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ABSTRACT

EMPIRICAL TESTING OF THE FRIEDMAN-
MEISELMAN HYPOTHESIS

By

Harland William Whitmore, Jr.

The purpose of this paper is to test empirically the Friedman-Meiselman hypothesis that the money supply is more important than government expenditures in determining changes in total spending. An apparent implication of this hypothesis is that monetary policy is more powerful than fiscal policy in bringing about desired changes in aggregate income. The significance of this issue is readily perceived. If monetary policy is in fact more effective, the monetary authority should assume the greater share of the burden in implementing economic stabilization policy.

The goal of this paper is to provide further evidence that might aid in making a decision as to the validity of the Friedman-Meiselman hypothesis. This extension of evidence is three-fold. First, we retain Friedman and Meiselman's equations and their statistical definitions of the variables, and we introduce revised data which are presumably better than theirs. Second, we discuss possible alternative statistical definitions of autonomous expenditures that were offered by

Friedman and Meiselman's critics and use these alternative measurements to find new correlation coefficients for the same equations. Third, the simplified versions of Friedman and Meiselman's equations of income determination are replaced by more extensive dynamic econometric models of the U. S. economy. The third extension comprises the major part of our investigation and involves an examination of the dynamic properties of Klein Models II and III. In this aspect of the study we base our test of the Friedman-Meiselman hypothesis on a comparison of the dynamic and long run multipliers for the money supply and government expenditures and on an analysis of causes of changes in real net national product over the sample period.

The simple single equation models we tested initially did not contradict Friedman and Meiselman's findings that the correlation between income and the money supply is greater than that between income and autonomous expenditures. Estimating Klein Model II, we found the long run and impact government expenditure multipliers to be greater than those corresponding to the money stock. Hence these estimates support the other side of the issue.

An analysis of the dynamic properties of Klein Model III yielded policy implications that are less apparent than those suggested by the simpler models.

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Our findings indicate that the relative efficacy of the money supply and government expenditures depends significantly on the particular definition of money we adopt and on whether a comparison is based on concurrent or cumulative effects of the policy instruments. In short and contrary to the claims made by Friedman and Meiselman, we did not find a clear answer to the relative effectiveness of monetary and fiscal policy.

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CHAPTER I

STATEMENT OF THE PROBLEM

The purpose of this paper is to test empirically the Friedman-Meiselman hypothesis that the money supply is a more important determinant of changes in total spending than are government expenditures. An apparent implication of this hypothesis is that monetary policy is a more powerful tool to bring about desired changes in aggregate income than is fiscal policy. The significance of this issue is readily perceived. If monetary policy is in fact more effective, the monetary authority should assume the greater share of the burden in implementing economic stabilization policy.

As will be shown in detail in Chapter II, Friedman and Meiselman tested their hypothesis using estimates of alternative single-equations of income determination. One such equation Friedman and Meiselman call "the quantity theory equation."[1] This expresses the level of income as a linear function of the size of the money stock; according to Friedman and Meiselman it also represents the view that "money matters."[2] Friedman and Meiselman choose as an alternative to "the quantity theory equation" a linear equation relating income to autonomous expenditures. They name this expression "the income-expenditure theory equation."[3] It allegedly represents the view that "money does not matter."[4] These equations were then fitted to the data for selected sub-periods between 1897 and 1958. Correlation coefficients for the fitted equations were then compared to ascertain whether the money supply or autonomous expenditure is more important in determining aggregate income.

The justification Friedman and Meiselman offer for proceeding in this manner is that in an initial investigation into the question of the relative effectiveness of monetary and fiscal policy, it is preferable "to rely on a wide range of evidence interpreted on a rather simple level than on the more indirect and longer chain of connections inevitable in a sophisticated analysis resting on a narrower base."[5] The authors freely acknowledge that since their approach is on a "simple level," their "results cannot be decisive."[6]

The goal of this paper is to provide further evidence that might aid in making a decision as to the validity of the Friedman-Meiselman hypothesis. We will extend the Friedman-Meiselman analysis. This extension will be three-fold. First, retaining Friedman and Meiselman's equations and their statistical definitions of autonomous expenditures and the money supply, we shall introduce revised data which are presumably better than theirs. These revised data cover a portion of the 1897-1958 period. Second, we shall discuss possible alternative statistical definitions of autonomous expenditures that were offered by Friedman and Meiselman's critics and use these alternative measurements to find new correlation coefficients for the same equations. Third, the simplified versions of Friedman and Meiselman's equations of income determination will be replaced by a more extensive dynamic econometric model of the U. S. economy. These extensions will help determine whether the conclusions reached by Friedman and Meiselman also hold using (a) revised data, (b) alternative statistical definitions of the variables, and (c) more sophisticated models of income determination.

The third extension will comprise the major part of our investigation into the Friedman-Meiselman hypothesis. A dynamic model allows inquiry into the stability of the time paths of the endogenous variables. It also permits an analysis of the changes which occur in the endogenous variables over the sample period. The change in an endogenous variable in a given period may be traced (a) to changes in the exogenous variables during that period, (b) to changes in the exogenous variables in each preceding period, and (c) to the initial conditions which prevailed at the beginning of the sample period. Dynamic analysis of linear models also permits a derivation of so-called dynamic multipliers for the policy variables. These dynamic multipliers indicate the relative effects of, for instance, a one billion dollar change in the stock of money and of a (sustained) one billion dollar change in government expenditures on aggregate income in the current period as well as in each succeeding period. From the dynamic multipliers it is also possible to determine the respective long run multipliers of the policy variables on income.

The central aspect of the Friedman-Meiselman technique is a comparison of correlation coefficients between the stock of money and the level of income on the one hand, and between the level of government expenditures and income on the other. It is clear that a full dynamic analysis of a general equilibrium econometric model provides a firmer basis upon which to judge the relative effectiveness of monetary and fiscal policy to influence income.

The plan of this study is as follows. Chapter II contains a discussion of the purpose and procedure of the Friedman-Meiselman analysis as well as of their results and conclusions. In Chapter III we review the criticisms of the Friedman-Meiselman analysis that are offered by Donald Hester, Ando and Modigliani, and DePrano and Mayer. We also include the results of the tests performed by the critics and the Friedman-Meiselman rebuttals. This chapter also includes our comments on points raised by all parties involved in the exchange.

Chapter IV consists of two parts. In the first part we test the Friedman-Meiselman equations using revised data for the policy variables, and in the second part we use the alternative definitions of autonomous expenditures that were offered by Friedman and Meiselman's critics. This provides an investigation of the resiliency of Friedman and Meiselman's conclusions to changes in statistical series and to alterations in definitions of the variables.

In Chapter V we look for alternative analytical frameworks within which to place our examination of the relative effectiveness of monetary and fiscal policy. After a brief discussion of econometric models that are not suited to this investigation, we re-estimate Klein Models II and III to cover longer sample periods than that for which they were originally constructed. Klein Model II is re-estimated for the period 1922-1941 and 1946-1965. Klein Model III is re-estimated for the period 1923-1941 and 1947-1965. In both models the most recently revised data are used. Also, some of the equations in Klein Model III have to be altered to allow a more direct comparison of the money supply and government expenditures as policy instruments.

In Chapter VI we begin with a general discussion of the elements involved in dynamic analysis and examine the dynamic properties of the relatively simple Klein Model II.

In Chapter VII we undertake the major aspect of our study with an examination of the dynamic properties of Klein Model III. We begin this chapter by deriving the fundamental dynamic equation for real net national product. Next we delineate the difficulties encountered in finding a stable fundamental dynamic equation displaying non-negative equilibrium policy multipliers. Proceeding, we examine the dynamic multipliers for the money supply and government expenditures. We then turn to an analysis of causes of changes in net national product over the sample period. This provides the basis upon which we test the Friedman-Meiselman hypothesis. We end the chapter with a general discussion of the coefficients in dynamic models which are important for stability and non-negative long run multipliers.

In Chapter VIII we offer our conclusions and the policy implications of the study.

FOOTNOTES--CHAPTER I

[1]Milton Friedman and David Meiselman, "The Relative Stability of Monetary Velocity and the Investment Multiplier in the United States, 1897-1958," _Stabilization Policies_ by E. C. Brown, _et al_. (A Series of Research Studies Prepared for the Commission on Money and Credit; Englewood Cliffs, N.J.: Prestice-Hall, 1963), pp. 170-171.

[2]_Ibid_., p. 166.

[3]_Ibid_., pp. 170-171.

[4]_Ibid_., p. 167.

[5]_Ibid_., p. 170.

[6]_Ibid_., p. 174.

CHAPTER II

THE FRIEDMAN-MEISELMAN ANALYSIS

Introduction

In their study prepared for the Commission on Money and Credit,[1] Friedman and Meiselman (hereafter referred to as FM) examine two stochastic relationships relevant to a comparison of the effectiveness of monetary and fiscal policy. One relationship expresses the level of aggregate income as dependent upon the size of the money stock; the other relates aggregate income to autonomous expenditures. Before we present a detailed account of FM's analysis of these relationships, it might be useful to review briefly some fundamental concepts involved in the estimation of stochastic relationships.

All stochastic relationships involving one independent or explanatory variable assign a conditional probability distribution to the dependent variable for each given value of the explanatory, or conditioning, variable. This means that given a value of the explanatory variable, probabilities are assigned to all possible values of the dependent variable. Each conditional probability distribution has a mean or expected value, referred to as the mean of the dependent variable.

In practice, interest is focused on how the mean of the dependent variable varies with the values assumed by the explanatory variable. The function which describes this variation is called the population regression function.[2] Denoting the dependent variable by Y and the explanatory variable by X, we may wish to postulate that the population regression function is linear. That is, we may assume that the expected value of Y given X, $E(Y|X)$, is of the form

$$\text{(a) } E(Y|X) = \alpha + \beta X$$

where α and β are unknown parameters.

In order to estimate these unknown parameters, a sample of pairs of observations of X and Y is taken--(X_1,Y_1), (X_2,Y_2),....(X_n,Y_n). Then, a straight line denoted by

$$\text{(b) } \hat{Y} = \hat{\alpha} + \hat{\beta}X$$

is fitted to the pairs of observations. This line serves as an estimate of the true regression, equation (a); $\hat{\alpha}$ and $\hat{\beta}$ are the corresponding estimates of α and β. As is well known, the least squares method is commonly employed to obtain equation (b). This method involves finding the pair of values $\hat{\alpha}$ and $\hat{\beta}$ which minimizes the sum of squares of deviations between the observed values, Y_i, and the estimated values, $\hat{Y}_i$.[3]

Associated with equation (b) is a statistic, denoted by R^2, called the coefficient of determination. Often this statistic is used as a measure of the "goodness-of-fit" of equation (b) to the sample data.[4] It is possible to interpret R^2 in this manner (if least squares estimation is used) because then R^2 is equal to the proportion of the sample variation in Y that is explained by the linear influence of X.[5]

The coefficient of determination plays a crucial role in the FM analysis. FM based their choice between "competing" models primarily on a comparison of values of the R^2's associated with these models. Now we turn to a discussion of the FM analysis. We begin with a statement of the purpose and procedure of this study.

Purpose and Procedure of the FM Analysis

Purpose of Study

FM proposed to examine the "relative stability"[6] of "a relation between income [Y] and the stock of money [M] suggested by the quantity theory of money . . . [and of] a relation between income [Y] and autonomous expenditures [A] suggested by the income expenditure theory. . . ."[7] These so called "relations," FM argue, are respectively the "marginal income velocity"[8] of money, V', and the "marginal [autonomous expenditure] multiplier,"[9] K', as expressed in the equations[10]

$$(1)\ Y = a + V'M$$

$$(2)\ Y = \alpha + K'A.$$

Equations (1) and (2) reflect, according to FM, ". . . the simplest form of the quantity theory . . . and . . . the simplest form of the income expenditure theory."[11]

FM readily acknowledge that the deterministic relationships embodied in a macroeconomic model could easily specify income as a function of both the money supply and the level of autonomous expenditures. However, they claim an important difference of opinion exists among those in the profession

> . . . about which set of relations in the more generalized theoretical system is (a) critical in the sense of being in practice the primary source of change and disturbance and (b) stable in the sense of expressing empirically consistent relations which can be depended on to remain the same from time to time. In other words, the crucial questions are (a) whether investment on [sic] the stock of money can better be regarded as subject to independent change, and to changes that have major effects on other variables, and (b) whether the multiplier (the ratio of the flow of income or consumption to the flow of investment) or velocity (the ratio of the flow of income or consumption to the stock of money) is the more stable.[12]

FM limit their study to the latter question: "The aim of this paper is to present some evidence bearing on the second of the two crucial issues--the relative stability of the multiplier and of velocity."[13] FM state further that their ". . . main approach to exploring the relative stability of velocity and of the multiplier will be to fit equations such as equations (1) and (2) to data for various periods of time in order to determine which of the two fits the data better."[14]

It seems desirable to recast FM's above statements in more formal terminology. This involves using the concepts presented in the introduction to this chapter. The purpose of the FM study is to determine whether a more significant functional relationship exists between the money stock and the level of income or between autonomous expenditures and income. Which of the two gives a better explanation of the level of income will be decided by "fitting" equations such as (1) and (2) to U. S. data for various sub-periods between 1897 and 1956 using the least squares method. The coefficients of determination, the R^2's, are calculated for each equation for each sub-period. The equation displaying the higher R^2 more often is the one which FM declared to "fit the data best." According to FM, if one relationship provides a higher R^2 for more sub-periods than does the other, the first is labeled "more stable." Further, FM argue, the one found to be more "stable" is to be stressed in economic theory and policy.

Procedural Considerations

FM consider several procedural questions. These pertain to the tasks of specifying alternative equations to be tested, grouping data into subperiods, and selecting the national income account categories to be included in the definitions of the variables. A detailed discussion of these procedural considerations follows.

<u>Equations To Be Tested</u>

Besides testing equations (1) and (2), FM consider four others. Two include both the money supply and autonomous expenditures as exogenous with one of these equations also including the price level as an independent variable. The remaining two equations have the price level added as an explanatory variable to equations (1) and (2). These additional equations are given below along with equations (1) and (2):

$$(1)\ Y = \alpha_1 + \alpha_2 M$$
$$(2)\ Y = \beta_1 + \beta_2 A$$
$$(3)\ Y = \alpha_3 + \alpha_4 M + \alpha_5 P$$
$$(4)\ Y = \beta_3 + \beta_4 A + \beta_5 P$$
$$(5)\ Y = \gamma_1 + \gamma_2 M + \gamma_3 A$$
$$(6)\ Y = \gamma_4 + \gamma_5 M + \gamma_6 A + \gamma_7 P$$

where Y = level of income, M = stock of money, P = an index of prices, and A = autonomous expenditures.[15]

It should be noted, though, that the dependent variable actually used by FM in testing each of the above equations is not the level of income, Y, but rather the level of "induced" expenditures, U. FM argue that the level of income can be defined as the sum of autonomous and induced expenditures, $Y = A + U$.[16] Therefore if, for instance, equation (2) were tested as presented above, the resulting "fit" would involve fitting Y "with part of itself."[17] This, FM claim, gives an unfair advantage to A over M in determining the level of Y since the R^2 for equation (2), for example, would tell "nothing about the stability of any economic relation."[18] According to FM, "the independent contribution of the multiplier analysis is to predict the other component of income, consumption [i.e. induced expenditures, U] from the known (or predicted)autonomous expenditure component."[19] FM argue further: "When the data are synchronous, that is, when no lagged responses are introduced, it therefore is preferable to replace equation (2) by one obtained by subtracting A from both sides of equation (2). . . ."[20] FM then argue that since U is the proper dependent variable in equations involving A, U must also replace Y as the dependent variable in equations containing the independent variable M. This is necessary because the purpose of the study is to compare the ability of M and A to influence a common dependent variable. We therefore re-write equations (1) through (6) as:

$$(1')\ U = \alpha_1^* + \alpha_2^* M$$

$$(2')\ U = \beta_1^* + \beta_2^* A$$

$$(3')\ U = \alpha_3^* + \alpha_4^* M + \alpha_5^* P$$

$$(4') \quad U = \beta_3^* + \beta_4^* A + \beta_5^* P$$

$$(5') \quad U = \gamma_1^* + \gamma_2^* M + \gamma_3^* A$$

$$(6') \quad U = \gamma_4^* + \gamma_5^* M + \gamma_6^* A + \gamma_7^* P$$

The reason FM give for testing equation (5') is that it allows them "to obtain a valid statistical test whether the correlation between C [our U] and M is significantly different from the correlation between C and A."[21, 22] FM argue that

> the partial correlations on each of the variables, M and A, keeping the other constant, indicate the net contribution of each to the explanation of C. . . . The simple correlations and the partial correlations necessarily differ in the same direction, so that the partial correlations add to our understanding of magnitude of effect but cannot reverse a conclusion [given by comparison of the sample correlations] about which variable is more highly correlated with consumption. If M and A were entirely independent of one another, in the sense that there was no statistical correlation between them, then, on the average, the partial correlations would equal the simple correlations. . . . However, in practice M and A are positively correlated <u>and that is to be expected under either of the theories under consideration</u> [emphasis mine]. . . . A positive simple correlation between A and C may simply be a disguised reflection of the effect of M on C; alternatively, a positive simple correlation between M and C may simply by a disguised reflection of the effect of A on C. . . . Presumably the disguised effect will be smaller and less consistent than the direct effect which is why a comparison of the simple correlations is relevant and will yield the same result with respect to direction as a comparison of the partial correlations. But only the partial correlation can indicate how much of either simple correlation is produced by the disguised effect of the other variable.[23]

Hence, FM argue that the simple correlation coefficients (the positive square root of the coefficient of determination) associated with equations (1') and (2') will provide measures of the total effect of M and A, respectively, on U. The partial correlation coefficients, between M and C on the one hand and between A and C on the other, of equation (5') provide measures of the "undisguised"portion of the total effects on U.

The price level, P, is added as a new variable to equations (1'), (2') and (5') to form equations (3'), (4') and (6'). This is done to check the degree to which the changes in money values of A and C and nominal values of M can be explained by variations in the price level.[24] Parenthetically, FM decide to add P as a new variable rather than replace A, C, and M by their "real" values because "dividing variables initially expressed in money terms by some index of prices introduces spurious correlation, since errors of measurement in the price index are introduced alike into both sides of the equation."[25]

FM's procedure involves making the following four comparisons.

(*i*) the correlation coefficient between U and M (r_{UM}) of equation (1') with that between U and A (r_{UA}) of equation (2')

($\underline{ii}$) the partial correlation coefficient between U and M ($r_{UM \cdot P}$) of equation (3') with that between U and A ($r_{UA \cdot P}$) of equation (4')

($\underline{iii}$) the partial correlation coefficients between U and M ($r_{UM \cdot A}$) and between U and A ($r_{UA \cdot M}$) of equation (5')

($\underline{iv}$) the partial correlation coefficients between U and M ($r_{UM \cdot AP}$) and between U and A ($r_{UA \cdot MP}$) of equation (6').[26]

Time Periods

FM propose to divide the period 1897 to 1958 into various subperiods and fit equations (1') through (6') to each subperiod rather than to the entire period. The reason FM give for this procedure is that they are primarily concerned with measuring "short-run stability of the realtions being compared."[27] FM reason further than "since the relations may differ at different phases of the cycle, it seems desirable that any one comparison should cover one or more complete cycles . . . "[28] Since only annual data are available for the pre-World War II period and because the cycles during this period were too short to provide a large enough "number of observations to yield statistically meaningful results,"[29] FM settle on a compromise. They decide

> . . . to divide the period [1897-1958] for which data are available into two sets of overlapping segments, one set marked out by the troughs of the major depressions during the period (1896, 1907, 1921, 1933, 1938) except for the post-World War II period, which we have marked off simply by the end of the war; a second set, by peaks intermediate between the troughs of major depressions, except again for dates separating out World War II. . . . The dates we have used are 1903, 1913 (to get a period excluding World War I), 1920, 1929, 1939 (to get a period excluding World War II), 1948, and 1957.[30]

In addition, the equations are tested for the periods 1938-53, 1929-58, and for the total period 1897-1958. The period 1929-58 is a "peak-to-peak" cycle of the last three sub-periods quoted above (1929-39, 1939-48, and 1948-57); 1938-53 is a "trough-to-trough" or depression cycle which completes the sequence of depression cycles covering the whole 1897-1958 period. Equations (1') through (6') are also tested with quarterly post-World War II data for period 1945_{III}-1958_{IV} and 1946_{I}-1958_{IV}.[31]

FM are primarily concerned with testing equations using the levels of the variables U, A, and M; but they supplement their study by testing equations containing first differences (i.e. year-to-year and quarter-to-quarter changes) of U, A, and M. The periods used in the tests of first differences differ only slightly from those used in testing equations (1') through (6'). FM also test equations with lagged explanatory variables of up to five periods for quarterly data 1945_{III}-1958_{IV}.[32]

Definition of the Variables

Before the comparisons (listed on pp. 16-17) are made, FM decide which national income and product account categories to include in the definitions of autonomous expenditures and the money supply. Induced expenditures comprise those residual items of the income concept not included in the autonomous category. Because FM feel a decision cannot be made a priori, they construct certain "conditions" which, when satisfied, determine the accounting items to be contained in the definitions. We will now discuss this construction in detail, since much of the criticism leveled at the FM analysis concerns their choice of items to be included.

Money Supply

In order to decide which accounting items the definition of the stock of money should contain, FM provisionally define the money supply as currency in circulation plus adjusted demand deposits. Because there is a degree of substitutability between the money supply as tentatively defined and time deposits at commercial banks, FM consider time deposits for inclusion as well. If time deposits, through investigation, are found to be "close substitutes for the other monetary items . . . it is preferable to treat them as if they were perfect substitutes than to omit them."[33] If time deposits are perfect substitutes for money, shifting a dollar's worth of time deposits to a dollar as money would have no effect on aggregate money income or induced expenditures.[34] Thus, FM argue that, "this suggests than an appropriate criterion whether time deposits are sufficiently close substitutes for other items [i.e. currency in circulation plus adjusted demand deposits] is whether income is more highly correlated with their sum than with each component separately."[35] In other words, FM regress money on income, time deposits, T, on income, and money plus time deposits, M+T, on income. Time deposits are accepted as a perfect substitute for money if the following "condition" holds:[36]

$$r_{Y(M+T)} > \begin{cases} r_{YM} \\ \text{and} \\ r_{YT} \end{cases}$$

where $r_{Y(M+T)}$ = simple correlation coefficient between income and the money supply plus time deposits

r_{YM} = simple correlation coefficient between income and the money supply

and r_{YT} = simple correlation coefficient between income and time deposits.

In effect, FM are saying that if M and T are perfect substitutes (and thus both can be considered as money) then the above condition holds; therefore, we should test to see if the condition holds and if it does, it can be concluded that M and T and perfect substitutes. If this condition does not hold, then M and T are not perfect substitutes and T cannot be accepted as part of the money supply.

Another alternative definition of money is considered, namely, currency in circulation plus adjusted demand deposits plus time deposits plus mutual savings bank deposits plus postal savings accounts plus savings and loan shares. Mutual savings bank deposits, postal savings accounts, and savings and loan shares are to be accepted as perfect substitutes for money plus time deposits if:

$$r_{YM_3} > \begin{cases} r_{YM_2} \\ \text{and} \\ r_{Y(M_3-M_2)} \end{cases}$$

where M_2 = currency in circulation plus adjusted demand deposits plus time deposits (same as M+T)

M_3 = M_2 plus mutual savings bank deposits plus postal savings accounts plus savings and loan shares.

FM use two alternative definitions of income in their tests. These are: Y_1 = personal disposable income plus statistical discrepancy, and $Y_2 = Y_1$ plus corporate retained earnings plus corporate inventory valuation adjustment.[37] Ten correlation coefficients are computed for each period taking simple linear regressions of M_1 (equals currency in circulation plus adjusted demand deposits), M_2 (equals M_1 and time deposits at commercial banks), M_3, (M_2-M_1), and (M_3-M_2) on each alternative definition of income. Six periods were tested, three of which (1929-1939, 1940-1952, and 1929-1952) were tested by fitting the regressions to annual data. The remaining three (1946-1958, 1946-1950, and 1951-1958) were tested with quarterly data.[38]

For all periods in which annual data were used and for the 1946-1958 period as well, FM found M_2 to be more highly correlated than either M_1 or M_3 with both Y_1 and Y_2. The correlations between M_2 and income are also higher than that of M_1 or (M_2-M_1) with each definition of income.[40]

For the periods 1946-1950 and 1951-1958 the results are mixed. The correlation between M_2 and income is less than that between M_3 and income for both Y_1 and Y_2 in both periods. In period 1946-1950 the correlation of M_2 is greater than that for either of its components, M_1 and (M_2-M_1), for definition Y_1 but less than the correlation of (M_2-M_1) for definition Y_2. In period 1951-1958 the correlation of M_2 is greater than that

TABLE 1.--Correlations between alternative definitions of money and income.[39]

	M_1	M_2	M_2-M_1	M_3	M_3-M_2
		I Annual data, 1929-1939			
Y_1	.498	.849	.592	.828	(-).351
Y_2	.512	.835	.548	.813	(-).361
		II Annual data, 1940-1952			
Y_1	.890	.891	.844	.869	.506
Y_2	.882	.886	.845	.858	.484
		III Annual data, 1929-1952			
Y_1	.958	.961	.882	.952	.765
Y_2	.955	.958	.880	.947	.752
		IV Quarterly data, 1946-1958			
Y_1	.961	.967	.888	.962	.941
Y_2	.956	.957	.873	.950	.928
		V Quarterly data, 1946-1950			
Y_1	.661	.758	.749	.893	.925
Y_2	.654	.779	.807	.904	.922
		VI Quarterly data, 1951-1958			
Y_1	.896	.959	.933	.977	.982
Y_2	.899	.950	.916	.967	.971

for either of its components for both definitions of income.[41]

Since the correlation of M_2 was greater than both of its components for definition Y_1 in all six periods and greater than both of its components for definition Y_2 in all but one period, and since the correlation of M_2 was greater than that of M_3 for both definitions of income in four periods; M_2 was chosen as the best empirical definition of money to use in the equations testing the relative stability of the velocity of money and the investment multiplier.[42]

Autonomous Expenditures

An approach similar to the one followed in deciding upon what items to include in the definition of the money supply is used to settle upon a definition of autonomous expenditures. However, when dealing with this particular topic, FM do a considerable amount of juggling of the national income and product account categories.

Income and Product Accounts

Because of the detailed way in which FM manipulate the income and product accounts when they discuss the various candidates for inclusion into "autonomous expenditures," it is necessary to insert a short presentation of these accounts. This way, we will be able to fit the FM analysis into a compact framework and give an orderly

presentation of their tests and accompanying results. We begin with the national income account known as the "foreign transaction account."

From the "foreign transactions account":

$$(7)\ E = I + T_f + F$$

where E = exports
I = imports
T_f = transfer payments to foreigners and
F = net foreign balance.

The "gross savings and investment account" shows:

$$(8)\ GDPI + F = S + W + R + D + GS_{IPA} + H$$

$$\text{or } K + F = S + W + R + GS_{IPA} + H$$

where K = GDPI-D
GDPI = gross private domestic investment
S = personal saving
W = excess of wages accruals over disbursement
R = corporate retained earnings after taxes plus inventory valuation adjustment
D = capital consumption allowance
GS_{IPA} = government surplus on income and product accounts and
H = statistical discrepancy.

From (7) and (8) personal saving, S, is equal to:

$$(9)\ S = K + (E-I) - T_f - W - R - H - GS_{IPA}.$$

From the "government receipts and expenditures account":

$$(10)\ G + T_f + T_g + T_i + Q + GS_{IPA} = T_p + T_c + T_b + T_s$$

where G = total government purchases of goods and services
T_g = government transfer payments
T_i = net interest paid by government
Q = subsidies less current surplus of government enterprises
T_p = personal tax and non-tax payments
T_c = corporate profit tax accruals
T_b = indirect business tax
T_s = contributions to social insurance.

Substituting GS_{IPA} from identity (10) into identity (9), S is equal to:

$$(11)\ S = K + (E-I) - T_f - W - R - H + G + T_f + T_g + T_i + Q - T_p - T_c - T_b - T_s = K + (E-I) - W - R - H + G + T_g + T_i + Q - T_p - T_c - T_b - T_s.$$

Most of the alternative definitions of autonomous expenditures which FM consider can be taken from identity (11). However, when FM conduct experiments on specific items of government expenditures and receipts, an implicit reference is made to the "Social Insurance Funds" account. Therefore, we need a further breakdown of some items found in identity (11). From the "Social Insurance Funds" account:

$$(12)\ T_s = T_{gg} + T_{g1} + T_{g3} + GS_{SIF} - T_{i2}$$

where T_{gg} = transfers to general government
T_{g1} = federal government benefits from social insurance funds
T_{g3} = state and local benefits from social insurance funds
GS_{SIF} = surplus of social insurance funds and
T_{i2} = investment income.

Our task now becomes one of incorporating the entries of the social insurance funds account into the government receipts and expenditures account. That is, we must break down government receipts and expenditures to show explicitly the items involved in the social insurance funds. Contributions to social insurance, T_s, is the only item in identity (12) which enters explicitly into identity (10). Transfers to general government, T_{gg}, is not capable of explicit treatment in equation (10) since these transfers to general government are intra-governmental. Thus, they ultimately become expenditures of either (a) purchase of goods and services (i.e. a part of G), (b) transfer payments (i.e. a part of T_f or T_g), (c) net interest paid (i.e. a part of T_i), or (d) a subset of subsidies less current surplus of government, Q. Using the following identity, items T_{g1} and T_{g3} can be integrated into equation (10):

$$(13)\ T_g = T_{g1} + T_{g2} + T_{g3} + T_{g4} + T_{g5}$$

where T_{g2} = federal transfer payments to persons other than federal government benefits from social insurance funds

T_{g4} = direct relief payments by state and local governments

T_{g5} = state and local government transfer payments to persons other than T_{g3} and T_{g4}.

Since FM consider the item "direct relief payments by state and local governments"[43] we are explicitly entering it into our accounting framework.

The surplus of social insurance funds could be shown by dividing the government surplus on the income and product account into two categories: (a) surplus of social insurance funds and (b) other government surplus; however, this division is not necessary for the FM analysis.

By means of the following identity T_{i2} can be explicitly entered into (10):

$$(14)\quad T_i = T_{i1} - T_{i2} - T_{i3}$$

where T_i = net interest paid
T_{i1} = total interest paid
T_{i2} = investment income on social insurance funds and
T_{i3} = interest received other than investment income on social insurance funds.

Making the above adjustments, identity (10) becomes:

$$(15)\quad G + T_f + T_{g1} + T_{g2} + T_{g3} + T_{g4} + T_{g5} + T_{i1} - T_{i2} - T_{i3} + Q + GS_{IPA} = T_p + T_c + T_b + T_s.$$

Substituting GS_{IPA} from (15) into (9) personal saving, S, becomes equal to:

$$(16)\quad S = K + (E-I) - W - R - H + G + T_{g1} + T_{g2} + T_{g3} + T_{g4} + T_{g5} + T_{i1} - T_{i2} - T_{i3} + Q - T_p - T_c - T_b - T_s.$$

Define T^* and G^* respectively as:

$$(16') \quad T^* = T_p + T_c + T_b + T_s$$

$$(16'') \quad G^* = G + T_f + T_{g1} + T_{g2} + T_{g3} + T_{g4} + T_{g5} + T_{i1} - T_{i2} - T_{i3} + Q.$$

(G^*-T^*) is equal to minus the government surplus on the income and product account (see identity (15)). If the government surplus is itself negative, (G^*-T^*) equals the government deficit and personal saving is equal to net private domestic investment plus exports minus imports plus the government deficit minus (1) excess of wage accruals over disbursements, (2) inventory valuation adjustment and corporate retained earnings, (3) the statistical discrepancy, and (4) transfer payments to foreigners.

Criteria Used in Experiments

Having outlined the accounting framework necessary for an orderly presentation of the FM quest for an appropriate definition of autonomous expenditures, we discuss, first, the criteria upon which they base their experiments. Next, we will provide a detailed account of FM's consideration of each of the alternative definitions. The definition which best satisfies the criteria is the one chosen as the "autonomous expenditures" variable used in equations (1') through (6'), which were discussed previously.

Two criteria are used to decide which items to include in the definition of autonomous expenditures. The first is similar to the one used in determining the appropriate definition of the money supply. Recall, first, that the criterion used for determining the appropriate definition of the money supply involved correlating income with (a) a tentatively accepted definition of the money supply plus (b) an item which is being considered for inclusion in the definition of the money supply. If this correlation coefficient is greater than the correlation coefficients between income and each of the items (a) and (b) separately, the conclusion is that the proper definition of the money supply is (a) plus (b). This criterion needs little alteration to be applied to the definition of autonomous expenditures:

> The application of this . . . approach to the definition of autonomous expenditures can be illustrated by considering the question whether durable consumer goods should be included in consumption or in autonomous expenditures. Let D stand for consumption expenditures or durable goods, N for non-durable goods, C for their total, and A for autonomous, according to some tentative definition that excludes durable consumer goods but settles other doubtful items. The question to be decided is whether D + A or A alone is a preferable definition for autonomous expenditures. If D and A were perfect substitutes as autonomous or income-generating expenditures, then a shift of $1 from D to A or from A to D would have no effect on N. Hence N would tend to have a lower

correlation with either D or A alone than with their sum. Consequently, this approach implies that a necessary condition for the inclusion of D in autonomous is that:

$$[\text{i}] \qquad r_{N(D+A)} > \begin{cases} r_{ND} \\ \text{and} \\ r_{NA}. \end{cases}$$ [44]

FM add a new twist, however, and develop a second criterion as well:

The requirement that the sum of autonomous and induced expenditures equal income gives rise to a similar test in the other direction, a possibility that did not arise for the simpler example of time deposits. Suppose [i] is not satisfied. If this occured because D was a part of induced expenditures along with N, one might expect shifts between D and N to be independent of changes in A. Changes in A would affect only their sum. But this would imply that

$$[\text{ii}] \qquad r_{A(D+N)} > \begin{cases} r_{AD} \\ \text{and} \\ r_{AN} \end{cases}$$

This approach therefore yields the following criterion:

Possibility	Condition [i]	Condition [ii]	Conclusion
(a)	Satisfied	Not satisfied	D autonomous
(b)	Not satisfied	Satisfied	D induced
(c)	Satisfied	Satisfied	Ambiguous
(d)	Not satisfied	Not satisfied	Ambiguous[45]

These passages are quoted in full to show that even though FM argue--that if D and A are perfect substitutes as autonomous expenditures, then condition (i) will be satisfied and that if D is induced, then condition (ii) will be satisfied--they are, in fact, testing conditions (i) and (ii) to decide whether D is autonomous or induced. For instance, if they find condition (i) to be satisfied they will then infer that D and A are autonomous. Thus, they are treating a conditional statement and its converse as equivalent.

Comparisons of Alternative Concepts

The alternative definitions of autonomous expenditures tested are:[46]

A = Net private domestic investment plus net foreign balance plus government deficit on income and product account

A_1 = A plus consumer durable expenditures

A_2 = A plus imports [FM call (E-I) "net foreign balance." Therefore, they say (A+I) "is equivalent to treating exports as autonomous . . . and imports as induced."[47]]

A_3 = A plus "some part of government receipts which is equivalent to treating most government expenditures as autonomous and all taxes and some government expenditures as induced."[48]

Before presenting the FM results from testing these alternative definitions, we tie definition A to the accounting framework we developed earlier. Identity (9) is:

$$S = K + (E\text{-}I) - W - R - H - GS_{IPA} - T_f.$$

If the government surplus is negative, (9) becomes:

$$(9)\ S = K + (E\text{-}I) - W - R - H + GD_{IPA} - T_f$$

where GD_{IPA} = government deficit on income and product account.

Therefore, FM's definition of A is:

$$(17)\ A = S + W + R + H + T_f.$$

From the "Personal Income and Outlay" account:

$$(18)\ Y_p = T_p + C + S$$

where C = personal consumption expenditures and
Y_p = personal income.

Re-writing (18) we have:

$$(18')\ C + S = Y^d$$

where $Y^d = Y_p - T_p$ = personal disposable income.

Therefore,

$$(19)\ C + A = Y^d + W + R + H + T_f = Y.$$

Thus, if A is the autonomous concept, Y is the corresponding income concept. Our journey through the details of the income and product accounts has yielded the implicit and unique income concept FM are assuming in their study.

Definition A_1 vs A (consideration of consumers durables as autonomous)

The test undertaken in comparing definition A_1 with definition A involves a consideration as to whether consumer durable expenditures is better looked upon as being autonomous or induced. Looking at (19) we see that durable consumer expenditures, D, is included in total personal consumption, C, which is the induced component. However if A_1 is the appropriate autonomous item, then the induced item is $C-D = N$ and we have $N+A_1 = Y$.

Criteria.--The criteria for this test are exactly those quoted above, i.e. if $r_{NA_1} > r_{ND}$ and r_{NA} then D is autonomous provided condition (ii), $r_{AC} > r_{AC}$ and r_{AN}, is not satisfied. The experiments were run with annual data for periods 1929-39, 1940-52, and 1929-52 and with quarterly data for 1946-58. Instead of using A for the periods using annual data, however, FM use $A^* = A$ plus surplus of government social insurance funds, GS_{SIF}, less excess of wage accruals over disbursements, W.[49] Thus $C + A^* = Y^d + R + H + T_f + GS_{SIF}$. This is the income concept FM are assuming. A_1 becomes $A^* + D = A_1^*$. Therefore, for the annual data, conditions (i) and (ii) are:

$$\text{(i)} \quad r_{NA_1^*} > \begin{cases} r_{ND} \\ \text{and} \\ r_{NA^*} \end{cases} \qquad \text{(ii)} \quad r_{A^*C} > \begin{cases} r_{A^*D} \\ \text{and} \\ r_{A^*N}. \end{cases}$$

Results of experiments.--Since r_{ND} was found to be greater than $r_{NA_1^*}$ for all periods in which annual data were used and since r_{ND} was also greater than r_{NA_1} for the period involving quarterly data, condition (i) was not satisfied in any of the periods.[50] As for condition (ii), r_{A*D} was not computed in any of the annual experiments; r_{A*N} was less than but close to r_{A*C} for periods 1929-39 and 1940-52 and greater than but close to r_{A*C} for 1929-52.[51] In the quarterly experiment, r_{A*D} was less than r_{A*C} and r_{A*N} was greater than r_{A*C}.[52] Thus, in a strict sense, condition (ii) was not satisfied for the quarterly data and for one annual period (1929-52). It was satisfied in part for the remaining two annual periods. Because the results were mixed for condition (ii) and rejected condition (i) in every case, FM decide that D is best considered as induced. Therefore, FM's tentative definition of autonomous expenditures remains as A.

Definition A_2 vs A (consideration of "net foreign investment," (E-I), as autonomous)

According to FM (E-I) might be autonomous either (a) because each item is or (b) "on its own account"[53] with imports induced and exports mixed (i.e. exports partly induced and partly autonomous).[54] They consider three possibilities.

<u>Three possibilities</u>.--(<u>1</u>) If imports, I, are autonomous (and exports, E, are assumed to be autonomous) the appropriate definition of autonomous expenditures would

be A and the induced component would be C. Condition (i) states that the correlation between the induced item C and the more inclusive autonomous item, A, must be greater than the correlations between C and each of the components of A, namely A_2 and I. It is to be noted that A is the more inclusive autonomous concept since I is included in the definition, though as a negative entity. This is different from the case we considered in the previous set of experiments since durable consumption is not included in A. From this we see that condition (i) would be:

$$r_{CA} > \begin{cases} r_{CI} \\ \text{and} \\ r_{CA_2}. \end{cases}$$

That is, I is autonomous (and exports autonomous) if condition (i) is satisfied and condition (ii) is not satisfied. FM argue, however, that since imports are a part of personal consumption, condition (i) involves correlating consumption with part of itself which leads to "spurious correlation." To remedy this they re-write condition (i) as:

$$r_{(C-I)A} > \begin{cases} r_{(C-I)I} \\ \text{and} \\ r_{(C-I)A_2}. \end{cases}$$ [55]

If I is induced (and exports autonomous), the appropriate definition of autonomous expenditures is A_2. Since induced expenditures plus autonomous expenditures must equal Y, and since C + A = Y and $A_2 = A + I$, the appropriate induced component becomes C-I. And condition (ii) becomes:

$$r_{A_2(C-I)} > \begin{cases} r_{A_2C} \\ \text{and} \\ r_{A_2I}. \end{cases}$$

FM neither test nor even state condition (ii), however. They only say that if I is autonomous, then $r_{(C-I)A}$ must be greater than $r_{(C-I)A_2}$. Since, in general, this was not found to be the case and since, also, $r_{(C-I)I}$ was found to be quite large, they decide imports are best treated as induced.[56]

(2) Next FM consider the possibility that (E-I) is "mixed," i.e. that E is autonomous but I is induced.[57] If E were autonomous, the appropriate definition of autonomous expenditures is A_2 and the appropriate induced concept is C-I. The components of A_2 are K + (G* - T*) and E. Letting K + (G* - T*) be equal to D_o, condition (i) is:

$$r_{(C-I)A_2} > \begin{cases} r_{(C-I)D_o} \\ \text{and} \\ r_{(C-I)E}. \end{cases}$$

If E were induced, the appropriate autonomous item would be D_o and the appropriate induced item would be (C-I) + E. From this, condition (ii) is:

$$r_{D_o[(C-I)+E]} > \begin{cases} r_{D_o(C-I)} \\ \text{and} \\ r_{D_oE}. \end{cases}$$

Again, FM consider only condition (i). FM did not mention condition (ii). Nevertheless, we include what we believe to be its correct formulation. In the three periods tested FM found $r_{(C-I)A_2}$ to be less than $r_{(C-I)E}$ which they say contradicts the hypothesis that E is autonomous.[58]

(3) The third and final possibility FM consider is the case in which (E-I) is "autonomous directly with imports induced and exports mixed."[59] If (E-I) is autonomous, then total autonomous expenditures is A and from identity (19), C is the corresponding induced expenditures. The components of A are K + (G*-T*) and E-I. Again letting K + (G*-T*) = D_o, condition (i) is:

$$r_{CA} > \begin{cases} r_{CD_o} \\ \text{and} \\ r_{C(E-I)}. \end{cases}$$

If (E-I) is induced, the total autonomous item is D_o and induced expenditures are C + (E-I). Condition (ii) becomes:

$$r_{D_o[C+(E-I)]} > \begin{cases} r_{D_oC} \\ \text{and} \\ r_{D_o(E-I)}. \end{cases}$$

Once again FM do not consider condition (ii).[60] Their full statement of what should be true if (E-I) is autonomous directly is:

> . . . then (C-m) [m=I] should be more highly correlated with (D_o+F) [F=(E-I) and (D_o+F) = A] than with D_o alone or F alone. These were, in fact, the results for all three annual periods, which were consistent with our decision to classify F as autonomous. In addition, if F is autonomous, C should be more highly correlated with D_o + F than with D_o alone or F alone. However, we have not made these calculations.[61]

Supposedly (but FM do not state this), the reason condition (i) was altered by correlating D_o, E-I, and A with C rather than with C-I is that I is induced and "spurious" correlation results from correlating C with "a part of itself."

