THE ALLOCATION OF PRODUCTION COSTS WITH THE USE OF LEARNING CURVES

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ABSTRACT

THE ALLOCATION OF PRODUCTION COSTS WITH
THE USE OF LEARNING CURVES

By

Wayne John Morse

For most firms in most industries the production costs of a product are higher when that product is first introduced than they are after that product has been produced for a period of time. A graphic representation of the decrease in production costs as total production increases is referred to as a learning curve. This research was devoted to developing a cost allocation model based on the learning curve phenomenon.

Current accounting techniques of cost allocation take a segmented view of the production life cycle of a product and charge inventory or the cost of goods sold in each period on the basis of the actual production costs of that period. The result is a relatively low level of reported income in early periods when production costs are high and a relatively high level of reported income in later periods when production costs are lower.

The learning curve (L-C) cost allocation model developed in this research takes the entire production life cycle of a product into consideration and attempts to reconcile the timing differences between this period and the normal accounting period. The effect of using the L-C cost allocation model is to decrease the early period production

costs charged to inventory and the cost of goods sold from their current levels, thus raising reported income, and to increase the later period production costs charged to inventory and the cost of goods sold, thus lowering reported income. These changes in reported income are accomplished by adopting production cost standards based on the learning curve phenomenon and using a cost equalization account to insure that, as production takes place, charges to inventory and the cost of goods sold are equal to standard unit costs based on the average cost of all anticipated production. As long as actual production costs proceed in accordance with the learning curve phenomenon, the reported cost of each unit is the same. If actual production costs differ from those projected by the learning curve, a period cost variance is recognized or the model is changed.

The primary difference between the L-C cost allocation model and most other cost allocation models is that they are concerned with matching costs to units while the L-C cost allocation model is concerned with matching costs to production ventures.

In the development of the model considerable attention was given to the accounting concepts of matching and materiality. The model was evaluated in the light of certain standards of materiality in order to determine the statistical properties of the underlying cost data required before the model should be implemented for external reporting. Some problems which can arise after the model has been implemented were considered and suggested solutions to these problems were presented.

Finally, the model was applied to an actual production venture and the income statements obtained from its use were compared with income statements developed by the use of normal accounting procedures.

It was concluded that the L-C cost allocation model could be a valuable tool in attempting to reconcile the differences between the accounting period and the production life cycle of a product. The L-C cost allocation model is able to allocate production costs over the entire production life cycle of a product while still retaining the accounting concepts of "cost" and "objectivity."

The primary impediment to the adoption of the L-C cost allocation model appears to be a lack of detailed production information. However, once a decision has been made to adopt the model, the additional information could probably be generated with relatively little cost as a part of the normal accounting process.

THE ALLOCATION OF PRODUCTION COSTS WITH THE USE OF LEARNING CURVES

By

Wayne John Morse

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If, despite the help of others, any conceptual, mathematical, grammatical, or typing errors exist in the finished product I claim full responsibility for them.

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THE LEARNING CURVE

INTRODUCTION:

For most firms in most industries the production costs of a product are higher when that product is first introduced than they are after that product has been produced for a period of time. A graphical representation of the decrease in production costs as total production increases has been referred to as a learning curve. This research effort is devoted to developing a cost allocation model based on the learning curve phenomenon.

Current accounting techniques of cost allocation take a segmented view of the production life cycle of a product and charge inventory or the cost of goods sold in each period on the basis of actual production costs of that period. A standard cost system usually charges the cost of goods sold with any excess of the actual costs of production over the steady state standard costs of production. The result is a relatively low level of reported income in early periods when production costs are high and a relatively high level of reported income in later periods when production costs are lower.

A cost allocation model based on the learning curve phenomenon would take the entire production life cycle of a product into consideration and reconcile the timing differences between this period and the normal accounting period. The effect of using such a cost allocation model would be to decrease the early period production costs charged to inventory and the cost of goods sold from their current levels, thus raising reported income, and to increase the later period production costs charged to inventory and the cost of goods sold above their current levels, thus lowering reported income. These changes in reported income are accomplished by adopting production cost standards based on the learning curve phenomenon and using a cost equalization account to insure that, as production takes place, charges to inventory and the cost of goods sold are equal to standard unit costs based on the average cost of all anticipated production. As long as actual production costs proceed in accordance with the learning curve phenomenon, the reported cost of each unit is the same. If actual costs differ from those projected by the learning curve, a period cost variance is recognized or the model is changed.

The cost allocation model developed in this research will be similar to the one described above. In the remainder of this chapter brief consideration will be given to the historical development, general characteristics, uses, and limitations of learning curves. In Chapter II their significance for accounting will be discussed. Attention will be given to such topics as the development of accrual accounting techniques and the matching concept. In Chapter III the basic elements of a cost allocation model based on the learning curve phenomenon will be presented. By way of a hypothetical corporation, the accounting reports which would have resulted from the use of this model will be compared with the accounting reports which would have resulted from the use of current accounting techniques of cost allocation or from the use of an economic value concept of income. In Chapter IV the statistical properties of the model will be studied. Attention will be given to the importance of the accounting concept of materiality in the determination of minimum levels of statistical accuracy required to implement the model. In subsequent chapters attention will be given to a number of problems which might arise when attempts are made to implement the model. Possible methods of handling these problems will be suggested. Finally, an application of the model will be made to an actual industrial situation.

THEORY OF LEARNING CURVES:

The total costs of a product are divisible into pre-, actual, and post-production costs. Pre-production costs include research and development, and investments in production facilities. Actual production costs consist mainly of direct labor, materials, and variable factory overhead. Post-production costs include product modifications and after-sales service.[1]

The theory of learning curves (L-C) concerns itself with actual production costs. More specifically, it deals with the direct labor element of such costs and other actual production costs associated with the incurrence of direct labor. Its foundation lies in the belief that "a worker learns as he works, and the more often he repeats an operation the more efficient he becomes, with the result that the direct labor hours per unit (of production) declines."[2] Application does not, however, deal with individual effort as much as with the efforts of the organization as a whole.

A number of factors effect the rate of decrease in the time it takes organizations or individuals to perform a task. Among these factors are the following:

(1) The human content of an operation. The greater the human

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[1] K. Hartley, "The Learning Curve and Its Application to the Aircraft Industry," Journal of Industrial Economics, (March, 1965), p. 122.

[2] Frank J. Andres, "The Learning Curve as a Production Tool," Harvard Business Review, (January-February, 1954), p. 87.

content, the greater the susceptibility of an operation to improvement.[1]

(2) The training, experience, and skill of the man on the job and related personnel whose coordinated efforts are required to complete a job.

(3) The supervisors and staff who coordinate production.

(4) The production rate. Learning will occur most rapidly when the number of units to be produced is large enough so that the production units are operating at capacity and production is continuous.[2]

Assuming that a number of these factors operate in a favorable manner to a sufficient extent, the production time required per unit of output will decrease as output increases in accordance with the theory of the learning curve. The most widely adopted model based on learning curve theory states that: "Whenever the total quantity of units produced doubles, the cumulative average cost per unit decline by a constant percentage."[3] This model is stated in terms of cost. The words "time" and "cost" are used interchangeably in L-C literature. This does not mean that cost and time bear a constant relationship to each other, but that learning curves are used to project both production times and production costs. The following example is presented

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[1] W. B. Hirschman, "Profit From the Learning Curve," Harvard Business Review, (January-February, 1964), p. 134.

[2] T. B. Sanders and E. E. Blystone, "The Progress Curve--An Aid to Decision Making," N.A.A. Bulletin, (July, 1961), pp. 81-86.

[3] Raymond B. Jordan, "Learning How to Use the Learning Curve," N.A.A. Bulletin, (January, 1958), p. 27.

in terms of cumulative average time.

If the total time to produce the first unit was 100 hours, the second unit was 80 hours, and the third and fourth units together were 144 hours, the production process would be said to be following a 90 percent learning curve, when the learning curve refers to cumulative average time. The decrease in cumulative average production time is further illustrated in table 1-1.

TABLE 1-1
Cumulative Average Production Times--
90% Learning Curve

Cumulative Production	Cumulative Average Hours Per Unit	Additional Production	Per Block Average Per Unit
1	100.0 Hours	1	100.0 Hours
2	90.0 "	1	80.0 "
4	81.0 "	2	72.0 "
8	72.9 "	4	64.8 "
16	65.6 "	8	58.3 "
32	59.0 "	16	52.4 "

This decrease in the cumulative average time required to accomplish a given task is the ratio between the cumulative average direct labor hours required at any unit of output and the cumulative average direct labor hours required at twice that output[1] thus

[1]Hartley, p. 123.

$\frac{90.0}{100.0} = \frac{81.0}{90.0} = \frac{72.9}{81.0}$, and so forth. The information presented in table 1-1 is frequently presented in a graphic manner as shown in figure 1-1.

FIGURE 1-1
Cumulative Average Production Times--90% Learning Curve

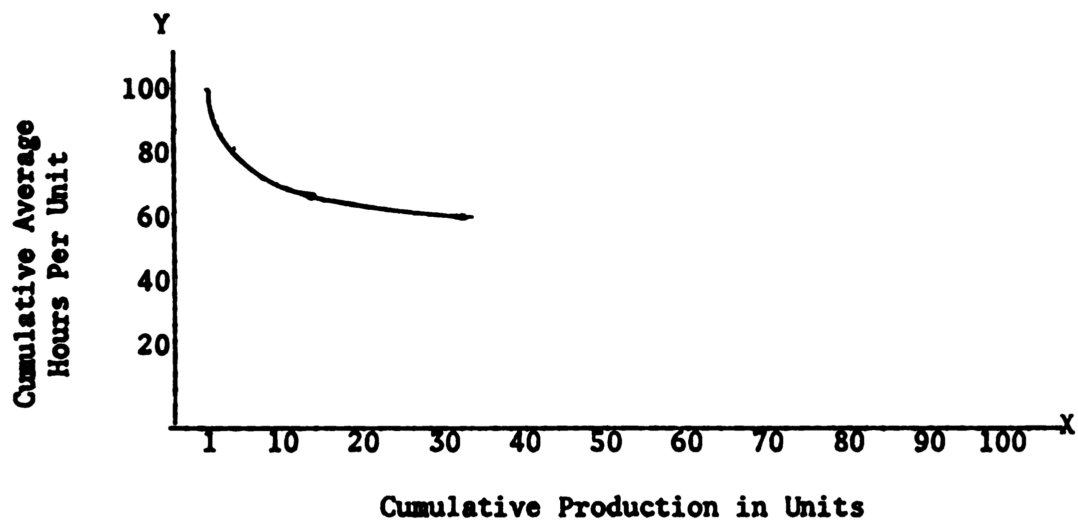

Table 1-1 and figure 1-1 show that the "economics of learning" are reaped in decreasing amounts. If the production run is long enough "learning" could cease for all practical purposes, and the cumulative average hours per unit could become almost constant. The relatively steep portion of the learning curve is referred to as the "startup phase," and the relatively flat portion of the learning curve is referred to as the "steady state phase."

The percentage used in reference to learning curves is in reality the complement of the rate of learning. For example, with no learning the learning rate is 0 percent, but the learning curve percent is 100.

With a 90 percent learning curve there is a 10 percent decrease in cumulative average hours between the first and second units of production.

Some authors prefer to use the terms "progress curve,"[1] "time reduction curve,"[2] "improvement curve,"[3] or "experience curve"[4] rather than "learning curve," because of their belief that a pure learning curve should reflect only the rate of the operator's learning, and not consider other possible causes of the curve's characteristic downward slope, such as equipment development, better tooling, improved materials and the development of management. They feel that their terminology can more accurately reflect the summation of all these factors. The semantically less accurate, but more widely used term, "learning curve," will be used throughout the remainder of this research. Here it will refer to both individual and organizational learning. It is intended to be a broad concept.

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[1]Sanders and Blystone, p. 81.

[2]S. A. Billon, Industrial Time Reduction Curves As Tools For Forecasting, (East Lansing, 1960).

[3]W. F. Brown, The Improvement Curve, (Wichita, 1955).

[4]A. W. Morgan, Experience Curves Applicable to the Aircraft Industry, (Baltimore, 1952).

EARLY RESEARCH:

Although the general concepts underlying learning curve theory have been known for many years it was not until the late 1930's that the rate of decrease in the time required to accomplish a task was observed to be regular enough to be predictable.[1]

T. P. Wright, of the Curtiss-Wright Corporation, is credited with formalizing the theory of learning curves. After observing aircraft production for some time, he found a consistent decrease in the cumulative average production time as output doubled. By studying previous production records he was able to determine the rate of decrease in production times for similar kinds of aircraft. Determining the rate of decrease in production time made it possible for him to predict production times and delivery schedules for future aircraft with a high degree of accuracy.[2]

The Stanford Research Institute undertook a similar study of the majority of aircraft produced during World War II. This study concluded that although the learning curves for different types of aircraft were different in terms of their starting points (i.e., the labor inputs for the first plane of a particular type), the great majority had one characteristic in common--their rate of improvement.[3]

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[1]Andres, p. 87.

[2]Carl Blair, "The Learning Curve Gets and Assist From the Computer," Management Review, (August, 1968), pp. 31-32.

[3]Andres, p. 88.

A third study of aircraft production experience, conducted by the British Ministry of Aircraft Production, led to the same conclusions.[1]

In 1943, France J. Montgomery reported on a study he made of the construction of liberty ships. "Between December 1941, when the first two ships were delivered, and the end of April, 1943, the average man-hour requirements per vessel delivered was reduced by more than one half."[2] Montgomery was one of the first to realize the potentially wide applicability of the learning curve phenomenon when he concluded that a study of the production figures of a company manufacturing a complex but standardized item would probably reveal a trend similar to that which occurred in the construction of liberty ships.[3]

In 1960, S. Alexander Billon conducted a study to see if the learning (time reduction) curve occurred in industries where a preconceived model of time reduction was not employed to set standards. His study of 54 products in 3 firms concluded that a "definite regularity in time reductions was observed in a majority of cases."[4]

A review of the literature since 1960 does not reveal any serious attempt to question or examine the theoretical foundations upon which

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[1]E. J. Broster, "The Learning Curve for Labor," Business Management, (March, 1968), p. 35.

[2]F. J. Montgomery, "Increased Productivity in the Construction of Liberty Vessels," Monthly Labor Review, (November, 1943), p. 861.

[3]*Ibid.*

[4]Billon, pp. 1-2.

the learning curve is based. The majority of writers have concerned themselves with a discussion of how to use the learning curve, the purposes for which its use is suitable, and a discussion of the limitations of its use.

PREVIOUS APPLICATIONS:

The ability of learning curve models to project production costs or times has resulted in their widespread application in the following areas of managerial decision making and evaluation:[1]

(1) Setting selling prices.

(2) Projecting labor loads in the factory.

(3) Determining manpower requirements.

(4) Controlling shop labor.

(5) Determining realistic prices for subcontracted items.

(6) Examining the training progress of new employees.

(7) Deciding whether to "make or buy."

(8) Determining break-even points.

(9) Planning finances, including cash flows.

Item (4) deserves special attention. By its very nature L-C theory refers to human operations and teamwork within an organization. During the startup phase of production the use of steady state phase performance norms will yield a stream of "unfavorable variances." These variances do not necessarily signal a departure from expectations nor act as a guide for corrective action.

> The motivational value of these variances is questionable. If the discrepancy between the steady state standard and actual performance is large, and remains so for several

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[1]For further readings in the area of previous application of learning curves see the bibliographic references to the works of Andres; Baloff & Kennelly; Brenneck; Broadston; and Jordan.

> months, the "goal" may lose all its motivational value. Aspiration level studies indicate that subjects may reject a goal as being unrealistic and unattainable if the differences between actual and target performance is large.[1]

Under such circumstances there have been cases where workers in the steel industry reacted by terminating the startup phase at an artifically low productivity level.[2] Instead of steady state standards, L-C theory has been used to develop "moving" or "sliding" standards.

Another interesting case is that in which management set its steady state standards at too low a level of productivity. Upon reaching shop standards labor terminated the startup phase.[3]

There are now four methods of cost projection:

(1) Recent experience data. Companies in mass production industries can use historical cost accounting data, modified for known changes to come, in order to produce standard costs.

(2) Similar parts data. Standard costs for parts similar to those which have been produced for an extended period of time can be developed in a manner similar to (1) above.

(3) Engineering standards and references. When one item or a very small number of items of a complex nature are to be produced,

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[1]Nicholas Baloff and John Kennelly, "Accounting Implications of Product and Process Start-Ups," Journal of Accounting Research, (Autumn, 1967), p. 141.

[2]Ibid.

[3]James D. Broadston, "Learning Curve Wage Incentives," Management Accounting, (August, 1968), p. 18.

the use of engineering standards and references is appropriate.

(4) Learning Curves. Industries whose products are neither mass produced over a long period of time nor produced in single item quantities are most susceptible to the application of learning curve theory for projecting their costs.[1]

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[1]Marvin L. Taylor, "The Learning Curve--A Basic Cost Projection Tool," N.A.A. Bulletin, (February, 1961), pp. 21-22.

BASIC MATHEMATICS:

The mathematics of learning curves concerns itself with the development of an equation which will fit the type of curve shown in figure 1-1, and certain modifications to that equation. The purpose of this section is to summarize the work of others[1] in terms of a common set of symbols.

Let

X = cumulative production (measured on the horizontal axis).
Y = cumulative average production time (cost) per unit (measured on the vertical axis).
a = time (cost) required to produce the first unit (vertical axis intercept). a used in text.
b = exponent which accounts for the slope of the L-C. b in text.

For the first unit of production in table 1-1 and figure 1-1:

$$Y = \frac{a}{X^b} = \frac{100}{1} = 100 \text{ hours}$$

No matter what value b assumes, the value of X^b will always be 1 when X = 1.

For the second unit of production a value of b must be found so that:

$$Y = \frac{a}{X^b} = \frac{100}{2^b} = 90$$

Solving for b

$$90 \cdot 2^b = 100$$

[1]For a more detailed discussion of the mathematics of learning curves see the bibliographic references to the works of Andres; Baloff & Kennelly; Blair; Broadston; Hein; Jordan; and Springer, Herliky, Mull, & Breggs; especially Hein.

$$2^b = \frac{100}{90} = 1.1111$$

$$b \log 2 = \log 1.1111$$

$$b = \frac{\log 1.1111}{\log 2} = \frac{.04375}{.30103} = .15198$$

Likewise when X equals 4:

$$Y = \frac{a}{X^b} = \frac{100}{4^b} = 81.0$$

$$81.0 \cdot 4^b = 100$$

$$4^b = \frac{100}{81.0} = 1.23457$$

$$b \log 4 = \log 1.23457$$

$$b = \frac{\log 1.23457}{\log 4} = \frac{.09152}{.60206} = .15201$$

Similar calculations could be made for the other values of X and Y in table 1-1 and figure 1-1.

Using the values of a and b developed above it is now possible to project values of Y for all values of X. Thus, for a cumulative production of 128 units the cumulative average hours per unit is:

$$Y = \frac{100}{128^{.152}} = 47.8$$

On the basis of production data for the first few units of output and the basic equation 1-1

$$Y = \frac{a}{X^b} \qquad (1\text{-}1)$$

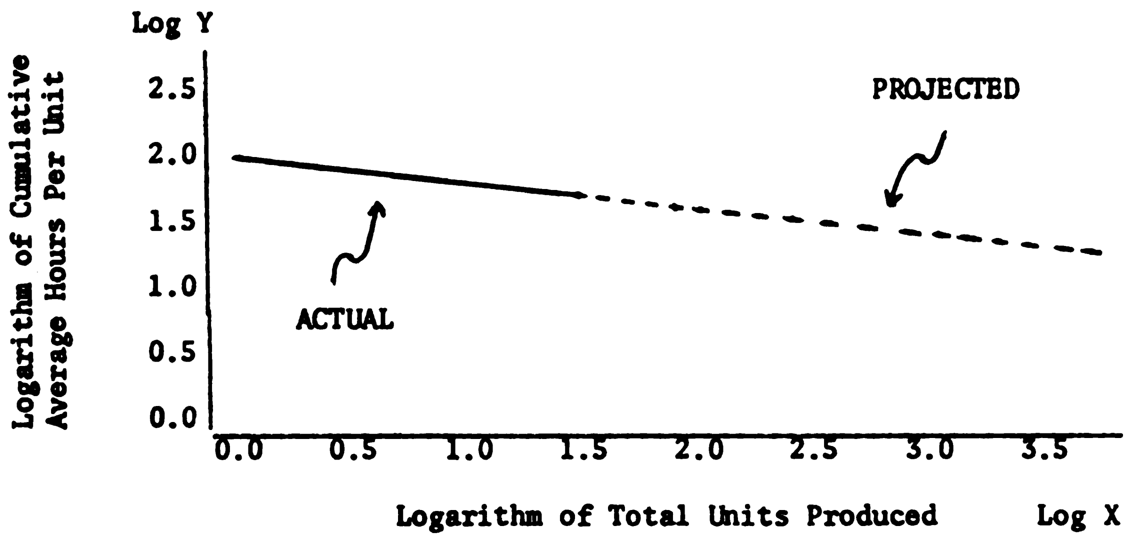
projections are made to determine future production time (or cost). It is this ability to project expected values of Y which has been the basis of most L-C applications.

The most widely used methods of finding the a and b parameters in (1-1) involve the transformation of this exponential function into a linear one by the use of logarithms. Table 1-2 and figure 1-2 show

TABLE 1-2
Logarmithmic Transformation of A 90% Learning Curve

Total Units X	Log X	Cumulative Average Hours Y	Log Y
1	0.00000	100.0	2.0000
2	0.30103	90.0	1.95424
4	0.60206	81.0	1.90849
8	0.90309	72.9	1.86273
16	1.20412	65.6	1.81690
32	1.50515	59.0	1.77085

FIGURE 1-2
Logarmithmic Transformation of A 90% Learning Curve



how the information contained in table 1-1 and figure 1-1 can be transformed into a straight line by the use of logarithms.

From equation 1-1 the linear relationship shown in figure 1-2 is found as follows:

$$Y = aX^{-b} \qquad (1\text{-}2)^{1}$$

$$\log Y = \log a - b \log X \qquad (1\text{-}3)$$

After transforming the X and Y data for the first few units into logarithmic form we can use the familiar formulas for the least squares regression analysis,

$$b = \frac{n\Sigma(\log X \log Y) - \Sigma\log X\Sigma\log Y}{n\Sigma(\log X)^2 - n(\Sigma\log X)^2} \qquad (1\text{-}4)$$

$$\log a = \frac{\Sigma\log Y}{n} - b\frac{\Sigma\log X}{n}, \qquad (1\text{-}5)$$

in order to determine the parameters of the equation. Applying equations 1-4 and 1-5 to the data in table 1-2 yields a b value of .1520 and an a value of 100.

The equation for the correlation coefficient,

$$R = \frac{n\Sigma(\log X \log Y) - \Sigma\log X\Sigma\log Y}{\sqrt{n\Sigma(\log X)^2 - (\Sigma\log X)^2} \cdot \sqrt{n\Sigma(\log Y)^2 - (\Sigma\log Y)^2}}, \qquad (1\text{-}6)$$

can be used to find the amount of change in Y as X varies which is accounted for by the solved values of a and b. For the data in table 1-2 all of the change in Y as X varies is accounted for by the solved values of a and b. Hence, the correlation coefficient is -1. In Chapter IV a considerable amount of space will be devoted to

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[1]Many authors prefer to use equation 1-2 instead of 1-1 to solve for cumulative average production time. They frequently present (1-2) as $Y = aX^b$ and specify that b is negative.

determining minimum absolute values of the correlation coefficient required to implement the cost allocation model developed in this research.

Rough estimates of future production times or costs can be obtained by the use of log-log paper. When this procedure is used the absolute values of the initial production data are plotted directly on log-log paper, a straight line is drawn through this data, and anticipated future values are read directly from the log-log paper. Because such a procedure lacks precision it is not used in this research.

The General Electric Company's Mark II Time Sharing Service has a number of computer programs available which calculate anticipated values of Y on the basis of data for the first few units of production.[1] Because of their cost and lack of availability for modification these programs were not used in this research. A number of computer programs which do meet the specific needs of this research are listed in the Appendices.

There are two basic learning curve models, one is primarily concerned with projection of cumulative average time or cost, the other is primarily concerned with projecting unit time or cost. The differences between these two types of learning curves is best explained by way of a brief example.

Assume 100 hours are required to manufacture the first unit of a product which has an 80 percent learning curve. A learning curve model

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[1]General Electric, Analysis Using Learning Curves, Mark II Time Sharing Service, Program Library Users Guide, (September, 1968).

based on cumulative average time applies the 80 percent curve to the cumulative average time for producing the units. Therefore, the cumulative average time for manufacturing two units will be 80 hours and the cumulative time for manufacturing two units is 160 hours. Because the first unit took 100 hours to produce, the time for the second must be 60 hours. A learning curve model based on unit time applies the 80 percent curve to the actual time it takes to produce each unit. Thus the production time for the second unit is 80 hours, and the cumulative time for manufacturing the first two units is 180 hours. Cumulative average time is thus 90 hours.

The cumulative average time model of learning curve theory is used, unless indicated otherwise, in the remainder of this research.

Two more formulas used in cumulative average time (cost) models can be derived from (1-1). Total production time (cost) to produce the first X units is developed from (1-1) by multiplying (1-1) by X,

$$T = X \cdot a/X^b \; . \qquad (1\text{-}7)$$

Equation 1-7 is simplified as follows:

$$T = aX^{(1-b)}$$

$$T = aX^c \qquad (1\text{-}8)$$

where:

T = total production time (cost) for the first X units;

c = (1-b).

Unit production time (cost) required to produce unit X is derived from (1-8):

$$U = aX^c - a(X-1)^c \qquad (1\text{-}9)$$

$$U = a(X^c - (X-1)^c) \tag{1-10}$$

where:

U = time (cost) required to produce unit X.

The unit time model of learning curve theory solves equation 1-11, below, for a and b.

$$U = aX^{-b} \tag{1-11}$$

This is done by transforming (1-11) into an equation similar to (1-3) and then applying a least squares regression analysis. The formulas for total production time (1-12) and cumulative average production time (1-13) are:

$$T = \int_1^N aX^{-b}dx = \frac{aX^{1-b}}{1-b} = \frac{a(N^{1-b}-1)}{1-b} \tag{1-12}$$

and

$$Y = \sum_{X=1}^{N} \frac{aX^{-b}}{N} \tag{1-13}$$

where:

N = number of units produced.

The cumulative average time model of learning curve theory is used in this research because the cumulative average time of all anticipated production is the most important value which must be calculated. There may be merit in developing a cost allocation model based on the unit time model of learning curve theory. This is mentioned in the Suggestions for Further Research.

LIMITATIONS OF LEARNING CURVES:

The limiting values of a cumulative average learning curve are 100 percent and 50 percent. If no learning occurs, the cumulative average time per unit does not change and the model follows a 100 percent learning curve. Given any level of output the cumulative average time per unit at that level of output is the same as that at any lower level of output. If the learning curve percent were 50, the model would indicate that the second unit took zero time to produce. If the cumulative average time for the first unit were 100 and the second were 0, the cumulative average time would be 50.[1] The mathematical properties of the learning curve makes a L-C of less than 70 percent difficult to envision.[2]

The industrial applications of learning curves fall between the limits of single units produced in accordance with special orders and items which have been mass produced for an extended period of time. As previously mentioned, engineering standards and references are employed when cost or production standards are set for a small number of items of a complex nature, and modified recent experience data is employed when cost or production standards are set for a product which has been mass produced for an extended period of time.

Figure 1-3, for an 80 percent learning curve, shows that as total output increases the L-C soon reaches a point where the difference

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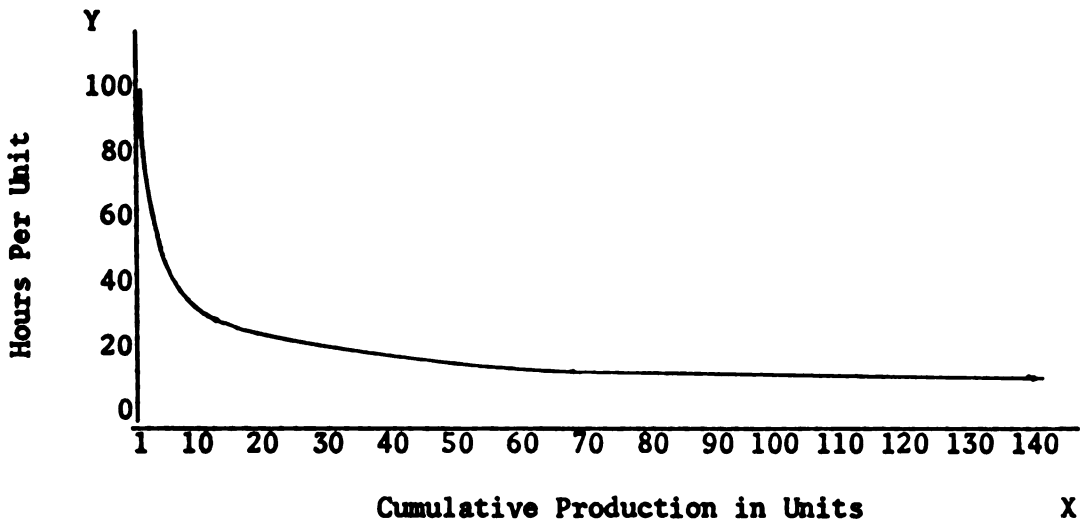
[1]Leonard W. Hein, The Quantitative Approach to Managerial Decisions, (Englewood Cliffs, 1967), p. 91.

[2]Jordan, p. 27.

FIGURE 1-3
Unit Production Time--80% Learning Curve



in production time between successive units approaches zero. In learning curve terminology, the learning curve is said to have reached a "steady state." Some authors have said learning curve theory is no longer applicable once the steady state is attained because no further decreases in production time (cost) take place.[1] Other authors contend that although the reduction in production time still takes place, it takes place over such a relatively large number of units and periods of time that it escapes notice.[2] In any event, the application of learning curve models to products which have reached the steady state is of little value.[3]

[1]Nicholas Baloff, "The Learning Curve--Some Controversial Issues," Journal of Industrial Economics, (July, 1966).

[2]Hirschman.

[3]Jordan, p. 28.

