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ELEMENTARY SCHOOL TEACHERS' CONCEPTIONS OF MATHEMATICS CONTENT AS A POTENTIAL INFLUENCE ON CLASSROOM INSTRUCTION

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ELEMENTARY SCHOOL TEACHERS' CONCEPTIONS OF MATHEMATICS CONTENT AS A POTENTIAL INFLUENCE ON CLASSROOM INSTRUCTION

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ABSTRACT

ELEMENTARY SCHOOL TEACHERS' CONCEPTIONS OF MATHEMATICS CONTENT AS A POTENTIAL INFLUENCE ON CLASSROOM INSTRUCTION

By

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Within an interview format, 19 teachers of grades three, four, and five were asked to describe the mathematics they hoped students might learn over the course of a full school year. The teachers were selected through the use of a questionnaire that asked faculty and administrators in a suburban school district to nominate teachers who might have strong, but different views about mathematics instruction. From the list of nominees, teachers were chosen so that a variety of viewpoints about mathematics instruction, and a distribution of building and grade level assignments were represented. The teachers each used one of five different commercially published mathematics programs in the classroom.

The purpose of this research was to describe: 1) the content areas teachers used to describe mathematics curriculum; 2) the topics teachers recognized as mathematics content; 3) the priorities teachers held for student learning in mathematics; and 4) the mathematics topics teachers associated with specific instructional activities. These dimension, collectively called a teacher's conception of mathematics content, were of interest because they were hypothesized to affect teachers' decisions about what to teach. The goal was to determine the extent to which teachers' conceptions of content were alike, and whether or not teachers' conceptions of content were consistent with modern trends in mathematics education.

Therese M. Kuhs

The data for this study were verbatim transcripts of a 2 to 4 hour interview with each teacher. The analysis was conducted in three stages and made use of a Taxonomy of Elementary School Mathematics Content to classify teachers statements about content. The taxonomy served as a standard for the comparison of content described by different teachers.

Similarities and differences in teachers' statements about mathematics instruction led to three general conclusions. First, elementary school teachers are primarily concerned with teaching content related to the arithmetic operations of addition, subtraction, multiplication, and division. Second, within general areas of mathematical content (e.g., multiplication, problem solving) teachers' perceptions of specific topics that should be taught vary. Finally, differences that were identified in teachers' conceptions of mathematics content suggested that content decisions teachers make, may result in variation in the content of instruction across classrooms, even when the same textbooks or instructional program is in use.

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Recognizing the importance of personal support and encouragement, further appreciation is extended to those who have been mentioned and to Dr. Gary Quast, to the Cole family, and to the faculty and students at the Institute. Gratitude is expressed to my family in Pittsburgh who were always available when I needed them, yet who didn't need me when I couldn't be there. Finally, special appreciation is expressed to Ms. Julie Burgess who completed the typing of this dissertation.

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Statement of the Problem

The autonomy teachers exercise in determining routines and activities for their classrooms has been characterized in recent research (e.g. Lortie, 1975; Jackson, 1978). These studies reveal that teachers receive few directives and little supervision in matters related to the daily instruction they present to children. Yet relatively little is known about how teachers use their autonomy to affect classroom lessons.

Research on classroom instruction in elementary school mathematics (e.g. Suydam & Osborne, 1977; Stake & Easley, 1978; Knappen & Floden, 1980) suggests that teacher autonomy may affect the content that is taught to students. In the classrooms studied, modern curriculum materials were in use. But the challenging and interesting topics presented in the modern materials were selectively omitted, as teachers emphasized traditional topics related to the skills of arithmetic. Further, within the narrow range of arithmetic content, different teachers sometimes were seen to teach different topics. Even in classrooms where the same mathematics textbook was used as the primary source of instructional material, Knappen and Floden (1980) found that teachers' reports of the content taught varied.

Observation of teachers' selective use of segments of written curricula suggests that the goals of a curriculum might be modified as teachers emphasize some topics and omit others. The failure of past attempts to influence the mathematics content taught in classrooms may be based on a failure to recognize the potential impact of teachers' autonomy in content selection. Innovative curriculum programs are likely to be modified by the teachers who use them, particularly when there is little consistency between teachers' instructional priorities and the goals of the newly developed curriculum.

The National Council of Teachers of Mathematics (NCTM) has recently announced recommendations for a major revision of the school mathematics curriculum (NCTM, 1980). These recommendations propose that problem solving become the focus of mathematics instruction, and that the acquisition of computational skills should no longer be the primary goal for classroom learning. The NCTM's formalized statement about the importance of problem solving content summarizes the rhetoric of the past decade (e.g., Conference on Basic Mathematical Skills and Learning, 1975; National Council of Supervisors of Mathematics, 1976). Yet little is known about whether or not teachers will accept problem solving as an important component of elementary school mathematics curriculum.

The success of the NCTM recommendations, and any future attempts to change school mathematics instruction, confronts two issues. First, widespread implementation of new curricula seems to require some level of consistency between the goals of the new programs and what teachers generally accept as important content to be taught. Second, dramatic differences in teachers' selection of topics for instruction may result in random modifications of new curricula that affect students in different classrooms in various ways. The importance of these issues can be considered through a study that reveals whether or not different teachers select the same topics for mathematics instruction, and if topics teachers select reflect concerns that are consistent with modern trends in mathematics education.

Conceptual Model

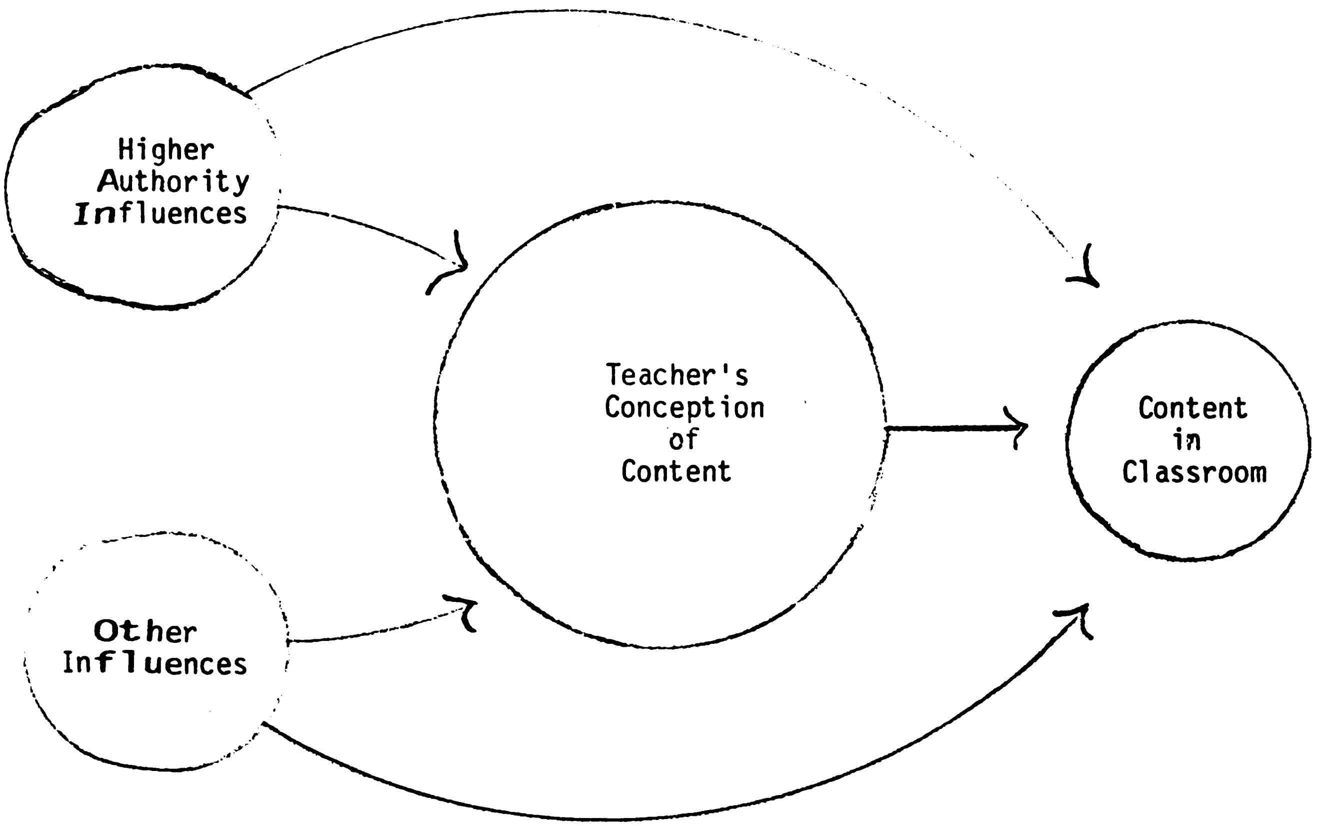
Over the past three years, the Content Determinants project in the Institute for Research on Teaching at Michigan State University has been investigating factors that influence the content of instruction in elementary school mathematics. A model of the teacher as a political broker has guided the consideration of teachers' decisions about the content of instruction (Schwille, Porter, Gant, 1980). From this point of view, teachers are thought to have enough discretion about their teaching to be influenced by their own beliefs about what schooling ought to be. But teachers may also choose (or be constrained to choose) to follow certain suggestions or pressures from without (e.g., school district instructional objectives).

The model for the study of content determination (Figure 1) considers three sources of influence. First, a formal component that transmits content decisions made by higher authorities in the school district includes such influences as school district instructional objectives and other guidelines and policies set by principals, curriculum coordinators, and the like. Second, other influences involve the recommendations of what to teach by those outside the classroom (e.g. the NCTM recommendations)and by conditions within the classroom (e.g. student's readiness or ability). The third component of the model recognizes the influence of the teacher's own conception of what are desireable and feasible goals for instruction.

Teachers' conceptions of content are hypothesized to be a significant variable in content determination. Influence from higher authority and from others, may have a direct effect on what students learn in

Figure 1

Model of Content Determination in Classrooms



classrooms (e.g., a mandated test may offer a particular learning experience to the students who take the test); but usually these influences communicate their recommendations to the teacher (e.g., school district objectives suggest what should be taught in the classroom). Thus, teachers' conceptions might result in the rejection or modification of the content implied by the message, before the message is translated into classroom instruction, or the teacher's conception itself may change.

Definitions

This study examines teachers' statements about mathematics instruction to consider the extent to which teachers' conceptions of mathematics content are alike, and whether or not teachers' conceptions of content are consistent with the curriculum prescribed by others who might influence the content of classroom instruction. The following definitions serve to elaborate the focus of this investigation.

1) Content of Mathematics Instruction is defined as those ideas, skills, and performances students might learn[1] from particular experiences with mathematics in the classroom.

2) Topic is defined as a single idea, skill or performance that students might learn.

3) A Content Area is made up of a collection of topics that represent some subdivision of the total subject-matter of mathematics.

4) Instructional Strategy refers to any classroom activity or task assigned to students. Instructional strategies are also characterized by the way in which the activity of task is presented (e.g., as independent work, or as the focus of group discussion).

[1]The researcher agrees with Skemp (1979) that content can be viewed from several perspectives. The teacher, the student, and outside observers might all identify different learning outcomes for the same mathematics lesson.

5) Curriculum includes the collection of specified topics as well as the instructional strategies that are associated with the various topics.

6) Teacher's Conception of Mathematics Content is defined to include several dimensions, each of which may influence classroom instruction. These dimensions are: a) the content areas a teacher uses to describe mathematics curriculum; b) the topics a teacher recognizes as mathematical content; c) the priorities a teacher holds for student learning in mathematics; and d) the mathematical topics a teacher associates with specific instructional strategies.

Some of the specified dimensions of a teacher's conception of mathematics content have obvious implications for content selection. For example, the topics a teacher recognizes as mathematical and a teacher's priorities for student learning, both imply a judgement that might influence instructional content. The recognition of content areas and the instructional strategy dimensions, however, need some elaboration.

A teacher's recognition of content areas within the curriculum might affect content selection by identifying the parameters of what might be taught in mathematics. An obvious case of influence would occur if a teacher failed to accept an area of content as part of what students should learn in elementary schools. For example, if a teacher thinks geometry is too difficult or that graphing is not really mathematics, topics in those areas might not be taught in that teacher's classroom.

Also, a teacher's judgement about what is content (i.e. what students should learn) and what is strateoy (i.e. ways to teach the content) might further affect content selection. For example, when using a published curriculum, teachers may identify certain aspects of the program as content and others as strategies. A textbook lesson might suggest that the teacher ask students to guess the weights of various objects and then measure the weight on a balance scale. One teacher might decide that, from the lesson, students should learn to estimate the weight of an object. The use of balance scales could be seen as a strategy to heighten student interest in the lesson. Another teacher might consider the lesson as useful for teaching children to use a balance scale and ignore the content, estimating weights of objects.

On the basis of these varying judgements about the lesson's content, students might be taught different topics. The first teacher could omit the weighing activity and merely tell students when their estimates are correct. The second teacher, on the other hand, would have students weigh objects, but might fail to ask them to estimate the weight of the objects. While yet a third teacher, who follows the textbook closely, would teach both topics to students.

Research Questions

The particular research questions that are addressed in this investigation of teachers' conceptions of mathematics content are:

Q_1: What content areas do teachers recognize as important components of the elementary school mathematics curriculum?

Q_2: Within selected content areas, what topics do teachers recognize?

Q_3: What mathematical topics do teachers view as important for students to learn?

Q_4: What mathematics topics do teachers associate with the instructional strategies they describe?

Q_5: What variation in the content of classroom instruction is implied by differences in teachers' conceptions of mathematics content?

Although teachers' remarks about mathematics instruction are likely to reflect conceptions of content that are idiosyncratic, some aspects of teachers' conceptions would be more likely to affect classroom instruction than others. Since teachers who use the same textbook or teachers who teach a particular grade level might be expected to teach the same content, the variables "text used" and "grade taught" may define groups of teachers who would discuss the same instructional content. Thus differences in remarks made by teachers within each text or grade level group would be of special interest. Such within group differences would reveal instances when teachers' conceptions of mathematics content are apt to result in the modification of written curricula and/or variation in the content of instruction across classrooms. Also, evidence that teachers who use different textbooks, or teachers who teach at different grade levels, discuss the same mathematics content suggests cases where teachers' content selections might not be influenced by a text or grade level effect. The consideration of teachers' conceptions of mathematics content will therefore be focused on identifying differences within groups of teachers who use the same textbook or teach at the same grade level, and in describing similarities that are evidence across such groups of teachers.

General Procedures

Within an interview format, teachers of grades three, four and five were asked to describe the mathematics they hoped students might learn over the course of a full school year if instruction was 100% effective. The interview schedule that was used required extensive probing to allow the content of classroom instruction to be described from the teacher's perspective. The interviews were conducted by seven researchers who participated in the development of the interview schedule as well as three stages of pilot interviews.

The teachers for this study were selected from a suburban school district with an enrollment of 4,825 students. A nomination questionnaire that was designed to identify teachers who have distinctive views about mathematics instruction was sent to teachers and administrators in the school district. From the list of nominees, subjects were selected for this study to represent a variety of views and a reasonable distribution of grade level (3 to 5) and building assignments in the district.

The data for this study were verbatim transcripts of the audio taped interviews. These transcripts were verified by the persons who conducted the interviews. The analysis of transcripts was completed by the author of this study and focused on the research questions that have been specified.

Purpose and Significance

The purpose of this research is to describe teachers on certain dimensions, collectively called their conceptions of mathematics content. These dimensions are of interest because they are hypothesized to affect the selection of content for classroom instruction. This investigation does not result in a summative description of individual teacher's conceptions of mathematics content. Rather, it reveals differences among teachers that suggest the effect of teacher autonomy on classroom instruction.

This research should be viewed as an effort designed to inform future attempts to study or improve mathematics instruction. The examination of teachers' conceptions of mathematics content provides insight into the issues that are confronted by those who wish to change elementary school mathematics curriculum. The identification of teachers' goals and priorities for mathematics instruction reveals the particular aspects of a written curriculum that might be modified as teachers implement the instructional program. Further, the description of variation in teachers' conceptions of mathematics content suggests those areas of mathematics curricula that are most difficult to standardize across classrooms.

The insights gained from this study serve several purposes. The study reveals content that might be more explicitly specified by future curriculum development projects. It also suggests areas of mathematics that might receive increased emphasis in preservice and in service teacher education. Finally, the evidence that the content of instruction may be affected by teachers' conceptions of content suggests the significance of that teacher variable for future studies of mathematics learning.

Overview

In this chapter the rationale and conceptual model for a study of teachers' conceptions of mathematics content has been presented. In Chapter 2 literature related to teachers' selection of content and teachers' beliefs and understanding of mathematics curriculum will be reviewed. Chapter 3 will describe the methods used for the study. The analyses of teachers' interviews will be presented in Chapter 4 and Chapter 5 completes the dissertation with conclusions, recommendations and suggestions for further research.

Chapter 2

Review of Literature

This chapter is divided into two sections. The first section contains a review of research in the areas of classroom instruction and teacher decision making. The literature discussed describes the teacher as decision maker, and considers factors that influence teachers' decisions. In the second section, the review focuses on research investigating teachers' beliefs about curriculum and mathematics. The research in this area considers differences in teachers' beliefs that may affect classroom instruction.

Curriculum and Teacher Decision Making

The content of elementary school mathematics is often considered as being non-controversial and clearly defined. Yet the various reports on the status of curricula in schools suggest that teachers use written mathematics programs in ways that result in the modification of the intended content.

A national survey involving 1,220 responses from teachers of grades two and five (Price, Kelley & Kelley, 1977) found that 82% of the teachers reported relying exclusively on some single source for instructional material in mathematics. Yet only 53% responded that they followed a textbook closely, and more than half the teachers claimed their students read less than two-fifths of the material presented in texts. The researchers concluded in a description of the "median" classroom,

> it seems likely that the text, at least as far as students are concerned, is primarily a source of problem lists. Teachers are essentially teaching the same way that they were taught in school. Almost none of the concepts, methods, or big ideas of modern mathematics programs have appeared in this median classroom. (Price et al., 1977, p. 330).

The absence of modern concepts in classroom instruction was also noted in a National Science Foundation project examining the status of mathematics education in the United States (Stake & Easley, 1978). In that effort, a national survey was supplemented by field observation in ten school districts selected to represent the spectrum of school size, geographic locations, community types, and curricular orientation. Stake and Easley reported various characteristics of the schools studied, including the observation that the content of mathematics courses from grades two to twelve focused primarily on computation. Further, noting the discrepancy between instructional goals expressed by the teachers' supervisors and the instruction that occurred in classrooms, the researchers commented that teachers had considerable autonomy in deciding what to teach and how to present the content in their classrooms.

Teacher autonomy is frequently cited as a factor that inhibits change in schools. Goodlad, Klein and Associates (1970) reported this finding from an investigation of the status of classroom curricula in 158 primary classrooms (grades K-3), within 67 schools in 26 school districts from 13 geographically distributed states. The study involved a series of classroom observations and interviews with principals and teachers.

The data were collected during the 60's, yet the impact of curriculum trends of that era was not apparent in the classrooms studied. The effects of a thrust for discovery learning, focus on the structure of the subject-matter, and the development of students' higher level cognitive skills were not witnessed in the research.

> The cognitive involvement called for was overwhelmingly the recognition of factual material... When higher intellectual processes were sought or elicited, they usually involved some display of reading or mathematical comprehension. There was little exploration, hunching, guessing, supposing, at any grade level (Goodlad et al., 1970, p. 53).

The innovative materials and curriculum guides were in use in classrooms, but the conventional instruction that was presented indicated that real change had not occurred. Reporting that the form but not the substance of new curricula was observed, the researchers suggested teachers' commitment to the familiar may have led to this incongruity.

> At all grade levels, teachers viewed themselves as having considerable flexibility with respect to curricular adaptation, especially with selecting activities to deal with bodies of content... Our impressions from conversations with teachers was that they felt free to adopt (sic) and enrich within a generally prescribed broad framework. (Goodlad et al., 1970, pp. 64-65)

The researchers' view of teacher autonomy in the classroom was reflected in interviews with the principals. About one principal in ten reported they had problems "getting teachers to abandon outmoded programs and practices". At the same time, teachers often expressed a desire for <u>more</u> freedom to adapt the curriculum. Attempting to explain teacher resistance to change Goodlad et al. comment,

> ...many of the assumptions underlying recent efforts at curriculum change run smack into long-established assumptions and procedures built into schooling as virtually a way of life. Thus, for example, the notion that the most important learnings are the fundamental concepts and processes to be developed over several years of inquiry tends to be negated by an emphasis on facts and topics "to be covered" in a semester or a grade (Goodlad et al., 1970, p. 103).

Teacher Decision Making

Each of the three studies that have been discussed suggest that teachers adapt the curriculum programs they use. The research on teacher decision making provides insight into how such adaptations occur. Although the literature in this relatively new area of research is limited, a review by Clark and Yinger (1977) identified a few efforts that have implications for a study of content decision making (e.g., Zahorik, 1975; Yinger, 1977).

Such investigations have often asked whether or not teachers follow the end-means model of curriculum decision making that is advocated by curriculum theorists (e.g., Tyler, 1950; Taba, 1962). The theorists employ a narrow definition of "content" and contend that content should be a means to accomplish an instructional objective; rather than being the goal of instruction. Hence, the end-means model recommends that decisions about the objectives of a lesson should precede the selection of content for instruction.

The end-means model, however, is based on a particular belief about what is to be learned in classroom lessons. Taba, in discussing the role of knowledge in learning and curriculum, explains that "each area of knowledge--a subject or a discipline--has at least two main characteristics: it has its own fund of acquired information (i.e., content) and a specialized method of inquiry" (Taba, 1962, p. 172). She explains that curriculum controversy today, sometimes results from different viewpoints about the role of subject matter in curriculum. One extreme considers the facts of subject matter as the goal for learning. At the other extreme, the method of inquiry that is used in a discipline, is the expected learning outcome.

Zahorik (1975) assessed teacher adherence to the end-means model through a study of their planning routines. He addressed the questions:

(1) what planning decisions do teachers make, and (2) what decisions are made first and most frequently. Teachers were asked to write a list of decisions they make before classroom instruction and to identify the decision that they consider first. These lists were anlayzed using eight classification categories, i.e. objectives, content, activities, materials, diagnosis, evaluation, instruction, and organization. Decisions classified as "objectives" related to goals, aims, or purposes of instruction while content decisions were about "the nature of the subject matter to be taught, such as the identification of facts, events, or other aspects" (Zahorik, 1975, p. 36).

Analysis of the lists written by 63 elementary school teachers revealed that no one decision type was listed by all of the teachers. The decision types most commonly listed were: activities, 81%; content, 68%; and objectives, 56%. Further, 46% of the teachers described content as the first area of decision making, while only 25% reported objectives as the focus of initial decisions. These findings suggested that, in many cases, a teacher's first concern in planning lessons is to identify the "facts" of the subject matter to be learned. Zahorik concluded,

> If proposed planning models are to become helpful tools for teachers, perhaps the place of content in the planning models ought to be more clearly delineated (Zahorik, 1975, p. 138).

A later Survey of Teacher Planning Practices (Clark & Yinger, 1979) used the same content-objective distinction in the analysis of teacher responses. This four part survey of 78 elementary school teachers who were enrolled in graduate courses at Michigan State University asked the subjects to describe various aspects of their planning practices. In one part, teachers were asked to write detailed descriptions of three actual plans they had made and implemented in the classroom. The results of this study suggested, among other things that:

- learning objectives are seldom the starting point for planning. Instead, teachers plan around their students and around activities.

