

A METHOD OF ESTIMATING MARGINAL VALUE
PRODUCTIVITIES OF INPUT AND INVESTMENT
CATEGORIES ON MULTIPLE ENTERPRISE FARMS

Thesis for the Degree of Ph. D.
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Christoph Beringer
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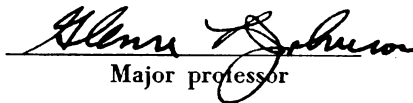
A METHOD OF ESTIMATING MARGINAL VALUE PRODUCTIVI -
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A METHOD OF ESTIMATING MARGINAL VALUE PRODUCTIVITIES
OF INPUT AND INVESTMENT CATEGORIES ON
MULTIPLE ENTERPRISE FARMS

By
Christoph Beringer

AN ABSTRACT

Submitted to the School of Advanced Graduate Studies of
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ABSTRACT

The purpose of the analysis was to modify presently used methods of productivity estimation so that they can be applied to the analysis of individual enterprises on multiple enterprise farms. Three multi-equational approaches were suggested as possible ways of solving the problem. These three approaches consisted of (1) a system of equations fitted by the method of simultaneous equations, (2) a system of equations, one equation for each major enterprise fitted independently to enterprise input-output data and (3) a system of equations, each equation fitted independently to data from specializing farms where the results of these estimates are applied on multiple enterprise farms.

Methods of grouping products into output categories and productive factors into input categories were considered. The conclusion was reached that generally products which are produced jointly can be grouped into one output category while products competing for resources should be analyzed separately.

Regarding the fitting of enterprise functions, it was concluded that fitting of one function to vertically integrated enterprises such as crops and hogs or crops and dairy is insufficient if it is desired to compare the productivity of various factors between crops and the livestock enterprises. Consequently, three separate functions were fitted, one to dairy, one to hog and one to crop-enterprise input-output data.

Regarding the grouping of inputs into input categories, it was concluded that an input classification which keeps intercorrelation among the independent variables and the errors of the regression coefficients at the lowest possible level is most desirable. This can be accomplished by choosing the sample purposively, thus, increasing the variance of the observations and by recombining input categories which are highly correlated. Furthermore, it is necessary to distinguish clearly in the accounts between investments and expenses as well as productive and nonproductive inputs.

In order to test the proposed methodologies independent enterprise functions and one aggregate function were fitted to detailed enterprise input-output data from 27 dairy-hog farms in northwestern Illinois. The Illinois records contained more detailed information than similar records kept at other experiment stations contacted in connection with this study.

A statistical analysis of the resulting enterprise function was carried out by testing the MVP of each production factor in each function against a minimum or reservation MVP which should have been earned by these factors in northern Illinois in 1950. Comparisons of the geometric mean organizations with these minimum MVP's revealed no serious maladjustments on the farms studied while a comparison of individual farms whose organization deviated from that on the geometric mean showed very serious madadjustments.

The productivity estimates carried out indicated that on the average farm in the sample the returns to labor in hogs are significantly below the price which has to be paid for labor indicating that less of this factor should be applied in hogs.

Regarding the productivity of feed, it was concluded that when compared at the geometric mean the returns in both livestock enterprises are just equal to the cost of feed.

The returns to land are high indicating that a possible expansion of the operations on these farms might be profitable.

A statistical comparison of the individual enterprise functions with the aggregate function indicated that the method of fitting individual enterprise functions furnishes more reliable information regarding individual enterprises than does the method of fitting one aggregate function to data from the entire farm business.

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CHAPTER I

INTRODUCTION

A brief reflection on the 2,000 year history of western civilization reveals that specialization has been recognized as the basic principle for achieving greater technical and intellectual skill and thus greater economic wealth for the individual as well as for the society as a whole.

Advocation of this principle has, however, not been equally strong at all times. Plato¹ was its most emphatic promoter in Greece of 400 B.C. making the principle of specialization the foundation on which he constructed his "Ideal State". While the Greek Empire fell, its intellectual and cultural heritage went into a hibernation lasting for over 1,000 years. The Renaissance ending the "dark" middle ages reawakened greek ideals of which the idea of specialization was at least an important part. Men like Adam Smith, David Ricardo and John Stuart Mill later extended the concept which had at the time of Plato referred only to the life within one city or a city state such as Athens, to cover various nations and even continents. Developments in the physical sciences through the discovery of physical laws also permitted the principle of specialization to become increasingly operative and effective.

¹ Plato, The Republic, Jowett Translation, New York: The Modern Library, p. 60 ff.

Thus, it is not entirely unjustified to look upon the Renaissance and ancient Greece as the intellectual father and grandfather of the present age of specialization with its rapid scientific and economic development.

While specialization is a powerful tool of achieving increasing wealth, it is not omnipotent. Even in economies as highly developed and specialized as those of the United States and Canada a number of industries, which are usually regarded as highly specialized, operate most efficiently when several products are produced in the same firm. Thus, shoe factories produce shoes of various sizes, colors and styles, automobile factories maintain a series of lines producing different models of cars of differing colors, shapes and sizes with the goal of using fixed resources in a way that the greatest total profit is achieved.

In agriculture, where considerable seasonal variation is present and where land, labor and/or other factors may be fixed, a greater total profit is often achieved if several enterprises are combined to permit full utilization of these fixed factors. In general, even in the most specialized economies, diversification will be present almost always although not to the same extent as and often for different reasons than in the economies which are less highly developed than those of North America and Northern Europe.

When enterprise combination is profitable, the economists major concern is to find in each particular case an enterprise combination which will maximize profit for each given quantity of resources.

This thesis deals with the problem of enterprise combination in agriculture. Its particular aim is to find a method with which it is possible to

(a) determine whether a farm should specialize in the production of one product or combine several enterprises and (b) determine the kind of enterprises which should be combined and the relative size in which they should be combined.

It is believed that this distinction is in line with the thinking of farm managers who are not only concerned about allocation of resources within one enterprise but also with the allocation of resources between enterprises. In fact, one can frequently observe that farmers are more concerned about inter- than intra- enterprise resource allocation which means essentially that their decisions are influenced more by output prices than by input prices. One possible explanation for this behavior, which is here merely suggested as an hypothesis, is that to the farm manager input prices vary on the average less widely than product prices.

This shall not imply that factor adjustments within one enterprise are unimportant or unnecessary; it merely means that the required adjustments due to factor price changes might on the average be considerably less significant than the changes required due to product price variations.

Past empirical workers estimating marginal productivities have concerned themselves almost exclusively with finding methods with which they could determine the best allocation of resources within one enterprise and have neglected to broaden their analyses to study inter-enterprise resource allocation.

The lack of empirical work in this area suggests the need to extend marginal productivity analysis to the problem of enterprise combination on multiple enterprise farms.

The following plan has been adopted to guide this investigation:

In Chapter II the static theory of production economics which furnishes the conceptual guide for the study will be explained. The relevant principles are presented in graphical as well as in mathematical terms, the latter permitting a generalization of the argument from one to several input variables used in one or several different enterprises.

The empirical part of the dissertation will be concerned mostly with an application of the theory of production economics to empirical estimation.

Chapter III contains a review of pioneer research studies dealing with the estimation of marginal productivities of inputs and investments in agriculture.

In Chapter IV, possible methods of estimating marginal value productivities on multiple enterprise farms and the conditions under which these methods are applicable are discussed.

In Chapter V and VI, the suggested methodologies are applied to actual farm enterprise data taken from the Illinois detailed Cost Account Records. With the help of the resulting estimates, a critical evaluation of the suggested methodologies is possible.

CHAPTER II

AN EXPOSITION OF THE THEORY SERVING AS THE CONCEPTUAL FRAMEWORK FOR THE EMPIRICAL INVESTIGATION

The present chapter will be concerned with the conceptual bases upon which the empirical analysis rests. The chapter is divided into two sections, one dealing with the relevant aspects of static general equilibrium theory, the other discussing the function used to obtain the empirical estimates and its mathematical characteristics. A discussion of sampling, combining of inputs into categories and combining of outputs into categories is reserved for Chapter IV.

A. The Relevant Aspects of Static General Equilibrium Theory

The theory guiding the empirical part of this study is the general equilibrium theory as developed by Walras,¹ Marshall,² Hicks³ et. al. This theory is deduced from the laws of diminishing marginal returns and diminishing marginal utilities.

The static theory of production economics which is an essential part of general equilibrium theory deduces from the law of diminishing returns under a set of static assumptions the conditions which have to be met if a firm wishes to maximize profits. Since the empirical estimation of production functions

¹ Walras, Leon, Elements of Pure Economics; or, The Theory of Social Wealth, Translated by William Jaffé, London: Allen and Brown, 1954.

² Marshall, Alfred, Principles of Economics, New York: The MacMillan Company, Eighth Edition, 1949.

³ Hicks, John R., Value and Capital, Oxford: The Clarendon Press, Second Edition 1946.

in succeeding chapters will be undertaken to establish whether a group of farm firms would meet these maximum profit conditions, it is necessary to discuss briefly the origin and nature of these conditions. First, the simple case of the firm with only one enterprise will be discussed. Then the case for the firm producing several outputs with a series of inputs will be considered.

1. The Theory Regarding Optimum Adjustment of Production

Factors within One Enterprise

From the law of diminishing returns, it follows that profit in any enterprise Y_i can be increased as long as the marginal value product (MVP)⁴ of any factor X_j , ceteris paribus, is not equal to the cost of this marginal unit (MFC).⁵ The proof for this proposition is obtained from the simplest form of the profit equation:

$$(2.1) \quad \pi = P_{Y_i} Y_i - \sum_{j=1}^n P_{X_j} X_j$$

which differentiated with respect to X_j yields

$$(2.2) \quad \frac{\partial \pi}{\partial x_j} = MPP_{X_j}(Y_i) P_{Y_i} - P_{X_j}.$$

Setting (2.2) equal to zero and assuming perfect competition

$$(2.3) \quad MVP_{X_j} = MFC_{X_j} \quad \text{where } j = 1, \dots, n$$

the profit maximizing condition for the firm producing one product using one

⁴ Marginal value product is defined as the value of the marginal product plus or minus the change in the value of the original total product which was caused by the marginal output.

⁵ Marginal factor cost is defined as the value of the marginal factor applied, plus or minus changes in the value of the original total quantity of the input used which resulted from the application of the last unit.

variable factor given one or several fixed factors is obtained.

2. The Theory Regarding Optimum Adjustment of Production Factors Between Enterprises

The static theory of enterprise combination can be explained both in graphical and in mathematical terms. Since this theory is used directly in the study, it will be explained first diagrammatically where it is limited to the analysis of two enterprises and then mathematically in the form of a generalized equation showing the optimum adjustment for any number of inputs in any number of enterprises.

a. The theory of enterprise combination in graphic terms. Two enterprises are assumed, each one using a set of factors $X_1, \dots, X_n$, fixed for the farm as a whole but variable between enterprises. The return obtained for these factors in both enterprises is smaller than the replacement cost of these factors and larger than their salvage value; for these factors the condition $P_{X_1, \dots, X_n} \text{ (Replacement)} > MVP_{X_1, \dots, X_n}(Y_1) =$

$MVP_{X_1, \dots, X_n}(Y_2) > P_{X_1, \dots, X_n} \text{ (Salvage)}$ holds.

Another set of factors $X_{n+1}, \dots, X_q$ are fixed in enterprise Y_1 . For these factors the condition $P_{X_{n+1}, \dots, X_q} \text{ (Replacement)} >$

$MVP_{X_{n+1}, \dots, X_q}(Y_1) > P_{X_{n+1}, \dots, X_q} \text{ (Salvage)}$ holds.

Analogously there is a set of factors $X_{q+1}, \dots, X_z$ which are fixed in enterprise Y_2 . For these factors the condition $P_{X_{q+1}, \dots, X_z} \text{ (Replacement)} >$

$MVP_{X_{q+1}, \dots, X_z}(Y_2) > P_{X_{q+1}, \dots, X_z} \text{ (Salvage)}$ holds.

Finally there are variable inputs $X_a, \dots, X_h$ which are neither fixed in one enterprise nor fixed for the farm as a whole. For them either the condition $P_{X_a, \dots, X_h} \text{ (Replacement)} < MVP_{X_a, \dots, X_h}$; or the condition $P_{X_a, \dots, X_h} \text{ (Salvage)} > MVP_{X_a, \dots, X_h}$ holds.

The two hypothetical production functions shown in Figure 1 and Figure 2 refer to enterprises Y_1 and Y_2 respectively. If $X_i, \dots, X_n$, the group of factors assumed fixed for the farm as a whole but variable between enterprises, is varied along with factors $X_a, \dots, X_h$ between the enterprises Y_1 and Y_2 , the surface of transformation curves shown in Figure 3 results.

A complication is introduced by the fact that the factors $X_i, \dots, X_n$ are priced in terms of their opportunity costs since they are fixed on the farm while the inputs $X_a, \dots, X_h$ are priced at market value. Thus, the condition specifying the best combination of enterprises for a given output is

$$\frac{MVP_{X_a, \dots, X_h}(Y_1)}{P_{X_a, \dots, X_h}} = \frac{MVP_{X_a, \dots, X_h}(Y_2)}{P_{X_a, \dots, X_h}} = \dots = \frac{MVP_{X_i, \dots, X_n}(Y_1)}{P_{X_i, \dots, X_n}} = \frac{MVP_{X_i, \dots, X_n}(Y_2)}{P_{X_i, \dots, X_n}}$$

cannot hold since $X_i, \dots, X_n$ are priced in the product-

ion of Y_1 at their MVP in Y_2 making

$$\frac{MVP_{X_i, \dots, X_n}(Y_1)}{P_{X_i, \dots, X_n}} = \frac{MVP_{X_i, \dots, X_n}(Y_2)}{P_{X_i, \dots, X_n}}$$

= 1 while the corresponding ratios for $X_a, \dots, X_h$, do not have to equal one.

Consequently, there are two equations which have to hold if the firm is in equilibrium assuming the iso cost line or opportunities line in a product-product dimension is specified. These equations are

$$(2.4) \quad \frac{MVP_{X_a}(Y_1)}{P_{X_a}} = \frac{MVP_{X_a}(Y_2)}{P_{X_a}} = \dots = \frac{MVP_{X_h}(Y_1)}{P_{X_h}} = \frac{MVP_{X_h}(Y_2)}{P_{X_h}} = c$$

a constant and

$$(2.5) \quad \frac{MVP_{X_1}(Y_1)}{MVP_{X_1}(Y_2)} = \dots = \frac{MVP_{X_n}(Y_1)}{MVP_{X_n}(Y_2)} = 1$$

The whole surface of iso cost curves, shown in Figure 3, shifts with each change in the relative prices of $X_a, \dots, X_h$. Thus, each additional iso cost curve drawn in Figure 3 refers only to one particular level of $\Sigma = P_{X_a} X_a, \dots, P_{X_h} X_h$.

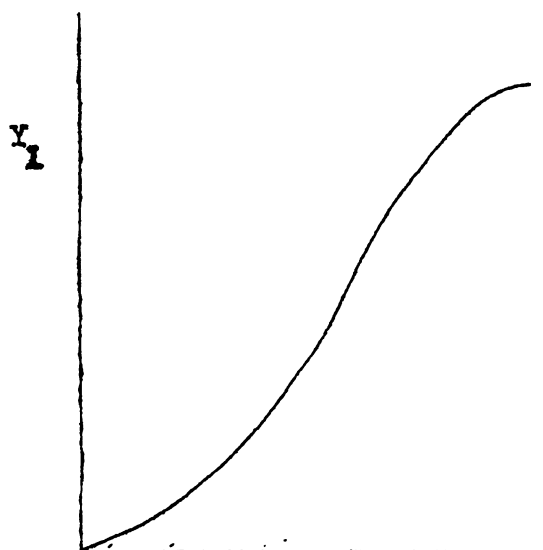
The iso revenue line which is tangent to the iso cost line in Figure 4 shows the relative prices of Y_1 and Y_2 and connects the various possible combinations of Y_1 and Y_2 yielding equal revenues. The iso revenue lines increase from the origin in the same way as the iso cost lines. The point at which an iso cost line is tangent to an iso revenue line represents that combination of Y_1 and Y_2 which will yield the greatest revenue and profit given a certain level of Σ and a certain set of relative prices among the $X_a, \dots, X_h$. In Figure 4, an amount A of Y_1 combined with an amount B of Y_2 represents the high profit combination.⁶

⁶ In Figure 4 any change $(-\Delta Y_1)$ in Y_1 must be equal to $-(MPP_{(X_1, \dots, X_n)}(Y_1))$

$(\Delta (X_1, \dots, X_n))$ and any change (ΔY_2) in Y_2 must be equal to

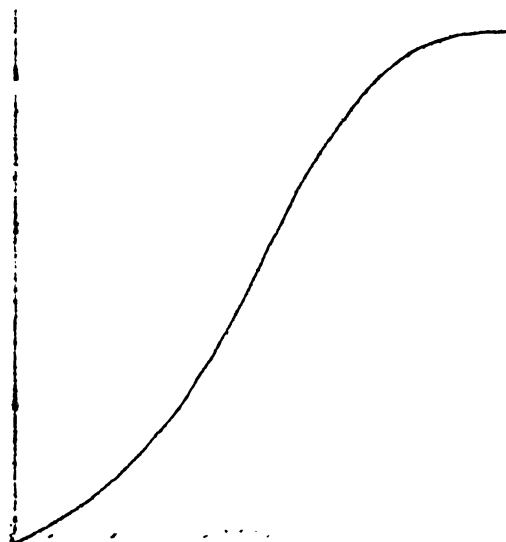
$(MPP_{(X_1, \dots, X_n)}(Y_2)) (\Delta (X_1, \dots, X_n))$. At the point of tangency between the iso cost and the iso revenue line their slopes are approximately equal $(-\Delta Y_1 / \Delta Y_2)$. The equation of the iso revenue line is $R = P_{Y_1} Y_1 + P_{Y_2} Y_2$ which, if solved for $Y_1 = \frac{R}{P_{Y_1}} - \frac{P_{Y_2}}{P_{Y_1}} Y_2$ and differentiated with respect

to Y_2 , $\frac{dY_1}{dY_2} = -\frac{P_{Y_2}}{P_{Y_1}}$ produces the slope of the iso-revenue line. Equating the expressions for the slope of the iso revenue line and the iso cost line



$X_a, \dots, X_n, X_1, \dots, X_n, X_{n+1}, \dots, X_q$

Fig. 1 Hypothetical Production Function for Enterprise Y_1



$X_a, \dots, X_n, X_1, \dots, X_n, X_{q+1}, \dots, X_z$

Fig. 2 Hypothetical Production Function for Enterprise Y_2

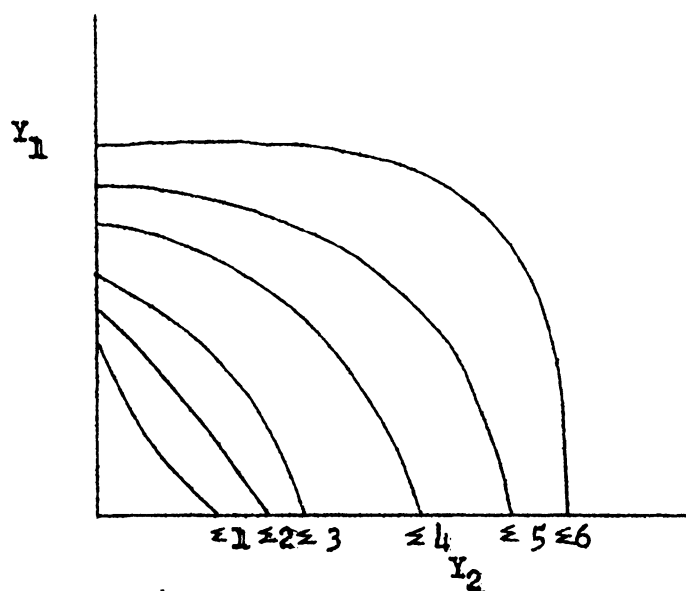


Fig. 3 Hypothetical Surface of Iso Cost Lines for Various Level of $X_1, \dots, X_n$

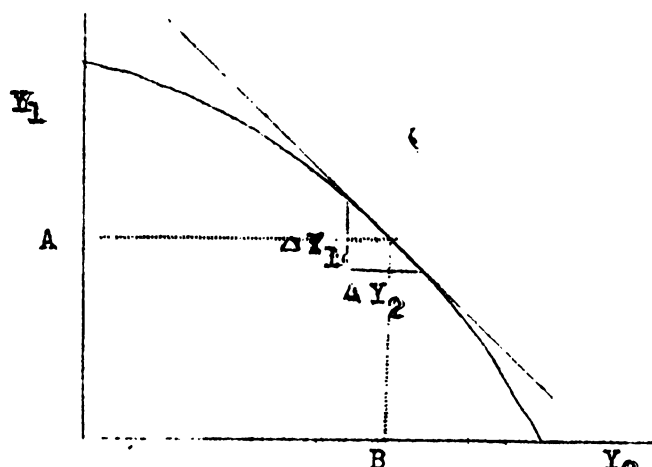


Fig. 4 Graphic Determination of the Optimum Combination of Y_1 and Y_2 ⁷

(cont.)

$$\frac{MPP(X_1, \dots, X_n)(Y_1)}{MPP(X_1, \dots, X_n)(Y_2)} = \frac{P_{Y_2}}{P_{Y_1}}$$

one obtains through simplification and division by MFC_{X_1} the equality

$$\frac{P_{Y_2} (MPP(X_1, \dots, X_n)(Y_2))}{MFC(X_1, \dots, X_n)} = \frac{P_{Y_1} (MPP(X_1, \dots, X_n)(Y_1))}{MFC(X_1, \dots, X_n)},$$

This expression can be generalized for any number of outputs and optimum adjustment for any given level of ξ is reached when the following condition is fulfilled

$$\frac{P_{Y_1} (MPP(X_1, \dots, X_n)(Y_1))}{MFC(X_1, \dots, X_n)} = \frac{P_{Y_2} (MPP(X_1, \dots, X_n)(Y_2))}{MFC(X_1, \dots, X_n)}, \dots,$$

$\frac{P_{Y_1} (MPP(X_1, \dots, X_n)(Y_1))}{MFC(X_1, \dots, X_n)}$. Ultimate adjustment cannot be reached until

the factors $X_1, \dots, X_n$ now fixed for the farm as a whole and variable between enterprises can become variable for the farm as a whole. Only then can they be freely bought or sold in such a way that the condition

$$\frac{P_{Y_1} (MPP_{X_1}) Y_1}{MFC_{X_1}} = \frac{P_{Y_1} (MPP_{X_j}) Y_1}{MFC_{X_j}}, \dots, \frac{P_{Y_1} (MPP_{X_1}) Y_1}{MFC_{X_1}} = \frac{P_{Y_2} (MPP_{X_1}) Y_2}{MFC_{X_1}} =$$

$$\frac{P_{Y_2} (MPP_{X_j}) Y_2}{MFC_{X_j}}, \dots, \frac{P_{Y_2} (MPP_{X_n}) Y_2}{MFC_{X_n}}, \dots, \frac{P_{Y_k} (MPP_{X_1}) Y_k}{MFC_{X_1}}, \dots, \frac{P_{Y_k} (MPP_{X_n}) Y_k}{MFC_{X_n}} = 1$$

is fulfilled.

7

A mathematical derivation of the equation of the iso cost line is presented in Appendix A.

The foregoing argument refers only to two enterprises and gives a graphical, intuitively appealing explanation of the theory of profit maximization in the farm with more than one major enterprise. A generalized theory of optimum combination of enterprises can be deduced from the profit equation just as this was done for the one variable case under (A) above.

b. The generalized theory of enterprise combination. Equation (2.4) is a generalized profit equation covering m outputs (Y_i), each one produced by any or all of n inputs (X_j).

$$(2.6) \quad \pi = \sum_{i=1}^m P_{Y_i} Y_i - \sum_{j=1}^n P_{X_j} X_j .$$

Assuming that changes in inputs or outputs which are made by the firm do not influence P_{Y_i} or P_{X_j} the system of differential equations (2.7) results upon successive differentiation of (2.6) with respect to the amount of each X_j used in producing each Y_i .

$$(2.7) \quad \frac{\partial \pi}{\partial X_j} = MPP_{X_j(Y_i)} P_{Y_i} - P_{X_j} .$$

Setting each of the $m \ n$ (2.7) equal to zero and assuming the second order conditions satisfied on the basis of the law of diminishing returns, results in $m \ n$ profit maximizing conditions which can be summed into

$$(2.8) \quad \sum_{i=1}^m MPP_{X_j(Y_i)} P_{Y_i} = \sum_{j=1}^n P_{X_j} .$$

Equation (2.8) is equivalent to the final equation in footnote 6 above.

B. The Empirical Procedures of Estimating Production Functions

The development of the theoretical concepts of marginal productivity analysis induced empirical workers to look for equations which would approxi-

mate actual production functions. Marginal Value productivities could be obtained from these equations through partial differentiation.

Generally, five names are associated with this work, all of them having suggested and used certain equations approximating the law of diminishing returns. These men are: Wicksell⁸, Mitscherlich⁹, Spillmann¹⁰, Cobb and Douglas.¹¹ The function originally developed by Cobb and Douglas was used at first to measure the respective contribution of labor and capital upon the gross national product of the United States economy. It is a power-function of the form $P = aC^k L^{1-k}$.¹² Cobb and Douglas did not use this function to estimate the MVP's for individual firms but confined themselves to the analysis of macro-economic relationships.

