

AN EVALUATION OF CONVENTIONAL
AND STATISTICAL METHODS OF
ACCOUNTING VARIANCE CONTROL

Thesis for the Degree of Ph. D.
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ROBERT W. KOEHLER
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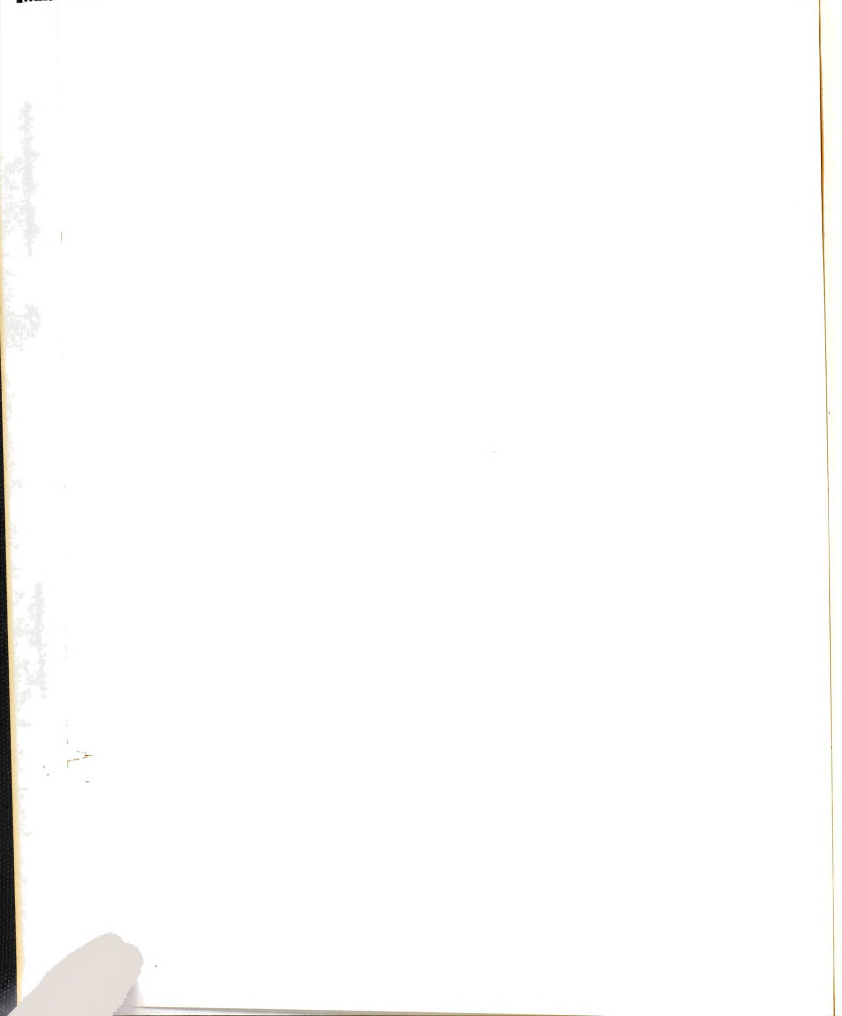
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ABSTRACT

AN EVALUATION OF CONVENTIONAL AND STATISTICAL METHODS OF ACCOUNTING VARIANCE CONTROL

by Robert W. Koehler

Standard costs are developed primarily to aid management in performance control. Accountants typically indicate the need for follow-up if the variance exceeds some selected percentage of the standard. These percentage cut-off points are subjectively determined by intuition, judgment, and experience. A 10 per cent variance is commonly designated as significant.

Lack of objective criteria for significance determination has hampered control. Furthermore, any summary report used as the principle control device permits significant variances to be averaged-out over time and to be offset between operations. In addition, of course, such a report does not facilitate timely control because those significant variances that are not averaged-out are still not detected until after the report is issued.

Accountants have not adequately considered the reasons for not investigating all variances. They ignore the fact that labor and overhead efficiency, material usage,

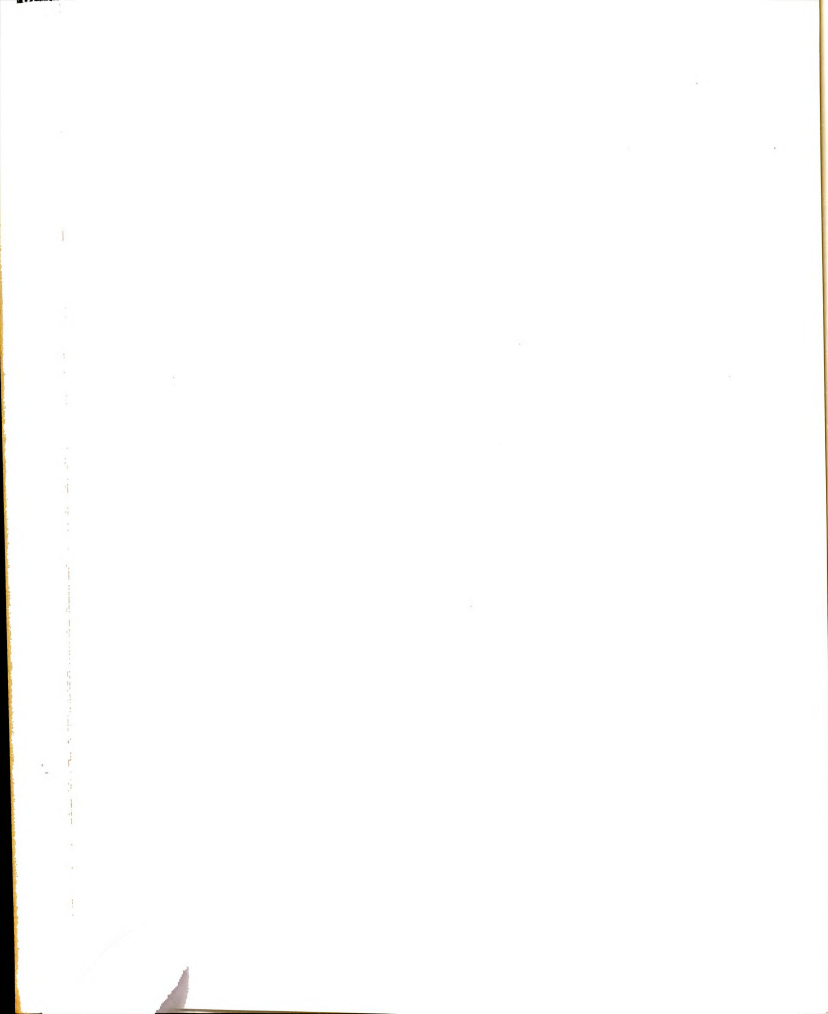


and volume as well as some manufacturing costs vary because of unexplainable factors which are identified as chance. Once chance is recognized as contributing to some variances, probability statistics evolves as a useful tool for significance determination because it involves procedures for evaluating patterns of chance occurrences.

The hypothesis that was tested in this dissertation is that new applications of presently developed statistical tools can increase the effectiveness of accounting variance control. In the test, all of the proposed statistical models resulted in significantly greater overall control than the commonly used 10 per cent cut-off point. Consequently, it is recommended that statistical procedures be adopted to aid in variance control.

Statistical models permit explicit consideration of various combinations of the following relevant factors:

1. Probability distribution of chance performances.
(The performances that vary for unexplainable reasons.)
2. Probability distribution for each assignable cause.
(These include faulty equipment, faulty materials, laziness, etc.)
3. Probability of making an unwarranted investigation
(Type I error).
4. Probability of accepting variance when an investigation is warranted (Type II error).



5. Opportunity cost of Type I error.
6. Opportunity cost of Type II error.
7. Prior probabilities of the occurrence of chance and each assignable cause.
8. Probability that any given variance is due to chance and the probability that it is due to each assignable cause.

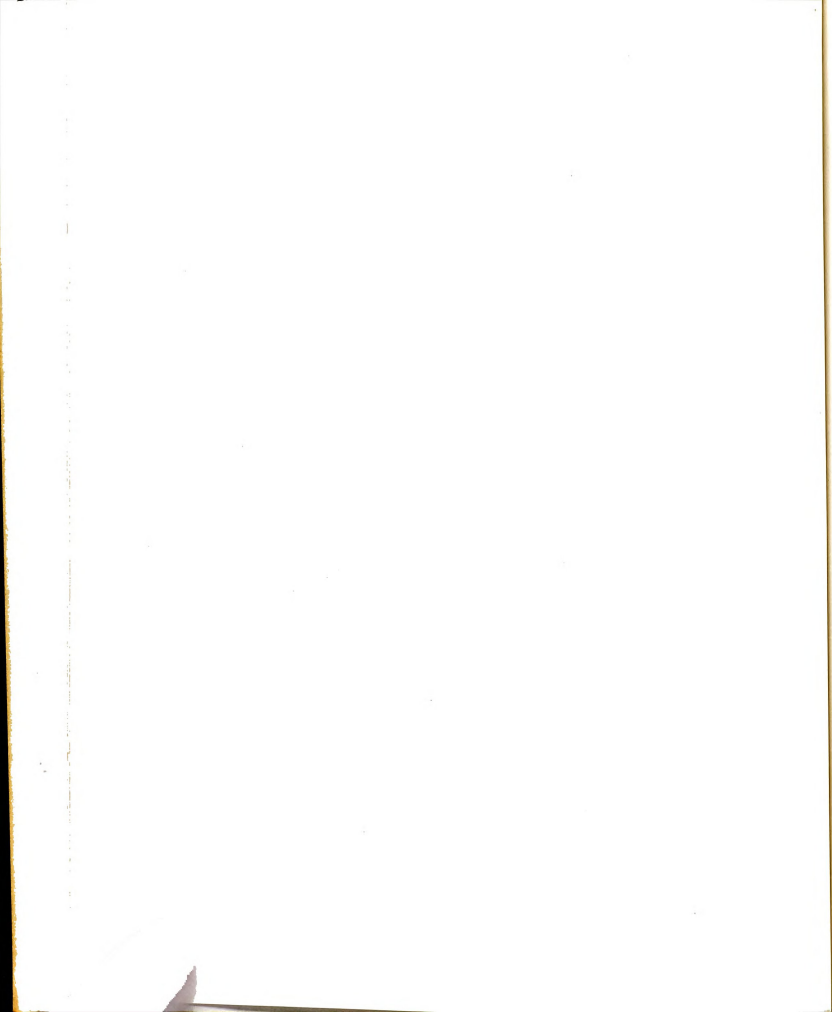
Initially four statistical models that had been proposed by others were examined. Each contained some questionable aspects. In an effort to counteract these, this writer constructed two additional models. One is an extension of Classical statistics which considered factors 1 through 6. The other, which is identified as the Minimization approach, contains an element of Bayesian statistics in that it incorporates factor 7 in addition to the first six. All of these statistical models in addition to the 10 per cent cut-off point were then tested to determine the best model for control purposes and to substantiate the hypothesis that statistical models are more desirable.

The test consisted of three parts. First, a hypothetical example was developed for which the causes and performance values of 1000 performances of a certain operation were assumed. Second, these values in conjunction with economic assumptions were used to compute the upper and lower control limits for each of the models under four

testing plans. The third phase of the test consisted of a financial analysis conducted to rank the approaches for control effectiveness for each corresponding control limit and testing plan.

The most significant conclusion is that all of the statistical procedures resulted in significantly greater overall control than the 10 per cent cut-off point. As expected, the Minimization approach which incorporated the largest number of relevant factors produced the greatest overall control. Factor 8 which was used in two of the models that had been proposed by others proved to be a sufficiently important determinant of effective control limits to outweigh some of the other deficiencies associated with these models. However, the individual rankings of the statistical approaches can be expected to vary somewhat depending upon the probability distributions of chance and assignable cause performances and also upon the testing plan with its corresponding control limit.

The example also illustrated how significant performances can be averaged-out so that they are not reflected in summary reports. To reduce this average-out effect and to facilitate more timely control, it is suggested that statistical models be applied at the performance level. However, because statistical procedures take cognizance of the degree of summarization, they can also be used for better interpretation of summary report.



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By

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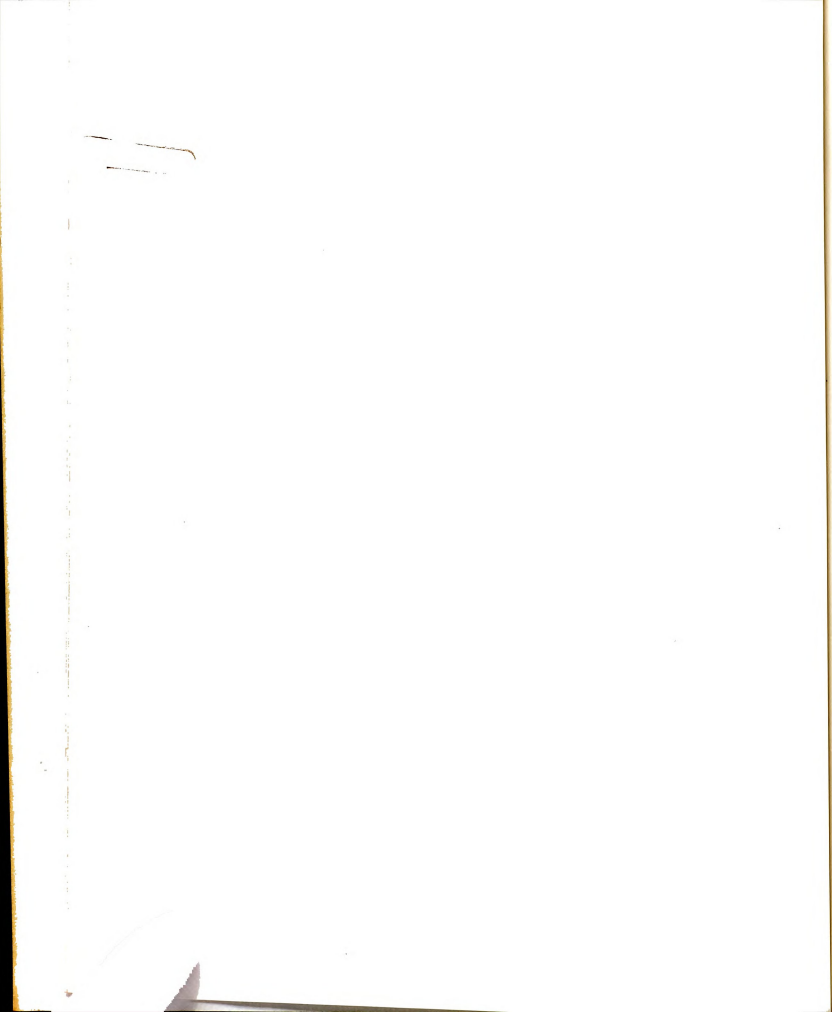
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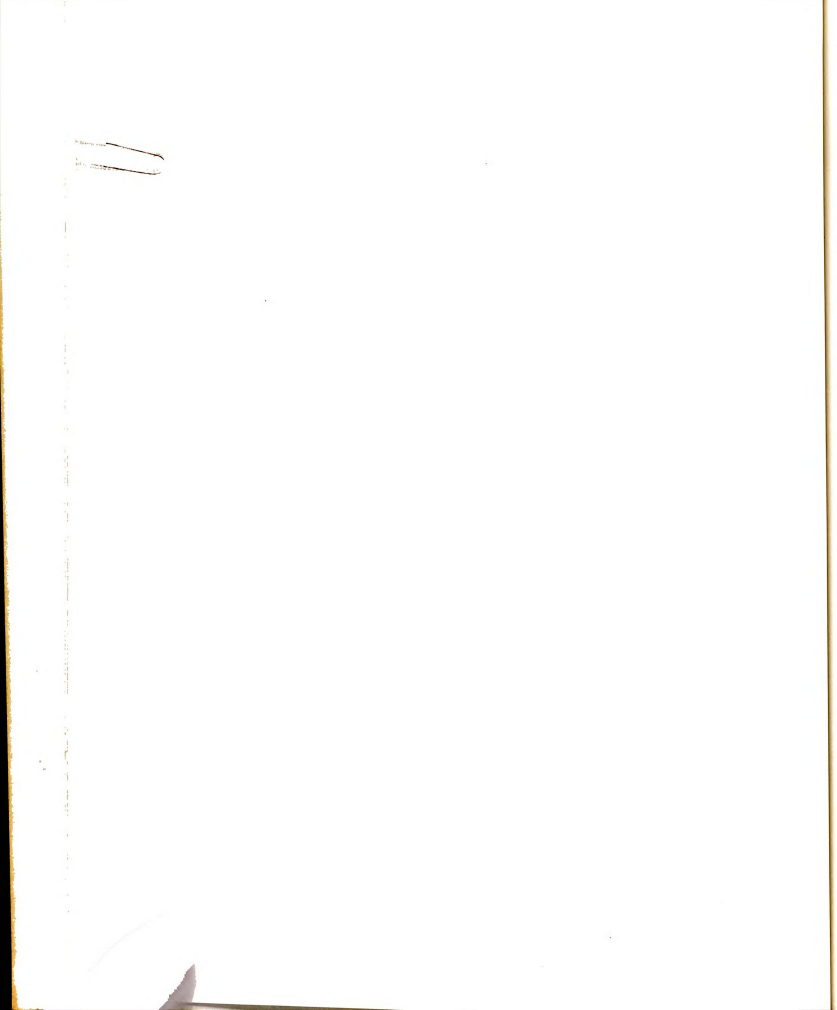
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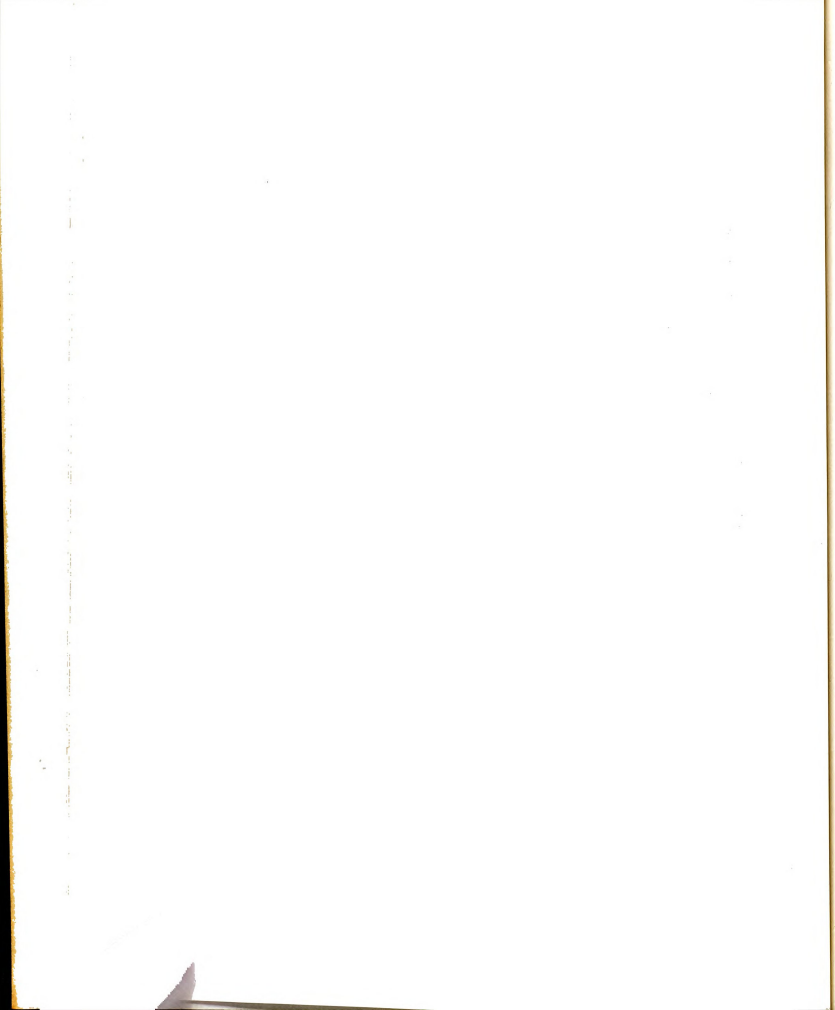
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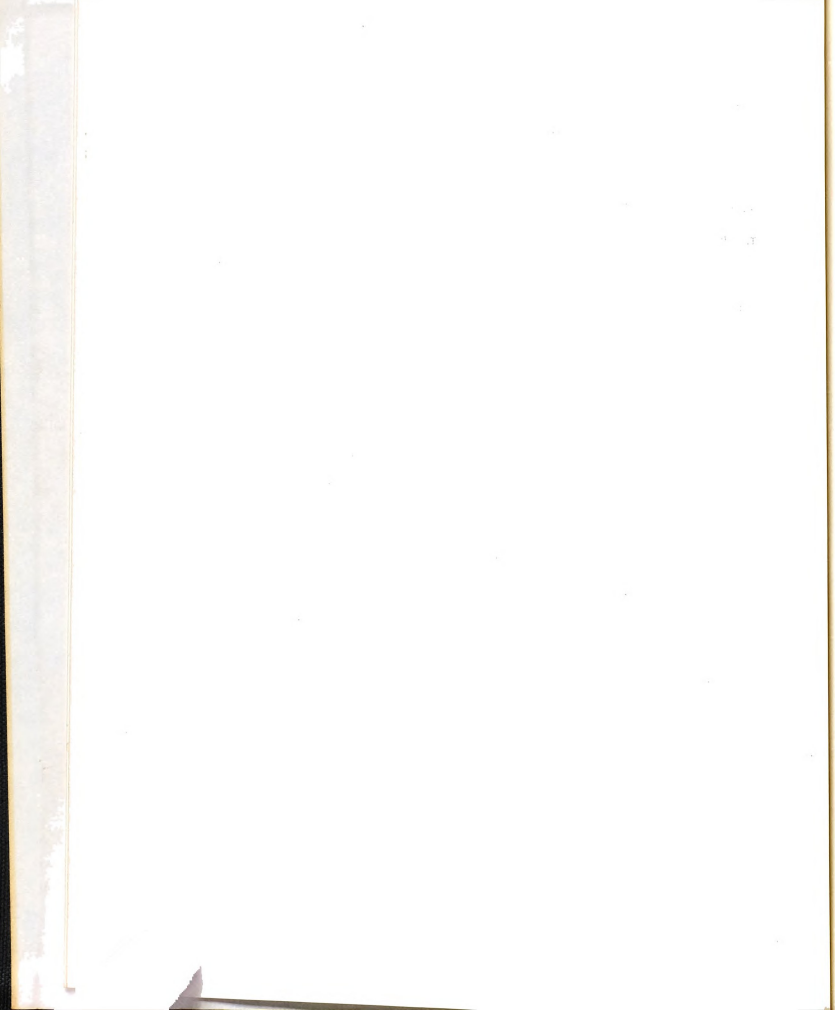
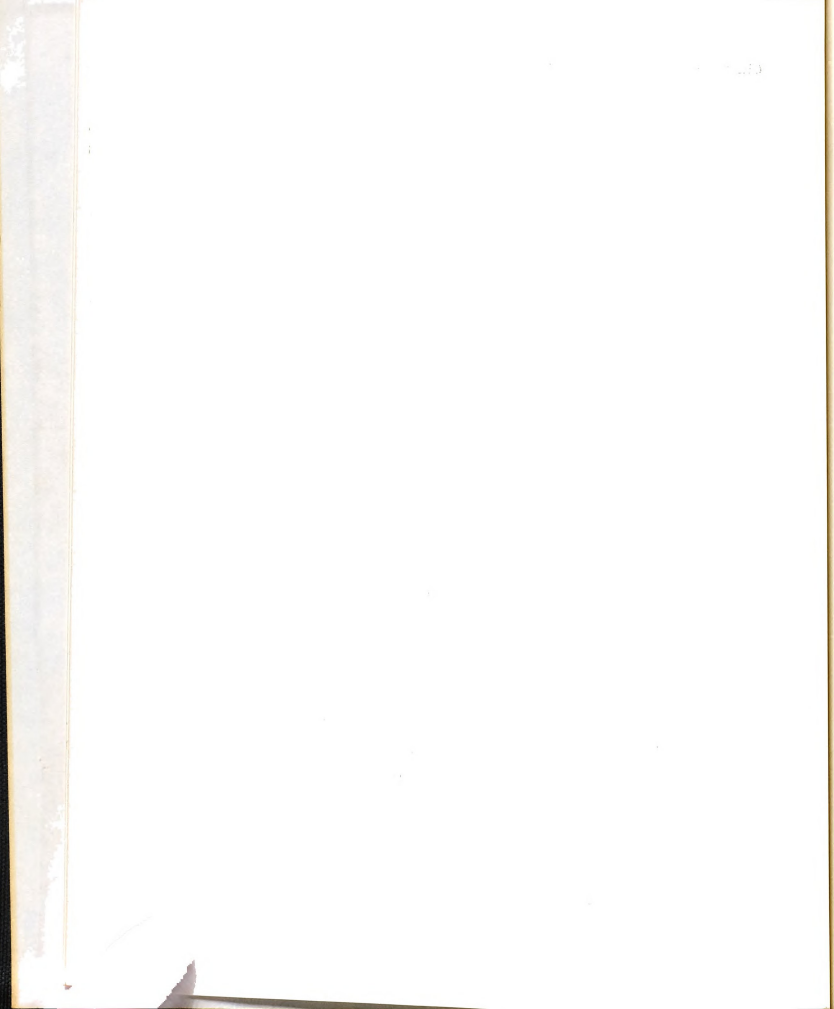


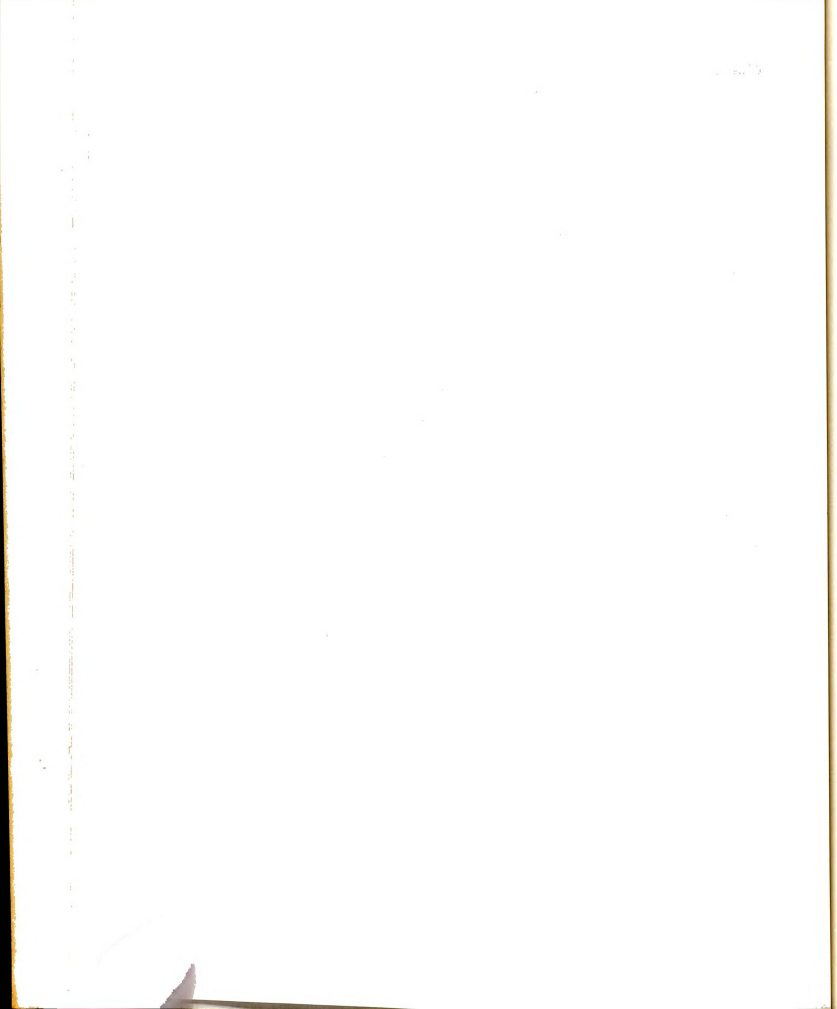
TABLE OF CONTENTS

	Page
ACKNOWLEDGMENTS	iii
LIST OF TABLES	vii
LIST OF FIGURES	xiii
LIST OF APPENDICES	xiv
 Chapter	
I. INTRODUCTION	1
Background	
The Problem	
Helpful Information	
Attempted Solutions	
Purpose of Dissertation	
Hypothesis	
Methodology	
Contributions	
II. AN EVALUATION OF CONVENTIONAL VARIANCE CONTROL	15
Conventional Significance Determination	
Treatment in the Literature	
Aggregation Problems	
Need for further Study	
Dissertation Objectives	
III. CONTROL AND CHANCE CONCEPTS.	28
Definitions of Control	
The Notion of Chance	
Illustration of Overlapping Populations	
Chance Influences on Individual	
Variance Classifications	
Conclusions	
IV. STATISTICAL CONTROL TECHNIQUES-- EVALUATION OF THREE PROPOSED METHODS . .	51
Hypothesis Testing	
The Basic Control Chart Approach	

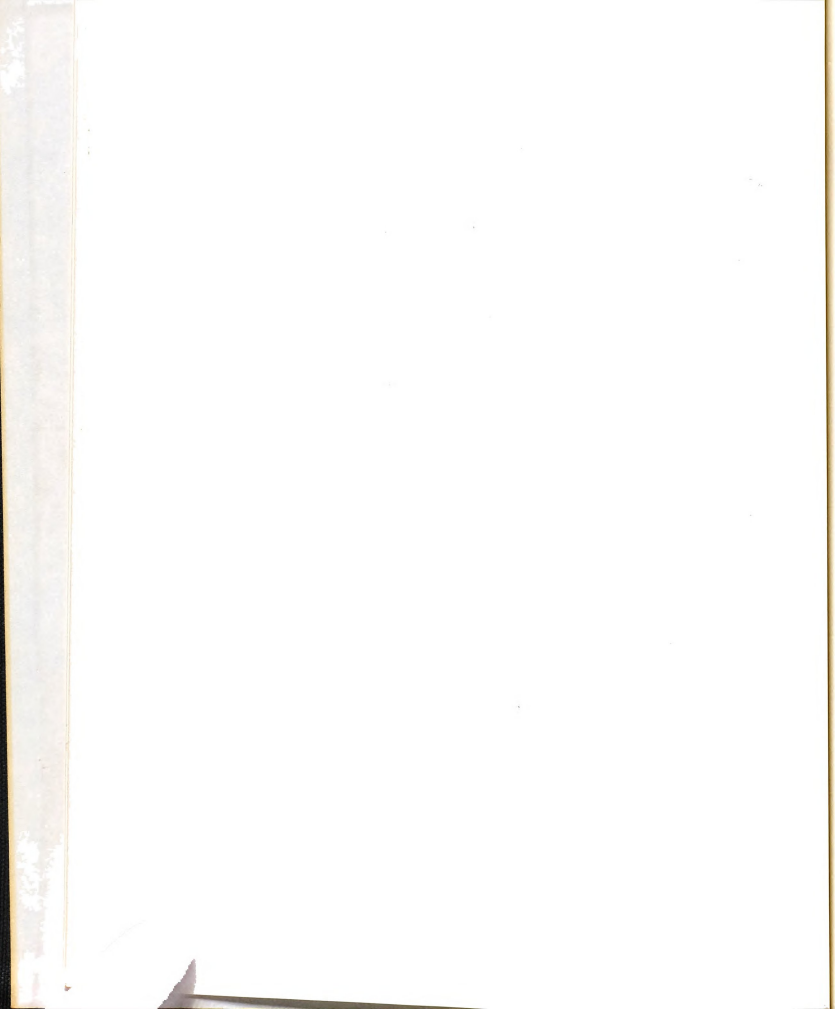


The Bierman, Fouraker, and
 Jaedicke Approach
 Analysis of the Bierman, Fouraker,
 and Jaedicke Approach
 McMenimen Approach
 Conclusions

V.	STATISTICAL CONTROL TECHNIQUES-- TWO MORE-REFINED METHODS	92
	An Equilization Approach Comparison of the Equilization Approach with the Bierman, Fouraker, and Jaedicke Approach Bayesian Statistics Bayesian Application to Quality Control Application of a Bayesian Concept to the Meat-Cutter Example--Minimization Approach	
VI.	A TEST OF THE ACCOUNTING AND STATISTICAL CONTROL TECHNIQUES	136
	Introduction The Example Derivation and Financial Analysis of Upper Control Limits for Single Observations--Each Performance Tested Derivation and Financial Analysis of Lower Control Limits for Single Observations--Each Performance Tested Derivation and Financial Analysis of Upper Control Limits for Single Observations--Every Tenth Performance Tested Derivation and Financial Analysis of Lower Control Limits for Single Observations--Every Tenth Performance Tested Derivation and Financial Analysis of Upper Control Limits Sample Size Five--Every Performance Included in a Sample Derivation and Financial Analysis of Lower Control Limits Sample Size Five--Every Performance Included in a Sample	



Derivation and Financial Analysis of Upper Control Limits--Sample Size Five--Sample Taken in Every Fifty Performances	
Derivation and Financial Analysis of Lower Control Limits--Sample Size Five--Sample Taken in Every Fifty Performances	
Conclusions	
VII. SUMMARY AND CONCLUSIONS.	280
Reasons for Study	
Conceptual Distinction between Significant and Insignificant Variances	
Examination of Statistical Models	
Examination Testing the Relative Control Effectiveness of the Conventional Accounting and the Various Statistical Methods	
Aggregation Problems	
Summary of Conclusions	
Summary of Recommendations	
BIBLIOGRAPHY	304
APPENDIX	311



LIST OF TABLES

Table	Page
1. Probability Distribution of Chance Performances for Table Assembly.	37
2. Probability Distribution of Chance Performances Resulting from Assignable Cause Due to Improvement	38
3. Probability Distribution of Chance Performances	54
4. Probability of Error for Various Parameters Given Single Observations and a .05 Level of Significance.	63
5. Comparison of the Probabilities of a Type II Error for Various Parameter Values Under Different Levels of Significance.	67
6. Conditional Cost Table	70
7. McMenimen's Illustration	86
8. Opportunity Costs of a Wrong Decision for Various Population Means	97
9. Conditional Average Opportunity Costs.	100
10. Weighted Opportunity Cost of Type II Error.	108
11. Expected Opportunity Costs of Two Alternatives	120
12. Unconditional Expected Opportunity Costs for Various Rejection Numbers.	122
13. Revision of Prior Probabilities.	125
14. Unconditional Expected Costs of Various Levels of Significance	130

Table	Page
15. Revision of Prior Probabilities.	132
16. Revised Probabilities for Performances 260 and 270	134
17. Causes--Their Frequencies and Means.	138
18. Distribution of Performance Values by Cause.	140
19. Application of McMenimen Technique	155
20. Decision Table for Equalization Approach.	159
21. Decision Table for Minimization Approach.	161
22. Extra Savings of Basic Control Chart Method	167
23. Financial Comparisons between Approaches . .	167
24. Decision Table for BF and J Application. First Interpretation of P.	171
25. Decision Table for BF and J Application. Second Interpretation of P	171
26. Decision Table for Equalization Approach.	174
27. Decision Table for Minimization Approach.	176
28. Financial Comparisons between Approaches . .	179
29. Decision Table for BF and J Application. First Interpretation of P.	182
30. Decision Table for BF and J Application. Second Interpretation of P	182
31. Application of McMenimen Technique	183
32. Application of McMenimen Technique	186
33. Derivation of Savings Values	188

Table	Page
34. McMenimen Technique--Incremental Application.	190
35. Decision Table for Equalization Approach.	191
36. Decision Table for Minimization Approach.	193
37. Extra Savings of Basic Control Chart Method	196
38. Financial Comparisons between Approaches . .	196
39. Decision Table for BF and J Application. First Interpretation of P.	199
40. Decision Table for BF and J Application. Second Interpretation of P	199
41. Application of McMenimen Technique	201
42. Decision Table for Equalization Approach.	202
43. Decision Table for Minimization Approach.	203
44. Financial Comparisons between Approaches . .	204
45. Decision Table for BF and J Application. First Interpretation of P.	210
46. Decision Table for BF and J Application. Second Interpretation of P	211
47. Determination of P's	213
48. Derivation of Savings Values	216
49. Application of McMenimen Technique	218
50. Decision Table for Equalization Approach.	220
51. Decision Table for Minimization Approach.	222
52. Additional Savings of Basic Control Chart Approach	226

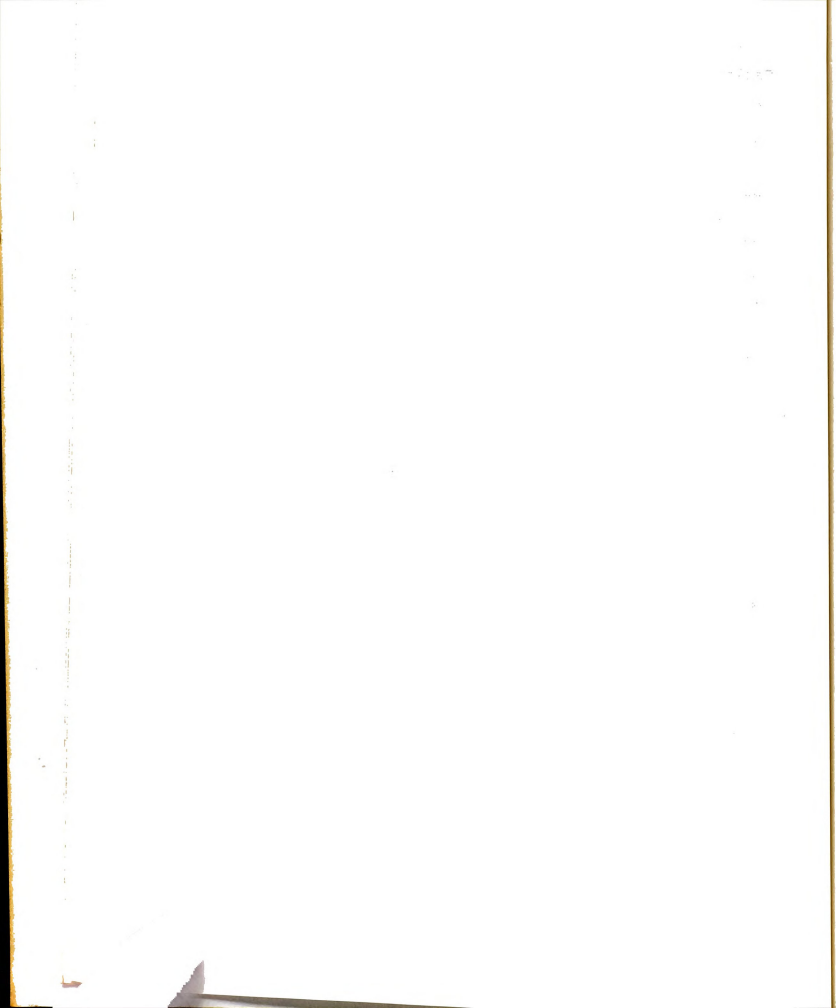


Table	Page
53. Financial Comparisons between Approaches . .	226
54. Decision Table for BF and J Application. First Interpretation of P.	228
55. Decision Table for BF and J Application. Second Interpretation of P	230
56. Determination of P's	230
57. Application of McMenimen Technique	232
58. Decision Table for Equalization Approach.	233
59. Decision Table for Minimization Approach.	234
60. Financial Comparisons between Approaches . .	237
61. Decision Table for BF and J Application. First Interpretation of P.	241
62. Decision Table for BF and J Application. Second Interpretation of P	241
63. Application of McMenimen Technique	242
64. Determination of P's	243
65. Decision Table for Equalization Approach.	244
66. Decision Table for Minimization Approach.	245
67. Financial Comparisons between Approaches . .	246
68. Decision Table for BF and J Application. First Interpretation of P.	249
69. Decision Table for BF and J Application. Second Interpretation of P	249
70. Application of McMenimen Technique	250
71. Decision Table for Equalization Approach.	251

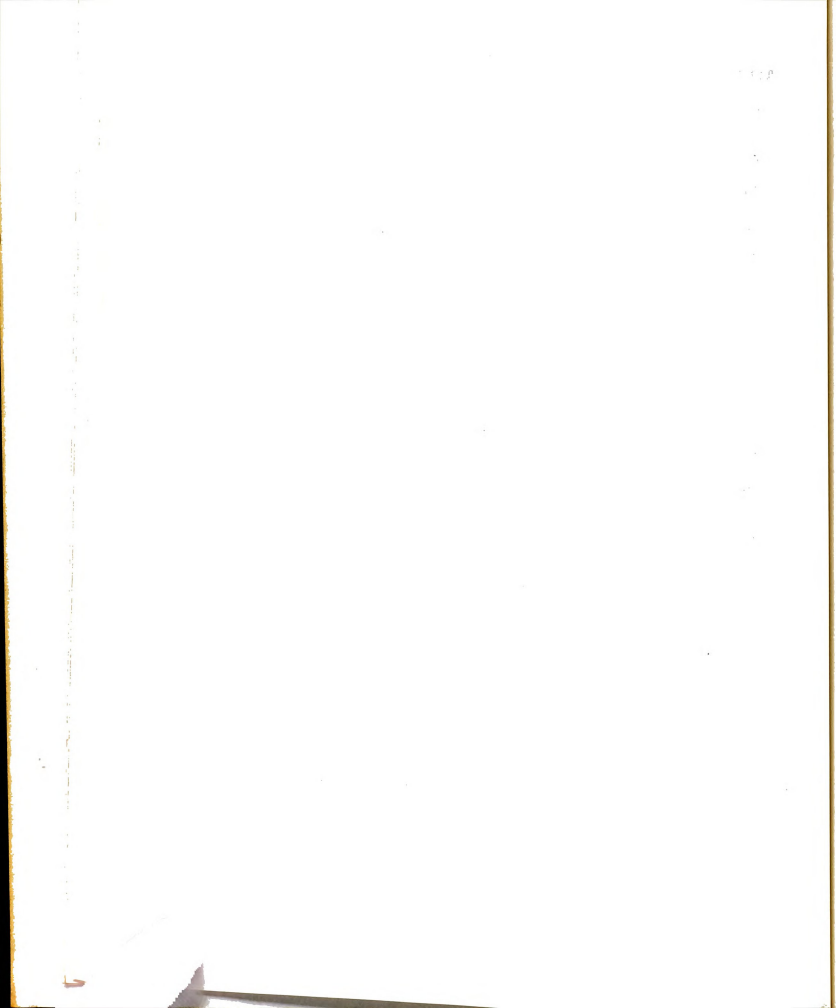


Table	Page
72. Decision Table for Minimization Approach.	253
73. Financial Comparisons between Approaches . .	255
74. Summary of Rankings.	256
75. Summary of Rankings.	259
76. Numerical Differences in Control Limits between the Top Ranking and the other Approaches for Testing Plan A.	272
77. Numerical Differences in the Control Limits between the Top Ranking and the other Approaches for All Testing Plans.	275
78. Summary of Upper Control Limits by Testing Plan.	278
79. Application of McMenimen Technique	316
80. McMenimen Technique--Incremental Application.	317
81. McMenimen Technique--Incremental Application.	318
82. Application of McMenimen Technique	321
83. Single Performance Opportunity Costs for Corresponding Assignable Causes. . . .	325
84. Weighted Opportunity Cost Associated with Poor Attitude Assuming UCL = 260. . .	326
85. Averaging Process to Find the Conditional Opportunity Cost of a Type II Error for Test Value 260	327
86. Weighted Conditionanl Opportunity Costs for Test Values 255 and 260.	328
87. Weighted Conditional Opportunity Costs for Selected Values Determined by Interpolation.	329

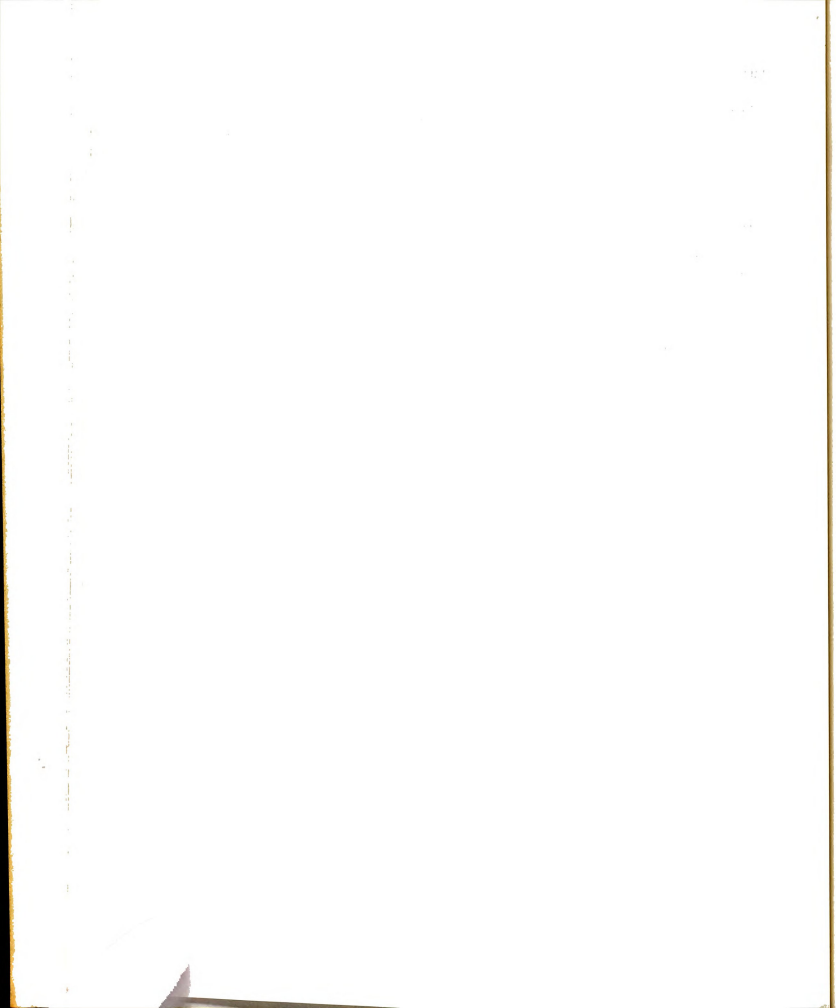
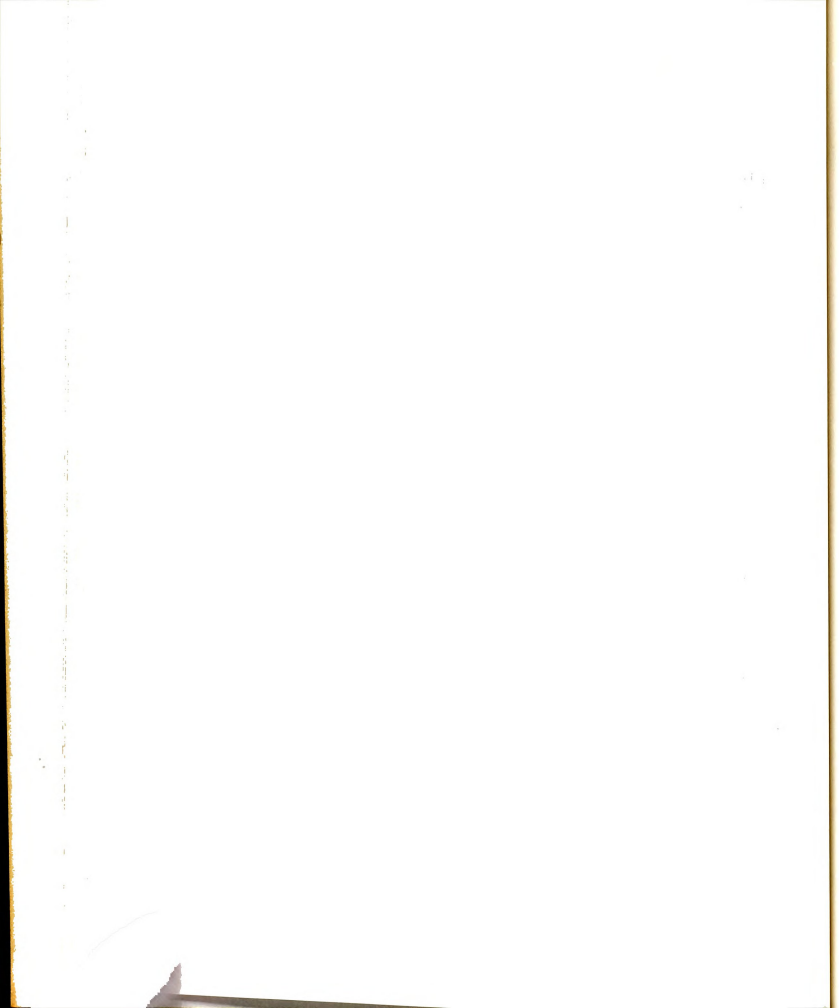
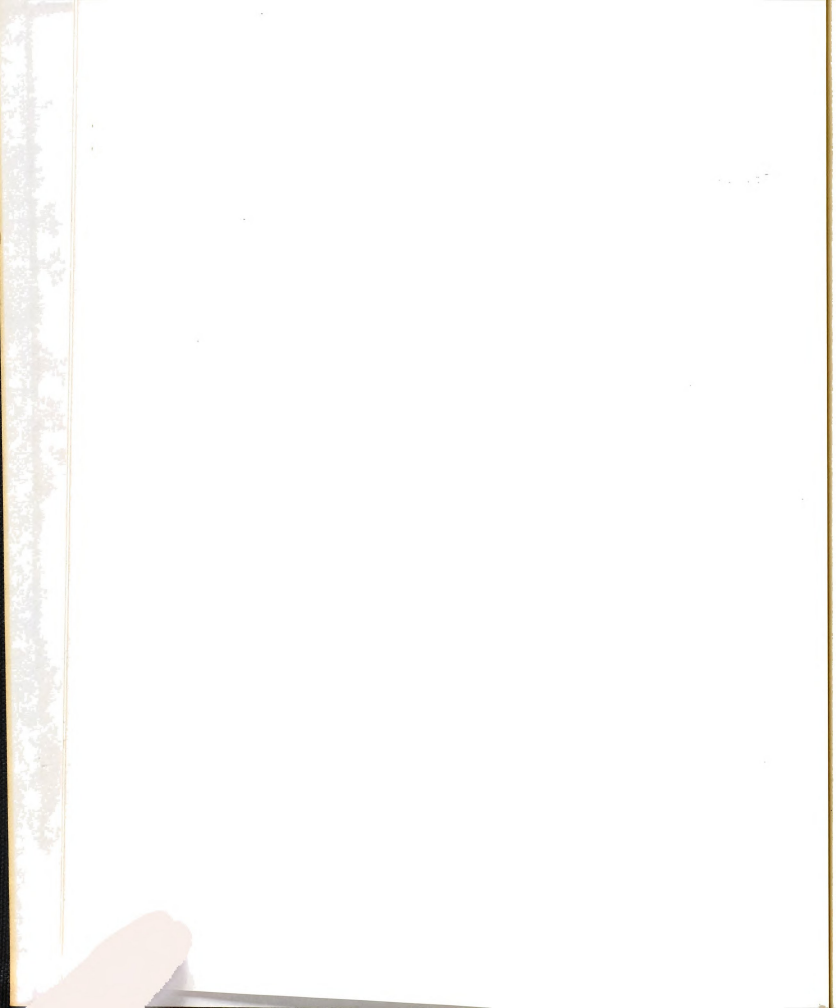


Table	Page
88. Averaging Process to Find the Conditional Opportunity Cost of a Type II Error for Various Test Values. . .	329
89. Relevant Prior Probability Distribution for Test Value 260.	330
90. Weighted Conditional Opportunity Costs for Test Values 265 and 260.	331
91. Weighted Conditional Opportunity Costs for Selected Values Determined by Interpolation.	332
92. Calculation of the Probabilities of a Wrong Decision for Each Assignable Cause Under Test Value 260	333
93. Differences in Weighted Conditional Opportunity Costs between Test Values 250 and 255	342
94. Weighted Conditional Opportunity Costs Determined by Interpolation.	342
95. Averaging Process to Find the Conditional Opportunity Cost of a Type II Error for Test Values 253 and 254 . . .	343
96. Weighted Opportunity Cost for Test Value 249	350



LIST OF FIGURES

Figure	Page
1. Gryna's Target Analogy	35
2. Figure Showing Overlapping Populations . . .	39
3. Illustration of a Control Chart.	54
4. Illustration of the Determination of the Probability of a Type II Error	65
5. Cost Control Decision Chart.	73
6. Diagram Indicating Direction of Desired Level of Significance.	103
7. Direction of Upper Control Limit	158
8. Outcomes of Financial Comparisons.	169
9. Outcomes of Financial Comparisons.	179
10. Outcomes of Financial Comparisons.	197
11. Outcomes of Financial Comparisons.	205
12. Outcomes of Financial Comparisons.	227
13. Outcomes of Financial Comparisons.	238
14. Outcomes of Financial Comparisons.	247
15. Outcomes of Financial Comparisons.	255



LIST OF APPENDICES

Appendix	Page
A. Bibliography of Statistical Applications to Accounting Variance Control	311
B. Computational Detail to Support Chapter VI	314



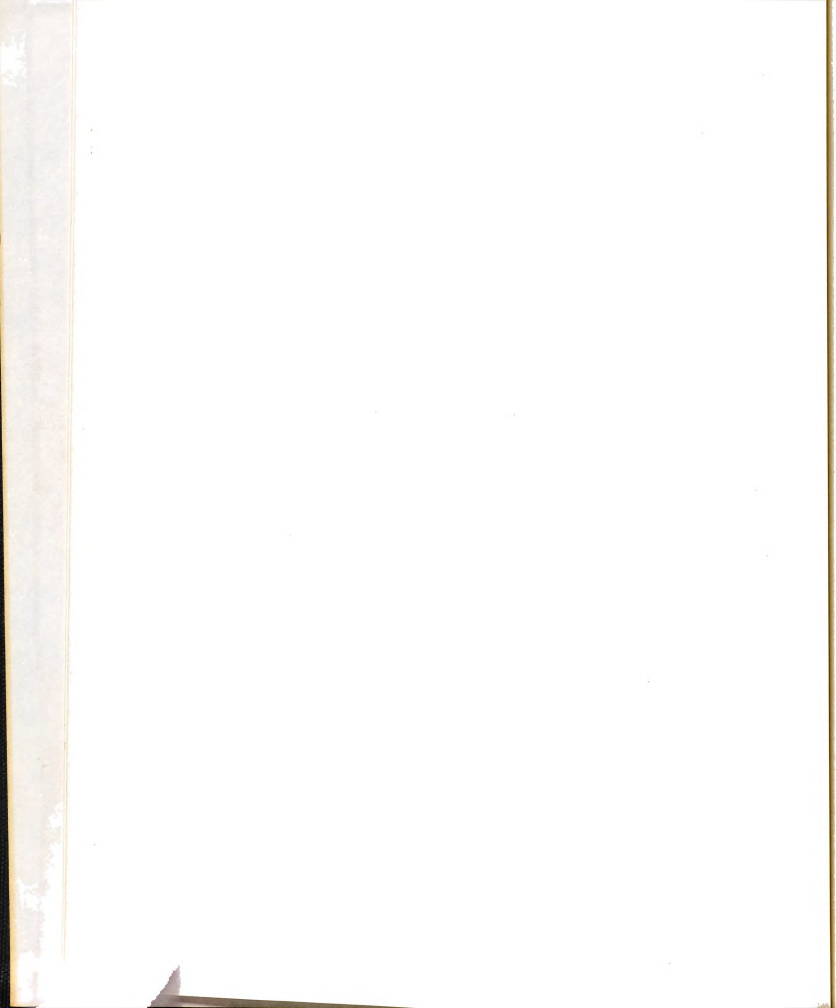
CHAPTER I

INTRODUCTION

In a standard cost accounting system standard unit product costs are established for materials, labor, and overhead. These standards are almost essential for the preparation of an adequate budget. Standard costs provide guidelines for pricing and expedite the valuation of inventories; but they are designed primarily to aid management in performance control. Possibilities for control emanate from the pre-determined standards. Actual performance is seldom equal to the standards because "persons and machines do not perform uniformly; there is always some variability in their work."¹ Accountants typically allow for this variability by an amount of variance between actual and standard which is termed "insignificant." A variance which is too large to be ignored is called "significant."

The accountant's function in variance control is to measure and report performance and to highlight "significant variances" so that management can initiate an

¹Lawrence L. Vance and John Neter, Statistical Sampling for Auditors and Accountants (New York: John Wiley and Sons, Inc., 1956), 148.



investigation and take corrective action. This imposes upon the accountant the need to develop criteria to determine when a variance is significant.

Background

It is appropriate to begin by considering answers to the following two questions:

1. Why are variances inevitable?
2. Why is some variability allowed and indeed expected?

In practice, it is unusual for performance exactly to equal standard; but there is little evidence to suggest that accountants have considered why this inequality inevitably emerges.² On the other hand, quality control literature does introduce the concept of chance to explain why variances are inevitable. Chance is "the absence of any known reason why an event should turn out one way rather than another."³ Chance variability can be explained only through an inherent omnipresent non-uniformity.

It is now useful to define a significant variance as one resulting from an assignable cause and an insignificant one as resulting from chance. Possible assignable

²The small amount of literature pertaining to statistical applications of accounting variance control has been devoted mainly to mechanics and has largely failed to consider these conceptual matters.

³C. L. Barnhart, ed., The American College Dictionary (New York: Random House, 1960), 200.

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causes include lack of training, illness, laziness, faulty materials, or improvement.⁴

Once this concept of "chance" is recognized, the answer to the second question follows logically. Variability due to chance should be allowed because it cannot be profitably reduced--it is inevitable. Chance variability can therefore be considered to be non-controllable for any given operational procedure.

There is also some chance variability present in the results due to an assignable cause. That is, a worker with faulty equipment will not always obtain the same results. In this case, the variability is attributed to

⁴Normally, the distributions of values from the several assignable cause populations will overlap with the distribution of values from the chance population. In these cases, it is not possible to select control limits so that results falling inside these limits are always due to chance and so that those falling outside result from an assignable cause. At this point, the statistically minded reader will note that chance performances falling outside of the control limits will signal the need for an investigation. This results in an error that statisticians identify as a Type I error. On the other hand, no action is indicated for assignable cause results that fall inside the control limits. This error statisticians refer to as a Type II error. The goal is to achieve a proper balance between the probabilities of committing Type I and Type II errors. This problem will be elaborated upon at greater length in the next section and also in Chapter III.

To summarize, the definition of a significant variance as one resulting from an assignable cause will be incorrect when a Type I error is committed. Also, the definition of an insignificant variance as one resulting from chance will be incorrect when a Type II error is committed. The definitions will, however, continue to be used because they are useful in defining the problem. Furthermore, since Type I and II errors cannot be eliminated when the populations overlap, there are no more precise definitions available.



both chance and the assignable cause. This variance is controllable because the average variance from the standard can be reduced by elimination of the assignable cause. After this elimination, however, non-controllable chance variability will still occur.

The Problem

Significance determination, then, properly involves distinguishing between variances due solely to chance and those due to assignable causes in conjunction with chance. On the surface it would seem relatively simple to obtain information regarding the set of values for which no assignable cause could be identified (e.g., to obtain an estimate of the distribution of values for the population of chance performances). Performance values falling outside this range of values could then signal the presence of an assignable cause. If, however, one were to obtain information pertaining to the set of values resulting from each assignable cause, he would find an overlap between the chance population values and the values of some of the assignable cause populations. That is, any specified deviation may be the result of either chance or several assignable causes. Without an investigation one cannot usually be sure which population a given performance came from.

Since the cost of an investigation for every performance is prohibitive, a decision must be made to investigate only those variances that are unlikely to have

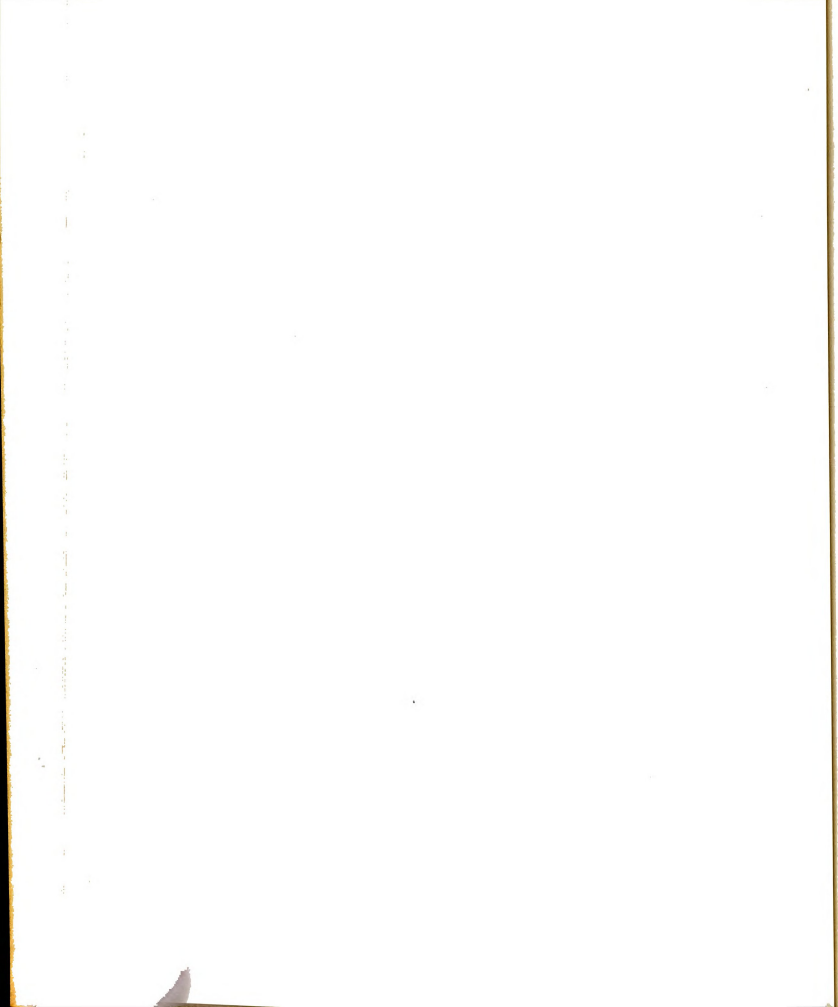


come from a chance population. The problem involves making a decision as to whether or not chance is operative on any given performance. More generally it involves setting up limits, called control limits, within which chance is likely to be operative. The use of probability statistics to help determine these limits seems logical because probability "is a statistical area dealing with the number of techniques for evaluating the possibilities and patterns of chance occurrences and the degree of effort needed to control them within pre-established limits."⁵

To reiterate, if a performance seems likely to have come from a population of chance performances, the cost of an investigation can be saved. On the other hand, if it seems unlikely to have come from a population of chance performances, it is important to investigate to determine the cause and to make the appropriate corrections if the cause is assignable, that is, due to factors other than chance. The problem involved is attempting to quantify the terms "likely" and "unlikely." Also involved is the determination of whether unlikely variances are worthwhile examining.

Because any given performance value may come from more than one population, two kinds of error are involved

⁵Arthur H. Smith, "Problem Solving Through Mathematical and Statistical Techniques: Use of Operations Research," N. A. A. Bulletin, XLII, No. 1, Section 3 (September, 1960), 10.



in making a decision. One may decide to investigate a performance that he later finds to have come from a chance population. This error is referred to as a Type I error. Conversely, one may decide to forego an investigation when, actually, an assignable cause is present. This error is called a Type II error. The risk of at least one type of error is present as long as there is an overlap between the values of the population of chance performances and those of the populations of some assignable cause performances. It will later be seen that the probability of a Type I error cannot be reduced without increasing the probability of a Type II error. Likewise, the probability of a Type II error cannot be reduced without increasing the probability of a Type I error. The solution lies in striking a balance between these types of error.

Helpful Information

For any value to be tested as a control limit, it is necessary to know (or estimate) the distribution of values of chance performances in order to evaluate the probability of a Type I error. Likewise, it is necessary to know (or estimate) the distribution of population values for each assignable cause in order to evaluate the probability of a Type II error. From the population distributions of each possible cause, one can determine the probability that any given variance is due to chance by dividing

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the number of times a given variance has occurred into the number of times it has occurred for chance causes. This is an important probability under some methods of striking the balance between the two types of error. Certainly the opportunity costs of incurring each type of error are an important consideration in striking the balance. In some analyses it may be helpful to know the probability that chance and each assignable cause will occur.

In summary, statistical models permit explicit consideration of various combinations of the following relevant factors:

1. Probability distribution of chance performances.
(The performances that vary for unexplainable reasons.)
2. Probability distribution for each assignable cause. (These include faulty equipment, faulty materials, laziness, etc.)
3. Probability of making an unwarranted investigation (Type I error).
4. Probability of accepting variance when an investigation is warranted (Type II error).
5. Opportunity cost of Type I error.
6. Opportunity cost of Type II error.
7. Prior probabilities of the occurrence of chance and each assignable cause.

1. The first of these is the fact that the number of cases of smallpox in 1870 was 1,000.

2. The second is that the number of cases of smallpox in 1871 was 1,000.

3. The third is that the number of cases of smallpox in 1872 was 1,000.

8. Probability that any given variance is due to chance and the probability that it is due to each assignable cause.

Attempted Solutions

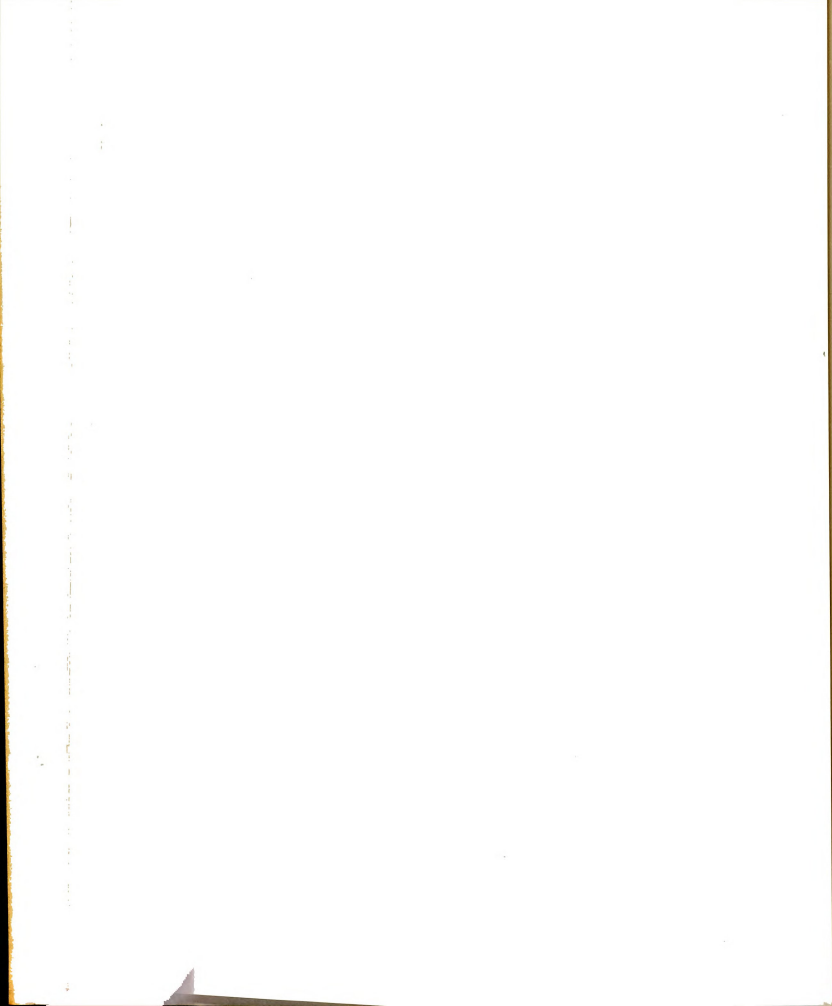
Conventional variance control does not explicitly consider any of the factors listed above although some combination of them may be considered on an intuitive basis. Several writers have suggested an application of the basic statistical quality control chart to analyze variances from accounting standards. This approach explicitly considers the distribution of chance performances and the probability of a Type I error. The analyst may intuitively consider some combination of the other factors. Both the Bierman, Fouraker, and Jaedicke⁶ and the McMenimen⁷ approaches have introduced into their models the economic aspects of the cost of an investigation and the present value of savings made possible by prompt detection of an assignable cause. (These economic aspects are similar to

⁶Harold Bierman, Jr., Lawrence E. Fouraker, and Robert K. Jaedicke, "A Use of Probability and Statistics in Performance Evaluation," Accounting Review, XXXVI, No. 3 (July, 1961), 409-417.

Harold Bierman, Jr., Lawrence E. Fouraker, and Robert K. Jaedicke, Quantitative Analysis for Business Decisions (Homewood, Illinois: Richard D. Irwin, Inc., 1961), 108-125.

Harold Bierman, Jr., Topics in Cost Accounting and Decisions (New York: McGraw-Hill Book Company, Inc., 1963) 15-23.

⁷Leo J. McMenimen, "Statistical Analysis of Cost Deviations," (Unpublished Master's Thesis, The Graduate School, The Pennsylvania State University, August, 1965).



factors five and six above.) McMenimen also included the cost of corrective action and factor eight into his analysis. In order to overcome some questionable aspects of these last two approaches, this writer developed an approach which explicitly considers the first six factors. He also introduced an application which minimizes expected opportunity costs by incorporating the first seven factors formally into the analysis.

Purpose of Dissertation

The purpose of this dissertation is to evaluate the accounting and statistical variance control procedures in an effort to ferret out the most adequate method of variance analysis for control purposes.

Hypothesis

The hypothesis to be tested is that new applications of presently developed statistical tools can increase the effectiveness of accounting variance control by providing a helpful analytical framework to determine the control limits.

Methodology

In Chapter II there is an evaluation of the conventional variance control procedures used by accountants. The purpose of this evaluation is to outline the limitations of these conventional procedures and, in this manner,



to establish the need for further study. Next, Chapter III inquires into the conceptual nature of control in order to provide clues to help establish more adequate control procedures. Chapter IV reviews several statistical procedures that have been proposed for variance control. These procedures are identified as the Basic Control Chart approach; the Bierman, Fouraker, and Jaedicke approach; and the McMenimen approach. Some aspects of each of these methods will be questioned. In an attempt to counteract these limitations, this writer has applied two additional methods in Chapter V. One which is referred to as the Equalization approach sets the control limit at that value where the probability of a Type I error times the opportunity cost of a Type I error is exactly equal to the probability of a Type II error times the opportunity cost of a Type II error. The other method developed in Chapter V establishes the control limit at that value which minimizes the expected opportunity costs. It is identified as the Minimization approach.

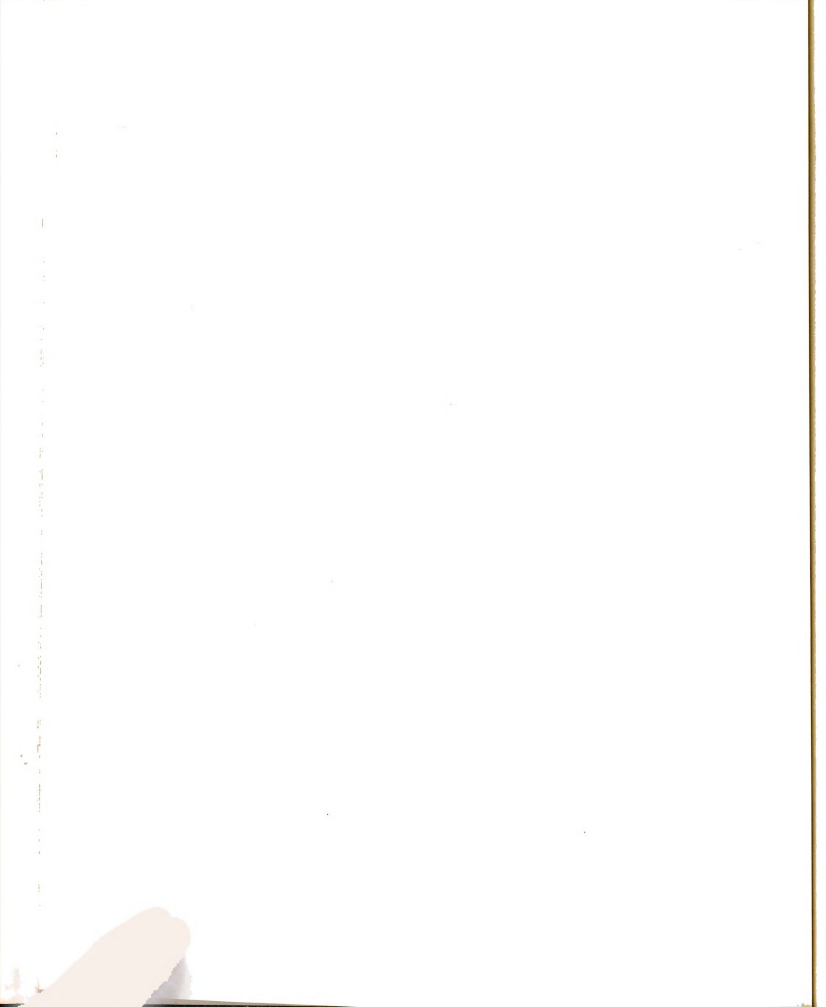
In Chapter VI all of the variance control methods indicated above are tested in an attempt to rank them in order of their usefulness for variance control. This test is accomplished through three major steps. In the first step a hypothetical example is developed which involves the time required for each of fifty meat cutters to butcher each of twenty cows. It is assumed that each of the 1,000



performances is investigated to determine the cause of its variance. The value of each performance is recorded and identified as to cause. In addition to chance the following assignable causes are assumed: dull knives, tough cows, lack of training, poor attitude, illness, improvement, and laziness. This detailed information of the values that occurred for each cause is used to complete the second step in the test of the variance control methods.

The second step involves the calculation of upper and lower control limits for each of the six above mentioned methods. Moreover, these control limits are calculated for each of four testing plans. The first two testing plans involve tests of single observations rather than samples. In the first plan the worker compares each performance with the control limits and reports any performances falling outside of these limits. The foreman compares every tenth performance on the average with the control limits in the second plan. The last two sampling plans consist of a comparison of a mean of a sample of five performances with the control limits based on these plans. In the third plan each performance is included in a sample. In the fourth plan one sample is taken for every fifty performances.

The third step in the test of the variance control methods involves a financial examination of the resulting differences found among the control limits associated with each method. This financial examination will consist of



analyzing the approaches by twos insofar as it is necessary to rank them in preferential order. Of any two approaches being compared, the one closer to the standard bears a greater investigation cost than the one farther from the standard. However, it also carries additional savings because of more timely detection of assignable causes. These investigation cost and savings figures are dependent upon the assumptions outlined in Chapter VI. A decision will be made on the following basis:

1. If the added savings is greater than the added investigation cost, the approach with the control limit closer to the standard will be regarded as more effective.
2. If the added savings is less than the added investigation cost, the approach with the control limit farther from the standard will be regarded as more effective.

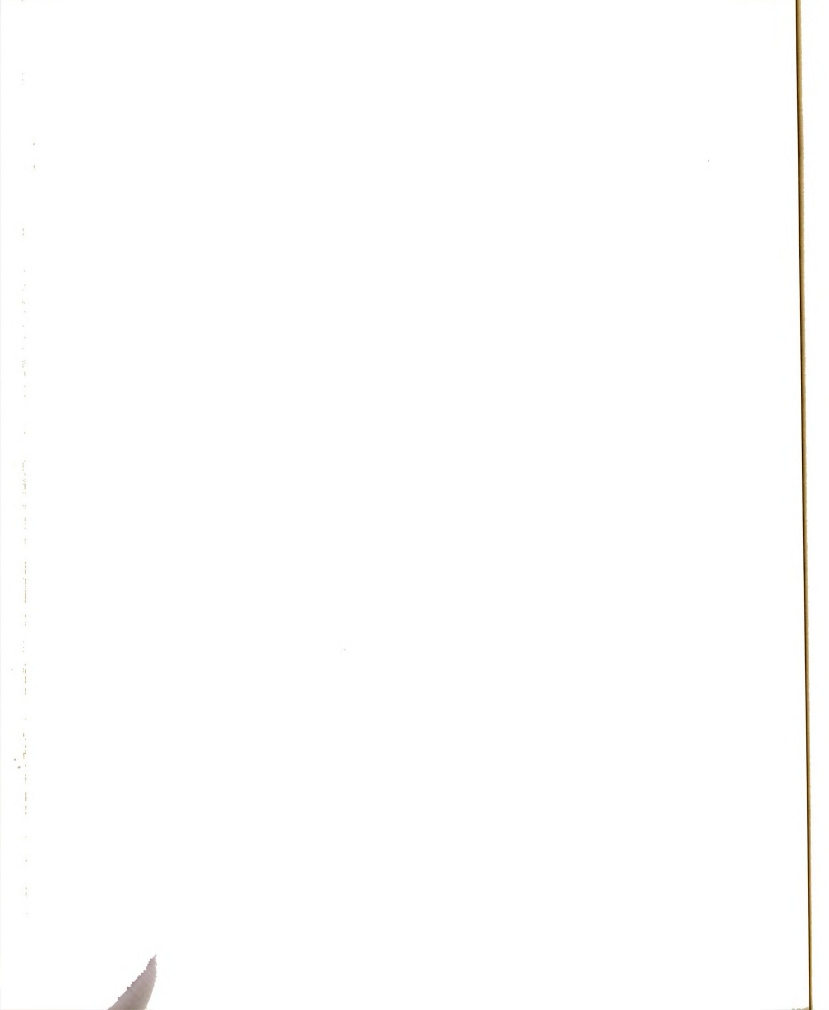
This analysis for each pair of approaches will be performed until it becomes possible to rank all of the approaches.

The conclusions are presented in Chapter VII.

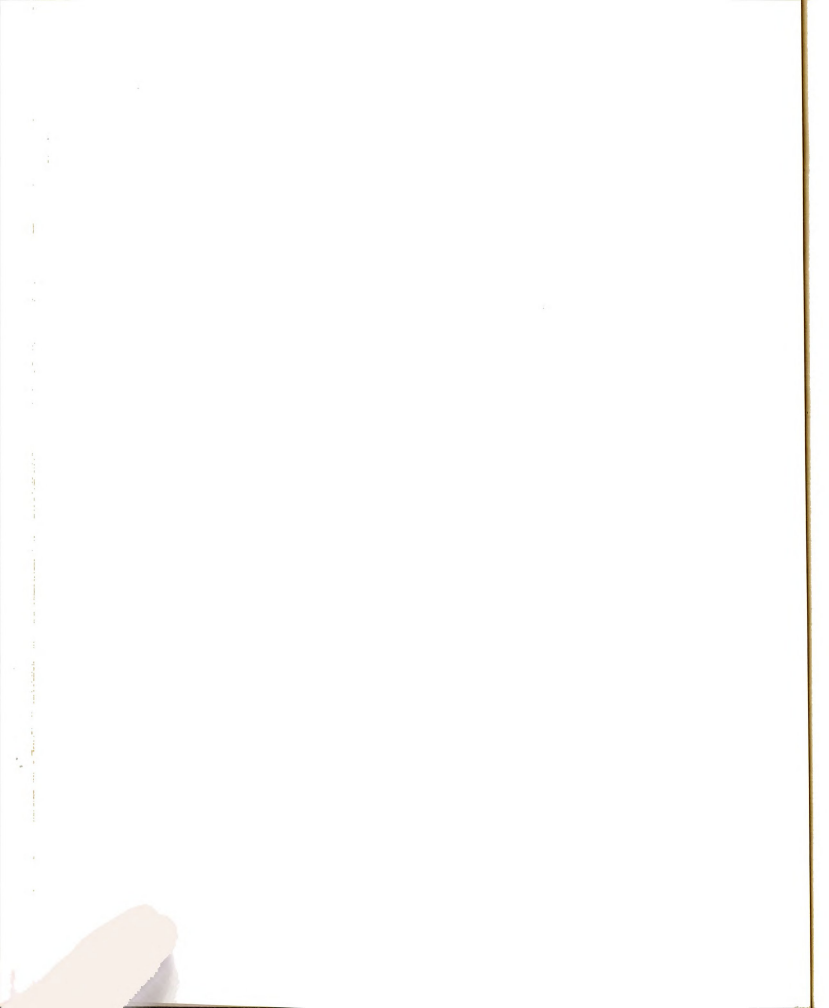
Contributions

The following contributions emerge from this study:

1. A conceptual distinction between significant and insignificant variances is established through specific recognition of chance factors.

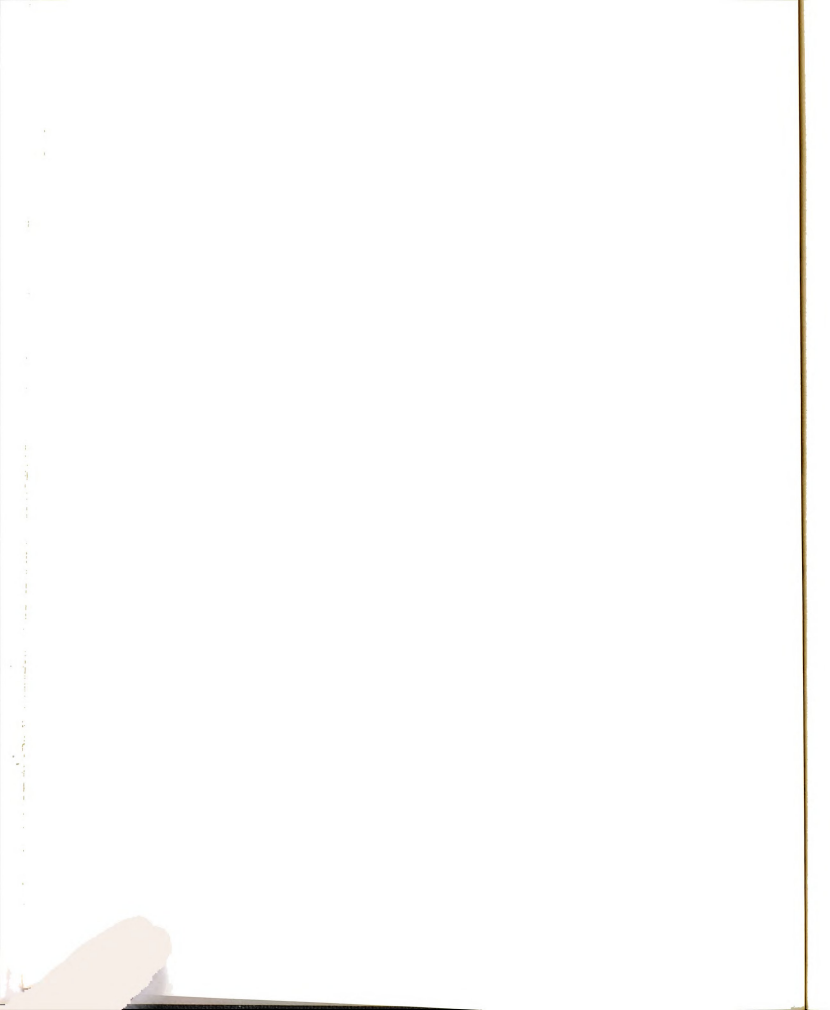


2. The chance concept is used to alter the definition of control to enable a clearer description of what is actually involved in the control process.
3. The statistical models that have been proposed by others are evaluated in order to clarify their strengths and deficiencies.
4. Two additional models are developed from available statistical concepts. These recognize factors not considered in the models previously developed. The Equalization approach, which explicitly incorporates relevant factors 1 through 6, contributes by using factors 2, 4, 5, and 6 which have not been proposed as a group for variance control within the context of a Classical model. The Minimization approach contributes by formally including relevant factor 7 in addition to the first six. For this reason it has some Bayesian overtones and will from time to time be classified as a semi-Bayesian approach.
5. A test is developed in order to ascertain the advantages of the statistical methods and to rank them in order of their control effectiveness. The test adds new insights into variance control by:
 - A. Delineating and developing a probability distribution for each relevant assignable cause. This enables a more scientific estimation of



the probability of committing a Type II error and of the related opportunity costs.

- B. Separating the Bierman, Fouraker, and Jaedicke model into two approaches in order to show the effect of the conflicting interpretations of their probabilities.
6. The possibility of significant variances being averaged-out is illustrated. This illustration depicts the importance of:
- A. Focusing control at the performance level.
 - B. Considering the sample size and the frequency of sampling when setting performance control limits.
 - C. Using statistical techniques to interpret the results reflected in summary reports. These techniques consider the degree of summarization.
7. The framework for making accounting decisions is provided via the integration of accounting and statistical concepts.



CHAPTER II

AN EVALUATION OF CONVENTIONAL VARIANCE CONTROL

Conventional Significance Determination

Conventionally accountants have indicated the desirability of an investigation when either the dollar amount of the variance or the ratio of the variance to the standard have exceeded some cut-off point. These cut-off points have been determined on the basis of "subjective judgments, guesses, or hunches."¹ While "guesses or hunches or feelings for situations are fundamental parts of managerial behavior," Horngren stresses that "these subjective methods often engender management disagreements, barren investigations, and a sense of frustration."²

In some cases intuition may be so keen that control of the variances will be adequate; however, conventional procedures do not provide an objective means to verify adequate control. Often barren investigations are undertaken with the result that time is wasted looking for causes that do not exist.³ Likewise, investigations are sometimes

¹Charles T. Horngren, Cost Accounting--A Managerial Emphasis (Englewood Cliffs, New Jersey: Prentice Hall, Inc., 1962), 748.

²Ibid., 155.

³This error has been referred to as Type I error.

not undertaken when they should be. This error, known as a Type II error, results in delay in detecting assignable causes. It was noted in Chapter I and will be illustrated in subsequent chapters that these errors cannot be eliminated. The best solution to the problem of significance determination lies in striking an optimum balance between the two types of error. A major difficulty with conventional variance control is that it does not provide a framework to evaluate the probability of error for any given decision.

Of the two conventional methods, the percentage of the variance to the standard cut-off point is more desirable because it allows a greater dollar variance for larger dollar amounts. In most cases there is a larger amount of variability inherent in operations involving larger expenditures. In spite of this, however, greater percentage variability is expected in some situations than in others. For example, Bierman, Fouraker, and Jaedicke⁴ indicate that a \$10,000 variance from a \$10,000 budget for snow removal might be uncontrollable during a bad winter so that an investigation would be unprofitable even though the deviation is 100 per cent of the standard. On the other hand, a \$10,000 deviation from a \$100,000 budget for fire insurance may be worthy of an investigation despite the fact that the deviation from standard is only 10 per cent.

⁴Bierman, Fouraker, and Jaedicke, 113.

In this hypothetical example neither conventional method helps to determine which variance or variances should be investigated. The dollar amounts of the variances are the same--yet it is possible that the variance pertaining to fire insurance should be investigated while the one pertaining to snow removal should not be. In this case a 10 per cent deviation for fire insurance should be investigated while nothing can be saved from an investigation of a 100 per cent deviation for snow removal.

A thoughtful management would understand the differences between these expense classifications and consequently would not make the drastic investigation errors suggested by this extreme example. In less obvious cases, however, cut-off points may be uniformly applied and result in costly errors.

Although the cut-off points may be varied in practice, this procedure has not been widely discussed in the literature. The N. A. A. Research Report 22 implies that standard percentage cut-off points are consistently used throughout the firm. On a suggested form for analyzing the reasons for variances, the first row under a column labeled "reason for variance" contains the following "reason": "No reason, variance less than 10 per cent."⁵

⁵ National Association of Accountants, The Analysis of Manufacturing Cost Variances, Research Report 22 (New York: National Association of Accountants, August 1, 1952), 12.

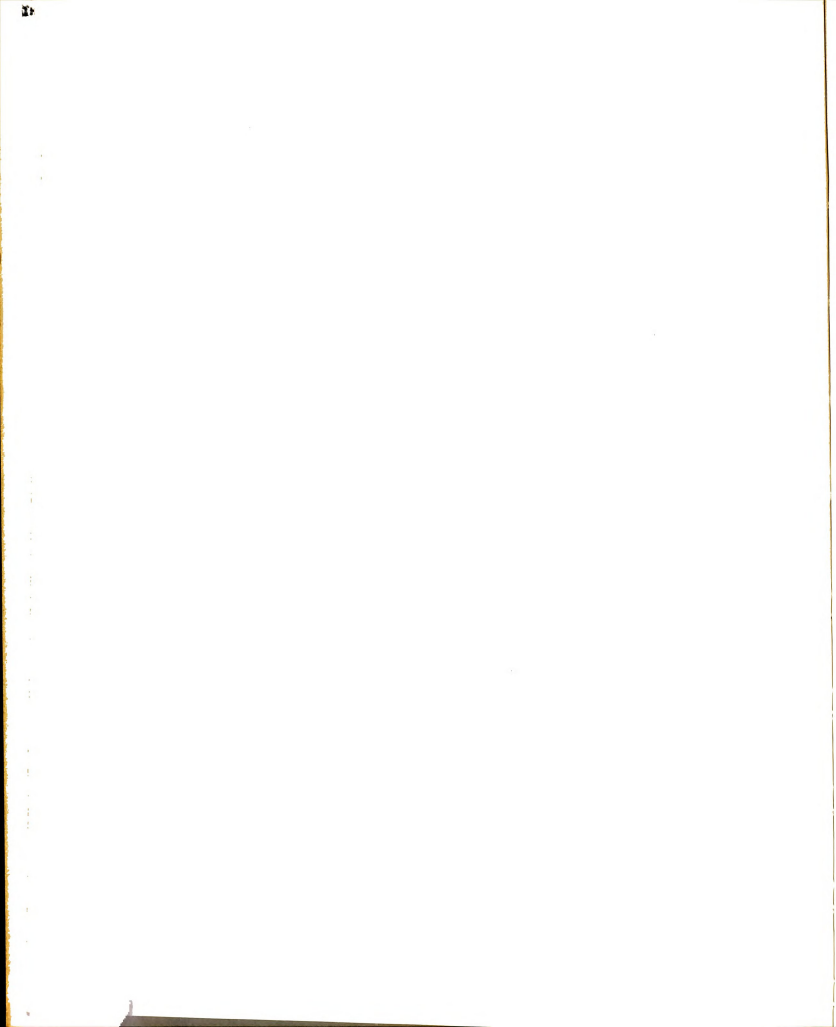
It is not logical that a variance less than 10 per cent is automatically due to chance nor that one larger than 10 per cent is automatically attributed to an assignable cause.

Allowance for different expectations of variability could be introduced by varying the percentage cut-off points. Percy Carter⁶ has recently suggested some guidelines for varying the cut-off points between 5 and 15 per cent depending upon the cost center and the type of expenditure. Even with this improvement, however, the cut-off points remain essentially arbitrary without explicit consideration of process variability.

Treatment in the Literature

In order to determine the treatment accorded to performance control in the literature, some forty cost and managerial accounting textbooks and numerous journal articles were reviewed. This review revealed that there is general agreement that performance control is a major benefit to be derived from the operation of a standard cost system. Detailed attention is devoted to the calculation of the following seven basic variances: material price, material usage, labor rate, labor efficiency, variable overhead efficiency, fixed overhead budget, and fixed overhead volume. In many cases further refinements are made to

⁶Percy C. Carter, "Maintaining the Adequacy and Accuracy of Standard Costs," N. A. A. Bulletin, XLV, No. 7 (March, 1964), 33-40.

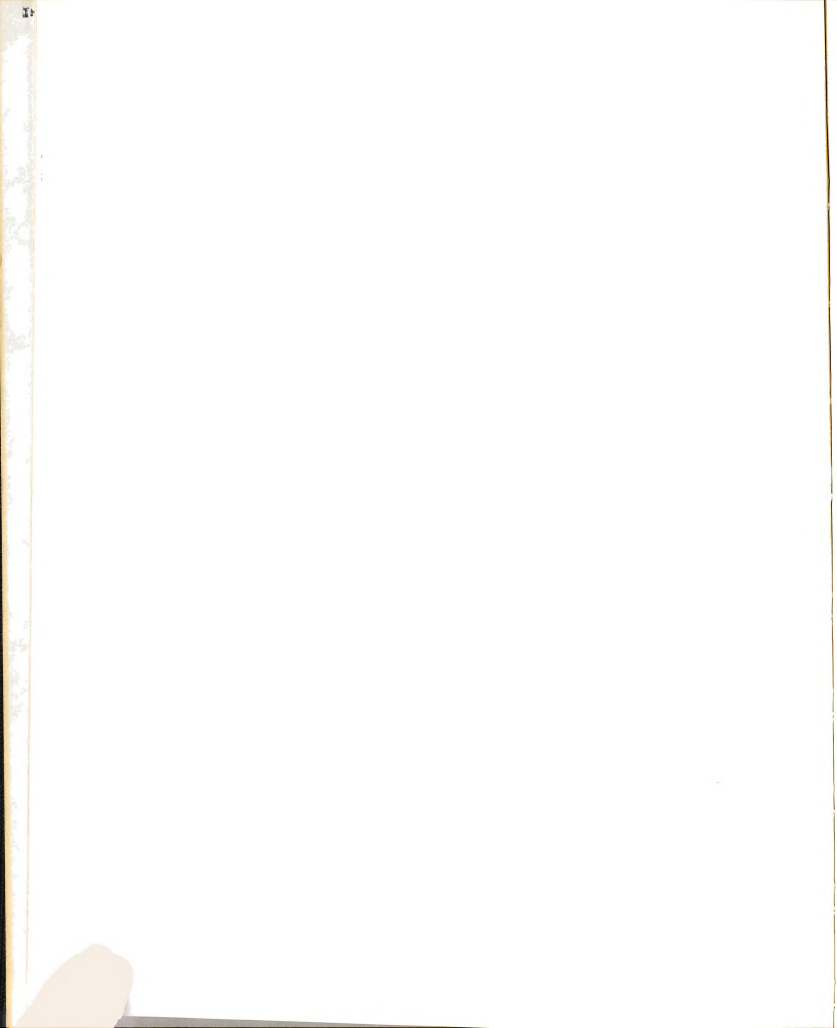


arrive at spoilage and grading variances. After the student occupies his time learning the techniques of these calculations, he reads that the accountant must highlight significant variances reflected in his report. There is a real danger that students go through these motions thinking that the report rather than control is the end product of a standard cost system.

Indeed, the accountant's preoccupation with the techniques of calculating the component breakdowns of variances has reduced his effectiveness in the control function. Allen Rucker warns that "there is a fascination about neatly tabulated figures and charts that needs to be resisted lest it lead managers to believe they are on top of their problems without thinking through them and coming to decisions."⁷

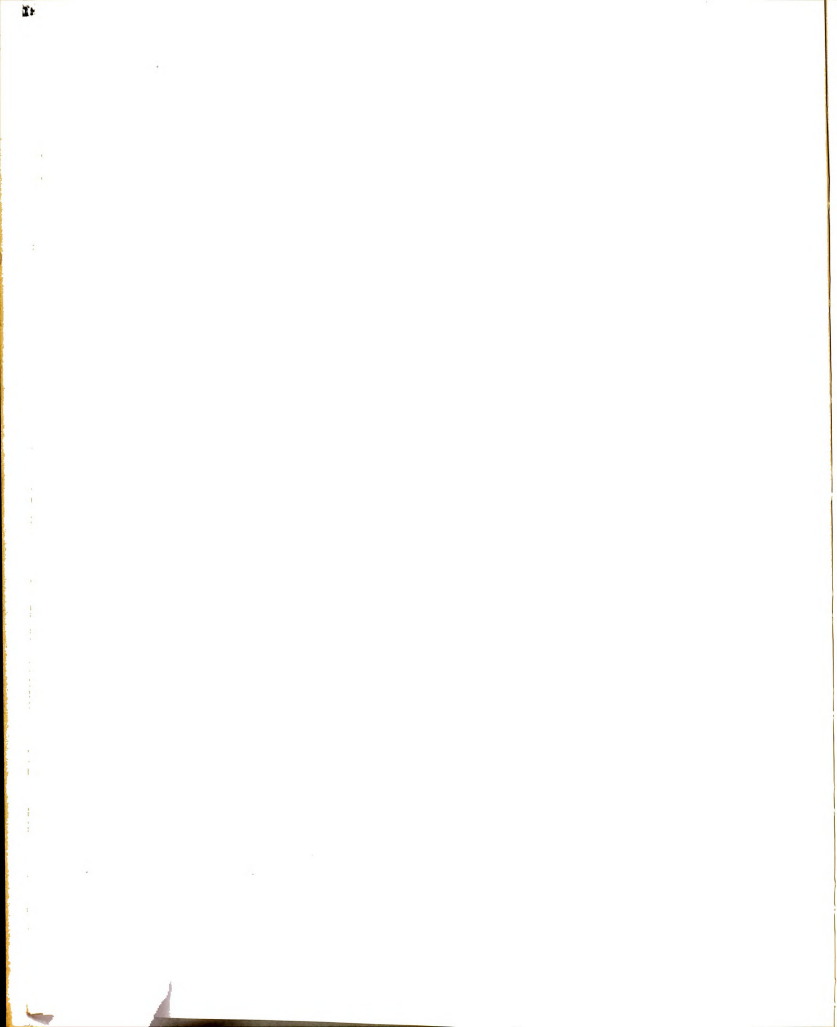
Most of the books and articles reviewed include no discussion of how a significant variance is recognized. However, a cursory comment regarding the importance of judgment in significance determination is often noted. Illustrative reports containing variances frequently do not identify those that are significant. A few of these books apply the 10 per cent criterion. They fail, however, to make it clear just why some variances are significant while others are insignificant. Reasons commonly advanced

⁷Allen W. Rucker, "Clocks for Management Control," Administrative Control and Executive Actions, eds. James Don Edwards and Bernhard Carl Lemke (Columbus, Ohio: C. E. Merrill Books, 1961), 329.

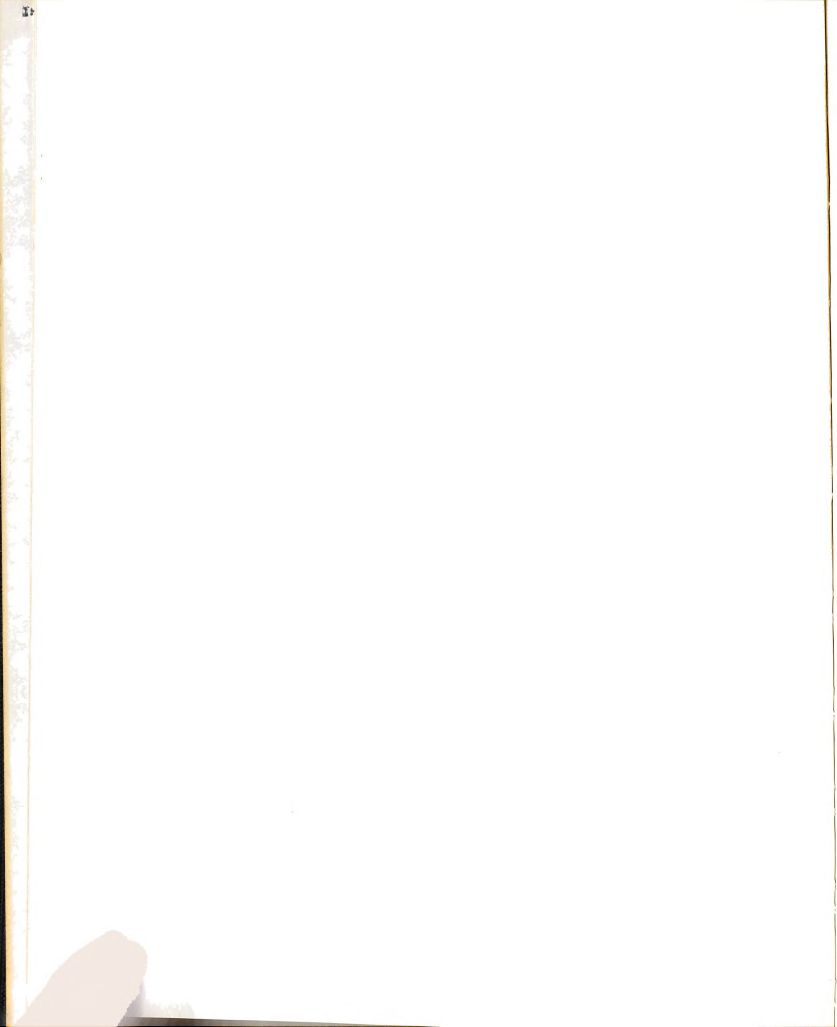


for an unfavorable labor efficiency variance were noted in Chapter I. They included faulty equipment, faulty materials, illness, laziness, poor attitude, and improper training. Since these reasons represent actual causes different from those involved in setting the standard, variances caused by these conditions are all significant. In none of the forty books and numerous articles that were reviewed is chance listed as a reason for a variance. Consequently, it appears as though accountants believe that all variances are the result of assignable causes. Their implicit distinction between significant and insignificant variances centers around a comparison of the cost of identifying and correcting the cause with the savings that will result from this action. How either the costs or the savings are determined is not specifically set forth.

In evaluating this conventional implicit approach, it is this writer's contention that a comparison of these costs with the resulting savings is an important consideration; but it is irrelevant if the variance is attributed to chance because nothing can be saved from an investigation. That is, if a variance is attributed to chance no assignable cause is present. Therefore, no savings would result from an investigation. Since accounting literature fails to mention chance as a possible cause of variances, it implies that all variances result from assignable causes. This would lead one to believe that accountants feel that



the savings figures should be derived by taking the present value of some multiple of the difference between standard and actual. This conclusion would be fine if, indeed, all variances did result from assignable causes. If this were true the present value of the savings should always be more than the present value of the cost of identification and correction because if a variance emanates from an assignable cause it should always be worthwhile to identify the specific cause and make the appropriate correction. The reasoning behind this is that the values of all assignable cause performances are eliminated when standards are established. That is, the standard should represent the mean of all performances which are due only to chance. In other words, only performances which are "in control" from the standpoint of management are included in setting the standard. If a cause subsequently appears, it should be worthwhile to eliminate it again if the standard is realistic. If the standard is not realistic it should be corrected. In no case should the known presence of an assignable cause be permitted to exist without reflection in the reports. If an assignable cause is reflected the variance is labeled as significant. The result of this reasoning is that the accountant is left without a way to explain the nature of insignificant variances until he recognizes the concept of chance.



Aggregation Problems

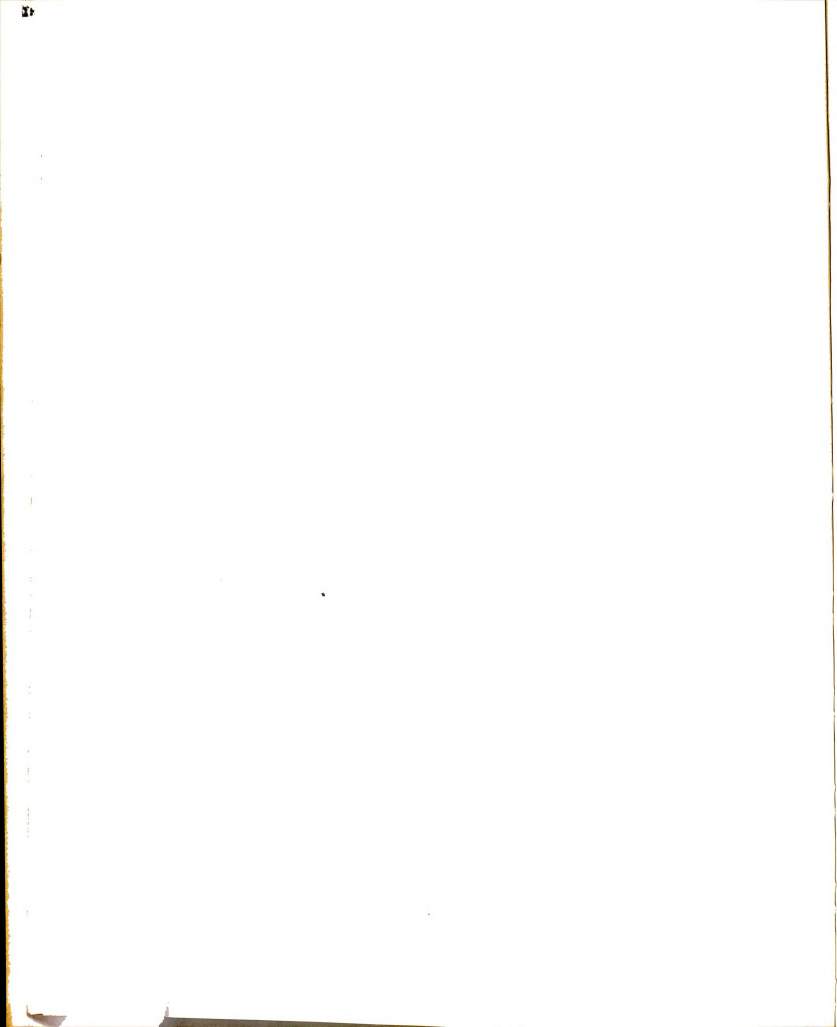
The accountant exercises his function in the control process primarily through departmental cost reports, which typically include time periods varying from a day to a month.

Some years ago the National Association of Accountants, then the National Association of Cost Accountants, published a report in which it indicated the following frequencies with which sixty-two companies reported labor performance variances:

Daily	7
Weekly	21
Monthly	25
Not at all	9
	62 ⁸

To the extent that the accountant waits for significant variances to show up on his report before he indicates the need for an investigation, his role in the control function is limited by a lack of timeliness. As C. E. Noble reports: "It certainly seems unwise to wave the red flag, to inform management that \$50,000 was lost last month

⁸National Association of Cost Accountants, How Standard Costs Are Being Used Currently, Complete N. A. C. A. Standard Cost Research Series (New York: National Association of Accountants, Not Dated), 40. (Records indicate that this publication was received at the Michigan State University Library in 1949.)



if the flag could have been raised the first day of the month and something done about the situation."⁹

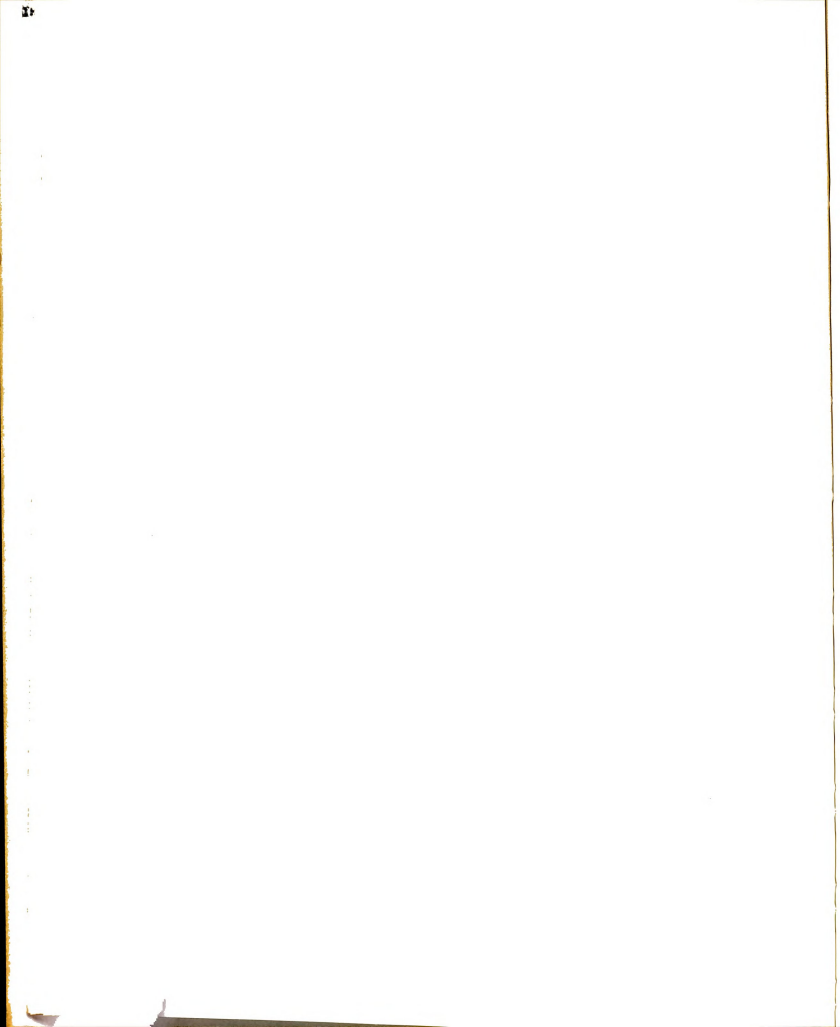
In addition to lack of timeliness, aggregation problems inherent in variance reports further hamper conventional variance control. Edwin Gaynor has recognized that these aggregation problems exist in conjunction with timing. He said:

Under conventional cost reporting methods periodic variances from the . . . standard are not discovered until the end of the day or week, or not until production and standard time are compared at the end of a payroll period--or perhaps not at all. Usually, an average for a relatively long period of time is composed of a great many compensating plus and minus variances completely overlooked simply because they are not apparent.¹⁰

These aggregation problems or, in Gaynor's terminology, problems of compensating variances, exist in the following ways: In cases where there is more than one operation in a department a significant variance in one operation may be off-set by the chance variances from the other operations. Even if variances are accumulated by operation, there is a danger, under conventional procedures, that significant variances occurring at one time during the accumulation process may be averaged-out by chance variances occurring during other times. These problems of "average-out"

⁹C. E. Noble, "Cost Accounting Potentials of Statistical Methods," N. A. C. A. Bulletin, XXXIII, No. 12 (August, 1952), 1477.

¹⁰Edwin W. Gaynor, "Use of Control Charts in Cost Control," N. A. C. A. Bulletin, XXXV, No. 10 (June, 1954), 1301.

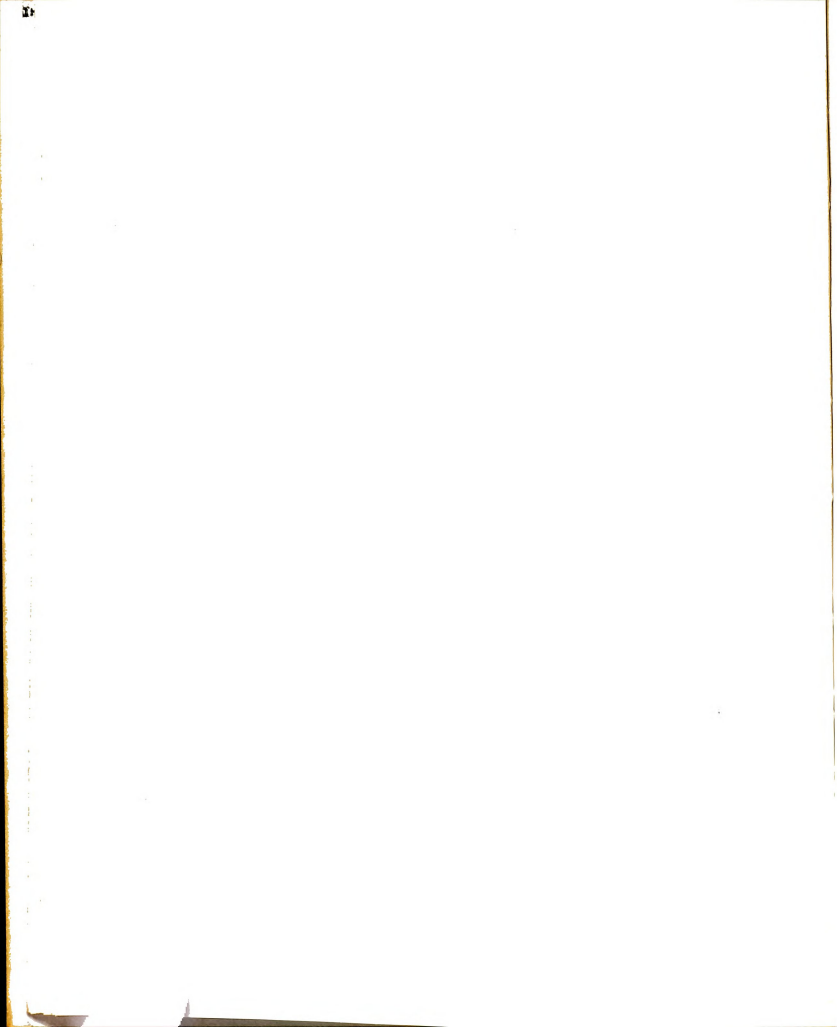


and "off-set" contribute further to the delay in detecting assignable causes.