Problems which may hinder all attempts to apply learning curve models include small orders, essential modifications, labor turnover,[1] strikes, and terminations of the startup phenomenon.[2] Increases in wages[3] are also a potential problem when the L-C is used to project production costs. This problem is considered in Chapter V.

Because the learning curve phenomena pertains to a large extent to improvement in the human element of productivity, and man's learning to control machines in more efficient ways, the slope of the curve depends to a large extent on the proportion of human and machine labor involved. In manual operations time reduction is limited only by the dexterity of the human hand. In manufacturing processes composed principally of machine operations much of the reduction in time and cost is limited by the feed and speed of the machine.

Jordan found the relationships shown in table 1-3 between the L-C percent and the proportions of operations performed by human and machine labor.[4] Table 1-3 indicates that the learning effect is more significant in production processes that have a greater element of human than machine labor, and that a change from one type of labor to another may have a significant effect on the learning curve. For example, a change from human to machine labor after a number of units are produced might result in a downward shift in the unit production time or cost curve (an essential modification) and a decrease in the L-C percent.

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[1]Broster, p. 35.

[2]Baloff & Kennelly, p. 141.

[3]Hein, p. 113.

[4]Jordan, p. 28.

TABLE 1-3
Relationship Between Labor Inputs and L-C Percent

Human Labor	Machine Labor	L-C Percent
75%	25%	80%
50%	50%	85%
25%	75%	90%

CHAPTER II

LEARNING CURVES AND ACCOUNTING

INTRODUCTION:

Before the advent of long lived business enterprise there was no need for the development of complex cost allocation systems to assist in reporting periodic earnings. Under the Italian bookkeeping methods there was no concept of the accounting period.

> Most business ventures were of short duration, or at least not continuous after a specific trading objective had been reached. As a result, profit was calculated only upon the completion of the venture. Without the concept of periodic profit, there was no need for accruals and deferrals.[1]

Today corporations have indefinite lives, and managers, investors, and creditors desire timely information about changes in the financial condition of businesses. Accounting reports are important in evaluating the past and planning for the future. Out of this need for timely information to assist in evaluation and planning has come an emphasis on the periodic reporting of business income. Accounting periods of equal length are used because they are "'consistent' and therefore promote comparability."[2] But merely having reporting periods of equal

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[1]Eldon S. Hendriksen, Accounting Theory, (Homewood, 1970), pp. 25-26.

[2]Maurice Moonitz, "The Basic Postulates of Accounting," Accounting Research Study No. 1, (AICPA, 1961), p. 17.

length is not enough.

It is generally recognized that reporting the results of business operations on a regular periodic basis is of little value for decision making unless some attempt is made to match costs with revenues.

> ...because revenue and expense transactions are reported separately, and because the acquisition and payment for goods and services do not usually coincide with the sales and collection processes related to the same product of the enterprise, matching has come to be considered a necessity or at least desirable. The leads and lags in the acquisition and use of, and the payment for, goods and services are assumed to be the reason for accruals or deferrals in order to match the expense with associated revenue.[1]

In 1964, the American Accounting Association's Concepts and Standards Research Study Committee defined "Costs" as resources given up or economic sacrifices made.[2] The committee stated that costs "are incurred with the anticipation that they will produce revenues in excess of the outlay"[3] and that "appropriate reporting of costs and revenues should therefore relate costs with revenues in such a way as to disclose most vividly the relationship between efforts and accomplishments."[4] In order to accomplish this the committee advocated that costs should be related to revenues within a specific period on the basis of some discernable positive correlation of such

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[1]Hendriksen, p. 183.

[2]Concepts and Standards Study Committee, "The Matching Concept," The Accounting Review, (April, 1965), p. 368.

[3]Ibid.

[4]Ibid.

costs with the recognized revenues.[1] In the case of costs incurred to produce goods intended for future sale, the committee recommends that the specific costs for material and labor be attached to specific units and that these costs be expensed when these units are sold.[2] The procedure described above, and the use of steady state standard costs are generally accepted accounting procedures for matching production costs with revenues.[3]

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[1] Ibid., p. 369.

[2] Ibid.

[3] In this research it is assumed that cost variances are written off to the cost of goods sold in the period incurred. In all of the examples presented in this research it is further assumed that production equals sales in each period and that there are no beginning or ending inventories. Therefore, in the examples presented in this research, the cost of goods sold is the same regardless of whether a standard cost system is used or the actual cost of each unit is expensed in the period it is sold.

SIGNIFICANCE OF L-C FOR ACCOUNTING REPORTS:

The use of either cost allocation (matching) procedure described above may lead to undesirable results when applied to firms whose unit production costs decline over the entire production life cycle of a product, (i.e., whose production costs follow the learning curve or similar phenomenon). The use of either accounting procedure may lead to significantly understated reported earnings in early accounting periods and overstated reported earnings in later periods. The actual cost of units produced in earlier periods will be above and the actual cost of units produced in later periods will be below the average unit cost of all units produced during the product's production life cycle. The accounting procedures described above make the reported earnings of such firms depend on the current stage in a product's production life cycle. The reported rate of return on investment will be highly variable.

The dangers of artificial volatility in reported earnings for investors were mentioned in a recent speech by Sidney Davidson:

> [Today] There is a widespread view among managers and accountants that the market responds directly to changes in reported earnings per share, that investors...cannot see through the reported earning data to the underlying economic facts which the reports are supposed to depict.[1]

A study reported on in 1967 by Hamil and Hodes indicated that companies with a history of highly volatile earnings tend to trade at a much lower price-earnings multiple than other comparable companies whose

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[1]Sidney Davidson, "Accounting and Financial Reporting in the Seventies," The Journal of Accountancy, (December, 1969), p. 30.

growth in earnings has been stable around a basic trend.[1] Assuming that investors react to increased variability by demanding an increased return, the variability in a firm's reported earnings which is caused by conventional methods of handling startup costs may lead to a lower P/E ratio for the firm's stock than the firm's underlying economic income trend warrants.

Sidney Davidson has also commented on the problems which current cost allocation techniques cause management when reported production costs decline over the production life cycle of a product and startup costs are high.

> It seems unthinkable that a wise decision by management, based on a careful consideration of probable future consequences and proceeding precisely according to plan, should have the effect of reducing reported net income. It is scant comfort for management to be told that, if the program continues according to plan, reported net income will ultimately be higher, indeed higher by an amount that compensates for the earlier reported losses or understatements of income. Income measures effectiveness, and judgements on managerial effectiveness are made too frequently for managers to take much solace from the thought of compensating gains sometime in the future. It is bad enough to think of the danger of being replaced by a new management as a result of troublesome accounting reporting practices, and worse to be told that the successor management will look especially good as the compensating effect for the losses charged against current management.[2]

Mr. Davidson concluded that, "If management comes to feel that accounting practices inhibit desirable action, this indeed will present

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[1]Hamil and Hodes, "Factors Influencing Price-Earnings Multiples," Financial Analyst Journal, (January, 1967), p. 90.

[2]Sidney Davidson, "The Day of Reckoning--Managerial Analysis and Accounting Theory," Journal of Accounting Research, (Autumn, 1963), pp. 18-19.

a day of reckoning for accounting."[1]

Despite the impact of startup costs on reported income, and their significance for investors, managers, and accountants, a review of the literature reveals only one article which indicates the potential magnitude of early period (startup) costs and their impact on financial statements, or suggests better methods of accounting for them.[2] Only the aircraft industry has made any attempt to systematically account for them. Baloff and Kennelly have noted that while accountants pay little attention to such costs and generally regard them as insignificant when averaged out over a one year period, scarcely a week goes by that some company is not using such costs as a rationale for disappointing earnings.[3] After reviewing write-ups on corporate earnings in the Wall Street Journal for a period of time, Baloff and Kennelly concluded that such costs are probably significant and "that the effects of start-ups on earnings provide many challenges for accountants."[4]

In accounting terminology, the problems discussed above center around income recognition and cost allocation. They are caused by the use of reporting periods which do not coincide with either the production life cycle of a firm's products or the life of the firm. In Accounting Research Study No. 1, Maurice Moonitz noted that, "The

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[1] Ibid., p. 19.

[2] B. R. Wyer, N.A.A. Bulletin, (July, 1958), Section 2.

[3] Baloff and Kennelly, p. 142.

[4] Ibid.

problem of recognition and allocation is made more difficult because the 'events' often take longer to work themselves out than the reporting periods customarily in vogue."[1] In this research effort the "event" is a venture whose duration is the production life cycle of a product. The problem is how to properly match all of the costs and revenues associated with this venture.

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[1]Moonitz, p. 33.

ECONOMIC VALUE AND ACCOUNTING INCOME:

On a theoretic level it might be argued that the real problem in income measurement is not one of achieving a proper matching of costs and revenues but one of asset valuation. If assets could be properly valued on the basis of the revenues they are expected to produce then income would consist of interest on these assets and changes caused by new asset acquisitions or investment programs undertaken by management. Persons who would make such statements are arguing for economic income and economic value as ideals toward which accounting should strive.

At this level of reasoning economic income has been defined to be

> ...the change, over some period of time, in the value of a firm's assets. The total value of a firm's assets at any point in time can be determined by discounting, at some normal rate of return, the expected net cash flows from asset utilization. The total economic income figure which results from a comparison of beginning and ending period asset values can be fragmented into two components: (1) expected income, and (2) unexpected income.[1]

The expected income component can be regarded as interest. It is the product of the normal rate of return and the net present value of the assets at the beginning of the period. The unexpected income component of economic income is equal to changes in asset net present value which developes as a result of changes in expectations regarding future cash flows.[2]

Indeed, "Economic income is generally defined as an ideal theoretical concept which is impractical to implement because of the

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[1]Lawrence Revsine, "On the Correspondence Between Replacement Cost Income and Economic Income," The Accounting Review, (July, 1970), p. 515.

[2]*Ibid.*, p. 516.

difficulty in an uncertain world of measuring cash flows."[1] Because of the subjectivity associated with the use of economic income and economic value such concepts cannot be used in practice. In this research they are retained as theoretically ideal concepts.[2] But, the objective of the cost allocation model which will be developed is not to approximate economic value and economic income, rather, it is to try and get accounting income to move in the same direction as economic income throughout the production life cycle of a product.

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[1]Keith Schwayder, "A Critique of Economic Income as An Accounting Concept," Abacus, (August, 1967), p. 28.

[2]Schwayder attacks economic income as an unsound theoretic model for accounting income measurement because all of the firm's important economic events except cash flow and rates of subjective income are ignored. He argues that economic income places too much emphasis on the future, does not consider the firm's past and current success in dealing with its economic environment, uses a subjective interest rate rather than the interval rate of return, and is limited by the certainty assumption. See Schwayder, pp. 34-35. In the example developed in Chapter III to compare economic income with the income derived from the use of the cost allocation model developed in this research, the interval rate of return is used, certainty is assumed, and the past is ignored.

LONG-TERM PROSPECTIVE:

What is needed is a better procedure for reporting costs and revenues. The 1964 AAA Concepts and Standards Research Committee felt that the "Appropriate reporting of costs and revenues should... relate costs with revenues in such a way as to disclose most vividly the relationships between efforts and accomplishments."[1] This researcher feels that attaching actual production costs to units or using steady state standard costs does not accomplish this objective as well as the use of a cost allocation model based on the learning curve phenomenon. What is needed is a long-term prospective of expenses which takes into consideration the probable decline in production costs as more units are produced.

Thomas R. Prince, in his book, Extension of the Boundries of Accounting Theory, argues for the use of a long-term income prospective in reporting the results of business operations.[2] According to Prince, "the long-term income perspective attempts to anticipate total cost and total revenue and match these two aggregates."[3] He notes that "the long-term approach would have more accruals and more deferrals which would result in the leveling out of reported business income."[4]

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[1]Concepts and Standards Study Committee, p. 368.

[2]Thomas R. Prince, Extension of the Boundries of Accounting Theory, (Cincinnati, 1963), p. 180.

[3]Ibid.

[4]Ibid.

The learning curve cost allocation model developed in this research will result in an additional deferred charge being shown on the balance sheet. It will not result in the addition of any accruals. It is primarily concerned with the allocation of production costs. It does not pay specific attention to revenue projection. The model will match costs to revenues on the basis of the percentage of total anticipated production sold. Attaching costs to specific units is of secondary importance.[1] Unit costs are important only for purposes of variance analysis.

During early periods of production the model will call for deferral of a portion of the incurred production costs through charges to the asset account "Improvements in Production Procedures." In later periods as the production life cycle of a product nears its end the account "Improvements in Production Procedures" will be reduced by charges to "Inventory" and "Cost of Goods Sold."

The choice of the title "Improvements in Production Procedures" is intended to convey the notion that these production costs are being deferred (capitalized) because of their relationship to the reduction in future production costs made possible by organizational learning through past production experiences. Increases in this account could be related to increases in the value of organizational learning and decreases in this account could be related to the decline in the cost

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[1]The 1964 AAA Concepts and Standards Study Committee advocated attaching costs to units. The researcher feels that this choice was made on the basis of the alternatives then available to the committee and that cost allocation procedures similar to those developed herein were not among those alternatives.

avoidance potential of this learning.

Improvements in Production Procedures is regarded as a cost equalization account whose use attempts to compensate for the fact that accounting periods are not symmetric with product production life cycles. When a new production venture is under consideration management evaluates it as a whole. Management's attention is not focused as much on the production cost or selling price of each individual unit as it is on the total production costs and total sales over the entire venture.

Similarly the L-C cost allocation model attempts to allocate costs over the entire production venture. The objective of the model is to avoid the write-off of costs incurred during a period as period expenses. The objective of the model is to consider all of the costs which will be incurred during the production life cycle of a product and match these costs with revenues on the basis of the percentage of total anticipated production sold.

INDUSTRIES WHERE L-C HAS GREATEST APPLICATION:

As previously mentioned, the original applications of learning curves were in the airframe industry. Other industries to which learning curves have been applied include: steel, electronic products, home appliances, glass, paper, shipbuilding, textiles, and defense.[1] In addition to these industries significant potential applications exist in residential home construction and computer assembly.

Use of the learning curve requires both room for improvement in the method of production, and an ability and desire to improve. As table 1-3 showed, the traditional area for improvement in the L-C is labor efficiency, the higher the percentage of labor in the production process, the more rapid the rate at which production times or costs can fall.

Another characteristic of industries to which learning curves are applicable is change, be it a change in product or production process. During the startup phase, the L-C phenomenon is most significant. One cannot profitably apply it to evaluate the future of items which have reached the steady state stage of production.

The consolidated earnings statements of large diversified corporations are probably not significantly affected by their failure to include the account "Improvements in Production Procedures." For them, new products are always being phased in while others are being phased out. Yet, even for these corporations, the use of the account "Improvements in Production Procedures" could be of value in reporting divisional,

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[1]See the bibliographic references to the works of Baloff, Baloff and Kennelly, and Jordan.

product line, or project earnings.

The learning curve model developed in this research will find its greatest potential application in companies or divisions which are characterized by product innovation and/or rapid improvement in basic production procedures. Ideally these companies or divisions would have product assembly as their basic task. Their production runs would be between 4 and 5,000 units over a one to five year period. These limits are not strict.

AN INDUSTRIAL APPLICATION:

The only industry which this researcher has found that gives any recognition to the learning curve phenomena in their published financial statements is the aircraft industry. The following quotation from the 1967 notes to the consolidated financial statements of the Boeing Corporation is typical:

> Work in process on military fixed-price incentive type contracts is stated at the total of direct costs and overhead applicable thereto, less the estimated average cost of deliveries based on the estimated total cost of the contracts. Work in process on straight fixed price contracts is stated in the same manner, except that applicable research, development, administrative and other general expenses are charged directly to earnings as incurred....
>
> To the extent that estimated program costs, determined in the above manner, are expected to exceed total sales price, charges are made to current earnings in order to reduce work in progress to estimated realizable value.

Boeing's auditors, Touche Ross and Co., give this company an unqualified opinion.[1]

Dispite the fact that Boeing's method of handling unit production costs appears to be similar to the method proposed in this research a comparison of the cost allocation model developed in this research with the cost allocation model described in Boeing's 1967 annual report reveals a number of significant differences.

(1) Boeing leaves costs in excess of average unit production costs in "Work in Process" and does not separate them from costs incurred to produce units still in production. The L-C cost allocation model uses a special asset account to show the unusual nature of this asset.

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[1]Boeing Corporation, Annual Report 1967.

(2) Boeing does not regard a cost as excessive, and to be written off in the period incurred, unless such a write-off is necessary in order to reduce work in process to estimated realizable value. The L-C cost allocation model requires a write-off of costs as excessive if the production cost of a unit exceeds the expected cost of that unit as determined by the model. In addition, the model recognizes favorable cost variances when unit production costs are less than expected.

(3) Boeing only produces after production orders have been received. Hence, the number of units to be produced is certain. The L-C cost allocation model does not require 100 percent certainty. Table 5-1 shows that there is considerable leeway in the accuracy with which the estimate of the number of units to be produced can be made.

(4) Boeing's cost projections are based on the unit curve version of learning curve theory. The L-C cost allocation model, for reasons stated previously, is based on the cumulative average curve version of learning curve theory.

PURPOSE AND METHODOLOGY OF RESEARCH:

The primary objective of this research is to develop a cost allocation model based on the learning curve phenomenon. Secondary objectives include analyzing the statistical properties of this model in order to determine its limitations, and testing it by actual application to an industrial situation. The rationale for the primary objective was set forth in the section "Significance of L-C for Accounting Reports" (page 29). It is the researcher's belief that the cost allocation model developed in this research results in a better matching of the costs and revenues associated with a production venture than cost allocation models based on traditional standard or actual costing procedures. In addition, under conditions of certainty, the L-C cost allocation model reports changes in accounting income which vary in the same direction as economic income while traditional cost allocation models result in changes in accounting income which vary in the opposite direction of economic income.

In connection with this last point a rate of return income model is presented in Chapter III. The income figures reported by a hypothetical corporation with the use of this rate of return income model are compared with the income figures this hypothetical corporation would have reported had it used either current accounting cost allocation techniques or a cost allocation procedure based on the learning curve. A comparison of the three income figures will show that allocating costs on the basis of the learning curve phenomenon results in changes in income in the same direction as those derived by the use of the rate of return income model while allocating costs on the basis of traditional cost allocation models results in changes in income in the opposite direction as those derived by the use of the rate of return model. Such an occurrence is significant if accounting reports are to be used to evaluate managerial effectiveness.

In the real world the relationship between cumulative average time (cost) and the number of units produced does not have a correlation of -1. Hence the statistical properties of the learning curve cost allocation model are important in any application of it. Chapter IV takes a close look at these statistical properties. Procedures are developed to determine whether or not the model can be applied to a particular situation, to determine the number of units which must be produced to define the model parameters before the model can be used, and to analyze production as it takes place to determine if a variance is of such a magnitude that it may indicate a need to change the model or the method with which it is applied.

If a variance between an actual and projected cost occurs, and an analysis of the situation reveals a change in any parameter underlying the L-C cost allocation model, special techniques can be applied to bring the model into line with this new situation. The necessary techniques are presented in Chapter V along with a discussion of some problems which can occur during application of the model. These problems include union production standards, essential modifications in the product or production process, changes in total anticipated production, and changes in hourly production costs.

In Chapter VI the model is applied to an actual production situation and comparisons are made between reported production costs derived by the use of the L-C cost allocation model and production costs derived by the use of traditional cost allocation models.

The application of learning curves to production problems and accounting procedures would be difficult without the use of the computer. The accuracy of reading data from log-log paper is low. The time consumed in hand or machine calculation is high. The computer programs required to implement the L-C cost allocation model are listed and documented in the appendices. Reference is made to these programs and the related documentation at various points throughout this research. They are written in the BASIC language. The programs in Appendices A and C have been run on the following machines: CDC 6500, IBM 360, GE 265, PDP 10. Those in Appendices B and E were run on a CDC 6500. Those in Appendices D and F were run on a GE 265.

In order to inform the reader of what is involved in the L-C cost allocation model, Chapter III begins with a brief description of the model and an example of how it operates.

CHAPTER III

THE LEARNING CURVE COST ALLOCATION MODEL

THE MODEL:

The learning curve (L-C) cost allocation model projects unit and cumulative average production costs over the entire anticipated production life cycle of a product on the basis of cost data for the first few units of production. Using this data comparisons are made between the projected cost of each unit and the expected average unit cost of all anticipated production. As production takes place any excess of the projected cost of each unit over the expected average cost of all anticipated production is charged to the asset account "Improvements in Production Procedures" and inventory is charged with an amount equal to the expected average unit cost of all anticipated production. Whenever the projected cost of a unit is less than the expected average unit cost of all anticipated production this difference is deducted from the account "Improvements in Production Procedures" and inventory is charged with an amount equal to the expected average unit cost of all anticipated production. As production takes place any differences between actual and projected unit cost are written off as a period variance unless a change in a parameter of the model occurs.[1] Figure 3-1 compares the flow of costs under a standard

[1] These changes are discussed in Chapter V.

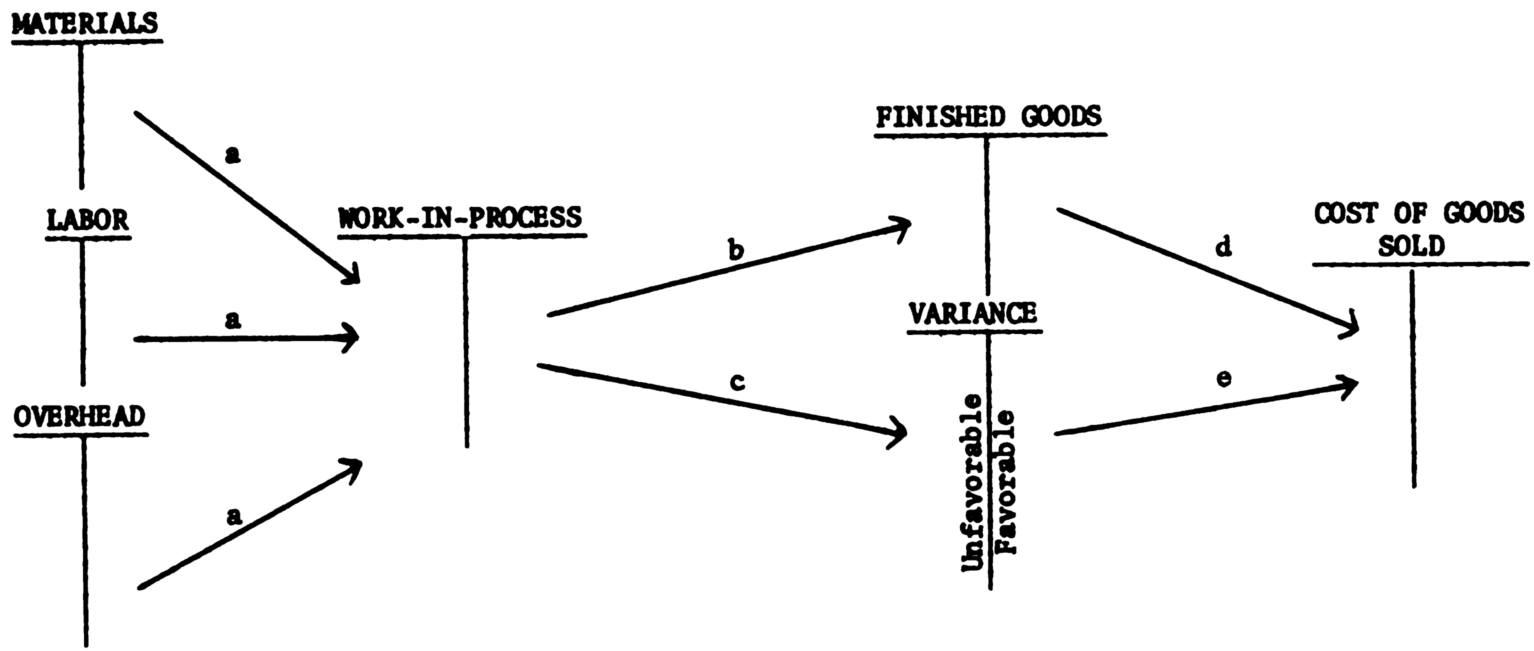
cost model and the L-C cost allocation model.

Figure 3-2 presents a series of cost curves for the example presented later in this chapter. Line 1 represents the cumulative average unit cost at various output levels. At N, the number of units of anticipated production, line 1 intersects line 2, the expected average unit cost of all anticipated production. Line 3 represents the calculated (projected) cost of each unit. Until point E is reached line 3 exceeds line 2. Until production reaches the output level represented by point E the account "Improvements in Production Procedures" is charged with the difference between lines 3 and 2. After output reaches point E the account "Improvements in Production Procedures" is credited with the differences between lines 2 and 3. At output level N the balance in Improvements in Production Procedures would be zero. In figure 3-2 this is true because area I is equal to area D.

The account "Improvements in Production Procedures" could be shown in any one of several places in the Balance Sheet. It could be shown in Current Assets near Work-In-Process in an attempt to indicate its value as an asset which is derived from the organization's ability to reduce future production costs because of its past production experiences. It could be shown in the Fixed Asset section of the balance sheet in an attempt to indicate the fact that the production venture which this asset account is associated with will include several accounting periods. It could also be shown as a Deferred Charge if the cost equalization nature of this account is stressed. Dispite

FIGURE 3-1
Diagram of Flow of Costs

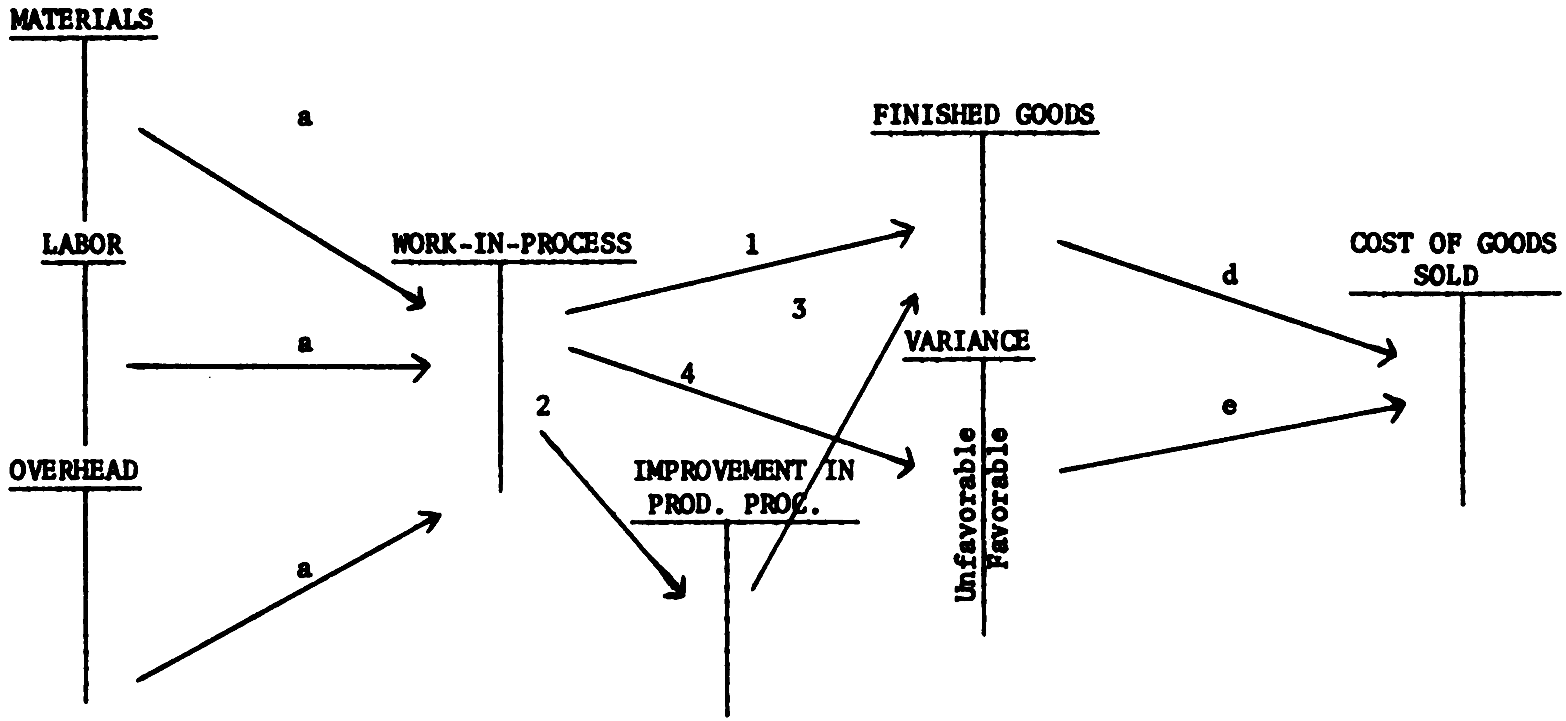
STANDARD COST ALLOCATION MODEL:



a. Actual Costs
b. Standard Costs
c. Actual Minus Standard
d. Period Sold
e. Period Incurred

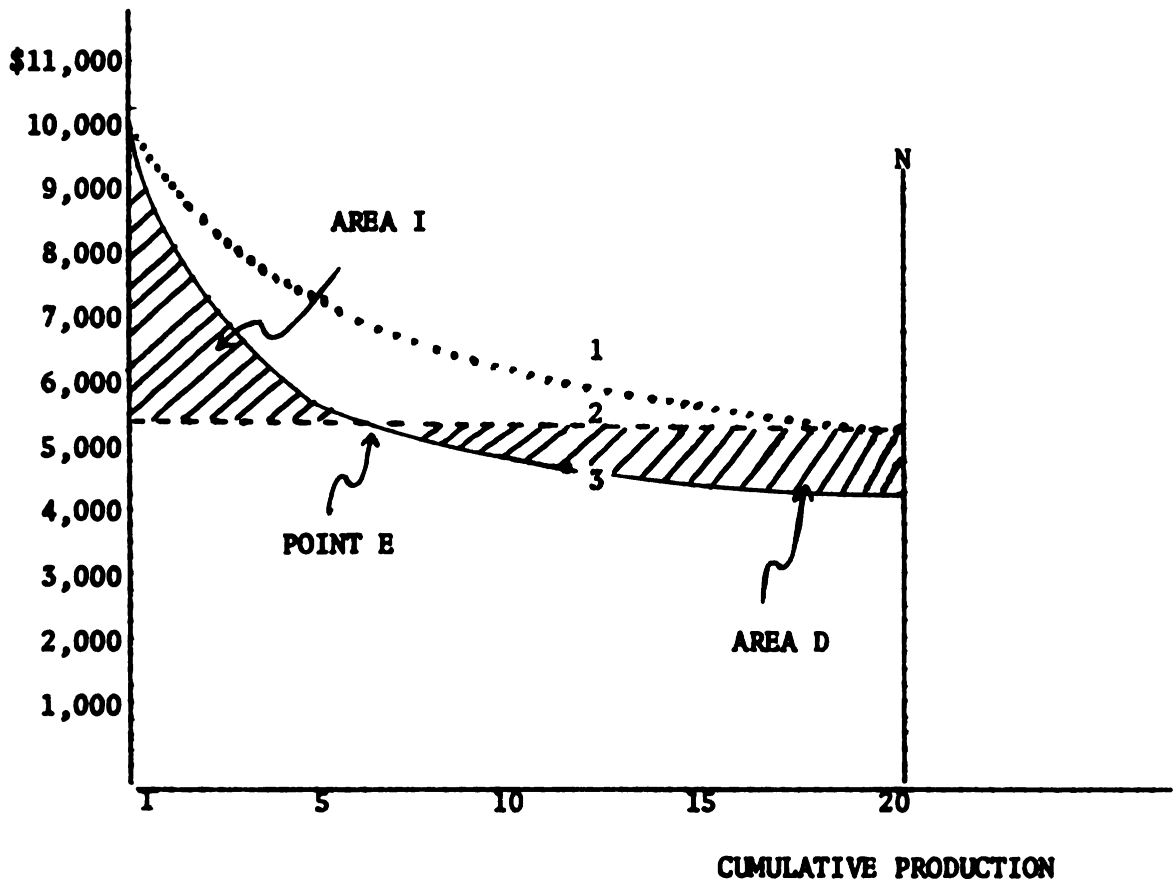
FIGURE 3-1 (con't)

L-C COST ALLOCATION MODEL:



1. Projected unit cost or expected average unit cost of all anticipated production, whichever is lower.
2. Excess of projected unit cost over expected average unit cost of all anticipated production.
3. Excess of expected average unit cost of all anticipated production over projected unit cost.
4. Actual less projected unit cost.

