A MODEL OF SELF GENERATING INFLATION: THE ARGENTINE CASE

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ABSTRACT

A MODEL OF SELF GENERATING INFLATION:
THE ARGENTINE CASE

by Dean S Dutton

This model was constructed to explain the rather high rates of inflation prevalent in most Latin American countries and other underdeveloped countries as well.

Some features common to these Latin American countries are: (1) The central government runs a large fiscal deficit each year. (2) The deficit is largely financed through the central bank. (3) Because of the method of deficit financing, a large increase in the money supply results. (4) The deficit is due partly to the government's unwillingness to levy sufficient taxes to cover planned expenditures and partly to the fact that the collection of taxes, which have been assessed sometime in the past, do not increase with the price level, but government expenditures do.

Because of these features the inflation tends to perpetuate itself. The purpose of our hypothesis then is to show how this process works, i.e., how the rate of change in the price level, the money stock, and the deficit are all interrelated.

The model comprises a system of four simultaneous equations wherein, (1) the rate of change of the price level is related to rates of change in the per capita money stock and to levels of real per capita money balances, (2) the rate of change of the money stock is

related to rates of change of the monetary base, (3) the rate of change of the monetary base is related to rates of nominal deficit expenditure, and (4) the rate of nominal deficit expenditure is related to the rate of real deficit expenditure (assumed to be constant), a previous period's price level, and the rate of change in the price level during said period.

Price level, money supply, monetary base (currency held by the public plus bank reserves), and increase in Central Bank claims on the government (used as a proxy for the fiscal deficit) data for Argentina for the period 1958-1966 were taken from International Financial Statistics. Population data comes from the Statistical Bulletin for Latin America.

The parameters in equations (1) and (4) were estimated by direct least squares and those in equations (2) and (3) by two stage least squares.

It was concluded that the system of four equations does reasonably well in describing a self generating inflationary process in Argentina for the period 1958-1966.

The dynamic analysis of the model indicates that the time path of the rate of change in the price level free of any external constraints or shocks, is one of damped oscillation which seeks an equilibrium rate of -3.26% per quarter.

A government plan to halt the inflation by eliminating its deficit thereby reducing the increase in the monetary base and the rate of increase in the money stock will have the desired effect only after some time lag. Elimination of the deficit will have its full impact on the monetary base within the current period. The adjustment process takes about a year, however for a 1% reduction in the monetary base to cause a 3/4% reduction in the money stock. It takes another year for a 1% reduction in the money stock to bring about a 1% reduction in the price level. No less than two years would be required to feel the effects of such a plan. However, it seems possible to reduce the rate of price increase from about 3.5% per quarter to 0.0% per quarter in about a year if the rate of increase in the monetary base is reduced to zero.

The results of the study also show that the one equation models of inflation, when applied to countries like Argentina, leave much of the process yet to be explained.

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By

Dean S Dutton

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CHAPTER I

INTRODUCTION

The purpose of this study is to formulate a hypothesis of inflation which is capable of explaining the rate of change in the price level in a particular country when its institutional structures conform to the requirements set forth in the hypothesis.

The inspiration for the study comes from the experiences of most Latin American countries and other underdeveloped countries as well, which have had substantial amounts of inflation.

Several features are common to all the Latin American countries which have had large amounts of inflation: (1) The central government runs a large fiscal deficit each year. (2) The deficit is largely financed through the Central Bank. (3) Because of the method of deficit financing, a large increase in the money supply results. (4) The deficit is due partly to the government's unwillingness to levy sufficient taxes to cover planned expenditures and partly to the fact that the collection of taxes, which have been assessed sometime in the past, do not increase with the price level, but government expenditures do. That is, government expenditures increase as the price of goods and services increase even if real expenditures remain fixed, but sales taxes, income taxes (not withheld), and property taxes are paid at their assessed level. The payment of these taxes is sometimes deferred a considerable length of time after their assessment.

Because of these features the inflation tends to perpetuate itself. The purpose of our hypothesis then is to show how this process works, i.e., how the rate of change in the price level, the money stock, and the deficit are all interrelated. The model is then tried on Argentine data.

Much of the research on inflation in underdeveloped countries has been directed towards the effect of the inflation on the rate of economic growth. The problem has been discussed from two diametrically opposed points of view: (1) the "structuralists" believe that inflation is a necessary companion of economic growth, and (2) the "monetarists" believe that inflation hinders economic growth.

The structuralists view inflation as a symptom of much deeper structural problems. One such theory views the economy as being divided into two principal sectors, i.e., agricultural and industrial. The industrial sector manufactures mainly consumer goods which are entirely absorbed within the country. The industrial sector must import industrial materials and equipment for its use. The agricultural sector provides the country's food supply and its export goods, which in turn determine its import capacity. The agricultural supply is very inelastic. The internal demand for food depends on real income, but is large even when real income is not very high. The expansion of the industrial sector under favorable terms of trade will increase real income which will increase demand for food which will increase its price since its supply is inelastic. The working class experiencing an increase in the cost of goods will demand higher wages through their unions, and manufacturers will pass the wage increase on entirely in the form of higher prices. Thus, the general price level increases. If the terms of trade are unfavorable and the country is desirous of maintaining the same level of imports, then periodic devaluation will probably occur which will increase the demand for the exported food raising its price, and the above transmission process is repeated so that the general price level rises. The inflation occurs because of the structural rigidity of the food supply combined with the policy of fostering a growing industrial sector.[1]

Other such theories include a few more variables, but the basis of all of them is the inability of the agricultural or some other sector to expand output due to an increase in the demand for its product. A steady increase in the price level is the necessary result. Even if the large budget deficits and rapid expansion of the money supply were eliminated, the price level would still continue to rise.

The monetarists, on the other hand, while recognizing structural rigidities in the less developed countries like Argentina, believe the major influences on inflation to lie in the manipulation of the monetary and fiscal apparatus. The monetarists believe price stability to be a necessary prerequisite for economic growth.

> In their view, a continued inflation diminishes the volume of resources available for domestic investment. Not only is community saving reduced, but a significant part of this saving is channeled to foreign rather than domestic investment, and simultaneously the flow of capital from abroad is discouraged. Furthermore, a substantial part of the reduced flow of resources for domestic investment is diverted to uses which are not of the highest social priority. The accumulation of large inventories is encouraged. There is a diversion of saving away from capital markets, where investment decisions would be subject to longer-term economic criteria, and toward

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[1]Javier Villanueva, The Inflationary Process in Argentina, 1943-60, 2d Edition, Working Paper Inst. Torcuato Di Tella Centro de Investigaciones Economicas, Buenos Aries: Editorial del Instituto Torcuato Di Tella, 1966, pp. 112-125.

> luxury housing and other kinds of investment which are socially less useful, but may be highly profitable because of the inflation. Balance of payments difficulties also result, and to reduce the foreign deficits the authorities are forced to resort to controls which in most cases protect uneconomic production. . . .[1]

While the connection between inflation and economic growth is certainly an important question, I shall not attempt in this study to consider whether inflation assists or hinders economic growth. My purpose is rather to formulate and test a theory which purports to explain a significant part of the inflation which has occurred in Argentina in the last several years.

Most of the work heretofore done in trying to explain inflation in Argentina or in other Latin American countries under similar conditions has been descriptive and non-analytical in nature.[2] That is, much has been said about the institutions and variables which might be important, in varying degrees, in determining the course of inflation, but little effort has been devoted to formulating tight hypotheses and in testing their explanatory power empirically. The work done for Chile by Arnold C. Harberger and for Argentina by Adolfo C. Diz are notable exceptions.[3]

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[1]Werner Baer and Isaac Kerstenetzky (eds.), Inflation and Growth in Latin America (Homewood, Ill.: Richard D. Irwin, Inc., 1964), pp. 3-4.

[2]For an excellent historical account of the Argentine economy and its inflation for 1943-60 see Villanueva. See also Aldo Ferer, The Argentina Economy (Berkeley: University of California Press, 1967).

[3]Arnold C. Harberger, "The Dynamics of Inflation in Chile" in Measurement in Economics: Studies in Mathematical Economics and Econometrics in Memory of Yehuda Grunfeld, ed. Carl F. Christ (Stanford, Calif.: Stanford University Press, 1963), and Adolfo C. Diz, "Money and Prices in Argentina 1935-1962," unpublished Ph.D. dissertation, Department of Economics, The University of Chicago, 1966.

The theory of inflation has been developed from two points of view--from the point of view of demand and from the point of view of supply. Further, those who have looked at inflation primarily from the demand side have espoused one of two positions.

> . . . The quantity theory of money, with centuries of tradition behind it, imputed price level changes to changes in the quantity of money (a stock). An increase in this stock generated excess supply of money at existing prices and interest rates, meaning excess demand for nonmonetary assets. The other position, following Keynes's How to Pay for the War, stressed the level of national expenditure (a flow) as the main determinant of the price level, with increasing expenditures opening an inflationary gap after full employment was attained.[1]

The model developed in this study is an application of the quantity theory of money, i.e., the rate of price change is related to the rate of monetary expansion. This is only one equation of the model, however. The money supply is not exogenous, but is related to the fiscal deficit. And the deficit is dependent upon the rate of change of the price level.[2]

Those who have used an adaptation of the quantity theory to explain the path of inflation have used a one equation model. That is, they have treated the money stock as an exogenous variable and used the reduced form equation for the price level in a simple macroeconomic model as the single equation.

Assuming all exogenous variables except the nominal money stock to be constant, the influence of changes in the nominal money

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[1]Martin Bronfenbrenner and Franklin D. Holzman, "A Survey of Inflation Theory," in Surveys of Economic Theory Vol. I (New York: The Macmillan Co., 1966), p. 52.

[2]For an excellent survey of the work that has been done in inflation theory in general--both demand and supply--see Bronfenbrenner and Holzman, ibid.

stock on the price level follows essentially from the aggregate demand for real cash balances. In a hypothetical society where the nominal money supply consists entirely of a purely fiduciary currency issued by a single monetary authority, the nominal money stock is whatever the authority decides it should be. The real stock of money is another matter, however. If people in the aggregate feel their real cash balances are too high, they will attempt to reduce them by increasing expenditures. As a result, nominal income and the price level will rise thereby reducing real cash balances to the desired level without altering the nominal money stock. Or if people in the aggregate feel their real cash balances are too low, they will attempt to increase them by reducing expenditures which leads to a reduction in nominal income and the price level thereby increasing real cash balances to the desired level. But again, the nominal stock of money remains unaffected--it being at whatever level the monetary authority has established. Given the level of real income and the nominal stock of money, the ratio of nominal income to the nominal money stock, or income velocity, is determined uniquely by the real stock of money. Income velocity then is a reflection of the real quantity of money that people desire to hold. "We can, therefore, speak more or less interchangeably about decisions of holders of money to change their real stock of money or to change the ratio of the flow of income to the stock of money."[1] And equilibrium between actual and desired real

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[1] Milton Friedman, "The Demand for Money: Some Theoretical and Empirical Results," The Journal of Political Economy, Vol. LXVII, No. 4, August, 1959, pp. 330-331.

cash balances or equivalently between the actual and desired income velocity is achieved through a movement in the price level.

Now if a stable functional relation between the real quantity of money demanded and the variables that determine it can be found, it is then possible to determine the effect of a change in the nominal supply of money on the price level. Assuming equilibrium initially between desired and actual real cash balances, an increase in the stock of money will raise real balances above their desired level. In their attempt to reduce them, the holders of money will increase the flow of expenditures and hence money income and the price level will rise, thereby reducing the real stock of money to its desired level. Just the opposite will occur in the case of a reduction in the nominal stock of money.

An important development illustrating the stability of the velocity (or demand for real cash balances) function, while velocity itself fluctuated tremendously, was Philip Cagan's study of seven hyperinflations.[1]

Of all the determinants of the demand for real cash balances specified in the theory of consumer behavior, Cagan makes a good case for the contention that in an environment of rapid inflation it can be explained largely by the cost of holding money. And the overshadowing portion of this cost is the rate of change in prices.[2] He then expresses the demand for real cash balances as a function of the expected rate of price change.[3] After specifying the differential

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[1]Philip Cagan, "The Monetary Dynamics of Hyperinflation" in Studies in the Quantity Theory of Money, ed. Milton Friedman (Chicago: The University of Chicago Press, 1956).

[2]*Ibid.*, pp. 27-33.

[3]*Ibid.*, p. 35.

equation giving the time path of adjustments of the expected to the actual rate of price change and making the assumption that actual and desired real money balances are equal, he was then able to estimate his demand for money function using observable monthly data.

In his study of the Argentine case, a much slower inflation than the seven studied by Cagan, Diz uses approximately the same functional form of the demand function, with the exceptions that he tries to explain real per capita money balances and adds real income (or permanent income) as an explanatory variable.[1] His assumption of how expectations are formed is also equivalent to Cagan's. He estimates his demand for money function using annual data and again demonstrates its stability.

Specifying the demand for real cash balances as a function of the rate of change in prices under the assumptions made by Cagan is logically equivalent to specifying either the price level or the rate of change in prices as a function of the level of and the rate of change in the money stock. An appropriate transformation of the demand for money function after substitution for the expected rate of price change is all that is required. Cagan did this and expressed the price level as a function of the current stock of money, the current rate of change in the money stock, and all rates of change in the stock of money from the initial to the current period.[2]

A similar transformation on Diz's demand for money function yields the rate of change in the price level expressed as a function

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[1]Diz, p. 40.

[2]Cagan, pp. 64-66.

of levels of the real per capita money stock, rates of change in per capita money stock, and rates of change in per capita real income.

Cagan does not attempt to estimate the parameters in his model using the transformed price level function, but he does show that in the German inflation his price level equation predicts pretty well the actual movements in the price level upon substituting values of the parameters into the equation which are close, but not exactly equal to, the ones he obtained in estimating his demand function.

When Diz proceeds to explain fluctuations in the rate of change in the price level, he follows the line of Harberger's study of the Chilean inflation and specifies a linear multiple regression of the rate of change in prices on the current and lagged rates of change in the money stock and some other variables.[1] He does not transform his demand for money function. In this part of his study, he uses quarterly data. Assuming that the demand for money function in a quarterly model would be similar to the one he uses in his annual model, a function explaining the rate of change in the price level would follow directly upon transformation.

