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ABOUTNESS:

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By

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ABSTRACT

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In this essay an attempt is made to clarify the concept of aboutness. More specifically, the question of what it means to say that a sentence is about a given thing is addressed. The answer offered draws heavily on the work of Nelson Goodman's 1961 paper "About."

In chapter I, after noting several examples of cavalier uses of the concept of aboutness by philosophers of such prominence as Russell, Frege, and Carnap, a number of conditions of adequacy for an acceptable analysis of aboutness are examined. After noting that the list of conditions is inconsistent, a shortened list of acceptable conditions is proposed. A critique of Goodman's analysis is provided, and it is suggested that an adequate evaluation of his views can only be advanced against the background of a formal language sufficiently rich in expressive power to allow for quantification over predicates.

In chapter II a second-order language _L_ is developed whose characteristic feature is that it contains, in addition to the usual apparatus of predicate variables and constants, _structured predicates_, where structured predicates are seen as the answer to several questions concerning

Goodman's analysis. Relative to L, a refined version of Goodman's definition of absolute aboutness is offered: a sentence ϕ is absolutely about an object if ϕ is equivalent to a sentence ψ that contains an essential occurrence of a term that designates that object, where a term is understood to occur essentially in ψ, if ψ does not imply its own generalization with respect to that term or with respect to any of its parts. A number of lemmas, theorems, and corollaries are proved the most important of which allows us to offer a simplified definition of essential occurrence: a term occurs essentially in ψ, if ψ does not imply its own universal generalization with respect to any of its atomic parts.

In chapter III two questions are addressed. The first concerns the question of which of the results obtained in chapter II remain in force with the introduction of definite descriptions. The second concerns the problem of how to analyze aboutness claims for sentences containing occurrences of nonreferring singular terms. The answer offered to the first question involves some surprising, but interesting results. Among these is the claim that no name ever occurs essentially in any monistic sentence, where a monistic sentence is one that entails '(x)(y)x = y'. The status of occurrences of descriptions in monistic sentences is examined and a theorem concerning occurrences of descriptions in nonmonistic sentences is proved. The answer

offered to the second question involves both a development of Goodman's concept of rhetorical aboutness and the use of free logic. In addition, it is noted that the question of which of the results previously obtained remain in force when the logic supposed is that of free logic is an open one.

For my parents: William L. and Hattie M. Hart

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CHAPTER I

THE PROBLEM OF ABOUTNESS

Section One: Some Historical Remarks

In its brief history semantic theory has treated successfully of such notions as reference and truth. Thus, e.g., Russell's theory of descriptions is rightly regarded as a triumph for the theory of reference, freeing us from the need to adopt an unwanted ontology; while Tarski's definition of truth seemed to make the very idea of a semantics for a language a respectable one. Curiously though, little attention has been paid to another, closely related, semantic notion, viz., aboutness. I say "curiously" because it is not difficult to find examples of semanticists who employ this notion at vital points in their work; and yet who do so without so much as a pause to explain systematically what they intend. Thus, e.g., Russell in "On Denoting" writes:

> If we say 'the King of England is bald', that is, it would seem, not a statement about the complex meaning 'the King of England', but about the actual man denoted by the meaning. But now consider 'the King of France is bald'. By parity of form, this also ought to be about the denotation of the phrase 'the King of France'. But this phrase, though it has a meaning provided 'the King of England' has a meaning, has certainly no denotation, at least in any obvious sense. Hence one would suppose that 'the King of France is bald' ought to be nonsense, but it is not nonsense, since it is

plainly false.[1]

Three premises appear to be concealed in this Parmenidean-looking argument. (1) A sentence is about what any of the denoting phrases occurring in it denotes. (2) If the only denoting phrases that occur in a sentence are those that fail to denote, then that sentence is about nothing. (3) If a sentence is about nothing in virtue of (2), then it is nonsense or meaningless. Whether Russell would endorse any of (1) through (3) is a question not answerable by examining the corpus of Russell's works, since he nowhere explicitly addresses himself to the question of what a sentence is about. One can, however, conjecture that Russell believed that (1) through (3) represent rather natural, or intuitive, ways of thinking about aboutness. That (1) through (3) should not be accepted as conditions of aboutness should be clear; acceptance of them leads directly to the problem that Russell is addressing, and while Russell's analysis of definite descriptions provides a way out of the problem, it is not the only way out. Indeed, one way out is to provide an acceptable analysis of aboutness that entails the falsity of at least one of (1) through (3). I am not, of course, suggesting that providing such an analysis automatically provides one with a solution to the problem with which Russell is concerned, viz., the problem of specifying truth conditions for

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[1]B. Russell, "On Denoting," in Logic and Knowledge, ed. R. C. Marsh (New York: Macmillan Co., 1956), p. 46.

sentences containing denoting phrases that fail to denote; that problem, I believe, can be reformulated without making any claims about what a sentence is about. I am, however, suggesting that the above formulation of that problem rests on a mistake, viz., the mistake of accepting (1) through (3) as a correct partial characterization of what it means to say that a sentence is about something. That the acceptance of any one of (1) through (3) constitutes a mistake is a matter that will be seen to follow from later results.

Frege is another whose comments concerning aboutness provoke a second reading. Witness:

> It is true that at first sight the proposition
> "All whales are mammals"
> seems to be not about concepts but about animals; but if we ask which animal then are we speaking of, we are unable to point to any one in particular. Even supposing a whale is before us, our proposition still does not state anything about it. We cannot infer from it that the animal before us is a mammal without the additional premise that it is a whale, as to which our proposition says nothing. As a general principle, it is impossible to speak of an object without in some way designating or naming it; but the word "whale" is not the name of any individual creature.[2]

Now I am not concerned with disputing Frege's view that "All whales are mammals" is about concepts. Rather, I am concerned with disputing the claim, implicit in the first sentence of the above, that if we suppose that "All whales are mammals" is about animals, that we are thereby, in some sense, obligated to answer the question "Which one(s)?" We are obligated, it seems to me, only if we

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[2] G. Frege, The Foundations of Arithmetic, trans. J. L. Austin (New York: Harper & Row, 1953), p. 60.

assume the following principle concerning aboutness: (4) If a sentence is about F's, then it is about some particular F. While acceptance of (4) *might* seem intuitively plausible, no argument for it has been offered. Indeed, if aboutness is like hunting, then the principle is false, since I can hunt lions without hunting any particular lion.

If (1) through (4) are unacceptable conditions of aboutness, surely the following *is* an acceptable principle: (5) Equivalent sentences are about the same things. Bearing this principle in mind, it is interesting to then observe that Frege's remarks in the latter part of the above would seem to suggest that he endorses the idea that a sentence can be about a thing only if it contains a name of that thing. Carnap, at any rate, did at one time endorse a like principle. Indeed, it appears that, for Carnap, a sentence is about a thing if and only if it contains an expression that designates that thing. He writes:

> We have here left out of account those logical sentences which assert something about the *meaning*, *content*, or *sense* of sentences or linguistic expressions of any *domain*. These also are pseudo-object-sentences. Let us consider as an example the following sentence, $\mathfrak{S}_1$ "Yesterday's lecture was about Babylon." $\mathfrak{S}_1$ appears to assert something about Babylon, since the name 'Babylon' occurs in it. In reality, however, $\mathfrak{S}_1$ says nothing about the town Babylon, but merely something about yesterday's lecture and the word 'Babylon'. This is easily shown by the following non-formal consideration: for our knowledge of the properties of the town of Babylon it does not matter whether $\mathfrak{S}_1$ is true or false. Further, that $\mathfrak{S}_1$ is only a pseudo-object sentence is clear from the circumstance that $\mathfrak{S}_1$ can be translated into the following sentence of (descriptive) syntax: "In yesterday's lecture either the word 'Babylon' or

> an expression synonymous with the word 'Babylon' occurred" ($\mathfrak{S}_2$).[3]

If (5) is correct, then both Frege and Carnap are in error. The sentence "All men are mortal," given its equivalence to "All non-mortals are non-men," would seem to be about non-men, though clearly it doesn't mention such things.

Ryle's account of aboutness appears to suffer from the same defect. He writes:

> 'The sentence S is about Q' often means 'in the sentence S 'Q' is the grammatical subject or nominative to the verb or main verb.' 'Q' will thus be any noun or pronoun or any phrase grammatically equivalent to a noun or pronoun.[4]

Even such contemporary writers as Lambert and Van Fraassen make claims concerning aboutness which, given the preceding discussions strike one as naive. In translating the argument "All Greeks are men; All men are mortal; Hence: All Greeks are mortal" as "$(x)(Gx \supset Hx)$, $(x)(Hx \supset Mx)$, $\therefore$ $(x)(Gx \supset Mx)$" they say the following concerning the translation:

> Notice that in the official idiom, each of these sentences is about <u>any</u> individual whatsoever. The first premise says that for any individual (in the domain), if he is Greek, then he is human. The conclusion says that for any individual (in the domain) it is the case that if he is a Greek, then he is mortal.[5]

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[3] R. Carnap, <u>The Logical Syntax of Language</u>, trans. A. Smeaton (London: Routledge & Kegan Paul, 1937), pp. 285-86.

[4] G. Ryle, "'About,'" <u>Analysis</u> 1 (1933):10-11.

[5] K. Lambert and B. C. Van Fraassen, <u>Derivation and Counterexample</u> (Encino: Dickenson Press, 1972), p. 88.

Now I have no qualms about reading "$(x)(Gx \supset Hx)$" and "$(x)(Gx \supset Mx)$" in the way suggested. However, I do have qualms about accepting the following principle which seems to be implied by the first sentence in the above: (6) A universal quantification is about each thing in the universe of discourse. We are to accept (6), presumably, because a universal quantification says of each thing in the domain that it satisfies a certain condition. However, since every sentence is equivalent to some universal quantification or other, acceptance of (6), along with (5), yields the unacceptable result that every sentence is about everything in the domain. Even supposing that our universe of discourse contains all prime numbers, I see no reason to believe that "Socrates is mortal" is about, e.g., the number eighty-nine.[6]

One might well attempt to defend the above authors by claiming that their remarks concerning aboutness are

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[6]The notion of aboutness appealed to in this essay is understood to be primarily that of a relation between a declarative sentence and an object. Although the question of what an interrogative or imperative sentence is about is not herein addressed, it is not difficult to advance a tentative answer. For direct questions, an interrogative sentence is about whatever its correct answer is about; an imperative sentence, one might conjecture, is about whatever the sentence obtained by attaching the name of the person, corporation, etc., to whom/which the imperative is addressed, to the imperative itself, is about. Thus, e.g., if the imperative "Close the door!" is addressed to John, we might suppose that this imperative is about whatever "John closed the door" is about. This is, of course, only a rough and ready analysis that obviously requires much refinement. As a first step, however, it at least has the virtue of appearing to be headed in the right direction.

merely "off the cuff," as it were, and were never intended to withstand the kind of critical scrutiny they are here being subjected to. Whether such a defense is necessary, or adequate, is not a matter that I am concerned to establish. I am merely concerned with pointing out that aboutness has frequently figured in discussions of other semantic concepts in writers as important as Russell, Frege, Carnap, Ryle, Lambert, and Van Fraassen.

One author whose comments concerning aboutness are clearly not intended to be off the cuff is Hans Herzberger. In his important article, "Paradoxes of Grounding in Semantics," he makes explicit use of the notion of aboutness. Operating with what he admits is only a rough and ready characterization of aboutness, Herzberger is able to show that the assumption that some sentence is about exactly the grounded sentences leads to paradox--where a sentence is said to be grounded just in case it is _not_ the first member of some infinite sequence of sentences, each of which belongs to the domain of its predecessor. The domain of a sentence, Herzberger suggests, comprises what a sentence is about. He writes:

> Momentarily conceding sense to the notion of aboutness, each sentence has a certain _domain_. For a simple sentence whose main verb is intransitive, the domain comprises everything that satisfies its underlying subject term . . . .
> . . . More complex sentences built up from connectives, transitive verbs, adverbial phrases, and the like will have correspondingly complex domains . . . .[7]

[7] H. Herzberger, "Paradoxes of Grounding in Semantics," _Journal of Philosophy_ 67:6:147.

It is clear from this, and other remarks of Herzberger, that he intends the above only as a rough and ready characterization of aboutness, and not as a formally adequate explication. Indeed, he suggests elsewhere that his results are ultimately independent of accepting the above characterization of aboutness. At any rate, it should be clear that Herzberger's characterization of aboutness is inadequate for the same reason that Carnap's is. Presumably, the domain of "All men are mortal" is the class of men. But, acceptance of (5) yields the consequence that this sentence is about not only that class, its domain, but about the class of non-mortals as well. Hence, Herzberger's analysis violates (5).

Section Two: Conditions of Adequacy

The foregoing considerations would seem to suggest that any formal explication of aboutness had best begin with some statement of the conditions of adequacy which such an explication should meet. Reflections on the arguments provided in the previous section, and on aboutness in general, suggest the following list of logical conditions of adequacy. The arguments provided for each condition in the list are not, of course, intended to be conclusive, but, rather, are to be viewed only as lending some *prima facie* plausibility to the condition in question.

CA(Eq): EQUIVALENCE CONDITION. *Equivalent sentences are about the same things*. If an argument is wanted for CA(Eq) it is perhaps the following: a sentence is about an object, presumably, if and only if it says something about that object. If we assume, as seems reasonable, that equivalent sentences say the same things about the same things, we are then committed to the view that equivalent sentences are about the same things, i.e., we are committed to CA(Eq).

CA(Neg): NEGATION CONDITION. A sentence and its negation are about the same things. CA(Neg) appears eminently plausible; if a sentence is about an object, then presumably it says of that object that it satisfies a certain condition, viz., a condition specified in that sentence. Clearly the negation of that sentence simply denies of that object that it satisfies that condition, or, alternatively, that it satisfies the "negation" of that condition. Hence, it would appear, the negation of that sentence is about that object. The failure of any intuitively plausible counterexample to CA(Neg) to come to mind also recommends its adoption.

CA(L-T): LOGICAL TRUTH. Logically true sentences aren't about anything. CA(L-T) asserts little more than is contained in the traditional doctrine that logically true sentences are sentences devoid of content; they tell us nothing about anything that they don't tell us about everything else. As Wittgenstein put it: "Propositions show what they say: tautologies and contradictions show that they say nothing."[8]

CA(L-I): LOGICAL INDETERMINACY. Logically indeterminate sentences are each about something. CA(L-I), it might be argued, lies quite in line with the tradition lately noted; logically indeterminate sentences do have content; hence do tell us something about the world, or at least parts of it.

CA(L-F): LOGICAL FALSEHOOD. Logically false sentences aren't about anything. CA(L-F) is motivated not only by the tradition discussed in connection with CA(L-T) and CA(L-I), but follows from CA(Neg) together with CA(L-T).

CA(Comp): COMPOSITION CONDITION. If a sentence is about a thing, then any L-indeterminate sentence of which it is a truth-functional component is about that thing. The necessity for the restriction to L-indeterminate sentences is best understood in the light of a supposed prior adoption of CA(L-T) and CA(L-F). Without such a restriction our list of conditions would obviously be inconsistent. The acceptability of CA(Comp) is perhaps best examined in the light of the arguments for its special cases CA(Lim Neg), CA(Conj) and CA(Alt).

CA(Lim Neg): LIMITED NEGATION CONDITION. If a sentence is about something and its negation is L-indeterminate, then its negation is about that thing. It is apparent that CA(Lim Neg) is a special case not only of

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[8] L. Wittgenstein, Tractatus Logico Philosophicus (London: Routledge & Kegan Paul, 1922), p. 97.

CA(Comp) but CA(Neg) as well. Hence, whatever plausibility CA(Neg) has is automatically conferred on CA(Lim Neg).

CA(Conj): CONJUNCTION CONDITION. A consistent conjunction is about whatever its conjuncts are about. The initial plausibility of CA(Conj) can be seen as follows: suppose that in asserting ϕ I have asserted something about k. If I then consistently complete my assertion by uttering 'and ψ', there would then seem to be no reason to deny that I had still said something about k.

CA(Alt): ALTERNATION CONDITION. An alternation that is not L-true is about whatever its alternates are about. CA(Alt) would appear to have as much, or as little, plausibility as CA(Conj). Indeed, since, as is well known, alternation is definable in terms of negation and conjunction, CA(Alt) follows from CA(Eq) together with CA(Neg) and CA(Conj). Similarly, since all truth functions can be defined in terms of negation and conjunction, corresponding conditions for all modes of truth functional composition follow from CA(Eq) together with CA(Neg) and CA(Conj).

CA(Dis): DISCRIMINATION CONDITION. It is false that all sentences are about the same things. CA(Dis) appears to be absolutely necessary. A theory of aboutness that had as a consequence that all sentences are about the same things would not be worthy of the name. The necessity of adopting CA(Dis) carries over, of course, to the following special cases.

CA(Non-Nul): NON-NULLITY CONDITION. <u>Some sentence is about something</u>.

CA(Non-Univ): NON-UNIVERSALITY CONDITION. <u>Some sentences are not about everything</u>.

The plausibility of the next condition is best viewed in the light of a supposed adoption of CA(Eq), CA(Neg), CA(L-T), and CA(L-I).

CA(Lim Dis): LIMITED DISCRIMINATION CONDITION. <u>It is not the case that all L-indeterminate sentences are about the same things</u>. Any theory adopting CA(Eq), CA(Neg), CA(L-T), and CA(L-I), but not CA(Lim Dis), would suffer from a defect similar to that suffered by a theory violating CA(Dis), i.e., any theory having the consequence that all L-indeterminate sentences are about the same things would not be a theory worthy of the name.

CA(Con): CONSEQUENCE CONDITION. <u>A consistent sentence is at least about whatever its consequences are about</u>. CA(Con) seems to embody the following intuition concerning aboutness. What a sentence says is embodied in the sentences that it implies, viz., its consequences. Hence, what a sentence says <u>about</u> something is also therein embodied. Therefore, if a consequence of a sentence is about something, so also is the sentence. To put it somewhat differently: if ϕ implies ψ, then ϕ is at least "as strong as" ψ. Hence, if ψ is about k, then ϕ is about k as well.

The above is by no means intended to be an exhaustive list of possible conditions of adequacy. Rather, it is merely a list of conditions that come naively to mind while precritically reflecting on aboutness. Actually CA(Eq) through CA(Conj) constitutes at best a redundant set of adequacy conditions. For example, CA(L-T) and CA(L-F) are easily seen to be equivalent under the assumption that CA(Neg) holds. Clearly also CA(L-T) and CA(L-F) imply CA(Non-Univ), while CA(L-I) implies both CA(Non-Nul) and CA(Non-Univ). Less obvious perhaps is the following: under the assumption that CA(Eq) and CA(Neg) hold, CA(Con), CA(Conj), and CA(Alt) are equivalent. The following three arguments constitute a proof of that observation.

CA(Alt) implies CA(Conj)

(1) $(\phi \;\&\; \psi)$ is consistent	Hypothesis
(2) ϕ is about k	Hypothesis
(3) $-\phi$ is about k	(2) CA(Neg)
(4) $(-\phi \text{ v } -\psi)$ is not L-true	(1)
(5) $(-\phi \text{ v } -\psi)$ is about k	(3), (4) CA(Alt)
(6) $-(-\phi \text{ v } -\psi)$ is about k	(5) CA(Neg)
(7) $(\phi \;\&\; \psi)$ is about k	(6) CA(Eq)

CA(Conj) implies CA(Con):

(1) ψ is about k	Hypothesis
(2) ϕ implies ψ	Hypothesis
(3) ϕ is consistent	Hypothesis
(4) $(\phi \;\&\; \psi)$ is consistent	(2), (3)

(5) $(\phi \ \& \ \psi)$ is about k	(1), (5) CA(Conj)
(6) ϕ is equivalent to $(\phi \ \& \ \psi)$	(2)
(7) ϕ is about k	(5), (6) CA(Eq)

CA(Con) implies CA(Alt):

(1) $(\phi \vee \psi)$ is not L-true	Hypothesis
(2) ϕ is about k	Hypothesis
(3) $-\phi$ is about k	(2) CA(Neg)
(4) $(-\phi \ \& \ -\psi)$ is consistent	(1)
(5) $(-\phi \ \& \ -\psi)$ is about k	(3), (4) CA(Con)
(6) $-(-\phi \ \& \ -\psi)$ is about k	(5) CA(Neg)
(7) $(\phi \vee \psi)$ is about k	(6) CA(Eq)

Not only do CA(Eq) through CA(Con) constitute a redundant list, as the above arguments show, they also constitute an inconsistent list. Granted that CA(Eq) and CA(Neg) imply the equivalence of CA(Alt), CA(Conj), and CA(Con), we can easily show that CA(Neg) and CA(Con) entail the negation of CA(Lim Dis).

(1) ϕ and ψ are L-indeterminate	Hypothesis
(2) ϕ is about k	Assume
(3) $(\phi \vee \psi)$ is not L-true	Assume
(4) $(\phi \vee \psi)$ is about k	(2), (3) CA(Alt), CA(Con)
(5) ψ implies $(\phi \vee \psi)$	Logic
(6) ψ is consistent	(1)
(7) ψ is about k	(4), (5), and (6) CA(Con)

(8)	If (ϕ v ψ) is not L-true then ψ is about k	(2) - (7)
(9)	(ϕ v $-\psi$) is not L-true	Assume
(10)	(ϕ v $-\psi$) is about k	(2), (9) CA(Alt) and CA(Con)
(11)	$-\psi$ implies (ϕ v $-\psi$)	Logic
(12)	$-\psi$ is consistent	(1)
(13)	$-\psi$ is about k	(10), (11), and (12) CA(Con)
(14)	ψ is about k	(13) CA(Neg)
(15)	If (ϕ v $-\psi$) is not L-true then ψ is about k	(9) - (14)
(16)	Either (ϕ v ψ) or (ϕ v $-\psi$) is not L-true	(1)
(17)	ψ is about k	(8), (15), and (16)
(18)	if ϕ is about k, then ψ is about k	(2) - (17)
(19)	if ψ is about k, then ϕ is about k	Similarly
(20)	ϕ is about k iff ψ is about k	(18) and (19)

Clearly some decisions must be made as to which of CA(Eq) - CA(Con) are acceptable. If a clear culprit is to be found surely it is CA(Con). A convincing counterexample to CA(Con) is easy to construct. Surely "Socrates is a philosopher or three is a prime number" is a consequence of "Socrates is a philosopher." Surely also "Socrates is a philosopher" is not about the number three, though, it would seem, the aforementioned consequence of this latter sentence is about the number three. Generalizing this simple counterexample provides the unacceptable result that any consistent sentence is about whatever any sentence is about. Hence, CA(Con) must be rejected. CA(Eq), though it is clearly a special case of CA(Con), remains in force.

