INSTRUCTION IN PROBLEM SOLVING AND PIAGET'S THEORY OF COGNITIVE DEVELOPMENT

Dissertation for the Degree of Ph. D.

MICHIGAN STATE UNIVERSITY
JERRY K. STONEWATER

1977

This is to certify that the

thesis entitled

INSTRUCTION IN PROBLEM SOLVING AND PIAGET'S THEORY OF COGNITIVE DEVELOPMENT

presented by

Jerry K. Stonewater

has been accepted towards fulfillment
of the requirements for

Ph.D. degree in Department of Administration and Higher Education

Major professor

Date March 14, 1977

I

Fo
are critic
without th
large perc
ever, are
research
the develo
instructio
formal th

P
as the ba
Reasoning
tive trea

T
imental g
at Michig

ABSTRACT

INSTRUCTION IN PROBLEM SOLVING AND PIAGET'S
THEORY OF COGNITIVE DEVELOPMENT

By

Jerry K. Stonewater

The Problem

For engineering students, formal reasoning skills are critical for mastery of an engineering curriculum; without them success in engineering is improbable. A large percentage of minority engineering students, however, are not formal-operational thinkers. In addition, research identifying instructional strategies that promote the development of formal thought is limited. Hence, instructional strategies that facilitate development of formal thought need to be designed.

Procedures

Piaget's theory of cognitive development was used as the basis for designing the course "Introduction to Reasoning and Problem Solving," hypothesized as an effective treatment for facilitating formal thought development.

The sample included 11 control group and 16 experimental group first-term minority engineering students at Michigan State University. The control-experimental

group pr

the expe

posttest

respond

Pendulum

measures

T

effective

there are

abstract

and vari

are trea

ses used

(b) Pear

and (c)

of vari

centage

were f

Eighty

Balanc

Pendul

group pretest-posttest research design was used with only the experimental group receiving the treatment. Pre- and posttest measures were obtained by asking students to respond to the Equilibrium in the Balance problem and the Pendulum Problem, which are theoretically both valid measures of abstract reasoning.

The study addresses whether (a) the treatment was effective in increasing abstract reasoning ability, (b) if there are significant correlations between both pretest abstract reasoning ability or change in abstract reasoning and various academic ability variables, and (c) if there are treatment by ability interactions. Statistical analyses used were (a) a two-sample, one-tailed t-test, (b) Pearson product-moment coefficients of correlation, and (c) Finn's multivariance technique for an analysis of variance with unequal cell size.

Conclusions

The major conclusions of the study are:

1. Upon entry into the University, a high percentage of the minority engineering students in the sample were functioning below the formal-operational level. Eighty-five percent were not formal-operational on the Balance test and 89% were not formal-operational on the Pendulum Problem.

followin

(

3

found:

(

(

to postt

group:

2. For the Equilibrium in the Balance test the following were found:

(a) the change in abstract reasoning from pretest to posttest was not significantly greater for the experimental group as compared to the control group;

(b) the posttest level of abstract reasoning was not significantly greater than the pretest level of abstract reasoning for the control group but was significantly greater for the experimental group.

3. For the Pendulum Problem the following were found:

(a) the change in abstract reasoning from pretest to posttest was not significantly greater for the experimental group as compared to the control group;

(b) the posttest level of abstract reasoning was not significantly greater than the pretest level of abstract reasoning for either the control or the experimental group.

4. Descriptive data indicate that from pretest to posttest the following are greater for the experimental group:

(a) the change in average level of abstract reasoning;

(b) the decrease in percentage of students who are not formal-operational;

(c) the percentage of subjects who increased in level of abstract reasoning.

ability

test are

(

(c

(c

6

academic

7

action ef

ing was n

within or

8

percentil

students.

5. The significant correlations for academic ability variables with the Equilibrium in the Balance test are:

(a) high school grade point average, arithmetic and mathematics placement scores, and ACT/SAT mathematics percentile are all positively correlated with entry-level abstract reasoning ability;

(b) ACT/SAT verbal percentile is positively correlated with amount of change in abstract reasoning for the experimental group;

(c) entry-level abstract reasoning ability is positively correlated with first-term MSU grade point average;

(d) MSU grade point average and grade in first-term mathematics course are both negatively correlated with amount of change in abstract reasoning for the control group.

6. There were no significant correlations for academic ability variables with the Pendulum Problem.

7. There were no treatment-by-ability interaction effects; the amount of change in abstract reasoning was not different for high or low ability students within or between groups.

8. High ability students on the ACT/SAT verbal percentile changed significantly more than low ability students.

I

in

Depart

INSTRUCTION IN PROBLEM SOLVING AND PIAGET'S THEORY OF COGNITIVE DEVELOPMENT

By

Jerry K. Stonewater

A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Administration and Higher Education

1977

To Barby and Benzy

doctora

in writi

Chairper

and Norm

upon my

like to a

Sweetland

T

of suppor

of the en

staff, the

involved i

dation, wh

study grew

I a

to my educa

to value le

have the be

I also than

ACKNOWLEDGMENTS

I would like to express my appreciation to my doctoral committee for their assistance and encouragement in writing this dissertation. Drs. George Van Dusen, Chairperson, Stephen Yelon, Director, Lou Stamatakos, and Norm Kagan helped me build upon my strengths, improve upon my weaknesses, and grow as a scholar. I would also like to acknowledge my original chairperson, Dr. William Sweetland; his untimely death was a great sadness to me.

The College of Engineering provided a great deal of support for this work. I am especially appreciative of the encouragement from the Engineering Student Affairs staff, the time spent by the engineering students involved in this study, and the Alfred P. Sloan Foundation, which funded the project out of which this study grew.

I am also grateful for my family's contribution to my education. My parents, Jean and Stoney, taught me to value learning; they sacrificed much so that I could have the best education possible. For this I thank them. I also thank my in-laws, Viv and Ray, for believing in me.

daughter

we have,

Lastly, I'm grateful to my wife Barb and our daughter Jennifer for the love we share, for the fun we have, and for life together.

TABLE OF CONTENTS

Chapter

Chapter

V. S

Chapter Page

Chapter

REFERENC

Table

LIST OF TABLES

Table

14. N

15. Pe

16. Pe

17. F-

18. Nu

Figure

1. Int
S

LIST OF FIGURES

One
his
grow
as t

of reali

develope

surpris

cannot

thought

their c

gies is

such as

problem

riculum

student

of psyc

ment ca

abstrac

strateg

CHAPTER I

THE PROBLEM

> One should conclude . . . we can aid a student in his intellectual growth and that his intellectual growth may be one of our ultimate accomplishments as teachers. . . . (Beistel, 1975, p. 151)

American higher education is falling far short of realizing its priority goal of educating intellectually developed students. This failure is evidenced by the surprisingly large percentage of college students who cannot utilize strategies based upon logic and abstract thought processes to solve problems basic to much of their coursework. The inability to utilize such strategies is especially troublesome for students in disciplines such as engineering, where the ability to solve complex problems is a prerequisite to a large part of the curriculum. Success in engineering is most improbable for students lacking these abstract reasoning skills.

Problem solving strategies based upon the work of psychologists who have studied intellectual development can be designed to promote the growth of logical, abstract thought. Taught early in the curriculum, these strategies can provide the necessary remediation for

stu
dev
suc

alt
hig
and
to
en
lo
it
ps
in
to
bi

de
in
po
ti
le
pr
as
("

students deficient in reasoning skills so that they can develop the intellectual capabilities required for success in engineering.

In this chapter, it will be established that although intellectual development is a major goal of higher education in this country and although logical and abstract reasoning capabilities are a prerequisite to the study of engineering, a large percentage of engineering students cannot reason on the basis of logical, abstract thought. To remedy this situation, it will be shown that, based upon the work of Swiss psychologist Jean Piaget, it is possible to design instructional strategies that directly teach students to develop abstract reasoning and problem solving capabilities.

The Problem-Intellectual Development

Promoting the intellectual development of students has long been a primary goal of higher education in this country. A recent Carnegie Commission report pointed out that one of the primary missions of universities should be to provide "opportunities for the intellectual . . . development of individual students, and the provision of campus environments which can constructively assist students in their more general developmental growth" ("The Purpose and Performance," 1973, p. 7). Properly selec

intel

The I
Stude

devot

ment.

as on

.
i
h
i
w
s

This

opera

who c

entir

will

opera

opera

Piage

menta

The i

level

selected, these "opportunities" should lead to the intellectually developed student.

The Intellectually Developed Student Defined

Jean Piaget, an eminent Swiss psychologist, has devoted a lifetime to the study of intellectual development. He defined the intellectually developed student as one who can draw

> . . . conclusions, not from a fact given in immediate observation, nor from a judgment one holds to be true without any qualifications, but in a judgment which one simply assumes, i.e. which one admits without believing it, just to see what it will lead to. (Piaget, 1952, p. 69)

This type of reasoning is defined to be formal-operational or abstract thinking. Thus, the student who can reason from "A" to "B" even though "A" may be entirely hypothetical, is intellectually developed and will be referred to hereafter as an abstract or formal-operational thinker. Students who are not formal-operational will be referred to as pre-formal-operational.

Piaget's Theory

Piaget's theory is a description of the developmental stages one moves through in cognitive growth. The invariant sequence and the age associated with each level are (Wadsworth, 1971):

I

It is

ment

of pr

an in

hiera

level

the

birth

conc

besi

find

two

new

into

thou

thou

as we

to co

---

used

be co

of th

O. Sensori-Motor (0-2 years)
I. Preoperational thought (2-7 years)
II a-b. Concrete operations (7-11 years)
III a-b. Formal operations (11-15 years)[1]

It is these four stages that describe cognitive development and level of abstract reasoning. Although the rate of progress through these stages may differ according to an individual's experience and heredity, the sequence is hierarchical, i.e. level O is prerequisite to level I, level I to IIa, etc.

During the sensori-motor period of development, the infant moves from performing only reflex actions at birth to coordinating vision and touch, developing the concept of object permanence, becoming aware that objects besides the self cause actions, and experimenting to find new ways of solving familiar problems. At about two years of age, the child begins to mentally invent new means of solving problems, which permits movement into the second stage of development: preoperational thought.

The major development during the preoperational thought stage is the acquisition of language. Language, as well as behavior, moves from egocentric and nonsocial to communicative and social. Thought is not restricted

---

[1]The numbering system O, I, IIa-b, and IIIa-b is used by Piaget and Inhelder in their descriptions. To be consistent, they are used here and in the description of the measurement instruments and their scoring.

by p
stag
beha
form
chil

atta
tran
logi
obje
verb
of s

one
the
is c
buil
with
deri
actu
theo

diff
abil
betw
cons

by perceptions and motor activities as in the previous stage, but is symbolic and representational, i.e., behaviors can be thought-out rather than actually performed. Thought, however, is restricted, in that the child cannot reverse operations or follow transformations.

During the concrete-operational stage, the child attains the ability to reverse operations and attend to transformations. Thought, for the first time, involves logical operations, but they are limited to concrete objects and problems rather than hypothetical or entirely verbal ones. Further, the child develops the operations of seriation and classification.

It is not until the formal-operational stage that one can solve problems that are hypothetical and involve the complete utilization of logical reasoning. Thought is characterized by scientific reasoning and hypothesis building and testing; problems are solved by logic without relying upon the specific content. Logically derived conclusions are seen as valid, independent of actual truth. The child is able to derive general theories and utilize them abstractly in solving problems.

These four stages of development are distinctly different in terms of how children think and what their abilities are. The process that brings about the changes between stages is described by Piaget in terms of four constructs: schema, assimilation, accommodation, and equilib

led to

are bri

structu

a perso

new info

schema d

if it is

ing one

tion. T

necessar

developm

satisfie

balance

An imbal

provides

to occur

balance.

next, ass

must all

<u>Intellect</u>
<u>to the St</u>

T

level is

engineeri

equilibrium (Wadsworth, 1971). These four constructs led to hypothesizing the treatment under study. They are briefly defined here and elaborated upon later.

Schema are defined by Piaget to be cognitive structures that house all of the intellectual information a person possesses. Assimilation is the process by which new information is taken into existing schema. If a schema does not exist for a new bit of information or if it is incomplete, a new schema is added or an existing one is modified. This process is called accommodation. Together, assimilation and accommodation are necessary but not sufficient conditions for cognitive development to occur. The sufficient criterion is satisfied when a state of equilibrium is reached and a balance exists between assimilation and accommodation. An imbalance between assimilation and accommodation provides the internal motivation for cognitive growth to occur because the child strives to reach a state of balance. Thus, in order to move from one level to the next, assimilation, accommodation, and disequilibrium must all occur.

Intellectual Development: A Prerequisite to the Study of Engineering

The ability to reason at the formal-operational level is especially critical for students enrolled in engineering curricula. As pointed out by Renner and

Lawson,

riculum

reason

> The
> stu
> and
> this
> hypo
> can
> fore
> In c
> form
> oper
> to i
> leve
> they
> lea
> leve

Thus, m

of the

courses

thinkin

the beg

summari

finding

subject

of thou

p. 262)

succeed

Lawson, successful mastery of a science-oriented curriculum such as engineering requires the ability to reason abstractly:

> The maximum educational gain that comes from the study of science is derived from the isolation and investigation of a problem. Quite obviously this involved the formulation and stating of hypotheses and using a form of thinking which can be described as, if . . ., then . . ., therefore. That is, of course, propositional logic. In other words, science teaching should promote formal thought. But it cannot do so if concrete operational thinkers (i.e. pre-formal) are asked to interact with science on a formal operational level and their teacher teaches them as though they think formally. Concrete operational learners must interact with science at that level; they cannot do otherwise. (1973a, p. 168)

Thus, mastery of engineering courses as well as mastery of the prerequisite mathematics, physics, and chemistry courses requires the ability to reason abstractly.

Students who have not yet developed formal thinking capabilities are severely handicapped from the beginning of their study of engineering. Kolodiy summarized Griffith's study and concluded that "his findings seem to suggest that if we are teaching formal subjects to students who are only at the concrete level of thought, basically nothing is getting through" (1974, p. 262). Thus, concrete-operational subjects cannot succeed in engineering course work.

student

(Elkins

1974; M

percent

abstrac

(Elkind

subject.

As Kohl

their r

> Not
> (65
> the
> dev
> Thi
> nev
> The
> of
> abs

educati

not be

student

to conc

develop

present

inadeq

do not

College Students and Formal Reasoning Ability

Although no existing study addresses engineering students in particular, it has been well documented (Elkins, 1962; Karplus & Karplus, 1970; Lawson & Renner, 1974; McKinnon, 1970, 1971) that a surprisingly large percentage of college students have not attained the abstract level of reasoning. Studies report from 42% (Elkind, 1962) to 78% (McKinnon, 1970) of the college subjects studied were pre-formal-operational thinkers. As Kohlberg and Gilligan point out in the summary of their research:

> Not until age 21-30 (does) . . . a clear majority (65%) attain formal reasoning. . . . They (i.e. the data) suggest that there is no further development of formal reasoning after age 30. This means that almost 50% of American adults never reach adolescence in the cognitive sense. The point, however, is that a large proportion of Americans never develop the capacity for abstract thought. (1971, p. 1065)

The literature clearly points out that higher education's priority goal of intellectual development is not being met. Because of the large percentage of students who are pre-formal-operational, it is reasonable to conclude that the "opportunities for intellectual development" recommended by the Carnegie Commission and presently utilized in higher education are extremely inadequate.

Although a large percentage of college students do not possess the abstract reasoning skills necessary for sur

in its

reasoni

that th

between

Althoug

abstrac

study as

A furthe

of a gro

course s

Analysis

not sign

a large

formal

the for

populat

based u

student

ities.

whether

are for

are not

enginee

enrolle

for survival in engineering, the literature is limited in its documentation of minority students' abstract reasoning abilities. Karplus and Peterson (1970) found that there was a large difference in reasoning abilities between urban and suburban eleventh- and twelfth-graders. Although 80% of the suburban students could reason abstractly, only 9% of the urban students could. (The study assumed the urban population was mainly minority.) A further study by Griffiths (1973) found that only 39% of a group of college students enrolled in a physics course scored at the highest abstract reasoning level. Analysis of racial subsamples indicated that they were not significantly different from the total group, i.e., a large percentage of minority students were also pre-formal thinkers.

Although no studies have been found which assess the formal reasoning abilities of minority engineering populations in particular, this research study is based upon the assumption that minority engineering students are deficient in abstract reasoning capabilities. Although the study will actually determine whether or not a group of minority engineering students are formal thinkers, there is reason to believe they are not. First, in 1972, 20 of 42 first-term minority engineering students at Michigan State University were enrolled in a remedial mathematics course, yet 40 of the

42 had

mathem

It is

tional

high sc

Secondl

in Engi

student

science

on Minc

if a la

operati

sive re

percent

in gene

centage

in forn

centage

operati

discip

or they

skills

studen

enginee

42 had completed at least three years of high school mathematics (College of Engineering, MSU, 1974, p. 3). It is doubtful that a student capable of formal-operational reasoning and who had completed three years of high school mathematics would enroll in a remedial course. Secondly, the nationally based Committee on Minorities in Engineering has recommended that minority engineering students need extensive instruction in "mathematics and science learning techniques and processes" (Committee on Minorities in Engineering, 1975, p. 23). Once again, if a large percentage of these students were formal-operational, they would probably not require such intensive remedial instruction.

Because of these reasons and because of the large percentage of pre-formal-operational college students in general, it will be assumed that a significant percentage of minority engineering students are deficient in formal reasoning skills.

The result of a situation in which a large percentage of minority engineering students are pre-formal-operational is that either they leave engineering for disciplines which require less rigorous reasoning ability, or they manage somehow to develop the formal reasoning skills necessary for survival. It is most probable that students who lack formal reasoning abilities leave engineering rather than develop the required skills.

Althou

of fac

famili

instruc

ment o

that "a

lectual

instruc

levels"

. .
ing
tha
abi
of
of

Hence,

facilit

student

print f

p. 18),

than or

neering

reasons

sizes t

to a la

likely

skills

Although this cannot be officially documented, a number of factors lead to this conclusion. First, educators familiar with Piaget's theory claim that current instructional procedures do not facilitate the development of formal reasoning. Renner and Lawson stressed that "at least some of this lack of development of intellectual capabilities can be traced to inappropriate instructional strategies at the secondary and college levels" (1975, p. 1). Kolodiy concurred:

> . . . present teaching techniques might be reaching fewer than one-half of our students. All that students are learning, apparently, is the ability to parrot back material for the purpose of attaining passing grades without learning any of the concepts involved. (1974, p. 262)

Hence, current instructional practice does little to facilitate the cognitive development of today's college student. Second, *Minorities in Engineering: A Blueprint for Action* (Alfred P. Sloan Foundation, 1974, p. 18), pointed out that between one-third and more than one-half of the minority students who enter engineering do not earn a degree. Although there are many reasons for this high attrition rate, this study hypothesizes that at least part of the problem can be attributed to a lack of formal reasoning ability. Thus, it is likely that minority students lacking formal reasoning skills leave engineering for other majors.

