AN INVESTIGATION OF THE RELATIVE EFFECTS OF TEACHING A MATHEMATICAL CONCEPT VIA MULTISENSORY MODELS IN ELEMENTARY SCHOOL MATHEMATICS

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ABSTRACT

AN INVESTIGATION OF THE RELATIVE EFFECTS OF TEACHING A MATHEMATICAL CONCEPT VIA MULTISENSORY MODELS IN ELEMENTARY SCHOOL MATHEMATICS

By

Pearlena Wallace

The central purpose of this study was to investigate, measure, and discuss the degree to which the teaching of a mathematical concept via multisensory models affects students' achievement in grades four, five, and six. In addition to the central purpose, answers to the following related questions were sought:

1. Will there be any difference among grades four, five, and six on the gain score?
2. Will there be any difference between the achievement of boys and the achievement of girls?
3. Will there be any difference between the achievement of the welfare and non-welfare recipients?
4. Will there be any difference between the achievement of students in the treatment and control groups on the manipulative?

Pearlena Wallace

The 154 subjects in the study were students in the fourth, fifth, and sixth grades at Morley Elementary School located in Saginaw, Michigan. Morley Elementary School is one of the eleven inner city schools in the Saginaw City School District. Approximately one-third of the students enrolled in Morley School are classified as Title I students or welfare recipients.

The study consisted of three treatment groups and three control groups. This allowed for one treatment group and one control group for each of the three grades (4, 5, and 6). Prior to the beginning of the experiment, the pre-test was administered to all students participating in the study. At the end of a three-week period of teaching, the post-test was administered to all students involved in the study. Also, at the end of the three-week period of teaching, a manipulative test was given to thirty students, fifteen students from the treatment group and fifteen students from the control group.

Data from the pre-test and the post-test were analyzed by the two way analysis of variance, the analysis of covariance, and the repeated measure design.

The findings of this study were as follows:

1. The mean performance of the students in the treatment group was superior to the mean performance of the students in the control group on both the achievement and manipulative tests.

2. There was no significant difference among grades four, five, and six.
3. The achievement of the boys was not significantly difference from the achievement of the girls.
4. The achievement of the welfare recipients was not significantly different from the achievement of the non-welfare recipients.

The results of the analysis led to the following conclusions:

1. Since the achievement of the students taught by the multisensory approach was significantly higher than the achievement of the students taught by the traditional approach, it seems reasonable to conclude that the use of multisensory models in learning activities increases meaning and understanding.
2. In-service training seems to be a sufficient means of preparing teachers to implement an instructional program that utilizes multisensory materials.
3. The program utilized in this study appears to be an effective means for providing for basic learning skills of individual children as well as groups of children.
4. From the data it cannot be concluded that sex is a determinant in the learning of fraction concepts when multisensory materials or traditional

instructions are used.

5. It can be inferred that any student, regardless of socioeconomic background, if given an appropriate learning environment, can evidence growth as was indicated by the parallel performance of the welfare and non-welfare recipients.

From the data, the following recommendations are presented:

1. The results of this study support the posture that the school systems should provide opportunities for in-service training for their classroom teachers. It is evident that the teachers, students, and school system as a whole would benefit from the participation of the classroom teachers in in-service programs designed to provide teachers with specific methodology and techniques in the multi-sensory approach. Further, since building principals and other administrators influence to a greater or lesser degree what goes on in the classroom, it seems logical that they should participate in the in-service training.
2. Colleges and universities will need to revamp their programs to provide the pre-service teachers with greater training in the utilization of the multi-sensory approach.

3. The elementary school mathematics curriculum should focus more attention on the psychomotor domain. For instance, more behavioral objectives in mathematics which require manipulative skills should become a necessary part of the curriculum.
4. The classroom teachers should have access to clearly stated, sequential objectives along with complementary materials and a repertoire of methods to implement the objectives.
5. School systems should spend more money on materials which are conducive to the multisensory method, since the findings support the position that students learn better from the multisensory method of teaching than they do from the traditional method of teaching. It is also apparent that the school systems will need to employ more mathematics consultants or specialists who can provide in-service training for teachers and other staff on the utilization of multisensory models in teaching mathematics.

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By

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Dedicated to my daughter, Sharon.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF APPENDICES

CHAPTER I

DESCRIPTION OF THE PROBLEM

Introduction

The low scholastic performace of a substantial number of students in mathematics is a continuing concern of parents and educators. The problem of turning students on to mathematics has plagued educators for decades. It is generally agreed that achievement is influenced by motivation and that the presence of appropriate activities will arouse motivation. Unfortunately, many schools have failed to provide the learner with the kinds of experiences which will appropriately motivate him to want to learn mathematics. Hence, the learner finds himself unable or unwilling to digest the mathematics desired of him.

If a child is to be inspired to learn mathematics, his state of mind will need to be influenced so that he feels a desire and a need to learn mathematics. The environment in which the child finds himself must be geared to his interest, and it must also offer a multiplicity of experiences and alternatives.

The educational process has basically involved the senses of sight and hearing, but the sense of touch and the kinesthetic sense should also play basic roles in the learning process. The use of the sense of touch and the kinesthetic sense in instruction should serve to strengthen and deepen conceptual development as well as make learning interesting. Consideration of these senses in the instructional process is particularly important for the child who does poorly with abstractions and symbols.

Multisensory models have often been excluded from the teaching-learning process. And too frequently, when multisensory models have been utilized in the classroom, the teacher, rather than the child, has been the prime manipulator. The child most often found himself playing the role of a spectator rather than a participant.

Even though educators are becoming more aware of the need to arouse interest in the child and make the subject of mathematics more meaningful, experimental evidence relating to techniques of learning mathematics is scarce. The challenge is to effectuate the achievement of students by fostering a desirability for learning.

Statement of the Problem

In spite of new teaching techniques, innovative materials, methodology, and special programs currently encompassing the mathematics curriculum, standardized test results still indicate many students are achieving at a drastically low level in mathematics. The abstract, verbal or paper-and-pencil teaching of mathematics has proven unproductive for many students. The major concern of this study was to investigate the "effects of teaching" a selected mathematical concept using multisensory models on student achievement in the fourth, fifth, and sixth grades.

Background and Rationale

Historically, elementary mathematics has been viewed as a discipline that provided computational skills per se. The teaching of mathematics has been more relentless drill than intuitive insight and, therefore, has created an attitude of resentment by many students and teachers.

Courses of study in mathematics have often ignored the importance of observation, discovery, and description, giving the erroneous impression that mathematics is simply the memorization of certain operations in accordance with certain memorized rules. This is an unimaginative and unfruitful way of considering mathematics. Many critics and mathematics educators believe that mathematics should begin with discovery of relationships and a growing mastery of the description of these relationships. Concrete models lend themselves admirably to this approach.[1,2]

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[1]Caleb Battegno, Mathematics with Numbers in Color (New York: Cuisenaire Company, Inc., 1966), p. 9.

[2]Julian Caparros and Marie Delgado, "First Report on the Canary Islands Mathematics Project," Journal of Structural Learning (New York: Gordon and Breach Science Publishers, 1970), 76-77.

The idea of using concrete models in the teaching of mathematics is historically dated. McLennan and Dewey, prior to the twentieth century, spoke of the need for balance between the manipulation of concrete objects and symbolization of concepts. This balance is necessary for effective learning.[3] Children learn by progressing from the concrete to the abstract, from the known to the unknown. During the process of learning mathematics, they utilize a variety of strategies depending upon their background, their environment, and their innate ability.[4]

It is known that numerous students have been turned off by the traditional method (teaching via textbooks, lecture or visual materials) of teaching mathematics. The fact is, abstract, rote drill has produced many students who later face great difficulty in arithmetic reasoning and the application of simple mathematics concepts.[5] A switch to a teaching style fostering experiences which are more tangible, interesting, and real to the learner is long overdue. Multisensory models help students to better

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[3]John Dewey and J. A. McLennan, Readings in the History of Mathematics Education (Washington, D.C.: National Council of Teachers of Mathematics, 1970).

[4]William Arnold and C. W. Schminke, Teaching the Child Mathematics (Hinsdale, Illinois: The Dryden Press, Inc., 1973), p. 31.

[5]Andrew Schreiber, An Empirical Approach at the Secondary Level: Building Number Skills in Dyslexic Children (Academic Therapy Publication, 1972), p. 4.

internalize concepts and principles. Because these learning aids help students get increased meaning and understanding, they tend to increase interest and promote the development of favorable attitudes. Moreover, multisensory models bring greater variety of experience into the learning situation. Their concrete and visual characteristics attract attention, and it must be remembered that attention precedes interest. Motivation is almost invariably improved when multisensory models are used.[6]

The widespread efforts to teach mathematics in a more activity-oriented approach represent an attempt to provide experience for individual children (1) to enhance curiosity, (2) to provide meaning to mathematical concepts, (3) to build the intuitions which make abstractions possible, (4) to make reasonable their applications, and (5) to nurture the healthy, natural development of intellect in children. Pupils remember best what they discover for themselves.[7,8]

The effective learning of mathematics follows a hierarchical pattern. Learning is more effective and

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[6]The National Council of Teachers of Mathematics, The Learning of Mathematics--Its Theory and Practice, Twenty-First Yearbook (Washington, D.C.: The National Council of Teachers of Mathematics, 1953), p. 54.

[7]Edith Biggs and James Maclean, Freedom to Learn (Ontario: Addison Wesley Publishing Co., 1969), p. 3.

[8]T. E. Kieren, "Activity Learning," Review of Educational Research, XXXIX (1969), 509-522.

generalizations are made more easily when teaching is done systematically rather than randomly. Therefore, consideration of subordinate skills by levels of difficulty must play an important role in the organization of the arithmetic curriculum.[9] The first priority in learning mathematics is concept development, which in turn enables the child to be more functional in the performance of skills and problem solving techniques. If there is a gap in the child's conceptual foundation in elementary mathematics, he will have difficulty performing the skills and problem solving tasks.[10] For instance, the learning of addition facts is essentially the assimilation of the results of combinations of objects which are first understood (conceptualized), second abstracted, and third verbalized.[11]

Both Ausubel and Gagné emphasize the importance of careful sequencing of instructional experiences so that any unit taught is closely related to those that preceded.[12]

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[9]Frances Flournoy, Elementary School Mathematics (New York: The Center for Applied Research in Education, Inc., 1964), p. 7.

[10]Fredericka K. Reisman, A Guide to the Diagnostic Teaching of Arithmetic (Columbus, Ohio: Charles E. Merrill Publishing Co., 1972), p. 16.

[11]Henry Syer, "Sensory Learning Applied to Mathematics," The Learning of Mathematics--Its Theory and Practice, Twenty-First Yearbook (Washington, D.C.: The National Council of Teachers of Mathematics, 1953), p. 105.

[12]Robert Travers, ed., The Second Handbook of Research on Teaching (Chicago: Rand McNally Co., 1973), p. 1117.