These considerations would seem to suggest the adoption of the following conditions: CA(Eq), CA(Neg), CA(L-T), CA(L-I), and CA(Lim Dis). Such an adoption requires, in the light of the unacceptability of CA(Con), the rejection of CA(Conj) and CA(Alt). As has been previously shown these latter are equivalent to CA(Con) in the presence of CA(Eq) and CA(Neg). As can be easily seen CA(L-F), CA(Lim Neg), CA(Non-Nul), CA(Dis), and CA(Non-Univ) follow from the recommended list.

It should be emphasized, of course, that our final list of conditions of adequacy is a list, not of <u>sufficient</u> conditions, but only <u>necessary</u> ones. It is easily verified that the following "theory" of aboutness satisfies all of CA(Eq), CA(Neg), CAL-T), CA(L-I), and CA(Lim Dis). Let $[\phi]$ be the class of sentences equivalent to ϕ, and let aboutness be so defined that

> ϕ is about k if and only if ϕ is neither L-true nor L-false and $k \in [\phi] \cup [-\phi]$.

Thus, according to this theory, a sentence is about a thing if and only if that thing is neither L-true nor L-false and is equivalent either to the first sentence or to its negation. Notice that according to this theory the only things which sentences are about are themselves sentences.

Barring such obviously incorrect theories as the above can, in the spirit of Hempel, be accomplished by the adoption of some suitable material condition of adequacy.

CA(Mat): MATERIAL CONDITION OF ADEQUACY. An explication of aboutness is adequate only if it is in sufficiently close agreement with the customary meaning of 'about' as that word is ordinarily used.

CA(Mat) suffers, of course, from all of the imprecision, vagueness, and ambiguity that seems attendant upon all statements of material conditions of adequacy in general, be they for aboutness, confirmation or whatever. Perhaps the following remarks by Goodman provide a more elegant expression of the intent of CA(Mat):

> The aim is not to describe in detail the everyday use of "about", but rather to define one or more technical counterparts that will be serviceable in precise discourse. Some sharp divergence from our ordinary notions concerning "about" is inevitable, since these notions are already shown to be inconsistent. Nevertheless, the acceptability of any definitions proposed, and of the way they resolve conflicts and ambiguities, will depend not only on their consistency and simplicity, but also upon how successfully they elicit and embody the most important features of the ordinary use of "about". The law must derive its authority from the people even though it must treat some of them harshly.[9]

It should be noted that the inconsistencies claimed by

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[9] N. Goodman, "About," Mind 70 (1961):1-24.

Goodman to be derivable from our ordinary notions concerning "about" are not the same as those lately considered in connection with CA(Con). The inconsistencies with which Goodman is concerned are derived by appealing to two principles concerning aboutness which are dubious at best and which will be examined in the next section.

Section Three: Goodman's Analysis

In his 1961 paper, "About," Nelson Goodman offers an interesting, if ambiguous, answer to the general question of what a statement is about.[10] Responsible responses to that paper have concentrated largely on the problem of refining and clarifying Goodman's definition of 'absolutely about' in such a way that his conception of logic as:

> . . . including the usual theory of statements and all of quantification theory (with identity), but as stopping short of the full theory of classes.[11]

is retained.

Such attempts at reconstruction are motivated by a number of factors, not the least of which is the uncharacteristically cavalier approach that Goodman takes in the expression of his theory. To see that this is so requires an examination of Goodman's theory. To that, then, let us now turn our attention.

Goodman begins his discussion of aboutness by noting a dilemma. Given what Goodman takes to be

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[10]Ibid.

[11]Ibid., p. 254.

intuitively plausible assumptions it is not difficult to show that any statement about anything is a statement about everything. He writes:

> The statement
>
> Maine has many lakes
>
> is obviously about Maine. Since Aroostook County is in Maine, the statement
>
> Aroostook County grows potatoes
>
> seems also to be about Maine. So also, since Maine is in New England, do the two statements
>
> New England is north of Pennsylvania
>
> New England States are small
>
> Apparently we speak about Maine whenever we speak about anything contained in (whether as part, member, member of member, etc.) Maine and whenever we speak about anything that contains Maine. But to accept this principle is to overlook an obvious Hempel syndrome and to be saddled with the conclusion that any statement about anything is a statement about Maine.[12]

The "Hempel syndrome" referred to in these remarks concerns, not aboutness, but confirmation. In his 1945 paper, "Studies in the Logic of Confirmation," Hempel was able to show, in his discussion of conditions of adequacy for a qualitative theory of confirmation, that the acceptance of the converse consequence condition, together with the consequence and entailment conditions, yields the unacceptable result that any sentence which

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[12]Ibid., p. 247.

confirms anything confirms everything.[13] Hence, the appropriateness of the appellation in the present context.

In reading these remarks one is struck by the oddity of the implicit claims that if a sentence ϕ is about k, then ϕ is about whatever is contained in k, and if ϕ is about k, then ϕ is about whatever contains k. That these two principles lead to inconsistencies is easy enough to grant. However, that they are somehow embodied in our intuitive conceptions of aboutness is a claim that this author finds dubious at best. That an analysis of aboutness is still needed, even without the dubious motivation of Goodman's two principles, should be clear from the discussion in the previous section concerning the inconsistencies derivable by the use of CA(Con).

Escaping the dilemma provided by these principles proves to be no easy task. Goodman's way out is to distinguish two senses of 'about'. The first sense of 'about' is that in which a statement is independently or <u>absolutely about</u> a given thing. The second is that in which a statement is, <u>relative to</u> certain other statements, about a given thing. The distinction between absolute and relative aboutness is thus intended to obviate the Hempel syndrome noted above.

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[13] C. G. Hempel, "Studies in the Logic of Confirmation," <u>Mind</u> 54 (1945); reprinted in <u>Aspects of Scientific Explanation and Other Essays in the Philosophy of Science</u> (Toronto: Collier-Macmillan Canada, 1965), pp. 31-32.

While

Aroostook County grows potatoes

is about Maine relative to

Aroostook County is in Maine,

it is not absolutely about Maine, and while

Maine has many lakes

is absolutely about Maine, it is about New England only relative to

Maine is in New England

Merely providing a distinction between two senses of 'about' does not, of course, constitute a theory of aboutness. Rather more is required. In particular, we need definitions of these two senses. As relative aboutness is ultimately defined in terms of absolute aboutness the latter proves to be the more fundamental concept; and, since, as we have already urged the introduction of the former concept has not been well motivated, its employment in this essay will be quite restricted. Absolute aboutness is ultimately defined in terms of designation, that concept being understood in such a way that a name or a description designates what, if anything, it refers to, and a predicate designates its extension: the class of those elements it applies to or denotes.

Having thus settled on this notion of designation, Goodman then characterizes the notion of a sentence mentioning a thing by claiming that a sentence may be said to mention whatever any expression in it designates. A sentence may mention a class or whole, he reminds us, without mentioning any particular member of that class or part of the whole. Thus, while a sentence might mention the New England States in virtue of containing an occurrence of the expression 'New England States' it does not thereby mention Maine. As a first step toward explaining absolute aboutness in terms of designation, Goodman offers us:

> . . . a statement S is absolutely about Maine if some statement that mentions Maine follows logically from S.[14]

As Goodman rightly notes, such an explication of absolute aboutness will not do. Let A be any statement whatsoever, clearly "A or Maine is in New England" follows logically from A. Since the disjunction mentions Maine, A satisfies the proposed criterion and is thus about Maine. But to accept this conclusion is to accept the reinstatement of the Hempel syndrome that the distinction between relative and absolute aboutness was intended to avoid. The problem, as Goodman argues, is that while A does yield the above disjunction it also yields any disjunction obtained by replacing "Maine is in New England" in "A or Maine is in New England" with any statement whatsoever. As he puts it:

> Now we must seriously raise the question whether a statement can properly be regarded as saying about any particular thing what it says about everything else. Or is a statement genuinely about Maine only

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[14]Goodman, "About," p. 249.

> if it says something about Maine that it does not say about something else.[15]

and

> In sum the trouble with our definition of absolute aboutness lies in the absence of any requirement of selectivity. That S yields logically a statement mentioning Maine is not a sufficient condition for S to be absolutely about Maine; S must, roughly speaking, yield such a statement without yielding a parallel statement for everything else.[16]

The obvious repair to our criterion of absolute aboutness that the above considerations suggest is expressed by Goodman thusly:

> Now a statement S is absolutely about Maine only if S yields logically some statement T in which some expression designates Maine, without so yielding the generalization of T with respect to that expression.[17]

Requiring that the generalization of T with respect to the expression designating Maine not also follow from S thus blocks the counterexample noted earlier. For while "A or Maine is in New England" does follow logically from A, so also does "(α)(A or α is in New England)." Hence, under the revised criterion A no longer qualifies as being about Maine.

Even with the above restriction in force we are still not out of the woods. Goodman offers us yet another counterexample:

From the statement

(4) Aroostook County grows potatoes

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[15] Ibid., p. 250.

[16] Ibid., p. 251.

[17] Ibid., p. 252.

follows the statement

> (5) Everything that is a State and contains Aroostook County contains a county that grows potatoes.

Our present formula rightly precludes taking (5) as evidence that (4) is about the class of States, for the generalization of (5) with respect to "State"--namely

> (6) (α)(Everything that is an α and contains Aroostook County contains a county that grows potatoes)

also follows from (4). However, (5) also has in it the expression "State and contains Aroostook County"; and the generalization of (5) with respect to this--namely,

> (7) (α)(Everything that is an α contains a county that grows potatoes)

--certainly does not follow from (4). Hence (5) may still be cited as showing that (4) is absolutely about whatever is a State and contains Aroostook County--that is, about Maine. Furthermore, by use of this same device, we could show that (4) is about anything that contains or is contained in Aroostook County. We must rule out statements like (5) as evidence that statements like (4) are about Maine (or about things containing or contained in Aroostook County) on the ground that the generalization of (5) with respect to a part of the expression designating Maine in (5) also follows from (4).[18]

Goodman's solution to this problem consists in the introduction of the notion of differential implication. A statement S is said to imply a statement T differentially with respect to k if S implies T, T contains an expression designating k and no generalization of T with respect to any part of that expression is also implied by S. Goodman's final definition of absolute aboutness is:

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[18] Ibid., pp. 252-53.

> S is absolutely about k if and only if there is a statement T such that S implies T differentially with respect to k.[19]

At the beginning of this section it was noted that the majority of the responses to "About" had focused on the problem of refining and clarifying Goodman's analysis in such a way that Goodman's expressed preference for a first-order quantificational logic (with identity) is retained within those refinements and clarifications. The need for such a focus should now be clear. Goodman's final definition of absolute aboutness makes essential use of the notions of designation, logical consequence (implication), and universal generalization. Such notions, as of this date, have precise characterizations only with respect to certain formal languages.

Goodman, beyond noting that the logic preferred by him is some standard system of first-order quantification theory (with identity), simply does not tell us what sort of language he has in mind. Moreover, even assuming that we are operating within a _first_-order language, it is difficult to see how one could verify the following claim by Goodman:

> The statement
>
> Men are earthbound
>
> if construed as of the form "$(x)(Mx \supset Ex)$", is about men and earthbound things; but if construed as of the

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[19] Ibid., p. 253.

> form "(x)(Mx ⊃ Bxe)", is about the earth, men, and the relation of being bound.[20]

Surely in applying the test of absolute aboutness to "(x)(Mx ⊃ Ex)" to determine whether it is about men one must check to see whether "(x)(Mx ⊃ Ex)" implies "(Q)(x)(Qx ⊃ Ex)." Such a determination can only be made, of course, within the confines of a second-order logic and not a first-order logic.[21] Conversely, taking Goodman's remarks concerning logic seriously provides us with the result that a logically true sentence such as "(x)(Fx ⊃ Fx)" is about the class of 'F's, given that this sentence does not imply itself differentially with respect to that class. It fails to do so, of course, simply because the universal generalization of this sentence with respect to 'F', as this notion is commonly understood, is not even a well-formed sentence of any standard first-order quantificational language. Readers of Goodman will, of course, note that this result is in direct conflict with Goodman's claim that one consequence of his definition is that logically true sentences are not about anything.

These considerations would seem to suggest that perhaps the adoption of a second-order language remains

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[20] Ibid., p. 254.

[21] It should be noted that the problems encountered in attempting to apply Goodman's theory while remaining within the confines of first-order logic are addressed by Ullian and Putnam in their paper, "More About 'About.'" Much of the present essay derives its inspiration from that paper. For a discussion of that paper see section four of chapter I.

the only reasonable course, and that Goodman's remarks concerning a "preferred" logic should simply be ignored. Goodman, himself, seems to encourage such a course, not only in the symbolic examples he employs, but also in these remarks:

> . . . a different specification of logic may perfectly well be used in conjunction with the above definition of absolute aboutness.[22]

In this context, it is particularly worthwhile to examine Goodman's response to Thomas Patton:

> Thomas Patton's criticisms seem to me quite mistaken. On my view "All ravens are black" is about non-ravens since from (1) "(x)(Rx ⊃ Bx)", the statement (2) "(x)(-Bx ⊃ -Rx)" follows differentially with respect to non-ravens. Patton argues in effect that this depends upon replacing "-R" by a universally quantified variable in making the test; and this treatment of an expression containing a logical constant he finds objectionable. What he quite overlooks is that a statement T follows from S differentially with respect to k if and only if T, but not the universal generalization of T with respect to <u>any part</u> of the expression designating k in T, follows <u>logically</u> from S. Thus, <u>if we allow universal generalization with respect to non-logical constants only</u> [my italics], the fact that (2) but not "(Q)(x)(-Bx ⊃ -Qx)" follows from (1) means that (2) follows differentially with respect to non-ravens, and hence that (1) is about non-ravens.[23]

The use of the predicate variable 'Q' in these remarks by Goodman is further evidence that a second-order language is necessary for the expression and application of his theory of aboutness. What is perhaps most striking in the above response to Patton is that while Goodman is willing to allow that universal generalization is to be restricted

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[22]Goodman, "About," p. 254.

[23]Ibid., p. 242.

to nonlogical constants only, "(Q)(x)(-Bx ⊃ -Qx)" still counts as a generalization of "(x)(-Bx ⊃ -Rx)." It so counts simply because it is a generalization with respect to a "part" of "-R." The point to notice here is that "-R" does count as a designator for Goodman. Thus, not only does Goodman's theory appear to require the employment of a second-order language, it also appears to require the employment of a second-order language which countenances what might well be called <u>structured</u> predicates. Presumably, structured predicates include not only expressions such as "-R" but also any expression that can be obtained from unstructured predicates (predicate letters) by quantification and truth functional composition.

The above considerations would seem to suggest three things concerning the possibility of evaluating Goodman's proposed definition of absolute aboutness: (1) Goodman's use of the terms 'designates', 'follows logically from', and 'universal generalization' requires that a definite answer be given to the question of what sort of language is to be employed in the expression and application of his theory. Terms such as these have precise characterizations only with respect to certain formal languages. (2) If Goodman's scruples concerning the type of logic to be employed in the application of his definition are to be observed, then some means of "generalizing" on predicates that does not involve the leap to a second-order logic must be secured. (3) Goodman's implicit use of "structured" predicates requires that the language employed in the application of his theory have within it the capacity for constructing such predicates. While the notion of a structured singular term is old hat, structured predicates are something of a novelty. Thus, an explicit theory of structured predicates must be developed within whatever language is offered by way of an answer to the question expressed in (1).

Before proceeding to a discussion of the views held by the friends and foes of Goodman's theory, it is worth pointing out that a somewhat simpler version of it can be offered.[24] Let us say that a designator δ occurs essentially in a sentence ψ if and only if ψ does not imply its own generalization with respect to any designator part of δ. Then our new definition of 'absolutely about' can be put thusly:

D ϕ is absolutely about k if and only if ϕ is equivalent to some sentence ψ containing an essential ocurrence of a designator δ that designates k.

The equivalence of this definition to Goodman's can be established as follows. Suppose that ϕ is about k in the sense of Goodman. Then there is a sentence ψ and a designator δ satisfying the following conditions

(1) $\phi \Longleftrightarrow \psi$

(2) δ designates k

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[24]This alternative definition of 'absolutely about' is due to Herbert E. Hendry.

(3) δ occurs in ψ

(4) $\phi \not\Rightarrow (\Delta)\psi\delta'/\Delta$ for any designator part δ' of δ

($(\Delta)\psi\delta'/\Delta$ is the universal generalization of ψ with respect to δ', where $\psi\delta'/\Delta$ is the result of replacing every occurrence of δ' in ψ by Δ.)

Suppose, for a contradiction, that ϕ is not about k in the sense of D. Then for any sentence χ equivalent to ϕ which satisfies (2) and (3) we have

(5) $\chi \Rightarrow (\Delta)\chi\delta'/\Delta$ for some designator part δ' of δ

From (1) we have

(6) $\phi \Leftrightarrow (\phi \; \& \; \psi)$

Thence, from (5) and (6) we have

(7) $(\phi \; \& \; \psi) \Rightarrow (\Delta)(\phi \; \& \; \psi)\delta'/\Delta$

Whence, from (6) and (7) we have

$\phi \Rightarrow (\Delta)(\phi \; \& \; \psi)\delta'/\Delta$

Thence,

$\phi \Rightarrow (\Delta)\phi\delta'/\Delta$ and $\phi \Rightarrow (\Delta)\psi\delta'/\Delta$

which contradicts (4).

Suppose now that ϕ is about k in the sense of D. Then there is a sentence ψ and a designator δ that satisfies these conditions:

(1) $\phi \Leftrightarrow \psi$

δ designates k

δ occurs in ψ

(2) $\psi \not\Rightarrow (\Delta)\psi\delta'/\Delta$ for any designator part δ' of δ

That ϕ is about k in Goodman's sense can be seen immediately. For from (1) and (2) we have

$\phi \Longrightarrow (\Delta)\psi\delta'/\Delta$ for any designator part δ' of δ.

The interest of D over Goodman's definitions is simply that it provides for a somewhat smoother working theory than can be obtained by the use of Goodman's definition. Indeed, in the chapters that follow, where the focus of our investigation will be that of proving certain general theorems concerning aboutness, it will be our practice to utilize D instead of Goodman's definition. The equivalence just established insures that the results therein obtained will apply to Goodman's definition as well as our own.

Section Four: Goodman's Critics

In this section I would like to focus attention on five articles. The first, by Nicholas Rescher, is an attempt, albeit a poor one, to discredit Goodman's definition. The second and third, by Joseph Ullian, and Ullian and Putnam respectively, are attempts to provide a positive reconstruction of Goodman's intentions. The fourth, by Thomas Patton, is both a survey of the attempts by Ullian and Putnam, and also an attempt to survey a range of plausible reconstructions of Goodman's theory. As Patton ultimately concludes that reconstructing Goodman's theory is either hopeless, or hopelessly complicated, we must place Patton, along with Rescher, in the camp of Goodman's detractors. The fifth, by Pavel Tichý, is in part an attempt to provide a counterexample to

Goodman's definition.

The purpose of reviewing the works of Rescher, Ullian, Putnam, Patton, and Tichý is twofold. First, such a review will likely increase our appreciation of the intricacies involved in the notion of aboutness. Second, it will increase our appreciation of Goodman's definition. In section one it was noted that writers of such eminence as Russell, Frege, and Carnap seem to employ conceptions of aboutness which, upon critical reflection, appear naive at best. It is instructive in the present context to note that even when operating with a fairly explicit definition of 'about' authors such as the five mentioned above are still wont to make mistakes.

In his "A Note on About" Rescher objects to Goodman's definition on the grounds that:

> . . . a statement predicating any property to any individual a is (in Goodman's sense) absolutely about any other individual (say b).[25]

The argument for this claim depends upon the assumption that the universe of discourse in question contains at least two individuals. Such an assumption, Rescher argues, warrants the following rule of inference:

> (R) Given F(k), we may infer $(\exists x)[x = k^* \,\&\, F(x)]$, where F is any predicate, k any individual, and k* any individual different from k.

Rescher now asks us to compare the following pair of statements:

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[25]N. Rescher, "A Note on 'About,'" Mind 72 (1963): 268-70.

(S_1) ϕa
(T_1) $(\exists x)[(x = b) \& \phi x]$,

and to observe that

(i) T_1 contains an expression designating b;
(ii) T_1 follows logically from S_1 (in consequence of (R));
(iii) The universal generalization of T_1 with respect to 'b', namely
$(y)(\exists x)[(x = y) \& \phi x]$
does not follow from S_1, for if it did, then 'ϕa' would entail '$(\exists x)[(x = a) \& \phi x]$, which is patently not the case.

Goodman's response to Rescher's argument is twofold. First, he insists, rightly I think, that "follows logically from" be employed in its standard sense, viz., as concerned simply with non-null universes, not universes with at least two members. Secondly, he argues:

> . . . Professor Rescher has concealed a premise in the words "for any other individual". Either he must adopt some such general assumption as that no two individuals have the same name--a drastic assumption that is false for English--or he must admit "a = b" as an explicit premise. From "Pa . a = b" the statement "$(\exists x)(x = b \,.\, Px)$" does follow logically; but this, far from showing that "Pa" will be absolutely about every individual, yields only the unobjectionable result that "Pa" is about b, relative to the statement "a = b", or that "Pa . a = b" is absolutely about b.[26]

Goodman's argument appears to be conclusive.