- the most common form of written plans was an outline or list of topics to be covered, although many teachers reported that the majority of planning was done mentally and never committed to paper. (Clark & Yinger, 1979, p.15)

Recognizing that many teachers may place a high priority on determining what content is to be presented in a lesson, it is important to consider various factors that influence teachers' content decisions. Yinger (1978), conducted a five month study of one (first-second grade) teacher's planning decisions investigating the mental processes engaged in while planning. Over a 12 week period, 40 full days of classroom instruction were observed using ethnographic methods. Later in the year the researcher observed the teacher's planning behavior and conducted additional classroom observations and interviews.

Two models of planning were developed from this research. The first model described planning at five levels: 1) yearly, 2) term, 3) unit, 4) weekly, and 5) daily. At the yearly level, the teacher was seen to establish the general scope and sequence of content to be presented during the school year. The second model characterized the teacher as problem solver and described three stages within the planning process: the Problem Finding stage, the Problem Formulation stage, and the Implementation, Evaluation, and Routinization stage.

Several factors were described by Yinger as influencing the Problem Finding Stage of planning. Two of these factors, the teacher's goal conceptions and the teacher's knowledge and experience, were of particular interest.

> ..."teaching goal conceptions" refers to the teachers' anticipatory notions of what effective teaching might be for a specific group of students, including conscious, explicitly stated goals and objectives (both cognitive and affective. It also refers to vague intutions, disposition or attitudes, toward teaching that a teacher might have.
>
> Knowledge and experience as portrayed in this model, refers to: (1) the ways in which the teacher has learned to perceive problem situations, and (2) the knowledge and methods the teacher can draw from hisoher memory. (Yinger, 1978, p. 11).

Yinger explained that these two factors served as "filters" for the teacher in the Problem Finding Stage of planning. Thus the focus of planning was directed to "problems" the teacher was prepared to solve. The teacher was seen to rely, to a great extent, on past experience. Yinger reported that, for the most part, planning consisted of the elaboration or modification of lessons that had been used in previous years. Content was identified as a salient focus in the teacher's planning and, although the school district's performance objectives were seen to influence the teacher's plans for instruction, she did not consult them while planning. Rather, her own knowledge about the content described in those objectives was the guiding factor for instructional planning.

What was observed in this teacher's planning practice was consistent with the conclusions of studies of classroom curricula that were discussed earlier in this chapter (e.g., Goodlad et al., 1970). The literature repeatedly identified teachers' reliance on past experience and their perceptions of what is to be learned as dominant influences on the content of classroom instruction. Yet it is hard to imagine that teachers deliberately inhibit attempts to improve or change instructional practices.

In fact, a series of interrelated studies that examined the effect of various hypothesized influences on teachers' content decision

(Porter et al., 1979), suggested teachers believe they are more compliant. This was evidenced in a policy-capturing study investigating teachers' perceptions of the effects of six sources of pressure to alter the content of fourth-grade mathematics (Floden et. al., 1979). Sixty-six teachers from five metropolitan areas in Michigan were asked to respond to descriptions of hypothetical situations in a school district that suggested either the inclusion or exclusion of certain topics of instruction. The sources of content messages considered were: (1) textbooks, supplied to the teachers, (2) standardized tests with results being published in a local newspaper, (3) school district instructional objectives, (4) pressure from upper grade teachers, (5) suggestions by the school principal, and (6) pressure from parents.

In response to a series of vignettes written to systematically vary the presence or absence of the six pressures in the hypothetical setting, teachers reported willingness to change what they teach, whatever the source of pressure. However, teachers did not seem to consider new topics as necessarily supplanting old ones. Although they reported that they were "fairly certain" to teach new topics, they still indicated that they would continue to teach all that had been taught before.

From a series of case studies, Knappen & Floden (1980) reported on teacher content decisions in a natural setting. The content of mathematics instruction for a full school year was monitored in seven classrooms located in six different schools within three school districts. Data for a case study of each classroom included daily logs kept by the teacher, supplemented by weekly debriefing sessions and a series of extensive interviews with the teacher, the principal and other influential persons in the school district (e.g. curriculum coordinators).

The preliminary report of this research revealed that, at the same grade level, different teachers reported teaching different mathematics content during the first 18 weeks of school. Even in classrooms where the same textbook was used as the primary source of instructional material, the content teachers said they taught differed.

Such studies of what occurs in classrooms, and the studies of teacher planning, imply that content decisions, or at least selections, are made by classroom teachers. In his discussion of teacher decision making, Shavelson (1976) explains that in planning, the teacher is essentially functioning as an instructional designer. Borko, Cone, Russo and Shavelson (1979) tested a model of preinstructional decisions in four laboratory studies. The factors that were considered as potential influences on teachers' decisions were: information or cues about students, teacher's inferences or estimates about students, beliefs and attitudes about education, the nature of the instructional task, and the availability of alternative strategies and materials.

One test of the model investigated teachers' planning decisions in reading and mathematics. The purpose of the study was to determine how student cues, teacher beliefs (as measured by a questionnaire), and lesson types, affect teachers' preinstructional decisions. Lessons were specified by the researchers and teachers were asked to make decisions about how the content would be presented. This procedure precluded the investigation of content decision making, but teachers' beliefs were seen to affect the selection of instructional strategies for the presentation of the lessons.

Summary

The research that has been discussed in this section suggests the potential influence of teachers' conceptions of content on classroom instruction. The studies of classroom curricula witnessed a modified version of the written programs teachers were using. The researchers contended that the autonomy teachers had in the classrooms allowed the distortion of the goals of written curricula.

The discussion of research on teacher decision making provided insight into how such adaptations occur. Since the interest in teacher thinking is relatively new, much of the work has been focused on the development and testing of various conceptual models. Teachers were found to consider the "facts" of the subject matter as an important focus for instructional planning, a view not commonly held by curriculum theorists who advocate a end-means model of curriculum development and who do not see the learning of facts as the goal of instruction.

Several factors have been found to influence content selection. Yinger's model of teacher planning indicates teacher goal conceptions and their knowledge and experience are influences. Teachers also report a willingness to comply with recommendations from outside sources (e.g. textbooks). However, they are also inclined to continue past classroom practices. In short, the literature suggested that teachers' conceptions of subject matter are likely to interact with any other factor that attempts to influence classroom instruction.

Teachers' Beliefs and Understanding

When the National Council of Teachers of Mathematics presented recommendations for school mathematics in the 1980's a concern was expressed by Max Bell.

> There is too little expertness in mathematics itself or in its teaching, directly available to working elementary school teachers. Furthermore, prospective elementary school teachers often begin their career so poorly trained in mathematics as to be already incompetent in the emphases that must come to characterize elementary school work. (Bell, 1980)

Research in mathematics education has often reflected this concern by investigating elementary teachers' levels of expertise and competence with the subject matter. However attempts to study this teacher variable do not agree on what might be a useful or appropriate index of "expertness".

Most often, some test of mathematics achievement is used to assesss teachers' preparation for teaching the subject, (e.g. Glennon, 1949; Callahan, 1967; Pigge et al, 1978). But such measures of mathematical achievement tell little about what a teacher might consider when deciding what to teach to students. This discussion will therefore focus on those few studies that investigated teachers' subject matter knowledge, or understanding, or beliefs using methods other than testing.

Bussis, Chittenden and Amarel (1976) investigated teachers' understanding and perceptions about curricula, students and schools. The researchers interviewed 60 elementary school teachers who were involved in the implementation of open or informal instructional approaches. An interview schedule was developed to get at the subjects' internalized assumptions about teaching. The transcripts of these interviews were coded using a specially devised classification system. Although this study did not focus on mathematics curriculum in particular, it provides a useful background to consider the other studies of mathematics.

The researchers examined two levels of teachers' "Curriculum Construct System". Teachers' descriptions of observable activities and events in the classroom were used to describe the "surface content". The "organizing content" consisted of the concerns and learning priorities teachers held for students, and was inferred from the teachers' statements in the interview. Classification of the organizing content resulted in the specification of three categories.

> <u>Narrow</u> priorities, for the most part are tuned in to a relatively narrow band of behavior that the teacher hopes children will exhibit, the behaviors in themselves constituting major goals of instruction.
>
> <u>Middle-range</u> priorities, on the other hand, are characterized by an interest in behavior as it reflects some internal state or quality of the child...(e.g. confidence and initiative).
>
> ...the teacher expressing <u>comprehensive</u> concerns reveals a higher order (presumably more synthesized and integrated) concept of internal resources. Thus, ideas of awareness, purpose, understanding, reflection, sensitivity and reciprocity appear in the thinking of these teachers. (Bussis et al., 1976, p. 54)

Analysis of: (1) the type of priority that characterized each teacher's comments, and (2) the surface content the teacher mentioned, resulted in the identification of four groups of teachers. Group 1 teachers (12% of the sample) expressed narrow priorities, and described classroom instruction that evidenced little experimentation or change from what they had done in past years. Group 2 teachers (22%) also expressed narrow priorities but were seen as cautious experimenters with the activities they mentioned. Both Group 3 teachers (39%) and Group 4 teachers (27%) described surface content that the researchers characterized as "potentially rich". The teachers in Group 3 most frequently mentioned middle-ranged priorities but also discussed priorities that were classified as narrow. The priorities expressed by teachers in Group 4, on the other hand, were characterized as comprehensive or middle-ranged and evidenced little concern for the narrow priorities that were at least mentioned by teachers in each of the other groups.

The teachers' ability to articulate the connection between their priorities and the surface content was also considered in the analysis. Teachers from Groups 3 and 4 did this most often; but the teachers in Group 4 were more inclined to talk about the connections between priorities and activities than teachers in the other three groups. Group 4 teachers also gave clearer descriptions of the connections between the surface and organizing contents.

In the sample of teachers interviewed, there was a wide range of variability in the priorities that were expressed, i.e. from narrow to comprehensive. The findings also suggest that many teachers (33% in Groups 1 and 2) were seemingly committted to a narrow view of what students might learn, and to using the instructional strategies they had used in the past.

The variation in teachers' priorities noted by Bussis et al. was also evidenced in four case studies of preservice teachers of secondary mathematics conducted by Shirk (1973). Using ethnographic methods the subjects were observed while student teaching. The first teacher's goals for instruction were described in a way that suggested "narrow-ranged priorities".

> Tom's conceptualizations were based on the premise that there existed a well ordered system of propositions which made up the field of mathematical knowledge and that it is the function of the teacher to transmit this system to students (Shirk, 1973, p. 160).

Two other teachers in the study seemed to have the same conception about the subject matter to be taught but one of these teachers, Lisa, did not press to "cover" all material. The fourth teacher, however, seemed to have a more comprehensive view of curriculum.

> Sally's objectives were more global, reflecting her conceptualization that it was her task to educate the "whole" person with mathematics being the particular vehicle whereby this could be accomplished (Shirk, 1973, p.163).

Since the teachers were described on the basis of both interview data and observations of their teaching, this research suggests that varying priorities can be reflected in the lessons teachers present. Thus an understanding of teachers' priorities, could provide insight into content decisions that might eventually affect classroom lessons.

One investigation of sixth grade teachers' priorities for student learning made use of a survey asking teachers about their beliefs concerning the importance of certain content in mathematics and the amount of time they spent teaching those content areas (O'Neil, 1971). The researcher wanted to identify teachers' priorities and examine the relationship between particular demographic characteristics and teachers' perceived priorities. Teachers were asked to respond to items describing eight content strands in mathematics: Sets-Numbers-Numerals, Operations and Properties, Numeration Systems, Geometry, Measurement, Mathematical Sentences and Problem Solving, Graphing and Functions, and Probability and Statistics.

In general, the priorities teachers expressed were found to be remarkably similar. Only a few of the demographics considered were found to correlate with teachers' priorities. For example, teachers with less than seven graduate credits in mathematics or mathematics education reported spending more time on teaching concepts from the Sets-Number-Numeral Strand, and placed greater importance on the teaching of measurement concepts, than did teachers with more graduate course work. Also, teachers from districts with larger pupil enrollment said they spent more instructional time on concepts in the Mathematical Sentences and Problem Solving strand than did teachers from districts with smaller enrollment. Although the textbook used by the teacher was considered as a possible influence, the effect of that variable was not found to be significant. O'Neil reported,

> The responses of all teachers indicated the assignment of a greater degree of importance to traditional mathematical content than to new content. Traditional content generally was rated very important or important. New content was generally rated important or somewhat importance. (O'Neil, 1971, p. 4845)

The repeated verification of teachers' conservative view of the goals of mathematics instruction raised questions about teachers' perceptions of the "modern" components of the elementary school mathematics curriculum. The apparent rejection of the importance of some "modern" content, by teachers, could be deliberate because this content is not valued by them. However, teachers might not present the activities related to modern goals for instruction, only because they do not recognize those activities as critical components of a modern mathematics program.

Collier (1970) conducted a study of "beliefs" about mathematics and mathematics instruction to determine if the preservice elementary school teachers' beliefs were consistent with "modern mathematics" thought. Recognizing that mathematics curricula were shifting from the traditional orientation that emphasized the "formal" content of mathematics to a view that children should experience the creative and imaginative side of mathematics (an informal view), Collier devised a questionnaire to assess beliefs of preservice teachers as either "formal", "informal" or "neutral".

Using a design that considered the level of course work in mathematics education that was completed, mathematics achievement, and the particular course section in which students were enrolled, the researcher found that the college course work seemed to influence preservice teachers' beliefs. The students who were just beginning college course work in mathematics and mathematics education were found to have neither formal nor informal views about the content or about how it should be taught. Students who had completed two content courses still had neutral beliefs about mathematics instruction but high achieving students' beliefs about mathematics were "slightly informal". After completing two content courses and a methods course, students had "moderately informal" beliefs about mathematics instruction and a "slightly informal view" of mathematics.

Another study investigating teachers beliefs about modern mathematics (Murphy, 1973) addressed two quesitons. First, do teachers and "experts" in mathematics education agree about what should be included in a program of modern mathematics instruction? Second, do teachers teach those concepts and ideas that they recognize as being "modern"?

A questionnaire was developed to ask if certain ideas and procedures were recognized as components of a program of modern mathematics. Eight college professors in mathematics education and 80 teachers completed the questionnaire. Murphy found,

> There is a significant difference between what experts in mathematics education believe is modern ideology concerning classroom instruction, and what a randomly selected sample of teachers believe. (Murphy, 1972, p. 80)

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The teachers in this study also completed a second questionnaire that asked about their use of the ideas and procedures that had been on the first instrument. The teachers' responses did not always evidence compliance with their perceptions of modern trends. Those ideas and approaches that the teachers viewed as being part of a modern mathematics program were not always the ones they said they would present in their classrooms. Those who would be concerned about teachers' failure to comply with the instructional procedures in written curricula might be skeptical about teachers' mathematical knowledge. Research often supports this skepticism. The studies that involve the use of tests to assess teachers' ability (e.g. Croswell, 1964; Pigge et al., 1978) have identified deficiencies in teachers' mathematical knowledge and provide some insight into how teacher's mathematical competence might influence classroom instruction.

A study of preservice teachers' learning of the mathematics concepts they would someday teach to children (Eisner, 1974) revealed critical deficiences in their understanding of the concepts. A sample of 16 student volunteers, enrolled in the first term mathematics course that was part of the required elementary education program at Michigan State University, participated. Eisner reported that students in the sample were better than average in mathematical ability when compared to other students in elementary education.

On a bi-weekly basis throughout the term, Eisner conducted unstructured interviews with each of the subjects. These sessions focused on a discussion of the concepts that had been taught in the preceding weeks of the mathematics class. A clinical problem solving exercise was included in each of the interviews.

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The purpose of the research was to examine students' level of attainment of concepts presented in class. However some of the behaviors Eisner noted during the interview provided insight into beliefs and dispositions held by these preservice teachers. First, he found them to be disinterested in learning new algorithms for the arithmetic operations. They prefered to rely on familiar methods. Second, in several topic areas (e.g. finding the greatest common factor) the perservice teachers understanding was purely functional. In other words, they were able to complete the problems without difficulty; but they were unable to explain the procedures used in solving the problems. Also, the subjects often over-estimated or inflated the value of certain teaching methods. They often expected geoboards or set language to be useful in providing explanations when such strategies inappropriately represented the concepts being discussed. The content preservice teachers associated with these instructional strategies was poorly described by the illustrations and examples they gave during the interviews.

Summary

The research related to teacher beliefs about curriculum suggests the significance of teachers' conception of content as an influence in content selection. Bussis et al. and Shirk have suggested the variability in teachers' perceptions of what should be taught to students. Murphy's research identified discrepancies between the views of mathematics experts and the views of the teachers in his study about the make-up of a modern mathematics program. Further, the teachers in that study did not always report compliance with their view of modern thought as they indicated what was done in their classrooms.

The influence of tradition and past experience on teachers' beliefs about mathematics learning was evidenced in the misconceptions identified by Eisner in his study of preservice teachers. O'Neil, also found that teachers reported traditional views about what should be taught to children.

The research reported in the first part of this chapter suggested that content decisions are made by teachers. The studies of classrooms revealed that written curricula undergo adaptations that sometimes modify the goals of the program. The studies of teacher planning and decision making have indicated that content selection may be influenced by teachers' conceptions of the goals of instruction and by their past knowledge and experience with subject matter and teaching. Recognizing that teachers make content selections when new curricula are implemented, a study of teachers' conceptions of mathematics content can be seen as potentially useful to understanding why the content of instruction varies in elementary school classrooms.

Chapter 3

Research Design and Descriptive Data

A description of the procedures used in the research is provided in this chapter. The first section details the method used to identify teachers who would be interviewed and describes the teachers who were selected to participate. The next section explains the development of an interview schedule and the data collection effort. Finally, in the last section the procedures followed for data analyses are outlined.

The Sample

The teachers for this study were selected from a single suburban school district that serves a university community. Selection of participants was accomplished through the use of a questionnaire designed to identify teachers who might have strong but different views about mathematics instruction. Respondents to the questionnaire were asked to give the names of as many teachers of grades 3 through 5 as possible, in connection with twelve characteristics including:

- has strong opinions about what should be taught in elementary school mathematics
- is very much against the back to basics in mathematics
- believes that one of the main benefits of mathematics for children is learning how to follow directions
- thinks mathematics is very well suited for creative children

Appendix A contains the questionnaire and cover letter as well as a followup letter that solicited response from those who had not completed the nomination after the first request. The questionnaire was sent to all teachers of grades three through five in the school district, to their building principals, and to three members of the school district central staff.

While the nomination survey identified a sufficient number of teachers for the interview study, the percentage of questionnaires returned was low. Thirty-five percent of teachers, 56 percent of principals and 100 percent of central administrators responded. In all, 40 teachers were nominated, though two were not elementary school teachers and seven were not teaching in grades 3 through 5. Twenty-eight of the 40 teachers were nominated by only one person. The largest number of people to nominate a particular teacher was 15, but only two other teachers were nominated by more than four respondents.

From the nominations, 21 teachers were selected with attention given to the characteristics they represented, their building, and their grade level assignment. For each characteristic, at least three teachers were included in the sample. Eight of the nine elementary schools in the district were represented and several teachers were selected from each of grades three through five. Teacher 11, who was selected initially, did not agree to participate in the study and the sample was reduced to twenty subjects.

A description of teachers who were interviewed is presented in Table 1. The column labeled "Grade 78-79" identifies the grade level the teacher taught during the year in which the interviews were conducted. The column labeled "Interview Focus" records the grade level the teacher discussed during the interview. Since teachers were requested to focus their remarks on the mathematics taught at a single grade level, teachers

Table 1

Description of Teachers Interviewed

Teacher I.D.	Sex	Grade 78-79	Interview Focus*	Years of Experience	School	Text
1	F	4	(4)	10	A	DMP
2	F	1-2-3	(3)	4	B	SF
3	F	3	(3)	11	C	SF
4	F	3	(3)	8	B	SF
5	F	4-5	(4)	15	D	SF
6	M	2	(5)	5	E	SF
7	F	2-3	(5)	5	C	SRA
8	F	4	(4)	8	E	SF
9	F	5	(5)	7.5	C	CPL
10	F	5	(4)	22	F	DMP
12	F	5	(5)	26	G	HRW
13	F	2-3	(3)	7	D	DMP
14	M	4	(4)	6	E	SF
15	F	5	(5)	6.75	B	SF
16	F	1-2-3	(1)	4	B	SF
17	F	3	(3)	17	E	CPL
18	F	4	(4)	19	F	DMP
19	M	4	(4)	4	H	SF
20	M	5	(5)	21	C	SF
21	F	3	(3)	18	F	DMP

*entries are grade level

assigned to a combination or split grade class only referred to one of the grades in the combined class setting during the interview. However, Teachers 6, 7 and 10 had just completed their first year's assignment at a new grade and each asked to discuss levels at which they had had more experience.

The remaining columns of Table 1 provide further information about each teacher. The number of years of classroom experience ranged from four to 26 years. Each teacher had taught at several different grade levels and four teachers (Teachers 9, 12, 14 & 20) indicated having taught middle school or high school mathematics. As was mentioned earlier, the subjects were selected from eight of nine elementary buildings in the district. They were not all, however, using the same instructional materials in their classrooms. Five teachers were using the Developing Mathematical Processes Curriculum (DMP), copyright 1976; eleven used the Scott Foresman Mathematics series (SF), copyright 1978; one teacher used a Science Research Associates mathematics book (SRA), 1976; one teacher used the Holt School Mathematics program (HRW), copyright 1978; and two teachers used the Continuous Progress Lab (CPL), copyright 1976, for mathematics instruction.

The Interview

The interview schedule used in this research was designed to ask teachers what they hoped students might learn in mathematics over the course of a full school year, if instruction was 100% effective. It started with some brief quesitons about teacher background, then asked a series of open-ended questions including:

> What in your opinion are the most important reasons for teaching mathematics in the elementary school;
>
> What do you hope students will learn in ___ grade mathematics;
>
> What other things could be taught in ___ grade mathematics?

The complete interview schedule is contained in Appendix B. Throughout the interivew, required and optional probes were specified to direct the line of questioning. If the respondent used certain a priori identified terms for describing content, the interviewer was to ask for a definition. Since relationships among topics were of particular interest, the interview schedule included a list of teacher statements to be pursued by further questions because these statements may have been indicative of relationships. Also, special instructions were provided for pursuing what teachers meant when they used the word "understanding" in reference to the content of mathematics.

The interview closed with questions designed to elicit teachers' thoughts about the structure of mathematics content they teach, i.e., what are the various content areas and how are these areas interrelated. This was accomplished by asking the teacher to think of all the content they teach and divide it into just a few categories. The question was repeated as many times as necessary until the teacher failed to suggest a new set of categories. Then the interviewer asked if the teacher could mention any common threads that run through most of the content they teach. The final question was designed to identify relationships teachers recognize that might be a part of the content of instruction. This series of questions will be referred to as the "category questions" in future discussion.