Tintner¹³, Brownlee and Heady¹⁴, following a suggestion made by Durand¹⁵, introduced to the analysis of input and investment productivities on farms

⁸ Wicksell, Knut, Lectures on Political Economy, Volume 1, London; G. Routledge and Sons Ltd., 1934-35, pp. 121-3 and pp. 127-30.

⁹ Mitscherlich, Eilhard A., Bodenkunde fuer Land und Forstwirte, Berlin; Paul Parey, 1905.

¹⁰ Spillman, W. I., Use of Exponential Yield Curves in Fertilizer Experiments, U.S.D.A. Technical Bulletin No. 318, 1933.

¹¹ Douglas, P. H. and C. W. Cobb, "A Theory of Production", American Economic Review, Vol. 18, 1928 supplement, pp. 139 ff.

¹² "P" in the original Cobb Douglas function represents Gross National Product, "C" represents Capital and "L" stands for Labor.

¹³ Tintner, G., and O. H. Brownlee, "Production Functions Derived from Farm Records", Journal of Farm Economics, Vol. 26, 1944, pp. 566 ff.

¹⁴ Heady, Earl O., "Production Functions from a Random Sample of Farms", Journal of Farm Economics, Vol. XXVIII, No. 4, November 1946, pp. 989-1004.

¹⁵ Durand, David, "Some Thoughts on Marginal Productivity with Special Reference to Professor Douglas' Analysis", Journal of Political Economy; Vol. XLV, December 1937, pp. 740-58.

a power function, which resembled the one used originally by Cobb and Douglas but did not require the sum of the exponents to equal one. The equation was of the form

$$(2.9) \quad Y = aX_1^{b_1}, \dots, X_2^{b_2}, \dots, X_n^{b_n}$$

where Y represents output, the dependent variable, and $X_1, \dots, X_n$ represents the independent inputs or investments which determine output. The following section will analyze the mathematics of this function which is commonly referred to as a Cobb-Douglas function. A treatment of the contents of past empirical studies using the Cobb-Douglas technique and pertaining to the subject matter of this study is reserved for Chapter III.

1. The Mathematics of the Cobb-Douglas Function

The Cobb-Douglas Function is a power function and has certain unique mathematical characteristics; some are helpful, others detrimental for empirical work. Among its positive features, the following are the most important: (1) The function is easily fitted to empirical data. In logarithmic form

$$(2.10) \quad \log Y = \log a + b_1 \log X_1 + b_2 \log X_2 + \dots + b_n \log X_n$$

the function is linear and can be fitted by the method of least squares. The only transformation required where the function is to be written in exponential form is the conversion of the constant a back into the natural numbers. The regression coefficients (b_i) are already in natural numbers and merely have to be raised to the exponent position. (2) The function gives immediately elasticities of the product Y with respect to the factors of production. (3) The power function allows the phenomenon of decreasing marginal returns to come into evidence without having to use up too many degrees of freedom.

If instead of the power function a quadratic function were used, decreasing marginal returns could also be shown. However, the number of regression coefficients which would have to be obtained increases substantially, thus reducing the number of degrees of freedom and requiring an increased number of observations. (4) If the errors in the data are small and normally distributed, a logarithmic transformation preserves the normality to a substantial degree. Even if the errors are not normally distributed and not independent the best linear estimate is obtained by the application of the method of least squares. Tests of significance, however, are no longer valid.¹⁶

Against these advantages stand a series of negative aspects which need to be understood: (1) the function can show increasing or decreasing returns but not both at the same time. Thus, the function increases at an increasing rate when the $\sum b_i > 1$, it increases at a constant rate when the $\sum b_i = 1$, it increases at a decreasing rate when the $\sum b_i < 1$, and it decreases absolutely when the $\sum b_i < 0$. (2) The function never reaches a maximum. (3) The function has constant elasticity throughout, which means that the scale lines will be straight lines resulting in the unrealistic feature that the optimum proportion in which the production factors are combined never changes as the enterprise increases in size. (4) The function in the form it is presently used must start at the origin and intersects the output-input planes at $Y = 0$.¹⁷

¹⁶ Tintner, Gerhardt, "A Note on the Derivation of Production Functions from Farm Records", Econometrica XIII, No. 1, January, 1944, p. 26.

¹⁷ Certain modifications of the function which would do away with some of its negative features have been developed by H. O. Carter at Michigan State University and are the subject matter of a forthcoming M.S. Dissertation.

2. The Marginal Value Products

The MVP's are obtained upon partial differentiation with respect to the factor for which the MVP is sought.

$$(2.11) \quad \frac{\partial Y}{\partial X_i} = aX_1^{b_1}X_2^{b_2}, \dots, b_iX_i^{b_i-1}X_{i+1}^{b_{i+1}}, \dots, X_n^{b_n}$$

Multiplying top and bottom of the right side of the differential equation

(2.11) by X_i , equation

$$(2.12) \quad \frac{\partial Y}{\partial X_i} = \frac{aX_1^{b_1}X_2^{b_2}, \dots, b_iX_i^{b_i-1}X_{i+1}^{b_{i+1}}, \dots, X_n^{b_n}}{X_i}$$

is obtained which reduces to

$$(2.13) \quad \frac{\partial Y}{\partial X_i} = \frac{b_i E(Y)}{X_i}$$

This partial derivative represents the empirical estimate of the MVP at the geometric mean and has the advantage that it can be computed for any level of Y and X_i which is not beyond the range of the data from which the original function was estimated.

CHAPTER III

PAST EMPIRICAL STUDIES ESTIMATING MARGINAL VALUE PRODUCTS OF INPUTS AND INVESTMENTS IN AGRICULTURE

The present chapter relates past empirical work in the field of productivity analysis to the present investigation. Most of the studies which have been conducted in the past concerned themselves with estimating productivities of inputs and investments on specializing farms. Only two research workers attempted to obtain productivity estimates of production factors in individual enterprises of multiple enterprise farms.

The chapter is divided into two sections, one of these will be concerned with the reasons why the analysis of multiple enterprise farms has been neglected, the other will discuss the more important studies which have been conducted and point out their relevance to the present investigation.

A. Reasons Why Past Productivity Analyses Have Concentrated Largely on Single Enterprise Farms

There appear to be three reasons why the analysis of individual enterprises on multiple enterprise farms has been neglected: (1) It was recognized by a number of empirical workers that fitting of one aggregate function to data from multiple enterprise farms would yield a general function which is frequently not applicable in any one enterprise on that farm, (2) accounts which were detailed enough to allow fitting of independent enterprise functions were not generally available, and (3) conceptual knowledge

regarding the dependence or independence of individual production functions was vague.

1. The errors introduced by fitting an aggregate function. An aggregate function in this context is defined as a production function estimated for the entire farm firm from input-output data taken from the entire farm, in contrast to a production function estimated for an individual enterprise on a farm, this function being fitted to data from that enterprise only.

Assuming that such an aggregate function is fitted to input-output data from farms with several equally important enterprises, unreasonable results must be expected: (a) If the nature of the returns to scale is different in the individual enterprises such that some of them would show increasing others decreasing returns to scale, the resulting estimate would be an average function which is not specifically applicable in any one of the enterprises, (b) the estimates of the regression coefficients would be meaningless if the true IVP of the factors for which the regression coefficient is estimated is not equal in the various enterprises on the farms studied. Estimates of marginal productivity resulting from such regression coefficients would not yield specific information about where in the firm the use of the factor should be changed, and (c) the errors of the regression coefficients which depend partially upon the correlation among the independent variables and the standard error of the regression line will sometimes be increased thus making the resulting MVP estimates more unreliable. In particular, fitting of an aggregate function should be avoided if the aggregation of inputs and outputs increases the standard error of the regression line and/or the correlation among the independent variables.

Two recent studies, one conducted by Glenn L. Johnson¹ in Calloway County, Kentucky, the other by Gerald Ion Trant² in northern Michigan may serve as illustrations of the complications which may result from fitting aggregate functions: Johnson fitted a function using 34 farm records from Calloway County farms. Most of these farms produced livestock, dark tobacco and popcorn in some combination. All the inputs and outputs were added and the resulting regression coefficients appeared unrealistic when compared to other estimates which were obtained from more specialized farms operating under similar production conditions as those found in Calloway County. In the Calloway County study, the MWP of land was unreasonably high, while those of forage-livestock investment and labor appeared unusually low. Johnson attributes these biases to a high intercorrelation among the aforementioned factors and suggests that some of the productivity of labor and livestock investment might have been reflected in the earning power of the land variable. This is in agreement with what has been pointed out earlier in this chapter. If two input categories are highly correlated, as might well be the case for forage-livestock investments and land, it is difficult to determine exactly the proper share of the product which is attributable to each one of these factors.

¹ Johnson, Glenn L., The Earning Power of Inputs and Investments on Upland Calloway County Farms, 1951, Progress Report No. 4, R-6 MA60, Kentucky Agricultural Experiment Station, University of Kentucky with Tennessee Valley Authority Cooperating.

² Trant, Gerald Ion, A Technique of Adjusting Marginal Value Productivities Estimates for Changing Prices, Unpublished M.S. Dissertation, Michigan State College, 1954.

Trant fitted two functions for thirty and thirty three farms respectively, both samples taken from 12 counties in northern lower Michigan. The regression coefficients for land and labor in the first sample turned out to be $-.232073 \pm .235095$ and $-.130584 \pm .225911$ respectively. The coefficient for labor in the second sample was $-.1738455 \pm .1417010$. These appeared unreasonable in view of a general knowledge of agriculture in that area. Trant points out "... while it is possible that increased quantities of land and labor might decrease gross income, it was not believed probable that they would do so."³ A number of the farms in both samples produced a highly heterogeneous product. A function fitted only to those farms in the sample which derived at least 40% of their total gross income from dairy produced regression coefficients which were positive and appeared to be more meaningful. Also the regression coefficients were associated with considerably smaller standard errors.⁴

From the foregoing considerations supported by the two studies mentioned, it appears that in many cases fitting of one aggregate function to data taken from a series of enterprises on a farm will not likely be a successful way of obtaining meaningful marginal value productivity estimates on multiple enterprise farms.

2. The availability of accurate input-output accounts. The second reason why empirical workers have avoided working with individual enterprises is the

³ Ibid, p. 37.

⁴ Ibid, p. 39.

difficulty of getting accurate input and output data. Detailed enterprise cost accounts are difficult to obtain by the survey method and farm account records ordinarily fail to contain enough detail by enterprises to fit such functions.

3. The conceptual difficulty of proving that production functions of individual enterprises are independent of each other. One solution for the multiple enterprise problem which appears, *prima facie*, to be a plausible one is that of fitting one independent function to each enterprise and then compare the resulting coefficients. This comparison would enable one to determine in which enterprise the application of any factor, X_i , is most profitable. However, there are conceptual and empirical questions which have to be answered before such a solution can be suggested as a reasonable one.

The conceptual question centers around the problem of determining the presence or absence of functional relationships among individual production functions on multiple enterprise farms. The empirical question centers around the accounting problem and asks if it is possible to devise an accounting scheme which is able to separate inputs which were used in enterprise Y_i from those used in enterprise Y_j in case production functions on multiple enterprise farms are independent of each other. The answers to these two questions will be sought in Chapter IV.

B. Review of Past Empirical Work Estimating Marginal
Value Productivities in Agriculture

Empirical work using the Cobb-Douglas technique to estimate Marginal Value Productivities of input and investment categories in agriculture was begun during World War II.⁵ This section will discuss some of the pioneer studies which can be classified according to whether they analyzed single or multiple enterprise farms. The first group of studies is by far the largest and within it the studies can be grouped according to the types of data upon which the analysis was based.

1. Studies which confined themselves to specializing farms. The design of most of the studies which estimated MVP's reflect the realization that data from multiple enterprise farms, if aggregated, yield unreliable results. Thus, in order to have only specializing farms in the sample, it was usually specified that a certain minimum percentage of the gross income on the farms included in the sample had to come from one enterprise.

The first applications of the Cobb-Douglas technique were made by Tintner⁶ and Tintner and Brownlee⁷ at Iowa State College. Farm business records from 5 types of Iowa farms were used: "Dairy", "Hogs", "Beef Feeders", "Crops" and "General". The independent variables were (A) Number of acres; (B) Labor

⁵ Tintner, Gerhard, "A Note on the Derivation of Production Functions from Farm Records" Econometrica XII, No. 1, January, 1944.

⁶ Ibid, pp. 26-34.

⁷ Tintner, Gerhard and O. H. Brownlee, "Production Functions Derived from Farm Records", Journal of Farm Economics, Vol. 26, August, 1944.

months; (C) Farm improvements (buildings, fences, etc.); (D) Liquid assets (livestock, feed, seed, fertilizer, etc.); (E) Working assets (farm machinery, including the farm share of the automobile, breeding livestock, equipment other than buildings and fences); and (F) Cash operating expenses (equipment repairs, fuel, oil, feed purchased). Management was not included as an input.

The standard errors of the regression coefficients of the farms called "General" were compared with those of all farms included in the study and were found to be higher than the average for each of the various input categories on the "General" farms. However, it is difficult to establish that these larger errors came about as a result of the aggregation of inputs and outputs from several enterprises. Tintner and Brownlee's sample included only 20 "General Farms" which reduces the number of degrees of freedom to 13 and the larger errors might be due to the smallness of the sample.

Drake,⁸ using Michigan Farm Account records fitted two general functions, one for dairy farms, the other for general farms. The reason for this separation is given as follows; (1) "the separation of the dairy farms from all farms permits a more specific analysis of the structure of the farm business..." (2) "The hypothesis was set up that the correlation of gross income with the factor categories should be greater for the more homogeneous groups of dairy farms than for the not-dairy farms. From this it should follow that the confidence intervals of the coefficients of elasticity of gross income for the

⁸ Drake, Louis S., Problems and Results in the Use of Farm Account Records to Derive Cobb-Douglas Value Productivity Functions. Unpublished Ph.D. Dissertation, Michigan State College, 1952.

different categories of factors should tend to be narrower..."⁹ Drake does not analyze in detail the reasons for his hypothesis. Presumably, however, his argument was similar to that developed in the preceding section of this chapter where it was shown that aggregation of input and output data can under certain circumstances increase the correlation between the independent variables and thus, make the estimates more unreliable. It turned out that Drake was unable to support his hypothesis with the data he used.¹⁰ However, he was the first one who recognized this difficulty and tried to establish the empirical proof.

Heady¹¹ fitted functions to random samples of farms and was the first one to use survey sampling data for Cobb-Douglas productivity analysis. A total of 13 functions were fitted to data which were grouped according to the enterprise which was most prevalent in the regions wherefrom the data were chosen.

The errors of the regression coefficients for the group called "General Farms" are not significantly higher than the errors which were found for the regression coefficients on specializing farms.

⁹ Ibid, p. 47.

¹⁰ Ibid, p. 47.

¹¹ Heady, Earl O., "Production Functions from a Random Sample of Farms", Journal of Farm Economics, Vol. 28 (1946), pp. 989-1004.

Johnson¹² was first to apply purposive sampling in devising sample surveys. The notion here is to obtain data covering the largest possible area of stage II on the production surface. Johnson also developed rules for input classification. Both the various possible sampling procedures and the rules for input classification as developed by Johnson will be discussed in the next chapter.

2. Productivity studies dealing with multiple enterprise farms. There are two studies which attempted to analyze individual enterprises on multiple enterprise farms.

Fienup¹³ fitted a crop and a livestock function to data from 150 Dry Land Crop and Livestock Farms in Montana. Fienup does not justify, conceptually, the fitting of independent functions nor does he attempt to treat the accounting problem in any systematic way. The data to which the individual enterprise functions are fitted are not the result of individual enterprise input-output accounts but are, in several cases, obtained by arbitrarily apportioning of the inputs used on the farm as a whole.

French¹⁴, leaning heavily on a paper by Marshak and Andrews¹⁵ employed simultaneous equations to estimate production functions for crop and livestock

¹² Johnson, Glenn L., Sources of Income on Upland Marshall County Farms, Progress Report No. 1, and Sources of Income on Upland McCracken County Farms, Progress Report No. 2, Lexington, Kentucky Agricultural Experiment Station, 1952.

¹³ Fienup, Darell, Resource Productivity on Montana Dry Land Crop Farms, Mimeograph Circular 66, Montana State College Agricultural Experiment Station, Bozeman, Montana, 1952.

¹⁴ French, Burton, Estimation by Simultaneous Equations of Resource Productivities from Time Series and Cross Sectional Farm Observations, Unpublished Ph.D. Dissertation, Iowa State College, 1952.

¹⁵ Marshak, Jacob, and William Andrews Jr., "Random Simultaneous Equations and the Theory of Production", Econometrica, Vol. 12, 1944, pp. 143-205.

enterprises on 15 Iowa farms. French treated these systems (crops and livestock) independently although he assumes that "... in actual operation we have two production functions each dependent upon the other."¹⁶

The reason why Marshak and Andrews suggest using simultaneous equations and not the single least squares approach to estimate production functions is mainly that the X_i in the production function are not considered to be truly independent variables since they are a function not only of prices - (Marshak and Andrews do not make the assumption of perfect competition) - but also of uncontrolled factors such as management for which no specific assumptions regarding their probability distribution can be made. Marshak's and Andrews' suggestions refer to the estimation of production functions not only for agricultural but also for industrial firms.

Since this study is concerned with production functions for agricultural firms only, it is believed that the essential assumptions for perfect competition can be made; thus Marshak's and Andrews' criticism of the single least squares approach is at least partially invalidated in this particular instance. The assumptions which were made in this study regarding the distribution of unexplained residuals such as management will be discussed in Chapter IV.

It should be pointed out also that Marshak's and Andrews' suggestion to use simultaneous equations is made because it is believed that by this method, the best fit of individual production functions is obtained. The simultaneous equations approach is not suggested to explain the relationships among production functions in multiple enterprise firms.

¹⁶ French, Burton, op. cit. p. 55.

Analagously French' study is concerned with the best way of fitting independent equations and does not attack the most important problem of multiple enterprise analysis namely the question of functional relationships between individual production functions.

The following chapter will investigate two very closely allied questions:

(1) Can the production functions within agricultural firms assumed to be independent of each other or are they functionally related? (2) If these functions are actually independent, is it possible to guard against pseudo-dependencies through accurate accounting methods?

CHAPTER IV

METHODS OF ANALYSIS AND THE PROBLEM OF ACCOUNTING

The present chapter will consider ways of analyzing enterprises on multiple enterprise farms. In particular, possible methods of estimating the parameters statistically and problems of input and output accounting will be considered.

A. Analytical Methods Available and the Choice of the Most Appropriate Procedure on the Basis of Existing Relationships on Multiple Enterprise Farms

It has been shown in the previous chapter that many past empirical studies have been able to circumvent the multiple enterprise problem but have not employed the necessary analytical apparatus to cope with it adequately. The problem was circumvented by fitting functions to data from specializing farms; the results in these instances were accurate but revealed nothing about optimum combinations of enterprises. No attempts were made in earlier studies to work out the conditions under which functions fitted to data from specializing farms could be used in handling problems on multiple enterprise farms.

Studies which fitted an aggregate function to farms with a variety of enterprises yielded results which revealed nothing about enterprise combinations and often produced meaningless estimates.

As a result, it is concluded that the single equational approach is not generally applicable to the solution of the multiple enterprise problem. The

alternatives which are suggested consist of (1) fitting a series of equations, one for each major enterprise, to data from multiple enterprise farms or (2) fitting a series of equations to enterprise data from specializing farms, ascertaining the situations in which synthesis of such fits is possible on multiple enterprise farms.

1. Definition of Multi-Equational Approaches

By multi-equational approach either one of the following is implied:

- (a) a system of equations in which all or several of the individual functions are functionally related to each other. This system would have to be solved by the method of simultaneous equation which, if the system is "just identified" involves a transformation of the equations into the "reduced form" where each endogeneous variable is expressed in terms of exogeneous variables only. The equations are fitted in this form by the least squares procedure and reconverted into their original "non-reduced" form. If the system is "overidentified" the limited information, maximum likelihood method yields more accurate estimates of the parameters.¹
- (b) a system of equations, each one fitted independently to data from farms on which those enterprises are combined, and
- (c) a system of equations each fitted independently to data from farms specializing in those enterprises which are combined on the multiple enterprise farms studied.

¹ Klein, Lawrence, A Textbook of Econometrics, Evanston: Illinois and White Plains, New York: Row, Peterson and Company, 1953, p. 254.

2. The Applicability of the Three Approaches

The applicability of any one of the three methods of analysis outlined is not immediately evident and depends upon the presence or absence of structural interrelations between enterprises.

Approach (a) a system of equations, one for each technical production function, solved by the method of simultaneous equations may be applicable in those instances where the physical production functions of the individual enterprises are functionally related to each other. In less technical terms, this means that the physical input-output relationships and therewith the parameters of the production function, in one or all of the enterprises $Y_1, \dots, Y_n$, would always change as a result of changes in inputs or outputs occurring in enterprise Y_m .

Physical interdependence of production functions implies that the production parameters in one enterprise change as changes in inputs or outputs in other enterprises occur even if all the inputs are measured in terms of physical units. Another kind of interdependence is economic and occurs if all or some of the inputs are measured in terms of their opportunity costs in other enterprises making the production functions involving inputs so measured dependent upon each other. The remarks concerning the applicability of simultaneous equations apply only if this first type of interdependence which is of a technical nature exists. Economic interdependency can be handled through avoiding pricing of inputs or investments in terms of their opportunity costs for, if no external economies or diseconomies are present, market prices will be constants in the sense that the farmer has no influence upon them. If, however, internal economies and diseconomies exist and on-farm opportunity costs are used to price inputs, biases of the quantities of

inputs actually used would result. Accounting in physical units or constant dollar terms is not affected by the existence of internal economies or diseconomies.

Approach (b) a system of equations each fitted independently to data from multiple enterprise farms, appears applicable if the physical production functions of the individual enterprises are not related to each other. In those cases where physical dependencies among production functions seem to exist, accurate accounting can determine whether a pseudo-dependency or a structural dependency is at hand.

Approach (c) appears useable if the production functions along which enterprises on multiple enterprise farms operate are independent, the same as those on specializing farms and involve the same inputs.

In the remainder of this section a number of conceivable relationships on multiple enterprise farms will be investigated in an effort to determine under what conditions the production functions should be expected to be physically related or unrelated and/or the same or different from those of specializing farms. After these investigations are completed, it will be possible to choose from among the approaches outlined above the most appropriate one.

3. The Existing Relationships among Enterprises

a. Input-input complementarity. Input-input complementarity exists if two inputs used in producing a certain product have to be combined in rather definite proportions. A great number of inputs used in agriculture are complements to each other although the proportion in which they are optimally combined varies from enterprise to enterprise. Complementarity is due mostly

to physiological requirements which permit substitution of factors only within a rather narrow range. For purposes of illustration, a beef-hog farm is assumed. The inputs are hay and grain. The supply of both these inputs is fixed on the farm at least within one year. If the output of hogs is reduced due to lower anticipated pork prices, both grain and hay (pasture) will be released which can now be used in the production of beef feeders. However, since the hay/grain consumption ratio for hogs is, in almost all cases, lower than that for beef feeders it is apparent that grain will now be available in relatively larger quantities and a beef feeding system leaning more heavily towards dry lot feeding would be indicated. This is especially true in those instances where hay and grain are fixed on the farm, i.e. their MVP's are higher than the price which would be obtained if they were sold on the market. The question relevant to this investigation is whether or not shifts such as the one described change the parameters of the production function of the enterprise in which the substitution has occurred. If the inputs are measured in dollar values based on alternative earnings, this question must be answered in the affirmative since each change in the relative sizes of the enterprises would be connected with a re-valuation of the total quantity of the factor used in the two enterprises. Thus, in the example above, after the shift of grain from hogs to beef has occurred the opportunity cost of grain in beef would have fallen while that of hay would have risen. If the total quantity of grain fed to feeders were valued at this new (lower) opportunity cost a downward bias in the grain variable would result producing an apparent upward shift in the production function of beef.

If, however, the factors are measured in terms of constant dollar values or in terms of physical units, changes such as the ones described above will appear to be what they really are, namely movements along the same physical production function. It appears on the basis of the foregoing argument that the simultaneous equations approach is not necessary as a result of shifts involving differences in input-input complementarities.

b. Product-product complementarities. Two subcases of product-product complementarity can be distinguished: True complementarity and byproduct complementarity. True Complementarity exists if variable factors of production i.e. factors whose MVP is larger than their replacement cost or whose MVP is smaller than their salvage value can be varied between enterprises in such a way that the output of one enterprise is increased without decreasing the output of other enterprises. Each one of the curves in Figure 5 which shows different combinations of two products Y_1 and Y_2 obtainable with a certain quantity of these variable factors is called an iso outlay curve and shows ranges of complementarity and competitiveness. Thus, if a farmer decided to apply $\frac{1}{2}$ ton of fertilizer worth \$30.00 on two acres of land, one of them producing corn the other oats, he might well be in stage III if he applied all the fertilizer in corn. If he began to reduce the application in corn and applied some of it in oats both his corn and oats yields might be increased. A different, higher or lower, iso cost curve exists for each total quantity of the variable factors used between the two enterprises.

