, was not significant at the .01 level.[55] Hence, since there was no significant difference in the correlation coefficients, the two test forms could be viewed as equivalent.

Validity.--It has often been said that a test can be adequately reliable and still not be valid. Bean, in distinguishing between validity and reliability, said:

> The validity of a test refers to its accuracy with reference to the particular trait it is intended to measure, whereas reliability refers to the consistency with which it does so.[56]

Noll has said that the validity of a test may be elucidated by such questions as:

1. What does this test actually measure?
2. To what extent does it measure this particular ability or quality?
3. In what situations or under what conditions does it have validity?[57]

There are several widely accepted methods of determining the validity of measuring instruments. Many authorities classify them

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[55] t = 2.81 at .01 level.

[56] Kenneth L. Bean, Construction of Educational and Personnel Tests (New York: McGraw Hill Company, Inc., 1953), p. 15.

[57] Noll, op. cit., pp. 73-74.

under three categories: (1) curricular validity, (2) logical validity, and (3) statistical or empirical validity.[58]

The validity of the tests used in this study was established in terms of their curricular validity. Since the tests were: (1) designed around a specific list of understandings, (2) analyzed by a jury composed of mathematicians, psychologists, and professors of elementary education, (3) examined and studied by experienced classroom teachers, and (4) scrutinized as to form of items, curricular or content validity could be assumed. Noll said that such a method of designing the test was, in itself, a validating procedure.[59]

Item analysis.--Another technique employed as a measure of validity was the determining of the difficulty index and the discriminating power of individual questions or items.

The first of these, the difficulty of the item, is the proportion of individuals who answer the item correctly while the discrimination index is a measure of how well the item separates two groups.

> . . . When item-discrimination is used as a means of establishing validity, the test, which is in this case the criterion of validity, is assumed to have curricular validity because it measures what it purports to measure. The second assumption that the scores on a particular valid item agree with scores on the whole test is tested by comparing results on a given test item with scores on the whole test. Since the test is assumed to be valid, an item which agrees with the test is valid. That is, an

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[58]Noll, op. cit., p. 74.

[59]Ibid.

> item which is answered correctly by a higher proportion of those who make higher scores on the test than those who do poorly on it is functioning in a manner consistent with the scores on the whole test.[60]

Most sources identify two general ways of demonstrating item discrimination: (1) a test of the significant difference between two proportions and (2) correlational techniques. In the first method, the one chosen to use in this study, the per cent or proportion of individuals in the high group who answered the item correctly was tested against the proportion of individuals in the low group who answered the item correctly. If the difference was a significant one, the item was accepted as being one which discriminated.

For this study, the scores obtained by the tryout group were arranged in numerical order. Then, for the item analysis, the highest fifteen papers and the lowest fifteen papers were studied. Using a chart produced by Guilford, a four-fold coefficient or phi was read directly when the chart was entered with the per cent answering the item correctly in the upper and lower groups.[61]

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[60]Noll, _op. cit._, p. 149.

[61]For this chart, it was not necessary for the high and low groups to be based exactly upon the upper and lower twenty-seven per cent of the cases. The only requirement was that there be an equal number of individuals in both groups. Had the sample been larger, the tetrachoric coefficient would have been used. See N. M. Downie and R. W. Heath, _Basic Statistics Methods_ (New York: Harper and Brothers, 1959), p. 176.

Garrett says that:

> The normal curve can be taken as a guide in selection of difficulty indices. Thus 50% of the items might have difficulty between .25 and .75, 25% indices larger than .75, and 25% smaller than .25. . . . Items passed by 0% or 100% have no differentiating value but should be included in the test for psychological effect.[62]

The difficulty index and discrimination index for both forms of the test used in this study are shown in Appendix K. Sixty-four items, approximately sixty per cent, had difficulty indices between .25 and .75; eleven, or ten per cent, had indices smaller than .25; thirty-three, or approximately thirty per cent, had difficulty indices larger than .75. Of this number, only one item was passed by 100% of the participants and no item was marked incorrectly by all persons taking the test.

Phi coefficients also were obtained for each item and tested for significance by the use of chi square.[63] When the five (.05) per cent value of chi square (df=1) was read from the chi square table as 3.841, it was noted that any phi equal to or greater than .36 was significant at the five per cent level.

<u>Using the Final Test</u>

After the two parallel forms of the test were constructed and tried out, additional copies with any revisions deemed necessary were prepared for use with the population sample.

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[62]Garrett, *op. cit.*, p. 364.

[63]$\chi^2 = N\phi^2$. Thus the phi coefficient, because of this relationship, may be tested for significance by the use of chi square.

Administering the test.--These tests, as well as the Dutton Attitude Inventory, were administered at the first and last sessions of the two mathematics workshops for elementary teachers. Form A was used as a pre-test and Form B was administered at the final session as a post-test.

The members of Group C, the control group, also were given the tests at or about the time of the first and last study sessions. After permission was secured from the Superintendent of School System Y to have this group of teachers take the test, the test maker, visited each elementary school to discuss the test in general and to outline the procedure for taking the test. Care was taken to issue clear directions.

Teachers were allowed to finish the tests at different times. They were asked, however, to seek no help or discuss either form of the test with other teachers. Then, when the teachers had completed the tests, all tests were returned to the principal who in turn mailed them to the test maker.

In addition to the directions which accompanied the test form, all three groups were informed that the test would in no way be used to appraise their teaching effectiveness since test results from individual teachers neither were identified by name nor were to be released.

Scoring the tests.--A first step in the processing of the data was the scoring of the test forms for the development of the TMU (Test of Mathematical Understanding) Score.

The same procedure that was used with the tryout tests was followed in the scoring of the tests. One point was allowed for each answer. And, if the individual taking the test marked more than one choice, even though one of the choices marked was the right answer, the item was considered to be answered incorrectly. In the calculation of a total score, all omitted items were eliminated. A correction for "guessing," as previously explained, was made. All work was checked by a person other than the original scorer in order to minimize scoring errors.

Analysis of data.--The data gathered through the administering of the TMU (Test of Mathematical Understandings) were recorded on IBM work sheets. Arrangements were made with the Data Processing Center of Southeast Missouri State College to punch IBM cards and to assist in tabulating and correlating the data. A part of Chapter IV will be devoted to the presentation of the results of the analysis of this data.

The Selection and Use of an Arithmetic Attitude Inventory

This phase of the study was begun by searching for an instrument for measuring attitudes toward school subjects. Instruments, ranging in form from the opinion poll such as those used in the typical opinion survey to the more objective instrument such as those developed by Thurstone or Likert for measuring attitudes, have been devised and used for measuring attitudes toward elementary school subjects.

Selection of attitude inventory.--After a careful study of various attitude scales and after several tryouts with small groups of teachers, it was decided that the Arithmetic Attitude Inventory, an attitude scale developed by Wilbur Dutton at the University of California, would be appropriate for the present study. For this instrument, Dutton, using a technique perfected by Thurstone and Chave, gathered together a large number of written statements regarding arithmetic. These statements were obtained from the papers of six hundred university students over a period of five years.

Judges were asked to sort the statements using a scale of one to eleven (extremely favorable to extremely unfavorable). The percentage of judges who placed each statement in the different categories constituted the basic data for computing the "scale values" of the statements.

Twenty-two of the statements, whose scale values were equally spaced along the attitude continum, were selected for incorporation into the evaluation instrument. This instrument was used with over two hundred eighty-nine students. A reliability of .94 was obtained through test-retest procedure.[64]

The present attitude inventory, a revision of the earlier attitude scale, contains fifteen items. A reliability of .84 was obtained for this revision by the test-retest procedure.

Dutton also checked the stability of the scale values by rescaling the opinions from the first instrument on attitude toward arithmetic. In spite of some systematic shift in values, the correlation of scale values from the two sources indicated a high degree of stability as to rank order or relative placement.[65]

A copy of the attitude inventory used in this study is included in Appendix D.

Use of the attitude scale.--After permission was granted to use the 1961 revision of the Dutton Arithmetic Attitude Inventory, the inventory was administered to the eighty-nine teachers who made up the population sample for this study. This attitude inventory was given at the first and last session of the mathematics workshops.

The members of Group C, the control group, also were given the tests at or about the time of the first and last study sessions. They

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[64]Dutton, "Measuring Attitudes toward Arithmetic," Elementary School Journal, LV (September, 1954), pp. 24-31.

[65]Dutton, "Attitude Changes of Prospective Elementary School Teachers Toward Arithmetic," The Arithmetic Teacher, IX (December, 1962), pp. 418-424.

were allowed to complete the inventory at different times but were asked not to discuss the inventory with other teachers. When these teachers had completed the inventory, it was returned to the principal who returned it to the investigator.

Scoring the attitude inventory.--A first step in the processing of this data was the scoring of the attitude inventory for developing the Arithmetic Attitude Score. Each item on the scale was assigned a scale value. These were the same scale values which had been suggested by Dutton. A value of 1.0 represented an extremely negative attitude while a scale value of 10.5 represented an extremely positive attitude. The individual's total score was obtained by finding the average or median scale value of the statements toward which he acted favorably. This information also was recorded on the IBM work sheets.

A composite report of the results from the administration of this arithmetic attitude inventory may be found in Appendix L. In Chapter IV, charts, graphs, and summaries are used to describe the results of this part of the study.

The Procurement of and the Utilization of a Classroom Interview Instrument

It was necessary, in order to investigate the third hypothesis of this study, to select and procure an instrument which might be used in interviewing a random selection of pupils from the classrooms of teachers in both the in-service education group and the control group. Such an instrument was needed in order to elicit responses from a group of students relative to the classroom practices of their teachers with respect to the teaching of arithmetic. Since the purpose of the state-sponsored in-service education workshops was to provide teachers with the opportunity for learning of new developments in the field of elementary mathematics, for examining new materials, and for observing new methods of presentation, the investigator felt that the major components of any interview instrument used should relate to: (1) materials, (2) techniques, (3) organization, and (4) evaluation.

In the search for an interview instrument, it was found that the use of the interview technique, one of the many, time-honored sources of information, could vary from the highly-structured interview (representing little more than an orally administered questionnaire), through patterned or guided interviews covering pre-determined areas, to non-directive and depth interviews. All three types of face-to-face encounters, so authorities report, in the absence of peers, make it possible for the interviewer to observe reactions more closely and with greater attention than was possible in the regular classroom.[66]

An Interview Question Schedule, an orally administered questionnaire, and a companion Rating Scale, which had been designed as

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[66]Anastasi, op. cit., p. 631-632.

a part of a research project at the University of Texas, was selected as the interview instrument to be used in the present study for measuring improvement in classroom practices of elementary school teachers with respect to arithmetic.[67]

This particular instrument asked pupils to respond with respect to two dimensions: (1) qualitative and (2) quantitative. The first dimension had to do with the type of instructional program: (1) text-centered, (2) text-teacher-centered, (3) teacher-centered, (4) teacher-child centered, or (5) child-centered; the quantitative dimension had to do with: (1) the amount of class discussion, (2) the number of non-textbook materials utilized, and (3) a number of non-textbook activities.[68]

Furthermore, this interview instrument, using a non-directive question technique, attempted to avoid value-laden clues and direct cues which might bias the results.

> Questions were designed to eliminate responses at the "feeling level" by avoiding the use of direct reference to the teacher. Questions of broad scope introduced each of the component areas of the instrument. Thus desired information was obtained with a minimum of the more direct "probing" questions.[69]

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[67]A copy of the interview instrument and the rating scale may be found in Appendices E and F.

[68]While the instrument, in some areas, would appear relatively weak, it was, as far as the writer could ascertain, the best possible.

[69]DeVault, Houston, and Boyd, Television and Consultant Services, p. 52.

Reliability of this interview instrument was obtained, both through rank-order correlation coefficients (using the scores obtained by two recorders on a series of pilot interviews) and correlations between teacher and pupil interviews. High agreement was found between pupil groups but lower correlations (.46 to .65) were reported between teachers and pupils.

The developer of the interview instrument determined validity by interviewing the teacher and two groups of pupils from each of two classrooms. Validity measures ranged from .65 to .89.[70]

Utilization procedure.--For the present study, a random selection of two pupils from each of eight classrooms of teachers in Group A_2, one of the groups participating in the in-service education workshop, and a random selection of an equal number of pupils from the classroom of teachers in Group C, the control group, were interviewed to elicit responses relative to the classroom practices of the teachers with respect to teaching arithmetic. A table of random numbers was used as an aid in the selection of the pupils to be interviewed. Pupils were interviewed in January, at or about the time of the first in-service session, and again in April at the culmination of the in-service education program. The second interview was held with the same randomly selected pupils from each class.

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[70] Ibid., pp. 52-54.

The interviews. --All interviews, using the Interview Question Schedule as a guide, were conducted by the investigator or an assistant who had been trained by the investigator in interviewing techniques. And, in order to insure the highest possible reliability in the scoring of information, a small tape recorder was used to record the interview. The following steps were used:

1. The interviewer introduced herself to the student and asked for the name of the pupil.
2. Rapport was established.
3. The purpose of the interview was explained to the pupils.
4. The Stenorette (tape recorder) was demonstrated.
5. The name of the teacher, the name of the school, and the grade were entered on the tape.
6. An introductory question was asked and this was followed by a brief discussion.
7. The introductory question and identification information were played back to check the operation of the machine, let the student hear his own voice, and assist in putting the pupil at ease.
8. Questions were asked according to the Interview Question Schedule.
9. The interviewer ended by asking each pupil to tell about his favorite day in arithmetic during the year.

10. The interviewer expressed her thanks for their cooperation.

11. After the pupil left, the interviewer made a short written summary of salient points of the interview.

12. Each tape was identified and filed for future analysis.

Analysis of interview.--The two interviewers individually listened to each interview and tabulated data from each of the interviews on the Interview Schedule. To insure the highest possible reliability in the scoring of the information obtained from the Stenorette tape, a coding schedule was used in the interpretation of interview data.[71]

Responses to twenty-seven items on the "Interview Question Schedule" were evaluated on a five-point rating scale while the amount of discussion was described in terms of per cent of class time reportedly spent in discussion. The other two subscores (number of non-textbook materials utilized and number of non-textbook activities) were represented by the total number of items tallied in each area. All data were recorded on IBM work sheets and later punched into IBM cards. The Data Processing Center used a 1620 Computer to assist in tabulating and correlating the data.

Mean scores and standard deviations for subscores and total score of the Qualitative and Quantitative Dimensions of classroom practices were computed from the pre- and post-interview data. A

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[71] For further information, see Appendix F.

comparison of means between the experimental and control groups in both the sub-total and total scores were made by the use of the t-tests. The analysis of this data is presented in Chapter IV.

Procedure for Analysis of Data

All data were transferred to IBM work sheets and later punched into IBM cards. Cards were checked for accuracy by checking the IBM print-outs against raw data. This verified data on the IBM punch cards then became the source of information for the analysis.

Data were used in four ways: (1) to describe the population, (2) to test the three major hypotheses, (3) to study the relationship between test scores and background factors, and (4) to note the effect of a pre-test upon the post-test scores.

Selection of statistical procedure.--The repeated measurements of covariance and the coefficient of correlation were the statistical procedures selected for treatment of the data. The analysis of covariance permitted the investigator to adjust outcomes of the experiment in terms of the source of initial variation.

Significance level chosen.--The five per cent level of acceptance or rejection of the hypothesis being investigated was selected as being sufficiently rigorous for the conditions of this study. Thus, if the probability was at or less than five times in one hundred that the observed difference could be attributed to chance, the hypothesis was accepted; but, if the observed difference was of such magnitude that it might arise more than five times in one hundred through the operation of the chance factor, the hypothesis was rejected.

IV. Summary

This chapter dealt with the procedures which were followed in carrying out the present investigation. It was divided into four sections. The first portion of the chapter described the 1963-64 series of state-sponsored workshops in mathematics for elementary school teachers, the in-service education program which was investigated in this study.

A report of the population sample used in the study was included in the second part of the chapter. It included a description, in tabular form, of the population sample as well as explanations of the procedure used for assignment of the sample to groups for purpose of analysis.

The third part of the chapter was concerned with the procedures used in the collection of data. It included a discussion on the development of or the selection of the evaluation instruments used and the preparation of the data for analysis. The procedure followed in writing items for a paper-and-pencil test which could be used for evaluating the mathematical understandings of elementary school teachers was described in detail. The method utilized in obtaining evidence of reliability and validity was presented. The correlation coefficient between the two equivalent forms of the test was reported to be .87.

The next chapter, Chapter IV, will present, through tables, graphs, and explanatory materials, an analysis relative to the study. In Chapter V, a summary will be presented; conclusions and implications for further research also will be discussed.

CHAPTER IV

ANALYSIS OF THE DATA

In Chapter IV, data are presented in sections determined by the dependent variables which include:

1. Growth in basic mathematical understandings of elementary school teachers.
2. Changes in attitude of elementary school teachers toward mathematics.
3. Improvement in teacher classroom procedures with respect to the teaching of arithmetic.

Within the first two sections, data were reported in two parts. In the first of each of these sections, data relative to the testing of one of the major research hypotheses were presented and each research hypothesis was accepted or rejected on the basis of these data.

In the second part of these first two sections, two related questions, one having to do with test-retest effect and the other having to do with the relationship of improvement in mathematical understandings and changes in attitude toward arithmetic to selected background factors, were examined. A third major section described the data collected with respect to changes in teacher classroom practices.

On the basis of the data presented in this final section, the third major research hypothesis was accepted or rejected. A summary concluded the chapter.

I. Growth in Basic Mathematical Understandings of Elementary School Teachers

The data collected through the administration of two equivalent forms of the test, "A Test of Mathematical Understandings," the evaluation instruments developed by this investigator for use in the present study, were utilized in an attempt to measure the mathematical understandings of a group of elementary school teachers.

Hypothesis I

The first hypothesis was stated as: There will be a significant difference on a test of mathematical understandings between the post-test scores of a group of elementary school teachers who have participated in an in-service education program and their pre-test scores or the scores of a similar group who have not participated in the in-service education program.

Findings

This hypothesis was tested through the interaction of the first dependent variable and the independent variables, in-service presentation and testing procedure.

Pre- and post-test means.--Pre- and post-test means, standard deviations, and changes (difference in means) on the tests of mathematical understandings were computed for each research group.[1]

In order to determine the significance of the difference between the pre- and post-test means (correlated), a t-test was used.[2]

The resulting t-ratio was compared with the "t" in a table designed for use in determining the significance of "t." These data are included in Table 6.

During the time of the in-service education workshop in mathematics, all in-service workshop groups made gains on this measure.

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[1]The following formulas were used to compute the significance of the difference between correlated means obtained from tests administered to the same group.

$$SE_D = \sqrt{\sigma^2_{M_1} + \sigma^2_{M_2} - 2r_{12}\sigma_{M_1}\sigma_{M_2}}$$

where SE_D = standard error of the difference between correlated means,

σ_{M_1} = standard error of mean on first test,

σ_{M_2} = standard error of mean on second test, and

r = correlation between initial and final tests.

$$t = \frac{\bar{X}_1 - \bar{X}_2}{SE_D}$$

where $\bar{X}_1 - \bar{X}_2$ = difference between means.

[2]For further information, see Garrett, Statistics in Psychology and Education, pp. 226-227.

TABLE 6

PRE- AND POST-TEST RESULTS ON THE TEST OF MATHEMATICAL UNDERSTANDINGS FOR THE RESEARCH GROUPS IN THIS STUDY[a]

Research Group	Grade Level	Pre-Test Mean	Pre-Test SD	Post-Test Mean	Post-Test SD	Change M_1-M_2	t^b
Group A_1 (Experimental)	Primary	25.3	6.66	29.0	7.00	4.3	1.99*
Group A_2 (Experimental)	Intermediate	30.27	7.38	34.86	5.52	4.59	2.72*
Group B_1 (Experimental)	Primary	28.00	6.67	30.30	7.17	2.30	1.70
Group B_2 (Experimental)	Primary[c]			26.27	8.44		
Group C (Control)	Intermediate	31.58	6.20	31.10	6.21	-.48	-.96

[a]A more comprehensive table of test results may be found in Appendix J.

