ACHIEVEMENT OF FIFTH, SIXTH, NINTH AND TENTH GRADERS IN COORDINATE GEOMETRY

Thesis for the Degree of Ph. D.

MICHIGAN STATE UNIVERSITY

MARY CATHERINE GALLICK

1970

This is to certify that the

thesis entitled

ACHIEVEMENT OF FIFTH, SIXTH, NINTH AND
TENTH GRADERS IN COORDINATE GEOMETRY

presented by

Mary Catherine Gallick

has been accepted towards fulfillment
of the requirements for

Ph.D. degree in Elementary Education

Major professor

Date May 8, 1970

O-169

025

JUL 15 '74 R43

JUL 25 '74 205

093

27 '75 NC 31

SS-127

69

ABSTRACT

ACHIEVEMENT OF FIFTH, SIXTH, NINTH AND TENTH GRADERS IN COORDINATE GEOMETRY

By

Mary Catherine Gallick

This study was designed to analyze the achievement of upper elementary and of secondary school students on a comparable unit in coordinate geometry. One hundred sixty pupils, seventy-seven fifth graders and eighty-three sixth graders, comprised the elementary school sample. They were pupils whose teachers had enrolled in a coordinate geometry workshop and consented to teach a unit on this topic to their classes. The two hundred thirty-eight secondary school students represented a stratified random sample with one first year algebra class drawn from each of the five junior high schools and five classes from the three senior high schools of the Lansing metropolitan area.

A unit pertinent to linear equations and their graphs was written in the form of a set of lesson plans. The content of this unit, suggested materials, and pedagogical techniques for its development formed the basis for the workshop for elementary school teachers. Secondary school students studied coordinate geometry from their regular algebra textbooks in which the content of one chapter was comparable to this unit.

To appraise achievement, the Test on Coordinate Geometry (TOCG) was developed from preliminary drafts employed in two pilot classes. The following seven subtests were embedded in the instrument to study achievement on the specific component concepts of the unit:

1. Plotting and Recognizing Points in the Coordinate Plane
2. Recognizing Members of a Truth Set
3. Intercept Relation to Open Sentence or Graph
4. Slope-Graph Relation
5. Operations with Signed Numbers
6. Graph-Open Sentence Relation
7. Extension of Concepts

The appropriateness of the coordinate geometry unit for elementary school pupils was examined with respect to achievement level. The Arithmetic Concepts subtest of the Stanford Achievement Test, Intermediate II Battery, Form X was administered to sixth graders. On the basis of the median score, these pupils were dichotomized into two groups, high and low, and their achievement compared with that of ninth and tenth graders.

Data from the TOCG were analyzed by using a three-way analysis of variance repeated measures design with proportional subclass frequencies. The independent variables in the design were (1) Grade, (2) Class, and (3) Subtest. The design provided for an overall grades main effect and an interaction between Grades and Subtest, the repeated measures dimension of the design. Where main effects or interactions were significant, Scheffé's post hoc comparison was computed to test specific hypotheses.

The following conclusions were drawn from the analysis of the data. At the .05 level of confidence, there is no difference in achievement on a unit in coordinate geometry between

(1) fifth graders and tenth graders.

(2) sixth graders and tenth graders.

(3) the upper half of the six graders as measured by a general mathematics achievement test and ninth graders.

(4) the lower half of the sixth graders as measured by a general mathematics achievement test and tenth graders.

(5) the upper half of the sixth graders as measured by a general mathematics achievement test and ninth and tenth graders combined.

At the .05 level of confidence, statistical tests rejected the hypotheses that there were no differences in achievement on a unit in coordinate geometry between

(1) fifth graders and ninth graders.

(2) sixth graders and ninth graders.

(3) fifth and sixth graders combined and ninth and tenth graders combined.

In the three preceding comparisons, the achievement of secondary school students was higher than that of elementary school pupils.

When achievement in coordinate geometry was analyzed by concepts, all the component tasks were understood as readily by elementary school pupils as by secondary school students.

The

additiona

ferences

geometry

Eig

coordinat

and tenth

ment of al

except the

accounted.

students w

while the

mathemati

school

A

geometry.

school

to their

the secon

are

textbook.

are

Though not of primary concern to this study, two additional hypotheses were analyzed. No significant differences existed in achievement on a unit in coordinate geometry between fifth graders and sixth graders.

Significant differences in achievement on the coordinate geometry unit did exist between ninth graders and tenth graders in favor of the ninth graders. Achievement of ninth graders was higher than all other classes except the upper half of the sixth graders. This may be accounted for by assignment procedures which placed students with higher mathematics aptitude in Algebra I while those with lower aptitude first completed a general mathematics course. Thus, ninth graders represented a select group.

A reactionaire assessed attitude toward coordinate geometry. Approximately two-thirds of the elementary school pupils preferred the unit in coordinate geometry to their regular mathematics program. Only one-third of the secondary school students liked the chapter in coordinate geometry more than the algebraic topics in their textbook. Similar proportions of each group rated coordinate geometry more interesting.

ACHIEVEMENT OF FIFTH, SIXTH, NINTH AND TENTH GRADERS IN COORDINATE GEOMETRY

By

Mary Catherine Gallick

A THESIS

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Elementary Education

1970

1971

Many

of this st

professor,

invaluable

phase of th

entire

standards

accomplish

Many

research pr

Burmester

on Coordin

assistance

William

extend thi

Dr.

complement

to the par

teachers.

Fin

with deep

whose help

ACKNOWLEDGMENTS

Many people contributed to the successful completion of this study. I am most deeply indebted to my major professor, Dr. W. Robert Houston, whose counsel has been invaluable not only in encouraging and directing every phase of the research for this study, but throughout my entire doctoral program. His leadership and the high standards to which he subscribes add more meaning to the accomplishment of my goal.

Many others also contributed to the success of this research project. I owe special thanks to Dr. Mary Alice Burmester for her able assistance in developing the Test on Coordinate Geometry, to Dr. Andrew Porter for his assistance in planning the research design, and to Dr. William Fitzgerald for his interest and suggestions. I extend thanks to my committee members, Dr. John Wagner, Dr. Calhoun Collier, and Dr. George Myers. Their diverse, complementary talents were appreciated. I am also grateful to the participating elementary and secondary school teachers.

Finally, disregarding his protest, I must recognize with deep gratitude my matchless husband, Harold Gallick, whose helping hand was ubiquitous. Without his unselfish

assumptio

completio

this disc

assumption of part of my responsibility in our home, the completion of my studies and the research and writing of this dissertation would have been impossible.

TABLE OF CONTENTS

Table

1.

2.

3.

4.

5.

6.

7.

8.

9.

10.

LIST OF TABLES

Table

LIST OF FIGURES

LIST OF APPENDICES

CHAPTER I

INTRODUCTION

Background

Today's mathematics curriculum reflects current understanding of learning theory, instruction, and mathematics. Major changes in the elementary school mathematics program occurred during the 1960's as a result of extensive efforts by mathematicians, educators, and psychologists, and followed similar endeavors directed toward improving the secondary school program.

Examination of the new programs revealed at least two trends: (1) introduction of mathematical topics previously not taught in the traditional curriculum, and (2) introduction of topics at earlier levels in the mathematics program. The mathematics program continues to change, influenced by the work of Edith Biggs, Bert Kaufman, Robert Davis, Jean Piaget and others, and by projects such as The Madison Project, the Nuffield Project, and the School Mathematics Study Group. In a dynamic, changing society, the mathematics content and instructional procedures must evolve continually.

One vision of a rigorous program for the future was provided by twenty-nine mathematicians, scientists, psychologists, and mathematics educators who met for the Cambridge Conference on School Mathematics (CCSM).[1] In this recommended program, the high school student would study the equivalent of three years college mathematics in today's curriculum. Junior high school students would study algebra and geometry while elementary pupils would encounter such topics as groups, matrices, logic, and geometry,[2] all of which represent radical changes from present programs.

Prior to recent endeavors to revise the mathematics program, little geometry was included in the elementary school curriculum. In fifth and sixth grades, pupils formed a nodding acquaintance with perimeters and areas of familiar polygons, chiefly squares and rectangles.

An analysis of contemporary material, however, revealed an increased emphasis on geometry at the elementary school level. In the primary grades, many informal activities develop such concepts as congruence, symmetry, and topics from non-metric geometry. One fourth grade textbook introduces measurement concepts of length, perimeter, volume, and surface area.[3] Other

---

[1]Educational Services Incorporated, Goals for School Mathematics, The Report of the Cambridge Conference on School Mathematics (Boston: Houghton Mifflin Co., 1963), p. 6.

[2]Ibid., pp. 31-67.

[3]Robert E. Eicholz, et al., Elementary School Mathematics, Book 4 (Reading, Mass.: Addison-Wesley Publishing Co., 1964), pp. 1-25.

geometric topics in the upper elementary school curriculum include parallelism, parallelograms, polygons and diagonals, right triangles, triangular pyramids, circles, and central and inscribed angles. These topics, many of which did not appear in traditional textbooks until the tenth grade, are evidence that many branches of geometry have become an established part of the elementary school mathematics program. They represent simultaneously new topics for the elementary school level and an earlier introduction of topics once delegated exclusively to higher levels.

Accompanying the innovations by curriculum revisers were the pleas of educators and researchers for criticism and evaluation of new content and its grade placement.[4] Research by Piaget and others substantiated that children develop through growth stages. To what extent do these stages influence the age placement of topics in the elementary school? On the other hand, Bruner, in an often quoted thesis, hypothesized that any

---

[4]Carl B. Allendoerfer, "The Dilemma in Geometry," The Mathematics Teacher, LII (March, 1969), 165; Kenneth E. Brown and John J. Kinsella, Analysis of Research in the Teaching of Mathematics: 1957-1958, Bulletin 1960, #8 (Washington, D. C.: United States Government Printing Office, 1960), pp. 23-26; J. Fred Weaver, "Non-metric Geometry and Mathematics Preparation of Elementary School Teachers," American Mathematics Monthly, LXXIII (December, 1966), 1115-1121; and Otto C. Bassler, Research Workers in Mathematics Education," American Mathematics Monthly, LXXIV (September, 1967), 859.

sub

hit

With

curr

vari

topi

was

of fi

geome

this

compa

the t

evalua

studen

Which

unders

element

the top

able pr

Op

hypothe

5.
(Cambri

subject can be taught effectively in some intellectually honest form to any child at any stage of development.[5] With the introduction of new ideas in elementary school curriculum, research which studied pupil learning at various stages of development was important. One of the topics introduced into the elementary school curriculum was coordinate geometry.

The present investigation studied the achievement of fifth and sixth grade pupils on a unit in coordinate geometry, a topic heretofore generally not taught at this level. How did their achievement on this unit compare with that of ninth and tenth graders to whom the topic had been taught traditionally?

The Problem

The purpose of this study was to examine and evaluate the achievement of elementary and secondary students on a comparable unit in coordinate geometry. Which concepts, if any, could elementary school pupils understand as well as secondary students? Could all elementary school pupils learn the concepts or should the topic be treated as enrichment material for more able pupils?

Specifically, the study investigated the following hypotheses:

---

[5]Jerome S. Bruner, The Process of Education (Cambridge: Harvard University Press, 1962), p. 33.

Hypothesis A. There is no difference in achievement on a unit in coordinate geometry between fifth graders and ninth graders.

Hypothesis B. There is no difference in achievement on a unit in coordinate geometry between fifth graders and tenth graders.

Hypothesis C. There is no difference in achievement on a unit in coordinate geometry between sixth graders and ninth graders.

Hypothesis D. There is no difference in achievement on a unit in coordinate geometry between sixth graders and tenth graders.

Hypothesis E. There is no difference in achievement on a unit in coordinate geometry between fifth and sixth graders combined and ninth and tenth graders combined.

Hypothesis F. There is no difference in achievement on a unit in coordinate geometry between the upper half of the sixth grade students as measured by a general mathematics achievement test and ninth graders.

Hypothesis G. There is no difference in achievement on a unit in coordinate geometry between the lower half of the sixth grade class as measured by a general mathematics achievement test and tenth graders.

Hypothesis H. There is no difference in achievement on a unit in coordinate geometry between the upper half of the sixth grade as measured by a general mathematics achievement test and ninth and tenth graders combined.

To answer the pertinent questions related to distinct concepts in coordinate geometry, achievement was further analyzed by the seven subtests listed below. For each subtest, Hypotheses A through H were tested for significant differences.

Subtest 1: Plotting and Recognizing Points in the Coordinate Plane

Subtest 2: Recognizing Members of a Truth Set

Subtest 3: Intercept Relation to Open Sentence or Graph

Subtest 4: Slope-Graph Relation

Subtest 5: Operations with Signed Numbers

Subtest 6: Graph-Open Sentence Relation

Subtest 7: Extension of Concepts

Importance of the Study

Dictated by tradition, concepts of coordinate geometry related to graphing linear equations have been reserved for first year algebra courses. However, experimental programs such as the Madison Project,[6] for example, have introduced certain concepts of coordinate geometry to fifth and sixth grade pupils. Since such experimental projects are frequently the harbingers of the content in future commercial textbooks, an analysis of the achievement of fifth and sixth grade pupils on a unit in coordinate geometry should provide information helpful in determining if the topic, or a portion of it, may be appropriate for these levels.

In this research project, a unit in coordinate geometry was taught to fifth and sixth graders and their

---

[6]Robert B. Davis, Discovery in Mathematics (Reading, Mass.: Addison-Wesley Publishing Co., 1964), Chapters 10, 11, 17 and 18.

achievement was compared with that of first year algebra students. Such studies are important prerequisites to more general dissemination of new content in the mathematics curriculum.

Another aspect of this study relates to the qualifications of teachers. Coordinate geometry, as a part of the first year algebra course, is taught by teachers who have majors or minors in mathematics. Theoretically, they possess the capability to handle these and more advanced concepts. On the other hand, fifth and sixth grade teachers are elementary generalists who may possess the most meager of mathematical backgrounds. Thus, an obvious, essential question must be asked: Are elementary teachers prepared to teach meaningfully the increasing amount of geometry appearing in the curriculum?

The fifth and sixth grade teachers in this study participated in a six-week workshop in coordinate geometry designed to teach content and to suggest methodology for an activity-oriented, discovery-learning program. Is such brief in-service training adequate to meet the needs of the elementary teacher so that the new topic can be taught effectively?

Theoretical Foundation of the Study

Hypotheses generated for this study stem from the theoretical framework of curriculum building vis à vis grade placement and the sequence of topics, and pupil readiness. Both content and its sequence, perennial problems in all fields of education, have been influenced by pedagogical theories in vogue at a specific time.[7] A brief review of these theories should provide insight into the motivating forces behind current mathematics curricular endeavors.

When the Faculty Theory was extant, content which was difficult and purported to strengthen the faculties of memory and reason comprised mathematics courses. Little thought was given to selecting topics relevant to the child's needs, of intrinsic interest, or which might whet the child's mathematical appetite. A hierarchy of difficulty determined the sequence of content instead of child maturation.

The Child Psychology Theory, in contrast to the harshness of the faculty school, greatly affected both the content and sequence of arithmetic. The expressionist wing of this movement favored inclusion of only those topics which related to the child's interest and needs. he essentialist wing favored the teaching of only those pics that were essential to the social utility of ults. Since little need for arithmetic was seen for e early elementary child and his readiness to deal with

[7]M. Vere Devault and Thomas Kriewall, <u>Perspectives</u> <u>Elementary School Mathematics</u> (Columbus, Ohio: rles E. Merrill Publishing Co., 1969), pp. 41-71.

the mathematics was questioned, the teaching of arithmetic nearly disappeared in many schools at the primary level between 1930 and 1960.[8] To the child psychologist, the mental hygiene of the learner was the most important factor in choosing content, with maturation providing the basis for sequence and social need the basis for grade placement.

Subsequent theories had little impact on content and grade placement and neither underwent much change until the late 1950's. Thorndike's Stimulus-Response Theory offered the notion of hierarchies as the basis of sequence. Gestaltism exercised its greatest influence on method, increasing emphasis on discovery-oriented teaching and sustaining the meaning theory of instruction. While the major impact of the Neo-behaviorists has been at the experimental and research level, their emphasis on the use of programmed learning materials and the statement of classroom goals as behaviorial objectives influenced teachers and instructors of teachers throughout the nation.

Massive curriculum reform projects of the 1960's stimulated a renaissance of American interest in the work of Jean Piaget. Piaget, a Swiss zoologist and psychologist, conducted experiments with children for more than half a century. He investigated verbal and conceptual

[8] Ibid., p. 53.

aspects of a child's thought, the organization of the sensory-motor schemata for assimilating intelligence, and the development of operations which give rise to number and continuous quantity.[9] Piaget theorized that nature and nurture interact in a dual way. Environment serves as nourishment for mental growth whose pattern of development follows a course laid down by genes.[10] With respect to growth of abilities, Piaget believed that nature provides the pattern and the time schedule of its unfolding, while nurture provides the nourishment for the realization of this pattern. With respect to the content of knowledge, the reverse is true; nurture determines what is learned while nature provides the requisite capacities.

According to Piaget, the development of intellectual capacity proceeds through stages whose order is constant but whose time intervals vary by individual and society. These maturational stages in the thought process determine readiness for cognitive learning. Readiness factors underlie any and all efforts to

---

[9]Jean Piaget, The Child's Conception of Number (New York: Norton and Co., 1962); and Jean Piaget, Barbel Inhelder, and Alina Szeminska, The Child's Conception of Geometry (New York: Basic Books, 1960).

[10]David Elkind, "Piaget and Montessori," Harvard Education Review, XXXVII (Fall, 1967), 538-544.

improve the school program.[11] The notion that a child moves through levels of development with a timing that is difficult to accelerate has focused attention on the problem of how, when, and what to teach in mathematics. Piaget's findings raised questions about recent curriculum reform, for various innovative groups (such as SMSG and The Madison Project) have accelerated introduction of topics in the elementary school with apparent disregard for children's cognitive development. The successful teaching of these topics to younger children forces the educator to consider the investigations of J. S. Bruner and his efforts to construct an alternate theory of cognitive growth.

Bruner, a professor of psychology at Harvard University, also believed that intellectual development moved through stages. Even though his theory reflects the strong influence of Piaget, Bruner nevertheless cautioned against rigid acceptance of the concept of stages. His previously quoted statement that any subject can be taught effectively to any child at any age in some intellectually honest form appears to contradict his acceptance of Piaget's consideration for children's readiness for particular learnings on the basis of their prevailing stages of development. Some educators who

---

[11]Harry S. Broudy, Othaniel B. Smith, and J. R. urnette, "New Look at Readiness," Theory Into Practice, II (December 28, 1963), 424-429.

interpreted Bruner's statement literally, disagreed.[12] Ausubel termed it a "generalization that has wrought incalculable mischief in an entire generation of over-eager curriculum reform workers."[13] Shulman viewed Bruner's remark as a suggestion that the conception of readiness should be modified to include not only the child but also the subject matter.[14] Research exists supporting both Piaget's and Bruner's views;[15] more will be required to resolve the conflict.

In his research with elementary school children, Robert Davis, Director of the Madison Project, practiced tenets suscribed to by both Piaget and Bruner. Davis believed, as did Piaget, that good pedagogy must involve presenting the child with situations in which he experiments, manipulates objects and symbols, poses questions, seeks answers and compares findings. Like Bruner, Davis was a proponent of the discovery method. He was firmly

---

[12] David P. Ausubel, "Can Children Learn Anything That Adults Can--and More efficiently?" Elementary School Journal, LXII (February, 1962), 270-271; and Alice Keliher, "Editorial," Childhood Education, XLII (May, 1966), 527.

[13] David P. Ausubel, "Review of Toward a Theory of Instruction, by Jerome S. Bruner," Harvard Education Review, XXXVI (Fall, 1966), 338.

[14] Lee S. Shulman, "Perspectives on the Psychology of Learning and the Teaching of Mathematics," in Improving Mathematics Education, ed. by W. Robert Houston (East Lansing: Michigan State University, 1967), pp. 14-17.

[15] Ausubel, "Can Children Learn Anything That Adults Can--and More Efficiently?" op. cit.; and Arthur F. Coxford, "The Effects of Instruction on the Stage Placement of Children in Piaget's Seriation Experiments," The Arithmetic Teacher, XI (January, 1964), 4-9.

convinced that the intellectual springs of children, whether they were culturally disadvantaged, normal, or gifted, have not been tapped by our traditional arithmetic curriculum.[16] Of similar conviction, the researcher modelled the coordinate geometry unit utilized in the current study on Davis' theory of instruction as he stated it in The Madison Project's Approach to a Theory of Instruction:

> The Madison Project attempted to bring the elementary student in contact with mathematics . . . and has chosen topics with the following aims:
> 1. Students must be ready for ideas and take an active role in developing them.
> 2. Concepts must rise naturally from some problem solving situation.
> 3. The concepts must be related to some fundamental mathematical ideas.
> 4. The concepts must lead to some significant patterns of generality.
> 5. The topics must be appropriate to the age of the child and must appear, in toto, to observer to amount to a significant experience.[17]

In teaching the pilot classes and conducting the in-service workshop, the researcher used a "low-keyed discovery approach"[18] emulating Davis' technique.

---

[16] Robert B. Davis, "Mathematics for Younger Children--The Present Status of the Madison Project," New York State Mathematics Teachers Journal, X (April, 1960), 75-79.

[17] Robert B. Davis, "The Madison Project's Approach to a Theory of Instruction," Journal of Research in Science Teaching, II (March, 1964), 148.

[18] DeVault, op. cit., p. 63.

Overview of Procedures

A synopsis of the procedures used to test the hypotheses of the study is presented below and described in detail in Chapter III.

Unit for Pupils and Teachers

A unit in coordinate geometry, focusing on linear equations and their graphs and traditionally a part of first year algebra, was designed for the upper elementary school level. Elementary school teachers taught the unit. Because they had not studied the concepts involved in coordinate geometry, a six-week workshop was organized. In the workshop both content and methodological approach were emphasized.

Sample

Pupils of the workshop participants comprised the elementary sample. Their counterparts, students of randomly selected algebra teachers, made up the secondary sample.

Pilot Class and Instrument

The researcher piloted the coordinate geometry unit with two elementary classes, a sixth grade and a combined fourth-fifth grade. These trials indicated that the unit would require about four weeks of classroom instruction.

P
t
li
q
Th
me
in
vi
th

up
st
lin
alg

an
em
is
Gra
were

Rober
York:

Preliminary forms of the instrument used to measure pupil achievement in coordinate geometry were administered to these pilot classes. An item analysis of these preliminary forms provided the basis for selecting those questions which were included in the final form of the Test on Coordinate Geometry (TOCG). To study achievement on specific concepts by grade level, the TOCG used in this study was subdivided into seven subtests, previously identified in the statement of the problem of the study.

The TOCG was administered to elementary classes upon completion of the unit and to first year algebra students when they finished the chapter, "Graphs of Linear Equations and Inequalities," in the regular algebra text.[19]

Design of the Study

A three-way analysis of variance repeated measures design with proportional subclass frequencies was employed to analyze the data. The scheme of this design is presented in Figure 1. Independent variables included Grade, Class, and Subtests, while the dependent variables were Subtest and Total Achievement on the TOCG.

---

[19] Donovan A. Johnson, John J. Kinsella, and Herman Rosenberg, Algebra: Its Structure and Application (New York: The Macmillan Co., 1967), Chapter 9.

Preliminary forms of the instrument used to measure pupil achievement in coordinate geometry were administered to these pilot classes. An item analysis of these preliminary forms provided the basis for selecting those questions which were included in the final form of the Test on Coordinate Geometry (TOCG). To study achievement on specific concepts by grade level, the TOCG used in this study was subdivided into seven subtests, previously identified in the statement of the problem of the study.

The TOCG was administered to elementary classes upon completion of the unit and to first year algebra students when they finished the chapter, "Graphs of Linear Equations and Inequalities," in the regular algebra text.[19]

Design of the Study

A three-way analysis of variance repeated measures design with proportional subclass frequencies was employed to analyze the data. The scheme of this design is presented in Figure 1. Independent variables included Grade, Class, and Subtests, while the dependent variables were Subtest and Total Achievement on the TOCG.

---

[19] Donovan A. Johnson, John J. Kinsella, and Herman osenberg, Algebra: Its Structure and Application (New ork: The Macmillan Co., 1967), Chapter 9.

Grade	Class	Subtests						
		S_1	S_2	S_3	S_4	S_5	S_6	S_7
5	1	x_{11}	x_{12}					
	2	x_{21}	x_{22}					
	3							
6	1 High							
	2 High							
	3 High							
	1 Low							
	2 Low							
	3 Low							
9	1							
	2							
	3							
	4							
	5							
10	1							
	2							
	3							
	4							
	5							

Fig. 1.--A three-way analysis of variance repeated
[...]sures design with proportional subclass frequencies.

To determine the appropriateness of the content of the coordinate geometry unit for achievement levels of elementary school pupils, the Arithmetic Concepts subtest of the Stanford Achievement Test, Intermediate II Battery, Form X, was administered to the pupils in the sixth grade sample. They were dichotomized into groups, high and low, on the basis of the median score of the Concepts subtest.

The class mean was used as the experimental unit rather than individual pupil scores. One of the assumptions of the analysis of variance is the independence of experimental units. Were individual pupil scores utilized in the study, this assumption could not be met and the resulting statistics potentially invalid. The assumption that pupils within a class have learned geometry independently of each other cannot be made; whereas, between-classes independence can be assumed. Therefore, class means were computed and used as the dependent variable in the study. While this procedure reduces the power which the increased number of indi-vidual scores would generate, the resulting statistics eflect greater precision of measurement.

The analysis design outlined in Figure 1 provided ests for an overall Grades main effect and an interaction etween Grades and Subtest, the repeated measures dimen-on of this design. Where main effects or interactions were significant, Scheffé's post hoc comparison was computed to test the specific hypotheses of the study. The .05 level of confidence was selected as being sufficiently rigorous for the purposes of this investigation.

Organization of the Study

Research studies related to the present investigation and upon which the generated hypotheses were based are reviewed in Chapter II. Chapter III describes the procedures in detail while the results of statistical analyses are included in Chapter IV. Conclusions and educational implications are the focus of Chapter V.

CHAPTER II

REVIEW OF THE LITERATURE

Readiness

Increasing demands of the knowledge explosion stimulated curriculum reform in all the disciplines at all levels of the school's program during the late fifties and throughout the decade of the sixties. Since the curriculum depends on the child's "readiness," this concept assumed increased importance.

Cronbach defined a pupil's readiness for any situation as the sum of all the characteristics which make him more likely to respond in one way than another.[20] Whereas some earlier psychologists regarded readiness as solely a function of physical maturation,[21] contemporary psychologists and learning theorists believe that in addition to maturation, readiness is a function of the child's needs, goals, learned ideas, and skills. Moreover, readiness changes day by day as a result of experience,

---

[20] Lee J. Cronbach, Educational Psychology (2d ed.; New York: Harcourt, Brace and Co., 1963), p. 89.

[21] Arnold L. Gesell, "The Ontogenesis of Infant Behavior," in Manual of Child Psychology, ed. by Leonard Carmichael (New York: John Wiley and Sons, 1954), pp. 355-356; and George E. Coghill, Anatomy and the Problem of Behavior (New York: Macmillan and Co., 1929), p. 113.

biological development, and development of the nervous system. Changes in one facet can alter the whole system of responses.

The basic idea involved in readiness is that an optimum time exists for the initiation of new concepts.[22] Different views of readiness evolved from the theories of learning referred to in the previous chapter and from research in child development.

In support of a curriculum designed to discipline the mind, the faculty theorists totally disregarded child readiness. Child psychologists used the lack of readiness, which they viewed as a product of maturation, to support the doctrine of educational postponement. In a recent yearbook on Theories of Learning, McDonald reviewed two general theories of learning: stimulus response (S-R) theories and cognitive insight theories.[23]

In general, S-R psychologists seldom considered readiness for several reasons. Among them, Tyler cited the following: (1) S-R theorists who investigated learning sought materials devoid of association value; in such a case questions about readiness were reduced to

---

[22] Wilbur H. Dutton, Evaluating Pupils' Understanding of Arithmetic (Englewood Cliffs: Prentice-Hall, Inc., 1964), p. 23.

[23] Frederick J. McDonald, "The Influence of Learning Theories on Education," (1900-1950), Theories of Learning and Instruction, Sixty-third Yearbook of the National Society for the Study of Education, Part I (Chicago: University of Chicago Press, 1964), pp. 1-26.

questions about learning sets, topics which they had not investigated. (2) The concept of "mediation" used in descriptions of rote verbal learning also implied some notion of readiness because the learner's experiential background affected the mediators he employed.[24]

Recently, some neobehaviorists have become interested in processes of cognitive learning. Gagné and his associates conducted extensive research on the acquisition of knowledge, in particular of mathematical knowledge. Gagné believed that the attainment of any entity of knowledge depended upon prior attainment of relevant subordinate knowledge.[25] To teach a particular concept, Gagné first determined the desired terminal behavior. He analyzed the knowledge necessary for its accomplishment and then constructed a pyramid of prerequisites to prerequisites to the objective or terminal behavior.

Gagné viewed readiness as a function of prerequisite knowledge. The child who could not attain the stated goal simply did not possess the necessary prior experience. Given those experiences, Gagné believed the

---

[24]Frederick Tyler, "Readiness," Encyclopedia of Educational Research (4th ed.; New York: The Macmillan Co., 1969), p. 1065.

[25]Robert M. Gagné, "The Acquisition of Knowledge," Psychological Review, LXIX (July, 1962), 355-365.

child could accomplish the goal. He was unconcerned with the development considerations of the cognitive psychologists.[26]

Foremost among the cognitive approaches to development are the theories of Jean Piaget. Piaget held that mental development was an extension of biological growth. Development progressed through four concept stages. The first is the sensory-motor stage lasting approximately the first two years of life. The concept of permanence of an object develops during this stage. During the preoperational or second stage, from two to seven years of age, language, symbolic function, and thought develop. In the third or concrete operational stage (from seven to eleven years of age), the child develops number concepts in terms of a collection--substance, weight, and volume which are always discovered in that order. In the fourth, the formal operational stage (from twelve to fifteen years of age), the child performs operations on abstract ideas. Mathematics can be understood and the child is capable of constructing chains of deductive reasoning. Four factors contribute to the development of these stages: (1) maturation of the nervous

---

[26]Shulman, op. cit., p. 27.

system, (2) encounters with experience, (3) social transmission, and (4) equilibration or auto regulation.[27]

Piaget recognized that the time of appearance of these stages varied from culture to culture, but he accumulated a vast amount of data substantiating that the sequence is unaltered. These stages aid in assessing the readiness of a child to learn particular concepts. Upon identification of the stage of intellectual development attained by pupils, both content and method of providing related experience can be selected. Piaget's theory redirected attention toward the use of concrete objects to develop basic mathematical ideas. The use of materials such as the Stern blocks, Cuisenaire rods, and Dienes' multiphasic blocks in the classroom has provided multisensory experiences for the child. Rather than teaching the structure of the subject area, Piaget favored providing the child with situations where he was active and created the structure.

Piaget doubted that these stages could be accelerated. In his address at a conference at Rochester, New York in 1964, he said that he asks investigators who have succeeded in accelerating the operations structure the following three questions:

---

[27]Jean Piaget, "Development and Learning," <u>Journal of Research in Science Teaching</u>, II (March, 1964), 176-178.

1) Is this learning lasting?
2) How much generalization is possible?
3) In the case of each learning experience, what was the operational level of the subject before the experience and what more complex structures has this learning succeeded in achieving?[28]

Nevertheless, a growing body of research supports the claim that young children can learn more difficult concepts than presupposed.[29] An outspoken proponent of this tenet was Jerome Bruner who presented another view of cognitive development. He theorized that all cognitive activity was dependent upon the process of categorizing.[30] Although he subscribed to levels of development (enactive, iconic, and symbolic, which paralleled Piaget's pre-operational, concrete and formal stages), he felt Piaget did not recognize the role of environmental forces.

An articulate advocate of the discovery method, Bruner believed that new concepts should be introduced by means of concrete manipulative activities with ample provision for self exploration. He emphasized the

---

[28] Ibid., p. 183.

