A PILOT PROJECT FOR THE INVESTIGATION OF THE EFFECTS OF A MATHEMATICS LABORATORY EXPERIENCE: A CASE STUDY

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ABSTRACT

A PILOT PROJECT FOR THE INVESTIGATION OF THE EFFECTS OF A MATHEMATICS LABORATORY EXPERIENCE: A CASE STUDY

By

Paul Henry Boonstra

This research reports the design and execution of a pilot study to develop and evaluate techniques for the investigation of the effects of a mathematics laboratory experience upon the teaching behavior of its recipients. It was the purpose of this study to record and analyze the classroom behavior of student teachers who had been given two mathematics laboratory experiences. At Michigan State University the course Foundations of Arithmetic is a required mathematics course for all prospective elementary school teachers. In the fall of 1967 the format of this course was changed so that each student received a two-hour mathematics laboratory experience each week. In addition to other benefits, it was expected that the laboratory experience would have an effect on the teaching technique which these students would use teaching mathematics. The study answered two basic questions: (1) Do student teachers who have been taught a concept in a

mathematics laboratory use manipulative materials as they teach the same concept? (2) Do student teachers who have experienced a student-centered learning situation in a mathematics laboratory employ a student-centered teaching approach as they teach?

The subjects of this study were students who were enrolled in an off-campus mathematics methods course and who were in the process of doing their student teaching. Two two-hour laboratory experiences, similar to those given at Michigan State University in connection with the Foundations of Arithmetic course, were given to the students in the methods course. One laboratory experiment was concerned with the concept of mathematical relations and the other with the concept of mathematical functions. Those students who were doing student teaching in the fourth, fifth or sixth grade were selected as the subjects of the study and were asked to teach some aspect of the function concept to the class in which they were student teaching.

A novel method of data-gathering and data-analyzing was used in the study. Data were collected from the classroom by means of a tape recorder and a movie camera. The camera exposed a single frame of film once every three seconds. An audio-oscillator and electronic timer were developed to activate the camera and simultaneously enter an audio signal into the tape recorder. When the data were analyzed the timer was utilized to advance the film in a Kodak MFS-8 projector. The film was advanced in synchronization with the audio tone in the tape recording. The audio tone also provided a signal to the analyzer to make a judgment concerning the verbal behavior.

The verbal behavior in the classroom was analyzed by means of the Flanders Interaction Analysis. ID ratios as computed from the Flanders instrument were used to measure the student-centeredness of the classroom.

On the basis of the case studies it was concluded that two laboratory experiences are not sufficient to effect the teaching behavior of student teachers. The laboratory experiences did not result in the use of manipulative materials and did not effect the teacher-centeredness of the classrooms observed.

The data-gathering and data-analyzing processes used in this pilot study were very successful. It is recommended that studies concerning the effectiveness of teaching techniques use these processes.

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Paul Henry Boonstra

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To

Phyl

Deb, Pat, and Cindy

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Even as history is not a set of isolated events, so too this study is not the result of one person. The author is indebted to many, particularly to:

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my God. May any insights into the process of education which acrue to this study be used so children better may know Him and His creation.

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CHAPTER I

PURPOSE

Introduction

This research reports the design and execution of a pilot study to develop and evaluate techniques for the investigation of the effects of a mathematics laboratory experience upon the teaching behavior of its recipients. The laboratory experience that is offered as a part of the Foundations of Arithmetic course at Michigan State University is part of a relatively recent renovation of that course. Since the course can be, and generally is, taken early in the student's college career, very few of the students who had had the laboratory experience were yet teaching. Because of this the subjects of this study were student teachers who had received a selected sample of laboratory experiences similar to those given to students in the Foundations of Arithmetic course at Michigan State University. The purpose of this study was to record and analyze the teaching behavior of the subjects as they taught the concepts that previously had been taught them in the mathematics laboratory. The investigator was concerned with two questions: (1) Do student teachers who are taught a concept through the use of manipulative materials use manipulative materials as they teach the concept? (2) Do student teachers who have experienced a student-centered learning situation in a mathematics laboratory employ a student-centered teaching approach as they teach?

A projected two-fold advantage of such a study anticipated that it would provide not only information for evaluating the program at Michigan State University, but also information concerning the effectiveness of the laboratory approach to teacher education. One of the goals of the study was to develop techniques which would provide guidelines for future studies of the effectiveness of such a laboratory approach to teaching mathematics and science in the elementary school.

Background

One of the important developments in recent years in the teaching of mathematics in the elementary school has been the increased use of manipulative materials. Through the use of such manipulative materials children are becoming involved in the "doing" of mathematics. The emphasis is shifting from a passive situation in which the students are told what the important mathematical facts are, to an active one in which the students become engaged in the use of physical materials in order to discover important relations. Leadership in this type of teaching of mathematics in the elementary school has been provided by the Nuffield Project in England, and it is still one of the better exemplars of it [2]. In the United States, the Madison Project, under the direction of Dr. R. Davis [26], is an illustration of a program which stresses the axiom that children learn by doing.

As attempts are made to introduce this style of teaching into the elementary schools, it becomes apparent that this new teaching technique dictates a re-evaluation of the teacher education program in the colleges. Educators who are responsible for the education of prospective elementary school teachers are becoming aware that it is just as important for prospective teachers to be doing mathematics as they learn as it is for elementary school children to be doing mathematics as they learn. Dr. E. L. Lomon puts it this way:

> The discovery approach should be part of the teaching technique used with prospective teachers. The 1966 conference advocated a correlation between the methods and content courses in college. This does not go far enough. This relation must be more than that the two types of courses talk about each other. Some, perhaps all, of these courses should be so integrated that content is taught in the same manner as is advocated for teaching children in the elementary school. While this approach illustrates effective teaching strategies, it also is the most effective way for college students to learn. They, too, should benefit from having to think, to reason, and answer questions for themselves [40].

Speaking to the same topic, Fitzgerald asks, "Might not teachers provide such a curriculum more effectively if they had personally experienced a learning situation in mathematics that was individually oriented and activity centered" [32]?

Concurrent with the increased emphasis on student discovery in the mathematics classroom is the rapid increase in the number of commercially available manipulative materials which can be used to teach mathematical concepts in the elementary school. Some textbook publishers market physical materials which are designed for use in correlation with their texts. It seems desirable that prospective elementary teachers should be made aware of the existence of such materials and instructed in their use.

In an attempt to meet the needs described above the mathematics department of Michigan State University altered the mathematics course, Foundations of Arithmetic, which is required of all prospective elementary school teachers. This course was formerly taught to large lecture sections by the traditional lecture method. The class met for one hour, four times per week. This format was changed so that the students now attend three one-hour lecture sections, and a two-hour laboratory session each week. The enrollment for this course is approximately 300 students per term. The students attend the lecture sessions en masse, but the laboratory sections are small, about twenty students to each section. The purpose of the laboratory is threefold: (a) to learn the mathematical concepts of the course, (b) to become familiar with manipulative materials and how they are used to teach a concept, and (c) to experience a student-centered rather than a teacher-centered classroom situation [3]. At the time of this study this altered form of the Foundations of Arithmetic course had been in operation for five terms, and the author had been an instructor in some of the sections in each of the five terms.

Definitions

The following are definitions of terms which are used frequently in this dissertation.

Throughout this study the idea of a mathematics laboratory is considered to be a learning situation which involves materials, instruments, and/or equipment, with the aim of deducing and abstracting therefrom certain mathematical concepts and understandings.

By the laboratory method the author means a teaching technique which utilizes activity by the students with materials other than blackboard, paper for writing, or library reference materials.

In this study two mathematical terms are used so frequently that they deserve to be stated here. A function is defined as follows:

> A function, F, of a set S to a set T is a subset of S X T with the properties:
>
> (a) For each $s \in S$ there is a $t \in T$ such that $(s,t) \in F$
>
> (b) If (s,t) and (s,r) are in F, then $s=r$ [6].

Zwier and Nyhoff [18] give a slightly different definition of function, which stresses the correspondence between sets.

> Given two sets A and B, a <u>function</u> f from the set A to the set B, is a correspondence that associates with each element a in A a unique element, which we denote by f(a), in B.

While these definitions vary slightly, no attempt was made in this study to emphasize one over the other. It is noted, however, that the second definition is more readily adapted to the language of animation which is typical of that employed in a mathematics laboratory.

The second mathematical term is <u>relation</u>. A subset R of S X S is a binary <u>relation</u> on S [6]. Though a relation is merely a special type of subset, one chosen from the Cartesian Product, it is often instructive to know how the subset was chosen from the Cartesian Product. In order to accomplish this a relation is often designated by describing the set S, and the relationship between elements of the subset. For example, consider the set of words on this page and the relation, "has more letters in its spelling than." The ordered pair (the,is) belongs to this relation.

One of the goals of a mathematics laboratory is that the student experience a student-centered rather than a teacher-centered learning situation. By a <u>student-centered</u> learning situation is meant a classroom situation in which the student is free to explore a problem along avenues of his own choosing. In such a situation the role of the teacher becomes that of a person who suggests ways of finding solutions to problems, and one who prods the students by asking key questions and suggesting related problems. The teacher is viewed as a resource person rather than a final authority. On the contrary, in a teacher-centered learning situation the teacher not only asks the questions but also presents the solutions. The teacher is viewed as the final authority.

In order to determine the amount of teacher-centered learning that had occurred in the classroom during the observations which were made, ID (Indirect-Direct) ratios and revised ID ratios were computed. ID ratios and revised ID ratios are measures defined by Flanders [1] as the amount of indirect teacher influence in verbal classroom behavior divided by the amount of direct teacher influence. The ID ratio for a given observation is obtained from the Flanders Interaction Matrix (see pages 17-18, and Appendix C) and is computed by dividing the total number of tallies in categories one through four of the matrix by the total number of tallies in categories five through seven. The revised ID ratios are computed by dividing the total number of tallies in categories one through three by the total number of tallies in categories six and seven.

Limitations

A purpose of this pilot study was to investigate the effects of a mathematics laboratory experience upon the teaching behavior of its recipients. The study is limited to the analysis of the teaching behavior of fourteen student teachers who were given two laboratory experiences: one concerning the concept of mathematical relations and a second concerning the concept of mathematical functions. The study is limited to an analysis of their teaching behavior as the student teachers made an initial attempt to teach the concept of function to their students.

A second purpose of the study was to develop techniques for the investigation of the effect of laboratory experiences of pre-service teachers. The study is limited to analysis of teaching behavior by applying the Flanders Interaction Analysis to analyze verbal behavior.

Assumptions

It was assumed that the verbal behavior of a teacher in the classroom is a reliable indicator of the total behavior of the teacher relative to her students. Also it was assumed that high ID ratios would indicate a student-centered learning situation, and that low ID ratios would indicate teacher-centered learning situations. It was further assumed that the behavior of the student teacher is indicative of the subsequent behavior of that person as a teacher, and, hence, observations made of student teachers can be used to determine the impact of the teacher education program on the classroom behavior of teachers.

Overview of Procedure

In order to determine the effectiveness of the laboratory approach as a teacher education method, a sample of fourteen student teachers was selected. These student teachers were enrolled in an off-campus mathematics methods course which was used as the vehicle to present them with a selected sample of laboratory experience. The fourteen teachers were teaching in schools scattered throughout the northern suburbs of Detroit, Michigan. These suburbs are predominately middle-class, white areas.

After the student teachers had received two laboratory experiences which closely paralleled those given at Michigan State University, they were asked to teach the function concept to their classes. The author observed the class while this concept was taught, photographing the activity once every three seconds and recording the verbal behavior of the teacher by means of a tape recorder. This observational record was analyzed by means of a Flanders Interaction Analysis.

The student teachers were asked to present the author with a record of the mathematics courses which they had taken and the grades which they had received in them. Following this, a search was made of their university records to substantiate and refine the information given by the student teachers.

The format of this investigation is that of a case study. The case study includes information about the subject's past experience with mathematics, a description of the activities in the classroom that was observed, and an analysis of the information gained by the Flanders Interaction instrument.

Chapter I of this dissertation has given the background and purpose of this study, definitions of terms used in it, limitations of the study, assumptions made in the study, and an overview of procedure. The second chapter contains the results of the review that was made of pertinent literature. Chapter III describes more completely the procedure used in this study. Chapter IV contains the case study of the fourteen subjects. Conclusions and recommendations are presented in Chapter V.

CHAPTER II

REVIEW OF LITERATURE

Introduction

The purpose of this study was to record and analyze the classroom behavior of student teachers who had received a selected sample of laboratory experiences. The student teachers were asked to teach the mathematical concept which was the basis for the laboratory experience, to students in their own teaching situation. In order to carry out this purpose it was necessary to gather data from a classroom, hence one section of this chapter is concerned with a review of some of the methods that have been used to collect data from a classroom setting.

Once the raw data have been collected they must be analyzed. The method of analysis may even dictate the data collecting technique. For this reason the literature was reviewed to determine what methods have been used to analyze student-teacher interaction and a section of this chapter is devoted to the results of that search.

It was hypothesized that the student teachers of this study would imitate the teaching technique used in their laboratory experience. This raised the question whether other teacher education methods had been studied to determine the effect they had on teachers' behavior. Hence a third section of this chapter is concerned with the effect of methods courses, content courses, pre-service courses, and in-service workshops upon the behavior and attitudes of classroom teachers.

Since a selected sample of mathematics laboratory experience was used in this study, some criteria for selection were necessary. The portion that was selected for the study was chosen because of the amount of manipulative material involved and because of the significance of the topic to the field of elementary mathematics. A fourth section of this chapter establishes a rationale for this significance.

Though the laboratory method of teaching mathematics is relatively new, there are indications in the literature that the use of this technique in the elementary school is desirable. The final section of this chapter reports these.

In summary, then, the sections of this chapter are: (a) Data Collecting Methods, (b) Methods of Analyzing the Interaction of Students and Teachers, (c) The Effect of Methods Courses, Content Courses, and Workshops, (d) The Importance of the Function Concept, and (e) The Mathematics Laboratory.

Data Collecting Methods

It has long been recognized that it is difficult to collect data from a classroom. Hughs observed, "Probably all interested parties would agree that the best way to secure a record of what is happening in classrooms would be with motion pictures and sound synchronized" [11]. She goes on to say that with only one camera and one microphone the recording still would be selective. It is virtually impossible to record all that transpires in a classroom. But in spite of the fact that complete recording is not possible, efforts have been made to obtain good selection samples. A method frequently used is verbatim recording, either by machine or by stenographic methods, of verbal behavior. Lewin, Lippitt, and White used this method in 1939 in their studies of the social-emotional climate of the classroom [35]. Anderson, Brewer, and Reed [3], in 1946, Withall [55] in 1949, and Gallegher [10], in 1962, used this method in studies of verbal interaction in the classroom.