In his 1962 paper "Corollary to Goodman's Explication of 'About'" Joseph Ullian offered a proof to the effect that if a statement S is absolutely about k, then the negation of S is also absolutely about k. In providing the proof, Ullian employed a first-order language

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[26]Goodman, "About," p. 273.

which contained only one predicate, a two-place predicate 'E', where 'Exy' is read as 'x is a member of y'. Thus satisfaction of at least some of the criteria necessary to a proper evaluation of Goodman's work seemed to be secured. He writes:

> Now regardless of its complexity S can be expressed set-theoretically. But since differential implication turns only on the logic of quantification theory and identity, we may signify membership by a 2-place predicate 'E' governed by no special axioms. (Example: 'The class of cows is amorphous and Hermione and Sappho are distinct cows' may be represented as 'Exw . Eyx . Exz x y ≠ z'.) Thus representation of S as a quantificational schema (possibly with identity) suffices for consideration of all questions of differential implication.[27]

The defectiveness of the last claim in the above is easily brought to light when one observes that it is a consequence of Ullian's analysis that a statement is about k only if it mentions k, i.e., contains a term that designates k, a claim explicitly denied by Goodman. The argument for this runs as follows: Suppose that S and T are such that S contains no designator that designates k while T does. Suppose further that $S \supset T$ is valid. Then $(x)(S \supset T)$ is valid, where this latter sentence is the universal generalization of $S \supset T$ with respect to the designator occurring in T that designates k. Therefore, $S \supset (x)T$ is also valid. Hence, S is not about k.

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[27] J. Ullian, "Corollary to Goodman's Explication of About," Mind 71:284:545.

This consequence is a result merely of the fact that in the language proposed by Ullian the only expressions that are allowed the status of a designator are occurrences of free variables within sentences. Thus, where '(x)(Exy ⊃ Exz)' represents "All cows are animals," '(x)(-Exz ⊃ -Exy)' would presumably represent "All non-animals are non-cows," where 'y' designates the class of cows and 'z' designates the class of animals. Unfortunately, nothing in '(x)(-Exz ⊃ -Exy)' corresponds <u>as a designator</u> to the English designator "non-cows." Thus, under Ullian's proposal, "All cows are animals" would not, contra Goodman, be about non-cows.

In a later paper, "More About 'About,'" after noting this defect of Ullian's earlier analysis, Ullian and Putnam offer two new interpretations. The first amounts to a reworking of Ullian's first approach. By adopting the following axiom schemata (a) $Er\bar{s} \equiv -Ers$, (b) $Er(s \cup t) \equiv (Ers \vee Ert)$, and (c) $Er(s \cap t) \equiv (Ers \cdot Ert)$, Ullian and Putnam are at least able to force 'E' to conform to Boolean laws. Since '$\bar{s}$', '$(s \cup t)$', and '$(s \cap t)$' are allowed the status of designators, the undesirable consequences of Ullian's earlier analysis are thus avoided.

Unfortunately, the above modifications of Ullian's analysis continue to leave untouched another serious problem. The difficulty in question stems largely from the apparent inability of Ullian's proposed language, either in its original or amended versions, to express relations in any very useful way. Perhaps the following considerations will make this clear. A statement such as "Hermione loves Sappho" ought, it would seem, to be understood as being about at least Hermione, Sappho, and love. Yet utilizing only the apparatus of Ullian's language, the symbolization of this statement yields the formula 'Exy' (Hermione is a member of the class of Sappho's lovers) which, though about the class of Sappho's lovers, is not thereby about love at all. One might, of course, reinterpret "Exy' as "The pair <Hermione, Sappho> is a member of the love relation" with 'x' designating the pair <Hermione, Sappho> and 'y' designating the love relation, i.e., {<x,y>: x loves y}. Such reinterpretation does yield the desired result that 'Exy' is about love, but, of course, only at the equally expensive cost of its not being about either Hermione or Sappho, but, rather, only about the pair <Hermione, Sappho>. The situation is yet worsened when we consider the notion of the converse of a relation. The statements "Hermione loves Sappho" and "Sappho is loved by Hermione" ought, given their equivalence, to be about the same things. In particular, they would seem to be about Hermione, Sappho, the love relation, and the converse of the love relation. Yet symbolization of the first statement as 'Exy' leaves us with no means of symbolizing the second short of the nonequivalent 'Ewz'.

In a second approach by Ullian and Putnam predicates as well as variables are viewed as designators (in the first approach, it will be remembered, even in the amended version, only variables are allowed this status) with truth functions of predicates viewed as designating Boolean compounds. They write:

> Now let us say that a quantificational schema S(P) containing the predicate letter 'P' implies its own generalization with respect to 'P' if S(P) $\supset$ S(Q) is valid, where 'Q' is a predicate letter not occurring in S(P), and S(Q) is the result of putting 'Q' for all occurrences of 'P' in S(P). Let us call a schema simple if it does not imply its own generalization with respect to any of its free variables or predicate letters.[28]

By an ingenious application of Craig's Interpolation Theorem Ullian and Putnam are able to prove that "Every quantificational schema is equivalent to a simple schema," which in turn enables them to offer the following "revised" definition of 'absolutely about':

> . . . S is absolutely about k if and only if the simple equivalents of S contain free occurrences of all the free variables and free occurrences of all the predicate letters that occur in some term designating k.[29]

Perhaps the first thing to notice concerning this proposal is that while it allows predicates the status of designators it involves no quantification over predicates or classes and so keeps us within the confines of first-order logic.

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[28] J. Ullian and H. Putnam, "More About 'About,'" Journal of Philosophy 62:12:307.

[29] Ibid., p. 307.

In his "Some Comments on "About"" Thomas Patton not only denies the above noted virtue of Ullian and Putnam's analysis, but seems to decry the very possibility of providing an adequate positive reconstruction of Goodman's theory. His criticism of Ullian and Putnam's analysis appears, however, to rest on a mistake. It is the same mistake that underlies his criticism of Goodman's theory proper. After noting some of the difficulties with Goodman's theory that we have already pointed out, Patton puts forth the following principle:

> (Q) Any U.G. of a valid formula is itself a valid formula.[30]

(Q), he claims, is violated by Putnam and Ullian's analysis

> For the U.G. of the valid $(x)(\exists y)(Ax \cdot By) \supset (\exists y)(x)(Ax \cdot By)$, for example, with respect to its subformula $Ax \cdot By$, would presumably be $(P)[(x)(\exists y)Pxy \supset (\exists y)(x)Pxy]$ or an equivalent, which is invalid in 2QT. Seeing no escape from this flaw, we must abandon the kind of U.G. in which it arises.
>
> This decision rules out one of the interpretations proposed by Putnam and Ullian. The authors, who make no reference to 2QT, are in effect using it, owing to the fact that where (P)T is a U.G. such that P is not free in S, $S \supset (P)T$ is valid in 2QT just in case S   T is valid in 1QT. (Thus there are obvious 1QT equivalents for conditions that I intend to frame in terms of 2QT.) On their scheme, in one of their examples, the U.G. with respect to $Bx \cdot Cx$ of the valid $(\exists x)(Bx \cdot Px \cdot Cx) \supset (\exists x)(Bx \cdot Cx)$ is $(\exists x)(Bx \cdot Px \cdot Cx) \supset (\exists x)Qx$ (or, put in terms of 2QT, $(Q)[(\exists x)(Bx \cdot Px \cdot Cx) \supset (\exists x)Qx]$, which is invalid. The authors not only adopt a kind of U.G. doomed not to satisfy (Q), but even admit and exploit such failures.[31]

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[30] T. Patton, "Some Comments on "About,"" Journal of Philosophy 62:12:319.

[31] Ibid., pp. 320-21.

Far from admitting or exploiting such a failure Ullian and Putnam commit none. Indeed, in their postcript to "More About 'About'" Ullian and Putnam are able to prove that under their analysis the extension of 'absolutely about' is the same whether we allow generalization with respect to compound terms or only with respect to lone predicate letters and free variables. The point to notice then concerning $(\exists x)(Bx \cdot Px \cdot Cx) \supset (\exists x)(Bx \cdot Cx)$ is that there are three U.G.s of this formula with respect to $Bx \cdot Cx$ when it comes to the question of differential implication, the one noted by Patton, $(\exists x)(Qx \cdot Px \cdot Cx) \supset (\exists x)(Qx \cdot Cx)$ and $(\exists x)(Bx \cdot Px \cdot Qx) \supset (\exists x)(Bx \cdot Qx)$. These latter formulae, as well as their 2QT versions, viz., $(Q)[(\exists x)(Qx \cdot Px \cdot Cx) \supset (\exists x)(Qx \cdot Cx)]$ and $(Q)[(\exists x)(Bx \cdot Px \cdot Qx) \supset (\exists x)(Qx \cdot Cx)]$, are, of course, valid. What Ullian and Putnam have shown is that only such U.G.s as these latter two need be considered for deciding questions of differential implication, and, hence, of absolute aboutness. Patton's criticisms and the work of Ullian and Putnam thus illustrate the subtlety and intricacy of Goodman's requirement of selectivity, viz., that the generalization of T with respect to an expression occurring in it that designates k, or any part of that expression, not also follow from S.

In his "What Do We Talk About?" Pavel Tichý, after expositing Goodman's definition of absolute aboutness claims:

> Goodman's requirement that T be generalizable with respect to no part of the expression designating k seems to me rather intuitively undermotivated.[32]

On the contrary, as we have already shown, ignoring this requirement results in the untenable view that any sentence about anything is about everything.

At any rate, Tichý now proceeds to argue that, even with this requirement, a sentence such as "Chicago is a city" is not only about Chicago, but about any city on the globe. He argues:

> To begin with, let us consider the class L of terrestrial coordinates with integral values of degrees and minutes. L is thus a finite class of items like <30.58N49.01E>, <68.64S172.12W>, etc. Each city t is geographically located at a unique element--call it $\ell(t)$--of L. For instance, ℓ(Chicago) = <41.50W87.45W>.
>
> Let us fix a linear ordering of L, say the lexicographic one. Thus for any two members a and b of L, we have a < b or b < a. Now we define three binary relations as follows:
>
> $$x <^* y \equiv \text{df. } C(x) \;\&\; C(y) \;\&\; x = y \;\&\; \ell(x) < \ell(y) \;\&\; -(\exists z)[C(z) \;\&\; \ell(x) < \ell(z) < \ell(y)]$$
>
> (where C is the predicate 'is a city').
>
> $$R_F\ (x,y) \equiv \text{df. } x <^* y \vee [x=y \;\&\; -(\exists z) x <^* z],$$
> $$R_B\ (x,y) \equiv \text{df. } y <^* x \vee [x=y \;\&\; -(\exists z) x <^* z].$$
>
> Now consider an arbitrary city Y other than Chicago. We shall show that according to (D4) [Goodman's definition of 'absolutely about'],
>
> (7) C(Chicago)
>
> is about Y.
>
> Case 1: ℓ(Chicago) < ℓ(Y). It is easy to see that there are cities x_1, x_2, ..., x_n such that Y (as a value of y satisfies.

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[32]P. Tichý, "What Do We Talk About?" Philosophy of Science 42:1 (March 1975):89.

(Φ) R_FChicago, x_1) & $R_F(x_1, x_2)$ & . . . & $R_F(x_{n-1}, x_n)$ & $R_F(x_n, y)$.

Moreover, for no city y other than Y it is possible to find x_1, . . ., x_n satisfying Φ. In consequence, the term

(Φ*) $(\iota y)(\exists x_1)(\exists x_2)\ . . .\ (\exists x_n)\Phi$

designates Y.

It is not difficult to show that

(8) $(\exists z)R_F(\Phi^*, z)$

follows logically from (7). Moreover, observe that 'R_F', 'Chicago', and Φ* itself are the only components of Φ* generalizable upon in (8). Writing $\Phi^*(R_F$, Chicago) for Φ*, the three generalizations take the respective forms

$(\forall r)(\exists z)r(\Phi^*(r, \text{Chicago}), z)$,
$(\forall u)(\exists z)R_F(\Phi^*(R_F, u), z)$, and
$(\forall f)(\exists z)R_F(f, z)$

But none of these follows logically from (7). Thus (8) follows from (7) differentially with respect to Y.

Case 2: $\ell(Y) < \ell(\text{Chicago})$. We get a parallel result by using R_B in lieu of R_F.

In either case, (7) is absolutely about the arbitrary city Y.[33]

The first thing one notices concerning Tichý's argument is that (8), if expanded into primitive notation, is monstrously long. The following is, e.g., only a partial such expansion:

(9) $(\exists z)\{\{C(\Phi^*)\ \&\ C(z)\ \&\ \Phi^* = z\ \&\ \ell(\Phi^*) < \ell(z)\ \&\ -(\exists y)[C(y)\ \&\ \ell(\Phi^*) < \ell(z)]\}\ v\ [\Phi^* = z\ \&\ -(\exists y)\Phi^* <^* y]\}$

The second thing to notice is that Tichy's claim that (8) follows logically from (7) is simply false. (8) follows, not from (7) alone, but only in conjunction

[33]Ibid., pp. 89-90.

with the definitions of '<*' and 'R_F'. At best Tichý has shown that the conjunction of (7) with these definitions is about any city on the globe. To put the matter somewhat differently, all that Tichý has shown is that (7) is about any city on the globe <u>relative</u> to the definitions of '<*' and 'R_F'. <u>This</u> claim, far from showing a defect in Goodman's theory, serves only to support it.

CHAPTER II

ABOUTNESS ANALYZED

Section One: Structured Predicates and Second-Order Logic

In the developments that follow we are to envisage a second-order language L. The characteristic feature of *L* is that it contains *structured* predicates in addition to the usual apparatus of predicate constants and predicate variables. Syntactically, the relationship between structured predicates and unstructured predicates is somewhat analogous to the relationship between definite descriptions (structured singular terms) and names (unstructured singular terms), viz., structured predicates, like structured singular terms, may contain logical, as well as extralogical vocabulary, as parts.

Structured predicates occur, of course, not only in artificial languages like *L*, but in natural languages as well. Compare, e.g., the following sentences:

(1) Alice is pretty and Alice is blonde

(2) Alice is a pretty blonde

Normally, of course, we regard (1) and (2) as equivalent. Syntactically, however, (1) appears to be the result of attaching the predicate 'is pretty' to the name 'Alice', and conjoining the result of that operation with the result of attaching the predicate 'is blonde' to the name 'Alice'. That is to say, <u>two</u> predicates appear to be involved in the generation of (1). (2), on the other hand, appears to be the result of attaching the <u>single</u> structured predicate 'is a pretty blonde' to the name 'Alice'. Translating (1) and (2) into some first-order quantificational language, say $\mathcal{L}$ of Mates' *Elementary Logic*, results in something like the following:

(3) (Pa & Ba)

If one were to give a syntactical description of (3), it would, no doubt, correspond to that provided for (1), and not that provided for (2).

The use of structured predicates in semantical investigations is, of course, not a novelty. Carnap, e.g., employed them in his *Introduction to Symbolic Logic*,[34] though not quite in the way that they will be employed here. Leonard, in his presidential address to the American Philosophical Association, "Essences, Attributes, and Predicates,"[35] also displays an awareness of their importance. He writes:

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[34] R. Carnap, *Introduction to Symbolic Logic* (New York: Dover Press, 1958), pp. 106-7.

[35] H. S. Leonard, "Essences, Attributes, and Predicates," *Proceedings and Addresses of the American Philosophical Association*, vol. XXXVIII (October 1964), pp. 31-32.

One binding operator that is not so often recognized, but that would do much to systematize discourse is an abstractor which we might call the simple abstractor, and which we might render in English by 'x such that'. When prefixed to an appropriate open singular sentence, the result is an attributive term. For example,

x such that either x is a horse or x is a dog

is an attributive term.

With the help of this attributive we could, for example, record the subject-predicate statements

Bucephalus is an x such that either x is a horse or x is a dog

and

Falla is an x such that either x is a horse or x is a dog.

The advantage of this abstractor and of the attributive terms which it generates is that with its assistance, we may convert sentences which are not subject-predicate sentences into synonymous subject-predicate sentences. For the above subject-predicate statements are synonymous with

Either Bucephalus is a horse or Bucephalus is a dog

and

Either Falla is a horse or Falla is a dog.

These latter statements are not subject-predicate sentences, but rather disjunctions of subject-predicate sentences.

Accordingly, our formalized language L will contain a counterpart to the binding operator which Leonard has herein called the "simple abstractor." The inclusion of such apparatus within L will thus allow us to achieve what Mates' does not, viz., the construction of structured predicates. The precise characterization of L's version of that apparatus, and therewith the syntactical and semantical characterization of L's structured predicates, is a matter that will have to be taken up in sections two and three.

Historically, semantical investigations of second-order logic have followed one of two traditions. In the first tradition the notions of an interpretation and of a logically true sentence are defined analogously to the manner in which these concepts are defined for first-order languages. By an L-true sentence of a second-order language is meant one which is true whenever the individual variables are interpreted as ranging over an arbitrary non-empty set, or domain, of individuals, while the predicate variables of degree n range over all sets of ordered n-tuples of members of the domain. Under this definition of L-truth, we must conclude from Gödel's results that the set of second-order logical truths is not axiomatizable. In the second tradition the notion of an interpretation is modified so that the predicate variables of degree n are allowed to range over certain "well-behaved" classes of ordered n-tuples of members of the domain. The manner in which these classes must be "well-behaved" is well expressed by Henkin:

> These classes cannot really be taken in an arbitrary manner, if every formula is to have an interpretation. For example, if the formula F(x) is interpreted as meaning that x is in the class F, then ~F(x) means that x is in the complement of F; hence the range for functional variables such as F should be closed under complementation. Similarly, if G refers to a set of ordered pairs in some model [interpretation], then the set of individuals x satisfying the formula $(\exists y)G(x,y)$ is a projection of the set G; hence we require that the various domains be closed under projection. In short each method of compounding formulas of the calculus has associated with it some operations on the

domains of a model, with respect to which the domains must be closed.[36]

As is well known, Henkin has demonstrated that a semantics based on this second tradition can be shown to yield a completeness theorem for second-order logic. However, it is a completeness with a price. The interpretations allowed for in this latter tradition seem, to some extent, "unnatural" or ad hoc. Indeed, in Computability and Logic Boolos and Jeffrey go so far as to reserve the expression "real second-order logic" only for those calculi whose interpretations are "full" in the sense of Robbin's Mathematical Logic: A First Course,[37] i.e., for those calculi whose semantics are based on the first tradition. Accordingly, our approach will be to follow the lead of Boolos and Jeffrey and, hence, to base our semantics for L on the first tradition. This decision need not be a matter of concern to those with a penchant for completeness theorems. As will become clear in section four the results to be obtained concerning aboutness can be obtained even with a semantics based on the second of the two traditions lately discussed. Indeed, as will become clear at the end of this chapter, our results can be obtained even within certain first-order fragments of L.

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[36] L. Henkin, "Completeness in the Theory of Types," Journal of Symbolic Logic 15 (1950):81.

[37] G. Boolos and R. Jeffrey, Computability and Logic (London: Cambridge University Press, 1975), p. 198; J. Robbin, Mathematical Logic: A First Course (New York: W. A. Benjamin, Inc., 1969), p. 140.

Section Two: Language L: Syntax

The primitive vocabulary of *L* consists of a countably infinite number of individual variables, a countably infinite number of individual constants or names, a countably infinite number of predicate variables of each degree, a countably infinite number of predicate letters (constants) of each degree, a one-place connective '-', a two-place connective '&', the identity sign '=', a quantifier key '∃', grouping indicators and the symbol ':'. The notions of a predicate of *L* and of a formula of *L* are then defined simultaneously as follows (references to *L* are suppressed for the sake of brevity):

(1) If Θ is an n-place predicate letter or predicate variable, then Θ is an n-place predicate.

(2) If ϕ is a formula and $\alpha_1, \ldots, \alpha_n$ are n distinct individual variables, then $(\alpha_1 \ldots \alpha_n{:}\ \phi)$ is an n-place predicate.

(3) If Θ is an n-place predicate and $\beta_1, \ldots, \beta_n$ are terms (names or individual variables), then $\Theta\beta_1 \ldots \beta_n$ is a formula.

(4) If β_1 and β_2 are terms, then $\beta_1 = \beta_2$ is a formula.

(5) If ϕ is a formula, then so is $-\phi$.

(6) If ϕ and ψ are formulas, then so is $(\phi \ \& \ \psi)$.

(7) If ϕ is a formula and α a variable, then $(\exists\alpha)\phi$ is a formula.[38]

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[38] The syntactical and semantical account of structured predicates of *L* is due to Herbert E. Hendry.

Predicates defined under clause (1) will be referred to as unstructured predicates, those defined under clause (2) will be referred to as structured predicates. We say that an occurrence of a variable α in a predicate or formula is bound if and only if the occurrence falls within a part of Ω that belongs to one of the following types: (1) $(\exists\alpha)\phi$, or (2) $(\alpha_1 \ldots \alpha_n\colon \phi\alpha)$ where $\alpha = \alpha_i$ for some i $(1 \leq i \leq n)$. Occurrences of variables that are not bound are free. Thus we see, in agreement with Leonard, that the operation of forming structured predicates is a variable binding operation. A predicate is open if and only if some variable occurs free in it; a predicate is closed if it is not open. A sentence is a formula in which no variable has a free occurrence.

In consequence, then, expressions such as '(x: Fx)', '(xy: Fx & Gy)', '(xyz: Fxy & Gyz)', '(x: Fy)', '(x: Yx)', count as structured predicates and '(x: Fx)a', '(xy: Fx & Gy)ab', '(xyz: Fxy & Gyz)abc' count as sentences. Note that neither '(x: Fy)a' nor '(x: Yx)a' count as sentences in virtue of the free occurrences of 'y' and 'Y'.

