MECHANISMS OF HEAT TRANSFER THROUGH ORGANIC POWDER IN A PACKED BED

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MECHANISMS OF HEAT TRANSFER THROUGH ORGANIC POWDER IN A PACKED BED

By

Albert Chun-yung Chen

Due to an ever increasing demand for convenience-type foods, many food products are being produced in dehydrated and powdered forms. Most of the dried and powdered products are subjected to either cooling or heating during processing. In order to effectively control the temperature level and quality of the products, reliable information regarding the thermal properties of the powdered food is needed. The objectives of this research is to describe and represent quantitatively the transfer of heat through a packed bed of small organic powder particles. A mathematical model was derived to predict the effective thermal conductivity of a powder which contained solids with known thermal conductivity. The basic parameters considered were: thermal and mechanical properties of dry milk solids, thermal conductivity of the interstitial gas, void fraction of the packed bed, particle size and size distribution, temperature level, moisture content and structure of the packed bed.

In order to measure the thermal and mechanical properties of dry milk solid, regular nonfat dry milk

was compressed by a hydraulic press until the bulk density of the compressed dry milk was equal to the density of dry milk solids (1.46 gm. per ml.). The thermal and mechanical properties of the specimen thus made was used to approximate the thermal and mechanical properties of the particle solids. The thermal conductivity of the specimen was measured by a transcient thermal property measuring facility. The mechanical properties were measured by an Instron testing machine. The effective thermal conductivity of powdered milk was measured by a steady-state thermal conductivity measuring apparatus.

The thermal conductivity of dry milk solids was found to depend on the temperature and moisture content of the solid. Its value was 0.2699 Btu/hr ft ^{O}F at 140 ^{O}F and 3.5% moisture content. The primary mechanism of heat transfer through organic powder in a packed bed was found to be conduction through the particle solids. In general, this contribution was 93.3% of the effective thermal conductivity. Conduction through the gas phase was 4.8% while conduction through contact points was 1.9% of the effective thermal conductivity. Due to the small contribution of the gas phase, different types of interstitial gases did not have a significant influence on the values of effective thermal conductivity of a powder bed. Temperature, moisture

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content and void fraction were the dominating parameters for the values of effective thermal conductivity of a powder bed.

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MECHANISMS OF HEAT TRANSFER THROUGH ORGANIC POWDER IN A PACKED BED

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For their personal sacrifice, understanding and encouragement, this thesis is dedicated

To my wife, Judy

new-born son, Mark

and parents, Mr. and Mrs. Ching Chuan Chen

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NOMENCLATURE

a	Radius of contact area between two particles, (in).
A	Cross-sectional area of a packed bed, $(ft)^2$.
B	Properties of solid material.
b	Distance between the geometrical center of a particle and the contact surface between particles, ft.
C_1, C_2	Constants defined in Eq. (A1).
d	Constant in Eq. (3.4).
D	Apparent particle density, g/ml.
E	Modulus of Elasticity, psi.
e	Porosity inside particles.
f(x)	Log - normal density function.
$f(\cdot)$	Function of $\cdot$
F(x)	Probability frequency distribution function of particle sizes.
G(s)	The distribution of particle sizes as they appear on a cross-section of a random bed.
$G(0^+)$	Number of Contact Points on the cross-section of a random bed predicted by Eq. (A7).
h_{rs}	Heat Transfer coefficient for thermal radiation between particles, Btu/(hr)(°F)(sq. ft).
H	Bulk modulus, Psi.
I	Distance between the geometrical center of a particle and an unit area on the cross-section, ft.
K_s	Thermal conductivity of solid material (milk), Btu/(hr.)(°F)(ft).

K_g Thermal conductivity of intersticial gas, /(hr.)(°F)(ft).

K_c Thermal contact conductance, Btu/(hr.)(°F)(ft).

K_e Effective thermal conductivity of a powder bed, Btu/(hr.)(°F)(ft).

K_p Thermal conductivity with pressure in Eq. (3.4), Btu/(hr.)(°F)(ft).

K_r Radiation contribution to the effective thermal conductivity of a packed bed, Btu/(hr.)(°F)(ft).

K_1 Contribution of solid phase on a cross-section to the effective thermal conductivity of a packed bed, Btu/(hr.)(°F)(ft).

K_2 Contribution of gasous phase on a cross-section to the effective thermal conductivity of a packed bed, Btu/(hr.)(°F)(ft).

K_{345} Contribution of a contact point on a cross-section to the effective thermal conductivity of a packed bed, Btu/(hr.)(°F)(ft).

L Length of a packed bed, ft.

Δl Length of the unit cell in Fig. 4.6(a), ft.

l_s Effective path length (Fig. 4.6) for solid particles, or, the thickness of a slab of solid material which would offer the same resistance to heat transfer as the spherical shaped particles, ft.

l_g Effective path length (Fig. 4.6) between adjacent solid particles, or, the thickness of a slab of stationary fluid which would offer the same heat transfer resistance as the filaments of fluid near the contact points between particles, ft.

M_{IF} First moment of F(x), ft.

MC Moisture Content, %.

P Pressure applied, psi.

q, q_{avg} Heat flux in Fig. 3.2, Btu/(hr.).

$\dot{q}$ Heat flux in a random packed bed.

r_o Average radius of particles in a packed bed, ft.

S Particle size as they appear on the cross-section of a random bed, (Fig. 4.5), ft.

T Temperature

ΔT, ΔT_s, ΔT_g Temperature drop in unit cell, solid phase and gas phase, respectively, °F, in Fig. 4.6.

u_o,u_1 Radius of stagnation area and contact area, respectively, in Fig. 4.6, ft.

W Constant in Eq. (4.10).

w Loading, lbs., in Eq. (3.1).

x Diameter of particles in a packed bed, ft.

x_o Average diameter of particles in a random bed, ft.

Greek

β,ϕ,γ,δ Parameters in Eq. (4.18).

ε Void between particles.

ε_F Void and porosity on the cross-section of a packed bed.

ε_2 Void on the cross-section of a packed bed for heat flux through the interstitial gas phase, Fig. 4.6.

ε_{45} Void on the cross-section of a packed bed for heat flux through the stagnation fluid around the contact points, Fig. 4.6.

ε_1 Fraction of the cross-section for heat flux through solid phase, Fig. 4.6.

ε_3 Fraction of the cross-section for heat flux through contact area of contact points, Fig. 4.6.

μ Micron, unit for particle size of powdered milk.

θ Contact angle in Fig. 4.6, radians.

σ Standard deviation of particle size.

ω Solid angle in Appendix V.

ν Poission Ratio.

ρ Bulk Density, g/ml.

CHAPTER I

INTRODUCTION

Due to an ever increasing demand for convenience-type foods, many food products are being produced in dehydrated forms. For example, milk powder, egg powder, coffee, powdered tea, etc. are well known powdered foods; some special products like powdered honey, molasses, caramel and a large variety of fruit and vegetable powders; tomato powder, orange powder, bean powder, pumpkin powder and apple sauce--the list is endless.

As the result of cooperative research by engineers and food scientists, the quality and quantity of these convenience-type foods has been highly improved for over a quarter of a century such that, in fact, powdered food is now one of the most important branches of the food industry.

Most of the dried and powdered products are subjected to either heating or cooling during processing. The quality of milk powder (solubility, bacteria count, flavor and appearance) is affected by the temperature of the product during manufacturing, packaging, and storage as well as the quality of milk.

In order to effectively control the quality and temperature level of the products, reliable information regarding the thermal properties of the powdered food is needed. However, there is only limited information available on this subject. Ojha et al. (1966) have done some preliminary measurements on the thermal conductivities of non-fat, regular, spray, milk powder and wheat flour. The effects of mean operating temperature and moisture content on the effective thermal conductivities of these powdered foods were determined. Farrall et al. (1968) continued this work on various types of milk powder. It was felt that the thermal conductivities of the powdered food might be affected by some other factors, such as bulk density, fat content, particle size and distribution and some geometric structures as well as moisture contents.

Since the usefulness of experimental data is limited to conditions of measurement, it would be desirable to formulate some mathematical models to describe the influence of these factors on the mechanism of heat flow through the powdered food, which is a particulate system.

Particulate systems are studied in many areas of chemical engineering. Particles may be catalyst pellets, ion exchange beads, etc. (Beresford, 1967). Chemical engineers have been mainly concerned with particulate systems in connection with the design and the analysis of

the performance of packed bed equipment, when the bed is used as a heat-exchange device and when it serves as a catalytic reactor. Obviously, the thermal conductivity of such a system is the property of the most concern. A system of powdered food in packed bed can be analyzed as a model of particulate systems.

CHAPTER II

OBJECTIVES

Justification

The thermal conductivities of inorganic granular media have been investigated by Wilhelm et al. (1948) experimentally on macroscopic basis. The results of Kazarian and Hall (1963) and Ojha et al. (1966) for thermal properties of food grains were also investigated by macroscopic approach.

Since the transfer of heat through granulated material is of considerable importance as discussed above, it therefore seems highly desirable to establish some models which can be used to analyze the measured effective thermal conductivities of granulated materials, or of any heterogeneous systems. In addition, with such models, it will also be possible to predict the effective conductivity of a heterogeneous system from the conductivities of its constituent parts.

It is evident, however, that the task of evolving a general theory for the conduction of heat through a heterogeneous system is very complicated. The difficulty is due to its complex bed structure. Even in the comparatively simple case of the conduction of heat through a system of

uniform sphere packed in a regular way, the mathematical difficulties are such that the problem has been studied only briefly to date (Masamune and Smith, 1963). In fact, many naturally or artificially granular materials have a random void structure. Where the system consists of granulated material of undefined shape and where the packing is irregular with a particle size distribution, a strictly mathematical analysis is entirely impossible. This fact, however, need not be discouraging entirely, because it may still be possible to obtain a mathematical formula, arrived at by unorthodox methods based on statistical probability theory for the bed structure, which might have sufficient accuracy and considerable value in practice.

Objectives of This Research

The overall objectives of this research are to describe and represent quantitatively the transfer of heat through a packing bed of small powder particles (Figure 2.1), and derive mathematical models to estimate the effective thermal conductivity of a powder, K_e in Eq. (2.1), made from a solid with known thermal conductivity.

$$q = -K_e \, A \frac{\Delta T}{\Delta X} \tag{2.1}$$

The primary parameters involved would be as follows:

1. The thermal conductivity of the solid material.

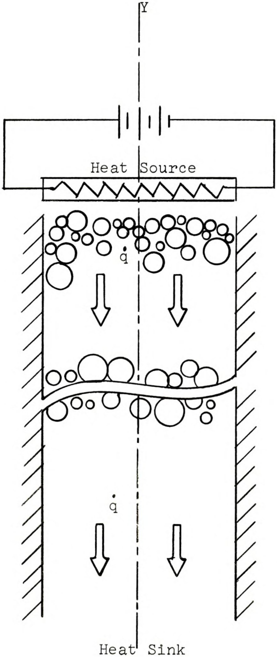


Figure 2.1.--One Dimensional Heat Flow through Particulate System in a Packed Bed.

2. The thermal conductivity of the interstitial gas between the granular particles.
3. The extent of void between the particles and, sometimes, the porosity within each particle.
4. The bulk density of the powder bed.
5. The mean particle size.
6. The particle size distribution.
7. The content number of a sphere in a random bed.
8. Mechanical properties of the solid material, such as modulus of elasticity and Poisson's ratio.
9. Temperature level of measuring.
10. Moisture contents.

Knowing these parameters for a heterogeneous powdered bed, the general mathematical models, as the result of this research, should be able to predict the effective thermal conductivity of the bed and the effect of each parameter for various types of powder, physical structure and processing conditions. The thermal properties as well as the qualities of the powdered food product could be controlled and improved by altering processing conditions.

CHAPTER III

LITERATURE SURVEY AND JUSTIFICATION

Scarlett (1967) presented a relatively complete review of the literature on the topic of thermal conductivity of inorganic powder.

The study of thermal conductivity of a heterogeneous medium was started by Maxwell (1881). It was found that, at atmospheric pressure, the conductivity of the powder was greater than that of the air. Smoluchowski (1910) conducted a systematic study on the thermal conductivities of several powders at various air pressures from 0.2 to 760 mm Hg. to investigate the effect of the air pressure on the thermal conductivity. He plotted the effective thermal conductivity value against the logarithm of the interstitial gas pressure and found that it was a S-shaped curve. He concluded that the conductivity of the powder depends mainly on pressure and nature of the interstitial gas. Some later investigation agreed with Smoluchowski's conclusion in general, but several authors did not agree and stated that gaseous phase is not the only factor in control. No theory was given to relate the conductivity of a heterogeneous system to the conductivities of its constituents satisfactorily. Some new

approaches were suggested. The importance of investigating the effect of each parameter on the heat transfer instead of overall changes due to several parameters was stressed (Rowley et al., 1951). The following literature review is grouped for each parameter involved.

The Conductivity of the Interstitial Gas

The importance of the insterstitial gas (called K_g in the following chapters) was first observed in 1910 by Smoluchowski as mentioned above. Kannuluck and Martin (1933) measured the conductivities of some inorganic powders when the powders were filled with helium, which has a conductivity higher than that of the air. As was expected, they found that the conductivity of the powder was higher than when this product was filled with air. Schuman and Voss (1934) observed the same response with hydrogen. Meanwhile, Kistler and Caldwell (1934) filled the silica aerogel powder with dichloro-difluoromethane, which has a lower conductivity, and found that the conductivity of the powder was lower than that with air, as was expected. Prins et al. (1951) confirmed the results of earlier workers that the effective thermal conductivity of a powder would be increased with a interstial gas of higher conductivity. Recently Deissler and Boegli (1958) conducted an experimental study to determine the effective thermal conductivities of magnesium oxide, stainless

steel, and uranium oxide powders in various gases. They concluded that the effective conductivity of a powder is a strong function of the conductivity of the interstitial gas.

The Effect of Temperature Level

The effect of temperature level on the effective conductivity of a powder is two fold: the thermal conductivities of the constituents and the contribution of radiation.

An organic powder could be considered as a solid-gas system or a mixture of amorphous materials. "If a mixture of amorphous materials forms a heterogeneous system, its thermal conductivity may be considered in first approximation as an additive property and calculated by summation of the contributions of constituents" (Kowalczyk, 1954). Furthermore, "Since K of both constituents increases with temperature, the thermal conductivity of a solid-gas system always increases with temperature" (Perry, 1950; Kowalczyk, 1954). A similar conclusion has been obtained experimentally (Deissler and Boegli, 1958; Kazarian and Hall, 1963).

As the temperature is increased, the radiation from solid to solid through the interstices plays an increasing part of overall heat transfer (Russell,1935; Laubnitz, 1959; Bretsznajder and Ziotkowski,1959). However,

Chapter IV shows the contribution of radiation to be nearly negligible for the practical situation of organic powders.

The Effect of Bulk Density

A packed bed of organic powder could be classified as a two phase system of solid and gas. "The thermal conductivity of a solid-air system at constant temperature is a function of apparent density" (Perry, 1950). "This proves that the thermal conductivity of heterogeneous system is an additive property" (Kowalczyk, 1954).

Farrall *et al*. (1968) conducted an investigation on the effect of bulk density on the thermal conductivity of various types of dry milk. The type of samples included non-fat and whole dry milk prepared by regular spray, roller spray and foam spray processes. An interesting result that the thermal conductivity increased linearly with the bulk density was observed for each type of powdered milk.

The bulk density in turn is influenced by spray drying conditions (Hayashi *et al*., 1967a). In general, bulk density increased with increasing solids content, preheat treatment, and pump pressure and decreasing inlet air temperature.

Thermal Conductivity of Solid Material and Contact Conductance

The powdered material can be characterized if the nature of the solid and the particle size distribution of the powder are known. The conductivity of the solid grains influences the conductivity of the powder, but it would seem to be a secondary effect (Scarlett, 1967). Deissler and Boegli (1958) conducted an experimental study to determine the effective thermal conductivity of magnesium oxide, stainless steel, and uranium oxide powder in various gases including the air. They found that the effective conductivity of a given void in a powdered system is greatly influenced by the arrangement of the material for high values of K_s/K_g, whereas for low value of K_s/K_g the arrangement of the material is of lesser importance. For a given value of K_g (air for example), the value of conductivity of the solid material thus determined the importance of the arrangement of the particulate system. This could be visualized from the characteristic of small regions or points of contact between the particles in the powder. For higher values of K_s/K_g such as the case of metal powder, most heat flow takes place in the vicinity of these points of contacts because the gas acts as an insulator at points where the particles are separated. Figure 3.1 shows the calculated temperature distribution

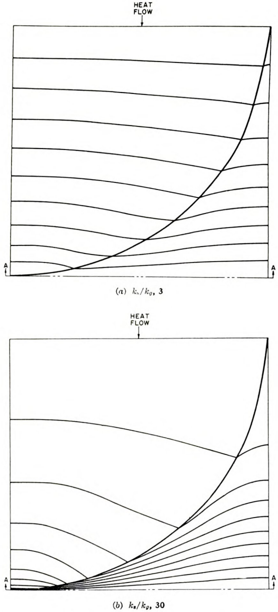


Figure 3.1.--Constant Temperature Lines in Representative Sample of Spheres in Cubical Array as Obtained by Relaxation Solution (Deissler and Boegli, 1958).

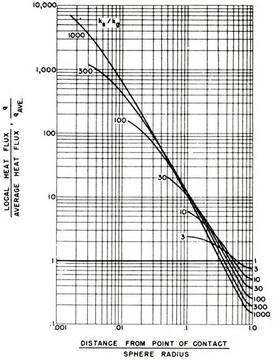


Figure 3.2.--Variation of Ratio of Local Heat Flux (per unit area) to Average Heat Flux Across Plane A-A in Fig. 3.1 for Various Values of k_s/k_g (Deissler and Boegli, 1958).

for heat flow through spheres in cubical array. As the K_s/K_g increases from 3 to 30, the contact temperature lines crowd together in the vicinity of the point of contact; that is, more of the heat flow across the gas spaces takes place near the point of contact. It is evident that for values of K_s/K_g on the order of 1000 nearly all the heat flow will take place through an extremely small area near the point of contact (Deissler and Boegli, 1958). This effect can be seen more clearly in Figure 3.2, where ratio of local heat flux to average heat flux across the plane containing the point of contact (plane A-A in Figure 3.1) is plotted against distance from the point of contact divided by sphere radius. Values of $q/q_{avg.}$ on the order of 5000 were indicated near the point of contact for high values of K_s/K_g. Therefore, for high values of K_s/K_g, the effective conductivity will be very sensitive to the exact way in which the particles make contact and to slight irregularities on the surface near the points of contact (Deissler and Boegli, 1958). However, for organic powder (nonfat dry milk) the K_s/K_g value is comparatively low so that the contact conductance, the arrangement of particles and the regularity of particle surface would not be as important as in the case of metal powder.