Harberger does not confine himself to a specific demand for money function, but rather suggests the variables that determine it. He then indicates that a simple transformation of the demand function will express the rate of change in prices as a function of the rate of change in the quantity of money, the rate of change of real income, and the rate of change in the expected cost of holding money. After choosing the difference between the rate of inflation in the past year and the rate of inflation in the year before that as the

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[1]Diz, p. 69.

variable to represent the rate of change in the expected cost of holding cash he runs his regressions. He uses quarterly and annual data.[1]

One common feature of the three studies just mentioned is that they attempt to explain the price level (Cagan) or the rate of change in the price level (Harberger and Diz) as a function of the current level or rate of change of the money stock and past rates of change in the money stock plus perhaps some other variables--the monetary variables being by far the most important.

For Argentina at least and probably for Chile and possibly for the seven countries of Cagan's study, I believe a one equation model which attempts to explain the rate of change in prices to be only part of the story. The evidence for Argentina suggests that the inflation is to a large degree self generating. It is necessary then to specify the rest of the model. That is, it is necessary to specify how the rate of change in prices, the rate of monetary expansion, and the federal deficit are all interrelated. Let us review briefly the heuristic arguments for such an interrelation beginning with the deficit.

Part of the deficit results simply because the government plans it. To explain the rest of the deficit, assume that inflation is in progress and that the central government makes a tax assessment as some proportion of current expenditures. Because a significant portion of the taxes are not collected until sometime after their assessment (In 1960 sales taxes in Argentina were paid a year

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[1]Harberger, p. 226.

after the sale, for example.),[1] and because they are based on the price level prevailing at the time of assessment, the actual budget deficit exceeds the planned deficit. This is so because of the increase in the price level between the time of the assessment and the time of the collection of taxes, i.e., expenditures increase with increasing prices, but non-collected taxes do not.

Given the institutional structure of the tax system the inflation generates a larger deficit than was planned by the government.

The major part of debt financing by the government originates in the Central Bank.[2] As a result, bank reserves and hence the monetary base (bank reserves plus currency outside banks) are increased. Fractional reserve banking[3] allows a dollar increase in the monetary base to cause an increase in the money supply of an amount greater than a dollar. At any rate, the increase in the base leads to an increase in the money supply.

The increase in the money supply causes the price level to rise. As described above, the increase in the price level leads to a deficit greater than the one planned by the government. The above process is repeated over and over again and the inflation continues until the entire deficit is eliminated or some other link in the chain is broken.

While a model describing the effect of the rate of change in the money stock on the rate of change in the price level is a

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[1]Stanley S. Surry and Oliver Oldman, "Report of a Preliminary Survey of the Tax System of Argentina," Public Finance, Vol. XVI, No. 2, 3, 4, 1961, p. 317.

[2]International Monetary Fund, "Argentina-1966 Article XIV Consultation," (Washington, D. C., 1967), p. 25.

[3]Diz, p. 16.

necessary part of an explanation of the inflationary process in Argentina (and in other countries with similar institutions) it becomes only one equation in a larger system. A better understanding of the inflation can be had if a hypothesis can be formulated which comprises the whole system.

In part one of Chapter II, the theoretical foundations of just such a hypothesis are laid out. It consists of four equations: First, from the theory of consumer choice, a demand for money function is deduced expressing the demand for real per capita money balances as a function of the expected rate of price change. After specifying how expectations of price changes are formed, it is possible to eliminate all non-observable variables by a simple transformation of the demand for money function. This gives us our first equation expressing the rate of change in the price level as a function of the current period's rate of change in the price level and last period's real per capita money stock.

Next, the second equation is developed from a simple theory of banking to explain how the money supply reacts to an increase in the monetary base. That is, a function to explain the money supply or in this case, the rate of change in the money supply, is incorporated into the model. The rate of change in the money supply is expressed as a geometric lag function of the current and all past values of the rate of change in the monetary base.

The third equation explains the relationship between the monetary base and the fiscal deficit. When any part of the deficit is financed by the sale of securities to the Central Bank, currency outside banks and bank reserves increase. This equation expresses the rate of change in the monetary base as a function of the current and the sum of past fiscal deficits.

In the fourth equation, the level of the fiscal deficit per period is explained. The deficit is the result of two related factors: (1) the inability or lack of desire on the part of the government to finance its expenditures through taxation, and (2) the increasing price level over the period causing the tax collections to be less in real terms than planned. Therefore, the deficit is expressed as a function of the planned real deficit, the price level at the beginning of the period in which the deficit expenditure was contracted, and the rate of change in the price level over said period.

In part two of Chapter II, the stochastic properties of each of the equations are discussed and the method of estimation of the parameters of the model is presented. The implications of various assumptions made in constructing the model and of the estimating technique are also discussed, i.e., the properties of the parameter estimates and their reliability are discussed.

In Chapter III, the empirical results of the estimation are presented. The implications of the parameters are discussed and some comparisons are made between the results obtained in this study and results obtained in other studies.

In Chapter IV, the data sources and a brief history of Argentina's inflation during the period of this study are given.

In Chapter V, the dynamic properties of the model are discussed, i.e., the time path of the rate of change in the price level, free of any external constraints or shocks, is computed and discussed.

The conclusions of the study and the policy implications are discussed in Chapter VI.

CHAPTER II

THE MODEL

Theoretical Development of the Model

The model to be developed consists of four simultaneous equations wherein, (1) the rate of change of the price level is related to rates of change in the per capita money stock and to levels of real per capita money balances, (2) the rate of change of the money stock is related to rates of change of the monetary base, (3) the rate of change of the monetary base is related to rates of nominal deficit expenditure, and (4) the rate of nominal deficit expenditure is related to the rate of real deficit expenditure (assumed to be constant), a previous period's price level, and the rate of change in the price level during said period.[1]

Each equation will be developed separately and then the entire model will be presented at the end of this section.

The rate of change in the price level.--Since the equation explaining the rate of change in the price level is derived from a demand for money function we shall first show the theoretical justification for the demand for money function which we use.

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[1]The money stock is defined to be currency outside banks plus demand deposits held by the public; the monetary base includes currency held by the public plus bank reserves; the deficit is expenditures minus tax receipts of the central government; and the price level is measured by an index of consumer prices. All stocks and the price index are end of quarter values.

Let us assume that the total time available to a consumer during any period is allocated to working, undertaking transactions, and leisure. Under these conditions the time allocated to undertaking transactions, T_t, leisure, L_t, and work, W_t, during any time period, t, will all sum to one, i.e.,

(i) $$T_t + L_t + W_t = 1$$

Therefore, we can write

(ii) $$W_t = 1 - (T_t + L_t)$$

Let us further assume that real income earned per period, y_t, is a function of the time spent working during the period.

$$y_t = G(W_t) = G(1-[T_t + L_t])$$

upon substitution for W_t. This can be rewritten as

(iii) $$y_t = G^*(T_t, L_t)$$

where $\frac{\partial G^*}{\partial T_t} < 0$, $\frac{\partial G^*}{\partial L_t} < 0$ and $\frac{\partial^2 G^*}{\partial T_t^2} = \frac{\partial^2 G^*}{\partial L_t^2} = 0$.

Now assume the time spent in undertaking transactions during the period to be a function of the rate of real consumption expenditures, c_t, and the average level of real cash balances, m_t.

(iv) $$T_t = T(c_t, m_t)$$

where $\frac{\partial T}{\partial c_t} > 0$, $\frac{\partial T}{\partial m_t} < 0$, and $\frac{\partial^2 T}{\partial c_t^2} > 0$, $\frac{\partial^2 T}{\partial m_t^2} > 0$.

Income can now be expressed as

$$\text{(v)} \qquad y_t = G^{**}(c_t, m_t, L_t)$$

upon substitution of equation (iv) into equation (iii) where

$$\frac{\partial G^{**}}{\partial c_t} < 0,\ \frac{\partial G^{**}}{\partial m_t} > 0,\ \frac{\partial G^{**}}{\partial L_t} < 0 \text{ and } \frac{\partial^2 G^{**}}{\partial c_t^2} < 0,\ \frac{\partial^2 G^{**}}{\partial m_t^2} < 0,\ \frac{\partial^2 G^{**}}{\partial L_t^2} = 0.$$

Assuming certainly the consumer's total wealth, Φ, over his horizon is equal to the capitalized value of his earned income stream plus the capitalized value of his initial assets minus the capitalized value of the opportunity cost of holding money, i.e.,

$$\text{(vi)} \qquad \Phi = \sum_{t=0}^{H} (1+r)^{H-t} G^{**}(c_t, m_t, L_t) + (1+r)^{H+1} a_o - \sum_{t=0}^{H} (1+r)^{H-t} (r+\pi) m_t$$

where H is his horizon, r is the real rate of interest assumed to be greater than minus one and constant, a_o is his initial asset holdings including money, and π_t is the rate of change in prices, i.e., π_t equals $d\,\frac{\log P_t}{dt}$ (where P_t is an index of prices) and is approximated by the difference in the logarithms of the successive values of the index of prices.[1] The rate of interest, r, measures the amount of real income foregone in period t by holding one dollar of real money balances, and the rate of price change, π_t, measures the depreciation in value in period t of one dollar of real money balances.

Given the rate of consumption expenditure, c_t, the rate of leisure, L_t, the horizon, H, the level of initial asset holdings, a_o,

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[1]This logarithmic difference is the relative rate of change, compounded continuously, for the price level, provided e is the base of these logarithms. Throughout this study, e is taken to be the base of the logarithms used.

the rate of interest, r, and the rate of change in prices, π_t, the consumer will adjust his money holdings so as to maximize his total wealth, Φ.

Assuming that G** is continuous and at least twice differentiable, the maximization of Φ requires that

$$\text{(vii)} \qquad d\Phi = \sum_{t=0}^{H} (1+r)^{H-t} \left[\frac{\partial G^{**}}{\partial m_t} - (r+\pi_t)\right] dm_t = 0$$

and

$$\text{(viii)} \qquad d^2\Phi = \sum_{t=0}^{H} (1+r)^{H-t} \frac{\partial^2 G^{**}}{\partial m_t^2} dm_t^2 < 0$$

$\frac{\partial G^{**}}{\partial m_t}$ equals the marginal revenue of a unit of money and $(r+\pi_t)$ is the marginal cost of a unit of money. Since dm_t is some arbitrary increment in m_t, for (vii) to hold it is necessary and sufficient that

$$\text{(ix)} \qquad \frac{\partial G^{**}}{\partial m_t} = r+\pi_t, \text{ for all } t \in [0,H]$$

Now since (1+r) is always positive and $\frac{\partial^2 G^{**}}{\partial m_t^2}$ is always negative, equation (viii), the sufficient condition for maximization, is always satisfied. Therefore, the solution of equation (ix) gives the level of m_t, which maximizes wealth.

Since I assume $\frac{\partial G^{**}}{\partial m_t}$ to be continuous, $\frac{\partial^2 G^{**}}{\partial m_t^2}$ to be negative, and all the arguments of G** except m_t to be held constant, equation (ix) can be solved explicitly for m_t giving the demand for real balances as

$$\text{(x)} \qquad m_t = \Gamma(c_t, L_t, r+\pi_t)$$

where $\frac{\partial \Gamma}{\partial c_t} > 0$, $\frac{\partial \Gamma}{\partial L_t} < 0$, $\frac{\partial \Gamma}{\partial (r+\pi)} < 0$, $\frac{\partial \Gamma}{\partial r} < 0$, $\frac{\partial \Gamma}{\partial \pi} < 0$.

Of all the variables in equation (x), m_t and π_t are the only ones on which we have adequate data over the period of study. However, the level and variability of the rate of inflation have been so high that it probably dominates the other variables in explaining the demand for real balances. With the goal in mind of a demand for money function which can be used in the empirical part of the study, let us assume that c_t, L_t, and r are constant. The demand for real money balances can then be expressed as a function of the rate of change in prices alone with c_t, L_t, and r entering as shifting parameters.

(xi) $$m_t = \Gamma^*(\pi_t; c_t, L_t, r)$$

Dropping the assumption of certainty, the consumer seeks to maximize his expected wealth--which is now a function of expected levels of consumption, leisure, the interest rate, the rate of change in prices, and real cash balances. We are assuming here that the consumer is interested only in expected values and not in variances around these expected values. We can proceed here as we did before in the case of certainty allowing us now to solve for the demand for real cash balances as a function of the expected levels of consumption, leisure, the rate of interest, and the rate of change in the price level. Assuming that the expected values of all variables except the rate of change in the price level are equal to their actual values at any point in time, and again assuming that c_t, L_t, and r are constant, the desired level of real balances can then be expressed as a function of the expected rate of price change.[1]

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[1]For a detailed development of the demand for money, see Boris P. Pesek and Thomas R. Saving, Money, Wealth, and Economic Theory. New York: The Macmillan Co., 1967.

In Argentina over the period of this study, income distribution remained quite constant. And it is quite reasonable to assume that roughly the same price changes and level of the interest rate were experienced by the entire population. Let us further assume that the distribution of leisure time consumption did not change over the period. It is then possible to aggregate the individuals' demands for real balances and divide by population to get the aggregate demand for real per capita cash balances. We can now write

$$\text{(xii)} \qquad \left(\frac{M}{NP}\right)_t = \Gamma^*\left(\pi_t; \left(\frac{C}{NP}\right)_t, \left(\frac{L}{NP}\right)_t, r\right)$$

where $\left(\frac{M}{NP}\right)_t$ is the aggregate demand for real per capita money balances, π_t is the expected rate of price change, $\left(\frac{C}{NP}\right)_t$ is the real per capita level of consumption (assumed constant), $\left(\frac{L}{NP}\right)_t$ is the real per capita level of leisure consumption (assumed constant), and r is the rate of interest (assumed constant).

The specific functional form of equation (xii) assumed here is approximately like the one used by Phillip Cagan in his study of hyperinflation.

$$\text{(xiii)} \qquad \log\left(\frac{M}{NP}\right)_t = \gamma - \alpha E_t$$

where E_t is the expected rate of price change at the end of quarter t, N is population at the same point in time, and γ and α are positive constants.[1]

If we further assume that desired and actual real per capita money balances are always equal, then $\left(\frac{M}{NP}\right)_t$ is an observable quantity.

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[1]Cagan, p. 35.