As can be seen from the above, our language L countenances only five kinds of sentences, viz., unstructured predications (these understood so as to include identity claims), structured predications, negations, conjunctions, and existential quantifications. Universal quantifications, disjunctions, conditionals, and biconditionals are then to be understood as having been definitionally introduced into L in accordance with some standard scheme of abbreviating such sentences in terms of those we already have at hand.

Rounding out our syntactical account of L we say now that a designator is any name, or closed predicate containing at least one occurrence of a name or predicate letter.[39]

In addition to the above syntactical account of L, the following notion will prove particularly useful. Where ϕ is any formula, $\alpha_1, \ldots, \alpha_n$ are distinct variables and $\delta_1, \ldots, \delta_n$ are designators which satisfy the condition that each δ_i is of the same category as α_i $(1 \leq i \leq n)$, $\phi\alpha_1 \ldots \alpha_n/\delta_1 \ldots \delta_n$ is the result of replacing each free occurrence of α_1 in ϕ by δ_1, ..., and each free occurrence of α_n in ϕ by δ_n. (δ_i and α_i belong to the same category if and only if either δ_i is a name and α_i is an individual variable or δ_i is a closed predicate and α_i is a predicate variable of the same degree as α_i).

Section Three: Language L: Semantics

By an interpretation I of L we understand an ordered couple <D,f> where D is any nonempty set and f is a function that

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[39]The motivation for requiring that a closed predicate contain at least one occurrence of a name or predicate letter, in order that it be counted as a designator, will become clear in section seven of this chapter.

(1) associates with each name of L a member of D, and

(2) associates with each n-place predicate letter of L an n-ary relation on D.

If Δ is any set of names or predicate letters and I and I' are arbitrary interpretations, then I' is a Δ-variant of I if and only if I' is exactly like I except (possibly) for what it assigns to the members of Δ.

With these notions at hand we are now in a position to offer simultaneous definitions of the notions of the designation of a designator ($des_I[\delta]$, I = <D,f>) under an interpretation and truth under an interpretation. Let δ by any designator, ϕ any sentence, and I any interpretation, then

(1) If δ is a name, $des_I[\delta] = f(\delta)$

(2) If δ is a predicate letter, then $des_I[\delta] = f(\delta)$

(3) If $\delta = (\alpha_1 \ldots \alpha_n\colon \psi)$ and $o_1, \ldots, o_n$ are members of D, then $\langle o_1, \ldots, o_n\rangle \in des_I[\delta]$ if and only if $\psi\alpha_1 \ldots \alpha_n/\beta_1 \ldots \beta_n$ is true under that $\{\beta_1, \ldots, \beta_n\}$-variant of I that assigns o_1 to β_1, ..., and o_n to β_n, where $\beta_1, \ldots, \beta_n$ are the first n alphabetically earliest names new to ψ.

(4) If $\phi = \Theta\beta_1 \ldots \beta_n$, then ϕ is true under I if and only if $\langle f(\beta_1), \ldots, f(\beta_n)\rangle \in des_I[\Theta]$.

(5) If $\phi = \beta_1 = \beta_2$, then ϕ is true under I if and only if $f(\beta_1) = f(\beta_2)$.

(6) If $\phi = -\psi$, then ϕ is true under I if and only if ψ is

not true under I.

(7) If $\phi = (\psi \& \chi)$, then ϕ is true under I if and only if both ψ and χ are true under I.

(8) If $\phi = (\exists\alpha)\psi$, then ϕ is true under I if and only if $\psi\alpha/\beta$ is true under some $\{\beta\}$-variant of I, where β is either the alphabetically earliest name not already occurring in ψ if α is an individual variable, or the alphabetically earliest n-place predicate letter not already occurring in ψ, if α is an n-place predicate variable.[40]

Finally, then, we say that a sentence is false under an interpretation if and only if it is not true under that interpretation.

The logical concepts of implication, equivalence, consequence, etc., are defined in the usual ways. For example, a sentence ϕ is said to imply a sentence ψ, $\phi \Longrightarrow \psi$, if and only if there is no interpretation under which ϕ is true and ψ is false. The symbol ' $\Longleftrightarrow$ ' will be used as a sign of equivalence between sentences, and '$\models$' will represent the consequence relation.

As was announced in section three of chapter I, our policy will be to employ a definition of 'absolutely about' different from, but equivalent to, that of Goodman's.

[40]Throughout this essay interpretations will be represented by upper case 'I' followed by zero or more occurrences of apostrophes. Thus, I = <D, f>, I' = <D', f'>, etc. For simplicity, it is assumed that the domain of an interpretation is a set of individuals.

As that alternative definition depended on the notion of an essential occurrence of a designator in a sentence, we offer now the following definition. Let ϕ be a sentence and δ a closed designator.

> δ occurs <u>essentially</u> in ϕ if and only if δ occurs in ϕ and $\phi \Longrightarrow (\alpha)\phi\delta'/\alpha$, where δ' is any designator part of δ and α is an individual variable if δ' is a name and an n-place predicate variable if δ' is an n-place predicate.

A designator is said to occur <u>vacuously</u> in a sentence if and only if it occurs in that sentence and does not occur essentially in that sentence. Roughly, an essential occurrence of a designator is one whose designation under an interpretation affects the truth value of the sentence in which it occurs under that interpretaion; a vacuous occurrence of a designator is one whose designation under an interpretation does not affect the truth value of the sentence in question under that interpretation. The latter claim can be easily seen from the following argument. Suppose that δ occurs vacuously in ϕ. Thus, $\phi \Longrightarrow (\alpha)\phi\delta'/\alpha$, where δ' is some closed designator part of δ. By universal instantiation we have $(\alpha)\phi\delta'/\alpha \Longrightarrow \phi$. Clearly then $\phi \Longleftrightarrow (\alpha)\phi\delta'/\alpha$. Since δ does not occur in $(\alpha)\phi\delta'/\alpha$, the designation of δ under an interpretation cannot affect the truth value of $(\alpha)\phi\delta'/\alpha$ under an interpretation. Since this latter sentence is equivalent to ϕ, the designation of δ under an interpretation does not affect the truth value of ϕ under an interpretation either. To put the matter succinctly, any sentence in which a designator occurs only vacuously is equivalent to one in which that designator does not occur at all.

Section Four: A Theorem Concerning Structured Predicates

Having completed our semantical account of L we are now in a position to prove the following important theorem concerning structured predicates:

THEOREM 1. The following three conditions are equivalent:

(1) $(\exists\alpha_1 \ldots \alpha_n)(\alpha_1 = \beta_1 \;\&\; \ldots \;\&\; \alpha_n = \beta_n \;\&\; \phi)$

(2) $\phi\alpha_1 \ldots \alpha_n/\beta_1 \ldots \beta_n$

(3) $(\alpha_1 \ldots \alpha_n\!: \phi)\beta_1 \ldots \beta_n$

PROOF. The proof of this claim will proceed for the case n = 1; it will be clear that the proof can be generalized. The strategy for this proof will be to demonstrate that (1) $\Rightarrow$ (2), (2) $\Rightarrow$ (3), and (3) $\Rightarrow$ (1).

To prove that (1) $\Rightarrow$ (2) (i.e., $(\exists\alpha)(\alpha = \beta \;\&\; \phi) \Rightarrow \phi\alpha/\beta$), assume first that $(\exists\alpha)(\alpha = \beta \;\&\; \phi)$ is true under some interpretation I. Then, where γ is the alphabetically earliest name new to ϕ and distinct from β $(\alpha = \beta \;\&\; \phi)\alpha/\gamma$ is true under some $\{\gamma\}$-variant of I, say I'. As I' is a $\{\gamma\}$-variant of I and not a $\{\beta\}$-variant of I, it follows that $f'(\gamma) = f(\beta)$; thus I and I' agree in their assignments to the designators that occur in ϕ; therefore, $\phi\alpha/\beta$ is true under I.

To prove that (2) $\Rightarrow$ (3) (i.e., $\phi\alpha/\beta \Rightarrow (\alpha: \phi)\beta$), assume that $\phi\alpha/\beta$ is true under some I. Then, by clause (3) of the definition of "designation under an interpretation," we have that $f(\beta) \in des_I[(\alpha: \phi)]$. Hence, by clause (4) of the definition of "truth under an interpretation," we have that $(\alpha: \phi)\beta$ is true under I.

To prove that (3) $\Rightarrow$ (1) (i.e., $(\alpha: \phi)\beta \Rightarrow (\exists\alpha)(\alpha = \beta \;\&\; \phi)$), assume first that $(\alpha: \phi)\beta$ is true under some I. Hence $f(\beta) \in des_I[(\alpha: \phi)]$. Thus, we have then that $\phi\alpha/\beta$ is true under that $\{\beta\}$-variant of I that assigns $f(\beta)$ to β. But as $(\beta = \beta \;\&\; \phi\alpha/\beta)$ is clearly also true under that interpretation, it follows that $(\exists\alpha)(\alpha = \beta \;\&\; \phi)$ is true under I.

Our proof of theorem 1 signals a noteworthy result. Having successfuly contextually defined structured predicates, we thus have at our disposal the means of eliminating (or introducing) structured predicates from (into) any sentences in which they occur (do not occur).

Section Five: Aboutness Defined

The introduction of our definition of absolute aboutness now takes place as follows. In compliance with the decision announced in section three of chapter I we utilize not Goodman's definition, but, rather, the definition therein proved equivalent to Goodman's. We suppose first that some fixed, but unspecified, interpretation of L has already been provided. In what follows references to that interpretation and to L will be suppressed for the sake of brevity. Let ϕ and ψ be sentences:

> ϕ is about k if and only if there is a ψ such that $\phi \Leftrightarrow \psi$ and ψ contains an essential occurrence of a designator δ that designates k.

With this definition at hand, and given that '(x)(Rx → Bx)' $\Leftrightarrow$ '(y)[(x: Rx & Bx)y v (x: -Rx & Bx)y v (x: -Rx & -Bx) y]', we see then that '(x)(Rx → Bx)' (our translation of "All ravens are black") is about not only the class of ravens and the class of black things but also about the class of black ravens, the class of black non-ravens, and the class of non-black non-ravens as well. This result is, of course, quite in compliance with Goodman's conceptions.

A claim not advocated by Goodman, but one that seems quite in harmony with intuitive conceptions of aboutness, is the following. If a sentence is about a relation, then it is about the converse of that relation. It will be recalled that one of the shortcomings of Ullian's analysis was the inability of the language proposed by him to systematically designate the converse of any relation that had previously been designated in that language. That a proof of the above claim is available for L therefore marks an advance.

Suppose, then that ϕ is about a binary relation ρ. Hence, there is a θ and a ψ such that des[θ] = ρ, $\psi \Leftrightarrow \phi$, and (1) $\psi \not\Leftrightarrow (\Delta)\psi\delta/\Delta$, for any designator part δ of θ.

Let $\psi' = (\psi \;\&\; -(\alpha\gamma\colon \Theta\gamma\alpha)\beta_1\beta_2)$. Note that $(\alpha\gamma\colon \Theta\gamma\alpha)$ designates the converse of des[Θ]. Clearly either (2) $\psi' \Leftrightarrow \psi$ or (3) $\psi' \not\Leftrightarrow \psi$. Suppose (2) and, for a contradiction, that $(\alpha\gamma\colon \Theta\gamma\alpha)$ does not occur essentially in ψ', i.e., $\psi' \Rightarrow (\Delta)\psi'\delta/\Delta$ for some δ a designator part of $(\alpha\gamma\colon \Theta\gamma\alpha)$. By (2), then $\psi \Rightarrow (\Delta)\psi'\delta/\Delta$, whence (4) $\psi \Rightarrow (\Delta)\psi\delta/\Delta$ and (5) $\psi \Rightarrow (\Delta)-(\alpha\gamma\colon \Theta\gamma\alpha)\beta_1\beta_2\delta/\Delta$. If δ is a proper part of $(\alpha\gamma\colon \Theta\gamma\alpha)$, then (4) contradicts (1). So suppose that δ is improper, i.e., $\delta = (\alpha\gamma\colon \Theta\gamma\alpha)$. Since $(\Delta)-(\alpha\gamma\colon \Theta\gamma\alpha)\beta_1\beta_2\delta/\Delta \Rightarrow -(\alpha\gamma\colon \Theta\gamma\alpha \vee -\Theta\gamma\alpha)\beta_1\beta_2$, and since this latter sentence is inconsistent, we have from (5) that ψ is inconsistent. But we know from (1) that this is impossible.

So suppose (3) and let $\chi = (\Theta\beta_2\beta_1 \rightarrow (\alpha\gamma\colon \Theta\gamma\alpha)\beta_1\beta_2)$. Clearly χ is L-true, hence $\psi \Leftrightarrow (\psi \;\&\; \chi)$. Suppose that $(\alpha\gamma\colon \Theta\gamma\alpha)$ does not occur essentially in $(\psi \;\&\; \chi)$, i.e., $(\psi \;\&\; \chi) \Rightarrow (\Delta)(\psi \;\&\; \chi)\delta/\Delta$. Thus, we have (6) $\psi \Rightarrow (\Delta)\phi\delta/\Delta$ and (7) $\psi \Rightarrow (\Delta)\chi\delta/\Delta$. (6) contradicts (1), if δ is a proper part of $(\alpha\gamma\colon \Theta\gamma\alpha)$. So suppose that δ is improper, i.e., $\delta = (\alpha\gamma\colon \Theta\gamma\alpha)$. Since $(\Delta)\chi\delta/\Delta \Rightarrow (\Theta\beta_2\beta_1 \rightarrow (\alpha\gamma\colon \Theta\alpha\gamma \;\&\; -\Theta\alpha\gamma)\beta_1\beta_2)$, it follows from (7) that $(\psi \;\&\; \Theta\beta_2\beta_1) \Rightarrow (\alpha\gamma\colon \Theta\alpha\gamma \;\&\; -\Theta\alpha\gamma)\beta_1\beta_2)$. Since $(\alpha\gamma\colon \Theta\alpha\gamma \;\&\; -\Theta\alpha\gamma)\beta_1\beta_2$ is inconsistent, it follows that $\psi \Rightarrow -\Theta\beta_2\beta_1$. By theorem 1, then $\psi \Rightarrow -(\alpha\gamma\colon \Theta\alpha\gamma)\beta_2\beta_1$. But note that $-(\alpha\gamma\colon \Theta\alpha\gamma)\beta_2\beta_1 \Rightarrow -(\alpha\gamma\colon \Theta\gamma\alpha)\beta_1\beta_2$. Thus, $\psi \Rightarrow -(\alpha\gamma\colon \Theta\gamma\alpha)\beta_1\beta_2$. But this is so only if $\psi' \Leftrightarrow \psi$, which contradicts (3). Hence $(\alpha\gamma\colon \Theta\gamma\alpha)$ occurs essentially in ψ', if $\psi' \Leftrightarrow \psi$, or it occurs essentially in $(\psi \;\&\; \chi)$, if $\psi' \not\Leftrightarrow \psi$. In any case,

$(\alpha\gamma\colon \Theta\gamma\alpha)$ occurs essentially in some sentence equivalent to ψ and thus to ϕ. Therefore, ϕ is about des[$(\alpha\gamma\colon \Theta\gamma\alpha)$], i.e., ϕ is about the converse of ρ.

Actually, the above claim, along with the earlier results concerning "All ravens are black," can be derived as corollaries of a rather general theorem to the effect that the test for aboutness can be successfully applied by considering only those generalizations on the sentence in question which are generalizations with respect to just the unstructured parts of the designator in question. The proof of that theorem requires the use of several lemmas to which we now turn our attention.

Section Six: The Main Theorem

In this section several lemmas are established that will enable us to prove the theorem whose description is the title of this section.

LEMMA 1. If Θ is an n-place predicate occurring essentially in ϕ, then $(\alpha_1 \ldots \alpha_n\colon \Theta\alpha_1 \ldots \alpha_n)$ occurs essentially in some $\phi' \Leftrightarrow \phi$.

PROOF. We prove this for the case n = 1; it will be clear that the proof can be generalized. Assume first that Θ occurs essentially in ϕ. Thus,

(1) $\phi \not\Rightarrow (\Delta)\phi\delta/\Delta$ for any δ a designator part of Θ.

Let $\phi' = [\phi \;\&\; -(\alpha\colon \Theta\alpha)\beta]$. Clearly either (2) $\phi' \Leftrightarrow \phi$ or (3) $\phi' \not\Leftrightarrow \phi$. Suppose (2) and, for a contradiction, that $(\alpha\colon \Theta\alpha)$ does not occur essentially in ϕ', i.e., $\phi' \Rightarrow$

$(\Delta)\phi'\delta/\Delta$ for some δ a designator part of $(\alpha\colon \Theta\alpha)$. From (2), then, $\phi \Longrightarrow (\Delta)\phi'\delta/\Delta$; whence (4) $\phi \Longrightarrow (\Delta)\phi\delta/\Delta$ and (5) $\phi \Longrightarrow (\Delta)-(\alpha\colon \Theta\alpha)\beta\delta/\Delta$. (4) contradicts (1), if δ is a proper part of $(\alpha\colon \Theta\alpha)$. So suppose that δ is improper, i.e., $\delta = (\alpha\colon \Theta\alpha)$. Since $(\Delta)-(\alpha\colon \Theta\alpha)\beta\delta/\Delta \Longrightarrow -(\alpha\colon \Theta\alpha \vee -\Theta\alpha)\beta$, we have from (5) that $\phi \Longrightarrow -(\alpha\colon \Theta\alpha \vee -\Theta\alpha)\beta$. But since this latter sentence is inconsistent, ϕ must be so also. However, given (1) this is impossible.

So suppose (2) and let $\chi = (\Theta\beta \rightarrow (\alpha\colon \Theta\alpha)\beta)$. Clearly χ is L-true; hence $\phi \Longleftrightarrow (\phi \;\&\; \chi)$. Suppose, for a contradiction, that $(\alpha\colon \Theta\alpha)$ does not occur essentially in $(\phi \;\&\; \chi)$, i.e., $(\phi \;\&\; \chi) \Longrightarrow (\Delta)(\phi \;\&\; \chi)\delta/\Delta$ for some δ a designator part of $(\alpha\colon \Theta\alpha)$. Thus, $\phi \Longrightarrow (\Delta)(\phi \;\&\; \chi)\delta/\Delta$; whence (6) $\phi \Longrightarrow (\Delta)\phi\delta/\Delta$ and (7) $\phi \Longrightarrow (\Delta)\chi\delta/\Delta$. (6) contradicts (1), if δ is a proper part of $(\alpha\colon \Theta\alpha)$. So suppose that δ is improper, i.e., $\delta = (\alpha\colon \Theta\alpha)$. Since $(\Delta)\chi\delta/\Delta \Longrightarrow (\Theta\beta \rightarrow (\alpha\colon \Theta\alpha \;\&\; -\Theta\alpha)\beta)$, we have from (7) that $\phi \Longrightarrow (\Theta\beta \rightarrow (\alpha\colon \Theta\alpha \;\&\; -\Theta\alpha)\beta)$. Hence, also, $(\phi \;\&\; \Theta\beta) \Longrightarrow (\alpha\colon \Theta\alpha \;\&\; -\Theta\alpha)\beta$. Since this latter sentence is inconsistent, we have that $(\phi \;\&\; \Theta\beta)$ is inconsistent. By theorem 1 it follows that $(\phi \;\&\; (\alpha\colon \Theta\alpha)\beta)$ is inconsistent. Thus $\phi \Longrightarrow -(\alpha\colon \Theta\alpha)\beta$. But clearly this is so only if $\phi \Longleftrightarrow \phi'$, which contradicts (3). Hence, $(\alpha\colon \Theta\alpha)$ occurs essentially in ϕ', if $\phi \Longleftrightarrow \phi'$, or it occurs essentially in $(\phi \;\&\; \chi)$, if $\phi \not\Longleftrightarrow \phi'$. In any case, $(\alpha\colon \Theta\alpha)$ occurs essentially in some sentence equivalent to ϕ.

LEMMA 2. If ψ is a formula and $(\alpha_1 \ldots \alpha_n\colon \psi)$ occurs essentially in ϕ, then $(\alpha_1 \ldots \alpha_n\colon -\psi)$ occurs essentially in some $\phi' \Leftrightarrow \phi$.

PROOF. Again, we prove this for the case n = 1. Assume first that $(\alpha\colon \psi)$ occurs essentially in ϕ. Thus, (1) $\phi \not\Rightarrow (\Delta)\phi\delta/\Delta$ for any δ a designator part of $(\alpha\colon \psi)$. Let $\phi' = (\phi \ \& \ -(\alpha\colon -\psi)\beta)$. Clearly either (2) $\phi' \Leftrightarrow \phi$ or (3) $\phi' \not\Leftrightarrow \phi$. Suppose (2) and, for a contradiction, that $(\alpha\colon -\psi)$ does not occur essentially in ϕ', i.e., $\phi' \Rightarrow (\Delta)(\phi \ \& \ -(\alpha\colon -\psi)\beta)\delta/\Delta$ for some δ a designator part of $(\alpha\colon -\psi)$. By (2) we have $\phi \Rightarrow (\Delta)(\phi \ \& \ -(\alpha\colon -\psi)\beta)\delta/\Delta$; hence (4) $\phi \Rightarrow (\Delta)\phi\delta/\Delta$ and (5) $\phi \Rightarrow (\Delta)-(\alpha\colon -\psi)\beta\delta/\Delta$. If δ is a proper part of $(\alpha\colon -\psi)$, then (4) contradicts (1). So suppose that δ is improper, i.e., $\delta = (\alpha\colon -\psi)$. Since $(\Delta)-(\alpha\colon -\psi)\beta\delta/\Delta \Rightarrow -(\alpha\colon \Theta\alpha \vee -\Theta\alpha)\beta$, and since this latter sentence is inconsistent, we have by (5) that ϕ is inconsistent. But this contradicts (1).