The Effect of Particle Size

The influence of particle size is two fold. First of all, it affects the bulk density. Secondly, it affects the mechanism of heat transfer.

Hayashi et al. (1967b) found that the bulk density of nonfat regular spray dry milk increased gradually as particle size was decreased. This result is as expected since interstice between larger product particles will be larger and will reduce the bulk density. Duffie and Marshall (1953) also indicated that small particles may be inherently more dense than larger ones dried under the same condition. For roller dried skim milk, it was found that its density varied from 0.3 to 0.5, depending primarily on the fineness of grinding (Whittier and Webb, 1950; Coulter et al., 1951). The increasing bulk density with smaller particles could result in higher thermal conductivity of the powder. For other inorganic materials, it was found that the thermal conductivity of a glass spheres-air system increased while particle size decreased (Schotte, 1960). However, for metallic powder where K_s is much higher than K_g and thus K_s is the maximum contribution to K_e, a fine powder will have a lower conductivity than a coarse powder packed to the same porosity because of the larger number of gas-solid interfaces which acts as the resistance in the heat path (Scarlett, 1967). This would not be the case for dry

milk powder, because K_s of organic powder would not be as high as that of metallic powder.

The second effect of particle size is on the mechanism of heat transfer. Waddams (1944) measured the conductivity of steel spheres and coarse calcite powder at atmospheric pressure. He found that the conductivity increased with particle size, and the result was contributed to the mechanism of convection heat transfer as the particle size was increased. Kennuluck and Martin (1933) had similar observations for carborundum powder, magnesium oxide, glass and diphenylamine powders. However, the particle size of the organic powder is not large enough to consider convection. More details will be discussed in Chapter IV.

Void and Porosity

Generally, the bulk density of dry milk includes two kinds of air when packed: porosity and void. Porosity is the air contained in each unit particle, while void is defined as the space contained between each unit particle in a container (Hayashi et al., 1967b). However, void defined above was usually called porosity in most works where there is no air inside each unit particle.

The dependence of conductivity on porosity can be explained very easily; the greater the proportion of the bed occupied by gas spaces, the higher the resistance of the bed (Scarlett, 1967). This could be visualized as

K_g is less than that of K_s. Marathe and Tendolkar (1953) measured the conductivities of marble haematite and copper powder and concluded that conductivity varied linearly with the porosity of the bed.

The Effect of Particle Size Distribution

The influence of the particle size distribution of a powder is not clear. Scarlett (1967) was the first to investigate this problem. He measured the conductivities of aluminum powder for various sizes and mixtures of different sizes. He concluded that the conductivity is a function of both the porosity and the mean particle size and; since the porosity depends in turn on the particle size distribution, the conductivity is a function of both mean particle size and the size distribution.

Having made the conclusion above, he demonstrated qualitatively by mixing the coarse particles of less than 150 mesh and the fine particles of greater than 300 mesh grades of powder together in different proportion to produce a range of porosities and mean sizes. Figure 3.3 shows the thermal conductivity of the powders at atmospheric pressure plotted against the percentage of the fine particles in the mixture. The porosity of the packed powder and the mean particle size are shown. The porosity decreased initially as the fine particles filled the pore spaces between the large particles and passed a minimum value at 40% by weight of fines. However, the maximum

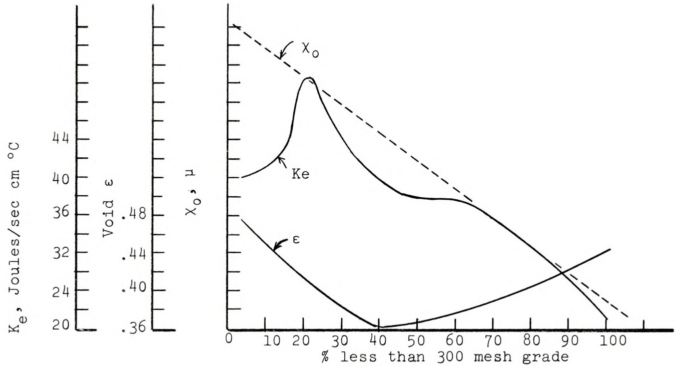


Figure 3.3.--Effect of Particle Size Distribution on Ke, ε and χ_o

conductivity occurs for the 20% mixture and this is clearly due to the opposing influence of the decreasing porosity and decreasing mean particle size. This experiment thus confirmed that a decrease in mean particle size can cause a substantial decreasing in effective conductivity.

The above result was for aluminum powder which has a high K_s. For organic powder which has a much smaller K_s, the effect of mean particle size and porosity (or particle size distribution) on effective thermal conductivity might be different from Scarlett's result.

The Effect of Moisture Content

The fact that the thermal conductivity of a particulate system increased with its moisture content has been reported (Kazarian and Hall, 1963; Ojha et al., 1966; Farrall et al., 1968).

This could be due to two reasons. First of all, moisture content is dispersed in dry milk solid (Hall and Hedrick, 1966; King, 1965; Hayashi et al., 1967a). It has been found that the thermal conductivity of solids increase with increasing moisture content (Patten, 1909). This is understandable since the conductivity of a liquid is much greater than that of a gas. It also could be visualized from the fact that a system in which the continuous medium is a liquid will have a higher conductivity (Scarlett, 1967). The second reason for K_e value

to be increased with moisture content might be due to the increasing of bulk density. Hayashi et al. (1967b) found that bulk density of nonfat regular spray dry milk increased gradually with its moisture content up to 6%.

After moisture content of non-fat dry milk is greater than 6%, some physical changes would be expected, such as lactose crystallization or clustering of particles (Hayashi et al., 1967a).

The Effect of Pressure and Mechanical Properties of the Solid Material

As the bulk density of a powder bed is increased either by application of pressure or by settling, void will be decreased. Meanwhile, the contact area and the number of contact points between particles will be affected.

Mohsenin (1966) suggested that the following equation be used to calculate the radius of the contact area between two particles under pressure: (Figure 3.4)

$$a = 0.721 \left(\frac{wB}{\frac{1}{x_1} + \frac{1}{x_2}} \right)^{1/3} \tag{3.1}$$

where

w = Loading, lbs.
x_1, x_2 = Diameter of two particles in contact, in.
a = Radius of the contact area, in.

$$B = \frac{1 - \nu_1^{\,2}}{E_1} + \frac{1 - \nu_2^{\,2}}{E_2} \tag{3.2}$$

$$= 2\,\frac{1 - \nu^2}{E}$$

if

$$\nu_1 = \nu_2 \text{ and } E_1 = E_2,$$

where,

E = Modulus of elasticity, psi

ν = Poisson's ratio.

Thus the radius of contact area of two identical spheres in contact could thus be expressed as:

$$a = 0.721 \left[\frac{W\ x_o\ (1 - \nu^2)}{E} \right]^{1/3} \tag{3.3}$$

Since the contact thermal conductivity, or sometimes called "Residual Thermal Conductivities," is related to the contact area between two particles, the effect of pressure and mechanical properties of the solid material on K_e could thus be visualized.

Also, according to Kowalczyk (1954):

> . . . Pressure exerted on amorphous solids increases the contact area between molecules and atoms and should, therefore, promote heat conduction by atomic vibrations. This conclusion was confirmed by experiments of Bridgman (1949) who has shown that for nonmetallic, amorphous solids (Pyrex glass, limestones, talc, etc.) the thermal conductivity increases proportionally to the pressure, according to the formula:

$$K_p = K_{1\ atm} + d\ \frac{p}{1000} \tag{3.4}$$

where p is the pressure in kilogram/cm^2 and d is a numerical constant, whose value was determined experimentally for each material.

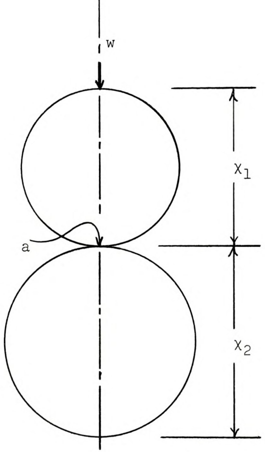


Figure 3.4.--Two Spherical Particles in Contact.

CHAPTER IV

DEVELOPMENT OF THEORY

Assumption

> Theoretically, it should be possible to calculate the effective conductivity of a given powder from the conductivities of the solid and gas by using Laplace's heat conduction equation in the solid and in the gas together with the boundary conditions at the interface. The actual calculation of the results by this method, however, appears to be impracticable because of the irregular shape and arrangement of the powder particles. (Deissler and Boegli, 1958.)

Also, Cetinkale and Fishenden (1951) claimed that direct analytical solution proved impracticable because of the many boundary conditions to be satisfied. Therefore, some reasonable assumptions for the practical situation are necessary in order to make the formulation of the theory possible. There are five assumptions for the case of organic powder:

1. The Mechanism of Radiation Heat Transfer is Negligible

The condition of radiation heat transfer was first pointed out by Smoluchowski (1910). Radiation may contribute significantly at high temperature, particularly when the particles are larger (Schotte, 1960; Wilhelm, et al., 1948; Strong et al., 1960). Russell (1935) and

Damköhler (1937) postulated some formula for calculating the contribution to heat transfer by radiation. However, the mean temperature range on which Laubnitz (1959) measured the conductivities was 100°C to 1000°C. Schotte (1960) pointed out that radiation would not be important unless the particle size is greater than 1 mm at temperatures above 400°C, or for 0.1 mm particles above 1500°C. For this reason the radiation was neglected in most research to date (Masamume and Smith, 1963; Wilhelm <u>et al</u>., 1948). The upper limits for Wilhelm <u>et al</u>. (1948) to neglect radiation were 3-4 mm particle size, and 8-10 atmospheres, 300°C. Therefore, the particle size and temperature level of the organic powder were practically within the range where radiation is negligible.

Argo and Smith (1953) simplified Damköehler's expression for radiation contribution and suggested the following empirical equation:

$$K_r = 4 \left(\frac{\varepsilon}{2 - \varepsilon}\right) x_o \; (0.173) \left(\frac{Ta^3}{100^4}\right) \qquad (4.1)$$

For example, non-fat, regular spray milk powder will give the following results:

Assume void $\varepsilon = 0.57$,

mean particle size $x_o = 55\,\mu = 0.18 \times 10^{-3}$ ft

(4.2)

$$\text{average temperature} = 150°F = 610°R \tag{4.3}$$

then, by Equation (4.1),

$$K_r = 4\left(\frac{0.57}{2 - 0.57}\right)(0.18 \times 10^{-3})(0.173)\left(\frac{610^3}{100^4}\right) \tag{4.4}$$

$$= 0.1125 \times 10^{-3} \text{ Btu/hr ft °F}$$

which is practically negligible as compared to the effective thermal conductivities, K_e, which ranges from 0.1 to 0.3 Btu/hr ft °F (see Table 6.6).

2. Free Convection is Negligible

As indicated in Chapter III when discussing particle size, convective heat transfer starts to play a role as the particle size of a particulate system becomes larger. However, convection cannot be an important mechanism of heat transfer through fine powders (Schuman and Voss, 1934; Russell, 1935). Deissler and Eian (1952), Deissler and Boegli (1958), Schotte (1960) confirmed this result and indicated that free convection was not an important factor in determining the effective thermal conductivity of a powder at high pressures up to 100 atm; the thermal conductivity being independent of gas pressure. If there were approciable free convection in the powder, the conductivity would continue to increase with

gas pressure inasmuch as free convection is a function of the density of the gas.

Wilhelm et al. (1948) indicated the limits within which free convection can be neglected by stating "Heat transfer is almost purely conductive provided the particle size, the gas pressure, or the temperature are not too high. Rough upper limits are 3-4 mm diameter, 8-10 atmosphere, and 300°C." Russell (1935) also pointed out that with pore space greater than 1/4 in., convection occurs. For organic powders, the particle size and temperature are certainly within this limit of negligible free convection.

3. Heat Flow is Assumed One Dimensional

Due to its complexity, most of the theoretical predictions of effective conductivity have been derived assuming plane isothermal or parallel lines of heat flow (Russell, 1935; Gemant, 1950; Deissler and Eian, 1952; Woodside, 1958; Deissler and Boegli, 1958; Masamune and Smith, 1963). All of these previous investigations have one factor in common, that is, they assumed a regular cubical array for the arrangement of uniform spheres to simplify the random arrangement and size distribution in the particle situation. Thus, the results can only be order of magnitude estimates.

In order to investigate the validity of one dimensional heat flow, Deissler and Eian (1952) applied relaxation method on several regular arrangements of cylinder packing to investigate the effect of bending heat-flow lines. However, they did not improve their prediction of effective thermal conductivities because the relaxation method still could not account for irregularity of particle arrangement. Therefore, they accepted this assumption of isothermal plane because it gave better agreement between the theoretical and the experimental results.

4. The Particles are Assumed Spherical and Smooth

Obviously the shape and surface conditions of the particles do influence the effective thermal conductivity. By Scarlett (1967), polishing the spheres decreased the effective conductivity, but he did not offer any explanation. "Spray dried particles are usually spherically shaped, but some may be elongated and generally range in diameter from 10 to 250μ." Also, "The surface of the spray dried milk particles is usually smooth" (Hall and Hedrick, 1966; Hayashi, 1962). Therefore, this assumption is made to simplify the complex situation.

5. Bulk Density is the Only Criterion for Mechanical and Thermal Properties of Solid

In order to investigate the mechanical and thermal properties of powder solid, the powder will be compressed

until the bulk density of the powder reaches the density of the powder solids. Figure 4.1(a) shows a cross-section of solid material and Figure 4.1(b) illustrates compressed powder until ρ is equal to ρ of solid. It is assumed here that the mechanical and thermal properties of Figure 4.1(a) can be approximated by that of Figure 4.1(b).

Since the particle size of organic powder is so small, it is impossible to measure the mechanical and thermal properties of particle solid directly. The above assumption suggests that a compressed solid block of powder with the same bulk density as particle solids would approximate the mechanical and thermal properties of particle solids.

Structure of Random Bed

In practice we are generally concerned with the behavior of spheres or irregularly shaped particles when packed at random. The arrangement of such a packing is very difficult to determine, especially for particles with size distribution. No comprehensive relationships between the variables describing such a packing arrangement has been developed to date. However, one possible approach to such a problem has been shown by means of statistical methods (Debbas and Rumpf, 1966).

(a) Powder solids
(ρ = 1.46 g/ml)

(b) Compressed powder solids
(ρ = 1.46 g/ml)

Figure 4.1.--Powder Solids of Regular Nonfat Dry Milk.

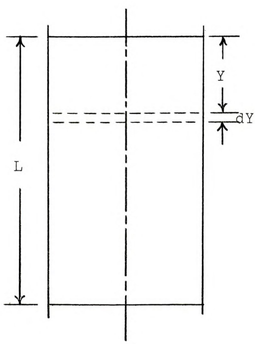


Figure 4.2.--A Packed Bed for Eq. (4.2).

1. Definition of a Random Bed

A random packing can be defined analogous to a random mixture in the following manner: "Every particle has the same probability to occupy each unit volume throughout the packed bed, and with regard to the orientation every direction has the same probability" (Leschonski, 1967). From this definition, the following two relations could be deduced.

2. Relation Between the Volume and Area Void

The volume void, ε, is defined as the ratio of the volume of void to the total volume of the packed bed including void and particles. The area void, ε_F, is analogously defined as the ratio of the area of the sectioned voids in a cross-section of the packing, to the total area of the sectioned voids and particles.

From the definition of randomness, no cross-section is preferred to the other, hence:

$$\varepsilon_{F_1} = \varepsilon_{F_2} = \cdots\cdots = \varepsilon_{F_n} \tag{4.5}$$

i.e., the mean area void of all cross-sections is equal. Since ε_F = constant, therefore

$$\varepsilon = \frac{\int_0^L \varepsilon_F \cdot A \cdot dY}{A \cdot L} = \varepsilon_F \tag{4.6}$$

(see Figure 4.2) i.e., the mean volume void is equal to the mean area void in a random bed (Masamune and Smith, 1963; Rumpf, 1958).

3. Particle Size and Size Distribution

Particle-size distribution studies are becoming increasingly important in numerous areas of agricultural engineering research. The particle size and size distribution of dry milk is a basic and important physical property influencing product reconstitutability, packing density and dustability. Hayashi et al. (1967c) conducted some investigations on the size distribution of nonfat dry milk. Their results could be properly described by log-normal distribution as following:

$$f(x) = \frac{1}{\sqrt{2\pi}\ \sigma_g} \exp\left\{- 1/2 \left[\frac{\ln x - \ln \bar{x}_g}{\ln \sigma_g}\right]^2\right\} \qquad (4.7)$$

where

$f(x)$ = log-normal density function,

$\bar{x}_g$ = geometric mean of particle sizes = x_o

σ_g = geometric standard deviation.

When a log-normal distribution may be approximately assumed, the cumulative frequency data may be fitted as a straight line on log-probability paper. The abscissa value corresponding to 50 per cent level is the geometric mean and the ratio of the 84.13 per cent to the 50 per cent values is the geometric standard deviation. (Cooke and Bowen, 1966; Orr and DallaValle, 1959.)

Figure 4.3 shows the log-normal distribution of regular nonfat dry milk plotted from the observation of Hayashi et al. (1967c). Also, the results of Mori (1964) for instant dry milk is shown. The abscissa values correspond to 50 per cent and 84.13 per cent are 45.5 micron and 76 micron, respectively. Therefore, for regular nonfat dry milk,

$$x_o = 45.5 \text{ micron}$$

$$\sigma = 76/45.5 = 1.67 \qquad (4.8)$$

Equation (4.7) along with Equation (4.8) gives the frequency distribution of particle sizes of regular nonfat dry milk, or f(x) in Figure 4.4

The area under curve in Figure 4.4 was found to be 50.1. By probability theory in statistics, the area under the curve should be equal to unity for the total range of particle sizes (Parzen, 1965). Then one unit on the ordinate scale will be determined as following:

$$\frac{1}{10 \times 50.1} = 0.00196 \simeq 0.002 \qquad (4.9)$$

Therefore,

$$F(x) = \frac{0.002}{0.1} \; f(x)$$

$$= W \; f(x)$$

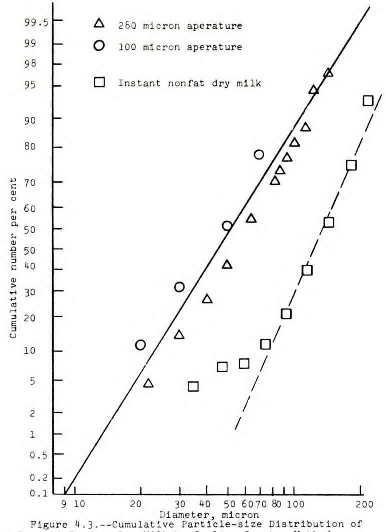


Figure 4.3.--Cumulative Particle-size Distribution of Regular and Instant Dry Milk by Coulter Counter Method.