FIGURE 3-2
Cost Curves for Example Company



LINE 1	....	Cumulative Average Unit Cost
LINE 2	------	Average Unit Cost of All Anticipated
Line 3	________	Calculated Unit Cost

the fact that there is precident in the aircraft industry for treating Improvements in Production Procedures as a current asset and placing it near Work-In-Process, this researcher feels that it should be placed under the "Deferred Charge" caption because of its intangible and nonmarketable nature.

The use of the account "Improvements in Production Procedures" would cause some income to be recognized for accounting purposes before it is recognized for tax purposes. The procedures for handling this situation are adequately described in Accounting Principles Board Opinion No. 11, "Accounting for Income Taxes."[1] The tax expense would be recorded on the basis of accounting income and the estimated tax liability would be recorded on the basis of taxable income. The difference between these two would be treated as a deferred tax liability until the situation reversed itself. Taxes are ignored in the remainder of this research.

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[1]AICPA, Accounting Principles Board Opinion No. 11, "Accounting for Income Taxes," (New York: AICPA, 1967).

EXAMPLE OF THE L-C COST ALLOCATION MODEL:

To explain and describe further the operational characteristics of the L-C cost allocation model a hypothetical corporation is used in this and the following chapters.

The Great Lakes Union Boat Co. began construction of a new product line of sailboats in 1969. The firm's management estimated that it could sell 20 of these boats over a three year period at a price of $10,000 each. The firm built and sold 5 of these boats in 1969. The last one was completed and sold of December 31. Hence, there was no inventory on 12/31/69.

A study of the firm's cost records revealed the following:

BOAT NUMBER	ACTUAL COST
1	$10,000
2	7,500
3	6,600
4	6,000
5	5,700
	$35,800

Assuming all other expenses for the firm totaled $15,000, the 1969 Income Statement, prepared using a standard cost allocation model would be as follows:

G.L.U.B. Co.
Income Statement (Std.)[1]
For the Year Ending 12/31/69

Sales	$50,000.00
Cost of Goods Sold	35,800.00
Gross Margin	$14,200.00
Other Expenses	15,000.00
Net Income (loss)	$ (800.00)

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[1]Std. indicates that the statement was prepared using an actual or standard cost allocation model. L-C indicates that the statement

If G.L.U.B. Co. had used the L-C cost allocation model developed in this research effort the income statement would have appeared as follows:

G.L.U.B. Co.
Income Statement (L-C)
For the Year Ending 12/31/69

Sales		$50,000.00
Cost of Goods Sold	$26,970.75	
Cost Variance	(163.37)*	26,807.08
Gross Margin		$23,192.92
Other Expenses		15,000.00
Net Income (loss)		$ 8,192.92

*Favorable

This was arrived at as follows:

(1) The production data for the first 5 units was transformed into logarithms. By the use of formulas 1-4 and 1-5, the values of A_c and B_c were found to be $10,045.70 and 0.20757 respectively. Based on a B_c value of 0.20757 the learning curve percent was determined to be 86.599.

(2) Given an anticipated total continuous production of 20 units and formula 1-1, the average unit cost of all anticipated production, Y_{cN}, was determined to be $5,394.15.

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was prepared using the L-C cost allocation model. Assuming that all cost variances are closed to the cost of goods sold in the period incurred, and that period sales and production are equal, the results obtained by the use of a standard cost allocation model would be the same as those attained by attaching actual costs to units sold with an actual cost allocation model. In this research the problems caused by the allocation of fixed overhead are ignored and it is assumed that all fixed costs are written off in the period incurred.

(3) By the use of equation 1-8 the formula-based cost of the first five units was calculated as $35,963.67. The expected average cost of five units is $5Y_{cN}$, $26,970.75. The actual cost of the first five units is $35,800.00.

(4) The summarizations necessary to make entries in the records are:

Actual Cost of Units Produced	$35,800.00
-Calculated Cost of Units Produced	35,963.67
Cost Variance-Unfavorable, (Favorable)	$ (163.67)
Calculated Cost of Units Produced	$35,963.67
-Number of Units Produced x Average Unit Cost	26,970.75
Change in Improvements in Production Procedures	$ 8,992.92

Assuming that the firm is on a periodic inventory system the following adjusting entry would be made on December 31, 1969:

Cost of Goods Sold	$26,970.75	
Improvements in Production Procedures	8,992.92	
Cost Variance-Favorable		$ 163.67
Various Cost Accounts		35,800.00

If 10 units were produced in 1970 at a cost of $50,000.00, and 5 units were produced in 1971 at a cost of $21,500.00, the following entries would be made at the end of each year: (Calculated cost of units 6-15 is $49,927.49. Calculated cost of units 16-20 is $21,991.95.)

1970:

Cost of Goods Sold	$53,941.50	
Cost Variance-Unfavorable	72.51	
Improvements in Production Procedures		$ 4,014.01
Various Cost Accounts		50,000.00

1971:

Cost of Goods Sold	\$26,970.70	
Cost Variance-Favorable		\$ 491.90
Improvements in Production Procedures		4,978.80
Various Cost Accounts		21,500.00

The calculated cost of units 6 to 15 can be arrived at in two ways:

First, formula 1-10 could be summed for values of X = 6 to 10:

$$\sum_{x=6}^{15} A_c(X^{C_c}-(X-1)^{C_c}) \text{ .}^{1}$$

Second, formula 1-8 could be used to find the difference between the values of T_c at X equals 5 and X equals 15:

$$T_{c15} - T_{c5} = A_c \cdot 15^{C_c} - A_c \cdot 5^{C_c} \text{ .}$$

The second procedure is easier and is recommended.

If all other expenses remained \$15,000 in 1970 and 1971 the comparative incomes would be:

	1969	1970	1971	Total
Standard or Actual Cost Model	\$ (800.00)	\$ 34,999.92	\$ 13,500.00	\$ 47,699.92*
L-C Cost Allocation Model	8,192.92	30,985.91	8,521.20	47,700.03

*Errors due to rounding.

In the first period, when G.L.U.B. Co. is making an investment in production procedures and organizational knowhow, the L-C income figure is higher than the Standard Cost one. In later periods when their knowledge has decreasing value, due to the production run's

[1] In the remainder of this research the symbol "c" indicates that this value or parameter was calculated by the use of regression analysis.

nearing completion, the L-C income figure is lower.

Figure 3-2 gives a graphic illustration of the cost data for the G.L.U.B. Co. Figure 3-2 can be used in conjunction with figure 3-1 to analyze the G.L.U.B. Company's flow of costs under standard and L-C cost allocation procedures.

RATE OF RETURN INCOME MODEL:

It was previously mentioned that many accountants regard economic income as a theoretical ideal toward which accounting income should strive. It was also pointed out that because of the amount of subjectivity required in estimating cash flows and determining the proper discount rate economic income cannot be determined in the real world.

In this section a rate of return income model will be presented as a surrogate for an economic income model. Under conditions of certainty the only differences between the rate of return model presented in this section and an economic income model is the discount rate. The rate of return model uses the internal rate of return on investment. Economic incomes models use a subjective or external rate of return. The effect of this difference is that the rate of return model has a constant percentage return on investment and does not recognize extraordinary gains at the time of investment, while economic income models do recognize immediate gains on investment, thereby causing variances in the rate of return.

The purpose of this section is to show that changes in income reported by the use of the L-C cost allocation model move in the same direction as changes in income reported by the use of the rate of return income model while changes in income reported by the use of a standard or actual cost model sometimes move in the opposite direction. The significance of this lies in the different impression various changes in income give the investor. When actions are being taken by management which will increase the excess of future cash receipts over future cash disbursements income should be rising. When the potential benefits of a past investment are nearing their end income on that investment should be falling.

The rate of return income model presented below is based on a model in Bierman's book, Financial Accounting Theory.[1]

Assume an investment of $15,000 is required at time zero to generate net cash flows of $5,500, $7,260, and $5,324 in periods 1, 2, and 3. Table 3-1 shows that the rate of return on this investment is 10 percent. Table 3-2 shows that the interest on unrecovered investment in periods 1, 2, and 3 at 10 percent are $1,500, $1,100, and $484.

If certainty is assumed the same model can also be applied to the G.L.U.B. Co. Assume that G.L.U.B. Co. invested $40,000 in the productive facilities needed to produce the 20 sailboats referred to in the example of the L-C cost allocation model, and further assume that G.L.U.B. Co. was able to sell these facilities for $40,000[2] in mid 1971 when construction on the sailboats was completed. The assumption of only a half year of operations is made because of the decreased amount of time required to produce the last five boats. The semi-annual rate of return is shown to be 18.5 percent in

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[1]Harold Bierman, Jr., Financial Accounting Theory, (New York, 1965), pp. 122-127.

[2]This assumption is made to avoid the problem of how to handle depreciation under the L-C cost and standard cost allocation models.

TABLE 3-1
Present Value of Cash Flows Discounted At 10%

YEAR	CASH PROCEEDS	PRESENT VALUE FACTOR	PRESENT VALUE OF CASH PROCEEDS
1	$5,500	0.909	$ 5,000
2	7,260	0.826	6,000
3	5,324	0.751	4,000
			$15,000

TABLE 3-2
Computation of Present Value and Interest

PERIOD	BASIC INVESTMENT	INTEREST YEAR 1-10%	INVESTMENT PLUS INTEREST	INTEREST YEAR 2-10%	INVESTMENT PLUS INTEREST	INTEREST YEAR 3-10%
1	$ 5,000	$ 500				
2	6,000	600	$ 6,600	$ 660		
3	4,000	400	4,400	440	$4,840	$484
	$15,000	$1,500	$11,000	$1,100	$4,840	$484

table 3-3. Table 3-4 shows that the interest on unrecovered investment in periods 1, 2, and 3 are $16,237.65, $23,070.38, and $8,353.00.

TABLE 3-3
Present Value of G.L.U.B. Co. Cash Flows Discounted At 18 1/2%

Year	Proceeds	Present Value Factor	Present Value of Cash Proceeds
1/2			
1	$ (800)	.712	$ (569.60)
1 1/2			
2	34,999.92	.507	17,744.96
2 1/2	53,500.00*	.428	22,898.00
			$40,073.36**

* $40,000 + 13,500
** Error caused by rounding present value factors.

Table 3-5 compares income derived from the rate of return, L-C cost, and standard cost models. Although the absolute level of income using the rate of return and L-C cost models differ, their reported incomes change in the same direction. Additionally, if the income reported by the rate of return model is accepted as a surrogate for economic income, and economic income is regarded as a theoretic ideal, then L-C cost allocation results in an income being reported which is closer to that ideal than that reported by the use of standard or actual cost allocation.

TABLE 3-4
Computation of G.L.U.B. Company's Present Value and Interest

YEAR	BASIC INVESTMENT	INTEREST 1/2 YR-18.5%	INVESTMENT PLUS INTEREST	INTEREST 1 YR-18.5%	INVESTMENT PLUS INTEREST
1/2					
1	$ (569.60)	$ (105.376)	$ (464.224)	$ (85.881)	
1 1/2					
2	17,744.96	3,282.818	21,027.778	3,890.139	$24,919.167
2 1/2	22,898.00	4,236.130	27,134.130	5,019.817	32,153.945
	$40,073.36	$7,413.572		$8,824.073	

YEAR	INTEREST 1 1/2 YR-18.5%	INVESTMENT PLUS INTEREST	INTEREST 2 YR-18.5%	INVESTMENT PLUS INTEREST	INTEREST 2 1/2 YR-18.5%
1/2					
1					
1 1/2					
2	$ 4,610.046	$29,529.213	$ 5,462.904		
2 1/2	5,948.480	38,102.425	7,048.949	$45,141.374	$ 8,353.004
	$10,558.526		$12,511.853		$8,353.004

TOTALS:
YEAR 1 $ 7,413.572 + 8,824.073 = $16,237.645
YEAR 2 $10,558.526 + 12,511.853 = $23,070.379
YEAR 3 $ 8,353.004

TABLE 3-5

Rate of Return, L-C Cost, Standard Cost Incomes Compared

YEAR		1	2	3(1/2 Yr)	Total
METHOD:	RATE OF RETURN	$16,237.65	$23,070.38	$ 8,353.00	$47,661.03*
	L-C Cost	8,192.92	30,985.91	8,521.20	47,700.03
	STANDARD COST	(800.00)	34,999.92	13,500.00	47,699.92

* Errors due to rounding.

CHAPTER IV

MATHEMATICAL RELATIONSHIPS

INTRODUCTION:

The primary purpose of this chapter is to study some of the mathematical properties of the L-C cost allocation model developed in this research and to suggest a number of minimum statistical requirements which should be met before the model is implemented. In connection with the development of these minimum statistical requirements, the accounting concept of "Materiality" will be examined briefly. A secondary purpose of this chapter is to suggest two alternative procedures for determining when a change in a parameter of the L-C cost allocation model has taken place. Procedures for handling such changes are developed in Chapter V.

In an attempt to determine how accurate the calculation of the average cost of all anticipated production should be the next section of this chapter examines the accounting concept of materiality. In that section it is concluded that most accountants do not regard a change in reported income of less than 10 percent as material. In this research tables are developed for 5 and 10 percent levels of materiality. Users of the L-C cost allocation model should first attempt to predict the cumulative average cost of all anticipated production with a

5 percent confidence interval and a 95 percent confidence level. If this is impossible they should then attempt to predict the cumulative average cost of all anticipated production with a 10 percent confidence interval and confidence levels of 95, 90, and 80 percent. If this cannot be done, and the use of additional data in the determination of the model's parameters is not fessible, use of the L-C cost allocation model for financial reporting is not recommended.

In the section "Confidence Interval For Specified b Confidence Level" (page 69) a t distribution is applied to a regression analysis of linear equations in the form of (1-3). This section shows that after some initial data is gathered and analyzed and a confidence level has been specified, a confidence interval for the b parameter is automatically determined. The determination of confidence levels and intervals for the b parameter is important because the b parameter is one of the major determinates of unit, total, and average cost. The following section shows that if a particular confidence level and interval are desired, the only variable item is the number of units of input data. Thus, given the amount of correlation in the initial data, and a specific confidence level and interval, the required number of units of input can be determined. In the section entitled "Minimum R Required to Obtain Desired b Confidence Level and Interval" (page 93) tables 4-7, 4-8, and 4-9 show how many units of initial data must be gathered in order to obtain various confidence intervals for b at confidence levels of 80, 90, and 95 percent. Intermediate sections develop tables that show the b confidence intervals which must be attained in the original data in order to project the average cost of all anticipated production, the unit cost at a particular output level, and the total cost at a particular output level with 5 and 10 percent confidence intervals under varying circumstances. These tables are used in conjunction with tables 4-7, 4-8, and 4-9 in order to determine the number of units of input needed to apply the L-C cost allocation model to a particular product or project.

The last section of this chapter summarizes verbally the significant points presented in this chapter and shows in a step by step manner how to use the concepts and tables developed herein.

MATERIALITY AND CONFIDENCE INTERVALS:

"Materiality" is a very often used and a very imprecisely defined word in accounting literature. Hendriksen defines materiality "in a positive sense to include all information that may be of significance to the user...materiality refers to the significance of a specific item in a specific context."[1] This throws the determination as to whether or not an item is material into the area of professional judgement, requiring each case to be evaluated on its own merits. Indeed, this is what Hendriksen intends when he indicates that "...without some assumptions regarding what decisions are likely to be made and what information is relevant to these decisions...[any] percentage criterion, is not likely to provide a meaningful guide."[2]

Unfortunately some percentage concept of materiality must be adopted in this research in order to determine minimum levels of statistical accuracy required to implement the L-C cost allocation model. Without some such objectively determinable guidelines the model would lack the objectivity and verifiability required to make it a viable accounting technique.

Writing in the Journal of Accounting Research, Ernest L. Hicks also concluded that ultimately:

> The materiality of an item entering into financial statements lies in its impact on the user. The question to be answered is: is it likely that an average prudent investor or a reasonable person would be influenced in his

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[1]Hendriksen, p. 562.

[2]Ibid.

> investment decision if the matter at issue were disclosed or if net income or some other significant financial statement item were increased or decreased by the amount under consideration?[1]

Despite this conclusion, in discussing extraordinary items Hicks expressed his view that "an item under consideration for exclusion in determining net income is seldom material if it is less than 10 percent of ordinary net income, and that it is ordinarily material if it is 20 percent or more."[2] Hicks also noted that one of the few authoritative definitions of materiality is found in the Securities and Exchange Commission's Regulation S-X. There, a "significant subsidiary" is defined as one whose assets or sales exceed 15 percent of the assets or sales, respectively, shown by the consolidated statements which are filed.[3]

Leopold A. Bernstein, in a recent article in The Accounting Review, suggested a "border zone of 10% - 15% of net income after taxes as the point of distinction between what is material and what is not."[4] Within this border zone the determination of whether or not an item is material would, presumably, rest on professional judgement.

Graham, Dodd, and Cottle, in their book on security analysis, in discussing noncurrent items "define 'small' as affecting the net

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[1]Ernest L. Hicks, "Materiality," Journal of Accounting Research, (Autumn, 1964), p. 170.

[2]Ibid., p. 165.

[3]Ibid.

[4]Leopold A. Bernstein, "The Concept of Materiality," The Accounting Review, (January, 1967), p. 93.

result by less than 10 percent in the aggregate."[1]

A study supervised by Robert K. Mautz for the Financial Executive Research Foundation selected a materiality standard of 15 percent of net sales.[2]

In 1954, S. M. Woolsey reported on a survey of the opinions of accountants, bankers, and others as to the materiality of several hypothetical items. With respect to an extraordinary loss whose materiality was gauged by most respondents in relation to current-year income, the replies received indicated that the loss would have been deemed material had it exceeded 10 percent of net income before deducting the loss.[3] The breakdown on the percentage of net income at which an item becomes material is presented below by classification of respondents.

	Percent of Net Income
National CPAs	13.7
Local and regional CPAs	9.5
Controllers	7.8
Robert Morris Associates	10.9
Investment bankers, etc.	8.3
Professors of accounting	8.8
Others	10.9
Weighted over-all average	10.0

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[1]Benjamin Graham, David L. Dodd, Sidney Cottle, Security Analysis Principles and Technique, (New York, 1962), p. 112.

[2]R. K. Mautz, Financial Reporting by Diversified Companies, (New York, 1966), p. 135.

[3]S. M. Woolsey, "Development of Criteria to Guide the Accountant in Judging Materiality," The Journal of Accountancy, (February, 1954), p. 170.

On the basis of this information Woolsey suggested a judgemental bracket of from 5 to 15 percent of current income before taxes be used in determining whether or not an item is material.[1]

While no precise standards to determine what is or is not material can be agreed upon, for the purposes of this research the following minimal standards for implementing the L-C cost allocation model appear to be appropriate. The L-C cost allocation model should not be implemented unless the average cost of all anticipated production can be predicted with at least a 10 percent confidence interval and an 80 percent confidence level disregarding changes which take place in the models parameters after it has been implemented. Because the cost of goods sold is charged with amounts equal to the average cost of all anticipated production as goods are sold this is equivalent to stating that a 10 percent level of materiality has been set up for the cost of goods sold. The model should not be implemented if there is more than a 20 percent chance that the cost of goods sold in any period will be in error by more than 10 percent.

In the remainder of this chapter criteria will be developed for attaining both 5 and 10 percent confidence intervals for the average cost of all anticipated production, and unit and total costs at various outputs with confidence levels of 80, 90, and 95 percent. Where possible, it is recommended that the tighter standards be used.

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[1]*Ibid.*

CONFIDENCE INTERVAL FOR SPECIFIED b CONFIDENCE LEVEL:

Procedures for determining confidence intervals for the b parameter of the L-C cost allocation model when the confidence level is specified are presented in this section. It is necessary to develop these intervals for b because the confidence intervals for b determine the confidence intervals for the average cost of all anticipated production, the cost of each particular unit, and the total cost of any given number of units.

In dealing with a linear regression of the form:

$$\log Y = \log a + b \log X$$

Kenney and Keeping have shown that the ratio given in equation 4-1 has a Student t distribution with n-2 degrees of freedom.[1]

$$t = \frac{R(B_c - b)}{B_c} \left(\frac{n-2}{1-R^2}\right)^{1/2} \quad \begin{matrix} R<1 \\ n>2 \end{matrix} \qquad (4\text{-}1)$$

where:

R = correlation coefficient based on a regression analysis of n pairs of Y and X values. See equations 1-3 and 1-6.

B_c = calculated value of b. See equation 1-4.

n = number of pairs of input (X and Y).

Tables of values for the t distribution appear in most statistics books. They show the values of t corresponding to specified levels of confidence and degrees of freedom, f, (f=n-2). If an n value were specified and we wanted to find a value of t such that we could be 95 percent confident that the value of t computed by the use of (4-1) was

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[1]J. F. Kenney and E. S. Keeping, Mathematics of Statistics, Part 2, (Princeton, 1951), p. 210.

less than or equal to this value, we would go down the column labeled .95 (.025 or .05 depending on the table) until we came to the line for the appropriate degrees of freedom, (n-2).

For a 95 percent confidence level let $t_{.95}$ represent the t value for n-2 degrees of freedom such that the probability of finding a deviation greater than $+t_{.95}$ or less than $-t_{.95}$ is 0.05. Then

$$t_{.95} = \pm \frac{R(B_c - b)}{B_c} \left(\frac{n-2}{1-R^2}\right)^{1/2} . \tag{4-2}$$

We can now solve equation 4-2 to find the b confidence interval within which we can state with a 95 percent confidenc level the true value of b can be found.

$$b = B_c \pm t_{.95} \frac{B_c}{R} \left(\frac{1-R^2}{n-2}\right)^{1/2} . \tag{4-3}$$

Equation 4-3 says we can be 95 percent confident that the true value of b will not vary from the value of b calculated by the use of equation 1-4 by more than

$$\pm t_{.95} \frac{B_c}{R} \left(\frac{1-R^2}{n-2}\right)^{1/2}$$

where:

B_c = value of b calculated by the use of equation 1-4.

In the remainder of this research B_c will represent the calculated value of b and b will represent the true value of b which could be found if all past and future values of X and Y associated with a particular production run were known. Thus, equation 4-3 is setting a confidence interval for the true value of b by use of B_c, the best available estimate of b.

Assuming that the variance of production cost remains the same over the entire production cycle, the confidence limits for b (highest possible b value and lowest possible b value determined from the use of equation 4-3) can be used to determine confidence intervals for the average cost of all anticipated production (Y_N), the cost of a particular unit (U_i), and the total cost at a particular level of output (T_i). This is done after the following section of this chapter. The assumption that the variance of production time remains the same over the entire production cycle is considered in Chapter VI.

DETERMINING n FOR DESIRED CONFIDENCE LEVEL AND INTERVAL:

If a specific confidence level and interval is desired for b the only item over which control is retained is n, the number of units of input. Because the required size of n varies with the correlation coefficient, it is necessary to gather a few units of initial data, determine the correlation coefficient, and then make the calculations necessary to determine the final value of n.

From (4-3) we know that a 95 percent confidence level for b is represented by:

$$b = B_c \pm t_{.95} \frac{B_c}{R} \left(\frac{1-R^2}{n-2}\right)^{1/2} .$$

Assume we wish to be 90 percent confident that the true regression coefficient, b, is within ±5 percent of B_c. This interval may be expressed as:

$$.95B_c \leq b \leq 1.05B_c \quad . \tag{4-4}$$

Equations 4-3 and 4-4 are now combined in order to express the desired confidence interval in terms of $t_{.90}$.

$$B_c - \frac{B_c}{R} t_{.90} \left(\frac{1-R^2}{n-2}\right)^{1/2} = .95B_c \leq b \leq 1.05B_c + \frac{B_c}{R} t_{.90} \left(\frac{1-R^2}{n-2}\right)^{1/2} \quad . \tag{4-5}$$

We desire to evaluate (4-5) in order to determine n, the number of units of input required to obtain a ±5 percent confidence interval, and a 90 percent confidence level for b. As the results of evaluating either side of (4-5) are the same we need only evaluate the right side. In terms of n the right side of equation 4-5 may be expressed as:

$$n = \frac{t^2_{.90}(1-R^2)}{.0025\ R} + 2 \quad . \qquad (4\text{-}6)$$

In equation 4-6 the value of n is dependent upon the value of R. The larger the correlation coefficient, R, the smaller n. A low level of correlation will require a large number of units of input in order to obtain the desired confidence interval and level for b. For ease of computation (4-6) is now restated as:

$$\frac{.05R}{\sqrt{1-R^2}} = \frac{t_{.90}}{\sqrt{n-2}} \quad . \qquad (4\text{-}7)$$

To find n solve the left side of the equation. Then go to a t table and select a t value under the column which yields a 90 percent confidence level. The value selected should be placed in the right side of (4-7) along with the appropriate n value. This may have to be done several times until both sides of the equation are approximately equal.

For the example developed in Chapter III R = .99931 and (4-7) may be expressed as:

$$\frac{.05 \cdot .99931}{\sqrt{1 - .99862}} = \frac{t_{.90}}{\sqrt{n-2}}$$

$$1.345 = \frac{t_{.90}}{\sqrt{n-2}} \quad .$$

Reading from a standard t table we can proceed down the .90 column of t values, evaluating the above equation until the two terms are approximately equal. At f = 3, $t_{.90}$ = 2.353 and:

$$\frac{2.353}{\sqrt{5-2}} = 1.357 \quad .$$

This is as close as we can come with integer values of n. Thus, the value of n required to obtain a 90 percent confidence level, and a 5 percent interval for b is 5.

If a higher confidence level were desired, say 98 percent, we could evaluate (4-7) for $t_{.98}$. At f = 6, $t_{.98} = 3.143$ and:

$$\frac{3.143}{\sqrt{8-2}} = 1.3 \quad .$$

Hence, to be 98 percent confident that B_c is within ±5 percent of b, 8 pairs of X and Y values must be used in the determination of B_c.

If a tighter confidence interval were desired it could be found by substituting the desired interval value in place of .05 in (4-7), e.g., suppose a 3 percent interval and a 90 percent confidence level were desired:

$$\frac{.03 \cdot .99931}{\sqrt{1 - .99862}} = .806 \ .$$

At f = 6, $t_{.90} = 1.943$ and:

$$\frac{1.943}{\sqrt{8-2}} = .793 \quad .$$

This is as close as we can come with interger values of n. Hence, with a correlation coefficient of .99931, to be 90 percent confident that B_c is within ±3 percent of b, 8 pairs of X and Y values must be used.

After the n units of data specified by equation 4-7 are gathered and a new correlation coefficient and B_c value are calculated, this information should be analyzed to determine the confidence level and interval attained.

The purpose of determining confidence intervals and levels for b is to determine confidence intervals and levels for Y_N, U_i, and T_i. The b confidence intervals necessary to obtain ±5 or 10 percent intervals for each of these varies. The following sections consider the b confidence intervals required to obtain ±5 or 10 percent confidence intervals for Y, U, and T at different levels of output.

RELATIONSHIP BETWEEN b AND ANY Y_N CONFIDENCE INTERVALS:

In the L-C cost allocation model the projected average cost of all anticipated production, Y_{cN}, is used to determine the cost of goods produced and sold during each accounting period. In order to insure that the cost of goods sold determined in this manner is not materially in error certain minimum levels of statistical accuracy are specified for the calculation of Y_{cN}. The cost of goods produced and sold is said to be materially in error when the projected value of Y_N differs from the actual value of Y_N by more than 10 percent. For purposes of this research the minimum level of statistical accuracy required to implement the L-C cost allocation model for use in external accounting reports is defined to be an 80 percent confidence level that Y_{cN} does not differ from Y_N by more than 10 percent, assuming the true parameters of the L-C cost allocation model remain the same.

In this section procedures are developed to determine the confidence interval for the regression slope coefficient, b, required to obtain a specified confidence interval for Y_N. Once this is done the procedures developed in the previous section can then be used to determine the number of units of input data, n, needed to implement the model with given values of B_c, N, and R.