So suppose (3) and let $\chi = (-\psi\alpha/\beta \rightarrow (\alpha\colon -\psi)\beta)$. Since χ is L-true, we have $\phi \Leftrightarrow (\phi \ \& \ \chi)$. Suppose now for a contradiction that $(\alpha\colon -\psi)$ does not occur essentially in $(\phi \ \& \ \chi)$, i.e., $(\phi \ \& \ \chi) \Rightarrow (\Delta)(\phi \ \& \ \chi)\delta/\Delta$ for some δ a designator part of $(\alpha\colon -\psi)$. Hence, $\phi \Rightarrow (\Delta)(\phi \ \& \ \chi)\delta/\Delta$. Thus, (6) $\phi \Rightarrow (\Delta)\phi\delta/\Delta$ and (7) $\phi \Rightarrow (\Delta)\chi\delta/\Delta$. If δ is a proper part of $(\alpha\colon -\psi)$, then (6) contradicts (1). So suppose that δ is improper, i.e., $\delta = (\alpha\colon -\psi)$. Since $(\Delta)\chi\delta/\Delta \Rightarrow (-\psi\alpha/\beta \rightarrow (\alpha\colon \Theta\alpha \ \& \ -\Theta\alpha)\beta)$, we have by (7) that $\phi \Rightarrow (-\psi\alpha/\beta \rightarrow (\alpha\colon \Theta\alpha \ \& \ -\Theta\alpha)\beta)$. Thus we have that $(\phi \ \& \ -\psi\alpha/\beta) \Rightarrow$

$(\alpha\colon \Theta\alpha \mathbin{\&} -\Theta\alpha)\beta$. Since this latter sentence is inconsistent, we have that $(\phi \mathbin{\&} -\psi\alpha/\beta)$ is inconsistent. By theorem 1, therefore, $(\phi \mathbin{\&} (\alpha\colon -\psi)\beta)$ is inconsistent. Hence, we have that $\phi \Rightarrow -(\alpha\colon -\psi)\beta$. But this is so only if $\phi \Leftrightarrow \phi'$, which contradicts (3). Hence, $(\alpha\colon -\psi)$ occurs essentially in ϕ', if $\phi' \Leftrightarrow \phi$, or it occurs essentially in $(\phi \mathbin{\&} \chi)$, if $\phi' \not\Leftrightarrow \phi$.

<u>LEMMA</u> 3. If $(\alpha_1 \ldots \alpha_n\colon \psi)$ and $(\alpha_1 \ldots \alpha_n\colon \chi)$ occur essentially in ϕ, then $(\alpha_1 \ldots \alpha_n\colon \psi \mathbin{\&} \chi)$ occurs essentially in some $\phi' \Leftrightarrow \phi$.

PROOF. We prove this for $n = 1$. Assume first that $(\alpha\colon \psi)$ and $(\alpha\colon \chi)$ occur essentially in ϕ. Thus

(1) $\phi \not\Rightarrow (\Delta)\phi\delta/\Delta$ for any δ a designator part of either $(\alpha\colon \psi)$ or $(\alpha\colon \chi)$.

Let $\phi' = (\phi \mathbin{\&} -(\alpha\colon \psi \mathbin{\&} \chi)\beta)$. Clearly either (2) $\phi' \Leftrightarrow \phi$ or (3) $\phi' \not\Leftrightarrow \phi$. Suppose (2) and, for a contradiction, that $(\alpha\colon \psi \mathbin{\&} \chi)$ does not occur essentially in ϕ', i.e, $\phi' \Rightarrow (\Delta)\phi'\delta/\Delta$ for some δ a designator part of $(\alpha\colon \psi \mathbin{\&} \chi)$. Thus, $\phi \Rightarrow (\Delta)\phi'\delta/\Delta$. Hence, (4) $\phi \Rightarrow (\Delta)\phi\delta/\Delta$ and (5) $\phi \Rightarrow (\Delta)-(\alpha\colon \psi \mathbin{\&} \chi)\beta\delta/\Delta$. (4) contradicts (1), if ϕ is a proper part of $(\alpha\colon \psi \mathbin{\&} \chi)$. So suppose that δ is improper, i.e., $\delta = (\alpha\colon \psi \mathbin{\&} \chi)$. Since $(\Delta)-(\alpha\colon \psi \mathbin{\&} \chi)\beta\delta/\Delta \Rightarrow -(\alpha\colon \Theta\alpha \vee -\Theta\alpha)\beta$, and since this latter sentence is inconsistent, we have by (5) that ϕ is inconsistent. But, given (1), this is impossible.

So suppose (3) and let $\xi = ((\psi \ \& \ \chi)\alpha/\beta \rightarrow$ $(\alpha\colon \psi \ \& \ \chi)\beta)$. Since ξ is L-true, we have that $\phi \Leftrightarrow$ $(\phi \ \& \ \xi)$. Suppose, for a contradiction, that $(\alpha\colon \psi \ \& \ \chi)$ does not occur essentially in $(\phi \ \& \ \xi)$, i.e., $(\phi \ \& \ \xi) \Rightarrow$ $(\Delta)(\phi \ \& \ \xi)\delta/\Delta$ for some δ a designator part of $(\alpha\colon \psi \ \& \ \chi)$. Thus, $\phi \Rightarrow (\Delta)(\phi \ \& \ \xi)\delta/\Delta$. Hence, we have (6) $\phi \Rightarrow$ $(\Delta)\phi\delta/\Delta$ and (7) $\phi \Rightarrow (\Delta)\xi\delta/\Delta$. If δ is a proper part of $(\alpha\colon \psi \ \& \ \chi)$, then (6) contradicts (1). So suppose that δ is improper, i.e., $\delta = (\alpha\colon \psi \ \& \ \chi)$. Since $(\Delta)\xi\delta/\Delta \Rightarrow$ $((\psi \ \& \ \chi)\alpha/\beta \rightarrow (\alpha\colon \Theta\alpha \ \& \ -\Theta\alpha)\beta)$, we have by (7) that $\phi \Rightarrow$ $((\psi \ \& \ \chi)\alpha/\beta \rightarrow (\alpha\colon \Theta\alpha \ \& \ -\Theta\alpha)\beta)$. Thus, $(\phi \ \& \ (\psi \ \& \ \chi)\alpha/\beta) \Rightarrow$ $(\alpha\colon \Theta\alpha \ \& \ -\Theta\alpha)\beta$. Since this latter sentence is inconsistent, $(\phi \ \& \ (\psi \ \& \ \chi)\alpha/\beta)$ is inconsistent. Therefore, by theorem 1, $(\phi \ \& \ (\alpha\colon \psi \ \& \ \chi)\beta)$ is inconsistent. Thus, $\phi \Rightarrow$ $-(\alpha\colon \psi \ \& \ \chi)\beta$. But clearly this is so only if $\phi' \Leftrightarrow \phi$, which contradicts (3).

<u>LEMMA</u> 4. If an n-place predicate Θ occurs essentially in ϕ, then for each i $(1 \leq i \leq n)$, $(\alpha_1 \ldots \alpha_{i-1}\alpha_{i+1} \ldots \alpha_n\colon$ $(\exists\alpha_i)\Theta\alpha_1 \ldots \alpha_n)$ occurs essentially in some $\phi' \Leftrightarrow \phi$.

PROOF. We prove this only for the case n = i = 2. Assume first that Θ occurs essentially in ϕ. Thus

(1) $\phi \not\Rightarrow (\Delta)\phi\delta/\Delta$ for any δ a designator part of Θ. Let $\phi' = (\phi \ \& \ -(\alpha\colon (\exists\gamma)\Theta\alpha\gamma)\beta)$. Clearly, either (2) $\phi' \Leftrightarrow \phi$ or (3) $\phi' \not\Leftrightarrow \phi$. Suppose (2) and, for a contradiction, that $(\alpha\colon (\exists\gamma)\Theta\alpha\gamma)$ does not occur essentially in ϕ', i.e., $\phi' \Rightarrow (\Delta)\phi'\delta/\Delta$ for some δ a designator part of

$(\alpha: (\exists\gamma)\Theta\alpha\gamma)$. Thus $\phi \Rightarrow (\Delta)\phi'\delta/\Delta$. Hence, we have (4) $\phi \Rightarrow (\Delta)\phi\delta/\Delta$ and (5) $\phi \Rightarrow (\Delta)-(\alpha: (\exists\gamma)\Theta\alpha\gamma)\beta\delta/\Delta$. If δ is a proper part of $(\alpha: (\exists\gamma)\Theta\alpha\gamma)$, then (4) contradicts (1). So suppose that δ is improper, i.e., $\delta = (\alpha: (\exists\gamma)\Theta\alpha\gamma)$. Since $(\Delta)-(\alpha: (\exists\gamma)\Theta\alpha\gamma)\beta\delta/\Delta \Rightarrow -(\alpha: \Theta\alpha\alpha \vee -\Theta\alpha\alpha)\beta$, we have by (5) that $\phi \Rightarrow -(\alpha: \Theta\alpha\alpha \vee -\Theta\alpha\alpha)\beta$. Since this latter sentence is inconsistent, we have that ϕ is also. But, given (1), this is impossible.

So suppose (3) and let $\chi = ((\exists\gamma)\Theta\beta\gamma \rightarrow (\alpha: (\exists\gamma)\Theta\alpha\gamma)\beta)$. Since χ is L-true $\phi \Rightarrow (\phi \;\&\; \chi)$. Suppose, for a contradiction, that $(\alpha: (\exists\gamma)\Theta\alpha\gamma)$ does not occur essentially in $(\phi \;\&\; \chi)$, i.e., $(\phi \;\&\; \chi) \Rightarrow (\Delta)(\phi \;\&\; \chi)\delta/\Delta$ for some δ a designator part of $(\alpha: (\exists\gamma)\Theta\alpha\gamma)$. Hence, we have $\phi \Rightarrow (\Delta)(\phi \;\&\; \chi)\delta/\Delta$; whence (6) $\phi \Rightarrow (\Delta)\phi\delta/\Delta$ and (7) $\phi \Rightarrow (\Delta)\chi\delta/\Delta$. If δ is a proper part of $(\alpha: (\exists\gamma)\Theta\alpha\gamma)$, then (6) contradicts (1). So suppose that δ is improper, i.e., $\delta = (\alpha: (\exists\gamma)\Theta\alpha\gamma)$. Since $(\Delta)\chi\delta/\Delta \Rightarrow ((\exists\gamma)\Theta\beta\gamma \rightarrow (\alpha: \Theta\alpha\alpha \;\&\; -\Theta\alpha\alpha)\beta)$, we have from (7) that $\phi \Rightarrow ((\exists\gamma)\Theta\beta\gamma \rightarrow (\alpha: \Theta\alpha\alpha \;\&\; -\Theta\alpha\alpha)\beta)$. Thus, $(\phi \;\&\; (\exists\gamma)\Theta\beta\gamma) \Rightarrow (\alpha: \Theta\alpha\alpha \;\&\; -\Theta\alpha\alpha)\beta$. Since this latter sentence is inconsistent, we have that $(\phi \;\&\; (\exists\gamma)\Theta\beta\gamma)$ is also. Therefore, by theorem 1, $(\phi \;\&\; (\alpha: (\exists\gamma)\Theta\beta\gamma))$ is inconsistent. But this is so only if $\phi' \Leftrightarrow \phi$, which contradicts (3).

The import of Lemma 4 can perhaps be better understood via the introduction of some new terminology. By the <u>ith derelativization</u> of an n-place predicate Θ $(n>1)$ let us understand $(\alpha_1 \ldots \alpha_{i-1}\alpha_{i+1} \ldots \alpha_n\colon (\exists\alpha_i)\theta\alpha_1 \ldots \alpha_n)$. Similarly, by the ith projection of an n-ary relation ρ let us understand the (n-1)-ary relation that results from the deletion from ρ of the ith member of every n-tuple that is a member of ρ. Clearly the ith derelativization of a closed n-ary predicate that designates an n-ary relation ρ designates the ith projection of ρ. Thus, e.g., the first derelativization of "is a father of" is "someone is a father of," while its second derelativization is "is a father of someone." Likewise, the first projection of the fatherhood relation is the set of things that have fathers, viz., the set of children. The import of lemma 4 can thus be put as follows. If a predicate occurs essentially in a sentence, then each derelativization of that predicate occurs essentially in some equivalent sentence. It follows, of course, that if a sentence is about a relation, it is also about each of its projections.

We come now to the main theorem of this chapter.

THEOREM 2. If $\delta_1, \ldots, \delta_n$ are designators occurring essentially in ϕ and δ is a designator constructed solely from $\delta_1, \ldots, \delta_n$, then δ occurs essentially in some $\phi' \Leftrightarrow \phi$.

PROOF. By strong induction on the number of occurrences of connectives, quantifiers, and colons in δ. Suppose that the claim holds for any such designator having fewer occurrences of connectives, quantifiers, and colons than δ

has. We have five cases to consider.

(1) $\delta = \delta_i$ for some i $(1 \leq i \leq n)$

(2) $\delta = (\alpha_1 \ldots \alpha_n\colon \Theta\alpha_1 \ldots \alpha_n)$

(3) $\delta = (\alpha_1 \ldots \alpha_n\colon -\psi)$

(4) $\delta = (\alpha_1 \ldots \alpha_n\colon \psi \,\&\, \chi)$

(5) $\delta = (\alpha_1 \ldots \alpha_{i-1}\alpha_{i+1} \ldots \alpha_n\colon (\exists\alpha_i)\psi)$

Case (1): If $\delta = \delta_i$, then let $\phi' = \phi$.

Case (2): Clearly Θ falls under the induction hypothesis. Thus, Θ occurs essentially in some $\phi' \Leftrightarrow \phi$. Therefore, by lemma 1, $(\alpha_1 \ldots \alpha_n\colon \Theta\alpha_1 \ldots \alpha_n)$ occurs essentially in some $\phi'' \Leftrightarrow \phi' \Leftrightarrow \phi$. So $(\alpha_1 \ldots \alpha_n\colon \Theta\alpha_1 \ldots \alpha_n)$ occurs essentially in some sentence equivalent to ϕ.

Case (3): Clearly $(\alpha_1 \ldots \alpha_n\colon \psi)$ falls under the induction hypothesis and thus occurs essentially in some $\phi' == \phi$. Therefore, by lemma 2, $(\alpha_1 \ldots \alpha_n\colon -\psi)$ occurs essentially in some $\phi'' \Leftrightarrow \phi' \Leftrightarrow \phi$. Thus, $(\alpha_1 \ldots \alpha_n\colon -\psi)$ occurs essentially in some sentence equivalent to ϕ.

Case (4): Clearly both $(\alpha_1 \ldots \alpha_n\colon \psi)$ and $(\alpha_1 \ldots \alpha_n\colon \chi)$ fall under the induction hypothesis. Thus, $(\alpha_1 \ldots \alpha_n\colon \psi)$ occurs essentially in some $\phi' \Leftrightarrow \phi$ and $(\alpha_1 \ldots \alpha_n\colon \chi)$ occurs essentially in some $\phi'' \Leftrightarrow \phi$. Let $\phi''' = (\phi'' \,\&\, \phi')$. Clearly $\phi''' \Leftrightarrow \phi'' \Leftrightarrow \phi' \Leftrightarrow \phi$. Clearly also, both $(\alpha_1 \ldots \alpha_n\colon \psi)$ and $(\alpha_1 \ldots \alpha_n\colon \chi)$ occur essentially in ϕ'''. Thus by lemma 3, $(\alpha_1 \ldots \alpha_n\colon \psi \,\&\, \chi)$ occurs essentially in some $\phi'''' \Leftrightarrow \phi''' \Leftrightarrow \phi$. Therefore, $(\alpha_1 \ldots \alpha_n\colon \psi \,\&\, \chi)$ occurs essentially in some sentence equivalent to ϕ.

Case (5): Clearly ($\alpha_1 \ldots \alpha_n$: ψ) falls under the induction hypothesis and thus occurs essentially in some $\phi' \Leftrightarrow \phi$. Therefore, by lemma 4, ($\alpha_1 \ldots \alpha_{i-1}\alpha_{i+1} \ldots \alpha_n$: $(\exists\alpha_i)$ ($\alpha_1 \ldots \alpha_n$: $\psi)\alpha_1 \ldots \alpha_n$) occurs essentially in some $\phi'' \Leftrightarrow \phi' \Leftrightarrow \phi$. Therefore, by theorem 1, ($\alpha_1 \ldots \alpha_{i-1}\alpha_{i+1} \ldots \alpha_n$: (α_i) ψ) occurs essentially in some $\phi''' \Leftrightarrow \phi'' \Leftrightarrow \phi' \Leftrightarrow \phi$. At any rate, ($\alpha_1 \ldots \alpha_{i-1}\alpha_{i+1} \ldots \alpha_n$: $(\exists\alpha_i)\psi$) occurs essentially in some sentence equivalent to ϕ.

As an immediate corollary to theorem 2 we derive the following:

<u>COROLLARY</u> 1. If $\delta_1, \ldots, \delta_n$ are <u>unstructured</u> designators occurring essentially in ϕ and δ is a designator constructed solely from $\delta_1, \ldots, \delta_n$, then δ occurs essentially in some $\phi' \Leftrightarrow \phi$.

PROOF. Suppose that $\delta_1, \ldots, \delta_n$ are unstructured designators occurring essentially in ϕ and that δ is a designator constructed solely from $\delta_1, \ldots, \delta_n$. Since any unstructured designator is a designator, it follows immediately from theorem 2 that δ occurs essentially in some $\phi' \Leftrightarrow \phi$.

The importance of corollary 1 will become clear at the end of this section. By its use we are able to offer a proof of theorem 5. Theorem 5, it will be seen, justifies the employment of a definition of absolute aboutness that is, at least in application, much simpler than that presently employed in this essay.

LEMMA 5. If δ is a designator constructed solely from designators δ_1, ..., δ_n and des[δ_1] = des[δ_1'], ..., des[δ_n] = des[δ_n'], then there is a designator δ' of the same degree as δ constructed solely from δ_1', ..., δ_n' such that des[δ'] = des[δ].

PROOF. The proof of this lemma is a straightforward induction on the number of occurrences of connectives, quantifiers, and colons in δ and is omitted.

We are now in a position to prove the following corollary of theorem 2.

COROLLARY 2. If ϕ is about des[δ_1], ..., and des[δ_n] and δ is a designator constructed solely from δ_1, ..., δ_n, then ϕ is about des[δ].

PROOF. Suppose that ϕ is about des[δ_1], ..., and des[δ_n] and that δ is a designator constructed solely from δ_1, ..., δ_n. Thus, there are designators δ_1', ..., δ_n' and sentences ϕ_1, ..., ϕ_n such that des[δ_1'] = des[δ_1], ..., des[δ_n'] = des[δ_n], where $\phi \Leftrightarrow \phi_1 \Leftrightarrow \ldots \Leftrightarrow \phi_n$, and each δ_i' ($1 \le i \le n$) occurs essentially in ϕ_i. Let ϕ' = (ϕ_1 & ... & ϕ_n), and suppose, for a contradiction, that some δ_j' ($1 \le j \le n$) does not occur essentially in ϕ'., i.e., $\phi' \Rightarrow (\Delta)\phi'\delta'/\Delta$ for some δ' a designator part of δ_j'. Clearly $\phi' \Leftrightarrow \phi_j$ for each j ($1 \le j \le n$). Thus, we have $\phi_j \Rightarrow (\Delta)\phi'\delta'/\Delta$. Hence, $\phi_j \Rightarrow (\Delta)\phi_1\delta'/\Delta$, ..., and $\phi_j \Rightarrow (\Delta)\phi_n\delta'/\Delta$. At any rate, we have $\phi_j \Rightarrow (\Delta)\phi j\delta'/\Delta$. But this contradicts the claim that each δ_i' occurs essentially in ϕ_i. By lemma 5, therefore, there is a designator δ'' constructed solely from δ_1', ..., δ_n' such that des[δ''] = des[δ]. By theorem 2, then, δ'' occurs essentially in some $\phi'' \Leftrightarrow \phi'$. Hence, $\phi \Leftrightarrow \phi''$ and δ'' occurs essentially in ϕ''. Thus ϕ is about des[δ'']. Since des[δ''] = des[δ], we have that ϕ is about des[δ].

We turn now to the proofs of certain claims which are in fact advocated by Goodman. These claims again take the form of corollaries.

<u>COROLLARY</u> 3. If ϕ is about a class Γ, then ϕ is about $\bar{\Gamma}$ (the complement of Γ).

PROOF. Suppose that ϕ is about Γ. Thus, there is a designator δ such that des[δ] = Γ and δ occurs essentially in some $\phi' \Leftrightarrow \phi$. We are supposing, of course, that δ is a predicate and not a name. Moreover, for simplicity, we suppose that δ is one-place. Clearly des[$(\alpha: -\delta\alpha)$] = $\bar{\Gamma}$. By corollary 2, therefore, ϕ is about des[$(\alpha: -\delta\alpha)$], i.e., ϕ is about $\bar{\Gamma}$.

<u>COROLLARY</u> 4. If ϕ is about Γ and ϕ is about Γ', then ϕ is about $\Gamma \cap \Gamma'$ (the intersection of Γ and Γ').

PROOF. Suppose that ϕ is about Γ and about Γ'. Thus, there are designators δ and δ' such that des[δ] = Γ and des[δ'] = Γ', where δ occurs essentially in some $\phi' \Leftrightarrow \phi$ and δ' occurs essentially in some $\phi'' \Leftrightarrow \phi$. Again, we suppose that δ and δ' are one-place predicates. By corollary 2, therefore, ϕ is about des[(α: $\delta\alpha$ & $\delta'\alpha$)]. Since des[(α: $\delta\alpha$ & $\delta'\alpha$)] = $\Gamma \cap \Gamma'$, ϕ is about $\Gamma \cap \Gamma'$.