Increa
in Eng

neerin

efforts

to incr

dation,

minorit

1982.

by 1982

enginee

ing. N

resourc

embarra

stood a

of reme

minori

on Min

Wh
ca
(B
Pu
pr
pp

To add

tive s

academ

relate

Increased Minority Involvement in Engineering

It is estimated that 5.1% of freshmen in engineering colleges in 1973 were minorities, while national efforts in minority engineering education are striving to increase this to 18% by 1982 (Alfred P. Sloan Foundation, 1974). This represents an increase from 2,640 minority students in 1973 to approximately 13,500 in 1982. With an attrition rate of one-third to one-half, by 1982, from 4,500 to 6,750 of the entering minority engineering students will not earn a degree in engineering. Not only is this a tremendous waste of human resources, time, and money, it is also an educational embarrassment that the phenomenon is not better understood and that at present there appears to be no means of remedying the situation.

The problem of the academic preparation of minority students was also pointed out by the Committee on Minorities in Engineering:

> What's more, as the most economically and educationally disadvantaged, these four minorities (Black, American Indian, Mexican American and Puerto Rican) often lack the rigorous academic preparation for engineering school. . . . (1975, pp. 1-2)

To address the problem, the committee recommended effective support programs, including methods to improve academic course success, teaching learning skills that relate to study habits, and teaching academic skills relate

should

and pr

ation,

Action

college

prepar

have d

P. Slo

dilemm

minori

pool

posse

engin

Instr
Facil
Devel

minor

Engin

three

to "

who

to e

Engi

related to analytical processes. Included in this should be "mathematics and science learning techniques and processes . . ." (1975, p. 23).

Also addressing mathematics and science preparation, Minorities in Engineering: A Blueprint for Action stated that "even the seemingly well-prepared college-bound minority youth feel insecure about their preparation in mathematics and science and expect to have difficulty with these subjects in college" (Alfred P. Sloan Foundation, 1974, p. 18).

Thus, engineering educators are faced with a dilemma: national efforts are calling for expanded minority involvement in engineering, yet of the potential pool of minority students, a large percentage does not possess the cognitive skills necessary for success in engineering.

Instructional Strategies that Facilitate Intellectual Development

In order to address the problem of insufficient minority involvement in engineering, the College of Engineering at Michigan State University received a three-year grant from the Alfred P. Sloan Foundation to "address the problems of ethnic minority students who have not mastered the technical skills necessary to enter traditional engineering programs" (College of Engineering, 1974, p. 3). The grant proposal stated:

W
H
e
l
m
r
c
s
n
i
r
t
c

to de
deve
rese
inst
of
inq
sci
thi
Saa
an
cor
rea
in
rea

as
and

> We have also experienced the problem of students having inadequate technical backgrounds to enter engineering directly from high school. In the 1972 fall term, twenty of the forty-two ethnic minority engineering freshmen were enrolled in remedial mathematics; forty of the forty-two had completed a minimum of three years of high school mathematics. Typically, a significant number of such students . . . take much longer in the technical foundation courses due to repeats, and tend to become discouraged with the academic rigor rather early in their college careers. (1974, p. 3)

One of the major goals of the Sloan Project was to develop instructional strategies to assist students in developing abstract reasoning abilities. However, little research has been conducted that isolates effective instructional strategies that facilitate the development of formal reasoning. McKinnon (1970) pointed to an inquiry-oriented strategy for instruction in a general science course and Renner and Lawson (1975) also found this technique useful in a science education course. Saarni's (1974) work, although not directly related to an instructional strategy, indicated a high positive correlation between problem-solving ability and abstract reasoning, leading to the speculation that instruction in problem solving may result in increased abstract reasoning skill.

The Treatment

This section will describe a course developed as part of the Sloan Project, "Introduction to Reasoning and Problem Solving," the instructional strategy designed to fa

and t

theor

The I
Conte

vation

conten

cation

variou

the de

(a) p

ing p

infor

drawi

apply

probl

probl

stra

were

appl

cour

cal

proc

to facilitate cognitive development. Both the content and the instructional method will be described and a theoretical basis for each will be discussed.

The Instructional Strategy: Content

Saarni's work (1974) provided the primary motivation for selecting problem solving as the particular content area for the instructional strategy. The implication of her study was that direct instruction in various problem-solving strategies should facilitate the development of abstract reasoning.

The content of the course fell into two areas: (a) preparing for problem solving, which included restating problems, defining unknown terms, specifying given information, deciding what is to be solved for, and drawing and labeling diagrams; and (b) learning and applying specific strategies to be used as methods in problem solving, including the simplification, subproblem, contradiction, inference, and working backward strategies. The preparation for problem-solving skills were included because they were prerequisite to the application of the specific strategies. Because the course was designed for students with limited mathematical background, the content and problems did not involve proofs or problems from theoretical mathematics, but rathe

did n

simpl

define

it out

cation

in a g

proble

origin

includ

and se

sizes

to the

a 2x2

Final

genera

organ

Accor

nitiv

a per

an al

of pr

In ad

sense

rather included the puzzle variety of problems, which did not require mathematical manipulations beyond simple arithmetic.

Each of the five problem-solving strategies was defined and explained by listing the steps used to carry it out as an algorithm. The algorithm for the simplification strategy instructed students to reduce the numbers in a given NxM array, attempt to solve the reduced problem, and generalize this method of solution to the original problem. For example, suppose the problem included a 5x7 array (i.e., a rectangle with five rows and seven columns) and asked how many rectangles of all sizes were included in the 5x7 array. First, according to the algorithm, the numbers must be reduced to, say, a 2x2 array, then this reduced problem is solved. Finally, the method used to solve the 2x2 problem is generalized to solve the 5x7 problem.

Thus, the algorithmic approach provided an organized method of adapting to problem solving. According to Wadsworth (1971, p. 9), Piaget views cognitive acts as acts of organization and adaptation to a perceived environment. By providing students with an algorithmic strategy for solving a particular class of problems, the course provided this organization. In addition, the strategy is a schema in the Piagetian sense and functions in one of two ways: either it fits into

equil

stude

enhanc

that h

that m

solved

of pr

W
w
a
l

In o

morn

Howe

avai

figu

this

and

of

for

seq

pro

pro

al

ad

into an existing schema or it creates a sense of disequilibrium because it does not fit. In the latter case, students are motivated to change and accommodate, thus enhancing cognitive growth.

The subproblem strategy utilized an algorithm that helped students identify parts of a given problem that must be solved before the major variable could be solved. The following is a simple example of this type of problem:

> What is the total income for the day if 3 hats were sold in the morning grossing $18 and twice as many hats were sold in the afternoon for $1 less than the morning price?

In order to solve this problem, the student must add morning income to afternoon income to obtain the answer. However, the value for "afternoon income" is not directly available from the problem statement. One must first figure out the cost per hat in the morning (X) and from this determine the cost per hat in the afternoon (Y). X and Y are each examples of subproblems. The major aim of this strategy was to provide students with a procedure for analyzing problems in order to identify unknowns and sequence the order in which they must be solved. The procedure results in an organized plan for solving the problem.

The algorithm for the contradiction strategy also provided students with an organized plan. In addition, the strategy was based upon reasoning with

propc
regar
Accor
reaso
critic
algori
solvin
what
that
assum
divis
opera
basis
reali
prov
conf
crea
to o

of t
algo
to o

to s
back

propositional logic on the basis of an assumption, regardless of the truth or falsehood of that assumption. According to Piaget (1952, p. 69), the ability to reason on the basis of hypothetical assumptions is a critical attribute of formal-operational reasoning. The algorithm specified that the assumption to be used in solving the problem should be the logical negation of what was to be proved. Thus, if the problem was "Show that all even integers are divisible by two," then the assumption was "There is an even integer that is not divisible by two." According to Piaget, the pre-formal-operational thinker cannot accept this assumption as a basis for reasoning to reach a contradiction because, in reality, it is untrue. Hence, the contradiction strategy provided the opportunity for the pre-formal thinker to be confronted with hypothetical assumptions that would create the disequilibrium necessary for cognitive change to occur.

The inference strategy was primarily an extension of the contradiction strategy. Although no specific algorithm was presented, instruction was based upon how to obtain (infer) additional information from the givens.

The working backward strategy taught students how to solve problems starting at the solution and working backwards to the givens as opposed to the traditional appro

follc

wards

Dc
a
wa

As opp

water

algori

and wo

cours

catic

worki

of Sa

abili

In ac

acts

prov

cont

tion

tion

The

sel

are

lea

approach of starting with the givens. For example, the following problem is much easier to solve working backwards:

> Describe a sequence of emptyings and fillings of a 3 qt. jar and a 7 qt. jar to obtain 5 qts. of water.

As opposed to starting with three and seven quarts of water and describing how five quarts are obtained, this algorithm directed students to begin with five quarts and work backwards until three and seven quarts are left.

Thus, the major emphasis of the content of the course was five problem-solving strategies: simplification, subproblem, contradiction, inference, and working backward. This content area was selected because of Saarni's implications that increased problem-solving ability leads to improved abstract reasoning ability. In addition, Piaget maintains that cognitive acts are acts of organization and adaptation; the algorithms provided this necessary organization. Finally, the contradiction strategy was included to provide instruction on reasoning on the basis of hypothetical assumptions.

The Instructional Method

The instructional method used was the mastery self-paced model. The critical features of this model are: (a) material to be learned was divided into small learning units called modules, which can be completed in a

incluc

materi

allowe

speed,

learni

object

level

requir

prerec

module

achie

small

remed

numbe

(Kul

self

lect

the

cour

is

of

mat

inc

wi

in a relatively short period of time. Each module included objectives to be mastered, printed instructional material and practice exercises. (b) Students were allowed to progress through the modules at their own speed, allowing for individual differences in rate of learning. (c) Students were expected to master the objectives in each module by performing at a specified level on each modular exam. Theoretically, this mastery requirement insures that students understand all material prerequisite to later modules. (d) Retesting on each module was allowed without penalty until mastery was achieved. (e) Extensive use was made of one-to-one or small group tutoring which permitted diagnosis and remediation of individual problems.

The mastery self-paced model was chosen for a number of reasons. First, it has been well documented (Kulik, 1975) that in 25 out of 31 studies, the mastery self-paced form was statistically superior to traditional lecture methods in terms of both student performance on the final examination and long-term retention. If the course "Introduction to Reasoning and Problem Solving" is to have an impact on the formal reasoning abilities of students, they must learn and retain the course material. It is well established that self-paced learning increases significantly the probability that students will learn and retain the material. K. Patricia Cross

put i
criti
(1976

was be
create
and tr
as cri
occur
and b
other
chanc
or a
beca
i.e.
the
dati
equ
Hen
soc
mer

le
fo
th
Pi

put it bluntly: "I believe mastery learning is the critical missing link in the education of low achievers" (1976, p. 78).

The second reason the self-paced model was chosen was because of its reliance on individual tutoring, which creates an environment of increased social interaction and transmission, one of four factors Piaget pointed out as critical for cognitive development. Since tutoring occurs between students, between students and tutors, and between students and instructor, interaction with others is increased. As a result, there is a greater chance that more can be assimilated into existing schema or accommodated into new schema. This occurs merely because of the increased amount of information flow, i.e., the greater the transmission of new information, the greater the occurrence of assimilation and accommodation. Accordingly, there is a greater chance for disequilibrium to occur, and subsequently, cognitive change. Hence, the self-paced approach, which permits increased social interaction, should facilitate cognitive development.

The third factor of the self-paced method that led to its selection was that it provides the opportunity for students to act upon objects in order to abstract their properties. This factor was also pointed out by Piaget as critical for the development of formal

reasor
defini
and nc
In adc
chance
accepta
of the
cannot
someth
strate
these
feedba
Thus,
by bot
facil

chose
terms
it cr
and t
lems

provi
reaso
parti

reasoning. The self-paced model of instruction demands by definition that the student act; there are no lectures and no way to learn unless the student does something. In addition, the repeated testing feature provided a chance for continued action until the standard of acceptable performance was met. Also, the very nature of the subject matter induced students to act: problems cannot be solved unless students act. They must do something to the problem to solve it. The specific strategies attempted to provide guidelines for what these actions should be and the homework exercises and feedback provided the opportunity for students to act. Thus, the opportunities to act upon objects brought about by both the instructional format and the content should facilitate cognitive growth.

In summary, the mastery self-paced model was chosen because it is superior to the lecture method in terms of student performance on final exams and retention; it creates an environment conducive to social interaction and transmission; and it forces students to act upon problems rather than passively receive information about them.

Statement of the Problem

It is a priority goal for higher education to provide opportunities for students to develop formal reasoning capabilities. For engineering students in particular, formal reasoning skills are critical for

maste
succe
of mi
these
resea
that
Hence
ment

Mich
Nati
Unit

fun
lev
gro

th
or

mastery of an engineering curriculum; without them success in engineering is improbable. A large percentage of minority engineering students, however, do not possess these necessary formal reasoning abilities. In addition, research in the area of effective instructional strategies that promote the development of formal thought is limited. Hence, instructional strategies that facilitate development of formal thought need to be designed and tested.

Terms and Their Definitions

Minority student--a student who is classified in Michigan State University records as Black, Chicano, Native-born or Asian-American and is a citizen of the United States.

Level of abstract reasoning, level of cognitive functioning or Piaget-level--refers to one of the four levels in the following invariant hierarchy of cognitive growth as defined by Piaget:

O. Sensori-motor

I. Preoperational thought

II a-b. Concrete operations

III a-b. Formal operations

A formal-operational student or a formal thinker--refers to a student who functions at level IIIa or b, formal operations.

stud

to th

in th

tion,

instr

forma

<u>refer</u>

metho

learn

allow

A mas

and

mast

tuto

on

Pre-formal-operational student--refers to a student who functions at levels O, I, IIa, or IIb.

Abstract reasoning or abstract thinking--refers to the abilities of a formal-operational student.

Problem solving--refers to the behaviors specified in the simplification, subproblem, inference, contradiction, or working backward strategies utilized in the instructional treatment.

Intellectually developed student--refers to a formal-operational thinker.

Mastery self-paced instructional model (also referred to as the self-paced or mastery model)--a method of instruction that divides the material to be learned into small learning units such that students are allowed to progress through each unit at their own speed. A mastery level is set for performance on each unit exam, and retesting is allowed without penalty until the mastery level is attained. Extensive use is made of tutoring.

Purpose

The purposes of this study are:

1. to design an instructional strategy based on Piaget's theory of cognitive development such that

the i

forma

tiona

devel

asses

cogni

data

scope

for d

suppo

wheth

thoug

to Re

ment

grou

(exp

were

leve

in a

ses

the instruction will facilitate the development of formal reasoning;

2. to assess the effectiveness of the instructional strategy;

3. to provide recommendations for further development of instructional strategies based on the assessment in (2); and

4. to measure minority students' entry-level cognitive abilities in order to (a) provide descriptive data on this population that could be useful beyond the scope of this study in providing the college information for developing further curricular, advising and general support programs for minority students, and (b) assess whether or not the age criteria for entry into formal thought as specified by Piaget is met by this population.

Research Hypotheses

In order to determine if the course "Introduction to Reasoning and Problem Solving" is an effective treatment for increasing abstract reasoning ability, a control group (no exposure to the course) vs. experimental group (exposure to the course) design was used. Both groups were pre- and posttested to measure individual student's level of reasoning according to Piaget's theory. "Change in abstract reasoning ability" referred to in the hypotheses is defined to be the difference between pretest and postt

resea

is gi

in th

studer

measu

reaso

chang

of ac

foll

Chap

Stud

var

rea

cha

posttest scores. A more complete description of the research design, the pre- and posttests and the scoring is given in Chapter III.

Three general research questions are addressed in this study:

1. Was the course effective in increasing students' abstract reasoning abilities?

2. Are there any relationships between various measures of academic ability and level of abstract reasoning upon entry into the university or amount of change in abstract reasoning after one term of instruction?

3. Is there an interaction effect between level of academic ability and treatment?

The null hypotheses for these research questions follow. A complete listing of the hypotheses appears in Chapter III.

<u>Question 1--Was the course effective in increasing students' abstract reasoning abilities</u>?

<u>Hypotheses 1-2</u>:

There will be no significant difference in change in abstract reasoning between the control and experimental groups.

<u>Question 2--Are there any relationships between various measures of academic ability and level of abstract reasoning upon entry into the university or amount of change in abstract reasoning after one term of instruction</u>?

the

reas

sign

be u

The

sign

in a

the

Such

Hypotheses 3-10:

For the experimental group only there will be no significant correlation between pretest scores measuring level of abstract reasoning and each of four measures of performance in the treatment.

Hypotheses 27-34:

For the experimental group only, there will be no significant correlation between amount of change in abstract reasoning and each of the four measures of performance in the treatment.

Hypotheses 11-26:

For both groups combined, there will be no significant correlation between pretest scores measuring level of abstract reasoning and each of eight measures of academic ability.

Hypotheses 43-50 and 59-66 (Control) and 35-42 and 51-58 (Experimental):

There will be no significant correlation between amount of change in abstract reasoning and each of eight measures of academic ability.

Hypotheses 3-10 and 11-26 provide an analysis of the correlations between pretest level of abstract reasoning ability and various ability measures. For significant correlations, the ability measures could be used as predictors of abstract reasoning ability. The remaining hypotheses provide correlations that, if significant, could be used to predict amount of change in abstract reasoning after one term in college for both the group receiving the treatment and the control group. Such descriptive data are useful as a first step toward iden

abili

sched

for t

acade

C

1.
2.
3.
4.

betw

pri

to

men

identifying variables that predict formal reasoning ability and can assist faculty in planning student schedules that reflect the needs of each individual.

Below is a list of the performance variables for the treatment course and a list of the measures of academic ability:

Course Performance	Academic Ability
1. grade received	1. high school grade point
2. total points earned	2. MSU first term grade point
3. final exam score	3. MSU arithmetic placement score
4. score on questions on final requiring application of a problem-solving strategy.	4. MSU mathematics placement score
	5. ACT/SAT mathematics percentile
	6. ACT/SAT verbal percentile
	7. ACT/SAT composite percentile
	8. grade received in mathematics course taken.

Question 3--Is there an interaction effect between level of academic ability and treatment?

Hypotheses 67-82:

There will be no significant difference in change in abstract reasoning between high and low ability students in the control or experimental group.

