

MATHEMATICAL UNDERSTANDINGS AND
MISCONCEPTIONS OF PROSPECTIVE
ELEMENTARY SCHOOL TEACHERS

Dissertation for the Degree of Ph. D.
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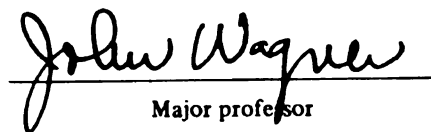


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MISCONCEPTIONS OF PROSPECTIVE
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Kilton Philip Eisner

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ABSTRACT

MATHEMATICAL UNDERSTANDING AND MISCONCEPTIONS OF PROSPECTIVE ELEMENTARY SCHOOL TEACHERS

By

Milton Philip

Mathematics 201, the one required undergraduate course for prospective elementary school teachers at Michigan State University, has been presented during four quarters over the last few years in a format consisting of three one-hour lectures and a two-hour laboratory period each week. This study was conducted at Michigan State in the winter quarter of 1974. Its purpose was to collect data which would suggest answers to the following two questions:

- (1) How do prospective elementary school teachers think about the mathematical concepts taught them in Mathematics 201?
- (2) How may instruction in this course, as it is offered at Michigan State University, be improved?

A sample of fifteen female students, chosen at random from volunteers, was interviewed every two weeks as they were progressing through Mathematics 201. (There were indications that the sample was mathematically superior to the class as a whole.) The interviews consisted of conversation about the

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course and problems which the subject was asked to solve aloud. The mathematical topics covered by the problems included sets, number bases, prime numbers, factors and multiples, rational numbers, decimals, and measurement; there were also three verbal problems loosely based on the course material. A profile of each subject was written, describing her feelings and her responses to the problems as she progressed through the course.

Significant findings included the following:

There was a widespread tendency among the subjects to confuse the ideas of matching (equivalence) and equality of sets. There also was a tendency to misuse the language of sets.

The subjects were generally proficient in converting a number expressed in a nondecimal base to base ten, and in recognizing the base of a worked-out example (though they found the latter easier in addition than in subtraction). They had some difficulty in translating a base ten numeral to a nondecimal base, particularly base two.

The subjects displayed a generally good understanding of the meaning of prime number. In testing numbers for primeness, there were tendencies either to try only a few numbers as possible divisors or to use inspection rationales. Few subjects knew that one need test only primes as divisors.

The subjects tended to take a mechanical, algorithmic approach when finding a least common multiple and greatest

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common factor; this suggests that they were unaware of the self-explanatory nature of the names of these concepts.

Subjects were quite proficient at finding a rational number between $1/3$ and $1/4$, and used various methods of doing so. They were generally able to find the decimal name for a fraction. In finding the fraction name for a repeating decimal, subjects generally knew the procedure but tended to make arithmetic errors.

A number of subjects had difficulty with the idea of square root, either drawing a blank when asked about it or thinking of square roots only in the context of the Pythagorean theorem.

Several subjects displayed the misconception that the area of a figure remains constant if its boundary is deformed.

Finally, the subjects as a group were deficient in their ability to read and understand verbal problems. In particular, they did not understand the relationship linking distance, rate, and time.

The study subjects tended to fall into four categories: (1) those deficient in arithmetic skills, who found themselves having to remedy these skills during the quarter, forcing them to neglect the course content to some extent; (2) those who had studied much mathematics earlier in their careers (usually four years in high school), and were bored by the course; (3) those who, though competent in arithmetic,

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bore a great hostility toward mathematics--the course did nothing to assuage this hostility and possibly even exacerbated it; and (4) the remainder of the class, made up mostly of students who had taken about three years of high school mathematics, earning average to above-average grades, and bearing a neutral or slightly positive attitude toward the subject, for whom the course functioned fairly well, frequently improving understanding and attitude. The investigator recommended that students in group (1) be excluded from the course until they have remedied their arithmetic deficiencies, that those in group (2) be offered an honors section which would hold their interest, and that the effects of alternative presentations of the course on group (3) be tested in further research.

Other recommendations made for improvement in the course included the incorporation of measurement and problem-solving into the subject matter of the course, rescheduling the two-hour laboratory session as two one-hour sessions, assigning only persons interested in the laboratory mode of instruction to teach laboratory sections, and various points of pedagogy.

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By

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The thesis committee, consisting of the following members and including Professors Robert H. Smith, John W. Smith, and Sonneborn, and Lauren Woodard, members of the committee, the investigator during the various stages of the study, which helped to shape the investigation into a research-oriented study.

The instructors of Mathematics, who have cooperated with the investigator at various stages of this study, include Professors Woodby, John Westerman, James Schultz, Glenda Lappan, and Mr. A DISSERTATION Special thanks are due to Professor Glenda Lappan for her gracious cooperation with the investigator. Submitted to study reported in this Michigan State University dissertation in partial fulfillment of the requirements of for the degree of Mathematics, Professor Joseph Adney, who helped to provide the study with financial DOCTOR OF PHILOSOPHY

The writer would like to express his appreciation to Department of Mathematics Professor Shlomo Libeskind of the University of Montana, who in his years at Michigan State took a strong personal

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The writer would like to express his appreciation to those who provided valuable assistance to him in the conduct of this research.

The thesis committee, headed by Professor John Wagner and including Professors Peter Lappan, William Mehrens, Lee Sonneborn, and Lauren Woodby, provided wise guidance to the investigator during the various stages of the research and helped to shape the investigation into a focused, coherent study.

The instructors of Mathematics 201 who cooperated with the investigator at various stages in this study include Professors Woodby, John Masterson, James Schultz, Glenda Lappan, and Mr. Stephen Snover. Special thanks are due to Professor Glenda Lappan for her gracious cooperation with the investigator during the main study reported in this dissertation, and to the chairman of the Department of Mathematics, Professor Joseph Adney, who helped to provide the study with financial support.

The writer would like to express his appreciation to Professor Shlomo Libeskind of the University of Montana, who in his years at Michigan State took a strong personal

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interest in the writer's studies and was largely responsible for his choice of a career in mathematics education.

Finally, the writer acknowledges his great debt to his wife, Gail, who provided substantial editorial and secretarial assistance in the preparation of this manuscript, and helped the writer through the difficult period while this research was being conducted.

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Many studies have been conducted to determine which mathematical topics are found difficult by pre-service and in-service elementary teachers. These studies will be summarized in Chapter 2. While these studies identified topics which elementary teachers do not understand, they were all based on test forms and therefore failed to describe how such teachers think about the various mathematical topics which they study.

Another factor motivating the present study was dissatisfaction among the mathematics education staff at Michigan State University with the required mathematics content course for prospective elementary school teachers. The situation involving this course will be described at the beginning of Chapter 4.

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CHAPTER 1

INTRODUCTION

The problem of the mathematical training of elementary school teachers has attracted much attention from the mathematics education community in recent years.

This attention is well justified because the influence of an elementary school teacher on her pupils' understanding of arithmetic and on their attitude toward mathematics is great.

Many studies have been conducted to determine which mathematical topics are found difficult by pre-service and in-service elementary teachers. These studies will be summarized in Chapter 2. While these studies identified topics which elementary teachers do not understand, they were all based on test forms and therefore failed to describe how such teachers think about the various mathematical topics which they study.

The format for the report of this study is as follows: Chapter 2 will summarize the literature on the mathematical understandings of elementary school teachers. Chapter 3 will review the literature on the use of the Michigan State University with the required mathematics content course for prospective elementary school teachers. Chapter 4 contains the report of the present study; it consists of a general description of the situation involving this course will be described at the beginning of Chapter 4.

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about Thus the study reported in this dissertation was designed to provide answers to the following questions:

on the (1) How do prospective elementary school teachers think about the mathematical concepts taught them in their required college mathematics course?

inferen (2) How do students react to the various features of the presentation of this course, and how might this presentation be improved?

for fu It was decided that the information needed to answer both questions could best be obtained from interviews with the prospective teachers themselves. (Literature on the use of the interview in research will be summarized in Chapter 3.) A sample of fifteen students was chosen in the winter of 1974 and interviewed every two weeks as they took their required mathematics course, Mathematics 201. The interviews consisted of conversation about the course and problems which the subjects were asked to solve aloud. In this manner data were obtained which provided some answers to both of the above questions.

The format for the report of this study is as follows: Chapter 2 will summarize the literature on the mathematical understandings of elementary school teachers. Chapter 3 will review the literature on the use of the interview in research. Chapter 4 contains the report of the present study; it consists of a general description of the situation leading to the investigation, a case study of each subject (describing in detail her reported thoughts

about the course and her attempts at problem solving), and finally a summary of how the subjects as a group performed on the problems presented to them. Chapter 5 contains an evaluation of the effects of the course on each subject, a discussion of each mathematical topic in the course with inferences drawn regarding the subjects' understanding of that topic, recommendations for the improvement of Math 201, an evaluation of the method of the study, and suggestions for further research.

first, followed by dissertations. This category will include studies whose primary focus was the evaluation of methods of teaching mathematics to prospective elementary school teachers (e.g.: Fuson, 1972; Glendon, 1972) known in the report of such a study the investigator mentioned specific mathematical topics or skills which elementary education majors found easy or difficult. Literature on the interview technique in research, its advantages and disadvantages, and its use in mathematics education research will be summarized in the following chapter.

Published Studies

According to Glennon (1949), the only study on the mathematical competencies of prospective elementary school teachers to appear before his own was one by Taylor (1938) which appeared in School Science and Mathematics some eleven years earlier. Taylor reported "some results from

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CHAPTER 2

REVIEW OF THE LITERATURE: MATHEMATICAL UNDERSTANDINGS OF PROSPECTIVE ELEMENTARY SCHOOL TEACHERS

In this chapter, the literature on the mathematical understandings of prospective elementary school teachers will be reviewed. Published studies will be summarized first, followed by dissertations. This listing will not include studies whose primary focus was the evaluation of methods of teaching mathematics to prospective elementary school teachers (e.g.: Fuson, 1972; Schultz, 1972) unless in the report of such a study the investigator mentioned specific mathematical topics or skills which elementary education majors found easy or difficult. Literature on the interview technique in research, its advantages and disadvantages, and its use in mathematics education research will be summarized in the following chapter.

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a test of meanings in arithmetic given to freshmen at Eastern Illinois State Teachers College [now Eastern Illinois University]." Presumably all those taking the test were prospective elementary school teachers. Listing percentages of students who missed some of the items on the test, Taylor concentrates on arithmetic with fractions and denominate numbers. Lamenting the decline in mathematics requirements which occurred during the Thirties, he recommends that prospective elementary teachers be required to take "not less than eight semester hours" of courses in the conceptual bases of arithmetic. These would be college-level, non-remedial, non-methods courses, giving "a new view, a teacher's view" of arithmetic.

Glennon's (1949) own study covered freshmen and seniors at three teachers' colleges as well as in-service teachers. Concentrating on the concepts underlying computations, Glennon describes the dismal state of teachers' understanding at that time. While he does not discuss specific mathematical topics, the few examples he gives of the 80 "understandings" measured by his test indicate that they dealt entirely with numeration and computation. This, of course, was appropriate to the curriculum at that time. The significance of Glennon's study is historical; it was one of the first calls for an emphasis on understanding in the teaching of mathematics. A few years later the curriculum reform movement of the late Fifties and Sixties commenced with this goal in mind.

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A study similar to Glennon's was reported four years later by Orleans and Wandt (1953). They surveyed in-service teachers during university summer sessions in 1951. After failing with a free-answer test form given to New York City undergraduate education majors, the investigators gave the in-service teachers a multiple-choice test form. Again reflecting the curriculum of that time, the focus of their test was on understanding the concepts which underlie the standard algorithms of arithmetic. They report the frequencies of response on different choices among the groups surveyed for three of their eighteen test items. Teachers of grades 1-3 had a mean score of 8.3 correct, while teachers of grades 4-6 had a mean score of 9.5 correct. The authors of this study added their voices to the cry for a curriculum which would emphasize understanding of arithmetic as a way of breaking out of the cycle of learning and then teaching arithmetic by rote.

Phillips (1953) administered the Schorling-Clark Hundred Problem Arithmetic Test along with algebra and geometry tests to students entering an Arithmetic for Teachers course at the University of Illinois. She remarks that their course background gave "little indication" of their competence at that time. Her other conclusions are:

4. Lack of achievement in mechanical mastery starts with the topic of fractions and continues with decimal fractions and percent. It is interesting to note that there is a greater negative reaction to arithmetic starting in the intermediate grades.

5. Problem solving achievement involving measurement, fractions, and percent is very low.

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6. There is a higher competence in Algebra than in Geometry. The reaction to the two subjects favors the former.

7. Achievement in the meaning and understanding of arithmetic is extremely low.

Weaver (1956) reported a study in which he administered Glennon's (1949) test to students in Methods of Teaching Arithmetic during four different semesters. The scores were "in quite close agreement" with the findings of Glennon. Weaver reports further that significant improvements in understanding occurred for one of his groups after it was given a course "organized around the nature of number and our number system and around the nature of the fundamental processes and operations in relation to integers, common fractions, and decimal fractions."

Buswell (1959) gave prospective elementary teachers an arithmetic test that previously had been given to eleven-year-old pupils in England and California. He found that 34 percent of the prospective teachers scored below the top third of the English pupils, and 10 percent below the mean of the California pupils. Noting that the prospective teachers had indicated their intelligence elsewhere, he recommended the wider use of "substantial and scholarly" courses in arithmetic.

Bean (1959) administered Glennon's test to 450 in-service teachers in Utah. He concluded that teachers were generally competent in understanding "the decimal system of notation and the operations of integers." On the

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other areas of the test--fractions, decimals, and the rationale of computation--there was a substantial difference between the scores of teachers of the primary grades and those of teachers of the intermediate grades, with a "progressive increase in mean score in Grades 4 to 6." Regarding understanding of arithmetic algorithms, Bean says "the results seem to point up conclusions of other researchers who have noted an emphasis on rote memorization and mechanical processes." He recommended a required college-level course for all elementary school teachers "to develop a working understanding of number systems."

Wozencraft (1960) reported the results of her administration of the Schorling, Clark, and Potter "Hundred-Problem Arithmetic Test" to 78 students enrolled in an elementary methods course. The median score was "just about equal to the estimated median raw score of the seventh graders!" This study also focused on arithmetic skills.

Fulkerson (1960) gave a 40-item arithmetic test to 158 prospective elementary school teachers at Southern Illinois University. He noted that performance was particularly poor on verbal problems and percent problems. In general, he found the prospective teachers' performance on the test rather poor, although those with some teaching experience and those with some mathematics course experience did better than the others.

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The studies discussed above were summarized and evaluated by Sparks (1961).

Nelson and Worth (1961) administered Phillips' and Glennon's tests to elementary education majors in Alberta, Illinois, and Massachusetts. The findings among the American students were comparable to those found by Glennon (1949), Weaver (1956), and Phillips (1959).

Dutton (1961) published a study which dealt with prospective teachers' understanding of arithmetical concepts rather than with skills. His instrument was the University of California Arithmetic Comprehension Test for sixth grade; it was given at the beginning and at the end of a one-semester mathematics course for prospective elementary teachers. By item analysis, Dutton identified those concepts which were of greatest difficulty. The six items which "continued to cause serious difficulty" at the end of the course were

- (1) What does a remainder mean in long division?
- (2) placement of quotient figures in long division;
- (3) placement of the decimal point in addition; (4) placement of the decimal point in multiplication of decimals; (5) meaning of gross and ream; (6) regrouping with denominate numerals.

Topics which caused considerable difficulty at the beginning of the course but not at the end included identification of partial product, multiplication terms (no explanation of this rubric is provided), using standard time zones, moving

Barber (1964) constructed an instrument to measure understanding of the basic concepts and symbols of

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the decimal point in division, drawing a picture of $3 \div 1 \frac{1}{2}$, reading a fractional part of a rectangle, drawing a picture of $2 \frac{1}{2} \div 1/2$, and using volume. Dutton feels that his data refute the contention of Orleans and Wandt (1953) that teachers have a generally poor understanding of arithmetic. He notes that "there are many basic arithmetical concepts which are understood by prospective teachers." He recommends an individualized "systematic approach to eradicate student misunderstandings of arithmetical concepts."

Corle (1963) gave a variety of tasks involving estimation of physical measurements to in-service teachers. He found that their average error of estimation on these tasks was 61.1 percent.

Creswell (1964) administered the Metropolitan Achievement Test (covering computation, concepts of elementary mathematics, and problem solving) to 313 graduating Georgia elementary education majors. He found that 81.6 percent of the sample scored at the ninth-grade level or above in computation, and 90 percent at this level in concepts and problem-solving. This compared favorably with the earlier studies by Weaver (1956) and Orleans and Wandt (1953). Creswell attributed this to increased emphasis on mathematics in the preparation of elementary school teachers since that time.

Harper (1964) constructed an instrument to measure understanding of the basic concepts and symbols of

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arithmetic and administered it to 396 in-service teachers. The results caused him to recommend "training in basic mathematics" for elementary school teachers. He recommends at least six hours of training, since those teachers with this much training outperformed those with less on every comparison. He also remarks (note the date) that "most teachers could profit from a course in 'modern mathematics.'" *for being too late*

course Dutton (1965) found that prospective elementary school teachers beginning a methods course had a poor understanding of the following concepts: "understanding of partial products in multiplication; meaning of remainders in division; reason for proper placement of quotient figures in division; rationalization of division of fractions; meaning of common measures such as ream, gross, ton, regrouping with denominate numbers; and understanding of the proper placement of the decimal point in work dealing with decimal fractions." However, he found, contrary to the findings of Glennon (1949) and Orleans and Wandt (1953), that about 50 percent of the students understood the concepts "place value, using partial products, placement of quotient figures, using decimals, and working with fractions." Dutton says he was able to remove most students' deficiencies by remedial instruction in his methods class. *out of*

38 correct Melson (1965) reported a study in which a test of 33 items in "modern elementary mathematics" was *written to* administered to 41 beginning elementary teachers. *the* The

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median score was 36 percent correct. The topics on which the teachers made the poorest showing were set theory and number bases, although performance was generally poor throughout. The author concludes that the teachers were "not adequately trained" to teach modern elementary school mathematics, and that since they all supposedly had been exposed to this mathematics in college, the colleges were at fault for being too lax in their standards both for course content and for grading the students.

In an answer to this study, Smith (1967) gave Melson's test as a pretest and as a posttest in an arithmetic methods course for prospective teachers. While the pretest data were similar to Melson's, the posttest revealed a substantial improvement in understanding of concepts at the end of the methods course. Smith uses this data to rebut Melson's charges of inadequacy in elementary teachers' college preparation.

Kenney (1965) administered an instrument to in-service elementary school teachers in California. He found them to be strongest in understanding of the place-value system of numeration, and weakest in percent and operations on whole numbers. Other topics examined were common fractions, decimal fractions, and measurement, graphs, and scales. The median score for all groups was 29.7 out of 50 correct. Kenney used a nonstandardized instrument in this study; a study of his instrument leads this writer to doubt its value somewhat. Several items were of the

multiple-choice select-the-best-answer type, with more than one valid choice. The author himself admits that some errors made were due "to inability to understand the language or vocabulary used in the test," and that "the extent to which [this] interfered with [the] primary purpose is not known."

Skypek (1965) examined the mathematical background of junior and senior women at a liberal arts college in order to compare education majors with other students. She found that both elementary and secondary education major groups had mathematics Scholastic Aptitude Test mean scores below the non-education sample. Both groups also were significantly lower on the college's mathematics entrance examination and in performance in the college's required mathematics course. Skypek concluded that "teacher education (elementary and secondary) majors are less competent mathematically than other student members of the junior and senior classes."

In evaluating a course for prospective elementary teachers, Todd (1966) was disappointed to discover that, although the course had improved the students' understanding, the test scores were about the same as those Glennon (1949) had obtained fifteen years earlier.

Weaver (1966a,b) administered to in-service elementary school teachers an instrument designed to test their recognition of examples and nonexamples of a polygon, quadrilateral, rectangle, simple closed curve, square, and

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triangle. While unsure of the representativeness of his sample, he reports that nearly always the scores of kindergarten teachers were lowest, primary (grades 1-3) teachers intermediate, and intermediate (grades 4-6) teachers, who were mathematics specialists, highest. Without drawing any statistical inferences, he merely notes that some teachers have a poor understanding of "rather simple aspects of nonmetric geometry," and that this should be borne in mind when curricula are planned.

Creswell (1967) administered a test of modern mathematics to in-service teachers who had participated in workshops in which they had studied these concepts. On a 120-item test, their mean score was 56.31, compared to a mean score of 65.25 for sixth-graders who took the same test. However, prospective elementary teachers who had taken two courses in modern mathematics attained a mean score of 93.9. Creswell concluded that the college course is superior to the in-service workshop as a device for teaching modern mathematics to teachers.

Kipps (1968) reported a study in which the subject matter areas important to elementary education were identified and a 42-item test developed and administered to in-service elementary teachers. In the area of numbers, numeration, and sets, teachers had the most trouble with the concepts of least common multiple, exponential notation, set relations, and modular arithmetic. They did well on basic operations but had some trouble with the topics of

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equivalent fractions and percent equivalents. In the area of algebra and logic two-thirds of the teachers could not give the coordinates of points on a grid and many had trouble with truth values of mathematical sentences; the author suggests symbolic logic be taught to teachers. In the area of geometry, measurement, and graphs, teachers' scores were "considerably lower than on the other three parts." Teachers had trouble finding areas of geometric figures both by formula and on graph paper.

Reys (1968a,b) conducted a study in which he attempted to ascertain both the extent of prospective teachers' mathematical competencies and the specific areas in which they were weakest. The test instrument used was the Contemporary Mathematics Test, Algebra Level, Forms W and X. The two forms were used as pretest and posttest, respectively, in three classes--a required undergraduate mathematics content course, a required undergraduate arithmetic methods course, and a graduate level methods course. All classes had significant score gains as a result of the course. Even so, "the post-test means for the mathematics content and undergraduate methods courses were significantly below the means of the eighth and ninth grade pupils completing a first year algebra course." The topics which were most troublesome for the subjects of this study were "the real number system, mathematical statements, and functions and graphs." Reys concluded that "the

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Moody and Wheatley (1969) ran a study at the University of Delaware which tested elementary education majors' ability to comprehend an article in The Arithmetic Teacher about a nondecimal numeration system (Brumfiel, 1967). Using a multiple choice questionnaire and a t-test, they found no significant differences between the mean scores of elementary education majors who had completed the first of three required mathematics courses and those of comparison groups. They recommend that such majors be given greater exposure to the professional literature.

Gibney et al. (1970) devised a 65-item test of "mathematical understandings in seven areas: (1) geometry, (2) number theory, (3) numeration systems, (4) fractional numbers, (5) structural properties for the set of whole numbers, (6) sets, and (7) the four basic operations on the set of whole numbers." The purpose of the test was to try to detect a difference in the degree of understanding of the topics between pre-service and in-service elementary teachers. Some significant differences appeared on t-tests; these were in favor of the pre-service teachers. However, this study did not attempt to find out how well the subjects knew the material, only which of the two groups knew more. The aim of the researchers was to demonstrate the desirability of different mathematics courses for pre-service and in-service teachers.

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In an analysis of this article, Reys (1972) gives many criticisms of this study, including shortcomings of method and lack of pertinent information. Among the former are the possible nonrepresentativeness of the sample and lack of data about the quality of the test instrument. The latter include sampling and procedural details not mentioned by the authors. These criticisms cast some doubt on the study's conclusions.

Dissertations

The earliest dissertation on this topic that the author was able to find was that of O'Donnell (1958). In contrast to most of the other studies in this area, he found that college seniors who were majoring in elementary education had a mean score in arithmetic achievement beyond the twelfth-grade level. He found that the greatest proficiency was in isolated computations, with more difficulty in problems requiring reading and interpretation.

E. C. Carroll (1961) developed an instrument, the Mathematical Understanding Inventory, to test the mathematical competence of prospective elementary school teachers. She found that the students tested possessed "a few more than half" of the understandings necessary for elementary school teachers. Topics on which students were particularly weak were fractions, decimals, percent, and mensuration.

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Jones (1963) ran a study to determine which mathematical concepts are least understood by elementary education majors. Unfortunately his conclusions are not presented in his abstract.

E. M. Carroll (1965) studied the mathematical competence of prospective elementary school teachers who were seniors at Negro private colleges. Their median scores on several mathematical topics were found to be at sixth to eighth grade norms.

Trine (1965) gave thirty-six elementary education majors a test designed to determine if they could recognize the commutative, closure, and identity properties in nine unfamiliar mathematical systems. He found that prospective teachers of kindergarten through second grade were less able to recognize these properties than the others. The identity property was found to be the most difficult of the three to identify.

Williams (1966) developed a test instrument and then determined a criterion score of mathematical competence based on the performance of sixth-grade students on the instrument. He administered this instrument to teachers of grades four, five, and six, principals, and supervisors. All groups except supervisors were below the criterion score. Those with negative attitudes toward "the new emphasis upon mathematics" tended to score lower. In-service training did not appear to make a difference in scores.

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Callahan (1967) developed an instrument of "Mathematical Knowledge" and administered it to prospective elementary school teachers who were freshmen, those who were seniors, and in-service teachers. He found significant declines in performance both from freshman to senior level and from senior to in-service level.

Griffin (1967) administered a test of mathematical understanding to in-service elementary teachers throughout North Carolina. He concluded that the teachers understood fewer than half the topics covered by the test and only one-third of those questions pertaining to "modern mathematics." He also concluded that teachers' understanding of topics was in the following descending order: place value, number bases, fractions, measurement, number postulates, "modern mathematics," geometry, percent, and sets.

Reys (1967) administered the Contemporary Mathematics Tests, Algebra and Upper Elementary Level, to prospective elementary school teachers both before and after their college content and methods courses. Although there were noticeable gains from pretest to posttest, the posttest scores still compared poorly to norms for ninth-grade pupils.

Withnell (1967), in the course of comparing the mathematical understandings of prospective elementary school teachers with respect to the number of courses they were required to take, found their understanding low

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regardless of the amount of preparation. He concluded that "prospective elementary teachers are not being satisfactorily prepared by a three, six, or nine semester hour mathematics requirement."

Garnett (1969) did a study which was inspired by the CUPM Level I recommendations. She constructed a test of mathematical understandings based on these recommendations, and used it as an instrument to test the effects of number of high-school mathematics courses and number of college mathematics courses upon elementary education major's understanding of mathematics. After accepting the null hypothesis regarding interaction between high school preparation and college preparation, Garnett's data suggested that each succeeding level of high school preparation led to greater understanding and that three college courses led to greater understanding than one or two. However, since the test was given to all subjects at the same time, the time lapse since completion of the last mathematics course could be a confounding variable in this study. Keith (1971) developed a geometry test based on the geometry. Backman (1970) tested 65 teachers of grades K-8 on their knowledge of geometry. Their mean score on his instrument was 46 percent. Bailey (1970) tested elementary education majors in Oregon on their knowledge of geometric concepts appearing in elementary school textbooks. He found that 70 percent of the subjects scored 70 percent or less on his

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Three 1970 dissertations used Callahan's Test of Professional and Mathematical Knowledge. Koeckeritz found no significant differences in scores on this test between in-service teachers, college seniors, college freshmen, and high school sophomores. He concluded that mathematics training programs for elementary teachers are ineffective. On the other hand, Hilton, in determining the increase in mathematics knowledge of prospective elementary school teachers in a methods course, observed that they "possessed greater mathematical knowledge" than students involved in previously reported studies. Greabell also reported that his subjects scored higher than the normative group for this test after taking some kind of mathematics content course.

Haggard (1971) tested Kentucky elementary education majors on various arithmetic concepts, but does not present his findings in his abstract.

Keith (1971) developed a geometry test based on the geometry recommendations for elementary school teachers issued by CUPM, SMSG, and the CEEB Commission on Mathematics. She found that in-service elementary teachers have a better knowledge of the geometry pupils are expected to learn than that recommended by these groups. She also reports that "in general, Virginia elementary teachers are weakest in their understanding of the following school

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geometric topics: (1) Points, lines, and planes; (2) Angles; (3) Polygons; (4) Polyhedrons; (5) Measurement; (6) Similarity; (7) Congruency; (8) Parallelism; (9) Perpendicularity; and (10) Pythagorean theorem." (One wonders which topics were their strong points.) Also, "the following variables correlated significantly with the geometry test scores: age (negative), male, female (negative), college training in mathematics, high school training in mathematics, training in geometry, years of teaching experience (negative), and grade taught." She concluded that "the Virginia elementary teachers' knowledge of geometry is somewhat deficient in some areas."

Ames (1972), in observing elementary education majors progress through a mathematics methods course, observed that the most difficult topics for them were numbers and numeration, operations, properties of operations, and geometry. The least difficult topics were sets and mathematical sentences.

Banning (1972) developed a geometry test and administered it to elementary education majors at Montana State University. She concluded that many of them were insufficiently prepared in geometry to teach it in elementary school.

Summary

If any general pattern emerges from the literature surveyed above, it is that prospective elementary school

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teachers are deficient in their understanding of the more complex ideas of arithmetic, such as fractions, decimals, and percent, and in their understanding of geometry. Evidence is mixed on such student's mastery of simpler topics such as sets and operations on whole numbers.

Studies such as those reported above generally use an instrument (either standardized or developed by the researcher) to determine which topics students do and do not understand. But to the knowledge of this investigator, no study has been done with teachers or prospective teachers to determine how they think about the various mathematical concepts they encounter. In order to learn this, the subject must be observed in the act of thinking about these mathematical ideas. The interview is a respected research technique well suited to accomplish this goal. The next chapter summarizes the literature on the use of the interview in research.

error in interview research--bias of the interviewer. Finally, we shall list examples of studies in mathematics education in which the interview was used fruitfully.

Advantages of the Interview Approach

The interview approach is used when the researcher wishes to obtain pertinent information from a sample of subjects. There are alternative means of doing this, such as the questionnaire. However, the interview has some advantages for certain types of studies.

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The written questionnaire is a very useful device for obtaining information sought can be readily obtained from the respondents' answers to specific questions.

CHAPTER 3

REVIEW OF THE LITERATURE: INTERVIEWING AS A RESEARCH TOOL

However, in certain types of research, the interview is an important method of obtaining data for research is the interview. General works on research methods such as Fox (1969) and Madge (1953) contain chapters on the interview as a research instrument. There are also good books on the subject by Merton et al. (1956), Richardson et al. (1965), and Gorden (1969). In this chapter we shall review the literature on the interview as a method of educational research. We shall discuss the advantages of the interview as well as its disadvantages. We shall also review the literature on a major source of error in interview research--bias of the interviewer. Finally, we shall list examples of studies in mathematics education in which the interview was used fruitfully. They discuss four dimensions which represent Advantages of the Interview Approach range, specificity, depth. The interview approach is used when the researcher wishes to obtain pertinent information from a sample of subjects. There are alternative means of doing this, such as the questionnaire. However, the interview has some advantages for certain types of studies.

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The written questionnaire is most useful when the information sought can be readily obtained from the respondents' answers to certain well-formulated questions. In this case it is certainly more economical of the time of all parties to administer the questionnaire.

However, in certain types of studies, the investigator wishes to obtain information which cannot be gathered by means of a questionnaire. Such cases would include situations in which it is impossible to formulate explicit questions that would elicit the desired information, situations where greater depth of response than a one-word or one-line answer is required, or situations in which one wishes to observe the subject over a period of time. In these cases an interview mode would be preferable.

A good book on the advantages of the interview approach is that by Merton, Fiske, and Kendall (1956). This book is a report on the authors' experience investigating the effects of propaganda films during World War II. They developed the technique of the "focused interview," in which they concentrated on the subject's reaction to the film. They discuss four dimensions which represent strengths of the interview approach: range, specificity, depth, and personal context.

The authors define range as "the extent of relevant data provided by the interview." The interview situation has the potential for eliciting a wide variety of comments from the respondent which may be of value to the

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interviewer. This flexibility is a major asset. In order to exploit it, the interviewer must not be too eager to ask only a specific schedule of questions; this reduces the interview to an oral questionnaire. "Unstructured" questions allow the interviewer to exploit the advantages of the situation and extend the range of the data obtained.

The interview also has the potential not only to determine the feelings and opinions of the respondents, but to explore in detail the causes of these phenomena. This is what is meant by specificity. The interviewer can probe his respondents' feelings until specific causes for them can be determined.

Depth is attained when the interviewer obtains "a maximum of self-revelatory reports on how the situation under review was experienced." Such reports can be gathered only in a person-to-person conversation, with the participants examining the subject's responses. "Depth responses" concentrate on the subject's feelings about the situation discussed in the interview. The level of depth attained in the interview is largely under the control of the interviewer.

Personal context consists of those features of the respondent's background which caused the feelings that arose. This can be either an "idiosyncratic context" caused by one person's experiences or a "role context" arising from a person's role in society or in the particular situation. The interview is well suited to the

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Possible Flaws in the Interview Approach

In any research in which data are obtained, the researcher is concerned that the data be both reliable and valid. Reliability is a measure of the probability that under similar conditions the same data will be obtained from a given subject. Validity is a measure of how well the data actually represent the variables that the researcher wishes to examine. Considerations of reliability and validity pose particular problems for the researcher who would use interviews as his instrument for data-gathering.

Gorden (1969) points out that "in any act of observation," four facets of the situation affect the reliability and validity of the reported observations. The first of these is the object of observation itself. When we are dealing with affective variables, these may be (in the terminology of Shulman) opinions, attitudes, or values. Of these, opinions fluctuate the most and are least reliable, while values are most stable and therefore most reliable. The complexity of the information sought may also affect the reliability of the reports.

In this area, Stember (1951) attempted to isolate characteristics of potential respondents which might be related to the reliability of their responses in public

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opinion surveys. He concluded that "the less-educated, and older people, are significantly less reliable in their responses" to the question he asked. ("Do you expect the United States to fight in another world war within the next ten years?") He also found that among the college-educated respondents a greater percentage shifted away from a "No" answer than shifted away from a "Yes" answer. He calls this an interview effect created by the intervening questions.

Returning to Gorden's exposition, the observer himself may be a cause of invalidity or unreliability. This will be discussed in a later section on interviewer bias.

The concepts involved in the discussion may be a source of invalidity if they are vague, leading in the respondent's mind to an unclear connection with the feelings desired. Invalidity also may occur if these concepts are sharp but still unrelated to the main information desired. The fourth source of unreliability and invalidity is the method of interviewing chosen.

Gorden further mentions that accuracy of measurement may be incompatible with that need for flexibility in discussion which maximizes the possibility of discovering interesting information. The scheduled interview, with questions constant in wording and all subjects asked the same questions, is the best format for accuracy of measurement, but obviously this only applies when the

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problems discussed above such as clarity of objectives and methodology have already been solved. In an exploratory study such as that reported in the following chapters, the investigator is only first obtaining the data he needs in order to solve these problems, and therefore may not be overly concerned about accuracy of measurement. Gorden demonstrates the independence of the criteria of reliability and validity. One example in particular is that in which the respondent has forgotten the experience under discussion; in this case the respondent will usually (reliably) report a response dictated by the interview situation, such as one she feels is expected by the interviewer.

Richardson et al. (1965) make some comments on the problem of validity. They mention that the best check for validity is external evidence known to the investigator. They caution against the interviewer using characteristics of the respondent as an index of the validity of the responses. This amounts to bias. As they put it, "The information of the cautious respondent may in fact be more valid than that given by the self-assured respondent." They also mention the respondent's motivation as a key factor in assessing validity. If there will be consequences for her as a result of her remarks, she will try to minimize the ill effects with her responses. The interview situation must be contrived so as to eliminate this problem. In a study planned to compare directly the personal interview with analysis of written tests on grounds of

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reliability and validity of the data gathered, Brownell and Watson (1936) examined children's solutions in the addition of proper fractions. They found that over several comparisons, "in every instance PI [the personal interview] proved to be noticeably more consistent, or more reliable [than analysis of written tests]." The authors reasoned from this that "since reliability contributes directly to validity, the greater reliability of PI argues for its greater validity in the detailed diagnosis of faulty procedures." Considering that the personal interviews in this study were restricted to pupils' statements with no probing permitted by the interviewer, the authors infer that "the advantage for PI is actually larger than that found in this study." They concluded that while the two techniques are "equally satisfactory . . . for the grosser types of diagnosis . . . When . . . diagnosis is that of the processes and difficulties of individual children PI is both more reliable and more valid."

Bias of the Interviewer

One of the factors which may undermine reliability and validity of interview data is the bias of the interviewer. In this section we shall survey the published literature on the effects of such bias. Most of this literature is concerned with political and sociological, rather than educational, surveys.

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The earliest scholarly article on interviewer bias found by this investigator was Rice's (1929) commentary on a survey of homeless men done in New York City in 1914. Rice noted that the responses gathered by a prohibitionist interviewer cited liquor as a cause of the respondent's problems substantially more often than those gathered by a socialist. This report lacks statistical analysis of the responses.

As long ago as 1940, Blankenship reported a study dealing with this problem as it affected public opinion surveys on the eve of World War II. His results showed that "in general the attitudes of the interviewers are correlated with the results they secured in these questions showing reliable differences." Deming (1944) also discussed the problem in a review of all types of sources of error in survey research.

Testing the effect of interviewers' expectations, Feldman et al. (1951) conducted a large-scale study involving 45 interviewers. They found few significant differences on "questions of the traditional closed type," but found "striking differences" on "field ratings" where interviewers rated "attributes of the respondent and his surroundings." On free-answer questions, experienced interviewers drew more data from the respondents, and the analysis supported "an earlier hypothesis that interviewer expectations affect the responses they get on open questions."

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Ferber and Wales (1952) mention two types of bias which are particularly threatening to multiple-interviewer social surveys. These are selection bias and answer bias. Selection bias consists of an interviewer's bias in selecting his sample, while answer bias is revealed in a consistent pattern of answers given a particular interviewer by respondents. The test for either is a chi-square statistic which reveals the probability under a null hypothesis that the variation between interviewers on the qualities mentioned is due to chance. While the statistical test here cannot be done in a one-interviewer study, both sample selection and effect of the interviewer can affect the data produced. The investigator can give data comparing the sample with the entire population as a way of dealing with the first question, but there is no way to determine the second in a study with only one interviewer.

Similarly, Boyd and Westfall (1955), in a marketing-oriented review, discuss the various errors that can be due to the interviewer. These fall into two major classifications--errors in selecting respondents and errors in collecting data. The first is typical of door-to-door poll-taking done in public opinion and marketing surveys. Errors in collecting data arise from the expectations and opinions of the interviewer and from subtle variations in the way questions are asked. The authors remark that "there is some indication expectations have a greater biasing effect than opinions." They also mention

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the problem in survey research of cheating, i.e., falsification of data or creation of false data by the interviewer; they recommend "proper management" to reduce its incidence.

One organization vitally concerned about the accuracy of survey results is the United States Bureau of the Census. Hanson and Marks (1958) report the results of a study done in connection with the 1950 census. They report the following "important factors" leading to significant interviewer effects:

(1) interviewer "resistance" to a given question--i.e., a tendency to omit or alter the question and/or to assume the answer; (2) relatively high ambiguity, "subjectivity," or complexity in the concept or wording of the inquiry; (3) the degree to which additional questioning ("probing") tends to alter initial respondent replies.

Their results emphasize the importance of the structure of the interview schedule and the construction of schedules for obtaining accurate results.

Bias can be introduced when the interviewer is asking information questions to which he knows the correct answers. Stanton and Baker (1942) found that interviewers reported more correct responses on items where the interviewer had been truly informed of the correct answer than on items where the interviewer had been falsely informed of the correct answer. However, in attempts to replicate this experiment, Friedman (1942) and Lindzey (1951) failed to obtain significant results.

The interviewer's verbal mannerisms can also bias the data obtained. Hildum and Brown (1956) ran a clever

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study comparing respondents' opinions toward the Harvard philosophy of General Education under four conditions--the interviewer said either "Good" or "mm-hmm" when the subject made either a pro- or anti-General Education response. The telephone was used to isolate the verbal effects. Multiple t-tests found that "Good" responses tended to bias the results while "Mm-hmm" did not.

Another source of bias is the interviewer's need to codify conversational data into a reportable form. Smith and Hyman (1950) conducted a study in which interviewers codified information on political attitudes taken from phonograph recordings. Interviewers were found to codify substantially equivalent responses differently, depending upon their expectations of what the respondent's opinion would be. The expectations arose as a result of the interviewer's prior exposure to the respondents' ideas.

Although these studies were concerned with political and sociological research, the problem they deal with may arise in educational research also. Nevertheless, for finding out students' misconceptions in mathematics, the interview can be a very productive technique. The next section lists some studies in which it has been used successfully.

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The Interview in Mathematics Education Research¹

The interview has long been recognized as an effective technique for research in mathematics education. Forty-five years ago, Brownell (1930) listed the personal interview in his summary of research techniques in arithmetic (it was the first technique to be mentioned), and cited several studies which had used the interview approach fruitfully. The earliest use of this approach in mathematics education research was in a study by Hall (1891) of children entering school; other very early examples were the studies by Gard (1907) and Judd (1909). Brownell (1941a,b) reiterated his support eleven years later, and the interview was also urged as a research technique by Buswell (1949). Brownell, Buswell and their collaborators were prolific producers of interview research in arithmetic during the first half of this century. (See Brownell, 1928; Brownell and Chazal, 1935; Brownell and Watson, 1936; Brownell, Kuchner, and Rein, 1939; Brownell, Doty, and Rein, 1941; Brownell and Carper, 1943; Brownell and Moser, 1949; Buswell, 1926; Buswell and John, 1931. Also Buckingham and MacLatchy, 1930.)

A strong influence on the use of the interview approach was the psychology of Piaget, developed through his use of observation and interview with children at

¹For the references in this section, the author is grateful to Marilyn N. Suydam, J. Fred Weaver, and Douglas A. Grouws for a bibliography prepared by them for the April 1973 NCTM Annual Meeting in Houston, Texas.

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various stages of development. (For examples of this related to mathematics, see Piaget, 1952 and 1960.) Many studies have been done along the lines of Piaget's thinking. Examples can be found in the book edited by Sigel and Hooper (1968) and in the studies of Brace and Nelson (1965) and of Steffe (1968).

Some of the more recent studies in mathematics which employed the interview technique were those of Brownell (1963, 1968), Buswell (1956), Dawson and Ruddell (1955), Erlwanger (1973), Gibb (1956), Gray (1965, 1966), Gunderson (1955), Grouws (1972), Kilpatrick (1968), Olander and Brown (1959), Pace (1961), Rea and Reys (1970), Ruddell (1959), Van Engen and Gibb (1956), Weaver (1955), and Zweng (1964). Nearly all of the above studies were concerned with the thought patterns of elementary school or pre-school children. To this author's knowledge there have been no published studies involving interviews with college students or teachers. Nevertheless there is no reason why the interview technique should not be equally useful in this context.

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CHAPTER 4

THE RESEARCH STUDY

The studies surveyed in Chapter 2 all dealt with the topics in mathematics which elementary school teachers do and do not know. It is important for those whose job is training elementary school teachers in mathematics to know which mathematical topics give them the greatest difficulty. However, if the instruction of these students is to be improved, it is also important to know how they think about the mathematical ideas presented to them, so that these thought patterns might be exploited by a skilled instructor. This was one consideration leading to the present study.

Another consideration was dissatisfaction among the mathematics education staff at Michigan State University with the one required mathematics content course for prospective elementary school teachers, Mathematics 201. Math 201 is a one-quarter (nine or ten weeks) course which usually has been offered in a format consisting of three one-hour lectures and a two-hour laboratory each week. All students (about 200-250 per quarter) would attend lecture on Monday, Wednesday, and Friday; the laboratory sessions would be held at various times throughout the week. The

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lecturer would be a faculty member in overall charge of the course, while the laboratory sessions would be taught by graduate assistants. Books used in the course were the text by Kelley and Richert (1970) and the laboratory manual by Fitzgerald et al. (1973); the material covered in the course will be discussed in later sections. (Exceptions to this format were made in the spring quarters of 1973 and 1974, when sufficient staff was available to offer the course in several small classes. However, it is expected that large lectures will be an economic necessity in this course in the fall and winter quarters for the foreseeable future.) Problems with this format included the difficulty of coordinating the laboratory work with the lecture presentation, and the situation in which a student has two different instructors in the two parts of the course. Considering this situation, the present study was conceived as a vehicle by which the instructional staff of Math 201 could obtain information on the nature of the students enrolled in the course and on the way they react to the course as presented. With this knowledge ways could be suggested to improve instruction in the course.

Thus the present study was conceived in the winter of 1973 with two objectives in mind: (1) to learn something about how prospective elementary school teachers think about the mathematical ideas presented to them, and (2) to see what were the reactions of these students to the lecture-laboratory presentation of Math 201.

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An interview approach was selected. It was decided to use only female subjects because females comprise the great majority of Math 201 students. The advantages of the interview as a research technique are described in Chapter 3; it was the best possible approach which could have satisfied the first objective. The investigator therefore needed to develop skill in interviewing; also, some practice was necessary to determine just what could and what could not be learned from such a study. With these aims in mind, pilot studies were conducted in the spring and fall terms of 1973.

Pilot Studies

The Spring 1973 study. In the spring of 1973, Math 201 was taught in small classes. Several of the instructors cooperated with the investigator in offering "favorable consideration" as part of their coursework to students who participated in the study. From those students who volunteered to participate, the investigator chose a sample of six subjects representing the various sections of the course as well as varying degrees of mathematics background. One subject quit the study after one interview but the other five participated through the last week of the course. Interviews were held weekly and tape recorded; the investigator took notes when playing the tape and then erased that tape.

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The investigator learned several things from this study. The first was that conversation with the subjects yielded insufficient information of the nature sought, so it was decided to have in the interviews both conversation about the course and a presentation of problems to the subjects for them to solve aloud. The investigator also learned that while presenting such problems to the subjects he should not inform them if their solutions were correct or not, or otherwise pass judgment on their work; aside from the possibility of creating antagonism between interviewer and respondent, this practice would have had the effect of making the session into a tutorial, in which the subject would receive help unavailable to her colleagues. The study sample would thus be unrepresentative of the course population in this respect. (It already was unrepresentative in that it consisted of volunteers only, but this was unavoidable.) Finally, since there was very little new information to discuss at intervals of one week, it was decided to schedule interviews at two-week intervals in future studies.

The Fall 1973 study. In this term another pilot study was conducted. Volunteers were solicited at the first lecture and promised financial compensation (the amount unspecified) for their participation. (At the end of the study they were paid five dollars each by the investigator personally, no other source of funds having

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been found.) Eighteen of these were selected at random to participate, of whom seventeen completed the study. After an introductory interview, the subjects met with the investigator every two weeks over the course of the term, and then once more after the term was over.

The conversation at these interviews was in a scheduled format. The investigator took the following topics from the organization of the text: sets, number bases, primes and factorization, integers, rational numbers, and decimals. After a subject had completed the study of one of these topics in the course, she would be asked the following six questions on that topic by the investigator:

Is this topic new for you or is it a review of something you've seen before? (If review) Where did you see it before?

Do you find this topic easy or hard to grasp? How well would you say you've learned it?

Do you like or dislike the method of presentation of this topic?

Do you think this topic is worthwhile for elementary school mathematics?

How do you feel about eventually teaching this topic?

Would you teach this topic as it has been taught to you in this course, or would you teach it some other way?

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The subject would then be presented with a series of exercises on that particular topic, which she would solve aloud. These exercises were written by the investigator.

Responses to the conversational questions were disappointing, so it was decided for the main study to change to a more open format in which the subject would initiate most of the conversation, with the interviewer following up on points raised by the subject or bringing up those areas which the subject failed to mention. The problems used in this study were refined for use in the main study. Many of the problems used in this study as well as the questions asked at the initial and final interviews were used again in the main study.

Description of the Study

The main study to be reported in this dissertation was conducted at Michigan State University in the winter quarter of 1974. During this term the course was given in the lecture-laboratory format described earlier. The lecturer was Professor Glenda Lappan and the laboratory instructors were Professor Lappan and Messrs. G. A. Badmus, Craig Nosal, and John Holzhauer.

The textbook was Kelley and Richert, but no manual was used in the laboratory. Instead, the lecturer wrote her own laboratory exercises. Students were required to attend the laboratory. The lectures covered most of the first six chapters of the text, including the topics of

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sets, numeration, number bases, fundamental operations, primes and factorization, some number theory, and the systems of integers, rational numbers, and real numbers. The first section of Chapter 3 and some of Chapter 6 were not covered in lecture. Tests were given on January 23, February 13, and March 6; the final examination was held on March 12. The nine laboratory sessions, identified below by date, included the following:

January 7-10. Attribute games, including A Blocks, People Pieces, and Color Cubes.

January 14-17. Dienes Multibase Arithmetic Blocks used to illustrate addition and subtraction.

January 21-24. Dienes Blocks used to illustrate multiplication and division.

January 28-31. Computation, including Napier's bones, lattice multiplication, Russian peasant multiplication, and the Whitney Mini-Computer (see Whitney, 1970).

February 4-7. Clock arithmetic.

February 11-14. Rational numbers, illustrated with GeoBlocks, tangrams, and Cuisenaire rods.

February 18-21. Real numbers, illustrated by ruler-and-compass constructions.

February 25-28. The geoboard.

March 4-7. The metric system: measurement of various objects using metric units.

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The subjects. Female volunteers were solicited at the first lecture on January 4, 1974; \$5.00 compensation was offered as an inducement. (This was paid by the Mathematics Department.) Thirty students volunteered, and these were invited to participate in a randomly determined order until sixteen subjects were obtained. In the following text, these sixteen subjects will be designated by the first sixteen letters of the alphabet, which were assigned to them in a randomly determined order. One volunteer, Student E, quit the study after her initial interview and will be excluded from the following discussion. Table 1 on pages 46-47 illustrates how the study subjects compared with the entire class in their responses to a questionnaire distributed at the first class session.

One can see from the table that in some respects the study sample was different from the class as a whole. The students in the sample tended to be younger, on the average, than their classmates. More of them had taken substantial coursework in education, and more of them had served as observers in elementary schools. A disproportionately large number were preparing for a career in special education. As might be expected of a sample of volunteers, they had stronger feelings about mathematics than did most of the class, and those of them who liked mathematics tended to like it very much. They averaged about as many years of high school mathematics as the class

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as a whole, but reported slightly better grades in this work. Only one subject, Student C, had taken any mathematics in college. All of the subjects were from Michigan except Students I and K, who were from New Jersey. It must also be mentioned that the study subjects differed from their classmates in their willingness to participate in a research study.

The profiles of the subjects which appear below were written in the following randomly determined order: L, F, J, I, N, A, G, D, O, H, B, C, M, P, K.

The interviews. The subjects reported for an initial interview during the period January 9-18. At this interview they were informed of the purpose and format of the study and were asked the following questions:

What kind of mathematics did you take in high school?

Would you call your high school mathematics a good learning experience?

What were your favorite subjects in high school?

Was mathematics among them?

What subjects did you dislike the most?

Do you have any thoughts about why you liked ----- and disliked -----?

In what extracurricular activities did you participate in high school?

Why did you take the mathematics courses that you did in college?

Question

2. Year:

- a) Fresh
- b) Soph
- c) Jun
- d) Sen

3. Age:

- a) 17-
- b) 19-
- c) 21-
- d) 23-
- e) 25

4. Sex:

- a) Male
- b) Female

5. Number

- a) 0
- b) 1-5
- c) 6-10
- d) 11-
- e) 16

6. Have
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- a) yes
- b) no

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Table 1

COMPARISON OF STUDY SUBJECTS WITH ENTIRE CLASS

Question	Percentage of Entire Class	Percentage of Study Subjects
2. Year:		
a) Freshman	.30	.53
b) Sophomore	.42	.40
c) Junior	.23	.07
d) Senior	.05	.00
3. Age:		
a) 17-18	.26	.47
b) 19-20	.53	.47
c) 21-22	.13	.07
d) 23-24	.03	.00
e) 25 or over	.04	.00
4. Sex:		
a) Male	.12	.00
b) Female	.88	1.00
5. Number of credit hours in Education before this quarter:		
a) 0	.31	.33
b) 1-5	.31	.27
c) 6-10	.19	.07
d) 11-15	.11	.00
e) 16 or over	.07	.27
6. Have you had a course in math. methods (how to teach mathematics)?		
a) yes	.02	.00
b) no	.98	.93
7. Which expresses your greatest level of involvement in the elementary classroom in the schools? (not counting when you were an elementary pupil!)		
a) no involvement	.25	.20
b) observer	.22	.53
c) student teacher	.04	.07
d) teacher	.01	.00
e) other (aide)	.54	.47

Table 1

Question

8. What

a) 3-

b) 4-

c) 7-

d) 11-

e) 12-

f) 15-

g) 17-

9. What

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c) no

d) a

11. How

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b) 1

c) 2

d) 3

e) 4

12. What

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a) A

b) B

c) C

d) D

e) F

Table 1 (Cont'd)

Question	Percentage of Entire Class	Percentage of Study Subjects
8. Which grades do you wish to teach?		
a) K-3	.47	.47
b) 4-6	.16	.13
c) 7-9	.05	.00
d) 10-12	.00	.00
e) undecided	.15	.07
f) special	.22	.47
g) pre-school	.01	.00
9. Which best summarizes your attitude toward mathematics?		
a) I strongly dislike it	.03	.07
b) I dislike it	.15	.20
c) neutral	.30	.20
d) I like it	.36	.13
e) I like it very much	.15	.40
10. How important do you feel this course is for <u>you</u> in preparing you to be a good teacher?		
a) extremely important	.53	.47
b) somewhat important	.44	.40
c) not very important	.03	.13
d) a waste of time	.00	.00
11. How many years of high school mathematics (starting with Algebra I) have you taken?		
a) 0	.00	.00
b) 1	.05	.00
c) 2	.34	.47
d) 3	.37	.27
e) 4 or more	.24	.27
12. What was your average grade in high school mathematics courses?		
a) A	.14	.13
b) B	.54	.73
c) C	.29	.13
d) D	.02	.00
e) F	.00	.00

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Did you enjoy these courses? Why or why not?

Why do you want to be an elementary school teacher?

Have you ever taught before? (If so) What subjects, level?

What are your feelings about eventually teaching mathematics in elementary school?

What do you feel is the purpose of teaching mathematics in elementary school?

The data obtained from responses to these questions are reported at the beginning of each of the following profiles.

Following this initial interview, each of the subjects was scheduled to meet with the investigator during the weeks of January 21, February 4, February 18, and March 4. At these interviews, the course and the subject's reaction to it were discussed. The investigator generally would allow the subject to initiate the discussion, asking questions which helped to clarify the subject's remarks. In case the subject did not mention her reaction to the most recent lectures, homework assignments, or laboratory sessions, the area she omitted would be brought up by the investigator. At the interviews of February 4-8, the subjects were asked their opinions on the pace of the course to that point; at the interviews of February 18-22, they were asked how they felt about eventually teaching the course material.

