INTRA-URBAN RESIDENTIAL MOBILITY IN LANSING-EAST LANSING: THE CONSTRUCTION, VALIDATION, AND APPLICATION OF A VACANCY CHAIN MODEL.

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This is to certify that the

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Thomas L. Conner
Major professor

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INTRA-URBAN RESIDENTIAL [illegible]
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The model indicates that some filtering-down of housing does occur in almost all sub-systems in both time periods. However, this is overshadowed by the fact that vacancy chains within Lansing-East Lansing are very short. If the length of chains is a function

ABSTRACT

INTRA-URBAN RESIDENTIAL MOBILITY IN LANSING-EAST LANSING: THE CONSTRUCTION, VALIDATION, AND APPLICATION OF A VACANCY CHAIN MODEL

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This study employs a vacancy chain model to examine intra-urban residential mobility. The purpose of the study is two-fold: to probe the utility of the vacancy chain model and to use the model to analyze the filtering process within a housing system.

The study area consists of the contiguous cities of Lansing and East Lansing, Michigan. Vacancies are inferred from changes in successive occupancies as reported in the Lansing City Directory, and a weighting procedure is used to derive an unbiased sample of vacancy chains. This procedure resulted in the selection of 1268 chains for 1969-1970, and 707 chains for 1964-1965.

The model fits the data extremely well in each time period, and in each of the housing sub-systems, though vacancy chains are extremely short. The excellent fit of the model lends support to the notion of the independence of vacancy transitions.

The model indicates that some filtering-down of housing does occur in almost all sub-systems in both time periods. However, this is overshadowed by the fact that vacancy chains within Lansing-East Lansing are very short. If the length of chains is a function

of housing system size, as it appears to be, then the major beneficiaries of housing vacancy creations in Lansing-East Lansing are not the residents of the study area, but the residents of the larger housing systems within which Lansing-East Lansing is embedded.

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1.1: Characteristics of Movers

Traditionally the analysis of residential mobility has been approached from two different perspectives: the analysis of migration streams, which is concerned with the volume and direction of the flow of people who move more or less permanently, between different places (see e.g. Stauffer, 1940; Zipf, 1946; McGinnis and White, 1967); and the analysis of differential migration which seeks to account for migration by differences in the attributes and characteristics of migrants and non-migrants, such as age, sex, occupational status, etc. (Rossi, 1955; Arminger, 1966; Rogers, 1966; Simmons, 1968; Straits, 1968; Morrison, 1971).

Residential mobility is seen as "the process by which families adjust their housing to the housing needs that are generated by shifts in family composition that accompany life-cycle changes (Rossi, 1955:9; see also Folger, 1957; Moore, 1966; Brown et al., 1970; and White, 1970). Because the process of adjustment formulated is a negative-feedback process, mobility potential is highest when living conditions and a family's desires are most discrepant (Rossi, 1955:76ff.), and much research has been devoted to the identification of persons or families most likely to move.

> The typical mover is a young person (or family) with a comparatively low income, who is currently renting an apartment. If this renter expects a rise in salary, or if he wants his own home, or if young children are part of the household - or all three - the mobility potential is increased (Abu-Lughod and Foley, 1970:471).

The emphasis on tenure status (the distinction between renters and owners) as the critical variable is widely supported (Butler et al., 1969; Cave, 1969; Moore, 1969; Brown and Holmes, 1971; McAllister et al., 1971), but there are several other variables that are associated with mobility. Those who are most likely to move tend to be: young adults; males; professionals; unemployed (Rogers, 1966:452); recently married; wage earners (Morrison, 1971:172); persons not rooted in the community (Arminger, 1966). Some authors have reported that Whites (Rogers, 1966:452) or Blacks (Lansing et al., 1969:52) are more likely to move than the other group, but recent findings suggest that the critical variable is tenure status (McAllister et al., 1971).

The identification of people likely to move is important because "a small proportion of frequent migrants accounts for a high proportion of all migration" (Taeuber, 1961:118). It would appear that approximately 20 percent of the population change residence each year (Taeuber et al., 1961:862n), but "measuring migration on the basis of the number of moves recorded ... overstates the number of migrants by about 80 percent" (Goldstein, 1964:1131).

> The findings reveal a substantial degree of chronicity ... a tendency for observed mobility rates to be the product of repeated and frequent movement by the same individuals rather than single moves by the observed population at risk (Morrison, 1971:172).

1.2: The Migration Process

The analysis of migration streams, the flow of movers, between different places has yielded a persistent finding: there is an inverse relationship between the size of the migrant flow between two places and the distance which separates them.

The bulk of this research has focused on intercounty or interurban movement (see Taeuber et al., 1961; Goldstein, 1964; Rogers, 1966; and Morrison, 1967). The use of the county or the metropolitan area as the smallest area amenable to analysis has been in large part determined by the availability of the data (Taeuber, 1961), or rather, the lack of data concerning intra-country or intra-urban mobility, despite the evidence which indicates that the highest proportion of residential movement occurs within a single metropolitan area or a single county (Butler, et al., 1969:2 et seq.; Simmons, 1968:622). Not only have research findings indicated this to be the case but this situation is an obvious conclusion of many of the theoretical and mathematical formulations which were put forward to account for interurban and intercounty movement.

This relationship can be expressed in its most general form as a probability density function. The probability of movement between two places a distance D apart

$$p\ (D) = D^{b}\ (ce^{-gD^{a}})$$

wherein b, c, g, and a are constants. This curve can be fitted with a high degree of success to almost all the empirical data regarding migration streams (Moore, 1966:19-20).

A more familiar form of this curve is the Pareto equation

$$p(D) = aD^{-b}$$

and its variants such as the gravitational model developed by Zipf (1940):

$$M = \frac{P_1 P_2}{D^{b}}$$

where

M = movement of population

P_1 = population at locale 1

P_2 = population at locale 2

Other ratios, such as are formed by Simmons (1968:641)

$$\frac{\text{in-migration}}{\text{population}} = \frac{a}{(\text{distance})^b}$$

are but simple variations of the basic function.[1] All of them yield similar positively skewed curves of population movement as a function of distance, as sketched in Figure 1.1, with the exact shape of the curve determined by the constants in the equation.

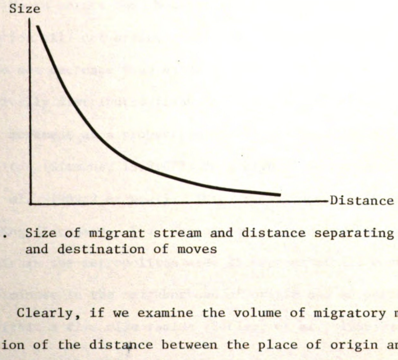

Figure 1.1. Size of migrant stream and distance separating the origin and destination of moves

Clearly, if we examine the volume of migratory mobility as a function of the distance between the place of origin and the

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[1]In fact, if we let population = P_1 and a = P_2 then Simmons' equation is identical with Zipf's.

place of destination

$$V(D) = \frac{a}{D^b}$$

and then extrapolate to include movements within a political boundary as well as movements between political entities, the great volume of intra-urban residential mobility should surprise no one. For $b \geqslant 0$, $V(D)$ increases at least as rapidly as D decreases ($V(D)$ increases linearly at $b = 0$, and exponentially at $b > 0$).

Moore points out, however, that within an urban area the Pareto function will not strictly hold if the opportunities for migration do not decrease monotonically, i.e., if the population is not symmetrically distributed (1966:21). But empirical estimates of intra-urban movement as a proportion of all residential mobility range from two-thirds (Simmons, 1968:622) to a high of 80 to 85 percent (Butler, et al., 1969:2 et seq.). This is consistent with the type of model being discussed. The consistency reaches further when we note that within the metropolitan area 25 percent of all moves are found to terminate in the neighborhood of origin and 60 percent terminate within a five mile radius (Butler, et al., 1969:9).

1.3: The Aim of the Dissertation

This paper will attempt to analyze intra-urban residential mobility within the framework of a vacancy-chain model (White, 1970).

By focusing on the structure of an urban housing system, and the movement of vacancies through it we will attempt to examine the opportunity structure within which residential mobility takes place.

The models of migration discussed above are largely "push" models (Butler, et al., 1969; Brown and Moore, 1970; Brown, et al., 1970). The impetus to move arises from some dissatisfaction with the existing dwelling, or from the emergence of certain needs which the existing dwelling does not fulfill. Only then is the decision made to move, and after that occurs the question "where to?" (Brown, et al., 1970:176). Regardless of whether the "push" is couched simply in terms of dissatisfaction (Butler, et al., 1969) or more rigourously in terms of maximizing place utility (Brown and Moore, 1970), in the language of the marketplace, these are analyses of housing "demand," i.e. "who is looking for new (different) housing? Why do people look for new housing? What type of housing are they looking for?"

The questions of housing "supply" are not dealt with, regardless of the fact that "the selection of a new home depends not only on demand conditions, but also on supply constraints" (Simmons, 1968:637).

Clearly the selection of a specific new dwelling can not be fully understood without knowledge of the existing available choices, and the vacancy-chain model is an attempt to examine those choice systems. Knowledge of the movement of vacancies through the housing system would allow for a fuller understanding of residential mobility, because the housing system is the system within which residential mobility occurs.

The vacancy chain model, after testing, will then be applied to the examination of the filtering process in a housing system. Filtering can be seen as "the changing of occupancy as the housing that is occupied by one income group becomes available to the next lower income group" (Ratcliff, 1949:321-22), and is widely considered to be the major mechanism for the provision of housing to lower income groups (see e.g., Forrester, 1969). The extent of house filtering, and even its existence, has been repeatedly questioned (Lowry, 1960; Grigsby, 1963; White, 1971), so the application of the vacancy chain model to the question of filtering not only probes the utility of the model, but may provide useful information regarding this important question.

2. MODELS OF MOBILITY

2.1: Markov Processes

"Mobility analysis is ... the study of families of temporal functions" (McGinnis, 1968:713), or "time dependent probability processes" (McGinnis, 1968:715) and the basic stochastic model used to describe and analyze residential and occupational mobility has been that of the population of movers as a Markov process. The Markov process describes an object moving according to some set of probabilities through a system of distinct and defined states. On the surface, a more appropriate model for the analysis of residential mobility could hardly be imagined.

Consider a population of objects initially distributed in a set of states $\{1, 2, \ldots, k\}$ and the matrix

$$\underline{P} = [p_{ij}]$$

of transition probabilities,[1] the Markov chain model can examine

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[1] p_{ij} is the probability that an object which was in state i at time t will be in state j at time t+1.

$$P\{X_t = j | X_0 = i\} \quad t = 1, 2, \ldots.$$

the probability that an object, X, will be in state j at time t, given that it began (t = 0) in state i. Under certain conditions it can also be used to examine the probability that an object beginning in state i will ever get to state j; how many moves it would take to get there, and other questions about the movement of objects in the population.

The tractability and usefulness of Markov chains as models of mobility processes cannot be denied, especially the tractability of first-order Markov chains. A first order Markov chain is one where

$$P\{X_{n+t} = j | X_0 = 1, \ldots, X_n = k, \ldots X_{n+t-1} = i\}$$

$$= P\{X_{n+t} = j | X_{n+t-1} = i\}.$$

This is the Markov property. It states that the probability of movement from any state i at time (n+t-1) depends only upon the state the object occupies at time (n+t-1). The transition probability p_{ij} is in no way affected by previous occupancies by X or by the route through the system whereby X came to be in state i at time (n+t-1). In other words, knowledge of X's history -- that it was in state 1 at time 0, state k at time n, and state i at time (n+t-1) yields the same p_{ij} (n+t-1) as knowing only that X is in i at (n+t-1). There are, of course, higher order Markov chains, whose transition probabilities do depend on the history of the process, but the mathematics becomes so cumbersome as to make them impractical (see, for example, Hua, 1973).

It is the Markov property that makes first-order Markov chains so attractive. If the transition probabilities remain constant over time, then the initial population distribution and the set of transition probabilities completely describe the system (see Anderson and Goodman, 1957:89ff). But for the Markov property to hold it must be specified that the objects moving within the system of states move independently of one another. That is to say, a uniform rate of movement from state i to state j at time t, $r_{ij}(t)$ is applied to the entire population of i at time t, and no selection process exists. What any object in the system does is in no way influenced by the action of any other object in the system. This requires that the population be homogeneous, and in the specific case of residential mobility that people have no friends, no relatives, no social contacts. At the very least the assumption requires that they ignore these contacts when they move, even though the most effective means of becoming aware of and taking advantage of residence vacancies is personal contact (Rossi, 1955:151; Moore, 1966:29).

The repeated use of first order Markov chains with constant transition probabilities by students of residential mobility attests to their attractiveness despite the stringent requirements imposed by the Markov property, and the assumptions regarding state classification, population homogeneity, and time stationarity. That the assumptions of the model are not met (or alternatively, that the model is simply not an adequate representation of mobility processes in human populations) is attested to by the constant revision, restatement, and refinement of such models.

> A great amount of energy and paper has been wasted attempting to "apply" various inadequate models to data when the models' inadequacy could more easily have been discovered - and perhaps remedied - by a careful theoretical analysis of the models' assumptions and/or their logical consequences (McFarland, 1970:472).

2.2: Refinements of the Simple Markov Model

Because Markov theory is concerned with state changes by an individual, not the movements of an entire population, population homogeneity must be assumed to exist. Because it does not exist the most persistent problem with simple Markov chain formulations of movement between residences or movement between jobs has been the failure of the predicted n^{th} - step transition matrix to coincide with the observed n^{th} - step transition matrix.

Blumen et al. (1966), in a study of occupational mobility were the first to note that the observed transition matrix $\underline{P}^{(n)}$ differed considerably from the predicted transition matrix $\underline{P}^{n}$ at the 8th step. In particular, they found that ${p_{ii}}^{8} < {p_{ii}}^{(8)}$ (1966:318). To reduce this discrepancy they proposed that the population be divided into two classes: those people who do not change jobs, the "Stayers," and those who do, the "Movers."

If $\underline{S}$ is the diagonal matrix,

$$\underline{S} = [s_{ii}]$$

the proportion of stayers in each industry class i; and $(\underline{I}-\underline{S})$ is the diagonal matrix of movers in each industry class i;[1] and, if "Stayers" remain in their current state with probability equal to 1 while "Movers" move according to the transition matrix

$$\underline{M} = [m_{ij}]$$

then it can be shown that

$$\underline{P} = \underline{S}+(\underline{I}-\underline{S})\underline{M}$$

and $$\underline{P}^{(n)} = \underline{S}+(\underline{I}-\underline{S})\underline{M}^{n}$$ (Blumen, et al., 1966:318-322).

In light of the theoretical similarity between residential and occupational mobility and the vast amount of empirical evidence that observed residential mobility is the product of a large number of moves by a relatively small number of people (Taeuber, 1961; Taeuber, et al., 1961; Goldstein, 1964; Morrison, 1967; Morrison, 1971; Spilerman, 1972b) the "Mover-Stayer" model was quickly adopted to study

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[1] I is the identity matrix.

residential mobility. However, while Myers, et al., (1967) found, as did Blumen, et al. (1966) that this partition of the population decreased the discrepancy between the observed $\underline{P}^{(n)}$ and the predicted $\hat{\underline{P}}^{(n)}$, the predicted n-step diagonal entries, the $\hat{p}_{ii}^{(n)}$, still underestimated the probability of an individual remaining in or returning to the same state in n steps.

Taeuber's conclusion:

> ... Residential mobility during a given time period is not independent of previous mobility experience. Persons who have not moved recently are less likely to move in the future than are those who have moved recently (1961:118)

was reformulated as the Axiom of Cumulative Inertia:

> The probability of remaining in any state of nature increases as a strict monotone function of duration of prior residence in that state (McGinnis, 1968:716).

In formal terms

$$ {}_{d}P_{ii}(t) > {}_{d-k}P_{ii}(t) \qquad d < \infty, \quad k = 1\ 2, \ldots $$

and

$$ \lim_{d\to\infty} {}_{d}P_{ii}(t) = 1 $$

where d is the prior duration in state i (Myers, et al., 1967:123).

It would appear that there are some theoretical as well as some practical problems in this line of enquiry. The questions being explored are those of proper occupancy-state classifications and proper partitioning of the population into homogeneous sub-populations and associating with each sub-population the appropriate transition matrix. The question is the same as the one McFarland raises regarding the Markov models of social mobility:

> ... intergenerational social mobility is not a Markov Chain when states are defined the way they defined them; the process might still be a Markov Chain if the states were defined differently (1970:464).

The practical problem inherent in this approach is the increase in the number of transition matrices and the number of transition probabilities which must be estimated. If a system initially contains k states, then k(k-1) transition probabilities must be estimated.[1] If we then classify our states into duration-specific states, with a maximum of h-1 prior-elapsed time periods, and allow the transition from any state S_i with occupancy-duration, d, to any state S_j with occupancy-duration 0, i.e., the transition from ${}_dS_i$ to ${}_0S_j$ then we must estimate k(k-2) transitions from ${}_dS_i$ to ${}_0S_j$ and k transitions from ${}_dS_i$ to ${}_{d+1}S_i$. For h time periods then there are $hk(k-1) = hk^2 - hk$ transitions to estimate. The simple model requires the estimation of only k(k-1) transition probabilities.

As the initial population is disaggregated into more homogeneous populations in an attempt to overcome the discrepancies of Markov Chain projections with heterogeneous populations, a second,

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[1]We need only estimate k(k-1) instead of k^2 transition probabilities because $\sum_{j=1}^{k} P_{ij} = 1$, for all i.

and perhaps more important, practical difficulty arises. Not only is there no guarantee that this particular disaggregation will fit the model[2], but the disaggregations are attained at great cost because of the information which must be gathered about the people in the system. As the subdivisions become more specific, more and more information must be gathered, at greater cost in time and money.

Even if we consider only the two population "Mover-Stayer" model, we must wait a sufficient length of time for "Stayers" to reveal themselves (Morrison, 1971:177-178). It is impossible to inquire of people whether they are one or the other. If more information is required as in the estimation of cohort-specific transition matrices (Rogers, 1966), or in the use of the exposure-residence concept (Taeuber et al., 1961), then the difficulties increase. If we assume, for example, that the Axiom of Cumulative Inertia has meaning, then we require a reliable measure of duration-of-residence. To obtain it we must resort to "individual histories of movement" (Myers et al., 1967:125).

Population partitions based on variables such as "rootedness in a community" (Arminger, 1966), or "satisfaction/dissatisfaction with current residence" (Butler, et al., 1969) or "place utility" functions (Brown and Moore, 1970) would be even more tenuous than would partitions based on the commonly used "demographic" characteristics of migrants (age, sex, occupational status, etc.)

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[2]Even in Spilerman's model, where virtually each individual in the system has his own transition matrix, the predicted diagonal entries $p_{ii}^{(n)}$ are still discrepant from the observed $p_{ii}^{(n)}$ (1972a:282n).

insofar as an

> analysis focusing on attitudinal questions about migratory behavior or intentions assumes that people understand their own complex behavior patterns -- an assumption which is probably unsound (Goldscheider, 1971:37).

These refinements of state classification and population disaggregation are an attempt to meet the requirement that individual transitions be made independently. However, they do not rectify the basic source of non-independence in the process of residential mobility. As stated above, the most effective means of discovering which dwellings are available for occupancy is personal contact (Rossi, 1955:161; Moore, 1966:29). As long as models of residential mobility are Markov Chain models of people moving through a system of occupancy-states, it does not appear that the independence requirement can be met.

3. THE VACANCY CHAIN MODEL

3.1: The Vacancy Chain Model

White (1970) has recently developed an interesting and elegant model for the analysis of mobility within systems of positions and occupants, which manages to avoid many of the problems discussed above. Although the model was originally formulated to deal with systems of men in jobs, its potential application to the study of residential mobility was quickly recognized (White, 1970:320-321, 390; White, 1971; Hua, 1973). In fact there are earlier indications of the development of such a model within the housing field (Kristof, 1965; and Lansing et al., 1969).

The model proposed is an embedded Markov Chain of first-order, but the crucial distinction is that the population of interest is not the people who move through the housing system, but rather the vacancies which appear when people leave the system. These vacancies then move through the system occupying successively different dwellings until they finally leave the system.

Consider the representation in Figure 3.1. Let A, B, and C represent addresses or dwelling units and a, b, c, and d represent people. The dashed line represents the boundary of the

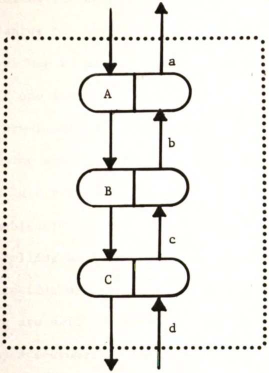

Figure 3.1. The vacancy chain model

housing system under scrutiny. Person a moves out of the system from position A, and b moves from B to A. In turn, person c moves from position C to B, and finally d fills C from outside the system. This sequence of linked moves by people can be regarded as a chain of vacancy movement. We can consider that a vacancy has moved in from outside the system to occupy A, and then moves from A to B to C and finally leaves the system.

In the housing system under consideration a vacancy can enter the system in several different ways. As in the above example, a vacancy is said to enter the system when the occupant of a dwelling leaves the system, either by moving away or dying, and leaves the dwelling vacant. If one spouse of a marriage were to die leaving the other in their previously joint residence, this would not create a vacancy. New housing may be built, or an existing house (1 residence) might be subdivided into r apartments, creating (r-1) new vacancies. The marriage or cohabitation of two or more persons each previously occupying his own dwelling would also create (n-1) new vacancies, where n is the number of residences previously occupied.

Vacancies are said to leave the system when an existing vacancy is filled by a newcomer to the housing system, usually a migrant. The formation of new households -- the marriage and establishment of a separate household by two people each previously living with his parents, or the separation of a married or otherwise cohabiting couple, causing one to seek a new separate residence -- also cause vacancies to leave the system. Vacancies also leave the housing system when the dwellings they occupy are destroyed or are converted to some non-residential use.

3.2: The Mathematics of Vacancy Chains

Mathematically, the model is straightforward, and can be presented very briefly:

Given a set of occupancy states,

$$i = \{1, 2, \ldots, s\}$$

where all states outside the system are denoted by $i = 0$, let the probability of a vacancy in stratum i moving to stratum k be q_{ik}. Then

$$\sum_{k=0}^{s} q_{ik} = 1 \qquad \begin{array}{l} i = 1,2,\ldots,s \\ k = 0,1,2,\ldots,s \end{array}$$

and

$$\sum_{k=1}^{s} q_{0k} = 1.$$

If we let $\underline{p} = \left[q_{i0}\right]$, a column vector and $\underline{Q} = \left[q_{ik}\right]$ $i,k = 1,2,\ldots,s$

it can be shown that the probability of a chain of length j beginning in stratum j,

$$\underline{P}_j = \underline{Q}^{j-1}\underline{p}. \tag{3.1}$$

To explicate the model by way of a simple example, let us consider the system with only 1 state. The probability of remaining in the system is equal to q, and the probability of leaving in any time period is equal to p, and $p+q = 1$.

The probability of remaining in the system for exactly one time period

$$P(1) = p.$$

The probability of remaining for exactly 2 time periods

$$P(2) = qp.$$

It follows that

$$P(3) = q^2p$$

and in general,

$$P(j) = q^{(j-1)}p. \tag{3.1a}$$

To determine the mean length of time a vacancy will spend in the system we simply compute the vector of mean chain lengths by stratum of arrival

$$\underline{\lambda} \equiv \sum_{j=1}^{\infty} j\underline{P}_j = (\underline{I} - \underline{Q})^{-1}\underline{1} \tag{3.2}$$

where $\underline{I}$ is the identity matrix and $\underline{1}$ is conformable column vector of 1's.

If $\underline{f}(t)$ is the row vector of proportions of vacancy arrivals in year t by stratum, then the overall distribution of chain lengths may be computed by $\underline{f}(t)\underline{P}_j$, for all $\underline{P}_j$. The overall mean length of a cohort of vacancy chains, j(t) becomes simply

$$j(t) \equiv \underline{f}(t)\underline{\lambda} \equiv \underline{f}(t)\ (\underline{I} - \underline{Q})^{-1}\underline{1} \qquad (3.3)$$

In the one-state model, the mean length of time spent in the system by a vacancy

$$\mu = \sum_{j=0}^{\infty} jP(j) = \frac{1}{p} = \frac{1}{1-q} \qquad (3.2a)$$

The vector $\underline{M}(t) = \left[m_i(t)\right]$ of the total number of moves ever made by vacancies entering the system in stratum i can be expressed as

$$\underline{M}(t) = \sum_{h=Q}^{\infty} \underline{F}(t)\underline{Q}^{h}$$

where $\underline{F}(t)$ is the vector of vacancy arrivals by stratum. This summation yields

$$\underline{M}(t) = \underline{F}(t)\ (\underline{I}-\underline{Q})^{-1}. \qquad (3.4)$$

The total number of moves made by the r vacancies which enter our one-state system is simply the product of the number of vacancies entering the system and the mean length of time (number of moves) each vacancy spends in the system.

$$m = r\left(\frac{1}{p}\right) = r\left(\frac{1}{1-q}\right). \qquad (3.4a)$$

Although the above equations are sufficient to describe, verify, and analyze the properties of the vacancy chain model, some additional comments are necessary. The matrix $(\underline{I}-\underline{Q})^{-1}$ is of great interest and importance in the study of housing vacancy chains. Just as in the simple model where $\frac{1}{1-q}$ is the mean number of moves a vacancy makes within the system, if we let

$$(\underline{I}-\underline{Q})^{-1} = [n_{ij}] \qquad i.j = 1,2,\ldots,s$$

then n_{ij} is the mean number of times that an object which began the process in state i will appear in state j before reaching an absorbing state (in this case, before leaving the system). For this reason $(\underline{I}-\underline{Q})^{-1}$ is called the multiplier matrix (White, 1970; White, 1971; Hua, 1972).

The model presented is an embedded Markov chain of first-order, although there are major differences between it and the "standard" Markov models of residential movers. Conceptually, the population of objects moving through the system is a population of vacancies, not people, and different questions are posed by the two models. We are concerned with chains of vacancies and their properties, such as length and persistence, and speed of movement from stratum to stratum. At the system level we are examining sequences of independent mobility acts which provide the framework within which residential mobility takes place.

Another important distinction is that the q_{ii} of the vacancy-chain model refer only to moves of vacancies within state i. The aggregation of address changes within state i and "no-moves" within i, which occurs in the p_{ii} of the "people" processes cannot occur here. The model is concerned only with moves of vacancies, because there is no vacancy without a move. Insofar as Butler, et al., show that one-quarter of all residential moves are to different places in the same neighborhood (1969:9) this distinction should prove quite useful. Furthermore, the consideration of moves only should alleviate the previously-mentioned persistent problem of the predicted n^{th} step $\hat{p}_{ii}$ underestimating the actual n^{th} step p_{ii}.

Finally, because the model is one of vacancies moving through a system of housing, vacancies are the entities which are assumed to move independently. This represents an attempt to reconcile the mathematical theory with physical and social reality, and this assumption is much more plausible than one requiring that people move independently.

3.3: Mean First Passage Times

One additional aspect of the vacancy chain model can be profitably studied to gain information about the structure of our housing system. The measure has been referred to as a measure of social distance (Beshers and Laumann, 1967) and functional distance (Brown and Horton, 1970), and reflects the degrees of connectedness which hold between the different housing strata in our housing system.

We will examine

$$\underline{M} = [m_{ij}]$$

the matrix of mean first passage times, i.e., the mean number of steps that will elapse before a vacancy starting its career in state i will arrive for the first time in state j.

> Mean first passage times provide a measure of a particular kind of contiguity -- one based on interchange probabilities rather than distance. Thus they may be viewed as indices of aspatial ... (interstrata) ... distance (Rogers, 1966: 454).

As before, let

$$\underline{Q} = \begin{pmatrix} q_{11} & q_{12} & \cdots\cdots & q_{1k} \\ q_{21} & q_{22} & \cdots\cdots & q_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ q_{k1} & q_{k2} & \cdots\cdots & q_{kk} \end{pmatrix}$$

$$\underline{p} = \begin{pmatrix} q_{10} \\ q_{20} \\ \vdots \\ q_{k0} \end{pmatrix}$$

and

$$\underline{f} = \begin{pmatrix} q_{01} & q_{02} & \cdots & q_{0k} \end{pmatrix}$$

Construct a matrix

$$\underline{\mathbf{P}} = \left(\begin{array}{c|c} \underline{Q} & \underline{p} \\ \hline \underline{f} & 0 \end{array}\right)$$

i.e.

$$\underline{P} = \left(\begin{array}{cccc|c} q_{11} & q_{12} & \cdots\cdots\cdots & q_{1k} & q_{10} \\ q_{21} & q_{22} & \cdots\cdots\cdots & q_{2k} & q_{20} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ q_{k1} & q_{k2} & \cdots\cdots\cdots & q_{kk} & q_{k0} \\ \hline q_{01} & q_{02} & \cdots\cdots\cdots & q_{0k} & 0 \end{array}\right)$$

Using the terminology and notation of Kemeny and Snell (1960: Ch.4), let the fundamental matrix of a regular Markov chain be

$$\underline{Z} = (\underline{I} - (\underline{P} - \underline{A}))^{-1} \qquad (3.5)$$

where

$$\underline{A} = [a_{ij}]$$

$$\underline{A} = \lim_{n\to\infty} \underline{P}^{(n)} = \lim_{n\to\infty} \underline{P}^{n}$$

Then, the matrix of mean first passage times

$$\underline{M} = (\underline{I} - \underline{Z} + \underline{E}\,\underline{Z}_{dg})\,\underline{D} \qquad (3.6)$$

where

$$\underline{E} = [1],$$

a square matrix with each element equal to 1; $\underline{Z}_{dg}$ is a diagonal matrix formed by setting the off-diagonal elements of $\underline{Z}$ equal to 0 and

$$\underline{D} = \left[d_{ij}\right]$$

is a diagonal matrix formed by setting the off-diagonal elements of $\underline{D}$

$$d_{ij} = 0 \qquad\qquad i \neq j$$

and setting the diagonal elements

$$d_{jj} = \frac{1}{a_{jj}}.$$ [1]

In the case of an independent trials process, a process at equilibrium, $\underline{M}$ is simply

$$\underline{M} = \frac{1}{p_{ij}}$$

If the process is at equilibrium then

$$\underline{P} = \underline{A}$$

---

[1] It should be noted that $\underline{A}$ has the form

$$\underline{A} = \begin{pmatrix} a_1 & a_2 & \cdots\cdots\cdots & a_k \\ a_1 & a_2 & \cdots\cdots\cdots & a_k \\ \vdots & \vdots & & \vdots \\ a_1 & a_2 & \cdots\cdots\cdots & a_k \end{pmatrix}$$

and

$$\underline{Z} = (\underline{I} - (\underline{P} - \underline{A}))^{-1} = \underline{I}^{-1} = \underline{I} \ ;$$

$$\underline{M} = (\underline{I} - \underline{Z} + \underline{EZ}_{dg})\underline{D}$$

reduces to

$$\underline{M} = (\underline{EZ}_{dg})\underline{D} = \underline{ED} \ ;$$

D becomes

$$\underline{D} = \left[\frac{1}{a_{jj}}\right] = \left[\frac{1}{p_{jj}}\right]$$

and

$$\underline{M} = \frac{1}{p_{ij}} \ .$$

Again, to explicate by means of a simple example. Consider the closed system with two states. Assume also that the system is at equilibrium so that

$$\underline{P} = \begin{pmatrix} q & p \\ q & p \end{pmatrix} \ .$$

The probability then of an object from state 1 going to state 2 in one step is equal to p.

$$p_{12}^{(1)} = p \ .$$

The probability

$$p_{12}^{(2)} = qp$$

is the probability of an object's going from state 1 to state 2 in 2 steps, i.e., staying in state 1 for 1 step and then moving to state 2. In general, the probability of an object's staying in state 1 for (n-1) steps and then moving to state 2 for the first time in the n^{th} step is

$$p_{12}^{(n)} = q^{n-1}p \ .$$

The mean of n, the mean first passage time

$$m = \sum_{n=0}^{\infty} nq^{n-1}p = \frac{1}{p} \ . \tag{3.6a}$$

The use of first mean passage times provides us with a measure of structural distance that allows us to consider all possible vacancy flows through the housing system from state i to state j, and also allows us to account for asymmetrical interstrata distances, i.e., $m_{ij} \neq m_{ji}$.

3.4: Testing the Model

Even though it is not clear that there are any statistical tests appropriate to processes of absorbing Markov chains such as are represented by $\underline{Q}$ (White, 1970:31n), the structure of the model lends itself to a relatively simple and straightforward examination of the fit of the model to the data. Once the q_{ik} are estimated, equation (3.1)

$$\underline{P}_j = \underline{Q}^{j-1}\underline{p}$$

yields a probability distribution of chain lengths which can be compared with the observed distribution. Other derivative statistics, such as $\underline{j}(t)$, $\underline{\lambda}$ and $\underline{M}(t)$ can also be compared with their observed counterparts.

That the single pool of data provides a valid test of the fit of the model is clear.

> The same sample of chains can yield both the observed length distribution and after decomoposition into constituent moves, the transition probability estimates (White, 1970: 33).

Because the predicted chain lengths are derived from the transition matrix $\underline{Q} = \left[q_{ik}\right]$, and there is no way for the q_{ik} to be inferred from the observed distribution of chain lengths, the test is a valid one.

4. ESTIMATION OF THE PARAMETERS OF THE VACANCY CHAIN MODEL

4.1: The Study Area

The contiguous cities of Lansing and East Lansing, Michigan were selected as the housing system for which a vacancy chain model of residential mobility was to be constructed. The vacancy moves which yielded the estimators of the q_{ij} of the transition matrix were derived (in the manner discussed below) from the Lansing City Directory published by R. L. Polk and Company. Two $\underline{Q}$ matrices were estimated, one for the period centered on 1969-1970, and the second based on vacancy moves of 1964-1965. City directories spanning the period 1961 to 1972 were required for the estimation of these parameters. The occupancy states through which these vacancies move were classified according to selected housing characteristics reported in U.S. Bureau of the Census, Census of Housing: 1970 Block Statistics Final Report HC(3) - 125 Lansing, Mich. Urbanized Area, (U.S. Government Printing Office: Washington, D.C., 1971).

Only the cities of Lansing and East Lansing, Michigan, were included in the housing system under consideration, even though the Lansing urbanized area contains several smaller towns (Dimondale, Haslett, Holt, and Okemos, to name a few) as well as other large, lightly-populated areas. However, the residents of these places have not been included in the Lansing City Directory until 1971, with the publication of the Lansing Suburban Directory (R. L. Polk and Co.). Consequently, because information regarding the mobility of residents to, from, and within these areas was not available, the areas were not classified into occupancy states. The area comprising the housing system umder consideration is shown in Map 1.

4.2: Classification of Occupancy States

Attempts to define and classify sub-areas of the city were the nineteenth-century precursors of one line of research in the field of Human Ecology (Levin and Lindesmith, 1961). The process is not a new one, but little agreement exists as to what criteria are necessary or even adequate for this process, though sophisticated techniques exist for manipulating, examining and measuring the variables that are selected (see, for example, Berry and Marble (eds.), 1968). Since Burgess' concentric-ring model of the city, several different sets of criteria have been posited which would allow one to classify the urban area into some number of meaningful sub-areas, i.e. a set of sub-areas which is indicative of social structure, in that the particular classification selected has behavioral consequences (Beshers, 1962:88).