It is difficult to evaluate the probability that significant variances are "averaged-out" or "off-set." Nevertheless, variances are sometimes found to be significant according to the accountant's conventional criteria for significance determination. Even, then, aggregation problems of the performance report contribute to delay in assignable cause detection. In order to locate the source of the significant results, it is necessary to sort through the detail that was used to build the report. The extent of this sorting varies in proportion to the extent of the summarization reflected in the report. As a possible solution to these aggregation problems L. Wheaton Smith suggested using the operation rather than the department as the unit of control. The importance of the operation as a unit is that "significant variations are localized in a particular operation and as occurring between certain times when the regular checks on that operation were made."¹¹

Keller and Ferrara assert that the accountant should begin the cost control process before the variances are accumulated. They introduce five lines of defense to protect against waste or off-standard conditions. These lines of defense consist of (1) workers, (2) foremen, (3) plant superintendent, (4) vice-president, and (5) president.

¹¹L. Wheaton Smith, Jr., "An Introduction to Statistical Cost Control," N. A. C. A. Bulletin, XXXIV, No. 4 (December, 1952), 511.



For the lower lines of defense, they stress the importance of observing variances as they occur so that significant ones can be eliminated long before the reports are issued.

They state:

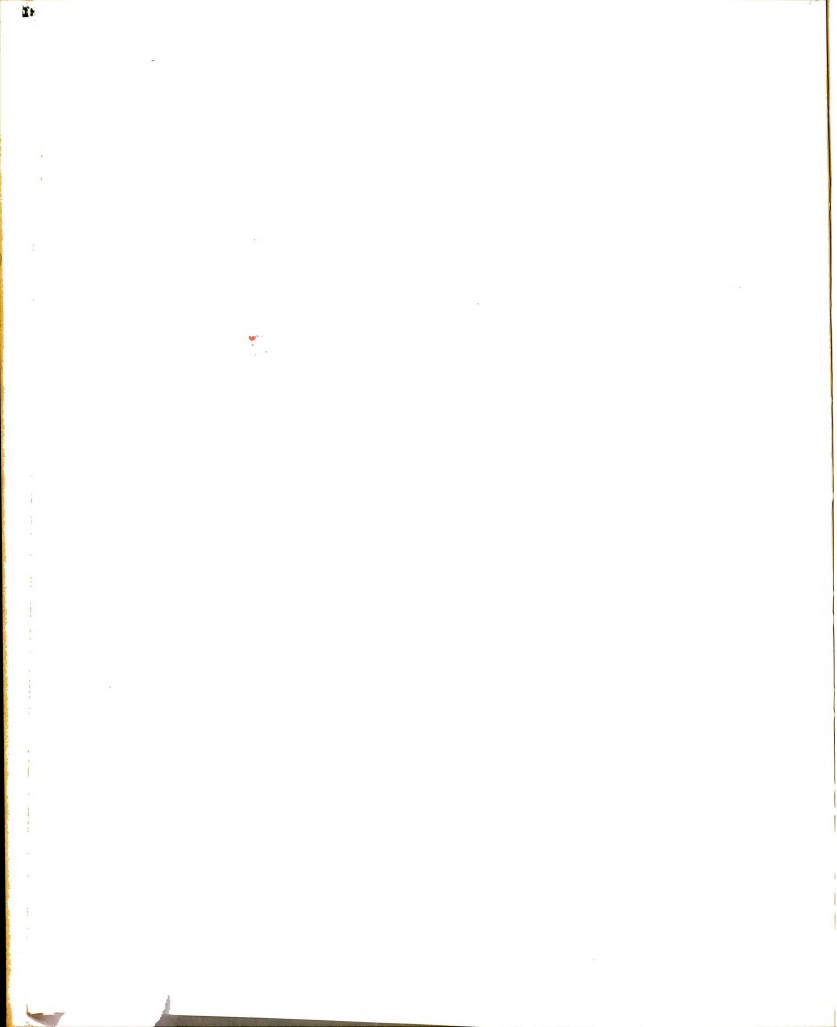
Workers, foremen, and supervisors should be aware of production standards, and thus it is not inconceivable to find that the root causes of some variances might be eliminated long before such variances are reported by the accountant, this is, by on the spot action taken by workers, foremen, and supervisors who observe variances as they are occurring.¹²

Need for further Study

It has been the purpose of this chapter to point out that while accountants have developed refined techniques for classifying variances (material price, material usage, labor rate, labor efficiency, variable overhead efficiency, budget, and volume components), their function in performance control is limited on three counts:

1. They have failed to explain conceptually the distinction between a significant variance and an insignificant one.
2. They have failed to utilize objective criteria for determining significant variances.
3. Their strict adherence to the report has caused delays in detection of significant variances because

¹²I. Wayne Keller and William L. Ferrara, Management Accounting for Profit Control (Second Ed.; New York: McGraw-Hill Book Co., 1966), 260.



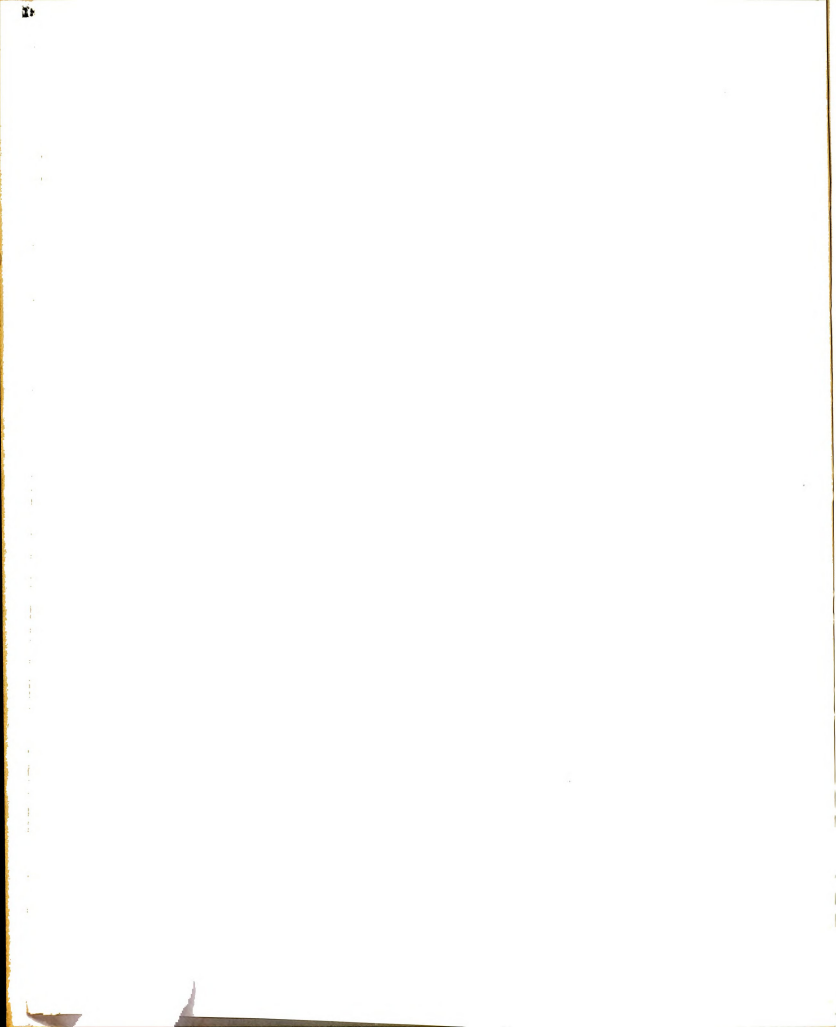
- A. An analysis is not made until the period covered by the report is completed.
- B. Significant variances can be off-set or averaged-out.
- C. Significant variances that do show up must be localized.

Dissertation Objectives

The purpose of this dissertation is to attempt to rectify these limitations by

1. Using quality control concepts to explain the distinction between significant and insignificant variances.
2. Examining more objective criteria for significance determination.
3. Illustrating through an hypothetical example the financial impact of employing these objective procedures at the individual performance level.
4. Showing through this example the tendency of significant variances to be off-set and averaged-out in the process accumulation used in developing the performance report.

By employing the statistical procedures at the performance level and by showing the financial impact of the aggregation problems, this writer hopes to persuade



the accounting profession to direct more attention toward observation on the performance level as a basis of control. The statistical procedures are intended to quantify observation by providing guidelines so that the worker and his foreman know by objective criteria when the variance is significant. Under this system the conventional summary report will not be the primary function of control; but it will be used to illustrate the financial impact of any efficiencies or inefficiencies.

CHAPTER III

CONTROL AND CHANCE CONCEPTS

Chapters I and II noted that accountants currently lack an analytical foundation for variance control because they are unable to conceptually explain the difference between significant and insignificant variances. Each analyst appears to make his own arbitrary distinction on an ad hoc basis.

It was pointed out earlier that the ideas developed by quality control engineers would be useful. With this recognition an insignificant variance is defined as one resulting solely from chance and a significant variance as one resulting from both chance and an assignable cause.¹

This chapter will examine in more detail the conventional notions of variance control. Chance concepts

¹It was noted in footnote 4, Chapter I, that Type I and II errors when committed will render these definitions incorrect. In spite of this problem, the definitions are conceptually useful and the best available. Of course, for the statistically sophisticated reader, the definitions could be qualified in the following manner so that they will always be correct. An insignificant variance is one resulting solely from chance unless, of course, a Type II error has been committed in which event assignable causes are also unknowingly present. A significant variance is one due to both chance and assignable causes unless a Type I error has been committed in which event only chance is present.

will be incorporated into the accountant's concept of variance control. This fusion will establish a logical basis for the use of statistical procedures. Finally, the variance classifications will be studied in order to select those that are influenced by chance. Only for these will statistical procedures be helpful.

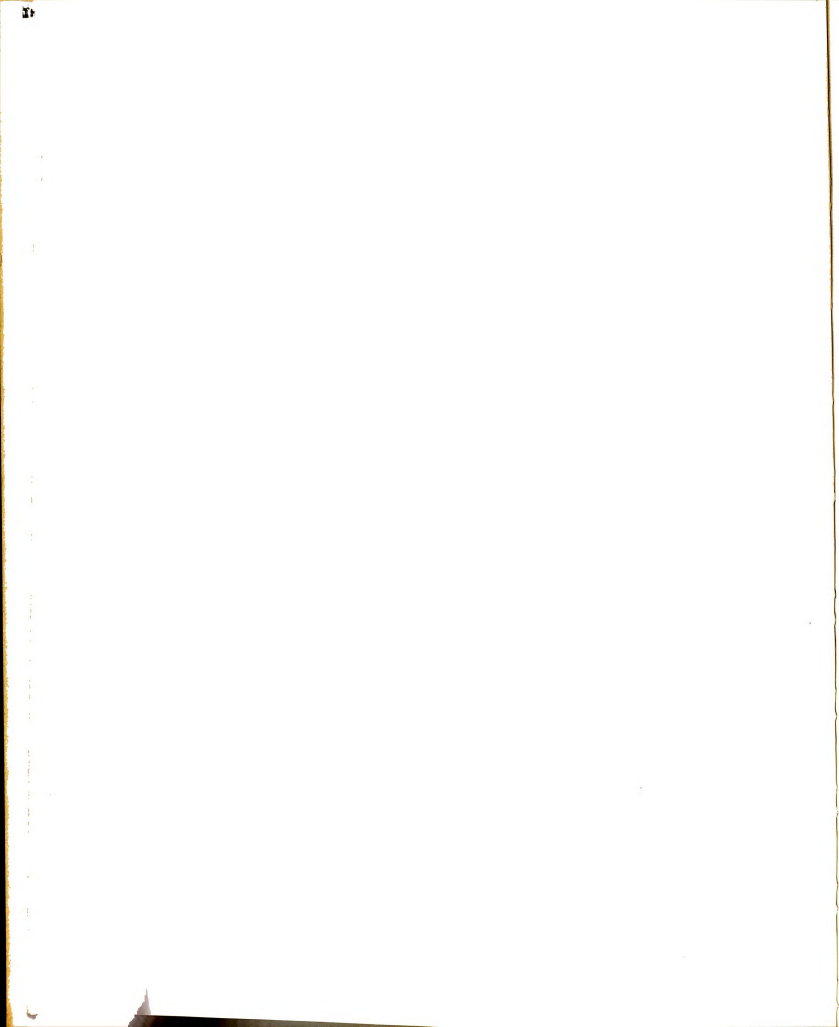
Definitions of Control

It is difficult to find a meaningful all-inclusive definition of control because there are so many different facets of control and many of these facets are exercised by different groups. Certainly there are differences between the kind of control exercised by the general stockholders over the board of directors and the kind which a mature man exercises over himself. Moreover, both of these facets are different than the control which an accountant exercises over variances. Even within the realm of accounting control there are differences in concept between control over variances, inventories, cash, and accounts receivable.

The following words are listed as synonyms of control: "authority," "influence," "power," "command,"² "regulate," "handle," "administer," "oversee," "look after," and "supervise."³ Of these words, all but the first two imply

²C. O. Sylvester Mawson, ed., Roget's Pocket Thesaurus (New York: Pocket Books, Inc., 1946), 44.

³Ibid., 201.



a line rather than a staff function. That is, they represent the kind of control exercised by the boss over his employees rather than the control exercised by the accountant over variances. In this regard the accountant has the authority, and indeed, the responsibility to determine the significance of the variances; but the command facet implies action which the accountant in his staff function would not (and probably should not) undertake.

While the accountant does not take action, he uses his influence to encourage management to take appropriate action where and when it is needed. This notion of influence is in accord with James L. Peirce's suggestion that control "does not take action, but it frequently impels action by turning a spot light on the pertinent facts."⁴ The words "authority" and "influence" might pertain to variance control, but they certainly do not adequately describe the function.

Webster's New Collegiate Dictionary defines the noun "control" as "anything affording a standard of comparison or means of verification; a check."⁵ Since control emanates from a comparison of actual with standard, this statement supplies a starting place on which to develop

⁴James L. Peirce, "The Planning and Control Concept," Administrative Control and Executive Action, eds. B. C. Lemke and James Don Edwards (Columbus, Ohio: Charles E. Merrill Books, Inc., 1961), 8.

⁵Webster's New Collegiate Dictionary (Springfield, Massachusetts: G. C. Merriam Co., 1956), 181.

the definition; but it does not hit at the heart of variance control. Webster's introduces the notion of limits in the following definition of the verb "control"--"to check or regulate, as payments; to keep within limits, as speed."⁶ However, this definition is not meant to apply specifically to accounting variance control. Moreover, it does not include the concept of chance. Finally, the dictionary definition of a controller as "an officer appointed to check expenditures"⁷ includes neither the notion of control limits nor the concept of chance. Consequently, neither synonyms nor dictionary definitions are very helpful in developing a conceptual foundation for variance control.

Accountants themselves have failed to develop an operationally meaningful definition of control. Eric Kohler has defined control as "the method and manner by which a person, or an organization, operation, or other activity is conformed to a desired plan of action."⁸ The objection to this definition is that the words "conformed to" seem to imply "made equal to." This suggests that accountants feel that standard and actual should be equal in order for control to exist. Since everyone knows that standard is rarely equal to actual, Kohler's definition is not

⁶Ibid. [Emphasis Mine.]

⁷Ibid.

⁸E. L. Kohler, A Dictionary for Accountants (Third ed.; Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1963), 127. [Emphasis Mine.]

operationally meaningful. It does, however, support an earlier contention that many accountants apparently feel that any deviation is the result of assignable causes; but that some are not worthy of action. Definitions similar to Kohler's typically appear in accounting literature.

One difficulty in developing a concept for variance control hinges on the fact that accountants have not decided what is meant by a significant variance. Carman Blough admits that the terms "significance" and materiality"

. . . are very important and yet we have no useful definitions of them. . . . Possibly these are terms which defy definition and whose meaning will have to be left to judgment in each situation, just as they have been in the past. However, if there are principles or criteria that may be used to interpret them, surely some effort should be made to develop and state them. If there are none, at least that could be stated.⁹

The accounting profession just has not identified chance as relevant to variance control. The books and articles listed in Appendix A pertain to accounting applications of statistical variance control; but they are concerned primarily with technique. They do not identify an insignificant variance as one due to chance nor do they discuss the fact that the use of probability statistics is logical because probability is a statistical area which evaluates patterns of chance occurrences.

It is, however, encouraging that Kohler defines a significant magnitude as

⁹Carman G. Blough, "Challenges to the Accounting Profession in the United States," Journal of Accountancy, CVIII, No. 6 (December, 1959), 38.

. . . measured by a departure from some norm or standard, to raise doubt that the deviation is the result of chance, random, or compensating factors; hence, indicating behavior calling for a better awareness or understanding of the cause, the removal of the cause, or a modification of the standard because of its inadequacy.¹⁰

On the other hand, it is informative to note that he defined statistical quality control as "the state of equilibrium reached when deviations from a given norm (such as the process average) are only random in character and without assignable cause."¹¹ A comparison of Kohler's definition of control with his definition of statistical quality control is interesting because he does not recognize the relevance of chance concepts for control but he does include them in his definition of statistical quality control. This writer contends that the accountant's failure to recognize chance concepts as they might pertain to accounting variance control has kept the profession from adopting statistical tools to aid in variance control. When chance is recognized, the usefulness of statistics becomes evident because statistics deals with an evaluation of the patterns of chance occurrences. Without the recognition of chance there is no apparent reason for using statistics.

To the extent that chance is relevant to accounting variance control, Kohler's definition of statistical quality

¹⁰Kohler, 446.

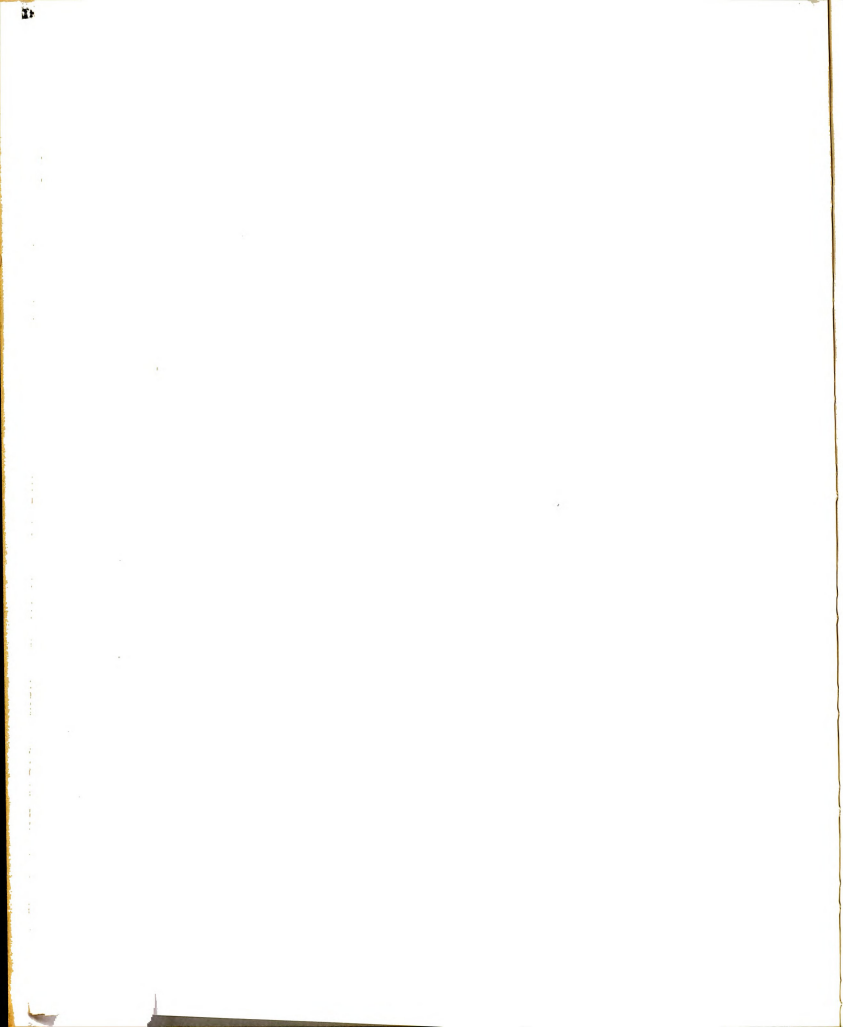
¹¹Ibid., 127.

control is operationally meaningful when applied to variance control. In the remainder of this chapter the nature of chance will be elaborated on further. This will be followed by an examination of the extent to which chance really pertains to accounting variances.

The Notion of Chance

Chance causes variations in the amount of time required to perform any activity even under substantially the same conditions. For example, a man does not consistently take exactly the same amount of time to shave. Some variation could be attributed to cold water, a dull blade, or a two-day growth instead of one; but if these assignable causes are eliminated, he still will be unable to shave in exactly the same amount of time. Likewise, there is a general lack of uniformity present in all natural phenomenon. Scientists agree that no two leaves, or snow flakes, or blades of grass are identical. This holds even when they are grown under the same conditions.

Chance explains differences in scores in sporting events, such as bowling or golf. Any bowler will agree that it is virtually impossible to continuously bowl the same score even though the same ball, shoes, and alley are used. In fact, many leagues award a prize to one who obtains the same score for three consecutive games.



Frank Gryna¹² has used a target analogy to illustrate the operation of chance patterns. In the left hand target of Figure 1 all shots have hit the bull's eye. Chance causes some variation in the shots; but the marksman still achieves a perfect score. Chance is also operative in the right hand target because again the marksman has failed to hit the same spot twice. Here, however, assignable causes are also operative because the marksman has not been hitting the bull's eye.

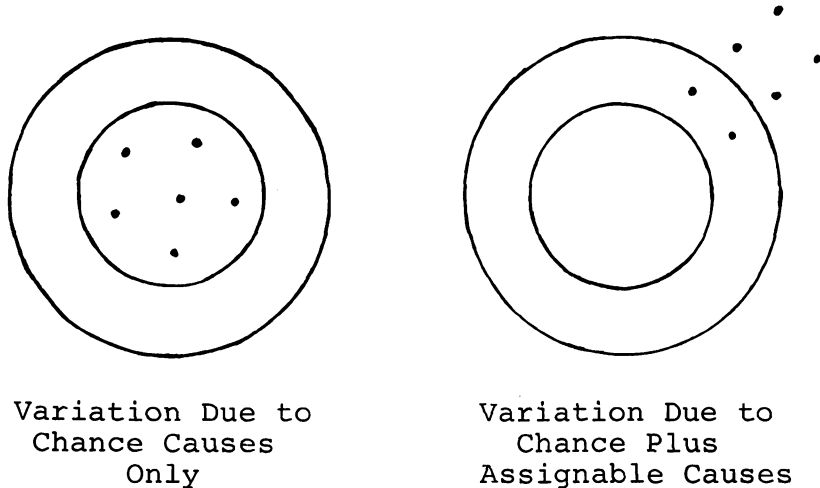
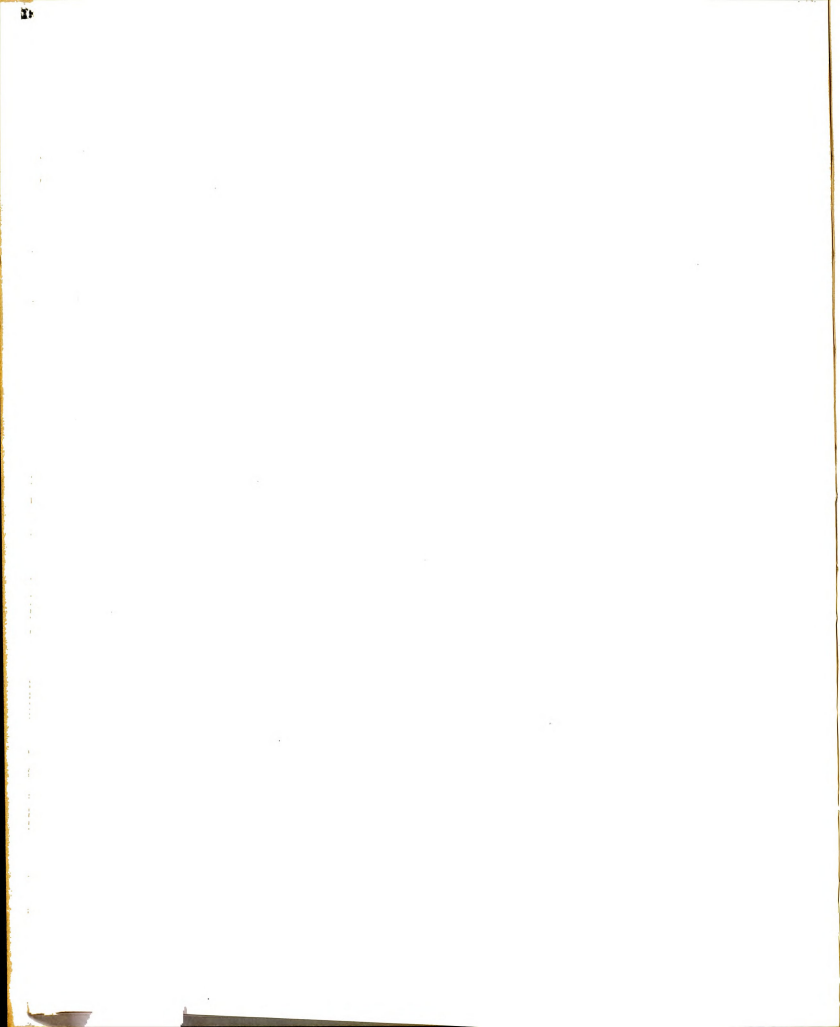


FIGURE 1.--Gryna's Target Analogy

Accounting variance control, like marksmanship and quality control, should be concerned with the distinction between variation due solely to chance and that due to chance plus assignable causes.

¹²Frank M. Gryna, Jr. "Statistical Methods in the Quality Function," Quality Control Handbook, ed., J. M. Juran (Second ed.; New York: McGraw-Hill Book Company, Inc., 1962), 13-42.



W. A. Shewhart who developed the control chart expressed variability and stability as the two characteristics of control. Variability is a characteristic because "a controlled quality must be a variable quality."¹³ Stability is a characteristic because results should vary only within pre-determined limits. In Shewhart's words, "The problem then is: how much may the quality of a product vary and yet be controlled?"¹⁴ The problem could be re-stated as follows to suit the accountants needs: How much may a variance vary and yet be in control?

Gryna's Target Analogy is oversimplified because the boundary between the chance population and the chance plus assignable cause population is clearly determined. In most variance control situations, the accountant is frustrated by the problem of overlapping populations.

Illustration of Overlapping Populations

In the following illustration of the problem of overlapping populations, a standard of 40 minutes has been established for the time to assemble a certain table. The probability distribution of chance performances shown in Table 1 indicates that chance performances have taken as long as 47 minutes and as few as 33 minutes. After the

¹³W. A. Shewhart, Economic Control of Manufactured Product (New York: D. Van Nostrand and Co., Inc., 1931), 6.

¹⁴Ibid., 3.

worker becomes familiar with the assembly operation, his skills improve. When his average time is reduced to 35 minutes, he is transferred to a more complex assembly operation and given a raise.

TABLE 1.--Probability distribution of chance performances for table assembly

<u>Minutes</u>	<u>Probability</u>
at least 33 but less than 35	.02
at least 35 but less than 37	.03
at least 37 but less than 39	.20
at least 39 but less than 41	.50
at least 41 but less than 43	.20
at least 43 but less than 45	.03
at least 45 but less than 47	.02
	1.00

Table 2 shows the distribution of chance performances after the improvement. Even though improvement is an assignable cause, chance also causes variation in performance values. Notice that the improved worker has performed his task in as few as 32 minutes; but that he has also taken as long as 38 minutes. The population of only chance performances (represented in Table 1) overlaps the population of performances due to improvement (represented in Table 2). The overlap indicates that only for results between 32 and 33 minutes is improvement conclusive because chance performances have been completed in as few as 33 minutes but never in as few as 32. Only for results over 38 minutes is it clear that improvement has not occurred because

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improved performances have taken as long as 38 minutes but never longer.

TABLE 2.--Probability distribution of chance performances resulting from assignable cause due to improvement

<u>Minutes</u>	<u>Probability</u>
at least 32 but less than 33	.03
at least 33 but less than 34	.07
at least 34 but less than 35	.40
at least 35 but less than 36	.40
at least 36 but less than 37	.07
at least 37 but less than 38	.03
	1.00

Figure 2 shows these overlapping populations graphically. The solid curve shows the distribution of chance performances and the dotted one shows the distribution after improvement has occurred. If 38 is selected as the lower control limit, the risk of a Type II error will not be incurred; but the risk of a Type I error is relatively high (equal to the proportionate area of the chance population, under the solid curve, below 38). As the control limit is reduced, the probability of committing a Type II error increases. It is equal to the proportionate area which is higher than the control limit under the dotted curve. At the same time, however, the probability of committing a Type I error is reduced because the proportionate area less than the control limit under solid curve will decline as the control limit declines. Thus, the probability of committing one kind of error can be reduced only at the expense of increasing

the other. The problem of determining significance involves striking an optimum balance between the probabilities of committing each of these errors. Certainly the opportunity cost of an investigation and the opportunity cost of failing to detect an improvement are relevant in striking this optimum balance.

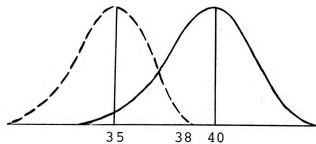


FIGURE 2.--Figure showing overlapping populations

The problem of determining the upper control limit is enhanced because significantly unfavorable variances may be caused by any number of assignable causes such as illness, laziness, lack of training, faulty equipment, faulty materials, etc. Most of the remainder of this dissertation will be devoted to an evaluation of various techniques for striking a balance between the probabilities of committing each of these errors. The intent of this evaluation is, of course, to discover that technique which yields the optimum balance.

The argument for the use of statistical procedures to determine control limits has been built around the

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premise that chance factors are expected to cause variances. The illustrations just covered which involve the time required for table assembly and the time needed to shave indicate the operation of chance on the labor efficiency variance. Since the extent of the possible usage of statistical techniques for variance analysis is dependent upon the extent to which chance factors cause variances, it is now appropriate to survey the other variance classifications to determine for each the extent to which chance is operative. Because the amount of the variance and whether it is favorable or unfavorable depend upon how the standards are established a brief discussion of the setting of standards will preface the examination of the presence of chance in the variance classifications.

Setting Standards

Standards fall into at least three categories:

1. The theoretical, ideal, or perfection standard.
2. The attainable good performance standard.
3. Average past performance standard.¹⁵

It is not expected that the ideal standard "will be attained in actual operations, but the standards are set up as goals toward which to work in the attempt to improve efficiency."¹⁶ The objection to this type of standard

¹⁵National Association of Cost Accountants, How Standard Costs . . ., 8.

¹⁶Ibid.

is that employees without an objective that they can reasonably be expected to meet may "cease to pay serious attention to the standards."¹⁷

The weakness of standards based on average past performance "lies in the implicit assumption--most often wrong--that what has happened in the past is what should continue to happen."¹⁸

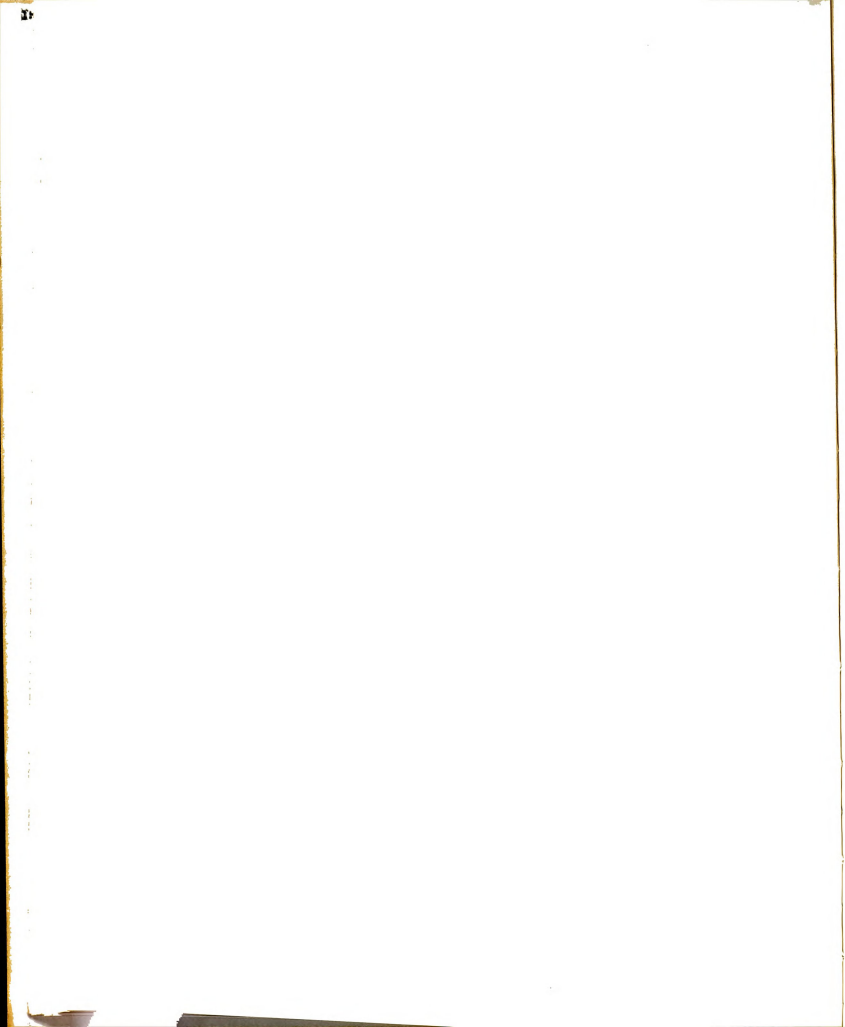
In fact, the Lybrand Newsletter recently reported that "experience of repeated instances indicates that work pace is rarely more than 60 per cent of what ultimately proves to be a reasonable standard."¹⁹

Consequently, standards based on attainable good performance are most effective. Good attainable performance should be established by chemists, engineers, and foremen who are familiar with the material and manpower requirements. The values are determined by a series of observations, revisions, and further observation until the mean of the performances coincides with what the experts consider to be good attainable performance. Performances attributed to assignable causes are not included in the set of values which are averaged in arriving at the standard.

¹⁷ Ibid.

¹⁸ Richard L. Smith, Management Through Accounting (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1962), 397.

¹⁹ Lybrand, Ross Brothers, and Montgomery, "Reducing White Collar Costs," The Lybrand Newsletter (November, 1964), 5.



Accordingly, the standard represents an average; but an average of current performances based on capabilities rather than an historical average.

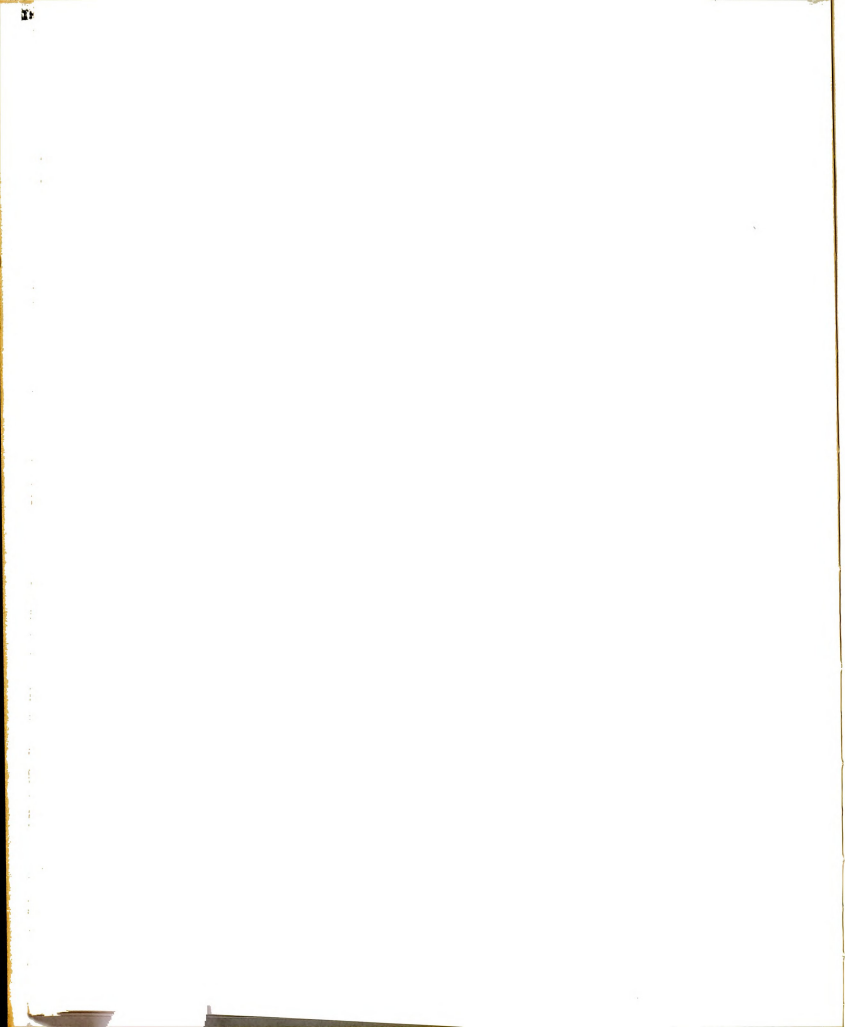
It is possible to have a satisfactory standard; but too much variability among the performance results. Continued performance, observation, and revision can reduce this variability. Also, workers become more uniform as they become familiar with their new tasks. Once the variability has been reduced as far as deemed profitable, it is the accountant's task to measure results and to highlight significant deviations. With the standards set according to the procedure just described, favorable variances will be expected to occur with the same frequency as unfavorable ones. Each will occur one half of the time when the operation is in control.²⁰

Chance Influences on Individual Variance Classifications

Material Quantity Variance

Specifications are established for the number of pages in a book, the board feet of wood in a piece of furniture, the pounds of metal in a typewriter, and the square feet of fabric in a suit. Thus, it might appear that the

²⁰ Usually, unfavorably significant variances will occur more frequently than favorably significant ones. However, significant variances are not included in the set of values which are averaged to obtain the standard. Moreover, the operation is not in control when significant variances are present.



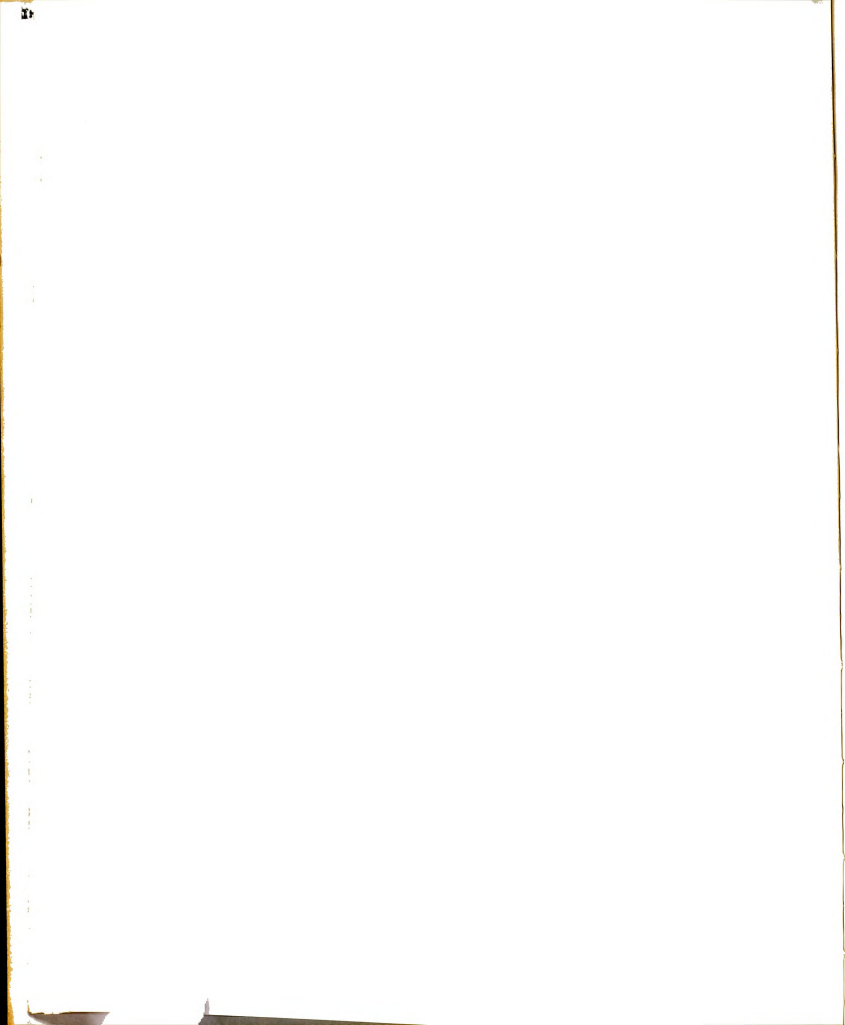
quantity of material used is not influenced by chance. It is doubtful, however, that the same amount of varnish is used for each piece of furniture (of the same style), or that the same amount of glue is used in assembly. A. C. Rosander²¹ reports chance variations in the number of grains of material used in the manufacture of apparently identical stockings. Accordingly, it seems reasonable that the number of grains of material used in any fabric might vary.

Perhaps more important than its influence on the amount of material appearing in good units is the effect chance has on the number of units spoiled while in process, the number rejected as finished goods, and the number that can be sold as seconds. Conventional standards properly allow for the expected amounts of these factors as well as the expected amount of material shrinkage. Sometimes separate variance accounts are established to isolate these various influences. What is now needed is the application of probability statistics to analyze the material quantity variance and its subdivisions.²²

As with the labor efficiency variance, control over the material quantity variance is truly effective only if

²¹A. C. Rosander, Industrial Quality Control, XI, No. 8 (May, 1955), 26.

²²For a control chart application for the analysis of material quantity variances see Dewey W. Neal, NAA Bulletin, XLII, No. 9 (May, 1961), 73-78.



it is applied at the performance level. Aggregate account balances are subject to the same average-out, off-set, and timing problems that hinder adequate control over labor efficiency.

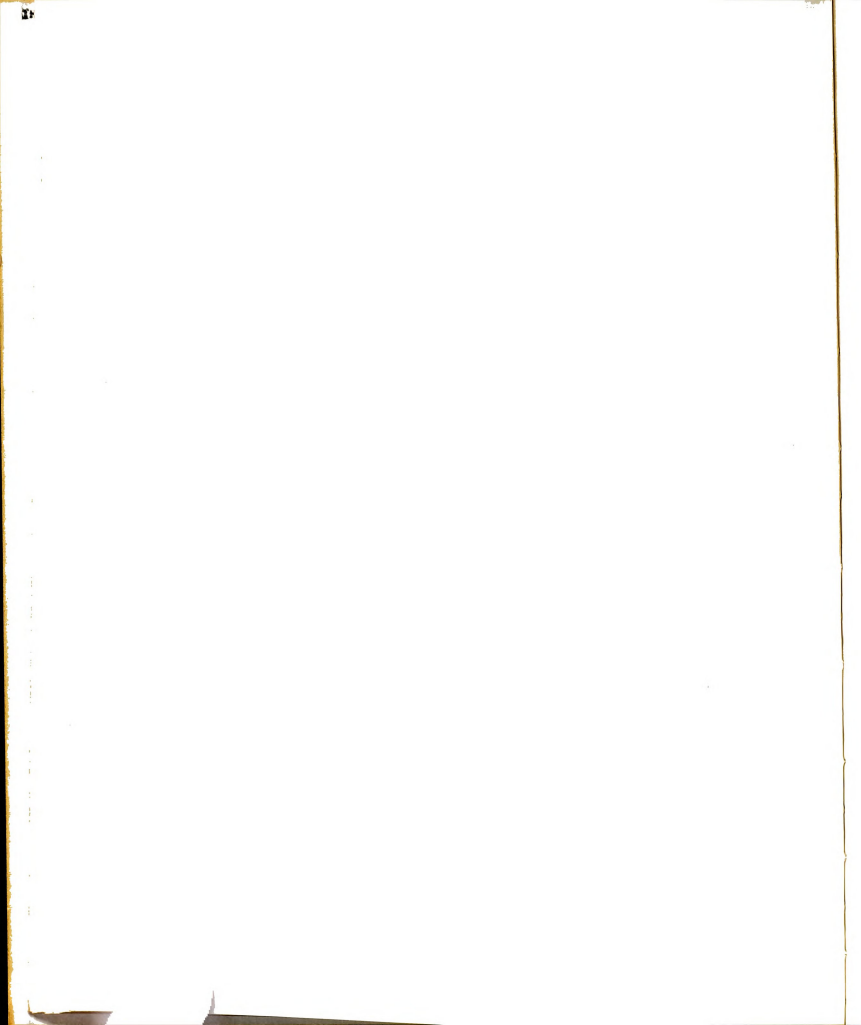
Labor Rate Variance

The labor rate variance does not often fluctuate randomly. Wage rates are generally negotiated and stated in labor contracts. When the rates change, the standards should be revised. Variances may arise from using a different labor classification than that established for a job or from using overtime. Both actions may be desirable in the short run in certain circumstances; but they should, nevertheless, be identified and explained. Accordingly, statistical procedures have extremely limited usefulness in analyzing the labor rate variance.

Material Price Variance

Similarly, the material price variance would not often be expected to occur randomly. The prices of many materials are administered. In cases where prices vary between suppliers, it is the responsibility of the purchasing department to make the most judicious purchases. Gillespie points out that in addition to negligence on the part of the purchasing department, a material price variance could reflect:

1. Failure of factory to anticipate needs.
2. Rush order accepted by sales department.



3. Transportation strike.
4. Error in forecasting costs.²³

As with the labor rate variance, it is advantageous to identify these causes so that responsibility can be established. Statistical variance analysis can only be helpful for prices which fluctuate randomly, such as those that truly reflect the conditions of supply and demand.

Variable Overhead Efficiency Variance

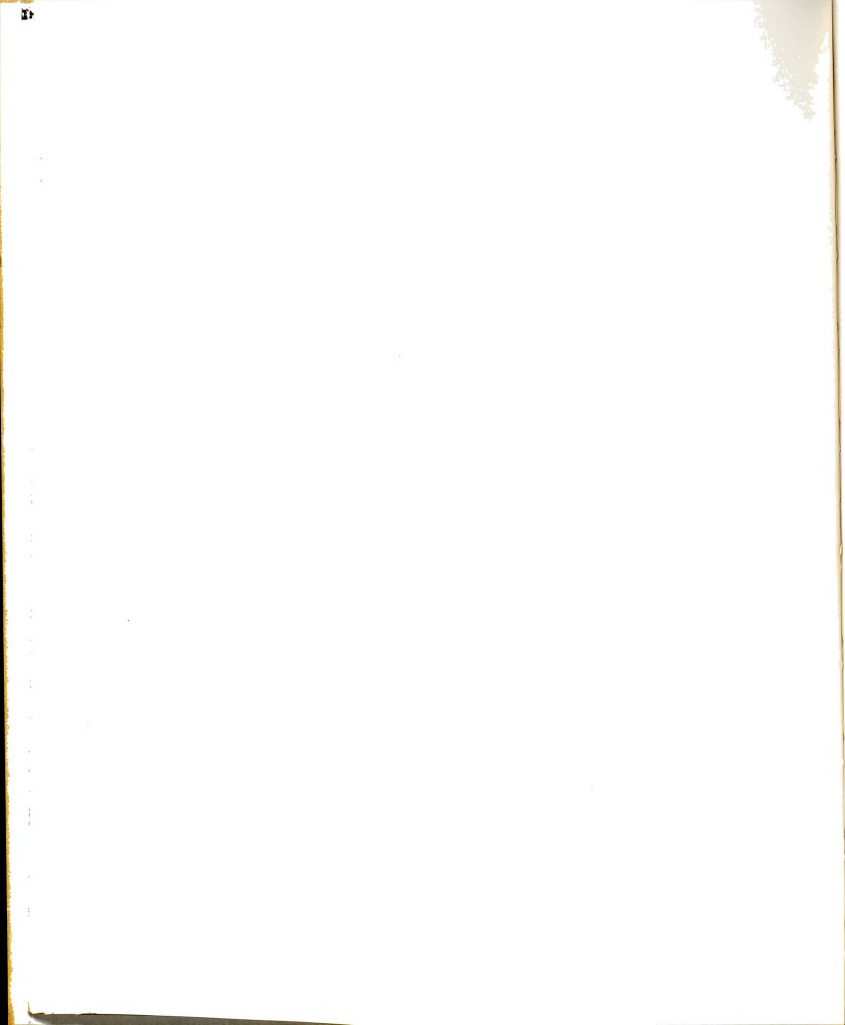
Overhead expenses can usually be identified with a particular cost center and, in this manner, responsibility for the various costs can be established. On the other hand, "physical standards exist for very few elements of factory overhead in the same sense that physical standards exist for direct materials and direct labor."²⁴ Therefore, the efficiency variance is usually analyzed in monthly departmental reports which represent a summary of the departmental expenses for the entire month. Thus the average-out, off-set and timing problems are present in this analysis. Keller and Ferrara report:

The summary nature of these variances for all practical purposes eliminates any control features, except perhaps the possibility of illustrating the overall profit realization of waste in factory overhead which could bring forth a fuller realization of waste and thus yield an important pressure for cost control on the prior lines of defense.²⁵

²³Cecil Gillespie, Standard and Direct Costing (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1962), 63.

²⁴I. Wayne Keller and William L. Ferrara, 179.

²⁵Ibid., 325.



It is suggested that physical standards be established for overhead in order to bring about more adequate control. The National Association of Cost Accountant's report How Standard Costs Are Being Used Currently states:

With overhead it is especially important that control be exercised at the source of the cost. After various prorations or distributions have been made the results of excess spending become diffused and it is virtually impossible to ascertain how much inefficiency has cost or who was responsible for it.²⁶

Physical standards could be expressed in terms of the time required to clean designated areas, to remove six inches of snow from the parking lot, to set up a machine, etc. Performance should then be checked on a sample basis by superiors. Phil Carroll suggests that time studies with incentives should be applied to indirect work. He writes:

You need some kind of work standards to control costs. Either you set standards or your people set their own. The difference is large. . . . It amounts to about 67 per cent excess costs when employees decide how much work to do. The 67 per cent is the difference between the 100 per cent you pay for and 60 per cent experts say you get on 'day work.'²⁷

The aggregate overhead variances should not then be relied upon to control overhead costs. Their purpose should be relegated to (1) show the total impact of inefficiencies, (2) review the adequacy of control, and (3) explain the difference between budgeted and actual costs for the period.

²⁶ National Association of Cost Accountants, 45.

²⁷ Phil Carroll, Overhead Cost Control (New York: McGraw-Hill Book Company, 1964), 79.

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Budget Variance

The budget variance results from subtracting actual fixed factory overhead from budgeted fixed factory overhead. Some expense classifications, such as rent and salaries, are arranged by contract; others, such as depreciation, are decided by company policy; and still others, such as insurance rates and taxes, are decided by outside agencies. Chance is not operative for any of these kinds of expenses; therefore, statistical procedures are not useful for analyzing any resultant variances. All variances should be explained.

Chance may contribute to some variation²⁸ in the fixed portion of heat, light, and power and therefore admit the possible usefulness of statistical procedures. On the other hand, since the aggregate account does not pinpoint the source of trouble, control may best be exerted by checks to see that machines are not running when they are not being used, that rooms are not overheated, that lights are turned off when the rooms are not in use, etc.

Volume Variance

To the extent that a pre-determined volume will never be precisely attained, chance is expected to operate on the capacity utilized. Consequently, statistical procedures can be helpful in analyzing the volume variance.

²⁸The term "fixed" does not mean that this portion of these expenses does not vary; but only that they do not vary in respect to productive activity.

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Non-Manufacturing Variances

Some have suggested that standards be established for clerical work. Charles H. Grady, Jr. contends that the clerical supervisor does not spend as much time planning and controlling the activities of his people as the factory supervisor. He offers this as partial explanation for the "continued trend toward larger proportions of clerical workers in relation to production workers."²⁹

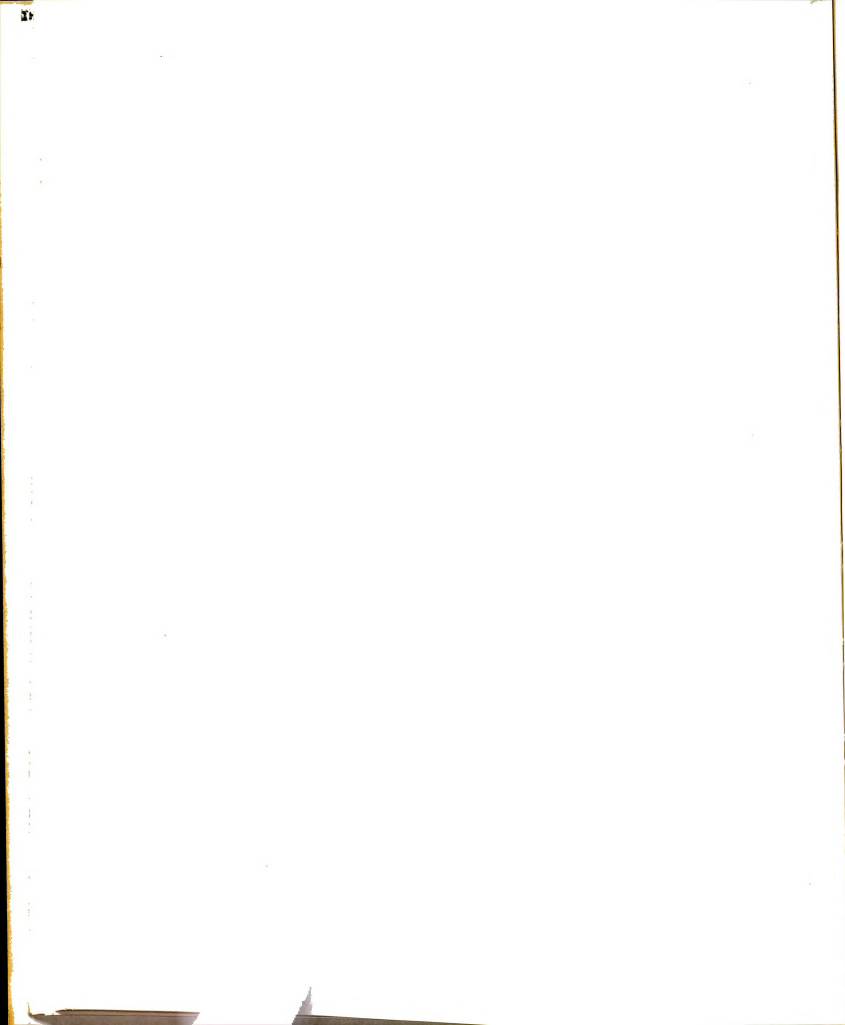
In the context of reducing white collar costs The Lybrand Newsletter reported that "without a rather clear knowledge of output per man, idle time will indeed tend to be invisible on the principle of Parkinson's Law: work expands to fill the time available for its execution."³⁰

Neither the Grady nor the Lybrand article recommended statistical variance analysis; but John L. Gable of the Industrial Engineering Division of Collins Radio Company inquired: "Would it be worthwhile for us to apply quality control procedures and techniques to some of our office and paper work functions?"³¹ He suggests that the routine paper work be organized and subjected to time and

²⁹Charles H. Grady, Jr., "Reducing Clerical Costs Through Improved Manpower Utilization," N.A.A. Bulletin, XLVI, No. 7 (March, 1965), 42.

³⁰Lybrand, Ross Brothers, and Montgomery, "Reducing White Collar Costs," The Lybrand Newsletter (November, 1964), 3.

³¹John L. Gable, "An Internal Audit Using Receiving Inspection Techniques," Industrial Quality Control, XIV, No. 7 (January, 1958), 15.

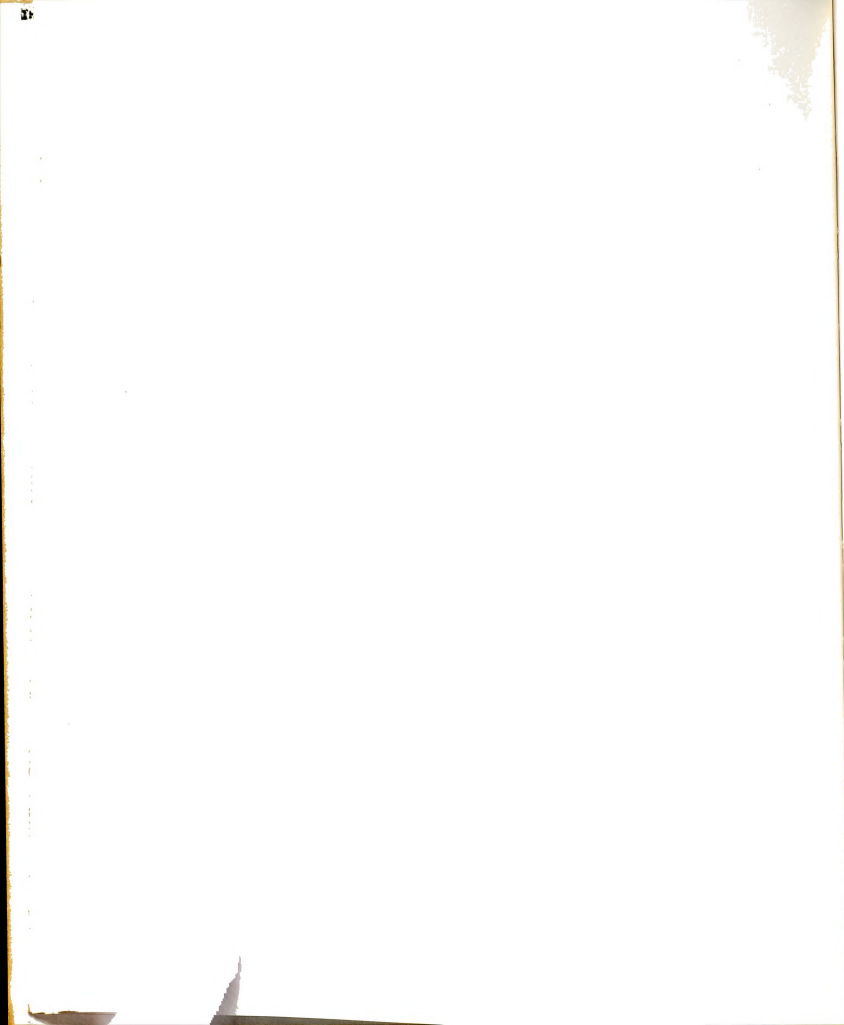


motion studies so that standards could be established. Since chance affects these performances much the same as it affects factory labor, statistical variance analysis should be equally applicable. Such a program should be initiated by experimenting first with a few of the most routine functions.

There are some non-manufacturing expense classifications for which physical performance standards are not relevant. For some, particularly salaries, control involves checking adherence to the budget. Statistical procedures are not helpful in analyzing variances from expense classifications that are not affected by chance.

Conclusions

The accountant's concept of control is limited because it does not give formal recognition to chance influences. Once chance is recognized, the logic behind using statistical tools to determine the significance of variances is evident from the fact that probability statistics is concerned with evaluating the patterns of chance influences. An examination of individual variance classifications revealed that chance influences definitely cause variations in labor and material usage. Chance is also operative on many elements of the overhead efficiency variance and the volume variance. Moreover, it causes variations in many non-manufacturing costs such as clerical work. Consequently, statistical procedures are helpful



for determining the significance of variances associated with the above items. Contrariwise, chance is not usually expected to have an effect on the material price, the labor rate, or the budget variances; therefore, statistical procedures would not be helpful in analyzing these variances.

Since statistical tools are helpful in analyzing some important variance classifications for which chance is expected to cause the variances, it is now worthwhile to find those statistical tools which are most helpful for variance control.

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CHAPTER IV

STATISTICAL CONTROL TECHNIQUES--EVALUATION OF THREE PROPOSED METHODS

Existing accounting literature involving statistical techniques for variance control is concerned mainly with an application of basic control chart procedures that were originally developed in 1924 by W. A. Shewhart¹ of the Bell Telephone Labs for purposes of quality control. While this method considers the distribution of chance performances in selecting the control limits, it has not typically considered the opportunity costs associated with investigative decisions. Recently, two approaches which have been identified as the Bierman, Fouraker, and Jaedicke Approach and the McMenimen Approach have formally considered these opportunity costs in their models for determining the appropriate control limits. This chapter evaluates all three methods for the purpose of isolating the strengths and weaknesses of each. The reader should refer to Appendix A for a bibliography of accounting literature pertaining to statistical techniques for variance control.

Hypothesis Testing

Throughout the remainder of this dissertation frequent reference will be made to the term "hypothesis testing."

¹W. A. Shewhart.

An hypothesis is simply any statement that is capable of being tested.

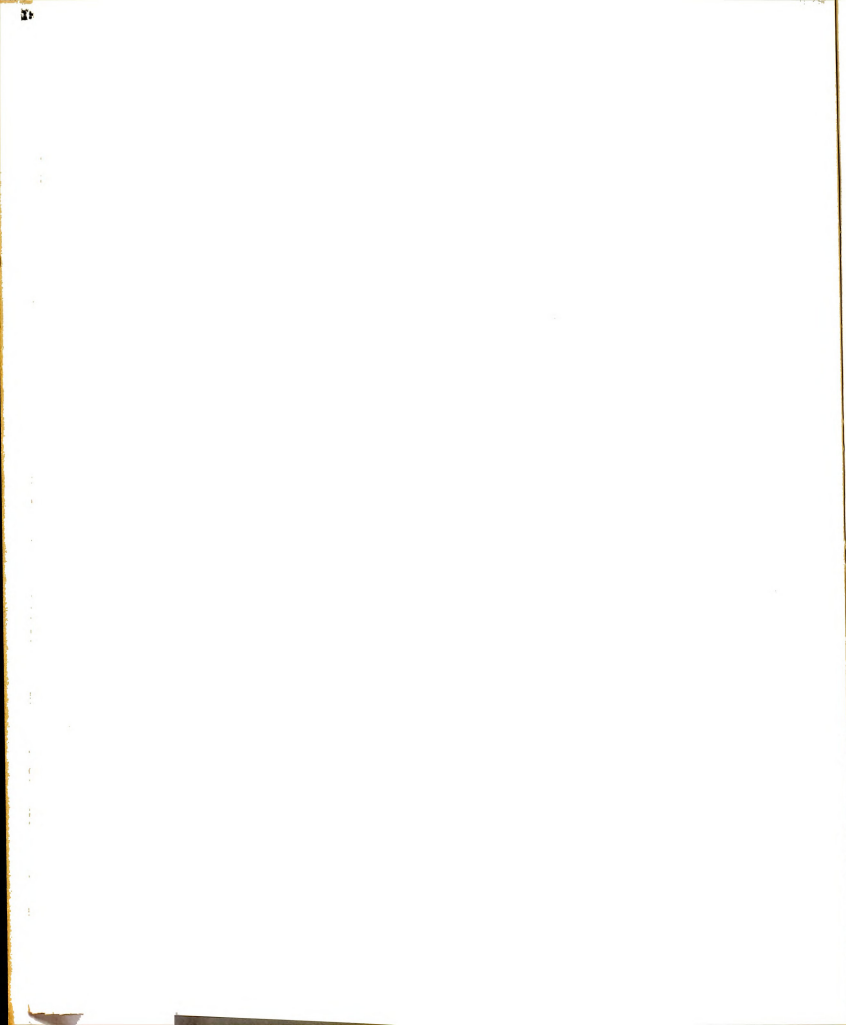
The hypothesis may be expressed in any of the following forms:

1. The operation is in control.
2. The standard is the mean of current performances.
3. The variance is attributed to chance.
4. The process has not changed because the same chance factors are contributing to variability among performances.
5. No assignable causes are present.

The term "the hypothesis" will be used to imply all of these forms of statement.

Acceptance of the hypothesis indicates that the test failed to provide sufficient evidence for rejecting these statements so that there is no reason for further investigation. Rejection of the hypothesis indicates that the sample variance would rarely be as large as that obtained if the hypothesis was true. Rejection, then, indicates negation of the above statements. It signals the need for

1. An investigation to determine the assignable cause.
2. Action to eliminate the assignable cause or revise the standard.



The Basic Control Chart Approach

The development of the control chart uses a combination of the theory of probability, which was formulated by Pascal and Pierre Fermat in 1654, and the subsequent theory of sampling which is dependent upon probability theory.² The control chart is really just a graphic presentation of the results of operations. It is used in situations where the same hypothesis must be tested over and over again. Hence, it is useful for accounting variance control where, ideally, the hypothesis that no assignable causes are present should be tested for frequent performance values. A numerical example will be used to illustrate this approach.

Assume that a standard of 245 minutes has been established for the time it should take to butcher a cow. This standard was established after all performances for some recent period of time were investigated. All performances with assignable causes were eliminated. Only the values pertaining to chance performances were averaged to arrive at the standard. The probability distribution on which this standard is based is represented in Table 3. The resulting standard is considered to represent good attainable performances.

²Douglas H. W. Allan, Statistical Quality Control (New York: Reinhold Publishing Corporation, 1959), 129.

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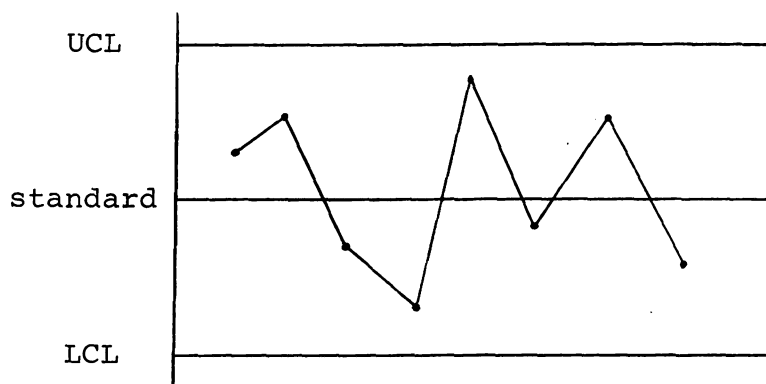
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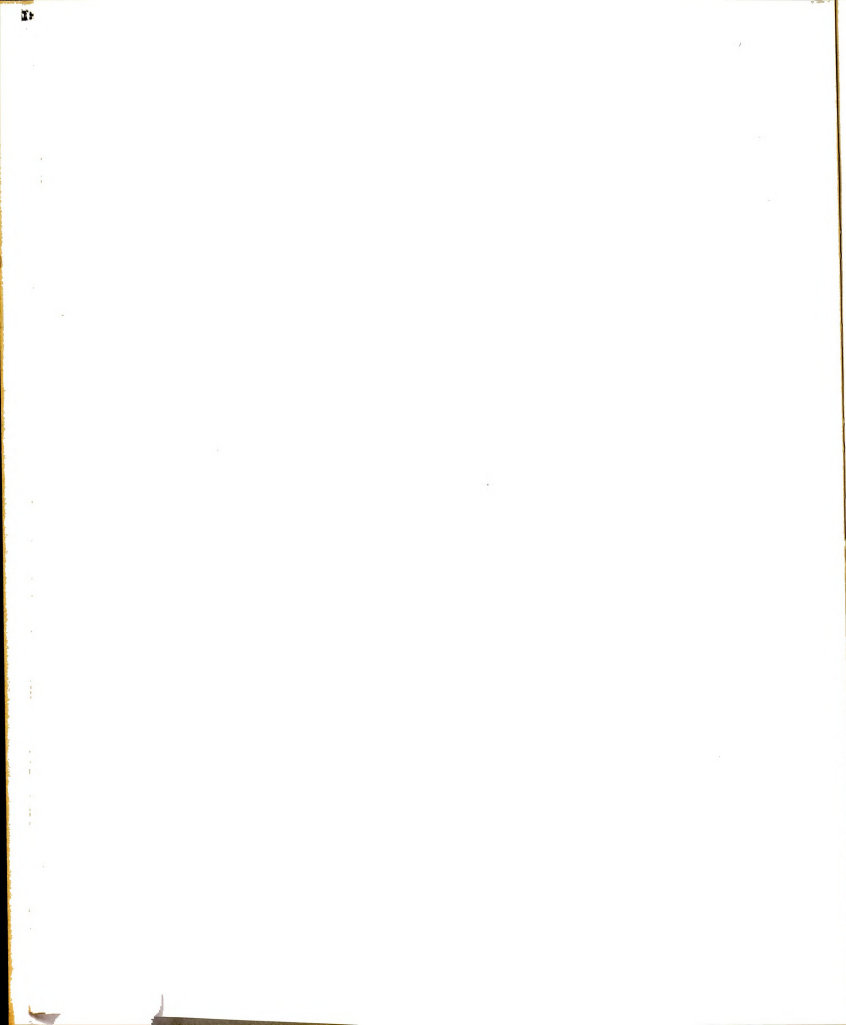
TABLE 3.--Probability distribution of chance performances

<u>Number of Minutes</u>	<u>Probability</u>
at least 220 but less than 225	.005
at least 225 but less than 230	.020
at least 230 but less than 240	.225
at least 240 but less than 250	.500
at least 250 but less than 260	.225
at least 260 but less than 265	.020
at least 265 but less than 270	.005
	1.000

The format of a control chart is depicted in Figure 3. The vertical scale contains a central line which represents the standard or the mean of the chance performances. The upper control limit is represented by the letters UCL and lower control limit by the letters LCL. The horizontal scale simply indicates the time sequence in which performances are tested.

FIGURE 3.--Illustration of a control chart





The values of individual performances³ are plotted on the control chart in the manner illustrated in Figure 3 and a decision to accept or reject the hypothesis is made according to the following general decision rules:

1. Accept the hypothesis for all observations falling between the upper and lower control limits.
2. Reject the hypothesis for all observations yielding values higher than the upper control limit or lower than the lower control limit.

The chart recognizes variability in that performances need not conform to a single value to be considered in control. It also recognizes stability because controlled performances may vary only within the control limits.

Actually, it is the method by which the control limits are determined that this writer has identified as the control chart approach. There is, however, no reason why the control chart could not be used to portray results regardless of the approach used to determine the control limits. Two elementary observations may be drawn from the distribution of chance performances in Table 3. First, any performance less than 220 has always been identified with a favorable assignable cause. Second, any performance

³The means of samples of four or five performances may also be plotted. In this event, of course, the control limits are based on means with this sample size. In order to simplify this presentation the testing of individual performances is assumed. Tests involving small samples will be introduced in Chapter VI.

over 270 minutes has always been the result of an unfavorable assignable cause. The hypothesis can, therefore, be automatically rejected for observations less than 220 because performances less than 220 have always been identified with a favorable assignable cause. Second, any performance over 270 minutes has always been the result of an unfavorable assignable cause. The hypothesis can, therefore, be automatically rejected for observations less than 220 minutes or for those over 270 minutes without the risk of incurring a Type I error. Control limits set at 220 and 270 would, however, carry an unusually high probability of incurring a Type II error.

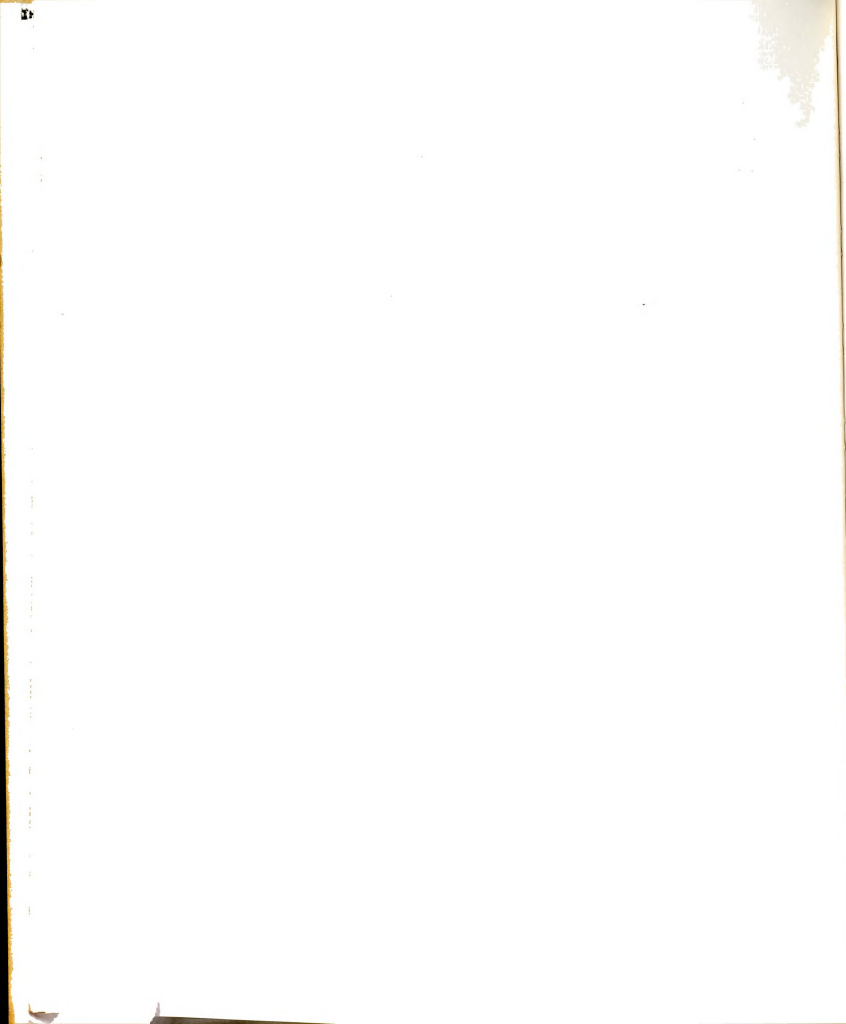
The control limits are generally set at points which permit a specified probability of committing a Type I error. The probability of incurring a Type I error for any limits is called the level of significance. Suppose .05 is chosen as the level of significance. Table 3 shows that the control limits would be 230 and 260 because 2-1/2 per cent of the chance performances are less than 230 and 2-1/2 per cent are over 260. The probability of a Type I error is .05--the same as the level of significance. By the same approach if .01 is chosen as the level of significance, the control limits would be 225 and 265 because 1/2 per cent of the chance performances are less than 225 and 1/2 per cent are over 265. The probability of a Type I error is now only .01; but, of course, the probability of

committing a Type II error is now greater than when the level of significance was .05 because more hypotheses, true as well as false, will be accepted with a .01 level of significance than with a .05 level.

Probability distributions, like Table 3, cannot give the control limits associated with a given level of significance unless the control limit happens to be one of the class limits of the distribution. For example, the control limits corresponding to the .03 level of significance would appear at those points where 1-1/2 per cent of the chance performances were less and 1-1/2 per cent of the chance performances were greater. Since the class intervals do not occur at these values, from reading Table 3 one can only learn that the lower control limit is between 225 and 230 and that the upper control limit is between 260 and 265. To help pin-point the control limits it is generally assumed that the distribution of chance performances is a normal one.

Normality is frequently assumed in statistical work; but it is rarely rigorously fulfilled. Since statistical decisions are based upon the laws of probability, inferences regarding the shape of a probability distribution are often necessary. If the shape of a given distribution does not differ significantly⁴ from normality,

⁴A chi square test can be used to test the hypothesis that the difference between the given distribution and a normal distribution is not significant.



useful, although not precise, conclusions will result. If the distribution in Table 3 were perfectly normal, the control limits corresponding to the .03 level of significance would be 229.2 and 260.8 instead of 230 and 260.⁵ It does not appear that these differences will greatly hamper the conclusions. The assumption of normality is, therefore, a practical one if it provides useful results. It is generally assumed in quality control work. Other distributions

⁵These figures can be verified by solving the following formulas for the lower control limit, LCL, and the upper control limit, UCL:

$$Z = \frac{LCL - \mu}{\sigma}$$

$$Z = \frac{UCL - \mu}{\sigma}$$

where: Z represents the number of standard deviation units between LCL or UCL and μ

μ is the standard or the mean of the chance performances

σ is the standard deviation of the distribution of chance performances.

Substitution yields the following:

$$-1.96 = \frac{LCL - 245}{8.06}$$

$$1.96 = \frac{UCL - 245}{8.06}$$

$$LCL = 229.2$$

$$UCL = 260.8$$

The Z value of 1.96 can be obtained from any table of Normal Curve Areas.

The table used by this writer pertained only to the area on one side of the mean. Since LCL in this example is to be that value which is greater than only 2-1/2 per cent of all chance values, 1.96 is that Z value corresponding to an area of .475 (.5 - .025) found in the body of the Table of Normal Curve Areas. (The table is constructed in such a way that it measures the area from μ to any specified Z value.)

can be used in cases where the assumption of normality is completely unrealistic.

With the assumption of normality it is possible to calculate the control limits corresponding to any level of significance. For example, solution of the formula below shows the control limits for the .03 level of significance to be 227.51 and 262.49 respectively. The meanings of the symbols are indicated in footnote 5.

$$Z = \frac{LCL - \mu}{\sigma}$$

$$-2.17 = \frac{LCL - 245}{8.06}$$

$$LCL = 227.51$$

$$Z = \frac{UCL - \mu}{\sigma}$$

$$2.17 = \frac{UCL - 245}{8.06}$$

$$UCL = 262.49$$

The Z value of 2.17 corresponds to the area of .485 (.5 - .015) found in the body of the Table of Normal Curve Areas.

Since the risks of error cannot be eliminated, the goal is to establish the control limits at those values which strike an economic balance between the possible risks associated with the two kinds of error. In this country, however, it is customary to use 2 or 3 sigma control limits. That is, the upper and lower control limits are drawn either at 2 or 3 standard deviations above and below the central line. The 2 sigma limit corresponds to the .05⁶ level of significance and 3 sigma limit corresponds to the .0026

⁶More accurately, the .05 level of significance is associated with a 1.96 sigma limit. The 2 sigma limit corresponds to a .0456 level of significance (.5 - 4772 = .0228 × 2 = .0456).



level of significance. (The reader can easily check for himself that 99.74 $[\cdot 4987 \times 2]$ per cent of the area under the normal curve lies between $Z = -3$ and $Z = 3$.)

The main objection to this customary practice is that the level of significance is arbitrarily selected⁷ without consideration of the other factors necessary to establish an economic balance. As Freund and Williams readily admit, "the use of 3- sigma control limits does not provide any guarantee, or for that matter any information, about the probabilities of committing Type II error. . . . Nevertheless," it is their opinion that "the use of 3- sigma control limits can be justified on the grounds of long experience and satisfactory performance in practice, and it is recommended that they be used unless there are very good reasons why other control limits should be preferred."⁸ It is this writer's contention, however, that without occasional tests of each control chart application, one cannot be sure that the 3 sigma, or for that matter the 2 sigma, control limits are satisfactory. At least, without such tests, one cannot be sure that they establish

⁷It should be emphasized that this approach is still not as arbitrary as that conventionally employed by accountants. At least this approach considers the distribution of chance performances and permits an evaluation of the probability of committing a Type I error.

⁸John E. Freund and Frank J. Williams, Modern Business Statistics (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1958), 478.



the best control limits for the given application. Such a test will be made in Chapter VI. It is hoped that it can be shown that, under certain circumstances, the customary 2 or 3 sigma levels do not provide the most adequate control over accounting variances.

Actually, the control chart is not essential for testing hypotheses. The control limits could be determined and the decision rules could be applied without plotting the values on the chart. The chart, however, serves as a visual guide to show the adequacy of control to both the worker and to management. Moreover, this visual presentation makes it easier to employ the theory of runs which serves to reduce the probabilities of not detecting a change in the cause system (i.e., to reduce the probabilities of committing a Type II error). A run is "any consecutive sequence of points falling above or below the process average."⁹ Probability statements can be constructed concerning the likelihood of runs of various magnitudes. If the probability of a given run is "small," an investigation is indicated despite the fact that all points fall within the control limits.

⁹ Richard M. Cyert and Justin H. Davidson, Statistical Sampling for Accounting Information (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1962), 183-185.



Cowden¹⁰ has indicated the following probabilities concerning the number of successive points which are expected to fall on the same side of the central line:

<u>Sequences</u>	<u>Probability</u>
7 straight	.016
10 out of 11	.012
12 out of 14	.013
14 out of 17	.013
16 out of 20	.012

Because these probabilities are all in the neighborhood of .01 the sequences are often used in addition to the control limits to indicate a shift in the parameter. Tests are also constructed which indicate the minimum number of runs to be expected in a long series of observations.¹¹

Although the Basic Control Chart approach does not usually consider the probability of committing a Type II error, it is possible to evaluate such probabilities under

¹⁰Dudley J. Cowden, Statistical Methods in Quality Control (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1957), 231-232.

¹¹For more information on the theory of runs, refer to the following sources:

Freund and Williams, 272-276.

Eugene L. Grant, Statistical Quality Control (New York: McGraw-Hill Book Company, Inc., 1952), 129.

F. Mosteller, "Note on Application of Runs to Control Charts," Annals of Mathematical Statistics, XII (1941), 229.

P. S. Olmstead, "Distribution of Sample Arrangements for Runs Up and Down," Annals of Mathematical Statistics, XVII (1946), 24.

S. Swed and C. Eisenhart, Tables for Testing Randomness of Sampling in a Sequence of Alternatives," Annals of Mathematical Statistics, XIV (1943), 66.

conventional statistical techniques. The probability of a Type II error is a function of both the unknown population mean (hereafter to be called the parameter) and the level of significance.¹² Table 4 shows these probabilities for selected parameter values for a .05 level of significance. They have been calculated under the assumption that the individual performances are normally distributed for each of the parameter values.

TABLE 4.--Probability of error for various parameters given single observations and a .05 level of significance

Parameter	Probability of Type II Error	Probability of Type I Error
210	.0066	0
215	.0314	0
220	.1075	0
225	.2676	0
230	.5000	0
235	.7314	0
240	.8859	0
244	.9352	0
245	0	.05
246	.9352	0
250	.8859	0
255	.7314	0
260	.5000	0
265	.2676	0
270	.1075	0
275	.0314	0
280	.0066	0

Figure 4 illustrates the probability of committing a Type II error for the alternative parameter 240 minutes.

¹²In cases where the test concerns sample means, rather than individual performances, the probability of a Type II error depends also upon the sample size. The probability of a Type II error can be reduced for a given level of significance, if the sample size is increased.



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The top curve shows the standard as the mean and the control limits 230 and 260. The shaded area, called the critical region, indicates the values for which the hypothesis would be rejected. The lower curve shows that the parameter has changed to 240. The hypothesis that the parameter is 245 will, however, be erroneously accepted if the test performance falls between 230 and 260. The probability of this happening equals the unshaded area under the lower curve. This area can be computed by converting each control limit into standard units and using the Table of Normal Curve Areas to find the corresponding area under the curve. The calculations appear below:

	Area Between the Control Limits and the Alterna- tive Parameter
$z = \frac{LCL - 240}{\sigma} = \frac{230 - 240}{8.06} = -1.24$	.3925
$z = \frac{UCL - 240}{\sigma} = \frac{260 - 240}{8.06} = 2.48$	<u>.4934</u>

Probability of Committing a Type II Error .8859

The probabilities of committing a Type II error for all other parameter values are computed in a similar manner.

Two general observations may be made from Table 4. First, only one type of error is possible for each parameter. For any value of the parameter other than the standard, acceptance results in a Type II error; rejection is a correct decision. If the parameter value and the standard



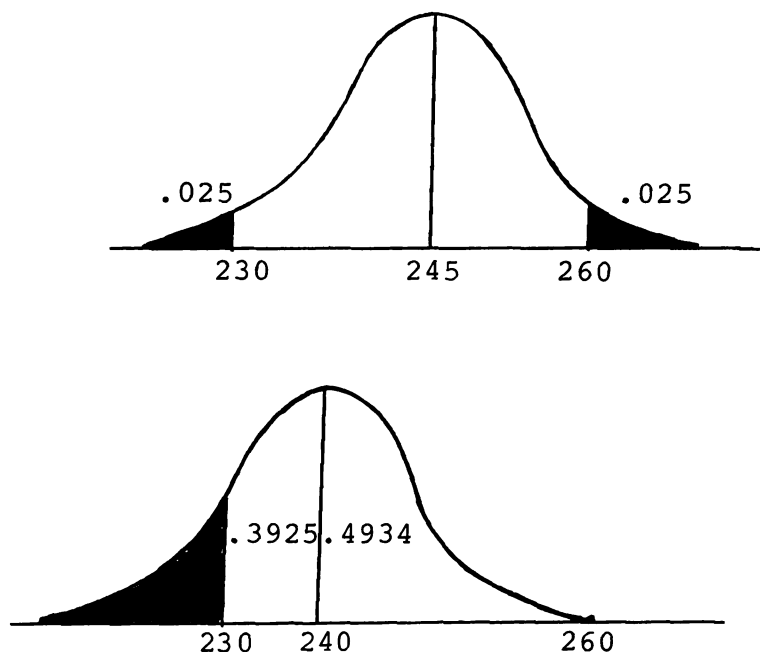


FIGURE 4.--Illustration of the determination of the probability of a Type II error

coincide, rejection, which is a Type I error, will occur with a probability equal to the level of significance. For this event acceptance is a correct decision--a Type II error is impossible. The second observation is that the probability of a Type II error is very high for parameter values close to the standard and becomes successively smaller for parameter values as they move away from the standard. In other words, small shifts in the parameter value are rarely detected; whereas, large shifts are almost always detected. This is counter-balanced by the fact that the error in failing to detect small shifts is

not costly relative to the error in failing to detect large ones. Consideration of the costs of these errors will be taken up in Chapter V.

In any given situation, the value of the parameter is unknown. Consideration of the figures in Table 4 permit a cursory evaluation of the level of significance. If the probability of a Type II error is considered to be too high for a parameter that is judged to be serious, the probability can be reduced by using a higher level of significance. The fact that a higher level of significance will result in lower probabilities for the Type II error and vice versa can be visualized by referring to Figure 4. If a higher level of significance is selected, the lower control limit will be higher than 230 and the upper control limit will be lower than 260. This will increase the shaded or critical region under both curves. Consequently, the unshaded region in the lower curve, representing the probability of a Type II error, will be less. Conversely, the selection of a lower level of significance will reduce the critical region under both curves and increase the unshaded region which in the lower curve portrays the probability of a Type II error.

This inverse relationship between the level of significance and the probability of a Type II error can also be observed in Table 5. The probabilities for the .01 level of significance were calculated in the same



manner as the probabilities for the .05 level that were previously listed in Table 4. The manner of calculation is illustrated in Figure 4. The reader will note that for any parameter value the probability of a Type II error is greater for the .01 level of significance than for the .05 level.

TABLE 5.--Comparison of the probabilities of a Type II error for various parameter values under different levels of significance

<u>Parameter</u>	<u>Probability of a Type II error</u>	
	<u>.05 Level</u>	<u>.01 Level</u>
210	.0066	.0314
215	.0314	.1075
220	.1075	.2676
225	.2676	.5000
230	.5000	.7324
235	.7314	.8925
240	.8859	.9685
244	.9352	.9862
245	0	0
246	.9352	.9862
250	.8859	.9685
255	.7314	.8925
260	.5000	.7324
265	.2676	.5000
270	.1075	.2676
275	.0314	.1075
280	.0066	.0314

Calculations for the above comparisons could be made for any desired number of levels of significance. These comparisons, however, do not automatically indicate the level of significance, although they do provide more

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objectivity than the arbitrarily selected level. The probabilities of greatest concern are those associated with alternative parameters (representing changes in the cause system) which would engender "serious" losses if they were not detected. The goal is to select a level of significance which will give a "low" probability for a Type II error for such alternative parameters without making the level of significance too "high." While this method makes use of more objective evidence than the arbitrarily selected level of significance, it supplies no objective way to evaluate this evidence. Without specific consideration of the costs of each type of error or without quantifying what is meant by the term "serious loss," both the selection of an alternative parameter and the final balance between the level of significance and the probability of a Type II error for an alternative parameter, once it is specified, are haphazardly determined.

It would appear that this approach is superior than the methods conventionally used by accountants because this method considers the distribution of chance performance which permits an evaluation of the level of significance. It is unfortunate, however, that the level of significance is generally chosen arbitrarily between .001 and .05. The probability of a Type II error is not often considered, although it can be evaluated for any given level of significance and alternative parameter as indicated in the above discussion.

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The Bierman, Fouraker, and Jaedicke Approach

Bierman, Fouraker, and Jaedicke¹³ consider the opportunity costs associated with each decision in addition to the probability distribution of chance performances.

The basic features of this model are illustrated in Table 6 which is a revised version of the Conditional Cost Table used by Bierman, Fouraker and Jaedicke.¹⁴ The following notation is used:

P is the probability that the hypothesis is true (i.e., that the deviations are caused solely by chance) given the occurrence of an unfavorable variance.

1-P is the probability that the hypothesis is false given the occurrence of an unfavorable variance.

C is the cost of an investigation.

L is the present value of the expected opportunity cost resulting from not taking corrective action on the basis of the present deviation.

Bierman, Fouraker, and Jaedicke would use the following table to analyze unfavorable variances only. A slightly different approach is used to analyze favorable variances. It is, of course, understood that no further

¹³Harold Bierman, Jr., Lawrence E. Fouraker, and Robert K. Jaedicke, Quantitative Analysis for Business Decisions (Homewood, Illinois: Richard D. Irwin, Inc., 1961), 108-125. See also Harold Bierman, Jr., Lawrence E. Fouraker, and Robert K. Jaedicke, "A Use of Probability and Statistics in Performance Evaluation," Accounting Review, XXXVI, No. 3 (July, 1961), 409-417, and Harold Bierman, Jr., Topics in Cost Accounting and Decisions (New York: McGraw-Hill Book Company, Inc., 1963), 15-23.

¹⁴Instead of using the term "events," Bierman, Fouraker, and Jaedicke refer to "states." State one they define as a variance attributed to random, noncontrollable causes; state two is a variance attributed to nonrandom, controllable causes. In this writer's terminology, state one is the same as the event that the hypothesis is true and state two is identical to the event that the hypothesis is false.

action will result from accepting the hypothesis; but that rejection signals the need for an investigation.

TABLE 6.--Conditional cost table

Events	Prob.	Acts			
		Accept Hyp.		Reject Hyp.	
		Conditional Opportunity Cost	Ex- pected	Conditional Opportunity Cost	Ex- pected
True Hyp.	P	0	0	C	CP
False Hyp.	1-P	L	$L(1-P)$	C	C-CP
Expected Cost of Acts			$L(1-P)$		C

The following explanation describes how the symbolic opportunity costs have been derived for various combinations of act and event. For combination of act-accept, and event-true hypothesis, the opportunity cost is zero because acceptance is a correct decision. For combination of act-accept and event-false hypothesis, the opportunity cost is equal to L because the hypothesis should be rejected. If act-reject, is chosen, the opportunity cost is C regardless of the event because the cost of the investigation is the same whether or not the decision is correct.¹⁵

The values in the expected column for each act are obtained by multiplying the conditional opportunity

¹⁵A fallacy in this logic will be noted in a subsequent subsection.



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costs for each combination of act and event by their respective probabilities. Only the totals in the expected columns have any meaning. These totals represent the expected cost of each act. The act with the lowest expected opportunity cost should be chosen. That is: if $L(1-P)$, the expected opportunity cost of accepting the hypothesis, is less than C , the expected opportunity cost of hypothesis rejection, the hypothesis should be accepted; but if $L(1-P)$ is greater than C the hypothesis should be rejected. When the expected costs of each act are equal, the decision maker is just indifferent between the two acts. Bierman, Fouraker, and Jaedicke equate the expected costs of the two acts to obtain the following formula for the critical probability, P_c .

$$C = L(1-P)$$

$$P_c = \frac{L - C}{L}$$

It is assumed that C is less than L . If P is larger than P_c the hypothesis is accepted; if P is smaller than P_c the hypothesis is rejected. Some general observations may be made from the above formula. When C is very small relative to L , P_c is close to 1 and most variances will be investigated. As C approaches L , the profitability of investigation decreases.

The following numerical example presented by Bierman¹⁶ illustrates the mechanics of the model.

¹⁶Bierman, 22-23.



The following assumptions are made:

1. The yearly budget for a certain expenditure is \$10,000.
2. The actual expenditure is \$13,000.
3. The standard deviation is \$6,000.
4. The cost of an investigation is \$40.
5. The condition, if off-standard, and not detected would continue for four years.
6. The discount rate is 10 per cent.

It is now necessary to calculate the conditional probability that a chance expenditure will deviate by $\$3,000/\$6,000 = .5$ standard deviations or more from its expected value, given that the deviation is an unfavorable one. The formula for calculating any conditional probability is:

$$P(B/A) = \frac{P(AB)}{P(A)}$$

In this case, B is the event that the deviation is .5 standard deviation units and A is the condition that the deviation is unfavorable. The probability that a deviation is unfavorable and at least .5 standard deviation units from its mean, $P(AB)$, is found from a table of normal curve areas to be .31. Therefore, the conditional probability of the \$3,000 deviation is:

$$P(B/A) = .31/.5 = .62$$



The critical value is:

$$P_C = \frac{L - C}{L} = \frac{\$9,000 - \$40}{\$9,000} = .996$$

L is determined by multiplying the \$3,000 deviation by 3-- the approximate present value of \$1 conveyed per period for four periods at a 10 per cent interest rate. Since P is less than P_C the hypothesis is rejected and an investigation is undertaken.

Bierman, Fouraker, and Jaedicke depict the decision process with a cost control decision chart similar to that shown in Figure 5. The curve, or critical path, can be drawn by plotting several combinations of P_C with its respective variance. With this chart, the calculation of P_C for every test can be avoided. If P lies above the critical path, the hypothesis is accepted. Otherwise, it is rejected.

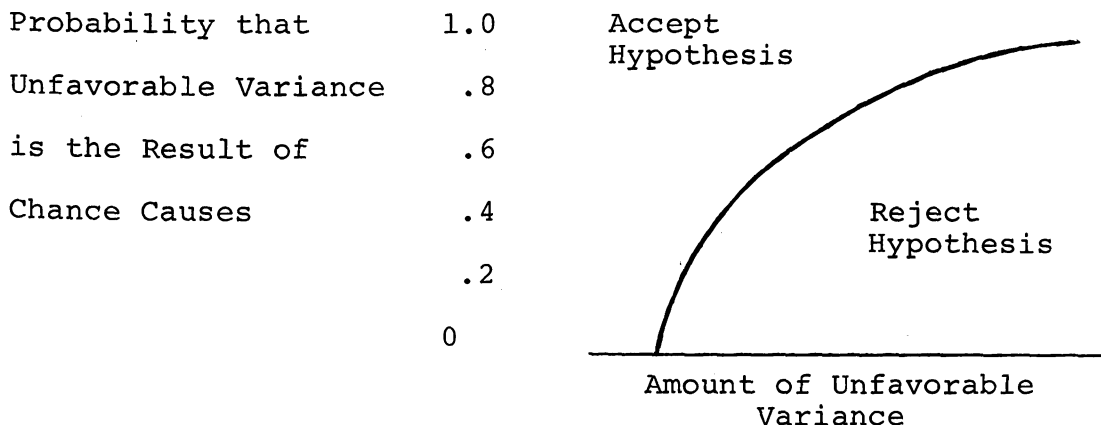


FIGURE 5.--Cost control decision chart

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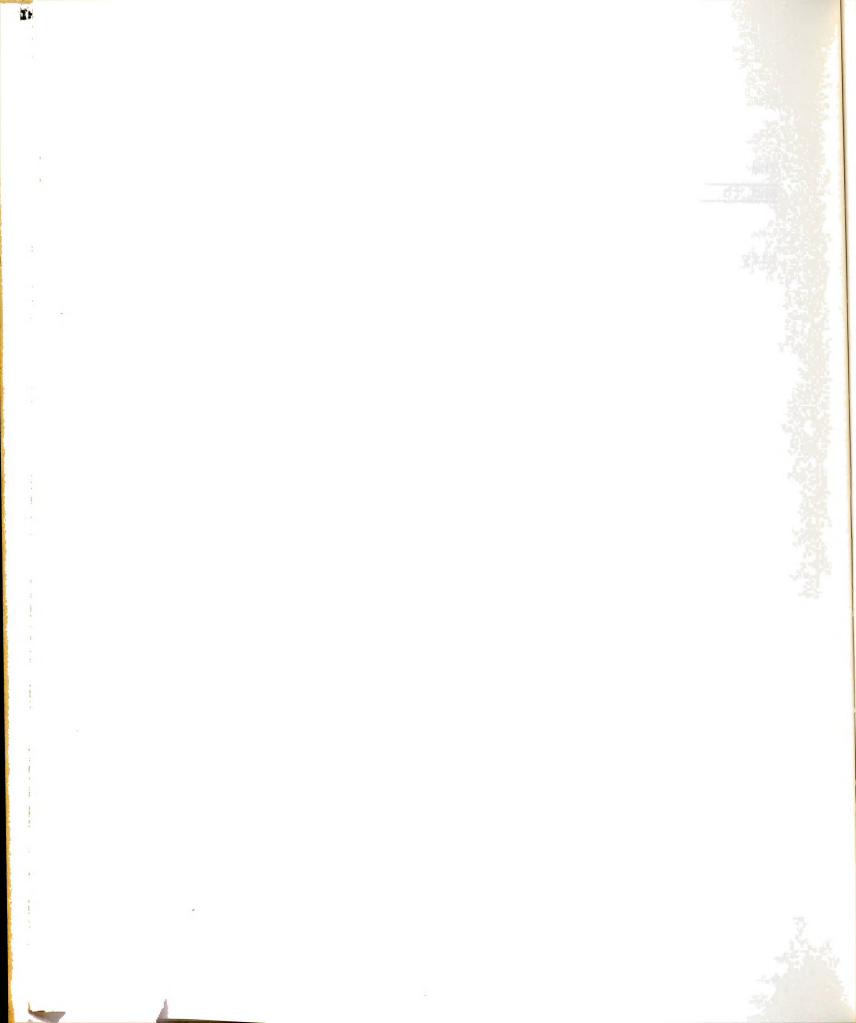
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Analysis of the Bierman, Fouraker,
and Jaedicke Approach

The Relationship between P_c
and the Level of Significance

While Bierman, Fouraker and Jaedicke do not identify it as such, P_c is the same as the level of significance. It is interesting to note that this critical value becomes larger as the amount of the unfavorable variance becomes larger. For most variances, Figure 5 shows P_c to be substantially higher than the conventional .05 or .001 values selected for the level of significance. In fact, with the above calculations a Type I error will be made 99.6 per cent of the time. The P_c indicates that 99.6 per cent of all chance variances will be investigated.

One explanation for this extremely high P_c resulting from Bierman's calculations is that his example pertained to yearly variances; whereas, the .05 or .001 levels which have been used in quality control work generally pertain to analyses of individual performances. P_c is higher for a yearly analysis because C is likely to be smaller in relation to L than it would be for an analysis of individual performances. The reason for this is that there is a certain minimum cost of an investigation so that one would not expect the investigation cost of a yearly variance to be proportionately higher than the investigation cost of an individual performance. These higher P_c 's which will result from yearly and even monthly



analyses illustrate even more dramatically the danger in applying an arbitrary level of significance.

Control of Performance vs.
Summary Expense Classifications

It has previously been noted that control is more timely and that the source of off-standard conditions can more easily be identified by analysis at the performance and operational levels. Nevertheless, the basic procedure is the same (although the level of significance is different) for the analysis of summary expense classifications. Actually, analysis of summary accounts should be encouraged because there are some cost items for which analysis by performance or operation is either not possible or not practical; but for which some review is desirable. It will be seen later, however, that these monthly and yearly analyses of entire expenditure classifications serve mainly to review the adequacy of control rather than to actually control costs.

A serious limitation of the Bierman, Fouraker, and Jaedicke example, along with most of the examples of the writers cited in Appendix A, is that they apply the control procedures at a level where control is too late and where off-set and average-out problems enter. Professor Ferrara¹⁷ contends that failure to identify the various levels where

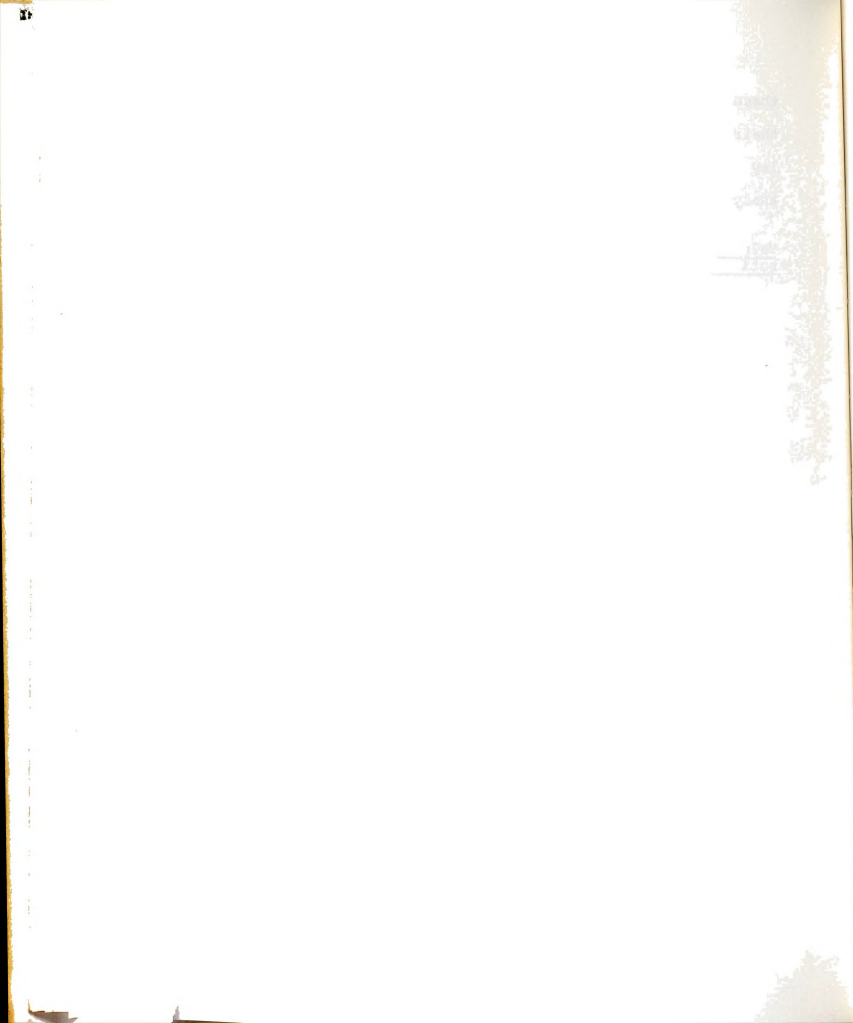
¹⁷Discussion, April 10, 1966, with William L. Ferrara, Professor of Accounting at the Pennsylvania State University.

these techniques should be applied and failure to indicate their usefulness at each level has contributed to the delay of acceptance of statistical procedures for variance analysis.

Value of the Alternative Parameter

Bierman avoids mention of the restraint imposed by selection of an alternative parameter. Instead, he implicitly assumes that the \$3,000 deviation, if significant, pertains to a parameter that is exactly \$3,000 more than the budget. This assumption, if true, would certainly be a coincidence. Part of any variance, whether or not it is significant, is due to chance. In the example cited by Bierman, the \$3,000 variance if significant, is not restricted, as he assumes, to a \$13,000 parameter. Just one of an infinite number of possibilities, concerning a \$13,000 actual cost is that the parameter is \$11,000. In this case \$1,000 (\$11,000 - \$10,000) of the variance is due to chance.

Because Bierman implicitly assumes that the alternative parameter coincides with the actual results, the alternative parameter depends upon the size of the variance which in turn causes P_c to depend upon the size of the variance. This explains why P_c increases as the size of the unfavorable variance increases as illustrated in Figure 5.



Time Interval before De-
tection of Inefficiency

An off-standard condition not detected at the end of the first year could be detected at the end of the second or third year in which event it would not proceed into the fourth year as Bierman assumes. Actually, the probability that it would continue into the fourth year is only .0310. This is calculated by the following procedure:

1. Re-calculate P_C with L equal to \$3,000.
This gives P_C equal to .987--only slightly less than the .996 obtained with L equal to \$9,000.
2. Compute the upper control limit corresponding to the revised P_C . The result is \$10,090.
3. Use the upper control limit to calculate the probability of making a Type II error given the alternative parameter \$13,000. This probability is .3139.
4. Take the third power of the probability of making a Type II error. The result is .0310.

Bierman's introduction of the present value approach into variance control is commendable; but, on balance, it appears that his example assuming arbitrarily that the inefficiency would last for four years is not well founded. It should be emphasized that there are no

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right or wrong values to use for L . From the probabilities, it appears that L should fall somewhere in the range between \$3,000 and \$9,000. Since both of these values are so high in relation to C , the actual value selected for L within this interval will not greatly effect P_C . (It has already been seen that P_C is .996 and .987 respectively when the corresponding L 's are \$9,000 and \$3,000 respectively.) In an analysis of individual performances, however, where the difference between L and C is not so great, the value of L will have a larger influence on the value of P_C . In the next chapter L will be estimated by first estimating the population variance (i.e., the difference between the standard and the alternative parameter). This estimate of the population variance will be weighted by the probability of failing to detect the change after n number of analyses.

One other possibility that Bierman's analysis failed to consider is that this cost expenditure would not be restricted to a yearly analysis. The inefficiency could, therefore, be detected by monthly or weekly analyses or by the analyses of individual performances. This extra consideration further reduces the probability that the inefficiency would continue for four years.

Inconsistency between Interpretation of P and its Calculation

In presenting their conditional cost table, Bierman, Fouraker, and Jaedicke define P as "the probability

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of an unfavorable deviation resulting from uncontrollable [chance] causes."¹⁸ This is the same thing as saying that P is the probability that the hypothesis is true given the occurrence of an unfavorable variance. Their calculation of P [or $P(B/A)$ in the numerical example just cited], corresponds to an earlier interpretation which differs substantially from the above interpretation. In their numerical examples they calculate P by converting the variance into standard units and using the table of normal curve areas. They correctly interpret this as "the probability of a deviation this large or larger occurring from random causes."¹⁹

Although the wording is similar, the method used to calculate P assumes that random or chance causes are prevailing. " P " then, is the probability that a deviation at least as large as that observed would result from the chance population. The interpretation of P in the Conditional Cost Table places the probability on whether the deviation came from the chance population (resulting in a true hypothesis) as opposed to coming from one of the assignable cause populations.

In order to determine "the probability of an unfavorable deviation resulting from uncontrollable causes,"

¹⁸ Bierman, Fouraker, and Jaedicke, 121.

¹⁹ Ibid., 113.

Leo McMenimen correctly contends that it would be necessary to know the following two values: (He assumes a \$500 unfavorable deviation.)

a = the number of times we have observed a \$500 cost deviation due solely to uncontrollable factors.

b = the number of times we have observed a \$500 cost deviation.²⁰

The ratio a/b would then be an estimate of P and the ratio $b-a/b$ an estimate of $1-P$, as P and $1-P$ are interpreted in the Conditional Cost Table (Table 6).

In an effort to clarify the distinction between these two interpretations, Leo McMenimen portrays a hypothetical company that he assumes never has and never will experience an assignable cause. The probability distribution of all results would, then, be due solely to chance causes. Assume that a given cost variance is \$500 and that by the method of converting to standard units and using the table of normal curve areas one gets $P = .3$. The proper interpretation of P associated with this calculation is that .3 is the probability that a deviation this large or larger will occur from chance causes. In other words, 30 per cent of all chance unfavorable deviations are larger than \$500. Bierman, Fouraker, and Jaedicke's second interpretation is that .3 is the probability that this unfavorable deviation results from chance causes.

²⁰McMenimen, 60.

Since, in this hypothetical case, all variances are due to chance, the probability that an unfavorable deviation will result from chance causes must equal one.

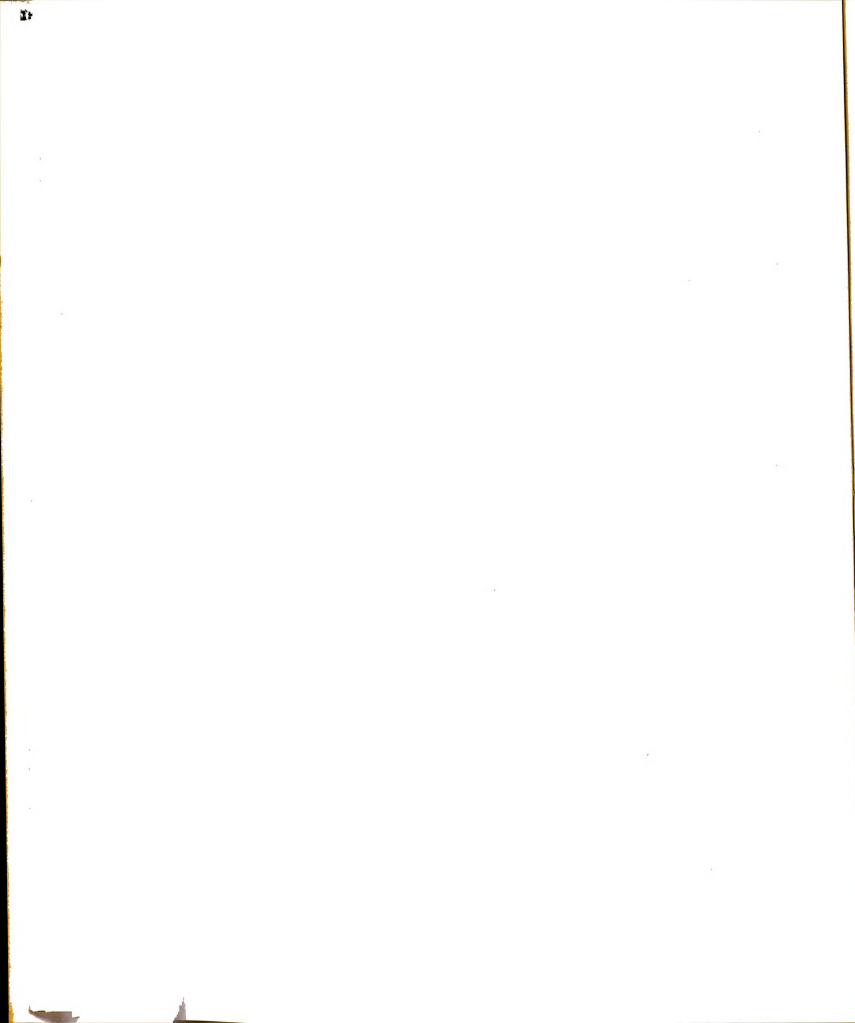
Cost of Control

McMenimen observed that

Bierman, Fouraker, and Jaedicke did not incorporate the cost of control into their analysis. They merely assumed that if you investigate the deviation and determine its cause, you can take corrective action without additional cost. This might be true in some cases, but probably not in all cases.²¹

This observation is of interest because there may be times when an assignable cause creates such a slight change in the parameter that it is not worth the cost of correcting. The logic behind the relevance of this recognition to the placement of the control limits is that there is no value in incurring the investigation cost to detect an assignable cause that one does not intend to correct. It will be seen shortly that the cost of control fits nicely into the McMenimen model to provide the decision maker with the expected value of his decision. It would, however, be difficult to incorporate this cost into the Bierman, Fouraker, and Jaedicke model because one cannot know the cost of correction until he knows the assignable cause. This, however, is determined by an investigation.

