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Virginia Sue Muraski

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A STUDY OF THE EFFECTS OF EXPLICIT READING INSTRUCTION ON READING PERFORMANCE IN MATHEMATICS AND ON PROBLEM SOLVING ABILITIES OF SIXTH GRADERS

By

Virginia Sue Muraski

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ABSTRACT

A STUDY OF THE EFFECTS OF EXPLICIT
READING INSTRUCTION ON READING PERFORMANCE
IN MATHEMATICS AND ON PROBLEM SOLVING
ABILITIES OF SIXTH GRADERS

By

Virginia Sue Muraski

A. Population

The population for this study was a sixth grade class at an elementary school located near Grand Rapids, Michigan. The population was randomly assigned to the experimental and control groups, with thirteen students in each group.

B. The Problem

This study explored the effects of a five-week (450-minute) program in the reading of mathematics on the problem solving abilities of thirteen sixth graders. The reading program consisted of three 30-minute lessons for each of five selected reading subskills of word recognition and comprehension. The five reading subskills selected for this study were:

1. Instant Recognition of Special Symbols of Mathematics and Their Voiced Equivalents (Word Equivalents)
2. Structural Analysis
3. Contextual Prediction

4. Inferential Reasoning
5. Evaluative Thinking

The fifteen reading lessons were designed and taught by the researcher.

The objectives of this experiment were:

1. Describe the behaviors of sixth graders who were being taught how to use the five selected reading subskills of word recognition and comprehension in reading mathematics
2. Determine the impact of the teaching of the five selected reading subskills on the abilities of these sixth graders to use these skills in
   a. Interpretation of mathematical symbols
   b. Mediation of an unrecognized mathematical word through its structural units
   c. Obtainment of factual comprehension of a mathematical message by contextual prediction
   d. Exploration of understanding by inferring relations among mathematical concepts
   e. Formulation of mathematical problems from given problem situations through evaluative thinking
3. Determine statistically the impact of teaching these five selected reading subskills on the problem solving abilities of sixth graders

C. Data Collected

Statistical data were collected from a pretest and a posttest to measure problem solving abilities. The Null and Alternate Hypotheses tested in this study were:

H_0: There will be no difference between the mean scores of the two sets of differences obtained by subtracting, for each group, each subject's score on the pretest from his score on the posttest.

H_A: The mean score for the experimental group will be greater than the mean score for the control group.

A t-test was used on the two mean scores to determine if the difference between them was significant.

Other measures consisted of a retest for retention, classroom teacher's assessment of the subjects' classroom behaviors, a daily log kept by the researcher and interviews for problem solving analyses conducted by the researcher.

D. Results and Conclusions

According to the test statistic, the gains on problem solving by the experimental group were significant at the 0.005 level. It became apparent through observation and interviews by the researcher that as the sixth grade subjects decoded a written mathematical passage into language and tried to resurrect the intended message, each of the selected reading subskills played important roles in the performance of these subskills in mathematics and problem solving.

Dedicated to:

My husband, Edward

and

My children, Edward, Jr. and Michél

ACKNOWLEDGMENTS

I wish to express my gratitude to the members of my doctoral committee--Professors William Fitzgerald, Chairman, Gerald Duffy, Glenda Lappan, William Cole and Bruce Mitchell--for their generous contributions of time and talent in assisting me in the writing of this dissertation.

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INTRODUCTION

Introduction

Rudolf Arnheim(173) has stated that language has been termed "a better vehicle for thought" than anything else. The more extensive a person's linguistic development the better this person would be able to articulate his thoughts clearly and succinctly to himself as well as to others. While detached, theoretical thinking--which is sometimes the case in mathematics--can possibly function without words, there seems to be no question that language helps thinking. Piercey(161) writes:

> Many ingredients make up successful learning and successful performing. But perhaps the major, the pivotal, ingredient is the ability to manage the specialized language peculiar to a special field. . . . Ability to manage the language of a subject area can be developed, and that development is a function of teaching. It is estimated that 80 percent of the learning students are asked to do involves the printed word.

Earle(73) maintains that any content objective in mathematics can be mastered by psycholinguistic activity at one or more of the following levels of reading:

1. Perceiving Symbols
2. Attaching Literal Meaning

3. Analyzing Relationships
4. Solving Word Problems

If our humanness is gauged in part by our ability to use written language, then the quality of reading and writing produced in mathematics classes ought to be a central social issue. Pauk(155) asserts, "Learn the language of mathematics, and learn to use the words precisely. Only then can you fully and accurately communicate with others, orally or in writing, in the realm of mathematics." The language of mathematics is composed of words, word phrases, numerical symbols, diagrams, graphic and tabular symbols, as well as other special mathematical symbols. This language is the language found in mathematics textbooks and articles in journals, newspapers, magazines and other material related to mathematics. Learning to read the language of mathematics would entail being able to understand messages encoded by using each of these symbol systems.

Research regarding what in the reading act prohibits or enhances the understanding of written messages in mathematics is difficult and has really just begun. Aiken(6) believes that "a significant degree of overlap between verbal and mathematical abilities remains unexplained" and that the pivotal variable may not be general intelligence but rather reading or linguistic ability. Research(6,7) has shown that reading is an important factor in the acquisition of mathematical knowledge - the how's and the why's have not been clearly defined or described. Many educational researchers(72,73) agree that the mathematical development of children correlates highly with their ability to read and that the more meaningful the approach taken in learning mathematics the more important reading ability becomes.

Thomas Barretts's(16) description of reading is:

> Reading involves the visual perception of written symbols and the transformation of the symbols to their explicit or implicit oral counterparts. The oral responses then act as stimuli for a thoughtful reaction on the part of the reader. The type or level of thought induced by the stimuli is determined, in part, by the intent and the background of the reader and the nature of the materials. In addition, the effort expended in the perceptual act and the intellectual impact of the written materials on the reader are influenced by his interest in the specific selection and by his attitude toward reading in general.

To read mathematics, one must know the language, both words and the way the words form thought patterns, as well as how the special symbols of mathematics fit into these patterns. Many factors probably influence the attitudes(70) or motivation of students in mathematics, but it is doubtful that mathematics can be meaningfully studied without reading as one essential means of subject mastery. An instructor who develops reading skills for problem solving would probably help prevent discouragement and frustration and could possibly sustain student interest. A student who learns how to cope with the language of a subject area will have the means for continued learning for a life time. As in the proverb:

> If you give a man a fish, he will have a single meal; if you teach him how to fish, he will eat all his life.

In the literature of Duffy and Sherman(68), it is maintained that reading is the most crucial of the functional skills and consists of thousands of different subskills that extend from the simple to the complex but which fall in to four categories: word identification, comprehension, efficient study and literary understanding. Identifying an author's scheme for stringing words together, thinking it through and then arriving at some conclusions are apparently behaviors necessary for a reader to interact with the author. Piercey(161) claims that all students need help with these behaviors some of the time and:

> . . . teachers do not give this help is both tragic and deplorable. It is tragic from the standpoint that some learners spend six or seven periods a day, five days a week spurning the printed word instead of learning with it; it is deplorable from the standpoint that without needed help during their school years odds are that they will continue to ignore or misuse the printed word as a resource for data-gathering and pleasure in adult years.

Duffy and Sherman(68) claim that a person, who is unable to read or who has difficulty reading, lacks one or more of the basic skills needed to be a successful reader; consequently, instruction in the basic skills of reading must be one of the major components of a good reading program. It seems that very few humans learn the basic concepts of mathematics with little or no direct instruction. Many humans find the understanding of mathematics a very difficult task--a mysterious process with no apparent logic; a foreign language with no order or understanding possible. Do such human beings, especially school students, need a systematic program emphasizing both reading skills instruction and how these skills can be applied specifically to reading mathematics?

According to Duffy and Sherman(68):

> . . . a primary cause of reading failure is that teachers have not been adequately trained in the skills of reading. Because teachers do not know the skills, they are unable to teach them, and even when a teacher does know the skills, instruction is too frequently haphazard and inefficient because the instructor lacks a system for determining what skill each learner needs at any given moment and also lacks an instructional strategy for teaching that skill.

Since it is likely that there are reading related cues which have an impact on learning mathematics, it seems important to identify them. Before specific instructional strategies can be devised for teaching teachers to teach reading in mathematics, attempts ought to be made to identify what in the reading act inhibits or enhances students' learning mathematics. What are the reading-mathematics related questions for which answers are desired?

Statement of Problem

Although reading is a complex activity in which many skills are apparently used, this study examined only a few of the beginning stages of reading that are frequently encountered in the elementary schools and explored their possible impact on the learning of mathematics.

This study was designed to investigate the effects of the explicit teaching of five selected reading subskills of word recognition and comprehension on the performance of these skills in mathematics and on the problem solving abilities of a group of sixth graders. The reading subskills of word recognition and comprehension selected for this study were:

1. Instant Recognition of Special Symbols of Mathematics and Their Voiced Equivalents (Word Equivalents)
2. Structural Analysis
3. Contextual Prediction
4. Inferential Reasoning
5. Evaluative Thinking

For the purpose of this study, Duffy and Sherman's(68) description of the reading process and of these five reading subskills were used. This description is discussed on pages 8 through 18.

This investigation attempted to answer the following question: Does the teaching of specific and explicit reading lessons in these five reading subskills effect performance of these skills in mathematics or problem solving abilities of sixth graders?

Purpose of the Study

Mathematics means different things to different people. Whorf(211) maintains that mathematics is a special language, and Bloomfield(24) claims that mathematics is a verbal activity and that logic is a study of verbal activities. Language, logic and much of mathematics apparently are inseparable. Linguists(93,136) maintain that for someone who really wants to learn a language, the best advice is that he go to an area where that language is used. Falk(82) writes, ". . . if the teacher himself does not speak the language fluently, his students will never learn it unless they are able to find some additional source from which to learn."

These observations have implications for the teaching of mathematics as well as the teaching of the reading of mathematics. In order to uncover one of man's most valuable creations, in an independent way, one must learn to read mathematical language. Streby(191) maintains that "the language of mathematics under-girds the decisions and facilitates the reports that citizens in all walks of life have to make."

One of the major goals of instruction in mathematics is to develop students' abilities to solve word problems and to confront problem situations successfully and independently. Research(6,73) supports the claim that most good problem solvers are also good readers.

Kane(123) suggests that reading is the perception and comprehension of a message transmitted by written language and maintains that in order for a student to become a competent reader in mathematics, special skills are required and the acquisition of such skills necessitates special instructional techniques in the reading of mathematical language.

The purposes of this study were:

1. Describe the behaviors of sixth graders who were being taught how to use the five selected reading subskills of word recognition and comprehension in reading mathematics
2. Determine the impact of the teaching of the five selected reading subskills on the abilities of these sixth graders to use these skills in
   a. Interpretation of mathematical symbols
   b. Mediation of an unrecognized mathematical word through its structural units
   c. Obtainment of factual comprehension of a mathematical message by contextual prediction
   d. Exploration of understanding by inferring relations among mathematical concepts
   e. Formulation of mathematical problems from given problem situations through evaluative thinking
3. Determine statistically the impact of teaching these five selected reading subskills on the problem solving abilities of sixth graders

Assumption

In teaching the basic reading skills, word recognition and comprehension are not to be isolated from each other and the interactions that exist between them should not be obscured(68). These interactions are crucial and can be more readily seen by defining the reading process as a "psycholinguistic guessing game"--actually three games played simultaneously by the reader, with each game consisting of a set of signals(68):

First--graphemic signals move the words from the page into the reader's head; alphabetic letters represent speech sounds.

Second--syntactic signals automatically group words into phrases, sentences or paragraphs according to the constraints of the grammar of the English language.

Third--semantic signals contribute the meaning of the printed message, meaning to the syntactic structure of the printed words.

In the words of Duffy and Sherman(68):

> . . . the interrelationship of word identification, syntactic constraints, and semantic prediction is apparent. The total process contains elements of both word recognition and comprehension, but these elements are never mutually exclusive in the good reader. . . . The good reader guesses or predicts his or her way through the message, using graphemic, syntactic and semantic signals.

Duffy and Sherman's description of the reading process is schematically explained in Figure 1, and details of this process may be found in Systematic Reading Instruction(68).

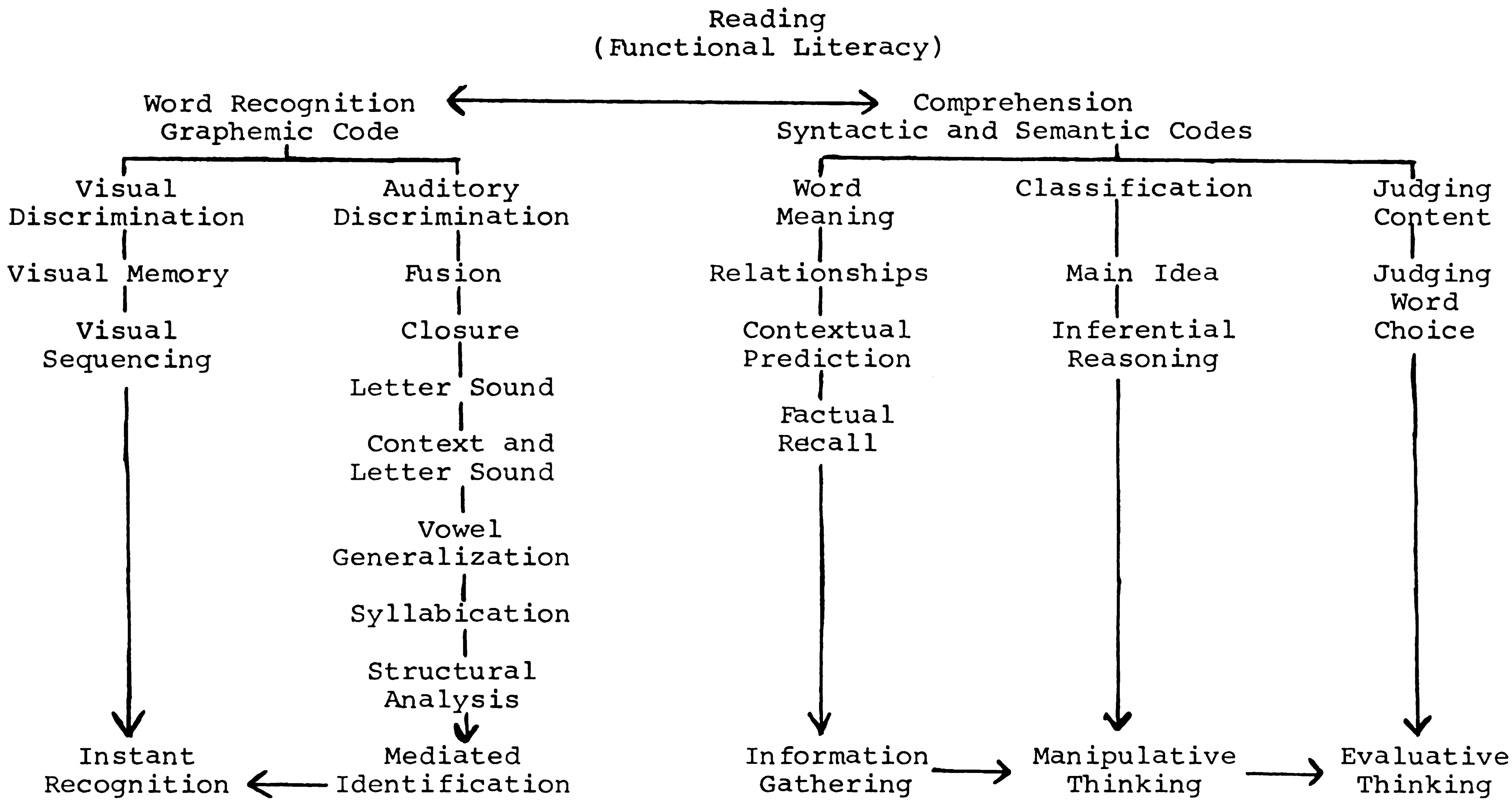


Figure 1 THE CHILD AND THE READING ENVIRONMENT

Figure 1 shows that the skills of comprehension reflect the syntactic and semantics signals and that the mastery of the graphemic signals of the English language gives the reader the skills of word recognition. An arrow goes from the mediated stream to the instant stream to reflect the fact that after an unknown word has been mediated several times, it should become a sight word. The third comprehension stream focuses on helping the reader make judgements about what they read and can only be valid if it follows both information and manipulative thinking. Duffy and Sherman(68) write, "It is absurd to ask children to do such an activity without the prerequisites of comprehension found in the preceding factual and manipulative levels." For this reason, arrows in the diagram go from the informational stream to the manipulative stream and from the manipulative stream to the evaluative stream to show a hierarchy of dependence. A double-headed arrow is found between the word recognition streams and the comprehension streams to reflect the interconnectedness and interrelatedness of the graphemic, syntactic and semantic signals.

Areas for This Research

Figure 2 schematically depicts those areas selected for exploration in this study.

WORD RECOGNITION

Strand		Specific Subskill
Instant Recognition	———	Recognition of Special Symbols of Mathematics
Mediated Identification	———	Structural Analysis

COMPREHENSION

Strand		Specific Subskill
Information Gathering	———	Contextual Prediction
Manipulative Thinking	———	Inferential Reasoning
Evaluative Thinking	———	Problem Posing

Figure 2 AREAS FOR RESEARCH

Word Recognition

Experts in the field of reading(68,161) insist that it is necessary for all teachers to recognize the importance of a child's developing effective word recognition skills and to support this commitment with a sophisticated understanding of the learning tasks and instructional techniques that influence their acquisition. Most of the skills in the area of word recognition reflect the graphemic code.

Instant recognition

With the mastery of a large number of sight words, decoding becomes an automatic process, hence, freeing the reader to concentrate on syntactic and semantic considerations. Call and Wiggin(48) write, "Words are linguistic signs which call up or reactivate a memory trace; they are symbols of concepts. Words derive their meanings from the context in which they are found."

Reading specialists(68,186) point out that instant word recognition is not a natural phenomenon for most children and requires instruction and that children learn to pair what is seen with what is said. Readers make the connection between the discriminated stimulus (a written word) and the spoken word.

In Robinson's(171) opinion, reading appears to be easier when one symbol system at a time is used. In mathematics, most often it is impossible to use only one symbol system--words, numerals, letters, graphics and special signs are intertwined--and the interrelationships among the symbol systems must be understood and utilized by the student in order to solve problems.

According to Wickelgren(212), problems often rely heavily on verbal language, and the first and most basic step in problem solving is to represent the verbal information concerning givens, actions and goals in special symbolic or diagrammatic form. What impact does instant recognition of special mathematical symbols and their voiced equivalents have on the learning of mathematics? It is likely that instant recognition of these special symbols and their voiced equivalents would be less of a natural phenomenon than instant word recognition and would require special attention and instruction. When should this experience with special mathematical symbols be started?

Mediated recognition

Mediating strategies serve as a bridge between not knowing words at all and recognizing them instantly. From the list of subskills for mediating recognition, structural analysis was selected for exploration in this study. What part does the understanding of the structures produced by the prefixes and stems used in mediating an unrecognized word play in the learning of mathematics? Does the history of a word or of its parts stimulate or enhance learning? Sawyer(174) feels that if one could trace the way in and progress by which mathematical terms were developed from every-day words, then one would understand and appreciate what mathematics really is. Could the time spent in developing the structural analysis reading subskill reap worthwhile benefits in mathematics?

Comprehension

Even though learning to read does demand the identifying of words, it is essentially a communication process in which obtaining meaning is of primary importance. The reader decodes the written page into language and tries to resurrect the intended message. Once the words are identified then they must be comprehended as they are used syntactically and semantically. In the words of Adler(2), "We express our thoughts in words. For this reason, the structure of thought is parallel to the structure of language."

Comprehension as an information gathering process

At this level of reading, the reader is trying to learn the factual content of a message. What does the message say literally? According to Duffy and Sherman(68), "As children listen to a communication, a portion of their understanding comes from their sentence sense that allows them simultaneously to know the idea of the message (its content) and to predict from both this knowledge and their grasp of grammatical relationships what word will follow what word and what idea will follow what idea."

The contextual clue is a strategy that is often employed by the efficient adult reader to obtain factual comprehension of a message(28). Since contextual prediction is a powerful tool in the comprehension of a written message, it has been selected as the subskill in this stream for exploration. What impact does it have on the learning of mathematics?

Comprehension as a manipulative thinking process

Once the informational process is secure, readers must think about this information and explore its implications and inferences. The reader moves from a literal to an inferential understanding.