From (1-1) we know that the value of Y_N depends on the value of b. Likewise, the determination of confidence intervals for Y_N depends on the determination of confidence intervals for b. If a 5 percent confidence interval is desired for Y_N this could be expressed as

$$.95Y_{cN} \leq Y_N \leq 1.05Y_{cN} \quad . \tag{4-8}$$

Let B1 and B2 represent extreme values of the confidence interval for b required to satisfy (4-8). Then, combining (1-1) and (4-8) we get

$$A_c/N^{B1} = .95Y_{cN} \leq Y_N \leq 1.05Y_{cN} = A_c/N^{B2} \qquad (4\text{-}9)$$

where:

A_c = calculated value of a, the cost of producing the first unit. See (1-5).

Solving for B1 and B2:

$$B1 = (B_c \log N + \log 100 - \log 95)/\log N \qquad (4\text{-}10)$$

$$B2 = (B_c \log N - \log 1.05)/\log N\,. \qquad (4\text{-}11)$$

Because the percentage interval $(B_c - B2)/B_c$ is smaller than $(B1 - B_c)/B_c$ for all values of N and B_c only (4-11) need be solved to determine the appropriate percent interval for b. The result is a slightly tighter interval than is necessary for application of the L-C cost allocation model. This difference is caused by the use of exponents.

After (4-11) is solved for the b confidence interval required to obtain the desired Y_N confidence interval, (4-7) can then be evaluated to determine the number of units of data, n, necessary to obtain the desired confidence interval and level for Y_N.

With n units of data gathered and analyzed by least squares regression analysis, equation 4-3 is then used to determine the confidence intervals and levels obtained for b. The values of B1 and B2 determined by (4-3) are then used in (1-1) to determine the confidence intervals attained for Y_N.

Assuming that the attained confidence intervals are determined to be sufficiently tight, and the confidence level sufficiently high, the L-C cost allocation model can be implemented. If the attained confidence level and interval for Y_N do not meet the desired standards four courses of action may be followed:

(1) The standards may be lowered to a minimum 80 percent confidence level and 10 percent confidence interval.

(2) Additional units of data may be gathered after determination of a new n by analysis of all gathered data through the use of equation 4-7.

(3) It may be decided that the potential variation of Y_N is so large that the L-C cost allocation model should not be employed.

(4) It may be decided that partial implementation of the L-C cost allocation model would be useful. In this case, costs could be capitalized until the output at which Y_N equals U is reached using a relatively high L-C percent. After that point the balance in Improvements in Production Procedures could be reduced through increasing charges to all remaining units of production.

The computer program listed in Appendix B1 was used to determine B2 in equation 4-11 for Y_{100}; Y_{500}; $Y_{1,000}$; $Y_{5,000}$; and $Y_{10,000}$, where b values ranged over L-C percents of 70 to 99. Table 4-1 lists the values of the percent confidence intervals for b required to obtain a 5 percent confidence interval for Y_N. Table 4-2 lists the values of the percent confidence intervals for b required to obtain a 10 percent confidence interval for Y_N.

Assume a 5 percent confidence interval is desired for Y_N, when B_c = .207572, and N = 100. From table 4-1 the percent confidence interval for b must be 5.273. Thus, we can have a 5 percent confidence interval for Y_N values up to N = 100 when the percent confidence interval for b is 5.273 or smaller, (B_c = .207572).

Tables 4-1 and 4-2 show that the required percentage confidence interval for b decreases as N becomes larger. This occurs because Y becomes smaller as N increases in accordance with the learning curve.

TABLE 4-1
Percent Confidence Interval for b Required to Obtain A ± 5% Interval for Y_N

L-C%	b	% ± b N=100	% ± b N=500	% ± b N=1,000	% ± b N=5,000	% ± b N=10,000
70	.51457	2.059	1.526	1.373	1.113	1.029
71	.49410	2.144	1.589	1.429	1.159	1.072
72	.47393	2.235	1.657	1.490	1.209	1.118
73	.45403	2.333	1.729	1.556	1.262	1.167
74	.43440	2.439	1.807	1.626	1.319	1.219
75	.41503	2.553	1.892	1.702	1.380	1.276
76	.39592	2.676	1.983	1.784	1.447	1.338
77	.37707	2.810	2.082	1.873	1.519	1.405
78	.35845	2.956	2.190	1.970	1.598	1.478
79	.34007	3.115	2.309	2.077	1.684	1.558
80	.32192	3.291	2.439	2.194	1.779	1.645
81	.30400	3.485	2.582	2.323	1.884	1.743
82	.28630	3.700	2.742	2.467	2.001	1.850
83	.26881	3.941	2.921	2.627	2.131	1.971
84	.25153	4.212	3.121	2.808	2.277	2.106
85	.23446	4.519	3.348	3.012	2.443	2.259
86	.21759	4.869	3.608	3.246	2.633	2.435
87	.20091	5.273	3.908	3.516	2.851	2.637
88	.18442	5.745	4.257	3.830	3.106	2.873
89	.16812	6.302	4.670	4.201	3.407	3.151
90	.15200	6.970	5.175	4.647	3.769	3.485
91	.13606	7.787	5.770	5.191	4.210	3.893
92	.12029	8.807	6.526	5.872	4.762	4.404
93	.10469	9.867	7.499	6.746	5.471	5.060
94	.08926	11.869	8.795	7.912	6.417	5.934
95	.07400	14.317	10.609	9.545	7.741	7.158
96	.05889	17.989	13.331	11.993	9.727	8.995
97	.04394	24.110	17.866	16.073	13.036	12.055
98	.02914	36.498	26.936	24.233	19.654	18.175
99	.01449	73.069	54.146	48.713	39.508	36.534

TABLE 4-2
Percent Confidence Interval for b Required to Obtain A ± 10% Interval for Y_N

L-C%	b	% ± b N=100	% ± b N=500	% ± b N=1,000	% ± b N=5,000	% ± b N=10,000
70	.51457	4.022	2.980	2.681	2.174	2.011
71	.49410	4.188	3.103	2.792	2.264	2.094
72	.47393	4.366	3.236	2.911	2.361	2.183
73	.45403	4.558	3.377	3.038	2.464	2.279
74	.43440	4.764	3.530	3.176	2.576	2.382
75	.41503	4.986	3.695	3.324	2.696	2.493
76	.39592	5.227	3.873	3.484	2.826	2.613
77	.37707	5.488	4.067	3.659	2.967	2.744
78	.35845	5.773	4.278	3.849	3.121	2.886
79	.34007	6.085	4.509	4.057	3.290	3.042
80	.32192	6.428	4.763	4.285	3.476	3.214
81	.30400	6.807	5.044	4.538	3.680	3.403
82	.28630	7.228	5.356	4.819	3.908	3.614
83	.26881	7.699	5.705	5.132	4.162	3.849
84	.25153	8.227	6.097	5.485	4.448	4.113
85	.23446	8.827	6.541	5.884	4.772	4.413
86	.21759	9.511	7.048	6.341	5.142	4.755
87	.20091	10.301	7.633	6.867	5.569	5.150
88	.18442	11.222	8.315	7.481	6.067	5.611
89	.16812	12.310	9.122	8.206	6.656	6.155
90	.15200	13.615	10.089	9.077	7.391	6.807
91	.13606	15.211	11.271	10.140	8.224	7.605
92	.12029	17.204	12.749	11.469	9.302	8.602
93	.10469	19.767	14.648	13.178	10.688	9.883
94	.08926	23.184	17.180	15.456	12.535	11.592
95	.07400	27.967	20.724	18.645	15.121	13.983
96	.05889	35.141	26.040	23.427	19.000	17.570
97	.04394	47.097	34.900	31.398	25.465	23.548
98	.02914	71.008	56.618	47.338	38.393	35.504
99	.01449	142.738	105.772	95.158	77.177	71.368

RELATIONSHIP BETWEEN b AND U_i CONFIDENCE INTERVALS:

The ability to predict Y_N with a minimum 80 percent confidence level and 10 percent confidence interval is necessary in order to minimize the probability of a material error in the computation of the cost of goods sold. If the correlation and size of the available data does not permit such confidence in the projection of Y_N the L-C cost allocation model should not be implemented for external reporting. This section deals with procedures for determining whether or not there is enough correlation in the original data to be able to predict the production cost of a particular unit, U_i, with specified confidence. It may be that it is impossible to obtain these levels of accuracy with any reasonable size n. If there is enough correlation in the original data to adapt the model other procedures could be used to determine whether or not a decision should be made to investigate for probable causes of a variance.[1] If U_i is to be used for variance analysis, the initial data must have enough correlation to be able to project U_i with at least a ±10 percent confidence interval and an 80 percent confidence level.

Assume it is desirable to be able to predict U_i with a 10 percent confidence interval. If this can be done with a high level of confidence a deviation of more than 10 percent between U_{ci} and U_i would

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[1]For example, Harold Bierman, Jr. develops a decision criteria based on the probability of a cost variance exceeding some critical percentage point, the cost of investigating the variance, and the cost of not investigating the variance. See Harold Bierman, Jr., Topics In Cost Accounting And Decisions, (New York, 1963), pp. 15-23.

indicate a need to investigate in order to determine whether or not a parameter of the L-C cost allocation model had changed. It could be assumed that very few variances in excess of 10 percent would be caused by random, nonrecurring factors.

Equation 1-10 shows that the value of U_{ci} depends on the value of B_c. Likewise, the confidence interval for U_i depends on the confidence interval for b. If a 10 percent confidence interval is desired for U_i this could be represented as:

$$.90U_{ci} \le U_i \le 1.10U_{ci} \quad . \qquad (4\text{-}12)$$

Let B1 and B2 represent the extreme values of the confidence interval for b required to satisfy (4-12). Then, combining (1-10) and (4-12) we have:

$$A_c(X^{C1}-(X-1)^{C1}) = .90A_c(X^{Cc}-(X-1)^{Cc}) \le U_i \le 1.10A_c(X^{Cc}-(X-1)^{Cc}) = A_c(X^{C2}-(X-1)^{C2}) \qquad (4\text{-}13)$$

where:

$$C1 = 1\text{-}B1$$

$$C_c = 1\text{-}B_c$$

$$C2 = 1\text{-}B2$$

$$X = i \ .$$

The values of C and B are indeterminate in (4-13). The computer program listed in Appendix B2 was used to find approximate values of B1 and B2 in equation 4-13 for U_{100}; U_{500}; $U_{1,000}$; $U_{5,000}$; and $U_{10,000}$, where b values ranged over L-C percents of 70 to 99. The percentage interval $(B_c - B2)/B_c$ was found to be smaller than the percentage interval $(B1 - B_c)/B_c$. The smaller percentage relationship is listed in tables 4-3 and 4-4. Table 4-3 lists the percent confidence intervals for b required to obtain a 5 percent interval for U_i. Table 4-4 lists the percent confidence intervals for b required to obtain a 10 percent interval for U_i.

A comparison of tables 4-1 and 4-3 reveals that the interval for b required to project U_i with a ±5 percent accuracy is much smaller than the interval for b required to project Y_i with a ±5 percent accuracy. This is because U_i is always smaller than Y_i, except at X = 1. Fortunately, for the L-C cost allocation model, projecting Y_N is much more important than projecting U_i. Even if the desired confidence interval for U were not attainable the L-C cost allocation model could be implemented as long as Y_N could be predicted with at least a 10 percent confidence interval and an 80 percent confidence level.

From table 4-4, the G.L.U.B. Co. must attain a b confidence interval of 8.112 percent in order to project U_{20} with ±10 percent accuracy. (The actual b confidence interval is somewhat larger because the closest value in the table is for U_{100}.) Previous calculations determined that an n of 5 was large enough to obtain a 5 percent confidence interval and a 90 percent confidence level for b, given an R of .99931. Thus, the confidence interval for U_{20} is tighter than the 10 percent desired and U_i may be used for variance analysis.

As production takes place, the G.L.U.B. Co. should analyze its unit production costs to determine whether or not they vary from the anticipated unit production costs by an amount sufficiently large to indicate a potential change in the parameters underlying the adopted

L-C cost allocation model.

Use of equation 1-10 gives a projected cost for U_{c5} of $5,828.75. The actual cost of U_5 is $5,700. Whereas $5,700 is less than -5 percent from the projected cost of U_5, it is concluded that the production process is following the L-C cost allocation model's original parameters, no significant changes have occurred, and no changes in the model are necessary.

Assume, as is frequently the case, that G.L.U.B. Co. did not keep records for individual unit production times or costs after U_5. Consequently some other procedures must be used to determine if the L-C allocation model and its original parameters are still appropriate for cost allocation. For this purpose the slightly less sensitive, but more readily available, measure of total production cost may be used.

TABLE 4-3
Percent Confidence Interval for b Required to Obtain A ± 5% Interval for U_i

L-C%	b	% ± b X=100	% ± b X=500	% ± b X=1,000	% ± b X=5,000	% ± b X=10,000
70	.51457	1.418	1.146	1.049	.893	.835
71	.49410	1.497	1.194	1.092	.930	.870
72	.47393	1.582	1.266	1.160	.970	.907
73	.45403	1.673	1.321	1.211	1.035	.969
74	.43440	1.749	1.404	1.289	1.081	1.012
75	.41503	1.855	1.469	1.349	1.132	1.060
76	.39592	1.970	1.565	1.439	1.187	1.111
77	.37707	2.068	1.644	1.511	1.272	1.193
78	.35845	2.203	1.729	1.590	1.339	1.255
79	.34007	2.323	1.852	1.676	1.411	1.323
80	.32192	2.485	1.956	1.801	1.491	1.397
81	.30400	2.631	2.073	1.907	1.611	1.480
82	.28630	2.829	2.235	2.025	1.711	1.571
83	.26881	3.013	2.380	2.194	1.822	1.711
84	.25153	3.259	2.544	2.345	1.948	1.828
85	.23446	3.497	2.729	2.516	2.089	1.961
86	.21759	3.814	2.987	2.711	2.251	2.114
87	.20091	4.131	3.235	2.936	2.438	2.289
88	.18442	4.500	3.524	3.253	2.711	2.494
89	.16812	4.996	3.866	3.568	2.974	2.736
90	.15200	5.526	4.342	3.947	3.289	3.026
91	.13606	6.173	4.850	4.409	3.674	3.454
92	.12029	7.066	5.486	4.987	4.156	3.907
93	.10429	8.118	6.303	5.730	4.775	4.489
94	.08926	9.521	7.393	6.721	5.601	5.265
95	.07400	11.486	8.918	8.243	6.756	6.351
96	.05889	14.602	11.376	10.357	8.489	7.980
97	.04394	19.570	15.246	13.881	11.605	10.695
98	.02914	29.506	22.987	20.928	17.497	16.125
99	.01449	59.312	46.208	42.070	35.173	32.414

TABLE 4-4
Percent Confidence Interval for b Required to Obtain A ± 10% Interval for U_i

L-C%	b	% ± b X=100	% ± b X=500	% ± b X=1,000	% ± b X=5,000	% ± b X=10,000
70	.51457	2.779	2.234	2.059	1.749	1.632
71	.49410	2.934	2.347	2.165	1.821	1.720
72	.47393	3.101	2.468	2.278	1.920	1.793
73	.45403	3.259	2.598	2.400	2.026	1.894
74	.43440	3.453	2.739	2.532	2.117	1.979
75	.41503	3.638	2.891	2.650	2.240	2.096
76	.39592	3.839	3.056	2.803	2.348	2.197
77	.37707	4.084	3.235	2.970	2.492	2.333
78	.35845	4.324	3.403	3.124	2.622	2.454
79	.34007	4.587	3.616	3.322	2.793	2.587
80	.32192	4.876	3.851	3.510	2.950	2.764
81	.30400	5.197	4.078	3.749	3.124	2.927
82	.28630	5.553	4.365	3.981	3.353	3.108
83	.26881	5.952	4.650	4.278	3.571	3.348
84	.25153	6.360	5.009	4.571	3.816	3.577
85	.23446	6.866	5.373	4.947	4.137	3.838
86	.21759	7.445	5.836	5.331	4.457	4.136
87	.20091	8.112	6.321	5.773	4.827	4.529
88	.18442	8.838	6.940	6.344	5.259	4.934
89	.16812	9.754	7.613	6.959	5.829	5.412
90	.15200	10.855	8.486	7.697	6.447	5.986
91	.13606	12.126	9.481	8.672	7.202	6.688
92	.12029	13.799	10.723	9.809	8.146	7.647
93	.10469	15.950	12.416	11.270	9.360	8.787
94	.08926	18.707	14.563	13.330	11.090	10.306
95	.07400	22.702	17.567	16.081	13.378	12.432
96	.05889	28.526	22.243	20.205	16.810	15.621
97	.04394	38.458	29.811	27.080	22.529	20.936
98	.02914	57.983	44.945	41.171	33.966	31.907
99	.01449	117.245	90.347	82.761	68.967	64.139

RELATIONSHIP BETWEEN b AND T_i CONFIDENCE INTERVALS:

In those cases where the production costs are not available for individual units, a comparison of actual and formula based total production costs may be used to determine whether or not the L-C cost allocation model and its parameters as originally determined are still appropriate. If T_i is to be used for variance analysis, the initial data must have enough correlation to be able to project T_i with at least a ±10 percent confidence interval and an 80 percent confidence level. As in the previous two sections of this chapter, we will now work back from the equation representing a percent confidence interval for total production cost to the underlying b parameter in equation 1-8 from which T is determined. The reason for doing this is to determine how tight a confidence interval must be obtained for b in order to be able to place a percent confidence interval around T. This confidence interval for b can then be used to assist in determining the number of units of initial input, n, needed to attain the desired level of statistical accuracy in the model. Once the model has attained the desired level of statistical accuracy, T_i may be used for variance analysis.

From (1-8) we know that the value of T_i depends on the value of b. Likewise, the determination of confidence intervals for T_i depends on the determination of confidence intervals for b.

A ±5 percent confidence interval for T_i is represented by equation 4-14.

$$.95T_{ci} \leq T_i \leq 1.05T_{ci} \qquad (4\text{-}14)$$

Let B1 and B2 represent extreme values of the confidence interval for b required to satisfy (4-14). Then, combining (1-8) and (4-14) we have:

$$A_c X^{C1} = .95 A_c X^{C_c} \leq T_i \leq 1.05 A_c X^{C_c} = A_c X^{C2} \qquad (4\text{-}15)$$

where:

$C1 = 1 - B1$

$C_c = 1 - B_c$

$C2 = 1 - B2$

$X = i$.

Solving for B1 and B2:

$$B1 = 1 - (\log 95 + C_c \log X - \log 100)/\log X \qquad (4\text{-}16)$$

$$B2 = 1 - (\log 1.05 + C_c \log X)/\log X \quad . \qquad (4\text{-}17)$$

The computer program listed in Appendix B3 was used to determine B1 and B2 in equations 4-16 and 4-17 for T_{100}; T_{500}; $T_{1,000}$; $T_{5,000}$; and $T_{10,000}$, where b values ranged over L-C percents of 70 to 99. The percentage interval $(B_c - B2)/B_c$ was found to be smaller than the percentage interval $(B1 - B_c)/B_c$, and is accordingly the one given in table 4-5. Table 4-5 lists the percent confidence intervals for b required to obtain a 5 percent interval for T_i. Table 4-6 lists the percent confidence intervals for b required to obtain a 10 percent interval for T_i.

For the G.L.U.B. Co., a b confidence interval of 4.924 is required for predictions of T_i with a ±5 percent confidence interval over 100 units. For the 20 units G.L.U.B. Co. is planning on producing the interval could be somewhat larger. Previous calculations showed that an n of 5 was large enough to obtain a 5 percent confidence interval and a 90 percent confidence level for b. Thus, T_i may be used by the G.L.U.B. Co. for purposes of variance analysis.

For B_c = .207572 and A_c = \$10,045.70, equation 1-8 yields T_{ci} values of \$35,963.60 at T_{c5}, \$85,891.00 at T_{c15}, and \$107,883.00 at T_{c20}. The actual values of T_5, T_{15}, and T_{20} were \$35,936.67, \$85,800.00, and \$107,300.00 respectively. As none of these actual values differed from the anticipated values by more than 5 percent we can conclude that the L-C cost allocation model originally applied to this product line remained appropriate throughout the line's entire production cycle.

TABLE 4-5
Percent Confidence Interval for b Required to Obtain A ± 5% Interval for $\bar{T}_i$

L-C%	b	% ± b X=100	% ± b X=500	% ± b X=1,000	% ± b X=5,000	% ± b X=10,000
70	.51457	1.922	1.424	1.281	1.039	.961
71	.49410	2.002	1.483	1.334	1.082	1.001
72	.47393	2.087	1.547	1.391	1.128	1.043
73	.45403	2.179	1.614	1.452	1.178	1.089
74	.43440	2.277	1.687	1.518	1.231	1.138
75	.41503	2.383	1.766	1.589	1.288	1.191
76	.39592	2.498	1.851	1.665	1.351	1.249
77	.37707	2.623	1.944	1.749	1.418	1.311
78	.35845	2.760	2.045	1.840	1.492	1.380
79	.34007	2.909	2.155	1.939	1.573	1.454
80	.32192	3.073	2.277	2.048	1.661	1.536
81	.30400	3.254	2.411	2.169	1.759	1.627
82	.28630	3.455	2.560	2.303	1.868	1.727
83	.26881	3.680	2.727	2.453	1.990	1.840
84	.25153	3.933	2.914	2.622	2.126	1.966
85	.23446	4.219	3.127	2.813	2.281	2.109
86	.21759	4.547	3.369	3.031	2.458	2.273
87	.20091	4.924	3.649	3.283	2.662	2.462
88	.18442	5.364	3.975	3.576	2.900	2.682
89	.16812	5.885	4.360	3.923	3.181	2.942
90	.15200	6.509	4.823	4.339	3.519	3.254
91	.13606	7.271	5.388	4.847	3.931	3.635
92	.12029	8.224	6.094	5.483	4.447	4.112
93	.10469	9.450	7.002	6.300	5.109	4.725
94	.08926	11.083	8.213	7.389	5.992	5.541
95	.07400	13.370	9.907	8.913	7.229	6.685
96	.05889	16.799	12.449	11.199	9.083	8.399
97	.04394	22.515	16.684	15.010	12.173	11.257
98	.02914	33.946	25.154	22.630	18.354	16.973
99	.01449	68.236	50.565	45.491	36.895	34.118

TABLE 4-6
Percent Confidence Interval for b Required to Obtain A ± 10% Interval for T_i

L-C%	b	% ± b X=100	% ± b X=500	% ± b X=1,000	% ± b X=5,000	% ± b X=10,000
70	.51457	4.022	2.980	2.681	2.174	2.011
71	.49410	4.188	3.103	2.792	2.264	2.094
72	.47393	4.366	3.236	2.911	2.361	2.183
73	.45403	4.558	3.377	3.038	2.464	2.279
74	.43440	4.764	3.530	3.176	2.576	2.382
75	.41503	4.986	3.695	3.324	2.696	2.493
76	.39592	5.227	3.873	3.484	2.826	2.613
77	.37707	5.488	4.067	3.659	2.967	2.744
78	.35845	5.773	4.278	3.849	3.121	2.886
79	.34007	6.085	4.509	4.057	3.290	3.042
80	.32192	6.428	4.763	4.285	3.476	3.214
81	.30400	6.807	5.044	4.538	3.680	3.403
82	.28630	7.228	5.356	4.819	3.908	3.614
83	.26881	7.699	5.705	5.132	4.162	3.849
84	.25153	8.227	6.097	5.485	4.448	4.113
85	.23446	8.827	6.541	5.884	4.772	4.413
86	.21759	9.511	7.048	6.341	5.142	4.755
87	.20091	10.301	7.633	6.867	5.569	5.150
88	.18442	11.222	8.315	7.481	6.067	5.611
89	.16812	12.310	9.122	8.206	6.656	6.155
90	.15200	13.615	10.089	9.077	7.361	6.807
91	.13606	15.211	11.271	10.140	8.224	7.605
92	.12029	17.204	12.749	11.469	9.302	8.602
93	.10469	19.767	14.648	13.178	10.688	9.883
94	.08926	23.184	17.180	15.456	12.535	11.592
95	.07400	27.967	20.724	18.645	15.121	13.983
96	.05889	35.141	26.040	23.427	19.000	17.570
97	.04394	47.097	34.900	31.398	25.465	23.548
98	.02914	71.008	52.618	47.338	38.393	35.504
99	.01449	142.273	105.772	95.158	77.177	71.368

MINIMUM R REQUIRED TO OBTAIN DESIRED b CONFIDENCE LEVEL AND INTERVAL:

Previous sections of this chapter have dealt with procedures for: (1) determining the b confidence intervals necessary to insure desired confidence intervals for Y_N, U_i, and T_i, (2) determining the number of units of input, n, needed to insure this b confidence interval at a desired confidence level, and (3) evaluating actual performance in order to determine whether or not the original parameters of the L-C cost allocation model are being followed. In all of these sections, the implied solution to the problem of an inability to obtain desired confidence intervals and levels due to a low correlation coefficient was to increase n. The correlation coefficient, R, may be so low that n would have to be infeasibly large, if it could be determined at all.

In this section, procedures are developed to determine whether or not the initial data has a correlation coefficient of sufficient magnitude to guarantee a desired confidence interval and level for b. If the correlation coefficient of the original data is not sufficient to guarantee a desired confidence interval and level for b, and it is probable that the correlation coefficient will not increase as n increases, the L-C cost allocation model developed in this research should not be used for external reporting. The model might still be of value for managerial decision making and could be used for internal reporting. The minimum standards for external reporting were stated to be an 80 percent confidence level that the project average cost of all anticipated production, Y_{cN}, would not vary from the actual average cost of all anticipated production, Y_N, by more than 10 percent unless some change took place in a parameter of the L-C cost allocation model after it was adopted.

Equation 4-6 is used in determining n when R and a b confidence level and interval are specified. By modifying (4-6) tables can be developed for this minimum n value. Then, all the user would have to do is go to the appropriate table (determined by the desired confidence level), the appropriate column in that table (determined by the desired b confidence interval), find R, and read n, the required number of units of input. This has been done in tables 4-7, 8, and 9 for 80, 90, and 95 percent confidence levels.

Table 4-8 was developed as follows:

From equation 4-6 we know that:

$$\frac{eR}{\sqrt{1-R^2}} = \frac{t_{.90}}{\sqrt{n-2}}$$

where:

a 90 percent confidence level is desired.

e = desired b confidence interval.

Solving for R:

$$R = t_{.90} / \sqrt{e^2(n-2) + t^2_{.90}} \qquad (4\text{-}18)$$

The computer program listed in Appendix B4 was used to solve equation 4-18 for e values of 1 through 10 percent and n values of 5 through 502.

Table 4-7 lists the minimum R required to be 80 percent confident that b has a desired confidence interval. Table 4-8 lists the minimum R required to be 90 percent confident that b has a desired confidence interval. Table 4-9 lists the minimum R required to be 95 percent confident that b has a desired confidence interval. Procedures describing how to use the tables presented in this chapter are given in the next section.