The accepted criteria of classification range from a broad set of indices of social rank, urbanization, and segregation (Shevky and Bell, 1955) to support of some cash rent or price measure as the sole criterion of housing level (Hua, 1972:122).

In attempting to apply the Shevky-Bell Social Area Analysis to some Australian data, Jones found that the three dimensions social rank, urbanization (type of housing and household composition), and segregation were not necessary. Almost as much predictive accuracy could be obtained with only two components -- a combined measure of socioeconomic status and ethnicity, and a measure of household composition (1968:438). In fact, in the housing field, where "housing conditions tend with few exceptions to correlate highly with all indices of socioeconomic status" (Michelson, 1970:18), one would expect the interchangeability of indices to hold:

> If we have a reasonable collection of indicator items then for most purposes it does not matter which subset we use to form our index (classificatory instrument). (Lazarsfeld, 1959:60).

The universe of items from which our classification scheme was chosen was determined by published census block data (U.S. Bureau of the Census, 1971), and, as Beshers has stated:

> We can only study the distributions of those characteristics that the census chose to gather information on and tabulate; we must rely on the census definitions for the characteristics.... (1962:90).

Consequently, the criteria finally chosen to classify urban sub-areas which could adequately stand for the occupancy states of a Markov process[1] were two: a measure combining the average value

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[1]That this problem is not limited to the question of residential mobility is seen in White (1970:132ff). The reader will also find a good general discussion of the difficulties involved in state classification.

of housing and the average contract rent, and a measure of tenure status -- the proportion of housing owner-occupied.

Average value of housing is the arithmetic mean of:

> the respondents' estimate of how much the property (house and lot) would sell for if it were for sale. Value data are limited to owner occupied one-family houses on less than ten acres (U.S. Bureau of the Census, 1971:viii).

Average contract rent is the arithmetic mean of:

> the monthly rental agreed to, or contracted for, regardless of any furnishings, utilities, or services that may be included. Contract rent data exclude one-family homes on a place of ten acres or more (U.S. Bureau of the Census, 1971:viii).

> A housing unit is "owner occupied" if the owner or co-owner lives in the unit, even if it is mortgaged or not fully paid for. A co-operative or condominium unit is "owner occupied only if the owner or co-owner lives in it. All other occupied units are classified as "renter occupied" including units rented for cash rent and those occupied without payment of cash rent (U.S. Bureau of the Census, 1971:viii).

Both of these measures are accepted as standard in defining housing sub-areas.

> The most relevant classificatory variables are price, tenure, size, and location. Any one or any combination of these variables defines the housing sector(s) of a housing system (Hua, 1973:4).

Additionally, tenure status is seen as strongly influencing mobility. Renters are almost universally found to be more mobile than homeowners (Cave, 1969; see also, Moore, 1969; Brown and Holmes, 1971; McAllister, et al., 1971; Hua, 1972; Pickvance, 1973). This relationship cannot be attributed solely to the monetary investment in an owned home, for it persists today when long-term (20 to 30 year) amortization mortgages have all but eliminated the financial distinction between owner and renter. The reduced mobility of owners seems to involve social and psychological factors as well as the legal and financial impedimenta of home ownership.

Moore (1969:23-24) finds the strongest correlation (r = -.72) between housing turnover rate and any other variable is the Private Home Index, the proportion of dwellings which are in single private units. In Lansing and East Lansing, we find the correlation between proportion of dwelling units owner occupied and proportion of single-family dwellings to be quite high (r = .944), so we would expect the relationship between tenure status and mobility in our sample to be quite high.

A non-metric measure of association, a Guttman-Lingoes smallest space analysis (Lingoes, 1973) based on zero-order correlation coefficients, shows that tenure status is associated with the variety of life-cycle and life-style variables considered important to mobility (Rogers, 1966: Moore, 1969; Pickvance, 1973) as well as to the determination of social areas within the city (Jones, 1968:41, see also Shevky and Bell, 1955; Beshers, 1962). Selected aspects of the smallest space analysis are presented in Figure 4.1. The distinction between the two variables used is indicated by the distance separating rent and value from the cluster of items surrounding the tenure variable.

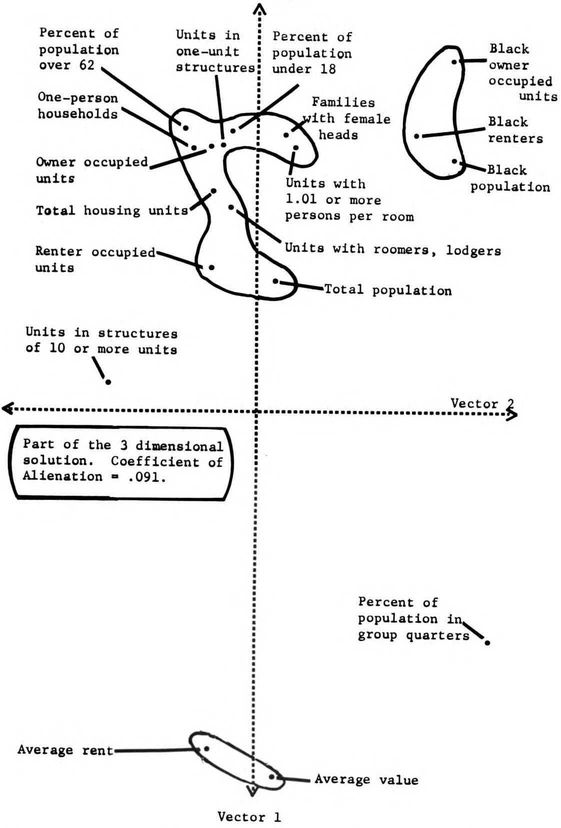

Figure 4.1. Smallest space analysis: characteristics of housing units and population

The correlation between average value and the proportion of owner occupied dwellings was .295, while the correlation between average rent and proportion owner occupied was -.052.

Some recent empirical findings and a theoretical consideration entered into the decision not to use race -- proportion of the population Black -- as a criterion of state classification. Although Blacks may appear to be more mobile than Whites, "the slightly greater mobility of Blacks is a result of their tenure status, rather than of racial, demographic, socio-economic, or attitudinal differences" (McAllister, et al., 1971:452).

As Table 4.1 shows, Black home owners are only slightly more mobile than White home owners and Black renters are actually <u>less</u> mobile than White renters.

Table 4.1

Mobility by Race and Tenure (McAllister, et al., 1971:451)

Moving Behavior	Owners		Renters	
	Black	White	Black	White
Stayed (%)	76.8	79.8	35.9	26.6
Moved (%)	23.2	20.2	64.1	73.4
Total	100.0	100.0	100.0	100.0
n	82	738	181	488

Further evidence for this view is provided by a dummy variable regression analysis of mobility behavior showing a β-weight of -.014 associated with the variable: "Race: non-White" (Morrison, 1971:175).

These findings, and the fact that in areas where the proportion of non-Whites is increasing, Black in-migrants tend to be of the same SES level as the Whites who are moving out, pointed to the conclusion that race would not be particularly useful in the determination of urban sub-areas as they would affect mobility behavior.

On theoretical grounds it was felt that the characteristic "race of occupant" was much less appropriate to the entity "vacancy" than were the characteristics "value of dwelling" (occupied by vacancy) and tenure status of dwelling. Consequently, "race" was not included as a variable of occupancy-state classification.

4.3: Sub-Areas of Lansing-East Lansing

The variables "average value and rent" and "tenure status" resulted in the establishment of four sub-areas of Lansing-East Lansing. These four occupancy states are shown in Map 2, and the states are labelled simply

1) Low
2) Lower Middle
3) Upper Middle
4) High.

While the establishment of four housing sub-areas is somewhat arbitrary, there are no rigid procedures for the establishment of such states (see again, White, 1970:132ff). Some have used simply deciles of housing value (Hua, 1972), but it was felt in this case that the 100 transition probabilities generated by that procedure would be far too great a number for stable parameter estimation.

Furthermore, the shapes of the distributions of average value and average rent in Figures 4.2 and 4.3 respectively, render the use of a measure such as deciles, quartiles, or even stanines inappropriate.

Table 4.2

Average Value of Housing

Average Value in 1,000's of Dollars	Number of Blocks	Relative Frequency (adjusted percent)	Cumulative Frequency (adjusted percent)
7	1	0.1	0.1
8	6	0.3	0.4
9	9	0.5	0.8
10	46	2.4	3.3
11	82	4.3	7.6
12	121	6.4	14.0
13	172	9.1	23.1
14	163	8.6	31.7
15	132	7.0	38.6
16	135	7.1	45.8
17	109	5.8	51.5
18	84	4.4	55.9
19	71	3.7	59.7
20	74	3.9	63.6
21	85	4.5	68.1
22	51	2.7	70.8
23	50	2.6	73.4
24	49	2.6	76.0
25	39	2.1	78.0
26	28	1.5	79.5
27	23	1.2	80.7
28	40	2.1	82.8
29	36	1.9	84.7
30	33	1.7	86.5
31	29	1.5	88.0
32	28	1.5	89.5
33	26	1.4	90.9
34	21	1.1	92.0
35	15	0.8	92.8
36	12	0.6	93.4
37	12	0.6	94.0
38	10	0.5	94.6
39	11	0.6	95.1
40	9	0.5	95.6
40+	83	4.4	100.0
Total	1895	100.0	100.0
No Value Given[1]	413		
Total	2308		

[1]No value is given for blocks which contain no owner occupied houses or for blocks which contain so few that to release average value information would actually be releasing specific information, in violation of the confidentiality guaranteed by the census.

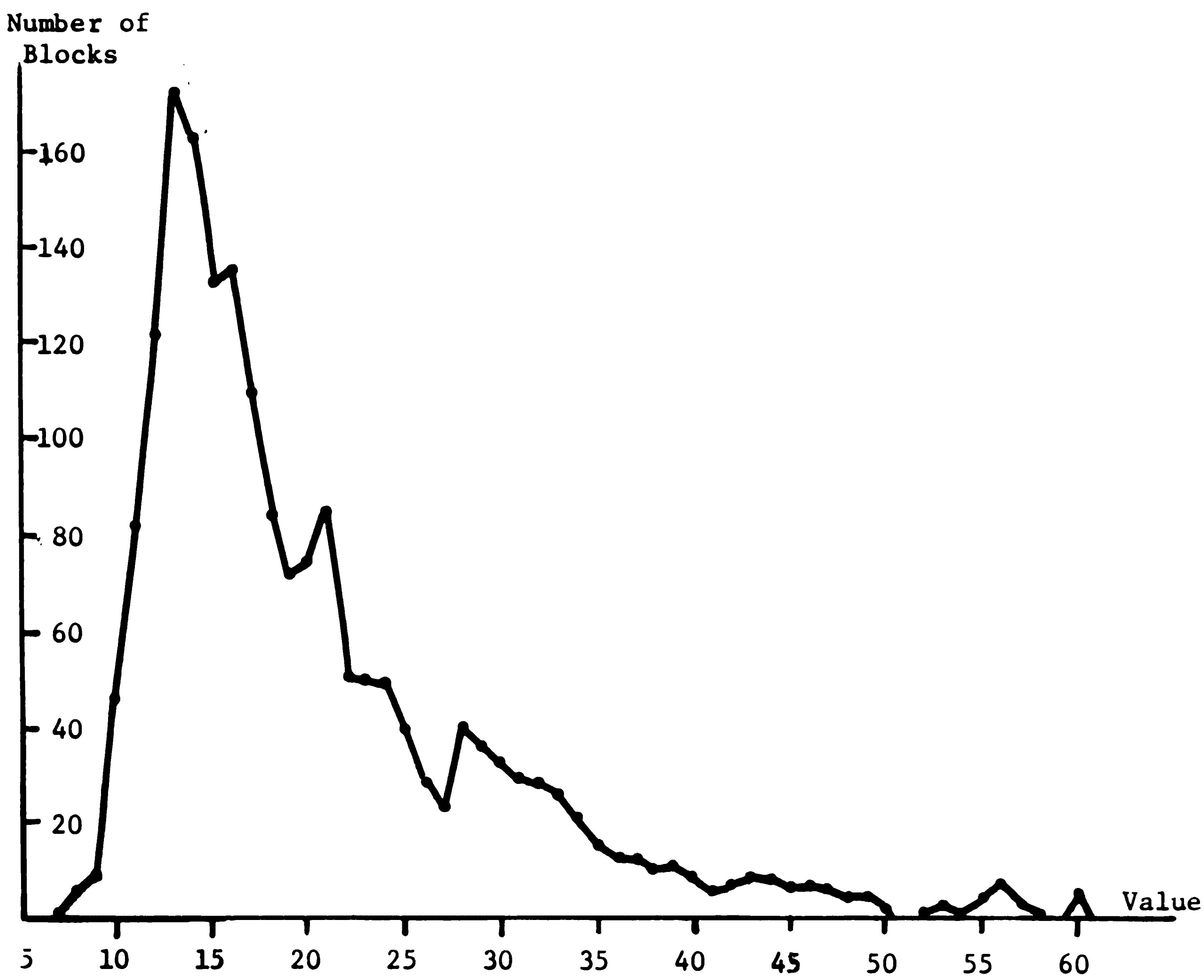

Figure 4.2. Average value of housing in thousands of dollars (Source: U.S. Bureau of the Census,1971)

Table 4.3

Average Monthly Rent

Average Rent (Dollars)	Number of Blocks	Relative Frequency (adjusted percent)	Cumulative Frequency (adjusted percent)
50	3	0.3	0.3
60	4	0.4	0.7
70	12	1.2	2.0
80	35	3.6	5.5
90	74	7.6	13.1
100	157	16.1	29.3
110	194	19.9	49.2
120	144	14.8	64.0
130	94	9.7	73.6
140	52	5.3	79.0
150	44	4.5	83.5
160	50	5.1	88.6
170	34	3.5	92.1
180	20	2.1	94.1
190	21	2.2	96.3
200	11	1.1	97.4
210	9	0.9	98.4
220	3	0.3	98.7
230	5	0.5	99.2
240	2	0.2	99.4
250	2	0.2	99.6
260	1	0.1	99.7
270	2	0.2	99.9
280	0	0.0	99.9
290	0	0.0	99.9
300	0	0.0	99.9
310	1	0.1	100.0
Total	974	100.0	100.0
No Value Given[1]	1334		
Total	2308		

[1]No value is given for blocks which contain no renter occupied dwellings or for blocks which contain so few that to release average rental information would actually be releasing specific information, in violation of the confidentiality guaranteed by the census.

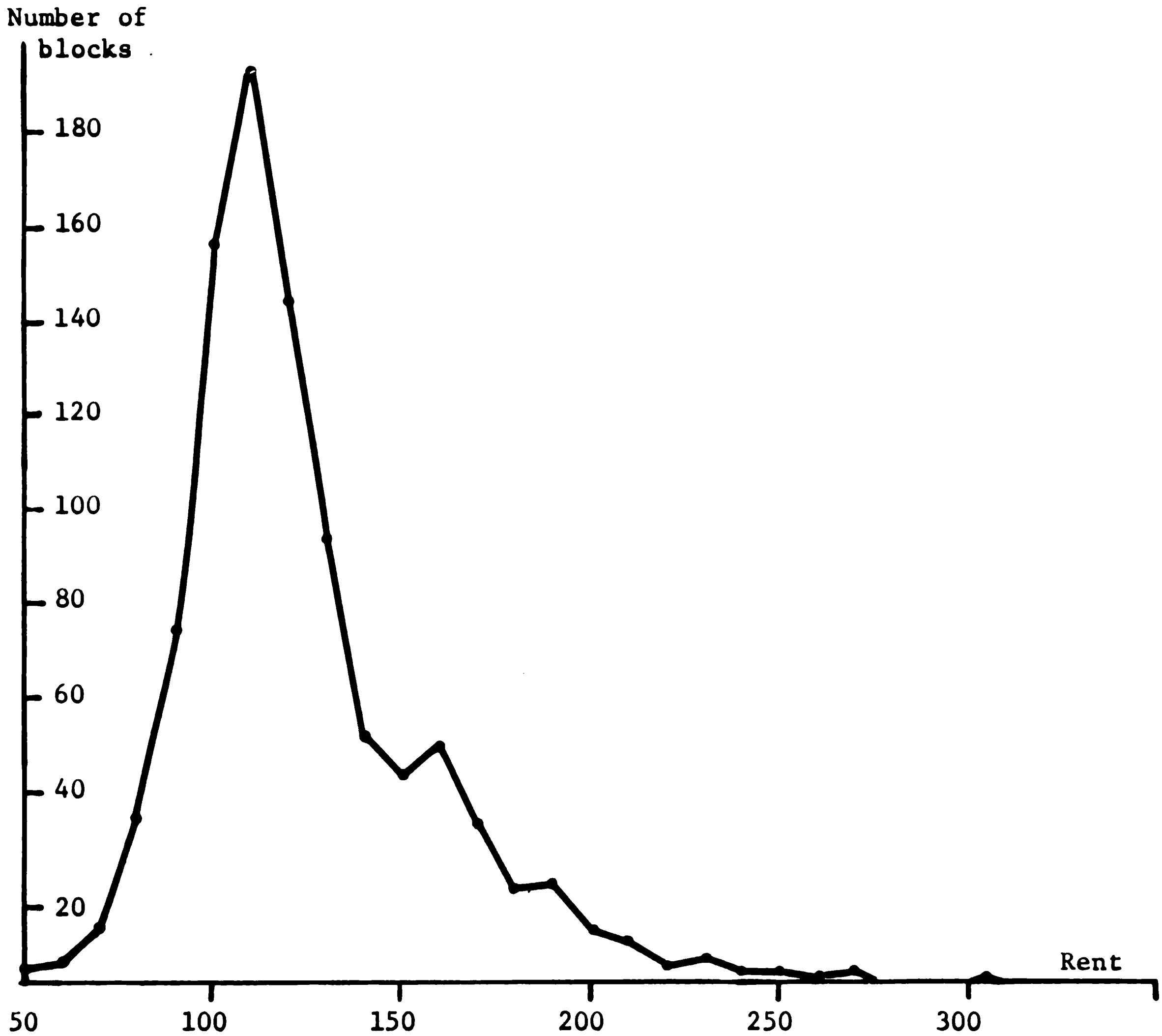

Figure 4.3. Average monthly rent in dollars (Source: U.S. Bureau of the Census, 1971)

Five levels of "average value" were determined, which were combined with four levels of "average contract rent," to yield the combined measure of rent and value.

$$\text{rent and value level} = \frac{\text{average value level} + \text{average rent level}}{2}$$

Table 4.4

Average Value Levels

Level	Number of Blocks	Relative Frequency	Cumulative Frequency
1. $ 6,500-14,499	600	31.7	31.7
2. 14,500-19,499	531	28.0	59.7
3. 19,500-27.499	399	21.0	80.7
4. 27,500-40,499	282	14.9	95.6
5. Over 40,500	83	4.4	100.0
Total	1895	100.0	100.0
No value given	413		
Total	2308	100.0	100.0

Table 4.5

Average Rent Levels

Level	Number of Blocks	Relative Frequency	Cumulative Frequency
1. $ 45- 94.99	128	13.1	13.1
2. 95-124.99	495	50.8	64.9
3. 125-149.99	190	19.5	84.4
4. Over 155	161	16.5	100.0
Total	974	100.0	100.0
No value given	1334		
Total	2308	100.0	100.0

The distribution of the tenure status indicator "proportion of housing owner occupied," was no more amenable to equal-sized divisions, as can be seen in Table 4.6 and Figure 4.4.

Table 4.6

Proportion of Housing Units Owner Occupied

Percentage Owner Occupied	Number of Blocks	Relative Frequency	Cumulative Frequency
0- 4.9	82	3.8	3.8
5- 9.9	42	1.9	5.7
10-14.9	42	1.9	7.6
15-19.9	51	2.3	9.9
20-24.9	49	2.3	12.2
25-29.9	43	2.0	14.2
30-34.9	43	2.0	16.2
35-39.9	52	2.4	18.6
40-44.9	38	1.7	20.3
45-49.9	69	3.2	23.5
50-54.9	65	3.0	26.5
55-59.9	89	4.1	30.6
60-64.9	92	4.2	34.8
65-69.9	136	6.3	41.1
70-74.9	119	5.5	46.6
75-79.9	161	7.5	54.1
80-84.9	208	9.6	63.7
85-89.9	222	10.2	73.9
90-94.9	264	12.2	86.1
95-99.9	75	3.5	89.6
100.0	226	10.4	100.0
Total	2173	100.0	100.0
No value given[1]	135		
Total	2308		

[1]No value is given for blocks where the release of general occupancy information would actually be releasing specific information, in violation of the confidentiality guaranteed by the census.

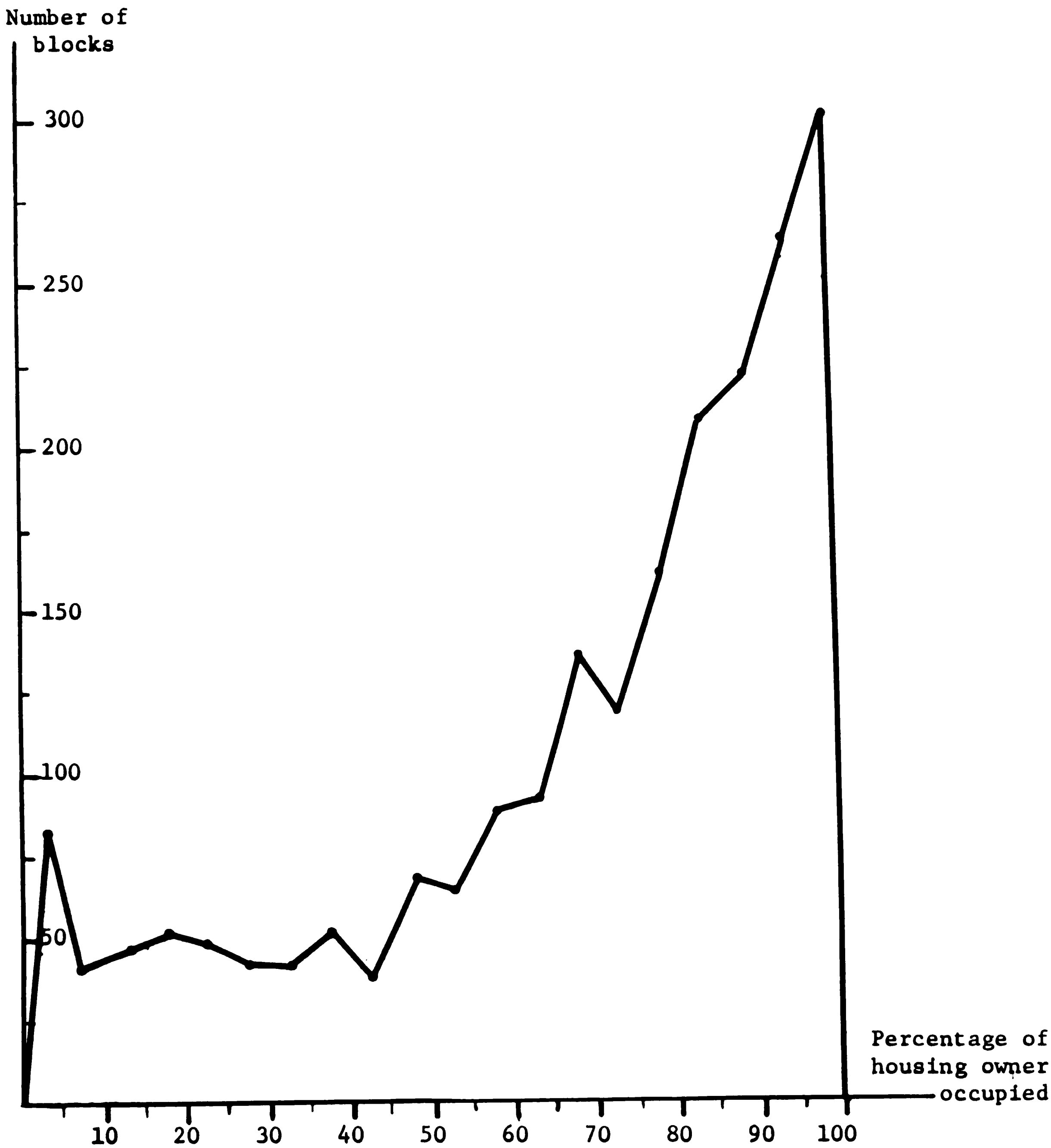

Figure 4.4. Proportion of housing owner occupied (Source: U.S. Bureau of the Census, 1971)

Table 4.7

Proportion Owner Occupied Levels

Level	Number of Blocks	Relative Frequency	Cumulative Frequency
1. 0-39.9%	409	18.8	18.8
2. 40-64.9	353	16.2	35.0
3. 65-84.9	624	28.7	63.7
4. 85-99.9	561	25.8	89.5
5. 100.0	226	10.4	100.0
Total	2173	100.0	100.0
No value given	135		
Total	2308	100.0	100.0

Five levels of "proportion of dwelling units owner occupied" were determined (Table 4.7) and combined with the above four levels of average rent and value to yield four housing sub-areas.

$$\text{sub-area} = \frac{\text{rent and value level} + \text{proportion owner occupied level}}{2}$$

This final combination resulted in the delimitation of the four housing sub-areas shown in Map 2. Certain "smoothing" procedures were followed in assigning all the blocks to a sub-area:

1) On the assumption that these characteristics are not randomly distributed in space, but rather that "sub-areas near one another have similar characteristics" (Hawkes, 1972:1219), blocks with

no information available were assigned to the stratum of the majority of their contiguous neighbors.[1]

2) Blocks for which there was information regarding only one or two of the criterion variables were assigned to strata according to the information available.

3) "Small islands" were not permitted. Groups of less than four continguous blocks of any stratum i, surrounded by stratum j, or strata j's, were converted to the appropriate stratum j by means of rule 1 above, applied recursively.

Characteristics of the sub-areas so defined are shown in Table 4.8 and graphically represented in Figures 4.5 through 4.10.

As can be seen in Table 4.8, the rank-ordering of each of the variables used in assigning city blocks to sub-areas is preserved, but it appears that some distinctions were based more on one variable than on the others. Sub-areas 1 and 2 differ not so much in terms of average value of housing ($14,400 and $15,300 respectively) or average rent ($110 and $121), but mostly in terms of

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[1]Blocks which shared a common border were considered contiguous. Blocks which shared a common point were not.

Table 4.8

Characteristics of Housing Sub-Areas

Area	Blocks Number	%	Housing Units Number	%	Population Number	%
1. Low	537	26.5	18,362	31.1	45,139	26.1
2. Lower Middle	864	42.6	24,535	41.6	73,783	42.7
3. Upper Middle	361	17.8	10,125	17.2	33,205	19.2
4. High	267	13.2	5,990	10.2	20,695	12.0
Total	2,029	100.1	59,012	100.1	172,822	100.0

Area	Mean Value of Housing	Mean Rent	Mean Proportion of Housing Owned
1. Low	$14,400	$110	.40
2. Lower Middle	15,300	121	.75
3. Upper Middle	23,700	169	.83
4. High	34,500	184	.88
Average	$19,100	$121	.69

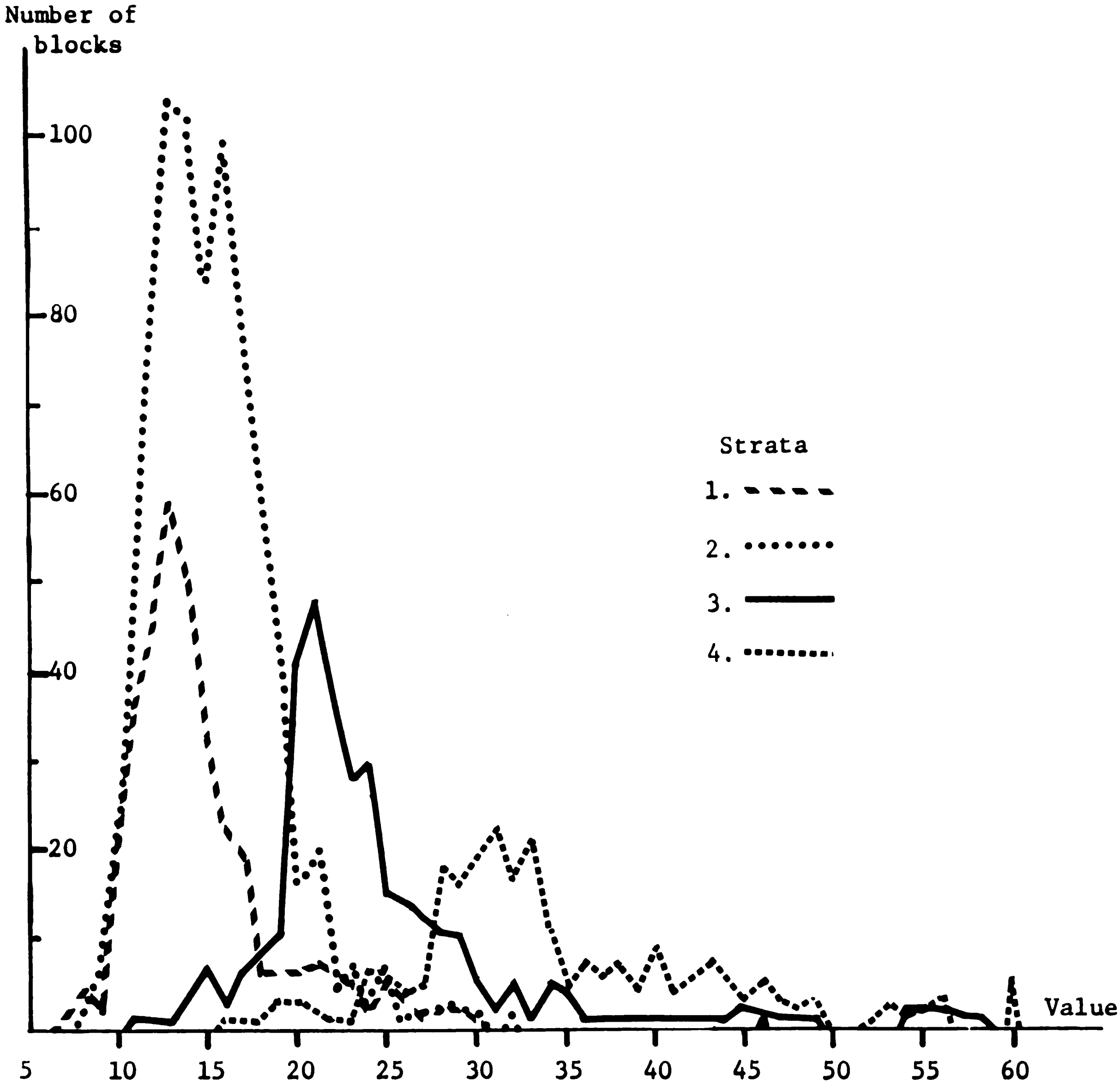

Figure 4.5. Average value of housing by stratum

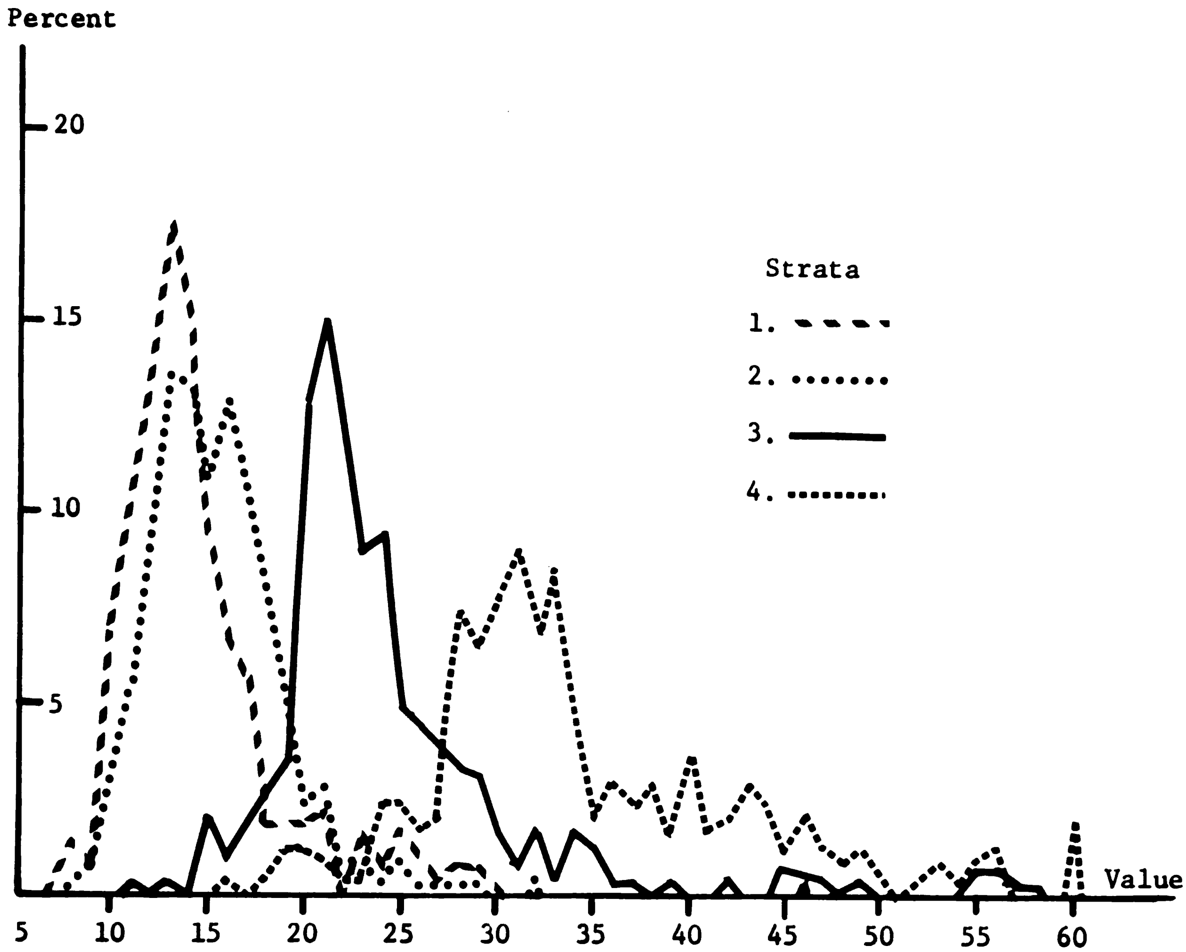

Figure 4.6. Average value of housing by stratum (in percent)