Schuler, Gold, and Mitzel [16], in 1962, used a kinescope recorder and television cameras mounted in classroom walls and operated by remote control. This technique has the obvious advantage that the observer is not in the classroom so that the class behavior would tend to be more normal than if the observer were physically present. However, for purposes of this study, the technique lacks the required portability. The installation of television cameras in the classroom would require that a prior visit be made to the classroom, and it would be difficult to handle the technical aspects of installation while the classes were being conducted.

Medley and Mitzel [42] in 1958, used an instrument designed for use by a single observer visiting the classroom. Their instrument, entitled OScAR--Observation Schedule and Record--is a listing of very many likely classroom activities. The observer records behavior by checking as many of the listed behaviors as he sees in a specified period of time. Another part of the instrument is a listing of the many types of social and administrative structures that can occur in a classroom. Still another part of the instrument is a listing of many different types of teacher behavior. The observer checks each of the areas for a prescribed length of time. This method seems to be highly dependent on the instantaneous judgment of the observer and it does not allow for any way in which these judgments can be checked for validity and reliability.

Kowatrakul [38], in 1959, introduced a variation into the recording of classroom behavior. He constructed a list of categories which describe student behavior. His technique was to observe one student just long enough to make one tally for him. He then moved to another subject and observed him long enough to make one tally. This was continued until a tally was recorded for each subject. Then the entire process was repeated, and this continued for the duration of the class period. While it is true that this process does produce data relative to every subject in the classroom, it has the disadvantage that much behavior goes unrecorded.

Schoggen [51], used a novel approach to the problem of collecting classroom data. Feeling that an observer who is constantly writing tends to miss much of the activity of the classroom, and hence to make a data record which lacks validity, he devised a method by which the observer recorded his observations by speaking rather than writing. The observer wore a face mask in which a microphone was housed, and he carried a battery powered tape recorder on a strap over his shoulder. The face mask was so designed that when the observer spoke the sound was trapped in the mask and was not audible even to persons nearby. Schoggen found that even with such curiosity arousing equipment as he used, an initial demonstration of the equipment to the children in the classroom was sufficient to satisfy their curiosity so that the observer could operate virtually unnoticed. Schoggen makes this comment concerning his equipment,

> Like other scientific specimens, a specimen record is not a perfect representation of the original. In our judgment, the described method improves substantially the quality of the records, with less discrepency between original event and the specimen which we are able to capture and preserve for scientific study.

On the basis of this review it was determined that the data for this study would be collected by means of a wireless microphone carried about the teacher's neck, and a movie camera which would record the activity of the classroom. Because of the analysis which was to be done, and which is described in the next section, the camera was equipped so that it would record a single frame every three seconds. Having the dialogue of the teacher and a photograph of the class activity, it was felt that a highly valid specimen would be obtained. Because of Schoggen's findings concerning the necessity of a brief demonstration of the equipment, the recording apparatus was explained to each class before the mathematics lesson was begun.

Methods of Analyzing Student-Teacher Interaction

Most of the methods used for analyzing classroom behavior are variations of what has been called the category system. A list of behavior categories is prepared which the observer then memorizes according to some code, or the list is printed in a fashion which will allow for easy checking in the classroom. In either case the observer notes each instance of class behavior by recording a tally by the behavior category which describes the observed behavior. This tally sheet constitutes the data for the analysis. The remainder of this section describes some of the ways these data can be analyzed.

One method of analysis is merely a count of the number of tallies in each category. Pucket [49], in 1928, essentially used this method of analysis. He analyzed his data by making a count of the number of tallies in each category. He found it advantageous to his study to combine categories into groups and compare these groups by tally count. He also weighted the tallies so that a tally in one group carried more significance than a tally in another.

It would seem that the category system would gain refinement by an increase in the number of distinct categories identified, but this process can be carried to the extreme. Jayne [36] identified 184 categories of behavior, but later reduced this list to eighty-four since many of the behaviors occurred seldom, if ever, in the classroom.

Withal [55], in 1949 used a category system in which the list of behaviors was limited to verbal behaviors. Later Hughes [11] found it advantageous to add non-verbal behavior to the list of Withal. Flanders [1] added the dimension of time to the category system of data analysis. Because the Flanders System of Interaction Analysis is the system that was used to analyze the data of this study, a more complete description of that system is presented here.

The Flanders System of Interaction Analysis added the time dimension to categorical systems by requiring that a judgment be made concerning verbal behavior in the classroom at three-second intervals. Emphasis is placed on verbal behavior with the assumption that verbal behavior is an adequate sample of an individual's total behavior. Also, verbal behavior can be observed with higher reliability than can non-verbal behavior.

Flanders has identified the following ten categories: (1) accepting student feeling, (2) giving praise, (3) accepting, clarifying, or making use of student's ideas, (4) asking a question, (5) lecturing, giving facts or opinions, (6) giving directions, (7) giving criticism, (8) student response, (9) student initiation, and (10) confusion or silence. The first seven of these categories describe teacher talk and categories eight and nine describe student comment. Table C-1 in Appendix C summarizes these various aspects of the Interaction Analysis.

To obtain an adequate sample of interaction, Flanders suggests that a mark for recording a number should be made approximately every three seconds, which would yield twenty instances per minute. Some ground rules that are suggested are as follows:

GROUND RULES

Rule 1: When not certain in which of two categories a statement belongs, choose the category that is numerically farthest from category five, <u>except</u> ten.

Rule 2: If the primary tone of the teacher's behavior has been consistently direct or consistently indirect, do not shift into the opposite classification unless a clear indication of shift is given by the teacher.

Rule 3: The observer must not be overly concerned with his own biases or with the teacher's intent.

Rule 4: If more than one category occurs during the three-second interval, then all categories used in the interval are recorded; therefore, record each change in category. If no change occurs within three seconds, repeat that category number.

Rule 5: If a silence is long enough for a break in the interaction to be discernible, and if it occurs at a three-second recording time, it is recorded as a ten [1, 17].

The observational record is, then, a sequence of numbers which indicates the behavior for successive three-second intervals. The sequence is translated into a set of ordered pairs by pairing each number with the number which follows it. For example, if the observation record contained the sequence 2, 3, 7, 8, 5, 9, the set of ordered pairs that would be generated is (2,3), (3,7), (7,8), (8,5), (5,9). Since ten categories of behavior are differentiated in the observation record, 100 distinct ordered pairs can be obtained in this manner. After the entire observation record is translated into ordered pairs, the number of each type of ordered pair is tallied. This tally is recorded in an interaction matrix, a sample of which appears in Appendix C. The value of this type of analysis is that not only is the number of occurrences of a particular type of behavior obtained, but also the number of times a particular behavior is followed by a certain other behavior. For example, the ordered pair (8,5) would indicate that a three-second interval of student response which was initiated by the teacher was followed by a three-second interval of teacher lecture.

The Flanders Interaction Analysis was used to analyze the data obtained in this study.

Methods Courses, Content Courses, Workshops and Teaching

The process of imitation is generally recognized as one of the means whereby people acquire a large portion of their own behavior. Cohen says, "I believe the majority of the teachers will teach the way they were taught. If teachers listen to lectures all the time, they will end up lecturing to children" [24]. In a study of student teachers of business subjects, Cooper found that attitudes toward classroom activities held by student teachers were more like those of their supervising teachers than those of their methods teachers [57]. However, Cooper goes on to report that when given a chance for second thought, the expressed attitudes of the student teachers became nearly parallel to those of their methods teachers. Price [48] concluded that student teachers acquire many of their teaching practices from their supervising teachers.

Gilbert [60] found that student teaching does not seem to be a contributing factor to a fuller understanding of arithmetic. He found that many future teachers do not possess an understanding of arithmetic which is consistent with that possessed by some seventh and eighth grade students, and that a lack of understanding seems to be a major cause for unfavorable attitudes.

Both Dossett [59] and Dickens [58] found that in-service training increased teachers' understanding of topics included in the training.

Williams [65] concludes that the most effective means of raising levels of mathematical comprehension among members of the instructional staff involve the use of experimental materials in the classroom.

In a study of the mathematical preparation of elementary school teachers at the University of Missouri, Reys [50] reports that one-third of the students did not find the content courses or the methods courses valuable to them. However, they preferred methods courses over content courses two to one.

Kanfer and Duerfeldt [37] studied vicarious learning. They found that subjects derive more benefit from observational learning during early, rather than later, stages of their attempts to master a technique. This suggested to them that "efficient use of such techniques . . . as training aids is dependent on the time of their presentation."

These studies seem to indicate that a laboratory experience must be given early in the training period. The methods courses and content courses are ineffective and should be changed--perhaps to include laboratory experiences.

It should be noted that the laboratory experience given as part of this study was given early in the term, before any of the subjects had actually begun their student teaching.

Basic Character of the Function Concept

When choosing a topic for the sample laboratory experience, the author was prompted by two considerations: the topic had to be one used in the mathematics laboratory connected with the Foundations of Arithmetic course as offered at Michigan State University, and the topic had to be one of mathematical significance. The function concept meets both of these criteria.

The function concept is one of the underlying and unifying concepts of mathematics in the primary school. E. H. Moore, in his retiring presidential address to the Mathematics Association of America in 1902, recognized this. He said,

> Would it not be possible for children in the grades to be trained in the power of observation and experimentation and reflection and deduction so that always their mathematics should be directly connected with matters of thoroughly concrete character? . . . They are to be taught to represent, according to the usual conventions, various phenomena in the pictures: to know, for example, what concrete meaning attaches to the fact that a graph curve at a certain point is going down, or is going up, or is horizontal. Thus the problems of percentage--interest, etc.--have their depiction in straight or broken line graphs [44].

Marshall Stone says about the function concept,

> In the teaching of mathematics, it is essential to start at an early stage to lay groundwork that will enable students, when they reach the stage between 15 and 18 years of age, to study this theory with understanding and master some of its numerous applications in arithmetic, algebra, geometry, and analysis [53].

Stone also says,

> To put the matter crudely, the concept of a function is included as a special case in the general concept of a relation. . . . the relations that seem most important to contemporary mathematicians are, in an overwhelming majority, functions [53].

And,

> In terms of the function concept algebra can be characterized as the study of systems of functions or operations that convert ordered sets of objects, all of the same kind, into an object of that kind [53].

Tracing the history of the function concept, H. R. Hamley writes,

> The idea that the function concept should be made the central theme of school mathematics may be said to have originated with Klein.* Others before him had advocated the inclusion of variables and functions in the school program, and even as early as 1873 Oettingen* had suggested that 'the notion of the function' should be an essential part of all mathematical work in the schools, but Klein was the first to press the view that functional thinking (functionales denken) should be the binding or unifying principle of school mathematics [33].

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*German mathematicians: Klein, (1845-1925), Oettingen, (1836-1920).

More recently D. W. Hight, discussing the history of the concept and the variety of ways that the concept has been defined, says, "In the last century much has been said about functions and the concept has become a central theme in secondary school curricula" [34].

It is established then that the function concept has been considered by mathematicians to be a topic basic to elementary mathematics.

The Laboratory Method

There is evidence in the literature that a laboratory method of teaching mathematics has long been advocated by leading mathematicians and educators. Note again the quotation from E. H. Moore, "Would it not be possible for children in the grades to be trained . . . so that always the mathematics should be directly connected with matters of a thoroughly concrete character" [44]? This quote indicates that in 1902 a leading mathematics educator felt that mathematics should relate to concrete matters. In 1927, C. A. Austin wrote,

> Geometry is essentially an experimental science, like any other, and . . . it should be taught observationally, descriptively and experimentally . . . the inherent nature of the subject matter demands a scientific and experimental treatment . . . the child to whom the subject is taught is fundamentally a scientist who lives and learns by experimentation and observation in a wonderful world laboratory [23].

A mathematics laboratory is an attempt to select from the world laboratory and enhance the child's experimental and observational powers.

In 1947, Howard Fehr wrote,

> The mathematics laboratory should be used to create a spirit of research and discovery. Field work should be a part of every mathematics course from Grade III to Grade XII, and even perhaps in junior college. It should not be play, but must consist of planned experiments and testing of desired outcomes. There is not a single topic in grade or high school that cannot be exemplified and put to work in a mathematics laboratory [30].

The Intermediate Mathematics Methodology Committee of the Ontario Institute for Studies in Education reports this about a mathematics laboratory,

> A mathematics laboratory places a student in a problem situation that requires him to go through an active exploratory phase before he can arrive at a conclusion. . . . Because the student . . . is learning by discovery . . . the math lab is one of the favorite techniques of teachers who are convinced that learning by discovery is the best approach to teaching [15].

While the literature review could uncover no studies concerning the effectiveness of mathematics laboratories--no doubt due to their relative newness on the American education scene--some studies were found concerning the merits of discovery learning, the technique used in a mathematics laboratory.

Swick [64] found, in a study conducted in Kingsport, Tennessee public schools in 1954, that there was "strong support to the desirability of using multi-sensory aids in teaching both arithmetical computation and reasoning." His findings give no support to the conjecture that such a program had special value either for pupils of high ability or for pupils of low ability. Interestingly, his study showed improvement in attitude by both pupils and teachers, and continued use of multi-sensory aids by the teachers.

Price [63], in 1965, prepared sample discovery lessons for tenth grade general mathematics students and found groups taught with these lessons showed a significant gain in inductive reasoning over the control group. Also the experimental group showed a positive attitude change toward mathematics while the control group showed a negative change.

CHAPTER III

EXECUTION

Procedure

In order to determine the effect of a series of selected mathematics laboratory experiences upon the teaching behavior of student teachers, it was necessary to obtain a group of student teachers who had not had a mathematics laboratory experience as part of their academic training. Such a group was found in an off-campus teaching center of Michigan State University. At the time this study was begun the group had not yet been assigned to schools for student teaching. Concurrent with the student teaching experience, the group was enrolled in a mathematics methods course which met once each week for a period of approximately three hours. The professors teaching the methods course allowed the author to use the better part of two successive class meetings in order to conduct the laboratory experience. Because very few of these students had had any experience with a mathematics laboratory, it was decided to spend the first of these periods using a unit on the topic of mathematical relations. At the time of this study these units were used in mimeographed form at Michigan State University. The mimeographed sheets are reproduced in Appendix B. Subsequently they have been published. (See Bibliography [9].) In the second class period the concept studied was the concept of function. It is the function concept that is the central theme of this study.

Three weeks later the members of the class had been assigned to their student teacher assignments. All members who had been assigned to teach in either the fourth, fifth, or sixth grade were asked to prepare a lesson concerning some aspect of the function concept and teach it to their assigned class. The subjects were told that the purpose of the study was to determine the level at which the function concept could be successfully taught. In keeping with the directions of Medley and Mitzel [43], the subjects were told precisely how the data were to be gathered. Each expressed a willingness to participate in the study. The subjects were asked to prepare a lesson plan in which their objectives were described in behavioral terms. In addition they were asked to supply the author with a listing of the mathematics courses they had taken in high school and college together with the grades they had received in them. They were asked to have this lesson plan and listing ready to give to the researcher at the time of his visit to their classroom. A schedule of visits was prepared and each of the fourteen subjects was notified at least one week in advance of the time for his visit.