Many mathematics educators and psychologists strongly urge greater use of manipulative models in the teaching of mathematics. For instance, Pestalozzi insists that whenever possible children should learn by touch and sensation. Further, Pestalozzi believes that nature must be organized for the child so that he can benefit from his sensory impressions.[13,14] The child, by physically acting upon concrete models, can more easily conceptualize and transmit the concept to the abstract. The learner's cognition level is more greatly influenced when he is personally involved with these models. Moreover, a recent report of a UNESCO conference also stated that the more the senses can be involved, the more efficient learning will become. Furthermore, if a child only hears but does not see, he does not learn as well as if he hears and sees at the same time. If he can touch as well as hear and see, he will learn far more soundly.[15]

The philosophy of the Chinese proverb,

> I hear, and I forget
> I see, and I remember
> I do, and I understand

is supportive of the use of concrete models in helping

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[13] Ibid., p. 531.

[14] Robert E. Reys, "Consideration for Teachers Using Manipulative Materials," The Arithmetic Teacher, XVIII (December, 1971), 552.

[15] Edith Biggs and James Maclean, Freedom to Learn (Ontario: Addison Wesley Publishing Co., 1969), pp. 3-4.

facilitate an understanding of that which is to be learned.

Piaget reported that initiating a concept with concrete models and keeping the child there until he is capable of dealing with symbols will conceptualize his understanding of the abstract.[16] It is erroneous to assume that a child acquires the notion of number and other mathematical concepts merely from formal instruction.

Further implication of Piaget's research is that physical action is one of the bases of learning. To learn effectively, the child must be a participant in events, not merely a spectator. To develop the child's concept of number, it is not enough that he looks at things. He must also touch them, move them and turn them. The child should be encouraged to maximize the manipulation of the objects.

Gagné maintains that concept learning requires the use of a variety of concrete examples of representative kinds of objects and pictures.[17] Because mathematics is structured on learning hierarchies, it is pedagogically sound to expose the child to concepts by direct observation and manipulation before he is introduced to concepts that are abstract.

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[16] Scientific American, 1953, pp. 74-79.

[17] Robert Gagné, The Conditions of Learning (New York: Holt Rinehart and Winston, Inc., 1970), p. 249.

Reisman contends that in learning basic mathematical concepts, it is important for the child to have motor experiences. Just as one cannot really get the idea of "hammer" without using a hammer, a child needs to manipulate sets of objects to abstract the number property of the set.[18]

Dienes states that when physical objects are used, there is a greater chance that the child will have a better understanding of the concept. Moreover, Dienes believes that the learning cycle should involve a large number of physical embodiments of mathematical notions to be learned.

Further, Dienes has experimented and found success with concrete models in many of his centers. He has also designed materials of his own, such as the multibase blocks and logic blocks, and has formulated a number of principles for the use of these aids. Dienes has also developed a K-6 curriculum which relies heavily on the use of concrete materials.[19]

Heidbreder advocated the use of manipulative models in concept attainment. To justify his belief, Heidbreder made the following observations. The concept "circle" is

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[18]Fredricka K. Reisman, A Guide to Diagnostic Teaching of Arithmetic (Columbus, Ohio: Charles E. Merrill Publishing Co., 1972), p. 40.

[19]Z. P. Dienes and E. W. Golding, Approach to Modern Mathematics (unpublished book, 1967), pp. 26-33.

attained significantly later than the concept "plate," because the circle is beyond the manipulability stage even though it is as perceptible as the plate. The plate can be manipulated with the hands, it can be felt, seen and weighed. But the circle drawn on the board cannot be manipulated. Even though its form can be traced in the air, this does not offset the advantage of manipulability from the standpoint of the attainment of the concept.[20]

Montessori has directed much of her work toward the sensorial area. She has stated that nothing comes to the intellect that is not first in the senses.[21] The importance of physical activity which will allow the child to utilize his hand can rarely be overemphasized. The human hand not only allows the mind to reveal itself but it enables the whole being to enter into special relationships with his environment.[22]

Moreover, Montessori advocated, as an essential part of the learning experience, materials which were designed to enable various sensory discriminations and classifications on the part of the child.[23]

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[20]E. Heidbreder, "The Attainment of Concepts," A Psychological Interpretation Transaction of the New York Academy of Science (New York: July, 1945).

[21]Maria Montessori, The Secret of Childhood (New York: Ballatine Books, Inc., 1972), p. 99.

[22]Ibid., p. 81.

[23]Robert Travers, ed., The Second Handbook of Research on Teaching (Chicago: Rand McNally Co., 1973) p. 561.

Similarly, other researchers have reported that knowledge cannot be directly acquired from the blackboard, textbook, or film by mere perception or acquired by drill; rather, these general ways of knowing have to be actively constructed by the child through interaction with the environment.[24]

In summary, one might expect those students who have been taught through the use of multisensory models to have more advanced knowledge of mathematical concepts. However, this assumption is tenuous and will need to be investigated.

Purpose of the Study

The purpose of this study was to compare the performance of fourth, fifth, and sixth grade students taught by a procedure utilizing concrete models with the performance of fourth, fifth, and sixth grade students who were taught by a traditional approach.

Hypothesis

In accord with the problem of this study, the major hypothesis with which this study is concerned is stated as follows:

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[24]Kevin Marjoribanks, "Environment Correlates of Diverse Mental Abilities," *Journal of Experimental Education*, XXXIX (1971), 64-68.

H: Students who are taught by the multisensory approach will achieve significantly higher scores on achievement tests than those students who are taught by the traditional approach.

Related Questions

The following related questions will also be investigated:

1. Will there be any difference among grades four, five, and six on the gain scores?
2. Will there be any difference between the achievement of boys and the achievement of girls?
3. Will there be any difference between the achievement of the welfare and non-welfare recipients?
4. Will there be any difference on the manipulative test between the achievement of students in the treatment and control groups?
5. Is there any interaction between treatment and sex?
6. Is there any interaction between treatment and grade?
7. Is there any interaction between grade and sex?
8. Is there any interaction among grade, treatment, and sex?

Clarification of Terms

For the purpose of this study the following terms are defined as stated below:

Multisensory, Concrete, and Manipulative

These three adjectives are used interchangeably in this study. They are used to refer to physical models which the child can see, feel, and manipulate. The child is involved in direct manipulation of these models. Examples of such models are the cuisenaire rods, attribute blocks, and multibase blocks.[25]

Concept Learning

Concept learning involves the meaningful labeling of a class of objects. Concepts usually have concrete referents as in abstracting the number property of equivalent sets. Examples of concrete learning include the fact that whales, dogs, and cats are mammals; understanding that the numerals 1, 2, 3, 7 are symbols which represent numbers; and that B and b are letter symbols for the same letter.[26]

Abstract Learning

Abstraction is the gathering together of a number of different events into a class using certain criteria

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[25]The National Council of Teachers of Mathematics, The Learning of Mathematics--Its Theory and Practice, Twenty-First Yearbook (Washington, D.C.: The National Council of Teachers of Mathematics, 1953), pp. 99-118.

[26]Fredricka K. Reisman, A Guide to Diagnostic Teaching of Arithmetic (Columbus, Ohio: Charles E. Merrill Publishing Co., 1972), p. 49.

that must be applicable to all these events and situations. For example, when one abstracts he draws out from many different situations that which is common to them, and he disregards those things which are irrelevant to this core.[27]

Inner City

This term refers to persons or areas which are characterized by the following:

Large number of welfare recipients.

High degree of transiency.

High rate of failure in school.

High proportion of dilapidated housing.

Low level of educational attainment among adults.

High crime rate.[28]

Limitations of the Study

The limitations of any study will be determined by the size of the population, the population sampling procedure, and the analysis process; therefore, one should be exceedingly careful in procuring the population.

The population for this study is not a

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[27]Z. P. Dienes, "Some Basic Processes Involved in Mathematics Learning," Research in Mathematics Education (Washington, D.C.: National Council of Teachers of Mathematics, 1967), p. 22.

[28]Kenneth Clark, Dark Ghetto (New York: Harper and Rowe Publishing Co., 1965), p. 19.

cross-sectional sample of students; therefore, the findings can not be generalized for all students. The total N consisted of only fourth, fifth, and sixth grade students. First, they were inner city boys and girls. Second, the sample was drawn from only one elementary school located in the Saginaw City School District.

This study was based on the teaching of only one mathematical concept. Hence, the results cannot be inferred to all mathematical concepts.

The non-paper-and-pencil test was not tested for reliability. However, the test was constructed from the achievement test which was tested for reliability. The objectives also served as a basis for the construction of the non-paper-and-pencil test.

Summary

In this chapter the problem, rationale, and purpose of the study have been reviewed. The statement of the problem, hypothesis, and related questions have been developed. Terms pertinent to the study have been defined and the limitations of the study stated. The remaining chapters will provide a description of the following:

In Chapter II, literature related to this study will be reviewed. Also, other research which had a bearing on this study was cited.

In Chapter III, the design, methodology, and test instruments will be discussed.

In Chapter IV, an analysis of the data and a discussion of those results will be reported.

In Chapter V, a summary, conclusions, and recommendations of this study will be reported.

CHAPTER II

REVIEW OF THE LITERATURE

The studies cited in this review are divided into three sections: (1) the learning of elementary school mathematics; (2) the teaching of elementary school mathematics; and (3) the elementary school mathematics curriculum. It is believed that these three factors are interdependent variables in the learning of mathematics.

The Learning of Elementary School Mathematics

An investigation of the literature leads to the assumption that learning mathematics is like learning language and the aesthetic subject. It is an active process and should call to play our imaginative and creative powers so that the subject is a delight to pupils of all ages.

Current research has shown that it is apparent that children learn mathematics through other avenues than their eyes and ears; for example, through the use of their hands. The more senses that are involved the better chance the child has to conceptualize what is to be learned.[1] Sensory learning is an essential part of all learning. In fact, all concepts are abstractions from simpler concepts or from perceptions experienced by the senses. One's perception is strengthened by the involvement of many senses. Hence on the basis of perceptions, one abstracts and generalized to form concepts in the memory, and combines concepts to form more advanced types of abstractions.[2]

Multisensory models are invaluable in the formation and conceptualization of mathematical ideas. The physical stimuli are also necessary in order to clarify the purpose for learning.[3,4] It seems reasonable to assume that, if the problem is made clear by physical manipulation, the child will develop a more positive attitude toward mathematics and his motivational level will be enhanced.

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[1] The National Council of Teachers of Mathematics, The Learning of Mathematics--Its Theory and Practice, Twenty-First Yearbook (Washington, D.C.: The National Council of Teachers of Mathematics, 1953), p. 99.

[2] Ibid.

[3] John G. Harvey and Thomas A. Romberg, "Educational Research in Mathematics at the University of Wisconsin Research and Development Center for Cognitive Learning," Journal for Research in Mathematics, IV (1973), 4.

[4] Robert E. Reys, "Consideration for Teachers Using Manipulative Materials," The Arithmetic Teacher, (December, 1971), p. 552.