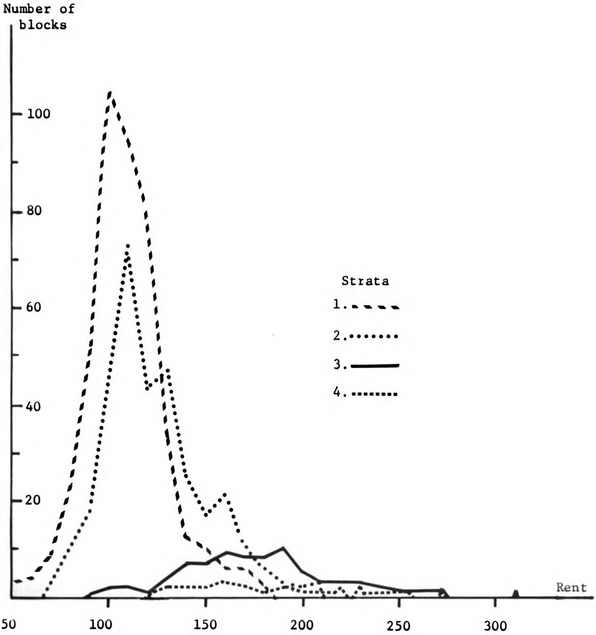

Figure 4.7. Average monthly rent by stratum

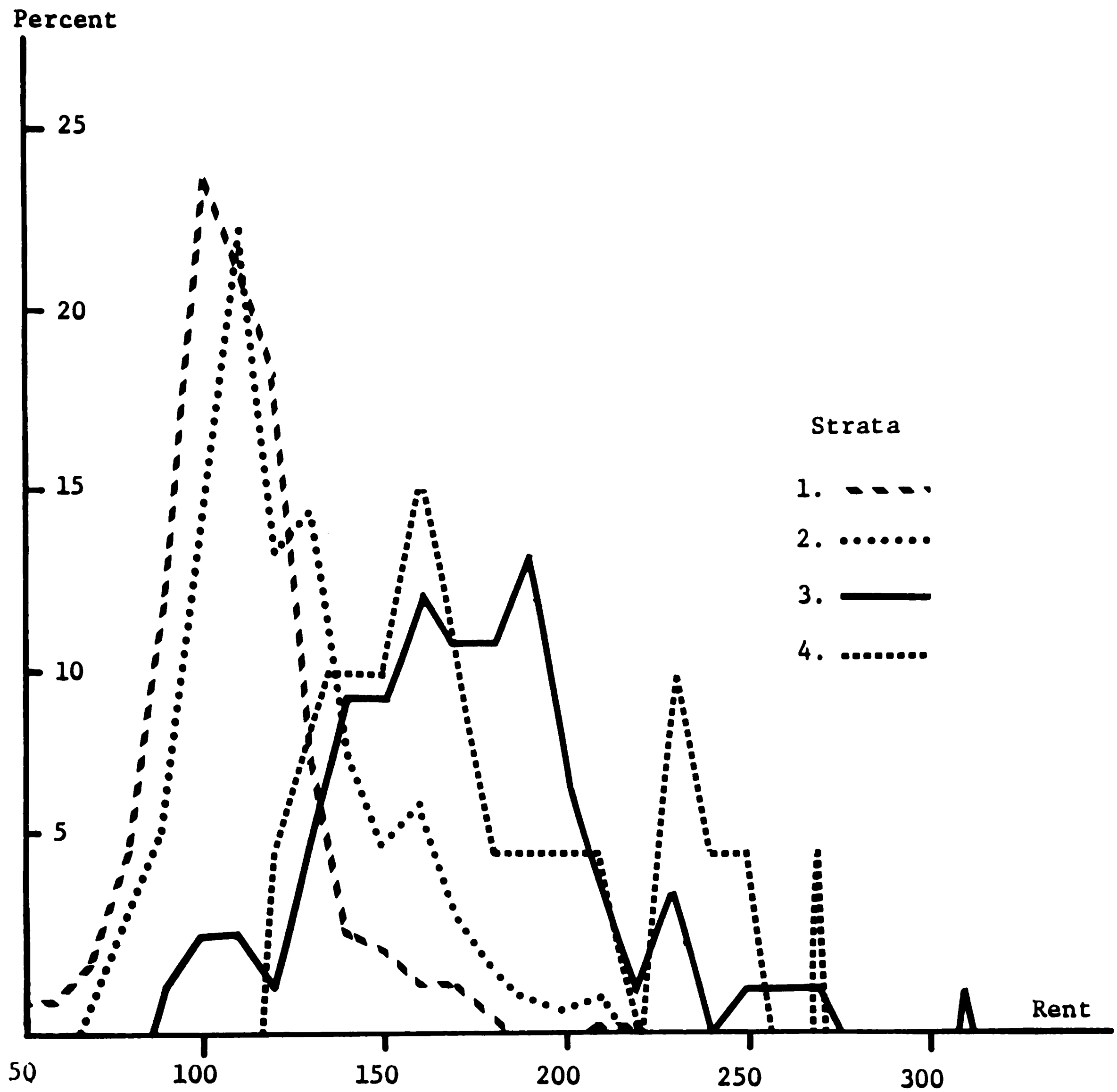

Figure 4.8. Average monthly rent by stratum (in percent)

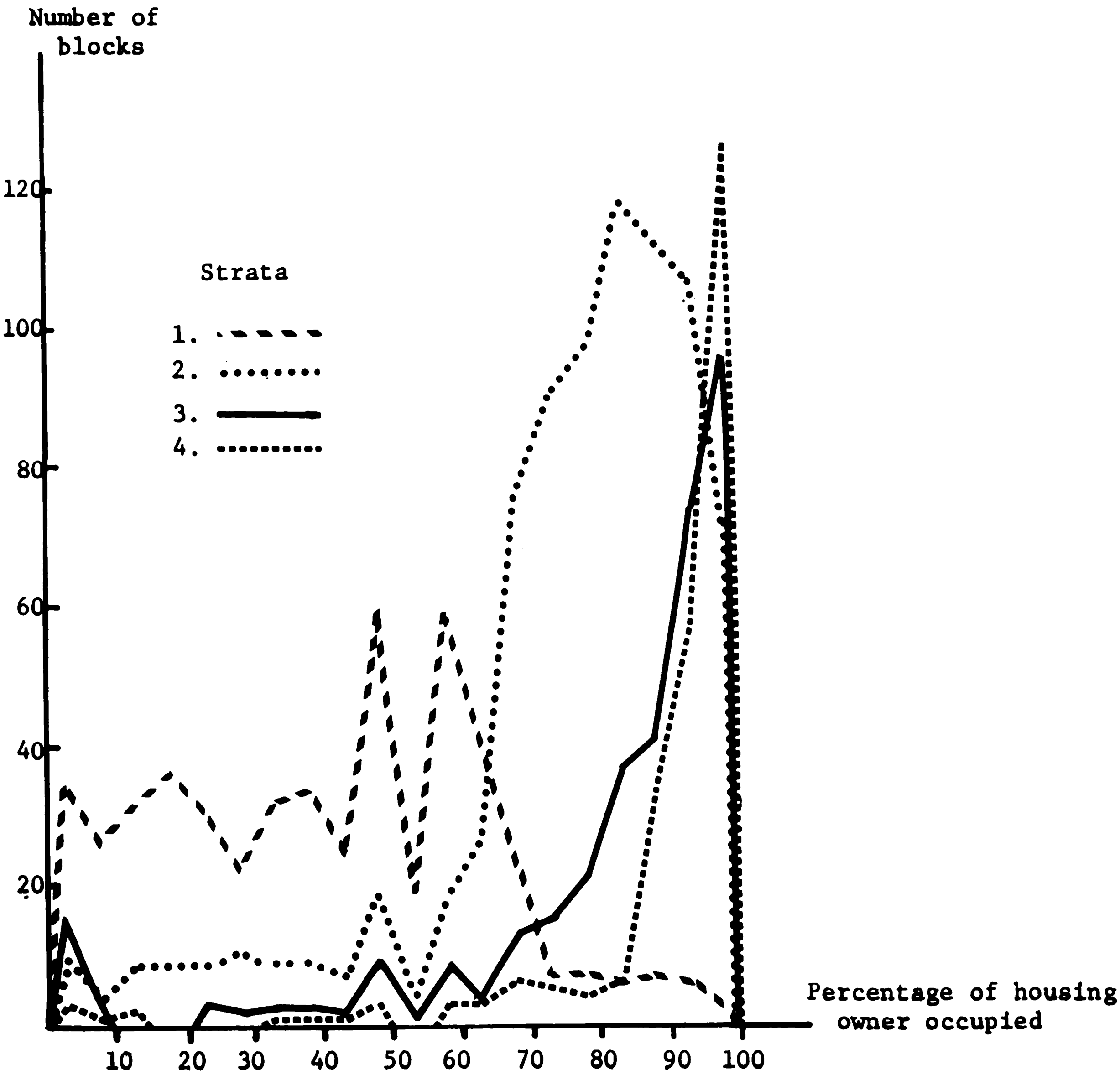

Figure 4.9. Proportion of housing owner occupied by stratum

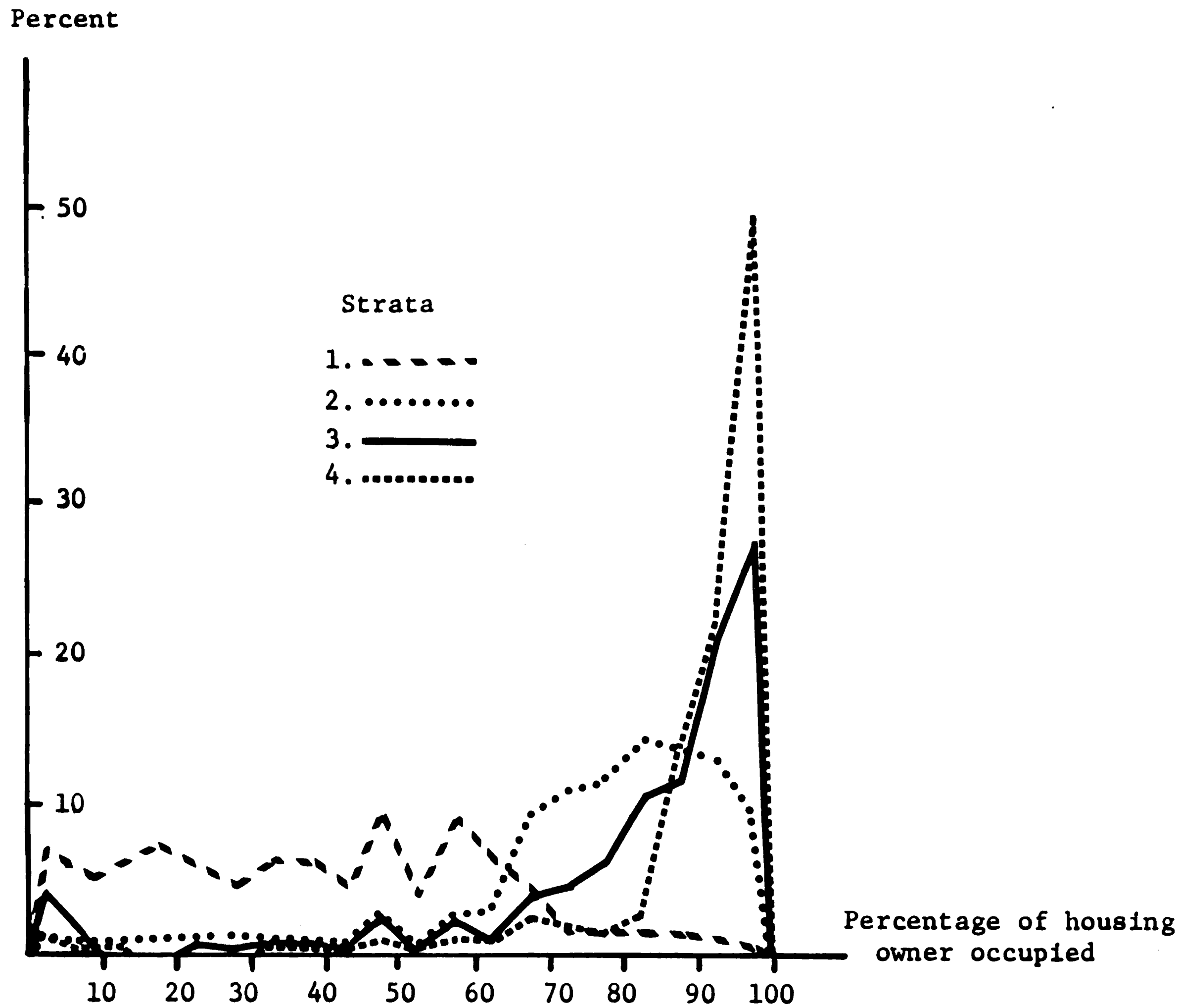

Figure 4.10. Proportion of housing owner occupied by stratum (in percent)

owner occupancy. Almost twice as great a proportion of homes per block are owned in sub-area 2, as are owned in sub-area 1 (75 percent compared to 40 percent). The differences in average value and rents are due, not surprisingly, to a greater proportion of homes in area 2 being valued between $16,000 and $20,000, and a greater proportion of rents between $130 and $160 per month.

There is a sizeable difference in average values and rents between sub-areas 3 and 2, however. On the average, homes in area 3 are valued at 55 percent more than homes in area 2 and rents are 40 percent more. Homes in sub-area 4 are valued at 45 percent more than homes in area 3, while rents in area 4 are less than 10 percent higher. Owner occupancy, on the average, is only 5 percent greater in area 4 than in area 3 and 13 percent greater than in area 2.

So it would appear than area 1 can be distinguished largely in terms of its lower proportion of homes owned -- almost 70 percent of all blocks have less than 50 percent of dwellings owner occupied, while the distinction between areas 2,3 and 4 is based largely on economic grounds. To a large extent this is true, but we cannot ignore the facts shown vividly in Figure 4.10: areas 3 and 4 both have more than 50 percent of their blocks with 90 percent owner occupancy -- in fact over 50 percent of blocks in area 4 are 95 percent owner occupied -- while area 2 has only 20 percent of blocks with 90 percent owner occupancy. The high average for area 2 is obtained with a high proportion of blocks of over 70 percent owner occupancy, and a very small proportion of blocks with less than 50 percent owner occupancy. The differences in these distributions should not be overlooked because of the similarities in their averages.

4.4: Estimation of the Transition Probabilities

The transition probabilities, the q_{ij} were estimated by the use of changes in the occupancy of dwelling units as reported in the Lansing City Directory (R.L. Polk and Co.). Sampling techniques were used which are felt to yield an unbiased sample of vacancy chains, even though the city directory is not a listing of vacancy chains, but addresses, and the population of vacancy chains from which our sample was drawn is "hidden."

Occupancy as reported in the city directory is generally considered to be an accurate "complete enumeration of the entire adult population of the community" (Goldstein, 1954:170). The data collection methods are quite thorough, involving, when necessary, two or more house calls, return postcards, telephone calls, and telephone calls to neighbors and reported places of work in order to identify occupants.[1] The accuracy of city directories is quite high, and there is substantial agreement among users that they

[1]Personal communication with R. L. Polk Detroit Production Manager, Mr. Head, who estimates the accuracy of the City Directory to be about 95% at the time of publication.

are reliable and useful sources of data (Albig, 1936; Goldstein, 1954; Ianni, 1957; Brown and Holmes, 1971). Comparisons of city directory and census counts of the adult male population of Norristown, Pennsylvania, show that "from 1930 on there is virtually 100 percent coverage by the directories" (Goldstein, 1954:172). In no year was the discrepancy more than 2.3 percent (Goldstein, 1954: 174). In the present case, an estimate of housing units in Lansing and East Lansing derived from the 1970 City Directory yields a total of 56,160 addresses. The census count of year-round housing units for 1970 is 56,494 (U.S. Bureau of the Census, 1971:1). The difference is slightly greater than one-half percent.

The directory lists by street address the names of household members over the age of 18 and indicates their occupation, place of work, marital status and tenure status. In addition, an alphabetical list of residents, with addresses and the above-mentioned other data, is provided.

In essence, the city directory is two directories, and it is this dual listing which permits us to infer vacancies and vacancy changes from changes in successive occupancies. The technique is as follows:

Consider that our sample of addresses consists of every nth address in the 1970 Lansing City Directory, and that the knth address is 123 First Street. The occupant of 123 First Street in 1970 is given as John Jones. This is then compared with the information reported in the 1969 directory. If we find that the occupant in 1969 is also John Jones, then no change and, consequently, no vacancy movement is said to have occurred.

However, if we find that the 1969 occupant of 123 First Street is someone other than John Jones, say, Peter Smith, then we conclude that a change of occupancy has occurred and that a vacancy must have passed through 123 First Street, and we proceed to trace out the complete vacancy chain. First we find the 1970 address for Peter Smith, and see that it is 456 Second Street. The 1969 occupant of 456 Second Street is given as Jane Johnston. Jane Johnston, however, is no longer listed in the 1970 directory and we infer that she left the housing system, and further, that this particular vacancy entered the system by her departure.

We then must complete the chain by tracing it out the other way, by finding John Jones' 1969 address. Let it be 789 First Street. The 1970 occupant of 789 First Street is Jack Wong, who is not listed in the 1969 directory, so we infer that he has just entered the housing system and it is by his entry that the vacancy leaves the system.

In this example, we have a vacancy entering the system at 456 Second Street (when Jane Johnston leaves the system), and moving then to 123 First Street, then to 789 First Street, and finally leaving the system from 789 First Street when Wong moves in. The process is represented pictorially (and perhaps more clearly) in Figure 4.11.

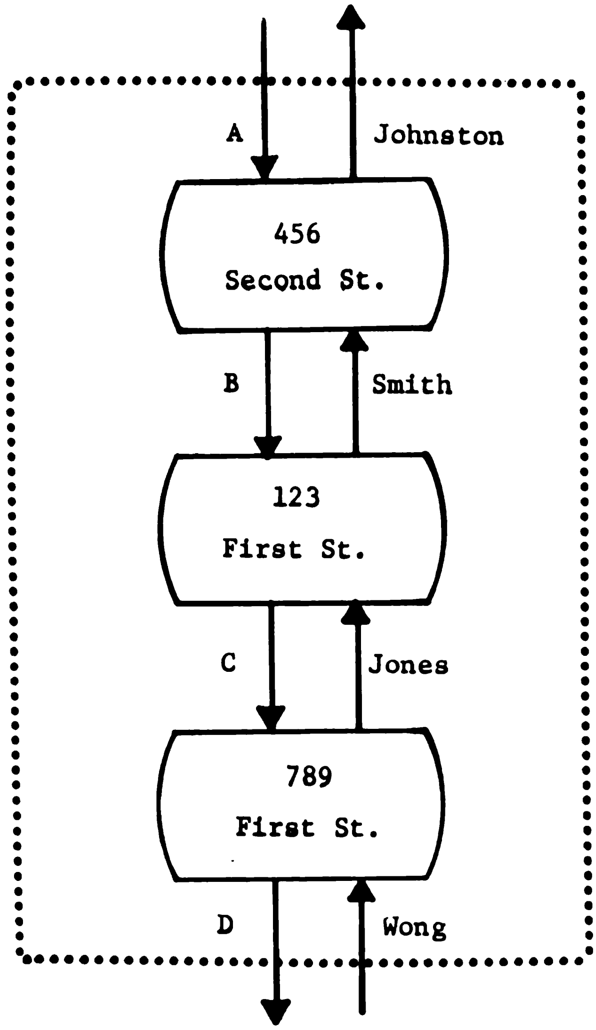

Figure 4.11. A residential vacancy chain

Each vacancy move (A, B, C, and D in Figure 4.11) is then assigned to a stratum of origin and destination according to its addresses of origin and destination, with the "outside" labelled stratum 0.

Information was also collected concerning the type of dwelling: apartment, house, townhouse; the marital status of the occupant: married, single, widow; tenure: owned, rented; and occupational status of movers. Occupational status was classified according to the Rice "modified white-collar, blue-collar code" (Robinson et al., 1969:342ff):

1) High status white-collar
2) Low status white-collar
3) High status blue-collar
4) Low status blue-collar
5) Farm occupations

with the addition of the codes

6) Student
7) Retired
8) Military.

The selection process described above insures that the address which is the beginning of a chain need not be the address initially sampled for the chain to be included in our sample of vacancy chains. If we find any address in a vacancy chain we must find the entire chain.

A 1/7 systematic sample of the addresses listed in the <u>1970 Lansing City Directory</u> was selected and the same proportion of addresses was selected from the <u>1965 City Directory</u>. If we can reasonably expect the proportion of addresses involved in moves, r, to be $.2 \leqslant r \leqslant .3$ (Taeuber, et al., 1961:826n) this would yield an estimated 1600 to 2400 vacancies in 1969-1970 and 1400 to 2100 vacancies in 1964-1965. Both sample sizes are large enough to permit the accurate estimation of overall vacancy rates ($r \pm .01$) as well as the q_{ik} (Cochran, 1963: Ch. 3, 4, 5, 5A).

The fact that our sample of vacancy chains was not derived from a sampling frame of vacancy chains, but from a sampling frame of vacancies, complicates only slightly the estimation of the transition probabilities. If the sample were drawn from a population of vacancy chains, such that all chains had an equal probability of being selected then the q_{ik} could be estimated by

$$\hat{q}_{ik} = \frac{a_{ik}(t)}{\sum_{h=0}^{s} a_{ih}(t)} \qquad (4.4.1a)$$

Where $a_{ik}(t)$ is the number of observed vacancy moves in a cohort from state i to state k.

$$\hat{f}_{k} = \frac{a_{0k}(t)}{\sum_{j=1}^{s} a_{0j}(t)} \qquad (4.4.2a)$$

could then be used as the estimator of the $f_k(t)$, the proportion of vacancy creations in stratum k.

However, because we are initially sampling vacancies, the chains do not have an equal probability of being selected. In fact, the probability of a chain of length γ being selected is exactly γ times as great as the probability of a chain of length 1, since there are γ times as many addresses in the chain of length γ. To compensate for this bias in the estimators of transition probabilities (4.4.1a and 4.4.2a), each vacancy move is assigned a weight equal to $\frac{1}{\gamma}$, where γ is the length of the chain the move appears in, so that the contribution of any particular move to both the numerator and the denominator of (4.4.1a) or (4.4.2a) is now $\frac{1}{\gamma}$.

Because of a small amount of non-response, the weighting factor $\frac{1}{\gamma}$ tends to underestimate the contribution of longer chains. Of the 8022 addresses sampled in 1970, 505 were listed as having submitted no return. This yields a response rate of $1 - \frac{505}{8022} = .937$. If "no-returns" are assumed to be independently distributed then the probability of a chain of length γ being completed is $.937^{\gamma}$. Thus, longer chains are more likely to be lost from the sample due to a failure to respond at any one of γ addresses, and a second weighting factor was introduced. Each vacancy move was therefore assigned a second weight, $\frac{1}{.937^{\gamma}}$, where γ is the length of the chain in which the move is found. The weighted contribution of any move from state i to state j was then

$$c_{ij} = \frac{1}{\gamma(.937^{\gamma})}$$

If we let $w_{ik}(t)$ be the sum of the observed weighted contributions of vacancy moves from state i to state k at time t, the resulting estimator of q_{ik} is

$$\hat{q}_{ik}(t) = \frac{w_{ik}(t)}{\sum_{h=0}^{s} w_{ih}(t)} \qquad (4.4.1b)$$

and the estimator of the proportion of vacancy chain creations is

$$\hat{f}_k(t) = \frac{w_{0k}(t)}{\sum_{j=1}^{s} w_{0j}(t)} . \qquad (4.4.2b)$$

Using these estimators, two vacancy models of residential mobility in Lansing - East Lansing were established, one based on $\hat{q}_{ik}$ for 1969-1970, and one based on the estimators for 1964-1965. These two models are discussed in Chapter 5.

5. VACANCY CHAINS IN LANSING-EAST LANSING

5.1: The General Model, 1969-1970

The sampling procedures described in Chapter 4 yielded a sample of 1397 complete vacancy chains distributed as shown in Table 5.1. The average length of chains was 1.378 moves with a standard deviation of .655 moves. The median length was 1.211 moves and the longest chains were 5 moves.

Table 5.1

Distribution of Unweighted Vacancy Chains by Length, 1969-1970

Length	Number of Chains	Proportion	Cumulative Proportion
1	983	.704	.704
2	318	.228	.931
3	81	.058	.989
4	12	.009	.998
5	3	.002	1.000
Total	1397	1.001	

Of these 1397 vacancy chains, 439 entered the system in stratum 1, while 560 arrived in stratum 2 and 289 and 109 chains made their entries in strata 3 and 4 respectively. The vector of vacancy arrivals by stratum, then

$$\underline{V}(t) = (439 \quad 560 \quad 289 \quad 109)$$

while the vector of vacancy departures,

$$\underline{D}(t) = (503 \quad 550 \quad 260 \quad 84).$$

As the two vectors show, slightly more vacancies left the system via stratum 1 than arrived there.

Table 5.2

Unweighted Vacancy Chain Arrivals and Departures by Stratum, 1969-1970

Stratum	Number of Arrivals	Proportion of Arrivals	Number of Departures	Proportion of Departures	Proportion of Total Housing
1. Low	439	.314	503	.360	.31
2. Lower Middle	560	.401	550	.394	.42
3. Upper Middle	289	.207	260	.186	.17
4. High	109	.078	84	.060	.10
Total	1397	1.000	1397	1.000	1.00

It should be noted that the proportion of vacancy arrivals and departures by stratum coincides very closely with the distribution of dwelling units within the strata. No stratum is undergoing a disproportionate inflow or outflow of vacancies, although stratum 4 appears slightly under-active.

Before these raw data can profitably be interpreted, the system of sampling weights must be taken into account. Because the sampling frame was not a frame of vacancy chains, chains of length γ were γ times as likely to be included in the sample as were chains of length 1. A second weighting factor was required to compensate for the loss of chains in the sample due to non-response. If the probability of collecting occupant information at any address is .937, then the probability of completing a chain of length γ is $.937^{\gamma}$. Consequently, a weighting factor, a function of chain length

$$W(\gamma) = \frac{1}{\gamma(.937)^{\gamma}}$$

was assigned to each vacancy move. Applying this weighting factor to our observed distribution of chain lengths yields the distribution of weighted chain lengths shown in Table 5.3. The frequency distributions of weighted and unweighted chain lengths are shown in Figure 5.1 below.

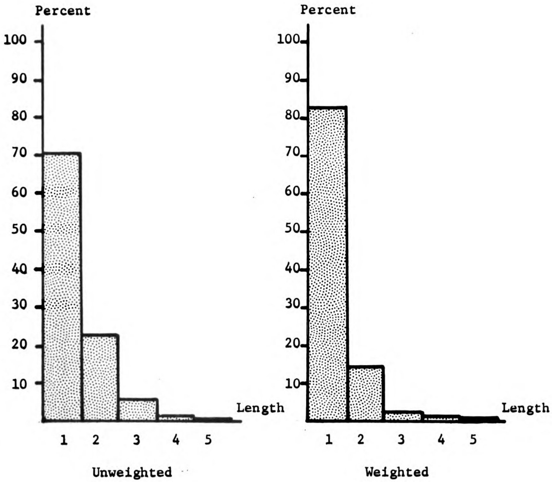

Figure 5.1. Unweighted and weighted chain length distributions, 1969-1970

Table 5.3

Distribution of Weighted Vacancy Chains by Length, 1969-1970

Length	Observed Number of Chains (A)	Weighting Factor (B)	Weighted Number of Chains (A x B)	Proportion	Cumulative Proportion
1	983	$\frac{1}{.937} = 1.0672$	1049.093	.829	.829
2	318	$\frac{1}{2(.937)^2} = .5695$	181.104	.142	.971
3	81	$\frac{1}{3(.937)^3} = .4052$	32.820	.024	.995
4	12	$\frac{1}{4(.937)^4} = .3242$	3.892	.004	.999
5	3	$\frac{1}{5(.937)^5} = .2769$	.831	.001	1.000
Total	1397		1267.739	1.000	

The mean length of weighted chains is 1.206 moves with a standard deviation of .490 moves. The median length is 1.104.

The pattern of weighted vacancy arrivals by stratum is similar to that of the raw chains,

$$\underline{F}(t) = (411.488 \quad 512.064 \quad 258.704 \quad 85.424),$$

as is the pattern of vacancy departures:

$$\underline{D}(t) = (441.564 \quad 508.141 \quad 244.896 \quad 73.072).$$

Though the patterns are similar, Table 5.4 shows slightly smaller differences between the number of arrivals and departures

Table 5.4

Weighted Vacancy Chain Arrivals and Departures by Stratum, 1969-1970

Stratum	Number of Arrivals	Proportion of Arrivals	Number of Departures	Proportion of Departures
1. Low	411.488	.325	441.564	.348
2. Lower Middle	512.064	.404	508.141	.401
3. Upper Middle	258.704	.204	244.896	.193
4. High	85.424	.067	73.072	.058
Total	1267.680[1]	1.000	1267.680[1]	1.000

[1]Discrepancy with Table 5.3 due to rounding.

within strata for the weighted chains, and also that area 4 is under-represented both in vacancy arrivals and departures. The implications of this latter fact will be discussed below.

Disaggregation of the 1397 raw vacancy chains into their 1925 constituent moves and weighting them as described in section 4.4 yielded the estimators of the matrix

$$\underline{Q} = [q_{ik}]$$

and the vector

$$\underline{p} = [q_{i0}] \; .$$

The 1397 entrance moves were weighted and used to estimate the proportion of vacancy arrivals by stratum, $\underline{f}(t)$. The vectors and matrices describing the 1969-1970 model are presented below:

$$\underline{F}(t) = (411.488 \quad 512.064 \quad 258.704 \quad 85.424)$$

$$\underline{f}(t) = (.325 \quad .404 \quad .204 \quad .067)$$

$$\underline{Q} = \begin{pmatrix} .095 & .032 & .012 & .004 \\ .057 & .084 & .019 & .006 \\ .050 & .062 & .050 & .020 \\ .049 & .098 & .080 & .083 \end{pmatrix}$$

$$\underline{P} = \begin{pmatrix} .857 \\ .835 \\ .817 \\ .690 \end{pmatrix}$$

$$(\underline{I}-\underline{Q})^{-1} = \begin{pmatrix} 1.109 & .040 & .015 & .005 \\ .070 & 1.096 & .023 & .008 \\ .065 & .077 & 1.057 & .024 \\ .073 & .126 & .095 & 1.093 \end{pmatrix}$$

$$\underline{P}_j = \begin{pmatrix} .857 & .121 & .019 & .003 & 0 \\ .835 & .138 & .023 & .004 & .001 \\ .817 & .150 & .027 & .005 & .001 \\ .690 & .246 & .052 & .010 & .002 \end{pmatrix}$$

The main tests of the fit of the vacancy model are the predictions of chain length distributions for the model taken as a whole, and predicted distributions across strata (White, 1970:33ff.). The prediction of vacancy chain lengths for the entire model is given by

$$\underline{F}(t)\underline{P}_j$$

and the vector of mean chain lengths by stratum of arrival is given by

$$\underline{\lambda} = \sum_{j=1}^{\infty} j\underline{P}_j = (\underline{I}-\underline{Q})^{-1}\ \underline{1}\ . \qquad (3.2)$$

The average length of vacancy chains for the entire model is given simply by

$$j(t) = \underline{f}(t)\ \underline{\lambda}\ . \qquad (3.3)$$

We can also compare the expected number of predicted vacancy moves ever made from a stratum in our system with the observed total moves made from strata by examining

$$\underline{M}(t) = \underline{F}(t)\ (\underline{I}-\underline{Q})^{-1} \qquad (3.4)$$

and $\underline{O}(t)$, the vector of observed vacancy moves by stratum. We can also compare the total number of moves predicted, M(t), with the total number of observed moves, O=(t).

The first test, the comparison of actual and predicted chain length distributions for the model is presented in Table 5.5. Although there are no statistical tests which can properly be applied to this data, an index of dissimilarity was computed to facilitate the comparison of the distributions. The index of dissimilarity is

$$\Delta = \frac{1}{2}\,\Sigma\,\left|(P_{1i} - P_{2i})\right|$$

where P_{1i} and P_{2i} are the proportions of cases found in state i in distributions 1 and 2 respectively. "Put as simply as possible, the Index of Dissimilarity indicates the minimum proportion in one or the other population which would have to change categories in order for the two distributions to be identical" (Matras, 1973: 157). The particular utility of the index of dissimilarity is that it allows the comparison of entire distributions and not simply central tendencies.

The congruence between observed and predicted chain length distributions seen in Table 5.5 is exceptional! There are virtually no differences between the two, and in fact, the predicted mean chain length, j(t), is identical with the observed mean chain length, $\bar{X} = 1.206$. The index of dissimilarity, $\Delta = .003$.

Table 5.5

Comparison of Observed and Predicted Chain Length Distributions for the Complete Model, 1969-1970

Chain Length	Observed Frequency	Predicted Frequency ($\underline{F}(t)\underline{P}_j$)	Observed Proportion	Predicted Proportion ($\underline{f}(t)\underline{P}_j$)
1	1049.093	1050.522	.828	.829
2	181.104	180.275	.143	.142
3	32.820	31.023	.026	.024
4	3.892	5.430	.003	.004
5	.831	.171	.001	.000
Total	1267.739	1267.421	1.001	.999

$\bar{X} = 1.206$ $j(t) = 1.206$

$\Delta = .003$

This extreme goodness-of-fit is reflected in the other tests of the model. A comparison of $\underline{\lambda}$, the predicted average length of chain by stratum of origin, and $\underline{L}$, the observed average length of chain by stratum, is shown in Table 5.6. In only one of the strata, stratum 4, does the discrepancy between predicted and observed values exceed one-half-percent, and this occurs in the stratum with the fewest vacancy creations (85.423).

Table 5.6

Comparison of Observed ($\underline{L}$) and Predicted ($\underline{\lambda}$) Mean Chain Lengths by Stratum of Origin, 1969-1970

Stratum	Mean Chain Length		Difference as a Proportion of $\underline{L}$
	Observed ($\underline{L}$)	Predicted ($\underline{\lambda}$)	($\lvert L_i-\lambda_i\rvert/L_i$)
1. Low	1.165	1.169	.003
2. Lower Middle	1.203	1.198	.004
3. Upper Middle	1.228	1.223	.004
4. High	1.447	1.388	.006

The last test, a comparison of the predicted total number of moves ever made from a stratum

$$\underline{M}(t) = \underline{F}(t)\ (\underline{I}-\underline{Q})^{-1}$$

with the observed total number of moves made from strata, $\underline{O}(t)$, also indicates an excellent fit.

$$\underline{M}(t) = (411.488 \quad 512.064 \quad 258.704 \quad 85.424) \begin{pmatrix} 1.109 & .040 & .015 & .005 \\ .070 & 1.096 & .023 & .008 \\ .065 & .077 & 1.057 & .024 \\ .073 & .126 & .095 & 1.093 \end{pmatrix}$$

$$\underline{M}(t) = (515.421 \quad 608.635 \quad 299.515 \quad 105.731$$

and the observed number of moves by stratum of origin

$$\underline{O}(t) = (515.135 \quad 608.573 \quad 299.813 \quad 105.891).$$

Obviously, the total numbers of observed and predicted vacancy moves coincide. $\underline{M}(t) = 1529.438$ and $\underline{O}(t) = 1529.412$

$$\frac{|\underline{O}(t) - \underline{M}(t)|}{\underline{O}(t)} = \frac{|1529.412 - 1529.438|}{1529.412} = .00002$$

In addition to the excellent fit of the model, one other finding should be noted here — the extremely high proportion of vacancy chains that leave the Lansing-East Lansing area in their first move. This proportion can be computed by

$$\underline{f}(t)\ \underline{p} = (.325 \quad .404 \quad .204 \quad .067) \begin{pmatrix} .857 \\ .835 \\ .817 \\ .690 \end{pmatrix}$$

$$\underline{f}(t)\ \underline{p} = .829$$

Over 80 percent of vacancies leave the system on their first move, so that very little vacancy movement is generated by the entrance of vacancies into the system. This low level is indicated by the value of $j(t) = 1.206$ -- each vacancy entrance generates, on the average, 1.206 vacancy moves. This is a result of the extreme shortness of vacancy chains originating in the lower strata, and the small numbers of chains originating in the strata with the highest average lengths or multipliers. Sub-area 4, has the highest multiplier, $\lambda_4 = 1.388$, but only 6.7 percent of vacancies enter the system at this point.