Definition A_3 vs A (consideration of various government budget items as autonomous)

Division of government budget into G' and T'.--In order to make a clear presentation of this set of FM's experiments, we bring forward identity (16) which we developed earlier.[62]

$$(16)\ S = K + (E-I) - W - R - H + G + T_{g1} + T_{g2} + T_{g3} + T_{g4} + T_{g5} + T_{i1} - T_{i2} - T_{i3} + Q - T_p - T_c - T_b - T_s.$$

The autonomous item FM select for the first set of experiments of this section if $A_3 = S + H + R + GS_{SIF}$.[63] In terms of identity (16):

$$(20)\ A_3 = K + (E-I) - W + G + T_{g1} + T_{g2} + T_{g3} + T_{g4} + T_{g5} + T_{i1} - T_{i2} - T_{i3} + Q - T_p - T_c - T_b - T_s + GS_{SIF}$$

or

$$(20')\ A_3 = K + (E-I) - W - T_f + (G^*-T^*) + GS_{SIF}$$

or

$$(20'')\ A_3 = K + F - W + (G^*-T^*) + GS_{SIF}$$

where G* and T* are defined as in (16') and (16'') above, and brought forward here:

$$(16'') \quad G^* = G + T_f + T_{g1} + T_{g2} + T_{g3} + T_{g4} + T_{g5} + T_{i1} - T_{i2} - T_{i3} + Q$$

$$(16') \quad T^* = T_p + T_c + T_b + T_s.$$

However, instead of breaking down the government budget in terms of receipts and expenditures, FM decide to categorize all items (save K, F, and W) on the right hand side of (20'') into an induced (by assumption) component and a residual component.

FM submit that the induced items of the budget are: state and local relief payments, T_{g4}; personal tax and non-tax payments, T_p; corporate profit tax accruals, T_c; and indirect business taxes, T_b.[64] Re-writing (20'') we have:

$$(21) \quad A_3 = K + F - W + [G + T_f + T_{g1} + T_{g2} + T_{g3} + T_{g5} + T_{i1} - T_{i2} - T_{i3} + Q + GS_{SIF} - T_s] - [T_p + T_c + T_b - T_{g4}]$$

where the items inside the first bracket are the residual components of the government budget; inside the second bracket are the items FM assume to be induced. Substituting for T_s from (12) where:

$$(12) \quad T_s = T_{gg} + T_{g1} + T_{g3} + GS_{SIF} - T_{i2}$$

into the first bracket of (21) we have:

$$(22)\ A_3 = K + F - W + [G + T_f + T_{g2} + T_{g5} + T_{i1} - T_{i3} + Q - T_{gg}] - [T_p + T_c + T_b - T_{g4}].$$

In terms of G* and T* the sums of the items in each bracket are respectively equal to:

$$(23)\ G^* - T_{g1} - T_{g3} - T_{g4} + T_{i2} - T_{gg}$$

$$(24)\ T^* - T_s - T_{g4}.$$

FM call item (23) G' and item (24) T'.[65] Identity (21) then becomes:

$$(25)\ A_3 = K + F - W + (G'-T').$$

FM, however, omit W from consideration. They also incorrectly find the residual, G', to be $G^* - T_{g1} - T_{g3} + T_{i2} - T_{gg}$.[66] That is, their G' is equal to our G' + T_{g4}. Thus, their definition of A_3 is actually:

$$(26)\ A_3' = K + F + (G'-T') + T_{g4}.$$

Because of this, $A_3' = S + W + R + H + GS_{SIF} + T_{g4}$. Since $C + S = Y^d$, we have $C + A_3' = Y^d + W + H + R + GS_{SIF} + T_{g4} = Y$. This is the income concept which FM are implicitly considering in this set of experiments.

Since FM's entire discussion of the definitions of G' and T' was limited to one short paragraph, it was necessary to develop the detailed income account relationships above so that we could keep track of and check their sketchy treatment.[67]

Having thus disposed of the means by which the definitions of G' and T' are derived, we now attempt to unravel the experiments which FM made using these variables.

<u>Criteria for tests on G' and T'</u>.--FM do not specify what conditions are to be satisfied in the experiments on G' and T'. However, from the above discussion of the derivation of G' and T' in which they assumed T' to be induced, and from the following paragraph we have some indication of the approach they used in this section.

> G' would be the appropriate government autonomous concept if T' were induced and total autonomous would be (A*+T') or A' [$A^* = A_3$ of (26)]. However, if T' were autonomous, A* would be the appropriate total autonomous. C was correlated with A* and the components of A*, namely T', G', I_{NP} [$I_{NP} = K + F$]. (C-T') was also correlated with A*, I_{NP}, and G'.[68]

Given that $A_3 = K + F + (G'-T') = I_{NP} + G' - T'$ and $C + A_3 = Y$, [i.e. using FM's definition of A_3' while dropping T_{g4} which was no doubt an oversight on their part in the first place] we submit FM first test for G' being autonomous, while assuming that T' is induced. In this case, the appropriate total autonomous item is

$I_{NP} + G' = A_3 + T' = A'$. Since induced expenditures plus autonomous expenditures must add to Y (= $C + A_3$), the corresponding induced item is $C - T'$. Condition (i), to be consistent with previous experiments, must be:

$$r_{(C-T')A'} > \begin{cases} r_{(C-T')I_{NP}} \\ \text{and} \\ r_{(C-T')G'}. \end{cases}$$

This, however, causes one to wonder if the last line of the last quote contains a misprint. (C-T') should be correlated with A' and not A* [our A_3]. Further confusion of FM's intentions is added by the fact that appendix Table II-A-4 has (C-T') correlated with A (and not with A* as the FM quote suggests). We grant FM the benefit of the doubt and assume they meant to say (C-T') is correlated with A'.

Results.--For periods 1940-52 and 1929-52 condition (i) is not satisfied. It is satisfied for 1929-39, however. For 1946-53 $r_{(C-T')A'} = .632$, $r_{(C-T')I_{NP}} = .040$, and $r_{(C-T')G'} = .645$. Therefore, condition (i) is not satisfied. Condition (ii) was not tested.[69] However, to be consistent with their previous experiments, condition (ii) must be $r_{I_{NP}(C-T' + G')} > r_{I_{NP}(C-T')}$ and $r_{I_{NP}G'}$.

Further experiments on G' and T'.--Next, we submit FM test to see if G' and T' are each autonomous. If this is the case, condition (i) should be:

$$r_{CA_3} > \begin{cases} r_{CI_{NP}} \\ \text{and} \\ r_{CG'} \\ \text{and} \\ r_{CT'}. \end{cases}$$

This follows from the fact that if G' and T' are both autonomous, the appropriate total autonomous item is A_3 and the corresponding induced item is C. Also, as FM stated when they initially outlined the condition (i) against which all the alternative definitions of autonomous expenditures were to be tested, the correlation between the induced item and the total autonomous item must be greater than the correlations between the induced item and each component of the autonomous item. However, the condition FM test violates their original statement that only one component is to be tested at one time. It seems they forgot this when considering the government budget even though they had remembered it in the previous experiments. Again, as has been the case in all experiments save the ones considering durable expenditures as autonomous, condition (ii) was not tested.

Condition (i) is not satisfied for periods 1940-52, 1929-52, and 1946-53. In period 1929-39 r_{CA_3} = .852 and is greater than both $r_{CG'}$ and $r_{CT'}$ while $r_{CI_{NP}} = .910$. Thus, strictly speaking, the condition is not satisfied for 1929-39, either. Thus, they conclude G' and T' are not separately autonomous.[70]

The final test for this set of experiments in this section is the one which considers (G'-T') to be autonomous directly. If this is the case, the appropriate total autonomous item is once again A_3 and condition (i) is:

$$r_{CA_3} > \begin{cases} r_{CI_{NP}} \\ \text{and} \\ r_{C(A_3-I_{NP})} \end{cases}$$

where $A_3 - I_{NP}$ is equal to (G'-T'). For the periods 1929-52 and 1940-52 this condition is not satisfied. However, it is satisfied for 1946-53. For the 1929-39 period r_{CA_3} equals .852 and is greater than $r_{C(A_3-I_{NP})}$; $r_{CI_{NP}}$ is equal to .910. FM state that r_{CA_3} "differs little from the correlation between C and I_{NP}."[71]

Let us summarize the results for the test of "(G'-T') autonomous directly" and the test of "G' autonomous and T' induced" in juxtaposition. In both cases, condition (i) is not satisfied for periods 1940-52 and 1929-52. For period 1946-53 condition (i) is satisfied for (G'-T') directly autonomous. Condition (i), strictly speaking, is not satisfied for 1946-53 period, for G' autonomous and T' induced since $r_{(C-T')A'} = .632$ and $r_{(C-T')G'} = .645$ for this period. In the 1929-39 period condition (i) for G' autonomous and T' induced is satisfied. For this period condition (i), strictly speaking, is not satisfied for (G'-T') directly autonomous since $r_{CA_3} = .852$ and $r_{CI_{NP}} = .910$. FM point out that r_{CA_3} is greater than $r_{(C-T')A'}$ for the periods 1929-39 and 1946-53, implying that this may be enough to swing the scales in favor of the definition of (G'-T') as directly autonomous. However, for periods 1940-52 and 1929-52, $r_{(C-T')A'}$ is greater than r_{CA_3}. Thus, neither definition seems to be preferable to the other.[72]

<u>Alternative division of government budget</u>.--Possibly because of the inconclusive results obtained by the above set of experiments, FM devise a second series based upon a different division of the government budget into induced and autonomous components. This set of experiments covers annual and quarterly data for the period 1946-58.[73]

The tentative (i.e. assumed) autonomous concept is equal to $A_4 = S + H + R + W = K + F + G^* - T^*$. All government expenditures, G^*, except state unemployment benefits, UB_S, were taken as the autonomous portion of the government budget. Defining $G'' = G^* - UB_S$ as the autonomous portion of the government budget, we have:

$$(27)\ A_4 = K + F + G'' + UB_S - T^*$$

$$= K + F + G'' - (T^* - UB_S).$$

Setting the induced part of the budget, $(T^* - UB_S)$, equal to T'', A_4 becomes:

$$(28)\ A_4 = K + F + G'' - T''.$$

Discussing the set of experiments they ran using definition A_4 FM claim:

> A^* [A_4] would be autonomous if T'' were autonomous. If T'' were autonomous then we should find that
>
> $$r_{(A^*+T'')(C-T'')} > \begin{cases} r_{(C-T'')A^*} \\ \text{and} \\ r_{(C-T'')T''}. \end{cases}$$
>
> However, $r_{(A^*+T'')(C-T'')}$ is less than $r_{(C-T'')T''}$ which suggests that T'' is not autonomous.
> On the other hand, if T'' were induced, the correlation between A^* [A_4] and C should be higher than the correlation between A^* and T'' alone, or A^* and $C-T''$ alone. However, this is not so for the 1946-58 period. A [A^*] is more highly correlated with T'' alone than with C. The results are therefore inconsistent and ambiguous.[74]

The above statement is highly confusing in that the conditions are inconsistent with all previous statements regarding condition (1). Also, nothing is said about G''. A_4 would be the appropriate definition of total autonomous expenditures if both G'' and T'' were autonomous. If we are testing whether T'' is autonomous, assuming

G'' to be autonomous, condition (i), in order to be consistent with what has been done previously, would be:

$$r_{CA_4} > \begin{cases} r_{C(A_4+T'')} \\ \text{and} \\ r_{CT''} \end{cases}$$

where $A_4 + T'' = K + F + G''$ and T'' are the settled and unsettled components of the definition of autonomous expenditures. Similarly, if T'' were induced, assuming G'' to be autonomous, the appropriate total autonomous item would be $A_4 + T''$ and the corresponding induced component would be $C - T''$. Condition (ii) would be:

$$r_{(A_4+T'')(C-T'')} > \begin{cases} r_{(A_4+T'')C} \\ \text{and} \\ r_{(A_4+T'')T''}. \end{cases}$$

It is to be noted that FM's above quoted claim cannot be referring to tests of whether T'' is autonomous while assuming G'' to be induced. The reason is that such an assumption precludes using A_4 (FM's A*) as the initial tentative definition of autonomous expenditures. However, if we are to assume G'' to be autonomous in interpreting the above quotation, we see that the positions FM gave correlations $r_{(C-T'')A*}$ and

$r_{(A^* + T'')(C-T'')}$ should be reversed. Secondly, all three correlations should involve C and not C - T''. We also see that the condition which FM submit must hold if T'' is induced is incorrect. They should have used A_4 + T'' [i.e. A*+T''] throughout instead of A_4 [A*]. Also, the relationships between the correlation coefficients are stated incorrectly.

The inconsistency of this section of the study can be verified by referring to the section "<u>Definition of A_2 vs A</u>" where the first possibility is discussed. If the reader substitutes T'' for I and abstracts from the proposition that I is to be considered as part of consumption, the inconsistency will be blatant. Also, the reader may compare this section with the original FM statement of the proper conditions to be tested.

We turn, now, to the conclusions reached by FM as to what part of the government budget should be considered as autonomous.

> Under the circumstances, we decided to adopt the usual treatment of the deficit alone as the autonomous contribution of the government, though we cannot demonstrate that this is the best treatment. The alternative would be to treat government expenditures alone as autonomous. This, too, is not supported by the data. Further, it would have led to the designation of C-T'' as the induced concept. The sum of consumer expenditures and essentially tax payments would make for a <u>rather novel consumption function</u> [emphasis mine]. In addition, it does not conform to the sense of the literature on money-income relations. The procuedure we followed therefore seemed the least bad.[75]

This paragraph points to the item which swung the pendulum in favor of treating the government deficit as the appropriate autonomous item as opposed to either government expenditures or receipts alone. For, taking the results of all experiments prior to this one into account, the treatment of the government deficit as autonomous leaves C as the only induced item and equations (1') through (6') represent consumption functions. FM's referral to a "novel consumption function" shows that they have become confused, at this point. They have become so accustomed to using the letter C and the term "consumption" in their study that they have forgotten they originally meant "consumption" to denote "induced expenditures" and not necessarily just personal consumption expenditures. Also, until now we had been implicitly assuming that FM were viewing the equation $U = a_3 + a_4A$ as a reduced form equation. But their above comment seems to indicate that they now view it as a "consumption function."

Before we outline the results of the tests FM made on equations (1') through (6'), it should be explained why the experiments which FM performed to arrive at "empirical definitions" of autonomous expenditures and the money supply were presented in such detail. Mainly, this was done to illustrate the degree to which their analysis deviates from the utilization of the conditions as originally stated. Secondly, since a significant source of criticism in the literature concerns the treatment of the government budget in the definition of autonomous expenditures, we presented the complete analysis of the alternative definitions considered so that a comparison could be made between the method used in considering the government budget with that used in considering the other items for inclusion in the definition of autonomous expenditures.

It has been shown that, even if we set aside the fact that conditions (i) and (ii) do not imply what FM contend they do, a) FM did not, except in one case, test condition (ii), b) their consideration of the various components of the government budget was inconsistent with their consideration of other items, c) the results were not clear-cut especially for the consideration of the government budget, and d) the concept tested for government budget was not expenditures minus receipts but rather autonomous budget items minus induced budget items.

Results of the Study

Having decided upon the following "empirical definitions" of the variables:

M = currency in circulation plus adjusted demand deposits plus time deposits in commercial banks

A = net private domestic investment plus the government deficit on income and product account plus exports minus imports

U = personal consumption expenditures (C)

P = index of consumer prices, 1954 = 100

equations (1') through (6') and their counterparts in terms of first differences are now tested. The results, FM argue, clearly indicate that "the income velocity of circulation of money in consistently and decidedly stabler [sic] than the investment multiplier . . . "[76] Comparing equations (1') and (2'), FM found r_{CM} to be greater than r_{CA} for both quarterly periods and for all annual periods except 1929-39. They say, however, that 1929 was an exceptional year and that if the 1929-39 period is altered to include only the years 1930-39, the correlation between consumption and money is greater than the correlation between consumption and autonomous expenditures for this period as well.[77] The same results are obtained when equation (5') is considered and a comparison is made between $r_{CA \cdot N}$ and $r_{CM \cdot A}$, the partial correlation coefficients between consumption and autonomous expenditures and between consumption and money, respectively.[78]

Since FM find the correlations between money and autonomous expenditures to be positive for all periods, quarterly and annual, they say that one of the simple correlations between consumption and money and between consumption and autonomous expenditures is probably the disguised effect of the other variable on consumption.[79] Since the partial correlation coefficients between consumption and money is greater than those between consumption and autonomous expenditures, FM argue that the effect of money on consumption is disguised in the simple correlation between consumption and autonomous expenditures rather than the other way around.[80]

In their comparison of equations (3') and (4') FM found the simple correlations between consumption and money to be greater than those between consumption and autonomous expenditures in all periods except for the period 1938-53 "for which both correlations are negative."[81] Testing equation (6') FM found the partial correlations $r_{CM \cdot P}$ to be consistently greater than $r_{CA \cdot P}$ and, also, $r_{CM \cdot AP}$ to be greater than $r_{CA \cdot MP}$.[82] These findings, FM argue, "clearly indicate that the results of the comparisons are even more one-sided when the statistical effects of the price level are held constant."[83]

The results of tests using first differences in equations (1') through (6') are too sketchy to merit any discussion. FM consider finally lagged values of M and A and regress them on C for the quarterly time period 1945_{III}-1958_{IV}. We shall not discuss the results of this part of their study, either, since it seems to be an after thought and violates credance of their treatment of equations (1') through (6') as reduced forms. The following tables show the results FM obtained from making the four comparisons of correlation coefficients listed in pp. 16-17.

TABLE 2.--Correlations between synchronous variables in nominal terms.[84]

Period	Income Expenditure Theory		Quantity Theory			
	r_{CA}	$r_{CA \cdot M}$	r_{CM}	$r_{CM \cdot A}$	r_{YM}	r_{AM}
1897-1958	.756	-.222	.985	.967	.988	.791
1897-1908	.587	-.496	.996	.996	.991	.622
1903-1913	.485	-.127	.997	.996	.987	.495
1908-1921	.672	.400	.995	.993	.975	.646
1913-1920	.791	.423	.991	.980	.975	.761
1920-1929	.569	.288	.968	.956	.933	.524
1921-1933	.843	.884	.897	.923	.810	.586
1929-1939	.937	.688	.912	.529	.915	.880
1933-1938	.935	.414	.991	.938	.985	.921
1938-1953	.397	-.328	.958	.955	.966	.500
1939-1948	.173	-.562	.963	.974	.967	.327
1948-1957	.747	.361	.990	.980	.986	.719
1929-1958	.705	-.424	.974	.957	.983	.784

TABLE 3.--Correlations between synchronous variables in real terms.[85]

Period	$r_{CA \cdot P}$	$r_{CA \cdot MP}$	$r_{CM \cdot P}$	$r_{CM \cdot AP}$	$r_{YM \cdot P}$
1897-1958	.157	.314	.878	.888	.901
1897-1908	.290	-.570	.911	.935	.910
1903-1913	.126	-.113	.918	.917	.757
1908-1921	-.673	-.443	.919	.880	.137
1913-1920	-.701	-.662	.863	.848	.059
1920-1929	.611	.190	.970	.954	.944
1921-1933	.611	.387	.956	.940	.917
1929-1939	.909	.807	.946	.887	.912
1933-1938	.442	.097	.952	.940	.896
1938-1953	-.513	-.472	-.342	-.261	-.010
1939-1948	-.904	-.929	.083	.505	.287
1948-1957	-.606	.203	.856	.771	.781
1929-1958	-.207	-.352	.222	.360	.485

Conclusion

The conclusions which FM draw from their results show they had no trouble selecting the "more stable relationship." This is evidenced by their following comments:

> The major implications of our findings are so obvious as to require little elaboration. For scientific analysis, they indicate that the quantity-theory approach to income change is likely to be more fruitful than the income-expenditure theory approach; that the first

> corresponds to empirical relations that are far more stable over the course of business cycles than the second. . . . For economic policy, our findings indicate that control over the stock of money is a far more useful tool for affecting the level of aggregate money demand than control over autonomous expenditures . . . [86]

FM conclude that ". . . it is what monetary policy does to the stock of money rather than what it does to interest rates that matters most."[87] Also, "changes in the stock of money have an effect on a much broader range of capital assets and correspondingly broader range of associated expenditure"[88] than is recognized by the income-expenditure approach.

Much of the procedure FM followed in this study has been subjected to criticism in the literature. Chapter III will fully present the objections others have found to this study as well as some criticisms not yet appearing in print and, so far as is known, are original with this author.

FOOTNOTES--CHAPTER II

[1]Milton Friedman and David Meiselman, "The Relative Stability of Monetary Velocity and the Investment Multiplier in the United States, 1897-1959," *Stabilization Policies* by E. C. Brown, *et al*. (A Series of Research Studies Prepared for the Commission on Money and Credit; Englewood Cliffs, N.J.: Prentice-Hall, 1963), pp. 165-268.

[2]Arthur S. Goldberger, *Topics in Regression Analysis* (New York: The MacMillan Co., 1968), p. 3.

[3]J. Johnston, *Econometric Methods* (New York: McGraw-Hill Co., Inc., 1963), pp. 9-11.

[4]Goldberger, p. 40.

[5]Johnston, p. 31.

[6]Friedman and Meiselman, p. 169.

[7]*Ibid*., p. 165.

[8]*Ibid*., p. 170.

[9]*Ibid*.

[10]*Ibid*.

[11]*Ibid*., p. 171.

[12]*Ibid*., pp. 168-619.

[13]*Ibid*., p. 169.

[14]*Ibid*., p. 174.

[15]*Ibid*., pp. 175-178.

[16]*Ibid*., p. 175.

[17]*Ibid*.

[18]*Ibid*.

[19] *Ibid.*

[20] *Ibid.*

[21] *Ibid.*, p. 177.

[22] FM consistently, when describing the equations to be tested, refer to induced expenditures as "consumption" and label this item with the letter C. This has caused a great deal of confusion as evidenced by the comments of the critics of FM. FM later acknowledged that the confusion was caused by their unfortunate choice of a substitute term for induced expenditures. In fact, the terminclogy even caused FM to stumble as we will demonstrate later in this chapter.

[23] Friedman and Meiselman, pp. 177-178.

[24] *Ibid.*, p. 178.

[25] *Ibid.*

[26] *Ibid.*, pp. 186-209.

[27] *Ibid.*, p. 174.

[28] *Ibid.*

[29] *Ibid.*, p. 175.

[30] *Ibid.*

[31] *Ibid.*, p. 190.

[32] *Ibid.*, pp. 234-241.

[33] *Ibid.*, p. 182.

[34] *Ibid.*

[35] *Ibid.*

[36] *Ibid.*

[37] *Ibid.*, p. 182. See, also, p. 242.

[38] *Ibid.*, p. 243.

[39]Each entry is the coefficient of determination. The signs in parentheses are the signs of the correlation coefficients. This table is reproduced from Friedman and Meiselman's Appendix Table II-A1, Experiments with Alternative Concepts of Money and Income, p. 244.

[40]Ibid., pp. 243-246.

[41]Ibid.

[42]Ibid., p. 246.

[43]Ibid., p. 243. See, also, p. 238.

[44]Ibid., pp. 182-183.

[45]Ibid., p. 183.

[46]Ibid., pp. 246-247.

[47]Ibid., p. 247.

[48]Ibid.

[49]Ibid., p. 249.

[50]Ibid.

[51]Ibid.

[52]Ibid.

[53]Ibid., p. 251.

[54]Ibid.

[55]Ibid.

[56]Ibid.

[57]Ibid.

[58]Ibid., p. 252.

[59]Ibid

[60]After the first draft of this paper was written, M. K. Lewis in a comment "Friedman and Meiselman and Autonomous Expenditures" in the American Economic Review, LVII (June, 1967), also points out the inconsistencies in the FM procedure. Lewis claims that "a correct application

of their [FM] criteria for selection provides support for the definitions they used" (p. 542). Since in the next chapter we are going to show the FM criteria are totally invalid in the first place, we will not concern ourselves with Lewis's detailed criticism of FM's application of these criteria.

[61]Friedman and Meiselman, p. 252.

[62]See p. 28 above.

[63]Friedman and Meiselman, p. 254.

[64]Friedman and Meiselman, p. 254.

[65]*Ibid.*

[66]*Ibid.*

[67]*Ibid.*

[68]*Ibid.*

[69]*Ibid.*, pp. 254-255.

[70]*Ibid.*, p. 257.

[71]*Ibid.*, p. 255.

[72]*Ibid.*, p. 257.

[73]*Ibid.*, p. 255.

[74]*Ibid.*, p. 256.

[75]*Ibid.*

[76]*Ibid.*, p. 186.

[77]*Ibid.*, p. 189.

[78]*Ibid.*, pp. 204-205.

[79]*Ibid.*, p. 204.

[80]*Ibid.*

[81]*Ibid.*, p. 207.

[82]*Ibid.*

[83]*Ibid.*

[84]This table is reproduced from FM's Table II-1 on p. 190.

[85]This table is reproduced from FM's Table II-3 on p. 228.

[86]Friedman and Meiselman, p. 213.

[87]*Ibid.*, p. 216.

[88]*Ibid.*, p. 217.

CHAPTER III

THE CRITICS OF FM

Introduction

In this chapter we shall review the criticisms of the FM analysis contributed by Donald Hester,[1] Albert Ando and Franco Modigliani[2] (hereafter referred to as AM), and Michael DePrano and Thomas Mayer[3] (hereafter referred to as DM). The three sets of authors frequently offer similar criticisms. We shall try to avoid repetition by including in our review of AM only those criticisms not fully discussed by Hester and in our review of DM only those criticisms presented by neither Hester nor AM.

FM have responded to the objections raised by their critics;[4] the critics, in turn, have issued rejoinders.[5] We shall include the pertinent remarks from these exchanges as we proceed, rather than place them in a separate section. Along the way, we shall also interject our own criticisms, comments, and interpretations.

Hester's Analysis

Objections to the FM Analysis

Hester limits his comments to FM's equations (1') and (2')[6] and, therefore, to only the first of the four comparisons listed above.[7] He objects to the national income and product account categories FM included in their concept of autonomous expenditures and to the "criteria" (i.e. conditions (i) and (ii)[8]) FM used to choose these categories.

Definition of Autonomous Expenditures

Hester submits that T and I "are not likely to be exogenous"[9] and that, therefore, A [= K + (G-T) + (E-I)] contains endogenous elements. Setting aside the fact that imports may be endogenous, Hester argues that if T is induced, the proper autonomous concept is L where L = K + G + (E-I). He then proceeds to show that it is possible for r_{CL} to take on a "high value"[10] without a correspondingly high value being assigned to r_{CA}.[11] Thus, if the proper autonomous item is L and not A we could arrive at a high value of r_{CL} which would support the "Keynesian" income-expenditure theory and a low value of r_{CA} which would reject its importance. Since FM used A as their measure of autonomous expenditure and since L is a more reasonable definition, Hester states that:

"Friedman and Meiselman have stacked the cards against the Keynesian model in their comparisons by ignoring the fact that taxes are a function of income."[12]

Hester considers next the dependency of imports on income and says that "by an argument completely analogous to that for taxes, failure to eliminate imports from L will serve to misrepresent the autonomous expenditure model."[13] Thus another possible and more "appealing" measure of autonomous expenditure, Hester feels, is L' where L' = G + GPDI + E = L + I + D where D is capital consumption allowances. Hester adds D to K because he feels that gross private domestic investment is more accurately measured by the national income statisticians than is K. This is because K is computed by subtracting D from gross investment and D is only an "imperfect approximation for actual depreciation."[14] Later he states that

> while in principle net investment is the ideal concept, it is well known that depreciation measures are highly imperfect and that measurement errors bias correlation coefficients toward zero. The fact that we don't know how to measure depreciation is not grounds for rejecting an autonomous expenditure theory.[15]

Hester proposes two more revisions in the autonomous concept. First, "spurious correlation exists between $\bar{M}$ [our I] and C"[16] since "part of imports is included in consumption."[17] Therefore, Hester subtracts imports from L' which results in a new concept, L'', equal to G + GPDI + E - I. Since L'' = L' - I and since L' = L + I + D, Hester defines L'' = L + D. Therefore, Hester claims the "spurious correlation" of imports with consumption will be removed by correlating C with L''.[18] This, of course, is nonsense! L' does not contain the variable I; subtracting I from L' re-introduces imports into the autonomous concept and simultaneously, therefore, re-introduces the so-called "spurious correlation" with C. In this case, Hester has done precisely the opposite of what he claims to have done. The second revision Hester proposes is L'''. L''' is defined as L'' minus inventory investment.[19] This adjustment is made because Hester feels it is proper to assume that inventory investment (which is included in GPDI) is endogenous; that is, "variations in consumption may cause negative variations in inventories."[20]

FM's reply.--In response to Hester, FM point out that their initial treatment does allow both T and I to be endogenously determined. FM contend that (G-T) and (E-I) are exogenous on their own account, with T and I each being endogenous (both are functions of income) and G and E each being "mixed" categories, i.e. "'the sum of an induced and autonomous item.'"[21] FM feel that "'foreign countries . . . spend on U. S. goods . . . [a certain amount which is determined exogenously] . . . plus what they earn for (U. S.) imports.'"[22] As for government e

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expenditures, FM say "we can regard the government as deciding that total expenditures shall equal what is raised by taxes plus (or minus) a specified sum to be financed by borrowing (or used to repay debt)."[23]

If we apply FM's rationale for defining a variable as falling into a "mixed category" to, for instance, consumption, we easily discover the absurdity of such a definition. The consumption function, $C = \alpha + \beta Y$, is the sum of an "autonomous" amount, α, and an induced amount, βY. Therefore, to be consistent with their above treatment of exports and government expenditures as "mixed," FM would have to regard consumption as "mixed" as well. Consumption could be considered as endogenous, according to their reasoning, only if the consumption function were $C = \beta Y$. Clearly, if a variable is influenced by other variables specified in the set of structural equations, it must be considered as an endogenous variable. If it is not endogenous, it must be exogenous since it is then completely determined outside of the model. All this is not to say that a model could not be constructed which would treat, for instance, the government deficit as exogenous while simultaneously treating G and T each as endogenous variables; but it seems it would be necessary to include in the model some type of "decision" or "reaction" functions which would explain how the government wi

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will behave in order to keep the deficit at its exogenously determined level.

FM attempt to show that their model is consistent with treating T as endogenous. This is done by defining the consumption function as $C = a + bY$ and defining income as $Y = C+A$ where $A = K + (G-T) + (E-I)$. These two equations form a "complete model";[24] the reduced forms for C and Y are

$$C = \frac{a}{1-b} + \frac{b}{1-b} A \text{ and } Y = \frac{a}{1-b} + \frac{1}{1-b} A.$$

FM then argue that "the value of T is not required for the solution";[25] but, setting $T = f + gY$, T can be derived as

$$T = (f + \frac{ga}{1-b}) + \frac{g}{1-b} A.$$

According to FM, "these equations demonstrate that, contrary to Hester's assertions, there is no inconsistency between our model and the treatment of taxes as induced."[26] This argument does not seem to be correct, since T enters the variable A. A is then equal to $K + G - f - gY + E-I$. Thus, A is induced and the FM reduced forms are not reduced forms at all. The model FM present is inconsistent with taxes being endogenously determined and A exogenous.

FM respond to Hester's demonstration that the correlation coefficient between C and L will be at least as great as the correlation coefficient between C and A with the assertion that "since $L = A + T$, it is easy to see that r_{CL} is the correlation of C with A plus part of itself and hence will be larger than r_{CA} if $r_{CA} < 1$."[27] This is disturbing on two counts. First, this statement is inconcsistent with FM's remark that "the value of T is not required for the solution" to the system of equations discussed in the above paragraph because they are now arguing that A, in fact, includes T. This verifies that our argument in the above paragraph is a proper one. Secondly, since $L = A + T = K + G + E - I$, it is not true that L is the sum of any item "plus a part of itself." L, in fact, only removes a variable previously included in the definition of autonomous expenditures. It seems FM refuse to admit the obvious fact that the government deficit plus taxes must equal government expenditures.

FM state that "we examined explicitly all but one of the alternative definitions he [Hester] proposes (we did not consider L''', which excludes inventory investment); . . . we presented statistical evidence on each; and . . . we explained why the evidence seemed to favor the concept we finally used."[28] Since FM based their choice of the definition of autonomous expenditure on the criteria presented above, we proceed to Hester's criticism of these conditions.

Criteria for Defining
Autonomous Expenditures

Regarding the criteria (conditions (i) and (ii)) FM used to decide which national income account categories to include in the concept of autonomous expenditures, Hester writes:

> Suppose there exist two doubtful components of autonomous expenditures, G and H. Somehow I is known to be autonomous. Then Friedman and Meiselman argue that a necessary condition for G to be autonomous is that $r_{C(I+G)} > r_{CI}$ and r_{CG}. Suppose in fact G is autonomous. Assume H is also autonomous and negatively correlated with G, but independent of I. In this case, r_{CI} may exceed $r_{C(I+G)}$ and G will be erroneously rejected as autonomous. Their test is sensitive to the variances and covariances of I, G, and H. The Friedman-Meiselman test is ill-suited for its task; components of autonomous expenditure will not be reliably selected by their procedure. Theory or "intuition" is necessary to specify components of autonomous expenditure.[29]

Some remakrs on Hester's criticism.--Hester is correct in his assertion that FM's criteria are invalid. This holds true not just in the case of finding a definition of autonomous expenditures, but also in the FM attempt to define the money supply.

We have shown above[30] that when FM consider a definition for money they state that if M and T are perfect substitutes then the condition, $r_{Y(M+T)} > r_{YM}$ and r_{YT}, holds. They argue that they should, therefore, test to see whether the condition holds. If it does they will claim M and T are perfect substitutes (implying both items can be considered as money). If the condition does not hold, then M and T are not perfect substitutes and T cannot be accepted as part of the money supply.

This method is invalid. If the condition is found to be satisfied, we cannot say anything about the substitutive nature of M and T. The condition, $r_{Y(M+T)} > r_{YM}$ and r_{YT}, is not a sufficient condition for M and T to be perfect substitutes. Therefore, even if the condition is satisfied, M and T may or may not be perfect substitutes. We will call the statement "M and T are perfect substitutes" p; and call the statement "the condition holds" q. The statement "M and T are not perfect substitutes" will be denoted by ~ p, and "the condition does not hold" we will call ~ q. The conditional statement "if p then q" is not equivalent to the converse "if q then p." The contrapositive "if ~ q then ~ p" is equivalent to "if p then q." Therefore, if the condition $r_{Y(M+T)} > r_{YM}$ and r_{YT} is not satisfied we can say that M and T are not perfect substitutes; but we can say this only if the original conditional statement is true. However, FM are completely incorrect in treating the conditional statement as equivalent to the converse.

We can now interpret Hester's criticism of FM's condition (i) concerning the definition of autonomous expenditures. (Condition (i) is directly analogous to the single condition for money.) Hester is saying that since the element we are testing can be autonomous even though condition (i) is not satisfied, the contrapositive and, therefore, the original conditional statement is not true. That is, he argues that G being autonomous is not sufficient for condition (i) to hold; or, in other words, it is not true that a necessary condition for G to be autonomous is that $r_{C(I+G)} > r_{CI}$ and r_{CG}. Therefore, he concludes that the FM experiments are based on an invalid statement causing an invalid statistical definition of autonomous expenditures.

Hester only argues that the conditional statement is not true. In Appendix A we offer a formal proof that A_1 and A_2 being perfect substitutes as autonomous expenditures does not imply that condition (i) holds. Obviously, this proof will also show, simultaneously, that the conditional statement for M and T is not true either.

Hester's Tests and Results

Hester's battery of tests is not very elaborate. It involves computing (using annual data) the correlation coefficients between C and each of the four definitions of autonomous expenditures which he feels are improvements upon FM's A. We repeat them here in order to aid the continuity of presentation. They are:

a) L = G + K + E - I

b) L" = G + GPDI + E

c) L'' = G + GPDI + E - I

d) L''' = G + GPDI + E - V

where V is change in inventories.

The correlation coefficients were computed for various subperiods between 1929 and 1958. We reproduce Hester's table which presents the results of his tests.

TABLE 4.--Correlations between consumption and the money supply and various measures of autonomous expenditure.[31]

Years	r_{CM}	r_{CA}	r_{CL}	$r_{CL'}$	$r_{CL''}$	$r_{CL'''}$
1929-1939	.912	.937	.903	.957	.933	.976
1933-1938	.991	.935	.995	.992	.997	.997
1938-1953	.958	.397	.755	.837	.809	.817
1939-1948	.964	.173	.471	.566	.519	.527
1948-1957	.990	.756	.925	.964	.961	.969
1929-1958	.974	.706	.915	.953	.943	.949

Hester finds, in general, that his four definitions performed better than the FM definition in the sense that "with the exception of the 1929-1939 period, the correlation between consumption and every proposed measure of autonomous expenditure exceeds r_{CA} as expected."[32]

Hester concludes his paper with an evaluation of the scope of the FM analysis: ". . . as both autonomous expenditure and quantity models predict a high correlation between Y and M (and hence C and M), high correlation between the money supply and consumption are of little value in discriminating between the models."[33]

FM's Reply

It is FM's contention that "the appearance of substantial difference between his [Hester's] results and ours derives primarily from the shorter period his calculations cover."[34] FM point out that Hester's alternative measures of autonomous expenditures display higher correlation coefficients than does the money supply only for periods 1929-1939 and 1933-1938.[35]

AM's Analysis

Critique of FM's Study

FM's Treatment of the Autonomous Expenditures Model

AM assert that the FM results from testing the correlation coefficients between autonomous expenditures and consumption are "irrelevant."[36] This irrelevancy, AM submit, arises from a misspecification of the consumption function; the inclusion of the war years, 1942-1946, in three of the six subperiods after 1929; the inclusion of induced components in the explanatory variable which results in a least squares bias; and the combination of exogenous variables to form a single variable.

<u>Misspecification of the consumption function</u>.--AM claim the misspecification of the FM consumption function is due to the structural equation

$$\text{(a) } C = a + bY + \delta$$

where δ = random disturbance.

We recall that equation (a) along with

$$\text{(b) } Y = C + A$$

leads to the reduced form equation for C

$$\text{(c) } C = \frac{a}{1-b} + \frac{b}{1-b} A + \frac{1}{1-b} \delta.$$ [37]

AM object to equation (a) and say it should be replaced by the more "conventional" consumption function

$$\text{(a') } C = a + bY^d + \delta$$

where Y^d = disposable income.