TABLE 4-7
Minimum R Required to Be 80% Confident that b Has A Desired Confidence Interval

		PERCENT CONFIDENCE INTERVAL DESIRED FOR b								
n	t	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
5	1.638	.99994	.99987	.9997	.9996	.9994	.9993	.9991	.9988	.9986
6	1.533	.99991	.99980	.9996	.9994	.9992	.9989	.9986	.9982	.9978
7	1.476	.99988	.99974	.9995	.9992	.9989	.9985	.9981	.9976	.9971
8	1.440	.99985	.99967	.9994	.9990	.9987	.9982	.9976	.9970	.9964
9	1.415	.99982	.99960	.9993	.9989	.9984	.9976	.9972	.9964	.9956
10	1.397	.99979	.99953	.9991	.9987	.9981	.9974	.9967	.9958	.9949
11	1.383	.99976	.99947	.9990	.9985	.9978	.9971	.9962	.9952	.9941
12	1.372	.99973	.99940	.9989	.9983	.9976	.9967	.9957	.9946	.9934
13	1.363	.99970	.99933	.9988	.9981	.9973	.9963	.9952	.9940	.9926
14	1.356	.99967	.99926	.9986	.9979	.9970	.9960	.9948	.9934	.9919
15	1.350	.99964	.99919	.9985	.9977	.9968	.9956	.9943	.9928	.9912
16	1.345	.99961	.99913	.9984	.9975	.9965	.9952	.9938	.9922	.9904
17	1.341	.99958	.99906	.9983	.9974	.9962	.9949	.9933	.9916	.9897
18	1.337	.99955	.99899	.9982	.9972	.9959	.9945	.9929	.9910	.9889
19	1.333	.99952	.99892	.9980	.9970	.9957	.9941	.9924	.9904	.9882
20	1.330	.99949	.99885	.9979	.9968	.9954	.9938	.9919	.9898	.9875
21	1.328	.99946	.99879	.9978	.9966	.9951	.9934	.9914	.9892	.9867
22	1.325	.99943	.99872	.9977	.9964	.9949	.9930	.9910	.9886	.9860
23	1.323	.99940	.99865	.9976	.9962	.9946	.9927	.9905	.9880	.9853
24	1.321	.99937	.99858	.9974	.9960	.9943	.9923	.9900	.9874	.9846
25	1.319	.99934	.99851	.9973	.9958	.9941	.9920	.9895	.9868	.9838
26	1.318	.99931	.99844	.9972	.9957	.9938	.9916	.9891	.9862	.9831
27	1.316	.99927	.99838	.9971	.9955	.9935	.9912	.9886	.9856	.9824
28	1.315	.99924	.99831	.9970	.9953	.9933	.9909	.9881	.9851	.9817
29	1.314	.99921	.99824	.9968	.9951	.9930	.9905	.9877	.9845	.9810
30	1.313	.99918	.99817	.9967	.9949	.9927	.9901	.9872	.9839	.9802
31	1.311	.99915	.99810	.9966	.9947	.9924	.9898	.9867	.9833	.9795
32	1.310	.99912	.99803	.9965	.9945	.9922	.9894	.9863	.9827	.9788
42	1.303	.99882	.99736	.9953	.9927	.9895	.9858	.9816	.9769	.9717
52	1.298	.99851	.99667	.9941	.9908	.9869	.9823	.9770	.9712	.9648
62	1.296	.99821	.99600	.9929	.9890	.9843	.9788	.9725	.9656	.9581
82	1.292	.99761	.99465	.9905	.9853	.9791	.9718	.9637	.9547	.9449
102	1.290	.99700	.99330	.9881	.9817	.9740	.9651	.9551	.9442	.9324
202	1.286	.99400	.98666	.9766	.9642	.9496	.9332	.9153	.8962	.8762
502	1.283	.98515	.96748	.9442	.9167	.8861	.8537	.8203	.7868	.7539
100,001	1.282	.37570	.26090	.1986	.1600	.1339	.1150	.1008	.0897	.0808

TABLE 4-7 (con't)

PERCENT CONFIDENCE INTERVAL DESIRED FOR b

n	t	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
5	1.638	.9983	.9979	.9976	.9972	.9968	.9964	.9959	.9955	.9949	.9944
6	1.533	.9974	.9969	.9964	.9958	.9952	.9945	.9939	.9931	.9924	.9915
7	1.476	.9965	.9958	.9951	.9944	.9936	.9927	.9918	.9908	.9898	.9887
8	1.440	.9956	.9948	.9939	.9929	.9919	.9908	.9897	.9884	.9871	.9858
9	1.415	.9947	.9937	.9926	.9915	.9903	.9889	.9876	.9861	.9845	.9829
10	1.397	.9938	.9927	.9914	.9901	.9886	.9871	.9855	.9838	.9820	.9801
11	1.383	.9929	.9916	.9902	.9886	.9870	.9852	.9834	.9814	.9794	.9772
12	1.372	.9920	.9905	.9889	.9872	.9853	.9834	.9813	.9791	.9768	.9744
13	1.363	.9911	.9895	.9877	.9858	.9837	.9815	.9792	.9768	.9743	.9716
14	1.356	.9902	.9884	.9864	.9843	.9821	.9797	.9772	.9745	.9717	.9688
15	1.350	.9893	.9874	.9852	.9829	.9805	.9779	.9751	.9723	.9692	.9661
16	1.345	.9884	.9863	.9840	.9815	.9782	.9761	.9731	.9700	.9668	.9634
17	1.341	.9876	.9853	.9828	.9801	.9773	.9743	.9711	.9678	.9643	.9607
18	1.337	.9867	.9842	.9816	.9787	.9757	.9725	.9691	.9656	.9619	.9580
19	1.333	.9858	.9832	.9803	.9773	.9741	.9707	.9671	.9633	.9594	.9553
20	1.330	.9849	.9821	.9791	.9759	.9725	.9689	.9651	.9611	.9570	.9527
21	1.328	.9840	.9811	.9779	.9746	.9710	.9672	.9632	.9590	.9546	.9501
22	1.325	.9832	.9801	.9767	.9732	.9694	.9654	.9612	.9568	.9522	.9474
23	1.323	.9823	.9790	.9755	.9718	.9678	.9636	.9592	.9546	.9498	.9449
24	1.321	.9814	.9780	.9743	.9704	.9663	.9619	.9573	.9525	.9475	.9423
25	1.319	.9805	.9770	.9731	.9691	.9647	.9602	.9554	.9504	.9452	.9398
26	1.318	.9797	.9760	.9720	.9677	.9632	.9585	.9535	.9483	.9429	.9373
27	1.316	.9788	.9749	.9708	.9664	.9617	.9567	.9516	.9462	.9406	.9348
28	1.315	.9780	.9739	.9696	.9650	.9602	.9551	.9497	.9441	.9383	.9323
29	1.314	.9771	.9729	.9685	.9637	.9587	.9534	.9478	.9421	.9361	.9299
30	1.313	.9763	.9719	.9673	.9624	.9572	.9517	.9460	.9400	.9338	.9275
31	1.311	.9754	.9709	.9661	.9610	.9556	.9500	.9441	.9379	.9315	.9250
32	1.310	.9745	.9699	.9649	.9597	.9541	.9483	.9422	.9359	.9293	.9226
42	1.303	.9661	.9601	.9536	.9468	.9396	.9321	.9244	.9163	.9081	.8996
52	1.298	.9579	.9505	.9426	.9343	.9257	.9167	.9074	.8978	.8881	.8781
62	1.296	.9499	.9413	.9321	.9225	.9125	.9021	.8915	.8806	.8696	.8583
82	1.292	.9345	.9235	.9119	.8999	.8875	.8747	.8618	.8487	.8355	.8222
102	1.290	.9198	.9067	.8930	.8789	.8645	.8498	.8350	.8201	.8052	.7903
202	1.286	.8556	.8346	.8135	.7924	.7714	.7508	.7305	.7107	.6914	.6727
502	1.283	.7219	.6911	.6617	.6339	.6076	.5828	.5594	.5375	.5169	.4976
100,001	1.282	.0735	.0674	.0622	.0578	.0539	.0506	.0476	.0449	.0426	.0405

TABLE 4-8
Minimum R Required to Be 90% Confident that b
Has A Desired Confidence Interval

		PERCENT CONFIDENCE INTERVAL DESIRED FOR b								
n	t	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
5	2.353	.99997	.99993	.9998	.9998	.9997	.9996	.9995	.9994	.9993
6	2.132	.99995	.99990	.9998	.9997	.9996	.9994	.9992	.9991	.9989
7	2.015	.99993	.99986	.9997	.9996	.9994	.9992	.9990	.9987	.9984
8	1.943	.99992	.99982	.9996	.9995	.9992	.9990	.9987	.9983	.9980
9	1.895	.99990	.99978	.9996	.9993	.9991	.9988	.9984	.9980	.9975
10	1.860	.99988	.99974	.9995	.9992	.9989	.9985	.9981	.9976	.9971
11	1.833	.99986	.99969	.9994	.9991	.9987	.9983	.9978	.9972	.9966
12	1.812	.99984	.99965	.9993	.9990	.9986	.9981	.9975	.9969	.9962
13	1.796	.99983	.99961	.9993	.9989	.9984	.9979	.9972	.9965	.9957
14	1.782	.99981	.99957	.9992	.9988	.9983	.9976	.9969	.9961	.9953
15	1.771	.99979	.99953	.9991	.9987	.9981	.9974	.9967	.9958	.9948
16	1.761	.99977	.99949	.9990	.9985	.9979	.9972	.9964	.9954	.9944
17	1.753	.99975	.99945	.9990	.9984	.9978	.9970	.9961	.9950	.9939
18	1.746	.99973	.99941	.9989	.9983	.9976	.9968	.9958	.9947	.9935
19	1.740	.99971	.99936	.9988	.9982	.9974	.9965	.9955	.9943	.9930
20	1.734	.99970	.99932	.9988	.9981	.9973	.9963	.9952	.9939	.9926
21	1.729	.99968	.99928	.9989	.9980	.9971	.9961	.9949	.9936	.9921
22	1.725	.99966	.99924	.9986	.9979	.9969	.9959	.9946	.9932	.9917
23	1.721	.99964	.99920	.9985	.9977	.9968	.9956	.9943	.9928	.9912
24	1.717	.99962	.99916	.9985	.9976	.9966	.9954	.9940	.9925	.9908
25	1.714	.99960	.99912	.9984	.9975	.9964	.9952	.9937	.9921	.9903
26	1.711	.99959	.99907	.9983	.9974	.9963	.9950	.9935	.9918	.9899
27	1.708	.99957	.99903	.9982	.9973	.9961	.9947	.9932	.9914	.9894
28	1.706	.99955	.99899	.9982	.9972	.9960	.9945	.9929	.9910	.9890
29	1.703	.99953	.99895	.9981	.9971	.9958	.9943	.9926	.9907	.9885
30	1.701	.99951	.99891	.9980	.9969	.9956	.9941	.9923	.9903	.9881
31	1.699	.99949	.99887	.9979	.9968	.9951	.9939	.9920	.9899	.9876
32	1.697	.99948	.99883	.9979	.9967	.9953	.9936	.9917	.9896	.9872
42	1.684	.99929	.99841	.9971	.9956	.9937	.9914	.9889	.9860	.9828
52	1.676	.99911	.99800	.9964	.9944	.9920	.9892	.9860	.9824	.9784
62	1.671	.99892	.99759	.9957	.9935	.9904	.9870	.9832	.9789	.9741
82	1.664	.99855	.99676	.9942	.9910	.9872	.9827	.9776	.9719	.9657
102	1.660	.99819	.99594	.9928	.9888	.9840	.9784	.9721	.9651	.9575
202	1.653	.99636	.99186	.9856	.9778	.9686	.9579	.9461	.9332	.9194
502	1.648	.99092	.97991	.9650	.9470	.9262	.9033	.8788	.8534	.8275
100,001	1.645	.46149	.32765	.2517	.2037	.1708	.1470	.1289	.1148	.1034

TABLE 4-8 (con't)

PERCENT CONFIDENCE INTERVAL DESIRED FOR b

n	t	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
5	2.353	.9991	.9990	.9988	.9986	.9984	.9982	.9980	.9978	.9975	.9973
6	2.132	.9986	.9984	.9981	.9978	.9975	.9971	.9968	.9964	.9960	.9956
7	2.015	.9981	.9977	.9974	.9969	.9965	.9960	.9955	.9950	.9944	.9938
8	1.943	.9976	.9971	.9966	.9961	.9955	.9949	.9943	.9936	.9929	.9921
9	1.895	.9970	.9965	.9959	.9952	.9945	.9938	.9930	.9921	.9913	.9903
10	1.860	.9965	.9958	.9951	.9943	.9935	.9926	.9917	.9907	.9897	.9886
11	1.833	.9959	.9952	.9943	.9935	.9925	.9915	.9904	.9893	.9881	.9868
12	1.812	.9954	.9945	.9936	.9926	.9915	.9903	.9891	.9878	.9865	.9851
13	1.796	.9948	.9939	.9928	.9917	.9905	.9892	.9879	.9864	.9849	.9833
14	1.782	.9943	.9932	.9921	.9908	.9895	.9881	.9866	.9850	.9833	.9816
15	1.771	.9937	.9926	.9913	.9899	.9885	.9869	.9853	.9836	.9818	.9798
16	1.761	.9932	.9919	.9905	.9891	.9875	.9858	.9840	.9822	.9802	.9781
17	1.753	.9926	.9913	.9898	.9882	.9865	.9847	.9828	.9807	.9786	.9764
18	1.746	.9921	.9906	.9890	.9873	.9855	.9836	.9815	.9793	.9771	.9747
19	1.740	.9916	.9900	.9883	.9865	.9845	.9825	.9803	.9780	.9755	.9730
20	1.734	.9910	.9893	.9875	.9856	.9835	.9813	.9790	.9766	.9740	.9713
21	1.729	.9905	.9887	.9868	.9847	.9825	.9802	.9778	.9752	.9724	.9696
22	1.725	.9899	.9881	.9860	.9839	.9816	.9791	.9765	.9738	.9709	.9679
23	1.721	.9894	.9874	.9853	.9830	.9806	.9780	.9753	.9724	.9694	.9663
24	1.717	.9889	.9868	.9845	.9822	.9796	.9769	.9740	.9710	.9679	.9646
25	1.714	.9883	.9861	.9838	.9813	.9786	.9758	.9728	.9697	.9664	.9630
26	1.711	.9878	.9855	.9831	.9805	.9777	.9747	.9716	.9683	.9649	.9613
27	1.708	.9872	.9849	.9823	.9796	.9767	.9736	.9704	.9670	.9634	.9597
28	1.706	.9867	.9842	.9816	.9788	.9757	.9725	.9692	.9656	.9619	.9581
29	1.703	.9862	.9836	.9808	.9779	.9748	.9714	.9679	.9643	.9604	.9564
30	1.701	.9856	.9830	.9801	.9771	.9738	.9704	.9667	.9629	.9589	.9548
31	1.699	.9851	.9823	.9794	.9762	.9728	.9693	.9655	.9616	.9575	.9532
32	1.697	.9846	.9817	.9786	.9754	.9719	.9682	.9643	.9603	.9560	.9516
42	1.684	.9793	.9755	.9714	.9671	.9625	.9577	.9526	.9473	.9418	.9361
52	1.676	.9741	.9694	.9643	.9590	.9534	.9474	.9413	.9348	.9282	.9213
62	1.671	.9690	.9634	.9574	.9511	.9445	.9376	.9303	.9229	.9151	.9072
82	1.664	.9589	.9517	.9440	.9359	.9274	.9186	.9095	.9001	.8906	.8808
102	1.660	.9492	.9404	.9311	.9214	.9113	.9008	.8900	.8791	.8679	.8565
202	1.653	.9048	.8896	.8739	.8579	.8416	.8252	.8087	.7923	.7760	.7598
502	1.648	.8014	.7755	.7499	.7250	.7009	.6775	.6551	.6335	.6129	.5932
100,001	1.645	.0941	.0863	.0797	.0741	.0691	.0648	.0610	.0577	.0546	.0519

TABLE 4-9
Minimum R Required to Be 95% Confident that b Has A Desired Confidence Interval

PERCENT CONFIDENCE INTERVAL DESIRED FOR b

n	t	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
5	3.182	.99998	.99996	.9999	.9999	.9998	.9998	.9997	.9997	.9996
6	2.776	.99997	.99994	.9998	.9998	.9997	.9996	.9995	.9994	.9993
7	2.571	.99996	.99991	.9998	.9997	.9996	.9995	.9993	.9992	.9990
8	2.447	.99995	.99988	.9998	.9996	.9995	.9993	.9991	.9989	.9987
9	2.365	.99993	.99985	.9997	.9996	.9994	.9992	.9990	.9987	.9984
10	2.306	.99992	.99983	.9996	.9995	.9993	.9990	.9987	.9984	.9981
11	2.262	.99991	.99980	.9996	.9994	.9992	.9989	.9985	.9982	.9978
12	2.228	.99989	.99977	.9995	.9993	.9990	.9987	.9983	.9979	.9974
13	2.201	.99988	.99974	.9995	.9992	.9989	.9986	.9981	.9977	.9971
14	2.179	.99987	.99971	.9994	.9992	.9988	.9984	.9979	.9974	.9968
15	2.160	.99986	.99968	.9994	.9991	.9987	.9982	.9977	.9971	.9965
16	2.145	.99984	.99965	.9993	.9990	.9986	.9981	.9975	.9969	.9962
17	2.131	.99983	.99962	.9993	.9989	.9985	.9979	.9973	.9966	.9958
18	2.120	.99982	.99960	.9992	.9988	.9984	.9978	.9971	.9964	.9955
19	2.110	.99980	.99957	.9992	.9988	.9982	.9976	.9969	.9961	.9952
20	2.101	.99979	.99954	.9991	.9987	.9981	.9975	.9967	.9958	.9949
21	2.093	.99978	.99951	.9991	.9986	.9980	.9973	.9965	.9956	.9946
22	2.086	.99977	.99948	.9990	.9985	.9979	.9971	.9963	.9953	.9943
23	2.080	.99975	.99945	.9990	.9984	.9978	.9970	.9961	.9951	.9939
24	2.074	.99974	.99942	.9989	.9984	.9977	.9968	.9959	.9948	.9936
25	2.069	.99973	.99939	.9989	.9983	.9975	.9967	.9957	.9946	.9933
26	2.063	.99971	.99936	.9988	.9982	.9974	.9965	.9955	.9943	.9930
27	2.060	.99970	.99933	.9988	.9981	.9973	.9964	.9953	.9940	.9927
28	2.056	.99969	.99930	.9987	.9980	.9972	.9962	.9951	.9938	.9923
29	2.052	.99968	.99927	.9987	.9980	.9971	.9960	.9949	.9935	.9920
30	2.048	.99966	.99925	.9986	.9979	.9970	.9959	.9947	.9933	.9917
31	2.045	.99965	.99922	.9986	.9978	.9968	.9957	.9944	.9930	.9914
32	2.042	.99964	.99919	.9985	.9977	.9967	.9956	.9942	.9927	.9911
42	2.021	.99951	.99890	.9980	.9969	.9956	.9940	.9922	.9902	.9879
52	2.009	.99938	.99860	.9975	.9961	.9944	.9924	.9902	.9876	.9848
62	2.000	.99925	.99831	.9970	.9953	.9933	.9909	.9882	.9851	.9817
82	1.990	.99899	.99773	.9959	.9937	.9910	.9878	.9842	.9801	.9756
102	1.984	.99873	.99715	.9949	.9921	.9887	.9847	.9802	.9752	.9696
202	1.972	.99743	.99426	.9898	.9843	.9776	.9699	.9612	.9516	.9413
502	1.965	.99358	.98574	.9750	.9618	.9463	.9290	.9101	.8900	.8691
100,001	1.960	.52682	.38188	.2960	.2406	.2023	.1743	.1531	.1364	.1230

TABLE 4-9 (con't)

PERCENT CONFIDENCE INTERVAL DESIRED FOR b

n	t	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5	10.0
5	3.182	.9995	.9994	.9993	.9992	.9991	.9990	.9989	.9988	.9986	.9985
6	2.776	.9992	.9990	.9989	.9987	.9985	.9983	.9981	.9979	.9976	.9974
7	2.571	.9988	.9986	.9984	.9981	.9978	.9975	.9972	.9969	.9966	.9962
8	2.447	.9984	.9982	.9978	.9975	.9971	.9968	.9964	.9959	.9955	.9950
9	2.365	.9981	.9977	.9973	.9969	.9964	.9960	.9955	.9949	.9944	.9938
10	2.306	.9977	.9973	.9968	.9963	.9957	.9952	.9946	.9939	.9932	.9925
11	2.262	.9973	.9968	.9963	.9957	.9950	.9944	.9937	.9929	.9921	.9913
12	2.228	.9969	.9963	.9957	.9951	.9943	.9936	.9928	.9919	.9910	.9900
13	2.201	.9965	.9959	.9952	.9944	.9936	.9928	.9918	.9909	.9899	.9888
14	2.179	.9961	.9954	.9947	.9938	.9929	.9920	.9909	.9899	.9887	.9875
15	2.160	.9958	.9950	.9941	.9932	.9922	.9912	.9900	.9889	.9876	.9863
16	2.145	.9954	.9945	.9936	.9926	.9915	.9904	.9891	.9879	.9865	.9851
17	2.131	.9950	.9941	.9930	.9920	.9908	.9895	.9882	.9868	.9854	.9838
18	2.120	.9946	.9936	.9925	.9913	.9901	.9887	.9873	.9858	.9843	.9826
19	2.110	.9942	.9931	.9920	.9907	.9894	.9880	.9864	.9848	.9832	.9814
20	2.101	.9938	.9927	.9914	.9901	.9887	.9872	.9855	.9838	.9820	.9802
21	2.093	.9935	.9922	.9909	.9895	.9880	.9864	.9846	.9828	.9809	.9789
22	2.086	.9931	.9918	.9904	.9889	.9873	.9856	.9837	.9818	.9798	.9777
23	2.080	.9927	.9913	.9899	.9883	.9866	.9848	.9829	.9809	.9787	.9765
24	2.074	.9923	.9909	.9893	.9877	.9859	.9840	.9820	.9799	.9776	.9753
25	2.069	.9919	.9904	.9888	.9870	.9852	.9832	.9811	.9789	.9766	.9741
26	2.063	.9915	.9900	.9883	.9864	.9845	.9824	.9802	.9779	.9755	.9729
27	2.060	.9912	.9895	.9877	.9858	.9838	.9816	.9793	.9769	.9744	.9717
28	2.056	.9908	.9891	.9872	.9852	.9831	.9808	.9784	.9759	.9733	.9705
29	2.052	.9904	.9886	.9867	.9846	.9824	.9800	.9776	.9750	.9722	.9694
30	2.048	.9900	.9881	.9861	.9840	.9817	.9792	.9767	.9740	.9711	.9682
31	2.045	.9896	.9877	.9856	.9834	.9810	.9785	.9758	.9730	.9701	.9670
32	2.042	.9892	.9872	.9851	.9828	.9803	.9777	.9749	.9720	.9690	.9658
42	2.021	.9855	.9828	.9799	.9768	.9735	.9700	.9663	.9625	.9585	.9543
52	2.009	.9817	.9784	.9748	.9709	.9668	.9625	.9580	.9533	.9483	.9432
62	2.000	.9780	.9740	.9697	.9651	.9603	.9552	.9498	.9442	.9384	.9325
82	1.990	.9707	.9655	.9598	.9539	.9476	.9410	.9341	.9270	.9196	.9121
102	1.984	.9636	.9571	.9502	.9430	.9353	.9274	.9191	.9106	.9019	.8929
202	1.972	.9302	.9185	.9063	.8937	.8806	.8673	.8538	.8401	.8264	.8126
502	1.965	.8476	.8258	.8039	.7821	.7606	.7394	.7187	.6986	.6790	.6601
100,001	1.960	.1119	.1027	.0949	.0881	.0823	.0772	.0727	.0687	.0651	.0618

SUMMARY OF STATISTICAL PROCEDURES:

By way of the example developed in Chapter III, this section summarizes the statistical procedures developed in this chapter. It illustrates how to use them rapidly, and gives a brief explaination to justify their use.

For the G.L.U.B. Co. the procedures necessary to determine the n required to insure a desired confidence level and interval for Y_N are as follows:

(1) Gather initial data (pairs of X and Y) and determine N, total anticipated production. (N = 20)

(2) Evaluate the initial data by the use of the equations 1-4, 1-5, and 1-6, or by the use of the computer programs listed in Appendix A1 or A2. Five pairs of X and Y were used. The correlation coefficient, R, was found to be .99931, and the regression slope coefficient, B_c, was determined to be .207572.

(3) Assume G.L.U.B. Co. desired to obtain a 90 percent confidence level and a ±5 percent confidence interval for Y_N. From table 4-1 a ±5.273 percent confidence interval for b is required to obtain a ±5 percent confidence interval for Y_{100}, (Y_{100} is the closest value to Y_{20} listed in table 4-1). From table 4-8 a minimum R of .9991 is required to be 90 percent confident that b has a ±5.5 (5.273 rounded) percent confidence interval when n equals 5. Thus, because the required R value is less than the computed R value only 5 pairs of X and Y are required to implement the L-C cost allocation model in this case.

If R had been .9975, 9 units of input would have been required. (See table 4-8.) After 4 additional pairs of X and Y data were gathered steps 2 and 3 would be repeated in order to insure that the desired confidence level and interval had been attained.

If R had been .9956, 12 units of input would be required to be 90 percent confident that Y_{cN} and Y_N would not vary by more than 5 percent. Since this is more than 50 percent of G.L.U.B. Co.'s anticipated production it might be desirable to lower the confidence level to 80 percent, and/or increase the confidence interval for Y_N. Lowering the b confidence level to 80 percent we would only need an n of 8. If we desire to increase the Y_N confidence interval to 10 percent we must go to table 4-2 (page 81) to find the required b confidence interval. It is 10.301 percent. For a 10 percent b confidence interval an n of 6 is required to attain a 90 percent confidence level.

(4) For purposes of variance analysis it is desirable to minimize the probability of large deviations between U_{ci} and U_i being caused by random, nonrecurring factors. One of the ways of doing this is to set up confidence intervals and levels for the prediction of U_i. If deviations larger than these previously determined intervals occur it will then be highly probable that they are caused by a change in an underlying parameter of the L-C cost allocation model. For purposes of this research, projections of U_i must be able to be made with a minimum 80 percent confidence level and a maximum 10 percent confidence interval if U_i is to be used for variance analysis.

Assume the G.L.U.B. Co. desired to use the differences between U_{ci} and U_i for purposes of variance analysis. In addition, assume they desired to be able to predict U_i with a 90 percent confidence level and a 5 percent confidence interval. From table 4-3 the b confidence interval necessary to predict U_{20} with a 5 percent confidence interval is 3.98 percent. Table 4-8 indicates an n of 6 is required to obtain a 90 percent confidence level and a 4 percent confidence interval for b when R = .99931. Table 4-7 indicates an n of 5 is required to obtain an 80 percent confidence level and a 4 percent confidence interval for b. Assuming that G.L.U.B. Co. did not desire to gather additional data before implementing the L-C cost allocation model, this lower confidence level would be accepted. (Because of rounding inaccuracies the higher confidence level could also be accepted.)

If U_i cannot be used as a variance indicator because of the lack of unit production data, an analysis can be made of total production time or cost. Use can be made of table 4-5 if a 5 percent confidence interval is desired for T_i. Use of this table shows that the G.L.U.B. Co. would have to obtain a 4.924 percent confidence interval for b in order to be able to predict T_{100} with a 5 percent confidence interval. Table 4-7 indicates that the G.L.U.B. Co. needs only 5 units of input to be able to predict T_i with a 5 percent confidence interval and a 10 percent confidence level.

CHAPTER V

PROBLEMS IN APPLICATION

INTRODUCTION:

The illustration of the L-C cost allocation model presented in Chapter III was straightforward. The correlation coefficient was high, actual production costs were close to those predicted by the model, and the actual number of units produced was the same as the anticipated total production. In the real world a number of factors may hinder application of the model. Startup termination, essential modifications in the product or production procedures, changes in total anticipated production (N), changes in production costs, and the existence of a low correlation coefficient may cause serious problems in applications of the L-C cost allocation model. The problems caused by a low correlation coefficient were considered in Chapter IV. The other problems mentioned above are considered in this chapter.

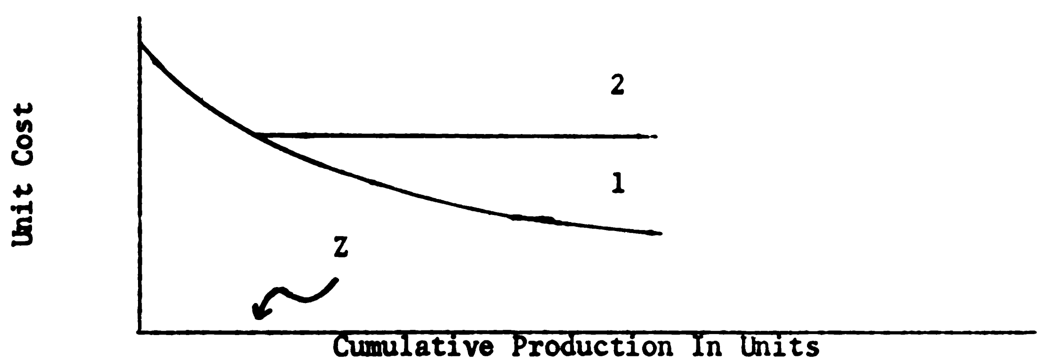
Management may anticipate some of these problems and allow for them in the L-C cost allocation model from its inception. Unanticipated changes, may however, also occur after the L-C cost allocation model has been implemented. If unanticipated changes occur the firm must modify its L-C cost allocation model to reflect these changes. If changes are anticipated, there are, in reality, no special problems, there are merely special procedures. If they are unanticipated there are special problems because the balance in the account "Improvements in Production Procedures" is inaccurate.

In most of the sections which follow, each potential problem in application is first presented as an anticipated change. Then it will be considered as an unanticipated change occuring after the L-C cost allocation model has been initially implemented.

STARTUP TERMINATION:

The startup phenomenon may be terminated due to union production standards, peer groups pressure among labor, or failure to set proper production standards. Figure 5-1 shows the effect of a startup termination on unit production cost. Line 1 represents unit production costs as they would normally occur with the L-C phenomenon. Line 2 represents unit production costs with the startup terminated at output level Z. Beyond output level Z unit production costs are constant.

FIGURE 5-1
Startup Termination



If management is aware of the existence of union production standards at the time the L-C cost allocation model is initiated, these standards may be incorporated into the model. Assume that the G.L.U.B. Co. has produced 5 units and has determined the values of R, A_c, and B_c from an analysis of this data using equations 1-4, 1-5, and 1-6 or the computer program listed in Appendix A1. With the values of A_c, B_c, and an N of 20, projections are made of the cumulative average cost of all anticipated production, the cost of each unit, the total cost at various output levels, and the cumulative average cost at various output levels by the use of equations 1-1, 1-8, and 1-10, or the computer program listed in Appendix A4. An analysis of unit production costs and the labor contract reveals that production costs (ignoring reduced materials usage) will reach their minimum at unit Z (Z<N) and unit production costs from Z through N will be constant. The unit cost of units 1 through Z is:

$$U = A_c(X^{C_c}-(X-1)^{C_c}) \qquad X = 1,2,\ldots,Z \qquad (5\text{-}1)$$

and the unit cost of units Z through N is constant at:

$$U = A_c(Z^{C_c}-(Z-1)^{C_c}) \quad . \qquad (5\text{-}2)$$

Equations 5-1 and 5-2 may be used to apply the L-C cost allocation model to situations involving startup termination, or the computer program listed in Appendix C1 may be used. In developing income statements with the L-C cost allocation model, G.L.U.B. Co. used the computer program listed in Appendix C1, a partial reproduction of the printout is presented below:

CUMULATIVE PRODUCTION	UNIT COST	TOTAL COST	AVERAGE COST	ACCOUNT BALANCE
5	5828.76	35963.6	7192.73	7918.17
10	4989.33	62288.5	6228.85	6197.62
15	4989.33	87235.2	5815.68	3098.81
20	4989.33	112182.0	5609.09	--

On the basis of this information, assuming no variances occur after the fifth unit, the following calculations and journal entries would be made:

	1969	1970	1971
Actual Cost of Units Produced	$35,800.00	$51,271.60	$24,946.80
-Calculated Cost of Units Produced	35,963.67	51,271.60	24,946.80
Cost Variance, Unfavorable, (Favorable)	$ (163.67)	$ -0-	$ -0-
Calculated Cost of Units Produced	$35,963.67	$51,271.60	$24,946.80
-Number of Units Produced x Average Unit Cost	28,045.45	56,090.90	28,045.45
Change in Improvements in Production Procedures	$ 7,918.22*	$(4,819.30)*	$(3,098.65)*

*Rounding errors cause these figures not to total to zero.

1969:

Cost of Goods Sold	$28,045.45	
Improvements in Production Procedures	7,918.22	
Cost Variance-Favorable		$ 163.67
Various Cost Accounts		35,800.00

1970:

Cost of Goods Sold	$56,090.90	
Improvements in Production Procedures		$ 4,819.30
Various Cost Accounts		51,271.60

1971:

Cost of Goods Sold	$28,045.45	
Improvements in Production Procedures		$ 3,098.65
Various Cost Accounts		24,946.80

Income statements for 1969-1971 using the L-C cost allocation model would be as follows:

G.L.U.B. Co.
Income Statements (L-C)
For the Years Ending December 31, 1969, 1970, 1971

		1969	1970	1971
Sales		$50,000.00	$100,000.00	$50,000.00
Cost of Goods Sold	$28,045.45			
Cost Variance	(163.67)*	27,881.78	56,090.90	28,045.45
Gross Margin		$22,118.22	$ 43,909.10	$21,954.55
Other Expenses		15,000.00	15,000.00	15,000.00
Net Income		$ 7,118.22	$ 28,909.10	$ 6,954.55

*Favorable

Using standard or actual costs the income statements for these years would be:

G.L.U.B. Co.
Income Statements (Std.)
For the Years Ending December 31, 1969, 1970, 1971

	1969	1970	1971
Sales	$50,000.00	$100,000.00	$50,000.00
Cost of Goods Sold	35,800.00	51,271.60	24,946.80
Gross Margin	$14,200.00	$ 48,728.40	$25,053.20
Other Expenses	15,000.00	15,000.00	15,000.00
Net Income (loss)	$ (800.00)	$ 33,728.40	$10,053.20

The characteristic differences between the L-C cost allocation model and current accounting practice are still apparent. The L-C cost allocation model gives a higher net income in earlier periods and a lower net income in later periods.