5.2: The General Model, 1964-1965

The frequency distribution of unweighted vacancy chains sampled in 1964-1965 is shown in Table 5.7. The 805 chains had an average length of 1.256 moves, with a standard deviation of .805 moves. The median length was 1.257 moves and the longest chains were of length 6. The arrival and departure distributions of the unweighted chains are shown in Table 5.8.

Table 5.7

Distribution of Unweighted Vacancy Chains by Length, 1964-1965

Length	Number of Chains	Proportion	Cumulative Proportion
1	532	.661	.661
2	189	.235	.896
3	59	.073	.969
4	19	.024	.993
5	4	.005	.998
6	2	.002	1.000
Total	805	1.000	

Table 5.8

Unweighted Vacancy Chain Arrivals and Departures by Stratum, 1964-1965

Stratum	Number of Arrivals	Proportion of Arrivals	Number of Departures	Proportion of Departures	Proportion of Total Housing
1. Low	264	.328	310	.385	.31
2. Lower Middle	328	.407	322	.400	.42
3. Upper Middle	147	.183	129	.160	.17
4. High	66	.082	44	.055	.10
Total	805	1.000	805	1.000	1.00

It should be noted that, just as in 1969-1970, the proportions of vacancy arrivals by strata follow very closely the interstratum distribution of housing units. The higher level of vacancy departures from stratum 1 in 1964-1965 seems to indicate a higher level of vacancy activity in this time period than in 1969-1970.

Weighting procedures identical to those used in 1969-1970 were applied to the observed distribution of chain lengths for 1964-1965 to yield the distribution of weighted chain lengths presented in Table 5.9. The mean length of weighted chains is 1.256 with a standard deviation of .60 moves. The median length is 1.123 moves. A comparison of the distributions of unweighted and weighted chains is shown graphically in Figure 5.2.

The relative distributions of the unweighted and weighted chains are not unlike their counterparts for 1969-1970, although the difference between them is somewhat greater for 1964-1965. This is due to the smaller porportion of unweighted chains of length 1 in 1964-1965.

Table 5.9

Distribution of Weighted Vacancy Chains by Length, 1964-1965

Length	Observed Number of Chains (A)	Weighting Factor (B)	Weighted Number of Chains (A x B)	Proportion	Cumulative Proportion
1	532	$\frac{1}{.937}$ =1.0672	567.769	.803	.803
2	189	$\frac{1}{2(.937)^2}$ = .5695	107.637	.152	.955
3	59	$\frac{1}{3(.937)^3}$ = .4052	23.906	.034	.989
4	19	$\frac{1}{4(.937)^4}$ = .3242	6.162	.009	.998
5	4	$\frac{1}{5(.937)^5}$ = .2769	1.108	.002	.999
6	2	$\frac{1}{6(.937)^6}$ = .2463	.493	.001	1.000
Total	805		707.075	1.001	

The distribution of weighted vacancy chain arrivals and departures is much the same as in the unweighted case, except that the discrepancy between rates of arrival and departure within strata has been reduced, though more vacancies still exit the system through stratum 1 than enter it there, and stratum 4 is still under-represented both in vacancy arrivals and departures.

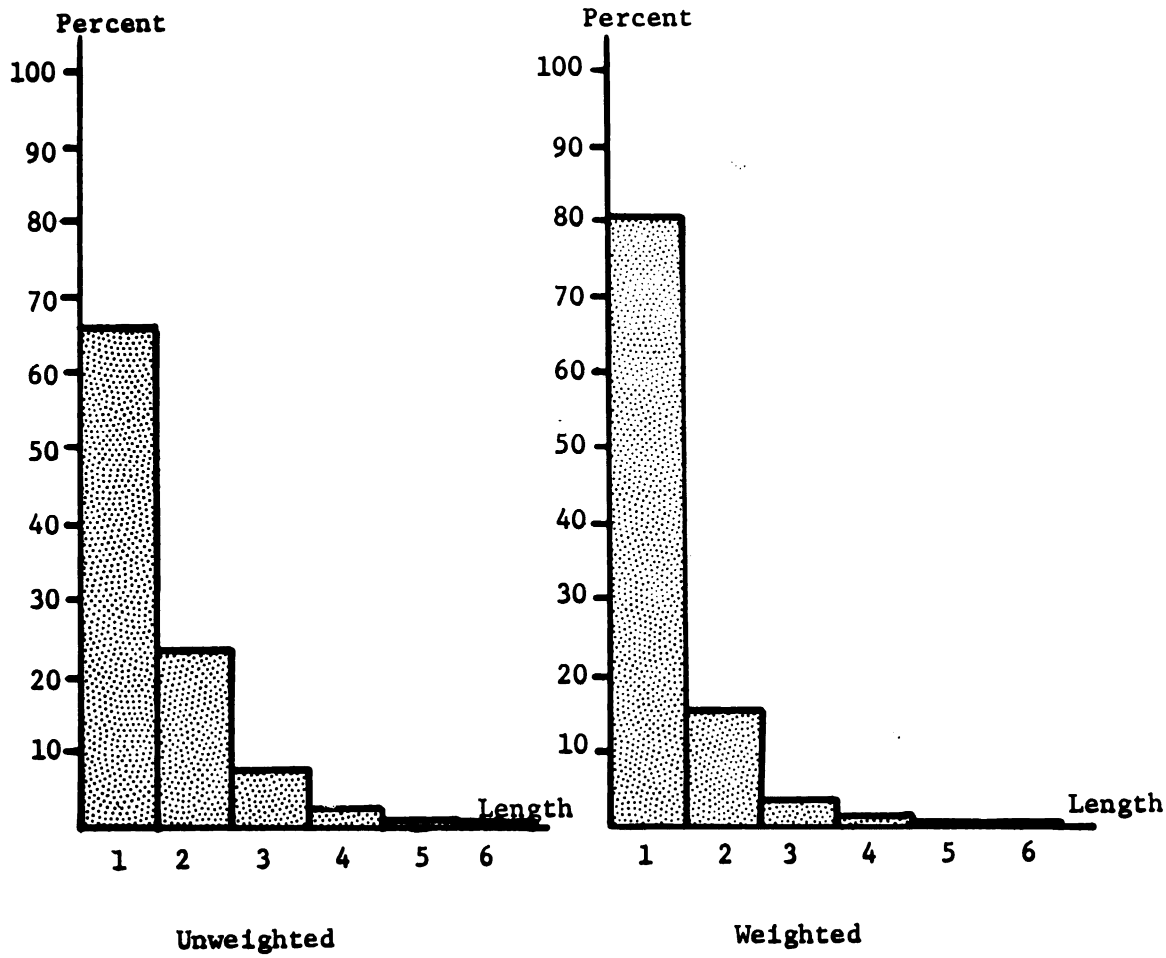

Figure 5.2. Unweighted and weighted chain length distributions, 1964-1965

Table 5.10

Weighted Vacancy Chain Arrivals and Departures by Stratum, 1964-1965

Stratum	Number of Arrivals	Proportion of Arrivals	Number of Departures	Proportion of Departures
1. Low	236.625	.335	259.159	.367
2. Lower-Middle	290.537	.411	286.725	.406
3. Upper-Middle	127.230	.180	118.366	.167
4. High	52.672	.074	42.812	.061
Total	707.064[1]	1.000	707.062[1]	1.001

[1]Discrepancy with Table 5.9 due to rounding.

The 805 chains were disaggregated and the 1195 vacancy moves and the 805 entrance moves were weighted and used to estimate the transition probabilities of the 1964-1965 vacancy chain model. The model has the following characteristics:

$$\underline{F}(t) = (236.625 \quad 290.537 \quad 127.230 \quad 52.672)$$

$$\underline{f}(t) = (.335 \quad .411 \quad .180 \quad .075)$$

$$\underline{Q} = \begin{pmatrix} .138 & .039 & .011 & 0 \\ .073 & .108 & .019 & .003 \\ .064 & .094 & .050 & .009 \\ .041 & .064 & .107 & .052 \end{pmatrix}$$

$$\underline{p} = \begin{pmatrix} .813 \\ .797 \\ .783 \\ .737 \end{pmatrix}$$

$$(\underline{I}-\underline{Q})^{-1} = \begin{pmatrix} 1.165 & .053 & .141 & 0 \\ .098 & 1.128 & .024 & .004 \\ .089 & .116 & 1.057 & .010 \\ .066 & .092 & .121 & 1.056 \end{pmatrix}$$

$$\underline{P}_j = \begin{pmatrix} .813 & .151 & .029 & .006 & .001 & 0 \\ .797 & .162 & .032 & .006 & .001 & 0 \\ .783 & .172 & .035 & .007 & .001 & 0 \\ .737 & .206 & .046 & .009 & .002 & 0 \end{pmatrix}$$

The comparison of predicted chain length distributions with observed is shown in Table 5.11.

Table 5.11

Comparison of Observed and Predicted Chain Length Distributions for the Complete Model, 1964-1965

Chain Length	Observed Frequency	Predicted Frequency ($\underline{F}(t)\underline{P}_j$)	Observed Proportion	Predicted Proportion ($\underline{f}(t)\underline{P}_j$)
1	567.769	562.374	.803	.796
2	107.637	115.531	.152	.164
3	23.906	23.035	.034	.033
4	6.162	4.528	.009	.006
5	1.108	.760	.002	.001
6	.493	0	.001	0
Total	707.075	706.228	1.001	1.000

$\bar{X} = 1.256$ $\quad$ $\mathrm{j}(t) = 1.256$

$\Delta = .011$

The goodness of fit indicated in Table 5.11 above is supported by the comparison of $\underline{\lambda}$ and $\underline{L}$ shown in Table 5.12. The largest discrepancy between observed and predicted chain lengths by stratum of arrival is only .016, and this discrepancy occurs in stratum 4, where the number of vacancy creations is smallest -- only 52.722 vacancy chains, 7.4 percent of the total, began their careers in stratum 4.

Table 5.12

Comparison of Observed ($\underline{L}$) and Predicted ($\underline{\lambda}$) Mean Chain Lengths by Stratum of Origin, 1964-1965

Stratum	Mean Chain Length Observed	Mean Chain Length Predicted	Difference as a Proportion of $\underline{L}$ ($\lvert L_i-\lambda_i\rvert / L_i$)
1. Low	1.232	1.215	.014
2. Lower Middle	1.253	1.254	.001
3. Upper Middle	1.272	1.276	.003
4. High	1.335	1.377	.016

Finally, the predicted total number of moves ever made from a stratum, $\underline{M}(t)$ was compared with the observed distribution of moves made from strata, $\underline{O}(t)$.

$$\underline{M}(t) = \underline{F}(t)\ (\underline{I}-\underline{Q})^{-1}$$

$$\underline{M}(t) = (318.954 \quad 359.725 \quad 151.096 \quad 58.098)$$

While

$$\underline{O}(t) = (318.939 \quad 359.660 \quad 151.100 \quad 58.118).$$

The predicted total number of moves M(t) is simply the sum of the $M_i(t)$, so that

$$M(t) = 887.873$$

and $$O(t) = 887.818.$$

This represents a discrepancy of

$$\frac{887.873-887.818}{887.818} = .0001.$$

The fit of the 1964-1965 model to the data is quite good.

In general terms, the model is very similar to the 1970 model. The proportion of vacancy chains leaving the system in their first move is very high:

$$\underline{f}(t)\ \underline{p} = .796$$

and the number of moves generated by entering vacancies is correspondingly low. The multiplier $j(t) = 1.256$. Although the multiplier varies among strata with a low of $\lambda_1 = 1.232$ and a high value of $\lambda_4 = 1.335$, the stratum with the largest multiplier has the lowest proportion of vacancy arrivals ($f_4 = .074$), so that the overall multiplier is only minimally influenced by it.

5.3: The Basic Models -- Discussion

In the most general terms, the two models described above, the complete vacancy chain models for 1969-1970 (to be called simply the 1970 model) and 1964-1965 (the 1965 model) are very similar. If one compares the two predicted chain length distributions, as in Table 5.13, there is little to choose between them. In fact the index of dissimilarity is equal to .026.

Table 5.13

Predicted Chain Length Distributions, 1969-1970 and 1964-1965

	1969-1970		1964-1965	
Length	Number	Proportion	Number	Proportion
1	1050.522	.829	567.769	.803
2	180.275	.142	107.637	.152
3	31.023	.024	23.906	.034
4	5.430	.004	6.162	.009
5	.170	.001	1.108	.002
6	0	0	.493	.001
Total	1267.420	1.000	707.075	1.001

$j(t) = 1.206$ $j(t) = 1.256$

$\Delta = .026$

The mean chain length for 1964-1965 is marginally longer, but only two and one-half percent of the population would have to change categories for the two distributions to be identical.

Both models are characterized by an extremely high degree of fit, indicated by all tests. The comparisons of predicted and observed chain length distributions, mean chain lengths by stratum, and moves made from strata are all very close. In none of these comparisons does the discrepancy between observed and predicted values exceed 3 percent. These findings lead one to conclude that the housing vacancy transfers occurring here can be modelled adequately by a first-order Markov chain. The problems of state classification and non-independence of transitions which seem to confound models of people as the population of movers (cf. Chapter 2) seem not to have arisen. In fact the fit is so good that one begins to suspect that state classification plays very little part in the determination of the model other than in terms of housing policy and that the more salient criterion for goodness-of-fit in the first-order Markov chain is the independence of the vacancy moves. If the moves are being made independently, then, the state classification is important only in terms of substantive theory. Mathematically, the classification is arbitrary.

That the vacancy moves are independent can perhaps be supported by the following oversimplification of the 1970 model: Let the housing system consist of only one state. The model can then be characterized as a series of Bernoulli trials with the probability of leaving the system on any trial

$$p = \frac{1}{\bar{j}(t)} = \frac{1}{1.206} = .829.$$

The probability of remaining in the system on any trial is then simply

$$q = 1-p = .171.$$

The distribution of chain lengths is then given by the function

$$P(j) = q^{j-1}p \ .$$

The number of chains expected at each length

$$N(j) = Nq^{j-1}p$$

where N is the total number of vacancy arrivals, 1267.739.

Table 5.14

Comparison of Observed and Expected Chain Length Distributions, Using $N(j) = Nq^{j-1}p$, 1970

Chain Length	Predicted Number ($Nq^{j-1}p$)	Predicted Proportion ($q^{j-1}p$)	Observed Number	Observed Proportion
1	1050.951	.829	1049.093	.828
2	179.713	.142	181.104	.143
3	30.731	.024	32.820	.026
4	5.255	.004	3.892	.003
5	.899	.001	.831	.001
6	.154	**	0	0
7 or more	.036	**	0	0
	1267.739	1.000	1267.739	1.001

$\Delta = .003$

** Less than .001

The findings for the 1965 model are identical. With

$$p = \frac{1}{j(t)} = \frac{1}{1.256} = .796$$

and N = 707.075, the Bernoulli trials model yields a predicted distribution of chain lengths which fits the data extremely closely.

The excellent fit of the simplified model serves not only to lend credence to the assumption of independent mobility of vacancies, but also to illuminate the extreme goodness-of-fit of the articulated model. To a certain extent the fit is an artifact of the low average chain length, or the high exit probabilities of vacancy chains in Lansing-East Lansing. The model is not tautological, to be sure, but

Table 5.15

Comparison of Observed and Expected Chain Length Distributions, Using $N(j) = Nq^{j-1}p$, 1965

Chain Length	Predicted Number ($Nq^{j-1}p$)	Predicted Proportion ($q^{j-1}p$)	Observed Number	Observed Proportion
1	562.832	.796	567.769	.803
2	114.818	.162	107.637	.152
3	23.423	.033	23.906	.034
4	4.778	.007	6.162	.009
5	.975	.001	1.108	.002
6	.199	**	.493	.001
7 or more	.050	**	0	0
Total	707.075	.999	707.075	1.001

$\Delta = .011$

**Less than .001.

an average chain length of 1.206 moves requires the vast majority of chains to be of length one. Since there is no such thing as a chain of length zero, there is no other way for this low average to be achieved. This, then, severely constrains the number of chains available for assignment, if you will, to the other four or five chain length categories. The result is that reasonable estimation of only the initial exit probabilities ensures that proportional discrepancies between observed and predicted numbers of chains of length 2 or more must be small. This domination of the model by the exit probabilities is seen in the accuracy of the predictions made assuming simply a Bernoulli process of vacancy movement.

The fit of the model does not guarantee that vacancies do move independently of each other, but it certainly makes that assumption much more attractive in this formulation than in traditional models of people moving through housing systems. What we are modelling is a housing system. The questions that we can ask and the answers that we get are different than the questions and answers we confront when we model flows of people through the system.

The $\underline{Q}$ matrix, the matrix of vacancy transitions between states of the housing system most closely resembles elements found in models of people. However, even $\underline{Q}$ differs from the more familiar transition matrix of movers. For example, in 1970

$$\underline{Q}_{70} = \begin{pmatrix} .095 & .032 & .012 & .004 \\ .057 & .084 & .019 & .006 \\ .050 & .062 & .050 & .020 \\ .049 & .098 & .080 & .083 \end{pmatrix}$$

The q_{ii} represent vacancy <u>moves</u> within the strata, and not the sum of vacancy moves within a stratum and the vacancy "not-moves" within a stratum. The diagonal entries in the simple models of people would represent both intra-stratum moves and "not moves." In fact, there is no such thing as a vacancy which doesn't move. The model is only concerned with entities that move. In $\underline{Q}_{70}$ above, for example, $q_{11} = .095$ is interpreted as .095 of vacancies arriving in stratum 1 move to another address in stratum 1 in the next time period; and .084 of the vacancies arriving in stratum 2 move to another address in stratum 2.

$$\underline{Q}_{65} \quad \begin{pmatrix} .138 & .039 & .011 & 0 \\ .073 & .108 & .019 & .003 \\ .064 & .094 & .050 & .009 \\ .040 & .064 & .107 & .052 \end{pmatrix}$$

$\underline{Q}_{65}$ is very similar to $\underline{Q}_{70}$. The proportion of vacancies which

remain within the system are low in both of them. The row sums of the matrices are on the order of .2. In addition, the flow of vacancy transfers within the system follows the same pattern: the proportion of intra-stratum moves is inversely proportional to the stratum of origin. In both time periods, in strata 1 and 2 over 50 percent of vacancy moves which remain within the housing system end in the stratum of origin. In fact in 1965, q_{11} accounts for over 90 percent of the within-system moves originating in stratum 1. In strata 3 and 4, however, intra-stratum moves account for less than 30 percent of the within-system moves originating in these strata. This indicates a flow, or rather, a trickle, of housing vacancies from areas of higher to lower housing level. The reverse flow of housing, not surprisingly, doesn't exist. But this downflow of housing from strata 3 and 4 to strata 1 and 2, mirroring the improving accomodation of previous residents of the lower housing strata, is almost overwhelmed by the flood of vacancies leaving the Lansing-East Lansing area at all strata.

This is perhaps more clearly seen in an examination of the $(\underline{I}-\underline{Q})^{-1}$ matrix, the fundamental (Kemeny and Snell, 1960) or the multiplier matrix (Kristof, 1965; White, 1970; 1971; Hua, 1973) of our vacancy chain model.

$$(\underline{I}-\underline{Q})^{-1}_{70} = \begin{pmatrix} 1.109 & .040 & .015 & .005 \\ .070 & 1.096 & .023 & .008 \\ .065 & .077 & 1.057 & .024 \\ .073 & .126 & .095 & 1.093 \end{pmatrix}$$

$$(\underline{I}-\underline{Q})^{-1}_{65} = \begin{pmatrix} 1.165 & .053 & .014 & 0 \\ .098 & 1.128 & .024 & .004 \\ .089 & .116 & 1.057 & .010 \\ .066 & .092 & .121 & 1.056 \end{pmatrix}$$

$(\underline{I}-\underline{Q})^{-1}$ is called the multiplier matrix because if $(\underline{I}-\underline{Q})^{-1} = [n_{ij}]$, then n_{ij} is the total number of moves generated in state j by a vacancy chain which began its career in state i. In other words, the vector of the row sums of $(\underline{I}-\underline{Q})^{-1}$ is equal to $\underline{\lambda}$, the vector of mean chain lengths by stratum of origin.

Inspection of the $(I-Q)^{-1}_{t}$ immediately reveals the low level of vacancy movement within the system. Inasmuch as the average chain lengths by strata are on the order of 1.2 to 1.25 and at least 1 move is accounted for by the stratum of origin (the minimum being 1.056 moves for n_{44} in 1965, with the maximum being 1.165 in n_{11} in 1965), there is very little room for moves to be generated elsewhere in the system.

What little internal movement there is seems to be in a downward direction. If it is not stretching a point, the matrices may be said to be lower triangular. Many more vacancy moves are generated in strata below than above the stratum of origin and, in that sense, vacancies seem to flow downward. The highest stratum appears to generate the greatest proportional level of vacancy movement and stimulates it in all 3 lower strata. Stratum 3 generates vacancy movements in both stratum 1 and stratum 2. It should be noted that in 1970, stratum 2 receives the greatest benefit of vacancies entering the system in stratum 4.

But the most striking finding has been the extremely high rate at which vacancy chains leave the cities of Lansing and East Lansing. In part this is due to the definition of the housing system used in this study. Because the urban area has been specified to include only the cities of Lansing and East Lansing, there is no way to estimate what proportion of chains leaving the system arrive in places such as Haslett, DeWitt, Okemos, or Holt, or other places comprising the Greater Lansing area. This omission is unfortunate in some senses, but the figures arrived at are important in terms of municipal housing policy. If the cities of Lansing and East Lansing are involved in housing programs, the benefits of these programs must be considered as they affect residents and taxpayers of the two cities. How much benefit do they derive if each unit created only generates 1.2 vacancy moves? How many households improve their situation as a result of such programs, and at what cost; or do the programs mainly allow outsiders to come into the city, with little housing relief for previous residents? In other words, what is the rate of new household formation from within the cities, compared to rates of household in-migration. Unless the former rates are high, the major beneficiaries of such housing programs, at least in the short run, would be non-residents of Lansing-East Lansing.

It was also considered that the shortness of chains might be due to the large number of students in the Lansing-East Lansing population — well over 10 percent of the population was comprised of Michigan

State University students. To examine this possibility, an analysis of the vacancy flows in chains containing no students was conducted and the results are presented below. For 1970:

$$\underline{F}(t) = (330.409 \quad 439.014 \quad 181.157 \quad 63.331)$$

$$\underline{Q} = \begin{pmatrix} .098 & .026 & .006 & .001 \\ .056 & .089 & .015 & .004 \\ .052 & .081 & .037 & .025 \\ .061 & .098 & .096 & .061 \end{pmatrix}$$

$$\underline{p} = \begin{pmatrix} .867 \\ .835 \\ .804 \\ .638 \end{pmatrix}$$

$$(\underline{I}-\underline{Q})^{-1} = \begin{pmatrix} 1.111 & .033 & .007 & .002 \\ .070 & 1.102 & .018 & .005 \\ .068 & .098 & 1.044 & .029 \\ .087 & .128 & .110 & 1.069 \end{pmatrix}$$

$$\underline{P_j} = \begin{pmatrix} .869 & .112 & .016 & .002 & 0 \\ .835 & .138 & .022 & .004 & .001 \\ .804 & .160 & .030 & .005 & .001 \\ .683 & .254 & .051 & .009 & .002 \end{pmatrix}$$

$$\underline{\lambda} = (1.153 \quad 1.196 \quad 1.239 \quad 1.392)$$

$$j(t) = 1.202$$

No students were found in 80 percent of vacancy chains occurring in 1969-1970. The average length of the 1014 chains was 1.202 moves - slightly shorter, in fact, than the average of 1.206 for all chains -- and $\underline{\lambda}$ for the model containing no students is

$$(1.153 \quad 1.196 \quad 1.239 \quad 1.392)$$

compared to

$$\underline{\lambda} = (1.169 \quad 1.197 \quad 1.223 \quad 1.387)$$

for the full model.

Clearly, then, in 1970, the presence of students in vacancy chains does not shorten the chains. The students' higher level of mobility is shown by their overrepresentation in chains, and while the $\underline{Q}$ matrices differ between the populations with and without students, the differences appearing in the values of j(t) and $\underline{\lambda}$ must be considered negligible.

The findings for 1965 are similar:

$$\underline{F}(t) = (209.722 \quad 268.014 \quad 114.761 \quad 49.320)$$

$$\underline{Q} = \begin{pmatrix} .139 & .044 & .121 & 0 \\ .067 & .112 & .018 & .002 \\ .068 & .100 & .053 & .009 \\ .038 & .053 & .115 & .050 \end{pmatrix}$$

$$\underline{p} = \begin{pmatrix} .805 \\ .801 \\ .770 \\ .745 \end{pmatrix}$$

$$(\underline{I}-\underline{Q})^{-1} = \begin{pmatrix} 1.167 & .060 & .016 & 0 \\ .090 & 1.133 & .023 & .002 \\ .094 & .125 & 1.060 & .011 \\ .063 & .080 & .131 & 1.054 \end{pmatrix}$$

$$\underline{P}_j = \begin{pmatrix} .805 & .156 & .031 & .006 & .001 & 0 \\ .801 & .159 & .032 & .006 & .001 & 0 \\ .770 & .182 & .038 & .007 & .002 & 0 \\ .745 & .198 & .045 & .009 & .002 & 0 \end{pmatrix}$$

There are 641.831 chains containing no students, and this comprises 90 percent of all chains. Chains without students have an average length, j(t)= 1.260 moves while the full model has an average of j(t)= 1.256. For chains containing no students

$$\underline{\lambda} = (1.243 \quad 1.248 \quad 1.290 \quad 1.327)$$

while, overall

$$\underline{\lambda} = (1.232 \quad 1.253 \quad 1.272 \quad 1.335).$$

Although the $\underline{Q}$ and $(\underline{I}-\underline{Q})^{-1}$ matrices show small differences between the two models, the shortness of chains cannot be accounted for by the presence of students in the population.

We must consider,however, the fact that the sample was drawn from city directories, from data collected at one-year intervals. The number of chains being discussed actually constitutes the minimum number of chains which could represent the data. Because we examine an address at only 2 points in time a year apart, we may miss several occupants of an address, i.e., several chains passing through an address. For example, if we sample address $_1$ at time$_1$ and find person A there, and then at time$_2$ we find person B at address$_1$, we infer simply that A moved out and B moved in. We do not consider the fact that persons C,D,E,F, and G may have successively occupied address$_1$ in the time interval between A's departure and B's arrival.

Despite these factors which may tend to shorten the chain lengths in our sample, the average length of chains appears to fit in with a general pattern. It seems axiomatic that "large cities provide more migration opportunities than small cities, and a large observation unit allows people to move farther without crossing a boundary (leaving the system)" (Simmons, 1968:627).[1] An examination of multipliers for various areas in Table 5.16 seems to support this reasoning. A regression equation of the form

$$\hat{j}(t) = a + b \log P$$

to predict the size of the multiplier yields the solution

---

[1]Parentheses mine.

$$\hat{j}\ (t) = -1.30Q + .577 \log P$$

with $r^2 = .36$. However, if we delete the data for Cleveland, 1938-40, from the analysis, because of the great temporal and economic discrepancies involved, our predictor becomes

$$\hat{j}\ (t) = -2.992 + .773 \log P$$

and $r^2 = .99$. Our computed value of $\hat{j}$ (t) = 1.208 fits in quite well with this model. Bearing in mind the differences in sample size and data collection methods in the 4 studies, this finding indicates a remarkable consistency in the size of multipliers as a function of population size.

Table 5.16

Multipliers and Housing Area Populations

Area	Multiplier	Population	log(Population)	Source
U.S.A., 1966	3.5	195,857,000	8.292	Lansing <u>et al</u>, 1969
New York, 1960	2.4	10,695,000	7.029	Kristof, 1969
Clydeside, 1970	1.7	2,000,000	6.301	Watson, 1974
Cleveland, 1939	3.7	1,195,000	6.077	Hua, 1972
Lansing, 1970	1.2	179,000	5.253	

Though the methods used may underestimate the number of vacancy chains in the system, there are advantages to the method which may counterbalance these disadvantages, especially in light of the accuracy of the model and its ability to describe the housing system. The advantages lie primarily in the realm of data collection, and provide some relief in terms of time and money costs. The data are readily available -- city directories are public information, as is the census. They are both relatively non-reactive data sources, and are largely free of contamination by researchers, i.e. the data are not altered by their use. Additionally, census data and city directory data is available for hundreds of cities in North America over a time-span of several decades, allowing comparative research to be conducted.

The use of city directory data also forces one to deal with human behavior. We only discuss moves as indicated by a change of occupancy at an address. Subjects are not required to respond verbally regarding their mobility behavior or their attitudes towards mobility. The study is one of mobility behavior not verbal behavior. The use of the vacancy chain model forces one to focus on system properties -- the system of housing opportunities within which we move -- and not individuals moving within a system considered as a given.

5.4: The Housing Sub-Systems, 1969-1970

Further exploration of the housing system requires that the system be decomposed into several sub-systems. In this way, we can examine the differences between vacancy chains beginning in houses and chains beginning in apartments; chains starting in newly constructed units and chains starting in existing units, etc. The 1397 unweighted chains can be classified according to the two dimensions of type of dwelling unit and age of unit. Age was considered to consist only of the two categories "new unit" and "existing unit." This classification is presented in Table 5.17.

We can also consider a second type of submarket in light of the effects of tenure and housing type on mobility. This is the class of pure chains -- chains which contain only houses or only apartments. It should be stated immediately that in 1969-1970 pure chains

Comprise 91.8 percent of all unweighted vacancy chains. Their distribution is seen in Table 5.18

Table 5.17

Distribution of Unweighted Chains by First Unit in Chain, 1969-1970

Age	House		Apartment		Other		Total	
# of Chains (% of Total)	149	(10.7)	86	(6.2)	10	(.1)	245	(17.5)
NEW								
(% of Column) (% of Row)	(17.1)	(60.8)	(16.9)	(35.1)	(58.8)	(4.1)	(17.5)	(100.0)
EXISTING	721	(51.6)	424	(30.4)	7	(.1)	1152	(82.5)
	(82.9)	(62.6)	(83.1)	(36.8)	(41.2)	(.1)	(82.5)	(100.0)
TOTAL	870	(62.3)	510	(36.5)	17*	(.1)	1397	(100.0)
	(100.0)	(62.3)	(100.0)	(36.5)	(100.0)	(.1)	(100.0)	(100.0)

Type of Dwelling

*The 17 "Other" dwellings consist of 5 old trailers and 10 new and 2 old townhouse units.

Table 5.18

Distribution of Unweighted Pure Chains by First Unit in Chain, 1969-1970

Age	House		Apartment		Total	
# of Chains (% of Total)	135	(10.5)	77	(6.0)	212	(16.5)
NEW						
(% of Column) (% of Row)	(16.6)	(63.7)	(16.4)	(36.3)	(16.5)	(100.0)
EXISTING	678	(52.8)	393	(30.6)	1071	(83.5)
	(83.4)	(63.3)	(83.6)	(36.7)	(83.5)	(100.0)
TOTAL	813	(63.4)	470	(36.6)	1283	(100.0)
	(100.0)	(63.4)	(100.0)	(36.6)	(100.0)	(100.0)

Type of Dwelling

Applying the weighting procedures described previously to correct for sampling biases, the distribution of weighted chains in

Table 5.19 is generated. Table 5.19 shows that of the approximately 1268 chains occurring in our sample in 1969-1970, about one-sixth originate in new units and the remainder in existing housing. Chains began in houses 60 percent of the time (single unit structures accounted for 57 percent of housing units in Lansing-East Lansing in 1970 (U.S. Bureau of the Census, 1970:1)). The one-sixth of chains beginning in new housing was proportionately distributed in apartments and houses.

Table 5.19

Distribution of Weighted Vacancy Chains by First Unit in Chain, 1969-1970

Age	Type of Dwelling			
	House	Apartment	Other	Total
# of Chains (% of Total) NEW	123.139 (9.7)	78.361 (6.2)	9.432 (.7)	210.932 (16.6)
(% of Column) (% of Row)	(16.0) (58.4)	(16.3) (37.1)	(59.9) (4.5)	(16.6) (100.0)
EXISTING	646.742 (51.0)	403.755 (31.8)	6.311 (.5)	1056.808 (83.4)
	(84.0) (61.2)	(83.7) (38.2)	(40.1) (.6)	(83.4) (100.0)
TOTAL	769.882 (60.7)	482.115 (38.0)	15.742* (1.2)	1267.739 (100.0)
	(100.0) (60.7)	(100.0) (38.0)	(100.0) (1.2)	(100.0) (100.0)

*Consists of approximately 5 old trailers, and 9 new and 2 old townhouses.

The distribution of pure chains in Table 5.20 is virtually identical, with pure chains accounting for 95% of all chains! Of chains beginning in houses, 93.1 percent are pure, as are 94.6 percent of chains beginning in new houses. Pure chains also account for 95.8 percent of chains beginning in an apartment and 94.7 percent of chains beginning in new apartments.

The implications of this high degree of separation between house and apartment sub-systems will be discussed in section 5.6 and

Chapter 6 below, but it should be remembered that this separation is a function of the shortness of chains. If 80 percent of chains leave the system in their first move, then only 20 percent of them can possibly contain both types of dwellings.

Table 5.20

Distribution of Weighted Pure Chains by First Unit in Chain, 1969-1970

Age	Type of Dwelling					
	House		Apartment		Total	
# of Chains (% of Total)	116.478	(9.7)	74.219	(6.2)	190.697	(15.8)
NEW						
(% of Column) (% of Row)	(15.7)	(61.1)	(16.1)	(38.9)	(15.8)	(100)
EXISTING	625.794	(52.0)	387.414	(32.2)	1013.208	(84.2)
	(84.3)	(61.8)	(83.9)	(38.2)	(84.2)	(100)
TOTAL	742.272	(61.7)	461.633	(38.3)	1203.905	(100)
	(100)	(61.7)	(100)	(38.3)	(100)	(100)

Perhaps the easiest way to inspect the sub-system models is to present a summary table of the eight mixed sub-system models and then another summary table of the six pure sub-system models. These tables show observed and predicted chain length distributions as well as $\overline{X}$, the observed mean chain length, $j(t)$, the predicted mean chain length, and Δ, the index of dissimilarity. This information for the mixed sub-systems is presented in Table 5.21. Pure systems are described in Table 5.22, where the same data for the complete model is also shown for comparative purposes. Clearly the sub-system models fit the data. In no case does Δ exceed .015 and in every case the predicted mean chain length matches the observed mean length exactly.