²¹Ibid., 60.



While the cost of control is an important consideration, it is this writer's opinion that this cost is more relevant in establishing the standard than in determining the significance of results. Most standards could be reduced by incurring more cost. This additional cost could take the form of more employee instruction, more time and motion study to facilitate greater efficiency, or the policy of hiring more highly skilled workers. The fact that a standard is set at a given level implies that it is worth reducing to that level but that further reduction is not profitable. Now if the parameter shifts, it should be profitable to re-establish the standard, if its level was profitable in the first place.

The Cost of an Investigation

The use of C as a constant for both events in Table 6 is questionable. If the hypothesis is false, rejection is a correct decision. An investigation, in this case, would be continued only until the particular assignable cause is determined. If, on the other hand, the hypothesis is true, an investigation would proceed until all potential causes were checked. Only then could one be reasonably certain that there were no assignable causes and that a Type I error was committed.

The logic of this reasoning leads to the conclusion that the value of C is higher if the hypothesis turns

out to be true than if it is false. Moreover, there is no unique cost of an investigation associated with a false hypothesis because some assignable causes can be detected more readily than others. Consequently, the use of C as a constant leads to questionable results from application of the Bierman, Fouraker, and Jaedicke model.

Another problem which should be made explicit is that C is more appropriately an opportunity cost concept related to the use of the investigator's time since a change in C is probably non-existent given a salaried investigating staff. On the other hand C as an opportunity cost is relevant in making an investigative decision because it is important that investigators spend their time in the most profitable endeavors. If one is spending his time in one way, he cannot be spending it in some other way.

Evaluation

In spite of the foregoing critique, the Bierman, Fouraker, and Jaedicke model has much to commend it. Not only did these authors deviate from traditional variance analysis by recognizing the probability of a chance deviation being at least as large as that observed (their interpretation corresponding to their calculation of P); but they were the first, known to this writer, to incorporate "the cost of investigation and expected benefits

of investigation explicitly into the analysis of cost variances."²² Third, they considered the expected value of future inefficiencies rather than just the cost inefficiency of one experiment. Finally, their second interpretation that P is the probability that the hypothesis is true, that is, that the deviation is the result of chance causes, would be useful information. Their limitation in this approach is that this is not the probability found by their calculations.

McMenimen Approach

McMenimen considered recognizing the possibility of more than two possible acts and two possible events. He wrote:

. . . it is possible to spend various amounts for the investigation of cost deviations before we either:

1. determine the cause of the cost deviation and the measures necessary to prevent its recurrence, or
2. designate the cost deviation as uncontrollable [i.e., due to chance].

We might also realize that the cost deviation may be reduced by various amounts depending upon how much control is exerted.²³

McMenimen's technique to handle more than two combinations of acts and events is shown in Table 7. For simplicity this table shows only three combinations of acts and events; but the approach can be adapted to consider any number of acts and events.

²²Bierman, 23.

²³McMenimen, 60.

Notice that the analysis is prepared for a deviation of a specific size, \$50 in this case. Savings consists of the "present value of the difference between the amount of the deviation eliminated and the cost of corrective action."²⁴

If the decision-maker selects act A_1 and does not investigate, it is obvious that nothing will be saved through the exertion of more control; therefore the probability of event E_1 given A_1 must be 1.00 (i.e., $P(E_1)/A_1 = 1.00$). If act A_2 is selected, there is a .5 probability that either the deviation is due to chance or that a \$10 investigation is not sufficient to discover the assignable cause. Therefore, $P(E_1)/A_2$ is .5. The other $P(E_1)$'s may be interpreted in a similar manner. Notice particularly that the probability of saving \$0 decreases as the amount spent investigating increases because the probability of overlooking an assignable cause decreases as the investigation becomes more extensive. If \$20 is spent on an investigation instead of \$10, there is a 20 per cent greater opportunity of detecting an assignable cause which, if corrected, would enable the savings of \$10.

The expected value is highest for act A_1 ; therefore, an investigation would not be undertaken for a \$50 deviation.

²⁴Ibid., 63.

TABLE 7.--McMenimen's Illustration^a (Given: \$50 Deviation)

Events	Acts	(A ₁) Spend up to \$0 Investigating		(A ₂) Spend up to \$10 Investigating		(A ₃) Spend up to \$20 Investigating				
		P(E ₁)	Cond. Value	Exp. Value	P(E ₁)	Cond. Value	Exp. Value	P(E ₁)	Cond. Value	Exp. Value
(E ₁)	Save \$ 0	1.00	\$0	\$0	0.50	\$-10	\$-5	0.30	\$-20	\$-6
(E ₂)	Save \$10	--	--	--	0.30	0	0	0.50	-10	-5
(E ₃)	Save \$25	--	--	--	0.20	15	3	0.20	5	1
		Exp. Value A ₁ = \$0		Exp. Value A ₂ = \$-2		Exp. Value A ₃ = \$-10				

Consideration of more than two acts introduces the idea that it might be profitable to begin an investigation but that it may also be profitable to terminate it short of completion. This idea is analogous to a decision to spend no time looking for a badly worn golf ball which has gone into the rough, but to spend up to ten minutes looking for a new ball costing \$1, and to spend up to twenty minutes looking for a new ball costing \$2. The golfer may stop searching for a dollar golf ball after ten minutes not solely because it is not worth another five or ten minutes to find a dollar ball; but, also, because he may subconsciously assign a low probability to his finding it in another five or ten minutes.

McMenimen's suggestion that it may be profitable to terminate an investigation short of finding the cause is a good one conceptually but in this writer's judgment it would not be feasible in practice unless:

1. The cost of an investigation is very high in relation to the present value of expected savings.
2. The cost of control is so high that no action would be taken even if the cause were determined.
3. The probability that the variance is attributed to an assignable cause other than those already investigated is very low.

The first of the above items it not likely to hold for analyses at the performance or operational levels

although it may hold for monthly or yearly analyses at a departmental or higher organizational level. With regard to the second item, it has already been noted that if it is worthwhile to establish a certain standard in the first place, it would be worthwhile to re-establish it unless conditions have changed in which case the standard should be revised. The third case may indeed frequently result. This will be illustrated in the example in Chapter VI.

The McMenimen technique would be clearer if it included a comprehensive numerical model to illustrate precisely how each value is derived. As it stands, some assumptions implied but not specifically stated by McMenimen must be set forth in order to employ his approach. The first of these assumptions concerns the determination of the values for the various amounts to be saved. The only directive given by McMenimen is his statement that savings consists of the "present value of the difference between the amount of the deviation eliminated and the cost of corrective action."²⁵ When he reviewed the Bierman, Fouraker and Jaedicke approach, McMenimen neither noted nor attacked their selection of an alternative parameter as being equal to the actual result. Moreover, McMenimen did not relate his savings values to the parameters pertaining to specific assignable causes. His suggestion that "the cost deviation may also be reduced by various

²⁵Ibid., 63.

amounts depending upon how much control is exerted" implies that the mean (parameter) of an off-standard condition might profitably be reduced but to some value still greater than the standard. However, it has already been noted that if the standard was realistic in the first place, it should be profitable to re-establish it unless conditions have changed to the extent that a revised standard is indicated.

The second assumption that must be made explicit in order to employ the McMenimen approach concerns his derivation of P . McMenimen said, "In order to obtain these values we would have had to either investigate all cost deviations (including cost deviations equal to zero) for a period of time, or sample all cost deviations for a period of time."²⁶ From this, it appears that McMenimen desires probabilities similar to the second interpretation used by Bierman, Fouraker, and Jaedicke. The following interpretation would apply to $P(E_1) = 0.50$ which corresponds to act (A_2) and event (E_1) in Table 5: Given the occurrence of a \$50 deviation and the fact that up to \$10 is spent on an investigation .5 is sum of (1) the probability that the hypothesis is true²⁷ and (2) the probability

²⁶Ibid., 63.

²⁷This is really the probability of a Type I error if an investigation is undertaken. The fact that $P(E_1)$ which corresponds to act (A_3) and event (E_1) is 0.30 means that the probability that the hypothesis is true given a \$50 deviation is at most .30.

that a \$10 investigation is insufficient to find an assignable cause.²⁸ The probability of detecting an assignable cause by spending up to \$10 investigating is 0.50 ($1 - 0.50$). This writer does not understand how McMenimen could allocate this 0.50 probability between events (E_2) and (E_3) without specifying assignable causes and their parameters (means). With this information, the interpretation, given a \$10 investigation, would be: (1) 0.30 is the probability of detecting an assignable cause which makes possible the saving of \$10 and (2) 0.20 is the probability of detecting another assignable cause which makes possible the saving of \$20.

There is a difficulty in implementing the procedure that McMenimen suggests. An analysis like that shown in Table 7 would have to be undertaken for each possible cost deviation. McMenimen, himself, points out that an extreme amount of information is needed for this technique and that this information must be constantly revised. Moreover, for some specific sized deviation there may be very few observations so that the probabilities assigned to the events would be largely a matter of guess. However, the application of this approach into the

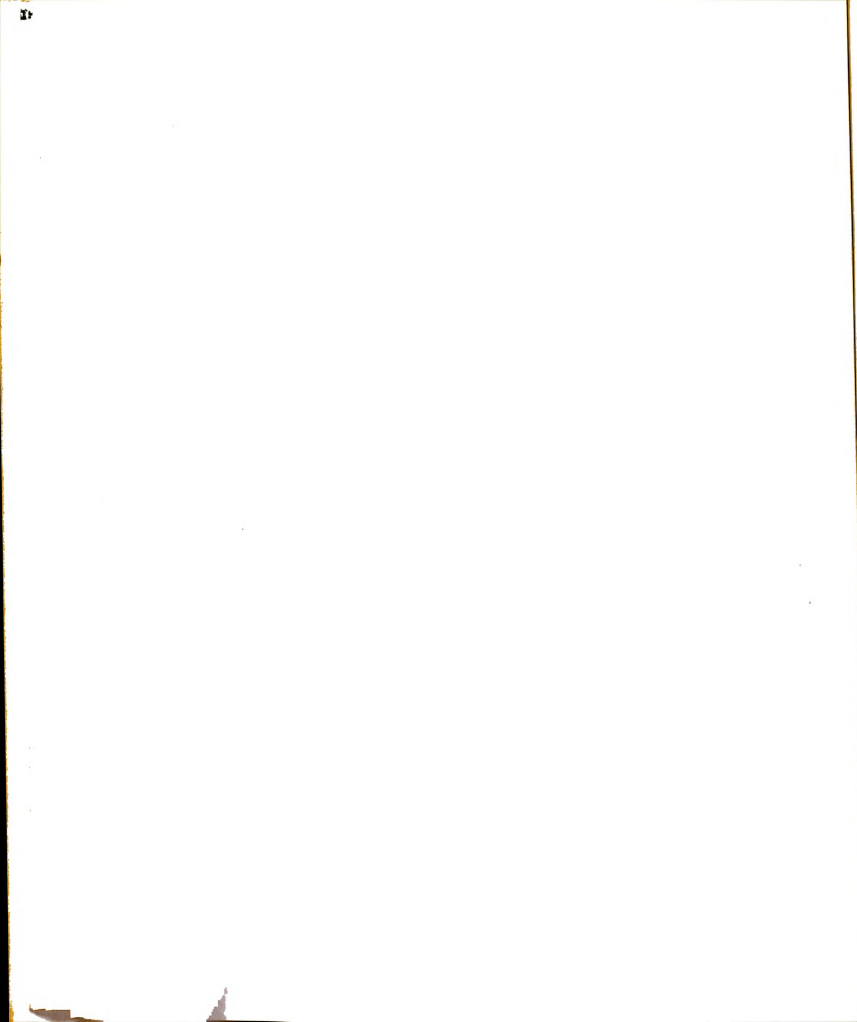
²⁸ This is the probability of a Type II error. It is at least .20 ($.50 - .30$) because 20 per cent of the time when the deviation is \$50 an assignable cause can be detected by spending an additional \$10 investigating (i.e., by spending \$20 on an investigation instead of \$10). Table 7 is not sufficiently detailed to determine the probability of detecting an assignable cause if more than \$20 is spent on an investigation.



comprehensive numerical illustration of Chapter VI in which assignable causes and their parameters are identified and an investigation procedure is established reveals several interesting insights into variance control.

Conclusions

Of the three approaches to statistical variance control that were evaluated in this chapter, the Basic Control Chart approach is the easiest to apply. All that is needed is the probability distribution of chance performances and some basic knowledge of probability statistics. This approach is, however, limited because it does not consider the economic aspects of decision making as the other approaches have attempted to do. The strengths and weaknesses as well as the similarities and differences of these approaches will become clearer in Chapter VI when they are all tested for their adequacy in variance control.



CHAPTER V

STATISTICAL CONTROL TECHNIQUES--

TWO MORE-REFINED METHODS

In Chapter IV, three statistical control techniques were evaluated. These techniques were identified as (1) the Basic Control Chart approach, (2) the Bierman, Fouraker, and Jaedicke approach, and (3) the McMenimen approach. Certain limitations were noted for each of these approaches. In an attempt to improve upon these limitations, this writer has applied two more approaches to variance control. The first is called an Equalization approach. The other method is referred to as the Minimization Approach because it minimizes the expected opportunity costs.

An Equalization Approach

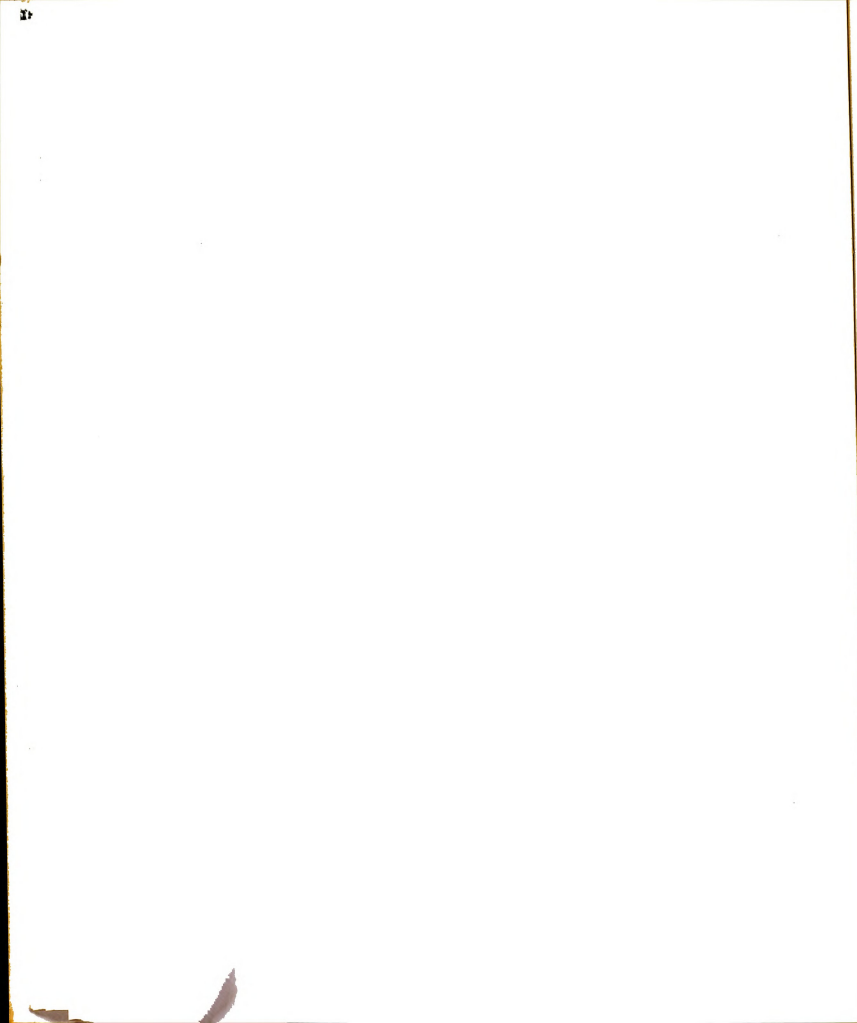
This approach establishes the control limits at those points where the probability of committing a Type I error times the opportunity cost associated with a Type I error is exactly equal to the probability of committing a Type II error times the opportunity cost associated with a Type II error. The probability of committing a Type I error and the probability of incurring a Type II error for a specified alternative parameter were developed in

Chapter IV in conjunction with a cow butchering illustration. These probabilities are shown in Tables 4 and 5 and in Figure 4. Accordingly, it is now appropriate to consider the opportunity costs associated with each type of error.

Opportunity Cost of a Type I Error

The opportunity costs of a Type I error have two aspects. One aspect is associated with the cost of an investigation which could be saved if the best act--not to investigate--were chosen for the event which occurred (the cause system has not changed). This cost is an opportunity cost because those making the investigation would normally be salaried. As an opportunity cost it is no less relevant, however, because it is important that salaried employees spend their time in the most profitable ways. Of course, if an increased number of such errors were incurred, at some point an additional supervisor would have to be added to make the additional investigations.

An investigation may take varying lengths of time depending on the cause of the variance. The longest time would be spent investigating a chance cause (which gives rise to a Type I error) because each other possible cause would be checked-out before the investigator could be reasonably sure that he had made a Type I error.



The other cost aspect associated with a Type I error is the cost of employee ill-will engendered by the implication that an employee is not performing according to standard. This cost is difficult to determine, but it can be greatly reduced by an educational program designed to explain the purpose of standards, control charts, sampling, and sampling errors. If employees understand that their wages ultimately depend on the success of the control program, greater cooperation can be elicited.

Opportunity Cost of a Type II Error

The opportunity cost of a Type II error is also composed of two elements. If the performance comes from an alternative parameter which is unfavorable the cost consists of the worker's time which could be used more productively if the assignable cause could be detected and corrected.

The other cost element of a Type II error concerns performances from favorable alternative parameters. In these cases, the cost involves wastes incurred by delays in revising the standard. This cost element is difficult to determine. The reader will recall that a hypothetical example involving the time required to butcher a cow was used in Chapter IV in conjunction with an explanation of the control chart approach. The standard for this operation was 245 minutes. Now, if because of a favorable

assignable cause the standard could be reduced to 240, the total manufacturing cost would be less than if the standard is 245. If, however, this change in the cause system is not detected, and Table 5 indicates that it will not be 88.59 per cent of the time with a .05 level of significance, the firm has no assurance that the meat cutter is not capable of performing at 235 or 230. Indeed, he may be aware of his increased skill and decide to reap the rewards by taking his time. An incentive plan for those who better their performance would partially eliminate this problem and make improvement detection easier; but there are still those who would prefer to work slower even at the expense of less pay. Therefore, it is important to determine a lower control limit so that improvement may be detected and the standard revised. An estimate of the opportunity cost of failing to detect favorable changes in the cause system is a factor involved in determining an optimum lower control limit.

Quantification of the Costs of a Wrong Decision

For illustrative purposes, the opportunity costs of error will be quantified by continuing with the cow butchering example. The investigation cost associated with a Type I error can be computed by:

1. Determining the time required to run through the complete list of procedures before chance, the residual cause, can be agreed upon.

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2. Converting this time into a dollar figure by taking an appropriate portion of the investigator's salary.

In this case, assume that (1) it takes one hour to run through a list of procedures before chance, the residual cause, can be agreed upon and (2) the prorated salary of the investigator is \$5 per hour. The cost of a complete investigation is, then, \$5. Assume, in this example, that educational programs have resulted in negligible employee ill-will associated with an investigation. The cost of a Type I error, therefore, is \$5.

The opportunity costs of a Type II error associated with an unfavorable change in the cause system are determined by: (1) dividing the difference between the standard and the alternative parameter by 60 to convert the difference into an hourly fraction and (2) multiplying this hourly fraction by the hourly wage. In this case it is assumed that the butcher receives \$3 per hour.

For the illustrative purposes of this problem, it has been assumed that the opportunity costs of a Type II error for favorable changes in the cause system are the same as the costs for equivalent unfavorable changes in the cause system. That is, the opportunity cost of a Type II error for a 240 parameter, representing a five minute favorable change in the cause system ($245 - 240$), is the same as the opportunity cost of a Type II error for a 250 parameter, representing a five minute unfavorable change in the cause system ($245 - 250$).

The opportunity costs of Type I and Type II errors are shown in Table 8 for various population means. Notice that a Type I error is made only when the population mean is 245. Since a Type I error is that error of rejecting a true hypothesis, it can be made only when the hypothesis is true. Moreover, a Type II error, that of accepting a false hypothesis, can be made only when the hypothesis is false. Therefore, it can be incurred for all non-chance parameters other than 245. Notice that the opportunity cost of a Type II error increases as the change in the cause system increases. That is, as the population mean moves away from the standard in either direction, the opportunity cost of a Type II error increases.

TABLE 8.--Opportunity costs of a wrong decision for various population means

Population Mean	Op. Cost of Type I Error	Op. Cost of Type II Error
210	\$5	\$1.75
215		1.50
220		1.25
225		1.00
230		0.75
235		0.50
240		0.25
244		0.05
245		
246		0.05
250		0.25
255		0.50
260		0.75
265		1.00
270		1.25
275		1.50
280		1.75

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In contrast to this, Table 5 shows that the probability of making a Type II error decreases as the population mean moves away from the standard. This means that as the change in the cause system becomes greater the probability of detecting the change, and thus avoiding a Type II error, also becomes greater.

Determining the Control Limits

Information to aid in the determination of the control limits has been marshalled in Table 9. The opportunity costs of a wrong decision are the same figures that were derived in Table 8 except that they are not identified as to the type of wrong decision. It is understood that the opportunity cost associated with parameter 245 pertains to the opportunity cost of a Type I error and that the opportunity costs of the other parameters represent the opportunity costs of Type II errors.

The figures appearing under the columns entitled "Prob. of Wrong Decision" were taken from Table 5 with the following exception. The probabilities listed in Table 5 are the probabilities of a Type II error; but those listed in Table 9 are the probabilities of a wrong decision whether it be a Type I or Type II error. Accordingly, the probabilities in Table 9 corresponding to the standard, or the 245 parameter, represent the probabilities of committing a Type I error-- $.05$ for the $.05$ level of significance and $.01$ for the $.01$ level of significance. The probabilities

corresponding to each of the other parameters represent the probabilities of committing a Type II error.

Conditional opportunity costs for each population mean and level of significance are obtained by multiplying each opportunity cost of a wrong decision by its probability of making a wrong decision. With the exception of the figures for the standard, these conditional opportunity costs represent, for each specified parameter, the expected opportunity cost of making a Type II error. For the standard, the conditional opportunity cost represents the expected opportunity cost of making a Type I error.

As a result of the interaction of the decreasing probability of a Type II error and the increasing opportunity cost of a wrong decision, the conditional average opportunity costs increase at first and then decrease as the alternative parameter moves further away from the standard in either direction.

For parameters other than the standard, the conditional average opportunity costs are higher for the .01 level of significance than for the .05 level because, of course, the probability of a Type II error is higher with the .01 level. For the standard, the conditional average opportunity cost is higher for the .05 level.

The conditional average opportunity cost figures are helpful in several ways. First, they help to determine when a change in the cause system becomes "serious."



For both the .05 and the .01 levels of significance, the highest conditional average opportunity costs occur for alternative parameters 230 and 260. Therefore, 230 and 260 will be specified as the alternative parameters in determining the control limits. Both of these parameters correspond to a 15 minute change in the cause system. Second, it will be seen that the conditional average opportunity cost figures aid in determining the desired level of significance for any specified parameter.

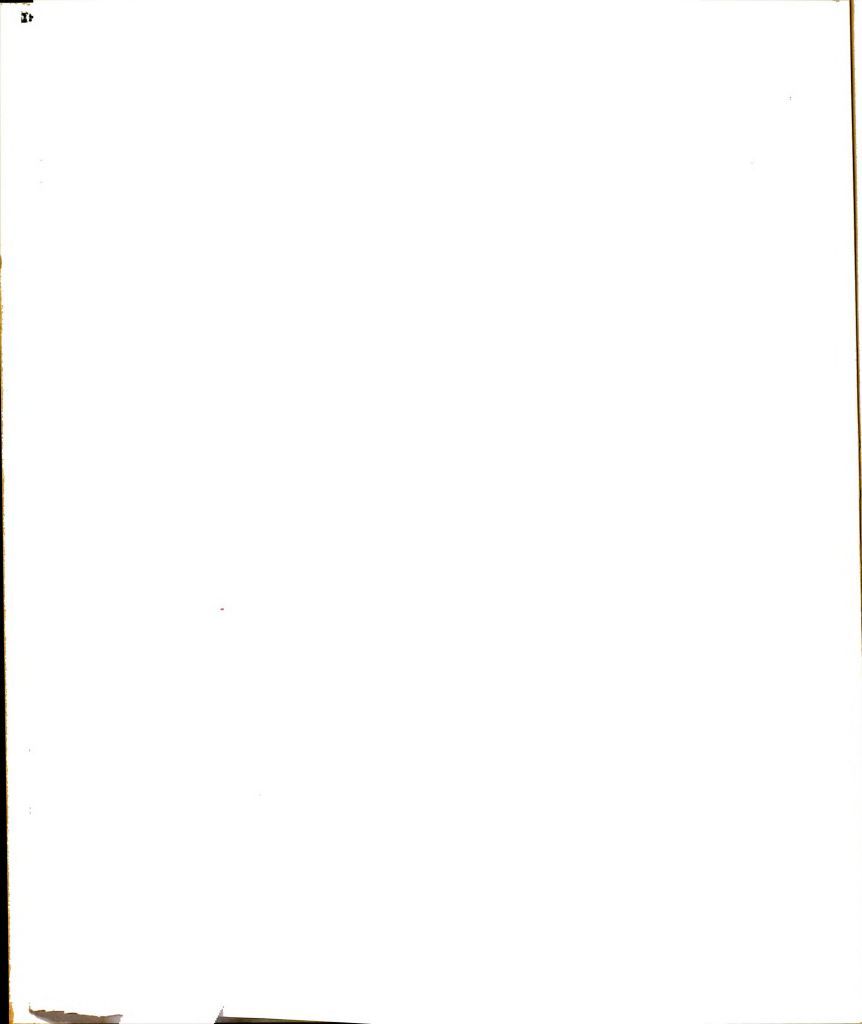
TABLE 9.--Conditional average opportunity costs

μ	Op. Cost of Wrong Decision	Level of Significance			
		.05		.01	
		Prob. of Wrong Decision	Cond. Ave. Op. Cost	Prob. of Wrong Decision	Cond. Ave. Op. Cost
210	\$1.75	.0066	\$.0116	.0314	\$.0550
215	1.50	.0314	.0471	.1075	.1612
220	1.25	.1075	.1344	.2676	.3345
225	1.00	.2676	.2676	.5000	.5000
230	.75	.5000	.3750	.7324	.5530
235	.50	.7314	.3657	.8925	.4462
240	.25	.8859	.2215	.9685	.2421
244	.05	.9352	.0468	.9862	.0493
245	5.00	.05	.25	.01	.05
246	.05	.9352	.0468	.9862	.0493
250	.25	.8859	.2215	.9685	.2421
255	.50	.7314	.3657	.8925	.4462
260	.75	.5000	.3750	.7324	.5530
265	1.00	.2676	.2676	.5000	.5000
270	1.25	.1075	.1344	.2676	.3345
275	1.50	.0314	.0471	.1075	.1612
280	1.75	.0066	.0116	.0314	.0550

For review, the Equalization control limit occurs at that value where the probability of committing a Type I

error times the opportunity cost of committing a Type I error just equals the probability of committing a Type II error times the opportunity cost of committing a Type II error. More simply, this can be expressed by saying that the Equalization control limit occurs at that value where the conditional average opportunity cost of a Type I error equals the conditional average opportunity cost of a Type II error. The word "conditional" is used because these costs are conditional on μ . The value where this equality occurs is located by trial and error. First, a level of significance is randomly selected near the value which the analyst expects to be the control limit. This level is, of course, related to a set of values which are being tested to see if they are the Equalization control limits. The level of significance can be converted to the values being tested for control limits by referring to the probability distribution of chance performances in Table 3. For example, if .01 is chosen as the level of significance, the values being tested for control limits are 225 and 265 because Table 8 shows .005 of the chance performances to be less than 225 and .005 to be more than 265 ($.005 + .005 = .01$).

The next step in the test is to see if the conditional average opportunity cost of a Type I error is equal to the conditional average opportunity cost of a Type II error for the level of significance or performance values



being tested for a specified alternative parameter. For testing the .01 level of significance, the reader can verify from Table 9 that the conditional average opportunity cost of a Type I error is \$.05. This appears opposite the population mean 245--the only value for which a Type I error could be made. The conditional average opportunity cost of a Type II error for alternative parameters 230 and 260 is \$.5530--the number corresponding to each of the parameters 230 and 260. Clearly .01 is not the desired level of significance (225 and 265 are not the Equalization control limits) because \$.05 is not equal to \$.5530. In testing for significance, the analyst should reject the hypothesis for either the value 225 or the value 265 and run the risk of incurring a Type I error because the conditional average opportunity cost of a Type I error, \$.05, is lower than the conditional average opportunity cost of a Type II error, \$.5530. A Type I error is the only type of error that can be made if the hypothesis is rejected. A Type II error is possible only when the hypothesis is accepted.

Since .01 is not the desired level of significance, another level must be tested. The direction of the appropriate level can be determined by the following line of reasoning. The desired level of significance determines the value of control limits which divide the area of hypothesis acceptance from the area of hypothesis rejection. If the test for a given level of significance

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shows the conditional average opportunity cost of rejection to be lower than the conditional average opportunity cost of acceptance, the best act is to reject the hypothesis for performance values corresponding to the level being tested. The performance value corresponding to this level should fall clearly in the area of rejection. In order to move toward the boundary, a larger level of significance is necessary.

This explanation can be visualized by reference to Figure 6 in which the shaded area represents the region of rejection. The boundaries marked with LCL and UCL represent performance values for which one would be just indifferent between the acts of rejection and acceptance. That is, they represent the Equalization control limits; but their values are unknown. If, however, the conditional average opportunity costs associated with the level of significance being tested indicate hypothesis rejection, the corresponding performance value falls in the shaded region and a move toward the boundary involves a larger level of significance.

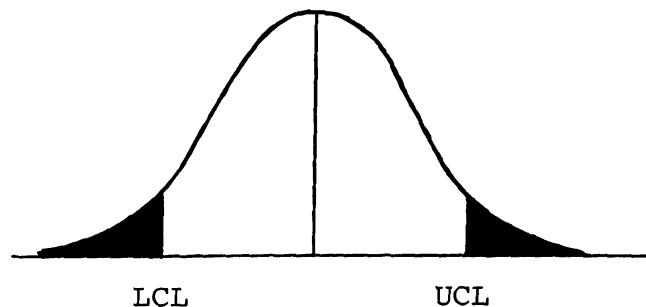


FIGURE 6.--Diagram indicating direction of desired level of significance

shows the conditional average opportunity cost of rejection to be lower than the conditional average opportunity cost of acceptance, the best act is to reject the hypothesis for performance values corresponding to the level being tested. The performance value corresponding to this level should fall clearly in the area of rejection. In order to move toward the boundary, a larger level of significance is necessary.

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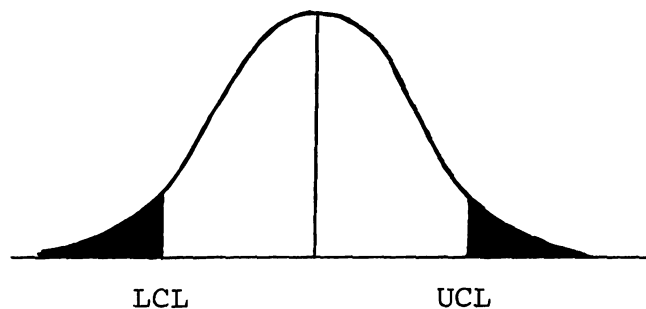
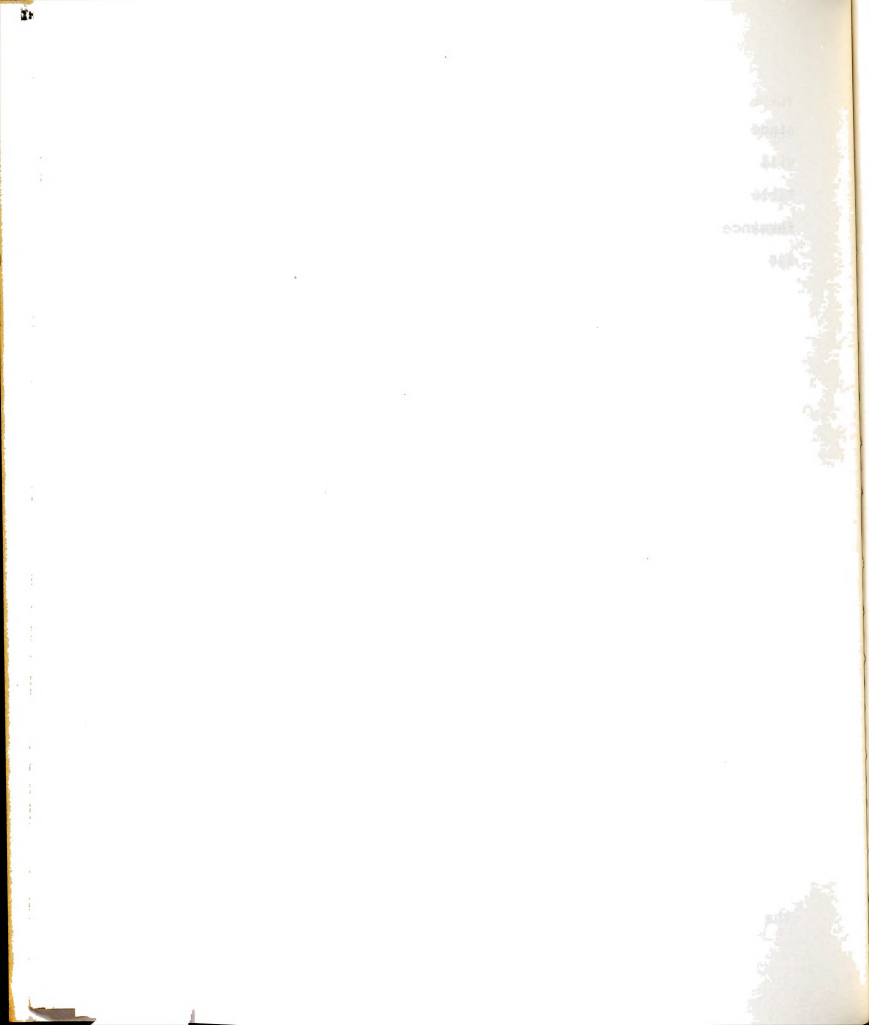


FIGURE 6.--Diagram indicating direction of desired level of significance

Since .05 is a larger level of significance and since the necessary information appears in Table 9, .05 will be tested for the desired level of significance. Table 3 shows 230 and 260 to be the corresponding performance values (.025 of the performances are lower than 230 and .025 are higher than 260). Table 9 indicates that the conditional average opportunity cost of a Type I error for a .05 level of significance is \$.25. The conditional average opportunity cost for a Type II error for alternative parameters 230 and 260 is \$.3750. The .05 level is still lower than that required because it is still cheaper on the average to reject the hypothesis for test values 230 and 260 and run the risk of incurring a Type I error.

It might now be appropriate to test the .07 level since it can easily be seen that for this level the conditional average opportunity cost of a Type I error is \$.35 ($.07 \times \5). To find the conditional average opportunity cost of a Type II error for alternative parameters 230 and 260 it is necessary to find the control limits corresponding to the .07 level of significance and to use these to calculate the probability of a Type II error. Since the probability distribution in Table 3, is not sufficiently detailed to permit reading these control limits directly from the table, they must be computed by assuming that the distribution is normal. Solution of the following



formula yields a lower control limit of 230.4 and an upper control limit of 259.6:

$$Z = \frac{LCL (UCL) - \mu}{\sigma}$$

where:

$$Z \text{ for LCL} = -1.81 \quad \mu = 245$$

$$Z \text{ for UCU} = 1.81 \quad \sigma = 8.06$$

The probability of making a Type II error for alternative parameters 230 and 260 is .48. This probability is computed in the same manner as illustrated in Figure 4. When .48 is multiplied by the \$.75 opportunity cost of a Type II error, \$.36 the conditional average opportunity costs for acceptance results. Since this is so close to the \$.35 conditional average opportunity cost of rejection, one can conclude that the Equalization level of significance is just slightly higher than .07.

The foregoing analysis is offered as evidence that the most desirable level of significance does not always fall in the .05 to .001 range as is generally assumed in the Basic Control Chart approach.

There is, however, one questionable aspect to the procedure just discussed. The opportunity costs of a Type II error are understated because an off-standard performance not detected on its first occurrence will extend opportunity costs into the future until the change in the cause system is detected and corrected. The reader will

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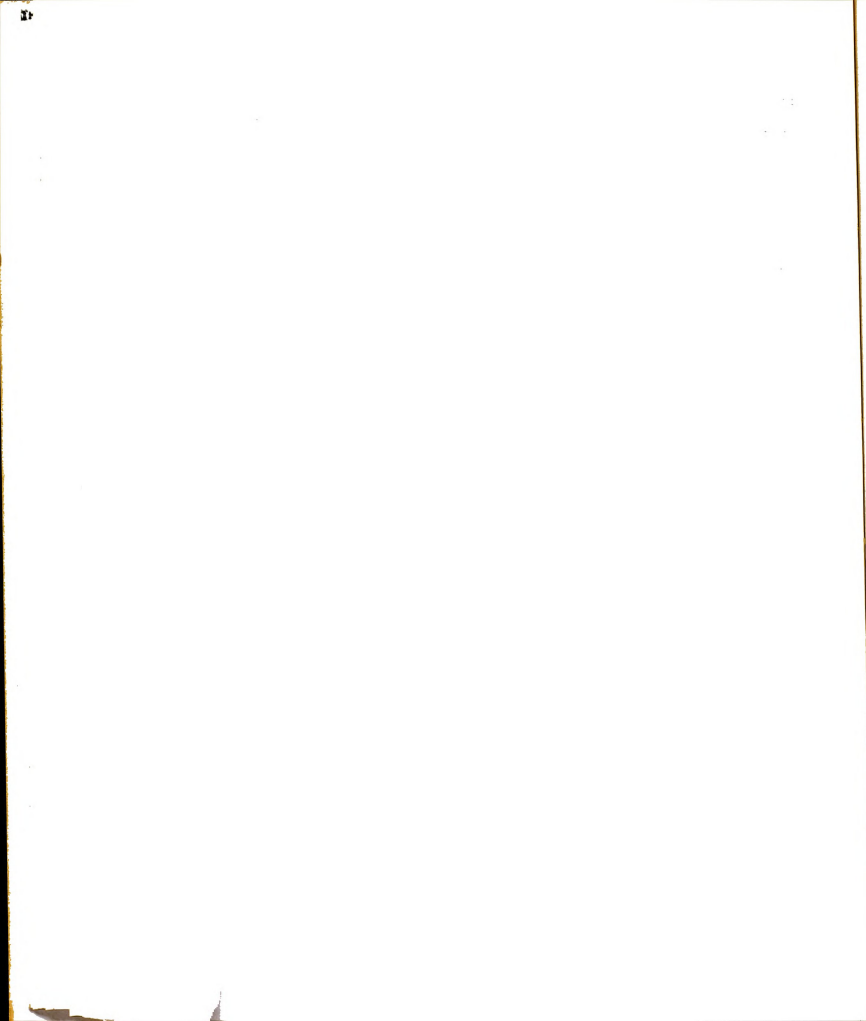
recall that Bierman's solution to this problem was to calculate the present value of an inefficiency which he arbitrarily assumed would continue for four years.¹ Since the cow butchering example pertains to individual performances rather than to yearly reports, the calculation of present values is not important because each performance is being tested at the present time. It could be assumed that an inefficiency would continue for four performances before being detected; but since four is an arbitrary value, a more scientific approach is illustrated in Table 10.

The purpose of this table is to develop a more realistic opportunity cost of a Type II error associated with parameters 230 and 260--more realistic, that is, than the \$.75 shown in Table 9. Column A represents the number of successive failures to detect a change in the cause system. The \$.75 opportunity cost in column B represents the opportunity cost of failing to detect a change in the cause system from a mean of 245 to a mean of 230 or 260 on its first occurrence. The other figures in column B increase successively by \$.75 for each additional failure to detect the change. The numbers in column C show the probability of failing to detect the assignable

¹See assumption five under the Bierman, Fouraker, and Jaedicke presentation in Chapter IV. This assumption was later questioned in the section entitled "Time Interval Before Detection of Inefficiency."

cause after the number of occurrences shown in column A. That is, the probability of failing to detect the change on its first occurrence is .5. The derivation of this probability was originally explained in conjunction with Table 4. The probability of failing to detect it on its second occurrence is .5 squared or .25. The probability of failing to detect the change on its third occurrence is .5 cubed or .125. The other figures in column C are determined by taking the power of .5 corresponding to the values in column A. As indicated on Table 10, column D results from multiplying the values in column B by those in column C. Only the summation of the values in column D is shown since it is the only value used in subsequent calculations. This summation of \$1.456650 is divided by the summation of the probabilities, .9960, in column C in order to get \$1.4625 as the opportunity cost of a Type II error. This value considers the fact that it takes on the average $(\$1.4625/$.75)$ 1.95 tests to detect an assignable cause once it has occurred.

Now that the opportunity cost of a Type II error has been increased, the Equalization level of significance will be increased. The increased opportunity cost of a Type II error increases the conditional opportunity cost of a Type II error. In order to raise the conditional opportunity cost of a Type I error to bring about the necessary equality, the level of significance (probability



of a Type I error) must be increased because the opportunity cost of a Type I error is constant at \$5.

TABLE 10.--Weighted opportunity cost of Type II error

Number of Tests	Accumulated Opportunity Costs	Probability of tests in Col. A	Column B Times Column C
(A)	(B)	(C)	(D)
1	\$.75	.5000	
2	1.50	.2500	
3	2.25	.1250	
4	3.00	.0625	
5	3.75	.0312	
6	4.00	.0156	
7	4.75	.0078	
8	5.50	<u>.0039</u>	
		.9960	<u>\$1.456650</u>
Weighted Cost = $\frac{\$1.456650}{.9960}$		\$1.4625	

As a start, a test will be made to see if .10 is an optimum level of significance. The conditional average opportunity cost of a Type I error is \$.50. This is determined by multiplying the probability of a Type I error, .10, by the \$5 opportunity cost of a Type II error. The conditional average opportunity cost of a Type II error assuming alternative parameters of 230 and 260 is \$.6028. This is determined by multiplying the probability of a Type II error, .4127,² by the \$1.46 opportunity cost of a

²This value was obtained by the following procedure:
1. Finding lower control limit corresponding to the .10 level of significance. This value is 231.74. It is

Type II error. Since the conditional average opportunity cost of a Type I error is lower than that for a Type II error, the desired level of significance is higher than .10. The level is, however, clearly less than .12 because the conditional average opportunity cost of a Type I error with a .12 level of significance is \$.6000 (.12 × \$5). The conditional average opportunity cost of a Type II error is \$.6028 for a .10 level of significance. It must be less for a .12 level since the probability of a Type II error is less for a .12 level than for the .10 level.

determined by solving the following formula for LCL:

$$-1.645 = \frac{245 - \text{LCL}}{8.06}$$

where: -1.645 is the normal deviate corresponding to the .10 level of significance (2 tailed test)
 245 is the standard
 8.06 is the standard deviation of the distribution of chance performances

2. Assuming that the cause system changed so that the parameter is now 230 instead of 245.
3. Finding the area under the normal curve between 230 and 231.74. This is .0871 determined by solving the following for Z and using the table of normal curve areas.

$$Z = \frac{231.74 - 230}{8.06} = .2159 \sim .22$$

4. Finding the area under the normal curve between 231.74 and the corresponding upper control limit of 258.26. This result of .4129 is obtained by subtracting .0871 from .5 (the area between 230 and 258.26). The probability of committing a Type II error, then, is .4129 because the hypothesis will be accepted if the test value is between 231.74 and 258.26 with a .10 level of significance. If, however, the population mean has changed to 230 acceptance would be a Type II error. The same value would be obtained by using 260 as the alternative parameter.

Consequently, a test will be made to see if .11 is the appropriate level of significance. Now the conditional average opportunity cost of a Type I error is \$.55 ($.11 \times \5). The \$.5802 conditional average opportunity cost of a Type II error is obtained by multiplying the $.3974^3$ probability of a Type II error by the \$1.46 opportunity cost of a Type II error. Accordingly, the desired level of significance is higher than .11 but less than .12. The Equalization lower control limit is then between 232.12 and 232.47 and the Equalization upper control limit is between 257.88 and 257.53. (232.12 and 257.88 are the control limits associated with a .11 level of significance and 232.47 and 257.53 are associated with a .12 level of significance.)

Effects of Changes in the Opportunity Costs of a Wrong Decision and/or Changes in the Probability of a Wrong Decision

If the cost of an investigation increases while everything else remains the same, the cost of false alarms becomes more costly. Consequently, a lower level of significance, giving fewer false alarms, becomes more desirable. For example, if the cost of an investigation increases to \$7.50, the .05 level of significance would be the Equalization level because this opportunity cost

³This value was obtained by following the same procedure outlined in footnote 2.

of rejection times the .05 level is just equal to the \$.75 opportunity cost of acceptance times the .5 probability of making a Type II error (for alternative parameters 230 and 260) at a conditional average opportunity cost of \$.375. Contrariwise, if the cost of an investigation is reduced, rejection becomes less expensive relative to acceptance, thus signaling the desirability for a higher level of significance.

Moreover, if meat cutter's wages are increased, while everything else remains constant, the opportunity cost of a Type II error is increased so that acceptance is more expensive relative to rejection. Therefore, a higher level of significance is desirable. If meat cutter's wages are reduced, the Equalization level of significance is lower by reverse reasoning.

The following generalizations can be drawn from this discussion.

1. The Equalization level of significance is increased if:
 - A. The cost of an investigation (i.e., the cost of a Type I error) is reduced.
 - B. The opportunity cost of a Type II error is increased.
2. The Equalization level of significance is reduced if:
 - A. The cost of an investigation is increased.
 - B. The opportunity cost of a Type II error is reduced.

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Comparison of the Equalization Approach with
the Bierman, Fouraker, and Jaedicke Approach

The Equalization approach concerned itself with an analysis of individual performances rather than an analysis of summary reports. While the Equalization approach could be applied to summary reports, it is used at the performance level because control is more effective at this level without the aggregation and timing problems inherent in summary reports.

The Equalization approach selected alternative parameters that could be serious--the ones that yielded the highest conditional average opportunity cost (see Table 9); whereas, the Bierman, Fouraker, and Jaedicke approach implicitly assumed that the alternative parameter would be equal to the actual performance value.

Table 10 shows a more scientific approach toward calculating the lapsed time interval before the detection of a change in the cause system; in contrast, Bierman, Fouraker, and Jaedicke arbitrarily multiply the single performance opportunity cost by four. The Equalization approach considers the cost of a Type I error, which is a constant, rather than the cost of an investigation, which is not a constant if an investigation is a correct decision.

In respect to the probabilities, the Equalization approach uses both the probability of Type I and Type II errors as they are defined in Classical Statistics.

The first interpretation of the Bierman, Fouraker, and Jaedicke approach uses the Classical probability of a Type I error. It does not consider the probability of a Type II error. Their second interpretation of P is neither Classical nor Bayesian but may yield useful results.

Bayesian Statistics

Without identifying their procedure as such Bierman, Fouraker, and Jaedicke made use of some aspects of the recently developed approach that is commonly labeled "Bayesian Statistics."⁴ This branch of statistics is so named after Bayes whose theorem "specifies how a prior distribution, when combined with additional sample evidence, leads to a revised distribution reflecting the most current information about the unknown parameter."⁵

Robert Schlaifer⁶ combined "explicit consideration of consequences [costs] of possible wrong decisions" and decision making "on the basis of expected monetary value"⁷

⁴Birnberg [J. G. Birnberg, "Bayesian Statistics: A Review," Journal of Accounting Research, II, No. 1 (Spring, 1964), 113.] did, however, recognize the Bayesian aspects of the Bierman, Fouraker and Jaedicke model.

⁵Robert Smith, "Quality Assurance in Government and Industry: A Bayesian Approach," Journal of Industrial Engineering, XVII, No. 5 (May, 1966), 256.

⁶Robert Schlaifer, Probability and Statistics for Business Decisions (New York: McGraw-Hill Book Company, Inc., 1959) and Robert Schlaifer, Introduction to Statistics for Business Decisions (New York: McGraw-Hill Book Company, Inc., 1961).

⁷Gerald H. Glasser, "Classical Versus Bayesian Method of Statistical Analysis," The Statistical News, XV, No. 6 (February, 1964), 3.

with Bayes' Theorem. This combination has been known as Bayesian Statistics although Robert Smith⁸ suggests that this second feature would more correctly be called the Schlaifer Method. The reader will recognize this second feature of the Bayesian approach as that adopted by Bierman, Fouraker, and Jaedicke.

To draw a distinction between Classical and Bayesian statistics, Morris Hamburg states that

in classical statistics, probability statements generally concern conditional probabilities of sample outcomes given specified population parameters. The Bayesian point of view would be that these are not the conditional probabilities we are usually interested in. Rather, we would like to have the very thing not permitted by classical methods--conditional probability statements concerning population values, given sample information.⁹

It has previously been mentioned that Classical statistics estimates the probability of obtaining a chance deviation as large or larger than that observed. The hypothesis is assumed to be true unless this estimated probability is smaller than an arbitrarily selected, and usually small, level of significance. The situation is somewhat analogous to a person charged with a crime who is assumed to be innocent until "proven" guilty. Regardless of whether the hypothesis is accepted or rejected,

⁸Robert Smith, Journal of Industrial Engineering, XVII, No. 5 (May, 1966), 256.

⁹Morris Hamburg, "Bayesian Decision Theory and Statistical Quality Control," Industrial Quality Control, XIX, No. 6 (December, 1962), 11.

no probability statements are placed on the truth or falsity of the hypothesis. This is why it was wrong for Bierman, Fouraker, and Jaedicke to calculate their probability according to the classical procedure and then to interpret this as the probability that the hypothesis is true.

In fact, the Bayesian approach does not even concern itself with the formulation of hypotheses. It begins by placing a probability on the existence of each parameter that might be possible. The resulting probability distribution is known as the prior distribution. The probabilities may be assigned on the basis of past information, intuition, or a combination of the two. A sample is then taken and the sample results are used to revise the original probabilities. As Hamburg indicates, this results in "conditional probability statements concerning population values, given sample information."¹⁰

The Bierman, Fouraker, and Jaedicke second interpretation of P corresponds to the prior distribution, although their P is calculated according to the Classical interpretation. They do not, however, carry through to revise these prior probabilities in light of sample information.

The Classical statistician objects to the Bayesian assignment of probabilities to possible parameter values.

¹⁰Ibid.

He claims that the parameter is a constant, in spite of the fact that its value is unknown, and that the assignment of probabilities implies that it is a random variable. The Bayesian retorts that the parameter is a random variable to the statistician, if he does not know the value. In this regard, Gerald H. Glasser makes the following distinction between Classical and Bayesian statistics. In reply to his own question "What is a random variable?"

Mr. Glasser noted:

Objectivist [Classical statistician]: A random variable is any sample quantity such as the sample mean the value of which will depend on the particular sample of observations that is obtained in a study. The quantity is a random variable in the sense its value would vary from sample to sample if we repeated our random sampling procedure many times.

Bayesian: If a decision-maker is uncertain of the value of some quantity (statistic or parameter or individual characteristic) it is a random variable to him. He may make personal probabilistic statements about the random variable. Once the value that the random variable assumes is known, it no longer is a random variable.¹¹

Bayesian Application to Quality Control

Robert Schlaifer has used an example from quality control to illustrate his application of the Bayesian approach. He assumes that a manufacturer uses an automatic machine to produce a particular part in production runs of 500. After each production run, the machine is taken down

¹¹Gerald H. Glasser, The Statistical News, XV, No. 6 (February, 1964), 3.

for the replacement of worn tools, etc., and then is readjusted by the operator. When the machine is properly adjusted, it will produce a process average fraction defective of .01. The machine is not capable of doing better, but there is no mechanical reason why it should do worse. From past records, the manufacturer computes the following frequency distribution of the fraction defective resulting from adjustments by the machine operator:

<u>Fraction Defective</u>	<u>Relative Frequency</u>
.01	.7
.05	.1
.15	.1
.25	.1
	1.0

(This is known as the prior distribution.) As an alternative to having the adjustment by the machine operator, the manufacturer can hire an expert mechanic who will always adjust the machine properly. The following information is needed to make a decision:

1. The mechanic charges \$6 for each adjustment.
2. Each defective part can be reworked at a cost of \$.40.
3. The operator can adjust the machine at no extra cost. (This is not, however, a realistic assumption; but the model could be adjusted to account for a charge.)

The expected opportunity cost of each alternative must be calculated. That alternative which yields the lowest expected opportunity cost should be selected. In order to find the expected opportunity costs, it is necessary first to find that fraction defective at which the firm is just indifferent between the alternatives. This break-even point can be found by equating the cost of accepting the operator's set-up with cost of rejecting his set-up. The cost of accepting the operator's set-up is 500, the number of parts in the run, times the unknown fraction defective, P , times \$.40 for re-working each defective part. (500 P represents the number of defective parts.) The cost of rejecting the operator's set-up is the \$6 cost of hiring the expert mechanic plus the quantity 5, the number of defective units that will inevitably result ($.01 \times 500$), times the \$.40 cost of re-working each defective unit. By equating these two costs and solving for P , one obtains the break-even point of .04 in the manner shown below.

Cost of Operator Acceptance = Cost of Operator Rejection

$$(500 P) \$.40 = \$6 + (.40 \times 5)$$

$$200 P = \$8$$

$$P = \frac{8}{200} = .04$$

If the manufacturer knew a priori that P on any given adjustment would be less than .04, he would allow

the operator to make the adjustment. Contrariwise, if he knew that P would be more than .04, he would hire the mechanic. Since he could not know this in advance, he must make his decision on the basis of expected opportunity costs.

Table 11 shows how the expected opportunity costs are derived. The conditional opportunity costs of acceptance and rejection are conditional upon the fraction defective. The individual values may be determined by reference to the following opportunity cost functions.

Opportunity cost of acceptance

<u>Event</u>	<u>Opportunity Cost</u>
if $P \leq .04$	0
if $P > .04$	$\$200 P - \8

Opportunity cost of rejection

if $P < .04$	$\$8 - \$200 P$
if $P \geq .04$	0

If the operator's set-up is accepted and if $P \leq .04$ the best decision was made for the event which actually occurred so there is no opportunity cost. Conversely, if $P > .04$, the manufacturer would spend $200 P$ by accepting the operator's set-up when he should have spent only \$8 by hiring the mechanic. The difference when $P = .05$ is $[\$200 (.05) - \$8] \$2$ --the conditional opportunity cost

of acceptance. When P is .15 and .25 respectively, the conditional opportunity costs of acceptance are \$22 and \$42 respectively.

Now if the operator's set-up is rejected and P is less than .04, the manufacturer spends \$8 by hiring the mechanic; whereas, he only had to spend $200P$ by allowing the operator to set-up the machine. The difference when $P = .01$ is $[\$8 - 200(.01)]$ \$6--the conditional opportunity cost of rejection. If $P > .04$ and the operator's set-up is rejected, the opportunity cost is zero because the best decision was made for the event which occurred.

TABLE 11.--Expected opportunity costs of two alternatives

Fraction Defective	Relative Frequency	Opportunity Cost of Acceptance		Opportunity Cost of Rejection	
		Cond.	Exp.	Cond.	Exp.
.01	.7	\$ 0	\$0	\$6	\$4.20
.05	.1	2	.20	0	0
.15	.1	22	2.20	0	0
.25	.1	42	4.20	0	0
	1.0		\$6.60		\$4.20

The following abbreviations were necessary:

Cond. for conditional

Exp. for Expected

The expected opportunity costs are the result of multiplying the conditional opportunity costs by the relative frequencies. In other words, the conditional figures

are averaged in order to find the expected ones. Since the expected opportunity cost of rejecting the operator's adjustment is less than the expected opportunity cost of accepting it, the manufacturer would hire the mechanic if he had to make a decision to either always hire the mechanic or always accept the operator's adjustment.

Fortunately, it is possible to reduce the opportunity costs still further by following the procedure indicated below:

1. Allow the operator to adjust the machine. It is assumed that this can be done at no extra cost to the manufacturer.
2. Take a sample of the first n pieces.
3. Record the number of defectives, r .
4. Make a decision on the basis of the following rules:
 - A. If $r \geq$ some pre-determined number, C , reject the operator's adjustment and call in the mechanic.
 - B. If $r < C$, accept the operator's adjustment.

Schlaifer begins by holding n constant at 20 and uses the probabilities and the opportunity costs to arrive at the best rejection number, C . The relevant information has been marshalled in Table 12 for rejection numbers one to three which fall in the relevant range. The table indicates that two is the best rejection number because its

expected opportunity cost, \$.71, is less than the opportunity cost for either of the other rejection numbers.

TABLE 12.--Unconditional expected opportunity costs for various rejection numbers

P	Prior Prob.	Op. Cost of Wrong Dec.	Prob. of Wrong Dec.	Ave. Op. Cost Given P	Expected Cost
C = 1					
.01	.7	\$ 6	.1821	\$1.09	\$.76
.05	.1	2	.3585	.72	.07
.15	.1	22	.0388	.84	.08
.25	<u>.1</u>	42	.0032	.13	<u>.01</u>
	1.0				\$.92
C = 2					
.01	.7	\$ 6	.0169	\$0.10	\$.07
.05	.1	2	.7358	1.47	.15
.15	.1	22	.1756	3.86	.39
.25	<u>.1</u>	42	.0243	1.02	<u>.10</u>
	1.0				\$.71
C = 3					
.01	.7	\$ 6	.0010	\$.06	\$.04
.05	.1	2	.9245	1.85	.18
.15	.1	22	.4049	8.91	.89
.25	<u>.1</u>	42	.0913	3.85	<u>.38</u>
	1.0				\$1.49

The following abbreviations are used:

P for fraction defective
 Prob. for probability
 Op. for opportunity
 Dec. for Decision

Some explanation of Table 12 might be helpful. The first two columns show the prior probability distribution. The opportunity costs of a wrong decision

were derived from the opportunity cost functions. The figures have already been shown in Table 11. If $P = .01$, rejection is a wrong decision and the opportunity cost is \$6. If $P = .05$, or $.15$, or $.25$, acceptance is a wrong decision and the opportunity costs are \$2, 22, and 42 respectively. The probabilities of a wrong decision are obtained from the table of binomial probabilities. For $C = 2$ and $P = .01$, a wrong decision consists of rejection so .0169 is the probability of obtaining two or more defective units in a sample of 20. The other probabilities are interpreted in a similar manner. The average opportunity cost given P is determined by multiplying the opportunity cost of a wrong decision by the probability of a wrong decision. The unconditional expected opportunity cost is a result of multiplying the average opportunity cost given P by the prior probabilities.

Schlaifer added the cost of sampling \$.65, to the \$.71 unconditional expected opportunity cost to arrive at \$1.36 total opportunity cost. He then found, by computer operation, the total opportunity costs for the best rejection numbers for other sample sizes. In this manner, Schlaifer found that the best sample size was 27 and the best rejection number was 2. This writer plans to use this same approach to determine the optimum level of significance to use for accounting variance control. So far, however, Schlaifer has considered only the opportunity

costs of wrong decisions and the probabilities that these wrong decisions will be made. The work of Bayes enters the picture only when the prior probabilities are combined with sample evidence in order to arrive at a revised probability distribution.

Schlaifer introduces the Bayesian aspect by assuming that a sample of 20 parts yielded 2 that were defective. Table 13 shows how this sample evidence is combined with the prior probabilities to yield the revised probabilities. As before, the first two columns represent the prior distribution. The conditional probabilities represent the probabilities of getting exactly 2 defective units in a sample of 20 given P . That is, .0159 is the probability that exactly 2 defective units will be found given that $P = .01$, .1887 is the probability that exactly 2 will be found if $P = .05$, etc. These probabilities are found in a table of the binomial distribution. The joint probabilities are obtained by multiplying the prior probabilities by the conditional probabilities for each respective P . The revised probabilities represent the ratio of the joint probability for each respective P to the summation of the joint probabilities. For example .187 is equal to $.0113/.05963$.

Now is it possible to make the kind of statement that distinguishes Bayesian statistics from Classical. The Bayesian would say given the sample evidence that 2

defectives were found and the prior probability distribution, .187 is the probability that $P = .01$. Likewise, the probabilities that $P = .05$, .15, and .25 are .316, .385, and .112 respectively. If the operator's set-up is accepted, the probability of a Type II error is .813 (.316 + .385 + .112). Conversely, if the hypothesis is rejected, the probability of a Type I error is .187. Since it has been determined that 2 is the best rejection number, the operator's set-up will be rejected. It is significant that the probability of a Type I error, .187, is higher than the .001 or .05 levels of significance that are customarily used in the application of the Basic Control Chart approach.¹²

TABLE 13.--Revision of prior probabilities

P	Prior Prob. of P	Conditional Probability	Joint Probability	Revised Probability
.01	.7	.0159	.01113	.187
.05	.1	.1887	.01887	.316
.15	.1	.2293	.02293	.385
.25	.1	.0670	.00670	.112
	1.0		.05963	1.000

Morris Hamburg indicates that the Bayesian approach criticizes Classical on these grounds:

(1) Classical does not provide a method for combining prior information with experimental evidence, and (2) too much burden is placed on significance levels as a

¹²The foregoing discussion was developed in Robert Schlaifer, Introduction to Statistics for Business Decisions, pp. 150-197.

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means of deciding between alternative acts--specifically, no formal method is provided for the inclusion of economic costs as a part of the decision making process.¹³

While the second of these grounds is historically quite true, there is no reason why economic costs cannot be incorporated into Classical statistics to determine the optimum level of significance. In fact, this is exactly what was done in the Equalization approach discussed earlier in this chapter. The inclusion of economic costs has been identified with the Bayesian approach; but these costs could just as well be included in the Classical approach. The level of significance is not necessarily unique to Classical statistics. The term "rejection number" used by Schlaifer is analogous to the level of significance. A study of the calculations indicates that the level of significance associated with the rejection number of 2 and the sample size of 20 is .0169. That is, .0169 is the probability of rejecting a hypothesis that should have been accepted. The first of Hamburg's criticisms, however, strikes at the heart of the difference between Classical and Bayesian statistics.

Application of a Bayesian Concept to the Meat-Cutter Example--Minimization Approach

Determination of Level of Significance

It is possible to incorporate the prior probabilities and the economic costs of a wrong decision into the

¹³Morris Hamburg, Industrial Quality Control, XIX, No. 6 (December, 1962), 14.

Table 12 format to determine a level of significance for the meat-cutter example. The first step is to prepare a prior probability distribution for all possible parameters. The results of this task will not be precise because one can never know the exact value of the parameter from which an individual performance is observed. One approach would be to:

1. Prepare a list of the causes of all performances.
2. Estimate the probability of the occurrence of each cause.
3. Estimate the value of the parameter associated with each cause.

Assume that this procedure results in the following information:

<u>Cause</u>	<u>Probability</u>	<u>Parameter</u>
Improvement in Skill	.05	235
Chance (Standard Performance)	.85	245
Dull Knives	.05	265
Laziness	.05	280

The estimates of the probabilities and the parameters may be made from past information, from intuition, or from a combination of both past information and intuition. This procedure permits the use of the most objective information available. When past information is not available or when it is incomplete, one must use his best judgment. The charge that the use of statistics replaces

judgment is not valid. Judgment is an integral part of statistics. If it is not explicitly incorporated into the analysis, it is implicit, as it must be, with an arbitrarily selected level of significance.

The advantage of using Bayesian statistics, however, is that it provides the procedure whereby intuitive judgment can be revised on the basis of experience. This procedure has just been indicated in Table 13. When past records are not available to estimate a prior probability distribution such a distribution can be based upon intuitive judgment. The sample results then serve as a basis for revising the prior distribution according to the procedure indicated in Table 13. Such revisions should be made frequently until the differences between the revised distributions are insignificant. At this time the probability distribution will be reasonably accurate. One could assume that the above probabilities were derived in such a manner. They will be used as the prior probabilities in Table 14.

The next step is to determine the opportunity cost of a wrong decision corresponding to each parameter. These figures are derived by the same procedure illustrated in Table 8. They are weighted according to the procedure followed in Table 10 to account for the fact that off-standard conditions are not always detected on their first occurrence.

Table 14 indicates how the relevant information is combined to determine the level of significance associated with the Minimization approach. This table is essentially the same as Table 12. For explanatory purposes attention will be directed first to just one level of significance--the .05 level. The figures for the opportunity cost of a wrong decision and the probability of a wrong decision are the same for their respective parameters as those shown in Table 9 except that the opportunity costs have been weighted for each parameter by the same procedure discussed in conjunction with Table 10. The average opportunity costs given the cause are the result of multiplying the weighted opportunity costs by the probability of a wrong decision. The contributions to the expected opportunity costs are determined by multiplying each average opportunity cost given the cause by the probability that its respective cause will occur. The sum of the expected opportunity cost column represents the unconditional expected opportunity cost for a .05 level of significance. This procedure should be carried out for other levels of significance within the relevant range. The desired level is the one with the lowest expected opportunity cost.

For the levels tested in Table 14, the decision maker is indifferent between the .01 and the .03 level. It is possible, however, that a lower expected opportunity cost could result from a level less than .01 or from one between .03 and .05.

TABLE 14.--Unconditional expected costs of various levels of significance

Chance	Prior Prob.	Weighted Op. Cost	Prob. of Wrong Decision	Ave. Op. Cost given Cause	Expected Op. Cost
.07 Level					
Improvement	.05	\$1.50	.7146	\$1.07	\$.05
Chance	.85	5.00	.07	.35	.28
Dull Knives	.05	1.30	.2514	.33	.02
Laziness	.05	1.75	.0057	.01	.00
					\$.35
.05 Level					
Improvement	.05	\$1.62	.7314	\$1.18	\$.06
Chance	.85	5.00	.05	.25	.21
Dull Knives	.05	1.31	.2676	.35	.00
Laziness	.05	1.75	.0066	.01	.00
					\$.29
.03 Level					
Improvement	.05	\$2.24	.8078	\$1.81	\$.09
Chance	.85	5.00	.03	.15	.13
Dull Knives	.05	1.52	.3557	.54	.03
Laziness	.05	1.75	.0129	.02	.00
					\$.25
.01 Level					
Improvement	.05	\$3.79	.8925	\$3.39	\$.17
Chance	.85	5.00	.01	.05	.04
Dull Knives	.05	1.72	.5	.86	.04
Laziness	.05	1.75	.0314	.05	.00
					\$.25

Space provisions necessitated the following abbreviations:

Prob. for Probability
Op. for Opportunity
Cond. for Conditional

The fact that this level of significance is lower than that obtained by the Equalization Approach, .11+, is obviously due to the introduction of the prior probabilities. In Chapter VI the financial impact of differences in the control limits will be examined in order to determine which of the methods of significance determination is most useful for variance control purposes.

Revision of the Prior Probability Distribution

The complete application of Bayes' Theorem requires that sample information be used to revise the prior probability distribution.¹⁴ Table 15 has been prepared under the assumption that a performance taking 250 minutes was observed. The first two columns represent the prior probability distribution. The conditional probabilities for each cause represent the probability of obtaining a performance of exactly 250 minutes given that cause. These probabilities are obtained by the method of "normal curve approximation." For example, .0085 is the probability of observing a performance of exactly 250 minutes if the cause is improvement. It is computed by finding the area under a normal curve between 250.5 and 249.5 given a parameter of 235 and a standard deviation of 8.06 (the mean and

¹⁴Here it can be assumed that the prior probability distribution has already been finalized by the revision process. The procedure is still useful, however, in attaching probabilities to the causes of specific variances.

standard deviation for assignable cause--improvement). The joint probabilities for each parameter are calculated by multiplying the original probabilities by the conditional probabilities. The revised probabilities represent the ratio of the joint probabilities for each parameter to the summation of the joint probabilities.

The revised probabilities may be interpreted in the following manner. Given a performance value of 250, the decision maker is .9752 confident that this is a chance variance from standard; he can assert with a probability of .0124 that there has been an improvement in the meat-cutter's ability. Finally, he can assert with a probability of .0124 that dull knives were used.

TABLE 15.--Revision of prior probabilities

Cause	Original Probability	Conditional Probability	Joint Probability	Revised Probability
Improvement	.05	.0085	.000425	.0124
Chance	.85	.0394	.033490	.9752
Dull knives	.05	.0085	.000425	.0124
Laziness	.05	.0000	<u>0.000000</u>	<u>0.0000</u>
			.034340	1.0000

For either the .03 or the .01 level of significance, 250 falls within the region of acceptance. (The upper control limits corresponding to the .03 and .01 levels of significance are 262 and 265 respectively.) The revised probability distribution indicates that acceptance will be a

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correct decision 97.52 per cent of the time. The probability that acceptance is a Type II error is $1 - .9752$ or $.0248$. Thus, the Bayesian is saying that the probability that the hypothesis is true is $.9752$; the probability that it is false is $.0248$.

One advantage of the Bayesian method is that it permits consideration of all possible alternative parameters which are worth the cost of control; in contrast, the Classical method permits consideration of only one alternative parameter or a pair of alternative parameters with each value being the same distance from the standard.

Another advantage is that the revised probabilities help to identify the cause of the variance. In the case just cited the betting odds of chance over assignable causes would be given as $.9752$ to $.0248$. Table 16 shows the revised probabilities for observed performances 260 and 270. Notice that 260, which falls within the control limits corresponding to either the $.01$ or the $.03$ levels of significance, still carries a high probability, $.8571$, of being attributed to chance. A performance of 270, however, is almost certain to be the result of an assignable cause. This is consistent with the distribution of chance performances in which a chance performance over 270 had never been observed. The odds favor dull knives ($\mu=265$) over laziness ($\mu=280$) 65 to 35 respectively. Not only do the revised probabilities help to determine the cause of

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the variance, but, in the case of significant variances, they determine the course of the investigation. For a performance of 270, it would be more profitable to check the sharpness of the knives before investigating laziness as a possible cause.

TABLE 16.--Revised probabilities for performances
260 and 270

Cause	Revised Probability Perf = 260	Revised Probability Perf = 270
Improvement	.0000	.0000
Chance	.8571	.0000
Dull knives	.1310	.6470
Laziness	.0119	.3530

Assume that the \$5 opportunity cost for a complete investigation consists of \$1 for dull knives and \$4 for laziness. If an investigation for dull knives reveals their condition to be satisfactory, laziness in this simplified problem, is identified as the cause by the process of elimination. Since the investigation for dull knives is a cheaper element of the investigation cost than the investigation for laziness, it is the only element that needs to be incurred for performances of 270 or more. The cost of an investigation, therefore, is only \$1. Had this model considered the parameters for other unfavorable assignable causes such as illness, improper training, or poor attitude, an investigation would have been continued

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until either the assignable cause was discovered or until all but one of the causes had been eliminated. In this more practical case, the Bayesian approach would be more helpful in establishing priorities for the investigation process. For performances between the upper control limit and 270, the complete investigation would have to be made since chance performances are possible in this interval.

(That is, a Type I error is possible between the upper control limit and 270; but performances over 270 always result from an assignable cause.)

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CHAPTER VI

A TEST OF THE ACCOUNTING AND STATISTICAL CONTROL TECHNIQUES

Introduction

In addition to the arbitrary methods conventionally employed by accountants, this dissertation has discussed five methods involving statistical procedures for determining the significance of variances. These methods have been identified as:

1. Basic Control Chart approach with an arbitrarily selected level of significance.
2. Bierman, Fouraker, and Jaedicke approach with two conflicting interpretations of the probabilities.
3. McMenimen approach.
4. An Equalization approach developed by this writer.
5. An Minimization approach which employs prior probabilities.

In this chapter an example will be developed and the upper and lower control limits will be calculated under each method according to the following testing plans:

1. Tests of single performances where
 - A. Each performance is tested

- B. Every tenth performance is tested on a systematic basis
2. Tests of the means of samples of five consecutive performances where
- A. Every performance is included in a sample
 - B. The frequency of sampling is adjusted so that on the average a sample of five is taken in every 50 performances.

The purpose of these calculations is to test the impact of the resulting differences in control limits in order to ferret out the method which is most effective for cost control.

The Example

Development

The hypothetical example involves the time taken for each of fifty meat cutters to butcher each of twenty cows. It is assumed that each of the 1000 performances was investigated to determine the cause. The value of each performance has been recorded and the mean has been computed for each cause. Table 17 summarizes the results. More detail is shown in Table 18 which depicts the number of performances occurring at each value under each cause. It is assumed that this information has been obtained without the knowledge of the butchers so that the frequency

1894-1895

1895-1896

1896-1897

1897-1898

of the causes represents what has been experienced in the immediate past without an unusual effort on the part of the butchers to reduce assignable causes. Such an unusual effort might be put forth on the part of the workers if they knew that each individual performance was being observed. Prior to this test it is assumed that control has taken the form of a comparison of actual weekly cost for the department (composed of all butchers) with budgeted cost for the cows butchered. An investigation has been undertaken when the variance exceeds 10 per cent of the budget.

TABLE 17.--Causes--their frequencies and means

Cause	Number of Performances	Mean
Dull Knives	120	270
Tough Cows	20	280
Lack of Training	40	285
Poor Attitude	60	255
Illness	20	265
Improvement	100	230
Laziness	40	275
Chance	<u>600</u>	<u>245</u>
Grand Mean	1,000	251

Note that the mean of the chance performances is 245. This value has previously been established as the standard. Note further that the grand mean is 251. Under the 10 per cent rule currently employed an investigation would not be undertaken unless the grand mean was 270

(245 + 10% of 245). In this case, the 10 per cent rule hardly seems adequate in view of the fact that 400 of the 1000 performances were due to assignable causes. This point will be taken up in more detail later.

The causes enumerated in Tables 17 and 18 are certainly not mutually exclusive. That is, it would be possible to have several combinations of causes present during any performance. However, to keep this analysis to a manageable level and to focus attention on concept rather than procedure, the cause of each of the 1000 performances has been assumed to be mutually exclusive in this example.

Before this plan was instituted it is assumed that two new apprentices were employed. Normally, it takes several months for the work performance of new employees to come up to that of the other butchers. These new men account for the forty performances in the lack of training category. This department contains five long-standing butchers whose mean performance has been known to be less than standard. The 1000 performances of these men are revealed in the improvement category. Even though the standard might be kept at 245 for double entry accounting purposes, it is this writer's opinion that it should be revised for control purposes to take cognizance of the improvement of these senior men.

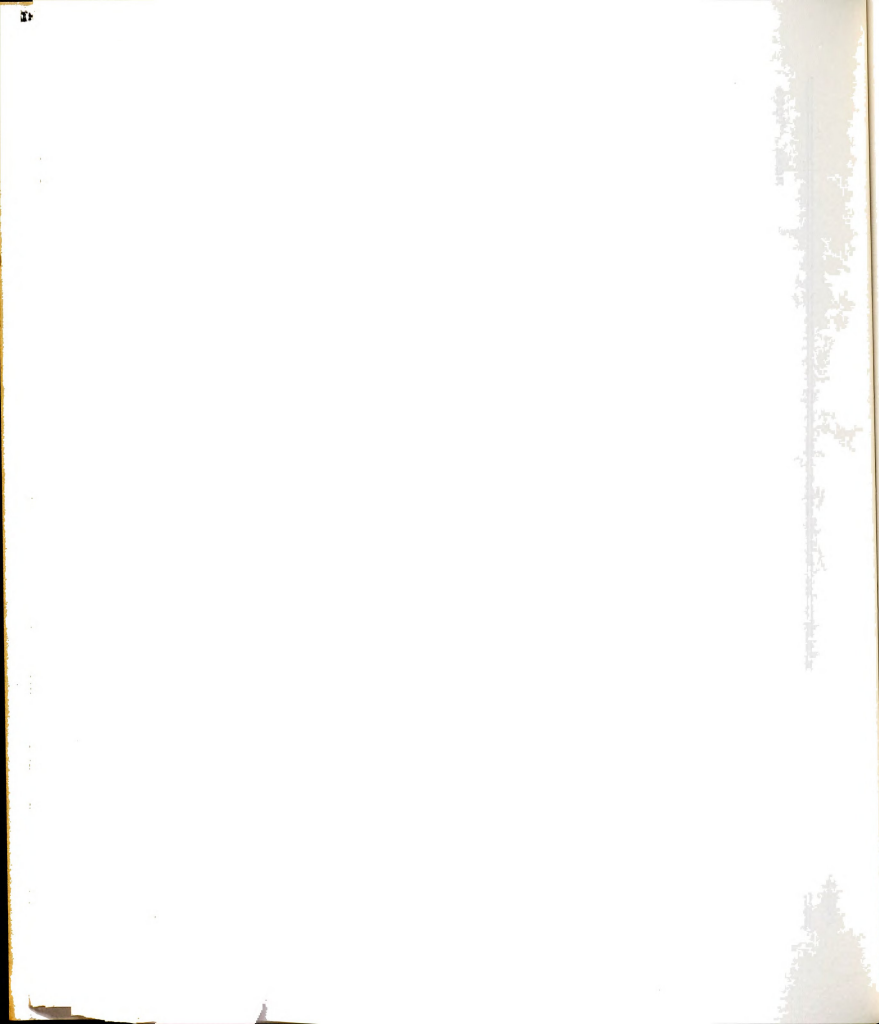
TABLE 18 (Continued)

Value	Improvement	Poor Attitude	Dull Knives	Tough Cows	Chance	Lack of Training	Illness	Laziness	Total
242	1	1			25				27
243	3	1			35				39
244	2				45				47
245	3	3			60				66
246					45				45
247	1	1			35		1		38
248	1	2			25				28
249	1	1			15				17
250	1	3			12				17
251		1	1		9				10
252		2			11		1		14
253		3	1		13				17
254		4			16		1		21
255		9	2		18				29
256		4	1		16				21
257		3	1		13		1		18
258		2	2		11			1	16
259		2	2		9		1	1	15
260		1	2		9				14
261		1	4		2				5
262		1	2		2		1		8
263		1	2	1	2			1	10
264		3	4		2		1	1	10
265		2	4		2		1	1	10
266		1	4		3		1	1	10
267		1	4	1				1	7
268		1	5					1	8
269		1	7		1			1	10
270		1	8	1			1	1	12
271		1	12		1			1	15
272			8				1	1	10
273			7	1				1	9
			5	1				2	9



TABLE 18 (Continued)

Value	Improvement	Poor Attitude	Dull Knives	Tough Cows	Chance	Lack of Training	Illness	Laziness	Total
274			4				1	3	8
275		1	4	1			1	4	11
276			4	1			1	2	8
277			4	1		1		2	8
278			4	1		1		2	8
279			4			1	1	2	8
280			1	2		1		1	5
281			1			2		1	4
282			1	2		3		1	7
283			1	1		3		1	6
284			2	1		4		1	8
285			1	1		5		1	8
286						3	1	1	4
287						3		1	4
288						3		1	5
289						3			4
290			1	1					2
291				1		1			1
292						1			2
293				1		1		1	2
294								1	2
295									0
296									0
297						1			1
298									0
299				1					0
300									1
301									0
302									0
303									0
304									0
305						1			1



It is assumed that the other information in Tables 17 and 18 was detected by the following procedures. The knives were all tested for sharpness before the study began. They were also tested periodically thereafter. If the knives were found to be dull, no action was taken until the butcher himself reported this condition. The reason for this is, of course, that the firm wanted to determine how frequently the butcher will not realize that his knives are dull and at the same time obtain an estimate of the probability that this assignable cause will be present. A psychological test was administered without explaining this study. It indicated that three men accounting for sixty performances had poor attitudes because of family problems. The same test showed that seven butchers are prone to laziness. This situation shows up in their work sporadically and contributed to forty performances during the observation period. The best way to improve this record is to detect the condition immediately and call it to the attention of the butcher involved. This might be accompanied along with the hint that his wage rate or other benefits might be adversely affected.

A physical examination revealed that one man accounting for twenty performances was ill although he had been unaware of his illness.

Use of the Example

The hypothetical information "discovered" through the procedures just described has been summarized in Table 18. It will be used throughout the remainder of this chapter to estimate the probabilities of committing Type I and Type II errors as well as to estimate the probabilities of the existence of the various causes.

Selection of the Example

This writer considered using an empirical example from an actual industrial situation. However, since statistical procedures other than the scant use of the Basic Control Chart approach are not actually employed anywhere to this writer's knowledge, complete information would not be readily available. Of course, estimates could have been made as they must be initially for a company adopting such procedures. It was, however, felt that the basic features of the model could be more clearly portrayed with the assumption of perfect knowledge through the construction of an hypothetical example.

The distribution of chance performances shown in Table 18 is symmetrical about the standard. It approximates a normal distribution but has not been fitted to a normal curve. The reader will note that it is not perfectly continuous. For example, no observations are reported for values 221 and 222 although there are observations listed

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at 220 and 223. Also, there are 11 performances at 238 but only 9 at 239; whereas, a perfectly continuous distribution would require more performances at 239 than at 238. The number of performances selected for each value were selected as this writer felt 600 performances might actually fall in practice. No attempt was made to bias the example to achieve any particular results. In fact, it will be noted in the conclusions that the results may change if the distribution of performance values changes.

Likewise, the distributions for each of the assignable causes are symmetrical about their respective means. Because there are fewer observations for each assignable cause than there were chance observations, the assignable cause distributions do not, in most cases, even approximate normality. The values were purposely set down so that there would be some overlap among the assignable cause distributions and the chance distribution. The reason for this was to include the possibility of Type I and Type II errors. Other than this, the values were not selected with any particular design in mind. They were selected so that they might give the appearance of reality. However, they were not selected in any effort to achieve any particular results.

Investigation Procedure

The purpose of this example is to calculate the upper and lower control limits for each testing plan under each

other hand, the investigation may be undertaken so that suppliers can be informed when the quality of cows is poor. When all these procedures fail to reveal an assignable cause, it is concluded that a Type I error has been made. The cost of the Type I error is \$6 which is determined as follows:

<u>Test For</u>	<u>Incremental Cost</u>	<u>Accumulated Cost</u>
Dull Knives	\$ 1	\$ 1
Attitude-Laziness	1	2
Illness	3	5
Tough Cows	1	6

If an investigation is not undertaken for tough cows, then, the opportunity cost of a Type I error is only \$5.

When a performance value observed by one of the butchers other than the five who are already known to have improved is smaller than the lower control limit, an investigation should be undertaken to ascertain whether improvement has occurred. This investigation would probably involve an analysis of some past performances for this worker as well as closer attention of his next few performances. It is assumed that the opportunity cost associated with this investigation is \$4.

The purpose of the foregoing discussion has been to explain the determination of the opportunity cost of a Type I error (or the cost of an investigation). These figures will be used to calculate the control limits.

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Derivation and Financial Analysis of
Upper Control Limits for Single
Observations--Each Performance
Tested

Accountant's Conventional Method

It has already been pointed out that this firm has historically designated a variance as significant when it exceeded 10 per cent of the standard. With this criterion the upper control limit would be 270 $[245 + 245 (.10)]$ minutes. It is interesting to note that this upper control limit would reduce the probability of a Type I error to zero because Table 18 shows that a chance performance has never taken longer than 270 minutes. If investigations are undertaken only for performance values over 270, it would be impossible to investigate a performance coming from a chance population.

Basic Control Chart Approach

This approach involves the arbitrary selection of a level of significance based on the distribution of chance performances shown in Table 18. This level is usually in the range from .05 to .01 based on a two-tailed test. The .05 level with a two-tailed test would place the upper control limit at that value which is exceeded by 2.5 per cent of the chance performances. Since 3.667 (22/600) per cent of the chance performances are at least 260 and 2.167 (13/600) per cent are at least 261, the upper control limit is between 260 and 261.

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Bierman, Fouraker, and Jaedicke
Approach

Chapter IV discussed the Bierman, Fouraker, and Jaedicke development of the following formula for the critical probability, P_c :

$$P_c = \frac{L - C}{L}$$

where:

L is the present value of the expected opportunity cost resulting from not taking corrective action on the basis of the present deviation

C is the cost of an investigation

The following decision rules were then adopted.

1. Accept the hypothesis if P is larger than P_c
2. Reject the hypothesis and investigate if P is smaller than P_c .

In order to find the upper control limit it is necessary to test values until P is equal to P_c . At this point the decision maker is indifferent between the acts of making an investigation and not making one. Therefore, the value at the point of equality represents the control limit.

A test will be run first to see if this equality exists at 260. In order to find the value for L it is first necessary to find the opportunity cost sustained on each performance if the operation is off-standard. In Chapter

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IV it was noted that Bierman, Fouraker, and Jaedicke considered the alternative parameter to be equal to the test value.¹ Therefore, in testing the value 260 they would say that the opportunity cost sustained on each off-standard performance is equal to $\frac{245 - 260}{60 \text{ min}} \times \3 (\$3 is the wage rate for butchers) or \$.75. To translate this into an L value it is necessary to consider how long the off-standard condition will prevail before it is detected. In their illustration which dealt with determining the significance of a yearly variance from an aggregate account Bierman, Fouraker, and Jaedicke assumed that the variance would continue for four years. Four was an arbitrarily selected number. Accordingly, it seems reasonable that Bierman, Fouraker, and Jaedicke would assume that an off-standard performance would continue for four more performances before detection.² With this assumption, L would be \$3.

¹This assumption was questioned in Chapter IV. Table 17 shows that none of the causes have a mean of 260.

²The use of the number four was also questioned in Chapter IV. The Table below shows that the probability that an off-standard performance would not be detected on the third performance (and thus extend the inefficiency to the fourth performance) is only .011. Column A represents the number of successive failures to detect a change in the cause system. Column B represents the probability that the change will not be detected after the number of tests indicated in the first column.

A	B
1	.207
2	.053
3	.011

The cost of an investigation, C , depends upon the investigation procedure and upon the cause. For example, according to the procedure explained in conjunction with this example the cost of an investigation associated with the various assignable causes is \$1 for dull knives, \$2 for poor attitudes or laziness, \$5 for illness, and \$6 for tough cows. Since Bierman, Fouraker, and Jaedicke did not discuss the steps of an investigation procedure, they must have had in mind the cost of a complete investigation which is \$6 in this case. However, since a tough cow cannot be butchered in less than 262 minutes, \$5 can be assumed to be the cost of a type I error for test values below 262.

The table is interpreted to mean that the probability that an off-standard condition will not be detected on its first occurrence is .207. This probability is determined in the following manner:

1. Find from Table 18 the number of unfavorable assignable causes less than the test value 260. This number is 62 itemized as follows:

Poor Attitude	45
Dull Knives	10
Illness	5
Laziness	2
	<u>62</u>

2. Divide this by 300--the total number of unfavorable assignable causes.

This is the proportion of unfavorable assignable causes below 260 and as such represents the probability that an assignable cause will not be detected with an upper control limit of 260.

The probability that an off-standard performance will not be detected on its second occurrence, if all performances are tested, is .207 squared or .053. Likewise, the probability that it will not be detected on its third occurrence is .207 cubed or .011.

Substitution in the formula yields a negative Pc value

$$(P_c = \frac{L - C}{L} = \frac{\$3 - \$5}{\$3} = -.667).$$

When 270 is tested, Pc is still negative if C is set at \$6.³ In this case, the Bierman, Fouraker, and Jaedicke approach will not yield a determinate solution within a range that makes sense. Since all performances higher than 270 resulted from assignable causes, 270 is the highest value that any wise person would use as an upper control limit. The reason for the failure to yield a determinate solution is that this testing plan is concerned with an investigation of individual performances--a case that Bierman, Fouraker, and Jaedicke did not discuss. When all performances are investigated, L is small in relation to what it is when a testing plan involving a sampling procedure is used. It is small because it consists only of the savings on an individual performance weighted by the fact that an off-standard condition may not be detected on its first occurrence--rather than a long range savings.

On the other hand, if C is set at \$5, Pc is zero for test value 270. This yields a determinate solution when P is defined as "the probability of a deviation this

$$^3P_c = \frac{L - C}{L} = \frac{\$5 - \$6}{\$5} = -\$.20$$

$$L = \frac{270 - 245}{60} \times \$3 \times 4 = \$5.$$

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large or larger occurring from random causes."⁴ It is also specified whether the deviation is favorable or unfavorable. This will be referred to as the Bierman, Fouraker, and Jaedicke first interpretation of P. Since there are no chance performances over 270 and only one at 270, P is equal to $1/600$ divided by 2 or .0034. Since this is almost zero, $P = P_c$ and 270 is the upper control limit.

The reader will recall, from Chapter IV, that at a later point, Bierman, Fouraker and Jaedicke define P as "the probability of an unfavorable deviation resulting from uncontrollable [chance] causes."⁵ This has already been identified as their second interpretation of P. They incorrectly used these two definitions of P synonymously; but all of their calculations were carried out with the first definition in mind. In order to evaluate P according to the second definition, it is necessary to divide: (1) the number of times that each deviation (or test value) has resulted from chance causes by (2) the total number of times that each deviation (or test value) has occurred. Table 18 contains the information to evaluate P in this manner. All that needs to be done to calculate the P for any test value is to divide the number of chance performances by the total number of performances for that test value. For example, for test value 270 P is $1/15$ or .0667.

⁴Bierman, Fouraker, and Jaedicke, 113.

⁵Ibid., 121.

It has just been found that P_c is at most zero for test value 270. Hence, 270 is not a control limit under this approach because .0667 does not equal zero. It has also been established that P_c is negative for test values lower than 270. Since P is either zero or positive for all test values, the upper control limit cannot lie below 270. A control limit above 270 would not be useful since no performances due to chance have ever taken longer than 270 minutes. Consequently, the Bierman, Fouraker, and Jaedicke second interpretation of P does not yield a determinate solution.

McMenimen Approach

Leo McMenimen proposes that various amounts could be spent on an investigation with the result that various amounts would be saved. He also insists that the cost of correcting an off-standard condition be formally involved in the investigative decision.

Since an investigative procedure has been established in this example, it is appropriate to begin by testing to see if it is worthwhile to spend \$1 investigating a performance value of 260 for dull knives. McMenimen did not specify a procedure for determining the amounts to be saved; but in this example a procedure seems clear. The knives are either sharp or they are dull. If they are sharp nothing can be saved by an investigation. On the other hand, if

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middle 21st
late 21st

they are dull the amount to be saved is equal to the opportunity cost less the cost of sharpening the knives. The single performance opportunity cost for dull knives is \$1.25 [$(\frac{270 - 245}{60})\3 where 270 is the mean of the performances with dull knives].⁶ Bierman, Fouraker, and Jaedicke arbitrarily assumed that four tests would lapse before an assignable cause would be detected. Since McMenimen neither questioned this assumption nor offered a weighting scheme of his own, the \$1.25 will be multiplied by 4 to arrive at a \$5 opportunity cost which considers the fact that dull knives may not be detected on their first occurrence. It costs \$1.50 to sharpen the knives; but since each of the four performances that lapse on the average before dull knives are detected would benefit from sharpening, the cost can be spread over these four performances. Consequently, the cost applicable to the sharpening of one set of knives

⁶Actually, McMenimen did not attack the Bierman, Fouraker, and Jaedicke assumption that the mean of the assignable cause is equal to the performance value being tested, i.e., Bierman, Fouraker, and Jaedicke would use 260 rather than 270 in the above calculation. While in his theoretical discussion McMenimen did not relate the amount to be saved to specific assignable causes, it is clear that in applying his procedure one must know the assignable cause before the cost of correcting the cause and thus the amount to be saved can be estimated. For example, it costs more to cure an illness than to sharpen dull knives. These two causes have different means even though either cause may produce some of the same performance results. Accordingly, in this application, this writer is taking the liberty of introducing specific assignable causes with their related means into McMenimen's work.

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applicable to a single performance is \$.375. Thus, the savings from detecting dull knives is \$4.625 (\$5 - \$.375).

The information necessary to make the investigative decision has been marshalled in Table 19. In this initial step only two events are considered. It would not make sense to have savings between \$0 and \$4.625 because it would not be rational to partially sharpen the knives as a result of the investigation. The knives would either be sharpened to enable parameter reduction to 245 with a savings of \$4.625 or they would not be sharpened so that nothing could be saved by the investigation.

TABLE 19.--Application of McMenimen technique

Event	Spend \$1 investigating		
	Pe Test Value 260	Cond.	Exp.
Save \$0	.7143	\$-1	\$- .7143
Save \$4.625	.2857	3.625	1.0357
Expected Savings			\$.3214

The $P_e = .7143$ is determined by dividing the 10 performances with 260 values (shown in Table 18) that are not attributed to dull knives⁷ by the 14 performances with 260 values. Since the 4 remaining performances are due to dull

⁷One is due to poor attitude and nine to chance. The one due to poor attitude is included in the save \$0 category in this case because it would not be detected by an investigation for dull knives.

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knives; $4/14$ or $.2857$ represents the probability that \$4.625 could be saved by an investigation. The conditional values result from subtracting the \$1 investigation cost from the savings values. The contributions to the expected value represent the product of each respective P_e by its conditional value. The sum of these contributions represents the expected savings as a result of the investigation. Since expected savings is positive, it would be worthwhile to investigate for dull knives.

Whether it is worthwhile to extend the investigation for poor attitude⁸ depends upon the expected savings for act spend up to \$2 investigating. McMenimen states that "we will choose that act with the highest expected value."⁹ This would be agreeable if the decision maker were faced with a number of independent alternative acts or if the particular act had to be chosen a priori. In this example, however, it has already been decided to spend \$1 to investigate for dull knives. If dull knives are the cause, the investigation is terminated and the knives are sharpened. If dull knives are not the cause, some guidelines are necessary to determine whether an additional \$1 should be spent investigating for poor attitude. As long as the

⁸This is the only other possible assignable cause with a 260 value.

⁹McMenimen, 63.

expected savings is positive more will be saved on the average by the investigation than will be spent; therefore, it is this writer's opinion that each respective investigation procedure should be undertaken as long as the resultant expected savings from employing the procedure are positive. Accordingly, this treatment will be followed throughout this dissertation.

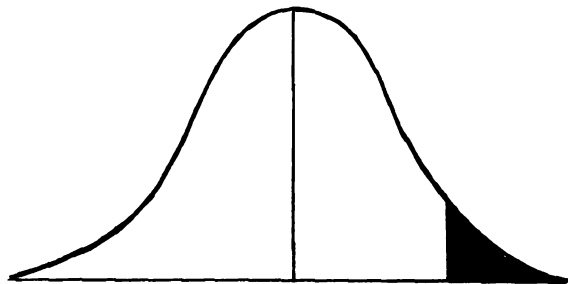
Appendix B contains the computational explanations to relieve the text from a plethora of such detail. The results from this appendix are summarized below. Considering only the incremental costs of additional investigation procedures, the upper control limit for poor attitude and laziness is between 261 and 262. Under these assumptions McMenimen would never find it profitable to investigate for illness. The upper control limit associated with dull knives is between 259 and 260.

Equalization Approach

For convenience, the probability of a Type I error times the opportunity cost of a Type I error has been designated as the expected opportunity cost of a Type I error. The probability of a Type II error times the conditional opportunity cost of a Type II error has been designated as the expected opportunity cost of a Type II error. Consequently, the Equalization control limit occurs at that test value for which the expected opportunity cost of a Type I error equals the expected opportunity cost of a Type II error.

Various values have been tested in an effort to find one which equates the expected opportunity costs of each type of error. The results of these tests are shown in Table 20. This table shows that 260 falls in the region of hypothesis rejection because the expected opportunity cost of committing a Type I error, \$.1853, is less than the \$.3111 expected opportunity cost of committing a Type II error. Hence, for the occurrence of performance value 260 it is cheaper in the long run to reject the hypothesis, undertake an investigation and run the risk of making a Type I error than to accept the hypothesis and run the risk of committing a Type II error. It is now appropriate to test a smaller value. The reason for the move in this direction is illustrated in the following diagram. Here 260 is shown to be in the shaded region of rejection. Obviously, the boundary (to be the upper control limit) must be less than 260.

FIGURE 7.--Direction of upper control limit



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 300
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The same conclusion (that of rejection) holds for test value 259. However, for test value 258, the expected opportunity cost of a Type II error is less than the expected opportunity cost of a Type I error. This fact indicates the desirability of accepting the hypothesis and refraining from an investigation for a performance value of 258. Thus, the Equalization upper control limit is between 258 and 259.

TABLE 20.--Decision table for Equalization approach

	Test Values		
	258	259	260
Probability of α	.0700	.0517	.0367
Opportunity Cost of α	\$5	\$5	\$5
Expected Opportunity Cost of α	<u>\$.3500</u>	<u>\$.2585</u>	<u>\$.1853</u>
Probability of β	.1700	.1867	.2067
Opportunity Cost of β	\$1.3726	\$1.5379	\$1.5053
Expected Opportunity Cost of β	<u>\$.2333</u>	<u>\$.2684</u>	<u>\$.3111</u>
Decision	Accept	Reject	Reject

α = Type I error

β = Type II error

Appendix B contains an explanation of the derivation of the probabilities and opportunity costs of each type of error.

Minimization Approach

It was pointed out in Chapter V that this approach involves the calculation of an expected opportunity cost

for each performance value in the range likely to represent the best control limit. The value yielding the lowest expected opportunity cost is the Minimization control limit. The expected opportunity cost is derived by adding the products of the prior probability, the weighted opportunity cost, and the probability of a wrong decision for each cause. Table 21 indicates the results. From the results of the other statistical approaches, the upper control limits appears to be around 260. Therefore, the results in Table 21 are shown first for test value 260. They are indicated next for test values 259 and 258 respectively. However, since the expected opportunity costs are higher for these values than for 260, it appears that the Minimization upper control limit is 260 or higher. Table 21 next shows the expected opportunity costs for test values 261 and 262. Since the expected opportunity cost for test value 261 is the lowest, it is designated at the Minimization upper control limit.

The reader should refer to Appendix B for an explanation of the derivation of the detail shown in Table 21. The reader will note, however, that this table bears the same format as Table 14.

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TABLE 21.--Decision table for Minimization approach

Cause	Prior Prob.	Weighted Op. Cost	Prob. of Wrong Decision	Cond. Average Op. Cost	Expected Opportunity Cost
Test Value 260					
Chance	.7143	\$5.0000	.0367	\$.1835	
Poor attitude	.0714	1.8036	.75	1.35	
Illness	.0238	1.3379	.25	.3345	
Dull Knives	.1429	1.3583	.0833	.1131	
Laziness	.0476	1.5825	.05	.0791	
	1.0000				\$.2554
Test Value 259					
Chance	.7143	5.0000	.0517	.2585	
Poor attitude	.0714	1.6127	.7167	1.1558	
Illness	.0238	1.3006	.2000	.2601	
Dull Knives	.1429	1.3410	.0667	.0895	
Laziness	.0476	1.5352	.0396	.0608	
	1.0000				\$.2890
Test Value 258					
Chance	.7143	5.0000	.0700	.3500	
Poor attitude	.0714	1.4218	.6833	.9715	
Illness	.0238	1.2632	.2000	.2526	
Dull Knives	.1429	1.3237	.0500	.0662	
Laziness	.0476	1.5000	-0-	-0-	
	1.0000				\$.3516

Table 21 (Continued)

Cause	Prior Prob.	Weighted Op. Cost	Prob. of Wrong Decision	Cond. Average Op. Cost	Expected Opportunity Cost
Test Value 261					
Chance	.7143	\$5.0000	.0217	\$.1085	
Poor attitude	.0714	2.1635	.7667	1.6588	
Illness	.0238	1.3770	.2000	.2754	
Dull Knives	.1429	1.3750	.1167	.1605	
Laziness	.0476	1.6084	.0500	.0804	
	1.0000				\$.2292
Test Value 262					
Chance	.7143	\$5.0000	.0183	\$.0915	
Poor attitude	.0714	2.5234	.7833	1.9766	
Illness	.0238	1.4161	.2000	.2832	
Dull Knives	.1249	1.3915	.1333	.1855	
Laziness	.0476	1.6344	.0500	.0817	
	1.0000				\$.2436

Comparison of Upper Control Limits
Among the Methods

The control limits that have just been derived under each of the approaches are shown below for review purposes:

<u>Approach</u>	<u>Upper Control Limit</u>
Accountant's Conventional	270
Basic Control Chart	260-261
Bierman, Fouraker, and Jaedicke	
First Interpretation of P	270
Second Interpretation of P	indeterminate
McMenimen	259-260 for dull knives 261-262 for poor attitude and laziness
Equalization	258-259
Minimization	261

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The listing of two figures such as 260-261 indicates that an investigation would not be undertaken for the occurrence of the first figure (260) but that one would be for the second one (261); therefore, the control limit is between the two.

Financial Analysis and Ranking

The approaches will be analyzed by twos insofar as necessary to rank them in preferential order of their desirability. The analysis takes the general form outlined below. Of the two approaches under consideration at any given time the one yielding the lowest upper control limit will involve larger total investigation costs. This additional cost can be measured by counting from Table 18 the number of chance performances between the two control limits (these performances would be investigated under the approach yielding the lower control limit but not under the approach yielding the higher one) and multiplying this number by \$5. If the tough cow investigation is made for performances equal to or greater than 262 the factor would be \$6 for those chance performances at least as large as 262 but less than the higher upper control limit. This approach gives the additional investigation cost per 1000 performances. It is, of course, true that an investigation would be undertaken for all performances between the control limits and not just those due to chance. Those

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investigations where assignable causes are present will not, however, be considered as additional investigation costs because the investigation would ultimately be undertaken even under the approach yielding the higher upper control limit.

The second part of the analysis considers the fact that the lower of the two upper control limits being evaluated will detect assignable causes which would not be detected until some later time with the higher of the upper control limits. This saving can be determined by counting from Table 18 for each assignable cause the number of performances having values between the two control limits. This number for each respective cause is multiplied by the weighted opportunity cost for the value representing the higher of the two control limits. The sums of these products for each cause are then added to obtain the incremental savings associated with the lower of the control limits.

If this savings is greater than the incremental investigation cost, the approach yielding the lower upper control limit is designated as more effective than the approach yielding the higher upper control limit. Conversely, if the savings is less than the incremental investigation cost, the approach yielding the higher upper control limit is designated as the more effective.

The following example illustrating the analysis between the Accountant's Conventional approach (yielding an

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upper control limit of 270) and the Basic Control Chart approach (yielding an upper control limit between 260 and 261) should clarify the procedure. Table 18 shows that there are 12 chance performances as large as 261 but less than 270. These would be investigated under the Basic Control Chart approach but not under the Accountant's Conventional method. The investigation would have an opportunity cost of \$5 each for the two performances at 261 and \$6 each for the remaining ten performances. Consequently, the added investigation cost under the Basic Control Chart method is \$70 per thousand performances.

Table 22 shows the derivation of the added savings per thousand performances associated with the Basic Control Chart method. The number of performances pertaining to each assignable cause with values between 261 and 269 inclusive represent the number of assignable causes that the Basic Control Chart method will detect that would not be detected under the Accountant's Conventional method. The sum of the products of each of these numbers by the weighted opportunity cost at 270 is the savings. Since the savings, \$266.25, is more than the extra investigation cost, the Basic Control Chart method is a more effective basis of control for this testing plan than the method conventionally employed by accountants.

The reader will note that the weighted opportunity costs are those developed by this writer in conjunction

with the Equalization approach. In making these comparisons it is necessary to employ the same weighting scheme. This one was selected because it is more scientific than the arbitrary selection of the number "four" used by Bierman, Fouraker, and Jaedicke. The reader will also note that no weighted costs are indicated for tough cows and lack of training. The fact that two new butchers are on hand is known in advance regardless of the control approach employed. In this case, savings emanates only from experience. By the same token, nothing can be saved after the tough cow has been butchered.

The results of all the comparisons are summarized in Table 23. The following abbreviations are necessary:

AC for Accountant's Conventional

BCC for Basic Control Chart

BFJ 1st for Bierman, Fouraker, and Jaedicke first

BFJ 2nd for Bierman, Fouraker, and Jaedicke second

McM for McMenimen

Equal for Equalization

Min for Minimization.

Since in the McMenimen approach the UCL varies, a slightly different analysis is undertaken. McMenimen would spend \$1 investigating nine chance performances at 260 that the basic control chart method would not investigate. This involves an extra cost of \$9. By investigating performances at 260, McMenimen would detect four dull knives at a weighted

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TABLE 22.--Added savings of basic control chart method

Cause	Performances between 261 and 269 incl. (F)	Opportunity Cost Weighted at 270 (C)	CF
Poor Attitude	12	\$12.5600	
Dull Knives	40	2.3058	
Tough Cows	2		
Illness	4	1.8447	
Laziness	8	1.9900	
Lack of Training	1		
Added Savings			\$266.2508

TABLE 23.--Financial comparisons between approaches

Approaches Tested		Added Inv. Cost of Lower UCL	Added Savings of Lower UCL	Most Effective Approach
BFJ 1st and AC	Min and BCC	\$ 70	\$266.25	Min and BCC
BFJ 2nd and AC	Equal	150	321.60	Equal
Equal	Min and BCC	90	17.73	Min and BCC
MCM	Min and BCC	41	3.11	MCM

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PAGE 1

cost of \$1.3583 for a total savings of \$6.4332. To continue, McMenimen would spend only \$1 investigating the two chance performances at 261 for dull knives; whereas, the control chart method would spend \$5 on these performances giving them the complete investigation. Therefore, the Control Chart approach would spend another \$8 here. For this, it would detect one performance at a weighted opportunity cost of \$2.1635 due to poor attitude. Finally, McMenimen would spend only \$2 investigating the eleven chance performances between 262 and 270, while the Control Chart approach would spend \$5 on the two at 262 and \$6 on the other nine for a total of \$64. Accordingly, the Control Chart approach would spend an additional \$42 for which it would detect four cases of illness at a weighted cost of \$1.8447 each for a total of \$7.3788. This discussion can be summarized as follows:

<u>Approach</u>	<u>Added Cost</u>	<u>Added Savings</u>
McM	(\$9)	(\$6.4332)
BCC	8	2.1635
BCC	<u>42</u>	<u>7.3788</u>
	\$41	\$3.1091

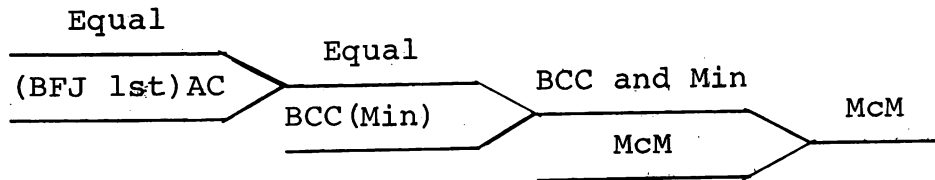
Since the Basic Control Chart approach spends more investigating relative to what it saves over the McMenimen approach, the McMenimen approach is more effective as a control method.

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As a guide to ranking the approaches, it is possible to depict the analysis in the form of a tree-diagram shown in Figure 8.

FIGURE 8.--Outcomes of financial comparisons



As a result of Figure 8, the following ranking now becomes obvious:

<u>Approach</u>	<u>Rank</u>
McM	1.0
BCC	2.5
Min	2.5
Equal	4.0
AC	5.5
BFJ 1st	5.5
BFJ 2nd	7.0

In cases of ties in ranks, it is customary to give each item the average of the ranks which they jointly occupy.

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Derivation and Financial Analysis of
Lower Control Limits for Single
Observations--Each Performance
Tested

Accountant's Conventional Method

With the 10 per cent rule, the lower control limit will be $[245 - 245 (.10)]$ 220 minutes. As with the upper control limit, this criterion results in a zero probability of committing a Type I error. In this case, however, the probability of making a Type II error is .83. This is determined by dividing the 83 improved performances with values over 220 by 100--the total number of improved performances.

Basic Control Chart Approach

A two-tailed test with a .05 level of significance was selected to determine the upper control limit. This same criterion will place the lower control limit at that value which is higher than 2-1/2 per cent of the chance performances. Table 18 shows that the probability that a chance performance will be 230 or less is 22/600 or .0367; whereas, the probability that it will be 229 or less is 13/600 or .0217. Since .025 is between .0217 and .0367, the lower control limit is between 229 and 230.

Bierman, Fouraker, and Jaedicke
Approach

First Interpretation of P. The necessary information for pin-pointing the control limit has been marshalled

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in Table 24 which shows that the lower control limit is between 224 and 225. Appendix B explains how the individual numbers which comprise the table are determined.

TABLE 24.--Decision table for B. F., and J. application
first interpretation of P

Test Value	L	C	Pc	P	Decision
220	\$5	\$4	.2000	.0034	Reject
221	4.80	4	.1667	.0034	Reject
222	4.60	4	.1304	.0034	Reject
223	4.40	4	.0909	.0068	Reject
224	4.20	4	.0476	.0136	Reject
225	4	4	0	.0167	Accept

Second Interpretation of P. Table 25 contains the information required to determine the control limit under this interpretation. Appendix B explains the logic behind placing it between 222 and 223. The appendix also explains some reservations concerning the utility of this approach under this particular set of assumptions.

TABLE 25.--Decision table for B. F., and J. application
second interpretation of P

Test Value	L	C	Pc	P	Decision
220	\$5	\$4	.2	.33	Accept
221	4.80	4	.1667	0	Reject
222	4.60	4	.1304	0	Reject
223	4.40	4	.0909	.25	Accept

McMenimen Approach

The savings value for the McMenimen approach is determined by multiplying the single performance opportunity cost by four and then subtracting the cost of correction. The single performance opportunity cost is \$.75 ($\frac{245 - 230}{60} \times \3 where 230 is the mean of the improved performances) regardless of the test value. The weighted opportunity cost is, then, \$3 which is less than the \$4 investigation cost even before the cost of correction is subtracted. Consequently, the McMenimen approach fails to yield a determinate solution for this situation.

The reason that this model does not yield a determinate solution and that the Bierman, Fouraker, and Jaedicke model yields a questionable solution is that these models were constructed with longer periods of time in mind than just the time required to complete one performance. Therefore, the savings were intended to be of a more long run nature. Actually, on the surface, it does not seem reasonable to spend \$4 on an investigation which may result in a savings of \$3 even though the other models give determinate solutions for this assumption. The paradox is that the \$3 reflects savings if we catch an off-standard condition now rather than at some other time in the near future. This figure assumes that each performance is tested. This writer believes that it is important to use a model which will yield a determinate solution at the performance level.