Inferential reasoning is an important comprehension skill because not all information worthy of obtainment is always stated literally. Children must be taught to think beyond the information level of a written communication by "reading between the lines" and to manipulate factual information in order to discover both implications and inferences(68).

Do I multiply or divide? Many human beings can perform basic arithmetic operations with success but cannot solve word problems or cope with problem situations. Friebel(88) gives a sixth grader's system for solving word problems:

> If there are lots of numbers, I add. If there are only two numbers with lots of parts, I subtract. But if there are only two numbers and one is littler than the other, it is a hard problem. I divide to see if they come out even. If they don't, then I multiply to get the answer.

A common difficulty of students in problem solving is the failure apparently to generate a map or a series of possible logical processes or inferences. What impact does the reading skill of inferential reasoning have on the learning of mathematics to solve problems?

Comprehension as an evaluative thinking process

According to Duffy and Sherman(68), comprehension in the informational and manipulative areas is passive in the sense that readers interpret what the author meant, but at the evaluative level, readers assert themselves and strike out on their own, making a judgement that may or may not be what the author intended. Are there comparable skills at this level of comprehension which would have an impact on the reading for successful problem solving?

Bagford(14) maintains that one of the biggest influences on comprehension growth in children is the nature of the questions asked by the teacher during reading class and during other parts of the day. Suppose the students were trained in asking the questions--recall or recognition type questions, as well as questions requiring evaluative thinking? What impact would this skill have on reading to solve problems in mathematics?

In this age of accountability, one of the major goals of mathematics instruction for many teachers is to prepare students to cope with the mathematical problems and problem situations which they will encounter during their lives. In a realistic problem situation, there may not be a clear statement of what problem is to be solved and the students will be forced to formulate a clear problem from the problem situation before formulating a solution procedure. In order for children to read successfully on the evaluative level, they must strike out on their own, making judgements and decision. Is it the case that problem posing activities could be effective in stimulating areas of creative thinking and decision-making skills necessary for problem solving and confronting problem situations?

Importance of the Study

To this investigator, the needs of today's children and young people do not differ radically from those of yesterday's--learning to cope with change and being equipped to deal with the future. Analytic reading--the reading used in mathematics--probably ought to be an integral part of a curriculum designed to develop students' abilities to acquire and use knowledge. Acquiring and using knowledge involve bringing order out of information chaos by finding a theme or pattern that can be communicated to others--behaviors used in problem solving, coping with change and dealing with the future.

In the words of Taschow(198), ". . . the language in these mathematical fields with their subdivisions is concise, and the reading is precise, particular, and critical . . . without mastering this complex reading process, the student is vexed even if he retreats to memorization." According to Samuels(173), ". . . any student, regardless of his experience, cultural background, or past performance can acquire the basic language skills, naturally and promptly, if he is properly motivated." Is this the case with the language of mathematics?

The implications are:

1. Much of mathematics and language are inseparable(24,25, 211)
2. Complex thought and language are inseparable(46,207)
3. Reading is a complex process made up of many skills (68,137)
4. Oral verbalization is an important behavior in successful reading and learning(178,186)
5. Effective handling of the printed word--critical reading--in any area is an important consideration for successful coping with a changing social and physical environment(161)
6. Curricula do not reflect this need for the teaching of reading in the mathematics area(10,80)
7. In teacher-training courses, very little is done in the area of the teaching of reading in mathematics(10,77)

8. The relationship between reading and problem solving in mathematics has not been clearly explicated(158)

The attempt to shed additional light on the relation of the learning of mathematics for problem solving to the learning to read mathematical language could be important.

Outline of Procedure

The main purpose of the study was to determine the effects of the explicit teaching of five selected reading subskills on the performance of these skills in reading mathematics and on the problem solving abilities of thirteen sixth graders.

The researcher planned three 30-minute lessons for each of the five selected reading subskills for a total of fifteen 30-minute lessons in reading mathematics. Each reading subskill was covered during one week with one 30-minute lesson being held on alternate mornings. The researcher did the instructing for five weeks in these five reading subskills as applied to mathematics.

A sixth grade population of twenty-six students was randomly divided into the experimental group and the control group. The experimental and control groups were pretested for problem solving ability and, after five weeks, were posttested for problem solving ability. Both statistical analysis and observations were used to describe the results of the program in reading and the effects that this reading program had on the problem solving abilities of the subjects participating in this study.

Limitations of the Study

Time was a limitation of this study. Any activity used for the reading instruction had to be within the confines of the 30-minute period assigned for this project. There was no extra time for remediation for those students who may have needed it. Three 30-minute lessons for each of the reading subskills seemingly would be an unreasonably short time to spend on apparently very powerful reading skills for problem solving. There were no regular or systematic evaluation nor corrective instruction. Individual deficiencies were not identified so that students who failed to reach mastery level of a subskill could be given additional assistance and time to master the objective. No attempt was made to learn what reading skills individual students already possessed.

Diagnostic instruction was not used because of time limitation, the absence of any established well-known reading cues related to the learning of mathematics, and the lack of knowledge in this area on the part of this researcher.

Another limitation of this study was the necessity of teaching the reading of mathematics in isolation from a quality mathematics curriculum involving comprehensive problem solving. In the words of Earle(73), ". . .

successful teaching of mathematical reading is most likely to occur in classes where both the content of mathematics and the process of reading are emphasized equally and constantly."

Other possible limitations of the study could be the presence of the researcher and the extra time spent on mathematical activities by the experimental group.

CHAPTER II

REVIEW OF RELATED LITERATURE

Language and Mathematics

Hogben(110) has pointed out that some of the difficulties with mathematics arise because of the language difficulties. According to Kolata(129), "Translating mathematics into English was described by one mathematician as being more difficult than translating Chinese Poetry."

In the opinion of Bruner(42), "There is one part of the picture in the building of mathematical curriculum now in progress where I see a virtual blank. It has to do with the investigation of the language and concepts that children of various ages use in attempting intuitively to grasp different concepts and sequences in mathematics." Madden(141) believes that mathematics is a language as well as a way of thinking with this language, and he feels that teachers must elicit from their children expressions of their language patterns because, according to him, there is great need to study differences between children's everyday language patterns and the more formal mathematical sentence models. Madden(141) maintains that "mathematics is internalized as language."

Davis(62) alludes to Bruner's experiment which claims that language provides the means of getting free of immediate appearance as the sole basis of judgement in dealing with mathematics tasks relating to things in the real world. Language makes possible the acquiring, recall, and growth of experience; it permeates intelligence; and it is a requisite vehicle in all methods of teaching and study, not excluding mathematics.

Chomsky(52) claims that there is reason to believe that the language-acquisition system may be fully functional only during a critical period of mental development. Lenneberg(136) writes:

> . . . language acquisition becomes more difficult when the physical maturation of the brain is complete. . . . There is evidence that children at this age [preteen years] are capable of developing language in the same natural way as do very young children [preschoolers]. . . . healthy children have a quite different propensity for acquiring foreign languages before the early teens than after the late teens. . . . Neurological material strongly suggests that something happens in the brain during the early teens that changes the propensity for language acquisition . . .

In the opinion of Cooper(56), children enter first grade with several years of oral language experience--the basic tool for learning to read, that is, to understand the meaning of something written or printed. Cooper(56) also believes that children's oral mathematics language is very limited. If the line of analysis of the reading process as described by reading specialists(68) and psycholinguists (178) is correct, before a student, in general, can begin reading mathematical language intelligently, he must learn to speak it.

Stauffer(186) feels that in the early childhood period activity and language need close association and that language development, as a part of maturation or all-around mental capacity, influences much of a child's progress from thought that is predominantly perceptual and intuitive to thought that is conceptual and logical. Brownell and Hendrickson(39) write:

> . . . the process of conceptualization has been related very closely to development in the ability to use language. . . . Not only does language enable us to escape the limitations of space and time in our search for meanings and relationships but it also enables us to think with understanding about things that do not exist and cannot exist, as, for example, the abstractions of mathematics. It is not without reason that language is our chief vehicle of instruction.

According to Bruner(42), language profoundly affects behavior and "Pupils progress when ideas of size, degree, and relationship shine clearly to them through mathematical language." Braunfeld(31) maintains that because the language of mathematics is more precise than that of ordinary English, students must have a good deal of experience in using this precise language so that an appreciation for the benefits, derived in mathematics from this very precise formulations of problems and results that mathematics demands, can be developed. It appears that, if a reading curriculum for mathematics is to be developed, the observations of these language specialists and researchers should be considered in light of the claim that:

1. Mathematics itself is a language
2. Oral language is the basic tool for learning to read that language in print

Rose and Rose(172) claim that childhood training in language is crucial for performing well in mathematics; actually suggest a sociopsychological variable of language style as one of the intervening explanatory variables in arithmetic performance. In the opinion of Aiken(6), this accounts partially for the relatively greater mathematical abilities of children from upper socio-cultural backgrounds. In Peters'(157) study of kindergarten children of low socio-economic backgrounds, he found verbal training to be significantly more effective than noncued, visual-cued, or no training when the criterion was immediate learning, but if delayed retention was the criterion, then both verbal training and visual-cued training were more effective than the other two procedures.

Sollee(182) concludes from her study of first and second graders that verbal competence is related to the achievement of stable and generalized levels of conservation and that this relationship remained significant when intelligence, measured nonverbally, was held constant. Weil's(210) study supports Piaget's claim that language is not a sufficient condition to advance operational thinking but found that the relationship between the language measures and the concept measures were significantly correlated. Pelfrey(156) concludes from his study that the lack of language in deaf children has significantly affected their cognitive development. Pullman(166) maintains that both verbal conditionality and cognitive structure could be used in explaining part of the variance in mathematical ability not explained by intelligence.

In learning algebraic symbols and syntax, Ausubel and Robinson(12) claim that the problems are the same as in learning a second language. The literature supports Aiken's(6) claim that many researchers maintain that mathematics is a language and should be taught as such(12, 33,56,69,110,133,141,149).

Verbalization and Mathematics

Irish(117) reports positive effects on problem solving by modifying instruction in mathematics to include emphasis upon children's verbalization. Palzere(153) and Stern(187) found no significant difference in requiring verbalization in mathematics; whereas, Greider(95) claims that verbalization does affect learning in a positive way.

Hendrix(102) claims:

> . . . correct verbalization of a discovery does emerge immediately after the dawning of awareness, but these cases are rare. They are confined to learners with unusual powers of correctness and precision in the use of a mother tongue, and they are confined to situations in which the learner happens to possess all the vocabulary and rules of sentence structure needed to formulate the new discovery.

Hendrix(102) also maintains that dialogue between teacher and students, as well as between the students, plays a very important role in the initial stages of mathematics learning. Aiken(6) feels that one of the important influences of verbalization is permanency in retaining learned concepts.

Piaget(160) claims that the needs a child tends to satisfy when he talks are neither linguistic nor logical but psychological. Piaget(160) feels that, "The mere act of telling ones thought--of telling it to others or telling it only to oneself--must be of enormous importance to the fundamental structure and functioning of thought in general and of child logic in particular."

Davis(62) writes:

> Anyone who watches children between the ages of three and about eight years old can hardly fail to be impressed by the child's tendency to verbalize about nearly everything. While he does something, he produces a gratuitous verbal description, aloud, concerning what he is doing. If our present line of analysis is at all correct, this talking-about-what-you-are-doing-while-you-are-doing-it behavior is one of the most basic learning activities of all.

Chase(51) maintains that children want to talk but that they, by 4th grade, have been turned off and trained not to talk about their academic subjects.

Vygotsky(207) claims that a child's intellectual growth is contingent on his mastering the social means of thought, which is language, and that verbal intercourse with adults becomes a powerful factor in the development of the child's concepts. Ausubel(11) writes:

> Verbalization, I submit, does more than verbally gild the lily of subverbal insight; it does more than just attach a symbolic handle to an idea so that one can record, verify, classify, and communicate it more

> readily. It constitutes, rather, an integral part of the very process of abstraction itself. When an individual uses language to express an idea, he is not merely encoding subverbal insight into words. On the contrary, he is engaged in a process of generating a higher level of insight that transcends by, in clarity, precision, generality and inclusiveness--the previously achieved stage of subverbal awareness.

Irish(117) reports to have obtained evidence that growth was obtained when instruction in computation was modified to include emphasis upon children's growth in ability to formulate verbal generalizations appropriate to the topic under study. Horn(114) maintains that investigations have shown that at higher grade levels students who cannot understand what they read usually cannot understand what they hear and that there seems to be close correspondence between the adequacy of a student's verbal statement of an idea and the clearness with which he grasps the idea.

Logic and Mathematics

Elder(81) reports that explicit instruction in logic improved the abilities of college students to verbalize generalizations discovered while working in a programmed algebra text. According to Rimoldi(168), there is an interaction between language and logical structure; that is, logical structure and language are not experimentally independent variable. Suppes(193) claims that although a class of fifth grade students proceeded much more slowly than the usual undergraduate class in college mathematical logic, the results indicate that children of this age were able to achieve a level of accomplishment comparable to college students in the skills of deduction essential to mathematical reasoning when instruction was geared to their level of experience.

Paris(154) concludes from his experiment that neither adults nor children interpret linguistic connectives according to formal propositional relationships prior to explicit instruction. Inhelder and Piaget(116) maintain that formal aspects of logic begin about age eleven. Hyram's(115) work suggests that direct teaching of the logical rules may bring about the improvement in children's performances in dealing with valid inferences. The recognition of valid inferences is one of the main skills in reading comprehension of mathematical materials. According to Hill(109), children of six, seven, and eight years old are able, to a marked degree, to recognize the validity of logical inferences and these same children can also demonstrate their understanding in reasoning from hypothetical premises.

Hyram(115) writes:

> . . . the fact that thinking does not appear to be inherently logical is of little significance with regard to the possibility that it can be developed into characteristically logical patterns through experience and habit formation. . . . Children in the upper grades of the elementary schools are, in general, mentally capable of acquiring the necessary understanding of logic and a proficiency in the use of its rules. . . . the most effective way of helping children to acquire the necessary working knowledge of the principles of logic is through direct instruction.

According to Suppes(195), "We must provide for and take advantage of the opportunity to pay more than incidental attention to the development of the student's reasoning powers. . . . The encouragement of critical thinking should be acknowledged as one of the legitimate objectives of our schools . . . critical thought requires the ability to make logically correct inferences, to recognize fallacies, and to identify inconsistencies among statements." These traits for critical thinking are also embedded in the skills required to read. Muscio(150) claims that the close relationship between language and thinking must be recognized as we develop appropriate learning experiences for children.

Special Symbols and Mathematics

According to Suppes(194), his empirical findings indicate that mathematical concepts generally can be understood by primary grade children provided that these concepts are presented precisely with the help of a consistent notation. Skemp(176) believes that the distinction between a symbol and its associated concept, being two different things, is not trivial in mathematics. He continues by claiming that making an idea conscious is closely related to associating it with a symbol and "Once the association has been formed, the symbol seems to act as a combined label and handle, whereby we can select (from our memory store) and manipulate our concepts at will. It is largely by the use of symbols that we achieve voluntary control over our thoughts."

Suppes'(194) experimental work also has shown that primary grade youngsters do not in general have any difficulty in grasping new notation. Earle(72) feels that the more times that students are exposed to the sight and sound of important mathematical symbols, the more likely they are to recognize those symbols accurately and rapidly. Taschow(198) maintains that sheer recognition of mathematical words and symbols without the ability to verbalize or pronounce them may restrain comprehension and later interfere with interpretation and evaluation. He suggests that writing the word or symbol will provide a third sensory attack on new or possibly forgotten terms and symbols. Skemp(176) maintains that "The power of mathematics is immense, and at all stages, symbols make a major contribution to this power. But without the ability of mathematicians to invest them with meaning, they are useless."

Phillips and Uprichard(158) found that translation from ordinary English to the special language of mathematics correlated consistently with Ninth Grade students' ability to solve verbal problems. Dahmus(59) claims that teachers must develop students' abilities to translate verbal situations into proper mathematical symbols. Polya(163) suggests that mathematical notation is a language and that:

> When we face a new problem, we must choose certain symbols. . . . Choosing a suitable notation may contribute essentially to understanding the problem. We need experience to choose the more suitable notation as we need experience to choose more suitable words.

Vocabulary and Mathematics

Georges(90) maintains that reading difficulties caused by mathematical terminology are predominant, with difficulties caused by symbols and notation being next. Suppes(194) claims that the use of explicit notation permits a clarity of definition of concepts that would otherwise be impossible.

Buckingham(46) writes, "Since reading and vocabulary are so intimately related, it follows that the setting for the learning process in a single problem may be so changed by an unknown word as to make the total problem incomprehensible to the student." This agrees with the findings of Bogolyubov(25), which indicate that if words are not understood or if they are misunderstood, the process of problem solving becomes extremely complicated and possibly impossible for some students. Bogolyubov writes, "Teachers work very little with children on words in the solution of problems, and in particular, do not teach the significance of certain words in everyday life. The latter fact frequently creates great difficulties in the study of arithmetic."

According to Bruner, Olver and Greenfield(43), the development of clear and adequate terminology is essential to cognitive growth. Johnson(120) found that, by development of a meaningful understanding of vocabulary beyond that which was provided by the textbook, an experimental group achieved significantly greater gains than did the control group in both vocabulary and problem solving. Treacy(203) reports a great need for stressing the meaning of terms, general and mathematical, as an approach to improving problem solving.

Vander Linde(205,206), from the results of his study, maintains that the study of the technical vocabulary of arithmetic cannot be left to chance but must be an essential part of the arithmetic program. Langer(134) maintains that the development of vocabulary is an inextricable part of concept development and difficulty in reading comprehension often stems from difficulty in the understanding of words and the concepts they represent. Drake(65) reports that students demonstrated superior achievement in algebra when the vocabulary of the subject was stressed. Johnson(121) found that in considering problem solving in relationship to general intelligence through the factors of vocabulary and reasoning that vocabulary rated highest in both problem solving and general intelligence. Johnson(121) writes:

> Perhaps it can be explained by knowing that words are our vehicle of thought and the better the vehicle the better the thinking. And the better the thinking one does the more intelligent he is and perhaps the better problem solver.

Dresher(67), Edwards(79), as well as Lyda and Duncan(140), found that their results show a considerable influence of vocabulary upon achievement in mathematics.

Bruner(42) maintains that teachers need clear and correct mathematical words to describe problem situations, words to question students' unreasoned statements in mathematics, words to encourage further student research and reading in mathematics. Bruner(42) continues by saying, ". . . good mathematical language challenges--relights the flame of curiosity. . . . Men use words to solve most of their perplexities, if not all of them. But it is not easy to use words properly in solving problems. . . . In helping students to think reflectively . . . the teacher should help them to understand the use of words . . ." Taschow(196) writes:

> Studies emphasize that without an understanding of words, comprehension is impossible . . . a large vocabulary is related to high comprehension. . . . Vocabulary should not be taught in isolation but should always be related to meaningful content . . . in order for the reader to achieve greatest possible comprehension he must become meaning-conscious. This skill is taught by means of word structure analysis and context clues.

Reading and Mathematics

Georges(90) suggests that accuracy of statement in mathematics, as well as accuracy of computation, is the only criterion of acceptable work and that this is impossible without accuracy of reading in mathematics. Call and Wiggin(48) report the results of an experiment in which second year algebra high school students were taught by an English teacher with some training in the teaching of reading but no experience in teaching mathematics and in which the control group was taught by an experienced mathematics teacher. The findings show that the experimental group did better on the criterion test in mathematics than the control group. The initial differences in reading and mathematics test scores were statistically controlled. Call and Wiggin write, "If by teaching reading, instead of mathematics, we got better results, it seems reasonable to infer that the competent mathematics teacher might get considerably better results if he were trained to teach reading of the kind encountered in mathematics."

Kress(130) found that nonreaders, when compared to readers, lacked (a) versatility, flexibility, and originality in devising suitable hypotheses for testing, (b) initiative and persistence in verifying solutions, (c) ability to draw inferences from relevant clues, (d) ability to analyze factors or to devise adequate labels, and (e) ability to form adequate concepts for dealing with language. Muscio(150) reports that arithmetic achievement depends on both general intelligence and verbal ability and that achievement on the measure of quantitative understanding is closely related to mathematical vocabulary and general reading ability. Gilmary's(91) study indicates a definite relationship between reading and arithmetic procedures, and the effects of the reading instruction had a significant transfer value for the arithmetic classes.