The remainder of this chapter is a description of the two laboratory experiences and a discussion of the method used to collect the data in the classroom.

The Relations Experiment

The relations experiment that was used in this study presupposes some exposure to the concept of relations and some knowledge of the properties attendant to it. At Michigan State University this knowledge is gained in the lectures which are given as part of the Foundations of Arithmetic course and which precede the laboratory experiment. At the time of the first meeting with the subjects the author was of the opinion that each of the students had taken a course that had been judged by admission officers to be equivalent to the Foundations of Arithmetic course taught at Michigan State University and, hence, the author assumed that the subjects were familiar with the technical terms used in the relations experiment. While reviewing these terms at the beginning of the laboratory period, the investigator felt that either they never had been taught the concept or they had forgotten the meaning of the terms connected with it. A subsequent review of the transcripts revealed that indeed many of the subjects had not had any equivalent course. Because of this situation, the first half-hour of the period was used explaining and illustrating the following terms: relation, reflexivity, symmetry, transitivity, and equivalence. When the author felt that the subjects had a sufficient knowledge of these terms to proceed, the group was divided into sets of four, Cuisenaire rods were distributed and mimeographed directions were given to each student. A copy of these directions appears in Appendix B. The purpose of this laboratory experiment was to have the students use the Cuisenaire rods to determine which properties obtain for a variety of relations. The students were encouraged to discuss the problems within their group and to manipulate the Cuisenaire rods in order to obtain answers to the questions posed in the mimeographed sheets. The author circulated about the room, answering questions about intent or procedure. This activity occupied the balance of the class period.

The Function Experiment

The class period was begun by playing the game, "Guess My Rule." The subjects were presented with a set of ordered pairs from which they had to determine some rule or function that associates the first member of the ordered pair to the second. The set of ordered pairs was obtained in this fashion: some member of the class presented a first member for an ordered pair. The leader, in this case the author, responded with the appropriate second member which was obtained by mentally applying the rule to the given first member. After each response the group was allowed to guess what rule of correspondence was being used. The author limited the functions to linear, quadratic and some simple exponential functions.

Next a set of six circular objects of different diameters was distributed. The students were asked to cut a piece of calculator tape of length equivalent to the distance around the object. The subjects came to the blackboard, marked the diameter of the object on the abscissa of a set of Cartesian coordinates, and at this point on the abscissa attached the tape in the ordinate direction. In this manner the class was led to discover that the circumference of a circle is a linear function of its diameter.

The third activity was a variation of the "Guess My Rule" game. Each student was given a 3 x 5 card upon which he could write some function of his own choosing. These cards were collected, shuffled, and redistributed. The class was divided into groups of about five students and "Guess My Rule" was played with one of the group being the leader and using the rule that he selected from the cards.

The remaining time, about one hour, was spent investigating Madison Project "shoe boxes." The boxes entitled "Centimeter Blocks," "Geoboard," "The Peg Game" and "The Tower Puzzle" were used. Each of these boxes contains materials from which data can be collected and a function discovered which describes the data. The boxes are described in detail in Appendix B.

The experiments described above were used in the mathematics laboratory at Michigan State University. The author has conducted these laboratory sections and it is his subjective judgment that the experiments conducted for this study very closely parallel that which was done on campus.

Data Collection and Analysis

A Flanders Verbal Interaction Analysis of verbal behavior in the classroom requires that judgments be made about the behavior at three-second intervals. It was decided that the data for this study would be collected from the classroom by means of a tape recorder and a movie camera which was capable of single frame exposure. In this way the judgments concerning the verbal behavior could be checked not only be replaying the tape, but also by viewing the photograph which accompanied the three-second interval in question.

The equipment which was developed for obtaining the data was an important part of this study and therefore it will be described in detail. The most essential part of the equipment was the audio-oscillator and electronic timer which were developed by Dr. Wayne Taylor of the Science-Mathematics Teaching Center at Michigan State University. The technical aspects of the audio-oscillator and electronic timer can be obtained by contacting Dr. Taylor. The equipment served two functions in the data collecting process: the audio-oscillator emitted a tone signal, or "beep," which was entered into the tape recording, and, simultaneously, the timer activated the mechanism which advanced the movie camera one frame. The audio-oscillator was constructed so that the amount of time between each "beep" could be adjusted. For this study the electronic timer was set so that there was a three-second interval between each "beep" from the audio-oscillator.

The teacher wore a wireless microphone about her neck. This microphone was essentially a very small, battery-powered, FM radio transmitter. Wireless microphones such as the one used are commercially available, however care must be exercised in storing and transporting these microphones since they are equipped with a mercury switch which is "on" whenever the microphones are in a vertical upright position. This researcher found it best to remove the battery whenever the microphone was not being used.

The radio signal from the microphone was received with a battery-powered portable FM radio located in the rear of the classroom. The radio was equipped with a jack which could receive a plug from the tape recorder so that the teacher's voice could be recorded without coming through the radio speaker. The wireless microphone contained an adjustment screw so that the radio signal was transmitted over a frequency which was not locally in use. Since all of the observations were made in the same geographic area, this adjustment was necessary only once.

A battery powered cassette-type tape recorder was used to record the verbal behavior of the teacher. The cassettes used were capable of recording for forty-five minutes on each side of the tape, hence it was not necessary to change the cartridge during the observation.

By using the wireless microphone the teacher had complete freedom to move about the room. All the data collecting equipment was at the rear of the room where the observer's presence had little effect upon the classroom behavior.

A battery-powered 8 mm movie camera was used to obtain the photographic record of the class. The camera was capable of single frame advance by means of a plunger-type cable attached to the side of the camera. The camera and the tripping mechanism were mounted on a small platform which was in turn mounted on a tripod in the rear of the room. Super-8 Tri-X black and white film was used in the camera. It should be noted that color film, (high speed Ektachrome), is now available which can be used in the light ranges that existed in the classrooms which were filmed for this study. Because of the speed of the Tri-X film, the classroom fluorescent lights provided sufficient illumination. A roll of Tri-X film contains approximately thirty-eight hundred frames and, since a forty-five minute class session used only nine hundred frames, it was not necessary to change film during any classroom observation.

The tripping mechanism created some noise as the relays opened and closed. To minimize this noise the mechanism was encased in a Styrofoam box.

A monitoring device was wired into the system so that the observer could adjust the volume of either the "beep" from the audio-oscillator or the sound from the FM radio.

It was not possible to include all of the classroom in the range of the camera, even though a wide angle lens was used. However, the observer always positioned his equipment to include as much of the class as possible. It is recognized that the procedures used produced an audio record which consisted mostly of the voice of the teacher, but, since the teacher was the center of the study, this selectivity did not seem disturbing. Before each data collecting session a brief demonstration of the equipment was made to the children. This was necessary since the mechanism which tripped the camera created some noise and this noise could have been a distracting factor. However, after the explanation there seemed to be little concern by the children for either the observer's presence or the noise of the equipment.

The following checklist indicates the order in which the equipment was put into operation:

1. Locate the power supply and run an extension cord from it to the spot selected for the camera.

2. Set the tripod.
3. Mount the camera and the tripping mechanism on the tripod.
4. Attach tripping mechanism to the electronic timer.
5. Focus camera and adjust for room light.
6. Attach audio-oscillator and timer to tape recorder.
7. Insert cassette into recorder.
8. Insert battery into wireless microphone.
9. Turn on radio and tune into the frequency of the wireless microphone.
10. Place microphone about teacher's neck.
11. Start the tape recorder.
12. Plug audio-oscillator and timer into extension cord.
13. Monitor system.

The flow chart, Figure 3.1, indicates how the parts of the data-gathering system are related.

The data collected by the method described in the above paragraphs were analyzed by means of a Flanders Interaction Matrix as described on pages 17-18 of this study. The analysis was primarily an analysis of the verbal record as obtained by use of a tape recorder, but the photographic record was used to aid the investigator in his choice of categories.

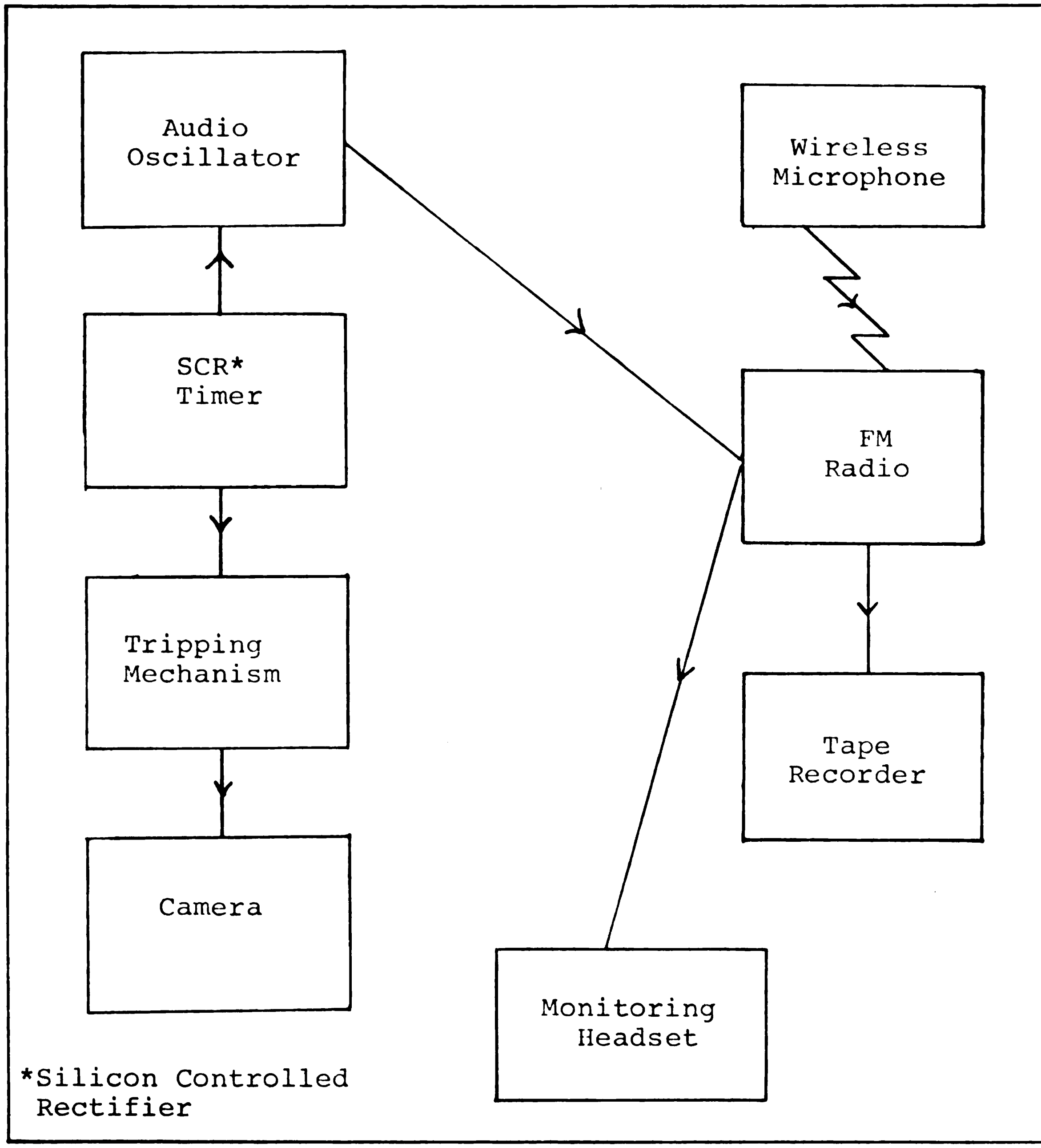

Figure 3.1   Flow Chart for data-collecting equipment.

The investigator used the following procedure to translate the data into the numerical record necessary for the analysis. The film was threaded into a Kodak MFS-8 Super-8 movie projector which was capable of single frame film advance. Using the manual control to advance the film at the rate of two or three frames per second, the analyzer previewed the film record of the classroom behavior. He then placed the cassette into the tape recorder and played back the recording of the teaching situation being analyzed.

Having thus acclimated himself to the classroom situation the investigator played the tape and ran the film in synchronization. This was accomplished in the following way. A timer similar to that used in the data gathering process, but modified to actuate the projector, was set at the approximate three-second recording speed. It was connected to the single frame advance mechanism of the projector in such a way that the film was advanced one frame for each "beep" on the tape. The technical aspects of this connection can be obtained from Dr. Taylor at Michigan State University. With the timer connected to the power supply, the projector was plugged into the timer as soon as the first "beep" was heard in the tape recording. With tape and film running simultaneously, the analyzer made judgments concerning the verbal behavior of the classroom. The audio "beep" served as a pacer for the analyzer so that a category number was recorded at each signal. After some practice this action became quite automatic. Each of the fourteen observations were analyzed in this way.

Each observational record was analyzed twice by the method described above, with at least one day, and often more, separating the analysis of a particular observation. Any discrepancies in the two numerical records obtained were resolved according to the directions given by Flanders: "When not certain in which of two categories a statement belongs, choose the category numerically farthest from category 5, except category 10" [1].

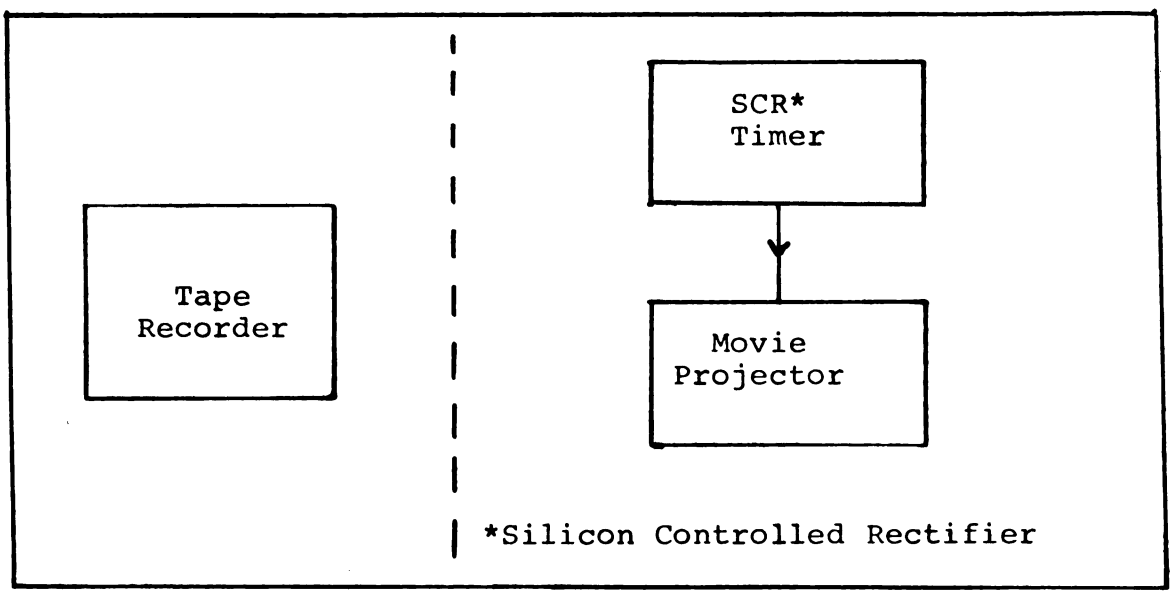

Figure 3.2 Flow Chart for data-analyzing equipment.