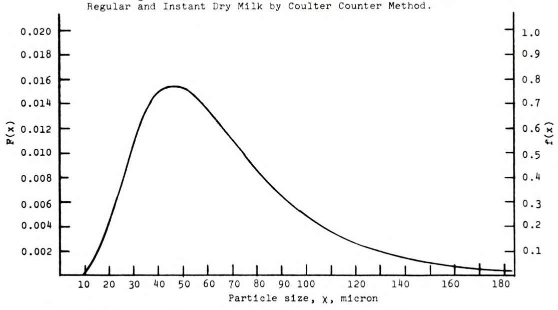


Figure 4.4.--Particle-size Distribution of Regular Nonfat Dry Milk.

$$= W \frac{1}{\sqrt{2\pi}\ \ln \sigma g} \exp \{-1/2\ [\frac{\ln x - \ln \bar{x}g}{\ln \sigma g}]^2\} \quad (4.10)$$

where

$$W = \frac{1}{50}$$

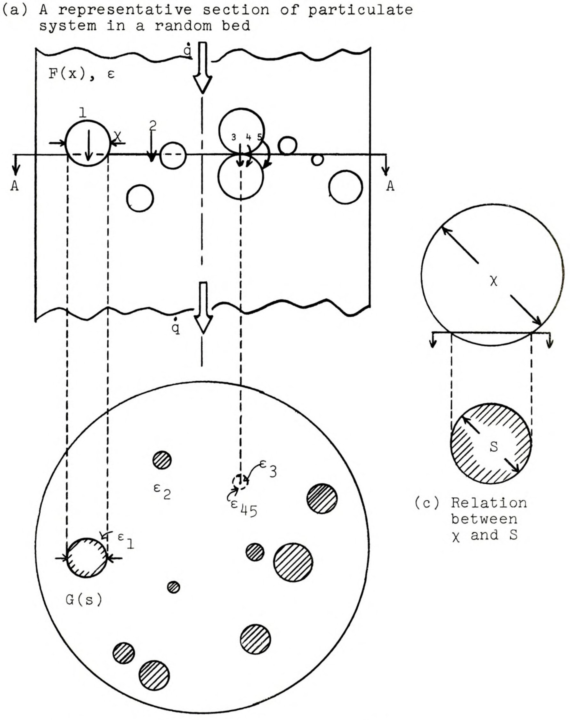
and $F(x)$ is the probability frequency distribution function of particle sizes of nonfat dry milk, with the following conditions:

$$F(x) \geqq 0 \text{ and } \qquad \text{for all } x \qquad (4.11)$$

$$\sum_{x=0}^{x_{max}} F(x) \cdot \Delta x = 1$$

4. The Distribution Function of the Sizes of the Particles Appearing on a Cross-section of a Random Bed

Figure 4.5(a) shows a section of a granular bed. The particles in the bed have a size distribution function F(x). The diameters, s, of the circles appearing on the cross-section of the granular bed with particles size distribution F(x) have a frequency distribution function G(s) (Figure 4.5[b]). The relation between F(x) and G(s) was suggested as follows



(b) Cross-section AA (Area = A = 1)

Figure 4.5.--A Section of Random Bed.

$$G(s) = \frac{s}{M_{1F}} \int_{x=s}^{x=x_{max}} \frac{F(x)}{(x^2 - s^2)^{\frac{1}{2}}} dx \qquad (4.12)$$

by consideration of probability (Debbas, 1966), where

$$M_{1F} = \int_{o}^{x_{max}} x\ F(x)\ dx \qquad (4.13)$$

is called the first moment of F(x). Notice that in Equation (4.12) the limits of integration are from s to x_{max} indicating that any sphere with size smaller than s will not contribute to the frequency distribution function G(s) on the cross-section.

Equation (4.12) could be reduced to the equation as follows for numerical integration:

$$G(s) = \frac{C_1 S}{M_{1F}} \left[\frac{1}{x} (x^2 - s^2)^{\frac{1}{2}} \cdot e^{C_2 \ln^2 \frac{x}{x_o}} \Bigg|_{s}^{x_{max}} - \int_{x=s}^{x=x_{max}} g(x)\ dx \right] \qquad (A7)$$

where C_1 and C_2 are constants for a given particle size distribution. Appendix I gives a complete derivation and definitions of Equation (A7).

There may be two applications for Equation (4.12). Since Equation (4.12) gives the distribution of sizes of

sectioned particles on the cross-section surface, the integration of Equation (4.12) should give the total sectioned area of solid. In addition, as s becomes very small, for example, 0.01μ, G(0.01μ) in Equation (4.12) would then possibly give the number of contact points on the cross-section surface A-A of Figure 4.5.

The Model of Heat Transfer Mechanisms

Due to the complexity of geometry involved, one dimensional heat flow in a powder bed has been assumed.

Under this assumption, any cross section of a random bed coulg be considered as an isothermal plane. Heat transfer through the isothermal plane (Section AA, Figure 4.5(a)) could be described by the following mechanisms.

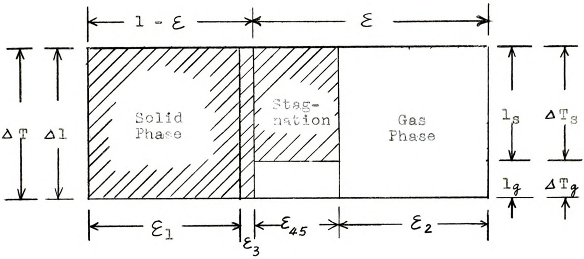
<u>Mechanism 1</u>: Heat Transfer through the Solid Phase.

<u>Mechanism 2</u>: Heat Transfer through the Interstitial Gas by Conduction and Radiation.

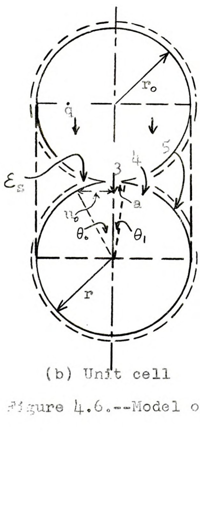
<u>Mechanism 3</u>: Heat Transfer through the contact area of the contact points on the cross-section.

<u>Mechanism 45</u>: Heat Transfer through the stagnation fluid around the contact points by conduction and radiation.

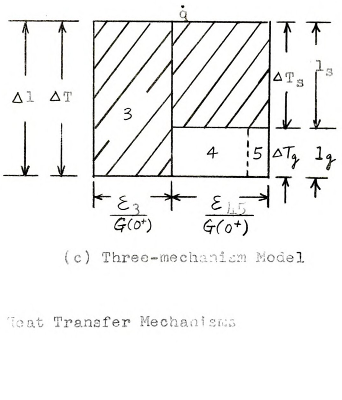
These four mechanisms are sketched in Figure 4.6(a). The effective area for each mechanism are denoted by ε_1, ε_2, ε_3 and ε_{45} respectively. They can be evaluated as follows:



(a) Model of Heat Transfer Mechanisms Through the Cross-section of a Random Bed



(b) Unit cell



(c) Three-mechanism Model

Figure 4.6.--Model of Heat Transfer Mechanisms

$$\varepsilon_1 = 1 - \varepsilon - \varepsilon_3 \tag{4.14}$$

$$\varepsilon_2 = \gamma\ \varepsilon \tag{4.15}$$

$$\varepsilon_3 = (\varepsilon_3 + \varepsilon_{45})\ \delta$$

$$= \varepsilon(1 - \gamma)\ \delta \tag{4.16}$$

$$\varepsilon_{45} = (\varepsilon - \varepsilon_2)\ (1 - \delta)$$

$$= \varepsilon(1 - \gamma)\ (1 - \delta) \tag{4.17}$$

where

$$\phi = l_g/\Delta l$$

$$\beta = \Delta l/x_o$$

$$\gamma = \varepsilon_2/\varepsilon$$

$$\delta = \varepsilon_3/(\varepsilon_3 + \varepsilon_{45}) \tag{4.18}$$

The parameters ϕ, β, γ and δ are evaluated in Appendix IV.

Since four mechanisms operate in a parallel fashion, their separate contribution may be added to obtain the total heat flow across the cross-section.

$$K_e \frac{\Delta T}{\Delta l} = \varepsilon_1\ K_s \frac{\Delta T}{\Delta l} + \varepsilon_2\ K_g \frac{\Delta T}{\Delta l} + \varepsilon_3\ K_s \frac{\Delta T}{\Delta l} + \varepsilon_{45}\ K_{45} \frac{\Delta T}{\Delta l} \tag{4.19}$$

where

K_{45} = Thermal conductivity of the series mechanism which represents heat flow through the solid phase and the stagnation fluid whose total length is Δl.

$$\frac{K_{45}}{l} = \frac{1}{\begin{matrix}\text{resistence} \\ \text{through} \\ \text{stagnation}\end{matrix} + \begin{matrix}\text{resistence} \\ \text{through} \\ \text{solid phase}\end{matrix}}$$

$$= \frac{1}{\frac{l_g}{K_g} + \frac{l_s}{K_s}}$$

$$= \frac{1}{\frac{\phi \Delta l}{K_g} + \frac{(1 - \phi) \quad \Delta l}{K_s}} \tag{4.20}$$

$$K_{45} = \frac{1}{\frac{\phi}{K_g} + \frac{1 - \phi}{K_s}} \tag{4.21}$$

Therefore Equation (4.19) can be written as:

$$K_e = (1 - \varepsilon) \ K_s + \gamma\varepsilon \ K_g + \varepsilon(1 - \gamma)\delta \ K_s$$

$$+ \ \varepsilon(1 - \gamma) \ (1 - \delta) \ \frac{1}{\frac{\phi}{K_g} + \frac{1 - \phi}{K_s}} \tag{4.22}$$

In Equation (4.22) the first term is the contribution of the solid phase, the second term is the contribution of the interstitial gas and the last two terms are the contribution of the contact point.

Let

$$K_1 = (1 - \varepsilon)\, K_s$$

$$K_2 = \gamma\varepsilon\, K_g \tag{4.23}$$

$$K_c = (1 - \varepsilon)\delta\, K_s + \frac{\varepsilon(1 - \gamma)\,(1 - \delta)}{\dfrac{\phi}{K_g} + \dfrac{1 - \phi}{K_s}}$$

Then Equation (4.22) is reduced to the following:

$$K_e = K_1 + K_2 + K_c \tag{4.24}$$

K_c in Equation (4.24) represents the contribution of total contact points on the cross-section. If G(0+) represents the number of contact points expected on the cross-section of a random bed, then the contribution of a single contact point to K_e will be as following:

$$K_{345} = K_c/G(0+) \tag{4.25}$$

Thus

$$K_e = K_1 + K_2 + K_{345} \cdot G(0+) \tag{4.26}$$

where K_1 can be calculated from Equation (4.23), K_2 can be calculated by Equations (4.23) and (A21), K_{345} can be calculated by Equations (4.25), (4.23), A18), (A31), A21), (A22), (A7), (A8), (A9) and (a1).

The value of K_s for regular nonfat dry milk solids will be determined experimentally in the following chapters. The values of K_g for various types of gases could be found. For the case of air, K_g is effected by temperature as follows (Kreith, 1964):

$$K_g = 0.0133 + 0.000021\ T \tag{4.27}$$

CHAPTER V

INSTRUMENTATION, EQUIPMENT AND EXPERIMENTAL PROCEDURES

There are several parameters in the mathematical models shown in Chapter IV which must be determined experimentally. They are the mechanical properties of powder particles, thermal conductivities of dry milk solid, number of contact points between particles in a packed bed and effective thermal conductivities of various powdered milk at various bulk densities and moisture contents.

Preparation of Specimens

1. Adjustment of Bulk Density

Since the size of dry milk particles is so small (usually from 10 to 250μ) the direct measurement of mechanical properties and thermal properties of particle solid is practically impossible. In order to prepare specimens of milk solids with workable dimensions, the bulk powdered milk was compressed mechanically until the bulk density of the specimen approached the density of particle solid. Then the mechanical and thermal properties of the specimen were measured to approximate the mechanical and thermal properties of the dry milk solids.

The powdered milk was compressed manually with a hydraulic press[1] shown in Figure 5.1. The maximum force available from the press is 60 tons. The cylinder and piston for making specimens are sketched in Figure 5.3(a). The cylinder, C in Figure 5.3(c), was made of steel AISI 4340 and the thickness was designed so that there was no appreciable expansion of the cylinder as the pressure of the piston reached as high as 70,000 psi (Lampi et al., 1965). The sliding collar was installed in order to eliminate the friction between the specimen and the cylinder wall. After the bulk density of the specimen approached the density of particle solid (1.44 to 1.48 g/ml; Hall and Hedrick, 1966), the specimen was removed from the cylinder by continuous application of hydraulic pressure. Specimens made by the 3 in-cylinder shown in Figure 5.3(b) were required for the transient thermal properties measurement facility. Figure 5.3(c) shows the tube used to prepare square specimens. Figure 5.2 shows the specimens made.

2. Adjustment of Moisture Content

The moisture content of commercial nonfat dry milk ranges from 3 to 4%. In order to investigate the effect of moisture level on the effective thermal conductivity of dry milk solids, the moisture content of the regular

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[1]Manufactured by K. R. Wilson Co., Buffalo, New York. Model 37E, Ser.: 2576.

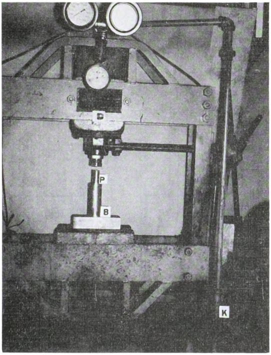


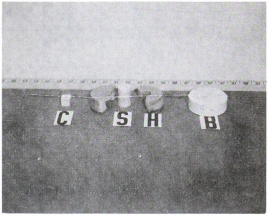
Figure 5.1.--Hydraulic Press

D = Hydraulic drum

P = Piston

B = Cylinder and Anvil

K = Hydraulic pump



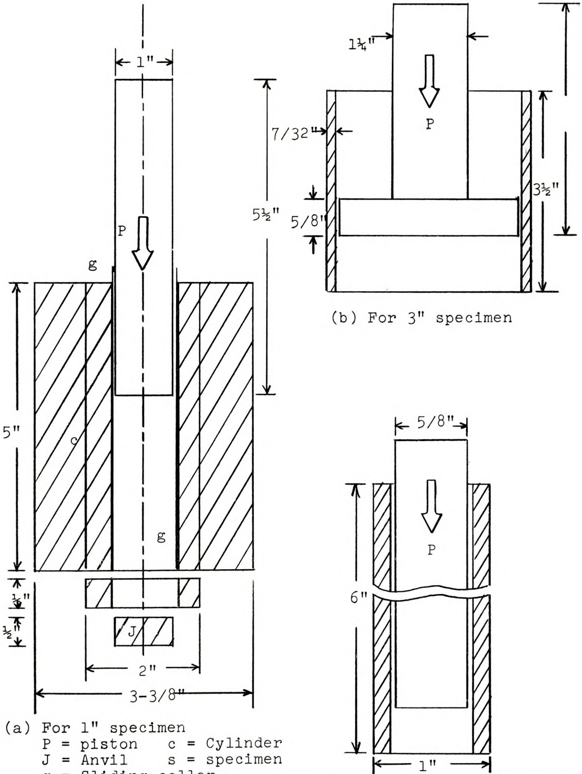
S = 1" Specimen

H = Insulation

B = 3" specimen

C = Cubic specimen

Figure 5.2.--Specimen for Measurement of Solid Thermal Conductivity.



(a) For 1" specimen
P = piston c = Cylinder
J = Anvil s = specimen
g = Sliding collar

(b) For 3" specimen

(c) For 5/8" cube specimen

Figure 5.3.--Specimen Press.

powdered milk was raised to a higher moisture level before measuring K_e. This was done by spraying a thin layer of dry milk (1/2 in. or less) on 2 ft by 2 ft square pans and then keeping these pans in a high moisture room for several hours. The high moisture room is available in the dairy plant of Michigan State University with room temperature of 61.5°F and 100% relative humidity. Then the powder was kept for a few days in a sealed container for moisture equilibration. The moisture content of the powdered milk was tested by toluene distillation method (American Dry Milk Institute, Inc., 1965).

Instron and Mechanical Properties of Particle Solid

The mechanical properties of the dry milk solid were measured using an Instron testing machine[1] available in the Agricultural Engineering Department (Figure 5.4). Figure 5.5 shows a sketch of the Instron System. The specimen was kept at position A of Figure 5.5(a). The cross-head moved downward at a constant speed, which could be adjusted. The balance gave the amount of loading so that by knowing the chart speed, the stress-strain relation of the specimen could be established. In addition, the deformations in both vertical and horizontal directions were measured by sensor C and D in Figure 5.5(a) and plotted by the X-Y Recorder after amplification.

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[1]Manufactured by Instron Co., Canton, Mass. Model No. TM, Ser. No.: 1687.