Cottrell(57) found that the strong relationship between reading, psycholinguistics and arithmetic are due to general linguistic ability. Skillman(177) concludes that it is possible to obtain improved achievement in mathematics by emphasizing reading techniques. Ziesemer's(218) study shows that the fifth graders, with a pattern of high reading and low arithmetic achievement, scored significantly higher than the students with high arithmetic and low reading achievement on a series of standardized tests measuring intelligence, mental abilities, picture vocabulary abilities, conceptualization, symbolic mediation, language association and application of verbal skills to new situations.

Buckingham(46) writes, "It cannot be assumed that an understanding of the problems as they occur in the text will materialize regardless of vocabulary or reading ability. . . . Mathematics has its own settings, words and meanings which must be taught in parallel with problem solving." Pribnow(165) maintains that the root of the inability of students to solve word problems is concerned with reading mathematics as opposed to the "Johnny can't read lament" because without the identification and comprehension of the given information or of what must be found, further analysis of the problem is impossible. There have been other studies dealing with the relation of reading, and language in general, to problem solving in mathematics(8,60,76,97,175).

Taschow(198) recommends that the reading of mathematics must be set in the curriculum in a systematic, direct and planned fashion and must not be left to incidental learning. Streby(191) claims:

> . . . reading is very complex, involving many skills and understanding. . . . Reading verbal problems in mathematics texts does require a different technique than reading descriptive material or fiction. The inability to grasp the full meaning of a statement is more serious in mathematics than in any other subject.

Georges(90) claims that the types of reading required in technical subjects, such as mathematics, are quite different from those required in courses in English. Georges(90) also found in his work that "any answer to a problem satisfies the careless reader whether it is the desired quantity or not."

Eagle(71) writes:

> The function of selecting and organizing the essential data is important in life situations to a greater extent than in most textbook problems. . . . More emphasis on this aspect of mathematics would relate the work of the schools more closely to practical applications, thereby making the mathematics courses more functional and probably better motivated. In such courses reading abilities . . . would be of even more importance in relation to achievement in mathematics, and they would be therefore developed to a greater extent than is the case in the usual mathematics courses.

Horn(114) asserts that low achievement in reading, in listening, and in oral or written composition is PRIMA

FACIE evidence of poor learning in other curriculum fields. Horn(114) also maintains that investigations have shown that at higher grade levels students who cannot understand what they read usually cannot understand what they hear. Communication of ideas depend upon the ability to associate written and oral symbols with the ideas.

Chase(51) writes:

> Every normal child uses his natural language quite fluently at a very early age, yet millions of normal ten- and twelve-year-olds are categorized as nonreaders. They find it difficult to transfer sounds into symbols. . . . homo sapiens are talking animals whose brain centers for writing and reading have only recently been opened up.

Stauffer(186) claims that critical reading and concept development can be taught from the first grade on, through a reading program which encompasses language and experience, reading and thinking, phonics, syntax, and semantics in a unified course of instruction and that word attack skills and concept development skills must be so taught that the reader can proceed in a self-reliant way regardless of the content. In the opinion of Skemp(176), the greatest and final contribution of an excellent mathematics teacher is to reduce the learner's dependence on him and this would entail the learner's being able to read the language of mathematics independently. According to Bruner(42), far too many students will not even attempt to read mathematics or to speak or to write about mathematics, and those who do attempt it, most often use it "muddyly, inconcisely, incorrectly and incogently."

Feeman(83) asserts that teachers of mathematics have long realized that many difficulties encountered by students in learning mathematics are reading related difficulties rather than intrinsic mathematical difficulties and that these reading difficulties are related to the reading of mathematics specifically. Feeman also claims that, in general, mathematics teachers do not see themselves as reading teachers, but strictly as mathematics teachers and that reading teachers per se seldom have much training in mathematics. Clark(53) writes, "Mathematical reading is highly specialized. The person best equipped to guide the reader is the mathematics teacher, who conceives of the teaching of reading of mathematical ideas as an integral part of the learning in his field."

The Harvard Survey(10) found that social studies and science were basically the only content areas in which reading skills were "sometimes" developed. The lack of time and teacher "know how" were cited as the chief barriers for this absence of helping students master the printed word in subject matter areas. According to Earp(77), there is very limited evidence that reading skills are being taught in the content areas, especially in the area of mathematics, and on examination of professional reading texts and basal reading series, he found reading recommendations are slanted toward the reading of informational materials more representative of social studies rather than mathematics. Austin and Morrison(10) claim that the graduating student in elementary education is not always adequately trained to undertake the responsibility of helping children master the printed word even in a general way much less for mathematics reading.

Research(6) which relates reading and mathematics seems to indicate that poor readers will probably be poor problem solvers and that the overall mathematical development of children correlates highly with their ability to read mathematics. Aiken(6) suggests that we do have enough evidence to conclude that instruction in reading in general and in reading in mathematics in particular improves performance in mathematics. He continues by writing, "It seems reasonable that attempting to cultivate the skill of reading carefully and analytically in order to note details and understand meanings, thinking about what one is reading, and translating what is read into special symbols would improve performance on many types of mathematics problems."

According to Bagford(14), Bormuth(27) and Bornick(28), a deeper understanding of the message of printed material can be fostered among students by use of the cloze procedure, since the students are required to read carefully, to use contextual clues and to become actually involved with what they are reading in order to predict correctly the words or the special symbols which have been deleted from the reading material. The cloze was developed by Wilson Taylor(199) in 1953 and is a contextually interrelated series of blanks to be completed by the students, with the word or special symbol that makes the most sense, given the context. Reading specialists (Duffy and Sherman(68), Earle(72), Stauffer(186), Taschow(197)), for example, agree that the contextual clue is a powerful strategy most often employed by efficient readers. Research (Kane(124), Hater(99), Culhane(58), Rankin(167)) has shown the cloze to be a very valid and reliable measure of reading comprehension as well as a means for teaching reading. According to Culhane(58), "Rarely does a method of instruction emerge that is as easily learned or as easy to put to use in the classroom as the cloze method."

Problem Solving and Mathematics

Treacy(203) suggests that students may be troubled in problem solving by the inability to retain significant facts as well as by the inability to read. Treacy's(203) results support the value of the pencil-paper reading skill in mathematics. According to Dahmus(59), texts and teachers fail to teach the vocabulary the students need and to develop the abilities of the students to translate a problem situation into proper mathematical symbols; hence, the students are unable to read or write mathematics. Dahmus(59) also claims that in mathematics classrooms, the answer is the "little god" and that the students' habit of trying to work problems mentally also makes teaching difficult. He suggests that the mental arithmetic exercises are partially responsible for creating this unfortunate situation. Dahmus'(59) work supports the need for children to be able to write the language of mathematics as well as to speak it, understand it spoken, and to read it, if successful problem solving is to be accomplished.

Henney(104,105) found that special instruction in reading verbal problems and supervised study in verbal problems both improved children's abilities to solve verbal problems. Linville(138) concludes from his study that syntactic structure and vocabulary level are important factors in solving verbal problems, with the vocabulary level perhaps the more crucial. He also found that children of high reading ability scored significantly higher on a verbal arithmetic problem test than did the children with low reading achievement. Pribnow(165) claims that the key to the inability of students to solve word problems appears to be organizing and analyzing the problem which apparently are skills related to reading for comprehension. Phillips and Uprichard(158) write, "Obviously, success in solving word problems depends upon skills in reading and computation, however, the relative contribution of these skills has not been clearly explicated." In their review of the literature, Phillips and Uprichard(158) conclude that problem solving has not been systematically investigated by mathematics educators and that studies of problem solving yield conflicting results.

Related Areas

Intelligence and Mathematics

According to Aiken(6), in 1958, Wrigley concluded from his study that high overall intelligence is the first requirement for success in mathematics, but Aiken feels that "a significant degree of overlap between verbal and mathematical abilities remains unexplained." Aiken goes on to demonstrate that the correlation between general intelligence and mathematical ability is appreciably reduced when reading ability or scores on other linguistic tests are partialed out. Aiken(6) concludes that the pivotal variable may not be general intelligence but rather reading or linguistic ability.

Verbalization in Mathematics Classrooms

Cooney(54), Fey(85), Gregory(94), Kysilka(131), and Lamanna(132) have investigated the verbal interactions of teachers and students in the mathematics classrooms. According to Aiken(6), these studies have "increased our knowledge of the kind, frequency and effects of verbal exchange between students and teachers in mathematics classrooms." According to Bagford(14), one of the biggest influences on comprehension growth in children is the nature of the questions asked by the teacher during reading class and during other parts of the day. Bagford(14) writes, "If teachers ask only questions which call for one word, factual answers, pupils form the habit of looking for and remembering only facts as they read. Consequently, the pupils do not develop the capacity to perceive and comprehend the larger ideas expressed in the reading material." Guzak(96) found that 70 per cent of the questions asked by teachers were either recall or recognition type questions.

Language Arts and Mathematics

Using

1. Similarities between teaching mathematics and the language arts(49,108,142)
2. Children's literature(159,190)
3. Anecdotal style of content presentation(151)

have been investigated and found to be effective motivational and instructional media for the teaching of mathematics.

Textbooks and Mathematics

Browning(40) maintains that there is a great disparity in terminology usuage from one mathematics textbook series to another and that the frequency of the usuage of most terms is very inadequate in all series examined. According to Willmon(214), the number of new mathematics words introduced in the textbooks for primary grades is too great for the textbooks to be the sole teaching vehicles. From Stevenson's(188) investigation of third grade mathematics textbooks and first and second grade readers, he concludes that the teachers must teach the new vocabulary in the mathematics books as no dependence upon the basic reading program is warranted. Stevenson also advises that teachers must devote time to teaching the reading of mathematics. Stauffer(185) reports that, in his investigation of primary textbooks, he found that even if a child in first grade mastered all the different words presented in all of the seven reading series, he would still need to learn to read at least one-half of the words presented in the arithmetic series in the arithmetic class. This means that he would need to be prepared to deal with these words semantically (meaning) and phonetically-structurally (speaking) in order to grasp and deal with arithmetic problems and discussions. Stauffer found a total of 1331 words, appearing in the arithmetic list made for grades 1, 2 and 3, which did not appear anywhere in the reading list made for the same grades. According to Stauffer, "It is apparent that every teacher is truly a teacher of comprehension or of reading."

Readability and Mathematics Materials

Hiatt(107) reports that a creative problem solving test measured certain mathematical thinking abilities not measured by standardized measures of achievement and intelligence. Kane(123,124,125) maintains that reading formulas for ordinary English are not appropriate for use in mathematics; they are valid only when applied to materials written in ordinary English. Hater(99,100) found the Cloze Procedure to be highly reliable and to be a valid predictor of the comprehensibility of mathematical language. Kane(124) claims that cloze scores exhibit high positive correlation with Shannon letter redundancy scores and that letter redundancy is a powerful determinant of the learning of verbal materials. According to Kane(123), "Extensive research shows that mean cloze scores are valid measures of reading difficulty for ordinary English. . . . cloze tests adapted for use with mathematical passages are measures of reading difficulty in the language of mathematics."

Instruments to measure reading in mathematics have been constructed by LeDuc(135) and Brunner(44). Brunner found her instrument to be more than an intelligence test or a general reading test. LeDuc's instrument would only measure some abilities that are important in the reading of mathematics as he left out of his study two very important aspects--technical vocabulary and symbolism. Byrne's(47) study reports the development of a measure of the familiarity of mathematical terms and of mathematical symbols for 7th and 8th grade students.

Loehrlein(139) suggests that readability formulas for ordinary English are inappropriate for rating the difficulty of mathematics materials. Her study confirms the theory that the language of mathematics possesses a number of unique characteristics which influence its comprehensibility and supports the hypothesis that teacher judgement is a highly reliable measure of the readability of materials written in the language of mathematics.

Bankston(15) and Thompson(202) both found that the readability of mathematics materials has a significant effect of achievement in mathematics. Smith(181) and Heddens(101) conclude that readability of mathematics materials is extremely important. In the words of Smith(181), "If the child cannot read the statement, then his abilities to think, analyze, and compute are hampered."

CHAPTER III

DESCRIPTION OF THE INVESTIGATION

The Problem

This study identified some questions which are related to the two reading skills of word recognition and comprehension, and what role these skills have in the acquisition of mathematical concepts from written descriptions of these concepts. Specifically, this study was designed to investigate the effects of explicitly teaching certain selected reading skills on the performance of these skills in mathematics and on the solving of verbal mathematical problems.

The five reading subskills, as described by Duffy and Sherman(68), selected for this study were:

1. Instant Recognition of Special Symbols of Mathematics and Their Voiced Equivalents (Voiced Word-Equivalents)
2. Structural Analysis
3. Contextual Prediction
4. Inferential Reasoning
5. Evaluative Thinking

The objectives of this experiment were:

1. Describe the behaviors of sixth graders who were being taught how to use these five selected reading subskills in reading mathematics
2. Determine the impact of the teaching of these five selected reading subskills on the abilities of these sixth graders to use these skills in:
   a. Interpretation of mathematical symbols
   b. Mediation of an unrecognized mathematical word through its structural units
   c. Obtainment of factual comprehension of a mathematical message by contextual prediction
   d. Exploration of understanding by inferring relations among mathematical concepts
   e. Formulation of mathematical problems from given problem situations
3. Determine statistically the impact of teaching these five selected reading subskills on the problem solving abilities of sixth graders

Procedure

Selection of Sample

The sample for this study was drawn from a sixth grade class in a suburban, K-6 elementary school near Grand Rapids, Michigan. The twenty-six students in the sixth grade class selected were randomly divided into two groups of thirteen for the control and experimental groups.

Development of Instructional Materials

The instructional materials for the explicit teaching of selected reading skills were designed by the researcher. There was a total of fifteen lessons--three thirty-minute lessons for each of the five selected reading subskills. Copies of the material covered in the reading lessons may be found in Appendix A. A description of the objectives and materials used for each of the five reading subskills selected for this study follows.

Subskill I: Instant Recognition of Special Symbols of Mathematics and Their Voiced Equivalents (Voiced Word-Equivalents)

Performance Objectives: Given a set of special symbols, provide word equivalents and voice them.

Given a word phrase or word sentence, provide special symbolic representation.

Most of the special symbols chosen for this subskill were those symbols judged to be essential for reading the sixth grade textbook being used by the subjects in this study. Some symbols, such as $\sqrt{}$, were chosen because of their importance to hand-held calculators. A special-symbol list, a special-symbol card game, construction paper models, and a list of word phrases and sentences for translation were used to accomplish the objectives for this subskill.

Subskill II: Structural Analysis

Performance Objective: Given an unknown content word which has structural units, identify and give meaning.

The mathematical words for this subskill were selected because of their structural units and their relation to the special symbols chosen for Subskill I. The material for this reading subskill consisted of word cards, pictures or diagrams demonstrating meaning of structural units of the content words, and a list of prefixes and their meanings.

Subskill III: Contextual Prediction

Performance Objective: Given a set of sentences (one or more) in which unknown words or special mathematical symbols are present, supply each with meaning.

For this subskill, the materials used were of three types:

1. A set of sentences using "mathematical" words in nonmathematical context
2. A set of mathematical sentences with certain key mathematical words underlined which were to be defined given that context
3. Cloze procedures constructed over mathematical passages. In these cloze procedures, every fifth word was deleted from the passages. The student was

required to complete each blank with the word or symbol that made the most sense in the given context

Subskill IV: Inferential Reasoning

Performance Objective: Given a story problem, determine the mathematical processes or operations to be used.

For the reading subskill of inferential reasoning, each of the three lessons consisted of a set of verbal problems in which the children, after reading the problem, inferred the types of mathematical processes or operations required to solve these problems.

Subskill V: Evaluative Thinking

Performance Objective: Given a situation, pose a problem.

To develop this fifth reading subskill, each of the three lessons contained a set of problem situations. From each of these problem situations, the children were to pose a problem.

Teaching Procedure

Preliminary arrangements

As hand-held calculators were to be used in some of the reading lessons, the researcher designed and presented, one week prior to the implementation of the reading lessons, five one-hour lessons on the hand-held calculator. The entire sixth grade class, participating in this study, received a daily one-hour lesson on the calculator for a total of five one-hour lessons.

Each student was provided a four-function calculator for these calculator lessons, the reading lessons, as well as for the pretest and the posttest. The calculator lessons were primarily given to this sixth grade class to eliminate any computational advantages of calculator use that the experimental group could possibly gleam from the use of the calculator in the reading lessons.

All twenty-six subjects participating in this study were instructed in certain aspects of a calculator to promote basic calculator literacy and were given activities in which to practice using the calculator to find out some of the things it can do and some things it can not do.

Teaching schedule

This study was carried out during the Fall Term, 1977. The researcher instructed the subjects in the experimental group in the five selected reading subskills while the control group was doing independent work such as individual reading of selected stories, homework or working on special projects in art. The explicit reading instruction in mathematics was given to the experimental group in addition to their regular mathematics instruction. The control group and the experimental group received the customary instruction in mathematics together as a class, instructed by the regular classroom teacher.

The explicit reading lessons were scheduled for three days per week: Monday, Wednesday and Friday--for thirty minutes each--9:00 A.M. to 9:30 A.M.--for five consecutive weeks. This schedule permitted three thirty-minute lessons for each of the five selected reading subskills to be investigated.

Presentation of instructional materials

Each reading subskill was described at the beginning of the first lesson for each subskill. Why the skill is important to reading mathematics was also discussed, as well as how the students would be asked to demonstrate their learning of the skill.

The teaching of the selected reading skills included for each subskill a series of three 30-minute lessons in a sequence designed to lead the student to develop the subskill as applied to reading mathematics. Each student had the opportunity to practice the skill being taught in completing the designed activities individually as well as going over them as a group. The students were instructed to collaborate on some of the tasks and to consult with each other after completing other exercises individually. Students were given the opportunity and encouraged to share as well as to defend their understanding. The exercises were designed to provide opportunities for experience in verbalizing orally in the language of mathematics as well as opportunities for interacting with the printed messages of mathematics. A description of how each subskill was developed follows.

Subskill I: Instant Recognition of Special Symbols of Mathematics and Their Voiced Equivalents (Voiced Word-Equivalents)

The lessons pertaining to this particular subskill illustrated that words and special symbols are used in mathematics as a way of encoding ideas and as a means of talking about mathematical situations. The stage for the idea of a special symbol representing a mathematical idea or an unknown was set via a short exercise, having students think about things which they could not see, hear, taste, smell or touch--mystery animals, mystery candy, or mystery persons, having certain characteristics or properties--the "guess-who" game or the "guess-which" game.

Example: (Clues given for dogfish)
I am a small animal.
I can be spiny or smooth.
I come from the North Atlantic, California coasts or southern European waters.
My "namesake" is a carnivorous domesticated mammal.
Part of my name is also used by humans to mean a worthless, wretched person.
My name also denotes a device for holding or fastening as well as an andiron.
My name also means to hunt or track.
I am a small shark.

At the beginning of the second lesson, dealing with unknowns was continued via having the students explore, in their minds, the future--the "what-if" game--what if it rained coke tomorrow? The future is not explicitly known and because they were dealing with an unknown, as

frequently required in mathematics, the subject was necessarily quite abstract. But, the students could explore in their minds!

The children were then reminded that people often in everyday life refer to an object in a way which indicates that they are speaking of an arbitrary element belonging to a certain set, for examples:

A car is an element of the set of automobiles.

A redbird is an element of the set of birds.

The unknowns and the arbitrary elements were used to introduce the idea of a variable, such as letting x represent a number.

Calculators, straightedges, construction paper models and cards were used to present the special symbols of mathematics and their corresponding word codes to the children. The children were:

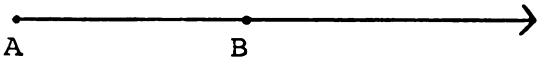
1. Asked to respond orally to special symbols on a card or to a construction paper-model by providing corresponding verbalizations
2. Given verbal commands, spoken and written, and asked to respond by using the correct special symbols on a hand-held calculator or by drawings or sketches on paper
3. Given words and asked to provide written symbolic representation

For Lesson 1, the following activities were used:

A. Each student was given a list of symbols and, in turn, provided oral verbalizations and examples

Examples: $\leq$; $x \leq 10$; The number x is less than or equal to ten; $7 \leq 10$.

↔; $\overleftrightarrow{AB}$; Ray determined by points A and B;



B. A deck of cards of the special symbols and of corresponding words were dealt

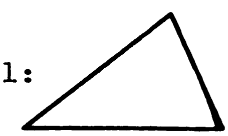
A student "read" orally a card from his hand; the card was placed on the table in the center of the group; all "related cards" were "read" and discarded. Students "decided" on the correctness of responses with supervision by the researcher.