[29] Betty Estes, "Some Mathematical and Logical Concepts in Children," Journal of Genetic Psychology, LXXXVIII (June, 1956), 219-222; and Marilynn J. Adler, "Some Educational Implications of the Theories of J. Piaget and J. S. Bruner," Canada Education and Research Digest, IV (December, 1964), 299.

[30] Jerome S. Bruner, Jacqueline Goodnow, and G. A. Austen, A Study in Thinking (New York: John Wiley and Sons, 1956), p. 246.

importance of a spiral curriculum in which the basic structural concepts of each subject are presented early and returned to again and again at higher cognitive levels as the child develops.[31]

Readiness, for Bruner, could be modified to include not only the child but also the subject matter. As the child grows, the same subject matter must be presented at a manipulative or enactive level, an iconic or representational level, and finally at a symbolic level.[32] Bruner argued that readiness is a function not so much of maturation but rather of the intentions and skill of the teacher to translate ideas into the language of those being taught. He warned that these intentions must be plain before teachers decide what can be presented to children of certain ages. "When we are clear about what we want to do . . . , I feel reasonably sure that we will be able to make rapid strides ahead in dealing with the pseudo-problem of readiness."[33]

Davis' perception of readiness incorporated ideas from both the S-R and cognitive insight schools of thought. Like Gagné, he believed that meticulous attention should be paid to prerequisites and that new

---

[31] Jerome S. Bruner, "On Learning Mathematics," The Mathematics Teacher, LIII (December, 1960) 617.

[32] Shulman, op. cit., p. 31.

[33] Bruner, op. cit., p. 619.

concepts should be subdivided into their "atomic constituents." But like Piaget and Bruner, he stressed that the child should be involved in a variety of concrete experiences relevant to new learning.[34]

Portions of all the preceding views of readiness are reflected in this study. The final task and all of the prerequisite skills were stated as behavioral objectives. Kinesthetic experiences were provided for pupils because they were in the concrete or enactive stage of development. By translating the ideas into iconic terms, the investigator sought to determine whether younger pupils could learn concepts traditionally taught to older students, thus raising the problem of grade or age placement, a problem which is inextricable from readiness.

Grade Placement

The most comprehensive research on grade placement was done by the Committee of Seven from 1924 to 1931. Using data obtained from controlled experiments in 255 cities, the Committee attempted to determine the optimal stage in child development at which various processes should be presented. When the Committee had ascertained the approximate grade placement from a comparative

---

[34]Robert B. Davis, "The Madison Project's Approach to a Theory of Instruction," op. cit., p. 148.

analysis of courses of study, it was taught at that grade, one grade lower, and one grade higher. Pre- and posttests were administered. After an analysis of test scores, the topic was placed at the grade level at which three-fourths of the children achieved 80 per cent accuracy.[35] Their recommended placement of arithmetic topics affected courses of study and textbook construction throughout United States and Canada until the current reappraisal in mathematics education. The work of the Committee of Seven was followed by many other studies dealing with the grade placement of arithmetic topics.[36]

Gibney and Houle pointed out that with the increased attention to geometry in contemporary mathematics texts, the topic of geometric readiness has been slighted.[37] D'Augustine attested to the need for

---

[35]Carleton W. Washburne, "The Grade Placement of Arithmetic Topics," Twenty-ninth Yearbook of The National Society for the Study of Education, Part II (Chicago: University of Chicago Press, 1930), pp. 641-670.

[36]Foster E. Grossnickle, "Experiment with One Figure Divisor in Short and Long Division," Elementary School Journal, XXXIV (March, 1934), 496-506; William A. Brownell and Harold Moser, "Meaningful vs. Mechanical Learning: A Study in Grade III Subtraction," Research Monograph No. 8 (Durham: Duke University Press, 1949); and William A. Brownell, "Arithmetic Readiness as a Practical Classroom Concept," Elementary School Journal, LII (September, 1951), 15-22.

[37]Thomas C. Gibney and William W. Houle, "Geometry Readiness in the Grades," The Arithmetic Teacher, XIV (October, 1967), 471.

definitive research to answer the questions: (1) At what grade levels are certain topics learned with a high degree of efficiency in terms of time and effort spent? (2) Which geometric and topological topics are appropriate at various levels to clarify and simplify other mathematical concepts? (3) What factors relate to student achievement with geometrical and topological topics?[38]

In the past, the only geometry included in the curriculum was metric in nature. Mensuration, for example, was the only geometric concept sequenced into the mathematics program by the Committee of Seven. But Piaget's work indicated that children learn topological concepts first, projective concepts next, and demonstrative concepts last--exactly the opposite order in which geometric concepts are taught in American schools.

Piaget described the following three stages in a child's conceptual construction of Euclidean space:

1. The child understands qualitative operations of distance, length, area, volume, and congruence. Stage one occurs when the child can distinguish between space as a container with fixed sides and space occupied by moving objects.

---

[38]Charles H. D'Augustine, "Factors Relating to Achievement with Selected Topics in Geometry and Topology," The Arithmetic Teacher, XIII (March, 1966), 192-197.

2. The child understands simple material operations of one, two and three dimensions, construction of coordinate systems, measures of angles, and areas. This stage depends on the ability of the child to apply operations of subdivision and change of position with recognition that wholes are conserved by these operations.

3. The child understands how to calculate areas and volumes. This stage is attained when the child comprehends that an area or volume is unaffected by surrounding space and conceives of space as a continuum.[39]

Piaget's findings in relation to the sequence in which children learn geometric concepts, along with the need for research on the placement of geometric topics cited in current literature, support an investigation such as the present one. Several studies which have examined the grade placement of geometric concepts will be described in the following section.

Related Studies

In investigating questions related to the hypotheses of this study, several researchers examined the teaching of a geometric unit at different grade levels. One of these studies was conducted by Corley who taught a one-week unit on reasoning followed by a four-week unit on demonstrative geometry to grades 6, 7, 8, 9 and 10.

---

[39] Piaget, Inhelder and Szeminska, pp. 389-408, passim.

The evaluative instrument was divided into three parts: knowledge of concepts, understanding of geometric proofs, and ability to use the syllogism for reasoning. Analysis of variance using the treatments by levels design tested the hypothesis that there was no difference in the mean achievement of the grades on each part of the geometry test as well as the total test. Corley's conclusions pertained to the comparative ability of pupils grade-by-grade to perform on this test. He concluded that sixth graders could learn simple geometric terms and concepts. A marked improvement in ability to learn methods of reaching conclusions and to understand logical systems occurred in seventh grade. He did not offer conclusions as to whether differences in the mean achievement of pupils by grade did or did not exist.[40]

Fitzgerald taught three units, one on numeration, one on negative numbers and one on non-metric geometry to fifth, seventh, and ninth graders. Only the results of the latter unit are of direct relevance to this study. Fitzgerald's primary interest was to study the degree to which classes of different grade levels were alike or different with respect to achievement on the experimental

---

[40] Glyn J. Corley, "An Experiment in Teaching Logical Thinking and Demonstrative Geometry in Grades 6 through 10" (unpublished Ph.D. dissertation, George Peabody University, 1959), Dissertation Abstracts, XX, p. 1375.

units. He also investigated the relative effect of various factors such as height, chronological age, mental age, educational age and total experimental success on the learning of mathematics. His treatment of data was purely descriptive. Test scores for the three grades were placed into the same distribution which was separated into six intervals by one-half standard deviations. The percentage of students from each class in each interval was computed. The mean achievement of the fifth graders on the geometry unit was slightly, but not significantly, higher than that of the seventh or ninth graders.

Fitzgerald found that the highest 10 per cent of the fifth grade learned an amount superior to that learned by the bottom ten per cent of the ninth grade, the bottom thirty per cent of the seventh grade was inferior to the top thirty per cent of the fifth grade; and that the top thirty per cent of the seventh grade was superior to the bottom thirty per cent of the ninth grade. He concluded that nonmetric geometric concepts were sufficiently easy for most upper elementary students to understand and that many elementary school age children could learn more about some of the basic ideas related to geometry than they were generally provided the opportunity to learn.[41]

---

[41]William Fitzgerald, "A Study of Some of the Factors Related to the Learning of Mathematics by Children in Grades 5, 7, and 9" (unpublished Ph.D. dissertation, University of Michigan, 1962).

Other studies support Fitzgerald's conclusion about the ability of elementary students to learn geometry. Denmark and Kalin concluded that fifth grade pupils learn a satisfactory amount of skills and concepts from the Hawley and Suppes' workbooks dealing with constructions. They found that the content of Book I was easy for the pupils as indicated by a mean score which represented 78.6 per cent of the total possible on the criterion test. In their opinion the class mean of 50 which represented 56.8 per cent of the total possible on the criterion test was not satisfactory in relation to the amount of instruction time used.

On the basis of their study, Denmark and Kalin concurred with the opinion of national curriculum groups such as the Greater Cleveland Program and the School Mathematics Study Group that more geometry could be taught in elementary school than was then taught. The researchers further speculated that most of Book I and some of Book II could be learned by either younger or less intellectually capable pupils than the children in the experimental class whose mean I.Q. was 126. Trial of these materials in an average class would seem desirable even for speculation.[42]

---

[42] Thomas Denmark and Robert Kalin, "Suitability of Teaching Geometry in Upper Elementary Grades: A Pilot Study," The Arithmetic Teacher, XI (February, 1964), 73-80.

D'Augustine investigated the factors which relate to a student's achievement with geometrical and topological topics. Fifth, sixth, and seventh grade pupils studied a programmed text which he developed. The text presented such topics as paths and their properties, simple closed paths, polygons, and classification of polygons based on nonmetric properties. An analysis of variance-covariance performed on test scores and scores from standardized achievement tests isolated reading and arithmetic as significant factors in achievement with topics of topology and geometry when taught with a programmed text. Mental age and chronological age did not significantly affect achievement. D'Augustine found no significant differences in the mean levels of achievement on the criterion post-test attributable to grade seven after the mean criterion posttests had been adjusted for mental age, chronological age, and reading and arithmetic achievement. D'Augustine, who pooled test results into one score, pointed to the need to investigate each independent subtopic.[43] The current study investigated the subtopics involved in the coordinate geometry unit, and compared the achievement of fifth and sixth grade elementary pupils to that of ninth and tenth grade first year algebra students.

---

[43] D'Augustine, op. cit., pp. 192-197.

In the study most closely related to the present one, Sair Ali Shah conducted research to determine to what extent certain content in geometry was satisfactory for children within the age range seven to eleven years. The mathematical content included the geometric concepts of:

1. Plane figures with three to twelve sides.
2. Nets of figures (obtained by opening models of cubes, rectangular blocks, etc.).
3. Symmetry of figures about a line.
4. Reflection of figures in a plane mirror.
5. Rotation of figures about a point.
6. Translation.
7. Bending-stretching of figures with no cutting.
8. Networks.

Shah reported satisfaction with the results of the tests on the various concepts listed above and felt that the results suggested the kind of geometry which children could study successfully. While the performance of the seven-to-eight age group was sometimes low, the performance by the eight-to-eleven age group was on the whole satisfactory and in some areas was high. He observed that performance became better as the age of students increased.

However, his main concern was the reactions of the children toward the content discussed. "Thus, to estimate the reactions, we used (1) distributions of percentage of candidates obtaining a given score interval, and (2) the means of scores obtained by different age groups."[44] These data, which reflected achievement rather than reaction, were presented for each of the nine tests on topological concepts. The results described offer in detail the performances of each age group on each concept. Since the data were not subjected to statistical analysis, two questions remain unanswered: (1) Does the performance of one age group differ from another on any or all of the concepts? (2) Is the difference, if any, significant?

The present study examined the performance of students of four different grade levels on various concepts of coordinate geometry--those pertinent to graphing linear equations. The results of the criterion instrument were subjected to analysis of variance to test for possible differences between grade levels and to Scheffé's post hoc test to determine if any existing differences were significant, thus extending Shah's study in the areas he considered as the most logical and necessary for further understanding of the curriculum and children's learning.

---

[44] Sair Ali Shah, "Selected Geometric Concepts Taught to Children Ages Seven to Eleven," The Arithmetic Teacher, XVI (February, 1969), 122.

CHAPTER III

PROCEDURES

The purposes of this study was to evaluate the relative achievement of elementary and secondary students on a comparable unit in coordinate geometry. This topic was being incorporated into experimental courses in elementary schools at the time of this study. The feasibility of introducing this topic four to five years earlier than students traditionally had studied it in first year algebra was explored. Eight hypotheses were posed and tested in the investigation.

The Study Sample

Elementary School Pupils

One hundred sixty of the three hundred ninety-eight students who comprised the sample population were elementary school pupils. Seventy-seven were fifth graders and eighty-three were sixth graders. They were pupils whose teachers enrolled in a coordinate geometry workshop and consented to teach a unit on this topic to their classes. Three classes, one fifth and two sixth grades, were from schools located in a low to lower-middle socioeconomic residential area. Three classes, two fifth and one sixth grade, were from schools which represented middle to upper socioeconomic residential areas.

Tables 1, 2 and 3 describe the mathematics achievement of the pupils in these classes as measured by the Stanford Achievement Test, Intermediate II Battery, Form X. Sixth grade data were from the test battery administered in October, 1968. Fifth grade data were from the test battery administered in October, 1967, when pupils were in fourth grade.

TABLE 1

DISTRIBUTION OF PUPIL ACHIEVEMENT BY CLASS AND STANINE ON THE STANFORD ACHIEVEMENT ARITHMETIC COMPUTATION SUBTEST

Class	Number of Pupils by Stanine									Class Mean Stanine
	1	2	3	4	5	6	7	8	9	
6-1	3	7	5	3	5	3	0	0	0	3.34
6-2	3	2	8	9	2	1	1	0	0	3.33
6-3	1	2	3	4	5	8	2	2	0	4.92
5-1	0	2	0	3	6	3	5	2	0	5.48
5-2	1	4	5	0	7	3	1	0	0	4.00
5-3	2	8	20	1	5	0	0	0	0	4.11

TABLE 2

DISTRIBUTION OF PUPIL ACHIEVEMENT BY CLASS AND STANINE ON THE STANFORD ACHIEVEMENT ARITHMETIC CONCEPTS SUBTEST

Class	Number of Pupils by Stanine									Class Mean Stanine
	1	2	3	4	5	6	7	8	9	
6-1	0	3	2	4	10	2	3	2	0	4.73
6-2	1	3	3	10	3	3	4	0	0	4.48
6-3	0	1	7	9	5	1	2	2	0	4.44
5-1	1	1	0	4	6	3	3	2	1	5.38
5-2	1	0	2	7	1	2	4	3	1	5.00
5-3	1	5	5	5	8	1	1	0	0	3.81

TABLE 3

DISTRIBUTION OF PUPIL ACHIEVEMENT BY CLASS AND STANINE ON THE STANFORD ACHIEVEMENT ARITHMETIC APPLICATIONS SUBTEST

Class	Number of Pupils by Stanine									Class Mean Stanine
	1	2	3	4	5	6	7	8	9	
6-1	3	2	1	3	5	8	1	3	0	4.84
6-2	1	2	4	7	5	3	5	0	0	4.56
6-3	0	4	2	6	7	4	3	1	0	4.67
5-1	0	0	3	0	7	7	1	1	2	5.66
5-2	3	7	2	9	2	2	1	0	0	3.38
5-3	0	2	4	4	2	2	2	2	2	4.85

The achievement scores of pupils in five of the six classes were skewed to low average in all three subtests; computations, concepts, and applications. In only one class, 5-1, did more pupils score above than below stanine five. More pupils scored above than below stanine five in class 6-1 and in class 6-3 on the applications and computations subtests respectively. The class mean stanine fell in the average range (fourth, fifth, and sixth stanines) on the applications and concepts subtests, while one class was below average in each of the subtests. Class 5-2 scored below average in the concepts subtests. The mean stanine of two classes, 6-1 and 6-2, were below average in the computation subtest, while the mean stanine of the four remaining classes was in the average range.

The mean stanines by grade are found in Table 4. Both the fifth and sixth grades placed in the low average range on all three subtests.

TABLE 4

MEAN STANINE BY GRADE IN ARITHMETIC SUBTESTS ON THE STANFORD ACHIEVEMENT TEST

Grade	Arithmetic Subtest		
	Computation	Concepts	Applications
5	4.53	4.76	4.63
6	3.86	4.55	4.69

The mathematical background and teaching experience of the teachers of these elementary classes are found in Table 5.

One of the elementary teachers who enrolled in the workshop was a beginning teacher. The remaining participants were experienced teachers who had taught from 6 to 18 years. None had taught at their currently assigned grade levels for more than 6 years.

Four of the teachers had studied mathematics for two years in high school and one for two and one-half years, while one teacher had not studied any mathematics in high school. Four teachers had earned 3 term hours of credit in college mathematics, one had earned 7.5 term hours and one had earned credit for 15 term hours of college mathematics. None of the teachers had studied geometry in college.

Secondary Students

To select a representative secondary student sample, a stratified random sample was drawn from each secondary school in the school district. Within each school, algebra classes were assigned identifying numbers. For the study one class was selected from each of the five junior high schools and five from the three senior high schools using a table of random numbers.[45]

---

[45] M. N. Downie and R. W. Heath, Basic Statistical Methods (New York: Harper and Row Publishers, 1965), p. 316.

TABLE 5

NUMBER OF YEARS TEACHING EXPERIENCE AND MATHEMATICS PREPARATION OF ELEMENTARY SCHOOL TEACHERS

Elementary Teacher	Grade Taught	Number Years Teaching Experience*		Mathematics Preparation (Term Hours)		
		Total	Present Grade	High School	College Geometry	College Other Mathematics
T1	6	0	0	2	0	7.5
T2	5	13	6	0	0	3
T3	5	18	2	2	0	15
T4	6	6	5	2.5	0	3
T5	5	13	1	2	0	3
T6	6	9	6	2	0	3

*Excludes year in which study was conducted.

Of the 238 secondary students included in the study, 135 were ninth graders and 103 were tenth graders.

The overwhelming majority of students in the Lansing school system studied algebra in either the ninth or the tenth grade. Mathematics was required in ninth grade where three courses (algebra, general mathematics, and basic mathematics) were available.

Eighth grade students were scheduled for one of these courses according to the recommendation of their eighth grade mathematics teacher. In placing a student in algebra in ninth grade, the teacher's recommendation was guided by the following factors:

1. The student's overall Stanford Achievement score (6th stanine or above)
2. The student's grades in seventh and eighth grade mathematics (A or B)
3. The student's average of concepts, computation and application scores on the mathematics subtests of the Stanford Achievement Battery (6th stanine or above)
4. Other factors (e.g., parental desires)

Ninth grade students were scheduled to study algebra in tenth grade on the basis of their performance in general mathematics. Students who studied basic mathematics seldom selected algebra in tenth grade. Consequently, ninth grade algebra students generally represented the upper third of the class, while tenth grade algebra students represented the middle third of the class in mathematical aptitude.

The mathematical background and teaching experience of the secondary teachers whose classes were included in the study are found in Table 6.

The number of years of teaching experience of the secondary teachers ranged from 4 to 17. Five of the teachers had studied mathematics four years in high school, and the remaining five from one to three and one-half years. College mathematics preparation ranged from 30 to 120 term hours. All ten secondary teachers had studied at least one college geometry course with a range of 3 to 11 hours. When the preparation of secondary school teachers is compared with that of the elementary school teachers, the far superior training in mathematics is evident.

Treatment

Elementary pupils, guided by their teachers, studied a unit in coordinate geometry which was designed by the researcher using the Madison Project's treatment of the topic as a model. The content of this unit, suggested materials, and pedagogical techniques for its development were presented in a workshop for elementary school teachers. Workshop participants volunteered to teach the content to their pupils. Four school weeks of

TABLE 6

NUMBER YEARS TEACHING EXPERIENCE AND MATHEMATICS PREPARATION OF SECONDARY SCHOOL TEACHERS

Secondary Teacher	Grade	Years Teaching Experience	Mathematics Preparation (Term Hours)		
			High School	College Mathematics	College Geometry
T11	Senior High	4	2	45	4
T12	Senior High	4	4	40	3
T13	Senior High	12	3.5	66	11
T14	Senior High	4	3.5	70	5
T15	Senior High	11	4	120	6
T16	Junior High	10	1	44	4
T17	Junior High	5	1	30	6
T18	Junior High	4	4	45	7
T19	Junior High	17	4	44	10
T20	Junior High	6	4	42	3

forty-five minute periods per day were allowed for classroom instruction.

Secondary students who elected algebra studied coordinate geometry as part of the regular course in Chapter 9, "Graphs of Linear Equations and Inequalities," in their textbook.[46] An average of eighteen fifty-minute periods was devoted to studying the chapter.

The Elementary Unit

A unit was written with the objectives of teaching fifth and sixth grade pupils to graph linear equations and to discover the slope and intercept relations betweeen graphs and open sentences. This unit was written in the form of lesson plans for teachers and employed the discovery approach.[47] Each lesson plan included student objectives in behavioral form, an overview for teachers (explaining the mathematics necessary for each lesson and indicating its present and future importance), suggested procedures, activities for pupils, and a set of exercises for each pupil. The objectives of the ten lessons are listed below:

Lesson 1. Pupils will learn to plot ordered pairs representing points in all four quadrants of the plane.

---

[46] Johnson, loc. cit.

[47] The complete set of lesson plans for teachers is found in Appendix A.

Lesson 2. Pupils will list some elements of the truth set of an open sentence with two variables by two methods; (a) by listing ordered pairs as elements of a truth set, and (b) by constructing a table.

Lesson 3. (a) Pupils will plot the graph of the truth set of an open sentence with two placeholders. (b) Pupils will recognize that the graph of an open sentence of the form $\Delta = \square + K$ is a straight line.

Lesson 4. Pupils will discover the pattern of the graph of an open sentence.

Lesson 5. (a) Pupils will discover the relation of the geometrical pattern of the graph and the open sentence. (b) Pupils will identify the intersection of the graph and the Δ axis from the open sentence.

Lesson 6. Pupils will discover the open sentence when the graph of the truth set is displayed.

Lesson 7. Pupils will compute sums of signed numbers.

Lesson 8. Pupils will compute products of signed numbers.

Lesson 9. Pupils will discover the pattern of the graph of an equation in which the Δ coordinate decreases when the $\square$ coordinate increases.

Lesson 10. Pupils will plot the graph of an equation as a continuous line instead of a set of discrete points.

Pedagogical approach.--Content of the coordinate geometry unit for elementary school pupils was developed by posing questions and creating challenging situations designed to guide pupil discovery by following the Madison Project's strategy:

1. Begin, if necessary, by recalling key words, experiences, notations, etc., from previous lessons that will be crucial in today's lesson.
2. Do something. Have the child actually carry out some sort of activity or happening.
3. As the occasion arises and as it becomes appropriate, discuss what the class has done.[48]

Pupils were involved in as many multisensory experiences as possible. The most valuable instructional aid was a 2 x 4 foot pegboard used to represent the coordinate plane. Yarn looped around golf tees inserted in the pegboard served as axes which could be moved to accommodate the teaching of particular mathematical concepts. Figure 2 illustrates the pegboard as it was used to represent the quadrants of the coordinate plane.

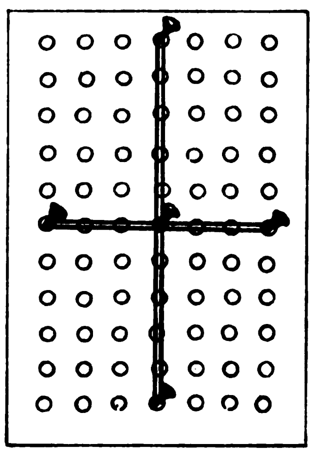


Figure 2.--Pegboard Divided Into Quadrants.

---

[48]Davis, Discovery in Mathematics, op. cit., pp. 19-22.

The pegboard afforded a means of involving pupils in many learning activities. They "plotted points" by inserting golf tees into specified holes. They discovered that the graph of the truth set of an open sentence with two variables was a straight line. They described the slopes of these lines by observing the pattern alignment of the golf tees. They saw the difference between a line composed of discrete points (whose domain was the set of integers) and a continuous line (whose domain was the set of real numbers) which was represented by looping yarn around the golf tees. When pupils participated in games, different colored tees conveniently identified teams. Other visual aids included grids for an overhead projector and a latticed chalk board.

The Secondary Unit

The content of one chapter (Graphs of Linear Equations and Inequalities) in the textbook studied by first year algebra students was comparable to that included in the elementary school unit. Table 7 compares the content of this chapter (by section) with that of the ten lesson plans constructed for the elementary classes.

Pilot Classes

The coordinate geometry unit was piloted in two elementary classes prior to use with the experimental population. Through such trials, the scope and sequence

TABLE 7

CORRESPONDENCE OF ELEMENTARY AND SECONDARY UNITS IN COORDINATE GEOMETRY

Topic	Chapter 9, Algebra Text, Secondary School Unit*	Coordinate Geometry, Elementary School Unit
Graphs of Integers on Number Line	Section 1	Lesson 1
Equations with Two Variables	Section 2	Lesson 2
Graphs of Pairs of Integers; The Lattice	Section 3	Lesson 1
Coordinates for Lattice Points	Section 4	Lesson 1
Lattice Graphs of Equations	Section 5	Lesson 3
Rational Numbers and Graphs	Section 6	Lesson 9
Drawing Graphs in the Rational Plane	Section 7	Lesson 3 and 9
Linear Equations	Section 8	Lesson 3, 4, 5
The Intercepts of Lines	Section 9	Lesson 5
Some Unusual Equations	Section 10	Lesson 9
The Slopes of Lines	Section 11	Lesson 5, 6, 9, 10

*Johnson, loc. cit.

could be tested with elementary pupils to assess the pacing of the unit. In addition, children's reaction to and interest in exercises and other materials could be ascertained. Preliminary editions of test items could be evaluated. Also, notes and ideas for elementary teachers could be gleaned from the sessions in pilot classes.

First Pilot Class

The first pilot group was a sixth grade class composed of thirty-one pupils, twenty-one of whom were boys and ten were girls. The school they attended was located in a low-income residential area. The general achievement of the pupils is described in Table 8.

The median percentile achievement in word meaning was 18.5, with a class range of from the fourth to the ninety-ninth percentile. The median percentile achievement for paragraph meaning was the seventeenth percentile with a class range of from the first to eighty-sixth percentile. The median percentile achievement on the arithmetic subtests measuring arithmetic computation, concepts and applications was 17.5, 25.5, and 22.5, respectively. On the basis of these low general achievement data, this class could adequately test the impact of a unit in coordinate geometry for low achievers.

TABLE 8

ACHIEVEMENT OF FIRST PILOT CLASS BY QUARTILES AND READING AND ARITHMETIC SUBTESTS AS MEASURED BY THE STANFORD ACHIEVEMENT TEST, INTERMEDIATE II, BATTERY X

	Percentile for Achievement Subtest				
	Word Meaning	Paragraph Meaning	Arithmetic Computation	Arithmetic Concepts	Arithmetic Applications
Total Range	4-99	1-86	2-66	2-86	2-92
First Quartile	7.5	5.0	3.7	7.9	10.3
Median	18.5	17.2	17.5	25.5	22.5
Third Quartile	54.0	27.5	30.0	39.7	47.5

The class studied the unit on coordinate geometry for 45 minutes each day for 13 consecutive school days. During this period, pupils explored the first five lessons in the unit. On the fourteenth day, a posttest was administered to measure their understanding of unit concepts. A summary of daily lessons is presented in Appendix C.

Revision of material after the first pilot class.--Because this pilot class was comprised of low achievers and the entire unit which had been prepared was not completed, a second pilot class was utilized during the trial period.

Modifications in lessons were made as a result of the experience gained from the first pilot class. Lesson 1, judged too long for an average class to comprehend in one day, was divided into two parts. Part one included leading students to see the necessity for a vertical axis and of two coordinates to locate a point. The importance of an agreement about the order of two points was also included. Part two developed the need for extending the axes and included practice in plotting points in all four quadrants.

Consideration was given to interchanging Lessons 4 and 5 because only one student discovered the relationship between an open sentence and the slope pattern and intercept of the graph. However, the order was maintained to see if the lack of discovery was due to the low mathematics achievement of the students.

Revision of the unit included several new features. A 2 x 4 foot pegboard and golf tees to represent points in the coordinate plane were employed. "For Fun" exercises were added. In these exercises, ordered pairs were listed which when plotted and joined consecutively produced familiar objects or optical illusions.

Second Pilot Class

To provide additional data on the unit selected for testing in this research study, a fourth-fifth grade combination class was selected as the second pilot group. In this class of twenty-two pupils, sixteen were fourth graders and six were fifth graders. The school that these pupils attended was located in a middle-class neighborhood. Table 9 describes the range and percentiles for the second pilot class by quartiles.

These statistics were computed from scores on the Stanford Achievement Test administered to the fourth graders of this class in October, 1968 and to the fifth graders in October of the previous year. The median percentile for word meaning was 30.5 and for paragraph meaning, 28.5. The median percentiles on the arithmetic subtests were 18.5, 50.7, and 42 for arithmetic computation, concepts, and applications, respectively.

TABLE 9

ACHIEVEMENT OF SECOND PILOT CLASS BY QUARTILES AND READING AND ARITHMETIC SUBTESTS AS MEASURED BY THE STANFORD ACHIEVEMENT TEST, INTERMEDIATE II, BATTERY X

	Percentile for Achievement Subtests				
	Word Meaning	Paragraph Meaning	Arithmetic Computation	Arithmetic Concepts	Arithmetic Applications
Total Range	1-96	4-96	1-50	10-98	10-96
First Quartile	9.5	6.0	8.0	26.5	25.2
Median	30.5	28.5	18.5	50.7	42.0
Third Quartile	84.5	66.5	27.5	73.5	63.5

Comparison of the scores of the two classes indicated that the second pilot class achieved at a higher level than the first.

The class studied the coordinate geometry unit for 40 minutes each day for twenty days while an additional three days were reserved for testing. During this period, the class completed all ten lessons and the accompanying exercises (Appendix B) in the unit.

An observer who attended the second pilot class tape-recorded the sessions and offered valuable criticism. The taped lessons, and his constructive suggestions were beneficial in teaching that particular class, revising the lesson plans, and preparing the workshop for teachers.

Elementary Teacher In-Service Education

Recruiting and Selecting Teachers

Announcements of the six-week workshop in coordinate geometry were sent to each elementary school where notices were posted on the teachers' bulletin board. The notice also stipulated that teachers who completed the workshop could participate in a research project that would culminate in a dissertation. Principals were asked to encourage fifth and sixth grade teachers to enroll. Of the nine teachers who registered, seven completed the workshop. One of these teachers was involved in a team teaching situation where mathematics was taught by the other member of the team, thus only six participants taught the material. The standard cost for an in-service workshop, $2.00, was paid by the teacher. One local school district professional growth unit was accredited to each teacher who completed the in-service training.

The Workshop Plan

Six two-hour sessions were planned which encompassed the content of ten pupil lessons, outlined in the previous section and detailed in Appendix A. The mode of instruction paralleled that utilized with the pilot classes. This method was chosen by the teachers who wished to experience the unit from their pupils' viewpoints. The choice met with the approval of the researcher, the instructor, who was afforded the opportunity to lead "teacher discovery" and to suggest pedagogical techniques.

Lesson plans were prepared, but were not distributed until teachers had experienced the lesson in the workshop. The first three sessions consisted solely of content and approaches to its development. After the third session, plans for lessons 1 and 2 were distributed to the teachers who began teaching the unit the following week. The remaining lesson plans were distributed at subsequent sessions.

Part of each of the last three sessions was devoted to a discussion of the teachers' experiences with their classes during the week. Teachers exchanged information about their problems, progress, successes and pupil reaction to the materials. The other part of these sessions continued with the explanation of content. For broadening teachers' backgrounds, concepts were extended beyond the content required of pupils. For example, the relationship of slopes of parallel and perpendicular lines, use of different visual aids, and other uses for the pegboard and golf tees, such as for finding perimeters and areas of polygons were discussed.