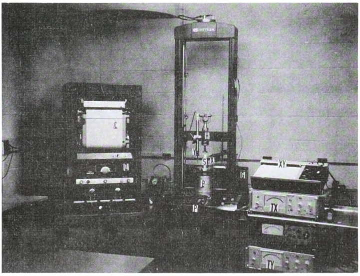

Figure 5.4.--Instron Testing Facility

C = Chart
Q = Control Panel

H = Cross-head
S = Specimen
B = Balance
X = X-sensor
Y = Y-sensor
P = Operation Panel

TX = X-Transducer
TY = Y-Transducer
XY = X-Y plotter

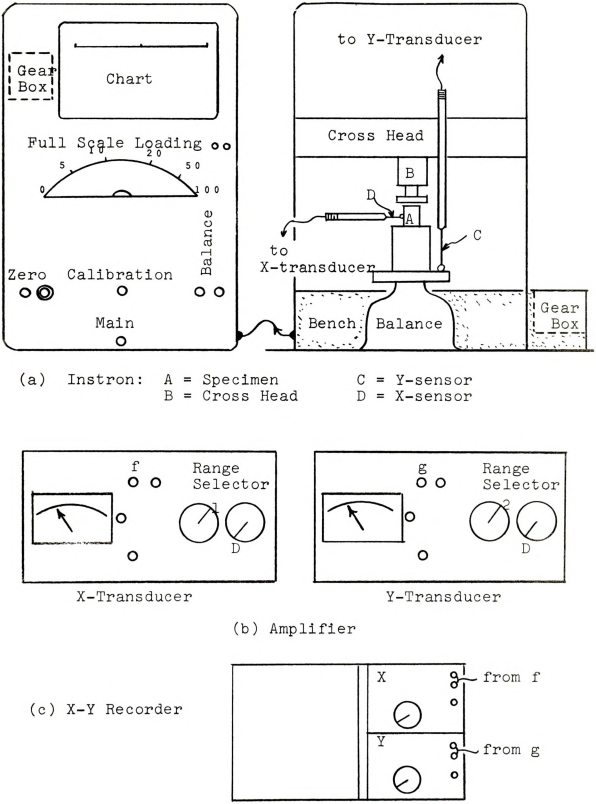

Figure 5.5.--A Sketch of Instron System.

Transient Thermal Properties Measurement Facility

The thermal conductivities of dry milk solids were measured using the facility shown in Figure 5.6. The major components of the facility are sketched in Figure 5.7. They are:

1. Specimen Shell; which includes sample specimen and supporting block (A and B in Figure 5.7).
2. Insulation-Heater mechanism; the solenoids control the Heaters which swing in and out as necessary.
3. Upper Specimen Assembly (D in Figure 5.7); the copper block at the center acts as the heat source during the experiment.
4. Hydraulic System;[1] the hydraulic cylinder C would raise the Specimen Shell and press the sample specimen against the heat source (U in Figure 5.7) during the experiment.
5. Control Panel;[2] which controls the temperature of the heat source and the combination action of solenoids, heater and hydraulic pressure cylinder.

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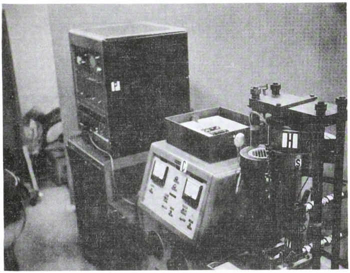
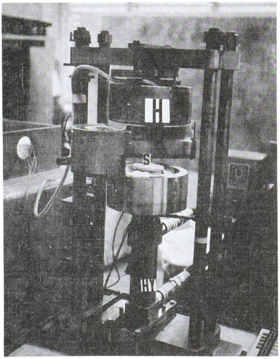
[1]Manufactured by Sperry Rand Co., Model No. CHJO 11609.

[2]Manufactured by Standard Electrical Prod. Co., Dayton, Ohio.

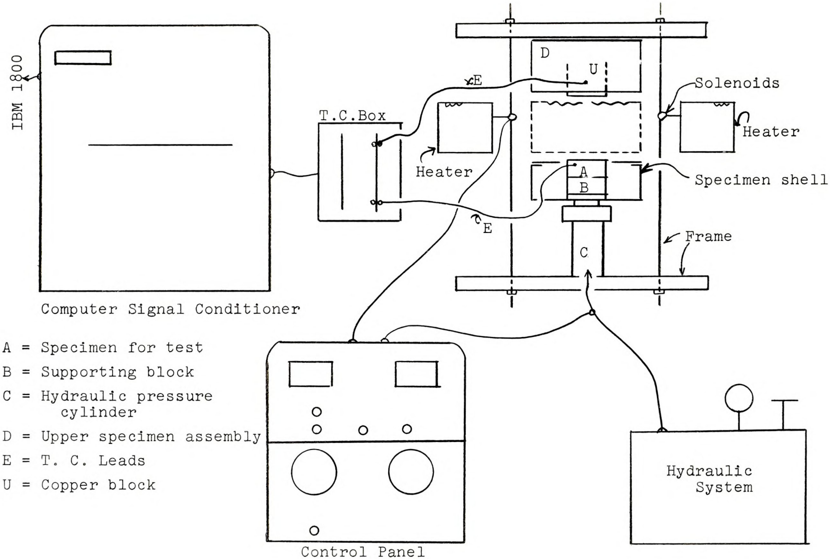


(a)
H = Heat Source
S = Specimen Shell
HY = Hydraulic Pressure Cylinder
C = Control Panel
P = Computer Signal Conditioner



(b)
H = Heat Source
S = Specimen in Position
HY = Hydraulic Press

Figure 5.6.--Optimun Transient Thermal Properties Measurement Facility.



A = Specimen for test

B = Supporting block

C = Hydraulic pressure cylinder

D = Upper specimen assembly

E = T. C. Leads

U = Copper block

Figure 5.7.--Sketch of Optimun Transient Thermal Properties Measurement Facility.

6. Computer Signal Conditioner;[1] which reads emf of each thermocouple and sends the temperature data directly to IBM 1800 Computer.

Before conducting a test, the Computer Signal Conditioner, Control Panel and Hydraulic System had to be well balanced and adjusted. Then the sample specimen with thermocouple leads were put in position (Figure 5.6[b]). The heater was swung in to heat the copper block (U in Figure 5.7). As the test was started, the heater swung out by solenoids and, meanwhile, the sample specimen was raised by the hydraulic pressure cylinder (C in Figure 5.7) and eventually was pressed against the heat source, U in Figure 5.7. The temperature history of the sample specimen was punched out by IBM 1800 Computer.

Figure 5.8 shows the dimensions of the specimen tested. The diameter of the copper block (U in Figure 5.6) is 3 in. The specimen shown in Figure 5.8(a) was prepared with the same diameter as the copper block and was tested without insulation around the side wall of the specimen. Computations in Appendix VI illustrate the amount of heat loss from the side wall. It is evident that the magnitude of the heat loss from the side wall is about 7.4% of the heat transferred from the copper block. Therefore, insulation (styrofoam) was placed around the specimen of

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[1]Manufactured by S. Sterling Co., Application Division, Southfield, Michigan.

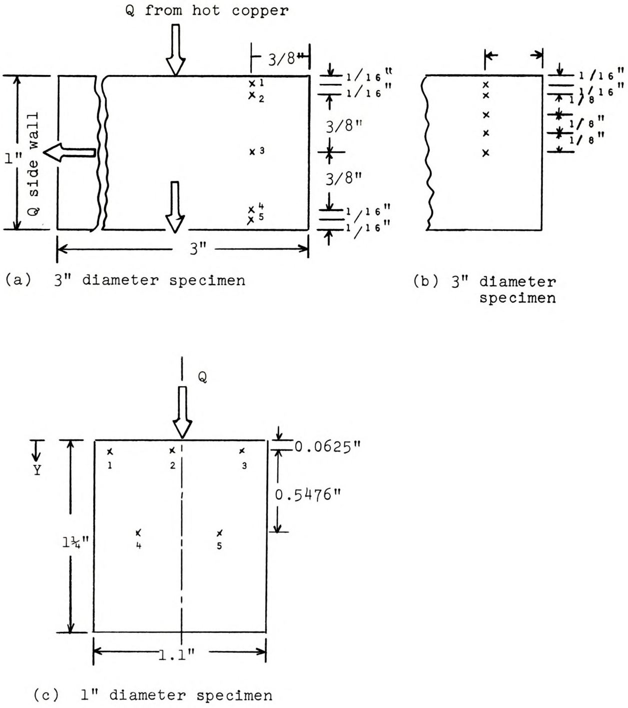


Figure 5.8.--Thermocouple Locations of Specimens for Measuring Thermal Properties.

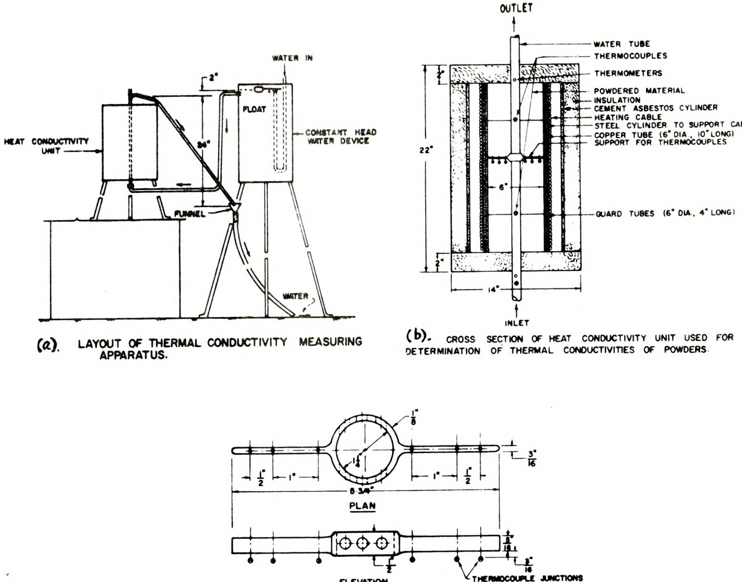
Figure 5.2 to eliminate the heat loss from the side wall.

The whole facility was located in the Engineering Building of Michigan State University.

Steady-State Thermal Conductivity Measuring Apparatus

Figure 5.9(a) shows the layout of the entire apparatus. It was originally designed by Ojha et al. (1966) and modified by Farrall et al. (1968). Powdered milk[1] was heated by the electrical cable around the copper cylinder, Figure 5.9(b). The voltage of the heating cable was regulated depending on the desired temperature level of the powder for conducting the experiment. Usually it was around 42 volts or less to increase the mean powder temperature to around 140°F. Heat was transferred through the powdered material radially into the center tube where cooling water was flowing. The amount of heat transferred from the temperature rise and flow rate of cooling water was calculated. The constant head water device in Figure 5.9(a) gave a constant flow rate of cooling water which was kept at room temperature. The thermocouples were distributed across the radius and were held in position by a wooden holder (Figure 5.9[c]).

[1]Manufactured by Mid-west Producer's Creameries Inc., South Bend, Indiana. Extra Grade, Hi-Heat, Bakery Quality, Spray Process.



(a). LAYOUT OF THERMAL CONDUCTIVITY MEASURING APPARATUS.

(b). CROSS SECTION OF HEAT CONDUCTIVITY UNIT USED FOR DETERMINATION OF THERMAL CONDUCTIVITIES OF POWDERS.

(c). THERMOCOUPLE JUNCTIONS HOLDER (WOOD)

Figure 5.9.--Steady-state Thermal Conductivity Measuring Apparatus.

Number of Contact Points Between Particles

In order to verify the number of contact points of each particle in a random packed bed, the value of n in Equation (A31) was measured by the apparatus shown in Figure 5.10. The "atom balls"[1] were packed in a cylindrical container. The balls were poured in randomly. For the random bed, the size of the container must be at least seven times larger than the size of spheres in order to avoid the effect of wall (Saunders and Ford, 1940). The sponge sheet at the top of the packed bed held the spheres in the bed undisturbed as the Poster-Tempera dye[2] was poured in through the funnel. The dye stayed in the void between balls for about 15 minutes before it was drained out. The dye was collected as it was drained for re-use. Then the sponge sheet was removed carefully and the whole bed was allowed to dry overnight.

After the whole bed dried completely, the "atom balls" were picked out manually one by one for examining the number of contact points on each sphere.

After each test, the dyed balls were washed by flushing in water with detergent and then collected for re-use. In addition, the porosity of the bed was measured by the ratio of the water volume to fill the void space between spheres to the container volume.

[1]Manufactured by Plasteel Corp., 26970 Princeton, Inkster, Michigan.

[2]Manufactured by Permanent Pigment Co., Cincinnati 12 Ohio.

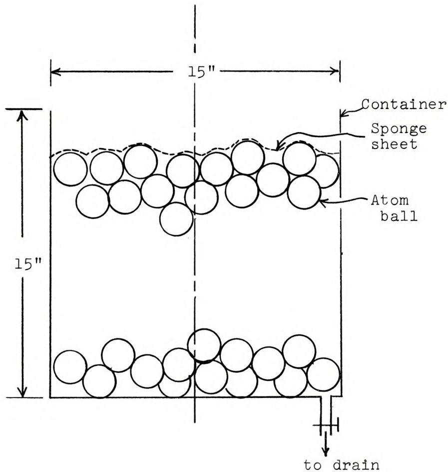


Figure 5.10.--The Container for Testing Contact Points.

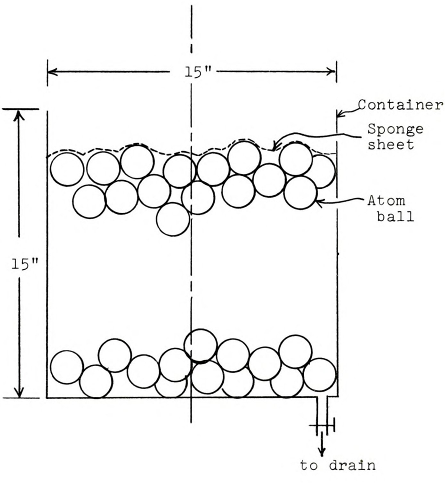


Figure 5.10.--The Container for Testing Contact Points.

Figure 5.10.--The [illegible] points.

CHAPTER VI

RESULTS AND DISCUSSION

Pressure, Bulk Density, Void and Porosity

1. Pressure and Bulk Density

In preparation of specimen by the hydraulic press shown in Figure 5.1, the relation between mechanical pressure and bulk density of dry milk specimen was observed.

There are few investigations reported on the relationship between pressure and bulk density of powdered materials. Vanden Berg (1958) found that the relationship for sand was linear on semi-logarithmic coordinates to some degree of confidence. Myklestad (1968) found that the relationship was exponential for peanuts kernal. However, all these results were limited to only a certain range of density.

"The packed bulk density of nonfat dry milk has a range of 0.18 to 1.25 gm. per ml., but regular spray dried milk generally is 0.50 to 0.60 gm. per ml. . . ." Also, for air free solids, "The true density of nonfat dry milk is 1.44 to 1.48 gm. per ml. . . ." (Hall and Hedrick, 1966). This gives the minimum and maximum, respectively, for the density range of nonfat milk powder.

Pressure was applied to the powder as follows (Figure 5.3[a]): For low pressure range, the pressure on the powder was created by the addition of weights to the piston. In order to avoid the effect of time, the weights were placed on the powder for a few hours before measuring the density of the powder (Myklested, 1968). For the medium pressure range, the pressure was created by the Instron machine (Figure 5.4). For high pressure range, the pressure was obtained by hydraulic press (Figure 5.1).

Table 1 shows the results for the 1 in. diameter cylinder. These results were plotted both on semi-log paper and on rectangular scale (Figure 6.2). The least square lines and coefficients of correlation were calculated by Mathatron 4280 Computer.[1] The results in Table 1 were correlated as follows: For the density

$\rho \geq 1$ g/ml:

$$\log p = 2.579875 + 1.400878\rho \qquad (6.1)$$

with coefficient of correlation = 0.994.
For density

$\rho < 1$ g/ml:

$$\log p = -24.801766 + 36.704806\,\rho \qquad (6.2)$$

---

[1]Manufactured by Barry Wright Corp., Waltham, Mass. Model No. 4280-T, Ser. No. 11-033.

TABLE 6.1.--Pressure Effect on Bulk Density.

Pressure, P (psi)	log P	Density (g/ml)	Source of Pressure	Date
1.52	0.18184	0.680	weight	1/7/69
4.09	0.61172	0.697	weight	1/7/69
52.8	1.72263	0.719	Instron	1/7/69
153.8	2.18696	0.731	Instron	1/7/69
309.0	2.48996	0.744	Instron	1/7/69
469.0	2.67117	0.752	Instron	1/7/69
10,470	4.01995	1.043	Hydraulic	2/2/69
16,710	4.22272	1.195	Hydraulic	2/2/69
20,900	4.32015	1.255	Hydraulic	2/2/69
31,380	4.49665	1.400	Hydraulic	2/2/69
37,620	4.57519	1.446	Hydraulic	2/2/69
41,800	4.62118	1.482	Hydraulic	2/2/69
48,100	4.68215	1.510	Hydraulic	2/2/69
52,250	4.71809	1.528	Hydraulic	2/2/69
62,700	4.79727	1.557	Hydraulic	2/2/69
73,200	4.86451	1.585	Hydraulic	2/2/69
10,470	4.01995	1.010	Hydraulic	2/3/69
16,710	4.22272	1.166	Hydraulic	2/3/69
20,900	4.32015	1.243	Hydraulic	2/3/69
25,100	4.39967	1.295	Hydraulic	2/3/69
31,380	4.49665	1.376	Hydraulic	2/3/69
41,800	4.62118	1.482	Hydraulic	2/3/69
52,250	4.71809	1.540	Hydraulic	2/3/69
62,700	4.79727	1.573	Hydraulic	2/3/69
73,200	4.86451	1.586	Hydraulic	2/3/69

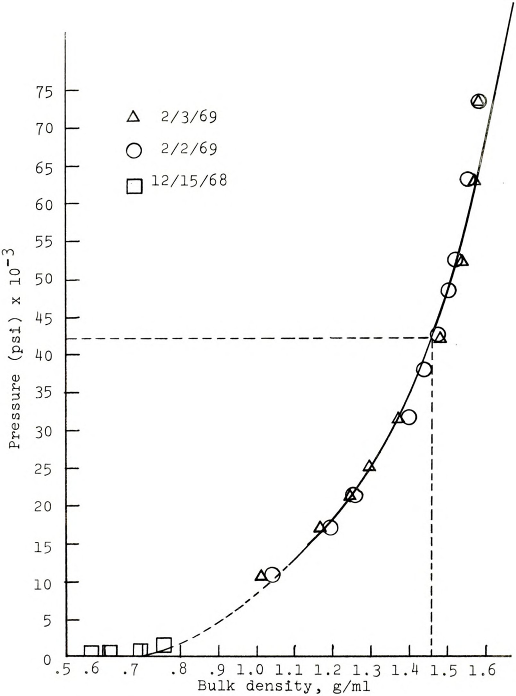


Figure 6.1.--Effect of Pressure on Bulk Density.