So far, this is the same argument as Hester's. However AM also replace (b) with

$$\text{(b') } Y^d = C + S.$$

Equations (a') and (b') yield a new reduced form equation for consumption

$$\text{(c') } C = \frac{a}{1-b} + \frac{b}{1-b} S + \frac{1}{1-b} \delta.$$ [38]

AM then argue that since $S = A - R - H - W - T_f$, the FM reduced form is valid only if Y^d is replaced by $Y^d + R + H + W + T_f$ in equation (a'). "But this surely involves a grievous misspecification of the consumption function . . . "[39]

The war years.--AM claim that during the war years

> . . . consumers may have been persuaded to consume abnormally small proportions of their income for patriotic reasons, and/or they may have changed their consumption habits in response to rationing and to unavailability of some goods. Hence any test including these years is worthless unless it has been shown that the results are largely invariant whether these years are included or omitted.[40]

AM tested four regression equations to discover whether this invariance existed. One equation regressed A on C for the 1929-1958 period; the second regressed A on C for 1929-1958 exclusive of the years 1942-1946; the third regressed M on C for 1929-1958; and the fourth regressed M on C for 1929-1958 exclusive of 1942-1946. The coefficient of determination between A and C rose from 0.49 to 0.92 when the war years were excluded; the R^2 between M and C changed from 0.94 to 0.98 when the same years were not included.[41] Because of the differences in the R^2's for the non-war period and the total period, AM claim "the omission of these years [1942-1946] makes an overwhelming difference."[42]

FM discount AM's results since FM based their conclusions on the results for shorter subperiods rather than on the period as a whole.[43] However, the lowest correlation coefficients FM obtained were for the periods 1938-1953 (r_{CA} = .397) and 1939-1948 (r_{CA} = .173) both of which include war years.[44]

Induced components in the explanatory variable.--The third objection AM raise concerns the explanatory variables A in equation (c) and S in equation (c'). Dropping H and W from the definition of personal saving and re-writing S as K + G + E - [T+I+R], AM argue that S (and, also, A) cannot be considered autonomous because the variables T, I, and R are not ". . . uncorrelated with the residual error of the consumption function . . . ."[45] AM decide to

> . . . call "autonomous" those variables that are expected to be uncorrelated with the error term of the test equation under consideration, and call "induced" all other variables. Autonomous variables in this sense are not necessarily "exogenous" in the usual sense of being determined entirely outside the economic system and therefore uncorrelated with the error term of any structural equation. Thus exogenous variables are autonomous, but not all autonomous variables are necessarily exogenous.[46]

Thus, AM conclude:

> The three components in the square brackets could not possibly be regarded as autonomous in the sense defined above. The movements of each of these three components are closely related to that of consumption (either directly as in the case of imports or through income as in the case of taxes) which in turn is clearly related with the error term ε [our δ] of the consumption function. Since S thus includes items correlated with ε [emphasis mine], it will in general be

> itself correlated with ε. It is well known that under these conditions direct regression of C on S will yield biased estimates of the coefficients as well as of the variance of the error term.[47]

Two comments are necessary before proceeding. First, by defining as autonomous "those variable expected to be uncorrelated with the error term of the test equation under consideration," AM are allowing lagged endogenous variables with no autocorrelation to be defined as autonomous. Secondly, AM's discussion of items being related to C and therefore with ε is valid only if "related" is synonomous with "dependency." The term, related, cannot mean merely that the components are correlated with consumption. For instance, we could assume G to be exogenous and find that G is also highly correlated with C; but the correlation with C would not imply that G was also correlated with ε.

AM argue, further, that since ε will be positively correlated with I and T, fluctuations in ε will tend to be negatively correlated with fluctuations in S (and also A). Thus, the coefficients of the regression equations associated with (c) and (c') will be biased downward. This may cause "the regression coefficient of S (or A) on C and hence also the correlation coefficient . . . [to be] zero or even negative . . . "[48] The relationship between R^2 and the coefficient of S (or A) in the regression equation is

$$R^2 = \left(\frac{b}{1-b}\right)^2 \cdot \frac{\Sigma(S_i-\bar{S})^2}{\Sigma(C_i-\bar{C})^2}$$

where S_i and C_i are the observation on S and C respectively and $\bar{S}$ and $\bar{C}$ are the corresponding means of the sample values.[49] Clearly the smaller $(\frac{b}{1-b})$ is in absolute value, other things remaining unchanged, the smaller R^2 is. However, the SLS estimate of $(\frac{b}{1-b})$ is equal to $\frac{\Sigma(S_i-\bar{S})(C_i-\bar{C})}{\Sigma(S_i-\bar{S})^2}$; it is extremely difficult to consider a smaller $(\frac{b}{1-b})$ without simultaneously allowing $\frac{\Sigma(S_i-\bar{S})^2}{\Sigma(C_i-\bar{C})^2}$ to vary as well. Therefore, AM's last statement is not obviously true.

Combining exogenous variables.--AM point out that the various exogenous variables FM consider as components of A, namely (G-T), K, and (E-I) can be combined to form A only if the coefficients of each of these variables explaining induced expenditures are equal to each other.[50]

The veracity of this statement can be illustrated by considering

$$(1)\ U_t = \beta_0 + \beta_1 A_{t1} + \beta_2 A_{t2} + \varepsilon$$

to be the true model, where A_{t1} and A_{t2} are two exogenous items which FM include in their one variable, A. Assume the estimated equation is

$$(2)\ U_t = b_0 + b_1(A_{t1}+A_{t2}) + t^*.$$

The estimates of b_1, $\hat{b}_1$, is

$$(3)\ \hat{b}_1 = \frac{m_{U(1+2)}}{m_{(1+2)(1+2)}} = \frac{\sum_t[(A_{t1}+A_{t2})-(\overline{A_1+A_2})](U_t-\bar{U})}{\sum_t[(A_{t1}+A_{t2})-(\overline{A_1+A_2})]^2}\ .$$

Since the number of observations of A_1 equals the number of observations of A_2, $\overline{A_1 + A_2} = \bar{A}_1 + \bar{A}_2$. Thus,

$$(4)\ \hat{b}_1 = \frac{\sum_t(A'_{t1} + A'_{t2})\ U'_t}{\sum_t(A'_{t1} + A'_{t2})^2}$$

where $A'_{ti} = A_{ti} - \bar{A}_i$ $\quad i = 1, 2$ and
$\bar{U} = U_t - \bar{U}$.
To find whether $\hat{b}_1$ is an unbiased estimator of β_1 and β_2, we calculate its expected value, $E(\hat{b}_1)$:

$$(5)\ E(\hat{b}_1) = \frac{1}{\sum_t(A'_{t1}+A'_{t2})^2}\ E[\sum_t(A'_{t1}+A'_{t2})U'_t]$$

$$= \frac{E[\sum_t A'_{t1}(\beta_1A'_{t1}+\beta_2A'_{t2}+\varepsilon'_t) + \sum_t A'_{t2}(\beta_1A'_{t1}+\beta_2A'_{t2}+\varepsilon'_t)]}{\sum_t(A'_{t1} + A'_{t2})^2}$$

$$= \frac{\beta_1(\sum_t A'_{t1}{}^2+\sum_t A'_{t1}A'_{t2}) + \beta_2(\sum_t A'_{t1}A'_{t2}+\sum_t A'_{t2}{}^2)}{\sum_t A'_{t1}{}^2+2\sum_t A'_{t1}A'_{t2}+\sum_t A'_{t2}{}^2}\ .$$

Therefore, $\hat{b}_1$ is a biased estimator of β_1 (i.e. $E(\hat{b}_1)-\beta_1 \neq 0$) unless $\beta_1 = \beta_2$. (Similarly, $\hat{b}_1$ will also, in general, be a biased estimator of β_2).

In Appendix B we discuss the comparative sizes of the R^2's associated with equations (1) and (2). We also consider the relative effects on the R^2 of equation (2) when positively correlated variables are added to form A versus the case in which negatively correlated variables form the composite. The latter consideration is necessary in light of AM's comment in the preceding section that FM's procedure biases the income expenditure correlations downward. The former is of interest since FM are more concerned with the correlation coefficient of equation (2) than with $\hat{b}_1$.

<u>AM's Autonomous Expenditure Model</u>

AM construct a model based upon a "conventional elementary form of the consumption function given by

$$[(6)]\ C = c_0 + c_1\, Y^d + \varepsilon.\text{"}^{51}$$

They define Y^d as

$$(7)\ Y^d \doteq C + S \equiv Z^a + X^a + Z^i + X^i$$

where Z^a and X^a are defined to be autonomous[52] and

$$Z^a = K_1 + G + E$$

$$X^a = T_{b2} + T_i + T_{g2} + Q - H - W$$

where K_1 = net investment in plant and equipment and in residential houses
G = government purchases of goods and services
E = exports
T_{b2} = property tax portion of indirect business taxes
T_i = net interest paid by government
T_{g2} = government transfer payments minus unemployment insurance benefits
Q = subsidies less current surplus of government enterprises
H = statistical discrepancy and
W = excess of wage accruals over disbursements.[53]

AM express the induced components Z^i and X^i as linear functions of the following variables: Y^d, C, X^a, and Z^a; X^i is also allowed to be a linear function of Z^i and Z^i is correspondingly permitted to be a linear function of X^i. According to AM, this yields the reduced form:

$$(8)\ C = \alpha_0 + \alpha_z Z^a + \alpha_x X^a + \varepsilon^*.$$

AM consider the possibility that $\alpha_z = \alpha_x$ and re-write (8) as

$$(9)\ C = \alpha_0' + \alpha_a' (Z^a + X^a) + \varepsilon^{**}.$$

Both (8) and (9) are tested for the period 1929-1958 (excluding the war years, 1942-1946) using annual series.

AM then state that if the consumption function were

$$(10)\ C_t = c_0 + c_1 Y_t^d + c_2 C_{t-1}$$ [54]

the reduced forms (8) and (9) would become

$$(11)\ C = \beta_0 + \beta_z Z^a + \beta_x X^a + \beta_c' C_{t-1} + \varepsilon'$$

$$(12)\ C = \beta_0' + \beta_a'(Z^a+X^a) + \beta_c' C_{t-1} + \varepsilon''.$$

Equations (11) and (12) are tested for the same period as equations (8) and (9). AM also carry out similar tests replacing C with $C^f = C + Z^i$ in equations (8), (9), (11), and (12), where $Z^i = K_2 - I$
$K_2 = K - K_1$ and
I = imports.[55]

This second set of tests AM claim "were inspired by a criticism kindly offered by Friedman to an earlier draft of this paper."[56] Since

> . . . the basic issue with which FM are concerned . . . is which of the two models [quantity theory of money versus income expenditure approach] does a better job of accounting for the behavior of a broad measure of income such as NNP . . . [and since NNP = $C + Z^i + Z^a$] . . . Friedman has . . . suggested that . . . the dependent variable in our tests should not be C, but rather the entire induced component, i.e. $C + Z^i$.[57]

The results of AM's tests are reproduced below:

TABLE 5.--Multiple correlation coefficients and unexplained variances of equations 8, 9, 11, and 12 with C and C^f as dependent variables.[58]

Equation	Using C as Dependent Variable		Using C^f as Dependent Variable	
	R^2	S_e^2	R^2	S_e^2
8	.995	41	.995	37
9	.992	69	.991	66
11	.998	16	.996	29
12	.997	23	.994	43

Variance of C = 7515
Variance of C^f = 6584

Recalling that when A is used for the autonomous concept for 1929-1958 (excluding the war years) the corresponding R^2 of equation (8) is .92 with $S_e^2 = 601$, AM comment that the results in Table 5 show "that replacing FM's variable A with the more relevant variable $X^a + Z^a$ . . . reduces the unexplained variance [of C, i.e. S_e^2 is reduced] from roughly 600 to 69, a reduction of nearly 90 per cent."[59] They note also that "the variance left unexplained by . . . [(8)] is less than half as large as that left unexplained by FM's money equation for the same period [the S_e^2 for the latter equation = 174; the corresponding $R^2 = .98$[60]]."[61] Alternative definitions of autonomous spending improve the fits with C since they all reduce the unexplained variance of C.

At this point we interject AM's rationale for comparing the S_e^2's rather than the R^2's. AM argue that

> even in terms of the issue posed by FM--which of the two "rival" models is more successful in accounting for the movement of NNP, given the relevant autonomous variables--the relevant measure of "success" is <u>the variance of the residual error and not the correlation coefficient</u> [emphasis mine], which depends on the ratio of this variance to another variance, and which can be radically changed by a mere transformation of variables in many cases.[62]

AM claim that

> for each of the equations containing the dependent variable lagged [i.e. C_{t-1} in this case], we can drastically reduce the correlation coefficient by changing the dependent variable to a first difference, without thereby changing the error variance or the estimates of the regression coefficients.[63]

AM fail to point out however that the "variance of the residual error" can be "radically changed" with even less effort than a transformation of variables. All that is needed is to express either an independent variable or the dependent variable in different units, say millions of dollars instead of billions. This will not alter the corresponding coefficient of correlation, however. AM's second statement is also open to question. Consider their equation (12):

$$(12)\ C_t = \beta_0' + \beta_a' (Z^a+X^a)_t + \beta_c' C_{t-1} + \varepsilon''.$$

AM submit that they can "drastically reduce the correlation coefficient by changing the dependent variable to a first difference" without altering S_e^2. To change the left hand side of (12) to a first difference we must transform (12) into:

$$(13)\ \Delta C_t = C_t - C_{t-1} = \beta_0^* + \beta_A^* (Z^a+X^a)_t + \beta_c^* C_{t-1} + \varepsilon'''.$$

Using the general formula which related R^2 to S_e^2:

$$(14)\ R^2 = 1 - \frac{S_e^2}{S_T^2}$$

where S_T^2 is the total sum of squares of the dependent variable we see that in order for S_e^2 to be of equal value in equations (12) and (13), the following must be true:

$$(15)\ S_{T_c}^2\ (1-R_c^2) = S_{T_{\Delta c}}^2\ (1-R_{\Delta c}^2).$$

It is not obvious that this will necessarily be the case when (12) is replaced by (13).

FM's reply.--FM offer several rebuttals. They argue that AM: (1) give no reason for using the autonomous expenditure variable, Z^a+X^a;[64] (2) do not once consider the criteria FM set up to choose among possible candidates for autonomous expenditures;[65] (3) are preoccupied with searching for the highest correlation coefficients;[66] (4) use different periods (i.e. they only look at 1929-1958) than FM do;[67] and (5) present an irrelevant and "unnecessary" analysis since it is based on a consumption function(s) that FM did not assume.[68]

Criticism of FM's Equations Involving the Money Supply

AM contend that the demand for money function "implicit in the FM tests"[69] is

$$(16)\ M^D = g_1N + g_0 + \varepsilon$$

where N = net national product.[70] AM point out that, in other words, Friedman has specified the demand for money function to be

$$(17)\ \frac{M^D}{P\pi_p} = \gamma \left(\frac{N_p^*}{P\pi_p}\right)^S + \eta^*\ ^{71}$$

where N_p^* = permanent net national product
P = population
π_p = permanent price level.

Assuming $\delta = 1$, $\gamma = {}_1$, $\gamma_0 = 0$, and $\eta = \eta^*\ P\pi_p$, AM derive the following from (17):

$$(18)\ M^D = \gamma_1\ N_p^* + \gamma_0 + \eta.$$

AM use as an approximation of N_p^*

$$(19)\ "N_{pt} \equiv \beta\ (1-\rho) \sum_{\gamma=0}^{\infty} \rho N_{t-\gamma}$$

where β is an adjustment factor for the time trend in N, a number slightly greater than unity. Friedman thinks that ρ is about .7."[72] Substituting for N_p^* into (18) and assuming that

$$(20)\ M_t^D = M_t$$

where M_t = currency plus demand deposits adjusted, AM derive

$$(21)\ N_t = \delta_0 + \delta_1\ M_t + \delta_2\ N_{p,t-1} + \eta'.^{73}$$

AM claim

> in order to make their test at least roughly consistent with Friedman's own model of how money affects income through the demand for money, FM should have added to their test equation the

> variable $N_{p,t-1}$ and judged the importance of money from the over-all fit of this equation and the partial correlation of M.[74]

Another possible reason the FM tests might be biased, according to AM, "is that, under the institutional arrangements prevailing during the period covered by the tests, M was at least partly induced, and in consequence positively correlated with the error term of . . . [equation 16] . . . or . . . [equation 21]."[75] Therefore AM suggest replacing M with the variable

> M*, defined as the estimated maximum amount of money (in the conventional definition) that could be created by the banking system on the basis of the reserves supplied by the monetary authority (except in response to commercial bank borrowings), account being taken of reserve requirements and currency-holding habits.[76]

That is, AM define M* as

$$\text{"}M^* = \frac{L-B}{L-E} M,$$

where L denotes currency in circulation plus member bank deposits minus reserves against time deposits minus reserves against U.S. Government deposits (when required); B denotes member banks borrowings from the Federal Reserve; and E, member bank excess reserves."[77] According to AM, M* is likely to be "more nearly autonomous than M and . . . the substitution is therefore a step in the right direction."[78]

FM's reply.--FM have no objection to the theoretical judgments AM used in this section. They do, however, object to the specific definition of M* which AM chose.

FM interpret the "reserve requirements" mentioned in M* as legal reserve requirements. This item should be altered, FM argue, to banks' desired reserve requirements; for if L were legal reserve requirements and were equal to zero, then "M* would approach infinity."[79]

Tests of AM's "Quantity Theory" Model

AM do not test equation (21) in the precise form given above. The variable M_t is never used. Instead a series of tests are run with either M* or M^f (where M^f = "currency outside banks plus demand deposits adjusted, plus time deposite in commercial banks"[80]) as an independent variable. AM also run a set of tests with C_{t-1} substituted for $N_{p,t-1}$ and another set with C_t as the dependent variable. Sometimes AM regress the dependent variable on the money stock only; at other times two independent variables are included in the regression equation. Table 6 reproduces the R^2's and S_e^2's which AM found. All tests covered the single period 1929-1958 excluding the war years and all used annual data.

AM contend that the results of the equation using N_t as the dependent variable and M^f and $N_{p,t-1}$ as the independent variables along with the equation using N_t as the dependent variable and M^f and C_{t-1} as independent variables "are consistent with the view that FM's high correlations are somewhat misleading and that money affects income through a mechanism which is quite different from the simple one envisaged by Friedman . . ."[81]

AM point out also

> that whether one uses M* or the more dubious variable M [M^f], the variance of the error of prediction of NNP [a label AM have given to the residual variance], though modest, is still three to four times larger than that resulting from using Z^a and X^a, and four to five times larger than that resulting from using Z^a, X^a and C_{t-1} . . .[82]

TABLE 6.--Multiple correlation coefficients and residual variances for AM's various regression equations on the money supply, 1929-1958, excluding 1942-1946.[83]

Independent Variables in Regression Equation	Using N_t as Dependent Variable		Using C_t as Dependent Variable	
	R^2	s_e^2	R^2	s_e^2
M^f	.974	438		
M^f, $N_{p,t-1}$	.991	160		
M^f, C_{t-1}	.992	147	.997	25
M*	.939	1,102	.935	524
M*, $N_{p,t-1}$	.992	139		
M*, C_{t-1}	.993	122	.997	23

Variance of N_t = 16765 Variance of C_t = 7515

AM's statement that their results show FM's high correlations to be "misleading" is partially based on the fact that the partial correlations between M^f and N_t are "dramatically smaller" than the simple correlations [in one case $r_{N_tM^f \cdot N_{t-1}}$ = .527 and in the other $r_{N_tM^f \cdot C_{t-1}}$ = .29]. At this point AM should be reminded that these partial correlation coefficients show only the additional proportion of the total variance of N_t explained by M^f after the effect of N_{t-1} (in one case) or of C_{t-1} (in the other) on N_t has been taken into account. The partial correlations do not show the proportion of the variance in N_t explained by M^f holding the other indepdnent variable constant. Parenthetically, FM should also be reminded of the limited use to which partial correlations can validly be subjected. In Chapter II (p. 15) we have quoted FM's statement as to what they believe partial correlation coefficients show. Clearly, they have contributed too much causal significance to this statistic.

AM's comparison of the "error of prediction of NNP" is questionable. The dependent variable regressed on Z^a, X^a, and C_{t-1}, for instance, was C^f which had a variance for the period of 6584. The dependent variable used in the tests with the money supply (which AM compare with the variance of C^f) is N, which has a variance of 16765. Therefore, to say that "the error of prediction of NNP" when money is the independent variable is "four to five times larger" than that when Z^a, X^a, and C_{t-1} are the independent variables has no relevance since the variance of the dependent variable is different in the two cases. They would have moved to firmer ground had they compared only the R^2's.

AM's Appendix

In their appendix AM attempt to show that high multiple correlation coefficients of regressions of income on autonomous expenditures and of income on money are consistent with the "general Keynesian system."[84] They begin with the following nine equation model expressed in terms of the endogenous variables C, Y^d, Z^a, M, X^i, Z^i, C^f, N, and r; two exogenous variables, X^a and M^*; and one lagged endogenous variable, C_{t-1}:

$$C = C(Y^d, r, C_{t-1}) + \varepsilon_c^* \quad \text{(consumption function)}$$

$$Z^a = f(r, C_{t-1}) + \varepsilon_z^* \quad \text{(investment function)}$$

$$M = L(N, r, C_{t-1}) + \eta_d^* \quad \text{(demand for money function)}$$

$$M = B(r, M^*) + \eta_s^* \quad \text{(supply of money function)}$$

$$Z^i = z_y Y^d + z_c C + z_x X^a + z_z Z^a + z_i X^i + z_0 + \eta_z$$

$$X^i = x_y Y^d + x_c C + x_x X^a + x_z Z^a + x_i Z^i + x_0 + \eta_x$$

$$Y^d \equiv C + Z^a + X^a + Z^i + X^i$$

$$C^f \equiv C + Z^i$$

$$N \equiv C^f + Z^a$$

where r is the rate of interest.[85] All other variables have been defined earlier. AM then assume that all equations are linear homogeneous of degree one (i.e. all are linear and there is no constant term) and proceed to reduce the system to four equations in the four endogenous variables C^f, r, M, and Z^a:

$$(1)\ C^f - c_1 Z^a + C_2 r = c_3 C_{t-1} + c_4 X^a + \varepsilon_c$$

$$(2)\ Z^a + f_1 r = f_2 C_{t-1} + \varepsilon_z$$

$$(3)\ -l_1\ C^f - l_1 Z^a + l_2 r + M = -l_3 C_{t-1} + \eta_d$$

$$(4)\ -b_1 r + M = b_4 M^* + \eta_s$$ [86]

According to AM "the first two equations summarize the real part of the system while the last two relate the monetary side representing, respectively, the demand and supply equations for money,"[87] and will represent the "general Keynesian system."

AM consider two views of income determination. The first one is that money "matters not at all" which AM call the "Effective Demand Only (EDO)" model. The second is that "money only matters" which they label the "Money Only (MO)" model.[88]

AM argue that the EDO model's "essence is that effective demand, be it for consumption, capital formation, or government outlay for goods and services, is totally unaffected by interest rates (or directly by the money supply)."[89] Therefore, they argue, the EDO model is derived from the general Keynesian system by setting c_2 and f_1 equal to zero. These coefficients, AM claim, "provide the link between money and commodity markets."[90] AM discuss the derivation of EDO from the general model further by stating:

> It is readily apparent from . . . [equation 2] that under these conditions Z^a can be regarded as a predetermined variable [emphasis mine]. Similarly, we see from . . . [equation 1] that Z^a together with the other predetermined variables will determine C^f and hence also $C^f + Z^a$ up to a random term ε_c, independently of money supply M or M* [emphasis mine]. On the other hand EDO has no special implications for equations . . . [3 and 4] except that the coefficient . . . [l_1] in . . . [3] must definitely be positive (or at least nonzero), for else the rate of interest would not appear in the system at all and the system would be overdeterminate.[91]

Before presenting AM's adjustments to transform the general model into the MO model, we first note they have not made sufficient adjustments to derive an EDO model. What is important for the EDO model is that only the real sector is allowed to affect C^f. However, by not specifying that l_1 equal zero, C^f is still influenced by the supply of money. Secondly, if $f_1 = 0$ as AM specify it must be, equation (2) becomes $Z^a = f_2 C_{t-1} + \varepsilon_z$. This does not mean, however, that Z^a "can be regarded as a predetermined variable." If we specified a consumption function, for instance, to be $C_t = \beta Y_{t-1} + \varepsilon$ following AM's reasoning, C_t would be regarded as predetermined. But C_t is neither exogenous nor lagged in our consumption function. Thus, of course, it is not predetermined. The EDO model does not imply that Z^a is a predetermined variable. The proper statement would have been that AM were now <u>assuming</u> Z^a to be exogenous and also dropping equation (2).

In order to transform the general system into the MO model, AM argue:

> . . . the equation obtained after eliminating M by a simultaneous solution of the demand and supply equations must contain only the endogenous variable N. In terms of our model this means that . . . [l_2] of . . . [equation 3] and . . . [b_1] of . . . [equation 4] must both be zero. . . . Finally, we may note that the MO model has no special implications concerning . . . [equation 1] and . . . [equation 2] except that . . . [c_2] and . . . [f_1] cannot both be zero, for otherwise r would appear nowhere in the system, making it overdeterminate.[92]

The adjustment AM make to derive the MO model are also incorrect. The MO model, we submit, means that real variables such as X^a do not influence C^f or Z^a. However, AM do not specify coefficient c_4 to be equal to zero. Therefore, both X_a and M* influence C^f and Z^a. This is not a "money only matters" model after all.

What caused AM to make such specifications on the general model? We submit they made the statements they did in order to justify testing the equations reported in the body of their work. In their Appendix, AM write:

> [Equation 1] will be readily recognized as identical in form with . . . [our equation 12 on p. 82], which was used in our test of the income-expenditure theory . . . , except that the variable r was omitted. Hence, the term . . . [$-c_2r$] must be regarded as included in the error term of . . . [12]. Similarly, by solving . . . [equation 3] for $(C^f + Z^a) = N$, obtaining . . .

$$[\ N = \frac{1}{l_1} M + \frac{l_3}{l_1} C_{t-1} + \frac{l_2}{l_1} r - \frac{n_d}{l_1}\ ]$$

it is immediately apparent that this equation is identical in form with . . . [an equation] used in our test of the money model . . . , except again for the absence of the term in r which is therefore implicitly lumped with the error term. Finally, if we use . . . [equation 4] to eliminate M from . . . [equation 3] and solve the resulting equation for $C^f + Z^a$, we obtain . . .

$$[\ N = \frac{b_4}{l_4} + \frac{l_3}{l_1}\ C_{t-1} + \frac{l_2+b_1}{l_4} r + \frac{n_s+n_d}{l_1}\ ],$$

which clearly corresponds to . . . [another regression equation used in their test of the money model], except again for the omission of the r term.[93]

In order for equation (1) to contain only one dependent variable, Z^a must be autonomous and c_2 must equal zero. It is then the equation (12) on p. 82. In order for the two money equations AM mentioned in the above quote to be the ones AM tested in the body of their paper, l_2 and b_1 must equal zero. It seems highly coincidental that in order for the equations tested to be free from biased estimates, they need precisely the same assumptions AM made in deriving the EDO and MO models. However, as we have shown, the assumptions which AM made still do not free the equations tested from the simultaneous equations bias if these equations come from the "general model." It seems ironic that AM, after taking considerable space in the body of their paper to criticize FM for the simultaneous equations bias in their equation, would turn right around and commit the same error.

FM's reply.--FM attack the "general Keynesian model" proposed by AM. Namely, FM argue (a) "if the simple models are to be elaborated, the introduction of a price level seems like the first and most important elaboration that is required,"[94] (b) "the equations . . . [(1) to (4)] . . . that AM use as their approximation to a more complex and generaly system are not homogeneous because of the presence of the interest-rate terms. According to that system, doubling all flow variables and the stock of money would imply doubling the interest rate,"[95] (c) M* is "logically defective and . . . empirically misleading,"[96] and (d) C^f "has no clear economic significance."[97]

DM's Analysis

Criticisms of FM's Study

Misspecifications of the FM Model

DM confine their attention to the misspecifications in the equation FM tested as representative of the autonomous expenditures model. DM show that the single equation tested by FM can be derived as a reduced form from the following model:

$$C = a + bY^d$$

$$N = C + P + G + E - I$$

$$Y^d = N - T$$

$$T = \bar{T}$$

$$P = \bar{P}$$

$$G = \bar{G}$$

$$E = \bar{E}$$

$$I = \bar{I}$$ [98]

where all variables except P were defined above. P equals net private domestic investment. "Transfers, corporate retained earnings, and some minor items are neglected for simplicity."[99] The reduced form equation for C is

$$(1)\ C = \frac{a}{1-b} + \frac{b}{1-b}(\bar{P} + \bar{G} + \bar{E} - \bar{I} - \bar{T}).$$

This equation, DM claim, "seems to be that tested in the FM study by $C = \alpha + KA$, since $A = (\bar{P} + \bar{G} - \bar{T} + \bar{E} - \bar{I})$."[100] DM's objections to FM treating A as exogenous are based on arguments similar to those presented by AM and Hester. Namely, DM argue that neither inventory investment, the government deficit, nor the net foreign balance is exogenous.[101] Above, we have thoroughly discussed the possible resulting bias, if these variables are in fact endogenous.[102] DM also point to the possible bias resulting from combining variables to form a single one

even if all components are exogenous.[103] We reviewed this objection when we surveyed AM's study and specifically showed the bias that could result.[104]

The alternative model DM propose is

$$C = a + bY^d$$
$$N = C + P + G + E - I$$
$$Y^d + N - T$$
$$T = c + dN$$
$$G = \bar{G}$$
$$E = \bar{E}$$
$$I = e + fN$$
$$P = F + B$$
$$F = \bar{F}$$
$$B = g + hN$$

where B is inventory investment and F is private fixed investment. From this model DM derive the following reduced form equation for C

$$C = \alpha + \beta\ (\bar{F} + \bar{G} + \bar{E}).$$ [105]

Comment on FM's "Criteria"

DM protest the statistical device FM used to select the components included in A, C, and M. DM declare "this practice of using the same data, or roughly similar data, both to choose the definitions of variables (the definition being, of course, really part of the over-all

hypothesis) and to test the hypothesis is particularly suspect."[106] In their rejoinder DM offer further criticism of FM's procedure. They call attention to the fact that FM frequently found "inconsistent and ambiguous" or "somewhat confusing" results and "then fell back on vague (and incorrect) references to the usual treatment in the literature."[107] Also, DM point out that FM's _modus operandi_ entails deciding all items to be included in the definition save one and ask how FM decided these items in the first place. DM are skeptical whether "components which are themselves in doubt [can] be used to test other components."[108]

FM's reply.--FM attempt to defend themselves against DM's first objection to their criteria by saying that (a) they [FM] used the same data only for the 1929-1958 period, (b) "in effect, . . . [FM] used different information for the same years in deriving definitions and in testing hypotheses"[109] because they "did not use the highest correlation as a criterion,"[110] and (c) FM were unable to use independent data because none exists.[111]

Statistical Tests on FM's Definition of A

Next, DM undertake a series of tests which correlate various components of FM's A with consumption "to show how the correlation coefficient falls as one adds components which are not exogenous."[112] The tests were run for annual data and using first differences as well as levels of the variables for the 1929-1963 period. One set excluded the years 1942-1945; another set did not. DM begin by correlating C with plant and equipment investment. The test using levels of the variables and excluding the war years yielded a correlation coefficient equal to .979. Adding non-residential and residential construction expenditures to plant and equipment investment increased the correlation with C to .993. Introducing inventory investment to form gross private domestic investment, L, lowered R to .985. Correlating C with private domestic investment plus exports, L+E, increased R to .990; but when L+E-I was used, R fell back to .985. Adding the government deficit to L+E-I yielded a value of R = .992; but after substituting net private domestic investment, P, for L, P + E - I + G - T = A correlated with C resulted in R - .946.[113]

DM claim these results are consistent with their contentions that (a) FM's model is misspecified because of the inclusion of endogenous items (inventory investment, imports, and taxes) in A and (b) subtracting an item such as taxes which are endogenous from other components to form A "can give a downward bias to the correlation coefficient."[114] In Appendix B we show it is not clear that an item positively correlated with the dependent variable, but subtracted from other items to form the independent variable, will in fact lower the correlation coefficient. DM did not show what happens to the correlation coefficient between C and the independent variable by first adding G to L + E - I and then subtracting T from L + E - I + G. Instead, they went directly from the concept L + E - I to L + E - I + G - T. This procedure causes one to wonder whether including G and T in a step-wise fashion does not verify their proposition that "minus taxes" lowers the correlation coefficient.

DM's Tests of "Rival Hypotheses"

DM suggest testing several alternative hypotheses of the determination of consumption:

> 1. Consumption can best be explained by the stock of money where money includes time deposits.
> 2. Consumption can best be explained by autonomous expenditures defined as net investment in producers' durable equipment, nonresidential construction, residential construction, inventory changes, government deficit on income and product account, and net foreign investment. This is the FM interpretation of the Keynesian hypothesis.
> 3. Consumption can best be explained by autonomous expenditures defined as investment in producers' durable equipment, nonresidential construction, residential construction, federal government expenditures on income and product account, and exports. One variant of this hypothesis subtracts capital consumption estimates, and the other does not. This is our hypothesis.[115]

Each corresponding regression equation is tested for levels of the variables as well as in terms of first differences; but DM stress the results from the latter sequence of runs. DM decide to include tests using two other definitions of autonomous expenditures, namely, gross private domestic investment and gross private domestic investment plus exports. All equations are tested for 1929-1963 and various subperiods of this interval. Two subperiods (1938-53 and 1929-58) include the war years; three subperiods (1929-39, 1948-63, and 1953-63) do not.

This battery of tests on the rival hypotheses evoked the following conclusions:

> 1. For the whole period excluding the war years, both autonomous expenditures as we define them and money give good fits, with money somewhat better.
> 2. Including the war years lowers the correlation coefficient for money to some extent and reduces the correlation coefficients for our autonomous expenditures much more.
> 3. For the periods before and after the war, our autonomous expenditures do better than money for first differences, while results are mixed when the levels of the data are used.
> 4. For the subperiods which include the war, money does much better than our autonomous expenditures.
> 5. In all periods fixed private domestic investment and fixed private domestic investment plus exports do extremely well. . . .
> 6. FM's concept of autonomous expenditures, with few exceptions, does worse than any of the other variables.[116]

DM also ran several multiple regression equations for 1929-1963 first excluding the war years and then including them. Only the total period was used in this set of tests. The multiple regressions contained the independent variables (a) money and (b) some measure of autonomous expenditures. The same five alternative definitions of autonomous expenditures were combined with money in this series of tests as were used in the simple regression equations. Again, the equations were tested both using levels of the variables and with the variables expressed in terms of first differences. DM claim their results show that "money and autonomous spending together do 'explain' consumption a great deal better than money alone--or autonomous expenditures alone."[117]

Further Comments

If equation (5') on p. 15 is the "true" model of income determination, the so-called "competing" hypotheses of income determination could be examined by testing the null hypotheses that the regression coefficients of M and A in equation (5') are not significantly different from zero. If M and A are positively correlated, as FM suggest in the quote cited on p. 15, little more can be accomplished than testing for statistical significance. For as Goldberger points out "when orthogonality is absent [i.e. if M and A are correlated] the concept of the contribution of an individual regressor remains inherently ambiguous."[118]

While FM did estimate regression coefficients, the major thrust of their work rested on a comparison of correlation coefficients in order to decide which "competing" hypothesis to accept. For instance, FM estimated the correlation coefficients between M and C and A and C in equations (1') and (2') respectively. They found the former to be consistently greater than the latter. Next they computed the partial correlation coefficients between M and C and between A and C in equation (5'). Because the partial correlation coefficient between M and C was found to be consistently greater, FM declared the "disguised" effect of A on C (which operates through M) to be smaller than the "disguised" effect of M on C (which operates through A). This approach is unacceptable however for if equations (1') and (2') are misspecified the distributions of the coefficients of determination associated with these equations are unknown and we cannot test whether they are significantly different from each other. Also, if equation (1') were the "true" model and equation (2') were then misspecified, a comparison of R^2's between (1') and (2') could cause acceptance of the misspecified model. This arises because we do not know the distribution associated with R^2 in (2') and it may be possible for this distribution to have a mean which is greater than that for the distribution of R^2 associated with equation (1').

A few other comments on the FM approach may be made. FM argued that the price index should appear in equations (3), (4) and (6) as an added variable rather than as a deflator of the other regressors. The latter FM contend would yield spurious correlation. However, this question must be decided on a theoretical basis. If the correct specification of the model relates "real" values of Y, M, and A then the "deflated" values of these variables should be used. Another questionable aspect of the FM approach is their choice of the time periods used in the sample. A necessary assumption for statistical inference is that the disturbance terms in the equations be randomly distributed. However, choosing sample periods according to phases of the "business cycle" and with relatively few sample observations may preclude the disturbances from displaying this property.

Also, FM equate "goodness of fit" with "stability." Usually stability refers to the nature of the movement about an equilibrium. For instance, an equilibrium position is defined as "'stable in the small' if for sufficiently small deviations from equilibrium all the variables approach this position in the limit as time passes."[119] FM's use of the term relates to whether the values of the parameters in the regression equation may be expected to change. If these values are expected to remain unchanged, FM call the regression equation "stable." Finally, the basis for the FM decision to adopt M_2 as the "best" empirical definition of money is

not sound. M_2 is the sum of two components, money and time deposits. Therefore, the covariance between M_2 and Y is the sum of the covariances between money and Y and between time deposits and Y. Since we would expect both of these covariances to be positive, it is highly likely that the covariance between M_2 and Y will be greater than that between Y and either component.

Conclusion

In this chapter we have shown that the major objections raised by FM's critics concern FM's definition of autonomous expenditures, the "criteria" FM used to decide on this definition, and the simplicity of FM's implicit models.

Neither Hester nor FM presented an explicit formal model of income determination. AM and DM did present such models in appendices; but we have expressed our suspicion that the former was developed to justify the regression equations used in the body of the paper. The latter is a very simple one.

One main issue which seemingly remains unresolved in the exchange is the method to use to decide upon the variables treated as "autonomous" i.e. determined outside the system. FM remain convinced that "the 'sensible' measure of autonomous is an empirical . . . question"[120] and state that "all of us use words to describe it [i.e. autonomous expenditures]--like 'independent,' 'uncorrelated with the residual error,' 'exogenous'--that are figurative rather than operational."[121] They submit further that "however useful 'autonomous expenditures' may be as a theoretical construct, it is still far from having any generally accepted empirical counterpart."[122]

Hester and DM have both objected to the specific statistical "criteria" FM use. The extent of DM's criticism is that the criteria were tested for part of the same period used in testing the equations. They also question using unsettled components to decide on other items. Both Hester and the present author have shown FM's criteria do not imply FM's concept of perfect substitutes as autonomous expenditures. FM's reply to Hester is that he did not offer a statistical test to replace the incorrect one.

The basic point which Hester makes, namely, that "a model embodies a unique set of judgments"[123] lies as the very core of this issue of defining exogenous items. The method of statistical inference must take the structural economic equations and the variables specified as determined outside the system as given.

Appendix A

According to FM, A_1 and A_2 being perfect substitutes as autonomous expenditures means that a shift of one dollar from A_1 to A_2 will have no effect on Y.[124] In terms of a regression equation,

$$(1)\ Y = \beta_0 + \beta_1 A_1 + \beta_2 A_2 + \varepsilon.$$

this says that A_1 and A_2 have equal regression coefficients. Therefore, A_1 and A_2 can be combined into one variable to form the equation

$$(2)\ Y = \beta_0 + \beta_A\ (A_1 + A_2) + \varepsilon.$$

The SLS estimator of β_A of equation (2) is

$$b_{Y(A_1+A_2)} = \frac{\sum_t (A'_{t1} + A'_{t2})\ Y'_t}{\sum_t (A'_{t1} + A'_{t2})^2}$$

where

$$A'_{ti} = A_{ti} - \bar{A}_i \quad i=1,\ 2$$

$$Y'_t = Y_t - \bar{Y}$$

The coefficient of determination of (2), $R^2_{Y(A_1+A_2)}$, is therefore

$$R^2_{Y(A_1+A_2)} = \frac{b_{Y(A_1+A_2)} \sum_t (A'_{t1} + A'_{t2})^2}{\sum_t Y'^2_t}$$

$$= \frac{[\sum_t (A'_{t1}+A'_{t2})\ Y'_t]^2}{\sum_t (A'_{t1}+A'_{t2})^2 \sum_t Y_t'^2} .$$

Define the following:

$\sum_t A'_{t1}\ Y'_t = m_{Y1}$ $\qquad \sum_t A'_{t2}\ Y'_t = m_{Y2}$ $\qquad \sum_t Y_t'^2 = m_{YY}$

$\sum_t A_{t1}'^2 = m_{11}$ $\qquad \sum_t A_{t2}'^2 = m_{22}$ $\qquad \sum_t A'_{t1}\ A'_{t2} = m_{12}.$

Then,

$$R^2_{Y(A_1+A_2)} = \frac{(m_{Y1} + m_{Y2})^2}{(m_{11} + m_{22} + 2m_{12})\ m_{YY}}$$

$$\text{or (3)}\quad R^2_{Y(A_1+A_2)} = \frac{m^2_{Y1} + m^2_{Y2} + 2m_{Y1}\ m_{Y2}}{(m_{11} + m_{22} + 2m_{12})\ m_{YY}} .$$

The SLS estimators of β_1 and β_2 of equation (2) are respectively

$$b_{YA_1 \cdot A_2} = \frac{m_{Y1}\ m_{22} = m_{Y2}\ m_{12}}{m_{11}\ m_{22} - m^2_{12}} \qquad \text{and}$$

$$b_{YA_2 \cdot A_1} = \frac{m_{Y2}\ m_{11} - m_{Y1}\ m_{12}}{m_{11}\ m_{22} - m^2_{12}}.$$

If A_1 and A_2 are perfect substitutes, then $b_{YA_1 \cdot A_2}$ will equal $b_{YA_2 \cdot A_1}$. This implies $m_{Y1}\ m_{22} - m_{Y2}\ m_{12}$ must equal $m_{Y2}\,m_{11} - m_{Y1}\,m_{12}$ in order to validly replace equation (1) with equation (2). This condition may be rewritten as:

$$(4)\quad m_{12} = \frac{m_{Y2}\,m_{11} - m_{Y1}\,m_{22}}{m_{Y1} - m_{Y2}} \quad \text{if } m_{Y1} \neq m_{Y2}.$$

The simple correlation coefficients, r_{YA_1} and r_{YA_2}, by definition are respectively

$$r_{YA_1} = \frac{m_{Y1}}{\sqrt{m_{11}}\sqrt{m_{YY}}} \quad \text{and} \quad r_{YA_2} = \frac{m_{Y2}}{\sqrt{m_{22}}\sqrt{m_{YY}}}$$

where all square roots are positive.