There is no reason to assume that movements along the scale line of each individual production surface occurring as factors are shifted between enter-

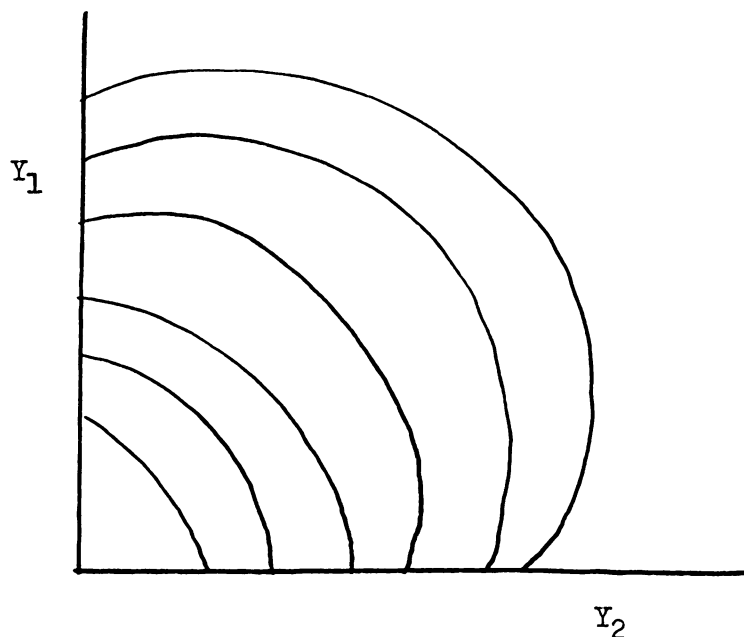
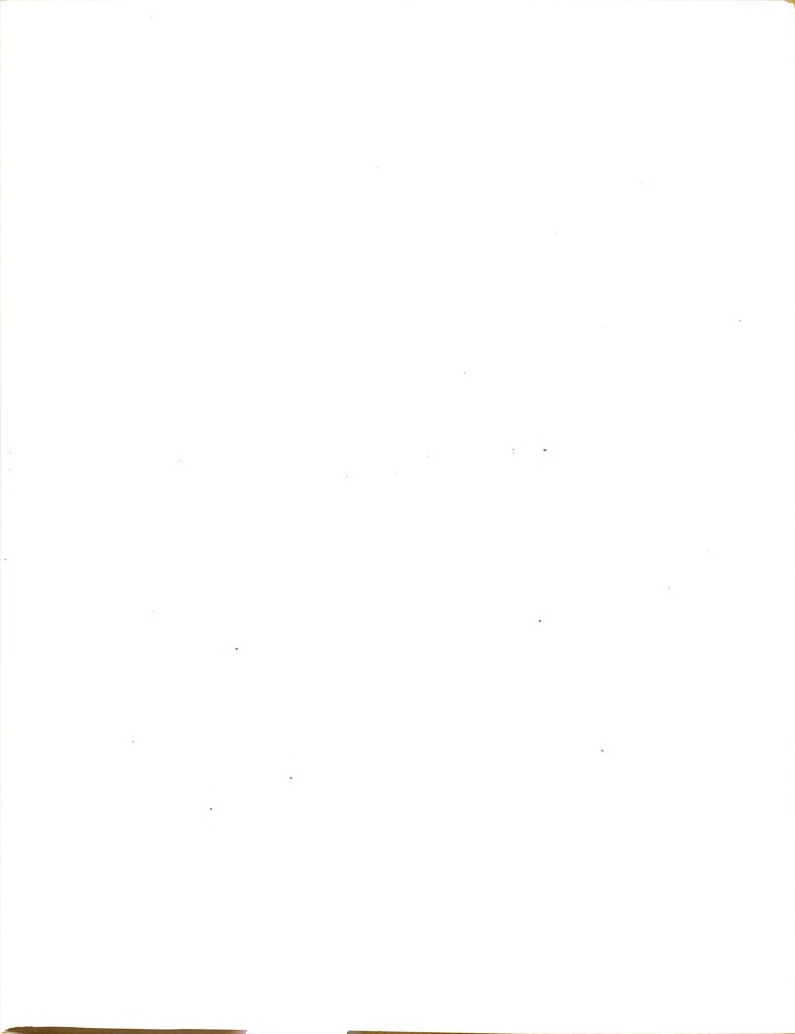


Fig. 5 , Hypothetical Surface of
Iso Cost Curves Showing Areas
of Complementarity and Com-
petitiveness

prises or movements along the scale line of the iso cost surface in the product-product dimension would make the individual production functions dependent upon each other. This is true only as long as inputs are measured either in constant dollar terms or in terms of physical units.

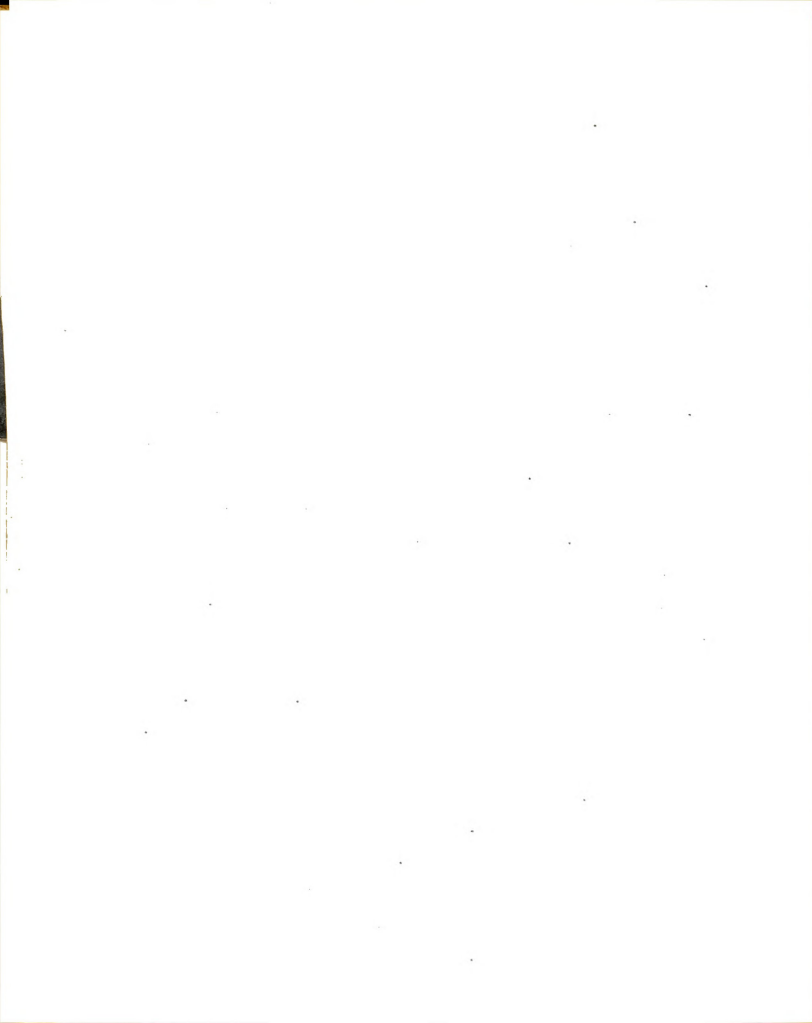
Supplementarity due to better utilization of fixed resources can be looked upon as a subcase of true complementarity closely allied with the one discussed above. The only difference is that part of the factors previously assumed to be variable have now become fixed inputs. While they are fixed for the farm as a whole they remain variable between enterprises. In such a case enterprises may be added and their size expanded until the returns of the fixed factors in the two or more enterprises in which they are used are



equal to each other. The important source of complementarity here is that the MVP of the fixed factors which might have been zero before additional enterprises were added has now increased thus reducing the average cost per unit of output. Again there is no reason to believe that this type of complementarity makes the physical production functions dependent upon each other.

Byproduct complementarity is a form of pseudo-complementarity occurring when part of the output coming from one enterprise is used as an input in the other enterprises in the firm without being considered in the accounting process. Failure, in accounting procedures, to credit such byproducts to the producing enterprise and charge them to the utilizing enterprise creates an apparent complementarity. An example of this is the situation which exists between corn and legume enterprises when the byproducts, nitrogen, humus and soil tilth are ignored. It appears as if, after the addition of the legume enterprise, the same amount or more corn can be grown in addition to the hay crop with the same total outlay used when no legumes were grown. Actually, however, the inputs to the two enterprises have increased if one considers the nitrogen added as well as the improvement in soil tilth which was brought about as a result of the addition of legumes. Figure 6, 7, and 8 show the situation which exists as a result of the presence of byproducts.

Assume that the total quantity of factors available are (a) of nitrogen and (b) of potassium. If these quantities were used in corn production, an amount A of corn would be produced. Assume now that a part of the total quantity of N and K_2O is applied to legumes. These legumes will build up nitrogen in the soil which might in succeeding years grow a larger quantity of corn with a smaller quantity of the commercial inputs N and K_2O than was used previously in this enterprise. The conclusion might then be reached



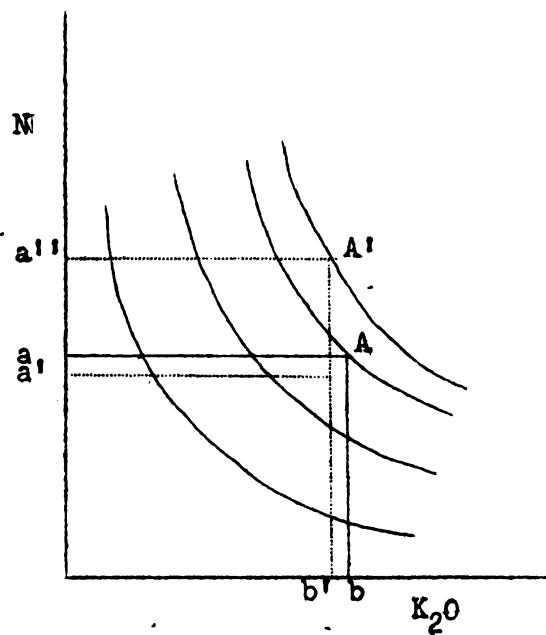


Fig. 6 Hypothetical Production Surface for Corn (Y_1) using N and K_2O

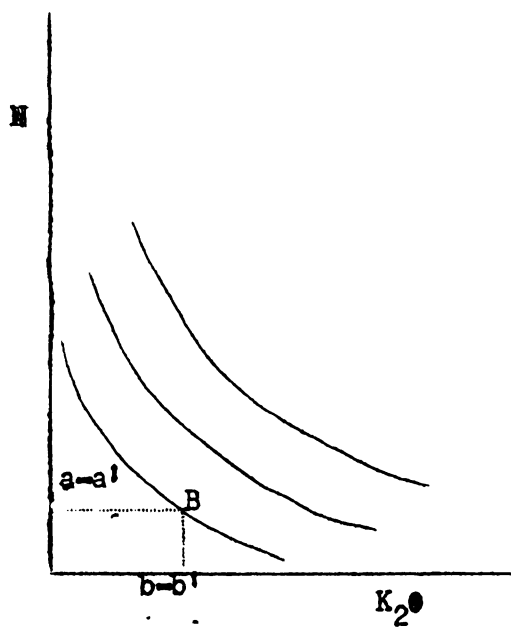


Fig. 7 Hypothetical Production Surface for Hay (Y_2) using N and K_2O

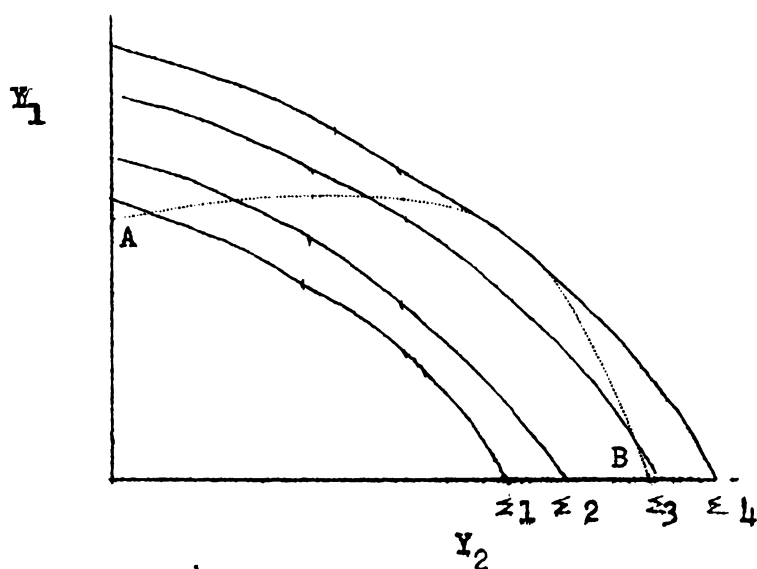
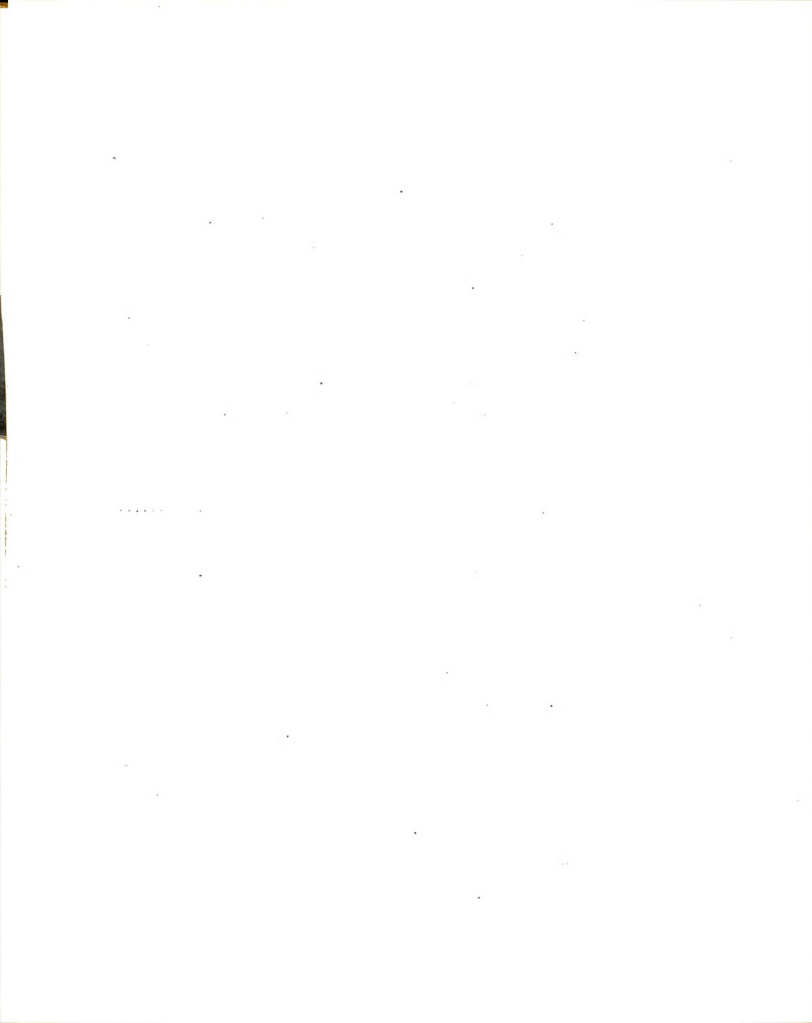


Fig. 8 Hypothetical Iso Cost Curve between Corn and Hay

that corn was produced in stage III since the production of both corn and legumes has been increased through a shift of the productive agents between enterprises. Figure 6 illustrates the situation for corn. If the total quantity of N and K_2O were applied to corn, a quantity A (Figure 8) might be produced. After the application of N and K_2O in corn has been reduced to a' of N and b' of K_2O an amount B of legumes is produced. However, since the legumes grown left nitrogen in the soil, the actual quantity of nitrogen available to corn might, in the following year, be a'' which combined with b' of K_2O produced a quantity A' of corn which is more than the original quantity A. Here then a peculiar type of discontinuity exists in the iso cost function of Figure 8, which can be explained if it is recalled (Chapter II) that a different iso cost curve in the product-product dimension exists for each new quantity of the factor N which becomes available. Thus, the true transformation curves $\Sigma_1, \Sigma_2, \dots, \Sigma_n$ are intersected by a pseudo-iso cost line AB which appears to be the relevant curve if no proper charges are made for the byproducts utilized. There appears to be no reason for expecting the production functions for byproduct producing and utilizing enterprises to be dependent upon each other if the byproducts are appropriately measured, charged and credited in either physical or constant dollar terms. Again, the conclusion is reached that simultaneous equation techniques are not required to fit the functions.

At this point in the investigation, it is concluded that production functions of individual enterprises on multiple enterprise farms can be regarded as being independent in a technical sense. Apparent dependencies are due to pricing in terms of on-farm opportunity costs and/or neglecting to charge and credit byproducts in a proper way.



Aside from the technical relationships discussed so far there are economic influences usually referred to as external economies and external diseconomies. Possible changes of production functions due to the influence of these factors still have to be considered.

c. The effects of economies and diseconomies upon production functions on multiple enterprise farms. Examples of external economies are reductions in factor prices due to larger quantities bought and increases in product prices resulting from a larger volume of products making it possible to enter a better market. If such economies exist, it is possible that as one enterprise is expanded, it becomes profitable to replace certain inputs by different ones which can be bought more profitably in larger quantities. In such cases it may be profitable to change the production processes in a number of enterprises.

In the case of external diseconomies, a similar argument applies, namely that the production processes will change as a result of relative increases in the price or opportunity cost of using one of the factors or a relative decrease in the price of the product.

Whether or not the individual production functions can be looked upon as being dependent upon or independent of each other depends upon whether or not the changes in the production functions are continuous changes or whether the new function, once determined, remains the same. If the changes appear continuous, stratified sampling might handle the difficulty without the use of simultaneous equations. If the change is not continuous, simultaneous equations are unnecessary.

The next step is to investigate the nature of production functions on specializing farms and compare them with those on multiple enterprise farms

in order to determine whether functions on these two types of farms are, in general, the same or different from each other.

d. Differences between production functions on specializing and on multiple enterprise farms. The third approach which was outlined above would utilize functions fitted to data from specializing farms to make inferences about the productivity of factors in multiple enterprise farms. This approach is not likely to be successful because the utilization of byproducts which are frequently wasted on specialty farms comes into play on multiple enterprise farms. Thus, when several enterprises are combined on the same farm, production processes may be altered substantially to provide for byproduct utilization. These alterations exist whether inputs are measured physically, in constant dollars or in terms of opportunity costs. An example is straw bailing which might occur on a wheat livestock farm, but might not on straight livestock or wheat farms.

Regarding the influence of external economies and diseconomies it has been shown in the previous section that these factors may be of considerable importance on multiple enterprise farms. Consequently, it should be expected that these factors also cause production functions on multiple enterprise farms to differ considerably from those found on specializing farms.

On the basis of these arguments, it is concluded that functions should not be fitted to data from single enterprise farms for use on multiple enterprise farms.

e. Conclusions from the foregoing arguments. In summary, the following conclusions are reached on the basis of the foregoing analysis:



Input-input relationships, product-product relationships and internal economies and diseconomies are unlikely sources of dependency among physical production functions on multiple enterprise farms if all inputs are measured in physical or constant dollar terms. Consequently, it appears unnecessary to use the simultaneous equations approach in fitting production functions.

From the investigation of the physical and economic relationships on multiple enterprise farms and from a comparison of the nature of production functions between specializing and multiple enterprise farms, it appears that functions fitted to data from specializing farms for use on multiple enterprise farms should not be expected to be appropriate. This leaves the approach of fitting each equation of a system of equations for multiple enterprise farms independently to data from such farms. This is the approach which has been used in the empirical part of this study.

B. Empirical Problems in Estimating Production Functions on Multiple Enterprise Farms

The remainder of this chapter will investigate problems which arise in connection with the empirical estimation of production functions. Problems of output and input classification, problems of byproduct accounting and pricing of fixed factors, methods of sampling and alternative fitting procedures are considered.

1. Output Classification

In multiple enterprise analysis, it often becomes necessary to combine enterprises into categories because even in the simplest farm organizations many products are frequently produced which cannot and need not be analyzed separately. Combination of products into output categories often makes the



analysis easier and more meaningful. Output classification should be undertaken in view of the objectives of a productivity analysis, which on multiple enterprise farms, are (a) determination of the optimum relative size of the present enterprises on the basis of the relative efficiency of the productive agents in these enterprises; (b) conclusions regarding the profitability of the enterprises presently found on the farms which are studied in comparison with enterprises not found on the farms under similar conditions; and (c) conclusions regarding the profitability of feeding crops to livestock versus selling these crops for cash.

a. Joint products. Bradford and Johnson define a farm enterprise as "... a line of production necessitating individual and distinct production treatment".² Following this definition, it appears that all products requiring the same production treatments can be called joint products even though these products differ in their nature. Examples are milk and calves, mutton and wool, or pork and lard. These pairs of products compete very little for resources and can thus be considered as one enterprise; they need not be analyzed separately in a productivity analysis.

b. Products competing for resources. All products requiring different production treatment can be considered as competing for resources and should be analyzed separately if it is the objective of the analysis to determine the relative efficiency of factors in the competing enterprises.

² Bradford, Lawrence A. and Glenn L. Johnson, Farm Management Analysis, New York: John Wiley and Sons Inc., 1953, p. 153.

c. Horizontally versus vertically fitted production functions. A multiple enterprise farm can be looked upon as consisting of a series of major enterprise groups. These groups are (a) the livestock enterprises and (b) the crop enterprises producing feed and/or cash crops. Each one of these enterprise groups contains a series of sub-enterprises which, as it was shown previously, can sometimes be treated as joint products not competing for resources and do not have to be analyzed separately.

In addition to integrating the enterprises within the major categories it is conceivable to combine the feed crop enterprises with the livestock enterprises and consider the proper share of the inputs in crops, this share being determined on the basis of the proportionate amount of feed consumed by each livestock enterprise, as direct inputs in the livestock enterprises. In such a function, livestock output would be considered as the dependent variable and the inputs used in crops and the livestock enterprise would be considered as the independent variables. It is apparent that in such a function feed produced on the farm cannot be charged as an input since this would amount to a double accounting of the factors used in producing feed.

This, in brief, is the approach which has been used in most productivity studies conducted on farms producing crops and specializing in the production of one type of livestock. The approach could be extended to the analysis of multiple enterprise farms by fitting a series of such vertical functions.

If, on the other hand, it is desired to obtain a productivity estimate for all major groups of enterprises and if it is to be determined whether feeding of crops rather than selling them is profitable the vertical connection between crops and livestock enterprises has to be interrupted and independent functions have to be fitted to the major livestock and crop enterprises. Since it was desired in this study to arrive at conclusions regard-

ing the profitability of feeding crops versus selling them and conclusions regarding the efficiency of other factors in crops as compared to their efficiency in livestock it was decided to fit one independent function each to the dairy enterprise, the hog enterprise and the crop enterprises.

2. Problems of Input Classification

The need for classifying inputs into groups in such a way that the categories are economically meaningful was expressed in every productivity study undertaken so far. The reasons for attempting to classify inputs into categories are the same as those given for output classification; the actual production processes are too complex to allow each input to be handled separately. Farm account data which were used in early productivity studies are not set up to permit meaningful groupings of inputs into categories because investments are not clearly distinguished from expenses and productive inputs are not kept separate from nonproductive inputs. Many of these shortcomings have been overcome in studies based on survey data which can be designed to permit a more meaningful grouping of the variables. Bradford and Johnson³ partly through learning from errors made in earlier studies, partly through analytical reasoning developed a set of rules which proved useful in the present investigation. In addition to these rules, new ones were developed in connection with the problem of input classification on multiple enterprise farms.

³ Bradford and Johnson, op. cit., p. 144.

The ideal in input classification is to have the inputs within one category combined in scale line proportions. Since this ideal is often difficult to attain, Johnson and Bradford suggest 3 rules which assure that it is at least approximated "... the inputs within one category be as nearly perfect substitutes or perfect complements as possible". If inputs within one category are perfect complements they can only be used in one specified combination and the estimated MVP will apply to them jointly. If the inputs within the category are good substitutes and measured in terms of their least common denominator the estimated MVP applies to the least common denominator which caused them to be good substitutes.

"... that categories made up of substitutes (a) be measured according to the least common denominator (often physical) causing them to be good substitutes and (b) be priced on the basis of the dollar value of the least common denominator unit."

"... that categories made up of complements (a) be measured in terms of units made up of the inputs combined in the proper proportions (which are relatively unaffected by price relationships) and (b) be priced on an index basis with constant weights assigned to each complementary input..."

Concerning the relationship of input categories among each other, Bradford and Johnson specify "... that the categories of inputs be neither perfect complements nor substitutes relative to each other". If input categories do not fulfill this condition the correlation among them will be high thus making the resulting MVP estimates for the individual categories unreliable. Combining such categories into one is the appropriate procedure in such a case.

"... investments and expenses should be kept in separate categories." The reason for this suggestion is that one expects to get a dollar in return for each dollar spent on items not depreciated, i.e. expenditures which have to be met annually. Examples are expenditures for fertilizer, animal care etc. Investments in turn are not expected to pay for themselves in one year but have to be replaced over a period of years. Thus, if a machine is expected to be worn out after 5 years one would be satisfied if each dollar invested in this machine returns approximately 20¢ plus interest on the investment. Finally, Bradford and Johnson suggest "... that maintenance expenditures and depreciation be eliminated from the expense categories because of the difficulty encountered in preventing duplication. This means that the earning of the investment categories must be large enough to cover maintenance and/or depreciation". The implication is that depreciation, if included as an expenditure and the depreciated item is also included as an investment would cause the b_i for both categories (expenditures and investment) to be biased downward.

Regarding the treatment of interest and taxes similar rules apply. It is erroneous to regard these factors as productive since really no return is expected to accrue from them. Consequently, such items have to be excluded from the input categories because they would bias the resulting marginal value productivity estimate downward.