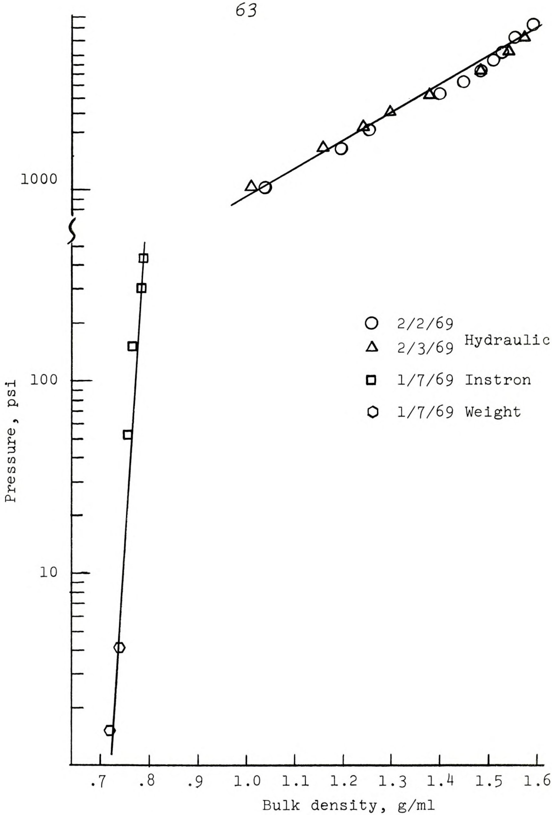


Figure 6.2.--Pressure vs. Bulk Density.

with coefficient of correlation = 0.992, where p is in psi. The formula used for calculating coefficient of correlation is shown in Appendix VII.

Figure 6.1 shows that for a specimen having the same density as the density of dry milk solids (1.46 g/ml), the pressure required is 44,000 psi. According to Lampi et al. (1965) this pressure level should not produce any detectable chemical changes (protein, liquid and etc.) of the food products under compression.

The relation between pressure and bulk density has considerable practical application. It can be used to predict the densities of milk powder at various vertical locations in large storage containers (silos, etc.). The powder at the bottom of a storage container will have higher densities than that at the top due to the weight of the powder above it. The effects of pressure are two-fold: to increase bulk density and also to increase the contact area between particles, Equation (3.1).

2. Bulk Density, Void and Porosity

The application of pressure could reduce the void between particles and also, at higher pressure, reduce the porosity inside particles. The relation between bulk density, ρ, and apparent particle density, D, could be expressed as following:

$$\rho = D\,(1 - \varepsilon) \tag{6.3}$$

$$= 1.46\,(1 - e)\,(1 - \varepsilon) \tag{6.4}$$

where

ε = void

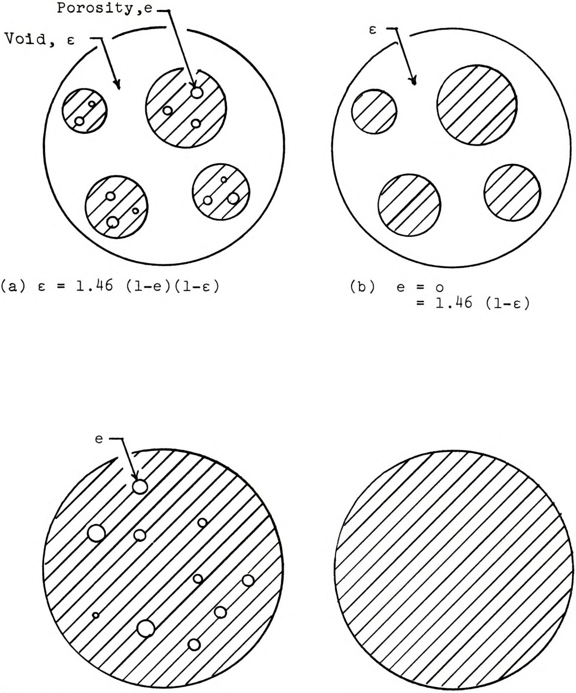
e = porosity

The apparent particle density, D, is defined as the density of milk powder particles which usually include some air space.

Figure 6.3 shows the values of void and porosity under various conditions. Figure 6.3(a) is the case of powdered milk where air is trapped inside the particles during the drying operation. Figure 6.3(b) is the case of a regular packed bed where there is no air trapped inside the particles. Figure 6.3(c) is the case of porous material and (d) is the case of solid bed. True density is defined as the density of solid free of air as the case of Figure 6.3(d).

Mechanical Properties of Powder Particles

In order to calculate the contact area between two powder particles, the modulus of elasticity and Poisson's ratio in Equation (3.1) were measured by compressing a solid specimen of powder in the Instron machine (Figure 5.4).



(a) $\varepsilon = 1.46\ (1-e)(1-\varepsilon)$

(b) $e = o$
$= 1.46\ (1-\varepsilon)$

(c) $\varepsilon = 0$
$\rho = 1.46\ (1-e)$

(d) $e = 0,\ \varepsilon = 0$
$\rho = 1.46$

Figure 6.3.--Void and Porosity on the Cross-section of Packed Bed.

Figure 6.4 shows one of the specimens tested and the force-deformation curve from the chart of the Instron machine. The elasticity of the specimens were well illustrated by the small amount of hysteresis observed in Figure 6.4(b).

The modulus of elasticity, E, was calculated from the definition:

$$E = \frac{\text{stress}}{\text{strain}} = \frac{P/A}{\Delta Y/Y_o} \tag{6.5}$$

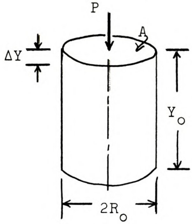
where P, A, ΔY and Y_o are defined in Figure 6.4(a). ΔY was calculated by the combination of chart speed (5 in./min) and cross-head speed (0.02 in./min).

Table 6.2 and Figure 6.5 present the effect of bulk density on modulus of elasticity. The least square line was found as following:

$$\log E = 2.59873 + 1.96356\ \rho \tag{6.6}$$

with coefficient of correlation = 0.94.

Equation (6.6) has considerable application. Since the apparent particle density is usually less than the true density of particle solids (1.46 g/ml for nonfat dry milk), the modulus of elasticity of a powder particle could be estimated from its apparent particle density by Equation (6.6). The numerical value obtained by this approach is the value of E for determining contact area between two particles in a packed bed, Equation (3.1).



(a) Specimen Tested

$A = 0.831$ sq. in.

$Y_o = 1.265$ in.

$2R_o = 1.03$ in.

(b) Force-deformation curve (11/30/68)

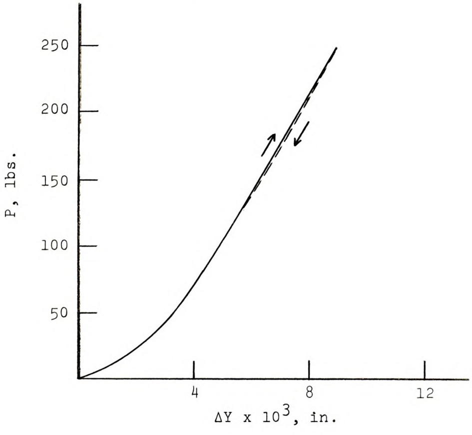


Figure 6.4.--Measurement of Mechanical Properties.

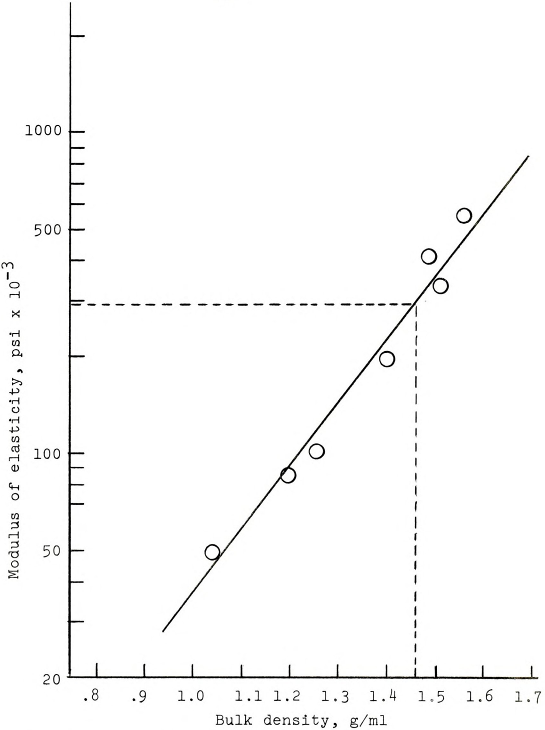


Figure 6.5.--Effect of Bulk Density on Modulus of Elasticity.

TABLE 6.2.--Modulus of Elasticity of Regular Nonfat Dry Milk (II 36).*

Bulk Density, g/ml	Modulus of Elasticity, psi
1.043	48,800
1.195	86,000
1.255	102,000
1.40	198,700
1.482	416,000
1.557	551,000

*Roman numbers refer to research notebook numbers; Arabic numbers refer to research notebook page number.

Figure 6.5 indicates that the modulus of elasticity could be 290,000 psi for the true density of dry milk solids. Table 6.3 shows the comparison of E values for dry milk solids with other common materials and agricultural products. Dry milk solids has a much higher modulus of elasticity than potato or apple but is lower than regular wood.

Poisson's ratio was the next mechanical property to be investigated. Poisson's ratio is defined as "the ratio of the relative change of diameter of a bar to a unit change in length under an axial load which does not stress it beyond the elastic limit. It varies from different material but is usually about 1/4 in. (Hudson,

TABLE 6.3.--Modulus of Elasticity of Dry Milk Solid and Some Common Materials.

Material	Density (g/ml)	Modulus of Elasticity (psi)	Poisson's Ratio	Source
Structural steel	7.85	30,000,000	0.303	Hudson (1963)
Wrought iron	7.69	28,000,000	0.278	Hudson (1963)
Cast iron	7.21	15,000,000	0.270	Hudson (1963)
Concrete	2.41	2,000,000	0.20	Hudson (1963)
Yellow pine	0.64	1,500,000	--	Hudson (1963)
White oak	0.77	1,500,000	--	Hudson (1963)
Dry milk solids	1.46	290,000	--	Figure 6.5
Apple	0.86	2,085	0.32	Mohsenin (1968)
Potato	1.12	543	0.492	Finney and Hall (1967)

1963). In addition, the value of Poisson's ratio is expected to range from 0.25 to 0.50 for elastic materials (Sokolnikoff, 1956). The tests to measure the Poisson's ratio of dry milk solid were conducted by the Instron Machine shown in Figure 5.5 . However, the results were beyond 0.5. New methods or approaches are needed to measure Poisson's ratio of milk solids.

One of the possible approaches for measurement or Poisson's ratio was suggested by Jastrzebski (1964) as follows:

$$H = \frac{E}{3\ (1 - 2\nu)} \tag{6.7}$$

where

E = modulus of elasticity, psi

$H = \frac{\text{stress}}{\Delta V/V}$ = bulk modulus, psi

ν = Poisson's ratio

In other words, Poisson's ratio can be calculated from modulus of elasticity and bulk modulus. However, the equipment to measure bulk modulus was unavailable at the moment of writing this thesis.

For the purpose of this investigation, the value for ν in Equation (3.1) is assumed to be between 0.25 and 0.50.

Thermal Properties of Particle Solid

Figure 6.6 shows the temperature profiles in measuring K_s of nonfat dry milk by Transient Thermal Properties Measurement Facility (Figure 5.6). The time of measuring in each test was 600 sec. which is much shorter than the time needed for measuring by steady-state methods. It usually took 6 to 10 hours to run a test by steady-state methods (Farrall et al., 1968). The advantage of transient method is that the migration of moisture in the specimen during the test is not as serious as the steady-state method due to short time intervale for measurement.

After the thermocouples were installed in the specimen as shown in Figure 5.8(c) and Figure 5.2, the specimen was held at room temperature overnight for uniform initial temperature. Figure 6.6 is an example of temperature profiles of the experiment. Y is the location of thermal-couples measured from the interface of the milk solids specimen and the hot copper block which acted as a heat source during the experiment (U in Figure 5.7). The temperature profiles were analyzed by the CDC 3600 computer on the Michigan State University campus.

The results were tabulated in Table 6.4. The rms (root mean square) indicated the difference between the theoretical temperature profile and the experimental one (See Figure 6.6).

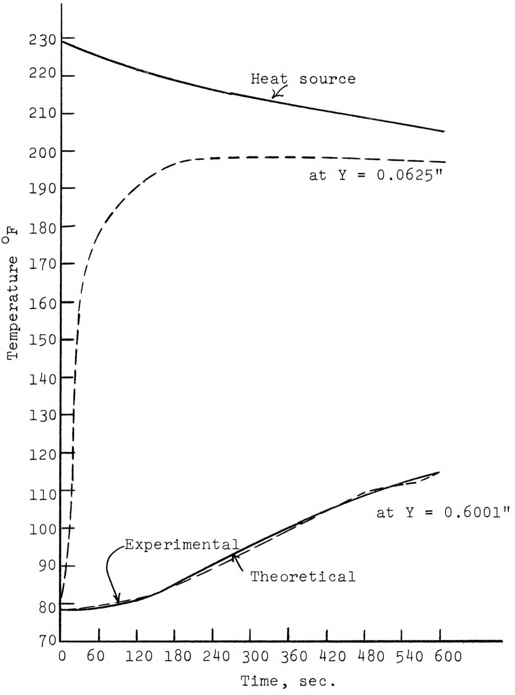


Figure 6.6.--Temperature Profiles in Measuring Thermal Properties of Dry Milk Solids.

TABLE 6.4.--Results for Solid Thermal Conductivity, K_s, of Regular Nonfat Dry Milk.

Specimen No. (Date)	ρ (g/ml)	α (sq.ft/hr.)	C_p* (Btu/lb.°F)	K_s (Btu/°F hr ft)	rms °F	error (%)
1/21/69	1.38	0.00657	0.45	--	0.66	0.56
	1.46	0.00695	0.45	0.285	--	
1/23/69	1.38	0.00674	0.45	--	1.04	1.06
	1.46	0.00713	0.45	0.292	--	

*Farrall, 1968

In addition, the computer predicted the effect of temperature on K_s at 1.38 g/ml as shown in Table 6.5. Assuming a linear relation between temperature and K_s, the results could be described by the following equation:

$$K_s = 0.322 - 0.000376\ T \tag{6.8}$$

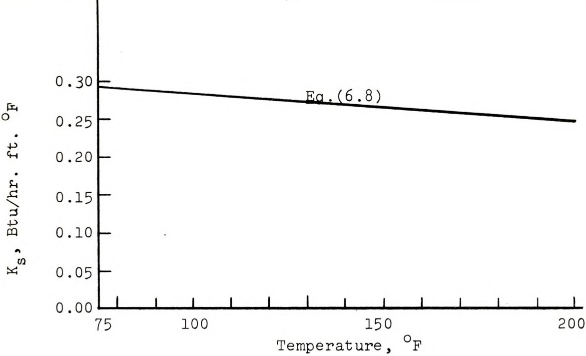
The moisture content of the dry milk used in the test was 3.5% as determined by toluene distillation method.

TABLE 6.5.--Effect of Temperature on K_s.

Temperature (°F)	ρ (g/ml)	K_s (Btu/(°F)(hr)(ft))
75	1.38	0.27993
	1.46	0.29400
200	1.38	0.23360
	1.46	0.24700

Figure 6.7 shows the effect of temperature on K_s and K_g. K_s decreases slightly as temperature increases.

Figure 6.8 shows the comparison of thermal conductivity of dry milk solids to other materials and agricultural products. Notice that thermal conductivities



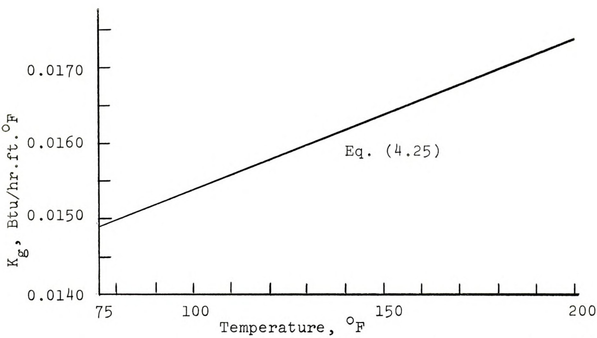


Figure 6.7.--Effect of Temperature on Thermal Conductivity of Milk Solids and Air.

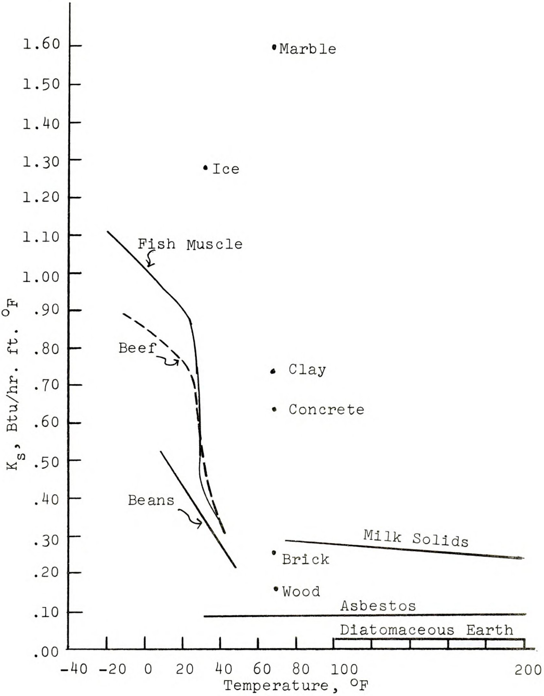


Figure 6.8.--Temperature Effect on Thermal Conductivities of Food Products and other Material.

of all food products decrease as temperature increases (Woodams and Nowrey, 1968).

Effective Thermal Conductivities of Dry Milk

Effective thermal conductivities of several types of powdered milk at various levels of bulk densities and moisture content were investigated by Farrall et al. (1968). The values of K_e were calculated by the equation as follows (Deissler and Boegli, 1958):

$$K_e = \frac{Q \log_e \left(\frac{R_2}{R_1}\right)}{2 \pi l (T_2 - T_1)}$$

where T_1, T_2 are the temperatures at radii R_1 and R_2, respectively, ℓ is the length of cylinder and Q is the heat flux transferred from the heating coil to the water flow at the center of the cylinder (Figure 5.9). Table 6.6 shows the results for K_e of nonfat dry milk.

Figure 6.9 shows the effect of moisture content on the K_e of nonfat dry milk. It was correlated as follows:

$$K_e = 0.114 + 0.0107 \text{ MC} \qquad (6.9)$$

by Mathatron 4280 computer with coefficient of correlation 0.95, where MC is the moisture content in percentage.