The interview schedule used in this research went through a number of revisions based on pilot studies. The first pilot focused on assessing the clarity of questions. The seven researchers each interviewed one teacher and then suggested modifications in the schedule of questions. In cases where all of the teachers interviewed had difficulty understanding or responding to an inquiry, the question was revised or deleted. The difficulties reported by only a few researchers were considered in light of the strategies used by the interviewer. Discussion centered on how questions were asked and which probes were most useful. The same procedures were employed during a second and third pilot of the interview schedule. Transcripts of the audio taped interviews were also used to determine if researchers were using the same strategies in asking teachers questions.

The seven researchers who developed and piloted the interview schedule conducted the interviews for this study (six conducted three interviews; one conducted two). Each interview was audio tape recorded. The basic data were verbatim transcripts from each interview. These transcripts were verified against the tapes by the persons who conducted the interviews. Fourteen of the interviews were completed in a single two-hour session. Six of the interviews lasted three to four hours and required two interview sessions.

Analysis of Interview Data

The analysis of interview transcripts for this research was conducted in three stages. The goal at each stage was to develop summaries of each teacher's statements that were: (1) representative of the remarks the teacher made about content or activities; (2) descriptive of the context in which the remark was made, i.e. what was the general topic of discussion when the teacher mentioned the content or activity; (3) reduced to a form that provided a basis for comparing the remarks made by different teachers; and (4) explicit enough to identify similarities and differences between the views expressed by teachers and the views of others in the field of mathematics education (e.g. curriculum writers).

The first stage of data analysis was planned to organize the interview data for further analyses. The interview schedule was specifically designed to avoid imposing a particular structure on teachers' remarks about mathematics. Also, the interview schedule required that, at certain intervals, content the teacher mentioned previously would be introduced into the discussion again, if the teachers' initial remarks were ambiguous or cursory. As a result, conversations within the interview often shifted from topic to topic. During the first reading of the transcripts, commments bearing on a particular theme were collected on pages coded with that theme. For example, if the interviewer asked, "Would you tell me more about what you teach in multiplication?", the teacher's verbatim response was added to the page containing his or her earlier remarks about multiplication. The original transcript page was recorded on each entry. Teachers' comments that had no direct bearing on the research interest were eliminated from the data. The types of discussion that were accepted as relevant were: remarks about mathematical content or classroom activities; comments about factors that might influence content selection (e.g., textbooks); and comments about student learning. Examples of remarks that were eliminated are: descriptions of individual students who were disciplinary problems, elaborate descriptions of classroom events outside of mathematics instruction, and statements a teacher made but later retracted when clarifying the meaning of his/her earlier comments.

The second stage of analysis was designed to provide a description of the mathematical content mentioned by teachers, from the researcher's perspective. The reduced form of the transcripts (from stage one) was used to compile lists of the ideas, skills and problem types teachers reported presenting to the students in their classrooms. The entries on the lists were then analyzed using a taxonomy of mathematics content that will be described in the next section of this chapter.

After completing the second stage of analysis, the decision to eliminate Teacher 16's interview from further analyses was made. Teacher 16 had focused her remarks on mathematics taught at the first grade level. The content she discussed was often different from that mentioned by other teachers. However, because no other teacher had been interviewed regarding first grade mathematics, it was impossible to assess if the unique aspects of Teacher 16's remarks were due to the grade level she described or indicative of her own conception of mathematical content.

The third stage of data analysis focused on particular subsets of the collected data. Considering only the summaries from the first two stages that dealt with multiplication, division, problem solving, and mathematical understanding, and teachers' responses to the categories questions, the teachers' comments were compared to identify variation in different teachers' conceptions of mathematics content. The rationale for the selection of these particular areas for in-depth analysis will be discussed in the following section of this chapter. The comparison of teachers' remarks was focused to address the research questions:

> What content areas do teachers recognize as important components of the elementary school mathematics curriculum?

> Within the content areas multiplication, division, problem solving, and mathematical understanding, what topics do teachers recognize?
>
> What methematical topics do teachers view as important for students to learn?
>
> What mathematics topics do teachers associate with the instructional strategies they describe?
>
> What variation in the content of classroom instruction is implied by differences in teachers' conceptions of mathematics content?

Since data was collected from only a single interview with each teacher, significant components of mathematics curriculum, from the teachers' perspective, may have been omitted from the discussion for a variety of reasons. The teacher's disposition and experiences on the day of the interview, the type of probes used by the interviewer, the teachers' interpretations of the line of questioning, all could have resulted in different omissions in each teacher's remarks. For this reason the analysis was focused on what each teacher mentioned, rather than on what the teacher failed to mention.

Identification of Areas for In-depth Analysis

The particular areas that served as a focus for the third stage of analysis were selected after second stage summaries were completed, but the selection was also informed by the review of literature. The apparent failure of teachers to implement modern curricula, as indicated in the literature, suggested that some component of modern programs should serve as a focus of analysis. Since stage 1 and 2 analyses revealed that "understanding" was frequently mentioned in all of the interviews, consideration of this area seemed promising. Further, understanding was seen by the researcher, as a learning outcome in mathematics that crosses most conventionally recognized content areas.

The review of the literature also suggested a second area of focus for the analysis. The consistent findings, that teachers place high priority on computation, suggested an investigation of teachers' remarks about one of the arithmetic operations. Multiplication was selected as the particular area to be considered because: (1) It was mentioned by every teacher in the study during the discussion of categories of mathematics, and (2) multiplication was the only operation that all teachers at the various grade levels described as being a significant part of their instructional program. Thus, both within and across grade level comparisons could be considered.

Several teachers, however, combined multiplication and division as a single category of mathematics content when responding to the categories questions. Others had mentioned the relatedness of operations as a common thread through mathematics content. Thus, the decision was made to analyze the combined content area, multiplication-division, to determine if teachers who suggested a recognition of the relationship between the operations, had different views than those who did not.

Finally, problem solving was selected as an area of special interest because it provided an opportunity to consider how teachers have interpreted the recent thrust for the study of problem solving in mathematics. The National Advisory Committee on Mathematics Education (1975) and the Euclid Conference on Basic Mathematical Skills and Learning (1975) both identified problem solving as an area of the elementary school curriculum that should be expanded. More recently, the National Council of Teachers of Mathematics (1980) announced that problem solving should be the focus of school mathematics in the 80's.

The stage 1 and 2 analyses revealed that teachers thought of this content area in different ways. For example, responding to the categories questions, some teachers mentioned problem solving as a common thread in mathematics content, others mentioned "story problems" or "applications" or "problem solving" as categories of content, while some teachers did not mention the area at all. Also, since several teachers in the study were using the DMP curriculum, which emphasizes the development of problem solving skills, the teachers' interpretation of that curriculum could be analyzed.

Taxonomy of Elementary School Mathematics Content

To identify unique aspects of teachers' conceptions of mathematics content, and to facilitate the comparison of different teachers' statements, some standard description of elementary school mathematics was needed. A taxonomy of elementary school mathematics content (Kuhs et al., 1979) that had been used to analyze the content of textbooks and standardized tests of mathematics, was selected as the standard.

An outline of the taxonomy is provided in Table 2. The taxonomy is formed by the intersection of the three factors shown in Table 2, and results in 1170 possible cells. Each cell was viewed as a topic of mathematical content from the researcher's perspective.

When using the taxonomy to classify content, one level from each factor is recorded. For example, the problem 1/2 + 1/3, would be classified as (4,7,3). At Factor I, the general intent was to develop a skill (level 4); at Factor II, the problem dealt with common fractions having unlike denominators (level 7); at Factor III, the operation was to add without carrying (level 3).

Table 2

Taxonomy of Elementary School Mathematics Content

Factor I: General Intent

1) Conceptual Understanding with Concrete or Pictorial Models
2) Conceptual Understanding Without Pictorial Models
3) Skill in Reading Graphs, Tables, Measurement Devices
4) Computation/Numeration Skills
5) Applications Involving Graphs, Tables, etc.
6) Applications Without Graphs, Tables, etc.

Factor II: Nature of Material

1) Single Digits or Basic Number Facts
2) Single & Multiple Digit Numbers
3) Multiple Digit Numbers
4) Number Sentences/Phrases
5) Algebraic Sentences/Phrases
6) Single or Like Fractions
7) Unlike Fractions
8) Mixed Numbers
9) Decimals
10) Percents
11) Measurement
12) Essential Units of Measurement
13) Geometry
14) Other

Factor III: Operations

1) Identify Equivalents
2) Order
3) Add Without Carrying
4) Add With Carrying
5) Column Addition
6) Subtract Without Borrowing
7) Subtract With Borrowing
8) Multiply
9) Divide Without Remainder
10) Divide With Remainder
11) Combination
12) Apply Concepts (terms)
13) Apply Properties
14) Identify Place Value
15) Estimate

Classification Conventions for Interview Data

Teachers' comments in the interview were not always sufficient to allow complete classification of the content. Several conventions were developed for use in the coding of the interview data.

Convention 1: If a teacher did not make any reference to one of the three factors, a zero code was used for the factor not mentioned.

Convention 2: If in discussing lessons to develop conceptual understanding, the teacher's remarks were unclear about whether or not pictorial models were used, the entry on the interview summary was coded 1' at Factor I, (i.e., the entry may or may not involve pictorial representation).

Convention 3: When describing the "nature of material," if the teacher did not specify which level of whole numbers might be involved, the entry was coded 1' at Factor II to indicate some type of whole numbers. If the types of fractions were not specified the entry was coded 6' at Factor II to indicate some type of fractions.

Convention 4: The prime notation was also used to indicate generic categories in Factor III. In discussing operations, if the teacher did not specify a particular level of addition, the Factor II code was 3'. If the type of subtraction (with or without borrowing) was not mentioned, a 6' code was used at Factor III. If division problems were not described as having remainders or not, the entry was coded 9' at Factor III.

Convention 5: When the teacher discussed computational problems or arithmetic operations, the comment was classified as a skill (level 4) at Factor I, unless understanding or application was specifically mentioned as the intent of the lesson.

The analysis of teachers' responses to the category questions was completed separately. First, teachers' responses to the category questions were coded to describe the particular factor of the taxonomy that included the content description implied by the teacher's response. Thus, if the teachers' response was descriptive of the General Intent as described by Factor I of the taxonomy, the response was coded "I". If the response identified the Nature of Material involved in the content area, i.e., the types of numbers or whether it was geometry or measurement, it was coded "II". Finally, when the teachers' remarks focused on attributes of content that are described by Factor III of the taxonomy, i.e., operations the student would perform, the response was coded "III". Those categories teachers mentioned that were not descriptive of any of the factors of the taxonomy were coded "0" to indicate that other dimensions of content were implied by the teacher's response.

Sometimes the categories teachers provided included descriptions that incorporated two different levels of the taxonomy (e.g. addition of whole numbers). For this reason, multiple classifications were allowed for a given response.

When teachers' categories were not consistent with labels in the taxonomy, the teachers' discussion about what would be included in that category was considered to facilitate appropriate classification. Thus certain categories (e.g. "computation" or "story problems") were accepted as synonyms for levels of the taxonomy (e.g. skills or applications) if the teachers' description of the category was consistent with a level specified by the taxonomy. After this initial coding of teachers' categories the analysis was directed to identify the particular levels within each factor of the taxonomy that were mentioned by teachers.

Chapter 4

Analysis of Interviews

The analysis of teacher interviews is presented in this chapter. First is a discussion of the content areas inferred from teachers' responses to the category questions. This analysis identified the content areas teachers recognized as important components of the elementary school mathematics curriculum. Sections two through four present a comparative analysis of teachers' conceptions of content in selected areas of the mathematics curriculum: section two, understanding in mathematics; section three, problem solving; and section four, multiplication and division.

In these three sections, the analyses considered comments made by teachers as representing individual viewpoints, but contrasts were also made to examine views expressed by different groups of teachers. Such comparisons deal with teachers grouped according to the grade level they were discussing or by whether or not they used the DMP curriculum. In other cases, comparisons of the content described by different teachers within such groups are discussed. The comparisons that are reported represent those instances when striking similarities and differences were evidenced by the remarks made by teachers in particular groups.

Instances are also cited when teachers' discussions of particular instructional strategies implied that content of instruction, from the teacher's perspective, was broader or narrower than the content from the perspective of the researcher. In other words, the discussion, in these cases, focuses on situations when the researcher and teachers do not agree on the content of particular classroom activities. At other times different teachers' discussions of an instructional strategy are compared to illustrate variation among teachers' perspectives of the content associated with instructional activities.

The chapter concludes with a summary of the data reported in earlier sections. Although this summary is organized to address the research questions for this study, interpretation of the analysis is left, for the most part, to the final chapter.

Content Areas

The category questions in the interview schedule were designed to elicit teachers' thoughts about the various areas of content in the mathematics curriculum and how these areas were seen to be related. Teachers were asked to think of all the content they teach in mathematics and divide it into categories. The question was repeated as many times as necessary until teachers failed to suggest a new set of categories. Then, they were asked to mention any common threads that run through most of the content they teach.

The analysis of teachers' responses to the category questions provided a description of content areas that teachers recognized as important components of the mathematics curriculum in elementary schools. By identifying the attributes of mathematics content that were salient in teachers' specifications of categories, the analysis revealed characteristics that teachers used to define segments of content. Then consideration of the particular categories teachers mentioned, identified areas that dominated teachers' comments when they thought about the subject matter holistically.

Teachers' reactions to the request that they subdivide the content they teach in several different ways often indicated rigidity in their thinking about the subject matter. In other words, the content was perceived by many teachers as being clearly delineated by some single schema. Only eight teachers (Teachers 1, 2, 3, 4, 12, 14, 15 & 21) reflected some level of flexibility in their thinking about mathematics by providing more than a single set of categories. Three of these eight teachers (Teachers 1, 3 & 21) were only able to describe two sets of categories. The reaction of the 11 teachers who had difficulty responding with more than one set of categories is characterized by a remark made by Teacher 9.

> Whole numbers--adding, subtracting, multiplying, dividing, and then fractions and decimals. I guess that's it... That's basically the curriculum in my head with all those little parts thereof that I've already talked about.

The later categories provided by those teachers who gave third and fourth responses to the question usually provided no additional insight into their recognition of characteristics of mathematics content. Often, previously mentioned content was given a new label. For example, Teacher 14 replaced "computation" and "conceptual" from his first set of categories, with "operating" and "conceptualizing" to form a fourth set. Two teachers made use of schema that focused on instructional concerns to create additional category sets. Teacher 15, in a final response to the question, divided content according to the type of student that should study it, i.e. content for accelerated, average, or low achieving students. In stating a third set of categories, Teacher 14 characterized content as central, border, or peripheral, delineating his priorities for student learning.

The only teacher who focused on different descriptions of content in a third set of categories was Teacher 12. This teacher's first set of categories described mathematical operations including addition, subtraction, multiplication, and division. The second set given by Teacher 12 included categories that referred primarily to skill development and applications, thus characterizing the general intent of content. In the third set, Teacher 12 focused on the nature of material giving fractions, decimals, money, and measurement as categories within the set. She had referred to the nature of material, however, in earlier sets mentioning "algebra" as a category in her first response and "measurement," in the second response. Since no additional information about characteristics of mathematics content was provided in any teachers' third and fourth sets of categories, further analyses focused only on the first and second responses teachers gave to the question. Appendix C summarizes the multiple sets of categories given by each teacher.

Attributes Used to Describe Mathematics Content

The taxonomy of elementary school mathematics described in Chapter 3 (see Table 2) is comprised of three factors that encompass four descriptors of content. Factor I both describes the general intent of the content, i.e., whether the content deals with concept development, skill acquisition, or the application of concepts or skills, and incorporates a distinction between concrete (or pictorial) and symbolic modes of presentation. Factor II of the taxonomy characterizes content according to the nature of material involved (e.g., does it deal with whole numbers, geometry). Finally, the operations students are to perform (e.g. add, identify place value) are described by Factor III of the taxonomy.

The analysis of teachers' categories first identified the factors of the taxonomy that include the attributes used when teachers specified a category of mathematics. The description of attributes used by teachers in either their first or second response to the categories question is summarized in Table 3. In general, categories given by the teachers were consistent with the characteristics of content that are described by the various factors of the taxonomy, i.e. general intent, nature of material, and operations. However, only seven teachers (Teachers 1, 2, 3, 4, 12, 19 & 20) made use of all three factors in their category labels. Further, only one teacher (Teacher 14) mentioned the distinction between the concrete and abstract in his description of categories of mathematics.

Every teacher did not mention categories from all three factors of the taxonomy. Teachers 5, 7, 8, 9, 10, and 18 did not describe the general intent of content within their responses. Teachers 6, 7, 15, and 17 gave no category that characterized the nature of material. Finally, Teachers 6, 13, 14 and 21 failed to describe specific operations in their sets of categories of mathematics. However, each of the teachers in this final group gave a category that suggested the general intent of content as skill development; Teachers 6, 13, and 14 mentioned "computation" and Teacher 21, "arithmetic". When asked to subdivide these categories, each of these teachers specified the operations that were given as categories by other teachers.

The two teachers who mentioned categories of mathematics that were not consistent with descriptions provided by one of the factors of the taxonomy (Teachers 15 & 20) each focused on different aspects of content. Teacher 20 included content from the affective domain, specifying "appreciation of mathematics" as a category of content that was necessary

TABLE 3

Description of the Attributes used by Teachers'
in Their Lists of Categories

Teacher I.D.[1]	General Intent	Nature of Material	Operation	Other
2	X	X	X	
3	X	X	X	
4	X	X	X	
17	X		X	
13	X	X		
21	X	X		
5		X	X	
8		X	X	
14	X	X		
19	X	X	X	
1	X	X	X	
10		X	X	
18		X	X	
6	X			
15	X		X	X
20	X	X	X	X
9		X	X	
7			X	
12	X	X	X	

[1]All tables in this chapter were organized to cluster teachers by grade level and textbook within grade level. The grade level and text variables are identified only when those factors were important to the interpretation of the data

for students to learn. Teacher 15 gave "content situation" as a descriptive feature of content. She explained this category telling the interviewer that the context of problem situations (e.g. problems about buying groceries) might be used to categorize mathematical content. These were the only two cases when the content areas suggested by teachers' responses to the category questions involved descriptions of content outside those provided by the taxonomy.

Content Areas Recognized as Important

The second issue addressed by the analysis of teachers' responses to the category questions was to identify the particular areas of mathematical content that were incorporated in teachers' sets of categories. This information provided some indication of the content areas that dominated teachers' thinking about mathematics curriculum.

Although most of the categories mentioned by teachers were described by levels of the taxonomy, not all levels in the taxonomy were recognized in the teachers' responses. For the most part, teachers' remarks classified at Factor I, general intent, involved the skills and applications levels. Categories that focused on the nature of material, Factor II, most often referred to measurement, geometry, or fractions. The levels of Factor III, operations, that were descriptive of several teachers' categories were the arithmetic operations (e.g. addition), and place value. Other levels of the taxonomy (e.g. estimation, whole numbers, conceptual understanding) were only mentioned by a few teachers, and some levels (e.g. properties) were not mentioned as a category by any teacher.

A summary of categories mentioned by four or more teachers in their first or second response to the question about categories of mathematics is presented in Table 4. Categories that were commonly mentioned, for the most part, referred to arithmetic operations. In fact, every teacher in the study included some characterization of computational skills in their lists of categories. Some teachers gave a single category including all the computational skills, while others gave each operation as a separate category. Geometry was mentioned as a category by only four teachers and these teachers were each discussing third or fourth grade mathematics. None of the fifth grade teachers included geometry as a category of the content they teach. Another category, measurement, was mentioned by only one teacher discussing grade 5 mathematics, while nine teachers from the grades 3 and 4 mentioned measurement as a category of content.

The table reports that eight teachers mentioned "problem solving" as a category of mathematics. These teachers, however, used a variety of referrents (e.g. story problems or practical math). Because their discussions suggested some level of agreement about, at least, the instructional activities included in that category, it is reported as a generally recognized content area.

Although a great deal of similarity was noticed in teachers' responses to the category questions, the content suggested by some teachers' lists of categories was sometimes unique. For example, Teacher 12 described "algebra" as a category that included missing factor or missing addend problems. Within the interviews many teachers discussed missing factor or missing addend problems; however, they did not refer to them as a separate content category in this part of the interview. Three

TABLE 4

Categories Mentioned By More Than Three Teachers

Category	Number of Teachers			
	Gr.3	Gr.4	Gr.5	Total
Computation	5	3	5	13
Problem Solving	4	1	3	8
Addition	1	1	3	5
Subtraction	2	1	3	6
Multiplication	2	2	3	7
Division	1	2	3	6
Add. & Subtraction	1	3	1	5
Multiply & Divide	2	3	0	5
Fractions	2	6	2	10
Measurement	4	4	1	9
Geometry	2	2	0	4

teachers referred to content related to money problems in their discussion of categories but later incorporated them under more general headings. Teacher 12 mentioned "money" as a category, while Teacher 3 included "money" under a category labeled "other". Finally, despite the current thrust for teaching skills of estimation, only two teachers mentioned estimation in this final stage of the interview. Teacher 2 gave a category "estimation" and Teacher 5 mentioned "estimation and place value" as a common thread.

Understanding in Mathematics

Throughout the interviews, teachers frequently described a concern that students "understand" various aspects of mathematics. The analysis of such remarks identified the topics teachers included in the discussion to determine what teachers meant when referring to understanding in mathematics. The instructional activities teachers described as being related to the development of understanding in mathematics were also analyzed to determine if the content of these activities, from the teacher's perspective, was consistent with the researcher's perspective of content. Finally, teachers' views about the importance of topics and activities related to mathematical understanding were also examined. In the discussion that follows, the results of each of these analyses will be reported.

Topics Teachers Related to Mathematical Understanding

Consideration of the ideas, skills and performances that teachers described in connection with the understanding of mathematics evidenced that teachers interpret "understanding" in different ways. Teacher 10 summarized three different dimensions to the concept when she was asked, "what would be involved in understanding multiplication?".

> I can see at least three different things. Understand ing multiplication as a short cut way of adding...the next thing is mechanically, how you do it...the third thing, of course, is knowing when to use it.

Most teachers, when discussinq understandinq in some area of mathematics, focused their attention on only one of the dimensions outlined by Teacher 10. Some discussions about understanding described concept learning as the content. Other remarks equated getting correct answers with understanding. Yet a third type of comment stressed the ability to use mathematics in given situations as the key to understanding. Further, an individual teacher's interpretation of understanding sometimes changed as the teacher discussed different content areas. The examples that follow are presented to illustrate the various interpretations of understanding.

Teacher 14 made a clear distinction between understanding, as concept learning, and other content he described during the interview. When asked what he meant by "concept," he responded,

> I guess I mean it is an idea as opposed to a skill--the idea of --oh, the concept of fraction being part of something, whereas the skill of working with fractions would be writin~ the fraction, as opposed to recognizing what it stands for. Where the concept of division would be repeated subtraction, or dividing something into a certain number of groups as opposed to the skill of actually being able to do it.

As was mentioned in the analysis of content areas, Teacher 14 was theact only teacher who mentioned the distinction between concrete and abstrhe content in his discussion of categories of mathematics. Throughout t the interview, the teacher's discussion of lessons offering experience in-concrete and pictorial levels indicated a concern that students under stand concepts of mathematics.

Teacher 12 also described instructional content that would be characterized as concept learning. Her interest in structuring activities, however, was focused on providing opportunities for students to verbalize their understanding of concepts. She criticized the use of self-paced instructional strategies in mathematics because such strategies do not encourage students to discuss the concepts they are studying.