3. Problems of Pricing

Pricing of inputs and outputs presents difficulties in those cases where either no good market for the factors exists or accounting in physical terms

is impossible.

a. Pricing of outputs. Most outputs produced by the various enterprises on a multiple enterprise farm can be priced rather easily because a well established market exists for them. Difficulties exist in pricing byproducts which are used only on the farm. These are often not handled commercially in large enough quantities and no well established market exists for them. Examples are manure, leguminous nitrogen etc. Since most of these items are substitutes in one way or another for commercially handled production factors, it seems reasonable to price them in terms of the least common denominator which causes them to be good substitutes.

b. Pricing of inputs. Difficulties similar to those discussed in the previous section exist with respect to pricing of fixed inputs. Some of these problems are eased since the Cobb-Douglas technique admits accounting of inputs in physical terms. Thus, items like labor, land and building space can be measured in physical units and the resulting MVP's state the returns for labor hour, acre of land, housing animal units rather than the dollar value of the investment. The problem is more difficult for such items as machinery investment which cannot be accounted in terms of physical units but have to be priced. Machinery investment can be established fairly accurately by resorting to Farm Implement Bluebooks which give current prices for most types of new and used machinery. Appropriate discounts or additions can be made depending upon the condition of the machine. Major repairs (new motors, tires, etc.) are added to the investment since they prolong the life of the machine.

4. Management and Unexplained Residuals

Not all of the factors contributing in the production process can be included among the independent variables. It is for this reason that the theory of regression analysis speaks of unexplained residuals which reflect the influence in the case of this study of such items as management, weather, different production systems in hogs and dairy. Regression analysis works with the assumption that these errors or unexplained residuals are normally and independently distributed with respect to the independent variables.

How justified is the aforementioned assumption regarding management on multiple enterprise farms?

Management obviously plays an important role in combining enterprises and farms which are well adjusted will have, in general, a high quantity and quality of management while the opposite is true for farms which are poorly adjusted. The implication here is that management is inversely related to the magnitude of the MVP_{X_i} ratios in the different enterprises. Since the percentage profit on well adjusted farms is higher than on poorly adjusted farms, a profit index of management could be incorporated as an independent variable in a productivity study. This, however, is only true if one is willing to accept monetary profit as the major objective and the sole criterion of management. If, however, a broader definition of management is adopted which bases itself not only upon profit maximization but upon maximization of satisfactions as well, the assumption that management is distributed randomly, normally and independently of other factors is well justified.



5. Problems of Sampling

It was pointed out in a previous section that most farm account data are not well suited for use in productivity studies in the way they are presently organized. Usually there is no clear distinction made between expenses and investment repairs and between productive and nonproductive inputs. As a result, several analysts preferred the survey sample and the data thus gathered proved to furnish more reliable estimates. In general, two different types of samples were used (a) the random sample, (b) the purposively chosen sample.

Random Sampling appears at first sight to be statistically a more accurate and commendable procedure. However, if the majority of the farms in the area are relatively well adjusted economically, a random sample would provide observations which are mostly concentrated around a small section of the scale line as indicated in Figure 9.

Such random samples might not provide good estimates of the entire production surface which of course extends beyond the small area in which a random sample might fall. In addition, the errors of the regression coefficients might be large due to the fact that the variances of the independent variables are small. Since only a small area of the surface is covered, the estimates of the iso product lines extend only over a small area thus preventing reliable conclusions about the least cost combination.

Purposive Sampling can overcome some of these shortcomings of random samples through selecting observations covering the largest possible area of Stage II of the production surface. Purposive sampling thus approximates closely the design of controlled field experiments where observations over

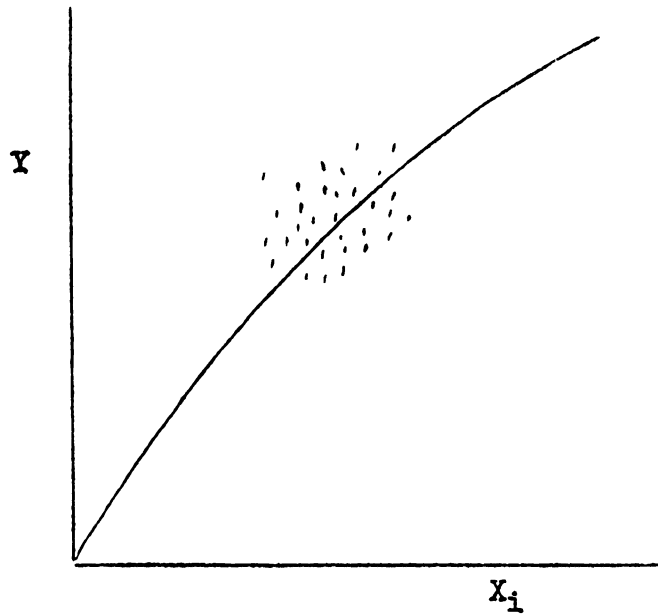


Fig. 9 Typical Distribution
of the Sample Observations
in the Case of a Random-
ly Chosen Sample

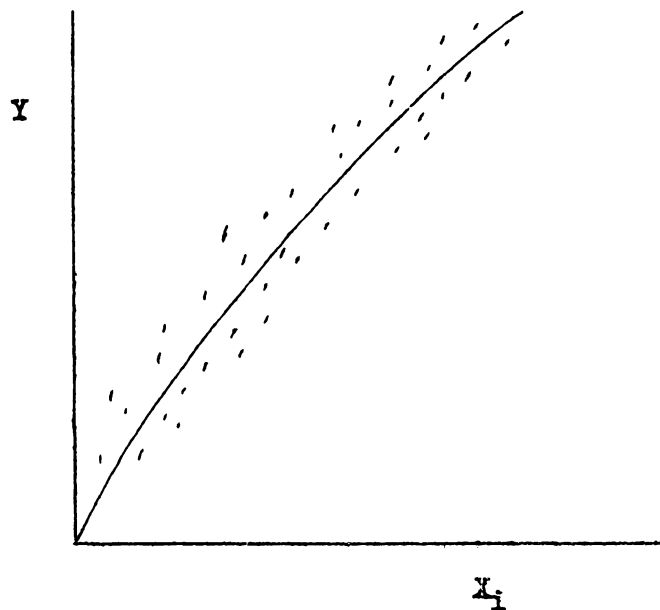


Fig. 10 Typical Distribution
of the Sample Observations
in the Case of a Purposi-
vely Chosen Sample



the entire production surface can be obtained. In addition, purposive sampling increases the variance of the independent variables; thus, a tendency to reduce the errors of the regression coefficients exists. The reduction of the inter-correlation among the independent variables which can be effected through purposive sampling is another positive feature of this type of sample design. Figure 10 illustrates such a purposive sample design.

A problem common to all sampling designs is that of finding enough farms which are on the same production function. For this reason, restrictions relating to the equality of land quality, equality of the types of milk production, climatic and market conditions are usually imposed when the samples are drawn. If these conditions are met, it is safe to assume that the estimated production function is intrafirm rather than interfirm.⁴ In selecting a sample of multiple enterprise farms, the same restrictions are applicable and care should be taken that the farms throughout the sample are on the same production function with respect to each enterprise.

6. The Choice of the Fitting Procedure

There have been discussions regarding the most expedient approach to use for fitting regression equations to non-experimental data. The principal

⁴

cf: Bronfenbrenner, Martin, "Production Functions: Cobb-Douglas, Inter-firm, Intrafirm," Econometrica, Vol. 12, January 1944, pp. 37 ff. Bronfenbrenner's argument is that the Cobb-Douglas function is an interfirm rather than an intrafirm function. His argument might well have been valid in those cases where a function was fitted to data from different industries throughout the United States Economy. (cf. Cobb and Douglas, op. cit.) However, since in agricultural productivity studies controls of the type explained above are used in drawing the sample, it is not likely that the estimated functions are interfirm functions.



antagonists are the methods of single least squares and the simultaneous equations technique the latter using both the limited information technique and the reduced form methods. The method of least squares has been applied in all but one agricultural productivity study, mainly because it requires only two assumptions, namely that the disturbance factors are normally distributed and uncorrelated with the regressors.⁵ It has been pointed out previously that there are grounds on which it can be assumed that this assumption is justified.⁶

The second method was suggested by Marschak and Andrews⁷ and has been applied to agricultural productivity studies by Burton French.⁸ French fitted equations with both the least squares method and the limited information method. The computational methods used in the limited information method were those given in Bronnfenbrenner and Chernoff⁹ and Anderson and Rubin.¹⁰ The results obtained from the limited information method compare unfavorably with estimates obtained from the same data by the simple least squares regression method,¹¹ as far as the economic meaning of the estimates is concerned.

⁵ Wold, Herman and Lars Jureen, Demand Analysis, New York: John Wiley and Sons Inc., 1953, p. 56.

⁶ Supra p.

⁷ Marschak and Andrews, op. cit.

⁸ French, Burton, op. cit.

⁹ Bronnfenbrenner, Jean and Hermann Chernoff, "Computational Methods Used in Limited Information Treatment of a Set of Linear Stochastic Difference Equations", Cowles Commission Discussion Paper: Statistics: No. 328. Cowles Commission for Research in Economics, University of Chicago, Chicago, Illinois, February 22, 1949.

¹⁰ Anderson, T. W. and H. Rubin, "The asymptotic Properties of Estimates of the Parameters of a Single Equation in a Complete System of Stochastic Equations, Annals of Mathematical Statistics 21: pp. 570-82, 1950.

¹¹ French, Burton, op. cit., p. 58.

The choice of the method of simple least squares in this analysis is only incidentally in agreement with French' conclusion since one empirical application can not establish the superiority of one method over the other. Rather, the choice has been made on the basis of the treatment given the subject by Wold and Jureen which, after comparing the various approaches, concludes that simple least squares regression is still the most efficient method since it can be employed with a minimum of assumptions.

CHAPTER V

DISCUSSION OF THE DATA USED
FOR THE EMPIRICAL PART
OF THIS STUDY

Since no detailed input output accounts for multiple enterprise farms are compiled in Michigan it was necessary to obtain data elsewhere in order to apply the foregoing analysis.

Among the experiment stations keeping detailed enterprise input - output accounts, Illinois had information which was well suited for the purposes of this project. To set up a questionnaire and obtain the data here in Michigan through a sample survey would have involved excessive costs in view of the exploratory nature of the study. Furthermore, farmers can not be expected to remember the required detail on how much of each input they used in each enterprise during the past year.

The Illinois detailed cost account project is part of the much larger Farm Bureau - Farm Management cost account project and uses data from multiple enterprise farms only. The project is moved every second year to a different area of Illinois. The number of farms studied varies between 30 and 50; however, not all of these farms have the same enterprise combinations.

In choosing one area to use for the empirical part of this study it was necessary to select a sample area which had a large enough number (at least 25 - 30) multiple enterprise farms with the same enterprises on them, was not subject to unusual weather conditions and had approximately the same types of land.

The 1950 data from the Blackhawk area¹ appeared to meet these conditions. In this area, financial and production records were kept by 37 farmers, 27 of which were dairy-hog farmers. These 27 dairy-hog farmers constitute the sample which was used in this analysis.

On the average, these farmers derived 46 per cent of their gross income from dairy sales, 36 per cent from hogs, 3 per cent from poultry and 13 per cent from feeds and grains sold for cash while 2 per cent came from work off farm and miscellaneous receipts.² Thus, the 27 farmers in the sample derived 89 per cent of their total gross income from the hog and dairy enterprises.

A. The Organization of the Records

Each cooperating farmer kept separate sheets for each of the enterprises on his farm. A fieldman supervising the project helped the cooperating farmers in summarizing the individual entries. A summary sheet was prepared annually by the University of Illinois.

Most of the entries in the input and investment categories met the requirements of input classification outlined in Chapter IV; the entries which had to be adjusted were mostly expense items which included nonproductive expenses such as taxes, depreciation, interest and similar items.

B. The Functions which were Fitted from the Data

Following the theoretical arguments outlined in the previous chapter it was decided to fit three independent functions one for dairy, one for hogs and one for the crop enterprises. To permit a more extended evaluation

¹ The Blackhawk area includes Carroll, Jo Daviess, Ogle, Stephenson, Whiteside and Winnebago Counties.

² A table showing the distribution of income by sources is included in Appendix B.



of the suggested procedure namely that of fitting individual enterprise functions, it was decided to fit one aggregate function to data from the entire farm and compare the results with those obtained from the enterprise functions.

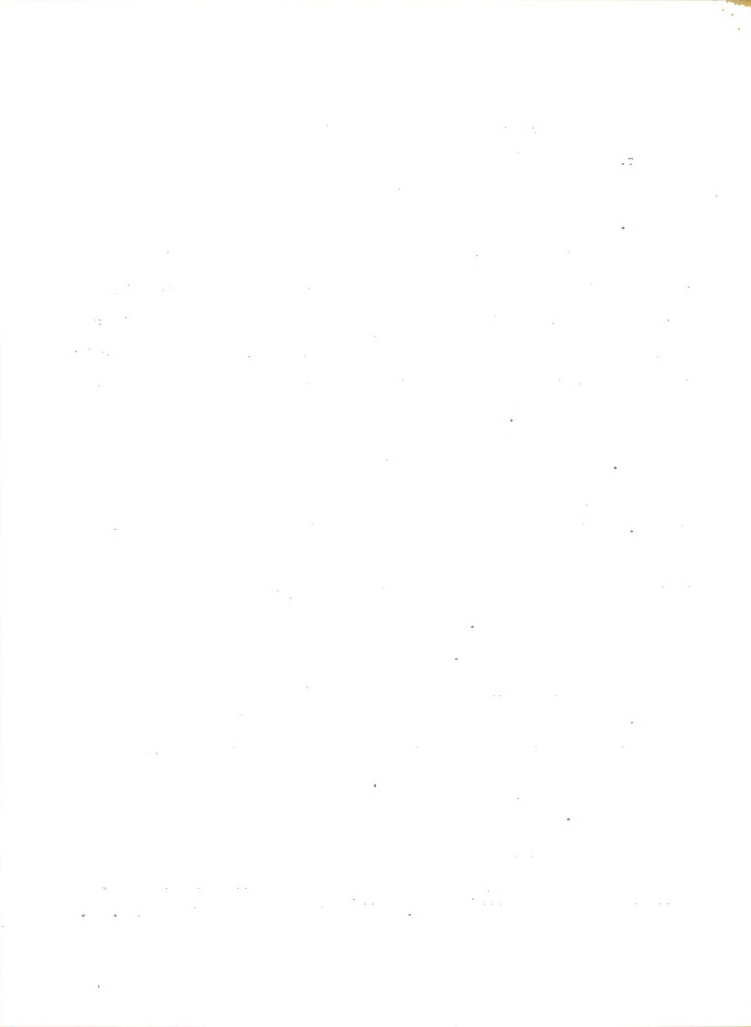
Regarding the crop function, it was assumed that the aggregation of data from various crop enterprises would not increase the intercorrelation among the independent variables as the major crop enterprises on these farms are on approximately the same production function. Thus, an aggregate function might still permit valid inferences about each particular enterprise included in the crop function.

1. The Variables Included in the Two Livestock Functions

(X_1) Dairy Gross Income was used as the dependent variables in the dairy function. It is determined in the Illinois Detailed Cost Accounts by subtracting the sum of the opening inventory and purchases from the sum of sales, closing inventory and value of dairy animals and dairy products consumed by the household or fed to hogs. The dairy enterprise is also credited for the manure produced by the animals.³

(X_1) Hog Gross Income was used as the dependent variable in the hog function. It is determined analogously to the determination of gross income in the dairy enterprise by subtracting the sum of the opening inventory and purchases from the sum of sales, closing inventory and value of pork consumed by the household.

³ This definition of Dairy Gross Income is taken from Detailed Cost Report for Northwestern Illinois, 1949 and 1950, Department of Agricultural Economics, University of Illinois. Publication (AE2871) April 1952, p. 17.



The following variables were considered as independent and causal variables in the analysis.

X_2 Labor. The Illinois cost accounts give "direct" and "indirect" labor hours spent in each enterprise. "Direct" labor includes time for milking, feeding and caring for the animals while "indirect" labor includes time spent in repairing barns, milking equipment etc. For the purposes of this analysis "direct" and "indirect" labor hours were added to obtain the total amount of labor hours used in each of the enterprises. Although some of the "indirect" labor is used in repairing investment items such as barns and should properly be added to the appropriate investment categories there was no possibility of making appropriate subtractions to eliminate this type of labor from the indirect labor categories.

X_3 Feed. Since feed is one of the most important inputs both in the dairy and hog enterprises feed expenses were separated from other expenses and the MVP of feed was estimated separately. The total value of the feed fed (both home grown and purchased feed) is included plus a 13 cent charge for each day the animals were pastured.

X_4 Expenses. "Power and Machinery Expense", "Equipment Expense", "Building Expense", "Cash and other Expense", "Interest on Investment" and "General Farm Expense" are items which are classified as cash expenses in the Illinois as well as in most other traditional farm accounts. Since not all of these items can be considered productive inputs for the purposes of productivity studies machinery and equipment expenses as given in the Illinois records were adjusted to eliminate the depreciation charges. The adjustment was made on the basis of the depreciation rate used by each individual farmer for his machinery. "Building Expenses" were excluded entirely because they



consist mostly of depreciation charges and/or expenses for improvements which are properly classified as building investments since they prolong the life of the building. "Cash and Other Expenses" were included while "Interest on Livestock Investment and "General Farm Expense", the latter including such items as taxes, magazine subscriptions etc., were not included. It is believed that the resulting figure for cash expenses reflects more closely the actual productive cash expenses incurred in the different enterprises on these farms.⁴

X₅ Machinery Investment. Determination of Machinery Investment in each enterprise was not particularly difficult since the Illinois cost data list each machine and its value in the machinery inventory.

Only beginning inventories have been used for this study. In two cases where new tractors were bought during 1950 appropriate credits were made. The values of the individual machines were taken from the account data at the beginning inventory value stated for them. This procedure was followed for all machines except tractors which were priced on the basis of the used tractor value stated in the Farm Implement Bluebook for 1950⁵. Admittedly these values are not entirely accurate because the actual value of one and the same tractor type varies from farm to farm depending upon the machine's condition. Nevertheless it is believed that this value is closer to the actual value of the machine since it is not based on a constant depreciation rate employed by most farmers in determining the values

⁴ Appendix C shows formally how these adjustments were made.

⁵ National Tractor and Farm Implement Bluebook, Chicago, National Market Reports Inc. Publisher.



of their machines for purposes of taxation. Determination of the share of the tractor investment in each enterprise was done on the basis of the number of tractor hours used in each enterprise. General farm equipment, i.e. items which were not separately listed, were also apportioned on this basis.

Silo and silo equipment was included under "Dairy Equipment". Fences and water systems were apportioned on the basis of pasture days which were consumed by dairy cows and hogs respectively. No fencing charges were made for crops. Tillage was charged to crops only.

X₆ Livestock Investment. Livestock investment was determined on the basis of the beginning inventory value of the animals plus proportional allowances for animals sold during the year less proportional charges for animals bought.⁶ No charges for forage investment were made since the value of "Pasture Days" consumed was included in the feed variable.

X₇ Housing. Following a suggestion made by Wagley⁷ it was decided to circumvent the difficulties of pricing farm buildings by using physical capacity measures. One dairy housing animal unit was defined as 1500 cu. ft. This includes space requirements for one mature dairy cow plus replacement and storage space for a one year supply of feeds for the animals.⁸ The

⁶ If a cow was sold in February at \$300.00, 10/12 of the beginning inventory value of this cow was subtracted from the livestock investment. If a cow was bought in July at \$300.00, 6/12 of this value, \$150.00, was added to the livestock investment.

⁷ Wagley, R. Vance, Marginal Productivities of Investments and Expenditures, Selected Ingham County Farms, 1952, pp. 45-46. Unpubl. M.S. Dissertation, Michigan State College, 1953.

⁸ This estimate of the space requirement is based on Farm Management Facts and Figures, Department of Agricultural Economics, Michigan State College, 1953, pp. 68-69.



dairy housing available on each farm included in the study was known in terms of cubic feet of barn space. For hog housing no physical data were available, consequently the values of permanent and temporary hog housing as indicated in the building inventory had to be used.

2. The Variables Included in the Crop Function

It was pointed out earlier that it is possible to fit an aggregate function to a number of enterprises if these enterprises can be assumed to be on approximately the same production function and if the aggregation of inputs from several enterprises does not cause the intercorrelation among the independent variables to increase. This assumption was made for the crop enterprises on the 27 farms and inputs as well as outputs from each crop enterprise were aggregated and one function was fitted to them. Almost all of the farms in the sample raised corn (both grain and silage), oats, and legumes as their major crops. Other crops were soybeans, barley and wheat. Pasture was not included in the function because no input data for permanent pasture were available.

The following variables were included in the crop function:

X_1 Gross Income from Crops. The yields obtained in each enterprise were multiplied by the appropriate prices which were stated by the farmers in the sample to have prevailed in their market. Thus all crops were valued at market prices.

The following variables were treated as independent (causal) variables:

X_2 Labor. As in the case of livestock the records indicated the number of labor hours which were used "directly" and "indirectly" in each crop enterprise. The total amount of labor used in all the crop enterprises was thus easily obtained through addition.



X_3 Land. Land was measured in acres rather than in monetary terms. The total number of acres in crops was obtained by adding the number of acres used in each enterprise. It should be pointed out here that not all land was of the same quality. Thus the resulting IVP estimates apply to the average quality land included in the sample.

X_4 Expenses. Regarding crop expenses considerable adjustments were made in the original data by applying a "depreciation factor" to eliminate machinery depreciation from the expense categories. Building expenses and general farm expense were left out entirely because they contain mostly depreciation charges or nonproductive expenses such as taxes, interest charges and magazine subscriptions.

X_5 Machinery Investment. The value of crop machinery and equipment had to be determined on the basis of the values given in the machinery inventory since neither age nor make of these machines were known to permit the use Farm Implement Bluebooks in determining their values. The values of tractors on the other hand for which this information was available were determined with the help of Farm Implement Bluebooks. The crop share of the tractor was computed on the basis of the number of hours the tractors were used in crops.

3. The Variables Included in the Aggregate Function.

In order to furnish an additional basis for an evaluation of the individual enterprise functions it was decided to fit an aggregate function to data from all the enterprises on the 27 farms.

In this aggregate function (X_1) was used as the dependent variable. It represents gross income including income received from dairy, income received



from hogs and income received from cash crop sales.

The following variables were treated as independent in the aggregate function: (X_2) labor, including all the labor hours spent "directly" and "indirectly" in dairy, hogs and the various crop enterprises; (X_3) land, measured in acres and including all crop acres used in producing feed and cash crops (pasture acres were not included since no input - output data were available for pasture; a pasture charge was made, however, under expenses); (X_4) cash expenses, including all productive cash expenses in livestock and crops, feed purchased and pasture consumed but not feed produced on the farm (non productive expenses included include, taxes, insurance, and depreciation); (X_5) machinery investment, including the value of machinery and equipment used in livestock and crop enterprises; (X_6) livestock investment including dairy cows, bulls and breeding hogs, (X_7) barn space and space in permanent hog housing, measured in dairy housing animal units; since no physical data for housing were available in the case of hogs the building inventory value indicated for hogs was converted to dairy housing animal units using the rate of 1 Housing Animal Unit = \$403.00. This is the rate which Wagley⁹ determined as the establishment cost of one dairy housing animal unit.

⁹ Wagley, R. Vance, op. cit. p. 82.



CHAPTER VI

INTERPRETATION OF FOUR PRODUCTION FUNCTIONS FITTED TO ENTERPRISE
INPUT-OUTPUT DATA FROM 27 DAIRY-HOG FARMS IN
NORTHWESTERN ILLINOIS

The present chapter examines results obtained from the statistical analysis of input-output data from 27 multiple enterprise farms in northwestern Illinois. Four separate functions were fitted, one for the hog enterprises, one for the dairy enterprises, one for the combined crop enterprises and one aggregate function for the entire farm.

In Part A of this chapter, the regression coefficients from each function and their economic meaning are discussed independently of the other functions fitted. In Part B the functions and the regression coefficients for like factors are compared. Inter-enterprise comparison of coefficients is the method by which the firm is brought towards internal adjustment. If each factor is used to equal advantage in the various enterprises, a maximum profit from a given amount of resources will be achieved.

In Part C the results obtained from the individual enterprise functions are compared with results obtained from an aggregate function in order to evaluate the methodology suggested and employed in this thesis.

A. Inter-Enterprise Comparison of Regression Coefficients
and Marginal Value Productivity Estimates

1. The Hog Function

a. The regression coefficients and the MVP estimates. The function fitted to the hog enterprises of the 27 farms included in the sample yielded the regression coefficients and marginal value productivities, estimated at the geometric mean, indicated in Table I.

The regression coefficients reported in column 2 of Table I are the elasticities of gross income with respect to each of the six inputs or investments which were used as independent or explanatory variables in the regression analysis.¹ Each one of these coefficients indicates the percentage change in gross income associated with a one per cent increase in the independent factors. Thus, if the feed input in the hog function were increased by one per cent, the hog output would be expected to increase .702417 per cent.