CHAPTER IV

CASE STUDIES

Introduction

There were fourteen subjects involved in this study. Those who received both the laboratory experience dealing with the concept of relations and the laboratory experience dealing with the concept of function are designated as B1, B2, B3, B4, B5, B6, and B7. Those who received only the relations laboratory are designated as R1, R2, R3, R4, and R5. Those who received only the functions laboratory are designated as F1 and F2.

Each subject was asked to teach some aspect of the function concept to the class in which he was doing his student teaching. The length of the lesson was not stipulated, but the subjects were told to limit themselves to forty-five minutes. This limit was imposed so that the observer would not have to change the cassette in the tape recorder during the observation.

Some of the subjects were not to the point in their student teaching at which they teach the entire class, but were teaching small groups of five or six students. These subjects taught the function concept to one of their small groups.

Subject B1

Subject B1 had taken Algebra and Geometry in high school, receiving grades of C. His college experience in mathematics was one course which was taken at a Community College and was judged by Michigan State University admission officials to be equivalent to the Foundations of Arithmetic course taught there. His college grade point average was 2.00 at the time of this study.

The function lesson was taught to a fifth grade class. The lesson was very brief, lasting only thirteen minutes. The lesson plan which B1 had prepared for this lesson was very sketchy. The only behavioral objective which he had listed was, "to have students understand functions." Although the subject had had experience in the mathematics laboratory with the concept of relations, he used the word "relation" incorrectly in his lesson. He used the word to describe the correspondence between the first and second members of the ordered pairs which he used as illustrations of functions. His work dealt exclusively with situations in which the students were given a rule of correspondence and the first member of the ordered pair, and then were asked to find the second member of the ordered pair.

Table 1. Verbal Interaction Matrix for B1

*	1	2	3	4	5	6	7	8	9	10	Totals
1											
2				1							1
3				2	7						9
4			1	6	6			31			44
5			1	23	145					4	173
6				1							1
7											
8		1	7	11	11			1	1		32
9					1						1
10					3	1					4
%		.3	3.3	16.6	65.2	.3		12.0	.3	1.5	265

Teacher Talk
Columns 1-7 = 228

Student Talk
Columns 8-9 = 33

Indirect (1-4) / Direct (5-7) = ID ratio
54 ÷ 174 = .31

Indirect (1-3) / Direct (6-7) = Revised ID ratio
10 ÷ 1 = 10.0

*See Appendix C for a description of the categories.

After subject B1 had indicated to the author that he had completed the lesson, and after the recording equipment was packed away, he returned to the lesson about functions and answered students' questions concerning them. No explanation was given for this behavior. The interaction analysis shows that B1 spent 82 per cent of the time lecturing and asking questions, whereas the students talked only 12 per cent of the time. Perhaps the subject sensed this imbalance and therefore gave the students an opportunity to ask questions.

Subject B2

Subject B2 had three years of mathematics in high school and had taken the Foundations of Arithmetic course at Michigan State University, receiving a grade of B for the course. The Foundations of Arithmetic course had no mathematics laboratory experience connected with it. At the time of this study subject B2 had a grade point average of 3.24.

This was a micro-learning situation, consisting of five fifth grade students. No manipulative materials were used by the students. Even though the class was very small the structure of the class was quite formal. Subject B2 used the entire period of time, about thirty-five minutes, talking and lecturing from the blackboard. At no time did students come to the board, nor did they do any written work at their tables.

Table 2. Verbal Interaction Matrix for B2

*	1	2	3	4	5	6	7	8	9	10	Totals
1											
2					1					1	2
3			2	5	10					2	21
4			1	26	12	3		58	2	10	112
5			3	26	153	6		3	3	26	220
6				3	1	16		3		12	35
7											
8		2	12	22	14	2		18	1	11	82
9			1	3	2				5		11
10			2	26	28	8				141	205
%		.2	3.0	16.2	31.9	5.0		11.9	1.5	29.7	688

Teacher Talk
Columns 1-7 = 390

Student Talk
Columns 8-9 = 93

Indirect (1-4) / Direct (5-7) = ID ratio
135 ÷ 255 = .53

Indirect (1-3) / Direct (6-7) = Revised ID ratio
24 ÷ 35 = .69

*See Appendix C for a description of the categories.

B2 began the lesson by having the students provide the second member of the ordered pair, having been given the first member and the rule of correspondence. Then a form of "Guess My Rule" was played. Each student was given a turn to present a set of ordered pairs from which the others had to determine the rule of correspondence. Most of the period was used playing this game.

The interaction analysis shows almost 30 per cent of the time was recorded in category 10, silence or confusion. This relatively high incidence was the result of time spent writing on the board, since during that time no verbal interaction was taking place.

Subject B3

Subject B3 had three years of mathematics in high school. She took the Foundations of Arithmetic course at Michigan State University during the summer of 1968. There was no laboratory experience connected with this summer course. At the time of this study subject B3 had a grade point average of 3.28.

The lesson was taught to a fifth grade class and lasted twenty-one minutes. No manipulative materials were used by the students. The students were presented with pairs of numbers and were asked to find the rule, or, in the words of B3, "write a mathematical sentence about the relation." There was some discussion about plotting points on graph papers. She had prepared some graphs of functions,

Table 3. Verbal Interaction Matrix for B3

*	1	2	3	4	5	6	7	8	9	10	Totals
1											
2			1		2	1			1		5
3		1	3	3	3			1			11
4			1	17	11		1	25	5	3	63
5			2	26	131	7		4	2	9	181
6				1	7	16			1	3	28
7					1						1
8		3	3	9	12			25	3	3	58
9		1		4	4	1			2	4	16
10			1	3	10	3		3	2	34	56
%		1.1	2.6	15.0	43.1	6.6	0.2	13.8	3.8	13.3	419

Teacher Talk
Columns 1-7 = 289

Student Talk
Columns 8-9 = 74

Indirect (1-4) / Direct (5-7) = ID ratio
79 ÷ 210 = .38

Indirect (1-3) / Direct (6-7) = Revised ID ratio
16 ÷ 28 = .55

*See Appendix C for a description of the categories.

which she hung on the front wall. Since these graphs were too small for the children to see from their seats, groups of children were allowed to come forward to see them. B3 then discussed how these graphs could be used to predict second members of ordered pairs. A ditto was distributed with problems for the students to do at their seats. The problems concerned finding the rule of correspondence when given a set of ordered pairs.

Subject B4

Subject B4 did not submit a record of her high school work in mathematics. A subsequent search of school records by the author revealed that B4 had a very weak background in mathematics. B4 had taken Foundations of Arithmetic at Michigan State University but had received a grade of F. It should be noted that during this course she did receive experience in a mathematics laboratory. Her grade point average at the time of this study was 1.76.

The lesson was taught to a class of fourth grade students, and lasted about thirty minutes. After a brief introduction to the conception of function, B4 had the children make rules which the other children had to guess from the given ordered pairs. The activity was not too successful since many of the numbers suggested for first members were so large the student could not correctly compute the second member. No manipulative materials were used by the students.

Table 4. Verbal Interaction Matrix for B4

*	1	2	3	4	5	6	7	8	9	10	Totals
1											
2											
3			3	5	1			2	3	2	16
4			2	31	11	5	4	67	11	19	150
5				25	54	3		4	4	6	96
6				14	3	27	1		11	7	63
7			1	7	1	5				1	15
8			3	24	16	7	5	46	4	21	126
9			6	15	2	12	3		19	6	63
10			1	29	7	4	2	8	11	41	103
%			2.5	23.7	15.0	10.0	2.4	20.9	10.0	16.3	632

Teacher Talk
Columns 1-7 = 329

Student Talk
Columns 8-9 = 200

Indirect (1-4) / Direct (5-7) = ID ratio
166 ÷ 173 = .96
Indirect (1-3) Direct (6-7) = Revised ID ratio
16 ÷ 78 = .21

*See Appendix C for a description of the categories.

Subject B5

Subject B5 had three years of mathematics in high school. She took one mathematics course at a Community College and the Foundations of Arithmetic course at Michigan State University. There was no laboratory experience connected with this course. Her grade for this course was D. At the time of the study her grade point average was 2.08.

B5 taught a class of sixth grade students a lesson which lasted about thirty-three minutes. In her presentation she talked about "function machines" and had a chart illustrating a "function machine." It was apparent from the comments she made in class that she did not understand the concept of function. She identified the function with the second member of the ordered pair. The following appeared on her lesson plan:

> f(2) in the rule $n+7=9$. No other number can be associated with 9 according to the rule. So 9 is the function of 2.

In her class presentation she called 9 the "purpose" of 2.

No manipulative materials were used by the students. All the work was of the form in which the student is given the rule and the first member of the ordered pair and then must find the second member.

The interaction analysis shows that although 5 per cent of the time was used in category 2 (accepting students' ideas), almost 3 per cent was used in criticism and/or defense of her authority.

Table 5. Verbal Interaction Matrix for B5

*	1	2	3	4	5	6	7	8	9	10	Totals
1											
2		1		15	9	2		4		2	33
3			11	4							15
4		6	1	51	11	3	1	70	3	5	151
5		1	2	35	182	5	1		2	2	230
6		1		3	7	22	1		1	5	40
7		2		5	1	2	4	3	1		18
8		24	1	27	14	3	7	35	1		112
9				1	3	3	2		13		22
10				9	3		2		2	34	50
%		5.0	2.2	22.3	34.3	6.0	2.7	16.7	3.4	7.5	671

Teacher Talk
Columns 1-7 = 486

Student Talk
Columns 8-9 = 135

Indirect (1-4) / Direct (5-7) = ID ratio
198 ÷ 288 = .69

Indirect (1-3) Direct (6-7) = Revised ID ratio
48 ÷ 58 = .83

*See Appendix C for a description of the categories.

Subject B6

Subject B6 did not follow the usual college preparatory track in high school. Until her senior year all of her mathematics courses were in general mathematics. She received grades of A in these courses. In her senior year she took Algebra and received a grade of D. At Michigan State University she took Foundations of Arithmetic, but she did not pass the course. She will have to repeat it in order to graduate. However, because she took Foundations of Arithmetic on campus, she has had almost a full term of exposure to the mathematics laboratory. The laboratory experiences given as part of this study were repeats of some of her earlier experiences. At the time of this study B6 had a grade point average of 1.59.

In her presentation subject B6 used an overhead projector and cardboard posters. The children used no manipulative materials. The entire period was used guessing functions from given sets of ordered pairs. First subject B6 presented some sets of data and then she had the students make sets of data from which the rest of the class had to guess the rule of correspondence. This lesson was with the entire class of fourth grade students and lasted about thirty-five minutes.

Table 6. Verbal Interaction Matrix for B6

*	1	2	3	4	5	6	7	8	9	10	Totals
1	1				2						3
2	1	6		11		1				3	22
3		2	5	15	2	1		2		2	29
4	1	2	2	8	4	1		60	6	23	107
5		1	2	6	28	7		4	1	19	68
6				6	2	19		6	2	12	47
7											
8		7	19	26	10	5		53	6	16	142
9		2	1	8	2				7	3	23
10		2	1	27	17	13		17	11	192	270
%	.4	3.0	4.2	15.0	9.4	6.6		19.9	3.2	37.9	711

Teacher Talk
Columns 1-7 = 276

Student Talk
Columns 8-9 = 165

Indirect (1-4) / Direct (5-7) = ID ratio
162 ÷ 114 = 1.42
Indirect (1-3) / Direct (6-7) = Revised ID ratio
55 ÷ 47 = 1.17

*See Appendix C for a description of the categories.

Subject B7

Subject B7 was a mathematics major in high school. The only mathematics course taken in college was Foundations of Arithmetic at Michigan State University. She received a grade of B in this course. There was no laboratory experience connected with the course that she took. At the time of the study the subject had a grade point average of 3.71.

The lesson was taught to about ten fifth grade students, the top third of the class in ability. The lesson was very short, lasting only ten minutes. This subject was primarily interested in that property of functions which guarantees a unique second member. At no time did she clearly define what a function is, but rather seemed to confuse functions with operations. Throughout the lesson her supervisor remained in the room. The subject found it necessary to refer to notes which she had in her hand during the lesson. No manipulative materials were used and at no time were the students actively involved in the learning situation. At the end of the lesson the subject distributed a ditto sheet containing three problems dealing with the function concept.

Subject R1

Subject R1 never presented the author with a record of the mathematics courses which she had taken in high school. Her college course was Foundations of Arithmetic,

Table 7. Verbal Interaction Matrix for B7

*	1	2	3	4	5	6	7	8	9	10	Totals
1											
2			1		2						3
3				3	3						6
4				2	1			20			23
5				14	95					7	116
6					1	3					4
7											
8		3	5	3	8			7		2	28
9											
10				1	5	1		1		8	16
%		1.5	3.0	11.7	58.6	2.0		14.2		8.6	196

Teacher Talk
Columns 1-7 = 151

Student Talk
Columns 8-9 = 28

Indirect (1-4) / Direct (5-7) = ID ratio
32 ÷ 119 = .27

Indirect (1-3) / Direct (6-7) = Revised ID ratio
9 ÷ 4 = 2.25

*See Appendix C for a description of the categories.

Table 8. Verbal Interaction Matrix for R1

*	1	2	3	4	5	6	7	8	9	10	Totals
1											
2			1	3	4	1				1	10
3		4	6	7	4	2		1	2	5	31
4			4	26	13	3		71	1	8	126
5			3	23	79	6		2		13	126
6			2	12	6	44	1	1	1	12	79
7						1	3	1	1	3	9
8		6	14	41	7	4		24	2	2	102
9			1		4	2			6	2	15
10			1	14	8	16	5	2	2	78	126
%		1.6	5.1	20.1	20.0	12.6	1.4	16.3	2.4	20.1	624

Teacher Talk
Columns 1-7 = 381

Student Talk
Columns 8-9 = 117

Indirect (1-4) / Direct (5-7) = ID ratio
168 ÷ 213 = .79
Indirect (1-3) Direct (6-7) = Revised ID ratio
42 ÷ 81 = .52

*See Appendix C for a description of the categories.

taken at Michigan State University. She received a grade of C in this course and had no laboratory experience in connection with it. Her grade point average at the time of the study was 2.70.