Several studies have noted a correlation between positive attitude and learning. For instance, Basham, Murphy, and others (1964) found that there was a low positive relationship between the pupil's attitude and his achievement in mathematics.[5]

Intimately associated with motivation and positive attitude is the existence of curiosity. It is essential that a child's natural curiosity be encouraged, for curiosity is a potentially powerful motivator of the elementary school child. If mathematical learning experiences are structured in a manner that capitalizes on his desire to explore, the child will want to learn.[6]

Cole and Bruce have made some observations which are particularly apt:

> The building of models, the drawing of the diagram, either in imagination or with physical materials, is a crucial step in the human thought process. The architect thinks with his drawing board and instruments, the machine builder with his model.... It is this ability to build a thought model which transforms the direct, bungling, blundering behavior of the child into the thoughtful, planful reasoning behavior of the adult.... What a thought model does in addition to giving us something to manipulate in our planning, is to provide a host of suggestions of possible manipulations. Science is a system of models, symbols, and relationships and once

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[5]Robert Travers, ed., The Second Handbook of Research on Teaching (Chicago: Rand McNally Co., 1973), p. 1151.

[6]C. W. Schminke and Norbert Maertens, Teaching the Child Mathematics (Hinsdale, Illinois: The Dryden Press, Inc., 1973), p. 18.

> we have mastered its network of relationships, we turn to it quickly when we are confronted with a problem.[7]

It seems to be an uncertainty among elementary school teachers that multisensory models stimulate the child to learn. But it has been stated that if the learner experiences the same thing through different channels, concepts are more easily attained, retention is improved, and the transfer of knowledge is more applicable to problem-solving situations. Sensory learning is necessary to the understanding of concepts at two levels: (1) when the concept is being developed, and (2) whenever the concept is being applied.[8]

Balow, et al. (1964) have cogently argued that concept development is a prerequisite for problem-solving skills. Children learn better when they develop an understanding of the process of problem-solving rather than rote memory of problem-solving.[9] Thiel found that children who had been taught by the generalization method, where meanings and relationships were stressed, were significantly superior to children taught by drill on a test of verbal

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[7] L. E. Cole and F. L. Bruce, Educational Psychology (New York: World Book Co., 1950), p. 26.

[8] Robert Travers, ed., The Second Handbook of Research on Teaching (Chicago: Rand McNally Co., 1973) p. 1098.

[9] H. I. Balow, "Reading and Computation Ability as Determinants of Problem-Solving," The Arithmetic Teacher, 1964, pp. 18-22.

problem-solving ability.[10]

Piaget outlines three stages through which the formation of a concept passes before finally being established within a person.

1. The first stage is an arbitrary kind of stimulus-bombardment responded to by playful activity without any conscious aim. This play furnishes the fundamental reality experience from which the concept will eventually be acquired.

2. The second stage is an intermediate one, in which it is to be realized that some structuralization of the play experience is needed, and some of the concepts are beginning to be put together. This is a more purposeful stage aiming at some final organization as yet unperceived.

3. The third stage is the formation of the final concept. The correct Gestalt of all the component parts is seen; in other words, a measure of unity is achieved in the way the parts are seen to build up the whole.[11]

Dienes and Golding (1967) maintain that the child should learn through activity and personal experiences. These experiences must involve "things" or people. Children cannot have any experiences without their being involved with something outside themselves.[12]

Since the days of Montessori, teachers have used, at various times, objects to demonstrate what they have

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[10]C. L. Thiele, "Contribution of Generalizations to the Learning of Additional Facts," Teachers College Contribution to Education, 1938.

[11]Z. P. Dienes, Concept Formation and Personality Leicester, England: Leicester University Press, 1959), pp. 6-7.

[12]Z. P. Dienes and E. W. Golding, Approach to Modern Mathematics (unpublished book, 1967), pp. 23-25.

been talking about. Unfortunately, these objects have been used more as "teaching aids" than "learning aids." This means that the teacher has usually manipulated the objects rather than the child. The child learns far more through his own personal experiences than he does through the related experiences of others.[13]

Piaget advocates learning through activity. He believes that the child will learn more effectively if he is an active participant in the learning process rather than a spectator. Too frequently, traditional learning has required that the child adopt a passive role, even to the extent that he has been expected to sit quietly, listen to the teacher, and, at the most, answer a series of questions designed to see if he can repeat what he has been told.[14]

After in-depth research, Piaget suggests that the development of a mental structure in a child parallels physical activity of a personal kind. The passive child learns far less than the active child, and the amount of activity is an important factor which determines the extent of learning.[15]

Bruner has also pointed out that an overly passive

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[13]*Ibid.*, p. 25.

[14]*Ibid.*, p. 24.

[15]*Ibid.*, p. 24.

approach to learning creates a situation in which the learner expects order to come from the outside, that is, from the material which is presented. But the learning of mathematics requires the learner to exert mathematical reasoning which includes unmasking, simplifying, and re-ordering. Transfer of learning becomes more effective when the problem-solving attitude precedes the rote-learning attitude.[16]

Ausubel and Gagné maintain that learning is more effective when there is a careful sequencing of instructional experiences so that each unit taught is clearly related to those that preceded. It is this continuity between the learner's existing cognitive structure and the new material to be learned that makes the new material meaningful.[17]

Gagné (1967) contends that any human learning task may be analyzed into a set of component tasks which are distinct from each other in terms of the experimental operations needed to produce them. Thus, the presence or absence of these task components affects positive transfer to the final performance. Further, Scandura and Wells have perpetuated the idea of sequencing in learning by developing approaches to sequencing variables.[18]

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[16]Jerome Bruner, The Process of Education (Cambridge, Mass.: Harvard University Press, 1960).

[17]Robert Travers, ed., The Second Handbook of Research on Teaching (Chicago: Rand McNally Co., 1973), p. 1117.

[18]Ibid., p. 1163.

Maria Montessori maintained that the young child can learn better by regulating his sensory motor interaction with selected objects in his environment. The central idea permeating this approach is that the child is an active inquisitive organism who can discover knowledge through guided exercise of the senses and his body.[19]

According to Montessori, the fundamental principle of learning is the liberty of the child. Liberty is activity, and free activity is balanced by order and control, not by the teacher, but rather, from the organization of the environment. This means that the child should be provided with tangible situations and materials which will encourage active learning.[20]

Montessori further believed that the human hand is a viable element to learning. The human hand not only allows the mind to reveal itself, but it enables the whole being to enter into special relationships with his environment.[21] The child gains a better understanding of mathematics by appealing to the mind through the hands as well as through the eyes and ears.

Bruner (1966) stated that learning should begin with a focus on the production and manipulation of materials.

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[19] _Ibid._, p. 531.

[20] _Ibid._, p. 561.

[21] Maria Montessori, _The Secret of Childhood_ (New York: Ballantine Books, 1972), p. 81.

Bruner further described the child as moving through three levels of representation as he learns. The first level is the enactive level, where the child manipulates materials directly. The child then progrsses to the ikonic level, where he deals with mental images of objects but does not manipulate them directly. Finally, the child moves to the symbolic level, where he is strictly manipulating symbols and no longer mental images of objects.[22]

Bruner's thinking, which is influenced by the Gestalt psychologists and most recently by Piaget, is that the whole is greater than the sum of its parts and that the process cannot be taught didactically but must be undergone in toto. In other words, we must elicit from the learner things that he has always known and thus restructure his knowledge.[23]

In recent years Dienes, together with various groups of experimentalists, has built upon and extended the work of Piaget in investigating the learning style of children. Dienes' experiments have been and are being conducted in many parts of the world so that the effects of various cultures and economic conditions can be taken into account. There is already sufficient evidence to

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[22]Lee Shulman, "Psychology of Mathematics," Mathematics Education, Sixty-Ninth Yearbook (Chicago, Illinois: National Society for the Study of Education, 1970), p. 29.

[23]Ibid., pp. 51-52.

support the principle of learning by multiple embodiment. Multiple embodiment means that every concept should be presented in as many different ways as possible.[24] Most teachers will agree that children learn at different rates and that some will reach higher levels than others. It is equally necessary for one to understand that children learn in different ways. The more differences we provide in the various embodiments the more we are likely to meet the needs of every child.

The use of a multiplicity of embodiments has also proven to be effectual to long term memory. Further, it has been helpful in prohibiting closure. It has been noted that children tend to form closures on materials of limited variability, and so they "learn the materials" rather than acquiring the concepts.[25]

Recent research has focused attention on another "learning style" which is referred to as verbal interaction and mathematics learning. Research has implied that the verbal behavior of students and/or teachers influences the learning of mathematics. An important principle in the psychology of learning is that learning with awareness (insight or understanding) is more permanent than learning without awareness. In addition, it seems reasonable to

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[24] Z. P. Dienes and E. W. Golding, Approach to Modern Mathematics (unpublished boo, 1967), p. 27.

[25] Ibid., p. 27.

assume that requiring the learner to verbalize a mathematical concept or principle after he appears to understand it might increase his degree of awareness of that particular abstraction and help to fix it in his mind.[26]

Many investigators have reported positive effects of verbalization on learning mathematics at the elementary school level. For instance, in a study by Irish (1964), fourth-grade teachers spent part of the class time, that was usually spent on computation, helping students state generalizations about number problems. The result was that these students made greater yearly gains on the STEP Mathematics Test than other students in the system.[27]

Retzer (1969) found that teaching certain concepts of logic not only had differential effects on eight-graders abilities to verbalize mathematical generalizations, but that students with high verbalization abilities could better transfer learned mathematical generalizations. Furthermore, results of research on the relationship between language and cognition indicate that the process of linguistic encoding, as in giving something a name, improves both recognition and recall of that "thing."[28]

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[26]Lewis R. Aiken, "Language Factors in Learning Mathematics," ERIC Information Analysis Center (1972), p. 19.

[27]*Ibid.*, p. 20.

[28]*Ibid.*, p. 21.

Peter (1970), in a study of 131 kindergarten children of lower socioeconomic status, found verbal training to be significantly more effective than non-cued, visual-cued, or no training, when the criterion was immediate learning.

As is shown in the studies cited, mathematics is best learned through human activity. Thus each child must be an active participant in the process of forming concepts and developing intellectual skills, rather than a spectator.

The Teaching of Elementary School Mathematics

Research related to elementary school mathematics instruction has steadily improved; however, there is still a great need for further improvement. There are a number of identifiable teaching strategies, but at the present time, the majority of studies have contrasted *discovery* or *guided discovery* teaching with *didactic* or *expository* teaching. In addition, there is a growing amount of research concerned with a "laboratory" approach.

Researchers favoring discovery learning have suggested that it (1) is a natural and preferred way of learning for many, (2) builds motivation, (3) promotes better learning and retention, and (4) leads the learner to be a constructionist and thus avoids the information drift that fails to keep account of the uses to which information might be put.[29]

In the act of teaching anything, a teacher may exercise nearly total guidance over the learner's behavior or practically no guidance. In the former case, we generally speak of expository or didactic teaching. And in the latter instance, in spite of the multiple meanings of discovery, one most likely would be inclined to call this method discovery teaching. However, these two characterizations merely highlight the extreme ends of a continuum. Much instruction occupies intermediate points between them. Wittrock[30] has suggested that we characterize the degree of guidance in terms of whether the rule and the solution to the problem being taught are given. Table 1 depicts the four possibilities.

Table 1. Wittrock's four methods of teaching.

	Rule	Solution
Exposition	Give	Given
Guided Discovery (deductive)	Given	Not Given
Guided Discovery (inductive)	Not Given	Given
"Pure Discovery"	Not Given	Not Given

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[29] Robert Travers, ed., The Second Handbook of Research on Teaching (Chicago: Rand McNally Co., 1973), p. 1163.

