THE DETECTION AND IDENTIFICATION OF COMPREHENSIVE PROBLEM SOLVING STRATEGIES USED BY SELECTED FOURTH GRADE STUDENTS

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ABSTRACT

THE DETECTION AND IDENTIFICATION OF COMPREHENSIVE PROBLEM SOLVING STRATEGIES USED BY SELECTED FOURTH GRADE STUDENTS

By

Joseph Jennings Shields

For the past seventy-five years many persons, within and without the academic community, have called for more real life problem solving in the elementary schools. During the past ten years some programs have been developed to teach problem solving for specific subject areas and, in a few cases, to teach problem solving of an interdisciplinary nature. However, these programs have, in general, assumed a model of problem solving which is appropriate for adult subjects. The purpose of this study is to examine the problem solving behavior of selected fourth grade youngsters to detect what strategies they use in the process of solving open ended problems, and to determine if the process they utilize coincides with that of the model appropriate for adult subjects.

A popular view of human problem solving is based on an informational processing approach to problems. The

solver processes the problem in four steps: (1) clearly define the goal of the problem, (2) generate a number of alternatives, (3) synthesize information concerning these alternatives, and (4) select the solution which is most consistent with the goal.

Once the problem is defined, one or more search strategies can be employed to aid the solver in reaching the optimal solution. These strategies include: (a) making inferences, (b) looking at related problems, (c) identifying subgoals, (d) use of contradiction, and (e) working backwards.

Procedures

Three male and three female subjects were randomly selected from each of two fourth grade classes, the classes being denoted the experimental and control classes. The particular classes were chosen by the researcher because of the students openness and independence shown in self-initiated activities.

The study involved five interviews for each of the twelve subjects, and was conducted at regular intervals over a four month period. The purpose of these interviews was to gather information concerning the manner in which the students solved comprehensive problems.

During the study period the experimental class received some problem solving training. The results of these interviews were analyzed to suggest answers to the following questions:

a) Is there an identifiable process which elementary school children use in solving open ended comprehensive problems?

b) Which heuristics are used by elementary school children engaged in solving these comprehensive problems?

c) Does problem solving training affect the process or strategies used by the successful fourth grade problem solver?

d) Do youngsters who are engaged in regular problem solving activities become more mature in their view of the nature of problems, solutions, and the means of attaining these solutions?

Problems and Results

For each of the five interviews conducted, the same four step problem solving process was apparent in those cases where the subject reached meaningful solutions.

The first interview involved finding a solution to an evaluation type of problem: Determine which of five given notebooks is the best for a school district to buy for all fourth graders. The generalized problem solving strategy of making inferences was used by all of the successful solvers. Designing a classroom suitable for fourth grade youngsters was the focus of the problem in the second interview. Subjects used contradiction, made inferences, identified subgoals, and looked at related problems in reaching solutions to this problem. These strategies were also evident in the solution to the third challenge of planning an ice skating party for twenty-five children, and in the last interview, concerned with planning and laying out a playground.

The fourth interview centered on the solution to a describing people problem: Determine which characteristics are most useful in describing a person. Due to the students inexperience with a sampling type of problem, subjects made little progress in reaching partial solutions, and did not employ generalized strategies in this problem.

Conclusions and Recommendations

The researcher concluded that fourth grade children do employ an identifiable process in addressing comprehensive problems, and this process sometimes involves the use of one or more problem solving strategies. There were no differences in the quality of solutions given by either the experimental or control class. The solving process and strategies became more refined as the study progressed, and indicates a growth in problem solving ability of these subjects.

Based in part on these conclusions, the researcher recommends that more problem solving activities be carried out at all grade levels. Secondly, new and existing programs in problem solving training should be structured to build on the natural processes and strategies of youngsters.

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By

Joseph Jennings Shields

A DISSERTATION

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Dedicated to,

My Father and Mother,
Joseph and Mary Shields, Sr.

My Wife,
Kathryn

My Children,
Joseph III, my son
Christine and Ann, my daughters.

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CHAPTER I

THE PROBLEM

Background and Need for the Study

The cry to make "real world" problems a prominent part of the curriculum has been heard at various times during the past seventy-five years. At the turn of the century the "Perry Movement" in this country received great impetus from E.H. Moore in his presidential address[1] before the American Mathematical Society at its annual meeting in 1902. During this address he stated: "The fundamental problem (in the pedogogy of elementary mathematics) is that of the unification of pure and applied mathematics." Later, the Progressive Education Association report of the Committee on the Function of Mathematics in General Education[2] in 1938 included a chapter on some concepts basic to problem solving. The Committee felt very strongly that "the major role of mathematics in developing desirable characteristics lies in

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[1]E.H. Moore's presidential address before the American Mathematical Society at its ninth annual meeting, December 29, 1902. The address was printed first in Science n.s. 17 (March, 1903).

[2]Progressive Education Association, Mathematics in General Education: A Report of the Committee on the Function of Mathematics in General Education. 1938. In Readings in The History of Mathematics Education, NCTM (Washington D.C.: 1970), pp. 534-566.

the contribution it can make to growth in the abilities involved in . . . problem solving."

To teach students to solve "real world" problems, educators have emphasized mathematical skills and problem solving techniques which could be learned by solving simplified models of "real" problems. This distinction between problems which are personal to the student and exercises presented in class has produced varied meanings to "problem solving."

Written material on the methods of problem solving has appeared at various times during the last three score of years. Perhaps the most widely read book on this subject is How To Solve It by G. Polya[1] which presented the basic concepts of problem solving and strategies for teaching problem solving. During the sixth decade of this century the twenty-first yearbook of the National Council of Teachers of Mathematics contained a chapter on "Problem Solving in Mathematics."[2] This chapter discussed the theory of problem solving and the implications of this theory for the classroom.

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[1] G. Polya, How to Solve It, (2nd ed., Garden City, N.Y., Doubleday and Company, Inc., 1957).

[2] Kenneth B. Henderson and Robert E. Pingry, "Problem Solving in Mathematics," The Learning of Mathematics: Its Theory and Practice, Twenty-first yearbook of the NCTM (Boston, Houghton Mifflin Co., 1953).

General educators and learning theorists such as Thorndike and Ausubel[1] have investigated problem solving; they state the need for problem solving practice cannot be overemphasized. O'Brien and Shapiro[2] interpret Piaget as saying that the child is liberated from his egocentric view of reality by social interaction and by confronting problems in his daily experience. Hence it is "problem (solving) . . that is a central vehicle for cognitive growth."[3] Finally, mathematics educators in the post "New Math" years continue to support and encourage problem solving activities. For example, Johnson and Rising[4] state, "learning to solve problems is the most significant learning in every mathematics class. . ."

The impact of this emphasis on problem solving, by virtually every major reform suggestion of this century, is

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[1]Robert L. Thorndike, "How Children Learn the Principles and Techniques of Problem Solving," Learning and Instruction, Forty-ninth yearbook of the National Society for the study of Education, (Chicago: University of Chicago Press, 1950), pp. 191-216; David P. Ausubel, "Learning by Discovery," Educational Leadership, XX (January, 1969), p. 117.

[2]Thomas C. O'Brien and Bernard J. Shapiro, "Problem Solving and the Development of Cognitive Structures," The Arithmetic Teacher, XVI (January, 1969), p. 11.

[3]Ibid., p. 12.

[4]Donovan A. Johnson and Gerald Rising, Guidelines for Teaching Mathematics, (Belmont, California: Wadworth Publishing Company, 1967), p. 104.

apparent in the Cambridge report of 1963 and in the School Mathematics Study Group (SMSG) report of a Conference on Secondary School Mathematics, 1966. The first of these reports states, ". . . the problem material (in textbooks) should get at least half of the time and attention of the authors."[1] And in the March, 1966 meeting of the SMSG Conference, the committee on problem solving stated, ". . . the activity of constructing and analyzing mathematical models of scientific and life situations is, apart from technical questions, the principle link between mathematics and the rest of civilization."[2]

The SMSG Secondary School Mathematics Units were developed during the late sixties to ". . . make clear to all students that mathematics is indeed useful, that it can help us in understanding the world we live in and in solving some of the problems that face us."[3] The fourteen units in this program are an integrated plan for high school mathematics containing substantial material on problem solving. The committee on problem solving wanted "understandable remarks and digressions on models and modeling and their various aspects (to) be liberally . . . sprinkled throughout the

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[1] Goals for School Mathematics, The Report of the Cambridge Conference on School Mathematics, (Boston: Houghton Mifflin Co., 1963), p. 28.

[2] SMSG: A Report of a Conference on Secondary School Mathematics, March 14-16, 1966.

[3] The Panel on Secondary School Mathematics of the School Mathematics Study Group, SMSG: Secondary School Mathematics, Leland Stanford Junior University, 1971, page i.

mathematical education process."[1] Problem solving material appears in nine of the twenty eight chapters and deals with problem formation, use of diagrams, tables, guesses, and deductive reasoning.

Other mathematics programs, developed during the sixties, utilized problem solving as an integral part of the teaching and learning process. For example, the Minnesota School Mathematics and Science Teaching Project (MINNEMAST) and the Madison Project both challenged the student to discover solutions to problems through experimentations, collecting data and making conjectures. However, neither SMSG, MINNEMAST, nor the Madison Project had comprehensive problem solving as a major theme. Indeed, the recent conference on the K-12 mathematics curriculum held in Snowmass, Colorado during June, 1973 concluded, "the process of solving problems of any description has not been given due attention in curriculum material."[2] Throughout the conference, the topic of problem solving was a continual and pervasive subject of discussion. The conference report contains the following suggestions:

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[1]Ibid.

[2]The Report of the Conference on the K-12 Mathematics Curriculum, Snowmass, Colorado, Mathematics Education Development Center, Indiana University, 1973, p. 33.

1. There is a need for a center or centers for basic research in how children (students) solve problems.

2. There is a need for this same center to develop curriculum material for both school students as well as teachers.[1]

There have been programs which attempted to cultivate problem solving skills and creativity in the learner. For example, the Creative Education Foundation at the State University of New York at Buffalo has concentrated on the improvement of adult creative problem solving through special courses.[2] This foundation published a student workbook, instructor's manual and visual aids for use in teaching creative problem solving courses for business, adult education, military and university groups.[3] A program developed at the University of Minnesota to nurture creativity in Elementary School children was titled "Invitation to Thinking and Doing" and was under the direction of R. E. Myers and E. Paul Torrance.[4] Currently, there are several universities such as University of California at Los Angeles

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[1] The Report of the Conference on the K-12 Mathematics Curriculum, p. 34.

[2] Alex F. Osborn, Applied Imagination: Principles and Procedures of Creative Problem Solving, (3rd Rev. ed.), New York: Charles Scribner's Sons, 1963.

[3] S. J. Parnes, Creative Behavior Guidebook, New York: Charles Scribner's Sons, 1967.

[4] R. E. Myers and E. Paul Torrance, Invitation to Thinking and Doing, (Bureau of Educational Research, University of Minnesota, Perceptive Publishing).

and Massachusetts Institute of Technology which offer courses in problem solving to undergraduates.

During the academic years 1974-1975 and 1975-1976 the American Association for the Advancement of Science, with support from the National Science Foundation, conducted several short courses for college teachers entitled "Patterns of Problem Solving."[1] This course is designed to provide a new dimension in higher education that crosses boundary lines between disciplines. It is designed to provide the attitudes and skills necessary in dealing with complex problems, from the statement of the problem to its creative solution. The National Science Foundation also supported a new program, located in centers at Indiana University, The University of Northern Iowa, and The Oakland Intermediate School District, Oakland County, Michigan. This new program is developing curriculum material for the teaching of problem solving to elementary school pupils, using the hand held calculator.

At the elementary level, the only current program of widespread implementation whose major theme is problem solving is the Unified Science and Mathematics for Elementary School (USMES) program. The formulation of the USMES project was in response to the recommendation of the 1967 Cambridge

[1] Announcement: NSF Chautauqua - Type Short Courses for College Teachers, Academic Year 1974-1975 (Office of Science Education, Washington, D.C.).

Conference on the Correlation of Mathematics and Science in the Elementary School.[1] Since its inception in 1970, USMES has developed units which center on long range investigation of real and practical problems taken from the local school and environment of the community.[2]

Purpose of the Study

The purpose of this study is to collect information about the problem solving behavior of selected fourth grade students, and analyze this information to detect and identify the strategies used by those youngsters in addressing comprehensive challenges. This case study of five interviews with twelve subjects, randomly selected from two classes, seeks to find regularities in their solving process. From these regularities the researcher will suggest answers to the following questions:

a) Is there an identifiable process which elementary school children use in solving open ended comprehensive problems?

b) Which heuristics are used by elementary school children engaged in solving these comprehensive problems?

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[1] Goals for the Correlation of Elementary Science and Mathematics, Houghton Mifflin Co., 1966.

[2] USMES Guide, Education Development Center, Inc., Boston, June 1973.

c) Does problem solving training affect the process or strategies used by the successful fourth grade problem solver?

d) Do youngsters who are engaged in regular problem solving activities become more mature in their view of the nature of problems, solutions, and the means of attaining those solutions?

Definition of Terms

No single definition of a meaningful problem and problem solving as a process is agreed upon by mathematics educators. For example, at the Snowmass Conference the meaning of problem ranged from "the simple arithmetical examples usually found in elementary texts, to the more formal mathematical problem sets associated with abstract mathematics."[1] Polya defines what he thinks it means to have a problem by saying that the individual searches ". . . consciously, for some action appropriate to attain a clearly conceived, but not immediately attainable, aim."[2] Dodson defines a problem by suggesting that it has the following characteristics:

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[1] The Report of the Conference on the K-12 Mathematics Curriculum, Snowmass, Colorado, p. 33.

[2] George Polya, Mathematical Discovery: On Understanding, Learning, and Teaching Problem Solving, (John Wiley & Sons, Inc., 1972), p. 117.

1) It is a situation or question for which the student does not already possess a solution, or answer, or a method of obtaining the solution.

2) It is solvable using previous learning.

3) It cannot be solved by simple recall from memory or the standard use of a computational algorithm.

4) Its solution does not depend on a special trick.[1]

Robinson, Tickle, and Brison define a comprehensive problem as "a question that leads to alternatives, and where the solver must synthesize additional information to decide which alternative is the best answer to his question."[2] Johnson and Rising define problem solving as "finding the appropriate response to a situation which is unique and novel to the problem solver."[3] All of these definitions agree that the subject has a goal, or aim, which is temporarily unattainable or blocked. Non-mathematical writers such as Vinacke and Williams[4] agree on this characterization of a problem.

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[1]Joseph Dodson, Characteristics of a Successful Insightful Problem Solver, (NLSMA Report No. 31, School Mathematics Study Group, 1972), p. 9.

[2]F.G. Robinson, J. Tickle, and D.W. Brison, Inquiry Training: Fusing Theory and Practice (Ontario Institute for Studies in Education, Toronto, Ontario, 1972), p. 4.

[3]Donovan A. Johnson, and Gerald Rising, Guideline for Teaching Mathematics.

[4]Edger W. Vinacke, The Psychology of Thinking, (N.Y.: McGraw-Hill Book Co., 1952), p. 160; Frank Williams, Foundations of Creative Problem Solving: Principles and Applications, (Ann Arbor: Edwards Bros., Inc., 1960), p. 20.

In this study, the researcher shall use problem solving to mean that activity the student engages in whenever he or she consciously looks for answers to a problem which:

1. Presents a situation or challenge for which the student desires a solution and to which there is no immediate correct answer.

2. The student can arrive at a solution compatible with his understanding of the problem and his maturation.

3. Solutions cannot be found by employing special tricks or a simple sequence of algorithms, i.e., it is complex.

This definition is global enough to include a wide variety of problems, and implies that each problem has some condition or conditions which are given, as well as transformations which can be used on the given to reach the goal or aim. It is assumed that the student desires a solution either as an end in itself or as a means to an end, since problem solving is an active cognitive process. The student who already possesses a solution or does not desire a solution will not act on the givens in a problem to arrive at a solution. Furthermore, comprehensive problems may have more than one "correct" solution, and thus, the choice of a solution is dependent on the solvers understanding of the problem and desired goal. Finally, the problem must be complex and require the solver to work at the higher cognitive levels. This third condition eliminates most of those problems referred to as exercises, in many texts, which can be solved by simple recall or the use of an algorithm presented in class. Each of the five problems solved by the subjects in this study satisfy the conditions listed above.

The solution of a problem is a complete plan which can be carried out to arrive at the precise goal the solver desired. Note that a problem has reached a solution when the learner is aware of the correct course of action leading to the goal, irrespective of whether the plan is implemented. Therefore, to evaluate the "correctness" of a solution it is necessary that the researcher have the solver report a solution, or indicate the plan which will lead to the solution. This definition of a solution is equivalent to the operational definition which Wickelgren gives, ". . . a solution is a sequence of allowable actions that produces a completely specified goal expression."[1] Polya also agrees with this characterization of a solution as a plan leading to the goal or aim.[2]

To reach a solution, the solver will often use procedures, called strategies, which aid in organizing data and eliminating alternatives. Kilpatrick[3] calls these

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[1]Wayne A. Wickelgren, How To Solve Problems: Elements of a Theory of Problems and Problem Solving, (W.H. Freeman and Co., San Francisco, 1974), p. 16.

[2]George Polya, Mathematical Discovery: On Understanding, Learning, and Teaching Problem Solving, p. 118.

[3]Jeremy Kilpatrick, Analyzing the Solution of Word Problems in Mathematics, Unpublished doctoral dissertation, Standord University, 1967, p. 19.

strategies, heuristics, and defines them ". . . as any rule, technique, rule of thumb, etc., that improves problem solving performance." Covington[1] lists thirty-eight of these rules of thumb for problem solving, and McEver,[2] in Strategy Notebook: Tools for Change, provides fifty-nine strategies for youngsters to practice.

In this study a generalized problem solving strategy, or simply a search strategy, will mean one of five general procedures which can be used in open ended problem solving. These five are: (1) inference, (2) using a related problem, (3) identifying subgoals, (4) contradiction, and (5) working backwards.

Procedures

Two fourth grade classes, from schools near a large metropolitan area, were selected based on the following criterion:

1. Students learned in an environment where individual feeling, creativity, and intellectual activity were highly valued.
2. Students could accurately express their opinion when questioned.
3. Students had never worked on, or participated in, any problem solving program.

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[1]Martin V. Covington, "An Experimental Program for Increasing Ingenuity in Visual Problem Solving," University of California, Berkley, 1968.

[2]Catherine McEver, Strategy Notebook: Tools for Change, (Interaction Associates, Berkley, California, 1969).

4. Students were male and female, and from diverse socio-economic backgrounds.

Three males and three females were selected randomly from each of the classes described above. A two-way analysis of variance was computed to help determine whether the classes were similar in problem solving ability, using scores from the Purdue Elementary Problem Solving Inventory as the independent variable. This analysis supported the hypothesis that there was no difference in the problem solving ability of the two classes, or between sexes, at the onset of the study.

One class was differentiated from the other by having the teacher integrate problem solving activities into the usual classroom work during the duration of the study. These activities took one of two forms: 1) a comprehensive challenge of the USMES type, or 2) a short puzzle type problem.

The study involved tracing the behavior of the twelve students during five interviews, distributed over a four month period, in which they attempted to solve open ended comprehensive problems. All of the interviews were conducted in a comfortable classroom of the school building, and during each interview only one subject and the researcher were in the room. The interviews were recorded on a cassett recorder, and observations made by the interviewer were transcribed. These sixty interviews were analyzed to find evidence to support or reject the claim that youngsters use an identifiable process and specific strategies when solving open ended problems.

The use of the clinical paradigm was appropriate for this study, as the problem solving process used by the subjects had to be analyzed in great detail to identify strategies, and because any variation in the process or strategies was of interest.

Assumptions and Limitations of the Study

For purposes of this study, the following assumptions were applied:

a) That the comprehensive problems used during the interviews were appropriate and interesting to the subjects.

b) That all of the subjects wanted to participate in the study.

c) That variations between the subjects have a random effect on the results, and do not produce erroneous conclusions.

d) That the setting and population in which the study was conducted was not so unusual that the outcomes, within limitation, could not be used to generate hypotheses for similar populations.

This study was designed and undertaken with the following restriction: Only students in the sample were in the study, and any generalization of the results is limited to population similar to the experimental group.

CHAPTER II

THEORY AND SUPPORTING RESEARCH

Introduction

The theory and research related to comprehensive problem solving shall be presented in two parts. Part one deals with a theory of problems and problem solving and is used as a rationale for those strategies which may be useful to elementary school children in solving problems. Part two reviews the literature on problem solving and creativity and those variables which may influence problem solving skill.

PART ONE

A Theory of Problem Solving

Learning theorists have provided a good deal of material on which to build sound hypotheses for problem solving. In 1956 Bloom[1] and his associates prepared a comprehensive model descriptive of the possible cognitive levels. Bloom's taxonomy classified the various thinking processes, from simple recognition and recall to the most general process of open search, into levels: Knowledge, Comprehension, Application, Analysis, and Synthesis.

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[1]B.S. Bloom, et al., Taxonomy of Educational Objectives: Handbook I: Cognitive Domain, (New York: McKay Publishing Co., 1956).

Problem solving is included in the highest cognitive level by Bloom. Avital and Shettleworth[1] also defined a taxonomy, somewhat simpler in appearance, analogous to Blooms and again insisted that solving open ended problems involved the highest cognitive processes. Gagne considers problem solving as the highest form of learning wherein the solver does not apply previously learned rules but must "discover a combination of previously learned rules that he can apply to achieve a solution."[2]

Although problem solving is the highest form of learning for Gagne, he asserts that it is possible to teach at this level provided requisite rules have been learned. According to Gagne, persons who engage in problem solving develop a collection of "higher-order rules, which are usually called strategies"[3] and which provide guidance to the thinking process. Bruner agrees that it is possible to teach children problem solving and states, "there are certain general attitudes or approaches towards subjects that can be taught in earlier grades that would have considerable relevance for later learning."[4] He continues by

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[1]Shmuel M. Avital and Sara J. Shettleworth, Objectives for Mathematics Learning: Some Ideas for the Teacher, (Bulletin No. 3, 1968, The Ontario Institute for Studies in Education).

[2]Robert M. Gagne, The Conditions of Learning, (2nd. edition, Holt, Rinehart and Winston, Inc., N.Y., 1970), p. 214.

[3]Ibid., p. 215.

[4]Jerome S. Bruner, The Process of Education, (Vintage Books Edition, August, 1963, Random House, Inc., N.Y.), p. 43.

saying, "it might be wise to assess . . . (which) heuristic devices are most pervasive and useful, and that an effort should be made to teach children a rudimentary version of them that might be further refined as they progress through school." The identification and teaching of strategies which are useful in solving open ended problems depends on ones conception of the various components of a problem.

Components of a Problem

All problems can be considered to be composed of three types of information: first, information concerning the givens; second, information concerning operations which can be used to transform the givens, and third, information concerning goals.[1] The givens are those expressions which are present when the problem is posed. In problems such as finding the distance from a point S(2,3) to the line given by $3x + y - 2 = 0$, the givens are the point and line in the plane as well as assumptions, definitions and theorems from Analytic Geometry. In puzzle problems, such as 'Instant Insanity,' the givens are the four cubes, each of which has one of four colors on each side. In comprehensive problems the givens are all those objects, materials, data, axioms, and definitions, stated or implied which are known to exist after the statement of the problem. Few problems explicitly specify all of the givens. However, it is usually the case

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[1]Wayne A. Winkelgren, How To Solve Problems: Elements of a Theory of Problems and Problem Solving, (W.H. Freeman and Co., San Francisco, 1974), p. 10.

that some of the givens are implied, for example, a knowledge of elementary Calculus is at one's disposal in solving physics problems even though this fact is not generally explicitly stated.

An operation is an action which you are permitted to perform on the givens which can transform the givens into expressions containing the goal or a subgoal. For example, in a chess game the problem is to find a way to checkmate your opponent as quickly as possible and the allowable operations are the ways each piece can move. For comprehensive problems the allowable operations permitted on the givens are often limited only by the solvers imagination. For example, in a problem to find ways to increase profit in a student business, students can examine past profits statistically, survey prospective consumers for possible improvements, and examine past marketing and manufacturing techniques, as possible transformations on the givens.

The goal is the terminal expression which the solver wishes to exist in the world of the problem. Note that the goal state may not be completely specified in the statement of the problem. This is the case in problems where the solver is "to find" a quantity. For example, 'find the value of x which satisfies the equation $x^3 + 2x^2 - x - 2 = 0$' does not specify the numerical value of the solution and yet the goal is clear to the solver. To achieve the goal state the solver may have to pass through one or more intermediate problem states. A problem state is the set of all the expressions that exist in the world of the problem at a particular time. A search strategy for solving a problem is a plan which directs the learner from initial representation, through any necessary intermediate problem states, to a final goal state.