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In addition to the conversational part of the interviews, the subjects were asked to solve problems aloud. The following problems were presented during the weeks listed:

January 21-25:

(Sets) Let $A = \{\text{Kennedy, Johnson, Nixon}\}$
 $B = \{0\}$
 $C = \{[], \Delta, 0, \}$
 $D = \{0, 1, 2, . . ., 99\}$
 $E = \{0, 1, 2, . . .\}$

For each of the following sets, which of the sets above, if any, does it match?
 the set of all living people
 the set of counting numbers from 1 to 100, inclusive
 the set of suits in a standard deck of cards
 the set of all aardvarks enrolled at M. S. U.

(Number Bases)

Give the base ten name for the number expressed by 405_{eleven} .
 Give the name of each number in the indicated base.
 39_{ten} in base two
 44_{ten} in base seven
 The following examples are correct in some base. Name the base that makes each example correct.

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$$\begin{array}{r} 211 \\ - 12 \\ \hline 122 \end{array}$$

(Problem 1) Suppose the only U. S. coins were quarters, nickels, and pennies. If I have 1 quarter, 3 nickels, and 3 pennies, and you have 2 quarters, 4 nickels, and 3 pennies, what is the least number of coins which expresses the total amount of money between us?

February 4-8:

(Problem 2) In East Lansing, a telephone number can begin with 332-, 337-, or 351-. How many different phone numbers can there be in East Lansing?

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February 18-22:

(Prime Numbers) For each of the following numbers, tell whether it is a prime number or not.

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247

(Factors and Multiples)

Find the greatest common factor of 63 and 105.

Find the least common multiple of 42 and 48.

(Problem 3) Driving east at 40 mph, Dan passed through the center of town at 12 noon. At 1 o'clock, Dick, driving in the same direction, passed him at 50 mph. If both drivers had maintained their speeds, how far was Dick from the center of town at 9 A.M.?

March 4-8:

(Rational Numbers) Is there a rational number between $\frac{1}{3}$ and $\frac{1}{4}$? If so, name one.

(Decimals)

Find a decimal name for the number represented by $\frac{7}{12}$.

Find a fraction name for the number represented by .37777777. . .

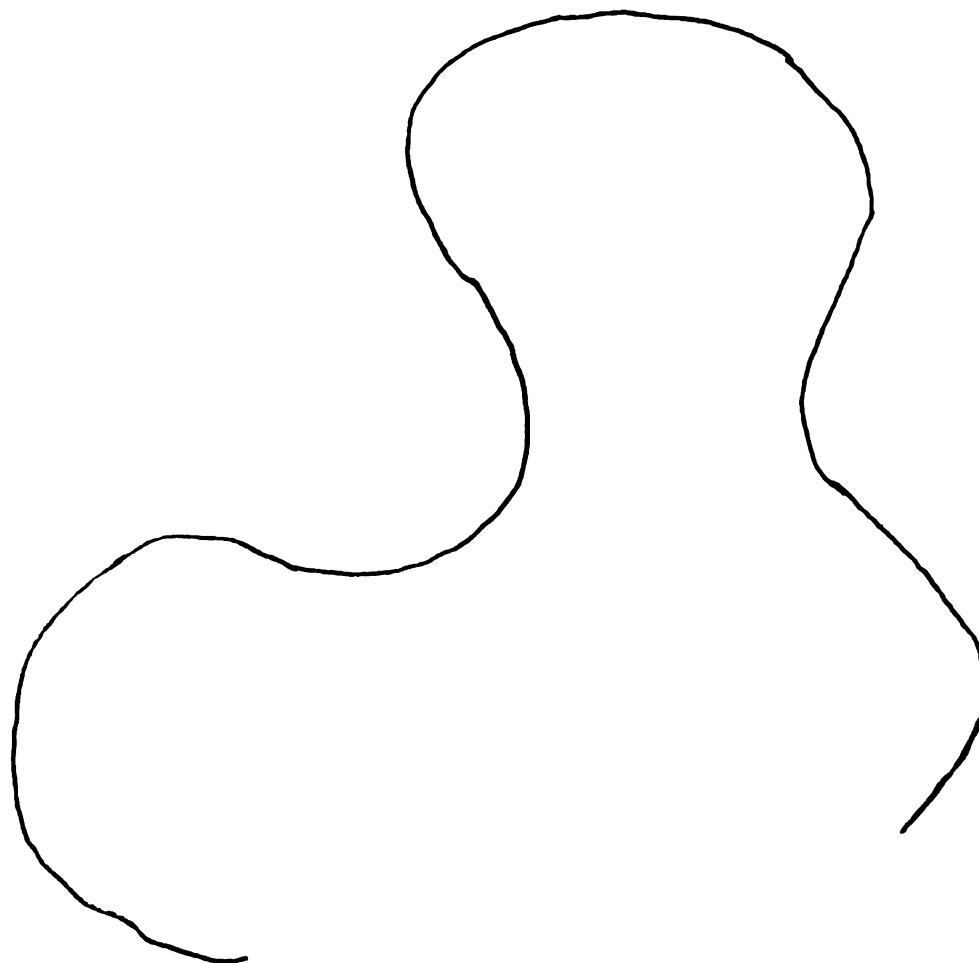
Find a terminating decimal approximation to $\frac{6}{11}$ that has an error less than .00001.

Find an approximate value for $\sqrt{7}$.

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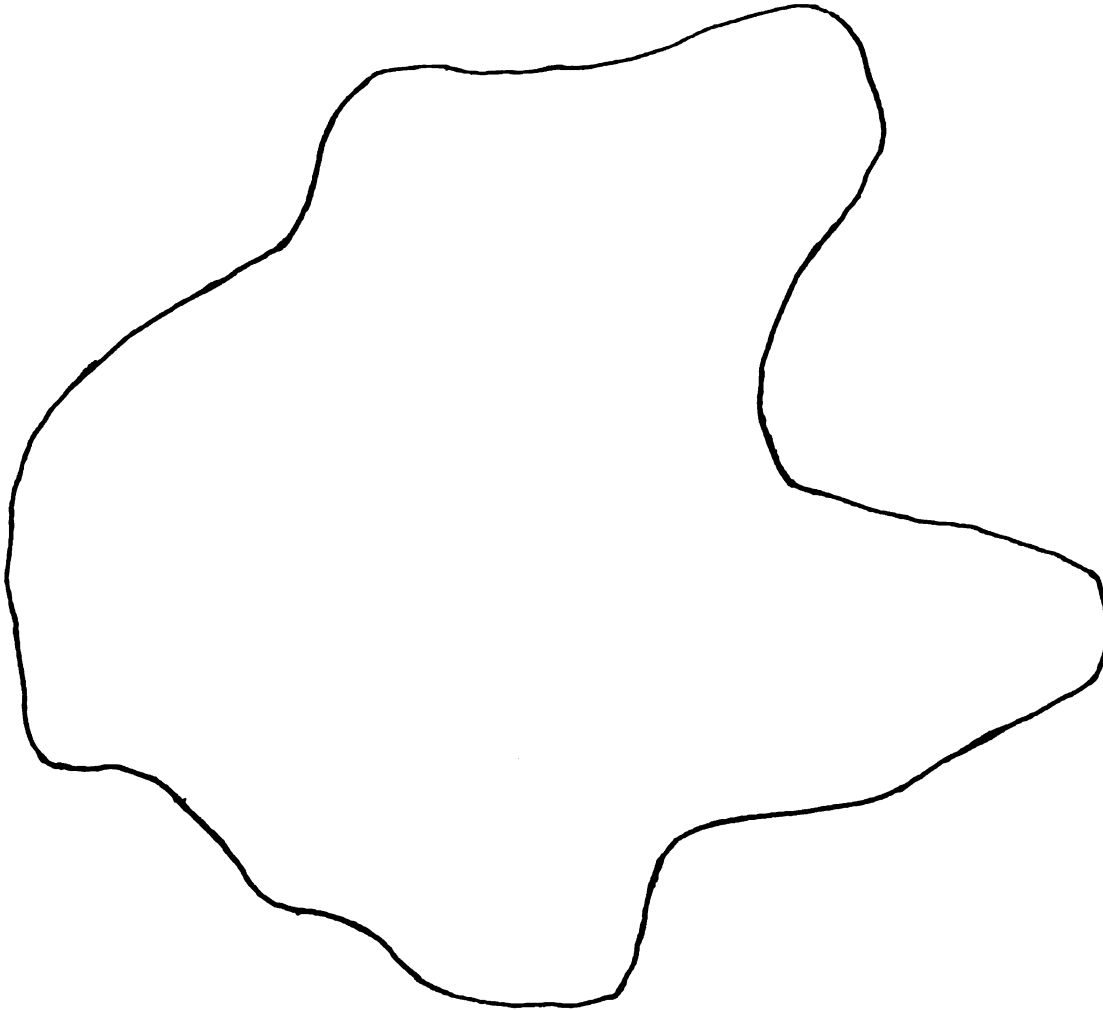
Find, as best you can, a measure of the length of the curve below. (Ruler and compass provided.)



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Find, as best you can, a measure of the area of the region below. (Ruler and compass provided.)



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(The last two problems on decimals and both measurement problems represented types of problems not seen by the students in the course, but the investigator decided to present them anyway.)

During the period March 26-April 4, at the beginning of the following quarter, each of the subjects met with the investigator for a final interview. At that time they were asked the following questions:

How have your feelings about mathematics changed, if at all, as a result of your experience in this course?

How have your feelings about eventually teaching mathematics changed, if at all, as a result of this course?

How have your feelings about the purpose of teaching mathematics changed, if at all, as a result of this course?

How have your feelings about your major changed, if at all, as a result of this course?

Was there anything in the course that you felt could have been done better?

For which topics were the laboratories most effective?

Was there anything about the course that hindered your learning of the material?

Would you take further mathematics courses? What type?

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What is your reaction to this study and to me as an interviewer?

If this study were to be run again, do you have any suggestions as to how it might be improved?

If you were doing this study, what kinds of questions would you ask?

Can you think of anything about the course that would be worth investigating that we did not discuss in this study?

Considering your experience in this course, do you think that elementary school mathematics should be taught by general classroom teachers or by mathematics specialists?

The remainder of this chapter will consist of a profile of each subject; these profiles will be followed by a summary of the subjects' solutions to the various problems.

Student A

Student A was a freshman from Detroit. She had taken a year and a half of algebra and a year of geometry in high school. She found both geometry and her later algebra course "difficult" and quit high school mathematics, after getting a C in her first semester of intermediate algebra, so as not to lower her average. She had no antipathy toward mathematics, saying, "I like math, but I don't like the grades when they come." Her favorite high

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school subjects had been sociology and psychology, where the relevance of the discussions to actual situations interested her. She had disliked history, which she had found to consist of only reading and memorizing. Student A also had taken a remedial arithmetic class (outside of school) while in high school. She had not taken any college mathematics prior to the study.

Student A intended to major in special education; her main interest was mental health. She had become interested in this field from television and from working at Wayne County General Hospital. She had previously worked with children in a recreational center, but said that she had not taught in that situation. When asked how she felt about eventually teaching mathematics, Student A replied, "I don't want to teach math . . . but if I know the math, I should be able to teach it." She felt the purpose of elementary school mathematics is "to help children get a better understanding [of] the methods of math, why we use those methods . . . basic arithmetic, to help the child get an understanding."

(Student A was by far the least articulate of the study subjects. In the following report the investigator has done his best to present her remarks accurately, although the meaning of these frequently is unclear.)

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January 21. At this interview, Student A said of the lectures, "They're explained very well. She gets it right down to the point. Then again, you can look at it and you might think of an easier way . . ." She mentioned bases specifically as a topic in which she could think of an easier approach: ". . . Now, we're doing the bases, and I can find easier ways to do the bases . . . besides moving [units, longs, etc.] . . . if the number's real long, you can work it out that way . . . If you can look at the problem and think, 'What is this number in the base that I'm doing?' it would be easier for you to take it . . . from there on [i.e., translate to base ten to do the problem]."

Student A said she was not sure if the book was oriented toward content or toward methods. She said it was "confusing . . . If I can't do it, I can't see how you're going to teach it to elementary students." In the book, she found the "wording of the questions" confusing, but said that they were clarified in the lectures. She cited as an example an exercise in the book asking the student to use the definitions of 2 and 4 to prove that 2 is not equal to 4. Student A had not realized that 2 and 4 had definitions until this was mentioned in lecture; at the time of this interview, she had not yet done this exercise. She also had difficulty converting a number in one nondecimal base to a different base without going through base ten. In general, she thought that the lectures

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presented material clearly, and that she understood them somewhat, but could understand the material better with more study.

Regarding the laboratory sessions which she had attended, Student A said that the session on attribute games was "kind of unnecessary . . . because . . . we know pretty much about sets." She found nothing of interest in this session. However, she found the second session, on Dienes blocks, "interesting" and helpful. While she had studied bases before, "it wasn't really understood," and "now I've got a better understanding of how to change from one base to another."

Asked to find the matching relationships, if any, of the four described sets to the five given sets in the first exercise, Student A said that each of the four matches none of those given. After giving these answers, she explained her reasoning. Regarding the set of all living people, she did not consider that it might match set A, B, or C; after considering sets D and E, she concluded that it matches none. In contemplating the set of counting numbers from 1 to 100, inclusive, she considered set D, noticed that one set went from 1 to 100 and the other from 0 to 99, and concluded that they do not match. Similarly, she saw no match between the set of suits in a standard deck of cards and set C. She said she saw no match for the set of all aardvarks enrolled at M. S. U. because she did not know what an aardvark is. Apparently,

Student A had confused the concepts of matching and of equality of sets.

Student A correctly translated 405_{eleven} to base ten, but could not do the reverse problem. To write 39_{ten} in base two, she made it 3 twos and 9 ones, which is fifteen, therefore 15_{two} . Similarly, 44_{ten} in base seven became 4 sevens and 4 ones, or thirty-two, so 32_{seven} . She was able to recognize that a twelve had been carried in the first missing-base example, and that therefore this example was in base twelve. However, in the second example, when she borrowed to subtract, she thought that the resulting subtraction was eleven minus two, and therefore looked for a base in which nine is written ____2. She hypothesized that base seven was the solution, but in attempting to complete the example found that it was not. She mentioned base three in her discussion of the problem, but never saw it as a solution because she continued to think of 11____ as eleven.

Student A read Problem 1 as asking for the difference between the two persons' amounts of money. After writing down the number of coins, she noted that this difference was 1 quarter and 1 nickel. She had wanted to compute the amounts of money involved, but was told by the investigator that the answer was requested in terms of coins.

At the conclusion of the interview Student A said she had no strong feelings about the interview situation and had been generally comfortable.

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February 6. Student A remarked at this interview that the material in the course was familiar to her from her junior high school education, and that she was merely learning a new approach to it. The lectures she found "not so much as help but . . . you can see it in a different way." As an example, she cited the two ways she had seen of finding the least common multiple of two numbers.

Student A said that she always did the assigned problems before they were discussed in the lecture. She usually used the methods she had learned in her previous education, then noted if the lecturer used a different method, and "if I come across it again, I might try it that way." Although she preferred to divide in the fashion in which she had been taught, she appreciated the power of the Greenwood algorithm to help the pupil's ability to guess a quotient. (She thought that the name of this method was "algorithm," because it was labeled thus in her notes.) She had been unable to relate the three descriptions of division presented, and had obtained help from her laboratory instructor. She said she knew how to determine if a number is prime, and how to find the factors of a number.

Turning to the laboratory sessions, Student A said of clock arithmetic, "I still don't see . . . exactly what it is. Is it a number? . . . I still can't see . . . what is the point of doing that method." Regarding the session

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on computation, Student A said, "It took me some time to catch on to it. [After] two hours, I finally got the hang of it." Her instructor did some sample problems to illustrate the techniques shown before the students worked on them. Using one of her favorite expressions, Student A said this was for students to "get an understanding." She remarked further, "After you've got it, it's pretty well on its way."

Student A said the pace of the course to this point had been "about right." She found the lectures interesting but the laboratory sessions too long at two hours.

In response to Problem 2, Student A said there was "an infinite number" of possible telephone numbers in East Lansing. Her reason for giving this answer was that "332- can have any other numbers from 0 to 9 in any order same as for the rest of them: After writing down two possibilities, she reiterated, "It's infinite . . . I'm looking at it as they can all start off with this number, and then use any other counting numbers from 0 to 9, and you can mix them in any order."

February 18. At this interview the first discussion was of the second course test, which had recently been returned to the students. Student A had thought that she knew the material well, but found herself "shook up" by the test, especially the questions on "properties and what methods." She had not been able to identify

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properties of number systems on the test, though she recalled their being mentioned in lecture. She had been able to find a least common multiple, but was confused about subtraction of integers, citing "all of steps, which I thought was unnecessary." Regarding the test question "Illustrate each of the following general statements about integers," Student A said, "When you look at it, you see a negative times a negative--automatically . . . you think of it as a positive, but we had to write down some sort of method where negative parentheses--you still come out with a positive. It's kind of unnecessary, but it took you a long time to see it . . ." She could not recall anything from the lectures which particularly helped her to understand the topic of integers.

Student A reiterated at this interview that she usually did the assigned problems by methods she had learned in her prior education. Although she customarily did them before they were discussed in lecture, she sometimes had to wait until after this discussion before she understood the material.

Student A did not appreciate the laboratory session on rational numbers. She said, "I don't know what it meant. It was just sticking . . . little blocks together." She could see some value in the exercise of finding the size of one block in terms of another. She said many of the laboratory sessions were "at the time, not that clear," but were explained later in lecture.

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Student A found it hard to say how she felt at this point about eventually teaching the material she encountered in this course. She remarked, "Math--as a whole--I don't think I want to teach it anyway." She added that she might work as an aide in a mathematics classroom.

Student A then proceeded to the problem sheet on prime numbers. She tried 2, 3, and 7 as divisors of 119. When carrying out a long division, if a known prime number appeared after a subtraction step, Student A knew she need not proceed further because the divisor would not go in evenly. For example, dividing 119 by 2, she did

$$\begin{array}{r} 5 \\ 2 \overline{)119} \\ \underline{10} \\ 19 \end{array}$$

Knowing that 19 is a prime, she realized that 2 would not divide it evenly. For 3, 29 appeared in this position. For 7, she made a subtraction error and 29 again appeared. As a result of this error she concluded that 119 is prime; she tried no divisors beyond 7. For 113, she tried only 2 and 3 as divisors; after obtaining similar results, she termed this a prime. In contemplating the prime numbers appearing in the division examples, Student A mentioned, "I was thinking of 63 . . . a number that wasn't prime that has 3 as the last digit." Student A tried 2, 3, and 9 (because $3 \times 9 = 27$) as factors of 227, calling it prime after none went in. She tried these same numbers as potential factors of 247; although she did not call 247 prime aloud, she found no factors of it.

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In finding the greatest common factor and least common multiple, Student A in both examples factored both numbers into primes and took the correct combination of factors.

After reading Problem 3, Student A wrote down the following data:

$$\begin{array}{rcl} D & = & 40 \\ & & 12 \end{array} \qquad \begin{array}{rcl} DK & = & 50 \\ & & 1 \end{array}$$

She then was stymied by the problem, because "I can't . . . like, subtract the miles, and still get . . . back to 9 o'clock." After she repeated that she couldn't figure out "how to get back to 9 o'clock," she remarked, "Only way I can see--if I was told . . . how long it took . . . till it get there, then I could subtract back [to] 9 o'clock."

March 6. (Due to Student A's inability to meet on March 4, this interview was held on a test day, immediately before the test.)

Student A said she had, willingly, "learned a lot" during the last part of the course, and remarked of the problems, "It's OK . . . You know how to do them, but, really, I never looked at them." Although the material was familiar, she had not understood it before. She later said this section of the course had been "much harder" for her than the previous two sections.

In the lectures, Student A had found helpful the discussion of percent problems and the illustration of the

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process of conversion between a fraction and a repeating decimal. She found story problems particularly difficult and therefore appreciated the sheet of percent problems that was distributed. She was happy about the lecturer's presentation of "a set equation that you can do it with [i.e., a formula for solving percent problems]," but added, "I still don't have the hang of it, really." Student A said she had never before seen repeating decimals (she had always rounded them off) or the representation of a decimal as a fraction. She criticized the lecturer's style of presentation as "always confusing at first . . . She makes it seem harder than what it is . . . She'll show you three different methods and she'll go all in a roundabout way . . . then she'll summarize it all up in two words . . . for the entire lecture period." Despite this impression, Student A was unable to name anything specific in the lectures which had confused her, except for the process of converting a repeating decimal to a fraction, in which Student A could not see any pattern in the use of r , $10r$, $100r$, etc.

Regarding the assigned problems, Student A said that she was "not too good on percent problems." She had had some difficulty with assigned problems concerning fractions and the multiplication of decimals, which she said she never had seen before. She found helpful an exercise in finding a fraction name for base two "decimals"

(page 219) and the book's explanation of the invert-and-multiply algorithm for division of fractions.

Student A said of the laboratory classes in general, "I don't think too much of any of the labs . . . I still can't see why we have lab." She suggested that the laboratory session be used as a review and recitation class. Regarding the session on ruler-and-compass constructions, Student A said, "I just did it," adding later that she thought she had understood what she had done. Noting that there had been nothing resembling this material in the lecture, she remarked that this exercise had been "not exactly a waste of time, but I would prefer doing something else." She said that the session on the geoboard was not worth the time spent on it, commenting, "I'm trying to find a meaning to it." Of the laboratory session on metric measurements, she said, "I don't know what that was all about," and "I can't relate it to what we've been doing."

Asked to find a rational number between $1/3$ and $1/4$, Student A first noted that the lecturer had pointed out that there were many rational numbers between any two given. Nevertheless, she was skeptical of this, saying "I don't want to say no [but] between $1/3$ and $1/4$ I would say no . . . but I know from what she said, that there were rational numbers in between." She then wrote the integers 1 2 3 4, and, on a line below, the numbers $1\ 1/2$ 2 $1/2$ 3 $1/2$. She then said $7/2$ was between $1/3$ and $1/4$; the

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investigator took this to mean the reciprocal of $7/2$. She was not certain if $7/2$ or its reciprocal were rational numbers.

Student A correctly found the decimal name for $7/12$ by long division. She had a general idea of what to do in the reverse problem. Considering $r = .3777 \dots$, she attempted to subtract $10r - r$, but stopped after finding a 4 in the first place after the decimal point. To avoid this situation, she then set up the subtraction $37.77 \dots - 3.77 \dots$; in doing this she correctly labeled the first of these numbers $100r$, but carelessly labeled the second $1r$, which resulted in the incorrect answer $r = 34/99$. In the next problem, Student A divided $6/11$ out to two decimal places and stopped, saying "I'm not exactly sure what this one means." She attempted to find $\sqrt{7}$ as the sum of two numbers, somehow involving the Pythagorean theorem.

In the first measurement problem, Student A said she would use a string, "stretch it on there," and measure its length with a ruler. To find the area of the given region, she would put a string around its perimeter, then deform the string into a square, measure a side, and find the area by "a-side plus b-side or length times width."

Student A received the grade of 3.0 in the course.

March 27.

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March 27. At this interview, Student A began by saying there had been no change in her feelings toward mathematics after taking Math 201. She said she had taken the course "not as a challenge [but] to see how good I could do" and that "I still like to work with math. If I can do it, I even like it better." However, regarding teaching, she said emphatically, "I don't want to teach math . . ." On the purpose of teaching elementary school mathematics, she said she had had no change of feelings. As she said, "In elementary, you're learning the basics. It's reasonable that it should be taught." However, she objected to the textbook's presentation, saying, "Some of it was--not difficult . . . not unnecessary--like going through all these changes to get a simple adding problem." Student A said she had not changed her feelings about her major as a result of her experience in Math 201.

Student A could not name anything about the course that could have been done better, except to suggest again that the laboratory sessions cover some of the lecture material. She praised the lecturer's explanations of how to solve the problems. She could not suggest improvements, other than decreasing the size of the lecture class. She said she had liked the laboratory session on Dienes Blocks, but said "a lot of them" were bad or boring. Nothing had hindered her learning.

Student A said that she would consider taking further mathematics courses; indeed, she had wanted to

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enroll in one for the Spring 1974 term but was dissuaded from this at registration by an adviser who thought the course would be too difficult for her. (It was unclear to the investigator to which mathematics course Student A was referring here.)

Student A said she did not mind participating in the study but had always wondered whether she had done the problems correctly or had been "making a spectacle of myself." She could not suggest any possible improvements in the study, different questions, or other possible areas of investigation.

She favored general classroom teachers as teachers of elementary school mathematics.

Evaluation. To the investigator, Student A seemed to be rather inarticulate; she was the most frustrating of the subjects to talk to, and could not communicate much information about her feelings towards the course. It seems that the laboratory classes were wasted on her; she rarely saw the point of such activity. It appears to the investigator that Student A's grade in the course was too high for the degree of mastery of the material that she displayed.

Student B

Student B was a sophomore from Allen Park. She had taken three years of high school mathematics. Although

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she disliked mathematics, she had been a capable student in it and, after an accelerated seventh-grade course, had started algebra in the eighth grade. Toward the end of Algebra II she had started to do poorly and therefore had taken no further mathematics. (She had taken geometry between these two years of algebra.) Her favorite high school subjects had been history and English, in which she felt "you learn things that are useful." She had disliked mathematics and science, which, beyond a "basic foundation," she had found useless. As she put it, "What are you going to know dissecting a worm . . . once you're married and have three kids?" Student B had been active in many high school extracurricular activities. She had taken no previous college mathematics, although she had wanted to take the equivalent of Math 201 in her freshman year at Western Michigan University but had been unable to schedule it.

Student B was majoring in special education of the deaf. She had become interested in this field from observing her aunt, a teacher of the deaf. While in high school, Student B had worked with handicapped people, she had done her sixty-hour practicum over the Christmas break immediately preceding the study. She also had worked as a teacher's aide in a regular second-grade classroom while taking Education 101A, and had tutored first-graders in reading when she was in high school.

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Despite her dislike for any mathematics beyond arithmetic, Student B said, "I wouldn't mind [teaching mathematics] . . . I love to work with numbers and I like to make them come out exactly right--and to be very precise with numbers . . . I like to use them for practical purposes." She said that the purpose of elementary school mathematics was that "they have to know the basic things . . . so they can go on."

By the time of her initial interview (January 14), Student B had already developed strong feelings about Math 201, which derived from her feelings about the utility of mathematics and the nature of teaching mathematics. The following quotations illustrate her feelings:

The course seems so silly for what you're going to be teaching the kids, because when you're teaching the kids you have a certain math book that you have to get through. You have certain ways that the guide tells you to go through and help the kids, and you instinctively know from all your own use of math how to tell the kids to add three plus five, which [was the topic of the] lecture this morning . . . It just seems like a waste of time.

I guess I just don't like the whys for everything. She will explain a certain thing . . . say three plus five, and then she has to go into why you would have three--because you have a set that contains three objects--and why you would have five--because you have a set B that contains five objects and N of B equals five. It just really seems dumb. We know you have three, and we know you have five, and we know that when you add it, it's going to make eight, and the kids can see it also, 'cause they're going to have three pieces of candy in front of them or five pieces of candy in front of them, and they're not going to be asking 'Why is this three?' and 'Why is this five?' most of the time. They're going to already have known it from another class or you're going to have already taught it. It just seems silly. She just repeats

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herself in so many ways, going through all the whys and hows, that the class becomes boring.

The main thing [in teaching] is finding the math workbook you want, the . . . auxiliary things to go with it, the sheets you want them to do, the dittos, making the dittos, all the kinds of manual stuff that have to be done, the correcting of all the papers . . . I don't think teaching it would be that hard. I've done it before--it isn't that hard. The kids can understand it if they've got someone helping them. It's not all as difficult as she's trying to make it in class. She's trying to make it into a full-blown scale where you're going to have to explain the why for every single thing, and the kids just don't ask that. Once they have the basic why down, they might ask it a couple more times, but you know the answer--you've had it told to you how many million times in how many million math courses you've taken ever since first grade, so why do you have to have her repeat it to you again while you're in college? It just seems like a waste of my money. It seems really stupid.

I'm not going to hold some kid back because I don't like math . . . I'll just give him more material to work with, pass him on to another teacher [who] can teach him much better than I can . . . he'll be [learning]--square roots, whatever he wants to learn.

January 25. Discussing the lectures at this interview, Student B said, "They're getting better--she's explaining more . . . It just seems to be going along with the stuff [the text] better . . . Sometimes she gets very elementary, and it gets rather boring--a lot of people fall asleep, but, other than that, it's usually pretty good and she's pretty good about answering questions, too. The lecture I don't have too much complaint about." Student B said that she preferred learning the material from lectures rather than from reading the book on her own. She could not name anything specific the lecturer had done which had helped her. While "some of the things she did totally

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confused me," "it all came together" by the time of the test. She mentioned the number of subsets of a given set and the Austrian method of subtraction as topics which had confused her; she noted that she had not been confused by bases, since she had seen them before. She said that the lecturer had presented the ideas of transitivity and trichotomy in the counting numbers clearly. She noted that it was easy to confuse the formulas for the number of subsets of a given set (2^n) and the number of one-to-one correspondences of that set with itself ($n!$).

Student B said that the book was "really confusing to me"; she resented having to "follow every little detail." She said, ". . . they're just really messing me up trying to explain it all . . . for me it's easier to skim through the book and to really listen to the lecture, to have someone live in front of me telling me the same thing." Student B added, "Usually the homework problems help . . . make you think it out and really work it out but sometimes they can really, really be confusing." She said she went to the library, consulted the answer book, and got all the answers before doing the problems, then would "use answers to find out how to do it." She said of the book, "Somehow there must be a clearer way--an easier way--to teach what they want to teach." Regarding specific problems, Student B said that one (page 17) involving constructing sets A and B so that A, B, $N(A)$, and $N(B)$ satisfy various relationships had helped her to understand the relationship

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between a set and its cardinal number. She also said that testing various true-or-false statements about sets and subsets (page 27) had helped her, as had a similar collection involving sets, subsets, finiteness, and matching (page 28). In the chapter on fundamental operations she named as helpful exercises in determining whether given sets are closed under addition (page 51) and a series of statements (page 52) asking which properties of addition were illustrated.

Student B said of the laboratory sessions, "I don't think they're worth anything at all. . . They're not helping me." She suggested that the laboratory period be used as a help session for the lecture material, rather than a presentation of new material. She pointed out the two-hour length of the laboratory period and the fact that her instructor explained topics in terms different from those of the lecturer as possible factors contributing to her dislike of the laboratory. Student B said, ". . . It really seems like a waste of time," and noted that she felt that her colleagues in the laboratory "just don't want to be here." Student B said she would not attend the laboratory if she were not required to do so. She also said that when she needed help she would go directly to the lecturer for help, rather than to her laboratory instructor or to the book. Later, Student B said that the laboratory work "got me more confused." She had used base four Dienes blocks to do base five problems, due to a shortage of

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materials. Student B recommended that her laboratory instructor give more help to individuals rather than talk to groups. She also felt that the laboratory session on Monday should have been used as a review session when the test was on Wednesday; she said that everyone in class was worrying about the test. Student B noted that the laboratory material was far in advance of the lectures and reading assignments. She said that the earlier laboratory sessions had not been helpful and that she usually learned more from lectures and homework problems than from the laboratory and reading the text.

Student B also recommended that the course instructional staff make clearer which parts of the course were mathematics content and which were methods of teaching; she felt that Math 201 students expected the course to be methods.

Student B was then presented with the problem sheet on sets. She said that the set of all living people matches set "E . . . the set of all counting numbers because you can keep counting people forever and ever." Regarding the set of counting numbers from 1 to 100, Student B said, "There wouldn't be a set to match it . . . because A has three members, B has one, C has four, D is going from 0 to 99--[we] want the set from 1 to 100--and E is going from 0 forever, and 1 to 100 [does not appear]." One might infer from this that Student B did not understand the idea of matching, but on the next question (the set of suits in a

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standard deck of cards), she said, ". . . well, there's a set that has four in it, if they want to relate that set," and indicated that it matches set C. On the last set asked, Student B said, "I don't think any would match . . . B would be the closest and it's zero, so--it's not the empty set and it would have to be the empty set." She therefore said that the set of all aardvarks enrolled at M. S. U. matches none of the given sets.

Student B correctly converted 405_{eleven} to base ten by marking her columns 121, 11, and 1, and then taking $4 \times 121 + 5$. She also performed both conversions from base ten (39 to base two, 44 to base seven) correctly, labeling columns in the base desired and placing the appropriate digit in each column. In the first missing-base problem she thought that nine plus seven is sixteen; since a 1 had been carried and 4 were left over, the 1 carried represented a twelve, which therefore was the base. (She accepted this after some thought.) She named base three in the second of these problems because she saw that if $11 - 2 = 2$, then 11 had to be four, making the base three; she then checked this by doing the rest of the example.

On Problem 1, Student B mentally added within coins and wrote down that the sum of both amounts was 3 quarters, 7 nickels, and 6 pennies. She then exchanged for the fewest coins.

Student B said at the end of this session that she had felt "fine" in the interview. She said that she liked

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to express her opinions to a disinterested party. She again stated that she enjoyed doing problems involving numbers but not involving symbolic mathematics.

February 8. Student B said of the most recent part of the course, "It hasn't been that bad." She told the investigator that she had neglected her homework recently because she had had five midterms that week. She made sure to attend the lectures. Student B said that she was somewhat confused by the topics of prime numbers and integers; although she had seen them before, she had never really learned them. She had not yet done the homework on these topics but planned to do it before the test on February 13. She thought she understood the topic of prime numbers, but needed more work on it to be sure.

Student B said that most of the lecture material was familiar to her and that she recalled it from her earlier education. She said of prime numbers that she "can't find a really good purpose for them--I get really mixed up with them, on anything." She had trouble understanding clock arithmetic and the Sieve of Eratosthenes. Mentioning factorization and the ideas of least common multiple and greatest common factor, she said, "I've had all that before, but I'm really confused about it right now . . . I'm hoping that if I read it [which she had not yet done] they'll explain it better--put it together." She said that she was unable to compute least common

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multiples and greatest common factors easily, but that her main difficulty was with prime numbers; she felt she understood the discussion of negatives in the lecture immediately prior to this interview. Student B said that she usually read the book's presentation of a topic before that topic was discussed in lecture. She had discovered when she stopped reading the book that "you definitely need both components [lecture and book] . . . I can't just do it on one." She found the book confusing "to just sit down and read it," noting that the lecture sometimes clarified the book's presentation, and therefore that attendance was necessary. Student B criticized the lecturer's discussion of the distance between two numbers-- "this long, drawn-out discussion of the number line, and how you flip it over and . . . you take the opposite of q . . . opposite of p . . ."--saying "everybody already understood it." Discussing this presentation (and perhaps the course in general), she complained, "The thing that I see as a waste of time is that you're going to go into a class--an elementary class or whatever--and get a math book there and a teacher's guide and you're going to read through it, and you're going to know how to do it, and you're going to have math methods--it's going to give you the methods to teach it, so why are we going through all this right now?"

Student B mentioned two items on the first test with which students had had difficulty. One was the distinction

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between subset and proper subset; the other was the formulas for the number of subsets of a set of n elements and for the number of one-to-one correspondences of such a set with itself, as she had mentioned earlier. She said that after the test the lecturer had explained both ideas well.

Student B said of the laboratory part of the course, "The lab obviously is not meant to help you, as far as understanding the book and the lectures. You are to go to the lectures and to read the book and if you don't understand it to ask for outside help." She now realized that the laboratory had been planned not as a help session, as she had wished originally, but as a place to introduce new material. She noted the relation between the laboratory and the lecture material--"I can see where it all kind of ties in in a thin line." Regarding specific laboratory activities, Student B found the session on computation "pretty interesting." She thought "it was good to see the different ways you could do it," and felt this material might be usable in the elementary classroom. On the other hand, she found clock arithmetic "useless." She criticized her laboratory instructor for not explaining the material, saying he was "really confusing" and had not improved as the lecturer had.

Student B said that the pace of the course to this point had been "fairly good"--not too rapid, with the tests well spaced. She liked having three tests and a final

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rather than only a midterm and a final. She said of the lecturer, "She seems to know how to teach very well." Student B said that all of the lecture material had been familiar from her earlier education, except for some algorithms such as the Austrian method of subtraction. She said that all of the students had thought the course very easy at first, but it had become more difficult later. She said that the first test "wasn't that hard," but that she had made "dumb" mistakes on it. Such mistakes included not being able to name the corresponding element to an odd number k in a one-to-one correspondence between the odd numbers and even numbers (she had put 2) and not being able to identify properties illustrated by numerical examples; she called these "picky things."

On reading Problem 2, Student B immediately identified it with one of the concepts discussed earlier, saying, "This is just what we were talking about--the two four-group sets . . . so you're going to have four-group numbers here, and you're going to have how many different possibilities . . . so you're going to have three four-group sets, and to do the problem I'd have to look it up . . ." She then skimmed her book looking for an appropriate formula, saying, "If I have n members in each of my sets, which I do--then you'd . . . 'cause I don't know how many members--well, . . . I do know there are four members in each set--except I don't know how many different four members I have in each set." After she failed to obtain

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any help from the book, she said, "I don't know how I would go about answering it . . . unless I used the formula $n(n-1)(n-2)(n-3)$, but what I would use as my n I don't know . . . I don't know how to do it." At this point she gave up on the problem. However, she added, "If I had [it] for homework, I would go to the Math Library and look up the answer, and once I had the answer I'd be able to figure out the problem--the steps in between. That's usually the easiest way for me to work it out."

February 25. Student B prefaced the main discussion at this interview by remarking that the lecturer's treatment of integers and rational numbers "hasn't been that bad"; Student B remembered most of the material from high school chemistry. She said that she used the lectures "more than anything" to learn the material and that she thought nobody would attend the laboratory if it were not required.

Proceeding to specifics of the lectures, Student B said that the explanation of the arithmetic of fractions had been helpful. She especially liked the depiction of fractions as portions of cut-up squares, calling this "an excellent way to describe why for fractions." She said that this "why," which she had not understood before, had been explained by the lecturer "very clearly." Student B had not understood the lecture (immediately before the interview) on decimals and scientific notation; she said

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she would try to learn this from the book and ask questions in the laboratory. She found unclear the lecturer's explanations of greatest common factor and least common multiple, as well as the explanation of the arithmetic of negatives. She had asked her laboratory instructor for help with these topics. Commenting on her varying responses to the lectures, she remarked, "Maybe it's just how it goes into your head . . . how well you're thinking that day and how well she's explaining. Those are probably the main factors, but some things go in a lot more clear than others."

Student B said that the most recent problem assignments had been "challenging," but that she had not read the book and had relied solely on her lecture notes (which she found sufficient) for help with the problems. She found a problem on putting a collection of fractions in order helpful in understanding the ordering of the rational numbers. She also liked the book's Supplement of pages reproduced from elementary school texts; this helped her to see more accurately the type of material she one day would teach. She said that she found some of the least common multiple and greatest common factor problems helpful after she had asked her laboratory instructor for assistance. She considered problems asking her to draw pictures illustrating equivalent fractions and to list fractions equivalent to $\frac{1}{3}$ to be a waste of time, since she understood these ideas. She had not done a problem asking her

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to argue that multiplication of integers distributes over subtraction; she did not understand it, and planned to look up the answer in the library, as was her custom. She also had been unable to construct sets of integers closed or not closed under addition and/or multiplication.

Student B said that the last two laboratory sessions had been "a waste of time." About the ruler-and-compass laboratory, she said that "nobody really did it" because they knew it could not appear on a test; this had been "a waste of two hours." Regarding the session on rational numbers, she said, "We didn't do that one either. Everyone was studying for the test." They had done some operations with Cuisenaire rods, such as making trains, so that "we got the general idea," but had not worked with GeoBlocks and tangrams, using the time for a study session instead. Student B said that her instructor had realized that the students were concerned about the test (which was two days off) and had not insisted on their doing the laboratory exercise. Student B felt that it was futile for the instructional staff to present new material on Monday (which would not appear on the test), a time when the students were concerned about the test scheduled for Wednesday. She said, "It just seems really foolish to even try to convert someone else's mind when the main thing you're at the university for is to get grades and you're worried about your grade in math, and there they're trying to give you something new. Nobody's going to work on it."

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[The instructor] knows it . . ." She said of the laboratory period, "It's a good review session if they would use it as a review session." Student B said that she would enjoy learning new things such as Cuisenaire rods if the laboratory periods were shorter.

On the problem sheet on prime numbers, Student B was able to identify correctly the prime and the composite numbers. Considering 119, she eliminated 2 from consideration, then divided by 3 and saw 3 does not go in. She then wrote $5\overline{)119}$, but immediately crossed it out, realizing it would fail. She then divided 119 by 7 and found 7 to be a factor. For 113, she eliminated 2 and 5 mentally as possible divisors, then in writing tried 3, 7, 11, 13, 17, and 19. When all of these failed she called 113 a prime. Similarly, for 227, she eliminated 2 and 5, then tried in writing 3, 7, 11, 13, 17, 19, and 23; when all of these failed to divide it she called 227 a prime. For 247, she mentally eliminated 2, 3, and 5, then in writing divided 247 by 7, 11, and 13, which went in.

On the greatest common factor and least common multiple examples, Student B in both cases factored the given numbers into primes. However, she took the least common multiple in the greatest common factor problem and vice versa. This indicates that she did not have a good intuitive idea of what the terms mean.

Student B first read Problem 3 aloud. After the investigator explained that the problem referred to 9 A.M.

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that morning, she again read the problem aloud. She thought she would be unable to do the problem. However, she drew a diagram indicating the town and placing both drivers 40 miles east of town at 1:00. She then gave as her answer 150 miles, saying, ". . . it's not right . . . I just don't feel like it's right." She said she obtained this answer by considering motion at 50 mph for 3 hours; in obtaining this she apparently thought Dick was in town at noon.

March 8. Student B said that the last part of the course had been "interesting" and "a good review" because "you keep hearing about [rational numbers] and you don't know what they are." She added that she could characterize the entire course as "a good review." She said that after feeling unsure at first of her understanding of decimals, she found this topic easy.

Student B said that the laboratory session on the geoboard had been "really helpful. I really enjoyed working with those." However, she found the session on the metric system "a waste of time," and again spent much of the laboratory time reviewing for the upcoming test. ("The two hours wasn't wasted as usual--it was put to good use.") Student B said, "You always keep getting the metric system thrown at you . . . This is coming, this is coming! When it finally comes, I think people will learn it. Until then, it's kind of a waste of time."

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Student B said of the last lectures of the course, "I think . . . at the end of the course she kind of got everything together, and she just presented things well, she didn't waste a lot of time. . . . It was very well presented--concise, and you understood what was happening." She later complimented the lecturer on "her organization [and] her clarity." The only topic which Student B did not understand was the circle; on the last test she had been unable, given the circumference, to find the diameter of a circle, although she knew how to do the reverse problem.

Student B said that in the last part of the course she had not read the book at all; she only had done the assigned problems with the aid of her lecture notes. She had found these problems helpful, and thought the lecturer had chosen them well. She noted that "some of them are rather simplified," but that "if I didn't do the homework I'd be lost." On specific problems, Student B found helpful a question on whether particular sets and operations had closure, and an exercise in arranging in order first a collection of positive fractions, then their opposites. She found troublesome several exercises involving decimals. Asked to find a fraction name for a repeating decimal (page 234), she said she did not understand the question, but knew the procedure from her laboratory work. She had trouble on problems (page 168) involving positive and negative multiples of positive and negative integers.

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Student B reminded the investigator that she had been unable to argue that multiplication of integers distributes over subtraction; since the previous interview, she had not read this section of the book but had tried to follow the explanation of the problem in the answer book and had failed. She said that "I guess I lucked out" because it did not appear on the test. Despite these difficulties, Student B said of the assignments, "I didn't have any problems, really."

Asked to find a rational number between $1/3$ and $1/4$, Student B first converted both fractions to 12ths. Finding the numerators adjacent, she chose 36 as an alternative common denominator. Writing the given fractions as $12/36$ and $9/36$, she gave $11/36$ as a fraction between them.

Student B correctly found the decimal name for $7/12$, catching an arithmetic error she made in the middle of her division. To find a fraction name for $r = .3777 \dots$, she first wrote $100r$ over r as if to subtract, but noticed the decimal parts of these expressions were not identical. She then wrote $10r$ and r , and after this $1000r$ and r ; as before, the decimal parts were not identical for either of these combinations. Noting that this had not been done in class and that she was therefore unsure of it, she subtracted $10r$ from $100r$ and obtained the equation $90r = 34$, and therefore she concluded that $r = 34/90$, which she reduced to $17/45$. On the next problem, she divided out

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6/11, obtaining an infinite decimal, and said, "That's the only answer I can give." On the problem of finding an approximate value for $\sqrt{7}$, she replied, "I don't know how to do that at all."

Student B's first response to the first measurement problem was to tell the investigator, "You're talking to the wrong person." However, she decided to "mess around with it." Her first attempt was to construct some circular fragments coinciding with the curve; after a while she decided she could not solve the problem this way. After the investigator reminded her that she could use any materials she wanted, she said, ". . . if I had a string I would put it around there, I'd pull it out and then I'd measure it." In response to the second problem, she said, "Well, the first material I'd use would be my notebook to look up the formula for area." She then wrote down the formulas $A = (c/d)\pi$ (sic) and $A = l \times w$, but did not use either of these, realizing they would not work for the given figure. She suggested surrounding the figure with a circle or square, computing its area, and then subtracting the excess, "except I don't know what you would use to compute those missing areas."

Student B received the grade of 3.0 in the course.

April 1. At this interview Student B said her feelings about mathematics had changed "not a bit. I told you when I came into the class--I didn't like math, although

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I could do it, and I still feel the same way . . . I got a 3-point out of the class, which isn't good--it isn't bad--but . . . I didn't flunk, I knew how to do the stuff, and I still don't like it. If anything, it made me dislike it just as much as I always did. It didn't help it grow at all . . . just another boring old class, same old stuff taught," a review of grade school mathematics. She said that her feelings about teaching mathematics had not changed--"unfortunately, because I know if you dislike a subject you tend to avoid it . . . with your class, which isn't good. Maybe they will [change] when I take math methods, or maybe the books will pull through when I'm teaching math in class, I don't know." Asked if her feelings about the purpose of teaching mathematics had changed, Student B replied they had not, and endorsed the idea of forcing pupils to do mathematics in the following soliloquy:

I think it's needed, very purposeful, but as I said before . . . I think you need to teach the basics . . . kids . . . need to know how to do percentages, and they need to know a little bit about decimals and stuff like that, although . . . it seems like they're taught so much more . . . they're shoved all this information--learn it, there'll be a test next Thursday . . . how much of it you actually retain--like looking at this class, I remembered the basics . . . and so maybe by shoving all this at them--learn all this right now . . . they will remember the basics too by the time they get out so that when they . . . start having a family or anything or start in the business world, if they haven't had to take a lot of math, and it's mostly on what they remember from their elementary education . . . they'll be prepared enough to do story problems--like, you're buying a dress . . . or, buying food at the grocery store, just simple

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Student B said there had been no change in her feelings about her major.

Asked what about the course could have been better, Student B suggested it be offered in small classes rather than the large lecture. She found lectures helpful, and in a change of opinion, admitted, "Much as I hated the labs, they are probably a pretty good idea, as far as introducing new things, although I think they should hold them to an hour . . . People just get disinterested at a length of two hours." Discussing specific laboratory exercises, Student B said she had found Dienes Blocks "very interesting," and conceded that the session on the metric system was "needed [but] I just didn't find it interesting." She could not recall another session she thought was interesting but specifically named clock arithmetic and Cuisenaire rods as not being interesting. She said nothing had hindered her learning, except possibly the length of the laboratory period.

Student B said that she would not take any further mathematics besides the required methods course.

She said her participation in the study "seemed very routine" and she had left the interviews hoping she had been helpful. As a possible improvement in the study, Student B suggested having some group sessions at which students could react to each other's ideas about the course;

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a study could consist of all group sessions or some group and some individual sessions. She said her questions would probably be the same, but suggested more emphasis on attitude, which she felt was an important characteristic of a teacher. As another area for investigation, Student B recommended the teaching of the laboratory classes. She felt the laboratory should be taught by very effective teachers skilled in presenting the material in an interesting way, not by teachers such as her instructor, who, while friendly and helpful, was indifferent toward the material he was teaching. She pointed out that the students of Math 201, as prospective teachers, were looking for models of good teaching.

Asked whether specialists or generalists should teach elementary school mathematics, Student B said, "It might be more effective if it were taught by math specialists, although here again they would have to be--they'd have to know how to work with children, they'd have to be good teachers or know what they're doing in the classroom. . . . They couldn't just be brains you pulled off the street and knew calculus and trig and algebra . . . They'd have to know what they were doing with young kids and that's probably why it's stuck with the classroom teacher." Student B went on to say that she liked the idea of team teaching in a school, with one teacher, familiar to the students, responsible for mathematics and others responsible for other subjects.

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As a final comment, Student B suggested that methods be emphasized more in the course. This was what she had expected at the beginning of the course.

Evaluation. Student B was quite cynical about the value of the course, and for her it did very little. She had some facility with figures and seemed to understand the concepts as well or better than students whose grades were higher. Her perceptive comments and suggestions about the laboratory in her final interview were in contrast with the mechanical view of teaching she expressed earlier in the term.

Student C

Student C was a freshman from Cheboygan. In high school she had taken two years of algebra, one year of geometry, some trigonometry, and a little calculus (which she had attempted to learn in an independent study course). She had enjoyed her mathematics courses; she also had liked history, while she had disliked English grammar. She could not explain the reasons for her likes and dislikes. She had participated in numerous high school activities, including working as a teacher's assistant in a ninth-grade general mathematics class.

Student C had taken Math 111 (a pre-calculus review course) at Michigan State in the fall of 1973. Despite her extensive high school mathematics background, her grade

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had been only a 2.5 because she had forgotten some things and had misconceived others when she had first tried to learn them, and the misconceptions had never been corrected. She generally had enjoyed this course and the study of mathematics in general; this struck her as being unusual because "it's supposed to be a class that girls aren't supposed to like."

Student C was pursuing a mathematics-science specialization within her elementary education major. She wanted to teach "because I really like young people and working with them . . . I'd like to be able to give them a good background . . . what they need." She was conscious of the possibility of school boring her students, saying she wanted to help them "learn to enjoy school instead of hating it." Her teaching experience, in addition to the work as a teacher's assistant, had been acquired in babysitting, tutoring, and working on a "recreation project." She also had taught her high school biology class for several hours when the teacher had been absent. She said, "I like the idea [of teaching mathematics] 'cause I've been teaching it all my life." She said the purpose of elementary school mathematics is "to give them the background for higher-on in their math, because even if they don't want to, they're going to end up having to do some more math." However, the students should also "see how it applies to their everyday life--not just you have to know it 'cause you have to know it."

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January 22. Student C suggested at this interview that the course should be presented in two sections--one of them an honors class for students who already knew all of the standard course material, as she did. She called the class "boring--I write my letters to home in that class . . . I don't like the way it's presented, I don't like the subject matter, and I don't like taking the class. If I had my way I wouldn't have taken this class--I'd have taken some other math . . . Since it's . . . required, they could try to make it interesting, but they don't. I haven't talked to anybody that's liked the class yet." She disliked the lecturer's teaching style, calling it "on a first-grade level . . . to get us to teach that way, I guess . . . I just don't like it. It makes you feel put down . . . to be taught at that level." Student C remarked that nothing had helped her since she already knew all the material, and that for this reason she also had found all the homework routine.

Student C said with a giggle that the laboratory sessions were "more funny than class." She said she had not learned anything new in the laboratory, but had not been bored. She was surprised to see that some students had difficulty with the laboratory.

The first problems presented to Student C at this interview were those on sets. She said the set of all living people matches set E because it is infinite, while a set matching the set of counting numbers from 1 to 100 "would be E because D stops at 99." However, she said the

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set of suits matches "C, because it's four different things." Considering the set of all aardvarks enrolled at M. S. U., she said, "It's not up there. It would be the empty set, wouldn't it?" She then said it matches none of the given sets. Her responses to the first two questions indicate that Student C did not have complete mastery of this material.

Student C correctly converted 405_{eleven} to base ten, multiplying 4 by 121 and then adding 5. She also performed the two reverse conversions without error, in each case labeling columns correctly in the base asked and then placing the appropriate digits in the columns. She solved the first missing-base problem by considering that "nine and seven is sixteen," so that a twelve must have been carried and that therefore this was the base. She solved the second of these problems by reading the subtraction example as an inverted addition example, and in this context reasoning as before, concluded that the base was three.

Student C was the only subject to recognize and solve Problem 1 as an explicit base five problem. She wrote the numbers of coins in three columns (quarters, nickels, pennies--from left to right) and added the two rows base five to get the answer.

Student C said that she had not known what to expect in the interview.

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February 5. Student C said that the most recent part of the course "was OK . . . basic review from a long time ago . . . didn't help me, but it helped some people." She still objected to the lecturer's style of presentation--"You're a little kid." She recommended that the material be presented at a college student's level; the student could then bring it down to a child's level while teaching. She said that nothing in the lectures had helped her, although "there were a couple of new ideas." She could not recall being confused about anything, except once when she had left class confused, but afterward cleared up her problem.

Although she had never seen lattice multiplication before, Student C said that she felt "negative" about the laboratory session on computation. She conceded that "it could help some people" and that "you have to have several approaches to everything," but felt that "to present too many different ways, too many different ideas at once--it tends to confuse a person." She pointed out that two students at her table had been confused, had done the exercise wrong and "went back to the old way." Regarding the session on multiplication and division, Student C said that she "didn't care for it, but I guess we had to cover it in lab, too." She continued, "Maybe I just don't like it because it's so elementary to me, and I've had it so many times before, and I know it, and--so I just don't care for having to go over it another time after I've gone

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through twelve, thirteen, fourteen years of math . . . I'm not tired of math, I'm just tired of learning the basics over again . . . so I don't really care for the course. I think it's a waste of my time, and a waste of the university's time, as far as I'm concerned . . . I guess it's been the wrong approach to the class."

Student C said she could not comment on the homework because she had not done any since the last test (two weeks before). She said that she had found no new "basic concepts" in the book, but that "a couple of the methods" were unfamiliar.

Student C said that the pace of the course to this point had been "slow--she isn't covering very much of the book." She recommended that more time be spent on topics which the lecturer was skipping.

In attempting to solve Problem 2, Student C first pointed out that there were ten ways to fill each blank space in the number. (She wrote both 10 10 10 10 and 10 9 10 10, the 9 perhaps because she thought there had to be at least two different digits in the number.) She said, "Right now I forget the mathematical way to do it--it's something factorial, I think." She then noted, "You can find out how many for each exchange and multiply that by 3." After some thought, she said, "I'd rather go back to last week's [problems]." Student C laughed at her inability to solve the problem. After some consideration, she decided that the number of possible endings is $40!$ and

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that therefore the number of phone numbers is $40! \times 3$, but said of this answer that "it just doesn't look right." After some further thought, she decided to leave this as her response.