The activity consisted of a set of cardboard cards, about two inches by four inches, on which were written either some numbers or some rule of correspondence. She had arranged the cards so that each student would get a rule of correspondence on the first distribution of the cards. She then passed out pairs of cards with numbers. The student had to determine if the numbers constituted an ordered pair which satisfied the function he had been dealt. Because R1 was not clear in the directions she gave, confusion occurred. Not knowing quite how to extracate herself, R1 dropped the activity. Though unsuccessful, this was an attempt to use manipulative materials with the students.

Subject R2

Subject R2 had four years of high school mathematics. Her transcript revealed that when she entered college she planned to be a mathematics major. At Michigan State University she enrolled in two pre-calculus courses and received a grade of D in both of them. Subsequently she decided to enter elementary education. She took the Foundations of Arithmetic course and received a grade of C. She did not have a laboratory experience in conjunction

Table 9. Verbal Interaction Matrix for R2

*	1	2	3	4	5	6	7	8	9	10	Totals
1										1	1
2		2		2	1		1		1	1	8
3			2	8	6	1		1	3	5	26
4		1	2	10	9	4	3	38	21	9	97
5		1	2	29	104	14	3	2	7	11	173
6			1	7	11	57	2	1	9	15	103
7			1	7	1	1	1	1	4	5	21
8	1	2	7	5	13	9	3	11		6	57
9		1	6	11	15	7	4		18	5	67
10		1	5	18	13	10	4	3	4	51	109
%	0.1	1.2	4.1	15.5	27.8	16.5	3.3	9.1	10.7	17.5	662

Teacher Talk
Columns 1-7 = 429

Student Talk
Columns 8-9 = 124

Indirect (1-4) / Direct (5-7) = ID ratio
132 ÷ 297 = .44

Indirect (1-3) / Direct (6-7) = Revised ID ratio
35 ÷ 124 = .28

*See Appendix C for a description of the categories.

with this course. At the time of this study R2 had a grade point average of 3.00.

Subject R2 taught the lesson to a class of fourth grade students. She began by playing guessing games which lead to a form of the "Guess My Rule" game. She drew a function machine at the board, showing input, rule, and output. She then had the children give the output when given the rule and the input. No manipulative materials were used by the students.

Subject R3

Subject R3 had four years of mathematics in high school, maintaining a B average. Because she planned to be a high school mathematics teacher she started the Calculus sequence in college. She received a grade of A for the first term and a grade of C for the second. After switching to elementary education she took a course equivalent to Foundations of Arithmetic and received a grade of B. All of her college work was taken in a Community College. At the time of this study R3 had a grade point average of 2.55.

The lesson was taught to a class of fifth grade students. Subject R3 had prepared a cardboard "function machine" which she used for demonstration at the front of the room. Her objective, as listed in her lesson plan, was to have "students discover by exploration and experimentation that a function corresponds each number of a

Table 10. Verbal Interaction Matrix for R3

*	1	2	3	4	5	6	7	8	9	10	Totals
1											
2		1	1	1						2	5
3		2	7	4	14			2		6	3
4			3	20	9			71	9	10	122
5		1	2	37	159	11	2	6	1	7	226
6				4	5	21	1	6		7	44
7				2			5		1	1	9
8		1	19	27	21	6	1	81	8	13	177
9			1	9	6	1		2	12	1	32
10			2	18	12	5		9	1	47	94
%		0.7	0.4	17.3	32.1	6.2	1.2	25.1	4.5	13.3	703

Teacher Talk
Columns 1-7 = 400

Student Talk
Columns 8-9 = 209

Indirect (1-4) / Direct (5-7) = ID ratio
130 ÷ 279 = .47
Indirect (1-3) / Direct (6-7) = Revised ID ratio
8 ÷ 53 = .15

*See Appendix C for a description of the categories.

given set with exactly one member from another set." The experimentation consisted of verbal questions and answers. The cardboard demonstrator was used only briefly and never by the students. No manipulative materials were used by the students in this lesson which lasted thirty-three minutes.

Subject R4

Subject R4 had three years of high school mathematics. Her college course was a community college equivalent of Foundations of Arithmetic. She had received no mathematics laboratory experience prior to this study. Her grade point average was 2.97 at the time of this study.

This was a micro-teaching experience, the lesson being taught to five fifth grade students who were taken from their regular classroom to another room. She began by having the students supply the second member of the ordered pair when given the rule of correspondence and the first member of the ordered pair. She then had students guess the rule when given a set of ordered pairs. The subject did all of her work at the blackboard and used no manipulative materials. The students did no work at the board and also used no manipulative materials. The lesson lasted about twenty-two minutes.

Table 11. Verbal Interaction Matrix for R4

*	1	2	3	4	5	6	7	8	9	10	Totals
1											
2		2		1	2			1			6
3		3	8	4	6			2			23
4			1	8	3			57	2	4	75
5			2	25	109	2		19	3	7	166
6					2	8		2	2	3	17
7					1	1					2
8		1	10	26	25	5	1	154	14	7	243
9			1	7	9	2	1	1	31	2	54
10			1	5	8			7	2	50	73
%		0.9	3.4	11.3	25.1	2.5	0.3	36.8	8.1	1.1	659

Teacher Talk
Columns 1-7 = 289

Student Talk
Columns 8-9 = 297

Indirect (1-4) / Direct (5-7) = ID ratio
104 ÷ 185 = .56

Indirect (1-3) / Direct (6-7) = Revised ID ratio
29 ÷ 19 = 1.53

*See Appendix C for a description of the categories.

Subject R5

Subject R5 had only two courses of high school mathematics, Algebra and Geometry. Her grades show a steady decline from an initial A to a final D. Her college course was Foundations of Arithmetic taken at Michigan State University. There was no laboratory experience connected with this course. She received a grade of B. Her grade point average at the time of this study was 2.49.

Her lesson was with a class of fourth grade students. She began by distributing a ditto sheet which contained a drawing of a function machine. She explained how the machine "worked" and then had the students guess the output when given the rule and the input. Then she had students try to guess the rule from a given set of data. Finally she had students come to the board with a rule in mind. The class supplied the student with input numbers, the student then supplied the output numbers, and then the class tried to guess the rule. No manipulative materials were used. The lesson took about thirty minutes.

Subject F1

Subject F1 had two years of high school mathematics. She did not do well in these courses but indicated on a questionnaire that she felt that she had had poor instruction. Her college mathematics course was a Community College version of Foundations of Arithmetic. At the time of this study her grade point average was 2.21.

Table 12. Verbal Interaction Matrix for R5

*	1	2	3	4	5	6	7	8	9	10	Totals
1				1	1						2
2		1	3	4	6			1	1	6	22
3			3	6	3				3	1	16
4		3	3	17	14	4	1	33	4	18	97
5	1	3		27	57	10	1	3	1	14	117
6			1	3	6	10	3	3	1	12	39
7				2	1			2		2	7
8	1	6	1	12	9	4	2	43	1	15	94
9		3		3	3	1		2	8	1	21
10		6	5	22	16	10		7	2	129	197
%	0.3	3.6	2.6	16.0	19.1	6.4		15.5	3.4	32.7	605

Teacher Talk
Columns 1-7 = 292

Student Talk
Columns 8-9 = 115

Indirect (1-4) / Direct (5-7) = ID ratio
137 ÷ 155 = .88
Indirect (1-3) / Direct (6-7) = Revised ID ratio
40 ÷ 39 = 1.03

*See Appendix C for a description of the categories.

Table 13. Verbal Interaction Matrix for Fl

*	1	2	3	4	5	6	7	8	9	10	Totals
1											
2										1	1
3		1	2	1	1						5
4				14	8	4		73		5	104
5				22	120	6	3	10	5	13	179
6				9	7	38	3	2	1	11	71
7				6	1	1	12	2	2	3	27
8			2	38	21	11	3	55	7	7	144
9			1	3	7	2	1	1	23	4	42
10		1		11	13	9	5	1	4	109	153
%		0.2	0.6	14.4	24.6	9.8	3.7	19.9	5.8	21.1	722

Teacher Talk
Columns 1-7 = 383

Student Talk
Columns 8-9 = 186

Indirect (1-4) / Direct (5-7) = ID ratio
107 ÷ 276 = .39

Indirect (1-3) / Direct (6-7) = Revised ID ratio
3 ÷ 98 = .03

*See Appendix C for a description of the categories.

The lesson was taught to a class of fourth grade students. This was the first time the subject had taught the entire class. Her supervisor was away for the day and the subject was to have the class for the entire day. She had the students push their desks together to form five groups of six children. She had prepared five cardboard boxes to act as function machines. She had prepared sheets of paper which fit into a slot in the box and which contained data for various function problems. She began the period with a brief discussion of the concept of function and then a brief explanation of how the children were to use the boxes. The noise level in the room seemed to disturb the subject. The use of the boxes was not altogether successful. The students did not seem accustomed to group work and they did not seem to understand how they were to work with the function machines. However, this did represent an attempt to use manipulative materials, and some of the students seemed to get the idea quite well. The lesson lasted thirty-seven minutes.

Subject F2

Subject F2 had two years of high school mathematics. Her college mathematics course was Foundations of Arithmetic at Michigan State University. There was a laboratory experience connected with this course, and F2 received a grade of D. F2 volunteered, on a questionnaire,

Table 14. Verbal Interaction Matrix for F2

*	1	2	3	4	5	6	7	8	9	10	Totals
1											
2											
3			1	1	5	1	1				9
4			1	11	3	4	4	9	4	1	37
5			1	7	8	5	4	5	1	3	34
6				1	7	15	2	2	6	7	40
7			1	3	4	7	6	1	3	4	29
8			2	4	1	1	5	7	6	2	28
9			3	4	4	5	4		21	4	45
10				5	3	2	3	4	4	14	35
%			3.5	14.0	13.6	1.5	11.2	10.8	17.5	13.6	257

Teacher Talk
Columns 1-7 = 149

Student Talk
Columns 8-9 = 73

Indirect (1-4) / Direct (5-7) = ID ratio
45 ÷ 104 = .43

Indirect (1-3) / Direct (6-7) = Revised ID ratio
9 ÷ 69 = .13

*See Appendix C for a description of the categories.

Table 14. Verbal Interaction Matrix for F2

*	1	2	3	4	5	6	7	8	9	10	Totals
1											
2											
3			1	1	5	1	1				9
4			1	11	3	4	4	9	4	1	37
5			1	7	8	5	4	5	1	3	34
6				1	7	15	2	2	6	7	40
7			1	3	4	7	6	1	3	4	29
8			2	4	1	1	5	7	6	2	28
9			3	4	4	5	4		21	4	45
10				5	3	2	3	4	4	14	35
%			3.5	14.0	13.6	1.5	11.2	10.8	17.5	13.6	257

Teacher Talk
Columns 1-7 = 149

Student Talk
Columns 8-9 = 73

Indirect (1-4) / Direct (5-7) = ID ratio
45 ÷ 104 = .43
Indirect (1-3) / Direct (6-7) = Revised ID ratio
9 ÷ 69 = .13

*See Appendix C for a description of the categories.

that she disliked mathematics. She had a grade point average of 2.77 at the time of this study.

The lesson was taught to about fifteen fourth grade students. The rest of the class was involved in a band activity that had taken them out of the room, and those who remained were not happy with this situation. The college supervisor chose this period to attend the subject's class. All of these factors made this a difficult learning situation.

F2 made no formal introduction to the concept of function. She chose three students to come to the front of the room to act as "computers." The class selected a number and the computers had to give an answer according to a given rule. The first group of computers did not understand the idea so a second set of students was selected, with no better success. After a third group also did not get the idea, the project was dropped and the students were given a ditto sheet to work at their seats. This lesson lasted twelve minutes.

Of all of the subjects in the study, F2 was the only one who complained to the author about the noise of the recording equipment.

Though no manipulative equipment was used in this lesson, an attempt was made to have the students actively involved.

Summary

Two items were of special interest in this study: (1) the amount of use made of materials which could be manipulated by the students in order to better understand the concept taught in the classes which were observed, and (2) the ID ratios and revised ID ratios of each of the subjects as obtained from the Flanders Interaction Matrix.

The case studies show that two subjects attempted to use manipulative materials. In neither case was the function concept inherent in the materials manipulated by the students.

The ID ratios also seemed uneffected by the amount of laboratory experiences given to the subjects. For ease of comparing the ID ratios and the revised ID ratios with the amount of laboratory experience, these measures are presented in the histograms of Figure 4.1 and Figure 4.2.

Though the limitations of a grade point average as a measure of a student's academic ability are recognized, the author felt that this measure could be used to indicate the diverse abilities represented in this study. In Figure 4.3 the grade point averages are superimposed on the ID ratios. This histogram shows that there is no apparent connection between ID ratios and grade point averages.