Example:

5 + 7	The sum of five and seven.	Five is added to seven.	7 + 5

Seven is added to five.	The addition of five and seven.	The terms of a sum are 5 and 7.

C. Construction paper models were identified orally by students and special symbol-labels written on paper

Example: Model:  Student Response: triangle

On Paper: $\triangle ABC$

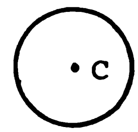
D. Verbal commands were given. Students produced special symbolization with pencil and paper and on a calculator

Example: Command: The sum of six and eight.

Pencil/Paper: 6 + 8

Calculator Sequence: 6, +, 8

E. Verbal commands were given. Students produced the special symbolization diagrammatically with labels

Example: Command: Circle C Diagram: 

Label: ⊙C

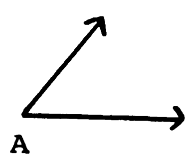
For Lesson 2, the students were presented a list of symbols and were asked to "read" them by giving voiced word equivalents. Verbal commands were given and students produced special symbolization with pencil/paper and calculator when applicable.

Examples: Command: Find the sum of five and nine.

Pencil/Paper: 5 + 9 = 14.

Calculator Sequence: 5, +, 9, =

Command: Draw an angle whose measure is less than 90°.

Pencil/Paper: 

$m(\angle A) < 90°$.

For Lesson 3, the children were given a list of word phrases and sentences and were asked to translate into special symbols. These translations were then discussed orally.

Example: The difference of eight and the second power of a number.

A possible translation: $8 - x^2$

The students were also given a list of verbal commands to "read" and respond by using the correct symbols on a calculator and recording results on paper.

Example: The square of 19.

Calculator Sequence: 19, ×, 19, =

Pencil/Paper: $19^2 = 19 \times 19 = 361$.

Subskill II: Structural Analysis

Structural clues rarely define a word completely. Pointing out structural clues to students was used to make them more sensitive to frequently used prefixes, suffixes and word roots in mathematics. This could also encourage them to draw on their own knowledge of word parts and to try making a relationship between what they know and a word which might be unknown or hazy to them or which might have multiple meanings.

The selected content words for developing this particular subskill were presented to the students on word cards, accompanied by the following oral questions:

Can you pronounce this word?

Do you know what it means? or

Does any part of the word look familiar to you?

Example: bisect

The students were also provided with "pictures" or diagrams in some cases to help demonstrate the meaning of the content words as gleamed from prefixes or word parts. The students were asked to "read" these diagrams to promote the verbalization of words with structural units as well as to promote the relation of special symbols to words and the interrelatedness of reading skills.

Example: bisect   midpoint

Diagram:

$$\overline{AC} \cong \overline{CB}$$

Reading: C is the midpoint of $\overline{AB}$.

C bisects $\overline{AB}$.

Line segment $\overline{AC}$ is congruent to line segment $\overline{CB}$.

On the third day (See Appendix A.), the students were given:

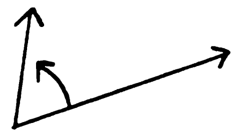
1. A list of some prefixes and their meanings
2. A list of expressions depicting meanings for mathematical words with these prefixes
3. A list of these mathematical words

The students were to match each word with the expression depicting meaning for the content word.

The children were also given a list of commands to "read" and to respond with pencil and paper sketches to demonstrate understanding.

Example: Draw an angle with counterclockwise rotation.

Possible response:



For some of the content words (See Appendix A.), completely unknown to any of the children or with interesting parts or origins, clues were given to develop the subskill of structural analysis. These new technical terms were taken apart to define their structural units. The students were asked to examine the examples carefully and to try to figure out a possible definition of the content word under consideration. It was conjectured that, if students had some experience with the history of the parts of a content word, they would possibly recall an interesting origin upon seeing the visual stimulus and then would be reminded of the meaning of the word.

Subskill III: Contextual Prediction

To illustrate the importance of syntactical structure and content in semantics, the first part of the first lesson for this subskill presented a set of "mathematical" words, each being used in a nonmathematical context. The children were asked to provide a sentence using each of these words in a mathematical context.

Example: Bring the clothes <u>line</u> into the house.

Response: A <u>line</u> is a geometric figure stretching endlessly in both directions.

In the second part of the first lesson for this subskill, the children were given a set of mathematical sentences with certain key items underlined. The children "read" the sentences and were then asked to explain the meaning of each of the underlined vocabulary items.

Example: Draw a <u>polygonal region</u> with a <u>triangular boundary</u>.

Response: The union of a polygon and its interior is called a closed polygonal region. If the polygon of this union has three sides (triangular), then it is said to be the boundary of the polygonal region, and is called a triangular boundary.

The other two lessons for this subskill used cloze procedures (See Appendix A.). The children were asked to read silently the entire cloze first before attempting to fill in the blanks. It was felt by the researcher that this would help the children make maximum use of redundant information and contextual clues throughout the passage. If the children were going to make use of context, it seems that it would be important that they read up to the unknown word and beyond it for additional clues. Students suggested words or special symbols which could semantically and syntactically fit into the blanks of the cloze and were then asked to offer reasons for their choices.

Example of a sentence taken from a cloze procedure:

The perimeter of a _______________ is defined as the _______________ of the lengths of _______________sides.

Response: polygon, sum, its

Subskill IV: Inferential Reasoning

For the reading subskill of inferential reasoning, the lessons were designed to develop the children's abilities to infer the mathematical processes or operations necessary to solve verbal problems.

Example: An airplane flew 3600 miles in 16 hours. What was its average speed per hour?

Oral response: To find the average speed, divide 3600 by 16.

Written response: Let S represent average speed. Then $S = 3600 \div 16$.

Some of the exercises were completed orally with each student being given the opportunity to respond. Others were completed as a group, with a student recording suggestions by the group on a small portable blackboard. Other exercises were written.

Not wishing to make the students feel locked into an inflexible progression of steps nor have them conclude that there is one and only one acceptable way to solve a problem, no rigid instructions were given by the researcher for the reading subskill of inferential reasoning. For the written exercises, the investigator circulated through the group to assess progress and to provide suggestions for those subjects who wanted to "get started" or to get "unstuck."

The aim of any offered suggestion was to get the students to read carefully and then to analyze the problem systematically, that is, to read critically and analytically. The suggestions given were:

1. Read the first time to obtain a general view of the situation
2. Read a second time for comprehension in full--read with a questioning mind and with a pencil and paper--translate the words into special symbols in order to see relationships more clearly
3. Read a third time to regain a comprehensive, full picture of the problem

Some of the students were encouraged to make a diagram or a flow chart so that they could "see" the problem situation visually as well as "see" in the mind--making it possibly easier to detect relationships so that a valid inference could be made.

Subskill V: Evaluative Thinking

The rationale for this reading subskill was motivated by presenting to the students the following questions:

Do you think it important:

1. For you to speculate about possible consequences of events

2. To develop abilities to imagine trying to survive as adults in the future

3. For you to develop your forecasting talents

To develop this fifth reading subskill, the lessons provided opportunities for the children to strike out on their own by creating or posing mathematical problems from given situations after evaluating the given situations. From the list of given situations, the children posed problems, both orally and in writing.

Example: On a 20-word spelling test, Denise missed 3 words.
Response: What per cent of words were missed?

Limitations of presentation

The entire experimental group received instruction in the subskills selected for this study. Individual deficiencies were not identified so that students who failed to reach mastery level of a subskill could be given additional assistance and time to master the objectives. No extra remedial exercises or activities were provided for those who may have needed them at the point where they were having difficulty. Even though each unit did not necessarily depend upon mastering a previous unit, there was a cumulative payoff in mastering sequential lessons.

The program in reading had to be usuable within the confines of the assigned 30-minute period, since it would have been unrealistic to assume that students requiring extra work would do it outside of the 30-minute session.

All sessions lasted 30 minutes, except the first lesson for the first reading subskill. This lesson lasted for approximately 50 minutes.

Testing and Evaluation Procedures

Testing instruments and schedule

Two equivalent instruments were designed by the researcher as a pretest and a posttest to measure the problem solving abilities of the sixth graders involved in this study and to determine the effects of the explicit teaching of certain selected reading subskills on the problem solving abilities of these sixth graders. The posttest items were obtained from the pretest by interchanging the order in which the problems were presented and, in some cases, by making other minor changes, such as substituting different numbers within a problem. The pretest was given 4 days before the start of the reading lessons and the posttest was given 2 days after the last reading lesson.

Previous to determining the final version of the items on the pretest, the instrument was given to a group of thirty-five prospective elementary teachers, enrolled in a preservice college mathematics course. This was done to obtain information and feedback in regards to the possible reading subskills required for solving the problems on the test. Copies of the pre- and posttests can be found in Appendix B.

Neither the pretest nor the posttest was timed but neither exceeded sixty minutes. A hand-held calculator was furnished each student for both tests.

Test validity and reliability were based upon the scholarly analyses of test items by mathematicians and mathematics educators and researchers in conjunction with a reading specialist at Michigan State University.

The tests were corrected by the investigator, using a predetermined score-analysis sheet. Credit was given in the following manner:

1. For a solution without error, the maximum point score of five was given
2. For a partially correct solution, a score of one to four was given
3. In cases of a totally incorrect solution or no solution, a score of zero was given

Statistical evaluation

Each subject's score on the pretest (y_i) and on the posttest (x_i); i = 1, 2, 3, 4, 5, 6, ..., 13; were paired for the same person to find the set of differences ($x_i - y_i$) for each of the two groups. The mean score of these differences for each group was determined. A t-test was used on these two mean scores to determine if the difference between them was significant.

The Null and Alternate Hypotheses tested in this study were:

H_0: There will be no difference between the mean scores of the two sets of differences obtained by subtracting, for each group, each subject's score on the pretest from his score on the posttest.

H_A: The mean score for the experimental group will be greater than the mean score for the control group.

<u>Other measures</u>

A set of test items from the posttest was selected for further detailed study. This study took the form of interviews with a randomly selected subset of seven subjects from the experimental group. The results of these interviews were used to help describe the effects of the reading instruction in mathematics.

A daily log, kept by the researcher, was used to help determine the impact of the reading instruction.

The classroom teacher's periodically written comments regarding the behaviors of the subjects in the experimental group in the regular classroom were read to obtain his observations of the effects of the reading lessons in mathematics on the subjects' classroom behaviors.

A retest for retention was given to the experimental group five weeks after the posttest. A copy of this retest for retention can be found in Appendix B.

Summary

This study explored the effects of a five-week (450-minute) program in the reading of mathematics on the problem solving abilities of thirteen sixth graders. The reading program consisted of three 30-minute lessons for each of five reading subskills as identified and described by Duffy and Sherman(68). The reading subskills selected for this investigation were:

1. Instant Recognition of Special Symbols of Mathematics and Their Voiced Equivalents (Voiced Word Equivalents)
2. Structural Analysis of Mathematical Words
3. Contextual Prediction in Mathematical Writings
4. Inferential Reasoning in Mathematical Writings
5. Evaluative Thinking in Mathematical Situations

The fifteen reading lessons were designed by the researcher. Researcher-designed pre- and posttests were used to measure performance of these skills in mathematics and problem solving abilities of the twenty-six subjects who participated in this study. The population in the study was divided randomly into control and experimental groups. A t-test was used to measure any significant difference in the gains of the two groups.

A daily log kept by the researcher, written comments of the regular classroom teacher, interviews with seven randomly selected subjects from the experimental group for certain problem-analyses and results of a retest for retention five weeks after the posttest were used to help describe the effects of the experiment.

CHAPTER IV

RESULTS

Introduction

Most reading specialists feel that the skills for reading can be categorized as skills of decoding the writing system to its associated language and skills in the use of this code for comprehension. According to these reading specialists, the thousands of different subskills for reading extend from the simple to the complex--from visual discrimination of symbols to judging content. The subskills described by Duffy and Sherman(68) and selected for investigation in this study were:

1. Instant Recognition of Special Symbols of Mathematics and Their Voiced Equivalents (Voiced Word Equivalents)
2. Structural Analysis
3. Contextual Prediction
4. Inferential Reasoning
5. Evaluative Thinking

The purposes of this study were:

1. Describe the behaviors of sixth graders who were being taught how to use the five selected reading subskills of word recognition and comprehension in reading mathematics

2. Determine the impact of the teaching of the five selected reading subskills on the abilities of these sixth graders to use these skills in
    a. Interpretation of mathematical symbols
    b. Mediation of an unrecognized mathematical word through its structural units
    c. Obtainment of factual comprehension of a mathematical message by contextual prediction
    d. Exploration of understanding by inferring relations among mathematical concepts
    e. Formulation of mathematical problems from given problem situations through evaluative thinking
3. Determine statistically the impact of teaching these five selected reading subskills on the problem solving abilities of sixth graders

Analysis of Data

Two random samples, the experimental group and the control group, were selected from a population of twenty-six sixth graders. For each group, each subject's score on the pretest (y_i) and on the posttest (x_i) were paired to find the set of differences ($x_i - y_i$). The mean score of these differences for each group was determined. A t-test was used on these two mean scores to determine if the difference between the two mean scores was significant.

Table 1.--Data for Experimental Group on Pre- and Posttests

Student	Pretest y_i	Posttest x_i	$x_i - y_i$	Deviations d_1	d_1^2
N	1	21	20	$^-12.923$	167.004
O	3	45	42	9.077	82.392
P	0	31	31	$^-1.923$	3.698
Q	11	55	44	11.077	122.700
R	12	75	63	30.077	904.626
S	2	44	42	9.077	82.392
T	10	24	14	$^-18.923$	358.080
U	6	29	23	$^-9.923$	98.466
V	0	15	15	$^-17.923$	321.234
W	8	66	58	25.077	628.856
X	8	31	23	$^-9.923$	98.466
Y	13	44	31	$^-1.923$	3.698
Z	7	29	22	$^-10.923$	119.312

From Table 1, $\sum_{i=1}^{13} (x_i - y_i) = 428.$

M_1 = 428 divided by 13 = 32.923--Mean Score for the Experimental Group.

Table 2.--Data for Control Group on Pre- and Posttests

Student	Pretest y_i	Posttest x_i	$x_i - y_i$	Deviations d_2	d_2^2
A	2	2	0	$^-1.615$	2.608
B	4	11	7	5.385	28.998
C	7	2	$^-5$	$^-6.615$	43.758
D	1	3	2	0.385	0.148
E	2	2	0	$^-1.615$	2.608
F	9	8	$^-1$	$^-2.615$	6.838
G	5	8	3	1.385	1.918
H	5	10	5	3.385	11.458
I	9	11	2	0.385	0.148
J	7	7	0	$^-1.615$	2.608
K	6	5	$^-1$	$^-2.615$	6.838
L	8	16	8	6.385	40.768
M	7	8	1	$^-0.615$	0.378

From Table 2, $\sum_{i=1}^{13} (x_i - y_i) = 21.$

M_2 = 21 divided by 13 = 1.615--Mean Score for the Control Group.

$$t = \frac{32.923 - 1.615}{\sqrt{\left(\frac{2990.924 + 149.074}{13 + 13 - 2}\right)\left(\frac{1}{13} + \frac{1}{13}\right)}}$$

$t = 6.979$ $\qquad$ $df = 24$

A $t = 6.979$ with 24 degrees of freedom is significant at the 0.005 level.

Based on the t-test for making comparisons among means, it can be concluded that the Null Hypothesis, H_O: $M_1 - M_2 = 0$, must be rejected in favor of the alternate, H_A: $M_1 - M_2 > 0$, at the 0.005 level of significance.

Observations

As a prelude to the discussion establishing the objective of the reading program, the following questions were asked:

1. Do you like mathematics
2. What do you think mathematics is
3. Do you like to read

In summary, the essense of the responses to these questions was:

Question 1: All the girls said that they did not like mathematics and that it was boring. The boys were less emphatic in their negative attitudes toward mathematics but most felt it was boring. One boy said that he really liked solving problems with the calculator. The other children then said, "Oh! But that was different!"

Question 2: The only comment received was that mathematics is, "Finding the answer with numbers."

Question 3: The girls enjoyed reading some things. On the other hand, the boys in general said that they did not like to read.

A daily log kept by the researcher was used to help describe the impact of the reading instruction for each of the five reading subskills.

Subskill I: Instant Recognition of Special Symbols of Mathematics and their Voiced Equivalents (Word Equivalents)

The students were not at all familiar with most of the special nondiagrammatical symbols of geometry or voiced equivalents of special symbols of geometry. For example, no student responded orally to the question as to the difference between the meaning of the term "circle" and the term "circular region." After being shown special diagrammatic symbols for these two mathematical concepts, all thirteen students demonstrated, in writing, a respectable intuitive understanding of the difference between these two items. In general, the responses were:

circle: "Just the line."

circular region: "What's inside the circle."

The students recognized the symbols such as +, -, × and ÷ as representing operations but were not familiar with "-" as in $^{-}3$, nor were they knowledgeable about the verbalizations of these symbols as operations. For example, they did not know the meanings for sum, product, quotient or difference. To all the students:

+ means add,

- means subtract,

× means times,

÷ means divide,

and all meant, "Find the answer."

These students were also totally unfamiliar with most of the other algebraic symbols such as: 3(2 + 7), 2b, $\sqrt{9}$, <, ≤, >, ≥, 8/2 (meaning divide), and 3^2. Two of the students, after some contemplation, remembered the meaning of the square root sign and of the exponent 2 from their lessons with the calculator. Once the calculator was mentioned, a discussion ensued between the children, whereby, all were able to demonstrate that they had remembered the meanings of these two symbols from the calculator lessons.

In giving examples of numbers, not one student volunteered an example of a nonpositive nor noninteger number. For example: For $3 < x < 9$, what are some possible values for the number x? The students agreed that the only values were 4, 5, 6, 7 or 8. The students were not aware that 3.85 represented the same number as 3.85000000, nor were they knowledgeable of the ordering relation of numbers such as 3.4563 and 3.4573. In fact, the students were very unfamiliar with decimal fractions and negative numbers.

The students seemed more relaxed in working with and talking about the geometric symbols and the ideas represented by these symbols. They seemed uncomfortable, being asked to talk about +, -, ×, and ÷ or about most of the algebraic symbols or about numbers in general. There were signs of frustration or embarrassment--tense body, high voice, twitching fingers and probably damp palms as some would wipe hands on their sleeves.

Generally the students seemed to grasp the ideas rather rapidly if paying attention when symbols and associated ideas were explained. Even though most of the students seemed easily distracted, it was amazing how much they remembered from one lesson to the next, especially concerning geometric items. There did not seem to be as much quality retention for the algebraic symbols.

In providing verbalizations for the special symbols in the second lesson, the students performed rather well. In many of the items, very little coaching was needed for most students to verbalize as well as to provide examples to demonstrate some comprehension.

Example: $\frac{1}{2}x$--"One-half of a number x."--"If x equals six, $\frac{1}{2}x$ is three."

Even though the students could verbalize expressions such as $4x - 3 < 5$ or $4x - 3 = 5$, as well as paraphrase (You multiply 4 times the number x, then you subtract 3 and that should give you a number less than 5 or you multiply 4 times some number x, then you subtract 3 and you'll get 5.), they did not know how to find specific values of x to produce true statements. No student suggested trial and error--guessing and checking. No time was spent on explaining how this was done specifically as this was classified as strictly mathematical concepts, according to this investigator. The suggestion of trial and error was offered to the students but not any other algebraic methods for solutions of these open sentences.

Some of the students had trouble distinguishing between the decoding of < (less than) and > (greater than). Some of the students verbalized ∠ (angle) as "less than" not using contextual clues to predict meaning. The other students quickly made the corrections.

The students in general did not perform well in translating word symbols to special symbols. Most all the students worked carelessly, made rather quick and rash decisions. In general, the girls did better than the boys. If this investigator worked individually, coaching by asking questions, each student could successfully translate most of the required expressions. Most of the students had trouble working individually, even the brighter students. They seemed to need constant assurance and frequent feedback. They were excited and enthusiastic if working with this investigator but seemed incapable or unwilling to do the required translations alone.

There appeared to be some perceptual errors such as in Exercise 13 of Lesson 3, as three students drew "two" lines instead of "three," but in Exercise 12, only two lines were called for. Upon being asked to reread Exercise 13, all three students "caught" their error.