February 19. Student C said that the most recent part of Math 201 had been the "same as the other part." She still felt that "it's all basic review" and found nothing interesting in the lectures. She said the lecturer "takes a long time covering it. I still think she should . . . go faster, but I guess she doesn't feel that way" While Student C was satisfied with her own performance on the second test, she inferred from the lecturer's repeated assurances that the scores would be curved that the class as a whole was not. She reiterated that she found Math 201 "a good class to write your letters home in."

Student C said of the assignments she had done before the last test, "They were mediocre . . . some of them helped on the test but a lot of them I think were just general busywork . . . I guess they've got to give us busywork . . . a couple of them did help, not much . . . if you were a person who doesn't understand it I guess it would help you." Her only comment on a specific problem was to call "interesting" a problem (page 100) containing an excerpt from Lewis Carroll's The Hunting of the Snark.

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Student C said "the last two labs have been pretty bad . . . we haven't been doing too much . . . we do it, and get it done, and that's it." She pointed out that clock arithmetic could be worked out without manipulating a physical clock. (She also protested the appearance on the test of clock arithmetic problems of a type not previously assigned; moreover, she felt it had been insufficiently covered but added that she had not studied it.) Student C had stronger feelings about the following session, in which GeoBlocks, tangrams, and Cuisenaire rods were used to illustrate ideas about rational numbers. Discussing GeoBlocks, she said, "It was strange trying to find the areas of the triangles in that [laboratory session]. You had to remember your formulas . . . half of a rectangle is a triangle . . . it was OK, but there were so many different shapes, you could be sitting there all day figuring that out . . . and some of them--you'd just say forget it . . . really, after a while, you get tired of it." She later complained, "You can [sort them out] for an hour." While she found none of these activities interesting, she said "the colored ones [Cuisenaire rods] were the best because they were colorful, and there were easier things to do with the colored ones . . ." Student C then criticized the whole manipulative approach itself; she continued, "If I had to do that when I was in first or second or third grade, I would have just said forget about math, forget about everything, and just take it, and

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leave it!" She felt that a child would find frustrating an exercise such as naming the sizes of various GeoBlocks. Recalling her own childhood, she remarked, "I liked blocks to play with, but not to work with. Blocks are supposed to be something that gives you pleasure, not frustration." Nevertheless, she did see the possibility of using the blocks in teaching. She said that the two hours in the laboratory went quickly, but that "I don't really know if I got anything out of it or not."

Student C said that she still wanted to teach mathematics; she was not yet frustrated enough to give up this goal. The course, however, had "stayed the same--mediocre."

On looking at the number 119, Student C said that she did not think it is prime. She then tried some divisors, mistakenly found 3 to be a factor, and said that 119 is not prime. For 113, she mentally tried as divisors 2, 3, 5, 7, 11, and 13, all of which failed to go in. Expressing her doubt that 17 would go in, she called 113 a prime without trying 17 as a divisor. For 227, she tried (again mentally) 2, 3, 5, 11, 13, and 7, all of which failed to go in. Squaring 17 and noting that its square is greater than 227, she called the latter a prime. For 247, she tried 2, 3, 5, 7, and 11 as divisors, all of which failed. She then tried 13; after subtracting 130 from 247, she stopped dividing, apparently sure that 13 is not a factor of 117 and therefore does not go into 247. She then

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called 247 a prime number. This exercise revealed Student C to be somewhat sloppy in arithmetic.

In doing the exercises on greatest common factor and least common multiple, Student C in each case factored the two numbers into primes, then took the correct combination of factors.

Student C was one of only two subjects who solved Problem 3 correctly, and she did it without a diagram. After first finding that Dick was 10 miles west of town at noon, she easily concluded that three hours earlier he was 160 miles west.

March 5. Student C said of the last section of the course, "It wasn't too bad. It wasn't too good. It was just--there . . . something that had to be done." She mentioned that while she had been bored, she was resigned to the class being boring, and this part had been neither better nor worse than what came before. She had seen no new concepts except in the laboratory. She said that nothing in the lectures had clarified her understanding of material she already knew; she felt that she knew the material as well before the course as she did at this point. She indicated that other students she knew had benefitted from the course, even though she herself had not. She said the only good that the course would do for her was to increase her average.

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Student C said about the laboratory exercises she had done that for some of them she could "see why I haven't been taught that way." Asked to name an example, she mentioned an exercise in which the student was to balance a cutout circle of radius r with squares of side r to see how many such squares would balance the circle. Her group had found that four such squares balanced the circle exactly! She said that this session had reminded her of high school geometry. She had disliked the session on the geoboard because "I can see it right away." She felt that she had not learned anything from either exercise. She criticized the lecturer's description of π as "3-plus," saying a student might become confused at the point where he had to switch from 3^+ to 3.14; she recommended using 3.14 from the beginning.

Student C said of the textbook, "I think somewhere there has to be a better book than that [because] some of its explanations confuse you more than they help you. . . ." She also criticized the book for emphasizing some items which she felt did not merit such emphasis. She could not say specifically what these items were, but said that she had skipped some sections of the book. She said that the assigned problems were "not really helpful and not really bad . . . busywork problems to do." She also said some of the book's answers were difficult to follow, and that these discouraged the student from seeking an explanation of a

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difficult topic; some of the book's explanations, she said, are "really bad."

To find a rational number between $1/3$ and $1/4$, Student C converted both of these fractions to 12ths; finding the numerators consecutive integers, she converted both to 24ths and found $7/24$ as a number in between. However, she said "that isn't a rational number." She then said that the lecturer had "goofed me up" on rational numbers--"I didn't like her explanations . . . I just didn't comprehend what I was reading last night." In the course of subsequent discussion, Student C said that $1/3$ and $1/4$ are rational numbers, while $2/3$ and $7/24$ are not. (The investigator surmised from the discussion that she thought a rational number is the reciprocal of an integer.) She said that while she could not remember how to find the rational number between these two, she knew "there has to be one."

To find the decimal name for $7/12$, Student C divided 70 (sic) by 12, obtaining an answer ten times the correct answer. She apparently had misplaced the decimal point in the quotient but did not notice the discrepancy between $7/12$ and $5.8\bar{3}$. On the next problem, she immediately wrote $100r - 10r$, and obtained $r = 34/90$, the correct answer. On the third question about decimals, Student C divided 6 by 11 and obtained a repeating decimal, then did no further work, thus failing to comply with the request in the problem. To find an approximate value for

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$\sqrt{7}$, Student C used the square root algorithm, carrying the calculation out to one decimal place. (If she had continued, she would have been in error, because she made a mistake at this point.) Student C was the only subject to attempt the problem in this way.

Student C gave several ideas for measuring the length of the given curve. (The investigator suspected that some of her suggestions were facetious.) Some of her ideas indicated she was thinking about area; therefore she may have been confused between the measurement of area and that of length. For example, she suggested that "you use that stupid board that I didn't like . . . the geoboard . . ." to compare the figure with square units or use Pick's theorem. After finding the area, she suggested that the perimeter be found using a value of "3-plus" for π . Also considering area, she drew a rectangle around the figure and suggested consideration of the interior and exterior of the curved figure in the rectangle. She also suggested using a string to measure the length of the curve. To find the area of the second figure, Student C said that the easiest way would be to use the geoboard, either enclosing the figure in a rectangle and subtracting the excess, or using Pick's theorem.

Student C received the grade of 4.0 in the course.

April 2. Asked how her feelings had changed with regard to mathematics, eventually teaching mathematics,

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the purpose of teaching mathematics, and her major, Student C reported no change of feelings toward any of these. In the course of answering these questions, she mentioned that she had seen some things which she "definitely will not do," and that her desire to teach had been "strengthened" because of her opinion of the lecturer: "I didn't think she was that good [as a teacher] . . . I'd like to get out and teach and show kids all teachers are not like that."

Asked what about the course could have been done better, Student C replied, "I think the presentation of most of the material could have been done better. I don't know exactly how for . . . each thing, but I didn't think it was presented very good and . . . I don't think it really helped kids understand it that much. It didn't bother me because I understood most of it, but some of the kids that I talked to that didn't understand it--they still do not understand it, or are more confused. And I think she [spent] too much time on most of the areas . . . I think we should have got through more of the book . . ."

Student C said that she did not remember any of the laboratory classes as being effective, and also reiterated her resentment at being asked to do activities which she considered intolerably juvenile, such as playing with "picture blocks." However, she also said that the only session which had been "really wasted" was that on the ruler-and-compass constructions. Asked if anything had

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hindered her learning, Student C answered, "Going to her lectures . . . just the way she presented it. And some of it's probably me. Some of it I know wasn't, because I talked to other kids, but--the way she presented it really turned me off and I don't really care whether I listened or not. I listened, but . . . it wasn't really something that I looked forward to. I think you learn more when you look forward to a class."

Student C expected to take more mathematics; she planned to take at least two quarters of calculus and, for her major, another elementary education mathematics course.

Regarding the research study, Student C remarked, "I'm just wondering what you're going to get out of the interviews. They're [sic] all been pretty much negative." She added that she had felt "just normal" in the interview situation. She could not suggest any possible improvements in the study, additional questions, or other possible areas of investigation.

Considering the question of general classroom teachers versus mathematics specialists, Student C said, "I'd like to say general classroom teachers, but knowing how some of the kids did in the class that don't care for math, and their attitude toward the class, I'd say math teachers, but I really hate to say that 'cause I think you can do better if the teacher knows the stuff in the general classroom."

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Evaluation. Of all the subjects, Student C was the most negative in her comments about the course. Despite her mastery of the course material as reflected in her grade, she was sometimes sloppy in arithmetic. In Student C we have an example of a student who was over-prepared for Math 201 and therefore bored and resentful. It is the author's judgment that there are enough such students in the course each term to justify a special honors section in which they would be free to pursue studies beyond the usual Math 201 material.

Student D

Student D was a freshman from Traverse City. She had taken three full years of high school mathematics, including algebra, geometry, and some trigonometry. She had a very favorable attitude toward her high school mathematics, remarking "I love math" at the interview. She had been in honors classes and said she had learned well in her high school math classes, except in ninth grade when she had had a personality conflict with her teacher.

While she had disliked English, government, and history, her favorite high school subjects had been mathematics, sciences, and music. She said that she had liked these subjects because they "were a challenge." She had come to like mathematics after writing a report on its applications in daily life.

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Student D was still unsure of her major. Uneasy about job prospects, she was taking background courses for both an elementary education major and a computer science major. She said she had always wanted to be a teacher, but had had no teaching experience of any kind prior to the study. Although somewhat unsure of her ability to teach mathematics eventually, she was confident that she could do it at the elementary school level. Responding to the question about the purpose of teaching elementary school mathematics, she said, "It's something we need in our life. Those kids are going to use it right away, and you've got to teach it right away. They've just got to know it . . . Everything they do, they've got to know math . . . The sooner they understand it, the better off they're going to be."

As she was leaving the initial interview, Student D remarked that she had just attended her first laboratory session in Math 201 and had loved it. The session had involved games with A Blocks and the accompanying loops and labels. Student D was proud that she had stymied the opposition in a guess-the-labels game by labeling two loops the same.

January 21. Student D said of the lectures at this interview, "I do enjoy them. I feel she's a very good teacher." She described what she liked about the lecturer's teaching style--"any time she'll stop and explain . . . She

went over it and over it ["base numbers"] until you did understand it . . . just like she knew that . . . most people have problems with base numbers because I never had them in grade school and I don't think very many of these kids have." (Student D had seen bases in a computer science course.) On the negative side, she said that the lecturer explained things too many times. Student D said she usually read the book after attending a lecture on that topic; otherwise she would not be interested in the lecture. She noted of the upcoming test that it had helped her to be forbidden to convert to base ten when working in a nondecimal base.

Student D said that she sometimes found the homework confusing. Although she felt that "you've got to try it" to see if you really understand the material, she sometimes skipped problems she found to be repetitive. Among those exercises she found helpful were working out "the long way" the number of one-to-one correspondences between a set and itself, and considering how to teach the meaning of "round" to a three-year-old. She found it laborious to invent numerous descriptions of the empty set. She had miscounted the number of subsets of a given set by one, neglecting the empty set. Student D had had trouble understanding the definition of proper subset, as well as with the relationship between set operations and number operations. She considered a long list of statements about subsets to be labeled true or false (page 27) to be a good

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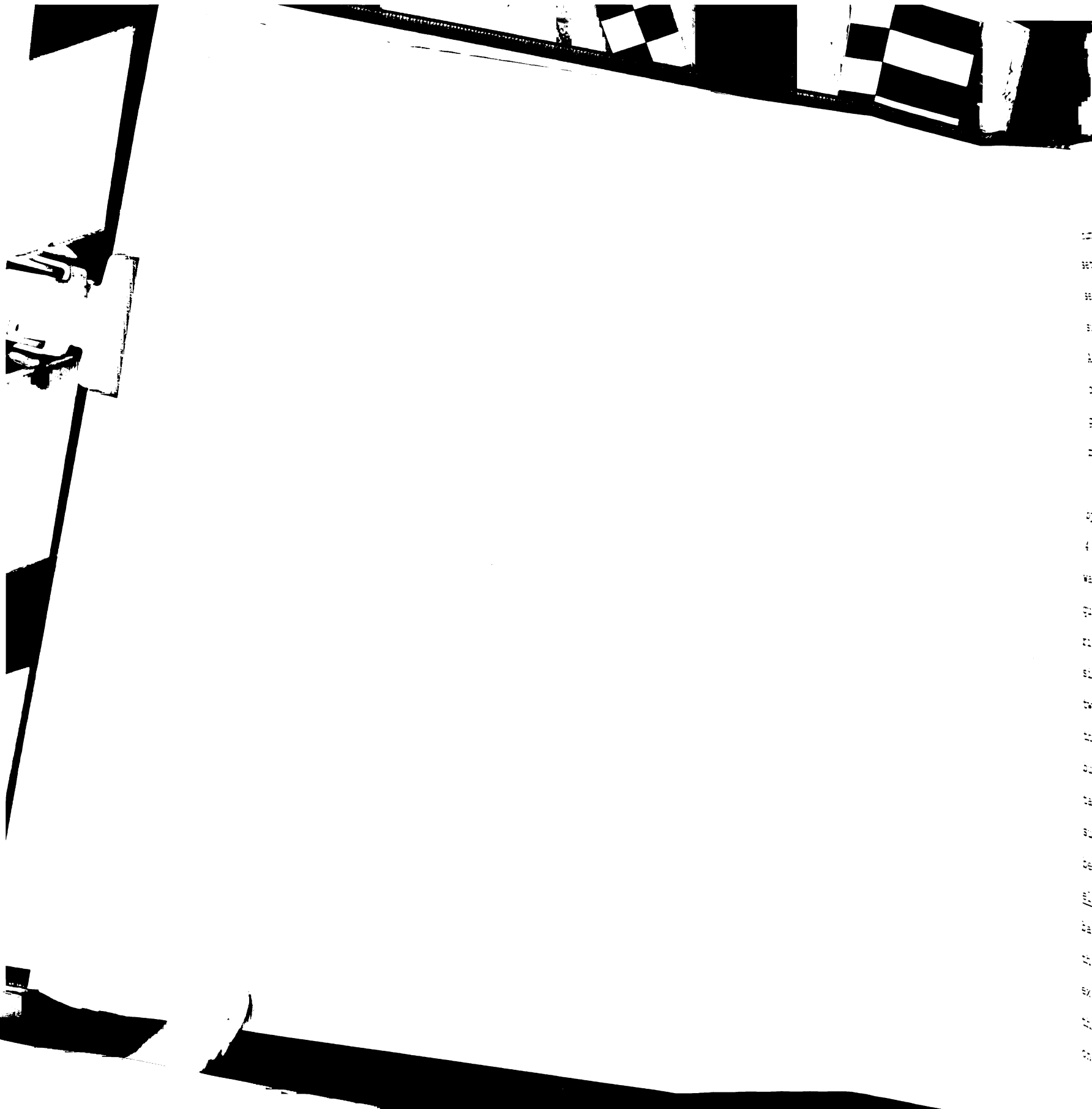
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problem. She enjoyed doing arithmetic in bases ("It's more like math to me doing the adding and subtracting . . .") and a problem in finding various set combinations under intersection and difference operations. ("You just have to sit down and figure out.")

Student D found the laboratory classes "very useful." Her group had required several attempts before understanding the addition and subtraction exercise with Dienes Blocks. She had enjoyed the guessing exercises with A Blocks; trying to get a fellow student to understand them had been a teaching experience for her. She found this activity "worthwhile--it's not games that you're playing; you're learning something by seeing it."

Student D said of the course as a whole that she enjoyed learning why mathematical processes work, as opposed to the mechanical approach of most mathematics courses.

The first problem sheet presented at this interview was that on sets. Student D said the set of all living people matches none of those listed "because to me match means match one member to another member exactly and having none left over, and Kennedy, Johnson, Nixon are three, and there [are] a lot more living people than that, and the rest have no meaning of living people at all." While this last phrase suggests that Student D may have confused equality and matching, the subsequent questions show she had not. After saying none of the above for the set of counting



numbers from 1 to 100, inclusive, she changed her answer to set D after noticing that 0 is an element of that set. The set of suits was said to match set C "because there's four suits in a deck of cards and there's four objects in there." After she was told what an aardvark is, she said the set of all aardvarks enrolled at M. S. U. matches none of the given sets because "it's the null set--there are no members," and none of the listed sets is empty.

Student D correctly converted 405_{eleven} to base ten by labeling her columns, computing 11×11 , and then taking $4 \times 121 + 5$. Her method for converting 39_{ten} to base two was to divide repeatedly 39 and its successive partial quotients by 2, placing the remainders in columns from right to left. She converted 44_{ten} to base seven similarly. On the first missing-base problem she wrote twelve and said, "but I'm guessing." She first thought that base twelve would not work in the left-hand column, but after some reconsideration noticed that it would. After taking a while to realize that the second example was subtraction, her first reaction was to say base seven because when she borrowed one she thought she was taking eleven minus two. Thus if nine was represented __2, the base was seven. However, this did not work in the rest of the example. After the investigator reiterated that the example was correct in some base, Student D thought of base three because the only digits in the example were twos and ones. She checked out this hunch and realized that it was

correct; in particular, she saw that 11_{three} is four. She later remarked that the laboratory session on Dienes Blocks had helped her with these problems.

In response to Problem 1, Student D said that "logically, quarters are going to contain more amount of money." She wrote down that there were 3 quarters. She then considered that there were seven nickels, wrote a 1 under the 3 quarters, then wrote 2 nickels. Considering the six pennies, she put a 1 under the 2 nickels, then wrote 1 penny. Adding the columns, she obtained the correct answer.

Student D remarked at the end of the interview that she had felt "afraid I'm always going to say the wrong thing." Asked what would be the wrong thing, she replied that some of the course material was boring--that material which was familiar to her. In her remarks Student D focused on the lecturer's style rather than on the material. She said that she "loves" the problem part of the interview, finding it a challenge. She was somewhat nervous about her problem-solving being preserved on tape, asking, "What if I don't get them right?" The investigator reassured her of both the nonpermanence of the tape recording and the importance of revealing her true feelings about the course.

February 4. Student D said of the lecture material at this interview that "still you know it, but it's not the basic facts like it was before and you knew it so well . . .

it's getting harder, so it's more interesting." She had had some difficulty with clock arithmetic. She was not sure whether or not she had seen it before, but even if she had, "I just don't remember anything of it at all." Student D had read the book's presentation before realizing that there would be alternative sources. She found multiplication and division in nondecimal bases "easy." She had seen the concepts of least common multiple and greatest common factor before; the treatment of these topics had refreshed her understanding.

In the book she had found confusing the treatment of multiplication; she had skimmed it in her eagerness to get to the problems--"my mind wasn't on it and then all of a sudden I went through something that I didn't know that well . . ." She also found confusing "this guy's chart--whatever his name is" (the Sieve of Eratosthenes), saying, "It's just numbers." She also mentioned "the Austrian method of division" (possibly meaning the Greenwood algorithm) as confusing, and was perplexed by her laboratory instructor's demonstration of division using Dienes Blocks, although "multiplication was easy." In the homework assignments, she had not understood some examples of greatest common factor and least common multiple and could not solve a problem asking for the relationship between them.

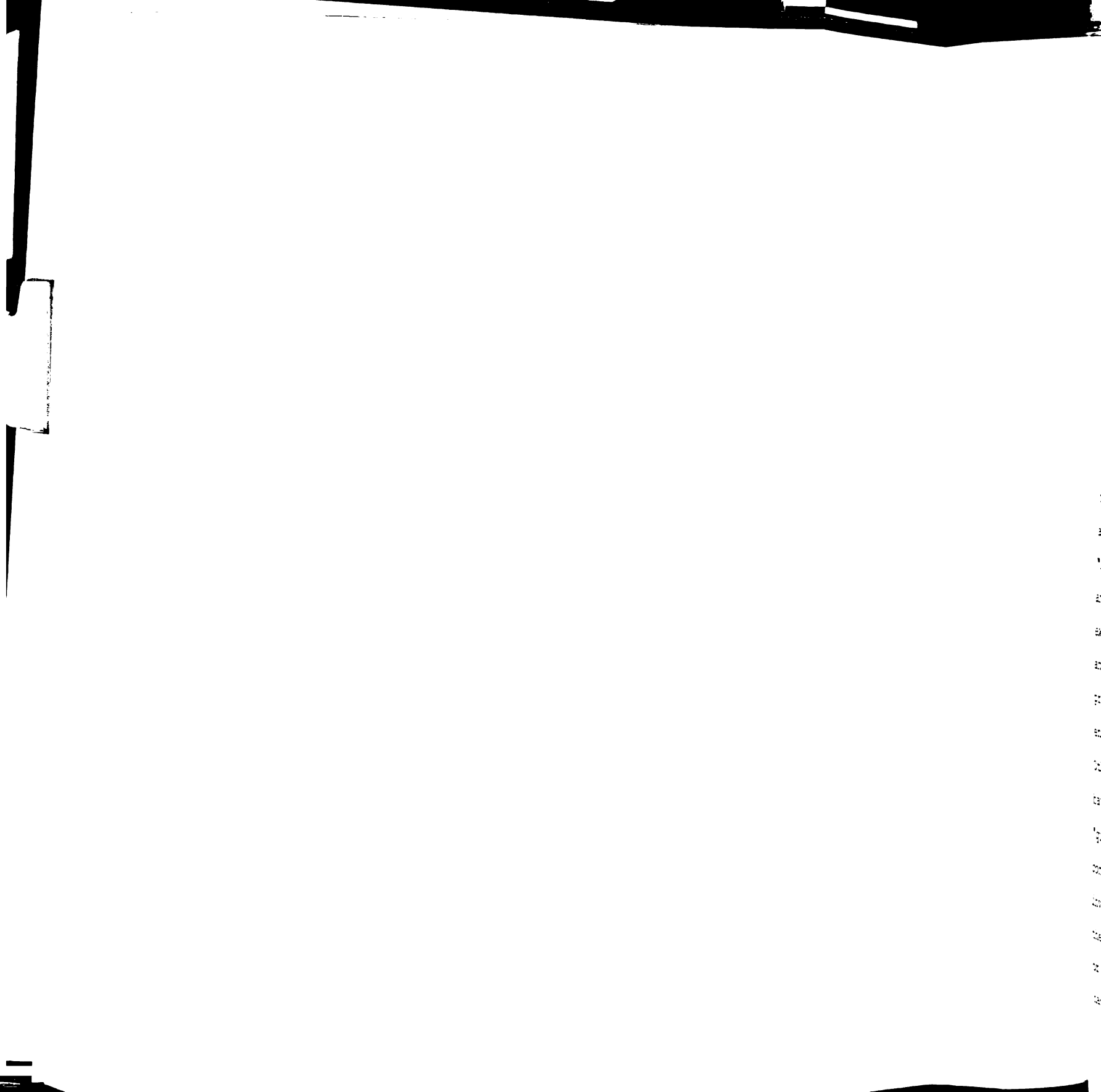
In the laboratory session on computation, Student D found some of the exercises confusing, but did not know their names. (The investigator surmised that they were the

Whitney Mini-Computer and Russian peasant multiplication.)

In general, she said the course was "pretty clearly put" and that the lecturer was "always willing to answer, or take time to make sure you understand."

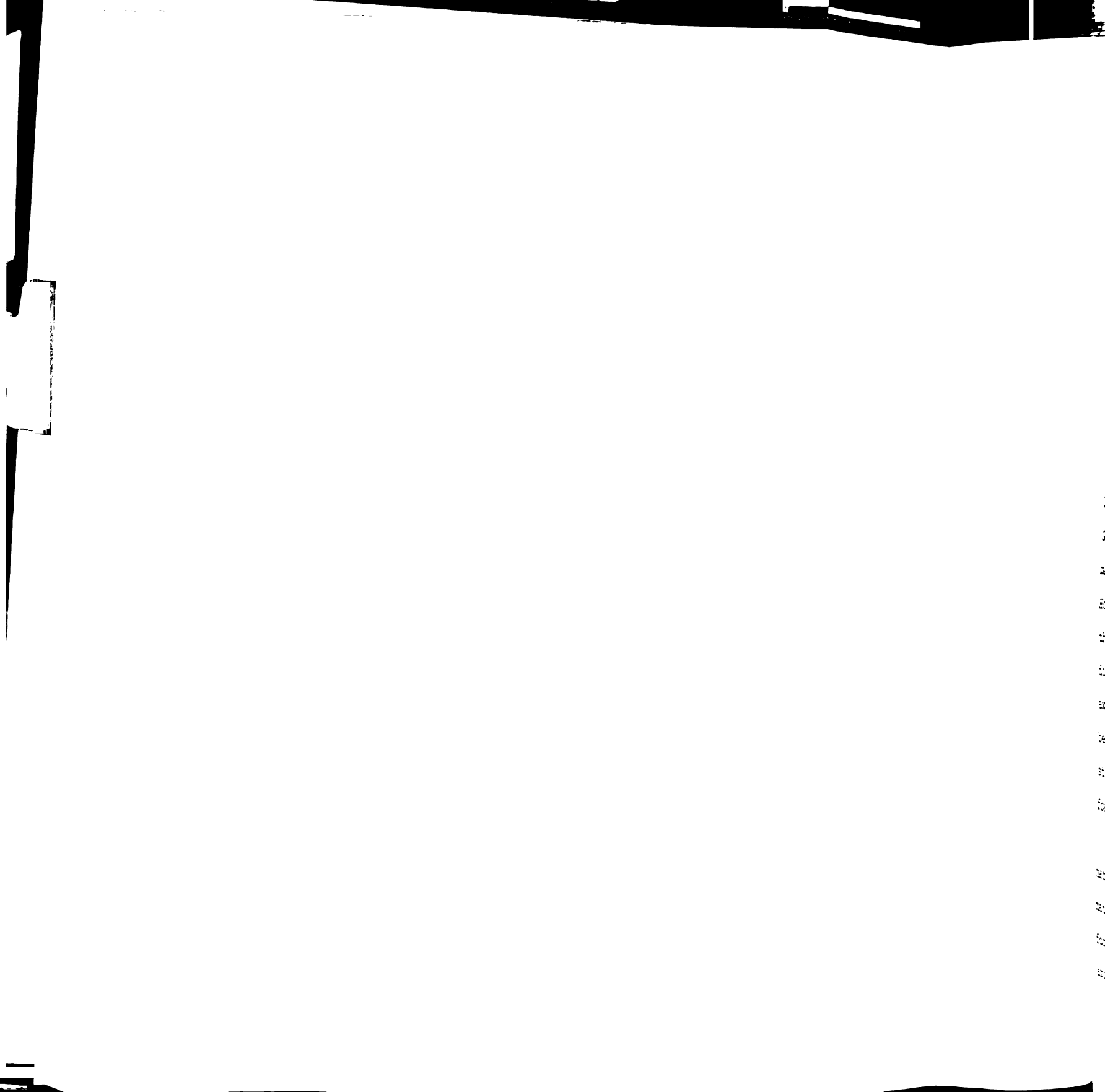
In response to Problem 2, Student D said, "First, you have to take the four numbers after the 332- . . . OK, there's ten numbers it could be, and four each time." She then said, "What I want to do is take my 10 and say $10 \times 9 \times 8$. . . all the way down, but the thing that tells me not to is because I have four numbers . . . it's going to be the second part of the number." Student D then proceeded to compute $10!$, apparently thinking this was the number of choices for each place, and then multiplied this number by 4 (for four places) and then by 3 (for three exchanges), obtaining as her answer 43,531,200.

February 18. At the beginning of this interview Student D praised the lecturer's presentation. She had liked especially the explanation of the order of integers, the lecturer's use of the number line, her clarification of the concepts of "opposite" and "negative," and her (and the book's) use of red and black pictures to illustrate negative and positive quantities. Student D found confusing only an occasional statement of a definition. She said the class was confused when $-a$ represented a positive number in the case where a is negative. In general she said that students "now have to think a little bit harder" and



that the course had become "more and more interesting, because it's getting difficulter."

In discussing the book and the assigned problems, Student D said that she liked the book's explanations of its statements--ideas which she previously had accepted without explanation. She was helped by the homework, which forced her to think, and by the lecturer's explanations of the problems. She added, "The problems are helping a lot more . . . than they were in the beginning." One of the problems Student D did not understand until it had been explained was why $7 \times 6 \times 5 \times 4 \times 3 \times 2 + 1$ had a prime factor greater than 7. On a problem involving positive and negative multiples of negative integers, she admitted, "I didn't really want to put that much thinking into it and I didn't." She was surprised by the lecturer's explanation of it ("I never would have thought of that") and resolved that "I'm going to know that now . . . That problem really helped." She had learned from this experience not to skip problems which appeared to be easy. Other problems provided her practice with concepts she may have forgotten. Student D said that the lecturer explained "questionable" problems "very well." She mentioned as confusing a problem about temperatures in Alaska (page 167) involving multiplication of integers ("I wasn't thinking then") and one asking her to find sets of integers closed or not closed under addition and/or multiplication. She had been unable to find a set of integers which is not



closed both under multiplication and under addition. She termed "more difficult" those problems which ask the student to explain something; she could not be sure her explanation was correct.

Student D said that she felt the laboratory classes were "a very good part of the course and I hope they don't get rid of them." She enjoyed the informality of the situation, noting, "You're having fun, but yet you're learning." She herself discovered the ideas taught, rather than waiting for the instructor's explanation. Student D found clock arithmetic interesting since she had never before seen it. She also had never seen materials like GeoBlocks; she said that working with GeoBlocks "really made you understand" the idea of volume measurement. However, in working with Cuisenaire rods, Student D found the idea of a train and the process of multiplication and division with the rods confusing until after it had been explained by her instructor. Division with the rods had been difficult because this had been the last exercise and students had been eager to leave class. Student D had found the tangram puzzle "challenging."

Student D said that she would like to teach this mathematics in elementary school. Previously she had thought mathematics would be too difficult to teach. She looked forward to the opportunity to help children understand arithmetic.

Student D said of the course at this point that it was "getting more interesting and difficult all the time . . . it's more challenging now . . . I just can't wait to see what's coming next, how she's going to explain it . . . how she's going to work her way out of it this time . . ."

Student D expected to enjoy the last part of the course even more than she had enjoyed the preceding parts.

In the problem set on prime numbers, Student D correctly identified the prime and the composite numbers. She mentally divided 119 by 2, 3, 4, 5, 6, and 7; about to stop trying and call it prime, she rechecked her division by 7 on paper, saw that it went in, and concluded that 119 is not prime. For 113, she first tried numbers through 8 as divisors, looking for 7 in particular. Later she tried 11 and 17. All of these divisions were mental. When none worked, she called 113 a prime number. For 227, she skipped 2, then divided 227 by 3 in writing. After this failed, she mentally divided 227 by 4, 5, 7, 11, and 17 (and perhaps 13). She called 227 a prime after none of these numbers divided it evenly. For 247, she eliminated 2 and 3 from consideration, using the divisibility test for 3. She then mentally divided 247 by other numbers. She wrote out the division by 13, apparently having seen it succeed mentally. She then said 247 is not prime.

In the exercise on least common multiple and greatest common factor, Student D in each part correctly

factored each number into primes and then took the correct combination of factors.

In attempting to solve Problem 3, Student D first wrote down some data from the problem. She then reasoned that since Dan was going 40 mph, he was 80 miles (sic) from town at 1 o'clock. To determine Dick's position at noon, she subtracted 50 from this, placing Dick 30 miles east of town at noon. Since 9 A.M. was three hours earlier, Dick had come 150 miles in that time, so he was 120 miles west of town at 9 A.M. This solution would have been correct but for the error in determining Dan's 1 o'clock position; it seems this error was careless. Student D tried to check her answer, but became confused trying to add integers with opposite signs and abandoned the check, leaving her original answer.

March 5. Student D said at the beginning of this interview that this part of the course had been "more difficult" than the preceding parts. She still did not understand that part of the book's discussion of real numbers which had not yet been discussed in lecture (dealing with square roots) and continued to have difficulty with percent, saying, "I can never get percent right anyway." One reason for this was that "I have lots of problems with story problems. I can't just sit down and read a story problem and figure out what they're asking [although] I know the formula." She tended to produce an

incorrect mathematical formulation of the problem because she "can't read the words right." She attributed the "scared" feeling she had when confronted with a story problem to her school experience, in which she had been told, "Story problems are very hard." Her difficulty with percent problems was not knowing how to arrange the numbers which appeared into an equation; she thought that she had never learned this. She thought that the course was helping her in this respect, but felt that "I'll have to wait for the test to see for sure."

Discussing the lectures, Student D first said that she could name nothing specific that she had found helpful, but later mentioned that she had been helped by the discussion of terminating and repeating decimals. She thought it good that the lecturer used examples which made her think beyond what she knew securely. The night before the interview, Student D had attended a review session for Test 3 but had been disappointed because she was familiar with all the material discussed.

Student D said that she did all of the assigned homework problems, but sometimes fell behind because she liked to wait until after a topic was covered in lecture to do the problems. (Otherwise she would be bored by the lecture.) She had tried to read the chapter on real numbers but had given up, hoping to pick up the material in lecture. Student D had enjoyed doing all of the percent problems. She also found helpful problems (page 219)

involving finding fraction names and missing bases for nondecimal "decimals." She did not like to draw pictures to illustrate operations with fractions such as $1/3 + 1/2$. She liked to check literal formulae by substituting numbers. She had had trouble understanding a wordy story (page 193) that was supposed to illustrate associativity.

Discussing the laboratory sessions, Student D said she had found the experiments in discovering the value of π "interesting," noting that she had never known why π is 3.14. She said she had enjoyed working with the geoboard--it had taught her "why an area was what it was." She mentioned that she had had to explain what she had learned to some other students who had walked in late; she felt this had been a good exercise in testing her own mastery of the lesson and suggested that the laboratory sessions be organized with half the students arriving late and the other half having to explain the lesson to them. Particular geoboard exercises she found interesting were the construction of squares of a given area and the illustration of the equality of triangles with the same base and height. She said that her laboratory instructor was "really good."

Student D responded to the question on rational numbers by first writing the word yes. She then expressed both fractions in terms of their least common denominator, 12. When she saw that the numerators were consecutive, she converted to 24ths and found $7/24$ in between.

Student D found the decimal name for $7/12$ by long division. To find a fraction name for $.377 \dots$, she called this r and then wrote what $10r$ would be. Realizing there would be a digit after the decimal point for $10r - r$, Student D then subtracted $10r$ from $100r$, finding the equation $90r = 34$. This led to the correct fraction $r = 34/90$. On the next problem, she divided out and stopped the decimal expansion after six places, and this was within the requested error bounds. In finding an approximate value for $\sqrt{7}$, she first squared 2.7; finding that the square was 7.29, she squared 2.6 and found 6.76. She then guessed that 2.65 would be a good approximation.

Student D's first idea on the first measurement problem was to take a string and superimpose it on the curve, then form it into a circle and compute the circumference. She later thought of measuring it in small segments, but felt it was "too curvy" for that; she also thought that it might be formed of circular fragments which would be measurable. After the investigator reminded her that she could use any materials, she got the idea of superimposing a string on the curve and measuring the string. Her final idea on how to find the area of the figure in the problem was to put a string along the boundary, form the string into a circle, and use the area formula to compute the area.

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Student D received the grade of 4.0 in the course.

March 27. Questioned about changes in her feelings
 d mathematics, Student D told about a change in her
 ption of mathematics: "It's not just rules now. Now
 why . . . I just . . . knowing how to use the stuff
 it's knowing where the stuff came from and how it
 nated . . . not just using the laws." (She later
 red the question, saying, "I like math. I love math.")
 aid that she felt more confident about teaching
 matics than she had at the beginning of the course,
 as still somewhat afraid of the idea. She said that
 ad been hostile to the idea of teaching mathematics
 e beginning of the course, but was now enthusiastic
 it. Student D reported no change in her feelings
 the purpose of teaching elementary school mathematics--
 ill felt that "it had to be taught." At the beginning
 course, Student D was undecided about whether to
 in elementary education or computer science. She
 ed that as a result of her Math 201 experience and
 perience in a computer science course, she had decided
 ndon computer science and opt for an elementary
 ion major. She said that Math 201 had had a strong
 upon this decision.

Asked what could be done to improve the course,
 t D recommended shortening the laboratory period to
 or ninety minutes. She noted that many students

ained about the laboratory, feeling it was "kid stuff."
 nt D felt that these complainers actually enjoyed the
 atory exercises but did not want others to know they
 ed them and so could not admit this psychologically.
 investigator cannot comment on this conjecture, but
 , contrary to Student D, that several of the subjects
 disliked the laboratory sessions.) Student D added
 her laboratory instructor had been helpful. She
 ed favorably the laboratory exercises on the geoboard
 attribute games (the latter "has always stuck in my
 . She had disliked those sessions on measurement
 boring for me") and on using blocks for counting
 investigator took this to mean Dienes Blocks). Overall,
 d enjoyed her laboratory work. Student D suggested
 he course proceed at a faster pace, noting that she
 have tolerated such a pace. She pointed out again
 eading the book in advance of lecture caused her to
 ttentive in lecture and thereby miss some important

Student D was working towards a mathematics major
 her elementary education program and had enrolled in
 11 (a precalculus review course) for the Spring 1974
 r. (In June 1974 she received the grade of 3.0 in
 11.) She had seen a lecture on absolute value in
 11 and understood the concept from Math 201. She
 ed about this, "I wondered how much I could use this
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think you can, but--you can." She said that she like to take even more mathematics, but that this depend upon how she found Math 111.

Asked her reaction to the study and to the investigation, Student D answered kiddingly, "All bad." Seriously, she said that she had not anticipated the investigator's questions; there had been some she would never have thought of asking. As an example, she mentioned the questions about specific problems in the book. She felt the investigator didn't know what to ask, and said, "I guess you've been a pretty good interviewer." She suggested having one or more group sessions as part of the study, in which the subjects could have a chance to respond to each other's ideas. She suggested no questions or other areas of investigation.

Student D felt general classroom teachers would be more able to specialists as teachers of elementary school mathematics. She said, "I feel . . . they could probably understand better than math specialists can. I mean, my gosh! a math specialist is going to get up there--maybe--I don't know. Really, what a math specialist is like--but I think--that an elementary education teacher you have to be able to understand the kids anyway and be able to tell when they are not getting it across, and I think they would be better. That's what we're learning now--we better

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Evaluation. If all Math 201 students were like Student D there would be few problems with the course. She was enthusiastic about mathematics and well prepared, but overprepared, for this course. She also enjoyed most of the laboratory work. She benefitted from the course in her understanding of mathematics and in her appreciation of what it means to teach mathematics in the elementary school.

Student F

Student F was a freshman from East Lansing. She had taken two years of high school mathematics. As she described this experience, "algebra I liked and I did well in geometry . . . it's as if I didn't even take it." She explained that "I like working with the numbers with algebra a lot more than with geometric figures." She also disliked her geometry teacher, and had spent that year of her life neglecting all her studies. Her favorite subjects were English, history, and social sciences such as psychology. Student F said she disliked mathematics in science; this contradicted her earlier remarks about algebra. She attributed these feelings to her family background--both her parents are on the Michigan State University, her father in English and her mother in social science, so "I've been oriented toward books all my life . . . there's really been no emphasis on math or scientific things in my whole family." She had not

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icipated in any extracurricular activities in high school. ("Maybe I wasn't a typical high school student.") She had not taken any college mathematics prior to the interview, and she said, "and I'm only taking this because I have to."

Although she had started out as an elementary education major, Student F was switching to child development in which she would be prepared to teach both elementary and nursery school. She hoped to work in a day school or day-care center. She chose this program because she said, "I really love children and I like working with them."

Her only teaching experience had been to help children with various subjects, including mathematics, while babysitting. She said the prospect of teaching mathematics "really scares me, 'cause I'm having so much trouble with this course [after less than two weeks in it] that I think it would be impossible for me to teach it." She said she was having trouble because "they assume you know some concepts, such as bases, powers, and exponents, which I could not recall.) Although she was competent in elementary education, she did not understand the "theory and principles behind it." Asked what she felt was the purpose of elementary school mathematics, she replied, "It's very fundamental. There are certain things that . . . you have to know--the very basics, working with numbers. It would be impossible [to] get along without it in everyday life. So . . . it has to be taught--but I don't know if I can teach it!"

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January 25. At this interview, the first discussion about the course lectures. Student F said, "I think she does a really good job . . . I like the way she explains things . . . in a lot of different ways . . . goes over and over it." Expressing her approval of the lecturer's examples, she also suggested more time be spent on questions and discussion of assigned problems. Although Student F had some trouble understanding bases, she said the explanations given had been sufficient to allay her confusion. Some of the things she had found difficult to follow in the lectures were the details of a choice tree (though she said she understood the principle) and the Austrian method of action.

Regarding the assignments, Student F said, "I don't like the book at all." Although she took notes on the assignments she read, she was unable to follow proofs. She said she was not able to understand the properties the book was discussing, and the explanations of these properties in the text were not clear. She said, "What I really don't understand is when they put everything in terms of sets. . . . I understand the definition but it confuses me when everything is always in terms of a set . . ." She reported having trouble with the assigned problems, usually looking at the answer before she attempted them because she had no idea "if I'm doing it the right way or not. Sometimes I don't even understand the wording of the questions." She said that looking at the answer sometimes helped. Of the assignments in general, she said,

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 ems, and I wouldn't really know what I was doing. It
 t until I went to class and heard the lecture which
 ined, like, the principles behind it, that I really
 stood what I was doing . . . I think she should
 in it before we do the problems."

With respect to the laboratory classes, Student F
 nted, "I guess they're helpful. I don't like them
 much." She said that Dienes Blocks had confused her
 hat she had ignored them; she had been unable to
 er a test question on the Blocks. She said that the
 atory classes should be optional rather than required,
 hat "they confuse me more than they help me . . . I
 o it much better in my head than I can with the
 s." She did say, though, that the Attribute Games
 loops had been "helpful, because you could see it."
 ver, in general, "I don't think I ever learned a new
 pt because of the lab."

The first group of problems presented to Student F
 is interview was that on sets. She said the set of all
 g people matches none of the sets shown because
 e's no set here . . . even a subset of the set of all
 g people." The set of counting numbers from 1 to 100,
 sive, was first said to match none; Student F misread
 as the numbers from 1 through 99 and noted set E is
 ite. In considering the set of units, however, she

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first said none, but then realized the meaning of the word "match," and said the set matches set C; after this, she returned to the previous question and decided the set of numbers from 1 through 100 matches set D. Student F said the set of all aardvarks enrolled at M. S. U. matches none of the above, since it is empty and no given set is empty.

Student F misunderstood the first problem on number bases and responded by attempting to translate 405_{ten} to base eleven. She correctly set up columns, but made an arithmetic error in subtracting $363 = 3 \times 11^2$ from 405. In the next two problems, Student F said that to convert 39_{ten} to base two, it would be 3 twos and 9 units, or 6 and 9 units, therefore 69_{two} . In converting 44_{ten} to base seven, it became 4 sevens and 4 ones, which was thirty-two, therefore 32_{seven} . When presented with the last two problems, Student F exclaimed, "Ucch! I hate bases!"

Nevertheless, she was able to recognize base twelve in the first example after some hesitation. In the second example, her first guess was base four because of the small numbers in the example. However, she soon realized that the problem was to find a base in which $11 - 2 = 2$, so 11 represents 4. She gave up on this problem, but after some review of her work by the investigator she realized that base three was the correct answer, and checked this by confirming the consistency of the rest of the example.

On Problem 1, Student F did no written work, but announced after some mental arithmetic that the answer was

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4 quarters, 1 dime (sic) and 1 penny. Asked how she got this answer, she said she had added up the sum of money, then converted to coins. This recollection caused her to realize her total was five cents short, so she revised her answer to 4 quarters, 1 dime, 1 nickel, and 1 penny.

At the end of the interview, Student F said, "I felt nervous 'cause I don't like those problems." However, she was unable to make any suggestions as to how the study might be improved.

February 5. At this interview Student F began by remarking that recently the course had become "very confusing," but that she expected this to end. The sections of the text on bases and prime numbers "[didn't] make sense to me." She usually did not do her homework until after the topic was discussed in lecture, but had made an exception to this the night before the interview, when she had read the first two sections on integers and done the assigned problems. (She said the material on integers was familiar to her from her previous education.) Student F said that it had been difficult for her to understand the section in the text on prime numbers after skipping the first part of that chapter (which had not been assigned). After attending the lectures on this topic, she understood it "better," but not "completely." Comparing the lectures and the book, she said she liked the lecturer's "method of explanation over the book's . . . they just go into proofs

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a paragraph long, and I just cannot follow them at all. She breaks it up into steps, and she goes over it a number of times, and it's just much easier to comprehend when she does it" Student F said she seldom used the book. She ignored items which she found confusing, including the Sieve of Eratosthenes and certain algorithms and definitions, which she could not name specifically in the interview. She said the pace of the lectures to this point had been about right.

Student F said that although she always read the textbook, "I usually feel completely frustrated after I read it." She repeated her criticisms of the text's style. She said she was "lost" on those problems whose answers were not provided in the text and which were not explained in lecture. With help from a friend she had been able to do all of the assigned problems on multiplication and most of those on division. She had been unable to do estimations in division of the type $\underline{\hspace{1cm}} \leq 386 \div 52 \leq \underline{\hspace{1cm}}$, unable to find the error in a "proof" that $1 = 0$, and unable to find the partial quotient and remainder in a division problem stated $314 = (q \times 6) + r$. She had been unable to apply the concept of closure to an arbitrary set of integers and also had difficulty with statements concerning absolute value, which had not yet been discussed in the lecture.

The only laboratory class Student F had attended since her previous interview was one on computation,

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including Napier's bones, lattice multiplication, and Russian peasant multiplication. Her reaction to this exercise was a grudging acknowledgment of the probable value of the experience, combined with a criticism of the amount of time spent on it.

At this interview the only problem presented to Student F was Problem 2. Her first reaction to this problem was to say, "I have no idea even how to begin to solve it." After she asked if she was expected to be able to do this, the interviewer replied, "I don't know. I just want to see how you would think about this problem--or not think about it, as the case may be." After some thought, Student F counted 9,999 possible suffixes (short by one) and gave the answer $9,999 \times 3 = 29,997$.

February 19. At this interview Student F remarked that the most recent section of the course had been somewhat easier for her than preceding sections because she recalled more of the present material from her previous education than she had before. However, she still said that the book's explanations and proofs were beyond her understanding. She reiterated her approval of the lectures, admiring the lecturer's discussion of a variety of examples of each concept. She had been unable to understand the array method of presenting rational numbers (as cut-up squares) and of depicting operations with them.

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Student F said that she was able to understand the statements in the text, but not its explanations. In particular, she could not understand the explanation of the invert-and-multiply algorithm and of one of the identity properties. She reiterated her dislike of the book and suggested that it should have more answers in the back, on which she depended for help. On a problem involving adjustment of a recipe, "I tried dividing everything by everything else, and the answers didn't make any sense." She was unable to recognize mathematical principles exemplified by verbal statements. She could not illustrate multiplication of fractions by a diagram. She had great difficulty in arranging a set of rational numbers in order. Student F said she had trouble applying the concept of closure to a given set, and giving an argument to support a particular line of reasoning. She did not do an assigned problem on determining an upper bound for the product of a bounded positive number and a bounded negative number. She said that the assigned problems were helpful in reinforcing what she knew, but not in clarifying concepts she found difficult. She also had trouble reading the problems.

Student F had not liked the laboratory class on clock arithmetic because she had found the directions unclear, and insufficient help had been provided by the instructor; however, she said this exercise had not been particularly difficult. She said that the session on rational numbers "wasn't bad," and that it was "more helpful

than the other one." She thought the GeoBlocks were the most beneficial of the materials used in this session. She said that in general she did not find the laboratory classes helpful for learning the lecture material.

Student F said at this interview that the prospect of teaching mathematics "scares me." She expected to work in a nursery school and thus not to teach mathematics. She thought she would still be poorly prepared for teaching mathematics after completing Math 201.

The first set of problems presented to Student F at this interview was that on prime numbers. In considering whether or not the given numbers are prime, Student F wrote nothing; all her responses were verbal. For 119, she said her first reaction on looking at the number was to call it a prime. Asked why, she said that 11 is prime and 9 is not. After some pause, she discovered that 119 is a multiple of 7 and thus not prime. For 113, she had the same first reaction; after trying some divisors mentally and finding no factors, she called it a prime. For 227, she said, "I don't know, 'cause I really don't know how to determine [for] such a large number whether it is or not." (She had succeeded in this for the first two numbers, however.) She divided it by 8 and by 11, finding neither to be a factor. She finally called it a prime, citing as her reason that it ends in 7, a prime. However, for 247, her first reaction was to say, "I don't think this is a prime." After considering that it too ends in 7, she changed

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her mind. She realized that any factor of 24 would leave 7 over and thus not be a factor of 247. Not finding any factors, she called 247 a prime.

On the problem of finding the greatest common factor and least common multiple, Student F did not split the given numbers into smaller factors. Her idea was to list the factors or multiples of the given numbers and then scan the lists for those that were common. She could not complete this procedure in either example. She said that she was able to factor the numbers into primes, but then would not know what to do to solve the given problems.

Student F said she could not even attempt to do Problem 3. She said, "I know that there is some relationship between all this stuff. [long pause] It seems like you'd set up some kind of a relationship between two rational numbers, but we haven't learned that yet, so I don't know if that's what it would be."

March 5. Concerning the material covered in the last section of the course, Student F remarked that she always had had difficulty with fractions and decimals, and that "I've never been able to do percent before in my life." She still expected never to understand these concepts, though she said she had a better idea of what they mean than she had before. She now was able to write a percent relation as a mathematical statement, replacing "%" by "1/100" (an equivalence she had not previously

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known), but doubted her ability to interpret a verbal problem correctly. In the area of fractions, the course repeated material she had known; concerning decimals, although she now could solve some problems she previously had been unable to solve, she still did not have a full understanding of the concept. She said she now was able to represent a fraction as a decimal, but not vice versa. (This was borne out somewhat by the subsequent problems.)

Student F attributed her increased understanding of fractions and decimals to the lecturer's repetitions of the principles involved. While she could not follow all of the steps in the lecturer's exposition, she still felt that she understood more than she had before. She found confusing the description of how to convert a repeating decimal into a fraction. She did not understand repeating decimals or scientific notation.

Student F said her homework assignments were "getting more difficult." She did not understand the statement that each repeating decimal represents a rational number. She "had a lot of trouble" with the problems in the chapter on rational numbers, though she said she presently understood some of them after obtaining help. She was unable to do any of the problems involving decimals herself, but after some help could manipulate powers of 10 and percents. She had not even tried some of the problems. Student F was able to compute sums and differences of fractions and illustrate these with diagrams, as well as

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do story problems and computations involving the multiplication and division of fractions; she could recognize principles expressed in verbal statements.

Regarding her most recent laboratory classes, Student F said she had liked the one on the geoboard. (She admitted in this context that she usually "hate[s] the labs.") She said this had been "helpful . . . the best lab," and that she had "learned a lot." She had not attended the session on ruler-and-compass constructions.

Student F said at this time that the course as a whole was becoming more confusing to her. She felt herself falling farther behind because of her difficulty with this material.

The first problem presented to Student F at this time was that on rational numbers. She said that she had learned that there is always a rational number between two distinct rational numbers. However, she had no idea of how to find it explicitly.

Student F first said she did not know how to write $7/12$ as a decimal. After some thought, she hesitantly divided 12 by 7, stopping after two decimal places. She did not know what to do with the remainder at this point, remarking that if she were dividing integers, she could add a fraction to the quotient. To the problem of converting a decimal to a fraction, she responded that she had no idea how to do it, saying, "You have to move the decimal, but

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I'm not sure where." She also said she had no idea of how to do the two subsequent problems on decimals.

Presented with the first measurement problem, Student F said, "I hate to say this again, but I've never done a problem even similar to that." She remarked that she had never before used a compass. On the second problem she recalled her geoboard exercise and suggested enclosing the figure in a measurable area and then subtracting the excess, though she made no moves to attempt to do this.

Student F received the grade of 2.0 in the course.

April 1. At her final interview, Student F said about mathematics: "I feel more capable of doing certain things. I certainly don't like it any better." She was quite happy about some of the new skills and understandings she had acquired in Math 201, remarking that she had been able to do some of a mathematics test in her natural science course, while before taking Math 201 she would have been unable to do any of it. She said she could now "look at something and it won't puzzle me as much as it used to." Regarding her eventual teaching of mathematics, she reported no change of feelings--"I'm still dreading it." She felt that Math 201 had not prepared her to teach, because "I started out so low that that class just taught me what I needed to know whereas most people already knew it, so they could concentrate on the concepts and teaching it. But I really couldn't because I was struggling to

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understand it myself." She said she felt mathematics is important, as she had before. She remarked on the poor quality of her previous education in mathematics, noting, "If I had to be a sophomore in college before I learned how to do percents--." Student F said she had no change of feelings about her major as a result of taking Math 201.

Regarding the course, her only complaint was that more time should have been spent going over the assigned problems and more outside help sessions held. (There had been only one of the latter, before the final.) While Student F said that in general, "I wasn't terribly impressed with the labs," she did recall those involving clock arithmetic, the geoboard, and GeoBlocks as being helpful to her. She was unable to name those she had disliked the most. She remarked that she had found the course to be "well organized" and that the lecturer "did a really good job." She said emphatically that she would not take any more mathematics courses.

Concerning the research study, Student F said that she had been "nervous at first," but not later. She had disliked having to do the problems, but had realized that they were essential to the study. She had no suggestions for improvements in the format of the study; while she thought the investigator had covered everything in his questioning, she suggested questioning in "more depth about the labs."

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Student F felt that elementary school mathematics should be taught by mathematics specialists, because with a superior grasp of the material they would be aware of a variety of pedagogical approaches which would enable them to deal better with the particular difficulties of individual pupils.

Evaluation. Student F was an intelligent person who had acquired the false impression that she could not understand or do mathematics. On some problems, she immediately said she could not do the problem, only to reveal some understanding after the investigator probed slightly.