Description of Content and Presentation Mode

Session 1.--Nine teachers attended. At the beginning of the session teachers were told that the workshop had two purposes: (1) to introduce the content of a unit in coordinate geometry, new to the elementary program, using concrete materials and to suggest techniques for instruction in an activity-centered, classroom setting, and (2) to determine the feasibility of teaching selected topics in coordinate geometry to fifth and sixth graders. All of the teachers agreed to teach the material to their pupils and granted the instructor permission to test their classes and use the data for a research project. During the week, one teacher dropped the course because he did not have adequate time and another teacher dropped because she had already taught similar material in her class. The content of student Lessons 1 and 2 was presented. Points were located on the horizontal number line, already familiar to teachers, and the need for a vertical number line was developed by placing a point above the number line. A series of leading questions for teachers to use in stimulating children to suggest the introduction of a vertical number line was offered. After the need for intersecting perpendicular lines was established, three golf tees were inserted into holes at the left top and both bottom corners of a 2 x 4 foot pegboard and yarn was wrapped around these golf tees to form a pair of axes perpendicular to each other. These axes, commonly called the x and y axis, were named the box (□) and triangle (Δ) axis respectively, using the Madison Project notation. Having demonstrated that the location of a point required two coordinates to designate its distance from the vertical (y) axis and its distance from the horizontal (x) axis, it was then illustrated that the location of a specific point depends upon the order in which these distances were stated. Teachers plotted points by inserting golf tees into the holes of the pegboard. Only points in the first quadrant were used until all teachers had grasped the concept of naming the coordinates in the proper order. Since points lie to the left and below these axes, the tees were placed so that the pegboard was divided into four quadrants. Yarn wrapped around the tees again formed the axes. Since teachers were familiar with negative numbers, locating points in quadrants II, III and IV followed readily.

The session closed with a discussion of mathematical sentences. Teachers gave examples of true, false, and open sentences.

Session 2.--The first session was reviewed by having teachers complete the exercises included in student lessons 1 and 2. Utilizing the pegboard, teachers plotted, with golf tees, sets of points listed in one of the pupil "for fun" exercises. The union of these consecutive "points," when joined with yarn, formed the image of a kangaroo.

Mathematical sentences were discussed. True, false and open mathematical sentences were illustrated. Open sentences were written in the form, $\Delta = \square + K$, where K was limited to 0 or a positive integer. The necessity for consistency in naming the order of coordinates was stressed with the agreement to first name $\square$, the horizontal distance, followed by Δ, the vertical distance.

Since mathematicians are most interested in values that make an open sentence true, concise methods for displaying the truth sets were desirable. Teachers were asked to suggest convenient methods for organizing pairs of coordinates that make a specific sentence true. The aim was to elicit the following suggestions: (1) the use of set notation, (2) the use of a table, and (3) the construction of a graph. Various teachers were asked to choose an open sentence and use the overhead projector to display its truth set by the three methods listed above. They noticed that the graph of the points was a straight line.

Teachers were eager to begin the unit with their pupils. One teacher had already bought a pegboard. All planned to finish the topics which their classes were studying so that they could begin the coordinate geometry unit after the next session.

Session 3.--Student lesson plans 1 and 2 and a set of exercises for each student were distributed to teachers. Content introduced in the two previous workshops was reviewed by discussing these lesson plans and the procedures suggested for their implementation. Concepts to be emphasized and some possible pitfalls, based on the instructor's experiences in pilot classes, were enumerated and demonstrated by playing portions of the tape of lessons recorded in the second pilot class. Lesson 1 was divided into two parts with the plotting of points in the first quadrant concluding part 1.

Teachers were taught to play a modified version of the Madison Project's game, tic-tac-toe, which when played by pupils served to strengthen, in an entertaining fashion, their ability to plot points.

An equation from pupil exercise 3 was plotted on the pegboard. Teachers were asked to add points to this line without doing any arithmetic. When a point was added and accepted by the class, its coordinates were named and substituted into the open sentence to verify that the ordered pair belonged to the truth set. As they graphed other equations on the pegboard, they quickly perceived the pattern of each. Informed that the pattern of a graph could be determined without plotting a single point, teachers were asked to do student exercise 4 to see if they could detect the clue. Five of them comprehended that the multiplier of □ revealed the slope pattern. Pupil exercise 5 was distributed with the encouragement that it contained stronger hints. These hints proved to be sufficiently plain so that upon completion of the exercise all teachers had discovered the relation between the number sentence and slope as well as the relation between the sentence and the Δ intercept.

All of the teachers planned to begin teaching the unit the following week.

Session 4.--This session began with teachers voluntarily discussing the experiences of their classes. All reported that their pupils were highly receptive of the material and that pupils especially enjoyed the "For Fun" exercises. In particular, they liked the picture of Snoopy. All teachers had completed Lesson 1 in their classes. One teacher's pupils had spent some time playing tic-tac-toe.

The topic which had terminated the previous in-service education session, the relation between the equation and the slope and Δ intercept of its graph, was continued. Teachers saw that the intercept and slope provided sufficient information to graph the sentence. The process was then reversed. Golf tees were inserted in the pegboard and teachers named the equation so represented.

Lesson plans 4, 5, and 6 with pupil exercises were distributed.

Session 5.--The discussion of lesson plans 4, 5, and 6 afforded the opportunity to review their content. Some teachers felt that pupils would need more exercises, especially for lesson 6. Others felt that the set of exercises was sufficient if, in addition to this set, they used the pegboard or overhead projector to represent graphs and let the pupils write or state their equations.

Most classes were finishing Lesson 3. Teachers reported that pupil interest continued high. They claimed that pupils wanted to play tic-tac-toe longer than their plans allowed. The most frequent problem reported was the tendency of students to name a point on one axis with only one coordinate. For example, they said some children were inclined to call (0,1) just 1, or (2,0) just 2, forgetting to name the 0 coordinate. In answer to one teacher's question about the slopes of the axes, the slopes of vertical and horizontal lines in general, which included the axes, were discussed. Teachers were asked to plot pairs of parallel lines and investigate the relation of the slopes of these lines. They also plotted perpendicular lines and "discovered" the relationship of their slopes.

In preparation for plotting lines with negative slopes, the Madison Project Postman Stories were introduced. In these stories the postman brings envelopes containing checks or bills. Considering his daily mail delivery as the only monetary transactions, pupils determine the family's immediate financial state which requires addition or subtraction of signed numbers.

Session 6.--In the discussion of the week's classes, three teachers felt that the content required "too much discovery" by children. Two teachers interchanged Lessons 4 and 5 for this reason. Students could see the pattern of a graph, they reported, but very few were able to relate the pattern and the open sentence. Nonetheless, student interest was still high.

After reviewing the "rules of the game" for Postman Stories that required adding signed numbers, Postman Stories that required multiplying signed numbers were created. Portions of the tape of Lessons 7 and 8 of the second pilot class were played so that teachers could hear typical questions and responses. Teachers examined the effect of negative multipliers of □ on the graphs of open sentences.

Rational numbers were then used as replacements for □. Ordered pairs were plotted on a grid projected on a screen to demonstrate that the graph of a linear equation is a continuous line if values selected for box were closer and closer together. To show that a line was continuous when discrete points were plotted on the peg-board, yarn was looped around the golf tees. This topic completed the content of the ten lessons to be presented to pupils.

Session 7.--Teachers reported losing student interest with the computation of the postman stories. Others related humorous stories told by their children. Arrangements were made with each teacher to administer the test on the unit of coordinate geometry within two weeks.

The Test Instruments

Two instruments were employed in the study: The Stanford Achievement Test and a Test on Coordinate Geometry (TOCG). To test Hypotheses F, G, and H, the sixth grade sample was divided into two groups, high and low, on the basis of their scores on the arithmetic concepts subtest of the Stanford Achievement Test. The

Test on Coordinate Geometry provided data for the dependent variables in the study (total score and seven subtests).

The Stanford Achievement Test

The Stanford Achievement Test is a series of comprehensive tests developed to measure the knowledges, skills, and understandings commonly accepted as goals of the major branches of the elementary school curriculum. First published in 1923, the test was revised in 1929, 1940, 1953, and 1964. To provide normative data descriptive of the current achievement of the nation's schools, more than 850,000 students from 264 school systems drawn from the 50 states were tested.

The Stanford Achievement Test measures reading, language, spelling, social studies, science and mathematics. The mathematics section consists of three subtests: arithmetic computation, arithmetic concepts, and arithmetic applications. The concepts subtest was chosen for dichotomizing the sixth grade into two groups because its measure of understanding was more relevant to the unit in coordinate geometry which did not require extensive computation or the application of arithmetic to practical problems. The reliability of the arithmetic concepts subtest obtained by the odd-even split-halves method for grade six was .85. Content validity was established by comparison of the test's content with the curriculum of the school.

The Test on Coordinate Geometry

To test the major hypotheses of the study, a reliable instrument with parallel forms valid for both the elementary and secondary content was required. A search for such an instrument was unsuccessful. Nor could a portion of any existing instrument be located which met the specifications of the research design; therefore, a suitable test was developed. In its final form, the instrument consisted of forty items selected from four preliminary forms. The pilot groups of elementary classes were used to test the preliminary drafts of the instrument.

Test administered to the first pilot class.--A preliminary instrument covering the content of Lessons 1 through 5 was administered to the first pilot class. Different types of objective questions were included to help determine the most desirable format for the instrument. These types included: (a) the plotting of points, (b) the completing of a table, (c) the graphing of an equation, and (d) responding to multiple-choice questions.

This test was scored by the researcher. Several types of items were eliminated from future revisions of the instrument because of subjective decisions involved in scoring. For example, in several instances, pupils located points in such a way that the response could be deemed either correct or incorrect. Legibility of numerals was another problem. After review of the administration and scoring problems, multiple-choice items seemed the most appropriate for future editions.

Tests administered to the second pilot class.-- Three instruments were administered to the second pilot class: Test I, consisting of 25 items, was administered at the end of Lesson 2; Test II, 20 items, was administered at the end of Lesson 6; and Test III, 25 items, at the end of Lesson 10.

Pupils marked their answers to all instruments on machine-scored 5-choice response sheets. Because fourth and fifth graders had limited experience with this means of response detailed instructions for marking the answer sheet were given. That the answers were numbered horizontally was emphasized. The rows of blanks for the answers were so close together that each pupil was given a sheet of paper to lay along a row in order to help him keep his place on the response sheet. Tests were scored by the Michigan State University Office of Evaluation Services.

The Data Processing Center of the University then computed an item analysis of each of these preliminary instrument drafts. The basic item statistics derived from this analysis included the index of difficulty and the index of discrimination. The index of difficulty is the proportion of the total group who answered each item incorrectly. The index of discrimination is the difference between the proportion of the upper and lower groups who answered each item correctly. Optimal discrimination was obtained by including 27 per cent of the total group in each of the upper and lower sub-groups.[49]

Choice of items for the instrument.--Forty items were selected from the three tests that were administered to the second pilot class. Several factors guided the selection of each item. Foremost, the question had to test at least one of the objectives of instruction. The indices of difficulty and discrimination were considered with the realization that these statistics may be unstable because they were computed from a small group. The number of items selected which focused on one concept was a function of the teaching time devoted to that concept. Table 10 lists the index of difficulty and the index of discrimination of the forty items selected.

Two items with negative discriminations were retained in the instrument. A correct response to these questions required pupils to extend and apply concepts they had learned to situations which had not been discussed in class. These items were retained because it was anticipated that correct solutions would be given by secondary students, but not by elementary pupils. Each of the questions presented a different challenge. Because spacial concepts were involved in Item 11 (shown on the following page), it was replaced by another item.

---

[49] Office of Evaluative Services, Item Analysis (East Lansing: Michigan State University, 1965), p. 4.

TABLE 10

INDEX OF DIFFICULTY AND INDEX OF DISCRIMINATION AS COMPUTED FROM SECOND PILOT CLASS FOR THE FORTY ITEMS INCLUDED IN THE TEST ON COORDINATE GEOMETRY

Instrument Item	Index of Difficulty	Index of Discrimination	Instrument Item	Index of Difficulty	Index of Discrimination
1	50	80	21	81	0
2	70	80	22	48	100
3	25	60	23	33	80
4	65	60	24	19	40
5	70	100	25	43	40
6	55	80	26	52	60
7	30	60	27	77	40
8	15	40	28	48	-20
9	25	80	29	50	60
10	5	20	30	36	80
11	80	-20	31	55	40
12	60	100	32	63	60
13	45	60	33	85	0
14	80	-20	34	80	60
15	43	60	35	90	20
16	40	40	36	90	20
17	new	new	37	50	0
18	10	20	38	65	60
19	39	80	39	80	60
20	67	20	40	70	20

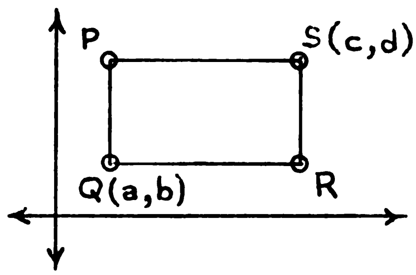
11. The location of a point in space requires

    A. 2 coordinates
    B. 3 coordinates
    C. 1 coordinate
    D. None of these

Item 14, listed below, was included to determine if the student could extrapolate from the specific instances to which he had been exposed in class to the more general case.

14. In the figure at the right, the coordinates of P are

    A. (a,d)
    B. (a,c)
    C. (c,a)
    D. (d,a)
    E. None of these



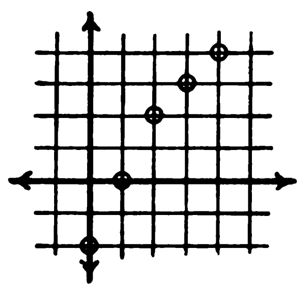
Item 28 required close scrutiny by the student of each point represented on the graph.

28. Does the graph at the right [below] represent the open sentence

    $\Delta = 2 \times \square - 2$ (elementary form)

    $y = 2x - 2$ (secondary form)

    A. Yes
    B. No



The instrument used in this study is found in Appendix D.

<u>Notation of variables employed in the instrument</u>.--Although elementary and secondary classes studied the same content, they used different notations for the axes and variables. Elementary school pupils used the □ - Δ notation of the Madison Project, while secondary school students used the conventional Cartesian x-y notation.

To determine if the different notations influenced the instrument, both forms were administered to two non-experimental tenth grade classes two days apart following their study of the chapter on linear equations and their graphs. A cover sheet on the test explained the use and definitions of the □ and Δ symbols. The means and variances of the results are reported in Table 11.

TABLE 11

MEANS AND STANDARD DEVIATIONS ON BOTH FORMS OF THE TEST ON COORDINATE GEOMETRY ADMINISTERED TO NON-EXPERIMENTAL TENTH GRADERS TWO DAYS APART

Form	$\bar{x}$	σ
□ - Δ Test	24.55	6.56
x - y Test	25.71	7.59

A t-test between these two means did not show any significant difference. The correlation between the two tests was .79. This correlation was similar to that expected in test-retest situations, and within expected tolerances for parallel tests measuring the same concept. Thus, the conclusion was drawn that the use of x - y and Δ - □ notation would not adversely affect study results.

Reliability of the instrument.--The reliability for both forms of the test taken by the two non-experimental classes was computed by the Kuder-Richardson Reliability #20. The reliability of the Δ - □ Test was .80. The reliability of the x - y Test was .81.

Validity of the instrument.--A professor of mathematics at Michigan State University examined the test for face validity vis à vis the unit in coordinate geometry which had been prepared for elementary teachers to use in their classes. A junior and a senior high school teacher examined the content of the test relevant to the content in the first year algebra textbook and judged that it tested the concepts included in Chapter 9, Graphs of Linear Equations and Inequalities.[50]

Subtests.--Seven subtests, determined by content groupings were embedded in the instrument. These categories and the relevant test items are listed below.

---

[50]Johnson, loc. cit.

Subtest 1. Plotting and Recognizing Points in the Coordinate Plane

Items 1, 2, 3, 4, 6, 7, 8, 9, 10, 12, 26

Subtest 2. Recognizing Members of a Truth Set

Items 5, 16, 17, 20, 34

Subtest 3. Intercept Relation to Open Sentence or Graph

Items 15, 19, 23, 25, 30, 40

Subtest 4. Slope-Graph Relation

Items 18, 24, 37, 39

Subtest 5. Operations with Signed Numbers

Items 11, 29, 31, 36

Subtest 6. Graph-Open Sentence Relation

Items 21, 22, 28, 32, 33, 35

Subtest 7. Extension of Concepts

Items 13, 14, 27, 38

Procedures of Testing

The test on the unit in coordinate geometry was administered to four junior high school classes (ninth grade) and two senior high school classes (tenth grade) on the same day. For uniformity of administration, the same individual administered all the tests. The following day, the test was administered to the remaining secondary classes. These tests were scheduled in the beginning of March, 1969, after the completion of the chapter on coordinate geometry.

The test on the coordinate geometry unit was administered to the elementary classes within a two-day period during the third week in March, 1969, following the experimental treatment period in which their teachers guided their learning the concepts of coordinate geometry. All tests were administered by the same individual to eliminate differences due to testing procedures. Since the instrument was designed as a power rather than a timed test, no time limits were set for completion; however, the typical time for completion was approximately 45 minutes, with a range from 35 to 55 minutes.

Analysis of Data

A three-way analysis of variance[51] was used to test the hypotheses of the study. The .05 level of significance was accepted as being sufficiently rigorous for the purposes of this study. Thus, the probability that any differences which were found were real rather than spurious was 95 out of 100 cases. Where significant differences occurred, Scheffé's post hoc comparison[52] was used to test the specific hypotheses involved.

---

[51]William Hays, Statistics for Psychologists (New York: Holt, Rhinehart, and Winston Publishing Co., 1963), p. 484.

[52]S. W. Greenhouse, "On Methods in the Analysis of Profile Data," Psychometrika, XXIV (June, 1959), 94-112.

CHAPTER IV

THE ANALYSIS OF THE DATA

Testing of the Hypotheses

The purpose of this study was to examine and evaluate the achievement of upper elementary and secondary school students on a comparable unit in coordinate geometry. The topic was traditionally studied in first year algebra, but had recently been introduced to elementary school pupils. To examine the feasibility of introducing this topic three to four years earlier than was customary, eight hypotheses were examined to determine differences in performance between elementary and secondary students. Achievement in coordinate geometry was further analyzed by seven subtests to determine if elementary school pupils could understand all or a part of the specific concepts embedded in the coordinate geometry unit as well as secondary school students.

Another facet, the degree of difficulty for individual items by subtest, was examined to determine if topics were appropriately placed. Finally, the results of a student questionnaire reflecting their interest and preferences for this material as opposed to the usual topics was analyzed.

The following eight hypotheses were tested:

Hypothesis A. There is no difference in achievement on a unit in coordinate geometry between fifth graders and ninth graders.

Hypothesis B. There is no difference in achievement on a unit in coordinate geometry between fifth graders and tenth graders.

Hypothesis C. There is no difference in achievement on a unit in coordinate geometry between sixth graders and ninth graders.

Hypothesis D. There is no difference in achievement on a unit in coordinate geometry between sixth graders and tenth graders.

Hypothesis E. There is no difference in achievement on a unit in coordinate geometry between fifth and sixth graders combined and ninth and tenth graders combined.

Hypothesis F. There is no difference in achievement on a unit in coordinate geometry between the upper half of the sixth graders as measured by a general mathematics achievement test and ninth graders.

Hypothesis G. There is no difference in achievement on a unit in coordinate geometry between the lower half of the sixth graders as measured by a general mathematics achievement test and tenth graders.

Hypothesis H. There is no difference in achievement on a unit in coordinate geometry between the upper half of the sixth graders as measured by a general mathematics achievement test and ninth and tenth graders combined.

These hypotheses and achievements by subtest were tested using analysis of variance and Scheffé's post hoc test. Table 12 summarizes the mean scores for the coordinate geometry test by subtests for grades 5, 6 High, 6 Low, 9 and 10. Table 13 reports the results of the analysis of variance of the data presented in Table 12.

TABLE 12

MEAN SCORES BY SUBTEST ON THE TEST ON COORDINATE GEOMETRY FOR STUDENTS IN GRADES 5, 6, 9 and 10

Subtest	Grade						
	5	6			9	10	Total
		High	Low	Total			
1. Plotting and Recognizing Points in the Coordinate Plane	6.65	7.99	6.23	7.11	8.27	7.53	7.45
2. Recognizing Members of a Truth Set	2.31	3.53	2.54	3.04	3.03	2.52	2.78
3. Intercept Relation to Open Sentence or Graph	2.72	3.81	2.21	3.01	3.39	2.63	2.96
4. Slope-Graph Relation	1.62	2.04	1.86	1.95	2.02	1.21	1.72
5. Operations with Signed Numbers	1.38	2.11	1.43	1.77	2.63	2.15	2.04
6. Graph-Open Sentence Relation	2.62	3.26	2.82	3.04	3.30	2.61	2.93
7. Extension of Concepts	1.19	1.98	1.10	1.54	2.09	1.51	1.62
TOTAL	18.49	24.72	18.19	21.46	24.73	20.16	21.50

TABLE 13

ANALYSIS OF VARIANCE SUMMARY

Source	df	Sum of Squares	Mean Square	F
Grade	4	21.7437	5.4359	5.092*
Class/Grade	14	14.9445	1.0675	
Subtest	6	461.6874	76.9479	
Subtest/Grade	24	8.7659	.3652	1.914
Subtest/Classes/Grade	84	16.0242	.1908	
TOTAL	132	523.1657	39.6333	

*Significant at the .05 level of confidence.

Table 12 indicates that the mean scores of the ninth graders exceeded the mean scores of fifth, low sixth, and tenth graders on all seven subtests. The mean scores of the ninth graders exceeded the high sixth on Subtests I, V, VI, and VII, and were inferior to the high sixth graders on Subtests II, III, and IV.

Results were significant at the .05 level of confidence in one of the main effects, Grade. Differences between subtests and differences among classes, the other two main effects, were not a primary concern in this study. Subtest-grade interaction was not significant.

To determine the source of significant differences among grade levels, each null hypothesis (H_o) was tested by the Scheffé confidence interval:[53]

$$C\ \{\hat{\psi} - \sqrt{(J-1)(F)(MS_E)(\Sigma\frac{c_i^{\,2}}{n_j})} \leq \psi \leq \hat{\psi}$$

$$+ \sqrt{(J-1)(F)(MS_E)(\frac{c_i^{\,2}}{n_j})}\} \geq 1 - \alpha$$

Where

J, the number of groups, equals 5

F, the critical value with 4 and 14 degrees of freedom, equals 3.11

MS_E, the mean square error, equals 1.0675, and

c_i is the weight given the ith group (grade)

n_j is the number of classes in the jth group (grade)

α, the level of confidence, equals .05

If the above interval spans zero, the test fails to reject H_o. If the above interval does not span zero, the test rejects H_o.

---

[53]Hays, loc. cit.

Hypothesis A. There is no difference in achievement on a unit in coordinate geometry between fifth graders and ninth graders.

Symbolically, H_o: $\psi = \mu_{G5} - \mu_{G9} = 0$

Estimated by $\hat{\psi} = \bar{X}_{G5} - \bar{X}_{G9}$

For this case: $\hat{\psi} = 18.49 - 24.73$

$$\hat{\psi} = -.624$$

When tested by the Scheffé confidence interval, where

$$\Sigma \left(\frac{c_i^2}{n_j}\right) = \frac{1}{3}^2 + \frac{1}{5}^2$$

$$= \frac{8}{15}$$

$$C\{-6.24 - \sqrt{4\ (3.11)(1.0675)(8/15)} \leq \psi \leq -6.24 + \sqrt{4\ (3.11)(1.0675)(8/15)}\} \geq 1 - .05$$

$$C\{-6.24 - 2.66 \leq \psi \leq -6.24 + 2.66\} \geq .95$$

$$C\{-8.9 \leq \psi \leq -3.58\} \geq .95$$

$\therefore$ reject the null hypothesis

Since the interval does not span zero, the null hypothesis was rejected. A significant difference in achievement on the unit in coordinate geometry did exist between fifth graders and ninth graders, favoring ninth graders. This result was consistent since the fifth grade sample included all fifth graders, whereas the ninth grade sample represented approximately the upper third of all ninth graders. They had been screened to study algebra on the basis of the overall Stanford Achievement Score (6th stanine or above), and the average of their seventh and eighth grades mathematics marks (A or B). Consequently, fifth graders representing the entire range of achievement could not be expected to equal ninth graders who not only represented the upper third of their class in mathematics achievement but had also completed four additional years of mathematics.

Hypothesis B. There is no difference in achievement on a unit in coordinate geometry between fifth graders and tenth graders.

Symbolically, H_o: $\psi = \mu_{G5} - \mu_{G10} = 0$

Estimated by $\hat{\psi} = \bar{X}_{G5} - \bar{X}_{G10}$

For this case: $\hat{\psi} = 18.49 - 20.16$

$$\hat{\psi} = -1.67$$

When tested by the Scheffé confidence interval, where

$$\Sigma \left(\frac{c_i^2}{n_j}\right) = \frac{1^2}{3} + \frac{1^2}{5}$$

$$= \frac{8}{15}$$

$$C\{-1.67 - \sqrt{4\ (3.11)(1.0675)(8/15)} \le \psi \le -1.67$$

$$+ \sqrt{4\ (3.11)(1.0675)(8/15)}\} \ge 1 - .05$$

$$C\{-1.67 - 2.66 \le \psi \le -1.67 + 2.66\} \ge .95$$

$$C\{-4.33 \le \psi \le .99\} \ge .95$$

$\therefore$ fail to reject the null hypothesis

This interval spans zero. The test therefore failed to reject the null hypothesis. Even though the elementary sample included all fifth graders, the secondary sample included only the second third (between the thirty-third and sixty-sixth percentiles) of the tenth grade population (the more able tenth grade students had studied algebra a year earlier). In contrast, fifth graders were challenged and motivated by the unit in coordinate geometry which was different from the content of their regular arithmetic book.

Hypothesis C: There is no difference in achievement on a unit in coordinate geometry between sixth graders and ninth graders.

Symbolically, H_o: $\psi = \mu_{G6} - \mu_{G9} = 0$

Estimated by $\hat{\psi} = \bar{X}_{G6} - \bar{X}_{G9}$

For this case: $\hat{\psi} = 21.46 - 24.73$

$$\hat{\psi} = -\ 3.27$$

When tested by the Scheffé confidence interval, where

$$\Sigma \left(\frac{c_i^{\ 2}}{n_j}\right) = \frac{1}{3}^2 + \frac{1}{5}^2$$

$$= \frac{8}{15}$$

$$C\ \{-3.27 - \sqrt{4\ (3.11)(1.0675)(8/15)} \leq \psi \leq -\ 3.27$$

$$+ \sqrt{4\ (3.11(1.0675)(8/15)}\} \geq 1 - .05$$

$$C\{-\ 3.27 - 2.26 \leq \psi \leq -\ 3.27 + 2.26\} \geq .95$$

$$C\{-\ 6.53 \leq \psi \nleq -\ 1.01\} \geq .95$$

$\therefore$ reject the null hypothesis

This interval did not span zero and the hypothesis that there was no difference in achievement on the unit in coordinate geometry between sixth graders and ninth graders was rejected. Ninth grade students achieved at a higher level than sixth grade students in the unit on coordinate geometry. The rationale pertinent to Hypothesis A applies to this hypothesis.

Hypothesis D. There is no difference in achievement on a unit in coordinate geometry between sixth graders and tenth graders.

Symbolically, H_o: $\psi = \mu_{G6} - \mu_{G10} = 0$

Estimated by $\hat{\psi} = \overline{X}_{G6} - \overline{X}_{G10}$

For this case: $\hat{\psi} = 21.46 - 20.16$

$\hat{\psi} = 1.30$

When tested by the Scheffé confidence interval, where

$$\Sigma \left(\frac{c_i^2}{n_j}\right) = \frac{1}{3} + \frac{1}{5}$$

$$= \frac{8}{15}$$

$$C \{-1.30 - \sqrt{4\ (3.11)(1.0675)(8/15)} \le \psi \le - 1.30$$

$$+ \sqrt{4\ (3.11)(1.0675)(8/15)}\} \ge 1 - .05$$

$$C -\{1.30 - 2.66 \le \psi \le - 1.30 + 2.66\} \ge .95$$

$$C\{- 3.96 \le \psi \le 1.36\} \ge .95$$

$\therefore$ fail to reject the null hypothesis

The Scheffé confidence interval failed to reject the null hypothesis. Sixth graders achieved as well as tenth grade algebra students on the coordinate geometry unit.

Hypothesis E. There is no difference in achievement on a unit in coordinate geometry between fifth and sixth graders combined and ninth and tenth graders combined.

Symbolically, H_o: $\psi = \mu_{G5} + \mu_{G6} - \mu_{G9} - \mu_{G10} = 0$

Estimated by $\hat{\psi} = \overline{X}_{G5} + \overline{X}_{G6} - \overline{X}_{G9} - \overline{X}_{G10}$

For this case: $\hat{\psi} = 18.49 + 21.46 - 24.73 - 20.16$

$$\hat{\psi} = -3.94$$

The difference between these means was tested by Scheffé confidence interval, where

$$\Sigma\left(\frac{c_i^2}{n_j}\right) = \frac{1}{3}^2 + \frac{1}{3}^2 + \frac{1}{5}^2 + \frac{1}{5}^2$$

$$= \frac{16}{15}$$

$$C\{-3.94 - \sqrt{4\ (3.11)(1.0675)(16/15)} \leq \psi \leq -3.94$$

$$+ \sqrt{4\ (3.11)(1.2675)(16/15)}\} \geq 1 - .05$$

$$C\{-3.94 - 3.76 \leq \psi \leq -3.94 + 3.76\} \geq .95$$

$$C\{-7.70 \leq \psi \nleq -0.18\} \geq .95$$

$\therefore$ reject the null hypothesis

The null hypothesis was rejected at the .05 level of confidence. Secondary students achieved at a higher level than elementary pupils.

Hypothesis F. There is no difference in achievement on a unit in coordinate geometry between the upper half of the sixth graders as measured by a general mathematics achievement test and ninth graders.

Symbolically, H_o: $\psi = \mu_{G6H} - \mu_{G9} = 0$

Estimated by $\hat{\psi} = \overline{X}_{G6H} - \overline{X}_{G9}$

For this case: $\hat{\psi} = 24.72 - 24.73$

$$\hat{\psi} = -.01$$

Tested by the Scheffé confidence interval, where

$$\Sigma\left(\frac{c_i^2}{n_j}\right) = \frac{1}{3}^2 + \frac{1}{5}^2$$

$$= \frac{8}{15}$$

$$C\{-.01 - \sqrt{4\,(3.11)(1.0675)(8/15)} \leq \psi \leq -.01$$

$$+ \sqrt{4\,(3.11)(1.0675)(8/15)}\} \geq 1 - .05$$

$$C\{-.01 - 2.66 \leq \psi \leq -.01 + 2.66\} \geq .95$$

$$C\{-2.67 \leq \psi \leq 2.65\} \geq .95$$

$\therefore$ fail to reject the null hypothesis

The test failed to reject H_o because the interval spanned zero. Any difference in achievement in coordinate geometry between the upper half of the sixth graders and ninth graders can be assumed to be chance occurrences. With respect to achievement, data collected in this study indicated that the upper half of the sixth grade class was as successful as ninth graders in the study of a comparable unit on coordinate geometry.

Hypothesis G. There is no difference in achievement on a unit in coordinate geometry between the lower half of the sixth graders as measured by a general mathematics achievement test and tenth graders.

Symbolically, H_o: $\psi = \mu_{G6L} - \mu_{G10} = 0$

Estimated by: $\hat{\psi} = \bar{X}_{G6L} - \bar{X}_{G10}$

For this case: $\hat{\psi} = 18.193 - 20.162$

$\hat{\psi} = - 1.969$

When tested by the Scheffé confidence interval, where

$$\Sigma \left(\frac{c_i^2}{c_j^2}\right) = \frac{1}{8}^2 + \frac{1}{5}^2$$

$$= \frac{8}{15}$$

$$C\ \{-1.97 - \sqrt{4\ (3.11)(1.0675)(8/15)} \leq \psi \leq -1.97$$

$$+ \sqrt{4\ (3.11)(1.0675)(8/15)}\} \geq 1 - .05$$

$$C\{-1.97 - 2.66 \leq \psi \leq -1.97 + 2.66\} \geq .95$$

$$C\{-4.63 \leq \psi \leq .69\} \geq .95$$

$\therefore$ fail to reject the null hypothesis

The test failed to reject the null hypothesis; the interval spanned zero. The lower half of the sixth graders and the tenth graders did not differ significantly in achievement on a unit in coordinate geometry.