For the activity of reading a list of verbal commands and responding by using the correct symbols on a calculator, one student finished long before the rest of the group, a feat which the student was obviously proud of. A quick check of his work showed several errors. The investigator asked if this student had "checked" his work. The response was, "No, I haven't."

"Why not?"

"I don't have the answer book!"

"Is that the only way to check your work?"

"Why yes! It is!"

"Do your results make sense? Can't you check this without an answer book?"

The student did not know what "making sense" meant in these calculator problems. As it turned out, none of the students really understood what "making sense" meant, so a short discussion about estimation was held--then the calculator lessons were remembered. Results making sense and estimation had been covered in the calculator lessons.

From the informal observations made by this investigator, these sixth graders are capable of dealing with the special symbols in the reading lessons for this study. They did not seem awed or afraid of these symbols. They did seem surprised to be asked to talk about the ideas represented by these symbols and to write about mathematics in word symbols. They appeared to be totally inexperienced in verbalizing in mathematics--that is, in expressing meaning and demonstrating understanding of mathematical concepts represented by the special symbols.

Subskill II: Structural Analysis

The students seemed totally unfamiliar with the reading subskill of structural analysis--not just in the area of mathematics. In answer to the question, "How does one obtain meaning of a word?", the response given was, "The teacher tells you what it means." Not one student even suggested the use of a dictionary. Yet, in spite of themselves or unconsciously, a few of the students would use contextual clues to give words a bit of meaning in some cases. Below is an approximation of the type of dialogue which evolved during this set of reading lessons.

Investigator: What does ascending mean?

Student 1: Don't know--never heard of the word.

Investigator: Does any part of the word suggest some meaning of the word to you?

Student 1: (After some thought) No--never heard of the word.

Investigator: What does, "I'm ascending the stairs," mean?

(The whole group is quiet. Student 1 smiles rather timidly but does not answer. Finally Student 2 volunteers.)

Student 2: Either you're "going up" or "coming down" the stairs.

Investigator: Why do you think that?

Student 2: That's what you do to the stairs!

Investigator: Don't we paint or carpet or sweep the stairs too?

Student 2: Yeah . . . I guess we do? (All were very puzzled now.)

After some discussion among themselves, it was unanimously decided by the students that "ascending" means "going up" but no student could or would explain why they felt this way. After the discussion of the etymology of the word, the students seemed to feel very confident and very proud of themselves for having made the correct decision.

The students acknowledged that they had studied prefixes and suffixes but could not define any of the prefixes presented to them even though all of the students knew where the prefix and the suffix belonged in the composition of a word.

After the explanation of this subskill and after the discussion of several examples of words with structural units, each student was given the chance to demonstrate understanding of this reading skill. For example:

Investigator: If "quad" means "four" and "lateral" means "side," in the context of mathematics, what do you suppose "quadrilateral" means?

Student: A four-sided figure.

When asked the meaning of "quadrant," the students responded that it had "something to do with four things." Upon being shown the diagrammatic symbol of a plane being separated into four quadrants, the students very quickly identified correctly a quadrant as applied to a plane. After the discussion of "bisect," the students were able to describe "trisect" quite accurately when reminded to think also about the meaning of "triangle."

The origins and historical notes seemed to be quite interesting to the students, and they clearly enjoyed trying to "guess" the meanings of words from the clues given. The students worked enthusiastically with the exercises of matching words containing structural units with the expressions depicting meaning for these content words.

The activity generating the most enthusiasm involved a list of commands to be "read" and to be responded to with pencil/paper symbols to demonstrate comprehension. The children enjoyed these exercises. Some of the comments overheard and recorded by this investigator as the students completed the activity were:

"Oh boy, this is fun!"

"Hey! Look at mine!"

"Boy, this is neat!"

"Wow! This is fun!"

"Once you get the hang of this, it is real easy!"

"I wish we did this stuff in math class!"

For this subskill of structural analysis, the students in general had very few decoding errors; that is, phonetic and syllabication difficulties were minimal. But the perceptual error of omission was rather strongly demonstrated in this subskill as in the first. This problem seemed to stem from careless reading and nonrereading. Once the students were instructed to reread the verbal command more slowly and to check against their response, the majority of students were able to discover most of their errors of omission.

In this section, processing errors were rather numerous but the students in general were able to perform successfully upon being encouraged to read and reread and then paraphrase or to talk over the command with the investigator or with each other. When an unknown word was related to something which the students were already familiar with--something in their everyday, ongoing lives--almost all the students could give successful responses. Students seemed to remember the unknown word if they remembered their own participation and contributions in group discussions.

Subskill III: Contextual Prediction

The children "read" the sentences in which "mathematical" words were used in a nonmathematical context with aplomb. That is, each student "called out" the words of these sentences perfectly. Most of the students had trouble providing, without coaching, a meaningful sentence, using the mathematical words in the context of mathematics. Most of the students had trouble explaining many of the meanings as used in the everyday sense. They were not very meaning-conscious readers. Reading to them appeared to be "the pronouncing of words"--a type of oral performance. The students seemed surprised, if not a bit agitated, to be asked to explain terms and discuss meanings, especially as applied to mathematics.

Even though the students in general seemed to be perturbed in attempting to explain meanings or to provide mathematical sentences, after being presented with clues, most of the students were able to provide mathematical sentences of ordinary or mediocre quality. Two of the students were able to produce sentences of a rather moderate to high excellence.

Examples: Here is an angle of 45°.

The answer to a problem is the solution to the problem.

This ray goes off in that direction without ending.

Two is a prime number.

A rectangular region whose length is 2 inches and width is 5 inches has an area of 10 square inches.

In the second part of the first lesson for this subskill, the children were given a set of mathematical sentences with certain key items underlined and asked to explain the meaning of each of the underlined vocabulary items. Their success with this activity was again accomplished with coaching in the form of cue or clue questions and sometimes with diagrammatic symbols where appropriate. Some of the students could demonstrate some intuitive comprehension by drawings or sketches, but did not possess the necessary mathematical vocabulary to give their explanations in words.

For their first cloze procedure, the children were very much at a loss and had to be not only coached frequently but also coaxed and wheedled. They were ready to quit almost immediately and again this group of nonperformers included, at the beginning, the brighter ones.

"This is too hard!" "I can't do this!"

"This seems stupid!"

They were very familiar with "fill in the blanks" nonmathematical tests with prejudged words being left out, in which, in the opinion of this investigator, it is hardly ever necessary to read critically, analytically or carefully. Also, in the ordinary "fill in the blanks" tests, knowledge of the syntactical structure of language is not, in the opinion of this investigator, as important as in the cloze procedure; nor is the use of the reading subskill of contextual analysis as prevalent. To supply missing words or special symbols in a cloze procedure, a person must "read" critically--must be acutely cognizant of the graphemic, syntactic and semantic signals as indicated explicitly or implicitly.

Even though the students admitted to having studied verbs, nouns and other parts of speech, they in general found it difficult to predict what part of speech was needed to complete a cloze. The students also found the situation of "more than one correct answer" uncomfortable if not actually intolerable.

"How can there be two right answers?"

After the completion and discussion of the first cloze procedures, the children commented that they could now understand the importance of the knowledge of parts of speech and how this knowledge could help them understand better the printed word, but "What has all this to do with mathematics?" During a discussion of the importance of reading in mathematics, all the children not only agreed that reading was important but could offer reasons why reading is important in mathematics and problem solving. Yet, it was demonstrated time and time again during this experiment that these children were not psychologically tuned to the realization of how important language and reading in particular were to mathematics.

Even though the situation of "more than one correct answer" still was not palatable to most of the children, by the third lesson, they did become rather self-assured and competitive. Before they would change their choice for a blank to agree with someone else's, they had to be "proven" incorrect. If the investigator agreed that two different responses were correct, then "Mine is the best," could be heard in many cases.

In the opinion of this investigator, the reading subskill of contextual analysis as developed through cloze procedures could have a positive effect on the reading abilities in mathematics of elementary children at the sixth grade level as well as at other elementary grade levels. This investigator feels that it is rather unreasonable to expect much progress within the short time spent on the cloze with these thirteen sixth graders, but through the observations made, these thirteen sixth graders demonstrated that progress can be made even in a very short time.

Subskill IV: Inferential Reasoning

With many of the students, their approach to the reading of the problems to infer the necessary mathematical processes for solution was a source of difficulty. Most of them, from their comments, had developed a deep-seated dread of "story" problems. After an inadequate first reading, most were inclined to panic. Some were impatient--dismissing a problem as "too hard for me" if they could not see as well as organize the relationships instantly and mentally. They hesitated to manipulate the definitions or other ideas and concepts to explore the relations so that an inference could be made.

None of the students used the pencil/paper reading at the onset--did not note important ideas and relations, or make sketches nor translate to special symbols. Some of the students seemed to slight preliminary reading, striking out blindly as if expecting to comprehend without reading all the way through. If they reached parts that they did not understand, some of the students simply stopped. They felt compelled to have meaning before going forward possibly to pick up contextual clues--until they were reminded of the previous reading subskill.

Few students thought that more than one reading would be necessary--they sat for the most part almost if waiting for a "bolt of lightning" to show them what had to be done. Most of the students were prone to take rather superficial approaches to the task of inferring the relations required. A popular belief seemed to be that "the last sentence in the problem always tells what is to be done." The students looked for signal words or symbols to tell them what processes to use.

The students seemed totally inexperienced in describing or in writing down sentences explaining the mathematical processes used in solving a word problem. If an inference was made, most of the students were prone to try to carry out any computations mentally without denoting which calculations were to be used. As the investigator called attention to this situation, most of the students were inclined to defend their actions with, "Well, I got the right answer, didn't I?" or "Well, that's the answer and that's the most important thing!"

Obviously these students were not experienced in using special symbols to describe "how to" in a problem. If they were reminded of their first reading lesson, some of the students would then try to translate the words into special symbols, whereby some success in ascertaining the correct procedures was accomplished. Other students had to have more explicit clues given to them, such as, "Suppose we let P represent the unknown product. Now, describe for me in special symbols how to find P."

Inferring "what to do" or "how to" seemed to have been an unusual experience to these children. All of the students experienced difficulties with this reading skill, partly because of their lack of knowledge and understanding of the concepts of mathematics and partly because of their lack of knowledge of the language of mathematics and its uses.

By the end of their third lesson in this reading subskill, a little progress had been made by most students. They apparently though could not proceed without the investigator's presence with coaching questions and immediate feedback as to the correctness of their inferences and their descriptions of these inferences. It was gratifying to notice that many of the students were discussing with each other the possible applications of the previous reading subskills. Some students had also developed a rudimentary mathematical language which gave them enough courage to debate timidly the possible mathematical processes needed for the exercise problems and to ask themselves and each other questions which led them into seeing the relations required for making the necessary inferences.

Subskill V: Evaluative Thinking (Posing Problems)

It did not seem difficult to convince the children that in a realistic problem situation, the students could be forced to formulate a precise problem from a problem situation and that posing problems could be helpful in developing forecasting talents. But the general response was, "What has this got to do with mathematics?" The students seemed unusually perturbed and rather uneasy trying to relate the objective of these lessons to what they had in mind as being classroom mathematics.

By the third lesson for this subskill, in general, the students exhibited very positive attitudes in trying to accomplish the task of posing problems. This task of posing problems seemed to be even more difficult than inferential reasoning. Some of the students would lose track of what was to be done and instead of trying to pose a problem they would be trying to solve a problem which was not even identified.

The difficulties experienced by the children in "striking out on their own" in posing problems from given problem situations probably were due to their lack of facility with mathematical language as well as to their lack of knowledge and understanding of mathematical concepts and their relations. Their own mental attitude toward mathematics apparently played an important part in this activity. The cases in which students managed to pose a meaningful problem demonstrated a tendency for the students to ask questions which called for a single computational process. Most of the students were impotent in posing meaningful problems from situations containing no numbers--"How can you make a problem if there're no numbers?"

Below are some examples of problem situations and the corresponding problems posed by some of the students.

Situation: Jane's father finds that his car travels 16.75 miles per gallon of gasoline.

Problems: How many miles can he drive a day?

How many miles can you get out of a gallon?

How many miles would Jane's father travel with 3 gallons?

How many gallons of gas would you have to use for 83.75 miles?

Situation: Douglas bought 4 pairs of socks for $4.76.

Problems: How much change would he get?

and he had 10 dollars, how much change would he get?

How much would one pair cost?

How much would a dozen pairs cost?

Situation: Mr. Cartwright bought a city lot 75 feet wide and 99 feet deep.

Problems: How many feet all together?

How wide could the store or the thing that he wants be?

What would the area of two be?

If Mr. Cartwright divided his lot in half, how wide would each half be?

Situation: The bottoms of some of the Great Lakes are below the level of the ocean.

Problems: How come?

Why is this so?

How does it do that?

Situation: When a boy tried to fly his kite, he found that it would not go up in the air.

Problems: How come the kite wouldn't fly?

Why doesn't it fly?

Why wouldn't the kite fly in the air? Circle one of these answers:

It wasn't a good kite.
There wasn't enough wind.

Problem Analyses

Problems 16, 17 and 18 from the posttest were selected for further analyses as to the reading cues which may have hindered the students in the solutions of these problems. These problems were chosen because the students did not give acceptable solutions to these problems. Seven randomly selected subjects from the experimental group were interviewed individually by this investigator for approximately thirty minutes. Each of these students were instructed:

Read the problem orally.

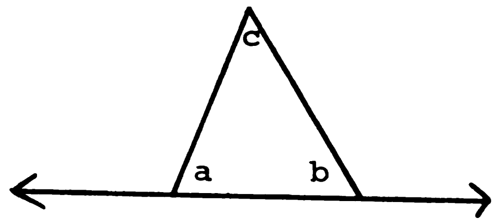
Verbalize your thinking as you solve the problem.

Describe what in the problem gave you trouble.

Problem 16: Consider the figure below with the given data. Find m($\angle$b) and the measure of the exterior angle at $\angle$a.

m($\angle$a) = 70° m($\angle$b) = ______________

m($\angle$c) = 50° m(exterior angle at $\angle$a) = ________________



Problem 17: The measures of the sides of a triangle are 4, 6, and 10 feet respectively. Find the perimeter of a similar triangle whose longest side is 60 feet.

Note: In constructing the posttest from the pretest, the decision to change measures of the sides of the triangle for this problem resulted in a triangle of area zero. This was an oversight. Even though the children had been made aware of the relation between the measures of the sides of a triangle, they overlooked, or were not bothered by this degenerate triangle.

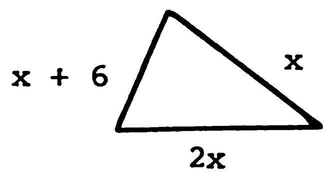
Problem 18: One side of a triangular flower bed must exceed a second side by 6 feet, while the third side must be twice the second side. If the perimeter of the flower bed is to be at least 18 feet, find the smallest possible measure of the shortest side.

Student R

Problem 16: Decoded the problem correctly and verbalized a correct solution. Did not know why he did not manage a correct solution on the test.

Problem 17: Decoded the problem correctly and verbalized a correct solution. Again, he did not know why he did not solve the problem correctly on the test.

Problem 18: Decoded the problem correctly. Sketched a triangle and verbalized the correct relation of sides but could not symbolize this relation. The student was coached with the question, "What if we let the second side have a measure of x feet?" The student set up the following procedure but did not know, "How to find x."



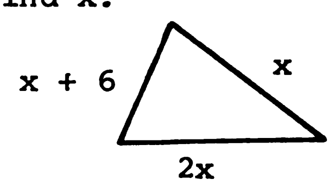
$P = x + 6 + x + 2x = 18$

<u>Student W</u>

Problem 16: Decoded the problem correctly and verbalized a correct solution. Did not know why he did not solve the problem correctly on the test.

Problem 17: Decoded the problem correctly and verbalized a correct solution. Guessed that if he had sketched a diagram as was done in the interview, he would have been able to solve the problem.

Problem 18: Decoded the problem correctly. Could not verbalize the correct relation of sides even though a sketch of a triangle was made. The student guessed that he just didn't "read good enough." With several coaching questions, the student finally was able to verbalize and symbolize the relation of sides. The following procedure was then set up by the student but the student did not know, "How to find x."



$P = x + 6 + x + 2x = 18$

<u>Student Q</u>

Problem 16: Was not able to decode problem correctly without coaching. Read the symbol for "the measure of" simply as an "m" and decoded the symbol for angle as "less than." After the first reading, the student said, "That doesn't make sense!" He was reminded of the reading subskills, especially of contextual analysis. The student then decided that "$\angle$" should be decoded as "angle" but still hesitated with

the "m". After being asked to reread the given data, the student was able to decode the "m" as "the measure of." Still could not verbalize a correct solution. The student could only offer "I don't know how to do the problem" as the reason for not being able to accomplish a solution. The interviewer conjectured that the student could not infer the relation of the measures a, b and c. After prompting, student guessed that the sum of these measures was 100°, therefore, $m(\angle b) = 100 - 120$. "No, no! That can't be right!"--exclaimed the student. After being given the correct relation, a correct measure for angle b was obtained by the student. Student could not then infer the relation of angle a to its exterior angle. After being coached about a straight angle, the student obtained a correct solution to this problem. Was not able to offer any reasons for difficulty, except, "I just didn't know it."

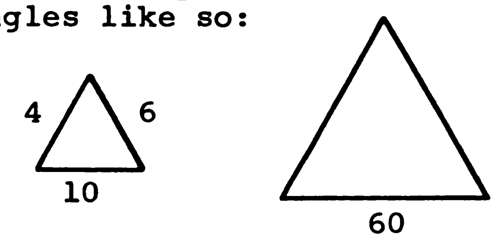
Problem 17: Decoded the problem correctly. Did not remember what perimeter or similar meant. Given clues concerning the parts of the word "perimeter," the student verbalized a correct definition. After several coaching questions, a correct diagram depicting the relation of "similar" was made by the student and the student then verbalized a correct solution. The student said that the meanings of perimeter and similar were the trouble spots.

Problem 18: Decoded the problem correctly. Even with coaching, could not infer the correct relation of sides. After a diagram depicting this relation was described by the interviewer, the student inferred that "Now the sides would be added together to find the perimeter" but could not verbalize precisely what this sum would look like. The interviewer verbalized the required inequality but the student did not know what to do with it. Again, the student responded to the question concerning the trouble spots with, "I just can't do the problem." It seemed that the big trouble was not being able to translate words to special symbols in order to see relations more clearly. Inferential reasoning was also an important reading cue.

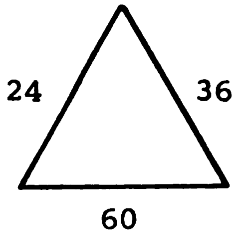
<u>Student O</u>

Problem 16: Decoded the symbols like "m(∠b)" as "m times angle b." After a period of silence, the student decided that he could not do the problem, but didn't know why, except that he "just didn't know what to do." Even after being encouraged to reconsider the decoding of "m," the student still maintained the "m times" meaning because "parentheses always means multiply!" He was then asked, "Does this make sense, in this context?" After a seemingly frustrating adjustment of his thinking and after being encouraged to use contextual analysis, the correct decoding was accomplished. The student then remarked that the measure of angle b could be found if he could just "remember the sum of the angles of a triangle." A correct measure for angle b was obtained by the student after being given the relation of the measures of angles a, b and c. The student then guessed that the other required measure was 110°. An explanation was asked for and the student described what he thought was the relation between angle a and its exterior angle. This turned out to be the correct relation.

Problem 17: Decoded the problem correctly. Described correctly the meaning of perimeter but had some difficulty with "similar." Drew two triangles like so:

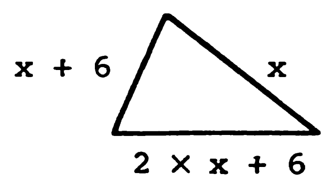


The student remarked that "60 is similar to 10." In attempting to explain this statement, he saw the light and quickly labeled the measures of the other sides of the second triangle as below:



This diagram was very proudly presented to the interviewer. After a pause and a puzzled look, the student asked, "Isn't that right?" He was then instructed to reread the problem, was able to discover that the problem was not finished and then effect a solution.

Problem 18: Decoded the problem correctly. Immediately drew a triangle and labeled as follows:



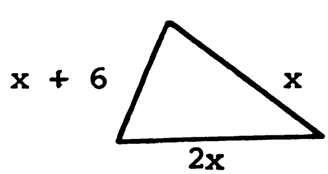
Remarked, "Now you add them all up!" After it was suggested that the problem be reread, the student claimed, "Oh, after you add them all up, you get 18." Student could not symbolize this sum. After several coaching questions, the student relabeled the triangle, showing the correct relation of sides, and after much encouragement, the student recorded $x + 6 + x + 2x = 18$, but could not find x.