[30] N. L. Gage, ed., Handbook of Research on Teaching (Chicago: Rand McNally and Co., 1963), p. 1159.

For Bruner, as well as Socrates and others, the focal point of student learning is a process called discovery. In discovery, the learner is confronted by problems. These problems may take the form of (1) goals to be achieved in the absence of readily discernible ways for achieving the goals, (2) contradictions among sources of information of apparently equal credibility, or (3) the quest for structure or symmetry in situations where such order is not readily apparent.[31]

The first step of discovery is a sensed incongruity or contrast. Bruner advocates building potential or emergent incongruity into the materials of instruction. Such contradictions are used to engage the child, because of the resulting intellectual discomfort, in an attempt to resolve this disequilibrium by making some new discovery in the form of a reorganization of his understanding, what the Gestalt psychologists would have called a "cognitive restructuring."[32]

Moreover, Bruner sees instruction as a roller-coaster ride of successive disequilibria and equilibria terminating in the attainment or discovery of a desired cognitive state.

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[31] Lee Shulman, "Psychology of Mathematics," Mathematics Education, Sixty-Ninth Yearbook (Chicago: National Society for the Study of Education, 1970), p. 51.

[32] Robert Travers, ed., The Second Handbook of Research on Teaching (Chicago: Rand McNally Co., 1973), p. 1117.

Many researchers have argued that discovery learning is more motivating. That is, children are more likely to be motivated when confronted with an enticing problem they cannot solve than when given some specific objectives to master on the promise that if they learn them well, at some indetermined future point they may be able to solve an exciting problem.

Edith Biggs (1969) contends that the child of the electronic age requires an educational environment that allows him maximum participation in discovery. The child needs to be provided with a multiplicity of stimuli that will spark his curiosity and engender a continuing desire to learn.

Worthen (1968) conducted an experiment with fifth and sixth grade students in which he compared a discovery-oriented approach and an expository presentation. Based on the evidence of the study, Worthen suggests that if pupils' abilities to retain mathematical concepts and to transfer the heuristics of problem-solving are valued outcomes of education, then discovery sequencing should be an integral part of the methodology used in teaching mathematics to elementary school pupils.

The concept of laboratory teaching, even though it is not new, has gained most of its popularity in recent years. There are numerous interpretations of the meaning of laboratory teaching, but it is generally agreed that the primary puspose of the laboratory idea is that students will discover and develop new concepts and understandings particularly well through experimental activities dealing with a variety of techniques and concrete models. The laboratory method, for instance, enables the students to observe the laws of geometry operating in concrete form before they are required to do logical abstract thinking.

The assumption of laboratory teaching is that the child will have first-hand experience in observing and manipulating the materials. Science assumes that this method is superior to other methods when the ultimate aim is to develop understanding and appreciation.[33]

From the standpoint of research, the activity of the student, the sensorimotor nature of the experience, and the individualization of laboratory instruction should contribute positively to learning. In addition to other advantages, research has shown laboratory teaching to be advantageous over other methods in the following ways: (1) greater ability to retain; (2) greater ability to apply learning; and (3) greater skills in observation and manipulation.[34]

Several decades ago, McLennan and Dewey initiated the idea of the mathematics laboratory. In keeping with

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[33]N. L. Gage, ed., Handbook of Research on Teaching (Chicago: Rand McNally and Co., 1963), p. 1144.

[34]*Ibid.*, p. 1144.

the theory "learning by doing," McLennan and Dewy emphasize the need to effectuate learning by providing a balance between the manipulation of concrete models and the symbolization of concepts.[35]

If the child is able to make discoveries and generalizations in quantitative situations by the use of symbols, he does not need manipulative models, but on the other hand, if the child is not able to deal understandingly with quantitative situations by use of symbols, he should use concrete models to discover relationships among quantitative situations.

Dienes (1967), E. Biggs (1969), R. B. Davis (1967), and others favor active programs which deal with discovery and laboratory teaching. Through research and observation, it is the premise that these methods of teaching provide more meaningful learning and produce a greater understanding of mathematical principles.

Teaching mathematics through the laboratory approach provides the learner with a multiplicity of alternatives. Learning, therefore, becomes more personalized. In the past, the teaching of mathematics has been more teacher-centered than student-centered. Table 2 contrasts these methods. The needs of the student in the affective and psychomotor

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[35]John Dewey and J. A. McLennan, Readings in the History of Mathematics Education (Washington, D.C.: National Council of Teachers of Mathematics, 1970).

Table 2. Comparison of student-centered and teacher-centered teaching methods.

Student-centered*	Teacher-centered
Goals	
Determined by the students	Determined by the teacher
Emphasis upon affective and attitudinal changes	Emphasis upon cognitive changes
Attempts to personalize	Emphasis on group instruction
Classroom Activities	
Much student participation	Much teacher participation
Student discovers for himself	Teacher discovers for student
Student manipulates objects	Teacher manipulates objects
Student decides upon own activities	Teacher determines activities
Student encouraged to verbalize	Student encouraged to listen
De-emphasis of tests and grades	Traditional use of tests and grades
Discussion of student's experiences encouraged	Discussion kept on course content

*Material in Table 2 was supported by Faw (1945), Mckeachie (1951), E. Biggs (1969), Glasser (1969), and others.

domains as well as the cognitive domain must encompass the teaching strategies in mathematics. From the thoughts of the Gestalt psychologists, it is believed that the considera- tion of the child as a total entity proves far more effective than viewing the child as separate entities.

Several other teaching strategies have been investigated with favorable outcomes. For instance, Wallach and Sprott (1964) provided first-grades with either no practice or practice in manipulation of objects to develop conservation of numbers. It was found that none of the no practice group achieved growth in conservation while fourteen of the fifteen trained children evidenced growth.

Crowder (1965) reported that a group of first-graders using the Cuisenaire Program learned more mathematical concepts and skills than pupils taught by a conventional program.[36] In a similar study, Lucus (1966) studied the use of Dienes' attricute blocks in the first grade and found that children taught with the attribute blocks showed greater ability to conserve cardinally and to conceptualize addition-subtraction relationships than those taught by a more conventional method.[37]

Palmer (1968) produced gains in children's ability to conserve numbers by producing cognitive conflict in two ways: (1) by expressing verbal surprise whenever a child responded as a conserver; and (2) by exposing non-conservers to the contradictory conclusion of their peers who were already conservers.[38]

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[36] Robert Travers, ed., The Second Handbook of Research on Teaching (Chicago: Rand McNally Co., 1973), p. 1157.

[37] Ibid.

[38] Ibid., p. 1152.

From the studies cited it is evident that children learn number, length, operations, and other concepts when teaching is based on natural-concrete experiences. It is through action that the mental operations are developed and coordinated.

The Elementary School Mathematics Curriculum

As the objectives of the elementary mathematics curriculum have been reevaluated, changed, broadened, and clarified, parallel changes in curriculum content have been taking place. Currently, various influences such as psychological, sociological, and mathematical have brought about many changes in the content of elementary school mathematics. Further, the inclusion of such topics as geometry and number theory points up the need to expand the content of the curriculum. Still another factor which is influencing the content of the curriculum is the inclusion of behavioral objectives which are sensitive to the affective and psychomotor domains as well as the cognitive domain.

In essence, these reforms are attempts to provide a more relevant curriculum, a curriculum which is consistent with the condition and pattern of experience of the learner.

Glennon and Callahan identified three major themes that can be the basis of the elementary school mathematics program: (1) The psychological basis for the curriculum. There are several psychological theories on which curriculum can be developed. One approach views curriculum selection on the basis of the development of cognitive learning by analysis of the nature of the learner and his development stages in learning in connection with the subject matter to be learned. Research conducted by Piaget, Brownell, Ausubel, Bruner, and Gagné is representative of this approach. (2) The sociological basis for the curriculum. In this approach, the content of the elementary school mathematics program would be the content most useful to adults in daily life and the adult world of business. (3) The logical or pure mathematics basis for the curriculum. This approach is concerned with building a mathematics hierarchy to show the relationship of different areas of mathematics.[39]

The rejuvenation of the elementary school mathematics curriculum has progressed through three phases. Initially, the basis for teaching mathematics was to provide the learner with mathematics which would prepare him for everyday living. This era encompassed the social basis for mathematics which extended through the forties. The second reformation in the mathematics curriculum occurred during the fifties and continued into the sixties.

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[39]Robert Travers, ed., The Second Handbook of Research on Teaching (Chicago: Rand McNally Co., 1970), p. 1155.

The primary emphasis during this phase was on the mathematical basis. The mathematical phase was greatly influenced by the launching of the first Russian satellite (age of Sputnik). The third phase of the elementary school mathematics curriculum was concerned primarily with the psychological basis. This era of change began during the late sixties and extended into the seventies.

At the present time, as indicated by research and theory, a concerted effort is being made to create a balance among the sociological, mathematical, and psychological bases for the elementary school mathematics curriculum. There seems to be general agreement that a curriculum is more relevant when it is constructed around these three factors.

Flournoy cogently stressed the importance of building the curriculum content around the psychological, sociological, and mathematical parameters. Flournoy viewed the psychological, sociological, and mathematical influences as follows:

The Psychological Influence: It is essential that the principles of child development and concept formation be considered in the determination and organization of the mathematics curriculum.

Another psychological consideration in the organization and determination of the mathematics curriculum is the fact that there are various degrees of understanding in the learning of any concept or skill. Hence, the learning of a topic should be spread over several grades rather than taught for mastery at one grade or level. For instance, it would not be feasible to try to teach the topic of numeration at one particular grade level; however, it is appropriate to teach certain objectives (from the topic) for mastery. Adherence to this principle has resulted in what is termed a "spiral" plan of organization.[40]

Bruner advocates a curriculum of spiral concepts around which factual information becomes a means of understanding concepts rather than an end in itself.[41]

The Sociological Influence: The social purpose of arithmetic is concerned with the pupil's learning those arithmetic concepts and skills which are useful in his immediate and future daily life. The learner is made sensitive to numbers in social situations; he learns to use arithmetic to solve the quantitative problems of daily life. The learner also comes to appreciate the way in which quantitative ideas are needed and used by the society in which he lives. Furthermore, it is important that a quantitative social situation be used frequently as a springboard to the

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[40]Frances Flournoy, Elementary School Mathematics (New York: The Center for Applied Research in Education, Inc., 1968), pp. 1-9.

[41]J. S. Bruner, The Process of Education (New York: 1960), p. 33.

introduction of a mathematical concept.[42]

The Mathematical Influnce: The content of the arithmetic program should be organized and presented in accordance with the sequential nature of the subject. This means that the content should be organized according to related skills and principles.

In the organization of the modern arithmetic programs at all levels, the attempt is being made to present the basic topics of mathematics so that unified ideas are stressed. The unifying of ideas should aid the learner to see mathematics not as a series of unrelated topics but as a systematic whole.[43]

Further, the mathematical aim of the curriculum is to develop in the learner a taste and inclination for the subject, flexibility in thinking, intellectual curiosity and independence, an attitude of discovery, logical thinking, and the ability to analyze and to generalize.[44]

Research in curriculum development in elementary school mathematics has been primarily concerned with four content areas:[45]

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[42]Frances Flournoy, Elementary School Mathematics (New York: The Center for Applied Research in Education, Inc., 1968), p. 6.