Hypothesis H. There is no difference in achievement on a unit in coordinate geometry between the upper half of the sixth graders as measured by a general mathematics achievement test and ninth and tenth graders combined.

Symbolically, H_o: $\psi = \mu_{G6H} - \frac{1}{2}\mu_{G9} - \frac{1}{2}\mu_{G10} = 0$

Estimated by $\hat{\psi} = \overline{X}_{G6H} - \frac{1}{2}\overline{X}_{G9} - \frac{1}{2}\overline{X}_{G10}$

For this case: $\hat{\psi} = 24.73 - \frac{1}{2}(24.73) - \frac{1}{2}(20.16)$

$$\hat{\psi} = 2.28$$

The difference between the means for the upper half of the sixth grade and the means of the ninth and tenth grades combined was tested by the Scheffé confidence interval, where

$$\Sigma \left(\frac{c_i^2}{n_j}\right) = \frac{1^2}{3} + \frac{\frac{1}{2}^2}{5} + \frac{\frac{1}{2}^2}{5} = \frac{13}{30}$$

$$C \{ 2.28 - \sqrt{4\ (3.11)(1.0675)(13/30)} \leq \psi \leq 2.28$$

$$+ \sqrt{4\ (3.11)(1.0675)(13/30)}\} \geq 1 - .05$$

$$C\{2.28 - 2.40 \leq \psi \leq 2.28 + 2.24\} \geq .95$$

$$C\{- .12 \leq \psi \leq 4.52\} \geq .95$$

$\therefore$ fail to reject the null hypothesis

Because the interval spanned zero, the test failed to reject the null hypothesis. No significant differences in achievement on a coordinate geometry unit existed between the upper half of the sixth grade class and the ninth and tenth grades combined. The upper half of the sixth grade pupils comprehended the concepts involved in the coordinate geometry unit as well as first year algebra students.

Two additional hypotheses were posed and tested to complete the analysis of possible differences between classes. The first:

There is no difference in achievement on a unit in coordinate geometry between fifth graders and sixth graders.

Symbolically, H_o: $\psi = \mu_{G5} - \mu_{G6} = 0$

Estimated by $\hat{\psi} = \bar{X}_{G5} - \bar{X}_{G6}$

For this case: $\hat{\psi} = 18.49 - 21.46$

$$\hat{\psi} = -\ 2.97$$

The difference between the fifth and sixth grade means was tested by the Scheffé confidence interval, where

$$\Sigma\ (\frac{c_i^{\ 2}}{n_j}) = \frac{1}{3}^2 + \frac{1}{3}^2$$

$$= \frac{2}{3}$$

$$C\ \{\ -2.97 - \sqrt{4\ (3.11)(1.0675)(2/3)} \leq \psi \leq -\ 2.97$$

$$+ \sqrt{4\ (3.11)(1.0675)(2/4)}\} \geq 1 - .05$$

$$C\{-\ 2.97 - 2.98 \leq \psi \leq -\ 2.97 + 2.98\} \geq .95$$

$$C\{-\ .95 \leq \psi \leq .01\} \geq .95$$

$\therefore$ fail to reject the null hypothesis

The interval spanned zero and the test failed to reject the null hypothesis.

The final hypothesis tested was:

There is no difference in achievement on a unit in coordinate geometry between ninth graders and tenth graders.

Symbolically, H_o: $\psi = \mu_{G9} - \mu_{G10} = 0$

Estimated by $\hat{\psi} = \bar{X}_{G9} - \bar{X}_{G10}$

For this case: $\hat{\psi} = 24.728 - 20.162$

$\hat{\psi} = 4.566$

When tested by the Scheffé confidence interval, where

$$\Sigma \left(\frac{c_i^2}{n_j}\right) = \frac{1}{5} + \frac{1}{5}$$

$$= \frac{2}{5}$$

$$C\{ 4.566 - \sqrt{4\ (3.11)(1.0675)(2/5)} \leq \psi \leq 4.566$$

$$+ \sqrt{4\ (3.11)(1.0675)(2/5)}\} \geq 1 - .05$$

$$C\{4.566 - 2.304 \leq \psi \leq 4.566 + 2.304\} \geq .95$$

$$C\{2.262 \nleq \psi \leq 6.870\} \geq .95$$

$\therefore$ reject the null hypothesis

This interval did not span zero and the null hypothesis was rejected. Significant differences existed between ninth and tenth graders in achievement in coordinate geometry, favoring ninth graders.

Comparable Difficulty by Subtest and Grade

The analysis of variance indicated no significant interaction between Subtests and Grade. Therefore, it was not necessary to test the difference in means between grades on each subtest for significance. Achievement scores by students at different grade levels verified no variations in mean subtest scores; thus no concepts were more difficult for students at one level than another. The remainder of this section examines the performance of each grade on specific concepts by subtest. Two tables are included for each of the seven TOCG subtests. The first lists the mean score, mean per cent and standard deviation by grade. The index of difficulty by item and grade are presented in the second table. Sixth grade data are presented by the groups 6 High and 6 Low, in order to compare the performance of the upper half of the sixth graders with that of the ninth graders and the performance of the lower half of the sixth graders with that of tenth graders. This comparison is of interest because the upper half of the sixth grade pupils will probably study algebra in ninth grade and the lower half in tenth grade.

The sixth high elementary pupils and the ninth grade algebra students scored the highest on Subtest 1, as indicated in Table 14. The mean score of the sixth-low elementary pupils was the lowest.

TABLE 14

MEAN SCORE AND STANDARD DEVIATION BY GRADES FOR SUBTEST 1: PLOTTING AND RECOGNIZING POINTS IN THE COORDINATE PLANE

	Grade				
	5	6 High	6 Low	9	10
Mean Score	6.65	7.99	6.23	8.27	7.53
Mean Per Cent	60.5	72.7	56.6	73.2	68.3
Standard Deviation	.94	.07	.27	.41	1.16

TABLE 15

INDEX OF DIFFICULTY BY ITEM FOR SUBTEST 1: PLOTTING AND RECOGNIZING POINTS IN THE COORDINATE PLANE

Item Number	Grade				
	5	6 High	6 Low	9	10
1	53	38	46	29	42
2	39	21	19	21	17
3	56	41	51	26	21
4	53	29	46	35	45
6	23	14	38	11	16
7	14	14	29	13	23
8	21	8	26	12	19
9	18	14	29	6	24
10	49	44	61	24	42
12	54	41	62	42	46
26	71	58	71	49	62

In the eleven items of Subtest 1, five items, 6 through 10, asked students to choose the coordinates of representations of points plotted on a graph. Three items, 1, 2, and 12, required the location of points in one of the four quadrants. The remaining items, 3, 4, and 26, related to the order of a pair of coordinates. Table 15 designates the index of difficulty for each item in Subtest 1 for each grade level. The concept, naming the coordinates of points was understood best and the concept related to the order designated by a pair of coordinates, least by students of all grades.

With a mean 70 per cent correct, the sixth-high elementary students scored the highest on Subtest 2. The performance of the fifth, sixth-low, and tenth graders was about the same, but the standard deviation of the sixth-low elementary pupils was the greatest.

All five items of Subtest 2, Recognizing Members of a Truth Set, involved the identification of ordered pairs that made an open sentence true. Data in Table 17 show that this subtest was easiest for the high-sixth graders and most difficult for the fifth graders. The greater mathematical experience of the secondary students did not aid them in choosing the sentence that contained the origin (item 20). Nor did their additional algebraic experience give them as much advantage as anticipated in choosing the correct equation for the table given in item 17, or for the point plotted in item 34.

TABLE 16

MEAN SCORE AND STANDARD DEVIATION BY GRADES FOR SUBTEST 2: RECOGNIZING MEMBERS OF A TRUTH SET

	Grade				
	5	6 High	6 Low	9	10
Mean Score	2.31	3.53	2.54	3.03	2.52
Mean Per Cent	46.00	70.6	50.8	60.6	50.4
Standard Deviation	.45	.34	.76	.40	.59

TABLE 17

INDEX OF DIFFICULTY BY ITEM FOR SUBTEST 2: RECOGNIZING MEMBERS OF A TRUTH SET

Item Number	Grade				
	5	6 High	6 Low	9	10
5	49	15	33	33	42
16	38	13	32	20	34
17	55	39	44	38	57
20	66	56	67	65	68
34	64	34	57	40	58

Both the mean score and the standard deviation for the sixth-low graders on Subtest 3 were the lowest. The largest standard deviation occurred in the tenth grade. The upper half of the sixth graders scored higher than all other grades with a mean of 3.81 or 63.6 per cent.

TABLE 18

MEAN SCORE AND STANDARD DEVIATION BY GRADES FOR SUBTEST 3: INTERCEPT RELATION TO OPEN SENTENCE OR GRAPH

	Grade				
	5	6 High	6 Low	9	10
Mean Score	2.72	3.81	2.21	3.39	2.63
Mean Per Cent	45.4	63.6	36.8	56.4	43.9
Standard Deviation	.45	.45	.30	.39	.59

Two items, 19 and 25, related to the y-intercept and the graph of the open sentence, and item 40 related to the x-intercept and the graph. Previous to this unit or to the comparable chapter in the algebra text, no class had had prior experience with this concept. Tables 18 and 19 show that the upper half of the sixth graders performed best. Both tables indicate the concept was equally difficult for the fifth and tenth graders and hardest for the lower half of the sixth graders.

TABLE 19

INDEX OF DIFFICULTY BY ITEM FOR SUBTEST 3: INTERCEPT RELATION TO OPEN SENTENCE OR GRAPH

Item Number	Grade				
	5	6 High	6 Low	9	10
15	59	40	62	60	71
19	36	18	42	26	42
23	54	36	64	58	72
25	61	44	69	42	53
30	58	43	72	43	57
40	64	47	73	42	51

The mean score of the ninth grade, 2.42 (61 per cent of the total), was the highest on Subtest 4. The mean score of the upper half of the sixth grade, 51 per cent, was second highest for this subtest. The scores of the lower half of the sixth graders were the least dispersed from the mean, and those of the ninth grade were the most dispersed.

Subtest 4 required the student to determine the slope of a line from its graph. Items 18 and 24 represented lines with positive slopes and items 37 and 39 represented lines with negative slopes.

Comparison of the index of difficulty for items 18 and 24 is interesting in that both graphs displayed sentences with the same slope but different domains. For

TABLE 20

MEAN SCORE AND STANDARD DEVIATION BY GRADES
FOR SUBTEST 4: SLOPE-GRAPH RELATION

	Grade				
	5	6 High	6 Low	9	10
Mean Score	1.62	2.04	1.86	2.42	1.21
Mean Per Cent	40.6	50.8	46.6	60.6	30.2
Standard Deviation	.43	.25	.12	.65	.37

TABLE 21

INDEX OF DIFFICULTY BY ITEM FOR SUBTEST 4:
SLOPE-GRAPH RELATION

Item Number	Grade				
	5	6 High	6 Low	9	10
18	39	21	22	38	62
24	65	51	57	51	69
37	51	49	63	52	63
39	81	71	83	59	85

item 18, the domain was the set of integers, thus its graph was a series of discrete points. For item 24, the graph was a continuous line; its domain was the set of real numbers. Slopes of continuous lines were harder for students to find than slopes of lines of discrete points. Items 37 and 39 represented graphs of lines with negative slopes. These two items were relatively harder for all grades than the items (18 and 24) which represented graphs of lines with positive slopes. The slopes of the graphs for items 18, 24, and 37 were rational numbers whose denominators were one (i.e., the change in x was one). The slope of the graph in item 37 was 5/2 (i.e., the change in x was two). This item was the most difficult of the subtest for students of all grades.

On Subtest 5, the performance of the upper half of the sixth grade pupils compared well with that of secondary students who were much more experienced with the concept (operations with signed numbers) tested. Both grades of secondary students had studied a chapter in the regular algebra textbook treating operations with signed numbers, as well as having had extensive experience with signed numbers acquired from the solution of equations. In addition, these students were introduced to signed numbers in the eighth grade textbook and tenth graders had studied the topic again in general mathematics. The means of the elementary students, except the sixth-high graders, were less than 50 per cent; that of secondary students and the sixth-high graders was above 50 per cent.

The greater mathematical experience of secondary students with signed numbers was evident in Subtest 5

TABLE 22

MEAN SCORE AND STANDARD DEVIATION BY GRADES FOR SUBTEST 5: OPERATIONS WITH SIGNED NUMBERS

	Grade				
	5	6 High	6 Low	9	10
Mean Score	1.38	2.11	1.43	2.63	2.15
Mean Per Cent	34.5	52.8	35.8	65.8	53.8
Standard Deviation	.34	.27	.37	.28	.42

TABLE 23

INDEX OF DIFFICULTY BY ITEM FOR SUBTEST 5: OPERATIONS WITH SIGNED NUMBERS

Item Number	Grade				
	5	6 High	6 Low	9	10
11	61	38	50	15	37
29	63	41	80	25	38
31	63	51	60	57	62
36	80	74	80	36	56

in which item 11 required addition of signed numbers, item 29 required multiplication of signed numbers, and items 31 and 36 used both operations. The limited experience of elementary pupils should be considered when appraising their performance on this subtest. From this standpoint, the upper half of the sixth grade pupils performed creditably.

The graph of an open sentence was a new concept for all students. Achievement on Subtest 6 of the sixth-high and the ninth graders was comparable. The performance of the fifth, sixth-low, and tenth graders was comparable. The standard deviation for the upper half of the sixth graders was much greater than for any of the other grades, indicating greater dispersion of scores.

In Subtest 6, items 21 and 33 displayed graphs for which students were required to select the correct open sentence. The graph of the open sentence for item 21, discrete points, was somewhat easier than the graph for item 33, a continuous line. Items 22, 32 and 35 gave an open sentence for which students selected the correct graph. The graph for the open sentence of item 35 was a continuous line and the other two graphs represented discrete points. Item 28 asked students to decide if the graph correctly represented the given open sentence. Table 23 lists the mean scores for this subtest for each group in the study.

Selecting the correct graph for an open sentence (items 22, 32, and 35) was easier for all students than selecting the correct open sentence for a graph (items 21 and 33). In the latter case, except for the lower half of the sixth grade, students found the concept slightly

TABLE 24

MEAN SCORE AND STANDARD DEVIATION BY GRADES FOR SUBTEST 6: GRAPH-OPEN SENTENCE RELATION

	Grade				
	5	6 High	6 Low	9	10
Mean Score	2.62	3.26	2.82	3.30	2.61
Mean Per Cent	43.7	54.3	47.0	55.1	43.4
Standard Deviation	.58	1.15	.48	.59	.71

TABLE 25

INDEX OF DIFFICULTY BY ITEM FOR SUBTEST 6: GRAPH-OPEN SENTENCE RELATION

Item Number	Grade				
	5	6 High	6 Low	9	10
21	64	45	72	49	63
22	57	27	36	41	59
28	40	20	27	33	39
32	53	33	64	33	59
33	73	61	69	55	61
35	53	54	57	56	72

easier for a graph of discrete points than for the graph of a continuous line (items 21 and 35). When the open sentence was given and students asked to select the correct graph, the continuous line was somewhat harder for all students except for the fifth graders for whom both concepts were equally difficult.

The mean of the ninth graders, who were the best performers on Subtest 7, was 2.09, and only 52 per cent of the total possible score. The mean of the upper half of the sixth graders (1.98) was not quite half of the total. The fifth graders had the lowest mean, 1.19 or 29.8 per cent. Table 27, likewise, reflects the low achievement on this subtest. The scores of the upper half of the sixth graders were the most dispersed and those of the lower half of the sixth graders the most homogeneous.

Each item of Subtest 7 extended one of the basic concepts of the unit. Items 13 and 14 related to naming the coordinates of a point, item 27 related to finding the truth set of an open sentence and item 38 explored the relation of the y-intercept and the open sentence.

Item 13 was intended to be the easiest of this subtest. A right triangle, for which the coordinates of the vertices of the acute angles were given, was pictured in quadrant III. Students were asked to choose the coordinates of the vertex of the right angle. The knowledge necessary to making the correct choice was tantamount to naming the coordinates of a point, yet more than 50 per cent of the fifth, sixth-low, and tenth graders missed the question.

TABLE 26

MEAN SCORE AND STANDARD DEVIATION BY GRADES FOR SUBTEST 7: EXTENSION OF CONCEPTS

	Grade				
	5	6 High	6 Low	9	10
Mean Score	1.19	1.97	1.10	2.09	1.52
Mean Per Cent	29.8	49.4	27.5	52.2	37.9
Standard Deviation	.42	.63	.18	.23	.34

TABLE 27

INDEX OF DIFFICULTY BY ITEM FOR SUBTEST 7: EXTENSION OF CONCEPTS

	Grade				
Item Number	5	6 High	6 Low	9	10
13	59	36	62	21	53
14	83	56	85	70	91
27	65	64	72	41	49
38	77	59	69	57	61

Item 14 pictured a rectangle in quadrant I, the coordinates of whose opposite vertices were listed as pairs of letters. Students were asked to select the coordinates of one of the other vertices. The choice required complete understanding of the meaning of ordered pairs of numbers. The anticipated outcome was that secondary students who had a semester's experience operating with variables, would be capable of the abstract thinking required to make the correct choice, but that elementary pupils would not. Instead, the item was easiest for the upper half of the sixth graders and most difficult for tenth graders for whom the number of correct responses may be attributed to chance alone. This item was the most difficult one of the subtest for every class except the upper half of the sixth graders.

Item 27 listed an open sentence with a positive slope. Students were asked to decide if y increased when x increased, decreased when x increased, etc. They could either substitute values for x and determine y, using their knowledge about open sentences, or visualize the graph of the open sentence. This item was easier for secondary students than for elementary pupils. It was the most difficult item of the subtest for the upper half of the sixth grade.

Item 38 listed an open sentence with letters for the coefficient of x and for the constant term instead of numbers. Students were asked to choose the coordinates of the y-intercept. Again, the anticipated outcome was that secondary students having had greater experience with variables and generalization would significantly outscore elementary pupils whose experience with letters as variables was meager. However, more than half of the students of all the grades missed the question. The upper half of the sixth graders performed as well as the ninth and tenth graders. Generalizing concepts appears to be as difficult for secondary students as for elementary pupils according to the results of Subtest 7.

When considering the mean achievement by grades for the complete test, the most outstanding result was that, on the same test, the upper half of the sixth graders emulated the upper third of the ninth graders who represented a more select subset of their class with three more years of mathematical experience. Moreover, these three years included one-half year of algebra.

Comparison of the means of the fifth, tenth, and lower half of the sixth grade may not be so obvious as the comparison of means of the upper half of the sixth grade with the ninth. However, the analysis of variance established that there was no significant difference in achievement between the fifth and tenth grades or between the lower half of the sixth grade and the tenth grade on this test on coordinate geometry.

TABLE 28

MEAN SCORE AND STANDARD DEVIATION BY GRADES FOR TOTAL TEST ON COORDINATE GEOMETRY

	Grade				
	5	6 High	6 Low	9	10
Mean Score	18.49	24.73	18.19	24.73	20.16
Mean Per Cent	46.2	61.8	45.5	61.8	50.4
Standard Deviation	8.76	3.32	3.57	2.46	1.98

Student Reaction to Coordinate Geometry

In order to appraise the reaction of the elementary pupils to the unit in coordinate geometry, two questions were reproduced and distributed with the final test. The questions were:

1. How well did you like the unit in coordinate geometry?

    a. More than mathematics from the regular textbook.

    b. As well as mathematics from the regular textbook.

    c. Less than mathematics from the regular textbook.

2. How interesting did you find coordinate geometry?

    a. More interesting than the mathematics in the regular textbook.

b. As interesting as the mathematics in the regular textbook.

c. Less interesting than the mathematics in the regular textbook.

Students were asked to respond honestly to the questions in order to evaluate the suitability of the materials for elementary pupils. For comparative purposes, secondary students were likewise asked to respond to similar questions with appropriately modified responses:

1. How well did you like the chapter on coordinate geometry?

    a. More than the previous chapters in the algebra textbook.

    b. As much as the previous chapters in the algebra textbook.

    c. Less than the previous chapters in the algebra textbook.

2. How interesting did you find the chapter on coordinate geometry?

    a. More interesting than previous chapters in the textbook.

    b. As interesting as previous chapters in the textbook.

    c. Less interesting than previous chapters in the textbook.

Table 29 presents the results of student response to the question "How well did you like the unit in coordinate geometry?"

The response of elementary pupils to the unit in coordinate geometry was much more favorable than that of secondary students to the comparable chapter in their

TABLE 29

STUDENT REACTION TO COORDINATE GEOMETRY

Reaction	Grade				
	5	6 High	6 Low	9	10
Liked more than regular mathematics	63.6	62.5	64.1	27.4	29.7
Liked as well as regular mathematics	23.4	25.0	20.5	34.1	34.7
Liked less than regular mathematics	13.0	12.5	15.4	38.5	35.6

textbook. While consideration must be given to the subjectivity of this appraisal and to factors which could have influenced opinions, such as teacher approval and the Hawthorne effect, still the results of the reactionaire reflected a highly positive attitude on the part of elementary pupils. About 60 per cent of the elementary pupils indicated they liked the coordinate geometry unit more than the mathematics in their regular textbooks. The fact that the unit differed from their regular mathematics program undoubtedly accounted for much of its appeal to the younger pupils. Less than 30 per cent of the secondary students liked the coordinate geometry chapter more than other chapters in the algebra textbook.

The favorable reaction of the elementary pupils agrees with teachers' opinions about the reception of the unit. They reported at workshop sessions that children were very enthusiastic about the unit and enjoyed it immensely. Moreover, even the sixth-low elementary school pupils participated with interest and learned the concepts. Sixty-four per cent of this group liked coordinate geometry more than their regular mathematics program.

Table 30 presents the results of student response to the question "How interesting did you find coordinate geometry?"

TABLE 30

STUDENT INTEREST IN COORDINATE GEOMETRY

Interest Rating	Percentage by Grade				
	5	6 High	6 Low	9	10
More interesting than regular mathematics	68.8	62.5	59.0	28.9	37.6
As interesting as regular mathematics	22.1	20.0	30.7	33.3	35.6
Less interesting than regular mathematics	9.1	17.5	10.3	37.8	26.7

About 65 per cent of the elementary pupils indicated that the coordinate geometry unit was more interesting than their regular mathematics. Tables 29 and 30 indicated that elementary pupils reacted more positively to the unit in coordinate geometry than did secondary students to the coordinate geometry chapter in their algebra textbook.

The final chapter of this dissertation will summarize the findings of this study, draw conclusions, and consider the educational implications.

CHAPTER V

CONCLUSIONS AND EDUCATIONAL IMPLICATIONS

The present study compared the achievement of elementary and secondary students on a comparable unit in coordinate geometry, a topic new at the elementary school level. A unit was written utilizing the discovery approach and piloted in two elementary school classes. Elementary school teachers who volunteered for a workshop taught by the researcher, introduced the material to their pupils. A test was developed and administered to these pupils upon completion of the unit. The same test was administered to the students of randomly selected first year algebra classes upon completion of the chapter in the first year algebra textbook dealing with the same concepts of coordinate geometry that were encompassed in the elementary school unit. The results were subjected to analysis of variance and Scheffé's post hoc test.

Conclusions

Achievement in Coordinate Geometry

Eight hypotheses were tested. At the .05 level of confidence, statistical tests failed to reject the following five hypotheses:

Hypothesis B. There is no difference in achievement on a unit in coordinate geometry between fifth graders and tenth graders.

Hypothesis D. There is no difference in achievement on a unit in coordinate geometry between sixth graders and tenth graders.

Hypothesis F. There is no difference in achievement on a unit in coordinate geometry between the upper half of the sixth grade students as measured by a general mathematics achievement test and ninth graders.

Hypothesis G. There is no difference in achievement on a unit in coordinate geometry between the lower half of the sixth grade class as measured by a general mathematics achievement test and tenth graders.

Hypothesis H. There is no difference in achievement on a unit in coordinate geometry between the upper half of the sixth grade as measured by a general mathematics achievement test and ninth and tenth graders combined.

At the .05 level of confidence, statistical tests rejected the following three hypotheses:

Hypothesis A. There is no difference in achievement on a unit in coordinate geometry between fifth graders and ninth graders.

Hypothesis C. There is no difference in achievement on a unit in coordinate geometry between sixth graders and ninth graders.

Hypothesis E. There is no difference in achievement on a unit in coordinate geometry between fifth and sixth graders combined and ninth and tenth graders combined.

In these three cases, secondary students achieved at a higher level than elementary school pupils.

Though not a prime concern of the study, the achievement of the fifth graders was compared to the sixth graders and the achievement of the ninth graders was compared with the tenth graders. Statistical tests failed to reject the hypothesis that there is no difference in achievement on a unit in coordinate geometry between fifth and sixth graders. On the other hand, statistical tests rejected the hypothesis that there is no difference in achievement between ninth and tenth graders on a comparable unit in coordinate geometry. Ninth graders achieved at a higher level than did tenth graders, due perhaps to the selection and placement procedures which assigned students with higher achievement to algebra in the ninth grade, while other students completed general mathematics in the ninth grade and algebra in the tenth.

Evidence compiled in this research project appeared to warrant the introduction of coordinate geometry into the upper elementary school mathematics curriculum. The upper half of the sixth graders, who in all probability would study algebra in the ninth grade where coordinate geometry has been taught traditionally, did just as well as ninth graders on the TOCG. In fact, the median for the upper half of the sixth grade was only .01 of a point below the median of the ninth grade. Not only were the upper half of the sixth graders as successful as the ninth graders, but the lower half of the sixth graders, those students who would probably study algebra in the tenth grade, did as well as tenth graders. Even the fifth graders, with still less mathematical experience, did not differ significantly in achievement from the tenth graders. These data indicate that upper elementary school pupils learn that part of coordinate geometry dealing with the graphs of linear equations as well as first year algebra students.

Achievement by Subtest

Achievement in coordinate geometry was further analyzed by concepts. Seven concepts were identified as the component tasks of the coordinate geometry unit. The instrument to measure achievement was constructed so that it could be divided into seven subtests related to the following concepts:

Subtest 1: Plotting and Recognizing Points in the Coordinate Plane

Subtest 2: Recognizing Members of a Truth Set

Subtest 3: Intercept Relation to Open Sentence or Graph

Subtest 4: Slope-Graph Relation

Subtest 5: Operations with Signed Numbers

Subtest 6: Graph-Open Sentence Relation

Subtest 7: Extension of Concepts

In writing the unit and constructing the test, the anticipated outcome was that elementary school pupils would find plotting points in the coordinate plane and recognizing members of a truth set (Subtests 1 and 2) easy, but that they would find such concepts as slope of a graph or the slope-intercept form of an open sentence (Subtests 4 and 6) too difficult. This hypothesis was accompanied by the speculation that all seven concepts would be comprehended more fully by secondary students. However, an analysis of variance established that there was no significant interaction between Subtests and Grade. In other words, elementary school pupils scored as well as secondary students on the Subtests.

Examination of the index of difficulty of individual items by subtest revealed some interesting similarities between elementary and secondary school students. For example, identifying the slope of the graph of the same open sentence was more difficult for both sets of students when the graph was represented by discrete points. Selecting the graph of an open sentence was more difficult for all students than selecting the correct open sentence for a graph. Likewise, graphs with negative slopes and graphs in which the change in x was not one was as hard for secondary school students as for elementary school pupils. However, operations with signed numbers were easier for secondary students who had much more experience with this concept than for the elementary school pupils who had no previous experience operating with signed numbers.

Contrary to expectation, all of the component concepts of the unit in coordinate geometry were understood as readily by elementary school pupils as by secondary school students. Caution should be exercised, however, in generalizing these results because of the number of items in some of the subtests.

Attitude Toward Coordinate Geometry

In responding to a reactionaire, approximately two-thirds of the elementary pupils preferred the unit in coordinate geometry to their regular mathematics program. Only one-third of the algebra students liked coordinate geometry more than algebraic topics. Likewise, a similar proportion of each group rated coordinate geometry more interesting.

Some reservations must be considered when evaluating these responses. Elementary school pupils may have responded as they perceived their teachers might hope that they would. The appeal of studying something different

from their textbooks and the operation of the Hawthorne effect may have influenced their opinions.

Implications of the Study

Implications for the Mathematics Curriculum

Data from this study indicated that the vision of the Cambridge Conference on School Mathematics (CCSM) may not have been unrealistic. In the recent past, geometry had not been viewed as a salient area of study for elementary school pupils. However, in this study, the upper half of the sixth graders performed as well as ninth graders and the lower half of the sixth graders performed as well as the tenth graders on comparable coordinate geometry concepts. Previously cited studies concluded that elementary school pupils were capable of learning other geometric concepts.[54] These studies supported the recommendations of many mathematics educators who favored the inclusion of more geometry in the elementary school curriculum.[55] With the recognition that young children can

---

[54]Fitzgerald, loc. cit.; Sair Ali Shah, loc. cit.

[55]Irvin Brune, "Geometry in the Grades," Enrichment Mathematics for the Grades, Twenty-seventh Yearbook of the National Council of Teachers of Mathematics (Washington, D.C.: National Council of Teachers of Mathematics, 1963), pp. 134-147; Lenore John, "Geometry for Elementary School Teachers," American Mathematics Monthly, LXVII (April, 1967), 374-376; Anne Peters, "Articulating Geometry Between Elementary and Secondary School," National Association of Secondary School Principals Bulletin, XLIII (May, 1957), 131-133.

learn more than had been supposed, such non-metric topics as matrices, probability, and modular arithmetic, which the CCSM proposed for the elementary curriculum and which teachers had questioned as appropriate, may be within the comprehension of elementary school pupils.

Introducing coordinate geometry at the elementary school level may be justified since it relates counting and number, bridges the gap between adding and subtracting and the properties of number, offers first-hand experience with positive and negative numbers and provides an informal introduction to important mathematical concepts as variable, slope, and function. Coordinate geometry also affords an opportunity for students to learn by discovery and to discover patterns. Both of these activities encompass the very essence of "modern mathematics" as this term applied to the K-12 curriculum.

Examination of the achievement of secondary students on the coordinate geometry chapter in the first year algebra textbook as measured by the TOCG showed that the mean of ninth graders, who represented the upper third of the class, was 24.73, or 60 per cent of the possible total. The mean of the tenth graders, who represented the middle third of the tenth grade class, was 20.16, or 50 per cent. These values may be low if the current emphasis on "quality education" also incorporates "quality performance." On the same TOCG, the mean of the upper half of the sixth graders, as determined by a general achievement test, was as high as that of the ninth graders and the mean of both the lower half of the sixth graders and of the fifth graders was not significantly different from the tenth graders. Thus previous exposure to coordinate and other geometric topics, beginning at the elementary school level and continuing through junior high school, could provide a foundation of knowledge which could be expanded and developed in greater depth and perhaps thereby increase achievement of secondary school students. Spiralling geometry throughout elementary school appears to be a practical and efficient procedure to increase understanding and competence of students.

At the time of this study, secondary school students studied coordinate geometry in the first year algebra course and Euclidean geometry a year later. Both of these courses were presented by the lecture method even though secondary students had had little previous experience with geometric concepts. Thus, secondary students met these concepts for the first time at the formal operations stage of learning according to Piaget's classification. Developments in learning theory indicate the desirability of modifying pedagogical techniques at the secondary school level so that students are encountering new concepts at the concrete and earlier stages of learning.

Implications for In-Service Education for Teachers

Research conducted by Hammond,[56] Houston,[57] and Ruddell and Brown[58] documented significant improvement in the achievement of pupils whose teachers were participating in in-service workshops. Weaver cautioned mathematics educators to be realistic in designing teacher preparatory programs. Materials must be developed that are related to the textbooks used by elementary personnel. These materials must begin at the prevailing level of understanding and be paced accordingly.[59]

Recent research implied that in-service education for elementary school teachers warranted treatment different from that of mathematics education courses designed

---

[56]Harry Hammond, "Developing Teacher Understanding of Arithmetic Concepts Through In-Service Education" (unpublished Ph.D. dissertation, University of California at Los Angeles, 1964), Dissertation Abstracts, XXV, 5138.