We now set out to find whether A_1 and A_2 being perfect substitutes implies FM's condition that $R^2_{Y(A_1+A_2)}$ is greater than both r^2_{Y1} and r^2_{Y2}. We fully recognize that the actual statement by FM is that A_1 and A_2 being perfect substitutes implies $R_{Y(A_1+A_2)} > r_{YA_1}$ and r_{YA_2}; however FM also argue that all correlation coefficients with Y should be positive so that our condition is equivalent to theirs.

Using the definitions of r_{YA_1} and r_{YA_2} and (3) we see that:

$$(5)\quad \frac{R^2_{Y(A_1+A_2)}}{r^2_{YA_1}} = \frac{m^2_{Y1}\,m_{11} + m^2_{Y2}\,m_{11} + 2m_{Y1}\,m_{Y2}\,m_{11}}{(m_{11} + m_{22} + 2m_{12})m^2_{Y1}}$$

$$(6)\quad \frac{R^2_{Y(A_1+A_2)}}{r^2_{YA_1}} = \frac{m^2_{Y1}\ m_{22} + m^2_{Y2}\ m_{22} + 2m_{Y1}\ m_{Y2}\ m_{22}}{(m_{11} + m_{22} + 2m_{12})m^2_{Y2}}$$

We see that since the denominators in (5) and (6) must be positive,

$$\frac{R^2_{Y(A_1+A_2)}}{r^2_{YA_1}} \text{ is greater than 1 if and only if}$$

$$(7)\quad m^2_{Y2}\ m_{11} + 2m_{Y1}\ m_{Y2}\ m_{11} \overset{?}{>} (m_{22} + 2m_{12})\ m^2_{Y1}$$

while $\dfrac{R^2_{Y(A_1+A_2)}}{r^2_{YA_2}}$ is greater than 1 if and only if

$$(8)\quad m^2_{Y1}\ m_{22} + 2m_{Y1}\ m_{Y2}\ m_{22} \overset{?}{>} (m_{11} + 2m_{12})\ m^2_{Y2}.$$

We now substitute (4) into (7) and (8) respectively. If A_1 and A_2 being perfect substitutes does imply $R^2_{Y(A_1+A_2)} > r^2_{YA_1}$ and $r^2_{YA_2}$ then this substitution will yield results which are always true. Substituting (4) into (7) we have

$$(7')\quad m^2_{Y2}\ m_{11} - m^2_{Y1}\ m_{22} \overset{?}{>}$$

$$2m^2_{Y1}\left[\frac{m_{Y2}\ m_{11} - m_{Y1}\ m_{22}}{m_{Y1} - m_{Y2}} - 2m_{Y1}\ m_{Y2}\ m_{11}\right].$$

Assuming $m_{Y1} > m_{Y2}$, this yields

$$(7'') \; m_{Y2}^2 \, m_{11} \, m_{Y1} - m_{Y2}^3 \, m_{11} - m_{Y1}^3 \, m_{22} + m_{Y1}^2 \, m_{22} \, m_{Y2} \overset{?}{>} 2m_{Y1}^2 \, m_{Y2} \, m_{11} - 2m_{Y1}^3 \, m_{22} - 2m_{Y1}^2 \, m_{Y2} \, m_{11} + 2m_{Y1} \, m_{Y2}^2 \, m_{11}$$

which reduces to

$$(7''') \; m_{Y1}^3 \, m_{22} + m_{Y1}^2 \, m_{22} \, m_{Y2} \overset{?}{>} m_{Y2}^3 \, m_{11} + m_{Y1} \, m_{Y2}^2 \, m_{11}.$$

Inequality (7''') becomes

$$(7^*) \; m_{22} \, m_{Y1}^2 \, (m_{Y1} + m_{Y2}) \overset{?}{>} m_{11} \, m_{Y2}^2 \, (m_{Y2} + m_{Y1}).$$

The direction of the inequality is not reversed if $m_{Y2} + m_{Y1} > 0$. Assuming $m_{Y1} + m_{Y2} > 0$, (7*) becomes

$$(7^{**}) \; m_{22} \, m_{Y1}^2 \overset{?}{>} m_{11} \, m_{Y2}^2$$

which is not always true.

Substituting (4) into (8) we have (assuming $m_{Y1} > m_{Y2}$)

$$(8') \quad m_{Y2}^2 \; m_{Y1} \; m_{11} - m_{Y2}^3 \; m_{11} - m_{Y1}^3 \; m_{22}$$

$$+ \; m_{Y1}^2 \; m_{22} \; m_{Y2} \overset{?}{<} -2m_{Y2}^3 \; m_{11}$$

$$+ \; 2m_{Y2}^2 \; m_{Y1} \; m_{22} + 2m_{Y1}^2 \; m_{Y2} \; m_{22}$$

$$- \; 2m_{Y1} \; m_{Y2}^2 \; m_{22} \; ,$$

which reduces to

$$(8'') \quad m_{Y2}^2 \; m_{11} \; (m_{Y2} + m_{Y1}) \overset{?}{<} m_{Y1}^2 \; m_{22} \; (m_{Y1} + m_{Y2}).$$

Assuming $m_{Y2} + m_{Y1} > 0$, (8'') becomes

$$(8^{**}) \quad m_{Y2}^2 \; m_{11} \overset{?}{<} m_{Y1}^2 \; m_{22}$$

which is precisely the inequality (7**). However, as was pointed out (7**) is not obviously true. If $m_{Y1} > m_{Y2}$, (8**) would not be satisfied if m_{11} were large enough relative to m_{22}. Therefore, FM's conditional statement that if A_1 and A_2 are perfect substitues then $R^2_{Y(A_1+A_2)} > r^2_{YA_1}$ and $r^2_{YA_2}$, is not true.

Appendix B

As stated on p. 81, we are interested in the comparative sizes of the R^2's in equations (1) and (2). As shown in Appendix A, the R^2 of equation (2), R_2^2, is

$$(9)\quad R_2^2 = \frac{(m_{Y1} + m_{Y2})^2}{(m_{11} + m_{22} + 2m_{12})m_{YY}} = \frac{m_{Y1}{}^2 + m_{Y2}{}^2 + 2m_{Y1}\, m_{Y2}}{(m_{11} + m_{22} + 2m_{12})m_{YY}}.$$

The coefficient of determination of equation (1), R_1^2, is

$$(10)\quad R_1^2 = \frac{b_{YA_1 \cdot A_2}\, m_{1Y} + b_{YA_2 \cdot A_1}\, m_{Y2}}{m_{YY}} =$$

$$= \frac{m_{22}\, m_{Y1}^2 + m_{11}\, m_{Y2}^2 - 2m_{12}\, m_{Y1}\, m_{Y2}}{(m_{11}\, m_{22} - m_{12}^2)\, m_{YY}}$$

where $b_{YA_1 \cdot A_2}$ and $b_{YA_2 \cdot A_1}$ are defined in Appendix A.

We set out, now to determine whether R_2^2 is greater than R_1^2. The ratio R_2^2/R_1^2 is greater than unity if and only if

$$(11)\quad \frac{(m_{11}\, m_{22} - m_{12}^2)\, m_{Y1}^2 + (m_{11}\, m_{22} - m_{12}^2)\, m_{Y2}^2}{m_{22}(m_{11} + m_{22} + 2m_{12})\, m_{Y1}^2 + m_{11}(m_{11} + m_{22}}$$

$$\frac{+\ 2(m_{11}\, m_{22} - m_{12}^2)\, m_{Y1}\, m_{Y2}}{+\ 2m_{12})m_{Y2}^2 - 2m_{12}(m_{11} + m_{22} + 2m_{12})\, m_{Y1}\, m_{Y2}} \overset{?}{>} 1$$

or

$$(11')\ [m_{11}\, m_{22} - m_{12}^2 - m_{22}(m_{11} + m_{22} + 2m_{12})]\, m_{Y1}^2$$
$$+ [m_{11}\, m_{22} - m_{12}^2 - m_{11}(m_{11} + m_{22} + 2m_{12})]\, m_{Y2}^2$$
$$+ 2[m_{11}\, m_{12} - m_{12}^2 + m_{12}(m_{11} + m_{22} + 2m_{12})]$$
$$m_{Y1}\, m_{Y2} \overset{?}{>} 0.$$

The coefficient of m_{Y1}^2 is > 0 when

$$m_{11}\, m_{22} - m_{12}^2 - m_{22}(m_{11} + m_{22} + 2m_{12}) > 0$$

or $m_{11}\, m_{22} - m_{12}^2 - m_{11}\, m_{22} - m_{22}^2 - 2m_{12}\, m_{22} > 0$

or $(m_{12} + m_{22})^2 < 0$ which is impossible.

The coefficient of m_{Y2}^2 is > 0 when

$$m_{11}\, m_{22} - m_{12}^2 - m_{11}(m_{11} + m_{22} + 2m_{12}) > 0$$

or $m_{11}\, m_{22} - m_{12}^2 - m_{11}^2 - m_{11}\, m_{22} - 2m_{11}\, m_{12} > 0$

or $(m_{12} + m_{11})^2 < 0$ which is impossible.

Since the coefficients of m_{Y1}^2 and m_{Y2}^2 are always ≤ 0 and since m_{Y1}^2 and m_{Y2}^2 are always ≥ 0, the left hand side of (11') is > 0 if and only if

$$2[m_{11}\ m_{22} - m_{12}^2 + m_{12}\ (m_{11} + m_{22} + 2m_{12})]\ m_{Y1}\ m_{Y2}$$

$$(12) > -\ [m_{11}\ m_{22} - m_{12}^2 - m_{22}\ (m_{11} + m_{22} + 2m_{12})]\ m_{Y1}^2$$

$$-\ [m_{11}\ m_{22} - m_{12}^2 - m_{11}\ (m_{11} + m_{22} + 2m_{12})]\ m_{Y2}^2$$

or

$$(13)\ 2(m_{11} + m_{12})(m_{12} + m_{22})\ m_{Y1}\ m_{Y2} \overset{?}{>}$$

$$(m_{12} + m_{22})^2\ m_{Y1}^2 + (m_{11} + m_{12})^2\ m_{Y2}^2$$

Setting $a = m_{12} + m_{22}$ and $b = m_{11} + m_{12}$, (13) becomes

$$(14)\ 2(a\ m_{Y1})(b\ m_{Y2}) \overset{?}{>} (a\ m_{Y1})^2 + (b\ m_{Y2})^2$$

or

$$(15)\ 0 \overset{?}{>} (a\ m_{Y1})^2 = 2(a\ m_{Y1})(b\ m_{Y2}) + (b\ m_{Y2})^2$$

or $\qquad 0 \overset{?}{>} (a\ m_{Y1} - b\ m_{Y2})^2$ which is impossible.

Therefore, inequality (11) cannot hold and we see that R_2^2 is less than R_1^2 except when $(m_{12} + m_{22})\ m_{Y1} = (m_{11} + m_{12})\ m_{Y2}$. In this latter case $R_2^2 = R_1^2$. But this equality is precisely the same as $m_{Y1}\ m_{22} - m_{Y2}\ m_{12} = m_{Y2}\ m_{11} - m_{Y1}\ m_{11}$ which holds if and only if $b_{YA_1 \cdot A_2} = b_{YA_2 \cdot A_1}$. Thus, if $\beta_1 \neq \beta_2$, R^2 from the estimated equation (2) is always less than the R^2 of the true model (1), unless the sample just happened to generate $b_{YA_1 \cdot A_2} = b_{YA_2 \cdot A_1}$.

Since FM also combine the components money, M_1, and time deposits, M_2, into a single variable, the R^2 obtained from the estimated equation:

$$(20)\ Y = a_0 + a_1\ (M_1 + M_2) + \varepsilon^*$$

$$\text{or}\quad Y = a_0 + a_1 M + \varepsilon^*$$

will always be less than or equal to the R^2 obtained from the true model:

$$(21)\ Y = \alpha_0 + \alpha_1 M_1 + \alpha_2 M_2 + \varepsilon.$$

The FM study compares the R^2 of equation (20) with that of equation (2). The question now becomes whether the R^2 of (2) is less than the R^2 of (1) by a greater or smaller amount than the R^2 of (20) is less than the R^2 of (21). That is, does combining autonomous expenditures cause the R^2 to be lower than that of model (1) by more or less than the combining M_1 and M_2 causes the R^2 to be lower than that of model (21)?

The key to this question rests on the fact that the A used by FM in equation (2) is composed of the elements K + G - T + E - I while the definition of M used in (20) is $M_1 + M_2$. We would expect K, G, T, E, and I to all be positively correlated with the dependent variable of equation (2). We would also expect M_1 and M_2 to be positively correlated with the dependent variable of equation (20).

And, we would expect -T and -I to be negatively correlated with the dependent variable of (2) as well as with K, G, and E.

Therefore, we examine R_1^2 and R_2^2 to see whether they are affected by the fact that one of the components, A_2, is negatively correlated with the dependent variable and with A_1.

We note that

$$R_1^2 = \frac{m_{22}\, m_{Y1}^2 + m_{11}\, m_{Y2}^2 - 2m_{12}\, m_{Y1}\, m_{Y2}}{(m_{11}\, m_{22} - m_{12}^2)\, m_{YY}}$$

We are assuming r_{12} and r_{Y2} to be negative.

Since $r_{12} = \dfrac{m_{12}}{\sqrt{m_{11}}\,\sqrt{m_{22}}}$

and $r_{Y2} = \dfrac{m_{Y2}}{\sqrt{m_{22}}\,\sqrt{m_{YY}}}$

and since m_{22} and m_{YY} are both positive (both being sums of squares), the assumption that r_{12} and r_{Y2} are negative leads to the condition that m_{12} and m_{2Y} are negative. Looking at R_1^2, we see that it is unaffected by negative values of m_{12} and m_{2Y}. When m_{12} and m_{2Y} appear separately in a term of the right hand side, they appear as squares. The size of R_2^2 is dependent on whether m_{12} and m_{2Y} are negative or positive, however.

We now check to see whether adding the two components A_1 and A_2 when $r_{12} < 0$, $m_{Y2} < 0$, and $m_{Y1} > 0$, lowers R_2^2 more than adding A_1 and A_2 when $r_{12} > 0$, $m_{Y2} > 0$, and $m_{Y1} > 0$. We call the R^2 connected with the first case $R^2_{2(-)}$ and the R^2 associated with the second $R^2_{2(+)}$. Since $R^2_{1(+)}$ equals $R^2_{1(-)}$ (that is, since the coefficient of determination of the true model is the same whether $r_{12} < 0$ and $m_{Y2} < 0$ or whether $r_{12} > 0$ and $m_{Y2} > 0$), the relative damage to the true model from adding components A_1 and A_2 when $r_{12} < 0$ and $m_{Y2} < 0$ versus when $r_{12} > 0$ and $m_{Y2} > 0$ will be measured by $R^2_{2(+)} - R^2_{2(-)}$. If this difference is positive, the addition of negatively correlated components causes the computed R^2 to be lower than the true R^2 (i.e. R_1^2) by more than does the addition of positively correlated components. We define

$$R^2_{2(+)} = \frac{m_{Y1}^2 + m_{Y2}^2 + 2m_{Y1}\, m_{Y2}}{(m_{11} + m_{22} - 2m_{12})\, m_{YY}}$$

$$R^2_{2(-)} = \frac{m_{Y1}^2 + m_{Y2}^2 - 2m_{Y1}\, m_{Y2}}{(m_{11} + m_{22} - 2m_{12})\, m_{YY}}$$

where all m's are positive.

Setting

$$m_{Y1}^2 + m_{Y2}^2 = a \qquad m_{11} + m_{22} = c$$

$$2m_{Y1}\, m_{Y2} = b \qquad 2m_{12} = d$$

the difference

$$R^2_{2(+)} - R^2_{2(-)} = \frac{a + b}{(c - d)\, m_{YY}} - \frac{a - b}{(c + d)\, m_{YY}} .$$

This difference is greater than zero if and only if

$$(a + b)(c - d) - (a - b)(c + d) \overset{?}{>} 0$$

$$\text{or} \quad bc \overset{?}{>} ad$$

$$\text{or} \quad m_{Y1}\, m_{Y2}\, (m_{11} + m_{22}) \overset{?}{>} (m_{Y1}^2 + m_{Y2}^2)\, m_{12}.$$

It is not clear that combining negatively correlated items will do greater damage to the R^2 than does combining positively correlated components.

FOOTNOTES--CHAPTER III

[1]Donald D. Hester, "Keynes and the Quantity Theory: A Comment on the Friendman-Meiselman CMC Paper," Review of Economics and Statistics, XLVI (November, 1964), 364-368.

[2]Albert Ando and Franco Modigliani, "The Relative Stability of Monetary Velocity and the Investment Multiplier," American Economic Review, LV (September, 1965), 693-728.

[3]Michael DePrano and Thomas Mayer, "Tests of the Relative Importance of Autonomous Expenditures and Money," American Economic Review, LV (September, 1965), 729-752.

[4]Milton Friedman and David Meiselman, "Reply to Donald Hester," Review of Economics and Statistics, XLVI (November, 1964), 369-376; and "Reply to Ando and Modigliani and to DePrano and Mayer," American Economic Review, LV (September, 1965), 753-785.

[5]Donald D. Hester, "Rejoinder," Review of Economics and Statistics, XLVI (November, 1964), 376-377; Albert Ando and Franco Modigliani, "Rejoinder," American Economic Review, LV (September, 1965), 786-790; Michael DePrano and Thomas Mayer, "Rejoinder," American Economic Review, LV (September, 1965), 791-792.

[6]Above, p. 14.

[7]Above, p. 16.

[8]Above, pp. 30-31.

[9]Hester, "Keynes and the . . . ," p. 365.

[10]Ibid., p. 366.

[11]Ibid.

[12]Ibid.

[13]Ibid.

[14]*Ibid*.

[15]*Ibid*., p. 368.

[16]*Ibid*., p. 366.

[17]*Ibid*.

[18]*Ibid*., pp. 366-367.

[19]*Ibid*., p. 367.

[20]*Ibid*.

[21]Friedman and Meiselman, "Reply to . . . Hester," p. 374.

[22]*Ibid*.

[23]*Ibid*., p. 375.

[24]*Ibid*., p. 372.

[25]*Ibid*.

[26]*Ibid*.

[27]*Ibid*., p. 373.

[28]*Ibid*., p. 370.

[29]Hester, "Rejoinder," p. 377.

[30]Above, p. 20.

[31]Hester, "Keynes and the . . . ," p. 367.

[32]*Ibid*.

[33]*Ibid*., p. 368.

[34]Friedman and Meiselman, "Reply to . . . Hester," p. 369.

[35]*Ibid*.

[36]Ando and Modigliani, "The Relative Stability . . . ," p. 696.

[37]Above, p. 68.

[38]Ando and Modigliani, "The Relative Stability . . . ," p. 696.

[39]*Ibid.*

[40]*Ibid.*, p. 697.

[41]*Ibid.*, pp. 697-698.

[42]*Ibid.*, p. 697.

[43]Friedman and Meiselman, "Reply to Ando . . . ," pp. 756-757.

[44]Friedman and Meiselman, "The Relative Stability . . . ," p. 190.

[45]Ando and Modigliani, "The Relative Stability . . . ," p. 697.

[46]*Ibid.*, pp. 697-698.

[47]*Ibid.*, p. 698.

[48]*Ibid.*, p. 699.

[49]Johnston, p. 31.

[50]Ando and Modigliani, "The Relative Stability . . . ," p. 702.

[51]*Ibid.*, p. 696.

[52]*Ibid.*, p. 701.

[53]*Ibid.*, pp. 695-696.

[54]*Ibid.*, p. 705.

[55]*Ibid.*, pp. 704, 695-696.

[56]*Ibid.*, p. 706.

[57]*Ibid.*

[58]These values were taken from AM's Table 3; *Ibid.*, p. 704.

[59]*Ibid.*, p. 703.

[60]These values were taken from AM's Table 2; *Ibid.*, p. 698.

[61] *Ibid.*, pp. 703-704.

[62] *Ibid.*, p. 714.

[63] *Ibid.*

[64] Friedman and Meiselman, "Reply to Ando . . . ," p. 756.

[65] *Ibid.*, p. 762.

[66] *Ibid.*, p. 766.

[67] *Ibid.*, p. 756.

[68] *Ibid.*, p. 767.

[69] Ando and Modigliani, "The Relative Stability . . . ," p. 708.

[70] *Ibid.*

[71] *Ibid.*

[72] *Ibid.*, p. 709.

[73] *Ibid.*, p. 710.

[74] *Ibid.*

[75] *Ibid.*, p. 711.

[76] *Ibid.*, p. 713.

[77] *Ibid.*, p. 695.

[78] *Ibid.*, p. 713.

[79] Friedman and Meiselman, "Reply to Ando . . . ," p. 780.

[80] Ando and Modigliani, "The Relative Stability . . . ," p. 695.

[81] *Ibid.*, p. 711.

[82] *Ibid.*, p. 713-714.

[83] These values were taken from AM's Table 4; *Ibid.*, p. 712.

[84]*Ibid.*, p. 719.

[85]*Ibid.*, p. 717.

[86]*Ibid.*, p. 718.

[87]*Ibid.*

[88]*Ibid.*, p. 716.

[89]*Ibid.*, p. 719

[90]*Ibid.*

[91]*Ibid.*

[92]*Ibid.*

[93]*Ibid.*, p. 718.

[94]Friedman and Meiselman, "Reply to Ando . . . ," p. 783.

[95]*Ibid.*

[96]*Ibid.*

[97]*Ibid.*

[98]DePrano and Mayer, "Tests of the Relative . . . ," pp. 747-748.

[99]*Ibid.*, p. 748.

[100]*Ibid.*

[101]*Ibid.*, pp. 732-734.

[102]Above, pp. 77-79.

[103]DePrano and Mayer, "Tests of the Relative . . . ," p. 732.

[104]Above, pp. 79-81.

[105]DePrano and Mayer, "Tests of the Relative . . . ," p. 749.

[106]*Ibid.*, p. 732.

[107]DePrano and Mayer, "Rejoinder," p. 792.

[108]*Ibid.*

[109]Friedman and Meiselman, "Reply to Ando . . . ," p. 766.

[110]*Ibid.*

[111]*Ibid.*

[112]DePrano and Mayer, "Tests of the Relative . . . ," p. 734.

[113]*Ibid.*, pp. 735-737.

[114]*Ibid.*, p. 737.

[115]*Ibid.*, p. 739.

[116]*Ibid.*, pp. 741-742.

[117]*Ibid.*, p. 745.

[118]Goldberger, *Econometric Theory*, p. 201.

[119]Carl Christ, *Econometric Models and Method* (New York: John Wiley & Sons, 1966), p. 169.

[120]Friedman and Meiselman, "Reply to . . . Hester," p. 373.

[121]Friedman and Meiselman, "Reply to Ando . . . ," p. 784.

[122]*Ibid.*, p. 754.

[123]Hester, "Rejoinder," p. 377.

[124]Friedman and Meiselman, "The Relative Stability . . . ," p. 182.

CHAPTER IV

TESTS OF ALTERNATIVE MEASUREMENTS

Introduction

Our discussion of FM's critics has uncovered several alternative definitions of autonomous expenditures. Hester proposed the concepts L, L', L'', and L'''; AM preferred $Z^a + X^a$; and DM offered a "gross"concept, A* = L''' minus state and local government purchases, and a "net" concept, A** = A* - D. When the critics fitted their definitions to the data they invariably found a higher correlation coefficient or coefficient of determination between their concepts and consumption, C, than FM found between A and C.

FM claim these results are suspect, first, because their critics should have used the same periods FM did in their regression equations and, second, because the regressions used by their critics included the wrong dependent variable. On this latter point, FM argue that their use of C was derived from the fact that it was the only induced item in their concept of income; hence (in order to be consistent with the FM procedure) when their critics altered the items included in autonomous spending, they should have simultaneously altered the dependent variable to include the new endogenous items.

In light of FM's comments, one of the objectives of this chapter is to re-estimate the coefficients of determination of regression equations of C on the various concepts of autonomous expenditures for the same periods FM used and compare their values. Since some of the proposed alternative concepts of A involve a detailed national income accounting framework, we will compare R^2's only for time periods including the years 1929-1965.

We have not heeded FM's second objection and their implicit advice to calculate another set of coefficients of determination for equations involving alternative concepts, A^{Δ}, as the explanatory variable and C^{Δ} ($Y - A^{\Delta}$, where A^{Δ} is the corresponding autonomous concept) as the dependent variable. Partially, the reason we have not done so is that FM's equations suffer from the greater malady of induced components contained in their so called autonomous concept. Also, since FM did not present a formal structural model of income determination in the first place and since they merely stumbled upon their concept of income (Y was found by summing U and A which were determined from statistical "experiments"), it seems we can do little more than compare what appear to be tests on alternative reduced form equations for consumption. The results of the refined estimation procedures and dynamic analyses applied to more sophisticated models in the chapters to follow are bound to overshadow any significance we could discover from manipulating the naive models we have reviewed so far.

While the major purpose of this chapter is to re-estimate equations offered by FM's critics using the same time periods FM considered, we have decided to test again FM's equations (1') through (6')[1] with revised data for M, A, C, and P. Secondly, since we object to FM's inclusion of time deposits in the definition of M, we will re-test equations (1'), (3'), (5'), and (6') using currency in circulation plus demand deposits as the definition of M. Also, we add the subperiods 1957-1965 and 1929-1965 to those suggested by FM.

Re-estimation of FM Equations Using Revised Data and an Alternative Definition of M

Equations Tested

The FM equations (1') through (6') were re-estimated with revised data. Equations (1'), (3'), (5'), and (6') were also tested with an alternative definition of the money supply. These equations are

$$(1'')\quad C_B = \alpha_1 + \alpha_2 M_{D+T} + \varepsilon_1$$

$$(2'')\quad C_B = \alpha_3 + \alpha_4 M_D + \varepsilon_2$$

$$(3'')\quad C_M = \beta_1 + \beta_2 A + \varepsilon_3$$

$$(4'')\ C_M = \beta_3 + \beta_4 A + \beta_5 p + \varepsilon_4$$

$$(5'')\ C_B = \alpha_5 + \alpha_6 M_{D+T} + \alpha_7 p + \varepsilon_5$$

$$(6'')\ C_B = \alpha_8 + \alpha_9 M_D + \alpha_8 p + \varepsilon_6$$

$$(7'')\ C_B = \gamma_1 + \gamma_2 A_B + \gamma_3 M_{D+T} + \varepsilon_7$$

$$(8'')\ C_B = \gamma_4 + \gamma_5 A_B + \gamma_6 M_D + \varepsilon_8$$

$$(9'')\ C_B = \gamma_7 + \gamma_8 A_B + \gamma_9 M_{D+T} + \gamma_{10} p + \varepsilon_9$$

$$(10'')\ C_B = \gamma_{11} + \gamma_{12} A_B + \gamma_{13} M_D + \gamma_{14} p + \varepsilon_{10}$$

Equation (1'') was tested for subperiods between 1897 and 1965, equation (2'') was fitted to subperiods between 1915 and 1965, and equations (3'') through (10'') were tested for subperiods with years between 1929 and 1965.

Definition of Variables

C_B = Personal consumption in billions of current dollars. For 1929-1965 $C_B = C_M/1000$. For 1897-1928 C_B was computed using the regression equation:

$C_{OBE} = 0.5821 + 0.9508\ C_K$ which was computed by regressing C_{OBE} on C_K for the period 1929-1941.

$C_{OBE} = C_{OBE}^{58} \times P_C^{58}$ where

P_C: Implicit price deflator for personal consumption expenditures (1958 = 100) for years 1929-65,

U. S. Department of Commerce, Bureau of the Census, <u>Long Term Economic Growth</u>, 1970-1965, Superintendent of Documents, U. S. Government Printing Office, Washington, D.C., October, 1966, pp. 200-201, Series B66.

C^{58}_{OBE}: Personal consumption expenditures in billions of 1958 dollars, <u>Ibid</u>., pp. 170-171, Series A24. And where

$C_K = C^{29}_K \times P^{29}_K$

P^{29}_K: Implicit price deflator for personal consumption expenditures (1929 = 100) for years 1899 to 1929, <u>Ibid</u>., pp. 200-201, Series B65. This price deflator was extended to the years 1930-41 by the formula: $P^{29}_{K_t} = P^{29}_{OBE} \times 100/55.3 \quad t > 1929$ where 100 is the index of P^{29}_K for the year 1929 and 55.3 is the index of P^{58}_{OBE} for the year 1929.

C^{29}_K: Personal consumption expenditures in millions of 1929 dollars, <u>Ibid</u>., pp. 170-171, Series A23. This series was changed to billions of 1929 dollars prior to running the regression.

C_M = Personal consumption expenditures in millions of current dollars. Data was taken from U. S. Department of Commerce, Office of Business Economics, "The National Income and Product Accounts of the United States, 1929-1965; (Statistical Tables)," A Supplement to the Survey of Current Business (Washington: Government Printing Office, .966), Table 1.1, line 2, pp. 2-3.

M_{D+T} = Money supply plus time deposits (currency in circulation, demand deposits at commercial banks other than those due to commercial banks and U. S. government, less cash items in process of collection and Federal Reserve float, and time deposits at commercial banks) in billions of dollars. For 1897-1946 data is taken from U. S. Department of Commerce, Bureau of the Census, Long Term Economic Growth, 1860-1965, Series B111, pp. 208-209 (data is twelve month centered means of seasonally adjusted data). For 1947-1965 data is twelve month centered means of seasonally adjusted monthly data of total money supply plus time deposits at commercial banks taken from data presented in Board of Governors, Federal Reserve Bulletin, Table--Money Supply and Related Data (in billions of dollars) June, 1964, pp. 682-692; June, 1965,

p. 978; May, 1966, p. 678; and April, 1967, p. 608.

M_D = Money supply in billions of dollars. For 1915-1946 data taken from U. S. Department of Commerce, Bureau of Census, Long term . . . , Series B109, pp. 208-209. (Data is centered means of seasonally adjusted data.) For 1947-1965 data is twelve month centered means of seasonally adjusted monthly data of total money supply taken from Board of Governors, Federal Research Bulletin, Table--Money Supply and Related Data (in billions of dollars), same issues and pages as for M_{D+T}.

A = FM's autonomous expenditure concept.

= $GPDI - CCA + E - I - GS^{F}_{IPA} - GS^{SL}_{IPA}$

Data is for 1929-1965 period where:

GPDI = Gross private domestic investment in millions of current dollars, U. S. Department of Commerce, Office of Business Economics, "The National Income . . . ," Supplement to the Survey of Current Business, Table 1.1, line 6, pp. 2-3.

CCA = Capital consumption allowance in millions of current dollars, U. S. Department of Commerce, OBE, Table 1.9, line 2, pp. 12-13.

E = Exports in millions of current dollars, U. S. Department of Commerce, OBE, Table 1.1, line 18.

I = Imports in millions of current dollars, U. S. Department of Commerce, OBE, Table 1.1, line 19.

GS^{F}_{IPA} = Federal government surplus on income and product account in millions of current dollars, U. S. Department of Commerce, OBE, Table 3.1, line 31, pp. 52-53.

GS^{SL}_{IPA} = State and local government surplus on income and product account in millions of current dollars, U. S. Department of Commerce, OBE, Table 3.3, line 33, pp. 54-55.

A_B = A/1000

p = Implicit price deflator for personal consumption expenditures (1958 = 100), U. S. Department of Commerce, OBE, Table 8.1, line 2, pp. 158-159.

Results

The results of the series of tests are given in Table 1 below. These results for the single variable equations show that the coefficients of determination are generally lowest when A is the explanatory variable and generally highest when M_{D+T} is the single explanatory

TABLE 7.--Coefficients of determination for various periods for EM equations using revised data

TABLE 7.--Coefficients of determination for various periods for FM equations using revised data and alternative definitions of the money supply.

Equations	Periods														
	1897-1965	1897-1908	1903-1913	1908-1921	1913-1920	1915-1965	1920-1929	1921-1933	1929-1939	1933-1938	1938-1953	1939-1948	1948-1957	1957-1965	1929-1965
(1'')	0.979	0.991	0.993	0.991	0.983		0.916	0.835	0.852	0.980	0.922	0.941	0.980	0.981	0.969
(2'')						0.895	0.867	0.923	0.488	0.980	0.896	0.916	0.964	0.964	0.871
(3'')									0.884	0.868	0.287	0.112	0.601	0.692	0.643
(4'')									0.954	0.930	0.985	0.996	0.953	0.980	0.934
(5'')									0.968	0.993	0.982	0.980	0.985	0.995	0.969
(6'')									0.988	0.991	0.985	0.980	0.967	0.994	0.938
(7'')									0.930	0.986	0.929	0.956	0.987	0.990	0.978
(8'')									0.885	0.981	0.912	0.950	0.964	0.972	0.877
(9'')									0.991	0.993	0.986	0.997	0.987	0.995	0.978
(10'')									0.991	0.992	0.987	0.996	0.969	0.994	0.938

variable. When the price level and one other variable are considered as independent, the lowest coefficients of determination arise when A is that variable and highest when the second independent variable is M_{D+T}. Equations with A and M as explanatory variables display higher R^2's when $M + M_{D+T}$ than when $M = M_D$. And, finally, when A, P, and M are the explanatory variables $M = M_{D+T}$ yields higher coefficients of determination than when $M = M_D$. Naturally, the R^2's increase as the number of variables is increased from one to two and from two to three.

Re-estimation of Alternative Definitions of A to Conform to the Periods Tested by FM

Answering FM's charge that their critics should have considered the same sample periods FM did when testing their equations, we fit the following equations to the data for various subperiods since 1929. We also add the two periods 1957-1965 and 1929-1965.

Equations Tested

The following seven equations proposed by FM's critics are taken as alternative reduced forms for consumption.

(11'') $C_M = \gamma_0 + \gamma_1 L + \varepsilon_{11}$

(12'') $C_M = \gamma_2 + \gamma_3 L' + \varepsilon_{12}$

$$(13'')\quad C_M = \gamma_4 + \gamma_5 L'' + \varepsilon_{13}$$

$$(14'')\quad C_M = \gamma_6 + \gamma_7 L''' + \varepsilon_{14}$$

$$(15'')\quad C_M = \gamma_8 + \gamma_9 A^{*} + \varepsilon_{15}$$

$$(16'')\quad C_M = \gamma_{10} + \gamma_{11} A^{**} + \varepsilon_{16}$$

$$(17'')\quad C_M = \gamma_{12} + \gamma_{13} A^{\Delta} + \varepsilon_{17}$$

Definition of the Variables

C_M = Personal consumption expenditures in millions of current dollars.

$L = A + GS^{F}_{IPA} + GS^{SL}_{IPA} + G_F + G_{SL}$

$= K + E - I + G$ = Hester's L

where A, GS^{F}_{IPA}, and GS^{SL}_{IPA} are defined above and

G_F = Federal government expenditures in millions of current dollars, U. S. Department of Commerce, OBE, Table 1.1, line 21, pp. 2-3.

G_{SL} = State and local government expenditure in millions of current dollars, U. S. Department of Commerce, OBE, Table 1.1, line 24, pp. 2-3.

$L' = L + CCA + I$

$= GPDI + E + G$ = Hester's L'

where data sources for CCA and I are shown in the above section.

$L'' = L + CCA$ (Hester's L'')

$L''' = L - INV$ (Hester's L''') where

INV = Net changes in inventory in millions of current dollars, U. S. Department of Commerce, OBE, Table 1.1, line 14, pp. 2-3.

$A^* = GPDI + E + G_F - INV$ (DM's "gross" concept)

$A^{**} = A^* - CCA$ (DM's "net" concept)

$A^{\Delta} = GPDI - CCA - INV + E + G_F + G_{SL} + T_{b2} + T_{iF} + T_{iS} + T_{g2} + Q - H - W$ (AM's $Z^a + X^a$) where

T_{b2} = Property tax portion of indirect business taxes in millions of current dollars, U. S. Department of Commerce, OBE, Table 3.3, line 19, pp. 54-55.

T_{iF} = Federal net interest paid in millions of current dollars, U. S. Department of Commerce, OBE, Table 3.1, line 29, pp. 52-53.

T_{iS} = State and local net interest paid in millions of current dollars, U. S. Department of Commerce, OBE, Table 3.3, line 31, pp. 54-55.

$T_{g2} = T_g - T_{g1}$ where

$T_g = T_{gF} + T_{gS}$ and $T_{g1} = SB + RB$ where

T_{gF} = Federal government transfer payments to persons in millions of current dollars, U. S.

Department of Commerce, OBE, Table 3.1, line 26, pp. 52-53.

T_{gS} = State and local government transfer payments to persons in millions of current dollars, U. S. Department of Commerce, OBE, Table 3.3, line 30, pp. 54-55.

SB = State unemployment benefits in millions of current dollars, U. S. Department of Commerce, OBE, Table 3.9, line 5, pp. 58-59.

RB = Railroad unemployment benefits in millions of current dollars, U. S. Department of Commerce, OBE, Table 3.9, line 7, pp. 58-59.

$Q = Q_F + Q_{SL}$ where

Q_F = Subsidies less current surplus of federal government enterprises in millions of current dollars, U. S. Department of Commerce, OBE, Table 3.1, line 30, pp. 52-53.

Q_{SL} = Subsidies less current surplus of state and local government enterprises in millions of current dollars, U. S. Department of Commerce, OBE, Table 3.3, line 32, pp. 54-55.

H = Statistical discrepancy in millions of current dollars, U. S. Department of Commerce, OBE, Table 5.1, line 14, pp. 78-79.

W = Wage accruals less disbursements in millions of current dollars, U. S. Department of Commerce, OBE, Table 5.1, line 7, pp. 78-79.

Results

Table 8 presents the coefficients of determination of these regressions for each of seven subperiods between 1929 and 1965.

TABLE 8.--Coefficients of determination for alternative definitions of autonomous expenditures.

Equations	Periods						
	1929-1939	1933-1938	1938-1953	1939-1948	1948-1957	1957-1965	1929-1965
(11'')	0.834	0.986	0.610	0.264	0.865	0.956	0.925
(12'')	0.919	0.980	0.729	0.355	0.936	0.961	0.964
(13'')	0.882	0.990	0.688	0.309	0.932	0.962	0.957
(14'')	0.959	0.993	0.699	0.312	0.949	0.965	0.961
(15'')	0.948	0.990	0.594	0.241	0.906	0.944	0.924
(16'')	0.913	0.993	0.492	0.199	0.806	0.915	0.851
(17'')	0.901	0.997	0.752	0.440	0.966	0.978	0.969

This table shows that, in general, all alternative concepts of autonomous expenditures yield higher coefficients of determination than does A. Except for the time periods 1938-1953 and 1939-1948, when all R^2's are rather low, the coefficients of determination are of "roughly the same magnitude" as DM have suggested.

Conclusion

Little insight into the relative effectiveness of monetary and fiscal policy has been offered by the refinements carried out in this chapter or by the entire exchange among authors we have reviewed to this point. As we have shown, FM's methodology is naive and not altogether sound.[2] Consequently their crowning the "quantity theory" with laurel seems somewhat persumptuous.

Fortunately, there is a better method at our disposal than that utilized by FM, namely, beginning at the beginning with a specific, more sophisticated, yet manageable, theoretical model of income determination.[3] This model is then tested statistically using econometric methods which account for the difficulties which arise in measuring economic relationships. Finally, but certainly not the least significantly, there is a large body of knowledge which can be brought to bear in analyzing the model (especially a dynamic one) after it has been estimated. It is this methodology which will guide us through the next three chapters.

FOOTNOTES--CHAPTER IV

[1]See above, pp.

[2]Not only is it unsound; it is contagious. See for instance R. H. Timberlake, Jr. and James Forston, "Time Deposits in the Definition of Money," American Economic Review, LVII (March, 1967), 190-194.

[3]This is in fact what FM suggested should be done. See Friedman and Meiselman, "Reply to Ando . . . ," p. 753 and Ando and Modigliani, "The Relative Stability . . . ," p. 716.

CHAPTER V

RE-ESTIMATION OF KLEIN MODELS II AND III

Introduction

Having demonstrated that the studies reviewed in Chapters II and III are likely to be inadequate explorations of the effects of money and autonomous expenditures on the level of income, we try to construct a more comprehensive framework to analyze the relative effectiveness of monetary and fiscal policy.