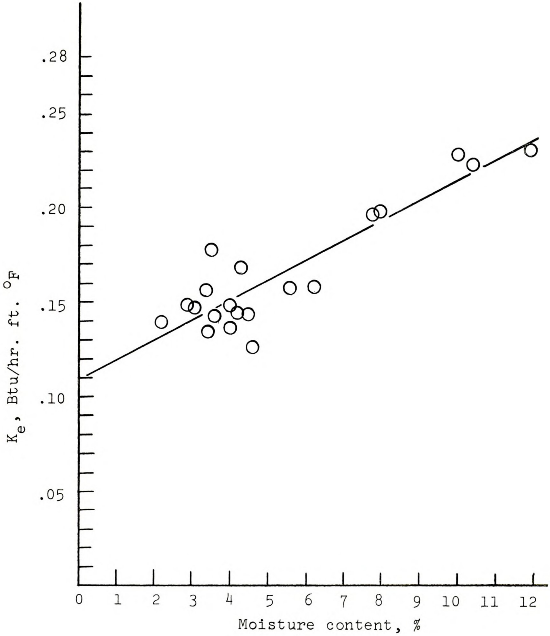


Figure 6.9.--Effect of Moisture Content on K_e of Nonfat Dry Milk.

The average temperature of the bed for experiments to measure effective thermal conductivities was shown in Table 6.6.

TABLE 6.6.--Effective Thermal Conductivity of Regular Non-fat Dry Milk.*

Moisture, %	K_e	Mean Temperature, °F	Bulk Density, (g/ml)
2.2	0.1391	144	0.594
3.4	0.1331	137	0.564
4.5	0.1430	140.4	0.605
4.6	0.1263	153	0.575
3.4	0.1331	137	0.564
4.0	0.1364	138	0.563
7.8	0.1966	151	0.533
7.9	0.1976	116	0.547
10.0	0.2311	125.7	0.588
10.4	0.2242	129.5	0.422
11.9	0.2327	118.6	0.544

*From Farrall _et al_. (1968).

Number of Contact Points of Spheres in a Random Bed

In Equation (A31) the parameter n is half of the average number of contact points of uniform spheres in a packed bed. Kunii and Smith (1960) obtained the following equation to estimate the parameter n:

$$n = 6.93 - 5.51 \frac{\varepsilon - 0.260}{0.476 - 0.260} \qquad (6.10)$$

which gives n = 1.42 for the most open packing (ε = 0.4760) of spheres and n = 6.93 for the most dense packing (ε = 0.260).

The agreement between experimental data and predicted values for 2 n in Table 6.7 suggests that Equation (6.10) might be valid for prediction of contact number of uniform spheres in a random bed from the void of the bed.

TABLE 6.7.--Number of Contact Points of Spheres in a Random Bed.

Size of Spheres	2n, Experimental	Porosity	2n, from Equation (6.10)	Number of Spheres Counted	Date
2"	6.73	.4256	6.42	22	2/ 4/69
1.5"	7.04	.4157	5.92	24	2/12/69
2"	7.16	.4255	6.42	18	2/18/69

CHAPTER VII

ANALYSIS

Combined Effects of Temperature and Moisture Content on K_s

Since the effect of temperature on the thermal conductivity of dry milk solids was shown as:

$$K_s = 0.3222 - 0.000376\ T \Bigg|_{\text{at MC}=3.5\%} \qquad (6.8)$$

from the results of transient state thermal properties measurement and the effect of moisture content on K_e was shown as:

$$K_e = 0.114 + 0.0107\ \text{MC} \Bigg|_{\text{at } T=140°\text{F}} \qquad (6.9)$$

from the results of steady state thermal conductivity measurement, the combined effect of temperature and moisture content on K_s can be analyzed as follows.

If the moisture content is assumed to have the same effect on K_s as on K_e, the slope in the relationship between K_s vs. MC would be the same as that of K_e vs. MC, Equation (6.9). "Water exists in the dry milk product in several forms. Water may be absorbed on the surface of

the particles of powder; water may be bound in the crystals of lactose; and water may be imbibed into the colloidal milk protein" (Hall and Hedrick, 1966). In other words, moisture in dry milk exists primarily in the particle solids. In addition, if it is assumed that y-intercept in Figure 6.9 could be adjusted to account for the void and porosity of the powder bed, then the effect of moisture content on K_s could be predicted as follows:

$$K_s = 0.114 \left(\frac{1}{1 - \varepsilon_F}\right) + 0.0107 \text{ MC} \Big|_{\text{at } 140°\text{F}} \qquad (7.1)$$

$$= 0.2328 + 0.0107 \text{ MC} \Big|_{\text{at } 140°\text{F}} \qquad (7.2)$$

for

$$\varepsilon_F = 0.51.$$

In order to verify the above adjustment, T = 140°F can be substituted into Equation (6.8) to obtain:

$$K_s = 0.3222 - 0.000376\ (140) = 0.2696 \Big|_{\substack{T=140°\text{F} \\ \text{MC}=3.5\%}} \qquad (7.3)$$

In addition, MC = 3.5% is substituted into Equation (7.2) for K_s value as follows:

$$K_s = 0.2328 + 0.0107\ (3.5) = 0.2702 \Bigg|_{\substack{T=140°F \\ MC=3.5\%}} \tag{7.4}$$

Since Equation (6.8) and Equation (6.9) were obtained from two unrelated experiments, the agreement between Equation (7.3) and Equation (7.4) gives some confidence for these two equations, (6.8) and (6.9).

The combined effect of T and MC on K_s could be derived as follows. Assume

$$K_s = X + Y \cdot T + Z \cdot MC \tag{7.5}$$

where X, Y and Z are the coefficients to be determined by Equation (6.8) and Equation (7.2). Since MC = 3.5%, coefficients of T and MC from Equation (6.8) and Equation (7.5) are compared. The result is

$$\begin{aligned} 0.3222 &= X + 3.5\ Z \\ Y &= -0.000376 \end{aligned} \tag{7.6}$$

Similarly, the coefficients of T and MC as T = 140°F are compared as follows:

$$\begin{aligned} 0.2328 &= X + 140\ Y \\ Z &= 0.0107 \end{aligned} \tag{7.7}$$

By Equation (7.6) and Equation (7.7) the correlation coefficients are found as:

$$X = 0.2851$$

$$Y = -0.000376$$

$$Z = 0.0107$$

Therefore, the equation for the combined effect of T and MC on K_s is as follows:

$$K_s = 0.2851 - 0.000376\ T + 0.0107\ MC \tag{7.8}$$

Since K_s is a function of T and MC, the solid thermal conductivity of regular nonfat dry milk can be predicted by knowing its temperature and moisture content.

Contact Conductance and Number of Contact Points Predicted on the Cross-Section of a Random Bed

In order to evaluate the influence of contact conductance, the contact conductance and number of contact points must be determined.

1. Contact Conductance on the Cross-Section

The contact conductance can be determined from:

$$K_c = \varepsilon(1 - \gamma)\ K_s + \varepsilon(1 - \gamma)\ (1 - \delta)\ \frac{1}{\frac{\phi}{K_g} + \frac{1 - \phi}{K_s}} \tag{4.23}$$

For regular nonfat dry milk, the values of each parameter in the above equation are:

$$\theta_o = \cos^{-1}\left(1 - \frac{1}{n}\right) = 0.714 \text{ radians for } n = 3.5 \qquad \text{(A31)}$$

$$\phi = 0.0193 \text{ for } \varepsilon_F = 0.51 \text{ and } n = 3.5 \qquad \text{(A18)}$$

$$\gamma = 0.9722 \qquad \text{(A21)}$$

$$\beta = 0.9825 \text{ for } n = 3.5 \qquad \text{(A31)}$$

$$x_o = 45\mu = 0.1493 \times 10^{-3} \text{ ft} = 0.179 \times 10^{-2} \text{ in.}$$

$$a = 0.25\mu \text{ for } w = 0.444 \times 10^{-6} \text{ lbs/particle}$$

$$E = 290{,}000 \text{ psi}, \ \nu = 0.3 \qquad \text{(7.9)}$$

$$\delta = 0.419 \times 10^{-3} \qquad \text{(A22)}$$

$$K_s = 0.2699 \text{ Btu/(hr)(°F)(ft) for } T = 140\text{"F and } MC = 3.5\% \qquad \text{(7.8)}$$

$$K_g = 0.01624 \text{ Btu/(hr)(°F)(ft) for } T = 140°\text{F} \qquad \text{(4.25)}$$

Substituting all of the parameters above into Equation (4.23) the result was found as:

$$K_c = 0.00292 \text{ Btu/(hr)(°F)(ft)} \qquad \text{(4.23)}$$

This represents the contact conductance for a single contact point on the cross-section of a random bed.

2. Frequency Distribution Function G(s) and the Predicted Number of Contact Points

Since G(s) is defined as the distribution of particle sizes as they appear on a cross-section, the value of G(s) should be a good estimate of the number of contact points on the cross-section as $s \to 0^+$.

$$G(s) = \frac{s}{M_{1F}} \int_{x=s}^{x=x_{max}} \frac{F(x)}{(x^2 - s^2)^{\frac{1}{2}}} dx \qquad (4.12)$$

$$= \frac{C_1 S}{M_{1F}} \left[\frac{1}{x} (x^2 - s^2)^{\frac{1}{2}} e^{C_2 \ln^2 \frac{x}{x_o}} \Big|_s^{x_{max}} - \int_{x=s}^{x=x_{max}} g(x)\, dx\right] \qquad (A7)$$

where M_{1F} and integration of g(x) were evaluated by Romberg numerical integration methods (Moursund and Duris, 1967). Appendix II and III shows the computer program used to evaluate M_{1F} and integration of g(x) respectively.

For regular nonfat dry milk,

$$x_o = 45.5\mu = 0.1493 \times 10^{-3} \text{ ft},$$

$$x_{min} = 5\mu = 0.164 \times 10^{-4} \text{ ft}, \qquad (7.10)$$

$$x_{max} = 180\mu = 0.59 \times 10^{-3} \text{ ft},$$

$$\sigma = 1.67,$$

the results of integration were:

$$M_{1F} = 35.22 \times 10^{-9}$$

$$\int_{x_{min}}^{x_{max}} g(x)\, dx = -1.255 \qquad (7.11)$$

Substituting Equation (7.10) and Equation (7.11) into Equation (A7), a simplified expression of G(s) for this particular particle size and size distribution of regular nonfat dry milk was obtained as following:

$$G(s) = 1.45\ s\ [0.027\ \frac{(180^2 - s^2)^{\frac{1}{2}}}{180} + 1.255] \qquad (7.12)$$

Equation (7.12) was plotted as shown in Figure 7.1. By varying x_o and σ, the effect of x_o and σ on G(s) is illustrated.

Since the frequency distribution function G(s) should be a good approximation of the number of contact points on the cross-section as $s \to 0^+$, the values of G(s) were calculated as $s = 0.50\mu$ for different combination of x_o and σ. Table 7.1 shows that the smaller particles and lower standard deviation (uniform particles) would have a larger number of contact points expected on the cross-section.

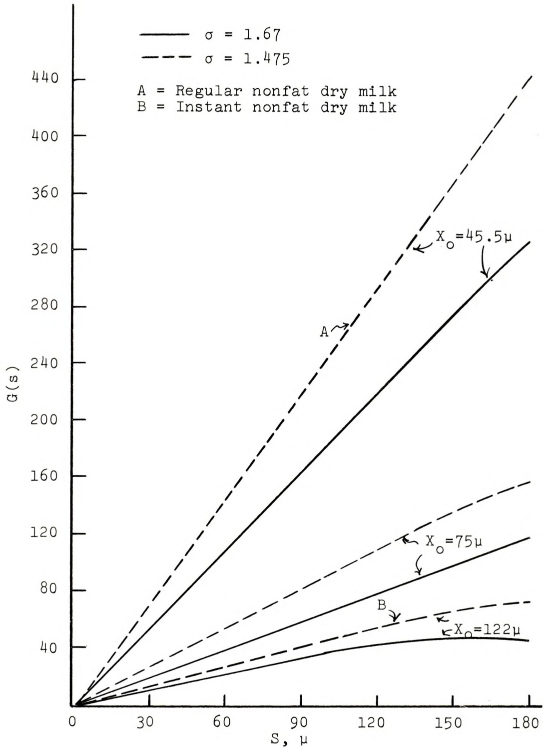


Figure 7.1.--Effects of Mean Particle Size and Standard Deviation of Size Distribution on Function G(s).

TABLE 7.1.--Predicted Number of Contact Points on the Cross-Section of a Random Bed (II 73).*

Standard Deviation, σ	Mean Particle Size, x_o		
	45.5μ	75μ	122μ
1.67	0.930	0.396	0.206
1.475	1.220	0.468	0.252

*Roman numbers refer to research notebook number; Arabic number refers to research notebook page number.

3. Contact Conductance

By knowing the contact conductance of total contact points, K_c, and the number of contact points on the cross-section, the contribution of the thermal conductance through a single contact point to K_e could be determined from the equation:

$$K_{345} = K_c/G(0.50\mu) \qquad (4.26)$$

where

$$K_c = 0.00292 \text{ Btu/(hr)(°F)(ft)} \qquad (7.9)$$

$$G(0.50\mu) = 0.930 \text{ from Table 7.1}$$

for regular nonfat dry milk. Thus

$$K_{345} = 0.00314 \text{ Btu/(hr)(°F)(ft)} \qquad (7.13)$$

which is negligible when compared to the K_e value in Table 6.6.

However, since K_c is a function of n, ε, x_o, h_{rs}, K_s, K_g, T and MC by Equation (4.18) and G(0.1μ) is a function of x_o, x_{min}, x_{max}, σ and s by Equation (4.12), the effect of these parameters on contact conductance, K_{345}, can be investigated independently for academic interest using the methematical model developed.

Effect of Each Parameter on K_e

There are eight basic parameters involved: E, ν, x_o, σ, N, T, MC and ε_F. The solid thermal conductivity, K_s, was expressed in terms of T and MC in Equation (7.8) for regular nonfat dry milk. Gas thermal conductivity, K_g, can be expressed in terms of T as Equation (4.25) for air.

Since the value of K_e is additive as shown in Equation (4.27), the effect of each parameter on K_e can be analyzed by the effect of that parameter on each components: K_1, K_2 or K_{345}.

In the previous section it has been shown that E, ν, x_o, σ, and N influence K_{345} only, which, in turn, has a negligible influence on K_e. Therefore, the effects of E, ν, x_o, σ and N on K_e are negligible.

Equation (4.27) is then reduced as follows:

$$K_e = K_1 + K_2 \tag{7.14}$$

where K_1 and K_2 are defined by Equation (4.21) and Equation (4.23) respectively. By Equation (7.8) and Equation (4.25):

$$K_1 = (0.2851 - 0.000376\ T + 0.0107\ MC)\ (1 - \varepsilon_F) \tag{7.15}$$

$$K_2 = (0.0133 + 0.0000\ 21\ T)\ \varepsilon_F \tag{7.16}$$

Figures 7.2 through 7.4 show the effect of temperature, moisture content and bulk density, respectively, on K_1, K_2 and K_e. Notice that in all cases K_1 is the dominant factor as far as the value of K_e is concerned. Table 7.2 shows the percentage contributions of the values of K_1, K_2 and K_{345} to K_e. By average,

$$\begin{aligned} K_1 &= 0.933\ K_e \\ K_2 &= 0.048\ K_e \qquad (7.17) \\ K_{345} &= 0.019\ K_e \end{aligned}$$

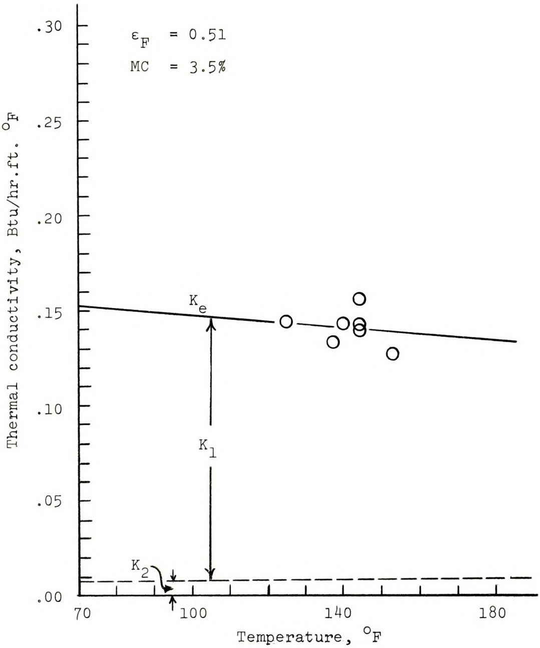


Figure 7.2.--Effect of Temperature on the Component and Effective Thermal Conductivities.

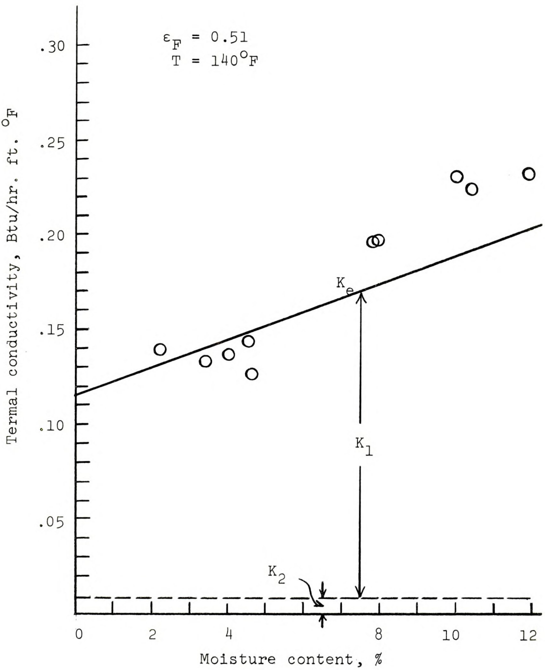


Figure 7.3.--Effect of Moisture Content on the Component and Effective Thermal Conductivities.

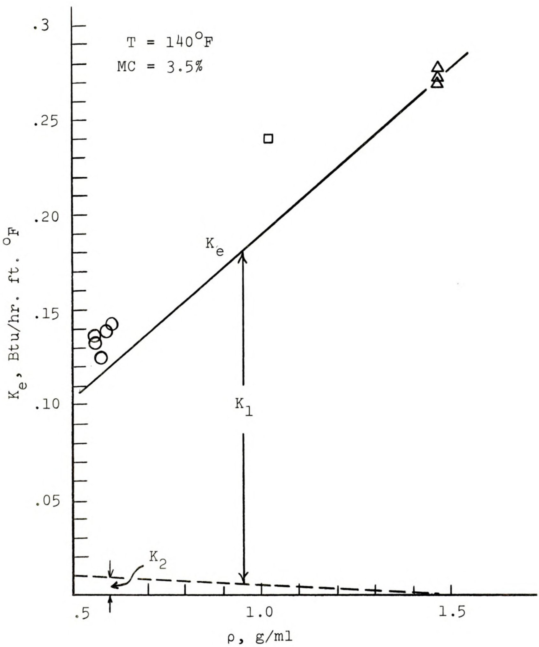


Figure 7.4.--Effect of Bulk Density on the Component and Effective Thermal Conductivities.