Student F herself admitted her need for remedial work in arithmetic. She was not really ready to take Math 201. It would be better for all concerned if such students were required to display competence in arithmetic before entering Math 201--they would then be better prepared and the instructional staff would be able to teach a better course.

Student G

Student G was a sophomore from Parchment. Although she had taken two years of high school mathematics, she described her background as "none, really." She had not done well in her courses, and, as she puts it, "I don't remember anything . . . I don't even remember like even

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Math 201, which is pretty simple." Summarizing her feelings, she said, "I really dislike math." Her favorite high school subjects had been biology and English. She had disliked mathematics, civics, and history the most, and had not taken any physical science courses, "but I know I'd have hated them." She disliked these subjects because they were difficult for her, not because of the nature of the subject matter; she said, "Sometimes I'll enjoy doing a math problem, but very simple. I have trouble even adding." In high school, Student G had been active in performing arts activities and in individual sports.

Student G had not taken any college mathematics prior to the study. Her intended major was special education--particularly the education of the deaf. Her first major had been sociology; she had hoped to be a social worker, but had found job prospects poor. While in tenth grade, she had observed deaf children with whom her mother worked. This had interested her and a Michigan State counselor advised her to go into deaf education rather than audiology because the former would be easier. Over the term break prior to the study she had done her sixty-hour practicum in a Kalamazoo special school, observing, helping individual deaf students, and playing games. She said she had enjoyed this experience, and remarked that the high school age deaf students were very

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poor in mathematics. Other than this she had had no teaching experience.

Student G was unenthusiastic about eventually teaching mathematics, but anticipated no problems teaching at the first- and second-grade levels, or with the generally slow deaf children. She admitted that she would be "scared" of teaching mathematics in an upper elementary grade. She felt that the purpose of elementary school mathematics is to allow the student the option of choosing a career where mathematics is necessary; she herself had wanted to be a nurse but had been deterred by the need for a strong mathematics and science background in that occupation.

January 21. Student G began this interview by remarking of the lecturer, "I think she explains most of the things pretty well." Later she said of the lecturer's style, "She simplified everything; I like that. I don't have to figure it out! At first it bothered me . . . like she's talking to second-graders, but I guess you have to . . ." Student G remarked that this style of teaching was necessary "so you'll keep it simple and you won't try to complicate it by putting in what you already know." She said that the lectures generally covered the material in the book, and that she usually read the book's treatment before attending a lecture on a particular topic. She pointed out that she knew a student who did not read the book at all. Her only criticism of the lecture was that

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Regarding the assigned problems, Student G said that "they helped." Although many students did not do the problems (according to Student G), she felt a need for the exercise. However, she skipped those problems which she felt she understood.

Student G criticized the laboratory classes, saying, "For two hours, it's so long." She suggested the session be shortened to one hour, adding, "I think it would be better without the lab. It takes up too much time." She remarked further that the laboratory had not helped her to understand anything and had not supplemented the lecture at all. Of the specific materials used, she found attribute games "less tedious" than Dienes Blocks, with which one could "figure it out in your head." Bases were a new concept for her, while sets were familiar.

In general, Student G found the course material "pretty easy." She felt her lack of a mathematics background was an advantage because it enabled her to look at the concepts being taught with a fresh eye, unencumbered by other learning experiences. She said that the course had "shown me how little I know. . . . I don't even remember my multiplication tables, 'cause I haven't used them at all for anything . . . my adding is really poor . . . terrible . . . [it] will be good for me to brush up on those things."

Like several other subjects, Student G displayed on the problem sheet on sets a confusion of the concepts of matching and equality and an unsureness of the correct use of the term "the empty set." Considering the set of all living people, she remarked, "These people [Kennedy and Johnson] aren't living." Not seeing a set which matches, she asked, "Do you mean . . . if it's not here, then it's empty?" She finally said that this set matches none of the listed sets. For the second set asked, she apparently was misled by the word "inclusive," saying set E, "because this is an infinite set, so it does include 1 to 100, plus more." She said that the set of suits in a standard deck of cards matches none "because none of these symbols [in set C] is from a deck of cards." The last set was said by her to match none of those listed "because there are no aardvarks" appearing in the listed sets.

Student G began the next sheet of problems by correctly converting 405_{eleven} to base ten by labeling columns 11×11 , 11, and 1, and taking the correct combination of values. In the reverse conversions, she was able to write 44_{ten} in base seven by setting up columns labeled 7×7 , 7, and 1, and placing the appropriate digits in the columns. However, she was unable to write 39_{ten} in base two. She correctly labeled the columns as far to the left as the 16's, but then decided to consider 39_{ten} as 3 tens and 9 ones and handle these separately. She noted that 3 tens was equal to 1 sixteen, 1 eight, 1 four, and 1

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two, but then did not know what to do with the 9 ones. She finally wrote 111111_2 , but said, "I'm sure this isn't right." In trying to name the correct base in the missing-base examples, Student G used the following approach: She added $49 + 37$ in base ten, obtaining 86_{ten} . She then attempted to find a base b in which $84_b = 86_{\text{ten}}$. She considered base eleven and base nine and found they did not work. She then gave up, saying, "It has to be one of the others." She attempted the second problem the same way, subtracting $211 - 12 = 189_{\text{ten}}$ (sic), and then looking for a base b in which $122_b = 189_{\text{ten}}$. She eventually gave up, saying, "I don't know how to do this at all."

Student G solved Problem 1 by adding mentally within coins and seeing that the totals were 3 quarters, 7 nickels, and 6 pennies. She then exchanged these for the fewest coins.

Regarding the interview, Student G said that she had felt comfortable in conversation, but that "I get a little nervous when I have to do problems . . . under pressure . . . I forget a lot of things." However, she could suggest nothing that might help her to be less uncomfortable.

February 4. At this interview, Student G began by discussing the recent lectures on the algorithms of arithmetic and on prime numbers. She said that just hearing the words "prime numbers" caused her to feel that this

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topic would be difficult. She said of the lectures on prime numbers, "It's a little more boring than addition and subtraction . . . I remember having prime numbers when I was younger and I always had a hard time understanding what they were . . . I think now, even though I understand what they are, I have trouble figuring out which ones are it . . . I have trouble picking them out." She added, "Maybe just the fact that I remember not liking them . . . earlier . . . had a lot to do with it [her discomfort with the topic]." She also had encountered difficulty with division, particularly the Greenwood algorithm, although she said the lecturer had satisfactorily explained division to her.

Student G said that she was beind in doing her assignments at this point. She still had trouble doing arithmetic with bases. She said she had tried to do division by the "long method" and felt that she understood it "as long as there's not an exact certain number that you're supposed to have." She remarked that she was "doing OK" as long as she could "do it your own way." (The investigator took this to mean not using the various algorithms discussed in the course.) She said that the reading assignments sometimes confused her.

Student G had missed the laboratory session on computation and expected to make it up soon. She said that "it always messes me up" to have the laboratory work before the lectures on a given topic, and suggested the

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order of these be reversed. Of the activities themselves, she said, "I hate doing that stuff with the little blocks. That confuses me . . ." However, she remarked of her partner in the laboratory that "the only way [he] can do it is with the little blocks. He said if he had little blocks on the test, he'd be fine." She complained about her laboratory instructor, saying that "he doesn't really explain anything--we just kind of read it and have to figure it out ourselves." She did not understand the reason for the laboratory exercises, although she realized that they had some relevance to the elementary school. She thought that she had been doing the exercises correctly but was unsure of what the lecturer expected.

Student G said that, to this point, the pace of the course had been about right, but that it had been too fast in isolated instances. Regarding the topics covered, she said the name "prime numbers" was familiar to her, but that she had forgotten the definition and theory of primes. The nonstandard algorithms for the operations of arithmetic had been new, as had been the variety of problems involving bases.

After some discussion with the investigator about Problem 2 in which the problem was clarified for Student G, she said, "I hate this kind of problems. I can never do this kind." Asked what kind this was, she replied, "Well, it's not really story, but . . . story-type problems. Unless I have it all written out like what kind of method,

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I have trouble--forget what I'm supposed to do to get the answer." Considering this problem, she said of the number of different phone numbers in East Lansing, "There's so many . . . It can't be infinite, but it's just about. I don't know how you'd figure it unless you wrote down all the different combinations for one [exchange] and figured it's that many for the next one, and then multiply by 3." Asked if she could do that, she wrote some down (1234, 5678, 9123--thinking there were nine possible digits) but had no idea of how to enumerate them. She knew, however, that the number was finite.

February 18. At this interview Student G mentioned first that she had not done well on the two tests she had taken in lecture. She said she had drawn a blank on "easy addition and subtraction" problems on the first test. For the second test, she had practiced multiplication and division and had done well on them, but had "slacked off on other stuff."

Discussing the most recent lectures, Student G said they generally had been "presented pretty well." She noted that she had forgotten the arithmetic of negatives and of fractions. She said that the lecturer's use of the number line "helps you see it" and that she had found most helpful the use of "examples and illustrations." Although she sometimes found lectures to be confusing toward the end of the period when the lecturer was pressed for time, the material was usually made clear at the next lecture.

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Student G said that at this time she was somewhat behind in her assignments. Although she found the lectures more helpful than the book, she noted that they clarified each other. When she forgot how to use the subtraction algorithm, she received no help from her book, but was aided by her lecture notes.

At this interview Student G expressed dislike of one laboratory exercise and enthusiasm for another. She said, "I don't like [clock arithmetic] at all . . . It just seemed dumb to sit there and try to figure out what plus what equals what on a clock . . . it seemed like a waste of time." She wished some time had been spent on it in lecture. However, she said of the session on rational numbers, that it "was one of the better ones as far as getting a point across and seeing something . . .". She had forgotten about GeoBlocks, not having spent much time with the, and said she "didn't have too much trouble" with tangrams. She had spent most of the period working with Cuisenaire rods, which she considered a useful activity for its potential applications to teaching. She did not find it difficult to do addition and subtraction with Cuisenaire rods, but found multiplication and division somewhat harder.

Student G said that at this time she felt "not too bad" about eventually teaching mathematics. She found the course relevant to elementary school mathematics but noted that it applied more to the late elementary grades.

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The first problem sheet presented to Student G at this interview was that on prime numbers. She called all of the given numbers prime, doing no written work. For 119, she tried mentally to divide it by "the lower numbers"-- 2, 3, 5, and 7. Missing 7, she said, "I would say it was prime just because I can't find anything to go into it evenly." She thought that "you have to have something that will go into either 11 or 9," and in this case only 3 would qualify, and it does not work. She mentally tried 2, 3, 5, 6, 7, and 8 as divisors of 113 before calling it prime; she was thinking of numbers that could be a factor of a number ending in 3. She called 227 a prime after trying 2, 3, 4, 5, 6, and 7 as divisors. She tried 2, 3, 4, 5, and 6 as divisors of 247; when all failed to go in, she called it a prime. She noted that many numbers were factors of 24 but none of these were factors of 7, and said, "I suppose we could go on forever and ever, dividing numbers into it." However, she stopped trying after 6 failed. She could not say why she stopped at this point or when one could stop trying divisors and call a number prime.

In taking the greatest common factor and least common multiple, Student G factored each number into primes and correctly took the greatest common factor. However, in trying to take the least common multiple of 42 and 48, she chose 2 as the least prime number going into both numbers--a sort of least common factor rather than least common multiple.

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Student G responded to Problem 3 by saying, "I hate these." After some silence, the investigator asked what she was thinking; she replied, "Nothing." After rereading the problem, she said, "I don't know how to do this." Asked what information she would need in order to solve it, she said she had to know how far Dan was from town at 1 o'clock, how far he had traveled in that hour. She could not determine how far Dan had traveled in an hour.

March 4. Returning to a familiar theme, Student G started this interview by saying, "I hate story problems . . . Even when I was younger I had trouble with story problems, and I haven't improved any." She expected to have trouble with story problems on the upcoming test. She mentioned that she had done all the outstanding homework recently in one night, before the lecture on the topic, "so I probably did it wrong" and "they probably didn't help me." She expected to go over these assignments again before the test, particularly the story problems, which she had skipped.

Regarding the lectures, Student G said that she could name nothing in them as being particularly helpful, although she said it was good that the lecturer had explained the problems on decimals and percent from the book and the handout sheet. Asked if anything had been confusing, she replied that she had been unable to follow an explanation of some problem this morning. She said that the lectures were generally clear.

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Student G described the laboratory session on the geoboard as "one of the better ones," and said that she understood it. This was due in part to her having attended a different class than that which she usually attended. She joined a group which she said worked together better than her usual group and made sure each one in the group understood the lesson. Her usual group worked as individuals on different parts of the assignment, and she rarely understood what she was supposed to do. She remembered having trouble with square roots, saying she had been "lost," and that "I can't remember what we were supposed to do with them." However, she didn't think much time had been spent on square roots. Student G recalled being "totally confused" by the problem of deciding which of the numbers $\sqrt{2}$, $\sqrt{3}$, $\sqrt{4}$, etc., are constructible as a side of a right triangle. Discussing the laboratory session on ruler-and-compass constructions, she said she had understood the exercises on constructing a line segment of length a given fraction of a given segment, and on finding the area and side of a square built on the diagonal of a square of side 1. She may have seen some of the other exercises done, but as if "one person . . . whipped through them."

The first problem at this interview was that on rational numbers. Student G first said, "Yes, there is a rational number between the two, because there's an infinite amount of numbers between any two numbers." To

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find the number, she converted both fractions to equivalent ones with a common denominator (12). Finding the numerators to be consecutive integers, she expressed both fractions with the denominator 24, then found $7/24$ in between.

Student G correctly found the decimal name for $7/12$ by long division. In trying to find a fraction name for $.3777 \dots$, she named this R and then correctly subtracted $100R - R = 99R = 37.4$. Not sure of what to do with the decimal point, she somehow dropped it, writing $99R = 37.4/10 = 374$, and concluding that $R = 374/99$. In the next problem, she divided out $6/11$ (making an arithmetic error in the division) but could not see how to get something terminating from the resulting repeating decimal. Asked to find an approximate value for $\sqrt{7}$, she said, "I know that the square root is what times what equals 7, . . . the same number, but I don't know how to get it at all." She did know, however, that it is between 2 and 3, and started to square 2.5 on the sheet, but then crossed out her work. She repeated that she had no idea how to find it.

Student G said that she would measure the length of the given arc by putting a string along the curve and measuring it. She noted that she could not use a small measuring unit because of the number of curves in the arc. She first read the area problem as a length problem. After the investigator pointed out that it was an area problem, she suggested enclosing it in a circle or, preferably, a

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square. She said the best way would be to look at it on a geoboard with a unit measure and count the square units contained in it.

Student G received the grade of 0.0 (failure) in the course.

March 27. At this interview Student G said her feelings toward mathematics had not changed much--"I still don't like math." She said there had been no change in her feelings about eventually teaching mathematics. "Depending on how much math changes . . . by the time I get there, if ever, and as long as it doesn't move too quickly," she anticipated no difficulty because she planned to teach very low level mathematics to deaf pupils. She admitted that she would have difficulty teaching mathematics at about the fifth-grade level. Regarding the purpose of teaching mathematics in elementary school, she said, "I always thought it was needed . . . I still think it is," and that her feelings had not changed. She also said that her feelings about her major had not changed.

Concerning the course, Student G said, "I didn't like this course that much . . . 'cause I don't like math." While she found the lectures "pretty well explained" (although she favored more explanation), Student G complained, "I didn't like the labs. I thought they were more or less a waste of time. Maybe they should have been shorter and just explain the material that was explained

in class . . . answer questions . . . a weekly review over what's been covered in class [rather than] doing things that we did. I find it pretty worthless. I don't think I learned very much from that at all." Although she protested the presentation of some of the algorithms shown in lecture ("It doesn't seem like it would help children learn that much"), she said, "It's mainly the lab I'd change." Asked which laboratory sessions she had liked or found helpful, she mentioned the session on Cuisenaire rods, which she had seen used in the schools. She could not remember any other laboratory activities.

Student G said that she would not take further mathematics courses, because "I just don't really care about math." She noted that she had to take methods. (She also had to repeat Math 201 because she had failed the course; this was not mentioned in the interview.)

Student G said she had found the research study interesting, but was not sure of its purpose. She considered the investigator's questions adequate, and could suggest neither other questions nor other possible areas of investigation.

In considering the question of generalists versus specialists, Student G said that it depended on the individual, but was inclined toward the generalist, saying, "I tend to think--maybe--people that don't know as much about math, just because it might be easier for them to explain on a simpler level. . . . If you had some math

genius that . . . knew calculus and everything else, it might be hard for them to explain [at] a very simple level to the child, whereas it would probably be easier for someone that doesn't know much more math.

Evaluation. Student G was poorly prepared in arithmetic and should not have been allowed into the course without doing some remedial work. Nevertheless, the investigator was surprised to learn she had failed the course, because her problem solving behavior in the study was not substantially worse than that of some other subjects, all of whom passed by a comfortable margin. This suggests that perhaps the behaviors observed in the study and those required on course examinations were somewhat different. Finally, it should be noted that the laboratory experience was wasted on Student G.

Student H

Student H was a junior from Bay City, the only upperclasswoman among the study participants. She had taken three years of mathematics in high school--two of algebra and one of geometry. She remembered very little of this mathematics at the time of the study, since it had been five years since she last had studied it. She felt that she had learned her high school mathematics well when she took it, but had not retained her knowledge. At the initial interview, she mentioned that she frequently had to ask her husband, an engineering student, for help with Math 201.

Student H's favorite subject in high school had been art, because she had been good at it and had always been encouraged to do it. She had most disliked history and current events. She said she had participated in no extracurricular activities until her senior year, when, after switching from a Catholic school to a public school, she had begun to participate in many activities.

Student H had taken no previous college mathematics. Her major was elementary education with specialization in fine arts. While in the fourth grade, she had decided that she would become a teacher, and she had not had occasion to change her mind since. At the time of the study she was taking Education 101A, and had had her ambition reinforced by a visit to a second-grade classroom the day before her initial interview, although she had not been able to explain the idea of one-fourth to a second-grader. Student H's only classroom teaching experience was when she and another student taught her high school art class in the teacher's absence. She felt confident of her ability eventually to teach second- or third-grade mathematics. She said the purpose of elementary school mathematics was to give the child the numerical skills he needs in everyday life.

January 25. At this interview Student H said of the lectures, "They just verify what I've studied . . . For me, they're just so boring, because it's repetition, it's so simple, and it's boring, but I think it's important

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that you go over that." She criticized the lecturer for using too many examples. Student H said that nothing in the lectures had helped her to understand the material, because she either knew it in advance or had her husband explain it to her; she said that "for me it's all review" and that the lecturer "just verifies." Student H had been confused by the lecturer's definition of one-to-one correspondence--she had not been sure if a one-to-one correspondence consisted of a set of ordered pairs or only one ordered pair. This had been explained by her laboratory instructor.

Student H said that the textbook had helped her, but that she wished it contained more problems and full solutions to all problems rather than selected numerical answers. She said that the "answers in the back are worthless." She found the problems helpful, but said that she needed the answers for "positive reinforcement."

Asked what she thought of the first few laboratory sessions, Student H replied, "That's a bunch of bull." She said that she had liked the most recent one (using Dienes blocks to illustrate multiplication and division) "because I could relate it to what we're doing, but the first two were just--I have no idea what I did. I played with blocks and . . . I can't relate it to how I'm going to teach children with them at all." Asked what had been different about the last one, Student H said that "we multiplied the blocks and it came out--you could relate the

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blocks to figures." Previously, she "couldn't relate the blocks to anything" except when her instructor had drawn Venn diagrams (for A Blocks, presumably). In general, she said, "I don't think they're helping at all. I think it's a waste of time," and, "It's so unrelated to what I'm doing."

In general, she said that she liked mathematics and liked the course. She had used her husband's HP-45 calculator to check herself on the test. She said, "I like to work problems . . . there's a definite answer for everything . . . you know when you're right, you're right."

The first problem sheet presented to Student H at this interview was that on sets. Comparing the set of all living people to those listed, she said, "It doesn't really match any of them . . .," because all but set E are too small, while set E, unlike the given set, is infinite. Of the set of counting numbers from 1 to 100, inclusive, she said, "That wouldn't match any of them either, 'cause that's [set D] from 1 to 99 [sic]; that wouldn't include 100." Regarding the set of suits, she noted, "That would be four, so you could match that with C," and of the set of all aardvarks enrolled at M. S. U., "That's empty, and there's no empty set in there."

Student H correctly converted 405_{eleven} to base ten. In converting 39_{ten} to base two, she labeled her columns 1, 2, 4, 16, and 32. Thus omitting the eights column, she wrote 39_{ten} as 10111_{two} . She correctly labeled columns in

writing 44_{ten} as 62_{seven} . In the first missing-base problem, Student H first thought that the base was ten because she thought that nine plus seven is fourteen; however, she quickly realized that this was incorrect. Guessing base eleven, she counted to four more than the base and obtained fifteen. She then guessed base twelve; confirming that four more than the base is sixteen, she concluded that the example was in base twelve. In the second problem, she realized that since $11 - 2 = 2$, 11 had to be four, so the example was in base three.

Student H responded to Problem 1 by saying, "I always hate story problems. I don't know why I can't do them." After reading the problem, she first thought it was asking which amount of money was smaller (apparently from the word "least") and answered 1 quarter, 3 nickels, and 3 pennies, then asked, "Is that what it's asking for?" The investigator responded by reading the question on the paper. Student H exclaimed, "Oh, I'm supposed to subtract those! . . . They didn't say subtract!" She then summed each person's money, then subtracted the smaller amount from the larger. After making and correcting an arithmetic error, she gave 30 cents as her answer. (The question had asked for coins.)

Asked how she had felt in the interview, Student H said she had been nervous working problems in front of the investigator, and also was very tired that day.

February 11. At this time Student H said of the course, "It's getting harder. I find now that I have to pay attention to my lectures. Finally it's getting interesting . . . getting to be a challenge." Previously it had been "more of a review than anything else."

Student H said that she was "really messed up" because she had missed class (and her interview appointment) on February 8 due to illness. She said that nothing in the lectures had especially helped her. She usually read the book before attending lecture, using the lectures to clear up those points she had not grasped in her reading. She also did all of the assigned problems before the lecture on a topic; she did them all again before the test for review. The only topic on which she had been confused was one-to-one correspondence. When she had difficulty she asked her husband for help. Student H said of the pace of the course to this point, "It could have been speeded up a hundred times faster . . . There's not enough work. I'm . . . just drifting . . . but there's some kids that haven't had . . . math . . . but for me, it's boring . . . if she assigned two or three chapters a night, I'd be happy . . . one, that's nothing." The only topic Student H had not seen before was clock arithmetic.

In discussing the homework, Student H complained about the following problem (page 153):

In a game of gin rummy, Mary is 25 in the hole and John is 75 in the hole. Who has the better score?

Later on, Mary has 50 points and John has 25 points.
Who gained the most points?

Student H exclaimed, "Now I don't know how to play gin rummy! So what am I supposed to do? I had to skip that one, 'cause I don't know . . . what the answer is . . . I've run across a couple of them like that." She added that while in the second grade, "you run across this kind of inconsistencies all the time," she had found "just a couple like that" in the Math 201 text. Regarding the problems, she said, "I wish they'd put more problems in like 'Tell what the property of this is,' or . . . [routine exercises] . . . I hate story problems. I'm really bad at them so it's probably good that they have them in there" She said that the book did not confuse her, and that "they could be a little harder--I'd like that."

Discussing the laboratory work, Student H said that clock arithmetic had been "really difficult for me, but after I read the book a couple of times I caught onto it." She then found the laboratory exercise on clock arithmetic easy because she had studied it in advance from the text; she said that "if I hadn't known it before I went into the lab I don't think the lab would have helped me." Student H said of the laboratory on computation, "I learned from that one . . . That was really interesting . . . really helpful . . . that was good." It had not been of assistance in understanding the lectures, however--only interesting. Of the laboratory sessions in general, Student H said, "It's

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kind of fun to sit and do math--I kind of like it--since I have to be there anyway. It's getting better than when we were first playing with little blocks. I didn't like that . . ."

In her response to Problem 2, Student H first noted that the ending of a telephone number has four digits. She thought each digit could be one of those from 1 through 9 (omitting 0), so that the ending would be "a combination of four numbers consisting of the numbers 1 through 9 for each one of those." If she could determine the number of possible endings, she would multiply this by 3 to get the answer. She had no idea of how to enumerate the endings, saying only, "It has to be a lot of them because there's a lot of people."

February 22. Student H said first at this interview that she had not done any reading or problems in over a week, but had attended lectures. Of the lectures, she said, "It's getting harder and I need to study, but the lectures are helping me considerably [because] I'm unable to read the book." The lecturer had done nothing which confused her--"It's all been very helpful." She liked the way the lecturer would do each problem twice and write everything down. Student H had looked at the book's discussion of absolute value but had forgotten it; this had been her only reading since the last interview.

Student H complained that her laboratory instructor was "horrible . . . He should be explaining this to us . . . The lab is directed towards a point . . . At the end, we're supposed to go, 'Wow! We discovered this!' and we never discover anything. We do the problems, hurry up, and rush out. I think that he should help lead us towards that and he doesn't. And he can't explain anything . . . He'd rather tell us the answer than explain it, and that's not helpful at all." She suggested the laboratory might be better if the instructor explained more. About the session on real numbers, whose objective she felt was for the student to discover π , Student H said, "It didn't help me at all, although it was fun working the problems. It was a riot, but I didn't learn anything." She added that it had been "an awful waste of time, I think. They could be teaching us something, but this discovery learning or whatever it's supposed to be isn't working." She felt that "most of us have forgotten" Euclidean geometry, while an understanding of the Pythagorean theorem and of certain constructions had been necessary in order to do the laboratory exercise. In contrast, Student H said that the session on fractions "got the point across" and had been "extremely helpful . . . well thought out." She said that when "we put the blocks [this could mean either Cuisenaire rods or GeoBlocks] together . . . it really got across what a fraction is." She added, however, that the lecturer's illustrations were even better.

Student H said that she did not expect to be able to teach this mathematics. She noted that she was not learning methods here and that she was not at all sure of her ability to teach this material.

The first problem set presented to Student H at this interview was that on prime numbers. Before beginning the set, she mentioned that she mistakenly had called 91 a prime number on the test, and so would be careful in these problems. Considering 119, she said, "I would think it was a prime number just because as I go through it I think-- . . . if I have something left over it's going to make a 19 or a 29 or a 39 [this is the result of the first subtraction in a division example; note she missed 49]-- there's just nothing that goes into those numbers, so I would say that this one is prime . . ." Suddenly realizing that 39 is not prime, she then wrote out a division of 119 by 3. When this failed, she finally called 119 a prime. About 113, she said, "Right off the bat I would think it was prime . . . 13's prime so 113 should be prime. But 91 was 7 times 13; I'll never forget that--I'll say it's prime. I don't know, maybe I should think about them more, huh?" Somewhere along the line she had attempted to divide 113 by 3; this was her only try at division. Seeing 227, Student H said, "Now that one I don't think is prime, but it could be." She tried 3 and 9 as divisors. After both of these failed, she said, "I would think this would be prime, too." Of 247, she remarked, "That looks like it

would be prime . . . That one [227] didn't look prime, but all the other ones did . . . To figure out if it's prime or not, . . . I'd just keep dividing into it." Asked when she would stop dividing, she replied, "Oh, after about three tries . . . the most logical tries . . . like 7 and 3 . . . I wouldn't try any even number because obviously they wouldn't go in . . . 3 times 9 is 27, so maybe . . . 9 would go in . . ." After trying 3, 7, and 9 as divisors, she concluded that 247 is prime.

Asked to find the greatest common factor of 63 and 105, Student H first said, "Geez, I don't know how to do this!" She then proceeded to do it correctly, factoring each number into primes and then taking the correct combination of factors. In finding the least common multiple, she again factored both numbers (though leaving 48 as $3 \times 2 \times 2 \times 4$) and took the correct combination.

Student H first read Problem 3 aloud, then drew a diagram depicting the town on an east-west axis. She labeled the town "12:00 40 mph," and a point to the east "1:00 50 mph." She said that Dick goes "10 miles per hour faster than the other dude" and commented that this was "10 miles per hour faster per hour." She noted that 9 A.M. was three hours before noon, then said, "I have a chance [to solve it] but it could take me forever," and added, "It's got to be simple . . ." After some thought, she gave the following recitation: "I don't know if this is right or not, but I would say if at 9 o'clock Dan would have

been . . . 9 o'clock is three hours away from 12 o'clock, which is 60 minutes an hour--I would say that at 9 o'clock he was 20 minutes from there, just because I would have said he made it in 10, 20, 30--30 minutes! . . . I would say that Dan was 30 minutes . . . from the center of town, and I would say that Dick . . . [going 10 mph faster] so OK, that's 4 hours, so I would just go . . ." She then focused on Dick. Considering that he was going 50 mph and it was 4 hours from 9 A.M. to 1 P.M., Student H exclaimed, "I was supposed to subtract twenty from there! Oh well, I'll just subtract 10, so that's 10, 20, 30, 40. I'd say Dick--40 minutes away from the center of town . . ." She then asked the investigator if this was correct; following the policy of the study, he would not say. However, he agreed to give her a copy of the problem so that she could ask her husband about it. (She mentioned at the next interview that her husband had solved the problem in his head.)

March 11. At this interview Student H said of the last part of the course, "I really thought it was hard. I don't have good background in that so it was really hard for me." She said that the book was unclear (especially in the chapter on real numbers, which was "extremely confusing") and that "I couldn't read it." Although she understood that the real numbers include both rationals and irrationals, she could not follow the book's

discussion of this topic and stopped reading it, instead asking her husband for an explanation, which she found satisfactory. She also felt that the lecturer's explanation of this area had been inadequate--especially the discussion of the meaning of irrational number. Student H said that the lecturer had assigned an insufficient number of problems on the topic of real numbers; she had been unable to do those which had been assigned. She also had attempted some unassigned problems--until she realized they had not been assigned, so she quit. (She pointed out that this was in contrast to her usual habit of doing all problems, assigned or not, because "I learn more that way.") She had found the handout sheet on percent problems "really, really helpful." Student H reiterated her criticism of the book for not providing a full answer set.

Student H said that the topic of rational numbers also had been confusing, but "a little easier to understand [than real numbers]." She continued, "I've never had a good background in fractions . . . we sat down and my husband went through what a fraction was and what a decimal was--I really didn't understand it my whole life. So we spent about three hours going through, figuring out what it was." Student H found the lectures helpful on this topic, but not the book. She termed drawing diagrams to illustrate operations with fractions helpful for herself and potentially useful for the elementary school classroom. She found practice with story problems helpful.

She considered the following problem (page 193) the hardest and had to ask her husband how to do it:

Suppose a recipe calls for $\frac{3}{4}$ cup of flour and $\frac{1}{2}$ cup of sugar. You have plenty of sugar but only $\frac{1}{2}$ cup of flour. You want to make as much as you can. How much sugar should you use?

Regarding the book's demonstrations and formulas, Student H said, "This stuff with all the letters . . . I never even pay attention to it." She thought the book had too many literal expressions; while she did some problems stated in terms of letters, she mostly did those involving numbers only. She mentioned that she had found a problem about a defective tape measure (page 205) a good problem, although initially hard.

Student H had found helpful the lecturer's use of a Venn diagram illustrating the inclusion relation between the counting numbers, integers, rationals, irrationals, and reals, although she said many other students in the class were confused by this. She criticized the lecturer for not discussing the metric system in lecture; although it had been treated in the laboratory, Student H had not learned it from the laboratory work. On the last test she had not answered a question asking for the approximate diameter of a circle of circumference 31 cm because she had assumed on seeing the expression "cm" that she could not do the problem, even though she knew the relationship between the circumference and the diameter of a circle. She liked the way the lecturer discussed homework problems and gave

examples of concepts, and said that, in general, "she really lectures well." Nevertheless, she reiterated that "she didn't go into [real numbers] enough," especially in explaining the difference between rational and irrational number.

Student H then talked about the things she had done in the laboratory session on the metric system, saying there had not been enough things to measure. She said of the session on the geoboard, "That was extremely helpful. I really liked that one I think it'll help me to teach children, too." She added that she had answered a test question on the geoboard incorrectly. Student H said that, in general, "the lab is a waste of time . . . I could have been studying or doing something really important . . . it didn't help that much at all." She pointed out that there had been few questions on the tests about the laboratory work, and recommended that if some activities were important, they could be done by the student at home.

Student H responded to the question on rational numbers by first saying, "Yes, there is." She then converted $\frac{1}{3}$ and $\frac{1}{4}$ to $\frac{3}{9}$ and $\frac{3}{12}$, respectively, and chose $\frac{3}{10}$ as a rational number between them.

Student H correctly converted $\frac{7}{12}$ to a decimal by long division. She remarked that she had not known how to do this before taking Math 201. To find a fraction name for .3777 . . . , she first took 100r - r (after some initial hesitation over whether to use 100r or 10r), and

obtained $99r = 37.4$, so $r = 37.4/99$. Noting the decimal point in the numerator, she said, "I don't think this is right." She explained the general procedure involved, but said it was incorrect to end up with a decimal point in the fraction as she had. Suddenly she got the idea to subtract $100r - 10r$. She did this, saying that she didn't think it was allowed, and ended up with $90r = 34$ and then $r = 34/90$. For the next problem, she divided out $6/11$ and obtained a correct decimal representation. She then said, "Now I don't know how to do this problem, but I would guess to say that you have to bring it out to five places." She therefore gave .54545 as her answer. After reading the problem involving $\sqrt{7}$, Student H said, "I don't even know how to do square root," although she added that she could do it using her calculator. She guessed at $2 \frac{1}{2}$ but mistakenly multiplied $5/2 \times 5/2 = 25/2$. She then said, "I'd just go through all of my numbers until I found something that multiplied by itself would be close to 7." She added, "I don't even understand square roots that well," and reiterated that she could do it using the calculator.

Student H said she would find the length in the first measurement problem by putting a string along the curve to be measured, then measuring the string. To find the area in the second problem, she would again run a string along the perimeter of the figure. Taking this length as the circumference, she would then find the diameter, radius, and area using the formulas for a circle.

Student H received the grade of 4.0 in the course.

March 26. Asked about the course's effect on her feelings toward mathematics, Student H replied, "I don't think they've changed. I've always liked math, and I enjoyed the course, because I like math. I thought it was fun to work the problems. So, if anything, it enhanced it." She said, however, that she expected to have difficulty teaching mathematics, unless she had a good methods course before teaching. About the purpose of teaching mathematics, Student H said, "It's just something that the kids have to learn . . . You just can't get along in the world without it." This showed no change in her feelings from the beginning of the course. She said that there had been no change in her feelings about her major as a result of her experience in Math 201.

Student H said that she had "really liked" the course, except for the laboratory work, which had been "a waste of time." She said, "I could have spent the time studying rather than being in there." While some of the laboratory sessions had been "nice" (she mentioned those on Dienes Blocks and on computation as those she had enjoyed the most), Student H still had found none of them helpful. She specifically called the ruler-and-compass session "a bummer." Nothing in the course had hindered her learning. She had had a feeling that she had retrogressed in mathematical skill when, in halving a recipe,

she computed $1/2 \times 2/3$, where previously she would have taken $1/3$ routinely. However, she noted that she could now take half of any fraction, which she could not do before, and that she also understood decimals now for the first time. She said that the course had gone "pretty smooth," and mentioned to the investigator that she had used her calculator on the final examination to check her answers.

Student H said that she might be interested in taking further mathematics courses, if I had the money and had the time--I like math." However, she expected not to take any more mathematics because her program did not require any.

Questioned about her reaction to the study, Student H said, "Oh, I just don't know! . . . I don't even think about it." She added, "It's been easy, and kind of fun, too." She wished that she could have seen the answers to the problems, though. She had been afraid of the interview situation at the beginning because she had not known what to expect, but this fear soon had disappeared. Student H could not suggest any improvements in the study, any different questions, or any other potential areas of investigation.

Student H thought it would be better to have elementary school mathematics taught by mathematics specialists, but considered this an unrealistic idea.

Evaluation. While Student H had some aptitude for figures, she had very little talent for making mathematical sense out of words, as evidenced by her responses to the gin rummy homework problem and to Problem 1 in the study. The fact that such a student was able to earn a grade of 4.0 in Math 201 suggests that more work in solving verbal problems should be included in the course so that such students may get some badly needed practice on these, and so that their ability to solve them be a component of their overall evaluation in the course.

Student I

Student I was a freshman from Point Pleasant, New Jersey. She had taken a great deal of high school mathematics, including two years of algebra, a year of plane geometry, a year of "pre-calculus," and a year of linear algebra and "space geometry" (vector geometry in n -space). Despite this extensive background, Student I said that she liked mathematics but didn't "love" it. She had taken so much mathematics because she had felt that the mathematics department was the only good department in her generally poor high school. (She had been so repelled by the nonacademic atmosphere and poor teaching in her high school that she had doubled up on subjects so that she could graduate after three years.)

She said she had learned her high school mathematics "extremely well" under a tyrannical teacher whose

high-pressure techniques forced students to learn, or, as many did, give up in despair. "He pushed, and he pushed hard, and he didn't care about anything except math. You learned your math, or else you were no good. He was very specialized." Although Student I had not been one of those who gave up, she said the material "wasn't that relevant to me." She felt she did not have the mathematical ability of her brothers who had previously studied under this teacher, and was not motivated, as was much of the class, by the desire for a high mathematics Scholastic Aptitude Test score. (The teacher set as a goal a class average of 750 on the mathematics SAT--and achieved it!)

Student I said that her favorite subject in high school had been English literature, but that she enjoyed nearly any subject if, "it's taught right and I can get something out of it." She said the subjects she had disliked the most were health ("making a marriage ceremony"), American literature, and French; she attributed all these dislikes to poor teaching in these courses. In high school, Student I had been involved with the yearbook, the band, dramatics, and sports. She had not taken any college mathematics prior to the study.

Student I was planning at the time of the study to major in special education of the emotionally disturbed. She had become interested in this from doing volunteer work at a special school near her high school where she had helped teach the pupils simple skills such as tying their

shoelaces, counting to ten, and color discrimination. She chose emotionally-disturbed over mentally-retarded education because of job prospects. She did not expect to teach very much mathematics beyond counting to her future students, and therefore did not anticipate any problems in her mathematics teaching. Asked the purpose of elementary school mathematics, she replied, "You need it for existence, just getting along in society . . . The younger you learn it, the better."

January 24. At this interview Student I rated the lectures she had attended "very, very dull." The material was all familiar to her; when she had trouble, she was able to get help by reading the book. She found the lecturer's style, which she described as "third-grade," "annoying." She said she had derived no benefit from attending the lectures.

Student I had done her assignments on a particular topic before that topic was discussed in the lecture. She suggested that the lecturer discuss problems whose answers do not appear in the book. She felt that reading the book was adequate preparation for doing the problems, which were "set up really well."

Student I described the laboratory sessions she had attended as "disorganized--nobody really knows what they're doing. You end up sitting there an hour . . . just waiting for him to get around to your table to show you what you're

supposed to be doing." She had enjoyed the session on Dienes Blocks for its pedagogical interest ("really excellent . . . a good way of teaching it") but called the use of color cubes to illustrate sets "senseless," remarking that it had confused further those students who were ignorant of sets. (Student I said she found it "shocking . . . how many people don't know about sets, yet they're here.")

Student I made the following responses to the exercise on sets: She said the set of all living people matches "set E, because set E is infinite, and the set of all living people is infinite." The set of counting numbers from 1 to 100, inclusive, was said to match "set D, because the set of counting numbers from one to zero [sic] inclusive does not include zero, but set D includes zero and doesn't include 100 so it would match." Student I said the set of suits in a standard deck of cards matches "set C, because the number of suits in the set . . . would be four, and the number of set C is equal to four." Regarding the set of all aardvarks enrolled at M. S. U., she remarked, "That would be the empty set, I hope, and it wouldn't match any set [given]. There's no empty set listed there."

In converting 405_{eleven} to base ten, Student I knew that it was $4 \times 11^2 + 5 \times 1$ but mistakenly took 132 as 11^2 , obtaining an incorrect answer. She translated each of the following two numbers correctly, labeling her columns and placing the appropriate digit in each one. She voiced these

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thoughts on the first missing base problem: "Nine plus seven equals four . . . it's going to be pretty close to base ten because adding them normally in base ten it comes out pretty close to eighty-four. It would be eighty-six . . ." She soon decided the base was twelve because a twelve had been carried. On the second of these problems, she thought silently for a long time before saying, "It's going to be a lot less than base ten," a conjecture she made from considering the digits that appeared. She finally obtained base three by inverting the problem to an addition problem, $2 + 2 = _1$, then checking her conjecture with the rest of the example.

Student I solved Problem 1 by adding within coins, obtaining 3 quarters, 7 nickels, and 1 penny, then exchanging for fewer coins.

Concerning the interview situation, Student I said she had been "at first a little worried because I didn't know what sort of problems they'd be . . . Otherwise . . . very casual."

February 7. Student I said at this interview that while all the lecture material was still familiar to her, she thought the course was being "presented in a good manner." Nevertheless, she maintained that "it's very, very slow paced . . . it could be a lot quicker . . . much, much too slow . . . it lags, it makes the lecture boring." She also complained of too many repetitions of a

demonstration by the lecturer even in the absence of questions, contending this and the slow pace caused a general boredom in the class.

While Student I could not name any features of the lecture as being particularly helpful or confusing, she did find some value in attending lectures: "For me, it judges . . . if I do actually really know and understand the concepts that are presented. I can sit there and listen to it and say, well, yeah, I understand, but until I actually sit there and try to do it and think it out I'm not going to really know if I know it or not . . . so I find it quite valuable." She remarked further that she found the lectures "usually quite clear" and that the other students especially appreciated the lecturer's discussion of assigned problems, particularly those whose answers were not provided in the book.

Student I could not name any of the assigned problems as being particularly helpful to her. She said that "the way that problems are arranged" was helpful; the logical progression of steps forced her into reasoning. The assigned reading was sufficient for her to understand the concepts taught. She could not name any assigned problems as being especially helpful or confusing, but said that some were perhaps too basic.

Student I again criticized the laboratory sessions at this interview, saying they were "just a get-together time for us . . . a waste of time." She said she wished

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attendance at these classes were not mandatory; as things were, she would fool around for an hour and leave. She said the laboratory classes were repetitive of ideas in the text, and that since she understood the text, she had no need for them. Indeed, she felt they were counter-productive, saying, "You just lose interest in it completely when you're just pushed into it that much." While she considered the course as a whole "a good refresher," she said only one of the laboratory sessions had been helpful for her. She had found nothing in the laboratory confusing, except when due to a shortage of equipment she had to do base six arithmetic using base five Dienes Blocks.

In response to Problem 2, Student I first said, "I imagine the things I'm going to have to work with is the fact that . . . the numbers are 0 through 9, which is ten numbers, the ten numbers are going to be arranged in groups of four, for the second [part] of the number . . ." Her first guess at the number of possible endings was 10×4 ; however, she did not think that was correct. She then said she did not know how to enumerate the possible endings, but once this number was determined, it should be multiplied by 3. After writing down some possible endings, she thought they might number $9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$, but after some reasoning discarded this hypothesis also and gave up.

February 25. This interview had been delayed due to Student I's illness, which had prevented her from attending class for more than a week. At this time, she was less critical of the lectures than she had been earlier, saying that they had improved compared to the "spoon-fed" presentation of the beginning. Although she was not as bored as she had been before, she still had not seen any material that was new to her. She still considered the course "a good refresher," but felt she would probably need additional refreshment if she ever had to teach this material. She could not recall any specific features of the lectures due to her prolonged absence. She also had not done any assignments since the previous interview.

Student I had a favorable response to the laboratory session on Cuisenaire rods; she found them a "valuable" technique for "conveying the whole idea" of fractions. She also liked the tangrams she worked with during this session, but "didn't think much of" GeoBlocks. She had found "interesting" the session on ruler-and-compass constructions, for which she had had to recall her high school geometry.

Student I said that on the whole, the course had improved and was not as boring and repetitious as it had been earlier. She still had not seen any material that she expected ever to teach.

In testing the four given numbers to see if they are primes, Student I did all her trial divisions mentally.

She found that 7 is a factor of 119 after mentally trying 2, 3, 4, 5, 6, and 7 as divisors. She tried the numbers 2 through 10 as divisors of 113 before calling it prime; for 227, she tried numbers through 15, stopping there because 15^2 is approximately equal to 227, then calling the latter prime. She tried divisors through 16 for 247; she called this one a prime also, apparently having erred when she tried to divide by 13.

In taking the greatest common factor and least common multiple, Student I correctly factored each number into primes and then took the correct combination in each case.

In considering Problem 3, she read "9 A.M." as meaning that time the following day. Miscounting 16 hours as the time between 1 P.M. and 9 A.M., she computed Dick's distance to be $16 \times 50 + 40 = 840$ miles.

March 7. At this interview Student I began by making favorable comments about the two most recent laboratory sessions. She had been completely unfamiliar with the metric system before, and found the laboratory session insufficient practice. She had enjoyed using the metric measurements and developing an intuition about their size. Concerning the session on the geoboard, she called it "excellent . . . a little bit more challenging, you had to use your head."

Student I described the most recent lectures by saying, "It seems to drag. Too picayune in some places, then gliding over other places, not really going over . . . what we might need but what she feels we need . . . It doesn't work." Asked which topics merited more class time, Student I said that there were none for which she personally needed more information. She felt the lecturer could have eliminated some repetition and some detail; she had gathered from conversations in the laboratory that the pace was considered slow by most of the students. She noted that lecture attendance was decreasing sharply due to a general boredom. She could name nothing in the lectures she had attended as being especially helpful or confusing to her.

Student I then proceeded to praise the textbook and the problems in it; her extensive mathematics background probably made it easier for her to read than it was for others in the class. She said, "I could have gotten along with just the book and the problems and said the hell with the lecture because the problems do go over basically everything . . . The book and the problems are fantastic." Student I said that if she were taking the course again, she would not attend lectures, but would read through the book at her own pace. Specific features that she liked included the depiction of rational numbers as cut-up squares (which she called "clear and simple to understand" and "fantastic") and base two "decimals." Student I found

confusing the book's description of the algorithm for finding a fraction name for a repeating decimal; she claimed to have a "block" about this procedure, continually forgetting it and having to look it up. (Nevertheless, she recalled it correctly in the problem part of the interview.) She could recall nothing else which gave her trouble.

To find a rational number between $1/3$ and $1/4$, Student I first converted both of these fractions to 12ths. She then took a "middle one," $3.5/12$, which she changed to $35/120$ and then reduced to $7/24$.

She found the decimal name for $7/12$ by long division, then in the second exercise executed the algorithm she had claimed to have difficulty remembering, taking $10r - r$ and obtaining $9r = 3.4$. Not upset by the 3.4 as were some other subjects, she simply said $r = 3.4/9 = 34/90 = 17/45$. Student I made an arithmetic error in finding a decimal name for $6/11$, but cut off her answer after five places to obtain the requested approximation. To approximate $\sqrt{7}$, she noted that it was between 2 and 3; then, feeling that it is closer to 3, she guessed 2.6, squared it, and got 6.76. She realized that this was an underestimate and took 2.7 and squared it. When this turned out to be 7.29, she guessed 2.64. Seeing that the square of 2.64 is 6.9696, she considered 2.64 to be an adequate approximation.

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string and measure the string. She also had the idea of obtaining an approximation by breaking the curve into circular arcs and measuring these. To find the area in the second measurement problem, Student I said that she would enclose it in a shape whose area she could find, then try to subtract the excess. Unsure of what shape would be best for this purpose, she said she would try several to determine the best one. She did not say how she would proceed in subtracting the excess area. She also suggested that the area could be found if one would "put shapes in it and add it up"; she was hazy about these details as well.

Student I received the grade of 4.0 in the course.

April 4. At her final interview, Student I began by saying that her feelings about mathematics had not changed as a result of her experience in Math 201. Indeed, she did not consider it a mathematics course, but arithmetic. Student I did not expect ever to teach this material and would not want to. She still considered elementary school mathematics "a necessary essential." She had undergone no change of feelings about her major.

Student I again emphasized the lectures as the most flawed part of the course. Suggesting that "they could have been a little bit less drawn out, and not repetitive," she added that they had been unnecessary for herself. Laboratory sessions that Student I recalled favorably were

those on the metric system ("really valuable"), the geoboard, Cuisenaire rods ("a good teaching method"), and tangrams. She said nothing had hindered her learning, but that attendance in the laboratory should not have been required, and that she had attended lectures only so that she would be able to answer the questions put to her in the study.

Student I said she would like to take more mathematics, and was considering taking calculus as soon as she had the time.

Regarding the research study itself, Student I said that she had felt comfortable in the interview situation and thought the idea of the study good if it could lead to constructive results. She could suggest no questions for such a study, but did suggest that actual teaching of mathematics be done by Math 201 students.

She found it "hard to say" whether elementary school mathematics should be taught by general classroom teachers or by mathematics specialists. She doubted the methodological usefulness of Math 201, since she had seen second-graders in East Lansing doing mathematics in individual notebooks. She therefore suggested deemphasizing teaching techniques in Math 201. She felt that overall, the course had been no more than "a good refresher."

Evaluation. Student I was one of those subjects who were mathematically overprepared for this course. Those

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few laboratory experiences she found worthwhile could easily be incorporated in the methods course. If students with the background of Student I are not permitted to waive the course, the department should feel obliged to provide them with a course that will not be a waste of their time.

Student J

Student J was a freshman from Rochester. She had taken three years of high school mathematics--two of algebra and one of geometry. She had enjoyed her first two years. Algebra I she had found "challenging" and "fun, but I didn't like the story problems." Geometry had been her favorite class, because she had had a good teacher who had "worked out logical problems [and] worked with physical things like we do in the lab in Math 201." In Algebra II she had had a mediocre teacher and had lost interest in mathematics; she later regretted not having taken Algebra III. Student J felt that "I just think math is a good thing . . . It's a good way to teach you how to think, how to use logic." She felt her mind was "rusty" from having taken no mathematics or science in her senior year. Although she felt she had learned her high school mathematics well, she was not sure how much of it she retained at the time of the study.

In high school Student J's favorite subjects had been English and history. She had liked science the least,

particularly biology. She felt that the study of English and history enabled her to communicate better and to understand the present better, while science was "just memorization" and did not apply to everyday life. Student J had participated in high school athletics; she had joined the mathematics-science club in order to help her in studying mathematics. She had not taken any college mathematics prior to the study.

At the time of the study Student J was still undecided about her major. She was taking Math 201 as part of a child development-elementary education major (aimed at a nursery school teaching position), but was also considering a major in television and radio. Asked to explain her interest in education, she said, "I like to see a child grasp onto something. I like to work with him and help him understand his world better." As a result of babysitting experience, she had realized that she loved preschool children. The term prior to the study she had taught four-year-olds in a religious school. She also had worked as a teacher's assistant in a Montessori school.

Student J said, "I feel scared [of eventually teaching mathematics] because it's very important. I don't think I have enough confidence in math. In fact, when anybody says math, I cringe . . . because I'm afraid I'm going to really confuse them." She hoped to alleviate her fears by taking more mathematics until she mastered the subject.

Student J said that elementary school mathematics is important because "math is a means of thinking." She then attempted to give an example, describing the oil shortage in terms of set theory.

January 24. Student J began this interview by commenting on the lecturer's style, one described by most of the subjects as the way one would address elementary school pupils. She expressed doubt about the appropriateness of this style for the lectures, saying, "I think we should learn that in the lab." About this style, Student J said that while "I think she's a very good explainer," too much time had been spent on explanation of concepts which did not warrant this time--on the whole, "it's a good style to have for slower people." In particular, Student J mentioned that she thought the Austrian method had been explained well, she liked the use of Venn diagrams to illustrate subtraction, and she liked the use of formulas for counting subsets and one-to-one correspondences. She had found the treatment of nondecimal bases boring; it had confused her to label columns "ones," "fives," "fives of fives," etc.

Student J said that in general she had found her assignments to be helpful; she mentioned particularly a series of true-or-false questions regarding subsets. In general, she found the book "a lot of wordiness" and disliked its habit of always qualifying its general statements.

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Student J enjoyed working in small groups in the laboratory; she found this to be "testing your ability to explain things . . . to a child." She noted that while she and her colleagues always used the laboratory materials as directed, many students did not, doing the problems by written computation instead. She rated the Attribute Games "kind of elementary . . . one hour of that would be enough." She considered the laboratory class a good place to ask questions on lecture material.

After this conversation, Student J answered the questions on sets. She said the set of all living people matches "none because this [set A] isn't all of the people anyway, and then two of them are dead . . . this [set E] could be if you're counting the people." She was not sure about set E, but considering that the set of all living people is finite, she said none of the above. The set of counting numbers from 1 to 100, inclusive, was said to match none of the given sets, since set D went only through 99. Student J said the set of suits in a standard deck of cards matches set C. She realized that the last set described was the empty set, and spent some time trying to decide whether this matches set B. Although at one point she said that set B has one element, she finally said the empty set did match set B.

On number bases, Student J correctly wrote 405_{eleven} as $4 \times 121 + 5 = 489_{\text{ten}}$. She also correctly carried out the two conversions from base ten by labeling columns for

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Student J solved Problem 1 by summing within coins, exchanging along the way. She then verified her answer by totalling the amount of money and checking that this matched her coin total.

Regarding the interview, Student J said that although "I felt dumb when I couldn't get that base three," she had liked the interview. She noted that her participation caused her to think about the class.

February 7. At this interview Student J remarked favorably on a perceived change in the style of the lecturer, who now "didn't talk to us as though we were children [but] as future teachers," though traces of the old

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style remained. Student J especially liked the lecture presentations of division and of prime numbers. She remarked that in general, "things are moving faster." Although she had found the Sieve of Eratosthenes confusing, "I know it's simple." She was unable to divide in non-decimal bases and converted the numbers to base ten in order to do such problems.

Student J voiced her approval of the lecturer's discussing more of the assigned problems in class. Of finding the least common multiple of $\{1, 2, \dots, 10\}$, she said, "that's an easy one but . . . I didn't go through the prime method to figure it out." She also had some trouble with the Sieve of Eratosthenes, as well as with arguing that $(7 \times 6 \times 5 \times 4 \times 3 \times 2) + 1$ has a prime factor greater than 7.

Student J had liked the laboratory session on computation; it had included lattice multiplication, which she had seen before and thought was a good way to do multiplication. In the session on clock arithmetic Student J had successfully challenged her instructor on a point (he had said division mod 8 was closed); she was proud of this. She had seen clock arithmetic in her previous education also. She considered both of these laboratory classes to be helpful to her.

Student J at this interview again complained about the book's verbosity, pointing out "all this big paragraph they explain just to say that an integer addition is an

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extension of a counting number addition." She recommended, "These little things are so simple to me . . . maybe they should just state it instead of going into detail."