<u>Student Y</u>

Problem 16: Decoded problem correctly. Verbalized a correct solution. Did not know why he did not solve problem correctly on test.

Problem 17: Decoded problem correctly. Verbalized a correct solution. Did not know why he did not solve problem correctly on test.

Problem 18: Decoded problem correctly. Verbalized the correct relation of sides but could not symbolize this relation. After being coached into introducing the variable x, the student symbolized the following procedure but could not effect a solution:



$$2x + x + x + 6 = 18$$

<u>Student S</u>

Problem 16: Decoded "the measure of" as simply an "m." After several rereadings, the student maintained that he did not know how to work the problem but could not pinpoint or describe the trouble spots. Even after being told of the meaning of "m," the student made a guess which could not be explained in any way by the student. It was "just a number which came to my head." After being reminded of the relation of the measures of angles a, b and c, the student was able to infer the correct procedure and effect a solution. With several coaching questions, the student was able to accomplish a correct solution for the measure of the exterior angle at angle a.

Problem 17: Decoded the problem correctly. The student could not decide the meaning of perimeter as opposed to area. Given clues concerning the parts of the word "perimeter," the student described perimeter correctly. The student described similar triangles as being "alike but one is bigger than the other." Even after the student sketched two triangles with the 60-foot side corresponding to the 10-foot side, he could not decide on the correct arithmetic to use. He tried to use an additive term of 50 instead of a multiplicative factor of 6. To the interviewer, this indicates a misunderstanding of the meaning of "similar triangles." After being given the correct arithmetic to use, the student was able to find the correct solution.

Problem 18: Decoded the problem correctly. Could not infer the correct relation of sides even with coaching. After a diagram was described by the interviewer, the student remarked that "Now, you add up all the sides." After several rereadings and coaching, the student still could not symbolize the required sum. The interviewer asked, "Do you know what this sum would be?" After another reading, "Oh yeah! You get 18!" The interviewer verbalized the correct inequality but the student did not know what to do with it. The student's explanation of his difficulty with this problem was, "I never could do story problems."

"Why not?"

"I just can't!"

Student P

Problem 16: Decoded the problem correctly except "the measure of" was simply read as an "m." Before any suggestions or coaching could be given, the "m" was decoded as "the measure of." The student inferred the correct process to be used for finding the measure of angle b but could not remember the sum of the measures of angles a, b and c. After obtaining a correct measure for angle b, the student had to be reminded that the problem was not totally solved. After reading again, the student verbalized a correct measure for the exterior angle at angle a. He was not able to shed any light on why this omission was made, except, "I was just careless."

Problem 17: Decoded the problem correctly. Even though the student was able to describe the meanings of perimeter and similar, no solution was verbalized. It was suggested that the student make a diagram, and after some hesitation and erasing, the student sketched a correct diagram and then verbalized the correct solution.

Problem 18: Decoded the problem correctly. Could not verbalize the relation of sides. "Picked up" some numbers, combining them in a way that did not correspond with the correct relation between what they represented. With several coaching questions, the student managed to label two sides x and 2x but could not translate "exceed" into "+" even though he paraphrased "exceed" to "more than." His translation was 6x. After the appropriate translation was made for the student, the student verbalized, "Now, we add up all the sides to get 18." The student had to be coaxed into translating this statement into special symbols, but even then, with the special symbolization at hand, the student could not find x.

Problem Posing on Pretest and Posttest

The description of the problems posed by the children on the pretest as compared to the ones posed on the posttest will be reported as they appeared on the children's papers for Problems 19 and 20. Even though in solving most problems the skill of problem posing may be employed, Problems 19 and 20 for both the pretest and posttest would demonstrate most effectively the students' abilities for using this subskill and would also demonstrate any change that may have occurred during the period in which the reading lessons were given. Besides inferential reasoning, reading for problem posing could be one of the most important reading skills to be used in a meaningful approach to studying mathematics.

Pretest	Posttest
19. A faucet is leaking. How much water is being wasted?	19. How many salmon go up the river during mating season at the dam in downtown Grand Rapids?
20. How many leaves on a tree?	20. How many deer hunting licenses would you issue each year in Michigan if you were in charge of issuing such permits?

CONTROL GROUP

Student B

19. Left Blank	19. thousands
20. Left Blank	20. 1

Student E

19. Left Blank	19. Left Blank
20. Left Blank	20. Left Blank

Student G

19.	Left Blank	19.	Left Blank
20.	Depends on how young or old the tree is.	20.	Left Blank

Student I

19.	Left Blank	19.	1000
20.	Left Blank	20.	50

Student L

19.	Lots	19.	Lots
20.	100	20.	0

Student J

19.	The amount of water that is leaking.	19.	Put electric eye beam on every river. They can count them for you.
20.	36	20.	Left Blank

Student C

19.	100 000 000 gallons	19.	A lot
20.	1 000 000 000 000 000	20.	A lot

Student A

19.	Enough	19.	A lot
20.	All of them	20.	A lot

Student F

19.	As much water we usually drink.	19.	More than 1000 fish
20.	If the tree is a healthy tree it has more than a sick tree would.	20.	About 500

Student H

19. Left Blank	19. 2000 a day
20. 11 000 000	20. Left Blank

Student D

19. Left Blank	19. 250
20. Left Blank	20. 800

Student M

19. It depends on how long the faucet was leaking.	19. Depends on how many fish there are.
20. It depends on how big or small the tree is or if it is dead or alive.	20. 150

Student K

19. It depends on how long it leaks.	19. Left Blank
20. There's a different number of leaves on each tree. Nobody knows.	20. Left Blank

EXPERIMENTAL GROUP

Student R

19. $[(12 \times 60) \times 48] \times 335 = 11\ 577\ 600$ drops a year.	19. 20 in an hour. $24 \times 20 = 580$ in a day.
20. Depends on how old the tree is.	20. Find the average of deer in about 15 places.

Student Q

19. I don't know.	19. First catch some fish and put tags on them. If you catch the same fish throw him back in. Then move down stream and do the same thing. Say you used 100 tags. You moved in 2 spots. You divide 2 into 100 and get 50. 50 would be the average.
20. Depends on the size--small, medium, large	20. You would tag each deer. If you caught 20 deer, you would hand out 10 licenses.

Student S

19. Not very much.	19. It depends on the season and how clean the water is.
20. 1000	20. Depends on how many deer there is.

Student U

19. Left Blank	19. It depends what time of year and how clean the water is.
20. Depends on how big the tree is.	20. It depends how many deer are in the woods and the weather.

Student X

19. Depends how long it was leaking.	19. Depends what time of year and how cold. Depends what season.
20. Could be 1 or 1 trillion.	20. Depends if it was deer season.

Student Z

19. Left Blank	19. Left Blank
20. Left Blank	20. Left Blank

Student O

19. Gallons	19. Depends on the time of year and how the weather is.
20. Left Blank	20. Depends on how many deer there is.

Student N

19. Enough	19. If 40 fish go up 24 hours, 120 go up in 3 days.
20. All of them	20. Same as 19 but with hunting licenses.

Student P

19. Left Blank	19. First you would have to see how pure the water is. And you would have to tag a lot of fish.
20. Left Blank	20. Just keep track of how many you issue. Also you would tag deer.

Student W

19. Left Blank	19. Depends on the weather and how many fish are there now.
20. Left Blank	20. It depends on how many deer there are.

Student T

19. Left Blank	19. 1000
20. 100	20. 700

Student V

19. Left Blank	19. Left Blank
20. Left Blank	20. Left Blank

Student Y

19.	Plug it, measure it often.	19.	Left Blank
20.	Left Blank	20.	Left Blank

Classroom Teacher's Observations

The cooperating classroom teacher in this study was asked at the onset of the project to be prepared to provide an assessment of the program through classroom observations of the subjects in the experimental group. This evaluation will be reported by listing the written descriptive comments made by the classroom teacher periodically during the experiment.

Oct. 3--Very positive response. Using calculators fun!

Oct. 5--It's going well because students who came back enjoyed it. Others who were not chosen wanted to go also.

Oct. 7--Students very enthusiastic!

Oct. 10--Very positive--other students wanted to go again!

Oct. 14--Liked learning about angles--congruent figures!

Oct. 17--Enthusiastic! No real improvement in attitude toward mathematics.

Oct. 19--Liked learning mathematics! Girls liked it! Boys said it was fun!

Oct. 26--Other students wanted to go also!

Oct. 28--Students enjoy going. Have seen no other enthusiasm for math in class.

Oct. 31--Student enjoy going!

Nov. 1--Excited!

Nov. 4--This has been a worthwhile experience for the students.

In summary, the teacher's comments indicate very positive affective results of the reading program in mathematics, which can somewhat be corroborated by the remarks written by the children on their papers, such as:

> I'm glad I got to go with you.
>
> Hi there! (A smiling face)
>
> Thank you!

Retest for Retention

A ten-item test (See Appendix B.) was given to the subjects in the experimental group five weeks after the posttest. The data for this test are recorded in Table 3. After the tests were corrected, each student was interviewed for approximately ten minutes in regards to their performance on this retention test.

Table 3.--Data on Test for Retention

Student	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Score %	85	70	85	70	100	75	75	85	55	100	80	85	60

The mean percentage score for the retention test was 78.8% and the range was 55%--100%. The median score was 80%.

In the interviews, the students were asked to read orally each of the problems, and then the problems missed were discussed. The decoding errors were minimal. Some

of the students decoded aloud "the measure of" simply as an "m" but it was apparent that in their minds they had comprehended "m" as "the measure of."

Some of the students, possibly feeling forced to come up with an answer, made vague leaps into an answer to some of the problems by performing meaningless arithmetic operations or by making an unrelated meaningless guess.

Many of the problems missed on the retention test were solved correctly by the students as they talked over the problems with the interviewer.

The results of interviews in regards to three problems presenting difficulty for many of the students are summarized below.

Problem 4: Give an example of 3 consecutive whole numbers if the first one is x.

Incorrect Responses: x, y, x
1, 2, 3
x + 1 + 2 + 3
Left Blank

Interviews: Most of the students had retained the idea of "consecutive" but admitted that the "x" disturbed them "left as an answer." They felt that they had to give one and only one specific answer with specific numbers. One student remembered the word "consecutive" but had "forgot what it means." After being coached, he gave the response of "x, y, z."

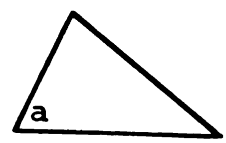
Problem 6: Give an example of a nonnegative number.

Incorrect Responses: Left Blank

Interviews: "Nonnegative" was the trouble spot. All the students had retained the idea of "nonnegative" as being "not negative" but were not knowledgeable of or comfortable with negative numbers to be able to give an example of a number that did not possess this quality. All admitted to having remembered discussions of these numbers in the reading lessons. Most of the students were able to give examples of negative as well as nonnegative numbers to the interviewer. "Why not on the test?"--insecure "giggles" were the popular responses to this question.

Problem 8: Given the figure below with the noted data. Find the m(exterior angle at $\angle a$).

$m(\angle a) = 65°$



Incorrect Responses: 20°, 65°, 180°, 30°, 70°

Interviews: All these students defined "interior" as "inside the triangle" and "exterior" as "outside the triangle" and explained their results on the retention test as, "I just guessed at it." Two of the students could not describe precisely which angle should be considered for measuring even though they "pointed" to the exterior region immediately adjacent to angle a. The children who had responded with "180°" on their tests added that they had remembered "it has something to do with a straight angle." After "talking it over" with the interviewer, two students were able to effect a correct solution. The six other students were able to verbalize a correct solution upon being coached to consider the problem's relation to a straight angle.

Summary

Previous instruction in the selected reading subskills for this study apparently had not produced mastery learning in these students in their reading classes as the children seemed rather unfamiliar with them in

general at the beginning of the study. Perceptual errors of omissions appeared more often than decoding errors. Phonetic and syllabication difficulties seemed minimal for these children in comparison to the processing errors for comprehension.

In inferring what mathematical processes are required in solving a problem, many of the students at the onset seemed to give no thought that more than one reading would be necessary--unaware that complete stops might be called for--thought time in addition to decoding time. This situation with the children's reading became rather apparent during the interviews.

The first set of problems for inferential reasoning did not require a substantial amount of abstract insights--just systematic sequential thought as to what the words were saying. In general, most of the children could not at the onset infer clearly without coaching the sequential steps as indicated by the message of the problem. They were not skilled in organizing information from the printed words of mathematics and skipped unsystematically back and forth through the entire problem.

The results of the posing of problems on the pretest compared to the results of the posing of problems on the posttest show improvement in the quality of the problems posed by the children in the experimental group. There seems to be an improvement in their willingness to attempt to pose a problem from a situation.

During the week of the reading lessons for the subskill of recognizing and verbalizing the special symbols of mathematics, the students demonstrated their abilities to learn this skill but were unable with the minimum training received to use this skill effectively in reading to solve problems. This skill appears to be important in solving problems as the students were easily thwarted by these symbols if not recognized immediately. Contextual analysis was not used to give some meaning to these special symbols until the students were coached to do so.

In analyzing some of the problems not solved on the posttest but which were solved by some of the students during the interviews, it was demonstrated that vocalizing thoughts as relationships are analyzed could be effective in teaching students to read for problem solving.

The statistical evidence is decidedly positive and encouraging--especially considering the relatively short period of time between pretest and posttest. The calculated value of the test statistic does fall in the rejection region; therefore, the Null Hypothesis of no difference between the change in the problem solving abilities of the subjects in the control group and the subjects in the experimental group is rejected. The data present sufficient evidence to indicate that the subjects in the experimental group did make significant gains.

CHAPTER V

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Summary

This study investigated the effects of explicit reading instruction in selected reading subskills of word recognition and comprehensions as described by Duffy and Sherman(68) on the performance of these subskills in mathematics and on verbal problem solving abilities of sixth graders.

The selected reading subskills were:

1. Instant Recognition of Special Symbols of Mathematics and Their Voiced Equivalents (Word Equivalents)
2. Structural Analysis
3. Contextual Prediction
4. Inferential Reasoning
5. Evaluative Thinking

Statistical analysis was used to determine the effects of the reading instruction on the problem solving abilities of sixth graders. Observations and interviews by the researcher were used to describe the impact of the instruction in these selected reading subskills on the abilities of sixth graders to use these skills in mathematics.

Statistical Results

All thirteen students in the experimental group made gains in their posttest scores on problem solving. The range of these gains was from 140% to 525%. The maximum gain for the control group was 175% and there were cases in which students regressed. A t-test indicated that the gains by the experimental group were significant at the 0.005 level.

Other Results

This study identified and obtained partial answers to some mathematics-reading-related-questions. These questions were:

1. What impact does the ability to recognize and voice special symbols of mathematics have on performance in mathematics
2. What role does the understanding of structures produced by word parts play in the learning of mathematics
3. What effect does the subskill of contextual prediction have on literal comprehension in mathematics
4. What impact does the training to infer logical processes have on manipulative thinking in mathematics
5. What impact does problem posing from evaluating problem situations have on evaluative thinking in mathematics

It became apparent through observations and interviews by the researcher that as the sixth grade subjects decoded a written mathematical passage into language and tried to resurrect the intended message, the selected reading subskills played important roles in the performance of these subskills in mathematics and problem solving.

Conclusions

Some of the students in the experimental group of this study claimed that they could read quite effectively but that they could not read mathematics. With the results of the observations and interviews, this investigator has concluded that these students could "read" orally with apparent facility even in mathematics but, at the onset of the study, comprehend only a small part of the material which they read. They seemed to "read" by pronouncing empty words and apparently felt little responsibility for knowing what they were reading, apart from some of the literal facts. As the study progressed, the students receiving explicit reading instructions in mathematics began to take on, faintly in some cases, some of the behaviors of critical readers. This conclusion is supported by the statistical evidence and the interviews for problem analysis. By the end of the five weeks of reading lessons, the children in the experimental group seemed more honestly aware of their abilities to cope with complex problems, to delve into a variety of mathematical concepts and to construct mental images of "things," not physically seen, through the manipulation of the language of mathematics--printed and spoken.

Interpretation of Mathematical Symbols

Verbal labels seemed to play an important role in enabling the children to discriminate between concepts. This investigator has concluded that not being able to distinguish clearly between the symbol and the concept represented by the symbol caused some difficulty initially for comprehension and for problem solving. As students became more comfortable with the concept, the symbol became more a part of the concept. Most of the students appeared to remember and be more comfortable with the geometric items or with the algebraic items integrated with geometric ideas as opposed to simply algebraic items. This could stem partially from the fact that they could become more "physically" involved by drawing special diagrammatic symbols representing these geometry-related ideas.

The special symbols were apparently used easily by the children if recognized with meaning. If a special symbol was not recognized at all or recognized without meaning or without the associated word equivalents, then comprehension difficulties were encountered. Translations from words to special symbols presented more difficulty for all the children than recognizing the special symbols and voicing them.

Word Mediation through Structural Units

Structural analysis seemed to be effective as well as affective in a very positive way. The children appeared to become less fearful of their inability to mediate meanings of mathematical words. The meanings of structural units as derived from their origins produced a clearly observable increased student participation and interest. Developing meaning of a word from its parts possibly lends a humanizing quality to mathematics and its language, hence enhancing interest and possibly comprehension.

Contextual Prediction

Contrasting mathematical meanings with nonmathematical meanings of words made the students more aware of meaning being obtained from context, as they seemed to demonstrate more comprehension of mathematical concepts denoted by these words. At the end of the contextual prediction lessons, the children in general seemed to be beginning at least to realize the tasks involved in "reading" mathematics for meaning. Taking the sentences apart by defining certain items seemed to be helpful in developing this realization. The cloze procedures were apparently effective in promoting not only critical reading but self-confidence in their own work and observations and in their abilities to read analytically and critically.

Inferential Reasoning

The results of this study show that in general sixth graders can be taught to read mathematics critically and to make logical inferences based on information contained in the readings. The inferential reasoning subskill of reading apparently encouraged the children to organize systematically the relations explicitly stated in a problem so as to be able to infer the implied relations.

Evaluative Thinking

This study supports the idea that sixth graders in general can learn to make critical judgements about statements which they read in mathematics. The subjects in the experimental group demonstrated that they could in some cases formulate mathematical problems from given problem situations. The lessons on posing problems seemed to encourage the children to pose questions in problem solving. This could partially explain why the children became more adept in seeing relations important in making inferences to solve problems. This observation indicates the dependency and interrelatedness of the reading subskills.

Problem Solving

Solving problems is a task which requires making sense of and bringing meaning to all kinds of signs and symbols. Before children can understand what they read, they must be able to understand the direct meaning of words, special symbols, phrases and sentences. They must recognize important details and understand the sequence of events. The statistical results of this study indicate that for these sixth graders the particular reading subskills selected for the explicit reading instruction were important in the reading related to problem solving.

It is concluded by this researcher that the oral-verbalizing component of the reading lessons and interviews was an important, effective and affective component. Some of the youngsters were initially shy about verbalizing mathematical thoughts. As the study progressed, all the subjects improved in their abilities to use mathematical language. The children clearly became more conscious of the control which they could have over their own thinking activities. In many cases, students exhibited enough of their ideas for the researcher to follow thought paths. The effectiveness of vocalized problem solving showed up in the interviews for the retest for retention. It is concluded by this researcher that the vocalized problem solving technique was effective partly because the children were able to read mathematics and think more clearly about the meaning of the printed words. Vague ideas could apparently be expanded and relations possibly seen more clearly. The human contact with the researcher in the vocalized activities could also have been an affective component.

Besides the vocalizations making an idea more clear and multiple ideas jell, vocalizing also seemed to provide an encouragement or an excuse to gesture. Both the vocalizing and the gesturing apparently aided in establishing the relations or processes needed to understand the message of the printed words of a problem. Once the verbalizations triggered the gesturing, communication appeared to be enhanced, and often times, the children were able to convey their basic intentions to the interviewer without words. But then, with encouragement, they were often able to describe these intentions with words.

The teacher-student dialogue seemed more effective than the student-student dialogue or the student-whole group dialogue. In the student-whole group dialogues, the children did not appear actually to listen to each other. If one student were reading aloud, it appeared difficult for the other children to listen courteously or inquiringly. It is concluded that verbal intercourse with adults does become an important factor in the development of a child's concept of mathematical ideas. When this verbal intercourse was one-to-one, adult-student, the result was clearly more positive than the teacher-whole group dialogue.