> I think when students go at their own pace as fast as they can, page by page--zoom through the book--they lose the verbal aspect of math where you need to be able to explain to somebody else so they can understand. Back to the old "what it is, I understand, and I can teach you.

Teacher 1's comments reflected a different view of "understanding." Unlike Teacher 12 who held verbal expression of concepts as a crucial element of classroom activity, Teacher 1 was a skeptic regarding children's ability to verbalize concepts. When asked what her students might say if they were asked to explain division, Teacher 1 commented, "..I have to say that you would have probably get (sic) as many answers as you have kids". Her explanation of what she meant when she said a student understands something was,

> Well, I guess, the fact that he would be able to take what he has learned, and be able to apply it somewhere else--would lead me to believe that he understands that. I guess, feeling that children learn certain things in isolation, I don't think it has much meaning to them--when a concept begins to have some meaning, where they can take that concept, apply it to whatever they might find a way to apply it, then I think you have a true understanding of that concept.

Although Teacher 1's view of understanding also focuses on concept learning, her expectation for student performance was extended to include the use or application of the concepts.

Another teacher (Teacher 2) described the use-interpretation of understanding as a starting place for concept development. In describing the sequence of instruction on multiplication, Teacher 2 explained,

> Well, usually it's not introduced at all much before the third grade level so the first thing is really getting them to understand what it's for--to know why you would multiply, showing the usefulness of that and doing just beginning multiplication problems.

Another teacher's remarks (Teacher 3) reflected the view that understanding is synonomous with getting right answers.

> Interviewer: Is there any way that you have that helps you to know when a child is understanding what he is doing?
>
> Teacher: Oh, yes. But I don't really know that I can tell you what it is: I guess it's more about how he looks.
>
> Interviewer: Well, perhaps we can make it more specific. How would you know that a child understands, let's say, fractions?
>
> Teacher: Well, I guess I would give him some fraction to do and see if he could do it.

Comments by Teacher 19, also evidenced this "doing it" interpretation of understanding. He described the understanding of large numbers as being "able to read and do calculations with those numbers". This perception of the content related to understanding is distinctly different from that expressed by Teacher 2 who explained, "they (students) have a hard time understanding large numbers...we do some things just getting them to conceptualize how big a million would be, you know, talking about how high a million sheets of paper would be".

From the perspective of the taxonomy and in the view of curriculum writers, this "doing it" interpretation of understanding does not incorporate topics related to mathematical understanding. Yet for two teachers,

(Teachers 3 & 19), students' demonstration of facility in completing problems precluded the need for further instruction related to concept development. Explaining that he did not require students to complete problems using a division algorithm that is intended to help understand why the algorithm works (the ladder method)[1], Teacher 19 told the interviewer,

> There's some kids that come to school understanding how to divide the way I divided when I was in school. But the way that they're taught how to divide in our new textbook is totally different, so it leads me to believe that they're getting some help from Mom and Dad, which is fine. I don't require them to do it the way that it is in the book, as long as they're coming up with correct answers.

The examples that have been cited illustrate how the topics teachers mentioned when discussing the understanding of mathematics reflected different interpretations of mathematical understanding. Although a teacher's remarks about understanding mathematics could sometimes be uniquely described by one of the categories discussed above, a teacher's view sometimes changed from content area to content area. Some of the inconsistencies evidenced in teachers' discussions about understanding in different content areas were influenced by their own mathematical understanding. This was noted in the interview with Teacher 15.

Throughout the interview, Teacher 15 expressed a concern that students learn <u>why</u> things work. For example, she explained that when teaching division of large numbers, she would begin with a discussion about the basic concept of division to be sure that students understood that division means the creation of equal groups. However, when this teacher

[1] The ladder method of division, requiring the repeated subtraction of the divisor or subtraction of multiples of the divisor, is recognized by the taxonomy as a procedure to develop understanding of the division algorithm.

described lessons and activities dealing with the content area of fractions, the concern for concept development seemed to disappear. As "understanding" took on a "being able to do it" dimension in her description of classroom lessons, the teacher recognized the change in her orientation. She then explained to the interviewer,

> By the time I got to fractions in Junior High (as a student), it was strictly, "Here's the way they're done. Do them." Very little background.

Earlier in the interview, this teacher had expressed the opinion that students might not "really understand" the concept of fractions until they get to hinher levels of math. The teacher's remarks evidenced her awareness that understanding of fractions was not a part of the content of instruction in her classroom.

The Content of Classroom Activities

The review of the literature reported an observation by Goodlad et al. (1970), that "the form, but not the substance," of modern curricula had reached the classroom. This finding was apparent in the analysis of teachers' discussions of the various activities that they used to promote understanding of mathematics. For example, different teachers viewed lessons involving the use of concrete materials as having different purposes. For some, like Teacher 14 whose views have already been discussed, such lessons represented a unique mathematical content to be learned.

Teacher 6's remarks about understanding in mathematics were similar to comments made by Teacher 14. When asked what he meant by basic understanding of fractions, Teacher 6 responded, "Well, I guess by example--to understand that when you have one-third of something, you have one of three parts, for instance. And you know what both of the numbers of the fraction stand for." But unlike Teacher 14, Teacher 6 was somewhat skeptical of the need for activities involving the use of manipulatives. He explained that the use of pictures accomplished "almost the same kind of thing". He then added, "Unfortunately, a lot of things that you do in the classroom--efficiency is a prime consideration."

This perception, that activities involving the use of manipulatives represented a strategy to teach content related to understanding, was also expressed by Teachers 12 and 17. Teacher 12 questioned the need for such activities, explaining that it would have been done at lower grade levels. Teacher 17 told the interviewer,

> I feel that they have so many things that they could do... I'm thinking of an example, using Cuisenaire rods to teach fractions. This could be used, you know. There are a lot of gimmicky type things that I have never...that I just never find time to do. There is so much crowded into a day's time if they are going to get the basic skills that I think they need in third grade math, that I just have never found time to bring a lot of additional things.

Teachers' comments indicating that activities involving the use of manipulatives were not presented because such materials were inefficient, used in previous grades, or gimmicks, demonstrated that these teachers did not recognize the content of such activities as unique from content presented in other lessons. The manipulatives were viewed as an instructional strategy and teachers felt free to exercise professional judgement regarding their use.

Teachers' discussions about understanding measurement evidenced the same variation in view points. Seven of the 19 teachers referred to particular concrete experiences to help students understand units of measurement. These included such things as: filling a box with cubes to establish the concept of cubic units; estimate distances and then pace them off; and marking off the area units when learning to find the area of shapes. For Teacher 17, however, such "strategies" were viewed as a last resort for students who could not learn to calculate conversions of measurement units. Teacher 17 told the interviewer,

> So they have to recognize the equivalent that four cups is equal to a quart and four quarts to a gallon. And they convert from one to the other. And we have--for a child that just cannot see this all, you know there are really good pictures and equivalents in their book. Then we actually have all those bottles.. and even some of them go to such lengths as pouring one from the other...

From the interviews it was impossible to conclusively determine the particular activities that might be modified for classroom use because teachers did not recognize the content implied by the activity. Examination of the priorities teachers expressed, however, gave some indication that such modifications occur.

The Importance of Mathematical Understanding

All of the teachers interviewed reported teaching lessons to help students understand mathematics. But the preferences they discussed concerning certain types of activities, and the priorities for student learning the teachers expressed, indicated that such lessons are viewed as being more important by some teachers than others. In the preceding discusion several teachers (Teachers 6, 12 & 17) were reported as viewing lessons involving the use of concrete objects as strategies that could be replaced by other activities. Yet eight other teachers (Teachers 2, 4, 7, 8, 10, 13, 15, & 18) characterized activities with concrete objects as an essential part of the mathematics curriculum in their classrooms. Teacher 7 explained that she sometimes had difficulty justifying such activities when parents questioned her. But her discussions of instruction in the areas of subtraction and multiplication reflected a concern that students acquire understnading of those mathematical concepts at the concrete level. Teacher 8 told the interviewer, that if students were working on materials independently without much help from the teacher, "they wouldn't have all the manipulatives that I scrounge around for to make them better understand what they're doing...". It was not clear from these two teachers' discussions that work with concrete manipulatives was a separate content area. It was clear, however, that activities using manipulatives were considered an important part of the mathematics program.

The two teachers whose remarks frequently indicated an interpretation of understanding related to the use of concepts (Teachers 1 & 2) each viewed such lessons as an important part of the curriculum. The three teachers who offered elaborate descriptions of understanding as concept learning (Teachers 8, 13 & 14) also placed a high priority on the content related to understandinq. Teacher 13 said, "I feel strongly that children should be taught why they're doing what they're doing. I mean, they should understand the process, not just memorize it."

Remarks made by Teachers 3 and 19 indicated less commitment to teaching the content of mathematical understanding. Teacher 19 explained that he taught lessons on place value only because such lessons appear n the textbook he uses. A stronger statement of compliance with the textbooks' views on the value of understanding was expressed by Teacher 3.

> There was a time, before I'd taught too long, where I thought we ought to eliminate almost everything we teach in math--because I couldn't see where some of it was going to matter to them. But for some reason, I'm very willing to teach them the theory that we teach them.

This teacher's inclusion of "theory" in mathematics instruction was at a conscious level. Her first response to the categories questions was "theory" and "computation". However, she never described a rationale other than one of compliance, for teaching the lessons on understanding of mathematics.

Teachers' remarks about when lessons on conceptual understanding should be presented, also reflected their priorities and suggested that different teachers attached different purposes to such lessons. Teacher 2 suggested a priority as she described her sequencing of such topics.

> I do want them to come up with the right answers and knowing (sic) which order to do this and that. But before I get into that I want them to understand why you'd be doing that in the first place, and what it means to be doing that.

Her plan for instruction is the reverse suggested by other teachers (e.g. Teachers 14 and 9). Teacher 9 commented,

> I have found that the most success I have with kids in teaching them concepts is first to teach them to compute--to be able to do the problem. Once they feel good about that which means that they can do the test problems--they can do the pages of whatever they're supposed to do. Then they are wide open to examining the concepts and the beauty of what they are doing.

Five teachers' interpretations of understanding (Teachers 4, 7, 10, 18, & 20), and their statements about the importance of the content, varied considerably as they discussed different content areas. For example, when talking about teaching fractions, Teacher 10 stressed the importance of students' learning the concept of fraction. She wanted them to also understand why fractions were manipulated as they are in finding equivalents. In discussing multiplication of large numbers, however, Teacher 10 expressed less interest in students' understanding.

> When you're multiplying large numbers, you have to put them in certain boxes and show the sub-products are this and this and this. I'm not so concerned about whether they understand. I don't know what the value of that is. If they are able and they use it in a situational problem where they have to solve it, I'm not so sure they have to go back and analyze...

Some of the inconsistencies in teachers' remarks may be due to the fact that "understanding" is not typically recoqnized as a content area. Responding to the category questions, only three teachers (Teachers 3,4,& 14) mentioned "understanding" as a category of mathematics. The eight other teachers who mentioned this aspect of the content in response to the category questions, described it as a common thread that runs through most of what they teach. Thus, the designation of the importance of understanding in mathematcs was considered further in the analysis of particular content areas, i.e. problem solving, multiplication and division.

Problem Solving

The investigation of teachers' conceptions of problem solving focused on their comments about three different types of activities. Teachers' descriptions of activities designed to teach the applications of mathematics in some situational context, their statements about "story problems"[1], and teachers' remarks about non-computational problem

[1]The label "Story problem" will be used in this discussion to refer to written problem statements that described situations and do not specify the arithmetic calculation that must be used to solve the problem. This type of problem is typically presented in textbooks, and is often called a "word problem" or "verbal problem". The "story problem" label is used here because it was the referent teachers used in the interviews.

solving content were analyzed. In the earlier discussion of the analysis of content areas, the similarity in activities teachers described under these three titles was reported. The analysis reported here reveals differences in the topics and learning outcomes the various teachers described as the activities were discussed.

Problem solving was selected as an area for in-depth analysis for two reasons. First, five teachers (Teachers 1, 10, 13, 18 & 21) had been using a published curriculum (DMP) that stressed the development of problem solving content. Second, the area of problem solving has been identified recently by mathematics educators as a part of the curriculum in elementary schools that should be expanded. The analysis reported here compared the remarks made by different teachers who used DMP. It also compared the content of problem solving described by DMP teachers, to that mentioned by teachers who used other instructional programs. These comparisons suggested the influence DMP might have on teachers' beliefs about content and identified ways in which different teachers interpreted the DMP curriculum. However, the question of whether teachers used DMP because it suited their beliefs, or their beliefs were influenced by the DMP, was not addressed.

Topics Teachers Related to "Problem Solving"

A summary of teachers' statements about problem solving is presented in Table 5. Section 1 of the table includes those ideas and performances that teachers associated with learning to analyze problems. The taxonomy classification for this section would be at the conceptual understanding level of Factor I. Section 2 details the various sources of problem solving experiences that teachers used to describe their activities. (Most of these entries would not be classified in the taxonomy.) Section 3 reports

TABLE 5

Teachers' Descriptions of Content and Activities in Problem Solving

Teacher I.D.	third grade 2	3	4	17	13	21	fourth grade 5	8	14	19	1	10	18	fifth grade 6	15	20	9	7	12
Section I - Problem Analysis																			
Write open sentence for story problem	*		*	*	*	*			*		*	*	*	*	*				
Learn: to read story problems						*													
how to look at story problems											*								
identify what is known			*								*		*		*				
identify what is unknown						*					*		*						
determine if info is sufficient						*													
identify operation needed	*		*	*	*	*				*	*	*						*	
difference between + and -										*	*	*							
difference between x and ÷								*											
difference between ÷ and -								*											
clue word for operations											*							*	*
terminology					*				*						*				
identify extraneous information												*							
Express relationships w/inequalities												*							
Logical process for story problems														*					
Represent problems in pictures																		*	
Add "difference" to concept of subtraction																			*
Difference between x and - of fraction																			*
Section II - Source of Problem																			
From textbooks:	*					*								*		*			
textbook w/graphic representations										*									
classroom situations	*															*			
other situations						*	*				*	*				*			*
newspaper ads	*					*										*			
Non-story problems					*														
True-false problem statements													*						
Logic problems (not in text)						*						*	*		*				
Puzzles without words											*								
Problem solving w/cuisenaire rods								*											
Student-made problems						*													
w/no. given by teacher													*						
in classroom lab experiences														*					

◯ DMP Teacher

* Entry Mentioned

TABLE 5 cont'd

Section III - Factor II & III Descriptions	Teacher I.D. third grade 2	3	4	17	13	21	fourth grade 5	8	14	19	1	10	18	fifth grade 6	15	20	9	7	12
Using proportions														*					
With one or two step solutions														*		*			
With whole number														*					
Add	*		*	*	*					*	*			*		*		*	*
Subtract	*		*	*				*	*		*					*			
Multiply	*		*	*	*			*		*	*								*
Divide	*			*				*	*	*	*					*			
With easy and hard numbers			*										*					*	
With measurement							*									*			*
With decimals																			*
With fractions																			*
With money					*				*										
With two unknowns													*						
With column addition					*														
With missing addend					*														
Find: day, hour, minute in one year								*											
averages								*	*						*	*			*
your age								*											
height of telephone pole														*					
distance on map, scores in phys. ed.																*			*
cost of meal				*															*
Read menu - compare costs				*															
Section IV - Other learning outcomes																			
Solve open sentence			*	*	*	*													
Label answers			*	*				*											
Assess solution						*					*	*							
Gives steps for solution					*	*					*								
Learn: mechanics of operations															*			*	
reinforce skills learned										*					*			*	
see purpose in computation																		*	

O DMP Teacher
* Entry Mentioned

teachers' comments that would be classified at Factor II (nature of material) or Factor III (operation) of the taxonomy. Finally, Section 4 describes learning outcomes teachers associated with this content area, with the exception of those outcomes that are listed in section 1. Because teachers' remarks rarely contained sufficient information for more complete classifications, complete taxonomy coding is not recorded on any section of the table.

The data recorded in Section 1 of Table 5 illustrated several differ ences between the content described by teachers using the DMP program (Teachers 13, 21, 1, 10, & 18) and the content mentioned by other teachers. This section of the table reports the content teachers said they taught related to the analysis of problem solving situations. The content mentioned by DMP teachers defined procedures for analysis of problems more specifically than content mentioned by other teachers. The procedures DMP teachers said they wanted students to learn were: identify what is known (Teacher 1 & 18); identify what is unknown (Teacher 21); and identify ex traneous information (Teacher 10). Although the DMP program contains activities dealing with each of these analysis procedures, no one DMP teacher described all of the steps for analysis as content. Furthermore, DMP Teacher 13 did not mention any of these procedures. However, with the exception of "identifying what is known" (mentioned by non-DMP Teachers 4 & 15), these procedures were not mentioned by teachers using other instructional programs.

Teachers using DMP also mentioned strategies that specifically applied to story problem solutions. For example, most of them (Teachers 13, 21, 1 & 10) wanted students to learn to identify the operation to be used in solving a problem, and all of them reported that students should learn to write an open sentence to represent a story problem situation.

Procedures of analysis that were mentioned by non-DMP teachers were usually less prescriptive than those mentioned by teachers who used DMP. For example, Teacher 2 told the interviewer students should learn to "write an open sentence" and "identify the operation needed", but she did not describe the procedures students would use to complete these performances. Also, the analysis procedures mentioned by non-DMP teachers usually applied only to problems that require an arithmetic calculation for solution. Among the procedures mentioned by teachers not using DMP were: writing an open sentence, identifying the operation to be used, and associating various terminology with the arithmetic operations. Only one non-DMP teacher (Teacher 7) mentioned teaching procedures of analysis that might apply to a broader range of problem types. Yet when Teacher 7 discussed that procedure, i.e., representing problems in pictures, her remarks focused on pictures that represented story problems.

Teachers who discussed topics related to problem analysis, from both the DMP and non-DMP groups, all mentioned or implied that students must learn to differentiate problems that dealt with the various operations. Teachers 7 and 19 mentioned the task of determining the operation to be used. Three teachers (Teachers 10, 7 & 12) mentioned teaching "clue words" or "key words" that can be used to signal the operation to be used in the solution (e.g. "difference" means subtract; "total" means add). Teachers 13, 14 and 15 also mentioned vocabulary building, but they wanted students to develop an understanding of the terminology used in problem statements, not just memorize the clues.

Further differences between teachers who used the DMP, and those who did not, are evidenced in section 2 of Table 5. Each DMP teacher mentioned some type of problem that would require other than an arithmetic calculation; while, with only two exceptions, the 14 teachers not using DMP only described problems that dealt with the calculation of some solution. Teacher 8, the first exception; mentioned problem solving involving Cuisenaire rods. The teacher's discussion of the Cuisenaire rod activities was too brief, however, to determine if the activities fit the noncomputation problem category. The second non-DMP teacher who described non-computational problem solving was Teacher 15. She mentioned "logic problems that are not in the text" as part of the instructional program in her classroom.

In short, with the exception of those teachers who had used the DMP program, there was little indication that teachers recognized topics within the area of problem solving other than those represented by the typical textbook problem. The examples discussed by non-DMP teachers, for the most part, were either story problems, or situations that could be written as story problems.

The Purpose of Problem Solving Lessons

In the analysis of teachers' discussions about understanding in mathematics, it was possible to contrast the content of particular activities from the perspectives of different teachers. The descriptions teachers gave of problem solving activities, however, were not as detailed as their descriptions about activities used to teach for understanding. The analysis of teachers' perceptions of the purpose of problem solving activities, therefore, addressed the generalized question, what do teachers see as the purpose of lessons on problem solving? Further insight into teachers' recognition of topics to be learned was gained through the analysis. Individual teachers were found to have dramatically different views about what students should learn when studying problem solving in an elementary school mathematics class. The different viewpoints will be illustrated in the discussion that follows.

Teacher 7 did not mention problem solving in her discussion of categories and common threads in mathematics. She did, however, mention story problems within the interview. When asked what she hoped students would learn from story problems, she responded,

> How to apply...they've learned-the mechanics of the operation. So now...you try to correlate it as you go along. Like you don't teach them all the mechanics, and say, "here is a problem to apply it to." As you are learning, you are <u>using written problems as a reinforcement activity of the skills that they are learning</u> so they can take it and use it in concrete situations. Taking what they've learned and applying it. Seeing why you have to learn how to do pages and pages of multiplication.

Thus Teacher 7 expressed the view that problem solving is a subset of the content to be learned when studying the arithmetic operations. The teacher's remarks suggested that the study of story problems provides some motivation and justification for acquiring computational skills.

Another teacher (Teacher 2) explained that story problem activities were helpful in assessing students' ability to <u>use</u> the computational skills they have learned. In a discussion about multiplication, Teacher 2 commented,

> The first thing is really getting them to understand what it's (multiplication) for--to know why you would multiply, showing the usefulness of that and doing just beginning multiplication problems. I guess what some people would call basic facts. And that's a good part of the third grade content level. Also, um, being able to use that in practical situations--story problems or other kinds of just situational kinds of things that I might describe to them and see if they can come up with an answer.

These remarks by Teacher 2, relate to the observation already made about this teacher, that she viewed "understanding" in an "applied" sense, i.e. to understand means to know how to use it. She also was a teacher who viewed "applications" as a high priority in her classroom. Yet the problem solving strategies she described as content students should learn, only involved writing an open sentence and identifying the operation to be used.

None of the other teachers interviewed discussed the purpose of story problems from the limited perspective expressed by Teachers 2 and 17. Four teachers (Teachers 1, 4, 13, & 21) who described "steps for solution" as content students should learn when studying problem solving are identified in section 4 of Table 5. This group of teachers is made up of three of the five teachers who used DMP, and one of the 14 teachers who used another type of instructional material. Problem solving, from the perspective of these teachers, included something more than the use of computational skills. The steps for solution, at least, represented additional content to be learned. However, each of these teachers also had unique views about the content of problem solving.

The solution steps suggested by Teacher 4 were: 1) identify numbers involved; 2) decide if the problem involves a comparison or a combination of quantities; 3) decide on the operation needed (comparisons require subtraction and combinations involve addition); 4) form a number sentence; and 5) get an answer and label it. These steps for solution were described as learning outcomes for students. Although the range of topics Teacher 4 discussed in the content area of problem solving was limited to story problems, this teacher recognized lessons on problem solving as having a purpose other than the extension of computational skills. Teacher 4 wanted students to learn to analyze and solve a story problem.

The three DMP teachers also gave steps when referring to the specific area, story problems, which is only a subset of the problem solving content of the DMP curriculum. While each of these teachers seemed to be influenced by DMP, their comments demonstrated ways that different teachers interpret a written curriculum.

In the earlier discussion, Teacher 13 was identified as the only DMP teacher who failed to mention procedures for the analysis of problems that might generalize to non-computational problem solving. The steps for solution mentioned by Teacher 13 would also be useful only for solving problems that involve computation. These steps were: (1) set up a number sentence, and (2) solve it.

During the interview, Teacher 13 praised the DMP curriculum because, "...an awful lot of the (DMP) approach is story problems". The first evidence of her interpretation of the curriculum was reflected in her attempt to equate story problems with problem solving. When asked what students might learn from story problems she responded,

> I think a lot of this is problem solving. They have to, uh, I mean it's not just set down for them, they have to think through it--and think about--what do I know in this, this situation. What am I looking for and how do I go about finding that out. I think it involves analyzing.