[b]See preceding page for procedure used for calculation of t. When t was significant at the .05 level, an asterisk (*) was used.

[c]Group B_1 was the experimental group used to measure test-retest effect. No pre-test was given.

However, when the obtained mean difference for each research group was tested against the hypothetical zero gain, a significance of the difference between pre- and post-test means was noted for Group A_1 and A_2; the resulting t for Group B_1 was 1.70 and approached significance ($t_{.05} = 1.73$).

Based on the results of this criterion measure, that part of the first hypothesis which stated that there would be a significant difference on a test of mathematical understandings between pre- and post-test scores of a group of elementary school teachers who had participated in an in-service education program was rejected.

Analysis of covariance.--To test the second part of the first hypothesis, that part having to do with a comparison of growth in mathematical understandings between participants in the in-service workshops and a similar group who were not participants, the analysis of covariance was utilized. This statistical technique, known to be particularly applicable to any experiment, such as the present one, in which groups could not be randomized or equated before treatment, made it possible for the investigator to adjust the outcomes of the experiment (gain in mathematical understandings) in terms of a source of variation (the pre-test).

The scores of forty-four teachers, twenty-two intermediate grade teachers from the experimental group (Group A_2) and twenty-two teachers who served as a control group (Group C), were used for this analysis. The data are presented in Table 7.

TABLE 7

SUMMARY OF THE ANALYSIS OF COVARIANCE FOR THE SCORES OF FORTY-FOUR INTERMEDIATE GRADE TEACHERS ON THE TEST OF MATHEMATICAL UNDERSTANDINGS[a]

Source of Variation	df	SS_x	SS_y	S_{xy}	$SS_{y.x}$	$MS_{y.x}(V_{y.x})$	$SD_{y.x}$
Among Groups	1	52	127	56	76.82	76.82	
Within Groups	41	1949	1762	590	66.69	1.63	1.31
Total	42	2001	1889	534	143.51		

$$F_{y.x} = \frac{76.82}{1.63} = 46.94$$

From F-ratio Table:
F at .05 level = 4.07
.01 level = 7.30

When the F-ratio was applied to the adjusted "among" and "within" variances, F was highly significant at either the .05 or .01 level.

Then, a significance of the difference between the adjusted post-test (Y) means was found to be 1.31. Sugsbituting for $t_{.05}$ (1.68) in the equation $t = D/SE_D$, an obtained difference of 2.66 at the .05 level was noted. It was concluded that, when initial differences were allowed for, the in-service education workshop in mathematics had made for significant changes in the final scores of the experimental group.

Based on this evidence, it was concluded that the two final means did differ significantly. That part of hypothesis one which stated that there will be a significant difference on a test of mathematical understandings between the post-test scores of a group of elementary teachers who participated in an in-service education workshop and the scores of a similar group who did not participate in the in-service program was accepted.

Related questions.--In connection with the testing of the first hypothesis, two questions were studied. These two questions and the procedure used for evaluation are discussed in the paragraphs that follow.

1. What effect, if any, does a pre-test have upon the post-test scores of teachers?

To determine the effect of a pre-test upon the post-test scores, the significance of the difference between mean scores of participants in Group B, a group of thirty-five participants in one of the in-service workshops who had been divided at random into two groups, one group given both pre- and post-test and the other given post-test only, was examined through the analysis of variance.

Records for the two groups are shown in Table 8.

Since the obtained F-ratio was not significant (less than the F at the .05 level), the mean differences were not significant; they could be expected to occur by chance more than once in twenty trials.[3]

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[3]Garrett, op. cit., pp. 280-295.

TABLE 8

ANALYSIS OF VARIANCE FOR RESEARCH GROUP B ON THE TEST OF MATHEMATICAL UNDERSTANDINGS

Source of Means Variation	df	Sums of Squares	Mean Squares (Variance)	SD
Among the of Conditions	1	140	140	
Within Conditions (Error)	33	1985	60.15	7.74
Total	34	2125		

$F = \frac{140}{60.15} = 2.33$

From Table:
$df_1 = 1$ and $df_2 = 33$
F at .05 = 4.19
F at .01 = 7.58

So, in answer to the first related question, this data indicated that a pre-test did not contribute significantly to the post-test scores of the evaluation instrument which was used to measure mathematical understandings. This is not to be interpreted to mean that teachers did not benefit from the in-service workshops. But, within the limitations of this study, it can be said that a pre-test did not contribute significantly to the post-test scores of a test designed to measure mathematical understandings of elementary school teachers.

2. What is the relationship of improvement in basic mathematical understandings to teacher background factors and information?

To study the degree of relationship between selected background factors and mathematical understandings, product-moment correlation coefficients for all background factors, except highest degree held, were obtained for both control and experimental groups.[4] The results are shown in Table 9.

Using a table of values of correlation coefficients for different levels of significance, it was noted that none of the correlation coefficients for the background factors and the dependent variable except the ones for semester hours credit in college mathematics and years of teaching experience either were significant or approached the level of significance at the five per cent level. The only significant correlation between mathematical understandings and background factors noted at the time of post-test was for post- "Test of Mathematical Understandings," and semester hours of college mathematics.

Recapitulation

The first dependent variable, growth in basic mathematical understandings of elementary school teachers, was measured by the administration of two equivalent forms of the test, "A Test of

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[4]Since the academic background of teachers had been described in terms of highest degree attained, classified as "Bachelors" or "Above," biserial correlation was used to obtain the correlation coefficient between highest degree attained and the evaluation instrument.

TABLE 9

CORRELATION COEFFICIENTS BETWEEN SELECTED TEACHER BACKGROUND INFORMATION AND SCORES ON TEST OF MATHEMATICAL UNDERSTANDINGS[a]

	Experimental (A_1)		Control (C)	
	Pre-	Post-Test	Pre-	Post-Test
Test Results				
Test of Mathematical Understandings (Pre)		.84*		.83*
Test of Mathematical Understandings (Post)	.84*		.83*	
Background Factors				
Years of Teaching Experience	.49*	.24	.06	.06
Credit in High School Mathematics	.17	.13	.23	.31
Semester Hours Credit in College Mathematics	.53*	.48*	.10	.06
Semester Hours Credit in Methods of Teaching Arithmetic	-.25	.05	.29	.30
Highest Degree[b]	-.07	-.06	-.05	-.09

[a]For df = 20, the value for the correlation coefficients obtained from the table was .423; data marked with an asterisk (*) were significant at the five per cent (.05) level.

[b]Biserial correlation was used to obtain this correlation coefficient for highest degree.

Mathematical Understandings," to eighty-nine teachers who participated in two in-service education workshops. The hypothesis was tested through the interaction of the independent variables, grade level, in-service presentation, and testing procedure. To determine the significance of the difference between pre- and post-test means, a t-ratio was computed. On this achievement measure, all in-service education workshop groups, with the exception of Group A_1, made gains which were statistically significant at the five per cent point.

In the analysis of covariance which was utilized for evaluation of test data, there was a significant difference between the experimental and the control group. The F-ratio was significant at the .05 level and, as a result, it was concluded that the in-service workshops in mathematics were effective in improving the mathematical background of the elementary teachers who participated.

There was no significant difference, however, between the two research groups which were used to evaluate the question of test-retest effect. Significant differences in correlation coefficients among the pre-test measure and the background factors were noted for years of teaching experience and semester hours credit in college mathematics. A significant correlation was noted at the time of post-test between mathematical understandings and the background factor, semester hours of mathematics.

II. Changes in Attitude of Elementary School Teachers Toward Mathematics

For this aspect of the present investigation, data collected through the administration of the 1961 Revised Dutton Arithmetic Attitude Inventory was utilized in an attempt to note changes, if any, in attitudes of elementary school teachers toward mathematics which occurred during the time of the 1963-64 series of in-service education workshops being conducted by the Missouri State Department of Education.

Hypothesis II

The hypothesis which had to do with this aspect of the study was stated as: There will be a significant difference on an arithmetic attitude test between the post-test scores of a group of elementary school teachers who have participated in an in-service education workshop and their pre-test scores or the scores of a similar group who have not participated in the in-service program.

This hypothesis was tested through the interaction of independent variables, in-service presentation and testing procedures, with change in attitude of elementary school teachers toward mathematics. Pre- and post-test means and standard deviations for achievement on the attitude inventory were computed for each research group. Any mean differences for the research groups were tested against the hypothetical zero gain through the use of the t-test. The analysis of covariance was utilized to compare the means between an experimental and control group.

Treatment of the Data

Responses to the fifteen statements on the arithmetic attitude inventory were tabulated according to item and group. Each item on the scale was assigned a scale value (from 1.0 which represented an extremely negative attitude toward arithmetic to 10.5 which represented an extremely positive attitude). The individual's total score was obtained by finding the average or median scale value of the statements toward which he acted favorably.

A composite report of the results from the administration of this arithmetic attitude inventory has been included in the Appendix L. Data included in the tables in this section were drawn from Appendix L.

Findings

The findings pertaining to the second hypothesis are discussed in the paragraphs that follow.

Pre- and post-test means.--Pre- and post-test means, standard deviations, and changes (difference in means) were obtained from administering the Revised Dutton Arithmetic Attitude Inventory to eighty-nine participants in two mathematics workshops for elementary school teachers and a similar group of teachers who were used as a control group.

To determine the significance of the difference between pre- and post-test means, a t-test was used.[5] The resulting t was compared with the t in a table designed for use in determining the significance of t. This data is presented in Table 10.[6]

All in-service education workshop groups made numerical gains between the pre-test and post-test. However, when the t-test was applied to the results, a significance of difference between pre- and post-test means was noted for Group A_1 and Group B_2; the resulting t for Group B_1 was 1.14 and did not approach the level of significance.

Based on the results of these data, that part of the first hypothesis which stated that there would be a significant difference on the arithmetic attitude inventory between the pre- and post-test scores of a group of elementary school teachers who participated in an in-service workshop was rejected. This should not be interpreted to mean that the teachers did not benefit from the workshop. But, within the limitation of this study, mean gains for all groups were not sufficient to be significant.

Analysis of covariance.--To test the second part of the second hypothesis, that part having to do with the comparison of change in attitudes

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[5]For an explanation of the formula used to compute the significance of the difference between these correlated means, see page 117.

[6]A more comprehensive table, including raw scores, group identification, and test form, may be found in Appendix K.

TABLE 10

PRE- AND POST-TEST RESULTS ON THE ARITHMETIC ATTITUDE INVENTORY FOR THE RESEARCH GROUPS IN THIS STUDY[a]

Research Group	Grade Level	Pre-Test Mean	Pre-Test SD	Post-Test Mean	Post-Test SD	Change M_1-M_2	t^b
Group A_1 (Experimental)	Primary	6.6	2.30	7.1	2.00	.50	2.17*
Group A_2 (Experimental)	Intermediate	6.58	2.30	7.75	.91	1.17	3.82*
Group B_1 (Experimental)	Primary	7.0	2.0	7.70	3.01	.70	1.14
Group B_2[c] (Experimental)	Primary			6.6	1.69		
Group C (Control)	Intermediate	8.72	.47	7.86	.24	-.86	-1.91

[a]A more comprehensive table of test results may be found in Appendix K.

[b]See page 117 for procedure used for calculation of t. When t was significant at the .05 level, an asterisk (*) was used.

[c]Group B_1 and Group B_2 were the experimental groups used to observe test-retest effect. No pre-test was given Group B_2.

toward arithmetic between participants in the in-service workshops and a similar group who were not participants, the statistical procedure of the analysis of covariance was utilized.[7]

The scores of forty-four teachers, twenty-two intermediate grade school teachers from the experimental group (Group A_1) and a similar group of twenty-two teachers who served as a control group (Group C), were used for this analysis. The data were presented in Table 11.

TABLE 11

SUMMARY OF THE ANALYSIS OF COVARIANCE FOR THE SCORES OF FORTY-FOUR INTERMEDIATE GRADE TEACHERS ON THE ARITHMETIC ATTITUDE INVENTORY

Source of Variation	df	SS_x	SS_y	S_{xy}	$SS_{y.x}$	$MS_{y.x}$ $(V_{y.x})$	$SD_{y.x}$
Among Groups	1	1	1	-6	3.30	3.30	
Within Groups	41	132	30	28	24.06	.59	.77
Total	42	133	31	22	27.36		

$$F_{y.x} = \frac{3.30}{.59} = 5.59$$

From Table F
df 1/41
F at .05 level = 4.07

[7]For description of procedure, see Edwards, Experimental Design in Psychological Research, pp. 295-296.

When the F-ratio was applied to the adjusted "among" and "within" variances, it was pertinent to note that the F was significant ($F_{y.x}$ = 3.30/.59 or 5.59 when $F_{.05}$ = 4.07) at the .05 level. Then, using the formula for the standard error of the difference between the adjusted post-test scores, significant difference at the .05 level (SE_D = 3.99 when $t_{.05}$ = 2.02) was noted.

Based on this evidence, it was concluded that the two final means, when initial difference was allowed for, did differ significantly; hypothesis two, that there will be significant difference on an arithmetic attitude inventory between the adjusted post-test scores of a group of elementary teachers who participated in an in-service education workshop and the adjusted post-test scores of a similar group who did not participate in the in-service program was accepted. This particular in-service education workshop in mathematics had made for significant changes in the final scores of the experimental group.

Related questions.--In connection with the testing of the second hypothesis, two related questions were studied. These two questions and the procedure used for evaluation are discussed in the paragraphs that follow.

1. What effect if any does a pre-test have upon the post-test scores of teachers on an attitude inventory?

To determine the effect of a pre-test upon the post-test scores, the significance of the difference between mean scores of Group B, a group of thirty-five participants in the in-service workshops who had been divided at random into two groups, one group given both pre- and post-test and the other given post-test only, was examined through the analysis of variance.

Data for the two groups are shown in Table 12.

TABLE 12

ANALYSIS OF VARIANCE FOR RESEARCH GROUP B ON ARITHMETIC ATTITUDE INVENTORY

Source of Variation	df	Sums of Squares	Mean Squares (Variance)	SD
Among the eans of Conditions	1	9	9	
Within Conditions	33	88	2.67	1.73
Total	34	97		

$F = \frac{9}{2.67} = 3.37$

From F-ratio table:
F at .05 level = 4.15

Since the obtained F-ratio was not significant (less than the F at .05 level), the mean differences were not significant; they could be expected to occur by chance more than once in twenty trials.

Thus, in answer to the first related question, this data indicated that a pre-test did not contribute significantly to the post-test scores made by the in-service group on a test designed to measure attitudes toward arithmetic. This is not interpreted to mean that teachers did not benefit from the in-service workshops. But, within the limitations of this study, it can be said that a pre-test did not contribute significantly to the post-test scores made by a group of elementary school teachers on an arithmetic attitude test.

2. What is the relationship between change in attitude toward arithmetic and selected teacher background factors?

To study the degree of relationship between selected background factors and attitudes toward arithmetic, product-moment correlation coefficients for all background factors, except highest degree held, were obtained for both control and experimental groups.[8] The results are shown in Table 13 on the following page.

Using a table of values of correlation coefficients for different levels of significance, it was noted that the correlations between the pre- or post-test measures and selected background factors were neither significant nor approached the level of significance.

Recapitulation

The second dependent variable, change in attitude of elementary school teachers toward arithmetic, was evaluated by the administration

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[8]Biserial correlation was used to obtain correlation coefficient for highest degree.

TABLE 13

CORRELATION COEFFICIENTS BETWEEN SELECTED TEACHER BACKGROUND INFORMATION AND SCORES ON ARITHMETIC ATTITUDE INVENTORY[a]

	Experimental (A_1)		Control (C)	
	Pre-	Post-Test	Pre-	Post-Test
Test Results				
Arithmetic Attitude Inventory (Pre)		.53		.31
Arithmetic Attitude Inventory (Post)	.53		.31	
Background Factors				
Years of Teaching Experience	.18	.10	.17	.19
Credit in High School Mathematics	.06	.06	.02	.16
Semester Hours Credit in College Mathematics	.25	.19	.17	-.05
Semester Hours Credit in Methods of Teaching Arithmetic	-.05	-.07	-.13	-.05
Highest Degree[b]	-.09	-.07	-.06	-.06

[a]Data marked with an asterisk (*) were significant at the five per cent (.05) level.

[b]Biserial correlation was used to obtain correlation coefficient.

of the 1961 Revised Dutton Arithmetic Attitude Inventory to two in-service education workshop groups, participants in a series of mathematics workshops being sponsored by the Missouri State Department of Education, and a similar group of teachers who were used as a control group. The hypothesis having to do with this aspect of the study was tested through the interaction of the dependent variable, change in attitude toward arithmetic, and the independent variables, in-service presentation and testing procedure.

To determine the significance of the difference between pre- and post-test means, t-ratios were computed. On this evaluation measure all in-service workshop groups made numerically higher scores; however, when the obtained mean difference for each research group was tested against the hypothetical zero gain, a significance of difference between pre- and post-test scores was noted for Group A_1 and Group A_2; the resulting t for Group B_1 was 1.14 and did not approach significance ($t_{.05} = 1.73$).

In the analysis of covariance, the statistical technique utilized for evaluation of test data having to do with a comparison of growth in attitude toward mathematics between participants in the in-service workshops and a similar group who were not participants, there was a significant difference between the adjusted post-test scores of the experimental and control groups. The F-ratio was significant at the .05 level and, as a result, it was concluded that the in-service workshop in mathematics was effective in improving the attitude of the workshop participants toward arithmetic.

There was no significant difference, however, between the post-test scores of the two research groups which were used to evaluate the question of test-retest effect. No significant differences in correlation coefficients between the pre-test measure and the selected background factors were noted.

III. Changes in Teacher Classroom Procedures with Respect to the Teaching of Arithmetic

The data collected through the use of the "Texas Elementary School Mathematics Classroom Practice Schedule," was utilized to measure changes in teacher classroom procedures, the third dependent variable of this study.[9] This particular instrument, developed for use in a research study at the University of Texas, was used to elicit responses from a random selection of two pupils from eight classrooms of teachers in one of the in-service (experimental) workshops, and an equal number of pupils from classrooms of a similar group of teachers who had not participated in the workshop. Using the non-directive question technique, the schedule of questions sought pupil responses concerning major components of an elementary school mathematics program.

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[9]A copy of this interview instrument may be found in Appendix E.

Hypothesis III

The third hypothesis was stated as: There will be a significant difference between the classroom practices of a group of elementary school teachers who had participated in an in-service education workshop and the classroom practices of a similar group who had not participated in the in-service education program.

This hypothesis was tested through the assessment of the independent variable, in-service presentation, with improvement in teacher classroom practices. Pre- and post-test means for subscores of both qualitative and quantitative dimensions of classroom practices were computed for experimental and control groups. Comparison of means between the experimental and control group on both the subscores and total scores were made by use of the t-tests.

Scores obtained in interviewing a random selection of twelve pupils from classrooms of eight teachers in the experimental group and a random selection of sixteen pupils from the classrooms of teachers in the control group are presented in Appendix M.[10] Data included in the tables in this section were drawn from Appendix M which may be founded on pages 248-251.

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[10] Originally the random selection of pupils (groups) to interview was equal. However, when four students from the experimental group were unable to complete the post-interview, it was necessary to withdraw data relative to their first interview.

Findings

Responses to the statements on the Interview Question Schedule were tabulated with respect to two dimensions of classroom practice: (1) qualitative dimension and (2) quantitative dimension.[11]

Qualitative dimension.--The qualitative dimension described the extent to which teachers' classroom practices were "textbook-centered" or "child-centered." Responses to twenty-seven items on the Question Schedule were evaluated by a five-point rating scale. This scaled rating permitted the classification of responses according to the type of instructional program represented:

1. Text-centered -- scaled rating of 1.
2. Text-teacher-centered -- scaled rating of 2.
3. Teacher-centered -- scaled rating of 3.
4. Teacher-child-centered -- scaled rating of 4.
5. Child-centered -- scaled rating of 5.