Since E_t, the expected rate of price change, is a non-observable variable, it is necessary to specify it as a function of observable variables. It is reasonable that individuals would base expectations of price change on past actual rates of change on the price level. Let us assume that the expected rate of price change is revised during the t^{th} quarter according to the following rule:

$$\text{(xiv)} \qquad \Delta E_t = \beta(\Delta \log P_{t-1} - E_{t-1})$$

where $\Delta E_t = E_t - E_{t-1}$ (the expected rate of price change at the end of quarter t minus the expected rate of price change at the end of quarter t-1), $\Delta \log P_{t-1} = \log P_{t-1} - \log P_{t-2}$ (the actual rate of price change during the $(t-1)^{st}$ quarter), and β is a positive constant less than one. The solution of difference equation (xiv) expresses the expected rate of price level change as a gemoetric lag function of all previous actual rates of price level change.

Upon taking first differences of equation (xiii) we get

$$\text{(xv)} \qquad \Delta \log(\frac{M}{NP})_t = -\alpha \Delta E_t$$

and upon lagging equation (xiii) one quarter and solving for E_{t-1} we get

$$\text{(xiiia)} \qquad E_{t-1} = \frac{1}{\alpha}[\gamma - \log(\frac{M}{NP})_{t-1}]$$

Now substitute equation (xiiia) into equation (xiv) and the result from this substitution into equation (xv) yielding

$$\text{(xvi)} \qquad \Delta \log(\frac{M}{NP})_t = -\alpha\beta[\Delta \log P_{t-1} - \frac{1}{\alpha}(\gamma - \log(\frac{M}{NP})_{t-1})]$$

which eliminates all expected rates of price change and reduces equations (xiii) and (xiv) to a single relation between observable variables. After rearranging variables in equation (xvi), the current rate of price change in quarter t is expressed explicitly as a function of the current rate of change in the per capita money stock, the previous quarter's rate of change in the price level, and the level of the previous quarter's prices and the per capita money stock.

$$(1) \qquad \Delta\log P_t = -\beta\gamma + \Delta\log(\frac{M}{N})_t + \alpha\beta\Delta\log P_{t-1} + \beta(\log(\frac{M}{N})_{t-1} - \log P_{t-1})$$

It can readily be seen that the current rate of price change can be expressed as a geometric lag function of the nominal and real per capita money stocks, i.e.,

$$(1a) \qquad \Delta\log P_t = -\frac{\beta\gamma}{1-\alpha\beta} + \sum_{i=0}^{\infty}(\alpha\beta)^i\Delta\log(\frac{M}{N})_{t-i} + \sum_{i=0}^{\infty}\alpha^i\beta^{i+1}\log(\frac{M}{NP})_{t-i-1}$$

where $\alpha\beta$ is assumed to be less than one. Equation (1) can be derived from equation (1a) by a Koyck transformation. Simply lag equation (1a) one quarter, multiply by $\alpha\beta$, and then subtract the resulting equation from equation (1a).

Several comments are in order regarding equation (1a). The sum of the weights on the $\Delta\log(\frac{M}{N})_{t-i}$ term is equal to $\frac{1}{1-\alpha\beta} > 1$, which implies that an upward and sustained shift in the rate of per capita monetary expansion will produce a somewhat greater percentage increase in the price level after all lagged adjustments have taken place. This is partially borne out in this study by the fact that the average rate of price increase over the period considered was about

7.5% per quarter while the average rate of per capita monetary expansion was about 5.9% per quarter. This is consistent with the theory of the demand for money specified in equation (xiii), which implies that a lower level of desired real cash balances is consistent with a higher expected rate of price change. The sum of the weights on the

$\log(\frac{M}{NP})_{t-i-1}$ term is equal to $\frac{\beta}{1-\alpha\beta} \gtrless 1$

as $\beta \gtrless \frac{1}{1+\alpha}$ <1 but the combined sums of the weights are equal to

$\frac{1+\beta}{1-\alpha\beta} > 1.$

The rate of change in the money stock.--An increase in the monetary base means an increase in the total of currency outside banks plus bank reserves. Let us assume that the increase in the monetary base is accomplished through an increase in currency outstanding which is the result of the government printing money to finance a deficit. The individuals whose currency holdings have increased find their portfolio balance between currency and demand deposits disturbed. They will proceed to reduce their currency holdings and thereby increase their holdings of demand deposits. These individuals would make this adjustment over a certain length of time, and the resulting increase in demand deposits would increase bank reserves.

The equal increase in both demand deposits and reserves will cause the actual reserves to deposits ratio to rise above the desired reserves to deposits ratio of banks operating in a fractional reserve system. Banks will respond by creating new demand deposits to equate the actual to the desired reserves to deposits ratio. This causes the money supply to increase.

Again, this expansion of the money supply through new demand deposit creation is not instantaneous, however, but can be accomplished only over some length of time. We would expect, though, that after some specific number of time periods most of the effect of an increase in the monetary base on the money supply would have been felt. Let us assume that the rate of change in the nominal money stock is a geometric lag function of the current and past rates of change in the monetary base, i.e.,

$$\text{(2a)} \qquad \Delta \log M_t = \lambda \sum_{i=0}^{\infty} (1-\lambda)^i \Delta \log B_{t-i}$$

where $0<\lambda<1$ and $\lambda \sum_{i=0}^{\infty} (1-\lambda)^i = 1$.

This specific formulation conforms with our expectations that in period t a 1% increase in the monetary base results in only λ% increase in the money stock, and the effect of this 1% increase in the monetary base on the money stock diminishes as time passes.

Again with the use of a Koyck transformation equation (2a) can be simplified. Lag equation (2a) one quarter and multiply by (1-λ). Subtract this from equation (2a) and rearrange the resulting equation to get

$$\text{(2)} \qquad \Delta \log M_t = \lambda \Delta \log B_t + (1-\lambda) \Delta \log M_{t-1}$$

Here we have the current rate of change in the nominal money stock being determined by the current rate of change in the monetary base and last quarter's rate of change in the nominal money stock.

<u>The rate of change in the monetary base</u>.--Where the government finances any of its deficit by the sale of securities to the Central Bank an increase in currency outside banks and bank reserves will be the result. That is, a fiscal deficit financed in this manner results

in an increase in the monetary base. In Argentina quite a large portion of the deficit is financed by the sale of government securities to the Central Bank. Therefore, the portion of the deficit financed in this manner would cause the monetary base to increase by that amount. This relationship can be expressed in the following equation form.

$$\text{(3a)} \qquad \Delta B_t = a + bD_t$$

where ΔB_t is the change in the monetary base during quarter t, D_t is the rate of deficit expenditure during the same quarter, and a and b are constants. b is the proportion of the deficit that is financed by the sale of securities to the Central Bank. a accounts for the other factors (assumed here to be constant) which cause changes in the monetary base. Solving equation (3a) for the total base in terms of an initial value $B_{(-1)}$ ($B_{(-1)}$ is the level of the monetary base at the beginning of quarter zero) and the increments in the base from the beginning of the initial quarter up to the end of quarter t gives

$$\text{(3b)} \qquad B_t = B_{(-1)} + \sum_{i=0}^{t} \Delta B_i = B_{(-1)} + \sum_{i=0}^{t} (a + bD_i) = B_{(-1)} + at + b\sum_{i=0}^{t} D_i$$

Dividing equation (3a) by equation (3b) gives the relative rate of change in B_t which is approximated by $\Delta \log B_t$. Therefore, we write

$$\text{(3)} \qquad \Delta \log B_t = \frac{a + bD_t}{B_{(-1)} + at + b\sum_{i=0}^{t} D_i}$$

which specifies the rate of change in the monetary base being determined by the current and all rates of nominal deficit expenditure since the initial period.

The rate of deficit expenditure.--The nominal deficit, which in total is caused by insufficient taxes to cover government expenditures, results from two sources: (1) the government plans a given positive deficit, and (2) a positive rate of price increase over the period in which the expenditure was contracted causes the nominal expenditure to rise but leaves the fixed tax assessment unaffected.

We have no data on planned real deficit expenditures per quarter, so to simplify the analysis, let us assume that the planned real deficit per quarter is constant ($=g_1$). The nominal deficit in quarter t, D_t resulting from this real deficit expenditure g_1, will depend on first, the price level at the beginning of the quarter in which the expenditures were initiated, second, the rate of change in the price level during the quarter, and third, the time path of the real deficit expenditure, g(x), during the quarter. Assuming a positive rate of price change, the nominal deficit would be greater if the entire real deficit, g_1, were spent on the last day of the quarter than if it were spent on the first day.

Government expenditures are almost always contracted at some date previous to the date of actual disbursement. That is, the government will arrange to buy something on a given date at the price prevailing on that date. The payment for the item will take place usually at some later date. The amount paid for the item then reflects the price level at the earlier date when the purchase was contracted. Therefore, a nominal deficit in the current period will reflect purchases of earlier periods. Let us assume that this lag is two quarters long. That is, the central government contracts to buy something on a given day at that day's prices but pays for it six months later.

Therefore, the nominal deficit expenditure during quarter t, D_t, will be given by evaluating the line integral of the price level function, $F(x,g) = P_{t-3}e^{x\Delta \log P_{t-2}}$, along the time path of deficit expenditures, g(x), i.e.,

$$(4a) \qquad D_t = \int_{g(x)} P_{t-3}e^{x\Delta \log P_{t-2}} dg(x)$$

where P_{t-3} is the price level at the beginning of the quarter six months before the current quarter t, and $\Delta \log P_{t-2}$ is the rate of price change over the quarter. $0<x<1$ (i.e., the length of the period over

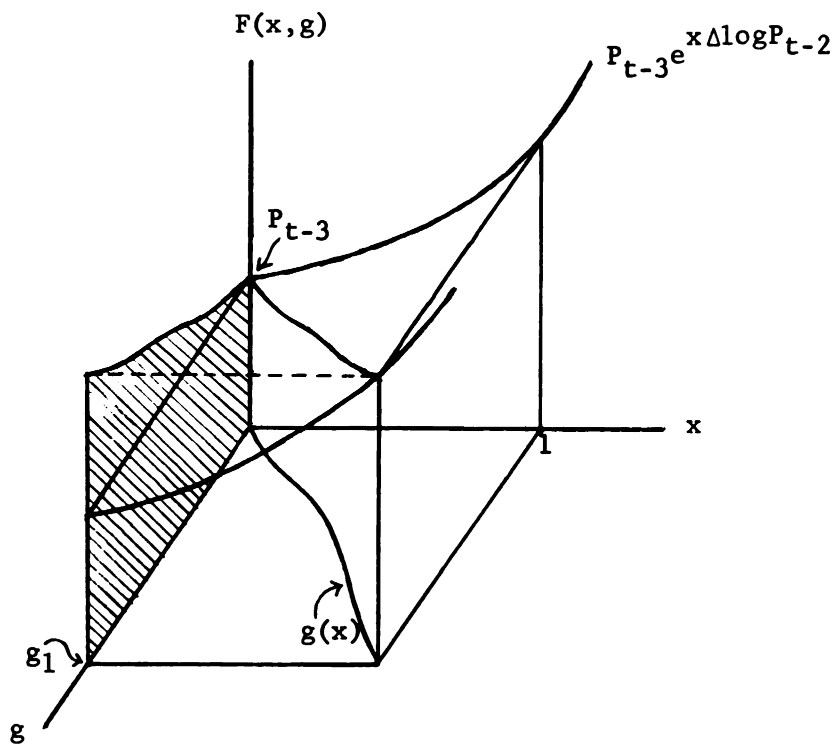

Fig. 1.--Nominal Deficit Expenditure

which the expenditure is made is one quarter), and $0 \leq g(x) \leq g_1$. The evaluated integral gives the area between the time path of real government expenditure line, g(x), and the $F(x,g)=P_{t-3}e^{x\Delta \log P_{t-2}}$ surface projected onto the F(x,g), g plane between g=0 and $g=g_1$ as shown in Figure 1 above. The continuity of g(x) and F(x,g) is sufficient for the existence of the line integral. Let us assume that real deficit expenditures are undertaken evenly over the quarter, i.e., $g(x)=g_1x$. Both g(x) and F(x,g) are therefore continuous.

Now $dg(x)=g_1dx$, and along $g(x)=g_1x$ when g=0, x=0 and when $g=g_1$, x=1. Substituting these values for the differential of g(x) and the upper and lower limits of x corresponding to the upper and lower limits of g(x) we get

(4b) $$D_t = \int_0^1 P_{t-3}e^{x\Delta \log P_{t-2}} g_1 dx$$

$$= P_{t-3}g_1 \int_0^1 e^{x\Delta \log P_{t-2}} dx$$

The integration yields

(4) $$D_t = \frac{P_{t-3}\ g_1}{\Delta \log P_{t-2}}(e^{\Delta \log P_{t-2}} -1)$$

which shows the explicit relationship between the current nominal deficit, the real deficit, the level of prices six months previous to the beginning of the current quarter, and the rate of change in prices over said quarter.

For simplicity in discussing equation (4), let us write $u = \Delta \log P_{t-2}$ so that

(4) $$D_t = P_{t-3}g_1 \frac{e^u -1}{u}$$

Note that $\lim_{u \to o} D_t = P_{t-3}\ g_1$. In other words, as the rate of price change approaches zero, the nominal deficit becomes equal to the price level at the beginning of the period when the deficit was contracted times the level of real deficit expenditures. Or in other words, the actual equals the planned deficit.

Note also that equation (4) is a strictly increasing function of u, which means that as the rate of price change increases, so does the nominal deficit. This is shown by taking the derivative

$$\frac{d\ Dt}{du} = \frac{ue^u - e^u + 1}{u^2}$$

which we must show to be positive. Using L'Hospital's rule, we establish that $\lim_{u \to o} \frac{d\ Dt}{du} = 0$. Now for u>o the denominator is positive. For u=o the numerator is equal to zero. So if the numerator increases as u increases, then we have shown that $\frac{d\ D_t}{du} > o$ for all positive u. This is so if

$$\frac{d}{du}\left(ue^u - e^u + 1\right) = ue^u > o$$

which is true for all u>o.