The areas under the normal curve corresponding to the "t" values in column 3, indicate the probability with which it can be assumed that regression coefficients obtained from a different sample of the same population would be as large or larger than the estimated regression coefficient if the true population coefficient were zero. Thus, a "t" of .066780 indicates that a regression coefficient as large as or larger than .005618 would be obtained

¹ The following is a simple proof that the regression coefficients in the Cobb-Douglas function are the elasticities of gross income (Y) with respect to the independent variable (X_i):

$$\text{let } e = \frac{\frac{\Delta Y}{Y}}{\frac{\Delta X_i}{X_i}} = \frac{X_i \Delta Y}{Y \Delta X_i} \quad \text{since } \frac{\Delta Y}{\Delta X_i} = \text{MVP}_{X_i} = \frac{b_i EY}{X_i} \quad \text{it follows that}$$

$$e = \frac{X_i b_i EY}{Y X_i} = b_i$$



TABLE I

REGRESSION COEFFICIENTS AND MARGINAL VALUE PRODUCTIVITY ESTIMATES

OBTAINED FROM A COBB-DOUGLAS FUNCTION FITTED TO HOG

ENTERPRISE DATA FROM 27 ILLINOIS FARMS, 1950

(1) Production factor	(2) Regression coefficient	(3) t value	(4) MVP at geometric mean
Labor (hours)	.005618 $\pm$.084127	.066780	Mean .0453 Upper .7245 Lower -.6338
Feed (dollars)	.702417 $\pm$.131202	5.353706	Mean 1.0087 Upper 1.1970 Lower .8203
Cash expense (dollars)	.106413 $\pm$.083474	1.274804	Mean 2.6426 Upper 4.7100 Lower .5696
Machinery (dollars)	.039436 $\pm$.053057	.743276	Mean 1.3693 Upper 3.2113 Lower -.4730
Breeding invest. (dollars)	-.059265 $\pm$.097410	.608408	Mean -.3443 Upper .2216 Lower -.9103
Housing (dollars)	-.002848 $\pm$.018586	.153234	Mean -.0875 Upper .4839 Lower -.6591

in 95 out of 100 cases if functions were fitted to different samples from the same population and the true population regression coefficient were zero.

Of greatest interest are the marginal value productivity estimates reported in column 4. The MVP's are a direct function of the b_i and indicate the return which might be expected from the last unit of each of the inputs used in the hog enterprise, *ceteris paribus*. The marginal value productivities reported here are estimated at the geometric mean of gross income and the inputs in the sample. Upper and lower confidence limits for the MVP's are computed to correspond to the upper and lower confidence limits for the regression coefficients.² It should be mentioned that MVP's can be estimated for any size of the input and output. For purposes of estimating MVP's for any particular farm, values of Y and X_i appropriate for this farm have to be used. Such a comparison requires (1) that the farm be within a probability distribution about the same production function as the farms in the sample and (2) that the data are not outside the range from which the function was estimated.

The "t" values in column 3 indicate that the regression coefficients for labor, machinery, breeding investment and housing are not significantly different from zero at the 68 per cent level. The relatively large standard errors are caused mostly by large intercorrelations of these variables with other independent variables.

The MVP estimate for labor at its geometric mean indicates that this factor returned about 5 cents per hour. This MVP varies from between 72 cents

² The upper and lower confidence limits of the MVP are computed using

$$MVP = \frac{[b_i \pm \sigma_{b_i}]E(Y)}{X_i} .$$

per hour when estimated at $b_2 + \sigma b_2$ to -63 cents per hour when estimated at $b_2 - \sigma b_2$. Obviously the mean estimate is very unreliable. The situation is similar for the estimated MVP's of Machinery Investment, Breeding Investment and Hog Housing.

The regression coefficients of feed and cash expenses are significantly different from zero at the 75 per cent level. The corresponding MVP's estimated at the geometric mean are 1.0087 dollars for feed and 2.6426 dollars for expenses. The MVP estimate of feed is not surprising since one would expect 1.00 dollar of feed input to return at least one dollar. The estimate for cash expenses indicates that at the geometric mean a dollar spent would return 2.64 dollars. Several factors aside from actual productivity of cash expenses on these farms might have caused this relatively large MVP; the following two appear to be the most likely causes: (1) the amount of cash expenses other than feed is relatively small in the hog enterprise, therefore, small errors in computing this figure might have affected the MVP estimate considerably. (2) Cash expenses (X_4) are very highly correlated with both feed (X_3), $r_{34} = .85$, and labor (X_2), $r_{24} = .68$, and might thus reflect some of the earnings of these categories.

b. Testing the regression coefficient against regression coefficients which would yield minimum MVP's. The only test of regression coefficients discussed so far tests the b_i against zero. A more meaningful test from an economic viewpoint compares the b_i of each input or investment against a "standard" b_i^* , capable of yielding a minimum or reservation MVP. The standard b_i^* is obtained by solving the equation
$$\text{MVP} = \frac{b_i E(Y)}{X_i}$$
 for b_i after the required minimum MVP has been determined. The MVP's assumed to be mini-

mum expected returns of inputs and investments in hog enterprises on northern Illinois farms are given in Table II, column 1.

The resulting b_i^* necessary to yield these MVP's on Illinois hog farms are indicated in Table III, column 3. In column 4, the estimated b_i is subtracted from b_i^* . Column 5 indicates the size of this difference in terms of the standard error of the b_i . Column 6 indicates the probability with which the b_i^* would be expected to assume that or any value farther away from the estimated b_i if the latter were the true population regression coefficient. It appears from this test that when compared at the geometric mean only the regression coefficients of labor and livestock investment are significantly lower, at the 68 per cent level, than the corresponding b_i^* yielding a minimum MVP. Consequently, it is concluded that a reduction in the use of these two inputs in the hog enterprise could be suggested.

While the farms appear to be in relatively good adjustment with respect to the remaining inputs if the comparison is made at the geometric mean, this does not necessarily hold for all the individual farms in the sample. Consequently, individual farms have been selected and their organization has been compared with an organization which would yield minimum or reservation MVP's for the various inputs and investments. Table III a makes a comparison using a farm whose hog enterprise appeared to deviate considerably from the scale line adjustment. The regression coefficients of labor and feed in column 1 appear to be significantly lower, on the 80 per cent level, than the b_i^* necessary to return a minimum MVP for these two inputs, indicating that too much labor is used and too much or too expensive feed is fed to hogs on this farm.

TABLE II

MVP'S FOR VARIOUS INPUTS AND INVESTMENTS WHICH ARE
 CONSIDERED MINIMUM EXPECTED RETURNS OR
 RESERVATION PRICES IN ILLINOIS, 1950

Input or investment category	(1) Hogs	(2) Dairy	(3) Crops
Labor	\$200.00/month = 74¢/hour	\$200.00/month = 74¢/hour	\$200.00/month = 74¢/hour
Land (acres)	-----	-----	$6\frac{1}{2}\% = \$14.50^1$ per acre
Feed (dollars)	1.00	1.00	-----
Cash expenses (dollars)	1.00	1.00	1.00
Machinery invest. (dollars)	20%	20%	20%
Livestock invest. (dollars)	50%	35% - 45%	-----
Housing (dollars)	5% - 8%	5% - 8%	-----

¹ This figure is based upon an average land value of \$223.00 for the sample.
 Source: Illinois Schedule of Land Values by Soil Productivity Ratings,
 Department of Agricultural Economics, University of Illinois, 1952.

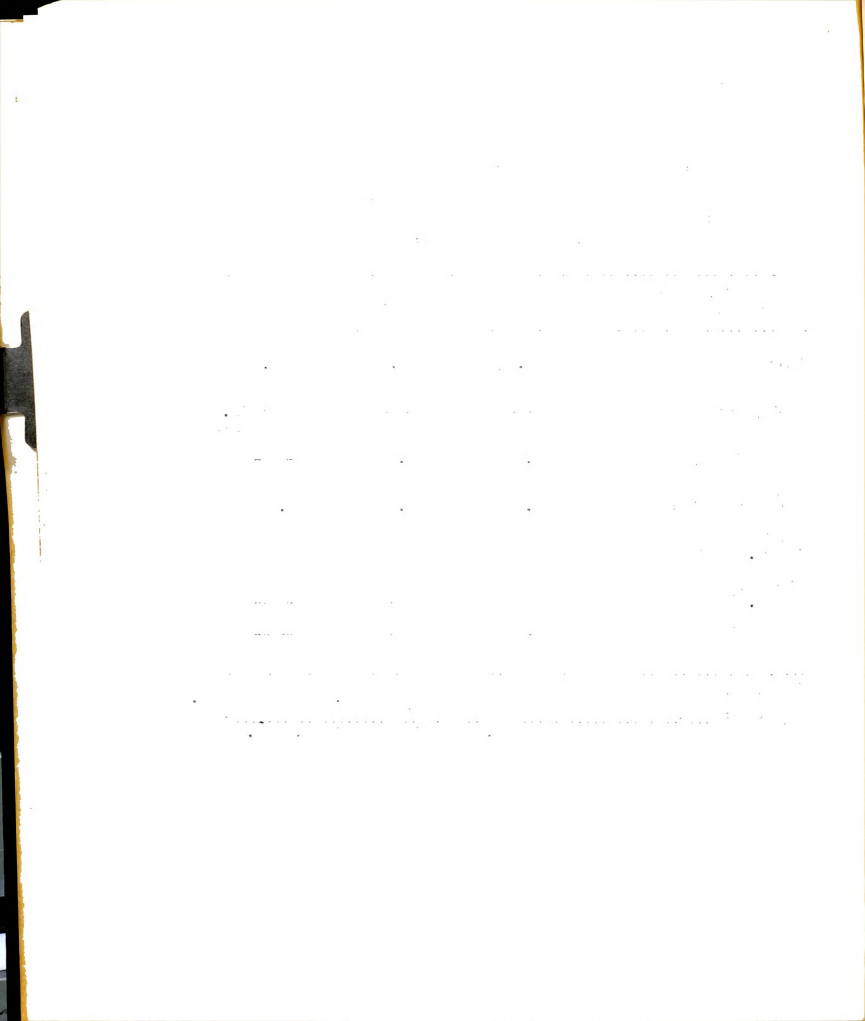


TABLE III

COMPARISON BETWEEN THE ESTIMATED b_i AND THE b_i^* NE-
CESSARY TO EQUATE MVP_{X_i} WITH MFC_{X_i} IN THE HOG
ENTERPRISES OF A SAMPLE OF 27 NORTHERN ILLI -

NOIS FARMS, 1950

Inputs or investment category	Geometric mean organ- ization	(1) Estimated b_i	(2) σb_i	(3) b_i^* necessary to return min. MVP	(4) $ b_i - b_i^* $	(5) $ b_i - b_i^* / \frac{\sigma b_i}{\sigma b_i}$	(6) Area of normal curve
Labor (hours)	669.32	.005618	.0841127	.09166	.086042	1.022760	.69
Feed (dollars)	3931.30	.702417	.131202	.69635	.006067	.046241	.04
Expenses (dollars)	227.33	.106413	.083474	.04026	.066153	.792490	.57
Machinery (dollars)	162.59	.039436	.053057	.00576	.033676	.634710	.47
Breeding inv. (dollars)	971.85	-.059265	.097410	.08607	.145335	1.491990	.86
Housing (dollars)	183.57	-.002848	.018586	.00211	.004958	.266759	.20

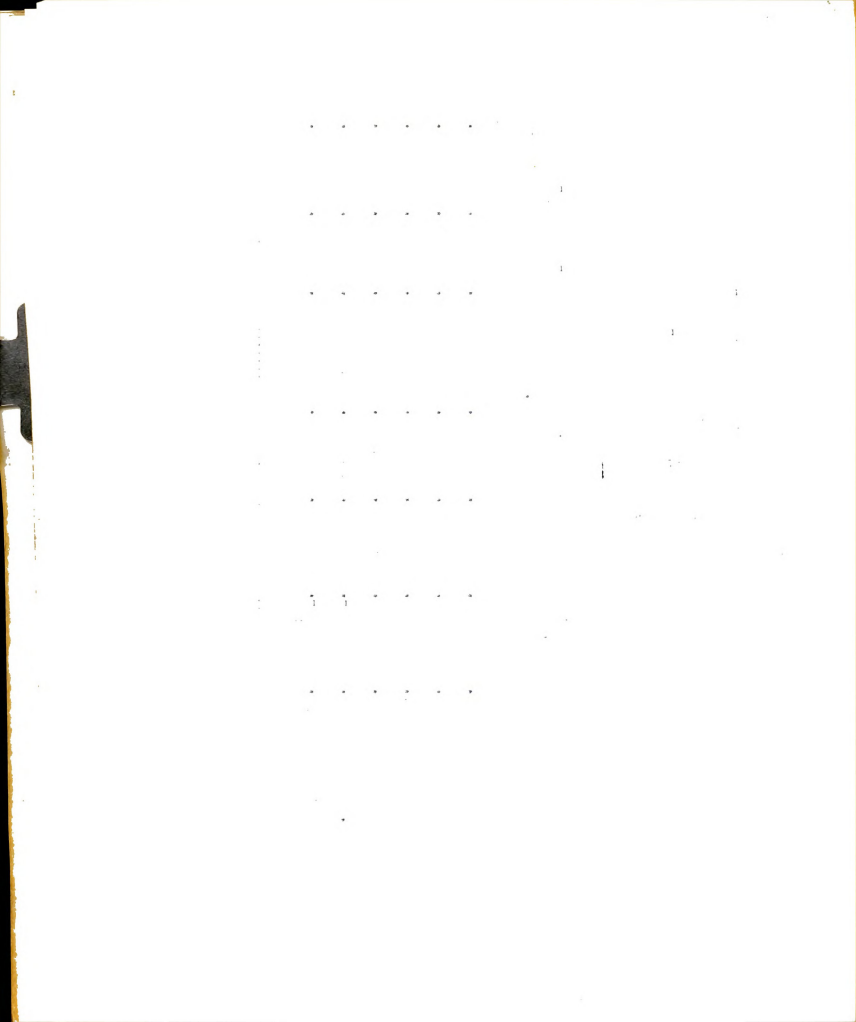


TABLE IIIa

STATISTICAL COMPARISON BETWEEN THE b_1^* REGRESSIVE TO YIELD A GIVEN MVP
ON A POORLY ADJUSTED FARM IN THE SAMPLE AND THE ESTIMATED b_1 ;
DAIRY ENTERPRISE, 1950

Input or investment category	MVP	(1) Estimated b_1	(2) σ_{b_1}	(3) b_1^*	Min. MVP	(4) $(b_1 - b_1^*)^2$	(5) $(b_1 - b_1^*)^2 / \sigma_{b_1}^2$	(6) Area under normal curve
Labor (hours)	.03	.005618	.084127	.115204	.74	.109536	1.302625	80%
Feed (dollars)	.60	.702417	.131202	.876725	1.00	.174306	1.328546	82%
Expenses (dollars)	1.65	.106413	.083474	.064211	1.00	.042202	.505570	40%
Machinery (dollars)	.92	.039436	.053057	.008566	.29	.030370	.561827	44%
Breeding inv. (dollars)	-.39	-.059265	.097410	.074696	.50	.133961	1.375228	84%
Housing (dollars)	-.10	-.002848	.018536	.001964	.07	.004812	.256904	20%

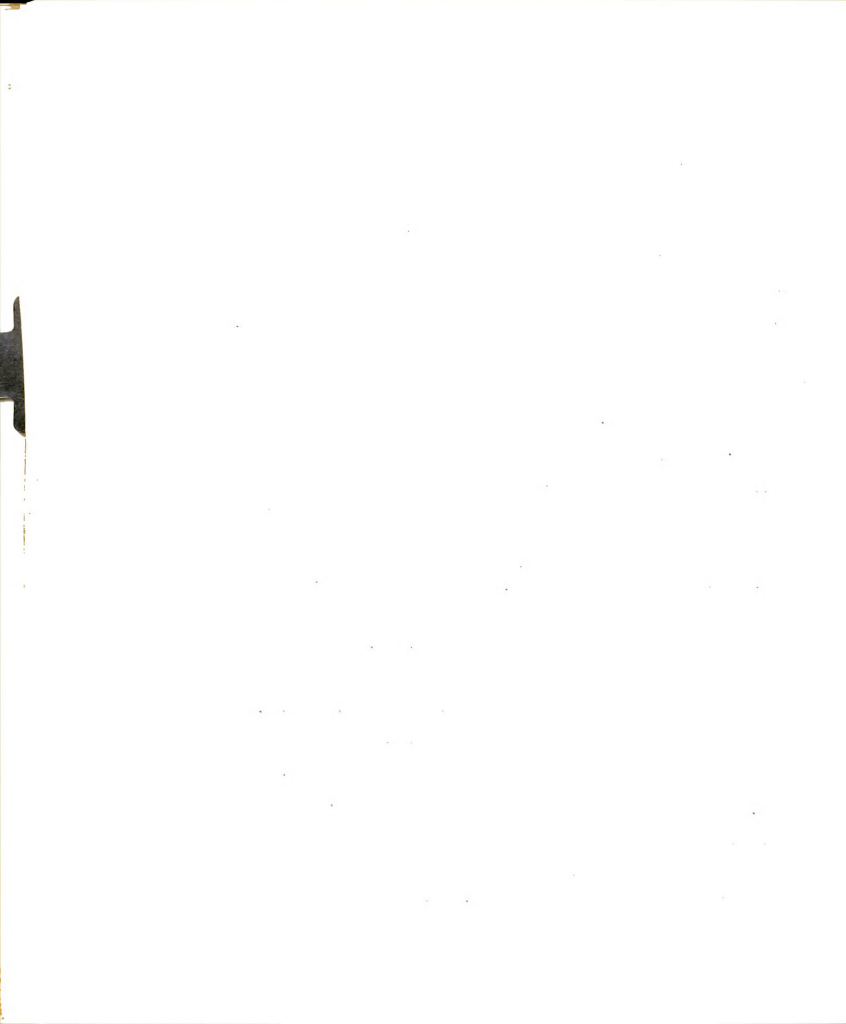
The foregoing analysis suggests that comparisons made at the geometric mean do not necessarily give reliable information regarding the adjustments of individual farms or the statistical and economic significance of productivity estimates at the geometric mean; however, a statistical comparison between the MVP's on a particular farm and minimum or reservation MVP's showed significant differences with respect to a number of variables for which the comparison at the geometric mean suggested no serious maladjustments. This result is not surprising if one considers that the farm at the geometric mean taken from a moderately representative sample should be expected to be relatively well adjusted.

c. The errors of the regression line and the coefficients of multiple correlation and determination. Regarding the error of the regression equation and the relationships between dependent and independent variables, the following quantities have been computed:

The standard error of estimate $\bar{S}_{1.234567}$ was computed to .066573 which indicates in logarithms the size of the standard error of hog output when estimated from the independent variables $X_2, \dots, X_7$.³ Expressed in natural numbers, this means that the probability that the mean of the true population income falls in the interval \$4,843.10 to \$6,734.00 is .68. The geometric mean of X_1 in this sample was \$5,645.50.

The adjusted coefficient of multiple correlation $\bar{R}_{1.234567}$ was .950682 while $\bar{R}_{1.234567}^2$ the Coefficient of Multiple Determination was .903796 indicating

3 Ezekiel, Mordecai, Methods of Correlation Analysis, New York: John Wiley and Company, Second Edition, 1953, p. 206.



that 90 per cent of the variation in the dependent variable X_1 is associated with variations in the independent variables $X_2, \dots, X_7$.

An evaluation of the hog function with the aim of making specific recommendations to the farmers included in this sample or to farms working under similar production conditions suggests that the farms are relatively well adjusted as long as the comparison is made at the geometric mean. Only the productivity of labor is significantly below the wage which has to be paid to a worker in agriculture. If, on the other hand, individual farms are investigated regarding their organizations it appears that a number of farms could be found whose organization deviates considerably from an optimal one. Consequently, it should be emphasized that in evaluating Cobb-Douglas productivity estimates comparisons of individual farms reveal more than comparisons made at the geometric mean.

2. The Dairy Function

A function using the same independent variables as the hog function was fitted to input-output data from the dairy enterprises of the 27 farms included in the sample. The only major difference between the two functions was that dairy housing ~~was~~ measured in physical terms while hog housing was measured in terms of its inventory values indicated by the farmers.

a. The regression coefficients and the MVP estimates. The regression coefficients which were obtained for the variables $X_2, \dots, X_7$ as well as the marginal value productivities, estimated at the geometric mean, are presented in Table IV. The area under the normal curve corresponding to the t values in column 3 indicates the probability with which it can be assumed that re-

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2. The second part of the report is a detailed description of the

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TABLE IV

REGRESSION COEFFICIENTS AND MARGINAL VALUE PRODUCTIVITY
 ESTIMATES OBTAINED FROM A COBB-DOUGLAS FUNCTION FITTED
 TO DAIRY ENTERPRISE DATA FROM 27 ILLINOIS FARMS, 1950

(1) Input or invest- ment category	(2) Regression coefficients	(3) t values	(4) Marginal value products
Labor (hours)	$.221717 \pm .141106$	1.571280	Mean .6367 Upper 1.1240 Lower .2499
Feed (dollars)	$.730461 \pm .192497$	3.794662	Mean 1.0187 Upper 1.2829 Lower .7478
Cash expense (dollars)	$-.014169 \pm .150443$	.094182	Mean - .1407 Upper 1.3532 Lower -1.6346
Machinery (dollars)	$.061646 \pm .096077$	.641631	Mean .4310 Upper 1.1026 Lower - .2407
Livestock investment (dollars)	$.009030 \pm .098352$	.091349	Mean .0245 Upper .2922 Lower - .2433
Housing (dairy housing animal units)	$.125801 \pm .123098$	.982068	Mean 24.6796 Upper 49.8100 Lower - .4500



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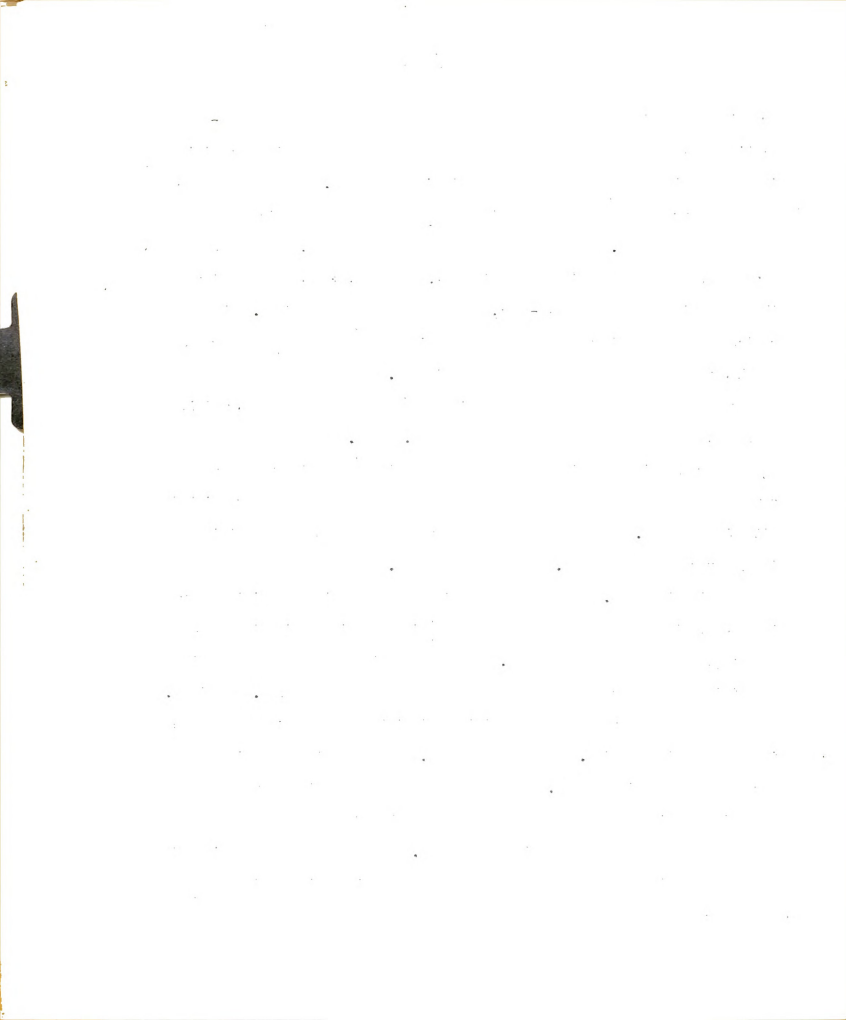
gression coefficients obtained from a different sample of the same population would be as large or larger than the estimated regression coefficient in column 2 if the true population coefficient were zero.

The marginal value productivities estimated at the geometric mean are reported in column 4. Three values of the MVP are computed, one at the mean value of the regression coefficient b_i , one at $b_i + \sigma b_i$ representing the upper limit and one at $b_i - \sigma b_i$, representing the lower limit. The choice of plus and minus one standard deviation for computing the upper and the lower limits respectively is quite arbitrary. If a 95 per cent rather than a 68 per cent confidence interval is desired the lower and upper limits would have to be recomputed at plus or minus $1.96 \sigma b_i$.

The regression coefficient of labor is significantly different from zero at the 88 per cent level owing to the relatively small error associated with the estimate. The MVP of labor in dairy estimated at the geometric mean is 69 cents per hour or 186.30 dollars per month.

The t value of 3.794662 associated with the regression coefficient for the feed variable indicates that this regression coefficient is different from zero at the 99 per cent level. The MVP of this input estimated at the geometric mean indicates that one dollars worth of feed returns 1.02 dollars.

The only other regression coefficient significantly different from zero is that of dairy housing. The " t " value of .982068 in column 3 indicates that a regression coefficient of .125801 or larger would occur in 67 out of 100 cases if functions were fitted to different samples from the same population and the true population coefficient were zero. The MVP of housing estimated at the geometric mean indicates that one "dairy housing animal unit" returns



24.68 dollars. Assuming that the replacement cost of one "dairy housing animal unit" is about 400.00 dollars, this MVP would correspond to a return of 6 per cent on the investment in dairy housing. The estimated percentage returns would increase correspondingly as one lowers the value of one housing animal unit to adjust for the age and condition of the buildings. The regression coefficients for cash expenses, machinery investment and livestock investment were not significantly different from zero as large errors were associated with these estimates. Regression coefficients, "t" values and MVP's of these inputs are indicated in columns 2, 3, and 4 of Table IV respectively.