The treatment of startup terminations is relatively straightforward if management can anticipate their occurance. The situation of an unexpected startup termination is somewhat more complex. Assume the G.L.U.B. Co. management did not anticipate a startup termination and, at the end of 1969, made the entry presented in Chapter III. The balance in Improvements in Production Procedures (IPP) after this entry was $8,992.92. As the year 1970 neared its end, it would become apparent that something was wrong with the original L-C cost allocation parameters. Unit number 15 cost $4,989.33, and the cost of the first 15 units was $87,071.53. This compares with an anticipated cost of $4,569.77 for unit number 15, and a projected cost of $85,891.00 for the first 15 units. Because the actual and projected costs of unit 15 differed by more than 5 percent a decision is made to investigate the situation.

After it is determined that the L-C terminated at unit 10, the computer program listed in Appendix C1 is run to determine the proper basis for cost allocation. A comparison of the previously presented printout of this program with the previous G.L.U.B. Co. calculations presented in Chapter III shows that the projected average cost of all anticipated production has increased from $5,394.15 to $5,609.09, and that the balance in IPP after 5 units have been produced should be $7,918.17 instead of $8,992.92. Because of the change in the G.L.U.B. Company's expected costs, something must be done to reconcile these differences.

Three alternatives are available:

(1) Amortize the $1,074.75 difference between $7,918.17 and

$8,992.92 over the remaining production life cycle of the product, increasing costs.

(2) Regard the $1,074.75 as a correction of prior year's earnings.

(3) Treat the $1,074.75 an an expense of the current period.

The first alternative is rejected because, in certain situations, it could result in an accounting loss on all subsequent production (suggesting production should be discontinued) when, in fact, income is being earned. This could occur if the difference between marginal revenue and the projected average cost of all anticipated additional production were small. The increase in unit cost caused by amortizing the $1,074.75 could cause reported unit costs to exceed unit revenues. While it is true that an unexpected loss on the entire venture may occur when a products startup is terminated (since the revised anticipated total production costs may now exceed anticipated total revenues), such a loss should not be spread over periods subsequent to the termination. If, after the termination, a review of projected additional costs and revenues reveals that additional revenues will exceed additional costs, accounting reports should show a profit for subsequent operations. The unexpected termination has resulted in a sunk cost. The firm is now in a new situation and should make the accounting entries necessary to have accounting reports reflect this new situation.

The second alternative, regarding the $1,074.75 as a correction of prior years' earnings, might seem proper in those situations where management should have been aware of the startup termination. Such a situation might be indicated when maximum production standards have been set by a previously existing labor contract, or where management is aware of informal production standards.[1]

If the startup termination could not have been anticipated it seems reasonable to have the accounting effects of the termination reported in the year the termination occurred. Accordingly, the $1,074.75 adjustment to the balance in IPP should be reflected in the current year's income statement.[2] For ease of presentation all adjustments to the balance in IPP will be treated as adjustments before arriving at the current year's net income.

Assuming that the G.L.U.B. Co. treated the $1,074.75 as an adjustment to 1970 net income the following journal entry would be made at the end of 1970:

Cost of Goods Sold	$56,090.90	
L-C Termination Adjustment	1,074.75	
Improvement in Production Procedures		$ 5,894.05
Various Cost Accounts		51,271.60

Under the L-C cost allocation model, with an unexpected startup termination at unit 10, the income statements for 1969, 1970, and

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[1]While an adjustment to prior periods' earnings may be desirable from a theoretic point of view, the criteria for prior period adjustments set forth in Accounting Principles Board Opinion No. 9 would rule out their use in this case. See AICPA, Accounting Principles Board Opinion No. 9, "Reporting the Results of Operations," (New York: AICPA, 1966), para 23.

[2]Paragraph 21 of APB #9 would require that adjustments to the balance in IPP be shown as an ordinary income item because of their reoccurring nature.

1971 would be as follows:

G.L.U.B. Co.
Income Statements (L-C)
For the Years Ending December 31, 1969, 1970, 1971

		1969	1970	1971
Sales		$50,000.00	$100,000.00	$50,000.00
Cost of Goods Sold	$27,970.75			
Cost Variance	(163.37)*	26,807.08	56,090.90	28,045.45
Gross Margin		$23,192.92	$ 43,909.10	$21,954.55
Other Expenses		15,000.00	15,000.00	15,000.00
Net Income Before L-C Adjustments		$ 8,192.92	$ 28,909.10	$ 6,954.55
Adjustment Caused by L-C Termination		-0-	1,074.75	-0-
Net Income		$ 8,192.92	$ 27,934.35	$ 6,954.55

*Favorable

Comparing these income figures with those presented earlier in this section shows that the L-C cost allocation model still shows a higher income in 1969 and a lower income in 1970 and 1971 than the income statements prepared using a standard or actual cost system.

INCREASE IN PRODUCTION COSTS:

Closely related to the problem of L-C termination is the problem of changes in hourly or unit production costs. Such changes may be caused by changes in the labor wage rate, the price of raw materials, or the cost of factory overhead.

These changes may be either upward or downward, anticipated or unanticipated. This section will first consider anticipated increases in production costs. Attention will then be given to the problems posed by unanticipated increases in production costs. The next section will deal with anticipated decreases in production costs, and unanticipated decreases in production costs. It should be noted that the only change is in cost. It is assumed that all other characteristics of the L-C cost allocation model including the reduction in material usage, and production time, and the number of units to be produced remain the same. In terms of cumulative average production time the learning curve remains intact.

As in the case of startup termination, an anticipated change in production costs does not present a special problem. It requires the use of special techniques. The necessary calculations can be performed by the computer program listed in Appendix C2.

Assume that G.L.U.B. Co. anticipated a 20 percent increase in the costs of all factors of production[1] starting with unit 11. The cost of the first 10 units is calculated by the use of equation 5-3.

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[1]Only variable costs are included in this example. Fixed costs are written off as a period expense.

$$U = A1_c(X^{C_c} - (X-1)^{C_c}) \quad (5\text{-}3)$$

where:

$A1_c$ = original A_c parameter. See (1-5). 10045.7

C_c = $1-B_c$

B_c = .207572 .

The cost of units 11 through 20 is calculated by the use of equation 5-4.

$$U = A2_c(X^{C_c} - (X-1)^{C_c}) \quad (5\text{-}4)$$

where:

$A2_c$ = $A1_c$ increased by the percent increase in production costs. 10045.7 x 1.20 = 12054.84.

The cost data for selected levels of production with the increase in production costs described above was calculated by the computer program listed in Appendix C2. A partial printout is presented below:

CUMULATIVE PRODUCTION	UNIT COST	TOTAL COST	AVERAGE COST	ACCOUNT BALANCE
5	5828.76	35963.6	7192.73	6713.18
10	4989.33	62288.5	6228.85	3787.63
15	5483.72	90611.5	6040.77	2860.17
20	5156.54	11700.2	5850.09	--

On the basis of this information, assuming no variances occur after the fifth unit, the calculations and journal entries presented on the following page would be made.

	1969	1970	1971
Actual Cost of Units Produced	$35,800.00	$54,647.90	$26,390.50
-Calculated Cost of Units Produced	35,963.67	54,647.90	26,390.50
Cost Variance, Unfavorable, (Favorable)	$ (163.67)	$ -0-	$ -0-
Calculated Cost of Units Produced	$35,963.67	$54,647.90	$26,390.50
-Number of Units Produced x Average Unit Cost	29,250.45	58,500.90	29,250.45
Change in Improvements in Production Procedures	$ 6,713.22*	$(3,853.00)*	$(2,859.95)*

*Rounding errors cause these figures not to total to zero.

1969:

Cost of Goods Sold	$29,250.45	
Improvements in Production Procedures	6,713.22	
Cost Variance-Favorable		$ 163.67
Various Cost Accounts		35,800.00

1970:

Cost of Goods Sold	$58,500.90	
Improvements in Production Procedures		$ 3,853.00
Various Cost Accounts		54,647.90

1971:

Cost of Goods Sold	$28,250.45	
Improvements in Production Procedures		$ 2,859.95
Various Cost Accounts		26,390.50

With the anticipated increase in production costs described above, the 1969, 1970, and 1971 income statements of the G.L.U.B. Co. using the L-C cost allocation model would be as presented on the following page.

G.L.U.B. Co.
Income Statements (L-C)
For the Years Ending December 31, 1969, 1970, 1971

		1969	1970	1971
Sales		$50,000.00	$100,000.00	$50,000.00
Cost of Goods Sold	$29,250.45			
Cost Variance	(163.67)*	29,086.78	58,500.90	29,250.45
Gross Margin		$20,913.22	$ 41,499.10	$20,749.55
Other Expenses		15,000.00	15,000.00	15,000.00
Net Income		$ 5,913.22	$ 26,499.10	$ 5,749.55

*Favorable

If a standard or actual cost allocation model was used the income statements would be:

G.L.U.B. Co.
Income Statements (Std.)
For the Years Ending December 31, 1969, 1970, 1971

	1969	1970	1971
Sales	$50,000.00	$100,000.00	$50,000.00
Cost of Goods Sold	35,800.00	54,657.90	26,390.50
Gross Margin	$14,200.00	$ 45,352.10	$23,609.50
Other Expenses	15,000.00	15,000.00	15,000.00
Net Income (loss)	$ (800.00)	$ 30,352.10	$ 8,609.50

Now assume that the G.L.U.B. Co. did not anticipate the change in production costs which took place during 1970 and, at the end of 1969, made the entry recorded in Chapter III. A comparison of either the actual and anticipated costs of unit number 15, or the actual and anticipated costs of the first 15 units would indicate the possible existence of a change in the parameters of the L-C cost allocation model, since in both cases actual and anticipated costs differ by more than 5 percent.

After it has been determined that the variance is caused by a 20 percent increase in production costs beginning with unit number 6 new cost projections would be made with the use of the computer program listed in Appendix C2. (See the partial printout presented earlier in this section.) An analysis of these cost projections reveals that the projected average cost of all anticipated production has increased from $5,394.15 to $5,850.09, and that the balance in IPP at the end of 1969 should have been $6,713.22 instead of $8,992.92. An adjustment to the balance in IPP will have to be made. Assuming that the 20 percent increase in production costs was an unexpected event, the following journal entry is made at the end of 1970:

Cost of Goods Sold	$58,500.90	
L-C Production Cost Increase Adjustment	2,279.70	
Improvements in Production Procedures		$ 6,132.70
Various Cost Accounts		54,647.90

The G.L.U.B. Co. income statements for the years 1969-1971, based on the L-C cost allocation model and the assumption that the change in production costs is unexpected are presented on the following page.

This set of statements should be compared with those presented earlier in this section. Compared to the statement developed using standard or actual costs 1969 income is higher, and 1970 and 1971 income is lower.

G.L.U.B Co.
Income Statements (L-C)
For the Years Ending December 31, 1969, 1970, 1971

		1969	1970	1971
Sales		$50,000.00	$100,000.00	$50,000.00
Cost of Goods Sold	$26,970.75			
Cost Variance	(163.37)*	26,807.08	58,500.90	29,250.45
Gross Margin		$23,192.92	$ 41,499.10	$20,749.55
Other Expenses		15,000.00	15,000.00	15,000.00
Net Income Before L-C Adjustments		$ 8,192.92	$ 26,499.10	$ 5,749.55
Adjustment Caused by L-C Production Cost Increase		-0-	2,279.70	-0-
Net Income		$ 8,192.92	$ 24,219.40	$ 5,749.55

*Favorable

DECREASE IN PRODUCTION COSTS:

An anticipated decrease in production costs should be handled in the same manner as anticipated increases in production costs are handled. Assume that the G.L.U.B. Co. expected a 20 percent decrease in production costs beginning with unit number 11. Production costs could be projected by the use of the computer program listed in Appendix C2. A partial printout of that program for the above situation is presented below:

CUMULATIVE PRODUCTION	UNIT COST	TOTAL COST	AVERAGE COST	ACCOUNT BALANCE
5	5828.76	35963.6	7192.73	11272.6
10	4989.33	62288.5	6228.85	12906.5
15	3655.81	81170.5	5411.37	7097.49
20	3437.69	98764.1	4938.2	--

On the basis of this information, assuming no variances occur after the fifth unit, the following calculations and journal entries would be made:

	1969	1970	1971
Actual Cost of Units Produced	$35,800.00	$45,206.90	$17,593.60
-Calculated Cost of Units Produced	35,963.67	45,206.90	17,593.60
Cost Variance, Unfavorable, (Favorable)	$ (163.67)	$ -0-	$ -0-
Calculated Cost of Units Produced	$35,963.67	$45,206.90	$17,593.60
-Number of Units Produced x Average Unit Cost	24,691.00	49,382.00	24,691.00
Change in Improvements in Production Procedures	$11,272.67*	$(4,175.10)*	$(7,097.40)*

*Rounding errors cause these figures not to total to zero.

1969:

Cost of Goods Sold	$24,691.00	
Improvements in Production Procedures	11,272.67	
Cost Variance-Favorable		$ 163.67
Various Cost Accounts		35,800.00

1970:

Cost of Goods Sold	$49,382.00	
Improvements in Production Procedures		$ 4,175.10
Various Cost Accounts		45,206.90

1971:

Cost of Goods Sold	$24,691.00	
Improvements in Production Procedures		$ 7,097.40
Various Cost Accounts		17,593.60

Using the L-C cost allocation model the income statements for 1969, 1970, and 1971 would be:

G.L.U.B. Co.
Income Statements (L-C)
For the Years Ending December 31, 1969, 1970, 1971

		1969	1970	1971
Sales		$50,000.00	$100,000.00	$50,000.00
Cost of Goods Sold	$24,691.00			
Cost Variance	(163.67)*	24,527.33	49,382.00	24,691.00
Gross Margin		$25,472.67	$ 50,618.00	$25,309.00
Other Expenses		15,000.00	15,000.00	15,000.00
Net Income		$10,472.67	$ 35,618.00	$10,309.00

*Favorable

If a standard or actual cost allocation model were used the income statements for 1969, 1970, and 1971 would be as presented on the following page.

G.L.U.B. Co.
Income Statements (Std.)
For the Years Ending December 31, 1969, 1970, 1971

	1969	1970	1971
Sales	$50,000.00	$100,000.00	$50,000.00
Cost of Goods Sold	35,800.00	45,206.90	17,593.60
Gross Margin	$14,200.00	$ 54,793.10	$32,406.40
Other Expenses	15,000.00	15,000.00	15,000.00
Net Income (loss)	$ (800.00)	$ 39,793.10	$17,406.40

If the G.L.U.B. Co. did not anticipate the decrease in production costs beginning with unit 11 a comparison of either actual and projected unit costs for unit 15 or actual and total costs of the first 15 units would show variances in excess of 5 percent. Assuming management decides to investigate these variances and determines that production costs decreased 20 percent beginning with unit number 11 a new series of cost projections could be made in a manner similar to those made earlier in this section.

A comparison of this new data with that previously developed shows that the balance in IPP after 5 units were produced should have been $11,272.67, not $8,992.92. G.L.U.B. Co. must therefore determine how to handle the $2,279.75 by which IPP is understated.

Three alternatives are available:

(1) Amortize the $2,279.75 over the remaining production life cycle of the product, decreasing cost.

(2) Treat the $2,279.75 as a correction of prior years' earnings.

(3) Treat the $2,279.75 as an income adjustment of the current period.

The first alternative is rejected because failure to correct the balance in IPP would result in charges to inventory and the cost of goods sold which do not reflect the average unit cost of all anticipated production. Such amortization is a step away from the cost allocation model developed in this research.

From a theoretical standpoint the second alternative seems appropriate when management should have been aware of the impending change in production costs. This alternative will not, however, be developed here because Accounting Principles Board Opinion No. 9 does not regard such action as a generally accepted accounting principle.[1]

In most situations, treating the $2,279.75 as an adjustment in arriving at net income seems appropriate. If the firm's expected profit potential has changed during a given period because of an unexpected event occurring during that period the accounting reports of that period should reflect the change.

Assuming that the G.L.U.B. Co. treated the $2,279.75 as an adjustment to 1970 net income the following journal entry would be made at the end of 1970:

Cost of Goods Sold	$49,382.00	
L-C Production Cost Decrease Adjustment		$ 2,279.75
Improvements in Production Procedures		1,895.35
Various Cost Accounts		45,206.90

[1]APB #9, paragraph 23.

Under the L-C cost allocation model, with an unexpected decrease in production costs starting at unit 11, the income statements for 1969, 1970, 1971 would be as follows:

G.L.U.B. Co.
Income Statements (L-C)
For the Years Ending December 31, 1969, 1970, 1971

		1969	1970	1971
Sales		$50,000.00	$100,000.00	$50,000.00
Cost of Goods Sold	$26,970.75			
Cost Variance	(163.37)*	26,807.08	49,382.00	24,691.00
Gross Margin		$23,192.92	$ 50,618.00	$25,309.00
Other Expenses		15,000.00	15,000.00	15,000.00
Income Before L-C Adjustment		$ 8,192.92	$ 35,618.00	$10,309.00
Adjustment Caused by L-C Production Cost Decrease		-0-	2,279.75	-0-
Net Income		$ 8,192.92	$ 37,897.75	$10,309.00

*Favorable

Comparing these income statements with those presented earlier in this section with a standard or actual cost allocation model shows that the L-C cost allocation model still results in higher incomes in earlier periods and lower incomes in later ones.

CHANGES IN N:

Total anticipated production, N, is likely to change after production begins. A change in N is, by its very nature, unanticipated. N changes as sales estimates become more precise. Unfortunately, a change in N does not automatically bring attention to itself by causing variances between U_{ci} and U_i or T_{ci} and T_i. Nevertheless, a change in N is important to the L-C cost allocation model because the projected average cost of all anticipated production Y_{cN}, is partially determined by N. If a confidence interval of ±5 (or 10) percent is desired for Y_N, it is desirable to determine how sensitive Y_N is to changes in N. If Y_N is relatively insensitive to changes in N, no special problems are caused by changes in N, unless such changes are major.

We previously determined that a ±5 percent confidence interval for Y_N is represented by equation 4-8:

$$.95Y_{cN} \le Y_N \le 1.05Y_{cN} \ .$$

Let N1 and N2 represent the extremes of the interval within which N can vary without changing Y_N by more than ±5 percent. Then, combining (1-1) and (4-8) we have:

$$A_c/N1^{B_c} = .95A_c/N^{B_c} \le A_c/N^{B_c} \le 1.05A_c/N^{B_c} = A_c/N2^{B_c} \qquad (5\text{-}5)$$

Solving for N1 and N2:

$$N1 = \text{antilog}\ ((B_c \log N - \log .95)/B_c) \qquad (5\text{-}6)$$

$$N2 = \text{antilog}\ ((B_c \log N - \log 1.05)/B_c) \qquad (5\text{-}7)$$

The computer program listed in Appendix C3 was used to solve equations 5-6 and 5-7 for the b values corresponding to L-C percents ranging from 70 to 99 at N values of 100; 500; 1,000; 5,000; and 10,000. The results were the same regardless of the N value used. Table 5-1 lists the percent change in N required to change Y_N by ±5 percent. Table 5-2 lists the percent change in N required to change Y_N by ±10 percent.

The first thing that should be obvious from tables 5-1 and 5-2 is that Y_N becomes less sensitive to changes in N as the L-C percent increases (the rate of learning decreases). In other words, the higher the L-C percent is the more the original estimate of N can be in error. For example, if a 5 percent interval for Y_N is desired, and the L-C percent is 90, the actual value of N can be 27.456 percent less than the estimated value before Y_N is off by 5 percent. If the L-C percent is 80, the actual value of N can be 17.273 percent greater than the estimated value of N before Y_N is off by 5 percent. Analysis of tables 5-1 and 5-2 indicates that the model is not so sensitive to N that the probability of changes in this parameter after production began would invalidate the model.

Despite the large leeway possible in estimating N, whenever circumstances indicate that a change in that estimate is necessary, that change should be incorporated into the L-C cost allocation model. This is especially desirable when the revised estimates are less than the previous ones. If this is not done a large balance may remain

TABLE 5-1
Percent Change in N Required to Change Y_N by ±5%

L-C%	b	-5% Y_N %ΔN	+5% Y_N %ΔN
70	.51457	10.482	- 9.046
71	.49410	10.939	- 9.403
72	.47393	11.430	- 9.783
73	.45408	11.960	-10.189
74	.43440	12.533	-10.624
75	.41508	13.155	-11.091
76	.39592	13.832	-11.594
77	.37707	14.572	-12.137
78	.35845	15.384	-12.726
79	.34007	16.280	-13.365
80	.32192	17.273	-14.063
81	.30400	18.380	-14.827
82	.28630	19.621	-15.668
83	.26881	21.023	-16.598
84	.25153	22.620	-17.632
85	.23446	24.454	-18.787
86	.21759	26.584	-20.087
87	.20091	29.085	-21.561
88	.18442	32.065	-23.245
89	.16812	35.675	-25.189
90	.15200	40.137	-27.456
91	.13606	45.788	-30.134
92	.12029	53.173	-33.342
93	.10469	63.218	-37.250
94	.08926	77.643	-42.106
95	.07400	100.000	-48.280
96	.05889	138.917	-56.327
97	.04394	221.818	-67.054
98	.02914	481.159	-81.250
99	.01449	3,338.340	-96.544

TABLE 5-2
Percent Change in N Required to
Change Y_N by ±10%

L-C%	b	-10% Y_N %ΔN	+10% Y_N %ΔN
70	.51457	22.722	-16.908
71	.49410	23.767	-17.543
72	.47393	24.895	-18.218
73	.45408	26.118	-18.935
74	.43440	27.448	-19.700
75	.41503	28.898	-20.518
76	.39592	30.487	-21.394
77	.37707	32.235	-22.335
78	.35845	34.168	-23.348
79	.34007	36.317	-24.441
80	.32192	38.718	-25.625
81	.30400	41.420	-26.912
82	.28630	44.484	-28.315
83	.26881	47.984	-29.851
84	.25153	52.023	-31.539
85	.23446	56.731	-33.402
86	.21759	62.288	-35.469
87	.20091	68.946	-37.773
88	.18442	77.056	-40.357
89	.16812	87.140	-43.272
90	.15200	100.00	-46.582
91	.13606	116.921	-50.365
92	.12029	140.094	-54.720
93	.10469	173.555	-59.761
94	.08926	225.531	-65.62
95	.07400	315.281	-72.416
96	.05889	498.360	-80.177
97	.04394	999.739	-88.570
98	.02914	3,615.000	-96.199
99	.01449	143,114.0	-99.860

in Improvements in Production Procedures when production is completed.

Referring to the example developed in Chapter III, assume that the G.L.U.B. Co. management revised their estimate of total production from 20 to 25 units in 1970. On the basis of an N of 25, the balance in IPP at the end of 1969 should have been \$10,213.70, not \$8,992.92. The increase is caused by a decrease in Y_{cN} from \$5,394.15 to \$5,150.00. Assuming there were no cost variances after the fifth unit the following journal entry would be made at the end of 1970:

Cost of Goods Sold	\$51,500.00	
L-C Increase in N Adjustment		\$ 1,220.78
Improvements in Production Procedures		351.84
Various Cost Accounts		49,927.40*

*Totals contain a two cent rounding error.

If, in 1971, it becomes apparent that 30 units will be produced instead of 25, the balance in IPP after 15 units had been produced should have been \$11,510.00, instead of \$8,641.08. This increase is caused by a decrease in Y_{cN} from \$5,150.00 to \$4,958.74. Assuming production on all 30 units was completed by the end of 1971, the following journal entry would be made at the end of 1971:

Cost of Goods Sold	\$74,281.10	
L-C Increase in N Adjustment		\$ 2,868.92
Improvements in Production Procedures		8,641.08
Various Cost Accounts		62,871.00*

*Totals contain a ten cent rounding error.

Using the L-C cost allocation model, and making the necessary adjustments in 1970 and 1971, the income statements for the G.L.U.B. Co. would be as follows:

G.L.U.B. Co.
Income Statements (L-C)
For the Years Ending December 31, 1969, 1970, 1971

		1969	1970	1971
Sales		$50,000.00	$100,000.00	$150,000.00
Cost of Goods Sold	$26,970.75			
Cost Variance	(163.37)*	26,807.08	51,500.00	74,381.10
Gross Margin		$23,192.92	$ 48,500.00	$ 75,618.90
Other Expenses		15,000.00	15,000.00	15,000.00
Net Income Before L-C Adjustments		$ 8,192.92	$ 33,500.00	$ 60,618.90
Adjustments Caused by L-C Increase in N		-0-	1,220.78	2,868.92
Net Income		$ 8,192.92	$ 34,720.78	$ 63,487.82

*Favorable

If a standard or actual cost allocation model was used the 1969, 1970, and 1971 income statements of the G.L.U.B. Co. would be as follows:

G.L.U.B. Co.
Income Statements (Std.)
For the Years Ending December 31, 1969, 1970, 1971

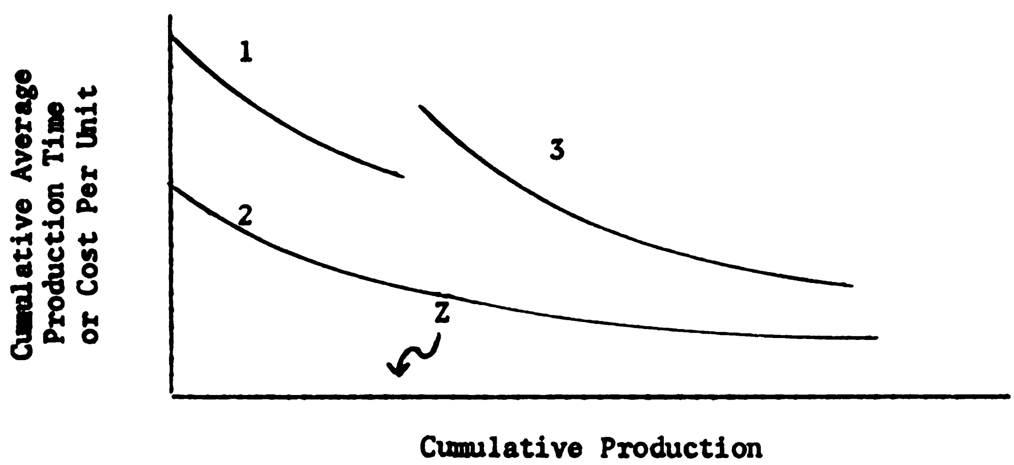
	1969	1970	1971
Sales	$50,000.00	$100,000.00	$150,000.00
Cost of Goods Sold	35,800.00	49,927.40	62,871.00
Gross Margin	$14,200.00	$ 50,072.60	$ 87,129.00
Other Expenses	15,000.00	15,000.00	15,000.00
Net Income (loss)	$ (800.00)	$ 35,072.60	$ 72,129.00

The procedures necessary to handle decreases in N correspond to those necessary to handle increases in N, and therefore do not appear in this chapter. A significant decrease in N, such as that which might be caused by a major buyer's bankruptcy or an economic downturn would presumably fulfill the requirements of APB No. 9 for presentation as an extraordinary loss.

ESSENTIAL MODIFICATIONS:

Changes in either the product or production procedures of other than a minor nature will affect the L-C cost allocation model. If a major change in production procedures takes place after implementation of the L-C cost allocation model, the cumulative average cost curve becomes a composite curve, one part made up of the cumulative average costs of the unchanged portion of the production process, the other part made up of the changed portion of the production process. Figure 5-2 shows the elements of this composite curve in a graphic manner.

FIGURE 5-2
Elements of Composite Curve Caused by Essential Modifications



In figure 5-2, line 1 represents the original cumulative average cost curve. Beyond output level Z, the output level at which the essential modifications took place, this curve is no longer applicable.

Line 2 represents the cumulative average unit cost of the unchanged portion of the production process. Line 3 represents the cumulative average unit cost of the new portion of the production process. It begins at output level Z. For it Z represents unit number 1.

To adequately account for modifications in the product or production process, it is necessary to:

(1) Determine what portion of the original product or production process has not changed;

(2) Determine learning curve parameters for the new portion of the product or production process;

(3) Combine the applicable portions of the original curve and the new curve to assist in determining the projected average cost of all anticipated production, and unit and total production costs at various output levels.

The portion of the original product or production process which has not changed may be determined by a comparison of the current product or production procedures with the original product or production procedures. Projections can then be made of the unchanged unit cost or cumulative average costs at various output levels. Separating costs incurred for the unchanged portion of the product or production process from the total unit or average costs incurred after the change, one can find the L-C parameters of the changed product or production process. First, make a projection of the unit costs associated with the unchanged portion of the product or production process. Second, subtract these unit costs from the unit costs incurred after the change.

The differences are the basic data necessary for calculating the L-C parameters of the changed portion of the product or production process. This data may now be analyzed by the use of equations 1-4 and 1-5 (after it is converted into cumulative average costs) or the computer program listed in Appendix A1.

The cost of units 1 through Z-1 is represented by:

$$U = A_c(X^{C_c} - (X-1)^{C_c}).$$

The cost of units Z through N is:

$$U = P \cdot A_c(X^{C_c} - (X-1)^{C_c}) + A_{2c}((X-Z+1)^{C_{2c}} - (X-Z)^{C_{2c}}) \qquad (5\text{-}8)$$

where:

Z = output level at which the essential modification took place

A_{2c} = calculated a parameter of changed portion of product or production process

$C_{2c} = 1 - B_{2c}$

B_{2c} = calculated b parameter of changed portion of product or production process

P = percent of original product or production process which has not changed

$X = Z.....N.$

Because of the computational difficulties involved in equation 5-8 the computer program listed in Appendix C4 performs all of the calculations necessary to implement the L-C cost allocation model after an essential modification has occurred and A_{2c}, C_{2c}, N, Z, and P are determined.