Summary.--In summary we can now write the system of four simultaneous equations which explain, (1) the rate of change in the price level, (2) the rate of change in the money stock, (3) the rate of change in the monetary base, and (4) the rate of deficit expenditure. From the model follows the self generating nature of the inflation.

$$(1) \qquad \Delta \log P_t = -\beta\gamma + \Delta \log\left(\frac{M}{N}\right)_t + \alpha\beta\Delta \log P_{t-1} + \beta\left(\log\left(\frac{M}{N}\right)_{t-1} - \log P_{t-1}\right)$$

(2) $$\Delta \log M_t = \lambda \Delta \log B_t + (1-\lambda)\Delta \log M_{t-1}$$

(3) $$\Delta \log B_t = \frac{a + bD_t}{B_{(-1)} + at + b\sum_{i=0}^{t} D_i}$$

(4) $$D_t = \frac{P_{t-3}g_1}{\Delta \log P_{t-2}} (e^{\Delta \log P_{t-2}} - 1)$$

$\Delta \log P_t$, $\Delta \log M_t$, $\Delta \log B_t$, and D_t are endogenous, $\Delta \log P_{t-1}$, $\Delta \log M_{t-1}$, D_{t-i} (i=1, ..., n, where n is the number of quarters since that quarter in which the amount of base money was equal to $B_{(-1)}$), $\Delta \log P_{t-2}$ and P_{t-3} are lagged endogenous, and only t and N are exogenous variables.

Estimation

The procedure in this section is to add a stochastic element at a spot in the development of each equation which seems to be reasonable and then to estimate the parameters. The properties of the parameters are then discussed.

The rate of change in the price level.--Let us assume that the demand for real per capita money balances is a random variable of the following form

(xiii') $$\log\left(\frac{M}{NP}\right)_t = \gamma - \alpha E_t + \varepsilon_{1t}$$

where $\varepsilon_{1t} \sim NID(0,\sigma_1^2)$. We continue to assume that

(xiv) $$\Delta E_t = \beta(\Delta \log P_{t-1} - E_{t-1})$$

With these assumptions, equation (1) becomes

$$\Delta \log P_t - \Delta \log(\frac{M}{N})_t = -\beta\gamma + \alpha\beta\Delta \log P_{t-1}$$

$$+ \beta(\log(\frac{M}{M})_{t-1} - \log P_{t-1})$$

$$- \varepsilon_{1t} + (1-\beta)\varepsilon_{1t-1}$$

or

$$\Delta \log(\frac{NP}{M})_t = -\beta\gamma + \alpha\beta\Delta \log P_{t-1} + \beta(\log(\frac{M}{N})_{t-1} - \log P_{t-1}) + \varepsilon^*_{1t} \qquad (1')$$

where $\varepsilon^*_{1t} = (1-\beta)\varepsilon_{1t-1} - \varepsilon_{1t}$.

For purposes of estimation, the dependent variable is now $\Delta \log(\frac{NP}{M})$. It is clear that ε^*_{1t} is not an independently distributed random variable, but is serially correlated.[1] The least square estimates of the parameters in equation (1') are therefore inefficient. We are also unable to express ε^*_{1t} as a simple first order autoregressive scheme and therefore the alternative procedures for obtaining efficient estimates of the parameters are not open to us.

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[1]The correlation coefficient of ε^*_{1t} and ε^*_{1t-1} is given by

$$\rho^*_1 = \frac{E\varepsilon^*_{1t}\varepsilon^*_{1t-1}}{E\varepsilon^{*2}_{1t}}$$

$$= \frac{E[(1-\beta)\varepsilon_{1t-1} - \varepsilon_{1t}]\ [(1-\beta)\varepsilon_{1t-2} - \varepsilon_{1t-1}]}{E[(1-\beta)\varepsilon_{1t-1} - \varepsilon_{1t}]^2}$$

It is clear that both $\Delta \log P_{t-1}$ and $\log(\frac{M}{NP})_{t-1}$ are correlated with the disturbance term, ε^*_{1t}, since $\Delta \log P_{t-1}$ is a function of ε^*_{1t-1}, and $\log(\frac{M}{NP})_{t-1}$ is a function of ε_{1t-1}, both of which are correlated with ε^*_{1t}. As a result, the least square estimates of α and β are also biased and inconsistent.

Another result of the autocorrelated disturbance terms is that the standard errors of the estimates are likely to be biased downward so that the inefficiency of the estimates is concealed.

The rate of change in the money stock.--Let us assume that the rate of change in the money stock is a stochastic variable of the following form

(2a') $$\Delta \log M_t = \lambda \sum_{i=0}^{\infty}(1-\lambda)^i \Delta \log B_{t-i} + \varepsilon_{2t}$$

where as before $0<\lambda<1$, $\lambda \sum_{i=0}^{\infty}(1-\lambda)^i = 1$ and $\varepsilon_{2t} \sim NID(0,\sigma_2^2)$.

$$= \frac{(1-\beta)^2 E\varepsilon_{1t-1}\,\varepsilon_{1t-2} - (1-\beta)E\varepsilon_{1t-1}\,\varepsilon_{1t-1}}{(1-\beta)^2 E\varepsilon_{1t-1}^2 - 2(1-\beta)E\varepsilon_{1t}\varepsilon_{1t-1} + E\varepsilon_{1t}^2}$$

$$- \frac{(1-\beta)E\varepsilon_{1t-2}\varepsilon_{1t} - E\varepsilon_{1t}\,\varepsilon_{1t-1}}{(1-\beta)^2 E\varepsilon_{1t-1}^2 - 2(1-\beta)E\varepsilon_{1t}\varepsilon_{1t-1} + E\varepsilon_{1t}^2}$$

$$= - \frac{(1-\beta)\sigma_1^2}{(1-\beta)^2\sigma_1^2 + \sigma_1^2} = - \frac{(1-\beta)}{1 + (1-\beta)^2}$$

Applying the Koyck transformation to equation (2a') we get

$$\Delta \log M_t - (1-\lambda)\Delta \log M_{t-1} = \lambda \Delta \log B_t - (1-\lambda)\varepsilon_{2t-1} + \varepsilon_{2t}$$

or

$$\Delta \log M_t - \Delta \log M_{t-1} = \lambda(\Delta \log B_t - \Delta \log M_{t-1}) + \varepsilon^*_{2t} \quad (2')$$

where $\varepsilon^*_{2t} = -(1-\lambda)\varepsilon_{2t-1} + \varepsilon_{2t}$.

For estimation purposes, the dependent variable now becomes $\Delta \log M_t - \Delta \log M_{t-1}$. Although the method of two stage least squares estimation of λ eliminates the correlation between $\Delta \log B_t$ and the error term, ε^*_{2t}, ε^*_{2t} is still serially correlated which introduces inefficiency, and correlation between $\Delta \log M_{t-1}$ and ε^*_{2t} introduces inconsistency. The autocorrelation of the disturbance terms again causes the estimates of the standard errors to be biased downward.

An alternative assumption about the form of equation (2a') will eliminate the theoretical problems in the estimation of λ. Let us assume that $\Delta \log M_t$ assumes the following functional form

$$\Delta \log M_t = \lambda_1 + \lambda_2 \Delta \log B_t + \lambda_3 \Delta \log B_{t-1} + \varepsilon_{2t} \quad (2'')$$

where $0<\lambda_2, \lambda_3<1$ and $\varepsilon_{2t} \sim NID(0,\sigma_2^2)$.

The two stage least square estimates of the λ's in equation (2") will be consistent.

<u>The rate of change in the monetary base</u>.--Let us assume that the change in the monetary base is a random variable of the following form

$$(3a') \qquad \Delta B_t = a + bD_t + \varepsilon_{3t}$$

where $\varepsilon_{3t} \sim NID(0,\sigma_3^2)$.

The two stage least square estimates of a and b are consistent. With the resulting estimates of ΔB_t, estimates of $\Delta \log B_t$ can be computed as is described in the derivation of equation (3) in part one above, i.e.,

$$(3') \qquad \widehat{\Delta \log B_t} = \frac{\widehat{\Delta B_t}}{B_{(-1)} + \sum_{i=0}^{t} \widehat{\Delta B_i}}$$

This estimate of $\Delta \log B_t$ is then used to get the two stage least square estimates of equations (2') and (2").

<u>The rate of deficit expenditure</u>.--Equation (4b) expresses the current deficit being determined by what happened to the price level six months previously. When we allow D_t to be stochastic we must add the integral of the random element of real deficit expenditure and what is happening to the price level during the current quarter, i.e.,

$$(4b') \qquad D_t = \int_0^1 P_{t-3} e^{x\Delta \log P_{t-2}} g_1 dx + \int_0^1 P_{t-1} e^{x\Delta \log P_t} \varepsilon_{4t} dx$$

where $\varepsilon_{4t} \sim NID(0,\sigma_4^2)$. The integration yields

$$D_t = \frac{P_{t-3}g_1}{\Delta \log P_{t-2}}(e^{\Delta \log P_{t-2}} -1) + \frac{P_{t-1}}{\Delta \log P_t}(e^{\Delta \log P_t} -1)\varepsilon_{4t} \text{ or}$$

$$(4') \qquad D_t = \frac{P_{t-3}g_1}{\Delta \log P_{t-2}}(e^{\Delta \log P_{t-2}} -1) + \varepsilon^*_{4t}$$

where $$\varepsilon^*_{4t} = \frac{P_{t-1}}{\Delta \log P_t}(e^{\Delta \log P_t} -1)\varepsilon_{4t}.$$

The least square estimate of g_1 in equation (4') is unbiased and consistent, but probably inefficient because ε^*_{4t} is almost certainly heteroskedastic.[1]

Finally, to take account of seasonal effects in the endogenous variable, a set of three seasonal (0,1) dummy variables is added to each equation.

So for estimation, the system of equations now becomes:

$$(1') \qquad \Delta\log(\frac{NP}{M})_t = -\beta\gamma + \alpha\beta\Delta\log P_{t-1} + \beta(\log(\frac{M}{N})_{t-1} - \log P_{t-1}) + d_{1i}S_i + \varepsilon^*_{1t}$$

$$(2') \qquad \Delta\log M_t - \Delta\log M_{t-1} = \lambda(\Delta\log B_t - \Delta\log M_{t-1}) + d_{2i}S_i + \varepsilon^*_{2t}$$

$$(3') \qquad \widehat{\Delta\log B_t} = \frac{\widehat{\Delta B_t}}{B_{(-1)} + \sum_{i=0}^{t} \widehat{\Delta B_i}}, \text{ which is derived from}$$

$$(3a') \qquad \Delta B_t = a + bD_t + d_{3i}S_i + \varepsilon_{3t}$$

$$(4') \qquad D_t = g_1 \frac{P_{t-3}}{\Delta\log P_{t-2}}(e^{\Delta\log P_{t-2}} - 1) + d_{4i}S_i + \varepsilon^*_{4t}$$

where S_i (i = 1, ..., 3) is set of three seasonal dummy variables in

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[1]To facilitate discussion let $\frac{P_{s-1}}{\Delta\log P_s}(e^{\Delta\log P_s} - 1) = \tilde{P}_s$.

So we have

$$\text{Var}(\varepsilon^*_{4i}) = E\,[\,\tilde{P}_i\varepsilon_{4i} - E\tilde{P}_i\varepsilon_{4i}\,]^2 = \sigma^{*2}_{4i}$$

and

$$\text{Var}(\varepsilon^*_{4_j}) = E[\tilde{P}_j\ 4_j - E\tilde{P}_j\ 4_j]^2 = \sigma^{*\ 2}_{4j}$$

ε_{4s} and $\tilde{P}_s$ are not independent. It is possible for $\tilde{P}_j$ and ε_{4i} to interact in the same manner in which P_j and ε_{4j} interact for $i \neq j$, but since most probably $\tilde{P}_i \neq \tilde{P}_j$, then most probably $\sigma^{*2}_{4i} \neq \sigma^{*2}_{4j}$.

each equation, and d_{ji} (j = 1, ..., 4) is the set of their coefficients to be estimated.

Conclusion

A caveat applying to the estimates of the parameters in all the equations is appropriate here. Since we cannot be sure that each equation is the true specification between the variables, there exists the possibility of misspecification which would cause the parameter estimates to be biased.

This danger exists on equation (1') where we assume real per capita consumption and leisure and the rate of interest to be constant, in equation (3a') where we assume the deficit to be the only source of change in the monetary base, and in equation (4') where we assume the real deficit to be constant.

Direct least square estimates are computed for equations (1') and (4'). The resulting estimate of D_t in equation (4') is used to compute the two stage least square estimate of equation (3a'). The resulting estimate of ΔB_t is used as indicated in equation (3') to obtain an estimate of $\Delta \log B_t$. The estimate of $\Delta \log B_t$ is used in the two stage least square estimate of equation (2'). Alternatively we also use the estimate of $\Delta \log B_t$ in the two stage least square estimate of equation

$$(2'') \qquad \Delta \log M_t = \lambda_1 + \lambda_2 \Delta \log B_t + \lambda_3 \Delta \log B_{t-1} + d_{2i}' S_i + \varepsilon_{2t}$$

where again S_i is the set of three seasonal dummy variables, and d_{2i}' is the set of three coefficients to be estimated.

Under the stated assumptions, the estimates of the parameters in equations (1'), (2'), (2"), (3a') and (4') have the previously mentioned properties.

CHAPTER III

EMPIRICAL RESULTS

The system of equations, whose paremeters are to be estimated is the following:

$$(1')\qquad \Delta\log(\frac{NP}{M})_t = -\beta\gamma + \alpha\beta\Delta\log P_{t-1} + \beta(\log(\frac{M}{N})_{t-1} - \log P_{t-1}) + d_{1i}S_i + \varepsilon^*_{1t}$$

$$(2')\qquad \Delta\log M_t - \Delta\log M_{t-1} = \lambda(\Delta\log B_t - \Delta\log M_{t-1}) + d_{2i}S_i + \varepsilon^*_{2t}$$

$$(2'')\qquad \Delta\log M_t = \lambda_1 + \lambda_2\Delta\log B_t + \lambda_3\Delta\log B_{t-1} + d'_{2i}S_i + \varepsilon^*_{2t}$$

$$(3a')\qquad \Delta B_t = a + bD_t + d_{3i}S_i + \varepsilon_{3t}$$

$$(4')\qquad D_t = g_1\frac{P_{t-3}}{\Delta\log P_{t-2}}(e^{\Delta\log P_{t-2}} - 1) + d_{4i}S_i + \varepsilon^*_{4t}$$

where S_i ($i = 1, \ldots, 3$) is a set of three seasonal dummy variables in each equation, and d_{ji} ($j = 1, \ldots, 4$) and d'_{2i} are the set of their coefficients, which are to be estimated.

Equation (2") is an alternative formulation of equation (2').