The experimental values of K_e are from Table 6.6 and Table 6.4. The agreement between prediction and experimental values is illustrated. Figure 7.5 shows the combined effect of temperature and moisture content on K_e of nonfat dry milk.

Since K_1 is the dominating component of K_e, Equation (4.21) implies that K_s and ε_F are the dominating factors as far as K_e value is concerned. Since K_s could be expressed in terms of T and MC as shown in Equation (7.8), therefore the dominating parameters of K_e values of regular nonfat dry milk are T, MC and ε_F.

Since K_2 is the minor component of K_e, the effect of K_g on K_e is minor as shown in Figure 7.6 Due to this fact, the different kinds of interstitial gasses will have only limited influence on the K_e value of nonfat dry milk.

TABLE 7.2.--Per cent of K_1, K_2 and K_c to K_e (ε_F = 0.51) (II 84).

Temp., °F	MC, %	K_1, %	K_2, %	K_c, %
75	3.5	93.6	4.4	2.0
140	3.5	91.8	6.3	1.9
180	3.5	91.5	6.4	2.1
75	8.0	94.1	4.0	1.9
140	8.0	93.5	6.4	1.9
180	8.0	92.8	5.2	2.0
75	12.0	94.9	3.3	1.8
140	12.0	94.2	3.9	1.9
180	12.0	93.5	4.6	1.9
	AVERAGE:	93.3	4.8	1.9

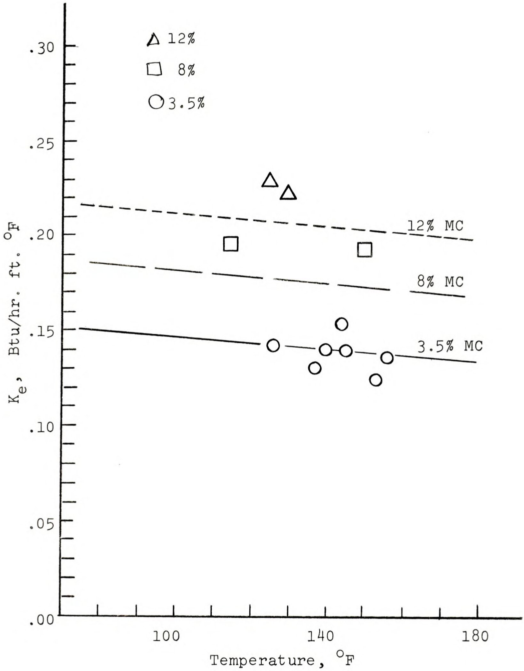


Figure 7.5.--Effect of Temperature and Moisture Content on Effective Thermal Conductivity.

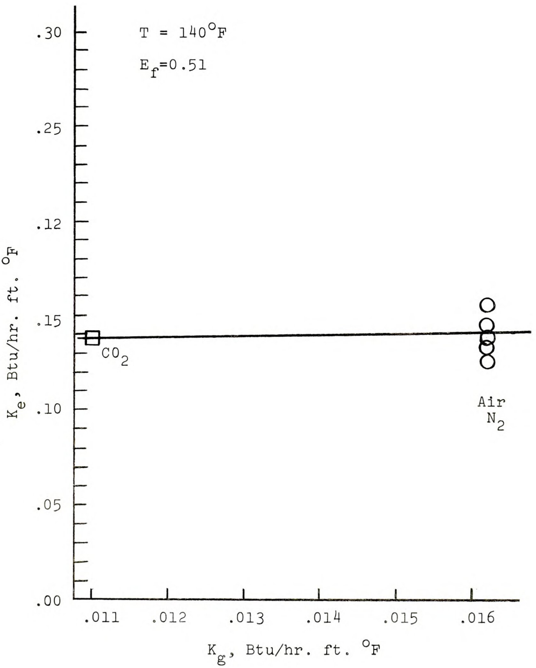


Figure 7.6.--Effect of the Thermal Conductivities of Interstitial Gases on the Effective Thermal Conductivity of Nonfat Dry Milk.

Influence of Component Thermal Conductivities on Effective Thermal Conductivity Using Dimensionless Groups

In order for the mathematical models developed to have broad application on various kinds of solid material and interstitial gases, two dimensionless groups, K_s/K_g and K_e/K_g, were plotted as shown in Figure 7.7 as a function of void fraction, ε. For this particular case (nonfat dry milk), the effect of moisture content at one temperature level, 75°F, is illustrated. For a given combination of K_s, K_g and ε, Figure 7.7 shows that K_e will be higher for higher moisture content as predicted.

Comparison of Predicted and Experimental Thermal Conductivities

Figure 7.8 shows the comparison between the predicted and experimental values of K_e. The mathematical model developed to predict the effective thermal conductivities of organic powder in a packed bed was verified by the agreement in Figure 7.8.

The experimental data were correlated by Mathatron 4280 computer. The result was as follows:

$$\log\,(K_e)_{\text{observed}} = 0.3480 + 1.4051 \log\,(K_e)_{\text{predicted}} \tag{7.18}$$

with coefficient of correlation equaled to 0.96.

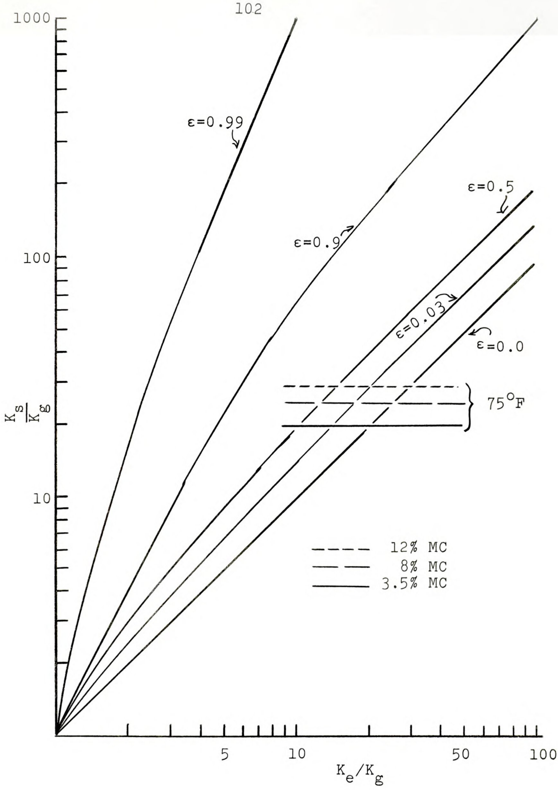


Figure 7.7.--Correlation for Thermal Conductivity of Nonfat Dry Milk in Packed Bed.

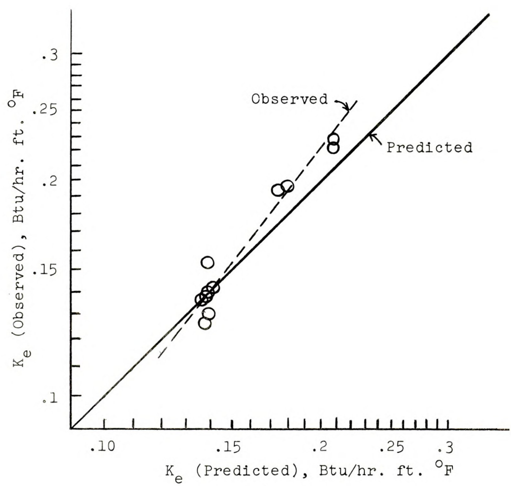


Figure 7.8.--Comparison of Predicted and Experimental K_e for Nonfat Dry Milk with Air as Interstitial Gas.

CHAPTER VIII

CONCLUSIONS

1. The primary mechanism of heat transfer through nonfat dry milk in a packed bed was found to be conduction. Convection and radiation were found to be negligible.

2. Heat conduction through nonfat dry milk in a packed bed was divided into three components in parallel, i.e., through particle solid, through interstitial gas and through contact points between particles. The contribution of the solid phase was found to be the dominating factor. The contribution of the gas phase was found to be a minor factor and that through contact points was found to be negligible.

3. A mathematical model was developed to predict the effective thermal conductivity of nonfat dry milk in a packed bed based on the three mechanisms mentioned above. The model was verified by comparing the predicted and the experimental data.

4. There are eight basic parameters considered in developing the mathematical model: mean particle size, particle size distribution, modulus of elasticity, Poisson's ratio, number of contact points, temperature level, moisture content and void fraction. The effects of mean particle

size, size distribution, modulus of elasticity, Poisson's ratio and number of contact points were found negligible because the contact conductance was negligible.

5. The number of contact points can be predicted from the porosity of the packed bed.

6. The effects of temperature, moisture content and void were significant because the solid thermal conductivity was the dominating factor.

7. The contribution of thermal conductivity of the interstitial gas to the effective thermal conductivity of the powder bed was found to be a minor factor. Different types of interstitial gases should not have a significant influence on the values of effective thermal conductivity of a powder bed.

8. The number of contact points appearing on the cross-section of a packed random bed will increase as particle size and standard deviation of particle size decreases.

9. By the relation established between bulk density, void and pressure, the distribution of the effective thermal conductivities of an organic powder along the vertical axis of a large container (silo, etc.) can be predicted, based on the mathematical model developed.

CHAPTER IX

LIMITATION OF THE MATHEMATICAL MODEL DEVELOPED

The mathematical model presented in Chapter IV was developed to predict the effective thermal conductivity of an organic powder in a packed bed. Since five assumptions were made in the development, the model has the following limitations:

1. The mean particle size must be less than one millimeter and temperature must be lower than 400°C in order to neglect the mechanism of radiation.

2. The mean particle size must be less than $3 \sim 4$ millimeter and the gas pressure must be lower than $8 \sim 10$ atmospheres in order to neglect the mechanism of convection.

3. The solid material of the powder must have a low thermal conductivity so that the assumption of one dimensional heat flow is valid.

CHAPTER X

RECOMMENDATIONS FOR FUTURE WORK

1. Determine the effect of moisture content on the thermal conductivity of dry milk solid experimentally.

2. Measure the Poisson's ratio by bulk modulus as suggested in Chapter VI.

3. Conduct an independent study on the effect of particle size and size distribution, modulus of elasticity, Poisson's ratio and contact number on the contact conductance.

4. Measure contact conductance for a packed bed of spheres with known solid thermal conductivity under vacuum and compare the value with the prediction by the mathematical model developed.

5. Verify the assumption 5 in Chapter IV by measuring the thermal conductivity of talc powder compressed to the same bulk density as talc block, and then compare the results with the thermal conductivity of the solid.

6. Measure the solid thermal conductivity of nonfat dry milk in the low temperature range to obtain the effect of temperature on solids thermal conductivity experimentally. This could be done by cooling the specimen by dry ice instead of heating the specimen by hot copper block as was done in this investigation.

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APPENDICES

APPENDIX I

REDUCTION OF EQUATION (4.12)

$$G(s) = \frac{S}{M_{1F}} \int_{x=s}^{x=x_{max}} \frac{F(x)}{(x^2 - s^2)^{\frac{1}{2}}} \, dx \qquad (4.12)$$

$$F(x) = \frac{W}{\sqrt{2\pi}\ \ln \sigma g} \; e^{-\frac{1}{2}\left(\frac{\ln \frac{x}{x_o}}{\ln \sigma}\right)^2}$$

$$(4.10)$$

$$= C_1 \left[e^{C_2 \left(\ln \frac{x}{x_o}\right)^2} \right]$$

where

$$C_1 = \frac{W}{\sqrt{2\pi}\ \ln \sigma}, \qquad C_2 = -\frac{1}{2(\ln \sigma)^2}, \qquad (A1)$$

$$G(s) = \frac{C_1 S}{M_{1F}} \int_{x=s}^{x=x_{max}} \frac{\left[e^{C_2 \left(\ln \frac{x}{x_o}\right)^2}\right]}{(x^2 - s^2)^{\frac{1}{2}}} \, dx \qquad (A2)$$

Notice that at the low limit of Equation (A2) function is not defined. The technique of integration by parts could be applied to Equation (A2) as follows:

Let $$u = \frac{1}{x} e^{C_2 (\ln \frac{x}{x_o})^2} \quad \text{and} \quad V = (x^2 - s^2)^{+\frac{1}{2}} \tag{A3}$$

then

$$dV = x(x^2 - s^2)^{-\frac{1}{2}} \tag{A4}$$

and

$$du = \frac{1}{x} \frac{d}{dx} \left(e^{C_2 (\ln \frac{x}{x_o})^2}\right) + e^{C_2 (\ln \frac{x}{x_o})^2} \frac{(-1)}{x^2}$$

$$= \frac{1}{x} e^{C_2 \ln^2 \frac{x}{x_o}} \cdot 2C_2 (\ln \frac{x}{x_o})(\frac{1}{x}) - \frac{1}{x^2} e^{C_2 (\ln \frac{x}{x_o})^2}$$

$$= \frac{2C_2}{x^2} (\ln \frac{x}{x_o}) \cdot e^{C_2 \ln^2 \frac{x}{x_o}} - \frac{1}{x^2} e^{C_2 \ln^2 \frac{x}{x_o}}$$

$$= (2C_2 \ln \frac{x}{x_o} - 1) \frac{e^{C_2 \ln^2 \frac{x}{x_o}}}{x^2} \tag{A5}$$

Apply Equation (A3) and (A5) to Equation (A2) as follows:

$$G(s) = \frac{C_1 S}{M_{1F}} \int_s^{x_{max}} u \, dv$$

$$= \frac{C_1 S}{M_{1F}} \left[uv \Big|_{s}^{x_{max}} - \int_{s}^{x_{max}} v \, du \right]$$

$$= \frac{C_1 S}{M_{1F}} \left[\frac{1}{x} (x^2 - s^2)^{\frac{1}{2}} \cdot e^{C_2 \ln^2 \frac{x}{x_o}} \Big|_{s}^{x_{max}} - \int_{x=s}^{x=x_{max}} \right.$$

$$\left. (x^2 - s^2)^{\frac{1}{2}} \left(2 C_2 \ln \frac{x}{x_o} - 1\right) \frac{e^{C_2 \ln^2 \frac{x}{x_o}}}{x^2} dx \right] \qquad \text{(A6)}$$

$$= \frac{C_1 S}{M_{1F}} \left[\frac{1}{x} (x^2 - s^2)^{\frac{1}{2}} \cdot e^{C_2 \ln^2 \frac{x}{x_o}} \Big|_{s}^{x_{max}} - \int_{x=s}^{x=x_{max}} \right.$$

$$\left. g(x) \, dx \right] \qquad \text{(A7)}$$

where

$$g(x) = (x^2 - s^2)^{\frac{1}{2}} \left(2 C_2 \ln \frac{x}{x_o} - 1\right) \frac{e^{C_2 \ln^2 \frac{x}{x_o}}}{x^2} \qquad \text{(A8)}$$

$$M_{1F} = \int_{x_{min}}^{x_{max}} x \cdot F(x) \, dx \qquad \text{(4.13)}$$

$$= \int_{x_{min}}^{x_{max}} x \cdot C_1 \, e^{C_2 \left(\ln \frac{x}{x_o}\right)^2} dx \qquad \text{(A9)}$$

Equation (A8) could be integrated by numerical integration as Appendix II. Also Appendix III shows the computer program for evaluating Equation (A9).

APPENDIX II

COMPUTER PROGRAM FOR INTEGRATION OF EQ. (A8)

```
'FOR,L,X
      PROGRAM ROMBERG
      DIMENSION P(11,11)
      READ 1,S,XMAX,K
    1 FORMAT (2F10.0,I2)
      PRINT 2,S,XMAX,K
    2 FORMAT (2E20.10,I3)
      H=XMAX-S
      P(1,1)=.5*H*(F(S)+F(XMAX))
      KP=K+1
      KC=1
      DO4 I=1,K
      V=0.
      DO 3 J=1,KC
      X=J
    3 V=V+F(S+(X-.5)*H)
      V=V*H
      P(I+1,1)=.5*(P(I,1)+V)
      KC=2*KC
    4 H=.5*H
      W=4.
      DO 8 I=2,KP
      WM=W-1.
      DO 6 J=I,KP
    6 P(J,I)=(W*P(J,I-1)-P(J-1,I-1))/WM
    8 W=4.*W
      DO 10 I=1,KP
   10 PRINT 11,(P(I,J),J=1,I)
   11 FORMAT (11E12.3)
      END
      FUNCTION F(X)
      S=5.0
      C2=-1.9
      XMEAN=45.5
      XCENT=X/XMEAN
      T=LOGF(XCENT)
      U=T**2
      F=SQRT(X**2-S**2)*(2*C2*T-1)*EXP(C2*U)/X**2
      RETURN
      END
'RUN,0,45,2100
        5.0        180.010
```