After Teacher 13 had described several types of story problems she presented to students when using the program, the interviewer asked her if she used any problem-solving activities other than story problems. She responded,

> I guess the story problem is the basic format. Uh, it just depends, sometimes you know, there are a lot of different ways that they might approach solving the problem. It might involve addition of a, you know, quite a few numbers. It can involve a lot of different things, but I guess the basic format would be a story problem.

Later, Teacher 13 explained her goal for student learning in the content area of problem solving; "...one thing that I want the kids to be able to do at the end of the year is to be able to look at a story problem and decide whether--what operation has to be used."

Teacher 13's only discussion of the non-story problem situations that were mentioned by other DMP teachers, occurred in the context of her explanation of the "Movement and Direction" unit contained within the DMP materials. In this case the teacher did not characterize the lessons as providing a problem solving experience. Rather, she suggested that students would learn to graph, to use a unit measure, and to know direction on a number line.

The solution steps mentioned by Teacher 1 were less focused on computational problems than the steps Teacher 13 described. The solution steps Teacher 1 wanted students to learn were: 1) how to look at it and break it down; 2) how to set up an equation; 3) how to decide if the answer makes sense. Although step 2 would apply to the more restricted problem type, the other steps could be viewed as more generalized problem solving strategies.

Teacher 1 also mentioned a broader range of problem solving activities than Teacher 13. When asked what she meant by problem solving Teacher 1 said,

> Well, I guess where children are given some information where they have to know a piece of what's missing. Maybe it's not a computational kind of problem. Sometimes they don't have words in them. But they're given certain pieces of a puzzle and you know they have to figure out what is missing, or figure out what is--how to put it all together, so to speak.

Yet the learning outcomes she described as the goal for instruction were much like those Teacher 13 had mentioned. Teacher 1 explained the types of problem solving experiences presented in her classroom.

> I think, for the most part, the--I guess I have to think of the problem solving based on what the functions are that we're going through. The addition and subtraction problem solving and then they're combined.. being able to tell the difference between an addition problem and a subtraction problem. I guess feeling that they had a good handle on that. There wasn't any confusion when they got to the multiplication and division...

When asked how problem solving in the DMP curriculum differed from what was presented in other programs, Teacher 1 responded that there were more problems in DMP and that the focus was on how to look at story problems. Reflecting on how her teaching had changed in using the DMP, Teacher 1 explained,

> I have to say that there are things that are different. Things that I'm teaching children are different. It's sort of like, a means to get at what I was teaching before. And that's, I think primarily what those things are. I feel like I'm doing much more teaching than I was before. Much more of, um, just much more of giving kids the skills to figure out problems... <u>Maybe my goal is the same, but the way I'm getting at that goal seems to be much more systematic... I don't think that the things are different, it's just maybe a different way of getting kids to look at you.</u>

The steps for solving story problems that were given by Teacher 21 could also apply to problems other than those that use computation: 1) reading and deciding what to do, 2) solving and assessing solutions.

What distinguished this teacher from the other teachers that have been discussed, was her recognition of other content to be learned through the DMP activities. In other words, Teacher 21 associated unique learning outcomes with such activities. Some of these outcomes were: ordering, drawing facts out of given information, knowing what constitutes a fact, finding unstated facts, learning strategies for games, thinking and reasoning, taking turns. This teacher viewed problem solving as being primarily associated with the computational content areas but she did recognize content from the broader spectrum of the area of problem solving as being mathematical content students should learn.

Teacher 21 also discussed computational story problems extensively, explaining that such problems were useful to expand students' understanding of the arithmetic operations. She expressed the view that story problem activities were often helpful in assessing students' understanding of mathematics.

> You get a lot more information as a teacher when you see a child with a problem like that of the boys walking the blocks and the child who subtracts, who takes nine minus six and tells you they walked three blocks--It gives me more insight into how their mathematical background is, than to just have--When you give a child a page of facts, and I don't use facts disparagingly--I mean I just--unrelated to anything else, if you get them all right, all you really know is that they know these. You don't know...any of the thought process that goes on. Whereas, if you take something that is related more to what we used to call a story problem, you can get a lot of insight into: 1) can they read; 2) can they set up the problem; 3) do they know which-what computation skill to use in solving it? Are they accurate in using that skill? And then, do (they) just understand mathematics enough to know, does that make sense.

As in the case of Teacher 1, discussed earlier, this view of using story problems to assess students' knowledge of arithmetic operations is consistent with Teacher 21's perception, that understanding requires correct use of a concept.

The detailed discussion of DMP teachers' conceptions of content in problem solving focused only on the three teachers (Teachers 13, 21 & 1) who had described steps for solution as a learning outcome. Table 5 evidences that the other two teachers using DMP (Teachers 10 & 18) were also concerned about students' learning how to solve story problems. These two teachers also included non-textbook logic problems in their discussion of content in problem solving.

The investigation of teachers' views of the purpose of problem solving activities evidenced teachers' interpretations of the goals of the DMP in different ways. The occurrence of such interpretations may be explained by Teacher 10's description of the confusion teachers and students confronted in using DMP.

> And the thing is, in the DMP program, a lot of kids couldn't relate to this/that--it was math, because there weren't very many numbers--And this is what one new teacher who didn't want to do the program was saying "(name), I really--they keep saying when are we going to do arithmetic" or math is what they call it. So you see, I think kids orientation, too, is manipulation of a lot of numbers. And, I like a lot of that too, but I like this other aspect of using numbers in some situations where you have to think out or analyze a problem.

The Importance of Problem Solving Content

Although the area of problem solving seems to be narrowly defined by most teachers, it is one that was viewed as important. In response to the questions about categories of mathematics and the common threads through the content, three teachers included "applications" in their response, one Teacher referred to "practical math", two teachers mentioned "story problems" and five teachers gave "problem solving". Teacher 14, who is not included in these groups, expressed the importance of this content area in his interview.

> About a third of what we do is...applied...applying computational skills or concepts to particular situations where it would be used. What word problems are, the most typical example, I guess, are story problems. Uh, creating situations that call for math and then figuring out, "Oh, what are you going to do to figure this out?".

Several teachers demonstrated their concern for students' learning problem solving and expressed frustration with the lack of instructional materials in this area. Teacher 20 told the interviewer,

> I think it is a serious omission in most textbooks. Like somebody said, "there's a crying need for a book to be published--just a story problem book"...I'm sure other people feel as I do, that you can't add columns and subtract columns and play with fractions page after page without knowing when, and why, and how.

Teacher 2, when asked to detail her reference to teaching "applications", explained,

> I think that's probably the weakest area in my instruction. It's the area...that I feel is the most important; yet the weakest just because there's such a shortage of materials to use with that kind of thing. You really have to come up with your own thing. Math books and other materials really stress computation such a lot.

Teachers' remarks about the nature of material and the operations students are to perform in problem solving activities (summarized in Table 5, section 3), also provided some insight into the particular topics that were viewed as important. For the most part, teachers characterized problem situations where solution required students to add, subtract, multiply, divide, or compare numbers. But they sometimes gave more detailed descriptions of these problems.

For example, seven teachers described particular contexts that framed the tasks by describing individual activities related to measurement problems (e.g. finding aaes or distances), or by referring to consumer situations (e.g. finding the cost of a meal in a restaurant). Teacher 19 differentiated textbook story problems according to whether they did, or did not offer graphic representations. Teacher 20 discussed problems involving map reading. Two teachers, (Teachers 21 & 6), reversed the problem situation for students by asking students to write their own story problems. Teacher 6, to illustrate this, explained that he gave students a set of numbers, like 4, 6, and 24 and asked students to write a story problem involving the numbers that were given.

It was not clear in the interviews whether or not teachers viewed the setting of a story problem as descriptive of unique content. For example, was the calculation of the total cost of a meal in a restaurant, somehow viewed as different by teachers from finding the cost of groceries. Only Teacher 15, who gave "content situation" as a category of mathematical content clearly described the context as a descriptor of content.

From the perspective of the taxonomy, however, the setting of a problem type does not characterize separate topics. The variety of different experiences teachers described to teach a single topic, from the taxonomy's perspective, might be viewed as an indication that such topics are seen by teachers as being important.

The data reported in section 3 of Table 5 reflects considerable agreement among teachers that the important problem solving topics involve situations where students add, subtract, multiply, or divide. More distinct descriptions of topics were provided by two teachers (Teachers 6 & 20), who mentioned that problems involving two-step solutions, and Teacher 12 who discussed story problems involving fractions as part of the content taught at the fifth grade level. Teachers 2 and 4 explained that story problems that require multiplication are only introduced, or are provided as experiences for accelerated students, at the third grade level. Thus, within the domain of problem solving experiences that require computation for solution, some problems were designated as more appropriate for students at particular grade levels.

Multiplication and Division

Teachers' descriptions of instruction in the areas of multiplication and division were analyzed to investigate teachers' views about content within computational areas. As in the analyses of other content areas, the questions addressed were: what topics do teachers recognize; and what are teachers' priorities for student learning.

The selection of multiplication as an area for analysis, allowed consideration of both between and within grade level contrasts of teachers' statements since all the teachers described multiplication as an important part of the curriculum in their classrooms. The between grade level comparisons investigated the assumption that teachers at different grade levels would teach different content, while the within grade level comparison examined whether or not teachers agree about what should be taught in a given grade.

By also considering teachers' statements about division content, the analysis identified teachers who recognized the relationships between arithmetic operations as content to be taught. For this analysis, contrasts between DMP and non-DMP teachers and comparison of statements made by teachers within the DMP group revealed differences. The findings of these analyses are presented in the discussions that follow.

The content and activities teachers mentioned when describing multiplication and division lessons are summarized in Table 6. To the left of an entry in the table the appropriate taxonomy classification is recorded. The conventions and notation used in this coding were described in Chapter 3. When more than one entry fits a given classification, the code is not repeated. Thus when lines are not coded, the classification for the preceding entry applies. An asterix (*) indicates an entry that was mentioned by the teacher as something done in his or her classroom. "A" codes specify items that a teacher said might be presented to accelerated students or used for enrichment lessons when students are doing well in mathematics. When the teacher reported that an entry was something to be "introduced" in the classroom to prepare students for further work on that topic in the next grade, an "I" code is used. "N" indicates entries that were mentioned by the teacher and described as topics not taught in that teacher's classroom.

The table is also divided into two sections. Section 1 contains a summary of remarks made during discussions about multiplication. The summary of teachers' comments about division is presented in section 2. Throughout the following discussion the taxonomy codes will be referenced to direct the reader's attention to the entries on Table 6 that are being considered.

Table 6

Teachers' Descriptions of Content and Activities in Multiplication-Division

Taxonomy Code		third grade						fourth grade							fifth grade					
	Teacher I.D.	2	3	4	17	13	21	5	8	14	19	1	10	18	6	15	20	9	7	12
	Section I - Multiplication																			
	Conceptual Understanding:																			
1,0,8	objects or pictures in groups						*		*			*		*						
	cuisenaire rods			*														*		
	number line		*		*															
	tree diagram								*											
1,0,8	As shortened form of addition															*				*
	As addition of equal groups		*										*							
1,1,11	Non-rectangular arragy												*							
2,0,8	As repeated addition	*	*	*	*	*	*		*		*		*	*						
	Through skip counting			*		*											*			
2,1,8	Learn the identity element				*						*									
	Learn about zero as a factor			*	*						*									
2,2,8	1 digit x 2-digit with carrying with expanded notation										*	*	*							
	partial product method	N	*	*	*															
2,3,8	2-digit x 2-digit w/repeat addition																	*		
	expanded for in matrix												*							
	describe verbally									*	*									
	group a number in different ways								*											
2,4,13	Is related to division						*	*				*								
	Communtative property	*				*	*											*	*	
2,0,12	Terms - "factor", "multiple"		*																	
	Use set language		*															*		
	Skills---																			
4,0,8	Do regrouping in multiplication			*								*								
	Multiply "mechanically"												*				*			
	Magic squares for practice								*											
	Cartesian products															*				
	Lattice-Napier's Bones										*					*				
4,1,8	Basic facts (not specified)			*			*	*	*	*	*	*	*	*	*			*	*	*
	to 3x9=		*		*															
	to 5x9=					*														
	to 9x9 or 10x10=	*										*	*	*						
	to 12x12													A		*	*			*
4,2,8	Single digit multipliers										*									
	1-digit x 2-digit	*	*		*												*	*		
	1-digit x 2-digit w/o carrying												*						*	
	1-digit x 2-digit (partial product)	N	*	*	*															
	1-digit x 3-digit	*	*		*															
	1-digit x 3-digit w/carrying												*						*	

Table 6 cont'd

Taxonomy Code	Teacher I.D.	third grade						fourth grade							fifth grade					
		2	3	4	17	13	21	5	8	14	19	1	10	18	6	15	20	9	7	12
	Section I - cont'd:																			
4,3,8	2-digit multipliers									*	*	*	*	*				*		
	2-digit x 2-digit	A															*			
	2-digit x 3-digit																*			
	2-digit w/o carrying																		*	
	2-digit w/carrying																		*	
	3-digit x 3-digit									I							*			
	4-digit multipliers															*				
	4-digit x 4-digit										*									
	by 10's, 100's, 1000's		*	*	*				*		*					*				
	Zero place holder (in partial prod.)												*							
	3-digit multipliers														*	*		*	*	*
	Tricks for 11 & "up-cross-up method"																*			
4,6;8	Multiply fractions																		*	*
4,11,8	Multiply measurements																		*	
4,8,8	Multiply mixed numbers																			*
4,9,8	Multiply in money problems									*										*
	Multiply decimals																		*	
0,4,8	Hard & easy number sentences													*						
4,8,15	Estimate costs								*											
4,0,15	Estimate products								*											
4,0,13	Check by dividing									*										
	Applications																			
5,11,12	Find areas							*		*			*							
6,0,8	Know when to use multiplication						*						*							
	Story problems						*		*	*			*						*	
	Show multiplication is useful	*																		
0,6;0	Cross product w/fractions										A									
	Find patterns in products						*						*							
6;6;8	Fractions in story problems																			*
	Section II - Division																			
	Conceptual Understanding																			
1;0,9	Understand division					*														
	as forming equal groups															*		*		
	using concrete objects																*	*		N
	parative model								*		*							*		
	measurement model							*	*		*									
	using cuisenaire rods		*																	
	using number line	*																		
1;0,10	Understand division w/remainder		*																	

Table 6 cont'd

Taxonomy Code	Teacher I.D.	third grade						fourth grade							fifth grade					
		2	3	4	17	13	21	5	8	14	19	1	10	18	6	15	20	9	7	12
	Section II - cont'd																			
1,3,9′	Large numbers w/pictures															*				
2,0,9′	As repeated subtraction	*	*	*	*		*	*	*	*	*	*	*	*			*			*
	Using expanded notation																		*	
2,2,9′	Repeated subtraction (divisor x 10)											*								
2,5,9′	Division as missing multiplier			*			*					*				*				
	No. sentence w/missing factor												*							
2,4,9′	Write no. sentences using division									*			*							
2,0,10	Division into = parts w/remainder															*				
2,0,12	Understand "average" & "mean"									*						*				
2,3,13	Family of facts	*						*					*							
2,4,13	Inverse of multiplication			*	*								*			*		*		
	Relationship with multiplication	*						*	*	*		*	*							
2,3,15	Round numbers																*		*	
	Skills																			
4,1,9	Basic facts		*	*				*	*	*	*	*	*	*	*		*		*	*
4,0,9	1-digit divisor, no remainder												*						*	
	w/out remainder (repeated subt.)	A			*															
	w/out remainder no subt. w/in)																		*	
4,0,9′	Repeated subtract (no. > divisor)			*	*		*													
	Ladder method															*				
	Mental division														*		*			
	"Goes into" method										*						*		*	
	Divide-multiply-subtract rule							*												
	Rule includes "bring down"													*						
4,1;9′	Single digit divisor								*	*		*		*						
	Short method (not repeated subt.)											*	A							
4,2,9′	3,4,5-digit ÷ 1-digit															*				
4,3,9′	By powers of ten																*	*		
	2-digit divisor							*	*		I	A	A	*	*	*	*			*
	2-digit ÷ 2-digit																		*	
	5 or 4-digit ÷ 2-digit																		*	*
	3-digit divisor																*	*	A	*
	4 or 5-digit ÷ 3-digit																			
4,0,10	Divide w/remainder (repeated sub.)	A			*															
	Ladder method w/remainder									*	*									
	W/1 remainder not within problem																		*	
4,1;10	1-digit divisor w/remainder												*							
	Facts w/remainder										*									

Table 6 cont'd

Taxonomy Code	Teacher I.D.	third grade						fourth grade							fifth grade					
		2	3	4	17	3	21	5	8	14	19	1	10	18	6	15	20	9	7	12
	Section II - cont'd																			
4,3,10	3-digit ÷ 2-digit w/remainder									*									*	
4,4,13	Check by multiplying				*					*									*	
	Check by multiplying and add remainder,									*						*	*			
4,0,15	Estimate quotients									A	*		A	*		*	*			
4,0,8	Multiply w/in division problem																*			
4,7,1	Divide to reduce fractions																*			
4,0,0	Put quotient digits in right place																*			
	Teach division sign						*													
	Applications																			
6,0,9´	Situations-measurement model	*		*			*													
	parative model		*				*			*										
	Story problems									*	*		*						*	*
6,9,9´	With money																			*
6,0,12	Compute average									*						*				

O - DMP Teacher
* - Taught
N - Not taught
A - For accelerated students
I - Only introduced

Topics Teachers Recognized in Multiplication

Teachers' comments about multiplication skills, (coded 4 in Factor 1) revealed differences and similarities in the content at different grade levels. Every teacher talked about the need for students to learn basic multiplication facts (code 4, 1, 8). But they differed in their expressed concern over speed and memorization, as well as in the number of basic facts they expected students to master. The school district objectives indicated that third grade students should learn basic facts through the three tables (i.e. multipliers 0 through 3, multiplicands through 9). Two third grade teachers (Teacher 13 & 17) set their expectations at the district standard, while two other third grade teachers set higher requirements. Teacher 2 wanted students to learn facts through 10 x 10 and Teacher 13 set facts to 5 x 9 as the goal. Teachers from other grade levels often did not specify their exact goals for students regarding the memorization of basic facts.

As indicated in Table 6, there were both between and within grade differences in teachers' emphasis of computational skills involving both single-digit times multiple-digit, and multiple-digit times multiple-digit problems (codes 4, 2, 8 and 4, 3, 8). In the third grade group, the teachers using DMP (Teachers 13 & 21) did not specify any multiplication problems beyond the basic facts. Teacher 2, on the other hand, said she teaches carrying within a multiplication problem and talked about problems having 3-digit numerals in the multiplicand. Teachers 3 and 17 also mentioned 3-digit type problems but it was not clear whether or not carrying was taught. Teacher 4 mentioned 2-digit times 1-digit problems but did not go beyond this level in her description of multiplication problems.

Most fourth grade teachers indicated a wider range of topics than third grade teachers. Five of the seven fourth grade teachers expanded the discussion of topics in this area to include two-digit multiplication, but they initiated the discussion of multiplication skills with the basic facts just as third grade teachers had.

At the fifth grade level, Teacher 6 mentioned only two topics, basic facts and problems with 3-digit multipliers. These two topics were described by all fifth grade teachers, except Teacher 20, as being appropriate for their students, but other teachers also mentioned subtopics (e.g. 2-digit multipliers) within the 4, 3, 8 code. Teacher 15 went beyond this to discuss 4-digit multipliers as content taught at the fifth grade level.

The analysis of these two topic areas (codes 4, 2, 8 and 4, 3, 8) called attention to differences among teachers in the level of detail used to distinguish topics. For example, Teacher 18 limited the content under code 4, 3, 8 to problems involving 2-digit multipliers, while the taxonomy includes larger multipliers as well. Other teachers identified additional subtopics within the two cells of the taxonomy. For example, the distinction made by some teachers between multiplication problems where carrying is or is not required, is not represented in the taxonomy.

There was reasonable consistency among the third grade teachers in the way whole number multiplication problems were described. Those who described topics other than the basic facts, specified the number of digits in both the multiplicand and multiplier for each problem type. In the fourth grade group, Teachers 1, 14, 18 and 19 described problems in terms of the multiplier only, i.e. one or two-digit multipliers; while only Teacher 10 made a distinction between problems with or without carrying and specified the number of digits in both factors. Teacher 5 did not describe topics in these two areas of the taxonomy and Teacher 8 only mentioned multiplication by powers of ten.

Analysis of fifth grade teachers' discussions of topics described by taxonomy codes 4, 2, 8 and 4, 3, 8 identified two teachers (Teachers 7 & 20) who gave especially detailed descriptions of the topics they mentioned. Teacher 7 was the only fifth grade teacher who distinguished problems that require carrying from those that do not. The three other teachers characterized multiple-digit multiplication problems by the number of digits in the multiplier, while Teachers 7 and 20 identified the number of digits in both the multiplier and the multiplicand. One fifth grade teacher (Teacher 12) did not mention content in this area of the taxonomy.

The question of whether or not teachers who failed to make content distinctions include all of the problem types in their classroom lessons, can not, of course, be addressed by this study. The focus here is on how teachers identify topics for a lesson and describe learning outcomes. The analysis revealed that teachers' specifications of distinct problem types were reasonably consistent with lessons presented in elementary school textbooks.

One teacher (Teacher 9), however, criticized such compliance with the text authors' conceptions of content.

> Well, generally the textbooks as I say, they package things in, you know, these neat little things...I just throw them in all the time. There isn't any progressing from neat little 2-digit numbers to add up first and one-digit numbers to add, then you add the 2-digit numbers, then you add three. I just throw them all in a big pot and we have fun with them.

The teacher was talking about problems where the two addends had different numbers of digits. She explained, "What I was trying to do was break down a fear of large numbers that I think textbooks foster..." Although these remarks were responses to questions about addition, this theme of objection to segmentation of content was frequently suggested by the teacher's discussion. It is further evidenced by the level of generality used in her description of problem types in both multiplication and division.

Comparing Teacher 9's specification of topics to those described by the taxonomy revealed that she sometimes described topics that included fewer problems. For example, the taxonomy's multiplication of multiple digit numbers was limited to two-digit multipliers by Teacher 9. Yet, she rarely defined more than one topic within a taxonomy category. Her topics were generally characterized by the most difficult problem in the taxonomy cell that she presented in the classroom.

Purpose of Activities and Lessons on Multiplication

Different teachers often mentioned using the same activities in the classroom lessons on multiplication. The analysis revealed, however, that teachers had different views about the content of such activities. At the same time, instances were also identified when several teachers recognized the same content that should be taught to students, and yet, they each described different purposes for students learning that content. Examples of both of these inconsistencies will be reported in the discussion that follows.

The use of counting sequences (code 2,0,8) during the instruction on multiplication provides an example of a topic serving different purposes. Two third grade teachers (Teachers 4 & 13) referred to students having learned to count by 2's, 3's, 5,'s and 10's and using this knowledge during lessons on multiplication. One of the teachers (Teacher 13) stated that counting was only introduced as a substitute until students memorized their multiplication facts. The other teacher (Teacher 4) discussed counting as a way students come to understand multiplication. She commented, "Once they understand it, they can count by 3's seven times--- They've done 3 x 7 and that's the answer".

Every teacher discussed the content of basic facts for multiplication (code 4, 1, 8). Within the various teachers' remarks about this topic three different purposes for having students learn the multiplication facts were expressed. Seven teachers placed an emphasis on speed and talked about using timed tests where students compete against themselves to improve their speed. Such teachers usually explained that not knowing the basic facts interferes with further progress. Teacher 17 expressed this view as follows.