Assume that after 10 units have been produced, the G.L.U.B. Co. finds it necessary to make major changes in its production procedures in order to prepare for an increased production run of 30 units. From an analysis of engineering reports, it is determined that 50 percent of the original production procedures will remain unchanged. Management decides to change the L-C cost allocation model after sufficient information has been gathered to make a composite model feasible. The following analysis would occur after 5 additional units have been produced.

UNIT NUMBER	(1) ACTUAL COST	(2) ORIGINAL PROJECTION	(3) RETAINED PORTION	(4) NEW PORTION
11	9,458.33	4,886.65	2,443.33	7,015.00
12	7,897.58	4,795.16	2,397.58	5,500.00
13	6,856.41	4,712.82	2,356.41	4,500.00
14	6,069.05	4,638.09	2,319.05	3,750.00
15	5,684.89	4,569.77	2,284.89	3,400.00

where:

Column 1 lists the actual cost of units 11-15.

Column 2 lists the original estimate of the cost of these units.

Column 3 is 50 percent of column 2, the portion of the original process retained.

Column 4 is column 1 less column 3, the costs applicable to the changed production procedures.[1]

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[1]In reality column 4 would contain a variety of things, including inefficiencies. Care would have to be taken to eliminate extraneous costs before an analysis is made. How important such extraneous factors are could be determined by looking at the correlation coefficient. In this example the correlation coefficient is high, indicating that the effect of extraneous factors is minimal and can be ignored.

An analysis of column 4 by the use of the computer program listed in Appendix A1 determines A_{2c} to be 7158.58, and B_{2c} to be .230141. The cost data for selected levels of production with the essential modification indicated above was calculated by the computer program listed in Appendix C4 to be.

CUMULATIVE PRODUCTION	UNIT COST	TOTAL COST	AVERAGE COST	ACCOUNT BALANCE
1	10045.7	10045.7	10045.7	4133.13
5	5828.76	35963.6	7192.73	6400.81
10	4989.33	62288.5	6228.85	3162.89
11	9601.9	71890.5	6535.5	6852.23
15	6185.75	98803.3	6586.89	10114.9
20	5431.65	127225.	6361.25	8973.6
30	4753.64	177377.	5912.57	---

Assuming that the procedures set forth in Chapter III were followed at the end of 1969, the balance in IPP would have been $8,992.92. With the increase in N and the essential modifications in production procedures incorporated into the L-C cost allocation model, the balance in IPP at the end of 1969 should have been $6,400.81. It may or may not be desirable to separate the difference caused by the essential modifications from the difference caused by the change in N when adjustments are made in the accounts. If the two changes were made independently they could be analyzed in their order of occurence. Assuming that the change in production procedures was necessary to attain an N of 30, the following journal entry would be made at the end of 1970:

Cost of Goods Sold	$59,125.70	
Improvements in Production Procedures	1,122.00	
L-C Increase in N and Essential Modifications Adjustment	2,592.19	
Various Cost Accounts		$62,839.70*

*Totals contain a seventeen cent rounding error.

Income statements for the years 1969, 1970, and 1971 using the L-C cost allocation model, and a standard or actual cost allocation model, assuming all variances after unit number 5 are zero, are presented below.

G.L.U.B. Co.
Income Statements (L-C)
For the Years Ending December 31, 1969, 1970, 1971

		1969	1970	1971
Sales		$50,000.00	$100,000.00	$150,000.00
Cost of Goods Sold	$26,970.75			
Cost Variance	(163.37)*	26,807.08	59,125.70	88,688.55
Gross Margin		$23,192.92	$ 40,874.30	$ 61,311.45
Other Expenses		15,000.00	15,000.00	15,000.00
Net Income Before L-C Adjustments		$ 8,192.92	$ 25,874.30	$ 46,311.45
Adjustment Caused by L-C Change in N and Essential Modification		-0-	2,592.19	-0-
Net Income		$ 8,192.92	$ 23,282.11	$ 46,311.45

*Favorable.

G.L.U.B. Co.
Income Statements (Std.)
For the Years Ending December 31, 1969, 1970, 1971

	1969	1970	1971
Sales	$50,000.00	$100,000.00	$150,000.00
Cost of Goods Sold	35,000.00	62,839.70	78,573.70
Gross Margin	$14,200.00	$ 37,160.30	$ 71,426.30
Other Expenses	15,000.00	15,000.00	15,000.00
Net Income (loss)	$ (800.00)	$ 22,160.30	$ 56,426.30

Use of the L-C cost allocation model still results in a higher reported income in 1969 and a lower reported income in 1971 than does use of a standard or actual cost allocation model. However, in 1970 it reports a higher income. This is caused by the L-C model's tendency to defer costs when a startup occurs, and the essential modification places a new startup in the model during 1970.

We have only considered essential modifications as events not planned for when the original L-C cost allocation model and its parameters were adopted. Essential modifications may be anticipated, but it is difficult to incorporate them into the original model because of the unavailability of the data needed for computation of parameters. For purposes of internal reporting and planning it may be desirable to estimate these parameters, but such estimates would usually lack a sufficient level of objectivity to justify their use for external reporting.

CONCLUSION:

There are many other problems which could occur in any attempt to apply the L-C cost allocation model. The basic procedures for handling other problems is the same as those developed to handle the problems considered in this chapter. After some sort of a change in a parameter of the L-C cost allocation model has occurred these changes are incorporated into the formula used to compute unit cost, cumulative average cost, total cost, and the balance in IPP at various output levels. A comparison is then made between the balance in IPP after the last recorded change in the account and the balance which should have been in IPP at that time given the change. IPP is then adjusted to the amount which should have been in it. The amount of the adjustment to IPP is usually treated as an adjustment to current income. After the adjustment is made the model, and its new parameters, are applied in the normal manner.

The solutions to problems caused by changes in parameters of the L-C cost allocation model is limited only by the ability of the problem solver to devise the proper mathematical formula and computer programs. The formulas presented in this chapter and the computer programs presented in the Appendices should serve as a guide.

CHAPTER VI

APPLICATION

INTRODUCTION:

This chapter concerns itself with an application of the L-C cost allocation model. The purpose of this application is to get an additioanl prospective of the model's operational characteristics and limitations.

The primary problem involved in testing the L-C cost allocation model was obtaining the appropriate data. Most firms contacted by this researcher did not maintain sufficient detail in their production records. The model requires consecutive production cost data for individual or small groups of units. Firms who appeared to have the appropriate information, for a variety of reasons, did not deem it advisable to let it be used. At least one firm that appeared to have the appropriate information and declined to let it be used, expressed interest in this research. They implied that their financial statements were influenced by the failure to consider the L-C phenomenon in their cost allocation procedures. One firm that did not have the appropriate data, but felt that the L-C phenomenon was important to them, invited the researcher to assist in setting up a system which would provide the data. Finally, with the assistance of a national accounting firm, the data required to test the model was obtained.

THE DATA:

The model was applied to a production venture undertaken by a midwest tool manufacturer. In this production venture 3,202 units were produced over a 19 month period. Table 6-1 lists the monthly production data, and the cumulative average labor cost, the cumulative average materials cost, and the cumulative average total cost of production as of the end of each month. The sequence of months and production cost data was supplied by the manufacturer. The production time period has been disguised by changing the starting date. Information as to the number of units sold in each period, the selling price of each unit, or other company costs was not available for this research. It is therefore assumed that all units were sold in the period produced for $600 each. The analysis will focus on the difference between the gross margin and the gross margin percent which would have been reported with the use of the L-C and the actual (or standard) cost allocation model during each half year of the total production period.

An inspection of table 6-1 indicates that labor costs and total costs were greatly influenced by the L-C phenomenon. Materials costs decreased for a while but then began to rise, apparently due to increases in the cost of materials purchased. This indicates that, in this case, the L-C cost allocation model might be more accurately applied to labor costs than to total costs. Accordingly, three sets of income statements are developed. The first is for those which would result from the use of actual or standard costs. The second is for those which would result from applying the L-C cost allocation model to total costs. The third is for those which would result from applying the L-C cost allocation model to labor costs alone.

TABLE 6-1
Data for Application of L-C
Cost Allocation Model

PERIOD NUMBER	PERIOD	CUMULATIVE PRODUCTION	CUM. AVG. LABOR COST	CUM. AVG. MATERIAL COST	CUM. AVG. TOTAL COST
1	Aug. 19x1	36	392.00	248	640.00
2	Sept.	131	322.00	235	557.00
3	Oct.	347	301.00	230	531.00
4	Nov.	499	294.00	230	524.00
5	Dec.	671	280.00	235	515.00
6	Jan. 19x2	868	273.00	230	503.00
7	Feb.	1123	259.00	230	489.00
8	Mar.	1322	252.00	230	482.00
9	Apr.	1543	254.45	230	484.45
10	May	1819	247.52	230	477.52
11	June	2019	243.81	235	478.81
12	Jul.	2219	241.36	235	476.36
13	Aug.	2386	239.33	240	479.33
14	Sept.	2425	238.84	240	478.84
15	Oct.	2643	235.90	240	475.90
16	Nov.	2795	233.94	240	473.94
17	Dec.	2917	231.98	240	471.98
18	Jan. 19x3	3119	229.67	245	474.64
19	Feb.	3202	228.41	245	473.41

ACTUAL (STANDARD) COST MODEL:

Assuming that the selling price is $600 per unit, the income statements presented on the following page are developed applying an actual (or standard) cost allocation model to the data presented in table 6-1.

As is typical of income statements developed with actual costs, when the L-C phenomenon is present, the gross margin and gross margin percent has a tendency to increase over time. In this case the tendency to increase is offset in the final period by the increase in materials cost. However, labor costs take up a continually decreasing portion of the sales dollar, decreasing from 46.4 percent to 31.9 percent.

Application
Income Statements (Std.)
For the 6-Month Periods Ending

	12/31/x1		6/30/x2		12/31/x2		6/30/x3	
	AMOUNT	%	AMOUNT	%	AMOUNT	%	AMOUNT	%
Sales	$402,600.00	100.0	$808,800.00	100.0	$538,800.00	100.0	$171,000.00	100.0
Cost of Goods Sold:								
Labor	$187,880.00	46.6	$304,372.39	37.6	$184,433.27	34.2	$ 54,683.16	31.9
Materials	157,685.00	39.1	316,780.00	39.1	225,615.00	41.8	84,410.00	49.4
Total	$345,565.00	85.8	$621,152.39	76.7	$410,048.27	76.1	$139,093.16	81.3
Gross Margin	$ 57,035.00	14.1	$187,647.61	23.2	$128,751.73	23.8	$ 31,906.84	18.6

L-C MODEL APPLIED TO TOTAL COST CURVE:

In December of 19x1, an evaluation of the relation between cumulative production, X, and cumulative average cost of each unit produced for each of the 5 production periods, n, in 19x1 determined A_c to be 815.612, B_c to be .0723382, and R to be -.976393. For the purpose of simplification, and because the original estimate is not known, it is assumed that total anticipated production, N, was estimated to be 3,202 units. Table 4-1, the "Percent Confidence Interval for b Required to Obtain A ±5% Interval for Y_N" indicates that the b confidence interval required to project a ±5 percent confidence interval for $Y_{5,000}$ when B_c is .0723382 is 7.741. For $Y_{1,000}$ the b confidence interval is 9.545. Because the estimate of N is 3,202, and the closest N in the table is 5,000, we can interpolate for the difference in the b confidence interval required for N = 1,000 and for N = 5,000 in order to find the approximate b confidence interval for N = 3,202. At N = 1,000 the b interval is 9.545. At N = 5,000 the b interval is 7.741. Thus:

$$\frac{(9.545 - 7.741)}{5,000 - 1,000} \times (3,202 - 1,000) = .9922$$

$$7.741 + .9922 = 8.7332$$.[1]

When B_c = .0723382 and N = 3,202 the percent confidence interval for b required to obtain a ±5 percent confidence interval for $Y_{3,202}$ is 8.7332.

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[1]Interpolation yields a slightly tighter confidence interval than would be obtained by solving equation 4-11. If application of the L-C cost allocation model is contemplated it would be desirable to expand all of the tables presented in Chapter IV and develop them in more detail. This is easily done by use of the computer programs listed in the Appendices.

Table 4-7 indicates that an n of 15 is required to obtain a 8.5 percent confidence interval for b when R is <.9772 and ≥.9751. But, in order to be of value the model should be implemented at this time because the greatest impact of the L-C cost allocation model is usually in the first accounting period. In general, three courses of action are available:

(1) Increase the confidence interval for Y_N from ±5 percent to ±10 percent.

(2) Break down the available data into smaller production groups or time periods in order to increase n, and then find A_c, B_c, and R for the new n.

(3) Decide not to apply the L-C cost allocation model.

Because the second course of action listed above was not available due to the lack of the raw data used to calculate table 6-1, a fourth option was added.

(4) Determine whether or not R was increasing over the production life cycle of the product. If R was increasing and B_c was not showing any drastic change the model could be provisionally implemented on the assumption that R would continue to increase and B_c would not vary by a significant percent.

Because of the limitations of the tables used in this research the first course of action is rejected. Table 4-2, for a ±10 percent confidence interval for Y_N, shows that the b interval for N = 1,000 is 18.645 percent, and the b interval for N = 5,000 is 15.121 percent. These values are out of the range of tables 4-7, 4-8, and 4-9. If a complete set of tables for implementing the L-C cost allocation model were developed, tables 4-7, 4-8, and 4-9 could be easily expanded to include these b confidence intervals.

Because the raw data used to develop table 6-1 is not available the second course of action is not available.

The fourth alternative was chosen because R increased over the production life cycle of this product.

Using the computer program listed in Appendix A4, the average cost of all anticipated production is determined to be \$454.891, the total cost of the first 671 units is computed to be \$341,764.00, and the proper balance in Improvements in Production Procedures is calculated to be \$36,531.60 after 671 units are produced. The actual cost of the first 671 units was \$345,565.00.

Using this data the following analysis and journal entry are made for the six-month period ending 12/31/x1:

Actual Cost of Units Produced	\$345,565.00
-Calculated Cost of Units Produced	341,764.00
Cost Variance - Unfavorable (Favorable)	\$ 3,801.00
Calculated Cost of Units Produced	\$341,764.00
-Number of Units Produced x Average Unit Cost	305,232.40
Change in Improvements in Production Procedures	\$ 36,531.60

Cost of Goods Sold	\$306,232.40	
Improvements in Production Procedures	36,531.60	
Cost Variance - Unfavorable	3,801.00	
Various Cost Accounts		\$345,565.00

The income statement for the 6-month period ending 12/31/x1, using the L-C cost allocation model, is:

Application - Total Cost
Income Statement (L-C)
For the 6-Month Period Ending 12/31/x1

		AMOUNT	%
Sales		$402,600.00	100.0
Cost of Goods Sold	$305,232.40		
Cost Variance	3,801.00*	309,033.40	76.7
Gross Margin		$ 93,566.60	23.3

*Favorable

For the 9-month period ending 4/30/x2 another analysis of cumulative production, X, and cumulative average cost, Y, is made. This analysis is made to determine A_c, B_c, and R on the basis of data for the previous 9 months (n = 9). The new values of A_c, B_c, and R are 813.486, .0717945, and -.989272. In order to obtain a ±5 percent confidence interval for $Y_{3,202}$ with an 80 percent confidence level 9 units of data are needed when R is <.9897, and ≥.9876. Therefore these new parameters can be used to predict Y_N with a ±5 percent confidence interval and an 80 percent confidence level. Using the computer program listed in Appendix A4 the data necessary to implement the L-C cost allocation model is determined to be:

CUMULATIVE PRODUCTION	UNIT COST	TOTAL COST	AVERAGE COST	ACCOUNT BALANCE
671	473.275	342,081	509.808	36,305.9
2019	437.129	951,033	471.042	30,972.2
2917	426.206	1,338,210	458.761	8,927
3202	423.028	1,459,150	455.701	--

This additional information indicates that the balance in IPP after 671 units had been produced should have been $36,531.60. Because the model was provisionally implemented due to the inability to get more detailed information and the assumption that R would rise as n increased, an adjustment must be made for the $225.70 difference between $36,531.60 and $36,305.90. Once the adjustment for this change is made, the L-C cost allocation model may be implemented in the normal manner. Using the above data, and the actual production costs shown in the income statements prepared by the use of a standard or actual cost allocation model, the following calculations are made for the periods ending 6/30/x2, 12/31/x2, and 6/30/x3:

	6/30/x2	12/31/x2	6/30/x3
Actual Cost of Units Produced	$621,152.39	$410,048.27	$139,093.16
-Calculated Cost of Units Produced	608,952.00	387,177.00	120,940.00
Cost Variance-Unfavorable (Favorable)	$ 12,200.39	$ 22,871.27	$ 18,153.16
Calculated Cost of Units Produced	$608,952.00	$387,177.00	$120,940.00
-Number of Units Produced x Average Unit Cost	614,285.70	409,222.20	129,867.00
Change in Improvements in Production Procedures	-$ 5,333.70	-$ 22,045.20	-$ 8,927.00
L-C Adjustment	- 225.70*		
Net Change in IPP	-$ 5,559.40	-$ 22,045.20	-$ 8,927.00

*Overstatement of balance in IPP at end of previous period.

The income statements for the 6-month periods ending 12/31/x1, 6/30/x2, 12/31/x2, and 6/30/x3, using the L-C cost allocation model are presented on the following page.

Comparing the gross margin percents in this set of income statements with those presented in the previous set shows that the

Application - Total Cost
Income Statements (L-C)
For the 6-Month Periods Ending

	12/31/x1 AMOUNT	%	6/30/x2 AMOUNT	%	12/31/x2 AMOUNT	%	6/30/x3 AMOUNT	%
Sales	$402,600.00	100.0	$808,800.00	100.0	$538,800.00	100.0	$171,000.00	100.0
Cost of Goods Sold	$305,232.40	75.8	$614,285.70	75.9	$409,222.20	75.9	$129,867.00	75.9
Cost Variance	3,801.00*	.9	12,200.39*	1.5	22,871.27*	4.2	18,153.16*	10.6
Total	$309,033.40	76.7	$626,486.09	77.4	$432,093.47	80.1	$148,020.16	86.5
Gross Margin Before L-C Adjustment	$ 93,566.60	23.3	$182,313.91	22.5	$106,706.53	19.8	$ 22,979.84	13.4
L-C Adjustment			- 225.70**					
Gross Margin	$ 93,566.60	23.3	$182,088.21	22.5	$106,706.53	19.8	$ 22,979.84	13.4

*Unfavorable

**L-C adjustment placed before rather than after gross margin because sufficient information is not available to complete the income statement.

gross margins reported with the L-C cost allocation model are higher in the first period and lower in subsequent periods than those reported with the actual or standard cost allocation model.

If variable costing is used the following points can be made about the L-C cost allocation model:

(1) In the absence of large variances or a change in a parameter the L-C cost allocation model will always report a higher gross margin in earlier periods and a lower gross margin in later periods than an actual or standard cost allocation model.

(2) In the absence of variances, or changes in a parameter of the L-C cost allocation model or the sales price, the L-C cost allocation model will report a constant gross margin percent in all periods and a constant percent for the cost of goods sold.

(3) If it is only applied to one element of the cost of goods sold the gross margin percent will vary, but the percentage cost to which it is applied will remain constant.

In the example presented above the unadjusted cost of goods sold is the same for all periods after the first (which used a slightly different B_c parameter). In this example the cost variance varied from a low of .9 percent of sales in the first period to a high of 10.6 percent of sales in the last. From the information presented in table 6-1 this appears to be caused by the high level of variability in the cost of materials. In the next section, when the L-C cost allocation model is only applied to labor costs, the highest cost variance is 2.5 percent of sales.

L-C MODEL APPLIED TO LABOR COST CURVE:

When a separate analysis is made of the materials and labor cost data contained in table 6-1 it soon becomes evident that the L-C cost allocation model could be implemented with greater accuracy to the labor cost curve than to either the total cost curve or the materials cost curve. Table 6-2 compares A_c, B_c, L-C percent, and R for each of these curves at 2 values of n.

TABLE 6-2
Comparison of Total, Labor, and Materials Cost Data

	n	A_c	B_c	L-C%	R
Total Cost Data	5	815.612	.0723382	95.10	-.976392
	10	811.186	.071246	95.18	-.98834
Labor Cost Data	5	567.757	.108667	92.74	-.983418
	10	581.089	.113462	92.43	-.992478
Material Cost Data	5	265.294	.0220332	94.48	-.823883
	10	258.452	.0166742	98.85	-.82267

For the materials cost data the value of R is and remains so low that the L-C cost allocation model should not be implemented for the materials cost alone. When the materials cost data is combined with the labor cost data the resulting total cost data has a sufficiently high R to permit implementation. Taken alone, the labor cost data has the highest values of R. Hence, it seems advisable to see what the results would be if the L-C cost allocation model is applied to the labor costs alone, with the actual materials costs written off in accordance with a standard or actual cost model. It should be noted that the relationships indicated in table 6-2 will not always occur. That is, a low R for one component of the total cost curve will not always offset a higher R for another component of the total cost curve and lower R for the total cost curve. There may be situations where two or more R values for the component cost curves will result in a higher R value for the total cost curve when the component curves are combined. This would happen if the variances in the component cost data offset each other when combined.

If, at the end of 19x1, the L-C cost allocation model is applied to labor cost data alone, a b confidence interval of 10.688 percent is required to insure that at $N = 5{,}000$ the actual value of Y_N will not differ from the computed value of Y_N by more than 10 percent. Table 4-7 indicates that an n of 9 is required to be 80 percent confident that Y_{cN} will not vary from Y_N by more than 10 percent.[1] Because R for the labor cost curve increases, and a higher R can be obtained, the model is implemented. If the raw data, from which table 6-1 was computed, were available the data for the first 5 months of production could be broken down to increase n without waiting for additional production periods.

Using the computer program listed in Appendix A4, the average cost of all anticipated production is determined to be $236.174, the total cost of the first 671 units is computed to be $187,808.00, and the balance in Improvements in Production Procedures should be $29,332.40

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[1]Additional refinement, which could be obtained by the use of interpolation, would be of little value here because the maximum confidence interval for b in tables 4-7, 4-8, and 4-9 is 10 percent.

after 671 units are produced. The actual labor cost of the first 671 units is $187,880.00.

Using this data the following analysis and journal entry are made for the period ending 12/31/x1:

Actual Labor Cost of Units Produced	$187,880.00
-Calculated Labor Cost of Units Produced	187,808.00
Cost Variance - Unfavorable (Favorable)	$ (72.00)

Calculated Labor Cost of Units Produced	$187,808.00
-Number of Units Produced x Average Labor Unit Cost	158,475.60
Change in Improvements in Production Procedures	$ 29,332.40

Labor Cost of Goods Sold	$158,475.60	
Improvements in Production Procedures	29,332.40	
Cost Variance - Unfavorable	72.00	
Labor Cost Accounts		$187,880.00

The income statement for the 6-month period ending 12/31/x1 using the L-C cost allocation model to determine labor costs is:

Application - Labor Cost
Income Statement (L-C)
For the 6-Month Period Ending 12/31/x1

		AMOUNT	%
Sales		$402,600.00	100.0
Cost of Goods Sold:			
Labor	$158,475.60		39.3
Materials	157,685.00		39.1
Cost Variance	72.00*	316,232.60	78.5
Gross Margin		$ 86,367.40	21.4

*Unfavorable

For the 9-month period ending 4/30/x2 another analysis of cumulative production, X, and cumulative average labor cost, Y, at the end of each month is made. This analysis is made to determine A_c, B_c, and

R on the basis of data for the previous 9 months (n = 9). The new values of A_c, B_c, and R are 580.458, .113253, and -.991038. In order to obtain a ±10 percent confidence interval for average labor cost at N = 3,202 with an 80 percent confidence interval, 7 units of data[1] are needed when R is -.991038. Because this is less than the number of units of data used, the confidence level is higher and the confidence interval is tighter than that previously specified. By the use of the computer program listed in Appendix A4 the data necessary to implement the L-C cost allocation model for labor costs is determined to be:

CUMULATIVE PRODUCTION	UNIT COST	TOTAL COST	AVERAGE COST	ACCOUNT BALANCE
671	246.298	186362	277.738	30229.7
2019	217.105	494983	245.162	25188.6
2917	208.602	685950	235.156	7203.69
3202	206.76	745062	232.686	--

This new information indicates that the balance in IPP after 671 units had been produced should have been \$30,229.70 instead of \$29,332.40. The \$897.30 difference between \$30,229.70 and \$29,332.40 is treated as an adjustment to gross profit in the second period. Using the above data, and the actual production costs shown in the income statements

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[1]For B_c = .113253, rounded to the nearest value in table 4-1, .12029, the $\underline{b}$ confidence interval for N = 1,000 is 11.469, and the b confidence interval for N = 5,000 is 9.302. Interpolating for $\overline{N}$ = 3,202

$$\frac{11.469 - 9.302}{5,000 - 1,000} \times (3,202 - 1,000) = 1.192$$

$$9.302 + 1.192 = 10.492$$

rounded to nearest figure in table 4-7, 10.0 percent.

prepared by the use of a standard or actual cost allocation model, the following calculations are made for the periods ending 6/30/x2, 12/31/x2, and 6/30/x3:

	6/30/x2	12/31/x2	6/30/x3
Actual Cost of Units Produced	$304,372.39	$184,433.27	$54,683.16
-Calculated Cost of Units Produced	308,621.00	190,967.00	59,112.00
Cost Variance-Unfavorable (Favorable)	$ (4,248.61)	$ (6,533.73)	$(4,428.84)
Calculated Cost of Units Produced	$308,621.00	$190,967.00	$59,112.00
-Number of Units Produced x Average Unit Cost	313,662.10	208,951.91	66,315.69
Change in Improvements in Production Procedures	-$ 5,041.10	-$ 17,984.91	-$ 7,203.69
L-C Adjustment	897.30*		
Net Change in IPP	-$ 4,143.80	-$ 17,984.91	-$ 7,203.69

*Understatement of balance in IPP at end of previous period.

The income statements for the 6-month periods ending 12/31/x1, 6/30/x2, 12/31/x2, and 6/30/x3, using the L-C cost allocation model for the determination of labor costs are presented on the next page.

In this set of statements the labor costs in all periods except the first is 39.7 percent of sales. The labor cost for the first period is 39.3 percent of sales. This difference is caused by the determination of a new B_c parameter for labor costs during the second. The variance between actual and calculated labor costs varies from a low of less than .05 percent of sales to a high of 2.5 percent of sales. This is considerably better than the low of .9 percent of sales and the high of 10.6 percent of sales which occurred when the L-C cost allocation model was applied to total production costs. The difference is caused by excluding the cost of materials from the L-C cost allocation

Application - Labor Cost
Income Statements (L-C)
For the 6-Month Periods Ending

	12/31/x1 AMOUNT	%	6/30/x2 AMOUNT	%	12/31/x2 AMOUNT	%	6/30/x3 AMOUNT	%
Sales	$402,600.00	100.0	$808,800.00	100.0	$538,800.00	100.0	$171,000.00	100.0
Cost of Goods Sold:								
Labor	$158,475.60	39.3	$313,662.10	38.7	$208,951.91	38.7	$ 66,315.69	38.7
Materials	157,685.00	39.1	316,780.00	39.1	225,615.00	41.8	84,410.00	49.3
Cost Variance	72.00*	.05	- 4,248.61**	.5	- 6,533.73**	1.2	- 4,428.84**	2.5
Total	$316,232.00	78.5	$626,193.49	77.4	$428,033.18	79.4	$146,296.85	85.5
Gross Margin Before L-C Adjustment	$ 86,367.40	21.4	$182,606.51	22.5	$110,766.82	20.5	$ 24,703.16	14.4
L-C Adjustment			897.30	.1				
Gross Margin	$ 86,367.40	21.4	$183,503.81	22.6	$110,766.82	20.5	$ 24,703.16	14.4

*Unfavorable

**Favorable

model. Because of the rising materials cost the gross margin percent fell over the production life cycle. The gross margin percents obtained by applying the model to labor costs does not vary by a significant amount from those obtained by applying the model to total costs. The greatest difference is 1.9 percent of sales. The preference for applying the model to labor costs alone has to rest on the reduction of variances, not on the change in gross margin percent. The model had a higher L-C percent when it was applied to total costs than it had when applied to labor costs because of the variability in materials costs.

Tables 4-1 and 4-2 show that the lower the L-C percent the tighter the confidence interval for $\underline{b}$ must be. This required an increase in the desired confidence interval for $\underline{b}$ (from 5 to 10 percent) when the model was applied to labor costs alone. In some situations it may be necessary to apply the model to a group of items which have a higher L-C percent. The increase in R, obtained by applying the model to data which more closely follows the learning curve, might not be enough to offset the higher value of R required to apply the model to the tighter confidence intervals for $\underline{b}$ necessitated by low L-C percents.

In both situations presented, the L-C cost allocation model resulted in a higher gross profit in the first period and a lower gross profit in the last 2 periods than did the use of an actual (or standard) cost allocation model.

CHAPTER VII

CONCLUSION

SUMMARY:

For most firms in most industries the production costs of a product are higher when that product is first introduced than they are after that product has been produced for a period of time. A graphic representation of the decrease in production costs as total production increases has been referred to as a learning curve. This research effort was devoted to developing a cost allocation model based on the learning curve phenomenon.

Current accounting techniques of cost allocation take a segmented view of the production life cycle of a product and charge inventory or the cost of goods sold in each period on the basis of actual production costs of that period. A standard cost allocation model usually charges the cost of goods sold with any excess of the actual costs of production over the standard costs of production. The result is a relatively low level of reported income in early periods when production costs are high and a relatively high level of reported income in later periods when production costs are lower.

The L-C cost allocation model developed in this research takes the entire production life cycle of a product into consideration and attempts to reconcile the timing differences between this period and the normal accounting period. The effect of using the L-C cost allocation model is to decrease the early period production costs charged to inventory and the cost of goods sold from their current levels, thus raising reported income, and to increase the later period production costs charged to inventory and the cost of goods sold, thus lowering reported income. These changes in reported income are accomplished by adopting production cost standards based on the learning curve phenomenon and using a cost equalization account to insure that, as production takes place, charges to inventory and the cost of goods sold are equal to standard unit costs based on the average cost of all anticipated production. As long as actual production costs proceed in accordance with the learning curve phenomenon, the reported cost of each unit is the same. If actual production costs differ from those projected by the learning curve, a period cost variance is recognized or the model is changed.