Now we shall examine the results of the estimation of the system of equations.[1]

$$(1')\qquad \Delta\log\left(\frac{NP}{M}\right) = \underset{(.033)}{.0093} + \underset{(.15)}{.35^*}\Delta\log P_{t-1} + \underset{(.054)}{.14^*}\left(\log\left(\frac{M}{N}\right)_{t-1} - \log P_{t-1}\right)$$

$$+ \underset{(.027)}{.057^*}S_1 + \underset{(.026)}{.055^*}S_2 + \underset{(.026)}{.044}S_3 + e_{1t} \qquad (R^2 = .44)\ \text{DLS}$$

$$(2')\qquad \Delta\log M_t - \Delta\log M_{t-1} = \underset{(.013)}{.044^*} + \underset{(.14)}{.31^*}(\Delta\log B_t - \log M_{t-1})$$

$$- \underset{(.020)}{.088^*}S_1 - \underset{(.018)}{.037^*}S_2 - \underset{(.019)}{.042^*}S_3 + e_{2t} \qquad (R^2 = .64)\ \text{2SLS}$$

$$(2'')\qquad \Delta\log M_t = \underset{(.021)}{.097^*} - \underset{(.22)}{.35}\Delta\log B_t + \underset{(.11)}{.64^*}\Delta\log B_{t-1}$$

$$- \underset{(.014)}{.069^*}S_1 - \underset{(.017)}{.075^*}S_2 - \underset{(.018)}{.061^*}S_3 + e'_{2t} \qquad (R^2 = .68)\ \text{2SLS}$$

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[1]Those estimates with an asterisk are significant at the five per cent level, and the method of estimation, whether direct least squares (DLS) or two stage least squares (2SLS) is indicated at the right of each equation.

The results show a constant term in each equation regardless of whether it was specified in Chapter II. In each of the equations except (3a') the constant term is to be interpreted as the seasonal effect of the fourth quarter.

$$(3a')\qquad \Delta B_t = \underset{(2.30)}{.86} + \underset{(.17)}{1.13^*}D_t + e_{3t} \qquad (R^2 = .57)\ \text{SLS}$$

$$(4')\qquad D_t = \underset{(2.55)}{2.73} + \underset{(.0040)}{.031^*}\ \frac{P_{t-3}}{\Delta \log P_{t-2}}(e^{\Delta \log P_{t-2}} - 1)$$

$$- \underset{(2.74)}{3.06}S_1 - \underset{(2.66)}{6.69^*}S_2 - \underset{(2.65)}{8.30^*}S_3 + e_{4t} \qquad (R^2 = /71)\ \text{2SLS}$$

Equation (3a') was estimated both with and without the seasonal dummy variables. The estimate with the seasonal dummies yielded a higher R^2 but a lower $\bar{R}^2$ (correlated for degrees of freedom) than the estimate without the seasonal dummies. None of the coefficients of the seasonal variables were significant at the 5 per cent level. The estimates derived from the equation without the seasonal variables was used, therefore, in the two stage least square estimates of equations (2') and (2").

In equation (1') both the estimates of $\alpha\beta$ and β are significant at the 5 per cent level. An estimate of .14 for β and .35 for $\alpha\beta$ imply an estimate of 2.5 for α. The estimate of .0093 for $-\beta\gamma$ is not significant at the 5 per cent level. This estimate and the estimate of β imply an estimate of -.066 for γ.

It will be remembered that the sum of the weights on the $\Delta\log(\frac{M}{N})_{t-i}$ term in equation (1a) is equal to $\frac{1}{1-\alpha\beta}$. Plugging in our estimate of $\alpha\beta$ gives a value of 1.54. This implies a 1% increase in the per capita money supply will produce a 1.54% increase in the price level after all lagged adjustments have taken place. A sustained increase in the per capita money stock of 5.9% per quarter would then produce a 9.1% per quarter increase in the price level. Over the period of this study, the per capita money stock expanded at a mean rate of 5.9% per quarter while the price level increased an average of 7.5% per quarter.

The hypothesis implies that the price level responds to an increase in the money supply only after some lag. Our estimate of $\alpha\beta$ would imply that after one quarter 65% $(= \frac{1}{1.54})$ of the adjustment has taken place, after two quarters about 88% $(= \frac{1.35}{1.54})$, after three quarters about 96% $(= \frac{1.47}{1.54})$, and after four quarters about 98% $(= \frac{1.51}{1.54})$, i.e., after a year we can expect that whatever changes occurred in the per capita money stock at the beginning of the year will have practically no more effect on the price level.

The response of the rate of price level change to a change in the rate of monetary expansion, other things being equal, depends on the desires of people to hold real cash balances--the response being different with different demands for real balances. If one assumes a specific demand for money function as we have in equation (xiii), one must be willing to accept all the logical conclusions which follow--one being the aforementioned response of the rate of change in prices to a sustained shift in the rate of monetary expansion. On the other hand, if one assumes a certain response of the rate of change in prices to an increase in the rate of monetary expansion, one is also obligated to abide by its logical conclusions--one being a specific form of the demand for money function. Diz's demand for money function is similar to the one used in this study and implies exactly the same conclusion concerning the effect of a sustained shift in the rate of monetary expansion on the rate of change in prices.[1] He goes on to say, though, that on theoretical grounds, other things equal, one would expect an upward and sustained shift in the rate of monetary expansion to bring about a similar increase in the rate of change in prices.[2] He then regresses the rate of change in prices on the current and lagged rates of change in the money supply plus some other variables, and argues that if all relevant lags were taken into account, the sum of the coefficients on the variables expressing the current and lagged rates of change in the money stock would equal one. That the sum of the coefficients is often significantly different from one is grounds for him to conclude that all relevant lags probably were not taken into account.[3] To argue this would be equivalent to arguing that the demand for real balances be independent of past rates of change in

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[1]Diz, pp. 39-40.

[2]*Ibid.*, p. 63.

[3]*Ibid.*, p. 67.

the price level, but on the same page Díz states that his demand for money function would imply a lower level of real cash balances to be consistent with a new and higher rate of change in prices after the upward and sustained shift in the rate of monetary expansion had fully worked its effects on the rate of change in the price level. Indeed, from equation (xiii)

$$\frac{\partial}{\partial \Delta \log P_{t-1-i}}\left[\log\left(\frac{M}{NP}\right)_t\right] = -\alpha\beta(1-\beta)^i < 0 \qquad (i=1, 2, \ldots)$$

where $E_t = \beta \sum_{i=0}^{\infty} (1-\beta)^i \Delta \log P_{t-1-i}$

from the solution of the difference equation (xiv).

Harberger argues similarly that general equilibrium theory would tell us to expect, in a truly "ceteris paribus" situation, that a 1 per cent per annum rise in the quantity of money would produce a 1 per cent per annum rise in the general price level, and that this would mean that in a regression of the rate of change in prices on the current and lagged rates of change in the money stock, the sum of the coefficients on the variables expressing the rates of change in the money stock should equal one.[1] Again this implies that the demand for real balances is independent of past rates of change of prices. But he assumes that people will attempt to reduce their real balances with rising expected cost of holding cash--the rate of change in the expected cost of holding cash being represented by the difference between the rate of inflation in the past year and the rate of inflation in the year before that.[2] Since the sum of the coefficients on the

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[1]Harberger, p. 222.

[2]*Ibid.*, p. 226.

variables expressing the current and lagged rates of monetary change is very close to unity, he concludes that the lags which he introduces fully capture the effect of money supply increases on prices.[1] This conclusion is not at all justified. Without exactly specifying the form of the demand for money function, which Harberger does not do, it is impossible to say what the exact value of the sum of the coefficients of the variables expressing the rates of monetary change should be. All we can say, under Harberger's assumptions about the demand for money, is that the sum should be greater than, not equal to, unity.

Some discussion of the economic meaning of β and its estimate is appropriate. From equation (xiv) we deduce that β gives the speed at which the expected rate of price change adjusts to the actual rate. When β is multiplied by the difference between last period's actual and expected rates of price change, the product gives the rate of change of the expected rate of change in the price level.

Cagan states that $\frac{1}{\beta}$ gives the "average length of each weighting pattern" and "serves as a measure of the average length of time by which expectations of price changes lag behind the actual changes."[2] Diz seems to be talking about the same thing when he mentions the "average length of the weighting pattern," but doesn't specify how he arrives at it.[3] The average length of the weighting pattern as defined to be equal to $\frac{1}{\beta}$ does not appear to have any significant meaning. If $\beta = .14$ as given by our estimate and an

[1] _Ibid_., p. 238.

[2] Cagan, p. 41.

[3] Diz, pp. 44-46, 73.

initial equality of expected and actual rates of price change is assumed, and if the actual rate of price change shifted to a new and constant level, this definition would seem to imply that the expected rate of price change would adjust to this new and constant level in about 7.1 ($= \frac{1}{.14}$) quarters.

As mentioned above, the solution of difference equation (xiv) is

$$E_t = \beta \sum_{i=0}^{\infty} (1-\beta)^i \Delta \log P_{t-1-i}$$

Let us assume that in the initial and all previous periods the expected rate of price change was equal to the actual, i.e., $E_o = \Delta \log P_o$, and in all subsequent periods, the actual rate of price change was equal to $C \neq \Delta \log P_o$. For any given number of quarters, n, from the initial period, it is now possible to compute the extent to which the expected rate of price change, E_n, has adjusted to the actual rate, C. We merely solve for X in

$$XC = \sum_{i=0}^{n} (1-\beta)^i C$$

where $XC = E_n$. This reduces to

$$X = \sum_{i=0}^{n} (1-\beta)^i = \frac{\beta[1-(1-\beta)^{n+1}]}{1 - (1-\beta)} = 1 - (1-\beta)^{n+1}$$

For = .14 we have

$$X = 1 - (.86)^{n+1}$$

This means that after one quarter the expected rate has adjusted to 26% of the actual rate of price change, after two quarters to 36%, after a year to 53%, after seven quarters to 70%, and after two years to 74%. Only after five years under these assumptions has the expected rate adjusted to 96% of the actual. Note that the full adjustment has not been made after 7.1 quarters.

Since the actual rate of price change is rarely constant, about the only meaningful interpretation of β would seem to be that the current quarter's expected rate of price change is revised at the rate of 100β% of the difference between last quarter's actual and expected rates of price change, whatever that difference might be. Our estimate of β equal to .14 would imply that this rate of adjustment is 14%.

In equation (2') the estimate of λ is .31 and is statistically significant at the 5 per cent level. The sum of the coefficients of $\Delta\log B_t$ and $\Delta\log B_{t-1}$ is equal to .50, implying that a sustained increase in the rate of expansion of the monetary base would, after two quarters, cause an increase in the rate of monetary expansion of half the magnitude. Further, the sum of the coefficients of $\Delta\log B_t$, $\Delta\log B_{t-1}$, and $\Delta\log B_{t-2}$ is .65 with the obviously similar implication. We notice that the total sum of the coefficients of the variables expressing rates of change in the monetary base is equal to 1 implying that after all adjustments have worked themselves out, a 1 per cent increase in the monetary base will affect a 1 per cent increase in the money supply.

In equation (2") the estimate of λ_2 is not statistically different from zero at the 5 per cent level, but the estimate of λ_3 is equal to .64 and is significant at the 5 per cent level. The implication is that a shift in the rate of change in the monetary base does not exert any influence on the rate of change in the money supply until the second quarter after the shift. Equations (2') and (2") give similar results in that both imply a lagged response in the rate of money expansion to changes in the rate of increase in the monetary base. They imply different lagging patterns, however. The mean rate of expansion of the money supply (as opposed to the per capita money supply considered in equation (1')) over the period of study was 6.2% per quarter and that of the monetary base was 5.9% per quarter.

Only the estimate of b equal to 1.13 in equation (3a') is significant at the 5 per cent level implying that a 1 peso increase in the deficit is associated with a 1.13 peso increase in the monetary base. Over the period of the study, the average quarterly increase in the Central Bank claims on the government (used as a proxy for the deficit) was 10.7 billion pesos and the average quarterly increase in the monetary base was 12.9 billion pesos. The results imply that a 10.7 peso per quarter increase in the Central Bank claims on the government will produce an increase in the monetary base of 12.1 pesos per quarter. Our a priori expectations would be that b be close to unity. Where the government finances most of its deficit by the sale of securities to the Central Bank we would expect that after completion of the expenditures the increase in the sum of bank reserves and currency would be approximately equal to the amount of the deficit.[1]

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[1]The rate of change in the monetary base can be deduced from equation (3a') as was done with equation (3a) in Chapter II, i.e.,

(3b') $$B_t = B_{(-1)} + \sum_{i=0}^{t} \Delta B_i + \sum_{i=0}^{t} \varepsilon_{3i} = B_{(-1)} + \sum_{i=0}^{t} (a + bD_i + \varepsilon_{3i})$$

$$= B_{(-1)} + at + b\sum_{i=0}^{t} D_i + \sum_{i=0}^{t} \varepsilon_{3i}$$

Equation (4') calls for the price index, P_{t-3}, to be expressed as a fraction of 1 (e.g., .89, 1.32, etc.) but instead, it was read

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and therefore after dividing equation (3a') by (3b') we get

$$(3'') \qquad \Delta \log B_t = \frac{a + b\, D_t + \varepsilon_{3t}}{B_{(-1)} + at + b \sum_{i=0}^{t} D_i + \sum_{i=0}^{t} \varepsilon_{3i}}$$

I attempted first to estimate a and b from equation (3") after taking a Taylor expansion around the points $a = 0$, $b = 1$, $\varepsilon_{3t} = o$, and $\sum_{i=0}^{t-1} \varepsilon_{3i} = 0$ and dropping the higher order terms. The linearized equation is

$$(3''') \qquad \Delta \log B_t = \frac{D_t \sum_{i=0}^{t} D_i}{(B_{(-1)} + \sum_{i=0}^{t} D_i)^2} + a \frac{B_{(-1)} - tD_t + \sum_{i=0}^{t} D_i}{(B_{(-1)} + \sum_{i=0}^{t} D_i)^2}$$

$$+ b \frac{B_{(-1)}\, D_t}{(B_{(-1)} + \sum_{i=0}^{t} D_i)^2} + \varepsilon'_{3t}$$

$${}'_{3t} = \frac{B_{(-1)} + \sum_{i=0}^{t-1} D_i}{(B_{(-1)} + \sum_{i-0}^{t} D_i)^2} \varepsilon_{3t} - \frac{D_t \sum_{i=0}^{t-1} \varepsilon_{3i}}{(B_{(-1)} + \sum_{i=0}^{t} D_i)^2}$$

Upon adding three seasonal dummy variables to equation (3''') direct least squares yielded an estimate of 1.04 for "b" with standard error .25 and an estimate of .68 for "a" with standard error 1.02. Direct least square estimates of equation (3a') with seasonal dummies added yielded an estimate of 1.09 for "b" with standard error .13 and an estimate of -.076 for "a" with standard error 3.11.