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Not only is control more timely at this level; but the aggregation problems of average-out and off-set are eliminated. Moreover, there is greater difficulty in objectively measuring long-run savings.

Equalization Approach

The information needed to make a decision by this method has been marshalled in Table 26. The table indicates that the Equalization lower control limit falls between 233 and 234. It is clear that the hypothesis should be rejected for test value 233 because the expected opportunity cost associated with act reject, \$.3668, is less than the expected opportunity cost associated with act accept, \$.4633. Likewise, it is clear that the hypothesis should be accepted for test value 234 because the expected opportunity cost associated with act accept, \$.4117, is less than \$.4732--the expected opportunity cost for act reject.

The derivation of the detail for this table is explained in Appendix B.

Minimization Approach

Table 27 shows that the lowest expected opportunity cost occurs for test value 229 which then becomes the lower control limit under this approach. The table is constructed in the same manner as Table 21 which was used to find the Minimization upper control limit.

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TABLE 26.--Decision table for Equalization approach

	Test Values			
	232	233	234	235
Probability of α	.0700	.0917	.1183	.1483
Opportunity Cost of α	\$4	\$4	\$4	\$4
Expected Opportunity Cost of α	<u>\$.2800</u>	<u>\$.3668</u>	<u>\$.4732</u>	<u>\$.5932</u>
Probability of β	.4000	.3600	.3000	.2700
Opportunity Cost of β	\$1.2871	\$1.1999	\$1.1127	\$1.0254
Expected Opportunity Cost of β	<u>\$.5148</u>	<u>\$.4320</u>	<u>\$.3338</u>	<u>\$.2768</u>
Decision	Reject	Reject	Accept	Accept

 α = Type I error β = Type II error

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Since improvement is the only assignable cause of concern, the prior probabilities are composed only of chance and improvement. Out of the 1000 original performances, 600 were due to chance and 100 to improvement; therefore $600/700 = .8572$ and $100/700 = .1428$ respectively represent the prior probabilities for chance and improvement.

The weighted opportunity cost, the probability of a wrong decision and the conditional average opportunity cost are the same for chance as the opportunity cost of a Type I error, the probability of a Type I error, and the expected opportunity cost of a Type I error respectively--all figures that were developed under the Equalization approach. For improvement the weighted opportunity cost, the probability of a wrong decision, and the conditional average opportunity cost are the same as the respective figures developed under the Equalization approach for the opportunity cost of a Type II error, the probability of a Type II error, and the expected opportunity cost of a Type II error. The weighted opportunity cost of a Type II error for test value 230 is \$1.4615. This same value is listed in Table 27 under improvement for test value 230. The weighted opportunity cost for improvement for test value 225 is \$2.2170. The weighted opportunity costs for test values 228 and 229 are found by interpolating between \$1.4615 and \$2.2170.

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TABLE 27.--Decision table for Minimization approach

Cause	Prior Prob.	Weighted Op. Cost	Prob. of Wrong Decision	Cond. Ave. Op. Cost	Expected Op. Cost
Test Value 231					
Chance	.8572	\$4	.0517	.2068	
Improvement	<u>.1428</u>	1.3743	.4400	.6047	
	1.0000				<u><u>\$.2636</u></u>
Test Value 230					
Chance	.8572	\$4	.0367	.1468	
Improvement	<u>.1428</u>	1.4615	.4600	.6723	
	1.0000				<u><u>\$.2218</u></u>
Test Value 229					
Chance	.8572	\$4	.0217	.0868	
Improvement	<u>.1428</u>	1.6126	.5200	.8386	
	1.0000				<u><u>\$.1942</u></u>
Test Value 228					
Chance	.8572	\$4	.0183	.0732	
Improvement	<u>.1428</u>	1.7637	.5600	.9877	
	1.0000				<u><u>\$.2038</u></u>

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The expected opportunity costs are calculated in the same manner described in conjunction with Table 21.

Comparison of Lower Control Limits
Among the Methods

The control limits that have just been calculated under each of the approaches are summarized below:

<u>Approach</u>	<u>Lower Control Limit</u>
Accountant's Conventional	220
Basic Control Chart	229-230
Bierman, Fouraker, and Jaedicke	
First Interpretation	224-225
Second Interpretation	222-223
McMenimen	Indeterminate
Equalization	233-234
Minimization	229

Financial Analysis and Ranking

The analysis takes the same general form as the analysis of the upper control limits. The approaches are paired off in twos and an evaluation is made to the extent necessary to rank the approaches. The only difference is that of the two approaches under consideration at any time the one yielding the higher lower control limit will involve larger total investigation costs while in the analysis of the upper control limit, the lower of the approaches being compared involved the larger investigation costs. As

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a general rule, it is safe to say that the control limits closest to the standard involve higher investigation costs than those farther away. It is also always true that the control limit involving the greatest investigation costs will detect some off-standard conditions sooner than the other.

For example, in the analysis between the Equalization approach (with a lower control limit between 233 and 234) and the Accountant's Conventional approach (with a lower control limit of 220) the Equalization approach will spend \$4 investigating each of 54 chance performances between 221 and 233 inclusive (itemized in Table 18) that the Accountant's Conventional approach would not investigate. This is an extra cost of \$216. For this extra investigation cost, the Equalization approach would detect 47 improved performances at a saving of \$4.1273¹⁰ each. The total savings of \$193.9831 is less than the extra cost of \$216 so the Accountant's Conventional method yields more profitable results in this situation.

In an analysis between the Accountant's Conventional approach and the Basic Control Chart approach, it is found that the Basic Control Chart approach leads to the investigation of 12 chance performances between 221 and 229 inclusive for an extra investigation charge of \$48. This approach

¹⁰This is the opportunity cost of a Type II error weighted for performance value 220.

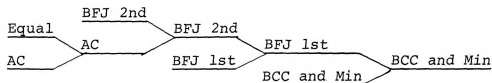
will, however, detect 31 improved performances at a savings of \$4.1273 each for a total savings of \$127.9463. Since the savings is greater than the charge, the Basic Control Chart approach is better than the Accountant's Conventional approach. Table 28 summarizes the results of all comparisons.

TABLE 28.--Financial comparisons between approaches

Approaches Tested		Added Inv. Cost of LCL Higher	Added Savings of Higher LCL	Most Effective Approach
Equal	AC	\$216	\$193.98	AC
AC	BCC	48	127.95	BCC
BFJ 2nd	AC	4	33.02	BFJ 2nd
BFJ 1st	BFJ 2nd	8	20.18	BFJ 1st
BFJ 1st	BCC and Min	40	44.38	BCC and Min

The Basic Control Chart approach must share honors with the Minimization approach since the investigative decisions are the same under either. Figure 9 depicts the results of this analysis in diagram form.

FIGURE 9.--Outcomes of the financial comparisons



From this diagram, the following ranking emerges:

<u>Approach</u>	<u>Rank</u>
BCC	1.5
Min	1.5
BFJ 1st	3
BFJ 2nd	4
AC	5
Equal	6
McM	7

Those approaches yielding indeterminate results are given the highest ranking.

Derivation and Financial Analysis of
Upper Control Limits for Single
Observations--Every Tenth
Performance Tested

Introduction

So far the coverage in this chapter has assumed that each performance is tested. In some cases this is a realistic assumption. The worker can be trained to compare the results of each performance with the control limits and to report cases in which his performance falls outside of the control limits. Of course, the complete success of this procedure depends upon the cooperation of the worker. Another procedure may involve the foreman's testing every n single performances. "N" may vary depending upon the number of workers per foreman, the length of

time needed to complete a task, and the degree of control already attained. In the following illustration, n is assumed to be ten.

Accountant's Conventional Method

Accountants have not made a distinction between testing every performance or testing every n^{th} performance in conjunction with the selection of control limits. Therefore, consistent application of the "ten per cent rule" results in the same upper control limit, 270, that resulted when every performance was investigated.

Basic Control Chart Approach

This approach like the Accountant's Conventional method also refrains from making a distinction between testing every performance and testing every n^{th} performance. Accordingly, the upper control limit with a two-tailed .05 level of significance remains between 260 and 261--the same interval that was found when each performance was tested.

Bierman, Fouraker, and Jaedicke Approach

First Interpretation of P . Table 29 shows that the upper control limit with this interpretation of P is 250. For this value P and P_c are equated. Some explanation of the values in the table is given in Appendix B.

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TABLE 29.--Decision table for BF and J application. First interpretation of P

Test Value	L	C	Pc	P	Decision
255	\$20	\$5	.7500	.2966	Reject
251	12	5	.5833	.4600	Reject
250	10	5	.5000	.5000	Indifferent
249	8	5	.3750	.5300	Accept
248	6	5	.1670	.6334	Accept

Second Interpretation of P. Table 30 shows that the upper control limit according to this interpretation is between 254 and 255; whereas, under the first interpretation it was 250. The only difference between Tables 29 and 30 lies in the calculation of P. This difference is explained in some detail in Appendix B.

TABLE 30.--Decision table for BF and J application. Second interpretation of P

Test Value	L	C	Pc	P	Decision
260	\$30	\$5	.8330	.6428	Reject
255	20	5	.7500	.6207	Reject
254	18	5	.7222	.7620	Accept
253	16	5	.6875	.7647	Accept

McMenimen Approach

To start, a test will be made to see if it is profitable to spend \$1 investigating a performance value of 260 for dull knives. The information necessary for an investigation decision appears in Table 31. The \$49.9625 savings figure is different than the \$4.625 used in Table 19 because

of the differences in the frequency of testing. The \$1.25 ($\frac{270 - 245}{60} \times 3$ where 270 is the mean of the dull knife performances) single performance saving is still multiplied by four--the arbitrarily selected number of tested performances which lapse on the average before an assignable cause is detected. The \$5 result must further be multiplied by 10 because under this testing procedure on the average only one performance out of ten is tested. The \$49.9625 savings is found by subtracting the prorated performance cost of sharpening the knives from the \$50 opportunity cost. In making investigative decisions where every performance was tested the \$1.50 cost of sharpening was spread over the four performances which allegedly lapse on the average before detecting the dull knives. Similarly, one may argue that the \$1.50 should be spread over the 40 (10×4) performances that are said to lapse before dull knives are detected. This averaging results in a \$.0375 ($\frac{1.50}{40}$) cost per performance which when subtracted from the \$50 leaves the \$49.9625 saving.

TABLE 31.--Application of McMenimen technique

Event	Spend Up to \$1 Investigating For Dull Knives		
	Pe	Cond.	Exp.
Test Value 260			
Save \$0	.7143	\$-1	\$ -.7143
Save \$49.9625	.2857	48.9625	13.9886
Expected Savings			\$13.2743

The probabilities and the mechanics of determining the expected savings are both found in the same way as that discussed in conjunction with Table 19. Since the expected savings is positive, it is worthwhile to spend at least \$1 investigating for dull knives. Decision tables like Table 31 for all performance values for which dull knives have been observed yield positive expected savings. This means that at least \$1 would be spent on an investigating for performance values of 250 or more for dull knives.¹¹ This is less than the 259-260 control limit determined under the McMeniman approach when all performances were tested. The reason for the decrease in the control limit is that it becomes more costly to fail to find an assignable cause when only one performance out of ten is tested. In McMenimen's terminology more can be saved by detecting an assignable cause when only one-tenth of the performances are tested.

To continue the McMenimen application, it is necessary to determine whether or not it is profitable to

¹¹Table 18 shows that dull knives have not resulted in performance times of 251, 252, or 254 so these may not be investigated for dull knives. The investigator may investigate them anyway, however, because there is no reason why these values could not be produced with dull knives. Just because they were not observed in the original 1000 values does not mean they could not occur in a larger population. The original 1000 values provide guidelines for decision making. These guidelines may be altered as a result of later observations or well-founded intuition.

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spend up to \$2 to administer the test for laziness and poor attitude. For values which have resulted from illness, it is also important to decide whether it is worthwhile to spend up to \$5 to require a physical examination. The model used to make these decisions is depicted in Table 32. In order to avoid confusion with too much arithmetic, the information is presented only for certain test values.

Table 33 shows how the savings values are determined. The single performance opportunity cost is calculated in the usual manner by the following formula $\frac{\bar{X}_a - 245}{60} \times \3 where $\bar{X}_a$ stands for the mean of the corresponding assignable cause. The multiplication weight is the result of multiplying the four performances that lapse on the average before an assignable cause is detected by ten. (Every tenth performance is tested.) The weighted opportunity cost results from multiplying the single performance opportunity cost by the multiplication weight. The savings values are, of course, determined by subtracting the correction cost per performance from the weighted opportunity cost. The cost of correction per performance is an estimated figure. Surprisingly, rather large changes in these figures do not produce drastic changes in the control limits.

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TABLE 32.--Application of McMenimen Technique

Event	P _E	Cond.	Exp.	P _E	Cond.	Exp.	P _E	Cond.	Exp.
Spend Up to \$1 Investi- Spend Up to \$2 Investi- Spend Up to \$5 Investi-									
gating for Dull Knives gating for Poor Attitude gating for Illness									
and Laziness									
Test Value 249									
Save \$0				.9737	\$-1	\$ -.9737			
Save \$19.75				.0263	18.75	.4931			
Expected Savings						<u><u>-.4806</u></u>			
Test Value 248									
Save \$0				.9286	\$-1	\$ -.9286			
Save \$19.75				.0714	18.75	1.3388			
Expected Savings						<u><u>+4.102</u></u>			
Test Value 252									
Save \$0				.8571	\$-1	\$ -.8571	.9167	\$-4	\$-3.6668
Save \$19.75				.1429	18.75	2.6794	.0833	35.50	+2.9572
Save \$39.50									
Expected Savings						<u><u>\$ +1.8223</u></u>			\$ -.7096

Table 32 (Continued)

Event	Spend Up to \$1 Investi- gating for Dull Knives			Spend Up to \$2 Investi- gating for Poor Attitude and Laziness			Spend Up to \$5 Investi- gating for Illness		
	P _E	Cond.	Exp.	P _E	Cond.	Exp.	P _E	Cond.	Exp.
Test Value 259									
Save \$0	.8667	\$-1	- .8667	.7692	\$-2	\$-1.5384	.9000	\$-5	\$-4.50
Save \$19.75				.1539	17.75	2.7317			
Save \$39.50							.1000	34.50	3.45
Save \$49.9625	.1333	48.9625	+6.5267						
Save \$59.75				.0769	57.75	4.4410			
Expected Savings			<u>+5.6600</u>			<u>5.6343</u>			<u>\$- .95</u>
Test Value 262									
Save \$0	.75	\$-1	- .75	.666	\$-2	\$-1.3320	.75	\$-5	\$-3.75
Save \$19.75				.167	17.75	2.9642			
Save \$39.50							.25	34.50	8.6250
Save \$49.9625	.25	48.9625	+12.2406						
Save \$59.75				.167	57.75	9.6442			
Expected Savings			<u>\$11.4906</u>			<u>\$11.2764</u>			<u>\$4.8750</u>

TABLE 33.--Derivation of savings values

Cause	Single Perf. Op. Cost	Multi- plication Weight	Weighted Op. Cost	Correction Cost per Performance	Savings
Poor attitude	\$.50	40	\$20	\$.25	\$19.75
Laziness	1.50	40	60	.25	59.75
Illness	1.00	40	40	.50	39.50

Some discussion of how the P_E 's are determined in Table 32 for act "spend up to \$2 investigating" might be enlightening. Since only one of the thirty-eight performances with value 249 resulted from poor attitude, $1/38$ or .0263 is the probability of saving \$19.75. Accordingly, the probability of saving \$0 from an investigation to detect poor attitude is $1 - .0263$ or .9737.

Because the expected savings is negative, an investigation would not be profitable for a performance value of 249.¹² The same analysis for all test values 248 and above yields a positive expected savings. Therefore, the

¹²The reader will note that the conditional values are only \$1 less than the amount of the savings instead of \$2 less. This is because no performances due to dull knives have ever occurred for value 249 so the investigator would by-pass checking the knives for sharpness and save \$1 in the investigation process. The same is true for test values 248 and 252.

upper control limit for the investigation for poor attitude and laziness¹³ is between 247 and 248.

The possibility of investigating for illness presents itself in conjunction with test value 252. Table 18 shows that two of the fourteen values at 252 are due to poor attitude. If the test fails to reveal poor attitude as the cause, the investigator reassesses the probability of finding the cause if he continues his investigation. The original sub-population shows that 12 performances occurred at 252 that were not due to poor attitude. One of these was due to illness and the other eleven to chance. The probability of saving \$39.50 is thus $1/12$ or .0833 by investigating for illness. The probability of saving \$0 is $1 - .0833$ or .9167. Since the expected savings for this act is negative, it would not be profitable to extend the investigation to the point of a physical examination in conjunction with a performance value of 252. If, however, the incremental approach is employed, Table 34 shows that the expected savings is positive. Consequently, the upper control limit in the incremental sense is 252 for illness.

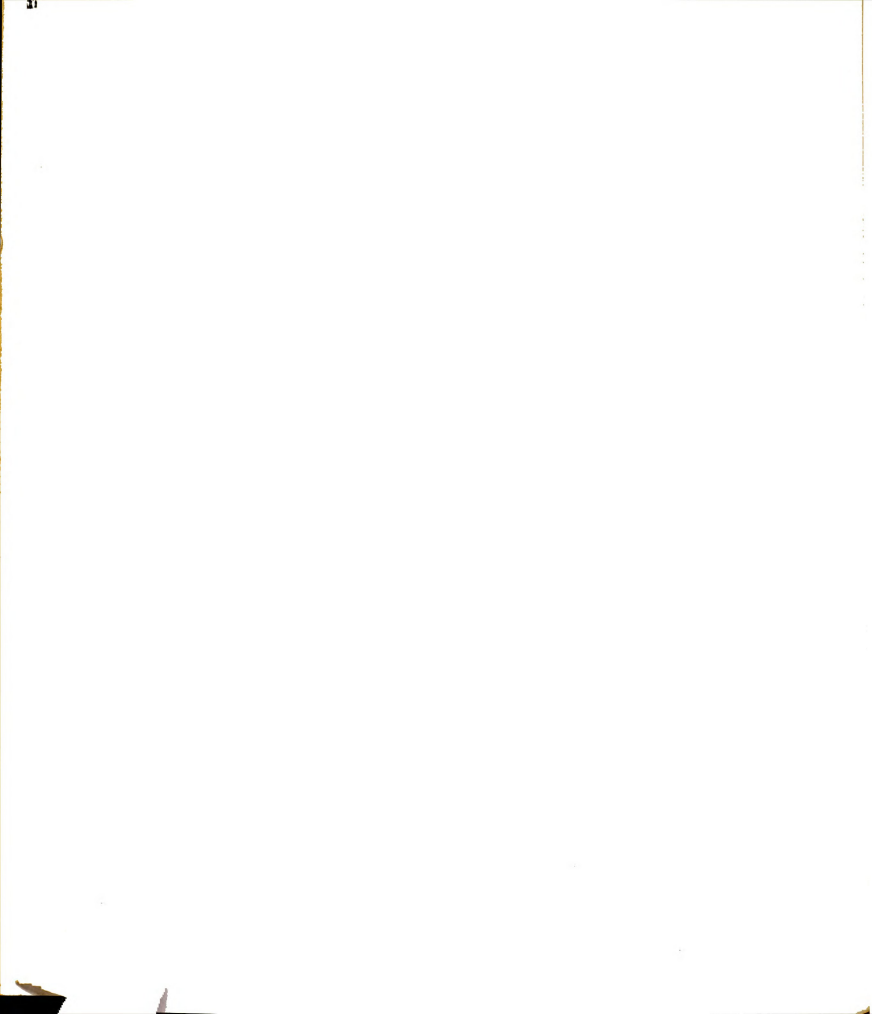
¹³The reader will note that for test values 259 and 262 laziness is also a possibility. Since it can be detected with the same test administered to detect poor attitude, its savings of \$59.75 is shown in conjunction with act "spend up to \$2 investigating."

TABLE 34.--McMenimen technique--incremental application

Event	Investigate for Illness		
	P _E	Cond.	Exp.
Test Value 252			
Save \$0	.9167	\$-3	\$-2.7501
Save \$39.50	.0833	36.50	<u>3.0404</u>
Expected Savings			\$+0.2903

The reader will recall from the discussion in conjunction with Table 80 that the incremental analysis considers only the additional opportunity cost of the next investigative step. In this case an investigation for illness is considered only if the investigation for poor attitude fails to reveal that as a cause. It requires only an additional \$3 to investigate for illness since the \$1 opportunity cost of an investigation for poor attitude is at that point a sunk cost. (Dull knives did not produce a value of 252.)

On the other hand, if the incremental approach is not employed, negative expected savings result for test values 254, 257 (not shown on Table 32) and for 259 (shown) as well as for test value 252. For test value 262, however, the expected saving is positive for an investigation for illness. Accordingly, the upper control limit for illness is 262 if the incremental approach is not applied.



Equalization Approach

The same type of information that appeared in Table 20 is marshalled in Table 35. All figures are determined in the same way with the exception that the opportunity cost of a Type II error for each corresponding test value is ten times larger. This accounts for the fact that ten performances now lapse between tests. Since Type II errors are now more expensive than they were when each performance was tested it is important that they be made less frequently. For this reason the upper control limit is closer to the standard. Table 35 indicates that the upper control limit is between 253 and 254.

TABLE 35.--Decision table for Equalization approach

	Test Values		
	255	254	253
Probability of α	.1483	.1750	.1967
Opportunity Cost of α	\$5	\$5	\$5
Expected Opportunity Cost of α	<u>\$.7415</u>	<u>\$.8750</u>	<u>\$.9835</u>
Probability of β	.1000	.0833	.0700
Opportunity Cost of β	\$11.331	\$11.160	\$10.989
Expected Opportunity Cost of β	<u>\$ 1.1331</u>	<u>\$.9296</u>	<u>\$.7692</u>
Decision	Reject	Reject	Accept

α = Type I error

β = Type II error

The process of interpolation has again been used to find the opportunity costs of committing a Type II error for test values 254 and 253. This interpolation process is explained in Appendix B.

Minimization Approach

Table 36 shows the expected opportunity costs for various test values. From this, it is apparent that 255 is the upper control limit under the Minimization approach because the expected opportunity cost for this test value is less than for any other. This table is basically the same as Table 21 except that the weighted opportunity costs for each combination of test value and assignable cause are ten times larger to account for the ten performances that lapse between tests. The weighted opportunity costs for test values 256 and 257 were derived in Table 87. These costs are shown in Tables 93 and 94 for test values 255 and 254 respectively. For the reason just indicated these weighted costs are multiplied by ten before they are entered in Table 36 under the weighted opportunity cost column. The values for the remainder of this table are calculated according to the same procedure explained in conjunction with Table 21.

TABLE 36.--Decision table for Minimization approach

Cause	Prior Prob.	Weighted Op. Cost	Prob. of Wrong Decision	Cond. Ave. Op. Cost	Expected Op. Cost
Test Value 257					
Chance	.7500	\$ 5.000	.0917	\$.4585	
Poor attitude	.0750	12.309	.6333	7.7953	
Illness	.0250	12.258	.1500	1.8387	
Dull knives	<u>.1500</u>	13.065	.0417	.5448	
	1.0000				<u>\$1.0562</u>
Test Value 256					
Chance	.7500	\$ 5.000	.1183	\$.5915	
Poor attitude	.0750	10.400	.5667	5.8937	
Illness	.0250	11.885	.1500	1.7828	
Dull knives	<u>.1500</u>	12.892	.0333	.4293	
	1.0000				<u>\$.9946</u>
Test Value 255					
Chance	.7500	\$ 5.000	.1483	\$.7415	
Poor attitude	.0750	8.492	.4167	3.5386	
Illness	.0250	11.512	.1500	1.7268	
Dull knives	<u>.1500</u>	12.720	.0167	.2124	
	1.0000				<u>\$.8965</u>
Test Value 254					
Chance	.7500	\$ 5.000	.1750	\$.8750	
Poor attitude	.0750	8.019	.3500	2.8066	
Illness	.0250	11.305	.1000	1.1305	
Dull knives	<u>.1500</u>	12.706	.0167	.2122	
	1.0000				<u>\$.9118</u>

Comparison of Upper Control Limits
Among the Methods

For review purposes the upper control limits that have been derived under the testing plan whereby a test is taken on the average for one out of every ten performances are listed below:

<u>Approach</u>	<u>Upper Control Limit</u>
Accountant's Conventional	270
Basic Control Chart	260-261
Bierman, Fouraker, and Jaedicke	
First Interpretation of P	250
Second Interpretation of P	254-255
McMenimen	250 for Dull Knives
	247-248 for Poor Attitude and Laziness
	252 for Illness
Equalization	253-254
Minimization	255

Financial Analysis and Ranking

This analysis is performed in the same manner that was illustrated when every performance was tested. The only difference is one of degree lying in the computation of the savings associated with the lower of the two upper control limits being compared at any given time. The opportunity costs weighted at the higher of the two limits are ten times higher for this analysis than they would be

for the same higher upper control limit when every performance is tested. The following explanation of the analysis between the Accountant's Conventional approach (yielding an upper control limit of 270) and the Basic Control Chart approach (yielding an upper control limit between 260 and 261) will serve to clarify this computational distinction.¹⁴

The added investigation cost under the Basic Control Chart method remains at \$70 per thousand performances tested ($2 \times \$5 + 10 \times 6 = \70). Table 37 shows the same number of performances between 261 and 269 inclusive for each assignable cause that were shown in Table 22. The only difference is that the opportunity costs weighted at 270 are ten times higher in Table 37 than they were in Table 22. The extra savings under the Basic Control Chart approach is thus ten times higher than when every performance was tested. Consequently, the Basic Control Chart approach is now even more strongly favored.

Table 38 shows the results for the comparison of the other approaches.

¹⁴Under both of these approaches the upper control limits remain the same as they were when every performance was tested. This condition does not hold for the other approaches which consider the opportunity costs of failing to detect a shift in the parameter (i.e., the opportunity costs of committing a Type II error). When a test is made only once in every ten performances it is relatively more costly to fail to detect a shift. Consequently, this fact serves to reduce the upper control limit.

TABLE 37.--Added savings of basic control chart method

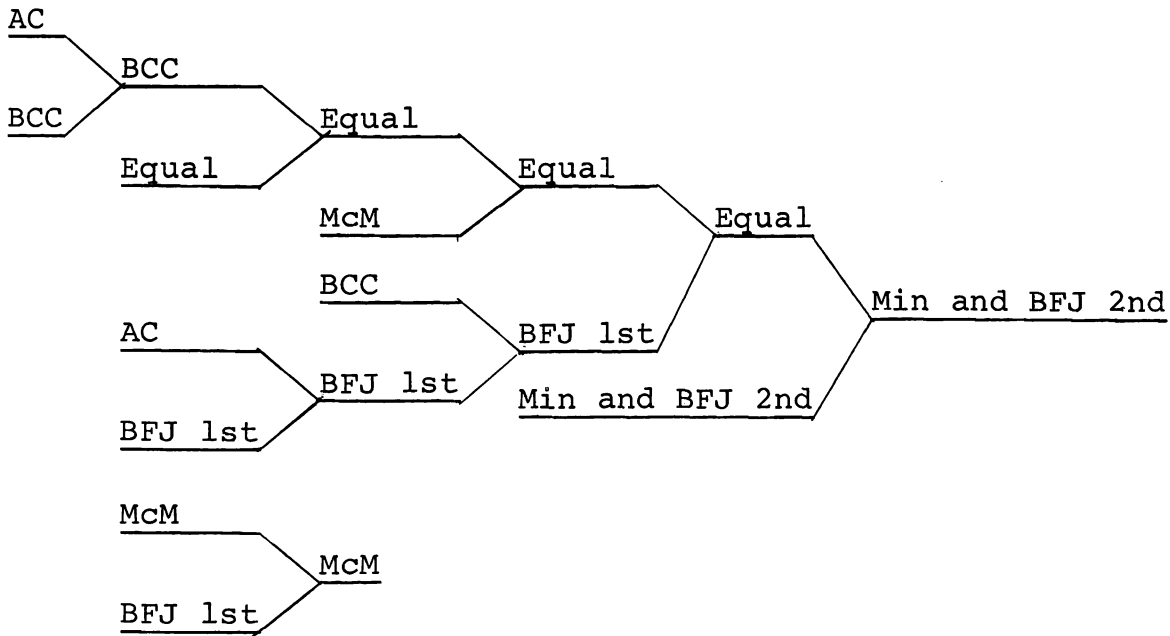
Cause	Performances Between 261 and 269 Incl. (F)	Opportunity Cost Weighted at 270 (C)	CF
Poor Attitude	12	\$125.600	
Dull Knives	40	23.058	
Tough Cows	2		
Illness	4	18.447	
Laziness	8	19.900	
Lack of Training	1	19.900	
Added Savings			\$2662.508

TABLE 38.--Financial comparisons between approaches

Approaches Tested		Added Inv. Cost of Lower UCL	Added Savings of Lower UCL	Most Effective Approach
AC	BCC	\$ 70	\$2,662.51	BCC
Equal	BCC	460	685.68	Equal
Equal	McM	148	126.84	Equal
AC	BFJ 1st	672	7,015.57	BFJ 1st
BCC	BFJ 1st	625	820.86	BFJ 1st
Equal	BFJ 1st	165	69.87	Equal
Equal	BFJ 2nd and Min	80	47.08	BFJ 2nd and Min
McM	BFJ 1st	15	22.64	McM

Figure 10 depicts this analysis in diagramatic form.

FIGURE 10.--Outcomes of the financial comparisons



The following ranking now becomes obvious:

<u>Approach</u>	<u>Rank</u>
Min	1.5
BFJ 2nd	1.5
Equal	3
McM	4
BFJ 1st	5
BCC	6
AC	7

Derivation and Financial Analysis of
Lower Control Limits for Single
Observations--Every Tenth
Performance Tested

Accountant's Conventional Method

The 10 per cent rule does not distinguish between tests of every ten performances and tests of every performance. Accordingly, the lower control limit remains at 220 [245 - (.10)245]--the same as it was when every performance was tested.

Basic Control Chart Approach

Neither does this method distinguish between tests of every performance and tests of every tenth performance. Consequently, the lower control limit for tests of every tenth performance assuming a two-tailed .05 level of significance will be between 229 and 230--the same interval into which it fell when each performance was tested.

Bierman, Fouraker, and Jaedicke
Approach

First Interpretation of P. Table 39 shows that the lower control limit under this approach is between 240 and 241. The values used in this table were derived in the same manner as those in Table 24 with the exception that the L values are ten times higher for each corresponding test value. Symbolically, L is now equal to

$$\frac{X - 245}{60} \times \$3 \times 4 \times 10$$

where:

X = the test value

245 = the standard

$\frac{X - 245}{60}$ = the fraction of an hour between the standard and the test value

\$3 = hourly wage rate of butchers

4 = assumed average number of tests that must be made before an assignable cause is detected

10 = number of performances that lapse between tests.

This factor did not pertain to the L values in Table 24.

TABLE 39.--Decision table for BF and J application. First interpretation of p

Test Value	L	C	P_c	P	Decision
240	\$10	\$4	.60	.50	Reject
241	8	4	.50	.55	Accept
242	6	4	.33	.63	Accept

Second Interpretation of P . Table 40 is constructed

in the same way as Table 39 except that P in Table 40 follows the second interpretation. The lower control limit according to this interpretation is between 235 and 236.

TABLE 40.--Decision table for BF and J application. Second interpretation of P

Test Value	L	C	P_c	P	Decision
234	\$22	\$4	.8182	.7273	Reject
235	20	4	.8000	.6000	Reject
236	18	4	.7778	.8889	Accept
237	16	4	.7500	.8125	Accept

McMenimen Approach

The information presented in Table 41 shows that the McMenimen lower control limit under this approach is between 235 and 236.

The savings of \$29.75 is found by subtracting the \$.25 per performance cost of correction¹⁵ from the \$30 opportunity cost. The \$30 is determined as follows:

$\frac{230 - 245}{60} \times \$3 \times 4 \times 10$ where 230 is the mean of the improved performances and the other numbers have the same meanings discussed in conjunction with Table 39.

The probabilities are determined in the same manner as the probabilities that were used to determine the upper control limits under the McMenimen approach in Table 19. That is, since three of the 22 performances at value 235 (see Table 18) were due to improvement, the probability of detecting improvement and thus saving \$29.75 by an investigation is 3/22 or .1364. The probability that nothing will be saved by the investigation is 1 - .1364 or .8636. Since the expected value is positive for test value 235 an investigation would be undertaken. On the other hand, it would be unprofitable to investigate a performance of 236 because the expected savings is negative. Consequently, the lower control limit is between 235 and 236.

¹⁵This \$.25 represents the raise that would be given the butcher when his improvement is recognized.

TABLE 41.--Application of McMenimen technique

Event	Spend Up to \$4 Investigating		
	Pe	Cond.	Exp.
Test Value 235			
Save \$0	.8636	\$-4	\$-3.4544
Save \$29.75	.1364	25.75	+3.5123
			<u>\$+ .0579</u>
Test Value 236			
Save \$0	.8889	\$-4	\$-3.5556
Save \$29.75	.1111	25.75	+2.8608
			<u>\$- .6948</u>

Equalization Approach

Table 42 shows that the lower control limit under this approach is between 241 and 242. The procedure followed in constructing the table is exactly the same as that discussed in conjunction with Table 26. In Table 42 the actual opportunity costs of committing a Type II error are calculated rather than interpolated because the calculations for these test values are not lengthy.

TABLE 42.--Decision table for Equalization approach

	Test Values		
	240	241	242
Probability of α	.2500	.2750	.3167
Opportunity Cost of α	\$4	\$4	\$4
Expected Opportunity Cost of α	<u>\$1.0000</u>	<u>\$1.1000</u>	<u>\$1.2668</u>
Probability of β	.15	.13	.12
Opportunity Cost of β	\$8.810	\$8.614	\$8.510
Expected Opportunity Cost of β	<u>\$1.3215</u>	<u>\$1.1198</u>	<u>\$1.0212</u>
Decision	Reject	Reject	Accept

α = Type I error

β = Type II error

Minimization Approach

Under this approach the lower control limit is 234 because the expected opportunity costs developed in Table 43 are less for this test value than for any other. The values in this table are obtained by the same procedure followed in Table 27 in which the Minimization lower control limit was derived for the situation in which every performance was tested. For previously explained reasons, the weighted opportunity costs associated with improvement are ten times higher in Table 43 than they would be for a corresponding test value in Table 27. If the reader wishes he can verify that the weighted opportunity costs for improvement for test values 233, 234, and 235 respectively

are exactly ten times more in Table 43 for each corresponding test value than they were in Table 26 for the opportunity cost of a Type II error. (The opportunity costs of a Type II error in Table 26 were derived in this same manner as the weighted opportunity costs of improvement in Table 27.)

TABLE 43.--Decision table for Minimization approach

Cause	Prior Prob.	Weighted Op. Cost	Prob. of Wrong Decision	Cond. Ave. Op. Cost	Expected Op. Cost
Test Value 233					
Chance	.8571	\$ 4	.0917	.3668	
Improvement	.1429	11.999	.3600	4.3196	
					<u>\$.9316</u>
Test Value 234					
Chance	.8571	\$ 4	.1183	.4732	
Improvement	.1429	11.127	.3000	3.3380	
					<u>\$.8826</u>
Test Value 235					
Chance	.8571	\$ 4	.1483	.5932	
Improvement	.1429	10.254	.2700	2.7686	
					<u>\$.9041</u>

Comparison of Upper Control Limits Among the Methods

The lower control limits just developed under this testing plan are listed on the next page:

<u>Approach</u>	<u>Lower Control Limit</u>
Accountant's Conventional	220
Basic Control Chart	229-230
Bierman, Fouraker, and Jaedicke	
First Interpretation of P	240-241
Second Interpretation of P	235-236
McMenimen	235-236
Equalization	241-242
Minimization	234

Financial Analysis and Ranking

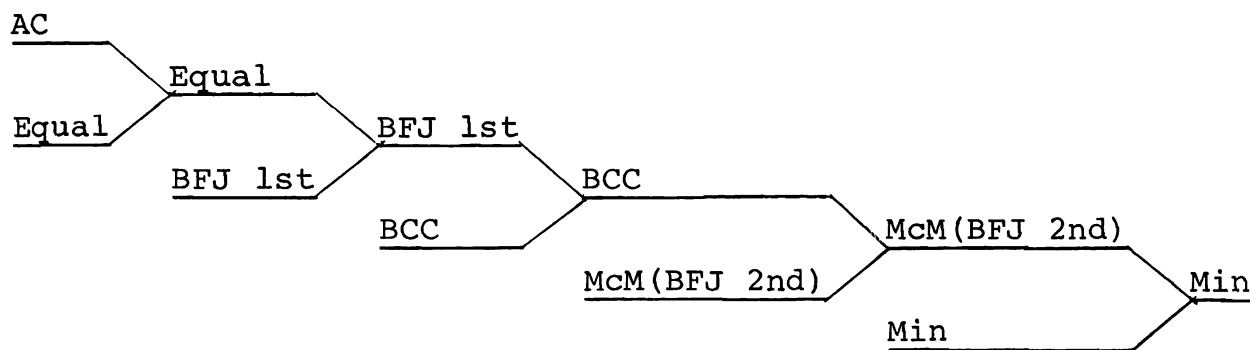
Table 44 shows the results for all the appropriate comparisons.

TABLE 44.--Financial comparisons between approaches

Approaches Tested		Added Inv. Cost of Higher LCL	Added Savings of Higher LCL	Most Effective Approach
AC	Equal	\$656	\$2,889.11	Equal
Equal	BFJ 1st	60	17.23	BFJ 1st
BFJ 1st	BCC	548	540.66	BCC
BCC	McM and BFJ 2nd	304	365.38	McM and BFJ 2nd
Min	McM and BFJ 2nd	72	30.76	Min

The results of these comparisons are depicted in Figure 11.

FIGURE 11.--Outcomes of the financial comparisons



From these comparisons, the following ranking

emerges:

<u>Approach</u>	<u>Rank</u>
Min	1
McM	2.5
BFJ 2nd	2.5
BCC	4
BFJ 1st	5
Equal	6
AC	7

Derivation and Financial Analysis of
Upper Control Limits--Sample Size
Five--Every Performance
Included in a Sample

Every performance is included in this testing plan. However, instead of comparing each performance against control limits developed for individual performances, the analyst groups the performances by fives and computes the mean of each group. These sample means are then compared with control limits developed for this testing plan. The derivation of these limits under each of the approaches to be considered is explained in this section.

Accountant's Conventional Method

In selecting control limits accountants have not made a distinction between testing individual performances and sampling groups of performances. Consistent application of the "ten per cent rule" results in the same upper control limit, 270, that resulted when every performance or every n^{th} performance was investigated.

Basic Control Chart Approach

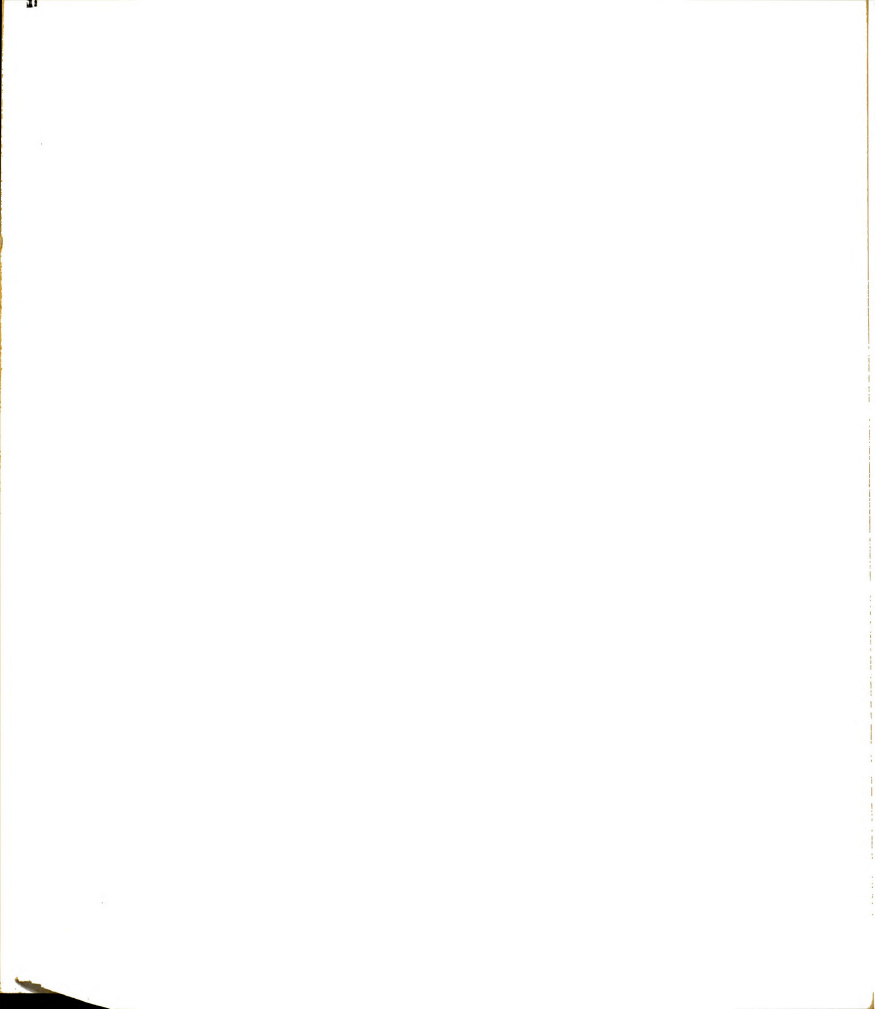
With the consistent use of the .05 level of significance with a two-tailed test, the upper control limit would be that value which is exceeded by 2.5 per cent of the sample means of five randomly selected chance performances. It is assumed that the distribution of sample means is Student-t distributed for sample sizes less than 30. The Student-t Distribution is symmetrical but flatter than the normal distribution. The upper control limit is found by solving the following formula:

$$t = \frac{UCL - 245}{\sigma_{\bar{X}}}$$

where:

t represents the number of standard deviation units between the standard and the upper control limit--that yet unknown value which is exceeded by 2.5 per cent of all sample means of size five.

UCL is the upper control limit.



245 is the mean of the distribution of chance performances

$\sigma_{\bar{X}}$ represents the standard error of the mean which is the standard derivation of all possible random sample means of size five that could be drawn from the distribution of chance performances.

The value for t can be found in a Student-t Distribution which is given in most statistics textbooks. The value in this case is 2.776 corresponding to four degrees of freedom (the sample size, 5, less one) and .025. The standard error of the mean is found by solving the following formula:

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n - 1}}$$

where:

σ is the standard deviation of chance performances which is 7.7846

n is the sample size which is 5

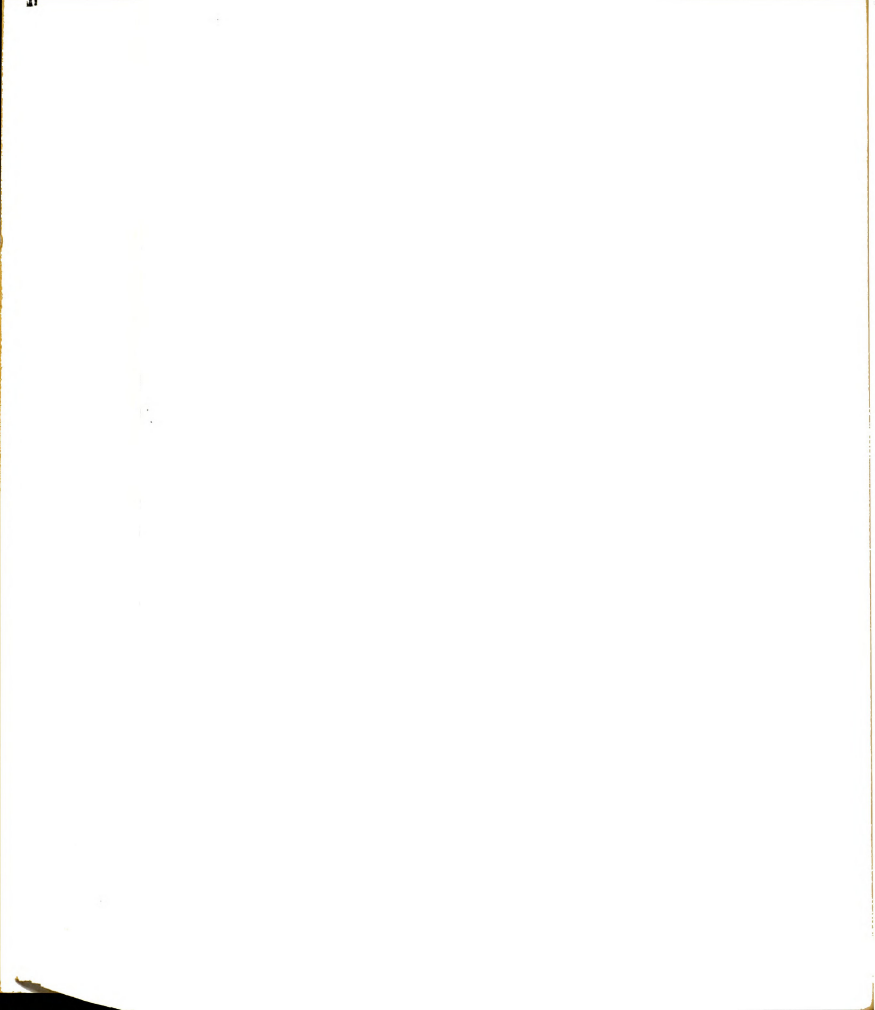
therefore:

$$\sigma_{\bar{X}} = \frac{7.7846}{\sqrt{5 - 1}} = 3.8923$$

The upper control limit is thus

$$t = \frac{UCL - 245}{\sigma_{\bar{X}}}$$

$$2.776 = \frac{UCL - 245}{3.8923}$$



$$UCL - 245 = 10.8050$$

$$UCL = 255.8050$$

Bierman, Fouraker, and Jaedicke
Approach

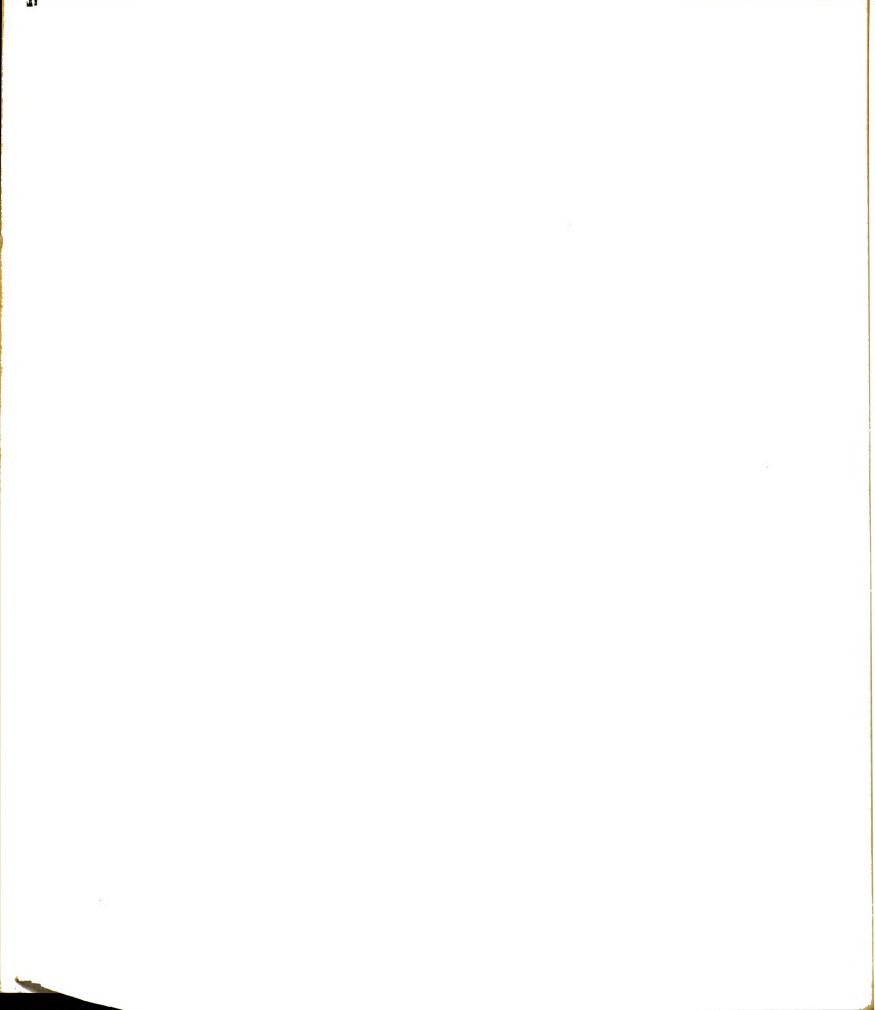
First Interpretation of P. Decision Table 45 shows the upper control limit to be between 250 and 251. In this table, "L" represents the single performance opportunity cost multiplied by twenty. As in previous applications of the Bierman, Fouraker, and Jaedicke approach the single performance opportunity cost is determined by the following formula:

$$\frac{X - 245}{60} \times \$3$$

where X represents the test value. The twenty is the result of multiplying the sample size of five by the four tests that these writers assume must be made on the average before an off-standard condition can be detected. "C" represents the cost of an investigation which is a constant in the Bierman, Fouraker, and Jaedicke system. Again, this cost is assumed to be \$5 rather than \$6 since there is no reason to investigate values in this range for tough cows because a tough cow never produced a value lower than 262. The reader will recall that the Pc values result from application of the following formula:

$$Pc = \frac{L - C}{L}.$$

"P" is the probability of obtaining a sample mean (from the



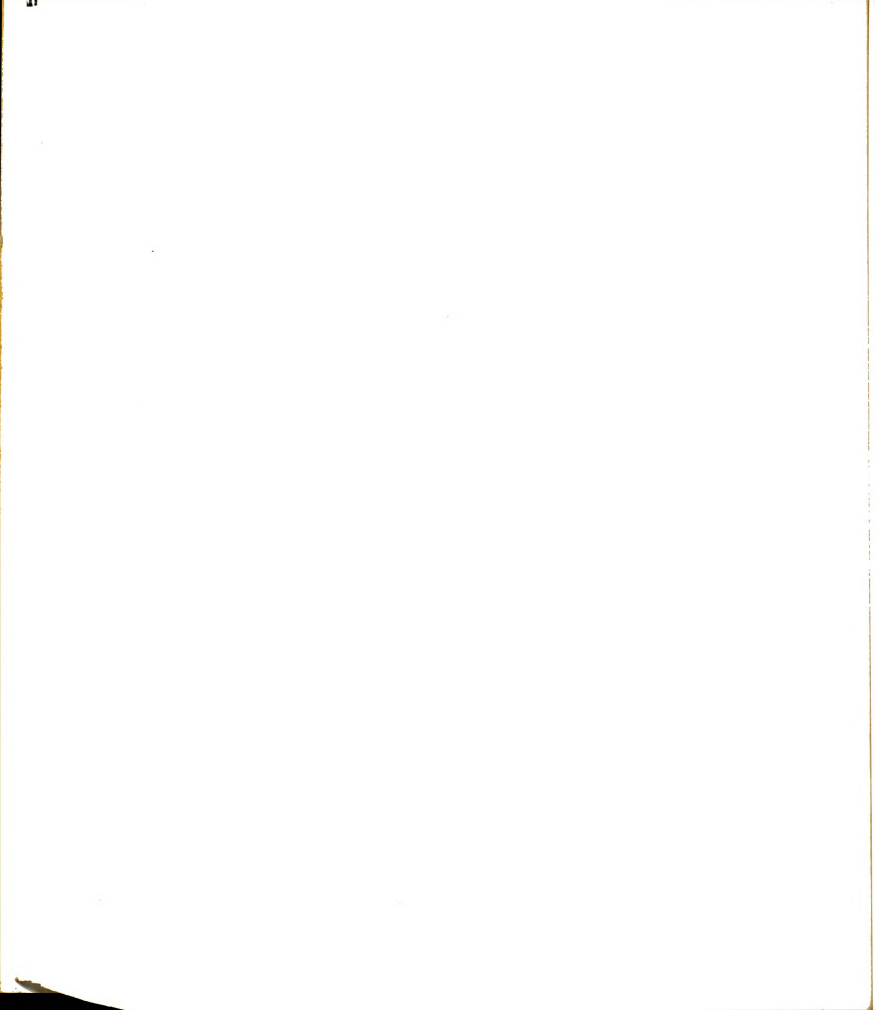
five randomly selected chance performances) at least as high as the test value given that the test value is unfavorable (i.e. greater than 245). For test value 252, "P" is determined by:

1. Finding the number of standard error of the mean between the standard and the test value.

$$Z = \frac{252 - 245}{3.8923} = 1.7984$$

2. Using the table of normal curve areas¹⁶ to convert the Z value into the area between the standard and the test value. In this case the area is .4641.
3. Subtracting this area from .5 to find the area larger than the test value. This area .0359 (.5 - .4641) may be interpreted as the probability that a sample mean will be at least as large as 252.
4. Dividing .0359 by .5 to obtain .0718--the probability that a sample mean will be at least as large as 252 given that the test value is unfavorable. This is simply a matter of limiting the sample space to only one-half the curve.

¹⁶ Actually since the sample size is under 30 it would be more appropriate to use the Student-t Distribution. However, most Student-t Distributions are not sufficiently detailed to provide the areas for all t or Z values. The normal distribution is used for convenience and because it provides a good approximation to the area. Moreover, it is customary for quality control engineers to use the normal distribution for sample sizes of five for the same reason indicated above.



The same process is followed to determine P for the other test values.

The reader will recall that a decision to accept the hypothesis and refrain from an investigation will be made as long as P is greater than P_c . Contrariwise, when P becomes smaller than P_c the hypothesis is rejected and an investigation is initiated. Accordingly, Table 45 indicates that the upper control limit calculated by this approach is between 250 and 251.

TABLE 45.--Decision table for BF and J application. First interpretation of P

Test Value	L	C	P_c	P	Decision
250	\$5.05	\$5	.0099	.2006	Accept
251	6.00	5	.1667	.1236	Reject
252	7.00	5	.2857	.0718	Reject
253	8.00	5	.3750	.0394	Reject

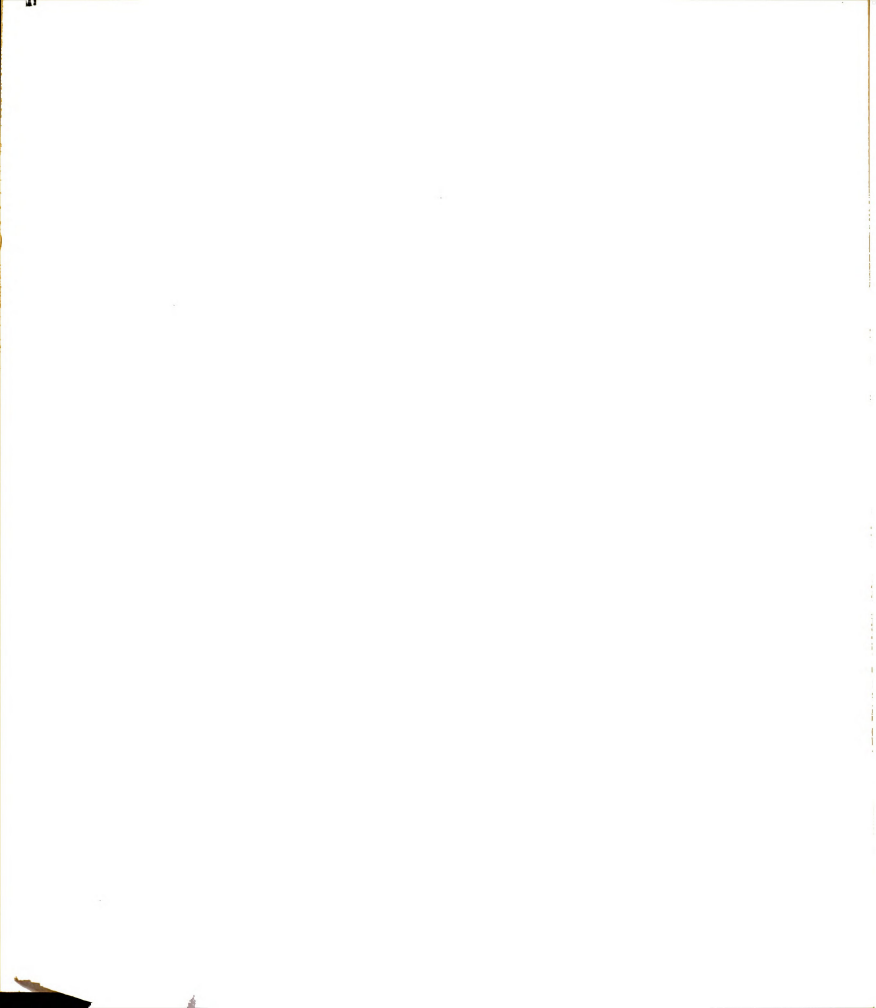
In dealing with testing plans involving individual performances it is sufficient to indicate a control limit as lying between two whole numbers. In this case, it is understood that an investigation will not be undertaken for the occurrence of a performance value closer to the standard; but that one will be undertaken for the occurrence of a value farther from the standard. No more precision is needed because the performances are not recorded in fractions of a minute.

However, in testing plans involving samples, the sample mean will rarely be a whole number. Therefore, it is necessary to pinpoint the control limit more exactly. This can be done by the following process of interpolation. The difference between P_c and P for test value 250 is .1907. The corresponding difference for test value 251 is .0431. The sum of these differences is .2338. The ratio $.0431/.2338$ indicates that only 18 per cent of the total difference is accounted for by test value 251. That is, P and P_c come much closer to being equated at 251 than at 250. Therefore, the control limit is much closer to 251 than to 250 and it can be determined by subtracting .18 from 251. Thus the upper control limit is 250.82.

Second Interpretation of P . The values in Table 46 depicting the second interpretation of P are determined in the same manner as the values in Table 45 except for P . " P " according to the second interpretation represents the probability that a sample mean equal to the given test value is due to chance. These probabilities can be estimated by a round about process shown in Table 47.

TABLE 46.--Decision table for BF and J application. Second interpretation of P

Test Value	L	C	P_c	P	Decision
252	\$ 7	\$5	.2857	.7330	Accept
253	8	5	.3750	.5807	Accept
254	9	5	.4444	.4307	Reject
255	10	5	.5000	.3684	Reject



In this table the conditional probabilities represent the probability of obtaining a sample mean exactly equal to the test value for each respective cause. These probabilities are obtained by the method of normal curve approximation. For example, .0207 is the probability of obtaining a sample mean of 252 from five chance performances randomly selected. The value .0207 represents the area under the normal curve between 251.5 and 252.5 with mean 245 and standard error of the mean 3.8923.¹⁷ Similarly, .0754 is the probability of obtaining a sample mean of exactly 252 from five randomly selected performances due to poor attitude. The value .0754 represents the area under the normal curve between 251.5 and 252.5 with mean 255 and standard error of the mean 4.0187.

(Table 17 shows that the mean of the performances due to poor attitude is 255.) The standard error of the mean, $\sigma_{\bar{X}}$, is calculated from the distribution in Table 2 by the following formula:

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n - 1}}.$$

It would be impossible to obtain a mean of 252 from a sample of five performances due to any of the other causes. The conditional probabilities for the other test values are interpreted in a similar manner.

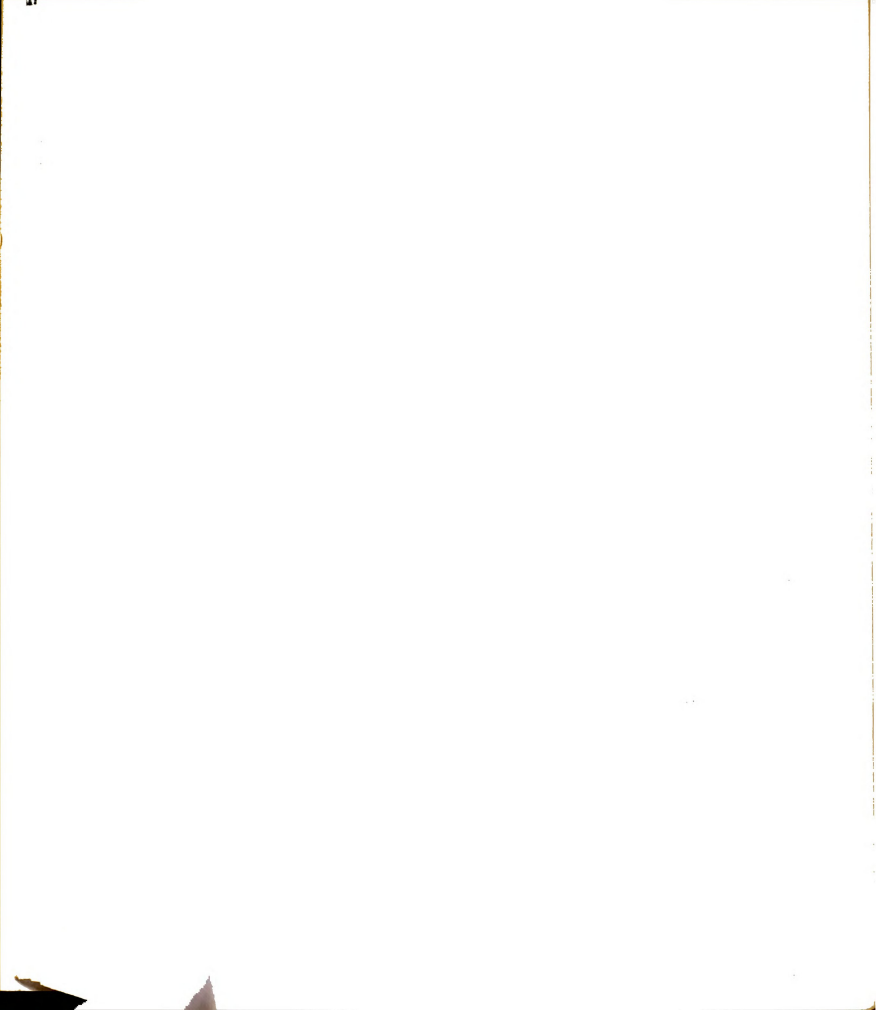
¹⁷The calculation of 3.8923 was just explained in conjunction with the Basic Control Chart approach according to this testing plan.

TABLE 47.--Determination of P's

Cause	Cond. Prob.	Number of Perf.	Number of Means	Prob. of Cause Given Occurrence of Test Value
Test Value 250				
Chance	.0477	600	28.6200	.9119
Poor Attitude	.0461	60	<u>2.7660</u>	<u>.0881</u>
			31.3860	1.0000
Test Value 251				
Chance	.0318	600	19.0800	.8395
Poor Attitude	.0608	60	<u>3.6480</u>	<u>.1605</u>
			22.7280	1.0000
Test Value 252				
Chance	.0207	600	12.4200	.7330
Poor Attitude	.0754	60	<u>4.5240</u>	<u>.2670</u>
			16.9440	1.0000
Test Value 255				
Chance	.0057	600	3.4200	.3684
Poor Attitude	.0956	60	5.7360	.6178
Illness	.0064	20	<u>.1280</u>	<u>.0138</u>
			9.2840	1.0000

The column in Table 47 labeled "number of performances" represents the number of performances attributed to each respective cause that occurred in the 1000 initial observations. The numbers are listed in Table 17 and their detail is shown in Table 18. They are used here only as weights. That is, the values in the column labeled "number of means" are the result of multiplying the conditional probabilities by the number of performances for each respective cause. The reason for weighting the conditional probabilities in this manner follows. The conditional probability column indicates that the probability of obtaining a sample mean of exactly 252, for example, is more than three times greater if poor attitude is the cause than if chance is operative. However the probability that chance is operative is ten times ($600/60$) the probability that poor attitude is operative. Consequently, the ratio $12.42/16.944 = .7330$ indicates the probability that a sample mean of 252 is due to chance (uncontrollable factors). This probability, .7330, is the P used for test value 252 in Table 46.

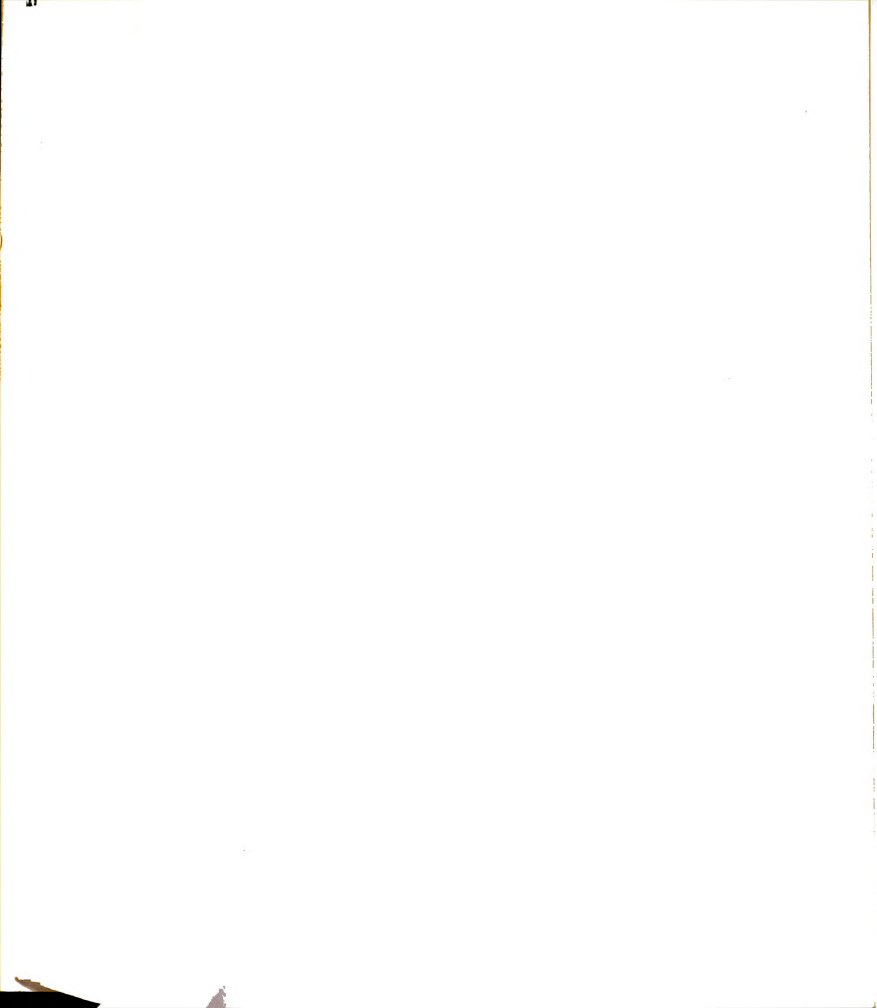
This decision table shows that the upper control limit according to this interpretation of P is between 253 and 254. The same process of interpolation that was applied for the first interpretation of P, establishes the control limit at 253.94.



McMenimen Approach

Use of this approach brings to light a very important consideration as far as the investigative procedure is concerned. From these calculations it appears that it is not always advantageous to begin by investigating for dull knives. In an attempt to find the upper control limit for a dull knife investigation, 250 is chosen first as a test value. Table 47, however, shows that a sample mean as low as 250 would never occur if dull knives were used. In fact, the same table indicates that a sample mean as low as 255 has never been observed with dull knives. By continuing this procedure for other test values, one would find that a random sample of five dull knife performances would rarely produce a sample mean less than 259. At the same time one would never obtain a sample mean from chance performances as high as 258. Thus, this is an ideal situation in which the chance sampling distribution does not overlap with the dull knives sampling distribution. Accordingly, by setting the dull knife upper control limit at 259, one can eliminate both the risks of committing a Type I and a Type II error. Therefore, it would not be profitable to investigate for dull knives until a sample mean of 259 appeared.

The McMenimen test does, however, indicate that it would be profitable to investigate sample means as low as 251 for poor attitude. Table 48 shows the derivation

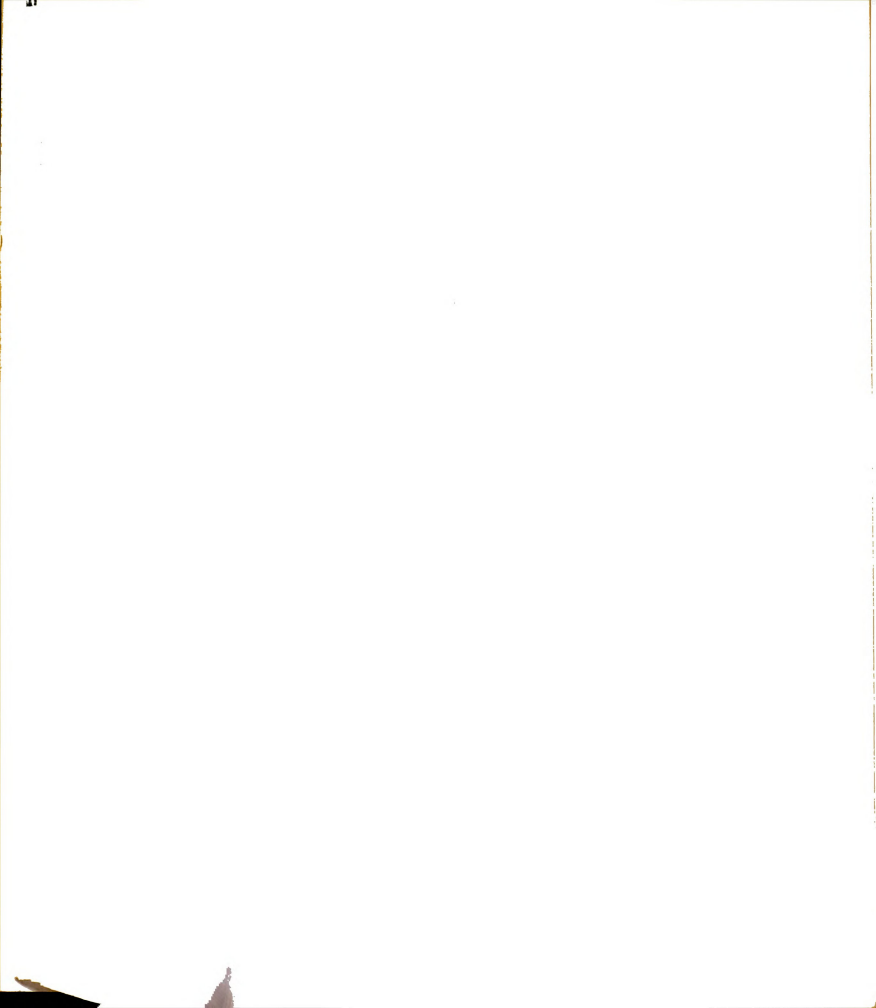


of the savings values for both poor attitude and illness. Laziness would be detected by the same test used for poor attitude but savings values for it are not indicated since a sample of these performances would never produce a mean in the 250 range. Essentially, this table is the same as Table 33 except that the multiplication weight is now 20 instead of 40. As the reader will recall, 20 results from multiplying the sample size of 5 by the 4 tests that are assumed to lapse on the average before an off-standard condition is detected.

TABLE 48.--Derivation of savings values

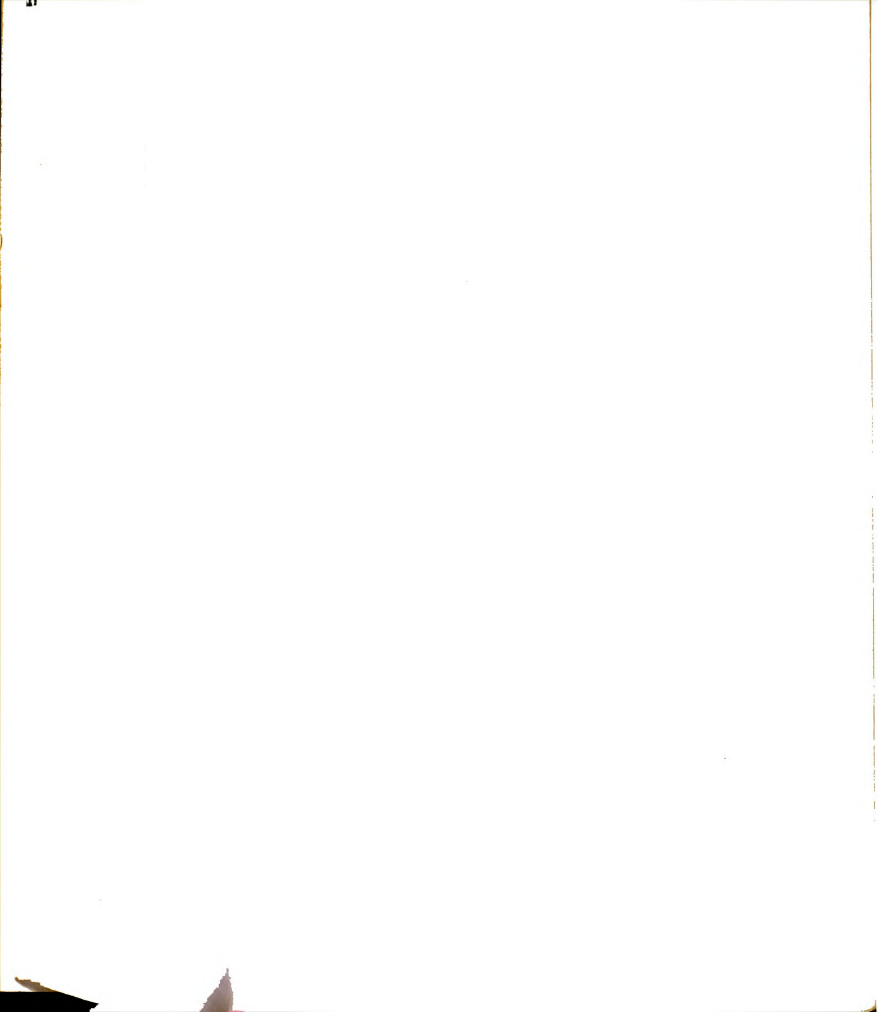
Cause	Single Perf. Op. Cost	Multi- plication Weight	Weighted Op. Cost	Correction Cost per Performance	Savings
Poor Attitude	\$.50	20	\$10	\$.25	\$ 9.75
Illness	1.00	20	20	.50	19.50

The probabilities indicated on Table 49 are the same for each corresponding test value as those derived in Table 47. For test value 250, the probability of saving \$0 from an investigation for poor attitude is .9119--the probability that the items sampled were drawn from a population of chance performances. The probability that \$9.75 can be saved is, of course, .0881--the estimated probability that poor attitude is prevailing. The application table



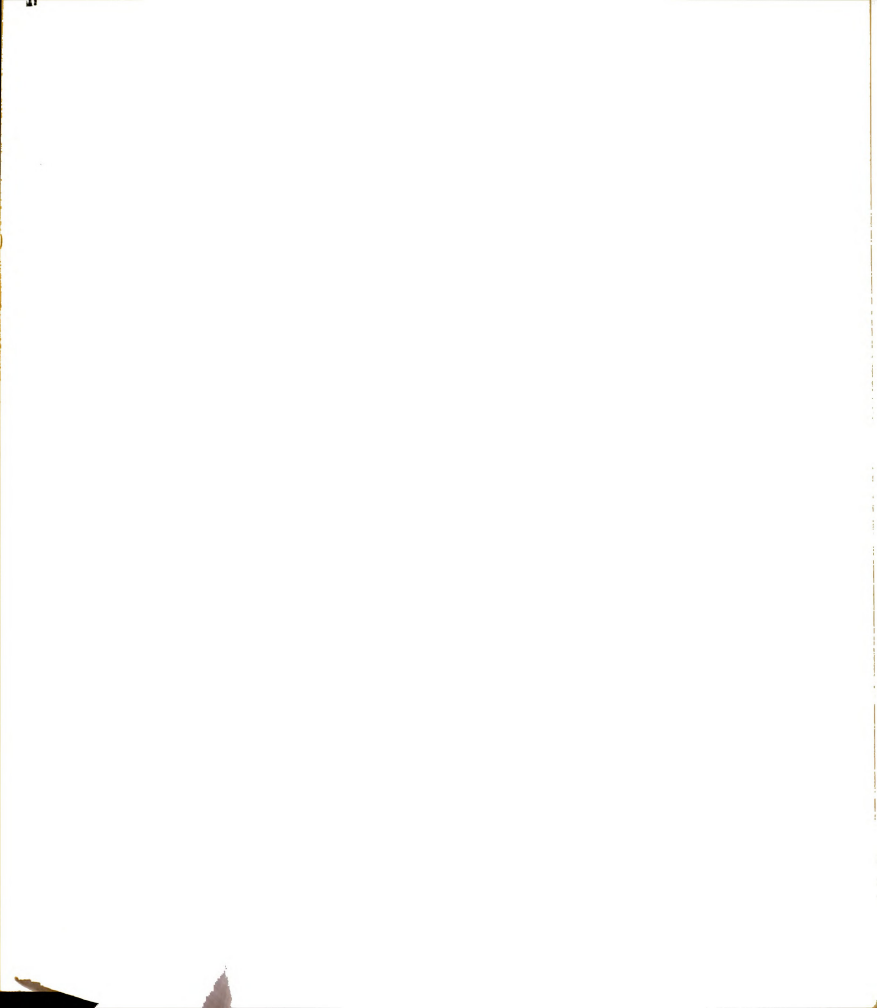
indicates that the upper control limit for poor attitude is between 250 and 251. Interpolation between these values results in an upper control limit of 250.2. This value was determined by the following technique. At an expected savings of zero, the analyst would be just indifferent between investigating and not investigating. For test value 250, the expected savings is .1410 from zero; for 251 it is .5649 from zero. Hence, the control limit is closer to 250. The sum of these differences is .7059. The ratio $.1410/.7059 = .20$ is the amount which should be added to 250 to arrive at the upper control limit of 250.2.

The possibility of investigating for illness is considered for test value 255. The probabilities are determined in the following manner. An investigation would be made first for poor attitude and an investigation for illness would be considered only if poor attitude was first eliminated as a possible cause. Therefore, the relevant probabilities associated with act-investigate for illness are found by reference to the "number of means" column in Table 47. The probability that \$0 can be saved is $3.4200/3.5480$ or .9639 and the probability that \$19.50 can be saved is $.1280/3.5480$ where 3.5480 is 3.4200 plus .1280. (The 5.73160 value for poor attitude has been eliminated.) The negative expected value indicates that it would not be profitable to investigate for illness for a sample mean as low as 255. The same test applied for



218

Event	Spend Up To \$1 Investigating For Poor Attitude		Spend Up To \$4 Investigating For Illness	
	Pe	Cond.	Pe	Cond.
	Test Value 250			
Save \$0	.9119	\$-1	\$- .9119	
Save \$9.75	.0881	8.75	+ .7709	
Expected Savings			<u>\$- .1410</u>	
	Test Value 251			
Save \$0	.8395	\$-1	\$- .8395	
Save \$9.75	.1605	8.75	1.4044	
Expected Savings			<u>\$+ .5649</u>	
	Test Value 255			
Save \$0	.3822	\$-1	\$- .3822	
Save \$9.75	.6178	8.75	5.4058	
Save \$19.50				
Expected Savings			<u>.0361</u>	<u>15.50</u>
				<u><u>\$-3.8556</u></u>
				<u><u>\$-3.2960</u></u>
	Test Value 257			
Save \$0	.1974	\$-1	\$- .1974	
Save \$9.75	.8026	8.75	7.0228	
Save \$19.50				
Expected Savings			<u>.2627</u>	<u>15.50</u>
				<u><u>\$+1.1226</u></u>



test value 257 indicated that an investigation for illness would be profitable for test value 257. The same information developed for test value 256 shows that an investigation at this value would not be profitable. Interpolation pinpoints the upper control limit at 256.43. Use of the incremental approach in which the additional \$3 rather than \$4 is subtracted from the savings in the conditional column yields an upper control limit of 255.94.

Equalization approach

Table 50 indicates that the upper control limit under this approach is between 250 and 251. The following interpolation process is employed. For test value 250, the difference between the expected opportunity cost of α and the expected opportunity cost of β is .0401. The corresponding difference for test value 251 is .0342. The sum of these differences is .0743. The ratio $.0401/.0743 = .54$ when added to 250 yields 250.54 as the upper control limit.

The derivation of the individual values shown in Table 50 is explained in Appendix B.

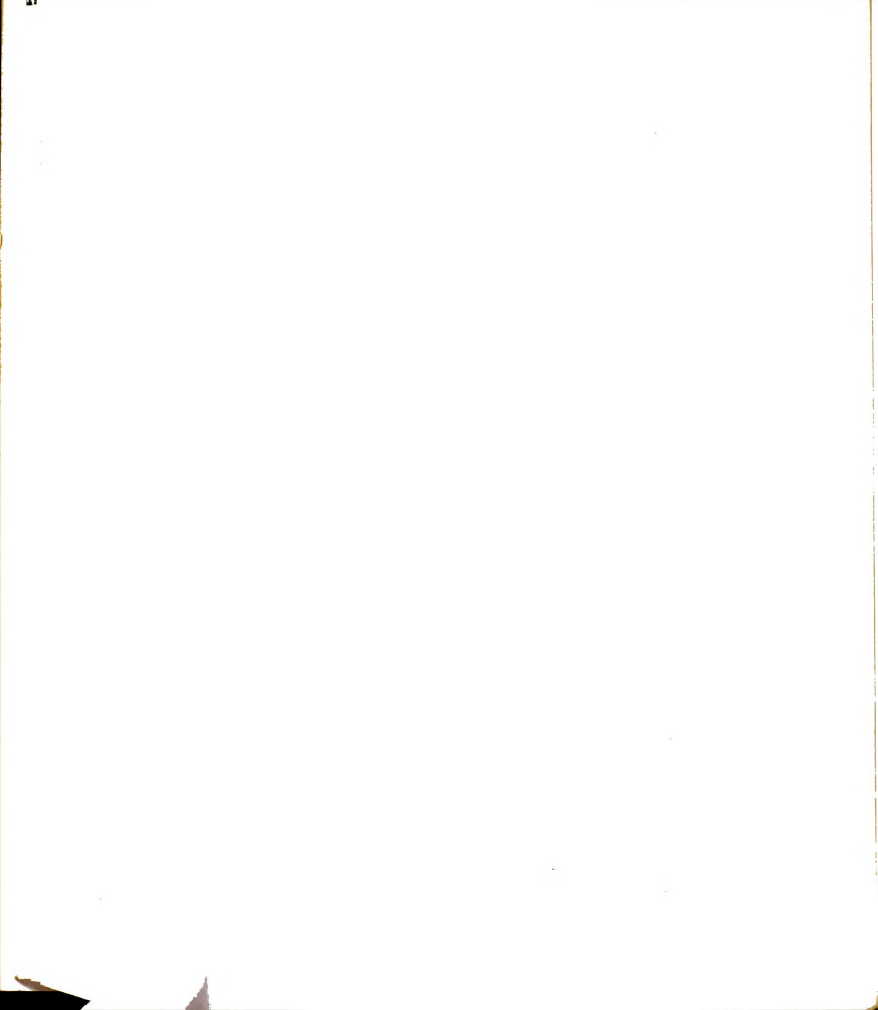


TABLE 50.--Decision table for Equalization approach

	Test Values	
	251	250
Probability of α	.0618	.1003
Opportunity Cost of α	\$1	\$1
Expected Opportunity Cost of α	<u>\$.0618</u>	<u>\$.1003</u>
Probability of β	.0322	.0215
Opportunity Cost of β	\$2.9800	\$2.7990
Expected Opportunity Cost of β	<u>\$.0960</u>	<u>\$.0602</u>
Decision	Reject	Accept

α = Type I error

β = Type II error

Minimization approach

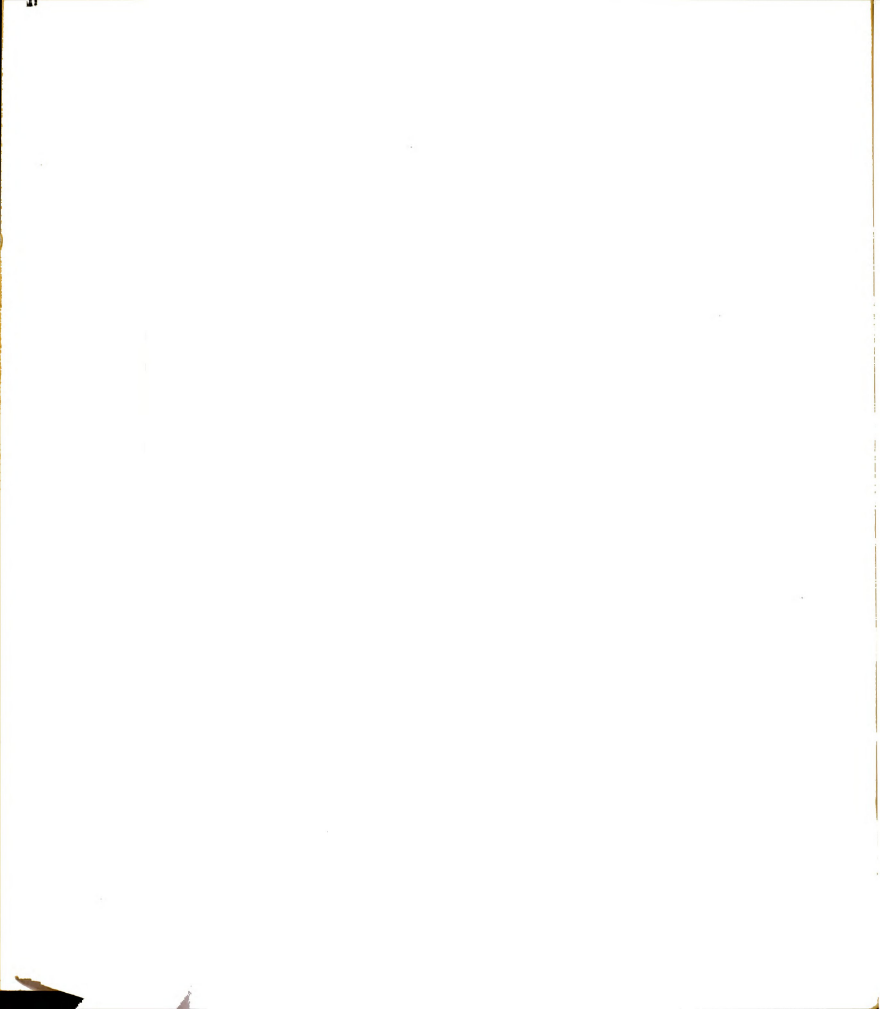
Table 51 indicates that 252 is the upper control limit because the expected opportunity cost is less for this test value than for any other. It has previously been established that only poor attitude could produce a sample mean in the low 250's--the range in which check tests indicate that the control limit will fall. Accordingly, only chance and poor attitude are considered in the prior distribution. Since 600 of the initial 1000 performances were due to chance and 60 to poor attitude the prior probabilities of .9091 and .0909 result from the ratios 600/660 and 60/660 respectively.

The weighted opportunity cost for a chance cause is \$1--the cost of a Type I error associated with the

investigation for poor attitude. The weighted opportunity costs for poor attitude are the same for each test value as the opportunity costs of committing a Type II error that were used in the Equalization approach.

Similarly, the probabilities of a wrong decision are the same for chance cause as the probabilities of committing a Type I error that were used for each test value in the Equalization approach. These probabilities for poor attitude for each test value are the same as the probabilities of obtaining a sample mean less than the test value.

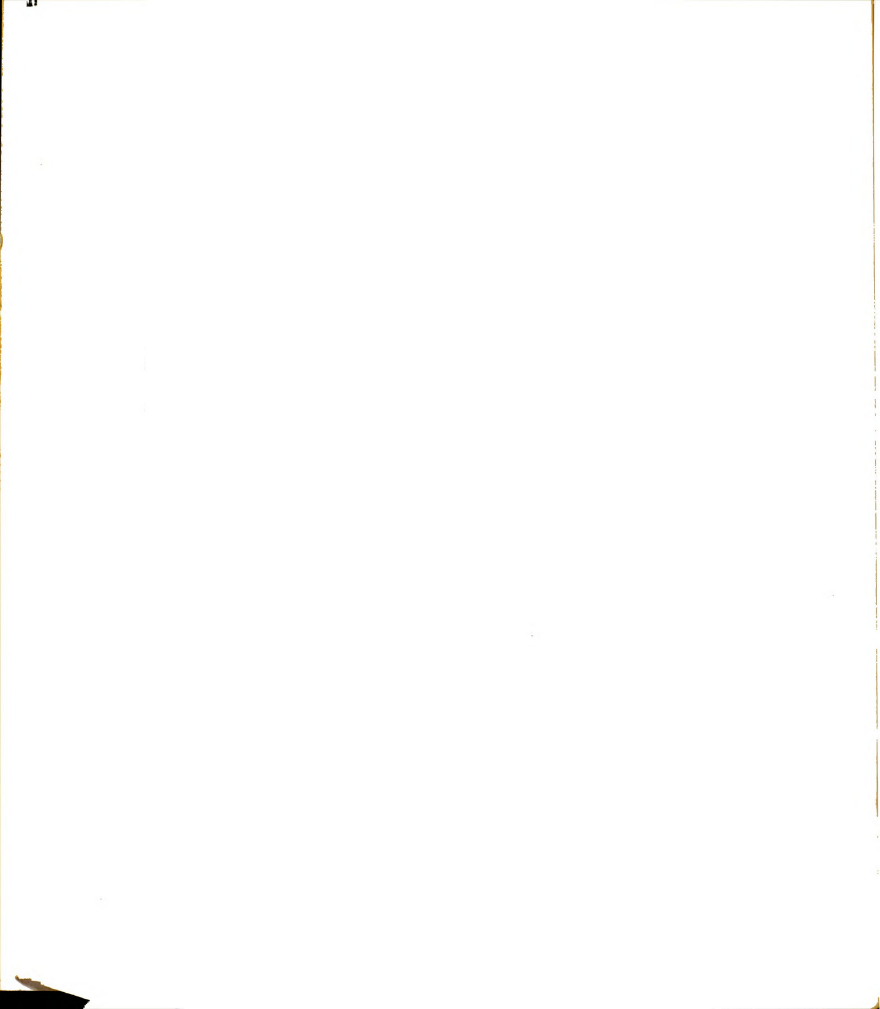
Such probabilities are determined, as the reader will recall, by finding the area that is less than the test value under the normal curve with mean 255 and standard error of the mean 4.0187. The probabilities of a wrong decision used here for poor attitude are not the same for each respective test value as the probability of a Type II error under the Equalization approach. The probability of a Type II error was calculated by adding the averaging step of multiplying by 60 and dividing by 300. No such averaging step is added here since one of the distinctive features of the Minimization approach is that it deals individually with each assignable cause possibility. As with previous illustrations of the Minimization approach the conditional opportunity costs result from multiplying each weighted opportunity cost by its respective probability of



a wrong decision. Also, the expected opportunity cost for each test value results from a summation of the products of the conditional opportunity cost and the prior probability for each cause.

TABLE 51.--Decision table for Minimization approach

Cause	Prior Prob.	Weighted Op. Cost	Wrong Decision	Cond. Ave. Op. Cost	Expected Op. Cost
Test Value 253					
Chance	.9091	\$1	.0197	\$.0197	
Poor Attitude	<u>.0909</u>	3.6260	.3121	1.1317	
	1.0000				<u><u>\$.1208</u></u>
Test Value 252					
Chance	.9091	\$1	.0359	\$.0359	
Poor Attitude	<u>.0909</u>	3.2575	.2266	.7313	
	1.0000				<u><u>\$.0991</u></u>
Test Value 251					
Chance	.9091	\$1	.0618	\$.0618	
Poor Attitude	<u>.0909</u>	2.9800	.1611	.4801	
	1.0000				<u><u>\$.0998</u></u>
Test Value 250					
Chance	.9091	\$1	.1003	\$.1003	
Poor Attitude	<u>.0909</u>	2.7990	.1075	.3009	
	1.0000				<u><u>\$.1185</u></u>



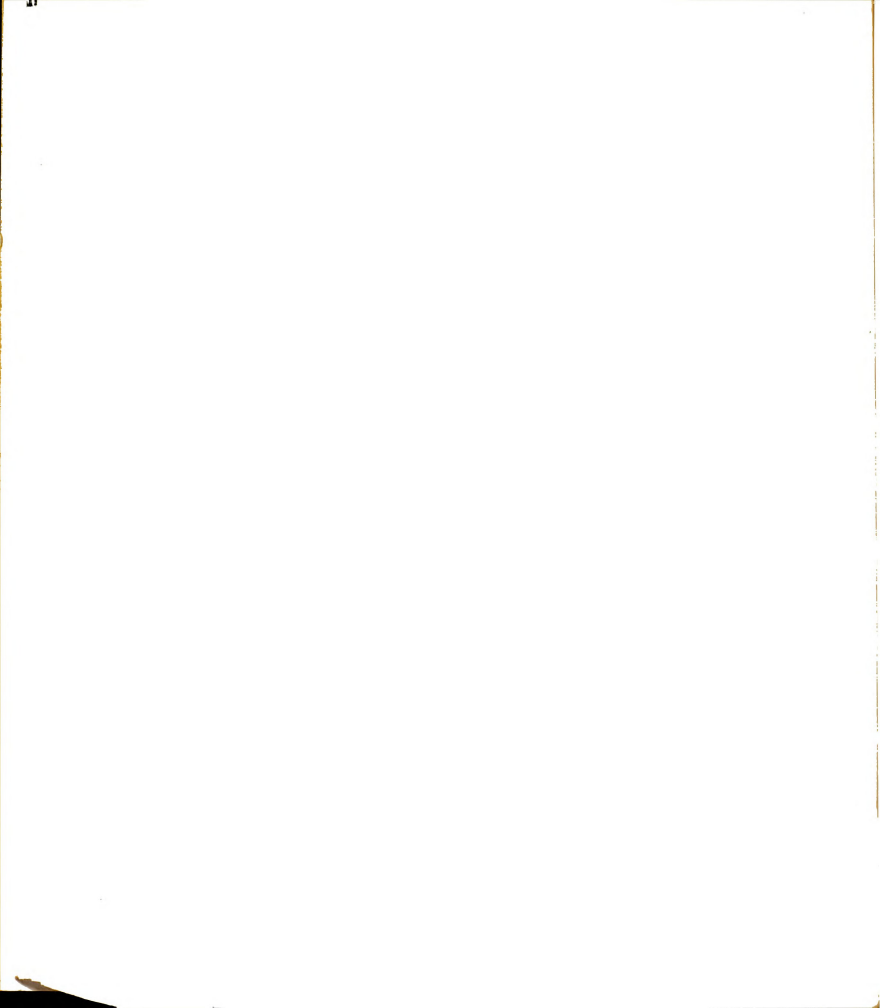
Comparison of Upper Control Limits Among the Methods

The control limits that have just been derived are itemized below:

<u>Approach</u>	<u>Upper Control Limit</u>
Accountant's Conventional	270
Basic Control Chart	255.8
Bierman, Fouraker, and Jaedicke	
First Interpretation of P	250.82
Second Interpretation of P	253.94
McMenimen	250.2 for poor attitude
	255.94 for illness (incremental)
Equalization	250.54
Minimization	252

Financial Analysis and Ranking

Under the sampling plans the approaches will continue to be analyzed by twos and ranked in preferential order according to their desirability for control purposes. Slightly different mechanics are necessitated by the introduction of sampling. It is not possible to count directly from Table 18 the number of sample means that are likely to fall within a specified range of values per thousand samples for any specified cause. In making the financial analysis for control limits based on individual performances, it was a simple procedure to count from Table 18 the number of individual performances falling within a specified range for any specified cause. Table 18 does,



however, provide the necessary information to make these calculations for sample means. The following analysis will illustrate the technique. It involves a comparison between the Accountant's Conventional approach and the Basic Control Chart approach.

The Basic Control Chart approach will incur additional investigation charges for sample means with values between 255.8 and 270 which are due to chance. The area between 255.8 and 270 under the normal curve with mean 245 and standard error of the mean 3.8923 is .0028.¹⁸ This probability that a sample mean will fall in this region if chance is operative must be multiplied by 600 in order to estimate the number of means per thousand samples that are likely to fall in this range. This product of 1.68 must be multiplied by \$5¹⁹ to arrive at \$8.40 as the additional investigation charge.

¹⁸The control limit under the Basic Control Chart approach was determined by choosing a value which .025 sample means would exceed assuming a student-t sampling distribution. It was easy to use the t distribution for this approach because the t values are always given for this level of significance. Under the other approaches used, the level of significance would rarely turn out to be a value for which the t values are customarily reported. Hence, the normal sampling distribution was assumed under these approaches. Now in comparing these approaches the areas must be calculated by the consistent use of a normal sampling distribution. This assumption results in an area of .0028 between 255.8 and 270 instead of the area .025 that would result from a t distribution.

¹⁹\$6 is not used in this calculation because a sample mean attributed to tough cows would almost always be greater than 270.

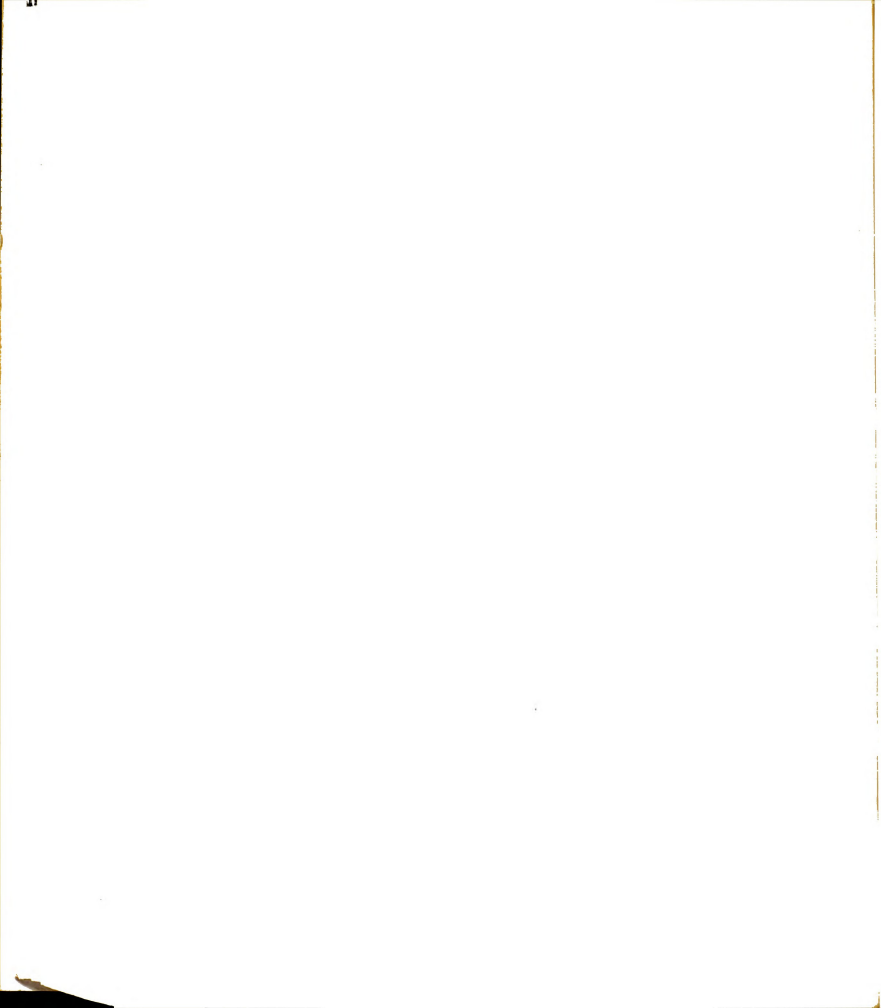
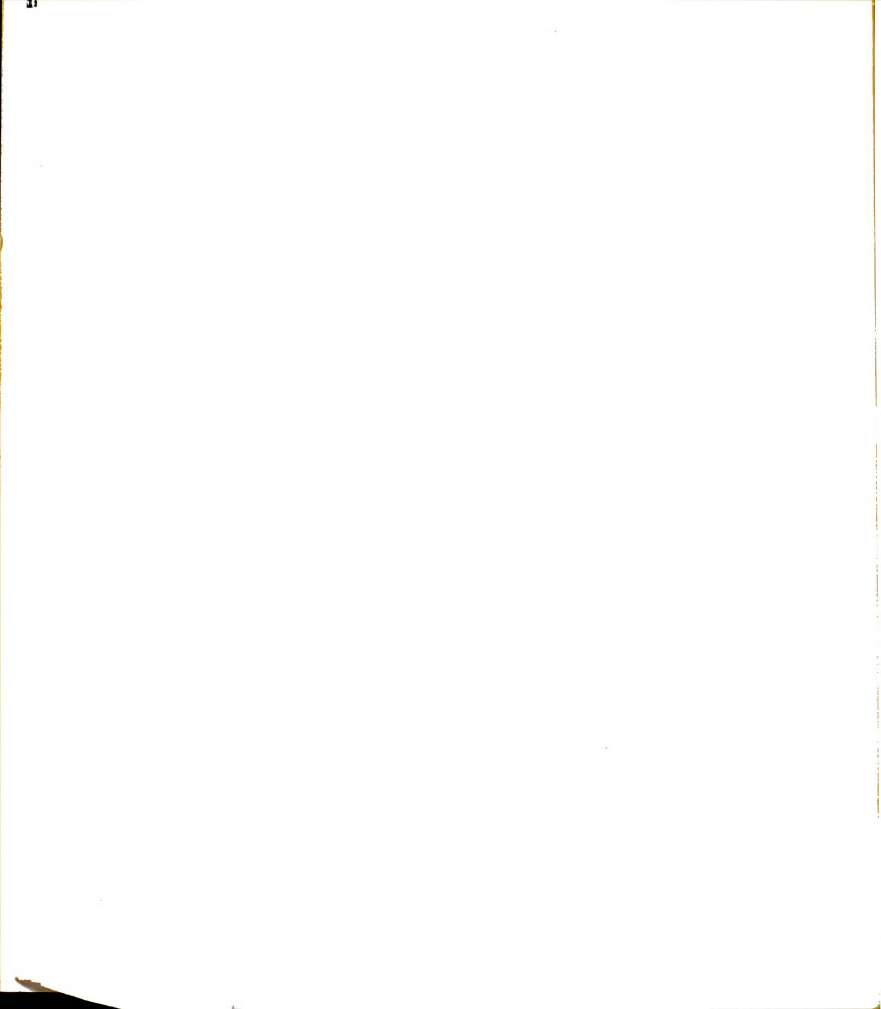


Table 52 illustrates the derivation of the additional savings brought about because the Basic Control Chart approach would involve investigations between 255.8 and 270 that would not be undertaken by applying the Accountant's Conventional method. For poor attitude the .4207 probability of getting a sample mean between 255.8 and 270 represents the area between 255.8 and 270 under the normal curve with mean 255 and standard error of the mean 4.0187.²⁰ The other probabilities represent the area under the normal curve between the same two values. The probabilities are different for each assignable cause because the means and the standard errors of the mean are different for each assignable cause. The number of performances for each corresponding cause are simply used as weights to enable an estimate of the number of sample means between 255.8 and 270 per thousand samples for each respective cause. These estimates are then multiplied by the weighted opportunity costs.²¹ The sum of the resulting products, \$2482.11, represents the additional savings under the Basic Control Chart approach. Since \$2,482.11 is larger

²⁰The mean of the poor attitude assignable cause is 255 and the standard error of the mean is 4.0187.

²¹These weighted opportunity costs represent the single performance opportunity costs weighted at test value 270 and then multiplied by five to recognize that an off-standard condition can only be detected at the conclusion of each five performances that are included in the sample.



than the \$8.40 additional cost, the Basic Control Chart approach is more effective for control purposes.

TABLE 52.--Additional savings of Basic Control Chart approach

Cause	Probability of getting x between 255.8 and 270	Number of Perf.	Number of Means	Weighted Op. Cost	Added Savings
Poor Attitude	.4207	60	25.2420	\$62.8000	
Dull Knives	.5000	120	60	11.5290	
Illness	.8555	20	17.1100	9.2235	
Laziness	.1190	40	4.7600	9.9500	
					\$2482.1137

The results of the other comparisons are summarized in Table 53.

TABLE 53.--Financial comparisons between approaches

Approaches Tested		Added Inv. Cost of Lower UCL	Added Savings of Lower UCL	Most Effective Approach
AC	BCC	\$ 8.40	\$2482.11	BCC
BCC	Equal	109.08	141.16	Equal
Equal	BFJ 1st	6.60	2.81	BFJ 1st
BFJ 1st	Min	92.70	14.99	Min
BFJ 2nd	Min	75.60	41.11	BFJ 2nd
Equal	McM	6.25	2.56	Equal
McM	BCC	52.38	82.66	McM

This analysis is depicted diagrammatically in Figure 12.

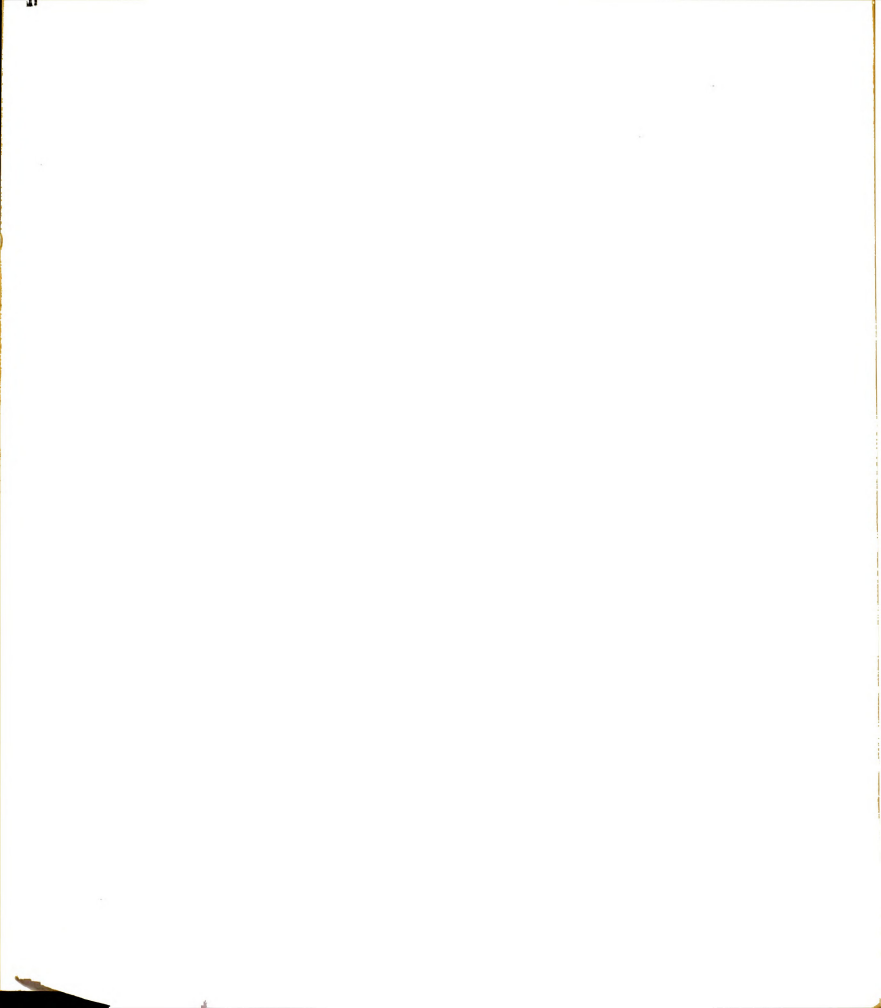
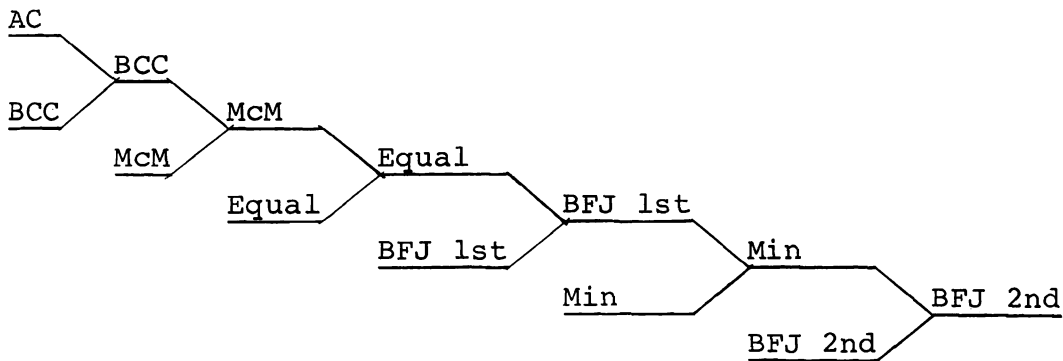


FIGURE 12.--Outcomes of financial comparisons



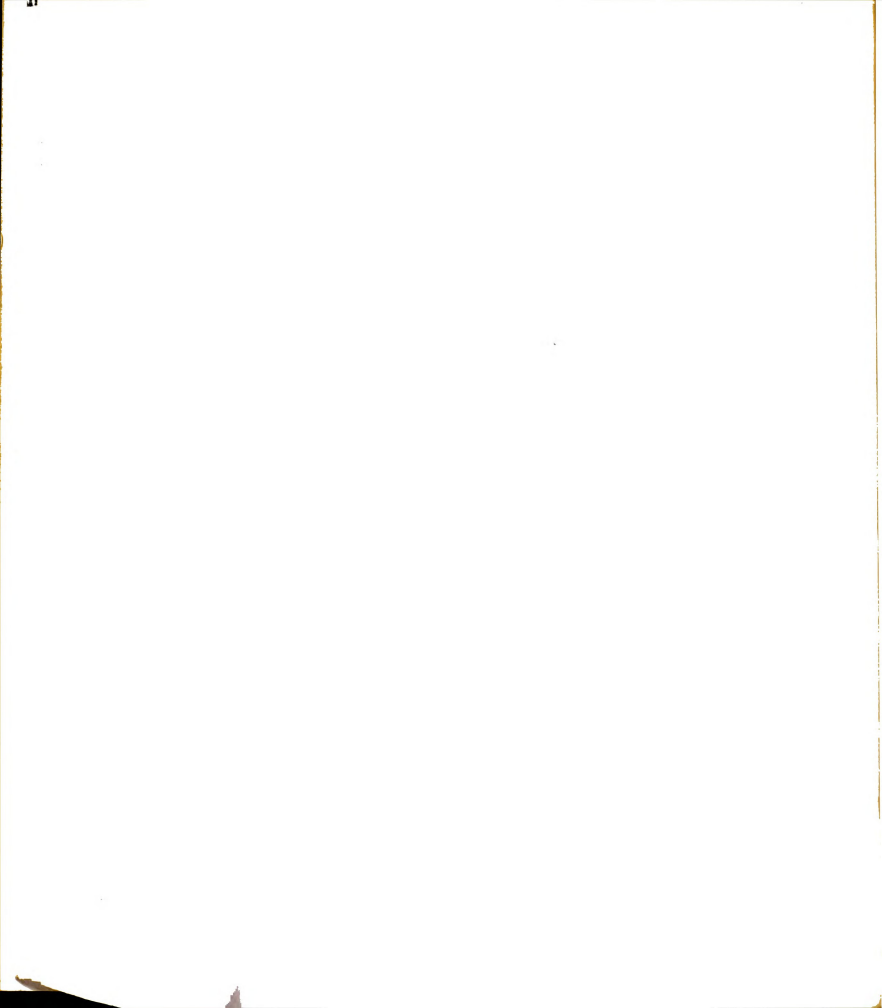
From the above presentation, the following ranking emerges:

<u>Approach</u>	<u>Rank</u>
BFJ 2nd	1
Min	2
BFJ 1st	3
Equal	4
McM	5
BCC	6
AC	7

Derivation and Financial Analysis of
Lower Control Limits--Sample Size
Five--Every Performance
Included in a Sample

Accountant's Conventional Method

Since the accountant does not make a distinction between testing individual performances and sampling groups of performances, the "ten per cent rule" still results in a lower control limit of 220.