[43]Ibid., pp. 8-9.

[44]Ibid., pp. 1-2.

[45]Robert Travers, ed., The Second Handbook of Research on Teaching (Chicago: Rand McNally Co., 1973), p. 1155.

1. Pre-number experience

   Serial correspondence
   Discontinuous quantity
   One-to-one correspondence

2. Whole numbers

   Principles of addition and subtraction
   Place value
   Multiplication of whole numbers

3. Fractions

   Understanding halves, thirds, fourths...
   Finding fractional parts of a set
   Addition and subtraction of fractions

4. Problem-solving

   Ability to express relationships as arithmetical sentences
   Ability to perform the four number operations
   Ability to discern the appropriate operation

In the reformation of elementary school mathematics, topics such as geometry, number theory, and probability theory are no longer considered primarily as college and pre-college courses. These topics are steadily finding a place in the elementary school mathematics curriculum. Several reasons why these topics are picking up momentum in the elementary school mathematics curriculum are that they are ideally suited in inquiry, informal exploration, and divergent thinking.

Moreover, the inclusion of geometry, number theory, and probability theory in the elementary school mathematics curriculum allows the child to make elementary observations which promote a rudimentary understanding of more sophisticated mathematical concepts and principles.

In spite of the past curriculum trends in elementary school mathematics, the inclusion of such topics as geometry, probability theory, and number theory in the content of the elementary mathematics curriculum is a clear attempt to alter past practice.

Several justifications have been given for the inclusion of geometry into the arithmetic content. The following are some of these justifications:

> It is evident that the child will learn by manipulating his environment, and geometry can provide the vehicle for this manipulation. Geometry provides unlimited opportunity for tangible and visual experiences which make things more interesting to students and thus increase motivation.[46,47]

> Geometry provides a rich area for an integrated study of abstract mathematics and the environment.[48]

> Geometry provides a sound mathematical background for children; many topics which are treated in the intermediate grades and secondary grades lend themselves to geometric interpretation.[49]

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[46] James E. Inskeep, Jr., "Primary-Grade Instruction in Geometry--Readings in Geometry," Arithmetic Teacher, 1970, 83.

[47] Ibid., p. 87.

[48] Report of the Cambridge Conference on School Mathematics (Boston: Houghton Mifflin Co., 1963), p. 33.

[49] James E. Inskeep, Jr., op. cit., p. 85.

Some justifications for including number theory in the content of the elementary mathematics curriculum are the following:

> The concepts and principles of number theory are ideally suited to inquiry that facilitates progression to higher levels of abstraction.[50]

> Number theory enables the students to make elementary observations which promote a rudimentary understanding of the natural numbers.[51]

Much of the content in today's mathematics programs is structured around behavioral objectives. A behavioral objective tells what the learner is to do, how he is to do it, and to what degree of success.[52] Sound behaviorally stated objectives are sensitive to the cognitive, affective, and psychomotor domains.[53] See Table 3 for examples of each domain. Behavioral objectives describe the methodology to be used in the implementation of the content of the curriculum.

Research cited indicates that the current reconstruction in the elementary school mathematics curriculum is sensitive to the sociological, psychological, and

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[50] C. W. Schminke and Norbert Maertens, Teaching the Child Mathematics (Hinsdale, Illinois: The Dryden Press, Inc., 1973), p. 410.

[51] Ibid.

[52] Educational Accountability (Lansing, Michigan: Michigan Department of Education, 1972).

[53] Ibid.

mathematical influences; these three factors are viewed as interdependent variables in selecting and organizing the content of the elementary school mathematics curriculum. Also, the inclusion of elementary content from the fields of number theory, geometry, and probability theory points up a profound change in the elementary curriculum. Further, the rejuvenated curriculum is concerned with behavioral objectives which reflect the affective, psychomotor, and cognitive domains.

Table 3. Components of a behaviorally stated objective.

Cognitive Domain Examples	Affective Domain Examples	Psychomotor Domain Examples
knowledge analysis comprehension application	value respond aware	manipulation articulation imitation precision

Summary

It appears from the literature reviewed in this chapter that:

1. Emphasis should be placed on meaning, understanding, and discovery rather than on rote learning of specific processes.

2. The creation of a positive attitude toward mathematics will enhance achievement in mathematics.

3. Concepts are learned better and retention is improved when the child has a chance to learn through the manipulation of physical models.

4. Personal involvement and learning are inseparable.

5. Inductive learning should precede deductive learning whenever possible.

6. Sensory learning is an essential part of all learning.

7. If there is a gap in the child's conceptual foundation in elementary mathematics, he will have difficulty performing the skill and problem solving tasks.

8. Multisensory models tend to increase interest and promote the development of favorable attitudes.

9. Discovery should precede the internalization of new facts, concepts, or principles.

10. The role of the teacher is that of a director; he directs the psychic activity of the child.

11. Instructional experiences should be carefully sequenced so that any unit taught is clearly related to those preceding it.

12. The content of the curriculum should reflect the affective and psychomotor domains as well as the cognitive domain.

13. The school program must always represent a compromise between ideas of teaching and human nature.

14. The effective curriculum is one which capitalizes upon the everyday needs of the children being served by it.

CHAPTER III

DESIGN OF THE STUDY

Introduction

This chapter contains a discussion of the selection and identification of the sample population, the procedure for procuring the data, the test instruments, and the statistical analysis used to test the hypotheses.

Description of the Sample Population

The sample for the study involved six classes randomly selected from the fourth, fifth, and sixth grades during the fall of 1973 at Morley Elementary School located in Saginaw, Michigan. Saginaw is an industrial city with a population of approximately 125,000 people.

Morley Elementary School is one of the eleven inner city schools in the city. There is a total of twenty-nine elementary schools in Saginaw. Morley Elementary School was chosen for the study because of its size and diverse population (socioeconomic status). Approximately one-third of the 477 students at Morley Elementary School are welfare recipients. The remaining two-thirds of the students are classified as children from predominantly black, middle socioeconomic class families.

Of the 235 fourth, fifth, and sixth grade students in the school, 171 were involved in the study. All of the students were third world (minority) persons with the exception of two students. The students were not grouped according to their mathematics ability; however, all students at the school were grouped according to their reading levels.

Seventeen pupils were eliminated from the study because they had moved or were absent during the post-testing. The final sample consisted of 154 subjects (75 girls and 79 boys) ages 9 through 12 years.

Six teachers participated in the study. Five of the teachers were female, and one was male. The average number of years of teaching experience for the six teachers was four years. All of the participating teachers taught mathematics and science during the regular school year.

Procedure for Data Procurement

The study consisted of three experimental units and three control units (see Table 4). The classes were randomly assigned to the experimental and control units. The random selection of the classes determined the assignment of the six teachers involved in the study.

Prior to the study, thirty students not included in the study, from grades four, five, and six at Morley Elementary School, were administered a pre-test. The pre-test

Table 4. The units for the experimental and control groups.

Grade	E	C
4	1 classroom	1 classroom
5	1 classroom	1 classroom
6	1 classroom	1 classroom

included fourteen items which were constructed from the four objectives used in the study (Table 5). A computer item analysis was compiled for the test. Table 6 reports the item analysis. The reliability of the pre-test was .74.

Table 5. The objectives used in the study.

Objectives	Number of Items per Objective
The student is able to	
1. name the parts of a region	6
2. identify portions of a set to represent a given fraction	3
3. identify the largest and smallest fraction	5
4. name fractions equivalent to a given shaded region	4

The Hoyt Procedure with the Fortap Computer Program was used to compute the reliability. Some revisions included the omission and rewriting of items which appeared either too easy or too difficult. Additional items were added to increase the reliability of the test. The pre-test also provided information as to the overall appropriateness of the test for the subjects of the sample, and the pre-test further indicated the time allowance for testing.

Each teacher was instructed to teach the same four objectives as shown in Table 5. The four objectives dealt

Table 6. The difficulty level and the discrimination index for the item analysis of the pre-test.

Item Number	Difficulty Level	Discrimination Index
1	.48	.78
2	.33	.44
3	.70	.77
4	.70	.83
5	.52	.69
6	.42	.68
7	.55	.59
8	.57	.40
9	.54	-.34
10	.39	.48
11	.48	.65
12	.33	.09
13	.39	.25
14	.09	.44

exclusively with developing the concept of fractions. The study was not concerned with computational or problem solving skills. For instance, the students were not expected to compute the sum or product of fractional numerals. Each teacher involved in the study was given instructions in the assigned approach (traditional or multisensory approach) and further assistance was available as required.

Three in-service training sessions were held prior to the beginning of the experiment. These sessions were conducted by the researcher, who is presently a mathematics specialist for the Saginaw School System. Both teachers of the control groups and the treatment groups were involved in the three in-service training sessions. However, the teachers of the treatment groups had separate in-service sessions. During the in-service training for the treatment groups, the teachers were given the objectives along with methods and strategies to be employed in the multisensory approach. These teachers were also provided with specific training in the utilization of the Cuisinaire Rods and the fractional parts. The teachers of the control groups were provided with the objectives (same objectives as the treatment groups) along with various methods of teaching the objectives. These teachers were also provided with materials, in addition to the textbooks, such as tapes, dittos, and worksheets.

The teachers of the experimental units were instructed to use the Cuisenaire Rods and magnetic fractional parts as their basic teaching materials. It was stressed that the students were to be the prime manipulators of the objects. The teachers of the experimental units were also allowed to use the textbook, dittos, workbooks, and the blackboard. The teachers of the control units were instructed not to use any manipulative materials. These teachers were told to use the textbook, workbooks, dittos, tapes, and the blackboard as their basic teaching materials.

The mathematics achievement test (pre-test), which is discussed in the section on test instruments, was administered to all of the students involved in the study. Teaching began on the day after the pre-test was administered. The teaching was conducted for fifteen, fifty-minute sessions. The post-test, which was the same as the pre-test, was administered the following day after the last teaching session.

To further invetigate the study, a non-paper-and-pencil test was administered to thirty students. These subjects were randomly chosen from the sample. Five students were randomly chosen from each of the six units. The non-paper-and-pencil test was administered within three days after the post-test. Each of the thirty students was individually administered the non-paper-and-pencil test.

Test Instruments

An achievement test, designed by the researcher, was constructed to measure the objectives of the study. An achievement test is a test that measures the extent to which a person has acquired certain information or mastered certain skills, usually as a result of specific instruction.[1] Numerous published tests were reviewed and found to be inappropriate for the sampling of the objectives of the study.

The achievement test contained 18 items. Seven of the test items were multiple choice, 11 items were completion. The objective-type items were used to obtain more objective information about the achievement of the students and because of the ease of scoring.

The reliability of the achievement test, obtained by using the Hoyt Procedure with the Fortap Computer Program, was .80. Since the test was constructed from the objectives of the study, the validity of the test was content and face validity.

A non-paper-and-pencil test (manipulative) was designed by the researcher and validated by a panel of mathematics experts (mathematics specialists, mathematics

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[1]William A. Mehrens and Irvin J. Lehmann, Standardized Tests in Education (New York: Holt, Rinehart and Winston, 1969), p. 299.

teachers, and mathematics professors). The mathematics experts reviewed the test and made suggestions. The test contained five items.