The total score ranged from a minimum of twenty-seven to a possible maximum of one hundred thirty-five.

Subscores on the qualitative dimension were obtained by grouping the twenty-seven item ratings under the following five subscores.

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[11] For interpretation of coding the "Texas Elementary School Mathematics Classroom Practices Schedule of Questions," see Appendix F.

1. Instructional Materials

2. Activities

3. Organization

4. Teaching Aids

5. Evaluation

The scores tabulated under instructional materials included the scaled ratings given the responses concerning the use made of single or multiple adoptions of textbooks as well as the utilization of library books, pamphlets, and mimeographed materials. The activities subscore was derived from compiling the scaled ratings concerning the use of "free time," homework, and activities outside the routine textbook approach. The organization subscore was obtained from the scaled ratings given to the responses which referred to the character or extent of individualized instruction and the use made of class discussions. The fourth subscore, teaching aids, pertained to ratings derived from questions concerning the source of graphic and manipulative teaching aids and the extent to which these aids were utilized for individual exploration. The fifth subscore, evaluation, considered the number, kind, and source of tests used and the extent to which pupils were involved in evaluating their own progress in mathematics.

A compilation of the scores obtained for both experimental and control groups on each of the five subscores of the qualitative dimension, as well as the total score, were included in Appendix M.

Quantitative dimension.--Data obtained from the pupil responses to six questions on the Interview Question Schedule were used to report a second dimension of classroom practices referred to as a quantitative dimension.[12] These questions, distributed throughout the interview instrument, described: (1) the amount of class discussion, (2) the number of trade books, library books, encyclopedias, pamphlets, films, slides, and graphic materials, and (3) the number of non-textbook or special "project" activities which were reported by the pupils at the time of the interview.[13]

The amount of discussion was described by the per cent of class time reportedly spent in discussion; the other two subscores, the number of non-textbook materials and the number of non-textbook activities, reported the total number of items tallied in each area.[14]

A compilation of responses obtained from the pupils of teachers in both the experimental and control groups on each of the three subscores of this quantitative dimension were included in Appendix M.

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[12]While the interview instrument, as previously discussed, was weak in some areas, it was, to the investigator's knowledge, the only instrument at the time which attempted to elicit responses from pupils with respect to classroom practices of teaching arithmetic.

[13]For further information on these questions, see those questions on the Interview Question Schedule in Appendix E which have an asterisk (*) before them.

[14]Since the amount of discussion was reported in per cent and the number of non-textbook materials and activities by item, no attempt was made to obtain a total score on the quantitative dimension.

Pre- and post-test means.--To evaluate the effectiveness of the in-service education program in improving the classroom practices of teachers, mean scores, standard deviations, and changes (difference in means) were computed. A t-test was used to determine the significance between the pre- and post-test means. These data are included in Table 14.

For the experimental group, the post-mean for the total score, as well as the post-mean for each subscore, was greater than the pre-test mean. Three of the five subscores on the qualitative dimension (utilization of instructional materials, variation in the use of activities, and organization of the classroom), as well as the total score, resulted in significant differences between pre- and post-interview measures.

The post-means for the control group were greater for each subscore except evaluation and teaching aids. Significant difference between pre- and post-interview measures was noted for one subscore, utilization of materials, in the control group.

Pupils reported that teachers in the experimental group were, at the end of the in-service program, using greater amounts of time in discussion, involving pupils in a greater number of non-textbook activities, and using a greater number of non-textbook materials than reportedly had been in use at the time of the pre-interview. One of the three quantitative dimensions for the experimental group, number

TABLE 14

PRE- AND POST-TEST DESCRIPTIONS OF TEACHER CLASSROOM PRACTICES AS DETERMINED BY PUPIL INTERVIEWS[a]

Research Group	Classroom Practices	Pre-Test		Post-Test		Change	t^b
		Mean	SD	Mean	SD	M_1-M_2	
Group A_1 (Experimental)	Qualitative						
	Materials	11.25	4.18	13.25	3.59	2.00	3.33*
	Activities	9.75	5.29	16.00	3.18	6.25	5.16*
	Organization	10.50	2.27	13.08	2.81	2.58	3.47*
	Teaching Aids	5.42	2.09	6.42	2.14	1.00	1.39
	Evaluation	7.25	2.30	8.17	1.95	.92	.83
	Total	44.17	5.43	56.92	6.91	12.75	5.10*
	Quantitative						
	Amount of Discussion[c]	36.66	11.76	38.75	11.62	2.08	1.50
	Number of Non-Textbook Activities	1.00	.95	1.17	1.42	.17	.79
	Number of Non-Textbook Materials	1.75	1.13	2.16	1.30	.41	1.60

TABLE 14 -- Continued

Research Group	Classroom Practices	Pre-Test		Post-Test		Change	t^b
		Mean	SD	Mean	SD	M_1-M_2	
Group C (Control)	Qualitative						
	Materials	7.19	4.89	8.88	4.04	1.69	1.81*
	Activities	11.31	2.72	12.37	1.89	1.06	1.17
	Organization	7.44	2.87	7.75	3.18	.31	.59
	Teaching Aids	5.13	1.71	3.31	2.01	-1.82	-2.35
	Evaluation	9.06	3.42	8.63	1.99	- .43	- .79
	Total	40.13	7.83	40.94	4.21	.81	.96
	Quantitative						
	Amount of Discussion[c]	34.37	10.01	35.47	8.90	1.04	.876
	Number of Non-Textbook Activities	.50	.51	.63	.62	.13	.73
	Number of Non-Textbook Materials	1.375	1.52	1.37	1.21	.00	.00

[a]A more comprehensive report of the interview results may be found in Appendix M.

[b]When "t" was significant at the .05 level (1.753), an asterisk (*) was used.

[c]Represents per cent.

of non-textbook activities, resulted in significant difference between pre- and post-test means.

On the total qualitative dimension, when a t-test was used to compare the pre-test means for the experimental and control groups, the resulting t (1.52) with 26 ($N_1 + N_2 - 2$) degrees of freedom was not significant at the .05 level.[15] Since significant difference was noted, it was concluded that the two pre-test means did not differ significantly.

The obtained value for t (2.26) for the post-test means for the two groups on the same dimension was significant at the .05 level. It was concluded that, within the limitations of this study, and for the particular group used, a mathematics workshop could be helpful in changing the classroom practices of the elementary teachers who participated.

Recapitulation.--The hypothesis was tested through the assessment of the independent variable, in-service presentation, with improvement in teacher classroom practices. To determine the significance of the difference between pre- and post-test means, a t-ratio was computed. For the experimental group, three of the five subscores, as well as

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[15]To determine whether the obtained "t" was significant, the procedure as outlined by Edwards on pages 273 and 274 in Statistical Methods for the Behavioral Sciences was followed.

[16]Since the amount of discussion was reported in per cent and the number of non-textbook materials and activities by item, no attempt was made to obtain a total score on the quantitative dimension.

the total score, on the qualitative dimension, resulted in significant differences between pre- and post-interview measures. Significant difference between pre- and post-interview measures for the control group was noted for only subscore.

In a comparison of means between experimental and control groups, there was a significant difference on the post-interview scores. The obtained value for "t" was significant at the .05 level and, as a result, it was concluded that, within the limitations of this study, and for the particular group used, a mathematics workshop could be helpful in changing the classroom practices of teachers who participated. One of the three quantitative dimensions for the experimental group also resulted in a significant difference between pre- and post-test means.

IV. Summary

This chapter has presented an analysis of the data in sections determined by the dependent variables:

1. Growth in basic mathematical understandings of elementary school teachers.
2. Changes in attitude of elementary school teachers toward mathematics.
3. Changes in teacher classroom procedures with respect to the teaching of arithmetic.

Table 15 is a summary of the conclusions reached for each of the three hypotheses of the study.

TABLE 15

SUMMARY OF ACCEPTANCES AND REJECTIONS OF THE RESEARCH HYPOTHESES OF THE STUDY

Hypothesis Related to	Dependent Variable[d]		
	I[a]	II[b]	III[c]
In-service Groups			
Group A_1	Rejected	Accepted	
Group A_2	Accepted	Accepted	
Group B_1	Accepted	Rejected	
Group B_2[e]	Accepted		
Experimental versus Control[d]	Accepted	Accepted	See footnote f

[a]Growth in mathematical understandings of elementary school teachers.

[b]Changes in attitudes of elementary school teachers toward mathematics.

[c]Improvement in teacher classroom practices.

[d]Dependent Variable III (Improvement in classroom procedure) applicable to experimental (Group A_2) and control (Group C) groups only.

[e]Group B_1 and Group B_2 were the experimental groups used to observe test-retest effect. No pre-test was given Group B_2.

[f]For explanation of acceptance or rejection of the third major hypothesis, see the procedure used for comparison of means in the recapitulation, pages 144 and 145.

CHAPTER V

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

The American people, operating on the premise that our welfare is dependent, among other factors, upon an enlightened citizenry, have focused a spotlight on public education. This spotlight has brought together groups of professional people representing different backgrounds of experience and training to study educational problems and to make suggestions for curriculum improvement. As a result, changes have been and are being made in the mathematics in the elementary school. These changes have stimulated new programs which are different both in content and in approach.

This call for new programs in mathematics has made it imperative that the elementary teacher know the mathematics necessary to teach the content of the new curriculum. Hardgrove, in a conference on the training of teachers in elementary school mathematics, stated that:

> Teachers need a mathematics education which makes it possible for them to understand and appreciate the structure of the mathematics they teach, which makes it possible for them to help children develop a problem-solving technique, and

> which makes it possible for them to have confidence in their teaching of arithmetic.[1]

An increasing concern for promoting the professional growth of all teachers, whether "neophytes" or "old-timers," has made the professional growth of teachers one of the most challenging and persistent problems faced by schools today. Many educators see the need for professional growth as far too pressing to be limited to the pre-service education of teachers.

> As teaching becomes more of a profession and less of a procession, more teachers are removed from their pre-service training than ever before. The training they received even a decade ago is inadequate today either as to substance or as to methodology. This underlines the demand for effective in-service education. . . . Without it, our schools cannot adequately prepare boys and girls for a dynamic society.[2]

Various forms of in-service education such as workshops, child study groups, cooperative curriculum study, and use of consultants, have been utilized by school systems and state departments of education; few reports, however, have attempted to evaluate objectively the effectiveness of any of the techniques being employed in the in-service education program of teachers. This, nevertheless, was the purpose of the present investigation.

It was hoped that this investigation, which utilized the participants in a state-sponsored series of workshops for mathematics for

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[1]Hardgrove, op. cit., p. 34.

[2]Ashby, op. cit., p. 271.

elementary school teachers, could push further "the knowledge claim" that an in-service presentation, with proper content emphasis, could raise the level of understandings of elementary school teachers with respect to mathematical understandings.

The preceding chapters were devoted to a discussion on the current significance of the problem, a delineation of its purpose, and a description of the procedure followed in evaluating a series of state-sponsored workshops in mathematics for elementary school teachers. Chapter V, the final chapter of this report, was devoted to: (1) a general summary of the study, (2) major conclusions, and (3) implications or recommendations for both curriculum and research.

I. General Summary

The purpose of this study was to analyze the effectiveness of the workshop as a means of in-service education for: (1) improving basic mathematical understandings of elementary school teachers, (2) changing the attitudes of elementary school teachers toward mathematics, and (3) improving the classroom practices of teachers with respect to the teaching of arithmetic. The two workshops used in this study were drawn from the 1963-64 series of workshops in mathematics for elementary school teachers, a part of an in-service education program being sponsored by the Missouri State Department of Education under Title III, National Defense Education Act.

As a basis for comparison, a control group of twenty-two teachers, a group who had also volunteered to participate in a mathematics workshop, was used.

These state-sponsored workshops, viewed as one means for helping teachers "to modernize the content and upgrade their teaching of arithmetic," were open to all elementary school teachers in the participating school districts. The teachers met for a period of ten weeks, one session per week. There were no fees and no college credit was given.

A bulletin issued by the State Department of Education set forth the following as objectives for these state-sponsored workshops in mathematics for elementary school teachers:

1. To provide teachers with the opportunity to raise their level of understanding of the concepts of mathematics and to lay the foundation for further self-improvement.
2. To illustrate through demonstration teaching some of the new approaches to, and methods of, presenting mathematical ideas to elementary students.
3. To create an atmosphere of enthusiasm around mathematics so that teachers and students alike may enjoy the pursuit of its excellence.
4. To acquaint the teachers with new teaching aids, manipulative devices and the laboratory approach to the teaching of elementary school mathematics.
5. To provide experiences and materials whereby teachers may broaden their horizons relative to experimental programs, pertinent literature, and extracurricular activities in the field of mathematics.

One hundred and twenty teachers in the two school districts where the workshops were held indicated an initial interest in participating in a mathematics workshop. Ten did not enroll, four attended only the first session, six did not attend sufficient sessions to obtain in-service credit, five did not take the post-test, and six returned their papers with incomplete data. The remaining eighty-nine teachers constituted the population sample for this study.

To test the three major hypotheses of the study and for the evaluation of the data relative to the background factors, a design which allowed the investigator to utilize features of both the one-group "Pre-test, Post-test Design" and the "Static Group Comparison Design" was used.

The following independent variables were used:

1. In-service presentation
2. Pre-testing
3. Post-testing
4. Grade level

Dependent variables were:

1. Growth in basic mathematical understandings of elementary school teachers.
2. Changes in attitude of elementary school teachers toward mathematics.
3. Changes in teacher classroom procedures.

The background information furnished by each teacher included: (1) sex, (2) years of teaching experience, (3) number of credits in high school mathematics, (4) number of semester hours credit in college mathematics, (5) number of semester hours credit in college mathematics teaching methods, and (6) highest degree attained.

Instruments selected or developed for use in the collection of data were:

1. Teacher Information Form and Problem Census Questionnaire.
2. A Test of Mathematical Understandings (two parallel forms).
3. Revised Form of Dutton Arithmetic Attitude Inventory.
4. Interview Question Schedule and Rating Scale.

All instruments used in the study were accepted by the mathematics consultant and the administrative-supervisory personnel from Title III, National Defense Education Act, Missouri State Department of Education, as being relevant to the objectives of the in-service education workshops involved.

The first task of this study involved a review of literature. Major research developments which were considered relevant to this study were organized under the following four categories: (1) improving the mathematical understandings of elementary school teachers, (2) utilizing in-service education procedures, (3) changing the attitudes of elementary school teachers toward mathematics, and (4) improving the classroom procedures of teachers with respect to the teaching of arithmetic.

Various sources indicated that a large number of prospective and experienced elementary school teachers simply do not know as much arithmetic as they should in order to teach it effectively. They saw the teacher's attitude toward the subject as a factor which might limit his or her effectiveness. The various states, the many colleges and universities, and the local school districts were urged to continue to study and evaluate the many forms of in-service teacher education.

Next, after a review of the literature, the investigator pursued the following course in the collection of data:

1. An information form entitled "Teacher Background Information" was prepared for distribution to all participants in the state-sponsored workshops. Teachers were asked to furnish the following data relative to background information: (1) grade level taught, (2) years of teaching experience, (3) highest degree attained, (4) number of high school credits in mathematics, (5) number of semester hours credit in college mathematics, and (6) number of semester hours credit in methods of teaching arithmetic.
2. Two parallel forms of a "Test of Mathematical Understandings" were constructed, tried out in experimental form, revised, and administered to both the experimental and control groups as either a pre- or post-test. These tests were administered at the first and last sessions of the two mathematics workshops.

The control group also was given the tests at or about the time of the first and last study session.

3. The 1961 Revised Dutton Arithmetic Attitude Inventory was selected as appropriate for studying the effectiveness of the workshops in changing the attitudes of classroom teachers toward arithmetic. It was administered to the experimental and control groups at the same time that the groups took the "Test of Mathematical Understandings."

4. The Texas Classroom Interview Question Schedule was selected as the interview instrument for eliciting responses from two pupils from each of eighteen classrooms. These pupils, interviewed in January, at or about the time of the first session, and again in April at the culmination of the in-service program, were asked to answer questions relative to the classroom practices of their teachers with respect to the teaching of arithmetic.

The analysis of covariance statistical technique was utilized in the analysis of the data relative to the effectiveness of the in-service presentation. Pearson Product-Moment Correlation Coefficient was used in the relationship analyses reported in the study.

Summary of Results

The summary of results is organized in terms of the hypothesis of the study and the questions related to these hypotheses.

Hypothesis I

The first hypothesis investigated was stated as: There will be a significant difference on a test of mathematical understandings between the post-test scores of a group of elementary school teachers who have participated in an in-service education program and their pre-test scores or the scores of a similar group who have not participated in the in-service education program.

Findings.--Testing this hypothesis, the investigator found that the mean scores for all teachers who participated in the in-service education workshop in mathematics were numerically higher; however, the gain for one of the groups, a group of primary teachers, was not statistically significant at the five per cent level. Based on these data, that part of the first hypothesis which stated that there would be a significant difference on a test of mathematical understandings between the pre- and post-test scores of a group of elementary teachers who had participated in an in-service education program was rejected.

When the second part of the first hypothesis, that part having to do with a comparison of growth in mathematical understandings between participants in the in-service workshops and a similar group who were not participants, was tested with the analysis of covariance, there was a significant difference between the adjusted post-test means. It was concluded that, when initial differences were allowed for, the in-service education workshop in mathematics had made for significant changes in the final scores of the experimental group and the second part of the first hypothesis was accepted.

Related Questions.--In connection with the first major hypothesis, two related questions were studied. These questions were:

1. What effect, if any, does a pre-test have upon the post-test scores of a group of teachers who participated in the in-service education program?
2. What is the relationship of improvement in basic mathematical understandings and change in attitude toward mathematics to teacher and background factors?

An examination of the data relative to the first question indicated that there was no significant difference between the post-test scores of a group of teachers who took both pre- and post-test and another group who took the post-test only. It was concluded that, within the limitations of this study, a pre-test did not contribute significantly to the post-test scores of a test designed to measure mathematical understandings of a group of elementary school teachers.

With regard to question two, few significant correlation coefficients were found between either pre- or post-test measures and the background factors. Semester hours credit in college mathematics was the only factor noted as significant on both occasions.

Hypothesis II

The second hypothesis investigated was stated as: There will be a significant difference on an arithmetic attitude test between the post-test scores of a group of elementary school teachers who have participated in an in-service education program and their pre-test scores or the scores of a similar group who have not participated in the in-service education program.

Findings.--When the second dependent variable, changes in attitudes of elementary school teachers toward arithmetic, was evaluated by the administration of the 1961 Revised Dutton Arithmetic Attitude Inventory to participants in two in-service workshop groups as well as a similar group of teachers who were used as a control group, it was noted that all in-service groups made numerically higher scores; these gains, however, were significant at the .05 level for Group A_1 and Group A_2 but not for Group B_1.

In the analysis of covariance, which was utilized for evaluation of this test data with respect to that part of the study having to do with the comparison of change in attitudes, there was a significant difference between the post-test scores of the experimental and control groups. The F-ratio was significant at the .05 level and, as a result, it was concluded that the in-service workshop in mathematics was effective in improving the attitude of the participants toward arithmetic.

Related Questions.--In connection with the second hypothesis, two related questions were studied. These questions were:

1. What effect, if any, does a pre-test have upon the post-test scores of a group of teachers who participated in the in-service education program?
2. What is the relationship of improvement in basic mathematical understandings and change in attitude toward mathematics to teacher and background factors?