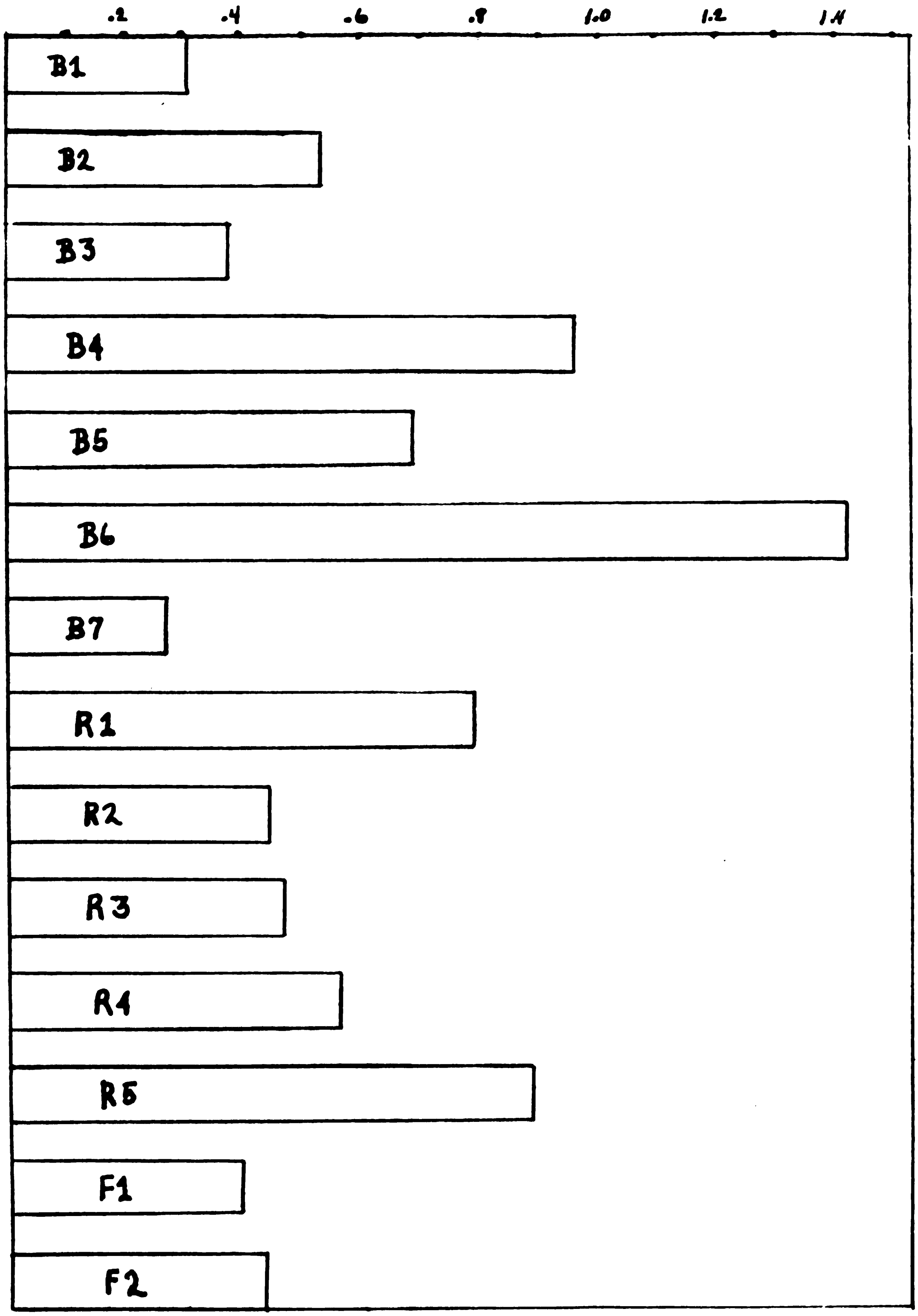

Figure 4.1 Subjects' ID Ratios.

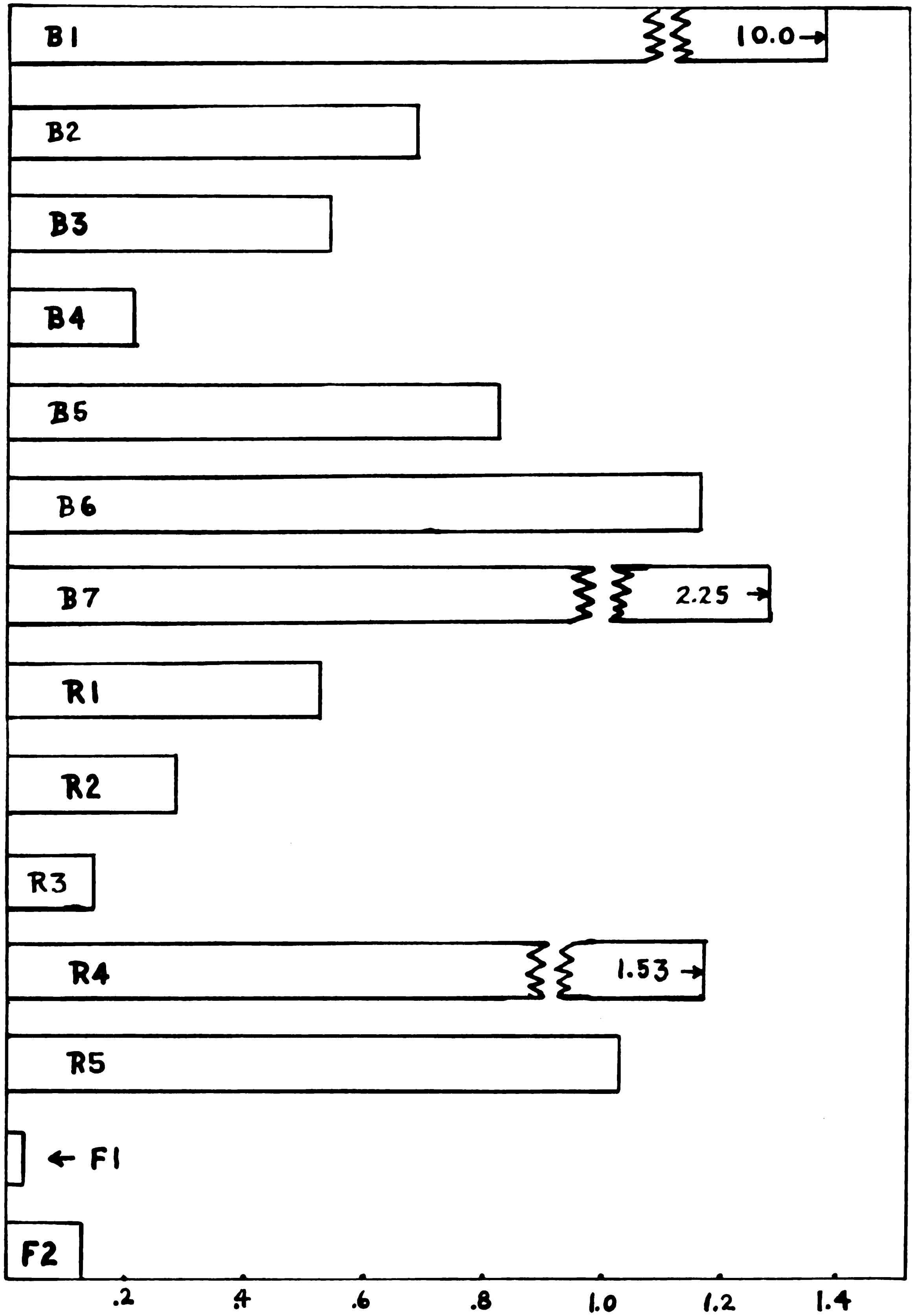

Figure 4.2 Subjects' Revised ID Ratios.

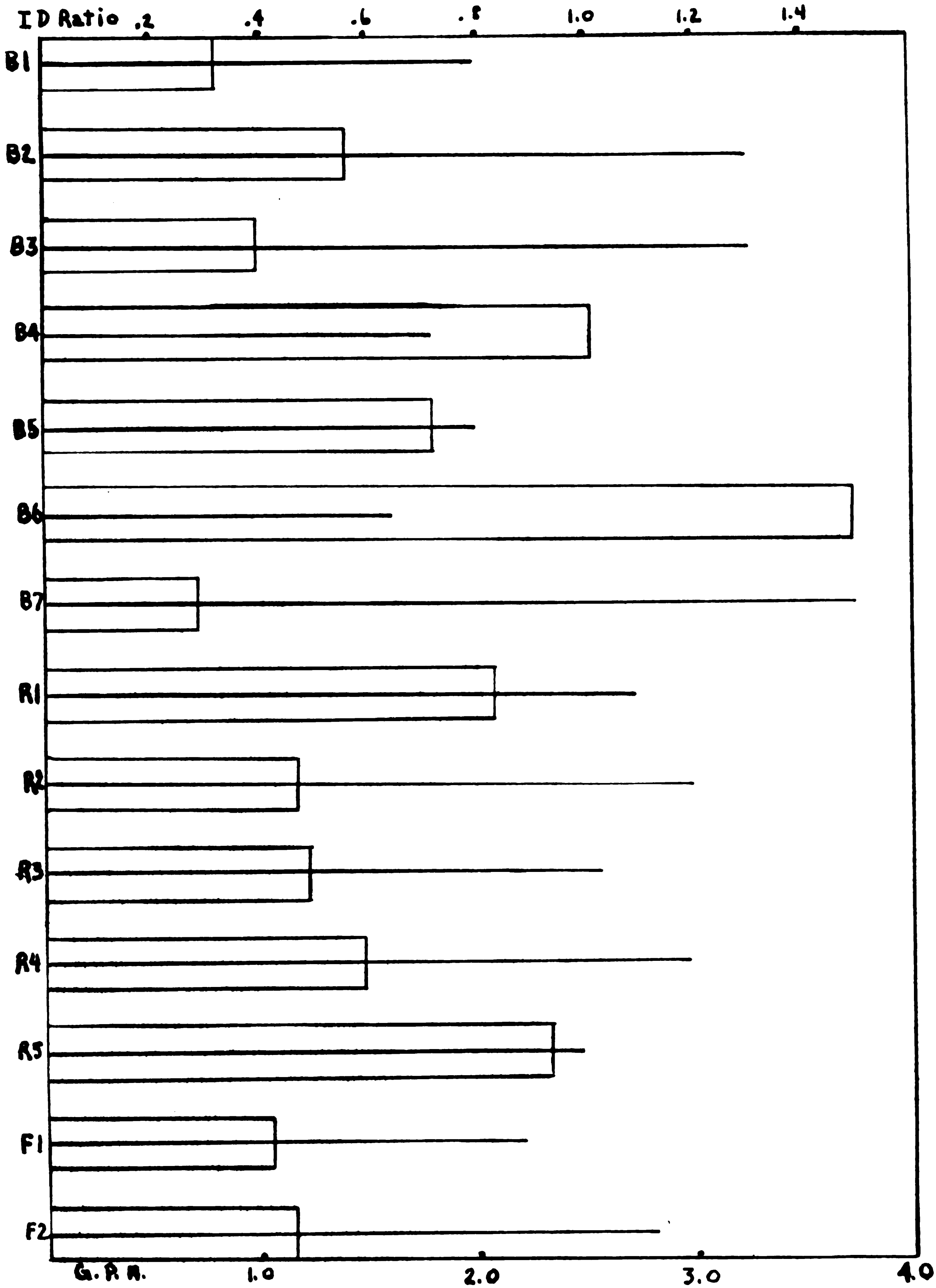

Figure 4.3 ID Ratios Compared With Grade Point Averages.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

Introduction

This research reports the design and execution of a pilot study to develop and evaluate techniques for the investigation of a mathematics laboratory experience upon the teaching behavior of its recipients. The researcher was concerned with two questions: (1) Do student teachers who are taught a concept through the use of manipulative materials use manipulative materials as they teach the concept? (2) Do student teachers who have experienced a student-centered learning situation in a mathematics laboratory employ a student-centered teaching approach as they teach?

The first section of this chapter is concerned with the answers which this study provided to the above questions concerning the effect of a mathematics laboratory experience upon the teaching behavior of student teachers.

The techniques used for the collection and analysis of classroom data were, as far as is known to the author, unique and therefore the second section of this chapter contains recommendations relative to the use of these techniques.

The Flanders Interaction Analysis was used to analyze the data collected from the classroom. The pilot study revealed some weaknesses in the instrument. A third section of this chapter is addressed to these weaknesses.

The chapter concludes with a summary of conclusions, implications, and recommendations of this pilot study.

The Effect of Laboratory Experience

A characteristic of learning mathematics by means of a laboratory experience is student involvement in an active, rather than passive, way. This active student involvement is attained by the use of physical materials which are manipulated and investigated by the learner. In each laboratory session conducted by the author with the subjects of this study manipulative materials were used in this manner. It was a matter of basic interest in this study to determine whether student teachers would in turn have their students discover the required concept by the use of manipulative materials. Only two of the subjects, F1 and R1, did, and in each case the materials they used did not have the concept of function inherent in them.

Another characteristic of the laboratory approach to the teaching of mathematics is that the learning process is student-centered rather than teacher-centered. ID ratios were used to measure the amount of student-centeredness in the classroom. Figures 4.1 and 4.2 indicate that the amount of laboratory experience had no apparent effect on the amount of student-centeredness since some subjects who had two laboratory experiences had ID ratios which were less than that of some subjects who had had no laboratory experiences.

Student-centered learning is encouraged by having the students work in small groups rather than having the teacher teach the entire class as one unit. A major portion of the laboratory experience given to the subjects of this study was characterized by investigations conducted by the subjects in small groups. Two of the subjects taught small groups, but used the traditional lecture method with them. Only one of the remaining twelve subjects divided the class into small groups and allowed the children in the group to work cooperatively.

It is concluded on the basis of this investigation that two laboratory experiences are not sufficient either to cause student teachers to adopt a student-centered approach to teaching or to cause student teachers to use manipulative materials in their teaching.

Since this is a pilot study to evaluate the laboratory approach to teaching mathematics it is proper to review the study to try to determine why no transfer of methodology occurred in this case. One reason for this lack of transfer may be the amount of exposure given to the subjects in a mathematics laboratory. It is recommended that a study be made to determine how many experiences are necessary to affect a change in the teaching behavior of student teachers.

It is possible that the author inadvertently caused the experiment to have the result that it did. When the subjects were given their instructions--following the laboratory experiments and prior to their classroom teaching--they were asked to teach the function concept to their classes. To disguise the fact that the primary concern of the study was their methodology, the author suggested that the purpose was to determine whether the function concept could successfully be taught at the level at which they were doing their student teaching. In so doing the author may have given the impression that the topic was not generally of significance to elementary school children, and therefore the subjects may not have been inclined to be very innovative in their approach. It should be noted also that the instructions given emphasize the teaching of the concept rather than the learning of the concept. The subjects may have been given the impression that the author was more interested in the performance before the class than the choice of methodology.

The timing of the experiment may have contributed to the result. The subjects had just begun their student teaching experience when they were asked to teach this lesson. Some had taught their class for only one week. Had the experiment been conducted some weeks later, the subjects may have felt more at ease in their classroom and, hence, been more disposed to use the laboratory approach.

It is also possible that the subjects were imitating the opening part of the mathematics laboratory functions experiment they received. Many of the subjects played "Guess My Rule" with their classes even as the author did at the outset of the functions experiment. It could be the topic of future research to determine if subjects given the opportunity to develop a series of lessons on the concept of functions would use the laboratory method in any of the later lessons.

It was assumed that the behavior of student teachers who had received a laboratory experience is similar to the classroom behavior of teachers already in the profession. Some of the behavior of the subjects of this study leads the author to question this assumption. It seemed that some of the subjects were quite concerned about conducting the class in a manner which was like that of the supervising teacher. If further research is to be done on the effect of a mathematics laboratory experience upon teaching behavior, prior research must be done to test the assumption that student teacher behavior parallels the behavior of in-service teachers.

Data Collection Process

The method for collecting the classroom data for this study involved the use of a tape recorder and a movie camera which was activated to photograph the classroom for a single frame once every three seconds. The author found the photographic record a great aid for proper interpretation of the voice record obtained on the tape recorder. Before each analysis of classroom data the author viewed the photographs by manually activating the single frame advance of the movie projector. This served to refresh the author's memory about the particular class being analyzed. He then connected the projector to the three-second timer so that the photographs were synchronized with the tape recording. The author feels that this procedure resulted in a more valid interpretation of the voice record.