Sequence of Events Involved in Problem Solving

The process of solving problems has been divided into stages by various authors. Dewey, Polya, Johnson and Gagne[1] generally agree that these stages include interpreting the problem, devising a plan, producing solutions, and deciding among solutions. Because of the vast differences in types of problems Gagne suggests that this model is not applicable in all situations nor is the solver totally aware of the occurrences of such stages in actual problem solving. Nonetheless, this sequence model is useful in organizing a discussion of problem solving and in identifying the strategies which will aid the solver in reaching the goal state.

Presentation of the Problem

Problems may arise as a result of some external stimuli such as finding a parking place as close to the theater as possible yet in a place free of parking

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[1] J. Dewey, How We Think, (Boston: D.C. Heath and Co., 1933); G. Polya, How To Solve It, (2nd, edition, Doubleday and Co., 1957); D.M. Johnson, The Psychology of Thought and Judgement, (New York: Harper & Row, 1955); R.M. Gagne, The Conditions of Learning, p. 217.

restrictions. On the other hand, the problem may arise as a result of some internal thought process as in the case of a good friend who adopts a moral view radically different from our own, making it necessary to reassess the friendship. Whether the problem is externally presented or not, the solver becomes aware that there is a problem which he wants to solve. Parnes refers to this first presentation of a situation requiring action as the "fuzzy problem"[1] which needs careful definition.

Problem Definition

The first step in problem solving involves carefully defining the problem in precise terms. Stating a problem forces some kind of representation. Posner[2] states, "the initial representation of a problem may be the most crucial single factor governing the likelihood of problem solution." Although the initial definition of the problem may be less important in a comprehensive problem, many books on problem solving such as the ones by Polya and Osborn,[3] emphasize that the initial representation should not be made too

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[1]Sidney J. Parnes, Student Workbook for Creative Problem Solving Courses and Institutes, (3rd. Rev., 1963, State University of New York at Buffalo).

[2]Michael I. Posner, Cognition: An Introduction, (Scott, Foresman Series, Scott, Foresman and Company, Glenview, Ill., 1973), p. 149.

[3]G. Polya, How To Solve It; Alex F. Osborn, Applied Imagination, (Rev. ed., N.Y.: Charles Scribner's Sons, 1957).

quickly. Parnes[1] suggests that the solver ask himself various questions to aid in viewing the problem from different perspectives. For example, suppose that a group of students are asked to solve the following problem:

> On one shool bus, an eight-year-old boy struck and bit other children and kept the entire busload in a noisy, confused state. Due to the disturbances, the driver nearly had several accidents. He pointed this out, but the boy jeered and continued behaving the same way. When the boy's victims complained, the school principal tried to punish the rowdy by keeping him in at recess. This had no effect, nor did it appeal to his parents. The driver reported that he was as bad as ever and was cautioned by the principal that anyone who spanked him might be sued or prosecuted. What should the bus driver do?

To aid the solver in viewing this problem from several vantage points Parnes suggests that the solver should carry out some "fact-finding" before attempting to define the problem. The students may wish to know: How does the boy act in the school building? What are his special interests? Does he have personal or family problems?

There is evidence to suggest that the initial representation of a problem is influenced both by what is in the problem and by what is in the problem solver. Humphrey and DeGroot[2] both concluded that the difference between average chess players and chess masters was the way the masters initially represented a problem in determining the next move

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[1]Sidney, J. Parnes, Student Workbook for Creative Problem-Solving Courses.

[2]B. Humphrey, Directed Thinking, (New York: Dodd, Mead & Co., 1948); A. D. DeGroot, Thought and Choice in Chess, (The Hague: Mouton & Co., 1965).

in a chess game. The less gifted players reasoned in the same way as the masters after the initial representation was reached. However, the time to this initial representation was considerably longer in the average player. This suggests that experience in solving problems of one type aids in initially representing problems of that type.

The factors which influence the way a problem is coded by the individual has been studied by several individuals. Duncker[1] concluded that if an object had one established use in a given situation, it was difficult to use the object in another way. He refers to this condition as functional fixity. Glucksburg, Weisburg and Danks[2] investigated this phenomenon for objects which were labeled and initially coded with that label. They concluded that subjects who coded the object in memory, by the label, were at a disadvantage in solving problems which called for uses not associated with the label.

This evidence suggests that for a period prior to careful definition of the problem the solver should consider different uses for the givens and view the problem from

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[1]K. Duncker, "On Problem Solving," Psychological Monographs, 1945, 58.

[2]S. Glucksberg and J. H. Danks, "Effects of discriminative labels and of nonsense labels upon availability of novel function," Journal of Verbal Learning and Verbal Behavior, 1968, 7, pp. 72-76; S. Glucksberg and R. W. Weisberg, "Verbal behavior and problem solving: Some effects on labeling in a functional fixedness problem," Journal of Experimental Psychology, 1966, 71, pp. 659-664.

various perspectives. After this initial encounter with the problem the solver should choose that formulation of the problem which most closely describes the precise goal the solver wishes to reach. For example, in the school bus driver problem, the students may decide that the best way to formulate the problem is: How might the bus driver achieve peace on the bus? This particular choice may come after several, less general, formulations of the problem such as: How might the bus driver settle this boy down? What might the bus driver do to isolate "problem children" from the others? What might the bus driver do to obtain a better understanding of the boy? How might the driver provide a safe and sane ride?

The first step in the problem solving process, interpreting the problem, is embodied in the definition of the problem. Any limitations on the givens and on possible transformations should be stated or implied in this definition. Since the definition of the problem describes the goal state, it is essential that any evaluation of the correctness of a solution be carried out in reference to the stated goal and its meaning to the solver.

Devising A Plan

The second step in the problem solving sequence is devising a plan or search strategy. Such plans are similar to hypotheses and tell the solver where to look in memory or what to examine in the external world in order to advance towards a solution. Posner[1] suggests that we "know very little about the development of these plans except that they appear to be a function of consciousness" and that the solver can often report them verbally. The development of a search strategy is at the heart of the problem solving process. At this stage of problem solving there are three strategies which will aid in formulating a plan of action; they are inference, identification of subgoals, and examining relations between problems.

Inference

All problems, clearly defined, present some of the relevant information in implicit, rather than explicit, form. This information is often critical in solving the problem. When presented with a clearly defined problem the solver must make inferences and draw conclusions from the given information which will render explicit statements from those implied by the givens. For example, consider the following problem from Analytic Geometry?

> Give an equation which describes the locus of points, C such that the line segment joining (-3,0) to any point of C is perpendicular to the line segment from that point of C to the point (3,0).

The solver could infer from the given information, 'the two line segments are perpendicular,' that the slopes of the two segments are the negative reciprocals of each other. Inferences can be drawn from implicit or explicit statements

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[1]Michael I. Posner, Cognition: An Introduction, p. 162.

of the givens or the goal state. The solver expands the goal or givens by bringing to bear all of the knowledge he has concerning the problem which is stored in memory.

Polya[1] suggests that the solver be guided in this search for relevant information by looking for conditions which relate the givens and the goal. Wickelgren states, "Drawing inferences (more generally, making transformations of the goal or the givens) is probably the first problem-solving method . . ."[2] the solver should employ. He continues by stating that the solver should continue to draw conclusions which satisfy one, or both, of the following criterion:

a) The inferences have frequently been made in the past from the same type of information;

b) The inferences are concerned with properties (variables, terms, expressions, and so on) that appear in the goal, the givens, or inferences from the goal or the givens;[3]

until it is difficult to draw new conclusions which seem to have any likelihood of being useful. Inferences from presented information also includes explicit symbolic or diagrammatic representation of the information that appears implicitly in the problem.

One method of generating new information from the givens or goal state is "brainstorming" the problem. This procedure which is espoused by the Creative Education

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[1]G. Polya, How To Solve It.

[2]Wayne A. Wickelgren, How To Solve Problems, p. 23.

[3]Ibid., p. 28.

Foundation, involves the deliberate stimulation of an individual or groups divergent thinking ability. During this procedure the flow of ideas is enhanced by withholding all criticism in the early stage so that solvers will feel free to propose ideas which may be judged as far off the subject. By deferring judgment, it is assumed that even "far out" ideas may suggest a realistic, creative solution or plan. As Osborn, the founder and chairman of Creative Education Foundation, states, "It is almost axiomatic that quantity breeds quality in ideation."[1]

Brainstorming is a generalization of the inference method appropriate for comprehensive problem solving. For example, in the problem of keeping peace on the school bus, brainstorming may lead to conclusions, inferred from the desired goal of peace, such as:

> Don't let the boy take the bus. Keep the boy separated from the other children. Arrange the route so the boy is picked up last and dropped off first. Arrange activities (on the bus) to keep the children busy. Provide books and magazines of interest. Equip the bus with TV.

Parnes suggests using "idea-spurring questions" such as; Can I modify, magnify, minify, rearrange or combine any

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[1]Alex F. Osborn, Applied Imagination: Principles and Procedures of Creative Problem Solving, (3rd rev. ed. Charles Scribner's Sons, New York, 1963), p. 131.

existing information, or suggestion, to produce new information which may be useful in solving the problem?[1]

Inferences can be made about the operations or transformations which are in a problem. Indeed many practical problems require the solver to think of a type of operation that will solve the problem. One example of this type of problem is the well known radiation problem of Duncker:

> Given a human being with an inoperable stomach tumor, and rays which destroy organic tissues at sufficient intensity, by what procedure can one free him of the tumor by these rays and at the same time avoid destroying the healthy tissue which surrounds it?[2]

The solver may infer several possible operations: avoid contact between rays and healthy tissue, desensitize the healthy tissue, or lower the intensity of the rays on their way through healthy tissue. As a second example, consider the one-heavy-coin problem:

> You have a pile of 24 coins, twenty-three of these coins have the same weight, and one is heavier than the others. Your task is to determine which coin is heavier and to do so in the minimum number of weighings. You are given a beam balance, which will compare the weights of any two sets of coins.

If the solver infers correctly that the beam balance actually has three different outcomes, then the solver is in a position to realize that the balance can provide him with the answer to which of three subsets contains the

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[1]Sidney J. Parnes, Student Workbook for Creative Problem Solving Courses.

[2]K. Duncker, "On Problem Solving," Psychological Monographs, 1945, 58.

heavier coin. Given two equal subsets of coins the left hand pan of the balance is either above, even with or below the right hand pan.

Inferences can be made from the givens, the goal or the operations and practice in drawing conclusions from these sources should be provided.

Identification of Subgoals

A second strategy useful in devising a plan to solve a problem is to identify one or more subgoals or subproblems. In essence, the purpose of this strategy is to replace a single difficult problem with two or more simpler problems. For example, in the Analytic Geometry problem posed in the last section a reasonable subgoal would be to find the slope of the segment from (x,y), a point of the locus, to the points (-3,0) and (3,0). A second subgoal would be to equate the negative reciprocal of one of these slopes to the other.

The subgoal method is advantageous for attacking problems that require a sequence of more than two or three actions to solve--which is the case in comprehensive problems. When a solver defines two or more subgoals to be achieved in getting from the given state to the goal he must decide whether one of these subgoals must be achieved before the others. In some problems it is clear that one of the subgoals (SG_1) is closer to the givens than a second (SG_2), which is closer to the goal. In such a case the solver should work to achieve SG_1 first. When this logical distinction can be made the problem contains ordered subgoals.[1] A diagram of a problem containing ordered subgoals is given in Figure One.

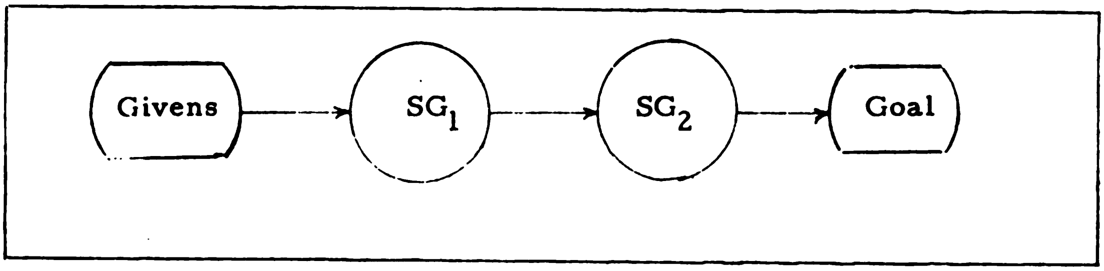

Figure 1. Ordered Subgoals

The locus problem is an example of an ordered subgoal problem. A second example is provided by the following problem:

> Nine men and two boys want to cross a river, using an inflatable raft that will carry either one man or the two boys. How many times must the boat cross the river in order to accomplish this goal?

An excellent subgoal is to find the minimum number of trips necessary to get one man on the other side and get the boat back to the starting side. Since there are nine men, the solver can simply multiply this minimum number by nine and subtract one to achieve the goal.

In comprehensive problems the solver usually defines several subgoals which are logically independent. These "subproblems" may often be broken into further sub-subproblems which lead to the goal. If it is known _a priori_ that

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[1]Wayne A. Wickelgren, _How To Solve Problems_.

either of two sequences of actions, through two equivalent subgoal chains, will lead to the desired solution the solver is free to choose to begin work on that chain which he feels will yield the goal more quickly. The problem is said to contain unordered subgoals. Refer to Figure Two for a schematic drawing of a problem containing unordered subgoals.

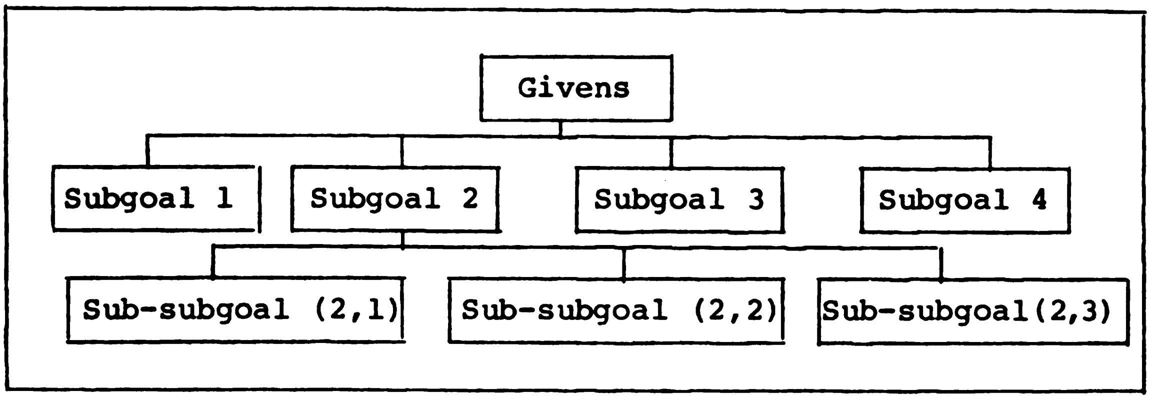

Figure 2. Unordered Subgoals

More often the solver does not know if a particular sequence of subgoals will lead to the most "correct" answer and should choose to work on that subgoal which contains the fewest subgoals to be solved.

Most often in comprehensive problem solving the careful statement of the problem, producing the problem definition, is quite broad. For example, consider the problem, "How might students increase sales in a student owned and operated store?" The solver might list ideas such as, "obtain more customers" or "step up advertising." These are not solutions to the problem, rather subproblems which will lead to an increase of sales in the store. The solver would choose one of these "approaches" at a time, and attempt to probe for more specific sub-subgoals. For example, in the subgoal 'obtain more customers,' the solver may identify sub-subgoals such as: How can we get more customers in the store during the morning and early afternoon hours? How can we get customers into departments with sagging sales? Once a number of subgoals have been identified, the solver chooses to work on that subgoal which he feels will produce the most rapid, or most dramatic, increase in sales.

The identification of subgoals may be one of the most useful problem solving strategies for the elementary school child, since it can be used throughout the school years to solve arithmetic, algebra and geometry word problems. For example, consider the following problem:

> Each day, Abe either walks to work and rides his bicycle home or rides his bicycle to work and walks home. Either way, the round trip takes one hour. If he were to ride both ways, it would take 30 minutes. How long would a round trip take, if Abe walked both ways.

The first subgoal the solver might define is to determine how long it takes Abe to ride one way. The solver can then determine, as a second subgoal, how long it takes to walk one way.

Wickelgren, Parnes and Polya[1] each identify the subgoal method as a generalized problem solving strategy which should be a part of problem solving training.

Examining Related Problems

A third generalized strategy for devising a problem-solving plan is to examine a related problem. Two problems can be related in the following ways: one is equivalent or completely analogous to the second, one may be similar in certain elements to the second or one may be a generalization of a second, that is, the second is a special case of the first.

If two problems are equivalent, they are the same except for the naming of the elements involved, that is, they are isomorphic. For example, if in a checker game the checker board is replaced with a chess board and the red checkers with quarters and the black checkers with pennies, without changing the operations or goal, the two problems (games) are equivalent. Recognizing that a problem is equivalent to another problem is usually a simple matter, and will only aid the solver in achieving the desired goal if the isomorphic copy has been solved. However, the solver who realizes that his problem is equivalent to another often gains new perspectives which aid in making inferences.

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[1]Wayne A. Wickelgren, How To Solve Problems; Sidney J. Parnes, Student Workbook for Creative Problem-Solving Courses; G. Polya, How To Solve It.

Two problems which are similar share some but not all of their essential characteristics. For example, the linear game of nim, involving the removal of counters from a line, is similar to the two dimensional game of nim, wherein the players remove all counters above and to the right of a selected counter. Two similar problems may be of equivalent difficulty, with the first simpler than the second or the first more complex than the second. For example, the "missionaries-and-cannibals" problem is as difficult as the similar "man, fox, goose and corn" problem. As in the case of equivalent problems, knowing how to solve either aids the solver in reaching the goal for the other.

Looking for a similar, simpler problem is a strategy Polya stresses throughout his book, How To Solve It.[1] A very common technique used to find a simpler problem (though not the only way) is to reduce the number of variables involved. For example, the one-heavy-coin problem presented earlier involved twenty-four coins, and is solved by weighing a subset of eight coins against any other subset containing eight coins. The solver who is unable to make the needed inferences when confronted with this problem might try the same problem with fewer coins. Note that examination of a simpler problem will only produce the insight needed to work the more difficult one if the simplified problem still contains the essential feature on which

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[1]G. Polya, How To Solve It.

the solution to the difficult problem depends. For this reason, simplifying the problem may not aid the solver if his simplification eliminates that facet of the problem which made it difficult. However, as Wickelgren states, ". . . it might still be good strategy to pose and solve . . . a simpler problem before you attempt to solve a more complex problem."[1]

Examination of special cases is especially useful in proof problems or in problems where the solver must make inferences concerning a more general problem. Mathematical induction involves looking at special cases. Special cases are simplified problems which, taken collectively, sometimes aid the solver by providing data for the intuition to work on. For example, determining the number of moves necessary to complete the "Tower of Hanoi" problem with k disks is most often approached by examining the problem with two, then three, then four disks and recognizing a pattern.

Teachers report that students at all level of instruction use the method of related problems to solve word problems. For example, in solving "age problems" in elementary algebra the learner is taught to mimic the procedures demonstrated by the teacher. In solving comprehensive problems, the student should be guided by previous experience gained from solving similar problems. This applies to making inferences which were helpful in similar problems,

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[1]Wayne A. Wickelgren, How To Solve Problems, p. 173.

identifying subgoals which have proved fruitful in similar situations, and gathering and interpreting data pertinent to the problem. For example, students who have learned to represent the mean time necessary for six different classes to cross a street by using bar graphs can use this method in the equivalent problem to represent the mean time necessary for these classes to pass through the lunch line. Indeed, Polya suggests that at each step of the solving process the student ask himself, "Do you know any problem with the same unknown?, . . . with a similar unknown?, . . . related to yours and solved before?"[1]

Producing Solutions

Once the search strategy has been devised, the solver follows the plan to its conclusion--the goal state. In comprehensive problem solving the solver must accumulate evidence which will support a decision concerning the problem. Indeed, Posner suggests, ". . . many problems can be viewed as involving the collection and evaluation of evidence."[2] For example, consider the simple problem of "Should I take a raincoat to work this morning?" In order to decide between the two hypotheses, "It will rain," and "It will not rain," the individual could consider whether it is raining at the present time. Does it look like it

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[1] G. Polya, How To Solve It, p. 11.

[2] M. I. Posner, Cognition: An Introduction, p. 176.

may rain? What is the weather report, the conditions of the wind, a barometer reading, the amount of rain during the past few days? Gathering and interpreting data is a necessary part of producing solutions.

The solver may find that, after working on a problem for some time that he is no closer to the goal state than he was soon after the initial representation. The solver must then concentrate on ways to redirect the search for a solution. Evidence suggests that once a particular strategy is chosen, it continues to direct the search in a direction which may not lead to the goal. Luchings[1] found that subjects continued to use a formula which had worked on previous problems even when a more direct method of solution was available. This tendency to repeat a solution once obtained is called set, or Einstellung. Tresselt and Leeds[2] demonstrated that, unlike functional fixity, Einstellung does not diminish even when the problem solving process is interrupted for a period of days. Ways of redirecting the search have been suggested by several individuals. DeBono[3] suggested the solver use a method called lateral

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[1]A. S. Luchings, "Mechanization in Problem Solving: The Effect of Einstellung," Psychometric Monographs, 1942, 54 (6), p. 95.

[2]M. E. Tresselt, and D. S. Leeds, "The Einstellung Effect in Immediate and Delayed Problem-Solving," Journal of General Psychology, 1953, 49, pp. 87-95.

[3]E. DeBono, New Think, The Use of Lateral Thinking in the Generation of New Ideas, (New York: Basic Books, Inc., 1968).

thinking, in which the solver looks at the problem from various standpoints. The Creative Education Foundation[1] suggests that the solver go back and repeat the brainstorming process. Crovitz[2] suggests thinking of action words which can be inserted into the problem to break old habits and produce new organizations. Since these methods are not mutually exclusive, there have been few attempts to compare them.

Wickelgren[3] suggests two methods which are useful in redirecting the search, working backwards, and the method of contradiction. These strategies provide more specific ways to reach a solution than the broad suggestions of DeBono, Crovitz or the Creative Education Foundation.

Contradiction

The method of contradiction involves drawing conclusions which contradict some piece of given information, and so proves that a goal can not possibly be obtained from the givens. This strategy gives us negative information which may imply the desired result. The indirect proof sometimes used in mathematics problems requires the solver to use contradiction. For example, the proof that $\sqrt{2}$ is irrational utilizes the indirect proof.

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[1] A. Osborn, Applied Imagination: Principles of Creative Problem Solving.

[2] H. F. Crovitz, Galton's Walk, (New York: Harper & Row, Publishers, 1970).

[3] Wayne A. Wickelgren, How To Solve Problems, pp. 109-151.

Another use of contradiction involves the elimination of alternatives in a small search space. Such elimination is useful in answering multiple choice problems; "Which of the following is a solution to the equation $x^3 + 4x^2 - 7x - 10 = 0$? x equals: (A) -2, (B) -5, (C) 4, (D) 3, (E) none of these." The solver can determine whether or not each of these numbers satisfies the equation more quickly than he could solve it by a direct approach. This particular strategy is used to solve logic problems such as the "brakeman, fireman and engineer problem," wherein the solver is asked to determine which of three trainmen is the engineer (given some information about the trainmen and three passengers on the train). Logic problems using contradiction in a small search space may provide practice for elementary school students in using techniques which could prove useful in later studies of algebra and geometry.

Solving open ended problems often involves a large number of alternative specific goals making it impractical, or impossible, to contradict them one at a time. In such a case, it is sometimes possible to eliminate large subgroups of alternative goals by contradiction. For example, in the school bus driver problem, several possible alternative hypotheses such as, provide TV on the bus, provide new games and puzzles regularly, provide entertainment for the children, provide movies or storytellers, can be eliminated because to assume such a solution contradicts the cost constraint the school board would impose. By drawing conclusions from an assumed goal, and contradicting some given information, the solver can gain considerable insight into the nature of a plausible solution.

Working Backward

The method of working backwards is a strategy for redirecting the solving process by focusing attention on the goal as the starting point. Unlike contradiction, which attempts to draw inferences from an assumed goal, the method of working backwards involves trying to guess what step must immediately precede the goal state. This method is helpful particularly when there is a unique goal and several given statements. In problems where there is a unique goal, Newell, Shaw and Simon[1] have demonstrated the superiority of working backward. In problems where there are a large number of givens and a unique goal, the givens may have a disjunctive relationship to one another. Hence, the solver, using a direct approach, would have to use a great deal of time trying various starting points (givens), some of which need not be related.

Students who attempt to solve comprehensive problems often come up with a large number of givens, or inferences from the givens, in the initial stages of problem solving. Because of the unique starting point used in working

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[1]A. Newell, J. C. Shaw and H. A. Simon, "The Processes of Creative Thinking," Contemporary Approaches to Creative Thinking, Eds. H. E. Gruber, G. Terrell, M. Wertheimer, (New York: Atherton Press, 1962).

backwards, namely the goal, the student is directed to just those aspects of the given information that are relevant to the solution.