Table 5.21

Observed and Predicted Chain Length Distributions, Mixed Sub-Systems, 1969-1970

All Chains	Chain Length 1	2	3	4	5	Total	% of All Chains	$\bar{X}$ j(t)	Δ
Observed	1049.093	181.104	32.820	3.892	.831	1267.739	100	1.206	
Predicted	1050.522	180.275	31.023	5.430	.171	1267.421		1.206	.003
Type of First Unit:									
New	164.354	35.870	8.104	2.595	0	210.932	16.6	1.284	
	163.751	37.038	7.953	1.680	.235	210.658		1.284	.007
Existing	884.739	145.225	24.716	1.297	.831	1056.808	83.4	1.191	
	887.265	142.234	22.579	4.001	.684	1056.764		1.191	.005
						1267.740	100.0		
House	615.795	126.431	23.906	2.919	.831	769.882	60.7	1.242	
	619.386	121.756	23.033	4.555	.976	769.706		1.242	.007
Apartment	419.424	53.534	8.509	.649	0	482.115	38.0	1.150	
	419.433	54.476	7.280	1.030	.033	482.252		1.150	.003
						1251.997*	98.8*		
New House	89.648	26.197	5.673	1.622	0	123.139	9.7	1.344	
	90.836	24.530	5.990	1.359	.320	123.035		1.344	.015
New Apartment	66.169	9.112	2.431	.649	0	78.361	6.2	1.203	
	65.487	10.411	1.910	.401	.090	78.300		1.203	
						201.5**	15.9**		
Existing House	526.147	100.233	18.233	1.297	.831	646.742	51.0	1.223	
	528.739	96.992	17.571	3.058	.670	647.030		1.223	.006
Existing Apartment	353.255	44.422	6.078	0	0	403.755	31.8	1.140	
	354.343	42.926	5.529	.728	.024	403.551		1.140	.005
						1050.497***	82.9***		
Total						1251.997*	98.8*		

* 15.742 chains start in units other than a house or apartment.
** 9.432 chains start in units other than a house or apartment.
*** 6.311 chains start in units other than a house or apartment.

Table 5.22

Observed and Predicted Chain Length Distributions, Pure Chains, 1969-1970

Houses only	Chain Length 1	2	3	4	5	Total	% of All Chains (1267.739)	$\overline{X}$ j(t)	Δ
Observed	615.795	108.776	15.802	1.622	.277	742.272	58.6	1.197	
Predicted	618.986	103.459	16.446	2.579	.189	741.659		1.197	.007
Apartments Only	419.424	38.157	4.052	0	0	461.633	36.4	1.100	
	419.834	37.916	3.532	.493	0	461.775		1.100	.002
						1203.905	95.0		
New First Unit:									
Houses	89.648	22.211	3.647	.973	0	116.478	9.2	1.278	
	90.415	20.861	4.261	.781	.137	116.456		1.278	.013
Apartments	66.169	6.834	5.267	0	0	74.219	5.9	1.125	
	66.304	6.755	.922	.164	.011	74.156		1.125	.005
						190.697	15.0		
Existing First Unit:									
Houses	526.147	86.565	12.156	.649	.277	652.794	49.4	1.182	
	528.964	82.288	12.416	1.861	.100	625.629		1.182	.007
Apartments	353.255	31.323	2.836	0	0	387.414	30.6	1.095	
	353.822	30.609	2.806	.323	0	387.561		1.095	.002
						1013.208	79.9		
Total						1203.905	95.0		

The agreement between the model and the data is also seen in the comparison of $\underline{\lambda}$, the vector of predicted mean chain length by stratum with $\underline{L}$, the vector of observed mean chain lengths by stratum, shown in Table 5.23.

Table 5.23

L and λ , Mixed Sub-Systems, 1969-1970

Type of First Unit		Stratum of Origin 1	2	3	4
New Unit	L	1.154	1.174	1.349	1.528
	λ	1.222	1.201	1.340	1.491
Existing Unit	L	1.150	1.210	1.200	1.367
	λ	1.162	1.197	1.200	1.320
House	L	1.179	1.228	1.358	1.560
	λ	1.182	1.222	1.332	1.515
Apartment	L	1.129	1.138	1.140	1.212
	λ	1.142	1.155	1.136	1.225
New House	L	1.185	1.170	1.591	1.666
	λ	1.168	1.232	1.472	1.614
New Apartment	L	1.250	1.184	1.150	1.125
	λ	1.320	1.183	1.173	1.205
Existing House	L	1.178	1.227	1.130	1.435
	λ	1.183	1.219	1.298	1.409
Existing Apartment	L	1.117	1.165	1.131	1.250
	λ	1.123	1.148	1.132	1.238

The predicted chain lengths agree very well with the observed distribution. The largest discrepancy occurs in chains beginning in new apartments where $\frac{L_1 - \lambda_1}{L_1} = .056$. However, we must consider that only 11.8 percent of chains originate in this cell.

Substantively, it should be noted in Table 5.21 that j(t) ranges from a low of 1.14 for chains beginning in existing apartments to a high of 1.344 for chains beginning in new houses. Chains beginning in houses are longer than those beginning in apartments and chains beginning in new units are longer than chains beginning in old ones. Not surprisingly this ordering of chain lengths is preserved for pure chains as well. Although the range is still narrow, the longest chains ($j(t) = 1.278$) in new houses and the shortest ($j(t) = 1.095$) originate in existing apartments.

5.5: The Housing Sub-Systems, 1964-1965

The distribution of unweighted vacancy chains for 1964-1965 according to the age and type of first unit is shown in Table 5.24 and the distribution of pure chains in Table 5.25. The marked separation of housing subsystems in 1965 is immediately evident. Over 95 percent of all chains are pure chains -- 86.8 percent of chains beginning in apartments and 97.7 percent of chains beginning in houses.

Table 5.24

Distribution of Unweighted Chains by First Unit in Chain, 1964-1965

Age	Type of Dwelling							
	House		Apartment		Other		Total	
# of Chains (% of Total)	183	(22.7)	35	(4.3)	4	(0.5)	222	(27.6)
NEW (% of Column (% of Row)	(26.3)	(82.4)	(33.0)	(15.8)	(100.0)	(1.8)	(27.6)	(100.0)
EXISTING	512	(63.6)	71	(8.8)	0	(0)	583	(72.4)
	(73.7)	(87.8)	(67.0)	(12.2)	(0)	(0)	(72.4)	(100.0)
TOTAL	695	(86.3)	106	(13.2)	4*	(0.5)	805	(100.0)
	(100.0)	(86.3)	(100.0)	(13.2)	(100.0)	(0.5)	(100.0)	(100.0)

*4 new townhouses.

Weighting the chains results in the distribution of chains shown in Table 5.26: 85 percent of chains begin in houses and almost 30 percent of chains begin in new units — almost twice as many as in 1970.

Table 5.25

Distribution of Unweighted Pure Chains
by First Unit in Chain, 1964-1965

Age	Type of Dwelling					
	House		Apartment		Total	
# of Chains (% of Total) NEW (% of Column) (% of Row)	179	(23.2)	31	(4.0)	210	(27.3)
	(26.4)	(85.2)	(34.1)	(14.8)	(27.3)	(100.0)
EXISTING	500	(64.9)	60	(7.8)	560	(72.7)
	(73.6)	(89.3)	(65.9)	(10.7)	(72.7)	(100.0)
TOTAL	679	(88.2)	91	(11.8)	770	(100.0)
	(100.0)	(88.2)	(100.0)	(11.8)	(100.0)	(100.0)

Table 5.26

Distribution of Weighted Vacancy Chains by First Unit in Chain, 1964-1965

Age	Type of Dwelling							
	House		Apartment		Other		Total	
# of Chains (% of Total) NEW (% of Column) (% of Row)	158.399	(22.4)	33.957	(4.8)	4.269	(0.6)	196.626	(27.8)
	(26.4)	(80.6)	(33.3)	(17.3)	(100.0)	(2.2)	(27.8)	(100.0)
EXISTING	442.387	(62.6)	68.063	(9.6)	0	(0)	510.449	(72.2)
	(73.6)	(86.7)	(66.7)	(13.3)	(0)	(0)	(72.2)	(100.0)
TOTAL	600.786	(85.0)	102.020	(14.4)	4.629*	(0.6)	707.075	(100.0)
	(100.0)	(85.0)	(100.0)	(14.4)	(100.0)	(0.6)	(100.0)	(100.0)

*4.269 new townhouses.

New chain starts are not evenly distributed in 1964-1965 as they were in 1970, either. New units account for one-quarter of the chains originating in houses, and for one-third of the chains beginning in apartments.

The pure chains in Table 5.27 account for 97.2 percent of weighted chains -- 92.1 percent of chains beginning in apartments and 98.8 percent of chains beginning in houses.

Table 5.27

Distribution of Weighted Pure Chains by First Unit in Chain, 1964-1965

Age	Type of Dwelling					
	House		Apartment		Total	
# of Chains (% of Total)	156.612	(22.8)	31.925	(4.6)	188.537	(27.4)
NEW (% of Column) (% of Row)	(26.4)	(83.1)	(34.0)	(16.9)	(27.4)	(100.0)
EXISTING	436.945	(63.6)	62.043	(9.0)	498.988	(72.6)
	(73.6)	(87.6)	(66.0)	(12.4)	(72.6)	(100.0)
TOTAL	593.557	(86.3)	93.960	(13.7)	687.525	(100.0)
	(100.0)	(86.3)	(100.0)	(13.7)	(100.0)	(100.0)

The distributions of observed and predicted chain lengths by sub-system along with the models' associated measures, $\overline{X}$, $j(t)$, and Δ, are presented in Table 5.28. The longest chains are found beginning in houses of both ages, and the shortest beginning in new apartments. The age of the first unit appears to have little bearing in 1965 on the average length of chains when compared to type of dwelling. The distribution of pure chains is seen in Table 5.29, where the patterns discussed above are preserved.

Table 5.28

Observed and Predicted Chain Length Distributions, Mixed Sub-Systems, 1964-1965

All Chains	Chain Length 1	2	3	4	5	6	Total	% of All Chains	$\bar{X}$ j(t)	Δ
Observed	567.769	107.637	23.906	6.162	1.108	.493	707.075	100	1.256	
Predicted	562.374	115.531	23.035	4.528	.760	0	706.227		1.256	.011
Type of First Unit:										
New	162.220	23.350	8.509	2.270	.277	0	196.626	27.8	1.246	
	157.662	31.368	6.153	1.144	.224	.0	196.551		1.246	.041
Existing	405.550	84.287	15.397	3.892	.831	.493	510.449	72.2	1.260	
	404.733	83.560	17.200	3.572	.727	0	509.791		1.260	.004
							707.075	100.0		
House	472.785	97.386	23.501	5.514	1.108	.493	600.786	85.0	1.279	
	468.476	104.249	21.984	4.725	1.045	.044	600.523		1.279	.011
Apartment	90.715	10.251	.405	.649	0	0	102.020	14.4	1.127	
	90.840	9.684	1.318	.329	.036	0	102.207		1.127	.009
							702.806*	99.4*		
New House	127.001	21.072	8.104	1.946	.277	0	158.399	22.4	1.279	
	123.342	27.828	5.834	1.166	.246	.025	158.441		1.279	.043
New	30.950	2.278	.405	.324	.0	0	33.957	4.8	1.120	
Apartment	30.404	3.108	.396	.049	0	.0	33.957		1.120	.024
							192.356*	27.2*		
Existing	345.784	76.314	15.397	3.568	.831	.493	442.387	62.6	1.279	
House	345.198	76.444	16.328	3.512	.722	.019	442.233		1.279	.002
Existing	59.765	7.973	0	.324	.0	0	68.063	9.6	1.131	
Apartment	60.526	6.632	.964	.150	.030	0	68.302		1.131	.023
Total							510.450	72.2		
							702.806*	99.4*		

*4.269 chains start in units other than house or apartment.

Table 5.29

Observed and Predicted Chain Length Distributions, Pure Sub-Systems, 1964-1965

	1	2	3	4	5	6	Total	% of All Chains (707.075)	$\bar{X}_j(t)$	Δ
Houses Only										
Observed	472.785	93.399	21.880	3.892	1.108	.493	593.557	83.9	1.262	
Predicted	469.106	99.483	20.000	4.028	.735	0	593.352		1.262	.011
Apartment	90.715	2.848	.405	0	0	0	93.968	13.3	1.039	
	90.475	3.344	.149	0	0	0	93.968		1.039	.005
							687.525	97.2		
New First Unit:										
House	127.001	19.933	8.104	1.297	.277	0	156.612	22.1	1.263	
	123.518	26.545	5.315	1.027	.181	0	156.586		1.263	.042
Apartment	30.950	.570	.405	0	0	0	31.925	4.5	1.043	
	30.588	1.267	.039	0	0	0	31.894		1.043	.022
							188.537	26.7		
Existing First Unit:										
House	345.784	73.467	13.776	2.595	.831	.493	436.945	61.8	1.262	
	345.751	72.595	14.883	2.996	.695	0	436.921		1.262	.003
Apartment	59.765	2.278	0	0	0	0	62.043	8.8	1.037	
	59.883	2.006	.115	0	0	0	62.004		1.037	.004
							498.988	70.6		
Total							687.525	97.2		

The fit of the model is extraordinarily good, especially when one considers that the occupancy states were classified according to 1970 data. The index of dissimilarity ranges from a minimum of .002 to a maximum of .043 in the sub-system beginning in new houses. This discrepancy is carried through the new unit sub-system (Δ=.041) because houses comprise over 80 percent of all new units. As well, there is a discrepancy of Δ=.024 in the new apartment sub-system which also contributes to the 4.1 percent difference in the observed and predicted distributions of chains beginning in new units.

Table 5.30

L and λ, Mixed Sub-Systems, 1964-1965

Type of First Unit		Stratum of Origin 1	2	3	4
New Unit	L	1.120	1.144	1.277	1.500
	λ	1.223	1.192	1.292	1.378
Existing Unit	L	1.223	1.290	1.222	1.320
	λ	1.235	1.284	1.258	1.293
House	L	1.209	1.278	1.350	1.477
	λ	1.233	1.276	1.322	1.410
Apartment	L	1.278	1.065	1.037	1.000
	λ	1.231	1.086	1.079	1.000
New House	L	1.200	1.169	1.361	1.600
	λ	1.219	1.221	1.334	1.424
New Apartment	L	1.112	1.063	1.200	1.000
	λ	1.238	1.081	1.141	1.000
Existing House	L	1.204	1.275	1.297	1.421
	λ	1.236	1.300	1.312	1.395
Existing Apartment	L	1.167	1.067	1.059	1.000
	λ	1.231	1.095	1.034	1.000

A comparison of L and λ, observed and predicted mean chain lengths by stratum is given in Table 5.30. Though there are some minor internal discrepancies originating in sub-systems with few chains, the general fit is quite good.

5.6: The Housing Sub-Systems -- Discussion

The decomposition of the complete models in order to examine selected aspects of the housing system structure, i.e., the nature of the housing submarkets, has yielded some interesting and useful information about both the housing system and the model. In general, the vacancy chain model fit the data very well in 1969-70 and only slightly worse in 1964-65. Perhaps this was due to the classification of occupancy states on the basis of 1970 data, and their inaccuracy when applied to the city of 1965. In any event, the largest index of dissimilarity for chain length distributions in 1964-65 was less than .05.

Overall predictions tended to be more accurate than were predictions of more specific values. Chain length distributions, mean lengths, j(t), and total numbers of moves were predicted more accurately than were values such as the λ_i. Values for the complete model were predicted more accurately than values for specific sub-systems. This pattern is due, in part,to the smaller cell frequencies that the more specific predictions are based on, and, in part, to the fact that in the more general predictions the smaller discrepancies of their more specific components tend to cancel each other out.

The most interesting discovery concerning the housing sub-system structure of Lansing-East Lansing is the extreme separation of the house and apartment sub-systems. Only 5 percent of all vacancy chains in 1970 contain both houses and apartments. Only 3 percent do so in 1965. Much of this phenomenon is attributable to the extreme shortness of chains found in this sample. With exit probabilities of .829 in 1970 and .796 in 1965, 83 percent and 80 percent of chains respectively are only one move long, and consequently contain only one dwelling type.

To control for the confounding influence of chains of length one on the proportion of pure chains in the sample, a second measure was calculated: pure chains as a proportion of all chains of length 2 or more. This information for 1970 is presented in Table 5.31 and for 1965 in Table 5.32.

Table 5.31

Pure Chains as a Proportion of All Chains of Length 2 or More, 1969-1970

Type of First Unit	Pure Chains (A)	All Chains (B)	Proportion (A/B)
House	126.477	154.087	.821
Apartment	42.209	62.291	.673
New House	26.830	33.491	.801
Existing House	99.647	120.595	.826
New Apartment	8.050	12.192	.660
Existing Apartment	34.159	50.500	.676

Table 5.32

Pure Chains as a Proportion of All Chains of Length 2 or More, 1964-1965

Type of First Unit	Pure Chains (A)	All Chains (B)	Proportion (A/B)
House	120.772	128.001	.944
Apartment	3.253	11.305	.288
New House	29.611	31.398	.943
Existing House	91.161	96.603	.944
New Apartment	.975	3.007	324
Existing Apartments	2.278	8.298	.275

While the elimination of 1-move chains from the analysis improves the situation somewhat in 1970, over four-fifths of chains beginning in houses are pure as are two-thirds of the chains beginning in apartments. This does not indicate a substantial flow of vacancies from houses to apartments.

The relative decrease in the proportion of pure chains for 1965 is much greater for apartments -- less than 30 percent of chains beginning in apartments are pure, but this is based on a sample of only 11.305 chains. As in 1970, the ratio of pure chains beginning in houses is extremely high.

This is surprising because previous research has shown substantial shifts in the tenure of occupants from renters to owners. This flow should have revealed itself in this study in a flow of vacancies from houses to apartments. A national sample indicates that one-third to one-half of renters in the early positions of a vacancy chain become owners (Lansing, et al., 1969:30), and Butler also shows that about 50 percent of previous renters have become owners in the last move (Butler, et al., 1969:10). Perhaps this shift was not found here because of the shortness of chains induced by the boundaries of our housing system. That is to say, that possibly a substantial proportion of vacancies beginning in houses which leave the system immediately move to apartments which are within the Lansing Metropolitan Area, but outside the Lansing-East Lansing municipal boundaries.

Also in contrast to the reported national situation is the proportion of vacancy chains created by new construction. New units accounted for only one-sixth of all chains in 1970 and one-quarter of the chains in 1965 while "sequences of new construction account for about half the sequences of moves in the nation" (Lansing et al., 1969:62). This discrepancy is either a reflection of sampling error or local differences in housing construction, or both.

The examination of the sub-systems yields some useful information regarding the vacancy multipliers -- the j(t), which describe the number of vacancy moves generated by each entering vacancy. While all the j(t) in our sample fall into a narrow range, 1.095 to 1.344 in 1970, and 1.037 to 1.279 in 1965, some patterns seem to emerge. Vacancy chains beginning in new units and houses tend to be longer, with chains beginning in new houses the longest of all. A rank-order correlation coefficient, Spearman's ρ was computed to compare the order of mean chain lengths by sub-system for the two time periods and $\rho = .633$ ($p<.10$).

Within sub-systems, chains beginning in higher strata tended to be longer. Tables 5.23 and 5.30 show 64 values of λ_i by stratum. Of these 64 values, 46 fit the descending pattern described above. Of the 18 non-conforming values of λ_i, 9 are caused by a single value -- in 1965, for chains beginning in apartments, $\lambda_4 = 1.000$. This pattern of chain length being proportional to the value of the original vacancy has been noted previously (Lansing et al., 1969: 17ff.) and seems a natural consequence of housing market structure. In fact, this process forms the subject matter of Chapter 6.

5.7: Summary of Findings

A sample of 1397 vacancy chains drawn from the Lansing City Directory (1970) and 805 chains drawn from the Lansing City Directory (1965) formed the basis for the construction of two vacancy chain models. The chains were disaggregated and the individual vacancy moves were weighted according to their relative sampling probabilities and models were developed based on 1528.039 weighted vacancy moves for 1970 and 888.086 weighted moves for 1965.

The most striking substantive finding was the extreme shortness of vacancy chains beginning in Lansing-East Lansing. The average length of chains was found to be 1.206 moves in 1970, and 1.256 moves in 1965. Much of the cause for this shortness can probably be found in the size of the housing system and the boundaries of this specific system. It is clear that the larger a housing system is, the more opportunities for movement within the system exist, and vacancy chains will be longer (see Simmons, 1968; Hua, 1972: 222n.). Defining the Lansing-East Lansing housing system in terms of the municipal boundaries of the two cities established a system with fewer than 60,000 addresses. Furthermore, such a definition causes vacancy moves to parts of the Lansing Metropolitan Area which are outside of Lansing and East Lansing to be counted as moves outside of the system. Both these factors tend to shorten chain lengths.

It should also be noted that the models being considered are minimal models, in that they represent the minimum numbers of chains and mobility rates which could fit the data. Insofar as vacancies are inferred from changes in occupancy reported at one-year intervals, the estimates of the numbers of chains are minimum estimates. If n people occupy one address during the course of a year, there would be n-1 moves through that address. Our sampling method would only find the first and last occupants and we infer only a single move through that address.

Vacancy arrivals and departures by stratum were found to match the distribution of housing units by stratum very closely, although slightly more vacancies left the system via stratum 1 than arrived there. This difference was only 2 percent in 1970 and 3 percent in 1965 and indicates a slight downward flow of vacancies through the system. The multiplier matrices, $(\underline{I}-\underline{Q})^{-1}$, show this trickle slightly more clearly and also show that the upward flow of vacancies is almost nil.

Examination of the housing sub-systems shows that, in general, vacancies created by new construction generated more vacancy moves than did vacancies entering via existing units; house entrances generated more moves than vacancies entering in apartments, and vacancies entering in higher strata generated more moves than vacancies entering the system at lower strata.

The sub-system analysis also shows that very few vacancy chains in our sample cross dwelling types. This is not merely a necessary conclusion of the fact that four-fifths of the chains in the sample are only one move long. In 1970, 78 percent of chains longer than 1 move did not cross dwelling type and 89 percent of chains longer than 1 move in 1965 did not.

The implications for housing policy of the very high exit provabilities, the slight downward flow of vacancies, the order of the multipliers by sub-system, and the separation of the house and apartment sub-systems form the substance of Chapter 6, below.

The major theoretical finding is the extraordinarily good fit which the vacancy chain model achieves with the data. Indices of dissimilarity based on the differences between observed and predicted chain length distributions for the complete models show Δ_{70} =.003 and Δ_{65} = .011. The model also predicts chain length distributions for the sub-systems extremely well. In no case does Δ exceed .05.

This goodness-of-fit is partially a function of the very high exit probabilities found in this sample. Exit probabilities on the order of .8 imply that if this one parameter is estimated with reasonable accuracy, then the predicted distribution must match the observed with considerable accuracy. The domination of the model by the exit probabilities is confirmed by the use of a simple Bernoulli trials model to represent the data. In this model, the probability that a chain will be of length j is

$$P(j) = q^{j-1}p$$

Where p is the probability of leaving the system on any turn, $q = 1 - p$, and the trials are independent. This model predicts a chain length distribution for 1970 which, when compared with the observed distribution yields an index of dissimilarity, $\Delta = .003$.

At the same time as it shows the domination of the model by the exit probabilities, the Bernoulli trials model lends support to the assumption of the independence of vacancy movement. What we have is a first-order Markov chain model of residential mobility where the independence assumption is met. The model is not a model of people moving through a housing system, but it is a model of that system itself, and of housing vacancies moving through it. It appears that the problems of independence when people are the population which moves have not arisen.

It also appears that when the independence assumption is met, the problems of state classification are minimized. This seems only logical, because state classification is critical only when the independence assumption is not met. Then the states must be classified so that the population within each state is homogeneous. It is this homogeneity that allows such Markov models of populations to assume the independence of transitions. If, however, the transitions are independent prior to state classification, then the classification of states can become a substantive theoretical, rather than a mathematical theoretical problem.

In addition to the advantages presented by the first-order Markov chain model in mathematical and analytical terms, the model discussed here has certain methodological advantages which should not be ignored. The model permits an analysis of mobility behavior, and need not resort to questionnaire data. This eliminates the specific problems of questionnaires about mobility (see, e.g., Goldscheider, 1971:37) and the more general problems of the discrepancies between verbal and actual behavior which have been documented since LaPiere's "Attitudes vs. Actions" (1934). The data sources used are available today for hundreds of cities in North America, and around the world. In North America, they are available more or less continuously, for at least 50 years. The data is public, non-reactive and relatively uncontaminated and uncontaminable.

To overcome the problem of year-long intervals between occupancy checks, more complete records might be used -- perhaps records of telephone connections or power connections, disconnections and reconnections. Such lists would certainly yield a more complete picture of vacancy movement.

To further examine the fit of the model, to determine the extent to which the shortness of chains affects the predictive accuracy of the model, larger housing systems should be studied. It would seem that the analysis of metropolitan areas with at least 150,000 housing units and preferably 250,000 units would allow the construction of models with a greater chain length. This longer average chain length would possess a larger maximum possible number of chain length distributions, and therefore allow a better evaluation of the fit of the model. It would also allow the further analysis of multipliers as a function of population size.

6. VACANCY CHAINS AND THE FILTERING OF HOUSING

6.1: The Filtering Process

Filtering, as a process which indirectly provides housing for lower-income groups when new, high-quality housing is built for higher income groups, has been a "well-recognized phenomenon"(Ratcliff, 1949:321) in the housing field since Hoyt first formulated it in his classic (1939) study of residential uses within a city (Smith, 1970:64). In the most general terms higher-income people moving into new high-quality homes leave their previous residences free for occupancy by lower-income people, who free their previous residences for occupancy by families of even lower incomes. The process continues, ideally, until this process has provided at least some new housing opportunities at all income levels in the community and consequently has resulted in the improvement of living standards at all economic levels.

The problems with the filtering process arise from a theoretical confusion regarding which of the several related processes mentioned above do we mean by filtering: the change in occupancy, the change in housing value, or the change in housing standards? The problem is further compounded by the fact that "filtering" is assumed to exist naturally in the housing market, and is often used as a tool of housing policy (see, e.g., Forrester, 1969). As a result, the questions regarding the existence and nature of some filtering process must often be dealt with at the same time as "value" questions regarding the filtering process: is it legitimate to tamper with the filtering process as it exists in the housing market?

Because of its widespread currency, the small number of attempts to rigourously define and explicate the concept of filtering is surprising. The first attempt at a more formal definition of the filtering process was made by Ratcliff in 1949 who said that filtering-down is "the changing of occupancy as the housing that is occupied by one income group becomes available to the next lower income group as a result of decline in market price" (1949:321-22). This definition, however, contains two elements: the change in occupancy and the change in value (Fisher and Winnick, 1951:49; Grigsby, 1963:85). In an attempt to develop a better measure of filtering (not just filtering-down), Fisher and Winnick eliminate what some consider to be the key element in Ratcliff's definition: the change of occupancy (Grigsby, 1963:88), and focus only on the change in relative housing value. "Filtering is defined as a change over time in the position of a given dwelling unit or group of dwelling units within the distribution of housing rents and prices in the community as a whole" (Fisher and Winnick, 1951:52).

This is a much more measurable quantity than Ratcliff's earlier definition yields and certainly it would correlate "filtering-down" with successive occupation of a housing unit by relatively lower classes (Fisher and Winnick, 1951:54). But it is clear that the definition "is not intended to answer the question of whether the filtering process is bringing dwellings within the range of low income groups" (Grigsby, 1963:90). Because the definition rests on changes in relative costs between housing units, substantial filtering could be indicated by this measure without it being available to lower-income groups.

Grigsby (1963) attempts to resolbe this problem by re-injecting occupancy into the definition of filtering, by speaking of an improvement in housing standards, a concept implicit in all discussions of filtering. "Filtering occurs only when value declines more rapidly than quality, so that families can obtain either higher quality and more space at the same price, or the same quality and space at a lower price than formerly" (Grigsby, 1963:97). Following Lowry's (1960) lead, Grigsby is talking in terms of real dollars, so that the question becomes: are <u>incomes</u> rising faster than housing costs (Grigsby, 1963:97).

As the above description illustrates, there is no concensus as to what constitutes housing filtering, even though we seem to have little trouble understanding and using the term <u>inter alia</u> (Grigsby, 1963:85-86). But the problem extends beyond even the resolution of the question of which process in the market constitutes filtering. There are other theoretical and empirical problems which must be overcome.

In assessing the efficacy of filtering as a mechanism for the provision of lower-income housing it is not enough to say that "the shortcomings of the filtering process ..(can be attributed to) ...the failure of the relatively well-to-do to place good quality existing housing on the market in such volume as to produce a significant reduction in its relative prices" (Winnick, 1960: 18). Given the structure of American society and its corresponding pyramid-shaped housing systems (a high proportion of housing units at the bottom and a very low proportion of units at the highest levels), it is ridiculous to assume that the 10 percent of population in the highest levels could ever place sufficient housing units on the market to accomodate the 50 percent of population in the lowest levels. The only way this could come about would be if the housing market took on the form of other consumer-durables markets (automobiles, heavy appliances, etc.) which would result in the high-income segments of society changing homes every 2 or 3 years. In the arguments by proponents of filtering models, "it is not altogether clear whether the argument is that the housing market <u>is</u> like the automobile market or that it <u>could become</u> like the automobile market" (Lowry, 1960: 364).

Nor is the problem simply one of untested assumptions in the filtering model: Do people only move to better housing? Do rapid rates of depreciation really have no adverse effects on housing quality (Grigsby, 1963: 96-97)? Does the required surplus of housing exist at all housing levels? At any housing levels (Ratcliff, 1949:323)?

The problems are such that the process called filtering might not even exist. Lowry's (1960) major contribution to the discussion was not his introduction of constant or real dollars to the value debate, but rather his analysis of the process adduced to result in filtering. The mechanisms which lead to price-filtering in Fisher's and Winnick's terms, could just as easily and as logically result in a deliberate program of housing under-maintenance and disinvestment resulting in an extremely rapid decline in the quality of housing. Such a decline in quality would provide no housing at the lower end of the value and income scale.

Lowry also questions the validity of attributing improvements in housing standards to the filtering process, as a part of the housing system. Analysis of variables endogenous and exogenous to the housing market shows that the causes of relative price declines of housing are exogenous ones. "If, for example, rising incomes cause filtering and result in an improved living environment, it is the increment to earning power, not the intermediate market consequence, which should be given credit" (Grigsby, 1963:94). If these kinds of events are occurring, does it seem reasonable to talk of filtering bringing about improvements in housing standards (Lowry, 1960: 366ff.).

This argument is carried one step further by White (1971). Not only must the flow of housing be considered as an exogenous variable, but the housing situation cannot even be considered a market in classical terms, because the mobility actions of each family change the context in which other households must act. What is required is a "model of continuing realignment between existing stocks of housing and families" (White, 1971:88-89).

6.2. Vacancy Chains and Filtering Effects

Regardless of the definition of filtering used -- changes in occupancy, changes in value, or changes in housing standards -- there seems to be agreement on the social effects of this process. When filtering is not directly cast in terms of changes in occupancy, the effect of the filtering process is "the succession of occupancy by lower-income classes" (Fisher and Winnick, 1951:49); see also Grigsby, 1963; Kristof, 1965; White, 1971). In such cases the decline in housing value is often seen as the mechanism which allows such a change of occupancy to take place.

The simplest way to assess this type of filtering with the vacancy chain model is to examine multipliers, the number of moves generated by each vacancy entrance and to examine the attributes of households at different positions in the sequences of moves (Kristof, 1965; Lansing et al., 1969; Watson, 1974). Kristof finds, for example, in New York that "at each successive link in the chain families with generally lower incomes than their predecessors moved into turnover units (1965:241) and in a national sample of housing "there is a strong tendency for monthly rents to decline from one position to the next" (Lansing et al., 1969:7).

If the vacancy chain model is more fully explicated, there are several structural aspects of the system which can be examined in order to study the flows of vacancies between strata. Ideally, filtering-down would be indicated by a lower-triangular multiplier matrix (Hua, 1972: 87), i.e. the multiplier matrix

$$(\underline{I}-\underline{Q})^{-1} = [n_{ij}]$$

would have the appearance:

$$(\underline{I}-\underline{Q})^{-1} = \begin{pmatrix} n_{11} & 0 & 0 & 0 \\ n_{21} & n_{22} & 0 & 0 \\ n_{31} & n_{32} & n_{33} & 0 \\ n_{41} & n_{42} & n_{43} & n_{44} \end{pmatrix}$$

Because we cannot expect the evidence to be so clear cut we will examine not only the multiplier matrices of the complete models for the two time periods, but we will also examine selected sub-system matrices. We will also examine the matrices of transition probabilities, the $\underline{Q}$ matrices, in order to gain a fuller understanding of the process, and $\underline{M}$, the matrix of mean first passage times, to gain another perspective on the structure of the housing system and flows within it.

As has been stated previously,

> mean first passage times provide a measure of a particular kind of contiguity -- one based on interchange probabilities rather than distance. Thus, they may be viewed as measures of aspatial ... (interstrata) ... distance (Rogers, 1966: 454).

6.3: Findings

A cursory examination of the complete models for 1970 and 1965 indicates low levels of filtering (up or down) occurring in Lansing-East Lansing. With multipliers, j(t) = 1.206 in 1970 and j(t) - 1.256 in 1965, we know there is only minimal vacancy movement within the city. But to determine the nature and direction of these flows we must study the $\underline{Q}$ matrices of vacancy transitions, and the $(\underline{I}-\underline{Q})^{-1}$, multiplier, matrices.

$$\underline{Q}_{70} = \begin{pmatrix} .095 & .032 & .012 & .004 \\ .057 & .084 & .019 & .006 \\ .050 & .062 & .050 & .020 \\ .049 & .098 & .080 & .083 \end{pmatrix} \quad (\underline{I}-\underline{Q})_{70}^{-1} = \begin{pmatrix} 1.109 & .040 & .015 & .005 \\ .070 & 1.096 & .023 & .008 \\ .065 & .077 & 1.057 & .024 \\ .073 & .126 & .095 & 1.093 \end{pmatrix}$$

$$\underline{Q}_{65} = \begin{pmatrix} .138 & .039 & .011 & 0 \\ .073 & .108 & .019 & .003 \\ .064 & .094 & .050 & .009 \\ .041 & .064 & .107 & .052 \end{pmatrix} \quad (\underline{I}-\underline{Q})_{65}^{-1} = \begin{pmatrix} 1.165 & .053 & .141 & 0 \\ .098 & 1.128 & .024 & .004 \\ .089 & .116 & 1.057 & .010 \\ .066 & .092 & .121 & 1.056 \end{pmatrix}$$

Inspection of the multiplier matrices shows that some filtering-down of housing seems to be occuring in the city. The matrices might charitably be said to be somewhat lower-triangular. In both time periods, all elements above the diagonal are less than .05. In 1965, the elements below the diagonal approach or exceed .1 moves, while in 1970, the lower elements all exceed .05 and n_{42} = .126 moves. Clearly, more vacancy moves are generated downwards, than upwards. A more precise measure of filtering-down might be computed by calculating the "filtering-down" ratio -- the ratio of the sum of vacancy moves below the diagonal in $(\underline{I}-\underline{Q})^{-1}$ to the sum of vacancy moves above the diagonal.

$$fd = \frac{\Sigma n_{ij}}{\Sigma n_{ji}} \qquad i > j$$

In 1970, this ratio was

$$fd = \frac{.506}{.115} = 4.40$$

and in 1965, it was

$$fd = \frac{.582}{.232} = 2.51.$$

In other words 4.4 times as many vacancy moves are generated downwards in 1970 and 2.5 times as many moves in 1965 are to lower strata. Obviously, fd < 1 indicates "filtering-up."