The estimates of "a" in both equations are not statistically significant at the 5 per cent level, but both the estimates of "b" are significant and of similar magnitudes. This experiénce was a testimony to me of the usefulness of a Taylor expansion in empirical work.

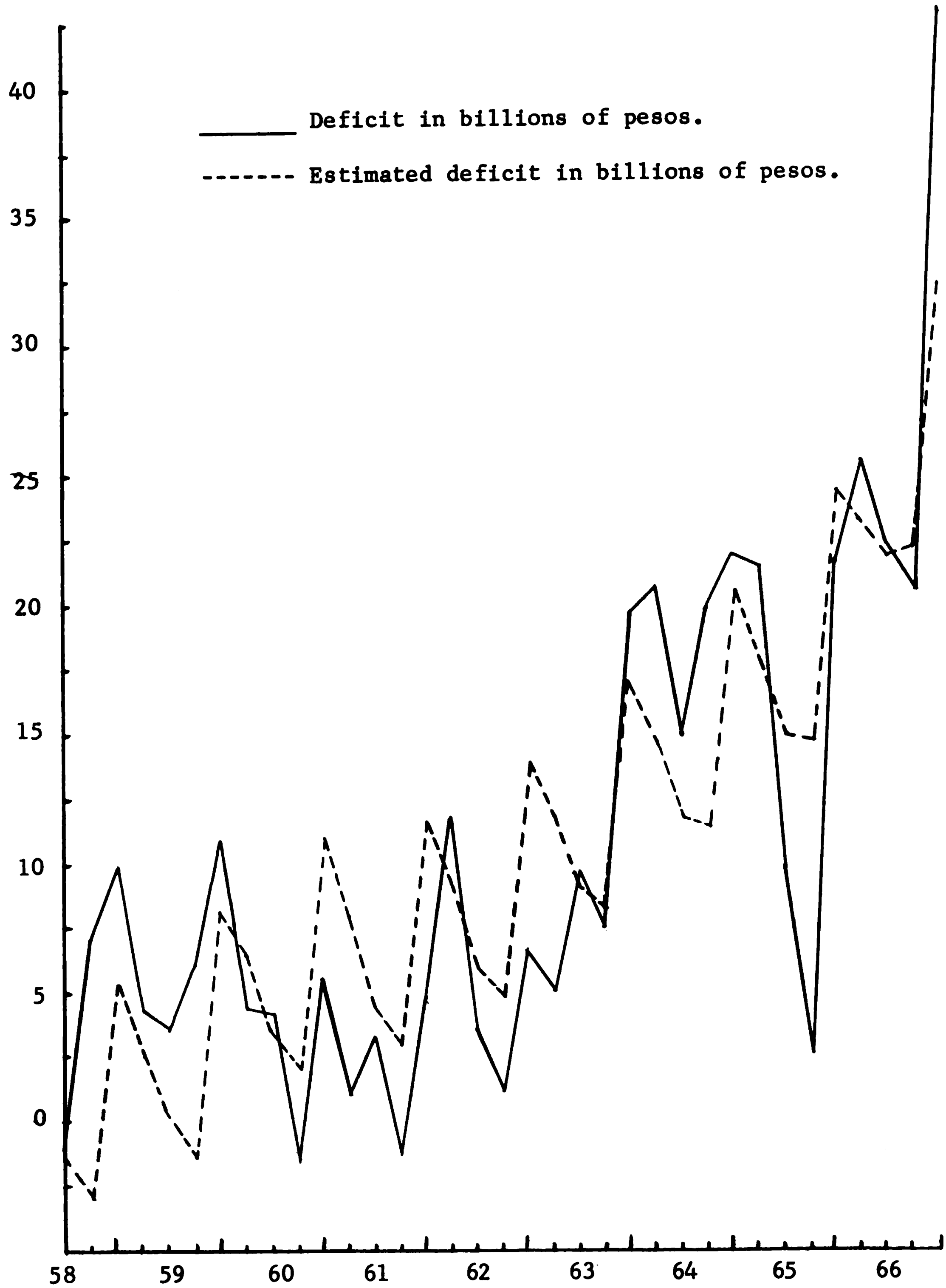

Fig. 2.--Actual and Estimated Deficit

into the computer as a fraction of 100 (e.g., 89, 132, etc.). It is therefore necessary to multiply the estimate of g_i in equation (4') by 100 to get the estimate of the real government deficit (increase in Central Bank claims on the government). This estimate is 3.1 billion pesos per quarter and is significant at the 5 per cent level.

The assumption of a constant real deficit is not a good one, but for lack of a better one describing the time path of real deficit expenditures, it was chosen. Even under this stringent assumption, it was possible to explain the quarterly deficit (increase in Central Bank claims on the government) with an $R^2 = .71$. Figure 2 shows the actual and estimated deficit (increase in Central Bank claims of the government).

Summary.--In this chapter, we have presented the results of the estimation of the parameters of the model.

The parameter estimates of equation (1') imply that a 1% increase in the per capita money supply will produce a 1.54% increase in the price level after all lagged adjustments have taken place, and that it takes about five years for the expected rate of price change to adjust to within 96% of some constant actual rate of price change.

The estimates of the parameters in equations (2') and (2") imply that after two quarters, a 1% increase in the monetary base will have caused the money stock to increase by .50% (equation (2')) or by .64% (equation (2")).

The estimate of 1.13 for b in equation (3a') supports our expectation that a one dollar increase in Central Bank claims on the government will lead to a one dollar increase in the monetary base. The estimate of 1.13 is reasonably close to the estimate of one which we would expect. If we had used deficit data instead of increases in Central Bank claims on the government, we would have expected our estimate of b to be less than one.

Equation (4') was reasonably good in estimating the actual deficit (increase in Central Bank claims on the government) even with the assumption of a constant real deficit (constant increase in Central Bank claims on the government per quarter) which was estimated to be 3.1 billion pesos per quarter in 1958 prices.

CHAPTER IV

DATA AND HISTORICAL SKETCH, 1958-1966

Data

The period of this study was the second quarter of 1958 through the fourth quarter of 1966. Because of the relevant lags, the data cover the period from the third quarter of 1957 through the fourth quarter of 1966.[1]

All of the data are end of quarter figures, and all of the data except that for population came from International Financial Statistics, a publication of the International Monetary Fund. The population data is from the Statistical Bulletin for Latin America, Vol. III, No. 7, Feb. 1966, United Nations.

Prices.--The "cost of living" price index was used.

Authorized by the Supply Law of 1964, the government relied quite heavily on direct price controls during 1964 and 1965 in an attempt to limit the rate of increase in the price level. The price controls were partially relaxed during 1966 and totally after the repeal of the Supply Law in November, 1966.[2]

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[1]This beginning period was chosen because deposits were returned to commercial banks in October, 1957 from the Central Bank where they had been since 1946. There is, therefore, no data for commercial bank reserves from the end of the first quarter, 1946 to the end of the third quarter, 1957. Diz, pp. 5, 7.

[2]International Monetary Fund, pp. 13-15.

The estimates of the parameters in equation (1') presented in Chapter III, will be somewhat biased because of this fact.

Money Supply.--The sum of currency outside banks ("a residual series obtained by subtracting commercial banks' holdings of Central Bank notes and coin from the total amounts issued. The figures in this series include currency held by the government and semi-fiscal agencies.")[1] plus demand deposits held by the nonbanking public. "This series excludes interbank deposits, government deposits, and items in process of collection."[2]

Monetary Base.--The sum of bank reserves ("amount of reserves held by commercial banks in the form of gold, currency, and deposits with the Central Bank.")[3] plus currency outside banks.

Deficit.--Expenditures of the central government minus tax receipts. I was unable to find quarterly data for this series. Annual data was available, but the two series which I checked were seemingly quite unrelated to each other. Since the central government relied heavily on the Central Bank for the financing of its deficit, I used the series of quarterly increases in the Central Bank's claims on the government as a proxy for the quarterly deficit.

As a result of this substitution, we would expect the estimate of b in equation (3a') to be very close to one, since a one dollar increase in Central Bank claims on the government would mean a one dollar increase in the monetary base. As we noted in Chapter III, the estimate of b is very close to unity.

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[1] Diz, p. 81.

[2] *Ibid.*

[3] *Ibid.*, p. 82.

However, the estimate of g_1 in equation (4') would not be the estimate of planned real fiscal deficit, but rather the estimate of planned real deficit financing through the Central Bank. The evidence suggests that deficit financing through the Central Bank and the deficit move together; therefore, this substitution probably won't affect the validity of our results.

Population.--The quarterly series was constructed by linearly interpolating the annual series of mid-year estimates.

Historical Sketch

Since the model outlined in Chapter II, has been used to describe the Argentine case, a brief history of the inflation over the period will be of interest.

Since the beginning of 1958, Argentina has experienced a substantial amount of inflation. Near the end of 1958, the government initiated a stabilization plan aimed at, among other things, reducing the rate of price increase. "The chief points of this plan were the abandonment of price controls and subsidies, a tight money policy, a balanced budget, wage controls, a further devaluation of the peso and thereafter a stabilization of the foreign exchange market."[1] Prices increased greatly in 1959 ". . . reflecting merely corrective price adjustments."[2]

An examination of Table 1 below indicates that a tight money policy was not followed in 1959 since the money supply increased by 40.5 per cent. In fact, the rate of monetary expansion was not

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[1]Villaneuva, p. 17.

[2]International Monetary Fund, p. 13.

TABLE 1

PERCENTAGE CHANGE (CONTINUOUS COMPOUNDING) IN THE PRICE LEVEL AND MONEY STOCK

Year	Cost of Living Price Index	Money Stock
1958	27.4	22.6
1959	76.1	40.5
1960	24.0	28.6
1961	12.8	16.1
1962	24.6	5.5
1963	21.8	16.1
1964	20.0	34.2
1965	25.1	27.9
1966	27.7	25.9

Source: International Financial Statistics by the International Monetary Fund.

stabilized until 1962. The rate of price increase reached its minimum in 1961, at which time the government decided to gradually abandon the stabilization plan.

The plan was criticized because it did not eliminate the inflation.[1] One should ask just how deflationary the plan really was. Between 1959 and 1962, the money supply more than doubled and the government achieved neither a substantial rise in taxation nor a significant reduction in expenditures.[2]

The period since 1962 has been characterized by substantial increases in prices and the money stock and large government deficits.[3]

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[1]Ferrer, p. 198.

[2]Villaneuva, p. 22

[3]International Monetary Fund, p. 20.

CHAPTER V

THE DYNAMIC PROPERTIES OF THE MODEL

It is of interest to note the time path of the rate of change in the price level in the absence of any external constraints or shocks. Although the model implies a self generating inflation, we still don't know just how self generating it is. That is, if the system is left alone, will successive periods generate the same rate of inflation, a lesser rate, a greater rate, or will the rate of inflation oscillate in some manner? To consider these questions, let us look at the model again in its deterministic form as presented in the first section of Chapter II.

$$(1) \qquad \Delta \log P_t = -\beta\gamma + \Delta \log(\tfrac{M}{N})_t + \alpha\beta\Delta\log P_{t-1} + \beta(\log(\tfrac{M}{N})_{t-1} - \log P_{t-1})$$

$$(2) \qquad \Delta \log M_t = \lambda \Delta \log B_t + (1-\lambda)\Delta \log M_{t-1}$$

$$(3) \qquad \Delta \log B_t = \frac{a + bD_t}{B_{(-1)} + at + b\sum_{i=0}^{t} D_i}$$

$$(4) \qquad D_t = \frac{P_{t-3}g_1}{\Delta \log P_{t-2}}(e^{\Delta \log P_{t-2}} - 1)$$

From equation (3) we note that the rate of change in the base decreases as time increases, all other things equal. We also note that

$$\frac{\partial\ \Delta\log B_t}{\partial D_t} = \frac{B_{(-1)} + a(t-1) + b\sum_{i=0}^{t-1} D_i}{(B_{(-1)} + at + b\sum_{i=0}^{t} D_i)^2}$$

which is probably always positive since the D_i are almost always positive. Therefore, $\Delta\log B_t$ decreases with time and increases with increases in the deficit, all other things constant. We cannot judge a priori which will dominate. If the net effect is an increase in $\Delta\log B_t$ as time passes, the rate of change in the price level could increase without limit depending on the magnitudes of α, β, and γ.