The method of working backwards is useful to students in less general problem solving contexts such as:

> Three people play a game in which one person loses and two people win each game. The one who loses must double the amount of money that each of the other two players has at the time. The three players agree to play three games. At the end of the three games, each player has lost once and each person has $8. What was the original stake of each player?

In this problem, the operation of exchanging money is destructive, that is, changes the givens at each stage of the problem. For this reason, by beginning with the goal and using the inverse operation at each step the solver can arrive at the solution that one player had $13, the second had $7, and the third had $4 at the beginning of the game. The problem of determining the optimal strategy for winning the game of "nim" is another example of a problem with a destructive operation, and hence can be worked using the method of working backwards from a unique goal.

The method of working backwards is extremely useful in redirecting the search strategy and it may be that letting children work with games or puzzles which involve this method will aid in their understanding of this procedure and encourage them to use it more often. As Polya states "Working backwards is a common-sense procedure within the reach of everybody. . ."[1] which should be cultivated into a powerful tool in problem solving.

Deciding Among Solutions

In comprehensive problem solving the solver must deal with a situation in which there is no optimal solution or in which the solution is unknown. Individuals generally do not attempt to find an optimal solution even when it can be computed. Simon[2] found that persons often find solutions short of optimality which will be good enough or acceptable. The subjects tended to avoid complex calculations in favor of short procedures on a fraction of the data. Posner suggests that this is the case because "limitations of operational memory make it difficult to deal with a long series of interrelated steps."[3]

When the solver has a great deal of data on which to base a decision concerning solutions, it has been found that these decisions are not made in an optimal or consistent way. Two studies, the first by Edwards[4] and the second by

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[1] G. Polya, How To Solve It, p. 230.

[2] H. A. Simon, The Sciences of the Artificial, (Cambridge, Mass.: MIT Press, 1968).

[3] M. I. Posner, Cognition: An Introduction, p. 175.

[4] W. Edwards, "Dynamic Decision Theory and Probabilistic Information Processing," Human Factors, 1962, 4, p. 59-73.

Slovic and Lichtenstein[1] indicate that persons often underweigh very strong evidence towards a particular hypothesis, and overweigh evidence which shows only weak support of a hypothesis. Furthermore, evidence which comes slowly is more heavily weighted than evidence which comes quickly and in great quantity. Peterson and DuCharme[2] showed that evidence introduced early will have greater impact on the solver than evidence which comes late in the solving process. These studies seem to suggest that the choice of the best solution is heavily influenced by how it was produced.

To provide the solver with a less subjective rationale for judging the correctness of a solution, Parnes[3] suggests that the solver list the evaluation criterion or "yardsticks" by which the solver can mentally test the effectiveness of each solution. The solver then grades each of his tentative solutions: Good, Fair, Poor, or Doesn't Pertain, for each of the criterion listed. The solution with the highest overall grade is selected as the most acceptable or satisfactory. This method of judging the quality of a solution is

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[1]P. Slovic and S. Lichtenstein, "Comparison of Bayesian and Regression Approaches to the Study of Information Processing in Judgment," Organic Behavior and Human Performance, 1971, 6, p. 649-744.

[2]L. R. Peterson and W. M. DuCharme, "A Primacy Effect in Subjective Probability Revision, Journal of Experimental Psychology, 1967, 73, p. 61-65.

[3]Sidney J. Parnes, Student Workbook for Creative Problem Solving Courses.

essentially a check to see how well it meets the goals set forth by the problem.

Another means of evaluating the correctness of a solution is to judge whether the solution produces insight or the "Eureka" experience. Wertheirmer[1] suggests that one measure of insight is the ability to transfer the learning on a problem to new situations. This may depend on the problem solution's ability to be represented in operational memory at one time. For example, a student claims to have insight into a mathematical proof when he is able to condense the steps in such a way that they are present at least implicitly in a representation that he can grasp all at once. Kohler[2] suggests that without this insight the solver will be less convinced of the correctness of a solution even when evidence supports this conclusion.

PART TWO

Review of the Related Literature

A search of the literature yielded many studies which related problem solving ability to other variables. Relevant material will be presented under one of the following classifications: (a) general problem solving, (b) creativity and problem solving ability, (c) problem solving as

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[1] M. Wertheirmer, Productive Thinking, (New York: Harper & Row Publishers, 1945).

[2] W. Kohler, The Task of Gestalt Psychology, (Princeton, N.J.: Princeton University Press, 1969).

related to other variables. In order to insure a broad survey of the literature, the researcher drew from the following sources: Educational Resource Information Center (ERIC), Dissertation Abstracts, Psychological Abstracts, books and articles suggested by the subject matter of problem solving.

The ERIC data base is composed of most of the education related material published during the past ten years, and is stored on computer tapes by System Development Corporation of Santa Monica, California. A computer search through the ERIC data base was completed to find all material which was related to teaching elementary school children methods of comprehensive problem solving. The researcher requested all material which contained the following three areas: (1) problem solving, productive thinking, convergent thinking, divergent thinking, creative thinking or creative activities; (2) elementary education, primary education, elementary grades, elementary school children, elementary school curriculum, primary grades or early childhood education; (3) teaching, creative teaching, instruction, concept teaching, teaching methods, teaching procedures or teaching techniques. Of the publications in the ERIC data base only 116 contained material involving all three of these areas. Of these 116, only 12 were actually related to teaching problem solving to children. To complement the ERIC search, a search through Dissertation

Abstracts and Psychological Abstracts was carried out to find studies which may not have appeared in the ERIC material.

General Problem Solving

The nature of the problem solving process is discussed in most general psychology and learning theory texts, and is usually presented as a procedure which generally involves four stages: Preparation, Definition, Hypthesis formation, and Solution selection. The early work of Maier[1] implied that the solver is usually not aware of intermediate stages in the solving process, that is, the perception of a solution is most often sudden. Later, Patrick[2] concluded that four stages of problem solving are apparent: 1) preparation, 2) incubation, 3) illumination, 4) verification. She agreed, however, with Maier that the creative solution most often appears as a whole. Simon and Newell[3] reviewed past and present theories of problem solving and currently regard problem solving as an information processing procedure

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[1]N. R. Maier, "Reasoning in Humans, The Solution of a Problem and its Appearance in Consciousness," Journal of Comparative and Physiological Psychology, Vol. XI, 1931, pp. 181-194.

[2]Catherine Patrick, "Whole and Part Relationship in Creative Thought," American Journal of Psychology, Vol. LIV, 1941, pp. 128-131.

[3]H. Simon and A. Newell, "Human Problem Solving: The State of the Theory in 1970," American Psychologist, 1971, (Feb.), Vol. 26 (2), pp. 145-159.

wherein the solver (processor) operates on the data (information) to arrive at a solution new to the solver. In this theory, the information processing system, the problem solver is confronted by a task, and the task is defined objectively in terms of a task environment or problem space. A search of the problem space in promising directions, avoiding the less promising ones, will lead to a solution. This is essentially the point of view that Wickelgren[1] takes in suggesting strategies or heuristics which can be employed to aid in the search through the problem space.

Simon and Newell[2] suggest that the current view of problem solving as information processing has enabled theorists to pin down terms like "memory" and "symbol structure" prevalent in prior problem solving theories. Previous theories depended, to a great extent, on the structure of the problem space in arriving at solutions. Miller and Slebodnick[3] demonstrated the superiority of training in strategies, over training in structure, in trouble shooting electronic equipment. This result was again

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[1]Wickelgren, <u>How To Solve Problems</u>.

[2]H. Simon and A. Newell, "Human Problem Solving: The State of the Theory."

[3]R. B. Miller and E. B. Slebodnick, "Research for experimental investigation of Transferable skills in electronics maintenance," Lackland AFB, Texas. Air Force Publication: <u>TRC TR - 2</u>, 1958.

demonstrated by Rundquist, et al.,[1] who showed that a strategy trained group did better than a structure trained group in terms of time to solution and the number of correct initial hypothesis for this type of electronic equipment problem.

There are some problem solving programs which have been developed and which emphasize teaching the solver strategies. The Creative Education Foundation program to train for problem solving processes emphasizes a process which relies heavily on a single heuristic-brainstorming. Osborn[2] suggests that research at the State University at Buffalo indicates a significant improvement in idea-producing skill. This improvement is due to the "defer judgment" aspect of brainstorming which provides the solver with a variety of tentative solutions.

The Covington, Crutchfield, Olton Project at the center for Productive Thinking at Berkley produced a program to teach problem solving. The program uses detective stories to teach strategies for problem solving

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[1]E. A. Rundquist, et al., "Preparation for Problem Solving: Structure vs. Strategy Pre-Training," San Diego: U.S. Naval Research Technical Bulletin 5: TB 66 - 1, July, 1966.

[2]Alex Osborn, The Creative Education Movement, (Creative Education Foundation, Buffalo, New York, 1964).

to fifth and sixth graders through programmed learning.[1] The program has been very successful in teaching children to solve these types of problems, but the amount of transfer to problems of a comprehensive nature has been limited. A third program which has been developed by Interaction Associates of Berkley, California is entitled "Tools for Change." This group has developed teacher manuals,[2] for primary and higher grades, and a strategy notebook[3] for student usage. "Tools for Change" is a problem solving course consisting of a series of units each dealing with a heuristic process. This program has identified thirty-eight strategies which are classified under one of the eight headings, such as strategies for manipulating information, strategies for information retrieval, and master strategies. The authors of this program have not developed problem solving tests which can provide quantitative results on which to judge the success of this program in teaching open ended problem solving. Since this program uses games and exercises to teach the heuristics, the amount of transfer in using these strategies to problems which are comprehensive, may be limited.

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[1] Covington, Crutchfield, and Olton, The Productive Thinking Program for Problem Solving, (Berkley, California, 1970).

[2] Lucinda L. Kindred, Tools for Change, Primary Version, (Second Ed. Interaction Association, Berkley, California, 1971).

[3] Catherine McEver, Strategy Notebook: Tools for Change, (Interaction Associates, Berkley, California, 1971).

The Unified Science and Mathematics for Elementary Schools (USMES) is a federally funded program to teach children to solve open ended problems. Students work with their classroom teacher several hours per week, throughout the year, on challenges such as weather prediction, designing for human proportions, and planning a safe school crossing.[1] Since its inception, USMES has used a variety of evaluation techniques such as teacher logs, classroom observations, and standardized tests. In 1971 Shapiro[2] studied the success of the USMES program in teaching students methods for attacking problems. From two to six students from thirty-one USMES classes and twenty-two control classes were tested using "The Notebook Problem:" choose the best of three notebooks for students to use as a science and mathematics notebook. Students were scored more favorably for responses which indicated consideration of measurable dimensions (e.g., size-volume, weight, quantity of paper, and cost). Shapiro concluded that USMES students made significantly more measurable responses than those in the control group. The USMES program neither identifies nor teaches specific strategies, but rather provides an environment in which the student may discover a general pattern of problem solving.

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[1]<u>USMES Guide</u>, Education Development Center, Boston, 1973.

[2]Bernard J. Shapiro, <u>The Notebook Problem: Report on Observations of Problem Solving Activity</u>, Education Development Center, Boston, 1973.

The Inquiry Training Project, begun in 1969, for sixth, seventh, and eight graders originated in Ontario, Canada under the direction of the Ontario Institute for Studies in Education. This project attempts to teach children a generalized problem solving process which involves defining the problem, finding alternatives, gathering information, synthesizing information, and reaching a conclusion.[1] This model is applied to six problem-solving areas: 1) logical inquiry, 2) physical science experiments, 3) experiments involving the principle of randomization, 4) correlational analysis, 5) case studies and 6) real life problems. Classes which are part of the project solve a problem of each type, using a variation of the generalized problem solving process indicated above.

Evaluation of the first two years of the project have involved both the subjective judgments of the teachers of the program and objective tests to insure that there is an effect and that it can be reasonably attributed to the new program. The authors of the program feel that the most reasonable interpretation of the results of these evaluations is that the program does have a large effect on certain types of problem-solving abilities.

There have also been attempts to teach problem solving within the framework of specific subject areas. For example,

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[1]F. Robinson, J. Tickle, and D. W. Brison, Inquiry Training: Fusing Theory and Practice, (The Ontario Institute for Studies in Education, Toronto, Ontario, Canada, 1972).

Schmiess studied success of a program to teach sixth grade students to solve scientific problems.[1] He concluded that by teaching students the methods of scientific investigation a significant gain can be made in the ability to solve new problems of a scientific nature. In another area, Schwab and Ambrose studied specific strategies for teaching decision making in the social sciences.[2] Although they found no significant difference between the treatment and control groups, the researchers suggested that the students were aware of the heuristics but that they were not permitted sufficient practice within the program to bring about the desired change.

In summary, the evidence seems to suggest that an informational processing theory of problem solving is most widely accepted, and the most consistent with, a strategy approach to problem solving. Within the past five years, some programs have been developed which have attempted to teach the skills involved in problem solving. Some of these programs have limited themselves to certain types of problems to be considered, some have limited success in teaching comprehensive problem solving, and some have not

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[1]Elmer G. Schmiess, An Investigation Approach to Elementary School Science Teaching, (Ed. D. Dissertation, University of North Dakota, University Microfilms, Ann Arbor, Michigan, 1970).

[2]Lynne Schwab and A. Ambrose, The Interaction of Decision-Making Style, Teaching Strategy, and Decision-Making Content Material in Social Studies, (American Educational Research Association, Washington, D.C., 1970).

identified specific strategies which can be strengthened in the high grades. The two most promising programs appear to be USMES, and the Inquiry Training Project.

Creativity and Problem Solving

Creativity is used in literature to mean a variety of thinking processes. Vernon[1] states, ". . . to the psychologist, creative thinking is merely one of the many kinds of thinking which range from autistic fantasy and dreaming to logical reasoning. . ." In general, American psychologists such as Gallagher and Wilson[2] agree with Guilford[3] who uses creativity, as a term, to describe divergent-productive thinking. Torrance views creativity as:

> A process of becoming sensitive to problems . . .; identifying the difficulty; searching for solutions . . .; testing these hypotheses; finally communicating the results.[4]

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[1]P. E. Vernon (Ed.), Creativity, (Baltimore: Penguin Books, 1970).

[2]J. J. Gallagher, Teaching the Gifted Child, (Boston: Allyn Bacon, 1964); R. C. Wilson, "The Structure of the Intellect," Productive Thinking in Education, Ed. M. J. Aschner and C. E. Bish, (Washington D.C.: National Education Association, 1965).

[3]J. P. Guilford, "Traits of Creativity," Creativity and its Cultivation, Ed. H. H. Anderson (New York: Harper and Row, 1959).

[4]E. P. Torrance, Torrance Tests of Creative Thinking: Norms-Technical Manual (Princeton: Personell Press, 1966).

Thurston[1] agrees that creativity involves reaching solutions that are unique to the individual. Fleigler suggests that the creative individual ". . . manipulates external symbols or objects to produce an unusual event uncommon to himself and/or his environment."[2]

Each of these definitions of creativity involves novel combinations or an unusual association of ideas. Some authorities require that these combinations be useful, that is to say that they solve some social or theoretical problem. Others merely require that the creative product is "new" to the individual doing the creating. These definitions suggest a very close relationship between problem solving ability and creativity.

Since creativity and problem solving are closely related, the characteristics of creative persons and ways of nurturing creativity are oftentimes examined. Drews[3] found that creative adolescents were sensitive to experiences both within and without themselves, and were open and searching in considering alternatives. A number of studies

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[1] L. L. Thurston, "Creative Talent," Application of Psychology, Ed. L. L. Thurston, (New York: Harper and Row, 1952).

[2] L. A. Fleigler, "Levels of Creativity," Educational Technology, 1959, 9 (21).

[3] E. M. Drews, "Profile of Creativity," NEA Journal, 1963, 52 (1), pp. 26-28.

were carried out by MacKinnon[1] and his colleagues during the sixties to determine the nature of a creative person. They found that creative individuals were above average in intelligence, more fluent, more alert, more independent in thought and action, and relatively free from conventional restraints. Guilford[2] identified aptitude traits which may have a direct bearing on creativity. These include the ability to see problems, fluency, flexibility, originality, redefinition (the ability to improvise), and elaboration.

Torrance and Razik[3] also summarized the traits of a creative person. They indicated that the creative person: (a) was less repressed, less inhibited, less formal, less conventional, (b) produced novel and unconventional solutions to problems, (c) was more intuitive and perceptive with an open, searching behavior, (d) more dedicated when solving problems, (e) showed flexibility in his tolerance for ambiguity, and (f) is not afraid of failure or being laughed at. Torrance[4] listed five principles which he

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[1]D. W. MacKinnon, "Personality Correlates of Creativity," Productive Thinking in Education, Ed. M. J. Aschner and E. Bish (Washington D.C.: National Education Association, 1965).

[2]J. P. Guilford, "Traits of Creativity," Creativity and Its Cultivation, Ed. H. Anderson (New York: Harper and Row, 1959).

[3]T. A. Razik, "Recent Findings and Developments in Creative Studies," Theory into Practice, 1966, 5, pp. 160-165; E. P. Torrance, Rewarding Creative Behavior (Englewood Cliffs, N.J.: Prentice Hall Publishers, 1965).

[4]E. P. Torrance, "Give the Devil His Dues. . . ," Gifted Child Quarterly, 1961, 5, pp. 115-120.

believed to be important in developing creative thinking: (1) be respectful of unusual questions, (2) be respectful of unusual ideas of children, (3) show children that their ideas have value, (4) provide opportunities for self-initiated learning, and (5) provide for periods of non-evaluated practice or learning. Enochs[1] examined the effects of these principles and found that creative thinking can be nurtured by applying these principles. Hallman, Durr, and Hughes[2] also suggest ways to nurture creativity which are analogous to those of Torrance. Each of these researchers agree that the classroom environment should be free from authoritarian control, encourage self-initiated learning and place a high value on the individual's feelings, creativity, and intellectual activity. In general, research during the past fifteen years, concerning creativity, supports the hypotheses that a suitable classroom and teacher environment can favorably affect measures of creativity and it is reasoned that this type of environment would be most conducive in producing an increase in problem solving ability.

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[1]P. D. Enochs, An Experimental Study of a Method for Developing Creative Thinking in Fifth Grade Children, (Unpublished Ph.D. Dissertation, University of Missouri, Dissertation Abstracts, 1965).

[2]R. J. Hallman, "Techniques of Creative Thinking," Journal of Creative Behavior, 1967, 1, pp. 325-330; W. K. Durr, The Gifted Child, (New York: Oxford University Press, 1964); H. K. Hughes, "The Enhancement of Creativity," Journal of Creative Behavior, 1969, 2, pp. 73-83.

Problem Solving as Related to Other Variables

There have been a few studies which have examined strategies for comprehensive problem solving. Most of the correlated information on variables involved in problem solving has been gathered in studies of problem solving in a particular subject area, such as mathematics or science, or problem solving as a cognitive activity.[1] These experiments most often limited their definition of a problem to those challenges which a student could work on and arrive at a solution for within a short period of time. Nonetheless, these studies provide a good deal of information about some variables which seem to influence problem solving ability.

The literature yields many studies which indicate a strong relationship exists between intelligence and problem solving ability. Max Engelhart[2] concluded that there is a strong correlation between intelligence and arithmetical problem solving ability. Studies by McCuen[3] and Erickson,[4]

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[1]P. C. Wason and P. N. Johnson-Laird (eds.), Thinking and Reasoning, (Penguin Modern Psychology Readings, Penguin Books, Ltd., Baltimore, Maryland, 1968).

[2]M. D. Engelhart, "The Relative Contribution of Certain Factors to Individual Differences in Arithmetical Problem Solving Ability," Journal of Experimental Psychology, I (September, 1932), p. 25.

[3]T. L. McCuen, "Predicting Success in Algebra," Journal of Educational Research, XXI (January, 1930), pp. 72-74.

[4]L. H. Erickson, "Certain Ability Factors and their Effect on Arithmetic Achievement," The Arithmetic Teacher, V (Sept., 1958), p. 291.

although some twenty eight years apart, showed a positive correlation between intelligence and arithmetic and algebra problem solving ability of .54 and .58 respectively. One of the most comprehensive studies dealing with the characteristics of an insightful mathematical problem solver was carried out by Dodson[1] in 1969 in which he analyzed the School Mathematics Study Group "Y" population. He concluded that particular measures of intelligence and thinking were strongly related to problem solving ability. Houtz, et al.,[2] also found that general problem solving ability was significantly correlated to several cognitive variables including intelligence. This high correlation between intelligence and problem solving ability could also be inferred from the assumed relationship between problem solving and creativity, as it is highly correlated to intelligence in itself.

Vos analyzed the problem solving heuristics of senior high school mathematics students and concluded that ". . . mathematical maturity was a definite factor in problem solving ability."[3] Maturation, in general, seems to affect

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[1]J. Dodson, Characteristics of Successful Insightful Problem Solvers, NLSMA Report No. 31, School Mathematics Study Group, 1972.

[2]J. C. Houtz, S. Ringenbach, J. F. Feldhusen, "Relationship of Problem Solving to Other Cognitive Variables," Psychological Reports, 1973, Vol. 33 (2), pp. 389-390.

[3]Kenneth E. Vos, "The Effect of Three Instructional Strategies on Problem Solving Behaviors in Secondary School Mathematics." Paper presented at the National Council of Teachers of Mathematics 53rd annual meeting, Denver, Colorado, April, 1975.

problem solving ability as interpreted by Magkaev and Rimoldi. Magkaev[1] concluded that, after studying 490 school children solving four types of reasoning problems (grades 1-4), the amount of planning used to obtain a solution increased with the subject's age. Rimoldi[2] found that the thought process during problem solving becomes more logical as the subject's age increased.

A third related variable is the perception time, that is the length of time necessary for the solver to encode the problem in memory and reach the first possible hypothesis. An early study by Rokeach[3] suggested that taking more time at the perception stage of solving enables the solver to think more abstractly concerning the problem and reduces his rigidity in suggesting possible solutions. Kennedy[4] also found a strong relationship between perception and

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[1]V. Magkaev, "An Experimental Study of the Planning Function of Thinking in Young School Children," Voprosy Psikhologii, 1974 (Sept.-Oct.), No. 5, 98-106. English summary in Psychological Abstracts, Vol. 54, No. 4, October, 1975.

[2]H. J. Rimoldi, "Language and Thought Processes," Rivista de Psicologia General y Aplicado, 1974 (May-June) Vol. 29 (128), 499-514. English summary in Psychological Abstracts, Vol. 53, 1975.

[3]M. Rokeach, "The Effect of Perception Time Upon Rigidity and Concreteness of Thinking," Journal of Experimental Psychology, XL, 1950, p. 206.

[4]M. M. Kennedy, Effect of Perception On and Information About Organization on Problem Solving, (Ph.D. Dissertation, M.S.U., 1973).

success at solving problems similar to the "fireman, brakeman, and engineer problem" (Chapter 2, Part One). This suggests that the student should not be in a hurry to begin using any heuristic in solving problems.

Many of the studies done on the mathematical problem solving ability of students have included the sex of the student as a factor in the design. The results of these studies which investigated the differences between the abilities of problem solving in boys and girls are non-directional. Some researchers such as Neill[1] and Sheehan[2] concluded that boys were better problem solvers. In other studies, such as one done by Cleveland and Bosworth,[3] no significant differences were found between their problem solving abilities. Schuner[4] found that boys do better on algebra problems, but that girls do better on geometry problems. Each of these studies recommends that more research be done on the question of the effect of sex on problem solving ability.

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[1]R. D. Neill, The Effects of Perception on and Information about Organization on Problem Solving, (Ph.D. Dissertation, M.S.U., 1973).

[2]T. J. Sheehan, "Patterns of Sex Differences in Learning Mathematics Problem Solving," The Journal of Experimental Education, XXXVI, p. 86.

[3]G. A. Cleveland and D. L. Bosworth, "A Study of Certain Psychological and Sociological Charactersitics as Related to Arithmetic Achievement," The Arithmetic Teacher, XIV, p. 387.

[4]J. Schunert, "The Association of Mathematical Achievement with Certain Factors Resident in the Teacher, in the Teaching, in the Pupil, and in the School," Journal of Experimental Education, XIX, p. 231.

Another variable which has an impact on problem solving is the solver's emotional state at the time the process is undertaken. Combs and Taylor[1] found that if the solver feels a mild degree of personal threat when problem solving, the time necessary to complete the problem will be increased. Cowen[2] concluded that reducing the stress in a problem solving situation by praising the solver during previous problem solving attempts resulted in significantly less rigid responses in suggesting possible solutions. Mohsin[3] studied the effects of frustration on problem solving behavior. He concluded that frustration affects each solver differently, and that the stronger the frustration the higher the interference with problem solving success. These studies suggest that the classroom environment, both during the presentation of the strategies and the practice using them to solve problems, be student oriented and free from authoritarian control.

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[1]A. Combs and C. Taylor, "The Effect of the Perception of Mild Degrees of Threat on Performance," Journal of Abnormal and Social Psychology, XLII, p. 420.

[2]E. L. Cowen, "Stress Reduction and Problem Solving Rigidity," Journal of Consulting Psychology, Vol. XVI, 1952, pp. 425-428.