Building a general equilibrium econometric model involves a myriad of problems and uncertainties as to performance. Therefore, it was decided to adopt and, if necessary, revise existing econometric models. The leisure consumed as a result of this decision and the freedom of responsibility for defects in the models used were felt to more than offset the accompanying costs. These costs include damages to the predictability of the models which may result from manipulation of the time periods, alteration of the status of variables as to their determination within or outside the model, and the deletion or addition of equations.

We considered several general equilibrium econometric models for adoption. The first condition we set down was that the model by dynamic and use annual data. Quarterly models cannot include the years prior to World War II since quarterly data are unavailable for the pre-war period. Secondly, the model chosen must have treated government expenditures and the money stock as exogenously determined. Thirdly, we were interested in finding a model that was originally tested for as many of the years 1897-1958 as possible--the interval tested by FM.

Klein's models I, II, and III and the Klein-Goldberger, Valavanis-Vali, and Morishima-Saito models comprise the field of candidates from which our selection was made.[1] Klein model I lacks a monetary sector, the Valavanis-Vail model lumps government spending with consumption into one endogenous variable, and the Morishima-Saito model specifies an endogenous variable which combines government spending with net private domestic investment. Therefore, these candidates do not contain the requisite exogenous categories for a comparison of the effectiveness of monetary and fiscal policy. While the Klein-Goldberger model contains a monetary sector, it exercises no influence over the real sector.

By the process of elimination, we decided to base our analysis on Klein's models II and III. Both treat the money supply and government spending as exogenous variables. However, both suffer from being previously tested for pre-World War II years only. Also, Klein

model III contains non-linear equations. Therefore, in order to carry out the dynamic analysis on Klein model III, other than by simulation methods, we must be satisfied with using linear approximations of the equations. Though significant, these deficiencies do not preclude employing Klein models II and III in a more sophisticated comparison of the relative effectiveness of monetary and fiscal policy than has been offered by the studies reviewed earlier.

Klein Model II

Klein Model II is a simple three equation model of income determination and is only a modest step toward a better analytical framework within which to analyze the issue at hand. This model comprises the following three equations:

$$(1)\ C/pN = \alpha_0 + \alpha_1 \frac{Y}{pN} + \alpha_2 \left(\frac{Y}{pN}\right)_{-1} + \alpha_3 \left(\frac{M}{pN}\right)_{-1} + u$$

$$(2)\ GNP = C + I' + G$$

$$(3)\ GNP = Y + T$$

where C = consumption in current dollars
Y = disposable income in current dollars
M = money supply in current dollars
I' = gross investment in current dollars
G = government expenditure plus foreign balance in current dollars
GNP = gross national product in current dollars
p = cost-of-living index
N = population in United States

T = government receipts plus corporate savings plus business reserves minus transfer payments minus inventory profits, all measured in current dollars.

The endogenous variables are C, Y, GNP. The exogenous variables are I'/pN, G/pN, T/pN and M/pN_{-1}.[2]

Data Revisions

Klein Model II was extended to cover the pre-war period 1922-1941 and the post-war period 1946-1965. The following presents the data and its revisions for the period 1920-41 and 1945-65.

C: Personal consumption expenditures in billions of current dollars. For 1929-65 data was taken from U. S. Department of Commerce, Office of Business Economics, "The National Income and Product Accounts of the United States, 1929-1965; (Statistical Tables)," A Supplement to the Survey of Current Business (Washington, D.C., 1966), Table 1.1, pp. 2-3, line 2. For 1920-28 data for personal consumption expenditures in billions of current dollars was computed as follows:

C_{OBE} = 0.5821 + 0.9508 C_K was computed using the years 1929-1941 and then C_{OBE} for years 1920 to 1928 was estimated from the regression where

$C_{OBE} = C_{OBE}^{58} \times P_{c}^{58}$ where

P_{c}^{58}: Implicit price deflator for personal consumption expenditures (1958 = 100) for years 1929-65, U. S. Department of Commerce, Bureau of the Census, Long Term Economic Growth, 1860-1965, Superintendent of Documents, U. S. Government Printing Office, Washington, D.C., October, 1966, pp. 200-201, Series B66.

C_{OBE}^{58}: Personal consumption expenditures in billions of 1958 dollars, Ibid., pp. 170-171, Series A24. And where

$C_{K} = C_{K}^{29} \times P_{K}^{29}$

P_{K}^{29}: Implicit price deflator for personal consumption expenditures (1929 = 100) for years 1899 to 1929, U. S. Department of Commerce, Bureau of the Census, Long Term . . . , pp. 200-201, Series B65. This price deflator was extended to the years 1930-41 by the formula:

$P_{K_t}^{29} = P_{OBE}^{29} \times \frac{100}{55.3}$ $\quad t > 1929$ where 100 is the index of P_{K}^{29} for the year

1929 and 55.3 is the index of P_{OBE}^{58} for the year 1929.

C_K^{29}: Personal consumption expenditures in millions of 1929 dollars, Ibid., pp. 170-171, Series A23. This series was changed to billions of 1929 dollars prior to running the regression.

G: Government expenditures in billions of current dollars. For 1929-1965, $G = G_F + G_{SL}$ where

G_F: Federal government expenditures, U. S. Department of Commerce, OBE, "The National Income . . . ," Table 3.1, pp. 52-53, line 19.

G_{SL}: State and local government expenditures, Ibid., Table 3.3, pp. 54-55, line 24.

For 1920-28 estimates of G were obtained using the regression of G_{OBE} on G_K for years 1929-41: $G_{OBE} = 2.92 + 1.104\ G_K$ where $G_{OBE} = G_{OBE}^{58} \times P_{GNP}^{58}$.

G_{OBE}: Government purchases of goods and services in billions of 1958 dollars, U. S. Department of Commerce, Bureau of Census, Long Term . . . , pp. 172-173, Series A34.

P_{GNP}^{58}: Implicit price deflator for GNP (1958 = 100), Ibid., pp. 200-201, Series B62.

and where $G_K = G_K^{29} \times p_{GNP}^{29}$

G_K^{29}: Government purchases of goods and services in billions of 1929 dollars, Ibid., pp. 170-171, Series A33.

p_{GNP}^{29}: Implicit price deflator for GNP (1929 = 100), Ibid., pp. 200-201, Series B61. Series extended using expression $p_{GNP}^{29} = p_{GNP}^{58} \times \frac{100}{50.6}$ where 50.6 is the value of p_{GNP}^{58} for 1929.

N: Population in billions for 1920-65 (converted from data in thousands), The U. S. Book of Facts, Statistics, and Information, (New York: Washington Square Press, Inc., 1966), Table No. 2--Estimated Population 1900 to 1966 (original source: U. S. Department of Commerce, Bureau of the Census, Current Population Reports, Series P-25, Nos. 331 and 340).

Y: Disposable personal income in billions of current dollars (converted from millions). For 1929-1965 data from U. S. Department of Commerce, OBE, "The National Income . . . ," Table 2.1, pp. 32-33, line 22. For 1920-1928 Y is regressed on Y_K for years 1929-41: $Y = 1.042\ Y_K$ where Y_K are Klein's data for disposable income.

I: Gross private domestic investment in billions of dollars (converted from millions of dollars).

1929-65 data from Ibid., Table 1.1, pp. 2-3, line 6. For 1920-1928 estimates of I were obtained using the regression of I_{OBE} on I_K for years 1929-41: $I_{OBE} = 1.238 + 0.8204\ I_K$ where

$$I_{OBE} = I_{OBE}^{58} \times p_I^{58}$$

I_{OBE}^{58}: Gross private domestic investment in billions of 1958 dollars, U. S. Department of Commerce, Bureau of Census, Long Term . . . , pp. 170-171, Series A28.

p_I^{58}: Implicit price deflator for fixed investment, (1958 = 100), Ibid., pp. 200-201, Series B68.

$$I_K = I_K^{29} \times p_I^{29}$$

I_K^{29}: Gross private domestic investment in billions of 1929 dollars (converted from millions of dollars), Ibid., pp. 170-171, Series A27.

p_I^{29}: Implicit price deflator for fixed investment, 1929 = 100, Ibid., pp. 200-201, Series B67. For 1930-1941 series extended by use of following expression:

$$p_{I_t}^{29} = p_{I_t}^{58} \times \frac{100}{39.4} \qquad t > 1929$$

where 39.4 = the value of p_I^{58} for the year 1929.

p_1: Implicit price deflator for GNP (1958 = 100). For 1929-1965 data from U. S. Department of Commerce, OBE, "The National Income . . . ," Table 8.1, pp. 158-59, line 1. For 1920-1928 $p_1 = p_K^{29} \times \frac{50.6}{100}$ where 50.6 is value of p_1 for 1929 and p_K^{29} and p_{GNP}^{29} are as defined above.

p_2: Implicit price deflator for personal consumption expenditures (1958 = 100). For 1929-65 data, Ibid., Table 8.1, pp. 158-59, line 2. For 1921-1928, $p_2 = p_C^{29} \times \frac{55.3}{100}$ where

p_C^{29}: Implicit price deflator for personal consumption expenditures (1929 = 100), U. S. Department of Commerce, Bureau of Census, Long Term . . . , pp. 200-201, Series B65; 55.3 is the value of p_2 for 1929.

M_1: Money supply billions of dollars, cf. M_1^D under Klein Model III.

M_2: Money supply plus time deposite in billions of current dollars, cf. M_S under Klein Model III.

Estimation Procedure

The stochastic equation of Klein Model II is

$$(1)\ C/pN = \alpha_0 + \alpha_1(Y/pN) + \alpha_2(Y/pN)_{-1} + \alpha_3(M/pN)_{-1} + u.$$

It was estimated several times, both for the period 1922-1941 and the extended period 1922-1941 and 1946-1965 using different measures of the implicit price deflator and the money supply. C and Y are endogenous variables in the system of equations comprising Klein Model II. We assume u_t is normally distributed with mean zero and variance σ^2 for all t, independent of past and future distrubance terms, and independent of the predetermined variables in the system. Applying simple least squares to equation (1) would yield the undesirable result of obtaining biased and inconsistent estimates of the α's. This arises from the fact that Y/pN is correlated with u_t. Therefore, we use two alternative methods to estimate equation (1) which take account of this problem. These methods are two stage least squares (2SLS) and limited information maximum likelihood (LI).

Results of Re-estimating Klein Model II

We now turn to the estimates of the coefficients we obtained using both the 2SLS and LI estimation methods for the years 1922-1941 and for the extended period 1922-1941 and 1946-1965. The figures in parentheses are the standard errors of the coefficients; those in brackets are the ratios of the coefficients to their standard errors.

Using $p = p_1$, $M = M_2$ (1922-1941)

Two Stage Least Squares

$$C/pN = \underset{\substack{(45.39866)\\ [4.04569]}}{183.66911} + \underset{\substack{(0.06344)\\ [8.82772]}}{0.56001}\, Y/pN + \underset{\substack{(0.07451)\\ [0.34591]}}{0.02577}\, (Y/pN)_{-1}$$

$$+ \underset{\substack{(0.08416)\\ [3.34981]}}{0.28193}\, (M_2/pN)_{-1}$$

All explanatory variables except $(Y/pN)_{-1}$ are highly significant at the 95 per cent level of confidence.

Limited Information

$$C/pN = \underset{\substack{(47.28288)\\ [4.01411]}}{189.79843} + \underset{\substack{(0.06607)\\ [7.81362]}}{0.51625}\, Y/pN + \underset{\substack{(0.07761)\\ [0.69298]}}{0.05378}\, (Y/pN)_{-1}$$

$$+ \underset{\substack{(0.08766)\\ [3.41628]}}{0.29946}\, (M_2/pN)_{-1}$$

Once again $(Y/pN)_{-1}$ is not statistically significant at the 95 per cent level of confidence.

Using $p = p_1$, $M = M_2$ (1922-1941 and 1946-1965)

Two Stage Least Squares

$$C/pN = \underset{\substack{(14.07767)\\ [2.85755]}}{40.22769} + \underset{\substack{(0.06063)\\ [11.39288]}}{0.69077}\, (Y/pN) + \underset{\substack{(0.06942)\\ [2.73289]}}{0.18972}\, (Y/pN)_{-1}$$

$$+ \underset{\substack{(0.03683)\\ [0.82195]}}{0.03027}\, (M_2/pN)_{-1}$$

Limited Information

$$C/pN = \underset{\substack{(14.55739)\\ [2.82895]}}{41.18210} + \underset{\substack{(0.06270)\\ [10.20692]}}{0.63995}\, (Y/pN) + \underset{\substack{(0.07179)\\ [3.38911]}}{0.24329}\, (Y/pN)_{-1}$$

$$+ \underset{\substack{(0.03809)\\ [0.69751]}}{0.02657}\, (M_2/pN)_{-1}$$

Extending the sample period causes variable $(M_2/pN)_{-1}$ to become not statistically significant and the variable $Y/pN)_{-1}$ to become significant.

Using $p = p_2$, $M = M_2$ (1922-1941)

Two Stage Least Squares

$$C/pN = \underset{\substack{(51.45664)\\ [4.08745]}}{210.32619} + \underset{\substack{(0.06758)\\ [7.74724]}}{0.52356}\,(Y/pN) + \underset{\substack{(0.07813)\\ [0.80972]}}{0.06326}\,(Y/pN)_{-1} + \underset{\substack{(0.08126)\\ [3.25406]}}{0.26442}\,(M_2/pN)_{-1}$$

Limited Information

$$C/pN = \underset{\substack{(53.76023)\\ [4.07593]}}{219.12311} + \underset{\substack{(0.07061)\\ [6.71825]}}{0.47434}\,(Y/pN) + \underset{\substack{(0.08163)\\ [1.18745]}}{0.09693}\,(Y/pN)_{-1} + \underset{\substack{(0.08490)\\ [3.28062]}}{0.27851}\,(M_2/pN)_{-1}$$

Using $p = p_2$, $M = M_2$ (1922-41 and 1946-65)

Two Stage Least Squares

$$C/pN = \underset{\substack{(16.57955)\\ [2.93047]}}{48.58589} + \underset{\substack{(0.06807)\\ [9.34564]}}{0.63614}\,(Y/pN) + \underset{\substack{(0.07514)\\ [3.11459]}}{0.23401}\,(Y/pN)_{-1} + \underset{\substack{(0.03799)\\ [1.08328]}}{0.04115}\,(M_2/pN)_{-1}$$

Limited Information

$$C/pN = \underset{\substack{(17.28810)\\ [2.86928]}}{49.60436} + \underset{\substack{(0.07098)\\ [8.11286]}}{0.57583}\,(Y/pN) + \underset{\substack{(0.07835)\\ [3.77367]}}{0.29565}\,(Y/pN)_{-1} + \underset{\substack{(0.03961)\\ [1.00849]}}{0.03995}\,(M_2/pN)_{-1}$$

This sub-group using $p = p_2$ and $M = M_2$ is the one which corresponds most closely to the definitions of the variables used by Klein. The results of extending the period are precisely those we found when $p = p_1$ and $M = M_2$. We now redefine M to be the money supply (rather than money supply plus time deposits).

Using $p = p_1$ and $M = M_1$ (1922-1941)

Two Stage Least Squares

$$C/pN = \underset{\substack{(46.97485)\\ [4.58278]}}{215.27562} + \underset{\substack{(0.07363)\\ [6.57828]}}{0.48434}\,(Y/pN) + \underset{\substack{(0.06969)\\ [1.72551]}}{0.12025}\,(Y/pN)_{-1} + \underset{\substack{(0.09484)\\ [3.41621]}}{0.32398}\,(M_1/pN)_{-1}$$

Limited Information

$$C/pN = \underset{\substack{(48.35036)\\ [4.56111]}}{220.53146} + \underset{\substack{(0.07734)\\ [5.85896]}}{0.45313}\,(Y/pN) + \underset{\substack{(0.07223)\\ [1.91845]}}{0.13857}\,(Y/pN)_{-1} + \underset{\substack{(0.09810)\\ [3.52489]}}{0.34580}\,(M_1/pN)_{-1}$$

Using $p = p_1$ and $M = M_1$ (1922-41 and 1946-65)

Two Stage Least Squares

$$C/pN = \underset{\substack{(14.37861)\\ [2.68163]}}{38.55807} + \underset{\substack{(0.06080)\\ [11.31334]}}{0.68786}\,(Y/pN) + \underset{\substack{(0.06536)\\ [3.29135]}}{0.21513}\,(Y/pN)_{-1} - \underset{\substack{(0.03148)\\ [-0.09688]}}{0.00305}\,(M_1/pN)_{-1}$$

Limited Information

$$C/pN = \underset{\substack{(14.67310)\\ [2.67084]}}{39.18944} + \underset{\substack{(0.06514)\\ [10.01547]}}{0.65242}\,(Y/pN) + \underset{\substack{(0.06976)\\ [3.60726]}}{0.25163}\,(Y/pN)_{-1}$$

$$- \underset{\substack{(0.03213)\\ [-0.15132]}}{0.00486}\,(M_1/pN)_{-1}$$

From these sets of regressions we see that considerable damage is done to the significance of money in Klein Model II when the sampling period is extended. The money supply variable loses its significance for this model in each case. This gives us further evidence that different specifications of models may be needed to explain pre-World War II and post-World War II macroeconomic behavior for the United States. We shall examine the dynamic properties of Klein Model II in Chapter VI. This model proves to be a very simple one to subject to dynamic analysis and will, therefore, provide a demonstration of the principles to be applied to the more sophisticated Klein Model III.

Klein Model III

Introduction

Klein Model III is a sixteen equation model in sixteen endogenous variables (W_1, p, X, I, K, H, C, Y, D_1, r, D_2, i, v, M_1^D, M_2^D, and R_1) and thirteen exogenous variables (E, t, q, q_1, ΔF, N^S, T, E_R, D_3, D'', G, W_2, and R_2).

Variables

I = net investment in private producers plant and equipment, measured in billions of constant dollars

q = price index of capital goods

p = price index of output as a whole

X = output of private sector of the economy (excluding housing services), measured in billions of constant dollars

E = excise taxes, measured in billions of current dollars

K = stock of business fixed capital, measured at the end of the year in billions of constant dollars

H = stock of inventories, measured at the end of the year in billions of constant dollars

W_1 = private wage-salary bill, measured in billions of current dollars

Y = disposable income, measured in billions of constant dollars

C = consumer expenditures, measured in billions of constant dollars

D_1 = gross construction expenditures on owner-occupied, single-family, non-farm residences, measured in billions of constant dollars

r = index of rent

q_1 = index of construction costs

D_2 = gross construction expenditures on rented, non-farm residence, measured in billions of constant dollars

i = average corporate bond yield

v = percentage of non-farm housing units occupied at the end of the year

N^S = millions of available non-farm housing units at the end of the year

M_1^D = demand deposits and circulating currency, averaged during the year, measured in billions of current dollars

M_2^D = time deposits, averaged during the year, measured in billions of current dollars

E_R = excess reserves, averaged during the year, measured in millions of current dollars

T = government revenues and corporate savings minus transfer payments minus government interest payments, all measured in billions of constant doolars

G = government expenditures on goods and services and net exports and net investment of non-profit institutions, all measured in billions of constant dollars

D_3 = gross construction expenditures on farm residences, measured in billions of constant dollars

D'' = depreciation on all residences (farm and non-farm), measured in billions of constant dollars

W_2 = government wage-salary bill, measured in billions of current dollars

R_1 = non-farm rentals, paid and imputed, measured in billions of current dollars

R_2 = farm rentals, paid and imputed, measured in billions of current dollars

p_0 = base year rent level, measured in thousands of dollars per annum

u_i = random distrubance

F = thousands of new non-farm families

t = trend variable

Equations

(1) $W_1 = \alpha_0 + \alpha_1(pX-E) + \alpha_2(pX-E)_{-1} + \alpha_3 t + u_1$

(demand for labor)

(2) $I = \beta_0 + \beta_1\left(\frac{pX-E}{q}\right) + \beta_2\left(\frac{pX-E}{q}\right)_{-1} + \beta_3 K_{-1} + \beta_4 t + u_2$

(demand for private producers' plant and equipment)

(3) $H = \gamma_0 + \gamma_1(X-\Delta H) + \gamma_2 p + \gamma_3 p_{-1} + \gamma_4 H_{-1} + \gamma_5 t + u_3$

(demand for inventories)

(4) $C = \delta_0 + \delta_1 Y + \delta_2 t + u_4$

(demand for consumer goods)

(5) $D_1 = \varepsilon_0 + \varepsilon_1\left(\frac{r}{q_1}\right) + \varepsilon_2(Y + Y_{-1} + Y_{-2}) + \varepsilon_3 \Delta F + u_5$

(demand for owner-occupied housing)

(6) $D_2 = \zeta_0 + \zeta_1 r_{-1} + \zeta_2 (q_1)_{-1} + \zeta_3 (q_1)_{-2} + \zeta_4 i +$

$+ \zeta_5 \Delta F_{-1} + u_6$

(demand for rental housing)

(7) $v = \eta_0 + \eta_1 r + \eta_2 Y + \eta_3 t + \eta_4 N^5 + u_7$

(demand-supply for dwelling space)

(8) $\Delta r = \theta_0 + \theta_1 v_{-1} + \theta_2 Y + \theta_3 \frac{1}{r_{-1}} + u_8$

(rent adjustment equation)

(9) $M_1^D = \lambda_0 + \lambda_1 p(Y+T) + \lambda_2 t + \lambda_3 p(Y+T)t + u_9$

(demand for active balances)

(10) $M_2^D = \mu_0 + \mu_1 i + \mu_2 i_{-1} + \mu_3 (M_2^D)_{-1} + \mu_4 t + u_{10}$

(demand for idle cash balances)

(11) $\Delta i = \xi_0 + \xi_1 E_R + \xi_2 i_{-1} + \xi_3 t + u_{11}$

(interest rate adjustment equation)

(12) $\Delta X = \pi_0 + \pi_1 (u_3)_{-1} + \pi_2 \Delta p + u_{12}$

(output adjustment equation)

(13) $Y + T = I + \Delta H + C + D_1 + D_2 + D_3 - D'' + G$

(definition of net national product)

(14) $X = \dfrac{p(Y+T) - W_2 - R_1 - R_2}{p}$

(definition of private output exclusive of housing)

(15) $\Delta K = I$

(definition of stock of capital)

$$(16)\ R_1 = p_0 r\left(\frac{vN^s}{100} + \frac{v_{-1}N^s_{-1}}{100}\right)\frac{1}{2}$$

(definition of rent payments).[3]

Adjustments in Structural Equations

Klein tested the above equations for the period 1922-1941. Before this model was tested for the extended period of 1922-41 and 1947-65, several minor adjustments were made in the structural equations. First, equation (16) was dropped and the status of R_1 was changed to that of an exogenous variable because of the degree to which the equation is non-linear. Second, the interest adjustment equation (11) was replaced by the equilibrium condition:

$$(11')\ M_s = M_1^D + M_2^D.$$

This adjustment allows a comparison to be made between the money supply and autonomous expenditure and is in the spirit of the FM analysis. Third, the variables p_{-1} and t were dropped from equation (3) and t was also dropped as an explanatory variable from equation (2). This third revision follows that of Klein who found these variables not to be statistically significant.

Data Revisions

Klein's raw data are for the period 1920-41. Several revisions and estimations were needed to extend the

sampling period to include the years 1945-1965 as well. Klein expressed some variables in constant 1934 dollars. In these instances we have changed the base year to 1958. The following are the data and sources used in the revised model for years 1929-1965.

C: Personal consumption expenditures (billions of 1958 dollars). For years 1929-1965 data taken from U. S. Department of Commerce, Bureau of Census <u>Long Term</u> . . . , Series A-24, pp. 170-171. To generate data for 1920-1928 consistent with that for 1929-1965 we used a regression of C_{OBE} on C_K for 1929-41 and estimated C_{OBE} from this regression for 1920-28:

$C_{OBE} = 0.5821 + 0.9508\ C_K$. See discussion of C under data revisions for Klein Model II.

I: Net fixed business investment in billions of 1958 dollars. For 1929-65:

$$I = \frac{I' - CCA_I}{p^I_{58}}$$

where

p^I_{58}: Implicit price deflator for non-residential fixed investment, (1958 = 100), U. S. Department of Commerce, OBE, "The National Income . . . ," Table 8.1, pp. 158-159, line 8.

I': Gross fixed business investment in billions of current dollars, <u>Ibid</u>., Table 1.1, pp. 2-3, line 8.

$CCA_I = CCA - CCA_D$: where

CCA: Capital consumption allowance in billions of current dollars, Ibid., Table 1.9, pp. 12-13, line 2.

CCA_D: Depreciation on all residences in billions of current dollars. This variable was calculated as shown below. For 1920-1928 we regressed our I on Klein's I for 1929-41 and estimated I for 1920-28 from

$$I_{58} = 1.031 + 3.702\ I_{34}.$$

q: Price index of fixed investment, (1958 = 100), Ibid., Table 8.1, pp. 158-159, line 7. For 1920-28 we regressed our q_{58} on Klein's q for 1929-41 and estimated q_{58} for 1920-28 from

$$q_{58} = .254 + .345\ q_{34}.$$

ΔH: Net change in inventories (billions of 1958 dollars), Ibid., Table 1.2, pp. 4-5, line 14. For 1920-28 we used regression $\Delta H_{38} = 0.838 + 2.506\ \Delta H_{34}$

D_1: Gross expenditure on construction of owner-occupied, single family, non-farm residences, billions of 1958 dollars.

$D_1 = \frac{D'}{q1}$ where

D': Estimated construction cost of single family, non-farm starts in millions of current dollars converted to billions, 1945-1965,

Department of Housing and Urban Development, Housing Statistics--Annual Data (Washington, D.C., Vol, XIX, No. 5, May, 1966), Table A-5, p. 4, column 5 (old series was extended to the years 1960 through 1965 by taking ratio of old series to new series figures for 1959 (0.890) and multiplying new series figures for 1960-65 by this ratio). For 1920-1941 Klein's D_1 (1934 dollars) was multiplied by Klein's q_1 (1934 = 100) and then divided by q_1 (1958 = 100).

D_2: Gross expenditure on construction of rented, non-farm residences, measured in billions of 1958 dollars.

$D_2 = D - D_1$

D = non-farm building--new construction, U. S. Department of Commerce, OBE, "The National Income . . . ," Table 5.3, pp. 82-83, line 5. For 1920-28 we used $D_2^{58} = .700 + 3.754\ D_2^{34}$.

D_3: Gross private domestic investment on residential structures--farm (billions of 1958 dollars), Table 1.2, line 13, pp. 4-5, Ibid.

D'': Depreciation of all residences (farm and non-farm) in billions of 1958 dollars. For years 1929-1965:

$D'' = \dfrac{CCA'_D}{p'_{D_{58}}}$ where

$p'_{D_{58}}$ = Implicit price deflator for construction expenditures on all residences (1958 = 100), Ibid., Table 8.1, pp. 58-59, line 11.

CCA'_D:

(a) For 1934: $CCA'_D = (67.6)\ (0.03) + (D'_{34})\ (.015)$ where

67.6 is the value in billions of dollars, Jan. 1, 1934, of the stock of residential dwellings in the United States, L. R. Klein, Economic Fluctuations . . . , p. 148.

0.03 represents 3% depreciation over the year of that housing in existence at beginning of year.

D': Gross expenditures on construction of residential structures, billions of current dollars, U. S. Department of Commerce, OBE, "The National Income . . . ," Table 1.1, pp. 2-3, line 11.

0.015 represents 1.5% depreciation (on the average) of housing build during the year. The implicit assumption is that housing is constructed uniformly over the year.

(b) For years 1935-1965: $CCA'_{D_t} = D_{A_t} + D_{B_t} + B_{C_t} + D^*_t$ where

$D_{A_t} = (67.6)\ (0.97)^{t-1934}\ (0.03).$

This is the depreciation during the year on that fraction still in existence on January 1 of the year t of housing in existence on January 1, 1934.

$D_{B_t} = (D_{1_t})\ (0.015)$

$D^*_t = (D'_{t-1})\ (0.985)\ (0.03).$ 98.5% of last years construction remains on Jan. 1 of year t. During year t it depreciates at 3% rate.

$D_{C_t} =$

$D_{C_{1935}} = .97D^*_{1934}$ but $D^*_{1934} = 0$

$D_{C_{1936}} = .97D_{C_{1935}} + .97D^*_{1935} = .97D^*_{1935}$; for $t > 1936$ $D_{C_t} = .97\ (D_{C_{t-1}} + D^*_{t-1})$

(c) <u>For years 1929-33</u>--Part of the value of 67.6 which was in existence on January 1, 1934 was not built by January 1, 1933. Klein begins by taking 67.6 and multiplying this value by $(0.97)^{-1}$, the

value which would have existed on January 1, 1933 if all of the 67.6 value were due to housing in existence prior to January 1, 1933. This product is then multiplied by 0.03 to find the 3% depreciation. However, $(0.97)^{-1}$ D' of the (67.6) $(0.97)^{-1}$ value was created during 1933 and should not have been depreciated by 3%. Assuming housing was added uniformly over 1933, the depreciation on D'_{1933} is only (D'_{1933}) (0.015) $(0.97)^{-1}$ and therefore the actual depreciation for 1933 is $CCA_{D'} = 67.6$ $(0.97)^{-1}$ $(0.03) - (D'_{1933})$ (0.015) $(0.97)^{-1}$.

1. For 1932: The value which existed on Jan. 1, 1933 is equal to the value existing on Jan. 1, 1934, 67.6, less the net value added during 1933, $D'_{33} - CCA_{D'_{33}}$. Therefore, if all the value existing on Jan. 1, 1933 were also in existence on Jan. 1, 1932, the depreciation during 1932 would be: $[67.6 - (D'_{33} - CCA_{D'_{33}})]$ $(0.97)^{-1}$ (0.03).

However, the amount D'_{32} was not in existence on Jan. 1, 1932 and therefore the depreciation for 1932 is not $[67.6 - (D'_{33} - CCA_{D'_{33}})]\ 0.97^{-1}\ (0.03)$, but rather $[67.6 - (D'_{33} - CCA_{D'_{33}})]\ (0.97)^{-1}\ (0.03) - D'_{32}\ (0.97)^{-1}\ (0.015)$.

2. <u>In general, for years prior to 1933</u>:

$$CCA_{D'_i} = [67.6 - \sum_{i=t+1}^{1933} (D'_i - CCA_{D'_i})]\ \frac{0.03}{0.97} - D_t(\frac{0.015}{0.97}).$$

This formula is the same as Klein's (p. 148) except that his involves D_1, D_2, and D_3 in 1934 dollars. Parenthetically, Klein's formula for years later than 1934 is incorrect. According to his expression, D'' for 1935 would be: (1) $D''_{35} = (67.6)\ (0.97)\ (0.03) + (D_1 + D_2 + D_3)_{1934}\ (0.985)\ (0.97)^{-1}\ (0.03) + (D_1 + D_2 + D_3)_{1935}\ (0.015)$. Substituting our notation for Klein's D'' and $(D_1 + D_2 + D_3)_i$

equation (1) becomes $CCA_{D'_{35}}$ = (67.6) (0.97) (0.03) + D'_{34} (0.985) $(0.97)^{-1}$ (0.03) + D'_{35} (0.015). However, the power of (0.97) in the second term should be zero and not minus 1.

(d) For years 1920-1929: Since the index of construction prices which we have been using as a deflator for CCA_D has been computed back to only 1929, a different approach must be followed to find D'' in 1958 dollars for the years 1920-1929. This is accomplished by regressing D'' in 1958 dollars for the years 1929-41 on D'' in 1934 dollars as computed by Klein and then using the resulting equation to estimate the earlier D'' (1958 dollars) figures using Klein's calculations. The equation used was $D''_{58} = 3.238 + 1.449\, D''_{34}$.

G: Government expenditures on goods and services and net exports (billions of 1958 dollars). For 1929-1965, U. S. Department of Commerce, Bureau of the Census, Long Term . . . , Series A32 and A34, pp. 170-173. For 1920-1928 used regression $G_{58} = 2.00 + 2.426\ G_{34}$.

Y + T: Net national product (billions of 1934 dollars).

$$Y + T = C + I + H + D_1 + D_2 + D_3 - D'' + G.$$

Y: Disposable income, measured in billions of 1958 dollars, Ibid., Series A41, pp. 172-173. For 1920-28 used regression $Y_{58} = 13.292 + 2.143\ Y_{34}$.

p: Price index of GNP (1958 = 100), U. S. Department of Commerce, OBE, "The National Income . . . ," Table 8.1, pp. 158-159, line 1; p_{58} for 1920-28 was generated using $p_{58} = 8.677 + 0.3440\ p_{34}$.

W_1: Private wage-salary bill in billions of current dollars, Ibid., Table 6.2, pp. 94-95, line 87, and pp. 96-97, line 85. For 1920-28 $W_{1_{58}}$ was found using the regression $W_{1_{58}} = 1.415 + 0.9139\ W_{1_{34}}$.

W_2: Government and government enterprises wages and salaries (millions of dollars converted to billions of dollars), Ibid., Table 6.2, pp. 94-95, line 73, and pp. 96-97, line 71. For 1920-28 used $W_{2_{58}} = -0.480 + 1.089\ W_{2_{34}}$.

R_1: Non-farm rentals, paid and imputed, for residential dwellings (billions of dollars).

$R_1 = R^{58}_{1,0} \times p^{58}_{R_{1,0}} + R^{58}_{1,T} \times p^{58}_{R_{1,T}}$ where

$R^{58}_{1,0}$: Space rental value of owner-occupied non-farm dwellings (billions of 1958 dollars), Ibid., Table 2.6, pp. 48-49, line 36.

$R^{58}_{1,T}$: Space rental value of tenant-occupied non-farm dwellings (billions of 1958 dollars), Ibid., Table 2.6, pp. 48-49, line 37.

$p^{58}_{R_{1,0}}$: Implicit price deflator for space rental value of owner-occupied non-farm dwellings (1958 = 100), Ibid., Table 8.6, pp. 162-163, line 36.

$p^{58}_{R_{1,T}}$: Implicit price deflator for space rental value of tenant-occupied non-farm dwellings (1958 = 100), Ibid., Table 8.6, pp. 162-163, line 37. For 1920-28 used $R_{1_{58}} = 0.352 + 1.037\ R_{1_{34}}$.

R_2: Farm rentals (billions of dollars).

$R_2 = R^{58}_2 \times p^{58}_{R_2}$ where

R^{58}_2: Rental value of farm houses (billions of 1958 dollars), Ibid., Table 2.6, pp. 48-49, line 38.

$p^{58}_{R_2}$: Implicit price deflator for rental value of farm houses (1958 = 100), Ibid., Table 8.6, pp. 162-163, line 38. For 1920-28 used $R_{2_{58}} = 0.100 + 0.939\ R_{2_{34}}$.

r: Index of rents (1958 = 100), Ibid., Table 8.6, pp. 162-163, line 35. For 1920-28 used $r_{58} = 3.423 + 0.523\ r_{34}$.

E: Excise taxes in billions of current dollars.

$E = E_{SL} + E_F$ where

E_{SL}: State and local sales taxes (millions of current dollars converted to billions of current dollars), Ibid., Table 3.3, pp. 54-55, line 11.

E_F: Federal excise taxes (millions of dollars converted to billions of dollars), Ibid., Table 3.1, pp. 52-53, line 11, For 1920-28 used $E_{58} = -0.02 + 0.9683\ E_{34}$.

q_1: Index of construction costs (1958 = 100), Ibid., Table 8.7, pp. 165-165, line 5. For 1920-28 used regression $q_{1_{58}} = 1.200 + 0.282\ q_{1_{34}}$.

i: Average corporate bond yield, Board of Governors, Federal Reserve Bulletin, Table--"Bond and Stock Yields," column entitled "average total corporate bonds yields."

K: End-of-year stock of fixed business investment, billions of 1958 dollars.

$K_t = \frac{107.8}{34.6} \times 100 = 311.6 \quad t = 1934$ where

107.8 = Klein's value for K for 1934 in current (i.e. 1934) dollars.

34.9 = Price deflator for the year 1934 for non-residential fixed investment (1958 = 100), U. S. Department of Commerce, OBE, "The National Income . . . ," Table 8.1, pp. 158-159, line 8.

$$K_t = 311.6 + \sum_{i=1935}^{t} I_i \qquad t > 1934$$

$$K_t = 311.6 - \sum_{i=t+1}^{1934} I_i \qquad t < 1934$$

H: End-of-year stock of inventories, billions of 1958 dollars.

$$H_t = \frac{21.8}{40.8} \times 100 = 53.4 \qquad t = 1934$$

21.8 = Klein's value for H for end of 1934 in current (i.e. 1934) dollars.

40.8 = Derived from wholesale price index, all commodities, BLS (1957-1959 = 100) as shown in series B69, U. S. Department of Commerce, Bureau of Census, <u>Long Term</u> . . . , pp. 201-202. Series shows 1958 index to be 100.4 and 1934 index to be 41.0. Index for 1934 to base 1958 was computed as $\frac{41.0}{100.4} \times 100 = 40.8$.

$$H_t = 53.4 - \sum_{i=t+1}^{1934} (\Delta H)_i \qquad t < 1934$$

$$H_t = 53.4 + \sum_{i=1935}^{t} (\Delta H)_i \qquad t > 1935$$

ΔF: Thousands of new non-farm families.

$\Delta F_t = F_t - F_{t-1}$ where

F: Number of non-farm families (converted to thousands), U. S. Department of Commerce, Bureau of the Census, Current Population Reports, Series P 20, "Population Characteristics," Households and Families by Type, various March issues, Table--Households by Type by Color of Head and Residence for the United States for 1945-1965. ΔF_t for 1920-41 were taken from Klein's data, p. 144.

N^s: Millions of available non-farm dwelling units at the end of the year.

N^s = Klein's data 1920-1928

$$N^s_t = 24.6 + \sum_{i=1929}^{t} S_i \quad \text{where}$$

24.6 = Klein's value for end of 1928.

S_i = New non-farm housing units started in thousands (converted to millions), U. S. Department of Commerce, Bureau of the Census, Construction Reports, Series C 20, various issues.

v: Percentage of non-farm housing units occupied at the end of the year.

For 1920-41 Klein's data was used.

For 1956-1965 data from "Vacant Housing Units in the United States: 1956 to 1965," Current Housing Reports--Housing Vacancies, Series H-111, No. 43, June, 1966, U. S. Department of Commerce, Bureau of the Census (Washington, D.C.), Table 1, Annual Average Vacancy Rates by Condition and Type of Vacancy for the United States, Inside and Outside Standard Metropolitan Statistical Areas and Regions: 1956 to 1965, p. 22, line 22.

For 1941 to 1956: "v" was looked upon as the ratio of the number of non-farm housing units occupied to the variable N^S. The numerator was calculated for the years 1956-1965 by $v_t \cdot N_t^S = O_t$. O_t was then regressed on F_t for 1956-1965 and, then, O_t was estimated for the period 1941 to 1956. The estimates of O_t along with the values of N^S were then substituted into $v_t \cdot N_t^S = O_t$ to obtain estimates of v_t for the years, 1941 to 1956.

M_1^D: Money supply billions of dollars. For 1920-1946, U. S. Department of Commerce, Bureau of the Census, Long Term . . . , pp. 208-209, Series B109. For 1947-1965 centered means of seasonally adjusted monthly data of total money supply, Board of

Governors, Federal Reserve Bulletin, June, 1964, pp. 682-692; June, 1965, p. 978; May, 1966, p. 678; and April, 1967, p. 608; Table--Money Supply and Related Data (in billions of dollars).

M^S: Money supply plus time deposits in billions of dollars. For 1920-1946 U. S. Department of Commerce, Bureau of Census, Long Term . . . , pp. 208-209, Series B111. For 1947-1965 centered means of seasonally adjusted monthly data of total money supply plus time deposits, Board of Governors, Federal Reserve Bulletin, June, 1964, pp. 682-692; June, 1965, p. 978; May, 1966, p. 678; and April, 1967, p. 608. Table--Money Supply and Related Data (billions of dollars).

Estimation Procedure

Let us consider the problem of estimating the structural coefficients by the method of 2SLS. For this we need the reduced form equations for Y, X, p, ΔH, i, and r. Beginning with a linear approximation of equation (14), we have

$$(17)\ X = (Y+T) + \phi_0 + \phi_1 p + \phi_2(W_2 + R_1 + R_2).$$

From (12) and (3), setting $\gamma_3 = 0$, we find

$$(18)\ p = \pi_0' + \pi_1' p_{-1} + \pi_2'(Y+T) + \pi_3'(W_2 + R_1 + R_2) + \pi_4' X_{-1} + \pi_5' H_{-1} + \pi_6' H_{-2}.$$

Also from (8), setting $1/r_{-1} = z$

$$(19)\ r = \theta_0' + \theta_1' v_{-1} + \theta_2' r_{-1} + \theta_3' Y + \theta_4' z.$$

Substituting for M_1^D from (9) into (11') (after taking a linear approximation of (9)), then substituting for M_2^D from (11') into (10), and then solving (10) for i we have

$$(20)\ i = \mu_0' + \mu_1' M_s + \mu_2' p + \mu_3'(Y+T) + \mu_4' t + \mu_5' i'_{-1} + \mu_6'(M_2^D)_{-1}.$$

Finally, from (3) setting $\gamma_3 = 0$, we see that

$$(21)\ \Delta H = \gamma'_0 + \gamma_1' X + \gamma_2' p + \gamma_3' H_{-1} + \gamma_4' t.$$

Viewing equations (17) - (21) it is now clear that finding the reduced form equation for Y will immediately yield the reduced forms of r and p. The reduced form for Y along with that for p will then give the reduced forms for X and i. Finally, substituting the reduced forms for X and p into (21) will result in the reduced form equation for ΔH. All that remains is the derivation of the reduced form for Y.