To get some idea, we solve the reduced form for $\Delta\log P_t$. To do this, it is necessary to linearize equation (3) and equation (4). Equation (3) was linearized by taking a Taylor expansion around $\bar{D}$, the average of all the D's, around $\overline{\sum^{t-1} D_i}$ (t = 35, the number of quarters during the study), the average of all the sums of the D's from the initial period to one period before the final period, and around $\bar{t}$, the average of the number of quarters. Equation (4) was linearized by taking a Taylor expansion around $\overline{\Delta\log P_{t-2}}$, the average of the rates of change in the price level from two quarters before the initial one to two quarters before the final one, and around $\overline{P_{t-3}}$, the average of the price level from three quarters before the initial one to three quarters before the final one. The system of equations now becomes

(1) $$\Delta\log P_t = -\beta\gamma + \Delta\log(\frac{M}{N})_t + \alpha\beta\Delta\log P_{t-1} + \beta(\log(\frac{M}{N})_{t-1} - \log P_{t-1})$$

(2) $$\Delta\log M_t = \lambda\Delta\log B_t + (1-\lambda)\Delta\log M_{t-1}$$

(3) $$\Delta \log B_t = A + BD_t - Ct - J \sum_{i=0}^{t-1} D_i$$

(4) $$D_t = - G + FP_{t-3} + K\Delta \log P_{t-2}$$

where $$A = \frac{(a + b\overline{D})\Delta - b(B_{(-1)} + a(\overline{t}-1) + b(\overline{\sum^{t-1} D_i}))\overline{D}}{\Delta^2}$$

$$+ \frac{(a^2+ab\overline{D})\overline{t} - b(a + b\overline{D})(\overline{\sum^{t-1} D_i})}{\Delta^2}$$

$$B = \frac{b(a(\overline{t}-1) + B_{(-1)} + b(\overline{\sum^{t-1} D_i}))}{\Delta^2}$$

$$C = \frac{a^2 + ab\overline{D}}{\Delta^2}$$

$$J = \frac{b(a + b\overline{D})}{\Delta^2}$$

$$K = \frac{g_1 \overline{P_{t-3}}}{\overline{\Delta \log P_{t-2}}} \left(e^{\overline{\Delta \log P_{t-2}}} - \frac{(e^{\overline{\Delta \log P_{t-2}}} -1)}{\overline{\Delta \log P_{t-2}}}\right)$$

$$F = g_1 \frac{(e^{\overline{\Delta \log P_{t-2}}} -1)}{\overline{\Delta \log P_{t-2}}}$$

$$G = g_1 \left(e^{\overline{\Delta \log P_{t-2}}} - \frac{(e^{\overline{\Delta \log P_{t-2}}} -1)}{\overline{\Delta \log P_{t-2}}}\right) \overline{P}_{t-3}$$

and $$\Delta = B_{(-1)} + a\overline{t} + b\overline{D} + b(\overline{\sum^{t-1} D_i})$$

By plugging in the estimated values of the parameters and the values of the other constants, we can solve for the coefficients in the structural equations. We get a value of 19.39×10^{10} for Δ and consequently a value of 375.792×10^{20} for Δ^2. The coefficients A, B, C, and J are essentially equal to zero, because a Δ^2 appears in the denominator of each of them and the highest magnitude in any one of the numerators is of order 10^{10}. The reduced form for the rate of change in prices is then simply derived from equations (1) and (2) after setting $\Delta \log B_t = 0$ in equation (2). This gives

$$(5.1a) \qquad \Delta \log P_t = -\beta\gamma + \alpha\beta\Delta \log P_{t-1} + (1-\lambda)\Delta \log M_{t-1} + \beta(\log M_{t-1} - \log P_{t-1} - \log N_{t-1}) - \Delta \log N_t$$

By lagging (5.1a) one quarter and subtracting the result from (5.1a) we get

$$(5.1b) \qquad \Delta \log P_t = (1+\alpha\beta-\beta)\Delta \log P_{t-1} - \alpha\beta\Delta \log P_{t-2} + (1-\lambda+\beta)\Delta \log M_{t-1} - (1-\lambda)\Delta \log M_{t-2} - \Delta \log N_t - \beta\Delta \log N_{t-1}$$

which becomes upon substituting in the estimated values of the parameters

$$(5.1) \qquad \Delta \log P_t = 1.21\Delta \log P_{t-2} - .35\Delta \log P_{t-2} + .83\Delta \log M_{t-1} - .69\Delta \log M_{t-2} - 1.14\Delta \log N$$

where we let $\Delta \log N_t = \Delta \log N_{t-1} = \Delta \log N$ = the average rate of increase in the population per quarter.

From equation (2), we note that the reduced form for the rate of increase in the money stock is

$$(5.2a) \qquad \Delta\log M_t = (1-\lambda)\Delta\log M_{t-1}$$

which becomes, upon substituting in the estimated value of λ

$$(5.2) \qquad \Delta\log M_t = .69\Delta\log M_{t-1}$$

Now we can simulate the time path of the rate of change in the price level. To evaluate the magnitudes of both endogenous variables for quarter t we insert into equations (5.1) and (5.2) the values for quarter (t-1) and for preceeding quarters, as required, along with the rate of change in the population. Using the values of the endogenous variables for quarter t thus obtained, we generate values of the endogenous variables for quarter (t+1) in like manner. We continue the process until we have proceeded far enough into the future to satisfy our curiosity.

In Table 2 below, we begin with the actual rate of price change in the third quarter of 1966, 3.49% per quarter. This rate of price change generates a 3.53% rate for the succeeding quarter. After three quarters, the rate of price increase has dropped to .41% per quarter, and all rates of price change after one year are then negative. We note that the time path of the rate of change in the price level is one of damped oscillation which reaches an equilibrium rate of -3.26% per quarter after 6.25 years.

These results follow from the fact that in the process of linearization of equation (3) the link between the deficit and the money supply is broken, so that the rate of monetary expansion falls

TABLE 2

TIME PATH OF QUARTERLY RATE OF CHANGE IN THE PRICE LEVEL

Quarters Since Initial Period	Rate of Price Change	Quarters Since Initial Period	Rate of Price Change
0	.0349	15	-.0346
1	.0353	16	-.0342
2	.0185	17	-.0339
3	.0041	18	-.0336
4	-.0141	19	-.0334
5	-.0242	20	-.0332
6	-.0306	21	-.0330
7	-.0342	22	-.0329
8	-.0360	23	-.0328
9	-.0367	24	-.0327
10	-.0367	25	-.0326
11	-.0364	26	-.0326
12	-.0360	27	-.0326
13	-.0355	28	-.0326
14	-.0350	29	-.0326

to zero after 4.75 years. The most dominant determinants of the current rate of price change become the two previous periods' rates of price change after the rate of monetary expansion has sufficiently diminished. After about a year, the rate of monetary expansion will have diminished sufficiently so that this will be the case.

This gives us an idea of what we might expect if the deficit were eliminated and the expansion of the money supply were halted.

If the rate of price increase were greater than 3.49% per quarter in the quarter in which these restrictive policies were initiated, it would take longer for the rate of price increase to fall to zero. This would also be the case if the rate of increase in the money stock were larger than the 6.28% per quarter in the initial period and the 5.26% in the period before that, which we used as initial conditions. It is clear that the system seeks a stable equilibrium, but the length of time that it takes to achieve that equilibrium depends upon the initial conditions.

For these results to hold, it is necessary that the coefficient of expectation, β, does not change. It is probable, though, that under protracted restrictive policies and diminishing rate of price increase that β would change.

CHAPTER VI

CONCLUSIONS

The purpose of this study was to formulate a model to explain inflation in a country whose institutions are such as to make the inflation self generating in nature. The model is designed to describe situations where there is a high rate of price increase, a high rate of monetary expansion and a large deficit which is financed partially or wholly through the Central Bank. Also, where tax collections, whose nominal value is fixed at the time of assessment, lag behind government expenditures so that the rising price level over the period in itself produces a deficit. That is, expenditures rise with the rising price level over the period, but tax collections do not. Argentina, being such a country, was chosen and its data were used to test the model.

The model comprises four equations. Equation (1) shows the relationship of the rate of change in the price level to the rate of change in the money stock. Equation (2) depicts the relationship between the rate of monetary expansion and the rate of increase of the monetary base (bank reserves plus currency outside banks). Equation (3) shows how the rate of fiscal deficit per period and the rate of change in the monetary base are related. Equation (4) shows the link between the rate of change in the price level and the rate of deficit expenditure per period.

From the theory of consumer behavior, it was shown that under specific assumptions, the aggregate demand for real per capita money balances can be expressed as a function of the expected rate of price change. Then a specific demand for money function similar to the one Phillip Cagan used in his study of hyperinflation was chosen and then transformed to express the rate of change in the price level as the dependent variable. This was labeled equation (1).

From a simple theory of banking behavior and consumer response to an increase in their currency holdings, the response in the rate of change in the money supply to variations in the rate of change in the monetary base was shown in equation (2).

Equation (3) explains the somewhat mechanical relationship between the rate of fiscal deficit per period and the rate of change in the monetary base.

In deriving equation (4) we indicated how the institutional structure of the fiscal mechanism establishes a link between the rate of change in the price level and the rate of fiscal deficit per period.

Estimates of the parameters of the model were obtained, most of which seem to be quite reasonable. The estimate of .35 for $\alpha\beta$ in equation (1) implies that a 1% increase in the per capita money supply will produce a 1.54% increase in the price level after all lagged adjustments have taken place. This result is implied by the specific demand for money function used in this study. The estimate of .14 for β implies that the expected rate of price level change would adjust to within 96% of a new and constant actual rate of price change only after five years.

The estimate of .31 for λ in equation (2) implies that a 1% increase in the monetary base will have produced a .5% increase in the money stock after two quarters.

The estimate of 1.13 for b in equation (3) implies that a one dollar increase in Central Bank claims on the government (proxy for the deficit) will cause a 1.13 dollar increase in the monetary base.

A constant real increase in Central Bank claims on the government (proxy for the deficit) of 3.1 billion pesos per quarter was obtained as the estimate of g_1 in equation (4).

The dynamic analysis of the model indicates that the time path of the rate of change in the price level free of any external constraints or shocks, is one of damped oscillation which seeks an equilibrium rate of -3.26% per quarter. The length of time which it takes to achieve this equilibrium depends upon the initial conditions, i.e., the initial rates of change in the price level and the initial rates of monetary expansion.

We can conclude that the system of four equations does reasonably well in describing the inflationary process in Argentina for the period 1958 to 1966. It is seen to be self generating in nature.

The analysis shows that a government plan to halt the inflation by eliminating its deficit thereby reducing the increase in the monetary base and the rate of increase in the money stock will have the desired effect only after some time lag. Elimination of the deficit will have its full impact on the monetary base within the current period. The adjustment process takes about a year, however, for a 1% reduction in the monetary base to cause a 3/4% reduction in the money stock. It takes another year for a 1% reduction in the money stock to bring about a 1% reduction in the price level. The results indicate that no less than two years would be required to feel the effects of such a plan. A reduction in the rate of increase in the monetary base to zero would produce quicker results, however. If such a measure were taken, it seems possible to reduce the rate of price increase from about 3.5% per quarter to 0.0% per quarter in about a year. But, barring such a strong measure, it is not surprising then that so many hold the view that a stabilization plan is ineffective when it is tried only for a short period and then abandoned because it did not halt the inflation. Results can be seen only if resolution and patience are exercised.

The results also show that a necessary part of any plan to halt the inflation is either elimination of the government deficit or the financing of the deficit by borrowing directly from the population. If the deficit is not eliminated, then it must not be financed through the Central Bank, i.e., this is necessary if it is desired to maintain a stable flow of bank credit to the private sector. If the monetization of the deficit were to continue, the money supply could be reduced by sufficiently restricting private credit, but this is probably not very desirable. This is what was done in 1959-1960, however, which was another criticism of the stabilization plan.

We have shown that a linear regression of the rate of change in prices on the current and lagged rate of change in the money stock, as Diz and Harberger do, introduces a simultaneous equation bias into the estimates of the coefficients. This is so because the equation for the rate of change in prices is only one in a system of equations wherein the current rate of money expansion is also an endogenous variable.

We have shown that the one equation models of inflation, when applied to countries like Argentina, leave much of the process yet to be explained. This study was an attempt, even though in a quite simplified manner, to explain the process.

APPENDIX A

In Chapter III, the results of the estimation of equations (1'), (2'), (2"), (3a'), and (4') were presented where equations (1') and (4') were estimated by the method of direct least squares and equations (2'), (2") and (3a') were estimated by the method of two stage least squares.

During the sequence of many computer runs, I obtained direct least square estimates of equations (2'), (2") and (3a') as well as their two stage least square estimates. The following are the results of those least square estimations. Partially for review and to maintain continuity the direct least square estimates of equations (1') and (4') are presented again as well.

Those parameter estimates with an asterik are significant at the five per cent level.

(1') $$\Delta\log\left(\frac{NP}{M}\right)_t = \underset{(.033)}{.0093} + \underset{(.15)}{.35^*}\,\Delta\log P_{t-1} + \underset{(.054)}{.14^*}\left(\log\left(\frac{M}{N}\right)_{t-1} - \log P_{t-1}\right)$$

$$+ \underset{(.027)}{.057^*}S_1 + \underset{(.026)}{.055^*}S_2 + \underset{(.026)}{.044}S_3 + e_1 \qquad (R^2 = .44) \text{ DLS}$$

(2') $$\Delta\log M_t - \Delta\log M_{t-1} = \underset{(.0089)}{.042^*} + \underset{(.099)}{.57^*}(\Delta\log B_t - \Delta\log M_{t-1}) - \underset{(.014)}{.079^*}S_1$$

$$- \underset{(.013)}{.028^*}S_2 - \underset{(.012)}{.053^*}S_3 + e_2 \qquad (R^2 = .80) \text{ DLS}$$

(2") $$\Delta\log M_t = \underset{(.011)}{.052^*} + \underset{(.12)}{.40^*}\Delta\log B_t + \underset{(.12)}{.37^*}\Delta\log B_{t-1} - \underset{(.012)}{.058^*}S_1$$

$$- \underset{(.013)}{.038^*}S_2 - \underset{(.012)}{.042^*}S_3 + e'_2 \qquad (R^2 = .74) \text{ DLS}$$

$$(3') \quad \Delta B_t = \underset{(3.11)}{-.076} + \underset{(.13)}{1.09^{*}}D_t + \underset{(3.40)}{2.36}S_1 - \underset{(3.42)}{.60}S_2 + \underset{(3.46)}{3.78}S_3 + e_3 \qquad (R^2 = .73) \text{ DLS}$$

$$(4') \quad D_t = \underset{(2.55)}{2.73} + \underset{(.0040)}{.031^{*}} \frac{P_{t-3}}{\Delta \log P_{t-2}}\left(e^{\Delta \log P_{t-2}} - 1\right) - \underset{(2.74)}{3.06}S_1$$

$$- \underset{(2.66)}{6.69^{*}}S_2 - \underset{(2.65)}{8.30^{*}}S_3 + e_4 \qquad (R^2 = .71) \text{ DLS}$$

APPENDIX B

ANNUAL MODEL

An attempt was made to explain the annual averages of the variables introduced in this study, i.e., the annual rates of price increase, money supply expansion, growth of the monetary base, and the annual deficit expenditures.

The annual model was developed along similar lines as the quarterly model but with some different assumptions. I was unable to specify a demand for money function and a function which specifies how price expectations are formed in such a way as to give, upon transformation of the demand function, reasonable estimates of the parameters. The other three equations gave quite reasonable estimates in comparison with the quarterly model.

The following is the development of the model in the framework of Chapter II. Then the results of direct least square estimation of the individual equations are given.

The rate of price change.--Let us assume that the demand for average annual per capita real money balances is expressed as a function of the expected annual rate of price change and is a random variable.

(i) $$\log(\frac{M}{NP})^a_t = \gamma - \alpha E^a_t + \varepsilon_{1t}$$

Where E^a_t is the expected annual rate of price change in year t, $\alpha > 0$, and $\varepsilon_{1t} \sim NID(0,\sigma^2_1)$.