Summary

The thesis of this research is that (a) a priority goal of higher education in this country is to provide students with opportunities for the development of formal reasoning as defined by Piaget; (b) that stude

ing;

engin

neces

instr

ment

a sou

gies.

area

Hence

teach

reaso

to de

a gr

form

and

tive

desc

the

dati

IV,

students must be formal thinkers to succeed in engineering; (c) that a surprisingly high percentage of minority engineering college students cannot reason at levels necessary for success in engineering; and (d) that instructional strategies that facilitate the development of formal reasoning need to be designed.

Piaget's theory of cognitive development provides a sound basis for the design of such instructional strategies. In addition, there is evidence that a content area for such instruction is problem-solving skills. Hence, it is hypothesized that a course designed to teach problem solving will increase students' abstract reasoning abilities. The major purpose of this study is to determine if such a course has the desired effect on a group of first-term minority engineering students.

In Chapter II the literature related to the formal reasoning ability of college students is reviewed and instructional strategies that have been found effective in developing formal reasoning capabilities are described. The design of the study, the analysis of the data, and a summary of this research and recommendations for further study are presented in Chapters III, IV, and V, respectively.

deter

Reasc

ment

term

Unive

deve

engi

engi

reas

engi

prob

ricu

with

faci

Piag

be c

CHAPTER II

REVIEW OF THE LITERATURE

One of the central purposes of this study is to determine if the self-paced course "Introduction to Reasoning and Problem Solving" facilitates the development of formal-operational thought in a group of first-term minority engineering students at Michigan State University. Instructional strategies promoting the development of formal reasoning capabilities of minority engineering students are needed because (a) success in engineering is improbable for the student without formal reasoning abilities; (b) a large percentage of minority engineering students are not formal thinkers and most probably cannot survive the rigors of engineering curricula; and (c) limited research has been conducted with the aim of isolating instructional strategies that facilitate formal reasoning development.

In this chapter it will be shown that although Piaget maintains that formal thought capabilities should be developed sometime between the ages of 11 and 15, a lar

by ac

pris

forma

revie

of ag

colle

lege

neces

stude

revi

to p

abi

Acc

sho

mai

in

Yea

a large percentage of students are not formal thinkers by age 15. In particular, it will be shown that a surprisingly high percentage of college students are not formal-operational. This section of the literature review will be divided into five categories on the basis of age: (a) pre-college students--general; (b) pre-college students enrolled in science courses; (c) college students; (d) college-age and older subjects not necessarily enrolled at a university; and (e) minority students.

In addition, the last section of this chapter will review the research on instructional strategies designed to promote the development of formal reasoning.

Piaget's Age Specification for Entry into Formal Operations

Success in an engineering program requires the ability to reason at the highest level of abstraction. According to Piaget, freshman engineering students should have no difficulty with abstract reasoning; he maintained that formal operations should be evidenced in adolescents sometime between the ages of 11 and 15 years:

> The same unity of behavior encountered earlier in the various stages is found again between eleven or twelve and fourteen or fifteen, when the subject succeeds in freeing himself from the concrete and in locating reality within a group of possible transformations. This final fundamental

c
H

Afte

menta

ing r

least

forma

of co

Pre-
Gene

cifi

in a

grou

57%

> decentering, which occurs at the end of childhood, prepares for adolescence, whose principal characteristic is a similar liberation from the concrete. (1969, p. 130)

After age 15 at the latest, students should have the mental prerequisites for engineering study. The following review of the literature will indicate that at least with respect to American students, very few are formal thinkers by age 15 and that a large percentage of college students cannot reason abstractly.

Research Concerning the Development of Formal Reasoning

Pre-college Students--General

Evidence to refute Piaget's 11-15 year age specification for entry into formal thought was indicated in a study by Higgins-Trenk and Gaite (1971). For a group of 13-18 year olds, the researchers found that 57% were pre-formal thinkers. They concluded:

> Most writers . . . have tended to tacitly accept Piaget's descriptions of intellectual development and have been content to describe the adolescent as typically attaining the formal-operational stage of thought during early adolescence, and then to leave the reader with the distinct impression that such adolescents thereafter habitually function at that level. However, . . . this is altogether too simplistic a view of the typical adolescent's thought processes and intellectual functioning.
>
> Thus, . . . formal operational thought may be a relatively foreign experience to the normal adolescent.
>
> What is clear is that . . . relatively few adolescents seem capable, or inclined, to use this formal mode of thought. This suggests that

to th

stude

and 6

think

stude

inte

sub

adu

> there is need for considerable revision of the commonly held view that the normal adolescent attains the level of formal operations soon after pubescence.
>
> Indeed, it would seem that the normal adolescent is unlikely to reach the level of formal-operational thought until his late teens or early twenties if he reaches it at all, and these results suggest that he may well not. (1971, pp. 201-202)

Lawson and Renner (1974) reported similar results to those of Higgins-Trenk and Gaite in a study of Oklahoma students. Seventy-one percent of the 99 eleventh graders and 66% of the 97 twelfth graders studied were not formal thinkers. To determine level of cognitive development, students were administered the following tests during interviews of approximately 20 minutes:

1. tests to assess concrete operational students--a) Conservation of Solid Amount--determined understanding of the principle that a change in shape does not alter the amount of material; b) Conservation of Weight--determined understanding of the principle that a change in shape of one of two objects of equal weight does not change weight of object; c) Conservation of Volume using clay--determined understanding of the principle that a change in shape of an object does not change its volume.
2. tests to assess formal-operational students--a) Conservation of Volume using Metal Cylinders--test for understanding of the principle that water displacement is a function of volume, not mass; b) Elimination of Contradiction--test to determine if an apparent contradiction could be explained; c) Exclusion of Irrelevant Variables--test to determine if irrelevant factors could be identified, controlled and excluded in explaining results of experiment.

In an extensive study of six different groups of subjects ranging from fifth and sixth grade students to adult physics teachers, Karplus and Karplus (1970) foun

subu

cour

was

D, i

but r

Abili

the c

Physi

with

and

Renn

thos

usec

abi

and

tes

obj

the

an

th

gr

re

t

c

t

found that 86% of the tenth through twelfth grade suburban students enrolled in college preparatory courses were pre-formal-operational. The test used was the Island Puzzle: given four islands A, B, C, and D, if planes could go between C and D and between B and D, but not between A and B, can they go between A and C? Ability to solve this problem required application of the contradiction strategy (American Association of Physics Teachers, 1975). The results were consistent with the 57% pre-formal thinkers found by Higgins-Trenk and Gaite and the 66% to 71% reported by Lawson and Renner.

In contrast to these results, however, were those reported by Karplus and Peterson (1970). They used the ratio test to measure the formal thinking ability to urban and suburban sixth graders and of urban and suburban eleventh and twelfth graders. In the ratio test subjects were asked to measure the height of an object in a specified unit of measure if they knew both the height of the object in a different unit of measure and a conversion factor between the two units. Although the difference in reasoning ability of the two sixth grade groups was statistically nonsignificant, the researchers found a very large difference between the two eleventh and twelfth grade groups. Twenty percent of the suburban students were classified pre-formal on the basis of the ratio task, while 91% of the urban stud

stud

form

the

in C

use

Utili

(197

Isla

Ther

did

thei

were

cen

ope

cur

ind

for

the

for

Pre
in

was

mer

students were pre-formal. The results for the suburban students indicated a surprisingly lower percentage of pre-formal-operational thinkers than the results reported by the other studies mentioned. However, as will be explained in Chapter II, these results are questionable due to the use of the Ratio Test as the measurement instrument. Utilizing principal components analysis, Lawson and Renner (1974, p. 556) found that the Ratio Test as well as the Island Puzzle possibly did not measure formal reasoning. Therefore, it is likely that Karplus and Peterson's study did not accurately report the formal reasoning ability of their subjects. In addition, Karplus and Karplus' results were questionable due to the use of the Island Puzzle.

Thus, three studies indicated that a large percentage of high school students had not mastered formal-operational tasks by age 15, while a fourth study concurred with this conclusion for urban students, but indicated a much lower percentage of pre-formal thinkers for suburban students. This later study, however, used the Ratio Test, a questionable instrument for measuring formal reasoning.

Pre-college Students--Enrolled in a Science Course

An interesting subset of high school students was the group enrolled in science courses. As was mentioned previously, Lawson and Renner (1974) found

that
in c
cate
only
cour
were
cent
larg
whom
stud
stud
form

test
were
cour
unpu
stud
scor
Engl
rela
for
rela
supp
test

that from 66% to 71% of eleventh and twelfth graders in general were classified below the formal-operational category. Additional research, however, indicated that only 4% of the students who enrolled in a chemistry course and 3% of those who enrolled in a physics course were pre-formal-operational. A somewhat higher percentage (29.5%) was reported for biology students. This large difference between students in general (some of whom could have been enrolled in a science course) and students enrolled in a science course indicated that students enrolled in a science course perform better on formal reasoning tasks.

To defend against the criticism that Piaget task tests gave science students an advantage because the tests were based upon the content of physics and chemistry courses, Lawson and Renner (1974) referred to a 1974 unpublished study by Lawson, Norland, and Devito. The study showed low correlations between Piaget task test scores and CEEB Achievement exams in science, math, and English. For the Conservation of Volume test the correlations were 0.32, 0.30, and 0.38 respectively, and for the Exclusion of Irrelevant Variables the correlations were 0.32, 0.37, and 0.33. These results supported the assertion that at least these two Piaget tests were content free.

Karp

twel

form

of t

prin

Laws

Rati

tior

Sin

it

on

en

Ka

in

of

a

ir

Co

(

me

ir

fe

o

In contrast to Lawson and Renner's results, Karplus and Karplus (1970) found that 72% of suburban twelfth graders enrolled in a physics course were pre-formal thinkers. The discrepancy between the results of these two studies can also be explained by the principal components analysis results of Renner and Lawson (1974). As was pointed out, in addition to the Ratio Test, the Island Puzzle was found to be a questionable instrument for measuring formal reasoning. Since Karplus and Karplus used only the Island Puzzle, it is doubtful that their results were accurate.

In summary, although the two studies reporting on formal reasoning ability of pre-college students enrolled in a science course were contradictory, the Karplus and Karplus study was questioned because of inadequate testing procedures. Therefore, on the basis of Lawson and Renner's results, it can be concluded that a very large percentage of pre-college students enrolled in a science course were formal-operational.

College Students

In a study involving college students, Elkind (1962) found that 42% were pre-formal thinkers as measured by the Conservation of Volume task. When the initial group was subdivided by sex, a significant difference was found between the performance of men, 26% of whom were pre-formal thinkers, and the performance of w

form

coul

with

fere

cons

expl

are

and

not

El

in

fe

th

cl

fo

h

i

e

c

a

s

r

a

of women, of whom almost twice as many (48%) were pre-formal. In addition, the percentage of subjects who could successfully conserve volume increased regularly with age for women, but not for men. This large difference between male and female subjects, which is found consistently throughout the literature, could be explained by our sex-stereotyped culture, in which males are rewarded for successful performance on mathematical and scientific problems while females are consistently not rewarded.

Tower and Wheatley (1971) attempted to replicate Elkind's 1962 study on college students, but their sample included only females. They found that 39% of the 71 females aged 17-27 could not conserve volume and were therefore considered pre-formal. These results are very close to the 42% pre-formal thinkers reported by Elkind for both men and women, but somewhat less than the 48% he found for the women-only subgroup.

As was the case with high school students, an interesting subgroup of college students was those enrolled in a science course. Griffiths (1973) studied chemistry and physics students at a state university and an inner-city community college and found that 61% scored below level IIIb, the highest of the two formal reasoning classification levels. He concluded that a majority of students enrolled in introductory science cou

the

col

Of

defi

the

grou

mal

fem

wer

and

sif

Mcl

thi

Ho

fo

ei

ab

la

in

51

Gr

hi

th

65

courses have a major handicap in the development of their cognitive structures.

McKinnon (1970) reported interesting results on college freshmen enrolled at the University of Oklahoma. Of 143 students tested, he found that only 22% were definitely formal thinkers, 27% were "probably" beyond the concrete level, and 51% were concrete. When the groups were divided by sex, he found that 44% of the males were classified formal, while only 8% of the females were formal. Approximately the same percentage were "probably" beyond concrete--27% of the females and 28% of the males, while 28% of the males were classified concrete and 65% of the females were concrete. McKinnon's finding that 28% of the males were not formal thinkers was very similar to Elkind's result of 26%. However, McKinnon's results for the percentage of pre-formal-operational females (65%) was much higher than either Elkind's (48%) or Tower and Wheatley's (39%).

In summary, the research on the formal reasoning abilities of college students clearly indicated that a large percentage were not formal-operational. Studies involving both males and females reported from 42% to 51% of the subjects were not formal-operational and Griffiths showed that 61% could not reason at the highest formal reasoning level. Studies by sex reported the range of pre-formal thinkers for females was 39% to 65% and 26% to 28% for males.

Coll
Not
Univ

reas
Subj
atte
repo
majo
Thus
rea
of
"th
Ame
the

an
fr
op
on
fo
pa
Te
hi
su
ca
in

College-age and Older Subjects--
Not Necessarily Enrolled in a
University

Two additional studies researched the formal reasoning capabilities of adults and young adults. Subjects were not identified on the basis of college attendance. Kohlberg and Gilligan (1971, p. 1065) reported that not until the ages of 21 to 30 was a clear majority (65%) of subjects classified as formal thinkers. Thus, 35% of the subjects were deficient in abstract reasoning. They also reported no further development of formal reasoning after age 30 and concluded that "the point, however, is that a large proportion of Americans never develop the capacity for abstract thought" (1971, p. 1065).

In addition, two groups analyzed in the Karplus and Karplus (1970) study cited earlier revealed that from 60% to 84% of the subjects were not formal-operational. In a group of participants at a conference on Piaget's work, 84% of the subjects were found to lack formal reasoning capabilities. For the other group--participants at an American Association of Physics Teachers conference--60% were not formal. A surprisingly high percentage of people who must teach physics, a subject which is based primarily upon formal thought, cannot reason at levels expected of the students enrolled in their courses.

you

ing

and

<u>Min</u>

out

has

par

esp

stu

and

tin

91

stu

ea

in

re

to

le

on

was

rev

a

was

lev

Hence, the two studies reported on adult and young adult populations revealed a lack of formal reasoning abilities. For the three groups analyzed, 35%, 60%, and 84% of the subjects were not formal-operational.

Minority Students

McKinnon and Renner (1971, p. 1048) have pointed out that there is "no indication that his [Piaget] work has been extended to include entering college students, particularly American students." This statement is especially true for minority college students; only two studies reported findings concerning "urban" students and in neither case was the sample defined to be distinctly minority. Karplus and Peterson (1970) reported 91% of the urban eleventh and twelfth graders in their study were not formal-operational, but as was discussed earlier, these results were questionable due to an inadequate measurement instrument. Griffiths (1973) reported that 61% of the college subjects in introductory chemistry and physics were not functioning at level IIIb, the highest formal-operational level. The only relationship to minority students in this study was that analysis on the basis of "racial" subsamples revealed no difference from the original sample, i.e., a large percentage of students in the racial subsamples was also performing below the highest formal-operational level.

reas

tan

of

tior

forn

maj

to

app

to

Adc

Th

sp

di

wo

Pi

of

pl

in

hi

The research to date only addressed the formal reasoning capabilities of minority college students tangentially. There is a clear need for an analysis of the abstract reasoning abilities of this population.

Research on Effective Instructional Strategies

Unfortunately, the research exploring instructional strategies designed to facilitate development of formal reasoning is extremely limited.

First, it is important to note that Piaget's major reason for researching cognitive development was to describe its sequence rather than to specify practical applications to improve learning. In their introduction to The Growth of Logical Thinking from Childhood to Adolescence, Inhelder and Piaget stated:

> This book has two aims: to set forth a description of changes in logical operations between childhood and adolescence and to describe the formal structures that mark the completion of the operational development of intelligence. (1958, pp. xxiii-xxiv)

They were describing the thinking of adolescents, not specifying ways to improve it. Further, Wadsworth, when discussing what he saw to be the implications of Piaget's work for education, stated, "It must be kept in mind that Piaget has not directed his efforts toward the solution of educational problems" (1971, p. 120). Thus, it is plausible that one of the reasons for the lack of research into the applications of Piaget's work is that Piaget himself did not interpret his work in this way.

der

McK

ins

reas

in s

form

prov

ing

ing

in

top

of

way

or

emp

abo

pro

A few educators, however, have attempted to derive practical applications from Piaget's theory. McKinnon attempted to determine if inquiry-based instruction has any impact on the development of formal reasoning. He found that an inquiry-oriented course in science had positive effects on the development of formal reasoning:

> The Forum for Scientific Inquiry course was designed to bring about a change in students' capacity to think logically; thus, in the short time of one semester, students did begin to accept the responsibility for their learning experiences; they did begin to contribute more mature views and judgments to the questions being evaluated. This effect was felt when the two groups were evaluated at the conclusion of the course as shown by the net increase in scores in the five Piagetian tests. (1970, p. 37)

By "inquiry," McKinnon meant that the course provided the opportunities for "questioning, hypothesizing, verifying, restructuring, interpreting, synthesizing, and predicting . . ." (1970, p. 37). Students met in small groups to discuss and research a particular topic and were asked to examine a particular aspect of a problem, to find out what was known, and to suggest ways to interpret data to either arrive at a solution or better understand the problem. Thus, the inquiry emphasis of McKinnon's course was not oriented to facts about science, but rather oriented to the process of problem solving in science. Although not designed to

tea
McK
pro

an
whi
ie
de
in
an
kn

re
pr
th
re
wh
pr
al

s
to
o
le
th
as

teach problem solving per-se, the objectives for McKinnon's course were certainly elements involved in problem solving.

Renner and Lawson (1975) studied the effects of an experimental program for elementary school teachers which attempted to provide an integrated science experience via an inquiry approach. The curriculum was designed to provide students with laboratory experiences in science that were concrete (in the Piagetian sense) and the emphasis on these experiences was "How do I know?" "Why do I believe?" and "What is the evidence?"

Using an experimental-control group design, the researchers found that the inquiry strategy of instruction produced significant movement into the formal stage for the experimental subjects. They concluded that these results support "the hypothesis that curricular materials which confront students with concrete materials and problems can promote the development of formal thinking abilities" (Renner & Lawson, 1975, p. 9).

Although not directly describing an instructional strategy, an interesting study by Saarni (1974) attempted to correlate problem-solving ability with Piaget-level of reasoning. She found that the higher the Piaget-level, the better the student was on productive thinking problems: " . . . individuals classified as formal-operational (or transitional) were generally

mor
pro
ati
int
bet
ab
in
pe
ak
Ho
o
m
s
c
"
a
S
T
c
e
a
a

v
s
s

more competent problem solvers on the productive thinking problems than those who were classified as concrete-operational" (1974, p. 342). Her research does not allow an interpretation that deduces a cause-effect relationship between levels of abstract reasoning and problem-solving ability: i.e., it cannot be determined if an increase in abstract reasoning causes better problem-solving performance or if an increase in problem solving abilities causes an increase in abstract reasoning. However, her research does raise the question of whether or not such a cause-effect relationship exists. She maintained, however, that an individual's problem-solving strategies can be understood in terms of developmental capacities as defined by Piaget and that Piaget's theory "appears to satisfy some of the more crucial demands of a theory of human problem solving as suggested by Newell, Simon, and Shaw, and Simon and Newell" (1974, p. 338). Thus, she definitely left open the possibility of a cause-effect relationship or at least an interaction effect between problem solving and development of abstract reasoning. The study being submitted here addresses this issue.