The primary difference between the L-C cost allocation model and most other cost allocation models is that they are concerned with matching costs to units while the L-C cost allocation model is concerned with matching costs to production ventures on the basis of the percentage of completion of the production venture.

In addition to developing the basic cost allocation model in Chapter III, a discussion of the significance of the model was presented in Chapter II. Chapter IV reviewed the accounting concept of materiality and evaluated the L-C cost allocation model in the light of this concept in order to determine certain statistical properties of cost data required before the model should be implemented for external reporting. Chapter V referred to a number of problems which can arise after the model has been implemented and mentioned some ways of handling these problems. Finally, in Chapter VI, the results of an application of the model to a production venture were presented.

The major points considered in Chapter II included a brief review of the matching concept. It was noted that despite the accountants' desire to match costs with revenues, current procedures attach costs to units. In industries which do not display the L-C phenomenon such a procedure leads to meaningful results. But, in industries which display the L-C phenomenon such a procedure may lead to artificial variability in reported earnings. It was further pointed out that this artificial variability in reported earnings may in turn lead to a lower P/E ratio for firms displaying the L-C phenomenon than their economic position justifies. It was also noted that because earnings are lower in early periods than in later periods for firms displaying the L-C phenomenon investors are apt to erroneously evaluate the firm and its management when management is undertaking a project which will eventually prove very profitable. It was concluded that a cost allocation model based on the L-C phenomenon could help eliminate these problems by taking a long term prospective of projects undertaken by management.

In Chapter IV, based on a review of the accounting concept of materiality, it was concluded that before the L-C cost allocation model is implemented for external reporting the model should be able to predict the average cost of all anticipated production with a ±10 percent confidence interval and an 80 percent confidence level. Procedures were developed, and tables presented, to assist in the implementation of the model with the desired degree of accuracy.

Chapter V was devoted to a consideration of some of the problems which can occur after the L-C cost allocation model is implemented. The problems considered included termination of the learning curve, changes in production cost, changes in the number of units to be produced, and essential modifications in the product or production process. It was noted that the model is fairly insensitive to changes in the number of units to be produced. The general solution to the above problems involved the computation of a new value for the average cost of all anticipated production and an adjustment to the cost equalization account, Improvements in Production Procedures.

In the application presented in Chapter VI gross margin was used as a surrogate for net income. In this application the L-C cost allocation model achieved its objective of raising gross margin in the first period while lowering it in the last. The L-C cost allocation model was applied to both total costs and labor costs. This was done because of the high level of variability in materials cost. Applying the model to labor costs alone resulted in a significantly lower variance between the costs projected by the model and the actual costs incurred.

CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER RESEARCH:

The L-C cost allocation model developed in this research could be a valuable tool in attempting to reconcile the differences between the accounting period and the production life cycle of a product. The L-C cost allocation model is able to allocate production costs over the entire production life cycle of a product while still retaining the accounting concepts of "cost" and "objectivity."

The primary impediment to the adoption of the L-C cost allocation model appears to be a lack of detailed production information. However, once a decision has been made to adopt the model, the additional information could probably be generated with relatively little cost as a part of the normal accounting process.

This research centered around the cumulative average time learning curve. Developing a cost allocation model based on the unit time learning curve might be of value. This latter version of the L-C cost allocation model could then be used whenever the correlation coefficient attainable with the unit time curve was higher than the correlation coefficient attainable with the cumulative average time learning curve.

In many ways this research is just a beginning step in attempts to incorporate dynamic production cost data into accounting reports. Frequently other curves or equations may relate more closely to the actual change in costs than the learning curve. There are many research topics in this area. "The study of dynamic data should not be constrained by the learning-curve 'theory' specifications."[1] The important fact is that dynamic data can be quantified and result in more meaningful accounting reports.

[1]Yezdi K. Bhada, "Dynamic Cost Analysis," Management Accounting, (July, 1970), p. 14.

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A P P E N D I C E S

APPENDIX A

Programs Used to Implement the L-C Cost Allocation Model

APPENDIX A1

Determination of Parameters With Unit Cost Data

Purpose: To determine A_c, B_c, L-C percent, and R from unit production time or cost data.

Requirements: Consecutive unit production time or cost data beginning with the first unit of production.

Input Data Format: As many DATA statements as required beginning with DATA statement number 500.

```
500  DATA  n, U1,
501  DATA  U1, U2, ... ,
502  DATA  ... , Un-1, Un
```

Output Data Format:

```
ACTUAL VALUE OF A = U1
COMPUTED VALUE OF A = Ac
REGRESSION SLOPE COEFFICIENT B = Bc
LEARNING CURVE PERCENT = L-C%
COEFFICIENT OF CORRELATION = R
COEFFICIENT OF DETERMINATION = R^2
```

Example: For the example presented in Chapter III the input data would be:

```
500  DATA  5, 10000,
501  DATA  10000, 7500, 6600, 6000, 5700
```

The output would be:

```
ACTUAL VALUE OF A = 10000
COMPUTED VALUE OF A = 10045.7
REGRESSION SLOPE COEFFICIENT B = .207572
LEARNING CURVE PERCENT = 86.5994
COEFFICIENT OF CORRELATION = -.99931
COEFFICIENT OF DETERMINATION = .99862
```

Mathematics: Based on equations 1-2, 1-3, 1-4, 1-5, 1-6, and the relationship:

$$L\text{-}C\% = 100 \ \frac{(A_c/2^{B_c})}{A_c}$$

R and R^2 is for the log-log slope.

LISTING OF A1

```
100  READ N,A
110  LET C=0
120  LET D=0
130  LET E=0
140  LET F=0
150  LET G=0
160  LET H=0
170  FOR X=1 TO N
180  READ U
190  LET C=C+U
200  LET Y=C/X
210  LET D=D+LOG(X)
220  LET E=E+LOG(Y)
230  LET F=F+LOG(X)*LOG(Y)
240  LET G=G+(LOG(X))↑2
250  LET H=H+(LOG(Y))↑2
260  NEXT X
270  LET B=(N*F-D*E)/(N*G-D↑2)
280  LET A1=(E/N)-(B*(D/N))
290  LET B=-B
300  LET A1=EXP(A1)
310  LET L1=100*(A1/(2↑B))/A1
320  LET R=(N*F-D*E)/(SQR(N*G-D↑2)*SQR(N*H-E↑2))
330  LET R2=R↑2
340  PRINT "ACTUAL VALUE OF A =" A
350  PRINT "COMPUTED VALUE OF A =" A1
360  PRINT "REGRESSION SLOPE COEFFICIENT B =" B
370  PRINT "LEARNING CURVE PERCENT =" L1
380  PRINT "COEFFICIENT OF CORRELATION =" R
390  PRINT "COEFFICIENT OF DETERMINATION =" R2
9999  END
```

APPENDIX A2

Determination of Parameters With Cumulative Average Cost Data

Purpose: To determine A_c, B_c, L-C percent, and R from cumulative average production time or cost data.

Requirements: A sequence of output levels and the cumulative average production time or cost of all output to each of the listed levels. Consecutive data is not required.

Input Data Format: As many DATA statements as required beginning with DATA statement number 500.

500 DATA n, X_1, Y_1,
501 DATA X_2, Y_2, ... ,
502 DATA ... , X_{n-1}, Y_{n-1}, X_n, Y_n

Output Data Format:

COMPUTED VALUE OF A = A_c
REGRESSION SLOPE COEFFICIENT B = B_c
LEARNING CURVE PERCENT = L-C%
COEFFICIENT OF CORRELATION = R
COEFFICIENT OF DETERMINATION = R^2

Example: For the original application to the total cost curve presented in Chapter VI the input data would be:

```
500 DATA 5, 36, 640, 131, 557,
501 DATA 347, 531, 499, 524, 671, 515
```

The output would be:

```
COMPUTED VALUE OF A = 815.612
REGRESSION SLOPE COEFFICIENT B = .0723382
LEARNING CURVE PERCENT = 95.1095
COEFFICIENT OF CORRELATION = -.976393
COEFFICIENT OF DETERMINATION = .953343
```

Mathematics: Based on equations 1-2, 1-3, 1-4, 1-5, 1-6, and the relationship:

$$L\text{-}C\% = 100\ \frac{(A_c/2^{B_c})}{A_c}$$

R and R^2 is for the log-log slope.

Comments: The essential difference between A1 and A2 is the presence of statements 170 through 200 in A1 which converts unit data into cumulative average data. In A2 the data is already in this form.

Similar programs could easily be devised for other types of data inputs such as total cost data.

LISTING OF A2

```
100  READ N
110  LET D=0
120  LET E=0
130  LET F=0
140  LET G=0
150  LET H=0
160  FOR I=1 TO N
170  READ X,Y
```

The remainder of A2 is the same as A1 except that lines 180, 190, 200, and 340 are omitted.

APPENDIX A3

Determination of Units Which Can Be Produced With Given Funds or Time

Purpose: To determine the number of units which can be produced within a given time period or with a specified amount of funds.

Requirements: Values of A_c, B_c, and an estimate of the available time period if A_c is in terms of time. An estimate of the available funds is required if A_c is in terms of money.

Input Data Format: One DATA statement is required for line 500.

500 DATA A_c, B_c, T

where:

T = total available time or funds

Output Data Format:

```
FOR A B VALUE OF B GIVEN T HOURS OF AVAILABLE
PRODUCTION TIME X UNITS CAN BE PRODUCED
```

Mathematics: The total production time of X units of production is represented by equation 1-8.

$$T = AX^C \qquad (1\text{-}8)$$

If T, A, and C are known, the equation can be solved for X:

$$X = \text{Antilog}((\log T - \log A)/C)$$

Comments: The major value of this program is to assist in the preparation of pro-forma financial statements. It could also be used in production scheduling and in sensitivity analysis when it is desired to determine how large a change in X will occur when either T, A, or C changes. Remember C=1-B, and B is the exponential representative of the learning curve percent.

LISTING OF A3

```
100  READ A,B,T
110  LET C=1-B
120  LET X=(LOG(T)-LOG(A))*(1/C)
130  LET X=EXP(X)
140  PRINT "FOR A B VALUE OF" B, "GIVEN" T "HOURS OF AVAILABLE"
150  PRINT "PRODUCTION TIME" X "UNITS CAN BE PRODUCED"
9999  END
```

APPENDIX A4

Model Projected Cost Data

Purpose: To make the cost projections and calculations required to implement the L-C cost allocation model, including the proper balance in Improvements in Production Procedures at various output levels.

Requirements: The previously determined values of A_c and B_c, an estimate of N, and the output levels, in terms of units, for which calculations are desired.

Input Data Format: As many DATA statements as required beginning with DATA statement number 500.

500 DATA A_c, B_c, N,
501 DATA X_1, X_2, ... , X_i, ... , X_N, 0

where:

X_i = an output level for which calculations are desired ($X_i \leq N$)

Output Data Format:

Y AT N = Y_N

CUMULATIVE PRODUCTION	UNIT COST	TOTAL COST	AVERAGE COST	ACCOUNT BALANCE
X_i	U_i	T_i	Y_i	$T_i - X_i \cdot Y_N$
X_N	U_N	T_N	Y_N	-0-

Example: For the example presented in Chapter III the input data would be:

```
500  DATA  10045.7, .207572, 20,
501  DATA  5, 15, 20, 0
```

The output would be:

```
Y AT N = 5394.15
```

CUMULATIVE PRODUCTION	UNIT COST	TOTAL COST	AVERAGE COST	ACCOUNT BALANCE
5	5828.76	35963.6	7192.72	8992.9
15	4569.77	85891.	5726.07	4978.83
20	4297.11	107883.	5394.14	0

Mathematics: Based on equations 1-1, 1-8, 1-10, and the following calculation for the account balance:

$$IPP_i = T_i - X_i \cdot Y_N \ .$$

LISTING OF A4

```
100  READ A,B,N
110  LET C=1-B
120  LET A1=A/N↑B
130  PRINT "Y AT N =", A1
140  PRINT "CUMULATIVE   UNIT   TOTAL   AVERAGE   ACCOUNT"
150  PRINT "PRODUCTION   COST   COST     COST     BALANCE"
160  READ X
170  LET U=A*(X↑C-(X-1)↑C)
180  LET T=A*X↑C
190  LET Y=A/X↑B
200  PRINT X,U,T,Y,T-X*A1
210  READ X
220  IF X=0 THEN 9999
230  GO TO 170
9999  END
```

APPENDIX B

Programs Used for Statistical Analysis of L-C Cost Allocation Model

APPENDIX B1

Confidence Intervals for $\underline{b}$ Required to Obtain Confidence Intervals for $\overline{Y}_N$

Purpose: To perform the calculations necessary to develop tables 4-1 and 4-2.

Mathematics: Determines B1 and B2 in equations 4-10 and 4-11 and indicates the percentage intervals $(B_c\text{-}B2)/B_c$ and $(B1\text{-}B_c)/B_c$.

The equations are solved for N values of 100; 500; 1,000; 5,000; and 10,000 while B_c varies from the B value corresponding to L-C percents of 70 to 99.

Comments: The program listed is for table 4-1. The program is easily modified for other N values.

LISTING OF B1

```
10    LET N(1)=100
20    LET N(2)=500
30    LET N(3)=1000
40    LET N(4)=5000
50    LET N(5)=10000
60    FOR I=1 TO 5
70    PRINT "N=",N(I)
80    LET A1=LOG(2)
90    FOR X=70 TO 99
100   LET F=100/X
110   LET D=LOG(F)
120   LET B=D/A1
130   LET B1=(LOG(100)+B*LOG(N(I))-LOG(95))/LOG(N(I))
140   LET B2=(B*LOG(N(I))-LOG(1.05))/LOG(N(I))
150   PRINT "B=",B,"B1=",B1,"B2=",B2
160   PRINT "(B1-B)/B=",(B1-B)/B,"(B-B2)/B=",(B-B2)/B
170   NEXT X
180   PRINT
190   NEXT I
9999  END
```

APPENDIX B2

Confidence Intervals for b Required to Obtain Confidence Intervals for U_i

Purpose: To perform the calculations necessary to develop tables 4-3 and 4-4.

Mathematics: Obtains approximate values of C1 and C2 in equation 4-13 and converts to terms of B1 and B2. B1 and B2 are listed along with the percent intervals $(B1-B_c)/B_c$ and $(B_c-B2)/B_c$. The solutions are accurate to 4 decimal places. The solutions are for N values of 100; 500; 1,000; 5,000; and 10,000 while B_c varies from the B values corresponding to L-C percents of 70 to 99.

Comments: The program listed is for a 10 percent confidence interval. In the BASIC programming language **is the same as ↑.

LISTING OF B2

```
10   LET X(1)=100
20   LET X(2)=500
30   LET X(3)=1000
40   LET X(4)=5000
50   LET X(5)=10000
60   FOR I=1 TO 5
70   PRINT "X=",X(I)
80   LET A=10000
90   LET A1=LOG(2)
100  FOR N=70 TO 99
110  LET F=100/N
120  LET D=LOG(F)
130  LET B=D/A1
140  LET C=1-B
150  LET U=A*(X(I)**C-(X(I)-1)**C
160  LET B1=B+.1
170  LET C1=1-B1
180  LET U1=A*(X(I)**C1-(X(I)-1)**C1)
190  IF U1>=.90*U THEN 220
200  LET B1=B1-.0001
210  GO TO 170
220  LET B2=B-.1
230  LET C2=1-B2
240  LET U2=A*(X(I)**C2-(X(I)-1)**C2)
250  IF U2<=1.10*U THEN 280
260  LET B2=B2+.0001
270  GO TO 230
280  PRINT "B=",B,"B1=",B1,"B2=",B2,
290  PRINT "(B1-B)/B=",(B1-B)/B,"(B-B2)/B=",(B-B2)/B
300  NEXT N
310  NEXT I
9999  END
```

APPENDIX B3

Confidence Intervals for b Required to Obtain Confidence Intervals for T_i

Purpose: To perform the calculations necessary to develop tables 4-5 and 4-6.

Mathematics: Determines B1 and B2 in equations 4-16 and 4-17 and indicates the percentage intervals $(B1-B_c)/B_c$ and $(B_c-B2)/B_c$. The equations are solved for N values of 100; 500; 1,000; 5,000; and 10,000 while B_c varies from the B value corresponding to L-C percents of 70 to 99.

Comments: The program listed is for table 4-6.

LISTING OF B3

```
10   LET X(1)=100
20   LET X(2)=500
30   LET X(3)=1000
40   LET X(4)=5000
50   LET X(5)=10000
60   FOR I=1 TO 5
70   PRINT "X=",X(I)
80   LET A1=LOG(2)
90   FOR N=70 TO 99
100  LET F=100/N
110  LET D=LOG(F)
120  LET B=D/A1
130  LET C=1-B
140  LET B1=1-(LOG(90)+C*LOG(X(I))-LOG(100))/LOG(X(I))
150  LET B2=1-(LOG(1.10)+C*LOG(X(I)))/LOG(X(I))
160  PRINT "B=",B,"B1=",B1,"B2=",B2
170  PRINT "(B-B1)/B=",(B-B1)/B,"(B2-B)/B=",(B2-B)/B
180  NEXT N
190  NEXT I
9999  END
```

APPENDIX B4

Minimum Value of R Required to Obtain Desired Confidence Level for b Confidence Interval

Purpose: To perform the calculations necessary to develop tables 4-7, 4-8, and 4-9.

Mathematics: Solves equation 4-18 for R. e varies between .01 and .10. t and F come from t-tables. In t-tables n=F+2.

Comments: The program listed is for table 4-8. The program could be easily modified to get more detailed listings of the relationship between required R and e, the percent confidence interval for b.

LISTING OF B4

```
10    PRINT "      T             N             E             R      "
20    READ M
30    FOR I=1 TO M STEP 1
40    READ T,F
50    FOR E=.01 TO .10 STEP .005
60    LET R=T/SQR(((E**2)*F)+(T**2))
70    PRINT T,F+2,E,R,
80    PRINT
90    NEXT E
100   PRINT
110   NEXT I
499   DATA 36
500   DATA 2.353,3,2.132,4,2.015,5,1.943,6,
501   DATA 1.895,7,1.860,8,1.833,9,1.812,10,1.796,11,
502   DATA 1.782,12,1.771,13,1.761,14,1.753,15,
503   DATA 1.746,16,1.740,17,1.734,18,1.729,19,
504   DATA 1.725,20,1.721,21,1.717,22,1.714,23,1.711,24,
505   DATA 1.708,25,1.706,26,1.703,27,1.701,28,1.699,29,
506   DATA 1.697,30,1.684,40,1.676,50,1.671,60,1.664,80,
507   DATA 1.660,100,1.653,200,1.648,500,1.645,99999,
9999  END
```

APPENDIX C

Programs Used to Handle Special Problems in Applying the L-C Cost Allocation Model

APPENDIX C1

Startup Termination

Purpose: To make the cost projections required to implement the L-C cost allocation model when the startup phenomenon is terminated.

Requirements: Values of A_c, B_c, N, and Z, the output level at which the startup termination occurred.

Input Data Format: As many DATA statements as required beginning with DATA statement number 500.

500 DATA A_c, B_c, Z, N,
501 DATA X_i, ... , X_N

Output Data Format:

VALUE OF Y AT N = Y_N

CUMULATIVE PRODUCTION	UNIT COST	TOTAL COST	CUMULATIVE AVERAGE	ACCOUNT BALANCE
X_i	U_i	T_i	Y_i	$T_i - X_i \cdot Y_N$

Example: For the startup termination presented in Chapter V the input data would be:

```
500  DATA  10045.7, .207572, 10, 20,
501  DATA  5, 10, 15, 20, 0
```

The output level at which the change occurred must be included. The output would be:

VALUE OF Y AT N = 5609.09

CUMULATIVE PRODUCTION	UNIT COST	TOTAL COST	CUMULATIVE AVERAGE	ACCOUNT BALANCE
5	5828.76	35963.6	7192.73	7918.17
10	4989.33	62288.5	6228.85	6197.62
15	4989.33	87235.2	5815.68	3098.81
20	4989.33	112182.	5609.09	-0-

Mathematics: Similar to those in Appendix A4 through unit Z. After Z unit costs are constant.

LISTING OF C1

```
100  DIM Y(100)
110  DIM T(200)
120  DIM U(100)
130  READ A1,B1,N,
140  LET Y(Z)=A1/Z↑B1
150  LET C1=1-B1
160  LET U(Z)=A1*(Z↑C1-(Z-1)↑C1)
170  LET T(Z)=A1*Z↑C1
180  LET T=U(Z)*(N-Z)
190  LET Y(N)=(T(Z)+T)/N
200  PRINT "VALUE OF Y AT N =",Y(N)
210  PRINT "CUMULATIVE   UNIT   TOTAL   CUMULATIVE   ACCOUNT"
220  PRINT "PRODUCTION   COST   COST     AVERAGE     BALANCE"
230  READ X1
240  IF X1=0 THEN 9999
250  IF X1>Z THEN 310
260  LET U=A1*(X1↑C1-(X1-1)↑C1)
270  LET T=A1*X1↑C1
280  LET Y=A1/X1↑B1
290  PRINT X1,U,T,Y,T-X1*Y(N)
300  GO TO 230
310  FOR X=Z+1 TO N
320  LET T=T+U(Z)
330  IF X1>X THEN 360
340  LET Y=T/X
350  PRINT X,U(Z),T,Y,T-(X*Y(N))
360  NEXT X
370  GO TO 230
9999  END
```

APPENDIX C2

Change in Production Costs

Purpose: To develop the cost projections required to implement the L-C cost allocation model when there is a change in the cost of the underlying factors of production.

Requirements: Values of A1, B, Z, A2, and N.

Where:

A1 = original estimate of <u>a</u>
B = regression slope coefficient
Z = output level at which costs increase
A2 = A1 increased or decreased by the percentage change in costs
N = total anticipated production

Input Data Format: As many DATA statements as required beginning with DATA statement number 500.

```
500 DATA A1, B, Z, A2, N,
501 DATA X1, ... , XN
```

Output Data Format: Same as in Appendix C1.

Example: For the decrease in production costs presented in Chapter V the input data would be:

```
500 DATA 10045.7, .207572, 11, 8036.56, 20,
501 DATA 5, 11, 15, 20, 0
```

The output would be:

VALUE OF Y AT N = 4938.2

CUMULATIVE PRODUCTION	UNIT COST	TOTAL COST	CUMULATIVE AVERAGE	ACCOUNT BALANCE
5	5828.76	35963.6	7192.73	11272.6
11	3909.32	66197.9	6017.99	11877.6
15	3655.81	81170.5	5411.37	7097.49
20	3437.69	98764.1	4938.2	-0-

Mathematics: Similar to those in Appendix A4 to Z. From Z on the value of A_c used in computations is changed. The total cost of all anticipated production is:

$$T=(A1(Z-1)^C)+(A2\cdot N^C)-(A2(Z-1)^C)$$

and

$$Y_N = T/N.$$

LISTING OF C2

```
100  READ A1,B,Z,A2,N
110  LET C=1-B
120  LET T1=(A1*(Z-1)↑C)+(A2*N↑C)-(A2*(Z-1)↑C)
130  LET Y2=T1/N
140  PRINT "VALUE OF Y AT N=",Y2
150  PRINT "CUMULATIVE   UNIT   TOTAL   CUMULATIVE    ACCOUNT"
160  PRINT "PRODUCTION   COST   COST     AVERAGE      BALANCE"
170  READ X1
180  IF X1=0 THEN 9999
190  IF X1>(Z-1) THEN 250
200  LET U=A1*(X1↑C-X1-1)↑C)
210  LET T=A1*X1↑C
220  LET Y=A1/X1↑B
230  PRINT X1,U,T,Y,T-X1*Y2
240  GO TO 170
250  LET T=A1*(Z-1)↑C
260  FOR X=Z TO N
270  LET U=A2*(X↑C-(X-1)↑C)
280  LET T=T+U
290  IF X1>X THEN 320
300  LET Y=T/X
310  PRINT X,U,T,Y,T-X*Y2
320  NEXT X
330  GO TO 170
9999  END
```

APPENDIX C3

Percent Change in N Required to Change Y_N by A Given Percentage

Purpose: To calculate the values presented in tables 6-1 and 6-2.

Mathematics: Computes N1 and N2 in equations 6-6 and 6-7 for N values of 100; 500; 1,000; 5,000; and 10,000; when B_C varies from those values corresponding to L-C percents of 70 to 99. Also computes the percentage intervals (N1-N)/N and (N2-N)/N.

Comments: The program listed is for table 6-2.

LISTING OF C3

```
100  READ M
110  LET A=LOG(2)
120  FOR I=1 TO M
130  READ N
140  PRINT N
150  FOR N1=70 TO 99
160  LET C=100/N1
170  LET D=LOG(C)
180  LET B=D/A
190  LET X1=(B*LOG(N)-LOG(.90))/B
200  LET X2=(B*LOG(N)-LOG(1.10))/B
210  LET X1=EXP(X1)
220  LET X2=EXP(X2)
230  PRINT B,X1,(X1-N)/N,X2,(X2-N)/N
240  NEXT N1
250  PRINT
260  NEXT I
500  DATA 5,
501  DATA 100,500,1000,5000,10000,
9999  END
```

APPENDIX C4

Essential Modifications

Purpose: To make the cost projections required to implement the L-C cost allocation model when there are essential modifications in the product or production procedures.

Requirements: Values of A1, B1, Z, A2, B2, N, and P. A1 and B1 refer to the original parameters. A2 and B2 are the parameters of the changed portion of the product or production procedures. Z is the output level at which the essential modification took place. P is the portion of the original product or production process which has not changed.

Input Data Format: As many DATA statements as required beginning with DATA statement 500.

Output Data Format: Same as C1.

Example: For the essential modification presented in Chapter V the input data would be:

```
500  DATA  10045.7, .207572, 11, 7158.58, .230141, 30, .5,
501  DATA  5, 10, 11, 15, 30, 0
```

The output would be:

VALUE OF Y AT N = 5912.57

CUMULATIVE PRODUCTION	UNIT COST	TOTAL COST	CUMULATIVE AVERAGE	ACCOUNT BALANCE
5	5828.76	35963.6	7192.73	6400.81
10	4989.33	62288.5	6228.85	3162.89
11	9601.9	71890.5	6535.5	6852.23
15	6185.75	98803.3	6586.89	10114.9
30	4753.64	177377.	5912.57	-0-

Mathematics: Similar to those in Appendix A4 through Z-1. After Z-1 the unit cost is:

$$U=P\cdot A1(X^{C1}-(X-1)^{C1})+A2((X-Z+1)^{C2}-(X-Z)^{C2})$$

LISTING OF C4

```
100  READ A1,B1,Z,A2,B2,N,P,
110  LET C1=1-B1
120  LET C2=1-B2
130  LET T1=A1*(Z-1)↑C1
140  FOR X=Z TO N
150  LET T1=T1+P*A1*(X↑C1-(X-1)↑C1)
160  NEXT X
170  LET T1=T1+A2*((N-Z+1)↑C2)
180  LET Y1=T1/N
190  PRINT "VALUE OF Y AT N =",Y1
200  PRINT "CUMULATIVE   UNIT   TOTAL    CUMULATIVE    ACCOUNT"
210  PRINT "PRODUCTION   COST   COST      AVERAGE      BALANCE"
220  LET T=0
230  READ X1
240  IF X1=0 THEN 9999
250  IF X1>(Z-1) THEN 310
260  LET U=A1*(X1↑C1-(X1-1)↑C1)
270  LET T=A1*X1↑C1
280  LET Y=A1/X1↑B1
290  PRINT X1,U,T,Y,T-X1*Y1
300  GO TO 230
310  LET T=A1*(Z-1)↑C1
320  FOR X=Z TO N
330  LET U=P*A1*(X↑C1-(X-1)↑C1)+A2*((X-Z+1)↑C2-(X-Z)↑C2)
340  LET T=T+U
350  IF X1>X GO TO 380
360  LET Y=T/X
370  PRINT X,U,T,Y,T-X*Y1
380  NEXT X
390  GO TO 230
9999  END
```

APPENDIX D

Relationship Between b Parameter and L-C Percent

APPENDIX D

It was previously determined that a L-C% of 90 had a b value approximately equal to .1520. This was arrived at by solving b in (1-1) when X equaled 2, a equaled 100, and Y equaled 90. Here, when cumulative output doubled from 1 to 2 units, cumulative average time fell from 100 to 90. This same basic relationship can be used to solve for the relationship between any other L-C% and b. Solving (1-1) for b in general terms we get:

$$Y = a_c / X^b$$

$$Y \cdot X^b = a$$

$$X^b = a/Y$$

$$b\log(X) = \log(a/Y)$$

$$b = \frac{\log(a/Y)}{\log(X)}$$

If X is set equal to 2, and a is set equal to 100, Y then represents the L-C percent and we can solve for b. Table D lists the b values corresponding to L-C percents of 51 through 100. The program used to compute the values presented in table D is listed on the page following table D.

TABLE D

b Values Corresponding to L-C Percents 51 to 100

L-C%	b	L-C%	b
51	0.97143	76	0.39592
52	0.94341	77	0.37707
53	0.91593	78	0.35845
54	0.88896	79	0.34007
55	0.86249	80	0.32192
56	0.83650	81	0.30400
57	0.81096	82	0.28630
58	0.78587	83	0.26881
59	0.76121	84	0.25153
60	0.73696	85	0.23446
61	0.71311	86	0.21759
62	0.68966	87	0.20091
63	0.66657	88	0.18442
64	0.64385	89	0.16812
65	0.62148	90	0.15200
66	0.59946	91	0.13606
67	0.57776	92	0.12029
68	0.55639	93	0.10469
69	0.53533	94	.0892673
70	0.51457	95	.0740005
71	0.49410	96	.0588937
72	0.47393	97	.0439433
73	0.45403	98	.0291463
74	0.43440	99	.0144996
75	0.41503	100	0.0

LISTING OF PROGRAM FOR D

```
10  LET A=LOG(2)
20  FOR N=51 TO 100
30  LET C=100/N
40  LET D=LOG(C)
50  LET B=D/A
60  PRINT N,B
70  NEXT N
9999  END
```