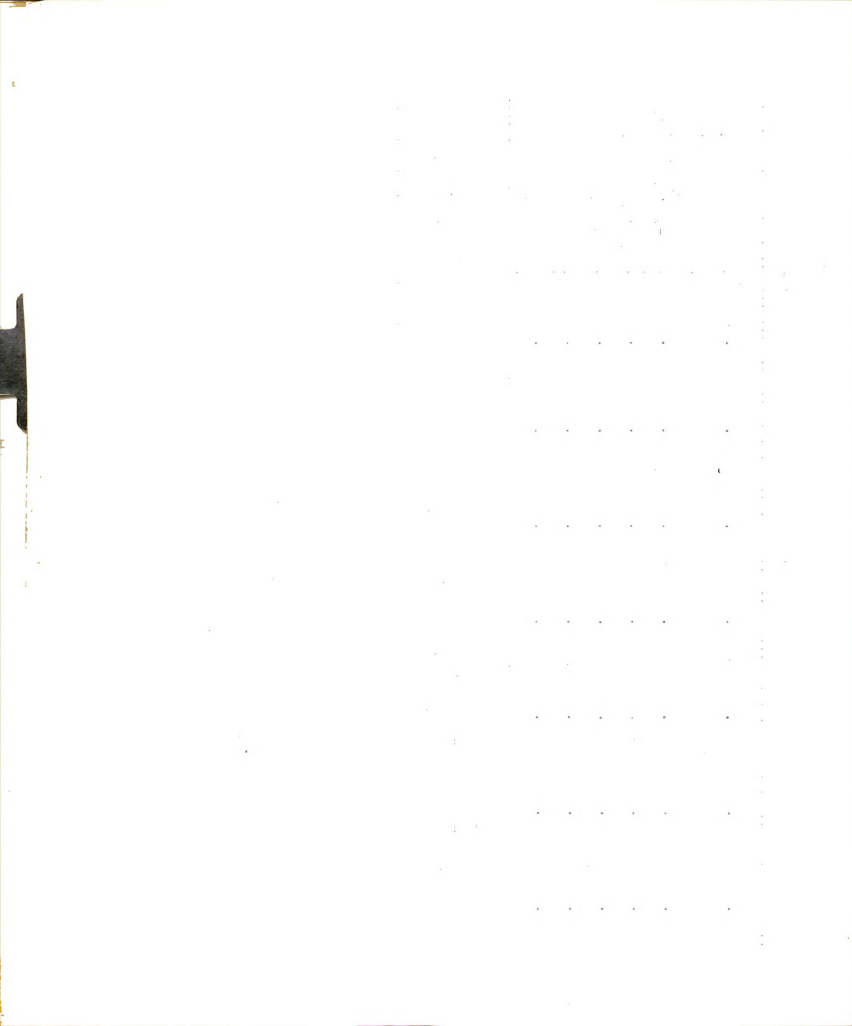
b. Testing the regression coefficients against regression coefficients which would yield minimum MVP's. As in the case of the hog function, the b_i were tested against regression coefficients which would yield a certain minimum expected MVP. These minimum or reservation returns have been presented in Table II column 2 above. The b_i^* necessary to yield these minimum marginal value products are presented in column 3 of Table V. Column 4 of Table V presents the difference between the b_i^* and the estimated b_i . Column 5 restates this difference in terms of the standard error of the b_i . The area under the normal curve corresponding to this standardized difference indicates the probability with which it can be expected that a regression coefficient which differs from b_i by more than $b_i^* - b_i$ would occur if b_i were the true correlation coefficient for the population. It is seen that only b_6 the coefficient of livestock investment can be expected to be significantly different from b_6^* (at the 80 per cent level) which makes these farms appear to be well adjusted when they are analyzed at the geometric mean.



TABLE V

COMPARISON BETWEEN THE ESTIMATED b_1 AND THE b_1^* NECESSARY TO EQUATE
 IVP_{X₁} WITH NEO_{X_1} IN THE DAIRY ENTERPRISES OF A
 SAMPLE OF 27 NORTHERN ILLINOIS FARMIS, 1950

Input or invest- ment category	Geometric mean organization	(1) Estimated b_1	(2) σ_{b_1}	(3) b_1^* necessary to return minimum IVP	(4) $ b_1 - b_1^* $	(5) $(b_1 - b_1^*) / \sigma_{b_1}$	Area under normal curve
Labor (hours)	2396.20	.221717	.141106	.238920	.017202	.121915	.10
Feed (dollars)	5321.50	.730461	.192497	.717033	.013423	.069730	.06
Expenses (dollars)	747.55	-.011169	.150443	.100727	.114696	.763717	.55
Machinery (dollars)	1061.60	.061646	.096077	.028600	.033046	.343953	.27
Livestock invest- ment (dollars)	2740.40	.009030	.098352	.147770	.136740	1.403512	.84
Housing (animal units)	37.83	.125601	.128098	.132530	.006729	.052530	.04



As before a farm whose organization appeared to deviate considerably from the scale line adjustment has been selected to determine if the sample regression coefficient b_i differs significantly from the b_i^* yielding a certain minimum or reservation MVP on this poorly adjusted farm. The comparison is presented in Table V a, and it indicates that on this particular farm the MVP's of several important variables differ considerably from the corresponding minimum MVP's. On the particular farm which was chosen for comparison it appears that the use of labor in dairy should be reduced while the use of feed, for which the MVP is high, could be increased or more expensive feed mixtures could be used. The MVP of dairy housing is significantly below the MVP which should be expected from one dairy housing animal unit suggesting that an over-investment in buildings exists on this farm.

Again the conclusion is reached that the analysis of individual farms on the basis of the estimated production function reveals considerably more about the location of these farms with respect to the scale line than do the comparisons which are made at the geometric mean.

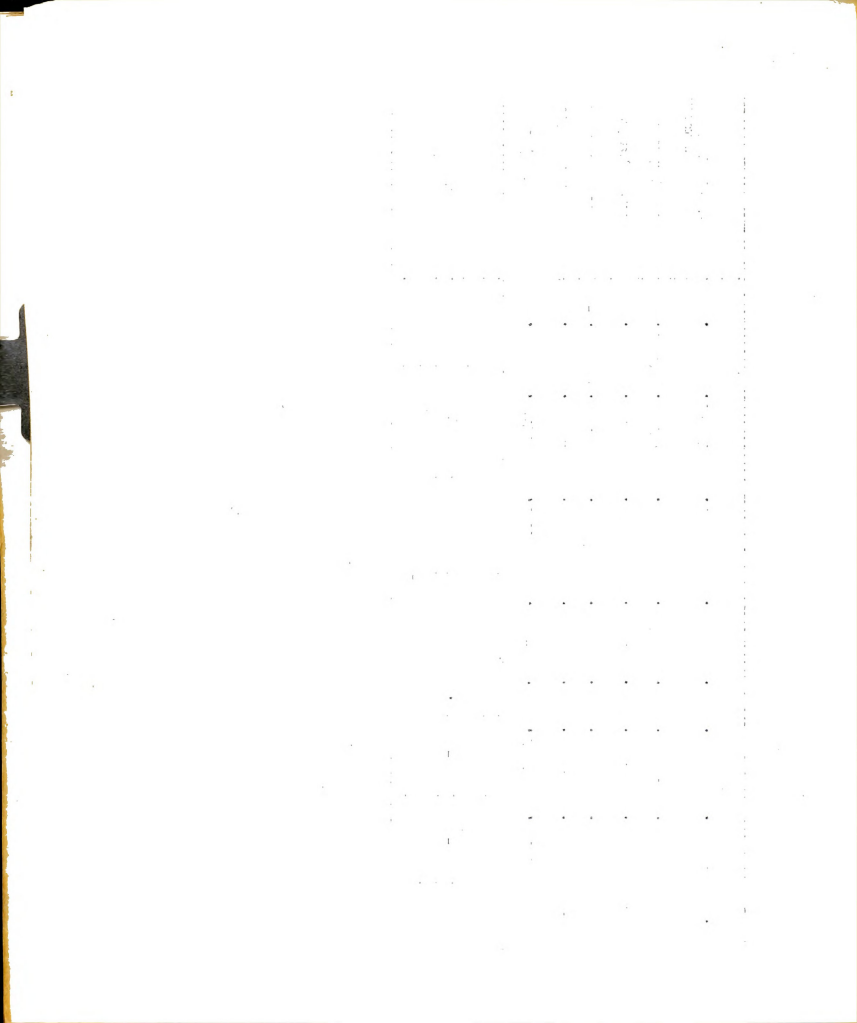
c. The errors of the regression line and the coefficients of multiple correlation and determination. The Standard Error of Estimate $\bar{S}_{1.234567}$ was computed to be .088679. It indicates in logarithms the size of the standard error of dairy output (X_1) when estimated from the independent variables $X_2, \dots, X_7$. Expressed in natural numbers this means that the probability that the geometric mean of gross income in the population falls in the interval \$6050.90 to \$9102.75 is .68. The geometric mean of gross income in this sample was \$7421.50.

TABLE Va

STATISTICAL COMPARISON BETWEEN THE b_1^* NECESSARY TO YIELD A MINIMUM
MVP ON A POORLY ADJUSTED FARM IN THE SAMPLE AND THE ESTIMATED b_1 ;

DAIRY ENTERPRISE, 1950

Input or investment category	MVP	Estimated b_1	σb_1	(3)		(4)	(5)	(6)
				b_1^*	Min. MVP	$ b_1 - b_1^* $	$ b_1 - b_1^* / \sigma b_1$	Area under normal curve
Labor (hours)	.46	.221717	.141106	.357350	.74	.135633	.961213	66%
Feed (dollars)	1.40	.730461	.192497	.519750	1.00	.210711	1.094619	72%
Expenses (dollars)	-.10	-.014169	.150443	.148303	1.00	.162972	1.083280	72%
Machinery (dollars)	.05	.061646	.096077	.073811	.20	.017165	.178658	14%
Livestock (dollars)	.16	.009030	.098852	.067951	.40	.053921	.596052	14%
Housing (animal units)	5.29	.125801	.128098	.570259	24.00	.444458	3.469671	99.9%



The Adjusted Coefficient of Multiple Correlation $\bar{R}_{1.234567}$ was computed to be .876584 while the coefficient of multiple determination $R_{1.234567}^2$ was .768400. The latter measures the amount of variation in X_1 which was associated with the independent variables included in the analysis. Again the omission of management, a factor which is very important in dairy farming, might be a serious shortcoming of the presently used input classification. However, no satisfactory measure of management is available so far.

3. The Crop Function

The function fitted to data from the crop enterprises of the 27 Illinois farms is an aggregate function in the sense that inputs used in several crop enterprises as well as outputs produced by them have been aggregated and one function was fitted to the resulting data. Crop output (X_1) valued at market prices was the dependent variable, Labor (X_2), Land (X_3), Cash expenses (X_4) and Machinery investment (X_5) are the explanatory or independent variables.

a. The regression coefficients and marginal value productivity estimates.

The regression coefficients which resulted from the crop function are indicated in column 2 and the marginal value productivity estimates are shown in column 3 of Table VI. The test against zero indicates that only two regression coefficients namely those of land and machinery investment are significantly different from zero at the 68 per cent level. The regression coefficients for labor and expenses are not significantly different from zero indicating that large standard errors were associated with these estimates.



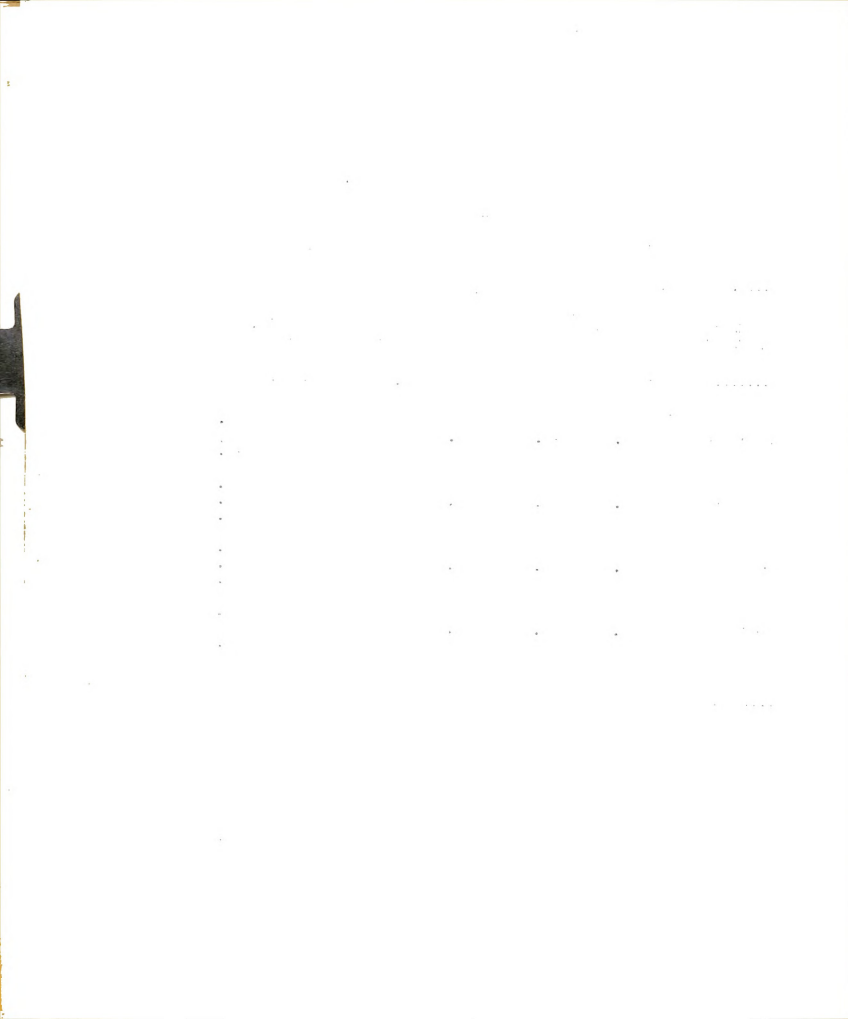
TABLE VI

REGRESSION COEFFICIENTS AND MARGINAL VALUE PRODUCTIVITY ESTIMATES

OBTAINED FROM A COBB-DOUGLAS FUNCTION FITTED TO

CROP ENTERPRISE DATA FROM 27 ILLINOIS FARMS, 1950

(1) Input or investment	(2) Regression coefficients	(3) t value	(4) Marginal value products
Labor (hours)	.128762 ± .166021	.775577	Mean .7182 Upper 1.6449 Lower - .2078
Land (acres)	.532656 ± .111129	3.774247	Mean 34.6660 Upper 43.8537 Lower 25.4826
Crop expense (dollars)	.128877 ± .218881	.588601	Mean .4305 Upper 1.1616 Lower - .3006
Machinery (dollars)	.098476 ± .097392	1.011130	Mean .1830 Upper .3640 Lower .0020



b. Testing the regression coefficients against regression coefficients yielding a minimum MVP. Again as done previously with the estimates of the livestock functions the estimated b_i in crops have been compared statistically with the corresponding b_i^* which would yield certain minimum or reservation returns. These minimum returns considered applicable for crop farms in northern Illinois in 1950 are indicated in column 3 of Table II above. The resulting b_i^* necessary to yield such minimum MVP's are indicated in column 3 of Table VII. The difference between b_i^* and b_i is presented in column 4. Column 5 states this difference in terms of the standard error of the b_i . Only the regression coefficient of land is significantly higher than the b_i yielding a minimum return suggesting that investments in land of the quality found on the average farm in the sample can be very profitable.

Again a farm has been selected whose crop organization appeared to deviate from the geometric mean organization and the estimated b_i have been compared with a b_i^* yielding a minimum or reservation MVP on this farm. This comparison is presented in Table VII a. On the particular farm which was selected the MVP of land is significantly higher, at the 98 per cent level, than the minimum MVP expected for land in this area, indicating that an addition of crop acres on this farm might be profitable. Regarding the other factors, it appears from the t test that this farm deviates considerably more, although not significantly at the 68 per cent level, from an optimum adjustment than did the farm with the geometric mean organization.

TABLE VII

COMPARISON BETWEEN THE ESTIMATED b_i AND THE b_i^* NECESSARY TO EQUATE $MVPX_i$ WITH $MFCX_i$ IN THE CROP ENTERPRISES OF A

SAMPLE OF 27 NORTHERN ILLINOIS FARMS, 1950

Input or investment category	Geometric mean organization	(1) Estimated b_i	(2) σb_i	(3) b_i^* necessary to return minimum MVP	(4) $ b_i - b_i^* $	(5) $ b_i - b_i^* / \sigma b_i$	(6) area under normal curve
Labor (hours)	1232.9	.129762	.166021	.125480	.003282	.019768	.02
Land (acres)	105.6	.532656	.141129	.222780	.313158	2.218948	.97
Expenses (dollars)	2059.0	.126877	.218881	.299377	.170500	.778962	.56
Machinery investment (dollars)	3700.7	.098476	.097392	.107610	.009134	.093785	.07

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TABLE VIIA

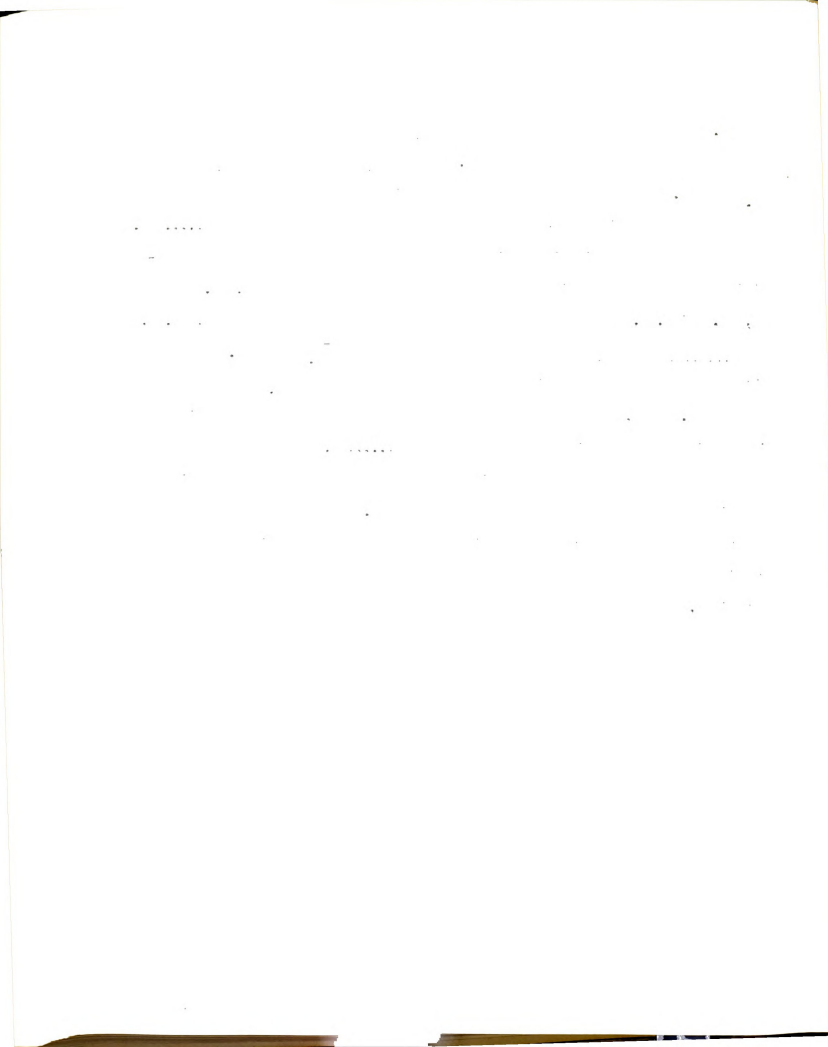
STATISTICAL COMPARISON BETWEEN THE b_1^* HEEDS AND TO YIELD A MINIMUM MVPON A POORLY ADJUSTED FACT IN THE SAMPLE AND THE ESTIMATED b_1

CROP ENTERPRISES, 1950

Input or investment category	MVP	(1) Estimated b_1	(2) σ_{b_1}	(3) $\frac{b_1}{\text{in. MVP}}$	(4) $ b_1 - b_1^* $	(5) $ b_1 - b_1^* \cdot \sigma_{b_1}$	(6) Area under normal curve
Labor (hours)	.30	.120762	.166021	.247360	.711	.118590	.711355
Land (acres)	37.12	.532656	.111129	.208079	14.50	.324577	2.299860
Expenses (dollars)	.49	.128677	.218361	.261533	1.00	.132656	.606064
Machinery (dollars)	.40	.090476	.097392	.040674	.20	.057602	.591144

c. The standard error of the regression line and the coefficients of multiple correlation and determination. The Standard Error of Estimate $\bar{S}_{1.2345}$ of .080566 indicates in terms of logarithms the size of the standard error of crop output X_1 when estimated from the independent variable $X_2, \dots, X_5$. A standard error of this size indicates that the probability of the true geometric mean of the population gross income falls in the interval \$5,713.60 to \$8,278.80 is .68. The geometric mean of gross income in this sample \$6,996.20.

The Adjusted Coefficient of Multiple Correlation $\bar{R}_{1.2345}$ was .832389 while its square the coefficient of multiple determination $R^2_{1.2345}$ turned out to be .692871. The latter measures the amount of variation in X_1 which is associated with the independent variables $X_2, \dots, X_5$. It is seen in this case that 69 per cent of the variation in the dependent variable is associated with the independent variables in the analysis. The omission of crop storage as an explanatory variable might have caused this coefficient to be considerably below the coefficient of multiple determination of the other two functions.



B. Inter-Enterprise Comparisons of Production Functions and Regression Coefficients

Up to this point, the three production functions have been analyzed individually and nothing has been said about the possibility of shifting particular production factors from one enterprise to another one or expanding one enterprise in favor of another one. Since determining optimum inter-enterprise resource allocation is one of the major purposes of analyzing multiple enterprise farms, this section will be concerned with comparing the three functions in general and the marginal value productivity of like factors in particular.

1. Comparison of the Slopes of the Production Functions

It has been pointed out in Chapter II that for a power function such as the Cobb-Douglas, the sum of the regression coefficients determines whether the function shows increasing, constant or decreasing returns depending upon whether the $\sum b_i$ is greater than, equal to, or smaller than one. For the economist, increasing returns to scale mean that each additional unit of production factors $X_2, \dots, X_n$ (combined in scale line proportions) returns more than the previous unit. Increasing returns to scale must always be uneconomical because they imply that some other production factor not included in the analysis yields negative marginal returns.⁴ Constant returns to scale mean to the economist that each additional unit of production factors $X_2, \dots, X_n$ (combined in scale line proportions) returns the same amount as the previous unit; in such a case profit also increases uniformly.

⁴ Lerner, Abba P., The Economics of Control, New York, the MacMillan Company, 1947, pp. 155-56.



Decreasing returns to scale mean that each additional unit of production factors $X_2, \dots, X_n$ (combined in scale like proportions) returns less than the previous unit. In agriculture where the law of diminishing returns is expected to come into play fairly quickly this is the only part of the production function along which an enterprise should be operated.

In order to prove that a certain sum of regression coefficients $\sum b_i$ is significantly greater (smaller) than one, a test is required which determines whether the sum of the regression coefficients in a particular sample is significantly larger than one. Olkin of Michigan State University, developed a test using the F-statistic which permits statistical testing of the $\sum b_i$ against any constant C .⁵ The test as well as the mechanics of computation adapted to the Doolittle method for computing multiple linear regressions are presented in Appendix D.

The test has been carried out for each function, the sum of the regression coefficients being tested against 1. None of the sums of the regression coefficients were significantly different from one. For the dairy function $\sum b_i$ was equal to 1.134406, the corrected F ratio was 1.092294 which was below the $F(.90), (1.21)$ of 2.36. For the hog function the $\sum b_i$ was .791771, the F ratio was .253460 which when tested against $F(.90), (1.21)$ showed that the sum of the regression coefficients of the hog function is not significantly different from 1.

⁵ Olkin, Ingram, "Unpublished report about a problem in testing sums of regression coefficients of linear multiple regression lines against a constant". This report has been made by the statistical group of the mathematics department to Professor Glenn L. Johnson, Department of Agricultural Economics, Michigan State University.

For the crop function $\sum b_i$ was .888772, the F ratio was .698248 indicating that the slope of the crop function is not significantly different from 1.

Thus, it is concluded that constant returns to scale prevail for the three functions included in the analysis.

In addition to testing the sum of the regression coefficients against one the $\sum b_i$ of dairy (1.134486) was tested against the $\sum b_i$ of hogs (.791771). The resulting F ratio was 7.093510 which indicates if compared with $F_{(1,21)}(.95)$, 4.33 that the slope of the dairy function is significantly different at the 95 per cent level from the slope of the hog function.

2. Comparison of Individual Estimates

It was shown previously that the reallocation of resources between enterprises is complete when the ratios MVP_{X_i} / MFC_{X_i} are equal in the different enterprises.

Direct comparisons of MVP's are difficult to make because the errors of the regression coefficients and thus also the errors of the MVP's differ from enterprise to enterprise. To circumvent this difficulty, it was decided to assume minimum values for the marginal factor costs and then determine in each function that b_i^* which would have yielded a MVP equating the ratio MVP_{X_i} / MFC_{X_i} . Using the t test it can be determined whether the estimated b_i is different from b_i^* . By this method, regression coefficients obtained in several enterprises become comparable even though they assume different absolute values and have different standard errors attached to them.