THEOREM 3. If ϕ is about $\Gamma_1, \ldots, \Gamma_n$, then ϕ is about each Boolean function of $\Gamma_1, \ldots, \Gamma_n$.
PROOF. Immediately from corollary 3 and corollary 4.

Readers of Goodman will note that the proof of theorem 3 constitutes a vindication of Goodman's claim that "In general, a statement absolutely about any class or classes is about each Boolean function of them."[41]

COROLLARY 5. If ϕ is about a relation, then ϕ is about each projection of that relation.
PROOF. Suppose that ϕ is about an n-ary relation ρ. Thus, there is a designator δ such that des[δ] = ρ and δ occurs essentially in some $\phi' \Leftrightarrow \phi$. We suppose that δ is an n-place predicate. Clearly the ith derelativization of δ designates the ith projection of $\rho (1 \leq i \leq n)$. By corollary 2, ϕ is about des[($\alpha_1 \ldots \alpha_{i-1}\alpha_{i+1} \ldots \alpha_n$: $(\exists\alpha_i)\delta\alpha_1 \ldots \alpha_n$)], i.e., ϕ is about the ith projection of ρ.

A Boolean function of $\Gamma_1, \ldots, \Gamma_n$ is, of course, understood to be any class obtainable from $\Gamma_1, \ldots, \Gamma_n$ by repeated applications of the operations of complementation and intersection. If to the operations of complementation and intersection we add the operation of projection, still

[41] Goodman, "About," p. 258.

further classes can be generated from $\Gamma_1, \ldots, \Gamma_n$. For lack of a better term let us call those classes generated from $\Gamma_1, \ldots, \Gamma_n$ by repeated applications of the operations of complementation, intersection, and projection classical composites. We have, then, the following theorem.

THEOREM 4. If ϕ is about $\Gamma_1, \ldots, \Gamma_n$, then ϕ is about each classical composite of $\Gamma_1, \ldots, \Gamma_n$.

PROOF. Immediately from corollaries 3, 4, and 5.

Our next theorem requires yet another lemma.

LEMMA 6. If $\delta_1, \ldots, \delta_n$ are the sole unstructured designators occurring essentially in ϕ, then $\delta_1, \ldots, \delta_n$ are all and only the unstructured designators occurring essentially in any $\phi' \Leftrightarrow \phi$.

PROOF. Assume that $\delta_1, \ldots, \delta_n$ are all of the unstructured designators occurring essentially in ϕ, i.e., $\phi \not\Rightarrow (\Delta)\phi\delta_i/\Delta$ for any i ($1 \leq i \leq n$). Let $\psi \Leftrightarrow \phi$. To establish the "all"-part of the theorem assume, for a contradiction, that $\psi \Rightarrow (\Delta)\psi\delta_i/\Delta$ for some i ($1 \leq i \leq n$). Thus, we have $\models (\Delta)(\phi \leftrightarrow \psi)\delta_i/\Delta$. Hence, $(\Delta)\phi\delta_i/\Delta \Leftrightarrow (\Delta)\psi\delta_i/\Delta$. Hence, $\psi \Rightarrow (\Delta)\phi\delta_i/\Delta$. So $\phi \Rightarrow (\Delta)\phi\delta_i/\Delta$. This last claim contradicts the assumption that each δ_i ($1 \leq i \leq n$) occurs essentially in ϕ. To establish the "only"-part assume, for a contradiction, that $\psi \not\Rightarrow (\Delta)\psi\delta'/\Delta$ for some unstructured designator $\delta' = \delta_i$ ($1 \leq i \leq n$), i.e., some unstructured designator distinct from each of $\delta_1, \ldots, \delta_n$ occurs essentially in ψ. Since δ' does not occur in ϕ, we have $\phi \Rightarrow (\Delta)\phi\delta'/\Delta$. We also have $\models (\Delta)(\phi \leftrightarrow \psi)\delta'/\Delta$. Hence, $(\Delta)\phi\delta'/\Delta \Leftrightarrow (\Delta)\psi\delta'/\Delta$. Thus, $\phi \Rightarrow (\Delta)\psi\delta'/\Delta$. Hence, $\psi \Rightarrow (\Delta)\psi\delta'/\Delta$. This last claim contradicts the assumption that δ' occurs essentially in ψ.

<u>THEOREM</u> 5. If ϕ and ψ have the same essentially occurring unstructured designator parts, then ϕ is about k if and only if ψ is about k, i.e., ϕ and ψ are about the same things.

PROOF. Assume that ϕ and ψ have the same essentially occurring unstructured designators and that $\delta_1, \ldots, \delta_n$ constitutes a complete list of them. Thus, there is a ϕ' and a δ such that $\phi' \Leftrightarrow \phi$, des[$\delta$] = k and $\phi' \not\Rightarrow (\Delta)\phi'\delta'/\Delta$ for any δ' a designator part of δ. By lemma 6 the only unstructured designators occurring essentially in ϕ' are $\delta_1, \ldots, \delta_n$. Thus, we have $\phi' \not\Rightarrow (\Delta)\phi'\delta_i/\Delta$ for any i ($1 \leq i \leq n$). By theorem 2, then δ occurs essentially in some $\psi' \Leftrightarrow \psi$, i.e., ψ is about k. This establishes that if ϕ is about k, then ψ is about k. Similarly, we can demonstrate that if ψ is about k, then ϕ is about k.

Our proof of theorem 5 signals something more than a merely routine result. With theorem 5 at our disposal we have a splendidly simple test for deciding whether two sentences are about the same things. We decide simply by checking to see whether they have the same essentially occurring unstructured designator parts.

Note, however, that the converse of theorem 5 does not hold. A simple example suffices to demonstrate this. Assume that Θ and Θ' are one-place unstructured predicates, and that des[Θ] = des[Θ']. Clearly then $(\exists\alpha)\Theta\alpha$ and $(\exists\alpha)\Theta\alpha'$ are about the same things. But supposing only that $\Theta \neq \Theta'$ we have a case then of two distinct sentences that are about the same things. More intuitively, "Something is a unicorn" and "Something is a centaur" are about the same things.

Corollary 1 now allows us to offer a definition of absolute aboutness that is more useful than that with which we have hitherto worked. Actually, our new definition is the same as the old with the exception that "essential occurrence" is now understood as follows. Where ϕ is a sentence and δ a designator:

> δ occurs essentially in ϕ if and only if δ occurs in ϕ and $\phi \not\Rightarrow (\alpha)\phi\delta'/\alpha$, where δ' is any unstructured designator part of δ, and α is an individual variable if δ' is a name, or an n-place predicate variable if δ' is an n-place predicate letter.

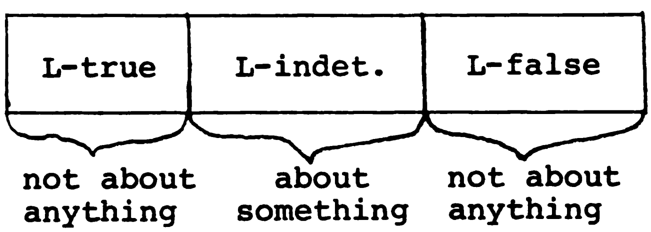
That the extension of 'absolutely about' remains the same whether we define it in terms of the conception of essential occurrence offered in section three or in terms of the conception offered above is demonstrated by the proof of the next theorem. Let us represent the claim "ϕ is about k," in the sense of the earlier conception of essential occurrence by "ϕ is about$_1$ k", and, in the above conception of essential occurrence by "ϕ is about$_2$ k."

<u>THEOREM</u> 6. ϕ is about$_1$ k if and only if ϕ is about$_2$ k.
PROOF. The direction from left to right is trivial. Hence, we prove only the direction from right to left. Assume, then, that ϕ is about$_2$ k. Hence, there is a designator δ and a sentence ψ such that des[δ] = k, $\phi \Leftrightarrow \psi$ and $\psi \not\Rightarrow (\Delta)\psi\delta'/\Delta$ for any unstructured designator part δ' of δ.
PROOF. Let $\delta_1, \ldots, \delta_n$ be a complete list of the unstructured designator parts of δ. Clearly each δ_i $(1 \leq i \leq n)$ occurs essentially (in the later sense) in ϕ. Hence, by corollary 1, δ occurs essentially (in the earlier sense) in some $\phi' \Leftrightarrow \phi$. Thus, $\phi \Leftrightarrow \phi'$ and $\phi' \not\Rightarrow (\Delta)\phi'\delta'/\Delta$ for any designator part δ' of ϕ. Hence, ϕ is about$_1$ k.

The usefulness of our new definition of absolute aboutness becomes obvious once one reflects upon the fact that the test for absolute aboutness can now be effected by considering only those generalizations which are generalizations of just the unstructured designator parts of the designator in question. Thus, e.g., in attempting to decide whether ϕ is about des['(uv: (xy: Fxa v Gyx)uv'], one need only check to see whether 'F', 'G', and 'a' occur essentially in some sentence equivalent to ϕ. Actually the test for aboutness can be further simplified. It can be easily verified that if an unstructured designator occurs essentially in any sentence equivalent to ϕ, then it occurs essentially in ϕ. Thus, the test for deciding whether ϕ is about des[δ] can be effected simply by determining whether the unstructured parts of δ occur essentially in ϕ.

<u>Section Seven: Identity</u>

In this section we focus our attention on the topic of identity. As a first step toward achieving such a focus we pose the question of what relationships obtain between aboutness and the logical concepts. In particular, are logical truths about anything; if so, are all or only some of them about something, and similarly for the concepts of L-indeterminacy and L-falsehood? As was noted in section three of chapter I Goodman claims that it is a consequence of his theory that no L-true sentence is about anything. Indeed, according to him, "A self contradictory or logically true statement is not absolutely about anything."[42] The remaining question of whether L-indeterminate statements are about something is, one might suppose, answerable in the affirmative. Such an answer is, no doubt, aesthetically appealing, since we are then provided with the following nicely symmetric picture of aboutness:

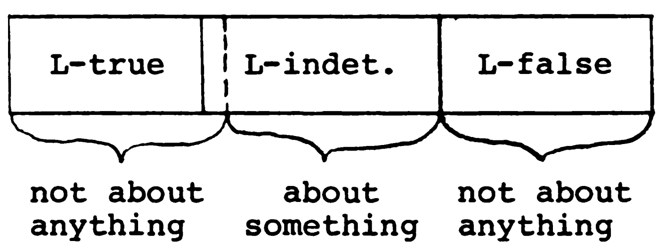


[42]Ibid., p. 256.

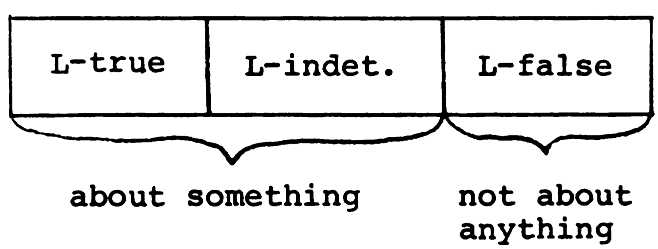
A vindication of the belief that aboutness is correctly depicted above, i.e., that logically determinate sentences are about nothing while L-indeterminate sentences are each about something (or other), must deal with the following obstacle. Consider any L-indeterminate sentence composed solely of logical symbols, say '$(\exists x)(\exists y)x = y$'. Since '=' is regarded by Goodman as a logical symbol, and, hence, as nondesignatory, such a sentence is not about anything. The proof that no L-indeterminate sentence composed solely of logical symbols is about anything is provided merely by reflecting on the fact that no such sentence contains a designator that designates anything; indeed, no such sentence contains any designator at all. This result is, of course, a consequence merely of the fact that most logicians adopt a somewhat ambiguous attitude toward the distinction between logical symbols and nonlogical symbols. On the one hand, predicates are classified as nonlogical symbols, while, on the other hand, '=', though clearly a predicate, is singled out as playing a special role in the language; a role that is best understood by counting '=' among the logical symbols. The result that '$(\exists x)(\exists y)x = y$' is not about anything may, then, simply be accepted as a natural result of this ambiguous attitude, or we may insist that '=', though a logical symbol, is still designatory, and, hence, sentences such as the above are about something after all, viz., the identity relation. Such an adjustment will, of course, accommodate the intuition that each

L-indeterminate sentence is about something or other, but only at the cost of abandoning the claim that no L-true sentence is about anything, i.e., only at the cost of a violation of CA(L-T). Consider, e.g., the L-true sentence '(x)x = x'. If we assume that '=' is a designator, then theis sentence is clearly about des['='], since '(x)x = x' $\not\Rightarrow$ '(Y)(x)Yxx'.[43] However, since any L-false sentence implies every sentence, L-false sentences will, even under the assumption that '=' is a designator, remain about nothing, thus spoiling the symmetry exhibited in the above picture and providing us with a violation of CA(Neg) as well. Our new choices, then, can be exhibited as follows:

'=' not a designator (Goodman)



'=' a designator



That all L-true sentences are about something, if '=' is counted as a designator is a consequence of the fact that,

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[43]A universal generalization of a sentence ϕ with respect to '=' is the result of replacing every occurrence of '$\alpha = \beta$' in ϕ by $\Theta\alpha\beta$, where Θ is a binary predicate variable, and prefixing this result with (Θ).

given their equivalence, they are all about the same things; hence, since one, at least, is about something, viz., the identity relation, they all are.

Neither of the choices depicted above appears satisfactory insofar as neither provides the symmetry displayed in the first picture. However, it would be a mistake to suppose from this that both of the latter pictures represent choices that are equally satisfactory. It is a mistake, anyway, if the following theorem is going to hold.

<u>THEOREM</u> 7. If ϕ is about k, then $-\phi$ is about k.

PROOF. Assume that ϕ is about k. Hence, there is a sentence ψ and a designator δ such that $\psi \Leftrightarrow \phi$, des[$\delta$] = k and (1) $\psi \not\Rightarrow (\Delta)\psi\delta'/\Delta$ for any δ' a designator part of δ. Suppose, for a contradiction, that $-\phi$ is not about k. Hence, for any $\chi \Leftrightarrow -\phi$, we have $\chi \Rightarrow (\Delta)\chi\delta'/\Delta$ for some δ' a designator part of δ. Since $-\psi \Leftrightarrow -\phi$, we thus have $-\psi \Rightarrow (\Delta)-\psi\delta'/\Delta$. Hence, $\models (-\psi \rightarrow (\Delta)-\psi\delta'/\Delta)$. Hence, we also have $\models (\Delta)(-\psi \rightarrow (\Delta)-\psi\delta'/\Delta)\delta'/\Delta$; whence $\models ((\exists\Delta)-\psi\delta'/\Delta \rightarrow (\Delta)-\psi\delta'/\Delta)$. Finally then we have (2) $(\exists\Delta)-\psi\delta'/\Delta \Rightarrow (\Delta)-\psi\delta'/\Delta$. From (1) we know that there is an interpretation I making ψ true and $(\Delta)\psi\delta'/\Delta$ false. Thus I makes $-(\Delta)\psi\delta'/\Delta$ and $(\exists\Delta)-\psi\delta'/\Delta$ true. By (2), then, I makes $(\Delta)-\psi\delta'/\Delta$ true. Hence, I makes $-\psi$ true, a contradiction.

That we have a counterexample to theorem 7, if '=' is counted as a designator can be seen from the following argument. Under that proposal, the L-true sentence '(x)x = x' is indeed about something, viz., des['=']. However, its negation '-(x)x = x' being L-false, is not about anything. Hence, theorem 7 is violated. It is easy to see where our proof of theorem 7 would break down if '=' were treated as a designator, viz., at the move from the claim that since $\models (-\psi \leftrightarrow (\Delta)-\psi\delta'/\Delta)$, $\models (\Delta)(-\psi \leftrightarrow (\Delta)-\psi\delta'/\Delta)\delta'/\Delta$. There, the principle employed is simply the principle that the universal generalization of an L-true sentence with respect to any of its unstructured designator parts is again an L-true sentence. The following, e.g., constitutes a counterexample. Clearly, '(x)x = x' is an L-true sentence. Clearly also the universal generalization of this sentence with respect to '=', viz., '(Y)(x)Yxx', is not L-true. Indeed, '(Y)(x)Yxx' is L-false. The remedy for this hypothetical failure of theorem 7 is obvious. Insist, as Goodman does, and as we have all along, that '=' is not a designator and the theorem holds.[44] The second of our three pictures of aboutness thus provides, I submit, the best picture. It is worth noting that in the proofs of all of our earlier lemmas, theorems, and corollaries we didn't appeal to the principle that a logical truth implies its own generalization with respect to its unstructured designator parts. Thus,

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[44]The motivation for requiring that a closed predicate contain at least one occurrence of a name or predicate letter, in order that it be counted as a designator, should now be clear. Without such a requirement '(x: x = x)' counts as a designator, and '(x: x = x)a', even though L-true, is about des['(x: x = x)'].

those results hold independently of our decision to not count '=' as a designator.

Though '=' is not, for us, a designator, the identity relation still receives designation. It is perhaps no surprise, then, that, as the proof of the following theorem demonstrates, any sentence about anything is about the identity relation.

<u>THEOREM</u> 8. If ϕ is about anything, then ϕ is about the identity relation.

PROOF. Assume that ϕ is about k. Thus, there is a sentence ψ and a designator δ such that $\psi \Leftrightarrow \phi$, des[$\delta$] = k, δ occurs in ψ and $\psi \not\Rightarrow (\Delta)\psi\delta'/\Delta$ for any unstructured designator part δ' of δ. If δ is a name β, then ψ is of the form $\ldots\beta\ldots$. Note that $\ldots\beta\ldots \Leftrightarrow \ldots\beta\ldots \;\&\; (\alpha\colon \alpha = \beta)\beta$. It should be clear that $(\alpha\colon \alpha = \beta)$ occurs essentially in $\ldots\beta\ldots \;\&\; (\alpha\colon \alpha = \beta)\beta$; otherwise $\ldots\beta\ldots \;\&\; (\alpha\colon \alpha = \beta)\beta \Rightarrow (\gamma)[\ldots\gamma\ldots \;\&\; (\alpha\colon \alpha = \gamma)\gamma]$; whence, $\ldots\beta\ldots \Rightarrow (\gamma)[\ldots\gamma\ldots \;\&\; (\alpha\colon \alpha = \gamma)\gamma]$. Thus, we would have that $\ldots\beta\ldots \Rightarrow (\gamma)\ldots\gamma\ldots$, which contradicts the assumption that β occurs essentially in ψ. Thus, we see that if a name β occurs essentially in ψ, then the one-place predicate $(\alpha\colon \alpha = \beta)$ occurs essentially in some $\psi' \Leftrightarrow \psi$. By theorem 2, then, $(\alpha\gamma\colon ((\alpha\colon \alpha = \beta) \vee -(\alpha\colon \alpha = \beta)) \;\&\; \alpha = \gamma)$ occurs essentially in some $\psi'' \Leftrightarrow \psi' \Leftrightarrow \psi$. Since des[$(\alpha\gamma\colon ((\alpha\colon \alpha = \beta) \vee -(\alpha\colon \alpha = \beta)) \;\&\; \alpha = \gamma)$] is the identity relation, we have then that ϕ is about the identity relation if δ is a name occurring essentially in some sentence equivalent to ϕ. Suppose now that δ is a predicate Θ and, for simplicity, that Θ is one-place. By theorem 2, then, $(\alpha\gamma\colon (\Theta\alpha \vee -\Theta\alpha) \,\&\, \alpha = \gamma)$ occurs essentially in some $\psi' \Leftrightarrow \psi$. Thus, ϕ is about des$[(\alpha\gamma\colon (\Theta\alpha \vee -\Theta\alpha) \,\&\, \alpha = \gamma)]$. Since des$[(\alpha\gamma\colon (\Theta\alpha \vee -\Theta\alpha) \,\&\, \alpha = \gamma)]$ is the identity relation, we have, then, that ϕ is about the identity relation.

Section Eight: Conditions of Adequacy Revisited

In this section we observe that the analysis of absolute aboutness offered in this chapter meets most of the conditions of adequacy proposed in section two of chapter I.

That it meets the condition CA(Eq) is an immediate consequence of the definition of 'absolutely about'. That our analysis meets the condition CA(Neg) is clear from theorem 7. Since any L-true sentence implies its own universal generalization with respect to any unstructured designator occurring in it, it follows that no designator ever occurs essentially in an L-true sentence; thus, no L-true sentence is about anything. Hence, CA(L-T) is satisfied by our analysis. That CA(L-I) is not satisfied by our analysis is clear from the discussion in the preceding section. That our analysis does meet the condition CA(Lim Dis) is evident from the fact that where Θ and Θ' are distinct one-place unstructured predicates and des[Θ] $\neq$ des[Θ'], ($\exists\alpha$)$\Theta\alpha$ and ($\exists\alpha$)$\Theta'\alpha$ are about different things, for ($\exists\alpha$)$\Theta\alpha$ is about des[Θ] while ($\exists\alpha$)$\Theta'\alpha$ is not.

Section Nine: First Order Fragments of L

The use of a second-order language to express Goodman's definition of 'absolutely about' is, as we have previously noted, in apparent conflict with Goodman's expressed desire of providing a first-order characterization of absolute aboutness. Accordingly, we now point out that the results obtained in this chapter can be obtained within the first-order fragment F of L that is obtained from L by deleting predicate variables from the vocabular of L, and deleting the appropriate clauses from the definitions of 'predicate' and 'truth under an interpretation'. Since no predicate variable occurs in F, the notion of an essential occurrence of a designator in a sentence is now explained as follows. Let δ be a designator that occurs in a sentence ϕ, and let δ_1, ..., δ_n be a complete list of the unstructured designator parts of δ.

> δ occurs essentially in ϕ if and only if δ occurs in ϕ and there is an interpretation I under which ϕ is true and at least one {δ_1, ..., δ_n}-variant of I under which ϕ is false.