A number of other researchers have suggested various instructional strategies that might increase students' abstract thinking, although none have been specifically studied. Beistel (1975, p. 151) suggested that a concrete student can develop formal thinking abilities through careful experimentation on the student's part and extensive critique of this experimentation by the instructor. As a result of this procedure, the student deduces the relevant principles involved. This approach seemed to involve the inquiry approach found helpful by McKinnon (1970) and by Renner and Lawson (1975) and also an extensive feedback mechanism. Further, Beistel claimed that in a lecture, intellectual development can be facilitated by using techniques such as introducing new topics in concrete terms, occasionally using an unsatisfactory hypothesis or incorrect conclusion for student evaluation, creating discrepant events, or describing how a process or concept is reversed (i.e., Piaget's reversibility concept). Kolodiy (1974, p. 262) hypothesized that programmed instruction specifically designed for developing formal thinkers may also be an effective mechanism for improving abstract reasoning.

Summary

A number of conclusions can be drawn about the level of abstract reasoning ability of various groups of students. First, a large percentage of pre-college-age students are pre-formal-operational. Two studies (Higgins-Trenk & Gaite, 1971; Lawson & Renner, 1974) indicated from 57% to 86% of the subjects studied were pre-formal thinkers. An additional study by Karplus and Peterson (1970) was not valid because of questionable measurement procedures.

In contrast to these studies were those analyzing pre-college students enrolled in a science course. Lawson and Renner (1974) reported only 3% to 29.5% of these subjects were not functioning at formal-operational levels. (A second study by Karplus & Karplus [1970] is discounted because of measurement problems.) These results indicated that either students enrolled in science-related courses are already formal-operational or learn formal-operational thinking in the course.

The third conclusion from the literature is that a large percentage of college students function at pre-formal-operational levels (from 42-60%). In addition, the literature consistently pointed out that a larger percentage of college females than college males were pre-formal-operational. When adult populations were studied, it was found that a large percentage (from 35-84%) were also pre-formal-operational.

The last population for which a review of the research was conducted was minority students. No studies reported on this particular population, indicating a need for analysis of minority groups.

Thus, it can be concluded that a large percentage of subjects from high school age to adult do not function at formal-operational levels. The only exception is students enrolled in a high school science course; they were primarily formal-operational. Hence, the research clearly pointed out that Piaget's age specification of 15 years for entry into the formal-operational stage is not accurate for American subjects. Effective means to remedy this situation are needed.

Unfortunately, the literature is limited in specifying effective instructional strategies that facilitate the development of formal reasoning. McKinnon (1970) and Renner and Lawson (1975) pointed to an inquiry approach as effective. Based upon synthesizing and evaluating science-type problems to predict solutions, the approach included skills necessary for problem solving. Saarni's (1974) results, indicating a high positive correlation between problem-solving ability and level of thinking, led to the speculation that improvement in problem-solving ability will result in an improvement in abstract reasoning. These studies pointed to direct instruction in problem solving as a strategy that should be tested.

Table 1

Summary of Piaget Studies

Group	Author/Date	Percentage Pre-Formal
Pre-college (General)		
13-18 years	Higgins-Trenk & Gaite (1971)	57
11th grade	Lawson & Renner (1974)	66
12th grade	Lawson & Renner (1974)	71
10-12th grade	Karplus & Karplus (1970)	86
11-12th grade (suburban)	Karplus & Peterson (1970)	20[a]
11-12th grade (urban)	Karplus & Peterson (1970)	91[a]
Pre-college (Science Course)		
11-12th grade (chemistry)	Lawson & Renner (1974)	4
11-12th grade (physics)	Lawson & Renner (1974)	3
11-12th grade (biology)	Lawson & Renner (1974)	29.5
12th grade (physics)	Karplus & Karplus (1970)	72[a]
College Students		
Male and female	Elkind (1962)	42
Male and female	McKinnon (1970)	51
Male and female	Griffiths (1973)	61 (below level IIb)

Table 1 (Continued)

Group	Author/Date	Percentage Pre-Formal
College Students (continued)		
Male	Elkind (1962)	26
Male	McKinnon (1970)	28
Female	Elkind (1962)	48
Female	Tower & Wheatley (1971)	39
Female	McKinnon (1970)	65
Adult and Young Adult		
21-30 years	Kohlberg & Gilligan (1971)	35
Adult (Piaget conf)	Karplus & Karplus (1970)	84
Adult (physics teachers)	Karplus & Karplus (1970)	60
Minority Students		
11-12th grade (urban)	Karplus & Peterson (1970)	91[a]
college (racial sub-samples)	Griffiths (1973)	61 (below level IIIb)

[a]Results questionable due to invalid testing.

CHAPTER III

THE RESEARCH DESIGN

In this chapter the research and statistical procedures for analyzing the data will be outlined. The procedures include a description of the population, the sample selection, details of the treatment, methodological procedures, reliability and validity concerns, and the research design and statistical procedures derived from the hypotheses.

The Population and Sample

The population in this research is defined to be all first-term minority engineering students in colleges of engineering. The sample includes all 60 first-term minority students enrolled in the College of Engineering at Michigan State University.

Assignment to Control and Experimental Groups

Of the 60 students in the sample, 38 were identified in June, 1975 prior to Michigan State University's Summer Orientation program. The remaining

22 students were admitted to the university during the summer. Because new freshmen must attend the orientation program to enroll for Fall courses, it was necessary to assign the original 38 students to the control or experimental group prior to orientation. This was accomplished by numbering the students alphabetically, selecting numbers from a table of random numbers, and assigning students who corresponded to the random number to the experimental group.

The remaining 22 students were alternately assigned to each group as they appeared at orientation sessions.

From the June, 1975 list of 38 students, 19 were assigned to the control group and 19 to the experimental group. Of the 19 assigned to the experimental group, three did not enroll in the university, two requested that they not be associated with the Sloan Project or its courses, and two had scheduling conflicts and could not enroll in the experimental section. These seven students were deleted from the study.

Of the 19 students assigned to the control group, six did not enroll for Fall term, two requested they not be associated with the Sloan Project, and two had scheduling problems forcing them to enroll in the experimental course but were not included in the data for the experimental group. Thus, 10 of the original control subjects were deleted from the study.

Therefore, on the basis of the original random assignment in June, 1975, 12 students were counted in the experimental group, 9 in the control group, and 17 were deleted from the study.

For those students who were admitted during the summer, 10 were placed in the experimental group and 12 in the control group. The final sample included 22 students in the experimental group and 21 in the control group. Table 2 summarizes this assignment procedure.

The Treatment

As was discussed in Chapter I, it is hypothesized that the treatment "Introduction to Reasoning and Problem Solving" will facilitate development of abstract reasoning. The content of the course included two areas: (a) Preparation for Problem Solving, which included restating problems, defining unknown terms, specifying given information, deciding what is to be solved for, and drawing and labeling diagrams, and (b) learning and applying the following problem-solving strategies: simplification, subproblem, contradiction, inference, and working backward.

Once these content areas were identified, the design of the course followed the principles of instructional design specified in Davis, Alexander,

Table 2

Number of Subjects Assigned to Group and Subsequent Mortality Prior to Testing

Group	June Assignment	Did Not Enroll	Schedule Problems	Declined Participation	Orientation Assignment	Total
Control	19	-6	-2	-2	+12	21
Experimental	19	-3	-2	-2	+10	22

and Yelon's Learning System Design (1974). This procedure included specifying and sequencing behavioral objectives (see Figure 1 for a complete listing), developing written instructional material for each objective, and writing evaluative questions to test accomplishment of each objective. In addition, the written materials were reviewed by an engineering professor for a critique of the content and its relationship to engineering, a mathematics professor for a critique of the content as it relates to mathematics, and an instructional development specialist for analysis from the viewpoint of instructional design. The reviews included suggestions for making the text clearer, ideas for alternative ways to explain a difficult point, and suggestions for reorganizing the order of the material. The reviews also indicated weaknesses in the text; changes were generally made, unless they mandated extensive reorganization of the course, which time did not permit. One such weakness was the lack of sufficient practice exercises throughout the modules.

Upon completing one of the modules in the course, students took exams based upon the specific objectives for the module. Students were required to repeat exams until 70% mastery was achieved. There was no penalty for repeated exams except after mastery was achieved; in this situation, students were required to keep the repeated grade.

Module 1--Preparing for Problem Solving: Understanding the Problem

1. State in proper order the steps used in the Preparation phase of problem solving.

2. Given a problem statement, properly define all terms used in the problem, recognize appropriate and inappropriate examples of the terms, and develop your own example of each term. (Criteria for accomplishing this specified in the module)

3. Given the statement of a problem, recognize proper and improper restatements of the problem. For improper restatements, tell why they are improper.

4. Given the statement of a problem, restate the problem in your own words according to the criteria specified in this module.

Module 2--Preparing for Problem Solving: Translation

5. Given a problem statement, determine the "Given" information.

6. Given a problem statement, write a Solution Statement according to the criteria listed in this module.

Module 3--Drawing Diagrams

7. Given a problem statement, determine which of the following diagrams is appropriate: (a) Given Situation Only, (b) Given and Solution Situation Same, (c) Given and Solution Situation Different, or (d) Two (or more) Givens and Solution.

8. After deciding which diagram is appropriate for a given problem statement, draw and label the diagram.

Module 4, 5, 6, 7, 8--Simplification, Subproblem, Drawing Inferences, Contradiction, Working Backward

9. List the steps to use in applying the strategy.

10. Given a problem that can be solved by the strategy, use the strategy to solve the problem.

11. List the criteria used to determine if the strategy should be tried in solving the problem.

12. Given a list of problems, select those for which a given strategy would be appropriate to try.

Figure 1. Introduction to Reasoning and Problem Solving: Course Objectives

In addition to the eight modular exams (each worth 50 points for a total of 400 points), a comprehensive final examination was administered during the university scheduled final exam session. No retests were allowed and the final counted 100 points or 20% of the total course grade.

Grades were assigned according to the point distribution in Table 3. The only exception made to this scale was when the final examination grade was higher than the grade corresponding to total points. In this case, the student received the final examination grade for a course grade. Final examination grades were based upon the same percentage range as total points.

Table 3

Grading Scale Used in the Treatment Course

Total Course Points	Course Grade	Percentage Range
470-500	4.0	94-100
440-469	3.5	88-93
410-439	3.0	82-87
380-409	2.5	76-81
350-379	2.0	70-75
320-349	1.5	64-69
290-319	1.0	58-63
0-289	0.0	0-57

In addition to the instructor, two senior undergraduate engineering students were involved as tutors.

Their function was to assist in one-to-one or small group instruction and in the administration of examinations.

The course met two days per week for two hours each day; there were 19 class sessions during the term for a total of 38 contact hours. Each student received two credits upon satisfactory completion of the course.

The Research Design

The experimental design for this research is the control-experimental group pretest-posttest form suggested by Campbell and Stanley (1963):

$$\begin{array}{llll} R & C_1 & & C_2 \\ R & E_1 & T & E_2 \end{array}$$

R stands for random assignment, C_1 for the control group, and E_1 the experimental group. T is the treatment, in this case exposure to the course, and the subscripted notation refers to pretest (1) and posttest (2). Subjects were randomly assigned to the control or experimental group, both groups were pretested to measure their abstract reasoning ability upon entry, the experimental group received the treatment, and both groups were posttested to measure abstract reasoning ability upon exit.

Methodological Procedures

This section will explain the procedures used to analyze the abstract reasoning abilities of the subjects in the study, including descriptions of the pre- and posttesting, the instruments used, and how they were scored.

Pre- and Posttesting

During the first seven days of the term, students in the experimental and control groups were asked to make one-half-hour appointments with the researcher for an "interview." When students appeared for the appointment, they were told they would do two experiments but that the results of their performance would not be discussed with them until the end of the term. Students who missed an appointment were rescheduled two additional times; those failing to appear at any of the appointments were deleted from the study.

Posttesting was conducted eight weeks later during the last week of Fall term classes. Once again, students scheduled one-half-hour interviews and were asked to repeat the same two experiments used on the pretest. After completing the posttest, the correct answer was explained to the student as well as the purpose of the research. They were asked not to discuss it with their friends. A follow-up of those who did not appear for testing was conducted the same way as during the pretesting.

Of the 22 experimental group and 21 control group subjects, 16 experimental subjects and 11 control subjects completed all testing. Table 4 indicates how many subjects were lost during the testing.

Table 4

Mortality of Subjects Due to Incomplete Testing

Group	Original Number	No Pretest	No Posttest	Completed All Testing
Control	21	6	4	11
Experimental	22	3	3	16

<u>Instrumentation</u>

In order for the researcher to measure level of cognitive development, each student was asked to perform two experiments described by Inhelder and Piaget. These were the Pendulum Problem (1958, pp. 67-79) and the Equilibrium in the Balance problem (1958, pp. 164-181).

In the Pendulum Problem, the student was confronted with three different length strings hanging from the ceiling. Attached to the end of each was a small pan on which weights could be attached. Each of the three pans were identical in size, shape, and weight. The student was also shown a number of 50 gram and 100 gram weights. The concept of frequency was then defined for the student as "the number of times the pendulum swings back and forth in a given time period."

With this information the student was told to experiment with the equipment in order to determine "what thing or things (if anything) affects the frequency of the pendulum."

In the Equilibrium in the Balance problem, the student was shown an apparatus that consisted of a yard stick with a balance point or fulcrum at its center. At each inch marking there was a small hole from which weights could be hung. The weights consisted of paper clips and it was explained that each clip constituted one unit of weight; thus two clips weighed two units, three clips weighed three units, and four clips weighed four units. The experiment proceeded as follows:

1. The experimenter balanced the yardstick and asked the subject if it was in balance.

2. The experimenter handed the subject two equal weights and asked if the subject agreed they were equal.

3. Next, the experimenter placed a weight at some point on the balance and handed an equal weight to the subject. The subject was then told, "Place this weight on the stick so it balances." Once the subjects did this, they were asked to explain why the weight was so placed.

4. In the second phase, the experimenter placed two equal weights at equal distances from the fulcrum

and asked the subject to agree the stick was in balance. Then the experimenter moved one weight toward the fulcrum and without letting go of the stick asked the subject "What happens when I let go of the stick and why?"

5. In the third part of the test the experimenter placed two unequal weights at unequal distances from the fulcrum in such a way that the stick balanced. The subject was then shown that the stick balanced and told "Take the weight on the right side of the fulcrum and place it on the left. Do the same with the weight on the left so that the stick balances." Once the subject did that, the experimenter held the stick and asked the subject to explain why the weights were so placed. After the subject gave a reason, the experimenter released the stick so the subject could see if the stick balanced.

6. In the last part of the experiment, the subject was given two unequal weights and told to place them on the stick so it balanced. Once the subjects did this, they were asked to explain why the weights were so placed. Then the experimenter released the stick.

Once the experiments were completed by each student, responses were translated into a score that indicated level of abstract reasoning ability. The criteria for each experiment, the associated level in Piaget's hierarchy, and the corresponding numerical

score are listed in Tables 5 and 6. A score of one indicates that the subject should be able to perform the criteria at the indicated level; a zero indicates performance is not expected.

Thus, the score received indicated the level of abstract reasoning, i.e., a score of 2 indicates the subject is at level IIB, while a score of 2.5 on the Pendulum Problem indicates the subject is in transition between IIB and IIIA. Hence, each subject was scored according to whether or not each criteria in each experiment was accomplished.

Reliability and Validity

The concepts of test reliability and validity are of importance to researchers using any type of testing instruments. Both of these concepts will be defined and discussed in terms of this study. In addition, Campbell and Stanley's (1963) criteria for internally and externally valid designs will be discussed.

Reliability

According to Anastasi, "reliability refers to the consistency of scores obtained by the same individuals when re-examined with the same test on different occasions . . ." (1968, p. 71). In addition, she points out that "test reliability indicates the extent to which individual differences in test scores are attributable to

Table 5

Criteria for Scoring Responses to the Pendulum Problem

Criteria	Piaget-Level				
	I	IIA	IIB	IIIA	IIIB
1. Orders weights serially	0	0	1	1	1
2. Discovers relationship between frequency and length (inverse)	0	1	1	1	1
3. Excludes variables when given combination in which only one variable varies	0	0	0	0.5	0.5
4. Produces combinations of variables in a systematic way	0	0	0	0.5	0.5
5. Varies only one variable while holding others constant	0	0	0	0	1
Total Points (score)	0	1	2	3	4

Table 6

Criteria for Scoring Responses to the Equilibrium in the Balance Test

Criteria	Piaget-Level				
	I	IIA	IIB	IIIA	IIIB
1. Discovers by trial and error relationship between length and width	0	1	1	1	1
2. Gives a qualitative explanation of relationship	0	0	1	1	1
3. Gives a quantitative explanation of relationship	0	0	0	1	1
4. Explains relationship in terms of height compensation	0	0	0	0	1
Total Points (score)	0	1	2	3	4

Note. The use of 0.5 for 3 and 4 is necessary in order to create a discrimination between the two criteria for level IIIA, B (i.e., criteria 3 & 4) and still maintain equal intervals between levels I through IIIB. In this way it is possible numerically to discriminate between a subject who can do criteria 1, 2, and 3 but not 4 (or 1, 2, and 4 but not 3) and is scored 2.5, indicating the subject is in transition between levels IIB and IIIA.

'true' differences in the characteristics under consideration and the extent to which they are attributable to chance errors" (1968, p. 71). Thus, a highly reliable test will produce scores which measure actual differences rather than scores that measure differences due to chance.

Traditionally, to determine test reliability, two equivalent forms of the same test are administered and a test-retest correlation coefficient is computed to provide an index of the reliability of the test. This procedure was not followed because equivalent forms of the tests used had not been identified in previous research.

For tests where the scorer must interpret the behavior of the subject in order to arrive at a score, Anastasi (1968, p. 86) recommends assessing the inter-rater reliability by having subject responses independently scored by two examiners. To obtain a measure of reliability, the resulting scores are correlated by use of one of the standard reliability coefficient techniques.

In this study a second scorer was not available; hence, an inter-rater reliability measure was not computed. This omission was due to the lack of availability of a trained scorer who could devote approximately 40 hours to analyzing subject responses to the tests. The lack of reliability information will be discussed in Chapter V as a limitation of this study.