Basic Control Chart Approach

The lower control limit is that value which is greater than 2.5 per cent of the sample means of five randomly selected chance performances. (Again, the .05 level of significance with a two-tailed test is used.) This value is found by solving the following formula for LCL:

$$t = \frac{LCL - 245}{\sigma_{\bar{X}}}$$

$$2.776 = \frac{LCL - 245}{3.8923}$$

$$LCL = 234.2$$

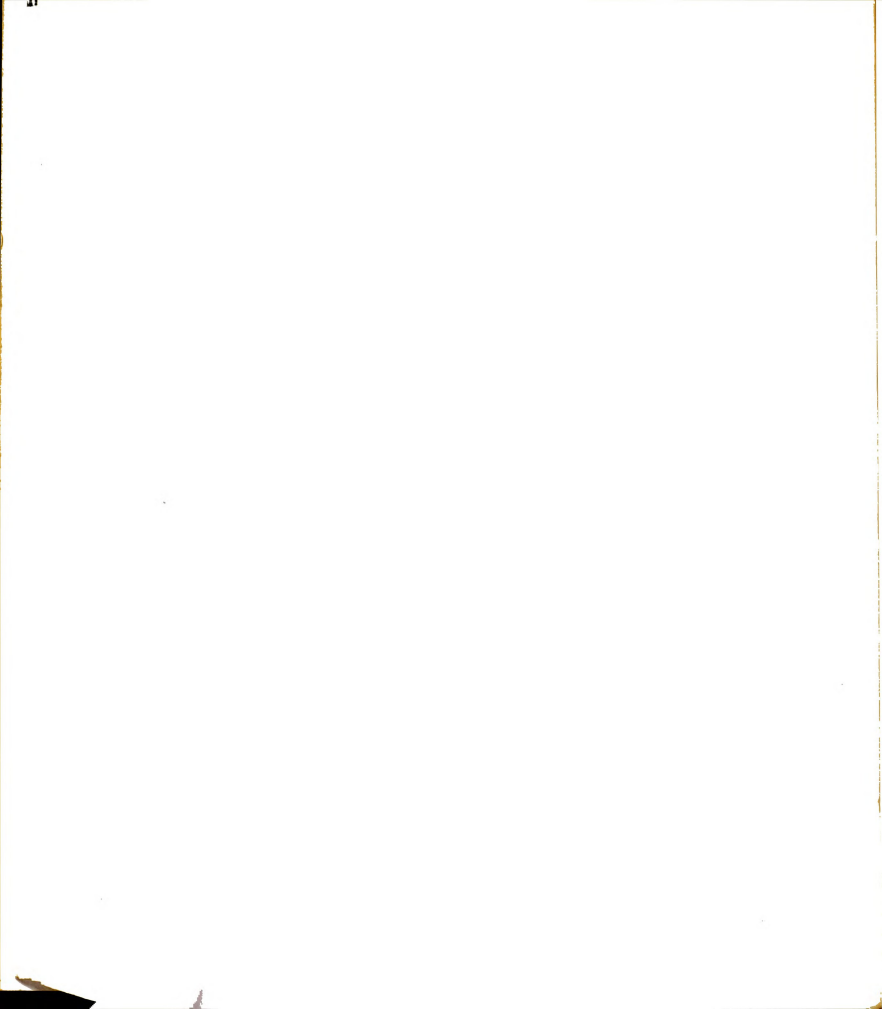
The symbols t and $\sigma_{\bar{X}}$ have the same meaning and their values are the same as those used to calculate the upper control limit under these circumstances.

Bierman, Fouraker, and Jaedicke Approach

First Interpretation of P. Decesion Table 54 shows that the lower control limit calculated under this approach is just slightly more than 239. That is, at test value 239 P_c is almost exactly equated with P .

TABLE 54.--Decision table for BF and J application. First interpretation of P

Test Value	L	C	P_c	P	Decision
239	\$6	\$4	.20	.2006	Accept
240	5	4	.33	.1236	Reject



In this table "L" and "P" are the same for values 240 and 239 as they were for values 250 and 251 respectively in Table 45 that was used to determine the upper control limit for this testing plan. The reason for this is, of course, that the normal curve is symmetrical and 240 and 239 are 5 and 6 values from the standard (the mean) just as 250 and 251 are. The cost of an investigation for a favorable assignable cause has already been specified to be \$4. The usual formula is used to calculate P_c .

Second Interpretation of P. Decision Table 55 indicates that the lower control limit is between 236 and 237 when this interpretation is followed. The same interpolation procedure that was applied by both Bierman, Fouraker, and Jaedicke approaches in the calculation of the upper control limit yields a lower control limit of 236.07.

In Table 55 "L" is determined in the same manner that is described for the calculation of the upper control limit and is the same for test values 236, 237, and 238 as for 254, 253, and 252 respectively because of the symmetry of the normal curve. "C" is a constant at \$4. The values for P_c are determined, as always, by $\frac{L - C}{L}$. The probabilities, P , are determined by the same system of calculations discussed in conjunction with their calculation for the second interpretation of the upper control limit when tests consist of sample of five. Table 56 shows these results.

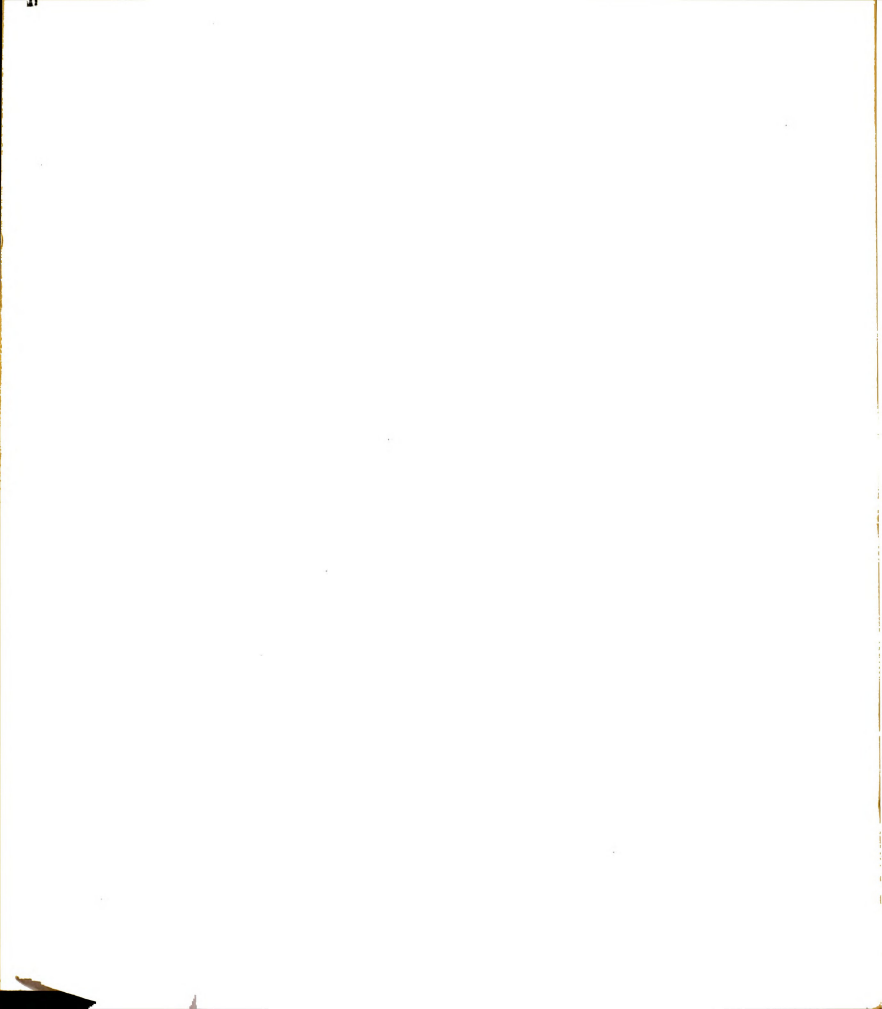
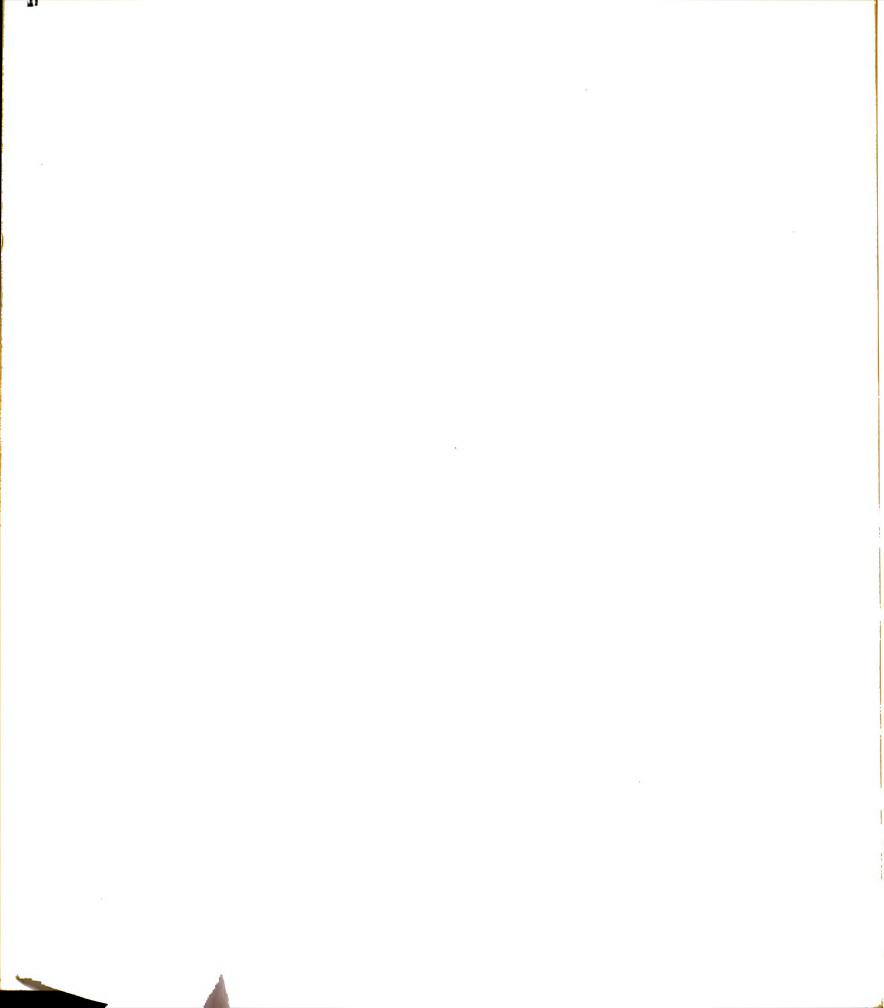


TABLE 55.--Decision table for BF and J application. Second interpretation of P

Test Value	L	C	PC	P	Decision
236	\$9	\$4	.5656	.5468	Reject
237	8	4	.5000	.7349	Accept
238	7	4	.4286	.8054	Accept

TABLE 56.--Determination of P's

Cause	Cond. Prob.	Number of Perf.	Number of Means	Prob. of Cause Given Occurrence of Test Value
Test Value 236				
Chance	.0073	600	4.3800	.5468
Improvement	.0363	100	3.6300	.4532
			8.0100	1.0000
Test Value 237				
Chance	.0122	600	7.3200	.7349
Improvement	.0264	100	2.6400	.2651
			9.9600	1.0000
Test Value 238				
Chance	.0207	600	12.4200	.8054
Improvement	.0300	100	3.0000	.1946
			15.4200	1.0000
Test Value 239				
Chance	.0318	600	19.0800	.9408
Improvement	.0120	100	1.2000	.0592
			20.2800	1.0000
Test Value 240				
Chance	.0437	600	26.2200	.9722
Improvement	.0075	100	.7500	.0278
			26.9700	1.0000
Test Value 241				
Chance	.0590	600	35.4000	.9874
Improvement	.0045	100	.4500	.0126
			35.8500	1.0000



McMenimen Approach

The savings value for improvement is determined as follows:

1. Calculate the savings on each performance by converting the difference in minutes between the mean of the improvement performances, 230, and the standard, 245, into a fraction of an hour by dividing by 60. The result is $1/4$. $\left(\frac{245 - 230}{60} \right)$.
2. Multiply the $1/4$ by \$3 - the hourly wage of the butcher. ($1/4 \times \$3 = $.75$)
3. Multiply this individual performance opportunity cost by the multiplication weight of 20.
 $20 \times .75 = \$15$.
4. Subtract the individual performance cost of correction \$.25 from the \$15. ($\$15 - .25 = \14.75).

The probabilities are the same as those determined in Table 56. The lower control limit indicated by Table 57 is between 236 and 237. Application of the same interpolation procedure used in conjunction with the calculation of the upper control limit under the McMenimen approach yields a lower control limit of 236.97.

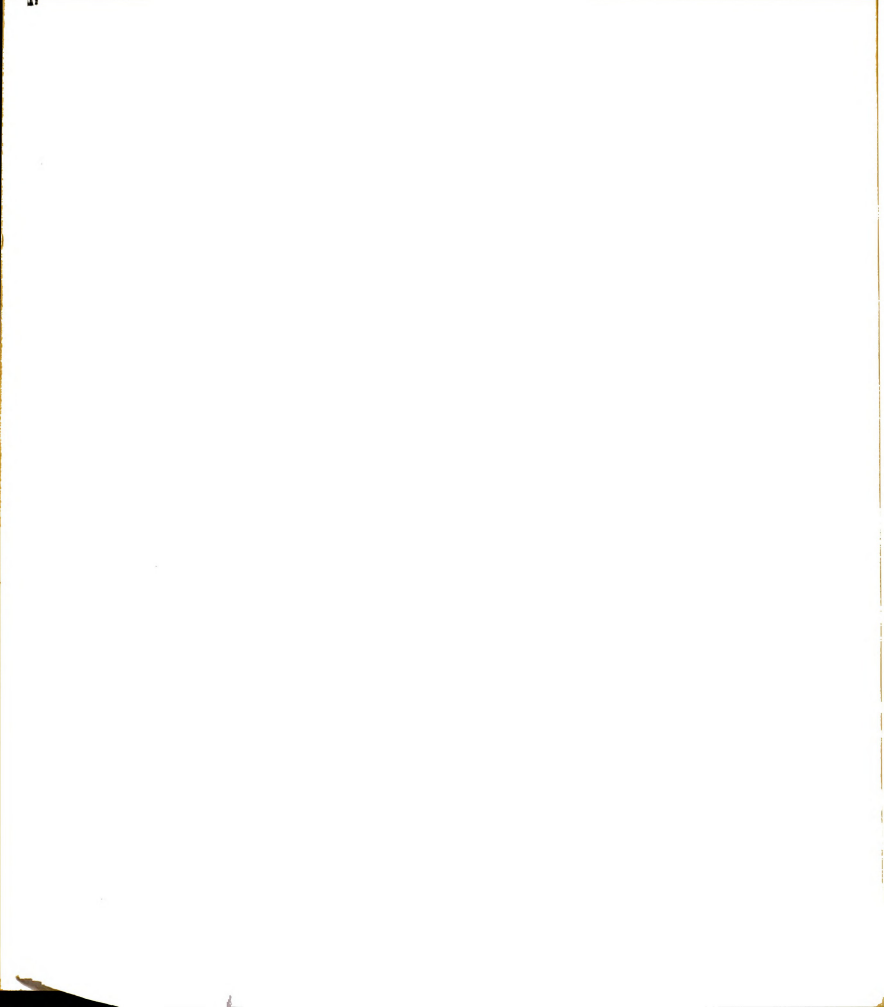


TABLE 57.--Application of McMenimen technique

Event	Spend Up to \$4 Investigating For Improvement		
	Pe	Cond.	Exp.
Test Value 236			
Save \$0	.5468	\$-4	\$-2.1872
Save \$14.75	.4532	10.75	+4.8719
Expected Savings			\$+2.6847
Text Value 237			
Save \$0	.7349	\$-4	\$-2.9396
Save \$14.75	.2651	10.75	+2.8498
Expected Savings			\$-0.0898

Equalization Approach

Table 58 indicates that the lower control limit for this approach is between 238 and 239. This control limit is pinpointed at 238.02 by the same interpolation procedure discussed in connection with the determination of the upper control limit for the Equalization procedure.

In the usual manner the probability of committing a Type I error is calculated by finding the area at least as small as the test value under the normal curve with mean 245 and standard error of the mean 3.8923.

The \$4 opportunity cost of a Type I error is the cost of an investigation to detect improvement.

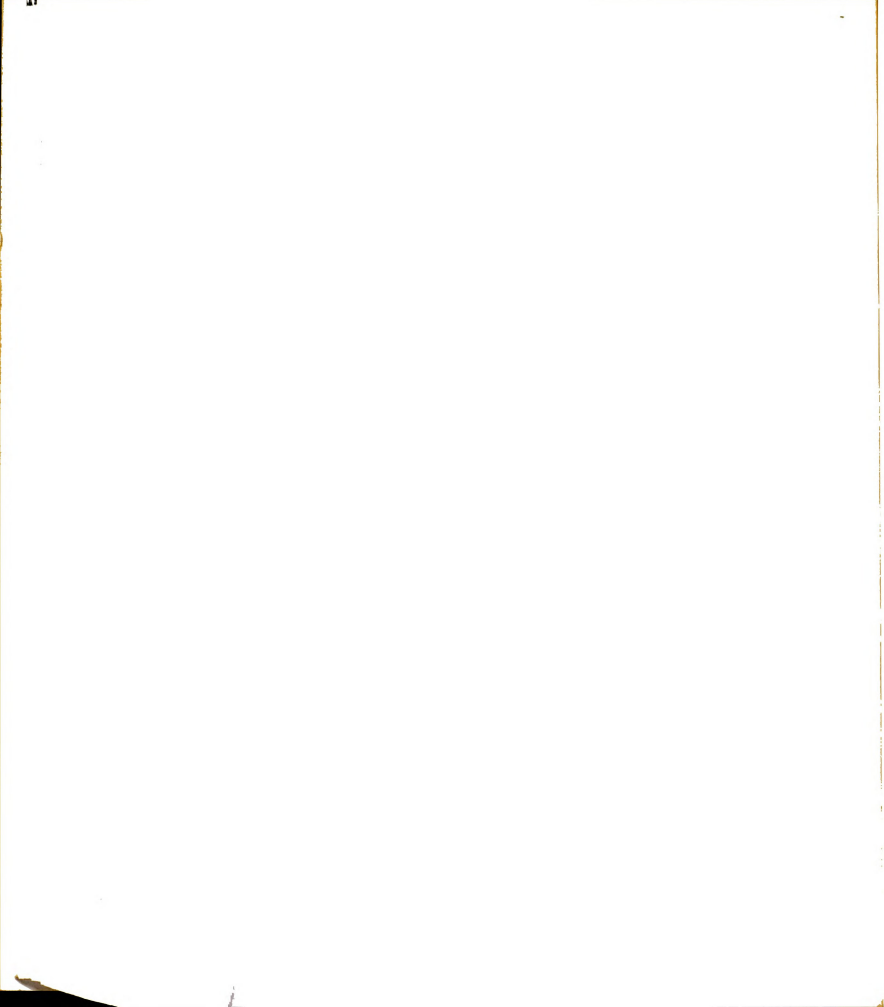


TABLE 58.--Decision table for Equalization approach

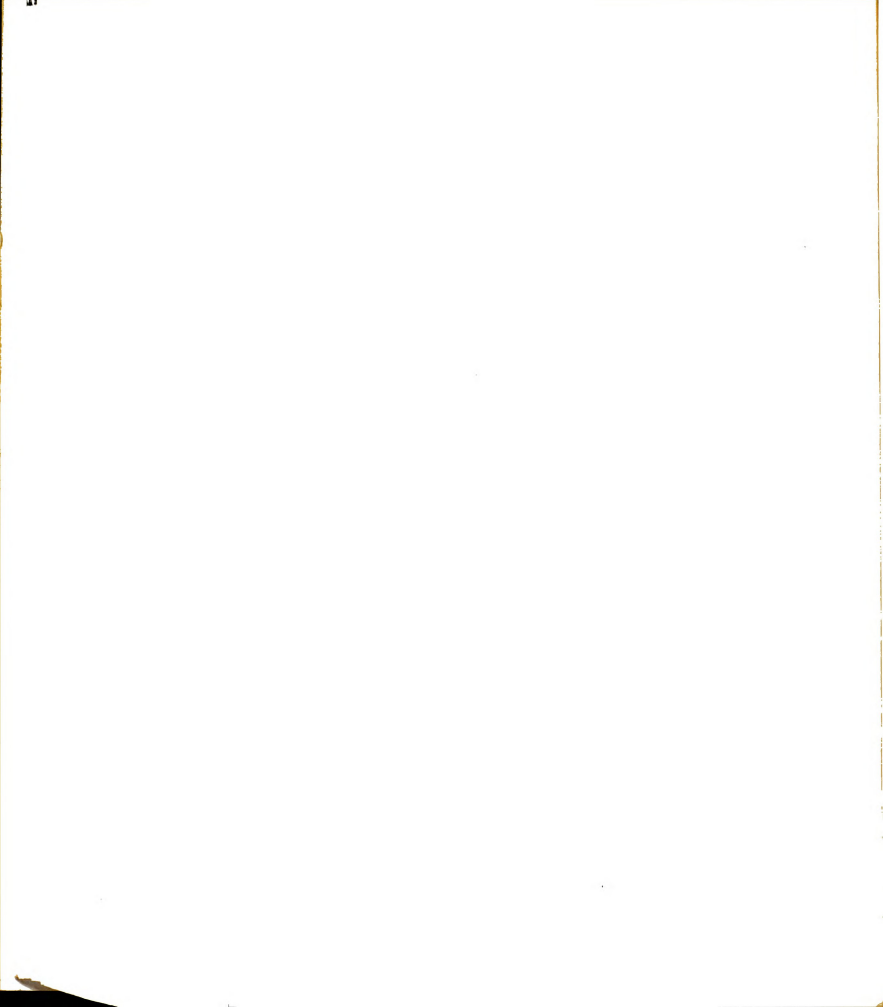
	Test Value	
	238	239
Probability of Type I Error	.0359	.0618
Opportunity Cost of Type I Error	\$4	\$4
Expected Opportunity Cost of Type I Error	<u>\$.1436</u>	<u>\$.2472</u>
Probability of Type II Error	.0375	.0228
Opportunity Cost of Type II Error	\$3.8975	\$3.8195
Expected Opportunity Cost of Type II Error	<u>\$.1462</u>	<u>\$.0870</u>
Decision	Reject	Accept

The probability of committing a Type II error is depicted by the area under the normal curve with mean 230 and standard error of the mean 4.5056²² which is greater than the test value. This is, of course, because the hypothesis will be accepted for a sample mean larger than the test value chosen as the lower control limit.²³ If, however, improvement has resulted a false hypothesis (Type II error) will have been accepted.

The opportunity cost of a Type II error results from weighting the \$.75 single performance opportunity cost

²²The mean of the 100 performances (Enumerated in Table 18) which are due to improvement is 230 and the standard error of the mean is 4.5056.

²³This holds only so long as the sample mean does not exceed the upper control limit.



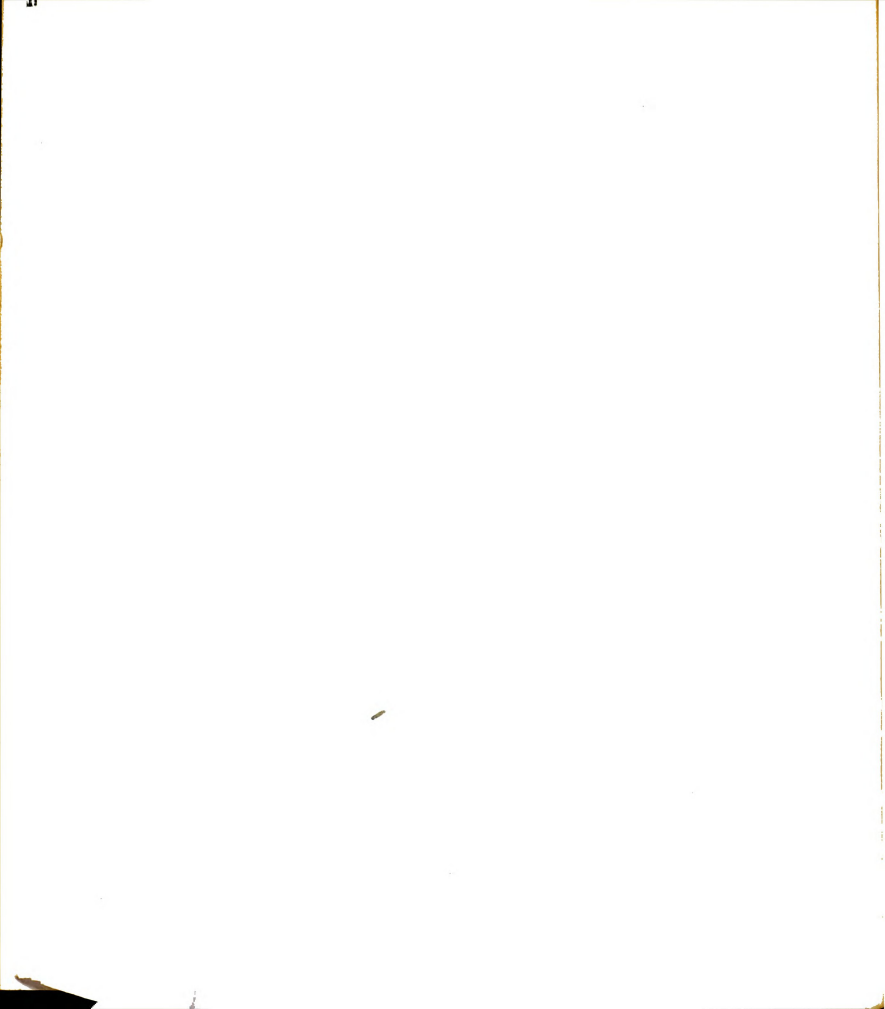
in the usual manner and then multiplying this by the sample size of 5.

Minimization Approach

Because the expected opportunity cost is less for test value 236 than for any other value indicated on Table 59, 236 is designated as the lower control limit.

TABLE 59.--Decision table for Minimization approach

Cause	Prior Prob.	Weighted Op. Cost	Prob. of Wrong Decision	Cond. Ave. Op. Cost	Expected Op. Cost
Test Value 234					
Chance	.8571	\$4	.0023	\$.0092	
Improvement	<u>.1429</u>	4.6035	.1867	.8595	
	1.0000				<u>\$.1307</u>
Test Value 235					
Chance	.8571	\$4	.0051	\$.0204	
Improvement	<u>.1429</u>	4.3215	.1335	.5769	
	1.0000				<u>\$.0999</u>
Test Value 236					
Chance	.8571	\$4	.0104	\$.0416	
Improvement	<u>.1429</u>	4.1345	.0918	.3795	
	1.0000				<u>\$.0899</u>
Test Value 237					
Chance	.8571	\$4	.0197	\$.0788	
Improvement	<u>.1429</u>	3.9845	.0606	.2415	
	1.0000				<u>\$.1020</u>

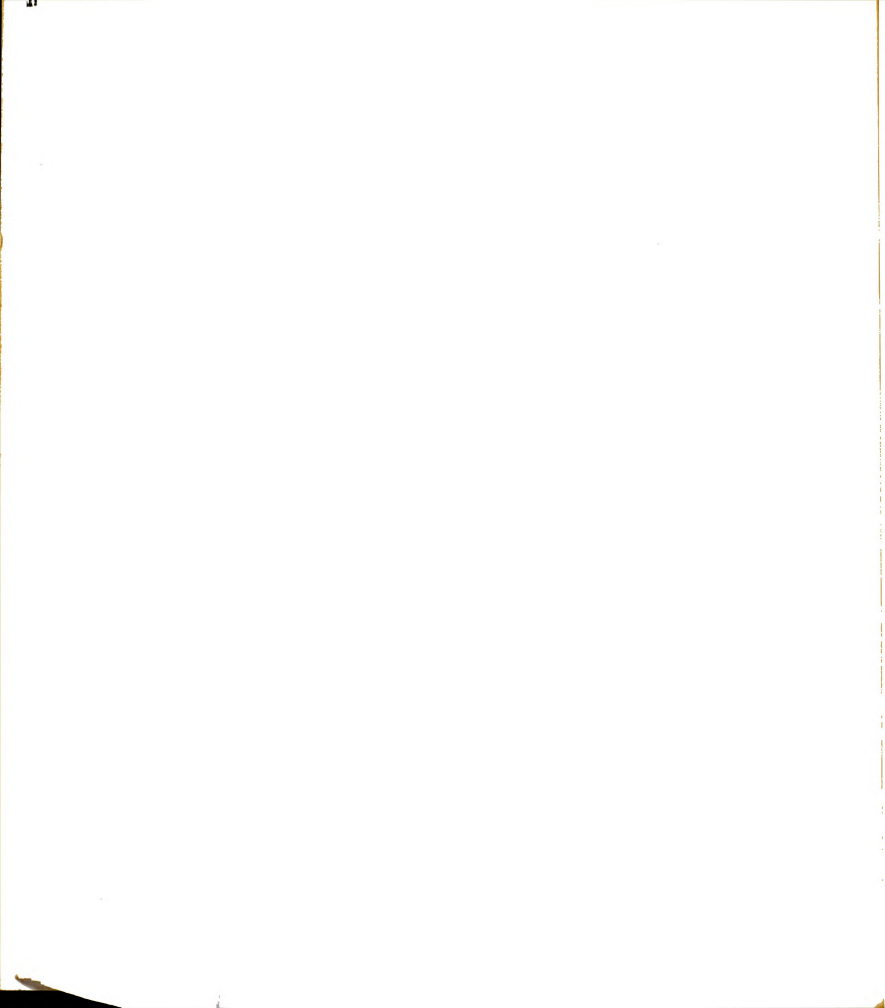


Chance and improvement are the only two causes of interest in setting the lower control limit. In the original distribution of 1000 values, 600 were due to chance and 100 to improvement. Therefore, the prior probabilities of .8571 and .1429 for chance and improvement respectively are determined from the ratios 600/700 and 100/700.

The \$4 weighted opportunity cost associated with the chance cause is the cost of committing a Type I error by investigating for improvement when in fact chance alone is cause the variation in the performances. The weighted opportunity costs associated with improvement result from:

1. Weighting the \$.75 single performance opportunity cost by the procedure indicated in Table 10 to account for the fact that improvement will not always be detected on the first test after its occurrence.
2. Multiplying this weighted value by 5 - the sample size.

The probabilities of a wrong decision corresponding to chance represent the probability of committing a Type I error. Accordingly, the figures are calculated by finding the area less than the test value under the normal curve with mean 245 and standard error of the mean 3.8923. In other words, if chance is the only prevailing cause of variation, the hypothesis will be falsely rejected if the sample mean is less than the test value that is selected as the lower control limit.



On the other hand, the probabilities of a wrong decision corresponding to improvement represent the probability of committing a Type II error. These probabilities are depicted by the area greater than the test value under the normal curve with mean 230 and standard error of the mean 4.5056.²⁴ This is because a sample mean greater than the test value selected as the lower control limit will lead to acceptance of the hypothesis which is a wrong conclusion (Type II error) if improvement has occurred.

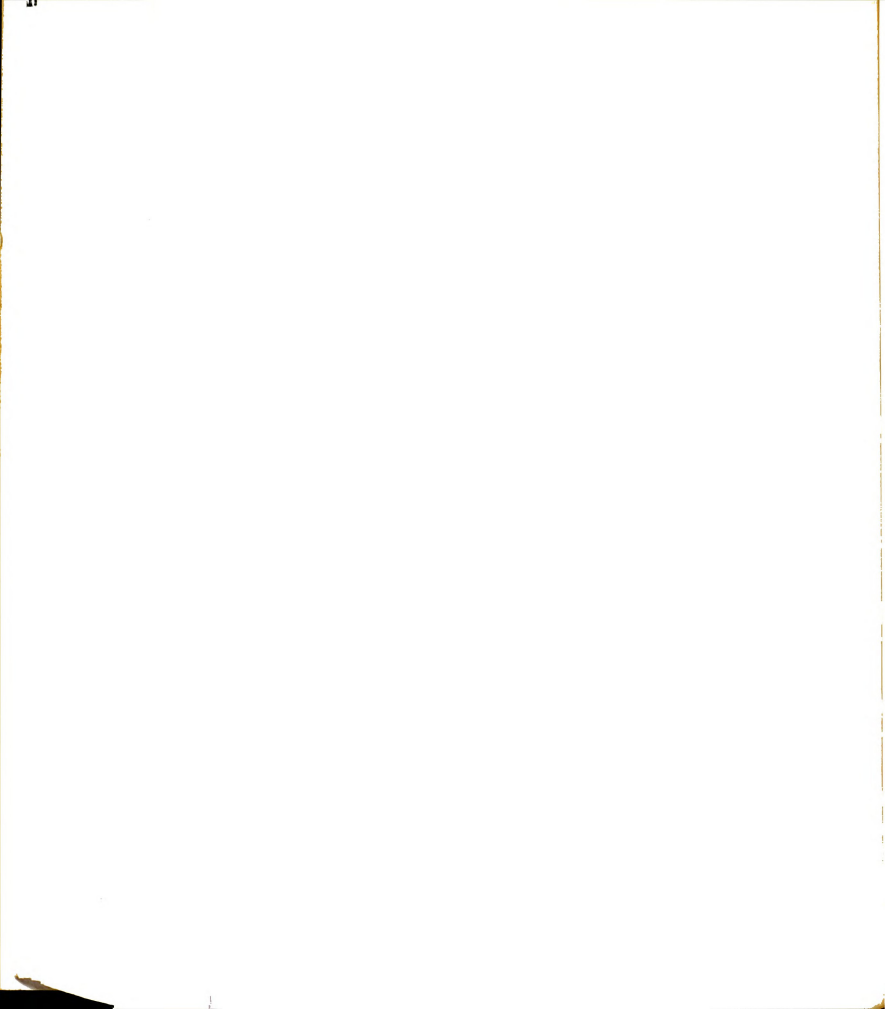
The figures in the last two columns in Table 59 are determined in the usual manner for the Minimization approach.

Comparison of Lower Control Limits
Among the Methods

For review, the control limits that have just been derived are listed below:

<u>Approach</u>	<u>Lower Control Limit</u>
Accountant's Conventional	220
Basic Control Chart	234.2
Bierman, Fouraker, and Jaedicke	
First Interpretation of P	239
Second Interpretation of P	236.07
McMenimen	236.97
Equalization	238.02
Minimization	236

²⁴As the reader will recall the mean of the 100 performances listed in Table 18 as being due to improvement is 230 and the standard error of the mean is 4.5056.

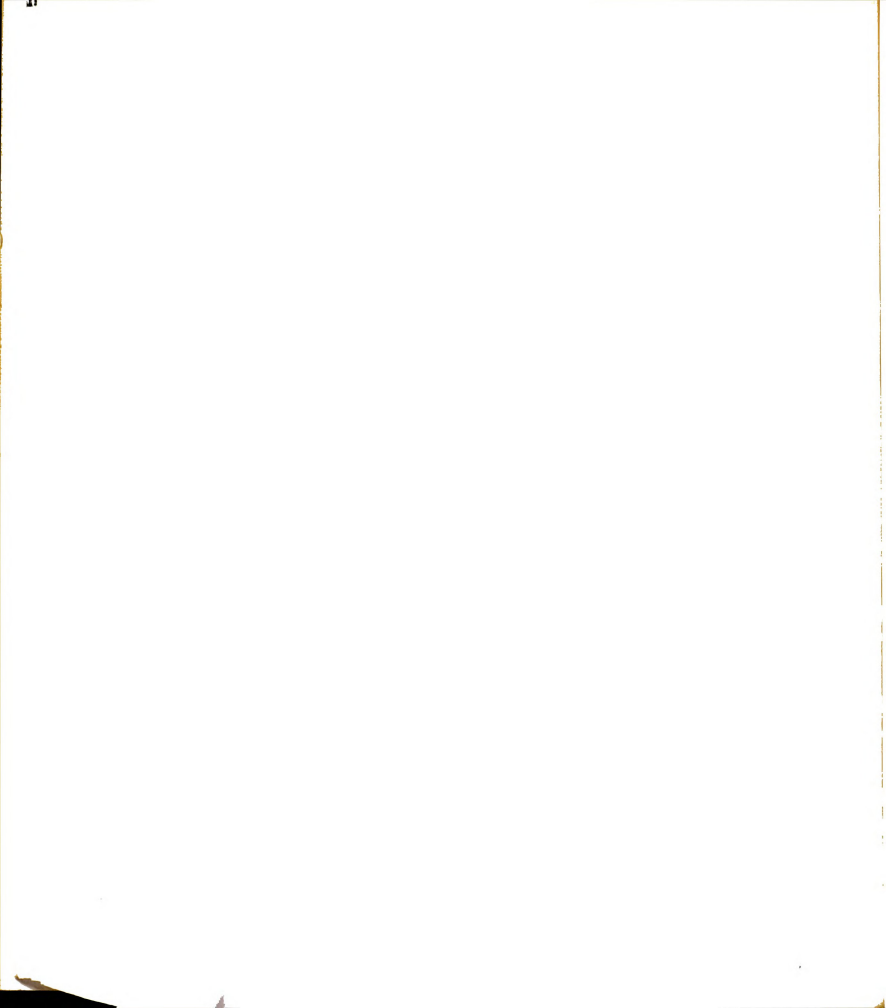


Financial Analysis and Ranking

The procedure for estimating the number of sample means per thousand samples falling within a specified range of values for a specified assignable cause was discussed in analyzing the impact of differences in upper control limits under this sampling plan. In analyzing the impact of the differences in lower control limits the same procedure is followed. These probabilities are used as they were in Table 52 in calculating the added savings associated with the control limit closer to the standard. The probabilities are also used in calculating the added investigation costs associated with the higher of the two lower control limits being compared at any given time. A summary of the added investigation costs and the added savings between the approaches is presented in Table 60.

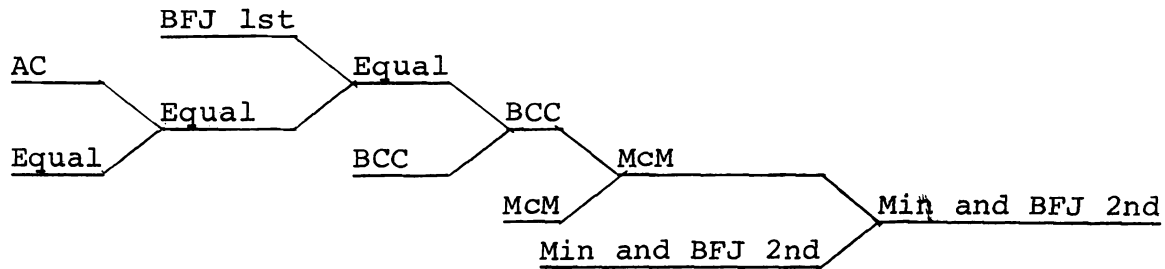
TABLE 60.--Financial comparisons between approaches

Approaches Tested		Added Inv. Cost of Higher LCL	Added Savings of Higher LCL	Most Effective Approach
Equal	AC	\$467.80	\$1959.02	Equal
BFJ 1st	Equal	60.24	5.73	Equal
Equal	BCC	136.84	63.85	BCC
McM	BCC	40.56	53.22	McM
McM	Min and BFJ 2nd	22.32	12.90	Min and BFJ 2nd



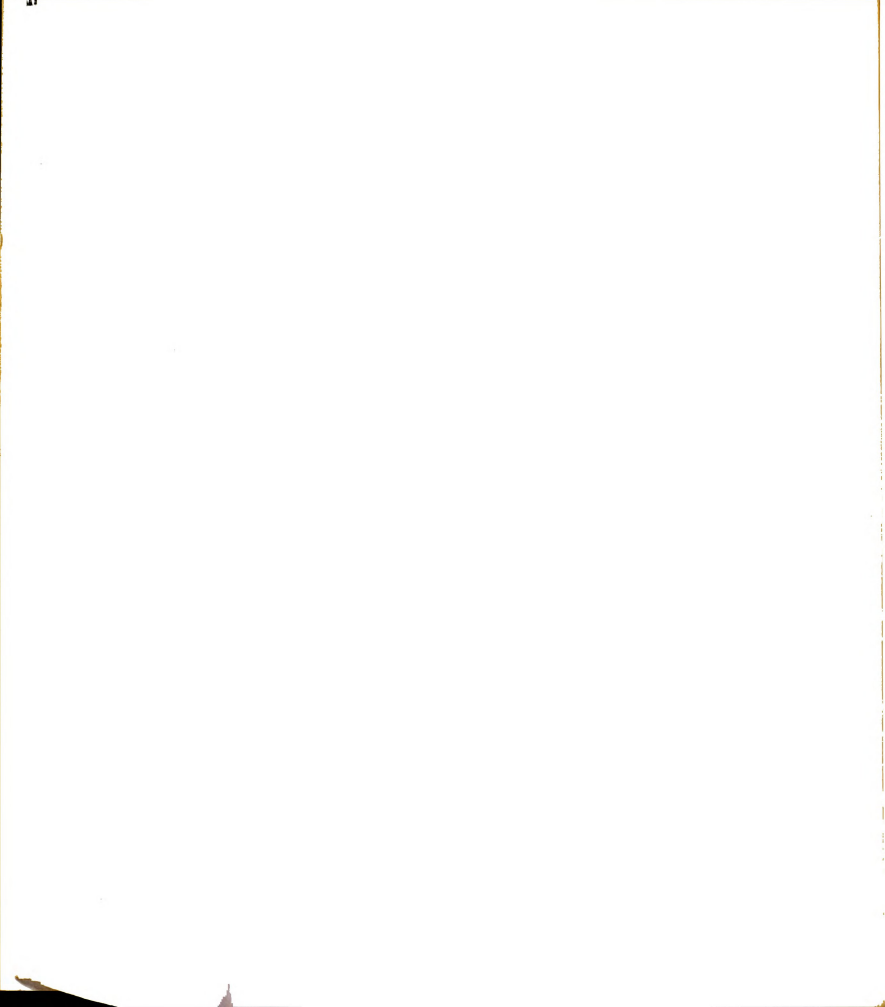
The results of the information presented in Table 60 are depicted in Figure 13.

FIGURE 13.--Outcomes of financial comparisons



From the above diagram, the following ranking emerges:

<u>Approach</u>	<u>Rank</u>
Min	1.5
BFJ 2nd	1.5
McM	3
BCC	4
Equal	5
BFJ 1st	6
AC	7



Derivation and Financial Analysis of
Upper Control Limits--Sample Size
Five--Sample Taken in Every
Fifty Performances

Introduction

In situations where sampling is used it is perhaps more common to take a sample every so often rather than to include every performance in a sample. Accordingly, control limits will now be calculated under the assumption that a sample of five is taken for every fifty performances.

Accountant's Conventional Method

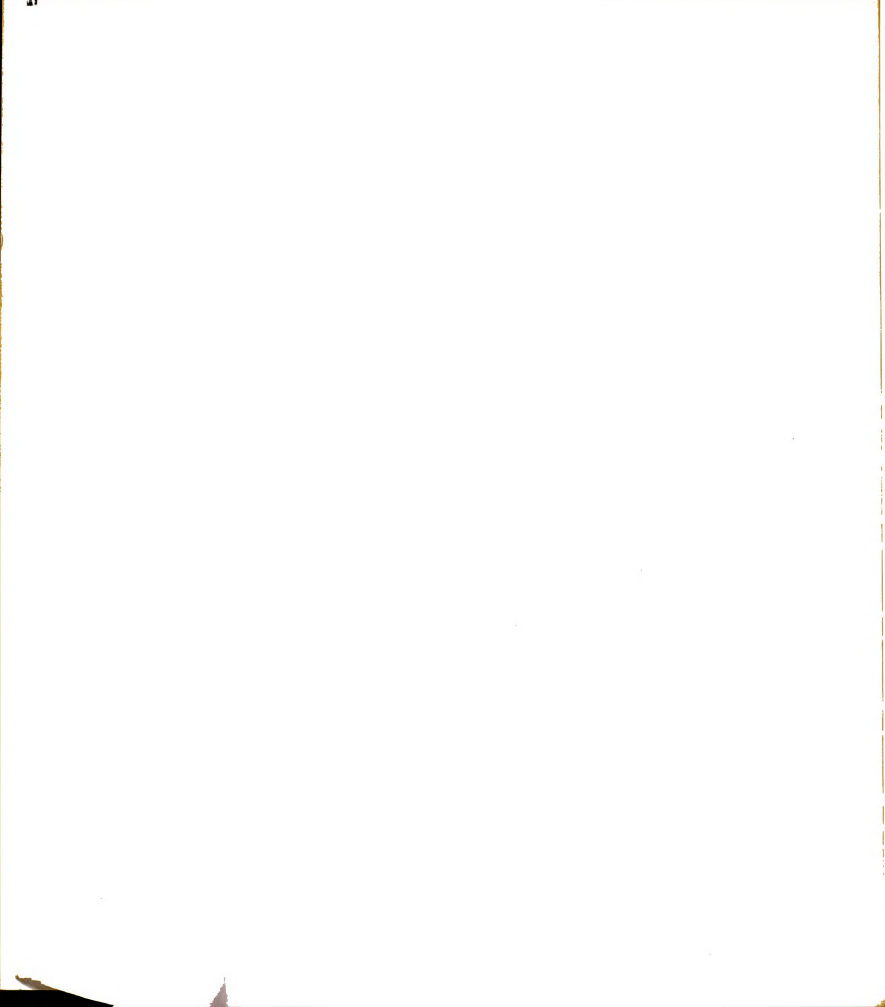
It has been noted before that the upper control limit remains at 270 regardless of the sampling plan.

Basic Control Chart Approach

The control limits resulting from this approach depend only upon the size of the sample and not upon the relative opportunity costs which vary with the frequency of sampling. Consequently, the upper limit is 255.8050 - the same as that determined when every performance was included in a sample.

Bierman, Fouraker and Jaedicke
Approach

First Interpretation of P. The upper control limit according to this sampling plan as indicated in Table 61 is between 246 and 247. The same interpolation procedure that has previously been applied for the Bierman, Fouraker,



and Jaedicke approaches establishes the control limit at 246.68.

Here "L" is the single performance opportunity cost multiplied by 200. The single performance opportunity cost is determined by the usual formula $\frac{X - 245}{60} \times \3 where X represents each respective test value. The multiplication weight of 200 results from multiplying the 4 tests that allegedly must be made on the average before an off-standard performance is detected by the 50 performances from which the 5 sample values for any single test are drawn. As with this approach for the other sampling plans, the cost of an investigation remains constant at \$5. "Pc" is determined by $\frac{L - C}{L}$. For this interpretation "P" is the probability of obtaining a sample mean at least as high as the test value given that the test value is unfavorable. These values are determined by the same procedure indicated in that section where every performance was included in a sample and illustrated in Table 47.

With Bierman, Fouraker, and Jaedicke approach, the decision rules, as the reader will recall, are made on the following basis:

1. If P is greater than Pc, accept the hypothesis and refrain from an investigation.
2. If P is less than Pc, reject the hypothesis that chance causes are prevailing and undertake an investigation.

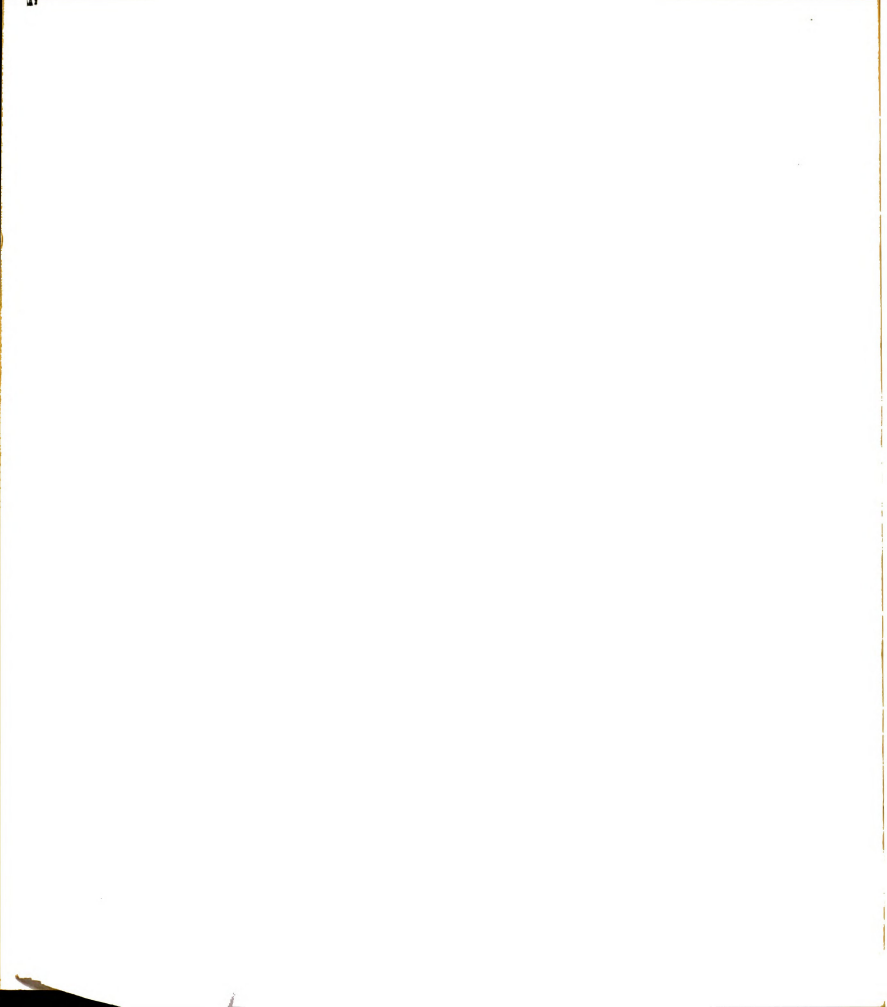


TABLE 61.--Decision table for BF and J application. First interpretation of P

Test Value	L	C	Pc	P	Decision
246	\$10	\$5	.50	.7948	Accept
247	20	5	.75	.6100	Reject
248	30	5	.83	.4412	Reject

Second Interpretation of P. The second interpretation of P yields an upper control limit between 250 and 251 as indicated in Table 62. This limit is further narrowed down to 250.13 by the process of interpolation which has been employed for the Bierman, Fouraker, and Jaedicke approaches.

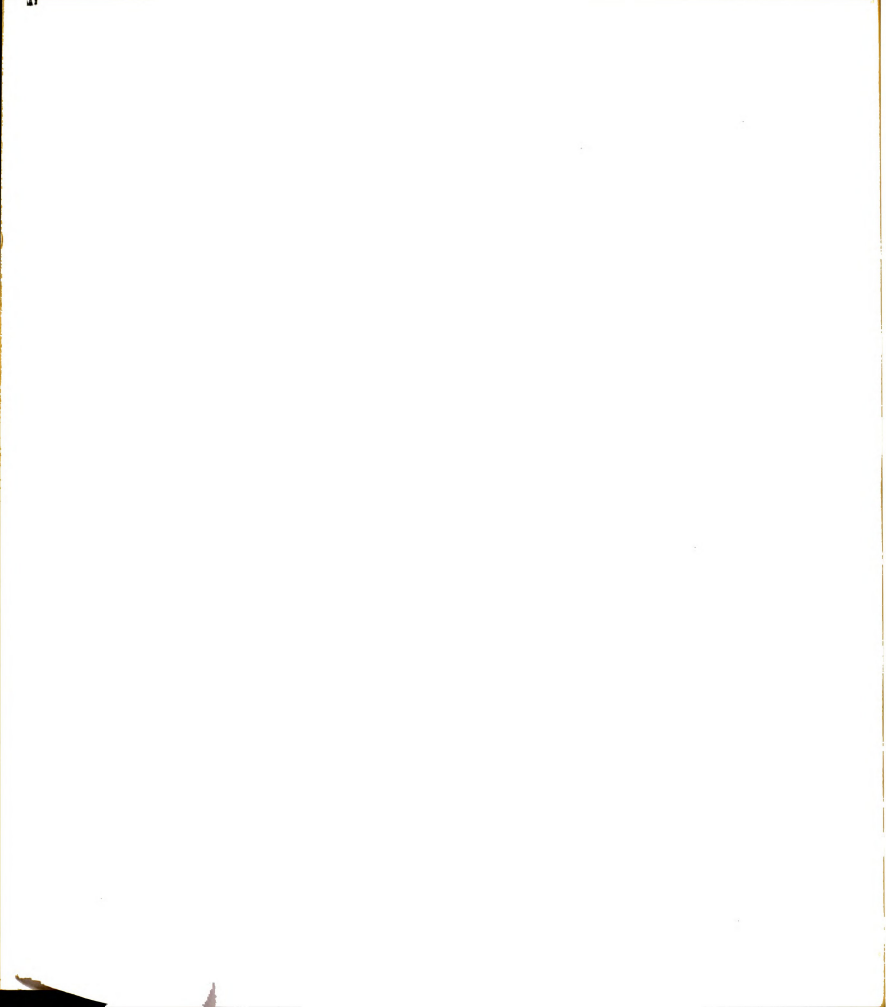
The values for P were obtained from Table 47. The other values are determined in the same manner used for the first interpretation of P.

TABLE 62.--Decision table for BF and J application. Second interpretation of P

Test Value	L	C	Pc	P	Decision
250	\$50	\$5	.90	.9119	Accept
251	60	5	.917	.8395	Reject

McMenimen Approach

The discussion of this approach where every performance was included in a sample size of five indicated that one would not logically investigate for dull knives until a sample mean at least as high as 259 was obtained. The



reasoning for this is that the probability of obtaining a sample mean less than 259 with the use of dull knives is almost zero. The same reasoning is equally valid for this sampling plan.

It is, however, profitable to begin an investigation for poor attitude with a sample mean of 247²⁵ as Table 63 indicates. The savings value for poor attitude is determined by multiplying the individual performance opportunity cost by the multiplication weight of 200 and subtracting the \$.25 cost of correction. The numerical values are $\frac{255 - 245}{60} \times \$3 \times 200 - \$.25 = \99.75 . The probabilities are determined in Table 64 which follows the same procedure as Table 56.

TABLE 63.--Application of McMenimen technique

Event	Spend Up To \$1 Investigating For Poor Attitude		
	Pe	Cond.	Exp.
Test Value 246			
Save \$0	.9920	\$-1	\$- .9920
Save \$99.75	.0080	98.75	+ .7900
Expected Savings			\$-0.2020
Test Value 247			
Save \$0	.9852	\$-1	\$- .9852
Save \$99.75	.0148	98.75	+1.4615
Expected Savings			\$+0.4763

²⁵Actually, the process of interpolation that was previously applied for the McMenimen technique yields an upper control limit of 246.30 as far as the investigation for poor attitude is concerned.

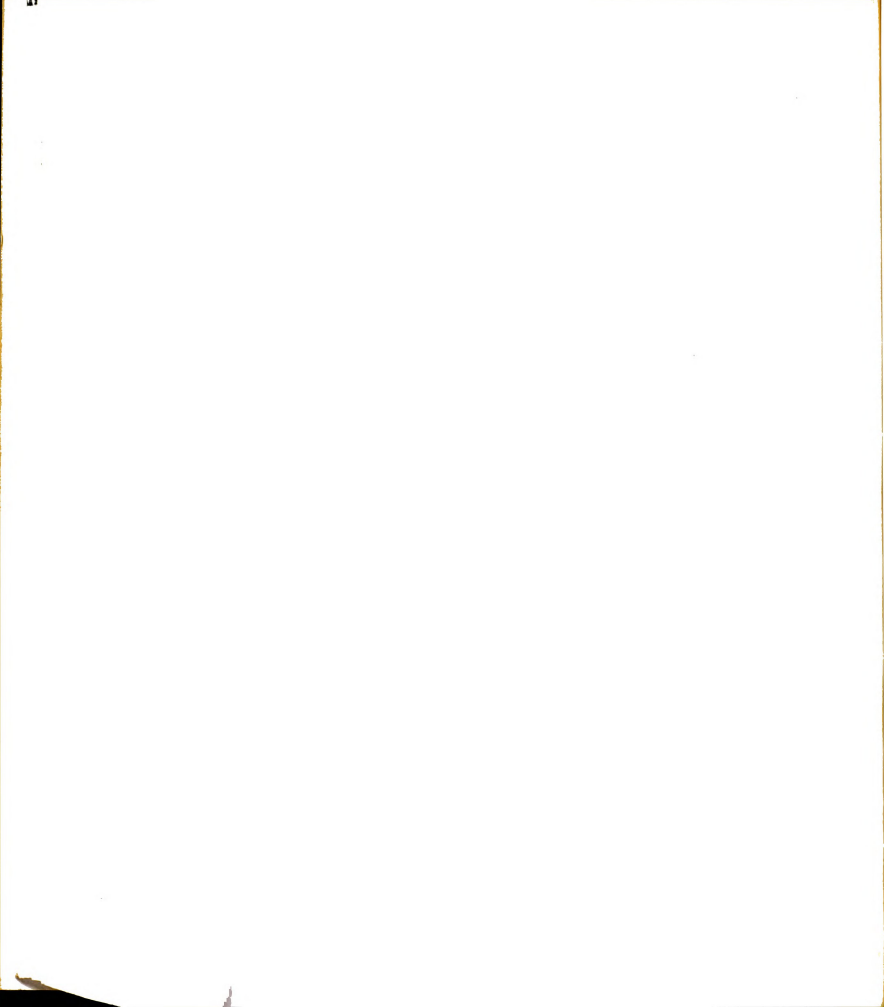


TABLE 64.--Determination of P's

Cause	Cond. Prob.	Number of Perf.	Number of Means	Prob. of Cause Given Occurrence of Test Value
Test Value 246				
Chance	.0987	600	59.2200	.9920
Poor Attitude	.0079	60	<u>00.4740</u>	<u>.0080</u>
			59.6940	1.0000
Test Value 247				
Chance	.0909	600	54.5400	.9852
Poor Attitude	.0137	60	<u>00.8820</u>	<u>.0148</u>
			55.3620	1.0000

Equalization Approach

Table 65 shows the upper control limit to be between 248 and 249. This limit is further narrowed down to 248.03 by the same interpolation procedure previously applied for the Equalization approach.

The individual figures that compose this table were derived in the same general manner as those for Table 50. This latter table was used in setting the upper control limit where a sample of five was chosen so that every performance was included in a sample. The derivation of these individual figures is explained in Appendix B.

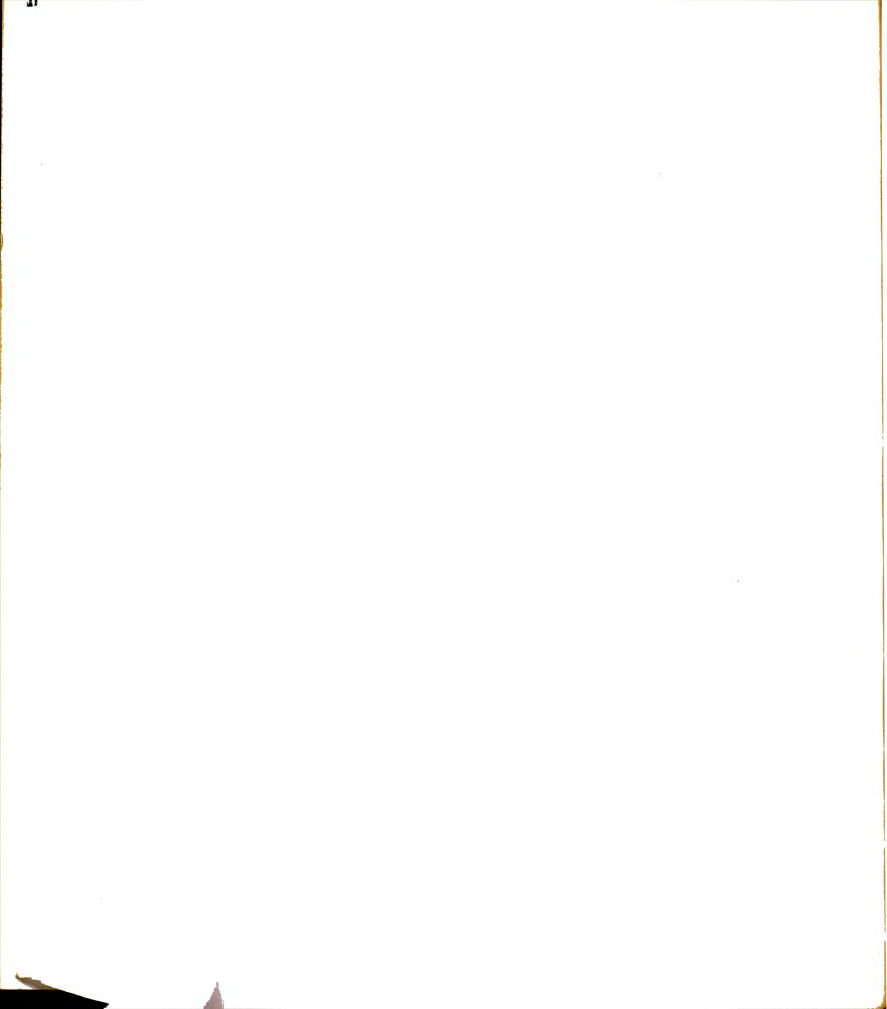


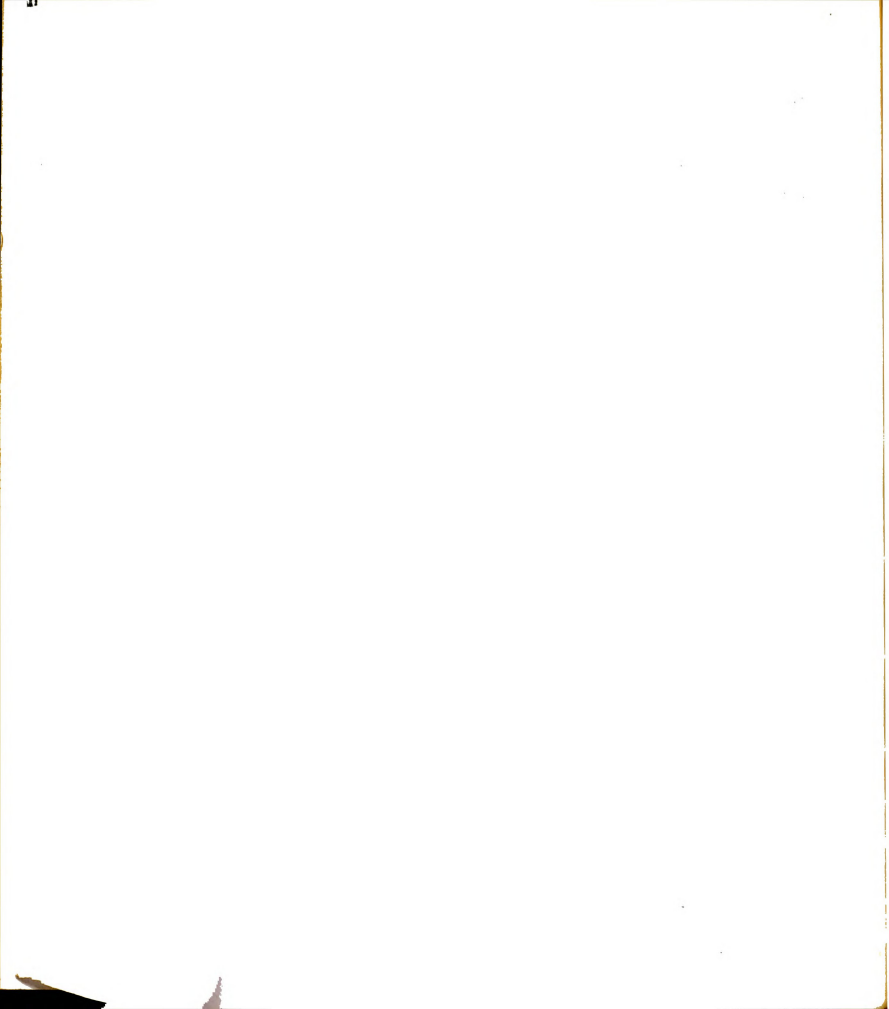
TABLE 65.--Decision table for Equalization approach

	Test Value	
	249	248
Probability of Type I Error	.1292	.2206
Opportunity Cost of Type I Error	\$ 1	\$ 1
Expected Opportunity Cost of Type I Error	<u>\$.1292</u>	<u>\$.2206</u>
Probability of Type II Error	.0136	.0082
Opportunity Cost of Type II Error	\$26.78	\$26.11
Expected Opportunity Cost of Type II Error	<u>\$.3642</u>	<u>\$.2141</u>
Decision	Reject	Accept

Minimization Approach

The upper control limit under this approach is 249 as shown in Table 66. With this test value the expected opportunity cost is less than for any other.

The prior probabilities are the same for each cause as they were in Table 51--the decision table used for the Minimization approach when every performance was included in a sample. Likewise, the opportunity cost for chance continues to be \$1 - the cost of committing a Type I error. Here, the weighted opportunity costs for poor attitude are calculated in the same manner as when every performance was included in a sample except that the results of weighting the single performance opportunity costs are multiplied by fifty instead of five. The result is that the weighted opportunity costs for poor attitude in Table 66 are ten



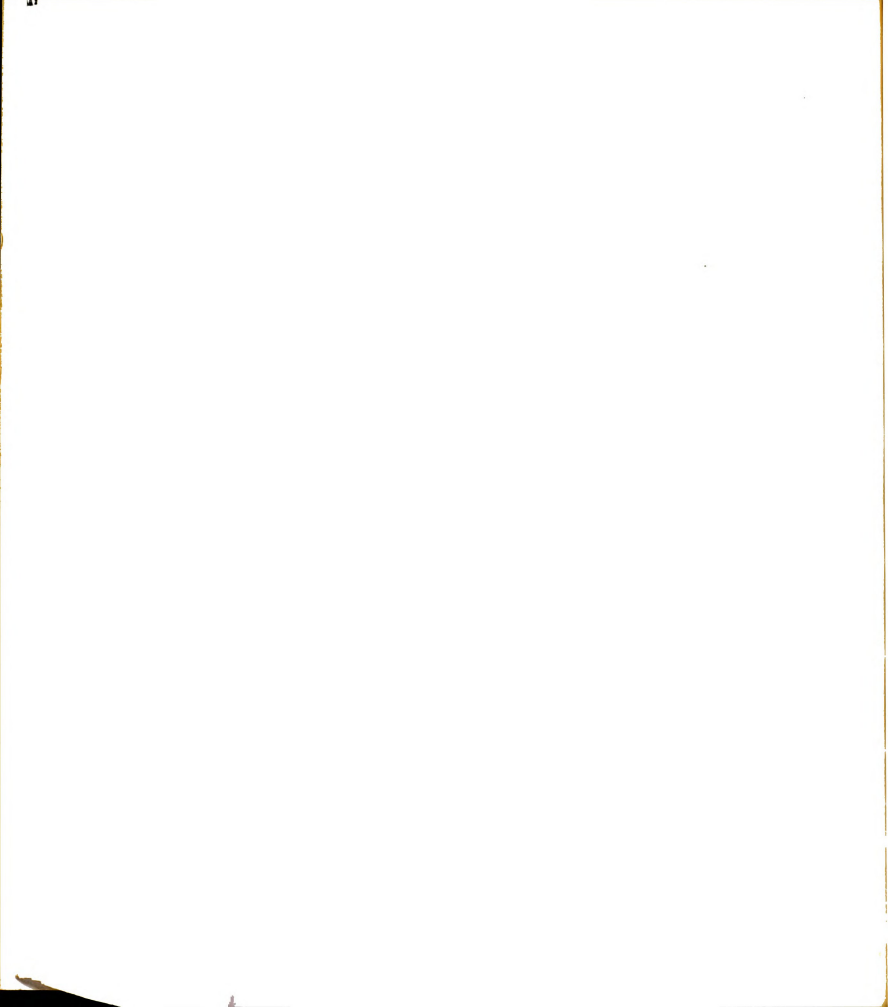
times more than those shown in Table 51 for each respective test value.

The probabilities of a wrong decision are the same for each respective test value and cause as those used in Table 51.

The figures in the last two columns of Table 66 are derived in the manner previously explained for the Minimization approach.

TABLE 66.--Decision table for Minimization approach

Cause	Prior Prob.	Weighted Op. Cost	Prob. of Wrong Decision	Cond. Ave. Op. Cost	Expected Op. Cost
Test Value 250					
Chance	.9091	\$ 1	.1003	.1003	
Poor Attitude	.0909	27.990	.1075	\$3.0089	
					\$.3647
Test Value 249					
Chance	.9091	\$ 1	.1292	\$.1292	
Poor Attitude	.0909	26.7800	.0681	1.8237	
					\$.2832
Test Value 248					
Chance	.9091	\$ 1	.2206	\$.2206	
Poor Attitude	.0909	26.1100	.0409	1.0679	
					\$.2976



Comparison of Upper Control
Limits Among the Methods

The control limits that have just been derived are itemized below:

<u>Approach</u>	<u>Upper Control Limit</u>
Accountant's Conventional	270
Basic Control Chart	255.8
Bierman, Fouraker, and Jaedicke	
First Interpretation of P	246.68
Second Interpretation of P	250.13
McMenimen	246.30
Equalization	248.03
Minimization	249

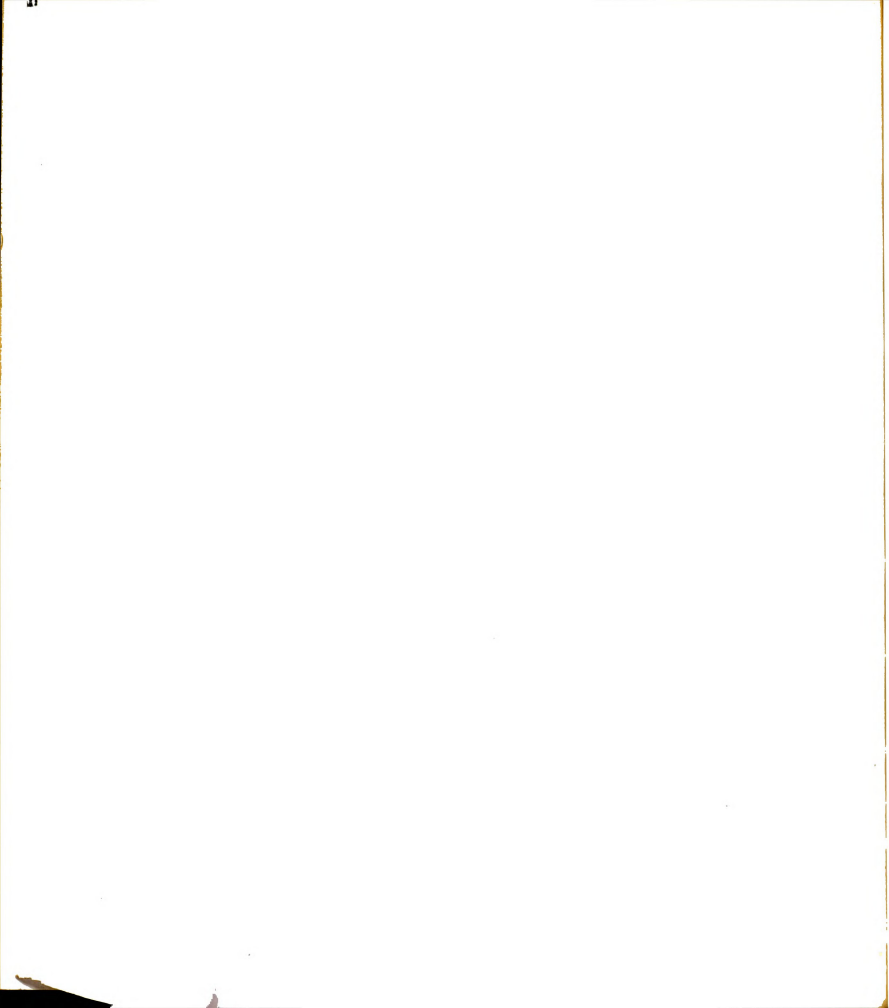
Financial Analysis and Ranking

The results of the comparisons between the methods are summarized in Table 67.

TABLE 67.--Financial comparisons between approaches

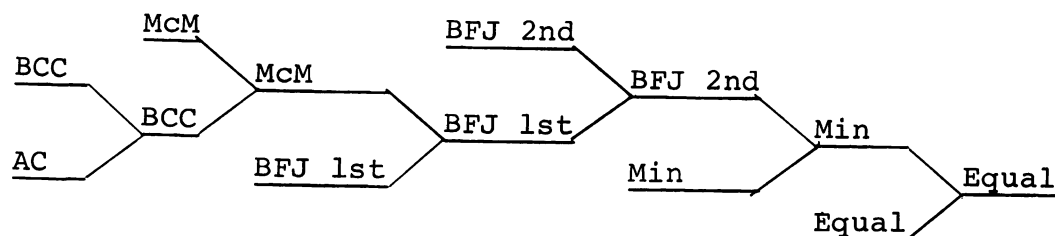
<u>Approaches Tested</u>		<u>Added Inv. Cost of Lower UCL</u>	<u>Added Savings^a of Lower UCL</u>	<u>Most Effective Approach</u>
AC	BCC	\$280.55	\$24,821.14	BCC
BCC	McM	232.08	1,780.05	McM
McM	BFJ 1st	22.26	5.65	BFJ 1st
BFJ 1st	BFJ 2nd	720.60	157.70	BFJ 2nd
BFJ 2nd	Min	34.86	75.57	Min
Min	Equal	39.72	42.26	Equal

^aThese savings figures are determined by the same general procedure indicated in Table 52. Now, however, the weighted opportunity costs result from the product of the weighted single performance opportunity costs for the appropriate test value and fifty.



These comparisons are depicted diagrammatically in Figure 14.

FIGURE 14.--Outcomes of financial comparisons



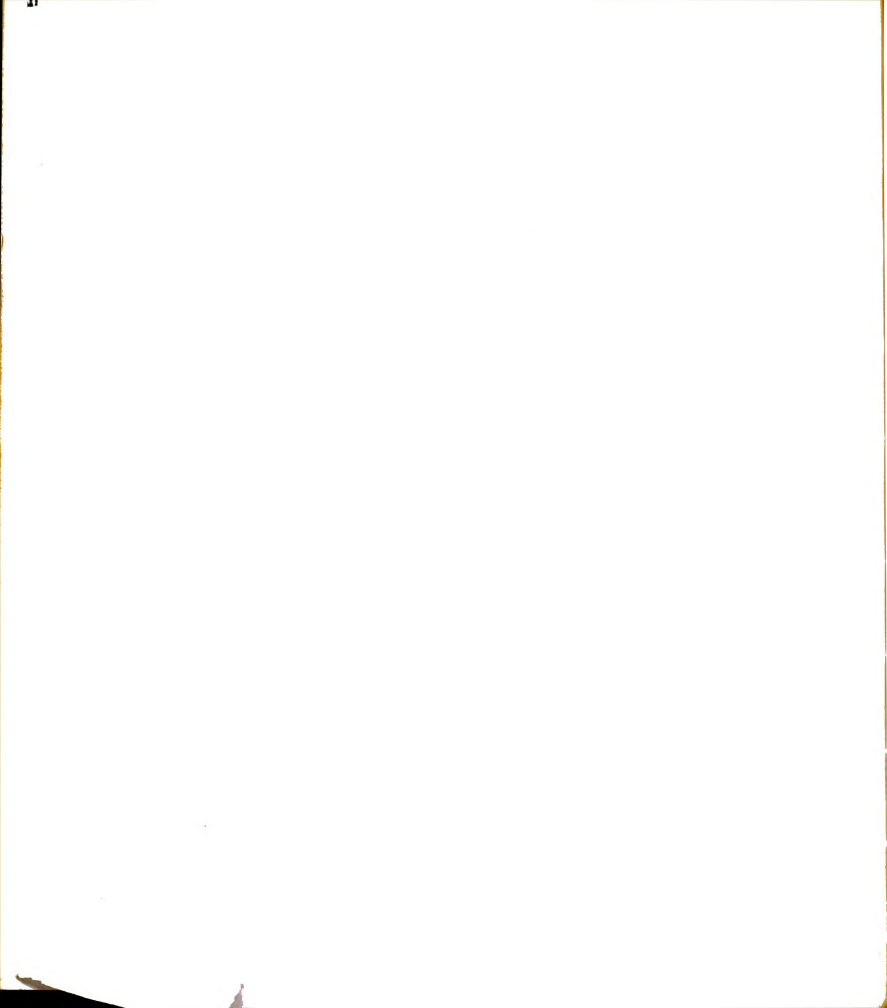
As a result of these comparisons, the following ranking emerges:

<u>Approach</u>	<u>Rank</u>
Equal	1
Min	2
BFJ 2nd	3
BFJ 1st	4
McM	5
BCC	6
AC	7

Derivation and Financial Analysis of
Lower Control Limits--Sample Size
Five--Sample Taken in Every
Fifty Performances

Accountant's Conventional Method

The lower control limit under the Accountant's Conventional method is 220 with the application of the



"10 per cent rule." It has repeatedly been noted that this does not change with the sampling plan.

Basic Control Chart Approach

Under this approach the lower limit is 234.2 - the same as the limit when every performance was included in a sample of five. The limit remains the same because it depends only upon the size of the sample and not upon the frequency of sampling.

Bierman, Fouraker, and Jaedicke Approach

First Interpretation of P. Table 68 indicates the lower control limit to be between 243 and 244. The interpolation procedure that has been used for both Bierman, Fouraker, and Jaedicke approaches narrows the control limit down to 243.49.

Because of the symmetry of the normal curve, "L" and "P" for 244, 243, and 242 are the same respectively as they were for 246, 247, and 248. These latter calculations were discussed in conjunction with the derivation of the upper control limit under the present sampling plan.

The cost of an investigation, "C", for improvement has previously been determined to be \$4. "Pc" continues to be found by $\frac{L - C}{L}$.

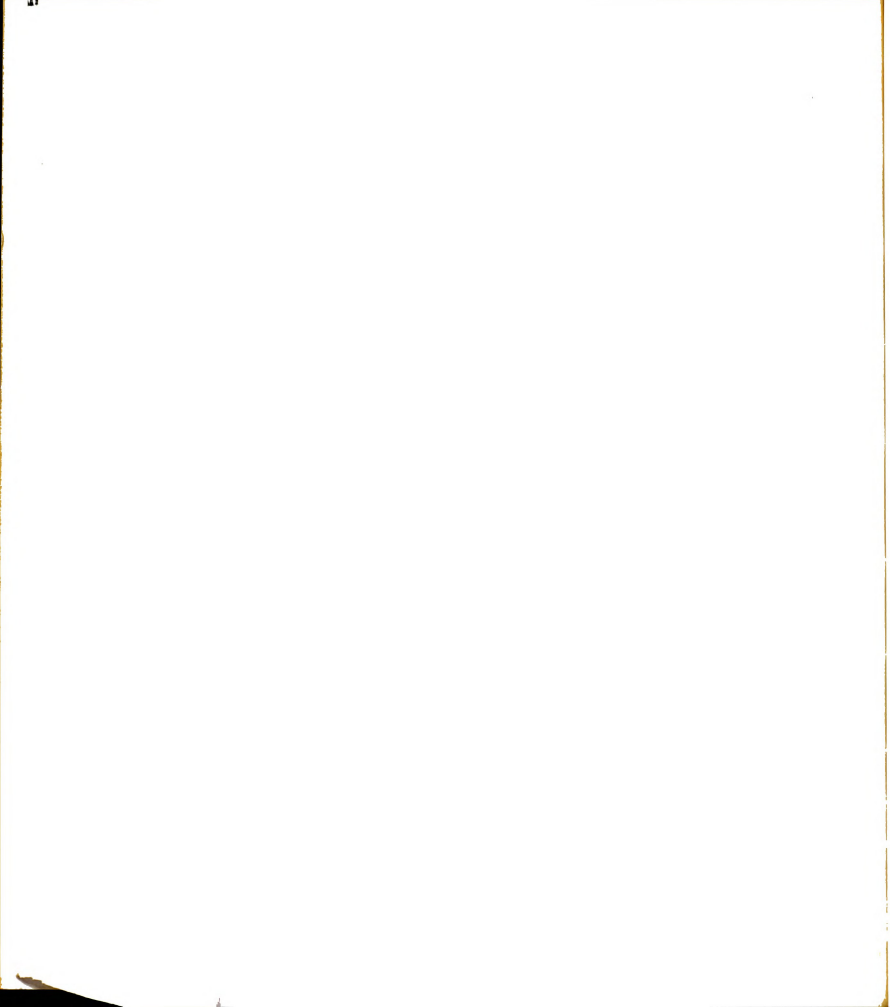


TABLE 68.--Decision table for BF and J application. First interpretation of P

Test Value	L	C	Pc	P	Decision
242	\$30	\$4	.8667	.4412	Reject
243	20	4	.8000	.6100	Reject
244	10	4	.6000	.7948	Accept

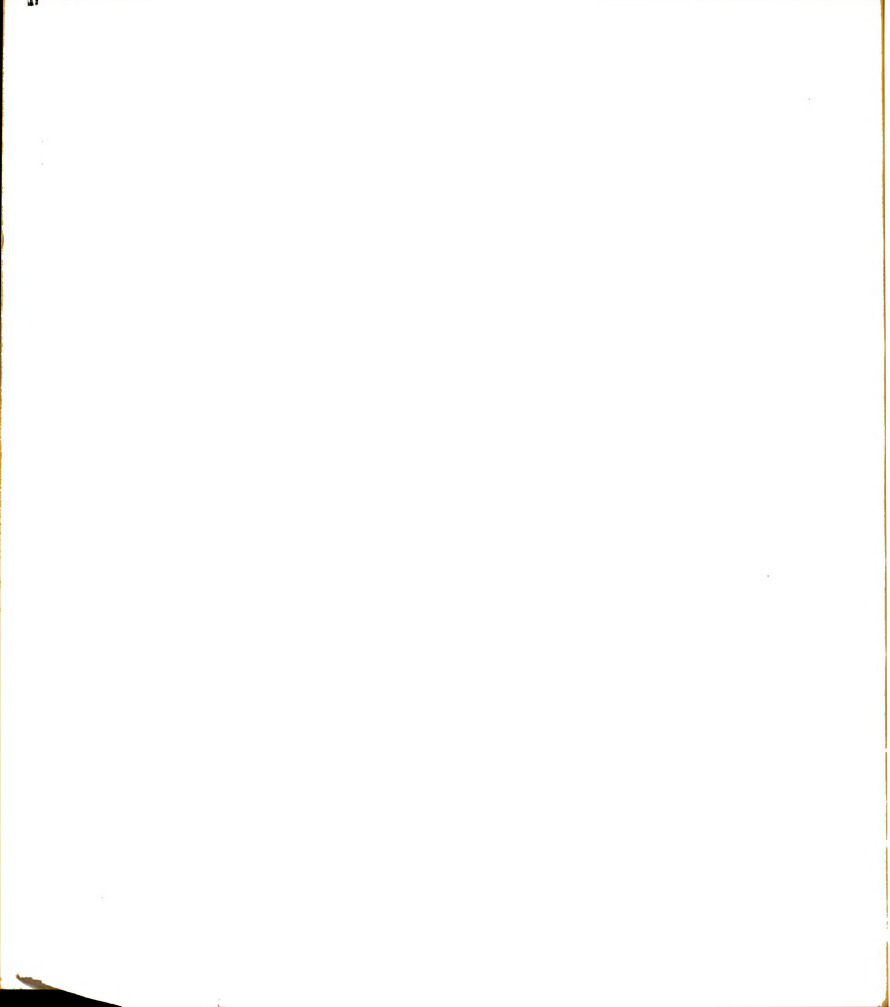
Second Interpretation of P. Decision Table 69 shows the control limit to be between 238 and 239 when this approach is followed. Interpolation yields a limit of 238.95. In this table, "L" and "P" are determined in their usual manner. (The multiplication weight in determining "L" is 200.) "C" is still a constant at \$4. The values for "P" were obtained from Table 56. The decision was made on the basis of the usual criteria.

TABLE 69.--Decision table for BF and J application. Second interpretation of P

Test Value	L	C	Pc	P	Decision
238	\$70	\$4	.9428	.8054	Reject
239	60	4	.9333	.9408	Accept

McMenimen Approach

The savings value is determined by multiplying the individual performance opportunity cost, $\frac{230 - 245}{60} \times \3 , by the multiplication weight, 200, and subtracting the individual performance cost of correction, \$.25. The



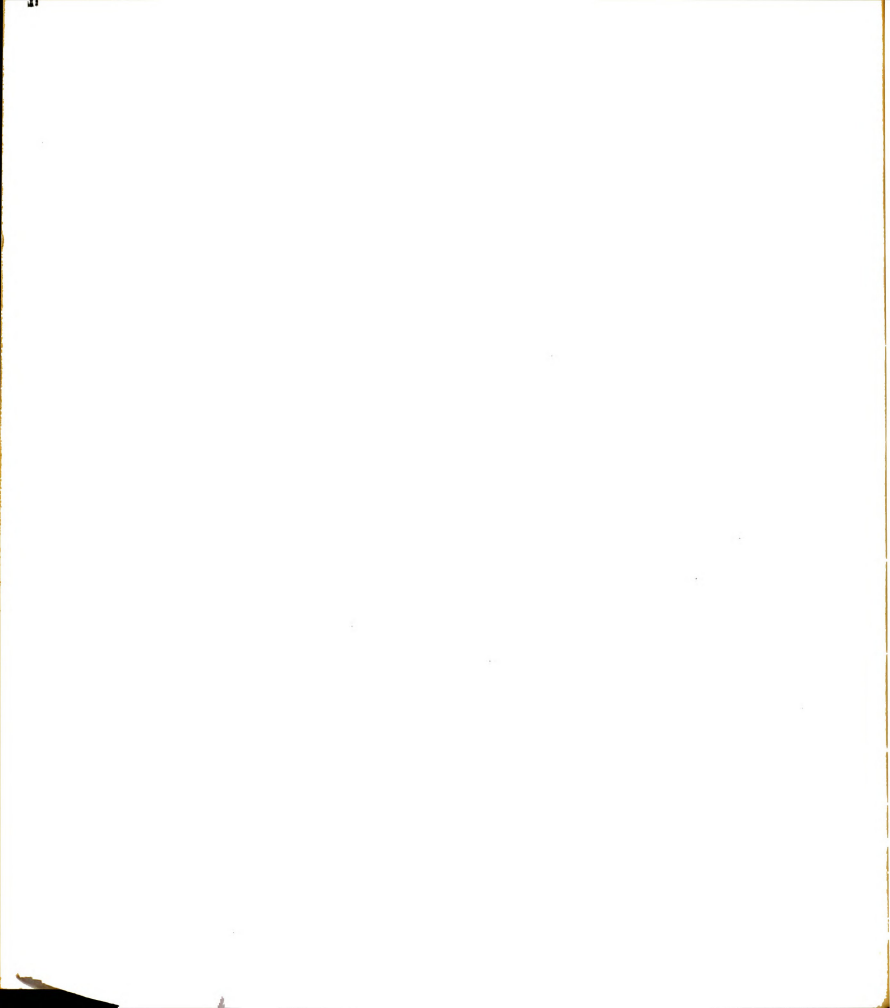
result is \$149.75. The probabilities used in this McMenimen application were obtained from Table 56. The lower control limit shown in Table 70 is between 240 and 241. The interpolation procedure previously employed for the McMenimen technique pinpoints the lower control limit at 240.07.

TABLE 70.--Application of McMenimen technique

Event	Spend Up To \$4 Investigating For Improvement		
	Pe	Cond.	Exp.
Test Value 239			
Save \$0	.9408	\$ -4	\$-3.7632
Save \$149.75	.0592	145.75	+8.6284
Expected Savings			\$+4.8652
Test Value 240			
Save \$0	.9722	\$ -4	\$-3.8888
Save \$149.75	.0278	145.75	+4.0518
Expected Savings			\$+ .1630
Test Value 241			
Save \$0	.9874	\$ -4	\$-3.9496
Save \$149.75	.0126	145.75	+1.8364
Expected Savings			\$-2.1132

Equalization Approach

Under this sampling plan, Table 71 shows the lower control limit to be between 240 and 241 for the Equalization approach. The interpolation procedure that has been



applied for the Equalization approach yields a limit of 240.30.

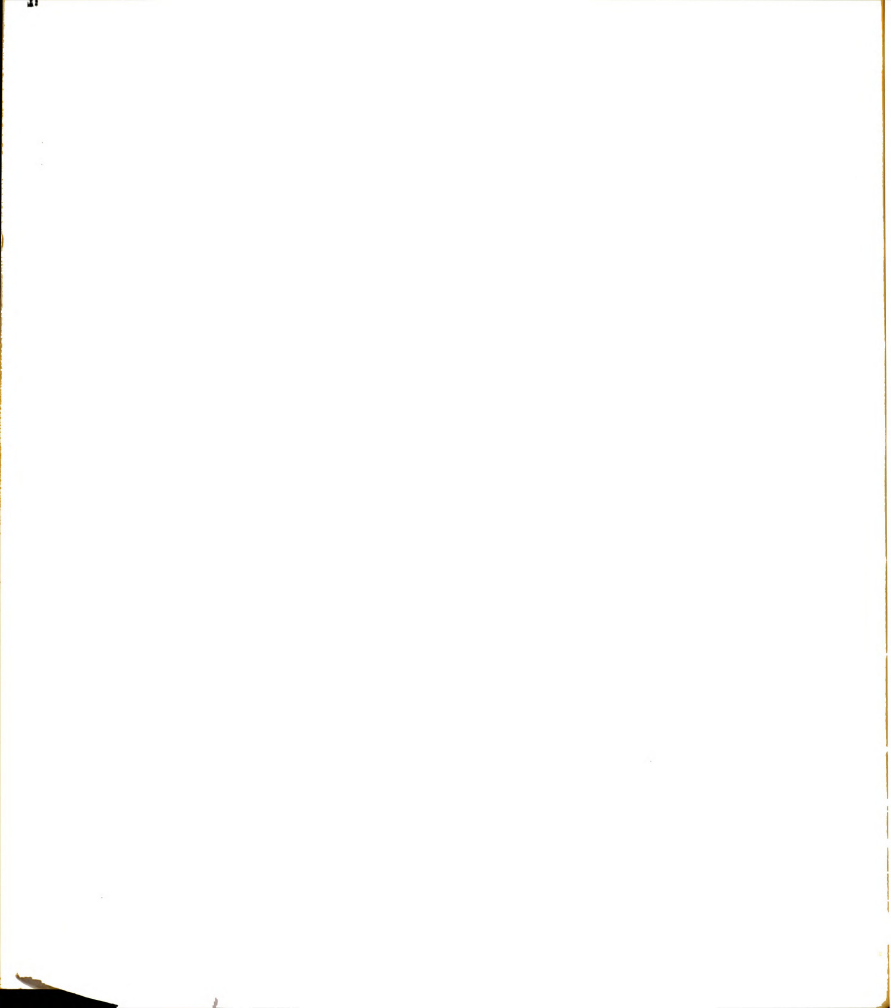
The individual values for this table were calculated in exactly the same manner as those for Table 58 with the following single exception. After weighting the single performance opportunity costs, they must be multiplied by fifty instead of five because a sample is drawn only once in every fifty performances. If improvement takes place immediately after a sample is drawn or if it is not discovered by any given test, the condition has no opportunity to be detected until another test is taken fifty performances later.

TABLE 71.--Decision table for Equalization approach

	Test Value	
	240	241
Probability of Type I Error	.1003	.1292
Opportunity Cost of Type I Error	\$ 4	\$ 4
Expected Opportunity Cost of Type I Error	<u>\$.4012</u>	<u>\$.5168</u>
Probability of Type II Error	.0132	.0073
Opportunity Cost of Type II Error	\$38.060	\$37.840
Expected Opportunity Cost of Type II Error	<u>\$.5024</u>	<u>\$.2762</u>
Decision	Reject	Accept

Minimization Approach

The expected opportunity cost is lowest in Table 72 for test value 238. Hence, 238 is designated as the lower control limit under this approach.



The prior probabilities are the same as those previously used in conjunction with setting the lower control limit under the Minimization approach.

For chance causes the weighted opportunity cost continues to be \$4 - the cost of a Type I error. For improvement these values represent the weighted single performance opportunity cost multiplied by fifty. Consequently, they are ten times higher for each respective test value than the weighted opportunity costs listed in Table 59.

The probabilities of a wrong decision are the same for each cause as they were for their respective values in Table 59.

Similarly, the figures in the last two columns are derived by the same procedure followed in the other Minimization models.

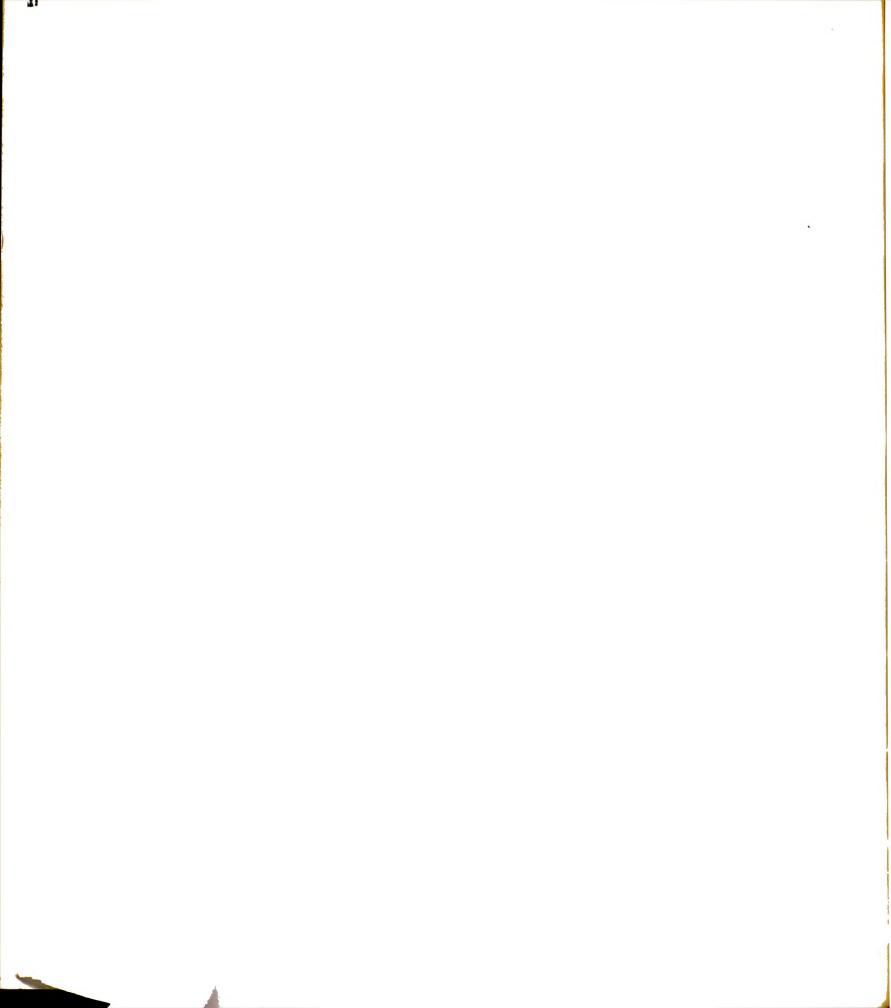
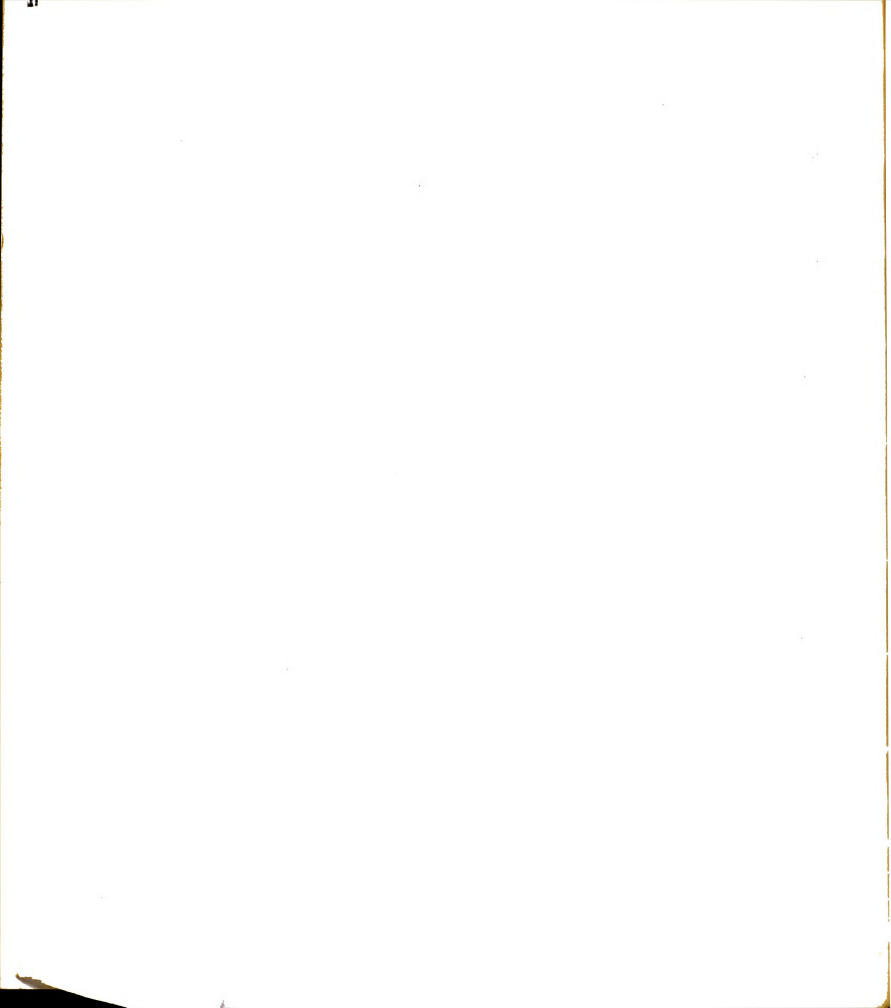


TABLE 72.--Decision table for Minimization approach

Cause	Prior Prob.	Weighted Op. Cost	Prob. of Wrong Decision	Cond. Ave. Op. Cost	Expected Op. Cost
Test Value 236					
Chance	.8571	\$ 4	.0104	.0416	
Improvement	.1429	41.345	.0918	3.7950	
					<u>\$.5780</u>
Test Value 237					
Chance	.8571	4	.0197	.0788	
Improvement	.1429	39.845	.0606	2.4150	
					<u>\$.4126</u>
Test Value 238					
Chance	.8571	4	.0359	.1436	
Improvement	.1429	38.975	.0375	1.4620	
					<u>\$.3320</u>
Test Value 239					
Chance	.8571	4	.0618	.2472	
Improvement	.1429	38.195	.0228	.8700	
					<u>\$.3362</u>
Test Value 240					
Chance	.8571	4	.1003	.4012	
Improvement	.1429	28.060	.0132	.5020	
					<u>\$.4156</u>

Comparison of Lower Control
Limits Among the Methods

For purposes of review the lower control limits pertaining to this sampling plan are indicated below:



<u>Approach</u>	<u>Lower Control Limit</u>
Accountant's Conventional	220
Basic Control Chart	234.2
Bierman, Fouraker, and Jaedicke	
First Interpretation of P	243.49
Second Interpretation of P	238.95
McMenimen	240.07
Equalization	240.30
Minimization	238

Financial Analysis and Ranking

In comparing the relative effectiveness of any two lower control limits, the higher of the two will carry a greater investigation cost; but will also bring about the detection of improvement sooner than the lower of the two. This more timely detection will bring about added savings. If the added savings is greater than the added investigation cost the higher of the two lower control limits is designated as more effective. Otherwise, the lower of the two is more effective.

A summary of the comparisons necessary to rank the approaches is presented in Table 73.

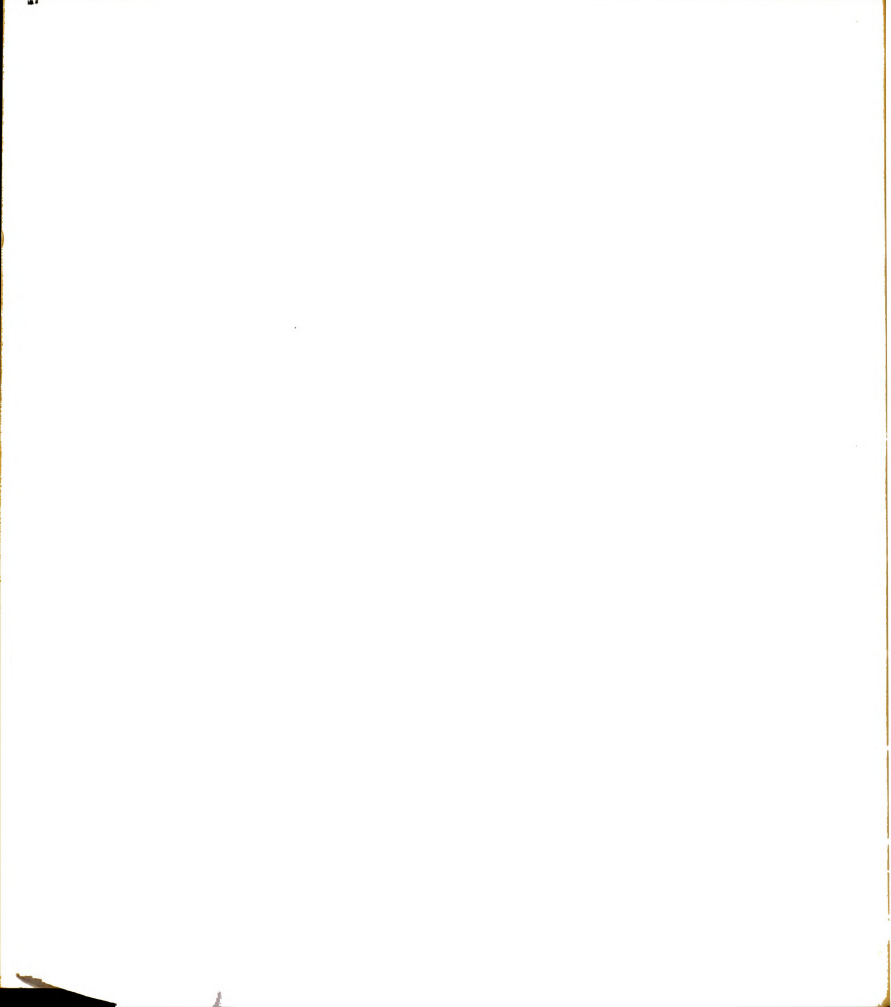
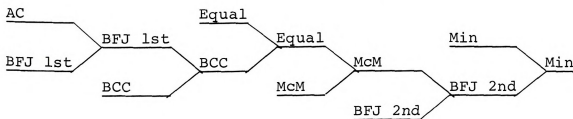


TABLE 73.--Financial comparisons between approaches

Approaches Tested		Added Inv. Cost of Higher LCL	Added Savings of Higher LCL	More Effective Approach
AC	BFJ 1st	\$835.93	\$20,364.10	BFJ 1st
BCC	BFJ 1st	829.20	811.14	BCC
BCC	Equal	264.72	760.50	Equal
Equal	McM	26.64	7.23	McM
McM	BFJ 2nd	99.36	39.99	BFJ 2nd
Min	BFJ 2nd	59.28	55.34	Min

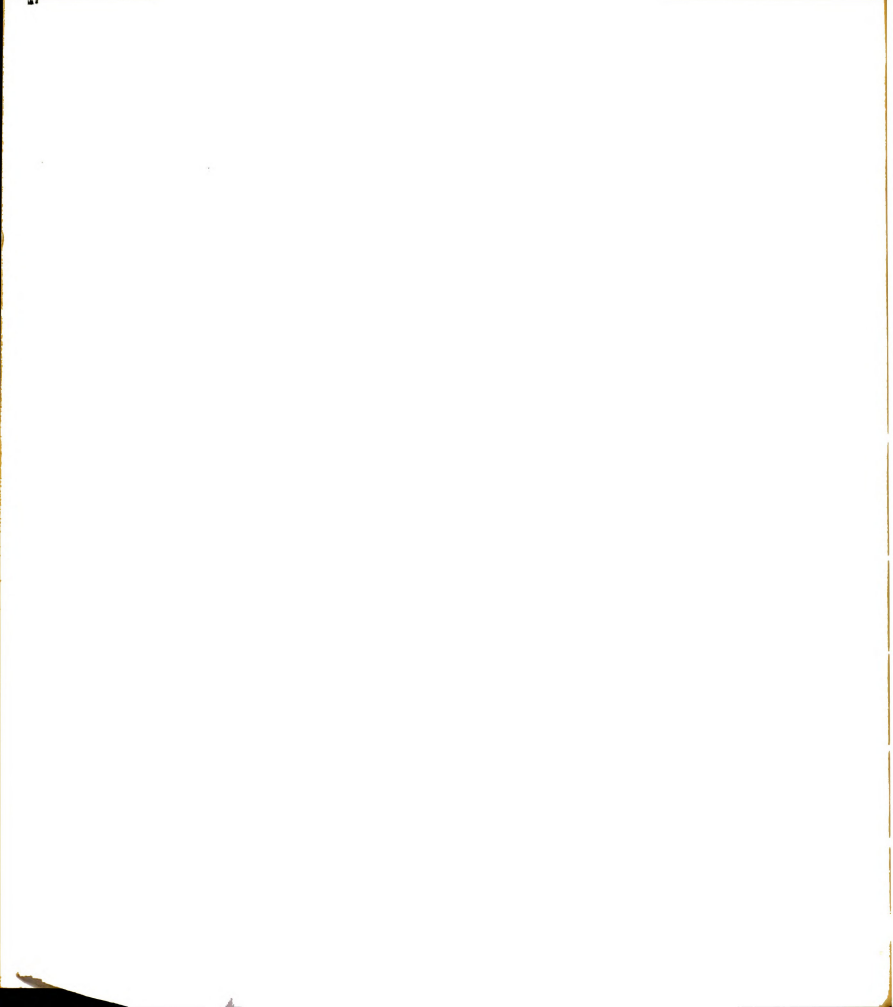
The above analysis is depicted in the tree-diagram in Figure 15.

FIGURE 15.--Outcomes of financial comparisons



From this diagram, the following ranking becomes obvious.

<u>Approach</u>	<u>Rank</u>
Min	1
BFJ 2nd	2
McM	3



<u>Approach</u> (Continued)	<u>Rank</u> (Continued)
Equal	4
BCC	5
BFJ 1st	6
AC	7

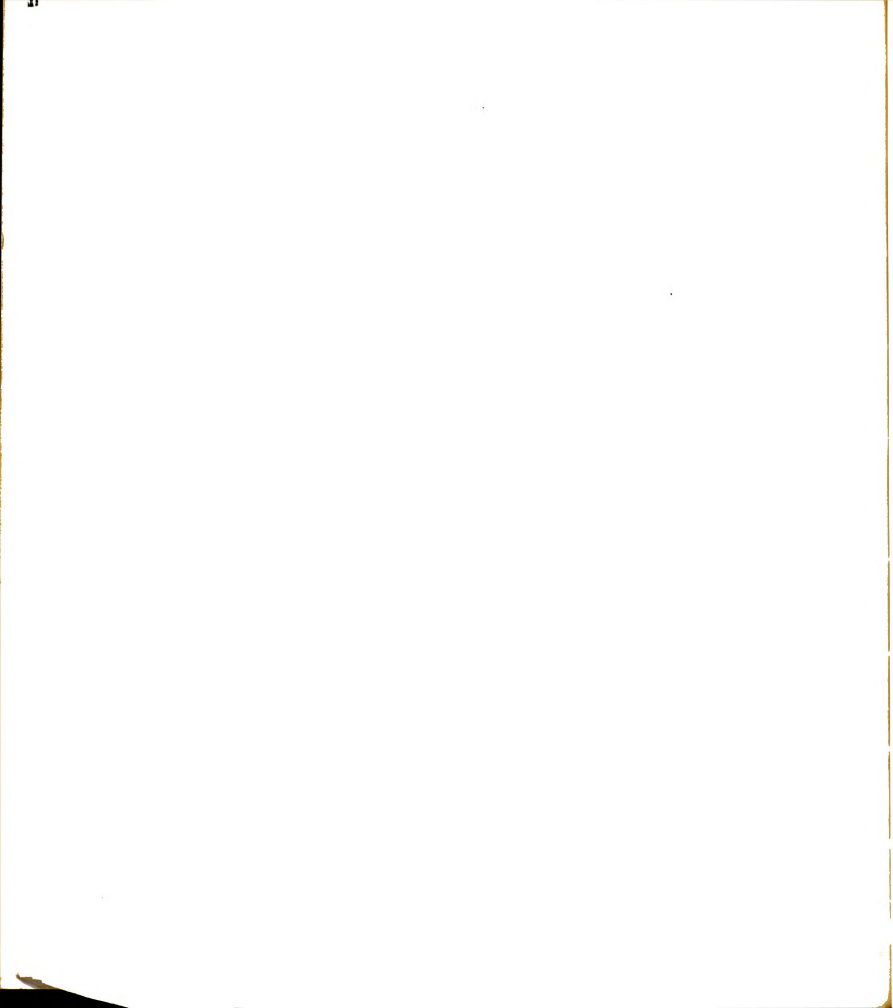
Conclusions

Summary of Rankings

In this chapter upper and lower control limits have been calculated for each of the seven approaches under four testing plans. After the derivation of each control limit for all the approaches according to each testing plan, a financial analysis was performed which enabled the ranking of the approaches in order of their effectiveness for control purposes. Table 74 summarizes these rankings as an aid in determining what generalization, if any, can be drawn.