[3]S. M. Mohsin, "Effect of Frustration on Problem Solving Behavior," Journal of Abnormal and Social Psychology, Vol. XLI, 1954, pp. 152.

CHAPTER III

IMPLEMENTATION OF THE STUDY

Introduction

The purpose of this research was to gather information concerning the problem solving process and the strategies used by elementary school children in arriving at meaningful solution to open end comprehensive problems.

A secondary purpose of this study was to compare the problem solving behaviors of children not engaged in a program of problem solving practice with children who regularly worked on comprehensive problem solving tasks. Based on this information compiled from a case study of twelve fourth grade youngsters, hypotheses for future experimental studies can be formulated. The use of this clinical paradigm was appropriate, as the problem solving process used by the subjects had to be analyzed in great detail in order to identify strategies and because any variation in the process or the strategies was of interest.

This study involved analyzing five interviews, conducted over a period of fourteen weeks, for each of twelve subjects. Students were selected from two fourth grade classes. The manner and rationale for choosing these particular classes is described in the first section of this chapter. Group description and individual characteristics of the students in this study is the subject of the second section of this chapter. The major factor which differentiated the students in the experimental class from students in the control class was the problem solving experience the experimental class received from their classroom teachers during the experiment. The nature of this practice in solving problems is discussed in the third section of this chapter.

Five interviews, each based upon a different comprehensive problem, were conducted by the researcher. A description of the comprehensive problems used in these interviews, and a description of the setting and interviewing procedure, is provided in sections four and five of this chapter.

Population and Classroom Selection

This experiment took place in one of the nine elementary schools in the Ypsilanti School District, Ypsilanti, Michigan. Located west of the Detroit metropolitan area, Ypsilanti is a community of about 70,000 persons, reflecting a variety of intellectual, socioeconomic, and racial representations. The district educates approximately 3730 elementary school age youngsters, of which 491 are fourth graders.[1] From this fourth grade population, twelve

[1]These figures are based on information received from the office of the Evaluator/Coordinator of the Ypsilanti School District.

students constitute the sample of this study.

Fourth grade students were chosen for this study since the researcher felt that at this grade level the students possessed minimal arithmetical, reading, and writing skills as are necessary to solve comprehensive problems. Furthermore, Silverman and Stone[1] showed that at the third grade level about half of the students were non-conservers, and that conservers prevailed over non-conservers[2] during interaction in problem solving groups.

Since three of the problems used in this study involve linear and area measurements, the researcher felt that the conservation of the properties of length and area were necessary to solve these challenges in a meaningful way. Wadsworth[3] suggests that children conserve both length and area by nine years of age. The researcher therefore concluded that by the fourth grade most of the students would be conservers, and less likely to acquiesce to the views of another student because of insufficient cognitive growth. On the other hand, since many individuals and organizations (e.g., Polya, Ausubel, and the Cambridge Conference on

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[1]W. Silverman and J. M. Stone, "Modifying Cognitive Function Through Participation in Problem Solving Groups," Journal of Educational Psychology, 1972, Vol. 63 (6), pp. 603-608.

[2]A child is said to be a conserver when he understands that a property (e.g., length, mass, area) of a body is preserved when the form or position of the body is altered.

[3]Barry J. Wadsworth, Piaget's Theory of Cognitive Development, New York: McKay Company, Inc., 1970, p. 31.

School Mathematics)[1] emphasize teaching problem solving at all levels, it seemed advisable to investigate the problem solving process as early as possible in the students schooling. Moreover, Torrance[2] reports that by the fourth grade much of the student's creativity has been lost due to the educational system itself. Vernon and Razik[3] agree that the wild questions and off-beat ideas of highly creative students are discouraged in favor of more conventional behaviors. These findings suggested that creative problem solving should be initiated as soon as possible, with the result being a reversal of this trend.

Pilot interviews conducted by the researcher four months prior to the beginning of this experiment suggested that the maximum number of students who could effectively respond to a challenge during an afternoon was six. If more than six students were chosen from one class, the interviews would take at least two days, and permit student discussion of the problem outside the interview period. Therefore, six students were chosen from each of two classes to provide the twelve subjects of this study.

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[1]G. Polya, How To Solve It (2nd edition, Garden City, New York: Doubleday and Co., Inc., 1957); David P. Ausubel, "Learning by Discovery," Educational Leadership XX (January 1969); Goals for School Mathematics, the Report of the Cambridge Conference on School Mathematics (Boston: Houghton Mifflin Co., 1963).

[2]E. P. Torrance, "Adventuring in Creativity," Childhood Education, No. 20, 1962, p. 83.

[3]P. E. Vernon, editor, Creativity (Baltimore: Penguin Books, 1970), T.A. Razik, "Creativity," Theory into Practice, No. 22, 1966, pp. 147-149.

Shapiro[1] concluded after examining children from seventy-four elementary school classes, grades 2 through 6, that youngsters who did not have experience in solving comprehensive problems made significantly fewer measureable responses in attacking a problem than students who had this experience. Shapiro's conclusion suggests that this study should investigate the problem solving processes used by both trained and untrained subjects. Therefore for the duration of the study, one class received some problem solving training, in addition to their usual school activities, from their classroom teacher.

The two classes used in this research were selected to meet the following criterion:

a) Students learned in an environment where individual feelings, creativity, and intellectual activity were highly valued.

b) Students could accurately express their opinion when questioned.

c) Students had never worked on, or participated in, any problem solving program.

d) Students were male and female, and from diverse socioeconomic backgrounds.

The researcher consulted the Coordinator/Evaluator of the Ypsilanti School System to obtain the names of teachers of classes which met the stated criterion. Principals were contacted and appointments made with those teachers

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[1]B. J. Shapiro, The Notebook Problem: Report on Observations of Problem Solving Activity, Education Development Center, Boston, 1973.

who expressed an interest in problem solving behaviors. Two fourth grade teachers were selected based on their interviews with the researcher, the suggestions of their principal, and observations of their interaction with their classes.

To provide a less subjective rationale for selecting these teachers and their classes, the researcher asked each teacher to complete a questionnaire. The questionnaire, which appears in Appendix A, contained requests for personal data and attitude assessment questions. The results of the survey indicated that the selected teachers were similar in attitudes concerning classroom management, student initiative, and their ability to teach problem solving.

Descriptions of Classes and Sample

The two selected fourth grade classes were from the same all white school with a student population from varied socioeconomic backgrounds. One of the teachers of the selected classes volunteered to provide problem solving experience for her class in addition to their usual activities. The class having the additional problem solving experience is designated as the experimental class throughout this study, while the other is referred to as the control class. The experimental class was composed of twelve male, and eleven female students, with a mean age of nine years, three months. The control class consisted of ten males and twelve females with a mean age of nine years, five months.

The classes used in this study were two of the three fourth grades in a school which ranked first and third respectively in mathematics and reading among the nine schools of the Ypsilanti School District on the State of Michigan Assessment tests. These tests were administered six weeks prior to the commencement of this study.

From each of the two classes described above, a random sample of three males and three females was drawn. Permission to include these students in the experiment was secured from their parents or guardians.

In an effort to show that the sample was homogeneous in problem solving ability at the onset of this study, the researcher attempted to assess this ability prior to the first interview. The only instrument available which is consistent with the theory outlined in Chapter Two and tests problem solving ability is the Purdue Problem Solving Inventory.[1]

The Purdue Problem Solving Inventory was developed to assess the specific behaviors which the developers felt were involved in the problem solving process. These behaviors are: (1) sensing that a problem exists, (2) defining the problem, (3) clarifying the goal, (4) asking questions, (5) guessing causes, (6) judging if more information is needed, (7) noticing relevant details, (8) using familiar objects in unfamiliar ways, (9) seeing implications,

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[1]J. Feldhusen, J. C. Houtz, and S. Ringenbach, "The Purdue Elementary Problem Solving Inventory," Psychology Reports, 1972, 31, pp. 891-901.

(10) solving single-solution problems, (11) solving multiple-solution problems, and (12) verifying solutions. The test presents problem situations which were judged to be interesting to students in various formats by teachers who had used these materials. The abstract form of the Inventory, requiring the student to make a multiple choice response to a problem presented in written form, was recommended for this study by the developer of the instrument in a personal correspondence.

The developers validated the Purdue Inventory by administering it to black, white and Spanish speaking second, fourth, and sixth graders from advantaged, and disadvantaged, backgrounds. Total scores and scores on the subclasses were obtained for 1073 children and a three way analysis of variance was computed. The ethnic factor accounted for only 3 percent of the total variance, social status accounted for 5 percent, and grade factor accounted for 3 percent. A principle factor analysis established that there are separate problem solving factors. Finally, the test retest reliability of the inventory[1] is .70 for advantaged students on the abstract form.

A week before the interviews began, all twelve subjects completed the abstract form of the Inventory under the supervision of the researcher. A two-way fixed design, sex

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[1]J. Feldhusen, J. C. Houtz, and S. Ringenbach, "An Abstract Test of Problem Solving Ability," A paper presented at a joint NCME-AERA session, New Orleans, 1973.

by treatment, Analysis of Variance was computed to detect differences between classes and/or sex. The results of the Analysis showed no significant differences between sexes and classes, and no significant interaction between variables, and therefore suggests that the two classes do not appear to be different in problem solving abilities.

As indicated earlier in this section, the treatment class received practice in problem solving, while the control class did not. The nature of this practice involved a combination of USMES activities and worksheets provided by the researcher. The twenty-two worksheets appear in the Teacher's Guide which can be found in Appendix B. The worksheets were problems modified from those found in puzzle books appropriate for nine year olds,[1] and embody one or more of the generalized problem solving strategies identified in Chapter Two. Approximately two of these worksheets were to be completed by the students each week of the experiment, under the direction of their teacher, as time permitted.

In addition to these problem puzzles, the entire class also worked on some comprehensive, teacher-supervised problems during the fourteen weeks of the study. The USMES challenge "Consumer Research-Product Testing" and a design problem planned by the researcher entitled "Design an

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[1]For example, Jack C. Crawford, Hooray for Play, (Doubleday & Co., 1958), Joan C. Weatherby, 100 Hours of Fun, (Doubleday & Co., 1962).

Underwater Habitat" were investigated by the experimental class during this period. The class spent eight weeks in product testing; selecting the "best" paper towel, glue, and notebook paper. The remaining six weeks were spent designing the underwater structure suitable for three persons to live in for seven days. In order that the teacher effectively guide the students through these activities, the researcher provided an in-service orientation to the comprehensive problems. The class was to devote about forty-five minutes twice a week to these activities.

Description of the Problems Used in the Interviews

Five interviews were conducted with each subject, at approximately three week intervals, to determine which problem solving strategies, if any, were employed in addressing five different challenges. The problems used in the interviews were taken from the Unified Science and Mathematics for Elementary Schools (USMES) materials since teachers using this program report that students find the material interesting and challenging. The particular problems were selected to represent four different types of problems: (1) product evaluation, (2) design, (3) event planning, and (4) description. The second and fifth interviews were design problems. The following paragraphs describe each of the five problems.

The Notebook Problem

The first problem, which was widely used to assess the USMES program, was to determine which of five notebooks would be best for use as a fourth grade mathematics and science notebook. The students were shown five spiral notebooks, each of which differed from the others in at least two ways. The notebooks had the following characteristics:

Notebook One - This notebook, denoted the "red" notebook, measured 10.1/2" by 8" and contained forty (40) pages of white, wide-ruled paper. The front cover, which was red, had an attached pocket inside the notebook facing the first page. This five-hole notebook cost 58¢.

Notebook Two - The second notebook, labeled the "pattern" notebook, had a multicolored floral design centered on the front cover which had a yellow border. This notebook contained one hundred and four 10.1/2" x 8" white, wide-ruled sheets. The pages were separated into four equal sections by unlined blue pages. The cost of this five-hole notebook was 97¢. The pages were bound together with a double wire binding which was heavier than the wire used in the others.

Notebook Three - The third notebook, referred to as the "yellow" notebook, contained forty-five sheets of 10.1/2" x 8" yellow paper with wide-ruled red lines. The yellow cover had a two-toned picture of two young adults, looking at a flower, with the words "Good Vibrations" below. This five-hole notebook cost 68¢.

Notebook Four - The "green" notebook measured 11" x 8.1/2" and contained one hundred (100) sheets of white, narrow ruled paper. The front cover of this three-hole notebook was green and the cost of this book was 97¢.

Notebook Five - This notebook, which is denoted the "light red" notebook, contained sixty-four sheets of 10.1/2" x 8" wide ruled white paper. The front cover of this five-hole notebook was light red and had a name space. The cost of this notebook was 67¢.

Each subject was seated at a desk with the five notebooks in front of him/her and given the following introduction and explanation to the problem:

> Suppose the city of Ypsilanti planned to buy each fourth grade student a notebook to be used for math and science work. There are more than 450 fourth graders in this city and the school board wants to leave the final selection to the students. I want you to tell me which of these notebooks they should buy, that is which is the "best" notebook to be purchased for all fourth graders in this city.

The student was also provided with pencil, pen, string, ruler, crayons, and scratch paper which they were told they could use in making their determination.

Classroom Design Problem

During the second interview, the students were challenged to design a classroom which would provide for all those activities that should go on there. The student was seated at a desk with a large poster board on it. The poster board had a scale drawing of a large 30' x 40' empty classroom which had windows located on two walls. A reduced copy of the classroom drawing is given in Appendix C. They were also provided with scale cutouts of desks for the students and the teacher. Blackboards, rugs, tables, chairs, bookcases, and shelves of various shapes and number were also provided, as well as a sink cutout and blanks which they could use to represent any other item they wanted to utilize in the room. Each student was given the following instructions:

> The city of Farmington is building a new elementary school and wants future students to like it. I want you to tell me what should go into the classroom and how the room should be arranged. Using any of these cutouts or others that we can make, plan a classroom to provide enough space and which contains the necessary furnishings to carry out all the activities which must go on there.

The interviewer explained the scale drawing of the classroom, indicating windows and doors, until the student said he understood the layout and the scale to which it was drawn. The interviewer asked and responded to questions until the student said he or she understood each ready-made cutout and that various sizes of certain pieces were available or could be cut out to suit the needs of the designer.

Plan an Ice Skating Party

The third problem of planning an ice skaing party for fourth graders is a variation of the "plan a picnic" challenge used by USMES as a part of their on-going assessment of the USMES program. The student was seated at a desk with pencil and paper. In front of the student were two large poster boards. On the first was a drawing of an aerial view of a school and three neighborhood ice rinks. A reduced copy of this drawing of the neighborhood is given in Appendix D. The rinks differed in various characteristics, and are described as follows:

Area One - This area represented a park, with an ice rink, six blocks northeast of the school. Within the park, two blocks from the rink, was a rest area containing a shelter, restrooms, and a concession stand where food could be purchased.

Area Two - Due west of the school two blocks was a small park with an ice rink slightly larger than that in area one. The park provided a place near the ice for a bonfire and had a concession stand.

Area Three - Eight blocks north of the school was an ice rink which had a 50¢ admission charge. Adjacent to this, the smallest of the three rinks, was a heated, enclosed building with restrooms, concession stand, and rest area.

The second poster board contained pictures of food which was available at each of the area's concession stands. Included on the menu were six types of sandwiches, pizza, soup, drinks, snacks, and desserts, with the price of each item listed next to it. The student was provided with the following background explanation and instruction:

> A school near my house has a fourth grade class of twenty-five students which has saved fifty dollars from bake sales they have sponsored. They would like to use the money for a skating party. They plan to go sometimes in January to one of these three areas and perhaps spend some of their money for something to eat. If you were on the planning committee how would you plan the party? Consider everything you think is important, like where and when to go, what to do, and whatever else they should think about.

The interviewer asked if the subject understood the drawing and pointed out the different aspects of each area. When the student indicated that he understood the problem, the interview began.

Describing People

The fourth interview challenged the youngsters to determine what information would be best included in the description of a person. After the student was seated in the interview room, he was shown a photograph of a person unknown to him/her. The interviewer asked how to describe the person in the photograph so that the interviewer's brother would recognize that person at the airport. After discussing the many describable attributes of a human being with the subject each was asked:

> Determine what is the best information to put in a description so that a person can be quickly and easily identified.

As the student responded to the problem, the interviewer interrupted only to ask for clarification of a tentative solution (e.g., "Why did you choose that characteristic?").

Playground Design

The last interview for each subject concerned the solution of the playground design problem: Design a playground suitable for an elementary school. The subject was provided with a large drawing of a play area of a school printed on poster board. A reduced copy of the scale drawing can be found in Appendix E. A second poster board contained pictures of various pieces of playground equipment, both conventional and free form items. The price of each unit was included with the picture. Finally, the student was given scale drawn cutouts of this equipment in each of the various available sizes, and poster board which could be cut to any size to represent playground equipment of the subjects own design. In addition, pencils, paper, scissors, and a ruler were available for the student to use.

After the explanation of the scale cutouts, the drawing of the playground and the catalog poster board was acknowledged as understood by the student, the following information and statement of the problem was given:

> An elementary school in Ann Arbor is planning a new playground for students from the first through the fifth grade to use. The school board has given the school $4,500 to be used for this purpose. Suppose you were helping to plan this playground. What equipment would you include, and where would you position it in the play area?

The only interjections made by the interviewer were to ask the students for clarification of what they were doing and why, or to answer student questions.

Interviewing Procedures

Beginning the week prior to the problem solving activities of the experimental group, and continuing at three week intervals thereafter, the researcher conducted interviews designed to detect patterns and strategies used by the subjects in solving the problems described in the previous section. While it is clear that the interpretation of the behaviors which were observed could be a function of the person making the inferences, the researcher felt that reporting these behaviors and specific strategies could be most effectively done by the researcher. The interviews were conducted during the afternoon of that day of the week which the classroom teacher felt would be least disruptive to learning activities. Each teacher wanted the interviews to take place in the afternoon.

All of the interviews and the pretest took place in a classroom of the school building containing approximately ten student desks of various sizes. The room was well lighted, comfortably warm, and free from outside distractions. The students were taken to the interview room by the researcher, one at a time, and returned to their classrooms at the end of each discussion. During the interview the only people in the room were the subject and the researcher.

For the first, second, and fourth interview the researcher sat facing the student and about five feet in front of the solver. During the third and fifth interviews the subject had a poster hanging on a blackboard in front of him, and the interviewer sat at the right side of the respondent.

Each interview was recorded on a cassette recorder, and observations made by the researcher were transcribed. Paper used by the student for computations were duplicated and added to the interview material. A description of these interviews is found in Chapter Four.

CHAPTER IV

STUDENT RESPONSES TO THE PROBLEMS

Introduction

This chapter contains a description of the problem solving behaviors demonstrated by the students during each of the five interviews. This material is organized, by problem, into five sections. Each section is subdivided into the following:

a) Setting and introduction to the problem

b) Responses of the experimental class

c) Responses of the control class

d) Identification of the strategies used by each class

Throughout this chapter, the students are referred to by first name, and a class designation; (E) for members of the experimental class, and (C) for members of the control class.

The Notebook Problem

The first interview took place during the second week of November, with the experimental class responding on Tuesday afternoon, and the control class on Thursday. Up until the time of this interview, neither class had been involved in any special problem solving activities. In order that the students become acquainted with the interviewer, they met with him twice, in their classrooms, during the last week of October. The Purdue Elementary Problem Solving Inventory was administered by the researcher to all twelve students, in the room to be used for the interviews, during the week prior to this first interview.

In the first interview each student was asked to respond to the following challenge:

> The school board is planning to purchase one thousand of one of these five notebooks for the use of each fourth grader in the district. Which notebook do you think is the best to buy?

While the interviewer gave this statement of the challenge, the student was shown five notebooks (described in Chapter Three), each of which differed in at least two attributes from the remaining four.

Meaningful solutions could only be reached by those students who were able to define "best" as it applies to notebooks, understand the aspects involved in the problem, examine the alternatives, and select the alternative most consistent with his/her definition.

After the problem was stated, it took an average of twenty-six seconds for the students in the experimental class, and twenty-nine seconds for the students of the control class, to reach a conclusion. The elapsed time to solution to the nearest five seconds, for each of the twelve students, is given in Table 1. The researcher feels that the short period of elapsed time before each subject arrived at their solution indicates a myoptic view of the problem.

Table 1. Elapsed Time to Solutions

Experimental Class		Control Class	
Student	Number of seconds	Student	Number of seconds
Trina	15	Shari	15
Jackie	30	Susan	30
Denise	30	Linda	25
Todd	20	John	30
Michael	30	Mark	30
Joey	30	James	40
Average	26	Average	28

<u>Responses of the Experimental Class</u>

After the challenge to select the best notebook was made, each student in the experimental class made a cursory examination of the notebooks before announcing his/her decision. The selection of each student, given in parentheses, is indicated below. Their choice is followed by their response to the question, "Why do you feel that is the best notebook?"

Jacquelyn (E): The one with yellow paper (yellow notebook). Has yellow paper and you might need it sometime.

Denise (E): This one (green notebook). Almost the most sheets and the biggest sheets. This one (pattern) has four more, but this one (pattern) the sheets are smaller.

Trina (E): This one (light red notebook). Has a space for your name.

Michael (E): That one, its more sturdy (pattern notebook). I narrowed it down to these two first (indicating the pattern and the green notebooks), then decided on this one. (Interviewer requested that Mike expand his answer.) First of all this one has yellow paper, and I don't think you need that (yellow notebook). This one has only sixty-four pages (light red notebook) and this one only has forty pages (dark red notebook). The binding is better (on the pattern notebook). The other (green notebook) may catch on something.

Todd (E): (dark red notebook) This one. You might have to do your work over if you lost it, and you won't lose it in here if she tells you to keep it.

Joey indicated his choice of the green notebook by pointing to it, but could give no explanation as to why it was the "best" notebook for the school district to buy.

The interviewer then asked questions to discover whether the students had considered any other aspects of the problem. Three youngsters were unable to expand on their answers [Joey (E), Jacquelyn (E), and Trina (E)]. Joey (E) was almost completely non-verbal during the interview, shaking his head or shrugging his shoulders to answer the questions which were directed at him. Jacquelyn (E) and Trina (E) investigated the problem superficially, as both seemed to evaluate the proposed challenge based solely on visual characteristics unique to one notebook so that an answer could be given. Neither could give a defense of their choice, nor in answer to further questions had they considered attributes such as cost, number of sheets, or ruling of the pages. Trina (E) did not respond to any further questioning, and the interview terminated five minutes after the statement of the problem. Jacquelyn (E) changed her mind four times during questioning, with each selection reflecting the direction given by the interviewer:

> Interviewer: What about the number of pages in the notebooks?
>
> Jacquelyn (E): I would pick this one (Indicating the green notebook after spending thirty seconds reviewing them a second time).
>
> Interviewer: How about the cost? Is that important?
>
> Jacquelyn (E): Very important.
>
> Interviewer: If you had to choose between the yellow notebook and the one which cost the least, which one would be the best?
>
> Jacquelyn (E): The one that cost the least.
>
> Interviewer: How about the number of pages?

Jacquelyn (E) again changed her mind. The interview was concluded with Jacquelyn (E) defending the notebook which cost the least. Her rationale for this decision was based on repeated attempts at finding the actual saving involved in purchasing the least expensive notebook as compared to the most expensive. Her frustration in finding the answer is reflected in her reply:

> Jacquelyn (E): Because, if you are going to get one thousand, you would save more than a dollar.

The other three respondents were able to expand on their initial answers. Michael (E) chose the pattern notebook based on several considerations: number of pages strength of the binding, color of the sheets. Todd (E) explained why he felt that a pocket in the cover was important and Denise explained that she wanted to maximize the amount of paper in the notebook.

Responses of the Control Class

Each subject within the control class perused the five notebooks before arriving at a decision as to what he/she considered the "best." Their selections, and the rationale for their choices, are as follows:

> Shari (C): (green notebook) This one. Its got a lot of pages. I like the big size. I have one this size (referring to the dimensions of the page).
>
> Linda (C): (pattern notebook was indicated by holding it up.) It has more paper, and it has two different kinds (color) in it.
>
> Susan (C): (pattern notebook) I pick this one. It has dividers, and they could tell which subjects and stuff. It would be a lot easier, and they wouldn't have to have as many notebooks. (This notebook has four blue pages in it which could serve as effective dividers.)
>
> James (C): (dark red notebook) I think this one. When you are finished with your math you could put your work in the folder. It cost less.
>
> Mark (C): (pattern notebook) This one, because you could keep different subjects separate. Its a thicker book.
>
> John (C): (pattern notebook) I'd say this one. I think so because its got a lot of paper and it also has a lot of colors on it and its got some blue paper to write on.

When compared to the responses of the experimental class, the initial replies of the students from the control class reflect a consideration of a greater number of aspects of the notebook challenge.

Identification of the Process and Strategies Used by the Students

Nine of the students, three experimental and six control, used an identifiable process in reaching a meaningful solution to this challenge.

In every instance, the student defined the problem by establishing priorities as to those attributes he deemed necessary in his choice of the "best" notebook.