Being able "to think" in complete sentences as demonstrated by the students' oral verbalizations of their thoughts seemed to play an important role in their comprehension and problem solving. It is concluded from this study that the sentence is probably the primary unit for the expression of thought in mathematics. If the children talked in single words or phrases, not in sentences, it was not only difficult for the researcher to find out what they were trying to say but, in most cases, the students themselves demonstrated that they were having difficulty with comprehension. The reading lessons apparently promoted the idea of complete sentences in the printed word and in the spoken word of mathematics. The students began to recognize the sentence structure even in the special-symbolic writings of mathematics.

Recommendations

Since mathematics has (or is) a language--a precise and efficient language for expressing ideas compactly--a language to describe succinctly the world around us and how this world might possibly be in the future--it seems important to this writer that children be given the opportunities and encouragements at all grade levels to develop their talents and abilities for handling this language and its uses.

The United States Government is two hundred years old; the English language is seven hundred years old; and mathematics is over 4000 years old. The study of mathematics and its language can help the human being learn something about thinking itself; how to state problems clearly, sort out the relevant from the irrelevant, argue coherently and abstract common properties from many individual situations. The heart of mathematics is problem solving--finding order, building models and creating abstractions. Mathematics with its language seems to be the ideal arena in which to develop skills in the areas of information organization, problem analysis and argument presentation. These skills are described by the reading specialists as being the necessary skills for critical reading.

In light of the results of this study, it is recommended that time and effort be spent in developing well-trained teachers who will be qualified in mathematics and in the teaching of the reading of mathematics so that "nonreaders" will be taught how to read if they are academically and mentally capable. Throughout their education, language--written and spoken--probably will be one of the most important ways in which our children will represent their mathematics, as well as having it represented for them.

The ability to translate physical imagery into significant mathematical symbols clearly could be one of the rarest gifts to be developed by our schools. The results of this study show that, in general, sixth graders can successfully deal with mathematical symbolism and can be taught to read nontrivial mathematics in order to solve nontrivial problems.

The types of reading required in mathematics are probably different enough from those required in courses in ordinary English to warrant the explicit teaching of skills necessary to obtain reading objectives in mathematics. Even the children in the experimental group of this study began to see that words often passed over quickly in informational writing or general prose had to be defined precisely and carefully in mathematics. The complex sentence structures, often ignored, had to be clearly examined in order to decide on the appropriate logical inferences.

Reading as an end product may be a generalized ability but it is suggested from the results of this study that reading should be taught as a composite of specific skills related to a specific discipline. This research suggests that the five subskills selected for this study ought to be included in a set of reading skills for mathematics. These five reading subskills were selected as a representative subset of the many reading subskills associated with word recognition and comprehension. There are many other reading subskills which could also play an important role in the learning of mathematics for problem solving and confronting problem situations.

In addition, it is recommended from the results of this study that:

1. This research be replicated with possible improvements in procedure by enlarging the sample and extending the study to include numerous and varied populations of elementary children
2. The contribution to problem solving abilities of each of these reading subskills be investigated by psycholinguists

3. The effects of explicit instruction in other reading subskills of word recognition and comprehension on the performance of these skills in mathematics and on problem solving abilities be studied, using similar and appropriate procedures on populations of various grade levels
4. The impact of vocalizing on problem solving abilities be investigated by psycholinguists and educational philosophers
5. The language of children of various ages, which is used in everyday language patterns and the more formal mathematical sentence models in attempting to grasp concepts of mathematics be compared
6. The contribution of cue questions on problem solving abilities be examined
7. Long range studies be done to investigate the effects of continuous and systematic reading instruction on the mathematical abilities of children through the elementary grades
8. Strategies for classroom implementation of explicit reading instruction in mathematics be investigated by educationalists possibly in conjunction with classroom teachers at various grade levels

Being able to communicate in the language of mathematics--printed and spoken--could be the most important mathematical activity, second only to individual thinking. As in the words of Henkin(103):

> Whatever philosophical view one adopts about the relation between mathematics and language, it is evident that the teaching of mathematics involves to some extent the teaching of new linguistic patterns.

The researcher readily admits that the growth by the experimental group as shown by the test statistic could be partially the result of the presence of the researcher or of the extra time on task required of the subjects in the experimental group. However, since this study involved the scientific method, it can be justifiably assumed that the results of this study are significant and the recommendations made are appropriate.

APPENDIX A

APPENDIX A

MATERIALS FOR READING INSTRUCTION

Subskill I: Instant Recognition of Special Symbols of Mathematics and Their Voiced Equivalents

Performance Objectives: Given a set of special symbols, provide word equivalents and voice them.

Given a word, word phrase or word sentence, provide special symbolic representation.

Lesson 1 Introduce some of the special symbols of mathematics, giving their voiced equivalents. These symbols will simply be used to talk about mathematical objects and their relationships. The children will be asked to respond orally to these special symbols placed on cards and to construction paper-models by providing corresponding verbalizations. They will be asked to produce uses of some of these symbols by displaying examples on a hand-held calculator or by diagrams on paper.

1. + as in $^{+}3$ or +3

2. + as in 7 + 3

3. - as in 9 - 4

4. - as in $^{-}3$ or -3

5. ab as in a × b (4t)

6. ( ) as in 4(2 + 5)

7. / as in 2/4

8. 2 as in 3^2 or 3 as in 4^3

9. $\sqrt{\ }$ as in $\sqrt{9}$

10. $=$ as in $4 = 8/2$

11. $\neq$ as in $1/3 \neq 0.3$

12. $<$ as in $5 < 6$

13. $\nless$ as in $6 \nless 5$

14. $>$ as in $6 > 5$

15. $\ngtr$ as in $5 \ngtr 6$

16. $\leq$ as in $x \leq 10$

17. $\geq$ as in $x \geq 10$

18. $\leq$ as in $3 \leq x \leq 9$

19. $<$ as in $3 < x < 9$

20. % as in 60%

21. m/s as in metres per second

22. π as in $C = \pi D$

23. $^\circ$ as in 90° angle

24. ... as in 1,2,3,4,...

25. $\angle$ as in $\angle A$ (angle A)

26. $-$ as in $\overline{AB}$

27. m as in $m(\overline{AB})$

28. $\leftrightarrow$ as in $\overleftrightarrow{AB}$

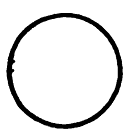
29. $\perp$ as in $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$

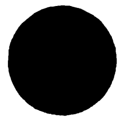
30. $\parallel$ as in $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

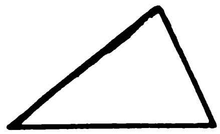
31. Δ as in ΔABC

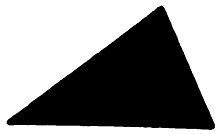
32. $\odot$ as in $\odot C$ (circle C)

33. $\rightarrow$ as in $\overrightarrow{AB}$

34.  circle

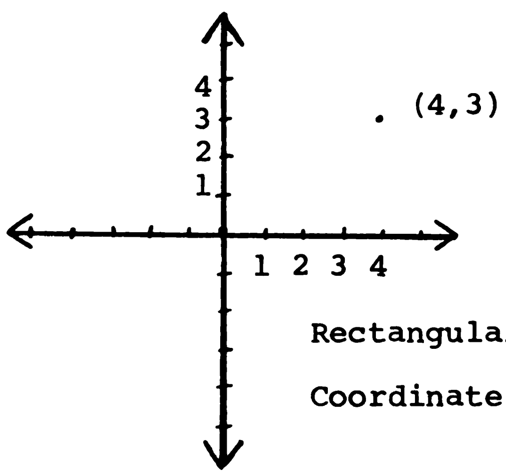
35.  circular region

36.  triangle

37.  triangular region

38. ≅ as in $\overline{AB} \cong \overline{CD}$

39. ~ as in $\Delta ABC \sim \Delta DEF$

40. 

Rectangular Coordinates

<u>Lesson 2</u> The children will provide word equivalents for the symbols below and voice them and will give examples for some of them by using a calculator.

1. □ + 5
2. $^{-}3 + x$
3. $x/2$
4. △ ÷ 2
5. xy
6. $\sqrt{4}$
7. x^2
8. $\frac{1}{2}x$
9. $4x^2$
10. $2^2 + 3^3$
11. $(4x)^2$
12. $5 - (6 + 7)$
13. 3□ - 4△
14. $29 + 72 = 101$

15. $\triangle > 0$

16. $6 < \pi + 3 < 7$

17. $\square \leq 7$

18. $y < 0$ and $y > 10$

19. $y < 0$ or $y > 10$

20. $4\triangle - 3 < 5$

21. $4x - 3 = 5$

22. $-4 \leq y < 5$

23. $2\triangle - 3 = 15$

24. $A = \pi r^2$

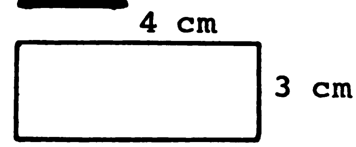
25. $C = \pi D$

26. $3\triangle - \frac{\square}{2} = 4$

27. 60% of $75 = x$

28. $\square$ + 2 km/h = 88 km/h

29. $\blacktriangle$

30. 

31. $\odot C \cong \odot C'$

32. $m(\overline{AB}) = 3$ cm

33. $m(\angle A) = 30°$

34. $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

35. $\overleftrightarrow{XY} \perp \overleftrightarrow{RS}$

36. 0,3,6,9,12,...

<u>Lesson 3</u> The children will translate the following word phrases or sentences into special symbols of mathematics. They will also be given verbal commands and asked to respond by using the correct special symbols on a calculator.

1. The product of seven and a number.
2. The difference of eight and the second power of a number.
3. Add eight to the quotient of a number and two.
4. A number increased by the square root of nine.
5. The measure of a side of a square is equal to the square root of the area of the region bounded by the square.
6. Eight diminished by the square of a number.
7. The sum of the nonnegative multiples of three.
8. Pi is a number between three and four.
9. A rectangle is twice as long as it is wide.
10. The product of two numbers.
11. The coordinate of the point on a number line halfway between zero and seven is three and one-half.

12. Two lines intersect to form four congruent angles.
13. Three coplanar lines do not intersect.
14. The perimeter of a polygon with sides measuring a, b and c units respectively.
15. The area of a rectangular region is six units and its length is one third of its width.
16. If a number is decreased by seven per cent, the result is twenty.
17. The speed of the car was eighty-five kilometres per hour.
18. The sum of the measures of two sides of a triangle is greater than the measure of the third side.

The children will respond to the following commands by using the correct symbols on a calculator.

1. seventeen decreased by 9
2. the product of 52 and 37
3. the sum of 82 and 49
4. the fifth power of 2
5. 39 increased by 42
6. the difference of 98 and 79
7. the quotient of 39 and 13
8. the square root of 196
9. the square root of 225
10. the fourth power of 3
11. the product of 22 and 73
12. the quotient of 94 and 47
13. 89 increased by 102
14. 117 decreased by 83
15. the difference of 712 and 526
16. the third power of seven
17. the square root of 256
18. seven times as large as 56
19. the sum of 412 and 713
20. the quotient of 2187 and 243
21. 8 diminished by 2^3
22. the product of 92 and 24
23. the quotient of 7434 and 18

24. the square of 19
25. the square root of 361
26. the sum of 1832 and 1478
27. half as large as 728
28. the double of 273
29. the triple of 413
30. 173 diminished by 3^4

Subskill II: Structural Analysis

Performance Objective: Given an unknown content word which has structural units, identify and give meaning.

The following three sets of content words will be used to develop abilities for using this particular subskill for successful reading in mathematics. Each of the selected content words will be presented to the children on word cards, accompanied by the following oral questions:

Can you pronounce this word?
Do you know what it means? or
Does any part of the word look familiar to you?

For some items totally unknown to any of the children and with interesting units or origins, clues will be given.

Lesson 1

1. ascending
2. associative
3. binary
4. binomial
5. bisect
6. boundary
7. circumscribe
8. circumference
9. clockwise
10. collinear
11. commutative
12. diagonal
13. diameter
14. displacement
15. distributive
16. concentric
17. concurrent
18. congruent
19. coplanar
20. counterclockwise
21. counterexample
22. descending

Lesson 2

1. decade
2. decagon
3. decimal
4. decimetre
5. equality
6. equiangular
7. equidistant
8. exterior
9. factorial
10. factorization
11. horizontal
12. identity
13. inequality
14. inscribe
15. interior
16. intersecting
17. midpoint
18. nonadjacent
19. nonmetric
20. nonnegative
21. nonparallel
22. nonpositive
23. nonterminating
24. nonzero

Lesson 3

1. octagon
2. octahedron
3. polygon
4. polygonal
5. quadrant
6. quadrilateral
7. reference
8. reflection
9. rotation
10. semicircle
11. subscript
12. subset
13. substitute
14. summation
15. terminating
16. transformation
17. translating
18. transversal
19. trichotomy
20. trinomial
21. trisection
22. variable

The following content words and their corresponding origins(9) will be given to the children to promote the use of structural analysis in reading mathematics.

geometry - Greek - ge meaning earth
metron meaning measure

angle - Latin - angulus meaning a corner

diagonal - Greek - dia meaning through
gonia meaning angle
It passes through the angles.

diameter - Greek - dia meaning through
metron meaning measure
It is a measure through the circle.

polygon - Greek - poluz meaning many
gonia meaning angle

isosceles - Greek - isos meaning equal
skelos meaning leg

perimeter - Greek - peri meaning around
metron meaning measure
Measure around a polygon.

circumference - Latin - circum meaning around
ferre meaning to carry
The measure is carried around the circle.

dimension - Latin - dimensio meaning apart
metiori meaning measure
A rectangle is taken apart into two measures - length and width.
A rectangle is a two-dimensional figure.

plane - Latin - planus meaning flat or level

circle - Latin - circulus meaning a little ring

fraction - Latin - fractus meaning broken

quadrilateral - Latin - quattuor meaning four
latus meaning side

parallel - Greek - para meaning beside
allelon meaning of one another

rectangle - Latin - rectus meaning right

The children will match each word with the expression below best depicting meaning of the word. The children will also be given a list of meanings suggested by certain prefixes or structural units.

____ 1. binomial
____ 2. bisect
____ 3. circumscribe
____ 4. circumference
____ 5. collinear
____ 6. concentric
____ 7. concurrent
____ 8. congruent
____ 9. coplanar
____10. decade
____11. decagon
____12. decimal
____13. diagonal
____14. diameter
____15. equiangular
____16. equilateral
____17. exterior
____18. factor
____19. factorization
____20. inequality
____21. inscribe
____22. interior
____23. invariant
____24. nonadjacent

____25. nonnegative	____34. substitute
____26. nonparallel	____35. quadrant
____27. nonpositive	____36. quadrilateral
____28. nonterminating	____37. translation
____29. nonzero	____38. transversal
____30. polygon	____39. trichotomy
____31. polygonal	____40. trinomial
____32. subscript	____41. trisect
____33. subset	____42. variable

1. Division into three parts or classes.
2. To divide into three congruent parts.
3. Consisting of three mathematical terms.
4. A straight line segment joining two nonadjacent vertices of a polygon.
5. A chord passing through the center of a circle.
6. To put a word, symbol or number value in place of another word, symbol or number value.
7. A distinguishing symbol or letter written immediately below and to the right of another character.
8. A mathematical set each of whose elements is an element of an inclusive set.
9. Outside, as the angle between a side of a polygon and an adjacent side extended.
10. A quantity that may assume any one of a set of values.
11. A closed plane figure made with three or more straight line segments as sides.
12. A closed plane figure bounded by a polygon.
13. To divide into two congruent parts.
14. Consisting of two mathematical terms.
15. A polygon of 10 sides and 10 angles.
16. A period of ten years.
17. A number expressible in powers of ten.
18. Something that actively contributes to the production of a result such as any of the numbers that when multiplied together form a product.
19. Process or result of finding numbers that when multiplied together form a product.
20. Any of the four parts into which a plane is divided by perpendicular reference lines used for graphing.

21. A polygon with four sides and four angles.
22. The distance around a circle.
23. To encircle.
24. The inside, as the inside region of a polygon.
25. Not equal.
26. Invariable quantity, unchanging.
27. To draw one figure within another.
28. Having all angles congruent.
29. Having all sides congruent.
30. Meeting in the same point.
31. Lying in the same plane.
32. Harmony or correspondence between things, like geometric figures having the same shape and the same size, so that they can be made to coincide.
33. Having a common center.
34. Lying in the same straight line.
35. To change over from one medium to another.
36. Lying across or intersecting a system of lines.
37. Lines which intersect on a plane.
38. Not next to.
39. Numbers x such that $x \leq 0$.
40. Numbers x such that $x \geq 0$.
41. Numbers x such that $x < 0$ or $x > 0$.
42. Not ending.

Meanings suggested by certain prefixes or structural units.

bi- ---- two parts

circum- ---- around

co-, con-, col- ---- with, together, jointly

deca-, dec- ---- ten

di- ---- twofold

dia- ---- through or across

equi- ---- equal

ex- ---- outside, outer

fact ---- a thing done

in- ---- not, into, within

lateral ---- side

linear ---- relating to a line

non- ---- not, an absence of

poly- ---- many

quad- ---- four

quadri- ---- four

sub- ---- under, below

trans- ---- across, beyond, through

tri- ---- three

vari- ---- diverse, changeable

The children will read the following commands and respond with the correct symbols to demonstrate comprehension.

1. Inscribe a triangle in a circle.
2. Draw a 90°-angle. Bisect it.
3. Draw a line segment. Denote its midpoint.
4. Draw a plane region with a quadrilateral as a boundary. Shade in the interior region of this figure.
5. Circumscribe a circle about a triangle.
6. Draw a circle and its diameter.
7. Draw a quadrilateral and its diagonals.
8. Sketch an equilateral triangle.
9. Sketch an equiangular triangle.
10. Sketch an angle rotated counterclockwise.
11. Draw three concurrent lines.
12. Sketch a nonsquare rectangle.
13. Sketch two congruent squares.
14. Sketch five collinear points.
15. Draw 2 nonparallel lines cut by 1 transversal.

Subskill III: Contextual Analysis

Performance Objective: Given a set of sentences (one or more) in which unknown words or special mathematical symbols are present, supply each with meaning.

Lesson 1

Part 1: The children will make a sentence, using the underlined word in each sentence, in a mathematical context.

1. Bring the clothes line into the house.
2. The Democrats are now in power in the federal government.
3. The intersection of Baldwin Avenue and 20th Street is not far from here.
4. The sun's rays are warm in July.
5. These chemicals form an explosive solution.
6. My father likes prime rib for dinner.
7. In Washington, D.C., the avenues form transversals for the streets.
8. The Capital building in Washington, D.C. is the center from which the avenues radiate.
9. What is your angle?
10. The area of knowledge in medicine is growing.

Part 2: The children will explain what the underlined vocabulary items mean in the following sentences.

1. The difference between the length and width of a rectangle is 10 inches and a diagonal of this rectangle is 12 inches. Find the dimensions of the rectangle.
2. Find the area of an equilateral triangle whose side is one unit.
3. Multiply the binomials (10 + 3)(10 - 3).
4. Is the ordered pair (2,3) a solution to 3x + 4y = 18?
5. One-third can be written as a nonterminating decimal.
6. Draw a polygonal region with a triangular boundary.
7. Draw three different concentric circles.
8. Consider the horizontal ray $\overleftrightarrow{AB}$ below. Draw $\overleftrightarrow{AC}$ representing a counterclockwise rotation of 90°.

9. Three hundred forty and sixty three hundredths written in decimal notation is 340.63, which means $3(10^2) + 4(10^1) + 0(10^0) + 6(10^{-1}) + 3(10^{-2})$. This notation shows groups of tens in descending powers.

10. Consider the set of four ordered pairs of numbers below as determining the consecutive vertices of a quadrilateral. Locate these four points on a graph and sketch the quadrilateral. Does this quadrilateral appear to be a parallelogram? Why?

    (2,1), (6,2), (4,3) and (3,4)

Lesson 2 The children will complete the following paragraphs by using the Cloze Procedure. They will be instructed to read silently the entire cloze first.

1. Carla looked at the ____________ on a shelf. She ____________, "There are six boxes ____________ each row. There are ____________ rows. How many boxes ____________ there in all?"

   Jay ____________:

   3 rows of 6 ____________ is equal to 1 ____________ boxes.