To compute $\underline{M}$, the matrix of mean first passage times, the vectors $\underline{p}$ and $\underline{f}$ were concatenated to $\underline{Q}$ in the following fashion to create

$$\underline{P} = \left(\begin{array}{c|c} \underline{Q} & \underline{p} \\ \hline \underline{f} & 0 \end{array}\right)$$

The matrices of mean first passage times were computed as described in section 3.3.

$$\underline{M}_{70} = \begin{pmatrix} 5.429 & 4.823 & 9.668 & 28.535 & 1.169 \\ 5.667 & 4.596 & 9.623 & 28.471 & 1.198 \\ 5.722 & 4.710 & 9.328 & 28.076 & 1.223 \\ 5.844 & 4.645 & 9.139 & 26.411 & 1.387 \\ 4.851 & 3.840 & 8.641 & 27.493 & 2.206 \end{pmatrix}$$

$$\underline{M}_{65} = \begin{pmatrix} 4.999 & 4.745 & 10.966 & 28.860 & 1.232 \\ 5.357 & 4.433 & 10.887 & 28.781 & 1.253 \\ 5.421 & 4.504 & 10.553 & 28.635 & 1.271 \\ 5.597 & 4.675 & 9.938 & 27.436 & 1.335 \\ 4.593 & 3.747 & 9.883 & 27.637 & 2.255 \end{pmatrix}$$

What is immediately evident is the complete domination of the $\underline{M}$ matrices by the exit probabilities. The first indication of this is seen in the relatively low values of the entries in the fifth columns of both matrices. Vacancies move much more quickly to the outside than to any other stratum in the system. The result of this is that the rows of the matrices become very similar, because arrival times are determined mostly by the exit and entrance probabilities — the $\underline{p}$ and $\underline{f}$ vectors — and not by internal vacancy transitions. The ratio of any element m_{ij} to any other element, m_{ik}, of the same row is very close to the ratio of f_k to f_j — the inverse ratio of their entrance probabilities. For example, in 1965

$$\frac{m_{13}}{m_{14}} = \frac{10.966}{28.860} = .380$$

and

$$\frac{f_4}{f_3} = \frac{.074}{.180} = .411$$

The construction of $\underline{P}$ with the assumption that vacancies leaving the system would be reflected back into it has rendered $\underline{M}$ virtually useless for the analysis of internal system structure, given the high exit probabilities found in this sample. Consequently, a second matrix of mean first passage times, $\underline{M}^*$, based only upon intra-system moves, was derived.[1]

To compute $\underline{M}^*$, a matrix, $\underline{R} = [r_{ij}]$, where

$$r_{ij} = \frac{q_{ij}}{\sum_{k=1}^{4} q_{ik}}$$

was defined. Substituting R for $\underline{P}$ in equation 3.5 (section 3.3) and

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[1] $\underline{M}$ and $\underline{M}^*$ for all sub-systems are presented in Appendix A.

following the identical procedure then yields the $\underline{M}^*$, the matrices of mean first passage times based on intra-city moves only. We now have our measure of intra-system "distances".

In 1970, for example,

$$\underline{R} = \begin{pmatrix} .666 & .225 & .085 & .025 \\ .342 & .507 & .113 & .038 \\ .274 & .340 & .275 & .111 \\ .158 & .317 & .258 & .267 \end{pmatrix}.$$

The mean first passage times are presented below:

$$\underline{M}^*_{70} = \begin{pmatrix} 2.088 & 4.132 & 9.982 & 25.601 \\ 3.117 & 2.939 & 9.548 & 25.003 \\ 3.443 & 3.472 & 7.828 & 22.784 \\ 3.923 & 3.478 & 7.648 & 18.848 \end{pmatrix}$$

$$\underline{M}^*_{65} = \begin{pmatrix} 1.775 & 4.454 & 14.320 & 116.391 \\ 2.867 & 2.961 & 13.443 & 113.352 \\ 3.107 & 3.203 & 11.386 & 109.968 \\ 3.684 & 3.714 & 8.070 & 91.190 \end{pmatrix}.$$

These two matrices clearly show that some downward flow of housing vacancies is taking place. The smaller values of m_{ij}^* when $j<i$ indicates the relatively more rapid flow of vacancies from areas of higher housing levels to areas of lower standards. It is also worth noting that in both years m_{41}^* and m_{42}^* are smaller than m_{43}^*, reflecting the tendency for vacancies created in stratum 4 to skip over stratum 3.

Although the absolute values of the m_{ij} cannot be readily interpreted in this model of a group process, their relative sizes are of interest (Kemeny and Snell, 1960: 193). In 1970, it takes roughly 3 times as long for vacancy to go from stratum 2 to stratum

3 as it does to go from stratum 3 to stratum 2, but it only takes slightly longer to go from 1 to 2 than from 2 to 1 (4.132 moves compared to 3.117 moves). The transition from state 3 to state 4 takes 3 times as long as the move from 4 to 3, as does the move from stratum 1 to stratum 3 compared with the reverse move. Moves from strata 1 and 2 to stratum 4 require 7 times as long as moves from state 4 to states 1 and 2.

The pattern in 1965 is similar. Vacancy moves from stratum 1 to stratum 2 take only 1.5 times as long as moves from 2 to 1, while moves from 2 to 3 take 4 times as long and moves from stratum 3 to stratum 4 take 12 times as long as their respective downward moves. Moves from state 1 to state 3 require 5 times as long as moves from state 3 to state 1 and moves from strata 1 and 2 to stratum 4 take 30 times as long as moves from state 4 to states 1 and 2. Again we can see vacancies from stratum 4 going directly to strata 1 and 2 without passing through stratum 3. Clearly vacancies flow downward at all levels more quickly than they flow up.

As Table 6.1 shows, relatively substantial filtering-down occurs in the sub-systems as well. In every case, except those chains

Table 6.1

Filtering-Down Ratio by Type of First Unit in Chain and Year

First Unit in Chain	Year 1969-70	1964-65
New Unit	3.65	3.22
Existing Unit	4.08	6.70
House	5.94	6.68
Apartment	2.12	.16
All Chains	4.40	2.51

starting in apartments, more than 3 times as many vacancy moves are generated in strata below the stratum of origin than are generated in strata above the stratum of origin.

The lowest value in the analysis, fd = .16 occurs for chains beginning in apartments in 1965. This one case of filtering-up can be largely attributed to the small sample size in this category (102 chains) and the extremely high exit probabilities, especially in strata 3 and 4. In most sub-systems the values of p decreased as the stratum increased, while in this case

$$Q = \begin{pmatrix} .166 & .008 & .017 & 0 \\ 0 & .045 & .035 & 0 \\ .011 & 0 & .043 & .019 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

and

$$\underline{p} = \begin{pmatrix} .809 \\ .921 \\ .927 \\ 1.000 \end{pmatrix}.$$

The values of p_3 and p_4, in particular, guarantee that no filtering-down will occur. This is confirmed by the multiplier matrix for this sub-system. Only 1 stratum, stratum 3, generates any downward vacancy movement at all.

$$(\underline{I}-\underline{Q})^{-1} = \begin{pmatrix} 1.199 & .009 & .022 & 0 \\ 0 & 1.047 & .038 & .001 \\ .014 & 0 & 1.046 & .020 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Chains beginning in houses or existing units, however, generate between 4 and 7 times as many moves downward as they do upward in strata, and account for the bulk of house-filtering in Lansing-East Lansing. Not only do such chains have the highest filtering-down ratios but they also comprise the vast majority of vacancy chains. In 1970, 60 percent of chains began in houses and 83 percent in existing units, while in 1965 the proportions were 85 and 72 percent, respectively.

The $\underline{M}^*$ matrics for these sub-systems for 1970 are presented below:

Chains beginning in new units:

$$\underline{M}^* = \begin{pmatrix} 2.013 & 4.370 & 21.192 & 15.872 \\ 3.259 & 2.774 & 20.631 & 15.786 \\ 3.929 & 3.754 & 15.306 & 11.417 \\ 3.666 & 3.522 & 16.816 & 12.909 \end{pmatrix}$$

Chains beginning in existing units:

$$\underline{M}^* = \begin{pmatrix} 2.099 & 4.081 & 8.904 & 33.307 \\ 3.099 & 2.974 & 8.422 & 32.521 \\ 3.307 & 3.391 & 7.063 & 31.490 \\ 4.443 & 3.571 & 5.926 & 21.859 \end{pmatrix}$$

Chains beginning in houses:

$$\underline{M}^* = \begin{pmatrix} 1.977 & 3.804 & 12.312 & 32.850 \\ 2.811 & 2.757 & 11.732 & 33.201 \\ 3.085 & 3.061 & 10.568 & 29.426 \\ 3.606 & 3.044 & 8.720 & 27.177 \end{pmatrix}$$

Chains beginning in apartments:

$$\underline{M}^* = \begin{pmatrix} 2.474 & 5.169 & 7.252 & 16.456 \\ 4.259 & 3.644 & 6.975 & 14.249 \\ 4.544 & 4.880 & 4.753 & 14.369 \\ 4.665 & 5.089 & 7.077 & 9.007 \end{pmatrix}$$

The relative magnitudes of the m_{ij}^* in the sub-systems are similar to those of the complete model, with the previously noted exception of chains beginning in apartments. Not only is the filtering-down ratio smaller, but the entire system seems more compact. Vacancies beginning in apartments seem to move more quickly to other strata above and below the stratum of origin. It only takes 3.5 times as long for a vacancy to move from stratum 1 to stratum 4 as it does for a vacancy to move from state 4 to state 1. In less extreme cases, it only takes 2 to 2.5 times as long for a vacancy to move from stratum i to stratum j above it, compared to the move from j to i. In the other sub-systems, with the exception of moves between strata 1 and 2, moves upward take anywhere from 3 to 10 times as long as the corresponding vacancy moves downward in strata.

It should also be noted that vacancies originating in stratum 4 move more quickly to stratum 2 than to stratum 1, and much more quickly than to stratum 3. This phenomenon is much less pronounced in the set of chains beginning in apartments.

This pattern of vacancy movement is found in 1965 in the three sets of chains which do not begin in apartments. Moves are made at more nearly equal speed between strata 1 and 2, than between any other pair of strata in the 3 models. With the exception of moves between strata 1 and 3 and strata 2 and 3 for chains beginning in new units (and strata 1 and 2 in all cases) moves upward through strata take 5 to 50 times longer than the corresponding moves downward. The "skip" phenomenon is more pronounced in 1965. Vacancies originating in stratum 4 move to stratum 1 or 2 rather than stratum 3 relatively more quickly than in 1970; furthermore, in 1965 they move to stratum 1 just as quickly as to stratum 2. The mean first passage times for the sub-systems in 1965 are presented below:

Chains beginning in new units:

$$\underline{M}^* = \begin{pmatrix} 2.731 & 3.252 & 4.843 & 53.136 \\ 3.206 & 2.497 & 5.394 & 53.687 \\ 3.622 & 2.761 & 4.811 & 48.294 \\ 3.992 & 3.895 & 3.064 & 39.361 \end{pmatrix}$$

Chains beginning in existing units:

$$\underline{M}^* = \begin{pmatrix} 1.644 & 4.778 & 21.668 & 148.924 \\ 2.758 & 3.079 & 20.176 & 144.146 \\ 2.839 & 3.419 & 16.995 & 147.565 \\ 3.617 & 2.834 & 15.098 & 125.992 \end{pmatrix}$$

Chains beginning in houses:

$$\underline{M}^* = \begin{pmatrix} 1.802 & 4.005 & 15.811 & 133.990 \\ 2.750 & 2.786 & 15.049 & 130.765 \\ 2.974 & 2.905 & 13.048 & 129.351 \\ 3.582 & 3.478 & 8.840 & 105.978 \end{pmatrix}$$

$\underline{M}^*$ for chains beginning in apartments could not be computed. Because one row of $\underline{Q}$ contained nothing but zeros ($q_{41} = q_{42} = q_{43} = q_{44} = 0$), the matrix $\underline{R}$ became the transition matrix of a Markov chain with an absorbing state.[1] Consequently, $\underline{M}^*$, the matrix of mean first passage times, could not be computed.

6.4: Filtering Effects in Lansing-East Lansing -- Discussion

The consideration of the filtering process in Lansing-East Lansing as a mechanism for the provision of housing at all levels of the housing system, and the effects of such a process must proceed at two levels. On a macroscopic scale, at the system level, we can only conclude that vacancies entering the cities of Lansing-East Lansing do not generate large numbers of vacancy moves within the city. With multipliers in both time periods not exceeding 1.3 moves, the system might be said to be relatively insensitive to vacancy creations.

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[1]See Appendix A, p. 188.

This low level of vacancy transfer within the system is further aggravated by the high degree of separateness of the housing sub-markets. Although not surprising (see White, 1971: 90) the limited exchange of vacancies between house and apartment sub-systems can only decrease the number of vacancy transfers in the system.

The vacancy chain model — the $(\underline{I}-\underline{Q})^{-1}$, multiplier matrix, and $\underline{M}^*$, mean first passage time matrix in particular — permits us to examine in detail the flows of vacancies through the system. These two matrices indicate that, in general, vacancies flow from higher strata to lower strata through the system. This downward flow of vacancies corresponds to an upward flow of people from lower to higher standards of housing. In 1970, more than 4 times as many vacancies travelled downward, and in 1965 more than 2.5 times as many vacancies travelled downward as travelled up.

Filtering-down ratios of this magnitude were found to exist in all the subsystems studied, except in the sequences of vacancy chains beginning in apartments (see Table 6.1). In this case, in 1970, only twice as many vacancy moves were generated downward as were generated upward, and in 1965 more vacancy moves were actually generated to strata above the stratum of origin. This extreme discrepancy in 1965 is largely due to the small number of chains entering the system via apartments in strata 3 and 4 (only 35.3 chains do so — and all but 2.9 exit the system immediately), but there definitely seems to be a difference between the house and apartment sub-systems.

$$\underline{M}^*_{70(\text{Apt.})} = \begin{pmatrix} 2.474 & 5.169 & 7.252 & 16.456 \\ 4.259 & 3.644 & 6.975 & 14.249 \\ 4.544 & 4.880 & 4.753 & 14.369 \\ 4.655 & 5.089 & 7.077 & 9.007 \end{pmatrix}$$

To use 1970 as an example, we can see that the sub-system is more compact than other sub-systems. The distances are relatively small and, with the exception of the m_{i4}*, the m_{ij}^* are of the same order. There is relatively little difference between m_{ij}* and m_{ji}*. Filtering-down in this model is indicated mainly in a negative way, by the length of time it takes vacancies to reach the highest stratum, and not by the speed at which vacancies flow downward. Even this span, as indicated by the m_{i4}* is not large when it is compared to other systems.

Several factors could account for the compactness of this system, the relative ease with which vacancies move both up and down between strata. The largely rental market of apartment units provides much greater potential for forced moves on short notice than does the house market, and this might account for some of the upwards vacancy movement. Perhaps, apartments have higher substitutability than houses, or perhaps apartment dwellers are more downwardly mobile than house dwellers.

A more reasonable explanation might be that apartment buildings, especially newer ones, might be non-conforming in our scheme of state classification. It is not uncommon for new apartments of high rent, status and dwelling standards to be built in the urban core or other urban redevelopment areas, resulting in an apartment building of much higher standard than its surrounding area. If this apartment occupied one, or at most, two blocks, the area would be too small to be classified as a high-level area, and the apartment would be classified as being, most likely, in stratum 1 or 2. Occupant moves to this apartment building from stratum 2 or 3 which are instances of upward housing mobility (i.e. vacancies filtering down, from the point of reference of the apartment building) would appear, because of state classification, to be instances of vacancies filtering up. The extent to which this occurs is an empirical question.

A second notable finding was the fact that vacancies originating in stratum 4 seem to skip stratum 3 in their downward movement through the system. In both time periods, in all sub-systems, vacancies beginning their careers in stratum 4 moved to stratum 2 and stratum 1 much sooner than they did to stratum 3. As Table 6.2 (excerpted from Table 4.8) shows, strata 3 and 4 contain housing of a much higher standard than strata 1 and 2. They also contain a much higher proportion of owner occupied homes.

Table 6.2

Selected Characteristics of Housing Sub-Areas

Sub-Area	Mean Value of Housing	Mean Rent	Mean Proportion of Housing Owned
1. Low	$14,400	$110	.40
2. Lower Middle	15,300	121	.75
3. Upper Middle	23,700	169	.83
4. High	34,500	184	.88

If, "most moves are undertaken voluntarily and are motivated by the changes in family size which rendered the old dwelling's space inadequate to its requirements" (Rossi, 1955:175), then it may be that the "skip" phenomenon can be accounted for in terms of household life-cycle. It is not unreasonable that occupancy moves from strata 1 and 2 to stratum 4 represent moves made by families entering the "child-bearing" or expansion stage of the life-cycle.

Residents of stratum 3 would in many cases already have made such a move to accomodate their increased need for space, and would be less likely to move to a new, larger home in stratum 4. Moves into strata 3 and 4 could well represent the same type of behavior — households entering their child-rearing periods — and result in the pattern of vacancy movement described.

Bearing in mind the dangers of ecological correlation, such a pattern of behavior with the lower levels of owner-occupancy in strata 1 and 2, could also indicate a shift of previous renters becoming home-owners in Lansing-East Lansing. Such tenure changes are very common (Lansing, et al., 1969: 30; Butler, et al., 1969: 10) and their absence would be curious.

We should also note that the lower ratios of reciprocal mean first passage times between strata 1 and 2 when compared with other adjacent strata, and the "skip" phenomenon differentiating them only slightly lends some support to the notion of cash rent or price as the sole criterion of housing levels (Hua, 1972: 122). As Table 6.2 shows, the major difference between the two strata is not the value of housing or rent, but the proportion of housing owner-occupied. The differences in rents and values are minimal when compared to the differences between strata 2 and 3 and strata 3 and 4.

The vacancy chain model has enabled us to examine closely the internal filtering processes of Lansing-East Lansing. But we should not let this micro-analysis blur the major finding: a multiplier of 1.3 moves per vacancy arrival does not yield many opportunities for occupants to change their housing standards, even if 70 percent of those who change, improve them.

7. CONCLUSIONS

The purpose of this study has been two-fold. The first aim was the development of a vacancy chain model of intra-urban residential mobility and the assessment of the adequacy of this model to represent the transfer of housing vacancies in the urban area. The second aim was to apply the model, if it proved adequate, to the analysis of the filtering process in the same urban housing system -- Lansing-East Lansing, Michigan.

Does the model fit the data? The model more than adequately represents the process of housing vacancy transfer in Lansing-East Lansing. The fit of the model to the data is exceptional. Comparisons of predicted and observed values for the complete model and numerous sub-system models in each of two time periods never yields a discrepancy of more than 5 percent, and only rarely do these discrepancies exceed 2 percent.

The transitions of individual vacancies appear to be independent of one another. As a result, we can construct a first-order Markov chain that represents vacancy movements extremely well. The problem of non-independence of transitions which arises in models of people as the population of movers does not arise when vacancies are the population of movers. This provides two distinct benefits:

the first is the use of first-order Markov chains in the analysis of residential mobility, and the second is to allow us to focus on state classification from a substantive or a policy perspective.

If the individual transitions are not made independently, as is the case with people changing addresses (Rossi, 1955; Moore, 1969), then one of the functions of state-classification is to ensure population homogeneity within occupancy states. This homogeneity allows us to assume the independence of transitions for practical purposes. If, however, the transitions really are made independently, then state classification forms an arbitrary partition of the occupancy space. Different partitions will obviously result in different transition parameters but will not affect the fit of the model. This allows states to be classified on the basis of substantive housing policy or theoretical concerns, free of any mathematical-theoretical constraints.

In our case, the model can be just as well represented by a single-state model of the residential system. This model predicts vacancy chain length distributions just as accurately as the 4-state model. While this lends support to our belief in the assumption of independent vacancy transitions (the 1-state model reduces to a series of Bernoulli trials), this fit also serves to explain the extraordinarily good fit of the complete model. Because the average length of chains is so low the vast majority of chains must be only 1 move long. This sharply limits the number of chains which could possibly be distributed at lengths other than 1 move. Consequently,

reasonably accurate estimation of the exit probabilities guarantees that overall differences between the model and the data will be minimal. That is to say, the high exit probabilities in this sample imply that the good fit of the model rests not on the estimation of 5x4 = 20 transition probabilities, but only on the estimation of the 4 exit probabilities -- especially when these are all so similar.

The low average chain length determined by these exit probabilities is a function of the size of the housing system under study. Lansing-East Lansing contains fewer than 200,000 people and the multiplier lengths of 1.2 to 1.3 fit quite well with other recent estimates of multipliers in areas of different sizes (Kristof, 1965; Lansing et al, 1969; Watson, 1974). This pattern is shown in Figure 7.1.

Chain lengths varied only slightly among the several sub-systems studied in the two time periods. Generally, chains beginning in houses, new units, and higher strata tended to be longer. The shortest chains were pure chains beginning in apartments in 1965 ($\overline{X}$ = 1.04) and the longest began in new houses in 1970 ($\overline{X}$ = 1.34).

The sub-systems also seem to be highly separated in that 78 percent of chains longer than 1 move in 1970 and 89 percent of such chains in 1965 contained only houses or apartments. There was a minimal flow of vacancies between these two dwelling types.

The vacancy chain model is an exceptional tool for the study of the filtering process. The use of mean chain lengths as an indicator

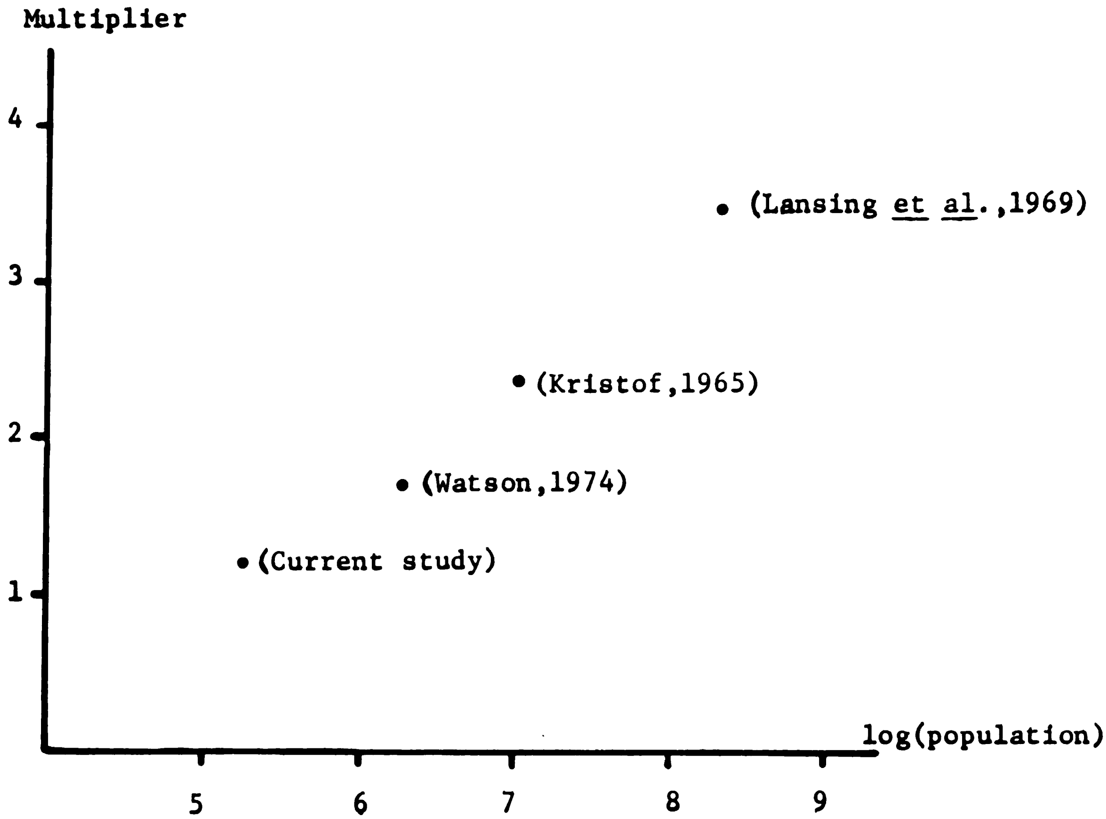

Figure 7.1. Multipliers and housing system populations

of the amount of filtering is not new (Kristof, 1965; Lansing et al., 1969; Watson, 1974). But the complete model yields a wealth of information regarding internal vacancy movement as well. In fact, the volume and extent of such micro-system data provided is such that one might ignore or forget the macroscopic findings with their system-level implications.

With the exception of chains beginning in apartments, some vacancy filtering was found to occur in all sub-systems. Overall, 4.4 times as many vacancy moves were generated downward in 1970 as were generated upward. In 1965, the ratio of moves downward in strata to moves upward was 2.5 : 1. Chains beginning in apartments, however, had the lowest filtering-down ratio in each time period. In fact, in 1965, filtering-up from apartments was indicated. How much of this apparent vacancy filtering-up is due to the construction of high-standard apartments in areas of low housing standards as parts of urban renewal programs must be determined.

Vacancies were also found to move more quickly from the highest housing level to the two lowest levels, skipping one stratum in the process, most likely reflecting life-cycle differences between the populations of strata 1 and 2 and stratum 3.

Reciprocal vacancy movement between the 2 lowest housing levels was the most nearly equal of any pair of housing strata. Insofar as these strata are also the most similar in terms of housing values and rents, we must consider again the notion of using only a measure of cash rent or price as the sole criterion of housing levels (Hua, 1972: 122).

The internal analysis of vacancy filtering points to the previously stated conclusion that anyone who moves benefits from such a move (Lansing et al., 1969: 65). The problem with the filtering process as a mechanism for the provision of new housing units throughout the housing system is that not many people move! Only 1.2 or 1.3 moves occur per vacancy creation. This pattern would surely change if we consider filtering on a regional or even national basis, for surely filtering need not only meet the needs of local residents (Watson, 1974: 349), especially in the light of recent flows of migrants from rural to urban areas. It is clear, however, that the major beneficiaries of the filtering process are not the residents of Lansing-East Lansing.

It would seem then that further study is required, both of the model and method employed, and of the Lansing-East Lansing housing system. Inclusion of the other portions of the Lansing Metropolitan Area would permit greater understanding of mobility processes in Lansing. Greater specificity of housing level classification, especially in the case of apartment buildings might allow the reconciliation of vacancy movements from apartments with other movements.

The model could be more fully explored by the selection of a larger study area. A larger system can be expected to have a longer average chain length. A longer average chain length reduces the extent to which the fit of the model is determined by the exit probabilities. This would allow a sounder evaluation of the fit of the model.

The use of directory data facilitates the study of larger metropolitan areas. The use of directory data in combination with questionnaires to trace the places of origin of in-migrants and the extension of our model to include all such places of origin, say, within a state, would increase costs, but would permit the analysis of filtering as a regional process.

The vacancy chain model is promising. It is a model of the housing system within which residential mobility occurs. It can be based on data which is now available for a large number of cities over a considerable time span. These factors together with the fit of the model within early studies, and the congruences between early studies call for its further exploration, and its application in the analysis of residential mobility.

APPENDICES

APPENDIX A

DATA MATRICES

This appendix presents the raw data from which the vacancy chain models of the complete housing system, and all the housing sub=systems, for both time periods were constructed. The unweighted and weighted vacancy moves are presented in matrix form. If $\underline{V} = [v_{ij}]$, the matrix of (weighted or unweighted) vacancy moves, then v_{ij} is the number of moves from stratum i to stratum j.

In addition to the raw data, the $\underline{f}$, $\underline{p}$, and $\underline{Q}$ matrices -- vacancy arrival, departure, and transition probabilities -- are presented, as are the derivative matrices and parameters of the model:

$(\underline{I}-\underline{Q})^{-1}$	-	the multiplier matrix;
$\underline{\lambda}$	-	the vector of mean chain lengths by stratum of origin;
$j(t)$	-	the mean chain length;
$\underline{P}_j$	-	the probability distribution of expected chain lengths by stratum of origin;
$\underline{M}$*	-	the matrix of intra-system mean first passage times;
$\underline{M}$	-	the matrix of mean first passage times including moves outside the system.

The appendix also presents a comparison of the observed and predicted chain length distributions.

ALL CHAINS, 1969 - 1970

Unweighted Vacancy Moves

	1	2	3	4	0
0	439	560	289	109	
1	98	32	13	4	503
2	72	101	22	7	550
3	31	37	31	12	260
4	10	22	16	20	84

Weighted Vacancy Moves

$\underline{F}(t)$	=	441.488	512.064	258.704	85.424	
		49.042	16.536	6.208	1.785	441.564
		34.380	50.895	11.334	3.822	508.142
		15.031	18.681	15.109	6.096	244.896
		5.202	10.396	8.457	8.764	73.072

Transition Probabilities

$\underline{f}(t)$	=	.325	.404	.204	.607	$\underline{p}$
		.095	.032	.012	.003	.857
$\underline{Q}$	=	.056	.083	.019	.006	.834
		.050	.062	.050	.020	.817
		.049	.098	.080	.083	.690

Multiplier Matrix

		1.109	.040	.015	.004
$(\underline{I}-\underline{Q})^{-1}$	=	.070	1.096	.023	.008
		.065	.077	1.057	.024
		.072	.126	.095	1.093

Mean Chain Length by Stratum

$\underline{\lambda}$	=	1.169	1.198	1.223	1.388

Mean Chain Length

$j(t)$ = 1.206

Probability Distribution of Chain Lengths

			1	2	3	4	5
			.857	.121	.019	.003	.000
$\underline{P}_j$	=		.835	.138	.023	.004	.001
			.817	.150	.027	.005	.001
			.690	.246	.052	.010	.002

Observed Chain Length Distribution

938	318	81	12	3

Weighted Chain Length Distribution

1049.093	181.104	32.820	3.892	.831

Mean First Passage Times

			1	2	3	4
		1	2.088	4.032	9.982	25.601
$\underline{M}$*	=	2	3.117	2.939	9.548	25.003
		3	3.443	3.472	7.828	22.784
		4	3.923	3.478	7.648	18.848

Mean First Passage Times (Including Outside)

			1	2	3	4	0
		1	5.429	4.823	9.668	28.535	1.169
$\underline{M}$	=	2	5.667	4.596	9.633	28.471	1.198
		3	5.722	4.710	9.328	28.076	1.223
		4	5.844	4.647	9.139	26.411	1.287
		0	4.851	3.840	8.641	27.493	2.206

CHAINS STARTING IN NEW UNITS, 1969 - 1970

Unweighted Vacancy Moves

	1	2	3	4	0
0	43	100	53	49	
1	16	4	1	1	65
2	13	22	2	2	108
3	7	8	8	7	41
4	8	13	7	8	31

Weighted Vacancy Moves

$\underline{F}(t)$	=	38.758	92.218	43.822	36.472	
		7.474	2.278	.324	.569	48.418
		6.092	10.888	.894	.975	96.364
		2.513	3.737	3.489	3.577	38.370
		4.227	6.092	3.496	3.244	27.778

Transition Probabilities

$\underline{f}(t)$	=	.184	.437	.206	.173	$\underline{p}$
		.127	.039	.005	.010	.820
$\underline{Q}$	=	.053	.095	.008	.008	.836
		.049	.072	.068	.069	.742
		.094	.136	.078	.072	.620

Multiplier Matrix

$(\underline{I}-\underline{Q})^{-1}$	=	1.150	.052	.008	.013
		.069	1.110	.011	.012
		.075	.102	.081	.082
		.133	.176	.093	1.088

Mean Chain Length by Stratum

$\underline{\lambda}$	=	1.223	1.201	1.340	1.491

Mean Chain Length

$j(t)$ = 1.284

Probability Distributions of Chain Lengths

		1	2	3	4	5
		.820	.146	.028	.005	.001
$\underline{P}_j$	=	.836	.133	.024	.005	.001
		.742	.193	.050	.011	.002
		.620	.294	.068	.147	.003

Observed Chain Length Distribution

154	63	20	8	0

Weighted Chain Length Distribution

164.354	35.87	8.104	2.595	0

Mean First Passage Times

			1	2	3	4
		1	2.013	4.370	21.192	15.872
$\underline{M}^*$	=	2	2.259	2.774	20.631	15.786
		3	3.929	3.754	15.306	11.417
		4	3.666	3.522	16.816	12.909

Mean First Passage Times (Including Outside)

			1	2	3	4	0
		1	8.157	4.447	9.878	11.282	1.223
$\underline{M}$	=	2	8.795	4.182	9.835	11.276	1.201
		3	8.884	4.354	9.322	10.655	1.340
		4	8.560	4.194	9.355	10.746	1.491
		0	8.157	3.440	8.733	10.120	2.284

CHAINS STARTING IN OLD UNITS, 1969 - 1970

Unweighted Vacancy Moves

	1	2	3	4	0
0	396	460	236	60	
1	82	28	12	3	438
2	59	79	20	5	442
3	24	29	23	5	219
4	2	9	9	12	53

Weighted Vacancy Moves

$\underline{F}(t)$ =	372.735	419.857	215.222	48.952	
	41.568	14.258	5.884	1.216	393.154
	28.289	40.007	10.440	2.847	411.790
	12.518	14.944	11.620	2.519	206.527
	.975	4.304	4.961	5.520	45.294

Transition Probabilities

$\underline{f}(t)$ =	.353	.397	.204	.046	$\underline{p}$
	.091	.031	.013	.003	.862
$\underline{Q}$ =	.057	.081	.021	.006	.835
	.050	.060	.047	.010	.832
	.016	.070	.081	.090	.742

Multiplier Matrix

$(\underline{I}-\underline{Q})^{-1}$	1.104	.039	.016	.004
	.071	1.093	.026	.007
	.063	.072	1.053	.012
	.030	.092	.096	1.101

Mean Chain Length by Stratum

$\underline{\lambda}$ = 1.162 1.197 1.200 1.320

Mean Chain Length

$j(t)$ = 1.191

Probability Distribution of Chain Lengths

		1	2	3	4	5
		.862	.117	.017	.003	.000
$\underline{P}_j$	=	.835	.139	.022	.004	.001
		.832	.140	.022	.004	.001
		.742	.207	.042	.007	.001

Observed Chain Length Distribution

829	255	61	4	3

Weighted Chain Length Distribution

844.739	145.225	24.716	1.297	.831

Mean First Passage Times

			1	2	3	4
		1	2.099	4.081	8.904	33.307
$\underline{M}^*$	=	2	3.099	2.974	8.422	32.521
		3	3.307	3.391	7.063	31.490
		4	4.443	3.571	5.926	21.859

Mean First Passage Times (Including Outside)