[57]W. Robert Houston, "Mathematics In-Service Education: Teacher Growth Increases Pupil Growth," Arithmetic Teacher, X (May, 1963), 243-247.

[58]Arden Ruddell and Gerald Brown, "In-Service Education in Arithmetic; Three Approaches," Elementary School Journal, LXVII (April, 1964), 417.

[59]Fred Weaver, "Non-Metric Geometry and the Mathematics Preparation of Elementary School Teachers," The American Mathematics Monthly, LXXXIII (December, 1966), 1115-1121.

for preservice teachers.[60] The present study indicated that when in-service training was planned to deal with content, methods and materials that teachers could use directly in their classroom programs, their students learned the intended concepts. The elementary school teachers who participated in the workshop conducted for this study did not have particularly strong mathematics backgrounds. Yet these teachers, with the help of a mathematics specialist, broadened their mathematical knowledge on a specific topic and developed and used materials so effectively that the achievement of their pupils was almost as great as the achievement of secondary students (whose teachers held mathematics majors or minors) on the same topic.

A potential implication of the findings of this study is that subject matter specialists might provide a method for improving classroom effectiveness until teacher training programs, such as those supported by CUPM and CCTT, can be effected or until some modification in the structure of elementary schools, such as team teaching, enables teachers to become specialists in certain fields and pool their strengths.

There is no reason to believe that the achievement of the pupils whose teachers participated in this workshop

---

[60]Thomas C. Gibney, John L. Ginther, and Fred L. Pigge, "The Mathematical Understandings of Preservice and In-Service Teachers," The Arithmetic Teacher, XVII (February, 1970), 155-162.

was peculiar to this special unit. The cooperative endeavors of elementary school teachers and subject matter specialists could improve the mathematics program by broadening the teacher's knowledge and keeping him abreast of current materials and methods just as the recent cooperation of mathematicians, educators, and classroom teachers improved the content, itself, of the mathematics curriculum.

Implications for Further Research

The fact that a specific topic in coordinate geometry can be learned as efficiently by fifth and sixth graders as by ninth and tenth grade algebra students does not imply that it should automatically be assigned to these elementary grades. This philosophical issue is so important and encompassing that it must be resolved before delegating this concept to the fifth or sixth grade. The achievement of pupils in other elementary grades on the graphing of linear equations and other geometric concepts should be researched. Nor should the introduction of new material at the elementary level be limited to geometry. The concepts of matrices, logarithms, trigonometric functions, and conic sections, to name but a few, are also worthy of investigation. Component or subordinate tasks of these concepts should be identified and assigned to the level appropriate to the child's comprehension.

Secondary school students studied a unit in coordinate geometry taken directly from their textbooks while elementary school pupils studied from specially produced materials. This procedure, recognized as a potentially limiting factor, may have influenced results through operation of what is commonly referred to as the "Hawthorne Effect." A replication of this study using special materials for the secondary sample should be undertaken to test the interaction between student and specially produced materials.

In this study, the alarmingly low achievement (50 to 60 per cent) of the secondary students on coordinate geometry may reflect the inadequacy of a one-chapter encounter with Cartesian geometry. The presentation of geometric topics at the elementary level, even if limited to the most rudimentary concepts, may favorably influence student achievement in subsequent grades. Therefore, a longitudinal research project is recommended to investigate the effect of earlier introduction of coordinate geometry on pupil achievement when they again encounter this topic in high school algebra.

The instrument utilized in the study was developed for one geographical area, and primarily with students whose general mathematics achievement was slightly below the national test norms. Data in this present study indicated that the instrument might be more widely applicable.

However, full range of its use and usefulness has not been tested, thus suggesting further study and refinement of instrument sub-categories and items.

The study of curriculum and the placement of topics are of utmost importance. Some learning theorists have hypothesized that topic placement is a function of prior learning in a particular field. This is especially true for mathematics, where a relatively linerar curriculum pattern pervades most textbook series, and continuous progress plans are being strongly recommended.

In contrast, other learning theorists profess that the method of learning is most important, and that topic placement is not the vital concern of the curriculum specialist. While the present study neither supports nor denies either position, it does focus attention upon the undergirding problem. Research must be undertaken to test these theories. Only through careful study of the many facets of curriculum and content placement can a more viable mathematics program evolve.

BIBLIOGRAPHY

BIBLIOGRAPHY

Adler, Marilynne J. "Some Educational Implications of the Theories of J. Piaget and J. S. Bruner." Canada Education and Research Digest, IV (December, 1964), 291-305.

Allendoerfer, Carl B. "The Dilemma in Geometry." The Mathematics Teacher, LII (March, 1969), 165-171.

Ausubel, David P. "Can Children Learn Anything That Adults Can--and More Efficiently?" Elementary School Journal, LXII (February, 1962), 270-272.

______. Review of Toward a Theory of Instruction, by Jerone S. Bruner. Harvard Education Review, XXXIV (Fall, 1966), 332-340.

Bassler, Otto C. "Research Workers in Mathematics Education." American Mathematics Monthly, LXXIV (September, 1967), 859.

Broudy, Harry S.; Smith, Othaniel B.; and Burnette, J. R. "New Look at Readiness." Theory Into Practice, III (December 28, 1963), 425-429.

Brown, Kenneth E., and Kinsella, John J. Analysis of Research in the Teaching of Mathematics: 1957-1958, Bulletin 1960 No. 8. Washington, D.C.: Government Printing Office, 1960.

Brownell, William A. "Arithmetic Readiness as a Practical Classroom Concept." Elementary School Journal, LII (September, 1951), 15-22.

Brownell, William A., and Moser, Harold. "Meaningful vs. Mechanical Learning: A Study in Grade III Subtraction." Research Monograph, No. 8. Durham: Duke University Press, 1949.

Brune, Irvin. "Geometry in the Grades." Enrichment Mathematics for the Grades. Twenty-seventh Yearbook of the National Council of Teachers of Mathematics. Washington, D.C.: National Council of Teachers of Mathematics, 1963.

Bruner, Jerome S. "On Learning Mathematics." The Mathematics Teacher, LIII (December, 1960), 610-619.

______. The Process of Education. Cambridge: Harvard University Press, 1962.

Bruner, Jerome S.; Goodnow, Jacqueline; and Austen, G. A Study in Thinking. New York: John Wiley and Sons, 1956.

Coghill, George E. Anatomy and the Problem of Behavior. New York: The Macmillan Co., 1929.

Corley, Glyn J. "An Experiment in Teaching Logical Thinking and Demonstrative Geometry in Grades 6 Through 10." Unpublished Ph. D. dissertation, George Peabody University, 1959. Dissertation Abstracts, XX, 1375.

Coxford, Arthur F. "The Effects of Instruction on the Stage Placement of Children in Piaget's Seriation Experiments." The Arithmetic Teacher, XI (January, 1964), 4-9.

Cronbach, Lee J. Educational Psychology. 2d ed. New York: Harcourt, Brace and Co., 1963.

D'Augustine, Charles H. "Factors Relating to Achievement with Selected Topics in Geometry and Topology." The Arithmetic Teacher, XIII (March, 1966), 192-197.

Davis, Robert B. Discovery in Mathematics. Reading, Massachusetts: Addison-Wesley Publishing Co., 1964.

______. "Mathematics for Younger Children--The Present Status of the Madison Project." New York State Mathematics Teachers Journal, X (April, 1960), 75-79.

______. "The Madison Project's Approach to a Theory of Instruction." Journal of Research in Science Teaching, III (March, 1964), 146-162.

Denmark, Thomas, and Kalin, Robert. "Suitability of Teaching Geometry in Upper Elementary Grades: A Pilot Study." The Arithmetic Teacher, XI (February, 1964), 73-80.

Devault, M. Vere, and Kriewall, Thomas. Perspectives in Elementary School Mathematics. Columbus, Ohio: Charles E. Merrill Publishing Co., 1969.

Downie, M. N., and Heath, R. W. Basic Statistical Methods. New York: Harper and Row Publishers, 1965.

Dutton, Wilbur H. Evaluating Pupils' Understanding of Arithmetic. Englewood Cliffs: Prentice-Hall, Inc., 1964.

Educational Services Incorporated. Goals for School Mathematics. The Report of the Cambridge Conference on School Mathematics. Boston: Houghton Mifflin Co., 1964.

Eicholz, Robert E., et al. Elementary School Mathematics. Book 4. Reading, Massachusetts: Addison-Wesley Publishing Co., 1964.

Elkind, David. "Piaget and Montessori." Harvard Education Review, XXXVII (Fall, 1967), 538-544.

Estes, Betty. "Some Mathematical and Logical Concepts in Children." Journal of Genetic Psychology, LXXXVIII (June, 1956), 219-222.

Fitzgerald, William M. "A Study of Some of the Factors Related to the Learning of Mathematics by Children in Grades 5, 7, and 9." Unpublished Ph. D. dissertation, University of Michigan, 1962.

Gagné, Robert M. "The Acquisition of Knowledge." Psychological Review, LXIX (July, 1962), 355-365.

Gesell, Arnold. "The Ontogenesis of Infant Behavior." Manual of Child Psychology. Edited by Leonard Carmichael. New York: John Wiley and Sons, 1954.

Gibney, Thomas C.; Ginther, John L.; and Pigge, Fred L. "The Mathematical Understandings of Preservice and In-service Teachers." The Arithmetic Teacher, XVII (February, 1970), 155-162.

Gibney, Thomas C., and Houle, William W. "Geometry Readiness in the Primary Grades." The Arithmetic Teacher, XIV (October, 1967), 470-473.

Greenhouse, Samuel W., and Geisser, Seymour. "On Methods in the Analysis of Profile Data." Psychometrika, XXIV (June, 1959), 94-112.

Grossnickle, Foster E. "Experiment with One Figure Divisor in Short and Long Division." Elementary School Journal, XXXIV (March, 1934), 496-506.

Hammond, Harry. "Developing Teacher Understanding of Arithmetic Concepts Through In-Service Education." Unpublished Ph. D. dissertation, University of California at Los Angeles, 1964. Dissertation Abstracts, XXV, 5138.

Hays, William. Statistics for Psychologists. New York: Holt, Rinehart, and Winston Publishing Co., 1963.

Houston, W. Robert. "Mathematics In-Service Education: Teacher Growth Increases Pupil Growth." Arithmetic Teacher, X (May, 1963), 243-247.

Houston, W. Robert. "Selected Methods of In-Service Education and the Mathematics Achievement and Interest of Elementary School Pupils." Unpublished Ph. D. dissertation, University of Texas, 1962. Dissertation Abstracts, XXIII, 157.

John, Lenore. "Geometry for Elementary School Teachers." American Mathematics Monthly, LXVIII (April, 1967), 374-376.

Johnson, Donovan A.; Kinsella, John J.; and Rosenberg, Herman. Algebra: Its Structure and Application. New York: The Macmillan Co., 1963.

Keliher, Alice. "Editorial." Childhood Education, XLII (May, 1966), 527.

Office of Evaluation Services. Item Analysis. East Lansing: Michigan State University, 1965.

Peters, Anne. "Articulating Geometry Between Elementary and Secondary School." National Association of Secondary School Principal's Bulletin, XLIII (May, 1957), 131-133.

Piaget, Jean. "Development and Learning." Journal of Research in Science Teaching, II (March, 1964), 176-186.

______. The Child's Conception of Number. New York: Norton and Co., 1962.

Piaget, Jean; Inhelder, Barbal; and Szeminska, Alina. The Child's Conception of Geometry. New York: Basic Books, 1960.

Ruddell, Arden K., and Brown, Gerald W. "In-Service Education in Arithmetic: Three Approaches." Elementary School Journal, LXIV (April, 1964), 377-382.

Shah, Sair Ali. "Selected Geometric Concepts Taught to Children Ages Seven to Eleven." The Arithmetic Teacher, XVI (February, 1969), 119-128.

Shulman, Lee S. "Perspectives on the Psychology of Learning and the Teaching of Mathematics." Improved Mathematics Education. Edited by W. Robert Houston. East Lansing: Michigan State University, 1967.

Tyler, Frederick. "Readiness." Encyclopedia of Educational Research. 4th ed. New York: The Macmillan Co., 1969.

Washburne, Carleton W. "The Grade Placement of Arithmetic Topics." Report of the Society's Committee on Arithmetic. Twenty-ninth Yearbook of The National Society for the Study of Education, Part II. Chicago: University of Chicago Press, 1930.

Weaver, Fred J. "Non-metric Geometry and Mathematics Preparation of Elementary School Teachers." American Mathematics Monthly, LXXIII (December, 1966), 1115-1121.

APPENDICES

APPENDIX A

TEACHER LESSON PLANS FOR ELEMENTARY SCHOOL UNIT ON COORDINATE GEOMETRY

LESSON PLANS IN COORDINATE GEOMETRY FOR ELEMENTARY TEACHERS

LESSON 1. PLOTTING POINTS

Pupil Objective

Pupils will learn to plot ordered pairs representing points in all four quadrants of the plane.

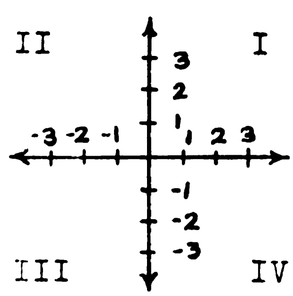
Overview for Teachers

Introducing the student to Cartesian coordinates (named after Descartes, who developed analytic geometry) may involve simultaneously, his first mathematical experience with signed numbers. Thus, two important concepts must be developed:

a. The concept that a sign is a vital part of a number, and

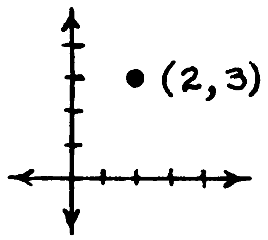
b. The concept of an ordered pair.

The union of a horizontal number line and a vertical number line forms a pair of axes which divide the plane into four quadrants. These quadrants are numbered counterclockwise, I through IV, beginning with the upper right quadrant. Numbers with positive signs lie to the right of 0 on the horizontal number line and above 0 on the vertical number line. Negative numbers lie to the left of 0 on the horizontal number line and below 0 on the vertical number line.

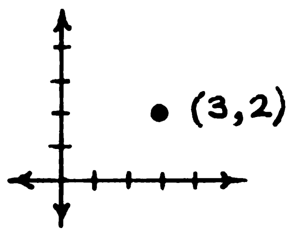


In advanced courses the horizontal number line will be called the abscissa and the vertical number line the ordinate. The intersection of the axes, (0,0), is called the origin.

Relevant to order, mathematical convention dictates that horizontal distance be cited first and vertical distance second. Thus, an ordered pair, called the coordinates of a point, is conventionally enclosed in parentheses and the numbers separated by a comma. For example, the ordered pair (2,3) designates the point which lies two units to the right of the ordinate and three units above the abscissa and is plotted:

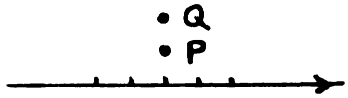


This is not the same point as represented by the ordered pair (3,2) which lies three units to the right of the ordinate and two units above the abscissa:

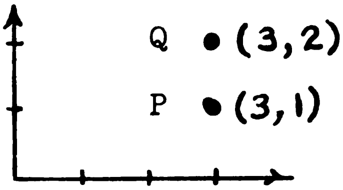


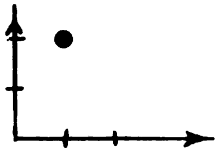
Suggested Procedure

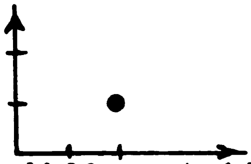
Let students review the location of points on a number line. Draw a number line on the board and have students represent points, such as 3, 5, 1, 7 and 4, on the number line. Call attention to the fact that for each number there corresponds one point and that for each point there corresponds only one number. Represent several points on the number line and let students name the coordinate. Now place a point P one unit above the number line as follows and ask students to name its coordinate.



If students respond 3, put a dot on the number line at 3. Ask if they think 3 adequately locates both points. If they answer yes, ask for the coordinate of Q, which is one unit above P and two units above the axis. Try to stimulate students to suggest a vertical number line if the suggestion has not arisen by now. You may want to remind them of the one-to-one correspondence between points and numbers. When the construction of a vertical number line has been suggested, name several points such as P with coordinates (3,1) and Q with coordinates (3,2).



Ask a student to plot (1,2). If he plots  see if the class agrees. Possibly someone will plot this point:



Here are two different ideas for the position of one set of coordinates. When the coordinates of a point are named, we must be sure that everyone plots the same point. Therefore, mathematicians have agreed to name the horizontal distance first and the vertical distance second. To emphasize that the order of naming the coordinates determines different positions for the point represented, have pupils plot several more points, interchanging their coordinates, as (4,3), (3,4); (1,5), (5,1), until they recognize the importance of order.

When students can plot points in the first quadrant, ask if they think that the horizontal number line ends at zero. (They probably already know that it continues to the left of zero). To motivate continuing the vertical number line below 0, try to guide the discussion to the consideration of temperature. When pupils suggest "2 below zero," ask if they know the mathematical names for these numbers. If they do not, name them "negative numbers" and discuss possible applications as below zero temperatures, below sea level, debit, and other ideas they should suggest. Treat positive and negative numbers as opposites--if positive numbers indicate "up," then negative numbers indicate "down." If positive numbers indicate "gain," then negative numbers indicate "loss."

Continue to emphasize order. Stress the location of positive numbers to the <u>right</u> of the ordinate and <u>above</u> the abscissa and the location of negative numbers to the <u>left</u> of the ordinate and <u>below</u> the abscissa. Thus, $(^{-}1,3)$ is located 1 unit to the left of the ordinate and 3 units above the abscissa; $(^{-}1,^{-}3)$ is one unit to the left of the ordinate and 3 units below the abscissa.

<u>Activities for Pupils</u>

1. Send pupils to the blackboard or overhead projector to

plot points.

2. Distribute geoboards, pegboards, or checkerboards so that a small group of pupils (3 or 4) has access to one of these devices. Cut plastic straws into little cylinders for use with geoboards. Place rubber bands around the middle row and center column of pegs to represent the axes. Use straight pins for the checkerboards and golf tees for the pegboard. Call out ordered pairs, as (3,4). Pupils with geoboards can place a plastic cylinder over the nail 3 units to the right of the vertical axis and 4 units above the horizontal axis. Pupils with pegboards can insert golf tees into the proper hole while pupils with checkerboards can stick a straight pin into the intersection of the correct horizontal and vertical line.

3. Distribute Exercise 1. Have pupils complete this exercise during the class period.

LESSON 2. LISTING TRUTH SETS

Pupil Objective

Pupils will list some elements of the truth set of an open sentence with two variables (1) by listing ordered pairs as elements of a truth set, and (2) by constructing a table.

Overview for Teachers

This lesson prepares the pupil for graphing linear functions. The truth set of an open sentence with two place holders (△ and □ represent the two variables) consists of an ordered pair. The ordered pairs which make a particular equation true can be listed as the elements of a set. For example, {(⁻1,⁻3),(0,0), (1,3),(2,6), (3,9)...} are a few elements of the open sentence △ = 3 x □. A table is another convenient device for displaying the truth set in an open sentence.

□	△
1	3
2	6
3	9

Suggested Procedure

If necessary (depending upon the class' background) discuss open sentences and their classification as true or false depending upon the numbers substituted for the placeholders. Ask pupils to consider the open sentence △ = □ + 1 where △ and □ both represent numbers.

Replacing the box and triangle requires a pair of numbers, one of which replaces box; the other replaces triangle. We agree to name the number that replaces box first and the number that replaces triangle second. Ask a pupil to name a pair of numbers. If he names (2,4), then 2 replaces box and 4 replaces triangle. The sentence becomes: $4 = 2 + 1$. A convenient method of demonstrating the replacement of $\triangle$ and $\square$ with numbers is to write the numbers within the symbols, as $\overset{4}{\triangle} = \boxed{2} + 1$. This sentence is false. Let other pupils name pairs of numbers. Substitute the first number named into the box, the second into the triangle and classify the sentence as true or false accordingly. Tell pupils to look for values that make the sentence true. Continue to emphasize the importance of order as pupils suggest pairs of numbers. The pair, (4,5), produces a true statement for the above open sentence: $\overset{5}{\triangle} = \boxed{4} + 1$, or $5 = 5$. But the ordered pair (5,4) makes the open sentence $\overset{4}{\triangle} = \boxed{5} + 1$, or $4 \neq 6$, false. Call the pairs of numbers which make the sentence true the <u>truth set</u> of the open sentence.

A table provides another convenient method for listing the members of a truth set. Consider another open sentence such as $\triangle = \square + 3$. Let the pupils make a table of ordered pairs of numbers which make this sentence true.

□	△
0	3
1	4
2	5

Activities for Pupils

1. Relay game. Divide the class into teams. If pupils are arranged in rectangular arrays, let each column be a team. (Shift pupils so that each column contains the same number.) Write the same open sentence on a piece of paper--one for each column. Include a blank table. Let each pupil fill in a pair of numbers on the table and pass it to the pupil sitting behind him. The column that completes the table first with every pair a correct element of the truth set wins the relay.

2. Distribute Exercise 2. Let pupils complete exercise during the class period.

LESSON 3. GRAPHING THE TRUTH SET OF AN OPEN SENTENCE

Pupil Objective

(1) Pupils will plot the graph of the truth set of an open sentence with two placeholders. (2) Pupils will recognize that the graph of an open sentence of the form $\triangle$ = __ x $\square$ + __ is a straight line.

Overview for Teachers

Pupils have already represented the truth set of an open sentence with two variables by listing the ordered pairs as elements of a set and by making a table. A third way to represent these truth sets is by graphing. Let $\square$ denote the abscissa (horizontal axis), and $\triangle$ the ordinate (vertical axis). After plotting several graphs, pupils should realize that the graphs of the truth sets suggest straight lines.

Suggested Procedure

1. Review the methods for writing the truth set of an equation, such as $\triangle = \square + 2$ by

   a. Listing the truth set of ordered pairs as elements of a set:

      {(0,2), (1,3), (2.4).(3,5)...}

   b. Making a table of some of the elements of the truth set.

□	△
0	2
1	3
2	4
3	5

2. Ask pupils if they can think of another way to represent the truth set of the open sentence, △ = □ + 2. If dead silence ensues, suggest that there is a method which relates to the new information they learned in the last few days. If further hints are needed, remind them of plotting points on the number line and the agreement to plot the horizontal distance first and the vertical distance second. Ask if any other agreements were made about open sentences. Hopefully, some pupil will recall that in finding the truth set of an open sentence, □ was named first and △ was named second. Ask if they can relate the two agreements. Try to stimulate pupils to suggest naming the horizontal axis, □, and the vertical axis, △.

3. Consider the open sentence △ = 2 x □. Send various pupils to the board to plot points representing elements of the truth set. Plot a few additional graphs, and leave all of the graphs on the board. Ask pupils what the picture of an open sentence of the form

△ = __ x □ + __ would be.

Activities for Pupils

1. Graph several open sentences by sending pupils to the blackboard to plot elements of the truth set. (Or mark points on an overlay.)

2. Play Tic-Tac-Toe.

a. On the blackboard.

b. With teams of boys against girls. Arrange desks or chairs in rectangular array. Number rows and columns. Vacate desks. Team captains will name coordinates of point. Team members, in turn, will sit at designated desks. Team placing four boys or four girls in row, column or diagonal wins.

3. Distribute Exercise 3. Have students complete the problems during the class period.

LESSON 4. DISCOVERING SLOPE PATTERN

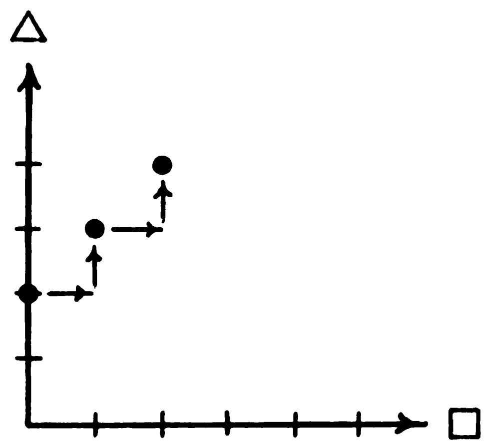
Pupil Objective

To discover the pattern of the graph of an open sentence.

Overview for Teachers

Pupils have discovered that the truth set of an open sentence (equation) which has two variables, □ and △ (x and y), can be displayed by a graph which will be a straight line. This graph has a "pattern." The pattern, which is called the slope of the graph, is not always the same. The slope depends upon the open sentence. In analytic geometry the slope pattern is defined as the ratio of the vertical change compared to the horizontal change. Since it is easier to see the slope pattern if only whole numbers are used, use whole numbers only when first introducing the concept. In the graph of △ = □ + 2, the slope pattern is: over one square to the right and up one square.

□	△
0	2
1	3
2	4
3	5

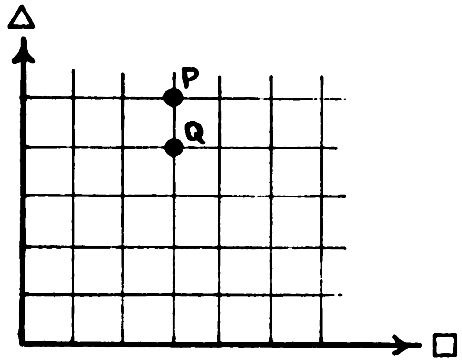


<u>Suggested Procedure</u>

List an open sentence, such as $\triangle = \square + 2$, with a table containing about three elements of the truth set. Have pupils verify that these elements do belong to the truth set. Plot the points on a pegboard with golf tees after verification. Then ask a pupil to "place" another point without doing any arithmetic. After a point has been plotted, let pupils determine its coordinates and verify that the point is an element of the truth set by substituting the coordinates into the equation.

For $\triangle = \square + 2$, the pattern is "over one square to the right and up one square." Do not tell your pupils this pattern. Lead them to discover it by asking some pupil to place another golf tee on the graph simply by observing the pattern without doing any arithmetic. After a point is placed, have another pupil name its coordinates and substitute them into the equation to see whether the resulting statement is true or false. Test all points. This process also enables the pupil to see the relation between the points on the graph and the numbers substituted for $\square$ and $\triangle$.

Graph the truth set of the open sentence, $\triangle = 2 \times \square - 1$. Have pupils determine if the graph is correct by verifying each point. Have a pupil plot a point on the pegboard by observing the pattern and not doing any arithmetic. (Pattern is one square right, one square up.)



Verify that the coordinates of this point makes the sentence true; e.g., the coordinates of Q when substituted into the sentence results in 4 = 2 x 3 - 1, or 4 = 5, which is false. The coordinates of P, (3,5), when substituted into the equation, gives: 5 = 2 x 3 - 1, or 5 = 5, a true sentence.

Do not tell the pattern to pupils. Moreover, as each child discovers the pattern, encourage him to keep it a secret so that the other pupils can discover it for themselves.

Activities for Pupils

1. Prepare several graphs for overhead projection. Have pupils place additional points on these graphs by determining the "pattern."

2. Plot the graph of an open sentence on a pegboard. Have pupils place golf tees to represent additional points without doing any arithmetic and then substitute the coordinates into the open sentence to verify that the point belongs to the truth set.

3. If floor is tiled, agree on number of tiles per unit. List open sentence and a table of a few elements of

truth set on the blackboard. Let pupils be "points" and stand on intersections of tiles corresponding to the coordinates of the points listed. Then ask another pupil to stand on intersection of "tile lines" so that he will be a member of the truth set (by observing the geometry of the slope pattern and not doing any arithmetic).

4. Distribute Exercise 4. Have pupils complete the problems during the class period.

LESSON 5. DISCOVERING RELATION OF SLOPE PATTERN AND Δ-INTERCEPT TO THE OPEN SENTENCE

Pupil Objective

(1) Pupils will discover the relation of the geometrical pattern of the graph and the open sentence. (2) Pupils will identify the intersection of the graph and the Δ axis from the open sentence.

Overview for Teachers

The pattern of the graph of the truth set of an open sentence can be determined without plotting a single point. One can also determine the point of intersection of the graph and the Δ axis without plotting points. The following exercises are constructed to direct the pupil's attention to the relation between these two concepts about the graph and the open sentence.

Suggested Procedure

Distribute Exercise 5. Each pupil will construct tables and plot the graphs on the exercise. Encourage pupils to work individually. Pupils will study each open sentence and its graph to see if they can see a relation between the two.

Activities for Pupils

Distribute Exercise 5.

LESSON 6. DETERMINING THE OPEN SENTENCE FROM A GRAPH

Pupil Objective

Pupils will discover the open sentence when the graph of the truth set is displayed.

Overview for Teachers

Pupils who have discovered the relation between the open sentence and the slope of its graph, and the relation between the open sentence and the △-intercept, should be able to look at a graph and write its equation. (For those who have not made this discovery, here is another chance to make it.) The equation will be in the form △ = __ x □ + __, or y = mx + b, known in analytic geometry as the slope-intercept form. Identification of the pattern of the graph gives the slope coefficient--the number preceding box. The multiplier of box determines the pattern (slope) of the graph. In the equation △ = 2 x □ + 1, the slope is found on the graph by counting over 1 to the right and up 2. Two, the multiplier of □ is the slope. The constant in the open sentence indicates where the truth set intersects the △ axis. In this case, it is 1.

Suggested Procedure

1. Use overlays on overhead projector. Project a few graphs. See if pupils can find their equations.

2. Plot graphs of lines on the pegboard with golf tees. Have pupils find the equations.

<u>Activities for Pupils</u>

Distribute Exercise 6. Have pupils complete the exercise in class.

LESSON 7. ADDING SIGNED NUMBERS

Pupil Objective

Pupils will find sums of signed numbers.

Overview for Teachers

Graphs whose △ (y)-intercepts are less than 0 require some understanding of sums with negative addends. Addition of signed numbers can be introduced in several ways. In one method, a cricket hops on the number line. Another method measures directed distances on the number line. We will use still another method, the Madison Project's "Postman Stories." In this scheme, (1) a check is represented by a positive number, a bill by a negative number. (2) Something brought to you is represented with an addition sign, something taken away is represented by the subtraction sign. (3) In a sum, something happens and then something else also happens.

Suggested Procedure

1. Invent an arithmetic for numbers with signs, i.e., the numbers left of zero on the horizontal axis, below zero on the vertical axis, and positive numbers that the pupils have been using. To aid this invention, make up postman stories. For example, suppose the postman brings a check for $4. We can represent this outcome as +4. If he brings a bill for $2, we can represent this as -2. A

story based on this information might be: the postman brings a check for $4 and a bill for $2. Are you richer or poorer? By how much? Or, if the postman brings a bill for $2 and a bill for $5, are you richer or poorer?

2. Let students make up postman stories about several problems.

Activities for Pupils

Distribute Exercise 7. Have pupils create stories for each problem and tell the stories to the class. Have pupils write the sum.

LESSON 8. MULTIPLYING SIGNED NUMBERS

Pupil Objective

Pupils will find products of signed numbers.

Overview for Teachers

Thus far, we have substituted only positive numbers for box and have considered only equations with positive slopes, i.e., as □ increased △ also increased. In order to graph any equation with two variables, pupils should be able to substitute negative integers for box and consider equations with negative slopes. Both situations force the consideration of products with at least one negative factor. The substitution of a negative number into an open sentence with a negative slope makes finding a product with two negative factors necessary. Again, the Postman Stories of the Madison Project will be used:

a. A check is represented by a positive number

b. A bill is represented by a negative number

c. Something brought to you is represented by an addition sign

d. Something taken away from you is represented by a negative sign

e. In a product the second factor is the bill or check, the first factor is how many bills or how many checks.

Suggested Procedure

Make up a Postman Story to suggest the use of signs. Example: In today's mail, suppose mother found that the postman had brought 3 checks for $5 each. We could make up this numerical problem about the checks: +3 x +5. Is mother richer or poorer? (Richer). By how much? +3 x +5 or +15. How did you get the answer? Suppose the postman brings 2 bills for $6 each. Are we richer or poorer? (Poorer). By how much? $12. How did you get the answer? (+2 x -6).