Assume that

(ii) $$E_t^a = E_t^q$$

where E_t^q is the expected annual rate of price change in the first quarter of year t. Let E_t^q be revised quarterly according to the following rule.

(iii) $$\Delta E_t^q = \beta(4\Delta \log P^q_{t-\frac{1}{4}} - E^q_{t-\frac{1}{4}})$$

where $0 < \beta < 1$ and $\Delta \log P^q_{t-\frac{1}{4}}$ is the quarterly rate of price change in the last quarter of year t-1, so that $4\Delta \log P^q_{t-\frac{1}{4}}$ is the annual rate of price change during the last quarter of year t-1. $\Delta E_t^q = E_t^q - E^q_{t-\frac{1}{4}}$ and $\Delta \log P_t^q = \log P_t^q - \log P^q_{t-\frac{1}{4}}$. The solution of the difference equation (iii) gives

(iv) $$E_t^q = 4\beta \sum_{i=0}^{\infty} (1-\beta)^i \Delta \log P^q_{t-\frac{i+1}{4}}$$

Now upon substituting this for $E_t^q = E_t^a$ into the demand for money function we get

(v) $$\log(\frac{M}{NP})_t^a = \gamma - 4\alpha\beta \sum_{i=0}^{\infty} (1-\beta)^i \Delta \log P_{t-\frac{i+1}{4}} + \varepsilon_{1t}$$

By lagging equation (v) four quarters and subtracting the lagged value from equation (v) and multiplying through by -1 we get

$$\text{(vi)} \qquad \Delta \log P_t^a - \Delta \log\left(\frac{M}{N}\right)_t^a = 4\alpha\beta \Delta \log P_{t-\frac{1}{4}}^q + 4\alpha\beta(1-\beta)\Delta \log P_{t-\frac{2}{4}}^q$$

$$+ 4\alpha\beta(1-\beta)^2 \Delta \log P_{t-\frac{3}{4}}^q + 4\alpha\beta(1-\beta)^3 \Delta \log P_{t-\frac{4}{4}}^q$$

$$+ 4\alpha\beta[(1-\beta)^4 - 1]\Delta \log P_{t-\frac{5}{4}}^q$$

$$+ 4\alpha\beta[(1-\beta)^5 - (1-\beta)]\Delta \log P_{t-\frac{6}{4}}^q + \dots + \varepsilon_{1t-1} - \varepsilon_{1t}$$

By lagging equation (vi) by one quarter and multiplying by (1-β) and subtracting the result from equation (vi) we get upon rearranging

$$\text{(vii)} \qquad \Delta \log P_t^a = \Delta \log\left(\frac{M}{N}\right)_t^a + \left(\Delta \log P_{t-\frac{1}{4}}^a - \Delta \log\left(\frac{M}{N}\right)_{t-\frac{1}{4}}^a\right)$$

$$- \beta\left(\Delta \log P_{t-\frac{1}{4}}^a - \Delta \log\left(\frac{M}{N}\right)_{t-\frac{1}{4}}^a\right)$$

$$+ 4\alpha\beta\left(\Delta \log P_{t-\frac{1}{4}}^q - \Delta \log P_{t-\frac{5}{4}}^q\right) + \varepsilon_{1t-1}$$

$$- \varepsilon_{1t} - (1-\beta)\left(\varepsilon_{1t-\frac{5}{4}} - \varepsilon_{1t-\frac{1}{4}}\right)$$

The annual rate of price change in year t is then expressed as a function of the current annual rate of per capita money change, the annual rate of price and per capita money change lagged one quarter, and the one quarter and five quarter lagged quarterly rates of price change.

For estimating equation (vii) becomes

$$\text{(1)} \qquad \Delta\log P^a_t - \Delta\log\left(\frac{M}{N}\right)^a_t - \left(\Delta\log P^a_{t-\frac{1}{4}} - \Delta\log\left(\frac{M}{N}\right)^a_{t-\frac{1}{4}}\right)$$

$$= \beta\left(\Delta\log P^a_{t-\frac{1}{4}} - \Delta\log\left(\frac{M}{N}\right)^a_{t-\frac{1}{4}}\right)$$

$$+ 4\alpha\beta\left(\Delta\log P^q_{t-\frac{1}{4}} - \Delta\log P^q_{t-\frac{5}{4}}\right) + \varepsilon^*_{1t}$$

where $\varepsilon^*_{1t} = \varepsilon_{1t-1} - \varepsilon_{1t} - (1-\beta)(\varepsilon_{1t-\frac{5}{4}} - \varepsilon_{1t-\frac{1}{4}})$.

<u>The rate of change in the money stock</u>.--Let us assume that the rate of change in the money stock is a stochastic variable of the following form.

$$\text{(2a)} \qquad \Delta\log M^a_t = \lambda \sum_{i=0}^{\infty} (1-\lambda)^i \Delta\log B^a_{t-i} + \varepsilon_{2t}$$

where $0<\lambda<1$, $\lambda \sum_{i=0}^{\infty} (1-\lambda)^i = 1$, and $\varepsilon_{2t} \sim NID(0,\sigma^2_2)$.

Applying the Koyck transformation to equation (2a) we get

$$\Delta\log M^a_t - (1-\lambda)\Delta\log M^a_{t-1} = \lambda\Delta\log B^a_t - (1-\lambda)\varepsilon_{2t-1} + \varepsilon_{2t}$$

or for estimation we write

$$\text{(2)} \qquad \Delta\log M^a_t - \Delta\log M^a_{t-1} = \lambda(\Delta\log B^a_t - \Delta\log M^a_{t-1}) + \varepsilon^*_{2t}$$

where $\varepsilon^*_{2t} = -(1-\lambda)\varepsilon_{2t-1} + \varepsilon_{2t}$.

<u>The rate of change in the monetary base</u>.--Let us assume that the change in the monetary base is a random variable of the following form.

(3a) $$\Delta B_t^a = a + bD_t^a + \varepsilon_{3t}$$

where $\varepsilon_{3t} \sim NID(0,\sigma_3^2)$.

With the resulting estimates of ΔB_t^a, estimates of $\Delta \log B_t^a$ can be computed as described in the derivation of equation (3) in Chapter II above, but with annual data, i.e.,

(3) $$\widehat{\Delta \log B_t^a} = \frac{\widehat{\Delta B_t^a}}{B_{(-1)}^a + \sum_{i=0}^{t} \widehat{\Delta B_i^a}}$$

where $B_{(-1)}^a$ is the average level of the monetary base in the year preceeding year zero.

<u>The rate of deficit expenditure</u>.--We assume here that the current deficit is determined by the price level at the beginning of the year, the rate of change in prices over the year, and the level of real deficit expenditures over the year, which again we assume to be constant. We will recall from Chapter I that the deficit results because the tax assessments are based on the price level at the beginning of the year, but the taxes are paid later on in the year when prices are higher. Expenditures increase with the price level, but tax collections do not, so a deficit results which is larger than was planned. The function expressing the level of deficit expenditure assumes the following form.

(4b) $$D_t^a = \int_0^1 P_{t-1}^a e^{x\Delta \log P_t^a} g_1 dx + \int_0^1 P_{t-1}^a e^{x\Delta \log P_t^a} \varepsilon_{4t} dx$$

where g_1 is the constant level of real deficit expenditure and

$\varepsilon_{4t} \sim NID(0,\sigma_4^2)$. The integration yields

$$D_t^a = \frac{g_1 P_{t-1}^a}{\Delta \log P_t^a}(e^{\Delta \log P_t^a} - 1) + \frac{P_{t-1}^a}{\Delta \log P_t^a}(e^{\Delta \log P_t^a} - 1)\varepsilon_{4t}$$

So for estimation we have

$$(4) \qquad D_t^a = \frac{g_1 P_{t-1}^a}{\Delta \log P_t^a}(e^{\Delta \log P_t^a} - 1) + \varepsilon^*_{4t}$$

where $\varepsilon^*_{4t} = \frac{P_{t-1}^a}{\Delta \log P_t^a}(e^{\Delta \log P_t^a} - 1)\varepsilon_{4t}$.

<u>Summary</u>.--Now to rewrite the four equations to be estimated in the annual model we have

$$(1) \qquad \Delta \log P_t^a - \Delta \log(\tfrac{M}{N})_t^a = (1-\beta)(\Delta \log P_{t-\frac{1}{4}}^a - \Delta \log(\tfrac{M}{N})_{t-\frac{1}{4}}^a)$$
$$+ 4\alpha\beta(\Delta \log P_{t-\frac{1}{4}}^q - \Delta \log P_{t-\frac{5}{4}}^q) + \varepsilon^*_{1t}$$

$$(2) \qquad \Delta \log M_t^a - \Delta \log M_{t-1}^a = \lambda(\Delta \log B_t^a - \Delta \log M_{t-1}^a) + \varepsilon^*_{2t}$$

$$(3a) \qquad \Delta B_t^a = a + bD_t^a + \varepsilon_{3t}$$

$$(4) \qquad D_t^a = \frac{g_1 P_{t-1}^a}{\Delta \log P_t^a}(e^{\Delta \log P_t^a} - 1) + \varepsilon^*_{4t}$$

<u>Empirical results of estimation</u>.--Direct least squares yields the following estimates. Those estimates with an asterisk are significant at the five per cent level.

(1) $$\Delta \log P_t^a - \Delta \log\left(\frac{M}{N}\right)_t^a = \underset{(.47)}{3.15^*}\left(\Delta \log P_{t-\frac{1}{4}}^a - \Delta \log\left(\frac{M}{N}\right)_{t-\frac{1}{4}}^a\right)$$

$$-.10\left(\Delta \log P_{t-\frac{1}{4}}^q - \Delta \log P_{t-\frac{5}{4}}^q\right) + e_1 \qquad (R^2 = .91) \text{ DLS}$$

(2) $$\Delta \log M_t^a - \Delta \log M_{t-1}^a = \underset{(.12)}{.77^*}\left(\Delta \log B_t^a - \Delta \log M_{t-1}^a\right)$$

$$+ e_2 \qquad (R^2 = .86) \text{ DLS}$$

(3a) $$\Delta B_t^a = \underset{(14.18)}{7.80} + \underset{(.25)}{.92^*} D_t^a + e_3 \qquad (R^2 = .69) \text{ DLS}$$

(4) $$D_t^a = \underset{(.0118)}{.107^*} \frac{P_{t-1}^a}{\Delta \log P_t^a}\left(e^{\Delta \log P_t^a} - 1\right) + e_4 \qquad (R^2 = .78) \text{ DLS}$$

BIBLIOGRAPHY

Books

Ackley, Gardner. Macroeconomic Theory. New York: The Macmillan Company, 1964.

Baer, Werner and Kerstenetzky, Isaac (eds.). Inflation and Growth in Latin America. Homewood, Ill.: Richard D. Irwin, Inc., 1964.

Bailey, Martin J. National Income and the Price Level. New York: McGraw-Hill Book Company, Inc., 1962.

Bronfenbrenner, Martin and Holzman, Franklin D. "A Survey of Inflation Theory." Survey of Economic Theory Vol. I. New York: The Macmillan Co., 1966.

Cagan, Philip. "The Monetary Dynamics of Hyperinflation." Studies in the Quantity Theory of Money. Edited by Milton Friedman. Chicago: The University of Chicago Press, 1956.

Ferrer, Aldo. The Argentine Economy. Berkeley: University of California Press, 1967.

Friedman, Milton. A Theory of the Consumption Function. Princeton: Princeton University Press, 1957.

________. "The Quantity Theory of Money--A Restatement." Studies in the Quantity Theory of Money. Edited by Milton Friedman. Chicago: The University of Chicago Press, 1956.

Harberger, Arnold C. "The Dynamics of Inflation in Chile." Measurement in Economics: Studies in Mathematical Economics and Econometrics in Memory of Yehuda Grunfeld. Edited by Carl F. Christ. Stanford, California: Stanford University Press, 1963.

Henderson, James M. and Quandt, Richard W. Microeconomic Theory, A Mathematical Approach. New York: McGraw-Hill Book Company, 1958.

Keynes, John Maynard. How to Pay for the War. New York: Harcourt, Brace and Company, 1940.

Lerner, Eugene M. "Inflation in the Confederacy 1801-65." Studies in the Quantity Theory of Money. Edited by Milton Friedman. Chicago: The University of Chicago Press, 1956.

Pesek, Boris P. and Saving, Thomas R. Money, Wealth and Economic Theory. New York: The Macmillan Company, 1967.

Selden, Richard T. "Monetary Velocity in the United States." Studies in the Quantity Theory of Money. Edited by Milton Friedman. Chicago: The University of Chicago Press, 1956.

Warburton, Clark. "The Misplaced Emphasis in Contemporary Business Cycle Theory." Readings in Monetary Theory. Edited by F. A. Lutz and L. W. Mints. Homewood, Ill.: Richard D. Irwin, Inc., 1951.

Articles and Periodicals

Adelman, Irma and Adelman, Frank L. "The Dynamic Properties of the Klein-Goldberger Model. Econometrica, XXVII, No. 4, October, 1959, pp. 596-625.

Friedman, Milton. "The Demand for Money: Some Theoretical and Empirical Results." The Journal of Political Economy, LXVII, No. 4, August, 1959, pp. 327-351.

Griliches, Zvi. "Distributed Lags: A Survey." Econometrica, XXXV, No. 1, January, 1967, pp. 16-49.

Kmenta, Jan and Smith, Paul E. "Autonomous Expenditures Versus Money Supply: An Application of Dynamic Multipliers." Econometrics Workshop Papers. Workshop Paper No. 6604. Michigan State University, 1957.

Kmenta, Jan. "An Econometric Model of Australia, 1948-61." SSRI Reprint Series. Social Systems Research Institute, University of Wisconsin, 1968.

Surrey, Stanley S. and Oldman, Oliver. "Report of a Preliminary Survey of the Tax System of Argentina." Public Finance, XVI, No. 2, 3, 4, 1961, pp. 155-182, 313-340.

Villanueve, Javier. The Inflationary Process in Argentina, 1943-60. 2nd edition. Working Paper Inst. Torcuato Di Tella Centro de Investigaciones Economicas. Buenos Aires: Editorial del Instituto Torcuato Di Tella, 1966.

Reports

International Monetary Fund. "Argentina-1966 Article XIV Consultation." Washington, D. C., 1967.

Unpublished Materials

Diz, Adolfo Cesar. "Money and Prices in Argentina, 1935-1962." Unpublished Ph.D. dissertation, Department of Economics, The University of Chicago, 1966.