Beginning with equation (13)

$$(13)\ Y + T = I + \Delta H + C + D_1 + D_2 + (D_3 - D'' + G)$$

and substituting for C and I, yields

$$(22)\quad Y + T = \alpha_0 + \alpha_1 p + \alpha_2 X + \alpha_3 E + \alpha_4 q + \alpha_5 X_{-1} + \alpha_6 K_{-1} + \alpha_7 t + \alpha_8 H + \alpha_9 H_{-1} + \alpha_{10} D_1 + \alpha_{11} D_2 + \alpha_{12}(D_3 - D'' + G) + \alpha_{13} T + \alpha_{14} p_{-1} + \alpha_{15} E_{-1} + \alpha_{16} q_{-1}.$$

Substituting for H:

$$(23)\quad Y + T = \beta_0 + \beta_1 p + \beta_2 X + \beta_3 E + \beta_4 q + \beta_5 X_{-1} + \beta_6 K_{-1} + \beta_7 t + \beta_8 H_{-1} + \beta_9 p_{-1} + \beta_{10} D_1 + \beta_{11} D_2 + \beta_{12}(D_3 - D'' + G) + \beta_{13} T + \beta_{14} E_{-1} + \beta_{15} q_{-1}$$

and then substituting for D_1, D_2, and r into (23) yields

$$(24)\quad Y + T = \delta_0 + \delta_1 p + \delta_2 X + \delta_3 E + \delta_4 q + \delta_5 X_{-1} + \delta_6 K_{-1} + \delta_7 t + \delta_8 H_{-1} + \delta_9 p_{-1} + \delta_{10} v_{-1} + \delta_{11} q_1 + \delta_{12} Y_{-1} + \delta_{13} Y_{-2} + \delta_{14} \Delta F + \delta_{15} r_{-1} + \delta_{16}(q_1)_{-1} + \delta_{17}(q_1)_{-2} + \delta_{18} i + \delta_{19} \Delta F_{-1} + \delta_{20} T + \delta_{21}(D_3 - D'' + G) + \delta_{22} q_{-1} + \delta_{23} E_{-1}.$$

Substituting for M_2^D from (11') and then substituting for i from (10) we have

$$(25)\quad Y + T = \eta_0 + \eta_1 p + \eta_2 X + \eta_3 E + \eta_4 q + \eta_5 X_{-1}$$
$$+ \eta_6 K_{-1} + \eta_7 t + \eta_8 H_{-1} + \eta_9 p_{-1}$$
$$+ \eta_{10} v_{-1} + \eta_{11} q_1 + \eta_{12} Y_{-1} + \eta_{13} Y_{-2}$$
$$+ \eta_{14} \Delta F + \eta_{15} r_{-1} + \eta_{16} (q_1)_{-1}$$
$$+ \eta_{17} (q_1)_{-2} + \eta_{18} i'_{-1} + \eta_{19} M_s$$
$$+ \eta_{20} M_1^D + \eta_{21} (M_2^D)_{-1} + \eta_{22} \Delta F_{-1}$$
$$+ \eta_{23} T + \eta_{24} (D_3 - D'' + G) + \eta_{25} E_{-1}$$
$$+ \eta_{26} q_{-1}.$$

Substituting for M_1:

$$(26)\quad Y + T = \eta'_0 + \eta'_1 p + \eta'_2 X + \eta'_3 E + \eta'_4 q + \eta'_5 X_{-1}$$
$$+ \eta'_6 K_{-1} + \eta'_7 t + \eta'_8 H_{-1} + \eta'_9 p_{-1}$$
$$+ \eta'_{10} v_{-1} + \eta'_{11} q_1 + \eta_{12} Y_{-1} + \eta'_{13} Y_{-2}$$
$$+ \eta'_{14} \Delta F + \eta'_{15} r_{-1} + \eta'_{16} (q_1)_{-1}$$
$$+ \eta_{17} (q_1)_{-2} + \eta_{18} i + \eta'_{19} M_s + \eta'_{20} (M_2^D)_{-1}$$
$$+ \eta'_{21} \Delta F_{-1} + \eta'_{22} T + \eta'_{23} (D_3 - D'' + G)$$
$$+ \eta'_{24} E_{-1} + \eta'_{25} q_{-1}.$$

Substituting for X:

$$(27)\quad Y + T = \rho_0 + \rho_1 p + \rho_2 (W_2 + R_1 + R_2) + \rho_3 E$$
$$+ \rho_4 q + \rho_5 X_{-1} + \rho_6 K_{-1} + \rho_7 t + \rho_8 H_{-1}$$
$$+ \rho_9 p_{-1} + \rho_{10} v_{-1} + \rho_{11} q_1 + \rho_{12} Y_{-1}$$

$$+ \rho_{13}Y_{-2} + \rho_{14}\Delta F + \rho_{15}r_{-1}$$
$$+ \rho_{16}(q_1)_{-1} + \rho_{17}(q_1)_{-2} + \rho_{18}i'$$
$$+ \rho_{19}M_s + \rho_{20}(M_2^D)_{-1} + \rho_{21}\Delta F_{-1} + \rho_{22}T$$
$$+ \rho_{23}(D_3 - D'' + G) + \rho_{24}E_{-1} + \rho_{25}q_{-1}$$

Substituting for X into (12), then substituting for p into (27):

$$(28)\ Y + T = \omega_0 + \omega_1 X_{-1} + \omega_2 H_{-2} + \omega_3 p_{-2} +$$
$$+ \omega_4(W_2 + R_1 + R_2) + \omega_5 E + \omega_6 q + \omega_7 E_{-1}$$
$$+ \omega_8 K_{-1} + \omega_9 t + \omega_{10} H_{-1} + \omega_{11} p_{-1}$$
$$+ \omega_{12} v_{-1} + \omega_{13} q_1 + \omega_{14} Y_{-1} + \omega_{15} Y_{-2}$$
$$+ \omega_{16}\Delta F + \omega_{17} r_{-1} + \omega_{18}(q_1)_{-1} + \omega_{19}(q_1)_{-2}$$
$$+ \omega_{20} i_{-1} + \omega_{21} M_s + \omega_{22}(M_2^D)_{-1} + \omega_{23}\Delta F_{-1}$$
$$+ \omega_{24} T + \omega_{25}(D_3 - D'' + G) + \omega_{26} q_{-1}.$$

Since equations (17) - (21) do not contain any more predetermined variables than those given by the reduced form for Y + T (and therefore for Y), the reduced form equations for X, ΔH, p, i, and r will contain precisely the predetermined variables given by equation (28). Equation (28) involves 26 predetermined variables. Because we wish to re-estimate Klein Model III for the period 1922-1941 and inspect the damage to Klein's structural parameters resulting from our manipulation of the variables, we cannot estimate the reduced form coefficients. We have data for only the 22 years from 1920-1941, thus there are too few observations to calculate all 27 coefficients of the reduced form. Also, two observations must be dropped to allow for the lagged variables.

Variables Participation Matrix (VPM)

What is needed, then, is a method to decide the subsets of predetermined variables in the original reduced forms to include in the moidfied reduced form equations of the endogenous variables. In the spirit of Franklin Fisher[4] we first set up a variables participation matrix. This matrix is a transform of the coefficients matrix, B, of the structural equations:

$$BY_t + \Gamma Z_t = u_t.$$

We transform B into the VPM by replacing the b_{ij}'s such that if $b_{ij} = 0$ we write 0, and if $b_{ij} \neq 0$ we write "$\pm$".

The variables participation matrix provides *a priori* structural information which can be used to decide which predetermined variables to include in the first stage of the 2SLS method. As can be seen from the VPM below, the Klein Model III is an integrated structure. An integrated structure is one for which the coefficients matrix, B, is neither block-diagonal nor block-triangular.

FIGURE 1.--Variables participation matrix for Klein Model III.

	W_1	p	X	H	K	I	Y	C	D_1	r	v	D_2	i	M_1^D	M_2^D
W_1	±	±	±	0	0	0	0	0	0	0	0	0	0	0	0
p	0	±	±	0	0	0	0	0	0	0	0	0	0	0	0
X	0	±	±	0	0	0	±	0	0	0	0	0	0	0	0
H	0	±	±	±	0	0	0	0	0	0	0	0	0	0	0
K	0	±	±	0	±	0	0	0	0	0	0	0	0	0	0
I	0	0	0	0	±	±	0	0	0	0	0	0	0	0	0
Y	0	0	0	±	0	±	±	±	±	0	0	±	0	0	0
C	0	0	0	0	0	0	±	±	0	0	0	0	0	0	0
D_1	0	0	0	0	0	0	±	0	±	±	0	0	0	0	0
r	0	0	0	0	0	0	±	0	0	±	0	0	0	0	0
v	0	0	0	0	0	0	±	0	0	±	±	0	0	0	0
D_2	0	0	0	0	0	0	0	0	0	0	0	±	±	0	0
i	0	0	0	0	0	0	0	0	0	0	0	0	±	0	±
M_1^D	0	±	0	0	0	0	±	0	0	0	0	0	0	±	0
M_2^D	0	0	0	0	0	0	0	0	0	0	0	0	0	±	±

If the coefficients matrix were "block-diagonal" the structure would be defined as "non-integrated"; if the matrix were "block-triangular" the structure would be defined as "causal" or "block recursive." These classifications pertain to linear structures. The Klein Model III is not linear. However, this situation can be remedied by taking a linear approximation of the structural equations in the system for the purpose of deriving the VPM.

While the VPM for Klein Model III falls under the integrated category, it is quite close to being block-triangular and therefore approximates the causal variety. In fact, it is nearly block-diagonal. Because of this the model may be viewed as composed of three sectors and this *a priori* information used to decide the variables to include as the predetermined variables in the first stage of the 2SLS method.

Predetermined variables included in the first stage.--The current endogenous variables p, ΔH, X, Y, r, and i appear as explanatory variables in the structural equations. We decide on the predetermined variables to include in estimating the reduced form equations of these current endogenous variables by noticing, first, the sector into which each variable falls in the VPM. Since the equations for H, p, and X fall in the first sector (numbering the sectors along the principal diagonal from left to right), we decide to include in the "modified reduced forms" for H, p, and X those predetermined variables appearing in the structural equations of sector one. Clearly q, t, E $(W_2 + R_1 + R_2)$, K_{-1}, E_{-k}, q_{-1}, ρ_{-1}, and X_{-1} belong to this set as can be seen from an examination of structural equations (1), (2), (3), (12), (14), and (15). Since (12) includes the term $(u_3)_{-1}$, we also include H_{-2} and p_{-2} (this time we allow γ_3 to be non-zero).

Since X is directly affected by Y + T which in turn is directly affected by $(D_3 - D'' + G)$, we include $(D_3 - D'' + G)$ as well in these modified reduced forms. H_{-1} was included in the modified reduced form for p, but was excluded from the first stage estimation of ΔH and X. The reason it was excluded from participating in the estimation of ΔH is obvious. ΔH is by definition $H - H_{-1}$; hence we would expect to find a very high dependency between H_{-1} and the distrubance term in the equation estimating ΔH. The reason H_{-1} was excluded from the reduced form for X is procedural rather than theoretical--as it happened, the reduced form estimates for ΔH and X were run simultaneously on the computer and it was easier to leave H_{-1} out altogether.

Sector two involves the structural equations for Y, C, D_1, r, and v as shown by the VPM. Even though we originally joined D_2 with the variables i, M_1^D, and M_2^D to form sector three, D_2 is a part of the housing market and therefore is closely mated to the variables D_1, r, and v.

Also, the nature of the VPM would not be altered significantly by constructing sector two to include D_2. All that happens is that the "$\pm$" in cell (Y, D_2) now becomes a part of the second diagonal matrix and the "$\pm$" in cell (D_2, i) falls above the third diagonal matrix. In either case, the influence of the monetary sector on the other sectors of the model will operate through only one variable. As the VPM was originally constructed, the monetary sector influenced Y through its effect on D_2. When sector two is expanded to include D_2, changes in the monetary sector influence sector two via the interest rate.

Nevertheless, the truncated reduced forms for r and Y were estimated using the predetermined variables appearing in the structural equations for Y, C, D_1, r, and D_2. Thus, r and Y were estimated using the variables: q_1, K_{-1}, t, ΔF, N^S, $(D_3 - D'' + G)$, T, H_{-1}, Y_{-1}, Y_{-2}, r_{-1}, $(q_1)_{-1}$, $(q_1)_{-2}$, ΔF_{-1}, and v_{-1}. The variable K_{-1} crept into these equations by substituting for I in equation (13) its identical counterpart, $K - K_{-1}$.

The truncated reduced form for i was estimated using the variables: M_s, $(M_2^D)_{-1}$, i_{-1}, r_{-1}, $(q_1)_{-1}$, $(q_1)_{-2}$, ΔF_{-1}, t, $(D_3 - D'' + G)$, K_{-1}, and H_{-1}. The first eight variables enter the equations of sector three--(6), (9), (10), and (11')--directly. The remaining variables entered the estimated equation through their direct in-influence on Y + T.

Second Stage of 2SLS

After the estimates of the current endogenous variables, r, p, Y, ΔH, X, and i, were found, they were substituted into the structural equations as explanatory variables. Then the second stage of 2SLS was taken by performing ordinary least squares with the resulting set of variables. The equations were first estimated for the period 1922-1941 in order to find some indication of the damage to Model III caused by the revisions discussed earlier.

Results

Klein's Results

The following are the estimates which Klein obtained using the "method of reduced forms" (now commonly referred to as limited information maximum likelihood).

$$W_1 = 5.04 + 0.41\ (pX-E) + 0.17\ (pX-E)_{-1}$$
$$+ 0.17\ (t-1931)$$

$$I = 2.59 + 0.12\ \left(\frac{pX-E}{q}\right) + 0.04\ \left(\frac{pX-E}{q}\right)_{-1} - 0.10\ K_{-1}$$

$$H = 1.17 + 4.60\ p + 0.12\ (X-\Delta H) + 0.50\ H_{-1}$$

$$C = 11.87 + 0.73\ Y + 0.04\ (t-1931)$$

$$D_1 = -9.03 + 3.74\ \left(\frac{r}{q_1}\right) + 0.02\ (Y + Y_{-1} + Y_{-2})$$
$$+ 0.0043\ \Delta F$$

$$D_2 = -2.14 + 2.81\ r_{-1} + 0.02\ (q_1)_{-1} - 0.44\ (q_1)_{-2}$$
$$+ 0.0016\ \Delta F_{-1} - 0.18\ i$$

$$v = 178.01 + 0.29\ Y - 2.62\ r + 1.42\ (t-1931) - 3.76\ N^s$$

$$\Delta r = -2.15 + 0.02\ v_{-1} + 0.00071\ Y + 0.17\ \frac{1}{r_{-1}}$$

$$M_1^D = 8.45 + 0.24\ p(Y+T) + 0.03\ p(Y+T)(t-1931) - 1.43\ (t-1931)$$

$$M_2^D = 15.37 + 0.28\ i - 1.90\ i_{-1} + 0.74\ (M_2^D)_{-1} - 0.18\ (t-1931)$$

$$\Delta X = 2.55 - 4.46\ (u_3)_{-1} + 82.76\ \Delta p$$

<u>Reestimation of Klein Model III for Period 1922-1941</u>

The following are the 2SLS estimates of Klein Model III derived from the use of revised data, the replacement of Klein's equation (11) with equation (11'), and the treatment of R_1 as exogenously determined. All equations except the output adjustment equation (equation (12)) were estimated for 1922-1941. Equation (12) was estimated for 1923-1941 since it contains as an explanatory variable the residual of equation (3) lagged one period.

$$(1^*)\quad W_1 = 7.26768 + \underset{(0.03373)}{0.40159}\ (pX-E) + \underset{(0.03886)}{0.09670}\ (pX-E)_{-1} + \underset{(0.05152)}{0.02953}\ t, \qquad R^2 = 0.9713$$

$$(2^*)\quad I = 7.03134 + \underset{(0.02284)}{0.12187}\left(\frac{pX-E}{q}\right) + \underset{(0.02435)}{0.06759}\left(\frac{pX-E}{q}\right)_{-1} - \underset{(0.01817)}{0.10617}\ K_{-1}, \qquad R^2 = 0.9343$$

$$(3^*)\quad H = \underset{}{-3.25596} + \underset{(7.54425)}{18.76474}\,p + \underset{(0.01601)}{0.14090}\,(X-\Delta H)$$

$$+ \underset{(0.06659)}{0.63565}\,H_{-1}, \qquad R^2 = 0.9500$$

$$(4^*)\quad C = 28.78560 + \underset{(0.04643)}{0.63131}\,Y + \underset{(0.14710)}{0.98523}\,t,$$

$$R^2 = 0.9685$$

$$(5^*)\quad D_1 = -24.76041 + \underset{(0.87043)}{5.20749}\,(r/q_1)$$

$$+ \underset{(0.00488)}{0.03038}\,(Y+Y_{-1}+Y_{-2}) + \underset{(0.0157)}{0.01106}\,\Delta F,$$

$$R^2 = 0.7913$$

$$(6^*)\quad D_2 = -5.36336 + \underset{(5.01210)}{22.81729}\,r_{-1} - \underset{(14.96885)}{6.89784}\,(q_1)_{-1}$$

$$- \underset{(7.91248)}{1.63128}\,(q_1)_{-2} + \underset{(0.00224)}{0.00507}\,\Delta F_{-1}$$

$$- \underset{(0.52135)}{1.30531}\,i, \qquad R^2 = 0.8731$$

$$(7^*)\quad v = 158.45360 + \underset{(0.02441)}{0.09550}\,Y + \underset{(6.84348)}{2.71645}\,r$$

$$+ \underset{(0.35635)}{1.69510}\,t - \underset{(0.44540)}{3.86616}\,N^s, \qquad R^2 = 0.9179$$

$$(8^*)\quad \Delta r = -1.16478 + \underset{(0.00116)}{0.01079}\,v_{-1} + \underset{(0.00017)}{0.00036}\,Y$$

$$+ 0.05138\,\frac{1}{r_{-1}}, \qquad R^2 = 0.8701$$

$$(9^*)\quad M_1^D = 19.92253 + \underset{(0.05200)}{0.00575}\,p(Y+T)$$

$$+ \underset{(0.00321)}{0.01715}\,p(Y+T)t - \underset{(0.26779)}{0.79942}\,t, \quad R^2 = 0.9781$$

$$(10^*)\quad M_2^D = 18.01618 - \underset{(0.36626)}{1.39971}\,i - \underset{(0.35205)}{0.75883}\,i_{-1}$$

$$+ \underset{(0.05848)}{0.81421}\,(M_2^D)_{-1} - \underset{(0.03272)}{0.32694}\,t, \quad R^2 = 0.9715$$

(11*) $M_s = M_1^D + M_2^D$

(12*) $\Delta X = 6.28263 - \underset{(1.81979)}{2.50434}\,(u_3)_{-1}$

$$+ \underset{(132.32889)}{635.70364}\,\Delta p, \qquad R_2 = 0.6340$$

(13*) $Y + T = I + \Delta H + C + D_1 + D_2 + D_3 - D'' + G$

(14*) $X = \dfrac{p(Y+T) - W_2 - R_1 - R_2}{p}$

(15*) $\Delta K = I$

Eleven coefficients appearing in the revised set of sample regressions appear to differ significantly from those Klein obtained. Aside from the coefficient tied to t in equation (4*) and the one associated with p(Y+T) in equation (9*), all coefficients which blatantly deviate from Klein's are associated with indexes of prices or interest rates. In equation (6*) the coefficients of r_{-1}, $(q_1)_{-1}$, $(q_1)_{-2}$, and i are not of roughly the same magnitude as those previously found. Other sizeable differences between Klein's estimates and those presented here are connected to p of equation (3*), Δp of equation (12*), r of equation (7*), $\frac{1}{r_{-1}}$ of equation (8*), and i of equation (10*).

While eleven coefficients contained in the eleven stochastic equations differ from the order of magnitude computed by Klein, the picture is brighter in terms of the number of variables which are no longer statistically significant at the 95 per cent level of confidence. The adjustments we have made in Klein Model III have caused only the coefficients of p(Y+T) in equation (9*), $(q_1)_{-1}$ and $(q_1)_{-2}$ in equation (6*), r in equation (7*), and $(u_3)_{-1}$ in equation (12*) to become not significantly different from zero. Because the damage inflicted seems to be relatively minor, we proceed to estimate Model III for the extended period 1922-41 and 1947-1965.

Results for the Extended Period

For the extended period, the sample regression equations are:

$$(1^{**})\quad W_1 = -1.10413 + \underset{(0.05223)}{0.41425}\,(pX-E) + \underset{(0.05400)}{0.22280}\,(pX-E)_{-1} + \underset{(0.11805)}{0.01825}\,t, \quad R^2 = 0.9985$$

$$(2^{**})\quad I = 10.85164 + \underset{(0.02101)}{0.10246}\left(\frac{pX-E}{q}\right) + \underset{(0.02211)}{0.07957}\left(\frac{pX-E}{q}\right)_{-1} - \underset{(0.00822)}{0.11434}\,K_{-1}, \quad R^2 = 0.9342$$

$$(3^{**})\quad H = 11.65666 + \underset{(0.02406)}{0.15125}\,(X-\Delta H) - \underset{(8.17592)}{9.04019}\,p + \underset{(0.07286)}{0.57622}\,H_{-1}, \quad R^2 = 0.9939$$

$$(4^{**})\quad C = 4.96737 + \underset{(0.02826)}{0.83644}\,Y + \underset{(0.19864)}{0.57315}\,t, \quad R^2 = 0.9971$$

$$(5^{**})\quad D_1 = 0.69715 - \underset{(1.03702)}{1.11282}\,(r/q_1) + \underset{(0.00203)}{0.01111}\,(Y+Y_{-1}+Y_{-2}) + \underset{(0.00091)}{0.00161}\,\Delta F, \quad R^2 = 0.8485$$

$$(6^{**})\quad D_2 = -2.94203 + \underset{(3.31570)}{23.04778}\, r_{-1} + \underset{(6.16504)}{3.88784}\, (q_1)_{-1}$$

$$- \underset{(6.02974)}{6.90503}\, (q_1)_{-2} + \underset{(0.00058)}{0.00117}\, \Delta F_{-1}$$

$$- \underset{(0.38517)}{1.82335}\, i, \qquad R^2 = 0.9047$$

$$(7^{**})\quad v = 107.44945 + \underset{(0.03111)}{0.09504}\, Y + \underset{(6.54262)}{19.87004}\, r$$

$$+ \underset{(0.23421)}{1.30607}\, t - \underset{(0.30316)}{2.15627}\, N^s, \qquad R^2 = 0.6292$$

$$(8^{**})\quad \Delta r = -0.80098 + \underset{(0.00081)}{0.00738}\, v_{-1} + \underset{(0.00003)}{0.00025}\, Y$$

$$+ \underset{(0.01166)}{0.03519}\, \frac{1}{r_{-1}}, \qquad R^2 = 0.7739$$

$$(9^{**})\quad M_1^D = -37.43881 + \underset{(0.04883)}{0.69677}\, p(Y+T)$$

$$- \underset{(0.00096)}{0.01133}\, p(Y+T)t + \underset{(0.20065)}{1.85179}\, t, \quad R^2 = 0.9896$$

$$(10^{**})\quad M_2^D = 2.20494 - \underset{(1.19702)}{3.21538}\, i + \underset{(1.00185)}{2.42517}\, i_{-1}$$

$$+ \underset{(0.03141)}{1.24003}\, (M_2^D)_{-1} - \underset{(0.07478)}{0.18928}\, t, \quad R^2 = 0.9971$$

$$(11^{**})\quad M_s = M_1^D + M_2^D$$

$$(12^{**})\quad \Delta X = 4.57200 - \underset{(0.84718)}{1.44730}\, (u_3)_{-1} + \underset{(101.88610)}{378.45320}\, \Delta p,$$

$$R^2 = 0.3778$$

$$(13^{**})\quad Y + T = I + \Delta H + C + D_1 + D_2 + D_3 - D'' + G$$

$$(14^{**})\quad X = \frac{p(Y+T) - W_2 - R_1 - R_2}{p}$$

$$(15^{**})\quad \Delta K = I$$

Equation (12**) was estimated for the years 1923-1941 and 1948-1965. All coefficients except those for t in equation (1**), $(u_3)_{-1}$ in equation (12**), $\frac{r}{q_1}$ and ΔF in equation (5**), and $(q_1)_{-1}$ and $(q_1)_{-2}$ in equation (6**) are significantly different from zero at the 95 per cent confidence level.

Conclusion

This chapter has outlined the data revisions and other alterations we made in Klein Models II and III. Klein Model II was estimated by the 2SLS and the LI methods for alternative definitions of the price index and the money supply. Klein Model III was estimated using 2SLS. We outlined the difficulties encountered as a result of the large number of variables present in the reduced form equations and our procedural remedy. Since the F values computed for all equations for the extended period show that a highly significant relationship exists between the variables in each equation, we retain Model III.

The building and estimation of the structural equations of a dynamic model, while considered by many to be the end product (because it is more or less all that is needed for forecasting), are only a start toward a thorough analysis of causes of change in the endogenous variables. In Chapter VI the fundamentals of dynamic analysis are presented and then applied to the naive Klein Model II. In Chapter VII the complexity of analysis increases somewhat with an exploration of the dynamic properties of Model III.

FOOTNOTES--CHAPTER V

[1]Klein's Models I, II, and III are found in L. R. Klein, Economic Fluctuations in the United States, 1921-1941 (Cowles Commission Monograph No. 11; New York: John Wiley and Sons, 1950), 168 pp.; L. R. Klein and A. S. Goldberger, An Econometric Model of the United States 1929-1952 (Amsterdam: North Holland, 1955), 165 pp.; Stefan Valavanis-Vail, "An Econometric Model of Growth, U. S. A. 1869-1953," American Economic Review, XXXXV (May, 1955), 208-221; Michio Morishima and Mitsuo Saito, "A Dynamic Analysis of the American Economy 1902-1952," International Economic Review, V (May, 1964), 125-164.

[2]Klein, Economic Fluctuations . . . , p. 80.

[3]Ibid., pp. 102-105.

[4]Franklin Fisher, "Dynamic Structure and Estimation in Economy-wide Econometric Models," in The Brookings Quarterly Econometric Model of the United States, ed. by J. S. Duesenberry, G. Froom, L. R. Klein, and E. Kuh (Chicago: Rand McNally, 1965), pp. 589-635.

CHAPTER VI

DYNAMIC PROPERTIES OF THE KLEIN MODEL II

Our objective is to collect further evidence pertaining to the effectiveness of monetary and fiscal policy. The policy variables we select for examination by virtue of our choosing Klein Models II and III are government expenditures, personal taxes, and the money supply.

In this chapter we subject Klein Model II to dynamic analysis. Since the model is a naive one, the results we obtain may be suspect. However, Kelin Model II will provide the training ground for applying similar maneuvers to Klein Model III. Before proceeding with an examination of Model II, we pause to explain the various facets pertaining to dynamic analysis *per se*. This digression will point out the several bases that dynamic analysis provides to compare the effectiveness of policy variables.

An Introduction to Dynamic Analysis

In our review of the studies of FM, DM, AM, and Hester, we found that these studies went no further than to compare the correlation coefficients associated with alternative reduced form equations for consumption. Any decisions as to the relative effectiveness of monetary and fiscal policy were based primarily on these comparisons. The major disputes arose over the definition of the variables included in the equations and the specification of this system. Much more can, in fact, be done by concentrating more fully on forms which are derivable from the structure.

A full dynamic analysis examines the reduced form equations of the jointly determined variables as well as the so called fundamental dynamic equations. Proper attention to these equations uncovers a wealth of information. First, it allows an examination of the stability of the system. Second, it enables us to estimate the impact and dynamic multipliers which in turn may be used to analyze the causes of changes in the endogenous variables.

The impact and dynamic multipliers allocate the portions of the changes in the endogenous variables which are caused by (a) changes in the exogenous variables including the trend variable; (b) the given initial values of the exogenous variables, the so called "starting values"; (c) the given initial values of the endogenous variables, the so called "initial conditions"; (d) changes in the constant term, the changes in exogenous variables not specified by the structure; and (e) the effects of disturbances and errors.

Dynamic models are systems of difference equations. As such, they are considered better approximations of macroeconomic phenomena than comparative static models since economic phenomena are likely to affect future events, and to be affected by past events. Comparative statics can reveal only the ultimate changes in static equilibrium positions resulting from changes in magnitudes of structural parameters and exogenous variables. The derivative of an endogenous variable with respect to a policy variable does divulge the long run or "equilibrium multiplier" of that policy variable on the endogenous variable. However, the multipliers derived in comparative statics do not offer the slightest insight into (a) the length of time it will take to reach the new equilibrium position, (b) the proportion of the ultimate change in the endogenous variable that will take place within a few unit time intervals, (c) the time paths of the endogenous variables given the values of exogenous variables and the initial conditions, (d) the stability of the system, and (e) the effects on the time paths of the endogenous variables resulting from a "one-shot" or impulse change in an exogenous variable. Any serious attempt to decide the comparative effectiveness of policy variables must consider these effects. A complete dynamic analysis does allow for consideration of the temporal properties of the structure.

Example

It may be easiest to illustrate the efficacy of dynamic analysis by use of an example. Assume that we have the following three equation deterministic structural model:

(1) $C_t = \alpha_0 + \alpha_1 Y_t$

(2) $I_t = \beta_0 + \beta_1 Y_{t-1}$

(3) $Y_t = C_t + I_t + G_t$

where C_t = consumption
I_t = investment
G_t = government expenditure, and
Y_t = income.

For simplicity we choose a model with only a one period lag and only one exogenous variable. Since the first two equations are stochastic, we are also abstracting from the presence of the disturbance terms in these equations.

Reduced Forms

First we derive the reduced form equations which express each endogenous variable in terms of the predetermined variables, G_t and Y_{t-1}. The reduced form equation for Y_t is found by substituting for C_t and I_t in equation (3):

$$Y_t = \alpha_0 + \alpha_1 Y_t + \beta_0 + \beta_1 Y_{t-1} + G_t \quad \text{therefore}$$

$$(4)\ Y_t = \alpha_0 + \frac{\beta_0}{1-\alpha_1} + \frac{\beta_1}{1-\alpha_1} Y_{t-1} + \frac{1}{1-\alpha_1} G_t .$$

Substituting for Y_t from equation (4) into equation (1) we find the reduced form for C_t:

$$(5)\ C_t = \alpha_0 + \frac{\beta_0}{1-\alpha_1} + \frac{\alpha_1\beta_1}{1-\alpha_1} Y_{t-1} + \frac{\alpha_1}{1-\alpha_1} G_t .$$

The reduced form equation for I_t is the same as its structural equation, i.e.

$$(6)\ I_t = \beta_0 + \beta_1 Y_{t-1} .$$

The coefficients of the predetermined variables in reduced form equations (4), (5), and (6) are expressed in terms of the structural parameters and are called the impact multipliers. They measure the direct (but not necessarily the total) effect on the endogenous variable in period t of a unit change in the value of the predetermined variable to which the coefficient is attached holding the magnitudes of all other predetermined variables fixed. Therefore, $\partial C_t/\partial G_t = \alpha_1/1-\alpha_1$ measures the direct effect (in this case, the total effect as well) on current consumption expenditures of a unit change of

current government expenditures, given the value of the level of income in the previous period.

Fundamental Dynamic Equations

The reduced form equations show part of the effect on the current levels of the endogenous variables to be due to the values which the endogenous variables took on in the past. These past values, however, were influenced in turn by previous magnitudes assumed by the exogenous variable, G. Hence we attempt to purge the reduced form equations of lagged values of endogenous variables other than the one explained by the reduced form equation. That is, we transform the reduced form equations so that each equation expresses a current endogenous variable only in terms of past values of that variable and current and past values of the exogenous variables. These new expressions are called "fundamental dynamic" equations. In the present example, the reduced form equation for Y_t is also the fundamental dynamic equation for Y since the only endogenous variable in this reduced form equation is Y.

To find the fundamental dynamic equation for C_t we first substitute equation (2) for I_t in equation (3). This gives an expression for Y_t in terms of C_t, Y_{t-1}, and G_t:

$$(7)\ Y_t = \beta_0 + C_t + \beta_1 Y_{t-1} + G_t.$$

Next, we eliminate Y_t from equation (1) by substituting into this equation the expression for Y_t given by (7):

$$(8)\ C_t = \alpha_0 + \alpha_1 [\beta_0 + C_t + \beta_1 Y_{t-1} + G_t].$$

Lagging equation (1) one period and multiplying by $-\beta_1$ we find:

$$(9)\ -\beta_1 C_{t-1} = -\beta_1 \alpha_0 - \beta_1 Y_{t-1}.$$

Next, we add equations (8) and (9):

$$(10)\ C_t - \beta_1 C_{t-1} = \alpha_0 + \alpha_1 \beta_0 - \beta_1 \alpha_0 + \alpha_1 C_t + \alpha_1 G_t \text{ or}$$

$$(11)\ C_t = \frac{\alpha_0(1-\beta_1) + \alpha_1 \beta_0}{1-\alpha_1} + \frac{\beta_1}{1-\alpha_1} C_{t-1} + \frac{\alpha_1}{1-\alpha_1} G_t.$$

This procedure of lagging equation (1), multiplying the lagged equation by minus the coefficient of Y_{t-1} in (1), and adding the new expression to equation (8) is called a Koyck transformation. This transformation has eliminated Y_{t-1} from equation (8) and has given the fundamental dynamic equation for C.

To find the fundamental dynamic equation for I_t, we proceed in a manner similar to that followed for C. First, we substitute for C_t into (3):

$$Y_t = \alpha_0 + \alpha_1 Y_t + I_t + G_t \text{ or}$$

$$Y_t = \frac{\alpha_0}{1-\alpha_1} + \frac{1}{1-\alpha_1} I_t + \frac{1}{1-\alpha_1} G_t.$$

Therefore,

$$I_t = \beta_0 + \beta_1 \left[\frac{\alpha_0}{1-\alpha_1} + \frac{1}{1-\alpha_1} I_{t-1} + \frac{1}{1-\alpha_1} G_{t-1} \right] \quad \text{or}$$

$$(12)\ I_t = \frac{\beta_0(1-\alpha_1) + \beta_1\alpha_0}{(1-\alpha_1)} + \frac{\beta_1}{1-\alpha_1} I_{t-1} + \frac{\beta_1}{1-\alpha_1} G_{t-1}.$$

Stability Conditions

We can now examine the system for stability by transferring all terms involving Y to the left hand side of the fundamental dynamic equation for Y and setting the right hand side equal to zero:

$$Y_t - \frac{\beta_1}{1-\alpha_1} Y_{t-1} = 0.$$

This gives the auxiliary equation

$$(13)\ \lambda - \frac{\beta_1}{1-\alpha_1} = 0.$$

Equation (13) is a first order dynamic equation; it is easily seen that the general solution is:

$$(14)\ Y_t = Y_0\left(\frac{\beta_1}{1-\alpha_1}\right)^t$$

The system is, therefore, inherently stable if $\beta_1/1-\alpha_1$ is less than one in absolute value. Equation (14) is the general solution to the homogenous equation. The particular solution depends on the time paths of the exogenous variables. But if the magnitudes of the exogenous variables are constant over time, the particular solution is either a constant or is proportional to t. Thus, the general solution of the non-homogenous equation will be

$$Y_t = \frac{1}{1-\alpha_1-\beta_1} + Y_0\left(\frac{\beta_1}{1-\alpha_1}\right)^t$$

and the second term will dominate. Thus, if $\beta_1/1-\alpha_1$ is less than one in absolute value, the non-homogenous system in said to be inherently stable. The inherent time path of Y depends on the values of the parameters, α_1 and β_1, associated with the endogenous and lagged endogenous variables, Y_t and Y_{t-1}.

Long-run Multiplier

The fundamental dynamic equation for Y can be used to find the long-run or equilibrium multiplier (if it exists) of a particular exogenous variable when the values of the other exogenous variables remain unchanged. To find this multiplier, we replace the subscripted Y's with $\bar{Y}$, and the subscripted exogenous variables with $\bar{G}$. We then find $d\bar{Y}/d\bar{G}$. In our example we have

$$(15)\ \bar{Y} = \frac{\alpha_0 + \beta_0}{1-\alpha_1} + \frac{\beta_1}{1-\alpha_1}\bar{Y} + \frac{1}{1-\alpha_1}\bar{G} \quad \text{or}$$

$$\left(1 - \frac{\beta_1}{1-\alpha_1}\right)\bar{Y} = \frac{\alpha_0 + \beta_0}{1-\alpha_1} + \frac{1}{1-\alpha_1}\bar{G} \quad \text{or}$$

$$\bar{Y} = \frac{\frac{\alpha_0 + \beta_0}{1-\alpha_1}}{1 - \frac{\beta_1}{1-\alpha_1}} + \frac{\frac{1}{1-\alpha_1}}{1 - \frac{\beta_1}{1-\alpha_1}}\bar{G} \quad \text{and}$$

$$(16)\ d\bar{Y}/d\bar{G} = \frac{\frac{1}{1-\alpha_1}}{1 - \frac{\beta_1}{1-\alpha_1}} = \frac{1}{1 - \alpha_1 - \beta_1} .$$

This long-run multiplier assumes that we begin at a static equilibrium position and indicates what will be the ultimate change in Y per unit change in G by the time the new static equilibrium position is reached. Since G is a flow variable, the new level of G would necessarily be sustained each period in order for equation (16) to represent the long-run multipliers.

Dynamic Multipliers and the "Final Form"

We can also use the fundamental dynamic equation for a given endogenous variable to derive an expression of the

current level of that variable entirely in terms of the exogenous variables. This representation is called the "final form."[1] All that needs to be done is to purge the lagged values of the endogenous variable from the fundamental dynamic equation.

The fundamental dynamic equation for the first unit time interval of the sample period, i.e. t=1, expresses the value of the endogenous variable for t=1 in terms of the initial conditions, the starting values of the exogenous variables (all assumed to be given), and current values of the exogenous variables. The value of the endogenous variable at t=1, Y_1, is then substituted into the fundamental dynamic equation for t=2. In our example, from the fundamental dynamic equation for Y, equation (4), we have:

$$Y_1 = \frac{\alpha_0 + \beta_0}{1-\alpha_1} + \frac{\beta_1}{1-\alpha_1}Y_0 + \frac{1}{1-\alpha_1}G_1 \quad \text{or}$$

$$(17)\quad Y_1 = a_1 + \frac{1}{1-\alpha_1}G_1$$

$$\text{where } a_1 = \frac{\alpha_0 + \beta_0}{1-\alpha_1} + \frac{1}{1-\alpha_1}Y_0.$$

Hence Y_1 is determined by the initial condition, Y_0; the starting value of the exogenous variable, G_0; and the

current level of the exogenous variable, G_1. Re-writing (17) as

$$(17') \quad Y_1 = a_1 + \gamma_2 G_1$$

we substitute (17') for Y_1 in the fundamental dynamic equation for t=2:

$$(18) \quad Y_2 = \frac{\alpha_0 + \beta_0}{1-\alpha_1} + \frac{\beta_1}{1-\alpha_1}\left[a_1 + \frac{1}{1-\alpha_1}G_1\right] + \frac{1}{1-\alpha_1}G_2 \quad \text{or}$$

$$(18') \quad Y_2 = a_2 + \frac{1}{1-\alpha_1}G_2 + \frac{\beta_1}{1-\alpha_1}\left[\frac{1}{1-\alpha_1}\right]G_1$$

$$\text{where } a_2 = \frac{\alpha_0 + \beta_0}{1-\alpha_1} + \frac{\beta_1}{1-\alpha_1}a_1.$$

Then:

$$(19) \quad Y_3 = \frac{\alpha_0 + \beta_0}{1-\alpha_1} + \frac{\beta_1}{1-\alpha_1}\left[a_2 + \frac{1}{1-\alpha_1}G_2 + \frac{\beta_1}{1-\alpha_1} \cdot \frac{1}{1-\alpha_1}G_1\right] + \frac{1}{1-\alpha_1}G_3 \quad \text{or}$$

$$(19') \quad Y_3 = a_3 + \gamma_2 G_3 + \gamma_1\gamma_2 G_2 + \gamma_1^2\gamma_2 G_1.$$

Continuing in this manner, we find:

$$(20) \quad Y_t = a_t + \gamma_2 G_t + \gamma_1\gamma_2 G_{t-1} + \gamma_1^2\gamma_2 G_{t-2} + \ldots + \gamma_1^{t-1}\gamma_2 G_1.$$

The coefficients of the G's in equation (20) are the dynamic multipliers. Each multiplier shows the effect on the time path of Y resulting from an impulse change in G in a particular period.[2] The dynamic multipliers are set down in tabular form here with the lag (denoted by k) shown in the first column and the corresponding dynamic multiplier of G_{t-k} in the second.