Validity

In general, the concept of validity refers to "what the test measures and how well it does so" (1968, p. 99). The type of validity most important to this study is construct validity, which is defined as the extent to which a particular test measures a theoretical construct or trait. The Equilibrium in the Balance test and the Pendulum Problem have construct validity if they in fact measure formal-operational reasoning according to the theory.

There is growing evidence that these two tests do measure formal-operational reasoning and thus do have construct validity. In their 1974 study, Lawson and Renner utilized the multivariate statistical technique of principal components analysis to examine the extent to which these and other tests actually measure formal reasoning. In principal components analysis, first the major factors which account for a significant amount of the variance in scores are identified. In the Lawson and Renner study there were two factors: formal- and concrete-operational reasoning. Hopefully, if the construct being analyzed hypothesizes N factors, then the analysis will identify exactly N factors. Next, correlations are computed between each of the tests being analyzed and each of the factors found. If all tests that are hypothesized to measure one of the factors have

significantly large correlations with that factor, then the test is said to have construct validity.

As a result of a principal components analysis, Lawson and Renner (1974) concluded that the Pendulum Problem does measure formal reasoning. They analyzed six Piaget-tests that were hypothesized to measure only two factors--formal- and concrete-operational reasoning. Results indicated that only these two factors were found, they accounted for a significant amount of the variance in scores (55.2%), and all correlations were as predicted. The correlation for the Pendulum Problem was 0.72 with the hypothesized factor, formal-operational reasoning.

In the same study, subsequent analyses of various combinations of Piaget-tests yielded results similar to the above: the hypothesized number of components was found, they accounted for a significant amount of the variance of the scores, and correlations between test and principal component were as predicted by the theory. (The Pendulum Problem had a correlation with the hypothesized component of 0.68 on one analysis; the Equilibrium in the Balance test had a correlation of 0.80 with the hypothesized component.) Thus, the authors concluded that "these results indicate that all the tasks . . . are measuring the same thing, i.e.: formal-operational thinking" (1974, p. 556).

Similar results using principal components analysis are also reported by Lovell and Butterworth and Hughes, in Measurement and Piaget (Green, et al., 1971, pp. 85-87).

Thus, there is evidence that both the Pendulum Problem and the Equilibrium in the Balance test do indeed measure what Piaget intended them to measure: formal-operational thought.

Internal and External Validity

Internal validity is defined to be the "basic minimum without which an experiment is uninterpretable" (Campbell & Stanley, 1963, p. 5). To be internally valid, an experiment must control for (a) history--what happens between first and second measures in addition to the experimental variable, (b) maturation of subjects, (c) testing--the effects of taking a first test upon a second test, (d) instrumentation--changes in observers who interpret tests, (e) statistical regression--operating when groups are chosen on the basis of extreme scores, (f) selection biases--differential selection for groups, (g) mortality of subjects--differential loss of subjects, and (h) selection-maturation interaction--differential maturation in multiple group designs.

External validity asks the question "To what populations . . . can this effect be generalized?"

(Campbell & Stanley, 1963, p. 5). The factors that must be controlled in order that an experiment be externally valid are: (a) reactive effect of testing--pretest sensitizes subjects to experimental variable, (b) interaction of selection biases and experimental variable, (c) reactive effects of experimental arrangements--the experiment itself creates the effect, and (d) multiple-treatment interference--multiple treatments affect subjects.

According to Campbell and Stanley, there are three sources of concern for the internal validity of this study: mortality of subjects, selection biases, and instrumentation biases.

There are a number of factors which contributed to the mortality of subjects from the original random assignment to control or experimental group. Nine subjects did not appear for classes during Fall term, four had scheduling difficulties, and four refused to participate resulting in 43 subjects remaining in the two groups out of the original 60. In addition, nine subjects from the control group and six from the experimental group did not complete all testing. Thus, there is a high mortality rate of subjects from the original random assignment to the final group which completed all testing. Since Campbell and Stanley pointed out that it can be assumed the mortality rate is randomly distributed over the two groups, a post-hoc analysis was conducted to determine if in fact this was the case. Two-sample, two-tailed t-tests ($p \leq .05$) were conducted to determine if (a) the original control group differed significantly from the original experimental group or from the final control group; (b) if the original experimental group differed from the final experimental group; or (c) if the final experimental group differed from the final control group. Both "original" groups were defined to be all 60 students who were randomly assigned to groups minus the nine students who did not enroll for Fall term, as data were not available for them. The "final" groups were comprised of only those students who completed all testing. The t-tests were conducted on the following variables: (a) ACT/SAT math percentile, (b) ACT/SAT verbal percentile, (c) ACT/SAT composite score percentile, (d) high school grade point average, (e) MSU arithmetic test score, and (f) MSU mathematics test score. The ACT or SAT percentiles were localized Michigan State University norms. Table 7 reports means, N's, and variances for the various groups on the six measures.

As Table 8 indicates, there was no significant difference between any two groups for any of the six variables. Thus, it can be concluded that although mortality of subjects was a troublesome factor in this

Table 7

Mean, Group Size, and Variance for Six Measures of the Original Sample and Final Sample

Group	Measure																	
	ACT/SAT Mathematics Percentile			ACT/SAT Verbal Percentile			ACT/SAT Composite Percentile			High School Grade Point Average			Arithmetic Placement Score			Mathematics Placement Score		
	N	$\bar{x}$	s^2	N	$\bar{x}$	s^2	N	$\bar{x}$	s^2	N	$\bar{x}$	s^2	N	$\bar{x}$	s^2	N	$\bar{x}$	s^2
Original Experimental[a]	26	41.5	714.66	26	26.27	496.52	26	31.35	662.16	26	2.96	0.21	26	28.92	48.07	26	14.62	36.97
Original Control[a]	25	30.48	591.76	25	22.20	576.00	25	21.44	583.59	25	2.97	0.20	25	28.52	35.68	25	13.08	27.49
Final Experimental	16	41.31	734.63	16	22.12	479.98	16	29.44	124.40	16	2.81	0.18	16	29.12	44.78	16	13.56	30.26
Final Control	11	28.45	733.07	11	29.18	1050.16	11	19.91	792.69	11	2.92	0.19	11	28.27	48.42	11	12.45	22.87

[a]Less those students who did not enroll for Fall, 1975.

Table 8

T-test Values for Analysis of Differences Between Original Sample and Final Sample

Comparison Groups	Degrees of Freedom	Measures					
		ACT/SAT Mathematics Percentile	ACT/SAT Verbal Percentile	ACT/SAT Composite Percentile	High School Grade Point Average	Arithmetic Placement Score	Mathematics Placement Score
Original Experiment --Original Control	49	1.54	0.69	1.42	0.10	0.22	0.97
Original Experimental --Final Experimental	40	0.02	0.59	0.23	1.04	0.09	0.56
Original Control --Final Control	34	0.22	0.72	0.17	0.32	0.11	0.34
Final Experimental --Final Control	25	1.22	0.68	0.93	0.66	0.32	0.54

* = significant for $p \leq .05$.

study, the mortality was randomly distributed between control and experimental groups with respect to the six variables analyzed. Hence, the final control and experimental groups on which data were collected and analyzed were not significantly different from the original randomly selected control and experimental groups. Further, the final control group was not significantly different from the final experimental group on the basis of the six variables. Thus, the contaminating effects of mortality are not a limitation of this study and do not affect its internal validity.

The second area of concern for the internal validity of this study is selection biases due to the four students who refused to participate in the study. It is possible that their lack of involvement skewed the results in one direction or the other. Although this is a contaminating factor, the evidence indicating no significant differences between original and final groups minimizes the effects of the selection biases.

The internal validity of this study could have been affected by instrumentation biases created by the researcher conducting all testing. Knowledge of who was in the control group and who was in the experimental group could have an effect upon how student performance was analyzed and scored. It is possible that the researcher compensated for student performance by

either scoring the experimental subjects higher than the control subjects--thus biasing the results in favor of the experimental group--or by scoring the experimental subjects lower than the control subjects--thus biasing results in favor of the control group. Because of the researcher's involvement in pre- and posttesting, instrumentation biases may have been introduced.

Of concern for the external validity of this study are interaction of testing and treatment, interaction of selection and treatment, and reactive arrangements.

In any design using pretests, the risk is present that subjects will be sensitized to the treatment, i.e., there will be an interaction of testing and treatment. According to Campbell and Stanley the effects of pretests sensitizing the subjects should be equal on both the experimental and control groups, thus the effects cancel out one another. In addition, the pretest, which involved performing specific Piaget experiments, had no <u>content</u> relationship to the treatment, which involved learning specific problem-solving strategies. Therefore, interaction of testing and treatment is not a significant problem in this study.

Campbell and Stanley also suggest that selection and treatment interaction could affect the external validity if a significant number of subjects refused to cooperate in the study. In this study, four students refused to be involved in the study, introducing interaction of selection and treatment biases. However, results of the post-hoc analysis indicate that this effect is minimal.

The term "reactive arrangement" refers to a situation in which subjects know they are involved in an experiment. As Campbell and Stanley pointed out, this effect is almost always present: attempts should be made to reduce its effects. In this research, subjects were aware that they were involved in an experiment, but did not know what the experiment was about nor whether they were in the control or experimental group. Thus, any reactive arrangement effects should be randomly distributed over all subjects cancelling out any particular effect in favor of either group.

Hypotheses and Statistical Analysis

There are three major research questions this study addresses. Each is listed below, followed by the specific hypotheses and statistical procedures relevant to the research question.

Question 1--Was the course effective in increasing students' abstract reasoning abilities? These hypotheses were stated to determine whether the amount of change from pretest to posttest was significantly

greater for the experimental group as compared to the amount of change for the control group.

Hypothesis 1:

H_o: There will be no significant difference in change in abstract reasoning ability as measured by the Equilibrium in the Balance test between the control and experimental groups.

H_1: There will be a significant difference in change in abstract reasoning ability as measured by the Equilibrium in the Balance test in favor of the experimental group.

Hypothesis 2:

H_0: There will be no significant difference in change in abstract reasoning ability as measured by the Pendulum Problem between the control and experimental group.

H_1: There will be a significant difference in change in abstract reasoning ability as measured by the Pendulum Problem in favor of the experimental group.

Statistical Analysis:
Hypotheses 1-2

Change in abstract reasoning (Δ) will be measured by the difference between the posttest score (P_0) and the pretest score (P_r):

$$\Delta = P_0 - P_r$$

for each subject.

The average change score ($\bar{x}_i$) for each group will be computed and a two-sample, one-tailed t-test with $p \leq .05$ will be conducted to determine significance, where:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

where:

$$s^2_p = \frac{s_1^2 (n_1 - 1) + s_2^2 (n_2 - 1)}{n_1 + n_2 - 2}$$

and the degrees of freedom are $n_1 + n_2 - 2$.

The null hypothesis is rejected when the computed t-value is greater than or equal to $t_{.05,\ n_1 + n_2 - 2}$. Otherwise, the null hypothesis is not rejected.

<u>Question 2--Are there any relationships between various measures of academic ability and either level of abstract reasoning upon entry into the university or amount of change in abstract reasoning after one term in college?</u> The hypotheses are stated to determine what, if any, relationships exist between each of the measures of academic ability and (a) level of abstract reasoning upon entry into the university, (b) amount of change in abstract reasoning after one term of instruction without exposure to the treatment, and (c) amount of change

in abstract reasoning after one term of instruction with exposure to the treatment.

The academic ability measures are listed below. They were chosen because they are standard measures of academic ability and commonly used:

1. GRADE--the grade received in the treatment course; computed on the basis of total points received in the course (see Table 3). Thus, the grade was composed of 50 points each for the eight module exams (400 points) plus 100 points for the final. Grades are on a four-point system and include 4.0, 3.5, 3.0, 2.5, 2.0, 1.5, 1.0, and 0.0.

2. TOTPTS--total points earned in the treatment course.

3. FINAL--points earned on the final examination for the treatment course; maximum possible is 100 points.

4. OBJPTS--points earned on the final examination questions requiring application of a problem solving strategy; students selected three of four problems on the final; each was worth 16 points for a total of 48 points on the final.

5. HSGPA--high school grade point average; used on a 0 to 4.00 scale as reported in the student's permanent university records.

6. MSUGPA--first-term MSU grade point average; a 0 to 4.00 scale is used; includes all courses student took during Fall, 1975.

7. ARITHP--MSU Arithmetic Placement score; an MSU exam administered to all incoming students; used in conjunction with the Mathematics Placement score to place students in appropriate math courses; scored from 0 to 40 points.

8. MTHPLC--MSU Mathematics Placement score; an MSU exam administered to all incoming students; used in conjunction with the Arithmetic Placement score to place students in appropriate math courses; scored from 0 to 30 points.

9-11. MTHPCT, VRBPCT, CMPPCT--ACT or SAT mathematics, verbal and composite percentiles; ACT or SAT raw mathematics, verbal or composite scores were converted to a percentile score. Percentiles are localized MSU Fall, 1975 norms.

12. MTHGRD--grade received in mathematics course taken during Fall, 1975; four-point scale used (same as for GRADE).

Hypotheses 3-6:

H_o: For the experimental group only, there will be no significant correlation between the Equilibrium in the Balance pretest and each of the following variables:

3. GRADE
4. TOTPTS
5. FINAL
6. OBJPTS

H_1: For the experimental group only, there will be a significant correlation between the Equilibrium in the Balance pretest and each of the variables 3-6.

Hypotheses 7-10:

H_o: For the experimental group only, there will be no significant correlation between the Pendulum Problem pretest and each variable in Hypotheses 3-6.

H_1: For the experimental group only, there will be a significant correlation between the Pendulum Problem pretest and each variable in Hypotheses 3-6.

Hypotheses 11-18:

H_o: For the experimental and control groups combined, there will be no significant correlation between the Equilibrium in the Balance pretest and each of the following variables:

11. HSGPA
12. MSUGPA
13. ARITHP
14. MTHPLC
15. MTHPCT
16. VRBPCT
17. CMPPCT
18. MTHGRD

H_1: For the experimental and control groups combined, there will be a significant correlation between the Equilibrium in the Balance pretest and each of the variables 11-18.

Hypotheses 19-26:

H_o: For the experimental and control groups combined, there will be no significant correlation between the Pendulum Problem pretest and each variable listed in Hypotheses 11-18.

H_1: For the experimental and control groups combined, there will be a significant correlation between the Pendulum Problem pretest and each variable listed in Hypotheses 11-18.

Hypotheses 27-30:

H_o: For the experimental group only, there will be no significant correlation between change in abstract reasoning ability as measured by the Equilibrium in the Balance test and each of the following variables:

27. GRADE
28. TOTPTS
29. FINAL
30. OBJPTS

H_1: For the experimental group only, there will be a significant correlation between change in abstract reasoning ability as measured by the Equilibrium in the Balance test and each variable 27-30.

Hypotheses 31-34:

H_o: For the experimental group only, there will be no significant correlation between change in abstract reasoning ability as measured by the Pendulum Problem and each variable listed in Hypotheses 27-30.

H_1: For the experimental group only, there will be a significant correlation between change in abstract reasoning ability as measured by the Pendulum Problem and each variable listed in Hypotheses 27-30.

Hypotheses 35-42:

H_o: For the experimental group, there will be no significant correlation between change in abstract reasoning as measured by the Equilibrium in the Balance problem and each of the following variables:

35. HSGPA
36. MSUGPA
37. ARITHP
38. MTHPLC
39. MTHPCT
40. VRBPCT
41. CMPPCT
42. MTHGRD

H_1: For the experimental group, there will be a significant correlation between change in abstract reasoning as measured by the Equilibrium in the Balance problem and each variable 35-42.

Hypotheses 43-50:

H_o: For the control group, there will be no significant correlation between change in abstract reasoning as measured by the Equilibrium in the Balance Problem and each variable listed in Hypotheses 35-42.

H_1: For the control group, there will be a significant correlation between change in abstract reasoning as measured by the Equilibrium in the Balance Problem and each variable listed in Hypotheses 35-42.

Hypotheses 51-58:

H_o: For the experimental group, there will be no significant correlation between change in abstract reasoning as measured by the Pendulum Problem and each of the variables listed in Hypotheses 35-42.

H_1: For the experimental group, there will be a significant correlation between change in abstract reasoning as measured by the Pendulum Problem and each of the variables listed in Hypotheses 35-42.

Hypotheses 59-66:

H_o: For the control group there will be no significant correlation between change in abstract reasoning as measured by the Pendulum Problem and each of the variables listed in Hypotheses 35-42.

H_1: For the control group there will be a significant correlation between change in abstract reasoning as measured by the Pendulum Problem and each of the variables listed in Hypotheses 35-42.

Statistical Analysis:
Hypotheses 3-66

A Pearson product-moment coefficient of correlation (r) will be computed. To test the significance of the correlation, the following one-tailed t-test statistic is used ($p \leq .05$):

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

where the degrees of freedom are n-2. The null hypothesis is rejected when the computed t-value is greater than or equal to $t_{.05;\ n-2}$. Otherwise the null hypothesis is not rejected.

Question 3--Is there an interaction effect between level of academic ability and treatment? These hypotheses were stated to determine if there was a difference in the amounts of change in abstract reasoning for high ability vs. low ability students within or between groups.

Hypotheses 67-74:

H_o: There will be no significant difference in change in abstract reasoning as measured by the Equilibrium in the Balance test between high or low ability students in the experimental or control group. Ability level will be measured by:

67. HSGPA
68. MSUGPA
69. ARITHP
70. MTHPLC
71. MTHPCT
72. VRBPCT
73. CMPPCT
74. MTHGRD

H_1: There will be a significant difference in change in abstract reasoning as measured by the Equilibrium in the Balance test between high or low ability students in the experimental or control group for each ability measure listed in variables 67-74.

Hypotheses 75-82:

H_o: There will be no significant difference in change in abstract reasoning as measured by the Pendulum Problem between high or low ability students in the experimental or control group. Ability level will be measured by the variables listed in Hypotheses 67-74.

H_1: There will be a significant difference in change in abstract reasoning as measured by the Pendulum Problem between high or low ability students in the experimental or control group for each ability measure listed in variables 67-74.

Statistical Analysis:
Hypotheses 67-82

A two-way analysis of variance was used for each hypothesis. Because of unequal cell size, Finn's

(Scheifley & Schmidt, 1973) multivariance technique was used; it utilizes an exact least squares analysis. The following matrix represents the manner in which the data were classified for each academic ability variable:

	Academic Variable	
Group	HI	LO
Control	$\bar{x}_{11}$	$\bar{x}_{12}$
Experimental	$\bar{x}_{21}$	$\bar{x}_{22}$

where $\bar{x}_{ij}$ represents the mean change score for the i-th row and j-th column. An F-test was computed and the null hypothesis was rejected for values greater than or equal to $F_{.05;\, n_1,\, n_2}$, where n_1 and n_2 are the degrees of freedom; otherwise the null form was not rejected. The degrees of freedom were 1 and 21 for "grade in math course taken" and were 1 and 23 for the remaining variables.