To determine the effect of a pre-test upon the post-test scores, the significance of the difference between mean scores of Group B, a group of thirty-five participants in the in-service workshops who had been divided at random into two groups, one group given both pre- and post-test and the other given post-test only, was examined through the analysis of variance.

It was noted that there was no significant difference between the post-test scores of the two research groups which were used to evaluate the question of test-retest effect; it was concluded that, within the limitations of this study, a pre-test did not contribute significantly to the post-test scores on an arithmetic attitude test made by a group of elementary school teachers participating in an in-service education program.

To study the degree of relationship between selected background factors and attitudes toward arithmetic, product-moment correlation coefficients for all background factors, except highest degree held, were obtained for both control and experimental groups. Using a table of values of correlation coefficients for different levels of significance, it was noted that the correlations between the pre- or post-test measures and selected background factors were neither significant nor approached the level of significance.

Hypothesis III

The third hypothesis investigated was stated as: There will be a significant difference between the classroom practices of a group of elementary school teachers who have participated in an in-service education program and a similar group who have not participated in the in-service education program.

Findings. --The "Texas Elementary School Mathematics Classroom Practice Schedule" was utilized to elicit responses from a random selection of pupils from the classrooms of teachers in one of the in-service (experimental) groups and an equal number of pupils from classrooms of a similar group of teachers who had not participated in the workshop. Using a t-test for comparison of means between experimental and control groups, it was pertinent to note that, within the limitations of this study, and for the particular group used, a mathematics workshop could be helpful in changing the classroom practices of teachers who had participated in the in-service education program in mathematics for the elementary school (K-8) teacher. Teachers in the experimental group were also reported by the pupils at the end of the in-service program as using a greater amount of time in discussion, involving pupils in a greater number of non-textbook activities, and using a greater number of non-textbook materials than reportedly had been in use at the time of the pre-interview. However, only one of these quantitative dimensions, number of non-textbook activities, was noted as significant.

II. Major Conclusions

The analysis of the data obtained in this study and presented in the preceding chapters appears to warrant a number of conclusions. These conclusions seem reasonable on the basis of evidence obtained from: (1) a review of literature and (2) the findings of the present study.

A review of literature pertinent to the present study revealed numerous concerns in the area of in-service education and the teaching of elementary school mathematics.

Some of the more important of these concerns are cited below:

1. As a part of a nationwide attempt to revitalize and update the schools, changes have been and are being made in the mathematics curriculum of the elementary school.

2. A careful preparation of elementary school teachers in mathematics subject-matter has become an important aspect of an improved program of arithmetic in the elementary school.
3. Appropriate provision for the in-service education of teachers is needed.
4. Curriculum revision of professional courses at the pre-service and in-service level should be concerned with the improvement of subject matter as well as with the principles of teaching the subject matter.
5. Evaluation of important mathematical outcomes has not kept pace with instruction in America; there is a need for more systematic and comprehensive research relative to evaluating the newer outcomes of arithmetic instruction.
6. Few research studies have attempted to evaluate objectively the effectiveness of the various techniques being employed in the in-service education of teachers.

Conclusions which were an outgrowth of the findings of this study were:

1. In-service education workshop participants (experimental), with the exception of one group of primary teachers, made gains between pre- and post-test on a test of mathematical understandings which were not only numerically higher but also statistically significant; however, the gains for one of the

workshop groups, a group of primary teachers, were not noted as significant.

2. When the scores made by a group of elementary teachers who had participated in an in-service education program were compared with a group of teachers who had not participated in the in-service education program, the participants in the mathematics workshops for elementary teachers made gains on a test of mathematical understanding which were not only numerically higher but statistically significant.

3. In-service education workshop participants, with the exception of one group of primary teachers, made gains between pre- and post-test on an attitude inventory which were not only numerically higher but statistically significant.

4. When the scores made by a group of elementary teachers who had participated in an in-service education program were compared with a group of teachers who had not participated in the program, the participants in the mathematics workshop for elementary teachers made gains on an arithmetic attitude inventory which were not only numerically higher but statistically significant.

5. Within the limitations of this study, and for the particular group used, a pre-test did not contribute significantly to the workshop participant's capacity to develop mathematical understandings.

6. Significant relationship was found to exist between the pre- and post-test scores of a test of mathematical understandings and the background factor, semester hours credit in college mathematics.

7. Within the limitations of this study, and for the particular group utilized, a pre-test did not contribute significantly to the participant's attitude toward arithmetic.

8. No significant relationship was found to exist between either the pre-test or post-test scores of an attitude inventory and selected teacher background factors.

9. Teachers participating in an in-service education program were reported by their pupils as making more significant changes in their classroom practices than did a similar group of teachers who had not participated in the in-service education program.

III. Implications and Recommendations for Curriculum and Research

Numerous sources in the field of education have pointed to the need for the critical, objective analysis on the effectiveness of various kinds of in-service education programs. The present investigation represented an attempt to analyze the effectiveness of the workshop as one means of in-service teacher education for developing needed mathematical understandings of elementary school teachers, for changing the attitude of elementary school teachers toward mathematics, and for improving classroom procedures of teachers with respect to the teaching of arithmetic. It has affirmed that such an in-service program is both feasible and practical.

By the design of the present experiment, the data were employed simply to indicate that, through a comparison of differences in mean scores for individuals or differences in scores between experimental and control groups, changes identified could be attributed to the workshop. The establishment of such a causal relationship, perhaps, is not as meaningful to educators as the explanations for these changes. Therefore, further research related to and growing out of this study should serve to facilitate in-service education programs and their effect upon people. Several suggestions for research seem warranted. They have been organized in two categories: (1) implication for in-service education and (2) implication for research.

Implication for In-service Education

Methods of in-service education.--There is a great need for the evaluation of the relative effectiveness of other forms of in-service education. Certainly the method or methods most effective and most expedient should be used. Therefore, studies which evaluate the effectiveness of the various forms of in-service education--child study programs, orientation sessions, teacher inter-visitation, and the use of printed materials, films, or videotapes --would be valuable.

Organization of in-service education. --Within the scope of the present study, it was not possible to manipulate or observe the effect of an in-service program which utilized various leadership roles. What would be the relative effectiveness of in-service education when the program is principal directed, teacher organized, or planned exclusively by the central office? Studies which attempted to analyze the effectiveness of one or all of these organizational patterns would be helpful.

Findings of the present study tended to indicate that an in-service education program which was concentrated into a short period of time could make for significant changes in improvement in mathematical understandings, change in attitude toward arithmetic and change in classroom procedure related to teaching of arithmetic. Studies are needed which would compare an in-service education program which is concentrated into a short period of time with one extended over a longer period of time.

Since the data used in the present study were obtained from one series of workshops, similar studies using other populations are needed. Is it possible that a higher degree of positive change would result from an in-service education program in which teachers less qualified participated? Would a higher degree of positive change result if college credit were given?

Evaluation instrument.--Since the examination of arithmetic tests revealed that no test commonly used at the present time contained more than a few items designed to measure mathematical understandings, further study and refinement of the instrument developed by this investigator for evaluating mathematical understandings seems feasible.

Further refinement and evaluation of the Interview Question Schedule used in the present study would also meet a growing need.

What anxieties are produced as a result of a testing program and to what extent are the outcomes of an in-service education program restricted by the nature of the test instruments? Even though most educators agree that paper-and-pencil tests contribute much useful data to the total evaluation program, studies are needed which would utilize the individual interview or conference, a procedure which would provide the investigator with the opportunity to: (1) observe what the individual does and (2) question the individual.

Implications for Research

In order to provide further support for the hypothesis concerning change in classroom procedure with respect to teaching of arithmetic, future research might elicit responses from not only the pupils of teachers of experimental and control groups but also, through some type of survey questionnaire or information form, obtain data from the teachers themselves. Thus it would be possible to note an agreement or lack of agreement between the data that the sample respondents (teachers) reported about themselves and that reported by their students.

Retention. --To what extent do teachers retain concepts, attitudes, and teaching procedures developed as a result of in-service teacher education programs? Do these gains continue to grow or decline in subsequent months and years?

Since one of the objectives of the state-sponsored workshops was related to building a foundation for further self-improvement, studies are needed which would attempt to determine if the understandings derived are more effective in subsequent years than during the actual time of presentation.

It was pointed out that one of the basic premises of in-service education is that professional growth of teachers will bring about increased achievement or learning in pupils; studies are needed which attempt to determine how the teachers' mathematical understanding before and after the in-service program is related to changes in pupil achievement.

In conclusion, it seems that the present study has pointed to the need for an objective and more critical evaluation of in-service education if in-service education is to contribute to the present-day demand for teachers who are trained in an up-to-date approach in mathematics and who are familiar with the many new tools to be used in teaching.

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APPENDIX A

TEACHER BACKGROUND INFORMATION

Name ______________________________ Sex __________

Address ______________________ Home Phone ______________

Teaching Position __________________ Location __________________

Years of Teaching Experience ______________

Check one:

M.A. degree or more	_____	No. of high school credits in mathematics	_____
B.S. degree or more but less than M.A.	_____	No. of semester hours of college credit in college mathematics	_____
B.S. or B.A. degree	_____	No. of semester hours of college credit in mathematics teaching methods	_____
Less than B.S. but more than 90 hours	_____		
Less than 90 hours	_____		

What bothers you most in teaching arithmetic?

____

____

____

____

APPENDIX B

A TEST OF MATHEMATICAL UNDERSTANDINGS*
Form A

Prepared by
Mildred Jerline Dossett
Southeast Missouri State College
Cape Girardeau, Missouri

With the assistance of: Calhoun C. Collier, James M. Drickey, Vincent J. Glennon, W. Robert Houston, Lois Knowles, and Joe L. Wise

Directions: This test is designed to measure your understanding of mathematics. Many of the items relate to the new content in present programs of mathematics for elementary school pupils.

Each of the fifty-five questions is of the multiple-choice type and includes four possible answers. Read each question carefully and decide which answer best fulfills the requirements of the statement. Then mark the space on the answer sheet to indicate your choice.

Mark only one answer for each question. If you change your choice, erase your original mark and mark the correct one.

<u>Sample</u> <u>Question</u>:

Which of the following shows the decimal form of the fraction $\frac{5}{4}$?

a. 125 c. 1.25
b. 12.5 d. .125

<u>Answer</u> <u>Sheet</u>:

a ( ) b ( ) c (▮) d ( )

Since (c) is correct, the space under c is blackened.

*Those items with an asterisk (*) are items from Glennon's "Test of Basic Mathematical Understandings." Permission was granted by the author to use any or all of the items on the "Test of Basic Mathematical Understandings."

1. When you write the numeral "5" you are writing
   a. the number 5.
   b. a pictorial expression.
   c. a symbol that stands for an idea.
   d. a Hindu-Babylonian symbol.

2. Bill discovered that $>$ means "is greater than" and $<$ means "is less than." In which of the following are these symbols not used correctly?
   a. The number of states in the United States $<$ the number of United States Senators.
   b. The number of states in the United States $>$ the number of stripes in the flag.
   c. $2^3 > 3^2$
   d. $3 + a < 5 + a$

3. When two Roman numerals stand side by side in a symbol, their values are added
   a. always.
   b. sometimes.
   c. never.
   d. if the base is X.

4. Which of the following describe/describes our own system of numeration?
   a. additive
   b. positional
   c. subtractive
   d. introduces new digits for numbers larger than 10
      1) a and b are correct.
      2) a and c are correct.
      3) a and d are correct.
      4) a, b, and d are correct.

5. Zero may be used
   a. as a place holder.
   b. as a point of origin.
   c. to represent the absence of quantity.
   d. in all of the above different ways.

6. 2,200.02 is shown by
   a. 2000 + 200 + 20.
   b. 2000 + 20 + 2/10.
   c. 2000 + 200 + 2/100.
   d. 2000 + 200 + 200.

7. 5840 rearranged so that the 8 is 200 times the size of the 4 would be
   a. 5840.
   b. 8540.
   c. 5048.
   d. 5403.

8. Which of the following does not show the meaning of 423_{ten}?
   a. (4 x 100) + (2 x 10) + 3(1) = 423
   b. 42 tens + 3 ones = 423
   c. 423 ones = 423
   d. 4 hundreds + 42 tens + 23 ones = 423

9. A numeral for the X's in this example can be written in many different bases. Which numerals are correct?

XX	X	XX	XX
X	X	X	X
X	XX	X	X

   a. 100_{four}
   b. 14_{twelve}
   c. 16_{ten}
   d. 31_{five}
      1) a and c are correct.
      2) b and c are correct.
      3) a, b, and c are correct.
      4) all four are correct.

10. A "2" in the third place of a base twelve number would represent
   a. 2×12^3
   b. 12×2^3
   c. 12×2^{12}
   d. 2×12^2

11. In this addition example, in what base are the numerals written?
   a. base two
   b. base three
   c. base four
   d. none of the above

$$\begin{array}{r} 120_? \\ +\ 10_? \\ \hline 200_? \end{array}$$

12. About how many tens are there in 6542?
   * a. 6540
   b. 654
   c. $65\frac{1}{2}$
   d. 6.5

13. Place or order in a series is shown by
   a. book no. 7.
   b. three boxes of matches.
   c. a dozen cupcakes.
   d. two months.

14. Which of the following indicates a group?
    a. 45 tickets
    b. track 45
    c. page 54
    d. apartment No. 7

15. The sum of any two natural numbers
    a. is not a natural number.
    b. is sometimes a natural number.
    c. is always a natural number.
    d. is a natural number equal to one of the numbers being added.

16. The counting numbers are closed under the operations of
    a. addition and subtraction.
    b. addition and multiplication.
    c. addition, subtraction, multiplication, and division.
    d. addition, subtraction, and multiplication.

17. If a and b are natural numbers, then $a + b = b + a$ is an example of
    a. commutative property.
    b. associative property.
    c. distributive property.
    d. closure.

18. If $a \times b = 0$ then
    a. a must be zero.
    b. b must be zero.
    c. either a or b must be zero.
    d. neither a nor b must be zero.

19. When a natural number is multiplied by a natural number other than 1, how does the answer compare with the natural number multiplied?
    a. larger
    b. smaller
    c. the same
    d. can't tell from information given

20. Which of the following is the quickest way to find the sum of
* several numbers of the same size?
    a. counting
    b. adding
    c. subtracting
    d. multiplication

21. How would the product in this example be affected if you put the
* 29 above the 4306 and multiplied the two numbers?

$$\begin{array}{r} 4306 \\ \times 29 \\ \hline \end{array}$$

a. The answer would be larger.
b. The answer would be smaller.
c. You cannot tell until you multiply both ways.
d. The answer would be the same.

22. An important mathematical principle can be helpful in solving the following example.

$$28 + 659 + 72 = \square$$

What principle will be of most help?
a. the associative principle
b. the commutative principle
c. the distributive principle
d. both the associative and distributive principles

23. The product of 356 x 7 is equal to
a. (300 x 50) x (6 + 7).
b. (3 x 7) + (5 x 7) + (6 x 7).
c. 300 x 50 x 6 x 7.
d. (300 x 7) + (50 x 7) + (6 x 7).

24. Which of the following is <u>not</u> a prime number?
a. 271
b. 277
c. 281
d. 282

25. Which of the following numbers is odd?
a. 18 x 11
b. 11 x 20
c. 99 x 77
d. none of the above

26. The inverse operation generally used to check multiplication is
a. addition.
b. subtraction.
c. multiplication.
d. division.

27. The greatest common factor of 48 and 60 is
a. 2 x 3.
b. 2 x 2 x 3.
c. 2 x 2 x 2 x 2 x 3 x 5.
d. none of the above.

28. Look at the example at the right. Why is the "4" in the third partial product moved over two places and written under the 2 of the multiplier?

$$\begin{array}{r} 157 \\ \times\ 246 \\ \hline 942 \\ 628 \\ 314 \\ \hline 38622 \end{array}$$

a. If you put it directly under the other partial products, the answer would be wrong.
b. You must move the third partial product two places to the left because there are three numbers in the multiplier.
c. The number 2 is in the hundreds column, so the third partial product must come under the hundreds column.
d. You are really multiplying by 200.

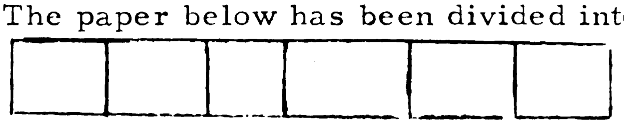
29. Which of the fundamental properties of arithmetic would you employ in proving that $(a + b) + (a + c) = 2a + b + c$?
a. Associative property
b. Commutative property
c. Associative and distributive properties
d. Associative and commutative properties

30. If N represents an even number, the next larger even number can be represented by
a. $N + 1$.
b. $N + 2$.
c. $N + N$.
d. $2 \times N + 1$.

31. Every natural number has at least the following factors:
a. zero and one.
b. zero and itself.
c. one and itself.
d. itself and two.

32. It is said that the set of whole numbers has a natural-order. To find the successor of a natural number, one must
a. add 1.
b. find a number that is greater.
c. square the natural number.
d. subtract 1 from the natural number.

33. The paper below has been divided into 6 pieces. It shows



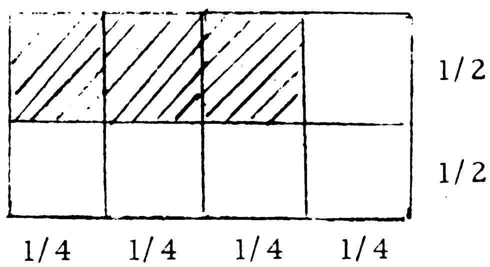
a. sixths.
b. thirds.
c. halves.
d. parts.

34. A fraction may be interpreted as:
   a. a quotient of two natural numbers.
   b. equal part/parts of a whole.
   c. a comparison between two numbers.
   d. all of the above.

35. When a common (proper) fraction is divided by a common fraction, how does the answer compare with the fraction divided?
   a. It will be larger.
   b. It will be smaller.
   c. It will be twice as large.
   d. There will be no difference.

36. Which algorithm is illustrated by the following sketch?

   a. $\frac{1}{2} \times \frac{3}{4} = ?$
   b. $\frac{1}{2} + \frac{3}{4} = ?$
   c. $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = ?$
   d. $\frac{4}{4} - \frac{3}{2} = ?$



37. Another name for the inverse for multiplication of a rational number is the
   a. reciprocal.
   b. opposite.
   c. reverse.
   d. zero.

38. Examine the division example on the right. Which sentense best tells why the answer is larger than the 5?

$$5 \div \frac{3}{4} = 6\frac{2}{3}$$

   a. Inverting the divisor turned the $\frac{3}{4}$ upside down.
   b. Multiplying always makes the answer larger.
   c. The divisor $\frac{3}{4}$ is less than 1.
   d. Dividing by proper and improper fractions makes the answer larger than the number divided.

39. The value of a common fraction will not be changed if
   a. we add the same number to both terms.
   b. we multiply one term and divide the other term by that same number.
   c. we subtract the same amount from both terms.
   d. we multiply both terms by the same number.

40. The nearest to 45% is
  a. 44 out of 100.
  b. .435.
  c. 4.5.
  d. .405.

41. The principal of a school said that 27 per cent of the pupils went to the museum. Which statement best describes the expression "27 per cent of the pupils went to the museum?"
  a. It means that 27 children out of every 100 children went to the museum.
  b. It means that you must multiply the number of children in the school by 27/100 to find the number who went to the museum.
  c. If the children were divided into groups of 100 and those who went to the museum were distributed evenly among them, there would be in each group 27 who went to the museum.
  d. 27 per cent is the same as .27 -- a decimal fraction written in per cent form.