The comparison is carried out in Table VIII. Column 1 lists the various production factors used in the three enterprises. Column 2 states the minimum MVP which should have been expected on Illinois farms in 1950.⁶ Sub-column a, b and c show the values for b_i , b_i^* and the t values for each of the three enterprises. t values below "one" indicate that the b_i^* is not significantly different from b_i at the 68% level. Thus, when the comparison is made at the geometric mean there are only two input categories which suggest possible readjustment between enterprises. The first is the b_i^* for labor employed in hogs which indicates that less labor should be used in the hog enterprise. The second is the b_i for land in crops which indicates that the application of land to grow crops is highly profitable and should probably be expanded.

There appear to be two reasons why the foregoing comparison of regression coefficients of like factors between functions did not suggest many significant changes. (1) The geometric mean organization of a sample of farms not purposively chosen is expected to be fairly close to the scale line adjustment. (2) The errors of most regression coefficients were large causing large confidence intervals for regression coefficients and marginal value productivity estimates.

A comparison of MVP's for one particular farm has not been undertaken, however, it is expected that here too, several farms could be detected on which significant inter-enterprise reallocations of factors would be required.

⁶ For detailed description of how these values are determined see Appendix E.

TABLE VIII

COMPARISON OF INDIVIDUAL ESTIMATES OF MARGINAL

VALUE PRODUCTIVITIES BETWEEN ENTERPRISES

(1) Production factor	(2) Minimum IVP	(3) Dairy			(4) Hogs			(5) Crops		
		(a) b_i	(b) b_i^*	(c) t	(a) b_i	(b) b_i^*	(c) t	(a) b_i	(b) b_i^*	(c) t
Labor (hours)	74¢/hour	.221717	.230920	.121917	.005819	.001660	1.022760	.120762	.125480	.013760
Land (acres)	6¢= \$14.50/A.	--	--	--	--	--	--	.532556	.222780	2.218948
Feed (dollars)	\$1.00	.730461	.717030	.069730	.702417	.696350	.046241	--	--	--
Cash expense (dollars)	\$1.00	-.014169	.100727	.753717	.106413	.040260	.792490	.128877	.299377	.778962
Machinery investment (dollars)	20%	.061646	.026600	.043953	.039436	.005760	.634710	.090476	.107610	.093705
Livestock investment (dollars)	--	--	--	--	--	--	--	--	--	--
Housing (dollars)	5¢-6¢	.125801	.132530	.052530	-.002048	.002110	.266759	--	--	--

C. Comparison of the Three Enterprise Functions with the Aggregate Function

In order to evaluate the methodology suggested in this thesis (namely to fit separate functions to individual enterprises) an aggregate function was fitted which employed the input-output data used in the three enterprise fits without regard to the enterprise with which they were associated. Regression coefficients as well as other statistics were then compared with those for the three enterprise functions.

1. The Estimates of Regression Coefficients and Marginal Value Productivities in the Aggregate Function

Regression Coefficients and Marginal Value Productivity estimates at the geometric mean for the aggregate function are presented in Table IX. The regression coefficients in column 2 are the elasticities of gross income with respect to each of the 6 independent variables listed in column 1. Column 3 lists the "t" values corresponding to each one of the regression coefficients. Areas under the normal curve corresponding to the "t" values indicate if the regression coefficients are significantly different from zero. The coefficients of land and expenses are significantly different at the 95 per cent level, the coefficient of labor is significantly different from zero at the 88 per cent level while the coefficient of livestock investment is significantly different from zero at the 70 per cent level. The remaining coefficients are not significantly different from zero at the 68 per cent level.

The marginal value productivities estimated at the geometric mean indicate that labor returns 70 cents per hour, land .33.80 per acre, expenses

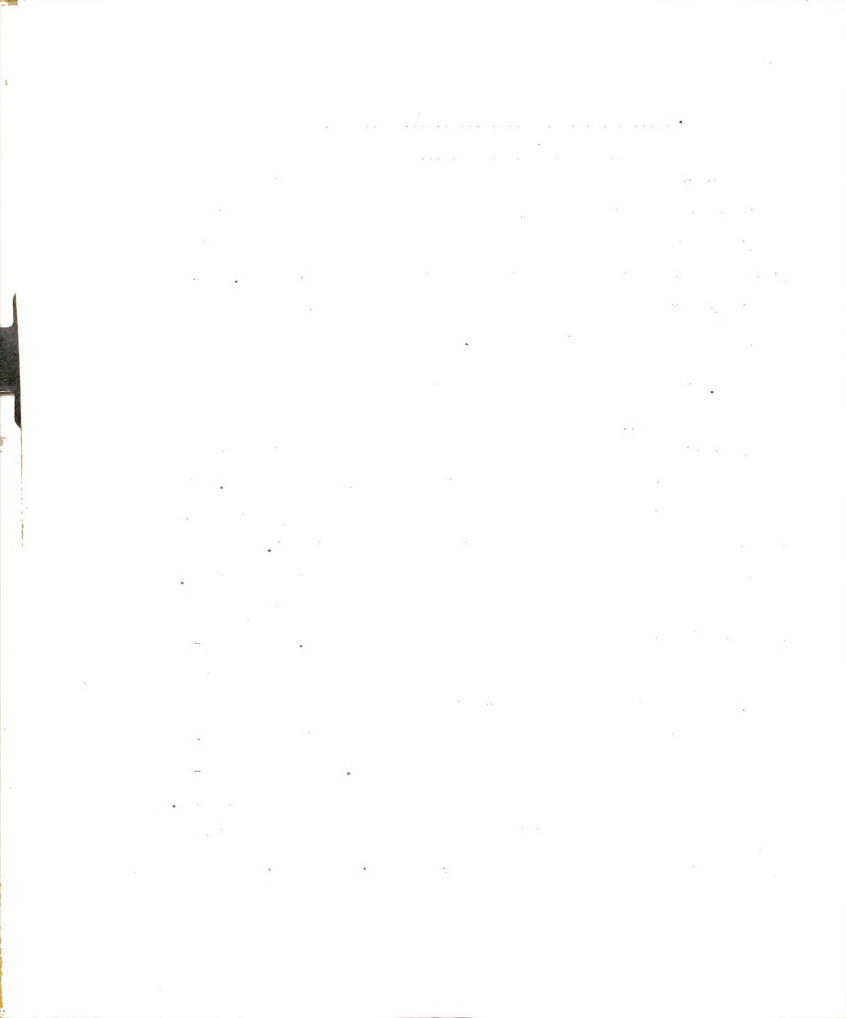


TABLE IX

REGRESSION COEFFICIENTS AND MARGINAL VALUE PRODUCTIVITY ESTIMATES

OBTAINED FROM A COBB-DOUGLAS FUNCTION FITTED TO

AGGREGATED DATA FROM 27 ILLINOIS FARMS, 1950

(1) Input or investment	(2) Regression coefficients	(3) t values	(4) Marginal value products
Labor (hours)	.223812 $\pm$.146651	1.560244	Mean .7056 Upper 1.1579 Lower .2534
Land (acres)	.258914 $\pm$.121503	2.130926	Mean 33.8006 Upper 49.6626 Lower 17.9387
Expenses (dollars)	.509587 $\pm$.175216	2.908334	Mean 1.2965 Upper 1.7423 Lower .8507
Machinery investment (dollars)	.075081 $\pm$.092824	.808854	Mean .2004 Upper .4481 Lower - .0473
Livestock investment (dollars)	.110422 $\pm$.093060	1.126071	Mean .3339 Upper .7362 Lower .0436
Housing (animal units)	.014524 $\pm$.087395	.166992	Mean 5.1740 Upper 36.1560 Lower -25.8085

\$1.29 for each dollar spent, machinery investment 20 per cent, livestock investment 39 per cent and housing investment \$5.17 per housing animal unit which is equal to 1.29 per cent if one housing animal unit is valued at \$400.00.

2. Comparison of Marginal Value Productivity Estimates in the Aggregate Function with the Corresponding Estimates in the Individual Enterprise Functions

Table X compares the MVP estimates obtained in the aggregate function with the corresponding estimates in each one of the individual enterprise functions. The coefficients of "expenses" are not comparable since this variable in the aggregate function necessarily included feed bought while in the livestock functions feed was considered as a separate variable.

With respect to the labor variable, the returns estimated in the aggregate function are the same as those estimated in crops and dairy. However, the low returns which this factor earned in hogs are not reflected in the estimate obtained through the aggregate function.

The returns to the land variable is the same in both the aggregate and the crop function. The standard error of this estimate is considerably smaller in the crop function indicating that the aggregation had a negative influence upon the reliability of this estimate.

Returns to machinery investment vary widely in the four functions. It appears that the estimate for this variable in the aggregate function is a weighted average of the returns estimated for this input in the three individual enterprises.

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TABLE X

COMPARISON OF MARGINAL VALUE PRODUCTIVITY ESTIMATES FROM THREE ENTERPRISE FUNCTIONS

WITH THE CORRESPONDING ESTIMATES OBTAINED FROM AN AGGREGATE FUNCTION

(1) Production factor	Level ¹	(2) Hog function	(3) Dairy function	(4) Crop function	(5) Aggregate function
Labor	M	.0454	.6867	.7182	.7056
	U	.7245	1.1240	1.6449	1.1579
	L	-.6338	.2499	-.2078	.2534
Land	M	----	----	34.6680	33.8006
	U	----	----	43.8537	49.6626
	L	----	----	25.4828	17.9387
Machinery investment	M	1.3693	.4310	.1830	.2004
	U	3.2113	1.1026	.3640	.4481
	L	-.4730	-.2407	.0020	-.0473
Livestock	M	-.3443	.0244	----	.3899
	U	.2216	.2922	----	.7362
	L	-.9103	-.2433	----	.0436
Housing	M	-.0375	24.6796	----	5.1740
	U	.4839	49.8100	----	36.1560
	L	-.6591	-.4500	----	-25.8085

"M" stands for "estimate at the mean value" of the regression coefficient; "U" stands for "estimate at the upper limit" $\pm b_i$; and "L" stands for "estimate at the lower limit" $\pm b_i$.

The estimates for livestock investment and animal housing cannot be considered separately. It is easily seen that the estimates for both variables are in complete disagreement with the corresponding estimates obtained in the livestock functions. In the aggregate function, the returns for livestock investment are very high (35 per cent) while the returns for housing are low. The opposite is true in the dairy function where the returns to dairy investment are unusually low while the returns to dairy housing are high. It is very likely that the high intercorrelation existing between these two variables caused the resulting estimates in both functions to be unreliable.

Generally the foregoing comparison indicates that the MVP estimates obtained from the individual enterprise functions are statistically as reliable as the estimates obtained through the aggregate function and are from an economic point of view superior inasmuch as they permit statements about the relative efficiency of production factors in individual enterprises whereas the aggregate function if considered applicable in individual enterprises would result in several instances in faulty inferences regarding the productivity of factors in individual enterprises.

3. Comparisons of the Constant a , Standard Errors of Estimate and Coefficients of Multiple Correlation and Determination in the Aggregate Function with the Corresponding Estimates in the Enterprise Functions

Function characteristics other than regression coefficients and marginal value productivity estimates are presented in Table XII. It is seen that the " a " values differ considerably among the 3 enterprise functions indicating

TABLE XI

INTER-FUNCTIONAL COMPARISON OF THE CONSTANT "a", STANDARD ERRORS OF ESTIMATES
AND COEFFICIENTS OF MULTIPLE CORRELATION AND DETERMINATION

(1)	(2) Hog function	(3) Dairy function	(4) Crop function	(5) Aggregate function
"a" value in natural numbers	11.334000	1.057100	28.260000	1.480000
Standard error of estimate $\bar{S}$	.066573	.088679	.080566	.064854
Coefficient of multiple correlation $\bar{R}$	.950682	.876584	.925090	.919098
Coefficient of multiple determination $\bar{R}^2$	.903796	.768400	.680770	.844741

that the steepness of the regression surfaces are not the same. The dairy function shows the least steepness, the crop function the highest. Again the "a" value estimated for the aggregate function would yield erroneous results if applied to crops or hogs.

The Standard Error of Estimate $\bar{S}_{1.234567}$ has not increased as a result of the aggregation process.

The Adjusted Coefficients of Multiple Correlation $\bar{R}_{1.234567}$ and Multiple Determination $\bar{R}_{1.254567}$ appear to be higher in the aggregate function than in the dairy and the hog functions they are, however, somewhat lower than the corresponding values in the hog function.

From this comparison, it is concluded that the aggregation of data did not affect the errors of the regression line which might also be the reason why the errors of the regression coefficients in the aggregate function have not increased compared to the corresponding coefficients in the individual enterprise functions.

CHAPTER VII

EVALUATION OF THE METHODOLOGICAL AND EMPIRICAL RESULTS OBTAINED IN THE FOREGOING ANALYSIS

The present chapter evaluates the results of the methodological and empirical investigation in the light of the major purpose for which the analysis was undertaken, namely to improve methods of estimating resource productivities of inputs and investments on multiple enterprise farms. In addition to evaluating the results, suggestions will be made regarding areas which need further investigation.

A. Evaluation of the Conclusions Reached in the Conceptual Part of the Analysis

1. Applicability of various multi - equational approaches. The conceptual part of the analysis began with the assumption that it is possible to estimate productivities of production factors in individual enterprises and suggested three multi - equational approaches. Their applicability was made dependent upon the existing relationships among individual enterprises on multiple enterprise farms. The three approaches were (1) a system of equations solved by the method of simultaneous equations, (2) a system of equations in which each function is fitted independently to data from multiple enterprise farms, and (3) a system of equations each one fitted to enterprises on specializing farms the results to be applied in individual enterprises on multiple enterprise farms.

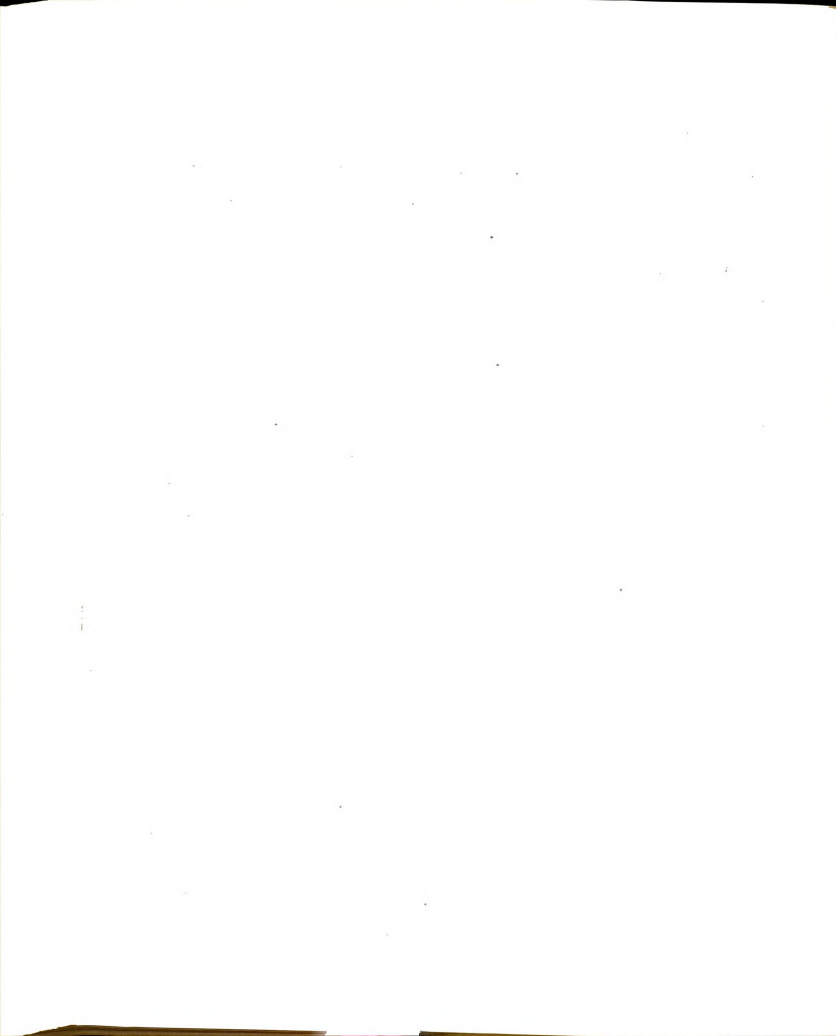
The suggested approach seems general and does not appear to depend upon the type of function chosen. Thus, if it were decided to use a function which is different from the Cobb Douglas, such as the quadratic, the same arguments would probably apply.

If the physical production functions on multiple enterprise farms are dependent upon each other approach (1) is the most appropriate one since it would determine the functional relationship existing between production functions of different enterprises.

If the functions are independent in a technical sense but different from those on specializing farms, approach (2) is applicable.

If the utilization of byproducts and existing external economies and diseconomies do not influence the shape of the production function on multiple enterprise farms and these functions appear to be the same as the corresponding functions on specializing farms approach (3) seems to be the most expedient one.

A detailed investigation of the relationships among production functions on multiple enterprise farms revealed that most production functions can be regarded as being technically independent and that apparent dependencies caused by economic factors and the presence of byproduct complementarity can be resolved through accounting in physical units or constant dollar terms and through appropriate accounting procedures involving charges and credits made for byproducts utilized and produced on the farm. At the same time it was concluded that due to byproduct utilization production functions on multiple enterprise farms should be expected to be different from the corresponding production functions on specializing farms. Thus approach (2) fitting of independent functions to enterprise data from multiple enterprise farms evolved



on an a priori basis as the most appropriate method.

2. Methods of grouping outputs and inputs into categories. Since most farms produce a series of outputs and use a great number of different inputs which cannot possibly be treated individually in a productivity analysis it was concluded that grouping of outputs and inputs into meaningful categories was necessary.

The investigation revealed that in general all products which are produced jointly, e.g., milk and calves, mutton and wool, or lean and lard, can be grouped into one output category and treated as an individual enterprise. Products which compete for resources should be treated as separate enterprises and should be analysed individually. With respect to the combinations of vertically integrates enterprises such as crops and hogs or crops and dairy it was decided to depart from the traditional procedures of fitting one aggregate function to such a vertical chain of enterprises. In this study, mainly in order to permit comparisons of the earning powers of feeds and other inputs in crops as compared to the livestock enterprises a separate function was fitted to the major livestock enterprises and the crop enterprises.

The question was also investigated if it is at all possible to fit one aggregate function to a group of enterprises even though these enterprises compete for resources. On the basis of a theoretical analysis of this problem it was concluded that if the enterprises included in the aggregate function can be assumed to be on the same production function showing the same nature of returns to scale and if the aggregation of the inputs used in the individual enterprises does not increase the intercorrelation among the input and investment categories, valid productivity estimates for individual

factors used can be expected.

Regarding the grouping of inputs into categories it was concluded that the estimates obtained from the Cobb Douglas analysis are unreliable if separate regression coefficients were computed for individual inputs which are highly correlated and should have been combined properly into one category. Thus Bradford's and Johnson's suggestion to use that input classification which keeps intercorrelation among independent variables low appears to be a useful guide ¹.

3. Methods of sampling and accounting procedures. The empirical evaluation of the estimates revealed several important conclusions regarding sampling methods and accounting procedures.

Most important among these conclusions is that in order to obtain a better estimate of the production functions it is necessary to use a purposively chosen sample which would insure that a larger area of the production surface is covered than if a random sample were used. Besides, the intercorrelation among the independent variables would likely be reduced through the use of this type of a sample.

To reduce the amount of disturbances and to insure their independence of the factors included in the analysis it is necessary that tight controls on non-studied variables be maintained when the sample is chosen. These controls can be imposed by choosing farms whose land quality, climate, milk and hog production systems and market location are approximately equal. The independence of management from the variables included in the analysis can be approximated by choosing farmers whose managerial ability and intellectual

¹ Bradford and Johnson, op. cit.



habits are approximately the same.²

Controls such the ones mentioned could not be imposed upon the sample which was used in this analysis since it was necessary to take the largest sample of multiple enterprise farms with the same enterprise combination which was available. Future detailed accounting projects which are set up with the objective to be used for productivity studies should provide a controlled sample in the sense outlined above.

It should be pointed out that the survey interview method is not likely to be a successful way of obtaining accurate enterprise input output information since the farmers interviewed can not be expected to remember enterprise inputs and outputs in sufficient detail. Consequently it is concluded that farm accounts will be necessary in order to obtain reliable information.

Regarding the accounting procedures used to obtain individual enterprise input output data the following remarks appear relevant: The Illinois enterprise accounts give reliable information in the way they are presently set up with the following exceptions: (1) The expense categories should include only productive expenses and not such items as interest, depreciation and taxes. (2) Investments and Investment repairs should be kept separate from operating expenses. (3) Physical measures of buildings including the number of animals which can be housed should be given.

B. Evaluation of the Conclusions Reached in the Empirical

Part of the Analysis

1. Comparison of an aggregate function with individual enterprise

² This suggestion is taken from a paper by Glenn L. Johnson entitled Certain Classification, Accounting and Sampling Problems in Fitting Production Functions to Farm Record and Survey Data. A paper presented Conference on Resource Productivity and Farm Size, Chicago, Ill., Oct. 19 - 20, 1954.

functions. In order to make it possible to evaluate the suggested procedure, namely that of fitting individual enterprise functions, it was decided to fit an aggregate function to the entire farm business and compare the resulting estimates with respect to their statistical reliability and economic meaning. On the basis of this comparison it is concluded that the estimates obtained from the aggregate function did not permit valid inferences regarding the profitability of factors in the individual enterprises. For example the profitability of labor in the hog enterprise was significantly below the minimum MVP which should have been expected for this factor while the aggregate function showed a labor return which was approximately the same as the minimum MVP expected for this factor. In general, MVP's and other function characteristics such as the multiple regression coefficient $\bar{R}_{1.234567}$, the standard error of the estimate $\bar{S}_{1.234567}$ and the sum of the regression coefficients appeared to be weighted averages of the corresponding estimates in the individual enterprise functions. Thus it appeared that the method of fitting independent functions represents a significant improvement over the method of fitting one aggregate function to the entire farm business. This result was anticipated on the basis of the theoretical discussion of this topic in the conceptual part of this analysis.

2. Inter-and intra-enterprise comparison of individual productivity estimates. A considerable part of the analysis was devoted to developing methods with which productivity estimates could be evaluated both within and between enterprises. The method of comparison was to compute a regression coefficient which would yield a certain minimum or reservation MVP and using the "t" Test establish whether this computed regression coefficient is significantly different from the regression coefficient estimated for the function.

The comparisons were made at the geometric mean of each function as well as for individual farms whose organization appeared to deviate from that of the geometric mean organization. The conclusion was reached that the comparison at the geometric mean revealed no serious maladjustments suggesting that in this sample the organization at the geometric mean did not deviate significantly from an optimum organization. The comparison made for individual farms evidenced very serious maladjustments on these farms suggesting that this last type of comparison should be given considerably more weight in future evaluations of productivity estimates.

3. Conclusions regarding the comparative efficiency of production factors on multiple enterprise farms in northern Illinois. Although the principal aims of the thesis were of a methodological nature the estimates obtained reveal several conclusions which can be of practical use to the farmers in northern Illinois.

From the "F Test" which was used to determine whether the returns to scale of the individual production functions are increasing, constant or decreasing it appears that none of the sums of the regression coefficients is significantly different from one. This result suggests that returns to scale which are not different from constant return to scale prevail on these production functions.

With respect to the individual estimates it appears that when compared at the geometric mean the farmers in the sample used feed to equal advantage in the dairy and the hog enterprises. Also the returns obtained for each dollar's worth of feed are equal to one dollar if estimated at the geometric mean indicating that farmers are correct in feeding rather than selling the



crop for cash.

With respect to the utilization of labor it is concluded that the farmers in the sample get relatively high returns for this factor in the dairy and crop enterprises while the returns of labor in hogs were significantly below a minimum wage indicating that a reduction of this factor in hogs is necessary.

The returns to land and labor in crops are high indicating that an addition of crop acres might be profitable. On this basis as well as on the basis of the nature of the returns to scale in the livestock enterprises it is concluded that the farms studied could profitably expand their operations in all the enterprises studied.

In concluding this evaluation of the results of the foregoing analysis it should be pointed out that additional valuable conclusions could probably have been reached if the analyst would have been personally acquainted with the farms included in the sample. Only practical inferences going beyond a mere statistical comparison would fully exploit the results of an investigation such as the one carried out in the latter part of this thesis.



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APPENDIX A

A MATHEMATICAL DERIVATION OF EQUATIONS FOR THE ISO COST (TRANS -
FORMATION) CURVES FROM TWO COBB - DOUGLAS EQUATIONS FIT -
TED TO INPUT - OUTPUT DATA FROM TWO DIFFERENT ENTER -
PRISES

**A MATHEMATICAL DERIVATION OF EQUATIONS FOR THE ISO COST (TRANSFORMATION)
CURVES FROM TWO COBB - DOUGLAS EQUATIONS FITTED TO INPUT - OUTPUT DATA
FROM TWO DIFFERENT ENTERPRISES**

Referring to section B of chapter II where the iso cost curve was explained conceptually as derived from two production functions, it can now be shown how the equations for the iso - cost curves can be obtained from two independently fitted Cobb Douglas Equations. For purposes of illustration, a function with only two independent variables will be used. The total quantity of each factor (X_1 , X_2) available on the farm is designated by Σ_1 . X_1 is the quantity of the factor which is used in enterprise I, $\Sigma_1 - X_1$ is used in Enterprise II. Equations I and II with the constants written as λ_1 and λ_2 read :

$$(I) \quad Y_1 = \lambda_1 X_1^{b_1} X_2^{b_2}$$

$$(II) \quad Y_2 = \lambda_2 (\Sigma_1 - X_1)^{b_1'} (\Sigma_2 - X_2)^{b_2'}$$

Since along the iso - cost lines the factors in each of the enterprises have to be combined in scale line proportions and since these proportions remain the same along the scale line of any Cobb Douglas function, the factors X_2 and $\Sigma_2 - X_2$ can be expressed as functions of X_1 and $\Sigma_1 - X_1$ respectively. Along the scale line the condition $\frac{MVP_{X_1}(Y_1)}{P_{X_1}} = \frac{MVP_{X_2}(Y_1)}{P_{X_2}}$ holds .