Activities for Pupils

1. Dictate some postman problems for students to multiply and answer orally.

2. Distribute Exercise 8. Have students complete the exercise in class.

3. Have students create a Postman Story for each problem.

LESSON 9. GRAPHING SENTENCES WITH NEGATIVE SLOPES

Pupil Objective

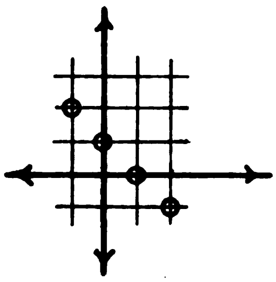
Pupils will discover the pattern of the graph of an open sentence in which the △ coordinate decreases when the □ coordinate increases.

Overview for Teachers

Here we take a second look at the pattern of a graph. This time, after the postman stories, we investigate negative slope patterns in which △ decreases as □ increases.

Suggested Procedure

Plot several graphs on the pegboard having negative slopes. Ask the children to find the slope patterns of these graphs. In the following example, the slope is over one to right and down one.



All the graphs plotted so far have had positive slope coefficients. That is, as □ increased, △ increased. Plot the graph of △ = 2 x □ + 3. Ask pupils what happens to values of △ as values for □ increase. Suggest using the arithmetic of the Postman Stories to plot the graph of $\triangle = -2 \times \square + 3$. Now investigate what happens to values of $\triangle$ when values for $\square$ increase.

<u>Activities for Pupils</u>

1. Send students to the pegboard to plot graphs with negative slopes.

2. Distribute Exercise 9.

LESSON 10. PLOTTING GRAPHS AS CONTINUOUS LINES

Pupil Objective

Pupils will plot the graph of an equation as a line instead of discrete points.

Overview for Teachers

By substituting fractions as well as whole numbers for box, the points of the graph get closer and closer together. If we allow all kinds of numbers--whole numbers, fractions and irrational numbers--the graph would become a continuous line. If any pupil suggests plotting intercept and then the pattern, hurrah!

Suggested Procedure

Use overlays or the balckboard to graph the equation $\triangle = 2 \times \square + 3$.

1. First with whole numbers
2. Then use halves
3. Then fourths
4. Then eights.

The slope pattern of a continuous line can be determined more easily from the graph when only whole numbers are substituted for box. To illustrate a continuous line, stick pins with large heads into the pegboard at appropriate distances between the golf tees.

Perhaps pupils may see that they need only two points to draw the graph of a continuous line--two very easy points are the Δ-intercept and the slope coefficient. If they do not make this observation, don't tell them.

Activities for Pupils

Distribute Exercise 10.

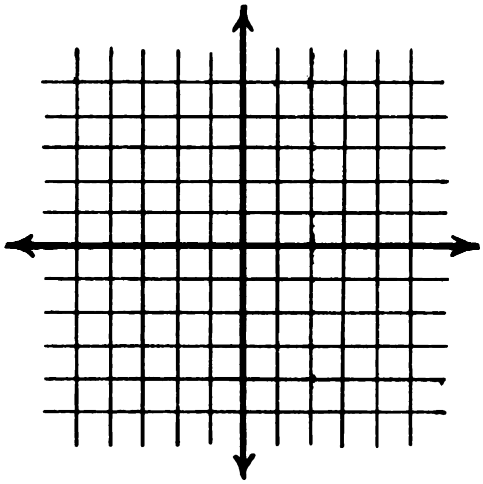
APPENDIX B

PUPIL EXERCISES FOR ELEMENTARY SCHOOL UNIT ON COORDINATE GEOMETRY

EXERCISE I

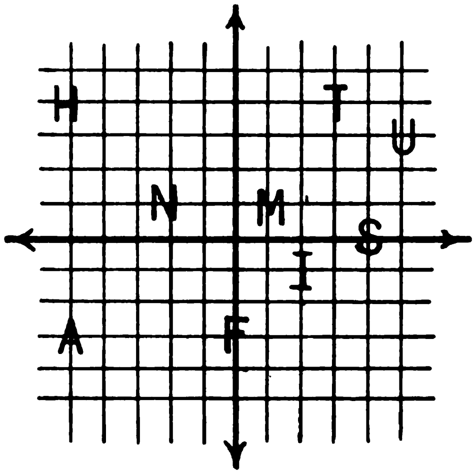
1. Plot the following points.

   a. (3,1)　　e. (2,5)
   b. (-1,3)　　f. (-5,-2)
   c. (-1,-3)　　g. (-1,-2)
   d. (1,-3)　　h. (2, -3)



2. The letters for the sets of coordinates listed below spell a special message for you. See if you can find the message.

   (1,1), (-5,-3), (3,4), (-5,4)
   (2,-1), (4,0)
   (0,-3), (5,3), (-2,1)



3. Write the coordinates of the ordered pair representing:

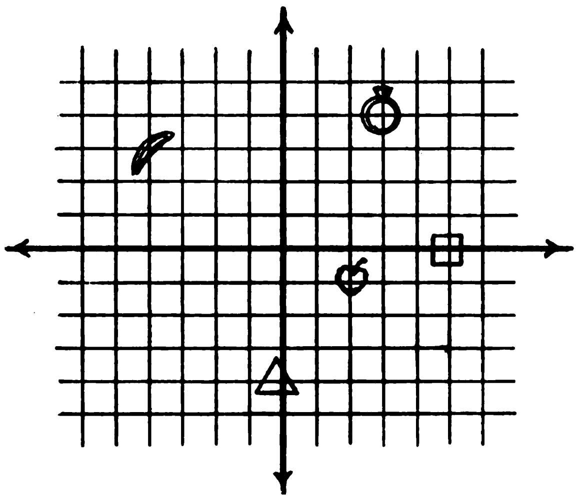
   a. Apple ________________

   b. Banana ________________

   c. Ring ________________

   d. Triangle ________________

   e. Box ________________



FOR FUN EXERCISES

Directions: Plot the points and join them with lines in the sequence plotted. That is, join a to b, b to c, c to d, and so on.

Exercise 1

1. (-10, 10)
2. (-11, 8)
3. (-12, 7)
4. (-8, 6)
5. (1, 9)
6. (9, 5)
7. (15, 6)
8. (22, 10)
9. (16, 4)
10. (5, 2)
11. (0, -4)
12. (-1, -7)
13. (-6, -10)
14. (-6, -9)
15. (-2, -6)
16. (-1, 0)
17. (-3, 0)
18. (-3, -2)
19. (-2, -4)
20. (-5, -2)
21. (-5, 0)
22. (-7, 0)
23. (-12, 4)
24. (-14, 2)
25. (-15, 3)
26. (-14, 6)

Exercise 2

a. (-5, -12)
b. (-4, -13)
c. (-3, -12)
d. (-3, 4)
e. (1, 4)
f. (1, 2)
g. (-1, 2)
h. (-3, 4)
STOP (do not draw a line from h to i)
i. (1, 2)
j. (1, -12)
STOP
k. (-1, 2)
l. (-1, -12)
m. (), -13)
n. (1, -12)
o. (0, -11)
p. (-1, -12)
STOP
q. (3, 7)
r. (5, 5)
s. (5, -12)
t. (4, -13)
u. (3, -12)
v. (4, -11)
w. (5, -12)
STOP
x. (-3, -12)
y. (-4, -11)
z. (-5, -12)
aa. (-5, -7)
bb. (3, 7)
cc. (3, -12)

Exercise 3.

a. (1, 11 1/3)
b. (3 1/2, 11)
c. (4, 10)
d. (3 2/3, 9 2/3)
e. (2 1/2, 10)
f. (-1/2, 10)
g. (-3/4, 12)
h. (-1, 12 1/2)
i. (-1 1/2, 14 1/2)
j. (-2 1/3, 15)
k. (-2, 15 1/2)
l. (-3, 15 1/3)
m. (-2 2/3, 15)
n. (-3, 15)
o. (-4, 14)
p. (-3 1/2, 12 1/2)
q. (-3, 12)

STOP: Do not draw line between q and r.

r. (8, 10)
s. (9 1/2, 10)
t. (11 1/2, -1/2)
u. (8 1/2, -1)
v. (8 1/2, -7)
w. (-6 1/2, -7)
x. (-6 1/2, -1)
y. (-9 1/2, -1/2)
z. (-7 1/2, 10)
aa. (-3 1/2, 10)
bb. (-4 1/3, 11)
cc. (-3 1/2, 12 1/2)

STOP: Do not draw line between cc and dd.

dd. (4, 10)
ee. (8, 10)
ff. (7 3/4, 12 3/4)
gg. (6 3/4, 13 1/2)
hh. (6, 12 3/4)
ii. (6 3/4, 11)
jj. (5 1/3, 11)
kk. (4 2/3, 11 3/4)
ll. (3 2/3, 13)
mm. (2 1/3, 13 2/3)
nn. (1 1/2, 13 1/2)
oo. (0, 12)
pp. (-3/4, 12)
qq. (-1/2, 10)
rr. (-3 1/4, 10)
ss. (-2 1/3, 7 1/2)
tt. (-2 1/2, 7)
uu. (-3, 7 1/2)
vv. (-3, 10)
ww. (-3 1/2, 10)

Exercise 4.

a. (8, 2)
b. (6, 0)
c. (-8, 3)
d. (-8, -2)
e. (10, -5)
f. (8, -7)
g. (-10, -4)
h. (-10, 5)
i. (8, 2)
j. (8, -3)
k. (-6, 0)
l. (-6, 2 1/2)

STOP: Do not draw line between l and m.

m. (-6, 0)
n. (-8, -2)

STOP: Do not draw line between n and o.

o. (6, 0)
p. (6, -2 1/2)

STOP: Do not draw line between p and q.

q. (
r. (-8, 7)
s. (10, 4)
t. (10, -5)

Exercise 5.

a. (-22, 9)
b. (-21, 8)
c. (-17, -6)
d. (15, -6)
e. (24, 6)
f. (11, 6)
g. (9, 10)
h. (5, 10)
i. (4, 14)
j. (-1, 14)
k. (0, 10)
l. (-3, 10)
m. (-4, 14)
n. (-9, 14)
o. (-8, 10)
p. (-12, 10)
q. (-14, 8)
r. (-21, 8)

Exercise 6.

a. (-3 1/2, -10)
b. (-3 1/2, -15)
c. (-5 2/3, -17)
d. (-5 2/3, 12)
e. (-3 1/2, -10)
f. (-3 1/2, -6)
g. (-2 3/4, -5)
h. (-2 1/2, 2 1/2)
i. (-2, 3 1/2)
j. (-1 1/2, 12 1/2)
k. (0, 14 1/3)
l. (1 1/2, 12 1/2)
m. (2, 3 1/2)
n. (2 1/2, 2 1/2)
o. (2 3/4, -5)
p. (3 1/2, -6)
q. (3 1/2, -15)
r. (5 2/3, -17)
s. (5 2/3, -12)
t. (3 1/2, -10)
u. (3 1/2, -14)
v. (-3 1/2, -14)

EXECISE 2

Find several pairs of numbers that will make the following open sentences true. List your answers as truth sets.

1. △ = □ + 2 Answer ________________
2. △ = 3 x □ Answer ________________
3. △ = 2 x □ + 1 Answer ________________
4. If □ is represented by each of the following numbers, what is △ using the open sentence △ = □ + 2?

□	△
0	
1	
2	
3	
4	

Make tables of the truth sets for the following open sentences:

5. △ = 2 x □ + 1

□	△

6. $\triangle = 3 \times \square + 1$

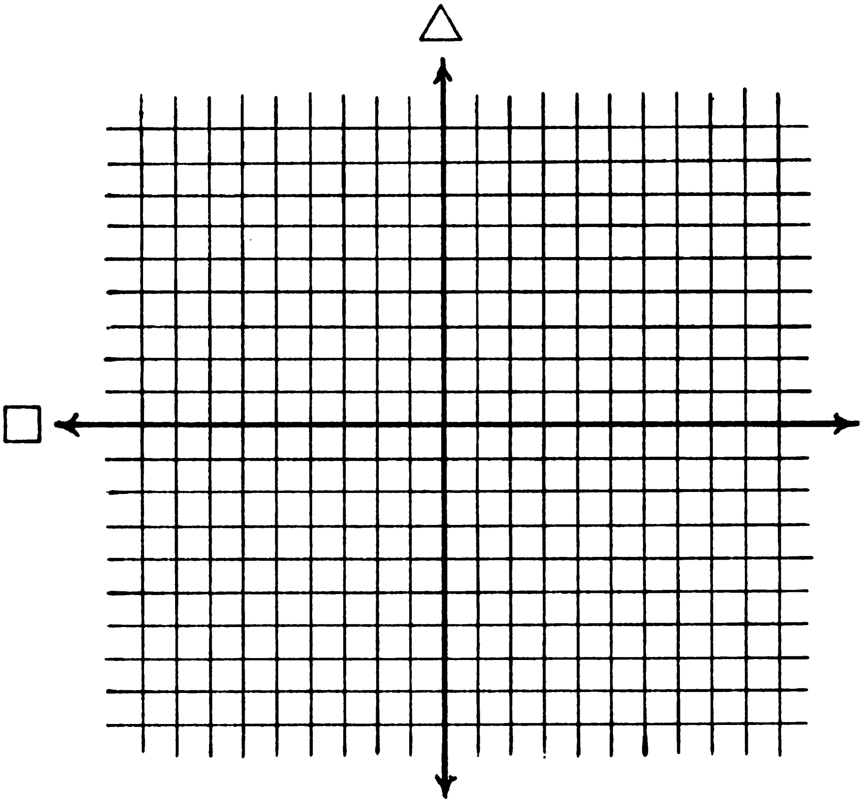
$\square$	$\triangle$

EXERCISE 3

Make a table and graph of the truth sets of the following open sentences.

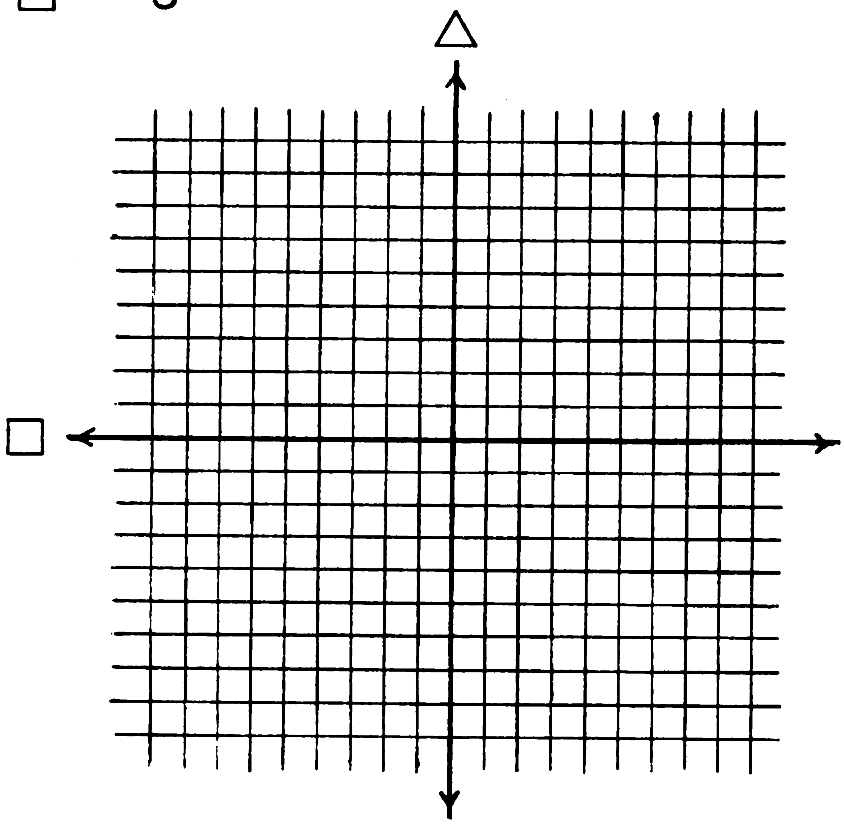
1. △ = □

□	△



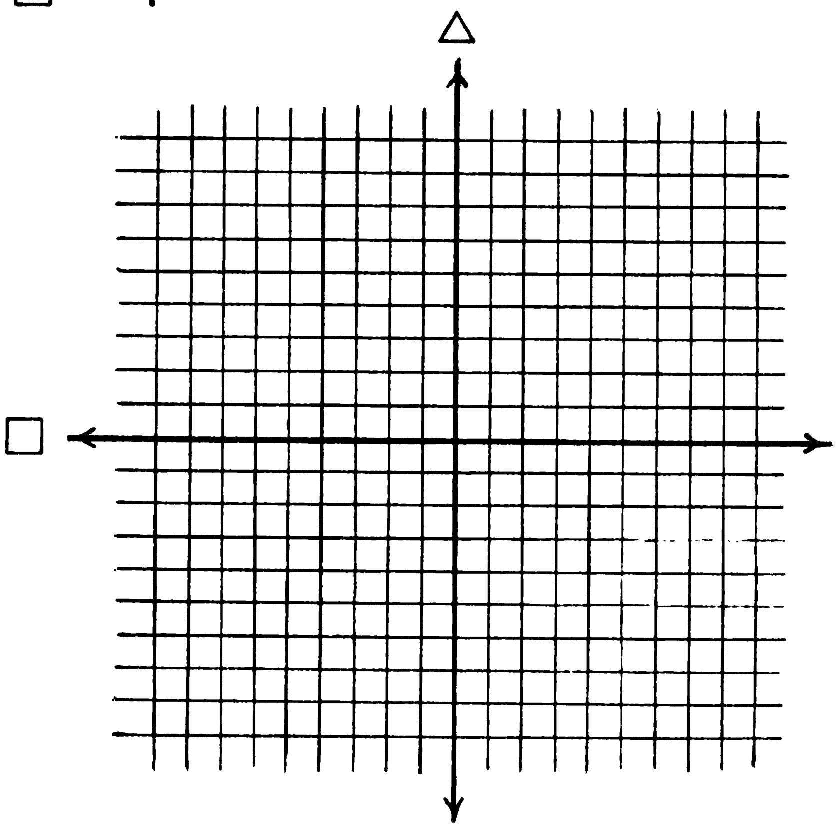
2. $\triangle = 2 \times \square + 3$

$\square$	$\triangle$



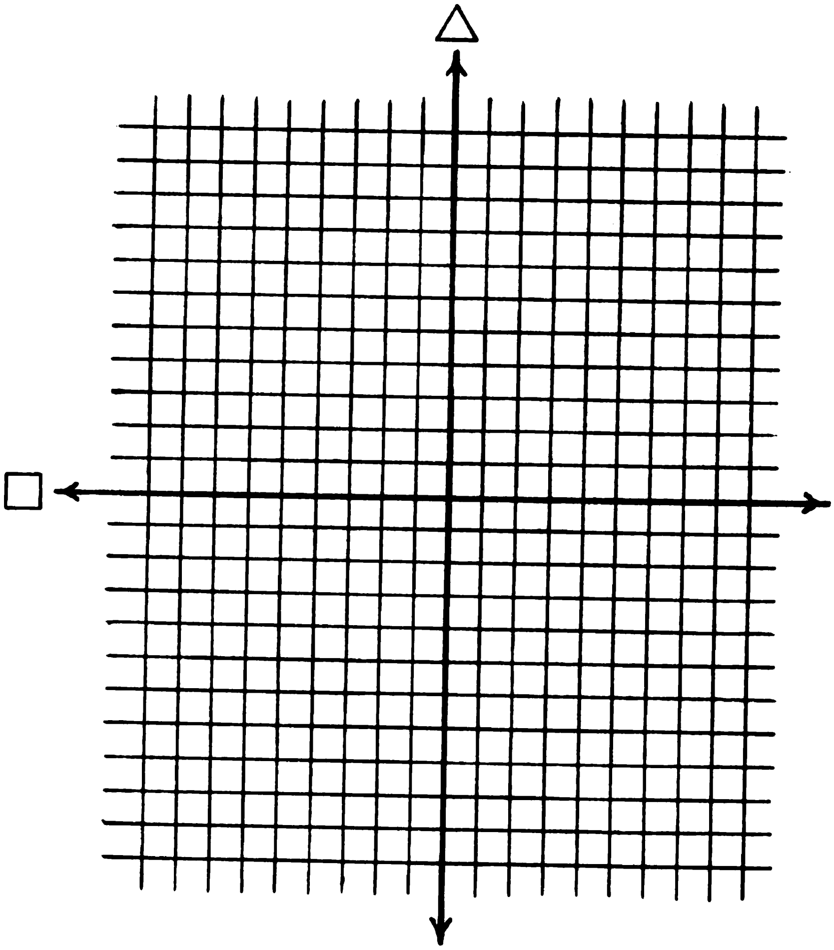
3. $\triangle = 2 \times \square + 1$

$\square$	$\triangle$



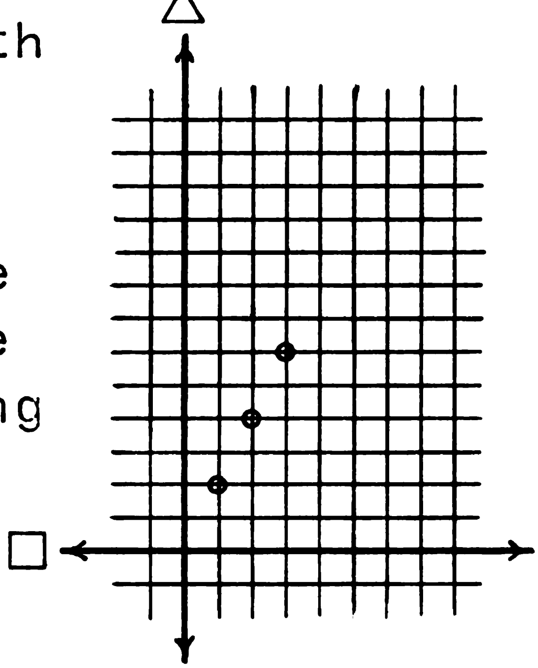
4. △ = 3 x □ - 2

□	△



EXERCISE 4

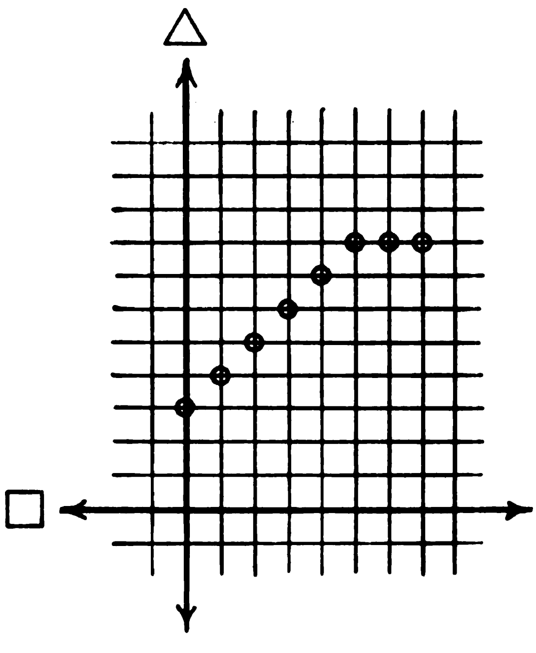
1. At the right is the graph for the truth set of $\triangle = 2 \times \square$. Is this correct?



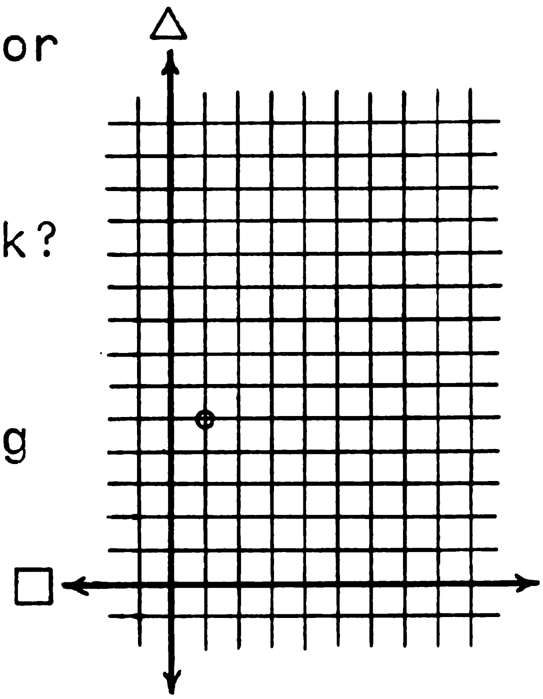
2. Can you mark three more points on the graph without doing the arithmetic? (By looking at the geometric pattern.)

3. Check your points by substituting. Are they right?

4. At the right is the graph for the truth set of $\triangle = \square + 5$. Is the graph right?

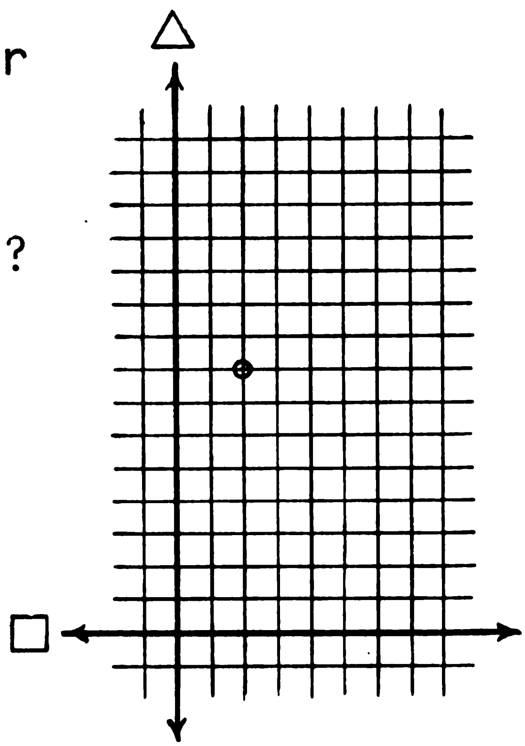


5. At the right is the start of a graph for the truth set of $\triangle = 3 \times \square + 2$. Does the point work?



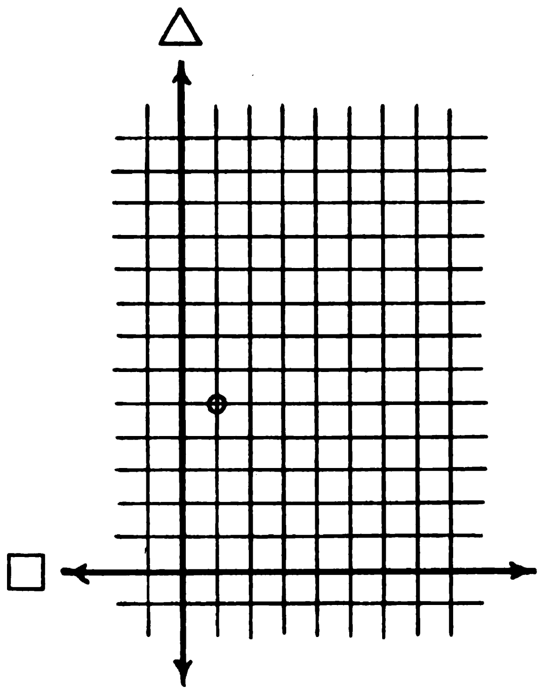
6. Can you mark three more points on the graph without doing the arithmetic?
7. Check your points by substituting into the equation.
8. Were your points right?

9. At the right is the start of a graph for the equation $\triangle = 5 \times \square - 2$. Does the point work?



10. Can you mark two more points on the graph without doing the arithmetic?
11. Substitute into the open sentence. Do you get a true statement?
12. Were your points right?

13. Below is the start of a graph for △ = 2 x □ + 3. Do you agree?
14. Can you mark three more points on the graph without doing the arithmetic?
15. Substitute into the equation.
16. Were your points right?

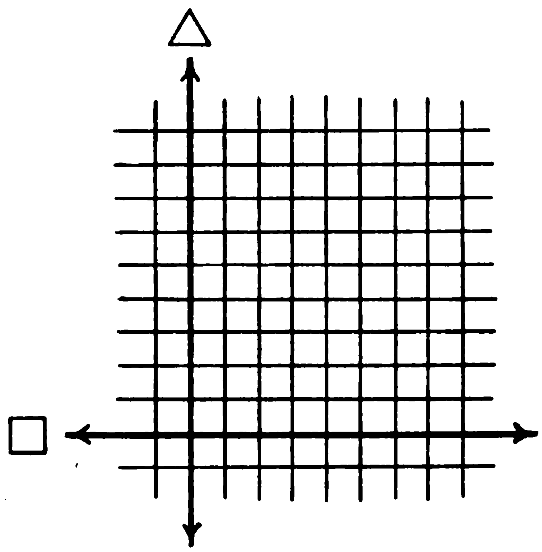


EXERCISE 5

Complete the table and plot the graph for each of the following equations.
Can you find the pattern for each graph?
Can you see the relationship between the pattern of each graph and its equation?

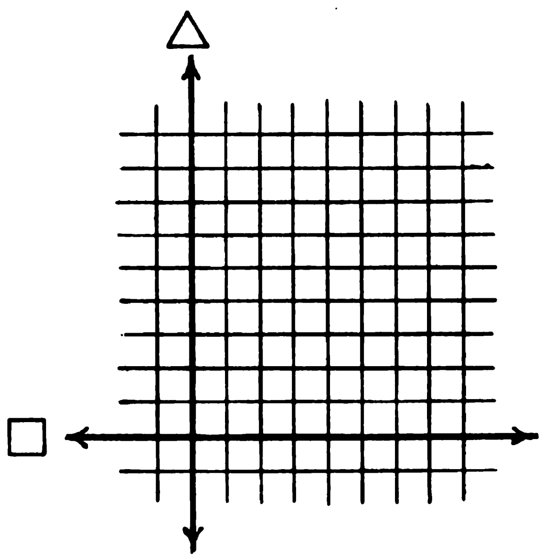
1. △ = 2 x □

□	△
0	
1	
2	
3	



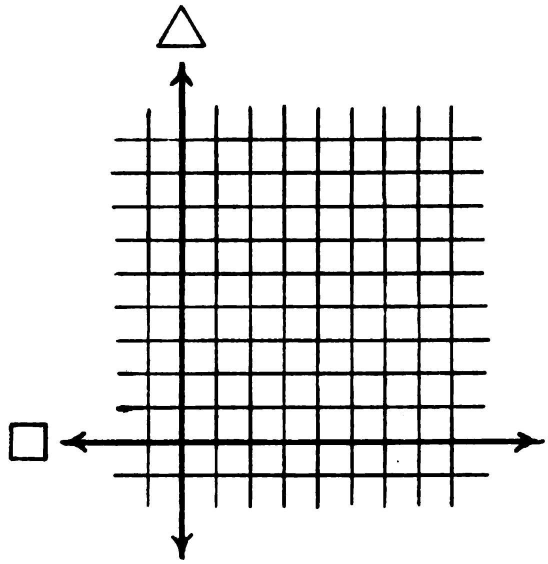
2. △ = 3 x □

□	△
0	
1	
2	
3	



3. △ = □

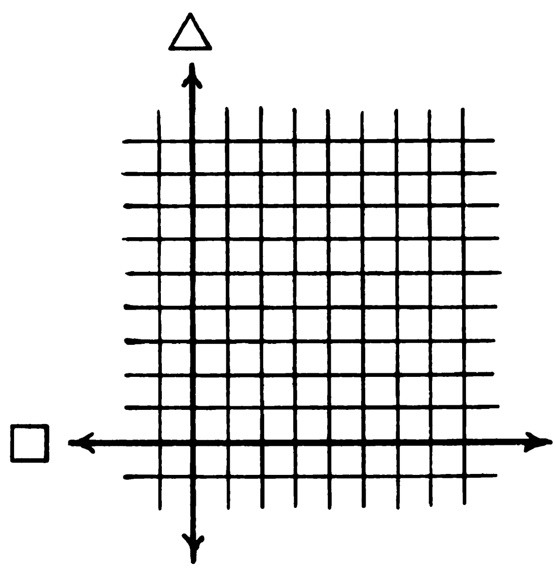
□	△
0	
1	
2	
3	



Complete the table and plot the graph for each of the following equations. Where does the graph cross the △ axis?
Do you see a relationship between the intersection and the equation?