Each of the eight academic ability variables was split into a high and a low group. Table 9 indicates the cut-off value and how many subjects in each group were classified high and low. Subjects with scores greater than or equal to the cut-off value were included in the high group and subjects with scores less than the cut-off value were included in the low group. For cases where there was more than one subject with the

Table 9

Cut-Off Values by Academic Ability Variables for Dividing Subjects into High and Low Groups

Variable	Cut-Off Value	Control		Experimental	
		High	Low	High	Low
High School Grade Point Average	2.77	6	5	7	9
MSU Grade Point Average	2.82	5	6	8	8
Arithmetic Placement Score	31	5	6	8	8
Mathematics Placement Score	13	4	7	10	6
ACT/SAT Mathematics Percentile	39	4	7	9	7
ACT/SAT Verbal Percentile	16	5	6	8	8
ACT/SAT Composite Percentile	13	3	8	10	6
Math Course Grade	3.00	5	5	7	8

cut-off value for a score, the subjects were randomly placed in the high or low group.

Summary

The original sample for this study was comprised of 60 first-term minority engineering students, while the final sample included 11 students in the control group and 16 in the experimental group. The control-experimental group pretest-posttest research design was used with only the experimental group receiving the treatment. Pre- and posttest measures were obtained on the basis of student responses to both the Equilibrium in the Balance test and the Pendulum Problem, which are both valid measures of level of abstract reasoning.

A limitation of this study is that no attempt was made to assess either test-retest reliability or inter-rater reliability for the tests used. There is, however, reasonable evidence from other research to conclude that the study has construct validity.

The concerns for the internal validity of this study are mortality of subjects, selection biases, and instrumentation biases. A post-hoc analysis of six entry-level variables indicated that the mortality of subjects was randomly distributed between groups.

The external validity concerns are interaction of testing and treatment, interaction of selection and treatment, and reactive arrangements.

This study addresses whether (a) the treatment was effective in increasing students' abstract reasoning ability, (b) if there are any significant correlations between both pretest abstract reasoning ability or change in abstract reasoning and various academic ability variables, and (c) if there are any treatment by ability interactions. Statistical analyses used were (a) a two-sample, one-tailed t-test, (b) Pearson product-moment coefficients of correlation, and (c) Finn's multi-variance technique for an analysis of variance with unequal cell size.

Chapter IV will report on the results of the analysis of the data.

CHAPTER IV

ANALYSIS OF THE RESULTS

The analysis of the data with respect to the hypotheses listed in Chapter III is reported in this chapter. The discussion is organized into three sections: (a) the results of the analysis for determining if the treatment, "Introduction to Reasoning and Problem Solving," facilitates the development of abstract reasoning ability; (b) the results of correlations between either pretest scores or change scores and numerous ability measures; and (c) the results of the analysis of variance for determining if there is an interaction effect between treatment (experimental or control) and ability level (high or low).

Analysis of the Effectiveness of the Treatment

The extent to which the treatment is an effective vehicle for improving abstract reasoning is addressed in two parts. The first part reports statistically significant or nonsignificant changes in abstract reasoning caused by the treatment, while the second part describes the magnitude of any changes observed but is not intended to be interpreted statistically.

Statistical Results

The main criterion used to assess the effectiveness of the treatment is the difference between the change in abstract reasoning for the experimental group and the change in abstract reasoning for the control group. Two null hypotheses are specified:

> Hypothesis 1:
>
> There will be no significant difference in change in abstract reasoning ability as measured by the Equilibrium in the Balance test between the control and experimental groups.
>
> Hypothesis 2:
>
> There will be no significant difference in change in abstract reasoning ability as measured by the Pendulum Problem between the control and experimental groups.

The two-sample, one-tailed t-tests used to compare the mean change in abstract reasoning from pretest to posttest for the experimental group to that of the control group are not statistically significant for both the Equilibrium in the Balance test and the Pendulum Problem (Table 10). The calculated t-values are $t = 0.41$ ($p < .35$) for the Balance test and $t = 0.46$ ($p < .33$) for the Pendulum Problem. Thus, the null forms for both Hypotheses 1 and 2 are not rejected for $p \leq .05$,

Table 10

Values of t-tests for Analysis of Differences Between Groups' Mean Piaget-Level by Test and by Measure

Group	Measure																	
	Equilibrium in the Balance									Pendulum Problem								
	Pretest			Posttest			Posttest minus Pretest			Pretest			Posttest			Posttest minus Pretest		
	N	$\bar{X}$	s^2	N	$\bar{X}$	s^2	N	$\bar{X}$	s^2	N	$\bar{X}$	s^2	N	$\bar{X}$	s^2	N	$\bar{X}$	s^2
Control	11	1.91	0.29	11	2.18	0.16	11	0.27	0.62	11	2.05	1.62	11	2.27	0.37	11	0.23	1.17
Experimental	16	1.94	0.46	16	2.31	0.23	16	0.38	0.38	16	2.22	0.60	16	2.66	0.72	16	0.44	1.56
t-Value	0.12			0.74			0.41			0.44			1.32			0.46		
$p <$	.40			.24			.35			.34			.10			.33		

indicating that for both measures, the amount of change for the experimental group was not statistically different from the amount of change for the control group.

This analysis compares the amount of change for the experimental group relative to the amount of change for the control group. Since this relative difference between amounts of change for the two groups is not significant, further analysis was conducted to determine if either the control or experimental group increased in abstract reasoning relative to itself. That is, (a) are the experimental group's posttest scores statistically different from its pretest scores, and (b) are the control group's posttest scores statistically different from its pretest scores?

To answer these questions, a one-sample, repeated measures t-test is used. For each group separately, posttest scores minus pretest scores were computed for each subject. The average of these differences ($\bar{d}$) is the unit of analysis for the dependent measure t-test.[1]

This analysis reveals that for the experimental group, the posttest scores are significantly greater than pretest scores for the Equilibrium in the Balance test ($t = 2.42$; $p < .02$), but not for the Pendulum Problem ($t = 1.40$; $p < .09$). For the control group,

---

[1] $t = \dfrac{\bar{d}}{s_d/\sqrt{n}}$, where s_d is the standard deviation of the difference scores.

posttest scores are not significantly different from pretest scores for either measure (Balance: $t = 1.39$, $p < .10$; Pendulum: $t = 0.71$, $p < .25$). These results are summarized in Table 11.

Hence, the statistical results indicate that for both measures, the amount of change from pretest to posttest for the experimental group is not statistically different from the amount of change for the control group. However, for the Equilibrium in the Balance test only, the posttest scores are significantly greater than the pretest scores for the experimental group, but not for the control group.

Descriptive Results

The descriptive results to be discussed are:

1. The average Piaget-level at which subjects are classified on the pretest and on the posttest;

2. The percentage of subjects functioning below the formal-operational level on the pretest (groups combined) and on the posttest;

3. The number of Piaget-levels changed from pre- to posttest.

1. Subjects' average Piaget-level. The statistical analysis is based upon an average level of abstract reasoning for each group and changes in these averages from pre- to posttest. These same

Table 11

Values of t-tests for Analysis of Differences Between Pretest and Posttest by Group and by Measure

Group	Measure									
	Equilibrium in the Balance					Pendulum Problem				
	Posttest minus Pretest					Posttest minus Pretest				
	N	$\bar{d}$	s_d	t^a	p<	N	$\bar{d}$	s_d	t^a	p<
Control	11	0.27	0.79	1.39	0.10	11	0.23	1.08	0.71	0.25
Experimental	16	0.38	0.62	2.42*	0.02	16	0.44	1.25	1.40	0.09

[a] Dependent, 1-tailed t-tests.

*Significant for $p \leq .05$

averages are useful in describing the average level of abstract reasoning of the subjects in each group. The individual scores from which the averages are computed and their corresponding Piaget-levels are:

Score	Level	
0	I	Preoperational
1	IIA	Concrete-Operational
2, 2.5	IIB	
3, 3.5	IIIA	Formal-Operational
4	IIIB	

As Table 10 shows, the pretest levels for both groups and for both tests are between 1.91 and 2.22, indicating the average Piaget-level for all subjects on the pretest is close to level IIB--the higher of the two concrete-operational levels. The average level for the control group is 1.91 on the Equilibrium in the Balance test and 2.05 on the Pendulum Problem; the experimental group scores are 1.94 and 2.22 on the two tests. As subjects were randomly assigned to group, it is expected that the pretest mean for the experimental group is not statistically different from the pretest mean for the control group on either measure used. This is in fact the case; Table 10 summarizes the two-sample independent t-test results, which were used to determine if pretest differences existed between groups (Balance: $t = 0.12$, $p < .40$; Pendulum: $t = 0.74$, $p < .24$).

Table 10 also indicates subjects' average Piaget-level on the posttest. The control group means are 2.18

on the Equilibrium in the Balance test and 2.27 on the Pendulum Problem; the experimental group means are 2.31 and 2.66 on the two tests respectively. These data indicate that according to each measure, both groups are well into the concrete-operational stage on the posttest, but still functioning below the formal-operational level.

The amount of change from pretest to posttest for the control group is 0.27 on the Equilibrium in the Balance test and 0.23 on the Pendulum Problem and the amount for the experimental group is 0.38 and 0.44 on the corresponding tests (see Table 10). Thus, the amount of change for the experimental group is 1.41 times greater than the amount of change for the control group on the Equilibrium in the Balance test and 1.91 times greater on the Pendulum problem. However, as has been discussed, the amount of change for the experimental group is not statistically greater than that of the control group on either test.

Thus, the descriptive results indicate that both groups increased in abstract reasoning from pretest to posttest and that the amount of increase is greater for the experimental group. Statistically, however, only the change from the Balance pretest to posttest for the experimental group is a significant increase.

2. Subjects functioning below the formal-operational level on the pretests. According to Piaget's

Table 12

Number and Percentage of Subjects at Each Piaget-Level by Group, Test, and Measure

Level	Score	Pretest												Posttest							
		Equilibrium in the Balance						Pendulum Problem						Equilibrium in the Balance				Pendulum Problem			
		Control		Experimental		Both		Control		Experimental		Both		Control		Experimental		Control		Experimental	
		N	%	N	%	N	%	N	%	N	%	N	%	N	%	N	%	N	%	N	%
I	0	0	0%	0	0%	0	0%	2	18%	1	6%	3	11%	0	0%	0	0%	0	0%	0	0%
IIA	1	2	18	4	25	6	22	0	0	0	0	0	0	0	0	0	0	0	0	0	0
IIB	2	8	73	9	56	17	63	7	64	14	88	21	78	9	82	11	69	10	91	11	69
IIIA	3	1	9	3	19	4	15	0	0	0	0	0	0	2	18	5	31	0	0	1	6
IIIB	4	0	0	0	0	0	0	2	18	1	6	3	11	0	0	0	0	1	9	4	25
Average Level[a]		1.91		1.94		1.93		2.05		2.22		2.15		2.18		2.31		2.27		2.66	

[a]Reflects scoring using 2.5 and 3.5.

theory, it is expected that all subjects in this study should be formal-operational. However, a surprisingly large percentage of subjects are not formal-operational upon entry into the university (pretest data). According to the Equilibrium in the Balance pretest, pre-formal-operational subjects comprise 91% of the control group, 81% of the experimental group, and 85% of both groups combined. For the Pendulum Problem, 82% of the control group, 94% of the experimental group, and 89% of both groups combined are not formal-operational thinkers (see Table 13). Posttest results indicate that on the basis of the Equilibrium in the Balance test, the percentage of students who are not formal-operational is 82% for the control group and 69% for the experimental group. For the Pendulum Problem, posttest results indicate 91% of the control group and 69% of the experimental group are not formal-operational. Scores reporting the percentages for the control and experimental groups combined are meaningless because of the possible treatment effect on the experimental group; hence, they are not reported.

For the Balance test, the number of subjects not functioning at formal-operational levels decreased nine percentage points from pretest to posttest for the control group and 12 percentage points for the experimental group. For the Pendulum Problem, the number of pre-formal operational thinkers actually increased nine

Table 13

Percentage of Pre-Formal-Operational Subjects by Test, Group, and Measure

Group	Pretest		Posttest		Posttest minus Pretest	
	Equilibrium in the Balance	Pendulum Problem	Equilibrium in the Balance	Pendulum Problem	Equilibrium in the Balance	Pendulum Problem
Control	91%	82%	82%	91%	-9	+9
Experimental	81%	94%	69%	69%	-12	-25
Both	85%	89%	74%	77.5%	-11	-11.5

percentage points for the control group and decreased 25 percentage points for the experimental group (see Table 13). Thus, on both measures, the percentage-point decrease in pre-formal-operational subjects is greater for the experimental group. Once again, it should be kept in mind that these percentage-point differences are only descriptive; the mean changes between the two groups are not statistically significant.

3. Number of Piaget-levels changed--Pre- to posttest. In order to provide a more complete analysis of how subjects changed from pre- to posttest, Table 14 summarizes the number of Piaget-levels subjects changed. The table indicates that a large percentage of subjects in each group and on each test do not change at all, although fewer subjects in the experimental group show no change than in the control group. That is, on the Equilibrium in the Balance test 64% of the control group did not change, while only 50% of the experimental group did not change. For the Pendulum Problem, 64% of the control group and 44% of the experimental group remains unchanged.

In addition, change in the positive direction is observed in the control group for 27% of the subjects on both tests, while a somewhat higher percentage is observed for the experimental group: 44% (Equilibrium in the Balance) and 30% (Pendulum). Conversely, higher

Table 14

Number and Percentage of Piaget-Levels Changed from Pretest to Posttest by Group and Measure

Change	Equilibrium in the Balance						Pendulum Problem					
	Control			Experimental			Control			Experimental		
+4										1	6%	
+3												
+2	1	9%	27%			44%	2	18%	27%	1	6%	30%
+1.5										2	12%	
+1	2	18%		7	44%							
+0.5							1	9%		1	6%	
0	7	64%		8	50%		7	64%		7	44%	
-0.5										3	19%	
-1	1	9%	73%	1	6%	56%			73%	1	6%	69%
-1.5												
-2							1	9%				

percentages of subjects who either do not change or change in the negative direction are observed for the control group (73% and 73% on the two tests) over the experimental group (56% and 69%).

Summary

The descriptive results of the analysis of the effectiveness of the treatment indicate that on both measures used (a) both the control and experimental groups increased in abstract reasoning from pre- to posttest and (b) the magnitude of the increase is greater for the experimental group than the control group. None of these changes, however, reach statistical significance for $p \leq .05$, except for the amount of change from pretest to posttest for the experimental group when the measure is the Balance test. The remaining changes observed are attributed to chance.

The results do indicate that a very large percentage of the sample entered the university at pre-formal-operational thinking levels. Depending upon the test used, from 85% to 89% of the entry group are pre-formal-operational. Their average Piaget-level is concrete-operational.

After one term at the university, from 82% to 91% of the control group and 69% of the experimental group are still not formal-operational thinkers. Their average Piaget-level is concrete-operational.

Correlations

To determine if there are any variables correlated with either pretest level of cognitive functioning or amount of change, a number of correlation coefficients were computed. First (a) the correlation between the pretest score for the Equilibrium in the Balance test and each of the following variables was computed:

a)	Grade received in the treatment	(GRADE)
b)	Total points received in the treatment	(TOTPTS)
c)	Final exam points	(FINAL)
d)	Points received on final exam questions requiring application of a problem-solving strategy	(OBJPTS)
e)	High school grade point average	(HSGPA)
f)	First term MSU grade point average	(MSUGPA)
g)	MSU arithmetic placement score	(ARITHP)
h)	MSU mathematics placement score	(MTHPLC)
i)	ACT or SAT mathematics percentile	(MTHPCT)
j)	ACT or SAT verbal percentile	(VRBPCT)
k)	ACT or SAT composite percentile	(CMPPCT)
l)	Grade received in MSU Math Course(s)	(MTHGRD)

Subsequently, each variable was correlated with (b) the pretest score for the Pendulum Problem, (c) the Equilibrium in the Balance change score (posttest minus pretest), and (d) the Pendulum Problem change score.

All correlations involving pretest data from the Balance and Pendulum Problems were computed on the combined experimental and control groups in order to increase the size of N and provide for a more stable and reliable correlation. This procedure is valid due to the random assignment of subjects to groups, which insures similarity in performance between groups. The change score

correlations were computed separately for each group because of the possible effects of the intervening treatment on the experimental group.

The only exception to using both groups for pretest correlations occurred for variables (a) through (d), for which only the experimental group was used. These variables measure performance in the treatment course which was taken by the experimental group only.

Correlations were not computed for posttest scores, since they provide no information about either level of cognitive functioning upon entry into the university or amount of cognitive change after one term, which are the major areas of interest of this study. Therefore, posttest correlations were omitted.

Correlations between Pretest Scores for the Experimental Group and Performance in the Treatment

Hypotheses 3-10 specify the possible correlations between pretest scores and four variables measuring performance in the treatment course: grade received (a), total points earned (b), score on the final exam (c), and ability to apply problem-solving strategies on the final exam (d). Table 15 reports these correlations and their corresponding probability (p) values for the two tests used.

None of the eight correlations are significant for $p \leq .05$, indicating that no relationships exist

Table 15

Pearson Product-Moment Correlations: Ability Measure by Pretest

Ability Measure	Equilibrium in the Balance			Pendulum Problem		
	Hypothesis Number	r	p<	Hypothesis Number	r	p<
			Experimental Only			
Course Grade	3	0.35	0.09**	7	-0.13	0.31
Course Total Points	4	0.38	0.07**	8	-0.04	0.45
Course Final Exam	5	0.40	0.07**	9	-0.06	0.41
Final-Strategy Application	6	0.36	0.08**	10	-0.02	0.47
			Both Groups			
High School Grade Point Average	11	0.37	0.03*	19	0.16	0.21
MSU Grade Point Average	12	0.40	0.02*	20	0.17	0.19
Arithmetic Placement	13	0.46	0.01*	21	0.06	0.38
Mathematics Placement	14	0.52	0.01*	22	0.23	0.13
ACT/SAT Math Percentile	15	0.43	0.01*	23	0.14	0.24
ACT/SAT Verbal Percentile	16	0.06	0.38	24	0.15	0.23
ACT/SAT Composite Percentile	17	0.26	0.10**	25	0.14	0.24
Math Course Grade	18	0.32	0.06**	26	0.06	0.39

Note. N = 27 Both Groups; N = 16 Experimental.