Student J greeted Problem 2 with a gasp. Her first mental reaction was to think of probability, which she had studied the previous term in natural science in relation to genetics. She stated, "You have three chances for the beginning, and for the end you have a chance of a combination of a group of four . . . from any ten digits, 0 through 9." However, she was unable to enumerate the possible endings. She said, "The only way I can think of is one of those long, tedious processes of pairing up numbers--ordered pairs." Asked how, she listed some 5-tuples (apparently she was now thinking five instead of four) of digits. In frustration, she exclaimed, "Oh, it's just infinite!" but apparently did not mean it literally. All she could say about the number of possible endings was that it was "more than forty."

February 21. At this interview the first part of the discussion concerned the lectures. Student J said that those on integers had been "all right." However, she had become generally bored with the lectures. She attributed this to the classroom environment--that of a large, cold hall with numerous distractions--rather than to the instruction itself. Student J said that the lectures were worth her attention, but that she was unable to give it.

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She conjectured that it might be the approaching end of the term that was causing her ennui. She had liked the array diagrams for depicting operations with rational numbers. The use of absolute value in defining operations had confused her.

Student J had not derived much benefit from her most recent laboratory session--that on real number constructions. Nearly an hour had been spent discussing the second test, there had been a shortage of materials, and her instructor had not been able to give any help to Student J's table. She remarked that most of the material in this session was appropriate for elementary school, with the exception of the Pythagorean theorem. Student J had difficulty recalling the details of the previous week's laboratory class. Reminded of them, she said that working with GeoBlocks "was OK, . . . really good," and that Cuisenaire rods were "pretty good . . . good for kids." She could not recall tangrams. She thought that session as a whole to be "pretty good" but remarked that it had not been helpful for actual computation.

Student J repeated her criticism of the book, saying that she would like a text consisting of important statements illustrated by examples. She had liked the book's use of "happies" and "sads" to illustrate positives and negatives. She thought the problem of naming sets of integers closed under addition and/or multiplication was good, but was disappointed when it was not discussed in

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lecture. She called the assigned computations of products of signed integers "simple." At this point, Student J indicated some part of the book and exclaimed, "See, look at all this writing--it just drives me bats!" While she would have liked to read only the emphasized statements, she was afraid she might miss something important this way. Student J was disappointed that assigned problems asking for examples of absolute value relationships had not been discussed; she said, "These really test your understanding of absolute value," and she had found them troublesome.

In the problem part of the interview, Student J correctly identified 119 and 247 as composite numbers and 113 and 227 as prime numbers. For 119, she discovered 7 is a factor after dividing by 3, 2, 5, and 7, all of these by long division. (She skipped 5 on the second number and skipped 2 and 5 on the third and fourth numbers.) She found the factors of 247 after having tried as divisors 3, 7, 37 (a wild hunch whose division she did not complete), 11, and 13. For 113, she tried 3, 7, 9, 11, 13, and 17, and for 227, these divisors and also 19, before calling the numbers prime. She said she could stop at this point because the "square of 19 is bigger than that."

Asked to find a greatest common factor and a least common multiple, she correctly factored both numbers into primes in both problems. While she took the least common multiple correctly, for the greatest common factor she took

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On Problem 3, Student J responded by writing $R \cdot T = D$ and attempting to construct a table incorporating the information in the problem, as she had done with motion problems in the past. She noted the rates of the two drivers, and represented their respective times as x (Dan) and $x+1$ (Dick). She then set up the equation $40x = 50(x + 1)$, but before solving it, said, "I don't think I'm doing this right, for some reason . . . the distance should be the variable here, not the time." She then wrote $R/T = D$, but could not proceed further with the problem.

March 7. At the beginning of this interview Student J expressed her approval of the lecturer's presentation of the topic of fractions, mentioning in particular the use of boxes to illustrate arithmetic operations with fractions (which Student J had trouble learning from the book) and the rationale for common denominators. Student J also mentioned that she had learned how to explain the multiplication of decimals. She felt that she attained a "better understanding of decimals by working through fractions," converting back and forth. She expected that the "abstract" idea of irrational number would be difficult to teach. Student J faulted the lecturer's discussion of scientific notation: "She made it so hard . . . just the way she explained it," although

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actually Student J found it easy. Student J explained that "you just count off the places" to put a number in scientific notation. She further faulted the lecturer's explanation of "decimals" in base two, which similarly "wasn't that hard." Student J commented that the lecturer recently had chosen to discuss the more difficult homework problems rather than the routine ones, and said that she approved of this.

Concerning specific homework problems, Student J remarked that most of the class had appreciated the lecturer's illustrations of the fact that division is not left distributive, but that she had not needed this demonstration. She also mentioned that many in the class had trouble with a verbal problem involving fractions. She considered a good exercise one in which she learned which fractions' decimal representations terminate; she had found this idea "confusing." Student J had found it easy to arrange a sequence of fractions in order but again noted that many in the class had had difficulty with this. She said the handout sheet on percent problems had been helpful. (During this discussion, Student J misused the term "irrational" to mean a nonterminating decimal.)

Student J had enjoyed her laboratory session on the geoboard involving the concepts of fraction and of area; her only reservation was to doubt the wisdom of using rubber bands in elementary school. She had also found helpful the session on the metric system, saying, "I can't

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wait till we switch over to it." However, she noted that many students in her class had had difficulty with this exercise, and that many had neglected it and instead studied for the test.

Student J mentioned that, although at the previous interview she had reported that it had become boring, the last segment of the course had been interesting.

In trying to find a rational number between $\frac{1}{3}$ and $\frac{1}{4}$, Student J's first idea was to multiply these two to get $\frac{1}{12}$, and then convert to the equivalent fraction $\frac{2}{24}$. She felt this was wrong, however, so she tried another approach. Saying she wanted to "chop this area up into $\frac{2}{3}$," she took $\frac{2}{3}$ of $\frac{1}{2}$ and obtained $\frac{2}{9}$; still not sure, she took $\frac{2}{3}$ of $\frac{1}{4}$ also. When she was not satisfied with these numbers, she began to multiply $\frac{1}{3}$ and $\frac{1}{4}$ by $\frac{5}{7}$. Asked where $\frac{5}{7}$ had come from, she replied, "I'm just trying a prime numerator over a prime denominator." She soon said, "That's probably wrong," and gave up on the problem. She explained her reasoning thus: "If I multiply $\frac{1}{3}$ by 1, it's just the same as $\frac{1}{3}$. So I thought, if I multiplied it by $\frac{2}{6}$. . . , well, that's $\frac{1}{3}$, so I thought, well, maybe by something prime." It seems that Student J was trying to find the right fractional multiple of $\frac{1}{3}$ that would fall between $\frac{1}{3}$ and $\frac{1}{4}$. Her confused attempts illustrate her lack of intuition about fractions.

Student J was able to find a decimal name for $\frac{7}{12}$. She also knew the algorithm for finding a fraction name for

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a repeating decimal, but, executing it in haste, took 100r - 10r to be 10r. In attempting to find a terminating decimal approximation to $6/11$, Student J divided out this fraction and obtained a repeating decimal, but did not understand the part of the problem concerning error. After the investigator explained the idea of error, Student J asked, "Is that like uncertainty factor in chemistry?" She was unable to proceed further with the problem. Asked to approximate $\sqrt{7}$, Student J first thought of the Pythagorean theorem, but could not use it in this context. She knew the meaning of square root, and said it must lie between 2 and 3, probably closer to 3; this was her best possible approximation.

On the first measurement problem, the only way Student J could imagine for finding the length of the curve was to "stretch it out"; she could not say how. She also thought of approximating segments of the curve by circular arcs, but was unable to do this. On the area problem, Student J suggested choosing an arbitrary square unit, then superimposing a grid of these squares over the figure. One could then estimate the number of square units covered by the figure.

Student J received the grade of 3.0 in the course.

March 28. Student J opened her post-course interview by describing this change of feelings toward mathematics: "I'm a little bit more confident now . . . I

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was afraid I wouldn't be able to communicate . . . the concept of it to the children, but now I think I know [how]." However, she expected to have to brush up again before actually teaching. Of teaching, she said, "I was afraid before . . . and I still would rather not do it, but now that I've learned all these games . . . I realize it can be fun, rather than just addition and multiplication tables." She did not expect to teach mathematics, though, because she was specializing in pre-school education. Student J reported no change in her feelings about the purpose of teaching mathematics. She felt that the study of mathematics should be required throughout high school and even in college, because "math is very important to get your mind thinking in a rational way, logical way." She was still unsure about her major, considering a change to communications. She was worried about possible competition for elementary education positions, portended by the large enrollments in her education courses. Math 201, however, had not influenced her choice of major.

Asked for suggestions for improving the course, Student J recommended the routine of the lectures--new exposition at the beginning followed by a discussion of homework--be reversed. The laboratory sessions she had enjoyed included those on attribute games, GeoBlocks, Dienes Blocks, and the geoboard. She had disliked the session on ruler-and-compass constructions. In general, she said "the labs were OK." She said that she felt the

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lecture class was too large a group, and that she could have learned more in a smaller class. Student J said that she possibly might take more mathematics; she was considering taking Math 108 (college algebra). She said that she was hoping to take two more mathematics courses, one in each of the following two years.

Of her participation in the research study, she remarked, "I didn't mind it; I enjoyed it," because "it helped me think about what I felt about the course. I never analyzed a course before." She also said it had been "interesting . . . challenging . . . to see if I could work out those problems." She thought that her comments about the course had been constructive. She could not suggest any possible improvements in the study or any areas that had not been investigated, other than to suggest that the investigator visit the lectures to get a better idea of what the subjects were discussing; the investigator explained that this had been considered and rejected because it had been felt it might harm his image as a disinterested outsider.

Asked if generalists or specialists should teach elementary school mathematics, Student J replied, "If this was my first meeting, I'd say math specialists, but now, I think [that] we're capable of doing it now, and if math specialists teach us, then we can relate to the kids because they get used to a teacher all day." She felt

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Student J's final comments were to say that she had liked the course. She thought the instructional staff had treated the students fairly, and said she had particularly liked her laboratory instructor.

Evaluation. Despite her respectable performance in the course, Student J still had trouble with several mathematical concepts, as well as a tendency to misuse mathematical ideas, as in her description of the oil shortage in terms of set theory. One would hope she would have further study in mathematics prior to her teaching career.

Student K

Student K was a sophomore from Middletown, New Jersey. She had taken four years of high school mathematics, including two years of algebra, one of geometry, and a full year of trigonometry. She had been in the higher track of those taking the mathematics sequence, and had had the same demanding teacher for four years. She had enjoyed this experience, and considered that she had learned it well because the things she had studied then came back to her as she was taking Math 201.

She had liked her high school teacher's style and remarked at her first interview that the lecturer taught

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in the same style. Asked to describe that style, she said, "She's going over what's in the book and she's repeating the rules and writing them down again--just teaching as it is in the book, and then of course when you do your homework you have to read that section first anyway, and so it makes it almost as review to what she said in class . . . and she assigned enough [problems] that even if you know it, if you do them, it's practice enough so you should be able to get it down." Saying this system worked for her, she said that to "go through it step by step" is "the only way it can be taught to people who maybe don't know it yet or have never had it."

Student K's favorite subject in high school had been mathematics; she attributed this feeling to the influence of her teacher. She had disliked science the most of all her subjects. Although she perceived the symbiotic relationship of mathematics and science, she felt that mathematics is "a structured, logical thing that you can sit down and learn even . . . on your own" (though with some help) while in science "there's a lot of abstract concepts . . . for me it's hard." She also said her feeling might be due to the teachers she had had. Student K had been active in a variety of high school extracurricular activities. She had not taken any mathematics in college prior to the study.

Student K was majoring in special education of the mentally retarded. She was deterred by job prospects from

majoring in general elementary education, although she was also considering acquiring the necessary background to be a teacher of remedial skills. She felt she had enough patience to be a good teacher of the mentally retarded. Student K had taught in a Saturday religious school while in high school. She had also worked with trainable children, trying to teach them everyday skills, such as brushing teeth, and some simple concepts. (It had been impossible for her to get the children to recognize the difference between a rectangle and a square.) In Education 101A she had worked as an aide in a regular fourth-grade classroom. The school was using IPI (Individually Prescribed Instruction) mathematics; Student K felt that the pupils were not learning well under this system due to insufficient pupil-teacher contact.

Student K said that she was "glad" about eventually teaching mathematics, although she was "not a brain in it--that's for sure." She felt confident that once she had mastered the material she would be able to present it well. Acknowledging the need for children to know the basic operations, she also said the purpose of elementary school mathematics was "to learn the logical system of it . . . work your mind into something logical."

January 22. At this interview, Student K called the lectures "good"; she said that they helped to explain the book, and felt that she understood the material.

She said that the lecturer explained problems step by step, choosing from each assignment at least one problem whose answer did not appear in the book. She found it helpful to convert from one nondecimal base to another without expressing the number in base ten.

Student K said that the homework problems helped to clarify the text, which she found unclear. She said if one did not do the problems, one would "be stuck." While she could name no problem as being particularly useful, she said "they all help." Student K usually did the problems before attending lecture; the lectures then clarified what she had done.

Student K said she had enjoyed both of the laboratory sessions she had attended to date. Regarding attribute games, she said, "They were hard," and commented, "Combining three and four sets and intersecting . . . made things more clear . . ." She said that the groupings of students, in which they explained the activities to one another, was a helpful device. Student K also liked Dienes Blocks. She said of her group, "We did good on that one," and felt that the blocks had helped her to understand bases.

Student K felt that she could not be sure of her learning until she took a test. She had seen bases before, but not in such detail as in the current presentation.

The problem sheet on sets was then presented to Student K. Regarding the set of all living people, she

said, "This doesn't really match any of these sets," because set E is infinite and set D "ends too soon." She said that the set of counting numbers from 1 to 100, inclusive, would not match any of the given sets, because set D includes 0 and does not include 100. She decided that the set of suits in a standard deck of cards does not match any of the listed sets, because "there's nothing that includes the four--hearts, clubs, spades, and diamonds--C is way off." After laughing at the last set to be considered, Student K said, "I hope it would [match] B." After some thought, she realized that set B has one member, and so does not match the set of all aardvarks enrolled at M. S. U. At this point, the investigator was about to proceed to the next group of problems, but Student K yanked back her problem sheet. She suddenly had realized that she had confused the concepts of matching and equality. After looking over the sheet again, she said that the set of counting numbers from 1 to 100, inclusive, matches set D, and that the set of suits in a standard deck of cards matches set C.

Student K correctly translated 405_{eleven} to base ten, noting that it represents 5 ones, 0 elevens, and 4 eleven-squareds and computing the sum of these in base ten. To express 39_{ten} in base two, she labeled her columns correctly and filled them from left to right, at each stage correctly determining which digit should fill that place. In the same way she correctly converted 44_{ten} to base

seven. In the first missing-base problem, Student K noted that since nine plus seven is sixteen, "you moved a group of twelve over," and therefore the base was twelve. She struggled for a while with the second of these problems, but eventually saw that the solution was base three. She said, "You need four up here to get two here [in the difference] so you need to bring over three units but it seems to me it does not work further." After more checking, she realized that base three was correct. Student K explained that her problem was that she actually had been working in base ten although she thought she had been working in base three.

Problem 1 was given next to Student K; she solved it by adding mentally within coins, obtaining 3 quarters, 7 nickels, and 6 pennies. She then exchanged these for the fewest possible coins. She checked herself by computing the amount of cash both before and after the exchange and verifying that they were equal.

Asked how she had felt in the interview, Student K replied that she had felt silly when she made mistakes on the problems, especially confusing matching and equality of sets.

February 7. Student K began this interview by saying that this section of the course had been "harder than the first part . . . working with primes is confusing." Nevertheless, she was still "doing OK," reviewing familiar

material. Commenting on the lecturer's style, Student K said, ". . . to take an idea, to go through why it works the way it does, to give you the reasons and the proofs for it, then to go over homework, and especially the ones that don't have the answers in the back, to show you why it would work, 'cause we always cover the problems in the back that go over properties and make you go through a proof . . . I like that. I think that's why I don't have any problem understanding it. It makes it much, much easier than if you just read it and vaguely understand it and try and do a problem and get it wrong and then not understand why." Student K had not yet encountered any problems with the material. She could not name any features of the lecture as particularly helpful or confusing because all of the material so far had been familiar to her.

Student K usually did her homework problems a week before the topic was discussed in lecture. She said of the problems, "I find them more than helpful, I . . . find them essential . . . it's putting into practice what I just read to make sure I really understand it If I didn't do the problems, I probably wouldn't have done well on the test or wouldn't understand . . . what she's talking about . . . because I understand things better through examples." Student K found it repetitious to do the elementary school exercises reproduced in the Supplement to the text. Presented (page 99) with a "phony 'proof'

that $1 = 0$," she "didn't know the exact reason" why it was not correct. She said she had trouble identifying number properties (such as distributivity, identity, etc.) illustrated by equations. Student K found division hard, but had no trouble with prime numbers or with the integers. She was befuddled for a while by a division algorithm in which partial quotients are stacked vertically, and said that she was sorry for children who had to learn this algorithm, although it was probably all right for people who grew up with it.

The conversation then turned to the laboratory work. Student K said that the session on clock arithmetic "went through the whole thing, from groups to fields" but "sort of dragged out, because we waited for every group to go through it." Nevertheless, she still had left this session early. Regarding the session on multiplication and division with Dienes Blocks, she commented, "The division--I could not figure out what he was doing and I had him go through it at least four times before I could figure out what he was doing. To do it with the blocks . . . I couldn't figure out where he was starting with his units, how did he figure out how many to start with . . . I finally got it!" Of the session on computation, Student K said, "I liked it. I think it was a good way of learning to teach younger kids the properties of subtraction and addition." She remarked that while other Math 201 students had said they would not use Napier's bones in an elementary school

classroom, she would use them to interest pupils who already had mastered paper-and-pencil multiplication. The work on division algorithms "helped me to do my homework. . . . It was when I did my homework that I finally understood what we were doing in lab." She added later, ". . . it probably would have taken twice as long to do the homework if I hadn't done the lab." Student K saw the session on computation as an illustration of teaching methods, not as an aid to her own learning. She said that she had found nothing confusing in the most recent laboratory sessions.

Student K said that the pace of the course was "a little slow" for her. She skipped lecture on Fridays. She thought the lecturer spent too much time discussing homework. Student K felt that the students in the class did not do their homework before it was discussed by the lecturer; she got this impression from hearing the same question asked repeatedly in lecture--about a problem which was explained in the textbook! Student K admitted that she too would do this if she did not understand some of the material. In general, she said she liked the lecturer's teaching style.

The only problem presented to Student K at this interview was Problem 2. (Hearing there was only one problem, she joked, "I can only make a fool of myself once?") Considering the problem, Student K pointed out that there were four digits in an ending, with ten choices

for each. Discarding 40 as an unreasonable total for these, she realized they must number $10 \times 10 \times 10 \times 10 = 10,000$. She therefore concluded that there are $3 \times 10,000 = 30,000$ possible numbers.

At the end of the interview, Student K told the investigator that she had scored 100 on the first test, and remarked that her fiance would have "killed" her if she scored any lower, since she knew all of the test material.

February 21. Student K said at this time that her main interest in Math 201 was in obtaining suggestions for future teaching; she had seen all of the material before. She had been most interested by the presentation of operations with fractions, which showed her to "work it out step by step . . . [not] teach the rules first"; this made clear the reasons why the algorithms for these fractions work. Student K was behind at this point in doing her problems, but had kept up with the reading. She said that she liked the book's presentation and intended to keep the book. Before Test 2 she had been able to read through one more section than the test covered; she remarked that the "test didn't give me any problems, other than I went blank." She said that this test had been comparable to the homework problems, but harder.

Commenting on the laboratory session on ruler-and-compass constructions, Student K said that she "didn't have that much of a problem . . . understanding it," although

"we had to review a lot." She said she had liked this session because "I hadn't thought of some things [such as areas of geometric figures and the Pythagorean theorem] in certain ways." She commented that seeing "different ways to arrive at the same thing . . . helps me" in preparing for teaching. She remarked that she "couldn't remember' how to evaluate expressions such as $1 + 1/2 + 1/4 + 1/8 + . . .$ and $1 + 1/2 + 1/3 + 1/4 + 1/5 + . . .$ and how to find the area of a figure built on the diagonal of a given triangle. Student K said that attendance at the laboratory sessions was becoming rather irregular, and pointed out that at the most recent session her instructor had moved among the various tables, while at other sessions he would address the class as a whole.

Student K complained that clock arithmetic had not been emphasized in the course to the extent that it appeared on the test. She said many of the students in her section answered the questions on "clock time" incorrectly. Her laboratory instructor had attempted to help them. The students were demanding an extra review session before the next test.

The first group of problems presented to Student K at this interview was that on prime numbers. Considering 119, Student K mentally tried as divisors 2, 3, and 5, all of which failed to go in. She then wrote out the division by 7; when this succeeded, she called 119 "not a prime." For 113, she rejected 2, 3, and 5 as potential divisors,

then tried 7, 11, and 13 mentally. She then realized that it had been unnecessary to try the last two of these since $11^2 > 113$. She therefore called 113 a prime. Similarly, she mentally eliminated 2, 3, 5, and 7 as divisors of 227, then in writing tried 11 and 13 as divisors. Student K then computed 13^2 and 17^2 ; since $17^2 > 227$, her previous attempts sufficed to call 227 a prime. In considering 247, Student K realized from the previous example that she needed only to try the primes through 13 as divisors to determine if it is prime or not. She did this, and discovered 13 to be a factor. It was clear from her solutions of these examples that Student K knew that she could stop trying numbers as divisors when she passed the square root of the number under consideration.

Student K said that she had "blanked out" on the test when asked about greatest common factor and least common multiple. In both of the examples presented to her by the investigator she factored each of the given numbers into primes, then took the correct combination of factors for the concept asked. However, in computing $2 \times 2 \times 2 \times 2 \times 3 \times 7$, she took 8 to be 2^4 and thus was off by half in her answer.

After reading Problem 3, Student K said, "You're not joking, are you?" Assured that the problem was serious, she drew several arrows indicating the motion of the drivers. She then synthesized her thoughts into one diagram indicating the town, a distance of 40 miles east

(labeled 1:00) and a distance of 200 miles west of this point, indicating Dick's position at 9 A.M. This was clearly 160 miles west of town.

March 7. Student K said that she had liked this part of the course because "there's a lot of working out to do. I like working with fractions. I like dividing and multiplying . . . making the drawings to show multiplication and division of fractions . . . I like explaining it, too. . . . So I really like this section . . . because I really like to play with math--[it's] fun."

Commenting on a problem (page 219) in which the student was asked to find decimal names for $23/625$ and for $23/620$, Student K remarked that she had never realized before that the prime factorization of the denominator of a fraction determines whether or not its decimal expansion terminates. She had enjoyed seeing this. Student K said that she knew percent well before encountering it here; nothing in the lecture had confused her.

Concerning the homework, Student K said that she was up to date in doing it. She had no special comments on problems besides the one mentioned above, but remarked, "I just like working them all . . . having part of the information and finding the rest. I like that. I like working things out." Student K said that she found the concept of nondecimal "decimals" tricky. She had no trouble with any other topic, and enjoyed working the assigned problems.

Regarding the last laboratory session, Student K said, "It just seems like--I think there are some better ways you could do the metric system" She suggested more practice in converting between metric units and activities designed to show the relations between the units, rather than only measuring as had been done. Student K called the session on the geoboard "fun," and found Pick's theorem "interesting," though difficult to discover. She also said she had had difficulty in working with triangles "where you worked out the diagonals and working backwards to find the other side"; it was not explicit which session she was referring to here, although it sounded like the Pythagorean theorem and the investigator conjectured that it referred to the session on ruler-and-compass constructions which Student K had discussed at the previous interview.

The first problem at this interview was that on rational numbers. Student K converted both fractions to 12ths. When she saw that in this form they had consecutive integers as numerators, she converted both to 24ths and found $7/24$ as a fraction in between.

Student K correctly found a decimal name for $7/12$ by dividing 7 by 12. To find a fraction name for $.3777 \dots$, she called this number r and first tried to subtract $100r - r$. This yielded the equation $99r = 37.4$. Saying, "That wouldn't do it," she crossed out this work. She then subtracted $10r - r$, and obtained the equation $9r = 3.4$.

Accepting this, she then said $r = 3\frac{4}{10} \div 9$, which became $34/10 \times 1/9$. In the last step, Student K lost the 10 and gave as her answer $34/9$. For the next problem, Student K divided 6 by 11, obtaining an infinite decimal, and then said, "I don't know how to do it." Asked to find an approximate value for $\sqrt{7}$, Student K first said, "It's two point something." She thought that a formula existed for finding such things, but she did not know it. She then squared 2.5, 2.6, and 2.7, and discovered that the square root lay between the latter two. She squared 2.65 and obtained 7.0225, so she settled on 2.65 as her approximate value for $\sqrt{7}$.

Student K said that she would solve the first measurement problem by putting a string over the curve, then measuring the string with a ruler. She first thought the other measurement problem also involved length and suggested solving it the same way. Told that area was requested, she suggested surrounding the figure as closely as possible with a circle. Although she realized that the area of the figure would be "still a lot less" than that of the circle, she said, "I'd stick to the outside circle, and just say that's an approximation that's a lot greater."

Student K received the grade of 4.0 in the course.

March 26. At this interview, Student K said of her feelings toward mathematics, "They haven't really changed. I still enjoy it." She said that the laboratories had been

the best part of the course for her, "mainly because I've had all the other information, so the labs were more enjoyable. I got more out of them." Regarding teaching mathematics, she said, "I'd still like to," and mentioned that she was considering earning the necessary credits to be certified as an elementary school mathematics specialist as well as a special education teacher. This new interest in a dual certification represented her only change of feelings with respect to her major. She also said there had been no change in her feelings about the purpose of teaching elementary school mathematics, calling it "necessary for everyone to have math, at least up to a certain point, as much as they can understand of it."

Student K said that she wished the laboratory part of Math 201 could be either incorporated into the required mathematics methods course or organized as a separate course, while the lecture part could be waived by examination. She resented being forced to take (and pay for, at out-of-state rates) a course whose material was completely familiar. However, she found the laboratory work worthwhile as a demonstration of methods and of some new ideas. Regarding the laboratory sessions, Student K said, "I liked them all." Asked for specifics, she mentioned as things she had liked the various multiplication algorithms that were shown, Napier's bones, and the comparison of circles and squares. She had disliked the repetitious measurement of GeoBlocks. She said that

nothing had hindered her learning, and that "I liked the way the whole course was set up, the way it was taught."

Student K was considering taking more mathematics courses. She said that if she decided to pursue the mathematics specialist certificate, she would take the other two elementary education mathematics courses. Otherwise, she would take precalculus algebra and trigonometry courses.

Student K had no feelings about the research study. While she was not sure of its purpose, she said she had not minded participating, and expressed curiosity about the results. She suggested that the participants in such a study be chosen not at random, but with attention to their mathematics backgrounds, for "more control" over the types of participants chosen. Student K said that she would ask the same questions as had the investigator, but would focus more on the amount of work the subject did in the course, where the investigator had been more interested in what they got out of the course. She could think of no features of the course that were not discussed in the study.

On the problems of whether specialists or generalists should teach elementary school mathematics, Student K said, "Oh, math specialists! . . . It should be by people who first know math . . . [and] can show many different ways of teaching it [as opposed to only one or two]." She assumed that specialists would be well qualified.

Evaluation. As Student K pointed out, the lecture part of the course was a waste of her time. However, the course had not been a total waste for her because of her interest in the laboratory. It is likely that a section of the course especially for students with preparation such as hers would provide them with a more worthwhile experience in Math 201.

Student L

Student L was a freshman from Grosse Pointe Woods. She had taken two years of high school mathematics--one year each of algebra and geometry. In discussing her background, she said, "I don't really like math." Her grades had been B's in algebra, and B's and C's in geometry. She had liked algebra somewhat better than geometry, where she had been "really bored." Asked why, she replied, ". . . algebra is numbers and concepts that I can understand while geometry was . . . things that are more abstract like planes . . . it doesn't interest me." She also had disliked science (though not so much as mathematics), while she had liked history, English, speech, and acting. She gave this reason for her preferences: "I like things that you can think about, that through reading you acquire knowledge that you can apply." She felt that her high school mathematics had been inapplicable to daily life. Student L had participated in student government, had acted in school productions, and had worked as a timer

of athletic events while in high school. She had taken no previous college mathematics.

Student L was a special education major with an interest in the education of the emotionally disturbed. She had worked with such children prior to attending college. She said, "I love working with special children and adults. [They] have something to offer me and I think I have something to offer them." In her volunteer work she had helped children of all ages with tasks such as counting and learning the alphabet.

She could not say how she felt about eventually teaching mathematics, because her perception of her relationship with mathematics was changing. However, she did say that elementary school mathematics "doesn't frustrate me. It's basic enough so I could learn it and digest it and give it back to children." She said that the purpose of elementary school mathematics is to "learn to associate through grouping . . . it's a higher form of being able to communicate . . . it's useful to be able to count, multiply . . ."

January 24. At this interview, Student L and the investigator discussed her reactions to the presentation of the course to date. Her comments on the lectures were mainly focused on the style of the lecturer: "She explains things to us as she would explain it to a child . . . which is really good, because . . . we would probably forget"

otherwise that these concepts eventually would be taught to children by the students. Student L especially liked the use of examples and materials, particularly pictorial ones, which can be used in explaining the material to children. She remarked also that lectures were boring when the approach used was "cut-and-dried--this is this, this is this, this is this, and this is this"; i.e., a sequence of definitions closely following one another. Student L also expressed approval of the book's presentation at this point, although regarding some of the assigned problems, she remarked that "it was like they were trying to trick you into asking you a question that you couldn't answer, which kind of frustrates me, especially in something which shouldn't be that difficult." Asked for an example of such a question, she said she was referring to "negative" questions of the form "What is wrong with such-and-such?" (In such problems, the text presents a fallacious argument or incorrectly worked out example, and asks the student to find the error.) She also criticized the book for not having enough answers in the back.

Student L severely criticized the laboratory sessions she had attended up to this point: "I think our lab stinks, because it's always so far ahead of what we're doing in the lecture that it's not relevant at all--for me, anyway." Although she said that a laboratory approach was "excellent" for children, she felt that for herself it was two hours spent on five minutes worth of work. She

found the laboratory work somewhat confusing because she herself had not learned that way, and she felt she had to "undo the things I've learned" in order to appreciate the laboratory method of instruction. She also criticized her laboratory instructor, saying that he was "not that competent" and "acts like it's a joke." She resented not being allowed to use the laboratory period immediately preceding a test in lecture as a review session. Concerning the laboratory sessions, she remarked, "This just reinforces that I don't like math a lot more. It's not helping me to like math any more."

Despite all these criticisms of the laboratory sessions, Student L remarked at this interview that she wished she had taken this course in the spring of 1973, as a friend of hers had, in a class which had met four days per week in the laboratory and had studied the material through laboratory work only. She felt that such a presentation would be superior to that which she was experiencing. Concerning the course in general, she said that she liked the lecturer personally and that the course as a whole was "OK, but it's not good." She expected it to improve.

At this interview, Student L was presented with the problems on sets and on number bases, as well as with Problem 1. She said that the set of all living people matches "none of the above, the empty set." (Like several subjects, Student L used the expression "the empty set"

changeably with "none of the above.") Asked what she
 by "the empty set," she said, "OK, these [in set C]
 bjects . . . well, B is the same thing as the empty
 . . Kennedy and Johnson are dead . . . so I would say
 The set of counting numbers from 1 to 100, inclusive,
 aid to match "B again, because this [A] is a set of
 , . . . these are objects [in C], [D] only goes to
 h wait! E would be." Changing her mind from B, she
 E because "E is an infinite set. Therefore I would
 ." Student L said the set of suits in a standard deck
 rds matches "none of the above . . . no, B . . . when
 none of them, I mean the empty set, which is B . . .
 se none of these [objects in C] are suits." To the
 f all aardvarks enrolled at M. S. U., "I would say B
 , because Kennedy, Johnson and Nixon aren't aardvarks,"
 he also saw nothing resembling either aardvarks or
 U. enrollees in any other set.

It can be seen from this set of questions that
 nt L still had several misconceptions about sets after
 ad studied this topic in Math 201. First, she
 derstood the term matching, which was used in both
 ecture and the textbook to mean equivalence of sets
 e-to-one correspondence. Student L thought that sets
 t match if they have different members; she confused
 ing with equality, as revealed by her comments that
 dy and Johnson are dead and that the objects in set C
 ot suits. Second, Student L both misused the

expression "the empty set" and mistakenly called set B empty. When she meant to say a set matched none of those given, she said "the empty set" to mean "none of the above." Last, although Student L correctly identified set E as an infinite set, she said that it matches a finite set; this is another indication of her lack of the concept of matching.

In contrast to her confused understanding of sets was Student L's performance on the set of exercises involving number bases. She correctly recognized that 405_{eleven} was equal to $4 \times 121 + 5 = 489_{\text{ten}}$; she also was able to do both conversions to a nondecimal base after correctly labeling her columns and then successively subtracting from the given number what could be put in each column, starting from the left. Student L also named the correct base in each missing-base example. In the first one, she thought "If there's a remainder of 4, what . . . group would have to be carried?" Since nine plus seven is sixteen, it had to be a twelve. In the second example she first suspected it was a small base "'cause they're all ones and twos." Her first hunch was that it was base four, but she saw that this did not work out, and then tried base three and checked that it worked.

On Problem 1, she first read the problem aloud. Even after the investigator explained that an answer in coins was wanted, Student L said, "I don't know what the question says. I don't understand the question." After

some thought, she said, "I think what you're trying to say is--to take mine minus yours--what do I have? I don't know if that's what you want, but--" She then proceeded to compute the amount held by each person, found the difference to be thirty cents, and wrote this as one quarter and one nickel.

After the problems, Student L was asked how she had felt in the interview. She said she had been "nervous" because "I hate people looking over my shoulder when I'm doing anything." However, she had no suggestions for improving this situation within the context of the study. She merely suggested to the investigator that he "take into account" the subject's possible discomfort.

February 5. At this interview Student L reported that "I'm getting to like it better." The lectures since the last interview had discussed the algorithms of arithmetic, including various nonstandard ones, as well as prime numbers. Student L said she had "no reaction" to the presentation; she was only trying to learn the material. This material was mostly review for her, although some nonstandard algorithms and modular arithmetic were new. Asked for specific features of the lectures that she found particularly helpful, she said she "couldn't pinpoint" any, but was usually able to follow the presentation, except sometimes when the lecturer would "leave out steps." At these times she would ask for, and

get, a personal explanation after class. She felt that the pace of the course up to this point had been about right on the average, although in spots it had been too slow or too fast.

Reiterating her feelings from the previous interview, Student L remarked that the book was "pretty good." However, she sometimes could not "conceptualize" how the book's discussion relates to elementary school mathematics; in this connection she found the Supplement of excerpts from elementary school textbooks useful. Page 14 on integers she mentioned particularly. Student L found it hard to deal with statements and formulas in which numbers are represented by letters; she said, "I can't picture it 'cause it isn't a number," although the lecturer "made it more easy to understand."

Student L's comments on the laboratory sessions largely repeated her remarks of two weeks earlier. She said she hadn't learned anything there because the material was presented too far in advance, it would "drag out in two hours what we could do in ten minutes," and her instructor's attitude was "I know this is a drag, but you've got to do it." She said attendance at laboratory classes was poor and that these classes had never helped her with anything. "It's always new mathematics . . . I can't understand it, and the guy can't explain it to us very well." She put part of the blame on her instructor, saying she wished she had her lecture instructor for the laboratory class.

Regarding the specific exercises, she said clock arithmetic "was a pretty simple concept, and we spent two hours on it . . . I just don't see the relevance for the class."

The only problem presented at this interview was Problem 2. Immediately after reading the problem, Student L answered that there were "an infinite amount" of possible telephone numbers in East Lansing. Asked why, she replied, "Just because you're giving three specific possibilities, there's still an infinite amount of other combinations of numbers"; she then named a few.

February 19. At this time Student L remarked that the content of the course had become "easier to conceptualize" because she had studied the current topics, integers and rational numbers, more recently in her education than she had the previous material. She was unable without her notes to name specific features of the lectures which she found particularly helpful or troublesome, but expressed general approval of the lecturer's style. In the book she especially liked the "array method" of illustrating operations with rational numbers by means of divided squares; she contrasted this approach with her own education, which, lacking pictorial aids to conceptualization, "was like memorization, it wasn't understanding." Such an education would cause one to dislike mathematics, according to Student L. She said that she had done most of the assigned problems without difficulty, and that the problems reinforced the lecture material.

Student L at this interview again made several derogatory comments about the laboratory sessions. She realized that their purpose was to familiarize prospective teachers with materials which will help elementary school children to learn mathematics, but berated the way the classes were organized. She again mentioned that her instructor "doesn't know what he's doing." Asked specifically about the last two laboratory classes she had attended, she said she had found nothing in them helpful or interesting. She described a typical class as follows: "They throw us these things on the table and say, 'OK, do this.' Then they say, 'Well, I don't really know how to do it, but let's see if we can figure it out.' . . . He doesn't know how to do it, we don't know how to do it, and the whole class ends up skipping over things . . . making believe that they're doing it, which is a normal reaction [to the situation]." The result was that "things like this do turn people off . . . they're getting reinforced that math is just confusing because no one knows what they're talking about." Student L, however, had learned from the experience that as an educator, "you do have an effect on people. You can de-motivate them a lot easier than you can motivate them."

Student L was asked at this time how she felt when she considered that some day she may teach mathematics. She replied that she did not expect ever to teach mathematics, since she intended to become a teacher of the

emotionally disturbed. She then remarked, "I don't think this course was that good, personally. It could be good, but . . . they herd five hundred people into a math class . . . Math is a very individual thing, so then they get to the lab, which is the individual part, and they screw it up. It's not as good as it could be . . . I think I could learn as much as I am right now by taking this book home and just doing it myself, 'cause if I'm not getting any personal help . . . it's all in the book. . . . You can just go to the lecture and do the homework and never read it and you'll slide through. . . . It's not really stimulating me. . . ."

The first set of problems presented to Student L at this interview was that on prime numbers. For 119, she tried as divisors 2 (using the division algorithm and obtaining a remainder), 3, and 7, which went in. For 113, she remarked at first glance that "I think 113 is prime." She divided by 3, and when 3 failed to go in she repeated this remark and explained, "I would assume because 13 is prime that 113 will be prime, and what I would do is I would sit here all day and I would go through all the numbers and after a long time I would say that's probably prime." Student L did not notice that the first example contradicted her theory. Similarly, her first reaction to the number 227 was to say, "I know this is not prime by looking at it. . . . Usually you can tell by the last two numbers, if they're divisible by a number--sometimes . . ."

However, after trying some numbers as divisors, particularly 3 and 9 (also 2, 4, 8, and 11), she was no longer sure. To check herself, she divided 200 by 3. After all tries failed, she said, "I guess that's prime." For 247, she tried as divisors 2, 3, 4, 5, 6, 7, 8, and 9 (all the divisions were written out); when all failed to go in, she said, "I guess I would say . . . that it's a prime, too."

After this exercise, Student L was asked to find the greatest common factor of 63 and 105, and then the least common multiple of 42 and 48. In both cases, she correctly factored each number into primes by means of a factor tree, then took the correct combination of factors.

After reading and considering Problem 3, she said, "I don't think there's enough information 'cause we don't know how far he was from the center at 1 o'clock. We only know that he [Dick] was passing him [Dan]. . . If the problem would have said, 'Driving east at 40 mph, Dan passed through the center of town at 12 noon, and then at 1 o'clock, Dick passed through the center going at 50 mph,' . . . I don't know . . . We can't know possibly from this 'cause we don't know where he was." Asked what extra information she would need in order to solve the problem, Student L replied, "You'd have to know how far Dick was from the center at 1 o'clock. . . . They don't even tell you about Dan; they just say he passes through the center but they don't tell you how far from the center he was . . ."

March 7. In the last section of the course, the topics of rational numbers, decimals, and percent were discussed. Student L said she did not find these difficult; most were familiar, and she had remembered some and forgotten others. Some, such as nondecimal "decimals," were new for her. She felt that the lectures on these topics were proceeding at too fast a pace, as if to cover the remaining topics hurriedly before the end of the term. As before, Student L could not name specific features of the lectures that were especially helpful or particularly confusing for her, except for the possibly confusing effect of the fast pace. She thought she had learned the material fairly well, and expected to retain her learning because "it's something that you learn. You can't memorize math; you've got to learn it. . . . Even though I dislike math the most, I probably retain it for the longest." Nevertheless, she was not doing as well as she felt she could. She said she was "not very motivated . . . If I really was interested and felt that I was really . . . being taught by somebody who was . . . not pushing me, . . . but-- . . . pulling me, . . . then I'd do more. . . . Just to go to class and have somebody sit and talk at you about math for an hour three times a week and go into a lab where no one's putting it together . . . it's a waste of time." She said that she wished she had postponed taking the course, since a better version might be offered in the future.

Asked how she felt about the book's presentation and the homework examples in this section, Student L explained that she did not have definite answers to the investigator's questions. (This had been evident all along.) She said, "It's so hard. It seems like you ask me, 'What do you think of this?' . . . and I don't really think that much . . . I don't really have any feelings. It was just there, I had to do it, and I did it. . . . I didn't think it was fun, I didn't think it was interesting . . ." She did say that she found helpful "those things in the red boxes [the emphasized sentences in the text]," as well as "when they work out a problem." Asked if she could name specific parts of the presentation or specific problems she had found helpful, she flippantly opened the book to a random page and pointed at some items on it.

Student L said she had liked the last two laboratory sessions, which had included the geoboard and measuring in metric units. However, she repeated her complaint about the lack of correlation between the lecture and laboratory classes. She remarked that this could not be done in elementary school. She did say that the laboratory classes had improved toward the end of the term, as had her instructor, although he still "hates it." She ended the conversational part of the interview by saying that she was entitled to the opportunity to voice her complaints since she had spent (and, in her opinion, wasted) so much money on the course.

The first problem presented at this interview was the single one on rational numbers. Student L said, "I'll just do it the way I did it on the test," and converted both fractions to equivalent ones with the least common denominator, 12. The resulting numerators were consecutive integers. After a long pause, she expressed both fractions in terms of 24ths rather than 12ths and was able to find a fraction between them. She was not sure her answer, $7/24$, was correct.

Student L could not remember how to write $7/12$ as a decimal. She said her first reaction was to write $.712$, but "I know that's wrong." She tried various other approaches, including attempting to solve the proportion $7/12 = x/100$, and dividing 12 into 70 and 1700. When none of these worked, she gave up. She also did not know how to find a fraction name for a repeating decimal, although she remarked, "I know you're supposed to isolate . . . the thing that repeats." She said this procedure was never discussed in lecture; this was not true. As for finding a terminating decimal approximation to a fraction, Student L said, "I've never seen anything like it and I don't know how to do it." Asked to find an approximate value for $\sqrt{7}$, she first asked, "What kind of a value?" and was told, "Any kind you want." Her only reaction was to recall the Pythagorean theorem from her laboratory class; she thought of constructing a right triangle with $a^2 + b^2 = 7$, whose hypotenuse would be of length $\sqrt{7}$.

Student L had no ideas about how to do the first measurement problem. She complained that she had not done this in her laboratory class and said of this problem (and perhaps of the study in general): "I get frustrated . . . I hate things like this 'cause I don't have a good math background and I don't know how to do them." Regarding the second problem (on area), she thought, "I could use a geoboard . . . divide it up . . . if we used the measure of the geoboard as one unit . . ." Although she was unsure at first that this could be done, she decided it could be, and said she would put the figure on a geoboard and count the number of square units it covered.

Student L received the grade of 2.5 in the course.

March 29. At this interview, Student L started by saying that she did not think her feelings about mathematics had changed as a result of her experience in Math 201. As she put it: "I don't hate any subject. I like to learn. I get . . . good grades here, and I did in high school. . . Mathematics is my least favorite subject, 'cause I don't think I've ever been taught math in a very good way I think I have the capability to do it. . ." She felt that while her mathematics teachers understood their subject, they were unable to communicate it to their students. She felt a slightly greater appreciation of mathematics as a result of her Math 201 experience. Student L said she had no change in her feelings about eventually teaching

mathematics; she did not expect to teach it since she was majoring in the education of the emotionally disturbed. She said she felt no change in her feelings about the purpose of teaching mathematics, although she had hoped for some change here. Student L said she was disappointed that the course had failed to excite her interest in mathematics; she was not sure if this had been due to her own antipathy toward mathematics or to the organization of the course. She specifically criticized the lack of methods orientation in the course.

Asked how the course might be improved, Student L responded with a tirade of sentence fragments, including the following: "[Math 201 was] presented too much like a 108 [precalculus] class . . . Math education for teachers isn't that. . . . Many people in mathematics get carried away . . . trying to flaunt their intelligence upon you . . . You have to be really good to be able to bring it down to such a simple level that somebody can go away with it and give it to someone else--it's a very difficult thing to do. And I think that is showing a high degree of intelligence and perception for a person to be able to do that, where I think a lot of people have the idea . . . 'Well, they wouldn't really know how much I know if I did it that way, so . . . let me impress them . . . let me throw around my credentials and show them how good I am,' which to me doesn't say very much at all." Regarding the laboratory sessions, Student L favorably recalled those

involving Dienes Blocks and the geoboard. She had resented being required to attend these classes, most of which she found irrelevant. As she put it, "If they could make it good enough without saying that [attendance was required], everybody would be there." While nothing had hindered her learning, she said, "I think I could have learned a lot more, had it been done in a different fashion." Student L said she was not interested in taking further mathematics courses, aside from the required mathematics methods course.

Asked her reaction to the research study, Student L replied that she had been comfortable in the interviews. However, she doubted that the study would have any beneficial effects, due to the predilections of mathematics teachers that she had mentioned. She thought that the interviews had been scheduled too frequently. She suggested a three-week interval between interviews and the inclusion of males as possible improvements in the format of the study. She could suggest no additional questions or areas of investigation for the study.

Asked whether specialists or generalists should teach elementary school mathematics, Student L replied that only the capability of the person mattered, not the title.

Evaluation. We have seen in Student L a student with a great amount of hostility toward mathematics and particularly toward mathematics teachers. Such a student

could not possibly derive much benefit from a mathematics course presented in a traditional lecture format. The presence of students like Student L argues for an activity-oriented presentation of the course; such a presentation, preferably integrated with the methods course, should be available to this type of student.

Student M

Student M was a sophomore from Oak Park. She had taken three years of high school mathematics--two consecutive years of algebra, then a year of geometry as a junior. She said she had had trouble understanding her first-year algebra teacher, but had enjoyed the subsequent classes and had learned the material well. Her favorite high school subjects had been history and government, while she had liked mathematics and science the least. Student M said that "math and science just kind of lost me"; science in particular "got too picky." She had been active in student politics and various sports.

Student M had taken no college mathematics prior to the study. Her major was elementary education, because "I enjoy working with children and being with them and I feel I would be really doing something good if I could teach them." As a student in Education 101A (Exploring Teaching), she had worked as a teacher's aide and occasionally taught a lesson. She also had worked as a camp counselor for preschool children, where she had taught

them swimming and arts and crafts, and at the time of the study was teaching English to bilingual children at Hannah Middle School in East Lansing. Student M hoped to teach grades two and three, and felt that "the math I'd be teaching . . . at [that] grade level won't be that complex where I shouldn't be able to understand it, so I shouldn't have any problems." She said that elementary school mathematics is "important, because the children will be using math all their lives. They'll always have to add and subtract and divide, so they should know it."

January 25. Student M said that the lecturer was "very thorough [and] covers everything." While Student M herself was sometimes bored by this approach, she felt that it was good for those students to whom the material was new. In any case, she preferred a teacher of mathematics who went too slowly to one who went too fast, since she had once had an unpleasant experience with one of the latter. She said that she felt free to ask questions in lecture, and that she was happy with her score on the first test. She liked the lecturer's use of pictures to illustrate mathematical ideas. However, she said that she was "totally lost" and "confused" by the Austrian method of subtraction, and had abandoned her attempts to learn it. Nothing else was particularly confusing to her.

Student M said that the problem assignments were "good." She usually checked her solutions with the printed

answers, finding this practice a good review for the test. She said that for some problems, she did not understand the problem until she looked at the answer; she then saw the point of the problem. She added that while she understood the concepts that were taught, sometimes questions were worded so that she couldn't grasp "what they were fishing for." She found helpful practice in changing bases, although at first she had difficulty with bases eleven and twelve. She criticized the book for referring to some arithmetic property as "Property 1," rather than giving it a name.

Because Student M had had a problem communicating with her first laboratory instructor, she had switched sections, and was pleased with the instructor of the new section. She liked the question-and-answer period in the laboratory where homework was discussed, and said she liked the coordination of the laboratory with the lecture. However, she had some negative comments about the activities themselves. She said that attribute games were "ridiculous, kind of worthless . . . I guess it was supposed to demonstrate a rule, but to me it was just playing with blocks." Regarding the exercise on multiplication and division using Dienes Blocks, Student M said, "I could not see taking kids and teaching them multiplying with [Dienes] blocks. . . . It was so confusing, and it was just the most ridiculous thing that I had ever seen. . . . If I had learned that way in the very beginning, maybe I wouldn't feel this way,

but I couldn't follow it, so I don't know how a kid could I wouldn't even consider teaching it that way." However, she said that she would use Dienes Blocks to teach addition and subtraction.

The first collection of problems presented to Student M at this interview was that on sets. Asked which of the given sets matches the set of all living people, Student M replied, "That would be the empty set," meaning none of the given sets matches. Asked why, she replied, "The only set with people in it is set A, and at the time, Nixon is the only one out of that set that's living, so therefore it wouldn't work." She concluded, "It's the empty set . . . none of them match." Responding to the question about the set of counting numbers from 1 to 100, inclusive, Student M said, "E . . . set E starts with zero and it goes all the way up. It's infinite, so it would include 100. This would be a subset of E." (Since she explicitly said it would be a subset, the investigator inferred that Student M had been misled by the word "inclusive" to look for a set which includes the set under discussion.) Student M said that the set of suits in a standard deck of cards matches "none . . . you would have to have hearts and spades and diamonds and clubs and you don't have them in any of these sets." She said of the set of all aardvarks enrolled at M. S. U., "That would have to be the empty set . . . none of the above." One can see from her responses that Student M had confused the idea of

matching with that of equality, and that she also was prone to misuse the expression "the empty set." These errors were common among the subjects.

Student M correctly translated 405_{eleven} to base ten, multiplying 4 by 121 and then adding 5. She also correctly performed the other two conversions, but using a method unique to herself among the subjects. After setting up the place values of her columns in base two, she divided 39 by 32, obtaining a quotient of 1 and a remainder of 7. The quotient told her that there was 1 32 in 39, and she put a 1 in the 32s column. She then divided the 7 by 4, obtaining 1 4, remainder 3, etc., putting the quotients in the proper columns. In the second of these problems she divided 44 by 7, obtaining a quotient of 6 and a remainder of 2, which became 62_{seven} .

Regarding the first missing-base example, Student M said, "I know it would have to be higher than base eight . . . because if this answer is correct in a base, it couldn't be base eight because there's an eight included in the answer." She then tried twelve as the base and saw that it was correct. Student M explained her reasoning: "Nine and seven is sixteen, and changing the answer to base twelve, that would make it a ten [sic] to carry over to the other column . . . and that would leave four ones, and then when you carry the ten [sic] over, seven and one is eight, so it's base twelve." She said in response to the second example, "It's base three . . . You have to borrow, and I

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noticed that in the ones column 2 is the answer, so I had to think . . . what minus two would leave me two, and it would be four, and if it was base three I would have to carry three over. Three and one would be four, and then four minus two is two, and then carrying the problem through, it would work."

Student M saw the connection between Problem 1 and bases, but did not use base five in her solution. She said, "I'm going to do it exactly like it was a base problem, and then . . . kind of carry the things over as I have enough to group into the next highest thing." She summed within coins, writing the total as 3 quarters, 7 nickels, and 6 pennies, and then exchanged for fewest possible coins.

Student M said that she had felt "all right" in the interview, but at first had felt "strange . . . weird" because she had not known what to expect.

February 8. Student M mentioned first at this interview that she had had difficulty with the concepts of least common multiple and greatest common divisor. While she recalled considering this topic "fun" when she first had learned it in elementary school, she said that the definitions in the book "thoroughly confused me." She felt that she understood it better now after obtaining help from friends, but was still "kind of vague" about it. She called the book's definitions of least common multiple and greatest common divisor (in terms of intersections of sets

of multiples or divisors) "totally misleading," and added for emphasis, "Some of the ways they explain things--I couldn't believe it!" Student M said she had not studied the Austrian method of subtraction in nondecimal bases, because "if we don't have to know any specific method, I'm not going to confuse myself." She said the book was "not very specific" in its explanations of base twelve; she could not learn it from the book and again asked friends for help, after which she understood it. She disliked the book's illustrations of operations as machines, saying, "There are better ways to illustrate that." Regarding the different definitions of division, Student M said, "I can't see the purpose of breaking it down like that." She said that when she saw a statement such as (page 58):

Suppose that $p, q, \bar{p}, \bar{q}$ are counting numbers and that $\bar{p} \leq p < \bar{p}$ and $\bar{q} \leq q < \bar{q}$. Then $\bar{p} - \bar{q} \leq p - q \leq \bar{p} - \bar{q}$ [sic] (of course the two inequalities only make sense if $\bar{p} \geq \bar{q}$).

she ignored it and tried only to get a general idea. Student M said that she had done "brilliantly" on the test despite having skipped these things. She pointed out some errors in the book and said that she had no trouble with integers or with primes up to least common multiple and greatest common divisor. She added that she had found helpful everything in the book which she had not mentioned.

Discussing the lectures, Student M said, "The last week I have been so bored in lecture!" She mentioned that

it seemed to her that the other students in the class also were bored. She was unable to pay attention to the lecturer, even though "I knew I was having difficulty." Student M said that this was not due to the teaching style of the lecturer, which she found "thorough." (She thought that the lecturer had sensed the restlessness of the class.) Student M said that she realized that other students needed an explanation of the material which she understood. She also pointed out that she tried to do her coursework before the material was covered in lecture, so that "by the time she got to them in class my mind just wasn't there." She said that nothing in the lectures had been confusing to her.

Student M said that she had no trouble in the laboratory and called her instructor thorough. She reiterated her criticism of the session on multiplication and division with Dienes Blocks, calling it "really ridiculous . . . you start multiplying these big numbers with these little tiny blocks--you can go crazy!" Regarding clock arithmetic (which she called "modulo math"), she said, "I understand it, but I can't really see where it would have a purpose." There was nothing else in the laboratory work which Student M found irrelevant. She mentioned as helpful the pencil-and-paper practice on work with bases and the laboratory on computation illustrating different ways of multiplication.

Student M said that the pace of the course to this point had been "fine, the work isn't that overwhelming." She thought that she had seen all of the material before "except this mod math."