Scoring

The eighteen-item achievement test, which was administered to the total sample, was scored by giving one point for each correct response. The manipulative test was also scored by giving one point for each correct response. The basic procedure was to compute each score by subtracting the wrong response from the total items on the test.

Time Allowance

The achievement test and the manipulative test were administered as a power test; hence, the subjects had as much time as needed to complete the test.

Method of Statistical Analysis

The hypotheses of this study were tested by means of the analysis of variance; the analysis of covariance was also used to test the hypothesis. The design used for this study was the "two-way analysis of variance" (2 X 3 design). This design (factorial design developed by Fisher) allows the researcher to investigate the interaction of variables. In the factorial design, two or more independent variables can be manipulated simultaneously.[2]

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[2] D. V. Dalen and W. J. Meyer, Understanding Educational Research (New York: McGraw-Hill Book Co., 1966), pp. 269-270.

The ANOVA Tables were used to report the results and analyze the data gathered in the experiment. The ANOVA Table contains five elements for analyzing the data: (1) source of variation; (2) degrees of freedom; (3) formulas for the sums of squares; (4) mean squares; and (5) expected values of mean squares. In addition, the ANCOVA (repeated measure design) Tables were used to report the results of the data. The ANCOVA Table also contains five elements for analyzing the data: (1) source of variation; (2) degrees of freedom; (3) mean square; (4) F-ratio; and (5) probability.

This study involved four computer programs: (1) the first program was the analysis of variance of the achievement test; (2) the second program was the analysis of covariance of the achievement test; (3) the third program was the analysis of covariance for the welfare and non-welfare recipients; and (4) the fourth program was the analysis of variance for the manipulative test.

CHAPTER IV

ANALYSIS OF THE DATA

Introduction

The purpose of this study was to investigate the effects of teaching a selected mathematical concept via the multisensory approach. The concept of fractions was arbitrarily chosen for this study. Data from the administration of the specially designed pre-test and post-test were obtained from the 154 subjects in grades four, five, and six at Morley Elementary School located in Saginaw, Michigan.

The statistical method employed in this study was the "two-way analysis of variance" (2 x 3 design). In addition to the analysis of variance, the analysis of covariance and the repeated measure design were used to further analyze the data.

The results of this analysis are reported in the following progression. Part one presents (1) a restatement of the major hypothesis and the subordinate hypotheses, (2) the tables for the analysis of gain scores, and (3) the tables for the analysis of covariance and the repeated measure design. Part two reports the means and the ANCOVA

Table for the welfare and non-welfare recipients. Part three reports the means and the ANOVA Table for the manipulative test. Finally, part four discusses the conclusive findings of this study.

The Major Null Hypothesis

The major null hypothesis upon which this study was based states that:

> $H_0 1$: There will be no difference in the scores on the achievement tests of students who were taught a selected mathematical concept via the multisensory approach and students taught via the traditional approach.

The Subordinate Null Hypotheses

The following null hypotheses were also tested. (Hypotheses were taken from related questions posed in Chapter I.)

> $H_0 2$: There will be no difference on the means of gain scores among grades four, five, and six.

> $H_0 3$: There will be no difference on the mean of gain scores between male and female.

Table 7 indicates the mean of the gain scores for the treatment groups and the control groups. The average mean of the gain scores for the boys in the treatment groups was 7.16, and the average mean of the gain score for the girls in the treatment groups was 7.35. The average mean of the gain scores for the boys in the control groups was 4.85, and the average mean of the gain score for

Table 7. The mean of gain scores on the achievement test.

Grade	Treatment		Control	
	Boys	Girls	Boys	Girls
4	7.50	8.19	4.09	5.58
5	8.09	7.29	5.70	7.11
6	5.88	6.56	4.75	5.60

the girls was 6.09. The average mean of the gain scores for the girls in the control was 1.24 higher than the average mean of the gain scores for the boys in the control groups. The overall (across groups) mean of the gain scores for the girls was 4.32 higher than the boys. The maximum gain score was 8.29, which corresponded to the fourth grade girls in the treatment group. The minimum gain score was 4.09, which corresponded to the boys in the control group in the fourth grade. The range of the gain scores for the treatment groups was 5.88 to 8.19, whereas the range of the gain scores for the control groups was 4.09 to 7.11.

The analysis of variance of the gain scores is summarized in Table 8. The following null hypotheses were tested:

$H_0 1$: There will be no difference between the treatment and control groups. (major hypothesis)

$H_0 2$: There will be no difference on the mean of the gain scores among grades four, five, and six.

$H_0 3$: There will be no difference on the mean of the gain scores between male and female.

Table 8. ANOVA of gain scores on the achievement test.

Source of Variation	df	Sum of Squares	Mean Square	F
Sex	1	3.11	3.11	5.654
Grade	2	0.61	0.30	0.545
Treatment	1	1.68	1.68	3.054
Error	2	1.10	0.55	
Total	6	6.50		

The F-value needs to be significant at .05. Hence, the F-value will be statistically significant at F>18.51 for treatment and sex, and the F-value for the grade will be statistically significant at F>19.00. As shown in Table 8, the F = 5.654 for variable S is not statistically significant; likewise, the F = 3.054 for variable T is not statistically significant (for 1 df, F must be greater than 18.51). The F = 0.545 for variable G is not statistically significant (for 2 df, F must be greater than 19.00). Because of the small sample size (six classrooms were employed in this study) the error term was large (1.10). The large error term was due to the limitation of the analysis of variance to compensate for a lack of original equivalency between groups in a small sample. Since there was only one observation per cell, the error contained the true error as well as the interaction (uncontrolled variation).

The analysis of variance of the achievement test yielded the following results:

$H_0 1$: There will be no difference between the treatment and control groups. (major hypothesis)
(Accepted)

$H_0 2$: There will be no difference on the mean of the gain scores among grades four, five, and six.
(Accepted)

$H_0 3$: There will be no difference on the mean of gain scores between male and female.
(Accepted)

The data will be further tested by employing the analysis of covariance. This procedure enables the researcher to control variation in the treatment and control groups. The use of this technique involves certain assumptions such as that of homogeneity and independency. For this portion of the analysis the students will be considered as a unit; thus, some of the assumptions of dependency will be violated. Table 9 reports the cell frequencies, means, and cell standard deviations.

The mean average of the pre-test for the treatment groups was 4.98; the mean average of the post-test for the treatment groups was 12.33. The mean gain for the treatment groups was 7.35. The mean average of the pre-test for the control groups was 3.45; the mean average of the post-test for the control groups was 8.92. The mean gain for the control groups was 5.47. Both the treatment and control groups showed gain; however, the gain for the treatment groups was substantially higher (1.88) than the control groups.

Table 9. Table of cell frequencies, means and cell standard deviation of the achievement test.

		Grade 4			Grade 5			Grade 6	
Treatment									
Boys		N = 12			N = 11			N = 16	
		Pre	Post		Pre	Post		Pre	Post
	X =	3.42	10.92	X =	3.64	11.73	X =	7.87	13.75
	S.D.	1.92	3.90	S.D.	3.11	2.97	S.D.	5.34	3.62
Girls		N = 16			N = 14			N = 9	
		Pre	Post		Pre	Post		Pre	Post
	X =	3.12	11.31	X =	4.13	11.43	X =	7.67	14.22
	S.D.	0.71	2.36	S.D.	3.01	2.47	S.D.	3.32	2.77
Control									
Boys		N = 11			N = 17			N = 12	
		Pre	Post		Pre	Post		Pre	Post
	X =	1.54	5.56	X =	4.53	10.24	X =	4.59	9.33
	S.D.	0.93	2.25	S.D.	2.35	1.64	S.D.	3.73	3.11
Girls		N = 12			N = 9			N = 15	
		Pre	Post		Pre	Post		Pre	Post
	X =	2.25	7.83	X =	2.78	9.89	X =	5.00	10.60
	S.D.	1.05	3.46	S.D.	1.71	1.76	S.D.	3.80	2.64

The mean average (across groups) on the post-test for the boys was 10.27; the mean average (across groups) on the post-test for the girls was 10.88. The difference (.61) between the average gain mean of the boys and girls was not significant.

Table 10 summarizes the analysis of covariance. The following null hypotheses were tested:

$H_0 1$: There will be no difference among grades four, five, and six.

$H_0 2$: There will be no difference between the treatment and control groups.

$H_0 3$: There will be no difference between male and female.

$H_0 4$: There will be no interaction between grade and treatment.

$H_0 5$: There will be no interaction between grade and sex.

$H_0 6$: There will be no interaction between treatment and sex.

$H_0 7$: There will be no interaction among grade, treatment, and sex.

Table 10. Analysis of covariance of the achievement test.

Sources of Variation	df	Mean Square	F	P
Grade	2	24.6939	3.6242	.0292
Treatment	1	262.7682	38.5649	.0001
Sex	1	11.5274	1.6918	.1955
G X T	2	20.3853	2.9918	.0534
G X S	2	5.27	.7741	.4631
T X S	1	7.9789	1.1710	.2811
G X T X S	2	.6338	.0937	.9106
Error	141	6.814		

[a]P is significant at $\alpha = .05$.

[b]P is highly significant at $\alpha = .01$.

The F-Test is statistically significant for $H_0 1$ and $H_0 2$. Null Hypothesis 1 was statistically significant at $P = .0292$ with $F = 3.6242$ and 2 df. Null Hypothesis 2 was statistically significant at $P = .0001$ with $F = 38.5649$ and 1 df. There was no significant difference between sex; between grade and treatment; between treatment and sex; and among grade, treatment, and sex. The test of the null hypotheses yielded the following results:

$H_0 1$: There will be no difference among grades four, five, and six.
(Rejected)

$H_0 2$: There will be no difference between the treatment and control groups.
(Rejected)

$H_0 3$: There will be no difference between male and female.
(Accepted)

$H_0 4$: There will be no interaction between grade and treatment.
(Accepted)

$H_0 5$: There will be no interaction between grade and sex.
(Accepted)

$H_0 6$: There will be no interaction between treatment and sex.
(Accepted)

$H_0 7$: There will be no interaction among grade, treatment, and sex.
(Accepted)

The F-Test shows that there is a highly significant difference ($P = .0001$ and $F = 38.5649$ with 1 df). Hence, the multisensory appraoch was superior to the traditional approach.

Table 11 summarizes the results of the repeated measure test.

Table 11. The repeated measure design for the achievement test.

Sources of Variation	df	Mean Square	F	P
Grade	2	258.587	22.045	.0001
Treatment	1	419.893	35.796	.0001
Sex	1	0.368	0.031	.8597
G X T	2	54.483	4.645	.0112
G X S	2	9.3484	0.797	.453
T X S	1	2.075	0.1769	.675
G X T X S	2	10.632	0.9064	.4064
Subj.: G X T X S	142	11.730		
Gains (R)	1	3111.82	600.67	.0001
R X G	2	10.59	2.045	.1333
R X T	1	60.781	11.732	.0009
R X S	1	10.486	2.024	.1571
R X G X T	2	7.136	1.377	.2556
R X G X S	2	0.9552	0.184	.832
R X T X S	1	5.091	0.983	.323
R X T X G X S	2	1.6591	0.320	.7265
R X Subj: G X T X S	142	5.180		

The repeated measure design (Table 11) was used to test the following null hypotheses:

$H_0 1$: There will be no difference on the mean of the pre-achievement test and the post-achievement test.