*significant for $p \leq .05$

**significant for $p \leq .10$

between abstract reasoning ability and performance in the course as measured by these four variables (see Table 15). It should be noted, however, that for $p \leq .10$, the Equilibrium in the Balance test is significantly correlated with each of the four variables measuring course performance. Although these correlations are not large, the general result for $.05 < p \leq .10$ is that the higher the level of abstract reasoning upon entry into the course (as measured by the Equilibrium in the Balance test only), then the higher the course grade, the higher the total points earned, the higher the final exam score, and the better the ability to apply problem-solving strategies on the final. For $p \leq .10$ no correlations were significant for the Pendulum Problem.

Correlations between Pretest Scores (Both Groups Combined) and Variables Measuring Academic Ability

Null Hypotheses 11-26 state there is no significant correlation between pretest scores for both groups combined and eight different academic ability variables: high school grade point average (e), first-term MSU grade point average (f), arithmetic placement score (g), mathematics placement score (h), ACT/SAT mathematics percentile (i), ACT/SAT verbal percentile (j), ACT/SAT composite score (k), and grade in math course taken (l).

The null form is rejected at $p \leq .05$ for Hypotheses 11-15 and not rejected for Hypotheses 16-26 using the Equilibrium in the Balance measure (see Table 15). Thus, there are significant positive correlations between pretest level of abstract reasoning using the Equilibrium in the Balance test and high school grade point average, first-term MSU grade point average, the arithmetic placement score, the mathematics placement score, and the ACT or SAT Mathematics percentile. There are no significant correlations between Equilibrium in the Balance pretest scores and the ACT or SAT verbal percentile, composite percentile, or grade received in math course taken. In addition no correlation is significant when the Pendulum Problem was the measure used.

These results indicate that the higher the high school grade point, the higher the MSU first-term grade point, the higher the arithmetic or mathematics placement score, or the higher the ACT/SAT mathematics percentile, the higher the subject's level of abstract reasoning as measured by the Equilibrium in the Balance pretest.

In addition, the composite ACT/SAT percentile and the grade received in math course taken are significantly correlated with the Balance test for $p < .10$ and $p < .06$ respectively, but not for the criterion level of $p \leq .05$.

Correlations between Change Scores for Experimental Group and Performance in the Treatment Course

No correlation between change scores and the four variables measuring course performance is significant for either measure (see Table 16). Since this indicates there is no relationship between success in the course and how much abstract reasoning changed, null Hypotheses 27-34 are not rejected for $p \leq .05$. However, for $p \leq .10$ there is a significant positive correlation ($r = .34$) between the grade received in the course and the amount of change in abstract reasoning as measured by the Pendulum Problem.

Correlations between Change in Abstract Reasoning for the Experimental Group or Control Group Separately, and Variables Measuring Academic Ability

For the experimental group, null Hypotheses 35-42 state that the correlations between change in abstract reasoning as measured by the Equilibrium in the Balance test and each of the eight academic ability variables (e through f) are not significant. Null Hypotheses 51-58 specify that the correlations between these variables and amount of change as measured by the Pendulum Problem are also not significant. None of these hypotheses are rejected for $p \leq .05$, except for Hypothesis 40, indicating that the amount of change in abstract reasoning as measured by the Equilibrium in the Balance test is

Table 16

Pearson Product-Moment Correlations: Ability Measure by Change Score

Ability Measure	Name	Equilibrium in the Balance						Pendulum					
		Group						Group					
		Experimental			Control			Experimental			Control		
		Hypothesis	r	p<	Hypothesis	r	p<	Hypothesis	r	p<	Hypothesis	r	p<
Course Grade	Grade	27	-.18	.25	-	-	-	31	.34	.10**	-	-	-
Course Total Points	TOTPTS	28	-.23	.19	-	-	-	32	.21	.22	-	-	-
Course Final Exam	Final	29	-.27	.16	-	-	-	33	.17	.26	-	-	-
Final-Strategy Application	OBJPTS	30	-.09	.38	-	-	-	34	.20	.23	-	-	-
High School Grade Point Average	HSGPA	35	.03	.46	43	.10	.38	51	-.03	.46	59	-.16	.32
MSU Grade Point Average	MSUGPA	36	-.21	.22	44	-.69	.01*	52	.14	.31	60	-.36	.14
Arithmetic Placement	ARITHP	37	-.15	.28	45	.18	.30	53	.28	.15	61	.12	.36
Mathematics Placement	MTHPLC	38	-.22	.20	46	.30	.19	54	-.21	.22	62	-.04	.45
ACT/SAT Math Percentile	MTHPCT	39	-.15	.29	47	.50	.06**	55	.03	.45	63	.14	.34
ACT/SAT Verbal Percentile	VRBPCT	40	.50	.02*	48	.33	.16	56	.10	.36	64	-.004	.50
ACT/SAT Composite Percentile	CMPPCT	41	.13	.31	49	.43	.10**	57	-.02	.48	65	.11	.38
Math Course Grade	MTHGRD	42	-.19	.25	50	-.70	.01*	58	.11	.34	66	-.09	.40

*significant for $p \leq .05$

**significant for $p \leq .10$

positively correlated with ACT/SAT verbal percentile ($r = .50$, $p < .02$). See Table 16 for a summary.

For the control group, null Hypotheses 43-50 specify nonsignificant correlations between change in abstract reasoning as measured by the Balance test and the eight academic ability variables; null Hypotheses 59-66 specify nonsignificant correlations between change in abstract reasoning as measured by the Pendulum Problem and these same variables. Of these, only null Hypotheses 44 and 50 are rejected for $p \leq .05$: the amount of change as measured by the Balance test is negatively correlated with first-term MSU grade point average ($r = -.69$), significant at $p < .01$, and also negatively correlated with grade in math course taken ($r = -.70$), significant at $p < .01$. No other correlations for either measure are significant. Thus, for the control group, the greater the amount of change according to the Balance test, the lower the MSU grade point average or the lower the grade in math course taken.

Although not significant for $p \leq .05$, the amount of change in abstract reasoning measured by the Equilibrium in the Balance test is positively correlated with ACT/SAT mathematics percentile ($r = .50$), significant at $p < .06$, and ACT/SAT composite percentile ($r = .43$), significant at $p < .10$.

Summary of Correlations

Of the 64 correlations computed, 8 are significant for $p \leq .05$ (Hypotheses 11, 12, 13, 14, 15, 40, 44, and 50); an additional 9 are significant for $p \leq .10$ (Hypotheses 3, 4, 5, 6, 17, 18, 31, 47, and 49).

The correlations significant for $p \leq .05$ are as follows: for both groups combined, the greater the high school grade point average, the greater the first-term MSU grade point average, the greater the arithmetic and mathematics placement scores, and the higher the ACT/SAT mathematics percentile, then the higher the level of abstract reasoning on the Balance pretest. Also, for the experimental group, the higher the ACT/SAT verbal percentile, the more the change in abstract reasoning as measured by the Balance test. However, for the control group, the higher the MSU grade point average or the higher the math grade, the less the amount of change in abstract reasoning on the Balance test.

For those correlations significant for $p \leq .10$ but not for $p \leq .05$, it was found that the higher the level of abstract reasoning on the Balance pretest, the better the performance in the course according to all four measures. In addition, for both groups combined, the higher the ACT/SAT composite score or the higher the math grade, the higher the level of abstract reasoning on the Balance pretest. For the experimental group only, the greater the amount of change in abstract reasoning as measured by the Pendulum Problem, the higher the course grade. For the control group only, the correlations between ACT/SAT mathematics percentile or ACT/SAT composite percentile and amount of change on the Balance test are positive.

Analysis of Variance

The last part of the analysis of the data (Hypotheses 67-82) addresses the question of whether there is an interaction between treatment (control or experimental) and level of academic ability (High or Low). Separate analyses were conducted for each of the following academic ability variables: high school grade point average (e), MSU first-term grade point average (f), arithmetic placement score (g), mathematics placement score (h), ACT/SAT mathematics percentile (i), ACT/SAT verbal percentile (j), ACT/SAT composite percentile (k), and grade in math course taken (l). A significant interaction effect indicates that the mean change in abstract reasoning scores for at least two of the following groups are significantly different: (a) experimental-high, (b) experimental-low, (c) control-high, and (d) control-low.

Analysis of the data reveals that none of the hypotheses are rejected for $p \leq .05$, indicating that mean change scores are not significantly different for the four groups. Table 17 summarizes these results per hypothesis.

Secondary analyses were conducted to determine if any of the high ability groups had significantly different change scores from any of the corresponding low ability groups. The data were divided into either the high or the low group without regard to membership in the experimental or control group. The only significant difference in change in abstract reasoning between high and low groups occurs for ACT/SAT verbal percentile for the Equilibrium in the Balance test. For this variable, $F = 4.81$ for $p < .04$. Thus, the mean change in abstract reasoning for those who scored high on the ACT/SAT verbal percentile (0.62) is significantly greater than the mean change for those who scored low (0.07).

For $p \leq .10$ the mean change in abstract reasoning is greater for the low group for the variable MSU grade point average (low = 0.57, high = 0.08) on the Equilibrium in the Balance Problem ($F = 3.94$, $p < .06$).

Two addition descriptions are helpful in synthesizing these data. First, it is interesting to observe whether the high or low ability group has the greater change in abstract reasoning score, irrespective of membership in the control or experimental group. Column 3 in Table 17 reports which group has the

Table 17

F-Values for Analysis of Variance Using Change Scores: Treatment by Ability Interaction

Equilibrium in the Balance

Ability Measure	Combined Groups						Experimental			Control			Statistic	
	Hyp.	Low	High	>	F	p<	Low	High	>	Low	High	>	F	p<
HSGPA	67	.43	.23	LO	0.52	.48	.44	.29	LO	.40	.17	LO	0.02	.90
MSUGPA	68	.57	.08	LO	3.94	.06*	.50	.25	LO	.67	-.20	LO	1.47	.24
ARITHP	69	.50	.15	LO	1.69	.21	.50	.25	LO	.50	.00	LO	0.21	.65
MTHPLC	70	.38	.29	LO	0.14	.71	.67	.20	LO	.14	.50	HI	2.19	.15
MTHPCT	71	.36	.31	LO	0.03	.86	.57	.22	LO	.14	.50	HI	1.59	.22
VRBPCT	72	.07	.62	HI	4.81	.04**	.00	.75	HI	.17	.40	HI	1.04	.32
CMPPCT	73	.29	.38	HI	0.13	.72	.33	.40	HI	.25	.33	HI	0.001	.98
MTHGRD	74	.38	.08	LO	1.63	.22	.38	.29	LO	.40	-.20	LO	1.12	.30

Pendulum Problem

Ability Measure	Combined Groups						Experimental			Control			Statistic	
	Hyp.	Low	High	>	F	p<	Low	High	>	Low	High	>	F	p<
HSGPA	75	.54	.15	LO	0.66	.42	.56	.29	LO	.50	.00	LO	0.06	.81
MSUGPA	76	.32	.38	HI	0.02	.90	.31	.56	HI	.33	.10	LO	0.25	.62
ARITHP	77	.29	.42	HI	0.09	.77	.25	.63	HI	.33	.10	LO	0.40	.53
MTHPLC	78	.42	.29	LO	0.09	.77	.20	.07	LO	.07	.50	HI	1.19	.29
MTHPCT	79	.39	.31	LO	0.03	.86	.22	.07	LO	.07	.50	HI	0.89	.35
VRBPCT	80	.00	.73	HI	2.68	.12	.94	.08	HI	.08	.40	HI	0.56	.46
CMPPCT	81	.18	.54	HI	0.59	.45	.50	.06	HI	.06	.67	HI	0.18	.68
MTHGRD	82	.15	.46	HI	0.41	.53	.42	-.40	LO	-.40	.50	HI	1.01	.33

Note. HSGPA = High School Grade Point Average; MSUGPA = MSU Grade Point Average; ARITHP = Arithmetic Placement Score; MTHPLC = Mathematics Placement Score; MTHPCT = ACT/SAT Mathematics Percentile; VRBPCT = ACT/SAT Verbal Percentile; CMPPCT = ACT/SAT Composite Percentile; MTHGRD = Math Course Grade; > = Group with greater change score.

*significant for $p \leq .10$

**significant for $p \leq .05$

higher change score. It indicates that the low group has a higher mean change score six out of eight times on the Equilibrium in the Balance test and only three out of eight times on the Pendulum Problem.

The second descriptive summary that is useful is to determine which group, high or low, has the greater change when subjects are divided by experimental and control group (columns 4-9, Table 17). On the Equilibrium in the Balance test, the experimental group-low scores are greater than the experimental group-high scores six out of eight times. The control group-low scores are greater than the control group-high scores only four out of eight times. For the Pendulum Problem, the experimental group-low subjects score higher four out of eight times, while the control group-low subjects score higher only three out of eight times. These data are summarized in Table 18.

Table 18

Number of Ability Measures out of Eight for Which Indicated Group's Change Score was Greater

Group	Measure			
	Equilibrium in the Balance		Pendulum	
	Low	High	Low	High
Control	4	4	3	5
Experimental	6	2	4	4
Both	6	2	3	5

Summary

The analysis of the data addressed three major issues:

1. Is the treatment effective in increasing the abstract reasoning of experimental group subjects?

2. What are the correlations between either pretest scores or change scores and various ability variables?

3. Is there an interaction between treatment and ability level?

The results of the analysis of the effect of the treatment were divided into two parts: statistical results and descriptive results. The statistical results indicate that (a) the change in abstract reasoning for the experimental group as measured by both the Equilibrium in the Balance test and the Pendulum Problem is not significantly different from the change in abstract reasoning for the control group; (b) for both measures, the posttest level of abstract reasoning for the control group is not significantly greater than its pretest level, but that (c) the posttest level of abstract reasoning for the experimental group is significantly greater than its pretest level for the Balance test, but not significantly different for the Pendulum Problem.

The descriptive results, which report the magnitudes of differences between the two groups rather than

statistically significant differences, indicate that (a) the average pretest level of cognitive functioning for both groups is concrete-operational, (b) the average posttest measure is higher than the pretest measure for both groups and for both tests, but it remains below the formal-operational level, (c) the change from pre- to posttest is greater for the experimental group on both tests used, (d) the percentage of students who are not formal-operational decreases more from pretest to post-test for the experimental group according to both tests, and finally (e) a larger percentage of students in the experimental group increase in level of abstract reasoning from pre- to posttest on both measures used.

The results of the pretest correlations show that high school grade point average, MSU first-term grade point average, arithmetic placement score, mathematics placement score, and ACT/SAT mathematics percentile are all positively correlated with the Equilibrium in the Balance pretest and are significant for $p \leq .05$.

The significant correlations for the change score are ACT/SAT verbal percentile for the experimental group ($r = .50$), MSU grade point for the control group ($r = -.69$), and grade received in mathematics course taken for the control group ($r = -.70$). These are all significant for $p \leq .05$ for the change score as measured by the Equilibrium in the Balance test.

The analysis of variance results indicate that there is no significant treatment-by-ability-level interaction, indicating that the mean change in abstract reasoning is not different for ability level by group. Irrespective of group, however, students with high ACT/SAT verbal percentiles change significantly more than those with low percentiles for the Equilibrium in the Balance test.

Chapter V will summarize this study, discuss its limitations, and offer recommendations for further research.

CHAPTER V

SUMMARY AND CONCLUSIONS

In this chapter the purpose and need for this research are summarized, the limitations of this study are discussed, the conclusions drawn from the data analysis are presented, and implications for further research are outlined.

Purpose and Need for the Study

The purpose of this study is to determine if a course, "Introduction to Reasoning and Problem Solving," increases the Piagetian-defined level of abstract reasoning of a group of first-term minority engineering students. The design of the course is based upon implications for instruction derived from Piaget's work. The major assumption underlying this research is that it is necessary for students to function at the highest levels of abstract reasoning in order to succeed in engineering. However, the literature clearly documents that a surprisingly high percentage of college students do not meet this criterion. In addition, limited work has been conducted to describe effective instructional strategies for increasing abstract reasoning.

Limitations of the Study

The limitations of this study are: (a) the mortality of subjects resulting in the small sample size, (b) instrumentation biases, (c) the lack of data concerning the reliability of the tests used to assess formal-operational thinking, and (d) the use of one-item tests.

Mortality of Subjects

Of the 60 students in the original sample, 27 completed all pre- and posttesting. The mortality of subjects was due to (a) students who did not attend MSU during Fall term (N=9), (b) students who had scheduling problems (N=4), (c) those who declined participation (N=4), and (d) those who did not complete all pre- or posttesting (N=16). The resulting sample size of 27 students is small and limits the scope of this study.

In retrospect, it would have been difficult to decrease the magnitude of the mortality of subjects. However, if the random assignment to the experimental course had been delayed until the day before classes began, the nine students who did not enroll for Fall could have been excluded entirely. In addition, this procedure would have enabled the researcher to conduct only one random assignment of subjects to group, instead of the procedure used of assigning students randomly before summer orientation, then alternating new students between groups as they appeared at orientation during the summer. Conducting only one random assignment would have introduced administrative problems for registration, but it would have provided a "cleaner" procedure.

Decreasing the mortality due to incomplete testing would also have been difficult, but a more persistent follow-up of students would have helped.

Although it was shown in Chapter III that there were no statistical differences between groups before mortality or after mortality on any of six variables, the mortality of subjects must be considered a limitation of this study because of the resultant small sample size of 27 students.

Instrumentation Biases

Biases due to instrumentation procedures are the second limitation of this study. Because the researcher conducted and interpreted all pre- and posttesting with knowledge of which subjects belonged to which group, there is a possibility that the procedure caused either over- or under-compensation for the experimental group, thus biasing the data.

The obvious means of avoiding this difficulty is to employ a person to administer and interpret all tests. This person should not be otherwise involved in the study and should not know to which group each subject belonged. This procedure was not used because of the lack of trained personnel to devote 40-60 hours testing students. Thus, the experimenter's involvement in the testing procedures could have biased the results and is a limitation of this study.

Reliability

The testing of students to assess level of abstract reasoning did not include a second scorer, whereby estimates of the inter-rater reliability of the tests used could be determined. The second scorer was not used because of the lack of trained personnel and money. Additionally, no test reliability data are available due to the use of one-item tests. None the less, since reliability information is unavailable, a limitation of this study is that it is not known if the tests are a reliable measure of abstract reasoning, nor if the rater was reliable in scoring.

Use of One-Item Tests

Both the Equilibrium in the Balance test and the Pendulum Problem can be considered one-item tests, that is, each presents only one problem for the student to solve. Students are classified as formal or pre-formal thinkers on the basis of their solution to the one problem. Generally, one-item tests are not sufficient to assess the presence or absence of such a complex construct as formal reasoning; one-item tests also tend to reduce reliability. Therefore, the one-item tests used in this study limit the scope of the interpretations of the results. Conclusions can only be specified with respect to the test used and should not be generalized to refer to the overall construct of formal-operational reasoning.