Let us represent the claim that a sentence ϕ is about k in the above sense of essential occurrence' by "ϕ is about$_3$ k." Corollary 1 guarantees that a sentence ϕ of F is

$about_3$ k if and only if ϕ is $about_2$ k, for the definiens of the above definition is just another way of saying that ϕ does not imply its own universal generalization with respect to any of the unstructured designator designators occurring in ϕ. Since our final conception of absolute aboutness makes no appeal, implicit or explicit, to second-order languages, Goodman's scruples concerning the type of logic to be employed in the application and expression of his definition of absolute aboutness are thus preserved.

Since, as was claimed above, a sentence ϕ of F is $about_3$ k if and only if ϕ is $about_2$ k, it follows, then, that the results obtained in this chapter hold for any fragment of F that includes '='. Our results thus hold for those fragments of F that, like natural languages, contain only a finite number of names or predicates. Theorem 8 fails, of course, for those fragments of F that fail to contain '='.

CHAPTER III

DEFINITE DESCRIPTIONS AND NONEXISTENTS

Section One: Introductory Remarks

In this chapter two questions are addressed. The first is the question of which of the results obtained in chapter II remain in force if definite descriptions are introduced into L. The answer to this question involves some surprising, but interesting, results. Among these is the claim that no name ever occurs essentially in any monistic sentence, where a monistic sentence is one that entails '(x)(y)x = y'. The second question concerns the problem of how to analyze aboutness claims for sentences containing nonreferring singular terms. In this context the resources of free logic are exploited for the purpose of providing a formal setting within which Goodman's notion of rhetorical aboutness can be examined.

Section Two: Definite Descriptions

The need for an examination of description theory in our investigation of aboutness can be seen from the following. Consider the sentence

(1) The man who shot Lincoln is an actor.

According to standard translation schemes, perhaps the best translation of (1) into L is

(2) $(\exists x)[(y)(Syl \leftrightarrow y = x) \& Ax]$,

where des['S'] = {<x,y>: x shot y}, des['l'] = Lincoln, and des['A'] = {x: x is an actor}. Given this translation, it is easily verified that (1) is about Lincoln, about the class of actors, about the shooting relation, about the class of shooters, about the class of things shot, etc. Intuition seems to dictate that (1) is also about <u>the man who shot Lincoln</u>. However, since (2) is equivalent to no sentence containing an essential occurrence of a designator that designates the man who shot Lincoln, (2) cannot be shown to be about this person. At best, (2) can be shown to be about {x: x = the man who shot Lincoln}. This latter claim can be seen by appealing to the equivalence of (2) with '$(\exists x)[(x\colon (y)(Syl \leftrightarrow y = x))x \& Ax]$'. The failure of (2) to be about the man who shot Lincoln can, of course, be traced to its failure to be equivalent to a sentence containing an essential occurrence of a designator that designates that man, while (1) clearly does contain such an expression, viz., "the man who shot Lincoln." Such a disparity between (1) and (2) does not, of course, constitute, in and of itself, a telling argument against accepting (2) as an adequate translation of (1). It is simply an inevitable by-product of the implicit acceptance of Russell's scheme of contextually defining descriptions. For Russell, no description designates; sentences in which descriptions occur are understood by producing paraphrases of them in which they do not occur, precisely as in the move from (1) to (2). Indeed, whether or not any string of symbols from some artificial language constitutes an acceptable or adequate translation of some piece of English discourse depends on many factors, not the least of which is the purpose or purposes that the artificial language, and thereby the "translation," is intended to serve. As our present purpose includes confirming that a sentence of the type "The F is G" is about the F where a unique F exists, (2) must, then, be viewed as an inadequate translation of (1) and Russell's program must be viewed as unavailable to us.

An adequate translation of (1) can be obtained by expanding L in such a way as to allow for the introduction of descriptions. The introduction of descriptions requires that we consider, among other things, the problem of scope. A sentence such as "The present King of France is not bald," to use Russell's famous example, is ambiguous. Under one reading, it says that there is a unique object satisfying the condition that it is a present King of France and, moreover, that object is not bald. Under another reading this sentence denies that there is any unique bald King of France. Since the former claim is false and the latter claim is true, the ambiguity is thus established. The former provides the *wide* scope reading of "The present King of France," the latter the *narrow* scope reading. In *Principia Mathematica* Russell and Whitehead employed scope operators to resolve the ambiguity.[45] Thus, e.g., '∿[(℩x)Px]B(℩x)Px' is the translation of the narrow scope reading, while [(℩x)Px]∿B(℩x)Px' is the translation of the wide scope reading. Happily, a notationally more economical device for resolving scope ambiguities, due to Sharvy, is available.[46] Quine, an ardent admirer of Russell, applauds Sharvy's device in the following remarks:

> Those who would use prefixes to distinguish scopes of descriptions will be pleased by Richard Sharvy's neat notation: Where Russell would write '[(℩x)ϕx].ψ(℩x)ϕx]', Sharvy writes '(℩x)ϕx.ψx'. The expression '(℩x)ϕx' becomes a complex quantifier, 'for the sole object x such that ϕx'.[47]

Thus, under Sharvy's scheme a sentence of the type "The F is G" is regarded as the result of prefixing the expression "The F" (The x such that x is F is such that) to "G" (x is G). The introduction of descriptions via Sharvy's notation requires the following syntactical modifications of L.[48] First, count the inverted iota '℩' among the primitive expressions of L. Second, modify the simultaneous inductive definition of "a predicate of L" and "a formula of L" by adding the following clause: an expression of the form

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[45]B. Russell and A. N. Whitehead, Principia Mathematica to *56 (London: Cambridge University Press, 1966), p. 173.

[46]R. Sharvy, "Three Types of Referential Opacity," Philosophy of Science 39 (June 1972):153-61.

[47]W. V. Quine, "Reply to Grice," Words and Objections, Essays on the Work of W. V. Quine (Dordrecht, Holland: Reidel, 1969), p. 327.

[48]The following syntactical and semantical development of Sharvy's device is due to H. E. Hendry.

$(\imath\alpha)\psi:\chi$, where α is an individual variable and ψ and χ are formulas, is a formula. Third, define a <u>description</u> as any expression of the form $(\imath\alpha)\psi$, where α is an individual variable and ψ is a formula. Fourth, modify the definition of a bound occurrence of a variable to read as follows: An occurrence of a variable in a description, predicate or formula Ω is <u>bound</u> if and only if the occurrence falls within a part of Ω that belongs to one of the following types: (i) $(\imath\alpha)\phi:\psi$, (ii) $(\exists\alpha)\phi$, or (iii) $(\alpha_1 \ldots \alpha_n: \phi)$ where $\alpha = \alpha_i$ for some i ($1 \leq i \leq n$). An occurrence of a variable in a description, predicate, or formula is said to be <u>free</u> if it is not bound. If no variable occurs free in a description, it will be said to be <u>closed</u>. And a description is said to be <u>open</u> if it is not closed. The class of designators of <u>L</u> is now characterized as follows. A designator is any name, closed predicate containing at least one occurrence of a name or predicate letter, or closed description.

The introduction of descriptions into <u>L</u> provides us with at least one semantic complication. The complication arises from improper descriptions, i.e., descriptions that fail to designate either because their base is satisfied by no object (e.g., "the winged horse captured by Bellerophon") or because their base is satisfied by more than one object (e.g., "the President of the United States"). Frege proposed to handle such descriptions by assigning the null set as designatum to each of them.[49] While such an approach does have the virtue of maintaining a simplified logical theory, it engenders what are, at the very least, counterintuitive results. Under Frege's approach, e.g., the sentence "The winged horse captured by Bellerophon is the President of the United States" is true.

Strawson approaches the problem from quite a different perspective. According to him it is a statement (use of a sentence) and not the sentence itself that has a truth value, is or is not about something, etc. Moreover, in the case where "The F" is improper, sentences of the form "The F is G" cannot even be used to make a true or false assertion (statement). This is so, Strawson argues, because

> . . . the question of whether they are being used to make true or false assertions does not arise except when the existential condition is fulfilled for the subject term.[50]

While some may believe that Strawson's approach has the merit of a closer conformity to certain intuitive standards than, say, Russell's, it has the defect of injecting serious complications into logical theory. Quine has put the matter nicely as follows:

> Mr. Strawson . . . ably shows the failure of Russell's theory of descriptions as an analysis of the vernacular usage of the singular 'the', but he shows no

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[49] G. Frege, Grundgesetze der Arithmetik, vol I, (Jena, 1893), p. 19.

[50] P. F. Strawson, "On Referring," Mind 59 (1950): 343-44.

> appreciation of the value of Russell's theory as a means of getting on in science without use of any real equivalent of the vernacular 'the'. Russell's '(ιx)' is to the vernacular 'the x such that ' as '$\supset$' is to the vernacular 'if-then'; in neither case do we have a *translation*, but in both cases we have an important means of *avoidance* for scientific purposes. And in both cases we therefore have solutions of philosophical problems, in one important sense of this phrase.[51]

Our problem, to reiterate, is this: since we want, contra Russell, to allow descriptions the status of designators, and since improper descriptions fail to designate, how are we to understand designation? It can no longer be understood as a function. The answer is to understand it as a *partial* function. Thus, we require the following new clauses in our simultaneous definition of "designation under an interpretation" and "truth under an interpretation":

(i) If $\delta = (\exists\alpha)\phi$, then, where β is the alphabetically earliest name not already occurring in ϕ, (a) if there is a unique $\{\beta\}$-variant $I' = \langle D', f'\rangle$ of I such that $\phi\alpha/\beta$ is true under I', then des[δ] = $f'(\beta)$; and (b) if there is no such unique $\{\beta\}$-variant, then des[δ] is undefined.

(ii) If $\phi = (\iota\alpha)\psi{:}\chi$, then ϕ is true under I if and only if $\psi\alpha/\beta$ is true under exactly one $\{\beta\}$-variant of I and $\chi\alpha/\beta$ is true under that $\{\beta\}$-variant, where β is the alphabetically earliest name occurring neither in ψ nor χ.

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[51] W. V. Quine, "Mr. Strawson on Logical Theory," *Mind* 62 (1953):446.

Since our concern with descriptions is, in part, to analyze aboutness claims for them, and since the test for aboutness is expressed in terms of the universal generalization of a sentence with respect to a designator occurring in that sentence, the notion of the universal generalization of a sentence with respect to a description needs to be explained. Obviously the universal generalization of '$(\imath x)Fx{:}Gx$' is not '$(x){:}Gx$'; this latter expression is not even a well formed sentence of L. Intuitively, the universal generalization of "The present King of France is bald" is "Everyone is bald." Accordingly, we adopt the following convention: let ϕ be a sentence and let ϕ' be the result of deleting every occurrence of $(\imath\alpha)\psi$ from ϕ. The universal generalization of ϕ with respect to $(\imath\alpha)\psi$ is $(\alpha)\phi'$.

Our translation of (1) now becomes

(3) $(\imath x)Sxl{:}Ax$,

where the interpretation function provides the same assignments that des does in (2). Verifying that (3) is about the man who shot Lincoln, as well as, of course, everything that (2) is about, now becomes an easy matter. Pending a consideration of which lemmas, theorems, and corollaries proved in chapter II are, with the introduction of descriptions, still available to us, we must employ the conception of aboutness offered in section five of that chapter. Surely (3) is equivalent to itself, yet (3) does not imply any of '$(y)(\imath x)Sxy{:}Ax$', '$(Y)(\imath x)Yxl{:}Ax$' or '$(y)Ay$'. Hence, '$(\imath x)Sxl$' occurs essentially in (3). Since des['$(\imath x)Sxl$'] =

the man who shot Lincoln, (3) turns out, as desired, to be about the man who shot Lincoln. Given the demonstrable equivalence of (2) and (3), (2) can now be seen to be about the man who shot Lincoln.

The extension of L to include descriptions naturally raises the question of which of the lemmas, theorems, and corollaries proved in chapter II still remain in force. It should be clear that lemmas 1 through 6 carry over unchanged. So also does theorem 1. Theorem 2, however, is a different story. As a matter of fact the introduction of descriptions permits the construction of a counterexample to theorem 2. By a monistic sentence let us understand one that implies '(x)(y)x = y'. A monistic sentence is thus true under an interpretation only if the domain of the interpretation contains at most one object. Suppose now that ϕ is a monistic sentence containing at least one occurrence of the name 'a'. Let ϕ = '...a...'. Granted only these assumptions, it is an easy matter to show that 'a' does not occur essentially in ϕ. We prove this in the form of a deduction:

(1) ...a...	Premise
(2) (x)(y)x = y	(1), since ϕ is monistic
(3) (y)a = y	(2) U.I.
(4) a = b	(3) U.I., where 'b' does not occur in ϕ
(5) ...b...	(1)(4) Identity
(6) (x)...x...	(5) U.G. b

The import of this proof is simply that if ϕ is any monistic sentence, then $\phi \Rightarrow (\alpha)\phi\beta/\alpha$ for any name β. This, of course, is tantamount to claiming that no name ever occurs essentially in any monistic sentence. This fact does not, of course, provide us with a counterexample to theorem 2, since if ϕ is monistic and δ is a name, the antecedent of theorem 2 is never satisfied and the theorem holds, albeit vaduously. However, with the introduction of descriptions a counterexample to theorem 2 can be constructed along the following lines. Let ϕ = 'Fa & (x)(y)x = y'. Thus ϕ is monistic. Suppose now that '(ɿx)Fx' occurs in some sentence ψ equivalent to ϕ, say '...(ɿx)Fx:Gx...'. Since $\phi \Leftrightarrow \psi$, ψ is also monistic. Note therefore, that $\phi \Rightarrow$ '[(ɿx)Fx:Gx ↔ (Fa & Ga)]'. Note also that $\phi \Rightarrow$ '[(Fa & Ga) ↔ Ga]'; hence, $\psi \Rightarrow$ '[(Fa & Ga) ↔ Ga]'. Thus, $\psi \Rightarrow$ '...Ga...'. Since no name ever occurs essentially in any monistic sentence, $\psi \Rightarrow$ '(x)...Gx...'. But note that this is tantamount to claiming that ψ implies its own generalization with respect to '(ɿx)Fx'. Hence, where ϕ is a monistic sentence and δ is a description constructed solely from designators occurring essentially in ϕ, theorem 2 is violated. The reader should not mistake this claim for the claim that no description ever occurs essentially in a monistic sentence; that claim is false. The following, e.g., is a monistic sentence in which a description does occur essentially: '(x)(y)x = y & -(ɿx)Fx:Gx'. To see that this is so consider the following interpretation I = <D, f>. D = {a}, f('F') = ∅ and f('G') = {a}. Clearly

'$(x)(y)x = y$ & $-(\iota x)Fx{:}Gx$' is true under this interpretation. Clearly also both '$(x)((x)(y)x = y$ & $-Gx)$' and '$(Y)((x)(y)x = y$ & $-(\iota x)Yx{:}Gx$' are false under this interpretation.

THEOREM 9. If every designator part of ψ occurs essentially in some nonmonistic sentence ϕ and $(\iota\alpha)\psi$ is a closed description, then $(\iota\alpha)\psi$ occurs essentially in some $\phi' \Leftrightarrow \phi$.

PROOF. Let ϕ and ψ be as above and let Θ be some one-place predicate foreign to ϕ. Let $\chi = \{\phi$ & $[(\iota\alpha)\psi{:}\Theta\alpha \vee -(\exists\alpha)[(\beta)(\psi \leftrightarrow \alpha = \beta)$ & $\Theta\alpha]]$. Clearly $\phi \Leftrightarrow \chi$. Suppose, for a contradiction, that $(\iota\alpha)\psi$ does not occur essentially in χ, i.e., $\chi \Rightarrow (\Delta)\chi\delta/\Delta$ for some δ a designator part of $(\iota\alpha)\psi$. Hence, $\chi \Rightarrow (\Delta)\phi\delta/\Delta$ and $\chi \Rightarrow (\Delta)[(\iota\alpha)\psi{:}\Theta\alpha \vee -(\exists\alpha)[(\beta)(\psi \leftrightarrow \alpha = \beta)$ & $\Theta\alpha]]\delta/\Delta$. Thus, we have (1) $\phi \Rightarrow (\Delta)\phi\delta/\Delta$ and (2) $\phi \Rightarrow (\Delta)[(\iota\alpha)\psi{:}\Theta\alpha \vee -(\exists\alpha)[(\beta)(\psi \leftrightarrow \alpha = \beta)$ & $\Theta\alpha]]\delta/\Delta$. If δ is a proper part of $(\iota\alpha)\psi$, then (1) contradicts the antecedent of theorem 9. So suppose that δ is improper, i.e., $\delta = (\iota\alpha)\psi$. From (2) we have that $\phi \Rightarrow \{(\alpha)\Theta\alpha \vee -(\exists\alpha)[(\beta)(\psi \leftrightarrow \alpha = \beta$ & $\Theta\alpha]\}$. Hence, $\phi \Rightarrow [(\alpha)\Theta\alpha \vee -(\iota\alpha)\psi{:}\Theta\alpha]$. Generalizing with respect to $(\iota\alpha)\psi$ once again we have $\phi \Rightarrow (\alpha)((\alpha)\Theta\alpha \vee -\Theta\alpha)$. Thus, we have (3) $\phi \Rightarrow ((\alpha)\Theta\alpha \vee (\alpha){-}\Theta\alpha)$. But note that, so long as ϕ is nonmonistic, (3) is false. To see this, consider any interpretation I under which ϕ is true and which meets the following conditions: the domain of I contains at least two objects and the interpretation function assigns some proper subset of the domain to Θ. Under these conditions $((\alpha)\Theta\alpha \vee (\alpha)-\Theta\alpha)$ is clearly false.

With theorem 9 at hand a positive result, analogous to theorem 2, which does hold for descriptions can be obtained.

<u>COROLLARY</u> 6. If $\delta_1, \ldots, \delta_n$ are designators occurring essentially in some nonmonistic sentence ϕ and δ is a designator constructed solely from $\delta_1, \ldots, \delta_n$, then δ occurs essentially in some $\phi' \Leftrightarrow \phi$.

PROOF. Immediately from theorem 2 and theorem 9.

Corollaries 1 through 5, depending as they do on theorem 2, must, with the introduction of descriptions, be understood to hold only for <u>nonmonistic</u> ϕ. Likewise, theorems 3, 4, 5, 6, and 8 must now be understood to hold only for nonmonistic ϕ. As the reader can verify, theorem 7 carries over unchanged.

The failure of corollary 1 to hold in the case of descriptions constitutes an important breakdown of the results proved in chapter II. It will be remembered that it was essentially through the use of corollary 1 that the adequacy of the simplified test for aboutness offered at the end of section six of that chapter was proved. It will also be remembered that in section nine of chapter II it was claimed, on the basis of this simplified test, that Goodman's scruples concerning the type of logic to be employed in the expression and application of his theory could be vindicated. In the light of the above failure such vindication must be seen as available, at least for the present, only in the case of nonmonistic sentences.

Earlier in this section we considered the semantic difficulties surrounding improper descriptions. The semantic difficulties associated with those improper descriptions which fail to designate have, of course, a parallel at the level of unstructured singular terms, viz., <u>names</u> that fail to designate. In the next section we consider the question of how to extend <u>L</u> in such a way as to allow for the introduction of names that fail to designate. Aboutness claims for sentences containing such names are seen as analyzable via Goodman's notion of <u>rhetorical aboutness</u>. As will become obvious the notion of rhetorical aboutness has utility in the case of improper descriptions as well. As should <u>be</u> obvious we have at present no means of establishing that "The present King of France is bald" ('($\imath$x)Fx:Bx') is about the present King of France, given that des['($\imath$x)Fx'] is undefined. Rhetorical, not absolute, aboutness will thus be the vehicle by which such a claim is vindicated.

<u>Section Three: Nonexistents</u>

In this section we consider the problem of how to evaluate questions of aboutness for sentences containing singular terms that fail to designate. The problem of nondesignating singular terms arises in part from the fact that absolute aboutness is, thus far, understood to be a *relation* between a sentence and an object or class of objects. Since, in the case of nondesignating singular terms, there is no object or class for the sentence to be about, it would appear that a sentence such as

(1) The messenger of the gods wears Adidas,

cannot be said to be absolutely about the messenger of the gods. Yet surely (1) *is*, in some sense, about the messenger of the gods. Our approach to this problem is twofold. First, we introduce Goodman's notion of *rhetorical aboutness* and show how to characterize that notion for *L*.[52] Second, we note that the extension of *L* to allow for the introduction of nondesignating names is best understood in the context of *free logic*. Accordingly, we offer a semantics for such an extended *L* based on that developed by Lambert and Van Fraassen in *Derivation and Counterexample*.[53]

As was noted in section two of this chapter, the introduction of descriptions into *L* requires that we construe designation as a *partial* function. Thus, if we translate (1) as

(2) $(\iota x)Mx{:}Ax$,

we still cannot say that (2), and thus (1), is about the messenger of the gods, given that des['$(\iota x)Mx$'] is undefined. Perhaps the best way to express our intuition that

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[52]Goodman, "About," p. 265.

[53]Lambert and Van Fraassen, *Derivation and Counter-Example*, pp. 180-81.

(1) and (2) are, in some sense, about "the messenger of the gods," is to introduce into our discussion a new sense of aboutness, viz., Goodman's notion of rhetorical aboutness. Rhetorical aboutness is in part employed by Goodman to handle sentences containing occurrences of nondesignating singular terms such as 'Pegasus', 'Pickwick', and, presumably, improper descriptions as well. In terms of our notation and terminology rhetorical aboutness will be referred to as "δ-aboutness" and is defined thusly. Where ϕ is a sentence and δ a designator:

> ϕ is δ-about if and only if δ occurs essentially in some $\psi \Leftrightarrow \phi$.