TABLE 74.--Summary of rankings

Approach	UCL				LCL			
	Testing Plan				Testing Plan			
	A	B	C	D	A	B	C	D
AC	5.5	7	7	7	5	7	7	7
BCC	2.5	6	6	6	1.5	4	4	5
BFJ 1st	5.5	5	3	4	3	5	6	6
BFJ 2nd	7	1.5	1	3	4	2.5	1.5	2
MCM	1	4	5	5	7	2.5	3	3
Equal	4	3	4	1	6	6	5	4
Min	2.5	1.5	2	2	1.5	1	1.5	1

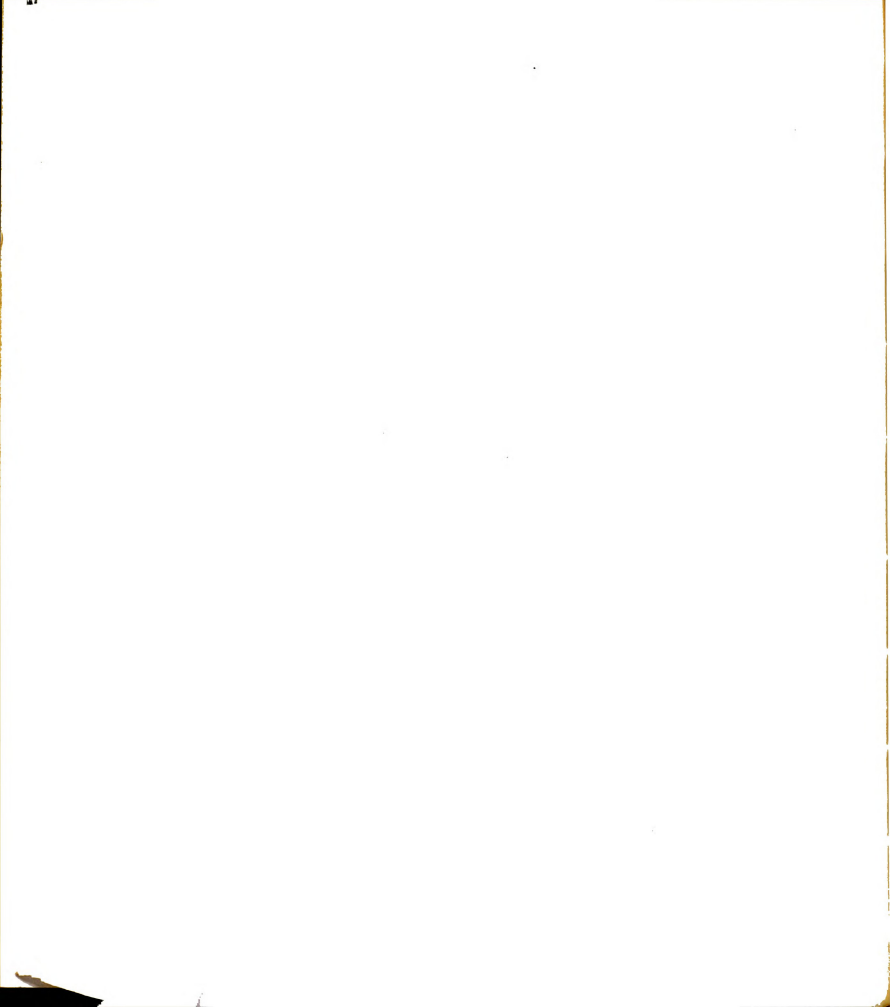


The approaches are listed in order of their presentation in this dissertation. The testing plans are identified by the letters A, B, C, and D as follows:

- A represents tests of single performances where each performance is tested
- B represents tests of single performances where every tenth performance is tested
- C represents tests of samples of five consecutive performances where every performance is included in a sample.
- D represents tests of samples of five consecutive performances where a sample is taken on the average for each 50 performances.

From a cursory glance at Table 74, the reader might conclude that the results do not consistently favor one approach over the others. For example, the number one ranking for the upper control limit is achieved by the McMenimen; Bierman, Fouraker, and Jaedicke second; and Equalization approaches for sampling plans A, C, and D respectively. There is a tie between the Bierman, Fouraker, and Jaedicke second and the Minimization approaches for testing plan B. In other words, the most effective approach is different for each of the testing plans.

Top ranking is more consistent for the lower control limit. In every case it is held by the Minimization



approach although honors must be shared with the Basic Control Chart approach for testing plan A and with Bierman, Fouraker, and Jaedicke second approach for testing plan C. For both control limits taken together, each approach except the Accountant's Conventional and Bierman, Fouraker, and Jaedicke first achieved a number one ranking at least once.

The results with regard to the least effective approach are more consistent. For each control limit, the Accountant's Conventional approach ranks last for testing plans B, C, and D. This approach would undoubtedly rank lower for testing plan A if it were not for the fact that the second interpretation of Bierman, Fouraker, and Jaedicke gave an indeterminate solution for the upper control limit and the McMenimen approach gave an indeterminate solution for the lower control limit.

To obtain a more comprehensive picture the rankings have been aggregated and these aggregations have been ranked in Table 75. The summation of ranks corresponding to UCL and AC is 26.5 which is obtained by adding 5.5, 7, 7, and 7. These are the values shown in Table 74 for UCL and AC under each of the testing plans. The grand total column represents the summations of the ranks for all testing plans for both control limits. These summations are then ranked from one to seven with the lowest summation receiving a rank of one.

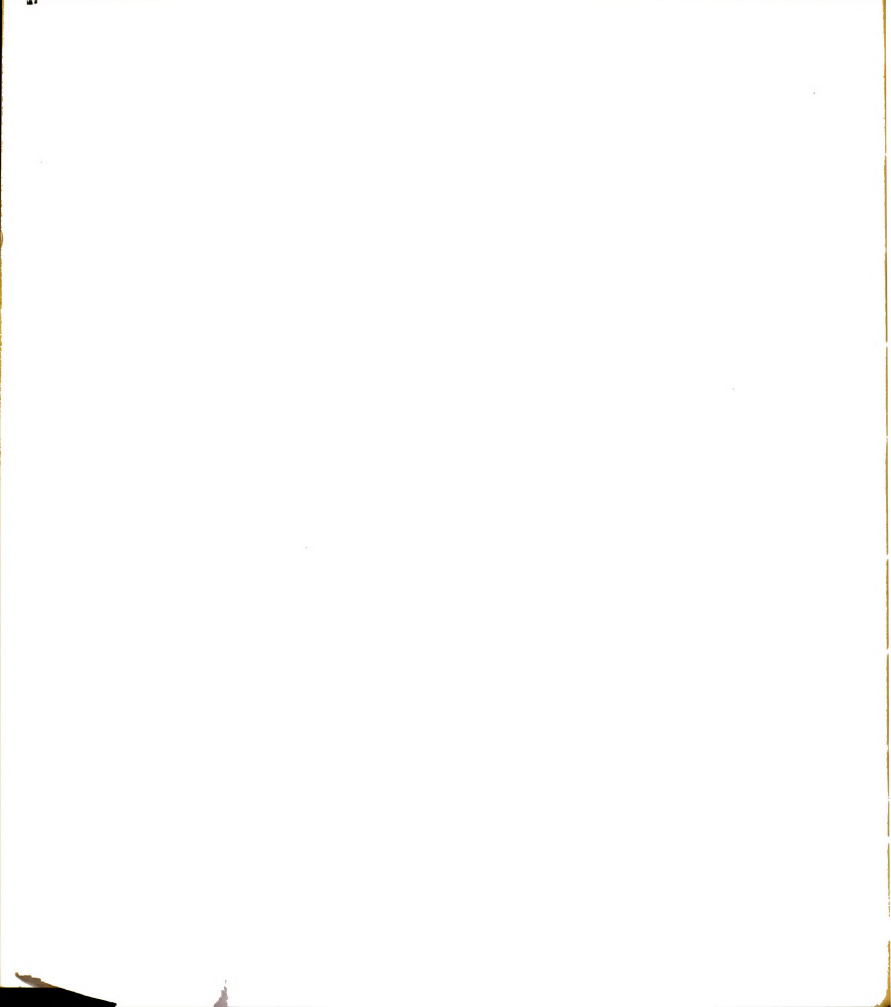


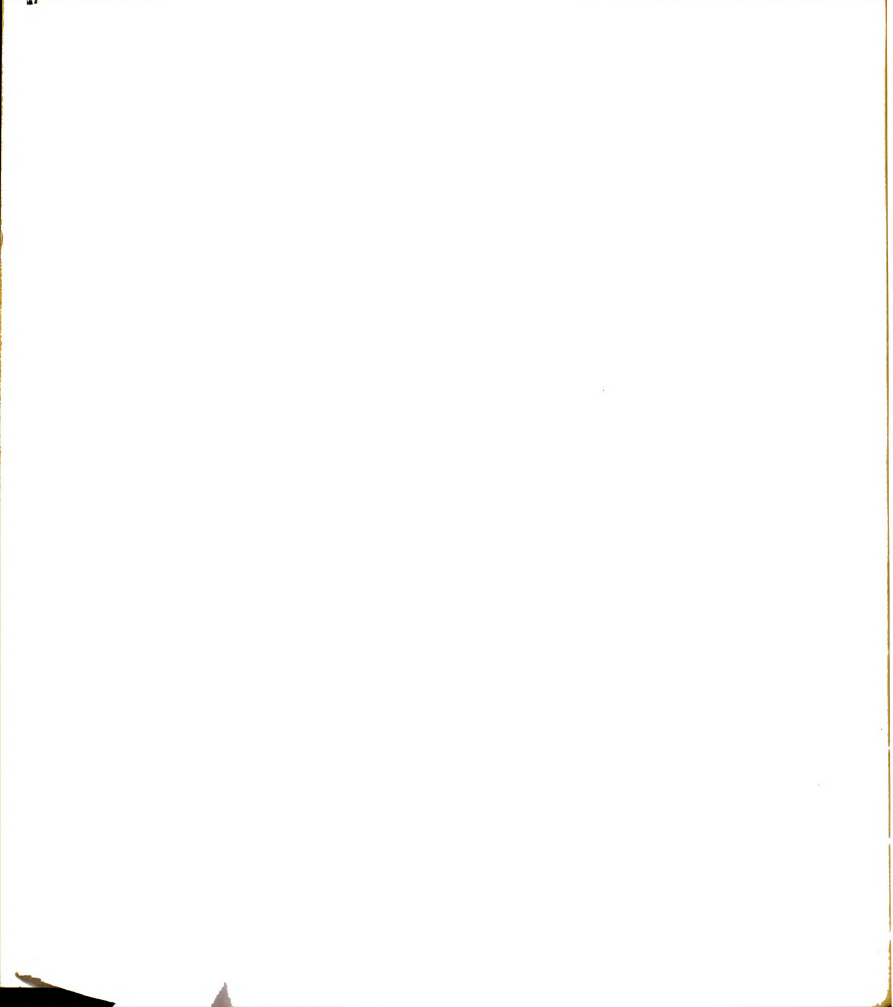
TABLE 75.--Summation of ranks and ranking of summations
by control limit

Approach	Summation of Ranks			Ranking of Summations		
	UCL	LCL	Grand Total	UCL	LCL	Grand Total
AC	26.5	26	52.5	7	7	7
BCC	20.5	14.5	35	6	3	5
BFJ 1st	17.5	20	37.5	5	5	6
BFJ 2nd	12.5	10	22.5	3	2	2
MCM	15	15.5	30.5	4	4	3
Equal	12	21	33	2	6	4
Min	8	5	13	1	1	1

Significant Generalizations Resulting
from the Summary of Rankings

The Minimization approach had the lowest summation for both control limits. This may have been anticipated on the grounds that the Minimization approach utilizes more information in deriving the control limits than the other approaches do.

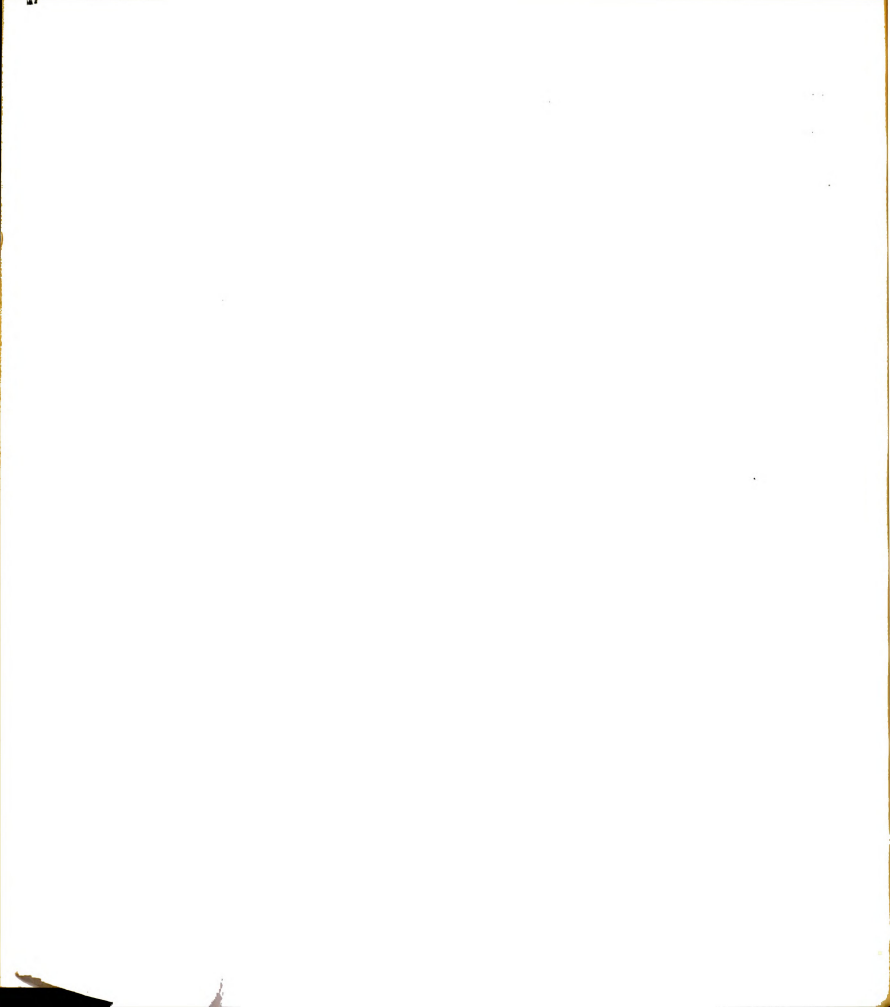
The grand summation is considerably higher for the Accountant's Conventional approach than for the next highest summation. This lends credence to this dissertation's hypothesis that new applications of presently developed statistical tools can increase the effectiveness of accounting variance control by providing a helpful analytical framework to determine the control limits. The hypothesis is further reinforced by the fact that the dollar difference between the added investigation cost and



the added savings is relatively greater for financial analyses between the Accountant's Conventional approach and the approach which occupies the next highest ranking than it is for analyses between approaches not involving the Accountant's Conventional approach. This can readily be observed by reference to Tables 23, 28, 38, 44, 53, 60, 67, and 73. This difference is especially large for testing plans B, C, and D. That is, the large difference is least noticeable in Tables 23 and 28. It is interesting to note that testing plans B, C, and D involve tests of less than 100 per cent of all performances. As such, they are more realistic in practice.

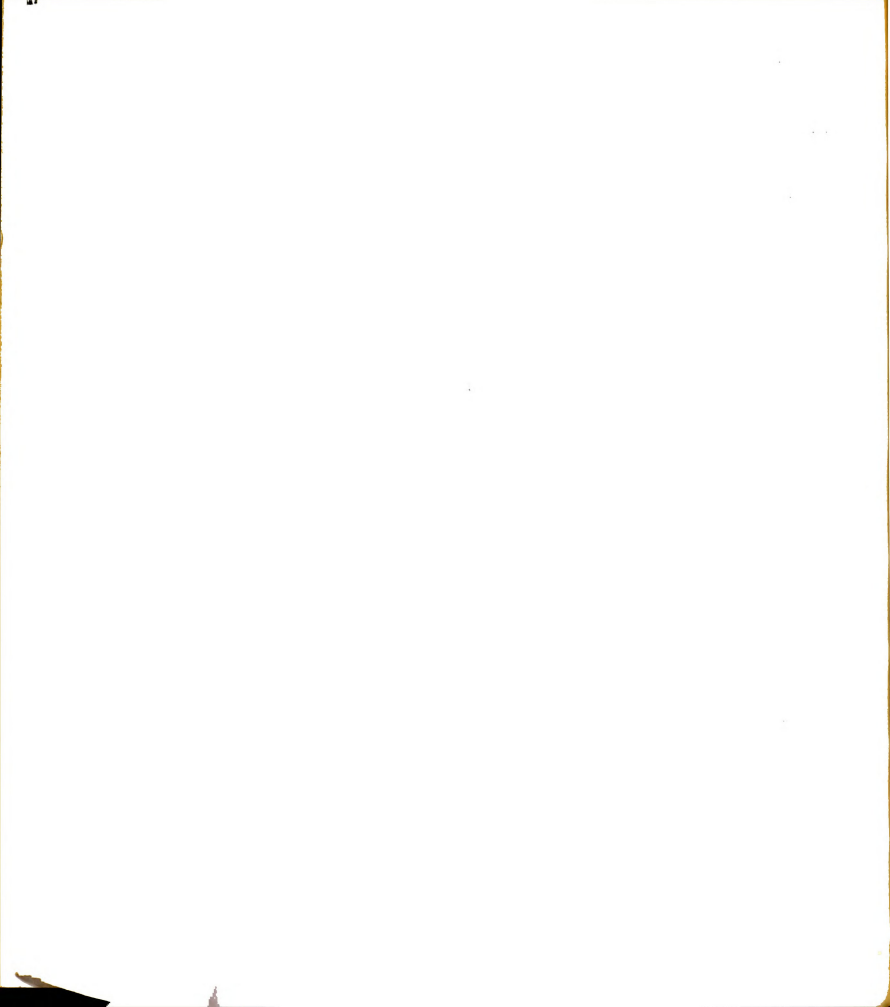
For example, if the reader will turn to Table 73, he will observe that for the comparison between AC and BFJ 1st the added savings associated with BFJ 1st is \$20,364.10. This is \$19,528.18 greater than the \$835.92 added investigation cost of this approach. The difference between the added savings and the added investigation cost for the other comparisons are all less than \$100 with the exception of the \$495.78 difference between BCC and Equal. The point to be emphasized is that the Accountant's Conventional approach is much less effective than the Bierman, Fouraker, and Jaedicke first approach--the approach with the new lower ranking in this case.

To summarize, two conclusions have thus far been reached as a result of the example which has been developed



in this chapter. One is that in the aggregate the Minimization application is the most effective. The other conclusion is that, also in the aggregate, the Accountant's Conventional is the least effective. The fact that these conclusions hold the aggregate and not for each individual testing plan and control limit signifies that the rankings can vary with different testing plans. Indeed, the arbitrarily selected per cent of the variance to the standard selected as a cut-off point according to the Accountant's Conventional approach need not be 10 per cent. In any actual or hypothetical situation, the per cent cut-off point could just happen to be selected so as to give the same control limit produced by the Minimization or the otherwise most effective approach. However, only coincidentally, would the cut-off point selected by accountants in the conventional manner be the same as that yielded by the most effective approach.

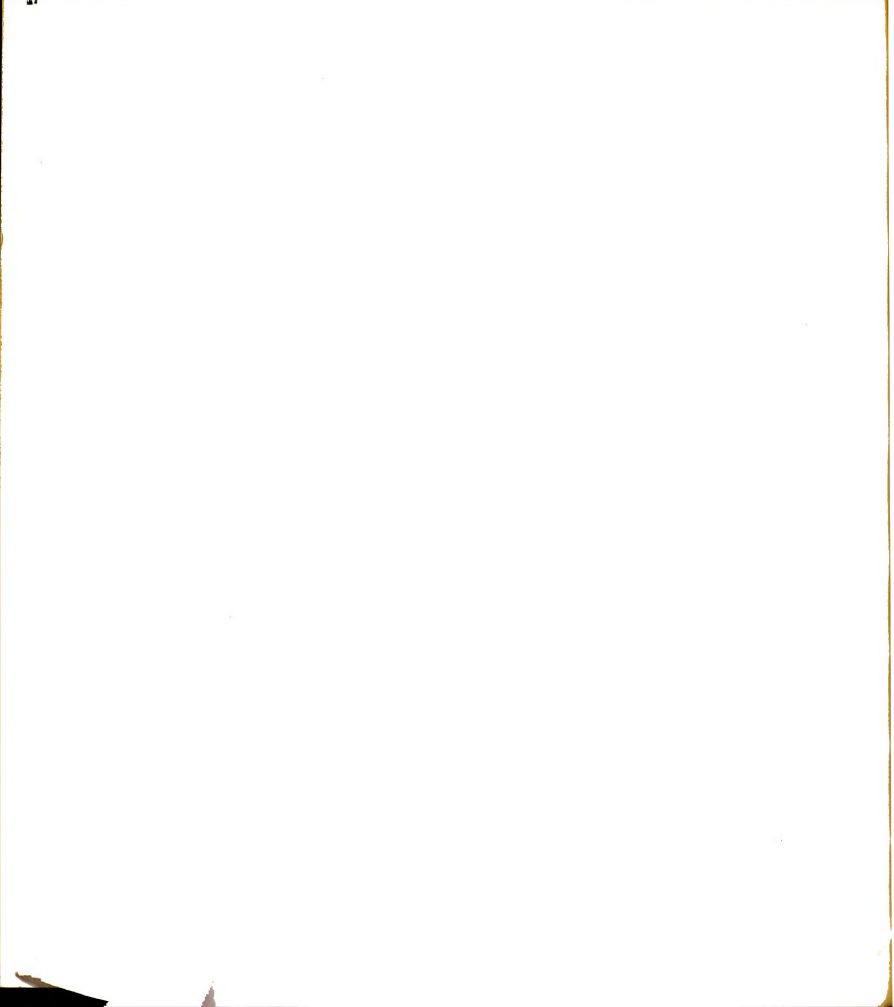
Some may argue that the accountant's experience and intuition may produce control limits very close to those that are statistically determined. Since this is a qualitative argument, it is difficult to refute analytically. It only needs to be re-emphasized, however, that statistics provides a systematic method for formally considering experience, judgment, and intuition to make it appear unwise to draw conclusions without the use of techniques that are already available--techniques that



have stood the test of time for use in other disciplines. Judgment and experience are needed to estimate probability distributions like those shown in Table 18. They are also needed to set down the opportunity costs of error. The chance and assignable cause populations and the costs of error exist whether or not they are explicitly considered in setting the control limits. If they are not explicit they are implicit in which case the analyst is not aware of the magnitude of their values. Judgment and experience certainly have a better basis when used to estimate the relevant factors in decision making than they have when used to make final decisions without considering what factors may be relevant. Surely judgment and experience are more productive when they are systematically rather than haphazardly used.

Secondary Generalizations Resulting from the Summary of Rankings

It is now appropriate to examine the rankings of the other approaches. In the aggregate the Bierman, Fouraker, and Jaedicke second and McMenimen approaches hold rankings two and three respectively. Because these approaches interpret P in the same manner it is interesting to note that their respective rankings follow the Minimization approach. In other words, the top three ranks are held by approaches that could not be identified as Classical statistics. Their identification as Non-classical

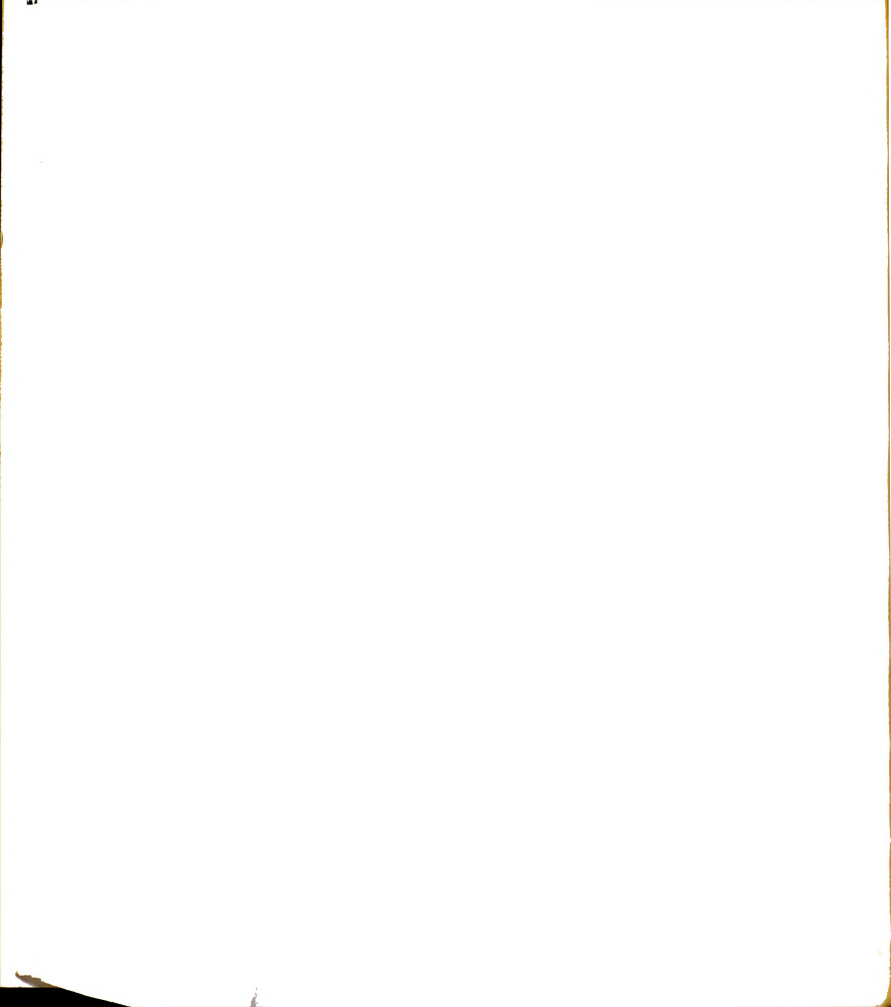


is not because they consider financial implications in addition to probabilities but because of the probabilities these methods obtain.

The reader will recall that Classical statisticians are interested in two types of probabilities--the probability of a Type I error and the probability of a Type II error. The probability of a Type I error is the probability that a chance value falls outside the control limits and thus leads to rejection of a true hypothesis. The probability of a Type II error is the probability that a non-chance performance falls inside the control limits and thus lead to acceptance of a false hypothesis.

The probabilities of concern in the Bierman, Fouraker, and Jaedicke second and McMenimen approaches are of an entirely different nature. These latter two approaches estimate the probability that a performance is due to chance and the probability that it is due to an assignable cause given the test value. The Bierman, Fouraker, and Jaedicke and McMenimen approaches are neither Classical nor Bayesian; nevertheless, they yield useful results.

The Minimization approach involves the use of Type I and Type II errors in the probability of a wrong decision column. Unlike Classical applications, however, the probability of a Type II error is developed for each assignable cause. Another distinguishing feature of the Minimization approach is its use of the prior probabilities.

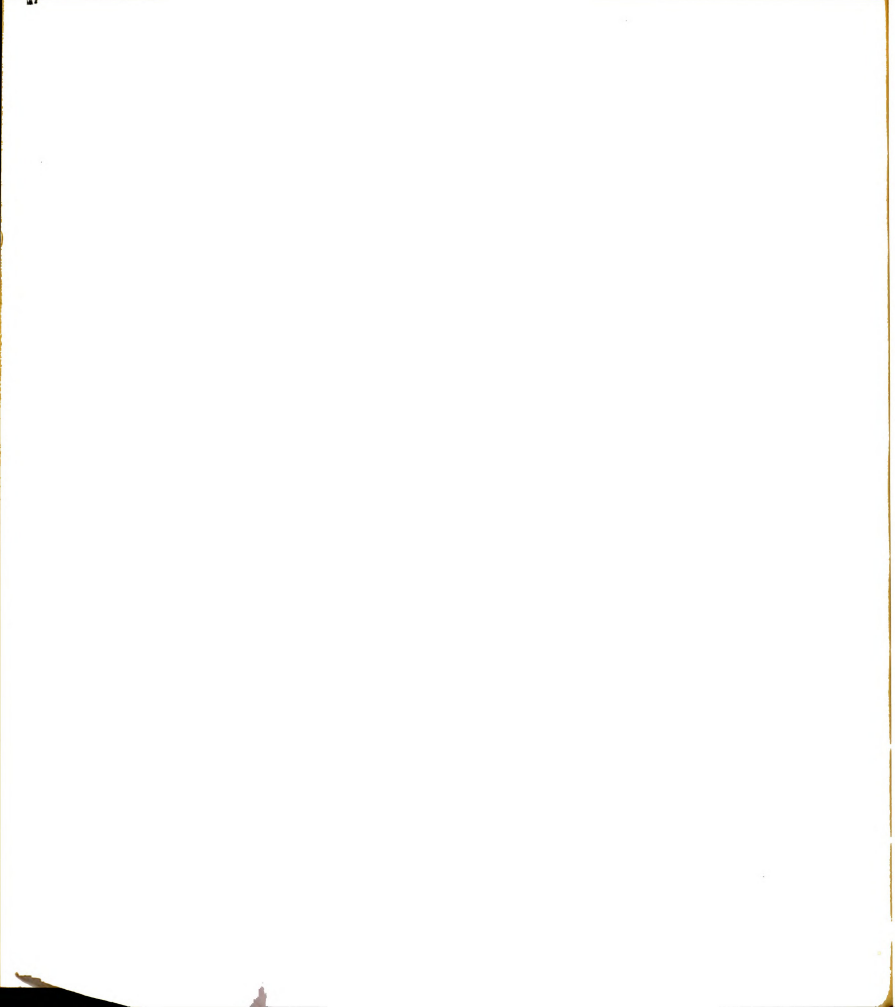


After the several questionable aspects of the Bierman, Fouraker, and Jaedicke approaches that were identified in Chapter IV, it appears rather surprising that the Bierman, Fouraker, and Jaedicke second approach rated second in the overall rankings. This ranking is sufficiently striking to compel a reconsideration of the following criticisms that were originally outlined in Chapter IV.

First, the numerical example used by Bierman, Fouraker, and Jaedicke involved control of a summary expense classification for a period of one year rather than control of individual performances. However, with one noted exception,²⁶ the approach also lent itself to application at the performance level for comparative utilization in this dissertation. The criticism levied in Chapter IV was not that the model was necessarily incapable of dealing with control at the performance level but only that Bierman, Fouraker, and Jaedicke did not apply it at this crucial level.

This writer criticized the Bierman, Fouraker, and Jaedicke model because it assumes that the mean of the assignable cause is equal to the test value (or actual results)--a condition which would only coincidentally be true. This criticism is still regarded as valid although

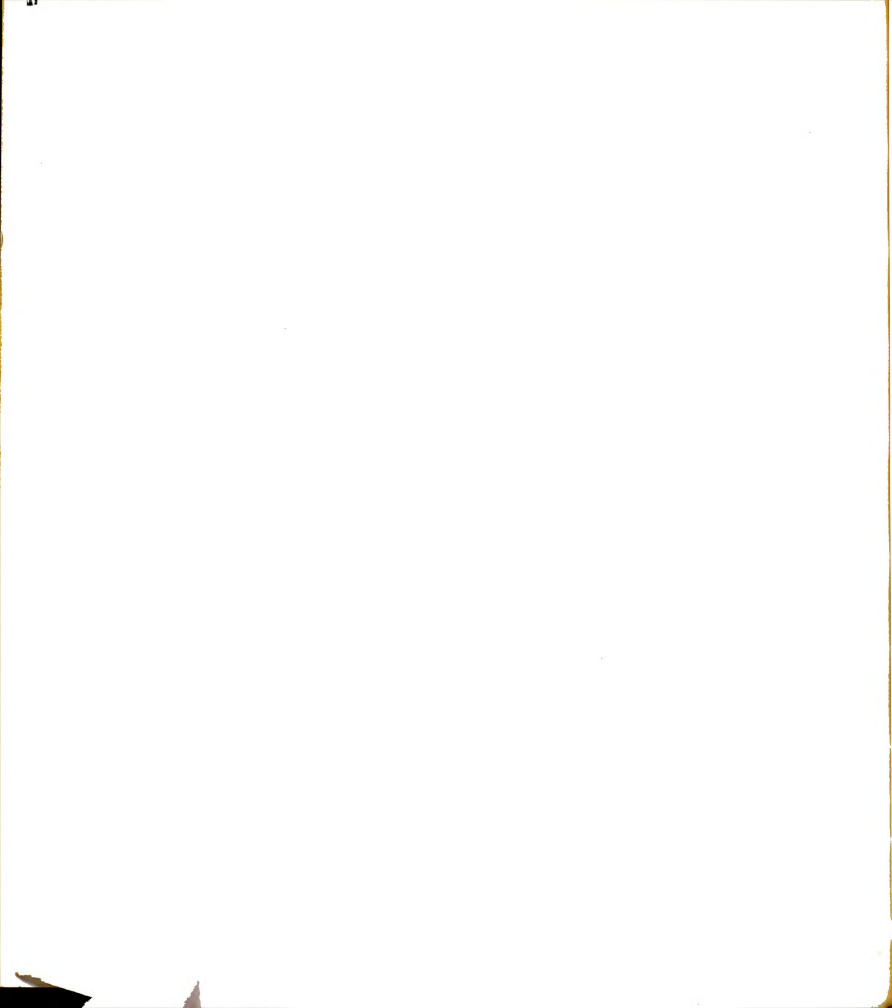
²⁶For UCL testing plan A.



the objection could easily be corrected without altering the basic model.

Third, the Bierman, Fouraker, and Jaedicke allegation that an off-standard condition requires four tests on the average before detection was questioned. It was shown that the number of tests required on the average for detection depends upon the control limit that is established. As the control limit moves away from the standard the number of tests required increases. For most statistically determined control limits the number will be less than four. This arbitrary selection of the number four remains a basic objection to the Bierman, Fouraker, and Jaedicke model. It would, of course, be possible to implement the weighting scheme proposed in this dissertation into this model.

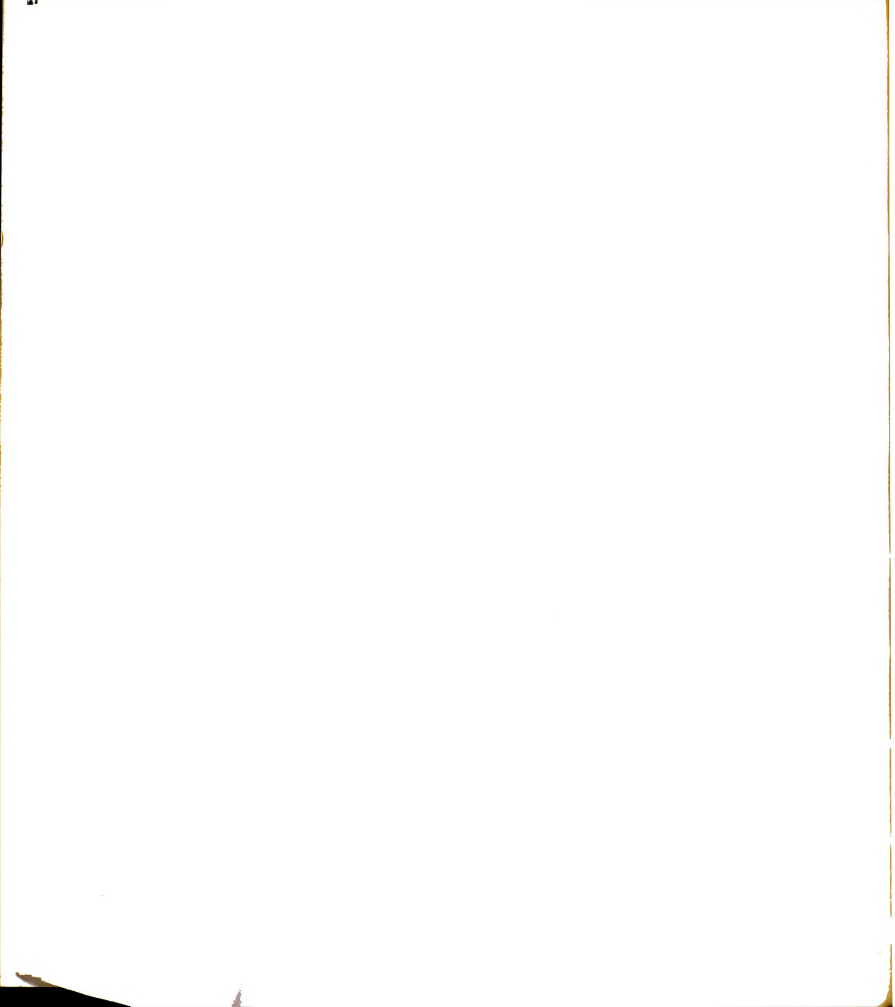
The cogency of this model is further diminished by their synonymous treatment of two different interpretations of P . The conceptual distinction between these interpretations has been established. Moreover, it has clearly been illustrated that each of these interpretations will yield different control limits with a second overall rating achieved by the second interpretation as compared to a sixth overall rating for the first interpretation. Chapter IV did not question the usefulness of either interpretation but merely noted the distinction which Bierman, Fouraker, and Jaedicke failed to make.



Finally, Bierman, Fouraker, and Jaedicke treated the cost of an investigation as a constant. The example in this chapter illustrates that this is clearly not the case. The cost of an investigation depends upon the particular assignable cause as well as the established investigation procedure.

Once it is agreed to use the second interpretation of the Bierman, Fouraker, and Jaedicke model for control at the performance level, the second, third, and fifth objections that were discussed above still hold. Why then, does this approach achieve a higher overall ranking than the Equalization approach that was designed to counteract the objections of the Bierman, Fouraker, and Jaedicke model? This reason is that the Equalization approach uses probabilities in the Classical manner; in contrast, the Bierman, Fouraker, and Jaedicke second approach uses the detail of Table 18 to estimate the probability that a performance is due to chance given the test value. On balance, it appears that these latter probabilities are sufficiently important determinants of effective control limits in this example to outweigh these stated objections of the Bierman, Fouraker, and Jaedicke approach.

The McMenimen approach also achieved a more effective overall ranking than the Equalization approach. The distinguishing features of the McMenimen approach that



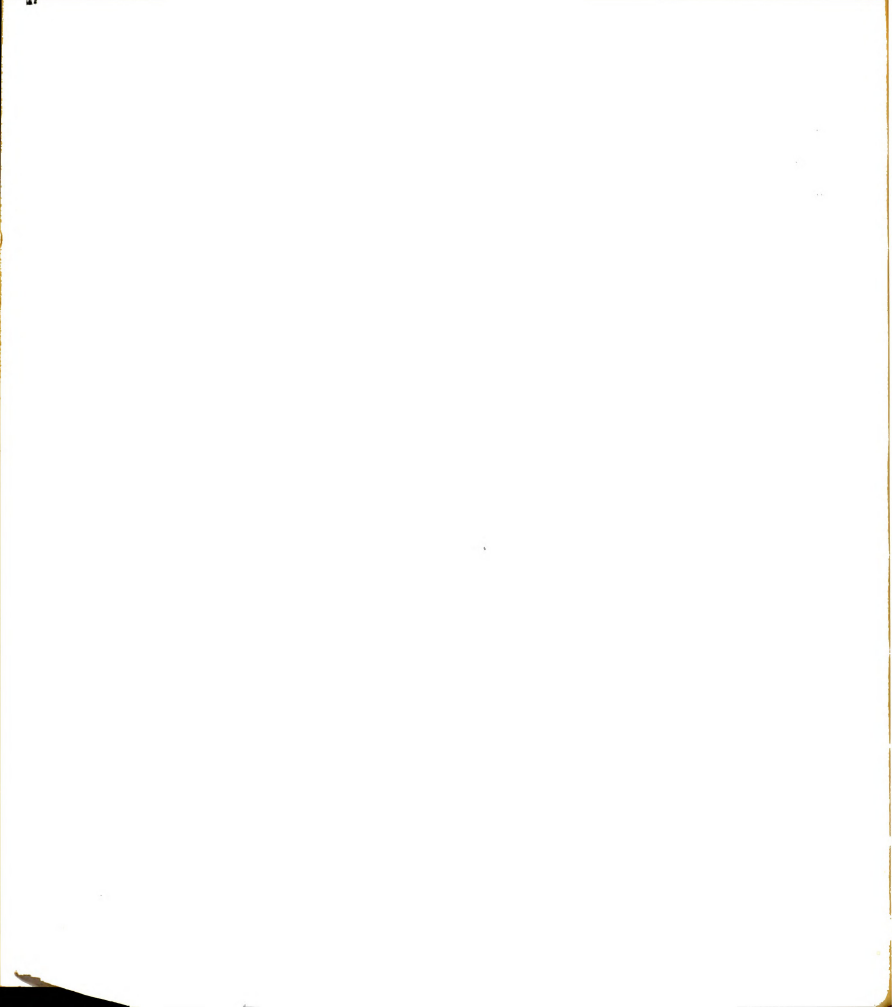
were noted in Chapter IV were that:

1. An investigation might be terminated before finding the cause.
2. A cost deviation might be reduced by various amounts.

In discussing this approach at that time it was this writer's belief that while it might be worthwhile to terminate an investigation short of finding the cause it would not be feasible in practice unless:

1. The cost of an investigation is very high in relation to the present value of expected savings.
2. The cost of control is so high that no action would be taken even if the cause were determined.
3. The probability that the variance is attributed to an assignable cause other than those already investigated is very low.

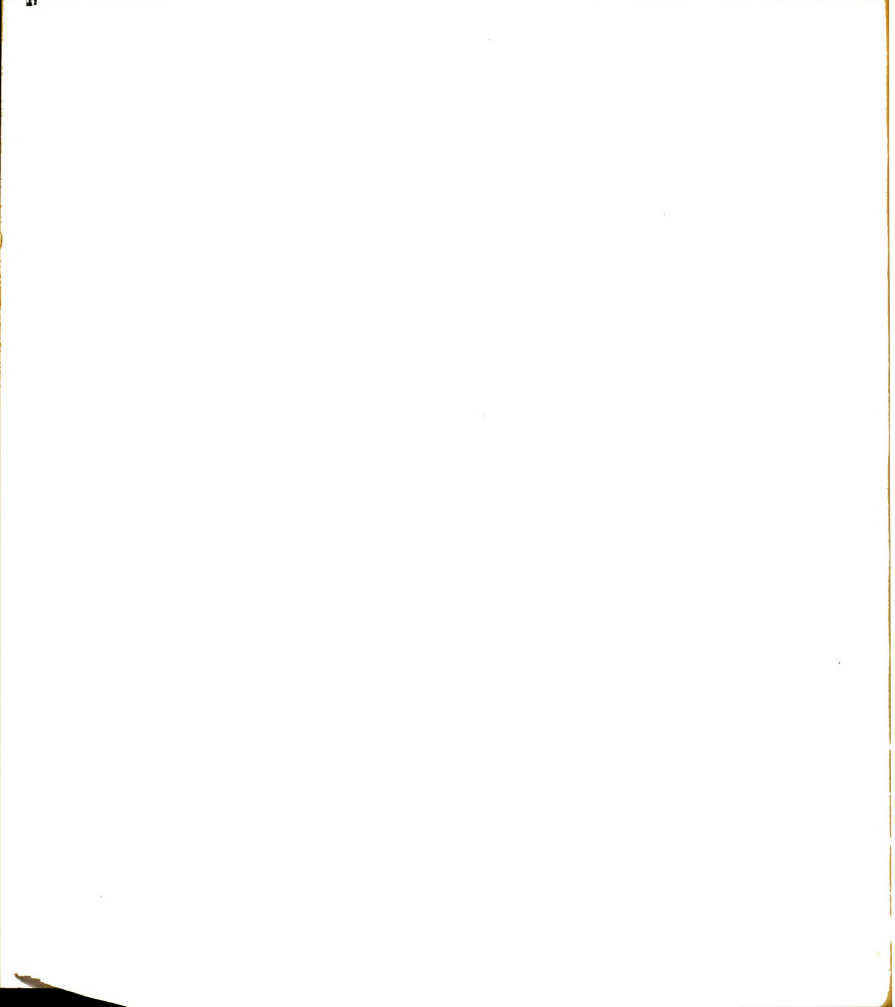
It was noted in Chapter IV that the first of the above items is not likely to hold for analyses at the performance or operational levels although it may hold for monthly or yearly analyses at a departmental or higher organizational level. With regard to the second item, it was also noted in Chapter IV that if it is worthwhile to establish a certain standard in the first place, it would be worthwhile to re-establish it unless conditions have changed in which case the standard should be changed. The example in



Chapter VI indicates that the third item will often hold. For example, Table 32 indicates that the probability that test value 252 is attributed to illness after an investigation for poor attitude is only .0833. For this reason, termination of an investigation before the cause is identified may be profitable.

The second feature of the McMenimen approach was questioned on the grounds that a realistic standard once developed should be maintained or else it should be changed. Stated simply, a deviation known to be attributed to an assignable cause should not be permitted to exist. In applying the McMenimen technique in the example developed in this chapter this objectionable feature was disregarded.

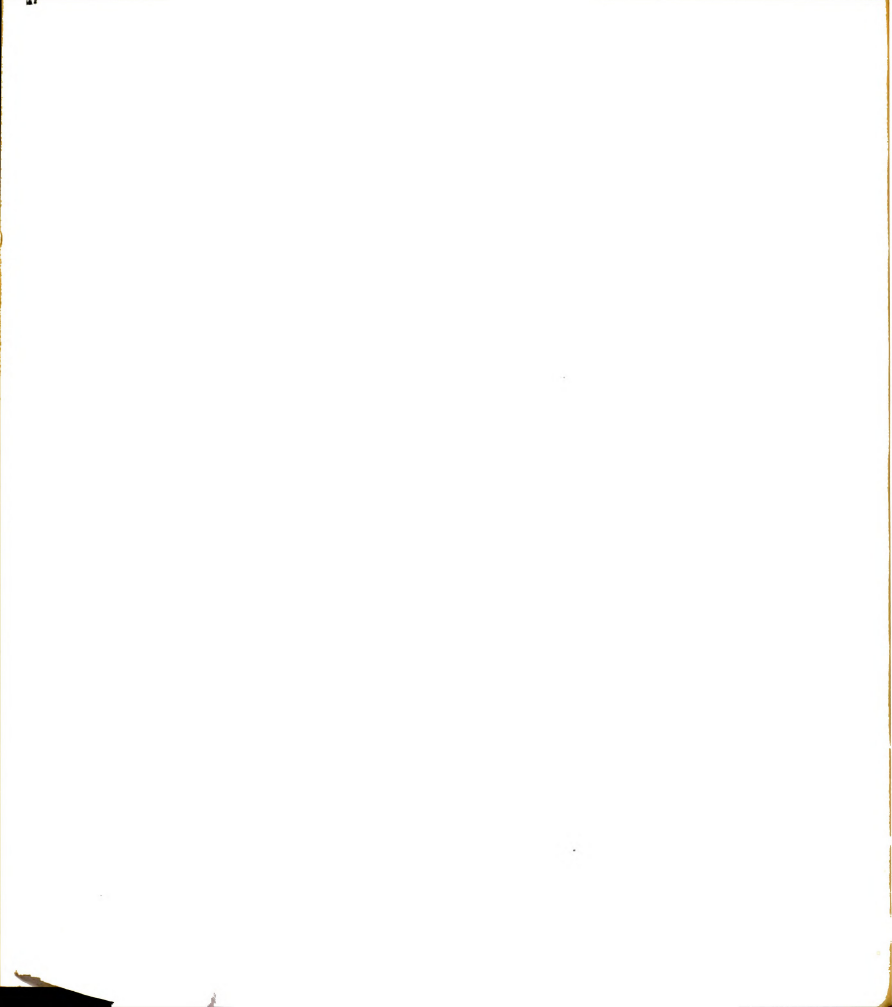
McMenimen developed his approach to overcome the weaknesses he noted in the Bierman, Fouraker, and Jaedicke approach. Since he criticized neither the Bierman, Fouraker, and Jaedicke use of the test value as the mean of the off-standard condition nor their allegation that four tests must be conducted on the average to detect an assignable cause, one might assume that he intended to make use of these ideas in applying his model. However, for reasons noted by this writer in his application of the McMenimen technique, the savings values would be difficult to obtain without consideration of specific assignable causes and their related means. Accordingly, this McMenimen oversight



caused by his failure to develop a numerical application was modified in this chapter. The four test assumption, however, was retained. In spite of this, the McMenimen application achieved a better overall ranking than that received by the Equalization approach. The reason for this is the same as the reason that the Bierman, Fouraker, and Jaedicke second approach had a better overall ranking than the Equalization approach. In the McMenimen application, Table 18 was used to estimate the probability that chance and each assignable cause was prevailing given the test value. As was true with Bierman, Fouraker, and Jaedicke first approach, it appears that these probabilities are, on balance, sufficiently important determinants of effective control limits to outweigh some other questionable aspects.

Following the Minimization; Bierman, Fouraker, and Jaedicke second; and McMenimen approaches respectively in the aggregate ranking are the three approaches that involve Classical statistics.²⁷ Of these approaches, Equalization ranks fourth; Basic Control Chart fifth; and Bierman, Fouraker, and Jaedicke first ranks sixth. From all the discussion, this order is not too surprising. The

²⁷It has already been noted that Classical statistics has not typically considered financial implications. These considerations such as found in the Bierman, Fouraker, and Jaedicke first and Equalization approaches do not, however, materially alter the conceptual basis of the Classical approach.



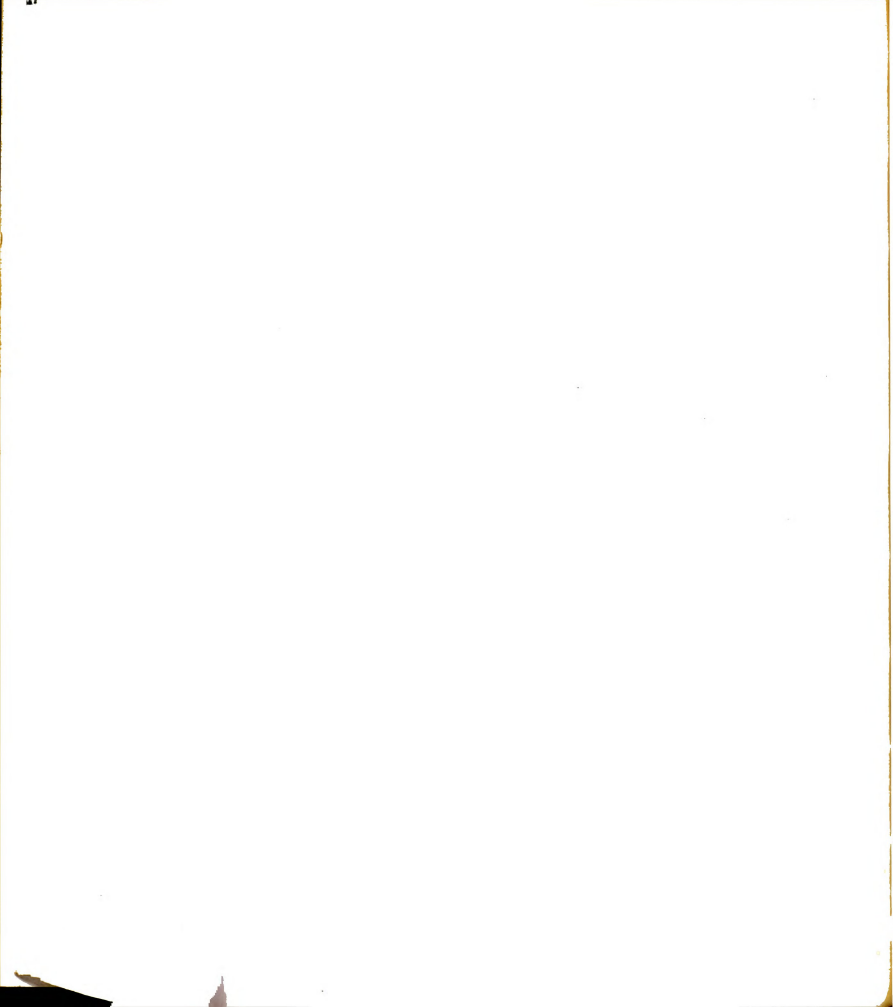
Equalization approach considers the financial implications not considered by the Basic Control Chart approach. At the same time it remedies some of the objections to the Bierman, Fouraker, and Jaedicke first approach. Hence, it might be expected to top the list of the Classical statistical methods.

Summary of Generalizations

All this discussion, then, leads to four general conclusions. First, and most significant, the approach conventionally employed by accountants is generally inferior to the statistical methods. Second, the Minimization approach tends to be most effective for control purposes. Third, the Bierman, Fouraker, and Jaedicke second and McMenimen approaches which consider the probability that chance is prevailing given the test value appear to rank next most effective after the Minimization approach. Their employment of these rather unconventional probabilities is sufficiently useful to counteract other previously designated deficiencies associated with these approaches. Fourth, the Classical approaches headed by the Equalization approach appear to be the least effective of the statistical approaches.

Stability of Generalizations

The third and fourth conclusions are not nearly as important or as valid as the first two. They could be



influenced by changing some of the assumptions of the problem. For instance, the Equalization approach ranks sixth for the lower control limit under sampling plans A and B. In both cases the Equalization control limits are closer to the standard than the approach which ranked first. One could tamper with the distribution in Table 18 in such a way as to reduce the lower control limit under the Equalization approach without greatly affecting the lower control limits under the other approaches. This could be achieved by including fewer performances due to improvement between the lower control limit for the Equalization approach and the standard thus lowering the probability of a Type II error which will cause the decision maker to accept the hypothesis until such tampering could be designed to give the Equalization approach a better ranking. If in another example this latter type of distribution would in fact prevail the Equalization approach might rank better. Conversely, tampering in the reverse direction could lead to a poorer ranking for the Equalization approach.

The number of assignable cause and chance performances occurring at any test value play an important role in the comparative financial analysis used to rank these approaches. This is illustrated for testing plan A in Table 76.

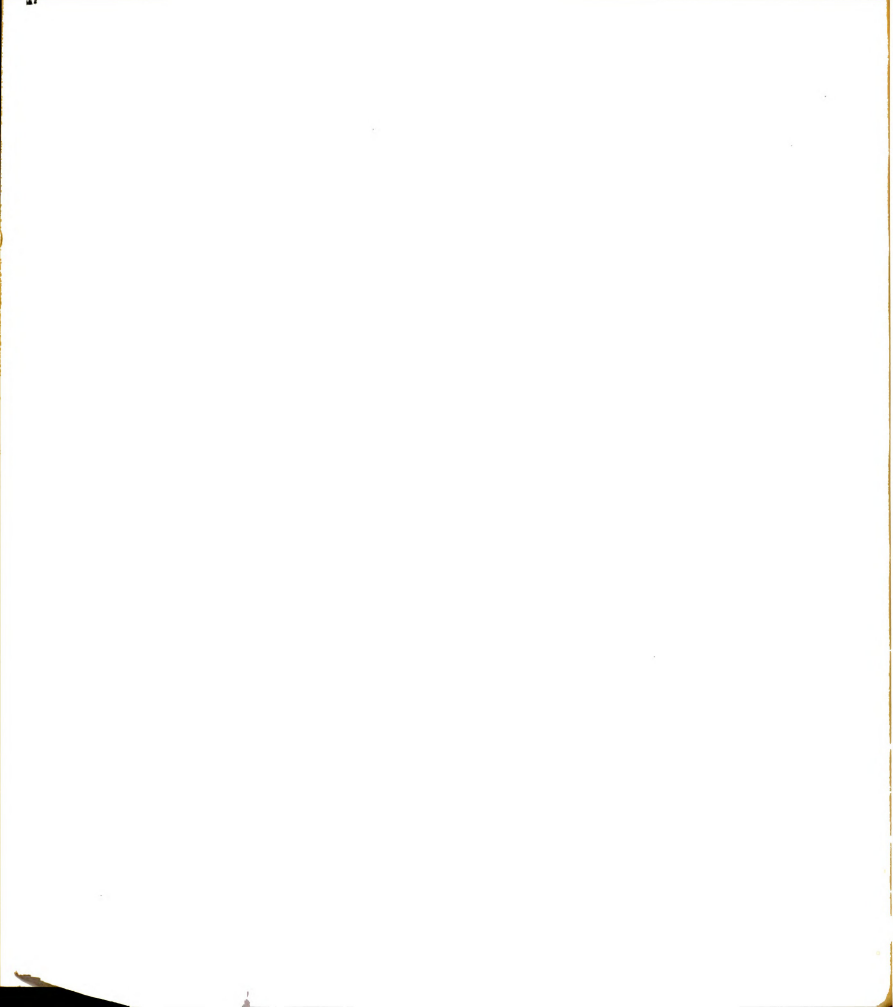


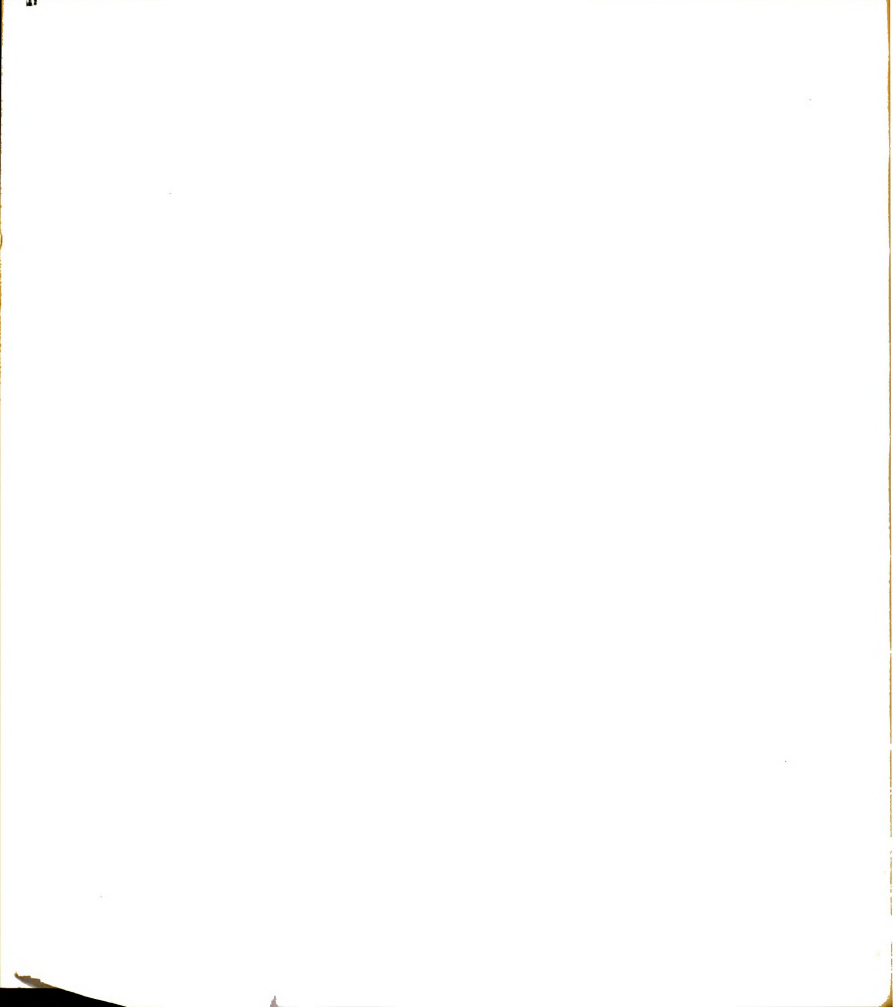
TABLE 76.--Numerical differences in control limits between the top ranking and the other approaches for testing Plan A

Approach	UCL		LCL	
	Numerical Difference	Ranking	Numerical Difference	Ranking
AC	+(10 - 11)	5.5	-(9 - 10)	5
BCC	+1	2.5	0	1.5
BFJ 1st	+(10 - 11)	5.5	-5	3
BFJ 2nd	a	7	-7	4
McM b	0	1	a	7
Equal	-1	4	+4	6
Min	+1	2.5	0	1.5

a = indeterminate

b = CL for dull knives is used (259-260)

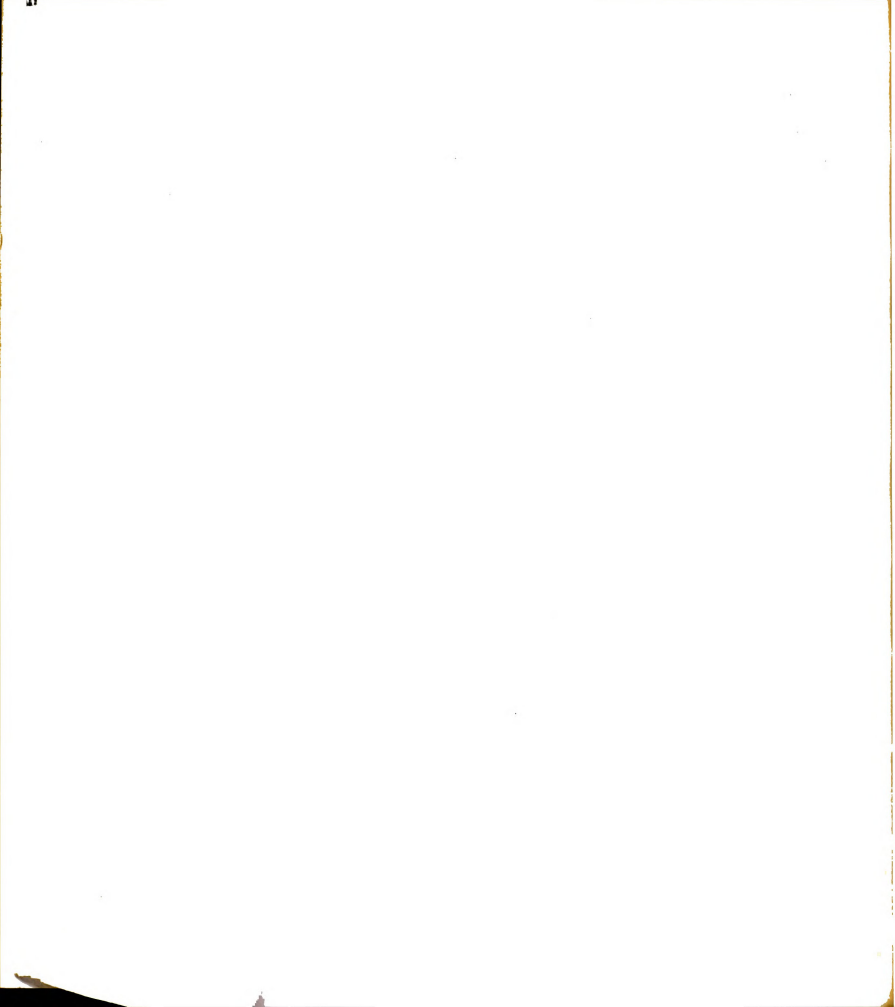
The column labeled "numerical difference" reports the difference between the control limit achieving a number one rating and each respective approach. The rankings are also shown. Three approaches had upper control limits only one minute from the most effective one. The Basic Control Chart and Minimization approaches had upper control limits one minute over the McMenimen upper control limit and the Equalization approach had an upper control limit one minute lower. In terms of numerical differences there would be a three way tie; but because there are more chance performances that would be investigated in the direction of the Equalization upper control limit, its additional investigation cost is higher. Consequently, the Basic Control Chart and Minimization approaches tie for second ranking and the Equalization approach ranks



fourth. Had the performances been distributed differently the rankings might have been altered. This comparison is complicated by the fact that the McMenimen upper control limit for poor attitude and laziness is in the direction of the Basic Control Chart and Minimization limits. The numerical differences reported for the lower control limits illustrate the influence of the concentration of performance values more clearly. The Equalization lower control limit is four minutes over the two approaches ranking first. Both Bierman, Fouraker, and Jaedicke interpretations and the Accountant's Conventional approach had larger numerical differences than the Equalization approach. In spite of this fact the Equalization approach received a poorer ranking. The reason is that there are a greater number of chance performances requiring investment between the lower control limits ranking first and the Equalization control limit than between the control limit ranking first and the control limits for the two Bierman, Fouraker, and Jaedicke approaches and for the Accountant's Conventional approach. A slightly different distribution of chance performances may cause different results.

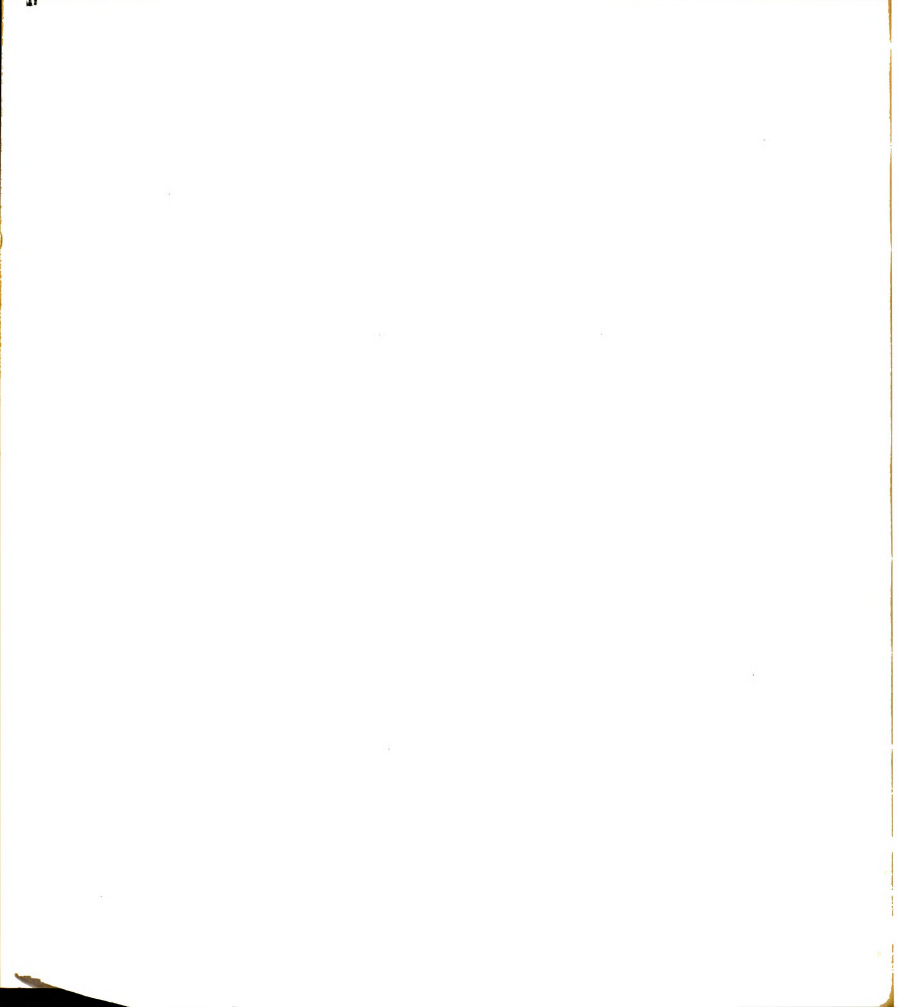
Final Conclusions

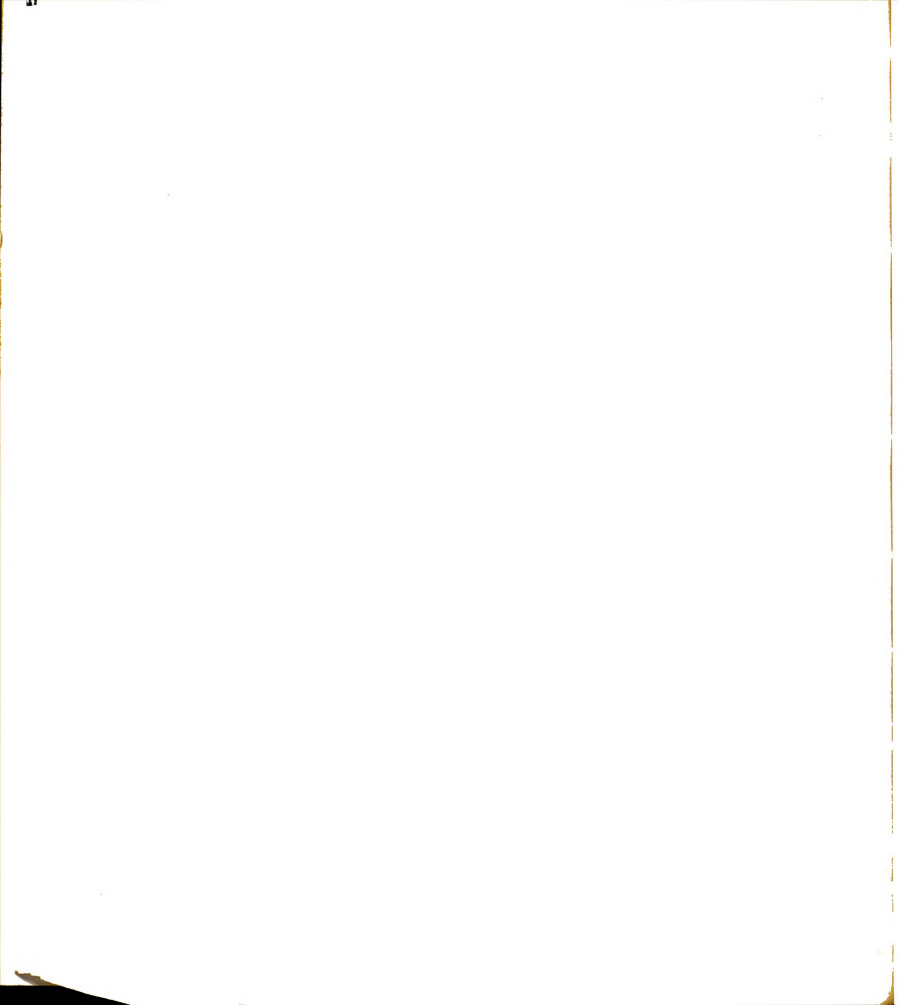
The point to be emphasized is that the control limits under the various statistical approaches are generally fairly closely grouped about the control limit achieving the number one ranking while the Accountant's Conventional



method produces control limits fairly far apart from this grouping. This is illustrated in Table 77. Because of this and because the rankings of the statistical methods seem to vary with differing testing plans and with different sets of assumptions, it is probably pre-mature to support any particular statistical method. Anyone of these would generally be an improvement over the Accountant's Conventional method.

A company planning to adopt statistical procedures for variance control might begin by taking certain key operations, calculating the control limits under each of the approaches, and running a financial analysis to determine the one best suited to their operation. For a beginning, the firm might be satisfied to use the Basic Control Chart approach which, of course, is the simplest of all the statistical methods. This would also make a smoother transition into the more sophisticated methods which should be instituted after more knowledge is obtained about the operation. Intelligent guesses about the relevant distributions and costs produce better control limits than arbitrarily selected ones for which the shape of the distributions and the costs are implicit but unspecified by the analyst. One might also begin by making the necessary estimates to employ the Minimization approach. This would be followed by careful tabulation of subsequent results according to the format indicated in Table 18. Application

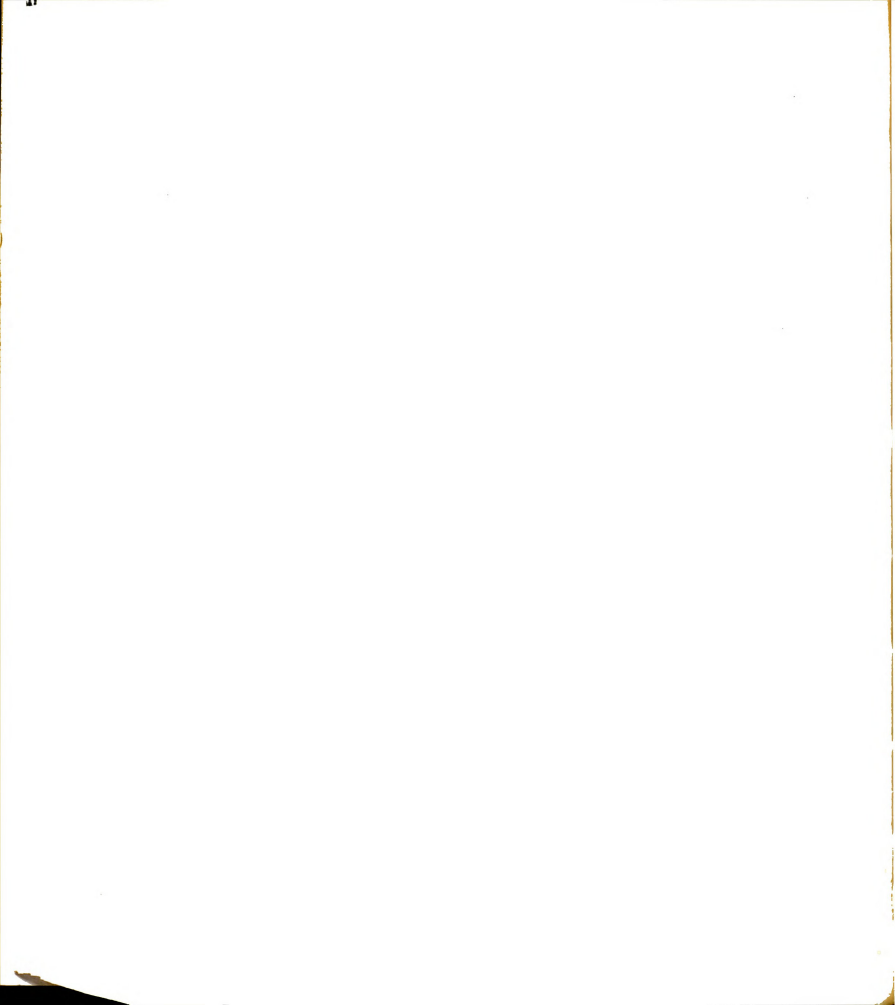




of Bayes' Theorem from time to time could be used to revise the original estimates until the differences between subsequent estimates become insignificant. This procedure assumes that the population mean to the operation remains constant.

While the Basic Control Chart approach might be easy to use in the absence of more detailed information, and while it may serve as a good transition into other statistical methods, this writer would not recommend its long continuance. The conventional practice of selecting an arbitrary level of significance between .001 and .05 just does not yield as satisfactory control limits as is commonly believed. In this example, the .05 level was used. Although the Basic Control Chart approach ranked fifth of the sixth statistical methods, the control limits under all statistical approaches were generally close together. However, if the .01 level had been chosen, the Basic Control Chart approach would not have ranked much better than the Accountant's Conventional approach. In this case the differences between the control limits under the Basic Control Chart approach and those of the other statistical approaches would be wider.

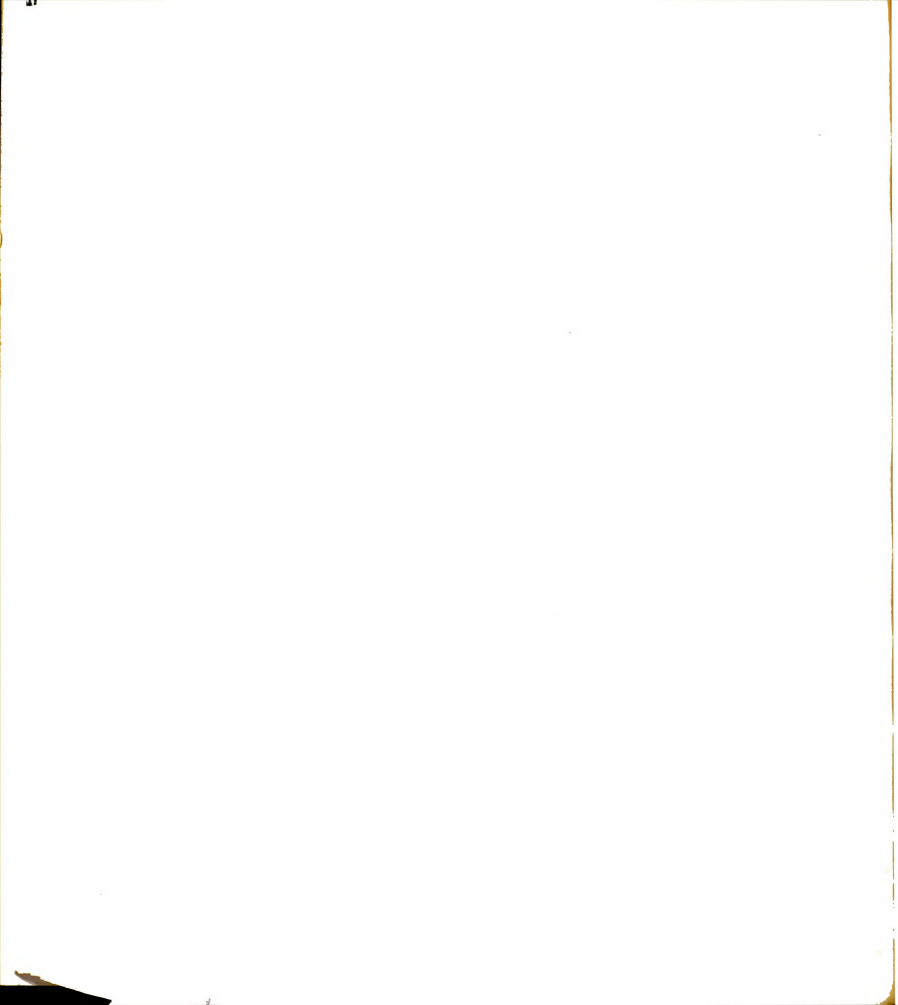
It might be noted in passing that the Basic Control Chart approach has been used in this dissertation to describe the conventional Classical statistical procedure. The control chart diagram could certainly be used under



any approach to aid the analyst in visualizing the results of the operation.

Before concluding this chapter, one more observation must be made. It has frequently been noted that the control limits utilized under the Accountant's Conventional approach are not altered by changes in the testing plan. The upper control limit in Table 78 is shown at 270 for each testing plan. For the other approaches, however, there is a tendency for the control limit to move closer to the standard as the testing plan moves from A through D.²⁸ This movement toward the standard takes cognizance of the average-out effect that was elaborated upon in Chapter II. That is, the existence of an assignable cause may not be too strongly suspected for a single performance as high as approximately 260 for testing plan A. However, an investigation would normally be undertaken should the mean of five performances (for testing plan D) reach as high as 260; it is not likely that five chance performances will average 260. This is analogous to saying that a tail is a likely occurrence in one flip of a coin; but five tails in five flips are not nearly so likely to occur. There is still another reason why the control limits move closer to the standard as the reliance on sampling is increased. This is that a Type II error becomes more

²⁸This also holds for the lower control limit.



costly. Consequently, the control limits are tighter so that Type II errors are made less frequently.

TABLE 78.--Summary of upper control limits by testing plan

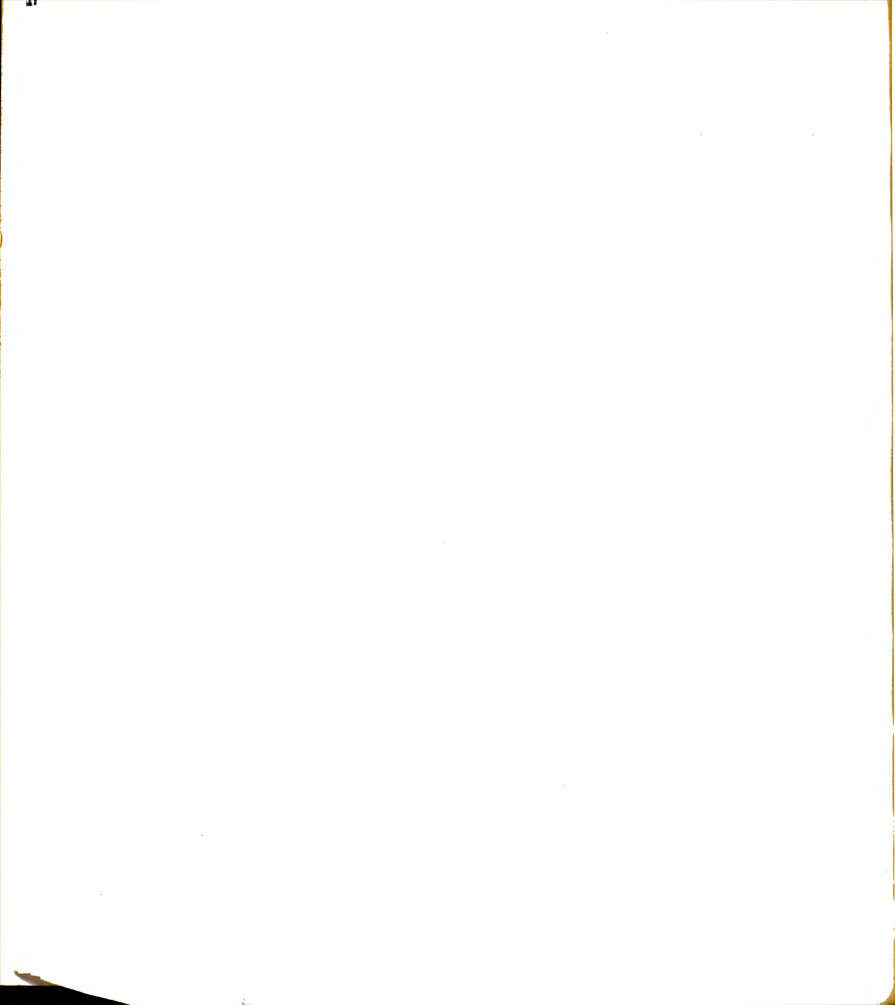
Approach	Testing Plan			
	A	B	C	D
AC	270	270	270	270
BCC	260-261	260-261	255.8	255.80
BFJ 1st	270	250	250.82	246.68
BFJ 2nd	a	254-255	253.94	250.13
McM	259-260 ^b	250 ^b	250.20 ^c	246.30
Equal	258-259	253-254	250.54	248.03
Min	261	255	252	249

^aIndeterminate

^bDull knives CL

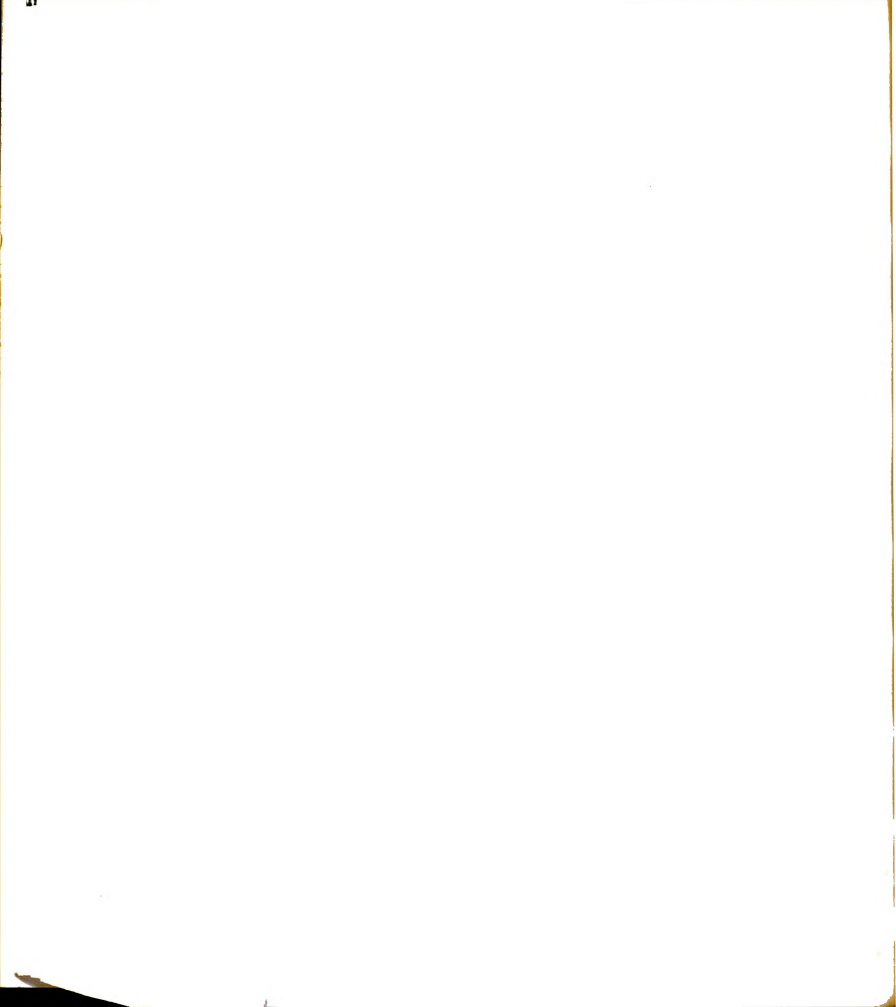
^cPoor attitudes CL

Failure to recognize the average-out problem is even more serious under the conventional control programs which analyze reported variances on a summary basis. Assume that the 1000 performances included in this example represent the performances to be included in the summary report. Table 17 shows that the mean of all these performances is 251. Since this is less than the 10 per cent rule would allow (even less than 5 per cent), accountants typically would regard the variance as insignificant. However, Table 17 also shows that 40 per cent of the performances were attributed to assignable causes--a situation which should certainly be considered significant. If no



assignable causes were prevailing, the accountant has 95 per cent assurance the mean of 1000 performances will lie within 244.4 and 245.6 [$245 \pm 1.96(7.7846/1000)$ where 1.96 is the normal deviate corresponding the middle 95 per cent of the curve and 7.7846 is the standard deviation of the chance performances.]

Therefore, it is recommended that greater emphasis be placed on control at the performance level because it is more timely. Since statistical procedures account for the degree of summary reflected in a report, they should be used to interpret summary results as well as individual performance results.



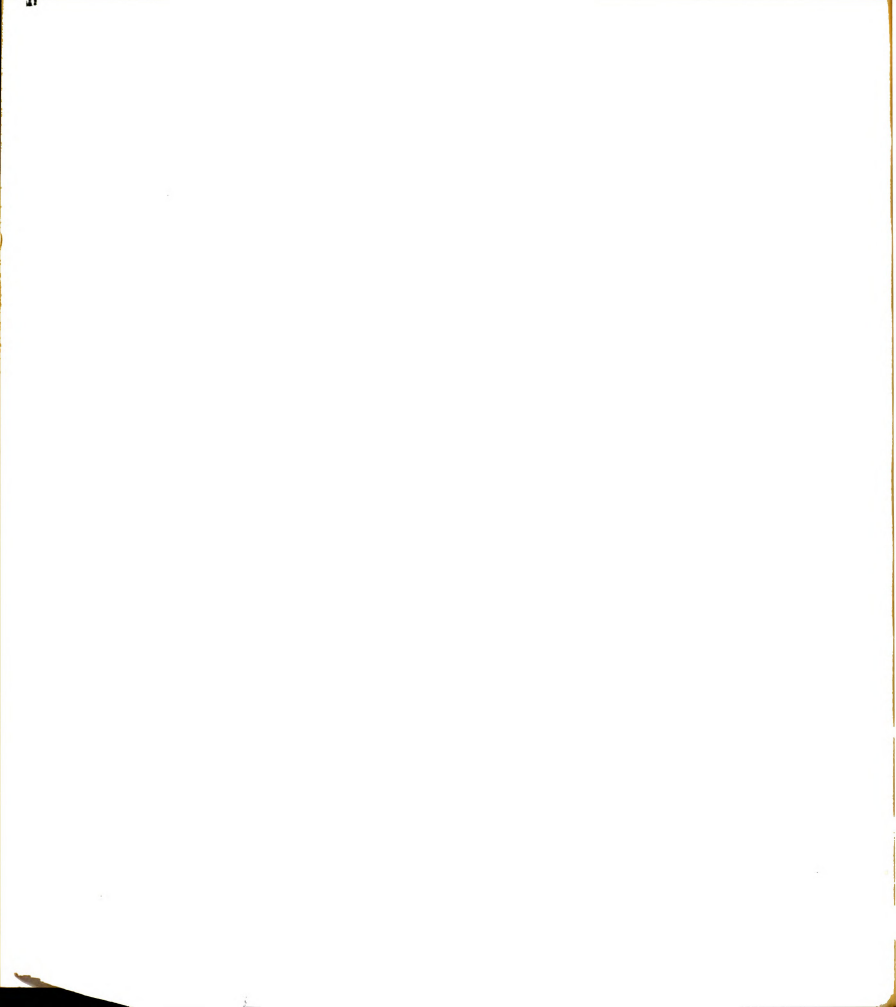
CHAPTER VII

SUMMARY AND CONCLUSIONS

Reason for Study

This study emanated from a general dissatisfaction with the lack of objective criteria utilized in determining the significance of variances from standard. The accountant confronted with variance reports has not employed any structured guidelines to distinguish between when an investigation should be undertaken, on one hand, and when no further action is warranted on the other. Moreover, it was felt that too much reliance is placed on the variance report as a control device. While on-the-spot observation of performance is also currently considered to be an important and timely aspect of control, it was this writer's belief that these control procedures are not of the utmost benefit without an organized and analytical framework to signal the need for follow-up.

Standard costs have been widely adopted. The ultimate goal, of course, is to pin-point areas where investigation is needed. To accomplish this, great effort is typically expended in developing realistic standards. Periodically, detailed procedures are employed to report

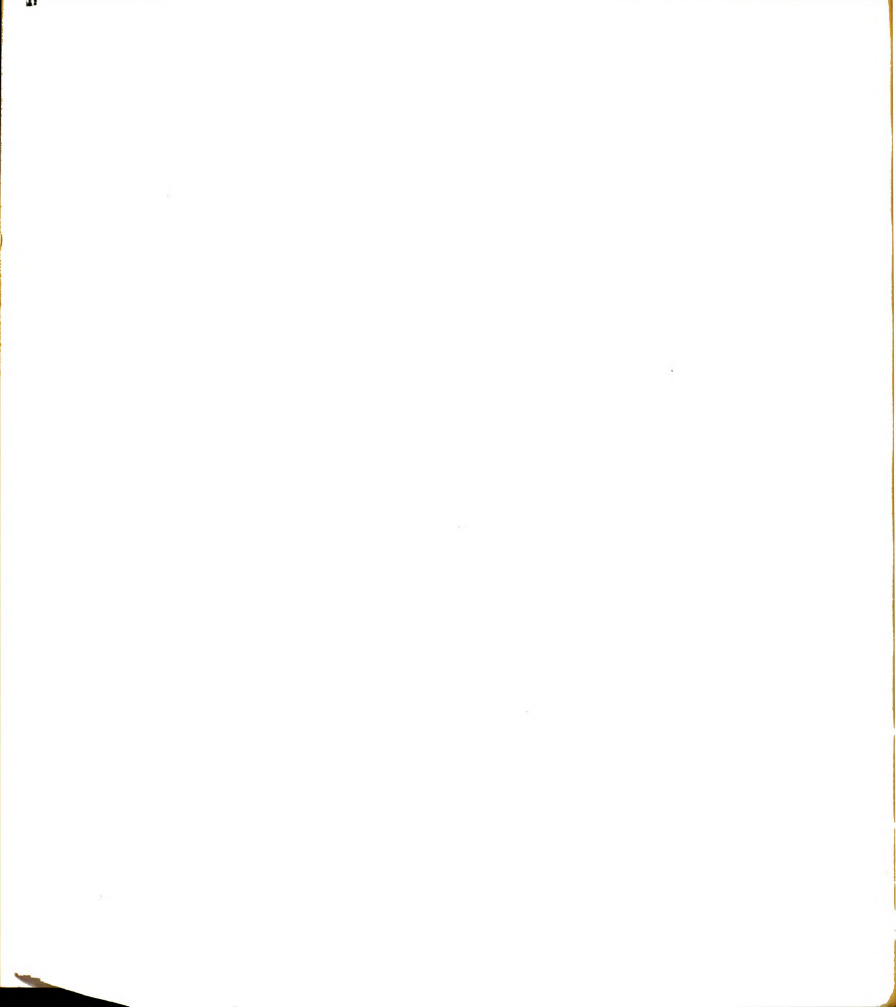


actual performance and to classify variances by source (that is, labor efficiency, material price, etc.). After all this precision this writer felt that it was ironic that the control limits are selected without a formalized framework for consideration of the relevant factors.

After consulting with numerous faculty and practitioners, it was felt that significance determination was a real problem of sufficient import to warrant further study. After a review of conventional variance analysis in Chapter II a number of dissertation objectives were specified. In the following sections, each objective will be reconsidered in light of how it was accomplished and what conclusions resulted.

Conceptual Distinction between Significant and Insignificant Variances

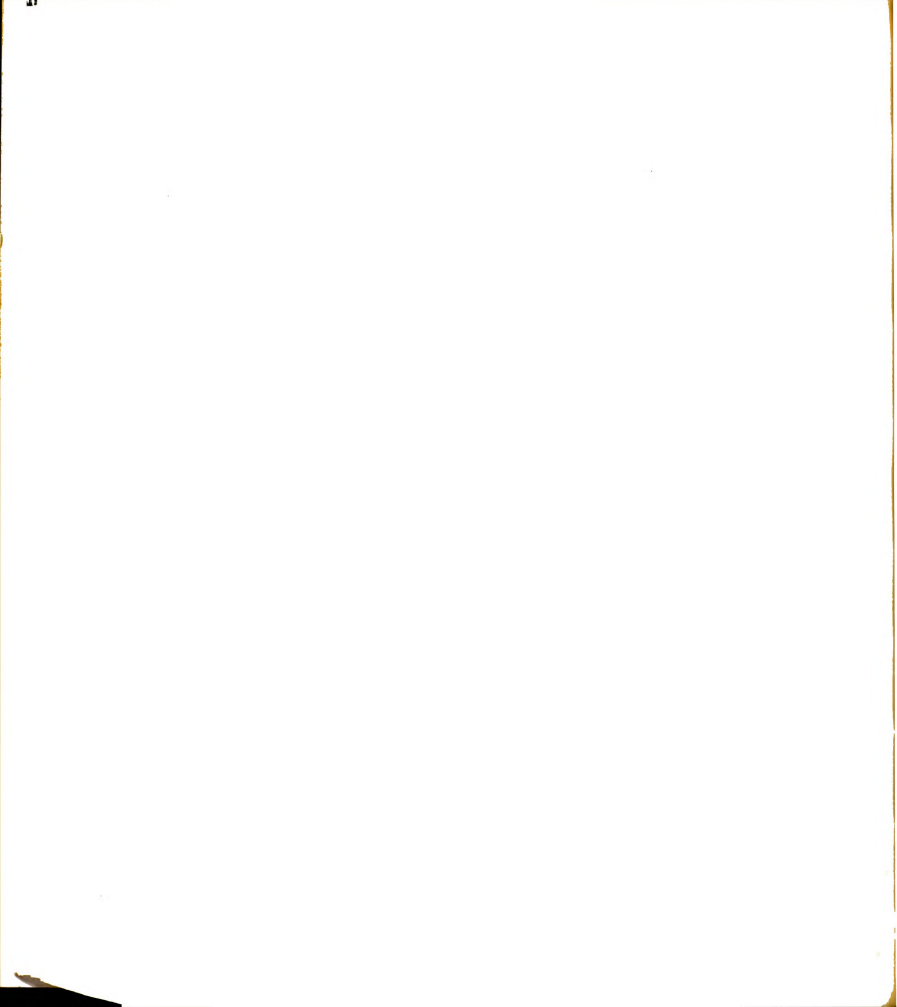
Accounting definitions of control take the following general form. Control entails those procedures designed to make actual results conform to the plan or standard. Such definitions do not account for any variance and certainly do not make a conceptual distinction between significant and insignificant variances. The first objective was to make such a distinction. It was felt that a conceptual framework might supply clues for determining how such a distinction might actually be made in practice. From reading literature in the field of quality control, it became apparent that many types of variances result



from a host of unexplainable factors which are identified as chance. In other words, there is an omnipresent non-uniformity which cannot be eliminated. This non-uniformity is a natural phenomenon--even plants or animals experimentally developed under the same conditions are not identical. Likewise, tasks performed by the same worker under the same conditions are not identical. The reasons for such variation are unknown to man and have been identified by quality control engineers and statisticians as chance. Thus, this objective was accomplished by interdisciplinary study.

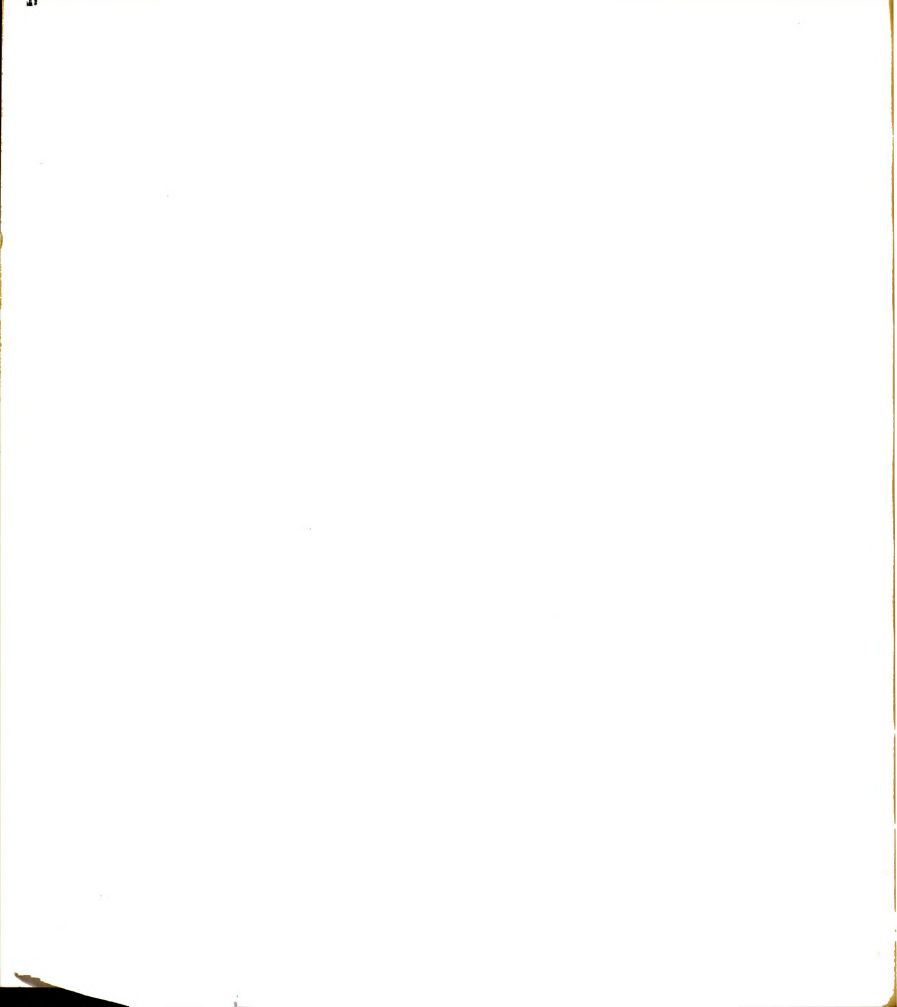
However, an examination of the typical variance classifications revealed that chance is not operative for all of them. For expenses which are either contractual, a matter of company policy, or determined by outside agencies, chance is not operative. In these cases any variance is significant in the sense that its cause should be identified. Thus, chance does not affect the material price, labor rate, or budget variance. It does, however, cause variation in labor and overhead efficiency, material usage, and volume as well as in some non-manufacturing costs.

Three recommendations result from these findings. First, an insignificant variance should be regarded as one due to chance factors. Since these variances can neither be explained nor eliminated, there is no reason to



undertake an investigation. Second, a significant variance should be regarded as one resulting from some cause which is capable of being identified (assignable cause). Accordingly, an investigation should be undertaken to identify the cause. Third, this distinction should be incorporated into the accountant's definition of control. For variance classifications which entail chance factors, control would be defined as consisting of those procedures designed to maintain variation within limits due to chance. For those variance classifications where chance is not operative, the conventional accounting definition as the procedures designed to make performance conform to the standard is satisfactory.

With all of this settled, it would now seem to be a relatively simple matter to observe performance under established conditions for the purpose of determining the limits within which chance is operative. The difficulty is that the distribution of values due to chance overlaps the distribution of values due to assignable causes. The dilemma remains: Where should the control limits be placed? Recognition of chance factors, however, provides the clue for confronting this problem and for utilizing more objective criteria for significance determination. Since probability statistics is an area concerned with procedures for evaluating the patterns of chance influences, its use is a logical extension from the recognition that chance



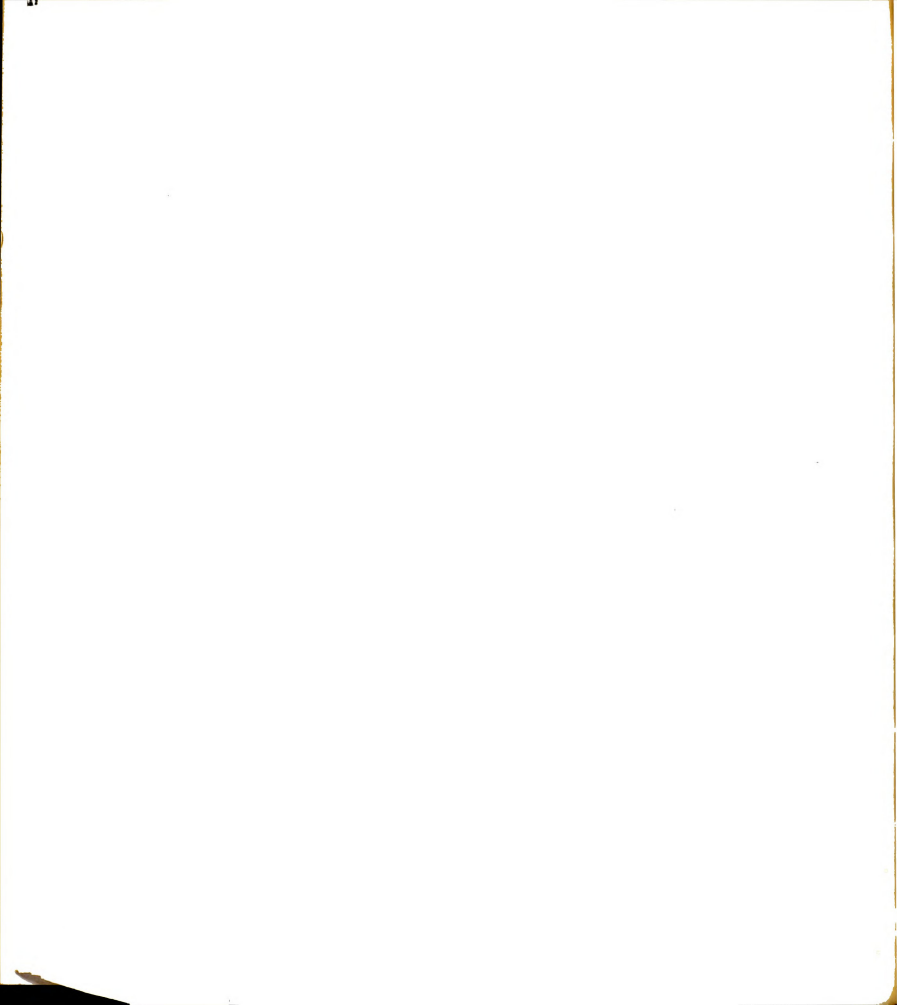
factors distinguish between significant and insignificant variances.

Examination of Statistical Models

The second dissertation objective was to examine more objective criteria for significance determination. This was accomplished through two major steps. First, three statistical models that have been proposed by others were evaluated. One of these, which has been identified as the Basic Control Chart approach, employs conventional Classical statistics. Another, that was devised by Bierman, Fouraker, and Jaedicke, actually involves two approaches. The third of these was conceived by Leo McMenimen in a Master's thesis from The Pennsylvania State University. Since each of these available models involves some questionable aspects, this writer constructed two additional models. These represent new applications of already developed statistical concepts.

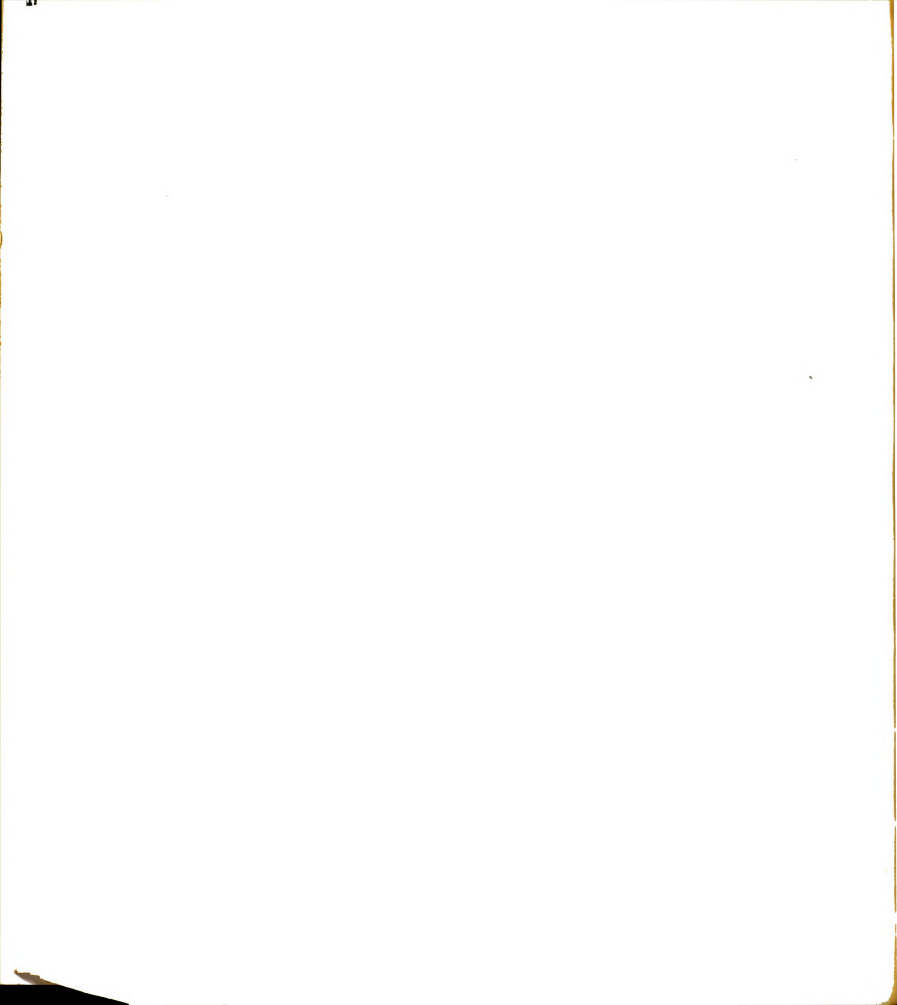
These statistical models are capable of considering various combinations are the following eight relevant factors:

1. Distribution of values of chance performances.
2. Distribution of values for each assignable cause.
3. Probability of making an unwarranted investigation (Type I error).



4. Probability of accepting variance when an investigation is warranted (Type II error).
5. Opportunity cost of Type I error.
6. Opportunity cost of Type II error.
7. Prior probabilities of the occurrence of chance and each assignable cause.
8. Probability that any given variance is due to chance and the probability that it is due to each assignable cause.

The Basic Control Chart approach formally considers the distribution of chance performances (factor 1). This in turn enables the evaluation of the probability of committing a Type I error (factor 3). This approach may in some undefined way also evaluate the probability of committing a Type II error (factor 4) for some alternative parameter that is considered serious. If this is done, an attempt would be made to select a control limit that would yield a "low" probability of a Type II error without making the probability of a Type I error too "high." The difficulty is that there are no available criteria for determining what is "high" and what is "low." With or without consideration of the probability of a Type II error, this approach would normally involve selection of a control limit that would yield a level of significance between .001 and .05. The major objection to this approach is



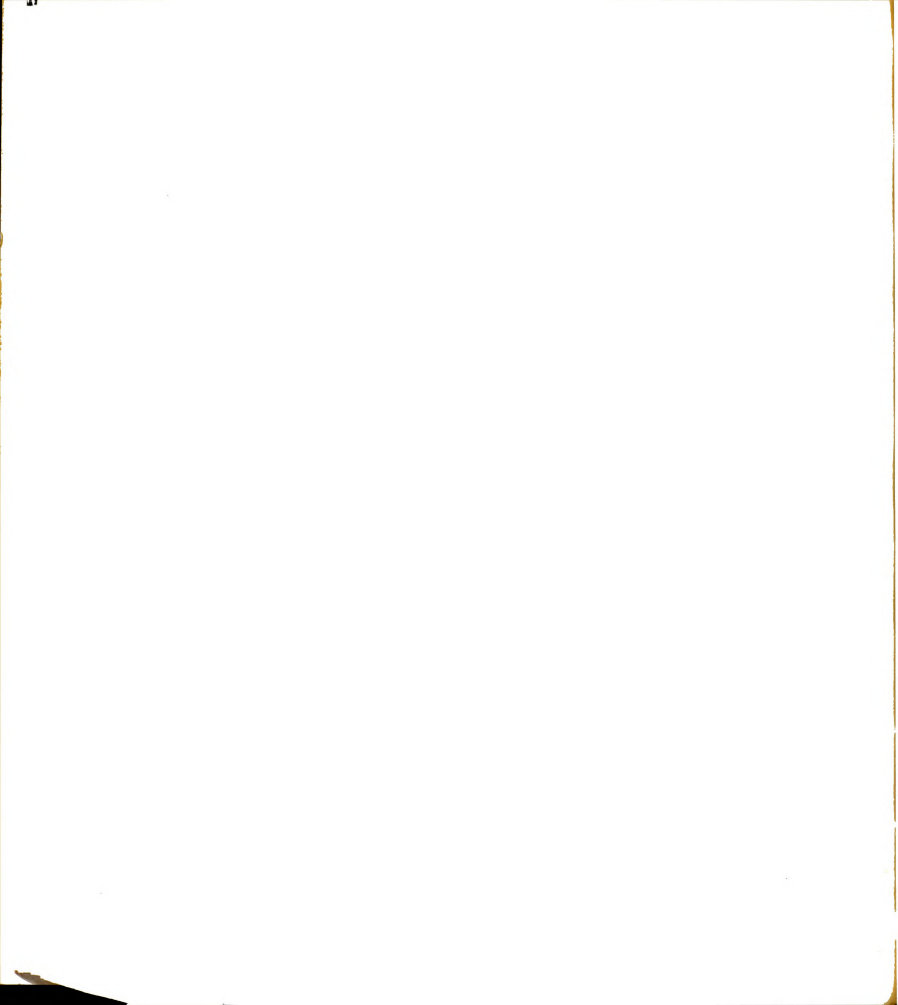
that it does not consider the opportunity costs associated with each type of error. Without this, the analyst cannot know when he has achieved a good balance between a Type I and a Type II error.

Bierman, Fouraker, and Jaedicke introduced a model which actually turned out to be two models because of their inadvertent dual definitions of the probabilities used in their model. From their numerical example, it is obvious that they intended P to be the same as the classical probability of a Type I error.¹ However, in an attempt to improve upon the Basic Control Chart approach these writers introduced the cost of an investigation and the expected opportunity cost resulting from failure to identify an assignable cause (factor 6) formally into their model. They did not, however, attempt to incorporate the probability of committing a Type II error into their model.

They apparently thought that they were restating the definition for the probability of committing a Type I error when they interpreted P as "the probability of an unfavorable deviation resulting from uncontrollable [chance] causes."² Unknowingly, then, they introduced into the

¹In this dissertation this approach has been referred to as the Bierman, Fouraker, and Jaedicke first interpretation of P .