After the definition stage, each student considered the different notebooks and eliminated those which were inconsistent with this ordering. This process resulted in a choice which the individual felt was the optimal selection.

Based on observation by the interviewer, it seemed that the elimination of alternatives was carried out based purely on perception. For example, to find the notebook with the maximum number of pages, seven students (four control and three experimental) chose one that was thick. Comparisons were made by laying or standing two notebooks side by side. Denise, for example, wanted the notebook with the maximum amount of paper, that is, the greatest area for writing. She didn't attempt to compare the total writing area available in each notebook analytically, but rather she held them up to check the dimensions of the pages visually. None of the youngsters attempted to count the number of pages, and only two of them, Michael (E) and Denise (E), gave evidence that they noticed that the number of pages was written on each notebook.

Except for James (C) none of the twelve students considered the price of each notebook until the interviewer directed their attention to this aspect of the problem. After such direction, five students (two from the experimental class and three from the control class) compared prices, but did not proceed to use this information to support or reject their original choice. When the interviewer asked about the cost of their selections, two typical responses were those of Denise (E) and Susan (C):

> Denise (E): (After noting the cost of the notebook) This one (green) is still better, because this one has more pages, and these two (green and pattern) have the same amount of pages. (The interviewer then challenged the subject to explain why she chose the green notebook over the less expensive ones.) Cause they don't have as many (pages) and they might not go through the whole year.

> Susan (C): They are both the same (pattern and green notebooks). You get more for your money. (The interviewer pointed out that any of the three notebooks cost less than either of the notebooks she was considering.) I just picked this one.

This first problem also provided evidence that children can and do make inferences. The problem solving strategy of making inferences involves drawing conclusions from the goal or givens. Each of the subjects who chose a notebook after defining "best" to themselves made conclusions based on the attributes (givens) of the notebooks.

John (C) felt that the best notebook should last as long as possible. He reasoned from this goal that maximizing the lifespan of a notebook implies that the student

must not abuse it. John (C) inferred that students were not as likely to misuse a notebook which they liked, hence he wanted the "best" notebook to be attractive. Michael (E) inferred from the goal of finding a durable notebook that the binding must be sturdy.

None of the twelve subjects considered the ruling of the pages, the number of holes (three or five), or the strength of the individual sheets. It was not clear from the interviews whether the subjects avoided experiments of testing the strength of the paper, or the visibility of marks on a sheet, because they had not thought of them, or because they were inhibited since they realized that the notebooks were to be used by other youngsters.

In this first interview, half of the experimental class gave meaningful responses, while each of the control class demonstrated a process which led to valid solutions. The researcher concludes that this variation is due to individual personality differences, that is members of the control class were more extroverted and willing to express opinions.

Classroom Design Problem

The second interview took place during the first week of December, and concerned the planning and arranging of a classroom. Since the first interview, the control class had engaged in their usual classroom activities. The experimental class had two periods in which they worked with the Consumer Research-Product Testing unit of USMES. In addition, this class had completed two of the problem solving worksheets during the last two weeks of November.

During the second interview the students were asked to respond to the following question:

> How would you plan and arrange a classroom to provide adequate space for all of the activities that might be carried on there?

Each student was shown an outline of an empty classroom and given a number of cutouts, drawn to scale, representing various items which might be found in a classroom. These materials were to be used by the student in exhibiting his/her proposed layout of the classroom.

In the researcher's judgment, this challenge provided an opportunity for the subjects to use several strategies: inference, contradiction, and the identification of subgoals. The nature of the problem allowed students to investigate and reach conclusions without much verbal response.

Responses of the Experimental Class

After the initial familiarization with the materials to be used in seeking a solution, every student began by choosing objects for the room without stating or requesting the number of students for whom the room was being planned. The interviewer interrupted the solving process to ask each student how many children would use their classroom. Their replies are as follows:

Jacquelyn (E): About twenty.

Denise (E): Twenty to twenty-five seemed like the right number.

Trina (E): Twenty-three. That's what most everybody elses class have.

Joey (E): Twenty-two. Our class has twenty-three.

Michael (E): Twenty-four. That's the number of students in our room, and every other room in this school.

Todd (E): Twenty-five. Most classes around here have about that many.

These students are from a fourth grade class which contains twenty-three students.[1] Their responses seem to reflect these students' experience with classroom sizes.

Every experimental child began the solving process by putting the desired number of student desks (or a desk for the teacher) into the classroom. The researcher posed the question: "Why did you begin with desks?" All of their replies were essentially the same--desks are vital.

This evidence seems to support the claim that these students did, in fact, establish an ordering of the component tasks involved in filling a classroom. That is, they were using ordered subgoals to reach a solution.

Jacquelyn (E), Denise (E), and Michael (E) arranged the desks in U-shaped formation. This is the same arrangement of desks found in the experimental classes' room.

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[1]The principal of this school, in a discussion with the researcher, indicated that the average class size had decreased during the past two years. The third grade classes the year prior to this study averaged twenty-four students.

Trina (E) placed her desks in six rows, four desks to a row. Todd (E) segregated the class into four rows of six desks each, and put them in two rows on each side of the room. Joey's (E) arrangement began with the teacher's desk in a corner of the room, and three rows of student desks, in semi-circles, facing that corner. This floorplan reminded the interviewer of the way seats face the stage in an auditorium. Joey (E) was asked to tell the interviewer why he chose this way to set up the class, but he could give no reply and simply shrugged his shoulders.

A variety of items (tables and chairs, rugs, bookcases, sink) were added to the classroom after the desks had been placed. Each of the six students rearranged some of the furniture to admit the placement of one or more additional pieces. However, Denise (E) and Trina (E) ignored obvious contradictory situations by placing chalkboards in front of windows, and thus prevented light from entering the room. Although both of these girls understood where the windows were in the drawing at the onset of the challenge, they must not have realized their location when they placed the chalkboards ten or fifteen minutes later.

Responses of the Control Class

Immediately after the statement of the challenge each member of the control group began putting items into the classroom outline. The interviewer interrupted the solving process to ask each student how many children would use the classroom they were designing. Their answers and rationales are indicated below:

> Shari (C): I don't know. I'm going to try to get in pretty many. (Her final arrangement contained twenty-four desks.) Because where I was we always had more than we have here. (Shari had transferred into the school during May of her third grade year.)

> Linda (C): About twenty. Every room I've been in had at least twenty.

> Susan (C): Twenty-eight. I don't know (why I chose that number).

> James (C): Twelve. For two rows. We had it a couple of years ago.

> Mark (C): Twenty-two. I don't know (why I chose that number).

> John (C): Thirty. It's a good number. They could teach more students in one class.

Except for James (C) each student used between twenty and thirty student desks. John (C), James (C) and Susan (C) planned for a class containing a significantly different number of desks than found in the control classroom (twenty-one).

Of the six students from the control class, only Linda (C) and Mark (C) began by placing anything other than student desks in the room. After placing two items in the classroom, both of these students started putting desks into the outlined classroom area. The interviewer asked each of the six students why they began as they did. Linda (C) could give no explanation for starting with a rug and a bookcase. The remaining five felt that desks were the most important furniture for the classroom, and should go into the room as soon as possible. For example, Shari (C) responded, "Well, you've got to have desks for the students."

Four students [Shari (C), James (C), Mark (C), and John (C)] arranged the desks in a row by column array. Initially, Susan (C) and Linda (C) grouped the desks into sets of four, however Linda (C) altered her arrangement to include two sets of two rows of desks. This use of rectangular array arrangement is interesting, since in the control class the position of desks were currently arranged in clusters (circular, square, rectangular), and had been for the six weeks prior to this interview. None of the students could explain why they thought their arrangements were the best possible.

After the desks had been placed, the students added items which were found in their control classroom (worktables, rugs, bookcases, sink). Each member included a rug and a bookcase in one corner of the room, an arrangement which coincides with that found in the control classroom.

Rearrangements of the furnishings were necessary when space became limited or when the position of an item contradicted its usage (e.g., a chalkboard which nobody could see).

Identification of the Strategies Used by the Students

The first strategy which became apparent during the interview was the identification and ordering of subgoals and subtasks which were to be completed. Six experimental and five control students clearly wanted to complete the task of arranging desks before planning any other aspect of the layout. This component of the design was therefore given preference before undertaking any other component task, such as planning a reading or art area.

A second strategy was detected as each student placed more and more things into the room outline. When placement of an item conflicted or interfered with the position or purpose of another cutout which had been previously situated, subjects would generally rearrange one or more of the pieces. For example, Mike (E), when he added the chalkboards, realized that his initial placement contradicted his goal that "every student could see the board," and so proceeded to rearrange the chalkboard location and some of the earlier set desks.

Further examples of the use of contradiction were apparent in the layouts of Mark (C) and Linda (C). Mark (C) moved the teacher's desk to permit youngsters a better view of the chalkboard. Linda (C) changed the complete arrangement of students' desks to avoid contradicting her goal of "not wasting space." Seven of the twelve students in this study (three control and four experimental) made some adjustments of the arrangements in their layouts as they encountered situations or objects (e.g., walls, windows) which precluded the attainment of a goal. The remaining five students did not rearrange the cutouts and it was not clear to the researcher whether these students used contradiction in planning their classroom design.

Since there was very little verbal response on the part of the subjects except when asked direct questions with respect to this second challenge, the interviewer was unable to detect behaviors which could only result from inferences made by the subject. However, the interviewer had the impression that some of the youngsters in this study did use inference, in conjunction with other strategies, in arriving at a solution. For example, Mark (C) placed his desks in rows so that students would not be seated behind each other. When asked why he did so, he replied, "So everyone can see." Clearly, his goal of providing an equal opportunity for all students to see what was going on implied that the line of sight from each desk (to the teacher's desk and to the chalkboard) must be unobstructed. Except in a few similar situations, the students worked without commenting on his or her reason for a particular placement, and hence the researcher can not suggest how much inference was being utilized.

Another problem solving strategy became apparent as the students completed their arrangements. All twelve of the planned layouts contained those objects which were found in their fourth grade classroom, that is bookcases, cabinets, tables, and a sink were found in every floorplan. The interviewer concluded that the subjects were recalling the solution (arrangement) to the equivalent problem of equipping the classrooms of their school when they planned the arrangement of the proposed new room. To support this conclusion, the interviewer asked each subject, after they had completed their classroom design, if there was anything else they wished to have included in the room. Three students, Joey (E), Jacquelyn (E) and Linda (C), answered no and their interviews were terminated. The other nine children began naming things as they looked around the classroom in which the interview took place. For example, Todd (C) suggested "two wastebaskets, flag, clock, thermometer (he meant thermostat), and closets," as he noticed these items were also found situated within the interview room. The researcher concluded that these students were using a related problem, to which they knew the answer, to respond to the present challenge of classroom format.

The solutions to this challenge also exemplified the lack of creativity demonstrated by the members of this study. As pointed out earlier, Joey (E) was the only student to give an arrangement of desks which was somewhat original. Moreover, no subject suggested items for the floorplan which were not found in either the interview room or in their fourth grade classrooms. This condition existed even though the researcher showed them templates, poster board, scissors, and rulers that were available, and also explained that scale cutouts of almost anything they suggested could be provided. The interviewer has the impression that the subjects tried to give solutions which were like an existing classroom or which was consistent with one they presumed the interviewer had in mind. Indeed, Jacquelyn (E) and John (C) asked "Is this okay?" when they had completed their solutions. Pointing to a specific cutout, Shari (C) was told twice, after she began the problem, that she could put whatever she wanted into the classroom after she asked "Can I use this?"

A second aspect of this challenge which was ignored or poorly handled by the children is the understanding and use of "scale" in the positioning of items within the classroom boundaries. Each subject utilized the entire floorspace of the scale drawing. There was no evidence that the placement of any item was determined a priori by considering the actual distance between fixed and movable objects. For example, Joey (E) put seven chairs around a table which was four feet in diameter. Denise (E) put a rectangular work table and three chairs between the teacher's desk and the blackboard--a distance of seven feet. Scale drawings are generally encountered during the post elementary school years, and neither the experimental nor the control class had completed a unit on this topic.

Planning a Party Problem

The third interview took place on Tuesday and Friday afternoons of the first week of January, four days after returning to school from a two week Christmas vacation. During the three class weeks of December, the control class was involved in their usual classroom activities. The experimental class worked on the USMES unit (Product Testing) once, and with the problem solving worksheets three times during these same three weeks.

The third challenge involved planning an ice skating party for twenty-five fourth grade youngsters, and assumed that $50.00 could be used to cover any costs incurred. The student was shown a map of the neighborhood surrounding a school on which three recreational areas, each containing an ice rink, were indicated. Each area differed from the other two in at least two ways (e.g., proximity to the school, enclosed shelter, restrooms). The student was instructed that food service was available at each of the areas. A large menu poster, listing and picturing food items which could be purchased at each area, was positioned in front of the subject after the introduction to the problem. The students were allowed to work in silence except when it was necessary for the researcher to interrupt and seek clarification of what was being planned. For example, the interviewer might ask, "Why did you select Area Two for the party?" The solution of this third problem involves answers to several subproblems which are essentially independent. In other words, the subjects must answer many questions before the plans for the ice skating party can be considered complete. For example, the questions of where and when the party will take place and what kind of snack or meal should be planned can be construed as unordered subgoals. These unordered subgoals must be resolved before a total solution to the challenge is achieved.

<u>Responses of the Experimental Class</u>

After the initial explanation of the problem by the interviewer, the students were asked to name some items which should be considered in planning a party. Jacquelyn (E) and Todd (E) and Michael (E) responded with answers which included when and where to go and what to do. Joey (E), Trina (E) and Denise (E) made no response to the interviewer's inquiry, and after thirty seconds the interviewer suggested a few things which they might consider.

After the student listed or was told at least three things to be considered in planning a party, the interviewer asked, "What would you like to plan first?" Denise (E), Trina (E), and Joey (E) did not answer, and after about twenty-five seconds the interviewer suggested a direction: "Where should we go?" The following responses were given:

> Denise (E): Area Two because its closest, and its got a big ice skating rink, and its got a place for a bonfire.
>
> Trina (E): Area Two because its closest. It doesn't cost anything--has a bonfire and food.
>
> Joey (E): Area Three because its bigger.

The interviewer had the impression that although these youngsters understood that there were several things to be planned, they were unable or unwilling to select one of these subproblems on which to commence work.

Jacquelyn (E), Todd (E), and Michael (E) were able to suggest where they would begin working to plan the party. Jacquelyn (E) began by solving the menu issue; Todd's (E) first task was the selection of the location for the party, and Michael (E) began by deciding the time aspect of the problem. It was not clear to the interviewer whether each of these children selected their answer because it was the first subgoal to occur to them, or because they realized that any of several equally valid starting points might be used. After some solution was found for their initial sub-problem, each of these six students directed their attention to another subtask. Thus, even those three students who needed some direction initially were aware of and chose a second aspect of the problem on which to work.

With guidance, all of the students were able to reach conclusions for several of the questions which make up this challenge. The location selected and rationale for those choices are given below:

> Jacquelyn (E): Area Two. The bonfire would keep you just as warm as three. It's closest.
>
> Denise (E): Area Two because it's closest, and it's got a big ice skating rink and it's got a place for a bonfire. If you went to the other ones, that one (Area One) would be further away, and that one (Area Three) cost fifty cents to get in.
>
> Trina (E): Area Two because it's close, it doesn't cost anything, and has a bonfire and food.
>
> Joey (E): Area Three because it's bigger. (After being asked if there is any other reason)--Has restrooms and it's enclosed.

Michael (E): I think I'd go to the Area One because it has the smallest rink and if you're trying to find the kids it would be easier. You would save some money.

Todd (E): Area One because you could just go there and ice-skate. Area Two doesn't have restrooms and Area Three cost 50¢ to get in and you probably wouldn't have enough money to get a real lunch.

Three students chose Area Two as the best place for the party based primarily on its proximity to the school. Trina (E) changed her mind when she noticed that there were no restrooms in Area Two. Denise (E) planned to stay "a couple of hours" and concluded that restrooms were not necessary. When the interviewer pointed out the fact that she planned to stay at Area Two for six hours, and that there were no restrooms available, Jacquelyn (E) replied that they "could go behind the bushes." Clearly what is a problem for so many people is not necessarily a problem for all fourth graders.

The selections of the best time for the party varied widely and are listed as follows:

Jacquelyn (E): Afternoon, 1:00 - 3:00 P.M. You could have twenty minutes to clean up. (School is dismissed at 3:20 P.M.)

Denise (E): Maybe at dinner time. Sometimes it's better to go at night because there is not a whole lot of people.

Trina (E): All day. Maybe until 1:00. You could come back and talk about it.

Joey (E): 5:30 - 7:00 P.M. (Joey did not justify his answer.)

Michael (E): Sometimes in the afternoon, because it's not as cold then. About two hours. If you stayed one half an hour you wouldn't have any time to skate. If you stayed longer, you might get cold.

Todd (E): From 11:00 A.M. to 1:30 P.M. Because when I go roller skating (which is easier), I can't roller skate for that amount of time. In the middle of the day you are rested.

Menus were planned by each youngster and all included hot chocolate and a dessert. Except for Joey (E), the students included some kind of hot sandwich or soup to provide a balanced warm meal.

Four of the students, Jacquelyn (E), Denise (E), Todd (E), and Michael (E), suggested some things to do while at the recreational area. The interviewer asked if there was anything else to consider in planning the ice skating party. Michael (E) was the only one to make a suggestion: "How are they going to get there?"

Responses of the Control Class

After the statement of the problem, the interviewer asked each student to list some things to consider when planning a party. Susan (C), Linda (C), and John (C) each named three or four things that needed to be explored. Shari (C), Mark (C), and James (C) did not respond within half a minute, and the interviewer made suggestions as he had with members of the experimental class.

Following this introductory discussion, the interviewer asked, "What would you like to plan first?" Each of the six members of the control class were able to suggest a starting point. John (C), Linda (C), and James (C) commenced work on the problem of deciding the location of the party. Susan (C), Mark (C), and Shari (C) planned for the optimal time before attacking another subtask.

With assistance, the six students worked through additional subproblems (e.g., what to do, what to eat, how long should the party last). The question of where to go for the party was resolved by each student, and their answers and rationale are as follows:

> Shari (C): Area Two. It's shorter and it's about the right size. (Lack of restrooms was pointed out to her.) Yeah, I just noticed. (After some thought about the length of time the party should last, Shari changed her mind to Area Three.) Because they have bathrooms; could buy food and go inside.

> Linda (C): Area One. It has restrooms. It has a covered shelter. No shelter here (indicated Area Two). Not Area Three because it cost fifty cents to get in.

> Susan (C): Area Three. It sounds better. It has a nice size rink, has an enclosed building in case it rains or something. (When asked why she rejected the others, she replied) I don't know. I still like Area Three.

> James (C): Area Three because it's enclosed, heated, and if you buy something to eat it would keep warm.

> Mark (C): Area Three. It's got restrooms. Because it has a building and benches and you get warmer faster.

> John (C): Area Three because it's enclosed. No one could bust in and steal the money.

The question of when to go was settled by each of the students. As in the case of the experimental class, the replies were varied and, when justified, considered only one aspect of the question of determining the optimal time. Consider the responses below:

Shari (C): From a quarter of nine until twelve.

Linda (C): 8:00 P.M. I would stay for two hours. (When asked if that was too late for some of the students, Linda replied) For some people. (She didn't alter her choice of the optimal time for the party.)

Susan (C): At 1:00 in the afternoon to 3:00.

James (C): Morning. When school started until 12:00.

Mark (C): 4:00 P.M. - 6:00 P.M. Not too late so they could see.

John (C): Lunch time. I'd stay until 3:30 because I like to ice skate.

The dietary plans of each of the students included three or four items. Each of them selected a hot sandwich or pizza. Susan (C) was the only member of the control class to choose soft drinks for the menu; the remaining five students selected hot chocolate. All of the menus included a dessert of fruit or candy bars.

Five students made suggestions when asked what additional things should be considered in planning a party:

Shari (C): Make some rules. Don't leave the area, all eat together.

Linda (C): Choose partners.

Susan (C): Permission slips. Make sure everyone has skates and dressed warm.

James (C): What to bring: warm clothing, an extra pair of socks.

Mark (C): Have some help in case someone doesn't know how to skate.

The interviewer could not determine why five students of the control class suggested additional questions to be resolved and only one member of the experimental class. The classroom teachers reported that the control class had made a field trip during the fall, but the experimental class had not.

Identification of Strategies Used by the Students

This challenge provided an opportunity for the students to solve a problem containing several subproblems, many of which were equally important. Six students, three from each class, were able to identify three or four of those subproblems. Three of the students from the experimental class, and six students from the control class, were able to select a subgoal to be achieved first. Hence, youngsters can recognize problems involving several component parts, and in some cases they can select a single aspect on which to begin work. None of the twelve students in this study were able to work through three successive subproblems without direction on the part of the interviewer, and the researcher concluded that this was because these subjects found it difficult to store several aspects of the challenge in immediate memory while working on a single component part.

With guidance, all of the subjects were able to reach conclusions for several of the questions which make up this challenge. In working through these questions, three problem solving strategies were demonstrated: inference, using a related problem, and contradiction.

Inference from a desired goal was used repeatedly in solving the subproblems of this challenge. For example, Michael (E) wanted to minimize the discomfort of the class, so he chose to have the party "sometime in the afternoon because it's not too cold then." Mark (C) planned the party to occur after school but "not too late, so they could see." He inferred from these constraints that the party should be held between 4:00 P.M. and 6:00 P.M. In planning menus for the party-goers, two subjects made inferences from the goal of providing good nutrition. Jacquelyn (E) chose hot soup, fruit, and hot chocolate because she wanted "something from the four food groups." John (C) also wanted "one from each group, because that means it's not a junky lunch." He selected a menu of sloppy joes, hot chocolate, fruit, and pudding. James (C) also included fruit in his dietary plans because it is "better for you." Finally, eleven youngsters inferred that hot chocolate or soup would be advantageous in reaching the goal of keeping warm on a winter's afternoon.

The choice of location for the ice skating party was made using inferences from the student's definition of the "best" skating area. The researcher construed from the responses that "best" meant many things to the students, such as warm, secure, having restrooms, and being close to school. Some typical responses are listed below:

Denise (E): Area Two because it's closest and it's got a big ice rink and it's got a place for a bonfire. The others are further away and that one (Area Three) cost fifty cents to get in.

Michael (E): I think I'd go to Area One because it has the smallest rink and if you were trying to find the kids it would be easier.

John (C): Area Three because it's enclosed. No one could bust in and steal the money.

Mark (C): Area Three because it's got restrooms, benches, and you get warmer.

As reported in the discussion of the notebook problem (challenge one) the subjects tended to focus on a single dimension of the problem. This myoptic view is evident in the solutions to several subproblems encountered in this challenge. Often, this leads them into a situation with conflicting alternatives. This conflict is resolved using the strategy of contradiction. For example, Shari (C) and Trina (E) had to reassess their solution to the location problem when their choice contradicted the goal of providing for the usual needs of the students. The following two examples are additional support of the claim that subjects use contradiction to eliminate alternative solutions to subproblems encountered in this challenge:

After Susan (C) selected Area Three as the place to have a skating party during the afternoon hours, she began planning a snack break:

Susan (C): Pizza, cookies, soft drink, and a candy bar.

Interviewer: Would you have enough money for all of those things after paying the admission charge?

Susan (C): (After finding the total cost) Yes.

Susan (C) realized that she hadn't used all of the available money, and considered adding to the list. When she discovered that the price of additional food would contradict the cost constraint of $2.00 per person, she rejected the alternative of increasing the snacks. Todd (E) also used contradiction in planning his menu for the party. As he selected food for the skaters, he kept a running total of the price of these edibles. His first four items cost $1.30 per person, and thereafter he tried alternatives and added or rejected them based on whether or not the total contradicted the $2.00 limit. Todd (E) was the only subject who considered the cost of the food as he was planning the menu. The remaining eleven subjects focused on this aspect of the problem only after the interviewer directed their attention to the question of whether or not they had enough money.

During Todd's (E) interview, another problem solving strategy became apparent--that of looking at a related problem. When Todd (E) planned the length of time the students would stay at the rink, he made inferences based on his solution to a similar skating problem:

> Todd (E): From 11:00 to 1:30 because when I go roller skating, which is easier since you have four rollers, I can't roller skate for that amount of time.

An upper bound of two and half hours for the ice skating party is established because it represents the maximum amount of time Todd (E) could spend on roller skates.

A more common use of referring to a related problem occurred when the students appeared satisfied that they had a fairly complete solution to the challenge. Just before the interview was terminated, in each instance, the interviewer asked if there was anything else to consider in planning the ice skating party. Six interviewees, five from the control class and one from the experimental class, responded postively. These subjects were asked why their considerations were important, and each responded that these arose when they had gone on other trips. Clearly each one was thinking of the solution to some previous planning problem which they had seen (experienced) on some class field trip, scouting outing, or family vacation.