			1	2	3	4	0
		1	5.076	4.912	9.634	41.460	1.162
$\underline{M}$	=	2	5.279	4.693	9.577	41.351	1.197
		3	5.320	4.794	9.331	41.167	1.200
		4	5.605	4.821	9.043	37.921	1.320
		0	4.440	3.932	8.622	40.436	2.191

CHAINS STARTING IN HOUSES, 1969 - 1970

Unweighted Vacancy Moves

	1	2	3	4	0
0	289	384	128	69	
1	69	25	7	3	342
2	60	79	15	2	379
3	21	28	12	8	107
4	7	19	14	10	42

Weighted Vacancy Moves

$\underline{F}(t)$	=	267.841	345.889	106.270	49.862	
		34.493	12.713	3.201	1.380	292.968
		28.203	39.352	7.593	1.139	344.240
		10.564	13.720	5.767	4.146	95.951
		3.658	8.851	7.318	4.219	36.700

Transition Probabilities

$\underline{f}(t)$	=	.348	.449	.138	.065	$\underline{p}$
		.100	.037	.009	.004	.850
$\underline{Q}$	=	.067	.094	.018	.003	.819
		.081	.105	.044	.032	.737
		.060	.146	.120	.069	.604

Multiplier Matrix

$(\underline{I}-\underline{Q})^{-1}$	=	1.116	.048	.012	.005
		.085	1.110	.022	.004
		.107	.133	1.054	.037
		.099	.194	.140	1.080

Mean Chain Length by Stratum

$\underline{\lambda}$	=	1.182	1.222	1.332	1.551

Mean Chain Length

$J(t)$ = 1.242

Probability Distribution of Chain Lengths

		1	2	3	4	5
		.850	.124	.021	.004	.001
$\underline{P}_j$	=	.819	.149	.027	.005	.001
		.737	.207	.045	.009	.002
		.604	.301	.075	.016	.003

Observed Chain Length Distribution

577	222	59	9	3

Weighted Chain Length Distribution

615.795	126.431	23.906	2.919	.831

Mean First Passage Times

			1	2	3	4
		1	1.977	3.804	12.312	32.850
M*	=	2	2.811	2.757	11.732	33.201
		3	3.085	3.061	10.568	29.426
		4	3.606	3.041	8.720	27.177

Mean First Passage Times (Including Outside)

			1	2	3	4	0
		1	5.006	4.319	13.668	30.208	1.181
$\underline{M}$	=	2	5.202	4.104	13.577	30.277	1.222
		3	5.199	4.120	13.260	29.461	1.322
		4	5.423	4.052	13.299	28.408	1.515
		0	4.406	3.334	12.652	29.179	2.242

CHAINS STARTING IN APARTMENTS, 1969 - 1970

Unweighted Vacancy Moves

	1	2	3	4	0
0	146	169	158	37	
1	27	7	5	1	157
2	12	22	7	5	163
3	9	8	19	4	151
4	3	3	2	8	39

Weighted Vacancy Moves

$\underline{F}(t)$	=	139.893	158.724	150.473	33.022	
		13.655	3.822	2.683	.405	144.845
		6.177	11.543	3.741	2.683	155.880
		4.142	4.392	9.342	1.949	147.553
		1.544	1.544	1.139	3.734	33.832

Transition Probabilities

$\underline{f}(t)$	=	.290	.329	.312	.068	$\underline{p}$
		.083	.023	.016	.002	.876
$\underline{Q}$	=	.034	.064	.021	.015	.866
		.025	.026	.056	.012	.882
		.037	.037	.027	.089	.809

Multiplier Matrix

	1.092	.028	.019	.004
$(\underline{I}-\underline{Q})^{-1}$	.041	1.071	.025	.018
	.030	.031	1.061	.014
	.047	.046	.034	1.099

Mean Chain Length by Stratum

$\underline{\lambda}$ = 1.142 1.155 1.136 1.225

Mean Chain Length

$j(t)$ = 1.150

Probability Distribution of Chain Lengths

		1	2	3	4	5
		.896	.109	.014	.002	0
$\underline{P}_j$	=	.866	.116	.016	.002	0
		.882	.103	.013	.002	0
		.809	.161	.025	.004	.001

Observed Chain Length Distribution

393	24	21	2	0

Weighted Chain Length Distribution

419.424	53.534	8.509	.649	0

Mean First Passage Times

			1	2	3	4
		1	2.474	5.169	7.252	16.456
$\underline{M}^*$	=	2	4.259	3.644	6.975	14.249
		3	4.544	4.880	4.753	14.369
		4	4.665	5.089	7.077	9.007

Mean First Passage Times (Including Outside)

			1	2	3	4	0
		1	6.267	5.995	6.455	27.095	1.142
$\underline{M}$	=	2	6.594	5.758	6.434	26.753	1.155
		3	6.645	5.969	6.193	26.828	1.136
		4	6.630	5.975	6.451	24.802	1.225
		0	5.699	5.012	5.433	26.043	2.150

CHAINS STARTING IN NEW HOUSES, 1969 - 1970

Unweighted Vacancy Moves

	1	2	3	4	0
0	29	52	29	39	
1	6	3	0	1	45
2	11	15	0	0	62
3	3	6	4	7	20
4	6	2	7	8	22

Weighted Vacancy Moves

$\underline{F}(t)$	=	26.968	47.039	22.336	26.795	
		3.088	1.708	0	.569	34.194
		5.117	7.231	0	0	51.915
		1.135	2.762	1.704	3.577	18.358
		3.253	5.522	3.496	3.244	18.671

Transition Probabilities

$\underline{f}(t)$	=	.219	.382	.181	.218	$\underline{p}$
		.078	.043	0	.014	.864
$\underline{Q}$	=	.080	.113	0	0	.808
		.041	.100	.062	.130	.667
		.095	.162	.102	.094	.546

Multiplier Matrix

		1.091	.057	.002	.018
$(\underline{I}-\underline{Q})^{-1}$	=	.097	1.132	0	.002
		.078	.155	1.083	.157
		.141	.225	.123	1.125

Mean Chain Length by Stratum

$\underline{\lambda}$ = 1.168 1.232 1.472 1.614

Mean Chain Length

$j(t)$ = 1.344

Probability Distribution of Chain Lengths

	1	2	3	4	5
$\underline{P}_j$ =	.864	.110	.020	.004	.001
	.808	.160	.027	.004	.001
	.667	.229	.078	.020	.005
	.546	.333	.091	.023	.005

Observed Chain Length Distribution

84	46	14	5	0

Weighted Chain Length Distribution

89.648	26.197	5.673	1.622	0

Mean First Passage Times

		1	2	3	4
$\underline{M}^*$ =	1	2.131	3.100	66.707	16.664
	2	2.413	2.285	69.121	19.077
	3	3.758	3.123	49.550	10.808
	4	3.421	2.976	50.044	13.718

Mean First Passage Times (Including Outside)

		1	2	3	4	0
$\underline{M}$ =	1	7.299	4.768	11.032	8.904	1.168
	2	7.316	4.493	11.114	9.104	1.232
	3	7.702	4.631	10.486	8.035	1.473
	4	7.383	4.455	10.212	8.446	1.614
	0	6.799	3.854	9.884	7.885	2.344

CHAINS STARTING IN NEW APARTMENTS, 1969 - 1970

Unweighted Vacancy Moves

	1	2	3	4	0
0	14	41	22	9	
1	9	1	0	0	20
2	2	7	2	2	38
3	3	1	4	0	20
4	2	1	0	0	8

Weighted Vacancy Moves

$\underline{F}(t)$	=	11.791	37.708	20.252	8.610	
		4.061	.569	0	0	14.224
		.975	3.658	.894	.975	36.409
		1.054	.405	1.785	0	19.687
		.975	.569	0	0	8.040

Transition Probabilities

$\underline{f}(t)$	=	.150	.481	.258	.110	$\underline{p}$
		.215	.030	0	0	.754
$\underline{Q}$	=	.023	.085	.021	.023	.848
		.046	.018	.078	0	.859
		.102	.059	0	0	.839

Multiplier Matrix

		1.276	.042	.001	.001
$(\underline{I}-\underline{Q})^{-1}$	=	.036	1.096	.025	.025
		.064	.023	1.085	.001
		.132	.069	.002	.002

Mean Chain Length by Stratum

$\underline{\lambda}$	=	1.320	1.183	1.173	1.205

Mean Chain Length

$j(t)$ = 1.203

Probability Distribution of Chain Lengths

		1	2	3	4	5
$\underline{P}_j$	=	.754	.188	.044	.010	.002
		.848	.126	.020	.004	.001
		.859	.116	.020	.004	.001
		.839	.127	.027	.006	.001

Observed Chain Length Distribution

62	16	6	2	0

Weighted Chain Length Distribution

66.169	9.112	2.431	.649	0

Mean First Passage Times

			1	2	3	4
$\underline{M}^*$	=	1	1.521	8.133	30.981	30.351
		2	4.233	4.251	22.849	22.219
		3	3.397	8.094	13.930	30.313
		4	2.561	6.139	28.983	28.353

Mean First Passage Times (Including Outside)

			1	2	3	4	0
$\underline{M}$	=	1	9.157	4.380	8.309	18.142	1.320
		2	11.213	4.023	7.992	17.573	1.183
		3	10.948	4.309	7.530	18.003	1.173
		4	10.360	4.155	8.189	18.015	1.205
		0	10.364	3.229	6.996	16.839	2.203

CHAINS STARTING IN OLD HOUSES, 1969 - 1970

Unweighted Vacancy Moves

	1	2	3	4	0
0	260	332	99	30	
1	63	22	7	2	297
2	49	64	15	2	317
3	18	22	8	1	87
4	1	7	7	2	20

Weighted Vacancy Moves

$\underline{F}(t)$	=	240.874	298.856	83.934	23.067	
		31.405	11.005	3.201	.810	258.778
		23.087	32.122	7.593	1.139	292.331
		9.429	10.958	4.063	.569	77.593
		.405	3.329	3.822	.975	18.030

Transition Probabilities

$\underline{f}(t)$	=	.372	.462	.130	.036	$\underline{p}$
		.102	.036	.010	.003	.848
$\underline{Q}$	=	.064	.090	.021	.003	.821
		.092	.107	.040	.006	.756
		.015	.125	.144	.037	.679

Multiplier Matrix

		1.120	.046	.014	.003
$(\underline{I}-\underline{Q})^{-1}$	=	.082	1.106	.026	.004
		.117	.128	.046	.007
		.046	.164	.160	1.040

Mean Chain Length by Stratum

$\underline{\lambda}$ = 1.183 1.219 1.298 1.409

Mean Chain Length

$j(t)$ = 1.223

Probability Distribution of Chain Lengths

	1	2	3	4	5
$\underline{P}_j$ =	.848	.127	.021	.003	.001
	.821	.147	.027	.005	.001
	.756	.199	.037	.007	.001
	.679	.250	.058	.011	.002

Observed Chain Length Distribution

493	176	45	4	3

Weighted Chain Length Distribution

526.147	100.233	18.233	1.297	.831

Mean First Passage Times

		1	2	3	4
$\underline{M}^*$ =	1	1.922	3.888	10.789	54.856
	2	2.804	2.813	10.059	54.829
	3	2.761	3.021	9.612	54.558
	4	3.760	2.866	6.140	49.445

Mean First Passage Times (Including Outside)

		1	2	3	4	0
$\underline{M}$ =	1	4.708	4.238	14.345	55.840	1.183
	2	4.918	4.033	14.208	55.837	1.218
	3	4.837	4.022	14.003	55.768	1.298
	4	5.281	3.991	12.524	54.099	1.409
	0	4.088	3.242	13.355	54.837	2.222

CHAINS STARTING IN OLD APARTMENTS, 1969 - 1970

Unweighted Vacancy Moves

	1	2	3	4	0
0	132	128	136	28	
1	18	6	5	1	137
2	10	15	5	3	125
3	6	7	15	4	131
4	1	2	2	8	31

Weighted Vacancy Moves

$\underline{F}(t)$ =	128.102	121.016	130.221	24.412	
	9.594	3.253	2.683	.405	130.621
	5.202	7.885	2.847	1.708	119.471
	3.088	3.986	7.557	1.949	127.866
	.569	.975	1.139	3.734	25.792

Transition Probabilities

$\underline{f}(t)$ =	.317	.300	.323	.060	$\underline{p}$
	.065	.022	.018	.003	.891
$\underline{Q}$ =	.038	.058	.021	.012	.871
	.021	.028	.052	.013	.885
	.018	.030	.035	.116	.801

Multiplier Matrix

	1.072	.026	.021	.004
$(\underline{I} - \underline{Q})^{-1}$ =	.044	1.063	.025	.016
	.026	.032	1.057	.017
	.024	.038	.044	1.132

Mean Chain Length by Stratum

$\underline{\lambda}$ = 1.123 1.148 1.132 1.238

Mean Chain Length

j(t) = 1.140

Probability Distribution of Chain Lengths

$\underline{P}_j$ =	1	2	3	4	5
	.891	.096	.011	.001	0
	.871	.112	.014	.002	0
	.885	.100	.013	.002	0
	.801	.166	.030	.004	.001

Observed Chain Length Distribution

331	78	15	0	0

Weighted Chain Length Distribution

353.255	44.422	6.078	0	0

Mean First Passage Times

$\underline{M}^*$ =		1	2	3	4
	1	2.908	4.829	5.974	14.879
	2	4.375	3.698	6.013	13.493
	3	5.095	4.655	4.246	12.890
	4	6.142	5.390	5.842	6.662

Mean First Passage Times (Including Outside)

$\underline{M}$ =		1	2	3	4	0
	1	5.896	6.513	6.187	30.157	1.123
	2	6.084	6.302	6.191	29.874	1.148
	3	6.175	6.483	5.982	29.826	1.132
	4	6.292	6.551	6.169	26.828	1.238
	0	5.196	5.553	5.192	29.142	2.140

PURE CHAINS, HOUSES, 1969 - 1970

Unweighted Vacancy Moves

	1	2	3	4	0
0	273	365	120	55	
1	55	16	4	0	322
2	52	65	8	1	357
3	11	26	10	7	97
4	6	11	9	7	37

Weighted Vacancy Moves

$\underline{F}(t)$	=	260.170	336.349	102.043	43.694	
		29.189	8.290	1.821	0	284.005
		24.762	33.324	4.227	.569	333.481
		5.936	12.874	5.038	3.577	90.585
		3.253	5.526	4.880	3.084	34.182

Transition Probabilities

$\underline{f}(t)$	=	.351	.453	.137	.059	$\underline{p}$
		.090	.026	.006	0	.878
$\underline{Q}$	=	.062	.084	.011	.001	.841
		.050	.109	.043	.030	.768
		.064	.109	.096	.061	.671

Multiplier Matrix

		1.102	.032	.007	0
$(\underline{I} - \underline{Q})^{-1}$	=	.076	1.096	.013	.002
		.069	.131	1.050	.034
		.091	.142	.109	1.068

Mean Chain Length by Stratum

$\underline{\lambda}$	=	1.141	1.187	1.284	1.410

Mean Chain Length

$j(t)$	=	1.197

Probability Distribution of Chain Lengths

		1	2	3	4	5
$\underline{P}_j$ =		.878	.105	.014	.002	0
		.841	.135	.020	.003	0
		.768	.189	.036	.006	.001
		.671	.262	.055	.010	.002

Observed Chain Length Distribution

577	191	39	5	1

Weighted Chain Length Distribution

615.795	108.776	15.802	1.622	.277

Mean First Passage Times

		1	2	3	4
$\underline{M}^*$ =	1	1.715	4,420	18.681	91.078
	2	2.669	2.967	18.022	89.451
	3	3.317	2.928	15.195	76.820
	4	3.490	3.325	12.968	70.601

Mean First Passage Times (Including Outside)

		1	2	3	4	0
$\underline{M}$ =	1	5.044	4.331	14.268	33.926	1.140
	2	5.219	4.114	14.232	33.914	1.187
	3	5.351	4.066	13.818	32.987	1.284
	4	5.369	4.146	13.126	32.020	1.410
	0	4.417	3.321	13.223	32.794	2.197

PURE CHAINS, APARTMENTS ONLY, 1969 - 1970

Unweighted Vacancy Moves

	1	2	3	4	0
0	142	145	149	34	
1	21	4	5	0	143
2	5	11	2	2	149
3	3	7	14	2	145
4	2	2	1	6	33

Weighted Vacancy Moves

$\underline{F}(t)$	=	137.779	145.794	146.414	31.642	
		11.302	2.278	2.683	0	138.184
		2.519	5.607	1.139	1.139	148.236
		1.708	3.822	7.316	.975	144.301
		1.139	1.139	.569	2.924	30.908

Transition Probabilities

$\underline{f}(t)$	=	.298	.316	.317	.069	$\underline{p}$
		.073	.015	.017	0	.895
$\underline{Q}$	=	.016	.035	.007	.007	.934
		.011	.024	.046	.006	.913
		.031	.031	.016	.080	.843

Multiplier Matrix

		1.079	.017	.020	0
$(\underline{I}-\underline{Q})^{-1}$	=	.018	1.037	.008	.008
		.013	.027	1.049	.007
		.037	.036	.019	1.087

Mean Chain Length by Stratum

$\underline{\lambda}$ = 1.117 1.072 1.096 1.179

Mean Chain Length

$\jmath(t)$ = 1.100

Probability Distribution of Chain Lengths

	1	2	3	4	5
$\underline{P}_j$ =	.895	.095	.009	.001	0
	.934	.060	.005	.001	0
	.913	.080	.007	.001	0
	.843	.138	.017	.002	0

Observed Chain Length Distribution

393	67	10	0	0

Weighted Chain Length Distribution

419.424	38.157	4.052	0	0

Mean First Passage Times

		1	2	3	4
$\underline{M}^*$ =	1	2.589	5.657	6.731	20.840
	2	4.702	3.419	7.536	17.322
	3	5.643	4.384	4.463	17.761
	4	5.034	5.165	7.736	10.289

Mean First Passage Times (Including Outside)

		1	2	3	4	0
$\underline{M}$ =	1	6.278	6.281	6.332	28.665	1.117
	2	6.618	6.112	6.358	28.412	1.072
	3	6.675	6.201	6.132	28.460	1.096
	4	6.605	6.227	6.402	26.434	1.179
	0	5.660	5.268	6.337	27.556	2.100

PURE CHAINS, STARTING IN NEW HOUSES, 1969 - 1970

Unweighted Vacancy Moves

	1	2	3	4	0
0	25	50	27	33	
1	4	1	0	0	40
2	9	12	0	0	56
3	1	6	2	6	19
4	6	8	5	6	20

Weighted Vacancy Moves

$\underline{F}(t)$	=	24.690	46.065	21.361	24.361	
		2.114	.569	0	0	32.085
		4.307	5.932	0	0	49.072
		.405	2.762	.975	3.008	17.789
		3.253	3.982	2.602	2.515	17.532

Transition Probabilities

$\underline{f}(t)$	=	.212	.395	.183	.209	$\underline{p}$
		.061	.016	0	0	.923
$\underline{Q}$	=	.073	.100	0	0	.827
		.016	.111	.039	.121	.713
		.109	.133	.087	.084	.587

Multiplier Matrix

		1.066	.019	0	0
$(\underline{I}-\underline{Q})^{-1}$	=	.086	1.113	0	0
		.046	.151	1.053	.139
		.144	.179	.100	1.105

Mean Chain Length by Stratum

$\bar{\underline{\lambda}}$	=	1.086	1.199	1.389	1.527

Mean Chain Length

$\bar{j}(t)$	=	1.278

Probability Distribution of Chain Lengths

		1	2	3	4	5
$\underline{P}_j$	=	.923	.070	.007	.001	0
		.827	.150	.020	.002	0
		.713	.205	.065	.014	.002
		.587	.322	.073	.015	.002

Observed Chain Length Distribution

84	39	9	3	0

Weighted Chain Length Distribution

89.648	22.211	3.647	.973	0

Mean First Passage Times

			1	2	3	4
		1	1.505	4.708	1.329	1.441
$\underline{M}^*$	=	2	2.378	2.980	1.329	1.441
		3	3.791	3.255	1.000	.739
		4	3.221	3.673	.978	1.000

Mean First Passage Times (Including Outside)

			1	2	3	4	0
		1	7.634	4.780	10.907	9.373	1.086
$\underline{M}$	=	2	7.596	4.475	11.020	9.487	1.199
		3	8.092	4.494	10.643	8.445	1.389
		4	7.485	4.509	10.283	8.882	1.528
		0	7.054	3.781	9.821	8.288	2.279

PURE CHAINS STARTING IN NEW APARTMENTS, 1969 - 1970

Unweighted Vacancy Moves

	1	2	3	4	0
0	13	36	19	9	
1	6	0	0	0	16
2	2	5	0	1	35
3	0	1	1	0	18
4	1	1	0	0	8

Weighted Vacancy Moves

$\underline{F}(t)$	=	11.221	35.270	19.118	8.610	
		3.088	0	0	0	12.765
		.975	2.519	0	.569	34.700
		0	.405	.569	0	18.712
		.569	.569	0	0	8.040

Transition Probabilities

$\underline{f}(t)$	=	.151	.475	.257	.116	$\underline{p}$
		.195	0	0	0	.805
$\underline{Q}$	=	.025	.065	0	.015	.895
		0	.021	.029	0	.950
		.062	.062	0	0	.876

Multiplier Matrix

		1.242	0	0	0
$(\underline{I}-\underline{Q})^{-1}$	=	.035	1.070	0	.016
		.001	.023	1.030	0
		.079	.066	0	1.001

Mean Chain Length by Stratum

$\underline{\lambda}$	=	1.242	1.121	1.054	1.147

Mean Chain Length

$j(t)$ = 1.125

Probability Distribution of Chain Lengths

	1	2	3	4	5
	.805	.157	.031	.005	.001
	.895	.091	.011	.002	0
$\underline{P}_j$ =	.950	.046	.003	0	0
	.876	.105	.015	.003	0

Observed Chain Length Distribution

62	12	3	0	0

Weighted Chain Length Distribution

66.169	6.834	5.267	0	0

Mean First Passage Times

		1	2	3	4
	1	1.000	*	2.403	*
$\underline{M}^*$ =	2	3.683	*	2.403	*
	3	6.086	0	1.000	*
	4	2.841	*	2.403	*

Mean First Passage Times (Including Outside)

		1	2	3	4	0
	1	9.949	4.477	8.439	17.296	1.242
$\underline{M}$ =	2	11.890	4.069	8.318	16.904	1.121
	3	12.160	4.196	8.012	17.101	1.054
	4	11.473	4.112	8.344	17.183	1.147
	0	11.114	3.235	7.197	16.053	2.125

*Value exceeds 10^{10}

PURE CHAINS STARTING IN OLD HOUSES, 1969 - 1970

Unweighted Vacancy Moves

	1	2	3	4	0
0	248	315	93	22	
1	51	15	4	0	282
2	43	53	8	1	301
3	10	20	8	1	78
4	0	3	4	1	17

Weighted Vacancy Moves

$\underline{F}(t)$	=	235.480	290.290	80.681	19.333	
		27.075	7.721	1.821	0	251.924
		20.455	27.393	4.227	.569	284.415
		5.531	10.111	4.063	.569	72.796
		0	1.544	2.278	.569	16.650

Transition Probabilities

$\underline{f}(t)$	=	.376	.464	.129	.031	$\underline{p}$
		.094	.027	.006	0	.873
$\underline{Q}$	=	.061	.081	.013	.002	.844
		.059	.109	.044	.006	.782
		0	.073	.108	.027	.791

Multiplier Matrix

		1.106	.033	.008	0
$(\underline{I}-\underline{Q})^{-1}$	=	.074	1.093	.015	.002
		.077	.127	1.049	.007
		.014	.097	.118	1.029

Mean Chain Length by Stratum

$\underline{\lambda}$ = 1.147 1.184 1.259 1.257

Mean Chain Length

$j(t)$ = 1.182

Probability Distribution of Chain Lengths

$\underline{P}_j$ =

1	2	3	4	5
.873	.109	.015	.002	0
.844	.133	.020	.003	0
.782	.183	.030	.005	.001
.791	.168	.034	.006	.001

Observed Chain Length Distribution

493	152	30	2	1

Weighted Chain Length Distribution

526.147	86.565	12.156	.649	.277

Mean First Passage Times

$\underline{M}^*$ =

	1	2	3	4
1	1.721	4.375	16.460	177.933
2	2.691	2.959	15.594	174.825
3	3.072	2.844	13.474	171.011
4	4.067	2.844	7.449	151.157

Mean First Passage Times (Including Outside)

$\underline{M}$ =

	1	2	3	4	0
1	4.731	4.255	15.156	66.630	1.147
2	4.920	4.050	15.085	66.544	1.184
3	4.981	3.987	14.669	66.307	1.259
4	5.277	4.108	13.651	64.883	1.257
0	4.087	3.242	14.122	65.490	2.182

PURE CHAINS STARTING IN OLD APARTMENTS, 1969 - 1970

Unweighted Vacancy Moves

	1	2	3	4	0
0	129	109	130	25	
1	15	4	5	0	127
2	3	6	2	1	114
3	3	6	13	2	127
4	1	1	1	6	25

Weighted Vacancy Moves

$\underline{F}(t)$	=	126.558	110.524	127.297	23.032	
		8.214	2.278	2.683	0	125.419
		1.544	3.088	1.139	.569	113.536
		1.708	3.417	6.746	.975	125.588
		.569	.569	.569	2.924	22.868

Transition Probabilities

$\underline{f}(t)$	=	.327	.285	.329	.059	$\underline{p}$
		.059	.016	.019	0	.904
$\underline{Q}$	=	.013	.026	.010	.005	.947
		.012	.025	.049	.007	.907
		.021	.021	.021	.106	.832

Multiplier Matrix

		1.064	.019	.022	0
$(\underline{I}-\underline{Q})^{-1}$	=	.014	1.027	.011	.006
		.014	.027	1.052	.008
		.025	.025	.025	1.119

Mean Chain Length by Stratum

$\underline{\bar{\lambda}}$ = 1.104 1.058 1.102 1.195

Mean Chain Length

$j(t)$ = 1.095

Probability Distribution of Chain Lengths

	1	2	3	4	5
$\underline{P}_j$ =	.905	.087	.008	.001	0
	.947	.049	.004	0	0
	.907	.085	.007	.001	0
	.832	.146	.020	.003	0

Observed Chain Length Distribution

331	55	7	0	0

Weighted Chain Length Distribution

353.255	31.323	2.836	0	0

Mean First Passage Times

		1	2	3	4
$\underline{M}^*$ =	1	3.134	5.075	5.183	20.716
	2	5.206	3.569	5.514	18.076
	3	6.058	4.463	3.631	18.045
	4	6.466	5.891	6.278	7.987

Mean First Passage Times (Including Outside)

		1	2	3	4	0
$\underline{M}$ =	1	5.857	6.876	6.043	32.943	1.104
	2	6.099	6.772	6.062	32.740	1.058
	3	6.143	6.816	5.864	32.699	1.102
	4	6.172	6.924	6.114	29.519	1.195
	0	5.125	5.898	5.067	31.846	2.095

CHAINS WITHOUT STUDENTS, 1969 - 1970

Unweighted Vacancy Moves

	1	2	3	4	0
0	349	479	205	81	
1	80	20	5	1	414
2	60	92	15	4	469
3	22	33	16	10	174
4	9	16	14	11	57

Weighted Vacancy Moves

$\underline{F}(t)$	=	330.409	439.014	181.157	63.331	
		40.560	10.897	2.438	.569	361.104
		29.246	46.555	7.840	2.114	434.881
		10.727	16.696	7.717	5.285	166.046
		4.633	7.471	7.318	4.624	51.878

Transition Probabilities

$\underline{f}(t)$	=	.326	.433	.179	.062	$\underline{p}$
		.098	.026	.006	.001	.869
$\underline{Q}$	=	.056	.089	.015	.004	.835
		.052	.081	.037	.026	.804
		.061	.098	.096	.061	.683

Multiplier Matrix

		1.111	.032	.007	.002
$(\underline{I}-\underline{Q})^{-1}$	=	.070	1.102	.018	.005
		.068	.098	1.044	.029
		.087	.128	.110	1.069

Mean Chain Length by Stratum

$\underline{\lambda}$ = 1.153 1.196 1.239 1.392

Mean Chain Length

j(t) = 1.202

Probability Distribution of Chain Lengths

		1	2	3	4	5
		.869	.112	.016	.002	0
$\underline{P}_j$	=	.835	.138	.022	.004	.001
		.804	.160	.030	.005	.001
		.683	.254	.051	.009	.002

Observed Chain Length Distribution

787	259	56	11	1

Weighted Chain Length Distribution

839.915	147.503	22.690	3.568	.277

Mean First Passage Times

			1	2	3	4
		1	1.806	4.653	15.603	42.043
$\underline{M}^*$	=	2	3.068	2.983	14.386	40.692
		3	3.399	3.345	12.454	35.799
		4	3.699	3.608	10.493	32.638

Mean First Passage Times (Including Outside)

			1	2	3	4	0
		1	5.371	4.543	11.117	31.117	1.153
$\underline{M}$	=	2	5.633	4.287	10.044	31.060	1.196
		3	5.686	4.350	10.811	31.409	1.238
		4	5.741	4.375	10.253	29.400	1.392
		0	4.813	3.531	10.045	30.022	2.202

ALL CHAINS, 1964 - 1965

Unweighted Vacancy Moves

	1	2	3	4	0
0	264	328	147	66	
1	98	28	8	0	310
2	56	81	14	2	322
3	20	30	17	3	129
4	6	8	13	6	44

Weighted Vacancy Moves

$\underline{F}(t)$	=	236.625	290.537	127.230	52.672	
		43.907	12.464	3.408	0	259.160
		26.382	38.787	6.697	1.139	286.725
		9.674	14.205	7.559	1.299	118.366
		2.353	3.737	6.209	3.008	42.812

Transition Probabilities

$\underline{f}(t)$	=	.335	.411	.180	.074	$\underline{p}$
		.138	.039	.011	0	.813
$\underline{Q}$	=	.073	.108	.019	.003	.797
		.064	.094	.050	.009	.783
		.040	.064	.107	.052	.737

Multiplier Matrix

		1.165	.052	.014	0
$(\underline{I}-\underline{Q})^{-1}$	=	.098	1.128	.024	.004
		.089	.116	1.057	.010
		.066	.092	.121	1.056

Mean Chain Length by Stratum

$\underline{\lambda}$ = 1.232 1.253 1.272 1.335

Mean Chain Length

j(t) = 1.256

Probability Distribution of Chain Lengths

	1	2	3	4	5	6
$\underline{P}_j$ =	.813	.151	.029	.006	.001	0
	.797	.162	.032	.006	.001	0
	.783	.172	.035	.007	.001	0
	.737	.206	.046	.009	.002	0

Observed Chain Length Distribution

532	189	59	19	4	2

Weighted Chain Length Distribution

567.769	107.637	23.906	6.162	1.108	.493

Mean First Passage Times

		1	2	3	4
$\underline{M}$* =	1	1.775	4.454	14.320	116.391
	2	2.866	2.961	13.443	113.352
	3	3.107	3.203	11.386	109.968
	4	3.684	3.714	8.070	91.190

Mean First Passage Times (Including Outside)

		1	2	3	4	0
$\underline{M}$ =	1	4.999	4.745	10.966	28.860	1.232
	2	5.357	4.433	10.887	28.781	1.253
	3	5.421	4.504	10.553	28.635	1.271
	4	5.597	4.675	9.938	27.436	1.335
	0	4.593	3.747	9.883	27.637	2.255

Probability Distribution of Chain Lengths

	1	2	3	4	5	6
$\underline{P}_j$ =	.813	.151	.029	.006	.001	0
	.797	.162	.032	.006	.001	0
	.783	.172	.035	.007	.001	0
	.737	.206	.046	.009	.002	0

Observed Chain Length Distribution

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	2	2.866	2.961	13.443	113.352
	3	3.107	3.203	11.386	109.968
	4	3.684	3.714	8.070	91.190

Mean First Passage Times (Including Outside)

		1	2	3	4	0
$\underline{M}$ =	1	4.999	4.745	10.966	28.860	1.232
	2	5.357	4.433	10.887	28.781	1.253
	3	5.421	4.504	10.553	28.635	1.271
	4	5.597	4.675	9.938	27.436	1.335
	0	4.593	3.747	9.883	27.637	2.255

CHAINS STARTING IN NEW UNITS, 1964 - 1965

Unweighted Vacancy Moves

	1	2	3	4	0
0	28	103	55	36	
1	9	4	4	0	44
2	13	21	6	0	104
3	7	14	7	3	51
4	4	2	10	4	23

Weighted Vacancy Moves

$\underline{F}(t)$	=	25.163	97.410	46.524	27.528	
		3.521	1.949	1.704	0	32.506
		6.044	9.419	2.762	0	97.406
		3.248	6.123	3.168	1.299	44.820
		1.704	.729	4.500	2.114	21.893

Transition Probabilities

$\underline{f}(t)$	=	.128	.495	.237	.140	$\underline{p}$
		.089	.049	.043	0	.819
$\underline{Q}$	=	.052	.081	.024	0	.842
		.055	.104	.054	.022	.764
		.055	.024	.145	.068	.708

Multiplier Matrix

		1.104	.065	.051	.001
$(\underline{I}-\underline{Q})^{-1}$	=	.065	1.096	.031	.001
		.074	.126	1.068	.025
		.078	.051	.170	1.077

Mean Chain Length by Stratum

$\underline{\lambda}$	=	1.223	1.192	1.292	1.378

Mean Chain Length

$j(t)$ = 1.246

Probability Distribution of Chain Lengths

	1	2	3	4	5	6
$\underline{P}_j$ =	.819	.147	.028	.005	.001	0
	.842	.130	.023	.004	.001	0
	.764	.190	.037	.007	.001	0
	.708	.224	.054	.011	.002	0

Observed Chain Length Distribution

152	41	21	7	1	0

Weighted Chain Length Distribution

162.220	23.35	8.509	2.27	.277	0

Mean First Passage Times

		1	2	3	4
$\underline{M}^*$ =	1	2.731	3.252	4.843	53.136
	2	3.206	2.497	5.394	53.687
	3	3.622	2.761	4.811	48.294
	4	3.992	3.895	3.064	39.361

Mean First Passage Times (Including Outside)

		1	2	3	4	0
$\underline{M}$ =	1	11.122	3.967	7.576	15.204	1.223
	2	11.539	3.819	7.705	15.181	1.192
	3	11.541	3.804	7.529	14.930	1.293
	4	11.573	4.174	6.838	14.273	1.378
	0	11.068	2.992	6.744	13.999	2.246

CHAINS STARTING IN OLD UNITS, 1964 - 1965

Unweighted Vacancy Moves

	1	2	3	4	0
0	236	225	92	30	
1	89	24	4	0	266
2	43	60	8	2	218
3	13	16	10	0	78
4	2	6	3	2	21

Weighted Vacancy Moves

$\underline{F}(t)$	=	211.462	193.132	80.706	25.144	
		40.386	10.515	1.704	0	226.655
		20.337	29.368	3.935	1.138	189.324
		6.425	8.081	4.391	0	73.547
		.649	3.008	1.708	.894	20.919