42. Sally completed 2/3 of the story in 12 minutes. At that rate how long will it take her to read the entire story?
  a. 18 minutes
  b. 12 minutes
  c. 6 minutes
  d. 24 minutes

43. There were 400 students in the school. One hundred per cent of the children had lunch in the cafeteria on the first day of school. On the second day 2 boys were absent and 88 children went home for lunch. Which of the following equations can be used to find the per cent of the school enrollment who went home for lunch?
  a. $400 - 88 = X$
  b. $\frac{x}{100} = \frac{88}{400}$
  c. $\frac{x}{88} = 400$
  d. $400 - 90 = X$

44. What can be said about y in the following open sentence if x is a natural number?

$$x + x + 1 = y$$

  a. $x < y$
  b. $x > y$
  c. $x = y$
  d. $x \neq y$

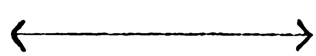
45. Which one of the following fractions will give a repeating decimal?
   a. 1/2
   b. 3/4
   c. 5/8
   d. 6/11

46. Which of the following is <u>not</u> an open sentence ?
   a. 7 + 2 = ☐
   b. h - 5 = 9
   c. c/1 - 30 = 6
   d. n - 3

47. For a mathematical system consisting of the set of odd numbers and the operation of multiplication,
   a. the system is closed.
   b. the system is commutative.
   c. the system has an identity element.
   d. all of the above are correct.

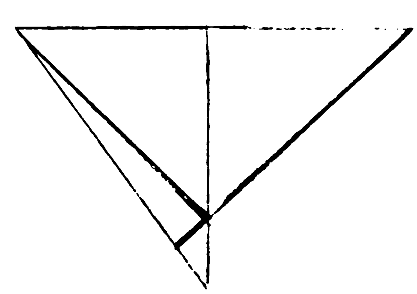
48. Measurement is a process which
   a. compares an object with some known standard or accepted unit.
   b. tries to find the exact amount.
   c. is never an exact measure.
   d. chooses a unit and then gives a number which tells how many of that unit it would take.
      1) a and b are correct.
      2) a and c are correct.
      3) a, b, and d are correct.
      4) a, c, and d are correct.

49. The set of points sketched below represents a



   a. line.
   b. ray.
   c. line segment.
   d. none of the above.

50. How many triangles does the figure contain?



   a. 4
   b. 6
   c. 8
   d. 10

51. The set of points on two rays with a common end-point is called
    a. a triangle.
    b. an angle.
    c. a vertex.
    d. a side of a triangle.

52. If a circle is drawn with the points of the compass 3 inches apart, what would be 3 inches in length?
    a. circumference
    b. diameter
    c. area
    d. radius

53. The solution set of an open sentence may consist of
    a. two or more numbers.
    b. no numbers.
    c. only one number.
    d. any or all of these.

54. Consider a set of three objects. How many sub-sets or groups can be arranged?
    a. nine
    b. eight
    c. seven
    d. six

55. If two sets are said to be equivalent, then
    a. every element in the first set can be paired with one and only one element in the second set.
    b. every element in one set must also be an element in the second set.
    c. they are intersecting sets.
    d. one must be the null set.

APPENDIX C

A TEST OF MATHEMATICAL UNDERSTANDINGS *
Form B

Prepared by
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With the assistance of: Calhoun C. Collier, James M. Drickey, Vincent J. Glennon, W. Robert Houston, Lois Knowles, and Joe L. Wise

Directions: This test is designed to measure your understanding of mathematics. Many of the items relate to the new content in present programs of mathematics for elementary school pupils.

Each of the fifty-five questions is of the multiple-choice type and includes four possible answers. Read each question carefully and decide which answer best fulfills the requirements of the statement. Then mark the space on the answer sheet to indicate your choice.

Mark only one answer for each question. If you change your choice, erase your original mark and mark the correct one.

Sample Question:

Which of the following shows the decimal form of the fraction $\frac{5}{4}$?

a. 125 c. 1.25
b. 12.5 d. .125

Answer Sheet:

a ( ) b ( ) c (❚) d ( )

Since (c) is correct, the space under c is blackened.

*Those items with an asterisk (*) are items from Glennon's "Test of Basic Mathematical Understandings." Permission was granted by the author to use any or all of the items on the "Test of Basic Mathematical Understandings."

1. Which of the underlined words or signs in the following sentences refer to symbols rather than the things they represent?
   a. 4 can be written on the blackboard.
   b. Regardless of what symbol we use, we are thinking about the number 2.
   c. A pencil is used for writing.
   d. The number 16 is the same as the number 7 + 9.

2. When we use the = symbol between two terms (as 2 + 2 = 4) we mean that both terms represent the same concept or idea. Which of the following is not correctly stated?
   a. 3 + 4 = 5 + 2
   b. 5 + 2 = 7 and 7 = 5 + 2
   c. (5 + 2) x 3 = 7 x 3
   d. 7 = 7
      1) a and b are correct.
      2) a and c are correct.
      3) a, b, and c are correct.
      4) a, b, c, and d are correct.

3. If the Roman system of numeration were a "place value system" with the same value for the base as the Hindu-Arabic system, the first four base symbols would be
   a. I, X, C, and M.
   b. I, V, X, and L.
   c. X, L, C, and M.
   d. X, C, L, and D.

4. Which of the following does not describe a characteristic of our decimal system of numeration?
   a. It uses zero to keep position when there is an absence of value.
   b. It makes ten a standard group for the organization of all numbers larger than nine.
   c. It makes 12 the basis for organizing numbers larger than eleven.
   d. It uses the additive concept in representing a number of several digits.

5. * In the numeral 7,843, how does the value of the 4 compare with the value of the 8?
   a. 2 times as great
   b. 1/2 as great
   c. 1/10 as great
   d. 1/20 as great

6. In the numeral 6,666 the value of the 6 on the extreme left as compared with the 6 on the extreme right is
   a. 6,000 times as great.
   b. 1,000 times as great.
   c. the same since both are sixes.
   d. six times as much.

7. Which of the following statements best tells why we write a zero in
* the numeral 4,039 when we want it to represent "four thousand thirty-nine?"
   a. Writing the zero helps to fill a place which would otherwise be empty and lead to misunderstanding.
   b. The numeral would represent "four hundred thirty-nine" if we did not write the zero.
   c. Writing the zero tells us not to read the hundreds' figure.
   d. Zero is used as a place-holder to show that there is no number to record in that place.
      1) a and b are correct.
      2) a and c are correct.
      3) a and d are correct.
      4) a, b, and d are correct.

8. Below are four numerals written in expanded notation. Which one is <u>not</u> written correctly?
   a. $4(\text{ten})^2 + 9(\text{ten})^1 + 3(\text{ones}) = 493_{\text{ten}}$
   b. $3(\text{seven})^3 + 6(\text{seven})^1 + 1(\text{one}) = 363_{\text{seven}}$
   c. $4(\text{twelve})^2 + 5(\text{twelve})^1 + e(\text{one}) = 45e_{\text{twelve}}$
   d. $2(\text{five})^2 + 2(\text{five})^1 + 4(\text{one}) = 224_{\text{five}}$

9. If you are permitted to use any or all of the symbols 0, 1, 2, 3, 4 and 5 for developing a system of numeration with a place value system of numeration similar to ours, a list of all possible bases would include:
   a. base one, two, three, four, five, and six.
   b. base two, three, four, five, and six.
   c. base two, three, four, and five.
   d. base one, two, three, four, and five.

10. About how many hundreds are there in 34,870?
* a. $3\frac{1}{2}$
   b. 35
   c. 350
   d. 3,500

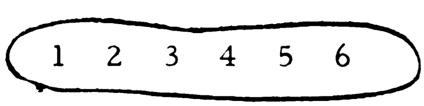
11. In what base are the numerals in this multiplication example written?
    a. base five
    b. base eight
    c. base eleven
    d. you can't tell

$$\begin{array}{r} 34_? \\ 23_? \\ \hline 124_? \\ 70_? \\ \hline 1024_? \end{array}$$

12. Which of the following are correct?
    a. In the symbol 5^3, 5 is the base and 3 is the exponent.
    b. In the symbol 5^3, 3 is the base and 5 is the exponent.
    c. $5^3 = 5 \times 5 \times 5$
    d. $5^3 = 3 \times 3 \times 3 \times 3 \times 3$
        1) a and d are correct.
        2) b and c are correct.
        3) a and c are correct.
        4) b and d are correct.

13. In the series of numerals 1, . . . 17, 18, 19, 20, 21, . . . , what term best applies to 19?
    a. nominal
    b. ordinal
    c. composite
    d. cardinal

14. Examine the following illustration:



Which of the following does the above best illustrate?
    a. The idea of a cardinal number
    b. The use of an ordinal number
    c. A means for determining the cardinal number of the set by counting with ordinal numbers
    d. None of the above

15. The quotient of any two whole numbers
    a. is not a natural number.
    b. is sometimes a natural number.
    c. is always a natural number.
    d. is a natural number less than one of the numbers being divided.

16. The integers are closed under the operations of
    a. addition.
    b. subtraction.
    c. multiplication.
    d. division.
        1) a and b are correct.
        2) a and c are correct.
        3) a, b, and c are correct.
        4) a, b, c, and d are correct.

17. A student solved this example by adding down; then he checked his work by adding up.

Add ↓		Check ↑	
	34		34
	52		52
	86		86

    It could be classified as an example of
    a. the distributive principle.
    b. the associative principle.
    c. the commutative principle.
    d. the law of compensation.

18. The statement "the quotient obtained when zero is divided by a number is zero" is expressed as
    a. $\frac{a}{0} = 0$
    b. $\frac{0}{a} = 0$
    c. $\frac{0}{0} = a$
    d. $\frac{a}{a} = 0$

19. When a natural number is divided by a natural number other than 1,
* how does the answer compare with the natural number divided?
    a. larger
    b. smaller
    c. one-half as large
    d. can't tell from information given

20. If you had a bag of 350 marbles to be shared equally by 5 boys, which would be the quickest way to determine each boy's share?
    a. counting
    b. adding
    c. subtracting
    d. dividing

21. If the multiplier is x, the largest possible number to carry is
   a. x.
   b. x + 1.
   c. 0.
   d. x - 1.

22. Which one of the following methods could be used to find the answer to this example?

$$17 \overline{)612}$$

   a. Multiply 17 by the quotient.
   b. Add 17 six hundred times.
   c. The answer would be the sum.
   d. Subtract 17 from 612 as many times as possible. The answer would be the number of times you were able to subtract.

23. * Which one of the following would give the correct answer to this example?

$$\begin{array}{r} 2.1 \\ \underline{\times\ 21} \end{array}$$

   a. The sum of 1 x 2.1 and 21 x 2.1.
   b. The sum of 10 x 2.1 and 2 x 2.1.
   c. The sum of 1 x 2.1 and 20 x 2.1.
   d. The sum of 1 x 2.1 and 2 x 2.1.

24. * Which would give the correct answer to 439 x 563?
   a. Multiply 439 x 3, 439 x 60, 439 x 5 and then add the answer.
   b. Multiply 563 x 9, 563 x 3, 563 x 4 and then add the answer.
   c. Multiply 563 x 9, 563 x 39, 563 x 439 and then add the answer.
   d. Multiply 439 x 3, 439 x 60, 439 x 500 and then add the answer.

25. Which of these numerals are names for prime numbers?
   a. 3
   b. 4/2
   c. 12_{five}
   d. 9 - 2
      1) a is correct.
      2) a and c are correct.
      3) a, b, and d are correct.
      4) a, b, c, and d are correct.

26. Let x represent an odd number; let y represent an even number. Then x + y must represent
   a. an even number.
   b. a prime number.
   c. an odd number.
   d. a composite number.

27. The inverse operation for addition is
    a. addition.
    b. subtraction.
    c. multiplication.
    d. division.

28. The least common multiple of 8, 12, and 20 is
    a. 2 x 2.
    b. 2 x 3 x 5.
    c. 2 x 2 x 2 x 3 x 5.
    d. 2 x 2 x 2 x 2 x 2 x 2 x 2 x 3 x 5.

29. Which statement best tells why we carry 2 from the second
* column?

$$\begin{array}{r} 251 \\ 161 \\ 252 \\ \underline{271} \\ 935 \end{array}$$

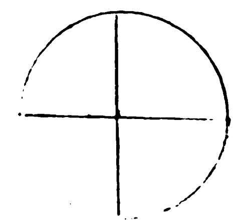
    a. If we do not carry the 2, the answer would be 20 less than the correct answer.
    b. Since the sum of the second column is more than 20, we put the 2 in the next column.
    c. Since the sum of the second column is 23 (which has two figures in it), we have room for the 3 only, so we put 2 in the next column.
    d. Since the value represented by the figures in the second column is more than 9 tens, we must put the hundreds in the next column.

30. The operations which are associative are
    a. addition.
    b. subtraction.
    c. multiplication.
    d. division.
        1) a and b are correct.
        2) a and c are correct.
        3) a, b, and c are correct.
        4) a and d are correct.

31. Which of the following is an even number?
    a. $(100)_{three}$
    b. $(100)_{five}$
    c. $(100)_{seven}$
    d. $(200)_{five}$

32. The fact that a + (b + c) is exactly equal to (c + b) + a is an example of
   a. distributivity.
   b. commutativity.
   c. closure.
   d. associativity.

33. Observe the drawing on the right. When the circle
* is cut into equal pieces, the size of each piece
   a. decreases as the number of pieces increases.
   b. increases as the number of pieces decreases.
   c. increases as the number of pieces increases.
   d. decreases as the number of pieces decreases.
      1) a and b are correct.
      2) a and c are correct.
      3) b and c are correct.
      4) b and d are correct.



34. The symbol 3/4 may be used to represent the idea that
   a. 3 is to be divided by 4.
   b. 3 of the 4 equal parts are being considered.
   c. 3 objects are to be compared with 4 objects.
   d. all of the above.

35. When a whole number is multiplied by a common (proper) fraction other than one, how does the answer compare with the whole number?
   a. It will be larger.
   b. It will be smaller.
   c. There will be no difference.
   d. You are not able to tell.

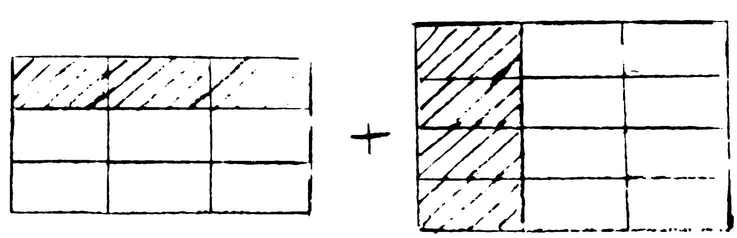
36. Which of the addition examples is best represented by the shaded parts of the diagram below?

   a. $\frac{1}{2} + \frac{1}{3}$

   b. $\frac{2}{3} + \frac{3}{4}$

   c. $\frac{2}{3} + \frac{1}{4}$

   d. $\frac{1}{3} + \frac{1}{3}$



37. We can change the denominator of the fraction $\frac{\frac{2}{3}}{\frac{4}{5}}$ to the number "1" without changing the values of the fraction by
   a. adding 5/4 to the numerator and denominator.
   b. subtracting 5/4 from the numerator and the denominator.
   c. multiplying both the numerator and the denominator by 5/4.
   d. dividing the numerator and the denominator by 5/4.

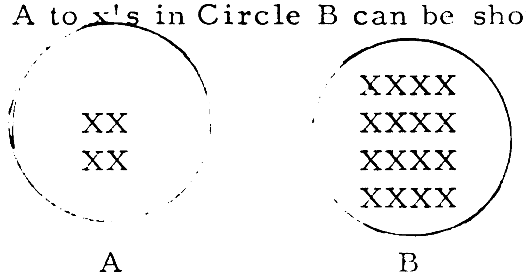
38. What statement best tells why we "invert the divisor and multiply" when dividing a fraction by a fraction?
   a. It is an easy method of finding a common denominator and arranging the numerators in multiplication form.
   b. It is an easy method for dividing the denominators and multiplying the numerators of the two fractions.
   c. It is a quick, easy, and accurate method of arranging two fractions in multiplication form.
   d. Dividing by a fraction is the same as multiplying by the reciprocal of the fraction.

39. If the denominator of the fraction 2/3 is multiplied by 2, the value of the resulting fraction will be
   a. half as large.
   b. double in value.
   c. unchanged in value.
   d. a new symbol for the same number.

40. 45% may also be written as
   a. .45
   b. 45/100
   c. 45 x 100%
   d. .450
      1) a and b are correct.
      2) a and c are correct.
      3) a and d are correct.
      4) a, b, and d are correct.

41. .5 and .27 are illustrations of "decimal fractions." They could be written as "common fractions" in the form of 1/2 and 27/100 respectively. What is a decimal fraction?
   a. It is another way of writing percentage.
   b. It is an extension of the decimal number system to the right of one's place.
   c. A number like $.37\frac{1}{2}$ which has both a decimal and a fraction as parts of it.
   d. A number like $\frac{.2}{.56}$ which is a fraction and has a decimal as either the numerator or denominator or both.

42. The ratio of x's in Circle A to x's in Circle B can be shown by
   a. 16/4.
   b. 1/4.
   c. 1/2.
   d. 4/16.



43. Sue paid 20¢ for 4 apples. Which of the equations below could be used to find the cost of 1 apple?
   a. $\frac{4}{20} = \frac{1}{x}$
   b. $x + 4 = 20$
   c. $\frac{x}{4} = 20$
   d. $x - 4 = 20$

44. The decimal for the numeral 6/17 will
   a. be a repeating decimal.
   b. not repeat or end since 17 is prime.
   c. repeat in cycles of less than 23 digits.
      1) a is correct.
      2) a and b are correct.
      3) a and c are correct.
      4) a, b, and c are correct.

45. Which of the following statements is <u>not</u> correct?
   a. $(-9) + 6 = -3$
   b. $(-5) + (-5) = -10$
   c. $-8 + 0 = -8$
   d. $(-8) + (9) = -1$

46. Which of the following is a list of all of the factors of 12?
   a. 1, 2, 3, 4, 8 & 12
   b. 1, 2, 3, 4, 6 & 12
   c. 1, 2, 3, 4 & 6
   d. 2, 3, 4, 6 & 12

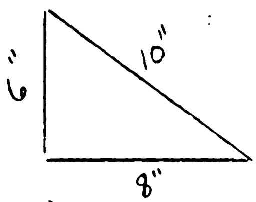
47. Modular arithmetic is
   a. commutative.
   b. associative.
   c. distributive with respect to multiplication over addition.
   d. all of the above.

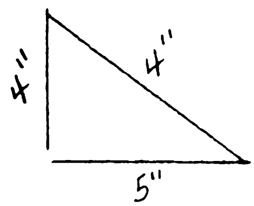
48. Which of the following is an approximate measure?
 a. 35 farms
 b. 12 buttons
 c. 7 1/2 inches
 d. 15 beads

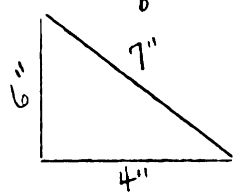
49. Which of the following does the sketch below represent?

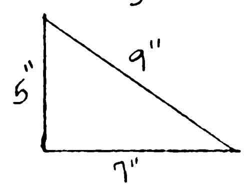
 a. line
 b. ray
 c. line segment
 d. set of points
  1) a is correct.
  2) a, b, and d are correct.
  3) a, c, and d are correct.
  4) b and d are correct.

50. Which of these triangles are right triangles according to the length of the sides given?

a.


b.


c.


d.


51. A distinct point is
 a. a point you can see.
 b. a sharp object.
 c. the intersection of two lines.
 d. a dot.

52. A clerk sold a lady a round tablecloth that had a radius of 14 inches. Which of the formulas can she use to determine the length around the cloth?

 a. $A = \pi r^2$
 b. $C = \pi d$
 c. $C = 2\pi r$
 d. $A = C/d$

53. Which of the following best defines a solution set?
   a. A solution set is a set which includes each and every member that gives a true statement.
   b. A solution set is a single sentence which identifies a variable that will give a true statement.
   c. A solution set is a set containing all the positive integers, zero, and the negative integers.
   d. A solution set is a set containing rational numbers.