This was the first of the five challenges, used in this study, which required the students to use simple arithmetic to insure the accuracy of a partial solution. Eleven of the students had to be asked if they had enough money to purchase the food they had planned for the party. Todd (E) was the only student who monitored his costs as he proceeded with the planning. Two students, John (C) and Trina (E), responded, "I don't know" to the interviewer's inquiry, and needed direction before they totaled the prices. Five students, Denise (E), Mark (C), James (C), Susan (C), and Linda (C), attempted to add the cost without the aid of pencil and paper, with mixed results. Two students, Joey (E), and Michael (E), were able to accurately estimate the cost by rounding off the individual prices to quarters, and adding. The final two participants in this study, Shari (C) and Jacquelyn (E), went directly to the paper provided them to add the prices of the items they were choosing. Of the eleven who received direction, five students (two from the experimental class, and three from the control) were unable to add the costs and arrive at a correct total. Since these youngsters are part of the classes which did fairly well on the state assessment test in mathematics, the researcher concluded that children, in general, are not given an ample opportunity to utilize mathematics in comprehensive problem solving situations.

Describing People Problem

The fourth interview took place on the last Tuesday and Friday of January. During this interview the students encountered the problem of describing themselves, and a person in a photograph, and to determine the best characteristics to use in describing any person. Since the last interview the control class had continued their usual classroom activities. During the month of January the experimental class completed four periods engaged in working on the design of an underwater habitat, and six class periods of work on the problem solving worksheets.

This interview began with the student describing himself. Following some discussion on whether they had ever tried to find someone using a description supplied by another person, the interviewer showed the subject a black and white photograph of an Air Force pilot getting into his plane. Each student was told that it was necessary for someone unfamiliar with the pilot to locate him at the airport, and that the student was needed to provide an accurate description of the pilot in the picture. This preliminary dialogue led to the statement of the general problem:

Which characteristics are the most useful in describing a person? Unlike the previous three challenges, this problem was posed, and partial solutions derived, without physical objects for the subjects to view and handle other than the photograph.

Responses of the Experimental Class

The initial interaction between the students and the interviewer involved their descriptions of themselves, and they were sketchy and only moderately accurate. The descriptions used most often frequently included color of hair, color of eyes, and what was being worn. The following listing gives some typical descriptions as initially provided by the students:

> Jacquelyn (E): Brown hair, blue eyes, black boots, curly hair, bit off fingernails. (Jacquelyn has straight brown hair and green eyes.)
>
> Trina (E): I don't know. Possibly brownish hair, brown or green eyes. (After a pause she added) I can't think of anything.
>
> Todd (E): Brown eyes, brown hair, about four feet ten, yellow sweatshirt, and green pants.

Each of the students was able to give some information concerning his/her appearance. The two least outgoing students, Trina (E) and Joey (E), gave the poorest descriptions. None of the students supplied descriptions which accounted for more than four attributes of their appearance.

The interviewer guided the discussion toward supplying the description of the pilot in the photograph. A collection of the student responses are as follows:

> Todd (E): Air Force clothes, brown eyes, straight teeth, dark hair, balding. Sort of a baby-face. I mean chubby.

> Michael (E): Tall, maybe overweight, five feet two maybe. Blue uniform, short hair, rather long legs.

> Joey (E): Black hair, blue suit, brown eyes, I think.

> Jacquelyn (E): He smiles a lot, short hair, dark (hair). What color eyes does he got?

Denise (E) and Trina (E) did not describe the pilot in the picture, but rather gave an indication of how they would go about giving such descriptions:

> Denise (E): Color hair, glasses or not, shape of face, smiling or not, his clothes. How tall, if he is tall or short, the way he wears his hair, how old he is.

> Trina (E): Tell what kind of hair he has, what he is wearing, what his name is.

After each student discussed his description of the pilot with the interviewer, the generalized problem was posed: "Which characteristics are most useful in describing a person?" To insure that each student was aware that there were many possible characteristics which might be used to describe a person, the interviewer and the student took turns suggesting attributes of a persons' appearance. After six or more characteristics were mentioned the challenge to determine which details were best, was issued.

The responses of the experimental class were given immediately and are listed below:

Trina (E): His name, clothes, color of his hair.

Denise (E): Probably color of hair, how tall he is.

Jacquelyn (E): Short or fat, what they wear.

Michael (E): How tall he is.

Todd (E): The face is best.

Joey (E) did not verbalize a response, rather shrugged his shoulders to suggest, "I don't know." The remaining five students were asked how they knew these were the "best" characteristics. Todd (E) was the only student who answered in a non-negative manner: "If it works, its a good way."

Each student was asked to suggest a test to verify that their descriptors were the best. Denise (E) recommended the following experiment: "Give one person four characteristics of an individual. Give a second person four characteristics of the same person, and see who could find him first." She added that she had not thought to test her choices before the interviewer asked her to prove she was correct in her selections of the best descriptors.

To determine whether the students used the solution to the same problem which might be supplied by someone else (e.g., family, teacher, police), the interviewer asked if the students knew anyone who describes people well. Jacquelyn (E) was the only youngster to respond affirmatively, "My friend is a good describer and she gives the same thing" (short or fat and what they wear). Under the interviewer's direction, each student suggested what policemen use in a description, however this information was not used to support or reject their claim that they had given the best descriptors.

Responses of the Control Class

The control class began the interview by providing descriptions of themselves which involved four or five attributes and turned out to be moderately accurate.

> Shari (C): Girl, bluish-green eyes, strawberry-blonde hair, lots of freckles, long hair.
>
> Linda (C): Sparkle teeth, bug eyes, and I usually wear my boots. (Linda is the only youngster in her class who wears glasses and has braces.)
>
> Susan (C): Blonde hair, light skin, about seventy-five pounds, about four feet eight inches tall.
>
> James (C): A boy, brown hair.
>
> Mark (C): Skinny, brown eyes, red-blonde hair, blue shirt, nine years old.
>
> John (C): About five feet tall, seventy-five pounds, wearing a coat and a yellow hat. (John is the shortest male member of his class. The interviewer estimated his height to be approximately four feet, four inches. He periodically wears his hat in the classroom.)

Johns's (E) inaccurate description may reflect his inability to estimate, or perhaps suggests that he described himself the way he would like to appear. Linda and Mark (C) provided accurate descriptions of themselves by including negative descriptors (e.g., bug eyes, skinny).

After a discussion of their descriptions, the interviewer showed the students the photograph of the pilot, and asked (after explaining the problem): "How would you describe this person?" Each student's response is listed below:

> Shari (C): I can't see his eyes. Would he be wearing that suit? Give his name and what he does, and ask somebody.
>
> Linda (C): Black boots, green suit. (Interviewer asked what would happen if he were not wearing the flight suit.) Color of their hair, how big they are, how old they would be, color eyes.
>
> Susan (C): Short hair, about five feet ten inches, and weighs about one hundred and sixty-five pounds and white skin. (Interviewer asked how she determined this accurate description.) He just looks like it.
>
> James (C): How many bags he has, rings, age, clothes.
>
> Mark (C): This clothes or others, (pause) that's the problem. Short hair. Maybe twenty-one.
>
> John (C): Boots, flying suit, about six feet three inches, dark hair. (John was asked how he determined the height of the pilot.) Most people are about seven feet five inches tall.

Only two of the members of the control class, John (C) and Linda (C), used the changeable attribute of clothing in their descriptions. Moreover, Mark (C) and Shari (C) gave descriptions in which they explicitly noted the possibility that the pilot's clothing could change. James (C) suggested descriptors (number of bags, rings) which, though changeable, could provide an accurate description.

In contrast to the three members of the experimental class, Linda was the only student in the control class to mention color in her description. Of all of the twelve students included in this study, Susan provided the description which was most consistent with the true appearance of the pilot.

Each student was then asked to determine which general characteristics were the best descriptors of a person. Shari (C) did not respond within a minute and was asked to name several characteristics which could be used in describing a person:

> Shari (C): How tall, do they usually wear a happy face, how he walks, color of hair, freckles or not, color of eyes, what he is wearing-- a watch.

The interviewer again asked for the best descriptors, but Shari (C) did not reply. When asked how descriptions were given on TV shows, she answered: "On Emergency they tell how old and what he is wearing." Shari made no further responses to the challenge of determining the "best" descriptors.

The remaining five students suggested what they felt were the best characteristics for describing a person immediately after the problem was posed:

> Linda (C): How tall, fat or skinny and what kind of luggage.

> Susan (C): How he dresses, height, and location of the person.

> James (C): Name, age, color of hair (pause). Is that right?

Mark (C): Slim, average, or husky, how tall, and what color of eyes.

John (C): Give a picture. (If you don't have a picture) Look for a uniform, check at desk, and see if he is in.

Linda (C) and John (C) were clearly trying to give the best descriptors to the person going to the airport. John's (C) suggestion of using a picture skirts the issue of determining the best characteristics. None of the students could suggest a way of proving their choices were the best descriptors to use, and the interviews were terminated shortly thereafter.

Identification of the Strategies Used by the Students

The initial challenges of providing self descriptions and a description of the pilot in the photograph were posed to the students to give a setting for the generalized problem of determining the best descriptors.

None of the twelve students of this study asked questions to clarify the problem. Every student responded based on their perception of the evidence found in the photograph without considering how long ago the picture had been taken, or whether the man was still in the Air Force. These responses support earlier conclusions concerning the inability of the subjects to consider several dimensions of a problem.

The inclusion of color by four of the students (three from the experimental group and one from the control) in their descriptions of the pilot suggests that they ascribed color to details they thought might be there. Nine students (five from the experimental class, and four from the control class) mentioned what the pilot was wearing as a part of their descriptions, and yet they failed to consider the possibility that this detail might change.

After the interviewer asked the question, "Determine which general characteristics are the best to be used in describing a person," eleven students gave immediate responses. There was no evidence that any students did, indeed, reach their conclusions by defining best, consider the alternatives, and select descriptors most consistent with the definition. None of the twelve students could amplify their previous answers. Eleven said they could suggest no test to verify that their descriptors were the best.

In order to provide some direction, the interviewer asked each student what the police use in describing a person, and although each supplied a correct answer, it was not used to support or reject their claim that they had given the best descriptors. The researcher could find no evidence that any subject had given, or intentionally referred to a solution of this challenge which could be given by a policeman, fireman, teacher, or parent. In other words, they did not consider how others had solved related problems.

This challenge was apparently very difficult for all of the subjects. The students appeared uncomfortable during the interview, fidgeting in their seats and playing with things in their hands. The responses to the generalized problem showed no evidence that a problem solving strategy or process was employed in arriving at their solutions.

The marked difference between the quality of solutions to the previous three challenges and this problem suggests to the interviewer that students must have some experience with at least some aspects of the problem they asked to solve before they can be expected to find meaningful solutions. Moreover, physical manifestations of the problem should be available for their consideration, a conclusion which is certainly consistent with Piaget's[1] interpretation of the concrete operational youngster. Furthermore, the inability to generalize from the specific problem of describing a particular person in this challenge may also be due, at least in part, to the lack of experience the youngster has in generalizing in problem solving situations.

The Playground Design Problem

The last interview occurred during the last week in February, and traced the behavior of the twelve students in solving the problem:

> Plan and layout a playground, costing less than $2500.00, suitable for an elementary school.

During February the control class continued with their usual classroom activities. The experimental class completed four periods working on USMES challenges (Product Testing and

[1]Barry J. Wadsworth, Piaget's Theory of Cognitive Development, (New York: David McKay Co., 1970), p. 67.

Designing for Human Proportions) and six periods solving puzzle sheets during the last month of this study. This final problem involved several aspects of the previous four problems and was utilized to determine whether growth in problem solving ability had occurred.

The students were shown a drawing of a rectangular play area, adjacent to a school, which was vacant except for a few small trees. A poster board with pictures and prices of playground equipment was positioned in front of the students. Scale drawn cutouts representing each item on the catalog poster were available for placement within the designated play area. The problem was posed only after the materials were explained and the students had the opportunity to investigate each piece. The subjects were also informed that other playground catalogs were available, and cutouts of any item which they wished to incorporate into their plan would be made.

After the initial period of familiarization and explanation, the students began to work immediately.

Responses of the Experimental Class

Michael's (E) initial response was unique. After he noted the cost constraint on the worksheet, he began adding the price of every item on the catalog poster in his head. After several seconds, he asked, "Did you add up everything on here?" suggesting that it might be possible to include everything and hence avoid the problem of selecting and monitoring cost. With help, he discovered he could not purchase everything with $2500.00. He then proceeded to select equipment for the playground without checking the total cost until asked to do so by the interviewer. When Michael (E) was asked how much the selected equipment would cost, he totaled the prices and exceeded the $2500 limit. Michael's (E) original sum was $3300, and realizing that he had exceeded the permissible maximum with that figure, he removed three picnic tables costing $120.00 each. He requested the help of the interviewer in finding his new total, $2940. Michael (E) remarked "If I take away something for $500;" he paused, and he then removed the $500.00 merry-go-round from the play area.

The remaining five students either kept a running total of their expenses as they made their selections for the playground or noted the prices on their worksheet and totaled the cost after four or five items had been chosen. This awareness of the financial aspect of this challenge is very much different from the way the budget was handled in the menu planning portion of the third challenge.

Each student in the experimental class except Todd (E) used the entire play area in positioning the equipment they chose. Todd grouped five of his six pieces safely together using a third of the available space because he didn't want the equipment "too far apart. No use in running a mile."

Todd (E) and Michael (E) were the only students to select equipment which was not represented on the poster catalog. Michael (E) wanted picnic tables, grills, and a basketball backboard and hoop. He commented that he "wanted more of a park, not just a playground." Todd (E) included a basketball court in his design and needed to adjust the size of the court twice in order to stay within the cost constraint established.

The other four students made use of the items listed on the poster. The equipment which was selected most often were those pieces the youngsters stated they enjoyed most or thought other children would enjoy.

<u>Responses of the Control Class</u>

The members of the control class began selecting equipment for the playground immediately after the problem was stated. Linda (C) and Susan (C) selected playground items without noting prices. Susan, however, was clearly aware of the cost limitation since she asked, "How much is that, or am I supposed to add it?" after she had chosen five items of equipment. The other four students monitored their costs as the students had done in the experimental class. Linda (C) incorrectly added the first time, and arrived at a sum of $18.85. Convinced she was correct, she began adding equipment to the play area, and although the interviewer would call out the price of each item as she chose it, she continued to write them with the decimal point misplaced. When she reached a total of $45.38 she realized that the school yard had become congested and announced, "That's enough." Linda's teacher later remarked that Linda was able to handle money problems given her in class, which suggests to the interviewer that she misread the prices, expecting to see a decimal point between the second and third digits. After the first five prices were totaled, a problem solving "set"[1] became apparent as she continued to make the same mistake repeatedly.

Susan (C) and James (C) added the price of the first few items (which totaled less than $2500.00) and began adding pieces, one at a time, checking after each addition to insure their cost total had not surpassed the limiting value. James (C) added his expenses and announced "$450.00 to spend yet" (his total at that point being $2150). He then looked at the catalog for something costing less than or equal to the available cash. His choice of a $400.00 castle climber permitted him to say, "That's it! I have $15.00 extra!" John's (C) initial cost total exceeded the $2500 limit and it was necessary for him to delete items from his playground until his costs were within that constraint.

All of the students except Shari (C) used the entire play area in planning the playground. Shari (C) left a region, elliptical in shape, measuring approximately fifty feet by thirty-five feet, empty and commented, "I left this area open for playing kickball or something."

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[1]During the problem solving process the solver may be unable to redirect his thinking and continues to use a strategy which has proven fruitless on previous attempts.

Linda (C) was the only youngster in the control group who included playground equipment which was not represented on the catalog poster. She requested the price and cutouts for five bounce spring animals which she included "for the younger kids." The remaining members of the control class chose equipment for which cutouts were readily available, and devoted their time to the positioning of these items.

Identification of the Strategies Used by the Students

In this problem, as in the Plan a Party Challenge, several students made arithmetical errors in computing the cost of the proposed expenditures. Those students who kept a running total of what was being spent made little use of estimation, but rather they treated the addition of each item as a trial and error procedure. This suggests that more practice in estimating answers would be helpful, and should be provided in the arithmetic classes.

There were instances where the subjects' comments suggest the problem had been defined, and from this definition of a goal inferences were made in choosing play things. For example, Linda (C) chose two sets of swings, "one for each age group." She also asked to see an additional catalog to find the cost of five bounce spring animals "for the younger kids." Michael (E) began with a climber and a swing, then requested an additional brochure and chose six picnic tables and three grills for the school yard to make it "more of a park, not just a playground."

Denise (E), Shari (C) and James (C) wanted benches so that children and teachers could "rest during the recess if they wanted." The remaining seven students in this study did not give reasons for their selections (four did comment that they thought a particular item would be fun), and conclusions concerning the use of a particular strategy cannot be made. Todd (E) and Shari (C) also used inference in establishing the position of the equipment which they chose. Todd (E) grouped items so that they wouldn't be "too far apart. No use in running a mile," and Shari (C) left an open area "for playing kickball or something." Clearly these two were planning the school yard with a goal in mind, from which they inferred the general location of the particular playthings they chose.

Further examples of the use of inference from the goal are provided by the six subjects who said they wanted a safe playground. These six placed equipment far enough apart to insure that the play action area of any item would not interfere with that of another fixed object. Some typical responses are provided by the following:

> Susan (C): If its crowded you can't run around. Not safe.
>
> James (C): So nobody gets hurt.
>
> Shari (C): So nothing hits anything else.

Todd (E) positioned his slide so that "If there was a line, it could go off this way," indicating the line would form away from the playing radius of the other equipment.

Michael (E) and Denise (E) did not state their concern over safety, but gave evidence that they wanted a safe playground by measuring distances and re-positioning equipment which they judged too close together. In all six of these interviews, the researcher concluded that the subjects inferred that a safe playground is an uncongested one.

Besides Michael (E) and Denise (E), James (C) and Mark (C) also used the ruler supplied them and the indicated scale to position objects and determine distances on the playground. These four children had not used the scale in the classroom design problem solved three months prior to the undertaking of this playground design challenge. The reason for this change in behavior is not clear. Perhaps congested classrooms don't seem as unsafe to these students as does a crowded playground area. One might conjecture that because there were comparitively fewer items which could be placed on the playground the subjects had more time to consider the position of each piece.

The use of contradiction was detected in the students control of the cost of the playground equipment which they chose. For example, Michael (E), Trina (E) and John (C) exceeded the $2500.00 limit on their first addition of their expenditures. Since this contradicted the amount they were given to spend, each of these students began subtracting pieces until a total less than $2500 was reached.

The student's use of a related problem and the identification of subgoals was not as evident as contradiction and inference in solving this final challenge. The researcher presumed that the subjects thought of their own school yard and local playgrounds in the planning of their own solutions to this problem. Supporting this assumption is Shari's (E) reference to another playground which she used in justifying the choice she made of benches for the school yard: "Because at our park they have to sit on stumps." Three of the students interviewed remarked that they had never seen some of the free-form playground equipment items pictured on the catalog poster before. Michael (E) wanted a particular kind of basketball backboard because "that's the kind we've got." The question of whether subgoals were established cannot be answered, since there is no evidence that the subjects viewed the problem as a combination of ordered or equivalent subproblems.

The subjects of this study spent more time in solving this challenge than any of the previous four presented to them. Perhaps this was because it was more real to them, or that they were more comfortable solving open ended problems at the conclusion of the experiment than they were earlier in the study. Michael (E) and Todd (E) supplied the school with basketball equipment. As noted earlier, Linda (C) wanted equipment for young children that was not listed on the poster catalog, and benches were added by four students. Michael (E) wanted picnic tables and grills. Blacktopping was requested three times. None of these items were listed on the available board, nor were cutouts available to them at the onset of the problem.

The interviewer had the impression that this was an enjoyable challenge, one which the subjects felt was reasonable. Mark (C) noted the similarity and differences between the classroom and playground design problems; in the classroom challenge the student "didn't have to worry about money." Mark also mentioned later, in his classroom, that this was easier to solve "because there were fewer things to move." The time taken in selecting pieces, the care with which they were positioned, and the facial expressions of concentration convinced the interviewer that the subjects wanted to find a good solution to this problem.

CHAPTER V

DISCUSSION, CONCLUSIONS AND RECOMMENDATIONS

Contrast Between Classes and Individual Growth

The purpose of this study was to detect (through an examination of problem solving behavior) the process and strategies used by subjects taken from two similar fourth grade classes. A secondary purpose of this research was to compare the problem solving behaviors of children who practiced comprehensive problem solving in the classroom with children who received no such practice. The experimental class was differentiated from the control class by having their teacher integrate problem solving activities into the usual classroom work during the duration of the study. These activities took one of two forms: 1) a comprehensive challenge of the USMES type, or 2) a short puzzle type problem. It was felt that differences between the two classes in problem solving behavior might suggest that certain classroom activities affect problem solving ability.

The solutions and procedures utilized to obtain these solutions, for the five problems involved in the interviews, suggest that neither class was better at problem solving than the other. The researcher concluded that any change in problem solving ability demonstrated by the experimental class was not due to the added problem solving practice they received. The apparent ineffectiveness of the problem solving worksheets and practice with USMES challenges may be due, in part, to the inconsistant manner in which it was applied. In order that the teacher not feel constrained and to encourage spontaneity on the part of the students, the researcher permitted a great deal of leeway in the selection of when and how often problem solving practice would occur in the experimental classroom. The teacher agreed that, if possible, she would spend two periods per week dealing with comprehensive problems and the same amount of time working on puzzle sheets. However, the four month period during which this study occurred contained several holidays and vacation days, and often other school activities took precedent over problem solving. Moreover, the experimental classroom teacher commented that she didn't feel comfortable with the comprehensive challenges, and she didn't like spending a great deal of time on them. Due to these conflicting demands for classroom time, and the lack of enthusiasm on the part of the teacher for USMES work, problem solving activities occurred twenty-eight times during the experimental period with varying regularity. Seventeen of these periods were used to work on a worksheet from the teachers guide.

Although there is no detectable difference between solutions provided by members of either class, there is strong evidence to support the claim that some subjects became more mature problem solvers during the experiment. All twelve students in this study became more at ease with the interviewer as the experimental period progressed. By the third problem each interviewee was willing to spend considerably more time on the problems, speculate on alternative solutions, and was candid in his replies to the researcher's questions. This willingness to spend time to find non-trivial solutions indicates a growth in the appreciation of the nature of comprehensive problem solving. Specifically, Joey (E), Trina (E), and Jackie (E) demonstrated essentially no problem solving in the first interview, but were able to silently arrive at a solution (a classroom layout) to the classroom design problem, the second challenge presented to them. Nine youngsters were able to control the cost variable in problem five, while these same subjects had ignored this dimension in the third problem. Mike (E), Denise (E), James (C), and Mark (C) displayed some growth in using scale drawing in the fifth problem as compared to the second challenge. Using this skill, which is not a generalized problem solving strategy, indicates a maturing in attitude towards problem solving, that is, these subjects recognized the many facets of the problem and realized that good solutions could be achieved by careful consideration of the component parts of the problem.

The researcher had the impression that, by the end of the study, all of the subjects understood that although many solutions existed and were different, they could be equally valid. By the completion of the study, no student tried to give the answer the interviewer might have had in mind or that his or her teacher would want as was often the case in the early interviews. This indicates a maturation in the concept of a solution to a comprehensive problem. By the fifth problem, subjects were able to work with a fluid definition of a problem which was modified as the subject became aware of other aspects of the problem. The researcher concluded that dealing with different problems and arriving at solutions brings a familiarity with the problem solving process which aids in cognitive growth.

Identification of Specific Strategies Used by Subjects

During the sixty interviews examined in this study the subjects (except for the three students mentioned in the previous section who failed to "solve" the notebook problem and the general failure to answer the "describing people" challenge) were able to reach some partial solutions to each problem. The process exhibited in reaching these goals followed the four steps outlined in Chapter Two. The solvers defined the problem in some way, generated alternative solutions, evaluated these alternatives, and selected the optimal solution. The steps were not clearly differentiated from one another, and the youngsters were not aware that they passed through these steps in solving these problems, but nonetheless these steps are identifiable. Furthermore, subjects used these steps in subtasks which were identified and contributed to the total solution.

During the second and third steps in the solving process the subjects utilized some generalized problem solving strategies. The strategies which were used are listed below by problem:

Notebook Problem: Inference from the goal and from the givens (notebooks) was widely used by each solver in this challenge.

Classroom Design: Inference from a goal; use of contradiction to eliminate alternatives; looking at a related (equivalent) problem for tentative answers; use of ordered subgoals to complete solutions.

Ice Skating Party: Inference from a goal; use of a related problem to suggest alternatives; use of contradiction to eliminate unacceptable solutions; identification of unordered subgoals.

Describing People: Solutions to the generalized problem were not found and no specific strategy was employed in an attempt to find such a solution.

Playground Design: Contradiction, inference, and looking at related problems were used.