$H_0 2$: There will be no difference among grades four, five, and six on the gain score.

$H_0 3$: There will be no difference between the treatment and control groups on the gain score. (major hypothesis)

$H_0 4$: There will be no difference between male and female on the gain score.

$H_0 5$: There will be no interaction on the gain scores between grade and treatment.

$H_0 6$: There will be no interaction on the gain scores between grade and sex.

$H_0 7$: There will be no interaction on the gain scores between treatment and sex.

$H_0 8$: There will be no interaction on the gain scores among grade, treatment, and sex.

In Table 11 the Repeated Measure Test reports a highly significant difference ($P = .0001$) on the gain scores (post-test scores). The probability for the repeated measure over treatment was highly significant at $P = .0009$. There was no significant difference in repeated measure by grade; sex; grade and treatment; grade and sex; treatment and sex; and grade, treatment, and sex.

The analysis of the repeated measure design yielded the following results:

$H_0 1$: There will be no difference on the mean of the pre-achievement test and the post-achievement test. (Rejected)

$H_0 2$: There will be no difference among grades four, five, and six on the gain scores. (Accepted)

$H_0 3$: There will be no difference between the treatment and control groups on the gain scores. (Rejected)

$H_0 4$: There will be no difference between male and female on the gain scores. (Accepted)

$H_0 5$: There will be no interaction on the gain scores between grade and treatment. (Accepted)

$H_0 6$: There will be no interaction on the gain scores between grade and sex. (Accepted)

$H_0 7$: There will be no interaction on the gain scores between treatment and sex. (Accepted)

$H_0 8$: There will be no interaction on the gain scores among grade, treatment, and sex. (Accepted)

It has been argued that students who are welfare recipients tend to achieve at a lesser rate than non-welfare recipients. To ascertain if there were differences in the achievement of the welfare recipients and non-welfare recipients, the mean of the pre-test and the post-test was reported, and the analysis of the gain scores was performed for both groups. The following null hypothesis was used to test the data:

$H_0 1$: There will be no difference between the mean scores of the welfare and non-welfare recipients.

Table 12 reports the means of the pre-test and the post-test for the welfare and non-welfare recipients.

As shown by Table 12, both groups made a substantial gain. The gain of the welfare recipients was 4.32; the gain of the non-welfare recipients was 4.58. The difference between the mean gains was .26, which was not substantial.

Table 12. The mean of the pre-test and the post-test of the welfare and non-welfare recipients.

	Pre-test	Post-test	N
Welfare Recipients	2.79	7.11	52
Non-Welfare Recipients	3.15	7.73	102

Table 13 summarizes the analysis of covariance of the gain scores.

Table 13. Summary of the ANCOVA of gain scores of the welfare and non-welfare recipients.

Source of Variation	df	Mean Square	F	P
Between Groups	1	2.2943	.4101	.5229
Within Groups	152	5.595		

P is significant at .05.

The results from Table 13 indicate that $H_0 1$ was not statistically significant at P = .5229 with F = .4101 and 1 df. Hence, the performance of the non-welfare recipients was not superior to the performance of the welfare recipients. The test of $H_0 1$ revealed the following result:

$H_0 1$: There will be no difference between the mean scores of the welfare and non-welfare recipients. (Accepted)

The manipulative test was constructed and administered in order to further verify this study. The data from the manipulative test were analyzed by reporting the mean scores and the ANOVA Table.

Table 14. Table of cell means of the manipulative test.

	Treatment	Control
4	4.20	1.80
5	4.80	2.40
6	4.80	1.40

The average mean score of the treatment groups was 4.80, wherease the average mean score of the control groups was 1.87. The difference between the average mean of the treatment groups and the control groups was 2.93; therefore, the achievement of the treatment groups was superior to the achievement of the control groups. The minimum mean score was 1.40, which corresponded to the control group in grade six. The maximum mean score was 4.80, which correponded to the treatment groups in grades five and six.

Table 15 summarizes the analysis of variance for the manipulative test. The following null hypotheses were tested:

$H_0 1$: There will be no difference among grades four, five, and six on the manipulative test.

$H_0 2$: There will be no difference between treatment and control groups on the manipulative test.

$H_0 3$: There will be no interaction between grade and treatment on the manipulative test.

Table 15. ANOVA table for the manipulative test.

Source of Variation	df	Sum Square	Mean Square	F
Grade	2	0	0	0
Treatment	1	73	73	109
Interaction	2	2	1	1.49
Error	24	16	.67	
Total	29	91	2.27	

The F-value needs to be significant at .05. Hence, the F-value will be statistically significant at F>3.40 for the grade and interaction, and the F-value for the treatment will be statistically significant at F>4.26. As shown in Table 15, F = 0 for variable G is not statistically significant (for 2 df F must be greater than 3.40). Likewise, F = 1.49 for I is not statistically significant. However, F = 109 for T is statistically significant (for 1 df F must be greater than 4.26).

The analysis of the data yielded the following results:

$H_0 1$: There will be no difference among grades four, five, and six on the manipulative test. (Accepted)

$H_0 2$: There will be no difference between the treatment and control groups on the manipulative test. (Rejected)

$H_0 3$: There will be no interaction between grade and treatment on the manipulative test. (Accepted)

A summary of the findings in this chapter is as follows:

1. There was no difference between the treatment and the control groups as shown by the analysis of variance.
2. There was a significant difference between the treatment and control groups as shown by the analysis of covariance.
3. There was no difference in the achievement of the welfare and non-welfare recipients as shown by analysis of covariance.
4. There was a significant difference on the manipulative test between the treatment and control groups as shown by the analysis of variance.

In general, the findings indicated that the multisensory approach to teaching mathematics was superior to the traditional approach.

CHAPTER V

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Introduction

The major concern of the investigation of this study was to determine if there was a significant difference in the achievement of students who were taught a selected mathematical concept by the multisensory approach as compared with the achievement of students taught by the traditional approach. In addition, several other concerns thought to be relevant to this study and further research were investigated.

The analysis of variance results indicated no significant difference between the treatment groups and the control groups. The statistically non-significant results can, at least in part, be attributed to the limitations of the analysis of variance to differentiate between the true error and the interaction (uncontrolled variables such as communication among students and variability among teachers), when the sample size is small (S = 6 for this study). The class was used as a unit; there was only one observation per cell. In Chapter 4, examination of Table 8 reveals an error term of 1.10, which is quite large.

Although the analysis of variance showed no significant difference between the treatment and the control groups, the mean average on the achievement post-test for the treatment groups was 1.29 higher than the mean average of the control groups; however, both groups showed a substantial gain (see Table 9). The analysis of variance also showed no significant difference among grades and no significant difference between male and female.

The analysis of covariance results revealed a highly significant difference between the treatment and the control groups. In employing the analysis of covariance, the students were used as a unit rather than the class. The ANCOVA method allowed for a large sample size (S = 154). It also yielded a significant difference among the grades (P = .0292 as indicated in Table 10). However, there was no significant difference between male and female as indicated in Table 10. The mean of the post-test results for the treatment group was signficantly higher (1.88) than the mean of the control group. In fact, each treatment subgroup (Grades 4, 5, and 6) was higher than the corresponding control group.

The repeated measure design for the pre and post achievement test yielded a highly significant difference (P = .0001, see Table 11) on the gain scores. This indicated that the treatment group was superior to the control group. The probability (P = .0009) for the repeated measure over treatment was also highly significant as shown in Table 11. However, there was no significant difference in the repeated measure by grade; sex; grade and treatment; grade and sex; treatment and sex; and grade, treatment, and sex.

The analysis of covariance was used to analyze the pre and post-test scores of the welfare and non-welfare recipients. There was no significant difference (see Table 13) between the performance of the welfare and non-welfare recipients.

The analysis of variance results for the manipulative test revealed that the treatment groups were highly superior to the control groups as indicated by the F-score in Table 15.

The analysis of variance yielded no significant differences among the grades.

In essence, the findings of this study supported the view that the teaching of elementary school mathematics is more effective when the learner is actively and physically involved in the learning process rather than assuming the role of a spectator. Hence, the results of this study favored the utilization of multisensory models in the teaching and learning of elementary school mathematics. However, the findings of this study are tenuous and will need to be further investigated.

Findings

1. The mean performance of the students in the treatment group was superior to the mean performance of the students in the control group.
2. There was no significant difference among grades four, five, and six.
3. The achievement of the boys was not significantly different from the achievement of the girls.
4. The achievement of the welfare recipients was not significantly different from the achievement of the non-welfare recipients.
5. There was no interaction between grade and treatment.
6. There was no interaction between grade and sex.
7. There was no interaction between treatment and sex.
8. There was no interaction among grade, treatment, and sex.

Conclusions

The results of the analysis, the sample employed, and the methodology of the study led to the following conclusions:

1. Since the achievement of the students taught by the multisensory approach was significantly higher than the achievement of the students taught by the traditional approach, it seems reasonable to conclude that the use of multisensory models in

learning activites increases meaning and understanding.

2. In-service training seems to be a sufficient means of preparing teachers to implement an instructional program that utilizes multisensory materials.
3. The data indicated that both the treatment and control groups made a substantial gain; hence, it can be presumed that the explicit, sequential objectives and the procedures provided the teachers were factors which accounted for the considerable gain in both groups.
4. Since there was no interaction between the grades and treatments, one can assume that the teachers adhered to the teaching procedures as instructed in the in-service training.
5. The data suggested that grades four, five, and six were fairly equal in their ability to grasp this concept, since there was no significant difference among the grades.
6. The program utilized in this study appears to be an effective means for providing for basic learning skills of individual children as well as groups of children.
7. From the data, it cannot be concluded that sex is a determinant in the learning of fraction concepts when multisensory instruction or traditional

instruction is used.

8. In can be inferred that any student, regardless of socioeconomic background, if given an appropriate learning environment, can evidence growth as was indicated by the parallel performance of the welfare and non-welfare recipients.

Recommendations for Research

These recommendations are based upon studies in the field of elementary school mathematics, related theoretical literature, observations of the researcher, and the findings of this study.

1. There is a need for this study to be duplicated to further verify the findings.
2. Several similar studies involving a larger sample should be conducted.
3. The multisensory approach needs to be examined in the teaching of other mathematical concepts and skills.
4. A program utilizing multisensory models needs to be researched at other grade levels, such as the early elementary grades.
5. More research on appropriate teaching method versus the didactic method of teaching is needed.
6. It would be beneficial to know whether the attributes of the multisensory models increased the

students' interest and in turn enhanced learning (affective domain) or whether the students' ability to feel, touch, and manipulate the models accounted for the increase in achievement (psychomotor domain).

Recommendations for Practice

1. The results of this study support the posture that the school systems should provide opportunities for in-service training for their classroom teachers. It is evident that the teachers, students, and school system as a whole would benefit from the participation of the classroom teachers in in-service programs designed to provide teachers with specific methodology and techniques in the multi-sensory approach. Further, since building principals and other administrators influence to a greater or lesser degree what goes on in the classroom, it seems logical that they should participate in the in-service training.
2. Colleges and universities will need to revamp their programs to provide the pre-service teachers with greater training in the utilization of the multi-sensory approach.
3. The traditional classroom must be reorganized to accommodate an environment which provides a wealth of manipulative models, the discovery approach,

and learning centers.