In empirical terms the foregoing condition can be written as

$$\frac{\frac{b_1 E(Y)}{X_1} / P_{X_1}}{\frac{b_2 E(Y)}{X_2} / P_{X_2}} = \frac{\frac{b_1 E(Y) P_{X_1}}{X_1}}{\frac{b_2 E(Y) P_{X_2}}{X_2}} = \frac{b_1 E(Y) P_{X_1} X_2}{b_2 E(Y) P_{X_2} X_1} = \frac{b_1 P_{X_1} X_2}{b_2 P_{X_2} X_1} = 1$$

From the foregoing it follows that $X_2 = \frac{b_2 P_{X_2} X_1}{b_1 P_{X_1}}$, similarly

$$Z_2 - X_2 = \frac{b_2' P_{X_2} (Z_1 - X_1)}{b_1 P_{X_1}}$$

Substituting these expressions back into equations I and II, equations

I' and II' are obtained :

$$(I') \quad Y_1 = \lambda_1 X_1^{b_1} \left[\frac{b_2 P_{X_2} X_1}{b_1 P_{X_1}} \right]^{b_2}$$

$$(II') \quad Y_2 = \lambda_2 X_1^{b_1'} \left[\frac{b_2' P_{X_2} (Z_1 - X_1)}{b_1' P_{X_1}} \right]^{b_2'}$$

Solving for X_1 in equation I' :

$$X_1 = \left[\frac{Y_1}{\frac{1}{\left(\frac{b_2 P_{X_2}}{b_1 P_{X_1}} \right)^{b_2}}} \right]^{\frac{1}{b_1 + b_2}}$$

and substituting into II', equation III' is obtained

$$(III') \quad Y_2 = \lambda_2 \left[\left(\frac{Y_1}{\lambda_1 \left\{ \frac{b_2 P_{X_2}}{b_1 P_{X_1}} \right\}^{b_2}} \right)^{\frac{1}{b_1 + b_2}} \right]^{b_1'} \left[\frac{b_2' P_{X_2} \left(Z_1 - \left(\frac{Y_1}{\lambda_1 \left\{ \frac{b_2 P_{X_2}}{b_1 P_{X_1}} \right\}^{b_2}} \right)^{\frac{1}{b_1 + b_2}} \right)}{b_1' P_{X_1}} \right]^{b_2'}$$

From equation III' the transformation curve can be plotted in the Y_1 and Y_2 dimension. The optimum point is obtained upon solving equation III' simultaneously with an iso revenue equation. In the example above the equation for the transformation curve has been worked out for two variables only. The extension to n variables is apparent however computationally difficult.

APPENDIX B

DISTRIBUTION OF GROSS INCOME BY ENTERPRISES ON THE 27 DAIRY-
HOG FARMS INCLUDED IN THE ANALYSIS

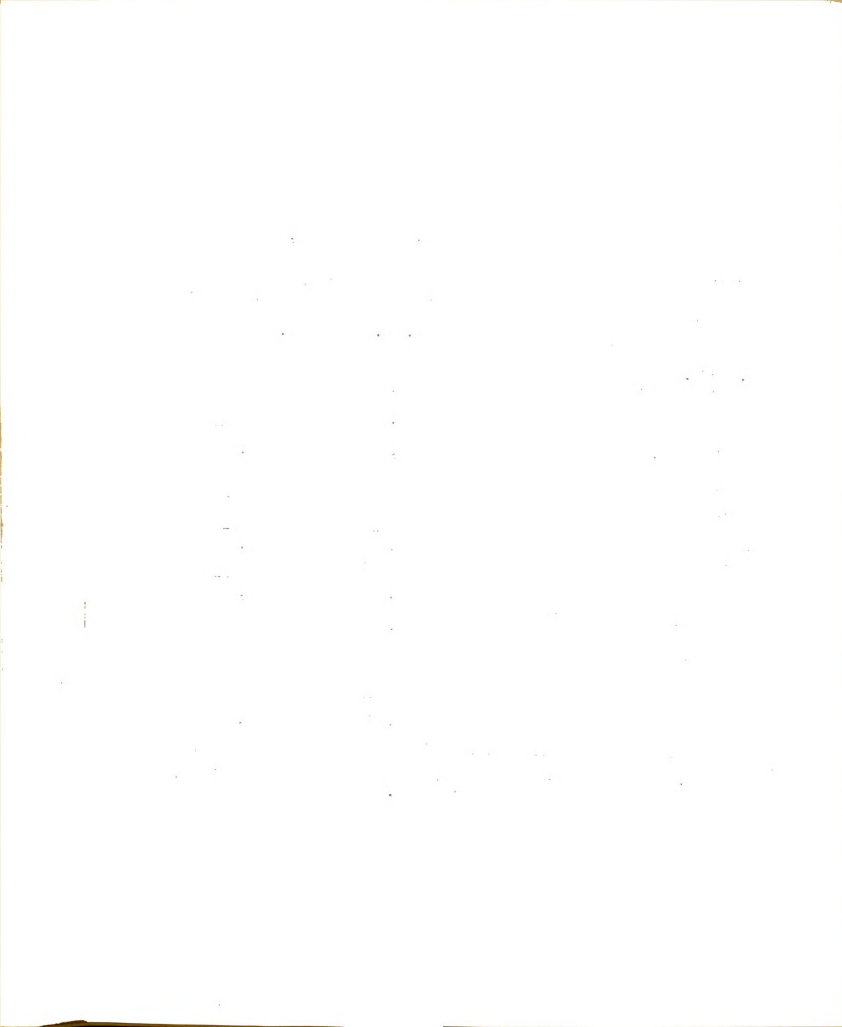
DISTRIBUTION OF GROSS INCOME BY ENTERPRISES ON THE 27 DAIRY-
HOG FARMS INCLUDED IN THE ANALYSIS

TABLE XII

Returns per Farm by Systems of Farming, Blackhawk Area, 1950

Item	Dairy-hog	
	Soil	Soil
	rating 2.0-3.9	rating 4.0 and over
Feed, Grain, and Seed		
Inventory increase	\$ 1,028	\$ 772
Consumed in the home	57	76
Sales	<u>1,784</u>	<u>1,143</u>
Total Feed, Grain and Seed Returns	\$ 2,869	\$ 1,991
Livestock		
Sheep	\$ 17	\$ --
Poultry	410	602
Beef herd	--	--
Dairy	9,166	7,718
Feeders	--	42
Other cattle ¹	--	--
Hogs	<u>7,359</u>	<u>5,603</u>
Total Livestock Returns	\$10,952	\$13,965
Labor and Machine Work off the Farm	214	137
All Other Receipts	<u>186</u>	<u>120</u>
Total Returns	\$20,221	\$16,213

¹ Other cattle include dual-purpose herds, mixed herds of breeding animals, and mixed herds of breeding and feeder cattle.



APPENDIX C

DETERMINATION OF THE DEPRECIATION RATE USED TO ADJUST
MACHINERY EXPENSES IN THE CASH EXPENSE ACCOUNTS

DETERMINATION OF THE DEPRECIATION RATE USED TO ADJUST
MACHINERY EXPENSES IN THE CASH EXPENSE ACCOUNTS

In order to eliminate depreciation from the machinery expense item adjustment factors were used which were determined on the basis of the following table stating the operating cost for different types of tractors:¹

TABLE XIII

Tractor Operating Cost by Drawbar Horsepower Ratings and
Hours Used During 1950, Blackhawk Area

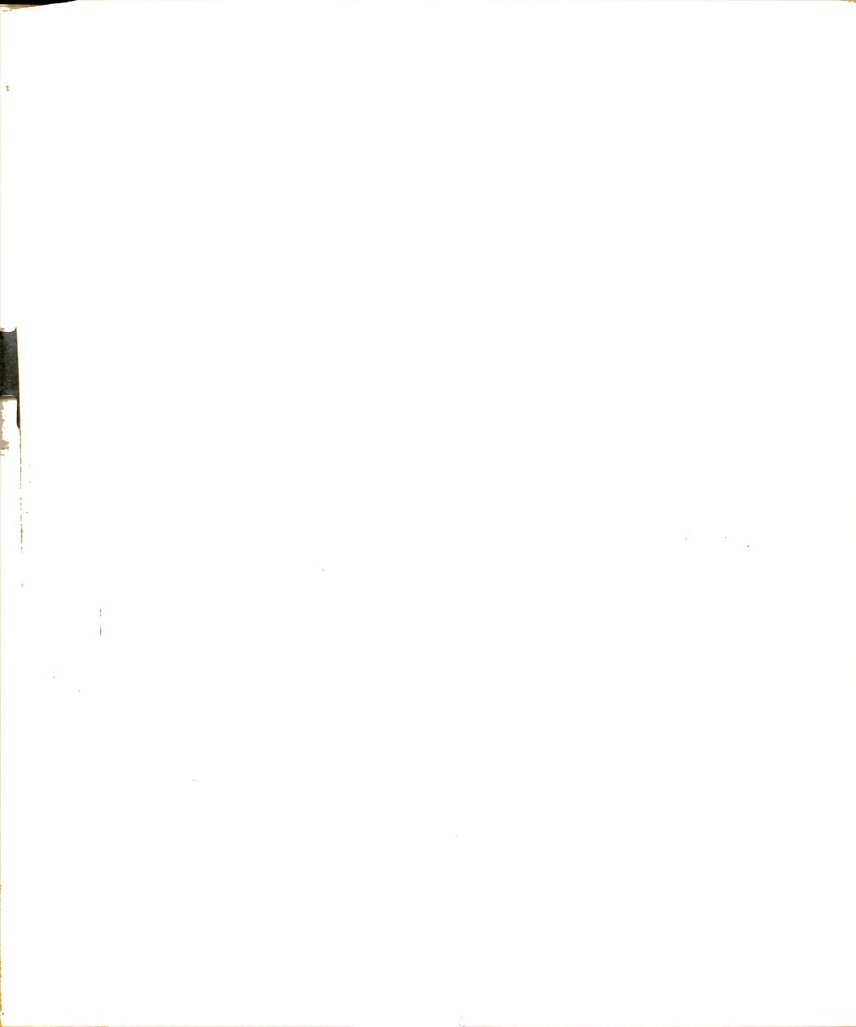
Item	Drawbar horsepower rating			
	10.0 through 20.9		21.0 through 31.9	
	Used fewer than 500 hours	Used 500 or more hours	Used fewer than 500 hours	Used 500 or more hours
Number of tractors	24	20	10	15
Average drawbar horsepower	16.9	17.2	26.3	26.2
Cost items per tractor				
Fuel, oil, and grease	\$128.38	\$249.77	\$196.32	\$295.95
Repairs	47.72	74.18	38.80	107.54
Labor	4.35	8.13	5.49	4.08
Shelter	9.07	10.46	10.65	20.55
Depreciation	108.43	109.36	205.50	183.99
Interest on investment	27.84	27.87	56.01	57.23
Miscellaneous	1.45	3.09	4.16	1.45
Total cost	\$327.24	\$482.86	\$516.93	\$670.79
Total cost minus depreciation and interest on investment	190.97=58%	344.86=71%	255.46=49%	429.57=64%
Hours tractors used				
Drawbar work	302	685	357	596
Belt work	13	32	9	59
Total hours used	3.5	717	366	655
Cost per hour of use	\$ 1.04	\$.67	\$ 1.41	\$ 1.02

¹ Taken from "Detailed Cost Report for Northwestern Illinois 1949 and 1950", Department of Agricultural Economics, Agricultural Experiment Station, University of Illinois College of Agriculture, Urbana, Illinois, April, 1952, AE 2871, p. 40.

The adjustment factor for depreciation in each one of the four classes was the percentage which "Total Cost Minus Depreciation and Interest on Investment" were of the "Total Cost". Thus, if a farmer's machinery cost in crops was given as \$1,000 and his tractors were used 500 or more hours and fell in the class 10.0 through 20.9 drawbar horsepower, the machinery cost included under cash expenses was \$710.

APPENDIX D

THE F TEST FOR TESTING THE SUM OF THE REGRESSION COEFFICIENTS
IN A LINEAR REGRESSION EQUATION AGAINST A CONSTANT



THE F TEST FOR TESTING THE SUM OF THE REGRESSION
COEFFICIENTS IN A LINEAR REGRESSION
EQUATION AGAINST A CONSTANT

The following is a method of testing the sum of the regression coefficients of a regression line against a constant. ^{1/} The test, developed by Dr. Ingram Olkin, Associate Professor of Statistics at Michigan State University, is applicable in all fitting procedures which use an $(n - 1) \times (n - 1)$ matrix when n parameters (including the (a) value) are to be estimated. The Doolittle method which was used in this study is a particular example of this kind of fitting procedure.

The Test: Consider a regression equation of the form

$$y = \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_p X_p + \epsilon$$

where ϵ is normally distributed with mean 0 and standard deviation σ .

A sample of N independent observation is taken and the hypothesis

$$H_0 : \sum_i \beta_i = c \text{ (same constant) is to be tested.}$$

Solution:

$$\text{Let } Y = \begin{bmatrix} Y_1 - \bar{Y} \\ \vdots \\ Y_n - \bar{Y} \end{bmatrix} \quad X = \begin{bmatrix} X_{11} - \bar{X}_1 & \dots & X_{1n} - \bar{X}_1 \\ X_{21} - \bar{X}_2 & \dots & X_{2n} - \bar{X}_2 \\ \vdots & & \vdots \\ X_{n1} - \bar{X}_n & \dots & X_{nn} - \bar{X}_n \end{bmatrix}$$

^{1/} It is recalled (Chapter II) that the sum of the regression coefficients in the Cobb Douglas function determines whether increasing, constant, or decreasing returns to scale are present depending upon whether the sum of the regression coefficients is greater than, equal to, or smaller than unity.

$A = XX'$, then A is a $p \times p$ matrix. The normal equations lead to

the least squares estimates of the β 's, namely

$$b = A^{-1}XY, \text{ where } b = \begin{bmatrix} b_1 \\ \vdots \\ b_p \end{bmatrix}. \quad \text{The test to be used is :}$$

$$(1) \quad \frac{(N - p) \left(c - \sum_{i=1}^p b_i \right)^2}{\sum_{i,j} a^{ij} s^2} = F(1, N - p)$$

where N = number of observations in the sample

p = number of regression coefficients (excluding a) which are estimated

c = some constant ($c = 1$ in case of linear hypothesis)

$\sum_{i=1}^p b_i$ = sum of the regression coefficients (excluding a)

a^{ij} = elements of the A^{-1} matrix. The a^{ij} are the c_{ij} values obtained in the back solution of the Doolittle method.

$$\text{and } s^2 = (Y - X^1b)^1 (Y - X^1b)$$

Observe here again that Y is the column vector of the adjusted values of the dependent variables, i.e.,

$$Y = \begin{bmatrix} Y_1 - \bar{Y} \\ \vdots \\ Y_n - \bar{Y} \end{bmatrix}$$

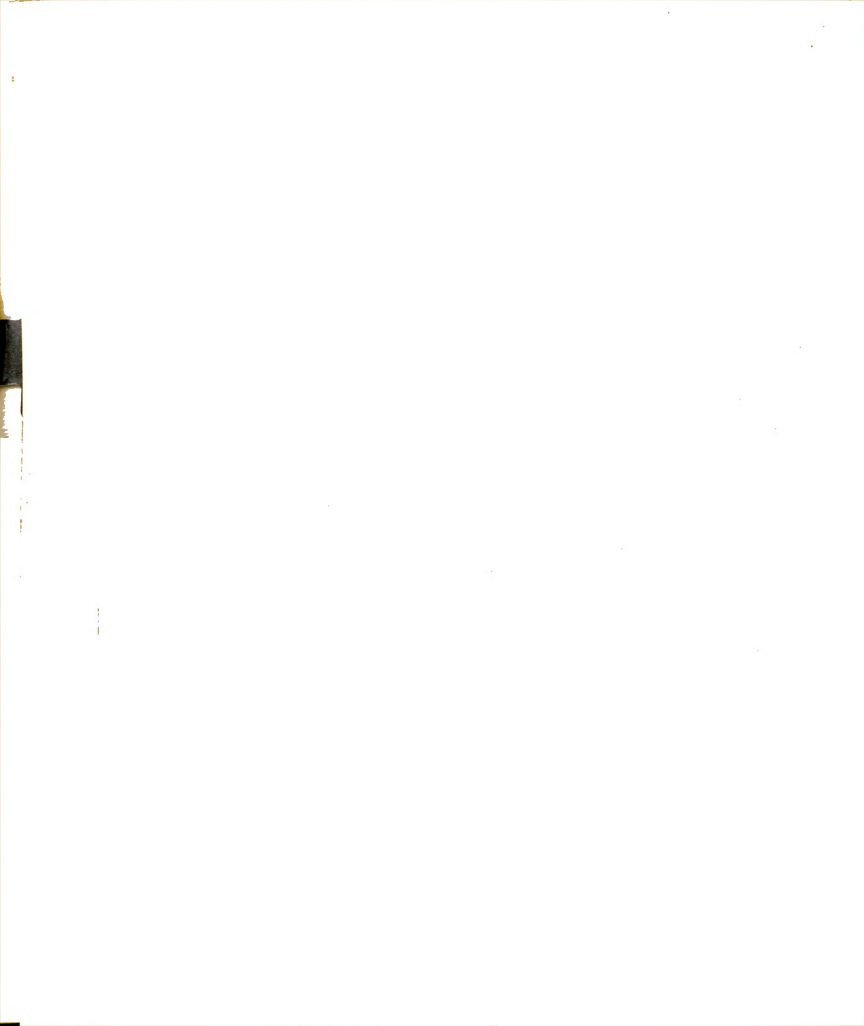
$$\text{Similarly, for the matrix } X^1 = \begin{bmatrix} X_{11} - \bar{X}_1 & \dots & X_{n1} - \bar{X}_n \\ X_{1n} - \bar{X}_1 & \dots & X_{nn} - \bar{X}_n \end{bmatrix}$$

In the case of the Cobb Douglas function, the X and Y matrices contain

logarithms rather than natural numbers. The statistic (1) has an F distribution with 1 degree of freedom in the numerator and $N - p$ degrees of freedom in the denominator. Large values of F are critical.

APPENDIX E

THE ORIGINAL OBSERVATIONS WHICH UNDERLY THE THREE
ENTERPRISE PRODUCTION FUNCTIONS



THE ORIGINAL OBSERVATIONS WHICH UNDERLY THE THREE

ENTERPRISE PRODUCTION FUNCTIONS

HOGS

Farm No.	X ₁ Gross inc.	X ₂ Labor hrs.	X ₃ Feed	X ₄ Exp.	X ₅ Mach. inv.	X ₆ Breed. inv.	X ₇ Perm. hog housing
1	3903.00	467.75	2666.70	171.12	126.16	938.50	184.00
2	3723.00	703.47	2570.02	231.86	69.32	315.00	1.00
3	6132.00	2118.01	3921.86	276.99	201.60	1020.00	325.00
4	10395.00	846.92	8271.63	629.77	812.12	1519.36	926.00
5	4335.00	873.55	4488.48	258.72	352.47	1065.00	220.00
6	6492.00	746.39	4209.89	291.81	122.34	935.00	501.00
7	20127.00	2745.92	15463.68	1132.56	755.48	2635.00	495.00
8	10630.00	1646.92	8124.83	842.87	562.33	2394.00	6359.00
9	6004.00	554.00	4338.00	253.77	77.28	1246.00	478.00
10	7228.00	510.92	5147.43	410.35	266.47	770.00	654.00
11	10647.00	1076.64	6229.16	769.04	939.96	1500.00	140.00
12	5942.00	349.39	4136.08	156.62	287.14	870.00	80.00
13	2426.00	406.70	2006.33	94.41	6.00	800.00	975.00
14	2043.00	494.57	1323.13	34.25	7.85	567.00	150.00
15	4956.00	657.94	4078.28	287.70	396.86	1451.00	158.00
16	5137.00	561.15	3657.50	98.27	252.14	1060.00	1.00
17	4040.00	774.18	2472.66	100.98	66.53	787.50	70.00
18	4460.00	422.46	2375.64	150.21	80.30	1100.00	330.00
19	8675.00	725.48	5805.22	702.23	854.71	990.00	358.00
20	4288.00	946.39	3554.77	170.52	187.16	1191.00	480.00
21	7577.00	555.78	4823.09	143.32	153.72	413.50	687.00
22	4356.00	349.01	3455.71	69.32	63.36	560.00	90.00
23	4717.00	399.37	2354.72	165.08	191.55	688.50	1.00
24	2724.00	333.44	1405.36	104.79	37.27	555.00	227.00
25	6691.00	839.34	5435.71	319.75	236.21	1353.50	514.00
26	7076.00	1087.32	4416.54	298.92	279.57	982.50	568.00
27	7603.00	846.40	5501.61	249.97	180.18	1531.50	525.00

DAIRY

Farm No.	X ₁ Gross inc.	X ₂ Labor hrs.	X ₃ Feed	X ₄ Exp.	X ₅ Mach. inv.	X ₆ Breed. inv.	X ₇ Dairy housing
1	6048.00	1985.30	4109.22	710.16	4302.44	578.38	41.47
2	5201.00	2304.80	4714.07	1069.08	2542.00	1013.40	21.09
3	4935.00	1500.60	5297.68	649.74	1667.00	446.28	20.13
4	9547.00	2748.00	8079.88	1018.13	6226.32	1840.82	52.40
5	5828.00	2074.80	3573.45	690.53	3792.77	1386.03	32.53
6	13350.00	4420.90	8704.39	1251.53	7436.75	1041.40	36.47
7	18115.00	4157.10	9358.93	1487.07	4187.27	2341.12	37.40
8	15546.00	5485.50	8231.80	1630.25	7361.92	1625.68	106.67
9	6619.00	2347.00	6296.91	1250.86	4390.82	981.12	51.37
10	14601.00	4131.70	8571.00	989.80	2317.73	2582.73	59.62
11	3604.00	1713.70	1844.46	491.63	602.86	1398.42	84.32
12	8594.00	1205.00	5633.05	504.00	5156.84	1674.13	50.53
13	13798.00	2686.40	9632.40	1568.05	8815.00	1269.00	63.20
14	3528.00	1264.40	3042.14	307.58	1834.16	142.35	9.39
15	5790.00	2923.50	4545.41	792.72	2357.00	923.78	42.03
16	7627.00	1853.70	5013.14	528.06	2079.93	1344.26	34.33
17	6204.00	1884.80	3845.54	454.86	1855.61	1821.59	44.80
18	7438.00	1294.80	6169.06	799.12	1840.98	1727.19	49.27
19	10578.00	3362.30	6530.37	832.15	1469.23	464.57	24.77
20	8195.00	2630.30	5001.20	525.29	2488.66	1337.09	35.20
21	5093.00	3140.70	4971.90	714.02	2619.18	1082.52	33.60
22	6681.00	2272.30	6511.98	830.03	2908.44	1162.64	36.39
23	7903.00	3245.60	7471.32	630.63	3272.44	1405.45	50.83
24	6529.00	2201.50	3955.26	517.45	2078.65	372.41	35.06
25	6748.00	2655.00	4483.91	993.78	1526.99	1021.91	30.71
26	6384.00	1713.70	3919.83	619.73	1613.95	1240.79	30.01
27	5042.00	2426.40	4134.47	314.10	1677.89	874.37	21.16

CROPS

Farm No.	X ₁ Gross inc.	X ₂ Labor hrs.	X ₃ Land	X ₄ Exp.	X ₅ Mach. inv.
1	3798.90	947.51	70.70	1236.09	3022.46
2	7296.03	954.61	120.60	1690.64	3924.28
3	7972.06	1297.22	95.40	1738.28	2732.12
4	8718.81	1406.78	121.30	1882.98	5780.06
5	5701.72	1200.77	68.00	2111.60	3654.50
6	4958.75	764.21	58.50	2340.90	5531.56
7	14258.00	2493.07	315.20	4228.13	8866.40
8	9350.20	2152.58	134.30	2803.60	4605.99
9	7634.81	1185.93	106.00	2125.42	4813.60
10	7502.42	1394.80	153.80	2865.66	8769.80
11	7574.06	773.84	120.50	1296.29	2053.62
12	5386.94	862.55	106.40	1970.52	5609.73
13	8236.50	1357.65	83.60	2586.67	8361.00
14	3588.00	1234.56	53.00	965.92	754.80
15	5379.02	820.46	71.00	1723.20	2285.60
16	5470.90	782.46	78.50	1874.86	1455.36
17	5376.90	1559.48	138.40	1982.46	2432.88
18	11331.54	1347.19	134.00	2592.95	5114.51
19	11017.00	1464.25	158.80	2435.67	6734.72
20	5769.00	1123.55	91.00	2598.89	5699.75
21	9472.94	1747.33	129.50	2630.79	3917.76
22	5234.25	1212.41	93.70	2186.43	2007.00
23	8907.83	1164.34	127.00	1919.71	7070.14
24	4305.15	1002.88	69.00	1679.28	4047.00
25	7954.50	1157.10	147.90	2193.02	3615.88
26	6450.00	1303.56	107.40	2008.45	2707.96
27	7304.38	2375.75	114.20	2198.97	1328.00



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