The degree to which any of these strategies were used varied between problems and from student to student. These strategies often appeared in primitive form, unnurtured by any training, but their presence is undeniable. The manifestation of these heuristics is most evident in the subjects manipulation of concrete materials. This kind of reasoning, using a physical model, is consistant with the cognitive development of a concrete operational youngster.

There was no evidence to support the claim that fourth graders can use the method of working backwards to solve problems. There are two obvious reasons why this evidence was not detected. Firstly, this strategy may require a higher level of cognitive development than found at the fourth grade level. A second reason working backwards may not have been used by the subjects in this study is that the problems did not lend themselves to utilization of this stategy.

The failure of every student to find a legitimate solution to the describing people challenge implies that the problem lacked meaning to the subjects. The subjects' inability to reach some meaningful conclusions suggests that experiencing the need to accurately describe a person is necessary for the development of schema which permits precise definition of the problem. This problem also lacked the physical embodiments which are necessary before the concrete operational youngster can act. The researcher concluded that this problem was inappropriate for the study and should not be used to evaluate the problem solving ability of any subject.

Recommendations

The author is aware that the nature of this type of case study permits conclusions which reflect the bias of the researcher. The following recommendations are made based on conclusions from the evidence presented in the previous chapter, and on the authors interpretation of students' feeling during the interviews. The second set of recommendations are intended for administrators, curriculum planners, and teachers so that they can suggest activities for the elementary school classroom.

Research Recommendations: Continuing research is essential to every empirical issue in mathematical education. This is especially true for the issue of Problem Solving; its nature, instruction, and learning. The following suggestions are by no means an inclusive listing, but rather represent areas which appeared notable during the implimentation and interpretation of this study.

The amount of time required to conduct a thorough clinical interview can be lengthy and therefore the researcher recommends that similar future studies be planned to allow at least an hour per interview. None of the sixty interviews reported in this study required more than thirty-five minutes to complete. However, based on the responses given during the last three interviews, the researcher feels that fourth grade children could work on a problem for a longer period of time and find more complete solutions without becoming tired or losing interest. To plan interviews lasting an hour or more implies that the number of students interviewed during one day must be limited. The researcher suggests that studies requiring extensive interviews should include no more than four students from a single class.

In order that concrete operational solvers reach meaningful solutions, the problems used during the interviews must be challenging comprehensive situations which have physical embodiments for the subjects to consider and consult. Interest must be stimulated quickly and maintained throughout the interview. Based on the responses to the classroom and playground design problems and the ice skating party problem, which contained such embodiments, the researcher suggests that future studies use large, colorful models. These can then be manipulated by the solver during the interview and can be used to exhibit solutions.

As noted earlier the students involved in this study became more verbal and willing to speculate concerning alternatives as the experimental period progressed. The researcher concluded that this willingness to search for meaningful solutions during an interview situation implies that familiarity with the interviewer provides a more comfortable environment for the solver. Therefore, the researcher suggests that future clinical studies be planned to permit each student three meetings with the interviewer before the students are asked to solve any problem. These meetings could be used to discuss the student's home, school, friends, interests, and hobbies.

A clinical study conducted over a four month period severely limits the opportunities to detect growth in problem solving ability. This research should be duplicated over a nine month period with a follow up interview conducted one year after the onset of the study. Although a long term study can present a great deal of difficulties, results from such a study could produce the most convincing evidence of growth in the problem solving process.

This study investigated the behaviors of youngsters who practice problem solving in the classroom as well as students not involved in any past or present problem solving program. As indicated elsewhere in this chapter, the researcher could detect no difference in the problem solving heuristics used by the two groups. To provide information concerning students' experiences in the problem solving process, this study should be replicated using subjects who have been actively involved in USMES for at least one year.

The researcher found several instances wherein the students failed to reach meaningful solutions because of weaknesses in mathematics and reading. The researcher was unable to suggest to what extent these limitations inhibited progress towards a solution. Therefore, studies should be carried out to determine the relationship between mathematics and reading ability and the problem solving strategies used by students.

Finally, the manner in which students at all grade levels solve problems is, for the most part, an unanswered question. Case studies to determine how students solve comprehensive problems should be conducted at each grade level, K-12. Collectively, such studies may suggest a pattern of growth in problem solving ability which would have implications in the instruction of problem solving skills.

Curriculum Recommendations: Worthwhile research in education must ultimately be responsible to the child in the classroom and provide partial answers to the questions of what and how things should be set up to provide that child with the "best" education. The following recommendations are intended to provide guidance to the teacher who wishes to improve the problem solving ability of his or her students. These recommendations are based on the student's responses to the challenges used in this study, supporting research on student achievement and creativity and the impressions the researcher had of the student's interest and enthusiasm. Some of these suggestions also reflect the feeling and experience of the researcher during three years of work with USMES classes.

Classroom activities should include at least three periods per week which are used for classroom investigation of open ended problems.

Provide short problem solving puzzles for students periodically and explore with them the manner in which solutions are found.

Encourage independent and creative work by every member of your class.

Design instructional strategies so that students will be practiced at using mathematics and reading in problem solving contexts.

Finally, allow each student to work on a comprehensive challenge, by himself, sometime during the year and report the results to an interested adult. Verbalizing results will increase the insight into the nature of a problem and its solution.

Problem solving is more than an enrichment activity for the schools or part of the subject matter of mathematics. It is an attitude and experience; the degree to which students master these learning experiences dictates the degree to which they will survive and grow in the future.

APPENDICES

APPENDIX A

TEACHER QUESTIONNAIRE

APPENDIX A

INFORMATION SHEET

Name______________________________ Phone number _______________

Address____

City_________________________ State_____________ Zip___________

Undergraduate College or University attended_________________

Major________________________ Minor____________________________

Number of years of full-time teaching experience_____________

Number of years teaching in Ypsilanti_________________________

Grade level you are teaching__________________________________

How many years have you taught this grade?___________________

How many hours do you have past your Bachelor's degree?_____

Do you have a Master's degree?________________________________

What area?____

HOW DO YOU FEEL?

The theoretical portion of my research has presumed certain attitudes in teachers are necessary in order that the child will want to solve problems which are posed.

The following questions are presented to support claims I have made concerning the similarity of attitudes between you and your colleague in this experiment.

PART 1:

Circle one of the following:

SD-strongly disagree, D-disagree, A-agree, SA-strongly agree

1. Almost every student could probably improve his or her class performance if he/she really wanted to.

   SD D A SA

2. It is unrealistic to expect students to show the same enthusiasm for their school activities as for their leisure activities.

   SD D A SA

3. Some form of grading system is necessary in schools because students are too immature to work for any higher motivation.

   SD D A SA

4. The teacher must at all times work to maintain order in the classroom. If you let up for one moment, students will misbehave and learning cannot take place.

   SD D A SA

5. Even though they like to talk about it, most students don't like to make decisions on their own. It is hard to get them to assume any real responsibility.

   SD D A SA

6. Being tough with students will usually get them to do what you want.

   SD D A SA

7. A good way to get students to do more work is to crack down on them once in a while.

   SD D A SA

8. Even when given encouragement by the teacher, very few students show a real desire to improve themselves in class.

SD D A SA

9. It weakens a teacher's prestige whenever he or she has to admit that a student has been right (and he or she has been wrong), or that a student has a better approach to a problem than the teacher.

SD D A SA

10. The most effective teacher is one who gets results, regardless of the methods used in handling students.

SD D A SA

11. It is too much to expect that students will try to do a good job without being prodded by their teachers.

SD D A SA

12. The teacher who expects students to set their own standards for superior performance will probably find that they don't set them very high.

SD D A SA

13. If students don't use much imagination and ingenuity in their classwork, it is probably because relatively few people have much of either.

SD D A SA

14. One problem in asking for the ideas of students is that their perspective is too limited for their suggestions to be of much practical value.

SD D A SA

15. It is only human nature for students to try to do as little as they can get away with.

SD D A SA

PART 2:

Rate each item in questions 16 and 17 using the following scale:

1-strongly disagree, 2-disagree, 3-not sure, 4-agree, 5-strongly agree

16. Teaching problem solving/reasoning is:

________something I do well with most children
________something I find difficult to do with children of experientially deprived backgrounds
________something I find difficult to do with children who lack concept acquisition
________something I find difficult to do with children who lack comprehension skills
________something I find difficult to do with children who have had a history of failure

17. I think the factors that most frequently appear to contribute to a students failure to make satisfactory progress in school are:

a)

b)

c)

Circle one of the following in answer to questions 18 through 22:

VW-very weak, NI-needs improvement, A-adequate, G-good E-excellent

18. My understanding of how non-institutional factors (such as home background, family, socioeconomic background, environment, etc.) affect a student's achievement in school is:

VW NI A G E

19. My ability to increase or decrease the affects of non-institutional factors on my students' achievement in school is:

VW NI A G E

20. How do you assess your ability to help pupils apply the mathematics you teach them to daily out-of-school mathematical tasks?

VW NI A G E

21. How do you assess your ability to develop instructional strategies for teaching concepts?

VW NI A G E

22. How do you assess your ability to develop instructional strategies for teaching problem solving/reasoning?

VW NI A G E

23. Which subjects do you enjoy teaching the most? The least?

24. How do you feel about your ability to get your pupils to work independently?

VW NI A G E

25. How do you feel about your ability to provide appropriate materials and activities for the pupils who will work independently?

VW NI A G E

APPENDIX B

TEACHER'S GUIDE

APPENDIX B

STRATEGIES FOR COMPREHENSIVE PROBLEM SOLVING

A Teacher's Guide

J.J. Shields
Fall, 1975

PREFACE

The Strategies for Comprehensive Problem Solving is a part of the author's doctoral research program at Michigan State University. This portion of the program involves several worksheets which introduce or reinforce strategies useful in comprehensive problem solving. It is hoped that you and your students will find these worksheets both enjoyable and beneficial. Your help in implementing this program in your classroom, and your cooperation in taking the time for the experimental evaluation to measure its success, is deeply appreciated.

J.J.S.

TO THE TEACHER

The purpose of this guide is to aid you in planning and incorporating problem solving worksheets into your ongoing classroom work. It is to be used in conjunction with the Unified Science and Mathematics for Elementary Schools (USMES) Teacher's Resource Books.

Since problem solving is such a diverse topic and involves many possible approaches, the strategies identified in each worksheet overview represents the attitude of the author as to which strategies children use in solving problems. In addition, the order in which these worksheets appear in this guide reflects the authors view of the problem solving process. Feel free to rearrange them, in whatever way you feel is most advantageous to your students' growth development, if necessary.

Finally, the introductory material is presented to provide you with some background in a theory of problem solving, and is not intended for your students.

PROLOGUE

Introduction to the Program

The Strategies for Comprehensive Problem Solving is a program which combines USMES challenges, found in their resource materials, with supplementary worksheets contained herein. It is designed for use with elementary school children in an effort to introduce them to strategies which they use to solve problems of a comprehensive nature. The program assumes an informational processing approach to problem solving, and suggests a series of steps which the solver passes through to arrive at a solution.

A Theory of Problem Solving

Every problem consists of three parts: the givens, the goal, and an indication of the methods we are permitted to use in getting from the first to the second. Problems arise everyday in a students life; they are situations in which the student wishes to affect some change in a present condition of things. When a person becomes aware of a situation which needs changing, he is in a position to clearly define the problem--what is given, what the goal is, and exactly what operations can be used to get from those givens to the goal. Defining the problem is the first step in problem solving. Secondly, a plan must be devised which can be followed and will lead the solver to the desired goal. At this stage of the solving process, the solver uses certain strategies which will aid in the formulation of a plan of approach. A strategy is a heuristic, or "rule of thumb," which has been useful in solving similar problems in the past. The strategies which youngsters most often use are: 1) making inferences, 2) identifying subgoals, 3) examining related problems, 4) using contradiction, and 5) working backwards. In the third step of the solving process, the plan is carried out to arrive at a tentative solution (or solutions if more than one is possible). The solver selects the best solution from those found in step three. These steps are summarized in Figure 1 below.

Awareness of a Problem	
Step One:	Define the problem precisely.
Step Two:	Devise a plan to find possible solutions to the problem.
Step Three:	Carry out your plan.
Step Four:	Select the optimum solution of those possible.

Figure 1: THE FOUR STEPS IN THE PROBLEM SOLVING PROCESS

The Strategies for Comprehensive Problem Solving program will take about seven weeks to complete, dependent on your scheduling within the framework of general classroom activities. Each worksheet in this guide contains a puzzle or problem which can be solved using a particular strategy or emphasizes one particular aspect of the solving process. The USMES activities provide an opportunity for your students to examine a comprehensive problem; they make observations, collect data, and draw conclusions.

Classroom Environment

Research suggest that the best problem solvers are those individuals who practice problem solving. The emotional and physical setting for this practice is very important, because the solver must feel free to suggest "wild" alternative approaches to a problem. These alternatives may stimulate another's imagination or be modified into a highly creative solution.

Choose a time during the day when the students are free from constraints such as the need to get math done (or science, reading, etc.); a time when all of your students can work together. Since each worksheet contains an activity which does not depend on any other worksheet, you can assign these sheets at any time during the day. Moreover, this time can be varied from day to day. There are enough worksheets so that you can give your class two or three different problem sheets during each week of the experiment.

In the classroom, as in real life problem solving, the solver has access to certain facts or techniques as he recognizes the need for them. Therefore, many of your students may ask for help with some of the worksheet problems. Do not hesitate, after a child has spent sufficient time on a problem, to answer specific questions he or she may pose. Also, it is important to permit students to interact with one another after each has toyed with the problem for a while.

A student's success in this program might be judged by his interest and his progress in finding some partial solution to any problem situation. To obtain these goals, the teacher should provide a classroom atmosphere in which each student can, in his own way, search out some solution to the problem or puzzle.

In summary, the following suggestions are made to the teacher:

1. Introduce the problem or puzzle situation in a way that allows your students to enjoy the worksheet.

2. Be patient, and let your students make their own mistakes and find their own way.

3. Assist individuals or groups of students as they investigate the problem by asking questions, making suggestions, etc.

4. Provide an opportunity for students to exchange ideas and perhaps draw conclusions concerning the particular strategy used in the worksheet.

THE PROBLEMS

The following pages contain a collection of challenges for your students. The problems are arranged so that consecutive problems are not similar, or workable, using the same strategy. If you feel that some other order is necessary (because of time limitations, for example) select any puzzle sheet you desire, since they are, for the most part, independent of each other.

Some of the problems or puzzles require the student to cut out figures, and move them around to find a solution. A few should be done orally, in which case you will have to take a leadership role in the discussion. Most of the worksheets can be done individually, and your students will gain the greatest insight into the solving process if they attempt them alone.

A brief introduction to each of the problems (including their solution as well as a mention of the type of strategy that will be used) is given before the individual worksheets of this guide.

Worksheet One: GETTING STARTED ON THE USMES CHALLENGE

The first worksheet is a very simple one for most students and can be done in a variety of ways. You might decide to read each one of the quotes from a TV advertisement to your class and have them guess the answer aloud or you may elect to have them work individually and try to be the first to identify all twenty-four (24) products.

No matter how you do this worksheet, it can lead to a discussion of fair advertising and the claims made by sponsors concerning their products. You can ask your students how they would choose the best pencil, or paper towel, etc. in a store, or how they would test the claims of a sponsor. After such a discussion, the USMES challenge, Consumer Research - Product Testing, will provide further opportunity to test various products and to learn something about solving comprehensive problems.

What product is associated with each of these lines from a TV advertisement?

1. Melts in your mouth, not in your hands.

2. Gets out the dirt kids get into.

3. Double your pleasure, double your fun.

4. You deserve a break today.

5. See the light.

6. Breakfast of champions.

7. Silly rabbit, ______ are for kids.

8. If it isn't fresh, I'm outta business.

9. M-m-m-m good.

10. Look up America.

11. Baked by elves.

12. No more tears.

13. I can be very friendly.

14. Your common sense cereal.

15. Baseball, hot dogs, apple pie and __________

16. Don't fiddle with the middle.

17. Fly the friendly skies.

18. Finger lickin' good.

19. Squeezably soft.

20. You can't beat ______ for fighting cavities.

21. Good to the very last drop.

22. Puts sex appeal in a tube.

23. Scrubbing bubbles.

24. How's your love life?

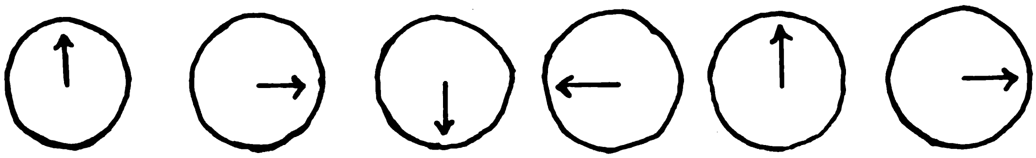
Worksheet Two: PATTERNS

The following worksheet is designed to challenge your students to examine given information and make the necessary inferences that will enable them to find a pattern in a sequence of geometric figures.

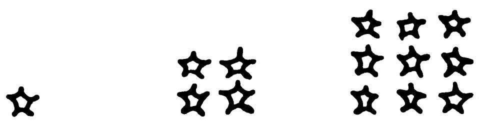
The problem solving strategy of making inferences is used more often than any other. Although inference alone can be used to solve puzzles or problems, more often it is used with other strategies in comprehensive problem solving. The USMES activities will provide many opportunities for your students to make inferences.

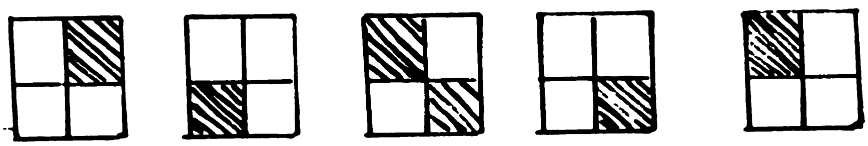
This worksheet utilizes the strategy of making inferences from the given to find a geometric figure that fits into the pattern. A solution to the patterns is indicated on your worksheet. The last three patterns are a little more difficult than the first four, and you might have to give some direction or clues as to what to look for in these series

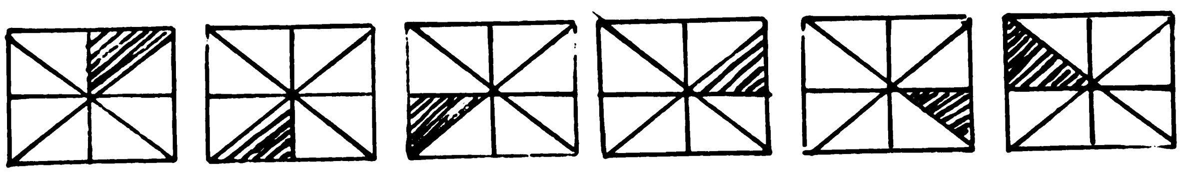
Find the pattern in each series of pictures and draw the next one or two pictures in the series.

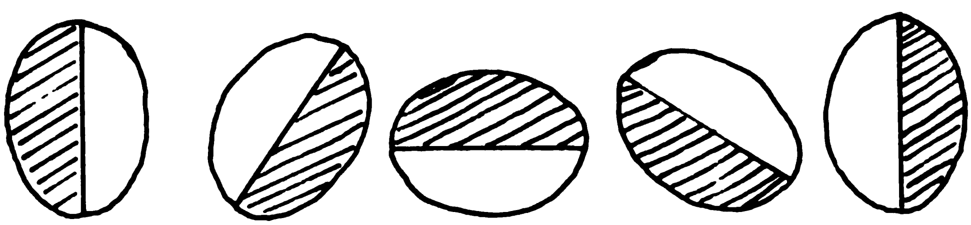
1. 

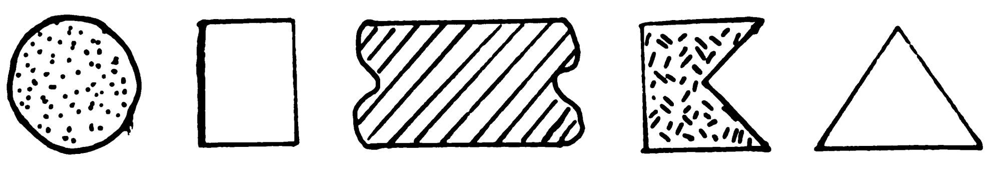
2. 

3. 

4. 

5. 

6. 

7. 

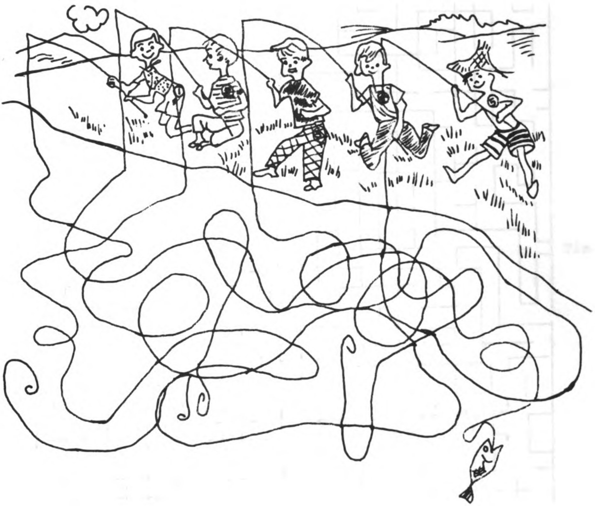
MAZE WORKSHEET

Starting with the goal in a problem and determining what problem state must have existed before the goal is reached is the strategy called working backwards. Whenever a solver has a problem in which there are several places to begin, each of which may lead to a desired unique goal, the method of working backwards is very useful. For example, solving maze problems is often accomplished by starting at the "finish" and working back to the "start." This worksheet provides the student with two maze-type problems on separate pieces of paper.

Have each student try the first maze puzzle without the help of any other student. Wait until almost every student has finished and then ask how many students have found the correct answer by working backwards. More than half of your students probably began with the fish and traced the line back to the correct answer--boy number 2. Ask some of these students why they decided to work backwards. Some may conclude that since there were many places to begin (boys) and only one goal (fish) it was easier to begin with the fish.

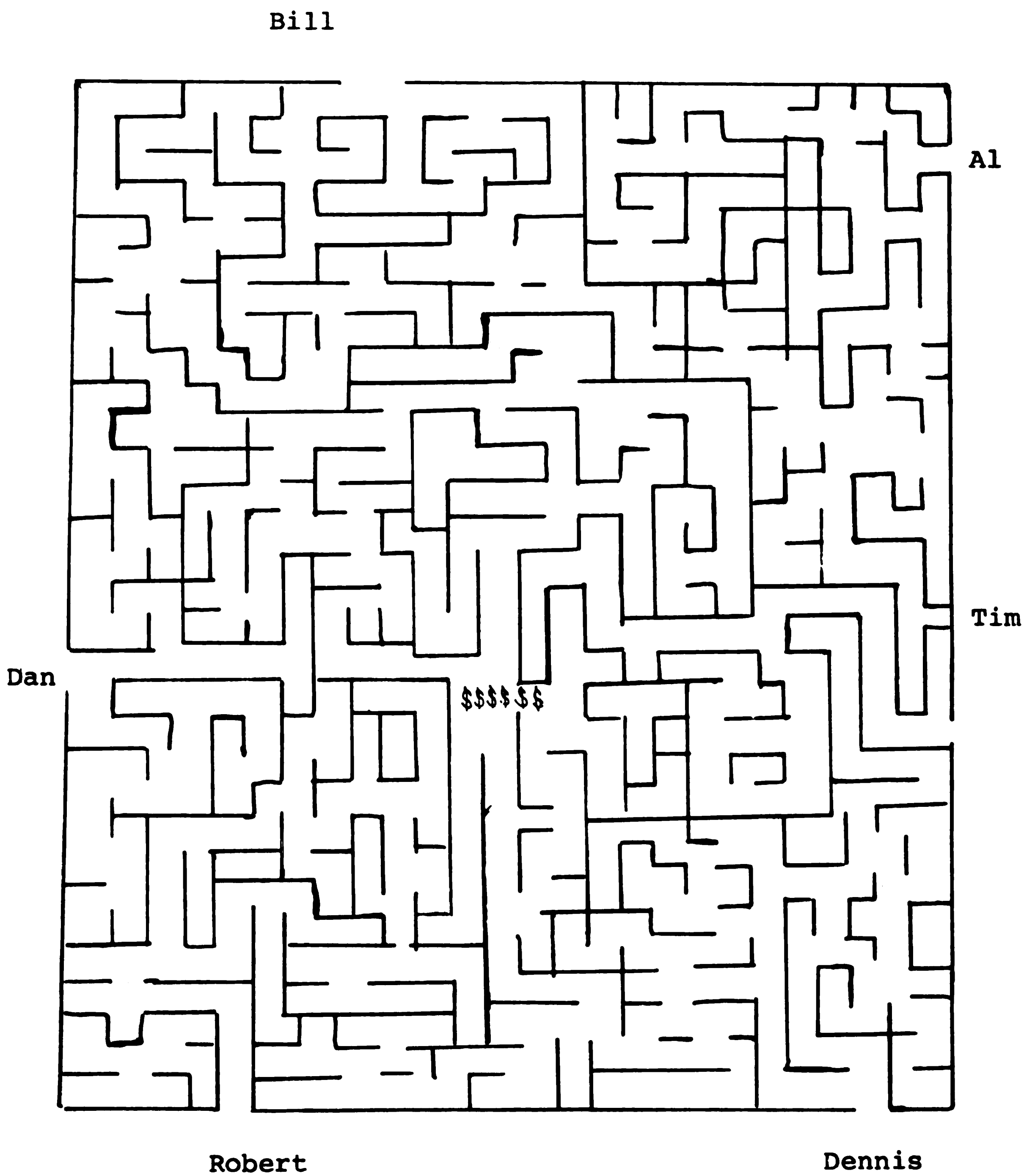
The second puzzle can be used to reinforce this idea. Divide your class into two groups. One group should be directed to begin at the money and the second should begin with one of the six boys. At your signal, tell all students to begin. Most of those beginning at the goal should complete the task before a majority of the other group. In this problem, Robert will find the money.