In considering Problem 2, Student M first noted, "There's three different beginnings a number can have . . . then there would be four other numbers that followed . . . the numbers that followed could be from 0 to 9, which would mean ten different numbers . . . [long pause] Uh-oh!" She said, "I know I've got to multiply something with the numbers by the number of different possibilities . . . then add the three together . . . I'm not sure what to multiply." After saying she didn't know what to do, Student M continued, "I know it's going to involve multiplication and then . . . adding them together in the end, but, wow! The only thing I could really think of would be ten, because that's the number of numbers that I can fit in, but I can only use four of those numbers at a time, and they can be in a million different orders . . . somehow I have to figure out the number of different arrangements of the numbers I can have . . . I don't know." Student M gave up at this point, frustrated because she recognized the resemblance of this problem to one which she had done for homework.

February 22. At this interview Student M began by discussing the homework problems. She said that when

attempting to solve a story problem, she would try to fit the data to a formula--"I know they want a formula, or some kind of structure, but I can't do it." While she was obtaining the correct answers, she was using her own methods, not doing the problems "the way they [the book] want me to do them." As an example, she cited a problem involving a recipe, and said, "I'm just flipping numbers around but . . . there's a reason to their madness." She said that she "was puzzled at first . . . sat down and I thought about it . . . and I went about it in my own way." After checking the book's explanation in the answer key, she realized that "I just thought about it differently." However, she was unable to describe her thought beyond saying, "It's different from the book." Regarding another problem from this set (page 193), she called it "more complicated . . . but, if I can understand it better that way, I'm going to do it!" Of the book's explanation of another problem in this set, Student M complained, "I just wish they'd get to the point." She also criticized the book for its typographical errors. However, she said she "was really impressed with" the treatment of fractions as cut-up squares, and that the treatment of percent had enabled her to "refresh myself." This part had been clear, as opposed to the "muddled" treatment of least common multiple and greatest common divisor.

Student M said that she was "delighted" with her score of 89 on the second test. Her comments on the lecture

were not as negative as they had been the previous time--
"Her lectures are OK now." While Student M was very interested in the presentation of fractions as cut-up squares, she could name nothing else in the lectures as being either particularly helpful or confusing.

Regarding the most recent laboratory sessions, Student M said that the one on ruler-and-compass constructions "was bad because . . . I had forgotten the constructions [from high school geometry]." She was also frustrated because the compasses were slipping. She said that a student should review Euclidean constructions before doing this exercise. When Student M said she did not remember the laboratory session on rational numbers, the investigator reminded her of the materials that had been used. She then said of GeoBlocks, "That was like an introduction to the fraction bit. . . a refresher thing. It was good." She also said that she had liked tangrams because she liked puzzles. She could not recall Cuisenaire rods.

Student M said that she felt "comfortable" about teaching this material and that "I feel that I understand it well enough to teach it." She anticipated no problems in teaching this material.

Presented with the set of problems on prime numbers, Student M first stated the definition of a prime number. Considering 119, she rejected 2, 5, and 10 as possible divisors, and tried 3, which failed. After a

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long pause, she said, "Maybe 7," tried 7 as a divisor, and found it to be a factor of 119. For 113, Student M eliminated 2, 5, 3, 6, and 7 as possible divisors. She said, "I think it's a prime number," and told the investigator she had gone up to 9 in trying divisors. Considering 227, she immediately eliminated 2 and 5 by inspection and 3 by the divisibility test for 3 (which she had not used earlier). She then divided 227 by 4, which failed to go in. She didn't think 6 would work so she skipped it, then divided 227 by 7, 9, and 13, all of which failed to go in. Student M said, "I think that's prime," then tried 11 and 17 as divisors, which also failed to go in. For 247, Student M again eliminated 2 and 5 by inspection and 3 mentally. She then divided 247 by 7, 9, 11, and 17, all of which failed to go in, and said, "I think it's prime." Asked how far one should go in trying divisors before calling a number prime, Student M said she was not sure, but thought "I should probably go through and try all the numbers that I know are prime [as divisors]." She thought one should try these numbers up to half of the number under inspection.

In doing the problems on greatest common factor and least common multiple, Student M factored each number into primes and then took the correct combination of factors for each problem.

On reading Problem 3, Student M remarked, "I can remember these and I can remember having trouble with

these." After absorbing the data in the problem, she noted that 9 A.M. was 3 hours before noon and that therefore Dan was 120 miles from town at that hour. Trying to relate this to Dick's movement, she said, "I know there's a simple formula to do it." She thought, "I know there's a difference of 10 miles in an hour, and I'm trying to work with that somehow and I don't know how I'm going to do it." After stating that she knew "there's some easy way to do it," Student M said that at 9 A.M. Dick was at least 120 miles from town, but could have been as far away as 150 miles. She then momentarily thought she had the solution, but did not. After noting that Dan had gone 160 miles between 9 A.M. and 1 P.M., she gave up, commenting, "That's frustrating."

March 8. Student M said that she had been confused by the book's presentation of the idea of absolute value: "It goes--if r is a positive rational number, then the absolute value of r equals r , and if r is negative, then the absolute value of r is equal to negative r . Well, . . . at first I didn't see it and I sat with the book and I was very, very frustrated because as far as I'm concerned . . . the absolute value of r is always going to be positive. And then it was explained to me that . . . we're just talking about--well, negative r isn't the number, it's just the symbol, and the symbol--after they take the absolute value of it they put in the negative sign. But to look at

that--that's very deceiving, and there can be better ways to word that, I'm sure." She said that while this concept "took a long time to get . . . through my head," she finally did learn it. Her final comment on this topic was, "If I was teaching and that was in my book, I don't know what I'd do, but [I] certainly wouldn't want the kids to sit and wonder about it and worry about it, because it's ridiculous."

The treatment of decimals in the book and in the lecture was difficult for Student M to understand; she said, "I couldn't understand it that way. I absolutely couldn't see it." As an example, she said that she could not follow the explanation of finding a fraction name for $.34_{\text{six}}$ by writing it as $3/6 + 4/6^2$. She considered this approach "extra work," compared to her approach of regarding $.34_{\text{six}}$ as $34_{\text{six}}/100_{\text{six}}$ and then converting the components of this fraction to base ten. She told the lecturer about her way of doing these problems, and the lecturer said it was acceptable. However, she found "helpful" the lecturer's presentation of converting a repeating decimal to a fraction.

Student M was glad to have received the handout sheet on percent problems because the book was deficient in these. She again said she had a method which she found superior to that of the lecturer, noting that "I couldn't follow her." For example, to find 35% of 84, the lecturer and book taught students to compute $35 \times 1/100 \times 84$.

Student M preferred to solve the proportion $35/100 = x/84$.

She called the other way "harder" and "more work."

Student M said that she had liked the lecturer's explanation of repeating decimals and the use of cut-up squares to depict operations with rational numbers. The material was all review for her. Student M said that she found most of the problems helpful because "it shows me what I know and what I need help in." She had some trouble when asked what principle was illustrated in a story (page 193), and called the book inconsistent because they worked out a solution rather than only naming a principle.

Student M said that the lecturer was "really fair," but was bothered by the lecturer's speaking style--"like we're three years old." However, she realized that this might be meant as a model for them to emulate as teachers in the future. Student M said that it seemed to her that she had a stronger background in mathematics than most of the class--that many other students were unfamiliar with the material of Math 201; they "don't understand anything [in the laboratory] and have never had anything before." She therefore realized that the lecturer's treatment may have been right for such students, although, it had "seemed so trivial" to her.

In brief comments on the laboratory exercises, Student M said that the geoboard had been "interesting, and I'll tell you, Pick's theorem came in handy on the test." She found the geoboard activities "kind of neat." She

thought it was a good idea to have a session on the metric system because of the coming conversion; she had learned the metric units before but had forgotten them. Student M reiterated that she was satisfied with her laboratory instructor.

Student M said that there had been a problem on the last test similar to that of finding a rational number between $1/3$ and $1/4$; at that time she had not been sure of how to do it. She said she would take "1 $1/2$ over 3" (which the investigator took to be a misstatement of 1 over $3 \frac{1}{2}$) but that she was unsure of whether or not this was a rational number and did not know how to proceed further.

Student M correctly found the decimal name for $7/12$ by long division. To find a fraction name for $.3777 \dots$, she called this r , wrote $100r = 37.777 \dots$, then subtracted r from $100r$, and not realizing that the infinite decimal vanished, obtained $99r = 37.4777 \dots$. At this point, she said, "It's not going to work." Since she knew that "this is how it's done," Student M did not know how to proceed from here. She said that if she had obtained an expression of finite length on the right side, she would then have put this expression over 99 as her value for r . After reading the next problem, she said, "I'm not sure what you want here." She proceeded to divide out $6/11$ as far as four decimal places and then stopped, saying she did not know how to answer the question. Asked to find an approximate value for $\sqrt{7}$, Student M first

stated the definition of square root, then pointed out it had not been discussed in lecture. She then tried to recall the square root algorithm, saying, "I don't remember how to do it, but I know I used to know how to do it." After this, she considered that $\sqrt{7}$ is between 2 and 3, and guessed 2.5 (whose square turned out to be too small), 2.7 (too large), and 2.6 (too small). Refining her estimate, she squared 2.65 and 2.64 and discovered that $\sqrt{7}$ is between them.

Student M said that she would solve the first measurement problem by putting a string over the curve, then measuring the string. She also considered approximating the length of the curve with fragments of circles. Her first reaction to the second problem was to say that if the figure went on a geoboard, she could use Pick's theorem. After a long pause, she suggested constructing as large a circle as possible inside the figure, then choosing a unit, measuring the area of the circle and the area of the excess of the figure over the circle, then adding these. After this, she suggested that one superimpose a grid of one-inch squares over the figure, then "figure out the area from there." She mentioned that she thought this the easiest possible way to solve the problem.

Student M received the grade of 4.0 in the course.

March 29. Student M said of her feelings toward mathematics, "Math has never been . . . my favorite subject,

but it doesn't bother me that much any more and I feel that I'm capable of teaching in the lower grades so long as it doesn't get really complicated." She said she had enjoyed the course "as a whole," and that "I think I've become more open towards [mathematics] and more positive towards it." Student M said that there had been no change in her feelings about eventually teaching mathematics; while she was now more aware of what she eventually would be doing as a teacher, she was as confident of her ability to do it as she had been at the start of the course. On the purpose of teaching mathematics, Student M said, "I'd still think it's really, really important and the children should have it, definitely. . . . It's something that they're going to use their whole life and they just can't be without it--there's just no way." Student M said that Math 201 had not affected her feelings about her major. As a result of a methods course in reading, she was moving toward language arts as a major area within elementary education. She was considering taking a minor in mathematics and science.

Asked what features of the course could be improved, Student M replied that "the lectures were boring." While she realized that other students had difficulty with topics that she found easy, she still felt that "on the whole, they were just so boring, it was horrible." She had no suggestions, though, remarking, "I don't think there's anything you can do to put life into a math lecture."

Regarding the book, Student M commented, ". . . as long as you don't read the explanations, you're doing OK--once you read the explanations . . . lots of luck!" However, she found this true of mathematics books in general, and could not suggest anything which might improve the situation. While she had been bored by the familiarity of the material, she had found the lecturer fair and concerned about students, and had liked her laboratory instructor. Student M said that she had liked the laboratory sessions on the geoboard and on lattice multiplication, and had hated attribute games. In the session on ruler-and-compass constructions, Student M had been annoyed by both the slipperiness of the compass and the expectation that she had remembered Euclidean constructions from high school. Asked if anything had hindered her learning, Student M replied that sometimes a method of solving a type of problem would confuse her when she was used to a different way. She realized that she was "supposed to be receptive to new ideas," but still found these difficult to learn. Regarding the lecturer's teaching style of addressing the students as if they were elementary school pupils, Student M said that she sometimes resented this, but at other times felt that it might be useful for her own teaching.

Student M said that she would not take further mathematics courses aside from the required mathematics methods course.

She said that she saw the value of this research study. She could make no suggestions for improvements in the study, additional questions, or other topics for investigation.

On the question of general classroom teachers versus specialists, Student M said, "I think a classroom teacher could teach it if she's had some experience and background in it." She added, "I think it's the job of the classroom teacher to get the children interested in . . . math and to make them realize how important it really is." She said that specialists should teach only special, mathematics-interested students.

As a final comment, Student M said that she was pleased with her grade in Math 201.

Evaluation. In Student M we have seen a student who, despite a mathematics background which could not be called strong, still managed to do a superior job in Math 201 and was bored by the material covered in the course. However, her erratic problem-solving behavior suggests that Student M's grade may indicate a degree of mastery over the course material which she did not possess.

Student N

Student N was a sophomore from Farmington. She had taken three years of high school mathematics--two of algebra and one of geometry. She had liked algebra better

than geometry, but had been essentially neutral toward both. She always had forced herself to catch up when she felt she was falling behind in a course (because "with math, . . . you get it or you don't. . . . If you don't get it, your're not going to remember it on a test"), and regularly had earned B's in mathematics. She said, "I didn't really like math or science, but math I'd take over science any day, because I could at least work with it with my hands . . . use a little bit of brainwork to try to figure things out." Student N felt that she had learned well, because the material was coming back to her as she took Math 201. Her favorite subjects had been what she called "creative things"--French, English, sociology, psychology, art, and theater. She had disliked science. She had been active in performing arts and in athletics.

Prior to the study, Student N had not taken any college mathematics, but she had taken a course in mechanical drawing--"which I consider math [apparently because of the geometry involved]." She had taken that course as part of a major in interior design, but had switched to education after one year because of her low opinion of the faculty in design. She had not yet decided at the time of the study whether to major in elementary or special education, but was working at both a regular elementary school and at the Michigan School for the Blind in order to explore both options. Her main concern in choosing a major was to find

one in which her desire for creative expression would be fulfilled.

Student N had tutored her younger siblings in various school subjects, including mathematics; this was the extent of her teaching experience. Asked how she felt about eventually teaching mathematics, she replied, "As long as that's not all I teach . . . I have to work with colors and stuff . . . I wouldn't mind teaching math at the elementary level at all. I think it would be fun." She said the purpose of teaching elementary school mathematics was that "math makes you think a lot. You have to use your brain . . . You really have to think . . . there's a right answer and a wrong answer--there's an answer! It's a cut-and-dried course."

January 21. Student N opened this interview by voicing several criticisms of the course. She said of lectures that she was "upset, because she teaches us like little children," and that they were "total boredom." She added later, "Math is--to-me--boring enough without making it more boring." She also termed the two-hour laboratory "utterly ridiculous" because it was "too long" and repetitive, and claimed that other students agreed with this assessment.

All of the concepts presented in the course up to this point were familiar to Student N, though some of the procedures were new. Her goal was to learn the material so

that she could "do it real quick" on the test. Student N liked the lecturer's use of examples to illustrate concepts and the particular examples used, although she criticized the amount of time spent in explaining them as excessive. She had trouble in converting from one nondecimal base to another without going through base ten, and also in using the Austrian method of subtraction.

Regarding the assigned problems, Student N said she found some of them helpful and some confusing; of the latter, she said, "The wording in them throws me off." She would prefer to work out all the problems in the book (rather than only those which were assigned) and have answers available to check herself. She had been confused by an exercise asking the number of one-to-one correspondences between n -member sets, and also by one involving the definition of a counting number. She said the book "hint[s] around" and that its examples are inadequate to illustrate the concepts presented. Student N said that she had to review the exercise on subset relationships.

In her laboratory class, Student N noted that at the first session the instructor was indifferent to the material being presented, while at the second session he was more enthusiastic. The attitude of the class tended to reflect that of the instructor in each case. Student N said that she found the exercises involving addition and subtraction with Dienes Blocks helpful. She remarked that a two-hour class was too long and that she felt an hour and

a half would be sufficient; she also said she felt most people would not attend the laboratory if it were not required. She had done some of the exercises mentally instead of using the manipulative materials as directed.

Some of Student N's remarks at this interview were concerned with the upcoming test in the course. She expected to be asked about the material in Chapters 1 and 2 of the text. Later she said she expected the test to be tricky, and either "really easy or really hard."

In considering the exercise sheet on sets, Student N said that each of the sets described matches none of those given; she first wrote this idea as $\{0\}$ under each description, then after having considered them all, went back and erased these marks and wrote the word "none." It was evident from the responses that Student N had confused the concepts of equality and matching. In considering the set of all living people, she noted that Nixon was the only living member of set A. For the second described set she pointed out that set D "only goes up to 99" while set E "goes on to infinity." She said the set of suits matches none of the given sets because she saw no suits. Considering the set of aardvarks enrolled at M. S. U., she asked, "What is an aardvark?" Then she said, "I'll just say the empty set." Student N also misused the idea of the empty set, thinking it equivalent to "none of the above," and at first mistakenly used $\{0\}$ to denote the empty set, although she did correct herself at the end.

Student N correctly converted 405_{eleven} to base ten by labeling columns in base eleven and then taking $121 \times 4 + 5$. Asked to convert 39_{ten} to base two, she labeled columns in base ten and entered the digits 3 and 9, then attempted to set up columns in base two. After setting up ones, twos, and fours columns, she did not know how to proceed further. She did know that she could use only the digits 0 and 1, but said, "I just can't remember how to set it up." However, she was able to do the translation of 44_{ten} to base seven, setting up ones, sevens, and forty-nines columns and putting in the correct digits. After doing this problem, she thought that she might be able to do the previous one, but could not. She easily recognized base twelve in the first missing base problem, realizing that twelve was the amount that had been carried, but was unable to solve the second example, which involved subtraction. She said, "I don't know. I can't see how to do it. . . I don't even know where to start." She said her difficulty was due to the fact that it was a subtraction, rather than addition, example.

Student N's first reaction to Problem 1, a story problem, was to exclaim, "Ooh, I hate those! . . . I can't ever figure those out." She did not understand what the problem said; the investigator then reformulated the question as, "Considering the total amount of money we have together, how could you express that in the fewest coins?" To this, she replied, "That sounds a lot easier." Still,

she complained, "I don't know if they're asking what the difference is--thirty cents--or if they're asking who has the most money or the least." She computed each person's holdings, then summed them, obtaining \$1.16, and represented this as 4 quarters, 3 nickels, and 1 penny.

Student N said that she had "really liked" the interview, especially the problem-solving part of it.

February 4. In the course of discussing the first class test at this interview, Student N mentioned those areas where she had had trouble. She had mislabeled a statement of the additive identity property as having illustrated commutativity, and she had not been able to illustrate a one-to-one correspondence between the odd and the even counting numbers. She also had not been able to tell which is larger, 4235_{six} or 4235_{eight} . In addition to these test errors, Student N said she was unsure of her ability to multiply and divide in nondecimal bases, and did not understand two of the three definitions of division presented (she understood the missing-factor description), saying these were "kind of confusing . . . and/or irrelevant, [but] maybe it's so easy that it's hard."

Regarding the lectures themselves, Student N said, "I get more out of the lectures than I do the book." She usually skimmed the book's presentation after attending a lecture on that topic. She liked the lecturer's discussions of assigned problems. In general, she felt that

she understood the lectures "really well." While Student N had liked the lecturer's presentation of the topic of prime numbers, she had been confused by the statement of the Fundamental Theorem of Arithmetic. She said the lecturer became "carried away" listing multiples in an example of least common multiple. Student N considered it "awkward" in teaching to give an example before stating the illustrated definition. She said she was "recalling a lot," and that all of the material was familiar to her. She expected to keep her textbook to use as a reference while teaching. She was worried about the final examination, where "you either know it or you don't."

Student N described the textbook as "kind of funky, 'cause it gets so involved with these crazy numbers." As an illustration, she cited the following statement (page 92):

Suppose that $p, \bar{p}, d, \bar{d}$ are counting numbers, such that $p \leq \bar{p}$, $d \leq \bar{d}$ and such that $p \div d, \bar{p} \div \bar{d}$, and $p \div \bar{d}$ are all defined. Then $p \div \bar{d} \leq p \div d \leq \bar{p} \div \bar{d}$.

"That's crazy," said Student N. Although she generally tried to read the emphasized statements in the book when reviewing, she had not understood this one and had skipped it. She ignored the book's machine diagrams, commenting, "I don't even look at that junk," and said that she understood the material anyway. To this point she had done the homework only through multiplication. She usually did her problems after the assignment had been discussed in lecture, with the discussion or the numerical answer as a guide; she said

she would have liked to have the answers to all of the problems available. Student N said that she now found the assigned problems easier to understand. She did not anticipate any trouble with the problems involving primes; of the other topics, bases had given her the most difficulty.

The laboratory sessions were "getting better," according to Student N. She found multiplication and division with Dienes Blocks easy. In the session on computation, she considered Napier's bones "kind of juvenile, but I guess it's important," and the Whitney Mini-Computer "boring." She said that her laboratory instructor was patient and helpful. She mentioned that a friend who had taken Math 201 during the previous summer term (without laboratory work) felt that Student N was having a better course than she had had. Student N said that the laboratory classes "definitely" helped her to understand the material. She felt that she learned a lot "just [from] interaction with the other students [because] I can relate to a student easier than to a teacher." She also appreciated the ability to ask questions in the small laboratory class, because she was too shy to ask them in lecture.

Student N said she felt the pace of the course to this point had been "nice." She always attended class, and liked the lecturer's thorough presentation and receptiveness to questions. Although she said the presentation was such

that "you can't miss anything if you have any ounce of brains," she had acquaintances who were not maintaining the pace. Student N forced herself to keep this pace, because, "If you don't get behind in math, you'll be all right."

Student N recognized the similarity between Problem 2 and one of the assigned homework problems. She saw that there were three possible choices of prefix. Because there were four places with ten possible digits in each, Student N enumerated forty possible suffixes for each prefix. Saying, "I hate these," she enumerated 120 possible phone numbers, but noted, "That doesn't seem like enough. I know there's a lot more." Nevertheless, she was unable to obtain any other answer, and so gave 120 as her answer. After she finished, Student N explained, "I hate story problems. I'm glad she didn't have many story problems on the test." (There had been one, and Student N said she had guessed the correct answer.) She continued, "I can't think logically. I read into [tests]."

February 18. At this interview the first discussion was about the laboratory sessions. Student N again mentioned the session on computation, saying she had enjoyed learning lattice multiplication and Napier's bones, but did not expect ever again to use the Whitney Mini-Computer. Concerning clock arithmetic, she said it "was stupid . . . We couldn't understand half the stuff on it. . . . I still don't know what a field is. I know it has to do with

prime numbers, but that's all I know about a field . . . I understand clock arithmetic . . ." She found Cuisenaire rods "interesting" since she had encountered them in her elementary school observation. She had seen their use in finding factors after her instructor had explained it. She had not spent much time with GeoBlocks, but called them "all right"; she had been able to work the tangram problem after receiving a clue from the instructor. She termed the session on rational numbers "a fairly good lab."

Student N said she was confused by the three descriptions of division and was glad they did not appear on the test; she felt they had been inadequately explained by both the book and the lecturer. Student N also reported having difficulty with the concept of closure. She was unable to construct examples of sets which were closed under various operations; however, once she had seen an example, she understood why it was an example. She said most of the assigned problems were easy for her. Nevertheless, she said of the following problem (page 161):

Give examples of integers p and q for which

$$|p + q| < |p| + |q|$$

Give examples of integers p and q for which

$$|p + q| = |p| + |q|$$

"I hate that. That drives me crazy," although she claimed to understand the concept of absolute value. She could illustrate Cartesian products but not when one factor was the empty set. She had trouble naming which of the field

properties was illustrated by a particular statement. She said she enjoyed doing arithmetic in nondecimal bases. When a problem required a proof, she would not attempt it; she would either skip it or look up the solution.

Student N appreciated the lecturer's hints on finding the greatest common factor and the least common multiple of two numbers. However, she had difficulty understanding the lecturer's general statements when they were expressed in terms of letters--"if I don't have numbers down there . . . I get confused." Nevertheless, she recognized the necessity of using letters in writing general statements. She said that the lectures were thorough.

Much of the discussion at this interview concerned past and future tests. Student N was oriented strongly toward the tests, always trying to anticipate what would appear.

Student N said that the course "all flows together." She had not yet needed to ask her laboratory instructor for help, though occasionally she would ask a fellow student.

Student N was able to identify correctly the prime and the composite numbers on the sheet presented to her. By long division, she divided 2, 3, 5, and 7 into 119, finding that 7 is a factor. For 113, she wrote out division symbols for 113 divided by 2, 3, 5, 7, 11, and 13, but only carried out the divisions through 7. Noting that $11^2 > 113$, she stopped at this point and called 113 a prime. For 227,

she tried as divisors 3 (she wrote 2 and quickly discarded it), 5, 7, 11, 13, and 17. Noting that $17^2 > 227$, she called the latter a prime. After trying several divisors for 247, she found 13 to be a factor. In both cases where she found a factor, Student N checked her result by multiplication.

In finding the least common multiple and greatest common factor, Student N factored each number into primes, and then took the correct combination of the factors. She had a number of mnemonic devices to help her remember which was which. The L in LCM reminded her that the LCM was larger than the given numbers (or perhaps merely larger than the GCF) while it followed from this that the GCF was smaller. To take the LCM, she said, "The L meant all so I put all of one down," then put down the appropriate factors of the other. She also said she used the G in GCF to remind her to put them all together, but the investigator could not see what she meant by this.

Student N did not know how to solve Problem 3. She was able to count 4 hours from 9 A.M. to 1 P.M., but could not use this information. She said she thought the problem might have been easier for her if the speed involved had been 60 mph, which is one mile per minute, but she didn't know. She then said she felt she had never been taught how to read a story problem and set it up for solution.

March 4. The first topic of discussion at this interview was the lecturer's homework sheet containing several verbal percent problems. Student N said that she had been unable to set these problems up for solution, but had followed the lecturer's explanations. She hoped that reading the book would help. As usual, Student N was thinking and worrying about the next test. She expected to have difficulty with verbal percent problems and with geometric problems involving area and the Pythagorean theorem. (Mentioning the formula $A = \pi r^2$, Student N said, "I don't see where she got that from. I guess it was from the geoboard.") Regarding the percent problems, she complained, "I never have understood story problems . . . I can't stand percent problems." She had learned from the explanation in lecture that "the percent sign is the same as writing one over a hundred, and I never knew that till now." Although she could do the necessary arithmetic and algebra once a problem had been formulated, she found difficulty in "setting it up." She feared that because of this "I'm going to blow my four-point [in the course]."

Student N did not like the situation in the laboratory sessions where she found herself unsure of which material was important. She suggested, "I think she should clearly state--before the labs--what's expected . . . You sit through that damn lab and you think, what does she want me to understand . . . that will be significant for the test. Nobody knows." Although Student N understood

clock arithmetic, she said many students in the class had missed these questions on the test and had not even realized that the questions were based on the activities they had done in the laboratory. Student N called the geoboard "interesting"; she had been able to compute areas of figures but could not see where square roots and the Pythagorean theorem were involved. The manipulative nature of the geoboard reminded her of Cuisenaire rods. She called the laboratory session on ruler-and-compass constructions "kind of dead . . . [it] wasn't that interesting." While she claimed to have understood it, she did not see its "relevance" until it was mentioned in lecture immediately prior to this interview. Student N thought it would be "a great idea" to have a laboratory activity after the topic had been discussed in lecture, rather than before. She felt disoriented without specific guidelines for the test: "These last few labs didn't have any meaning . . . If she would just state what she wants us to know . . ."

One of the features of the lectures that Student N found most helpful was the lecturer's use of a Venn diagram to illustrate the inclusion relations among the counting numbers, integers, rational numbers, irrational numbers, and real numbers. This somewhat clarified for her the distinctions between the various types of number; previously she had not understood these distinctions. Two items which Student N found "interesting" were scientific notation and nondecimal "decimals." Concerning the

procedure for finding a fraction name for a repeating decimal, Student N said, "I didn't really see the significance in finding this fraction. It seemed kind of stupid." She then mentioned that she was afraid she might misplace a decimal point on the test. She had figured out herself that $5/7$ and $7/5$ were the same; to her memory, she had not done this in elementary school. Student N remarked that she saw how elementary school pupils might have trouble with decimals. In further comments on the lectures, Student N said that the lecturer had "explained integers and reciprocals really good"; the lecturer had also explained cross-multiplication well and done many examples of rational number arithmetic by the array method. Nevertheless, Student N said that it "bugs me [that] she treats us like little kids." She noted that many students were restless during lectures.

In finding a rational number between $1/3$ and $1/4$, Student N represented $1/4$ as .25 and $1/3$ as .33 . . . Although she had some doubt that .25 is rational ("I'll say yes . . . because I just figure it is . . . I'm just guessing."), she then took .26 as a rational number between the original numbers.

Student N found the decimal name for $7/12$ by dividing out 7 by 12. However, she knew no procedure for finding a fraction name for a repeating decimal. Her only idea was to attack this problem by trial and error, choosing a fraction she thought might work, checking the

decimal name, then adjusting the numerator and denominator to attempt to make the decimal conform to that asked. In the next problem, Student N found the decimal name for $6/11$ but did not know the meaning of the word "error." After the investigator explained the meaning of the word, she was unable to proceed further with the problem. To find a value for $\sqrt{7}$, she drew a right triangle, labeling the legs 1 and 1 and the hypotenuse $\sqrt{1}$. From this she deduced that $\sqrt{7} + \sqrt{7} = 7$, so she said that $\sqrt{7}$ is half of 7. She was not sure if this answer was correct.

Student N said she would find the length of the curve in the first measurement problem by superimposing a string on the figure and then measuring the string. To find the area of the given region, she would again superimpose a string on the boundary of the figure, then deform the string into a circle and use the formula $A = \pi r^2$ to find the area.

Student N received the grade of 4.0 in the course.

March 27. Asked the effect of Math 201 on her feelings toward mathematics, Student N said, "I feel positive towards it because I did well." She felt she had learned much in the course, and that it would be easier for her to recall this material in the future because she now understood the procedures of arithmetic rather than merely knowing them mechanically. Regarding her eventual teaching of mathematics, she remarked, "I feel a little bit

stronger. I feel I could do it now. Before I felt like I couldn't do it at all, 'cause I hated it so much. But I've learned to like it . . . a lot more . . . I feel that I could teach it and not have it be boring . . . I can see now that you can make it interesting." She said she was "very sure" of her ability to teach elementary school mathematics. She strongly favored an activity approach as used in the laboratory sessions of Math 201; she considered such an approach best suited to her own personality. She called the question on the purpose of teaching elementary school mathematics a "hard question," then said, "The purpose of teaching math is so that they learn it," adding, "That's why I want to become a teacher--to learn myself and to teach to learn." Student N said that she planned to major in language arts within her elementary education program, and was considering a minor in mathematics. Concerning possible minors, she commented, "I hate science, because there isn't any answer. But math is . . . a little bit different because you know there's an answer--you know there's a right and a wrong way . . . So I feel like I can do it." She was not aware of the requirements for a mathematics minor. Student N said of mathematics, "I have come around to liking it a whole lot better. I don't know why, but it just made more sense." She had been impressed by the clarity of the explanations provided in Math 201. Student N felt that a career as a teacher of the blind would be an intolerable emotional

strain for her, and therefore was leaning more toward general elementary education as a major.

Student N's only suggestion for improving the course was to have the laboratory exercise after a topic had been discussed in the lecture rather than before. She had the impression that the lecturer "doesn't think of us as adults," but thought, "Maybe she was doing it for a purpose." Student N thought the laboratory sessions were a "necessary" part of the course. She had liked best the sessions on clock arithmetic, Cuisenaire rods, and computation (Napier's bones and lattice multiplication). She had disliked the session on the metric system and was upset because she had to learn it, but accepted this as a necessity. She could name nothing about the course that had hindered her learning, but did criticize the book as unreadable in places.

Student N said she would like to take more mathematics courses. She did not yet know specifically which courses she might consider, but said they would probably be in algebra or geometry.

She said the research study had been "interesting" and had forced her to think about the course. She had found the interviews a welcome outlet for her feelings. She also noted that the study showed that the department cared about the course. She suggested the possibility of having some group interviews rather than individual

interviews only. She could suggest no further questions or areas of investigation.

Student N generally favored mathematics specialists as teachers of elementary school mathematics, due to their superior preparation in the teaching of mathematics. She suggested that a general classroom teacher be observed occasionally by a mathematics specialist.

Evaluation. Student N appears to have derived some benefit from the course in improvement of her understanding and of her attitude toward mathematics. She was favorably impressed by the laboratory. However, her inability to solve verbal problems should have been a factor in determining her grade in the course.

Student O

Student O was a sophomore from Kentwood. She had taken a year of algebra and a year of geometry in high school. About this experience, she said, "I liked geometry a lot better than I liked algebra." These feelings had been reflected in better grades--B+/A- in geometry versus B- in algebra. Asked why she had felt this way, Student O said, "I liked the logical proof where you had to think about it and line up all your arguments." Her attitude toward mathematics had been and was one of neutrality. She had taken the minimum that she felt was expected of a college preparatory student. Her favorite high school

subjects had been physiology (which she had found "fascinating") and English. She had sung in musical productions and in the high school choir. She had not taken any college mathematics prior to the study.

Student O's major was special education of the mentally retarded. She chose this major out of a desire to work with people in close situations and a desire to help those who might otherwise be wards of society to live a productive life. She never had taught in a classroom situation, but as a high school student had tutored slow-learning elementary school students in various subjects. She hoped to be admitted to an interdisciplinary major in which she could work in a school while taking her methods courses.

Student O said she was "glad" to be taking Math 201 "because I knew they'd go over in detail what I need to know." She was aware of the dangers of teaching mathematics poorly from her sister's recent junior high school experience; Student O had had to give her remedial tutoring. Student O did not anticipate difficulty in learning the mathematics required for teaching the fifth and sixth grades or lower, but realized that "what I know, I'll have to know!" She said she liked the way "they get behind things" in Math 201.

Student O saw two purposes in elementary school mathematics. One is that "mathematics is something they're going to be needing in their everyday life on a practical

level"; the other is that "math . . . is a different way of thinking. It's a very logical, ordered way of thinking. It helps you look at things in a very structured, precise, clear way."

January 25. At this interview Student O said of the lectures, "I usually pretty much enjoy them. She presents everything pretty logically, right down the line." Student O usually read the book in advance of attending class, letting the lecture "emphasize" what she had read. She liked the way the lecturer clarified the arguments in the book. She mentioned that the lectures had made clear for her the ideas of matching sets, one-to-one correspondence, and the distributive law. She could not name anything else from the lectures which specifically had helped her.

Regarding the book, Student O said that she "understood their reasoning," but that "really detailed proofs just aren't my thing. I don't especially like them, but I can deal with that." She described her reaction after asking her roommate to explain a definition in the book: "Well, why didn't they just come out and say that, instead of going through all that jargon?" She proudly pointed out a typographical error she had discovered in the book and thought she had found another error in the answer to a problem. (She was wrong; the book was correct.) The problem (page 11) was to compare the sets {Honest Abe,

the Rough Rider, Lincoln} and {Teddy Roosevelt, the Rough Rider, Honest Abe}. Noting that the book called these equal, she said, "If that's true, I don't understand sets at all." Student O said of the problems in general, "They're good problems . . . they make you think." She said that she used the difficult problems as a guide to sections of the book which she would want to reread.

Student O said that her laboratory instructor had had some difficulty at the beginning of the course ("He'd be thinking at a more complicated level than we were at.") but had improved somewhat over the first few weeks. She described her difficulty as an incompatibility of the instructor's teaching and her way of thinking, saying, "When I do math, and I want to know how to do something, my minds works weird--I want to go through it step by step and know exactly what I have to do, 'cause if I know what I'm supposed to do--what a problem calls for . . . then I don't get uptight . . . But if they throw a problem at me and I don't know what they're asking . . . then I get really nervous . . ." She went on to say that once she had "a good grip on it, then I can go ahead and . . . see something and leave off from there, but . . . he didn't know all these particular weird things about how my mind works anyway." Rather, her laboratory instructor started at "step three, and I need to go one, two, three." Student O saw the point of working with Dienes Blocks, and said that "it helps sometimes with conceptualization [particularly]

multiplication and division," and with doing arithmetic in nondecimal bases, but added, "After a while . . . two problems . . . you get tired of working with the wood . . . we do the rest [of the problems] in our head." Student O said that the laboratory had not clarified any concepts for her because she had read ahead in the book and "I already had a pretty good grip on it." It had helped her to explain to another student how to use Venn diagrams. The only confusion she had encountered in the laboratory occurred when her instructor "contradicted himself."

The first problem sheet at this interview was that on sets. Student O said the set of all living people "is a lot bigger than 99 [while still finite], so I'd say it doesn't match any." Apparently overlooking set D, she said the set of counting numbers from 1 to 100 "would be like a subset of E . . . there's no one-to-one correspondence . . . it doesn't match any [but is a subset]." Student O said that the set of suits in a standard deck of cards matches set C because both have four elements. She said that the set of all aardvarks enrolled at M. S. U. is the empty set and matches none of the listed sets. Except for her oversight of set D in considering the second set, her responses indicate that Student O had learned the meaning of matching of sets.

In converting 405_{eleven} to base ten, Student O first noted that "in base ten, that number should be bigger." She then proceeded to convert the number by labeling her

columns 11 x 11, 11, and 1, and then taking $484 + 5 = 489_{\text{ten}}$. Student O converted 39_{ten} to base two by repeatedly dividing 39 and its successive partial quotients by 2, placing the successive remainders in columns from right to left. She converted 44_{ten} to base seven similarly. In considering the first missing-base problem, Student O first recognized that the base had to be greater than nine because the digit nine appeared in the example. Noting that it was not base ten and that "we only went up to base twelve," she concluded that the base must be eleven or twelve. Considering these two possibilities, she recognized it to be base twelve. Similarly, her first reaction to the second example was to say that the base has "got to be bigger than base two." After confirming that it was not base ten, Student O looked for a base in which $11 - 2 = 2$, and quickly realized that base three was the answer.

To solve Problem 1, Student O labeled columns Q, N, and P, put the appropriate numbers of coins under each one, added, and obtained 3 7 6 in her three columns. She said, "I get three-seventy-six in base ten . . . I could use any base . . . I could have done it in base five which would have been nice." Thinking that she had \$3.76, she expressed this amount as 15 quarters and 1 penny.

Student O said of the interview that she had felt inadequate when she could give no specific comments about the lecture. She had felt nervous about the problems

briefly, but this feeling had disappeared once she had started work on them.

February 8. Student O said this part of the course had been "kind of interesting." She had forgotten (or had not thought about) the basis for thinking about negative numbers and found this a worthwhile topic of study. She said that she obtained ideas for teaching from the course. Student O thought that the lectures were "good--because . . . she goes over everything and she hits all the main essentials, and she usually does the homework problems that I had trouble with . . ." Student O usually found the book's answers and the lecturer's explanations sufficient to clarify the problems for her. She also said that the lectures explained those parts of the book which she did not understand, and that nothing in the lectures had confused her.

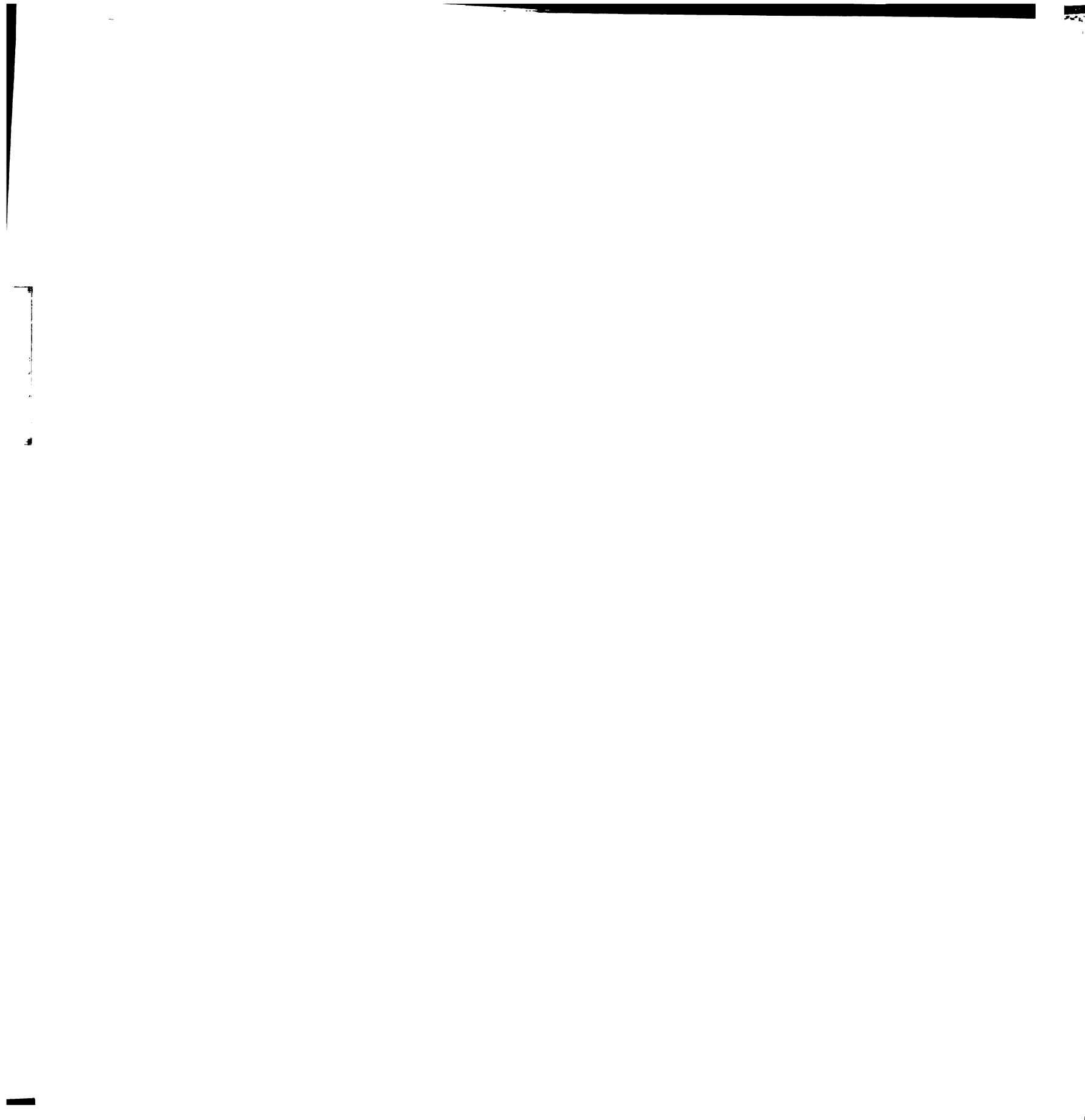
Discussing her experience with the textbook, Student O said, "I think I'm getting better at reading proofs . . . The only proofs that really get hairy for me are the ones when they talk about inequalities--the others I have to read like two or three times to just make sure I've got everything right. Some of the other ones . . . I just skip . . . I read the stuff in pink [and] went on . . . I'm getting more used to reading symbolic language." She again proudly pointed out a typographical error she had discovered. The problems about prime numbers helped her

to recall the Fundamental Theorem of Arithmetic. Student O perceptively noted that some of the book's problems anticipated the ideas in the following section.

In the laboratory, Student O was confused by the use of the division algorithm in nondecimal bases. She said, "I got the hang of it after a while, but I'd still rather work in base ten." She found the session on computation "interesting," thinking Russian peasant multiplication, Napier's bones, and lattice multiplication would be good ideas for the elementary school classroom. She also called clock arithmetic "interesting," saying the laboratory exercise had illustrated the idea of field better than the book's discussion, which she had read although it had not been assigned.

Student O said of the pace of the course to this point, "I don't think it's been too fast." She said that she tried to keep one section ahead of the lecture and worked on mathematics two nights a week. She said of the course content, "A lot of it's been review of stuff I've forgotten--like bases." Student O had seen bases while in the fifth grade and absolute value while in algebra. She had forgotten the meaning of square root and had been unable to do a problem where the book assumed knowledge of it.

In considering Problem 2, Student O first said, ". . . it's got to be something times three because . . . there's three basic groups that it can go into . . . I believe that there's four digits on the end . . ." She



realized that each of these four spaces could be occupied by one of ten digits. Drawing three lines with four blanks in each line, she put a "10" in each blank of the first line, then concluded that there were forty possible endings for each exchange. To get the total of possible numbers, she added the three 40's, obtaining 120, and then multiplied this by 3, obtaining 360, which she gave as her answer. Student O realized that this was not a reasonable answer to the problem, so she tried to enumerate the possibilities for each exchange. Her consideration confirmed for her that there were forty of these! She therefore let her answer of 360 stand, but remarked that since there obviously were more than 360 phone numbers in East Lansing, there must be more exchanges in reality than were mentioned in the problem.

February 22. Student O described the lectures in the most recent part of the course as follows: "A lot of material . . . theorems . . . properties have been thrown at you, like rational numbers is an extension of integers. There's a lot of complicated proofs we're going through right now." As an example of what she meant, she cited the topic of fractions. Student O said she knew how to do arithmetic with fractions and why the procedures worked, but was not sure that the book and the lecture had explained the topic clearly. She thought that the lectures were "a little weak" in anticipating the problems of elementary

school students. Student O said that the lecturer's explanation of the density of the rational numbers, using numerical illustrations, was much clearer than that of the book, which used letters. She said she was confused between the concepts of dense ordering and density. Student O agreed with the lecturer's stress on building models for children to rely on if they forget an algorithm. She said that she liked the book's advice on how to present the material to elementary school pupils, and remarked that she would have liked to take the joint section of this course with the methods course, but had been unable to arrange this. She then made the following remarks about story problems: "I used to hate story problems . . . [I had difficulty] finding out what they wanted . . . After a while [it] was drilled into my head . . . to watch for certain words, like if it said of, I know I have to multiply. Three percent of something, or one-third of something else . . . One thing I remember from grade school math is that I didn't like story problems. If they gave me straight . . . add 35 and 42--fine, no problem--but if they said John has blah-blah, I'd panic as soon as I saw it. I don't really know why--it's just that it was easier for me to see numerically than it was verbally. I guess maybe it was because I could see verbal stuff verbally, and math numerically, but when they start mixing the two together, after I get used to working in just straight numbers, then I have problems . . ."

Student O said that the only assigned problem with which she had had difficulty was the following (page 193):

You are going to do a bulk mailing of $22 \frac{1}{2}$ pounds of letters. You weigh 20 letters and find that they weigh 7 ounces. How many letters are there altogether?

Feeling unsure that the solution she had obtained was correct, she had looked up the answer, then had reworked the problem to obtain that value. Overall, she had found the variety of fraction exercises a good review. She had been able to arrange a set of fractions in order, but had used a more complicated method than that of the lecturer. Student O had been interested in the topic because she had "always liked fractions." She had found the book's presentation of this topic generally good, but "shaky on . . . the common denominator thing." She had had some trouble with division of fractions, and had asked her roommate to explain the idea of division as the inverse operation of multiplication. She said that she expected to use the book in her methods course. Student O had had no difficulty with the problems on integers and absolute value.

Student O said of the laboratory session on ruler-and-compass constructions, "I didn't really like that." Asked why, she replied, "It was hard," and added, "I'm still a little leery of square roots." Although her laboratory instructor had explained the material more than once, she felt "I could have used a couple more [explanations]." While she remembered from high school geometry

how to divide a line segment into equal pieces, she found difficult the construction of segments of length $\sqrt{2}$, $\sqrt{3}$, etc. Discussing the session on rational numbers, Student 0 said that comparing the sizes of the various GeoBlocks had been "fun" and "really interesting." She also had found interesting the exercises with tangrams and Cuisenaire rods. She had seen Cuisenaire rods in use in the elementary schools, and had explained them to her partner in the laboratory. She noted that her partner had been able to make a square from the tangram pieces but that she had not; however, she thought that she would have been able to do this with more thought.

The first problem sheet presented at this interview was that on prime numbers. Student 0 began by reminding herself of the definition of a prime number. She said of 119, "It kind of looks prime, but sometimes . . . that can fool you." She then tried to divide it by 2, 3, 4, 5, 6, and finally 7, which went in. For 113, she tried as divisors 3, 4, 5, 6, 7, and 8, then said, "I think that's prime . . . I wouldn't go any bigger than 11." (The investigator could not ascertain whether she actually had gone beyond 8.) For 227, her first reaction was to try 3 as a divisor; she was surprised when it failed. She tried 2, 4, 6, 7, 8, 9, 10, and 11; when all of these had failed, she called 227 prime. For 247, she tried 2, 3, and 5 mentally, then in writing tried 4, 6, 7, 8, 9, 10, 11, and 12. Stopping here, she called 247 a prime. She said that

she had not tried 13 because considering that $2 \times 13 = 26$, she thought that 13 could not go into 247.

Student 0 correctly factored 63 and 105 into primes and found their greatest common divisor. In the second problem, she wrote 42 as 6×7 and 48 as 6×8 , then wrote the former as $2 \times 3 \times 7$. She wrote as the least common multiple $2 \times 2 \times 2 \times 3 \times 7$, apparently because this was the smallest combination which was a multiple of 6, 8, and 7.

After Student 0 read Problem 3, the investigator explained to her that the "9 A.M." in the problem referred to 9 A.M. that morning. She was unable to put together the data in the problem in order to solve it. Her only statements were "He's [Dick] going to be one hour less than Dan is at 9 A.M." and that if Dan were 3 hours away at 40 mph, then Dick would be "negative 4 hours" away. Both statements illustrate her confusion of the relationship of distance and time.

March 11. Student 0 said of the last section of the course that some of it had been easy and some complicated. She mentioned as particularly complicated the chapter on real numbers (which had never been completed) in which the students had been asked to learn only some "basic ideas." Student 0 had had some difficulty in grasping the inclusion relations among the counting numbers, the integers, the rationals, the irrationals, and the reals. A misleading

Venn diagram had caused her to think that there are real numbers which are neither rational nor irrational; the lecturer had "straightened that out." She said that the explanation in lecture of converting a repeating decimal to a fraction had been clear, while the explanation in the book had not. The lecturer also had cleared up for her the distinction between scientific notation and expanded notation. Student O had trouble with decimals and with percent. She disliked the book's treatment of percent, but with the help of the lecturer and of her roommate obtained a universal formula for solving percent problems no matter which component was asked. Previously, she felt she needed a different model for each percent problem, so she appreciated having the formula, because "you can't carry the book around with you all your life." She was confident enough to offer to the investigator to solve a percent problem on the spot, "even . . . out of a story problem," and "for me to be able to do that out of a story problem, I'd have to know what I'm doing."

In discussing the book, Student O said that the chapter on decimals "really got me mixed up." She remarked that "the percent problems showed me that I didn't know that much about percent," but that she had not obtained any help here from the book. She considered the question (page 219) $2.3______ = 2.75_{\text{ten}}$ "a neat problem." She had found it "fun" to draw boxes to illustrate ideas about fractions, and considered this a good idea for presenting

the topic to children. She had found worthwhile the exercises on putting fractions in order and finding the distance between them. Student O had not been able to do problems in the multiplication and division of decimals until the lecturer explained them. She recalled having no other difficulties, even with story problems.

Student O said that the laboratory on ruler-and-compass constructions had been unclear, particularly the relationship between circumference and diameter (she also had discussed this laboratory session at the previous interview). She noted that she had answered this question wrong on the test, but would have been right if she had kept her original guess, which she rejected because "it looked too easy." She called the laboratory on the metric system "interesting" and "kind of neat." She disliked computing volumes in cubic units and noted that she frequently forgot to attach "cubic-" or "square-" to the name of the linear unit when it was appropriate. She also found it "neat" to compare Celsius and Fahrenheit temperatures, and planned to memorize the conversion formulas for the final examination. She termed the laboratory on the geoboard "fun." She had learned Pick's theorem (and used it on Test 3) but was "still a little hazy on that square root stuff [the Pythagorean theorem]," which she had seen in both the ruler-and-compass and geoboard laboratory sessions.

In finding a rational number between $1/3$ and $1/4$, student O noted that it could be done by computing the

average of the two, but that she preferred to rewrite the given fractions as $10/30$ and $10/40$, and then take $10/31$ as a rational number between them.

To find a decimal name for $7/12$, Student 0 first divided 12 by 7 (correctly). Before going on to the next problem, she realized that she had done the problem wrong, and then went back and divided 7 by 12, obtaining the correct answer. To find a fraction name for $.3777 \dots$, Student 0 called this r and wrote the decimal expressions for $10r$ and $100r$. She then took $100r - r$ and noticed there were still digits to the right of the decimal point. She noted that the lecturer had pointed out that one could still obtain a solution in this situation, but that she preferred to have no fraction parts remaining after subtraction. She then took $100r - 10r$, and obtained $r = 34/90$, which she reduced to $17/45$. She then checked her answer by dividing 45 into 17 and obtaining the original decimal. Student 0 said that she was not sure of the meaning of the next problem, but that she thought she should divide out $6/11$ as a decimal and stop after as many decimal places as as the error bound specified. She later said she did not know how to obtain a terminating decimal from the nonterminating expansion of $6/11$. To find an approximate value for $\sqrt{7}$, Student 0 thought of half of 7 (whose square she quickly saw to be greater than 9) and also of using the Pythagorean relation among the sides of a right triangle. She knew, though, that this number was between

2 and 3. She guessed 2.7, and found the square of this number to be 7.29. She then estimated it at 2.68, squared this, and found the square still larger than 7. She then commented, "There's got to be an easier way to do it, but I don't what it is," and noted, "It's got to be close to that number [2.68]."

Student O said that she would solve the first measurement problem by putting a string along the curve, then measuring the string with a ruler. In considering the second problem, she first noted that she could not use the formula for the area of a circle, nor could she use Pick's theorem since the figure was not on a geoboard or even rectilinear. She suggested putting a square around it, finding the area of the square, then subtracting the excess, but did not know how to find the amount of the excess. Similarly, she thought of putting a circle inside the figure, finding its area, and then adding the difference. She knew she could not deform the figure without changing its area. She said that her final resort would be to ask her roommate.

Student O received the grade of 3.5 in the course.

April 1. Student O said at this time that she felt "more positively" about mathematics than she had at the start of the course. She said that while before she had been able to tell her younger sister what to do on a mathematics problem, she could now explain why a procedure

works. She specifically mentioned that she could now do percent problems, which she had been unable to do before. Student O said she was now more confident about eventually teaching mathematics. While she had experience answering elementary school pupils' questions, she now felt that she could present material capably (with some reservations about the requirements for teaching the mentally retarded). Student O said that there had been no change in her feelings about the purpose of elementary school mathematics (she always had thought it worthwhile) or about her major.

As a possible improvement in the course, Student O said that "being able to work directly with kids would have been nice," although she realized this might be difficult to arrange. She also suggested incorporating more ideas that can be used in the elementary classroom. She was enthusiastic about the laboratory part of Math 201, naming the geoboard, computational methods, Cuisenaire rods, Dienes Blocks, and the metric system as laboratory activities she had enjoyed. Asked if she had disliked any of the laboratory sessions, she said the one on ruler-and-compass constructions had been poorly run and so was her "least favorite." She said nothing had hindered her learning, but that some parts of the presentation (both book and lecture) could have been "a little clearer."

Student O said she would "probably not" take further mathematics courses.