4. △ = □ + 1

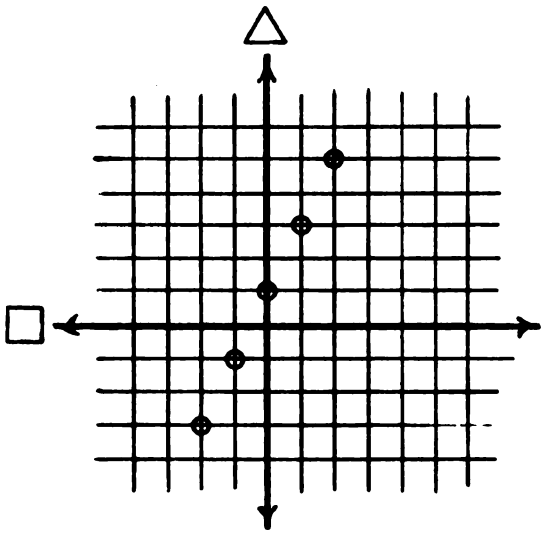
□	△
0	
1	
2	
3	



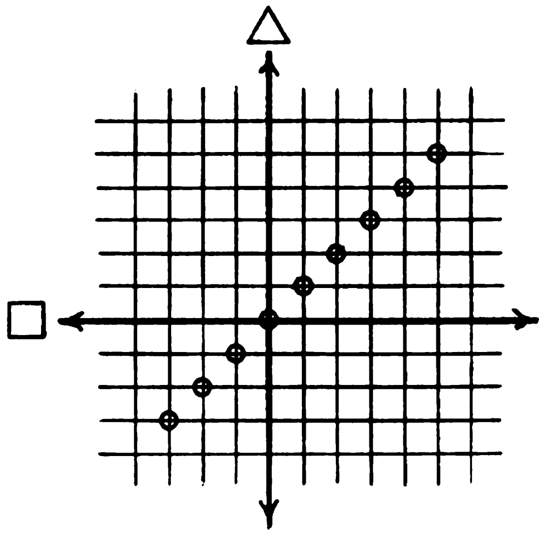
EXERCISE 6

Complete the open sentences for each of the following graphs.

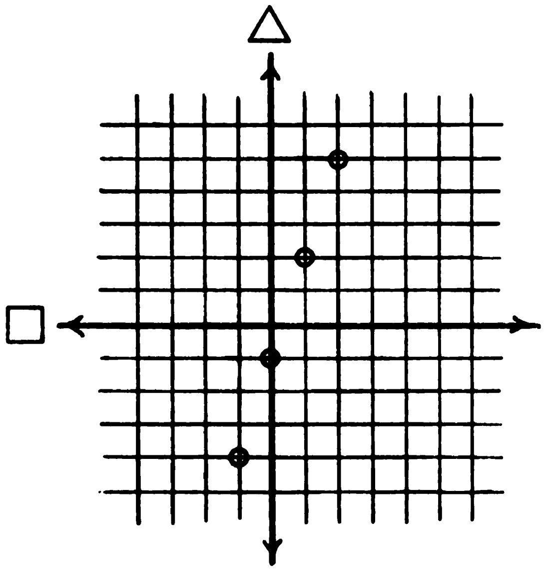
1. △ = __ x □ + __



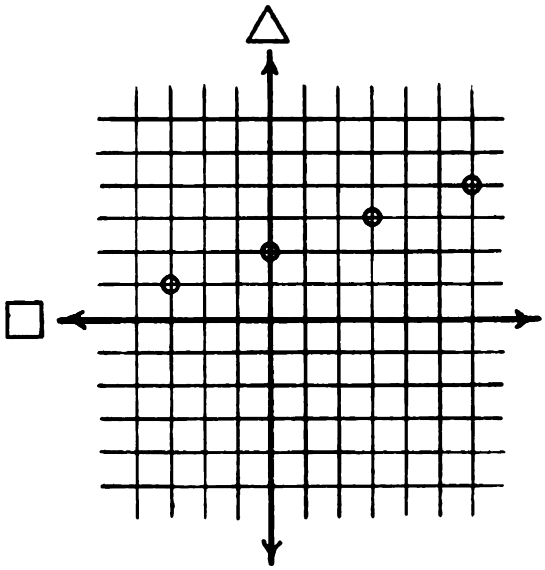
2. △ = __ x □ + __



3. △ = __ x □ + __

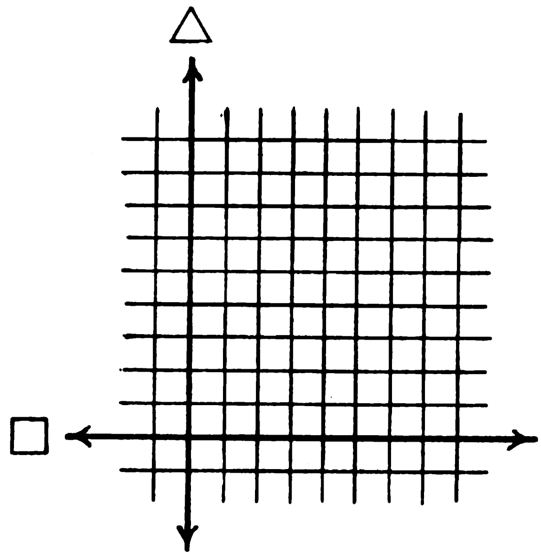


4. △ = __ x □ + __



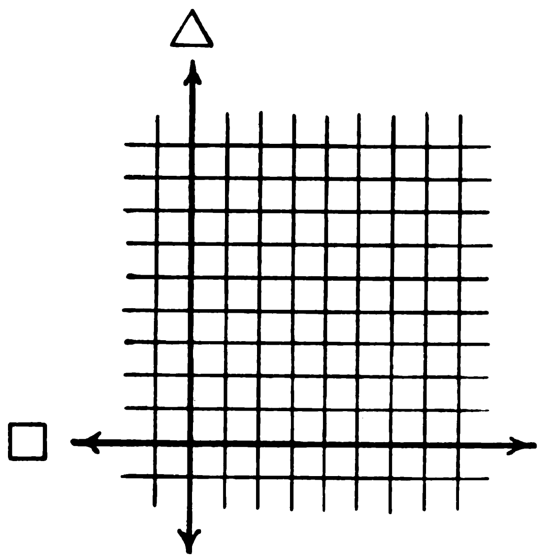
5. △ = □ + 2

□	△
0	
1	
2	
3	



6. △ = □ + 3

□	△
0	
1	
2	
3	



EXERCISE 7

Make up stories about the following number sentences and compute the outcome (result) of your story. You may tell your stories orally and then write the answers.

1. $^{+}5 + ^{-}2 =$
2. $^{-}2 + ^{-}2 =$
3. $^{+}5 + -6 =$
4. $^{-}7 + ^{+}9 =$
5. $^{-}5 + ^{+}1 =$
6. $^{-}3 + ^{+}0 =$
7. $^{-}2 + ^{-}5 =$
8. $^{+}6 + ^{+}3 =$
9. $^{+}8 + -1 =$
10. $^{-}3 + ^{-}6 =$
11. $^{+}1 + ^{+}12 =$
12. $^{+}7 + -9 =$
13. $^{+}2 + ^{+}17 =$
14. $^{+}5 + ^{-}10 =$
15. $^{+}10 + ^{-}5 =$

EXERCISE 8

Make up stories about the following number sentences and compute the outcome (result) of your story. You may tell your stories orally and then write the answers.

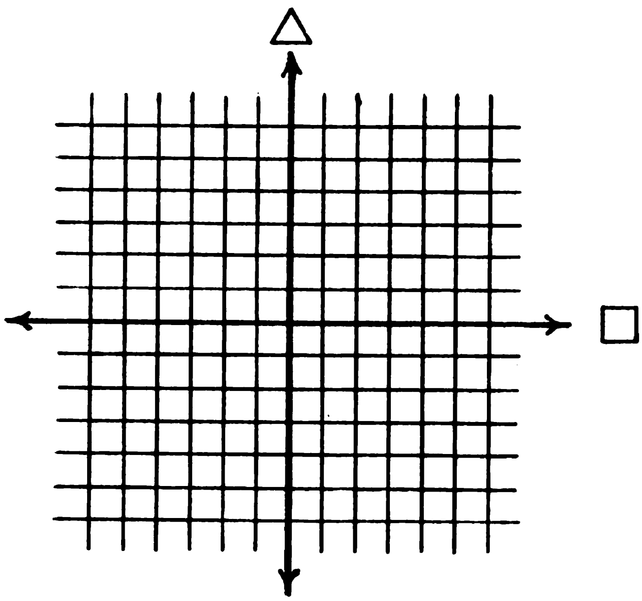
1. $^{+}2 \times {}^{-}1 =$
2. $^{-}3 \times {}^{+}2 =$
3. $^{-}3 \times {}^{+}1 =$
4. $^{-}1 \times {}^{+}6 =$
5. $^{+}1 \times {}^{-}3 =$
6. $^{+}2 \times {}^{+}12 =$
7. $^{-}2 \times {}^{-}4 =$
8. $^{-}1 \times {}^{-}6 =$
9. $^{+}3 \times {}^{-}8 =$
10. $^{-}2 \times {}^{+}7 =$
11. $^{+}3 \times {}^{-}5 =$
12. $^{+}4 \times {}^{-}7 =$
13. $^{+}3 \times {}^{+}11 =$
14. $^{-}4 \times {}^{+}5 =$
15. $^{+}6 \times {}^{-}7 =$

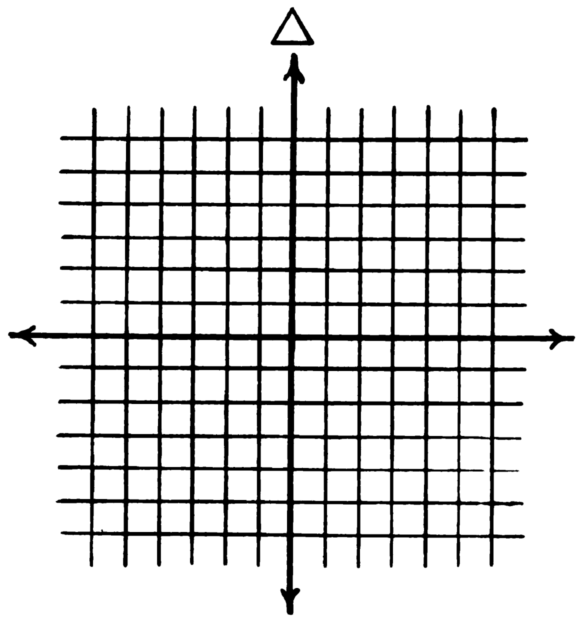
EXERCISE 9

Plot the graphs of the following equations.

1. $\triangle = {}^{+}2 \times \square + {}^{-}1$

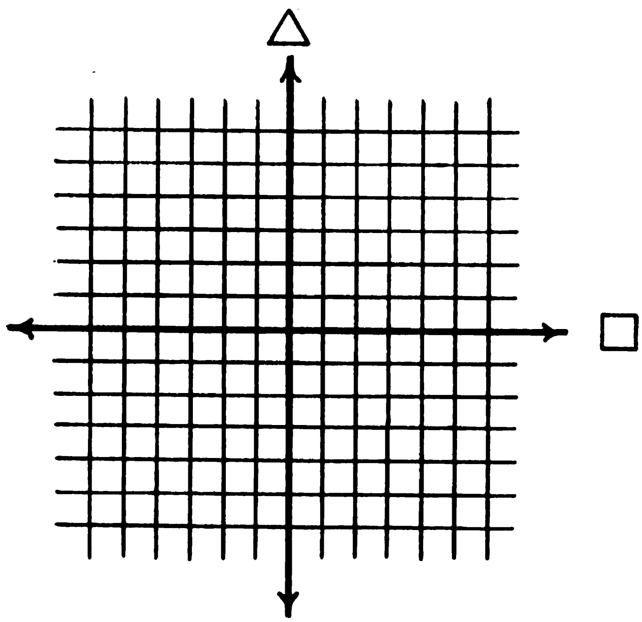
2. $\triangle = {}^{-}2 \times \square + {}^{-}1$

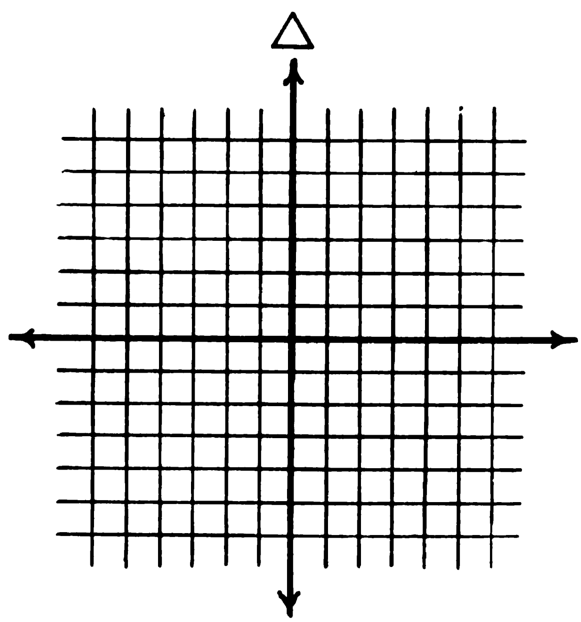




3. $\triangle = {}^{+}1 \times \square + {}^{+}1$

4. $\triangle = {}^{-}1 \times \square + {}^{+}1$

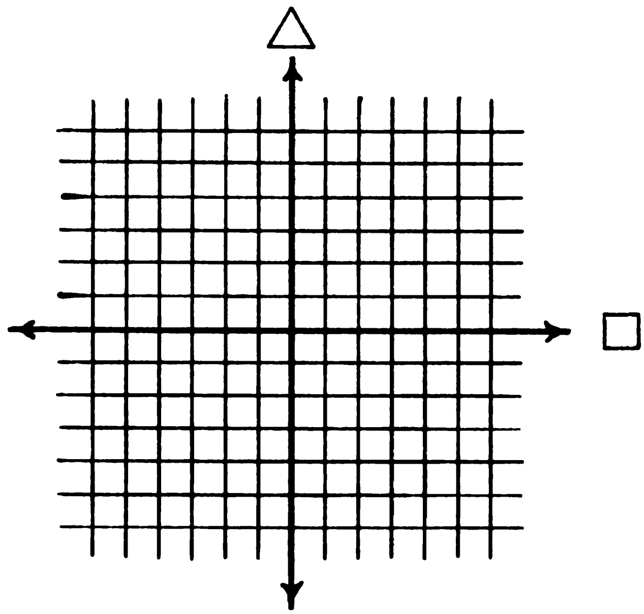




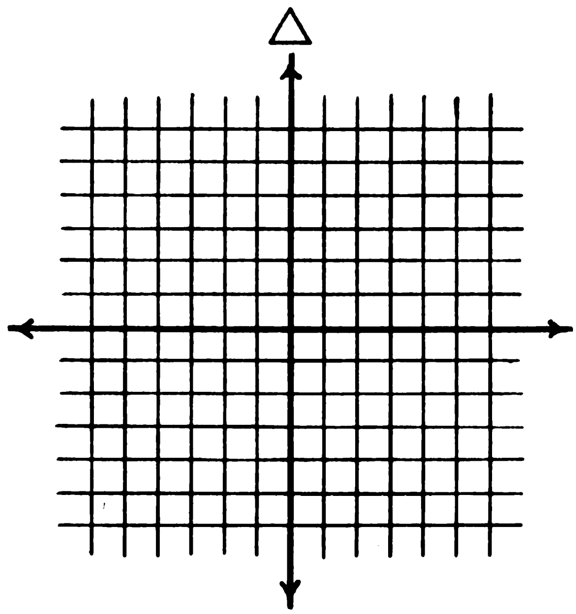
EXERCISE 10

Draw continuous graphs of the following equations.

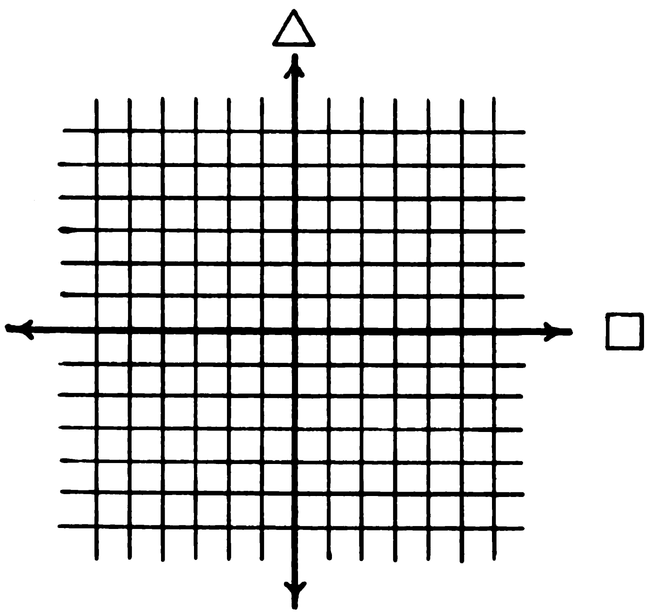
1. $\triangle = {}^{-}2 \times \square + {}^{+}3$



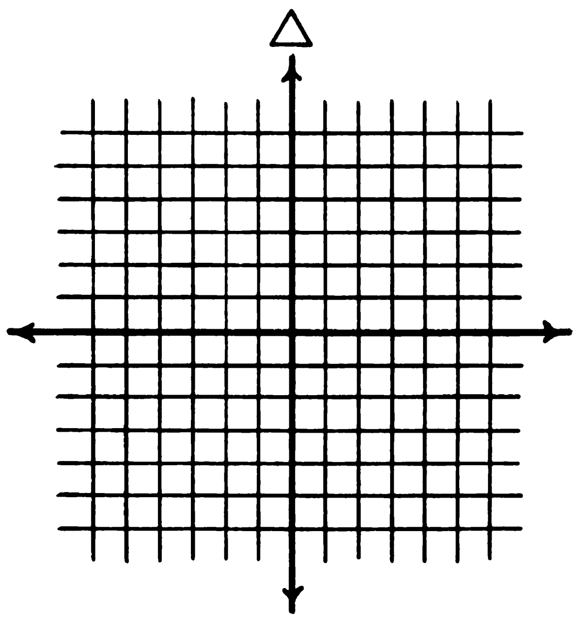
2. $\triangle = {}^{+}3 \times \square + {}^{-}2$



3. $\triangle = {}^{-}3 \times \square$



4. $\triangle = {}^{+}1 \times \square + {}^{-}2$



APPENDIX C

SYNOPSIS OF PILOT CLASSES FOR ELEMENTARY SCHOOL UNIT ON COORDINATE GEOMETRY

PART I

FIRST PILOT CLASS

APPENDIX C

SYNOPSIS OF DAILY SESSIONS

Part I: First Pilot Class

Session 1

Pupils were familiar with the horizontal number line. One student drew a number line on the board. Individual pupils went to the board and made dots with colored chalk to represent points designated by the instructor. The instructor marked points on the number line and asked pupils to name these points. Attention was directed to the fact that each point corresponded to one number and that to each number there corresponded one point.

When pupils could place points and name them unerringly, a point was placed approximately one unit above (3,0). When asked to name this point, pupils answered three. Reminded of the one-to-one correspondence between points on the number line and numbers, pupils decided to draw a line through the new point parallel to the original number line. This line was drawn and a point placed about one unit above the 3 on the newly constructed number line. If the point which represented 3 was to be discussed, the class was asked, "which point would you choose?" One student suggested that the original point be called "3 on number line 1," the second, "3 on number line 2," and the third, "3 on number line 3." By questioning the labelling of a point midway between the first two number lines, the first number line was renamed from 1 to 0. From this suggestion, the idea of a vertical number line emerged. Through these and other leading questions, pupils grasped the requirement of two numbers to locate a specific point on a plane. Stimulating pupils to build a vertical number line required the entire period.

Session 2

Using an overhead projector, a 7 x 7 grid of the first quadrant was projected on a screen. A student was asked to place a dot representing the point (1,2). He placed a dot (P_1) on the grid as shown in Figure 1. Many pupils agreed that the point was located correctly. One of the few who did not agree, marked a second point (P_2) on the grid (Figure 2). The importance that a specific point, such as (1,2) has a single location was evident.

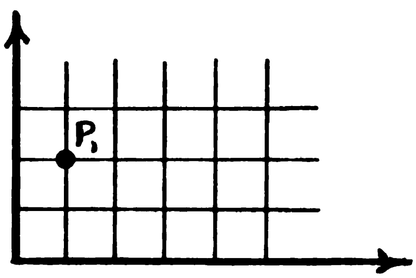


Figure 1

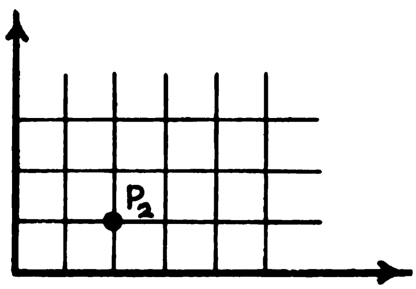


Figure 2

The class was told that consent as to which point should designate (1,2) had already been reached by mathematicians who agreed to name the horizontal distance first and the vertical distance second. The word "coordinates" of a point was defined, the notation described, and the significance of each number in the pair stressed.

Placing a point underneath the horizontal number line elicited the idea of extending the vertical number line below 0. Pupils called the numbers below 0, minus 1, minus 2, etc. Because "minus" implies subtraction and may cause confusion, students were informed of the accepted negative 1, negative 2, etc. nomenclature. The lesson ended with a discussion of positive and negative numbers connotating opposites.

Session 3

Content of the two previous lessons was reviewed. Pupils were asked to plot specific points in quadrants I and IV and to explain the meaning of each number in the pair of coordinates. Pupils saw that the location of points to the left of the vertical axis required the extension of the horizontal number line to the left of 0. The division of the plane into four sections by the number lines was pointed out. These sections were named quadrants I through IV in counterclockwise direction and each number line (horizontal and vertical) was referred to as an axis. Identifying points in all four quadrants was accomplished by having students plot points on a 16 x 16 grid on the overhead projector and calling on other students to name them.

Pupils in this class were seated with their desks pushed together in groups of four. Each group of pupils was supplied with four geoboards and small cylinders made by cutting straws into pieces. The geoboards were laid adjacent to each other so that each represented one quadrant. Pupils took turns placing the straw cylinders over the nails representing the points whose coordinates were called.

A distinct disadvantage accompanied the use of four geoboards to simulate the coordinate plane. The suggested line segments where the boards joined were considered to be the axes. As there were no nails on these "lines," points lying on the axes could not be plotted. If only one geoboard were used, a different difficulty resulted. Each geoboard was only 5 x 5. If rubber bands were stretched around the middle row and column to represent axes, little latitude remained for the choice of points. Because both situations were undesirable, geoboards were not used again even though pupils enjoyed this activity.

Session 4

Exercise 1 was distributed. The low reading level of the pupils was overwhelmingly evident. Without reading directions, students, with hands raised at every table, said they did not know what to do. Both the regular teacher and the researcher moved from pupil to pupil for the entire time allotted. The majority of the class could not work independently. Only a few worked without assistance. They were allowed enough time to complete Exercise 1 which was collected for diagnostic purposes.

Sentences, true, false, and those which were impossible to judge as either, were discussed. The latter were classified as open sentences. Pupils readily volunteered examples of all three kinds. The discussion was guided toward mathematical sentneces. Examples, as $3 \times 5 = 15$ and $2 + 1 = 4$, were written on the board and classified as true or false. The open sentence, $\triangle = \square + 1$, was introduced and students suggested pairs of numbers for $\triangle$ and $\square$. The discussion was guided toward choosing pairs of numbers whose coordinates differed by 1. Pupils could see that for a specific pair such as (2,3), if 2 replaced $\square$ and 3 replaced $\triangle$, a true sentence was produced, whereas if 3 replaced $\square$ and 2 replaced $\triangle$, a false sentence was produced. Therefore, an agreement was made whereby pupils named the number replacing $\square$ first and that replacing $\triangle$, second. Those pairs of numbers that made a true statement were identified as belonging to the truth set of the open sentence. Students were encouraged to find as many pairs as they could that would make the sentence true. Consequently, the truth set was shown to have so many members that all of them could not be enumerated. Since pupils were unfamiliar with set notation, the use of braces to enclose the truth set, the separation of these elements by commas, and the use of ellipsis (...) to indicate that the set did not end was explained.

Session 5

Exercise 1 was returned and discussed. Pupils had especially enjoyed decoding the "secret message." Five pupils missed six or more of the 22 parts. An open sentence was considered and some members of its truth set listed in set notation. The possibility of other methods for displaying truth sets was posed. One pupil suggested making two columns, one for the pairs of elements that made the sentence true, the other for elements that made the sentence false. This suggestion was followed. Then it was explained that the true and not the false sentences were the main interest. Forgetting the column of elements that produced false sentences, the other column offered a second method of displaying the truth set of a sentence, namely, a table. Tables for several open sentences were constructed on the blackboard.

Exercise 2 was given to students to complete in class. Again, many pupils depended on the teacher for additional directions and encouragement. The greatest difficulty lay in finding a number for $\square$. (Some were concerned because their work differed from that of their neighbors, yet both were correct.) They hesitated to perform the process--select <u>any</u> number, "plug" it into $\square$, and find $\triangle$.

Session 6

Pupils were allowed time to finish Exercise 2 which was collected. Various tables, randomly selected from this set of papers, were copied on the board and verified for accuracy.

The class could not visualize a method other than listing the elements in set notation and the use of tables for representing truth sets of open sentences until asked if they thought open sentences and plotting points could be linked. The same student who offered the idea that led to constructing a table immediately suggested plotting the number pairs listed in the tables on a grid. The set of elements for each table was plotted. Such a display of points, students were told, was called a graph. They now knew three methods for representing the truth set of an open sentence.

Session 7

Exercise 2 was returned. The three pupils who did poorly were given individual assistance later in the period. Constructing a graph for an open sentence was reviewed. Exercise 3 was given to the class to work for the remainder of the period.

Session 8

The class was divided into two teams for the purpose of playing a modified version of the Madison Project's tic-tac-toe. The game, which took the entire period, was enthusiastically accepted by the pupils whose display of competitive excitement was gratifying.

Session 9

Exercise 3, collected at the end of Session 7, was returned. One student copied his table for $\triangle = 2 \times \square + 3$ (No. 2 from Exercise 3) on the board and proved that each ordered pair was a member of the truth set of this equation. Another student plotted the points on a grid which was projected. The class was asked if anyone could place another point on the graph without doing any arithmetic. A point was plotted by one pupil, its coordinates were noted and used as replacements for $\triangle$ and $\square$ for verifying the sentence. A second pupil added another point and the process was repeated. These pupils were asked to tell the class the pattern that determined where they placed a new point. Pupils placed points confidently because, as they remarked, the graphs for exercise were straight lines.

Exercise 4 was distributed. The questions contained therein were designed for challenging pupils to find the "secret" of correctly placing additional points on graphs without performing arithmetic. Verbal assistance was not given. Pupils able to discover this secret were urged not to tell their classmates.

Session 10

Pupils continued to work Exercise 4. Most of the pupils could add points to a graph that contained more than one point, but they were unable to complete the three graphs in the exercise that contained only one point. Some pupils disregarded directions and completed the graphs by substituting numbers for $\square$.

Session 11

Pupils were given Exercise 5 which directed their attention from a graph to its sentence. The class performed the activities suggested in the exercise and answered the questions, but only one pupil perceived that the multiplier of $\square$ determined the pattern of the graph and that the constant denoted the $\triangle$-intercept.

Session 12

The pupil who saw the relationship between the graph and its open sentence told the class this "secret." Several pupils added points to graphs taken from Exercises 4 and 5. They then completed the exercises. "Telling" the class this information violated the Madison Project technique of proceeding to a different activity and returning to the same topic later. Revealing "discovery" information was justified as the limited time allotted for presenting materials did not permit the redevelopment of ideas.

Session 13

Using the overhead projector and chalk board, students graphed linear equations by plotting the Δ-intercept and counting out the slope which they determined from the sentence. The entire unit was reviewed.

Session 14

A test on the material covered in the first five lessons was administered, completing the first pilot class.

PART II

SECOND PILOT CLASS

Part II: Second Pilot Class

An observer attended the second pilot class sessions, recorded them, and offered valuable criticism. The taped lessons, and his constructive suggestions, were beneficial in teaching that particular class, revising the lesson plans and preparing for the workshop for teachers. Portions of the tape were played in some sessions of the workshop to demonstrate typical situations that teachers might encounter.

In describing the second pilot class only differences between it and the first pilot class will be elaborated, including those revisions which contributed to improving the presentation of material. Lessons 6 through 10, which were not covered in the first pilot class, will be discussed in detail.

Lessons 1-3

A number line was drawn on the chalk board as in the first pilot class. However, when pupils suggested that a vertical line be constructed, three golf tees were inserted into the pegboard and positioned so that when yarn was wrapped around them, perpendicular line segments were represented. The tees were placed such that a right angle was formed whose interior was the first quadrant. When the thought of plotting points in the exterior of this angle occurred to the students, the golf tees were removed and inserted midway on each of the four edges and in the center of the pegboard. Yarn was wrapped around the upper and lower tees and around the two side tees to form axes. The interiors of the four right angles represented quadrants I, II, III, and IV. The pegboard stimulated interest and proved to be a much more effective aid than either a latticed chalk board or a grid for the overhead projector. Moreover, it created an activity that children thoroughly enjoyed.

Red, yellow, white, and blue golf tees made possible the representation of points in each quadrant by a different color. When playing games, such as tic-tac-toe, the use of different colors made the identification of teams easy.

An unanticipated problem arose from the use of the pegboard. When plotting points on graph paper, a few pupils did not perceive their loci accurately. Some pupils plotted points in the middle of spaces when the coordinates were integers, as e.g., (1,2) in Figure 1.

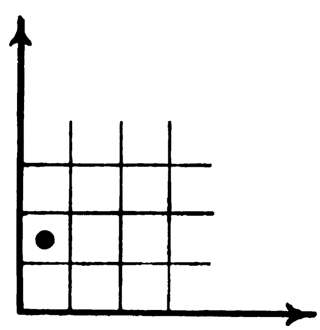


Figure 1

Others represented the points in the upper right corner of the space, as in Figure 2.

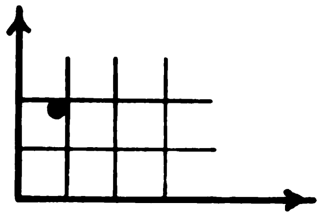


Figure 2

Two corrective measures eliminated the confusion: (1) horizontal and vertical lines were painted on the peg-board such that the holes fell on their intersections, and (2) pupils were shown that the loci of points with integral coordinates were the intersections of lines parallel to both axes at the distances indicated by the ordered pairs.

The nine days spent on Lessons 1, 2, and 3 included a day for the administration of a 25 item multiple-choice test upon the completion of Lesson 2.

Lessons 4 and 5

Lessons 4 and 5 were more successful in the second than the first pilot class. Exercise 3 was returned and discussed individually while pupils worked on Exercise 4. One pupil quickly discovered the relationship between the slope pattern and the equation. Another pupil perceived the clue the second day he worked on Exercise 4 (11th session). With the help of Exercise 5, two additional pupils found the secret.

If a graph contained at least two points, most pupils could find the pattern and place additional points correctly. In fact, most pupils completed both exercises. In answer to the question in Exercise 5, "Can you see the relationship between the pattern of each graph and its equation?", they answered, "yes." When asked, however, they could not describe the relationship. Therefore another question, "What is this relationship?", was added to Exercise 5 for future classes.

After working two and one-half class periods on Exercises 4 and 5, the first pupil who discovered that the multiplier of box indicated the graph's slope was permitted to relate the "secret" to the class. The second discoverer explained the relation between the constant in the equation and the graph. Again, "telling" the class was accepted for the reason stated in the first pilot class.