> Now in their daily work, I let them use a multiplication chart. I say, they can't use it on a test, so they are just going to have to learn their multiplication facts. But so their daily work isn't slowed down, I do let them resort to the chart that is in the back of their book.

The strategy of allowing students to consult a table of basic facts was not mentioned by the other seven teachers but their purpose in having students memorize the basic facts was essentially the same, i.e. knowing the basic facts will aid progress on other topics.

Two teachers, however, had a broader hope for the outcome of learning basic facts. Teachers 10 and 18 spoke of students acquiring a "feeling". Teacher 18 suggested that learning the multiplication facts is more than memorization, that the memorization led to understanding.

> I want them to understand it--that they see it, that the--to me the understanding is a visual concept. Because 3 x 4, they can mechanically say, "that's twelve", ", or they can look at a multiplication fact chart...But to really see, and understand that I mean three groups of four--I want them to see it and visually have this feeling about it. So that when I am doing it, I am not just--it is not just a mechanical process of cold memorization that really means nothing. I really want it to be imprinted more than that because then I think they will learn it better and they will know it better.

Teacher 10's discussion of the "feeling" she hoped students might acquire from studying basic facts was not focused on understanding and suggested a third purpose for learning basic facts. She recognized that fluency with multiplication facts would help students estimate answers.

> I just think it (drill & practice) makes you a better math student in anything you do. I mean--it's just like basic drill in athletics. It's--for some kids it's dull--but the outcome is that when you're working on a problem, if you know the relationship of numbers and your facts, you have a much closer feel of whether you're on the right track or not.

The methods teachers' described using to teach students to multiply represented an area where teachers recognized different content. Some textbooks teach students to multiply problems that require carrying (e.g. 52 x 8) using a partial product approach. The problem would be done as follows:

$$\begin{array}{r} 52 \\ \times\ 8 \\ \hline 16 \\ 400 \\ \hline 416 \end{array}$$

Four of the six third grade teachers discussed this technique (code 2, 2, 8). Collectively they suggested two different interpretations of the purpose of using the method. Two teachers (Teachers 4 & 18) characterized the method as an easier algorithm that eliminates the need to carry numbers. Teacher 18 described the method saying,

> First they start out with multiplying. They multiply their ones and they put it down. Then they multiply their tens and put it underneath the ones. And then they add them together. So they are not really actually regrouping there.

Another teacher (Teacher 2) said she did <u>not</u> teach this method, and was even more careful in her explanation of the second partial product, "...then, knowing that the "5" in 52 is really fifty they would then do 50 x 8 and write "400...". Teacher 2 viewed the partial product method as leading toward an understanding of carrying in multiplication and chose not to separate instruction related to understanding carrying in a problem and the actual doing of it. She continued,

> I make sure my children understand, you know, that the "5" is fifty and not five. But I...teach them to carry with it because it gets very bulky...I skip over the way they (textbooks) suggest presenting it and present it my own way and use their pages to practice.

Some of the activities teachers viewed as useful for drill and practice often had content implications the teacher did not mention. In such cases the activity was viewed as a strategy to make drill more interesting for students and the methods taught to students within the activity were not recognized as content. For example, the use of Napier's Bones could be viewed as content to be learned; but both teachers who talked about using Napier's Bones in the classroom (Teachers 15 & 19) explained that it was a "fun" activity used to drill basic facts. As Teacher 19 explained, "I don't show this to the class so they remember it. It's more of an enjoyable thing than it is a --used for multiplication. They get reinforcement of the facts is the main goal". Other practice activities (code 4, 3, 8) for multiplication that were described as having this purpose included: Magic Squares, mentioned by Teacher 8, and special tricks discussed by Teacher 20. In each of these cases the method used in the activity was considered secondary to the drill and practice students would experience in the "fun" context.

Two teachers using the DMP program (Teacher 10 & 21) remarked that students enjoyed drill and practice experiences in that program. Within such activities students solved a set of problems, then were asked to find a numerical pattern in the answers. A set of problems where every product had the same digit in the first and last place (e.g., 242), was one example given. During the discussion the teachers did not describe the activity as teaching students to recognize patterns. Rather, the activity was viewed as especialy good because of its motivational value, and because it helped students quickly spot problems where they may have made an error.

The strategies different teachers described when they talked about teaching students to multiply by 10's, 100's and 1000's (code 4, 3, 8) also implied different content. Although six teachers (Teachers 3, 4, 5, 17, 8 & 19) discussed this topic, only three of them also mentioned that students should understand the use of zero as a factor. When talking about this, Teacher 17 also described the concept of an identity for multiplication, explaining,

> They have to know the concept of multiplying one by any number is one, or anything multiplied by zero is zero. We stress that. And when we get into tens and hundreds, they realize that anything multiplied by a hundred, they add two zeros.

This teacher's discussion illustrated her interest in students learning about both the properties of zero and one as well as how to multiply by powers of ten. Such an approach was focused on students' understanding the process of multiplication by powers of ten. Another teacher's approach (Teacher 3) to this topic was more rule oriented.

> Oh, another thing they learned in third grade is how to multiply by 10's, 100's, and 1000's---adding zeros...see before you can do this well, you really need to know that when you multiply by 100, all you do is add two zeros...And so I try to make them (students) really appreciate what a favor I'm doing them (by showing this to students)...

Teacher 4, like Teacher 17, talked of teaching about zero as a factor. Although her discussion of lessons to teach multiplication by powers of ten reflected some confusion about how such problems are solved, she described such lessons as teaching more than a "trick".

> If you're multiplying 700 x 800, instead of writing all that out, (students learn to) understand if you put 7 x 8 you'd get 56 and all the zeros would work out and there's kind of a process that I can't really explain right now, but it's very clear in their books of how you do it. They take 700 and they put "7" in the hundreds column eight times and then you add it all up...

A fourth teacher's description (Teacher 8) of lessons on this topic associated a different learning outcome with the instruction. For Teacher 8, the purpose of the lesson on multiplication by powers of ten was beyond that problem type itself. The teacher viewed it as a skill that would allow students to find estimated products.

> They get into multiplication computation again relating to numbers multiplied by 100's, 10's and 1000's. Learning to estimate within multiplication..If it's 98 times three, and you don't need an exact answer--how can you quickly get that answer.

Five teachers' discussions about instruction related to the commutative property for multiplication also reflected recognition of different purposes. Three teachers talked about showing students that the property applies to some operations but not others. However, Teacher 21 said she taught children the commutative property of multiplication because a "hard problem" like 9 x 3 =? can be made shorter, i.e. 3 x 9 =? (the students were using the repeated addition model). Another teacher (Teacher 9) explained that she wanted students to learn to "appreciate" mathematics. This goal was reflected in her discussion of instruction on commutativity.

> ...the fact that it's (multiplication) commutative which I think is beautiful...any kind of a concept there that I think is really beautiful I always start out by telling them to think that mathematics is beautiful...I think...the whole concept of mathematics even at the lower grades is really like a symphony. It is just beautiful the way it interrelates.

Topics Teachers Recognized in Division

Table 6 shows that, at every grade level, teachers discussed topics related to understanding the concept of division (code 1 or 2 in Factor 1). However, differences among teachers at different grade levels were identified in their discussion of division skills (coded 4 in Factor 1). Only two third grade teachers (Teachers 3 & 4) talked about students learning the basic division facts (code 4, 1, 9); and the only algorithm third grade teachers discussed was the use of a method of repeated subtraction. Problems involving multiple-digit divisors (code 4,3,9) were mentioned as introductory or enrichment topics by fourth grade teachers and were part of the content all fifth grade teachers said they taught.

Such differences indicated that teachers did not expect students to master division of whole numbers before the fifth grade level.

Teachers sometimes expressed different expectations for student learning when discussing instruction on the division algorithm. Five of the nine teachers who mentioned teaching the long division algorithm, reflected a traditional approach. Despite any concerns they may have expressed related to understanding the concept of division, or the process of dividing, the final stage of the instruction they described, evidenced an attempt to "program" students to compute. Three teachers (Teachers 19, 15, & 7) described the "goes into" method as the ultimate performance they expected students to learn. In this, a problem such as 42 divided by 3 would be completed by finding how many times "3 goes into 4". The recognition of "4" as representing 40 was not a concern for these teachers in the development of the students' computational performance. Teachers 5 and 18 further explained how they define the pattern of rules for students by specifying a sequence of steps. For Teacher 5, the pattern was "Divide-Multiply-Subtract," while Teacher 18 used the memory device "Dumb Men Sink Boats" to remind students of a pattern, "Divide-Multiply-Subtract-Bring down."

Only four of seven teachers at the fourth grade level (Teachers 4, 19, 10, & 18) and two of the six from the fifth grade (Teachers 15 & 20), specifically discussed the need for students to learn how to estimate quotients while learning the algorithm. Teacher 20 also discussed topics dealing with how to round numbers as part of instruction related to division. Teacher 1, who cited a topic where students learned to do repeated subtraction using groups equal to multiples of the divisor (code 2, 2, 9'), suggested this would help them understand how to estimate quotients later. A similar topic (code 4, 0, 9') was mentioned by Teachers 4, 3, and 12 in their discussions of skills students should learn. The first entry in section 4, 3, 9' of Table 6 also could be construed as content related to the estimation of quotients. However, teachers' descriptions of this topic revealed a perception of division by powers of ten as "easy" cases of multiple-digit division problems.

Although teachers may have failed to stress estimation and content related to why the division algorithm works, they often recognized skills and performances students must learn that are not generally specified in written curricula. Teacher 20 explained a difficulty encountered when students learn the long division algorithm.

> I think one of the hardest things that people don't think about when you're dividing, traditionally, we have previously just done multiplying. Say it's "6" times "16" and the answer is "96". They go up and they put in their "six" (in the product) and carry their "3" and then they come down with a "96". All of a sudden you have this same problem and what you've got, for instance, is say a 16 out here (the divisor) and you've chosen "6" up there as a possible quotient answer there. They suddenly have to make themselves see that it's the same thing as they just previously did...I find this to be one of the most difficult things for kids to perceive. They previously can do this kind (the multiplication problem) of thing over and over without a mistake. Put it in a division problem--it's suddenly a new problem--a difficult one.

This explanation, that the notation or form used in presenting a problem implies something to be learned, guided the teacher's instruction. He told the interviewer..."readiness in math must be just the same thing as readiness in reading. It's symbology; and it's either there or it isn't."

Other teachers identified similar concerns for the content related to mathematical symbols by describing recognition of numerals (Teacher 15) or learning different symbols (Teacher 9) as common threads in the content of mathematics. Teacher 21 was also sensitive to the importance of learning notation. After using the DMP program with her third grade students, she commented that they had learned a great deal about division without having seen a division problem. Before administering the standardized achievement test at the end of the year, she "taught" the division symbol, telling students the division frame was a special way to represent a partitioning problem. During the interview she described her satisfaction in seeing students complete these problems on the test using repeated subtraction. A second DMP teacher (Teacher 13) had not anticipated this possibility and did not "teach" the symbol to her class. She expressed surprise that some of her students had been able to make the connection themselves and complete the problems.

To identify teachers who specified topics with a narrow range of problem types, i.e., at a finer level of distinction than the taxonomy, the teachers with the largest number of entries in this part of Table 6 were considered. At the third grade level, Teacher 4 had eight entries, but each of them was classified with a different code. Teachers at the fourth grade level, for the most part, also mentioned topics that were coded as separate topics from the perspective of the taxonomy.

As the fifth grade level, Teacher 7 was found to describe narrowly defined topics (as she had done in multiplication). The sequence of division skills she described included: (1) one-digit divisor, small dividend, no remainder; (2) one-digit divisor, no remainder, no subtraction within the problem (e.g., 22 / 2); (3) problems with no remainder but subtraction within the problem (e.g., 56 / 4); (4) problems with remainders and problems with larger divisors. Other teachers did not identify topics like numbers 3 and 4 in this teacher's list. Such problems were included as examples in more generally described topics given by other teachers.

Purpose of Activites in Division

As in the previously considered areas of content, teachers sometimes disagreed about the content or purpose of various activities they described. However in this area, the analysis also identified differences in teachers' recognition of topics related to the concept of division.

Textbooks on the methods of teaching elementary school mathematics (e.g. Copeland, 1972) frequently suggest that students should be taught two interpretations of the concept of division. First, division can be thought of as partitioning a set of objects into a specified number of subsets (the parative model); the quotient tells how many elements are in each set. The second interpretation refers to finding how many subsets can be created having a given number of elements (measurement model). One teacher (Teacher 1) described this duality within the concept and explained the need to illustrate two models of the concept of division, she explained,

> If you had 32 objects, how many different groups of eight could I make (measurement model). Of if I put them in eight groups, how many would be in each group (the parative model). So it would be grouping things into, <u>or</u> putting so many items into groups.

Teachers' descriptions of activities used to teach the division concept were analyzed to determine if they mentioned using both the parative and measurement models in the classroom. Since this distinction is not represented in the taxonomy, teachers' comments about activities with concrete objects based on either model are recorded in the section of Table 6 coded 1, 0, 9'. The situations teachers described to children in verbal problem statements to illustrate either model are coded 6, 0, 9'. With the exception of Teacher 1, teachers did not refer to the two models as distinct interpretations of the division concept. The analysis merely considered if the activities discussed by a teacher were biased toward a particular interpretation of the concept.

Only Teachers 8, 5, 9 and 1 described individual activities using concrete objects that could clearly be classified as representing one or the other model. Other teachers' descriptions of activities were classified in other lines of 1, 0, 9' for they were not complete enough to determine whether the activity was one of finding an unknown number of groups, or finding the number of objects that would be contained in a given number of groups. Only two teachers (Teachers 8 & 1) described an activity from both of the models.

The description of teachers' references to situations they might describe to students to illustrate the concept of division (code 6, 0, 9') revealed that only two of the five teachers who described this topic (Teachers 4 & 12) mentioned examples that illustrated both models. Teachers who have learned about the two models of division may view them as strategies for instruction, as does the taxonomy, and not recognize a potential content implication. Thus, the multiple interpretations of such division concepts may not be the focus of instruction.

The potential significance of ignoring the multiple interpretations of the division concept was suggested by a fifth grade teacher's report (Teacher 12) of students' failure to recognize the multiple interpretations of another operation, subtraction. Teacher 12 remarked, "The concept of subtracting is taking away something to a kid. If you're going to subtract it, you're going to take it away." Teacher 12 explained that when students were asked to find a difference, they did not realize that subtraction should also be used when making comparisons. Despite her recognition of the various functions of the subtraction operation, however, Teacher 12 did not discuss instruction in the area of division in sufficient detail to determine whether or not she recognized both aspects of the division concept.

Relatedness of Multiplication and Division as Content

The analysis of content areas found that 10 of the 19 teachers either mentioned the relatedness of operations as a common thread in mathematics content, or gave categories of mathematics that combined two operations. This observation raised two questions. First, do teachers giving such responses recognize the relationships between operations as content that should be taught to students; and second, are these teachers' conceptions of content in any way different from teachers who did not make such statements in their discussion of categories of mathematics?

In order to address these questions the analysis focused on certain entries in Table 6. Considering the first entry under 2, 0, 8 (multiplication as repeated addition) and the line coded 2, 0, 9' (division as repeated subtraction) revealed that all third and fourth grade teachers mentioned one or both of these topics. Because these models are commonly used to teach the concepts of multiplication and division, however, the teachers' discussions of these topics might not indicate deliberate interest in teaching the relatedness of operations.

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The code 2, 4, 13 (which is found in the part of the table that records teachers' comments about multiplication and appears again in the section reporting teachers' discussions of division), was also considered. Although six teachers (Teachers 2, 5, 8, 14, 1, & 10) mentioned teaching students that division is related to multiplication during their lessons on division, only three teachers (Teachers 21, 5, & 1) discussed this topic in the context of their remarks about teaching multiplication. Two of these teachers (Teachers 21 & 1) also discussed the concept of division as a missing multiplier (code 2, 5, 9') as did Teachers 4 and 15.

Two of the five fifth grade teachers (Teachers 15 &19) did not mention teaching the basic division facts. However, both teachers talked about lessons on the inverse relationship of multiplication and division. Teacher 15 explained,

> They (the text) don't teach division facts exactly although they spend time in the introduction and reversing the multiplication facts. Then division... they use it more in terms of large numbers being divided up and mostly with things--recognizable things--objects at that level and then just straight use of problems.

Of the four teachers who gave the relatedness of operations as a common thread (Teachers 5, 19, 1, & 15) and the five who specified a category of mathematics as "Multiplication and Division" (Teachers 2, 4, 5, 8, & 1), only Teachers 5, 1, 15 and 21 talked about more of these topics than other teachers. Two of the teachers (Teachers 21 & 1), who seemed to be especially interested in teaching the relationship between operations, were using the DMP program. DMP does not refer to multiplication and division separately, the content is presented in a Unit entitled, "Grouping and Partitioning". Thus, examination of DMP teachers' discussions illustrated their interpretation of the program's attempt to teach the relatedness of operations.

Throughout the interview Teacher 21 spoke of multiplication and division together. There was never a period when she went into elaborate remarks about one operation. She even used the title "Grouping and Partitioning" regularly, often translating it for the interviewer.

> ...grouping and partitioning which, in the third grade, is important because this is the introduction to multiplication and division.

Teacher 21 also made frequent defensive remarks about the program's failure to keep pace with traditional programs in teaching computational skills. When talking about the Grouping and Partitioning unit she explained, "And I say it's important because I think parents are very used to having their children learn to multiply in the third grade. If you didn't do it then, they would be very nervous."

The other DMP teacher (Teacher 1) who emphasized relationships in her remarks, applauded the concept development that occurred in the Grouping and Partitioning unit saying, "They've (DMP) gone about things in a whole different fashion, and I think that the kids have really gained just a better understanding of it." But when talking about division she commented,

> You know, I still--when I think about long division--you should never teach long division that way...I hope that they're (the students) going to be able to follow the process that they learned.

A third DMP teacher (Teacher 10) was also cautious in her endorsement of the new program. For her, the language, "Grouping and

Partitioning", was always her second referent when it was used. She most frequently discussed the operations using the traditional titles, and her discussions were easily separated by their focus on one or the other operation. Teacher 10 endorsed the DMP curriculum because of its' emphasis on story problems, yet she was skeptical about the instruction on computation presented in the program. She expressed concern over the limited development in computation saying,

> In fourth grade, we didn't have very much (division) really at all. We just simply--if you know how to do it in the inverse, or they would simply be able to write it two ways--as a multiplication sentence or a division...They had very little tacked in there. That's why I don't know, cause when we did traditional fourth grade before, the kids had to do a lot of algorithms.

She went on to explain that she was reserving judgement about the program until this year's standardized test scores were returned. Then she would know if the scores on the applications subtest had improved and would learn if the new program resulted in poor student performance on the computation subtest.

DMP Teachers 13 and 18 had failed to express topics that indicated an interest in teaching the relationship between multiplication and division. Neither teacher's responses to the category questions suggested a focus on relationships as content in mathematics. In fact, both Teacher 13 and 18 evidenced a rather traditional view of content in the areas of multiplication and division.

Importance of Multiplication and Division Content

Some of the teachers' comments that have already been reported suggest the importance placed on the content of multiplication and division in general, and also various topics within that content area. They also suggested that teachers most often viewed the areas of multiplication and division separately with only a few who treated the relatedness of these operations as the approach to develop an understanding of both operations.

The analysis of teachers' statements that implied priorities within this instructional area, addressed questions related to the expectations for student learning and emphasis placed on topics. Variation was noted among teachers at each grade level and across the three grades.

Certain paradoxes were evidenced in this comparison of teachers' expectations and priorities. Teacher 17, who was cited as mentioning that the use of concrete objects seemed "gimmicky" and who said she usually didn't have time for such activities, did express a priority for the development of understanding. "I mean, this is what they see multiplication as at the beginning. It's a series of additions and this is the only way they can multiply is to see it." When considering division, however, her view focused primarily on the content area as a skill, where multiplication is the prerequisite.

> They do get into some division but...that's not as important in third grade because they will--they know their multiplication facts in third grade, learn their basic facts. But I tell them they can't do division until they've learned to multiply.

Later in the interview she remarked, "...if they get multiplication we feel they've accomplished a lot in 3rd grade. And I don't worry too much about it. Division and fractions, I feel they can begin next year."

Teacher 4, on the other hand is a third grade teacher who was less intent on the mastery of certain areas at the cost of experience with other content.

> I feel very strongly that kids ought to be exposed to as many different kinds of experiences in math as they can, not for mastery, but simply for exposure

> for developmental purposes--so that math will have continuity for them if they can build upon experience, rather than have them wait till say sixth grade to encounter fractions for the first time. That they have to be exposed to these periodically and increasingly over time.

On the other hand, Teacher 21, who discussed the relatedness of multiplication and division and seemed to think of "Grouping and Partitioning" as a unified content area, had different expectations. In her discussion of teachinq the division sign before the standardized test she explained, "I mean, they don't really know how to divide but they know...they knew how to solve it. So, to me, the understanding of the solution was more important than having the formal algorithm of how do you divide".

Consideration of the fourth grade teachers' expectations and priorities reflected the same variation of thought among teachers. Teacher 5, was much like Teacher 17. She seemed to focus on multiplication as a prerequisite to divison saying, "I think they need to know their multiplication first." At the same time, Teacher 5 mentioned a narrower range of problem types for multiplication than other teachers.

Teacher 1 was another teacher using DMP who did not associate concept development with skill acquisition. She admitted the influence of the new curriculum on her priorities saying,

> I have to say my thinking has changed a great deal with this program this year. Because in the past, up to this year, I have spent most of my time doing just computational math. We had really--I had, I guess gone the full gamut of having kids introduced to theory that I never felt carried over to what they were doing in computation...So I think in the last couple, maybe three years, I have really run a pretty computational kind of thing...Well, this proqram has really gotten me to rethink the whole thing.

The discussions of fifth grade teachers reflected expectations and priorities that seemed to be influenced more by tradition than by beliefs about the subject matter and learning. Teacher 12 explained,

> I think I am not satisfied to leave long division at all until they can do it well and accurately. I want them to be able to--so that if they see those big numbers sometime, and somebody says, divide this for me, they can to that. And I won't leave it until they can.

Teacher 15 confessed the influence of tradition on her beliefs about what students should learn.

> I think I've been programmed so strongly through my education and my experience--through my own elementary school experience that is when I was in elementary school, and through my teaching experience that that's (division) what fifth graders do. It's like fourth graders learned how to multiply. Fifth graders learn how to divide. Sixth graders learn how to do fractions. And that's unfortunate that we've gotten that much of a mindset, but I think that's really true.

Evidence of "mindset" was presented earlier in the interview when the teacher explained problems done by students in her class. "Then we'd go through multiplication probably three by four-place, and I'm not--I think there's a case where I teach that mainly because I learned that when I was in school. I'm not sure of the value of that, but there seems to be one..."

Summary of Findings

Q_1: What content areas did teachers recognize as impor tant components of the elementary school mathematics curriculum?

The category questions in the interview schedule asked teachers to think of all the mathematics content they teach and divide it up into a few categories. Then teachers were asked to mention common threads they saw runninq through the content. The analysis of teachers' responses to these questions identified the content areas that came to teachers' minds when they thought of the mathematics curriculum holistically.