This process is in need of technical refinement in two areas. The mechanism which was used to trip the movie camera was somewhat noisy and occasionally its thrust was not sufficient to advance the film. Also, it was somewhat difficult to begin the tape recorder and the movie projector so that the two were properly synchronized. However, in spite of these difficulties the author found the photographic record a great aid in interpreting the verbal record to make a Flanders Interaction Analysis.

This method of collecting data from the classroom has many advantages. The cassettes used in the tape recorder and the rolls of film used in the movie camera were of ample length to collect the data from a forty-five minute class period without having to change either. The equipment is very portable and can be set up in the classroom in less than five minutes. Also, this technique allows one to review the photographs in order to study other aspects of the classroom than the amount of interaction between the teacher and the class. One could select a particular child and determine the amount of time he was interacting with the teacher or with other students.

An important factor which should not go unmentioned is the relatively low cost of the equipment used in this data-gathering process. The procedure is much less expensive than producing either a sound film or a video tape of the class, yet the process has many of their advantages.

It is recommended that the process be refined in the two areas mentioned above. It is also recommended that this method be used to collect and analyze data from classrooms in which the students are actively engaged in the learning process.

Flanders Interaction Analysis

The Flanders Interaction Analysis was used in this study to analyze the verbal interaction in the classroom. It was assumed that the verbal interaction was a reliable measure of the total interaction of the teacher with her students. The author was particularly interested in the ID ratios, since it was assumed that high ID ratios were indicative of student-centered learning situations, and that low ID ratios were indicative of teacher-centered learning situations.

When using the Flanders Interaction Analysis, a tally is made every three seconds in one of ten categories. Category 10 is used to indicate a three-second period of silence or confusion. Tallies in this category are not used in the computation of the ID ratio.

By using the data-collecting process described in this study one can determine whether a period of silence indicates direct or indirect control. For example, if the teacher is walking about the room while the students are busy with activities in groups about tables, this period of silence or confusion would be a period of indirect control. If the silence is the result of the teacher using the chalkboard to demonstrate a method of computation, this would would indicate the same kind of direct control of behavior as is indicated by category 5, lecturing.

It is therefore recommended that when the Flanders Interaction Analysis is used to analyze data obtained from a classroom by means of a tape recorder and a movie camera, category 10 be subdivided into three categories: one category to be used when the silence represents a form of indirect control, one category to be used when the silence represents direct control, and one category to be used when the silence represents neither.

The Flanders instrument assumes that verbal interaction in a classroom is a reliable indicator of the total interaction in the classroom. It is the judgment of the investigator that in an activities-oriented learning situation some of the non-verbal behavior is of as much significance as the verbal behavior. With the data-collecting process used in this study it would seem possible to develop instruments like the Flanders instrument to measure many kinds of non-verbal behavior. It is the judgment of the researcher that such instruments are necessary to analyze properly the learning situation that exists in a mathematics laboratory.

Summary

On the basis of the case studies of the fourteen subjects that participated in this study the following conclusions are reached relative to the main problems of the study:

1. Two laboratory experiences are not sufficient to cause student teachers to adopt a student-centered approach to teaching.
2. Two laboratory experiences are not sufficient to cause student teachers to adopt a teaching technique in which children learn through the use of manipulative materials.

The study has the following implications for the elementary teacher education program:

1. The teacher education program should provide more than two experiences in a mathematics laboratory.
2. More emphasis should be made in methods courses upon the teaching of mathematics by means of a mathematics laboratory.

Because this is a pilot study assessing the effects of a mathematics laboratory experience, the author presents the following recommendations for future studies.

1. A study should be made to determine the effect of laboratory experiences on the teaching behavior of in-service teachers.
2. The data-gathering process pioneered in this this study should be refined and used to collect data from classrooms in which the laboratory method is being used.
3. Instruments similar to the Flanders instrument should be developed to measure non-verbal interaction in the classroom.

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APPENDICES

APPENDIX A

APPENDIX A

Four Madison Project "shoe boxes" were used in the laboratory experience dealing with the function concept: the "Tower Puzzle," the "Peg Game," the "Centimeter Blocks," and the "Geoboard."

The "Tower Puzzle" box contains a sheet of graph paper, a board with three pegs, six circular wooden discs of graduated diameter and a hole in the center to fit on the pegs, and a set of instruction cards.

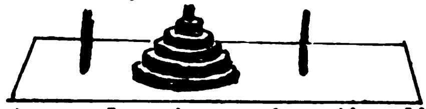
The "Peg Game" box contains a sheet of graph paper, a board with nine holes in a row, four white golf tees, four red golf tees, and a set of instruction cards.

The "Centimeter Blocks" box contains ten colored rods, each having a base one-centimeter by one-centimeter but with heights which are different integral multiples of one centimeter. It also contains a sheet of graph paper and a set of instruction cards.

The "Geoboard" box contains a geoboard, some rubber bands, and a set of instruction cards. A geoboard is a piece of wood 4 1/2" x 4 1/2" and about 3/4" thick. Nails are driven partially into the board at the intersections of a four by four grid of one-inch squares.

The instruction cards are four by six file cards. The instructions for each box are reproduced in the figures on the following pages.

Three students were examining the contents of this box one day. Marilyn said, "This is a puzzle I've seen before. The object of the puzzle is to transfer the discs from the center peg to either of the other two pegs, ending with the discs arranged in the same order as at the start (smaller discs on top of larger discs).



There are only two rules in moving the discs: 1) only one disc may be moved at a time and 2) a larger disc may never be placed on top of a smaller disc. "Frank and Mark said they would like to try it.
CAN YOU DO IT STARTING WITH 5 DISCS?

Figure A1.1 Tower Puzzle instruction card no. 1

CAN YOU DO THE PUZZLE STARTING WITH ONLY 3 DISCS?

To transfer 3 discs, Frank said it took him 11 moves. Mark did it in 7 moves, and Marilyn said it took her 10 moves. They each tried again. Mark said, "This is the shortest way to do the puzzle with three discs; it should take 7 moves. DO YOU AGREE?

IS THERE A WAY TO TELL WHETHER YOU HAVE THE MINIMUM (SMALLEST) NUMBER OF MOVES OR NOT?

CAN YOU DO THE PUZZLE STARTING WITH 2 DISCS? 4 DISCS? 6 DISCS?

Figure A1.2 Tower Puzzle instruction card no. 2

Frank asked, "Is there any relation between the number of discs and the minimum (smallest) number of moves needed to transfer the piles?" Mark suggested they make a table to keep track of the numbers. "Let the number of discs be the □ number and let the minimum number of moves to transfer all the discs be the △ number," he added. Marilyn said, "This is something like the game Guessing Functions, when you put a number in and get a number out and figure out a rule that works."

Number of discs → □	△ ← Minimum number of moves needed to transfer the discs
1	
2	
3	7
4	
5	

CAN YOU COMPLETE THE TABLE ABOVE? FILL IN THE NUMBER ON THE PAPER INCLUDED IN THE BOX.

Figure A1.3 Tower Puzzle instruction card no. 3

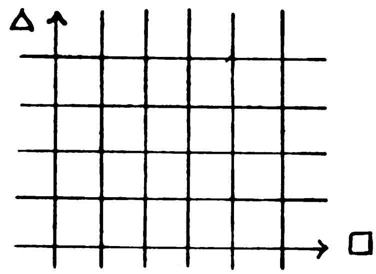
CAN YOU FIGURE OUT ONE RULE THAT WORKS FOR ALL THE PAIRS OF NUMBERS IN THE TABLE?

WRITE YOUR RULE (USING □ AND △), THEN CHECK IT BY TRYING VARIOUS PAIRS OF NUMBERS FROM THE TABLE.

Figure A1.4 Tower Puzzle instruction card no. 4

"I think the graph of these pairs of numbers will lie along a straight line," said Mark.

DO YOU AGREE? CAN YOU MAKE A GRAPH OF THE PAIRS OF NUMBERS IN THE TABLE



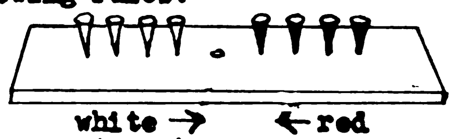
MAKE YOUR GRAPH ON THE GRAPH PAPER INCLUDED IN THE BOX.

Figure A1.5 Tower Puzzle instruction card no. 5

IF YOU STARTED WITH 100 DISCS AND IT TOOK ONE SECOND FOR EACH MOVE, HOW LONG WOULD IT TAKE TO TRANSFER THE PILE?

Figure A1.6 Tower Puzzle instruction card no. 6

Brian and David were examining the contents of this box one day. Brian said, "I've seen this game before. The object of the game is to interchange the red and white pegs. You must move the pegs according to the following rules:



i) The white pegs (tees) must move only to the right: the red pegs (tees) must move only to the left.
ii) You can only move one peg at a time.
iii) You can move a peg into an adjacent hole.
iv) You can jump, but only a single peg of the opposite color (you can't jump two pegs).

See if you can do it.

CAN YOU INTERCHANGE THE FOUR PEGS USING THE RULES ABOVE?

Figure A2.1 Peg Game instruction card no. 1

David said that if he started with two pegs on each side of the center hole, he needed 8 moves to interchange them.
DO YOU AGREE?
Brian suggested that a table be made to keep track of the number of pegs on each side (the number of pairs of pegs) and the corresponding number of moves. "Let the number of pairs of pegs be the number and let the number of moves to interchange them be the number," he suggested further.

number of pairs of pegs → □	Δ ← number of moves
1	
2	8
3	
4	
5	
6	

CAN YOU FILL OUT THE REST OF THE TABLE (ON THE PAPER INCLUDED IN THE BOX)?

Figure A2.2 Peg Game instruction card no. 2

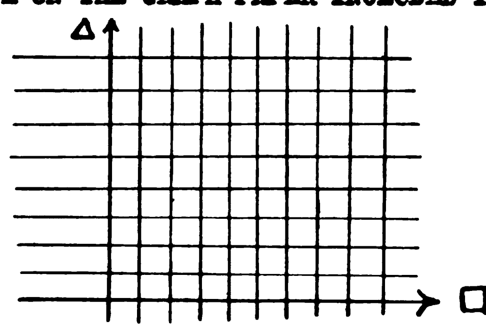
David said, "This is like the game, Guessing Functions; you give me a number (□), I use a rule on that number and get a number out (△)." CAN YOU FIND A RULE FOR THE PAIRS OF NUMBERS IN THE TABLE?

WRITE YOUR RULE (USING □ AND △), THEN CHECK IT BY TRYING VARIOUS PAIRS OF NUMBERS FROM THE TABLE.

Figure A2.3 Peg Game instruction card no. 3

David tried to graph the pairs of numbers from Brian's table. CAN YOU MAKE A GRAPH OF THE PAIRS OF NUMBERS IN THE TABLE?

MAKE YOUR GRAPH ON THE GRAPH PAPER INCLUDED IN THE BOX.



Brian said, "I bet the points lie on a straight line!" DO YOU AGREE?

Figure A2.4 Peg Game instruction card no. 4

If you took a white rod and used on side of it like a rubber stamp, what is the least number of times you could use it and cover the whole surface of a yellow rod?

Don said he got 17. DO YOU AGREE?

Marilyn said she got 22. WHO IS RIGHT?

Figure A3.1 Centimeter Blocks instruction card no. 1

Marilyn, Don and Jerry made this table with the various rods. (They called the "number of times stamped" the surface area)

Color of rod	Surface area
White	6
Red	
Light Green	
Purple	
Yellow	22

CAN YOU COMPLETE THIS TABLE?

Figure A3.2 Centimeter Blocks instruction card no. 2

Martha and Frank made their table a different way. Martha decided that since it took 5 white rods to make the same length as a yellow rod, she would call that one "5" and make her table like this:

	Number of white rods → □	△ ← Surface area
(white)	1	6
(red)	2	
(light green)		
(purple)		
(yellow)	5	22

CAN YOU FINISH THIS?

Figure A3.3 Centimeter Blocks instruction card no. 3

Ida Mae said that the table on card 3 reminded her of the game of "Guessing Functions." The "number of white rods" would be the □ number, and the "surface area" would be the △ number. Ida Mae said that she could even figure out a rube which would tell her the "△ number" if she knows the □ number.
CAN YOU FIND THE RULE?

Figure A3.4 Centimeter Blocks instruction card no. 4

CAN YOU GRAPH THE PAIRS OF NUMBERS IN THE TABLE ON CARD 3?

Figure A3.5 Centimeter Blocks instruction card no. 5

USING THE GRAPH CAN YOU FIND THE SURFACE AREA FOR THE BLUE ROD? THE ORANGE ROD?

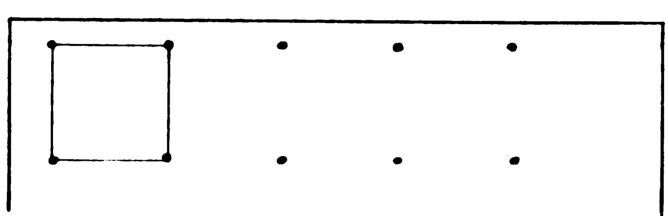
USING YOUR RULE CAN YOU FIND THE SURFACE AREA FOR THE BLUE ROD? THE ORANGE ROD?

Figure A3.6 Centimeter Blocks instruction card no. 6

How many different shapes and sizes can you make by stretching a rubber band around some of the nails on this geoboard?

Figure A4.1 Geoboard instruction card no. 1

Let's try something. Stretch a rubber band around four nails, like this:

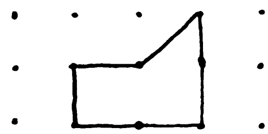


(the black dots are supposed to be the heads of nails)

If this is said to have an area of "one square," can you make a shape that you think would have an area of "2 squares"? "3 squares"? "$1\frac{1}{2}$ squares"?

Figure A4.2 Geoboard instruction card no. 2

Don says that <u>this</u> shape has an area of $2\frac{1}{2}$ squares.
DO YOU AGREE?



That <u>this</u> shape has an area of $2\frac{1}{2}$ squares. IS HE RIGHT?

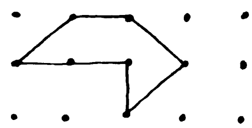
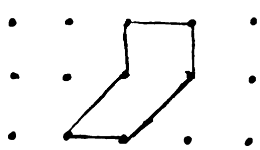


Figure A4.3 Geoboard instruction card no. 3

Frank says that this shape has an area of $1\frac{1}{2}$ squares. What do you think?



Frank's friend Martha says Frank is wrong. She says the area is 2 squares. WHO IS RIGHT?

Figure A4.4 Geoboard instruction card no. 4

CAN YOU FIGURE OUT THE AREAS WITHIN THESE SHAPES?