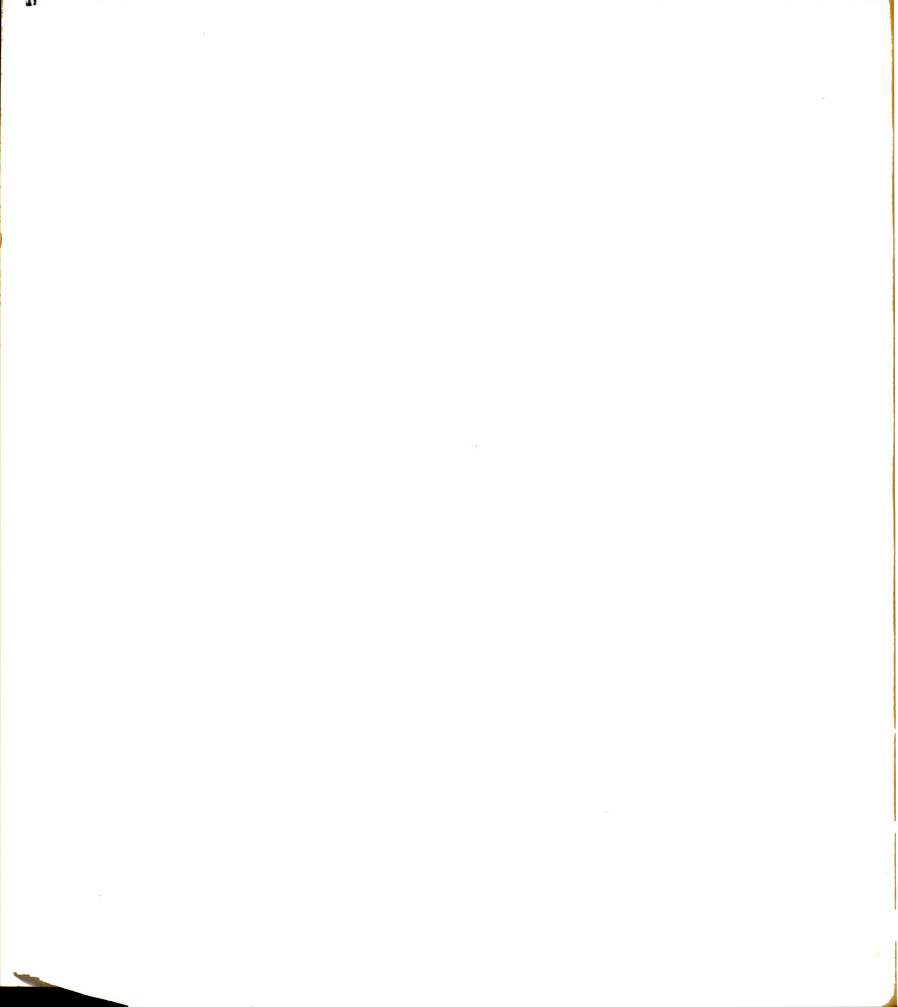
²Bierman, Fouraker, and Jaedicke, 121.



model a different kind of probability (factor 8) which this writer considered separately in an approach which he identified as the Bierman, Fouraker, and Jaedicke second approach. This approach explicitly considers factors 1, 2, 6, and 8. It also considers the cost of an investigation.

Even after these different interpretations of P are noted, both approaches still involved several questionable aspects which are summarized briefly below. First, in their numerical example Bierman, Fouraker, and Jaedicke used a summary expense classification for a period of one year rather than individual performances. Second, they assumed that the mean of the assignable cause is equal to the test value (or actual result)--a condition which would only coincidentally be true. Third, they arbitrarily assumed that an off-standard condition requires four tests on the average before detection. The second and third aspects result in a poor evaluation for the opportunity cost of a Type II error (factor 6). Fourth, they treated the cost of an investigation as a constant when, in fact, the cost of an investigation depends upon the cause and the order of the investigation procedure followed.

Because McMenimen did not include a numerical example it is difficult to tell what his precise treatment would be. It does, however, appear that his model

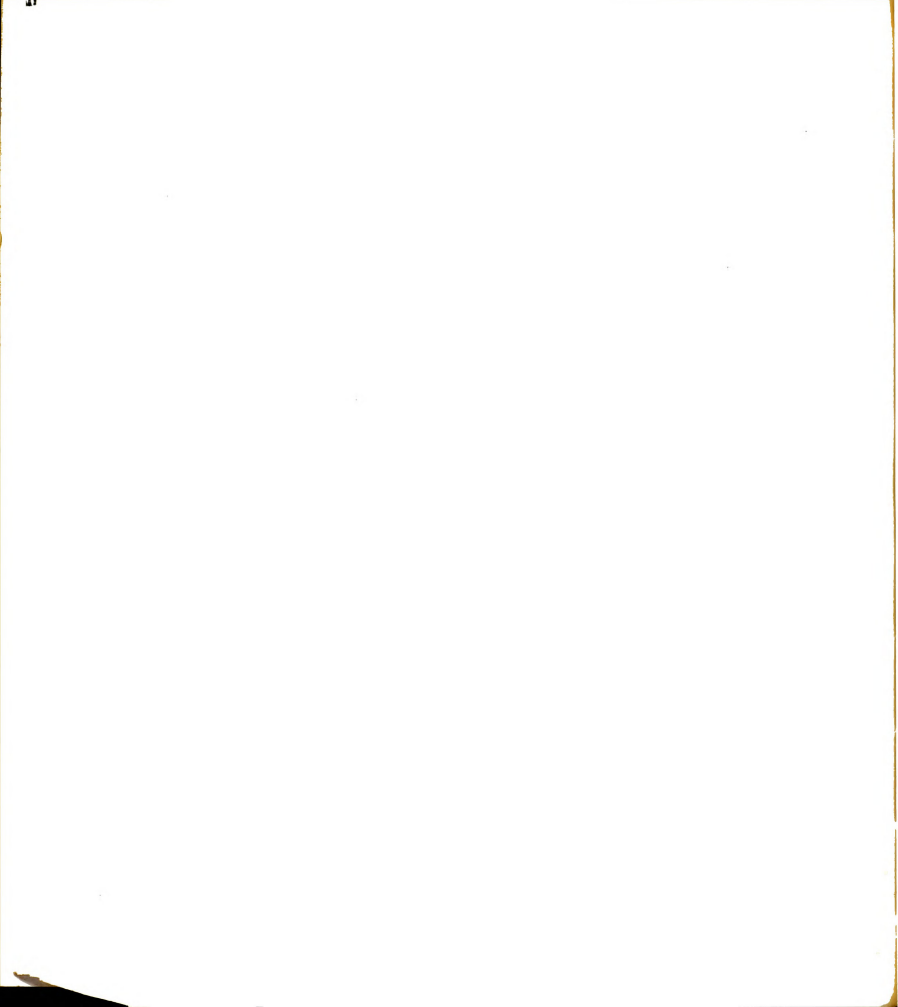


would consider factors 1, 2, 5, 6, and 8. This approach has two distinguishing features:

1. An investigation might be terminated before finding the cause.
2. A cost deviation might be reduced by various amounts.

It is this writer's opinion that the first of these features is a good one, particularly if at some point in the investigation process there appears to be a low probability that the variance is attributed to an assignable cause other than those already investigated. Conversely, the second feature is not regarded as valid. It should be worthwhile to maintain the standard if it was realistically established. If it was not or if conditions have changed, the standard should be revised. Stated simply, a deviation known to be attributed to an assignable cause should not be permitted to continue. This writer also implied that McMenimen would assume that the mean of the assignable cause is equal to the test value and that an off-standard condition requires four tests on the average before detection. These are also assumptions of Bierman, Fouraker, and Jaedicke but neither is valid.

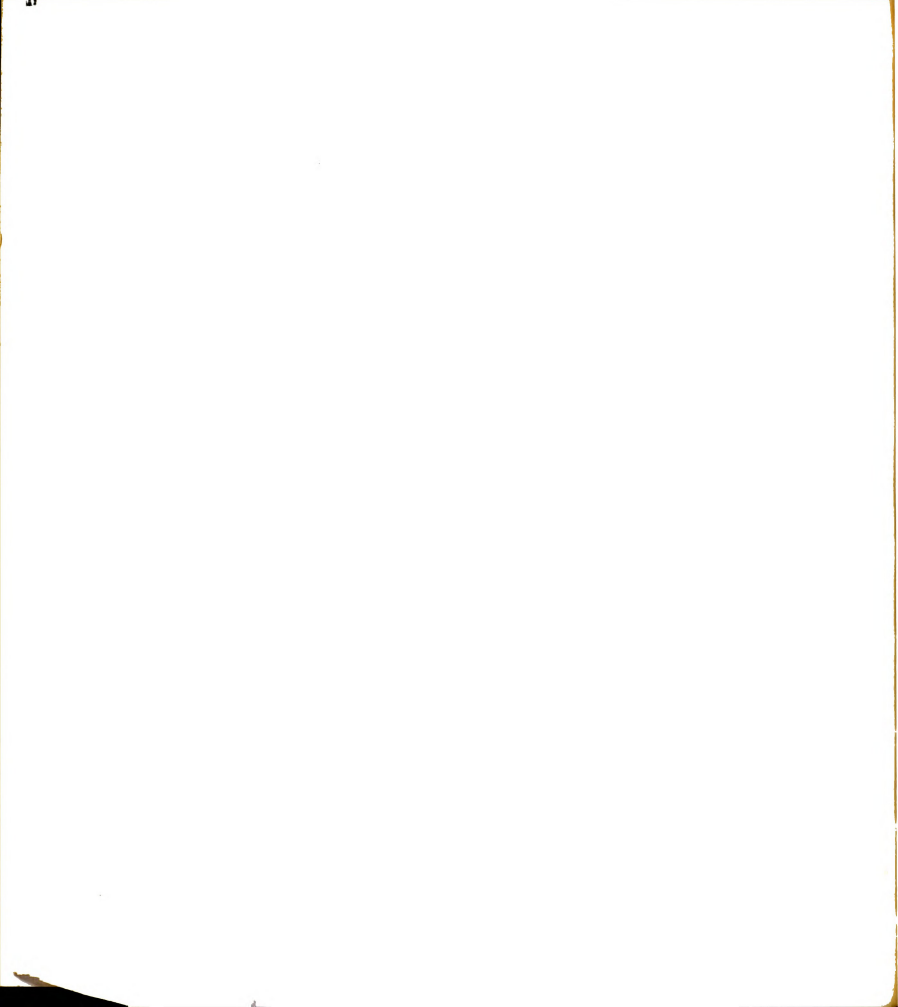
In an effort to counteract the limitations just noted for the Basic Control Chart; Bierman, Fouraker, and Jaedicke; and McMenimen approaches, two additional models



were constructed. These have been identified as the Equalization and the Minimization approaches respectively. The Equalization approach considers factors 1 through 6 inclusive. The Minimization approach considers factors 1 through 7 inclusive.

Example Testing the Relative Control
Effectiveness of the Conventional
Accounting and the Various
Statistical Methods

The third objective was to illustrate through an example the superiority of the statistical models over the procedures conventionally employed by accountants. The test consisted of three parts. First, a hypothetical example was developed for which the causes and performance values of 1000 performances of a certain operation were assumed. Second, these values in conjunction with economic assumptions were used to compute the upper and lower control limits for each of the models under four different testing plans. These models included all of the statistical models in addition to the 10 per cent cut-off point which was selected to represent the Accountant's Conventional approach. The third phase of the test consisted of a financial analysis conducted to rank the approaches for control effectiveness for each corresponding control limit and testing plan. The financial analysis consisted of analyzing the approaches by twos insofar as it was necessary to rank them in preferential order. This analysis took the following general form. Of any two approaches

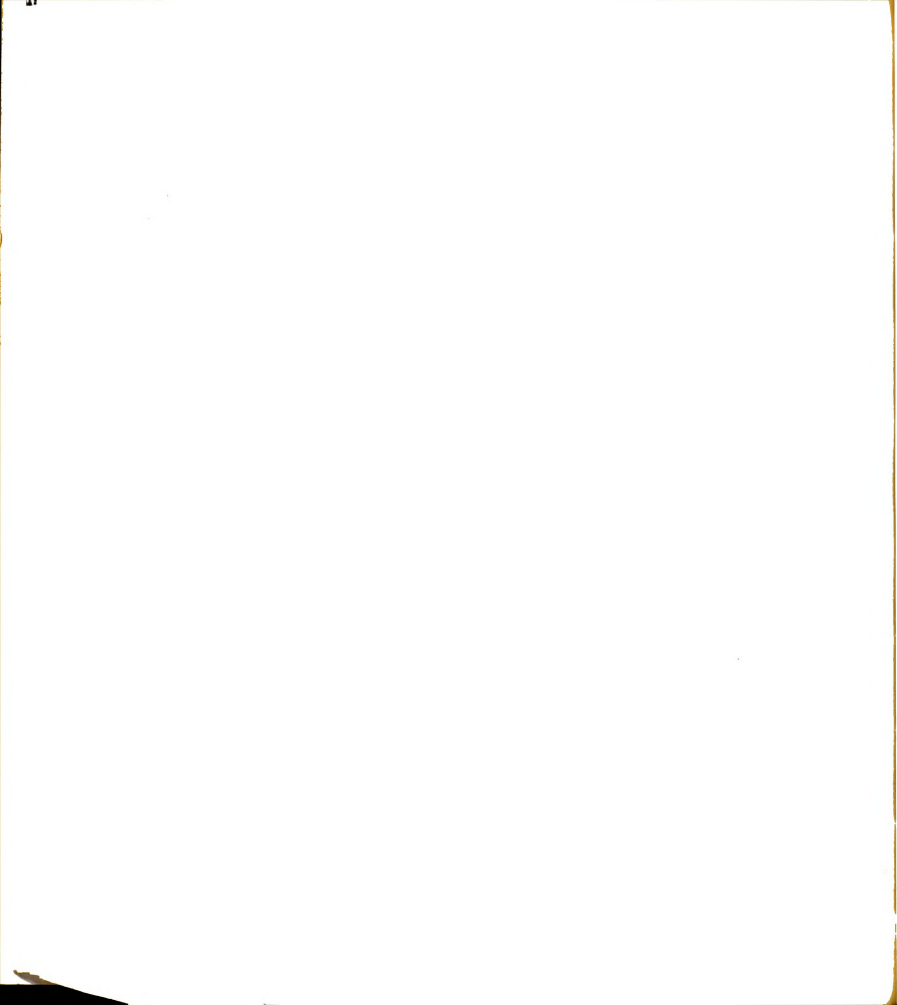


being compared, the one closer to the standard bears a greater investigation cost than the one farther from the standard. However, it also carries additional savings because it signals more investigations and thus detects assignable causes earlier. The additional investigation costs and the additional savings are computed by a rather technical process which is explained in Chapter VI. A decision is made on the following basis.

1. If the added savings is greater than the added investigation cost, the approach with the control limit closer to the standard is regarded as more effective.
2. If the added savings is less than the added investigation cost, the approach with the control limit farther from the standard is regarded as the more effective approach.

This analysis for each pair of approaches was performed until it became possible to rank all of the approaches.

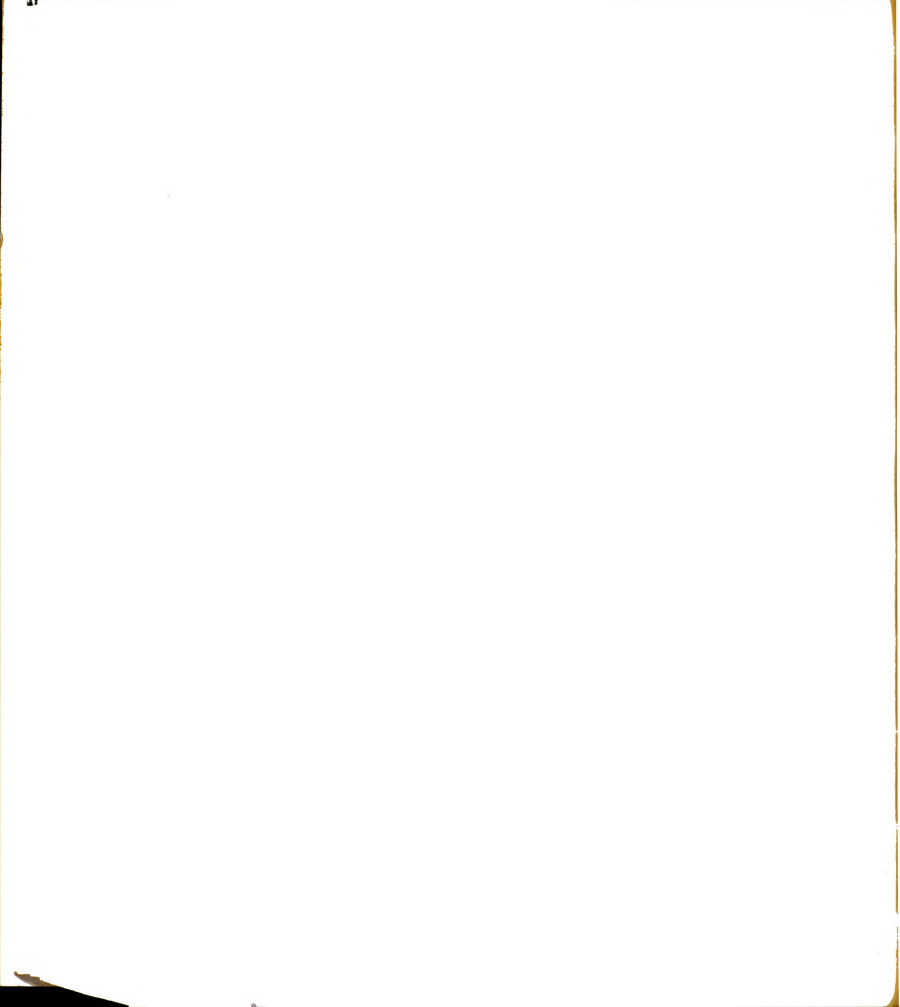
After summarizing the rankings, it was concluded in Chapter VI that no one approach ranked first for each control limit under each testing plan. The Minimization approach was, however, either first or tied for first in five out of the eight cases. Moreover, it was either second or tied for second in each of the remaining three cases. (Refer to Table 74.) Furthermore, the sum of the



rankings was lowest for the Minimization approach. It, then, was generally, but not always, the most effective. The superiority of this approach was anticipated because it considered more (seven) of the eight relevant factors than any of the other approaches. Also, it was constructed to eliminate the questionable aspects involved with the statistical models that have been proposed in the literature.

Unexpectedly, the Bierman, Fouraker, and Jaedicke second and McMenimen approaches achieved overall rankings of two and three respectively. These were the only two approaches to incorporate factor 8 directly into the model. The test, then, indicated that the probability that any given variance is due to chance and the probability that it is due to each assignable cause is an important determinant of effective control limits. In fact, in this example it was sufficiently important information to outweigh some noted questionable aspects associated with these approaches.

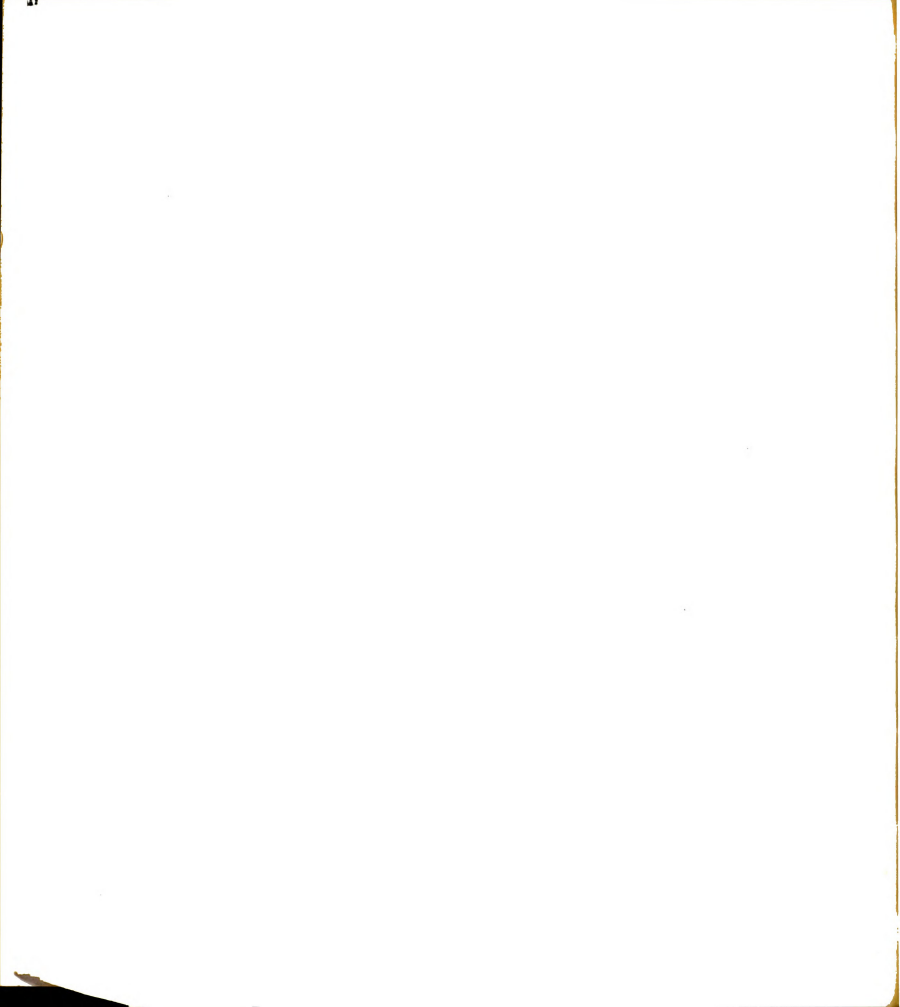
The three remaining statistical models involve either Classical statistics or extensions of Classical statistics. As expected, the Equalization approach which formally considers factors 1 through 6 outranked both the Bierman, Fouraker, and Jaedicke first and Basic Control Chart approaches.



It should be noted that these conclusions are true in general and not for each individual control limit with its corresponding testing plan. The "Stability of Generalizations" section in the conclusions to Chapter VI explained how it might be possible to tamper with the probability distributions in Table 18 in order to achieve different results for individual circumstances.

The reader will recall that the objective of this test was to illustrate the superiority of the statistical models over the procedures conventionally employed by accountants. The most significant conclusion, therefore, is that the Accountant's Conventional approach was designated as generally the least effective method of control. The following four findings support this conclusion:

1. The Accountant's Conventional approach achieved the least effective ranking for six out of the eight cases.
2. It obtained the highest sum of rankings.
3. The control limits under the various statistical approaches are generally fairly closely grouped about the control limit achieving the number one ranking while the Accountant's Conventional approach produces control limits fairly far apart from this grouping. As a result, differences in the rankings of the statistical approaches are



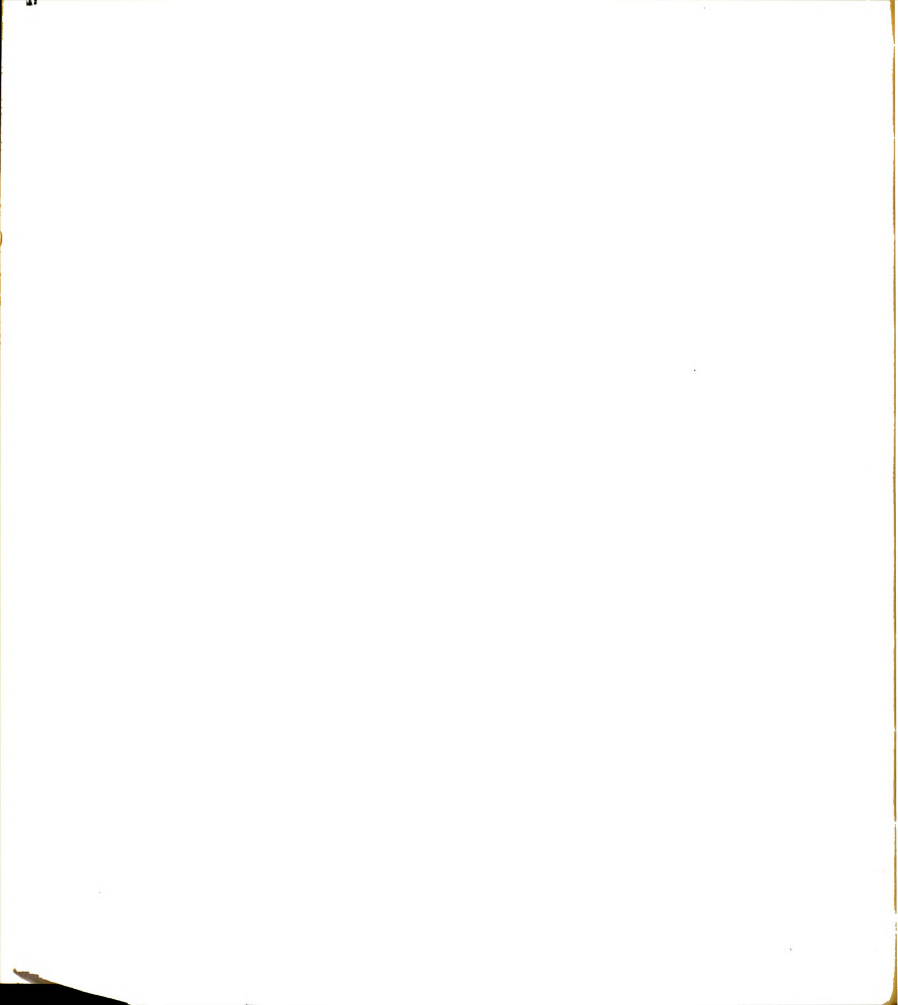
not nearly so significant as the difference between the statistical approaches taken together and the Accountant's Conventional approach.

4. The dollar difference between the added investigation cost and the added savings is relatively greater for financial analyses between the Accountant's Conventional approach and the approach which occupies the next highest ranking than it is for analyses between approaches not involving the Accountant's Conventional approach.

Aggregation Problems

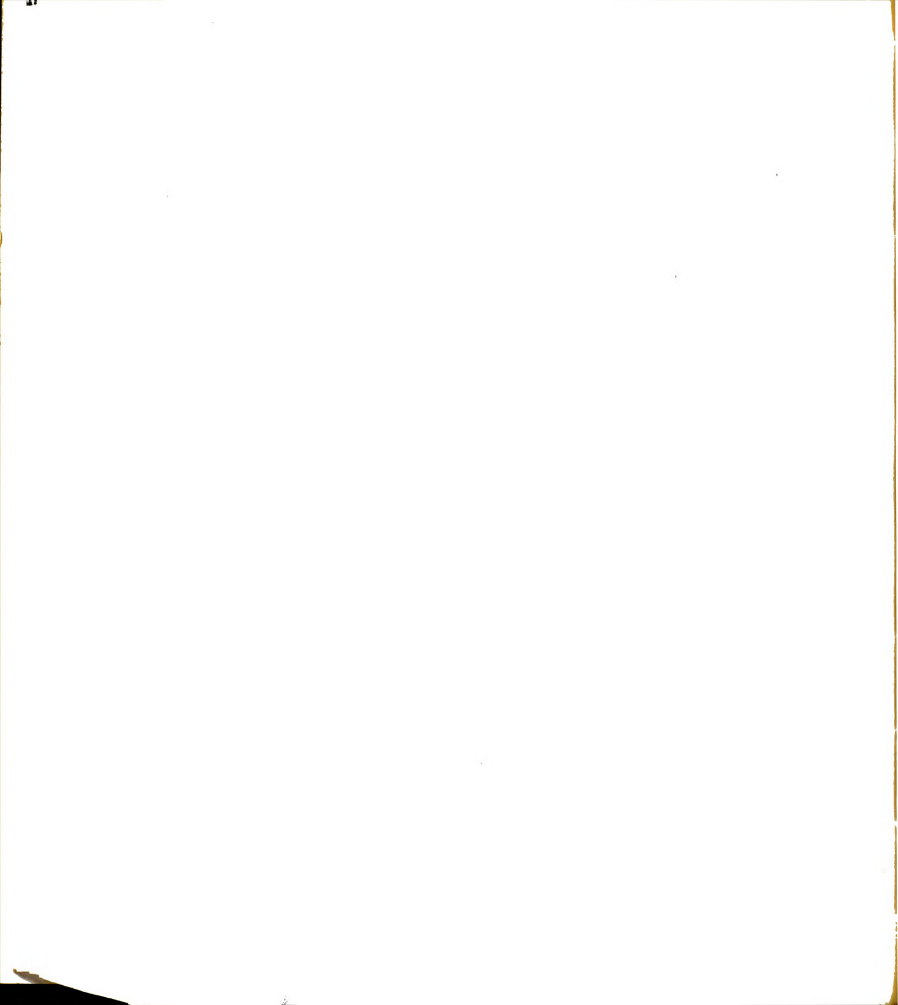
The fourth objective was to show through this example the tendency for significant variances to be averaged-out in the process of accumulation used in developing the typical performance report. Table 17 shows that 40 per cent of the 1000 performances upon which the example was based are attributed to assignable causes.

From this, it would appear that the operation is really not in control. However, an analysis typically employed by accountants might fail to disclose this fact. This type of analysis involves a comparison of the actual dollar cost of the operation for some period of time with the budgeted dollar cost. For convenience, assume that this period of time is coincidental with the time required to complete the 1000 performances. In physical terms,



this same analysis would compare the mean of 1000 performances with the standard. Table 17 indicates that the actual mean of these 1000 performances was 251. The difference between 245--the standard--and 251 would frequently be regarded by the accounting analyst as insignificant. In fact, under the 10 per cent rule conventionally employed significance would not be recognized unless the mean of the 1000 performances was as high as 270. Even the Accountant's Conventional approach applied on an individual performance basis would provide more adequate control by detecting significance more readily than the common sole reliance on the summary report.

This is an example of what may happen when aggregate reports rather than the individual performances are the basis of control. Aggregate reports may be useful for reviewing how effective control has been; but, here again, accountants should make use of statistical concepts. The more performances that are represented in the report, the closer the mean should be to the standard. In this situation significance would be indicated if the mean of the 1000 performances fell outside the range from 244.4 to 245.6. This range represents 245 ± 1.96 times $7.7846/1000$ where 7.7846 is the standard deviation of the 600 chance performance and 1.96 is the normal deviate which includes 95 per cent of the chance performances. Even if the mean of the 1000 performances fell within the

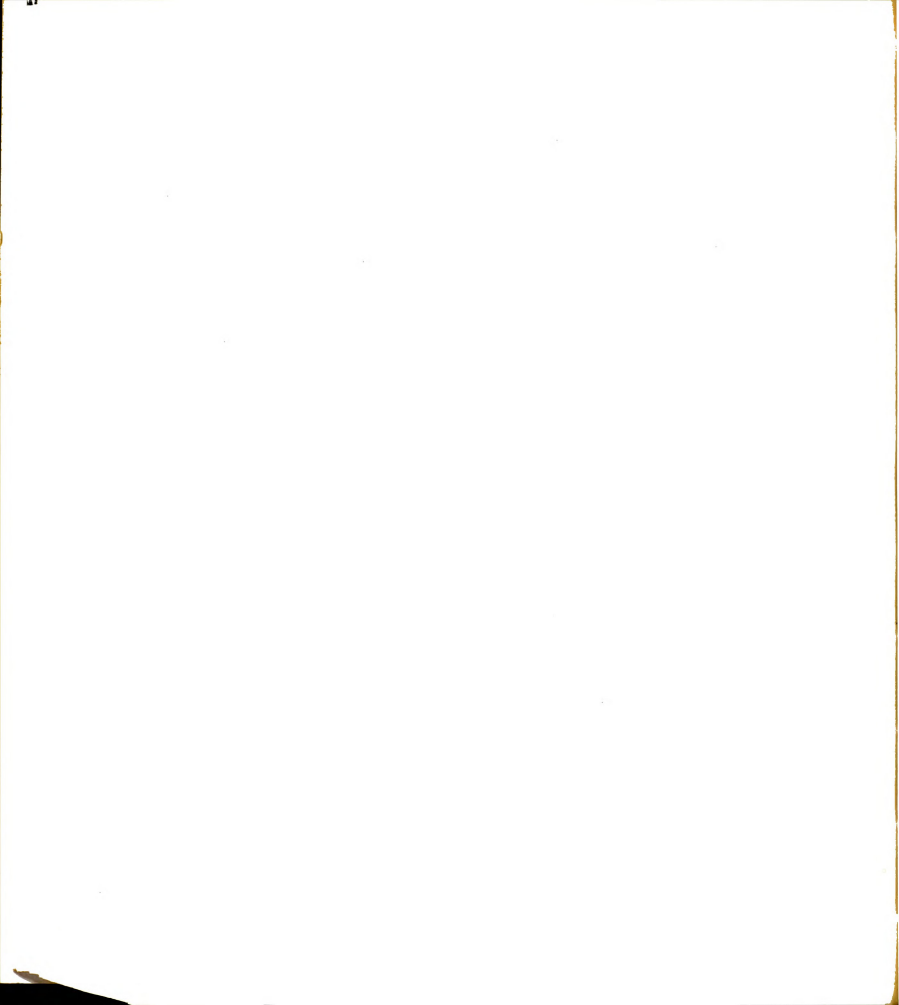


above interval some unfavorably significant variances may have been off-set by favorably significant ones. For this reason, and also in order to achieve more timely control, it is suggested that the major focus of control be addressed to individual performances rather than to weekly or monthly reports. This could, of course, be accomplished through sampling at the performance level in a manner similar to testing plan D.

Along these same lines, it should be noted that the control limits under the Accountant's Conventional approach remain the same regardless of the sampling plan. There is a strong tendency, however, for the control limits under the statistical approaches to move closer to the standard as fewer performances are tested. In general, two factors account for this:

1. Type II errors become more expensive so it is important that they be made less frequently.
2. As the sampling of more than one performance is introduced allowance is made for the fact that occasional extreme chance performances will be averaged-out by the more frequent performance values closer to the standard.

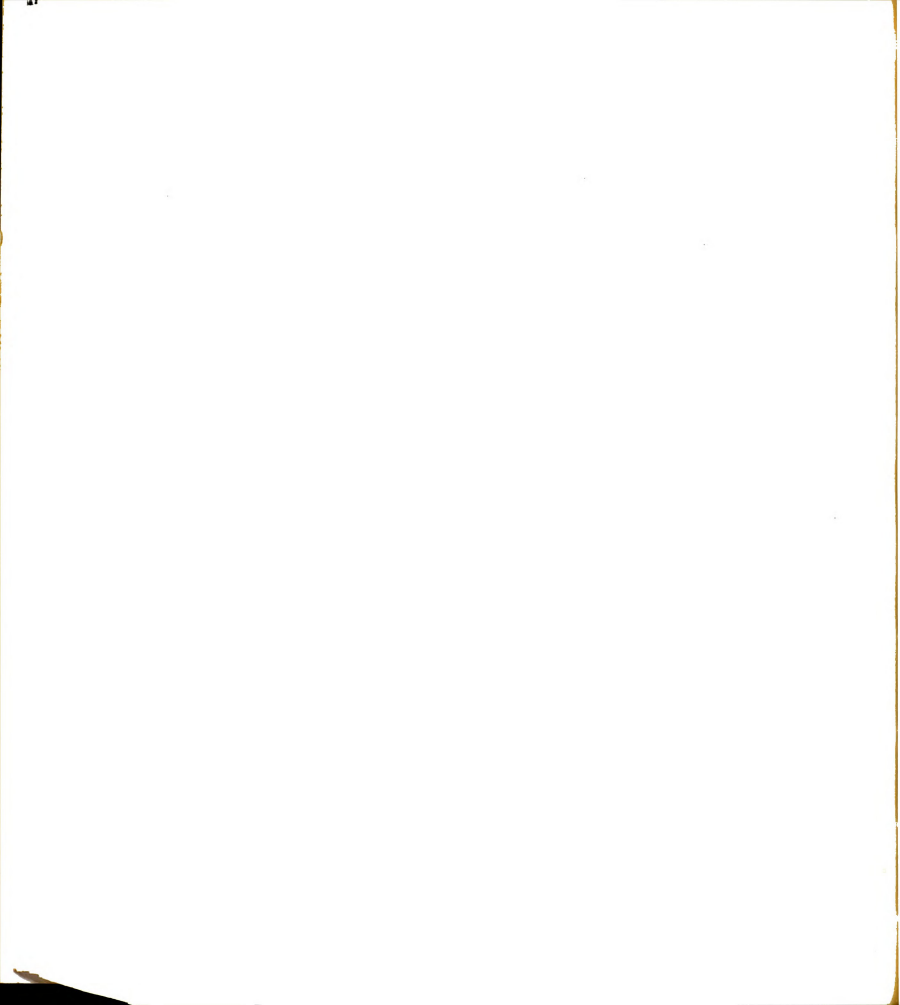
Mere recognition of the average-out problem will not eliminate it. However, it could be greatly reduced under any method of significance determination by focusing



greater control attention at the performance level. Further reduction would result from using statistical methods of significance determination which select the control limits to account for the average-out effect. Moreover, statistical concepts can be used at the summary report level. These account for the average-out effect by reducing the amount of allowable variation as the degree of report summarization is increased. For a report covering the 1000 performances, the mean of these performances should fall in the range between 244.4 and 245.6. This range could easily be expressed in dollar terms by multiplying 244.4 and 245.6 respectively by the standard³ wage rate per minute.

Some may argue that control at the performance level would be more expensive. There is, of course, the initial cost involving the time required to estimate the values pertaining to the relevant factors involved in calculating the control limits. For the Minimization approach, Bayes' Theorem should be applied periodically thereafter to revise factor 7. Once this is done, the extra cost of maintaining control at the performance level should be small. Procedures are currently used to accumulate information by performance for the summary report.

³The standard wage rate is used because any labor rate variance should be removed from this analysis.



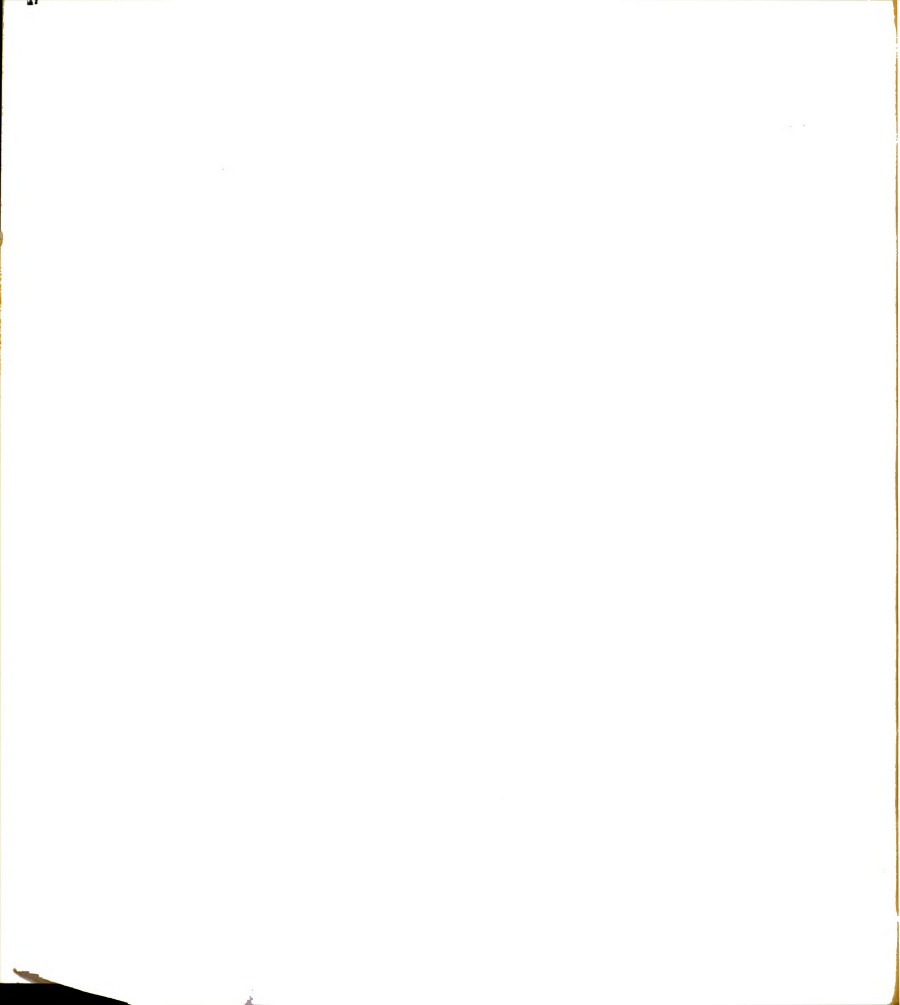
The additional time required to compare performance results with control limits at this point would not appear to be great and, of course, sampling can be used. The control limits simply provide guidelines for performance observation which is currently used by the worker and his foreman and sometimes even by those higher in command.

Under current procedures off-standard conditions continue until they are arbitrarily deemed significant on summary reports.⁴ Then time must be spent localizing them. Therefore, continued savings resulting from more timely detection of assignable causes in conjunction with increased detection resulting from reduction of the average-out effect should compensate for the increased analysis at the performance level.

Justin Davidson who has been active in applying statistical techniques to auditing and accounting problems estimates that the set-up costs would approximate \$5,000 to \$10,000--a range which he regards as modest for a system change.⁵ These costs would include the cost of establishing control limits, explaining the details in non-mathematical terms to those in charge, and writing a simple

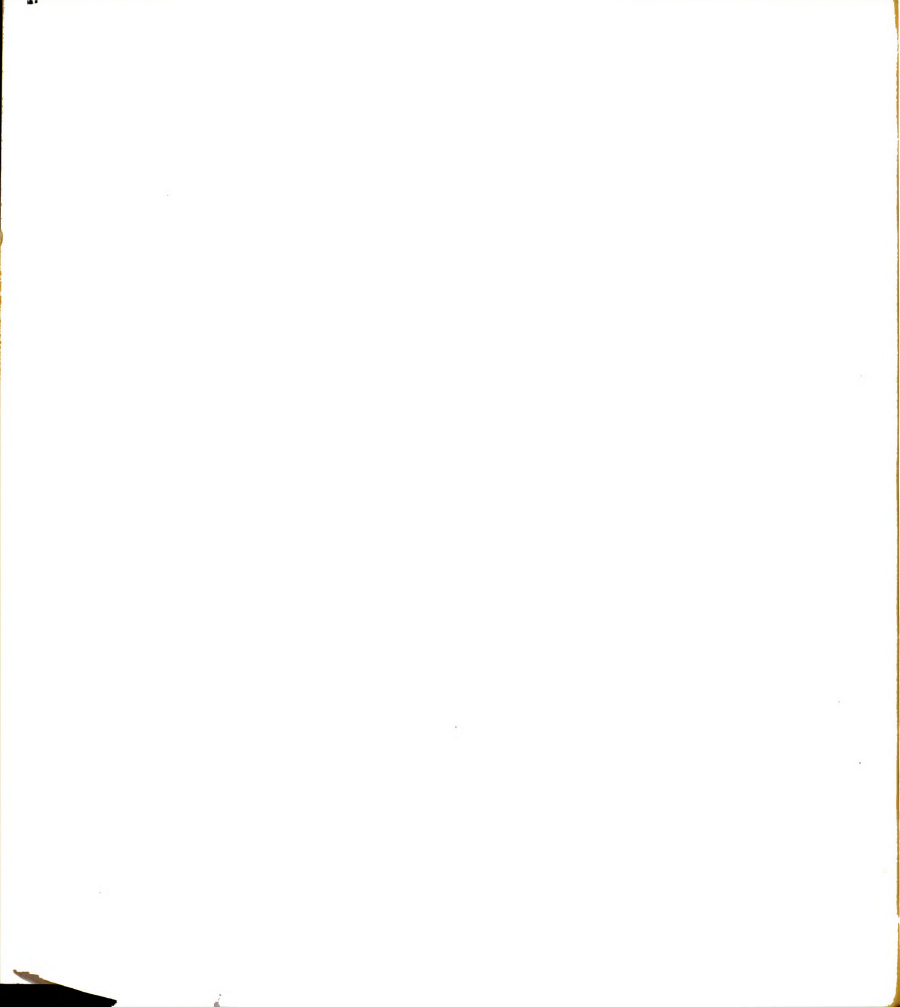
⁴Off-standard conditions may also be detected by observation but there are currently no organized criteria employed to detect assignable causes on this basis.

⁵Reported on telephone conversation on July 21, 1967. Mr. Davidson is a partner with Touche, Ross, Bailey, and Smart.



set of instructions for the worker and foreman who must maintain the system. The costs would be lower for companies whose controllers have used statistical procedures for other purposes. Mr. Davidson feels that the savings would soon compensate for these costs. For ten years he has believed that accountants should begin to establish statistical control limits for use in accounting variance analysis.

The accountant may now wonder whether there are a sufficient number of statistically competent accountants available to instigate such procedures. In Mr. Davidson's opinion, 25 per cent of the companies that have a standard cost accounting system have internal talent capable in this area. The remainder would need outside help. Mr. Davidson thinks that all of the big eight Certified Public Accounting firms have staff skilled in statistical applications. Some of the smaller national firms also have personnel proficient in this field. There is a growing awareness in the business community of the advantages of statistical and mathematical applications. Accordingly, at the college level business curricula are requiring heavier emphasis in these areas. At the post graduate level, there have been an increasing number of mathematics and statistics seminars to better acquaint professional people with the advantages of applications in these areas. There are, then, at least a



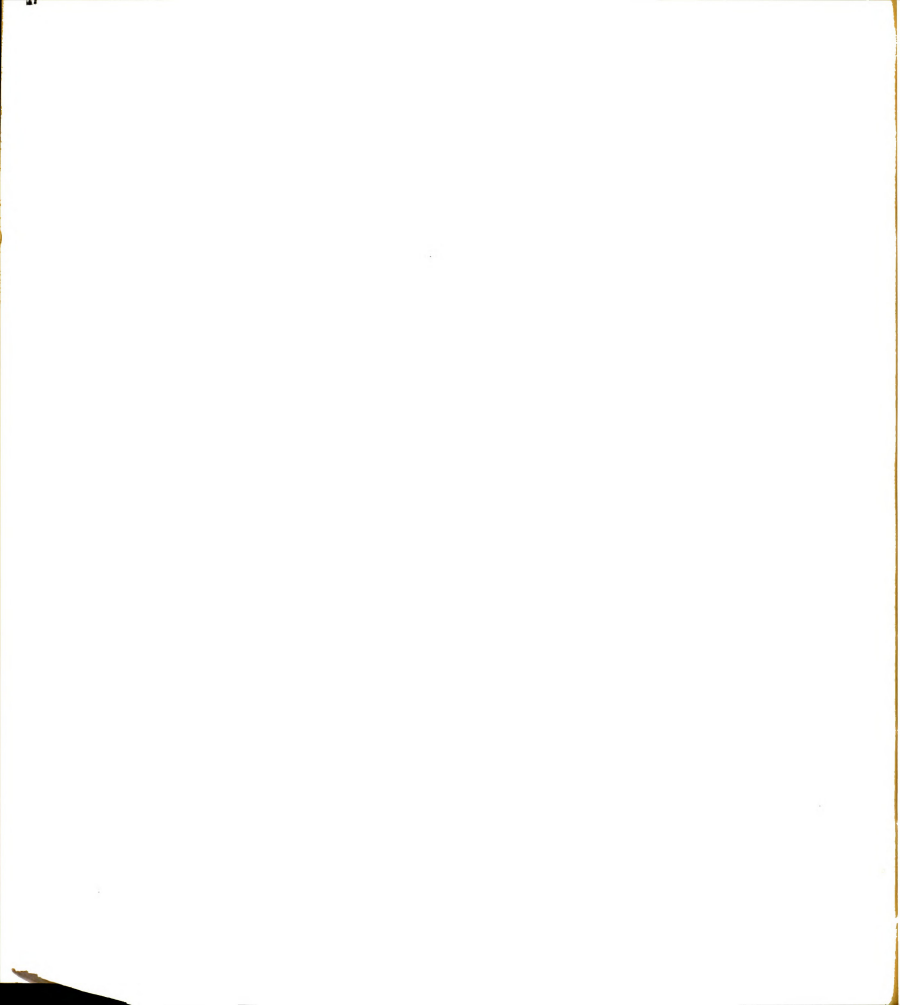
sufficient number of personnel available to begin establishing systems for statistical variance control and others are being educated for such work.

It is, therefore, recommended that statistical control limits be established at the performance level. Perhaps it would be wise to begin with a few of the more important operations. If the savings readily compensate for the set-up costs as Mr. Davidson and this writer feel they will, these procedures would logically be extended to include more operations.

Summary of Conclusions

The following conclusions result from the study:

1. The accounting definition that control consists of those procedures designed to make actual results conform to the standard does not explain why some variances are not investigated.
2. Unexplainable factors called chance cause variation in labor and overhead efficiency, material usage, and volume as well as in some non-manufacturing costs. Variances that result solely from chance should be identified as insignificant. Those resulting from chance and assignable causes should be identified as significant. The recognition that chance factors cause variable performance leads to the testing of statistical models since



probability statistics deals with evaluating patterns of chance occurrences.

3. There is very little literature dealing with statistical applications to accounting variance analysis⁶ and most of it deals with procedure rather than concept. Moreover, there is little evidence to suggest that the procedures that have been proposed are used. In fact, after some inquiry this writer has been unable to find a single case of their usage. This could be accounted for because the accountant not having recognized chance factors has no logical reason to search for statistical models. Another possible explanation is that many accountants are not currently statistically sophisticated. Still another reason might be that they have discovered some of the questionable aspects of the proposed models and have discarded them.
4. As a group, the statistical methods produce significantly more effective control than the Accountant's Conventional approach. This is illustrated in the example. Thus, the hypothesis is confirmed.

⁶What literature is available is listed in Appendix A.

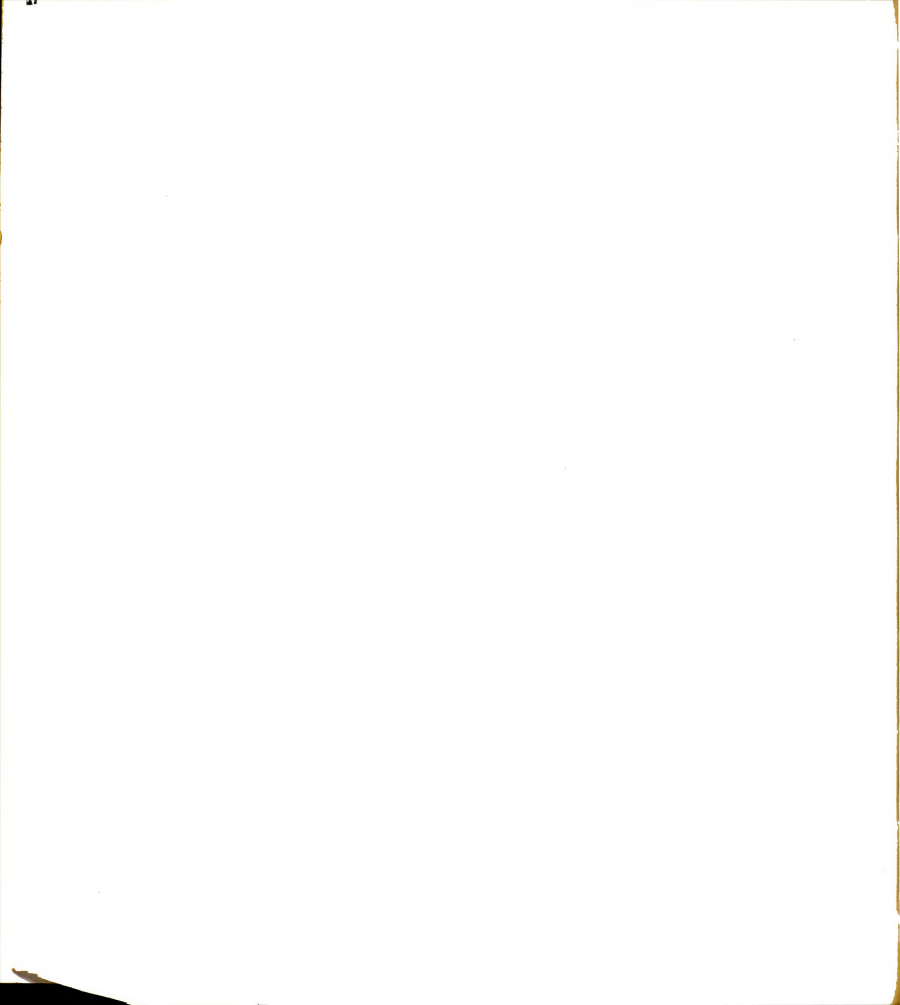


5. In general, the statistical models which incorporate the largest number of relevant factors achieve the most effective control. Thus, the statistical method identified as the Minimization approach which considers the first seven of the eight relevant factors (more than any other) was generally the most effective. However, factor 8, which it did not consider, proved to be a sufficiently important determinant of effective control limits to enable the Bierman, Fouraker, and Jaedicke second and McMenimen approaches to achieve overall rankings of two and three respectively in spite of some questionable aspects of these models.
6. The individual rankings of the statistical approaches can be expected to vary somewhat depending upon the probability distributions of chance and assignable cause performances and also upon the testing plan with its corresponding control limit.
7. Significant variances can be averaged-out in the summary report.

Summary of Recommendations

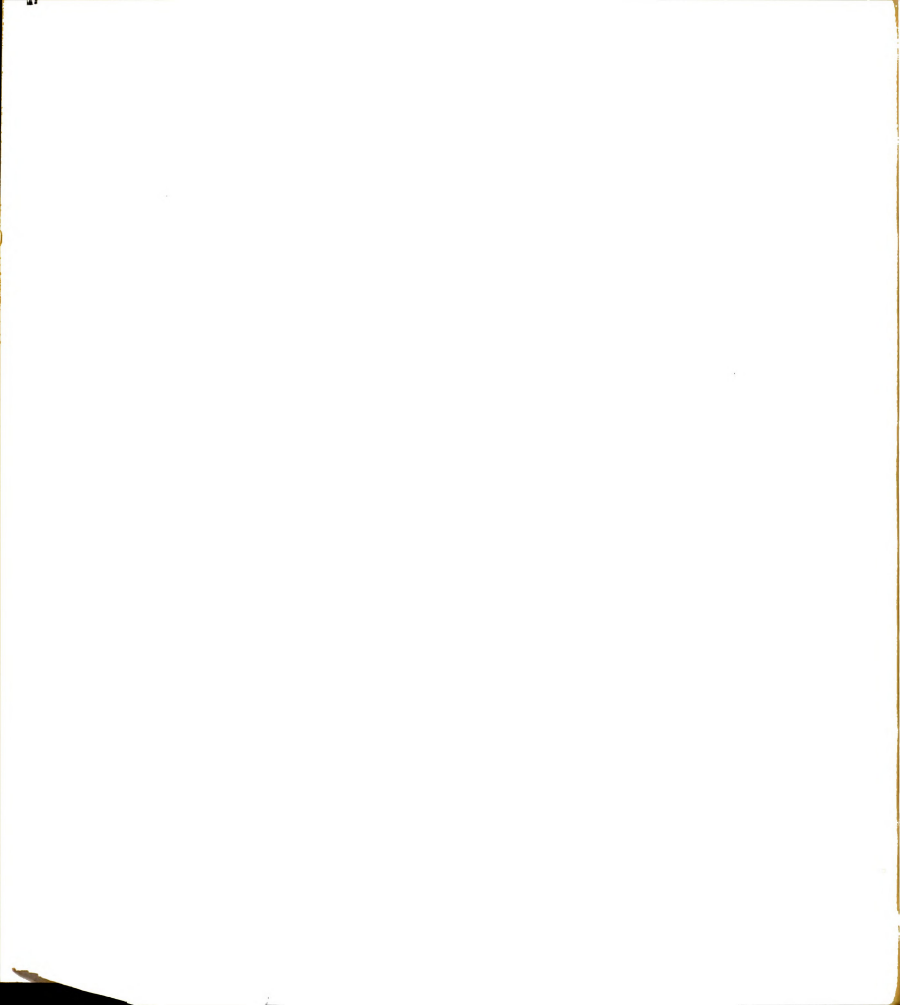
From these conclusions it is recommended that:

1. Accountants recognize the existence of chance factors and incorporate this concept into their



definition of control. For those variance classifications for which chance is operative, control would then be defined as consisting of those procedures designed to maintain actual results within limits due to chance. This definition explains why insignificant variances (due to chance) are not investigated.

2. Experience, judgment, and intuition be used to develop the information required for the eight relevant factors for several important operations. This information could be developed for overhead efficiency and material usage as well as for labor efficiency. Surely experience and judgment will be more useful if applied in an organized rather than in a haphazard way.
3. Various statistical models be tested according to a plan similar to that outlined in Chapter VI to determine the one most feasible for a given operation and testing plan. For this test, the analyst need not confine himself to those methods discussed in this dissertation. Indeed, other variations may prove to be more satisfactory.
4. The most desirable statistical model be directed toward control at the performance level for those operations where the benefits of more timely control and surer detection (without the average-out



problem) are thought to outweigh the additional costs.

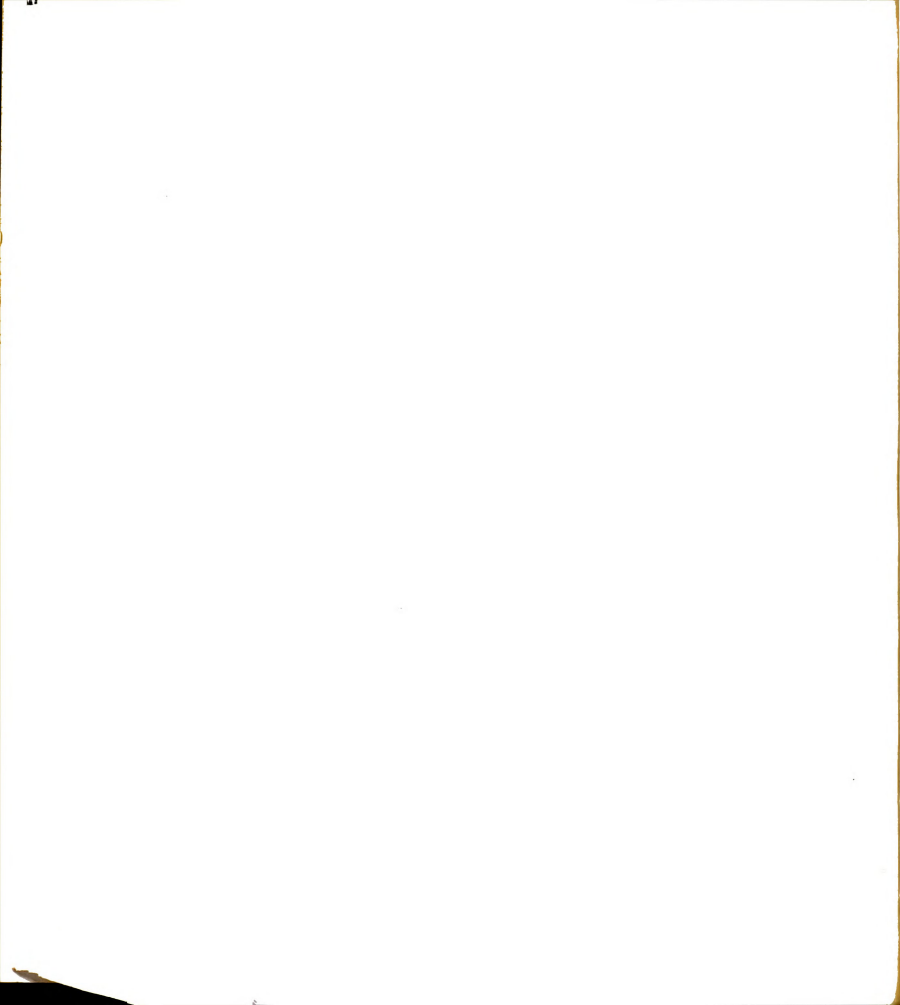
5. Statistical procedures be employed for analyzing the summary report. These procedures account for the degree of summarization and thus reduce the average-out effect.



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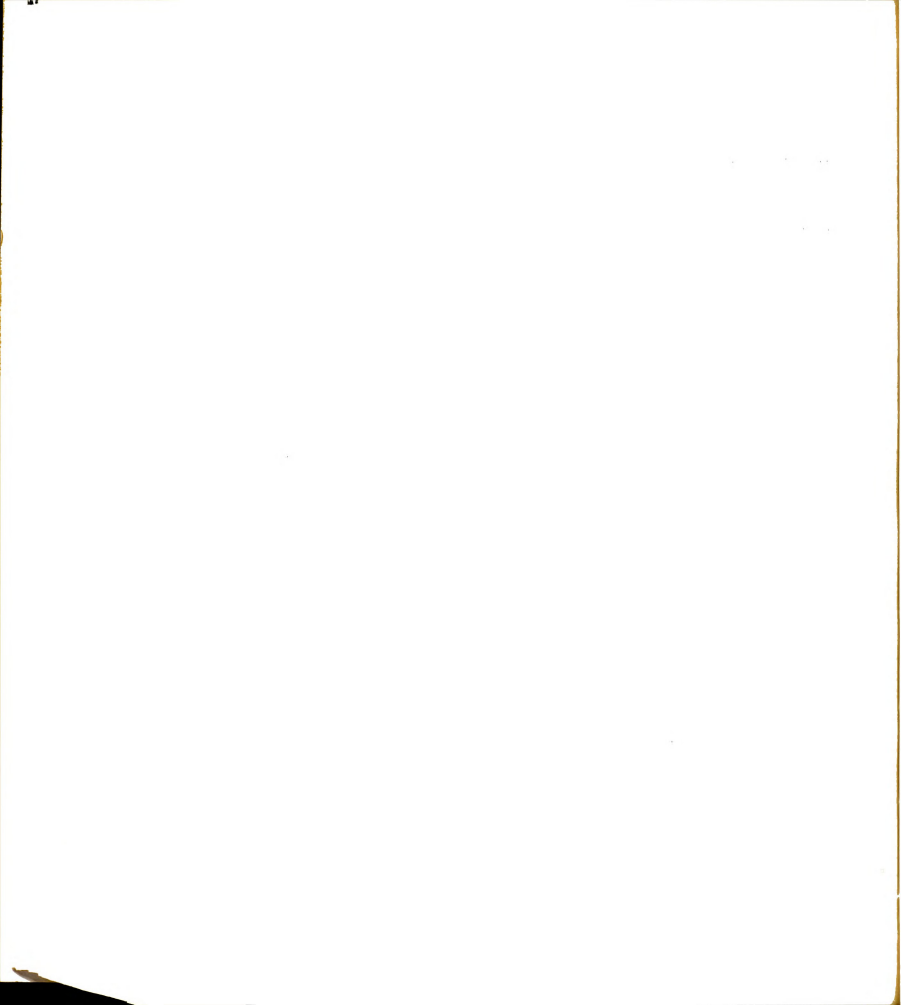
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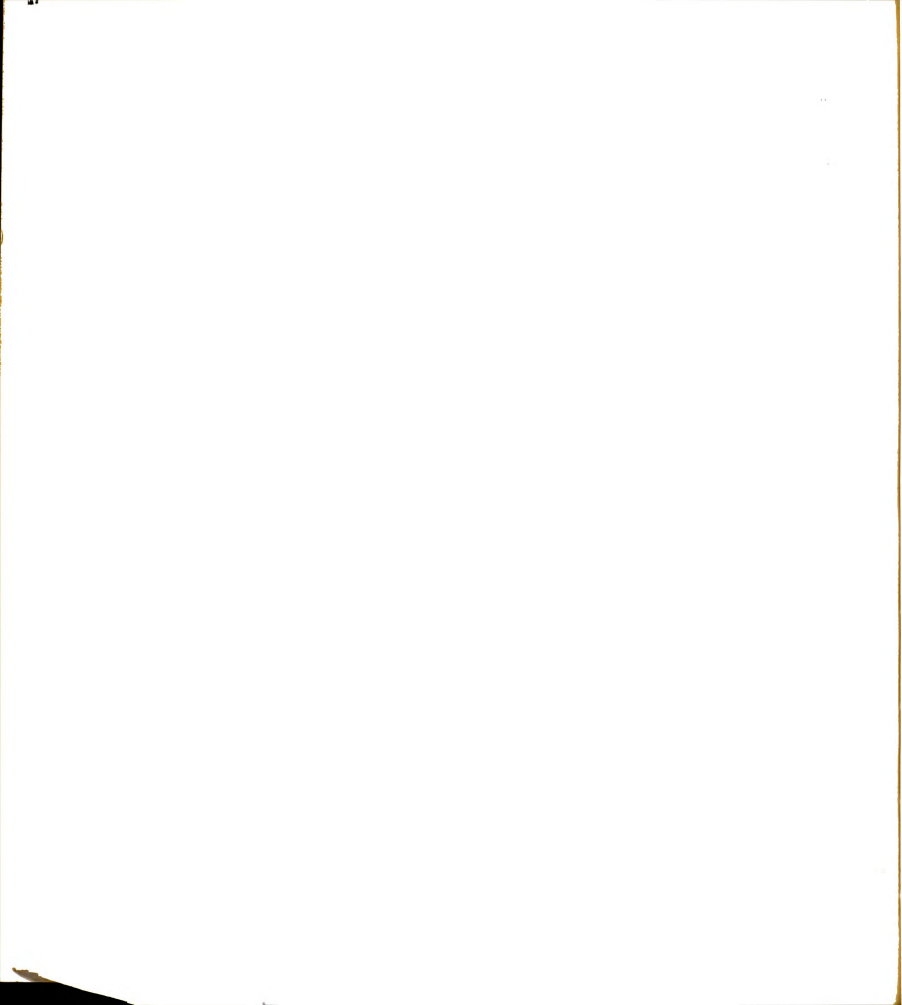
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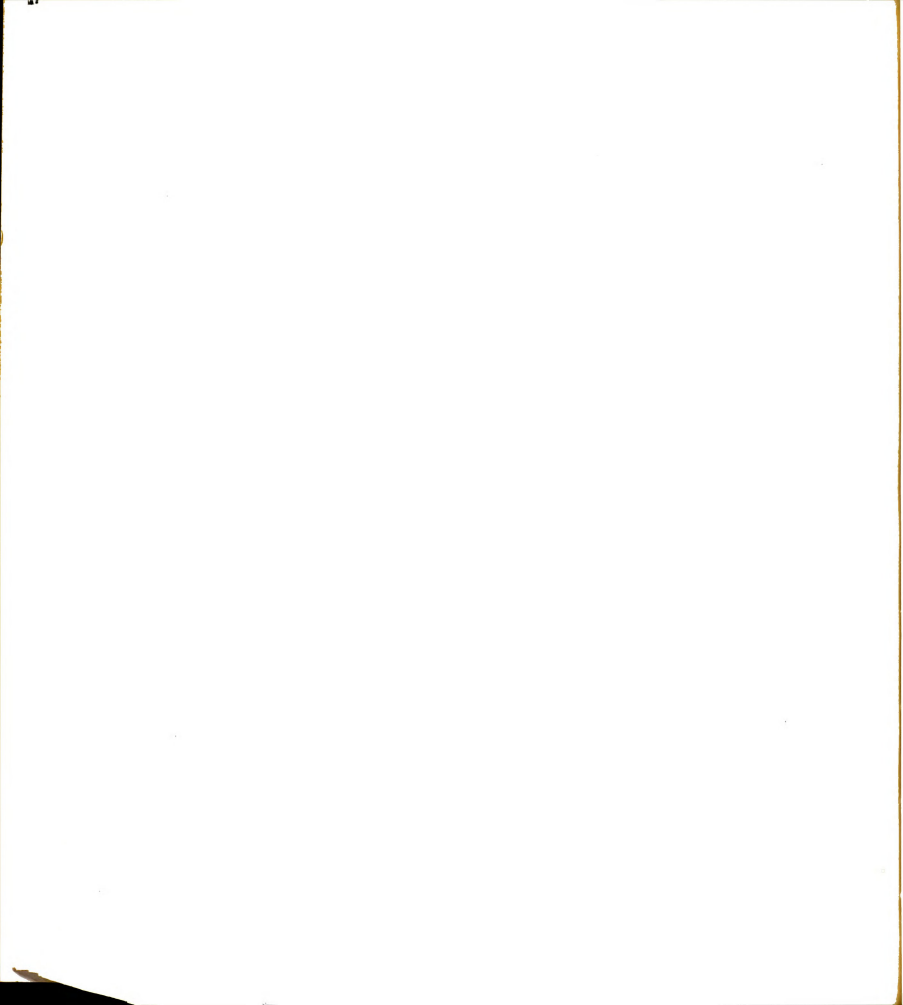
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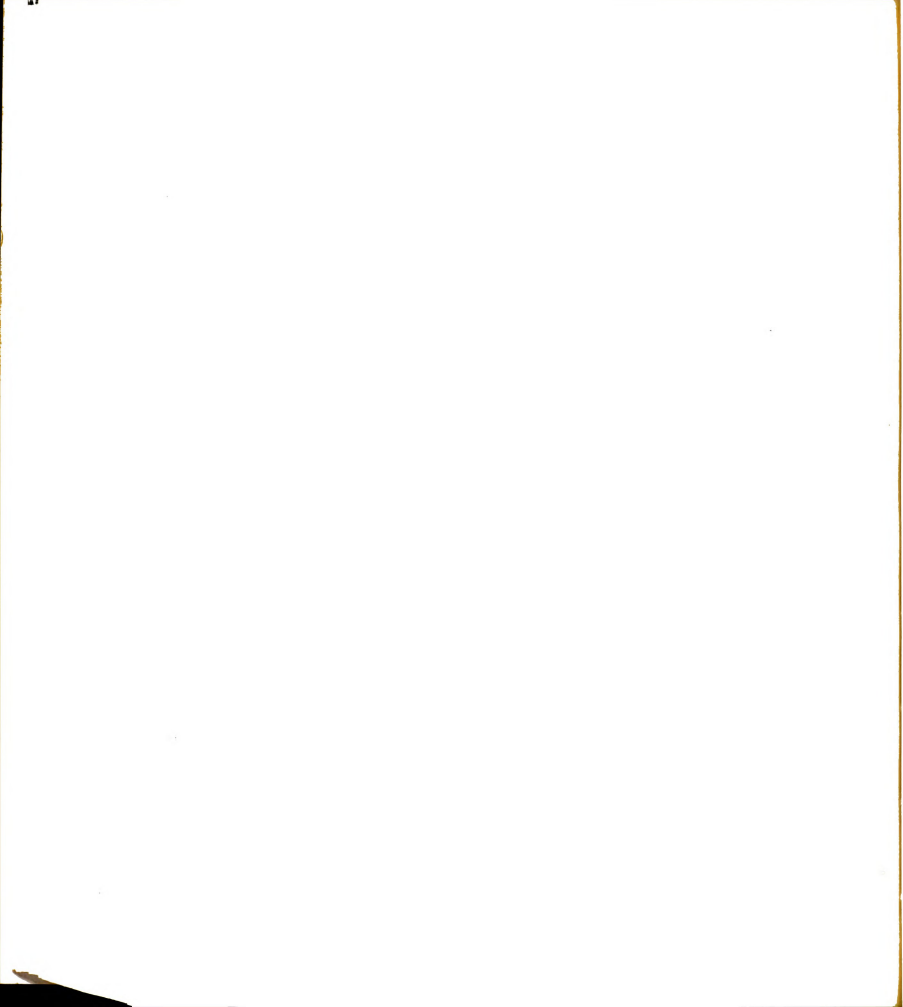
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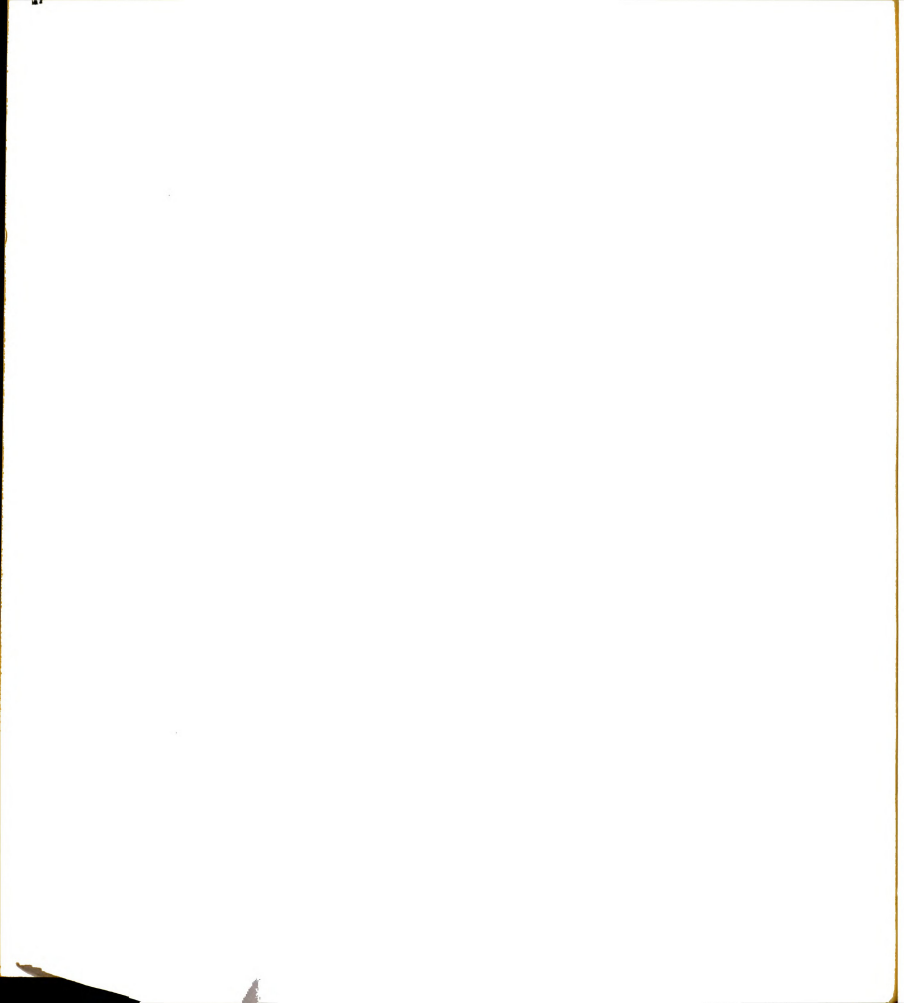
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APPENDIX A

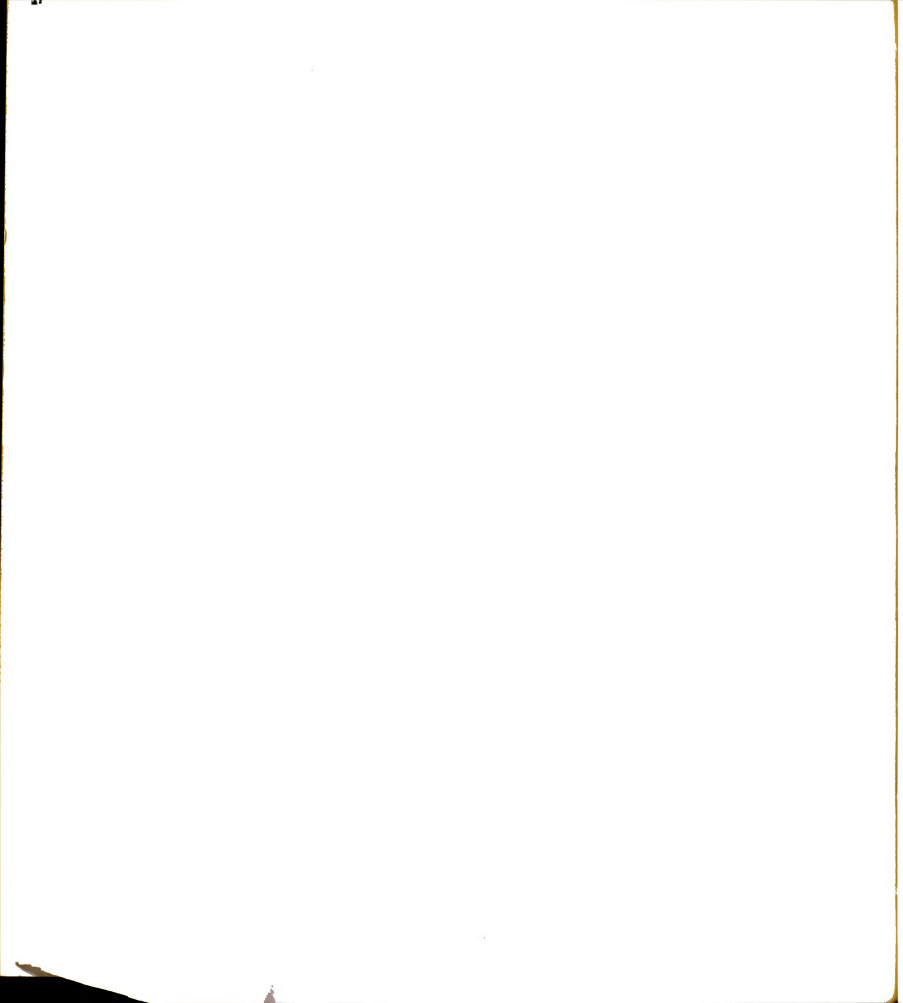
BIBLIOGRAPHY OF STATISTICAL APPLICATIONS TO ACCOUNTING VARIANCE CONTROL

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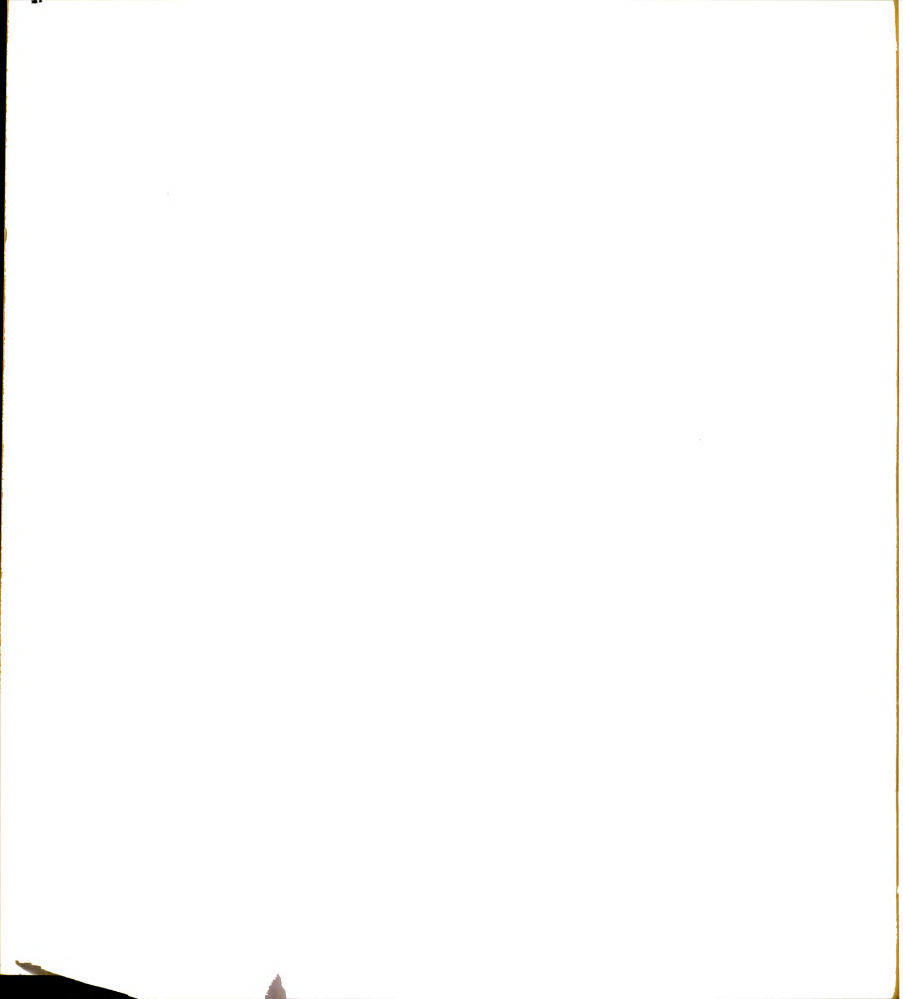
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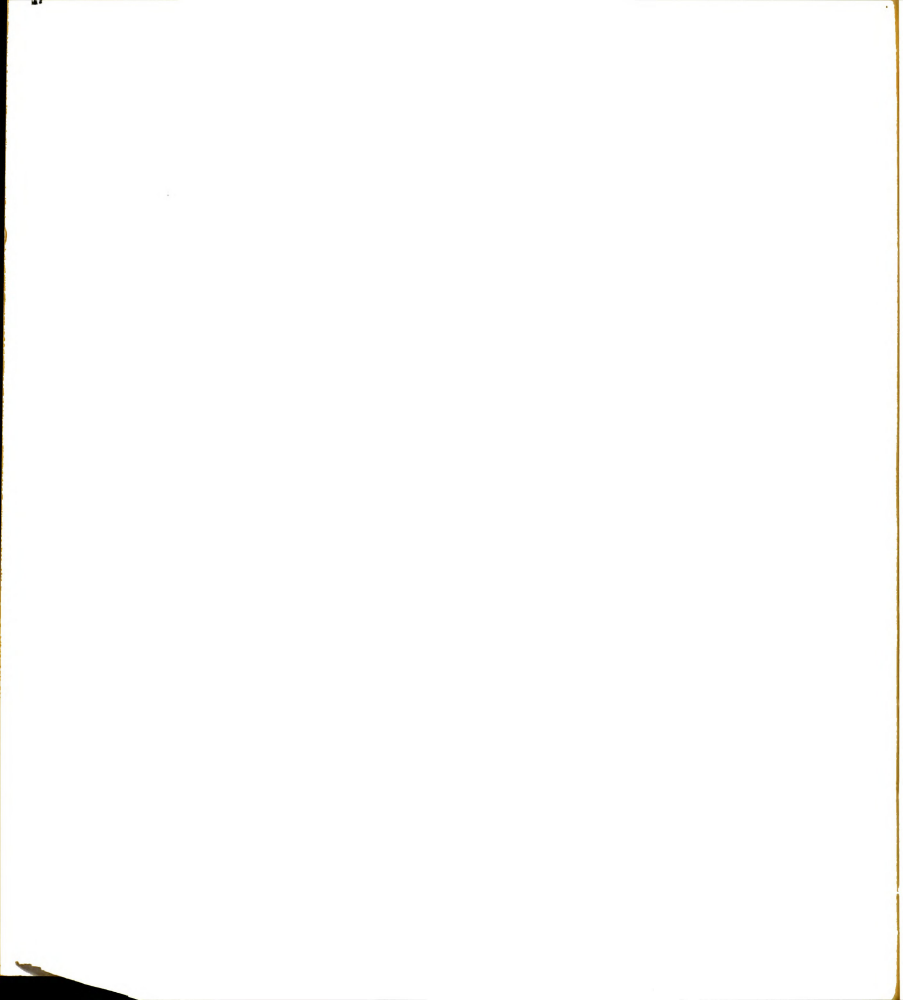


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APPENDIX B

COMPUTATIONAL DETAIL TO SUPPORT

CHAPTER VI

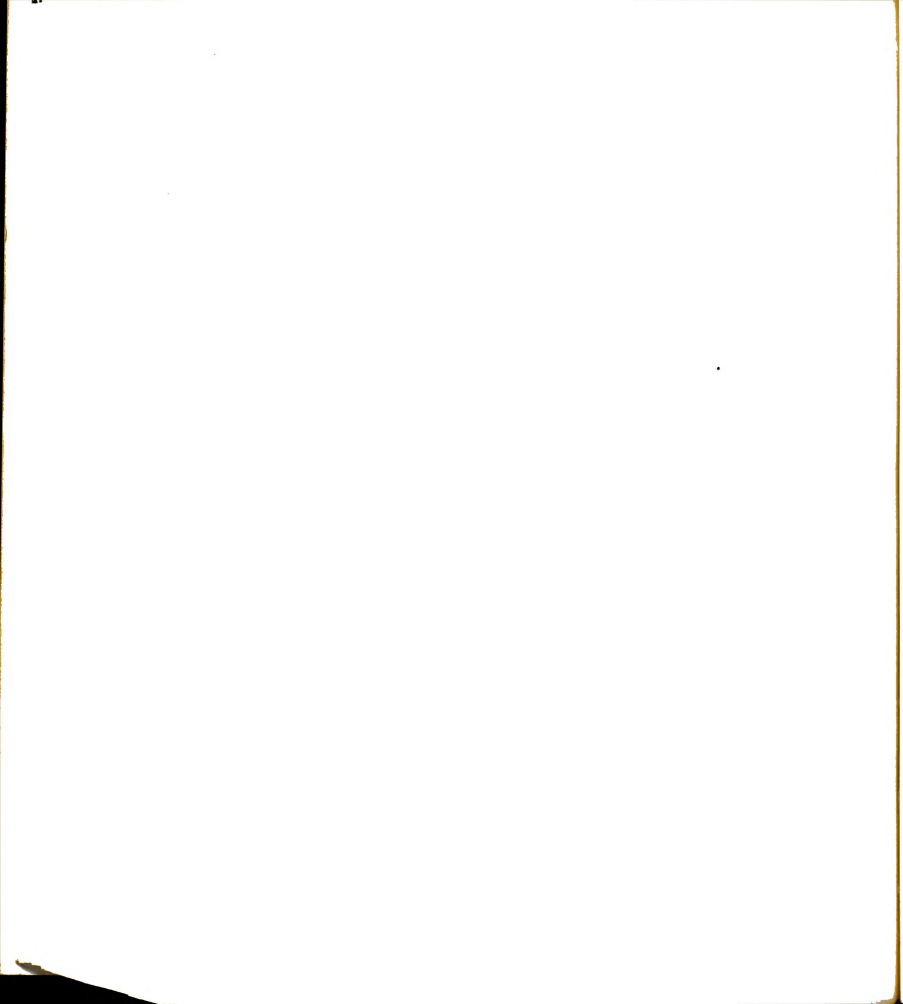
Derivation and Financial Analysis of Upper Control Limits for Single Observations--Each Performance Tested

McMenimen Approach

It has just been determined in Chapter VI that it would be worthwhile to spend \$1 investigating for dull knives upon the occurrence of a performance value of 260. The question now confronting the analyst is whether it is worthwhile to spend up to \$2 investigating for poor attitude--the only other assignable cause that was observed for test value 260 in the original 1000 performances.

The savings figure associated with poor attitude is determined by the following procedure:

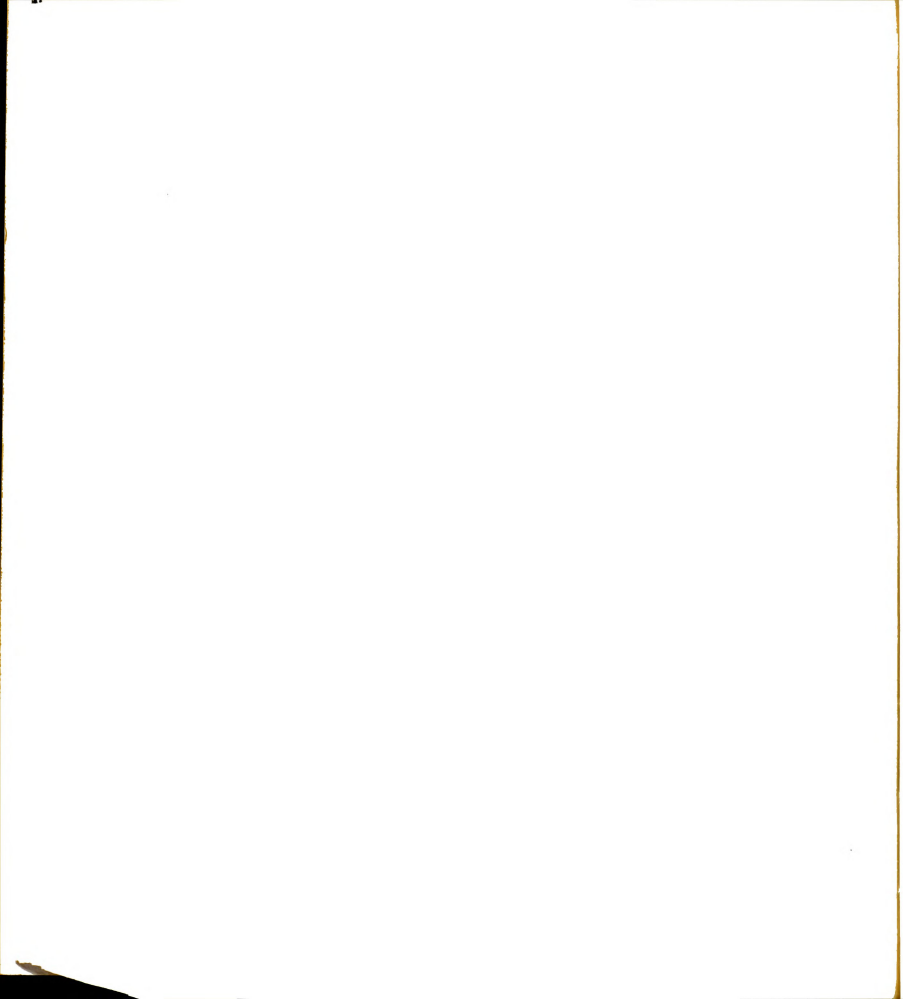
1. Find the opportunity cost associated with each performance. This is $\frac{255 - 245}{60} \times \$3 = \$.50$ where 255 is the mean of the poor attitude performances.
2. Multiply the \$.50 by 4. Result \$2.
3. Subtract the opportunity cost of correcting poor attitude from the \$2 weighted opportunity cost.



The opportunity cost of correcting poor attitude would be difficult to determine. More than one performance would benefit from any procedure aimed at attitude improvement. Assume that studies indicate that the cost of such procedures would average out to \$.25 on each performance. The savings if this one performance is investigated and attributed to poor attitude is then $\$2 - .25$ or \$1.75.

Here, one could conceive of instituting procedures to improve attitude a little; but not enough to reduce the mean to 245. In this case, various amounts other than \$0 or \$1.75 could be saved. It is this writer's opinion, however, that it should be worthwhile to re-establish the standard if 245 was a realistic standard to start with. If it was not, it should be revised. If circumstances have changed the standard should also be revised. Accordingly, only two events will be considered in conjunction with act "spend up to \$2 investigating."

Since the savings figure of \$1.75 is less than the \$2 cost of an investigation, the conditional value is $\$-.25$. It is obvious, then, that the expected savings will be negative so that an investigation could not be worthwhile regardless of the probabilities. However, because probabilities will be calculated in the same

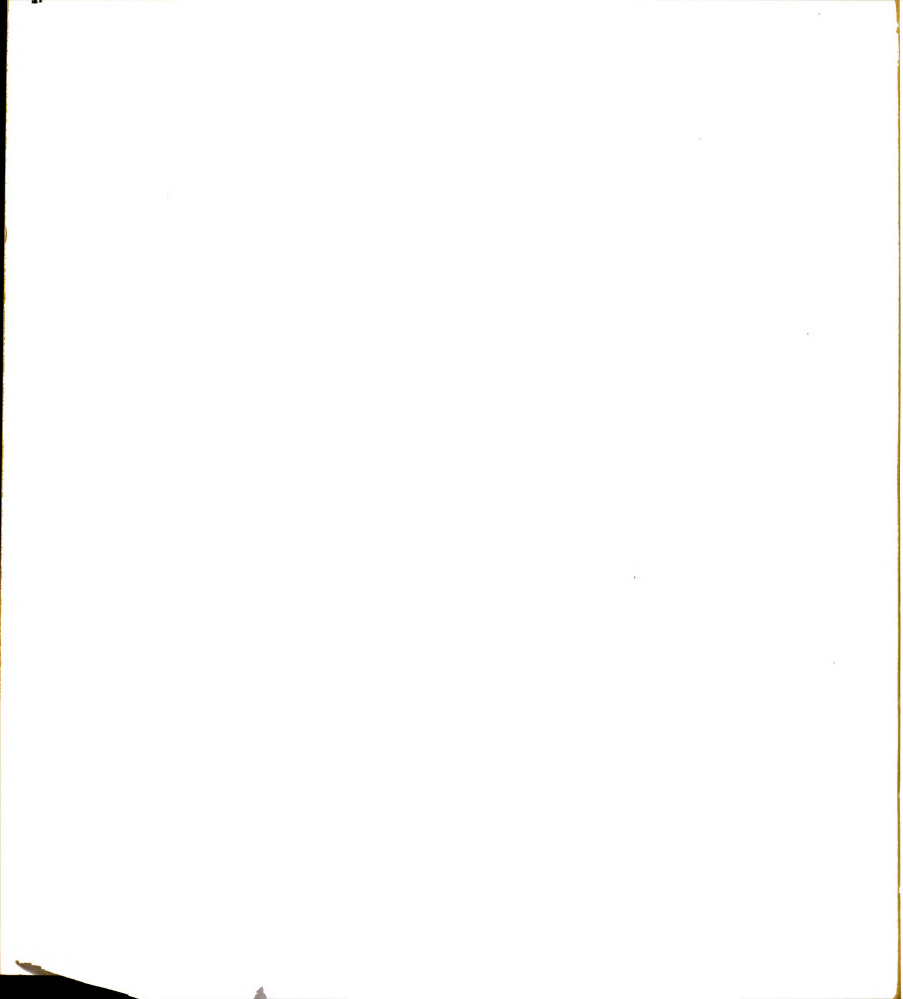


manner for further applications of this technique, it is instructive to discuss their derivation and complete the analysis by determining an expected savings value for this act. The results are shown in Table 79.

TABLE 79.--Application of McMenimen Technique

Event	Spend up to \$1 Investigating			Spend up to \$2 Investigating		
	Pe	Cond.	Exp.	Pe	Cond.	Exp.
Test Value 260						
Save \$0	.7143	\$-1	\$-.7143	.9	\$-2	\$-1.80
Save \$4.625	.2857	3.625	1.0357	.		
Save \$1.75				.1	-.25	- .02
Expected Savings			\$.3214			\$-1.82

The probabilities associated with each savings value for this act are estimated from the original distribution of 1,000 values shown in Table 18. These estimates are made according to the following line of reasoning. Fourteen of the 1,000 performances sampled had values of 260 minutes. Four of these were attributed to dull knives and their cause would be detected by the investigation for dull knives. Thus, ten performances remain for the second phase of the investigation. Of these, nine were due to chance so $Pe = .9$ for event "save 0" act "Spend up to \$2 investigating." One of the ten performances was due to poor attitude so $Pe = .1$ for event "save \$1.75."



Since the expected savings associated with act "Spend up to \$2 investigating" is negative, an investigation for a performance value of 260 would be undertaken only for dull knives.

Some may, however, wish to consider the fact that the \$1 spent investigating for dull knives is at this point in the decision process a sunk cost and that the concern now is in the incremental sense with whether an additional \$1 should be spent. Table 80 shows the effects of this incremental application for test value 260. The act is now labeled "investigate for poor attitude" rather than "spend up to \$2 investigating." The conditional values are only \$1 less than the savings figures.

TABLE 80.--McMenimen technique--incremental application

Event	Investigate for Poor Attitude		
	Pe	Cond.	Exp.
Test Value 260			
Save \$0	.9	\$-1	\$-.90
Save \$1.75	.1	.75	<u>+.75</u>
Expected Savings			\$-.15

The expected savings is still negative so an investigation would not be undertaken for poor attitude with a performance value of 260. The upper control limit for this cause is somewhat higher.

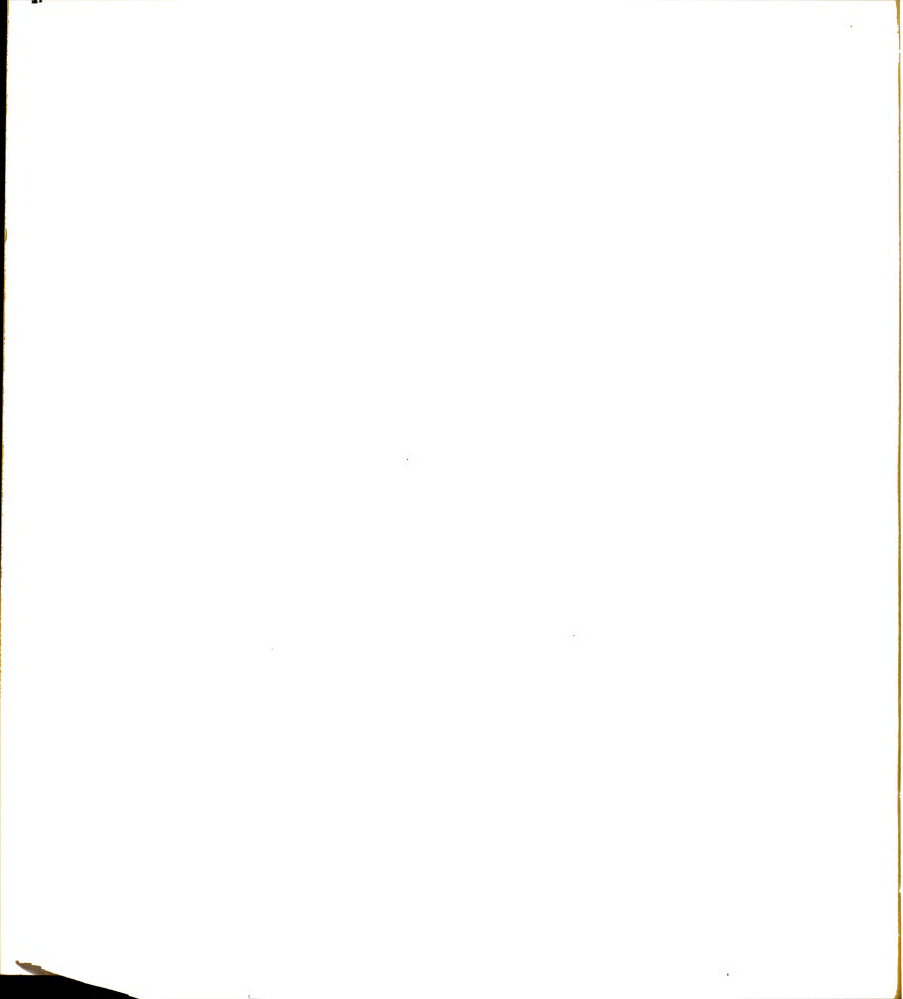
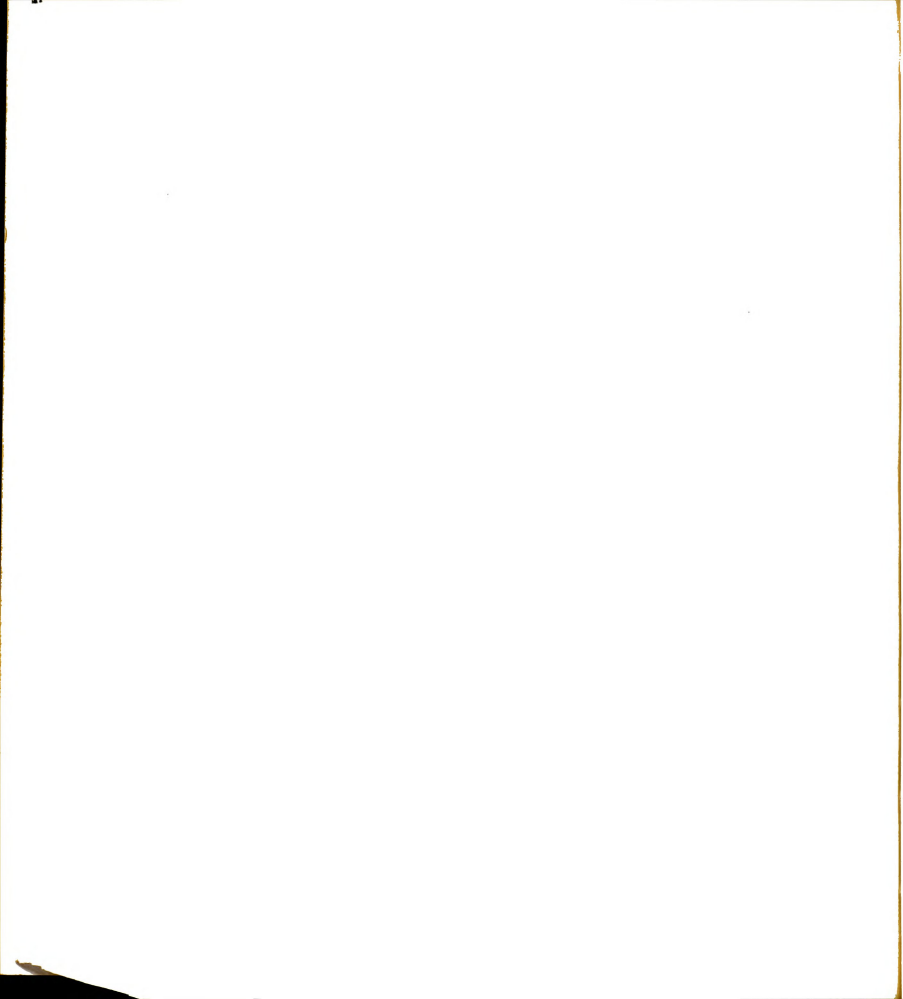


Table 81 shows that the upper control limit is between 261 and 262 for poor attitude or laziness. The figures are determined in the same way they were in Table 80. Of the 5 performances with values at 261, 2 are due to dull knives whose cause would have been detected by the first step in the investigation process. Now 3 performances remain. Two of these are due to chance so Pe of saving \$0 is $2/3$ or .667; one is due to poor attitude so Pe of saving \$1.75 is $1/3$ or .333. It would still not be worthwhile to administer the psychological test because the expected savings is still negative.

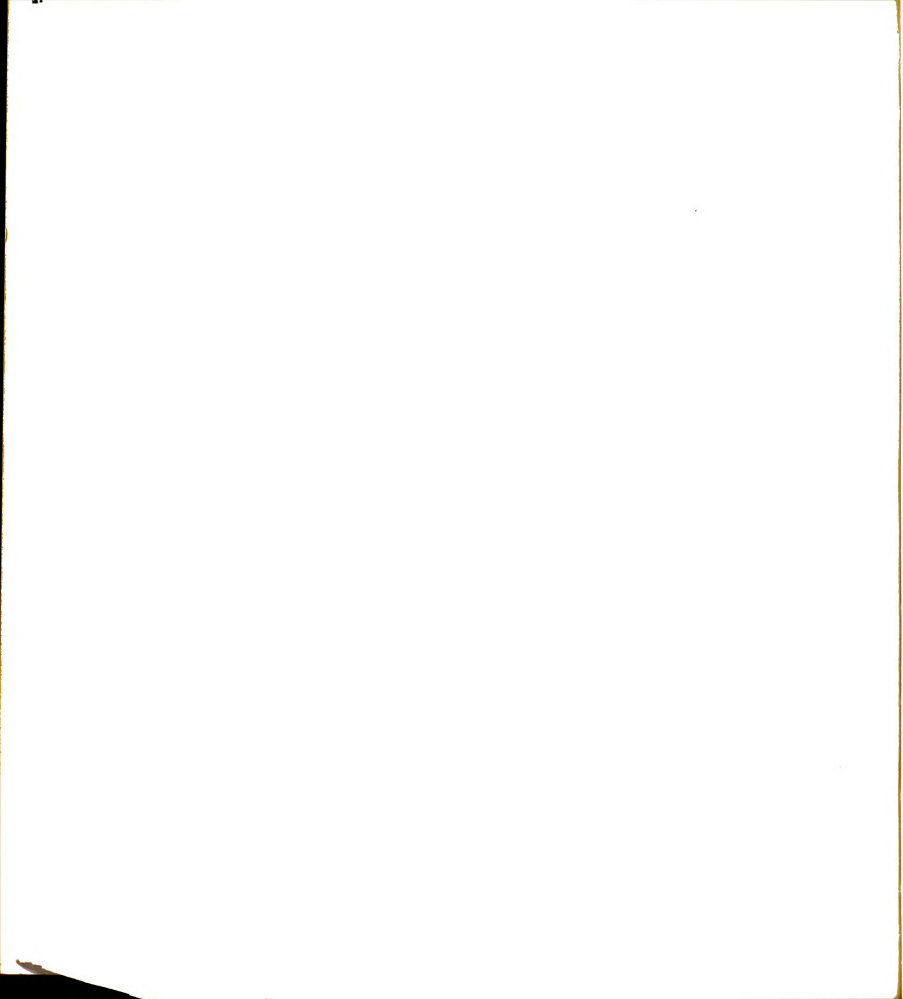
TABLE 81.--McMenimen technique--incremental application.

Event	Investigate for Poor Attitude and Laziness			Investigate for Illness		
	Pe	Cond.	Exp.	Pe	Cond.	Exp.
Test Value 261						
Save \$0	.667	\$-1	\$-.673			
Save \$1.75	.333	.75	+.25			
Expected Savings			<u>\$-.423</u>			
Test Value 262						
Save \$0	.666	-1	-.666	.75	\$-3	\$-2.25
Save \$1.75	.167	+ .75	+.125			
Save \$5.75	.167	4.75	+.7917			
Save \$3.50				.25	.50	+.125
Expected Savings			<u>\$+0.2507</u>			<u>\$-2.125</u>



Two new dimensions are added to the analysis for test value 262. First, one of the eight performances in the original 1,000 values was due to laziness; therefore there is an opportunity to make a savings from this cause in conjunction with the psychological test. The amount of the savings, \$5.75, is calculated by multiplying the single performance opportunity cost of \$1.50 $(\frac{275 - 245}{60} \times \3 where 275 is the mean of the performances due to laziness) by the 4 performances that allegedly lapse on the average before an assignable cause is detected. From this product of \$6, the estimated per performance cost of correction, \$.25, is subtracted to arrive at the savings of \$5.75. Since the expected savings is now positive, it would be worthwhile to administer the psychological test as the investigation for poor attitude and laziness. The upper control limit for these causes is thus between 261 and 262.

The other new dimension for test value 262 is the possibility of investigating for illness which involves an incremental cost of \$3. After dull knives, poor attitude, and laziness have been eliminated as causes, only 4 performances remain. Of these, one is due to illness so the Pe of saving \$3.50 is 1/4 or .25 and the Pe of saving \$0 is .75. The \$3.50 savings is determined by multiplying the \$1 single performance opportunity cost $(\frac{265 - 245}{60} \times \3 where 265 is the mean of the performance due to illness)



by the 4 performances that will lapse before the assignable cause is detected and subtracting from this product, \$4, the \$.50 estimated per performance cost of correction. The expected savings is negative so this aspect of the investigation is not profitable. Moreover, the investigation for illness will yield negative expected savings for all test values. Thus, McMenimen would never investigate for illness under these assumptions.

Chapter VI indicated that an investigation would be undertaken for dull knives for a performance value of 260. Since 260 falls in the region of hypothesis rejection, 259 would be in the direction of the control limit. Table 82 shows the expected savings of an investigation for dull knives for test value 259 to be \$.385.¹ Therefore, the investigation would not be profitable. The upper control limit would be between 259 and 260, as far as dull knives is concerned.

¹The probability of saving \$4.625 is $2/15 = .133$. Table 18 shows that two of the fifteen performances with values of 259 were due to dull knives. The remaining thirteen performances were due to other causes (including chance) for which an investigation for dull knives would result in \$0 savings. Consequently, P_e corresponding to event "Save \$0" is $13/15 = .867$.

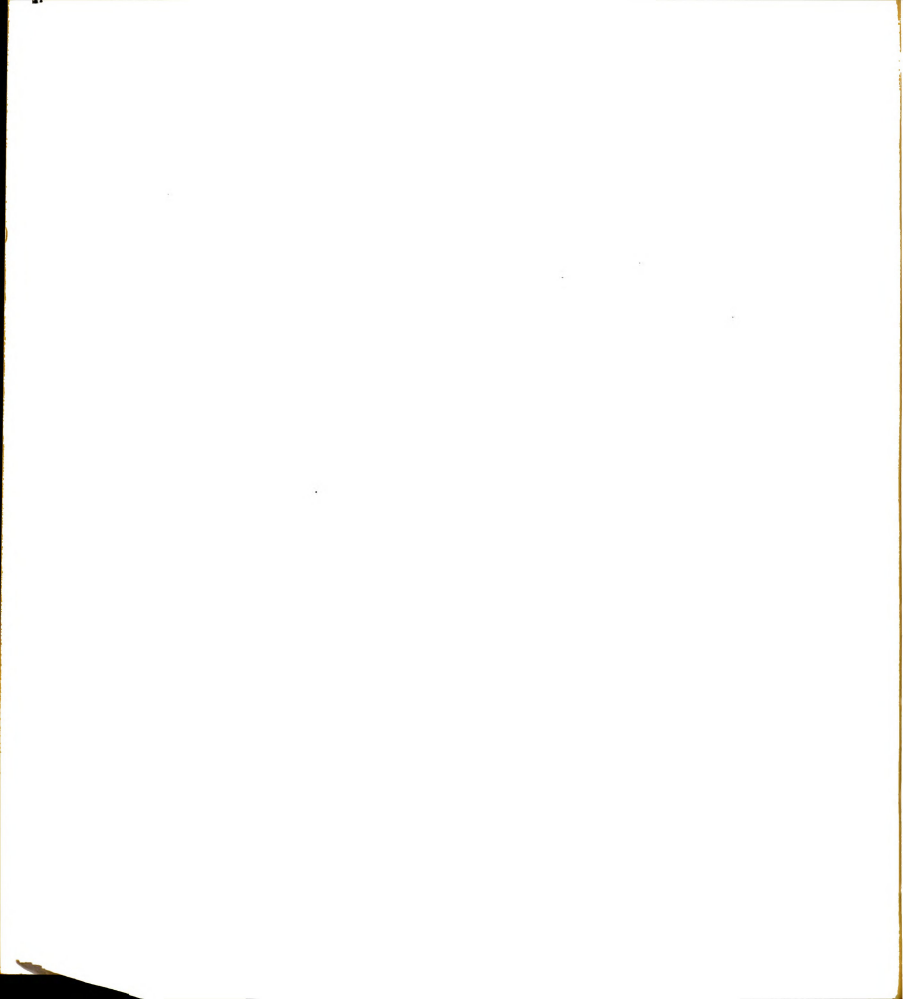


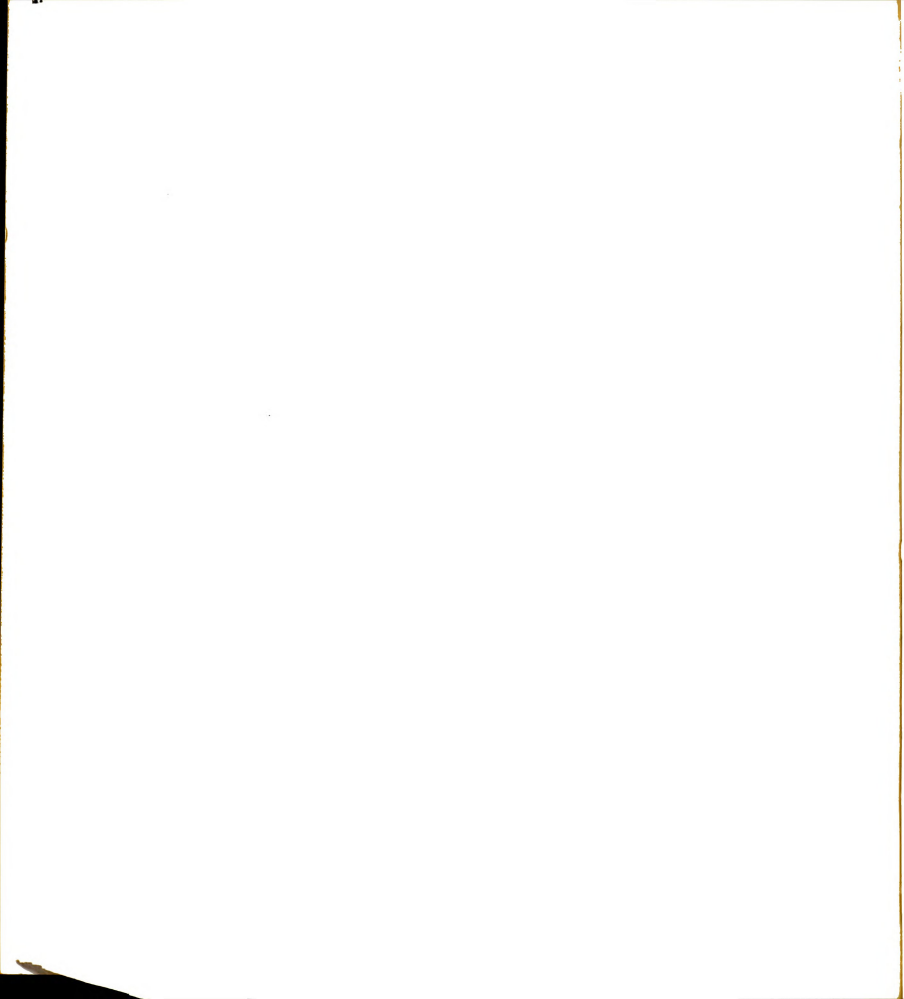
TABLE 82.--Application of McMenimen technique

Event	Spend Up to \$1 Investigating		
	Pe	Cond.	Exp.
Test Value 259			
Save \$0	.867	\$-1	\$-.867
Save \$4.625	.133	3.625	<u>+.482</u>
Expected Savings			\$-.385

Equalization Approach

The following explanation pertains to the determination of the probabilities and opportunity costs of each type of error for Equalization Decision Table 20.