   3 × 6 ____________ 18

   ↑ factor   ↑ factor   ↑ ____________

2. The perimeter of a ____________ is defined as the ____________ of the lengths of ____________ sides. Since a circle ____________ not ordinarily called a ____________, the distance around it ____________ its circumference. Perimeter is ____________ by the number of ____________ units it contains. The ____________ of any polygon can ____________ found by adding the ____________ of the sides. If ____________ knows that a figure ____________ a polygon and that ____________ has all congruent sides, ____________ the perimeter can be ____________ by multiplying instead of ____________.

Lesson 3 The children will complete the following paragraphs by using the Cloze Procedure.

1. A "dip" in the ______________ is the opposite of ______________ "rise." A temperature "below ______________" is the opposite of ______________ temperature "above zero." A "______________" is the opposite of ______________ "surplus." "Below the sea" ______________ the opposite of "above ______________ sea." A "loss" is ______________ opposite of a "gain." "______________. C." is the opposite ______________ "A. D." Can you ______________ of other opposites to ______________ to this list?

2. The object of the ______________ of Geometry is to ______________ about the properties of ______________, lines, planes, triangles, squares, ______________, circles and other figures ______________ up of points. Just ______________ the aim of algebra ______________ to learn about the ______________ of sets of numbers, ______________ aim in studying geometry ______________ to learn about the ______________ of sets of points. ______________ numbers, geometric points and ______________ of points such as ______________, planes and circles, are ______________ concepts, not concrete objects. ______________ helps his thinking by ______________ points by dots, lines ______________ strokes, and planes by ______________ surfaces.

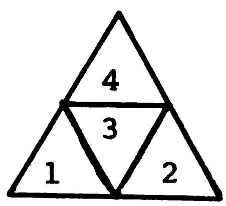
Subskill IV: Inferential Reasoning

Performance Objective: Given a story problem, determine the mathematical processes or operations to be used. (This one type of inferential reasoning is not synonymous in all aspects to the inferential reasoning associated with classical mathematical logic.)

In each of the three sets of exercises, the children will explain how they would solve the following problems.

Lesson 1

1. Find the sum of 142 and 48.
2. What is the product of 14 and 21?
3. An airplane flew 3600 miles in 16 hours. What was its average speed per hour?
4. At the start of a vacation Mr. Watson's car speedometer registered 39 624 miles. When he returned home it registered 42 383 miles. How far had he driven?
5. Mr. Moore rents an apartment for \$225 per month. How much rent does he pay in a year?
6. John wants to buy a bicycle for \$95.75. He has \$28.98 in savings and he received \$5.00 for his birthday. How much more money does he need?
7. One Saturday Bob caught six fish. They weighed 2 lb., $3\frac{1}{2}$ lb., $1\frac{1}{4}$ lb., $2\frac{1}{4}$ lb., 1 lb. and $3\frac{1}{4}$ lb. What was their average weight?
8. What is the perimeter of a rectangular flower bed that is 25 feet long and 12 feet wide?
9. Approximately how far would a jet plane fly in 12 hours if its average speed was 550 miles per hour?
10. If the middle of three consecutive whole numbers is y, the first is _______________ and the third one is _______________.
11. Each triangle in this figure is equilateral. Name as many pairs of congruent triangles as you can.



12. A quadrilateral ABCD is inscribed in a circle. $m(\angle A) = 8x$, $m(\angle B) = 7x$, $m(\angle C) = 2x$ and $m(\angle D) = 7x$. If the sum of the interior angles of a quadrilateral is 360°, find the measures of the four angles of this quadrilateral. Sketch a diagram.

Lesson 2

1. Kate Johnson bought a $44.78 plane ticket for herself and one for her niece at half price. How much did she have to pay altogether for the tickets?

2. In a certain city normal rainfall for July is 4.01 inches. One year 3.17 inches of rain fell in July. How much below normal was the average rainfall that year?

3. Ruth's uncle went by plane from New York to Honolulu, 5051 miles, then to Juneau, Alaska, 2825 miles, and back to New York, 2874 miles. How far did he travel in all?

4. How many square yards are there in the area of a circular flower bed that is 50 feet in diameter?

5. The closest that the moon comes to the earth is about 221 000 miles. About how long would a rocket traveling 26 000 miles an hour take to reach the moon at this distance?

6. Sound travels about 1100 feet per second. Suppose you hear the thunder 5 seconds after you see a flash of lightning. How far away is the lightning?

7. One aspirin tablet weighs about 325 milligrams. How many aspirin tablets weigh 4875 milligrams?

8. One barrel of crude oil produces about 72 litres of gasoline. How many barrels are needed to produce 11 160 litres of gasoline?

9. A golfer had a total score of 288 in a 72 hole tournament. What was the golfer's average score per hole?

10. The exterior angle at vertex C of ΔABC has a measure of 105° and $m(\angle A) = 60^\circ$. Find $m(\angle B)$, if the sum of the measures of the interior angles of a plane triangle is 180°.

11. Suppose ΔABC has two angles with equal measures and the third angle is twice as large as either of the two congruent angles. Find the measures of the angles of this triangle.

12. Consider the case in which each side of an equilateral triangle is doubled. Describe what happens to its perimeter by identifying the factor by which it is multiplied.

Lesson 3

1. If the sun is 93 million miles away from the earth and if light travels about 186 000 miles per second, how many minutes does the light from the sun take to reach the earth?
2. The sun's diameter is about 864 400 miles, which is about 100 times greater than the diameter of the earth. What is the diameter of the earth?
3. Uranus was discovered in 1781. How many complete revolutions has it made since then?
4. What is the speed in land miles per hour of a ship going 20 knots?
5. An empty ship weighs 10 000 tons. How many tons of water does it displace? The ship is loaded with 3000 tons of fuel and cargo. What is the total weight of the water it displaces?
6. How many years would it take an airplane flying 500 miles per hour to reach the sun?
7. How many Saturday nights will there be by the time you are 100 years old?
8. If you drove around the equator of the earth, how long would a round trip take?
9. How much does the air in your classroom weigh?
10. If the perimeter of a triangle is 18 decimetres and one side is twice another, is it the case that the only possible whole number lengths of the sides are 4, 6 and 8 decimetres? Explain.
11. How high is a flagpole which casts a fifty-foot shadow when a man six feet tall casts a $7\frac{1}{2}$-foot shadow?
12. Describe how you would mathematically find the largest possible diameter of a circular table top which can be carried through a doorway 7 feet by 4 feet. Between what two whole numbers do you think this diameter would lie?

Subskill V: Evaluative Thinking

Performance Objective: Given a situation, pose a problem.

The following three sets of situations will be given to the children. The children will pose a problem from each situation.

Lesson 1

1. Mike spelled 40 out of 50 words correctly in a spelling test. Jim took the same test and made 5 mistakes.
2. Jane's father finds that his car travels 16.75 miles per gallon of gasoline.
3. Mr. Cartwright bought a city lot 75 feet wide and 99 feet deep.
4. Douglas bought 4 pairs of socks for $4.76.
5. Don gained 5.25 pounds in 6 months.
6. On a 20-word spelling test Jane missed 3 words.
7. A gallon of water weighs 8.33 pounds.

Lesson 2

1. Since pioneer days, wild plants and animals have been disappearing in our country.
2. The first United States manned suborbital space flight reached a height of 601 920 feet.
3. The total cost to a club of 160 members for a charter flight to Europe was $62 880.
4. A school has 3538 students and 122 teachers.
5. Most of our rain and snow comes from water that was once in the oceans.
6. Turning off electric lights that are not needed can help conserve fuel.
7. During a thunderstorm, the lightning is always seen before the thunder is heard.

Lesson 3

1. The average depth of the soil is from 3 to 6 feet; 200 000 acres of good soil are washed into rivers every year and 500 to 1000 years are needed to form a layer of good soil 1 inch deep.
2. Much of the soil along the lower Mississippi River comes from higher land far to the north.
3. The bottoms of some of the Great Lakes are below the level of the ocean.
4. You have a block of oak wood and a block of pine wood.
5. Good eggs sink to the bottom of a jar of water, but spoiled eggs float.

6. The water in the ocean is salty.

7. When a boy tried to fly his kite, he found that it would not go up in the air.

APPENDIX B

APPENDIX B

EVALUATION INSTRUMENTS

Pretest

Name____________________

1. Draw a quadrilateral with nonparallel opposite sides.

2. Give an example of three numbers x, y and z which satisfy the given data: $x > y$ and $y = z$.

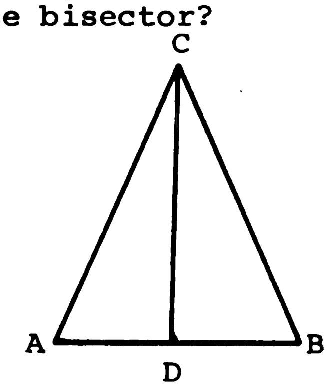
3. Write the next five terms of this sequence:

   0, 1, 0, 0, 1, 0, 0, 0, 1, ...

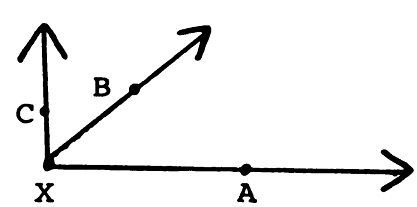
4. If the quotient of two numbers is 1, what would be their difference?

5. If m/3 represents a whole number, then describe the relation of m to 3.

6. Consider the figure below. What would be true if $\overline{CD}$ were an angle bisector?

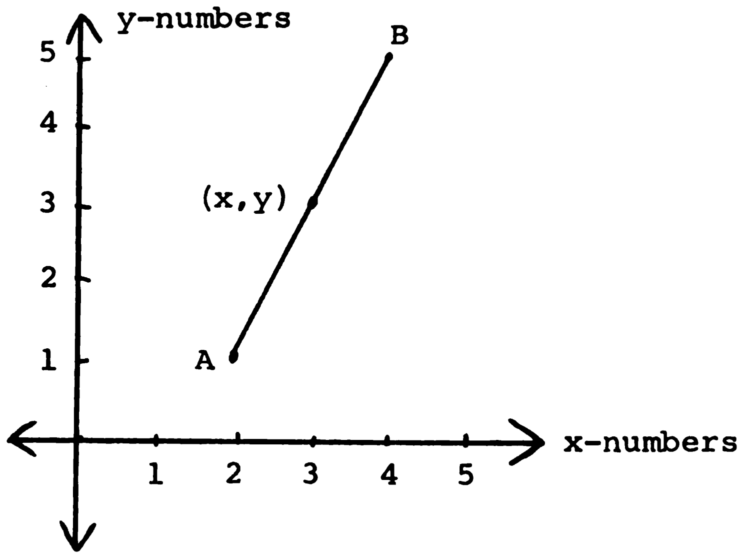


7. In the figure below, if $m(\angle BXA) = 40^\circ$ and $m(\angle CXB) = 50^\circ$, find the number of degrees in $\angle CXA$. Draw $\overrightarrow{XD}$ such that $m(\angle AXD) = 180^\circ$.

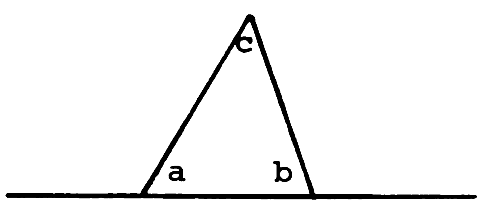


8. Draw a diagram of three collinear points, A, B and C. What seems to be the relation between $m(\overline{AB})$, $m(\overline{AC})$ and $m(\overline{BC})$?

9. Examine the diagram below. What are the specific numbers represented by the symbols x and y if the point labeled (x,y) is the midpoint of $\overline{AB}$?



10. Consider the figure below with the given data. Find $m(\angle b)$ and the measure of the exterior angle at $\angle a$.



$m(\angle a) = 80°$

$m(\angle c) = 50°$

11. Draw a figure showing a circle circumscribed about a square. If the radius of the circle is 4 inches, find the length of a diagonal of the square.

12. A certain polygon has a total of five diagonals. How many sides does this polygon have? Sketch a diagram of this polygon.

13. Mr. Jones buys stones to cover a rectangular patio of 160 square feet. If the patio may be between 8 and 10 feet wide, what limitations are there on its length, assuming that Mr. Jones uses all the patio stones?

14. a. The number twenty-four (24) has eight positive whole number factors. Find these eight factors of 24.

    b. There are four different-shaped rectangular regions of area 24 and with whole number lengths and widths. Identify these four rectangular regions by describing the dimensions of the boundary of each region.

15. The measures of the sides of a triangle are 8, 10 and 12 feet respectively. Find the perimeter of a similar triangle whose longest side is 84 feet.

16. One side of a triangular flower bed must exceed twice a second side by 5 feet, while the third side must be twice the second side. If the perimeter of the flower bed is to be at least 15 feet, find the smallest possible measure of the longest side.

17. Draw at least four different quadrilaterals, each having all sides congruent and at least two of them being equilateral and equiangular. Draw each figure's diagonals. What seems to be the common feature of each set of diagonals?

18. If $y = \frac{1}{x - 1}$ and if x increases from 2 on, what happens to y? (Remember x and y represent numbers.) Does it increase or decrease? Does it appear to be approaching a specific number as a limit?

19. A faucet is leaking. How much water is being wasted?

20. How many leaves on a tree?

Posttest

Name____________________

1. Give an example of three consecutive whole numbers if the first one is x.

2. Write the next six terms of this sequence:

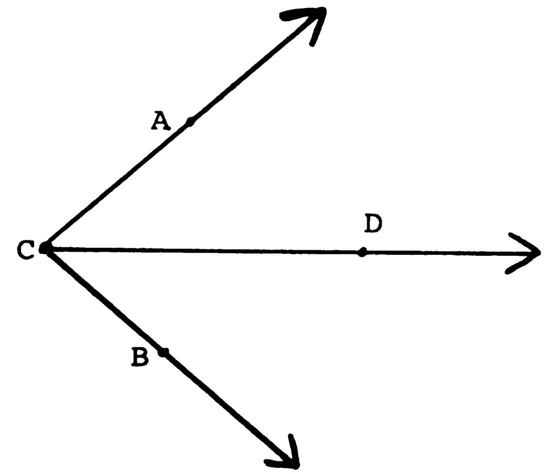
   2, 0, 2, 0, 0, 2, 0, 0, 0, 2, 0, 0, 0, 0, 2, ...

3. If the quotient of two numbers is 1, what would be their difference?

4. If y/5 represents a whole number, then describe the relation of y to 5.

5. Sketch a quadrilateral with nonparallel sides.

6. Consider the diagram below. What conclusion can be drawn if $\overrightarrow{CD}$ were an angle bisector?

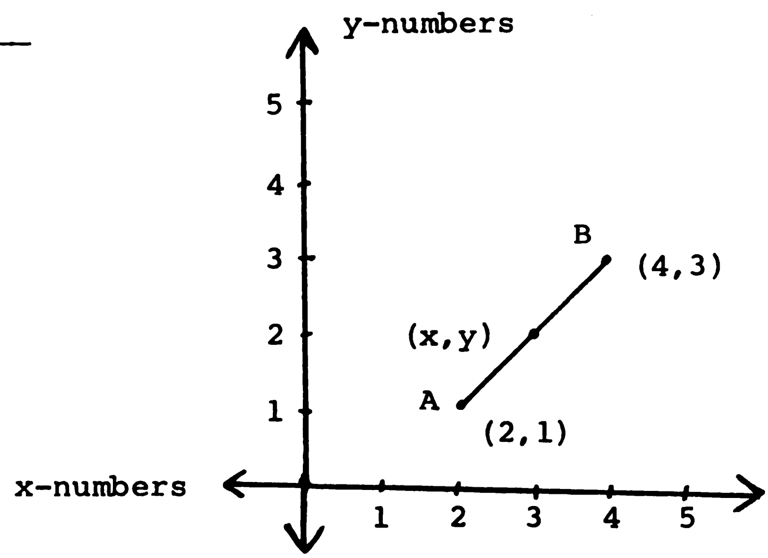


7. Draw a diagram of three collinear points, X, Y and Z. What appears to be the relation between m($\overline{XY}$), m($\overline{YZ}$) and m($\overline{XZ}$)?

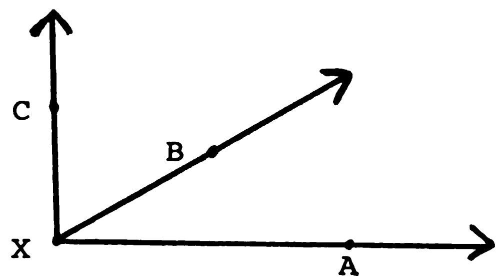
8. Examine the figure below. What are the specific numbers represented by the symbols x and y, if the point labeled (x,y) is the midpoint of $\overline{AB}$?

x = ________

y = ________



9. In the diagram below, if m(∠AXB) = 30° and m(∠BXC) = 60°, find the number of degrees in ∠AXC. Draw $\overleftrightarrow{XD}$ such that m(∠AXD) = 180°.



10. Draw at least four different quadrilaterals, making at least two of the quadrilaterals equilateral and at least two of them equiangular. Draw each quadrilateral's diagonals. What seems to be the common feature of each set of diagonals?

11. Circumscribe a circle about a square. If the length of a diagonal of the square is 8 inches, find the length of the radius of the circle.

12. If $y = \frac{1}{x - 1}$ and if x increases from 2 on, does y increase or decrease? Does y appear to be approaching a specific number as a limit?

13. a. The number twenty-four (24) has eight positive whole number factors. Find these eight factors of 24.

    b. The number of different-shaped rectangular regions of area 24 and with whole number lengths and widths is 4. Identify these four rectangular regions by describing the dimensions of each.

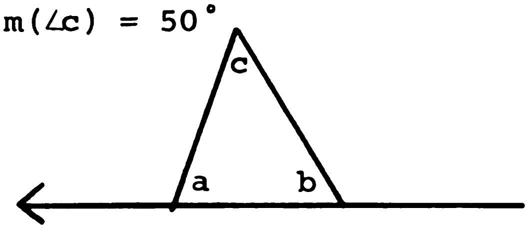
14. Mr. Smith buys stones to cover a rectangular patio of 160 square feet. If the patio may be between 8 and 10 feet wide, what limitations are there on its length, assuming that Mr. Smith uses all the patio stones?

15. A certain polygon has a total of five diagonals. How many sides does this polygon have? Sketch a diagram of this polygon and its diagonals.

16. Consider the figure below with the given data. Find m(∠b) and the measure of the exterior angle at ∠a.

m(∠a) = 70°

m(∠c) = 50°



m(∠b) = ______________

m(exterior angle at ∠a) =

____

17. The measures of the sides of a triangle are 4, 6 and 10 feet respectively. Find the perimeter of a similar triangle whose longest side is 60 feet.

18. One side of a triangular flower bed must exceed a second side by 6 feet, while the third side must be twice the second side. If the perimeter of the flower bed is to be at least 18 feet, find the smallest possible measure of the shortest side.

19. How many salmon go up the river during mating season at the dam in downtown Grand Rapids?

20. How many deer hunting licenses would you issue each year in Michigan if you were in charge of issuing such permits?

Test for Retention

Name____________________

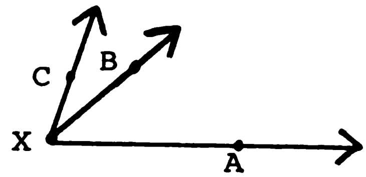
1. Sketch a quadrilateral.

2. Draw two nonparallel line segments.

3. Draw an angle whose sides are perpendicular.

4. Give an example of 3 consecutive whole numbers if the first one is x.

5. Consider the diagram below. If m(∠AXB) = 40° and m(∠BXC) = 30°, find m(∠AXC).

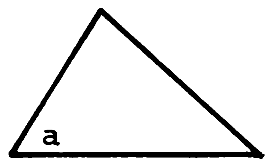


6. Give an example of a nonnegative number.

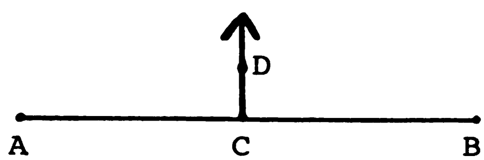
7. Draw 2 lines which are parallel and then draw a third line which is a transversal for the first two lines.

8. Given the figure below with the noted data. Find m(exterior angle at ∠a).

m(∠a) = 65°.



9. If $\overleftrightarrow{CD}$ bisects the following line segment and if m($\overline{AB}$) = 6 units, then find m($\overline{AC}$).



10. Sketch a figure to denote that a circle is circumscribed about a triangle.

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