Transition Probabilities

$\underline{f}(t)$	=	.414	.378	.158	.049	$\underline{p}$
		.145	.038	.006	0	.812
		.083	.120	.016	.005	.776
		.070	.087	.047	0	.796
		.024	.111	.063	.033	.770

Multiplier Matrix

$$(\underline{I}-\underline{Q})^{-1} = \begin{matrix} 1.175 & .051 & .008 & 0 \\ .113 & 1.144 & .020 & .005 \\ .096 & .109 & 1.052 & .001 \\ .048 & .139 & .071 & 1.035 \end{matrix}$$

Mean Chain Length by Stratum

$\underline{\lambda}$ = 1.235 1.284 1.258 1.293

Mean Chain Length

j(t) = 1.260

Probability Distribution of Chain Lengths

		1	2	3	4	5	6
		.812	.151	.030	.006	.001	0
$\underline{P}_j$	=	.776	.177	.037	.008	.002	0
		.796	.162	.034	.007	.001	0
		.770	.181	.039	.008	.002	0

Observed Chain Length Distribution

380	148	38	12	3	2

Weighted Chain Length Distribution

405.550	84.287	15.397	3.892	.831	.493

Mean First Passage Times

			1	2	3	4
		1	1.644	4.778	21.668	148.924
$\underline{M}^*$	=	2	2.758	3.078	20.176	144.146
		3	2.839	3.419	16.995	147.565
		4	3.617	2.834	15.098	125.992

Mean First Passage Times (Including Outside)

		1	2	3	4	0
	1	4.130	5.116	13.000	43.836	1.234
	2	4.432	4.724	12.899	43.662	1.283
$\underline{M}$	3	4.477	4.867	12.475	43.848	1.258
	4	4.711	4.758	12.278	42.434	1.293
	0	3.617	4.123	11.871	42.613	2.259

CHAINS STARTING IN HOUSES, 1964 - 1965

Unweighted Vacancy Moves

	1	2	3	4	0
0	223	293	121	58	
1	85	2[illegible]	6	0	271
2	56	78	12	2	288
3	19	30	14	2	101
4	6	8	13	6	35

Weighted Vacancy Moves

$\underline{F}(t)$	=	200.989	255.180	100.477	44.134	
		36.749	12.140	2.679	0	224.254
		26.382	37.323	5.558	1.139	252.182
		9.350	14.205	6.260	.730	90.639
		2.353	3.737	6.208	3.008	33.705

Transition Probabilities

$\underline{f}(t)$	=	.335	.425	.167	.073	$\underline{p}$
$\underline{Q}$	=	.133	.044	.010	0	.813
		.082	.116	.017	.004	.782
		.077	.117	.052	.006	.748
		.048	.076	.127	.061	.688

Multiplier Matrix

$(\underline{1}-\underline{Q})^{-1}$	=	1.161	.060	.013	0
		.110	1.140	.022	.004
		.109	.146	1.059	.007
		.083	.115	.145	1.067

Mean Chain Length by Stratum

$\underline{\lambda}$	=	1.23[illegible]	1.276	1.322	1.410

Mean Chain Length

$j(t)$ = 1.279

Probability Distribution of Chain Lengths

		1	2	3	4	5	6
		.813	.150	.029	.006	.001	0
$\underline{P}_j$	=	.782	.172	.036	.008	.002	0
		.748	.197	.043	.009	.002	0
		.688	.236	.060	.013	.003	.001

Observed Chain Length Distribution

443	171	58	17	4	2

Weighted Chain Length Distribution

472.785	97.386	23.501	5.514	1.108	.493

Mean First Passage Times

			1	2	3	4
		1	1.802	4.005	15.811	133.990
$\underline{M}^*$	=	2	2.750	2.786	15.049	130.765
		3	2.974	2.905	13.048	129.351
		4	3.582	3.478	8.840	105.978

Mean First Passage Times (Including Outside)

			1	2	3	4	0
		1	4.964	4.541	11.732	29.613	1.233
$\underline{M}$	=	2	5.528	4.244	11.668	29.541	1.276
		3	5.310	4.261	11.278	29.505	1.321
		4	5.526	4.481	10.413	27.934	1.410
		0	4.527	3.561	10.646	28.389	1.279

CHAINS STARTING IN APARTMENTS, 1964-65

Unweighted Vacancy Moves

	1	2	3	4	0
0	41	31	26	8	
1	13	1	2	0	39
2	0	3	2	0	30
3	1	0	3	1	28
4	0	0	0	0	9

Weighted Vacancy Moves

$\underline{F}(t) =$	35.636	31.093	26.753	8.538	
	7.158	.324	.729	0	34.906
	0	1.463	1.139	0	30.279
	.324	0	1.299	.569	27.727
	0	0	0	0	9.107

Transition Probabilities

$\underline{f}(t) =$	.349	.305	.262	.084	$\underline{P}$
	.166	.008	.017	0	.810
	0	.045	.035	0	.921
	.011	0	.043	.019	.927
	0	0	0	0	1.000

Multiplier Matrix

$(\underline{I}-\underline{Q})^{-1} =$	1.199	.009	.022	0
	0	1.047	.038	.001
	.014	0	1.046	.020
	0	0	0	1.000

Mean Chain Length by Stratum

$\underline{\lambda}$	1.231	1.086	1.079	1.000
$j(t) =$	1.127			

Probability Distribution of Chain Lengths

	1	2	3	4	5	6
$\underline{P}_j$ =	.810	.157	.028	.005	.001	0
	.921	.073	.006	0	0	0
	.927	.068	.005	.001	0	0
	1.000	0	0	0	0	0

Observed Chain Length Distribution

85	18	1	2	0	0

Weighted Chain Length Distribution

90.715	10.251	.405	.649	0	0

Mean First Passage Times

	1	2	3	4
1				
2				
3	$\underline{M}^*$ is undefined			
4				

Mean First Passage Times (Including Outside)

		1	2	3	4	0
	1	5.034	6.992	7.581	24.056	1.231
$\underline{M}$ =	2	5.890	6.602	7.318	23.904	1.086
	3	5.818	6.902	7.255	23.440	1.079
	4	5.807	6.824	7.507	23.835	1.000
	0	4.807	5.824	6.507	22.835	2.128

CHAINS STARTING IN NEW HOUSES, 1964 - 1965

Unweighted Vacancy Moves

	1	2	3	4	0
0	21	83	45	34	
1	8	4	2	0	38
2	13	21	4	0	86
3	6	14	5	2	39
4	4	2	10	4	20

Weighted Vacancy Moves

F(t)	=	19.595	77.061	36.349	25.394	
		2.952	1.949	.975	0	27.343
		6.044	9.419	1.623	0	78.196
		2.924	6.123	2.438	.730	33.670
		1.704	.730	4.500	2.114	19.189

Transition Probabilities

l(t)	=	.124	.487	.229	.160	P
Q	=	.089	.058	.029	0	.823
		.063	.099	.017	0	[illegible]
		.064	.133	.053	.016	.73[illegible]
		.060	.026	.159	.075	.680

Multiplier Matrix

$(I-Q)^{-1}$	=	1.105	.077	.036	.001
		.079	1.118	.023	0
		.087	.164	1.065	.018
		.089	.065	.186	1.084

Mean Chain Length by Stratum

[illegible]	=	1.239	1.221	1.334	1.424

Mean Chain Length

l(t)	=	1.279

Probability Distribution of Chain Lengths

	1	2	3	4	5	6
	.823	.143	.027	.005	.001	0
$\underline{P}_j$ =	.821	.146	.027	.005	.001	0
	.734	.212	.044	.009	.002	0
	.680	.239	.064	.014	.003	.001

Observed Chain Length Distribution

119	37	20	6	1	0

Weighted Chain Length Distribution

127.001	21.072	8.104	1.946	.277	0

Mean First Passage Times

		1	2	3	4
	1	2.537	2.792	7.380	121.222
$\underline{M}^*$ =	2	2.945	2.210	8.050	121.891
	3	3.378	2.347	7.039	113.842
	4	3.807	3.515	3.966	90.276

Mean First Passage Times (Including Outside)

		1	2	3	4	0
	1	10.870	3.943	7.983	13.650	1.219
$\underline{M}$ =	2	11.154	3.790	8.088	13.654	1.220
	3	11.184	3.730	7.869	13.539	1.334
	4	11.250	4.197	7.003	12.787	1.425
	0	10.796	3.017	7.045	12.438	1.280

CHAINS STARTING IN NEW APARTMENTS, 1964-1965

Unweighted Vacancy Moves

	1	2	3	4	0
0	7	16	10	2	
1	1	0	2	0	6
2	0	0	2	0	14
3	1	0	2	1	12
4	0	0	0	0	3

Weighted Vacancy Moves

$\underline{F}(t)$	=	5.568	16.080	10.175	2.134
		.569	0	.730	0
		0	0	.139	0
		.324	0	.730	.569
		0	0	0	0

Transition Probabilities

$\underline{f}(t)$	=	.164	.474	.300	.063	$\underline{p}$
		.088	0	.113	0	.799
		0	0	.071	0	.929
		.025	0	.057	.045	.873
		0	0	0	0	1.000

Multiplier Matrix

$(\underline{I}-\underline{Q})^{-1}$	=	1.100	0	.132	.006
		.002	1.000	.075	.003
		.030	0	1.064	.047
		0	0	0	1.000

Mean Chain Length by Stratum

$\underline{\lambda}$	1.238	1.081	1.141	1.000

Mean Chain Length

$j(t) = 1.120$

Probability Distribution of Chain Lengths

		1	2	3	4	5	6
		.799	.169	.028	.004	0	0
$\underline{P}_j$	=	.929	.062	.008	.001	0	0
		.873	.115	.011	.001	0	0
		1.000	0	0	0	0	0

Observed Chain Length Distribution

29	4	1	1	0	0

Weighted Chain Length Distribution

30.950	2.278	.405	.324	0	0

Mean First Passage Times

	1	2	3	4
1				
2	$\underline{M}$* is undefined			
3				
4				

Mean First Passage Times (Including Outside)

			1	2	3	4	0
		1	11.139	4.633	5.351	26.700	1.238
$\underline{M}$	=	2	12.076	4.476	5.511	26.610	1.081
		3	11.829	4.536	5.635	25.497	1.141
		4	12.018	4.395	5.855	26.619	1.000
		0	11.018	3.395	4.855	25.619	2.120

CHAINS STARTING IN OLD HOUSES, 1964-1965

Unweighted Vacancy Moves

	1	2	3	4	0
0	202	210	76	24	
1	77	23	4	0	233
2	43	57	8	2	202
3	13	16	9	0	62
4	2	6	3	2	15

Weighted Vacancy Moves

$\underline{F}(t)$	=	181.394	178.119	64.128	18.741	
		33.798	10.190	1.704	0	196.912
		20.337	27.905	3.935	1.139	173.987
		6.426	8.081	3.822	0	56.969
		.649	3.008	1.708	.894	14.515

Transition Probabilities

$\underline{f}(t)$	=	.410	.403	.145	.042	$\underline{p}$
		.139	.042	.007	0	.812
		.089	.123	.017	.005	.765
		.085	.107	.051	0	.757
		.031	.145	.082	.043	.699

Multiplier Matrix

$(\underline{I}-\underline{Q})^{-1}$	=	1.169	.057	.010	0
		.122	1.150	.022	.006
		.119	.135	1.057	.001
		.067	.187	.095	1.046

Mean Chain Length by Stratum

$\underline{\lambda}$	=	1.236	1.300	1.312	1.395
$j(t)$	=	1.279			

Probability Distribution of Chain Lengths

		1	2	3	4	5	6
		.812	.151	.030	.006	.001	0
$\underline{P}_j$	=	.765	.183	.040	.009	.002	0
		.757	.190	.042	.009	.002	0
		.699	.228	.057	.013	.003	.001

Observed Chain Length Distribution

324	134	38	11	3	2

Weighted Chain Length Distribution

345.784	76.314	15.397	3.568	.831	.493

Mean First Passage Times

			1	2	3	4
		1	1.701	4.296	19.676	139.067
$\underline{M}^*$	=	2	2.684	2.930	18.472	134.771
		3	2.759	3.166	16.039	137.937
		4	3.551	2.694	13.904	117.840

Mean First Passage Times (Including Outside)

			1	2	3	4	0
		1	4.156	4.781	13.945	50.587	1.236
$\underline{M}^*$	=	2	4.414	4.435	13.839	50.374	1.300
		3	4.439	4.511	13.389	50.643	1.311
		4	4.738	4.362	12.968	48.531	1.394
		0	3.621	3.799	12.839	49.366	2.279

CHAINS STARTING IN OLD APARTMENTS, 1964 - 1965

Unweighted Vacancy Moves

	1	2	3	4	0
0	34	15	16	6	
1	12	1	0	0	33
2	0	3	0	0	16
3	0	0	1	0	16
4	0	0	0	0	6

Weighted Vacancy Moves

$\underline{F}(t)$ =	30.068	15.013	16.578	6.403	
	6.589	.324	0	0	29.744
	0	1.463	0	0	15.337
	0	0	.569	0	16.578
	0	0	0	0	6.403

Transition Probabilities

$\underline{f}(t)$ =	.442	.221	.244	.094	$\underline{p}$
	.179	.009	0	0	.811
$\underline{Q}$ =	0	.087	0	0	.913
	0	0	.033	0	.967
	0	0	0	0	1.000

Multiplier Matrix

	1.219	.012	0	0
$(\underline{I}-\underline{Q})^{-1}$ =	0	1.095	0	0
	0	0	1.034	0
	0	0	0	1.000

Mean Chain Length by Stratum

$\underline{\lambda}$	1.231	1.095	1.034	1.000

Mean Chain Length

$j(t)$ = 1.131

Probability Distribution of Chain Lengths

		1	2	3	4	5	6
		.811	.154	.028	.005	.001	0
$\underline{P}_j$	=	.913	.080	.007	.001	0	0
		.967	.032	.001	0	0	0
		1.00	0	0	0	0	0

Observed Chain Length Distribution

56	14	0	1	0	0

Weighted Chain Length Distribution

59.765	7.973	0	.324	0	0

Mean First Passage Times

	1	2	3	4
1				
2	$\underline{M}$* is undefined			
3				
4				

Mean First Passage Times (Including Outside)

			1	2	3	4	0
		1	3.958	9.493	8.948	22.888	1.231
$\underline{M}$	=	2	4.690	8.635	8.813	22.752	1.095
		3	4.629	9.398	8.461	22.691	1.034
		4	4.594	9.364	8.717	22.657	1.000
		0	3.594	8.364	7.717	21.657	2.132

PURE CHAINS, HOUSES ONLY, 1964 - 1965

Unweighted Vacancy Moves

	1	2	3	4	0
0	217	291	116	55	
1	74	25	6	0	262
2	52	77	10	2	285
3	19	26	12	2	98
4	5	7	13	4	34

Weighted Vacancy Moves

$\underline{F}(t)$ =	198.065	254.286	98.365	42.835	
	32.042	11.410	2.679	0	220.112
	24.758	36.999	4.909	1.139	250.883
	9.350	12.662	5.366	.730	89.420
	2.029	3.332	6.208	2.114	33.135

Transition Probabilities

$\underline{f}(t)$ =	.334	.428	.166	.072	$\underline{p}$
	.120	.043	.010	0	.827
$\underline{Q}$ =	.078	.116	.015	.004	.787
	.080	.107	.046	.006	.761
	.043	.071	.133	.045	.708

Multiplier Matrix

	1.143	.057	.013	0
$(I-Q)^{-1}$ =	.103	1.139	.020	.004
	.107	.134	1.052	.007
	.074	.106	.148	1.049

Mean Chain Length by Stratum

$\underline{\lambda}$	1.213	1.266	1.301	1.377

Mean Chain Length

$j(t)$ = 1.262

Probability Distribution of Chain Lengths

	1	2	3	4	5	6
	.827	.141	.026	.005	.001	0
$\underline{P}_j$ =	.787	.170	.034	.007	.001	0
	.761	.190	.040	.008	.002	0
	.708	.225	.053	.011	.002	0

Observed Chain Length Distribution

443	164	54	12	4	2

Weighted Chain Length Distribution

472.785	93.399	21.880	3.892	1.108	.493

Mean First Passage Times

		1	2	3	4
	1	1.861	3.826	15.143	121.607
$\underline{M}^*$ =	2	2.795	2.666	14.669	118.589
	3	2.906	2.920	12.853	117.267
	4	3.547	3.420	8.063	101.133

Mean First Passage Times (Including Outside)

		1	2	3	4	0
	1	5.042	4.505	11.782	29.895	1.213
$\underline{M}$ =	2	5.299	4.212	11.754	29.831	1.266
	3	5.309	4.268	11.422	29.781	1.300
	4	5.552	4.462	10.401	28.673	1.377
	0	4.550	3.532	10.717	26.691	2.262

PURE CHAINS, APARTMENTS ONLY, 1964 - 1965

Unweighted Vacancy Moves

	1	2	3	4	0
0	30	29	24	8	
1	3	0	1	0	29
2	0	1	1	0	28
3	0	0	1	0	26
4	0	0	0	0	8

Weighted Vacancy Moves

$\underline{F}(t)$	=	29.862	29.954	25.614	8.538	
		1.708	0	.405	0	29.457
		0	.569	.569	0	29.385
		0	0	.405	0	26.588
		0	0	0	0	8.538

Transition Probabilities

$\underline{f}(t)$	=	.318	.319	.273	.091	$\underline{p}$
$\underline{Q}$	=	.054	0	.013	0	.933
		0	.019	.019	0	.963
		0	0	.015	0	.985
		0	0	0	0	1.000

Multiplier Matrix

$(\underline{I}-\underline{Q})^{-1}$	=	1.057	0	.014	0
		0	1.019	.019	0
		0	0	1.015	0
		0	0	0	1.000

Mean Chain Length by Stratum

$\underline{\lambda}$	=	1.071	1.038	1.015	1.000
$\underline{i}(t)$	=	1.039			

Probability Distribution of Chain Lengths

		1	2	3	4	5	6
		.933	.063	.004	0	0	0
$\underline{P}_j$	=	.963	.036	.001	0	0	0
		.985	.015	0	0	0	0
		1.000	0	0	0	0	0

Observed Chain Length Distribution

85	5	1	0	0	0

Weighted Chain Length Distribution

90.715	2.848	.405	0	0	0

Mean First Passage Times

	1	2	3	4
1				
2	$\underline{M}$* is undefined			
3				
4				

Mean First Passage Times (Including Outside)

			1	2	3	4	0
		1	6.069	6.430	7.165	22.514	1.071
		2	6.384	6.277	7.093	22.481	1.038
$\underline{M}$	=	3	6.361	6.374	7.098	22.458	1.015
		4	6.346	6.359	7.191	22.443	1.000
		0	5.346	5.539	6.191	21.443	2.039

PURE CHAINS STARTING IN NEW HOUSES, 1964 - 1965

Unweighted Vacancy Moves

	1	2	3	4	0
0	21	83	42	33	
1	8	4	2	0	38
2	13	20	2	0	86
3	6	12	3	2	36
4	4	2	10	3	19

Weighted Vacancy Moves

$\underline{F}(t)$ =	19.595	77.061	35.131	24.824	
	2.952	1.949	.975	0	27.343
	6.044	9.095	.975	0	78.196
	2.924	5.475	1.544	.730	32.452
	1.704	.730	4.500	1.544	18.620

Transition Probabilities

$\underline{f}(t)$ =	.125	.492	.224	.159	$\underline{p}$
$\underline{Q}$ =	.089	.059	.029	0	.823
	.064	.096	.010	0	.829
	.067	.127	.036	.017	.753
	.063	.027	.166	.057	.687

Multiplier Matrix

$(\underline{I}-\underline{Q})^{-1}$ =	1.106	.077	.035	.001
	.079	1.11[illegible]	.014	0
	.090	.15[illegible]	1.045	.019
	.092	.064	.187	1.064

Mean Chain Length by Stratum

$\underline{\lambda}$	1.217	1.208	1.306	1.406

Mean Chain Length

$j(t) = 1.263$

Probability Distribution of Chain Lengths

		1	2	3		5	6
$\underline{P}_j$	=	.823	.144	.027	[illegible]	.001	0
		.829	.140	.025	.005	.001	0
		.753	.200	.039	.007	.001	0
		.687	.238	.060	.012	.002	0

Observed Chain Length Distribution

119	35	20	4	1	0

Weighted Chain Length Distribution

127.001	19.933	8.104	[illegible].297	.277	0

Mean First Passage Time

			1	2	3	4
$\underline{M}^*$	=	1	2.414	2.788	8.[illegible]28	127.494
		2	2.723	2.187	9.[illegible]71	[illegible] 637
		3	2.082	2.336	8 [illegible]32	1[illegible]9.166
		4	3.509	3.423	4 [illegible]64	101 042

Mean First Passage Times (Including Outside)

			1	2	3	4	0
$\underline{M}$	=	1	10.670	3.907	8.213	13.717	1.218
		2	10.938	3.758	8.[illegible]	[illegible].712	1.208
		3	10.926	3.709	8.219	13.569	1.307
		4	11.005	4.144	7.152	1[illegible].080	1.406
		0	10.578	2.978	7.280	[illegible]2.507	2.263

PURE CHAINS STARTING IN NEW APARTMENTS, 1964 - 1965

Unweighted Vacancy Moves

	1	2	3	4	0
0	5	15	9	2	
1	0	0	1	0	4
2	0	0	1	0	14
3	0	0	1	0	11
4	0	0	0	0	2

Weighted Vacancy Moves

$F(t)$ =	4.674	15.511	9.605	2.134	
	0	0	.405	0	4.269
	0	0	.569	0	14.941
	0	0	.405	0	10.580
	0	0	0	0	2.134

Transition Probabilities

$\underline{f}(t)$ =	.146	.486	.301	.067	$\underline{p}$
	0	0	.087	0	.913
$\underline{Q}$ =	0	0	.037	0	.963
	0	0	.037	0	.963
	0	0	0	0	1.000

Multiplier Matrix

	1.000	0	.090	0
$(\underline{I}-\underline{Q})^{-1}$ =	0	1.000	.038	0
	0	0	1.038	0
	0	0	0	1.000

Mean Chain Length by Stratum

$\underline{\lambda}$	1.090	1.038	1.038	1.000
$\underline{l}(t)$ =	1.043			

Probability Dist[illegible] Lengths

		1	2			5	6
$\underline{P}_j$	=	.913	.083	.[illegible]	0	0	0
		.963	.035	.001	0	0	0
		.963	.036	.001	[illegible]	0	0
		1.000	0	0	0	0	0

Observed Chain Length Distribution

29	1	1	0	0	0

Weighted Chain Length Distribution

30.588	.569	.405	0	0	0

Mean First Passage Times

	1	2	3	4
1				
2	$\underline{M}$*is undefined			
3				
4				

Mean First Passage Times (Including Outside)

			1	2	3		0
		1	13.954	4.256	5.6[illegible]2	30.646	1.090
$\underline{M}$	=	2	13.902	4.204	5.938	30.594	1.038
		3	13.902	4.205	5.937	30.594	1.038
		4	13.864	4.167	5.126	30.556	1.000
		0	12.864	3.167	5.127	29.556	2.043

PURE CHAINS STARTING IN OLD HOUSES, 1964 - 1965

Unweighted Vacancy Moves

	1	2	3	4	0
0	196	208	74	22	
1	66	21	4	0	224
2	39	57	8	2	199
3	13	14	9	0	62
4	1	5	3	1	15

Weighted Vacancy Moves

$\underline{F}(t)$ =	178.470	177.225	63.234	18.011	
	29.090	9.461	1.704	0	192.769
	18.714	27.905	3.935	1.139	172.688
	6.426	7.187	3.822	0	56.969
	.324	2.602	1.708	.569	14.515

Transition Probabilities

$f(t)$ =	.408	.406	.145	.041	$\underline{p}$
Q	.125	.041	.007	0	.822
	.083	.124	.018	.005	.770
	.086	.097	.051	0	.766
	.016	.132	.087	.029	.736

Multiplier Matrix

$(\underline{I}-\underline{Q})^{-1}$ =	1.149	.054	.010	0
	.112	1.150	.023	.006
	.116	.122	1.057	.001
	.045	.168	.098	1.031

Mean Chain Length by Stratum

$\underline{\lambda}$ =	1.213	1.291	1.296	1.341

Mean Chain Length

$\underline{j}(t)$ = 1.262

Probability Distribution of Chain Lengths

	1	2	3	4	5	6
	.822	.140	.026	.005	.001	0
$\underline{P}_j$ =	.770	.182	.039	.008	.002	0
	.766	.185	.039	.008	.002	0
	.736	.203	.048	.010	.002	0

Observed Chain Length Distribution

324	129	34	8	3	2

Weighted Chain Length Distribution

345.784	73.467	13.776	2.595	.831	.493

Mean First Passage Times

		1	2	3	4
	1	1.777	4.095	17.794	127.269
$\underline{M}^*$ =	2	2.809	2.784	16.740	123.174
	3	2.764	3.214	14.458	126.388
	4	3.719	2.593	11.768	111.995

Mean First Pasaage Times (Including Outside)

		1	2	3	4	0
	1	4.241	4.750	13.830	51.507	1.213
$\underline{M}$ =	2	4.475	4.404	13.738	51.298	1.291
	3	4.463	4.535	13.283	51.572	1.296
	4	4.809	4.377	12.794	50.115	1.341
	0	3.659	3.777	12.749	50.309	2.262

PURE CHAINS STARTING IN OLD APARTMENTS, 1964 - 1965

Unweighted Vacancy Moves

	1	2	3	4	0
0	25	14	15	6	
1	3	0	0	0	25
2	0	1	0	0	14
3	0	0	0	0	15
4	0	0	0	0	6

Weighted Vacancy Moves

$\underline{F}(t)$ =	25.188	14.444	16.009	6.403	
	1.708	0	0	0	25.188
	0	.569	0	0	14.444
	0	0	0	0	16.009
	0	0	0	0	6.403

Transition Probabilities

$\underline{f}(t)$ =	.406	.233	.258	.103	$\underline{p}$
	.064	0	0	0	.936
	0	.038	0	0	.962
	0	0	0	0	1.000
	0	0	0	0	1.000

Multiplier Matrix

$(\underline{I}-\underline{Q})^{-1}$ =	1.068	0	0	0
	0	1.039	0	0
	0	0	1.000	0
	0	0	0	1.000

Mean Chain Length by Stratum

λ	1.068	1.039	1.000	1.000
$\underline{\lambda}(t)$ =	1.037			

Probability Distribution of Chain Lengths

	1	2	3	4	5	6
$\underline{P}_j$ =	.936	.059	.004	0	0	0
	.962	.036	.001	0	0	0
	1.000	0	0	0	0	0
	1.000	0	0	0	0	0

Observed Chain Length Distribution

56	4	0	0	0	0

Weighted Chain Length Distribution

59.765	2.278	0	0	0	0

Mean First Passage Times

	1	2	3	4
1				
2	$\underline{M}$* is undefined			
3				
4				

Mean First Passage Times (Including Outside)

		1	2	3	4	0
	1	4.698	8.777	7.961	19.803	1.068
$\underline{M}$ =	2	4.989	8.417	7.933	19.774	1.039
	3	4.949	8.710	7.894	19.735	1.000
	4	4.949	8.710	7.894	19.735	1.000
	0	3.949	7.710	6.894	18.735	2.037

CHAINS WITHOUT STUDENTS, 1964 - 1965

Unweighted Vacancy Moves

	1	2	3	4	0
0	236	302	134	61	
1	88	28	8	0	272
2	48	78	13	1	303
3	19	29	16	3	117
4	5	6	13	5	41

Weighted Vacancy Moves

F(t)	=	209.729	268.014	114.761	49.320	
		39.193	12.464	3.408	0	227.550
		22.316	37.488	6.128	.569	268.108
		9.350	13.799	7.234	1.299	106.057
		2.029	2.843	6.208	2.683	40.108

Transition Probabilities

f(t)	=	.327	.418	.179	.077	p
		.139	.044	.012	0	.805
Q	=	.067	.112	.018	.002	.801
		.068	.100	.053	.009	.770
		.038	.053	.115	.050	.745

Multiplier Matrix

		1.167	.060	.016	0
$(I-Q)^{-1}$	=	.090	1.133	.023	0
		.094	.125	1.060	.010
		.063	.080	.131	.053

Mean Chain Length by Stratum

		1.243	1.249	1.290	1.327
i(t)	=	1.260			

Probability Distribution of Chain Lengths

	1	2	3	4	5	6
$\underline{P}_j$ =	.805	.156	.031	.006	.001	0
	.801	.159	.032	.006	.001	0
	.770	.182	.038	.008	.002	0
	.745	.198	.045	.009	.002	0

Observed Chain Length Distribution

481	173	58	15	4	2

Weighted Chain Length Distribution

513.340	98.525	23.501	4.865	1.108	.493

Mean First Passage Times

		1	2	3	4
$\underline{M}^*$ =	1	1.889	4.130	13.563	148.277
	2	3.046	2.701	12.864	146.007
	3	3.218	3.073	10.896	140.039
	4	3.827	3.720	7.030	116.373

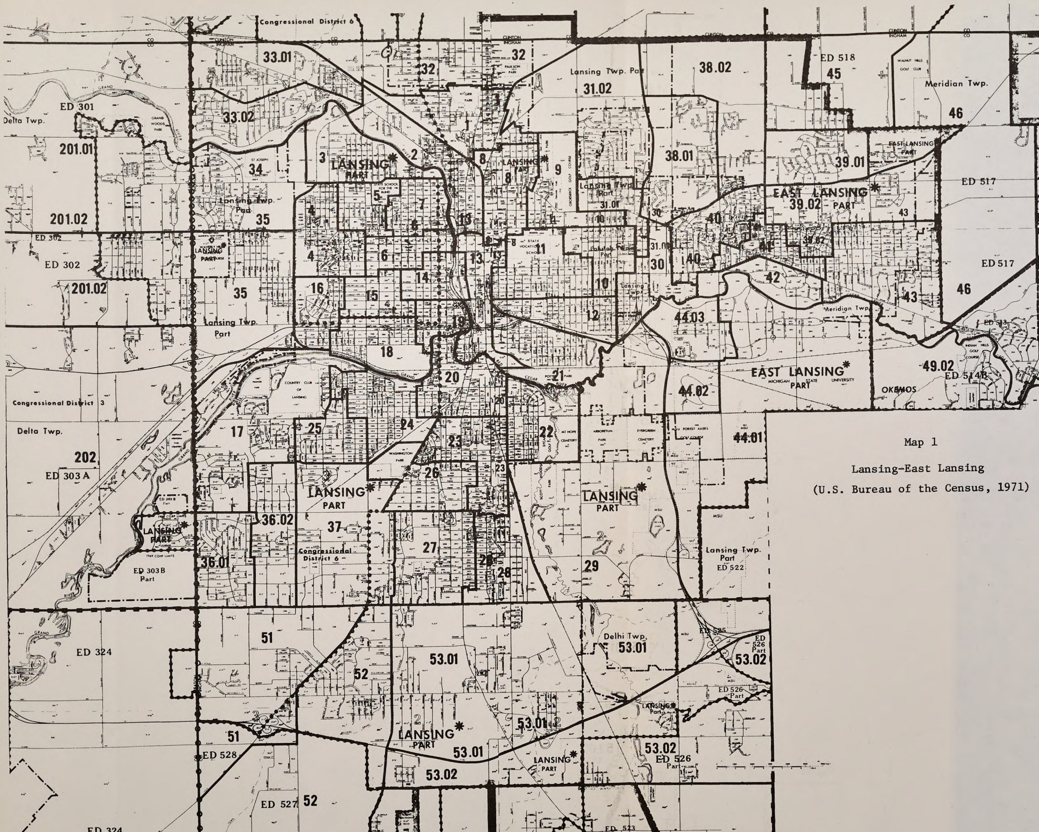
Mean First Passage Times (Including Outside)

		1	2	3	4	0
$\underline{M}$ =	1	5.132	4.648	10.950	28.282	1.243
	2	5.534	4.335	10.879	28.234	1.248
	3	5.555	4.412	10.530	28.047	1.290
	4	5.752	4.643	9.829	26.924	1.327
	0	4.746	3.664	9.876	27.046	2.260

APPENDIX B

MAPS

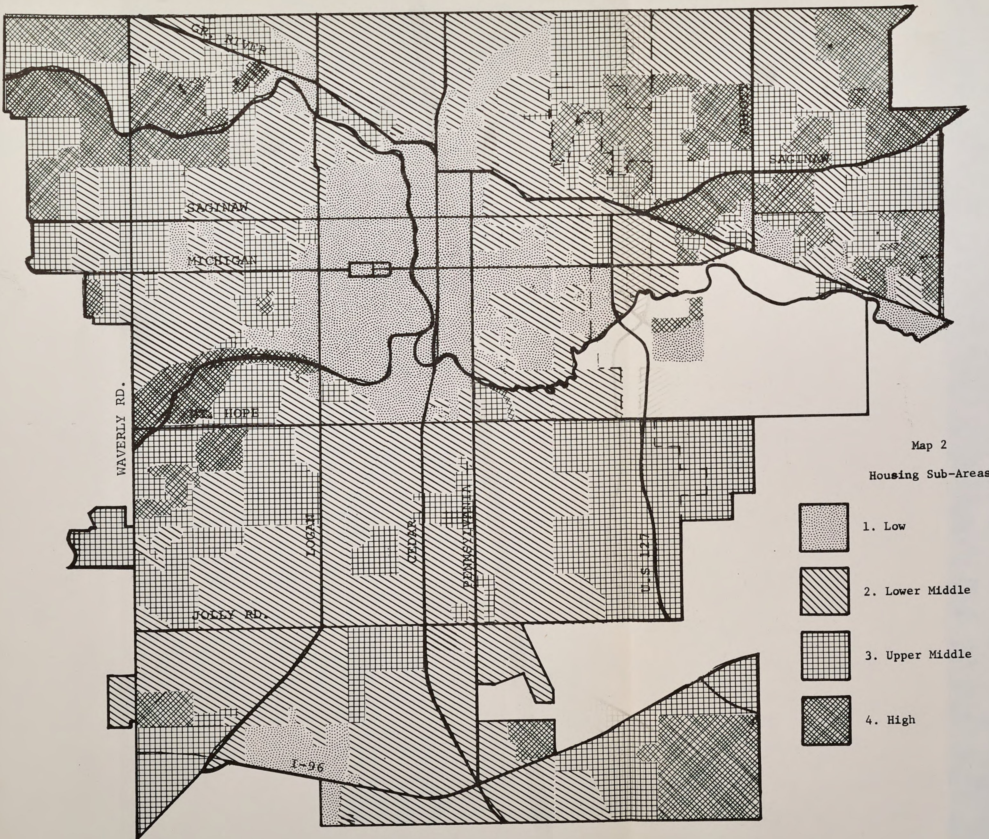
This appendix contains the maps showing the study area and the housing sub-areas.



Map 1

Lansing-East Lansing

(U.S. Bureau of the Census, 1971)


GR. RIVER
SAGINAW
MICHIGAN
SAGINAW
WAVERLY RD.
HOPE
LOGAN
CEDAR
PENNSYLVANIA
U.S. 127
JOLLY RD.
I-96
Map 2
Housing Sub-Areas
1. Low
2. Lower Middle
3. Upper Middle
4. High

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