54. Examine the following illustration.

   $S = \{0, 1, (-1), 2, (-2), 3, \ldots 10\}$

   Which one of the following is not a subset of S?
   a. $\{+9, +10\}$
   b. $\{0, (-2), 5\}$
   c. $\{3, (-3), 10\}$
   d. $\{1, (-1), 6, 10\}$

55. If we use the set concept to define the operations for the counting numbers, addition would be defined in terms of
   a. the intersection of disjoint sets.
   b. the union of intersecting sets.
   c. the intersection of sets with common elements.
   d. the union of disjoint sets.

APPENDIX D

DUTTON ARITHMETIC ATTITUDE INVENTORY

Directions: (Check (x) only the statements which express your feeling toward arithmetic.

	Agree	Disagree
1. I avoid arithmetic because I am not very good with figures.	( )	( )
2. Arithmetic is very interesting.	( )	( )
3. I am afraid of doing word problems.	( )	( )
4. I have always been afraid of arithmetic.	( )	( )
5. Working with numbers is fun.	( )	( )
6. I would rather do anything else than do arithmetic.	( )	( )
7. I like arithmetic because it is practical.	( )	( )
8. I have never liked arithmetic.	( )	( )
9. I don't feel sure of myself in arithmetic.	( )	( )
10. Sometimes I enjoy the challenge presented by an arithmetic problem.	( )	( )
11. I am completely indifferent to arithmetic.	( )	( )
12. I think about arithmetic problems outside of school and like to work them out.	( )	( )
13. Arithmetic thrills me and I like it better than any other subject.	( )	( )
14. I like arithmetic but I like other subjects just as well.	( )	( )
15. I never get tired of working with numbers.	( )	( )

APPENDIX E

TEXAS ELEMENTARY SCHOOL MATHEMATICS CLASSROOM PRACTICES SCHEDULE OF QUESTIONS

* Tell me about your arithmetic class.
* Where do your arithmetic lessons come from?

TEXT

How many arithmetic textbooks do you use?

1. How do you work through your arithmetic textbook?
   EPg : SkPg : Skpt : SkMst : SkAll
2. Are all of you working in the same place in your textbook?
   AllSm : : 2-3 Grps : : AllDf

WORKBOOK

*Why do you think you use your arithmetic workbook?

How many different kinds of arithmetic workbooks are used in your class?

1. How do you work through your arithmetic workbook?
   : RegAll : IrgSmtAll : TerNd : PplNd
2. How often do you use it?
   : Daily : Weekly : TerNd : PplNd
3. Are all of you working in the same workbook?
   : All : DfntGrps : IndfntGrps : Indiv
4. Are all of you working in the same place?
   : All : DfntGrps : IndfntGrps : Indiv

TRADE

What other kinds of books about arithmetic have you used?
How many?

1. Who chooses these books for you to use?
   : TerTxtSgstd : Ter : Ter-Chld : Chld
2. How have you used them?
   : ReRpts : Terplnsgrp : RptTer-Ppl : IndivInt
   : : IndvrptTer : :

APPENDIX E -- Continued

3. How often do you use these books?
   : Over Month : Month : Week : AlmstDly

FREE-TIME ACTIVITY

1. What do you do when you finish your assignments in class?
   : NxtAsgnmt : TerChs : ArithChs : FrChs
2. What kinds of special activities or projects have you done in arithmetic class?
   Text : TerElabText : TeachSgstd : Ter-ChlfDrv : Chld

HOMEWORK

1. What kind of homework do you have?
   Txt : WkBk : DittoCkBd : AsgnExpPrb : ChldExp Prb

FILMS

*Why do you see films or movies about arithmetic?
How many have you seen this year?

1. What did you do after the film was over?
   : Tst : TerSmy : RptDscs : IndvExplr

INTEGRATION

1. In what other subjects do you most often use arithmetic?
   None : Isubj : 2Subj : 3Subj : AllSubj
2. Do you also have a regular arithmetic lesson that day?
   : Yes : Usually : Not Often : Never

TEACHING AIDS

*What teaching aids and supplies do you think are important in the classroom for learning arithmetic?
Which of these have you used in your classroom?

1. Tell me how you used the ___________ to learn a new idea?
   : Skills : Abst : Expl : Appl Prep

APPENDIX E -- Continued

✓ 2. Where did these aids come from?

: Comm : Teacher : Teach-Child : Child

3. How does your class use arithmetic charts and graphs (posters and bulletin boards)?

: Alkls-Abst : Prep : Appl : Expl

: Alks-Abst : Prep : Appl : Expl

4. Where do they come from?

: Comm : Teach : Teach-Child : Child

ORGANIZATION

1. Do you spend the same amount of time studying arithmetic every day?

Yes : FwExcep : SmExcep : FreqExcep : Nvrsm

2. Do all of you study the same lesson every day?

No : 2 Grps : 3 Grps : 4 Grps : Indiv

3. Do some pupils have longer or harder assignments in arithmetic?

No : Length : : Level : Lgth&Lvl

4. How do pupils get individual help from the teacher?

Little : Terrexpcls : GrpHp : TerChsIndv : Ppl-Indiv
or None : : : :

DISCUSSION

How much of your arithmetic is spent talking about ideas in arithmetic?

1. Do most pupils take part in the discussion?

Ter : Ter : Ter : Joint : Child

*Do you ever have problems without pencil and paper? That is, mental arithmetic tests?

2. Where do your mental arithmetic problems come from?

Text : TextEpnd : Terdretd : : Ppl-Ppl

EVALUATION

1. How do you know how well you are getting along in arithmetic?

RptCrd : Tests-DiagAch : Tertes : TerChlConf : ChlPrgCut

2. Where do your tests come from?

Txt : WkBk : TerMade : Ter-Child : Child

APPENDIX E -- Continued

3. How often do you have tests?
   Dly : Wky : 6 wks : Tpk : Slf
4. Who usually grades your papers?
   Ter : : OthrPpl-Ter : : Slf
5. Where are your grades usually written or kept?
   Ter : : : Joint : Ppl

*Tell me about your favorite day in arithmetic this year.

APPENDIX F

INTERPRETATION OF CODING THE TEXAS ELEMENTARY SCHOOL MATHEMATICS CLASSROOM PRACTICES SCHEDULE OF QUESTIONS

A. TEXT

(Number) -- If use out of adopting text or other text to supplement, indicate total number of kinds.

1. Use

*(1) Every page --if progress through text from front to back (with the exception of practice sets) and for all practical purposes cover every page

(2) Skip page --if a few pages are skipped frequently --at the first of year skipped but not now

(3) Skip part --if skip is from part of part, or parts omitted; not (1) or (2)

(4) Skip most --if the text is used primarily as a reference or only as a source of practice problems

(5) Skip all --if a textbook is not used at all or rarely if ever

2. Assignment

(1) All the same --if everyone in the class has the same assignment in the textbook

(2) Only one or two persons have different assignment or textbook

(3) 2-3 groups --have different assignments or work at different places in the textbook

(4)

(5) All different --for all practical purposes assignments are made on an individual basis

B. WORKBOOK

(Number) -- Count tear sheets, ditto practice sheets as workbook type materials

1. Use

(1) None

(2) Regular assignment for all --work daily, no differentiation in assignments

APPENDIX F -- Continued

(3) Irregular assignment sometimes for all
 * Irregular assignments for all
 * Same but permitted to work at own rate
 * One person has a different assignment
(4) Teach ident. --with groups or individual assignments
(5) Pupil selects needed material

2. Frequency
 (1) None
 (2) Daily
 (3) Weekly--used regularly once a week, or seemingly without need--as "to keep us busy while he has other work to do." Also for extra credit.
 (4) Teacher identifies need of individual or group--when the teacher sees the need for practice, etc. and assigns to either the class, group, or individual as the case may be
 (5) Pupil selects needed material--the pupil selects which material he will work on in the workbook

3. Same workbook
 (1) None
 (2) All--all workbooks and materials are the same for all members of the class
 (3) Definite groups--one group uses different materials than the other; one or two students may have a different workbook or related materials
 (4) Same place--
  (1) None
  (2) All--all the class is working in the same place in the workbook
  (3) Definite groups--where one group is in a different place in the workbook than the other group/s. Or one or two students may be working in a different place than the remaining group
  (4) Indefinite groups--the groups change from time to time; groups have different assignments. Also if the class has no common assignment, but individually permitted to work at own rate
  (5) Individual--for all practical purposes, assignments are made on an individual basis

APPENDIX F -- Continued

C. TRADE

(Number) -- includes library books, pamphlets, books in the teacher's personal library, trade books brought from home, encyclopedias used, etc.

1. Choice
   (1) None
   (2) Teacher chooses text suggested--includes books ordered from textbook company (adopted text)
   (3) Teacher--she chooses the books herself, either from the library or other sources. If teacher has child check out specific book from library
   (4) Teacher-child--teacher selects the books and the children select from her selection; teacher selects some and children select some from library, home, etc.
   (5) Child--the child selects the books to be used, and may bring them to the classroom

2. Use
   (1) None
   (2) Regular reports, drill, tests--if they are required to report at regular intervals on reading; if the books are a source of drill or practice materials; if a test is the only follow-up on the reading of the books. If teacher uses books and children mention that she lectures on materials
   (3) Teacher plans groups of individual's report--assigned report
   (4) Report as a result of teacher-pupil planning--allows for pupil choice in topic, but the teacher has a hand in planning, assigning, or suggesting. Also includes extra-credit reports
   (5) Report as interest to individual--may report if he wants to, strictly voluntary--however, usually evidence needed that someone has or evidence that there is ample opportunity to read and not report if he chooses to

3. Frequency
   (1) None
   (2) Over month--once or twice in a six months period
   (3) Month--average of once a month
   (4) Week--not every week yet more often than monthly
   (5) Daily--evidence that some pupils use the books daily and that there is opportunity for many students to use them

APPENDIX F -- Continued

D. FREETIME ACTIVITY

1. Finish text-workbook assignment
   (1) Don't finish
   (2) Next Assignment --when they say they go on to homework, etc.
   (3) Teacher choice --teacher either tells them what to do or "to sit and be quiet until the others finish."
   (4) Arithmetic choice -- students can do anything in arithmetic such as go to the arithmetic table, read trade books, etc. --students choice
   (5) Free choice --students choose to do anything

2. Special arithmetic activities
   (1. List all. 2. Record total number of special activities. 3. record number of project related activities.)

   (1) None
   (2) Teacher elaboration on text --teacher takes a topic from the text and initiates some activity suggested there -- such as making a chart, graph, etc.
   (3) Teacher suggested --or assigns something outside of the text, discusses a topic outside of the text, initiates an activity not in the text (or related materials)
   (4) Teacher-child developed --teacher makes broad assignment or gives broad suggestion and the children have the freedom to choose and develop an activity themselves
   (5) Child on his own --child has freedom to choose what he does and how to develop it; evidence that he has originated and developed the activity brought to class

E. HOMEWORK

1. Source
   (1) Text --almost solely the text
   (2) Workbook --about half from the workbook and half from the textbook some ditto, some text and/or workbook
   (3) Ditto-Chalkboard
       * Not from textbook
       * Sometimes experiential problem and somefimes text or workbook
       * Make up problems similar to textbook problems

APPENDIX F -- Continued

(4) Assigns experiential problem -- source outside of text where child has to find facts as well as to work the problem

(5) Child brings experiential problem -- brings something from his own experience to class; brings a "real" problem to class -- voluntary on the part of the student

F. FILMS
(Number) -- includes filmstrips, slides, etc.

1. Follow-through

(1) None

(2) Text -- only follow through

(3) Teacher summary -- teacher asks questions and students answer or teacher simply summarizes

(4) Report or discussion -- children report on film; discussion must be a 50-50 one where pupils take an active part -- not merely a teacher questions-child answers session

(5) Individual exploration -- where the children try some of the things shown in the film or do some additional activity or reading

G. INTEGRATION -- Omit -- Not applicable to Dallas because of platoon organization

H. TEACHING AIDS -- list all mentioned. tabulate total number of manipulative and graphic aids.
(Number) -- charts, etc. (felt board) manipulative, etc.

1. Use

(1) None

(2) Skills -- games, etc. for drill or practice (little use, or teacher used)

(3) Abstraction -- when aid is used concurrently with pencil and paper, etc., when we get mixed up

(4) Exploration -- use of teaching aid before they work problems -- children actually use it

(5) Application, preparation -- preparation: used in introducing a topic; application: after abstraction to relate the abstract back to the concrete

APPENDIX F -- Continued

2. Source
   (1) None
   (2) Commercial
   (3) Teacher --either the teacher makes or some source other than the students in this particular class
   (4) Teacher-child --half and half
   (5) Child --children make practically all

3. Charts, use of
   (1) None
   (2) Skills, abstraction --to develop skill through practice or use --of a concept already learned
   (3) Preparation --to prepare students for the new concept --stimulate interest
   (4) Application --where the children (or teacher) make a chart to show application of concept learned
   (5) Exploration --actually explore when making the chart; or use a chart to explore --for example, making a Base 5 chart.

I. ORGANIZATION

1. Equal time daily --not applicable to Dallas

2. Grouping
   (1) None
   (2) 2 groups --two major groups or if only one or two persons have a different assignment
   (3) 3 groups
   (4) 4 plus groups
   (5) Individual

J. DISCUSSION -- Record per cent of class period spent in discussion

1. Who talks
   (1) (Teacher) --virtually does all the talking
   (2) (Teacher) --children talk a little -- teacher asks some questions
   (3) (Teacher) --teacher questions, children answer
   (4) (Joint) --both contributing equally and asking questions

2. Mental arithmetic
(Topics from text)--if not true mental arithmetic or text used only for mental arithmetic problems
(Text expanded)--text plus teachers personal books or plus some made up and estimation
(Teacher directed) --she makes most of them up or some source other than the text, workbook or teachers manual
(Pupil and teacher directed)--pupils bring some and teacher brings some
(Pupil-pupil)--where pupils take initiative and administer them

K. EVALUATION

1. Progress
(1) Report card only
(2) Tests (diagnostic or achievement tests) --teacher sends tests or notes home to parents
(3) Teacher tells --The child --or daily papers she keeps -- progress chart up for everyone. Include notebook or folder if just keep it, and no evidence of real use. Sometimes the class averages their grades, sometimes class keeps some grades and sometimes individuals keep all grades
(4) Teacher-child conference --talks with child privately
(5) Child's own progress chart --child keeps a record of his own grades, texts or otherwise

2. Evaluation devices
(1) Test --if directly, or from teacher manual
(2) Workbook --if sometimes text, sometimes made up, sometimes other tests
(3) Teacher-made --primarily made up by teacher
(4) Teacher-child
(5) Child

3. Frequency
(1) Daily
(2) Weekly--set day
(3) 6-weeks
(4) Topic
(5) As needed

APPENDIX F -- Continued

TEXAS ELEMENTARY SCHOOL MATHEMATICS CLASSROOM PRACTICES INTERVIEW SCALE

TOPIC		SUBJECT OF QUESTION	TEXT	TEXT-TEACHER	TEACHER	TEACHER-CHILD	CHILD
TEXT	1	Use	Every page	Skip page	Skip part	Skip most	Skip all
No:	2	Assignment	All the same		2-3 Groups		All different
WORKBOOK	1	Use		Reg. assignment for all	Irregular assignment sometimes for all	Teacher identifies need of Indiv. or group	Pupil selects needed material
No:	2	Frequency		Daily	Weekly	Teacher assigns as needed	Pupil selects as needed
	3	Same workbook		All	Definite groups	Indefinite groups	Individual
	4	Same place		All	Definite groups	Indefinite groups	Individual
TRADE	1	Choice		Teacher chooses	Teacher	Teacher-child	Child
No:				Text suggested			
	2	Use		Regular reports / Drill / Tests	Teacher plans groups or individual's report to teacher	Report as result of teacher-pupil planning	Report as interest to indiv
	3	Frequency		Over month lapse	Month	Week	Almost Daily
FREE TIME ACTIVITY	1	Finish text-workbook assignment		Next assignment	Teacher choice	Arithmetic choice	Free choice
a.	2	Special arithmetic activity	Text	Teacher elaboration text	Teacher suggested	Teacher-child developed	Child on his own
HOMEWORK	1	Source	Text	Workbook	Ditto-chalkboard	Assigns experiential problem	Child brings experiential problem
FILMS							
No:	1	Follow through		Test	Teacher summary	Report or discussion	Individual exploration
INTEGRATION	1	How many subjects	None	1 Subject	2 Subjects	3 Subjects	All subjects
a.							
b.	2	Regular lesson		Yes	Usually	Not often	Never
c.							
TEACHING AIDS	1	Use		Skills	Teacher abstraction	Exploration	Application Preparation
No:							
a.	2	Source		Commercial	Teacher	Teacher-child	Child
b.	3	Use of charts		Skills-abstractions	Preparation	Application	Exploration
c.	4	Source charts		Commercial	Teacher	Teacher-child	Child
d.							
ORGANIZATION	1	Equal time daily	Yes	Few exceptions	Some exceptions	Frequent exceptions	Never the same
	2	Grouping	No	2 Groups	3 Groups	4+ Groups	Individual
	3	Different assignments within groups	No	Length only		Level only	Length and level
	4	Individual help	Little or none	Teacher re-explains to entire class	Group help	Teacher choice - individual help	Child choice - individual help
DISCUSSION	1	Who talks	Teacher	Teacher	Teacher	Joint	Children
Amt:	2	Mental arithmetic	Topics from text	Text expanded	Teacher directed		Pupil-pupil directed
EVALUATION	1	Progress	Report card only	Test-diagnostic and achievement	Teacher tells	Teacher-child conf.	Child own progress chart
	2	Evaluation devices	Text	Workbook	Teacher-made	Teacher-child	Child
	3	Frequency	Daily	Weekly	6 Weeks	Topic	As needed
	4	Grading-checking	Teacher		Other pupil & teacher		Self
	5	Recording	Teacher			Joint	Self

APPENDIX F -- Continued

4. Grading and checking
   (1) Teacher --all
   (2) Teacher and helper
   (3) Other pupil and teacher
   (4) Self and teacher
   (5) Self

5. Recording
   (1) Teacher
   (2) Occasionally otherwise
   (3) Folder or notebook
   (4) Joint-group and teacher
   (5) Self

* Numerals in parentheses:
   (1) "Text" col. TESMCPIS
   (2) "Text-Teacher" TESMCPIS
   (3) "Teacher" TESMCPIS
   (4) "Teacher-Child" TESMCPIS
   (5) "Child" TESMCPIS

APPENDIX G

OUTLINE OF BASIC MATHEMATICAL CONCEPTS DEVELOPED FOR THE ELEMENTARY SCHOOL MATHEMATICS (K-8) IN-SERVICE WORKSHOPS

I. Introduction to Sets

A. Set vocabulary and operations with sets

1. The set concept has a language and symbolism that has applications in all mathematics.

2. The operations on sets are unique ways of thinking of two sets to get one set.

B. Applications of sets to elementary mathematics

1. Sets in the elementary grades will not be a radical departure for most teachers.

2. The set concept may be used to develop the concept of whole numbers.

3. The set concept and set operations may be used to give meaning to the operations of whole numbers.

4. We classify whole numbers into sets such as even, odd, composite and prime numbers, factors and multiples of numbers, etc.

5. We classify numbers by sets such as sets of natural, whole, integers, rational and real numbers.

II. Number and Numeral

A. Historical background of the number concept

1. Number is an invention of man.

2. Number is an abstract idea; it is something that exists in the mind; a number has many names.

B. Pre-Numeral recordings (matching)

1. Primitive number concepts probably were limited to a one-to-one matching of objects with fingers, pebbles, etc.

2. Man learned to count before he learned to symbolize the counting numbers.

C. Ancient systems of numeration

1. Systems of numerations are man made.

2. Scientists have made it possible for us to study the numeration systems developed by the Egyptians, Greeks, Babylonians, Mayan Indians and other ancient civilizations.