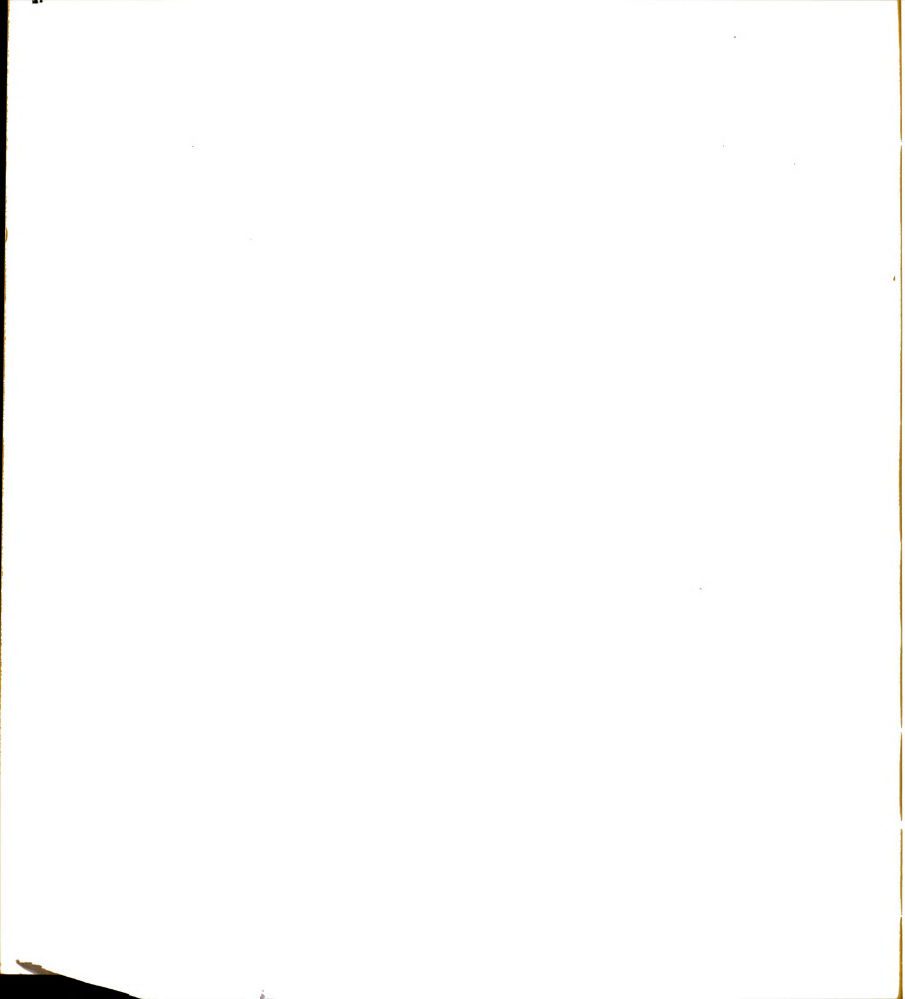
The probabilities of a Type I error are determined by dividing the number of chance performances with values at least as great as the test value by 600--the total number of chance performances. The reason for this is that the hypothesis will be rejected for any performance value greater than the test value selected as the control limit. If the performance value is attributed only to chance, a Type I error will be made. The number of chance occurrences at least as great as 258, 259, and 260 are shown in Table 18 to be 42, 31, and 22 respectively and their ratios to 600 are .0700, .0517, and .0367 respectively. These are the values shown for the probabilities of a Type I error. The reader will notice that the probabilities decline as the test value increases. This



follows from the fact that there are fewer chance performances at least as great as the test value present for higher test values.

The opportunity cost of a Type I error resulting from a barren investigation is \$6. However, since a tough cow has never been butchered in less than 262 minutes, the final aspect of the investigation--spend \$1 testing for tough cows--can be eliminated for test values less than 262. Thus, the opportunity cost of a Type I error shown in Table 20 is reduced to \$5. The expected opportunity cost of a Type I error for each test value results from multiplying the probability of committing a Type I error by \$5--the opportunity cost of committing a Type I error.

By the same token, the expected opportunity cost of a Type II error results from multiplying the probability of committing a Type II error by the opportunity cost of a Type II error. The reader will recall that the probability of committing a Type II error was determined in Chapters IV and V by selecting an alternative parameter, to represent an assignable cause assuming normal distributions for both the chance and the alternative populations, and computing the proportionate area under the curve representing the alternative population that falls within the control limits. The reasoning for this approach is, of course, that values falling within the control limits would lead to hypothesis acceptance--a Type II error when an assignable

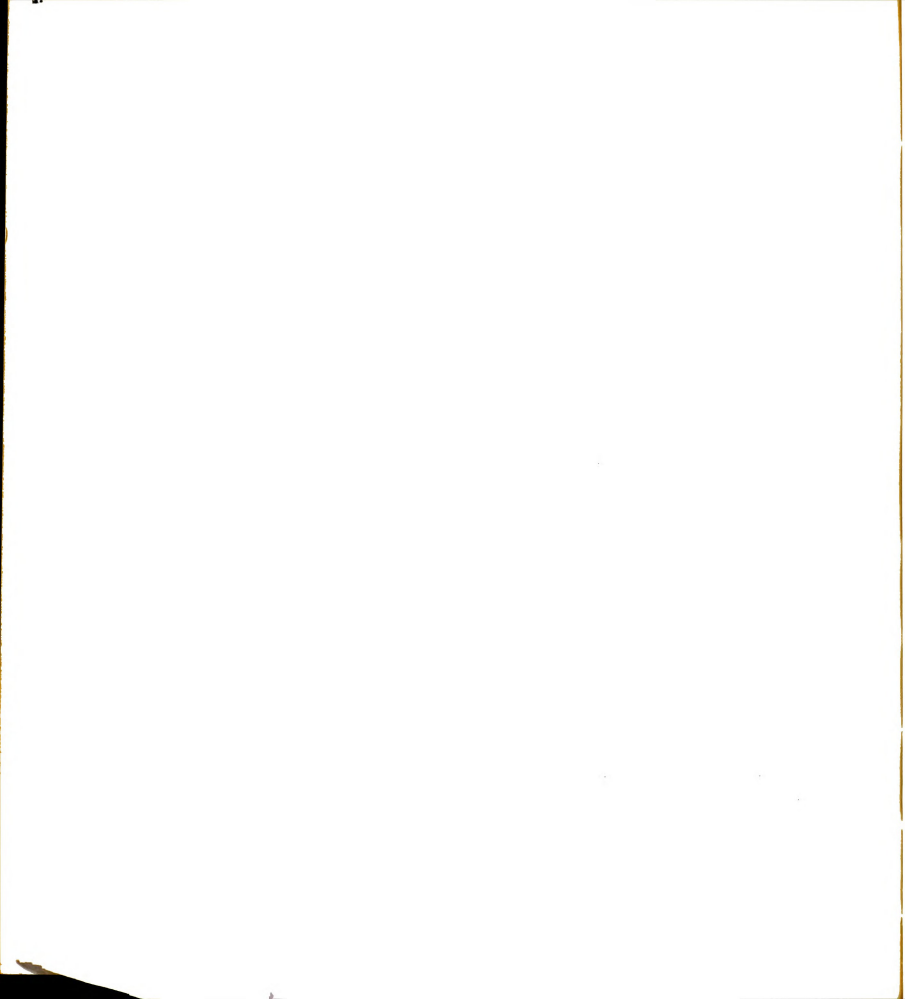


cause is operative. No attempt was made in Chapter V to identify the alternative parameter with a particular assignable cause. The probability of assignable causes with different parameters such as exist in this example was not considered. Here, however, six unfavorable assignable causes and one favorable one--each with different parameters are possible. Consequently, a different approach must be used to estimate the probability of committing a Type II error. Moreover, since detailed information regarding 1000 past performances is assumed, the assumption of normality is not necessary.

The probability of committing a Type II error will be estimated by dividing the number of assignable cause performances (other than improvement)² with values less than the test value³ by 300--the total number of unfavorable assignable cause performances. Given that some unfavorable off-standard condition exists, this ratio represents the probability that a performance will be executed in a time less than the test value. If the test value is selected as the upper control limit, this ratio

²Since improvement represents a favorable assignable cause, it is considered in determining the lower control limit.

³The hypothesis will be accepted for performance values less than the test value chosen as the upper control limit. If an assignable cause is operative, acceptance will result in a Type II error.



is an estimate of the probability of committing a Type II error. For test value 260 the ratio is $62/300$ or .2067. The numerator, 62, is determined by referring to Table Two and adding the 45 performances due to poor attitude, the 10 performances due to dull knives, the 5 caused by illness and the 2 caused by laziness that had performance values less than 260.

Because the conditional opportunity cost of a Type II error also depends upon the assignable cause, the cost will be determined for each assignable cause. These will then be averaged in order to arrive at a representative single figure to be used in the determination of the upper control limit. This is important because the control limit is used for decision making when the cause is unknown and it is important to have a single value that can be used to signal an assignable cause regardless of what that cause happens to be.

The single performance opportunity costs have already been calculated by dividing the difference between the mean of the assignable cause and 245 the mean of the chance performances by 60 to convert the difference into a fraction of an hour. This fraction is then multiplied by \$3--the hourly wage rate for butchers. The results for the assignable causes of interest are shown in Table 83.

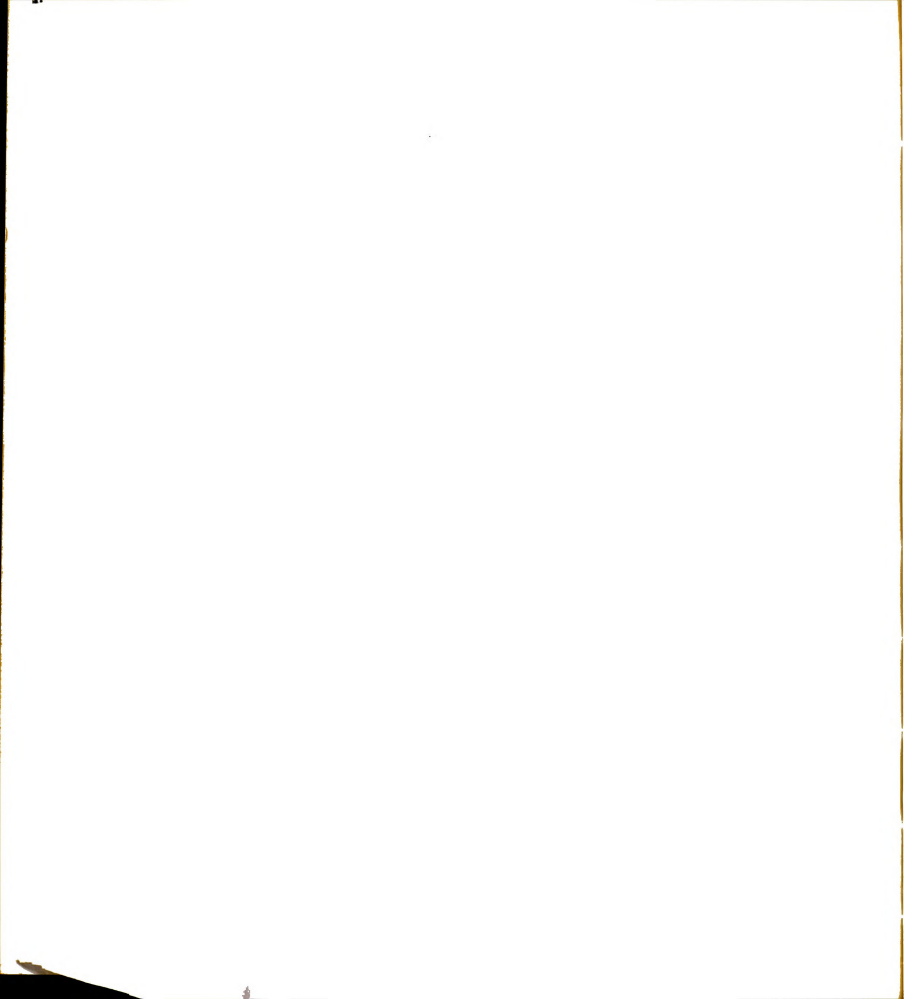
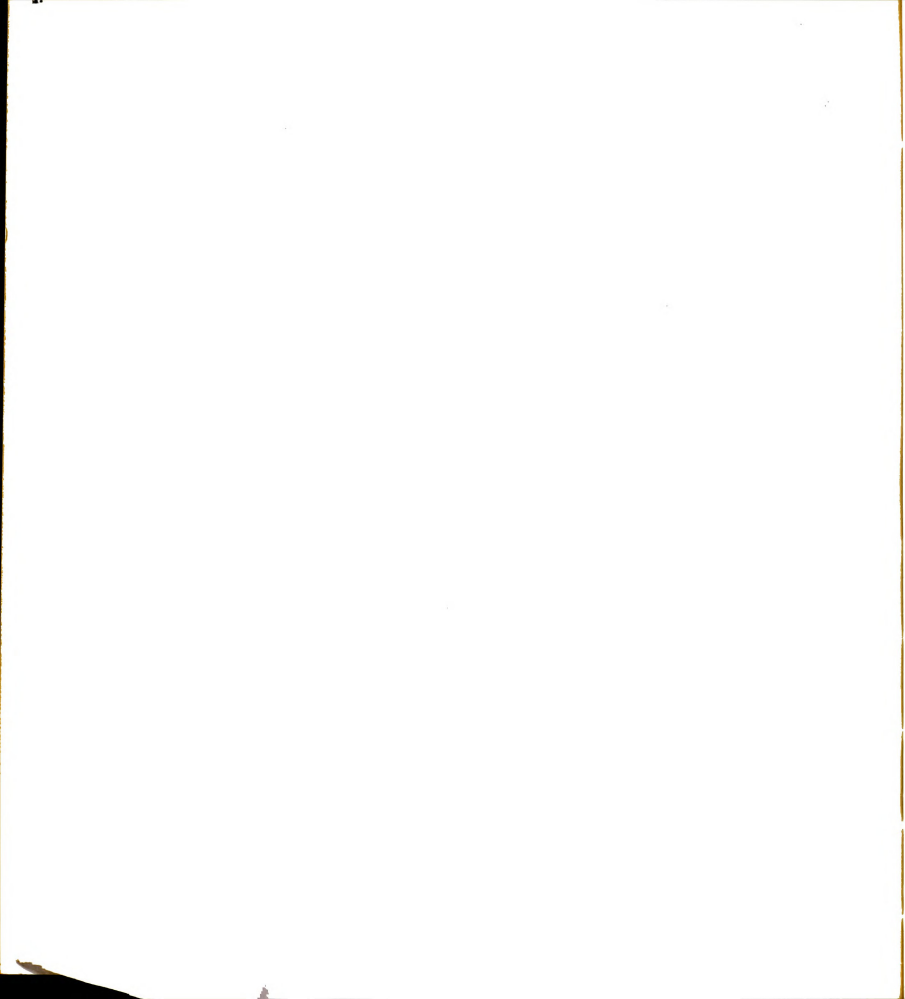


TABLE 83.--Single performance opportunity costs for corresponding assignable causes

<u>Assignable Cause</u>	<u>Opportunity Cost</u>
Illness	\$1.00
Laziness	1.50
Poor Attitude	.50
Dull Knives	1.25

These single performance opportunity costs must now be weighted by the same procedure illustrated in Table 10 to recognize the fact that an assignable cause might not be detected on its first occurrence. The weighting procedure associated with poor attitude is shown in Table 84. Again, Column A represents the number of successive failures to detect a change in the cause system. The derivation of the \$.50 opportunity cost corresponding to the first failure to detect a change in the cause system was explained in conjunction with Table 83. The other figures in Column B increase successively by \$.50 for each additional failure to detect the change. Column C shows the probability of failing to detect poor attitude for the numbers corresponding to Column A. Since 45 out of the 60 performances attributed to poor attitude are less than 260 (shown in Table 18) the probability of failing to detect an assignable cause of poor attitude on its first occurrence, if the control limit is 260, is $45/60$ or .75. The other figures in Column C are determined by taking the power of .75 corresponding to the values in Column A. The result of the weighting, \$1.8036, is the conditional



opportunity cost which recognizes that it takes on the average $(\$1.8036/.50)$ 3.6 tests to detect poor attitude once this assignable cause has appeared if 260 is selected as the upper control limit.

TABLE 84.--Weighted opportunity cost associated with poor attitude assuming UCL = 260

Number of Tests (A)	Accumulated Opportunity Losses (B)	Probability of Tests in Column A (C)	Column B times Column C (D)
1	\$.50	.75	
2	1.00	.5625	
3	1.50	.4219	
4	2.00	.3164	
5	2.50	.2373	
6	3.00	.1780	
7	3.50	.1335	
8	4.00	.1001	
9	4.50	.0751	
10	5.00	.0563	
11	5.50	.0422	
12	6.00	.0316	
		<u>2.9049</u>	<u>\$5.2392</u>

$$\text{Weighted cost} = \frac{\$5.2392}{2.9049} = \$1.8036$$

The conditional weighted opportunity costs for the other assignable causes are determined in a similar manner. The values corresponding to each assignable cause are shown in Table 85.

These weighted opportunity costs for each assignable cause are averaged in Table 85 in order to find the conditional opportunity cost of a Type II error. The number of times

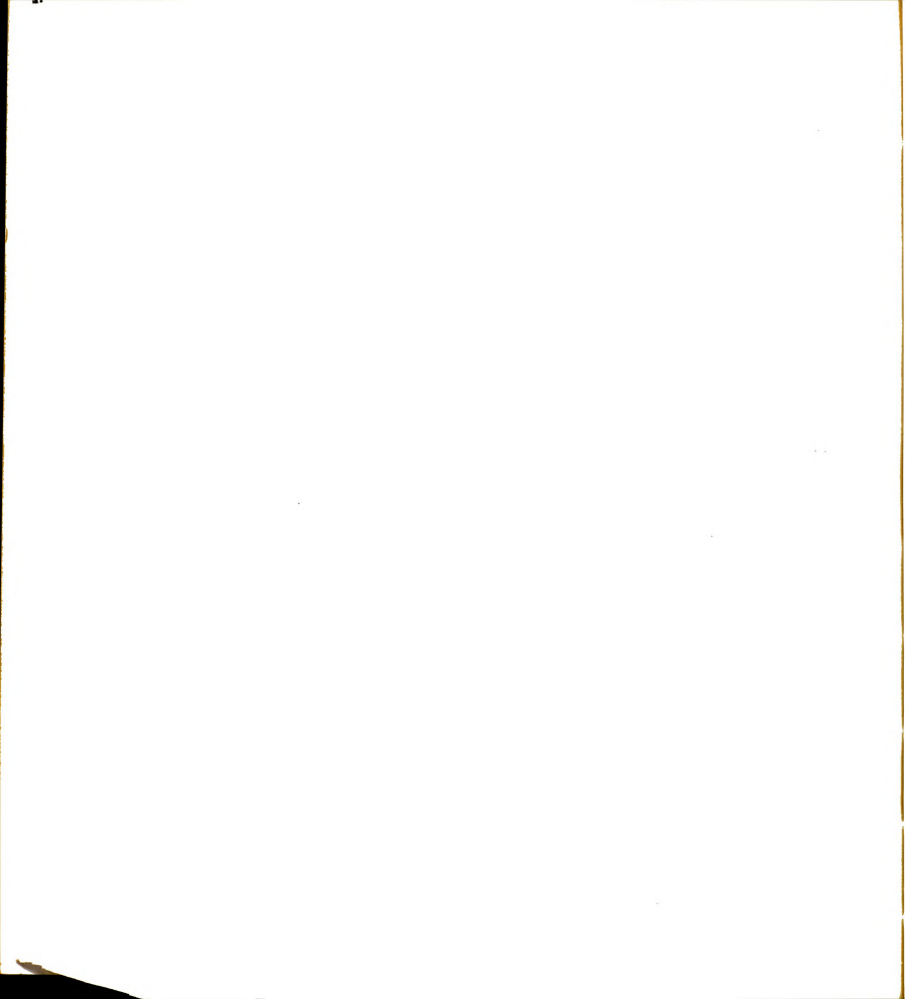
each assignable cause occurred are used as weights in this averaging process. These frequencies were originally indicated in Table 17. The averaging process yields \$1.5053 as the conditional opportunity cost of a Type II error.

TABLE 85.--Averaging process to find the conditional opportunity cost of a Type II error for test value 260

Relevant Assignable Cause	Number of Performances (F)	Weighted Conditional Opportunity Cost (C)	CF
Illness	20	\$1.3379	
Laziness	40	1.5825	
Poor Attitude	60	1.8036	
Dull Knives	120	1.3583	
	<u>240</u>		<u>\$361.2700</u>
$\text{Average} = \frac{\sum CF}{\sum F} = \frac{\$361.2700}{240} = \$1.5053$			

These weighted conditional opportunity costs shown in Table 85 are unique to the test value 260. Separate figures must be calculated for each test value. Since the work involved in calculating these figures is fairly tedious, it is preferable to make a good approximation of the figures by performing the calculations for another test value, 255, for example, and interpolating for the values between 255 and 260.

The weighted conditional opportunity costs for test value 255 are shown in Table 86 along with the same costs



for test value 260. The difference between the opportunity costs at 255 and 260 is also shown. Now in order to find the weighted conditional opportunity cost for test value 256 by interpolation, it is only necessary to add one-fifth of the difference to the test value for 255. To find the cost for 257, two-fifths of the difference is added to the test value for 255. Three-fifths is added for 258 and four-fifths for 259. The results for each assignable cause are indicated in Table 87. These values are averaged in Table 88 in the same manner used in Table 85. The resulting averages are those used in Table 20 for the opportunity cost of a Type II error.

TABLE 86.--Weighted conditional opportunity costs for test values 255 and 260

Cause	Weighted Conditional Opportunity Cost			One-Fifth of Difference
	260	255	Difference	
Illness ⁴	\$1.3379	\$1.1512	\$.1867	\$.03734
Laziness ⁴	1.5825			
Poor Attitude	1.8036	.8492	.9544	.19088
Dull Knives	1.3583	1.2720	.0863	.01726

⁴There were no values attributed to laziness as low as 255. Therefore, a Type II error could not be made with an upper control limit of 255 if laziness were the assignable cause.

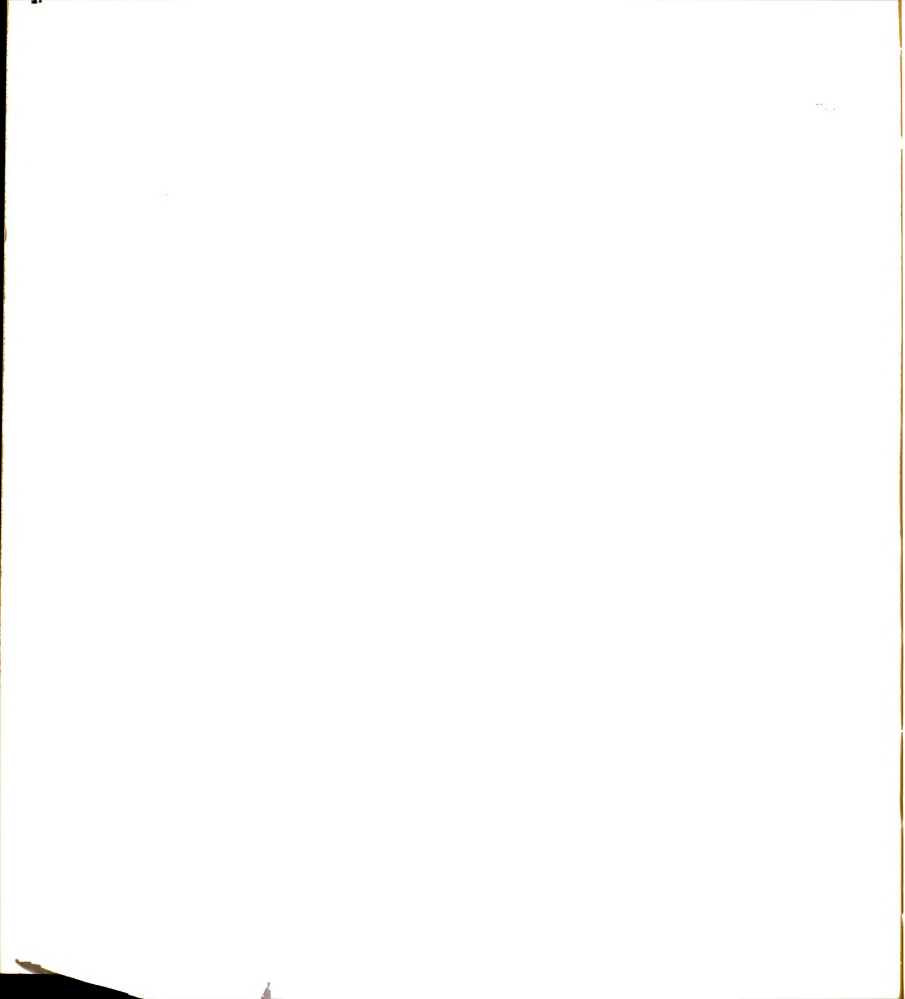
TABLE 87.--Weighted conditional opportunity costs for selected values determined by interpolation

Cause	Weighted Costs			
	256	257	258	259
Illness	\$1.1885	\$1.2258	\$1.2632	\$1.3006
Laziness*				
Poor Attitude	1.0400	1.2309	1.4218	1.6127
Dull Knives	1.2892	1.3065	1.3237	1.3410

TABLE 88.--Averaging process to find the conditional opportunity cost of a type II error for various test values

Relevant Assignable Cause	Number of Perf. (F)	255		256		257	
		(C)	CF	C	CF	C	CF
Illness	20	\$1.1512		1.1885		1.2258	
Laziness	40	None					
Poor Attitude	60	.8492		1.0400		1.2309	
Dull Knives	120	1.2720		1.2892		1.3065	
	240		226.6160		240.8740		255.1500
		258		259			
	F	C	CF	C	CF		
Illness	20	1.2632		1.3006			
Laziness	40	1.5000		1.5352			
Poor Attitude	60	1.4218		1.6127			
Dull Knives	120	1.3237		1.3410			
	240		329.4160		345.1020		
Test Value		Averages = $\Sigma CF / \Sigma F$					
255		226.6160/200 = \$1.1331					
256		240.8740/200 = 1.2044					
257		255.1500/200 = 1.2758					
258		329.4160/240 = 1.3726					
259		345.1020/240 = 1.4379					

*See footnote 4 on page 328.



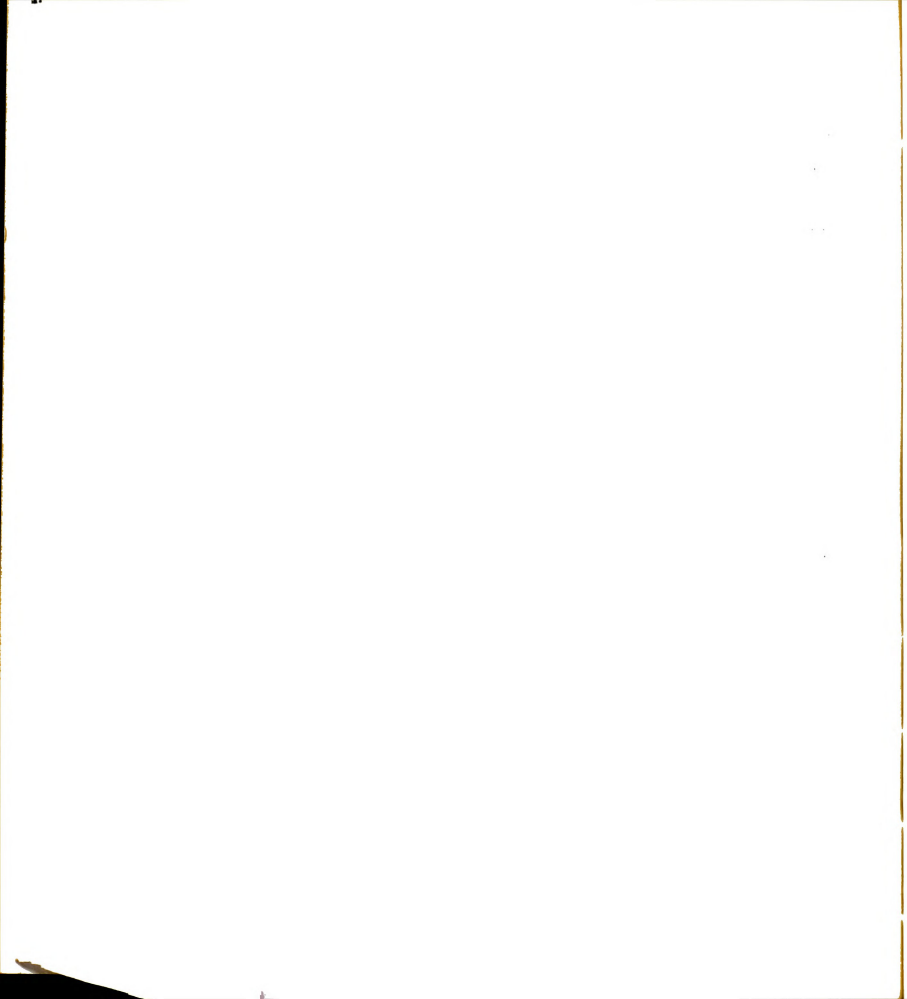
Minimization Approach

The following explanation pertains to the determination of the detail for Table 21.

The number of performances associated with each cause shown in Table 17 can be used to estimate the prior probabilities. Three causes can be eliminated in testing for an upper control limit of 260. These are improvement, tough cows, and lack of training. Improvement pertains to the establishment of the lower control limit. Tough cows and lack of training have not had values as low as 260. Accordingly, the relevant prior probability distribution for testing 260, is shown in Table 89. Of course, the probabilities represent the ratio that the number of performances for each cause bears to the total number of performances. For example, .7143 is equal to 600/840.

TABLE 89.--Relevant prior probability distribution for
test value 260

Cause	Parameter	Number of Performances	Probability
Chance	245	600	.7143
Poor Attitude	255	60	.0714
Illness	265	20	.0238
Dull Knives	270	120	.1429
Laziness	275	40	.0476
		840	1.0000



The weighted conditional opportunity costs are the same figures indicated in Tables 85 and 87 for test values 258, 259, and 260. They are estimated by interpolation for test values 261, 262, 263, and 264. The actual amounts were determined for test value 265. These amounts are shown in Table 90 in conjunction with the weighted conditional opportunity costs for test value 260. Notice that the weighted costs are higher for test value 265. This is because the probability of not detecting the shift to the assignable cause parameter is higher for a control limit at 265 than for a control limit at 260.

TABLE 90.--Weighted conditional opportunity costs for test values 265 and 260

Cause	Weighted Conditional Opportunity Cost		Difference
	260	265	
Illness	\$1.3379	\$1.5335	\$.1956
Laziness	1.5825	1.7122	.1297
Poor Attitude	1.8036	3.6031	1.7995
Dull Knives	1.3583	1.4408	.0822

That is, the probability of committing a Type II error is higher for each assignable cause at control limit at 265; therefore, the weights are higher in determining the weighted costs by the procedure indicated in Tables 10 and 84.

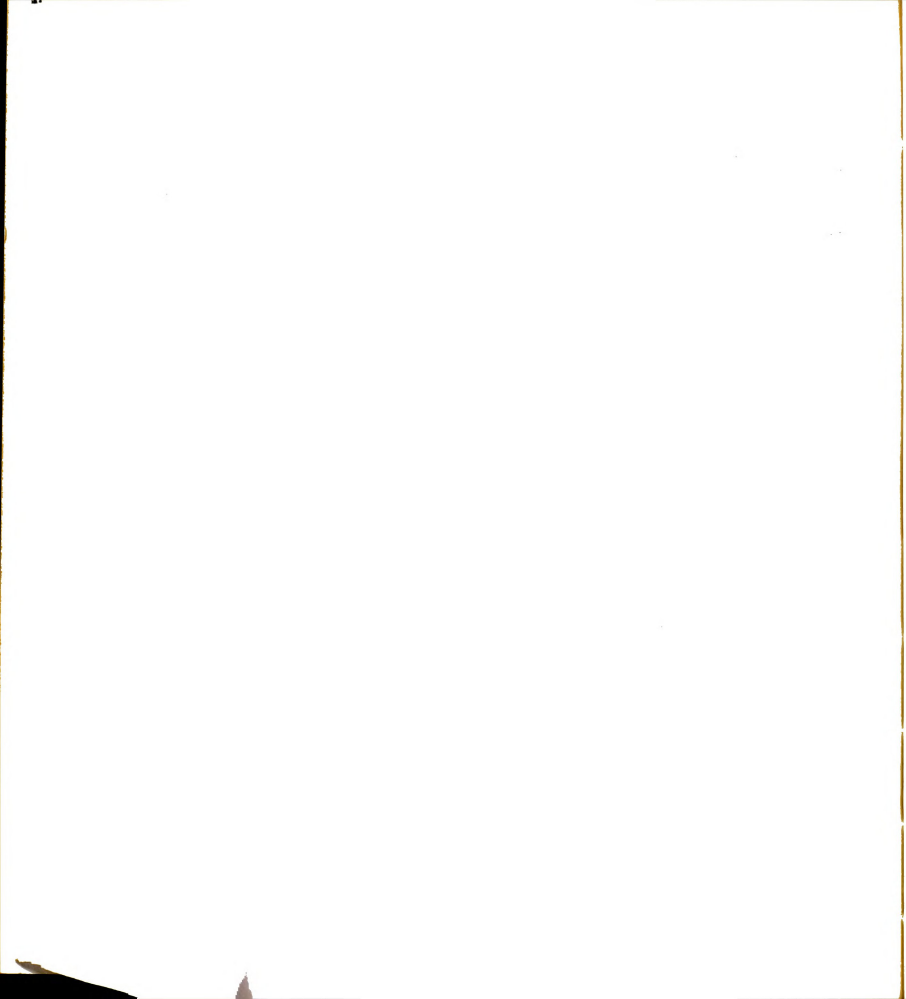
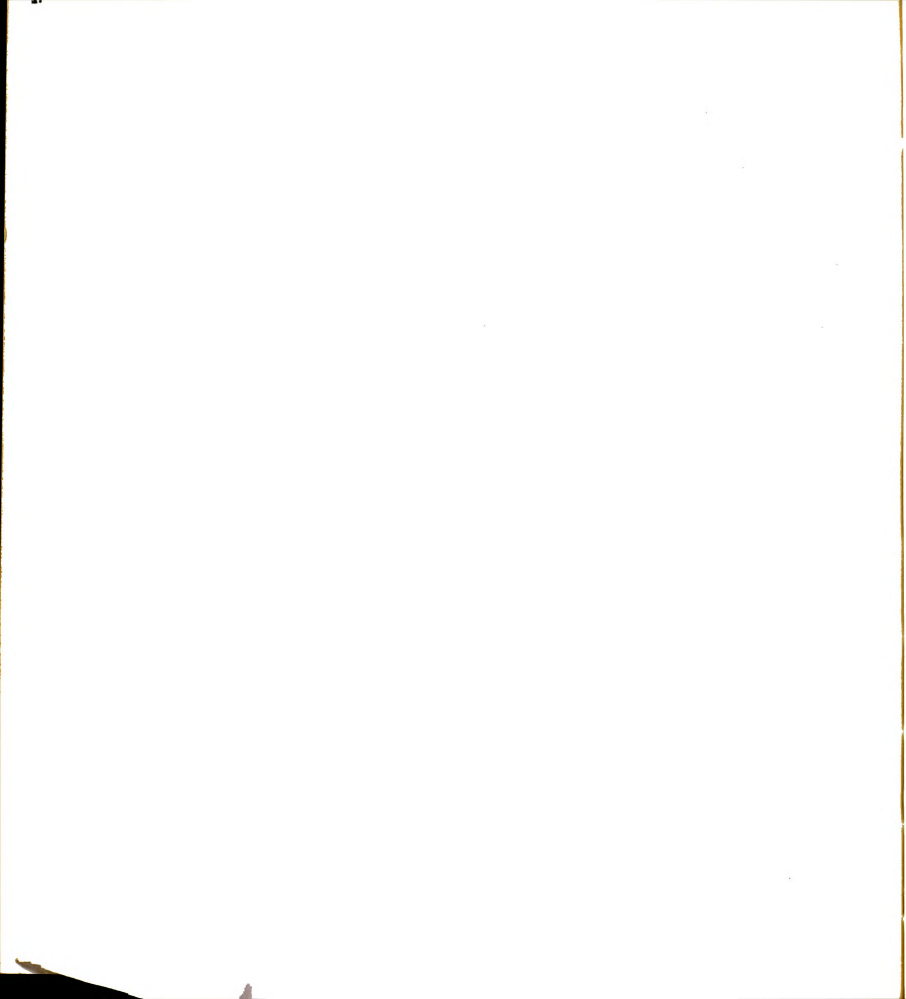


Table 91 shows the interpolated weighted conditional opportunity costs for test values 261, 262, 263, and 264. The weighted cost for test value 261 is determined by adding to the weighted value for 260 one-fifth of the difference between the weighted costs at 260 and 265. The weighted cost for test value 262 is found by adding two-fifths of the difference, etc.

TABLE 91.--Weighted conditional opportunity costs for selected values determined by interpolation

Cause	Weighted Costs			
	261	262	263	264
Illness	\$1.3770	\$1.4161	\$1.4553	\$1.4944
Laziness	1.6084	1.6344	1.6603	1.6863
Poor Attitude	2.1635	2.5234	2.8833	3.2432
Dull Knives	1.3750	1.3915	1.4079	1.4244

For the chance parameter, a wrong decision consists of rejecting the hypothesis for values of 260 or more when indeed chance caused the variation. This is the probability of a Type I error which is found for test value 260 by obtaining the number of performances of at least 260, from Table 18, and placing this number over 600--the total number of chance performances. The result is 22/600 or .0367 which is indicated in the probability of a wrong decision column. This same figure was determined in conjunction with the Equalization approach.



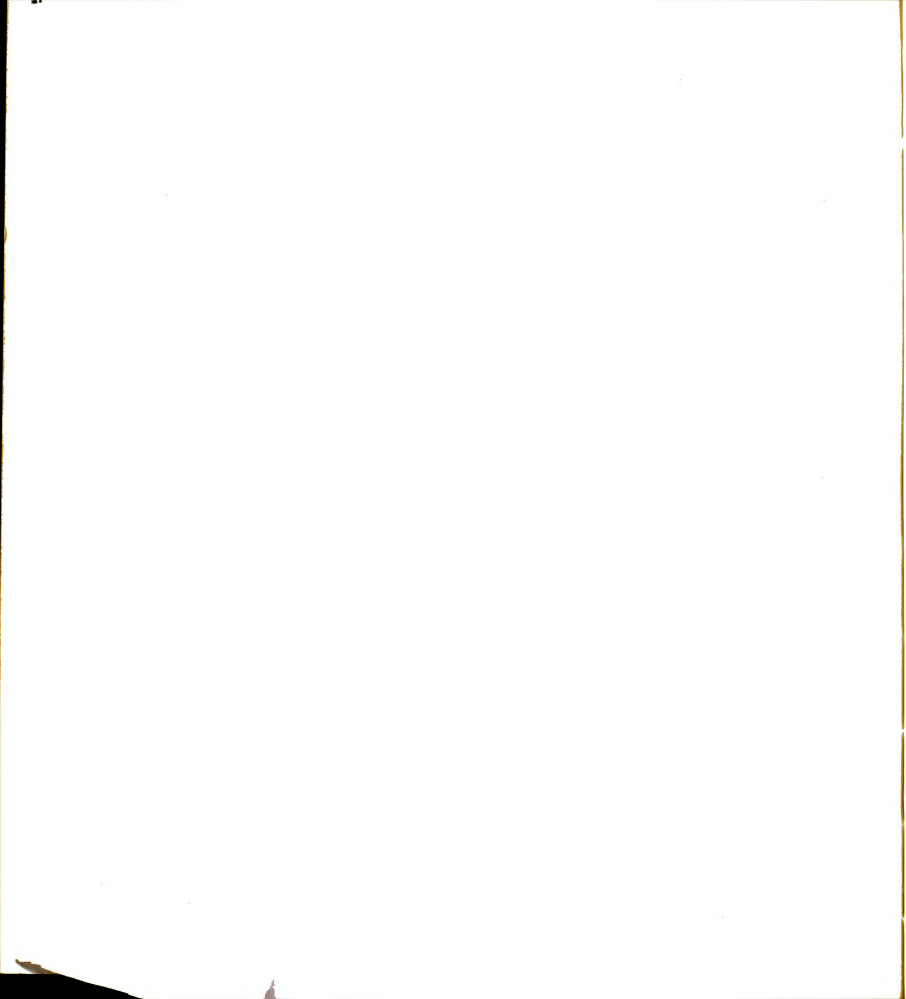
For the assignable cause parameters, a wrong decision is made when the hypothesis is accepted for values less than 260 when one of the assignable cause parameters is operating.

The derivation of the probabilities of a wrong decision for each assignable cause for test value 260 are shown in Table 92. The probabilities of a wrong decision are determined by dividing the number of performances less than 260 by the total number of performances corresponding to the given assignable cause.

TABLE 92.--Calculation of the probabilities of a wrong decision for each assignable cause under test value 260

Cause	Number of Performances Less than 260	Number of Performances	Probability of A Wrong Decision
Poor Attitude	45	60	.75
Illness	5	20	.25
Dull Knives	10	120	.0822
Laziness	2	40	.05

The conditional average opportunity cost figures in Table 21 are determined by multiplying the weighted opportunity costs for each cause by the corresponding probability of a wrong decision. These figures represent the average opportunity cost given the occurrence of each respective parameter.



Derivation and Financial Analysis of
Lower Control Limits for Single
Observations--Each Performance
Tested

Bierman, Fouraker, and Jaedicke
Approach

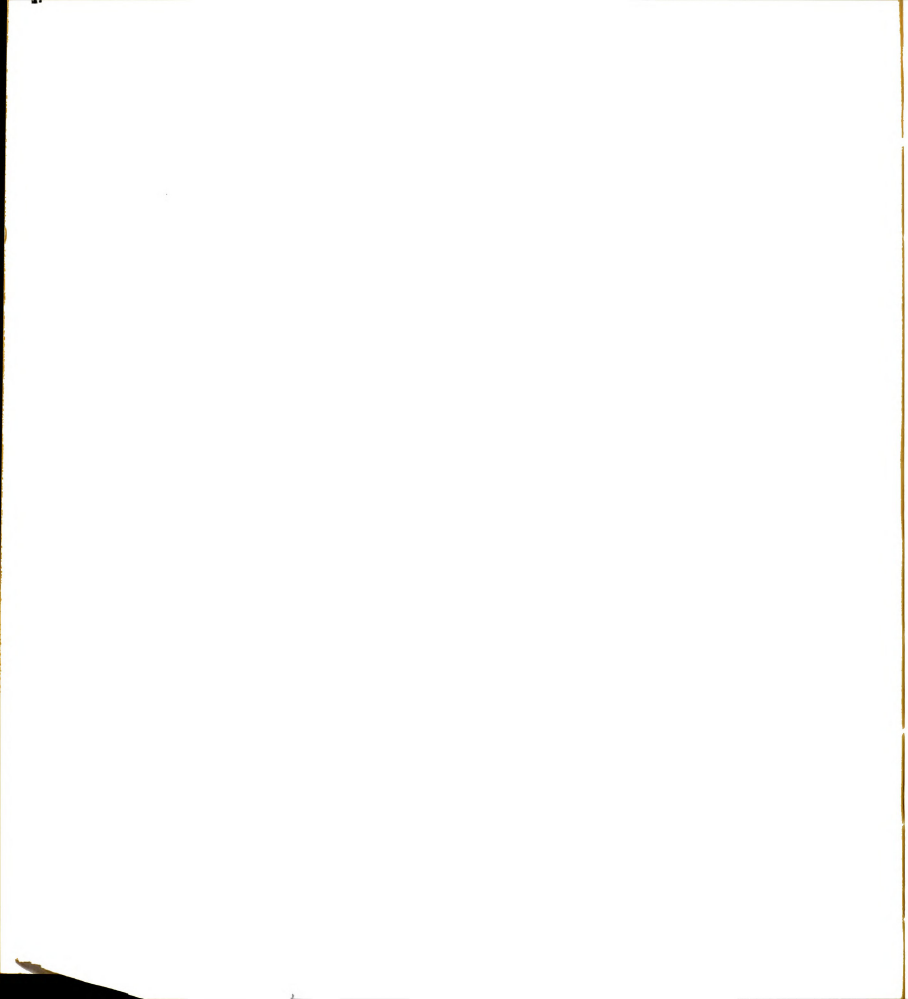
First Interpretation of P. The following explanation pertains to the derivation of the figures used in Table 24.

The "L" value for the Bierman, Fouraker, and Jaedicke approaches is calculated as it was for the derivation of the upper control limit by multiplying the single performance opportunity cost by four. The single performance opportunity cost associated with test value 220 is \$1.25 ($\frac{245 - 230}{60} \times \3). Hence, L is \$5. The cost of an investigation is given at \$4.

Accordingly, P_c is .2 ($P_c = \frac{L - C}{L} = \frac{\$5 - 4}{5}$) for test value 220.

For any test value, P is derived by dividing the number of chance performances at least as far from the standard as the test value by 600--the total number of chance performances. This result is then divided by .5 to limit the sample space to only one-half the curve which, of course, takes cognizance of the fact that any deviation is either favorable or unfavorable.

To find P for test value 220, it is necessary to refer to Table 2 to discover that only one chance performance is as far from the standard as 220. Hence, P is



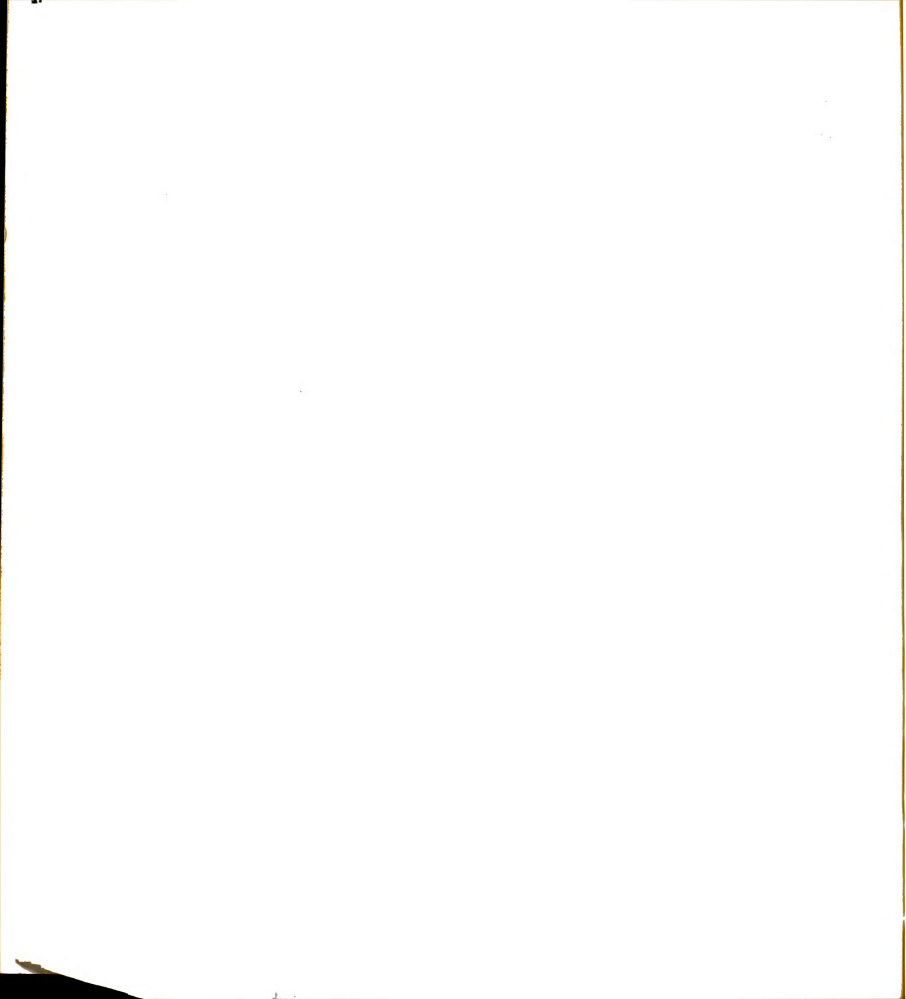
(1/600) divided by 2 or .0034. Since this is smaller than P_c , which is .2, the hypothesis should be rejected. The lower control limit which is on the boundary between the region of acceptance and rejection is thus higher than 220. It is, in fact, between 224 and 225 as indicated in Table 24.

Second Interpretation of P. In order to evaluate P according to the second definition, it is necessary to divide (1) the number of times that each deviation (or test value) has resulted from chance causes by (2) the total number of times that each deviation (or test value) has occurred. Table 18 contains the information to evaluate P in this manner.

For test value 220, P is thus 1/3 or .33. Since this is greater than P_c , the hypothesis should be accepted. Normally, acceptance indicates that the control limit is in the direction of a value farther from the standard. Since there are no chance values less than 220, P for each of these values is 0. On the other hand, P_c is .2307⁵ for test value 219 and it becomes increasingly greater as the test value moves farther away from the standard. Hence, the model indicates that the hypothesis should be rejected for all values less than 220. Rejection signals the need

$$^5 P_c = \frac{L - C}{L} = \frac{5.1996 - 4}{5.1996} = .2307$$

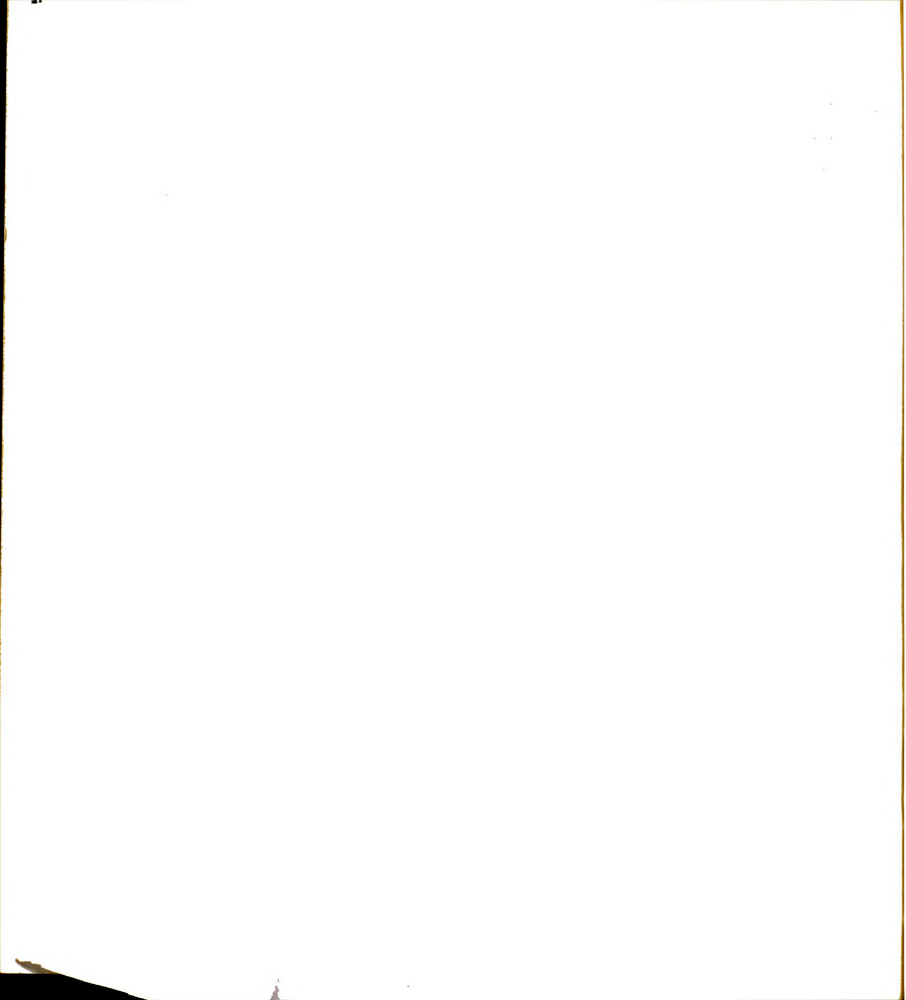
$$\text{where } L = \frac{245 - 219}{60} \times \$3 \times 4 = 5.1996.$$



for an investigation. However, for values less than 220, one might logically conclude that improvement has taken place without undertaking an investigation to verify it. This logic stems from the fact that all performances less than 220 have in the past been due to improvement.

The decision to reject but not to investigate is made solely on the basis of logic regardless of the model employed. Indeed, the other models would also indicate rejection for values less than 220. The point here is that this model does not appear to yield a control limit in the range 220 to 245 and is not then helpful as a model for decision making in this case. Simple inspection of the 1000 performances indicates that all values less than 220 are due to improvement.

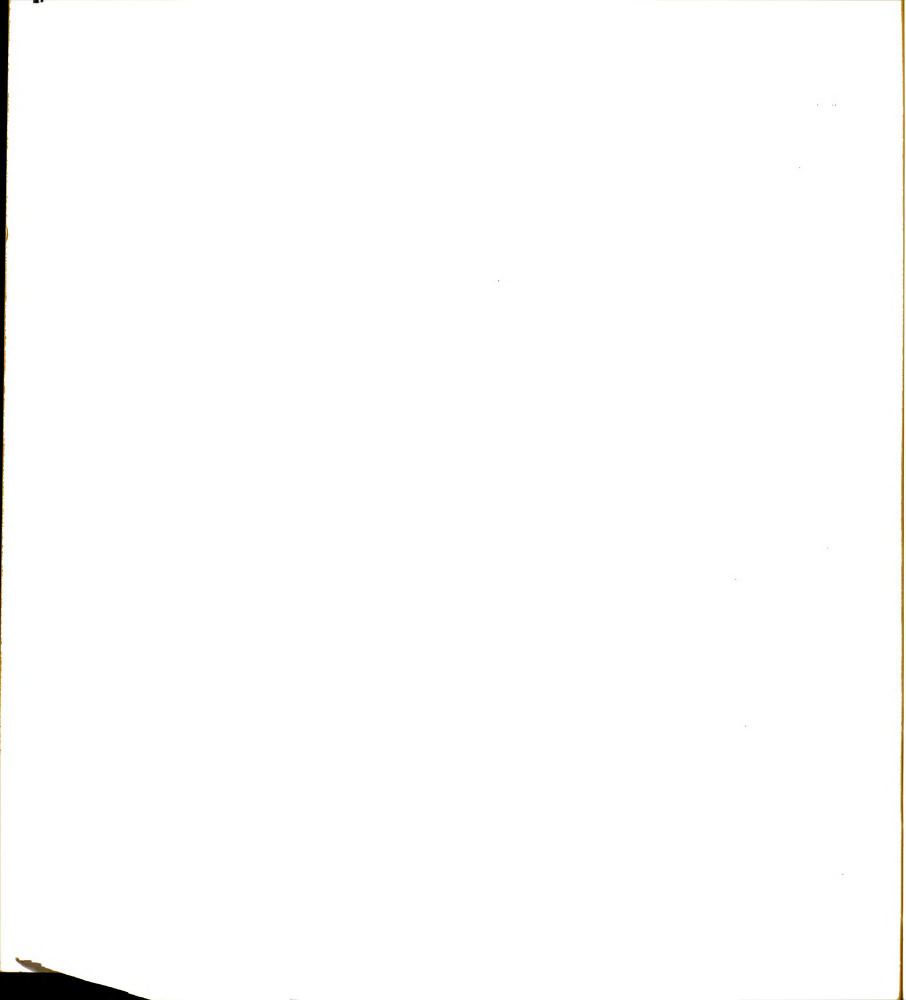
There remains, however, another consideration. While normally acceptance of a hypothesis for any test value indicates that the control limit is in the direction of a value farther from the standard, the actual results of 1000 performances compiled on Table 18 do not form a completely continuous series. The reader will note that no chance values were observed with values 221, or 222. Therefore, P for both of these values is 0. Tables 24 and 25 show that the P_c values for 221 and 222 are .1667 and .1304 respectively--both larger than 0. Table 25 is not very helpful in placing the control limit. Indeed



it appears as if there may be two control limits--one between 220 and 221 and another between 222 and 223. Of course, there would also arithmetically be one between 219 and 220. Perhaps, it would be best to think of the control limit as being between 222 and 223 with the one between 220 and 221 resulting only because 220 is a stray value which appeared after the continuous series of chance performances had ended. This conclusion is not reached without serious reservations concerning the utility of this model in this situation. If more than 1000 performances had been observed initially, a continuous series would probably have resulted. This would have rendered this interpretation of P useless in calculating a lower control limit under this testing plan.

Equalization Approach

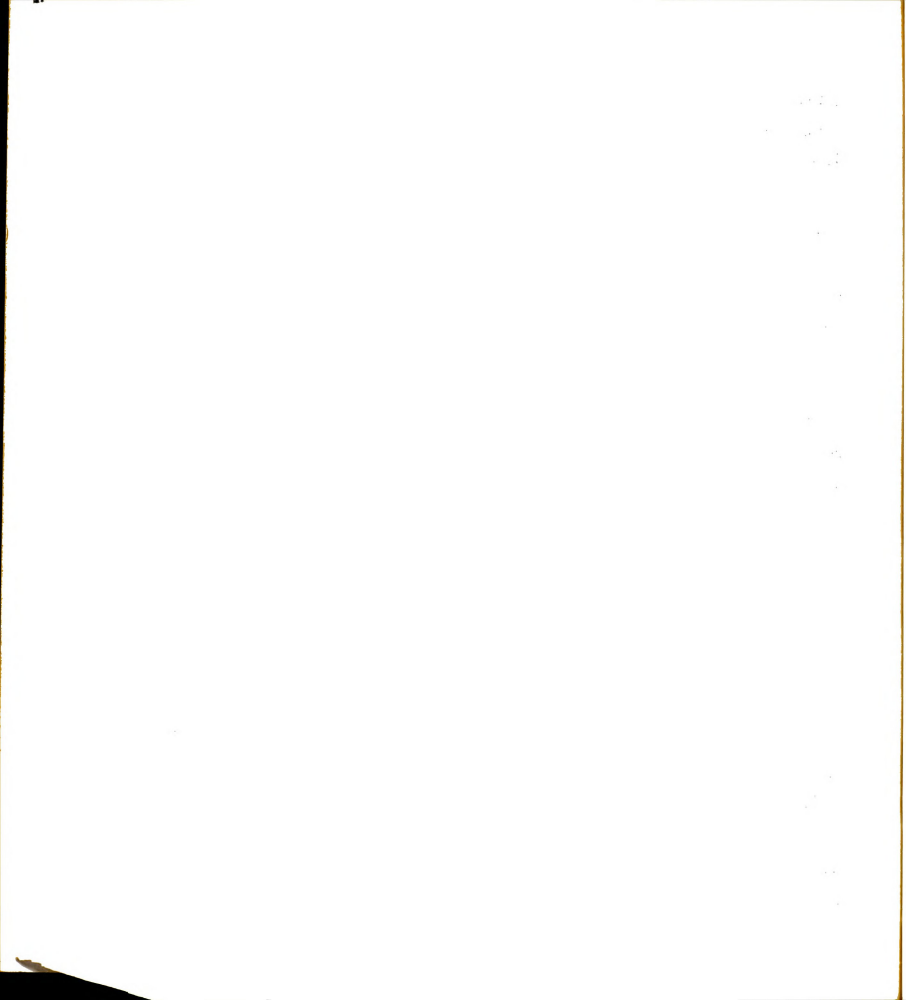
Some explanation of the determination of the individual values in Table 26 might be helpful. The probability of a Type I error represents the number of chance performances at least as small as the test value divided by 600, the total number of chance performances. That is, a Type I error is committed when a chance performance is less than the control limit. The opportunity cost of a Type I error, \$4, is the cost of an investigation. The derivation of this figure was explained early in this chapter under the sub-title "Investigation Procedure."



Of course, the expected opportunity cost of a Type I error is the product of the probability of a Type I error and the opportunity cost of a Type I error.

The probability of a Type II error for any test value is the number of improved performances higher than the test value divided by 100, the total number of improved performances. That is, if the improved performances is more than the test value chosen as the lower control limit, the improved situation will not be detected, and a Type II error will be made.

The cost of a Type II error is the cost engendered by failure to recognize improvement. It may result in the worker leaving the firm to take another opportunity that appears to have more potential for recognition. The firm would then have to incur the cost of hiring a replacement. Alternatively, the error may result in the worker in question influencing the morale and hence the performance of his co-workers. Finally, without proper recognition further efforts for self-improvement might be stifled. It is difficult to defend an analytical quantification of these costs. In the example in Chapter V, the opportunity costs were determined in the same way as the opportunity costs for equal unfavorable changes in the cause system. For convenience, the same method will be used here. The mean of improved performances is 230. This represents a 15-minute or 1/4 hour $(\frac{245 - 230}{60})$ change in the cause



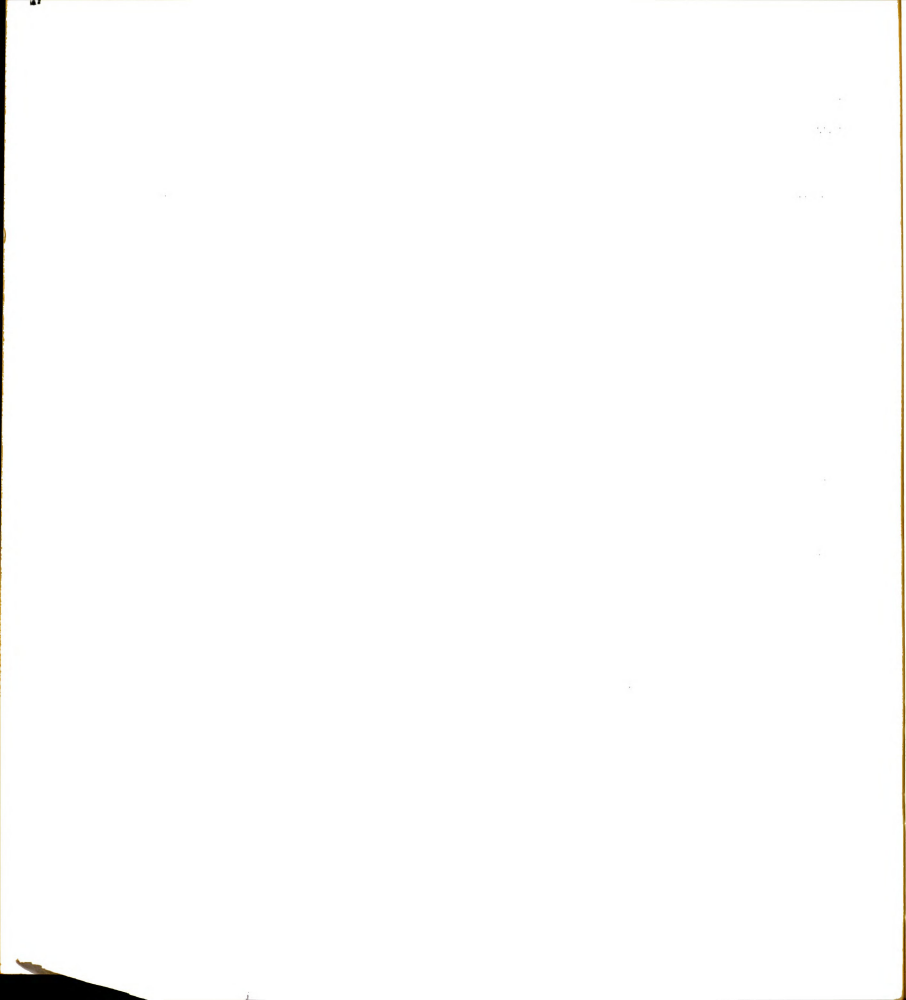
system. The opportunity cost associated with one improved performance is then \$.75 ($1/4 \times \3). This \$.75 is weighted for test values 230 and 235 by the same procedure as that discussed in conjunction with Tables 10 and 84. This weighting accounts for the fact that improvement will not always be detected on its first occurrence. The weighted opportunity costs are \$1.4615 and \$1.0254 for test values 230 and 235 respectively. The costs for the test values between 230 and 235 are determined by interpolation.

According to established procedure, the expected opportunity cost of a Type II error results from multiplying the probability of a Type II error by the opportunity cost of a Type II error.

Derivation and Financial Analysis of
Upper Control Limits for Single
Observations--Every Tenth
Performance Tested

Bierman, Fouraker, and Jaedicke
Approach

First Interpretation of P. The value for L is the product of three factors--the single performance opportunity cost, ten which represents the ten performances which lapse on the average before another test is taken, and four the number of tests which must be made on the average before an assignable cause is detected according to the Bierman, Fouraker, and Jaedicke assumption. The single performance opportunity cost for test value 255 is \$.50 ($\frac{255 - 245}{60} \times \3).



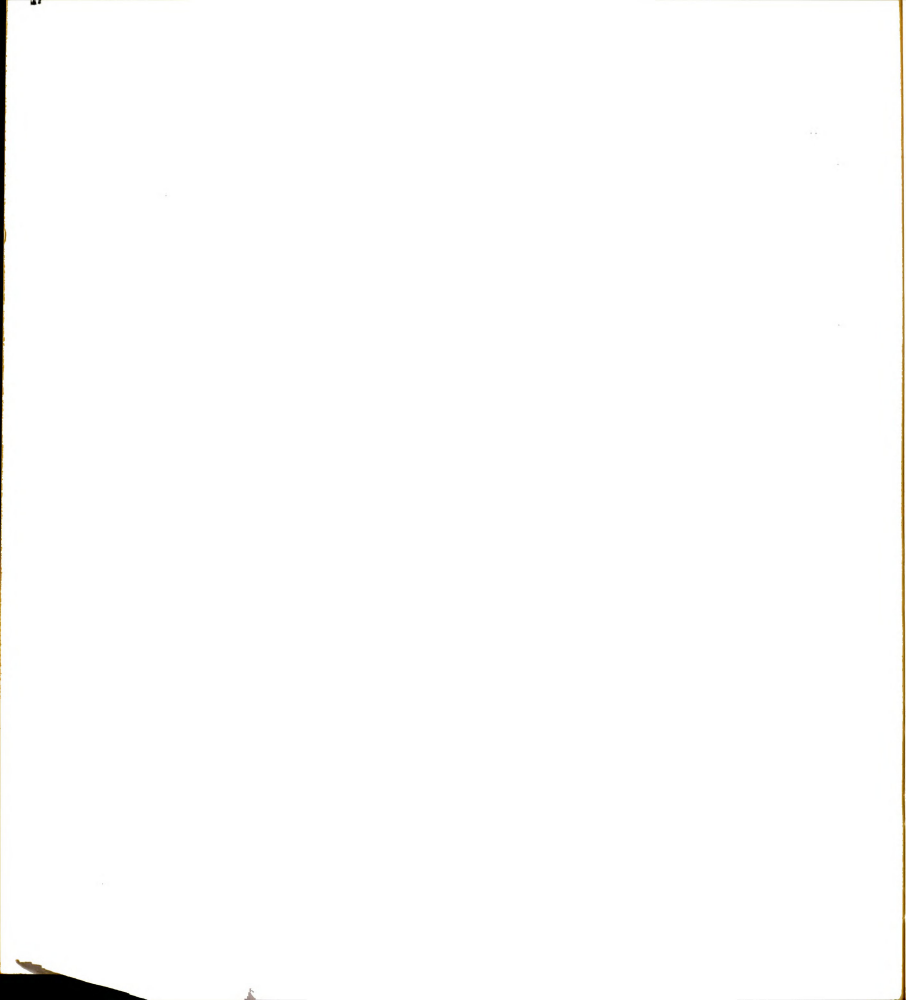
Hence, L for this test value is $\$.50 \times 10 \times 4$ or \$20. The other L values differ among test values only because each test value is used to calculate its corresponding single performance opportunity cost.

In Table 29, C remains constant at \$5 regardless of the test value. Pc is determined by the formula $\frac{L - C}{L}$. Two steps are necessary to calculate P. First, the total number of chance performances, 600, is divided into the number of chance performances at least as large as the given test value. Second, the above result is divided by .5 to limit the sample space to only one-half the curve. This second step emanates from the directive that this interpretation of P is the probability of a deviation this large or larger occurring from random causes given the occurrence of a favorable [or unfavorable] deviation.⁶ The underlined portion of this sentence necessitates the second step.

Second Interpretation of P. According to this interpretation, P represents "the probability of an unfavorable deviation resulting from uncontrollable [chance] causes."⁷ In Table 30, P is calculated by dividing the number of chance performances for each respective test value by the total number of performances observed for the given test

⁶Bierman, Fouraker, and Jaedicke, pp. 113 and 119.

⁷Ibid., 121.



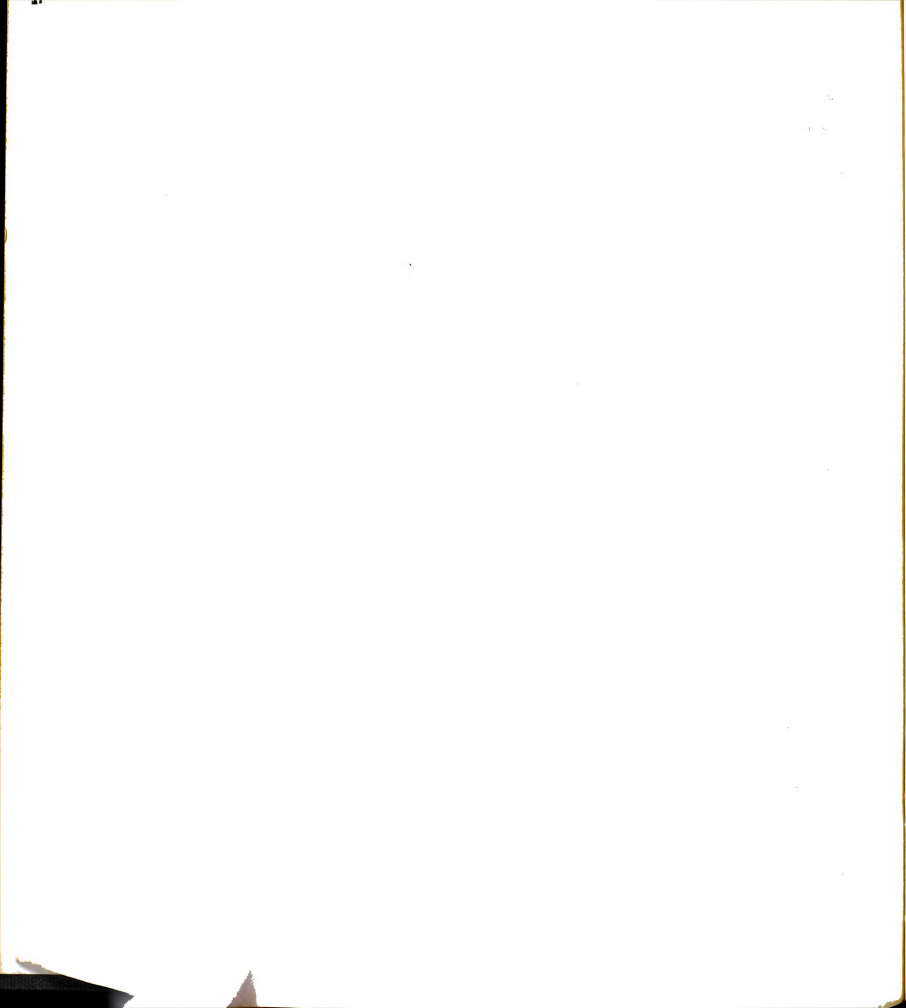
value. This information is obtained from Table 18. For example, for test value 255, 18 performances out of 29 total performances at that value were attributed to chance. Hence, P is $18/29$ or $.6207$. This represents the probability that the deviation results from chance causes. This second interpretation is certainly different from the probability of a deviation this large or larger occurring from random causes given the occurrence of an unfavorable deviation⁸ (the first interpretation). For test value 255, Table 29 indicates that P under the first interpretation is $.2966$.

Equalization Approach

The following explanation pertains to the process of interpolation used to find the opportunity costs of committing a Type II error for test values 254 and 253 which appeared in Table 35.

The weighted conditional opportunity costs have already been calculated for test value 255. These values were listed in Table 86. They are again shown in Table 93 along with the same costs for test value 250. The difference between the costs at these two values is also shown along with one-fifth of this difference. Then, in Table 94 the weighted costs are determined for each assignable cause for test value 251 by adding one-fifth of the difference to the weighted costs for test value 250. For

⁸Ibid., pp. 113 and 119.



test value 253, three-fifths of the difference is added to the cost for test value 250, etc. for the other values.

TABLE 93.--Differences in weighted conditional opportunity costs between test values 250 and 255⁹

Cause	Weighted Conditional Opportunity Cost			Diff	1/5 Diff
	255	250	254		
Illness	\$1.1512	\$1.0476		\$.1036	\$.02072
Poor attitude	0.8492	.6129		.2363	.04726
Dull knives	1.2720		\$1.2706	.0014	

TABLE 94.--Weighted conditional opportunity costs determined by interpolation

Cause	Weighted Costs			
	251	252	253	254
Illness	\$1.0683	\$1.0890	\$1.1098	\$1.1305
Poor attitude	.6601	.7074	.7547	.8019
Dull knives			1.2692	1.2706

The weighted costs for each assignable cause are averaged in Table 95 to find the conditional opportunity cost of a Type II error. This averaging process takes the same form as that previously indicated in Tables 85 and 88.

⁹ Since there are no performances attributed to dull knives below 250--the weighted conditional opportunity cost of a Type II error cannot be evaluated for test value 250. Instead it is evaluated for test value 254 and the difference between the costs at 255 and 254 is taken. This difference of .0014 is then subtracted from the cost at 254 to estimate the cost shown in Table 94 for test value 253.

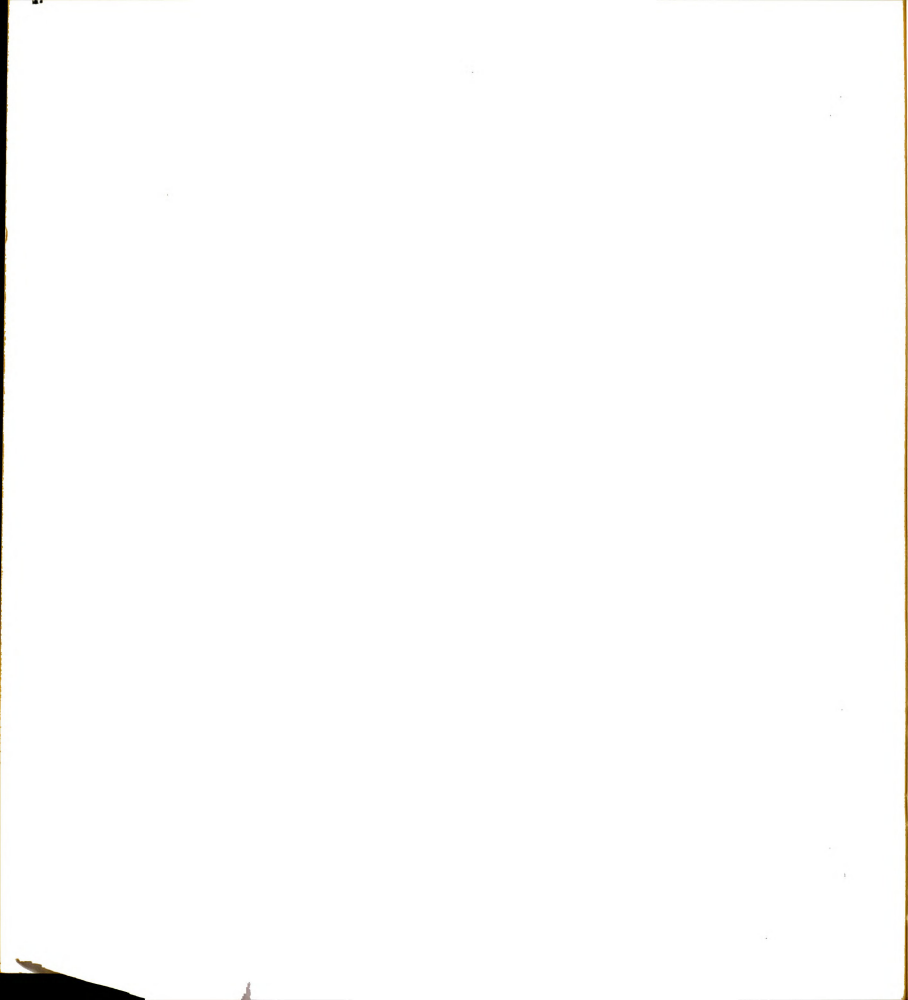
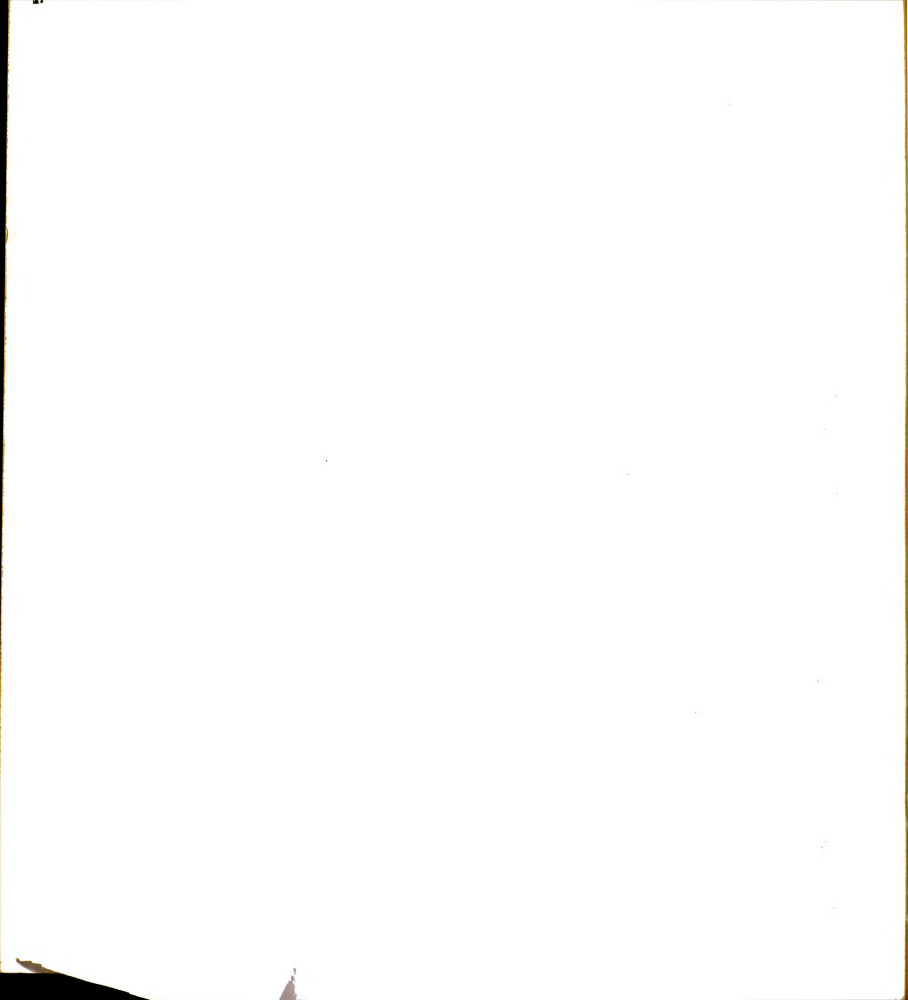


TABLE 95.--Averaging process to find the conditional opportunity cost of a Type II error for test values 253 and 254

Relevant Assignable Cause	Number of Perf. (F)	253		254	
		Weighted Cost (C)	CF	Weighted Cost (C)	CF
Illness	20	\$1.1098		\$1.1305	
Poor Attitude	60	.7547		.8019	
Dull Knives	<u>120</u>	1.2692		1.2706	
	200		<u>\$219.7820</u>		<u>\$223.1960</u>
<u>Test Value</u>		<u>Averages = $\Sigma CF / \Sigma F$</u>			
253		$\$219.7820 / 200 = \1.0989			
254		$\$223.1960 / 200 = \1.1160			

The averaged values derived in Table 95 must be multiplied by ten before entering them in Table 33 as opportunity costs of committing a Type II error. The reason for this is, of course, because a test is taken on the average once in every ten performances. The weighted costs used in Tables 93, 94, and 95 are weighted to account for the fact that an assignable cause may not be detected on the first test after its occurrence. They are not, however, weighted to consider the fact that a test is taken only once in every ten performances. Therefore, these values must be multiplied by ten before entering in Table 35.



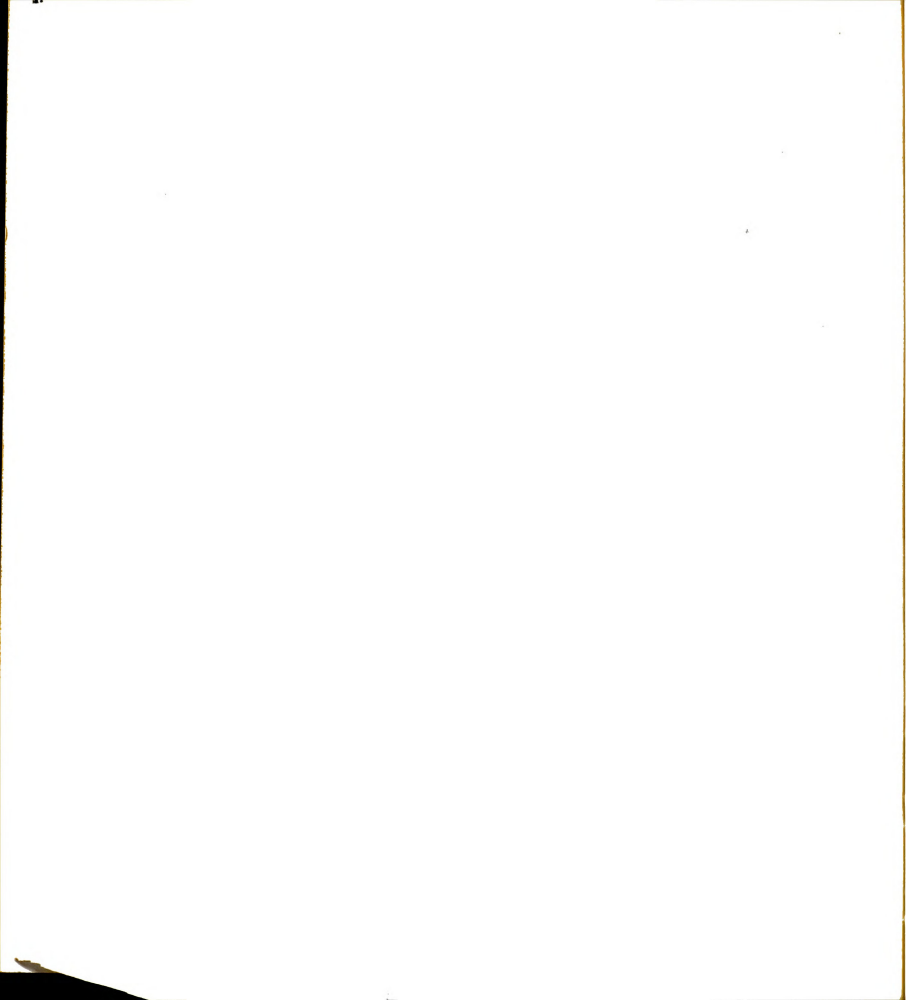
Derivation and Financial Analysis of
Upper Control Limits--Sample Size
Five--Every Performance Included
in a Sample

Equalization Approach

The following explanation pertains to the determination of the detail shown in Table 50.

The probability of a Type I error represents the probability that a sample mean calculated from five randomly selected chance performances will be at least as great as the test value. These probabilities are determined in the same manner outlined in conjunction with the first interpretation of the Bierman, Fouraker, and Jaedicke approach except that here the fourth step which consists of dividing by .2 has been omitted. That is, each probability here is interpreted as the probability that a sample mean will be at least as large as the test value; whereas, the Bierman, Fouraker, and Jaedicke approach interprets each as the probability that a sample mean will be at least as large as the test value given the occurrence of an unfavorable (or favorable) deviation. Consequently, the probabilities under the Equalization approach are exactly one-half as large for each respective test value as those indicated in the column labeled "P" in Table 45.

Since a sample mean as low as the test values 250 or 251 is possible only for assignable cause poor attitude,



the opportunity cost of a Type I error is shown as \$1. That is, the investigation for these values would include only the \$1 test for a poor attitude investigation since other assignable causes would not produce sample means this low. The reader should keep in mind that the goal here is to determine the upper control limit. Certainly, sample means in a range likely to be the result of other assignable causes would induce the investigative procedures specified at the beginning of this chapter. These means, however, would be well within the significant range so the cost of the complete investigative procedure is not relevant in determining the control limit for this sampling plan.

Under the sampling plans in which every performance or every tenth performance was tested, the probability of a Type II error used in computing the upper control limit was determined by counting the number of unfavorable assignable cause performances listed in Table 18 that had values less than the test value. This number was then divided by 300--the total number of unfavorable assignable causes in the original subpopulation that is being used to estimate the probabilities of error. With the advent of sampling more than one performance at a time, sample means are used to determine the control limits. The number of assignable cause performances below the test value are no longer helpful in estimating the probability of committing Type II errors so the procedure for making these estimates must be modified somewhat.

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It is possible to compute the probability of obtaining a sample mean from five randomly selected performances pertaining to each assignable cause which is less than any test value. For instance, if 251 is the test value, poor attitude is the only assignable cause which can produce a sample mean less than 251. The probability of this happening can be calculated by finding the area less than 251 under the normal curve with mean 255 and standard error of the mean 4.0187.¹⁰ The result is .1611. The probability of committing a Type II error can be expressed in the following way: Given that the unfavorable assignable cause is present the probability that a performance will produce a value less than the test value chosen as the upper control limit is the probability of a Type II error. In this chapter the probability has been calculated by an averaging process so that it does not apply to any particular assignable cause. In this case, .1611 is the probability that a sample of five will produce a value less than 251 (the test value) given that poor attitude is present. This probability can be averaged so that it can be expressed in terms of a Type II error by first multiplying it by 60--the total number of performances due to poor attitude. The result of 9.666 represents the

¹⁰The mean of the 60 performances due to poor attitude listed in Table 18 is 255 and the standard error of the mean is 4.0187.

200

100

50

25

12

6

3

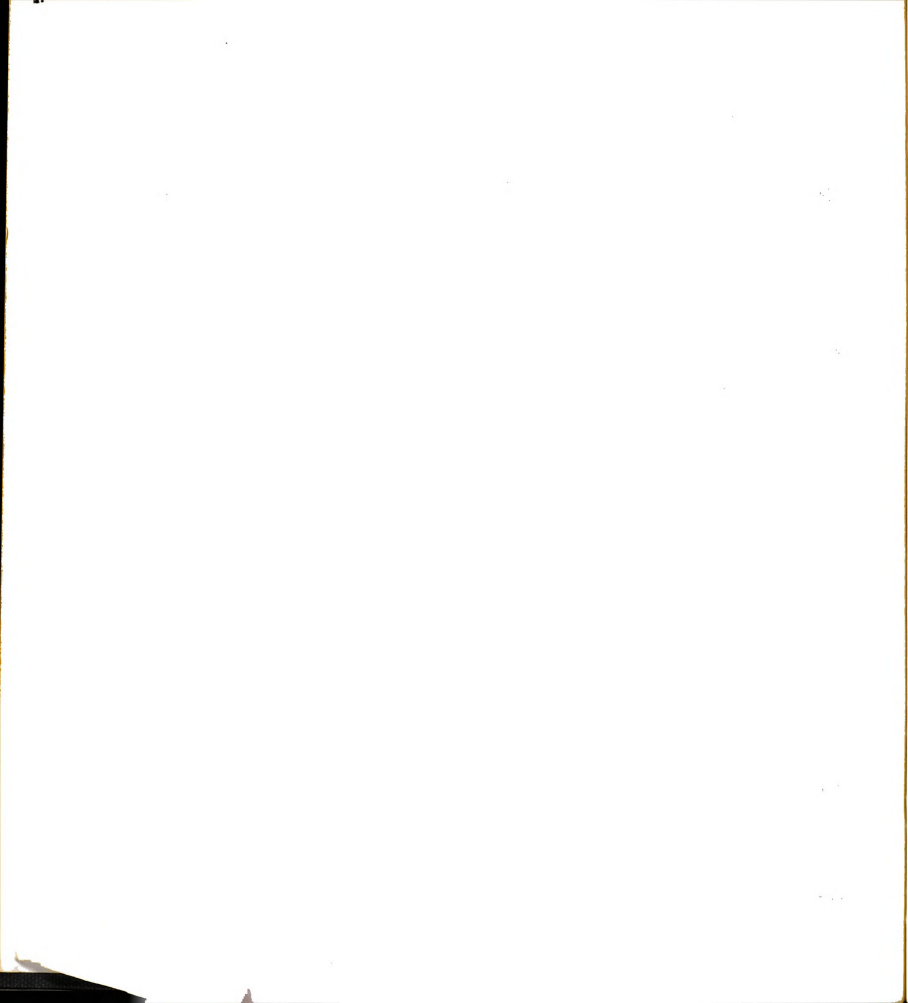
number of sample means less than 251 and due to poor attitude that could be expected from 1000 (the number used in the subpopulation from which the estimates of error have been made) samples. This number, 9.666 is then divided by 300 (the number of unfavorable assignable causes in the subpopulation) in order to arrive at the .0322 probability of a Type II error associated with test value 251.

The opportunity cost of a Type II error is found by:

1. Weighting the \$.50 single performance opportunity cost according to the procedure indicated in Table 10. The result is \$.5960.

2. Multiplying this \$.5960 by 5--the sample size. Since the sample must be completed before the test is made, five performances must lapse at the very minimum before significance can be established. This results in an opportunity cost of a Type II error of \$2.98 which is shown in Table 50.

The \$2.98 doesn't have to be averaged by similarly obtained figures for other assignable causes because it is impossible for other assignable causes to produce sample means as low as 251.



Derivation and Financial Analysis of
Upper Control Limits--Sample Size
Five--Sample Taken in Every
Fifty Performances

Equalization Approach

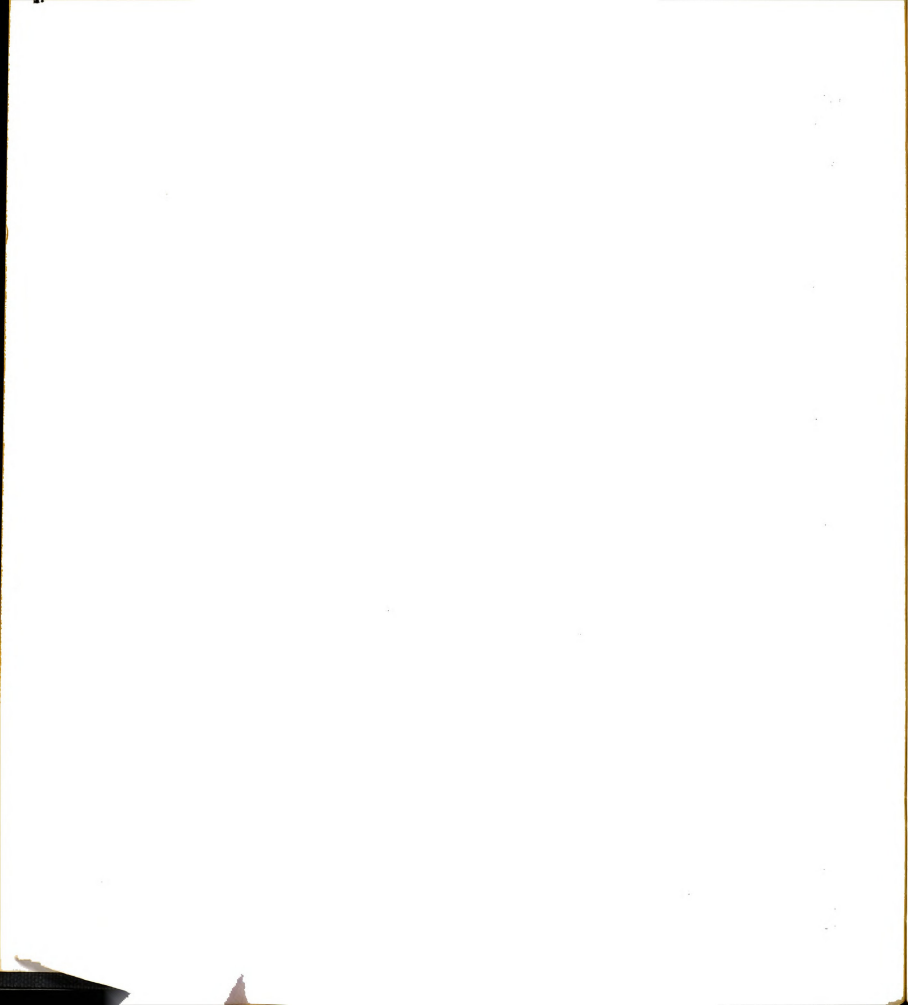
The following explanation pertains to the development of the figures in Table 65.

The probabilities of a Type I error are found by calculating the area under the normal curve with mean 245 and standard error of the mean 3.8923¹¹ that is equal to or larger than the test value. The procedure followed for test value 249 is enumerated below:

1. Find the area between the standard and the test value by converting the difference between 245 and 249 into standard units and using the table of normal curve areas. $z = \frac{249 - 245}{3.8923} = 1.03$. The corresponding area is .3708.
2. Subtract .3708 from .5 to find the area larger than 249. Thus, the probability of a Type I error corresponding to test value 249 is .1291 as shown in Table 65.

The opportunity cost of a Type I error remains at \$1, because the only investigation that would be undertaken for sample means in this range would be for poor

¹¹The mean of the chance population consisting of the 600 values listed in Table 18 is 245 and the standard error of the mean is 3.8923.



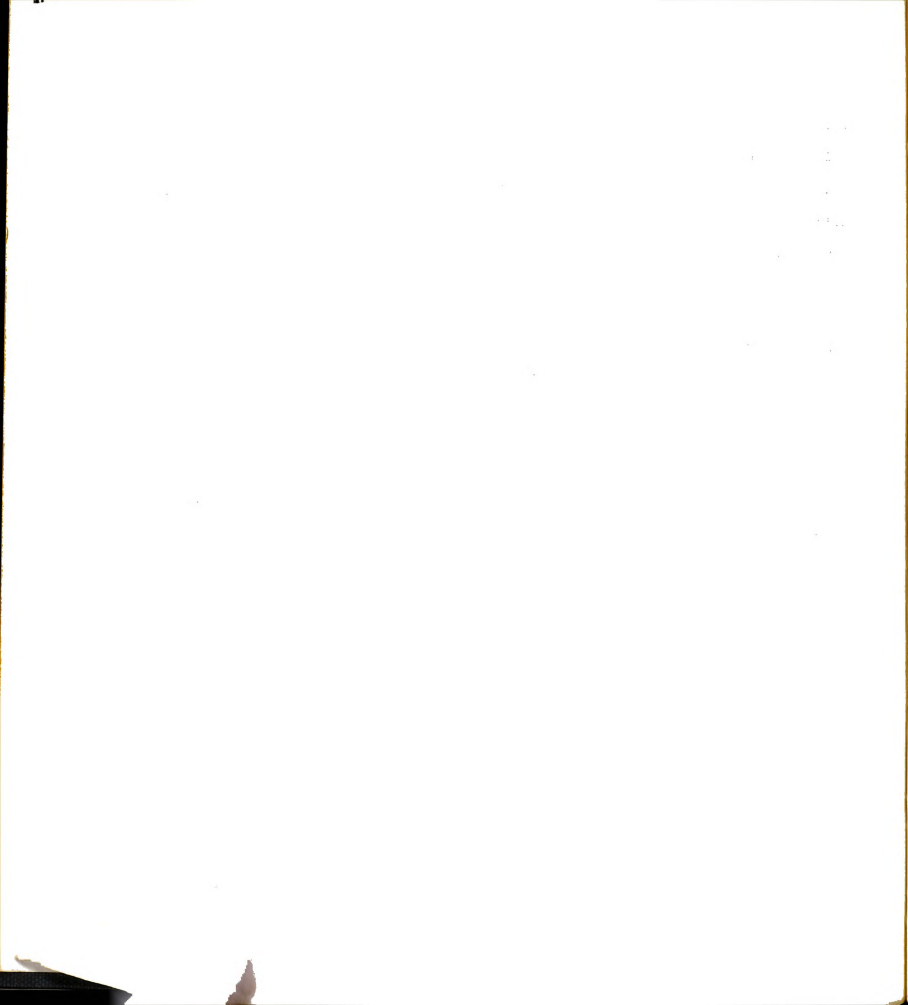
attitude. This is, of course, because it is impossible to obtain a sample mean this low when other assignable causes are prevailing.

The probabilities of committing a Type II error are estimated by the following procedure: (the relevant numbers corresponding to test value 249 are inserted.)

1. Find the area less than the test value (249) under the normal curve with mean 255 and standard error of the mean 4.0187.¹² The result is .0681.
2. Multiply the .0681 by 60 and divide that result by 300. $\frac{.0681 \times 60}{300} = .0136$.

The logic behind step two is to complete the averaging process so that the probability of a Type II error can be expressed in its usual manner as the probability of accepting a false hypothesis given the existence of an assignable cause (i.e., it can be expressed as the probability that five randomly selected performances due to an assignable cause will produce a sample mean less than the test value). If step two were not performed, the probability of a Type II error would be expressed only as the probability of accepting a false hypothesis given the existence of poor attitude. This issue was elaborated upon at greater length where the averaging process was first employed in Table 85.

¹²It has previously been noted that the mean of the 60 performances due to poor attitude is 255 and the standard error of the mean is 4.0187.

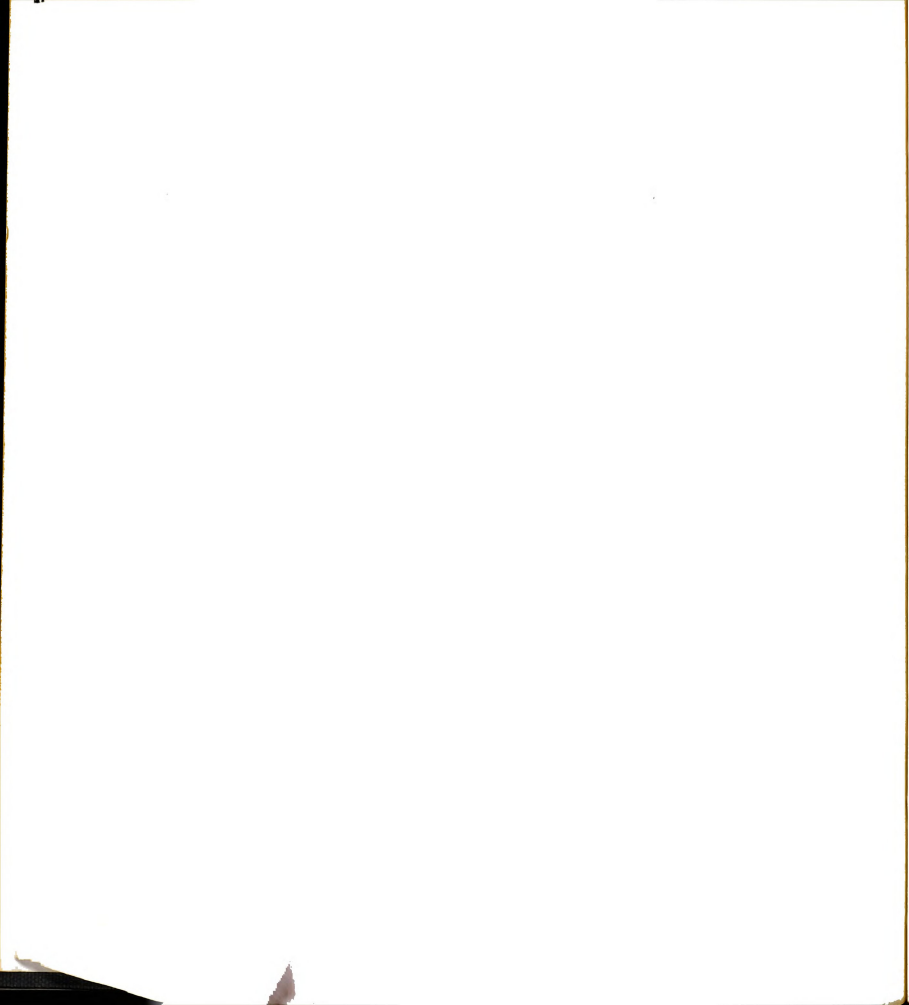


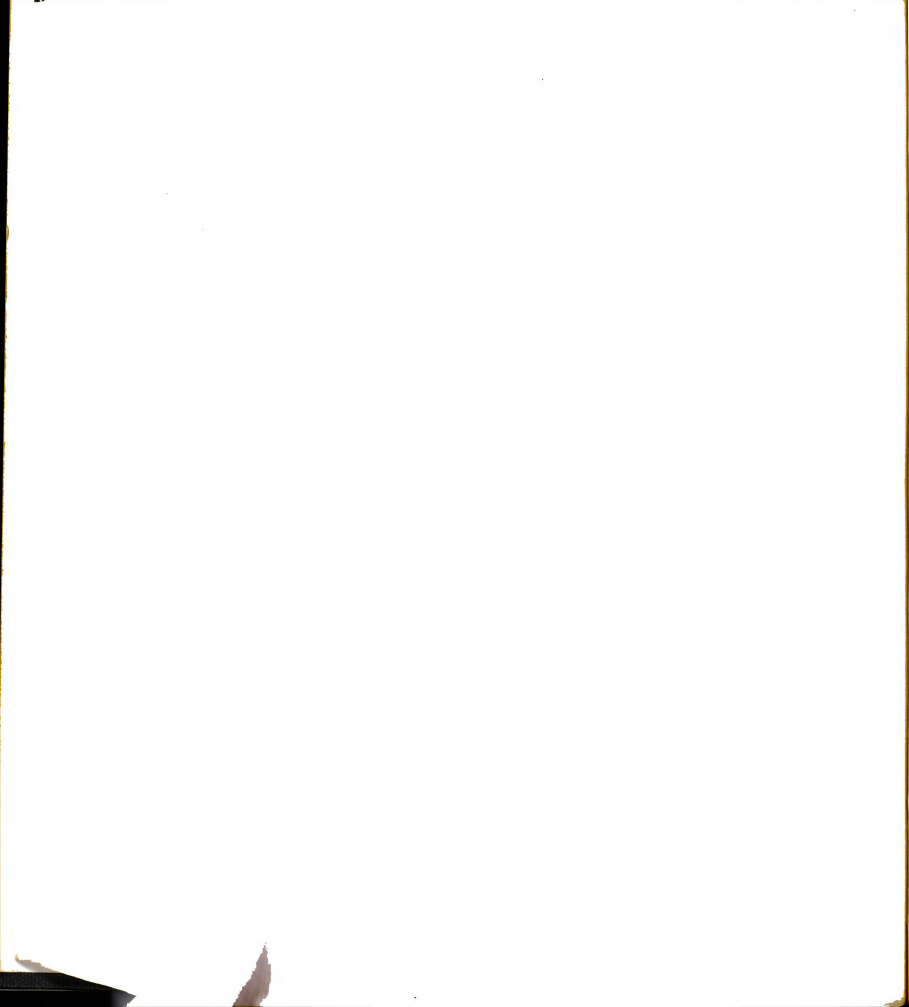
As before, the \$.50 single performance opportunity cost has been weighted (as illustrated in Table 96 for test value 249) to account for the fact that poor attitude may not be detected on its first occurrence. This weighted value is then multiplied by 50 to obtain the opportunity cost of a Type II error. This procedure is followed because a sample is drawn only once in every 50 performances; therefore, if poor attitude is not detected on its first occurrence 50 performances will lapse before another test is taken.

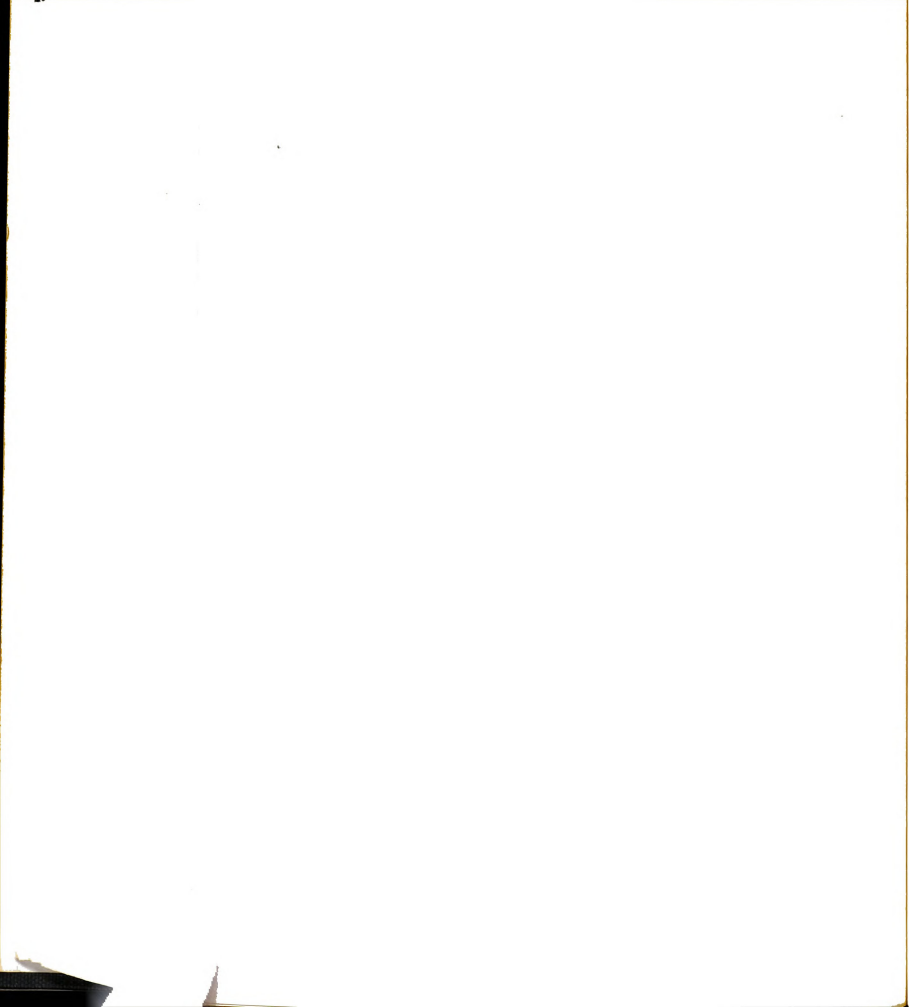
TABLE 96.--Weighted opportunity cost for test value 249

Number of Tests (A)	Accumulated Opportunity Losses (B)	Probability of Tests in Column A (C)	Column B times Column C (D)
1	\$.50	.0681	
2	1.00	.0046	
3	1.50	.0003	
		<u>.0730</u>	<u>\$.0391</u>

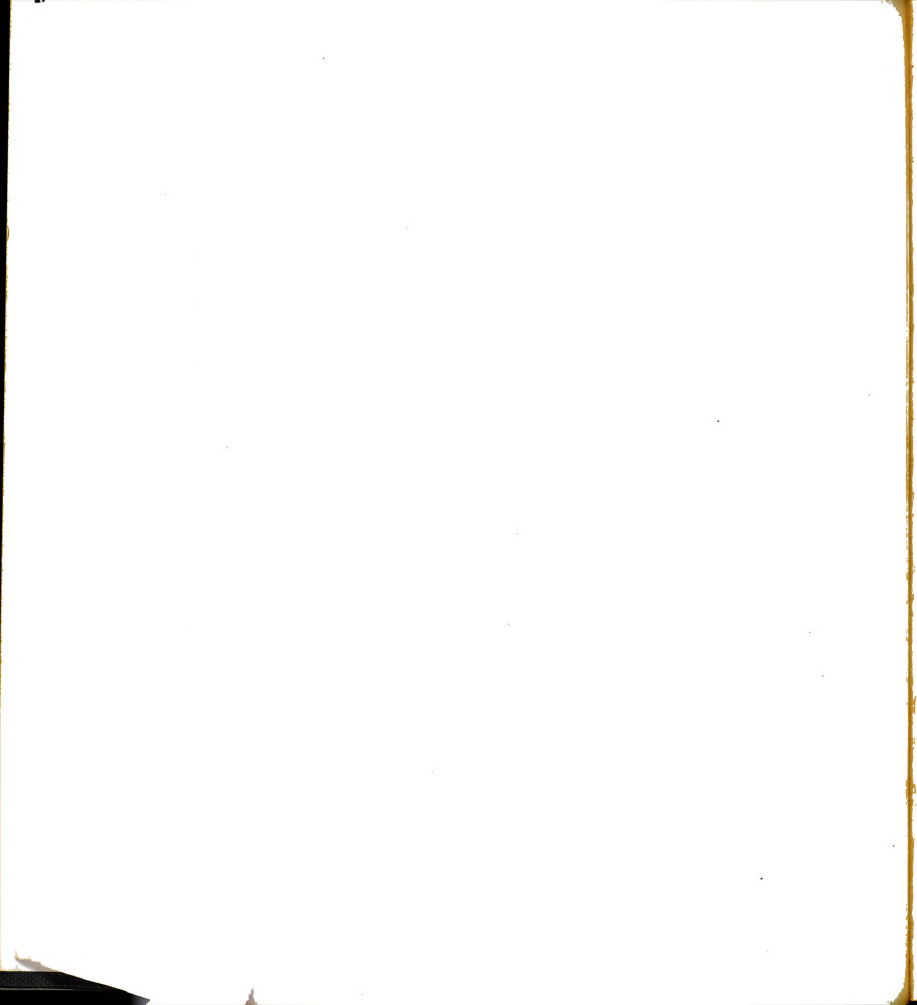
$$\text{Weighted Cost} = \$.0391 / .0730 = \$.5356$$











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