Student O said of the research study, "It was kind of neat." She said that she had enjoyed participating but had resented somewhat the effort required to come. She had found it "frustrating" when she was not told if her problem solutions were right or wrong, but appreciated the investigator's explanation of this policy. She said that it had been good that the investigator had had many specific questions to guide her comments, and remarked that such a study was better than the written evaluation she had done in the course. She could not suggest any improvements in the study, although she did suggest that the investigator might ask how the subject felt about particular mathematical topics. As an example, if she had been asked how she liked bases, she would have replied, "I like bases. I think they're neat." She suggested a study on how Math 201 might work as a self-paced course.

Asked whether elementary school mathematics should be taught by mathematics specialists or by general classroom teachers, Student O first answered, "Maybe a math specialist would know more than I would . . . and I suppose that would be good for the kids, but I'd kind of like to have a crack at it myself." She felt a general classroom teacher would have a better opportunity to display the uses of mathematics in her teaching of other school subjects. Although she thought the mathematics content of the upper elementary grades might be handled better by a specialist, she felt that teaching experience was probably

a more important variable in this regard than coursework in mathematics.

Evaluation. Student O was a type of student well suited to the course as given. She had good mathematics aptitude, yet was not overprepared for a course at this level. (Student O had a very good intuition about numbers--she could sense when her answer to a problem was not correct.) She was also receptive to the laboratory exercises and generally enjoyed them.

Student P

Student P was a freshman from Jackson, attending Michigan State for the first time at the time of the study. She came for an initial interview on January 9, the first of the subjects to be interviewed.

Student P had taken five full years of high school mathematics, starting in the eighth grade. Her background included two years of algebra, one year of geometry, one year of "trigonometry and analysis," and one year of "precalculus." She characterized her high school mathematics classes as highly motivated; this had been due to ability grouping. She remarked that mathematics was her favorite subject, and that she had enjoyed her high school courses, particularly those taught by one "very good math teacher." Her other favorite subjects had been English and chemistry, while she had disliked history. When asked

why she had these particular likes and dislikes, she replied, "I like doing things. [In] history you can't really do something; you just have to read and take it in. . . . I like to do things like problems--solve them, work with it. . ." She had participated in band and choir in high school, and in her last year had attended high school for half a day and Jackson Community College the other half, earning eighteen college credits. Like all but one of the study subjects, she was taking her first college mathematics course in Math 201.

Student P was an elementary education student who intended to major in mathematics-science. Her goal was to teach in a Montessori school (a psychology teacher had introduced her to Montessori education), but she wanted to have a regular elementary certificate in reserve. She wanted to teach the early grades. Her only teaching experience was tutoring some second graders as a high school junior. She said that the prospect of teaching mathematics "pleases me, because I think that's what I'd probably do the best in." Her idea of the purpose of elementary school mathematics was "to get kids to see . . . numbers, and groups. They should know how to add and subtract, too."

January 29. At this interview, Student P said of the lectures, "They're very elementary. It's hard to get excited about them. . . . It seems like we're the

elementary kids." Asked if she could name specific features of the lectures which had been either helpful or confusing, Student P, appearing to be straining for examples, mentioned the lecturer's explanations of "the way everything is related," and also said that "she did a really good job explaining bases." Student P also liked the lecturer's references to her daughter. When the lecturer had taught two different ways to convert from one base to another--the second just before the test--Student P had ignored the second method to avoid confusion. She said that all of the material so far was familiar to her.

Student P said that the lecturer assigned problems on a particular topic before it was discussed in lecture. She said that she liked the textbook's Supplement, in which pages from elementary school mathematics books are reproduced. She could name no particular problems as being especially helpful, but said that she liked those that asked how one would explain something to a child. While she had been somewhat confused by the book's problems involving the set operation $\sim$ (difference), the lectures had cleared up this difficulty.

Student P said of the laboratory sessions she had attended thus far that it was "very frustrating for me" to wait for her colleagues to understand what they were doing. She mentioned that she had found Dienes Blocks helpful in her work on bases.

The first sheet of problems presented to Student P at this interview was that on sets. After noting that "it's hard to define the set of all living people," Student P said she was not sure whether this set is finite or infinite. She said that if it were finite it would match none of the given sets, as sets A through D are all too small and set E is infinite. In considering the set of counting numbers from 1 to 100, she first thought that it matches none of those given, but then noticed the 0 in set D and realized that this set matches. She said that the set of suits in a deck of cards matches set C because set C contains four objects, and that the set of all aardvarks enrolled at M. S. U. matches none of the given sets because it is empty and none of the latter is.

Student P made two errors on the problems on bases. On the first problem, she wrote as a translation to base ten of 405_{eleven} the sum $44 + 11 + 5$, or 60_{ten} . Apparently she read the 4 as meaning four elevens, but the source of the 11 in the sum is not clear. In translating 39_{ten} to base two, she revealed that she did not know how to set up column values properly and that she was unaware of which digits are allowed in base two. She did this by writing 39_{ten} as 431_{two} , representing 4 eights, 3 twos, and 1 one. However, in the next problem, she correctly wrote 44_{ten} as 62_{seven} . Student P also solved both missing-base problems correctly. She concluded that the base of the

first example was twelve by reasoning that $9 + 7 = _4$ could happen only if the base were twelve. In the second, she reasoned that to have $11 - 2 = 2$, 11 had to be four, implying that the base was three; she corroborated this by working out the rest of the example.

On Problem 1, Student P did all her work mentally. She first added the pennies, exchanging their sum for 1 nickel and 1 penny. She then added this nickel to her other nickels and obtained 8 nickels, which she exchanged for 1 quarter and 3 nickels. Finally, she summed the quarters and gave as her answer 4 quarters, 3 nickels, and 1 penny.

Student P said at the end of the interview that she had been comfortable in the interview situation.

February 7. At this time, Student P said of the course, "It's still easy." She said that the lectures "were relevant. A couple of times I thought she . . . did it like we would try to do if we were going to teach it. . . . But still it's like we're the kids and she's the teacher." Student P said the lecturer was still teaching "pretty much" as one would to elementary school students. She said that the lecturer was discussing more homework problems than she had been earlier; while this made no difference to Student P, she thought that it "might be good for some people . . . it's reassuring." She said she had seen no new material yet and had not been confused by

anything. She was not having difficulty although she had not been keeping up to date in her assignments.

Student P said she had not been confused on any of the assigned problems, and reiterated that "I like those that ask how you would teach a kid." She mentioned that she had scored a 99 on the first test, a test which she considered reasonable and fair.

Student P said of the laboratory on computation that it had been "interesting to see all the different ways of multiplication." Regarding clock arithmetic, she commented, "I didn't really enjoy it, but . . . it was good for some of the girls there, though, I guess. . . . I guess I like to be challenged more." Student P was familiar with modular arithmetic from her earlier education. She had found it helpful to set up tables to determine whether a particular system satisfied the group or field properties.

Concerning the pace of the course, Student P said that it "could go faster, for me," although she was not sure about others. She also said, "I really think it's too bad that this course has to be offered as a lecture-type thing. . . . I think . . . it would be a lot more beneficial if it could be small groups, where everybody could give their ideas . . . be more open. I don't like it like this." As an example, she suggested, "Maybe introducing prime numbers, maybe ask a student how he

would do it, how he would teach a kid that, and then the rest of the students in the class could offer criticism, or offer help. . ."

Considering Problem 2, Student P first asked, "What concept is this supposed to use?" The investigator replied that he would prefer not to say and was only interested in how she thought about the problem. Student P then said, "There'd probably be 9,999 different ones for that . . . wait--" Unsure of this count, she continued, "Numbers go from 0 to 9--that's ten numbers--this is difficult--I've never thought of this before . . ." After facetiously suggesting writing them all down, she said, "I'm thinking this is like sets. Like say this were a big set, only you have like ten different choices for each space." She continued, "It's going to be a big number--it really is . . . first, there's going to be 0 all the way to 9 here--that's nine [sic] right there--then there's going to be all the way to 19--and those are all going to be different . . ." Finally, she conjectured, "It's probably about 3,000--no, it's probably about 30,000." Asked how she had obtained this answer, she replied after a pause, "Because there would be 10,000 numbers to choose from--3 times. Is that right?" (The investigator declined to answer this question as a matter of policy.) Student P explained that there were 10,000 numbers in the list of four-digit endings from 0000 to 9999, and said, "Either

that or you count the phone book!" However, she remained unconvinced of her answer.

The interview with Student P scheduled for February 21 was first postponed, and then cancelled, due to the subject's illness. The problems presented to the other subjects during this week were presented to Student P on March 7.

March 7. Student P missed about two weeks of the course due to illness, and returned to class on February 25. She said that in the last section of the course, the lecturer had "explained it exceptionally well . . . it came over a lot easier and a lot better, and . . . the students got the point better." She said that the topic of decimals in particular was clearly presented, without any problems or misconceptions arising. She said the lecturer "went step by step in powers of ten," and then to different bases; "she just showed everything . . . she didn't leave anything out or take it for granted that you knew something." Student P said the handout on percent problems was "a good idea." She said of the lecturer, "I think she's really a fantastic professor. She's so willing to help people. I don't see how anybody could have problems with that class." She said nothing had been confusing to her and that the lecturer had "brought everything together at the end of the course."

Concerning the homework, Student P said, "I still like story problems the best, because when you're going to teach little kids you've got to relate to things they can understand, and using story problems usually does that." She had found story problems particularly helpful to her in making up her missed work. She especially had liked the percent problems on the handout and the pictorial representation of operations with rational numbers. Student P sometimes did unassigned problems in addition to those assigned. She said, "I just don't have trouble with [the assignments]."

Discussing the laboratory work, Student P said that the session on the metric system had been "all right." She was familiar with the metric system from previous courses in physics. She thought it was a good idea for prospective teachers to know how to teach the metric system in view of the movement toward conversion. Regarding the geoboard, she said, "That was good. I liked that . . . it shows you, and you do it yourself, how you come up with area and so forth. . . . It's easier than drawing all those things. For kids, it would be something they can see and do." Student P had not yet done the laboratory on ruler-and-compass constructions. Of the session on rational numbers, she remarked, "I like those little rods," and said that she liked the idea of creative attempts at showing multiplication. Student P said that "on the whole,

labs were pretty good. I still wish we could work with kids." She felt that the laboratory was "a reinforcement for what's been going on [in lecture]" and would not recommend that they be given separately as a distinct course.

Student P was then presented with the problem sets which would have been given her at the cancelled interview. The first of these sets was that on prime numbers. Student P did no written work and ended up calling all of the given numbers prime. Regarding 119, she said, "I'd do it by trial and error to see if there's any factors in it or not." She tried as divisors 3 and 9, both of which failed to go in. Noting that 119 did not look like a square, Student P said that she would say it is prime. For 113, she tried as divisors 3, 9, 7, and 11. When all of these failed, she called 113 a prime. She also said 227 is prime; asked her reasoning, she replied, "I went through the numbers . . . like 1, 2, 3 . . . I skipped the evens." Regarding 247, she said, "I think it's a prime. I'm not sure, though." After some further consideration, she concluded that 247 is prime.

In the problems on greatest common factor and least common multiple, Student P in both cases factored each number into primes, making an error in the second problem in the case of 48 (which she factored as $2 \times 3 \times 3 \times 2 \times 2$). She then took the correct combination of factors in each

problem; her factorization error in the second example led to a corresponding error on the least common multiple.

After reading Problem 3, Student P drew arrows to indicate the motion of the two drivers. But after a long pause, she gave up, saying, "I don't know what this problem involves. . . . I really don't."

Student P was then presented with the problems which all of the subjects saw during this week. The first of these was that on rational numbers. Student P solved it by converting the two given fractions to equivalent ones with denominator 12; finding the numerators consecutive integers in this case, she converted both to 24ths and found $7/24$ to be in between.

Student P approached the problem of finding a decimal name for $7/12$ by dividing 7 by 12; she made an arithmetic error in her division and so did not obtain the correct answer. To find a fraction name for the number represented by $.3777 \dots$, Student P called this number r , then wrote $10r = 3.777 \dots$. However, under this, she sloppily wrote $r = .7777 \dots$, so that when she subtracted, she obtained $9r = 3$, leading her to conclude that $r = 3/9 = 1/3$. She noticed nothing unusual about this result. On the next problem, she divided 6 by 11, obtaining a repeating decimal, but did not understand the clause "that has an error less than .00001." Asked to find an approximate value for $\sqrt{7}$, Student P said that she knew

it was between 2 and 3--probably closer to 3. She said that she would find it by using a table of roots or a slide rule. She said that $\sqrt{7}$ is greater than 2.5 because 7 is closer to 9 than to 4.

Presented with the first measurement problem, Student P asked, "Is this the lab I missed?" She later said that she would put a string along the curve and then measure the string. She said that she would solve the second problem by putting a string "on the exact outline of this," then deforming the string into a shape whose area could be measured.

Student P received the grade of 4.0 in the course.

March 28. Student P said there had been no change in her feelings toward mathematics. She still planned to major in mathematics, and said that she still liked it. She also said that there had been no change in her feelings about eventually teaching mathematics. She thought the "course was beneficial . . . it should have been a smaller class, though . . . it was very well organized for being that way." She said there had been no change in her feelings about the purpose of teaching mathematics in elementary school--"I still think it's very important. I don't think my feelings about math will ever change." Student P had experienced no change of feelings about her major; she had wanted to teach since she was in the second

grade, and she still expected to major in mathematics-science within the elementary education program.

Asked what features of the course might be improved, Student P said, "The labs . . . were too long . . . you had too much time for what was to be done . . . The lab size would have been an ideal class size . . . more discussion, more ideas from the students . . . a smaller class." She said the homework had been "fine."

Student P said that her favorite laboratory sessions had been those on the geoboard and on Dienes Blocks. The session she called the worst was that on the metric system. Asked if anything in the course presentation had hindered her learning, Student P replied, "I could have gone a lot farther, but I wouldn't say it hindered me. I wasn't bored or anything, I just didn't have much to do. . . I like to work with math, so I think we could have got more done."

Student P expected to take more mathematics courses--either calculus or additional elementary education courses.

Regarding the research study, Student P said, "I don't know . . . it's meaningful, if you use the information . . . a good idea." She could not suggest any improvements in the study, commenting, "I think it was pretty good." She liked the idea of choosing study subjects randomly from volunteers. Student P said that if

she were running the study, her questions would be "basically the same kind." However, she suggested additional questions in the form "if you were teaching this class" She could suggest no other potential areas of investigation.

Asked whether elementary school mathematics should be taught by specialists or by general classroom teachers, Student P replied, "I'd say math specialists . . . if you just took the one class [Math 201] you really wouldn't know it that well. You'd tend to skip over things . . . if you couldn't explain them, but if you had a specialist . . . you know it." She compared mathematics to a subject like band or choir, where a specialist must be the instructor.

Evaluation. Student P entered Math 201 with a strong background in mathematics. She derived very little from the lectures as all of the material was familiar to her. Fortunately, she generally enjoyed the laboratory work. It is this writer's judgement that an honors section of Math 201 would be beneficial to students such as Student P.

Summary of Types of Problem Solutions

In this section we shall consider each problem individually and examine the attempts of the fifteen subjects to solve the problem.

Sets. The problem on sets presented to the subjects was:

Let $A = \{\text{Kennedy, Johnson, Nixon}\}$

$B = \{0\}$

$C = \{\square, \Delta, 0, *\}$

$D = \{0, 1, 2, 3, \dots, 99\}$

$E = \{0, 1, 2, 3, \dots\}$

For each of the following sets, which of the sets above, if any, does it match?

the set of all living people

the set of counting numbers from 1 to 100, inclusive

the set of suits in a standard deck of cards

the set of all aardvarks enrolled at M. S. U.

Considering the set of all living people, ten subjects said that it matches none of the given sets, some of these using the expression "the empty set" to mean "none of the above." Student P was not sure if the set is finite or infinite; she said that if it is a finite set, it matches none of those given. Student L meant "none of the above," thought the empty set, and said set B, which she thought to be the empty set. Students I, B, and C thought the set of all living people is infinite and matches set E. A remark made very often by the subjects in considering this set was that Kennedy and Johnson are dead; as in the other examples, this was an indication of their confusion of the concepts of matching and equality.

Five subjects said that the set of all counting numbers from 1 to 100, inclusive, matches set D; they were students I, K (after an error), P, D, and F. Four subjects--Students L, M, C, and G--said that this set matches set E; at least one of these (Student M) may have

been led by the word "inclusive" to look for a set including the given set. The six other subjects said none of the above; three of these (Students J, N, and H) remarked that set D goes only to 99, but either did not notice the 0 in set D or were confused between matching and equality.

Set C was seen to match the set of suits in a standard deck of cards by ten subjects, but five (Students L, M, N, A, and G) said none of the above because they did not see the suits among the given sets. As before, Student L said set B to mean none of the above.

Considering the set of all aardvarks enrolled at M.S.U., Student J was not sure if it matches set B. All of the others said it matches none of the above, some because they saw no aardvarks in the given sets. (Student L, as before, said "set B" for "none of the above.")

It was clear from this discussion that two misconceptions were common among the students. One was the confusion between the concepts of matching (equivalence) and equality. The other was misuse of the expression "the empty set."

Number Bases. The first problem in number bases was:

Give the base ten name for the number expressed by 405_{eleven} .

Twelve of the fifteen subjects correctly formulated this number as $4 \times 121 + 5 = 489_{\text{ten}}$. Of the remaining three, Student I mistakenly used 132 as 11^2 , Student P took the number to mean $44 + 11 + 5 = 60_{\text{ten}}$, and Student F attempted to translate 405_{ten} into base eleven (her result was incorrect).

The subjects were then asked to perform the reverse translation:

Give the name of each number in the indicated base.

39_{ten} in base two

44_{ten} in base seven

Six of them (Students B, C, I, J, K, and L) correctly solved both examples by properly labeling columns in the requested base, then successively subtracting multiples of these values from the given number, as in: 44 has 6 sevens, $44 - 42 = 2$, so $44_{\text{ten}} = 62_{\text{seven}}$. Three other subjects (Students D, M, and O) also correctly solved both examples, but by division rather than subtraction.

Students D and O divided the original number and its successive partial quotients by the base, placing remainders in the columns from right to left. Student M did this in the second example, but in the first she divided by 32, 16, 8, etc., and when the divisor went in she put a 1 in the appropriate column. Student H solved the second example correctly, and followed a generally correct procedure in the first, but left out the eights column.

Three subjects (Students G, N, and P) correctly solved the second example by means of successive subtraction, but failed to solve the base two example.

Student P said that 39_{ten} consists of 4 eights, 3 twos, and 1 one, so is 431_{two} . Student N drew a blank, while Student G broke 39_{ten} into 3 tens and 9 ones. She wrote the 3 tens as 1 sixteen, 1 eight, 1 four, and 1 two, but was then unsure about what to do with the 9 ones. She finally wrote 111111_{two} , but said she knew that this was not correct.

Two subjects could not solve either example.

Student A said that 39_{ten} becomes 3 twos and 9 ones in base two, which is fifteen, so that $39_{\text{ten}} = 15_{\text{two}}$; similarly $44_{\text{ten}} = 32_{\text{seven}}$. Student F did this in the base seven example, but in the base two she said 39 becomes 3 twos and 9 ones, which is 6 and 9 ones, or 69_{two} .

Finally the subjects were asked to recognize a missing base:

The following examples are correct in some base. Name the base that makes each example correct.

$\begin{array}{r} 49 \\ +37 \\ \hline 84 \end{array}$	$\begin{array}{r} 211 \\ -12 \\ \hline 122 \end{array}$
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Only one subject, Student G, was unable to recognize base twelve in the first example; she added 49 and 37 in base ten, obtained 86, and then attempted to find a base b in which $84_b = 86_{\text{ten}}$. Two subjects (Students J and M) used the size of the digits as a clue that the base was large.

In the second example, Student G attempted to solve it as she had attempted to do the first one. Student N drew a blank on this problem; Student A, after borrowing a one, tried to find a base in which eleven minus two is two. The other subjects all found base three as the solution. Students C and I inverted the problem to an addition problem to do this. Students D, F, I, and L said the size of the digits was a clue to them that the base was small.

Since the subjects' greatest difficulty was in writing numbers in nondecimal bases, they should be given more practice in this. Perhaps both methods of conversion (by subtraction and by division) should be taught.

Problem 1. The last sheet presented to the subjects at the first interview contained Problem 1, which was:

Suppose the only U. S. coins were quarters, nickels, and pennies. If I have 1 quarter, 3 nickels, and 3 pennies, and you have 2 quarters, 4 nickels, and 3 pennies, what is the least number of coins which expresses the total amount of money between us?

The most common method of solution of this problem was to add within coins, obtaining 3 quarters, 7 nickels, and 6 pennies, and then exchange for fewer coins. Nine subjects solved the problem essentially in this way; of these, only Student C perceived it as a base five problem and performed base five addition explicitly. Students M and O were the only other subjects to note a resemblance to base five arithmetic. Students D, G, I, J, K, P, and B

also solved the problem in this way. Students F and N first computed each person's money, added these, then converted to coins. Students H, L, and A read the problem as calling for subtraction, while Student O, after obtaining 3 quarters, 7 nickels, and 6 pennies, read this as \$3.76, which she then expressed in the required coins.

Problem 2. At the second interview session the only problem presented to the subjects was Problem 2:

In East Lansing, a telephone number can begin with 332-, 337-, or 351-. How many different phone numbers can there be in East Lansing?

Only two subjects were able to obtain the correct answer to this question, and of these, Student P was unsure of her answer and Student K made a false start before computing it correctly. Student F came close to a correct answer, counting 9,999 possible suffixes and multiplying this number by 3.

Several subjects knew the correct answer was three times the number of suffixes, but could not enumerate them. Students M, G, H, and I had no idea how to enumerate these, while Students N, O, C, and D used various incorrect formulae to obtain an answer.

Students L and A said that there are infinitely many possibilities. Student J had no idea how to begin to attack it. Student B looked in her book to find an appropriate formula; when she couldn't find one she said

that if this were a homework problem she would look up the answer and try to figure out how it had been obtained.

Prime Numbers. The first problem set presented at the third interview was that on prime numbers:

For each of the following numbers, tell whether it is a prime number or not.

119

113

227

247

All of the subjects correctly named 113 and 227 as primes, although the limit of their persistence in attempting divisors for 227 ranged from 7 (Student G) to 23 (Student B), while some subjects (Students F and H) pronounced three numbers prime by inspection.

Ten subjects found the factorization of 119; one, Student C, mistakenly found 3 to divide it and called it composite. Students P, A, G, and H said that 119 is prime; these subjects either failed to try 7 as a divisor (Students P and G), did not carry out the test far enough (Student H), or made an error along the way (Student A).

The subjects had less patience with 247, with only five of them (Students J, K, N, B, and D) finding that it is composite. The rest called it prime. At least three of these (Students I, M, and C) tried divisors through thirteen or greater but failed to note that 13 is a factor. The others either stopped trying short of 13 or used some inspection rationale to call it prime.

Some of the failures to recognize composite numbers may have been due to subjects' frustration with repeated divisibility tests. Nevertheless, this investigator would recommend that Math 201 students see larger as well as smaller primes in their experience with this topic.

Factors and Multiples. The subjects were then presented with the following two problems:

Find the greatest common factor of 63 and 105.

Find the least common multiple of 42 and 48.

Nine subjects did both examples correctly, factoring the numbers into primes and taking the appropriate combination of factors in each case. The other six had a variety of difficulties. Student J took the LCM correctly, but for the GCF she took the product of the two numbers, sort of a "greatest common multiple." Student G did the reverse of this, correctly finding the GCF but taking the "least common factor," 2, as the LCM. Student B took the LCM when asked for the GCF and vice versa. Student F did not know how to find factors of a number, and attempted to find the LCM by listing multiples of the given numbers and comparing the lists. Students O and P took the GCF correctly, but misplaced a factor of 2 when taking the LCM.

These concepts were included in the study because students in the previous term's study said they had had

difficulty with them. A possible remedy for this will be suggested in the section on recommendations.

Problem 3. The last problem presented to the subjects at their third interview was Problem 3:

Driving east at 40 mph, Dan passed through the center of town at 12 noon. At 1 o'clock, Dick, driving in the same direction, passed him at 50 mph. If both drivers had maintained their speeds, how far was Dick from the center of town at 9 A.M.?

Only two subjects, Students C and K, obtained the correct answer to this problem. Two others came close-- Student D, who mistakenly took Dick's position at 1 o'clock to be 80 miles east of town, and Student I, who in taking the problem to mean 9 A.M. the next day, miscounted the number of hours between 1 P.M. and 9 A.M.

Two subjects, Students G and L, said they couldn't finish the problem because they did not know the drivers' location at 1 o'clock. Student M was able to determine Dan's location at 9 A.M. but was unable to proceed to relate his movement to that of Dick. Students O and H gave an answer in terms of units of time--hours or minutes. Student A drew some arrows representing the drivers' movement but was unable to proceed. Student B guessed at 150 miles, from 3 hours at 50 mph, and said that she did not think this was correct. Student F drew a complete blank and said that she could not even attempt to solve the problem.

There was an amusing contrast between two other responses. Student N, after noting that it is four hours from 9 A.M. to 1 P.M., said she couldn't proceed further because she had never been taught how to set up a story problem. Student J, on the other hand, had been taught only too well how to set up a story problem; she routinely wrote the formula $D = r \times t$ and attempted to set up a table involving the movement of the two drivers, putting in unknowns and known values. When this device failed, she was forced to give up on the problem.

The subjects' nearly universal failure to solve this simple problem leads this investigator to recommend that a portion of the course be devoted to discussion of and practice in the solving of verbal problems.

Rational Numbers. The first problem presented to the subjects at the final interview was that on rational numbers:

Is there a rational number between $1/3$ and $1/4$?
If so, name one.

Most subjects obtained a correct solution to this problem; a variety of methods was used. The most common method was to express both numbers as equivalent fractions with common denominator 12; when the fractions were found to have consecutive numerators, the subjects converted them to 24ths and found $7/24$ in between. This method was used by Students G, K, L, P, and D. Student I said a solution

would be $3\frac{1}{2}/12$; she then doubled numerator and denominator to obtain $7/24$. Student B converted to 36ths and got $11/36$.

Students O and H converted the given numbers to equivalent fractions with a common numerator, then took a fraction with this numerator and an intermediate denominator. Student N expressed both fractions as decimals and named .26 as an intermediate value.

Student C found the value $7/24$, but she said this is not a rational number. This misconception was also held by Student A, who suggested the reciprocal of $3\frac{1}{2}$, but was not sure whether this is rational. Apparently both these subjects had the idea that a rational number is a fraction with numerator 1.

Student M also suggested the reciprocal of $3\frac{1}{2}$ (she actually said " $1\frac{1}{2}$ over 3," which the interviewer took to be a misstatement of 1 over $3\frac{1}{2}$), but could not express this as a fraction. Student J tried a variety of manipulations on the given numbers without success. Student F said that she knew there is one, as she had learned this in class as a fact, but could not find any explicitly.

Nearly all the subjects had learned as a fact that between any two rational numbers there is another rational number. One recommendation made here is that this fact be retaught in the unit on decimals; this would reinforce

students' mastery of the idea and illustrate the power of decimal representation.

Decimals. Following the single problem on rational numbers, the subjects were asked to solve four problems involving decimals. The first two of these were similar to material that had been assigned in their classwork, while the last two were of a type the subjects had not encountered in the course. The problems were:

Find a decimal name for the number represented by $7/12$.

Find a fraction name for the number represented by .3777777. . . .

Find a terminating decimal approximation to $6/11$ that has an error less than .00001.

Find an approximate value for $\sqrt{7}$.

On the first problem, twelve subjects obtained $.58\bar{3}$ by long division. (Student H remarked that she had first learned to do this in this course.) Student P did the division but made an arithmetic error, while Student F stopped after two decimal places, uncertain of how to proceed. Only Student L did not use division. Her first reaction was to say it was .712; realizing this was wrong, she then attempted to solve the equation $7/12 = x/100$, and failed in this also.

For the reverse problem, all but two subjects used some variety of $100r - 10r$, $10r - r$, etc. There was a general feeling that it is not right to subtract $10r$ from

100r (the subtrahend had to be r), but several subjects, while expressing this reservation, went ahead and did this anyway, obtaining the correct answer. Many subjects made arithmetic errors while carrying out this procedure, such as writing $100r - 10r = 10r$ (Student J), $100r - 10r = 99r$ (Student A), $r = .7777 \dots$ (Student P), or correctly setting up the problem and then not cancelling the repeating 7's (Student M). Only two students did not use this technique--Student F, who had no idea how to approach the problem, and Student N, who said that she would try to find the fraction name by repeated trial-and-error divisions until one gave that decimal expansion. In view of the great number of arithmetic errors made by the subjects in this process, this writer would recommend not only that it be taught carefully, but also that students be urged to check their results by division so that they will be alerted if a mistake has been made.

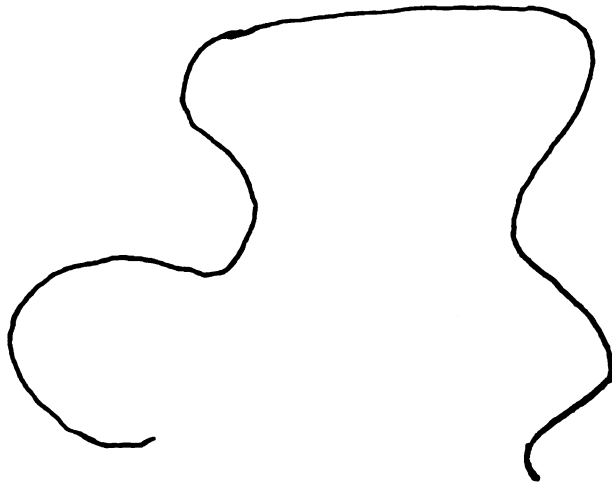
The last two problems on decimals were of a type which the subjects had not encountered in the course due to time limitations. The investigator decided to present these problems to the subjects anyway to see how they thought about them. On the third problem, only Student D correctly divided 6 by 11 and stopped after reaching the fifth decimal place. Student O said that she would do this, but did not actually carry it out, while Student I carried it out but made an error along the way. Ten other subjects did some

division but were not sure how to answer the question. Students F and L said that they had no idea how to solve the problem.

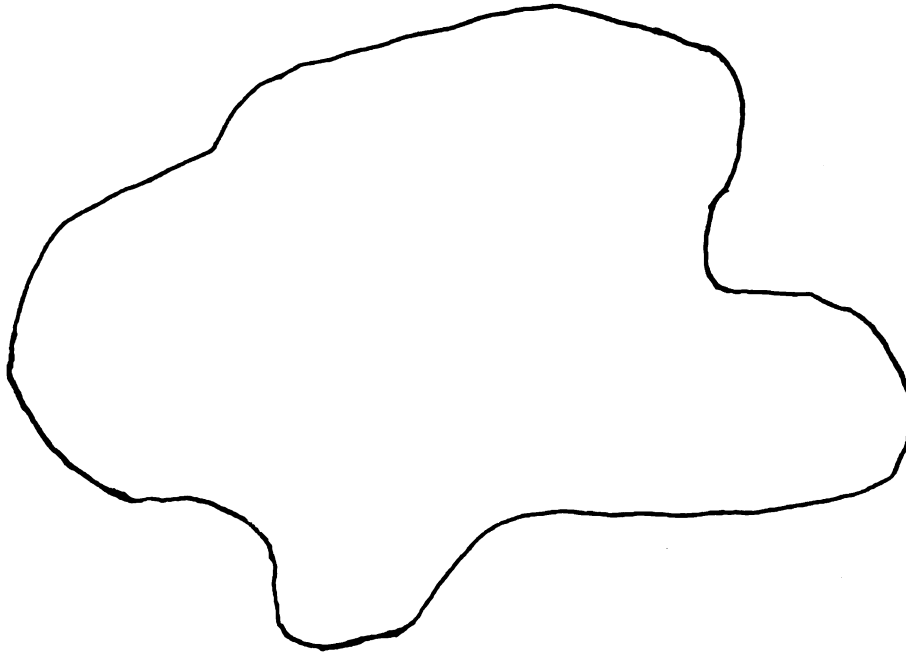
Ten subjects were able to answer in some correct form the problem of approximating $\sqrt{7}$. Students I, K, M, O, and D used trial and error, squaring numbers between 2 and 3, finally arriving at a two-decimal-place estimate between 2.6 and 2.7. Students G, H, J, and P were less accurate; they noted that the value was between 2 and 3, but could not estimate much beyond this (Student P did find that $\sqrt{7}$ was greater than 2.5; Student H gave 2.5 as her estimate and said that for greater accuracy she would use her calculator). Student C executed the square root algorithm to obtain an approximate value. Of the five subjects who could not give an approximation, Students L and A could think of only the Pythagorean theorem as a device for working with square roots. (Student J mentioned this also.) Student N said that $\sqrt{7}$ was half of seven, while Students F and B said that they had no idea of how to solve the problem.

Measurement. Although the subjects had not studied measurement in a systematic way, the investigator presented them with two problems. The subjects were told to imagine they had any materials they needed to solve the problems, and they were requested to state how they would use these materials to obtain a solution. The problems were:

Find, as best you can, a measure of the length of the curve below. (Ruler and compass provided.)



Find, as best you can, a measure of the area of the region below. (Ruler and compass provided.)



Twelve subjects, some of whom also mentioned other ideas, said they would obtain a measure of the length of the curve in the first problem by superimposing a string over the curve, then removing it and measuring its length with a ruler. Of the others, Student J talked vaguely

about stretching it out without mentioning the use of string, while Students L and F said they had no idea how to solve it.

Responses to the second problem were more varied. Seven subjects (Students K, O, B, C, F, G, and I) as their main response suggested approximating the area with a circle or rectangle contained in it or surrounding it. Three (Students L, M, and J) suggested superimposing a grid and counting the square units covered. Five others (Students N, P, A, D, and H) suggested superimposing a string on the outline of the figure, then deforming the shape into a circle or rectangle, whose area could be measured. While these were the primary responses of the subjects, several of them had more than one idea.

While time limitations prevented it in this instance, this writer recommends that the nature of measurement be treated in greater depth in the course.

CHAPTER 5

DISCUSSION AND RECOMMENDATIONS

In this chapter, the data reported in Chapter 4 will be examined and conclusions will be drawn regarding the mathematical understandings and misconceptions of the subjects. The overall effect of the course on individual subjects will be assessed, and recommendations will be made for improvement of instruction in Mathematics 201, with respect to both the pedagogy of specific mathematical topics and the general format of the course. These will be followed by an evaluation of the method used in the study and suggestions for further research.

Table 2 on pages 324-327 lists the problems presented to the study subjects followed by a listing of those subjects whose responses can be characterized as correct, correct but for a trivial error, partially conceptually correct, and completely incorrect. Table 3 on page 328 compares the subjects' performances on the problems in the study with their final grades in the course. Those entries in Table 3 which indicate an anomaly between a particular subject's study performance and her course grade will be discussed in the following section in the report on that subject.

Table 2

CHARACTERIZATION OF SUBJECTS' RESPONSES TO PROBLEMS

Correct	Correct but for Trivial Error	Partially Conceptually Correct	Completely Incorrect
Let $A = \{\text{Kennedy, Johnson, Nixon}\}$ $B = \{0\}$ $C = \{\square, \Delta, O, *\}$ $D = \{0, 1, 2, 3, \dots, 99\}$ $E = \{0, 1, 2, 3, \dots\}$			
For each of the following sets, which of the sets above, if any, does it match?			
the set of all living people			
A, D, F*, G*, H, J, K, M, N*, O		L, P	B, C, I
the set of counting numbers from 1 to 100, inclusive			
D, F, I, K, P	H, J	O	A, B, C, G, L, M, N
the set of suits in a standard deck of cards			
B, C, D, F, H, I, J, K, O, P			A, G, L, M, N
the set of all aardvarks enrolled at M. S. U.			
A*, B, C, D, F, G*, H, I, K, M, N*, O, P		L	J
Give the base ten name for the number expressed by 405_{eleven} .			
A, B, C, D, G, H, J, K, L, M, N, O	I		F, P
Give the name of each number in the indicated base.			
39_{ten} in base two			
B, C, D, I, J, K, L, M, O	H	G, N, P	A, F
44_{ten} in base seven			
B, C, D, G, H, I, J, K, L, M, N, O, P			A, F

Table 2 (Cont'd)

Correct	Correct but for Trivial Error	Partially Conceptually Correct	Completely Incorrect
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The following examples are correct in some base. Name the base that makes each example correct.

$$\begin{array}{r} 49 \\ + 37 \\ \hline 84 \end{array}$$

A, B, C, D,
F, H, I, J,
K, L, M, N,
O, P

G

$$\begin{array}{r} 211 \\ - 12 \\ \hline \end{array}$$

B, C, D, F,
H, I, J, K,
L, M, O, P

A

G, N

Suppose the only U. S. coins were quarters, nickels, and pennies. If I have 1 quarter, 3 nickels, and 3 pennies, and you have 2 quarters, 4 nickels, and 3 pennies, what is the least number of coins which expresses the total amount of money between us?

B, C, D, G,
I, J, K, M,
N, P

F

A, H, L, O

In East Lansing, a telephone number can begin with 332-, 337-, 351-. How many different phone numbers can there be in East Lansing?

K, P

F

C, D, G, H,
I, M, N, O

A, B, J, L

For each of the following numbers, tell whether it is a prime number or not.

119

B, D, F, I,
J, K, L, M,
N, O

C

A, G, H, P

113

B, C, D, G,
I, J, K, M,
N, O, P

A, F, L

H

Table 2 (Cont'd)

Correct	Correct but for Trivial Error	Partially Conceptually Correct	Completely Incorrect
---------	-------------------------------------	--------------------------------------	-------------------------

227

B, C, D, I,
J, K, L, M
N, O, P

A, F, G, H

247

B, D, J, K,
N

C

A, F, G, H,
I, L, M, O

P

Find the greatest common factor of 63 and 105.

A, C, D, G,
H, I, K, L,
M, N, O, P

B, F, J

Find the least common multiple of 42 and 48.

A, C, D, H,
I, J, L, M,
N

K, O, P

B, F, G

Driving east at 40 mph, Dan passed through the center of town at 12 noon. At 1 o'clock, Dick, driving in the same direction, passed him at 50 mph. If both drivers had maintained their speeds, how far was Dick from the center of town at 9 A.M.?

C, K

D, I

B, M

A, F, G, H,
J, L, N, O,
P

Is there a rational number between $1/3$ and $1/4$? If so, name one.

B, D, G, H,
I, K, L, N,
O, P

A, C, M

F, J

Find a decimal name for the number represented by $7/12$.

A, B, C, D,
G, H, I, J,
K, M, N, O

P

F

L

Find a fraction name for the number represented by $.37777\dots$

B, C, D, H,
I, O

A, G, J, K,
M, P

F, L, N

Table 2 (Cont'd)

Correct	Correct but for Trivial Error	Partially Conceptually Correct	Completely Incorrect
Find a terminating decimal approximation to $6/11$ that has an error less than .00001.			
D, H, I, O		A, B, C, G, J, K, M, N, P	F, L
Find an approximate value for $\sqrt{7}$.			
C, D, G*, I, J, K, M, O, P	H		A, B, F, L, N
Find, as best you can, a measure of the length of the curve below.			
A, B, C, D, G, H, I, K, M, N, O, P		J	F, L
Find, as best you can, a measure of the area of the region below.			
C, G, J, L, M		B, F, I, K, O	A, D, H, N, P

Notes on Table 2

An asterisk indicates a response which, while correct on the surface, reveals that the subject did not really understand the idea behind the question.

For the last problem (area), the first column contains responses suggesting the use of a grid or geoboard, the third responses suggesting approximation by another figure, and the fourth responses suggesting deformation of the boundary of the region.

Table 3
SUBJECTS' PROBLEM SCORES AND COURSE GRADES

Subject	Problem Score	Course Grade
A	58	3.0
B	90	3.0
C	104	4.0
D	116	4.0
F	57	2.0
G	78	0.0
H	90	4.0
I	109	4.0
J	94	3.0
K	117	4.0
L	63	2.5
M	99	4.0
N	81	4.0
O	102	3.5
P	90	4.0

Notes on Table 3

The Problem Score is derived from Table 2 by assigning 5 points for each entry in the first column, 4 points for each entry in the second column, 2 points for each entry in the third column, and 0 points for each entry in the fourth column.

The correlation between the two columns is .58. The equation for the regression of course grade (y) on problem score (x) is $y = .0332x + .2818$.

The Subjects

In this section the effects of the course on the individual subjects will be reviewed. In particular, their feelings about the various facets of the course presentation, and about mathematics itself, as these feelings were revealed in the interviews, will be discussed. Also, the mathematical understandings and misconceptions which each subject revealed in the interviews will be identified.

Student A. On the information sheet that she filled out at the beginning of the term, Student A said that she liked mathematics and was confident of her ability to teach it eventually. When asked the purpose of elementary school mathematics, she responded vaguely about the need for "a better understanding." This suggests that Student A possibly had a lesser appreciation of the utility of mathematics than did most of the other subjects.

Student A occasionally praised the lectures, but she criticized the text as confusing and as too complicated in its treatment of simple ideas. She was perplexed by most of the laboratory exercises, expressing praise only for Dienes Blocks. Nevertheless, she disapproved of the lecturer's use of Dienes Blocks as a model for arithmetic in nondecimal bases, saying she preferred to think in terms of the base ten equivalents of the nondecimal numerals. This was one indication among several of Student A's strong resistance to new ideas about arithmetic: she usually attempted to work in

patterns she already had formed, and rejected unfamiliar algorithms and procedures.

Student A was the second poorest performer among the subjects on the study problems. She was confused between matching and equality of sets, could not translate a base ten numeral to another base correctly, was sloppy in testing numbers for primeness, and could not solve correctly even the first of the three verbal problems. (Her failure to understand this problem indicated that Student A had difficulty in making mathematical sense out of a verbal statement.)

Despite these difficulties Student A received the respectable grade of 3.0 in the course. The investigator finds this instance the hardest to explain of those subjects whose study performance did not correspond with their course performance. It is impossible without an examination of Student A's class tests to analyze how she revealed on these tests knowledge which she did not show in the interviews.

At Student A's final interview she reported no change in her feelings toward mathematics itself. However, in contrast to her early confidence, she was strongly opposed to the idea of herself eventually teaching mathematics.

It is clear that Student A benefitted not at all from the laboratory sessions; these evoked only confusion and resentment in her, which could have affected her attitude negatively (although, according to her, it did not). Her response to the laboratory was probably a facet of Student A's resistance to new insights into elementary mathematics.

Student B. Student B revealed at her first interview that she always had hated mathematics; during the interviews she repeatedly discoursed at length about the uselessness of all mathematics beyond arithmetic. (Her favorite example of a useless topic was square roots.) Despite this hostility toward "higher" mathematics, she was aware of the importance of arithmetic and proud of her competence in computation.

Student B had general praise for the lectures. About halfway through the course she stopped reading the text, having found it too confusing. Thereafter she depended on lectures to learn the material. Her attitude toward the laboratory changed over the course of the term: at first she found it worthless, resented the presentation of ideas apparently unrelated to the lectures, and thought the period could be used better as a problem-solving and help session for the lectures. After attending several sessions, she realized that the purpose of the laboratory was to expose the students to new material. The only session which Student B found helpful was that on the geoboard. At the final interview she said that the laboratory sessions were a useful device for introducing new material; her only suggested change was that they be shortened to one hour.

Student B did well on the study problems involving computation, easily solving all the base problems and identifying the primes and composites without difficulty. She also displayed competence with fractions and decimals on those problems which were based on material covered in the course.

(The last two decimal problems were not.) She had some trouble with the set exercises and revealed a lack of understanding of the meaning of least common multiple and greatest common factor by taking each one when asked for the other. Her performance on the verbal problems indicated some ability to translate such problems into mathematical terms, though not necessarily the ability to perform the operations required to solve such problems. Her grade of 3.0 came as no surprise after this performance.

After completing the course, Student B reported that her dislike of mathematics was as intense as before. She had found Math 201 to be similar to the mathematics courses she had taken before and hated. She was somewhat disturbed because she was aware that as a teacher she would transmit her attitude to her pupils, but she accepted this as inevitable.

Student B considered the mathematical content of Math 201 to be overly formal and unnecessary to explain the meanings of arithmetic, which she considered to be self-evident. (For example, rather than take the union of a set of three elements with a disjoint set of five elements to illustrate three plus five, Student B would illustrate with apples or pieces of candy.) It appeared to the investigator that Student B benefitted very little from the course with respect either to understanding or to attitude. This experience was similar to that of Student L, another subject with an intense dislike of mathematics. An alternative treatment should be available to such a student.

Student C. Student C was a student who liked mathematics. In high school she had taken an independent study course in calculus, and in the fall term of 1973 she had taken a precalculus course at Michigan State. (She was the only subject with any previous college mathematics.) Her first response to the question on the purpose of elementary school mathematics was to say that its purpose is to prepare the student for higher studies in mathematics; only as an afterthought did she mention the utility of mathematics in daily life.

Like some other subjects with a strong mathematics background, Student C was bored by the lectures throughout the course. Although not bored by the laboratory, she frequently found the exercises assigned there trivial; she was surprised that other students had difficulty in the laboratory. Student C did not express a positive opinion of any of the laboratory sessions.

Her problem-solving behavior in the study suggests that Student C had not thoroughly mastered the mathematics she had studied. (This tendency had been indicated the previous term when she received the mediocre grade of 2.5 in a course which covered material she had already seen. She explained this by saying she had picked up some misunderstandings in her earlier studies.) In the study problems Student C displayed some misunderstandings about sets, was careless in her arithmetic when testing numbers for primeness, and said that $7/24$ is not a rational number. She correctly

solved all the base problems, found a least common multiple and a greatest common factor, and converted between a fraction and a repeating decimal. She was the only subject to use base five arithmetic to solve the first verbal problem, and one of only two who correctly solved the third problem. While she failed to solve the second problem correctly, she had enough intuitive understanding of the problem to realize that her answer was not correct. Despite her difficulties in the interviews, she had sufficient skill to earn a 4.0 in the course.

After completing the course, Student C reported no change in her feelings about mathematics or its purpose. In fact, she was more determined to teach it than she had been before--in order to demonstrate that it could be made more interesting than it had been made in Math 201.

Student C had studied too much mathematics to be interested in the mathematical content of Math 201. The course was of little benefit to her. Her attitude toward the laboratory classes indicates that she would have found an all-manipulative approach equally uninteresting. Such a student either should be permitted to waive the course or given a course whose level of difficulty is proportionate to her mathematical skill.

Student D. Student D had the most positive attitude toward mathematics of any of the subjects. She said that she loved mathematics because it provided a challenge. At the start of the course she was not sure whether she would major

in elementary education or in computer science. She was fairly confident of her ability to teach mathematics.

Student D generally enjoyed the lectures, but found them too repetitive. She said that she enjoyed seeing why the procedures of arithmetic work, and, as the course progressed, she reported that she was enjoying it more and more as the material became more challenging. Student D enjoyed the laboratory experience more than any of the other subjects: even at her initial interview, when the course itself had not yet been discussed, she spontaneously remarked that she had loved the first laboratory session, that on attribute games. Later in the course she said that she found the labs very good--a learning experience which also was fun. In addition to attribute games, she had specific praise for the sessions on Dienes Blocks (though she found division with the Blocks confusing), the geoboard, GeoBlocks, and real numbers (particularly, discovering the value of π). She had some difficulty with the session on computation and with Cuisenaire rods.

Student D ranked second among the subjects in her performance on the study problems: her only significant failures were on the second verbal problem, where she obtained a ridiculously large answer, and on the second measurement problem, where she suggested deforming the figure into a circle to find its area. Elsewhere, she displayed a mastery of the material covered in the problems that naturally corresponded with her grade of 4.0 in the course.

At the end of the course Student D said that her warm feelings toward mathematics had not changed. She said that she now understood why mathematical procedures worked, whereas before they had been "just rules" to her. While still somewhat afraid, Student D said that she was now more confident and more enthusiastic about eventually teaching mathematics. She still felt that mathematics is essential in daily life. Student D had changed her feelings ~~about~~ her major: as a result of taking a course in computer science, she had decided to drop that program in favor of elementary education.

Student D enjoyed the course and benefitted from it, because she was enthusiastic about mathematics but not over-prepared for the course. Her experience could be taken as evidence that the present format is acceptable for this type of student; however, Student D had such overflowing enthusiasm for mathematics that she probably would have enjoyed the course in any conceivable format.

Student F. Student F was one of two subjects who entered the course with a very weak mathematics background. She was disturbed that the course required prior knowledge which she did not possess. She reported on her information sheet that she strongly disliked mathematics. Although she could compute competently, she had no understanding of the meaning of arithmetic. Student F was very disturbed by the idea of teaching mathematics. She said that the reason for teaching elementary school mathematics was its utility in daily life.

Student F remarked several times during the study that she could not understand the book's explanations. When reading it, she would try to cover only the important statements. She frequently looked at the answers for help in solving the problems. She said that she did not understand the problems until after they had been explained in lecture; she would not even try to solve them until she had heard the solution. Student F generally liked the lectures, although she sometimes found the explanations insufficient. She had a low opinion of the laboratory sessions, with a few exceptions; she said that she had learned a lot from the session on the geoboard and had enjoyed working with GeoBlocks. At the end of the course she recalled clock arithmetic as a good exercise; this contradicted her opinion expressed earlier in the term. She found all of the other laboratory sessions useless.

Student F compiled the worst record of any subject on the study problems. She was the only subject who did not factor numbers in order to find their greatest common factor or least common multiple, instead trying to list their factors or multiples. She was inept at working with bases and failed to solve any of the problems involving rationals and decimals. She did all her arithmetic mentally in testing numbers for primeness, resulting in some sloppy work. On the meager positive side, she understood the idea of matching of sets (though she still answered one item incorrectly), understood the first verbal problem, and came very close to

solving the second verbal problem, which only two subjects solved correctly. Student F's problem-solving behavior may not have corresponded with her mathematical knowledge for two reasons. One was the subject's nervousness in the problem-solving situation. Another was Student F's tendency (possibly because of this nervousness) to underrate her own mathematical ability. On occasion, she would claim to be completely unable to solve a problem, only to make some progress after the interviewer had recapitulated the known facts about the problem (but had not given any hints for solution). A perusal of Student F's problem attempts might lead one to think that she would do quite poorly in the course, perhaps even fail. Yet Student F's course grade of 2.0 is actually below the prediction that is given by a regression equation based on Table 3! (That is, based on all fifteen subjects, a student with Student F's problem-solving record would tend to receive a grade above 2.0.) On the basis of this analysis, Student F's course grade appears fairly consistent with her problem-solving behavior.

At the end of the course, Student F reported that she now felt more capable in mathematics than she had felt at the start. She essentially had used Math 201 as a remedial skills course; as a result of the course she could do more types of arithmetic problems (such as percent) which she could not have done before. Despite this, she sensed no improvement in her understanding of arithmetic and continued to dread the idea of teaching it. She also said that at

this point she did not like mathematics any more than she had earlier.

Student G. At her initial interview Student G said that she "really" disliked mathematics. She said that she remembered none of the mathematics she had studied previously; however, she thought that she could teach mathematics capably at the grade-one or grade-two level, but not higher. At the final interview, her feelings on all these issues were unchanged.

Student G several times expressed a positive opinion of the lectures. She made only one comment on the text, remarking that it was sometimes confusing. She later said that she used her lecture notes, rather than the book, for reference. Most of Student G's comments on the laboratory were negative. She was unfortunate enough to find herself in a laboratory section with an instructor who could not communicate with her and with a group of students who worked on the lesson individually, rather than as a group. She would not have thought this unusual if she had not done the geoboard exercise in a different section and found that she learned the material better when she worked on it with other students. The only laboratory exercises which Student G enjoyed were those on the geoboard and on Cuisenaire rods, which she found useful for teaching. At the end of the course she said that the laboratory had been a waste of time and recommended that it be a shorter period used to review the lecture material.

Student G's failure in the course would not have been anticipated by her problem solving behavior in the study. This was the greatest such anomaly among the subjects. Student G performed better on the problems than did three other subjects (who received a 2.0, 2.5, and 3.0 in the course) and did nearly as well as a subject who received a 4.0. The anomalous position of Student G is further emphasized by the fact that the correlation between scores and course grades in Table 3 is .58, but when Student G is excluded it rises to .76. Although her problem score was inflated somewhat by credit for correctly answering items she clearly did not understand (some of the items on sets), Student G certainly did not display any less understanding of the problems than did Students A or F. She solved some of the base problems correctly, and seemed to understand those problems on rational numbers and decimals which were based on material covered in the course. She was sloppy in working with primes and did not know the meaning of least common multiple, although she found a greatest common factor correctly. She did not understand the idea of matching sets. Student G expressed great hostility toward all of the verbal problems, although she solved the first one correctly. She reiterated these sentiments in discussing the topic of percent, the treatment of which, of course, featured verbal problems.

It is this writer's opinion, based on remarks made by Student G, that she failed the course because of her

incompetence in the processes of arithmetic. She remarked after the first test that she had done poorly because of arithmetic mistakes. After the second test she said that she had again done poorly because she had concentrated on practicing arithmetic instead of on studying the content of Math 201. (Student G probably was a poor test-taker.)

Student G was similar to Student F in her background and attitude, but Student F had the advantage of being able to compute capably, allowing her to concentrate on the substance of the course. It is clear from the interviews that Student G was capable of passing Math 201; unfortunately, there was too much material for her to master in one quarter.

Student H. Student H said on her information sheet at the start of the course that she liked mathematics very much. She felt, however, that she had retained little of the mathematics she had learned in high school. She thought that elementary school mathematics is essential in daily life, and she was confident of her ability to teach it one day.

Student H's opinion of the lectures changed over the course of the study. At first, when all of the material was familiar to her, she found them repetitive and boring. She had a positive opinion of the lectures during the middle of the term when the material was more challenging, but by the end of the course she was saying they were inadequate to explain the material to her. (She required a long study session with her husband to understand fractions and

decimals.) Her opinion of the text similarly declined throughout the term; she felt that it overused literal expressions, which she would ignore in favor of numerical examples. She also criticized the text for including only numerical answers, rather than worked-out solutions, to the problems. At the end of the course, Student H characterized her laboratory experience as a waste of time. She said that some of the sessions had been "nice," but that none had helped her with the material of the course. Among those she had enjoyed were the second session on Dienes blocks (involving multiplication and division), the session on computation, the session on rational numbers, and the session on the geoboard (which she called good for children and "extremely helpful"--a contradiction to the opinion she expressed later).

Student H's performance on the study problems was not consistently good. She seemed to understand the idea of matching sets and the ideas of nondecimal bases. She was very sloppy in testing numbers for primeness and even used inspection rationales here, but found the greatest common factor and least common multiple of a pair of numbers without difficulty. She performed well on the rational number and decimal problems, including those based on material not covered in the course. Student H was very weak in translating a verbal statement into mathematical terms: she read the first verbal problem as calling for subtraction, and made a variety of fruitless attempts to solve the third. She also quickly gave up on simple problems involving

unfamiliar terms, such as the game of gin rummy and measurement in centimeters. However, her weakness in verbal problem solving was not reflected in her course tests, and she received the grade of 4.0 in the course.