TABLE 9.--Dynamic multipliers for G on the variable Y for three equation model.

Lag (k)	Multiplier
0	$\gamma_2 = 1/1-\alpha_1$
1	$\gamma_1\gamma_2 = \beta_1/(1-\alpha_1)^2$
2	$\gamma_1^2\gamma_2 = \beta_1^2/(1-\alpha_1)^3$
$\vdots$	$\vdots$

In this example, the multiplier associated with a particular lag is γ_1 times the dynamic multiplier one period earlier. Since γ_1 is equal to $\beta_1/1-\alpha_1$ and since $\beta_1/1-\alpha_1$ must be less than one in absolute value if the system is stable, the multipliers become smaller in

absolute value the greater the lag from the current period for models that are inherently stable.

The long-run multiplier discussed earlier can be computed in an alternative manner by summing all the dynamic multipliers for G. That is, we set all subscripted G's in equation (20) equal to $\bar{G}$ and sum up the coefficients.[3] In our case, the long-run multiplier is equal to:

$$(21)\quad \frac{1}{1-\alpha_1}\left[1 + \frac{\beta_1}{1-\alpha_1} + \left(\frac{\beta_1}{1-\alpha_1}\right)^2 + \ldots + \left(\frac{\beta_1}{1-\alpha_1}\right)^{t-1}\right]$$

$$= \frac{1}{1-\alpha_1} \quad \frac{1 - \left(\frac{\beta_1}{1-\alpha_1}\right)^t}{1 - \left(\frac{\beta_1}{1-\alpha_1}\right)}$$

As t approaches infinity, $[\beta_1/(1-\alpha_1)]^t$ will approach zero. Hence the long-run multiplier will equal

$$(22)\quad \frac{\frac{1}{1-\alpha_1}}{1 - \frac{\beta_1}{1-\alpha_1}} = \frac{1}{1 - \alpha_1 - \beta_1}$$

which is what we found earlier. The long-run multiplier will be finite if and only if the system is stable.

These results show that it is possible to determine how many periods are needed before, say, 90 per cent of 99 per cent of the equilibrium multiplier is attained.

Also, because the first few dynamic multipliers are the most important--seldom is a given policy variable altered and then sustained at the new level for more than a few periods before being changed again--we can sum the dynamic multipliers for the first few lags and use the ratio of this sum to the long-run multiplier as a measure of comparison of the effectiveness of policy variables.

The final form for Y is useful in finding the inherent time path of Y. In our example:

$$Y_t = a_t + \gamma_2 G_t + \gamma_1 \gamma_2 G_{t-1} + \gamma_1^2 \gamma_2 G_{t-2}$$

$$+ \ldots . + \gamma_1^{t-1} \gamma_2 G_1 .$$

The constant term a_t captures the movement of Y_t not due to government expenditures. Thus the inherent time path of Y is captured in the values of a_t. The value of a_t expresses the influence of the initial conditions on the time path of Y holding the levels of the exogenous variables constant. In terms of the initial conditions, the a_t's are

$$a_1 = \gamma_0 + \gamma_1 Y_0$$

$$a_2 = \gamma_0 + \gamma_1 a_1 = \gamma_0 + \gamma_1(\gamma_0 + \gamma_1 Y_0)$$

$$= \gamma_0(1 + \gamma_1) + \gamma_1^2 Y_0$$

$$a_3 = \gamma_0 + \gamma_1 a_2 = \gamma_0[1 + \gamma_1(1 + \gamma_1)] + \gamma_1^3 Y_0$$

$$a_4 = \gamma_0 + \gamma_1(\gamma_0[1 + \gamma_1 + (1 + \gamma_1)] + \gamma_1^3 Y_0)$$

$$= \gamma_0[1 + \gamma_1^2(1 + \gamma_1)] + \gamma_1^4 Y_0$$

$$\vdots$$

$$a_t = \gamma_0[1 + \gamma_1 + \gamma_1^2 + \ldots. + \gamma_1^{t-2}(1 + \gamma_1)] + \gamma_1^t Y_0.$$

Thus, the effect of the level of Y in the initial period, Y_0, disappears as t approaches infinity; the $\lim_{n\to\infty} a_t$ equals a constant that is independent of the initial conditions.

Actual Movements in Y

Until now we have been concerned with hypothetical changes in policy variables and their effects on the level of income. We can also trace the observed changes in income that were caused by actual changes in the exogenous variables. In our example:

$$(23)\ (Y_t - Y_{t-1}) = (a_t - a_{t-1}) + b_0(G_t - G_{t-1}) + b_1(G_{t-1} - G_{t-2}) + \ldots. + b_t G_1.$$

If we had more than one exogenous variable in our system and therefore on the right hand side of (23) as well, we could, by substituting in the actual values, find the role that each policy measure played in affecting the observed change in the level of income.

Dynamic Analysis of Klein Model II

The previous section outlined the nature of a complete dynamic analysis for a linear structural system. At this point we apply this analysis to the estimates of the Klein Model II we obtained in Chapter V.

The reduced form equations for this model were presented in Chapter V; but we shall reproduce the reduced from for Y/pN here using the estimated coefficients of the stochastic equation with $p = p_2$ and $M = M_2$ for the period 1921-1941 and 1945-1965 derived by 2SLS. The reduced form equation for per capita real disposable income is

$$Y/pN = \frac{48.58589}{1 - 0.63614} + \frac{0.23401}{1 - 0.63614} (Y/pN)_{-1} + \frac{0.04115}{1 - 0.63614} (M/pN)_{-1} + \frac{1}{1 - 0.63614} (I + G - T/pN$$

$$\text{or (24)} \quad Y/pN = 133.52908 + 0.64313\ (Y/pN)_{-1} + + 0.11309\ (M/pN)_{-1} + 2.74831\ (I + G - T/pN).$$

Equation (24) is also the fundamental dynamic equation for Y/pN. Solving the homogeneous auxiliary equation

$Y/pN - 0.64313\ (Y/pN)_{-1} = 0$ we see that the general solution is $(Y/pN)_t = k(0.64313)^t$. Therefore, the system is inherently stable.

The long-run multipliers for G and M are respectively

$$\partial\bar{Y}/\partial\bar{G} = 2.74831/0.35687 = 7.70115$$

$$\partial\bar{Y}/\partial\bar{M} = 0.11309/0.35687 = 0.31689.$$

The results of the estimation of the stochastic structural equation, which show the coefficient of the money supply to be not statistically significant, indicates that the Klein Model II may not provide a good description of the U. S. economy. Therefore, we shall not carry out a dynamic analysis of this model and, instead, turn to Klein Model III.

FOOTNOTES--CHAPTER VI

[1]Arthur S. Goldberger, Econometric Theory (New York: John Wiley and Sons, 1964), p. 374.

[2]*Ibid*., p. 375.

[3]*Ibid*.

CHAPTER VII

DYNAMIC ANALYSIS OF KLEIN MODEL III

Introduction

In this chapter our main purpose is to examine the dynamic properties of Klein Model III. Of particular interest is the relative effectiveness of the policy variables--the money supply, government expenditures, and (to a lesser extent) taxes--to induce changes in net national product over time. Since Klein Model III is more intricate than either Klein Model II or the models proposed by FM and their critics, it should describe more comprehensively the dependency of the time path of net national product on exogenous forces than the models we discussed heretofore. Thus, this model should place an analysis of the relative effectiveness of monetary and discal policy on firmer ground. Consequently, the analysis of the dynamic properties of Klein Model III constitutes the major thrust of our study.

Because we are primarily concerned with the exogenous forces influencing the time path of net national product, we begin with a derivation of the fundamental dynamic equation for this variable. After finding the fundamental dynamic equation, we are able to test the inherent stability of the system and find the dynamic multipliers associated with the policy variables. Also, the dynamic multipliers along with the actual changes in the policy variables and net national product over the sample period enable us to analyze the relative importance of the policy variables to cause changes in the level of net national product. Finally, we search for the cricial variables that determine the stability or instability of the system and the size of the policy multipliers.

Fundamental Dynamic Equation for Y + T

Initial Revisions in the Structural Equations

The re-estimated structural equations presented in Chapter V furnish the starting point of this dynamic analysis. The equations are reproduced here after dropping those variables from the structure which have re-estimated coefficients that were found to be not significantly different from zero. Including only those variables in the structure with statistically significant coefficients makes our task easier in two respects. First, it reduces the number of exogenous variables included in the fundamental dynamic equation as well as the number of lags for some of those exogenous variables which remain. Secondly, since many of the structural equations are non-linear, dropping the variables whose coefficients are not significantly different from zero reduces the number of linear approximations to be performed.

The revised re-estimated Klein Model III becomes:

$$(1') \quad W_{1_t} = -1.10413 + 0.41425\,(pX-E)_t + 0.22280\,(pX-E)_{t-1} + u_{1_t}$$

$$(2') \quad I_t = 10.85164 + 0.10246\,\left(\frac{pX-E}{q}\right)_t + 0.07957\,\left(\frac{pX-E}{q}\right)_{t-1} - 0.11434\,K_{t-1} + u_{2_t}$$

$$(3') \quad H_t = 11.65666 + 0.15125\,(X-\Delta H)_t + 0.57622\,H_{t-1} + u_{3_t}$$

$$(4') \quad C_t = 4.96737 + 0.83644\,Y_t + 0.57315\,t + u_{4_t}$$

$$(5') \quad D_{1_t} = 0.69715 + 0.0111\,(Y_t + Y_{t-1} + Y_{t-2}) + u_{5_t}$$

$$(6') \quad D_{2_t} = -2.94203 + 23.04778\,r_{t-1} + 0.00117\,\Delta F_{t-1} - 1.82335\,i_t + u_{6_t}$$

$$(7') \quad v_t = 107.4495 + 0.09504\,Y_t + 19.87004\,r_t + 1.30607\,t - 2.15627\,N_t^s + u_{7_t}$$

$$(8') \quad \Delta r_t = -0.80098 + 0.00738\,v_{t-1} + 0.00025\,Y_t + 0.03519\,1/r_{t-1} + u_{8_t}$$

$$(9') \quad M_{1_t}^D = -37.43881 + 0.69677\,p_t(Y+T)_t - 0.01133\,tp_t(Y+T)_t + 1.85179t + u_{9_t}$$

$$(10') \quad M^D_{2_t} = 2.20494 - 3.21538\ i_t + 2.42517\ i_{t-1} + 1.24003\ M^D_{2_{t-1}} - 0.18928\ t + u_{10_t}$$

$$(11') \quad M_{s_t} = M^D_{1_t} + M^D_{2_t}$$

$$(12') \quad \Delta X_t = 4.57200 + 378.45320\, \Delta p_t + u_{11_t}$$

$$(13') \quad (Y+T)_t = I_t + \Delta H_t + C_t + D_{1_t} + D_{2_t} + D_{3_t} - D_t'' + G_t$$

$$(14') \quad X_t = \frac{p_t (Y+T)_t - W_{2_t} - R_{1_t} - R_{2_t}}{p_t}$$

$$(15') \quad \Delta K_t = I_t .$$

We will be concerned only with the deterministic part of the structural equations. That is, we will abstract from the disturbance terms in the above eleven stochastic equations.

The Quest for a Suitable Fundamental Dynamic Equation

Our fundamental dynamic equation for net national product must exhibit certain properties if it is to be representative of the United States economy as well as consistent with economic theory. Namely, the experience of the United States economy dictates that the fundamental dynamic equation not exhibit explosive properties. Also, economic theory imposes the condition that the long run multipliers for government expenditures and the money supply must be non-negative. With these a priori restrictions in mind we proceed to a derivation of the fundamental dynamic equation for net national product starting with the revised re-estimated Klein Model III given at the beginning of this section.

The First Attempt

The initial step in the derivation involves substituting for C_t in (15') its equivalent as given by equation (4'):

$$(1)\ (Y+T)_t = 4.96737 + 0.83644\ Y_t + 0.57315\ t + I_t + \Delta H_t + D_{1_t} + D_{2_t} + D_{3_t} - D_t'' + G_t.$$

Next we eliminate D_{1_t} and D_{2_t} from (1) in the same manner that we eliminated C_t. This yields:

$$(2)\ 0.15245\ (Y+T)_t - 0.01111\ (Y+T)_{t-1} - 0.01111\ (Y+T)_{t-2} = 2.72249 + 23.04778\ r_{t-1} + 0.00117\ \Delta F_{t-1} - 1.82335\ i_t - 0.84755\ T_t - 0.01111\ T_{t-1} - 0.01111\ T_{t-2} + 0.57315\ t + I_t + \Delta H_t + D_{3_t} + D_t'' + G_t.$$

Substituting for v_t from (7') into equation (8'), finding a linear approximation of the term $1/r_{t-1}$ in (8') by taking the Taylor's expansion around the sample mean of r_{t-1} (= 0.82015), and subsequently solving for r_t, we find:

$$(3)\ r_t = 0.06817 + 0.00025\ Y_t + 0.0007\ Y_{t-1} + 1.09433\ r_{t-1} + 0.00964\ t - 0.01591\ N^s_{t-1}.$$

Performing a Koyck transformation on (2) in order to eliminate r_{t-1} yields:

$$\begin{aligned}(4)\ & 0.15245\ (Y+T)_t - 0.18370\ (Y+T)_{t-1} - 0.01512\ (Y+T)_{t-2} \\ & + 0.01216\ (Y+T)_{t-3} \\ & = 1.71947 + 0.00117\ \Delta F_{t-1} - 0.00128\ \Delta F_{t-2} \\ & - 1.82335\ i_t + 1.99534\ i_{t-1} - 0.84755\ T_t \\ & + 0.91062\ T_{t-1} - 0.01512\ T_{t-2} + 0.01216\ T_{t-3} \\ & + 0.16809\ t + I_t - 1.09433\ I_{t-1} + \Delta H_t \\ & - 1.09433\ \Delta H_{t-1} + (D_3 - D'' + G)_t - \\ & - 1.09433\ (D_3 - D'' + G)_{t-1} - 0.36677\ N^s_{t-2}.\end{aligned}$$

Next I_t is eliminated from (4) by substituting its definitional counterpart, $K_t - K_{t-1}$. In order to eliminate the current and lagged K variables from the resulting equation, we substitute $K_t - K_{t-1}$ for I_t in equation (2') and solve equation (2') for K_t. In the process of solving equation (2') for K_t, we "linearize" the terms involving $(\frac{pX-E}{q})_t$ and $(\frac{pX-E}{q})_{t-1}$ by the Taylor's expansion about the sample means of X_t, q_t, p_t, E_t, X_{t-1}, q_{t-1}, p_{t-1}, and E_{t-1}. The result is the following expression for K_t:

$$(5)\ K_t = 8.55846 + 36.53032\ p_t + 0.11282\ X_t - 38.15468\ q_t - 0.16111\ E_t + 27.99212\ p_{t-1} + 0.08831\ X_{t-1} - 29.49701\ q_{t-1} - 0.12786\ E_{t-1} + 0.88566\ K_{t-1}.$$

Next, K is eliminated from (4) by the same procedure as mentioned above.

After K is purged from (4) we set out to eliminate i from the resulting equation. We begin this operation by re-writing (10') as:

$$(6)\ 3.21538\ i_t = 2.20494 - M^D_{2_t} - 0.18928\ t + 2.42517\ i_{t-1} + 1.24003\ M^D_{2_{t-1}}.$$

From (11') $M^D_{2_t} = M_{s_t} - M^D_{1_t}$. Therefore, we substitute for $M^D_{2_t}$ into (6). This results in an expression for i_t in terms of M_{s_t}, $M^D_{1_t}$, t, i_{t-1}, $M_{s_{t-1}}$, and $M^D_{1_{t-1}}$.

Next, linearizing equation (9') gives:

$$(7)\ M^D_{1_t} = -65.60966 + 118.08776\ p_t + 0.29405\ (Y+T)_t - 0.42359\ t.$$

To find the expression for i_t in terms of (a) the endogenous variables yet to be purged from the expression for (Y+T) and (b) the exogenous variables, we substitute for $M^D_{1_t}$ into the revised expression for i_t:

$$(8)\ i_t = 5.42019 - 0.31101\ M_{s_t} + 0.38566\ M_{s_{t-1}} - 0.02725\ t + 36.72591\ p_t - 45.53528\ p_{t-1} + 0.09145\ (Y+T)_t - 0.11340\ (Y+T)_{t-1} + 0.75424\ i_{t-1}.$$

Then i_t is eliminated from the expression for (Y+T) by performing a Koyck transformation.

From (14'), after linearizing, we have:

$$(9)\ X_t = -64.18528 + (Y+T)_t - 1.42805\ (W_2+R_1+R_2)_t + 91.65968\ p_t.$$

By direct substitution of (9) into the new expression for (Y+T), the X_t's are purged from the expression leaving only the endogenous variables p and H. To eliminate the p's we recast equation (12') as

$$(10)\ 378.45320\ p_t = -4.57200 + X_t - X_{t-1} + 378.45320\ p_{t-1}.$$

Substituting (9) into (10) we have:

$$(11)\ 378.45320\ p_t = -4.57200 + (Y+T)_t - (Y+T)_{t-1} - 1.42805\ (W_2+R_1+R_2)_t + 1.42805\ (W_2+R_1+R_2)_{t-1} + 91.65968\ p_t - 91.65968\ p_{t-1} + 378.45320\ p_{t-1}.$$

Solving (11) for p_t will obviously make the coefficient of p_{t-1} equal to unity. Therefore, if a Koyck transformation were to be made to purge the p's from the expression for (Y+T), the multiplying constant would be unity causing the long run multipliers of M and G to be indeterminate. This result follows from the fact that the Koyck transformation, first, multiplies all the coefficients in the expression for (Y+T) lagged one period by the coefficient of p_{t-1} in the expression for p_t. In this case the coefficient in unity. Then the altered lagged equation for (Y+T) is subtracted from its unlagged version. In this particular case, all coefficients appearing in the altered lagged equation are precisely equal to the coefficients appearing in the unaltered equation one period later. Thus, adding all coefficients connected with either M or G in the expression for (Y+T) (after the Koyck transformation is completed) causes both sums to equal zero. The same sum is reached when the coefficients of the (Y+T)'s are added--first, because of the Koyck transformation and, second, because the coefficient of $(Y+T)_{t-1}$ is equal to minus the coefficient of $(Y+T)_t$ in the expression for p_t. Thus the long run multipliers for M and G are each equal to 0/0. It should be noted that this will be the result no matter what the values of the estimated coefficients. In equation (10)

the value of the coefficient of p_t will be equal to the coefficient of p_{t-1}. Whether (14') is linearized to obtain (9) or kept in its non-linear form will not alter the fact that p_t will have the same coefficient as p_{t-1} in equation (11) after X_t has been purged from equation (10). The reason is that X_t will be expressed in terms of current values of p only and that the coefficient of X_t in equation (10) is minus the coefficient of X_{t-1}. Thus, when X_t is purged from equation (10) and equation (11) is solved for p_t, the coefficient of p_{t-1} will equal unity. Thus, the Koyck transformation performed to purge p from the expression for Y+T will yield indeterminate long run multipliers for <u>all</u> samples.

To circumvent this indeterminacy we could either (a) stop with the elimination of the X's and change p and H to exogenous variables, (b) change p to an exogenous variable and proceed to eliminate H from the expression for (Y+T), or (c) reinstate the variable $(u_3)_{t-1}$ as it appears in (12**) of Chapter V even though the coefficient of this variable is not statistically significant at the 95 per cent level of confidence. We choose to reinstate $(u_3)_{t-1}$, the lag of the residual term of equation (3'), in equation (12'). Thus (11) becomes

$$(12)\quad 378.45320\, p_t = -4.57200 + X_t - X_{t-1} + 378.45320\, p_{t-1} + 1.44730\, (u_3)_{t-1}$$

where, from (3'), $(u_3)_{t-1}$ is:

$$(u_3)_{t-1} = -11.65666 - 0.15125\ X_{t-1} + 1.15125\ H_{t-1} - 0.72747\ H_{t-2}.$$

Substituting for $(u_3)_{t-1}$ into (12) and solving for p_t yields:

$$\begin{aligned}(13)\ p_t = {} & -0.02577\ 5552 + 0.93003\ 7879\ p_{t-1} \\ & + 0.00348\ 6820\ (Y+T)_t - 0.00425\ 01104\ (Y+T)_{t-1} \\ & + 0.00580\ 97691\ H_{t-1} - 0.00367\ 11685\ H_{t-2} \\ & - 0.00497\ 93606\ (W_2+R_1+R_2)_t \\ & + 0.00606\ 93631\ (W_2+R_1+R_2)_{t-1}.\end{aligned}$$

When (13) is used to purge p_t from the quasi-fundamental dynamic equation for (Y+T), the left hand side of this equation becomes

$$\begin{aligned}(13a)\ & 2.7643\ 37382\ (Y+T)_t - 1.42698\ 05652\ (Y+T)_{t-1} \\ & + 2.81804\ 09537\ (Y+T)_{t-2} - 2.63533\ 96798\ (Y+T)_{t-3} \\ & + 1.13355\ 46647\ (Y+T)_{t-4} - 0.15832\ 50781\ (Y+T)_{t-5} \\ & - 0.00755\ 33216\ (Y+T)_{t-6}.\end{aligned}$$

The right hand side contains only one endogenous variable, namely, H.

From equation (3'), after substituting for X_t and ignoring the residual $(u_3)_t$, we arrive at

$$(14)\ H_t = -1.94863\ 63788 + 0.15125\ (Y+T)_t - 0.21599\ 23105\ (W_2+R_1+R_2)_t + 12.04215\ 1539\ p_t + 0.63189\ 5765\ H_{t-1}.$$

After eliminating p_t from (14), we have

$$(15)\ H_t = -0.44672\ 3837 + 1.63189\ 5764\ H_{t-1} - 0.63189\ 5764\ H_{t-2} + 0.19323\ 8815\ (Y+T)_t - 0.19184\ 8703\ (Y+T)_{t-1} - 0.27595\ 45254\ (W_2+R_1+R_2)_t + 0.27396\ 92205\ (W_2+R_1+R_2)_{t-1}.$$

Equation (15) could be used to eliminate H from the quasi-fundamental dynamic equation for (Y+T). However, to purge H, it is necessary to perform two Koyck transformations. First, we would lag the expression for (Y+T) one period and multiply by 1.63189 5764. Second, we would lag the expression for (Y+T) two periods and multiply by -0.63189 5764. Then each equation would be subtracted from the original expression for (Y+T) after substituting into the original expression for the H's their equivalents as given by (15). However, performing these two Koyck transformations will result in the sum of the coefficients of M and G being equal to zero. This is because we are first multiplying each coefficient by 1.63189 5764 and then by -0.63189 5764; the net effect is multiplication by unity. Then these coefficients are subtracted from the original ones causing the sum of the coefficients for each variable, M and G, to equal zero. In the Appendix to this chapter we show that this result is independent of the sample chosen.

The effect on the coefficients of (Y+T), after performing the above Koyck transformations, is twofold. The first effect is the same as that on M and G. However, $(Y+T)_t$ and $(Y+T)_{t-1}$ enter (15) with the sum of the coefficients not equal to zero which by itself would imply that the sum of the coefficients of (Y+T) in the final expression would be non-zero. However, the H's appear in the quasi-fundamental dynamic equation in the form of first differences (see equation (4)). Therefore, when we rewrite the expression for (Y+T) with H's instead of ΔH's, the coefficients of the H's sum to zero. Thus the partial sum of the coefficients of (Y+T) in the final equation arising from (15) will be equal to zero after (15) is substituted for each H. Thus the combined effect is for the sum of all coefficients of (Y+T) in the fundamental dynamic equation of (Y+T) to equal zero. Consequently, the long run multipliers of the policy variables on (Y+T) will be indeterminate since the sums of the coefficients of each of these variables are also equal to zero. The equilibrium point of income is also indeterminate because the sum of the coefficients of (Y+T) is equal to zero.

The Second Attempt

Since the elimination of H from the expression for (Y+T) results in indeterminate long run multipliers, we did not test the resulting fundamental dynamic equation for stability. Instead, we dropped equation (3') from the model and changed H to an exogenous variable. This made the expression for (Y+T), which was found after eliminating p, the new fundamental dynamic equation. This equation (the left hand side of which is given in (13a)) was tested for stability. Since the largest root of the equation formed by setting (13a) equal to zero is 1.70188, the system is unstable. Parenthetically, the long run money multiplier is also negative. Thus, we have yet to find a suitable fundamental dynamic equation for (Y+T).

It is not our intention to describe at this juncture the remaining abortive attempts to find a fundamental dynamic equation displaying stability and positive long run money and government expenditure multipliers. The first two attempts were outlined here to show that making no adjustments in Klein Model III (other than dropping some coefficients that are not statistically significant) before undertaking a derivation of the fundamental dynamic equation for (Y+T) is fruitless. Secondly, making an initial "dry run" allowed us to set down the equations relevant to a discussion of the adjustments in the model which are necessary if the fundamental dynamic equation is to be stable and exhibit suitable policy multipliers. At this point we turn to the fundamental dynamic equation that was finally adopted, delaying until the last section of this chapter a full discussion of other trials.

Derivation of the Adopted Fundamental Dynamic Equation for (Y+T)

The derivation of the adopted equation begins as in the first attempt with the elimination of C_t, D_{1_t}, and D_{2_t} from the definitional equation (15'). A variant of equation (3) is found by dropping r_t from equation (7') and Y_t from (8'). This yields, instead of equation (3):

$$(3'')\quad r_t = 0.06817\,17151 + 0.00070\,13952\, Y_{t-1} + 0.94768\,42221\, r_{t-1} + 0.00963\,87966\, t - 0.01591\,32726\, N^{s}_{t-1}$$

and (4) becomes (setting $I_t = K_t - K_{t-1}$):

$$(4'')\quad 0.15245\,(Y+T)_t - 0.15558\,44597\,(Y+T)_{t-1} - 0.01674\,68306\,(Y+T)_{t-2} + 0.01052\,87717\,(Y+T)_{t-3}$$
$$= 2.03464\,82224 + 0.00117\,\Delta F_{t-1} - 0.00110\,87905\,\Delta F_{t-2} - 1.82335\, i_t$$

$$+ 1.72796\ 00264\ i_{t-1} - 0.84755\ T_t$$

$$+ 0.79209\ 97624\ T_{t-1} - 0.00058\ 12283\ T_{t-2}$$

$$+ 0.01052\ 87717\ T_{t-3} + 0.25213\ 76516\ t$$

$$+ 0.36676\ 56060\ N^{s}_{t-2} + \Delta H_t$$

$$- 0.94768\ 42221\ \Delta H_{t-1} + G_t - 0.94768\ 42221\ G_{t-1}$$

$$+ D_{3_t} - 0.94768\ 42221\ D_{3_{t-1}} - D''_t$$

$$+ 0.94768\ 42221\ D''_{t-1} + K_t$$

$$- 1.94768\ 42221\ K_{t-1} + 0.94768\ 42221\ K_{t-2}.$$

Using equation (5) the K_t's are eliminated from (4'') via a Koyck transformation. Before i is eliminated from the resulting equation, the variables $M_{s_{t-1}}$ and $(Y+T)_{t-1}$ are dropped from equation (8). Then the X's are removed from the expression for (Y+T) by direct substitution using equation (9). Finally, equations (3') and (12') are dropped from the system and p and H converted to exogenous variables.[1]

The resulting fundamental dynamic equation for (Y+T), after rounding to five decimal places, is:

$$(16)\quad 0.20637\ (Y+T)_t - 0.49477\ (Y+T)_{t-1} + 0.44614\ (Y+T)_{t-2}$$

$$- 0.19872\ (Y+T)_{t-3} + 0.03467\ (Y+T)_{t-4}$$

$$+ 0.00703\ (Y+T)_{t-5}$$

$$= 0.08237 + 0.00738\ t + G_t - 2.58758\ G_{t-1}$$

$$+ 2.22218\ G_{t-2} - 0.63305\ G_{t-3} + \Delta H_t$$

$$- 2.58758\ \Delta H_{t-1} + 2.22218\ \Delta H_{t-2} - 0.63305\ \Delta H_{t-3}$$

$$+ D_{3_t} - 2.58758\ D_{3_{t-1}} + 2.22218\ D_{3_{t-2}}$$

$$- 0.63305\ D_{3_{t-3}} - D_t'' + 2.58758\ D_{t-1}''$$

$$- 2.22218\ D_{t-2}'' + 0.63305\ D_{t-3}'' + 0.56707\ M_{s_t}$$

$$- 1.03964\ M_{s_{t-1}} + 0.47596\ M_{s_{t-2}}$$

$$- 20.09274\ p_t + 115.23867\ p_{t-1} - 192.65024\ p_{t-2}$$

$$+ 123.39447\ p_{t-3} - 25.79407\ p_{t-4} + 0.36677\ N_{t-2}^s$$

$$- 0.60146\ N_{t-3}^s + 0.24500\ N_{t-4}^s - 0.84755\ T_t$$

$$+ 2.18200\ T_{t-1} - 1.86571\ T_{t-2} + 0.54061\ T_{t-3}$$

$$- 0.01765\ T_{t-4} + 0.00703\ T_{t-5} - 0.16111\ E_t$$

$$+ 0.30745\ E_{t-1} - 0.04389\ E_{t-2} - 0.19385\ E_{t-3}$$

$$+ 0.09139\ E_{t-4} - 38.15468\ q_t + 73.59406\ q_{t-1}$$

$$- 12.50995\ q_{t-2} - 44.01336\ q_{t-3} + 21.08393\ q_{t-4}$$

$$- 0.16111\ (W_2+R_1+R_2)_t + 0.30921\ (W_2+R_1+R_2)_{t-1}$$

$$- 0.04862\ (W_2+R_1+R_2)_{t-2} - 0.18961\ (W_2+R_1+R_2)_{t-3}$$

$$+ 0.09014\ (W_2+R_1+R_2)_{t-4} + 0.00117\ \Delta F_{t-1}$$

$$- 0.00303\ \Delta F_{t-2} + 0.00260\ \Delta F_{t-3} - 0.00074\ \Delta F_{t-4}.$$

Dynamic Properties of the Adopted Fundamental Dynamic Equation for Net National Product

Stability of the System

The auxiliary equation associated with (16) is: $0.20637\ (Y+T)_t - 0.49477\ (Y+T)_{t-1} + 0.44614\ (Y+T)_{t-2} - 0.19872\ (Y+T)_{t-3} + 0.03467\ (Y+T)_{t-4} + 0.00703\ (Y+T)_{t-5} = 0$. The three real roots are 0.96565, 0.86737, and -0.11145; the pair of conjugate complex roots is $0.33796 \pm 0.50070\ i$. Since all three real roots and the modulus of the conjugate complex roots are less than unity in absolute value, the system is inherently stable.[2] The largest root, 0.96565, dominates. Hence the inherent time path of net national product is not highly damped. Also, since there is a pair of complex roots and a negative valued real root, the inherent time path of (Y+T) is oscillatory.

Long Run Multipliers

Setting $(Y+T)_{t-k} = (\overline{Y+T})$ for $k = 0, \ldots., 5$ and $(M_s)_{t-k} = \bar{M}_s$ for $k = 0, 1, 2$, we find the long run (equilibrium) money multiplier associated with equation (16) by first adding the coefficients for both $(\overline{Y+T})$ and $\bar{M}_s$ and then taking the partial derivative of $(\overline{Y+T})$ in (16) with respect to $\bar{M}_s$. In this case,

$\frac{\partial(\overline{Y+T})}{\partial \bar{M}_s} = 4.615$. Similarly $\frac{\partial(\overline{Y+T})}{\partial \bar{G}} = 2.101$ and $\frac{\partial(Y+T)}{\partial \bar{T}} =$ -1.741. All three long run multipliers have signs consistent with economic theory.

Dynamic Multipliers

In order to form the dynamic multipliers, equation (16) is solved for $(Y+T)_t$. Hence our alternative form of the fundamental dynamic equation becomes:

$$
\begin{aligned}
(17)\quad (Y+T)_t = {} & 2.39742\ (Y+T)_{t-1} - 2.16180\ (Y+T)_{t-2} \\
& + 0.96290\ (Y+T)_{t-3} - 0.16799\ (Y+T)_{t-4} \\
& - 0.03408\ (Y+T)_{t-5} + 0.39911 + 0.03577\ t \\
& + 4.84557\ G_t - 12.53833\ G_{t-1} + 10.76775\ G_{t-2} \\
& - 3.06750\ G_{t-3} + 4.84557\ D_{3_t} - 12.53833\ D_{3_{t-1}} \\
& + 10.76775\ D_{3_{t-2}} - 3.06750\ D_{3_{t-3}} \\
& + 4.84557\ \Delta H_t - 12.53833\ \Delta H_{t-1} + 10.76775\ \Delta H_{t-2} \\
& - 3.06750\ \Delta H_{t-3} - 4.84557\ D_t'' + 12.53833\ D_{t-1}'' \\
& - 10.76775\ D_{t-2}'' + 3.06750\ D_{t-3}'' + 2.74779\ M_{s_t} \\
& - 5.03764\ M_{s_{t-1}} + 2.30629\ M_{s_{t-2}} - 97.36082\ p_t \\
& + 558.39731\ p_{t-1} - 933.50067\ p_{t-2} \\
& + 597.91680\ p_{t-3} - 124.98704\ p_{t-4}
\end{aligned}
$$

$$+ 1.77719\ N^{s}_{t-2} - 2.91441\ N^{s}_{t-3} + 1.18716\ N^{s}_{t-4}$$

$$- 4.10686\ T_{t} + 10.57303\ T_{t-1} - 9.04043\ T_{t-2}$$

$$+ 2.61954\ T_{t-3} - 0.08555\ T_{t-4} + 0.03408\ T_{t-5}$$

$$- 0.78069\ E_{t} + 1.48979\ E_{t-1} - 0.21267\ E_{t-2}$$

$$- 0.93929\ E_{t-3} + 0.44286\ E_{t-4} - 184.88125\ q_{t}$$

$$+ 356.60533\ q_{t-1} - 60.61788\ q_{t-2}$$

$$- 213.26990\ q_{t-3} + 102.16371\ q_{t-4}$$

$$- 0.78069\ (W_2+R_1+R_2)_{t} + 1.49828\ (W_2+R_1+R_2)_{t-1}$$

$$- 0.23559\ (W_2+R_1+R_2)_{t-2} - 0.91879\ (W_2+R_1+R_2)_{t-3}$$

$$+ 0.43679\ (W_2+R_1+R_2)_{t-4} + 0.00567\ \Delta F_{t-1}$$

$$- 0.01467\ \Delta F_{t-2} + 0.01260\ \Delta F_{t-3}$$

$$- 0.00359\ \Delta F_{t-4}.$$

The inherent stability of the time path of (Y+T) discussed above relates to the movement of (Y+T) when all exogenous variables are held constant over time. We found that the movement of (Y+T) was in fact stable. Thus any instability of the time path must be caused by the actual levels of the exogenous variables over the period, the starting values of the exogenous variables, or random disturbances.

We, therefore, turn to a study of the effects of changes in the exogenous variables on the time path of

(Y+T). This is facilitated using the fundamental dynamic equation as the point of departure. For it is possible to derive from the fundamental dynamic equation the effect of a unit change in a given exogenous variable at time (t-k) on the expected value of $(Y+T)_t$ while holding constant the rest of the time path of this exogenous variable (that is, the initial change is unsustained), as well as fixing the values of all other exogenous variables. The resultant effects are called the dynamic multipliers.

To reveal these multipliers we derive the "final form" of (Y+T) by purging the lagged values of (Y+T) from the fundamental dynamic equation as shown in Chapter VI. The resulting coefficients of the "final form" are the dynamic multipliers. In Table 10 the multipliers are presented for the money supply, government expenditures, and taxes.

Several observations are possible as to the nature of these policy multipliers. First, all multipliers show damped oscillatory movement and converge to zero as the time lag increases. Second, government expenditures alternate between being stimulating and depressing for the first eleven lags and then are stimulating for the remainder of the period. Third, taxes alternate between being depressing and stimulating also for eleven lags and from then on are depressing. Fourth, the money supply

TABLE 10.--Dynamic multipliers for the time path of real NNP.

Lag k	Coefficient of $M_{s_{t-k}}$	G_{t-k}	T_{t-k}
0	2.74779	4.84557	-4.10686
1	1.54997	-0.92146	0.72716
2	0.08105	-1.91653	1.58109
3	-0.50817	-1.00443	0.88361
4	-0.36481	0.03383	0.00492
5	-0.05105	0.39671	-0.32408
6	0.11034	0.26414	-0.22715
7	0.10617	0.04228	-0.04157
8	0.04547	-0.05913	0.04841
9	0.00673	-0.04660	0.04146
10	0.00328	-0.00109	0.00393
11	0.01550	0.02510	-0.01887
12	0.02530	0.02615	-0.02053
13	0.02761	0.01722	-0.01325
14	0.02566	0.01069	-0.00764
15	0.02347	0.00942	-0.00639
16	0.02260	0.01078	-0.00748
17	0.02265	0.01201	-0.00855
18	0.02280	0.01217	-0.00874
19	0.02264	0.01165	-0.00834
20	0.02219	0.01105	-0.00786
21	0.02165	0.01064	-0.00754
22	0.02112	0.01039	-0.00735
23	0.02063	0.01017	-0.00720
24	0.02013	0.00991	-0.00702
25	0.01962	0.00962	-0.00681
26	0.01910	0.00932	-0.00659
27	0.01857	0.00903	-0.00638
28	0.01803	0.00875	-0.00618
29	0.01750	0.00848	-0.00599
30	0.01698	0.00821	-0.00580
31	0.01646	0.00795	-0.00561
32	0.01595	0.00769	-0.00543
33	0.01545	0.00744	-0.00525
34	0.01496	0.00720	-0.00508
35	0.01448	0.00696	-0.00491
36	0.01401	0.00673	-0.00474
37	0.01355	0.00650	-0.00458
38	0.01311	0.00628	-0.00443
39	0.01267	0.00607	-0.00428
40	0.01225	0.00586	-0.00413

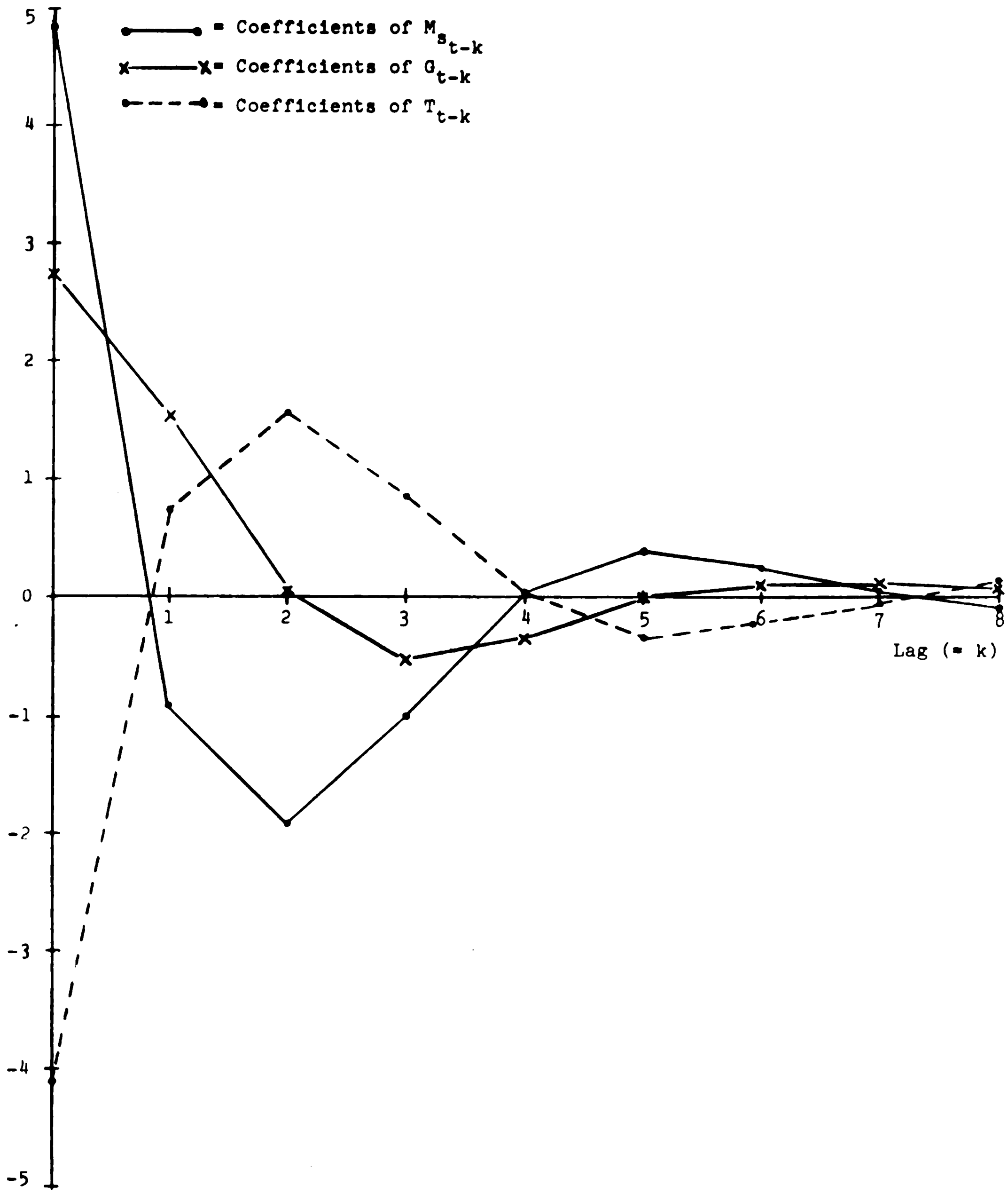

FIGURE 2.--Dynamic multipliers of M, G, and T for current period and first eight lags.

alternates between being stimulating and depressing for the initial six periods (the first three are stimulating and the next three depressing) and then is stimulating for the remainder of the time period. After twelve years the cumulative money supply multiplier is 3.74327 (81.1% of its long run value) while the cumulative government expenditure multiplier is 1.65839 (78.9% of its equilibrium multiplier). These compare with cumulative multipliers after six years of 3.45778 (74.9% of the equilibrium multiplier) for money and 1.43369 (68.2% of the long run multiplier) for government expenditures.