Agreement between the researcher's and teachers' methods of describing content areas was demonstrated by a high level of consistency between teachers' characterizations of content and the taxonomy. Teachers' categories were, for the most part, descriptive of the general intent, the nature of material, and the operations involved in the content. However, some of the content areas specified within these factors of the taxonomy were not mentioned by any teachers, while other content areas were mentioned by only a few teachers. The categories teachers mentioned suggested content areas teachers viewed as important.

Teachers' categories specifying the general intent of content usually referred to the content areas of skill development and applications. However, the referrent they used in mentioning the area of applications differed, i.e., problem solving, or story problems, or applications. Only three teachers mentioned a category describing conceptual understanding as content, but eight other teachers described "understanding" as a common thread that runs through most of the content of elementary school mathematics.

Categories mentioned by at least four teachers to describe the nature of material in content were geometry, measurement, and fractions. However, no fifth grade teacher and only two third and two fourth grade teachers gave "geometry" as a cateoory. Of the nine teachers who described "measurement" as a category of mathematics, only one was discussing the content of fifth grade mathematics.

Every teacher whose categories did not describe a unified area of skill development, (e.g., using labels "computation" or "arithmetic") mentioned the arithmetic operations, i.e., addition, subtraction, multiplication, division, in their sets of categories. Five teachers mentioned categories combining two operations (e.g. multiplication and division), while seven teachers described each operation as a separate category. The only other levels of the operations factor of the taxonomy that were mentioned by teachers as categories of mathematics were "place value," mentioned by three teachers, and "estimation," given by one teacher.

Finally, some categories that are not recognized by the taxonomy were mentioned by a few teachers. One teacher focused on affective outcomes specifying "appreciation of math" as a category; one teacher gave problem context as a category of content, and "time" and "money" were mentioned as categories by two teachers.

Q: Within the content areas of mathematical understanding, problem solving, multiplication, and division, what topics did teachers recognize?

Teachers' discussions of the topics they associated with mathematical understanding suggested three different interpretations to understanding in mathematics. The content described by some teachers focused on concept learning, and one teacher expressed specific concern that students learn to verbalize concepts. When other teachers described lessons to develop understanding they indicated a focus on students learning to use or apply concepts. These teachers described methods to assess whether or not students "understand" mathematics that were based on activities requiring the use of concepts. Yet a third qroup of teachers accepted students' facility with computational algorithms as an index of understanding. Individual teacher's interpretations of understandinq, however, sometimes varied from content area to content area.

The analysis of teachers' statements about problem solving evidenced differences between topic recognized by teachers using the DMP program and topics described by teachers who used other instructional materials in their classrooms. The DMP teachers described learning outcomes related to problem solving analysis in a more prescriptive and generalized format than other teachers. In other words, the strategies DMP teachers said students should learn specified how students would conduct the problem analysis, and also were strategies that could be used on problems other than those requiring computations for solution. There was little evidence that non DMP teachers recognized topics other than those dealing with problems solved by computation. Only two non DMP teachers mentioned other types of problems. One teacher mentioned logic problems. The other teacher discussed problems using Cuisenaire rods, but this teacher's disussion did not detail what would be involved in solving such problems.

In the area of multiplication, the skill topics teachers specified were defined at a finer level of detail than the taxonomy's topics. Topics specified by teachers were consistent with problems that appear in textbook lessons. For example, teachers used the number of digits in the multiplier (sometimes the multiplicand also) and the need to "carry" in a multiplication problem to distinguish topics. One teacher, however, was critical of such segmentation of content. The analysis also identified differences in topics mentioned by teachers at different grade levels. Teachers in successive grades expanded the content to be learned by discussing problems involving larger numbers, i.e., numbers having more digits. Further, the distinction between problems with or without carrying was made by four of six third grade teachers, three of seven teachers of fourth grade, and only one of the six fifth grade teachers.

Grade level differences were further evidenced in teachers' comments about division. These differences revealed that teachers did not expect mastery of division before fifth grade. Although teachers at every grade level discussed teaching students to divide, third grade teachers only mentioned the method of repeated subtraction and fourth grade teachers described problems involving multiple-digit divisors as enrichment topics at grade four. Three of the six fifth grade teachers and two of seven teachers from grade four communicated a rather traditional view of the content, describing mechanical algorithms as the ultimate expectation for student performance. Only six teachers (four fourth-grade and two fifth) discussed estimation of quotients in their description of division topics. Three fifth grade teachers and one at third grade teacher described topics not recognized by the taxonomy, explaining that notation and symbols used in division must be taught.

The analysis also considered whether or not teachers viewed the relationship between multiplication and division as content. The nine teachers whose responses to the category questions suggested recognition of topics dealing with the relatedness of operations, failed to discuss topics that reflected a unique recognition of relationships as content. Only four of these nine teachers described more topics dealing with relationships than other teachers and, although the DMP focuses on the multiplication-division relationship, only two DMP teachers expressed particular interest in teaching the relatedness of operations.

Q_3: What mathematical topics did teachers view as important for students to learn?

Teachers' views about the importance of mathematical understanding often varied. Some teachers thought it was important for students to understand an arithmetic operation before teaching the skill, while others believed that if students first learned to compute accurately they would then be able to have a richer experience in exploring the underlying concepts of that operation. On the other hand, two teachers admitted that they only taught lessons on understanding because such lessons were in the textbook. For those two teachers, students' facility with the algorithm was the dominating priority. Both teachers felt that, if students could find answers to problems, the alternative algorithms (leading to the understanding of the process), were not necessary experiences for students. Finally, a teacher's perception of the importance of understanding in mathematics sometimes varied from content area to content area.

There was greater agreement among teachers about important topics in problem solving. Every teacher's discussion about problem solving, specified the same limited subset of topics as the focus of instruction. Topics related to the use of arithmetic operations in a problem solving context were stressed in all teachers' remarks. Although seven teachers made particular reference to the need to present computational problem solving experiences from a variety of contexts, the analysis was not conclusive about whether or not teachers saw context as a descriptive feature of topics. However, three of the seven teachers confessed frustration with the lack of instructional materials that offer problem solving experiences in a variety of contexts. Despite DMP teachers' description of a broader range of problem solving that dealt with computation, the DMP teachers' remarks identified problem solving that dealt with computation, as the most important component of the problem solving content area.

Every teacher discussed multiplication and division as an important component of the curriculum in the classroom. As was mentioned earlier, in the discussion of topics teachers recognized, grade level differences were evidenced. Third grade teachers did not describe instruction on division as extensively as other teachers; and teachers in successive grades described different problem types as the focus of instruction.

Although every teacher discussed multiplication and division as important areas in the curriculum, differences in the content teachers might teach were suggested by teachers' priorities for student mastery. Two teachers expressed strong convictions about a hierarchy of prerequisite skills. These teachers said they would not progress to division, or to more difficult topics in mulitiplication, until certain topics had been mastered by students. One teacher, on the other hand, was particularly committed to exposing students to the whole spectrum of topics in the content area. The scope of this teacher's instructional program was not prescribed by assessments of student mastery of individual topics. The dominating priority expressed by fifth grade teachers was students' facility with the long division algorithm. Three of the six fifth grade teachers described mechanical steps for dividing ignoring the modern orientation, that students should understand the algorithms used for computation.

Q_4: What mathematics topics did teachers associate with the instructional strategies they described?

When discussing understanding in mathematics teachers' discussions of the purpose of instructional activities suggested varying perceptions of the content to be learned from the activity. For some teachers the ladder method of division was an alternative algorithm students might learn to use when having difficulty with the traditional algorithm. For other teachers, the ladder method was a strategy to teach content related to understanding the division process. Finally, in two instances, teachers explained that they taught the ladder method in an effort to comply with the design of the instructional materials used in schools. These two teachers' discussions did not reveal a recognition of any particular content that might be taught when the ladder method was presented in a classroom lesson.

The analysis of teachers' statements about the content of problem solving lessons focused on identifying teachers' justifications for the types of problem solving experiences they described. Some teachers saw problem solving in an applied sense, as the extension of the study of arithmetic operations. One of these teachers explained that story problems would motivate students to learn the arithmetic operations. Six teachers, on the other hand, saw the lessons dealing with story problems as the opportunity to develop problem solving strategies, and mentioned topics related to problem solving skills. Only four of these six teachers, all of whom were using DMP, discussed activities that incorporated problems that did not require the student to do a calculation. But only one of these four teachers described more generalized problem solving strategies as learning outcomes for such activities.

The activities teachers mentioned in the discussion of instruction on multiplication were often associated with different content by different teachers. Considering the two teachers who described the use of counting sequences, one teacher viewed the activity as helping students understand the multiplication concept, while the other teacher saw counting sequences as an aid to learning the basic facts. Although every teacher expected students to learn the basic multiplication facts, only seven teachers placed an emphasis on speed and accuracy, explaining that mastery of the basic facts was essential to future work in mathematics. Two teachers, on the other hand, felt students might acquire a "feeling" for mathematics when learning basic facts that would lead to better understanding of mathematics or be useful in the estimation of answers.

Teachers' descriptions of other activities in multiplication instruction demonstrated their failure to acknowledge the implied content of the activity. Two of the three teachers who described the partial product method for single and multiple-digit multiplication saw the activity as eliminating the need to carry in a multiplication problem. Although, the third teacher recognized the method as a strategy to help students understand carrying in multiplication, this third teacher explained that she did not use the method because it was cumbersome for students. Various drill and practice activities teachers discussed, i.e., Napier's Bones and searching for patterns in answers, were always described as motivational strategies ignoring the method of Napier's Bones or the search for patterns as content students should learn. Finally, of the six teachers who told the interviewer they taught the topic, multiplication by powers of ten, only three teachers included the of zero and one as factors. The other teachers described a rule oriented algorithm in discussing the topic.

The strategies teachers mentioned using for instruction on the division concept generally illustrated failure to recognize two interpretations of division, i.e. parative model and measurement model. Only one teacher explicitly described each interpretation as a distinct topic. Further, both interpretations were illustrated by only two of the four teachers who described concrete activities in sufficient detail to assess their interpretation of the concept, and by only two of the five teachers who gave story problems illustrating an interpretation of the concept. Although this distinction is not recognized by the taxonomy, it is one frequently made in mathematics education. And, in a discussion about subtraction, one fifth grade teacher mentioned students' failure to recognize multiple interpretations of subtraction, as a weakness in students' background.

Q_5: What variation in the content of classroom instruction was implied by differences in teachers' conceptions of mathematics content?

For the most part, this research question will be addressed in the chapter that follows. At this point, however, the question offers a structure to summarize all that has been reported. Consideration of the five dimensions that were specified in Chapter 1 as defining a teacher's conception of mathematics revealed:

1) In general, those characteristics of mathematics that teachers used to describe content areas in the curriculum were consistent with the dimensions and levels of the taxonomy. However, their failure to mention certain levels within each dimension demonstrated that the arithmetic operations were seen by teachers as the most important content areas.

2) Different teachers were seen to hold different beliefs about what is to be learned within a content area. This was illustrated by differences: a) in teachers' interpretation of understanding in mathematics, b) in the learning outcomes teachers described for problem solving, c) in the level of detail teachers used to specify topics for multiplication and division, and d) in teachers' attention to the relatedness of operations as instructional content.

3) Within each content area, some teachers were found to have unique priorities. However, in the areas of multiplication and division, fifth grade teachers expressed a dominating concern about students developing facility with the division algorithm.

4) Teachers' descriptions of instructional strategies often suggested differences in the content they associated with classroom activities. This was evidenced by teachers' explanations of the purpose of various classroom lessons and activities in each content area.

The analysis of teachers' descriptions of what they hoped students would learn in elementary school mathematics evidences similarities among teachers conceptions of mathematics content. However, the differences that were identified among teachers suggested that the teachers probably differed also in the content of their instruction.

Chapter 5

Summary, Conclusion, and Implications

The purposes of this research were to describe: 1) the content areas elementary school teachers used to describe mathematics curriculum; 2) the topics teachers recognized ~~as~~ mathematical content; 3) the priorities teachers held for student learning in mathematics; and 4) the mathematical topics teachers associated with specific instructional strategies. These dimensions were collectively called teachers' conceptions of mathematics content. The study examined teachers' statements about mathematics instruction to consider the extent to which elementary school teachers' conceptions of mathematics content were alike, and whether or not teachers' conceptions of content were consistent with current trends in mathematics education.

Teachers were selected for the study through use of a questionnaire that asked faculty and administrators in a single school district to nominate teachers who might have strong, yet different, views about mathematics instruction. From the list of nominees, teachers were chosen to represent a distribution of nominating characteristics, building, and grade level assignments, i.e., grades 3, 4 and 5. The data for the study were verbatim transcripts of an interview with each teacher that lasted from two to four hours.

The data analysis made use of a taxonomy of elementary school mathematics to describe the mathematics content mentioned by each teacher. The taxonomy served as a standard to identify similarities and differences in the teachers' discussions about mathematics curricula. However, content teachers mentioned that was not consistent with topics in the taxonomy was also considered in the analysis.

Findings and Conclusions

Similarities and differences in teachers' statements about mathematics instruction led to three general conclusions. Each of the conclusions is reported below with a summary of the findings that support the conclusion.

1. Elementary school teachers are primarily concerned with presenting instrucion they believe will result in students learning the arithmetic operations of addition, subtraction, multiplication, and division.

   a) When teachers were asked to subdivide all the mathematics they teach into categories, every teacher's response identified the arithmetic operations as a primary component of the mathematics curriculum.

   b) Other content areas (e.g., geometry, measurement, estimation, place value) were only mentioned as categories of content by a few teachers.

   c) Regardless of the text used or grade level taught, teachers expressed a strong concern that students learn basic arithmetic facts and algorithms.

   d) Although the content descriptions teachers used were consistent with the dimension of the taxonomy of content used in this research, most teachers did not discuss certain topics the taxonomy specifies that are outside the area of computational skills.

e) Teachers specified topics related to arithmetic calculations at a finer level of detail and therefore, identified more distinct topics in that area than the taxonomy describes.

f) Teachers' descriptions of topics in the problem solving content area focused on the solution of story problems that require a calculation for solution.

2. Within content areas, teachers' perceptions of topics that should be taught to students vary.

a) Although all teachers identified the arithmetic operations as important content to be taught, some teachers expressed concern that students "understand" the concepts and algorithms associated with each operation while others did not.

b) Instances were cited where some teachers said they did not require students to learn algorithms presented in textbooks, and where some teachers described modifications of the prescribed scope and sequence of school district objectives.

c) Within grade level variation was found as teachers described topics taught in lessons on problem solving, understanding mathematics, multiplication and division.

3. Teachers' varying ~~interpretations~~ of the content associated with instructional strategies and the goals of written curricula may result in variation in the content of instruction across classrooms.

a) Teachers' descriptions of the content taught when particular instructional activities are used varied.

b) Teachers using the same instructional materials in their classrooms, often discussed different topics as the content they hoped students would learn.

c) Teachers' discussion of lessons to teach problem solving and mathematical understanding suggested that some teachers thought they would be teaching topics in those areas when curriculum writers and experts in mathematics education might not agree that the topics were being taught.

Implications and Recommendations

In Chapter 1, teacher's conception of mathematics content was hypothesized as a variable that might affect the content of mathematics instruction in elementary schools. Based on research that revealed that teachers are autonomous in matters related to classroom instruction, two conjectures were suggested. First, widespread implementation of new curricula seems to require some level of consistency between the goals of new programs and the topics teachers generally accept as content that should be taught to students. Second, dramatic differences in teachers' selections of topics for instruction could result in random modifications of written programs that affect students across classrooms in various ways.

Although this study did not assess the effect of teachers' conceptions on content in classroom situations, the investigation identified considerable variation in teachers' descriptions of what should be taught in elementary school mathematics lessons. Thus the study provides insight into the particular aspects of the mathematics curriculum that may be most difficult to influence on any widespread basis. Those insights can be described by an examination of recent recommendations of the National Council of Teachers of Mathematics (NCTM, 1980).

In April, 1980, the NCTM presented An Agenda for Action: Recommendations for School Mathematics in the 1980's. Consideration of the NCTM recommendations regarding problem solving and basic skills, in particular, highlights the discrepancy between what teachers, as a group, see as important and what is advocated by NCTM. The first recommendation in the Agenda for Action states:

> Problem solving must be the focus of school mathematics in the 1980's. The mathematics curriculum should be organized around problem solving.
>
> - The current organization of the curriculum emphasizes component computational skills apart from their application. These skills are necessary tools but should not determine the scope and sequence of the curriculum...
>
> - Mathematics programs of the 1980's must be designed to equip students with the mathematics methods that support the full range of problem solving...
>
> (NCTM, 1980, p.2)

The focus and reorganization of curriculum proposed by NCTM represents a rejection of the goals and priorities expressed by every teacher interviewed in this study. Not only did all teachers view elementary school mathematics as primarily the study of arithmetic operations, many teachers also judged facility with computational algorithms as the most important outcome of mathematics instruction. Thus, from the perspective of these teachers, the content of mathematics is organized around the arithmetic operations.

The analysis also identified aspects of teachers' conceptions that varied from teacher to teacher. The varying viewpoints have further implications for the NCTM's agenda. Teachers who interpreted understanding in mathematics as being able to <u>use</u> concepts, and those who commented about the need for more materials to teach applications, may have views that are somewhat consistent with the NCTM recommendation. However, the priorities specified by those teachers also were dominated by a concern that arithmetic skills must be taught. For example, one teacher, who described the area of applications as the most important area of mathematics content, told the interviewer she refused to use DMP because the program doesn't emphasize computation. Teachers who view mathematical applications as topics embedded in content areas defined by arithmetic operations, are apt to reject or modify a curriculum organized by areas of problem solving.

The DMP curriculum is in many ways consistent with the NCTM recommendations. Problem solving experiences are a major component of the program and the curriculum is not organized around the separate arithmetic operations. The interviews with teachers who used the DMP program demonstrate the modifications that occur as teachers use a curriculum that does not focus on computation. Only one of the five DMP teachers mentioned non-computational problem solving activities as content and described unique learning outcomes that are incorporated in the DMP program. The other teachers' discussions of activities and learning outcomes did not reflect an interest in problem solving topics other than those dealing with the solution of problems using computation. Further, every DMP teacher described activities they used to supplement DMP so that children would learn to compute.

The DMP teachers' remarks also evidenced a rejection of content areas as specified by the DMP curriculum. Only one of the five teachers described the unified content area of multiplication and division that is implied by the DMP "Grouping and Partitioning" unit. For that teacher, the relationship between the multiplication and division was content to be taught. Further, only that teacher seemed to realize that students might learn the concepts and master the skills of both multiplication and division through the unified lessons. The one other DMP teacher who spoke of the relatedness of operations as content, discussed the need to teach the algorithms separately.

The various teachers' interpretations of the DMP program demonstrated that a curriculum package alone does not result in major revision of teachers' priorities. Although each DMP teacher explained that the curriculum had been selected because of its' focus on problem solving, use of the program did not circumvent the influence of teachers' conceptions of mathematics content.

Teachers' views of the importance of the arithmetic operations is challenged also by the second NCTM recommendation:

> The concept of basic skills in mathematics must encompass more than computational facility.
> (NCTM, 1980, p.6)

The "Position Paper on Basic Skills", written by the National Council of Supervisors of Mathematics (NCTM, 1976), is described by NCTM as identifying the nucleus for a new definition of basic skills. However, the study of teachers' conceptions of content illustrates the limited influence of that earlier document.

The NCTM paper defined basic skills to include estimation and approximation, geometry, measurement, and the broader view of problem solving. Yet the teachers interviewed did not describe estimation, or geometry, or measurement as a basic component of the instructional program in their classrooms. Further, the broader view of problem solving was only evidenced in one teacher's discussion of that content area.

The second NCTM recommendation further suggests a decreased emphasis of classroom activities that were described by teachers as a major part of the classroom curricula. NCTM advocates a deemphasis of:

> -Isolated drill with numbers apart from problem contexts
>
> -Performing pencil and paper calculations with numbers of more than two digits.
>
> (NCTM, 1980, p.7)

The deemphasis of these problem types challenges teachers' narrow specifications of topics. The in-depth analysis of multiplication and division revealed that teachers described skill topics at a finer level of detail than the taxonomy. Further, teachers's descriptions of the difference in multiplication topics from third to fourth grade, and in division topics from Grade 4 to Grade 5, involved teaching students to deal with problems when numbers were greater than 99. Thus, the NCTM recommendation suggests the deemphasis of problem types that the teachers saw as separate topics to be learned at a particular grade level.

In requesting teachers to change their priorities, redefine content areas, and rethink the specification of topics, the Agenda for Action suggests a revolution in mathematics education that parallels the "New Math" of the 1960's. Yet, teachers' varying interpretations of understanding in mathematics demonstrate that the goals of the curricula of the 60's were translated in different ways by classroom teachers. It is ironic that the thrust of the NCTM proposal for the 80's is based on problem solving, another area that was subject to diverse interpretations by teachers in this study.

Efforts to change mathematics instruction often focus on the development of instructional materials and textbooks. Yet programs that present clever activities are often perceived by teachers as offering innovative instructional methods to teach traditional content. If teachers modify or skip such activities, the goals of the program might be devastated. Yet, teachers might see themselves as implementing the program, and only exercising professional judgement about strategies that will help students learn.

To counteract this dilemma, textbooks and instructional programs must be written to more clearly define the content to be learned. More explicit descriptions of the purpose of instructional activities must be presented, and a clearer distinction should be made between the intended content of written lessons and the recommended instructional strategies. When the purpose of an activity is related to other than traditional goals, alternative instructional methods should be described that would allow teachers to adapt the curriculum for classroom use in ways that would not modify the goals of the program.

Teachers cannot be criticized for the ways they have interpreted innovative curricula in the past. Perhaps, the traditional view of mathematics content is perpetuated, not by teachers' mathematical ignorance, but by the continual reinforcement of that traditional view by those who hold teachers accountable. Although the NCTM recommendations also challenge the focus of current standardized test instruments, until these achievement measures are changed, teachers' traditional conceptions of mathematics content will continue to receive support. Those who write innovative curricula should provide standardized instruments to supplement or replace the tests teachers now administer to their students.

Further, these new measures of student learning must be accepted as legitimate by administrators who evaluate curriculum programs and supervise classroom teachers.

Teacher Education

Repeatedly, teachers in this study described classroom activities without alluding to content implied by the activity. Frequently, two teachers' discussions of the same classroom activity communicated different perceptions of the content to be learned in the lesson. Although a particular instructional procedure may serve multiple purposes, teachers' discussions of most activities suggested that, the full range of outcomes associated with particular activities were not recognized, or viewed as important.

Current debate in mathematics education recognizes this deficiency in teacher preparation. When considering ways to improve teacher education, one faction advocates preservice programs that combine courses in mathematics content with courses in methods of teaching mathematics. Others hold to the separation of content and methods courses with additional content courses as the recommended action to be pursued. The evidence that teachers do not recognize the implied content of some classroom activities gives some support to the idea of combined content-methods courses. However, the study also identifies a third critical dimension to the mathematical preparation of teachers. That third dimension involves the recognition of what students should learn in elementary school mathematics. The specification of teacher competencies should be expanded to include recognition of the goals of instructional programs and the purpose of instructional activities as a fundamental component of subject matter competence.