Unfortunately, acceptable alternatives to the one-item tests are not available. It is possible to use more than a single one-item test and sum the scores on each test. Many researchers employ this technique to avoid one-item test limitations. This is, however, questionable procedure and was not used in this study because summed scores are difficult to interpret. For example, a summed score of six from two one-item tests could be obtained by a score of three on each test or by a score of two on one test and four on the other. In the first case, a score of three on each test corresponds to the first level of formal-operational thought and is not difficult to interpret; the scores of two and four, on the other hand, refer to concrete-operational thought and formal thought, respectively. It is unclear how the summed score of six should be interpreted in the later case. Thus, summed scores are ambiguous and are not used in this study.

Although not a limitation of the study, it should be noted that the experimental design did not control for the possible contaminating effects of self-paced instruction. It was pointed out in Chapter I that there are a number of features of self-paced instruction that are theorized to facilitate the development of abstract reasoning (it is superior to the lecture method in terms of final examination performance, it increases the probability of occurrence of social interaction and transmission, and it forces students to be active rather than passive learners). These features were evident in the treatment course. However, students from both the experimental and control groups were taking as many as two additional self-paced courses. The possible contaminating effects of the other self-paced courses are of course unknown. Theoretically, an alternative experimental design could have been used by introducing a second control group that was neither taking the problem-solving course nor any other self-paced course. This would have provided a comparison that eliminated the self-paced effect. However, in reality this was not possible as the mathematics courses required of everyone were self-paced. Thus, the possible effects of self-paced instruction upon abstract reasoning must be left for further research.

In conclusion, the limitations of this study are the mortality of subjects, instrumentation biases, lack of reliability information, and the use of one-item tests. Further, because of the experimental design used, a possible contaminating effect due to the self-paced mode of instruction was introduced.

Conclusions

The conclusions of this study will be presented in five parts:

1. For the students in this study, what is the average level of abstract reasoning upon entry into the university?

2. What are the results of the statistical analysis of the data to determine the effect of the treatment upon abstract reasoning?

3. What are the results of the descriptive analysis of the effects of the treatment?

4. Are there any correlations between various academic ability measures and either level of cognitive ability upon entry into the university or amount of change after receiving the treatment?

5. Are there any interaction effects between treatment and ability?

Levels of Abstract Reasoning Upon Entry into the University

The literature pointed out that from 42% to 51% of college students have not developed formal-operational thinking abilities. At least with respect to the students in this study, a much larger percentage was classified at levels below formal-operational upon entry into the university: for the Equilibrium in the Balance test, 85% of all subjects were not formal-operational thinkers, while on the Pendulum Problem, 89% were not formal-operational. In addition, when the average pretest Piaget-level for all 27 subjects is considered, the Equilibrium in the Balance test score is 1.93 and the Pendulum Problem score is 2.15. Both of these scores put the average level of cognitive ability at II-B, or the higher of the two concrete-operational levels. Thus, these results indicate that this sample is extremely ill-equipped to handle an engineering curriculum.

Effect of the Treatment--Statistical Results

The results of the analysis of the data indicate that when the amount of change in abstract reasoning is compared between groups, no statistically significant treatment effect is found. However, when the amount of change within each group is analyzed, the experimental group did change significantly from pretest to posttest on the Equilibrium in the Balance test but not on the Pendulum Problem. The control group did not change significantly on either test.

Effect of the Treatment--Descriptive Results

The descriptive analysis of the treatment reports magnitudes of change rather than statistically significant or nonsignificant changes. The magnitudes of all descriptive results are greater for the experimental group: (a) the amount of change in average level of abstract reasoning from pretest to posttest is greater for the experimental group on both tests used; (b) the decrease from pretest to posttest in percentage of students who are not formal-operational is greater for the experimental group, or conversely, the percentage increase from pretest to posttest of formal-operational students is greater for the experimental group; and (c) a larger percentage of the experimental subjects increased in level of abstract reasoning from pretest to posttest.

Correlations

Sixty-four correlations were computed to determine if there are any relationships between various academic ability variables and either level of abstract reasoning upon entry into the university or amount of change observed. It was found that high school grade point average, arithmetic placement score, mathematics placement score, and ACT/SAT mathematics percentile were all positively correlated with the Balance pretest (significant for $p \leq .05$). Thus, these scores could be used to predict level of cognitive ability upon entry as measured by the Balance test. First-term MSU grade point average was also positively correlated with the Balance pretest (significant for $p \leq .05$), but obviously cannot be used as a predictor of pre-first-term cognitive ability.

For the experimental group, ACT/SAT verbal percentile was significantly correlated in the positive direction with amount of change according to the Balance test and could be used as a predictor. Only MSU grade point average and grade in first-term math course were negatively correlated for the control group on the Balance test (significant at $p \leq .05$). Once again, these grades cannot be used for pre-term predictors. This negative correlation could be an artifact due to a ceiling effect: the better math students were also those who scored higher on the pretest; hence, the amount of change possible was limited by high pretest scores.

Interaction Effects--Treatment vs. Ability

There were no significant differences between treatment-by-ability means, indicating that there was no difference in change in abstract reasoning regardless of ability level or treatment. However, it was found that for those subjects who scored high on the ACT/SAT verbal percentile (irrespective of group), their mean change in abstract reasoning on the Balance test was significantly greater than the mean change for those who scored low.

One possible explanation for the lack of differences in the four means (experimental-high, experimental-low, control-high, and control-low) is that, in general, the ability measures on these students were low to begin with. Thus, splitting the already low ability groups into high and low subgroups produced an artifact: the high students were not that much academically superior to the low students. Thus, it remains possible that for a group of students with a wider range in academic ability, a treatment-by-ability interaction might occur.

The significant difference between amount of change on the Balance test between high and low ACT/SAT verbal percentile subgroups is surprising in light of the fact that for all of the other ability measures, change scores between the high group and the low group were not significantly different. It is possible that since most classroom learning is dependent upon the ability to read and comprehend the written word, that those students with the high verbal percentiles were better able to assimilate new information into existing schema and thus had a greater probability for cognitive change. Therefore, their change scores were significantly higher than the change scores of those who were less able to read and comprehend. It is suggested that an area for further study address possible changes in abstract reasoning between high and low verbal ability students.

Implications for Further Research and Instruction

Recommendations for further research and instruction focus upon three issues: (a) changes in the instructional format, (b) changes in the research design, and (c) questions left unanswered by this study.

Changes in the Instructional Format

Changes in the instructional format of the problem-solving course should be explored. Based on McKinnon's (1970) findings that inquiry-oriented instruction facilitates the development of abstract reasoning, an attempt should be made to introduce inquiry-oriented sessions into the course. Although the course is self-paced, one class hour per week could be used as an instructor-lead inquiry session, in which the instructor could utilize inquiry techniques to assist students in discovering processes of solving different classes of problems. Not only would students discover solutions to problems, but they would also see how others attempt to solve them. In the event of a correct solution, students would have a model to follow; for incorrect

procedures students could see where the errors were and possibly receive feedback on their own methods of attack.

A second change based upon Renner and Lawson's (1975) findings would be to introduce laboratory-type "hands-on" experiences. Many of the problems in the course are the kind that can actually be worked as an experiment, providing the student with the opportunity to handle and manipulate concrete rather than hypothetical or verbal problems.

A third change would be to add to the text more intermittent practice frames with immediate feedback, increasing the amount of time students spend practicing the skills that are descriptive of the formal stage of reasoning (e.g., hypothesis building and testing, "If . . ., then . . ., therefore" reasoning, etc.). The immediate feedback would function as a catalyst to create the disequilibrium necessary for cognitive growth to occur.

It is suggested that inquiry sessions, "hands-on" experiences, and additional practice frames with feedback should be included in the course.

Changes in the Research Design

The major question that was left unanswered by this study concerns the possible effects self-paced instruction has upon the development of abstract

reasoning. To research this issue, it is suggested that an analysis of variance design be used with four groups: (a) students taking the problem-solving course self-paced, (b) students taking the problem-solving course by the lecture method, (c) students not taking the problem-solving course but one other self-paced course, and (d) students taking neither the problem-solving course nor a self-paced course. This type of design would provide information about the change in abstract reasoning between all combinations of these four groups. Conclusions could be drawn about the effects of problem-solving instruction and the self-paced format.

Questions Left Unanswered

An additional question that was not addressed in this study concerns the general rate of cognitive development and whether or not university instruction in general has an impact. For example, it was noted in this study that the magnitude of the mean Piaget-level increased for both the control and experimental group on both tests from the pre- to post-measures. The question to be answered is whether or not university students change any more or any less than a comparable group of subjects not enrolled in a university. Answers to this question would provide information concerning the impact of university instruction in general on the development of abstract reasoning.

A second issue that should be addressed concerns the low correlations between scores on the Equilibrium in the Balance test and the Pendulum Problem. The correlation between the two tests is 0.27 for the pretest scores and 0.11 for the posttest scores. Since both tests do in fact measure formal-operational reasoning, it is expected that each correlation would approach one. Since the correlations do not approach one, it is suggested that students may not have completely understood all terminology used in the tests. Without understanding the problem to be solved, it is doubtful that it would have been solved, resulting in classifications of less than the formal category. It is suggested that research be conducted to determine if this in fact was the case.

Another area that should be addressed in more detail is the interaction effects between treatment and ability. As was pointed out previously, a possible reason for the lack of differences between the mean abstract reasoning changes between treatment by ability groups is that the average score on all ability measures was relatively low. Thus, dividing an already low group into high and low subgroups did not produce any real contrast between high and low ability students, as they were all "low." Further research should address the issue of treatment-by-ability interactions with groups that range over the entire continuum of academic ability, rather than only considering the lowest percentiles.

The last question that needs to be answered concerns the measurement of abstract reasoning. Because Piaget specified one-item tests to use in assessing abstract reasoning, traditional means of assessing the reliability of the tests are not adequate as they depend upon utilizing multiple tests that are equivalent. It is not clear at the present time that all Piaget tests are equivalent for reliability purposes. In fact, the correlations between the two tests used in this study were low. Although this low correlation could have been due to students not completely understanding the problem, it could also have been due to the tests measuring different aspects of the construct formal reasoning.

Thus, additional research needs to be conducted on the reliability of the Piaget tests and the equivalency of various tests.

Speculations

To some extent the results of this study are disappointing--there is only slight evidence that the problem-solving course provided a catalyst for intellectual development. Aside from the careful statistical analysis presented here, the author's personal reaction to the effects of the course are mixed. On one hand, some students were never able to transcend their

concrete-operational mode of thinking. To them the techniques of problem solving were something to be memorized and strategies remained foreign to their mode of thinking. On the other hand, some students did appear to be able to internalize the various techniques and in the process were able to accept and experiment with hypothetical and abstract thinking.

One can speculate as to the reasons why some students changed and others did not. On a purely intuitive level, the author feels that the major differentiating factor between those students who changed and those who did not was the extent to which students had mastered the abilities that define concrete-operational thinking (decenters perceptions, attends to transformations, reverses operations, etc.). The more of these abilities that were mastered, the greater the probability that change occurred; conversely, students who had not mastered sufficient prerequisite abilities did not change. In short, the course did not remediate for insufficient knowledge of the prerequisite abilities from the concrete-operational stage.

The obvious course of action required to remedy this situation is that a diagnostic system needs to be developed that not only identifies at which level a student is, but also specifies what skills within the level the student has mastered. Once students are specifically diagnosed, instruction can be prescribed specific to the diagnosis.

Summary

The limitations of this study are the mortality of subjects resulting in a limited sample size, instrumentation biases, the lack of information about the reliability of the tests and scores, and the use of one-item tests. Keeping these in mind, the major conclusions of this study are:

1. Upon entry into the University, an extremely high percentage of the minority engineering students in this sample are functioning below the formal-operational level. Thus, these students are not equipped with the abstract reasoning capabilities needed for success in engineering. Eighty-five percent are pre-formal-operational on the Balance test and 89% are pre-formal-operational on the Pendulum Problem.

2. For the Equilibrium in the Balance test, the following findings apply:

   (a) the change in abstract reasoning from pretest to posttest is not significantly greater for the experimental group as compared to the control group;

   (b) the posttest level of abstract reasoning is not significantly greater than the pretest level of abstract reasoning for the control group; but

   (c) the posttest level of abstract reasoning is significantly greater than the pretest level of abstract reasoning for the experimental group.

3. For the Pendulum Problem, the following apply:

(a) the change in abstract reasoning from pretest to posttest is not significantly greater for the experimental group as compared to the control group;

(b) the posttest level of abstract reasoning is not significantly greater than the pretest level of abstract reasoning for the control group; and

(c) the posttest level of abstract reasoning is not significantly greater than the pretest level of abstract reasoning for the experimental group.

4. Descriptive data indicate that:

(a) the change in average level of abstract reasoning from pretest to posttest is greater for the experimental group on both tests used;

(b) the decrease from pretest to posttest in percentage of students who are not formal-operational is greater for the experimental group; and

(c) a larger percentage of the experimental subjects increase in level of abstract reasoning from pretest to posttest.

5. The significant correlations for academic ability variables with the Equilibrium in the Balance test are:

(a) high school grade point average, arithmetic and mathematics placement scores, and ACT/SAT mathematics percentile are all positively correlated with entry-level abstract reasoning ability;

(b) ACT/SAT verbal percentile is positively correlated with amount of change in abstract reasoning for the experimental group;

(c) entry-level abstract reasoning ability is positively correlated with first-term MSU grade point average;

(d) MSU grade point average and grade in first-term mathematics course are both negatively correlated with amount of change in abstract reasoning for the control group. (As was pointed out, this may be due to a ceiling effect on the better students.)

6. There are no significant correlations for academic ability variables with the Pendulum Problem.

7. There are no treatment-by-ability interaction effects; the amount of change in abstract reasoning is not different for high or low ability students within or between groups.

8. High ability students on the ACT/SAT verbal percentile change significantly more than low ability students.

A number of suggestions have been presented to improve the course, to improve the research design, and to answer questions left unanswered by this research. When implemented, these recommendations may provide a more definitive understanding of changes in abstract reasoning.

REFERENCES

REFERENCES

Alfred P. Sloan Foundation. *Minorities in engineering: A blueprint for action*. New York: Author, 1974.

American Association of Physics Teachers. Workshop on physics teaching and the development of reasoning. Unpublished, 1975.

Anastasi, A. *Psychological testing* (3rd ed.). London: MacMillan, 1968.

Beistel, D. W. A Piagetian approach to general chemistry. *Journal of Chemical Education*, 1975, *52*(3), 151-152.

Campbell, D. T., & Stanley, J. C. *Experimental and quasi-experimental design for research*. Chicago: Rand McNally, 1963.

College of Engineering, Michigan State University. *An experimental engineering student development project for ethnic minority students*. Proposal to the Alfred P. Sloan Foundation, E. Lansing, MI, August 1974.

Committee on Minorities in Engineering. *Building effective minority programs in engineering education*. Washington, D.C., 1975.

Cross, K. P. *Accent on learning*. San Francisco: Jossey-Bass, Pub., 1976.

Davis, R. H., Alexander, L. T., & Yelon, S. L. *Learning system design*. New York: McGraw-Hill, 1974.

Elkind, D. Quantity concepts in college students. *Journal of Social Psychology*, 1962, *57*, 459-465.

Green, D. R., Ford, M. P., & Flamer, G. B. (Eds.). *Measurement and Piaget*. New York: McGraw-Hill, 1971.

Griffiths, D. H. The study of the cognitive development of science students in introductory level courses. Unpublished doctoral dissertation, Rutgers University, 1973.

Herron, J. Piaget for chemists--explaining what "good" students cannot understand. Journal of Chemical Education, 1975, 52(3), 146-150.

Higgins-Trenk & Gaite, A. J. H. Elusiveness of formal operational thought in adolescents. Proceedings of the 79-th Annual Convention of the American Psychological Association, 1971, 6, 201-202.

Inhelder, B., & Piaget, J. The growth of logical thinking from childhood to adolescence. New York: Basic Books, 1958.

Karplus, F., & Karplus, R. Intellectual development beyond elementary school I: Deductive logic. School Science and Mathematics, 1970, 70, 398-406.

Karplus, R., & Peterson, R. W. Intellectual development beyond elementary school II: Ratio, a survey. School Science and Mathematics, 1970, 70, 813-820.

Kohlberg, L., & Gilligan, C. The adolescent as a philosopher: The discovery of self in a post-conventional world. Daedalus, 1971, 100, 1051-1084.

Kolodiy, G. Piagetian theory and college science teaching. Journal of College Science Teaching, 1974, 261-262.

Kulik, J. A. PSI: A formative evaluation. Paper presented at the National Conference on Personalized Instruction in Higher Education, Los Angeles, March 1975.

Lawson, A. E., & Renner, J. W. A quantitative analysis of responses to Piagetian tasks and its implications for curriculum. Science Education, 1974, 58, 545-559.

McKinnon, J. W. The influence of a college inquiry-centered course in science on student entry into the formal operational stage. Unpublished doctoral dissertation, University of Oklahoma, 1970.

McKinnon, J. W., & Renner, J. W. Are colleges concerned with intellectual development? American Journal of Physics, 1971, 39, 1047-1052.

Piaget, J. Judgment and reasoning in the child. New York: The Humanities Press, 1952.

Piaget, J., & Inhelder, B. The psychology of the child. New York: Basic Books, 1969.

Renner, J. W. Personal phone conversation. August 20, 1975.

Renner, J. W. Piaget theory and instruction in physics. The Physics Teacher, 1973, 11, 165-169.

Renner, J. W. Promoting intellectual development through science teaching. The Physics Teacher, 1973, 1.

Renner, J. W., & Lawson, A. E. Intellectual development in pre-service elementary school teachers: An evaluation. Unpublished, 1975.

Saarni, C. I. Piaget operations and field independence as factors in children's problem solving performance. Child Development, 1974, 45, 338-345.

Scheifley, V. M., & Schmidt, W. H. Jeremy D. Finn's multivariance--univariate and multivariate analysis of variance, covariance, and regression; modified and adapted for use on the CDC 6500. Occasional paper #22, Office of Research Consultation, College of Education, Michigan State University, October 1973.

Siegler, R. S., Liebert, D. E., & Liebert, R. M. Inhelder and Piaget's pendulum problem: Teaching pre-adolescents to act as scientists. Developmental Psychology, 1973, 9(1), 97-101.

Tanner, J. W., & Inhelder, B. (Eds.). Discussions on child development (Vol. 4). London: Tavistock, 1960.

The purpose and performance of higher education in the United States: Approaching the year 2000. Chronicle of Higher Education, June 4, 1973, p. 7.

Tower, J. O., & Wheatley, G. Conservation concepts in college students: A replication and critique. Journal of Genetic Psychology, 1971, 118, 265-270.

Wadsworth, B. J. Piaget's theory of cognitive development. New York: McKay Co., 1971.