The dynamic multipliers given in Table 10 illustrate the relative efficacy of a unit change in monetary and fiscal policy variables to influence changes in the level of real net national product. For instance, a comparison of the values listed in Table 10 indicates how effective a one billion dollar increase in the money supply this year will effect real NNP in each succeeding year compared to a similar increase in government expenditures. These multipliers are relevant in an analysis of the Friedman-Meiselman hypothesis for they demonstrate the relative effectiveness of monetary and fiscal policy after all cross-temporal and inter-temporal effects are considered. They also allow for a comparison of the speed at which the policy multipliers approach their respective equilibrium or long run values.

Analysis of Causes of Changes in NNP

The dynamic multipliers we estimated in the previous section demonstrate the effect on real NNP of hypothetical unit changes in the exogenous variables. These multipliers are an aid in a discussion of the broader issue to which we now turn. To attack the question of the relative effectiveness of monetary and fiscal policy from 1925-41 and 1947-65 we need to consider also the actual changes that occurred in the policy variables over the sample period. The dynamic multipliers and the actual changes in the variables, coupled with the assumption that the policymakers' only objective was to induce desirable changes in real NNP over the period, will provide the needed framework to delineate the historical performance of the fiscal and monetary authorities.

The "final form" of the fundamental dynamic equation for (Y+T) is the starting point of our probe into the causes of changes in NNP. Lagging this "final form" one period and subtracting the lagged version from the equation for year t yields:

$$(18)\quad \widehat{(Y+T)}_t - \widehat{(Y+T)}_{t-1} = (a_t - a_{t-1}) + b_0(M_t - M_{t-1}) + b_1(M_{t-1} - M_{t-2}) + \dots + b_{t-1}M_1 + c_0(G_t - G_{t-1}) + c_1(G_{t-1} - G_{t-2}) + \dots + c_{t-1}G_1 + d_0(T_t - T_{t-1})$$

$$+ d_1 (T_{t-1} - T_{t-2}) + \ldots$$
$$+ d_{t-1} T_1 .$$

Equation (18) expresses the change in estimated NNP, (Y+T), during a given year in terms of (a) the change in the constant term, (b) the actual changes in the exogenous variables--the coefficients of these changes are dynamic multipliers--since the beginning of the sample period, and (3) the starting values--i.e. the levels of the exogenous variables in the first year of the sample period. The change in the constant term represents the effect of the initial conditions (the values of NNP and the exogenous variables prior to the beginning of the sample period) on the change in NNP. The terms containing the changes in the exogenous variables show the effects on the current change in NNP of each exogenous variable in each preceding year. The terms representing the starting values relate the change in NNP to the beginning values of the exogenous variables. Since the coefficients associated with these starting values are dynamic multipliers lagged t-1 years and since these multipliers decrease in absolute value as the time lag increases, the effects of the starting values will decrease with the advance of time.

Equation (18) provides a basis for a comparison of both the cumulative and concurrent effects of the money supply and autonomous expenditure on the time path of NNP over the sample period. A comparison of the absolute values of the cumulative effects will describe the relative importance the policy instruments played in influencing economic activity. The signed values of the cumulative effects and, alternatively, the signed values of the concurrent or impact effects will provide two measures of the relative abilities of the monetary and fiscal authorities to realize some "optimal" (arbitrarily determined) rate of growth in real net national product. Hence equation (18) facilitates an analysis of the relative effectiveness of monetary and fiscal policy in several respects.

Analysis of Cumulative Effects

In Table 11 we present the cumulative effects of (1) the policy variables--money supply "broadly" defined, M_s; money supply "narrowly" defined, M_1; government expenditures plus net exports, G; and "net" taxes (net national product minue disposable income), T; (2) the other exogenous variables combined (i.e. other than M_1, G, and T); (3) the starting values; and (4) the initial conditions on current NNP. The cumulative effect of each exogenous variable is obtained by adding all terms on the right hand side of equation (18) which are associated with the concurrent and past changes in that exogenous variable. The residuals presented in column (10) are found by subtracting from the actual changes in NNP the estimated

TABLE 11.--Analysis of causes of annual changes in real NNP: 1926-1965. (constant 1958 dollars)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Time in Years	Changes in NNP (Y+T)	Sources of Change							
		ΔM_s	ΔM_1	ΔG	ΔT	Δ Other Exog. Var.	Δ Constant	Starting Values	Residuals
1926	10.7	4.396	1.374	1.454	-21.356	7.904	-148.975	200.164	-29.9
1927	1.1	5.503	0.775	12.807	5.013	- 7.072	-111.115	132.874	-32.3
1928	0.4	6.233	0.865	- 1.125	2.254	- 2.996	- 15.396	26.317	- 9.5
1929	17.8	2.307	0.760	0.939	-27.716	29.320	37.738	- 24.102	0.9
1930	-21.0	- 3.174	- 2.321	5.902	46.396	-18.596	37.381	- 18.961	-70.8
1931	-15.6	-11.742	- 6.228	- 1.994	47.664	- 9.669	16.955	2.841	-65.2
1932	-26.3	-24.988	-11.383	-12.280	27.583	-32.525	2.549	15.650	-15.9
1933	- 2.1	-21.793	- 7.820	- 8.314	-48.540	- 1.053	- 0.363	16.267	47.7
1934	15.4	1.752	4.344	21.930	-55.518	8.841	2.379	11.905	21.5
1935	10.6	21.752	16.136	- 2.191	6.396	13.234	4.822	8.625	-36.4
1936	28.0	25.236	18.013	18.404	-27.495	37.076	5.060	7.869	-30.9
1937	11.7	14.125	9.121	- 9.044	- 8.799	10.179	3.972	8.422	- 2.1
1938	-10.3	- 0.827	- 2.122	19.204	27.137	-43.169	2.841	8.929	-23.1
1939	16.4	4.549	5.046	- 4.638	0.019	22.519	2.204	8.923	-17.7
1940	17.8	17.874	17.669	- 0.536	-25.165	33.102	1.953	8.582	-17.8
1941	36.1	28.867	27.595	81.290	-42.651	22.420	1.814	8.201	-62.6
1942		35.430	35.058	262.350	-19.435	-43.272	1.636	7.911	
1943		62.670	57.644	122.473	-92.056	-12.132	1.414	7.705	
1944		70.823	58.006	-54.179	-12.806	40.709	1.197	7.513	
1945		75.995	52.865	-269.163	92.697	55.740	1.017	7.306	
1946		52.773	29.582	-513.[illegible]	186.278	111.[illegible]	0.875	7.085	
1947		27.010	16.998	122.561	- 5.874	-36.124	0.759	6.860	
1948	22.4	4.700	1.910	246.259	-119.182	20.835	0.658	6.642	-134.7
1949	- 2.0	- 8.094	- 7.338	159.411	-30.277	-24.699	0.567	6.431	-106.1
1950	29.4	2.828	3.183	-32.906	-35.746	77.948	0.487	6.226	10.2
1951	26.5	20.349	19.116	63.180	-97.972	9.057	0.417	6.026	-13.3
1952	9.8	35.967	28.003	16.747	31.253	-29.399	0.358	5.829	-43.0
1953	15.3	33.610	20.647	-31.595	29.059	21.237	0.306	5.637	-30.1
1954	- 8.6	25.794	9.244	-89.382	66.922	46.213	0.262	5.450	-47.3
1955	28.1	23.472	11.255	-25.379	-47.306	79.215	0.224	5.268	4.8
1956	7.3	15.290	8.915	35.469	3.801	5.080	0.190	5.091	-51.2
1957	5.2	16.445	5.619	49.848	1.158	- 9.885	0.161	4.920	-46.6
1958	- 6.3	30.921	5.508	4.838	35.640	33.501	0.137	4.753	-90.7
1959	27.0	38.897	15.877	-22.604	-62.392	86.540	0.115	4.591	4.9
1960	10.2	15.620	3.788	9.900	-19.013	45.186	0.097	4.435	-34.2
1961	7.8	29.641	4.994	28.190	25.047	24.694	0.081	4.283	-79.5
1962	28.5	56.129	11.514	22.192	-34.952	78.397	0.067	4.136	-52.9
1963	19.0	76.566	18.641	- 3.284	-10.848	72.871	0.055	3.994	-62.4
1964	26.7	82.445	24.307	1.853	11.223	63.560	0.045	3.856	-78.1
1965	31.9	91.280	27.111	-12.108	-15.907	100.235	0.036	3.722	-71.2

changes in NNP caused by the exogenous variables, the starting values, and the initial conditions. The years 1942-47 are not included for comparison since we eliminated 1942-46 in our estimation of Klein Model III. The first year estimated in the second half of the time period is 1947 and the change between 1947 and 1948 is given in the row corresponding to 1948.

Absolute values of the cumulative effects.--According to Table 11, the cumulative effect of net taxes was greater than that associated with the broad definition of money in 20 out of 34 cases. The item net taxes was also a more important influence on real NNP than the money supply "narrowly" defined in 26 of the 34 years. Government expenditures plus net exports had a greater absolute cumulative effect on income than did the money supply "broadly" defined in only 14 cases while "autonomous" expenditures were more important in influencing income than currency plus adjusted demand deposits in 24 cases. Since T is no doubt misspecified in Klein Model III as an exogenous variable we limit our remaining discussion to the effects of the money supply and autonomous expenditures on NNP.

Signed values of the cumulative effects.--We now impose an arbitrary "goal function"[3] on the time path of NNP by assuming that a desirable rate of change in real

NNP over the period was $10 billion (in constant 1958 dollars). The residuals as given in column (10) represent the changes that would have occurred in NNP had there been no concurrent or past changes in any of the exogenous variables. If the residual in any given year was greater than $10 billion the policy makers should have, according to the "goal function," attempted to decrease NNP; if the residual was less than $10 billion the cumulative effects of each of M and G should have been expansionary.

Applying the above criterion to the residuals of column (10), we find that changes in the money supply (defined either way) affected the level of NNP in the wrong direction only 7 out of 34 times while the changes in autonomous expenditures produced the wrong effect 13 times. When the criterion is changed to a $15 billion (in constant 1958 dollars) increase in real NNP, column (10) shows that the effects on NNP were in the wrong direction 6 times in the case of money and 14 times in the case of autonomous expenditures.

Analysis of Impact Effects

In Table 12 we present the estimated changes of the concurrent effects of the money supply and autonomous expenditures on NNP. In columns (2), (3), and (4) the concurrent effects on NNP are listed for the broadly

TABLE 12.--Annual changes in real NNP cue to current and cumulative changes in exogenous variables (in constant 1958 dollars).

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Time in Years	Change in Y+T	Sources of Change				
		Current Change in M_s	Current Change in M_1	Current Change in G	Residual (M_s, G)	Residual (M_1, G)
1926	10.7	4.396	1.374	1.454	4.8	7.9
1927	1.1	3.023	0.000	13.083	- 15.0	- 12.0
1928	0.4	4.396	0.824	1.938	- 5.9	- 2.4
1929	17.8	0.550	0.550	6.784	10.5	10.5
1930	-21.0	- 2.473	- 2.473	10.660	- 29.2	- 29.2
1931	-15.6	- 9.068	- 4.671	2.907	- 9.4	- 13.8
1932	-26.3	-19.235	- 8.518	- 7.268	0.2	- 10.5
1933	- 2.1	-11.266	- 3.297	- 7.268	16.4	8.5
1934	15.4	6.320	5.221	17.444	- 8.4	- 7.3
1935	10.6	13.464	10.991	- 4.361	1.5	4.0
1936	28.0	12.640	9.892	22.290	- 6.9	- 4.2
1937	11.7	6.320	3.847	- 2.423	7.8	10.3
1938	-10.3	- 0.550	- 1.374	27.620	- 37.4	- 30.5
1939	16.4	10.167	9.617	3.392	2.8	3.4
1940	17.8	15.662	14.563	9.691	- 7.6	- [illegible]
1941	36.1	20.059	18.960	98.189	- 72.1	- 71.0
1942	--	24.455	24.730	282.4[illegible]7	--	--
1943	--	51.658	46.438	210.782	--	--
1944	--	46.712	36.546	84.313	--	--
1945	--	54.406	37.920	-112.992	--	--
1946	--	32.973	18.410	-464.200	--	--
1947	-13.9	21.158	17.311	- 22.290	- 12.8	- 8.9
1948	22.4	6.869	2.198	0.969	14.6	1[illegible].2
1949	- 2.0	- 1.923	- 3.297	35.373	- 35.4	- 34.1
1950	29.4	8.518	7.419	- 20.351	41.2	42.3
1951	26.5	15.113	14.014	122.108	-110.7	-109.6
1952	9.8	23.356	16.487	69.776	- 83.3	- 76.5
1953	15.3	17.586	9.068	28.104	- 30.4	- 21.9
1954	- 8.6	16.212	4.946	- 43.610	18.8	30.1
1955	28.1	18.135	11.541	- 16.960	26.9	33.5
1956	7.3	8.793	4.396	9.207	- 10.7	- 6.3
1957	5.2	13.464	2.473	25.197	- 33.5	- 22.5
1958	- 6.3	24.730	3.572	4.361	- 35.4	- 14.2
1959	27.0	24.455	12.915	- 6.784	9.3	20.[illegible]
1960	10.2	0.824	5.221	20.351	- 11.0	- 15.4
1961	7.8	30.775	5.770	31.012	- 9.0	- 29.0
1962	28.5	42.866	8.518	31.012	- 45.4	- 11.6
1963	19.0	51.384	11.816	15.506	- 47.9	- 8.3
1964	26.7	53.857	27.203	22.290	- 49.4	- 22.8
1965	31.9	66.222	17.311	2.907	- 37.2	11.7

defined money supply, the narrowly defined money supply, and autonomous expenditures respectively. Column (5) shows the effects on real NNP given the values of all past exogenous variables and the current values of the exogenous variables other than M_s and G. That is, if the current changes in M_s and G had been equal to zero, the changes in real NNP would have been given by column (5). Similarly, column (6) indicates the changes in real NNP which would have occurred had there been no changes in either M_1 or G.

Applying the goal of a $10 billion increase in yearly NNP (measured in constant 1958 dollars) to the changes given in column (5), we see that current changes in government expenditures were of the wrong sign 7 of the 34 years while the current changes in the money supply plus time deposits were of the wrong sign in 9 cases. If we look instead at column (6), changes in government expenditures were in the wrong direction 7 times and the alterations in the size of the money stock were in the wrong direction 13 times.

Conclusions

The relative effectiveness of the policy instruments is not clear cut when we examine the actual changes which occurred in these variables over the sample period. The cumulative absolute influence of government expenditure on current income was less in a majority of cases when compared with the money supply defined as currency in circulation plus adjusted demand deposits plus time deposits; the absolute influence was greater, however, in the majority of cases when compared to the money supply defined as excluding time deposits. The "errors" found in monetary policy were somewhat smaller in number than those found in fiscal policy. In any case, our results do not seem to give strong support to the Friedman-Meiselman hypothesis that monetary policy has a decided advantage over fiscal policy as a tool for economic stabilization.

Search for Crucial Coefficients that Determine Whether System is Stable and Whether Long Run Multipliers of M and G are Positive

In an attempt to find a stable auxiliary equation of a fundamental dynamic equation that yields positive long run multipliers for the money supply and government expenditure, numerous variation of Klein Model III were considered. These variations involved combinations of the following: (a) truncating the model by allowing some endogenous variables to become exogenous, (b) dropping variables from structural equations, and (c) dropping lagged variables from equations which express endogenous variables in terms of exogenous variables and endogenous variables not yet purged from the quasi-fundamental dynamic equation. These experiments revealed some factors influencing stability and the sign of the multipliers. In this section we first delineate the experiments that were run on Klein Model III and then summarize what they indicate as to the crucial coefficients in the model. Next we offer a more abstract analysis in terms of dynamic models in general.

Variations of Klein Model III

As shown earlier in this chapter, the first and second attempts at finding a suitable fundamental dynamic equation revealed, first, that retaining all statistically significant variables (and only those variables) leads to indeterminate long run multipliers for M and G. Second, allowing $u_{3_{t-1}}$ to remain in equation (12**) also produces indeterminacy if both p and H are eliminated from the fundamental dynamic equation for (Y+T). Third, if only p is eliminated (H treated as exogenous) then (13a) represents the left hand side of the fundamental dynamic equation. The time path of (Y+T) is unstable.

The next attempt paralleled the first two except that neither p nor H were purged from the fundamental dynamic equation for (Y+T). This resulted in an auxiliary equation with a largest root equal to 1.43. Truncating Klein Model III further by altering the status of X to an exogenous variable constituted the fourth trial. In this case the largest root of the corresponding auxiliary equation was 1.23. So, both cases gave unstable time paths of (Y+T); the money supply long run multipliers were both negative, as well. Hence it became clear that merely truncating Klein Model III would not produce stability nor generate positive long run multipliers for M.

Further investigation into Klein Model III involved dropping, once again, $u_{3_{t-1}}$ from (12**). Thus, p must be treated as exogenous if the long run multipliers are not to be indeterminate. We also convert H to an exogenous status. Hence, the relevant structure in this set of trials consists of (1'), (2'), (4') through (11'), (13'), (14'), and (15').

Trial 1

The first case in this new series of trials sets the coefficient of r_t in equation (7') equal to zero, yielding a new expression for r_t:

$$(3^I)\ r_t = 0.06817\ 17151 + 0.00025\ Y_t + 0.00070\ 13952\ Y_{t-1} + 0.94768\ 42221\ r_{t-1} + 0.00963\ 87966\ t - 0.01591\ 32726\ N^s_{t-1}.$$

Equation (3^I) differs from the equation (3) developed earlier in that the coefficient of r_{t-1} in (3^I) is less than unity. Since it is this coefficient which is involved in the Koyck transformation to purge r from the expression for (Y+T), it was felt that a Koyck multiplier less than unity might contribute to stability. In fact, this adjustment did cause the new expression for (Y+T) to have a largest root equal to 1.38 after further eliminating I, K, i, and X from the expression for (Y+T). This compares favorably to the largest root, 1.43, obtained above when equation (3) was used rather than (3^I). Since setting the coefficient for r_{t-1} in (7') reduces the degree of instability of the time path of (Y+T), we retain a variant of (3^I) and seek other adjustments which will reduce the degree of instability even further.

However, before we proceed we should note that eliminating r_{t-1} from (2) using (3^I) has caused the sum of the coefficients of (Y+T) to change from positive to negative while keeping the sum of the coefficients of G positive. This arises from the fact that the long run multiplier (just after r_{t-1} is eliminated) is 1(1 - 0.94768 42221) while that for (Y+T) (just after r_{t-1} is eliminated) is (0.15245 - 0.01111 - 0.01111) (1 - 0.94768 42221) - 23.04778 (0.00025 + 0.00070 13952). The second term of the sum of the coefficients of (Y+T) enters through the substitution of (3^I) for r_{t-1} in (2) and then transferring terms involving (Y+T) to the left hand side. Since the "Koyck constant" (= 0.94768 42221) is very close to one, the negative second term more than offests the first term causing the sum of the coefficients of (Y+T) to become negative. The sum of the coefficients of G, however, remain positive, causing the long run multiplier for G to be negative. Since the new sum of coefficients of i is -1.82335 (1 - 0.94768 42221) and since the money supply enters the expression for (Y+T) via i with a positive sum of coefficients (see equation (8)), the net effect is a negative sum of coefficients of M in the expression for (Y+T). Naturally, this means the long run multiplier for money is then positive.

The signs of these multipliers do not change as we move to eliminate I, K, i, and X from the expression for (Y+T). This is due first of all to the "Koyck constants" involved in these elimination processes being positive but less than one. Secondly, eliminating I and K introduces no new terms involving either (Y+T) or M into the expression for (Y+T) which might offset the first effect. (New terms for G are never brought into the expression for (Y+T) since G appears in only the structural equation (13').) Thirdly, when i is eliminated, the negative sum of coefficients in the expression for (Y+T) is reinforced. Fourthly, when X is eliminated, the sum of the coefficients of (Y+T) is not affected.

We explore the third and fourth effects further since they are not as apparent as the first two. First

we look at the elimination of i from the quasi fundamental dynamic equation. When i is eliminated the coefficients of (Y+T) in (8) are multiplied by each coefficient of i appearing in the expression for (Y+T) obtained just prior to the elimination of i. These new coefficients are then transferred to the left hand side of the new expression for (Y+T) and when added together form part of the total sum of coefficients of (Y+T). To determine the sign of this partial sum we (a) note the sign of the sum of coefficients of (Y+T) in (8), (b) note the sign of the sum of the coefficients of i appearing in the quasi fundamental dynamic equation just prior to the elimination of i, and (c) change the sign of the product of (a) and (b) to transfer this product to the left hand side of the new expression for (Y+T). Since the sum of coefficients of (Y+T) in (8) is negative and the sum of coefficients of i in the quasi fundamental dynamic equation is also negative, the negative sum of coefficients of the previous quasi fundamental dynamic equation (i.e. the equation which contained the variable i) is reinforced upon the elimination of i.

The effect on the sign of the sum of coefficients of (Y+T) resulting from the elimination of X is more complex than that evolving from the elimination of i. First, X does not enter the expression for (Y+T) until K is eliminated from that expression. However, because I_t is defined as $K_t - K_{t-1}$, the sum of coefficients of K in the expression for (Y+T) is equal to zero. Thus, even though X enters the new expression for (Y+T) when K is eliminated, the sum of the coefficients of X appearing in this new expression is equal to zero. Eliminating i from the expression for (Y+T) does not bring in any other terms involving X. Therefore, performing a Koyck transformation on the expression for (Y+T) does not alter the value of the sum of coefficients of X. Eliminating X from the quasi fundamental dynamic equation for (Y+T) yields a sum corresponding to (b) in the previous paragraph which is equal to zero. Thus, the product of (a) (which is now the sum of coefficients of (Y+T) in (9)) with (b) is also equal to zero and, therefore, X has no effect on the sign of the sum of the coefficients of (Y+T) appearing in the final fundamental dynamic equation.

Trial 2

Next, it was decided to drop the variable Y_t from equation (8'). This causes the expression for r_t to lose the term 0.00025 Y_t in (3^I). That is, the expression for r_t is now:

$$(3^{II})\quad r_t = 0.06817\ 17151 + 0.00070\ 13952\ Y_{t-1} + 0.94768\ 42221\ r_{t-1} + 0.00963\ 87966\ t - 0.01591\ 32726\ N^s_{t-1}.$$

This expression is the same as (3'') in the previous section. Upon eliminating I, K, i, and X using the expressions for these variables outlined above, we obtained a largest root of 1.33 which makes the system only slightly less unstable. Also, the equilibrium multiplier for G remains negative.

Trial 3

In the third attempt we retained equation (3'') as the expression for r_t and altered equation (8) by deleting the variable $(Y+T)_{t-1}$. This alteration caused the lagged coefficients of (Y+T) in the fundamental dynamic equation to be smaller in absolute value. It was felt that reducing these coefficients would aid stability. In fact, this alteration did result in a stable system with the largest root equal to 0.96565.

A problem arose, however, when we dropped $(Y+T)_{t-1}$ from equation (8). The sum of coefficients of the (Y+T)'s in the fundamental dynamic equation became positive. Thus dropping $(Y+T)_{t-1}$ was enough to alter the sign of the sum of coefficients. The result of this change in sign was that the government expenditures long run multiplier became positive (which is fine), but the money supply equilibrium multiplier became negative.

Trial 4

We then attempted to reverse the sign of the equilibrium money multiplier without simultaneously (a) changing the sign of the long run government expenditure multiplier and (b) causing the system to become explosive. We truncated equation (8) this time by deleting $M_{s_{t-1}}$ as well as $(Y+T)_{t-1}$. Thus, the expression for i_t became:

$$(8^{II})\ i_t = 5.42018\,7806 - 0.31100\,5231\, M_{s_t} - 0.02724\,59254\, t + 36.72590\,965\, p_t - 45.53528\,182\, p_{t-1} + 0.09144\,97031\,(Y+T)_t + 0.75424\,0556\, i_{t-1}.$$

Using (8^{II}) in the derivation of the fundamental dynamic equation gave the desired positive long run multiplier for M_s and did not affect either the corresponding G multiplier or the stability of the time path of (Y+T). Thus, Trial 4 yielded the fundamental dynamic equation we decided to adopt and which we outlined in the previous section.

Trial 5

Next, we tested the fundamental dynamic equation derived using (3^{I}) and (8''). The system once again was found to be stable and exhibited positive long run multipliers for M and G. However, since the largest root

in this case was 0.97225, we chose to stay with the equation derived in Trial 4.

Trial 6

Another attempt was made to include the price level as an endogenous variable in the model. This was done by taking the fundamental dynamic equation of Trial 4--given as equation (16) in the previous section--and then eliminating p using equation (13). It was to no avail for the largest root was complex with a modulus of 1.152.

Trial 7

Once again we tried to find a stable fundamental dynamic equation for (Y+T) which did not contain p. This attempt involved dropping the variable $(Y+T)_{t-1}$ from (13) and then proceeding as usual. However, the largest root associated with the resulting auxiliary equation was 1.4989.

Other Trials

Several other trials were run, but do not merit much attention. Three involved changing I to an exogenous status; all three gave an unstable time path for (Y+T). Two other trials are worthy of mention. Both were computed from an estimated structure for Klein Model III which was different from that given in Chapter V. However, the size of the coefficients in this version were close enough to the estimated structural equations given in this paper so that the results of the two trials seem to be relevant. One trial attempted to find a fundamental dynamic equation while retaining all coefficients--even those in the structure that were not statistically significant. Once again the system was found to be explosive. The second trial on the different version of the estimated structure indicated that the variable $(Y_t + Y_{t-1} + Y_{t-2})$ in equation (5') contributes to stability.

Initial Findings of the Crucial Coefficients Which Determine Stability and Size of Policy Multipliers

On the basis of the trials outlined above it may be tempting to assert that (a) the crucial variables for inherent stability of Klein Model III are the coefficients of (Y+T) in the expression for r, the coefficient of $(Y+T)_{t-1}$ in the equation for i, and the coefficients of r_{t-1} in the expression for v; (b) the crucial variables that determine the sign of the long run multiplier for money are the coefficients of $(Y+T)_{t-1}$ and $M_{s_{t-1}}$ in the expression for i; and (c) the crucial variables determining the sign of the equilibrium multiplier for government expenditures are the coefficient of r_{t-1} in the structural equation for v and the coefficient of $(Y+T)_{t-1}$ in the expression for i. However, these variables altered stability and the signs of the multipliers only when we

held all other coefficients constant. There may in fact be other cases which set an entirely different group of coefficients equal to zero and which would also yield a suitable fundamental dynamic equation. That is, intuitively it is not any one or two coefficients that are crucial but rather it is the combinations of coefficients (particularly the sums) which matter most for the multipliers and it is the relationships among the coefficients of (Y+T) in the fundamental dynamic equation that matter for stability.

Theil and Boot[4] have shown that in general stability of the system depends on the coefficients of the current and lagged endogenous variables in the structure and that the long run multiplier associated with any given policy instrument depends on the structural coefficients of that policy instrument as well as the structural coefficients of all current and lagged endogenous variables.

Appendix

On p. 227 we found that reinstating the variable $(u_3)_{t-1}$ as it appears in equation (12**) in Chapter V and then eliminating H from the quasi fundamental dynamic equation for (Y+T) resulted in the sums of the coefficients of each of M and G to equal zero. This arose because the coefficients of H_{t-1} and H_{t-2} in equation (15) sum exactly to one. We now show that these coefficients will always sum to one no matter what the estimated values of the structural coefficients are. To show this, we develop equation (15) analytically. Since the coefficients of H_{t-1} and H_{t-2} in (15) depend only on the coefficients of H, X, p, and u_3 in the structural and derived equations for these variables, we simplify the derivation by abstracting from the constant terms and other variables which appear in these equations.

We begin by expressing the structural equation for H_t as:

$$H_t = \alpha_1 X_t - \alpha_1 H_t + \alpha_1 H_{t-1} + \alpha_2 H_{t-1}$$

or:

$$\text{(a)} \quad (1 + \alpha_1)H_t = \alpha_1 X_t + (\alpha_1 + \alpha_2)H_{t-1}$$

Writing the structural equation for X_t as

$$\text{(b) } X_t = \beta_1 \, p_t .$$

Therefore,

$$\text{(c) } (1 + \alpha_1) \, H_t = \alpha_1 \, \beta_1 \, p_t + (\alpha_1 + \alpha_2) H_{t-1} .$$

It should be kept in mind that we are trying to solve for H_t in terms of lagged values of H only. That is, p and X have been already eliminated from the quasi-fundamental dynamic equation for (X+T).

To eliminate p from (c) we write the structural equation for p as

$$\text{(d) } \sigma_0 \, p_t = X_t - X_{t-1} + \sigma_0 \, p_{t-1} + \sigma_1 \, (u_3)_{t-1} .$$

From equation (3'),

$$\text{(e) } (u_3)_{t-1} = -\alpha_1 \, X_{t-1} + (1 + \alpha_1) \, H_{t-1} - (\alpha_1 + \alpha_2) \, H_{t-2} .$$

Using (e) to eliminate $(u_3)_{t-1}$ from (d) yields

$$\sigma_0 \, p_t = \beta_1 \, p_t - \beta_1 \, p_{t-1} + \sigma_0 \, p_{t-1} - \sigma_1 \, \alpha_1 \, \beta_1 \, p_{t-1} + \sigma_1 \, (1 + \alpha_1) \, H_{t-1} - \sigma_1 \, (\alpha_1 + \alpha_2) \, H_{t-2}$$

or:

$$\text{(f) } p_t = \frac{\sigma_0 - \beta_1 - \sigma_1 \alpha_1 \beta_1}{\sigma_0 - \beta_1} p_{t-1} + \frac{\sigma_1 (1 + \alpha_1)}{\sigma_0 - \beta_1} H_{t-1} - \frac{\sigma_1 (\alpha_1 + \alpha_2)}{\sigma_0 - \beta_1} H_{t-2}$$

Solving (c) for H_t:

$$\text{(c') } H_t = \frac{\alpha_1 \beta_1}{1 + \alpha_1} p_t + \frac{\alpha_1 + \alpha_2}{1 + \alpha_1} H_{t-1}.$$

Performing a Koyck transformation to eliminate p_t from (c') we obtain:

$$\text{(g) } H_t = [(\frac{\sigma_0 - \beta_1 - \sigma_1 \alpha_1 \beta_1}{\sigma_0 - \beta_1}) + \frac{\alpha_1 \beta_1 \sigma_1}{(1 + \alpha_1)} (\frac{1 + \alpha_1}{\sigma_0 - \beta_1}) + \frac{\alpha_1 + \alpha_2}{1 + \alpha_1}] H_{t-1} - [\frac{\alpha_1 \beta_1 \sigma_1 (\alpha_1 + \alpha_2)}{(1 + \alpha_1)(\sigma_0 - \beta_1)} + (\frac{\sigma_0 - \beta_1 - \sigma_1 \alpha_1 \beta_1}{(\sigma_0 - \beta_1)})(\frac{\alpha_1 + \alpha_2}{1 + \alpha_1})] H_{t-2}$$

It is shown below that the sum of the coefficients of H_{t-1} and H_{t-2} is identically equal to unity--provided that $\alpha_1 \neq -1$ and $\sigma_0 \neq \beta_1$.

We now see if

$$[(\frac{\sigma_0 - \beta_1 - \sigma_1 \alpha_1 \beta_1}{\sigma_0 - \beta_1}) + \frac{\alpha_1 \beta_1}{1 + \alpha_1} (\frac{\sigma_1 (1 + \alpha_1)}{\sigma_0 - \beta_1}) + (\frac{\alpha_1 + \alpha_2}{1 + \alpha_1})]$$

$$- [\frac{\alpha_1 \beta_1}{1 + \alpha_1} (\frac{\sigma_1 (\alpha_1 + \alpha_2)}{\sigma_0 - \beta_1}) + (\frac{\sigma_0 - \beta_1 - \sigma_1 \alpha_1 \beta_1}{\sigma_0 - \beta_1})(\frac{\alpha_1 + \alpha_2}{1 + \alpha_1})$$

equals one. That is, does the following equality

$$\frac{(\sigma_0 - \beta_1 - \sigma_1\alpha_1\beta_1)(1+\alpha_1) + \alpha_1\beta_1\sigma_1(1+\alpha_1) + (\alpha_1+\alpha_2)(\sigma_0-\beta_1)}{(\sigma_0 - \beta_1)(1 + \alpha_1)}$$

$$= \frac{(\sigma_0-\beta_1)(1+\alpha_1) + \alpha_1\beta_1\sigma_1(\alpha_1+\alpha_2) + (\sigma_0 - \beta_1 - \sigma_1\alpha_1\beta_1)(\alpha_1+\alpha_2)}{(\sigma_0 - \beta_1)(1 + \alpha_1)}$$

always hold? The above expression is equivalent to:

$$(\sigma_0 - \beta_1 - \sigma_1\alpha_1\beta_1)(1+\alpha_1) + \alpha_1\beta_1\sigma_1 (1+\alpha_1) + (\alpha_1+\alpha_2)(\sigma_0-\beta_1)$$

$$= (\sigma_0-\beta_1)(1+\alpha_1) + \alpha_1\beta_1\sigma_1 (\alpha_1+\alpha_2) + (\sigma_0 - \beta_1 - \sigma_1\alpha_1\beta_1)(\alpha_1+\alpha_2)$$

or: $\alpha_1(\sigma_0-\beta_1) + \alpha_2 (\sigma_0-\beta_1) \overset{?}{=} (\sigma_0-\beta_1)(\alpha_1+\alpha_2)$

or: $(\alpha_1+\alpha_2)(\sigma_0-\beta_1) = (\sigma_0-\beta_1)(\alpha_1+\alpha_2)$ which is always true.

Therefore, the sums of the coefficients of each of M and G in the fundamental dynamic equation are always equal to zero.

FOOTNOTES--CHAPTER VII

[1]We did find another stable fundamental dynamic equation after we had completed the analysis of the dynamic multipliers and the causes of changes in NNP given later in this chapter and associated with equation (16). This fundamental dynamic equation resulted from retaining H as an endogenous variable but converting p to an exogenous status. Equation (14) was used to eliminate H from the expression for (Y+T) after removing the X's. This new equation had a largest root approximately equal to 0.9641 and long run money and government expenditure multipliers equal to 4.59 and 2.07 respectively.

[2]The reader should be cautioned that the figure 0.96565 represents an estimate, not the true value, of the largest root. If the standard error of this estimate is not rather small, the true value of the largest root may be more than one. See H. Theil and J. C. G. Boot, "The Final Form of Econometric Equation Systems," Readings in Economic Statistics and Econometrics, ed. by Arnold Zellner (Boston: Little, Brown and Company, 1968), pp. 624-627.

[3]Thomas R. Saving, "Monetary-Policy Targets and Indicators," The Journal of Political Economy, LXXV (No. 4, Part II; Supplement: August, 1967), 447.

[4]Theil and Boot, pp. 611-630.

CHAPTER VIII

CONCLUSION

In this study we have extended the FM analysis to find whether FM's hypothesis (that the money supply is a more important determinant of total spending than is autonomous expenditure) is verified for revised data, alternative definitions of the policy variables, and more refined models of income determination. The focus of the study was on sample periods of as long a duration as possible in order to retain the flavor of FM's analysis.

The simple one-equation models we tested initially did not contradict FM's findings that money is more highly correlated with income than are autonomous expenditures. However, this aspect of the study did reveal that FM's definition of the money supply consistently produced higher coefficients of determination than did a narrower definition of the money stock. It also showed that, in general, FM's definition of autonomous expenditures evoked lower coefficients of determination when regressed against income than did the alternative definitions offered by FM's critics.

Since we questioned FM's criteria for deciding the definitions of the policy variables and pointed to likely biases resulting from their single-equation tests, we proceeded to carry out empirical tests with the help of more complete models of income determination. The first step, admittedly, was a small one for it was based on the highly simplified three equation Klein Model II. Testing this model for the period 1922-1941 and 1946-1965 and performing a dynamic analysis of the system disclosed evidence supporting the other side of the issue. The estimated long run government expenditure multiplier was greater than seven and its impact multiplier was nearly three. On the other hand, the equilibrium money supply multiplier was less than one half and the concurrent effect of a hypothetical increase in the money supply was precisely zero.

An analysis of the dynamic properties of a revised version of Klein Model III yielded policy implications that were less apparent than those provided by the simpler models. While the estimates of the intermediate and long run multipliers for the money supply were larger than those estimated for government expenditures, the impact multiplier of the latter was greater. When we analyzed the causes of the actual changes in income during the sample period we found further evidence that it may be necessary to temper the FM claim that monetary policy is a more powerful stabilization device. Our findings suggested that the relative efficacy of the money stock and government expenditures depends significantly on the particular definition of money adopted and on whether a comparison is based on cumulative or concurrent effects of the policy instruments. In short, we did not find a clear answer to the question of the relative effectiveness of monetary and fiscal policy as FM have claimed to have done.

Possible Shortcomings of Klein Model III and Suggestions for Further Research

The dynamic analysis of Klein Model III, which occupied the greatest share of our attention in this study, gave results that are, of course, only as good as the estimated structural equations used to launch the analysis.

A possible shortcoming of the structure is that it specifies the money stock in terms of nominal units while expressing government expenditures and income in real terms. Because of this, the comparisons made in Chapter VII may not have placed the policy instruments on equal footing. For instance, the relative absolute sizes of the cumulative effects of money on real income may have differed significantly from those presented in Table 11 had the money stock been deflated by a price index. The reason is that for a given increase in real income and a given increase in the nominal money stock, the effect of real money on income will be greater than that for nominal money in years which experienced increases in the price level. The opposite would be the case in those years during which the price level fell.

Several possible specification errors exist in Klein Model III. For instance, permanent income may be more appropriate than the current level of income as an explanatory variable in the consumption function. Also, the money market seems rather sterile. A model of income determination incorporating a more elaborate theory of the supply of and demand for money may alter the findings substantially.

The most damaging criticism of Klein Model III appears to be that it lacks a bond market. From Walras' law it is justifiable to delete the bond market from a model of effective demand when we are concerned with static equilibrium analysis. However, in dynamic analysis the focus is on the time paths which the variables assume in approaching their equilibrium values. Thus, a complete dynamic analysis would consider the movements in the bond market and their influences on other components of economic activity.

Of the models presently in existence, we found Klein Model III to be the most appropriate one to examine the FM hypothesis for a time period of as long a duration as possible. It was not entirely adequate, however.

Some of the most recent efforts to build models of income determination for the U. S. economy are exemplified by the Brookings and Federal Reserve-MIT quarterly econometric models. The Brookings model contains approximately 150 non-linear equations.[1] Therefore it is extremely difficult to examine its properties except by using simulation techniques. The Federal Reserve-MIT model, unlike the Brookings model, "has as its major purpose the quantification of monetary policy and its effect on the economy."[2] The formulators of the former model have tried to avoid the "puzzling results"[3] found in other econometric models

> . . . either in their financial sectors or in the responses to financial variables in other sectors . . . by concentrating most of . . . [their] . . . efforts on the treatment of financial markets and on the links between finaicial markets and markets for goods and services.[4]

DeLeeuw and Gramlich report that:

> the preliminary results suggest that both monetary and fiscal policy have powerful effects on the economy though monetary policy operates with a longer lag. We also find that the response of money income to both monetary and fiscal policy changes is stronger than that implied by other large-scale econometric models.[5]

Hopefully, the further development of this model, and others like it, will contribute significantly to our understanding of the effects of policy instruments on the economy.

FOOTNOTES--CHAPTER VIII

[1]James S. Duesenberry et al., ed, The Brookings Quarterly Econometric Model of the United States (Chicago: Rand McNally & Company, 1965).

[2]Frank deLeeuw and Edward Gramlich, "The Federal Reserve-MIT Econometric Model," The Federal Reserve Bulletin, LIV (January, 1968), 11-40.

[3]Ibid., p. 11.

[4]Ibid.

[5]Ibid., p. 12.

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