After the course was over, Student H said that she had enjoyed it, as she had enjoyed mathematics courses in the past. She had found it fun to work the problems presented in the course. Her perception of the value of mathematics had not changed. She did not yet feel prepared to teach, saying she still needed a methods course.

One would hope that a student as weak in problem solving as Student H would not emerge from Math 201 with the highest possible grade. The experience of Student H suggests that there is a need for more verbal problem solving in Math 201.

Student I. Student I had one of the strongest mathematics backgrounds among the subjects. She liked mathematics a great deal, more for its intellectual challenge than for its substance. She had taken much of it in high school and thought she had learned it "extremely well." She said that elementary school mathematics is necessary to get along in the world.

Student I characterized Math 201 a number of times as "a good refresher." She found the lectures dull and boring; although presented clearly, they were too slow in covering familiar material. At the end of the course Student I said that she had attended the lectures only because of her

participation in the research study. In contrast to most other subjects, Student I liked the textbook. It was not overly symbolic for a student with her mathematics background. She said that the book and its problems were "fantastic" and completely sufficient for her to understand the material--lectures were unnecessary. Student I praised several of the laboratory sessions, including those on Dienes Blocks, Cuisenaire rods, tangrams, the geoboard, the metric system, and real numbers, but she disliked attribute games and Geo-Blocks. Despite her enjoyment of most of the laboratory sessions, she recommended that attendance at them be optional; she herself would have preferred not to attend.

Student I ranked third among the subjects in solving the study problems, displaying a mastery of the material consistent with her grade of 4.0 in the course. Among her few difficulties were her belief that the set of all living people is infinite, her inability to solve the second verbal problem, and careless work in testing 247 for primeness.

At the end of the course Student I said that it had not affected her feelings toward mathematics because she had not considered it a mathematics course. She did not expect ever to teach this material (her major was special education) and did not want to. She still felt that elementary school mathematics is necessary in everyday life.

Student I, like Student C, was mathematically over-prepared for this course. Necessarily aimed at the student

with less mathematical sophistication, it was largely a waste of her time.

Student J. Student J reported at the start of the course that she had a neutral attitude toward mathematics. She had enjoyed some of her high school mathematics courses, and felt that the study of mathematics helps one to think logically. (She herself had a tendency to assume that mathematical ideas apply in situations where they in fact do not.) Student J had not yet decided whether she would major in elementary education or in communications. She was afraid to teach mathematics, feeling inadequate for the job.

Student J's expressed opinions of the lectures fluctuated throughout the term between disparagement and approval. At first she called the lecturer's style "a good style for slower people"; later in the term she found herself inexplicably bored in lecture, although she did not blame this on the lecturer; at other times during the term she enjoyed the lectures and found them helpful. She found the book too wordy and said that she would prefer a book which contained only the important ideas of the course followed by numerical examples. Student J had a generally favorable opinion of the laboratory sessions. She especially liked those on Dienes Blocks, GeoBlocks, and the geoboard, and also spoke favorably of those on attribute games, computation, clock arithmetic, Cuisenaire rods, and the metric system.

Student J performed well on the study problems, outscoring three subjects who earned a 4.0 in the course. She

demonstrated an understanding of the idea of matching sets (but misused the expression "the empty set"), solved all but one of the base problems easily (and also solved that one after some difficulty), correctly identified primes and composites, correctly took a least common multiple, and understood the procedures for converting between a fraction and a repeating decimal. She did not understand the meaning of greatest common factor and was especially inept at finding a rational number between $1/3$ and $1/4$. She also was unable to make much progress on solving the second and third verbal problems. Student J's course grade of 3.0 was perhaps lower than one would expect from her study behavior, but not so low as to constitute an anomaly.

Student J reported some affective changes at the end of the course: she said that she felt more confident of her knowledge of mathematics and her ability to teach it. The activities she had seen had demonstrated to her that teaching mathematics could be fun, but she still had not decided whether she would major in elementary (preschool) education or in communications. She was still convinced of the power of mathematics to teach reasoning skills and felt that it should be included in the curriculum at all educational levels.

The course in its present form functioned adequately for Student J. She was well prepared, but not overprepared, for mathematical work at this level. The course improved both her understanding and her attitude; the laboratory served to demonstrate the relevance of the course material

to elementary school teaching. Her experience does not suggest any changes that should be made in Math 201.

Student K. At the start of the course Student K reported on her information sheet that she liked mathematics very much. She told the investigator it was her favorite subject. She had taken four years of it in high school in an upper-track sequence. She was enthusiastic about the prospect of teaching elementary school mathematics and felt capable of filling this role. Her first response to the question on the purpose of elementary school mathematics was to say that its purpose is to foster logical thinking; afterward she mentioned the necessity of knowing the basic operations.

Student K's main interest in the lectures was in the lecturer's suggestions for teaching, since all of the content was familiar to her from her earlier education. She liked the lecturer's style and thought it appropriate for students first learning the material; however, at the end of the term, she was resentful of having been forced to take and pay for a course whose substance was completely familiar to her. Student K said that the homework problems were essential in order for her to be sure of her understanding of the material; she made no comments on the book's presentation. She called the laboratory the most enjoyable part of the course, and said at the end that she had enjoyed all of the laboratory sessions. She suggested that students be given an opportunity to waive the lecture while doing the laboratory work either

as a separate course or as part of the required methods course.

Student K produced the best performance of any subject on the study problems: on all of those problems based on the course material she made only two arithmetic errors. Not surprisingly, her grade in the course was 4.0.

At the end of the course, Student K reported no change in her feelings toward mathematics. She was still enthusiastic about teaching it--so much so that she was considering earning an elementary mathematics specialist's certificate in addition to her special education certificate. She expressed the opinion that all students should study mathematics up to the limit of their understanding.

Her own comment at the last interview (cited above) emphasizes that the lecture part of the course was unnecessary for Student K; she felt it a waste of her time and money. Fortunately, she enjoyed the laboratory sessions, so she got something positive from the course.

Student L. Student L's attitude toward mathematics was one of dislike. She was not upset by the idea of teaching mathematics and felt capable of doing the job. In high school she had preferred algebra to geometry because algebra dealt with numbers, while she had found geometry too abstract. She felt that elementary school mathematics is taught because of its usefulness.

From the start Student L said that she would have preferred to take the course in a different format; a friend

of hers had taken it in an all-manipulative version offered in the spring of 1973. Although she usually expressed approval of the lecture presentation, Student L never changed this opinion. She had consistently negative reactions to the laboratory sessions, criticizing them for not being relevant to the lectures. While she considered manipulative materials to be "excellent" for children, she found learning with them a waste of time for college students--two hours spent on what could be shown in a few minutes. She recommended that the laboratory period be used to review lecture material. Her hostility to the laboratory increased as the term progressed. At the end she said she had been "insulted" by the policy of mandatory laboratory attendance--if the laboratory were good enough, she said, students would attend voluntarily. She made almost no comments on the specific laboratory exercises.

Only two subjects performed worse than Student L on the study problems; they earned grades of 2.0 and 3.0 in the course. Compared with these, Student L's grade of 2.5 is not surprising; in fact, it is quite close to the value predicted by a regression equation based on Table 3. Student L revealed a misunderstanding of ideas about sets, did sloppy work--including the use of inspection rationales--in testing numbers for primeness, and was particularly inept with decimals (she was the only subject not to divide 7 by 12 to find a decimal name for $7/12$). Also, Student L could not make correct mathematical sense out of any of the three verbal problems. On the positive side, she worked all of the base problems correctly, found the greatest common factor and

least common multiple of a pair of numbers, and found a rational number between $1/3$ and $1/4$.

At the end of the course Student L found that her feelings about mathematics had not changed. She seemed to bear a greater hostility toward mathematics teachers than toward the subject itself--she had found her teachers unable to communicate with students. She felt that these teachers were more interested in impressing the students with their intelligence than in effective teaching. She was disappointed that Math 201 had not improved her attitude toward mathematics. However, she did say she had a greater appreciation of mathematics as a result of her course experience.

Student L entered Math 201 with a hostility toward mathematics which had been building up over a long period of time. She was aware of this and hoped that her attitude could be changed. However, a course like Math 201, with its lecture mode of instruction similar to the type of instruction she had received in the past, could not be expected to accomplish this goal.

Student M. Student M said at the start of the course that she had a neutral attitude toward mathematics. Although she had enjoyed her high school courses and thought she had learned well in them, she ranked mathematics and science (which "lost me") as her least favorite high school subjects. She anticipated no difficulty in teaching elementary school mathematics, which she felt was taught because of its utility in everyday life.

Throughout the course Student M commented that the lectures were well presented, particularly for students who, unlike herself, were not familiar with the material. However, after the course was over, she admitted that she usually had found them very boring. She attributed this to her own familiarity with the material, and also to the nature of mathematics itself, saying that mathematics lectures could not help but be boring. She found the book's explanations unreadable and characterized this too as typical of mathematics. One trait of Student M which was reflected in her discussions of the lecture and the book was the strength of her mathematical habits, problem-solving techniques which she had mastered and used to the extent that she could not understand why a different method works equally well. Student M had mixed reactions to the laboratory sessions. She enjoyed most the sessions on computation and on the geoboard. She also enjoyed the sessions on rational numbers and on the metric system. She liked the use of Dienes Blocks to illustrate addition and subtraction, but found them impossible to work with in multiplication and division. She disliked the session on real numbers and hated attribute games.

Student M did fairly well on the study problems. She handled all the base problems capably, and made only one careless error on the prime number exercise. She correctly found the greatest common factor and least common multiple of a pair of numbers. She made some errors on the decimal problems and was unsure of herself in working with fractions.

She confused matching and equality of sets, and could solve only the first of the three verbal problems. Despite these difficulties, Student M did better on the study problems than three other subjects who also received a 4.0 in the course. Her course grade cannot be viewed as an anomaly.

After the course was over, Student M said that her feelings toward mathematics had become more positive. She said that mathematics "doesn't bother me that much any more," and remarked that she felt capable of teaching it in the lower elementary grades. She had a greater awareness of what it is like to teach elementary school mathematics.

Student M was a capable student whose most interesting characteristic was her rigid adherence to mathematical habits and her trouble in understanding different procedures. It is difficult to suggest how such an attitude can be overcome by a particular mode of instruction in Math 201. Even if such an instructional mode exists, it is probably impossible to identify the students who would need this treatment.

Student N. Student N reported on her information sheet that she liked mathematics. In high school she had earned consistent B's and learned her mathematics well. She enjoyed solving problems logically. She felt that the purpose of the study of mathematics was to improve one's ability to think.

At the start of the course Student N characterized the lectures as "total boredom," but as the course progressed she

seemed to become interested in them. She found the book's problems good, but its text hard to read; nevertheless, she expected to keep it for future reference. She usually skimmed the book after attending the lecture on a particular topic. She would have preferred a set of worked-out solutions to all problems to the book's list of selected numerical answers. At the end of the course, Student N said she had found the laboratory sessions helpful, but thought they should follow, rather than precede, the lecture on a particular topic. She liked best the sessions on computation and on Cuisenaire rods, and had a favorable opinion of most of the others.

Student N did the worst on the study problems among those subjects who earned a 4.0 in the course. In fact, only four of the subjects performed worse than Student N on the problems. Student N did not grasp the distinction between matching and equality of sets. She did some base problems but could not handle base two or recognize the base of a worked-out subtraction example. She correctly identified prime and composite numbers, and accurately (but very mechanically, without real understanding) took the greatest common factor and least common multiple of a pair of numbers. She did not know the algorithm for converting a repeating decimal to a fraction, but was able to do the reverse problem as well as find a rational number between $1/3$ and $1/4$. (She was the only subject to use decimals in order to solve the latter.) She hated verbal problems, solved the first of

them only unsurely, and failed to solve the others. It is somewhat anomalous that Student N was able to earn a 4.0 in the course despite such a record of mediocrity in her study behavior. This writer feels that an explanation of this is that Student N was a very skilled test-taker. Her entire behavior in the course was oriented toward the test, and for every new item she encountered, she considered whether it could appear on the test, in what form, how many times, etc. This was a skill which Student N had practiced throughout her career, and it served her well in this course. It is also possible, of course, that in her last-minute studying for the course tests, Student N actually learned material which she had not yet learned at the time of her interviews.

Student N said at the end of the course that she liked mathematics a lot more as a result of her Math 201 experience. Her knowledge of arithmetic had been refreshed and had expanded to include an understanding of why the processes of arithmetic work. She had liked the examples of activity teaching that she had seen in the laboratory and was very confident of her ability to teach successfully in this manner.

The course as it was presented worked well for Student N. She thought she had improved in both attitude and understanding at the end of the course. As with Student H, the experience of Student N suggests that problem-solving ability be considered in determining a student's grade in the course.

Student O. At the beginning of the course Student O said that she had a neutral attitude toward mathematics. In high school she had taken only the minimum necessary for a college preparatory program. She said she was glad to be taking a course which would prepare her to teach, and remarked that she was glad to see why the processes of arithmetic work. She said that elementary school mathematics is taught because of its uses in daily life, and also in order to foster the development of logical thinking.

Student O several times made positive comments about the lectures; it seems that her attitude toward them was positive. Her reaction to the book was mixed, consisting of a variety of comments, both positive and negative. She also generally enjoyed the laboratory sessions, remarking at the end that she had liked those on the geoboard, computation, Cuisenaire rods, Dienes Blocks, and the metric system, while referring to the session on real numbers as her "least favorite."

Student O performed quite well on the study problems. She seemed to understand the idea of matching of sets. (Her one error may have been due to misreading one of the given sets.) She solved all of the base problems. She identified 119 as a composite number but called 247 prime (her rule was to try numbers through 11 as possible factors). She understood the procedures for taking the greatest common factor and least common multiple of a pair of numbers. She performed well on all of the fraction and decimal problems. She had a

weakness in solving verbal problems, making a careless error on the first of these while failing to make a correct mathematical model of either of the others. In view of her generally good record here, it is not at all surprising that Student O received the grade of 3.5 in the course.

At the end of the course Student O reported that she had more positive feelings toward mathematics than she had had before. She understood some topics for the first time, and was more confident about her ability to teach elementary school mathematics. She felt that she could now explain why the processes of arithmetic work.

Student O was well suited to the course as it was presented. She was competent in mathematics but had not learned so much that she would be bored in this course. She enjoyed the laboratory and finished the course with improvements in both understanding and attitude.

Student P. At the beginning of the course Student P said that she liked mathematics very much; it was her favorite subject and her major within the elementary education program. In high school she had been in small, highly able mathematics classes. She was enthusiastic about eventually teaching elementary school mathematics; she said its purpose is for the student to learn to work with numbers.

Student P at no time said that she was having any difficulty with the course. She praised the lecturer for her presentation, especially of the topics treated toward the

end of the course. Student P's favorite problems were story problems and questions asking how one would explain an idea to a child; she considered these the most relevant to the preparation of an elementary teacher. Student P thought the laboratory sessions in general were good and only regretted not having the opportunity of actually working with children. She termed the sessions on the geoboard and on Dienes Blocks the best, and also expressed approval of those on computation and rational numbers. She had a negative reaction to the sessions on clock arithmetic and on the metric system, but realized that these had been useful to other students.

It is not surprising, in the light of her repeated comments about how easy she found the course, that Student P received the grade of 4.0. However, the grade is less consistent with her problem-solving behavior in the study. Three subjects who earned less than a 4.0 performed better than or equal to Student P on the study problems. Student P understood the idea of matching of sets, but revealed a flawed understanding of finite and infinite sets. She was sloppy in her work with bases and with primes. She understood the procedures for taking the greatest common factor and least common multiple of a pair of numbers, but miscalculated the latter because of an error in factoring. Student P made careless errors in both conversions between a fraction and a repeating decimal. She effortlessly found a rational number between $1/3$ and $1/4$, and was one of only two subjects to find the answer to the second verbal problem.

She easily solved the first problem mentally, but could do nothing with the third. It seems that, compared to other subjects who earned a 4.0, Student P displayed a stronger tendency to make careless errors in trying to solve the problems. She was probably more careful when doing work which counted toward her grade.

At the end of the course, Student P reported that her feelings about mathematics and about teaching it had not changed. She expected to take more mathematics in college--either calculus or elementary education mathematics courses. She said that she had benefitted from taking Math 201 and recommended only that the course be offered in smaller sections and that the laboratory sessions be shorter.

Despite her careless work on the study problems, Student P obviously understood all of the material taught in Math 201. This conclusion is reinforced by the observation that she received the grade of 4.0 despite having missed about two weeks of the course due to illness.

An overview. In retrospect, it appears that the subjects (and, by implication, Math 201 students in general) comprise four different types of student. (Of course, there may be other types which were not represented in the study.) One type is the student who does not have an adequate command of arithmetic, represented here by Students F and G. A student who cannot do arithmetic can no more study its foundations than a student who cannot read can study grammar.

One can even understand the concepts taught in Math 201 in and of themselves (as Student G did to an extent) yet fail to grasp their relationship to arithmetic. Such students should not be permitted to enroll in Math 201 until they have completed appropriate remedial work.

At the other end of the spectrum is the student who has been overprepared for Math 201 by a long and successful study of high school mathematics. Such students (represented in the study by Students C, I, K, and P) have been exposed to all of the ideas before and have already mastered them. They therefore find the course quite boring. If an honors section of Math 201 were available these students could be presented the type of challenging material they have learned to expect (and to enjoy) in a mathematics course.

A third type of student that could benefit from a different type of Math 201 course is the student who is capable in mathematics at least through arithmetic, but bears a great animus toward the subject. This type of student was represented in the study by Students B and L. A course presentation similar in style to those they had encountered earlier will probably only exacerbate these students' ill feelings. A radically different type of course is required to improve the attitudes of such students. An experiment should be conducted to see if either a joint content-and-methods course or an all-manipulative approach to Math 201 could effect such an improvement.

Finally, there are those students who do not dislike mathematics and have taken two or three years of it in high school, earning grades such as B and C. They are the majority of Math 201 students. During the quarter this study was conducted, 66 percent of the students in the course reported having a neutral or mildly favorable attitude toward mathematics, 71 percent had taken either two or three years of it in high school, and 83 percent had averaged either B or C in their high school work. The fact that study subjects in these categories (such as Students A, J, M, N, and O, who fell into all three categories, and Students D and H, who had a strongly favorable attitude) generally responded favorably to the course suggests that this format (with some modifications, to be discussed later) is probably acceptable for them, and that efforts should therefore be concentrated on steering the exceptional students into alternative presentations.

The Mathematical Topics

In this section, the various mathematical topics taught in Math 201 (including the laboratory sessions) will be surveyed. Inferences will be drawn from the interview data about whether Math 201 students understand a given topic or, if not, what misconceptions they tend to possess. The reader should be reminded at this point that the fifteen study subjects, considered as a group, scored substantially higher than their classmates on their course tests and that

therefore any misconceptions possessed by them are probably possessed in even greater proportion by the population of the course as a whole. Recommendations for the improvement of pedagogy of the various topics will be made in each section.

Sets. Data from the interviews indicate that the subjects misunderstood several important ideas about sets. Only three of them (Students D, F, and K) were able to give correct answers to all four of the questions about matching (equivalence). Furthermore, some of those who answered the first and fourth items correctly (the correct answer was "none of the above" in both cases) revealed in their comments that they did not actually understand the concept of matching of sets; examples of this are the responses of Students G and N (both of whom missed the middle two items) to those items and the responses of Student A to the fourth item and of Student F to the first item. These responses reflected a misconception which caused many of the subjects' errors--confusion of the idea of matching of sets with that of equality. If the subject did not see living people, card suits, or aardvarks in the listed sets, she would say they do not match the sets described, regardless of the number of elements of the sets in question. Students B, C, and J missed the second item but answered the third correctly; this could not have been due to a misunderstanding of matching, but probably was caused by the nature of the sets involved in this item.

Another common error was misuse of the term "the empty set." Several subjects (including Students, B, J, L, and N) used this expression in place of "none of the above." It is this writer's opinion that this practice resulted from these subjects' belief that set language can be used to describe situations which it cannot in fact describe. This belief is probably not surprising when one considers that the subjects have probably seen most of their school mathematics expressed in terms of sets. For example, Student J, at her initial interview, in giving an example of the importance of mathematics, described the oil shortage in terms of set theory, in words such as: there is a set of oil and a set of countries, etc. Such an inclination to the overuse (and misuse) of set-theoretic terminology could lead a subject, when none of the given answers to a question is correct, to say "the empty set" rather than "none of the above." A similar phenomenon can be seen in students' reaction to the geoboard. Both of these are cases where, mathematically, a little learning is a dangerous thing.

Other difficulties encountered by the subjects in their study of sets included the definition of numbers (Students A and N), trouble with the formulas for the number of subsets of a given set and for the number of one-to-one correspondences between a set and itself (Students B and M), trouble with finite and infinite sets (Students B, C, I, L, and M), difficulty in describing explicitly a one-to-one correspondence (Students B and N), enumerating all the subsets

of a given set (Student D), understanding the definition of proper subset (Student D), understanding the relation between set union and number addition (Student D), enumerating the one-to-one correspondences of a set with itself (Students F and N), confusion by the widespread use of set language in mathematics (Student F), understanding the definition of one-to-one correspondence (Student H), describing the Cartesian product of a set with the empty set (Student N), trouble with determining the equality of two differently described sets (Student O), and trouble in working problems involving the set difference operation (Student P). Of these, the only one which occurred frequently enough to merit a recommendation is the problem of finite and infinite sets. Since this topic tends to confuse Math 201 students and is not relevant to the elementary school curriculum, this writer recommends that it be discussed in honors sections only.

The writer recommends further that materials be used in the course which will help the students to distinguish between matching and equality of sets. If the same text is to be used, the staff should develop materials with the aim of accomplishing this goal.

Number Bases. All but two subjects (Students F and P) knew how to convert a number from a nondecimal base into base ten. Also, the subjects as a group displayed proficiency in recognizing the base of a worked-out example; only Student G failed to do this in the addition example,

and only Students A, G, and N in the subtraction example. The latter example also served to illuminate two subjects' (Students C and I) understanding of the relationship between subtraction and addition--they converted the example to an addition example in order to identify the base. Although twelve subjects correctly solved both items, their work indicates that the subtraction example was considerably more difficult for them than the addition example. One surprising observation here was that only a minority of the subjects (Students D, F, I, J, L, M, and O) used the digits in either example as a clue to the base.

The most interesting responses in the problem set on number bases were to the problems of converting a base ten numeral to a nondecimal base. Students A and F displayed a complete misunderstanding of what was required; both of them responded as follows: 44_{ten} in base seven is 4 sevens and 4 ones, which is thirty-two, so 32_{seven} . Students G, N, and P solved the base seven example correctly but not the base two. This writer feels that Math 201 students find it easier to convert to a larger base than to a smaller base because of the greater number of digits a number requires in a base such as two or three. Furthermore, students are somewhat befuddled by the fact that base two numerals contain only ones and zeros. The subjects who solved both examples correctly used different methods to do it. Most set up place-value columns in the requested base, then filled in the columns from left to right, subtracting the respective place

values from the original number as they proceeded. However, some subjects used a division approach. Students D and O repeatedly divided the original number and the resulting quotients by the desired base, and wrote the remainders in columns from right to left to represent the number in that base. Student M used a third approach: she repeatedly divided the original number, and then its remainders, by the place values of the columns, going from left to right. In this fashion she determined the correct digit for each column.

Some subjects reported difficulties with bases other than those revealed in the problems. These included converting from one nondecimal base to another without going through base ten (Student N), comparing in size identical numerals in different bases (Student N), multiplying a nondecimal numeral by the square of its base (Student N), and doing multiplication and division in nondecimal bases (Students J, N, and O).

Two subjects were vocal in expressing their opinions of this topic. Student D said that she enjoyed working with bases, while Student F exclaimed, "Ucch! I hate bases!" The investigator was surprised by the response of Student P (a subject with a strong mathematics background) to the base two problem; she left out columns and also used the digits 4 and 3. Student O remarked that 405_{eleven} would "look" bigger in base ten; the investigator was surprised that

no other subject mentioned the "look" of a number in a non-decimal base.

In summary, it would seem that the subjects' understanding of bases was adequate. However, due to the mathematical superiority of the subjects to the class as a whole, it is possible that misconceptions are more common among Math 201 students than is indicated by the subjects' responses.

Operations of Arithmetic. Although no study problems asked the subjects explicitly to perform arithmetic operations, some of them revealed in conversation their troubles with the course work in this area. The strongest reaction was to the Austrian method of subtraction. Five subjects (Students B, F, G, M, and N) displayed adverse reactions ranging from mild confusion to absolute frustration and total rejection. Students A and M were particularly unresponsive to the presentation of unfamiliar algorithms, rejecting them in favor of familiar methods. Several subjects reported difficulty with division: Student K encountered a general difficulty, Student F could not find q and r where $314 = 6q + r$ and $r < 6$, and Students A and N could not understand the three definitions of division (missing factor, partitioning into d subsets, partitioning into subsets of size d) presented in the course. Student O was confused by division in non-decimal bases, and Student G had "a little trouble" with division, especially with the Greenwood algorithm.

As long as different school arithmetic programs use different algorithms, it will be important that prospective elementary school teachers be exposed to those in use. However, the proliferation of personal calculators insures that calculation with large numbers will cease to be a vital skill in the near future.

Prime Numbers. The subjects were asked to say whether or not the numbers 119, 113, 227, and 247 are prime. All correctly identified the primes, but some used fallacious reasoning in arriving at this conclusion. Students A and G did not try enough divisors to establish the primeness of the numbers, while Students F, H, and L used inspection rationales to justify their calling these numbers prime. While only four subjects failed to recognize 119 as composite, ten failed to recognize 247. This probably was due to the greater amount of patience required to find a factor of the larger number. In testing these numbers for primeness the subjects stopped trying divisors at a variety of places. Only Students B, C, K, and N (and possibly Student J) knew that only primes need be tried as divisors.

Some interesting phenomena appeared in the subjects' problem responses and in the discussion. Four subjects (Students B, D, F, and J) said that they had had trouble understanding (or else had simply ignored) the Sieve of Erathosthenes, a device which this writer feels should have been clear and self-explanatory. Several subjects (including Students H, L, and P) were sloppy in their problem solutions,

obviously wishing to avoid the tedium of repeated trial divisions. Student C, who was bored throughout the course by the familiarity of the material, revealed herself here to be rather sloppy in arithmetic, dividing 3 evenly into 119 and later stopping her trial divisions, certain that 13 is not a factor of 117. Student N reported that she was confused by the Fundamental Theorem of Arithmetic; this writer suspects that such confusion is common among Math 201 students.

It is clear from the subjects' responses that they had a good understanding of the meaning of prime number, but tended to be sloppy when presented with the problem of determining whether a given number is prime. Regarding the teaching of prime numbers, this writer feels that inspection rationales should be actively discouraged. The use of examples such as 227 and 247 in the classroom will help to dissuade students from embracing the idea that the primeness of a number is related to its last two digits.

Greatest Common Factor/Least Common Multiple. These are concepts with which Math 201 students tend to have difficulty. While eleven of the subjects knew what to do when asked to find both items, several of these (Student N is the best example) revealed that their understanding was strictly mechanical and that they were blind to the self-explanatory nature of the names of the concepts. This phenomenon was particularly obvious in the responses of those

subjects who made errors. Student B took the LCM when asked for the GCF and vice versa. Student J correctly took the LCM, but when asked for the GCF took a sort of "greatest common multiple"--the product of the two numbers. In the exact reverse of this procedure, Student G correctly took the GCF, but when asked for the LCM, took a sort of "least common factor"--the smallest single prime factor common to both numbers. Student F did not even take prime factorizations in her responses to these examples.

It seems that this is a case where the instructional staff of Math 201 is tempted to think that what is self-explanatory to them is also self-explanatory to the students. This clearly is not the case here. This writer would recommend that the topic be taught by first emphasizing what is meant by a factor of a number, then explaining the idea of a common factor of two numbers, and finally introducing the concept of greatest common factor. Then, separately, the same treatment should be used to explain least common multiple. By emphasizing separately the meaning of each word in the names of these concepts, it should be possible to achieve a nonmechanical understanding of them in the students.

Integers. In the course the topic of negatives was introduced after completion of the study of the counting (whole) numbers. No study problem explicitly tested the subjects' mastery of this topic; Problem 3 was intended to

test it implicitly, but ended up testing their understanding of distance, rate, and time instead. Nevertheless the subjects did make during the interviews various comments on their course experience with this topic which should be included in this discussion. In their comments they reported encountering the following difficulties with the topic of integers: trouble with subtraction of negatives (Student A), difficulty in understanding why a negative number times a negative number is a positive number (Student A), not having learning this material earlier (Student B), trouble in understanding the order of negatives (Student D), confusion between the ideas of opposite and negative, or confusion because these are represented by the same symbol (Students D and M), and trouble with the concept of absolute value (Students F, J, M, and N). This writer feels that the distinction among the ideas of minus, negative, and opposite is important and should be stressed in teaching this topic. Clear use of language would help to alleviate students' difficulties with negatives.

Rational Numbers and Decimals. The only problem on rational numbers which was presented to the subjects asked them if there is a rational number between $\frac{1}{3}$ and $\frac{1}{4}$, and if so, to name one. Most subjects were able to do this problem; a variety of methods was displayed. Only Students F and J were unable to do the problem at least partially. A great surprise, for which this writer can find no explanation, was the idea of some subjects (particularly Students A and C)

that $7/24$ is not a rational number. One noteworthy feature of the responses is that only one subject (Student N) used decimals to solve the problem, even though all had studied decimals before it was presented. This writer recommends that after the relationship between fractions and decimals is taught, the density of the rationals be retaught via a demonstration utilizing decimals. There was general approval among the subjects for the "array method" of presenting rational numbers as cut-up squares, and this writer recommends that this method of presentation be continued.

Nearly all subjects knew the algorithm for converting a fraction to a repeating decimal. The only exceptions were Student F, who, unsure how to proceed further, stopped her division after two decimal places, and Student L, who tried to solve the proportion $7/12 = x/100$. In contrast, the subjects had much difficulty with the reverse operation. Only six of them (Students B, C, D, H, I, and O) were able to complete the process without error. Six more (Students A, G, J, K, M, and P) knew the procedure but made arithmetic errors along the way. It seems from this pattern of responses that the algorithm for converting a repeating decimal to a fraction is one which carries a strong chance for the student to make an arithmetic error. As a partial check on this tendency, it would be a good idea for students to acquire the habit of always checking their fraction answer by performing a long division and comparing the resulting decimal with the original.

Two other problems involving decimals were of types not seen by the subjects in the course due to lack of time. The first of these involved finding a terminating decimal approximation to a fraction. Thirteen subjects (all but Students F and L) divided out the fraction as a decimal as in the first problem, but of these only four (Students D, H, I, and O) grasped the meaning of the phrase "an error less than .00001" and terminated the resulting decimal after five places. Most of the subjects were able to give some answer to the item "Find an approximate value for $\sqrt{7}$." The responses varied in accuracy from the mere statement that $\sqrt{7}$ is between 2 and 3 to two-decimal-place approximations verified by squaring. Student C used the square root algorithm here. Five subjects could not answer this item correctly; they were Students A and L, who could think only of the Pythagorean theorem, Students B and F, who had no idea how to answer the question, and Student N, who said that $\sqrt{7}$ is half of 7. This number of subjects having difficulty with this item is cause for concern.

There were some surprising items among the subjects' responses to the various items involving decimals. Student A, although she carried out the procedure correctly but for an arithmetic error, said that she could see no pattern in the use of r , $10r$, $100r$, etc. Several subjects (including Students A, D, F, G, N, and O) said that they earlier had had difficulty with the idea of percent, and that Math 201 had helped them in this area. (Of course, competency in

arithmetic, including percent, is supposed to be a prerequisite for Math 201.) Student P made an arithmetic error in finding a fraction name for $.3777\dots$; the investigator was surprised when she obtained the fraction $1/3$ but did not see anything wrong with this answer.

There were other reports of difficulty among the subjects' responses to the ideas of rational numbers and decimals. These included difficulty in performing operations with decimals (Student B), difficulty in understanding scientific notation (Students B, F, J, and O), trouble with ordering fractions (Students B, F, and Student J referring to other students), difficulty with the ideas of terminating and repeating decimals (Students, D, F, and J), trouble in understanding the invert-and-multiply algorithm for division of fractions (Student D referring to other students), trouble with verbal problems involving fractions or percent (Students D, F, G, H, N, and O), prior ignorance of these topics (Students H, N, and O), a tendency to reject the procedures presented in the course in favor of personal habits (Student M), trouble with the idea of density (Student O), and difficulty with the concepts of rational and irrational number (Student O).

Field Properties. The closure, associative, commutative, distributive, and identity properties of the various number systems were discussed explicitly in the course. Test questions required the student to name the

properties illustrated by various equations. Several subjects (including Students A, B, H, K, and N) remarked that they had had difficulty in identifying these properties in illustrations. Some subjects (including Students B, F, and N) had trouble with an assigned problem asking them to construct sets of integers closed or not closed under addition and/or multiplication. Finally, two subjects (Students F and K) remarked that they could not see the flaw in a "phony 'proof'" in the book that $1 = 0$. (As is customary in such "proofs," it included a division by zero.)

Measurement. Measurement is sometimes taught explicitly in Math 201 but was not taught in the term under study because of time limitations. The subjects' responses to the second measurement problem indicate that it should be taught. Five subjects (Students A, D, H, N, and P), four of whom earned 4.0's in the course, thought that one could deform the boundary of a plane figure and leave the area unchanged. Measurement is now an important part of elementary mathematics, and in the near future, due to the effect of the calculator on the need for computational drill, geometric ideas may increase in importance in the elementary curriculum. In Math 201, laboratory time should be devoted to measurement, since it is best learned by actively measuring.

Problem Solving. Some of the most interesting findings of the study resulted from the investigation of the subjects' ability to solve three verbal problems. The investigator was surprised by the vehement hostility with which some subjects greeted his requests for their solution. Even subjects who were sufficiently facile with numbers to finish the course with a grade of 4.0, such as Students H and N, commented that they hated this type of problem; Student G, who failed the course, expressed the same sentiments. Other comments made by the subjects included that of Student D that she had been told long ago by her teacher that "story problems are very hard," and that of Student O that she used to hate story problems, but had learned how to use the words in the problem as clues to what mathematical ideas would be used in the solution.

Problem 1 was written to be a very easy problem; the investigator was interested in how the subjects attempted to solve it (i.e., whether they would recognize base five) rather than in whether or not they would understand it. Only three subjects (Students C, M, and O) even mentioned base five. Three other subjects (Students A, H, and L), however, failed to understand the language of the problem, concluded that the operation required was subtraction, and proceeded to subtract one amount of money from the other. The other subjects correctly solved the problem without mentioning base five; among these, only Student F made an error, which was not of a mathematical nature.

Problem 2 was thought by the investigator to be the most difficult of the three verbal problems in the study. It was intended as a multiplication problem, given after the subjects had studied the operations of arithmetic in the course. Only two subjects (Students K and P) correctly solved this problem; Student F came very close to a correct solution. Otherwise no subject could correctly say more than that the number required was three times the number of possible suffixes on one prefix.

The results most surprising to the investigator were the responses to Problem 3. This was intended to be a number line problem. It was also a motion problem, though, and the responses indicated that many subjects did not grasp the relationship among distance, rate, and time. Only Students C and K were able to solve this problem correctly; both solved it straightforwardly. Students D and I had the correct idea here but were off due to arithmetic errors. Students B and M displayed a partial understanding of the situation. The other nine subjects' responses were studies in incompetence. Some, such as Students H and O, used units of time to express distances. Others, such as Students G and L, said that they could not locate Dan's position at 1 o'clock. The rest made a variety of false starts or simply gave up.

The general incompetence of the subjects in solving the last two verbal problems is a cause for serious concern. (Their lack of understanding of the relationship of distance,

rate, and time leads one to wonder how well they would have done on verbal problems involving percent, interest, mixtures, and measurement; this would be an interesting topic for future research.) They will be certified to teach elementary school mathematics, and are likely to transmit their incompetence to their future students, even as they inherited the incompetence of their own former teachers. Since problem solving is so vital a mathematical skill, an attempt must be made to break this vicious cycle, and it can be done in Math 201. This writer recommends that a portion of Math 201 be devoted to the solution of verbal problems. This could be done in the laboratory if necessary.

Laboratory Sessions. Two subjects (Students D and K) said they had enjoyed the exercise on attribute games. Student F said it had been helpful. Student J first called it too elementary but at the end of the course recalled it as good. Student A called it unnecessary and Student G would say only that it was less tedious than that on Dienes Blocks. Student H said she had been lost here. Student M was the most negative, calling this exercise ridiculous and worthless.

Two sessions were devoted to Dienes Blocks, the first emphasizing addition and subtraction, the second multiplication and division. The great majority of subjects expressed a favorable opinion of this activity in the first session, but fewer of them cared for it in the second. Student M expressed the most sharply contrasting opinions here, calling

the second session ridiculous but emphasizing that she herself would use Dienes Blocks to teach addition and subtraction. Student K also had these opposite reactions to the two sessions. Students D and O particularly mentioned the division exercise as confusing to them. In contrast, Student H liked the second session but not the first. The first laboratory on Dienes Blocks was the only laboratory session on which Student A expressed a favorable opinion. Student I, who was generally bored in the laboratory, found the Blocks interesting pedagogically. Students C, F, and G seemed to have a generally negative reaction to both sessions.

Regarding the session on computation, which had included lattice multiplication, Russian peasant multiplication, Napier's bones, and the Whitney Mini-Computer, subjects' opinions were generally favorable, with eight subjects (Students F, H, J, K, M, N, O, and P) having positive comments as opposed to only three (Students A, C, and D) expressing negative opinions. The subjects had the opposite opinion of the session on clock arithmetic, with nine of them (Students A, B, C, F, G, K, L, M, and N) expressing negative opinions (although Students F and N recalled this session favorably at the end of the course) and only four (Students D, H, J, and O) positive ones.

The session on rational numbers, which featured GeoBlocks, tangrams, and Cuisenaire rods, coincided with test week, so that several subjects did not get to work with all of these materials. Opinions of Cuisenaire rods tended to be

favorable (including that of Student G, who rarely had good things to say about the laboratory), while those of GeoBlocks included both neutral and favorable ones. Only two subjects (Students D and K) had anything favorable to say about the exercise on ruler-and-compass constructions; this session evoked the most negative opinions from the subjects. The final session, on the metric system, produced mixed reactions.

Subjects' feelings were overwhelmingly positive toward the geoboard; only the generally negative Students A and C had unfavorable comments. It seems to this investigator that the geoboard was the most popular of the laboratory activities. Certainly it should be used in prospective elementary teachers' training in measurement and geometry; yet this writer feels there is some danger in using only the geoboard for this purpose. Math 201 students tend to overestimate the power of the geoboard, using it as a crutch, or, as with set language, thinking it is useful where in fact it is not. In one of the pilot studies conducted prior to this study, a subject, when asked to find the area of a plane rectilinear figure with only right angles and integer-length sides, told the investigator that she could do areas only on the geoboard. In a similar vein, several subjects told the investigator in the present study that they would attempt to find the area of the irregular shape in the second measurement problem by placing the figure on a geoboard, and some even said they would use Pick's theorem! (After all, this was how they had learned to find an area.) Student N said

that she thought the formula $A = \pi r^2$ came from the geoboard. It is clear from these examples that the geoboard is capable of creating as much confusion as understanding. It should be used only as part of a coordinated unit on measurement and geometry in which such topics as the sizes (length, area, volume, weight) of irregular figures should be covered and the meaning of measurements defined.

General Recommendations for Mathematics 201

Specific recommendations regarding the pedagogy of various topics taught in Math 201 were made in the preceding section. In this section general recommendations regarding the overall format of the course will be considered.

Regarding the laboratory, it is a mistake to grade students' laboratory work by putting on lecture tests items based on the laboratory activities. This practice stifles whatever curiosity students may have about the laboratory method of instruction by forcing them to worry about what types of items will appear on the next test, and also causes the reaction expressed by Student B in her comments on the ruler-and-compass laboratory; this cannot possibly be asked on the test, so I am free to ignore it. As an alternative, this writer would suggest grading the laboratory activities by requiring the student to submit for each session a written report describing in detail how she used the laboratory materials to solve various exercises and problems. This approach would entail much more work for both the students

and the laboratory instructors, but it would force students to contemplate the pedagogical effectiveness of the materials while freeing them from worry over future test items.

Another problem is the scheduling of the laboratory sessions. As is evident from the interviews, students are repelled by its two-hour length (especially when it begins at 8 A.M.) and therefore have some hostility in them even as they enter class. Students also resent not being able to use the laboratory as a review session prior to a test; in some instances, the laboratory is used as a review session, and the students miss a lesson. Instructors are frustrated because it is impossible to coordinate the laboratory assignments with the lectures. This writer suggests re-scheduling the Math 201 laboratory as two one-hour sessions per week; students would attend one session between the Monday and Wednesday lectures and another between the Wednesday and Friday lectures. This arrangement would solve all of the problems mentioned above. The laboratory period before a test could be used as a review session, and another laboratory lesson could still be held the same week. Also, it would be possible for the lecturer to coordinate laboratory activity with the lectures, because all students would have a given laboratory lesson between the same two lectures. A disadvantage of this plan is that one hour is sometimes insufficient to develop a good laboratory lesson. However, the writer feels that this disadvantage does not outweigh all of the advantages mentioned. If two laboratory hours

were desired for a particular lesson, the lesson could be resumed at the following session.

One other important point concerning the laboratory must be made. It has been the practice of the Mathematics Department to assign the instruction of these classes to graduate assistants who are untrained and sometimes even uninterested in elementary level mathematics and in the laboratory method of instruction. This practice should be halted. The purposes of the laboratory are (1) for students to learn some mathematics, and (2) for students to become acquainted with the laboratory method of instruction and to appreciate its power. These purposes are frustrated when students encounter an instructor who is uninterested in such instruction. The Math 201 laboratory can serve as an opportunity for mathematics education graduate students to become familiar with the use of manipulative materials in mathematics instruction; it did for this writer. But it is imperative that laboratory instructors be interested in elementary mathematics; otherwise their indifference will be transmitted to the students. This writer suggests that instruction of the Math 201 laboratory be restricted to mathematics education faculty or graduate assistants; enough such assistants should be hired to assure adequate staffing.

Something should be done to deal with the problem of students who are overprepared for Math 201. The results of the questionnaire distributed in the winter quarter of 1974 (reported earlier as Table 1) reveal that 24 percent of the

Math 201 students that term had taken four years or more of high school mathematics, and that 14 percent of the class (a total of 26 students) had received an average grade of A in their high school work in mathematics. (It is reasonable to assume that these groups largely overlap.) These groups of students (represented in this study by Students C, I, K, and P) are likely to be bored with Math 201 in its present form and to resent the expenditure of their time and of their money on the course. They deserve an alternative. One possibility would be to allow them to waive the course by examination. Another possibility is to offer an honors section of the course each quarter. Such a section would cover the usual topics of Math 201 at a level substantially above that of the main section of the course. For example, in the area of set theory, students could study the theory of infinite sets or perhaps Boolean algebra, which could then be compared with symbolic logic. In the area of number theory, students could see a proof of the fundamental theorem of arithmetic and study various properties of primes. In the area of mathematical systems, students could study the properties of a group and a field explicitly and see numerous examples of these concepts. They also could study arithmetic in a historical perspective. The opportunity to use Math 201 to develop well-trained elementary mathematics teachers should not be neglected.

There is also the problem of students (such as Students F and G) who are underprepared for Math 201. Such

students must of necessity pay more attention to sharpening their deficient arithmetic skills than to the substance of Math 201. (This phenomenon also occurs in some instances among those students who generally are prepared. Student D said that before taking this course she never had understood percent, and Student H said that she never before had understood the meaning of fractions and of decimals.) They have little hope of mastering the mathematical content of the course. Unfortunately, there are many such students. This writer taught Math 201 in the spring of 1974 and administered a placement test of 40 arithmetic questions. A student is supposed to score 32 out of 40 correct even to enroll in Math 201, yet the median score in the writer's class was 26. A student who cannot do arithmetic will do poorly in Math 201, and the presence of substantial numbers of these students cannot but lower the quality of the course. (It should also be kept in mind that those who pass Math 201 will be teaching arithmetic to children.) This writer recommends that admission to Math 201 be restricted to those who have passed a suitable arithmetic placement test. Such a procedure would place those students needing remedial work in the proper courses, and should significantly raise the mathematical level of instruction in Math 201.

Finally, there is the problem of those students (such as Students B and L) who are competent in arithmetic but bear a very negative attitude toward the subject. The lecture-laboratory format of Math 201 did nothing to improve

the attitudes of the study subjects in this group. Efforts should be made to identify such students and to direct them into alternative presentations of the course. Research studies should be undertaken to determine whether any alternative format of Math 201 (such as an all-laboratory format or a joint content-and-methods format) leads to an improvement in the attitudes of such students.

Evaluation of Method

In order to evaluate the method used in this study, the two questions for which the study was designed to provide answers must first be recalled:

(1) How do prospective elementary school teachers think about the mathematical concepts taught them in their required college mathematics course?

(2) How do students react to the various features of the presentation of this course, and how might this presentation be improved?

It must then be asked how successful the method used in this study was in providing answers to these two questions.

The advantages and disadvantages of the interview method were discussed at length in Chapter 3. It must be concluded here that this method proved to be fruitful in providing answers to the first of the two questions above. A written instrument, which would have been far more economical of the time of all concerned than were the interviews, can be effective in determining whether students do

or do not understand a particular topic. However, as the above question indicates, the goal in this study was to ascertain not only which concepts students do and do not understand, but to probe into how they actually think about these concepts. Faced with a test question which she does not understand, a student will likely leave the item blank or else display written work which gives little insight into the misconceptions held by the student. (This is particularly true of a course like Math 201 whose primary emphasis is on the understanding of concepts rather than on the mastery of various algorithms.) However, a probing interviewer can determine those misconceptions held by the student who has not mastered a particular concept.

The following findings of the study illustrate this advantage of the interview approach:

1. There was a general tendency among the subjects, when asked to find a set matching a set of living people, card suits, or aardvarks, to look explicitly for those objects in the sets which were supposed to match. This revealed dramatically that these subjects misunderstood the meaning of the word "match," confusing it with "equal."

2. In the same set of exercises, many subjects used the expression "the empty set" to mean "none of the above." This revealed a tendency to misuse the language of sets. It is unlikely that a written instrument would have revealed this misconception.

3. Subjects displayed various reactions to the problem of determining whether or not a given number is prime. A few used inspection rationales, based on the "look" of the number. Others displayed varying degrees of tenacity in testing divisors, ranging from a few perfunctory attempts with small divisors through an exhaustive testing routine up to the square root of the given number. In this area, it was also surprising that a number of subjects found the Sieve of Eratosthenes confusing.

4. Even though most of the subjects knew how to find a greatest common factor and a least common multiple, their solutions to these problems revealed that they had a purely mechanical, algorithmic view of these processes and were generally ignorant of the self-explanatory nature of the names of these concepts. A written instrument in this case would have revealed only the mechanically correct solutions, concealing the lack of understanding behind them.

5. Some subjects' idea that the area of a plane figure remains constant under boundary deformations was revealed in their responses to the second measurement problem. This type of question could not be given in a written format.

6. Some subjects felt that the geoboard explained some ideas which it in fact does not. Among these ideas were the areas of the circle and of irregularly shaped plane figures.

In addition to the above general trends, the interview method also provided insight into the idiosyncrasies of the various subjects. Student A displayed a stubborn

resistance to new mathematical ideas. Student B repeatedly revealed her view of the uselessness of post-arithmetic mathematics and her tendencies to depend on formulas and to work backward from given solutions. Student C showed that despite a good understanding of the course material, she was capable of sloppy work in arithmetic and of acquiring such egregious misconceptions as the one revealed in her statement that $7/24$ is not a rational number. Student D told of the impression made upon her by a teacher who told her that "story problems are very hard." Student F was the only subject who did not factor numbers when trying to find their greatest common factor or least common multiple; she unsuccessfully tried to find these by listing factors or multiples of the given numbers. Student H, in addition to her very sloppy work in testing numbers for primeness, displayed an astonishing inability to see simple or familiar mathematical ideas in verbal problems containing language describing situations with which she was unfamiliar, as revealed in her comments on the problems involving gin rummy and measurement in centimeters. Student L revealed deep feelings of hostility toward mathematics teachers at her final interview. Student M, like Student A, strongly resisted new mathematical ideas, solving problems by habits which had become mechanical to her. Student N revealed her obsession with tests, an obsession which apparently led her to become a very skillful test-taker; she was the only subject to use decimals in finding a rational number between

1/3 and 1/4, but also tried to find a fraction name for .3777... by trial and error, thought that the formula $A = \pi r^2$ was derived from the geoboard, and thought that $\sqrt{7}$ is half of 7. Student P could not say whether the set of all living people is finite or infinite, displayed an incompetence in working with bases which was unusual for a student with her ability and background, and was not disturbed when, due to an error, she found 1/3 to be the fraction name for .3777....

With regard to the second question, it is not as clear that a written instrument would not have been as fruitful as was the interview. However, the interview did provide much data regarding the subjects' reactions to various features of the lecture, textbook, and laboratory. Although the investigator was frustrated by the subjects' general failure to respond to his repeated questions about whether any specific features had helped them, there were some features which were widely commented upon, such as the "array" depiction of rational numbers as cut-up squares (where comment was mostly positive), and nonstandard algorithms such as Austrian subtraction (where comment was mostly negative). There were more comments on the laboratory exercises (these reactions were discussed earlier in this chapter). The investigator feels that it is probable that more information of this type was gathered in this study than would have been gathered by a written instrument. The study format--an interview every two weeks in which the interviewer

could probe the subject's feelings--allowed the subject to discuss features of the course which were fresh in her memory and which probably would have been forgotten by the end of the term. However, even if written evaluations were to be filled out frequently by the students, it is likely that they would reveal less information than they did verbally to a disinterested interviewer. For one thing, talking is easier than writing--and responses can be probed by the interviewer. Furthermore, students would probably feel more inhibitions over expressing (even anonymously) a negative opinion of the course presentation to one who is responsible for that presentation (i.e., a member of the instructional staff) than to a disinterested party. For these reasons, this writer suspects that a written instrument would not have produced as much data in response to the second question as did the interviews.

Nevertheless, any research method has limitations. One of the limitations of the present study was that it was possible to work with only a small sample of students because the interviews consumed a great deal of time. This leads to the question of how representative the study sample was of the course population. The differences between the study sample and the course population as a whole were enumerated at the beginning of Chapter 4. It is arguable whether or not the sample obtained accurately represents this population. At the end of the course, eight of the fifteen subjects received the maximum grade of 4.0, with one receiving a 3.5,

three a 3.0, one a 2.5, one a 2.0, and one a 0.0; according to the lecturer, this distribution was substantially higher than that of the class as a whole. This could have been due to the study subjects' greater mathematics aptitude as reflected in their reported high school grades. However, it must be mentioned that participation in the research study may have affected the subjects' performance in the course. Even though their participation was completely separate from their course work, the knowledge that they would have to solve problems in front of the investigator every two weeks may have caused them to be more diligent in their studies than they would have been otherwise. Considering this situation and the fact of their generally higher grades, one should read this dissertation with the reasonable supposition that these subjects, as a group, had a greater mastery of the course material than their classmates.

It also must be assumed that the subjects were honest in their comments on the course and in their problem-solving behavior. When the investigator thought he detected insincerity in a subject, this feeling was reported in the appropriate place in the profile of the subject. It must also be said that all the information in the reports of the interviews was gathered, organized, and presented by the investigator, so that the reader cannot but view the subjects through his eyes. The investigator has strived to be as objective as possible in reporting the results of the interviews.

Another limitation of the study which must be mentioned is that the particular mathematical strengths and weaknesses shown by the subjects are to a considerable degree a function of the particular personnel teaching the course at that time. One instructor may teach one topic well and another topic poorly, while a different instructor may reverse the situation. It must be assumed that Math 201, when presented in this format, is sufficiently consistent in nature that the results of the study can be applied to future offerings of the course. This was a basic assumption of the present study, and the investigator feels that the study was productive in that it identified areas in which instruction needs to be improved.

It also must be mentioned that many mathematical ideas taught in Math 201 were not investigated in this study. This was due in part to time limitations and in part to the difficulty of writing problems which would reveal students' thoughts on these topics. Such topics include operations of arithmetic, field properties, and negative numbers.

Despite the limitations mentioned above, this study did provide adequate data to suggest some answers to the two questions under consideration. The implications of the investigator's experience with this method for future research must now be discussed. First, the interview remains the best method for observing the mathematical thought patterns of the subjects, as was concluded by Brownell and Watson as long ago as 1936. This method should be used in such studies whenever feasible. The fruitfulness of a

particular study depends upon both the skill of the interviewer and the items about which the subjects are asked to think out loud. The investigator feels that he personally, after two pilot studies, had acquired enough skill in interviewing to perform this role competently in the main study. Hindsight suggests that more items and problems should have been presented to the subjects; the investigator would like to have seen their reactions to items based on the field properties and on various types of verbal problems. These questions must await further research.

Furthermore, the findings of the study regarding the second question should be acted upon before further research is undertaken. Evaluation of the recommendations for Math 201 made in this study should be performed by future researchers. In such studies, the interview method may not be necessary; written instruments may exist or may be developed which will be adequate to the task. However, the interview method proved useful in this study in identifying the areas of the course in need of revision. After a number of such revisions have taken place and have been fully evaluated, another interview study would be useful to detect any flaws that still exist or that may have been created.

Suggestions for Further Research

First, experimental studies should be conducted to test the feasibility and the effects of the reforms in Math 201 proposed earlier in this chapter.

Second, further research should be conducted into the mathematical thought of prospective elementary school teachers. The responses to Problem 3 revealed that many subjects had difficulty with the relationship among distance, rate, and time. Several subjects said they always had had difficulty with the idea of percent. A study such as the present one in which all the exercises were verbal problems of varying types (such as percentage, motion, mixture, interest, and measurement) would reveal how well prospective elementary teachers understand the uses of elementary mathematics in the world. It could also be used to examine the problems themselves to see which features of them tend to cause difficulty. Other mathematical topics not fully covered in the present study, such as negatives and field properties, should be investigated more fully in future studies.

Finally, studies should be undertaken to gauge the effects of alternative treatments of Math 201 (with respect to both understanding and attitude) on students who dislike mathematics. The findings of the present study indicate that such students had a negative response to the format used in the winter of 1974. An attempt should be made to identify such students and direct them into alternative formats, such as an all-manipulative format or a joint content-and-methods format. A research study could test if they showed improvements in understanding and in attitude over a similar group taking the course in a lecture-laboratory format. In addition, a study such as the present one using students taking Math 201

in such alternative formats should be conducted to see how the mathematical understandings and misconceptions which prospective elementary school teachers may acquire from such presentations tend to differ from those acquired in the lecture-laboratory format.

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