Lesson 6

Lesson 6 was completed in one session. Graphs were plotted on the pegboard. Pupils stated the equations of these graphs using the information they had discovered (or were told) in Lessons 4 and 5. Exercise 6 involved the same activity in written form.

Some graphs whose slopes were not integers were plotted. Students could determine the slope pattern, but lack of experience with rational numbers was very evident. To describe the slope of the graph in Figure 3, students said, "Write 2. Draw a line. Put a three underneath it."

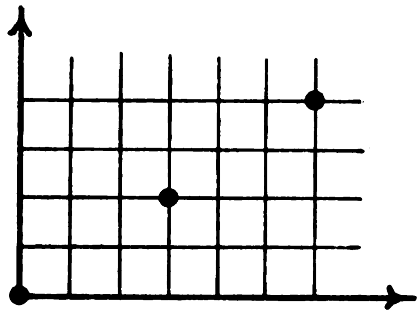


Figure 3

A 20 item multiple choice test was administered on Lessons 4, 5, and 6 during the 14th class session.

Lesson 7

Addition of signed numbers was presented through the use of the Postman Stories suggested in the Madison Project. Pupils participated in two different activities: (1) they figured the amount of money resulting from the postman's delivery of checks and bills, and (2) they made up stories that involved positive and negative numbers. Their first stories were similar to the stories of the instructor. But as they grasped the ideas, their stories displayed more imagination. One boy described a "weird" baseball game in which a team scored -4 runs. When his classmates protested, he retorted, "Well, I said it was weird, didn't I?"

The Postman Stories led to the plotting of equations with a negative constant (Δ - intercept). Two class sessions were devoted to Lesson 7.

Lesson 8

Replacing box with negative numbers required the ability to multiply signed numbers. Again the Postman Stories were used to convey the concepts. Pupils participated in the same activities for multiplying signed numbers as they did for adding signed numbers. The two activities are listed in Lesson 7. In pupil Exercises 7 and 8, stories were related orally and numerical results were written.

Operations with signed numbers were unquestionably difficult for some pupils. Insufficient command of arithmetic combinations was a stumbling block.

Two class sessions were spent on Lesson 8 and one additional class session reviewed the combination of Lessons 7 and 8.

Lesson 9

Pupils practiced graphing equations with negative slopes and negative intercepts. Equations which were dictated were graphed by one pupil on the pegboard and simultaneously by another pupil on the chalkboard. They enjoyed working with the pegboard again. It had not lost its appeal.

Lesson 10

The elements, (0,0), (1,8), and (2,16), of the number sentence △ = 8 x □ were plotted on the pegboard. Students were asked to compute the value for △ if □ were 1/2. The locus, (1/2, 4), was estimated and a pin with a large red head was stuck into the pegboard. Similarly, the points (1/4, 2) and (1/8, 1) were plotted. The class approximated △ when values for □ were chosen between two known points. They could see that if many points were chosen, the pins would soon touch each other. The conclusion of the class that the graph would be a continuous line was accepted because they had no knowledge of irrational numbers. When students plotted other equations, yarn was looped around the peg that represented the △ – intercept and the last inserted peg (after the slope had been counted out several times) to show that the graph was a continuous line if box were not restricted to integers.

The class spent the rest of the session on Exercise 10. This exercise was discussed and Lessons 7 through 10 were reviewed during the twenty-second session. A test on these lessons was administered in the following (last) session.

APPENDIX D

TEST ON COORDINATE GEOMETRY

(TOGC)

PART I

ELEMENTARY SCHOOL TOCG

COORDINATE GEOMETRY TEST FOR ELEMENTARY STUDENTS

DIRECTIONS: Choose the response that correctly completes the sentence. Mark the letter corresponding to this response on the answer sheet.

1. Point P whose coordinates are (2,-1) lies in
   A. Quadrant I B. Quadrant II
   C. Quadrant III D. Quadrant IV
   E. None of these

2. What is the □ coordinate of every point on the △ axis?
   A. 1 B. -1
   C. 0 D. None of these

3. In plotting a point, we agree to name first the
   A. △ intercept B. △ coordinate
   C. slope D. □ coordinate

4. Point P whose coordinates are (-2,1) lies in
   A. Quadrant I B. Quadrant II
   C. Quadrant III D. Quadrant IV
   E. None of these

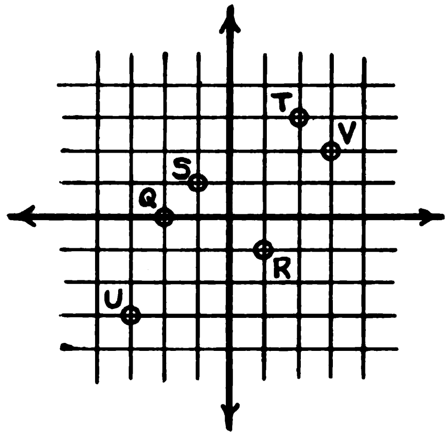
5. Some members of the truth set of the open sentence $\triangle = 2 \times \square + 4$ are

   A. (5,14) B. (0,4)

   C. (6,16) D. A and C

   E. All of these

Answer questions 6 through 10 from the graph at the right.



6. The coordinates of point R are

   A. (2,3) B. (-1,1)

   C. (3,2) D. (1,-1)

   E. None of these

7. The coordinates of point S are

   A. (-3,3) B. (-1,1) C. (1,-1)

   D. (1,1) E. None of these

8. The coordinates of point T are

   A. (3,2) B. (-1,1) C. (2,2)

   D. (2,3) E. None of these

9. The coordinates of point U are

   A. (2,-3) B. (3,2) C. (3,-3)

   D. (-3,-3) E. None of these

10. The coordinates of point Q are

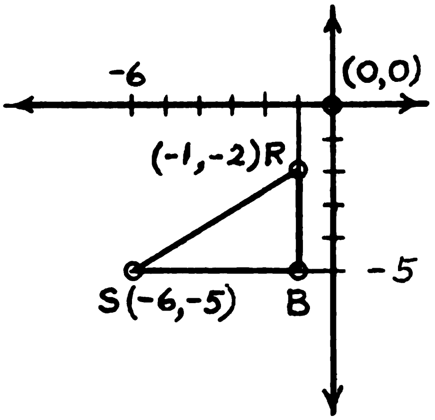
    A. (0,2) B. (2,0) C. (0,-2)

    D. None of these

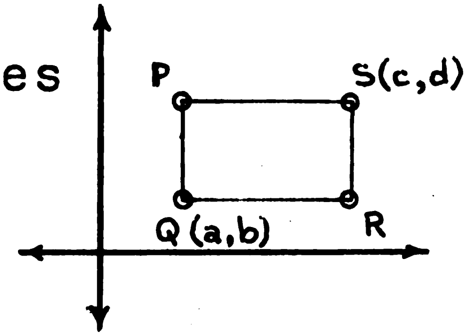
11. The coordinates of point A are (0,8). The coordinates of point B are (0,-5). The distance from A to B is
   A. 3   B. 13   C. 5
   D. None of these

12. A point whose □ coordinate is negative lies in
   A. Quadrant I or II
   B. Quadrant II or III
   C. Quadrant III or IV
   D. Quadrant I or IV

13. The coordinates of B in the figure at the right are
   A. (-5,-2)
   B. (-1,-5)
   C. (-2,-5)
   D. (-5,-1)
   E. None of these



14. In the figure at the right, the coordinates of P are
   A. (a,d)   B. (a,c)
   C. (c,a)   D. (d,a)
   E. None of these



15. If the △ intercept is 3, the open sentence is

A. △ = 3 x □ B. △ = 3 x □ -3

C. △ = □ + 3 D. △ = 3 x □ +1

E. None of these

16. Which of the following ordered pairs is not an element of the truth set of △ = 2 x □ + 1?

A. (0,1) B. (2,3)

C. (3,7) D. (1,3)

E. None of these

17. Study the following table. The open sentence for the table is

□	△
2	3
3	5
4	7

A. △ = □ +1

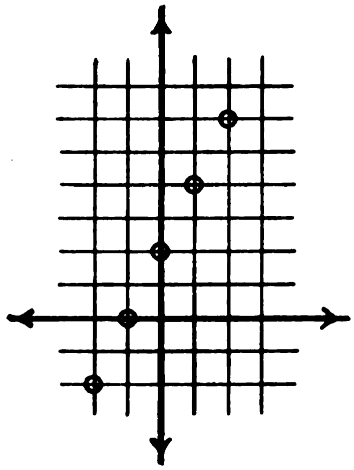
B. △ = □ +2

C. △ = 2 x □ - 1

D. △ = 3 x □ - 3

E. None of these

18. In the graph at the right, the slope pattern is



A. Over one to the right, up one
B. Over one to the right, up two
C. Over one to the left, up two
D. None of these

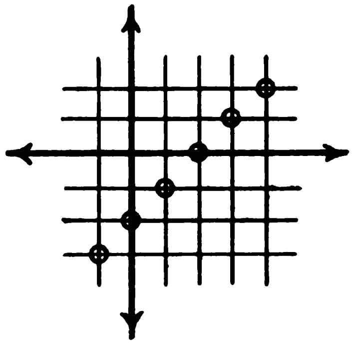
19. In the graph for number 18, the △ intercept is

A. (0,1) B. (-1,0)
C. (0,2) D. None of these

20. Which of the following represents a graph of an open sentence that does not contain the origin?

A. △= 2 x □ - 1 B. △= 2 x □
C. △= 3 x □ - 2 D. A and C
E. A, B and C

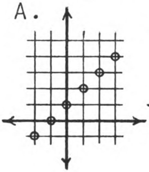
21. The equation of the graph at the right is



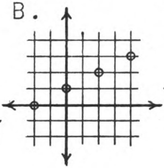
A. △ = 2 x □ - 1
B. △ = 1 x □ + 2
C. △ = 1 x □ - 2
D. △ = 2 x □ + 2
E. None of these

22. Which of the following is the graph of the open sentence △ = 3 x □ + 1?

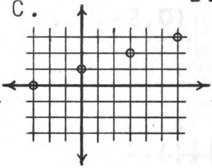
A.



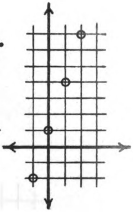
B.



C.



D.



23. Which of the graphs of the following open sentences has a intercept of 3?

A. △ = □ - 2

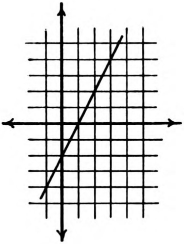
B. △ = 2 x □ + 3

C. 3 x △ = □ - 2

D. 2 x △ = 3 x □ + 1

E. None of these

24. The slope of the graph at the right is



A. Over one to the right, up one

B. Over one to the right, up two

C. Over two to the right, up one

D. Over two to the right, up two

E. None of these

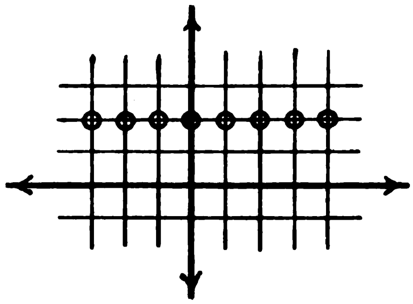
25. The △ intercept for the graph in number 24 is

A. (0,2) B. (1,0)

C. (2,0) D. (-2,0)

E. None of these

26. In the graph at the right, the △ coordinate is always



A. 0

B. Any number

C. 3

D. None of these

27. Inthe open sentence △ = 2 x □ - 1

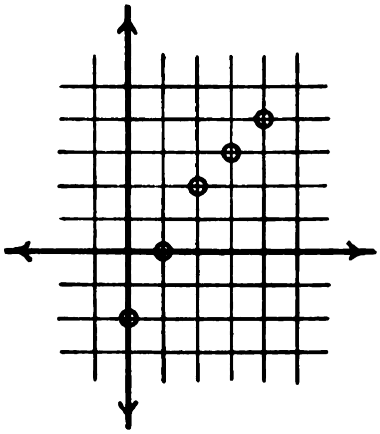
A. △ increases when □ increases

B. △ increases when □ decreases

C. △ decreases when □ increases

D. None of these

28. Does the graph at the right represent the open sentence △ = 2 x □ - 2?



A. Yes

B. No

29. The product of $^{-}2 \times {}^{-}5$ is

A. $^{-}7$ B. $^{+}7$

C. $^{-}10$ D. $^{+}10$

E. None of these

30. The △ intercept of the equation △ = 3 x □ - 2 is

A. The origin B. (3,0)

C. (0,2) D. (-2,0)

E. None of these

31. In the graph of △ = $^{-}3$ x □ + 2,

A. △ increases as □ increases

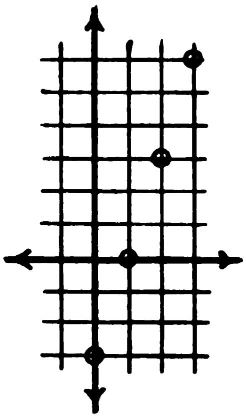
B. △ decreases as □ decreases

C. △ decreases as □ increases

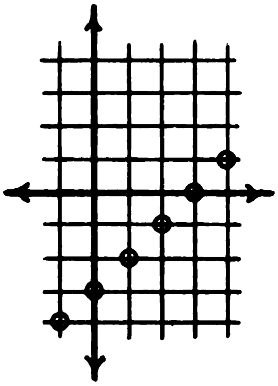
D. None of these

32. Which of the following is the graph of the equation △ = 3 x □ - 3

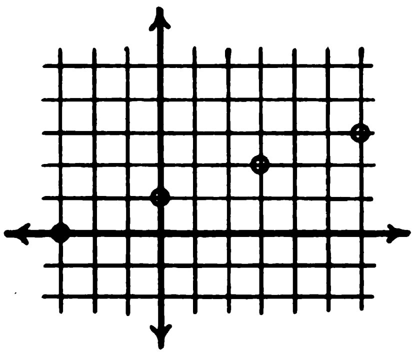
A.



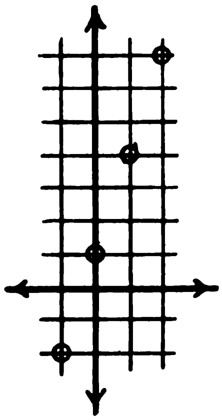
B.



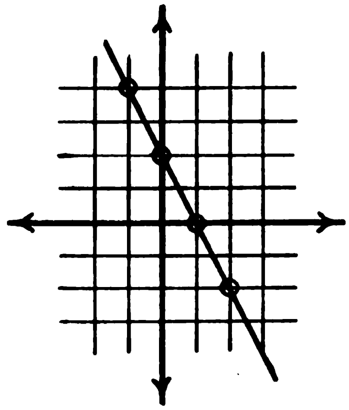
C.



D.



33. The equation of the graph at the right is



A. $\triangle = 2 \times \square + 2$

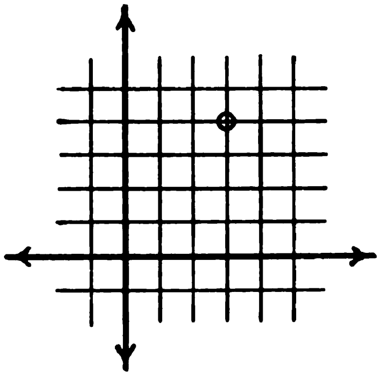
B. $\triangle = {}^{-}2 \times \square + 1$

C. $\triangle = {}^{-}2 \times \square - 2$

D. $\triangle = 2 \times \square + 1$

E. None of these

34. The point represented on the graph at the right is a member of the truth set of



A. $\triangle = 2 \times \square + 1$

B. $\triangle = \square + 1$

C. $\triangle = \square$

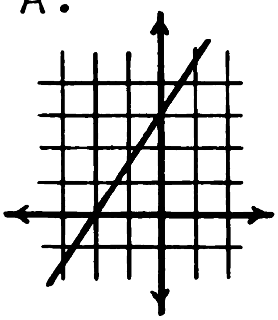
D. $\triangle = {}^{-}2 \times \square + 1$

E. None of these

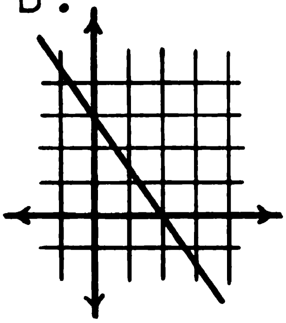
35. Which of the following is the graph of the equation

$\triangle = \frac{3}{2} \times \square - 2$?

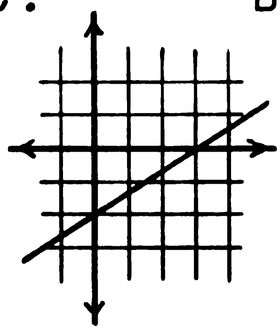
A.



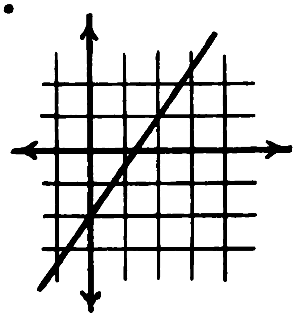
B.



C.



D.



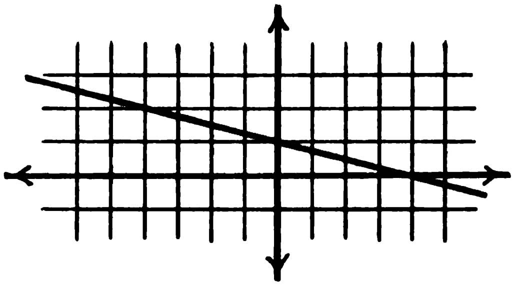
36. In the open sentence △ = ⁻4 x □ - 2, if □ = 4, △ =

A. -18 B. 14

C. 18 D. -14

E. None of these

37. In the graph at the right, the slope pattern is



A. -4 B. 4

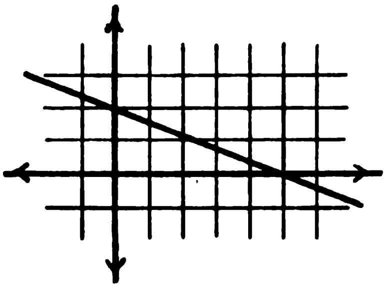
C. $-\frac{1}{4}$ D. None of these

38. In the open sentence △ = T x □ + P, the △ intercept is

A. (0,T) B. (T,0)

C. (T,P) D. None of these

39. In the graph at the right, the slope is



A. $\frac{^-5}{2}$ B. $\frac{5}{2}$

C. $\frac{2}{5}$ D. $\frac{^-2}{5}$

40. The □ intercept for the graph in number 39 is

A. (2,0) B. (5,0)

C. (0,5) D. None of these

PART II

SECONDARY SCHOOL TOCG

COORDINATE GEOMETRY TEST FOR SECONDARY STUDENTS

DIRECTIONS: Choose the response that correctly completes the sentence. Mark the letter corresponding to this response on the answer sheet.

1. Point P whose coordinates are (2,-1) lies in

   A. Quadrant I B. Quadrant II
   C. Quadrant III D. Quadrant IV
   E. None of these

2. What is the x coordinate of every point on the y axis?

   A. 1 B. -1 C. 0 D. None of these

3. In plotting a point, we agree to name first the

   A. y intercept B. y coordinate
   C. slope D. x coordinate

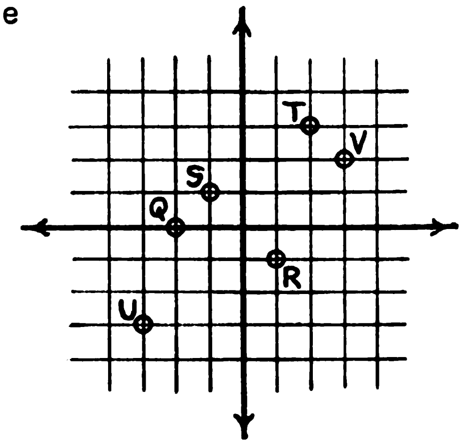
4. Point P whose coordinates are (-2,1) lies in

   A. Quadrant I B. Quadrant II
   C. Quadrant III D. Quadrant IV
   E. None of these

5. Some members of the truth set of the open sentence $y = 2x + 4$ are

   A. (5,14) B. (0,4) C. (6,16)
   D. A and C E. All of these

Answer questions 6 through 10 from the graph at the right.



6. The coordinates of point R are

   A. (2,3) B. (-1,1)
   C. (3,2) D. (1,-1)
   E. None of these

7. The coordinates of point S are

   A. (-3,3) B. (-1,1)
   B. (1,-1) D. (1,1)
   E. None of these

8. The coordinates of point T are

A. (3,2) B. (-1,1) C. (2,2)
D. (2,3) E. None of these

9. The coordinates of point U are

A. (2,-3) B. (3,2) C. (3,-3)
D. (-3,-3) E. NOne of these

10. The coordinates of point Q are

A. (0,2) B. (2,0) C. (0,-2)
D. None of these

11. The coordinates of point A are (0,8). The coordinates of point B are (0,-5). The distance from A to B is

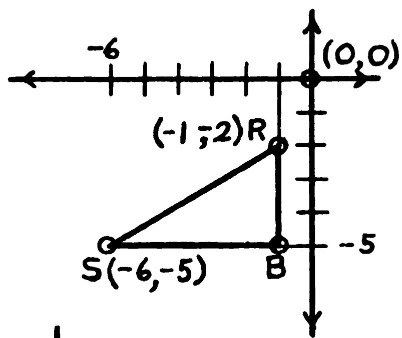
A. 3 B. 13 C. 5
D. None of these

12. A point whose x coordinate is negative lies in

A. Quadrant I or II B. Quadrant II or III
C. Quadrant III or IV D. Quadrant I or IV

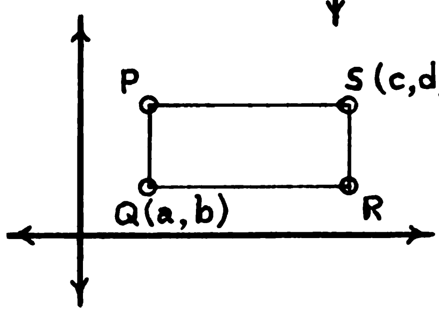
13. The coordinates of B in the figure at the right are

A. (-5,-2) B. (-1,-5)
C. (-1,-5) D. (-5,-1)
E. None of these



14. In the figure at the right, the coordinates of P are

A. (a,d) B. (a,c)
C. (c,a) D. (d,a)
E. None of these



15. If the y intercept is 3, the open sentence is

A. $y = 3x$ B. $y = 3x - 3$
C. $y = x + 3$ D. $y = 3x + 1$
E. None of these

16. Which of the following ordered pairs is not an element of the truth set of $y = 2x + 1$?

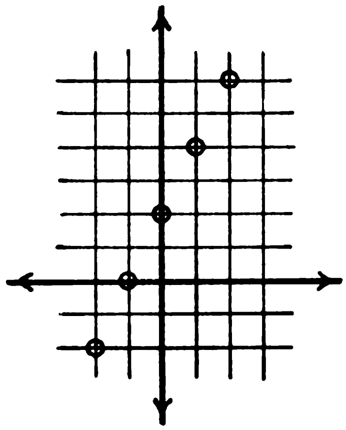
A. (0,1) B. (2,3) C. (3,7)
D. (1,3) E. None of these

17. Study the following table. The open sentence for the table is

x	y
2	3
3	5
4	7

A. $y = x + 1$ B. $y = x + 2$ C. $y = 2x - 1$
D. $y = 3x - 3$ E. None of these

18. In the graph at the right, the slope pattern is



A. Over one to the right, up one
B. Over one to the right, up two
C. Over one to the left, up two
D. None of these

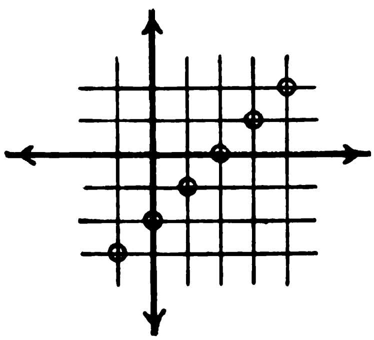
19. In the graph for number 18, the y intercept is

A. (0,1) B. (-1,0 C. (0,2)
D. None of these

20. Which of the following represents a graph of an open sentence that does not contain the origin?

A. $y = 2x - 1$ B. $y = 2x$ C. $y = 3x - 2$
D. A and C D. A, B and C

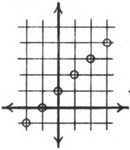
21. The equation of the graph at the right is



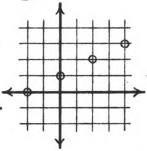
A. $y = 2x -1$ B. $y = x + 2$
C. $y = x - 2$ D. $y = 2x + 2$
D. None of these

22. Which of the following is the graph of the open sentence $y = 3x + 1$?

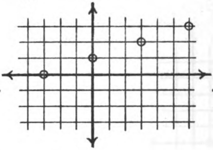
A.



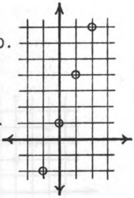
B.



C.



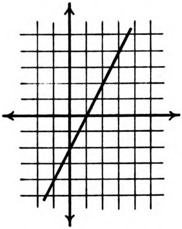
D.



23. Which of the graphs of the following open sentences has an intercept of 3?

A. $y = x - 2$ B. $y = 2x + 3$ C. $3y = x - 2$
D. $2y = 3x + 1$ E. None of these

24. The slope of the graph at the right is

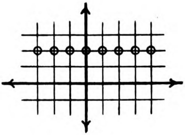


A. Over one to the right, up one
B. Over one to the right, up two
C. Over two to the right, up one
D. Over two to the right, up two
E. None of these

25. The y intercept for the graph in number 24 is

A. (0,2) B. (1,0) C. (2,0)
D. (-2,0) E. None of these

26. In the graph at the right, the y coordinate is always



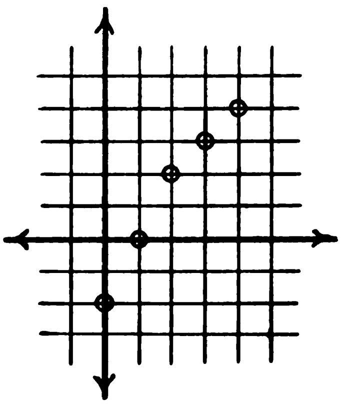
A. 0 B. Any number
C. 3 D. None of these

27. In the open sentence $y = 2x - 1$,

   A. y increases when x increases
   B. y increases when x decreases
   C. y decreases when x increases
   D. None of these

28. Does the graph at the right represent the open sentence $y = 2x - 2$?

   A. Yes B. No



29. The product of -2 x -5 is

   A. -7 B. +7 C. -10 D. +10

30. The y intercept of the equation $y = 3x - 2$ is

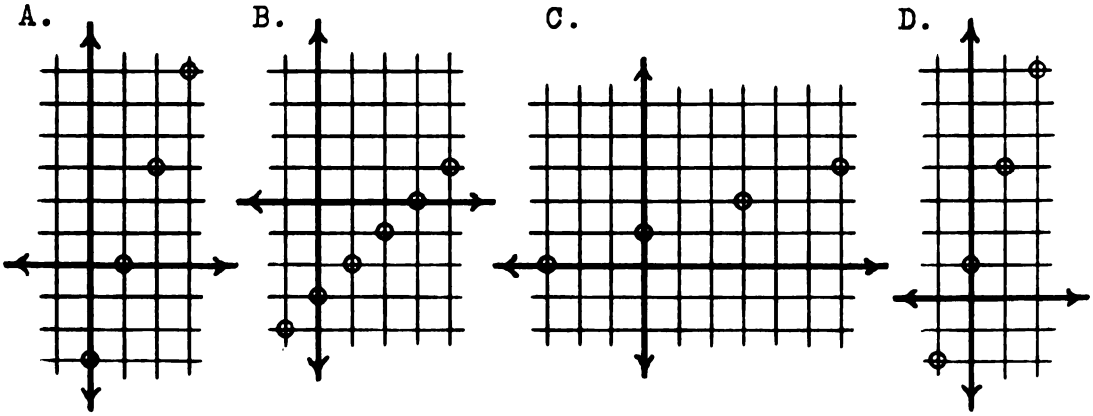
   A. The origin B. (3,0) C. (0,2)
   D. (-2,0) E. None of these

31. In the graph of $y = -3x + 2$,

   A. y increases as x increases
   B. y decreases as x decreases
   C. y decreases as x increases
   D. None of these

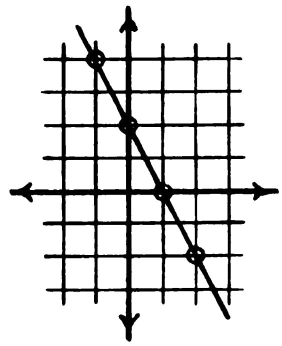
32. Which of the following is the graph of the equation $y = 3x - 3$?

A. B. C. D.



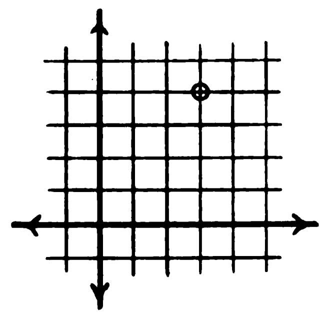
33. The equation of the graph at the right is

A. y = 2x + 2
B. y = -2x + 1
C. y = -2x - 2
D. y = 2x + 1
E. None of these



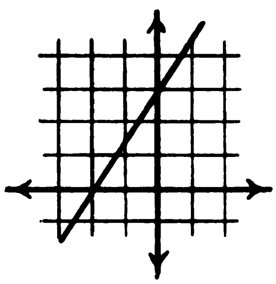
34. The point represented on the graph at the right is a member of the truth set of

A. y = 2x + 1
B. y = x + 1
C. y = x
D. y = -2x + 1
E. None of these

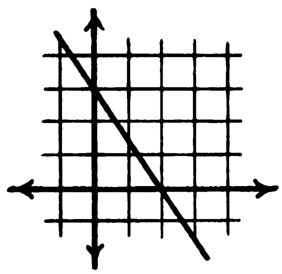


35. Which of the following is the graph of the equation $y = \frac{3}{2}x - 2$?

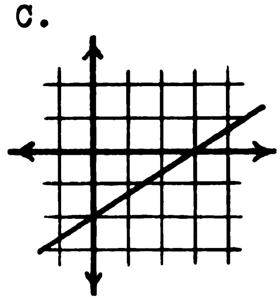
A.



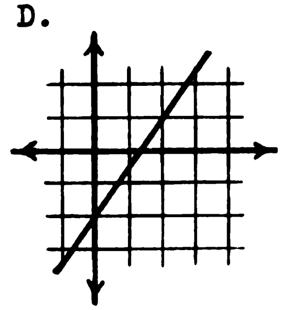
B.



C.



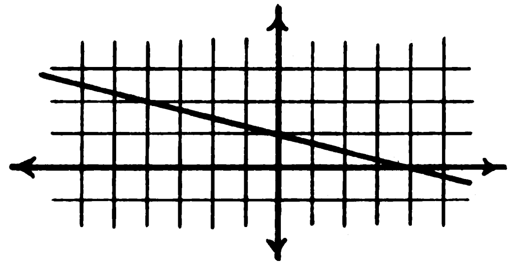
D.



36. In the open sentence y = -4x - 2, if x = 4, y =

A. -18 B. 14 C. 18 D. -14
E. None of these

37. In the graph at the right, the slope pattern is

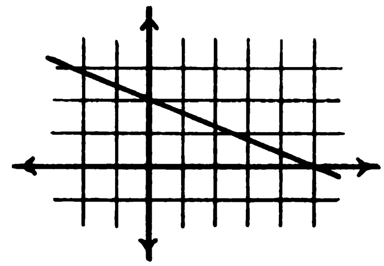


A. -4
B. 4
C. $-\frac{1}{4}$
D. None of these

38. In the open sentence $y = Tx + P$, the y intercept is

A. (0,T) B. (T,0) C. (T,P)
D. None of these

39. In the graph at the right, the slope pattern is



A. $-\frac{5}{2}$ B. $\frac{5}{2}$

C. $\frac{2}{5}$ D. $-\frac{2}{5}$

40. The x intercept for the graph in number 39 is

A. (2,0) B. (5,0) C. (0,5)
D. None of these