The assumption that elementary school teachers have a deficient background in mathematics sometimes fosters attempts to "program" teachers' behavior in classrooms. Workshops focus on providing the teacher with something to do in the classroom the next day. Such practices only add to the teachers' collection of activities without purpose. Programs of inservice education should focus less on how to use the instructional materials and more on what is to be learned from the classroom lessons. Such inservice should focus on identifying critical components of the curriculum, i.e., content areas, topics and priorities. Teachers should also be provided with criteria to determine whether or not they modify the goals of a program when prescribed activities are adapted for use in the classroom.

Classroom Instruction

Teachers' descriptions of what they hope students will learn in mathematics revealed important differences in teachers' views. Although this research did not involve the observation of classroom lessons, the findings suggest that the content of mathematics instruction may vary across classrooms where teachers have different conceptions of mathematics content. Variation was found in the priorities teachers expressed, the content teachers associated with classroom activities, teachers' interpretations of understanding in mathematics, and their recognition of the scope and purpose of problem solving. Each of these factors is likely to translate into differences in the content of mathematics lessons.

The effect of priorities is evidenced by the teacher who didn't require students to use alternative algorithms as long as the students could get correct answers. The students in that teacher's classroom might never be taught why the algorithms work. Yet other teachers described work with alternative algorithms as a major focus of instruction.

Different teachers' recognition of the implied content of classroom activities may also result in differences in instructional content. The teachers who viewed concrete objects as a strategy to develop understanding might choose to skip such activities, while one teacher clearly described these activities as unique content to be learned by students. Furthermore, those teachers who interpret understanding in terms of the use of concepts might be more inclined to skip the concrete activities to allow more time to discuss situations where the concepts are employed.

Finally, the contrast between teachers using the DMP curriculum and those who did not, revealed differences in the scope of activities teachers might present in their classrooms. Yet the differences among DMP teachers' views of the purpose of problem solving activities evidence the translation of written curriculum that could result in variation in the content of instruction in different classrooms when the same curriculum is used.

Future Research

This study of teachers' conceptions of mathematics content has revealed that teachers think about the content of mathematics curricula in different ways. Focusing on only a few dimensions of the total elementary school mathematics curriculum, the findings indicate considerable discrepancy in teachers' selection of topics for instruction. Teachers' beliefs about what should be learned in elementary school mathematics curriculum should be explored further. The failure of efforts to change classroom instruction that was reported in the review of the literature, may be due in part to teachers' priorities and their interpretations of the curricula they use. The improvement of mathematics instruction may require changes in the beliefs teachers hold, despite their willingness to comply with new programs. A first step in changing teachers' beliefs is to understand what they are.

Research on classroom instruction should also recognize the potential influence of teachers' conceptions of content. The review of the literature on teacher planning evidenced the impact of teachers, prior experiences and teachers' instructional goals on the preactive decisions teachers make. Although the teacher is usually recognized as a variable in classroom research, characteristics of teachers that correlate with student achievement have not been identified. The research on teacher decision making is pursuing a whole range of teacher variables that have not been explored previously. Teachers' conceptions of content in subject areas should not be ignored. The effect of teachers' conceptions of content on classroom instruction should be explored.

Finally, this research evidences the complexity of content as a variable in classroom research. The study made use of a taxonomy that was usually consistent with teachers' descriptions of content. Teachers did, however, describe content and instructional goals that could not be described using the taxonomy. Better measures for the content of classroom instruction should be developed. And, considering the discrepancies among teachers when describing the content of a particular activity, research should not assume that the content of instruction is the same in different classrooms, only because the same instructional materials are in use. Content, from the teachers' perspective, may be a critical variable. Further, the investigations of the students' perspectives of content may also have implications for studies of classroom learning.

This study examined only a few aspects of teachers' conceptions of mathematics content. In many ways, the findings of this study will come as no surprise to mathematics educators who are critical of elementary school teachers' mathematical competence. The research, however, suggests new dimensions of teachers' subject matter expertise that should be considered by those who wish to study or improve mathematics instruction in elementary school classrooms.

APPENDICES

APPENDIX A

Dear Educator:

You are invited to participate, through responding to the enclosed brief questionnaire, in one of the studies being conducted by the Institute for Research on Teaching (IRT). The Institute was established in 1976 as a national center for research on teacher perception and judgment. A better understanding of these processes, we believe will lead to improvements in teaching and teacher preparation. The Institute is funded by the National Institute of Education.

The study in question is one of a series which ask what factors influence teacher decisions to teach one set of topics rather than another. This focus is based on the belief that differences in content account for a good deal of the variation in student achievement from classroom to classroom.

One of the problems we face is to devise a precise measure of content. Our tentative solution to this problem is a method for classifying elementary school mathematics at a very detailed level. Before using this classification in further research, it is important to find out whether it takes into account the various ways in which teachers think about the content of mathematics. We therefore would like to interview some teachers in grades 3-5 who have strong and distinctive views on mathematics instruction. Your help in identifying such teachers would be greatly appreciated. The teachers you nominate would, of course, be under no obligation to participate. They would decide for themselves after learning about our research.

If you are willing to assist us, please take a few minutes now, or at your earliest convenience, to fill out the questionnaire and return it in the envelope provided. This task should take no more than ten to fifteen minutes of your time.

Note that you are not to sign the questionnaire; the responses will be processed anonymously. The questionnaire itself and our invitation to you have been approved by the (school district's) Superintendent's Office.

I very much hope to receive your help.

Sincerely,

MICHIGAN STATE UNIVERSITY
INSTITUTE FOR RESEARCH ON TEACHING
TEACHER NOMINATION QUESTIONNAIRE

Instructions:

- Please look over each of the following characteristics of teachers and consider whether you are aware of any teacher in the elementary school who fits the description particularly well.

- If you do, write in the teacher's name and school next to the characteristic.

- If you have no name to suggest, simply skip that characteristic.

- If a particular teacher fits more than one of the characteristics, list this person more than once.

- Likewise, if you can think of more than one teacher to fit any given characteristic, list as many persons as come to mind.

- Do not be concerned about your attitude toward the characteristic in question. We are interested in obtaining a wide variety of views. even if some of the views are not widely endorsed by other professional educators.

CHARACTERISTIC	NAME	SCHOOL
A teacher who comes to mind when you think of teaching mathematics in elementary school	________________	________________
A teacher who has strong opinions about what should be taught in elementary school mathematics	________________	________________
A teacher whom colleagues consult when trying to get new ideas or help for mathematics class	________________	________________
A teacher who believes that one of the main benefits of mathematics for children is to develop a systematic approach to problem solving	________________	________________
A teacher who to an unusual extent teaches without the use of published texts or curricula materials	________________	________________
A teacher who believes that one of the main benefits of mathematics for children is learning how to follow directions	________________	________________
A teacher who often attends math conferences and/or workshops	________________	________________
A teacher who is noted for telling children why mathematics is important	________________	________________
A teacher who thinks mathematics is particularly well suited for creative children	________________	________________

CHARACTERISTIC	NAME	SCHOOL
A teacher who is very much against back to basics in mathematics	______________	______________
A teacher who is very much in favor of back to basics in mathematics	______________	______________

There may be additional characteristics that we should have given to facilitate your identifying teachers with distinctive points of view regarding the teaching of mathematics. Please briefly describe any important characteristics you feel we have overlooked and indicate teachers' names and schools if appropriate.

____________________________	______________	______________
____________________________	______________	______________
____________________________	______________	______________
____________________________	______________	______________
____________________________	______________	______________

Dear Colleague:

About three weeks ago you received a short questionnaire asking for nominations of teachers with distinctive views on the teaching of mathematics. Since the winter months are a very busy time in schools, we are not surprised that a number of teachers have not been able to complete the questionnaire. However, it is important to our project to receive as many completed questionnaires as possible.

Therefore, we are sending a second request to all the teachers and principals who were given the earlier questionnaire. Note that, because of the need for anonymity, we do not know who has responded and who has not. Please disregard this letter if you have already mailed your questionnaire.

Completing this questionnaire will take but a very few minutes of your time. We would very much appreciate your help and hope that you can respond within the next few days. We realize that some teachers may have no persons in mind who fit the characteristics on the questionnaire. In that case or if you have another reason for not completing the questionnaire, simply return a blank questionnaire with a note on it saying why the questionnaire was not appropriate for you. An extra copy of the questionnaire and return envelope are enclosed in case you no longer have the one sent earlier.

We will be very grateful for your help in locating teachers who can help us learn more about what is involved in teaching mathematics in elementary school.

Sincerely,

APPENDIX B

Michigan State University
Institute for Research on Teaching
Teacher Survey of Mathematics Content
Instructions to Interviewers

General Instructions

The primary purpose of the interview is to obtain as complete a statement about the content of elementary school mathematics as seen by the respondent as is possible. Of particular interest are: 1) the level of detail (fineness of distinction) made when describing content, 2) content which would not be well described by the current version of the taxonomy, 3) the meaning teachers give to certain key terms, and 4) relationships among topics. A secondary interest is to become more knowledgeable about several external factors which might influence teachers' decisions about what to teach. Finally, there are a host of questions about which we hope to become more knowledgeable through these interviews, but which are not given explicit attention in the interview schedule, e.g., the content/strategy distinction, reasons other than external factors that teachers give for making decisions about the content of instruction. In pursuing these multiple ends, the interviewer is given some latitude:

1) All questions in the interview schedule are to be asked in the order given;

2) Variation in the wording of these questions is allowed;

3) All questions are to be asked in reference to the subject's current teaching assignment. When the assignment involves multiple grades, greatest attention should be given to the grade level closest to fourth grade;

4) From Section III on, all statements about content should be probed until the subject can think of nothing further to say;

5) The interviewer must decide on a case by case basis whether or not to pursue areas other than content and which are not explicit in the interview schedule. The primary concern in these decisions should be centrality to the objectives of the Content Determinants Group and interview time. The following are some general guidelines:

   a) the impact of external factors should not be pursued, although if a subject reports not making decisions consistent with an "external factor", the subject should be asked why;

   b) subjects should not be asked to describe content for grade levels they are not currently teaching;

   c) time permitting, statements relevant to the content/strategy distinction should be pursued;

The interviewer may wish to make note of interesting points which might be pursued at the end of the interview given sufficient time.

At the start of the interview the following should be done:

1) Have the subject complete the statement of informed consent, complete with social security number and address;

2) The subject should be told that the session will be audiotaped. Since a transcript of the audio tape is the basic data, the interview should be terminated immediately if the subject objects to the session being taped.

The interview is to last approximately two hours or less. If at the end of two hours the subject is still providing useful data, a second interview should be scheduled, unless the interview can be completed in 15 minutes.

Interview Schedule

I am primarily interested in understanding the content of mathematics which you teach to children in your classroom. One way of thinking about this is to answer the question, "What would a youngster have learned if instruction was 100% effective for him?" While I am interested in the full school year, I realize that a complete description for that period of time might be difficult to provide. Thus, I will try to stimulate your thinking by asking questions.

Let me first ask a few questions to put your description of content into perspective.

This section of the interview (prior to III) is not designed to elicit detailed statements about the content of instruction. With that in mind, the following rules apply:

1) **Probe for the meaning the teacher gives to any key terms which you do not find self-explanatory, e.g.,**

concepts	**basic number facts**
understanding	**basic skills**
skill	**problem solving**
applications	**properties**
place value	**relationships**
measurement	**transfer**
geometry	**_________ concept**
estimate	**concept of ________**
basics	**enrichment program**
processes	**enrichment activities**
story problems	**comprehension**

2) **Probe no further on content in this section. Instead list topics or areas of content the are volunteered by the subject so that these may be pursued in later sections of the interview.**

3) **If "understanding of _________" is mentioned by the subject, it should be probed in depth.**

I. Background

A) Teacher background

1) How many years have you taught in elementary school?

2) At what grade levels have you taught?

3) What grade levels do you currently teach?

B) When is mathematics taught?

1) Are regular time periods set aside for math instruction each day? When do these time periods begin? How long do they last?

2) Are there other times that students might be studying mathematics?

II. Teacher's reasons for teaching mathematics

A) What in your own opinion are the most important reasons for teaching mathematics in elementary school:

Probe: You say mathematics should be taught because _________. How is the importance you put on this reason reflected in your own teaching? Do you try to get children to understand the reasons you have just given?

This should not be probed too hard at this point. The purpose of putting the why questions here is to attempt to create a context for what follows that will generate responses that may push our boundaries for what constitutes content. After learning what is taught we may decide to ask teachers to relate that content to their answers here.

III. Teacher's descriptions of math content

A) **Skip the following transition and ask the next question if the teacher has not mentioned any math content up to this point.**

Transition: "In our conversations so far you have mentioned a number of things in mathematics that you hope students learn, for example--(interviewer gives a few examples from his notes). I would like to get a complete list of the things you hope students learn in _____grade mathematics. If you think that it would be easiest, you could start with some of the topics that you have already mentioned. My question is:

Question: What do you hope students will learn in ____grade mathematics?

1) Attempt to get as complete and detailed a response to this general question as is possible. The following two probes should be used:

a) Tell me all the different ways that _____ comes up in the content that you teach.

The probe should be used with the four main arithmetic operations and selectively with other general content areas about which you would like the subject to say more, e.g., fractions.

b) What are some other things that you hope students will learn?

2) Keep a list of all areas of content that have been satisfactorily described. (This list will be given orally to the subject just prior to the external factors section of the interview.)

3) Probe when any of the key words listed earlier are used by the subject;

4) Probe "understanding of _____" whenever it is volunteered. If "understanding of _____" is not volunteered but "understanding" is mentioned, make a note, since questioning at the end of this section is conditioned on past use of "understanding".

5) Probe all areas of content mentioned regardless of whether it is content taught;

6) Be particularly sensitive to any indications of relationships among topics:

a) When a teacher has indicated a relationship among topics or a structure for topics, the teacher should be asked for the logic behind that relationship or structure only once, i.e., the interviewer is to accept that teachers first response to a probe for the underlying logic to any indicated structure or relationship.

b) When probing the logic behind a relationship the following should be used, "How would you explain why a student would have to _____?".

c) Be alert for indications of topics which are crossed or nested within each other and look for indications of prerequisites. Probe the teacher on the following terms or phrases:

common thread	instructional sequence
difficulty	concrete to abstract
easy	basic
reach a certain point	cuts across
up through	links
take them further	tied
transfer to _____	coupled
inverse operations	

B) What other things could be taught in _____ grade mathematics? (Ask this question once.)

The following are a list of optional probes that may be used to encourage teachers to expand their remarks about content:

1) **How do you feel with the area in your teaching?**

2) **How do (would) the children proceed through this area? How is it (would it be) sequenced?**

3) **What kinds of assignments do (would) the children do and how do (would) these assignments differ in content?**

4) **What do (would) you hope the students learned?**

5) **What is it that students find difficult about this area?**

6) **If you had less time to teach the area, what would you omit?**

7) **If you had more time to teach this area, what would you add to increase the depth of coverage?**

8) **Can you tell me what you mean by that (their response)?**

9) **What kinds of errors do students typically make on ___?**

10) **Can you give me an example of a problem that illustrates _________? (Number the examples given and state number for tape.)**

C) Bring up any topics mentioned prior to III and that were not adequately described in III.

D) Attempt to summarize content that the teacher has mentioned up to this point and ask: "Are there any other areas of content of _____ grade mathematics that come to mind?"

E) If "understanding of _____" has not yet been sufficiently explored in the interview,

1) but "understanding" has been mentioned one or more times by the teacher without reference to a particular area of content, say:

You mentioned "understanding" earlier in the interview. How does understanding come up when you are teaching a particular area like fractions?

If the subject does not respond, the question should be repeated. If the subject still does not respond, the question should be dropped.

<u>or</u> 2) If "understanding" has not been mentioned by the teacher, say,

Earlier in the interview, you mentioned some things you hope students learn having to do with (fractions). I'd like to go back there for a minute to ask a further question. How do you know when a student understands (fractions)?

F. External Factors

The primary purpose for asking the six sets of questions on external factors is to elicit more descriptions of content. At the same time, however, responses may provide further understanding of the external factors. With this secondary purpose in mind, the following should serve as guidelines to the interviewer:

1) **Do not ask questions about the impact of external factors;**

2) **Do not ask questions designed to identify additional external factors;**

3) **If the subject mentioned something that sounds as though it might be an additional external factor, it is up to the interviewer to decide whether to probe. If the decision is in favor of probing, the following should be used to determine whether or not the teacher thinks of _____ as an external factor:**

"Is _______ something you might think about when trying to decide what to teach?"

4) **The interviewer must decide whether or not to find out how external factors are communicated. Communication is of greatest interest for the three personal factors, i.e., teachers, parents, school officials. If communication is to be pursued, the following questions may be helpful:**

"Who initiated the communication?"
"Have there been other similar incidents?"

5) **It is up to the interviewer to decide whether or not to press the subject on how knowledgeable they are about the content of tests and objectives.**

IV.

A) What advice or suggestions have you received from parents about what you teach in mathematics?

   1) What mathematics have parents asked you to stress in your instruction?

   2) What mathematics have parents asked you to delete from your instruction?

   3) Are there any aspects of mathematics about which you feel strongly but about which parents are apathetic?

B) What suggestions have you received from other teachers about what should be covered in mathematics?

   1) Do they place higher priority on some mathematics than you do?

   2) Is there mathematics which you teach that they would not?

C) Has anyone on the school district staff given you advice or direction on what to cover in mathematics?

   1) Have you been advised to provide more coverage of certain areas of mathematics than you do now?

   2) Is there mathematics which you teach that they have suggested might be omitted?

D) What materials are you using this year?

   1) What mathematics is included that you don't teach?

   2) Is there mathematics not covered which you wish were?

   3) Do you re-order the presentation of material?

   4) Would good students learn the same mathematics if they worked by themselves on the materials with hardly any help from you?

E) Does your district have a set of objectives for mathematics at your grade level?

   1) Which objectives do you wish were deleted?

   2) What objectives would you like to see added?

   3) Are you expected to teach the objectives in a particular order? If yes, what changes, if any, would you like to see made in the sequencing of objectives?

4) Are there objectives that you would like to see made more detailed and explicit?

F) What tests in your school are expected or required to be given?

1) What do you want your students to learn that is not reflected on these tests?

2) Is there mathematics tested which you do not teach?

IV. Teacher's Structures of Mathematics

A) If you had to divide (up) all the mathematics that you teach into just a few categories, which categories would you choose?

1) How would you subdivide the content within each of these categories?

2) Can you think of another set of categories that might be used? If yes, repeat (1) above and follow with (2) again until the respondent is done.

B) Do you see any common threads that run through many of the topics you teach? For each thread identified, ask the respondent to describe and indicate which topics.

APPENDIX C

Summary of Teachers' Responses to Category Questions

Teachers Using Scott Foresman Textbook

#2 (Grade 3)

	OR	OR
(I) Computation	(III) Addition & subtraction	(I) Computation
(II) Measurement	(III) Multiplication & division	(III) Estimation
(I) Applications	(II) Whole # & their relations	(Teacher unable to complete this set)
(III) Place Value	(II) Measurement	

Common threads: (III) basic operations; (II) numbers

#3 (Grade 3)

	OR
(I) Theory	(III) Addition
(I) Computation	(III) Subtraction
	(III) Multiplication
	(III) Division
	(II) Other (Measurement, Roman Numerals, Money)

Common threads: (II) addition

#4 (Grade 3)

(I) Concept formation	(III) W/regrouping (+,-)	(III) Addition & multiplication
(I) Computation	(III) W/out regrouping (+,-)	(III) Division & subtraction
*(II) Time	(III) Multiplication & division	(II) Divide by size # used
(II) Money		
(II) Measurement		
(II) Fractions		

*In discussion of the first two categories teacher commented that those two encompass the last three

Common threads: (III) place value; (II) fractions to money or time

#5 (Grade 4)

(III) Addition & subtraction
(III) Multiplication & division
(II) Measurement & graphing
(II) Fractions & decimals
(III) Place value

Common threads: (III) (I) relatedness of operations;
(II) fractions to money or time

#8 (Grade 4)

(III) Multiplication & division
(III) Review add, subtract, place value
(II) Measurement
(II) Fractions

Common threads: (I) problem solving; (I) learning to reason

#14 (Grade 4)

(I) Computation	(II) Whole numbers	(0) Central	(I) Operating
(I) Conceptual	(II) Fractions	(0) Border	(I) Conceptualizing
(I) Manipulative	(II) Decimals	(0) Peripheral	
	(II) Number theory		

Common thread: (I) hands on to abstract; doing things in sequence; (III) (I) relationships

#19 (Grade 4)

(III) Multiplication
(III) Division
(II) Fractions
(III) Addition
(III) Subtraction
(I) Problem solving techniques

Common thread: (III) (I) relatedness of operation

#6 (Grade 5)

(I) Computation
(I) Applicatin

Common thread: (None given)

#15 (Grade 5)

(I) Process (doing problems)	(III) Addition	(0) Chapters in Book
(0) Content situation	(III) Subtraction	OR
(I) & Applications of	(III) Multiplication	For accelerated students
(III) +,-,x,÷	(III) Division	For average students
		For low achieving students

Common thread: (II) recognition of numerals; (III) (I) relatedness of operation

#20 (grade 5)

(II) Fractions
(I) Computation
(II) Decimals & percents
(III) Addition & subtraction
(0) Appreciation for math

Teachers Using Continuous Progress Lab

#17 (Grade 3)

(III) Subtract
(III) Multiplication
(I) Story problems

Common thread: (I) (III) understanding place value

#9 (Grade 5)

(II)& Whole # (+,-,x, ÷)
(III) Fractions
Decimals

Common threads: (III) +,-,x, ÷ ; (III) (II) comparative size;
(II) different symbols

Teacher Using SRA

#7 (Grade 5)

(III) Addition
(III) Subtraction
(III) Multiplication
(III) Division

Common thread: (II) whole numbers, fractions, decimals, percents, negative numbers

Teacher Using Holt Textbook

#12 (Grade 5)

(III) Addition	(II) Measurement	(II) Fractions
(III) Subtraction	(I) Computation	(II) Decimals
(III) Multiplication	(I) Practical math	(II) Money
(III) Division		(II) Measurement
(II) Algebra		(I) Practical things

(When asked to subdivide categories, teachers gave third list as crossing the first set given.)

Common thread: (I) (II) understand number language; (0) math is either right or wrong

Teachers Using DMP

#13 (Grade 3)

(I) Computational skills
(I) Story problems
(II) Measurement

Common thread: (0) my approach to teaching

#21 (Grade 3)

(I) Problem solving	(II) Measurement
(II) Measurement	(II) Geometry
(II) Geometry	(II) Fractions
	(I) Arithmetic
	(II) & (I) Numeration

Common threads: (I) understanding numbers; (I) concrete experiences; (III) computation

#1 (Grade 4)

(III) Place value	(I) Measuring
(III) Addition & subtraction	(I) Computing
(III) Multiplication & division	
(II) Fractions	
(II) Geometry	

Common threads: (I) (III) relatedness of operations; (I) problem solving strategies

#10 (Grade 4)

(II)& (III) Whole number operation
(II) Fractions
(II) Measurement

Common threads: (0) appreciation; (I) problem solving strategies

#18 (Grade 4)

(III) Multiplication
(III) Division
(III) Addition & subtraction (w/place value)
(II) Fractions
(II) Geometry
(II) Measurement

Common threads: (I) preparation for life; (I) drill & computation; (0)fun

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