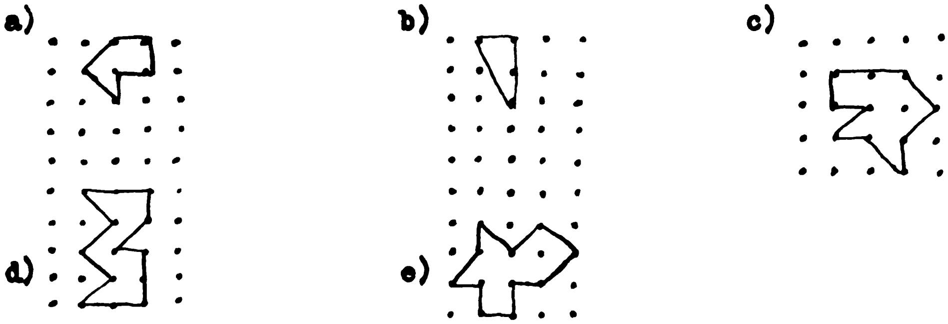


Figure A4.5 Geoboard instruction card no. 5

CAN you make up some interesting shapes and find the areas within them?

Can a friend of yours make up a shape that is too hard for you to find the area of?

Can you make up one that <u>he</u> can't find the area of?

Figure A4.6 Geoboard instruction card no. 6

Have you ever played the game "Guessing Functions?" If you have you might like to try this. Suppose you were told a certain number of nails, and you were told that you <u>must</u> touch all the nails with a rubber band.--CAN YOU PREDICT WHAT THE LARGEST POSSIBLE AREA WITHIN THE FIGURE WILL BE? (Nails within the figure are illegal)

Pat says that if you tell her "3 nails," the area will be $\frac{1}{2}$. DO YOU AGREE?

Bill says that if you tell him "5 nails," the area will be $1\frac{1}{2}$. IS HE RIGHT?

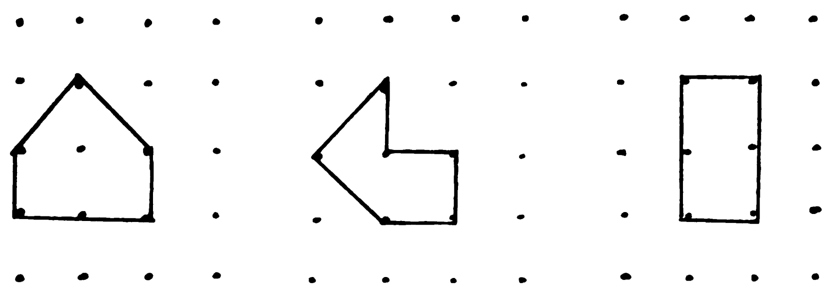
Louise says that if you tell her "6 nails," the area will be $2\frac{1}{2}$. WHAT DO YOU THINK?

Figure A4.7 Geoboard instruction card no. 7

Gary and Bob made this table for their answers.

(Nails)	(Area)
4	1
5	$1\frac{1}{2}$
6	2

For example, if you were told "6 nails"



This is illegal! This is legal! This is legal!

DO YOU THINK THEY ARE RIGHT?
CAN YOU FILL IN ANY MORE PAIRS OF NUMBERS?
CAN YOU GRAPH YOUR PAIRS OF NUMBERS?
CAN YOU GUESS THE RULE?

Figure A4.8 Geoboard instruction card no. 8

WHAT HAPPENS IF YOU NOW PLAY WITH DIFFERENT RULES AND ALLOW ONE NAIL WITHIN THE FIGURE, NOT TOUCHED BY THE RUBBER BAND?
DOES IT MAKE ANY DIFFERENCE HOW MANY NAILS THERE ARE WITHIN A FIGURE NOT TOUCHED BY THE RUBBER BAND?
CAN YOU WRITE A NEW RULE TO FIT <u>THIS</u> IDEA?

Figure A4.9 Geoboard instruction card no. 9

APPENDIX B

APPENDIX B

At the time of this study mimeographed instruction sheets were used in the mathematics laboratory that was used in conjunction with the Foundations of Arithmetic course taught at Michigan State University. These sheets were used for the "relations" laboratory experience that is described in this study. The sheets are reproduced on the following pages.

Unit 3. RELATIONS

3.1 CONCEPTS AND OBJECTIVES

The concept of a relation is one of the most fundamental concepts in mathematics. It is the foundation for the study of functions which in turn is the backbone of much of the study in higher mathematics. Thus, the main objective of this unit is to make the abstract notion of a relation one which the student can understand by exposing him to various relations defined on a set of manipulative blocks. Another objective is to exhibit the independence of the reflexive, symmetric and transitive properties. After examining these properties, you will be introduced to equivalence relations and the resulting equivalence classes.

3.2 MATERIALS NEEDED

The only materials needed for this unit is a set of colored rods. It is suggested that each student have a set available so that he may manipulate them at his own pace. A substitute for colored rods (sketched in unit 8) is a set of colored strips of paper whose lengths are in the same ration as the lengths of the rods.

3.3 PROCEDURE

Because of the nature of this material it is suggested that the basic definitions be discussed with the class before the students begin to manipulate the rods.

Def. 3.3-1 Let A represent an arbitrary set, and A x A be the Cartesian product of A with itself. A <u>relation</u>, call

Figure B1. Relations instruction sheet no. 1

-2-

it R, is a subset of A x A. Thus, all elements of the relation R are ordered pairs. Suppose that an ordered pair (a,b) is a member of the relation R. Two methods of denoting this membership are $(a,b) \in R$ and a R b.

1. If A is the set of colored rods, can you decide how many elements are in A x A?
2. {(red,red), (red,white), (white, blue)} is an example of one relation on the set of colored rods. Can you find three other relations on that set?
3. How many relations are there on the set of colored rods?

After examining questions such as these and arriving at appropriate answers, the student is ready for the definitions of the various properties which certain relations exhibit.

Def. 3.3-2 A relation R defined on the set A is said to be reflexive, or have the reflexive property, if $(x,x) \in R$ for each $x \in A$. One should note that if an element of A can be found for which this is not true, the relation is not reflexive.

Def. 3.3-3 A relation R defined on the set A is said to be symmetric, or have the symmetric property, if $(a,b) \in R$ implies $(b,a) \in R$. One should note that if he can find some element of R, say (p,q), for which $(q,p) \notin R$, the relation is not symmetric.

Def. 3.3-4 A relation R defined on the set A is said to be transitive, or have the transitive property,

Figure B2. Relations instruction sheet no. 2

-3-

if $(a,b) \in R$ and $(b,c) \in R$ together imply $(a,c) \in R$. Again, one should note that should the first two conditions be met, but $(a,c) \in R$, the relation is not transitive. The student should take special note of the fact that in order to meet the first two conditions the second coordinate of the first pair must be identical to the first coordinate of the second pair, but the first coordinate of the first pair need not be different from the second coordinate of the second pair. Many students interpret this to mean that transitivity and symmetry imply reflexivity.

DO SYMMETRY AND TRANSITIVITY IMPLY REFLEXIVITY?[1]

Suppose that R is a relation defined on a set A so that R is both symmetric and transitive. Also suppose that for an arbitrary element x of A, $(x,y) \in R$. Because R is symmetric, $(y,x) \in R$ also. Since (x,y) and (y,x) are both elements of R, and R is transitive, $(x,x) \in R$ also!!!

However, there must be something fishy about this argument because equivalence relations are typically defined as those which have all three of the mentioned properties. If the argument were valid, it would be enough to say that equivalence relations are symmetric and transitive.

The difficulty in the argument lies in the assumption that an element y exists such that $(x,y) \in R$. The following example should help to clairfy the situation.

Figure B3. Relations instruction sheet no. 3

-4-

Suppose A = {1, 2, 4, 6, 10}, $(p,q) \in R$ if and only if $p + q$ has a remainder zero when divided by 4. Obviously this is symmetric, and one can verify that it is also transitive. Therefore, by the argument given above, this should also be reflexive. However, we note that $(1,1) \notin R$. Thus R cannot be reflexive, and we see that symmetry and transitivity do not imply reflexivity. Questions and properties.

Students often wonder if it is possible to exhibit relations depicting all eight possible combinations of the properties. By working through the following exercises, you should see that the answer is yes. In each of the exercises the set will be the set of colored rods and R will be as defined. You are to determine which, if any, of the three properties are possessed by the defined relation.

4. $(x,y) \in R$ if and only if x has the same length as y.
5. $(x,y) \in R$ if and only if x "is shorter than" y.
6. $(x,y) \in R$ if and only if x "has a different length than" y.
7. $(x,y) \in R$ if and only if the length of x exceeds the length of y by an amount equal to the length of the white rod.
8. $(x,y) \in R$ if and only if neither x nor y are purple.
9. $(x,y) \in R$ if and only if the difference between the lengths of x and y is less than the length of the light green rod.
10. $(x,y) \in R$ if and only if x "is shorter than or has the same length as" y.

Figure B4. Relations instruction sheet no. 4

-5-

11. $(x,y) \in R$ if and only if x and y have the same color or the length of x exceeds the length of y by an amount less than the length of the yellow rod.

EQUIVALENCE RELATIONS

Def. 3.3-5 An equivalence relation is any relation which has the reflexive, symmetric and transitive properties. Some of the more common equivalence relations are:

1. The relation R, where R means "is equal to", as defined on the set of natural numbers. Reflexive - each number is equal to itself. Symmetric - if one number is equal to a second, the second is equal to the first also. Transitive - if a number is equal to a second, and the second is equal to the third, then the first is equal to the third.
2. The relation R, where R means "is congruent to" as defined on the set of squares. The reader should check the properties.
3. The relation R, where R means "has the same birthday month as" as defined on the set of students at your school. Again the reader should verify.

Def. 3.3-6 If R is an equivalence relation, R partitions the set upon which it is defined into disjoint subsets, called equivalence classes. It should be noted that every element of the original set falls into exactly one of the equivalence classes. In example 3 above, the equivalence relation R establishes 12 equivalence classes, namely:

Figure B5. Relations instruction sheet no. 5

-5-

1. The set of students in your school who were born in January.
2. The set of students in your school who were born in February.

3-12. etc.

Questions on equivalence relations.

Determine whether or not the following two relations as defined on the set of colored rods are equivalence relations. If you find that either or both are, list the equivalence classes.

12. $(x,y) \in R$ if and only if the length of x differs from the length of y by an amount equal to a multiple of the length of the red rod. (A multiple of the length of the red rod also includes the zero multiple.)
13. $(x,y) \in R$ if and only if the names for the colors of x and y begin with the same letter.

3.4 EVALUATION

The teacher may decide to evaluate student progress throughout the unit by creating equivalent exercises or merely evaluating those listed. The following exercises allow a little more flexibility in evaluation.

Because relations are merely subsets of Cartesian products, they may be written using the roster method.

Determine which property(ies) is exhibited by each of the following relations. If any relation is an equivalence

Figure B6. Relations instruction sheet no. 6

-7-

relation, list its equivalence classes.

1. R = $\{(1,2), (2,1)\}$ as defined on $\{1,2\}$
2. R = $\{(1,1), (2,2), (1,2), (2,1)\}$ as defined on $\{1, 2\}$
3. R = $\{(1,1), (2,2), (2,1)\}$ as defined on $\{1,2\}$
4. R = $\{(1,2), (2,3), (3,4)\}$ as defined on $\{1, 2, 3, 4\}$
5. R = $\{(1,2), (2,3), (1,3)\}$ as defined on $\{1, 2, 3\}$
6. R = $\{(0,0), (1,1), (2,2), (1,2), (2,1), (0,1), (1,0)\}$ as defined on $\{0, 1, 2\}$
7. $(x,y) \in R$ if and only if the last name of x and y begin with the same letter, as defined on the set of students in your school.
8. Suppose you are given the set of natural numbers through 9999. Can you define an equivalence relation on this set? Can you define an equivalence relation in such a way so as to have four equivalence classes emerge from your relation? Exhibit the four equivalence classes.

3.5 REFERENCES

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Cuisenaire Company of America
Mt. Vernon, New York

Colored Rods
Creative Playthings
Princeton, New Jersey

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[1] see "Remark on Equivalence Relations by R. A. Rosenbaum in the Nov., 1955 issue of the American Mathematical Monthly.

Figure B7. Relations instruction sheet no. 7

APPENDIX C

TABLE C1

INTERACTION ANALYSIS CATEGORIES

Ned A. Flanders

TEACHER TALK	INDIRECT INFLUENCE	1.* ACCEPTS FEELING: Accepts and clarifies the feeling tone of the students in a nonthreatening manner. Feelings may be positive or negative. Predicting or recalling feelings included.
		2.* PRAISES OR ENCOURAGES: Praises or encourages student action or student behavior. Jokes that release tension, not at the expense of another individual, are included. Nods head or says, "Um hm?" or "go on" also included.
		3.* ACCEPTS OR USES IDEAS OF STUDENT: clarifying, building, or developing ideas suggested by a student. As teacher brings more of his own ideas into play shift to category five.
		4.* ASKS QUESTIONS: Asking a question about content or procedure with the intent that a student answer.
		5.* LECTURING: Giving facts or opinions about content or procedure: expressing his own ideas.

TABLE C1--continued

	6.*	GIVING DIRECTIONS: Directions, commands, or orders to which a student is expected to comply.
DIRECT INFLUENCE	7.*	CRITICIZING OR JUSTIFYING AUTHORITY: Statements intended to change student behavior from non-acceptable to acceptable pattern; bawling someone out; stating why the teacher is doing what he is doing; extreme self-references.
STUDENT TALK	8.*	STUDENT TALK-RESPONSE: Talk by students in response to teacher. Teacher initiates the contact or determines type of student statement. As a student exponds his own ideas, shift to Category 9.
	9.*	STUDENT TALK-INITIATION: Talk initiated by students. The ideas expressed are created by students: statement content not easily predicted by previous action of teacher.
SILENCE OR CONFUSION	10.*	NONE OF ABOVE: Routine administrative comments, silence or confusion; interaction not related to learning activities.

*Note: The category numbers are purely nominal, no scale is implied.

Table C2. Sample Verbal Interaction Matrix

	1	2	3	4	5	6	7	8	9	10	Totals
1											
2			1	3	4	1				1	10
3		4	6	7	4	2		1	2	5	31
4			4	26	13	3		71	1	8	126
5			3	23	79	6		2		13	126
6			2	12	6	44	1	1	1	12	79
7						1	3	1	1	3	9
8		6	14	41	7	4		24	2	2	102
9			1		4	2			6	2	15
10			1	14	8	16	5	2	2	78	126
%		1.6	5.1	20.1	20.0	12.6	1.4	16.3	2.4	20.1	624

Teacher Talk
Columns 1-7 = 381

Student Talk
Columns 8-9 = 117

Indirect (1-4) / Direct (5-7) = ID ratio
168 ÷ 213 = .79
Indirect (1-3) / Direct (6-7) = Revised ID ratio
42 ÷ 81 = .52