APPENDIX III

COMPUTER PROGRAM FOR EQ. (A9)

```
•FOR,X,L
      PROGRAM GAU M1F
      CALL GAUSSN(-1,5.0,180.0,Y,1.0E-6,10)
      PRINT 1,Y
    1 FORMAT(1H1,E14.7)
      END
      SUBROUTINE GAUSSN(INIT,XO,XL,Y,REL,NP)
C     TO CONVERT FROM GAUSS16 TO GAUSSN, CHANGE THE CARDS WITH
C     COMMENTS, WHERE N = ORDER OF FORMULA.
      DIMENSION AA(16),HH(16),YBAR(10),BYB(10)
      IF(INIT)1,1,2
1     INIT = -INIT
      AA(1) = -.98940093499
      AA(2) = -.94457502307
      AA(3) = -.86563120239
      AA(4) = -.75540440836
      AA(5) = -.61787624440
      AA(6) = -.45801677766
      AA(7) = -.28160355078
      AA(8) = -.950125098 38E-01
      AA(9) = -AA(8)
      AA(10) = -AA(7)
      AA(11) = -AA(6)
      AA(12) = -AA(5)
      AA(13) = -AA(4)
      AA(14) = -AA(3)
      AA(15) = -AA(2)
      AA(16) = -AA(1)
      HH(1) = .27152459412E-01
      HH(2) = .62253523939E-01
      HH(3) = .95158511682E-01
      HH(4) = .12462897126
      HH(5) = .14959598882
      HH(6) = .16915651940
      HH(7) = .18260341504
      HH(8) = .18945061046
      HH(9) = HH(8)
      HH(10) = HH(7)
      HH(11) = HH(6)
      HH(12) = HH(5)
      HH(13) = HH(4)
      HH(14) = HH(3)
      HH(15) = HH(2)
      HH(16) = HH(1)
      NG = 16
```

```
2      Y = 0.
       XLGTH = XL-XO
       IF(XLGTH)201,105,201
201    NPP = NP
       DO 103 K = 1,10
       Y = 0.
       ENP = NP
       DO 200 L = 1,NP
       AREA = 0.
       AL = L
       X1PX2 = (2.*AL-1.)*XLGTH/ENP + 2.*XO
       X2MX1 = XLGTH/ENP
       DO 100 J = 1,NG
       X = (X1PX2 + AA(J) * X2MX1)/2.
       CALL FOFX(X,FMFX)
 100   AREA = AREA + HH(J)*FMFX
       Y = Y + AREA
200    CONTINUE
       Y = XLGTH/(2.*ENP) * Y
       YBAR(K) = Y
       IF(K-1)104,104,144
144    BYB(K-1) = ABSF(YBAR(K-1) - Y)
       IF(BYB(K-1) - REL*ABSF(Y))105,105,104
104    NP = 2*NP
103    CONTINUE
       DO 108 L = 1,10
       REL = 2.*REL
       DO 107 K = 2,10
       IF(BYB(K-1) - REL*ABSF(YBAR(K)))106,106,107
107    CONTINUE

108    CONTINUE
       K = 10
106    NP = (2**(K-1)) * NPP
       Y = YBAR(K)
105    RETURN
       END
       SUBROUTINE FOFX(X,FMFX)
       XMIN=5.0
       C2=-1.9
       XMEAN=45.5
       XCENT=X/XMEAN
       T=LOGF(XCENT)
       U=T**2
       C1=0.78*W
       W=1.0
       FMFX=X*C1*EXP(C2*U)
       RETURN
       END
'RUN,0.45,2100
```

APPENDIX IV

EVALUATION OF β, γ, ϕ AND δ

EQUATION (4.18)

A. For β,

$$\beta = \frac{\Delta l}{x_o} \tag{A10}$$

1. "Most Open" Packing, $\beta = 1$

2. "Most Close" Packing, β was evaluated by either

   a. Volume of Unit-Cell = $0.71\ x_o^{\ 3}$ (Dallavalle, 1948) (A11)

$$\Delta l = (0.71\ x_o)^{1/3} \tag{A12}$$

$$= 0.892\ x_o$$

$$\beta = \frac{\Delta l}{x_o} = 0.892 \tag{A13}$$

or

   b. The average value of β was calculated by (Kunii and Smith, 1960)

$$\beta = \frac{\Delta l}{x_o} = \frac{1}{x_o}\,\frac{1}{3}\left[\left(\frac{2}{3}\right)^{\frac{1}{2}} + 1 + \frac{(3)^{\frac{1}{2}}}{2}\right] x_o = 0.895 \qquad \text{(A14)}$$

Both methods agree well

B. For ϕ:

$$\phi = \frac{l_g}{x_o} \qquad \text{(A15)}$$

Masamamune and Smith (1963) determined the volume of pendular ring associated with each contact point, in terms of the particle radius, r_o, to be

$$\Delta V_g = 2\,\pi\, r_o^{\,3}\,(\sec\,\theta_o - 1)^2\left[1 - \left(\frac{\pi}{2} - \theta_o\right)\tan\,\theta_o\right] \qquad \text{(A16)}$$

The total heat flux per particle in series mechanism will pass through the number of rigns associated with a hemispherical surface of a particle, and hence, a volume $n(\Delta V_g)$. For one-dimensional heat flux, this volume will have a cross-sectional area equal to the projected area of the particle, $\pi\, r_o^{\,2}$, and a thickness l_g. Hence,

$$l_g = \frac{n(\Delta V_g)}{\pi\, r_o^{\,2}} \qquad \text{(A17)}$$

Applying Equation (A16), Equation (A17) and Equation (6.10), Masamune and Smith (1960) obtained the following result:

$$\phi = \frac{l_g}{x_o} = [6.93 - 5.51\ (\frac{\varepsilon - 0.260}{0.216})]\ x$$

$$\{(\sec\ \theta_o - 1)^2[1 - (\frac{\pi}{} - \theta_o)\ \tan\ \theta_o]\} \qquad (A18)$$

where

$$\varepsilon < 0.532, \qquad (A19)$$

$$\theta < \frac{\pi}{2} \qquad (A20)$$

The contact angle, θ_o, could be evaluated by the result of Appendix V as follows:

$$\theta_o = \cos^{-1}\ (1 - \frac{1}{n}) \qquad (A31)$$

where 2n is the average number of contact points of each sphere in a packed bed.

C. For γ:

Basing on the assumption that volume and area void fraction are equal, the parameter γ, like ϕ, was determined from geometry considerations alone. From the standpoint of the volume of the pendular rings, Masamune and Smith (1963) obtained the following expression:

$$\gamma = 1 - \frac{3}{2}\ (\frac{1 - \varepsilon}{\varepsilon})\phi \qquad (A21)$$

D. For δ:

For soft, rough particles the area of contact between two particles would be expected to be larger than for hard, smooth spheres. The contact angle, θ_1, in Figure (4.6a) is a function of the mechanical properties of the particle solids. By considering the area of contact between two particles to be $\pi r_o^2 \sin^2\theta_1$ for one contact and $n \pi r_o^2 \sin^2\theta_1$ on the hemispherical surface of a particle, δ was approximated by Masamune and Smith (1963) as follows:

$$\delta = \frac{n \pi r_o^2 \sin^2 \theta_1}{\pi r_o^2}$$

$$= n \sin^2 \theta_1 \qquad \text{(A22)}$$

APPENDIX V

THE RELATION BETWEEN THE CONTACT NUMBER, CONTACT AREA, CONTACT ANGLE AND PARTICLE SIZE

If 2n is the average number of contact points of each sphere in a random bed we assume that all of particle surface is covered by the pendular rings around each contact point. Figure A-1 shows the center of a particle and one of the contact area with radius u_o.

Since

$$dA = dz \cdot zd\rho$$

$$dA_{proj} = dA \cos \theta = dz \cdot zd\psi \cos \theta$$

Define $\quad d\omega$ = solid angle subtended by dA at o

$$= \frac{dA_{proj}}{I^2} \tag{A23}$$

$$\omega = \int \frac{dA_{proj}}{I^2} \tag{A24}$$

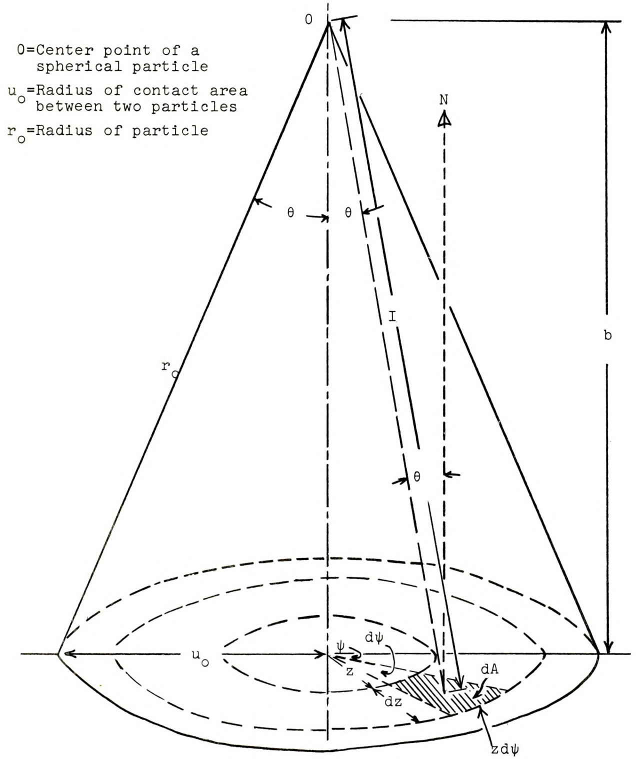

Figure A-1.--Contact Area and Contact Angle.

$$= \int_{\psi=o}^{2\pi} \int_{z=0}^{u_o} \frac{dz \cdot z d\psi \cos}{I^2}$$

$$= 2\pi \int_{z=o}^{u_o} \frac{z\, dz \cos \theta}{I^2} \tag{A25}$$

But

$$\cos \theta = \frac{b}{I} = \frac{r_o \cos \theta_o}{I} \tag{A26}$$

$$\therefore \omega = 2\pi b \int_{z=o}^{u_o} \frac{z\, dz}{I^3}$$

where

$$I^2 = z^2 + b^2 \tag{A27}$$

$$I^3 = (z^2 + b^2)^{3/2}$$

$$\therefore \omega = 2\pi b \int_{z=o}^{u_o} \frac{z\, dz}{(z^2 + b^2)^{3/2}} \tag{A28}$$

Let

$$z^2 + b^2 = \xi$$

$$2z\, dz = d\xi \quad \text{or } z\, dz = \frac{d\xi}{2} \tag{A29}$$

$$\omega = 2 \pi b \frac{1}{2} \int_{b^2}^{b^2 + u_o^2} \frac{d\xi}{\xi^{2/3}}$$

$$= \pi b \left[\frac{\xi^{-\frac{1}{2}}}{-\frac{1}{2}} \right]_{b^2}^{b^2 + u_o^2}$$

$$= -2 \pi b \left[\frac{1}{\sqrt{\xi}} \right]_{b^2}^{b^2 + u_o^2}$$

$$= -2 \pi b \left[\frac{1}{\sqrt{z^2 + u_o^2}} - \frac{1}{b} \right]$$

$$= 2 \pi b \left[\frac{1}{b} - \frac{1}{\sqrt{z^2 + u_o^2}} \right]$$

$$= 2 \pi \left[1 - \frac{b}{\sqrt{b^2 + u_o^2}} \right] \qquad (A30)$$

But $\omega = \frac{2\pi}{n}$ due to the solid angle of a sphere is 4π

$$\therefore \frac{1}{n} = 1 - \frac{b}{\sqrt{b^2 + u_o^2}} = 1 - \cos \theta_o$$

$$\therefore \quad \theta_o = \cos^{-1}\left(1 - \frac{1}{n}\right) \tag{A31}$$

$$u_o = r_o \sin \theta_o = r_o \sin \left[\cos^{-1}\left(1 - \frac{1}{n}\right)\right] \tag{A32}$$

APPENDIX VI

JUSTIFICATION OF HEAT LOSS FROM THE SIDE WALL OF SPECIMEN IN FIGURE 18(a)

In order to verify the availability of the Transient Thermal Properties Measurement Facility described in Chapter V, the one dimensional assumption for heat flow through the specimen has to be examined. This was done as follows by comparing the amount of heat loss from the side wall of the specimen, which had no insulation around during the test, to that of heat flow through the specimen.

Test No. 4 (done on 12/16/68) was picked to serve as an example. The size of the milk specimen was three inches in diameter and one inch thick (Figure 5.8). The locations of thermocouples were 1/16 inch, 1/8 inch and 1/2 inch from the top surface of the specimen. Thermocouples No. 1, 2, 3, 4 and 5 represented the temperature profiles of surface area A_1, A_2, A_3. A_4 and A_5, respectively, on the side wall, where

$$A_1 = A_2 = A_4 = A_5 = (1/8)\ 3\pi \text{ sq. in}$$

$$A_3 = (4/8) 3\pi \text{ sq. in}$$

The temperature profile of each thermocouple and the hot copper block (heat source) are shown in Figure A2 and Table A1. The error analysis could be carried in the following steps:

<u>Step 1</u>

Find $(\Delta T)_{avg}$: ΔT is defined as the temperature difference between the side wall surface of the specimen and room air. ΔT of each surface region in each time interval was calculated in Table A24.3. $(\Delta T)_{avg}$ was then weighed properly by the surface area associated as following:

$$(\Delta T)_{avg} = \frac{1}{8}(105.1) + \frac{1}{8}(68.8) + \frac{4}{8}(24) + \frac{1}{8}(10) + \frac{1}{8}(8) = 36°F \qquad (A33)$$

<u>Step 2</u>

Find h: The convection coefficient, h, for vertical wall was approximated by the equation as follows (Rohsenow and Choi, 1961):

$$h = 0.29\left[\frac{(\Delta T)_{avg}}{L}\right]^{1/4} \qquad (A34)$$

$$= 0.29\left[\frac{36}{1/12}\right]^{1/4} = 1.32 \text{ Btu/(hr)(°F)(sq. ft.)}$$

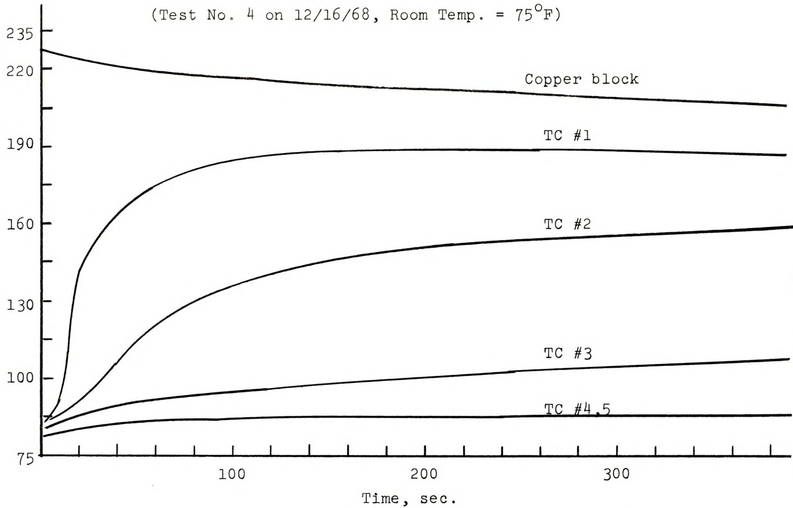

Figure A-2.--Example of Temperature Profiles for Transient Thermal Properties Measurement.

TABLE A1.--ΔT of Each Surface Region in Fig. 5.8(a).

Time Interval Sec.	ΔT, °F (TCNo.1)	ΔT, °F (TCNo.2)	ΔT, °F (TCNo.3)	ΔT, °F (TCNo.4)	ΔT, °F (TCNo.5)
0- 20	35	--	--	10	8
20- 40	78	24	13	10	8
40- 60	96	38	15	10	8
60- 80	103	49	16	10	8
80-100	107	56	18	10	8
100-120	110	62	18	10	8
120-140	112	66	20	10	8
140-160	113	70	22	10	8
180-200	113	75	25	10	8
200-220	113	76	25	10	8
220-240	113	77	26	10	8
240-260	113	78	26	10	8
260-280	112	78	27	10	8
280-300	112	79	28	10	8
300-320	112	80	29	10	8
320-340	112	80	30	10	8
340-360	112	81	31	10	8
360-380	112	82	32	10	8
380-400	111	83	32	10	8
Average	105.1	68.8	24	10	8

Step 3

Find the heat loss by convection:

$$\dot{q}_{\substack{\text{conv.}\\ \text{loss}}} = h\ A\ (\Delta T)_{avg} \tag{A35}$$

$$= 1.32\ \left(\frac{3\pi}{12}\right)\ \left(\frac{1}{12}\right)(36)$$

$$= 2.1\ \text{Btu/hr.}$$

Step 4

Find the heat loss by radiation:

$$\dot{q}_{\substack{\text{rad.}\\ \text{loss}}} = F\ \sigma\ \varepsilon\ A\ ({T_w}^4 - {T_a}^4) \tag{A36}$$

where

F = shape factor = 1

σ = Stefan-Boltzmann constant = 0.1714×10^{-8} Btu/(hr)(sq. ft.)($°R)^4$

ε = emissivity $\cong$ 0.9 (Kreith, 1964)

A = area of side wall = $\frac{3\ \pi}{144}$ sq. ft.

T_w = absolute temperature of specimen = 75°F + $(\Delta T)_{avg}$ = 111°F = 571°R

T_a = absolute temperature of room air = 75°F = 535°R

$$\dot{q}_{\substack{rad. \\ loss}} = 0.1714 \cdot 0.9 \cdot \frac{3}{144} (5.71^4 - 5.35^4)$$

$$= 2.47 \text{ Btu/(sq. ft.)(°F)(hr)}$$

Step 5

Find the heat transferred from the copper (heat source)

$$\dot{q}_{\substack{from \\ copper}} = m\ c\ \frac{\Delta T}{\Delta t} = \rho(\Delta V)\ C\ \frac{\Delta T}{\Delta t} \qquad \text{(A37)}$$

where

ρ = density of copper = 558 lbm/cu. ft

ΔV = volume of copper block = $(1/4)\pi(\frac{3}{12})^2 \cdot \frac{1.5}{12}$ cu. ft.

C = specific heat of copper = 0.091 Btu/(lbm)(°F)

t = time interval of testing = 400 sec = $\frac{400}{3600}$ hr.

T = temperature drop of copper block = 22F (Figure A2)

$$\dot{q}_{\substack{from \\ copper}} = 558\ [\frac{1}{4}\ \pi(\frac{3}{12})^2 \cdot \frac{1.5}{12}] \cdot (0.091)\ \frac{22}{(\frac{400}{3600})}$$

$$= 61.7 \text{ Btu/hr}$$

Step 6

$$\text{Error}_{\substack{\text{heat}\\ \text{loss}}} = \frac{\dot{q}_{\substack{\text{conv}\\ \text{loss}}} + \dot{q}_{\substack{\text{rad.}\\ \text{loss}}}}{\dot{q}_{\substack{\text{from}\\ \text{copper}}}} = \frac{2.1 + 2.47}{61.7} = 7.4\% \tag{A38}$$

This is the approximate experimental error due to heat loss from the side wall of the milk specimen. Obviously this error could be reduced to a minimum by covering the side wall with proper insulation material.

APPENDIX VII

FORMULA USED FOR CALCULATING COEFFICIENT OF CORRELATION

$$R = \frac{\Sigma XY - \frac{\Sigma X \cdot \Sigma Y}{N}}{N \ \sigma_x \ \sigma_y} \qquad \text{(A39)}$$

where

$$\sigma_x = \sqrt{\frac{\Sigma X^2}{N} - \frac{(\Sigma X)^2}{N^2}} \qquad \text{(A40)}$$

$$\sigma_y = \sqrt{\frac{\Sigma Y^2}{N} - \frac{(\Sigma Y)^2}{N^2}} \qquad \text{(A41)}$$

(See "Statistics" manual of "Mathatron 4280 TD, Tape Set Instructions".)