D. Characteristics of numeration systems

1. Most early numeration systems had single symbols for groups, the principle of repetition of symbols, and the principle of addition.

2. The development of the concept of place value and a symbol for zero gives us the numeration system that we use today.

III. Structure of a place value system of numeration

A. Using base other than 10

1. In a place value system each symbol has an absolute value and a place value depending upon the base.

2. Ten is probably the most common base because man has ten fingers.

3. Any whole number greater than one may be used for a base.

B. Fundamental operations with bases other than 10

1. In our every day activities we sometimes use other bases.

APPENDIX G -- Continued

2. Working with other bases should give greater insight into the structure of our base 10 system.

C. Hindu-Arabic system of numeration (decimal system)

1. Our decimal system has place value, a base of ten, and a symbol for zero.

2. The Hindu-Arabic system of numeration was brought to Europe by the Arabs.

3. The present form of the digits have for the most part evolved in Europe.

IV. Addition and multiplication of whole numbers

A. Addition and multiplication defined

1. Addition may be considered in terms of the union of disjoint sets.

2. Multiplication may be considered as repeated addition or in terms of sets.

B. Properties of addition and multiplication: Closure, commutative, associative, distributive and the identity elements

1. When we add or multiply two whole numbers we are guaranteed an answer and that answer is a whole number.

2. We have the right to change the order of addends of factors without affecting our answer.

3. When adding or multiplying more than two numbers, we may group or associate different pairs of numbers together without affecting our answer.

4. When we multiply numbers, it is possible to use the operation of addition to simplify the process.

5. Addition of zero to any number gives that number as an answer. Multiplication of any number by one gives that number as an answer.

C. Algorisms for addition and multiplication

1. The algorisms may be developed by using sets of physical objects and the characteristics of the numeration system.

2. The algorisms may be developed by using the commutative, associative, and distributive properties of numbers.

3. The algorisms that we teach today are not the only possible algorisms for the whole numbers.

V. Subtraction and division of whole numbers

A. Subtraction and division defined

1. Subtraction and division may be considered as unique ways of separating or partitioning sets.

2. Subtraction and division are secondary operations in arithmetic.

B. Subtraction and division as inverse operations

1. Addition and multiplication may be considered basic operations in terms of which the inverse operations, subtraction and division, are defined.

2. Subtraction may be considered as finding a missing addend when one addend and the sum is given.

3. Division may be considered as finding a missing factor when one factor and the product is given.

C. Algorisms for subtraction and division

1. The algorisms may be based on the numeration system and sets or the inverse properties of addition and multiplication.

2. A method of successive subtraction is the most simple division algorism.

VI. Equations and inequalities

A. Operation, relation and place holder symbols

1. Mathematicians have defined many symbols to indicate number operations.

2. To indicate that two expressions name the same thing we use the symbol "=."

3. To indicate that two expressions do not name the same thing we use the greater than $>$, less than $<$ or $\neq$ unequal to symbol.

4. In mathematics a place holder is a symbol (○, △, X, A, ?, etc.) for which one substitutes the names of numbers.

B. Number sentences and solution sets

1. The language of mathematics has a grammar and symbolism of its own just as the English language does.

2. It is possible to have true, false, or neither true nor false sentences in mathematics.

3. There may be a set of none, one, two or many numbers that will make an open number sentence true.

C. Picturing number sentences

1. Numbers may be associated with the points on a line.

2. A pair of numbers may be associated with a point on a grid.

D. Ordered pairs, ratio and rate

1. The concept of ordered pairs of numbers may be structured to produce a useful tool for solving number problems.

APPENDIX G -- Continued

VII. Rational numbers

A. Historical developments of fractions

1. A new kind of number resulted in a need to describe, measurements, parts of a whole or a part of a group of physical objects.

B. Structural development of rational numbers

1. To insure closure under the operation of division new numbers have to be invented.

C. Algorisms for rational numbers

1. The algorisms for fractions may be developed from physical models such as the number line, parts of circles, squares, etc.

2. The algorisms for rational numbers may be developed by inventing new numbers and assuming that they obey the basic properties of the whole numbers.

3. A decimal fraction is a different symbol for a rational number.

VIII. Integers

A. Physical models for the integers

1. The number line may be extended to the left of zero to picture negative numbers.

2. Number stories may be used to develop the operations for the integers. For example, profit and loss stories or stories of receiving and taking away bills and checks.

B. Structural development of the integers

1. New numbers (the integers) may be invented to insure closure under the operation of subtraction.

2. The rules of operations for the integers may be developed by assuming that the integers obey the basic properties of the whole numbers.

IX. Geometry

A. Introduction to the language and symbolism of intuitive geometry

1. We cannot see a point, line or geometric figure; we only draw pictures to represent the idea.

2. Unique symbols have been defined to represent a point, line, line segment, ray, angle, etc.

B. Simple constructions

1. The straight edge, compass, and protractor are the tools of geometry.

2. Teaching devices and methods should lean heavily on the senses of sight and touch.

C. Measurement

1. Standard units of measurement are arbitrarily chosen.

2. Numbers obtained by measuring represent approximations of the quantity.

X. Patterns in Arithmetic

A. Finite number systems

1. Clock arithmetic or modulo number systems have many properties in common with our real number systems.

2. Children have had some experience with modular or clock arithmetic.

B. Mathematical games and number puzzles

1. Many of the concepts of arithmetic may be discovered from patterns of numbers.

2. Mathematical games using grids, magic squares, number lines, number arrays, number puzzles, etc. have many applications to number concepts.

XI. Experimental programs

A. School Mathematics Study Group

B. Madison Project

C. University of Illinois Arithmetic Project

D. Stanford Elementary Mathematics Project

APPENDIX H

A LIST OF ARITHMETIC TEXTBOOKS AND EXPERIMENTAL MATERIALS EXAMINED

A. TEXTBOOKS

Buswell, Brownell, Sauble, and Weaver. Arithmetic We Need (Grades 1-8). Boston: Ginn and Company, 1963.

Clark, Junge et al. Growth in Arithmetic (Grades 3-8). Harcourt, Brace and World, 1962.

Deane, Kane, and Osterle. Modern Mathematics Series (Grades 1-8). Cincinnati: American Book Company, 1963.

Fehr, et al. Learning to Use Arithmetic (Grades 1-8). Boston: D. C. Heath and Company, 1962.

Hartung et al. Seeing through Arithmetic (Grades 1-6). Chicago: Scott, Foresman and Company, 1963.

McSwain, Brown, Gundlach et al. The Laidlaw Mathematics Series (Primer through Grade 8). River Forest, Illinois: Laidlaw Brothers, 1963.

Merton, Brueckner, and Grossnickle. Moving Ahead in Arithmetic (Grades 1-6). New York: Holt, Rinehart and Winston, 1963.

Morton et al. Modern Arithmetic through Discovery (Grades 1-6). Morristown, N. J.: Silver Burdett Company, 1963.

B. EXPERIMENTAL MATERIALS

Eicholtz, Robert et al. Elementary School Mathematics (outgrowth of Ball State Experimental Project) Reading, Mass.: Addison-Wesley Co., 1963.

Davis, Robert B. Discovery in Mathematics (Madison-Webster Project) Syracuse, N. Y.: Syracuse University, 1963.

APPENDIX H - Continued

Greater Cleveland Mathematics Research Council. Elementary Mathematics Series (now SRA). Cleveland: The Council, 1961.

Page, David. "University of Illinois Project in Elementary Mathematics." Urbana, Illinois: University of Illinois, 1960. (Mimeographed.)

School Mathematics Study Group. Mathematics for the Elementary School (Grades 4-6, Student Texts and Teacher's Commentaries).

Suppes, Pat. Sets and Numbers. New York: Blaisdell Publishing Co., 1962.

APPENDIX I

NAMES AND ADDRESSES OF PERSONS USED FOR TEST REVIEW

Dr. William Allison, Assistant Superintendent
Cape Girardeau Public Schools
Cape Girardeau, Missouri

Dr. Calhoun C. Collier, Professor of Education
Michigan State University
East Lansing, Michigan

Dr. James M. Drickey, Professor of Psychology
Southeast Missouri State College
Cape Girardeau, Missouri

Dr. Vincent J. Glennon, Director
Arithmetic Studies Center
Syracuse University
Syracuse, New York

Mr. Neal Holmes, Science-Mathematics Consultant
Parkway Public Schools
St. Louis, Missouri

Dr. W. Robert Houston, Associate Professor of Education
Michigan State University
East Lansing, Michigan

Dr. Lois Knowles, Professor of Education
University of Missouri
Columbia, Missouri

Mr. Joe L. Wise, Mathematics Consultant
Missouri State Department of Education
Jefferson City, Missouri

APPENDIX J

OUTLINE OF BASIC MATHEMATICAL UNDERSTANDINGS

I. Numerals and Numeration System
  A. History of number and numerals
    1. Numerals are man-made, created to stand for concepts which men have.
    2. Man had some sense of number long before he was able to count or use numerals to stand for number.
    3. Numbers are the same, regardless of the symbols used to represent them.
    4. Man now has both symbols (1, 2, 3, ...) and words (one, two, three, ...) which may be used to represent numbers.
    5. Some instruments, like the abacus, were used by ancient people to do computation.
    6. The Roman numerals were used for recording numbers and not for computation.
    7. Many of the early systems, such as the Roman, Greek, and Egyptian, were based upon 10 but did not use place value or have a symbol for zero.
    8. The Babylonians are said to have developed the first symbol to use for zero.
    9. A decimal system of numeration is used in most of the world today because it is less cumbersome and more concise than earlier systems.
  B. Place value and bases
    1. Our numerals are called Hindu-Arabic numerals.
    2. Since the Hindu-Arabic system uses groups of ten, it is called a decimal system; values decrease in a tenfold ratio to the right and increase in a tenfold ratio toward the left.
    3. Since grouping is by ten in the decimal system, its base is ten.
    4. The decimal system of numeration uses the idea of place value to represent the size of the group; the number that the numeral represents depends upon the position of the symbol and the value of the digit used to represent it.
    5. The decimal point indicates a change in the character of the digits. All numbers to the left are integral; all numbers to its right are fractional.

6. The individual must have a clear understanding of place value in order to carry out the operations of addition, multiplication and division.
7. To understand the meaning of a number represented by a numeral such as 123, the individual must add the numbers represented by each symbol (1 x 100) + (2 x 10) + (3 x 1) or 100 + 20 + 3.
8. There is no limit to the size of the number which can be represented by the decimal system of numeration.
9. The numeral "0" has the special property of always representing zero; there is no number to record in that place; zero is used to fill a place which would otherwise be empty and might lead to misunderstanding.
10. In an expression as 10^2, the number 10 is called the base and the number 2 is called the exponent.
11. A number such as 10^2 is called a power of ten.
12. In many everyday activities and in modern electronic computing machines, bases other than ten are used; the most frequently used bases other than ten are two, twelve, and sixty.
13. The base in any system of numeration is the number of units which must be reached before a change is made in the pattern used to denote the number.
14. In counting, we can group in any way; the selection of the base is arbitrary.
15. A number can be represented by different numerals.
16. Numeration systems, such as base four, do not represent a different number system, but only a different way of writing the same number.

II. Symbols.

A. A symbol is a mark, an object, a sign, or a word that represents another object or idea.

1. Usually there is little resemblance between a symbol and the object, or idea, which it represents.
2. Many symbols are used in mathematics.
    a) Some symbols represent numbers, some represent operations, and some compare numbers.
    b) In using a symbol, the individual must know the object or idea which it represents.

B. There is a difference between a number and a numeral.

1. A numeral is a symbol which is used to stand for a number.
2. The number is the idea for which the symbol stands.

C. The individual characters we use in writing a numeral are called digits.

D. A symbol for a number is different from the name of the number.

III. Introduction to Sets

A. The concept of sets is one of the unifying concepts of mathematics.

1. The word "set" can be used to mean any particular collection or group of objects.
2. The objects contained in a set are called the elements or the members of the set.
    a) We can describe sets in words (the set of all cities in the U. S. with a population greater than 100,000).
    b) We can also list or tabulate the elements of a set within braces -- {1, 2, 3, 4, 5} or {the set of counting numbers from one through five}.
3. The set which contains no elements is called an empty set or null set and is represented by the symbol { }.

B. When it is written that Set A = Set B (A = B), this means that "A" and "B" are different names for the very same set; the symbol "=" is used to mean "the same as."

C. If two sets belong to the same family, they are equivalent.

D. The number of elements in a set is often called the cardinal number of the set.

E. Subsets of a given set are those sets whose elements are members of the given set.

1. Every set is a subset of itself.
2. The empty set is a subset of every set.
3. Subsets which are different from the original set are called proper subsets.

F. A solution set for the sentence contains those elements which give us a true statement.

G. An equation is a sentence which states that two expressions represent the same number.

H. In dealing with sets, it is often necessary to consider the set composed of the two sets lumped together, or combined.

1. We can form a set by combination-union of two sets (U).
2. The intersection of two sets (written $\cap$) is the set consisting of all objects in both sets.

IV. Principles underlying Number Operations

A. Addition and multiplication are commutative.

1. The order in which two numbers are added does not affect the sum (a + b = b + c).
2. The order in which two numbers are multiplied does not affect the product (a x b = b x c).

B. Addition and multiplication are associative.
1. Grouping numbers differently does not affect the sum (a + b) + c = a + (b + c).
2. Grouping numbers differently does not affect the product (a x b) x c = a x (b x c).

C. Multiplication is distributive with respect to addition -- 3 (a + b) = (3 x a) + (3 x b).

D. Property of closure
1. The set of counting numbers is closed under addition and multiplication.
2. The set of counting numbers is not closed under division-- (20 ÷ 6 = 3 1/3).

E. Properties of zero and one
1. The number one is a special number.
   a) It is the smallest of the counting numbers.
   b) Because the product of any counting number and one is the original counting number, the number one is called the identity element for multiplication.
   c) The sum of a counting number and one is always the next larger counting number.
2. The number zero is also a special number.
   a) The number zero is the number of elements in the empty set; it is used to indicate the absence of quantity.
   b) Zero may also be used as a point of origin from which one may move in either direction.
   c) The sum of a counting number and zero is always that whole number; zero is an identity element for addition.
   d) The difference between the same two rational numbers is the special number zero.
   e) A counting number cannot be divided by zero (because 0 x 7 = 0).
   f) If the product of two or more whole numbers is zero, then one of the numbers must be zero.

F. Addition and multiplication are binary operations; we operate with just two numbers at a time. There is exactly one whole number which is the sum of every two whole numbers. This is referred to as the principle of uniqueness.

APPENDIX J -- *Continued*

V. Relationships and Generalizations
   A. Inverse operations
      1. Addition and subtraction are related processes. Subtraction is the *inverse* of addition.
      2. Multiplication and division are also related processes. The *inverse operation* of multiplication is division.
   B. Betweeness on the number line
      1. The manner in which whole numbers are related may be shown with a number line

         a) Any whole number is smaller than any of the numbers on the right side of it.
         b) Any whole number is greater than any of the numbers on its left.
      2. The number line can be used to determine how many whole numbers there are between any two whole numbers.
   C. The sum or difference is not altered if one number is decreased or increased provided the other number is increased or decreased by the same amount. This is referred to as the *principle of compensation*.

VI. Operations with Whole Numbers
   A. Whole or counting numbers[a]
      1. The numbers first used by man were the whole numbers.
      2. The whole or counting numbers are the numbers used to answer the question "how many?"
      3. Primitive man developed the idea of number by *matching* objects or things in one set with objects in another set; two groups are said to be in one-to-one correspondence when each item in the first group corresponds to one and only one item in the second group.
      4. There is a standard set of counting numbers which can be used to note that there are "just as many" in one set as in the other set.
      5. Counting numbers and zero are called *the set of whole numbers*.

---

[a]Brumfiel says that the whole numbers are counting numbers and includes zero; the SMSG materials state that the counting numbers and zero make up the whole numbers.

APPENDIX J -- Continued

6. There is no largest counting number.
7. When a number is used in its serial (or order) meaning, it is referred to as ordinal.
8. When a number is used to indicate the size of a group (collected meaning), it is referred to as cardinal.

B. Basic combinations and algorisms
1. Addition of whole numbers
   a) Addition is a binary operation which assigns a number called the sum to an ordered pair of numbers called addends.
   b) The sum of any ordered pair of numbers also can be determined by joining two disjoint sets and then counting the members in their union.
   c) The plus sign (+) is the sign of operation for addition.
   d) The sum may be recorded in various ways.
      (1) The equation $3 + 4 = 7$ is one way to state the sum of an ordered pair.
      (2) Sums of ordered pairs of numbers may also be written in vertical form.
$$\begin{array}{r} 3 \\ +4 \\ \hline 7 \end{array}$$
      (3) The sums of the ordered pairs of numbers may be summarized in table form.
   e) An algorism is a method of arranging numbers so as to reduce the number of steps in determining the correct result; it produces answers through numeral manipulation based on the use of number properties and definitions.
   f) All "carrying" or regrouping in addition is based on a combination of place value notation, associativity and commutativity.
   g) The commutative and associative properties also are utilized in checking a sum by adding "up" a column after adding "down."
2. Subtraction of whole numbers
   a) Subtraction, also a binary operation, assigns to an ordered pair of numbers a number that is called the difference.
   b) Subtraction is related to addition; it "undoes" the additive operation.
   c) The difference may be written in either equation or column (vertical) notation form.

APPENDIX J -- Continued

d) Differences of whole numbers do not always exist as whole numbers; the set of integers makes it possible to perform such an operation as $3 - 8 = -5$.

e) A thorough understanding of the addition algorism makes clear the meaning of a subtraction algorism such as:

$$\begin{array}{r} 47 = 40 + 7 = 30 + 17 \\ \underline{-19} = \underline{10 + 9} = \underline{10 + \ \ 9} \\ 20 + \ \ 8 \end{array}$$

The computer did not borrow; he has regrouped.

3. Multiplication of whole numbers
   a) Multiplication is a binary operation -- a regrouping process through which equal measuring units are combined in terms of the number system.
   b) Multiplication of whole numbers can be interpreted as repeated addition when the addends are the same.
   c) Multiplication of whole numbers is both commutative and associative.
   d) Both multiplier and multiplicand may be given the name factors.
   e) The product of any two whole numbers (other than zero and one) is a whole number.
   f) The number one is the identity number for multiplication.
   g) The number zero, when used as a factor, always gives a product that is zero.
   h) Multiplication is distributive with respect to addition -- $2(a + b) = 2a + 2b$.
   i) Multiplication combinations can be summarized in a multiplication table or a matrix.
   j) The multiplication algorism can be explained in terms of the properties of multiplication and addition; it can be used to find the product of any two whole numbers.
4. Division of whole numbers
   a) Division is a binary operation -- a regrouping process which finds either of two factors when their product and the other factor are known; it assigns to an ordered pair of numbers a number called the quotient.
   b) Division is the inverse of multiplication.
   c) Division is sometimes thought of as repeated subtraction.

d) The division algorism may be written as $12 \div 3$ or $\frac{12}{3}$ or $3\overline{)12}$.

e) The quotient of some whole numbers (as $8 \div 3$) does not exist as a whole number.

f) Mathematicians avoid division by zero because there is no number other than zero to satisfy the equation $\square \times 0 = 8$.

VII. Rational Numbers

A. History of fractions

1. Man has not always known about fractions; it is said that he began to use fractions when he began to measure as well as count.
2. Unit fractions, which were used first by the Egyptians, are fractions with a numerator of 1.
3. Our present fractional notation (e. g. $\frac{1}{3}$) came into general use in the 16th century.
4. A symbol "$\frac{a}{b}$" where a and b represent numbers, with b not zero is called a fraction; it represents a fractional number; the numerator tells how many parts of the whole are to be considered and the denominator tells the number of parts into which the whole has been divided.
5. A number which can be represented by fractional symbols is called a rational number ($\frac{1}{1}$, $\frac{5}{2}$, $\frac{3}{4}$, etc.).
6. A fraction can have different names ($\frac{63}{21} = \frac{9}{3} = \frac{3}{1}$); it is the measure or value of one or more equal parts of a unit expressed as a relation of the parts to the whole.
7. Two fractions which represent the same number are called equivalent fractions.