4. The elementary school mathematics curriculum should focus more attention on the psychomotor domain. For instance, more behavioral objectives in mathematics which require manipulative skills must become part of the curriculum.
5. The classroom teachers should have access to clearly stated, sequential objectives along with complementary materials and a repertoire of methods to implement the objectives.
6. School systems should spend more money on materials which are conducive to the multisensory method since the findings support the position that students learn better from the multisensory method of teaching than they do from the traditional method of teaching. It is also apparent that the school systems will need to employ more mathematics consultants or specialists who can provide in-service training for teachers and other staff on the utilization of multisensory models in teaching mathematics.
7. Publishers should place more emphasis on developing programs which will maximize the use of the multisensory approach, rather than programs which are designed primarily for paper-and-pencil activities.

Summary

Even though the findings of this study are tenuous, the data tend to support the premise that concepts in elementary school mathematics can be grasped more readily by students (the findings may also have implications for teaching adults) when multisensory models are utilized which will allow the students to become personally and actively involved in the teaching-learning process. Further, teaching by the multisensory approach seems to have resulted in greater retention and transfer of knowledge than teaching by the traditional approach. Moreover, the use of manipulative models provides for a diversity of learning environments, which permits a greater number of individuals to learn more readily than does the traditional approach.

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APPENDICES

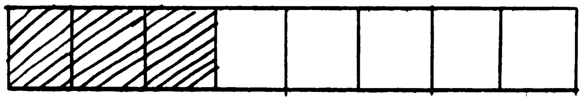
APPENDIX A

OBJECTIVES

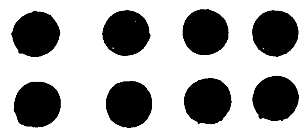
APPENDIX A

OBJECTIVES

1. Given a region divided into fractional parts, the learner can name the fraction for the shaded portion of the region.

   Example: 

2. Given a set of objects, the learner can identify a portion of the set to represent a given fraction.

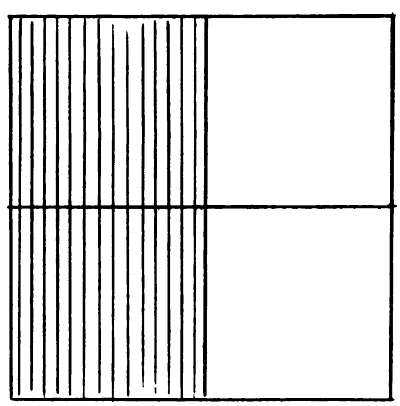
   Example: Circle 1/4 of the dots. 

3. Given three or more fractional numerals, the learner can identify the largest and/or smallest fractional numeral.

   Example: $\frac{1}{2}, \frac{1}{5}, \frac{1}{3}, \frac{1}{4}$

4. Given a region with a portion shaded, the learner can name fractional numerals equivalent to the shaded portion.

   Example:



APPENDIX B

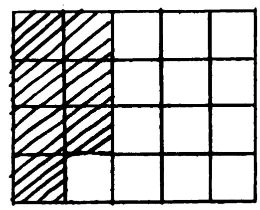
PRE-TEST

APPENDIX B

PRE-TEST

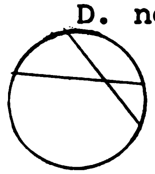
Answer Column

1. What part of the region is shaded? 1. ________

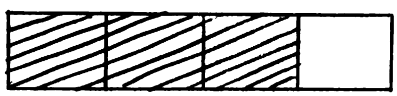


2. The circle is divided into 2. ________

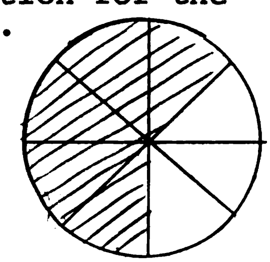
A. halves C. thirds
B. fourths D. none of these



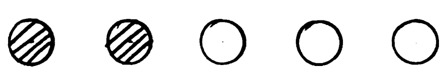
3. What part of the rod is shaded: 3. ________



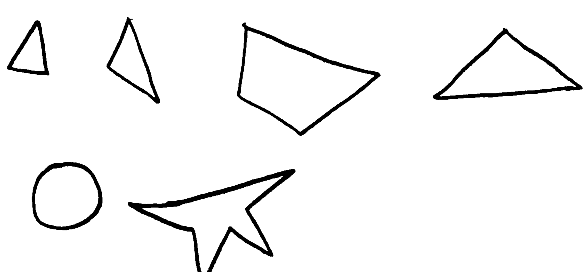
4. 5 of the 8 parts of the circle are shaded. Write a fraction for the region that is shaded. 4. ________



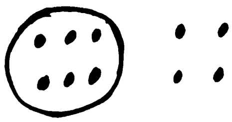
5. What part of these circles are shaded? 5. ________



6. What part of these figures are triangles? 6. ________



7. Give a fraction to tell what part of the dots is inside the circle. 7.____________



8. John had 9 pieces of candy. He ate 1/3 of them. How many did John eat? 8.____________

9. Which fraction is the greatest? 9.____________

1/3 3/3 2/3

10. Which of these fractions is the smallest? 10.____________

1/2 1/4 1/5

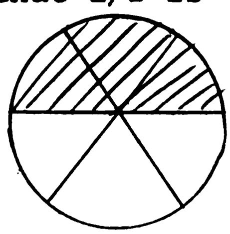
11. Which of these fractions is the largest? 11.____________

1/3 1/2 1/6

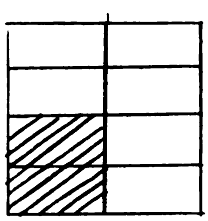
12. Study the figures below. Which statement is true? 12.____________

A. 1/2 is greater than 5/6
B. 3/4 is less than 1/2
C. 1/2 is greater than 1/4

13. This figure shows that 1/2 is equivalent to ?/6. 13.____________



14. From the picture we see that 2/8 is equivalent to ?/4. 14.____________



APPENDIX C

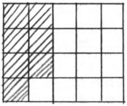
ACHIEVEMENT TEST

APPENDIX C

ACHIEVEMENT TEST

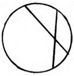
Answer Column

1. What part of the region is shaded? 1.__________

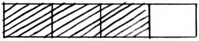


2. The circle is divided into 2.__________

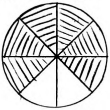
   A. triangles C. thirds
   B. fourths D. none of these



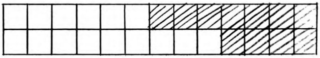
3. What part of the rod is shaded? 3.__________



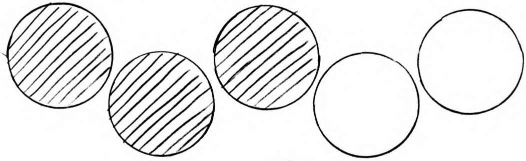
4. 6 of the 8 parts of the circle are shaded. Write a fraction for the region that is shaded. 4.__________



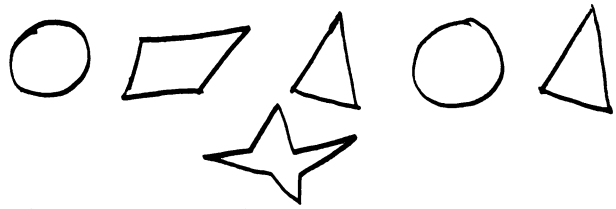
5. 11 of the regions are shaded. Write a fraction for the part that is shaded. 5.__________



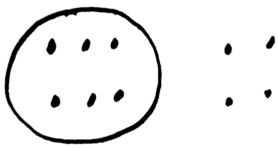
6. Give a fraction to tell what part of these circles is shaded. 6.__________



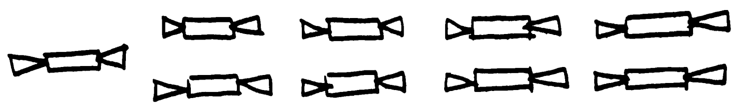
7. Write a fraction to tell what part of these figures is triangles. 7.__________



8. Give a fraction to tell what part of the dots is inside the circle. 8.__________



9. Mary had 9 pieces of candy. She ate 1/3 of them. How many did mary eat? 9.__________

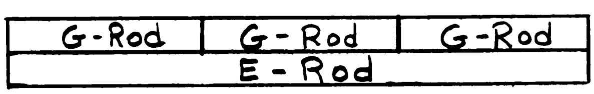


10. Which fraction is the greatest? 10.__________
A. 1/3 B. 3/3 C. 2/3 D. 1/4

11. Which of these fractions is the smallest? 11.__________
A. 1/2 B. 1/5 C. 1/4 D. 1/3

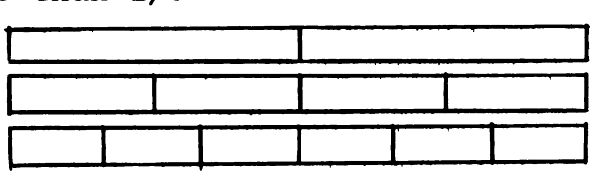
12. Which of these fractions is the largest? 12.__________
A. 1/8 B. 1/2 C. 1/6 D. 1/4

13. The G-Rod is what part of the E-Rod? 13.__________

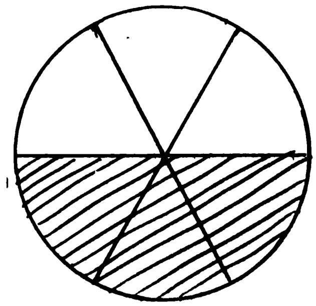


14. Study the figure below. Which statement is true? 14.__________

A. 1/2 is greater than 5/6
B. 3/4 is less than 1/2
C. 1/2 is greater than 1/4
D. 1/3 is less than 1/5

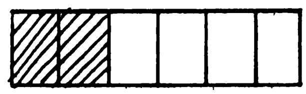


15. This figure shows that 1/2 is equal to ?/6. 15.__________

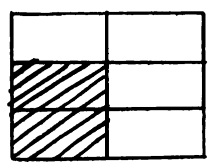


16. This picture shows that 2/6 is equal to what fraction. 16.__________

A. 1/6 B. 1/3
C. 1/2 C. 1/4

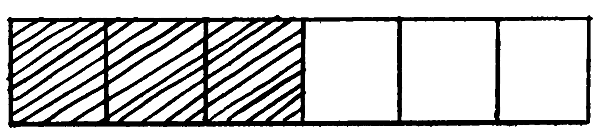


17. This picture shows that 2/8 is equal to ?/4. 17.__________



18. Study the picture. How much of the rod is shaded? 18.__________

A. 2/3
B. 3/6
C. 3/5
D. 1/6



APPENDIX D

Manipulative Test

APPENDIX D

MANIPULATIVE TEST

1. Show 1/5 of the set of 15 counters.

2. Show 3/4 of the set of 20 counters.

3. Use the Cuisenaire Rods to show 1/3 in three different ways.

4. Fold the paper rectangle into fourths; then cut away 2/4 of the paper.

5. All of the five cardboard circles are divided into three parts. Pick out the circles that are divided into thirds.

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