Perhaps the first thing to notice concerning this definition is that δ-aboutness, unlike absolute aboutness, is simply a <u>property</u> that a sentence may have or fail to have, and is <u>not</u> a <u>relation</u> between a sentence and a thing. Thus, to say that a sentence is Pegasus-about is not to say that it is absolutely about Pegasus. The above definition of δ-aboutness is, of course, similar to the definition of absolute aboutness. The crucial difference concerns the nature of δ. In the case of absolute aboutness, we require that des[δ] be defined, i.e., we require that δ actually designate; in the case of δ-aboutness that requirement is suspended. The positive relationship between absolute aboutness and δ-aboutness may thus be summarized by the following principle:

If ϕ is δ-about, then des[δ] = k if and only if ϕ is absolutely about k.

With this new apparatus at hand, (2) can be seen to be '$(\iota x)Mx$'-about; hence, (1) can be seen to be the messenger of the gods-about. That this is so is a result simply of the fact that (2) implies neither '$(Y)(\iota x)Yx{:}Ax$' nor '$(x)Ax$'; thus '$(\iota x)Mx$' occurs essentially in (2), and (2) is '$(\iota x)Mx$'-about. While this does not show that (1) is *absolutely* about the messenger of the gods, it does provide a vindication of the claim made earlier that (1) is, *in some sense*, about the messenger of the gods.

Aside from the present problem the notion of δ-aboutness has utility also in the case of certain general terms. The sentence "Every unicorn has a horn" is absolutely about unicorns. But given that all and only unicorns are centaurs, this sentence is also absolutely about centaurs. Notice, however, that while this sentence is unicorn-about, it is not centaur-about.

Since δ-aboutness, in contrast with absolute aboutness, is a property and not a relation, the question of which lemmas, theorems, or corollaries concerning absolute aboutness proved thus far hold also for δ-aboutness does not arise. However, the question of whether or not there are provable analogues of those results for δ-aboutness does arise. The answer to this latter question is "Yes." The theorems and corollaries in question consist of theorems 3 through 8 and corollaries 2 through 5; none of the lemmas is concerned with absolute aboutness. The formulation and proof of the analogues of these theorems and corollaries is a relatively routine matter. For example, the formulation of the analogue of theorem 6 (ϕ is about k if and only if $-\phi$ is about k) is: ϕ is δ-about if and only if $-\phi$ is δ-about. The proof of this latter claim parallels most of the proof of theorem 6. The formulations and proofs of the remaining theorems and corollaries constitute equally routine tasks and are therefore omitted.

While the introduction of δ-aboutness thus allows us to provide a vindication of the claim that (1) is, in some sense, about the messenger of the gods, we still have no means of confirming that

(3) Pegasus flies

is, in some sense, about Pegasus. We cannot, e.g., argue that (3) is Pegasus-about, since (3) is not, at this point, translatable into L. It is not translatable simply because, as L is presently understood, all of the names of L actually designate, while 'Pegasus' does not. We could, of course, deal with this latest complication by adopting the Quinean ploy of translating 'Pegasus' as a description, say "the thing that pegasizes." Following this line, (2) would be translated as '$(\imath x)Px{:}Fx$', and (3) could thus be shown to be the thing that pegasizes-about. While such a ploy is available, it still leaves us with no means of showing that (3) is Pegasus-about, nor for that matter, can we show that (3) is, in any interesting sense, about Pegasus.

The above considerations would seem to suggest that the most "natural" way of demonstrating that (3) is Pegasus-about is to allow for the introduction into _L_ of names that do not designate, or, what amounts to the same thing, liberalizing 'des' in such a way that it need not be defined for each name of _L_, and then proving that (3) is Pegasus-about. Allowing into one's language a stock of nondesignating names is only one of the ways in which one's logic becomes a free logic. Another way to achieve a free logic is to allow for the possibility that the domain of an interpretation contains no object. The first approach provides a way of responding to complaints that _standard_ logic cannot successfully represent inferences involving nonreferring names except via some such method as the Quinean ploy discussed above. The second approach provides a way of responding to complaints that standard logic is committed to an empirical assumption. It is so committed, the argument goes, because of the insistence in standard logic that the domain of an interpretation be nonempty. Since it is only an empirical assumption that at least one thing exists, standard logic is committed to an empirical assumption.

While neither of the above arguments is endorsed by us we propose to adopt both of these approaches in order to accommodate the introduction into _L_ of nondesignating names. Thus, the logic to be developed is what is known as _universally free logic with identity_.

While our syntactical account of L (with descriptions) remains unchanged, our semantical account receives a dramatic reformulation. Instead of first defining the notion of an interpretation and then simultaneously defining the notions of designation under an interpretation and truth under an interpretation, we must now simultaneously define all three notions. The reason for this additional complication is simply that the notion of an interpretation now involves a new element, viz., the notion of a story. As an adequate characterization of this latter notion must appeal to the concept of designation under an interpretation, and since 'des' is defined in terms of truth under an interpretation, a triple inductive definition is required.

An interpretation is thus construed as a triple containing a domain, a story, and an interpretation function. The domain of an interpretation constitutes the set of things that, from the "point of view" of the interpretation, actually exist. The interpretation function provides us with information concerning the extensions of predicates of the language and information concerning which names of the language name actually existing things, or perhaps better, concerning which names actually name. The function of a story in an interpretation is to provide us with information, consistent with the information provided by the rest of the interpretation, concerning those names which, under the interpretation, do not name actually existing things, i.e., members of the domain. Accordingly, we say that an interpretation I of L is an ordered triple $\langle D, S, f\rangle$ that satisfies conditions (1) through (3):

(1) D is a set (empty or nonempty)

(2) f is a function that associates with each member of some subset (possibly empty) of the set of names of L some member of D, and associates with each n-place predicate of L some n-ary relation on D.

(3) S is a set (possibly empty) of sentences, called a story, that satisfies the following conditions:

(i) If $\phi \in S$, then either (a) $\phi = \Theta\beta_1 \ldots \beta_n$ where Θ is a predicate letter, $\beta_1, \ldots, \beta_n$ are names, and des[β_i] is undefined from some i ($1 \leq i \leq n$) or (b) $\phi = (\imath\alpha)\psi{:}\alpha = \beta$, where des[$(\imath\alpha)\psi$] and des[$\beta$] are undefined.

(ii) If $(\imath\alpha)\psi{:}\alpha = \beta_1 \in S$ and $(\imath\alpha)\psi{:}\alpha = \beta_2 \in S$, then $\beta_1 = \beta_2 \in S$.

(iii) For each name β, if des[β] is undefined, then $\beta = \beta \in S$.

(iv) Where β_1 and β_2 are names, if $\beta_1 = \beta_2 \in S$ and $\phi \in S$ and ψ is the result of replacing any number of occurrences of β_1 in ϕ by β_2, then $\psi \in S$.

(v) If $(\imath\alpha)\phi{:}\alpha = \beta \in S$ and $\phi\alpha/\beta' \in S$, then $\beta = \beta' \in S$.

(4) For each designator δ:

(i) If δ is a name β and f(β) is defined, then des[β] = f(β); otherwise des[β] is undefined.

(ii) If δ is a predicate letter Θ, then des[Θ] = f(Θ)

(iii) If δ is a structured predicate ($\alpha_1 \ldots \alpha_n$: ϕ) and $o_1, \ldots, o_n$ are members of D, then $<o_1, \ldots, o_n>$ ε des[δ] if and only if $\phi\alpha_1 \ldots \alpha_n/\beta_1 \ldots \beta_n$ is true under that $\{\beta_1, \ldots, \beta_n\}$-variant of I that assigns o_1 to β_1, ..., o_n to β_n, where $\beta_1, \ldots, \beta_n$ are the first n alphabetically earliest names new to ϕ.

(iv) If δ is a description $(\iota\alpha)\psi$ and $\psi\alpha/\beta$ is true under exactly one $\{\beta\}$-variant of I (where β is the alphabetically earliest name new to ψ), say <D', S', f> then des[$(\iota\alpha)\psi$] = f'(β), otherwise des[$(\iota\alpha)\psi$] is undefined.

The notion of a variant interpretation remains the same. We do insist, of course, that 'des' be defined for that (those) designator(s) with respect to which one interpretation is a variant of another. Suppose now that ϕ is a sentence, Θ is an n-place predicate letter, $\beta_1, \ldots, \beta_n$ are names and ψ and χ are formulas.

(5) If $\phi = \Theta\beta_1 \ldots \beta_n$, then ϕ is true under I if and only if either $<f(\beta_1), \ldots, f(\beta_n)> \varepsilon$ des[Θ] or $\phi \varepsilon$ S.

(6) If $\phi = \beta_1 = \beta_2$, then ϕ is true under I if and only if $f(\beta_1) = f(\beta_2)$ or $\phi \varepsilon$ S.

(7) If $\phi = -\psi$, then ϕ is true under I if and only if ψ is not true under I.

(8) If $\phi = (\psi \;\&\; \chi)$, then ϕ is true under I if and only if

both ψ and χ are true under I.

(9) If $\phi = (\exists\alpha)\psi$, then ϕ is true under I if and only if $\psi\alpha/\beta$ is true under some $\{\beta\}$-variant of I, where β is either the alphabetically earliest name not already occurring in ψ if α is an individual variable, or the alphabetically earliest n-place predicate letter if α is an n-place predicate variable.

(10) If $\phi = (\iota\alpha)\psi{:}\chi$, then ϕ is true under I if and only if either (i) $des_I[(\iota\alpha)\psi]$ is defined and $des_I[(\iota\alpha)\psi] \in des_I[(\alpha{:}\ \chi)]$ or (ii) there is some ψ', some α', and some β such that $(\iota\alpha')\psi'{:}\alpha' = \beta \in S$, $\psi'\alpha'/\beta' \Leftrightarrow$ (<u>classically</u> equivalent) $\psi\alpha/\beta'$, where β' is the alphabetically earliest name not in either ψ' or ψ, and $\chi\alpha/\beta$ is true under I.

We hasten to offer explanations. It might be supposed that instead of (ii) we could have offered the following simpler formulation:

(ii') $(\iota\alpha)\psi{:}\alpha = \beta \in S$ and $\chi\alpha/\beta$ is true under I.

However, (ii') will not do. This can be seen from the following example. Suppose that $\phi =$ '$(\iota x)Fx{:}Gx$', des_I['$(\iota x)Fx$'] is undefined, and that $S = \{$'$(\iota y)Fx{:}y = a$', 'Ga'$\}$. Clearly the "intent" here is that ϕ be counted as true under I. Yet with only (ii') at our disposal ϕ is <u>not</u> true under I, since '$(\iota \underline{x})F\underline{x}{:}\underline{x} = a$' $\notin$ S. The problem, of course, is simply that the description in our story employs a different variable than that occurring in ϕ. Were this the only problem a simple emendation concerning alphabetic variants would set things in order. Unfortunately, this is not the only problem as the following example shows. Let ϕ = '(ιx)[(Fx & Gx) v (Fx & -Gx)]:Hx'. Let des_I['(ιx)[(Fx & Gx) v (Fx & -Gx)]'] be undefined and let S = {(ιx)Fx:x = a, Ha}. Here again the intent is that ϕ be counted as true under I. However, since '(ιx)[(Fx & Gx) v (Fx & -Gx)]' $\notin$ S, ϕ is not true under I according to (ii'). The problem here, of course, is not one of alphabetic variance; rather, the base of the descriptions in ϕ and in S, though different, are, in a sense, equivalent; hence the need for the clause in (ii) concerning α', ψ' and the equivalence of ψ'α'/β' and $\psi\alpha/\beta$'. Concerning the example given immediately above, note that, since '[(Fa & Ga) v (Fa & -Ga)]' $\Leftrightarrow$ 'Fa', ϕ <u>does</u> count as true under I according to (ii).

(11) If $\phi = (\alpha_1, \ldots, \alpha_n: \psi)\beta_1 \ldots \beta_n$, then ϕ is true under I if and only if $\phi\alpha_1 \ldots \alpha_n/\beta_1 \ldots \beta_n$ is true under I.

The role of S in our notion of an interpretation is simply that of providing us with the means of evaluating sentences containing occurrences of nondesignating singular terms. The matter is nicely explained by Lambert and Van Fraassen:

> We must note here that not all the constants need have a designation in the domain; some may be nonreferring terms. How can we find out whether "Pegasus flies" is true in M [I] if "Pegasus" does not designate anything in M? The answer to the question is: we cannot find out. Since Pegasus does not exist there are no facts to be discovered <u>about</u> [my italics] him. What we can do is arbitrarily assign that sentence a value. Or we can say that due to its occurrence in some story (say,

> Greek mythology), the name "Pegasus" has acquired a certain connotation. Due to this connotation, we may feel "Pegasus swims" is false and "Pegasus flies", true. To get all the true sentences in the language, then, we need as part of a model [interpretation] M also a story. This story has to be consistent with the facts in M, of course; if M is the real world, the story may say that Pegasus flies, but not that Pegasus exists, nor that Pegasus is identical with some real horse.[54]

The motivation for restrictions (ii) through (v) on the notion of a story can be seen as follows. (ii) through (iv) are required in order to insure that our laws concerning identity remain in force. That (v) is required can be seen via the following example. Suppose that S = {'($\imath$x)Fx:x = a', 'a = a', 'Fb', 'b = b'}. Since our intent is that we construe not only the members of S as true, but also the "consequences" of S, (v) is necessary in order to insure the truth of 'a = b'. In standard (non-free) logic we could establish that S $\models$ 'a = b' by appealing to the equivalence of '($\imath$x)Fx:x = a' and '($\exists$x)[(y)(Fy $\leftrightarrow$ x - y) & x = a]'. But since $(\imath\alpha)\psi:\chi$ is not in general equivalent to $(\exists\alpha)[(\beta)(\psi \leftrightarrow \alpha = \beta) \& \chi]$ in free logic, such a strategy is not available to us.

(11) is, of course, markedly different from the characterization of the truth conditions for structured predications offered in chapter II. The motivation for adopting (11) is simply that our proof of theorem 1 no longer works due to the failure of existential generalization in free logic. However, since it is possible to establish the equivalence of $(\alpha_1 \ldots \alpha_n\colon \psi)\beta_1 \ldots \beta_n$ to $\psi\alpha_1 \ldots$

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[54] Ibid., p. 180.

$\alpha_n/\beta_1 \ldots \beta_n$ independently of theorem 1, (11) is not only available in the present context but could have been adopted in chapter II as well.

Consider now any interpretation $I = \langle D, S, f \rangle$ meeting the following conditions, D = the set of animate things, f('F') = {x: x flies} and S = {'Fp', 'p = p'}, where 'p' translates "Pegasus." Here 'Fp' is understood to be our translation of "Pegasus flies." Since it is not difficult to show that 'Fp' $\not\Rightarrow$ '(x)Fx', and since 'Fp' meets all of the requirements of δ-aboutness, 'Fp' can be seen to be p-about, i.e., "Pegasus flies" can be seen to be Pegasus-about, though not (absolutely) about Pegasus.

The question of which lemmas, theorems, and corollaries hold in the case of free logic is, at this writing, an open one. It is not difficult to establish that our proofs of lemmas 1 through 4 break down. The crucial steps, e.g., in the case of lemma 1, are the claim that $(\Delta)-(\alpha\colon \Theta\alpha)\beta\delta/\Delta \Rightarrow -(\alpha\colon \Theta\alpha \vee -\Theta\alpha)\beta$ and the claim that $(\Delta)\chi\delta/\Delta \Rightarrow (\Theta\beta \rightarrow (\alpha\colon \Theta\alpha \,\&\, -\Theta\alpha)\beta)$. Both steps are applications of the principle of universal instantiation, the principle that a universal quantification implies each of its instances. That this principle breaks down in the case of free logic is obvious when the instantiation is, unlike the above, to a name. That it also breaks down when the instantiation is to a predicate can be seen from the following. Let ϕ = '(Y)Ya'. Clearly '(x: Ax & -Ax)a' is an instance of ϕ. Clearly also, this instance of ϕ is L-false. Notice, however, that ϕ is true under I = <D, S, f>, where D = $\emptyset$ and 'Aa' ε S. Thus, $\phi \Rightarrow$ '(x: Ax & -Ax)a'. Thus universal instantiation for predicates breaks down and therewith the proof of lemma 1. Similarly, the proofs of lemmas 2 through 4, making use of U.I., also break down. The proofs of lemmas 5 and 6, making no use of principles objectionable to free logic, do not break down. The proofs of theorems 2 through 9 and corollaries 1 through 6 all either depend on lemmas 1 through 4 or employ universal instantiation, and, hence, break down. While the proofs of these results do not go through, I have not been able, as of this date, to discover any counterexamples. Thus, the question of which of lemmas 1 through 4, theorems 2 through 9, and corollaries 1 through 6 hold, in the case of universally free logic, is an open one. It is interesting to observe that these results, with the exception of theorem 7, hold, if the empty domain is prohibited from entering an interpretation. This can be seen from the fact that the principle of universal instantiation for predicates holds for such interpretations and, hence, the proofs of these results, with the exception of theorem 7, hold. The proof of theorem 7, appealing as it does, to universal instantiation for names, continues to fail.

Section Four: Concluding Remarks

In this essay an attempt has been made to clarify the concept of aboutness. In particular, an attempt has been made to clarify Goodman's concept of absolute aboutness. In chapter I, it was observed that there is an apparent conflict between the expression of Goodman's theory and the type of logic endorsed by him for its application. On the one hand, the expression of his theory seemed to require not only quantification over predicates, and, thus, an ascent to second-order logic, but also quantification over formulae. On the other hand, Goodman expressly advocates the employment of first-order logic in the application of his theory. In addition to the goal of clarifying aboutness, a reconciliation of these apparently conflicting demands constitutes an important goal of chapter II. In that chapter the notion of a structured predicate is introduced in order to answer the question, properly raised by Patton, of what the universal generalization of a sentence with respect to a formula could possibly mean. The principle result of chapter II is the substantiation of the claim that, at least with respect to first-order classical languages with structured predicates, the question of whether a structured predicate occurs essentially in a sentence can be decided by considering only those generalizations of the sentence in question which are generalizations with respect to just the atomic or unstructured designator parts of the predicate in question. This result paves the way not only for a simplification of the test for aboutness, but also for a vindication of Goodman's implicit claim that first-order logic is all that is necessary for the application of his theory. This vindication comes in the form of a reduction of one question to another: the first question is that of whether a sentence implies its own generalization with respect to the unstructured parts of a designator, the second question is that of whether, given an interpretation under which a sentence is true, there are variants of that interpretation, with respect to the unstructured parts of the designator in question, under which the sentence is false. Since no more set theory is involved in the notion of a variant interpretation, with respect to a predicate letter, than is already involved in the notion of a first-order interpretation, the claimed vindication is at hand.

The introduction of descriptions is seen as providing a complication in our picture of aboutness. The complication arises primarily in the context of monistic sentences. The complication is, however, systematic in nature and the results previously obtained are, with certain qualifications, seen to obtain for descriptions.

The question of whether the results obtained thus far can be extended to languages whose semantics is based on that of free logic is, as of this writing, an open one.

The question of how the concept of aboutness fares in languages containing modal operators is a question which, it is hoped, is now mature enough to receive systematic research.

BIBLIOGRAPHY

BIBLIOGRAPHY

Boolos, G., and Jeffrey, R. Computability and Logic Cambridge: University Press, 1975.

Carnap, Rudolph. The Logical Syntax of Language. Translated by A. Smeaton. London: Routledge & Kegan Paul, 1937.

________. Introduction to Symbolic Logic. New York: Dover Press, 1958.

Frege, Gottlob. The Foundations of Arithmetic. 2d ed. Translated by J. L. Austin. New York: Harper & Brothers, 1953.

________. Grundgesetze Der Arithmetik. Vol. I. Jena, 1893.

Goodman, Nelson. "About." Mind 70 (1961):1-24.

Hempel, Carl. "Studies in the Logic of Confirmation." Mind 54 (1945):1-26 and 97-121.

Henkin, Leon. "Completeness in the Theory of Types." Journal of Symbolic Logic 15 (1950):81-91.

Herzberger, H. "Paradoxes of Grounding in Semantics." Journal of Philosophy 67 (1970):145-167.

Lambert, K., and Van Fraassen, B. C. Derivation and Counterexample. Encino: Dickenson Press, 1972.

Leonard, Henry S. "Essences, Attributes, and Predicates." Proceedings and Addresses of the American Philosophical Association 38:25-51.

Mates, Benson. Elementary Logic. 2d ed. New York: Oxford University Press, 1972.

Patton, Thomas. "Some Comments on "About"." Journal of Philosophy 62:12 (1965):311-25.

Putnam, H., and Ullian, J. S. "More about 'About'." Journal of Philosophy 62:12 (1965):305-10.

Quine, W. V. "Reply to Grice." In Words and Objections. Essays on the Work of W. V. Quine. Dordrecht, Holland: Reidel, 1969 326-27.

________. "Mr. Strawson on Logical Theory." Mind 62 (1953): 433-51.

Rescher, Nicholas. "A Note on 'About'." Mind 72 (1963): 268-70.

Robbin, J. Mathematical Logic: A First Course. New York: W. A. Benjamin, Inc., 1969.

Russell, Bertrand. "On Denoting." Mind 14 (1905): 479-93.

Ryle, Gilbert. "'About'." Analysis 1 (1933):10-12.

Sharvy, Richard. "Three Types of Referential Opacity." Philosophy of Science 39 (1972): 153-61.

Strawson, Peter F. "On Referring." Mind 59 (1950):320-44.

Tichý, Pavel. "What Do We Talk About." Philosophy of Science 42 (1975:80-93.

Ullian, Joseph S. "Corollary to Goodman's Explication of About." Mind 71:284 (1962):545.

Whitehead, A. N. and Russell, B. Principia Mathematica to *56. Cambridge: University Press, 1962.

Wittgenstein, Ludwig. Tractatus Logico Philosophicus. London: Routledge & Kegan Paul, 1922.