B. Properties of rational numbers

1. The set of rational numbers is closed with respect to the operation of addition and multiplication.
2. The operations of addition and multiplication for the rational numbers are both associative and commutative.
3. The operation of multiplication is distributive over addition for the rational numbers.
4. Zero is the identity element for addition of rational numbers; one is the identity element for multiplication of rational numbers.

5. If the numerator and denominator of a fraction are multiplied or divided by the same counting number, the number represented is not changed.
6. If the numerator is multiplied by a number, the value of a fraction will be changed; if the denominator of a fraction is multiplied by a number, the value of a fraction also will be changed.
7. To write a fraction in the simplest form, find the greatest common factor of both numerator and denominator and multiply both terms by that number.
8. A fraction in which the numerator is greater than the denominator often is called an improper fraction.

C. Reciprocals
   1. If the product of two numbers is 1, the numbers are called reciprocals of each other ($\frac{a}{b} \times \frac{b}{a} = 1$ if neither is zero).

D. The number line can be very useful in working with fractions (i.e., divide the line segment with endpoints 0 and 12 into four equal parts); equivalent fractions can be illustrated with the number line.

E. Fractions also can be ordered.

F. A fraction may express the ratio of one number to another.

G. Per cent may be written as a fraction or as a decimal.

VIII. Decimal Notation

A. Differences between common fractions and decimal fractions are differences in notation only; decimal fractions may be expressed as common fractions.
   1. When written in decimal notation, each digit represents a certain value according to its place in the numeral.
      a) The form for place value in base ten shows that the value of each place immediately to the left of a given place is ten times the value of the given place; each place value immediately to the right of a given place is one-tenth of the place value of the given place.
      b) The places to the right of one's place are usually referred to as the decimal places.
      c) The decimal point locates the one's place.

B. You can operate with decimals in much the same way you handle whole numbers.
   1. Numbers can be carefully written directly under one another for addition and subtraction.

2. To find the number of decimal places in the product when two numbers are multiplied, add the number of decimal places in the numerals.
3. The usual form of division may be used; then the same number of decimal places may be pointed in the quotient as the dividend.
4. You can also divide by a decimal fraction by multiplying the dividend and divisor by such a power of ten as will make the divisor a whole number.

IX. Real Numbers

A. The entire set of numbers represented by decimals is called the set of real numbers.
   1. A real number is rational if its decimal representation repeats.
   2. An irrational number is represented by non-repeating decimals.

B. Close approximations of square roots of numbers can be made; a table of square roots may also be helpful.

C. Numbers may be represented by many different kinds of numerals.
   1. The decimal representation is an important one.
      a) Some decimal symbols "come out even."
         (terminating e.g. $\frac{1}{2} = 0.5000$)
      b) Other decimals never "come out even."
         (repeating e.g. $\frac{1}{3} = 0.333$)
   2. Repeating decimals represent rational numbers.
   3. Numbers represented by non-repeating decimals are called irrational numbers.
      (e.g. $\sqrt{2} = 1.414213$)

D. The decimal terminates if and only if the rational number can be expressed as the quotient of two natural numbers in which the decimal is the power of 10. These will be only 2 or 5 (or both).

X. Measurement

A. Measurement, or the process of measuring, involves assigning a number to some physical quantity; the number is called the <u>measure of the quantity</u>.

B. A measure can be assigned to the quantity on the basis of either direct or indirect comparison of the object to be measured with a <u>standard unit</u>.

1. In direct measurement, the number assigned to the quantity is determined by direct comparison of the object to be measured with a standard unit of measurement -- choose a unit of measure and then give a number which tells how many of these units it will take.
2. In direct measure (as) the quantity being measured cannot be compared directly with the unit of measurement.

C. The choice of a selection of a unit of measure is arbitrary; the selection of a unit of measure is made in terms of the physical quantity to be measured and in terms of the precision desired.

D. A given quantity can be measured in terms of different standard units.

E. When a physical quantity is described by a number, it is essential that the unit of measure be designated.

F. When measuring continuous properties, one can never be assured of arriving at an exact measure (all measures are an approximation).

G. A special symbol ($\overset{m}{=}$) read as "the same amount as" has been adopted to use with units of measure; it has been found that the symbol "$\overset{m}{=}$" avoids considerable confusion if used until both the idea of unit measurement and the idea of equality are well established.

XI. Geometry

A. Points, lines, planes, and space

1. A geometric point is thought of as being so small that it has no size; it is represented by a dot.
2. The set of all points is called space; there is an unlimited number of points in space.
3. Mathematicians think of a plane as a special set of points.
4. A line is a set of points in space; it extends without limit in each of two directions; it contains an unlimited number of points.
5. Through any two different points in space there is exactly one line.
6. Every line segment has exactly two endpoints.
7. A ray has only one endpoint; the endpoint is named first.
    a) A ray is the union of the endpoint and all points on a line in one direction from the point.
    b) A ray is always a part of a line.

APPENDIX J -- Continued

B. Geometric figures
   1. A geometric figure is any subset of the set we call space.
   2. The set of points consisting of two rays from a single point is called an angle; the rays are called the sides of the angle.
   3. The set of points consisting of three points not all on one line and all the points between the pair of points is called a triangle.
   4. Congruence ($\cong$) means the same length or same size.

C. Plans and space figures
   1. All geometric figures may be separated into two groups.
      a) Plane figures are those figures whose points all lie in one plane.
      b) Space figures are those figures whose points do not lie in one plane.
   2. Some examples of space figures are cubes, prisms, pyramids, cones, and cylinders.
   3. Some familiar plane figures are triangles, squares, and parallelograms.
   4. The set of points in a plane at a given distance from a point is called a circle; the distance around a circle is called the circumference.

D. Perimeter, area, and volume
   1. The perimeter of a polygon is the sum of the length of the sides.
   2. The area of a set of points is a number which measures the "amount of surface" in the set.
   3. The equation for the volume is: V = lwh.

APPENDIX K

ITEM ANALYSIS FOR FORM A, A TEST OF MATHEMATICAL UNDERSTANDINGS[a]

Form A Item	P of Successes Upper 27%	P of Successes Lower 27%	Difficulty Index	ϕ (phi)[b] or Item Discrimination
1[c]	100	72	86	.28
2	83	83	83	.00
3	66	72	69	-.05
4	50	46	58	.16
5	94	78	86	.17
6	94	78	86	.17
7	78	52	67	.22
8	89	66	78	.22
9	61	28	44	.33
10	83	39	61	.44
11	72	22	47	.50
12	83	83	83	.00
13	78	78	78	.00
14	94	89	92	.05
15	83	61	72	.22
16	56	06	31	.50
17	50	11	31	.39
18	89	89	89	.00
19	89	50	69	.39
20	100	88	94	.11
21	89	11	50	.78
22	39	27	33	.11
23	100	83	92	.17
24	67	50	58	.17
25	67	52	61	.11
26	94	72	83	.22
27	11	0	05	.11
28	61	22	42	.39
29	39	22	31	.17
30	89	61	75	.28
31	72	17	44	.17
32	94	33	64	.61
33	28	22	25	.05

APPENDIX K -- Continued

Form A Item	P of Successes Upper 27%	P of Successes Lower 27%	Difficulty Index	ϕ (phi)[b] or Item Discrimination
34	72	33	53	.39
35	89	22	56	.67
36	44	39	42	.17
37	89	52	72	.33
38	39	17	28	.22
39	67	33	50	.22
40	83	28	56	.55
41	25	17	11	.11
42	89	73	83	.11
43	94	61	78	.33
44	56	39	47	.39
45	89	67	78	.22
46	39	56	47	.17
47	44	28	36	.17
48	11	28	19	-.16
50	61	11	36	.50
51	67	28	47	.17
52	94	61	78	.33
53	72	28	50	.11
54	56	22	39	.33
55	22	22	22	.00

[a]Since a printing error was made in Item 49, this item was voided and not used in obtaining either a total score or item analysis.

[b]When phi (ϕ) is equal to or greater than .28, the item is significant at the .05 level; phi was tested for significance by the use of chi square ($\chi^2 = N\phi^2$).

[c]Reading the table from left to right, on the first item, 100 per cent of the upper group and 72 per cent of the lower group gave correct answers to the test items; the difficulty index was 73 and the discrimination index was 55.

APPENDIX K -- Continued

ITEM ANALYSIS FOR FORM B, A TEST OF MATHEMATICAL UNDERSTANDINGS[a]

Form B Item	P of Successes Upper 27%	P of Successes Lower 27%	Difficulty Index	ϕ (phi)[b] or Item Discrimination
1[c]	67	39	53	.28
2	94	94	94	.00
3	72	50	61	.22
4	94	77	86	.16
5	89	83	86	.05
6	83	72	78	.11
7	89	38	64	.50
8	83	50	67	.33
9	39	0	19	.40
10	94	83	89	.11
11	61	28	44	.22
12	83	78	81	.05
13	67	33	50	.22
14	48	33	31	.15
15	33	22	27	.11
16	11	16	14	-.05
17	44	17	30	.28
18	100	39	69	.61
19	100	39	69	.61
20	100	100	100	.00
21	67	17	42	.50
22	78	72	75	.05
23	83	56	69	.28
24	94	56	75	.39
25	33	17	25	.17
26	83	39	61	.44
27	67	61	64	.06
28	28	17	22	.11
29	89	67	78	.22
30	39	17	28	.22
31	83	50	67	.33
32	39	22	31	.05
33	94	89	92	.05

APPENDIX K -- Continued

Form B Item	P of Successes Upper 27%	P of Successes Lower 27%	Difficulty Index	ϕ (phi)[b] or Item Discrimination
34	89	72	81	.17
35	67	33	50	.22
36	92	83	78	.21
37	94	55	75	.39
38	100	78	89	.22
39	78	56	67	.22
40	05	33	19	-.22
41	33	17	25	.17
42	33	33	33	.00
43	94	50	72	.44
44	58	39	33	.21
45	100	67	83	.22
46	83	61	72	.22
47	78	44	61	.22
48	83	83	83	.00
50	89	61	75	.28
51	89	61	75	.28
52	56	39	47	.17
53	56	33	44	.22
55	28	22	25	.05

[a]Since a printing error was made in Item 54, this item was voided and not used in obtaining either a total score or item analysis.

[b]When phi (ϕ) is equal to or greater than .28, the item is significant at the .05 level; phi was tested for significance by the use of chi square ($\chi^2 = N\phi^2$).

[c]Reading the table from left to right, on the first item, 100 per cent of the upper group and 72 per cent of the lower group gave correct answers to the test items; the difficulty index was 73 and the discrimination index was 55.

APPENDIX L

RAW SCORES, RESEARCH GROUP IDENTIFICATION, AND TEST FORM FOR EIGHTY-NINE ELEMENTARY SCHOOL TEACHERS UTILIZED IN THIS STUDY

Research Group	Identification Number	Test of Mathematical Understandings		Dutton Arithmetic Attitude Inventory	
		Pre-test	Post-test	Pre-test	Post-test
A_1	87	36	40	8.2	8.4
	82	33	25	7.7	8.5
	35	33	39	8.7	8.2
	88	27	38	8.2	8.2
	22	25	27	5.6	6.3
	77	24	51	7.9	8.2
	80	18	13	2.8	3.1
	83	15	18	2.5	7.0
	01	26	29	7.3	7.4
	10	35	37	7.6	7.9
A_2	13	46	55	8.4	8.7
	20	43	33	8.4	8.5
	08	33	36	8.2	8.2
	14	31	33	7.9	8.2
	42	33	25	7.7	8.5
	26	30	37	7.7	8.4
	27	30	39	7.0	7.9
	52	30	37	7.6	8.2
	16	29	37	7.7	7.9
	39	28	31	7.9	8.2
	34	27	35	6.3	7.6
	11	20	27	3.7	7.3
	37	36	40	8.2	7.4
	45	33	39	8.7	8.2
	38	27	38	8.2	8.2
	12	25	27	5.6	6.3
	17	24	51	7.9	8.2
	50	18	13	2.8	3.1
	33	15	18	2.5	7.0
	31	35	43	2.5	8.2
	51	36	42	2.5	8.2
	47	18	18	8.2	8.2

APPENDIX L -- Continued

Research Group	Identification Number	Test of Mathematical Understandings		Dutton Arithmetic Attitude Inventory	
		Pre-test	Post-test	Pre-test	Post-test
B_1	H24	44	25	7.0	7.7
	H14	40	41	7.7	7.9
	H21	37	38	7.0	7.7
	H31	37	35	4.9	7.0
	H13	35	27	7.0	3.7
	H07	32	37	7.7	8.7
	H28	34	34	6.3	7.0
	H17	31	29	6.3	7.3
	H22	27	28	6.3	7.3
	H30	24	27	5.6	7.9
	H08	29	29	6.3	7.0
	H20	29	32	7.4	8.2
	H16	28	25	7.7	8.2
	H37	20	25	7.0	7.9
	H36	25	29	8.2	7.9
	H23	26	39	7.0	7.3
	H34	21	44	6.3	8.2
	H32	18	18	7.0	7.7
	H29	19	22	7.0	3.2
	H09	11	18	8.2	8.2
B_2[a]	H27		24		7.3
	H11		43		7.2
	H19		46		7.9
	H35		25		7.7
	H26		30		7.0
	H02		17		7.9
	H12		17		6.3
	H33		23		3.4
	H25		22		7.7
	H01		23		2.8
	H18		30		3.1
	H03		29		7.3
	H04		25		7.9
	H05		21		8.1
	H06		19		7.9

APPENDIX L -- Continued

Research Group	Identification Number	Test of Mathematical Understandings		Dutton Arithmetic Attitude Inventory	
		Pre-test	Post-test	Pre-test	Post-test
C	IH17	44	40	7.9	8.2
	IH26	44	46	7.9	7.7
	IH04	32	25	7.3	7.3
	IH01	35	36	7.7	7.3
	IH14	34	35	8.2	8.2
	IH19	33	28	8.4	8.4
	IH	33	35	7.9	8.2
	IH03	23	23	7.9	7.7
	IH25	33	37	8.2	7.9
	IH24	33	28	8.2	7.3
	IH23	30	26	7.7	7.7
	IH02	22	19	7.3	7.3
	IH13	31	32	8.4	8.4
	IH16	31	36	7.9	7.7
	IH21	30	32	8.4	7.9
	IH33	30	24	8.4	7.9
	IH06	30	33	7.9	7.9
	IH22	24	28	8.2	8.2
	IH27	39	34	7.9	7.9
	IH29	26	27	7.7	7.3
	IH15	24	25	8.2	8.4
	IH28	32	34	7.9	8.2

[a]Group B_1 and Group B_2 were the experimental groups used to observe test-retest effect. No pre-test was given Group B_2.

APPENDIX M

RAW SCORES FOR RESPONSES OF TWENTY-EIGHT STUDENTS TO CLASSROOM PRACTICE SCHEDULE OF QUESTIONS

Group	Identification Number	Pre-test						Post-test					
		Qualitative Classroom Practices						Qualitative Classroom Practices					
		Materials	Activities	Organizations	Teaching Aids	Evaluation	Total	Materials	Activities	Organizations	Teaching Aids	Evaluation	Total
A_1	14K	10	16	9	4	7	46	13	19	14	8	9	63
	15R	18	8	8	8	6	48	16	17	10	4	9	56
	15B	15	5	10	4	6	40	15	16	15	4	7	57
	25B	15	5	10	4	6	40	15	16	15	4	7	57
	15S	8	3	11	4	10	36	12	15	15	4	8	54
	4SC	14	11	11	4	5	45	18	18	15	8	8	67
	25S	8	3	11	4	10	36	12	15	15	4	8	54
	4TS	8	17	7	8	5	45	8	17	7	8	5	45
	6WG	3	15	16	9	12	55	6	15	9	9	12	51
	6WB	12	7	13	8	8	48	13	7	13	8	8	49
	24K	10	16	9	4	7	46	13	19	14	8	9	63
	4SC	14	11	11	4	5	45	18	18	15	8	8	67
Sub-total		135	117	126	65	87	530	159	192	157	77	98	683

APPENDIX M -- Continued

Group	Identification Number	Pre-test: Qualitative Classroom Practices						Post-test: Qualitative Classroom Practices					
		Materials	Activities	Organizations	Teaching Aids	Evaluation	Total	Materials	Activities	Organizations	Teaching Aids	Evaluation	Total
C	5BS	15	14	9	4	11	53	12	13	10	4	12	51
	1TS	7	12	6	4	9	38	11	11	5	4	9	40
	1LR	4	12	12	4	5	37	8	11	14	4	7	44
	1KW	8	9	3	4	8	32	2	13	7	0	7	29
	2TJ	3	6	5	9	7	30	5	14	5	0	6	30
	2MW	8	9	3	4	8	32	12	11	7	0	7	37
	2TR	7	12	6	4	9	38	11	11	5	4	9	40
	1CB	2	17	8	4	12	43	4	16	8	4	11	43
	1TS	3	6	5	9	7	30	5	14	5	0	6	30
	1CS	4	7	7	6	14	38	11	10	5	5	9	40
	1BS	15	14	9	4	11	53	12	13	10	4	12	51
	2W	2	17	8	4	12	43	4	16	8	4	11	43
	2LT	4	12	12	4	5	37	8	11	14	4	7	44
	1LD	15	16	7	6	8	52	15	12	8	5	8	48
	2CS	4	7	7	6	14	38	11	10	5	5	9	40
	1LD	14	11	12	6	5	48	11	12	8	6	8	45
Sub-total		115	181	119	82	145	642	142	198	124	53	138	655

APPENDIX M -- Continued

Group	Identification Number	Pre-test			Post-test		
		Quantitative Classroom Practices			Quantitative Classroom Practices		
		Amount of Discussion	Number of Non-Textbook Activities	Number of Non-Textbook Materials	Amount of Discussion	Number of Non-Textbook Activities	Number of Non-Textbook Materials
A_1	14K	50	2	4	50	4	4
	15R	50	0	2	50	1	2
	15B	50	2	2	50	1	3
	25B	25	1	1	33	1	3
	15S	25	2	0	25	2	0
	4SC	33	0	2	25	0	2
	25S	33	2	2	33	2	2
	4TS	33	0	0	33	0	0
	6WG	50	0	1	50	0	3
	6WB	25	1	4	33	1	3
	24K	33	0	2	33	0	2
	4SC	33	2	1	50	2	2
Sub-total		440	12	21	465	14	26

APPENDIX M -- Continued

Group	Identification Number	Pre-test: Quantitative Classroom Practices			Post-test: Quantitative Classroom Practices		
		Amount of Discussion	Number of Non-Textbook Activities	Number of Non-Textbook Materials	Amount of Discussion	Number of Non-Textbook Activities	Number of Non-Textbook Materials
C	5BS	33	1	1	33	1	0
	1TS	33	1	1	33	1	1
	1LR	33	0	0	33	1	2
	1KW	33	1	2	33	1	2
	2TJ	50	0	5	60	0	4
	2MW	50	0	0	50	0	1
	2TR	25	0	1	25	1	2
	1CB	25	1	1	25	0	0
	1TS	50	1	1	50	1	0
	1CS	50	1	1	50	1	1
	1BS	25	0	5	25	1	4
	2W	25	1	0	25	1	1
	2LT	33	0	2	33	0	2
	1LD	25	0	1	25	0	2
	2CS	30	0	1	33	1	2
	1LD	30	1	1	33	0	0
Sub-total		550	8	22	566	10	22