Here are five boys sitting on the edge of a stream. Though they don't know it, their fishing lines have become tangled. One of them has caught a fish.
Can you tell which one?



Mr. Adams, the sixth grade teacher, has lost the money for the class party coming up, and he has asked six boys to help him find it.

Can you tell who will find the money?



Worksheet: CREATIVITY

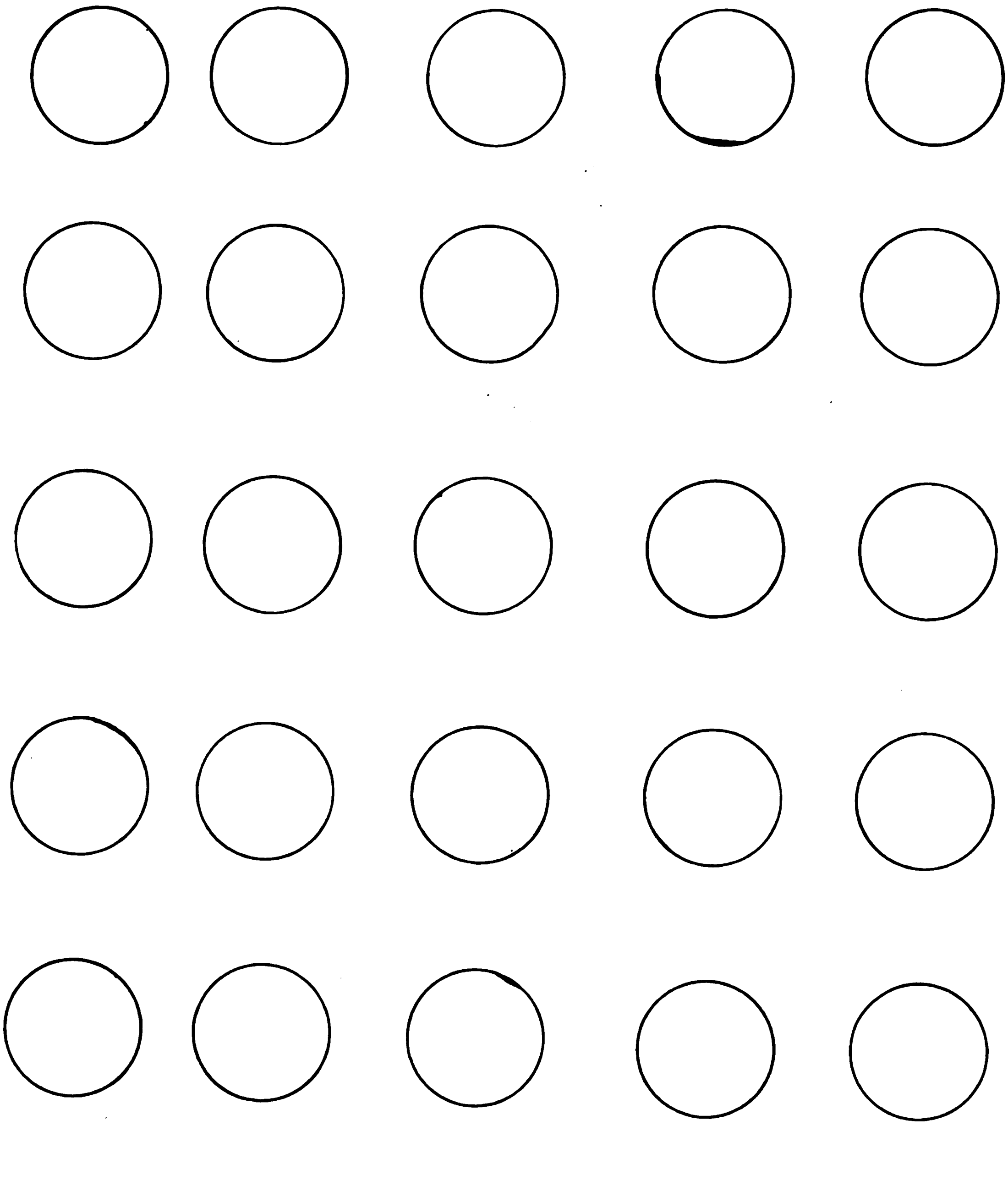
The process of problem solving is closely related to creativity. Research suggest that the highly creative person is usually a better problem solver than the less creative person. In this worksheet (as in some of the others) the student can generate many possible responses, each as good as any other.

Give each student a worksheet, and explain that each circle contained on it should be transformed to something within a specific time period; say two minutes.

Some may automatically decide to draw as many faces as they can think of, while others may try to approximate a series of drawing which are made of the same shape (a base-ball, an orange, a bowling ball, etc.). Some of your students might draw pictures of objects which are all different yet have a circular component. Finally, some students might use a group of circles together to form a train, an insect, etc. There is no single correct response, and hence each student can accomplish the task with complete accuracy.

Give your students the opportunity to share their responses with other members of the class, and by so doing new ideas or uses for the circles may be suggested.

CLASSROOM EXERCISE



Worksheet Five: USING CONTRADICTION

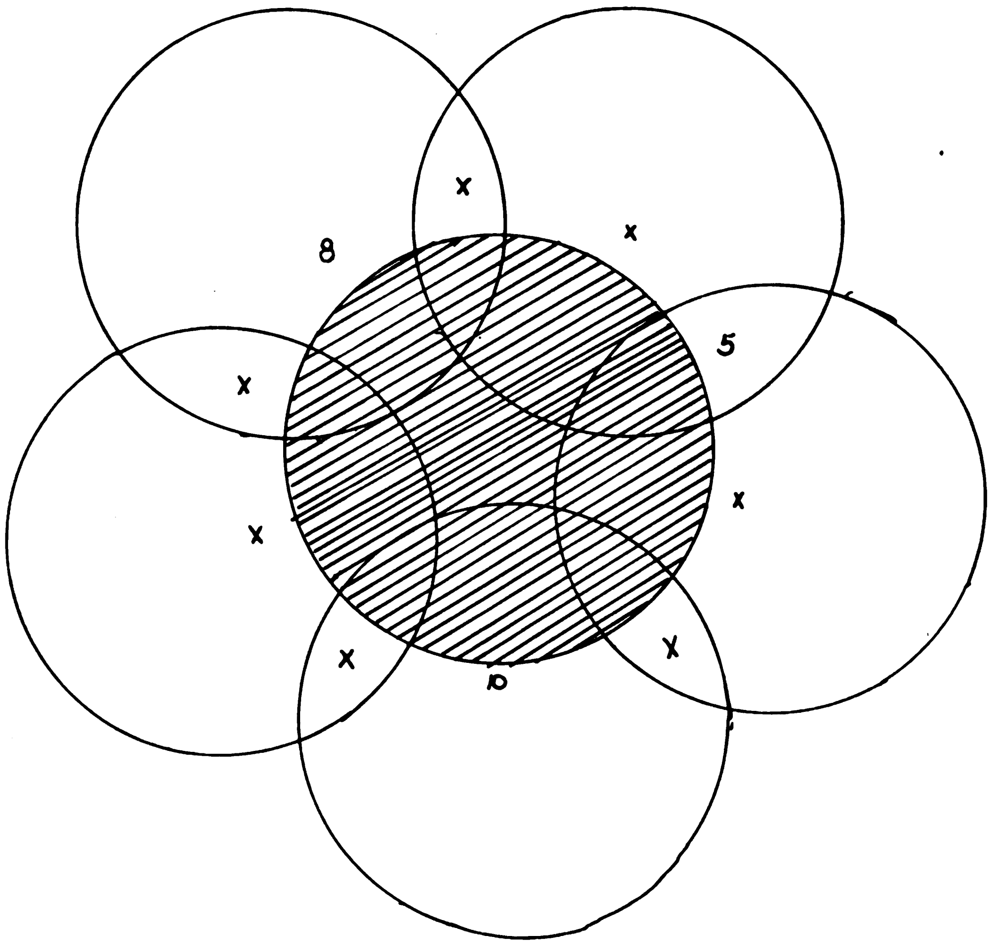
The method of utilizing contradiction is a useful one in problems where alternative solutions are readily available. The individual assumes one of these tentative solutions and checks to see whether such a solution contradicts some piece of given information. If the solver reaches a contradiction, then that solution is discarded and another alternative is considered.

This worksheet contains five overlapping circles which enclose ten regions. The object is to place a number from 1-10 in each region so that the sum of the numbers in each circle is 14. Three numbers are given, and the student should simply try various alternatives until he or she discovers the correct way the numbers can be inserted into the regions.

A better way would be to note that the top circle, which contains a 10, can only use a 1 and a 3 in adjacent regions. The circle with the 8 in it can only have a 2 and a 4 in adjacent regions. Now the number of possibilities have been reduced substantially, and the method of contradiction will yield a solution quickly.

The correct solution is indicated on your worksheet.

Using each number just one time, place the numbers 1 through 10 in the spots marked by X's in the figure below so that each small circle of the figure will total 14. Some of the numbers have already been inserted to help you get started.

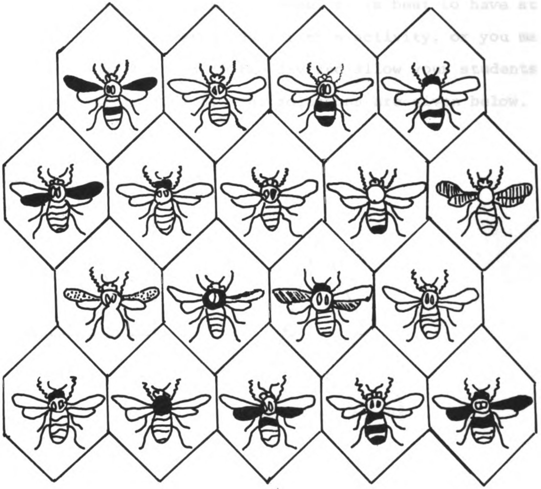


Worksheet Six: LOOK CAREFULLY

This sixth worksheet is a very short activity and you can insert it into your day whenever you have a few minutes.

Problem solving often involves looking at data very carefully and then drawing conclusions concerning the nature of the given information. In this problem, your students are asked to find the two bees in the group of eighteen which look alike. The two which most closely resemble each other are the second one in the second row, and the first one in the fourth row. Each has clear wings and abdomen, but blackened head.

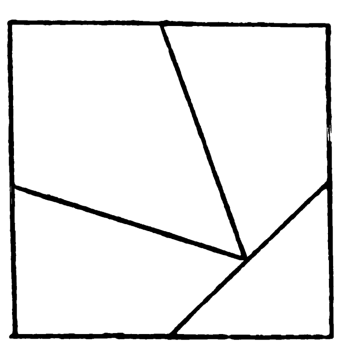
There are eighteen bees in the picture below. They all look similar. Sixteen are different from one another in some way, but two of them are exactly alike. Can you find the two that are identical?

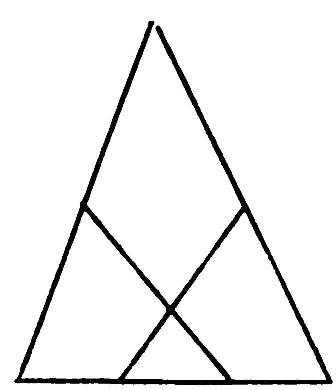


Worksheet Seven: CONSTRUCTING A SQUARE

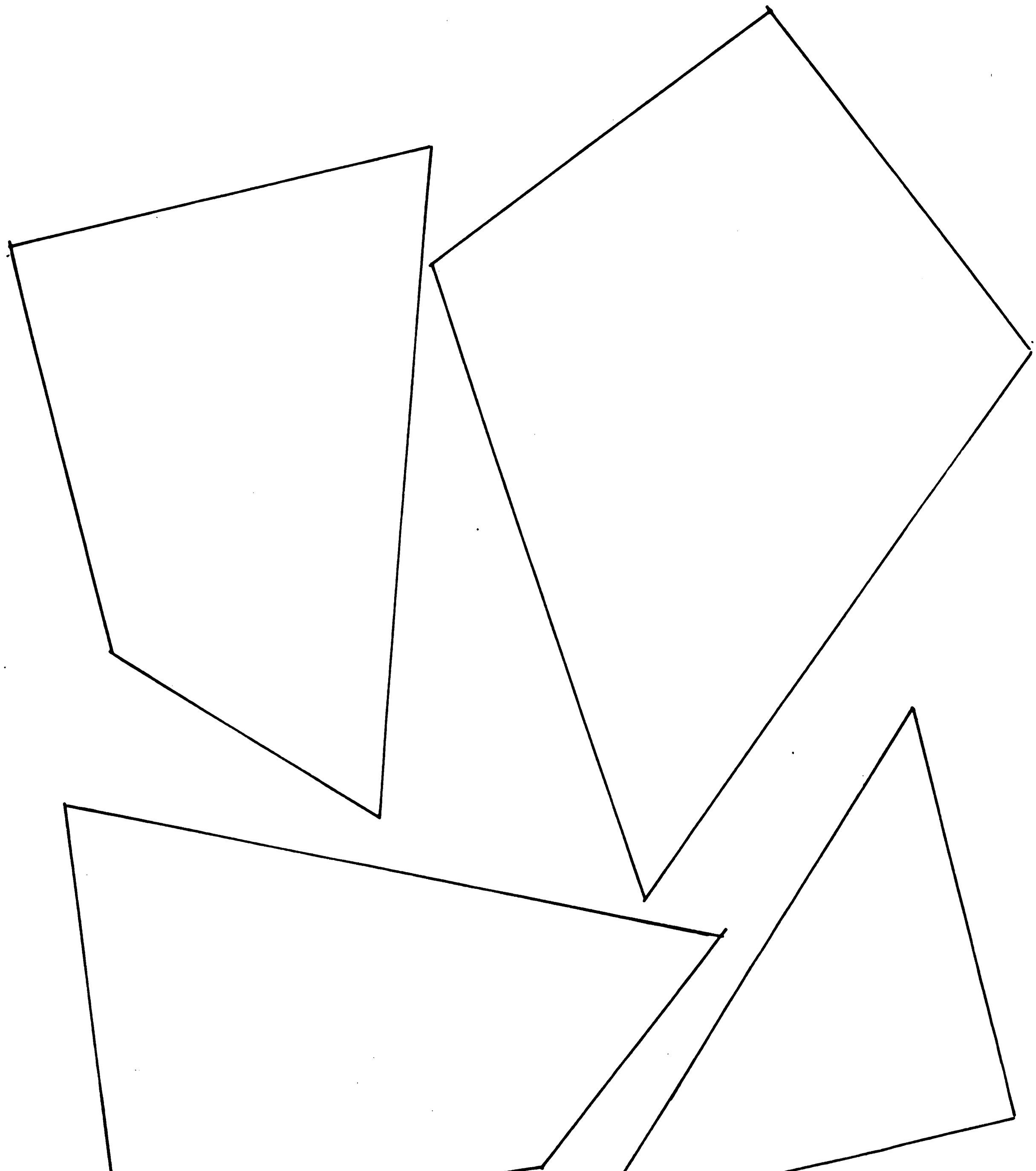
This worksheet, found on page 21 of your guide, involves arranging four pieces into a square and a triangle. Since there are only four pieces, the student can use the method of contradiction (page 16) to eliminate configurations which can't lead to the desired shapes.

The worksheet involves cutting out four shapes and then moving them around to arrive at the solution and will take a few minutes to get started. Hence, it is best to have at least twenty minutes set aside for this activity, or you may give it out near the end of the day and allow your students to take it home to finish. The solutions are given below.





Cut out the four pieces below and arrange them so that they fit together to form a square. They can also be arranged to form a triangle.

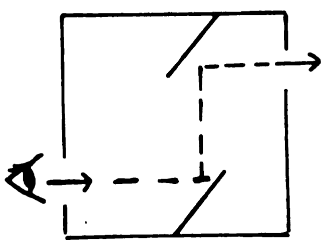


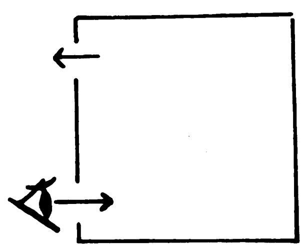
PERISCOPE WORKSHEET

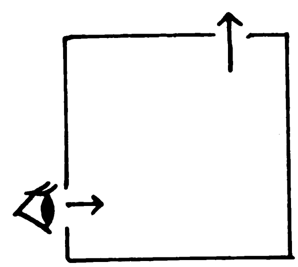
Here is a worksheet that will test your students ability and provide an opportunity to learn some natural science. The sheet contains six cross section drawings of periscopes in which your students are to place mirrors in order that the viewer will see out the indicated slot. Since some of your students may be unfamiliar with how a periscope functions, the first drawing is completed to act as a sample. Moreover, you may have to remind them that light travels in a straight line, and so to "bend" the light rays they will need a mirror. The correct arrangements of mirrors appear on your worksheet (page 23).

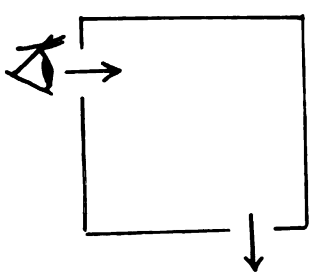
A periscope is a devise by which you can see in one direction by looking in another direction. Each box below represents a different kind of periscope. The eye looks into the box through one hole and sees things outside the box through the other hole. Draw an arrangement of mirrors inside of each box to show how the box works.

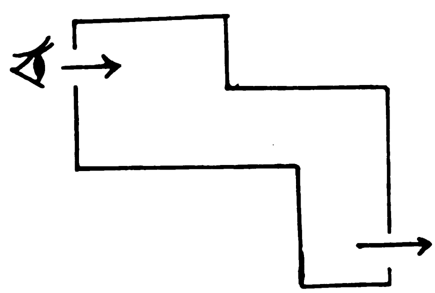
SAMPLE:

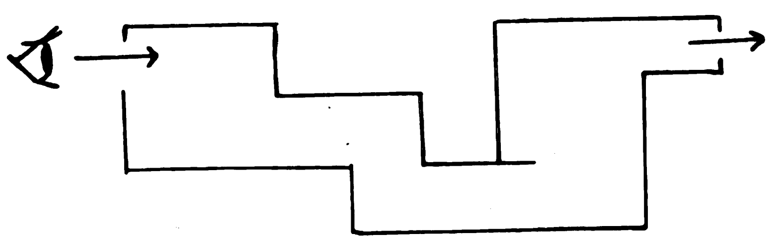










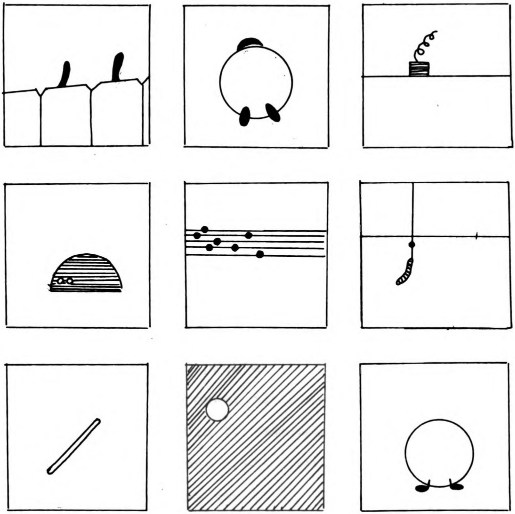


Worksheet Nine: DROODLES

"Droodles" are drawings which are made up of a few simple lines and suggest different things to different people. Each student is given a copy of the worksheet and is to respond with "It looks like . . ." Droodles is more than a child's game. It is an activity which can stimulate your students imagination and give him or her important practice in recognizing more than one point of view in problem situations.

The object of this worksheet is to introduce your students to the techniques of "Brainstorming." Take each picture and have your students give their opinion as to what each Droodle is. Try to generate as many ideas as possible. One student may see one thing and his ideas may stimulate another child to see many more things in these pictures. Some possible things to see are indicated under the pictures of your worksheet.

"Droodles" are drawings which are made up of a few simple lines and suggest different things to different people. Look at each of these pictures and decide what you think is pictured. You and your classmates may see different things in the same picture.



Worksheet Ten: BREAKING THE CODE

This worksheet contains two puzzles. The first is easy enough that most students will solve it with minimal assistance. The second is more difficult, and will only be solved by students who work at it for a long time and receive your help.

The challenge is to determine digits which represent the given letters. Each letter represents exactly one digit. The strategies used to work this type of problem are inference and contradiction.

The solutions are given on your worksheet. To obtain these solutions you might reason as follows:

1) the subtraction problem

Since A - A = B we know that B must equal 0. Since B equals 0 (and in checking we find that C + A = AB in column two; that is a two digit number ending in 0) we can conclude that A = 1. Finally, since 10 + CA = 101, C must equal 9.

2) the addition problem

All letters represent distinct digits and no column can add up to more than 26 (assuming column two has A = 9 and B = 8). Now A is a one digit number and the most which can be carried from column two is 2, we can conclude that DE must be 10. Hence, D = 1, E = 0, and A = 9 or A = 8. Consider column three: the sum C + B + A ends in an A so B + C = 10. Try various alternatives for B and C such as B = 8 and C = 2, or B = 7 and C = 3 until you arrive at the complete solution.

In the following subtraction problem, each digit has been replaced by a certain letter. Can you reconstruct the original problem?

```
   A B A
 -   C A
 -------
     A B
```

In the following addition problem, each digit has been replaced by a certain letter. Can you determine which digit each letter represents, and reconstruct the original problem?

```
   A B A
     A B
 +   A A
 -------
 D E E A
```

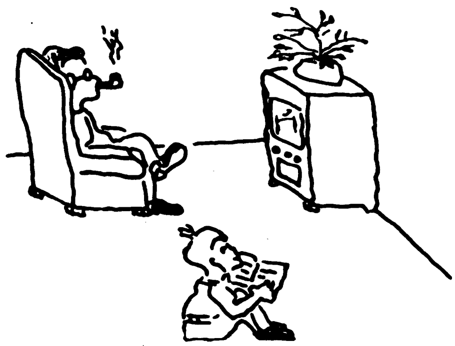
PROBLEM SENSITIVITY WORKSHEET

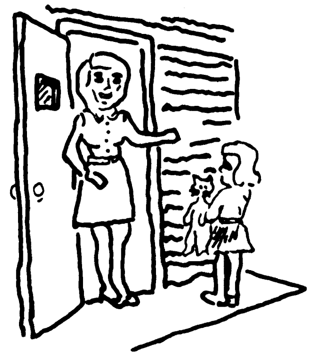
This worksheet is presented to provide practice in drawing inferences from given information or situations. Your students are to give as many answers as are possible to the question, "What could happen in each of the following situations? What is the problem and what are some possible solutions?"

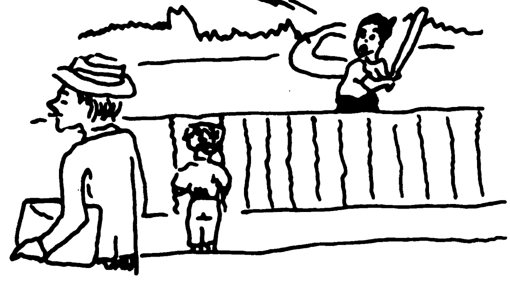
Give each student one of the worksheets and have them consider each cartoon. For example, in the first cartoon, the two people in the picture could have a problem deciding what TV program to watch. There are various suggestions your students could make to the question "What could happen in this situation?" They might suggest that one or the other will give in and watch whatever is on at the moment. They may suggest that the persons could flip a coin to see who gets their choice, or they might ask a third person to decide who should get to choose a television program. Your students might suggest many different feelings are possible for the people in the cartoon; for example, kindness and consideration, or anger and hostility.

The objective of this activity is to aid the student in becoming aware that real world situations most often involve the interaction of many people, places, and things. Some students may begin to realize that solving a comprehensive problem does not imply finding a unique correct solution, but rather involves selecting a solution which will be the best solution "most of the time." Your concern should be to get most of your students involved, and get them to view the situation creatively from various points of view.

What could happen in the following situations? What is the problem, and what are some possible solutions?







Worksheet Twelve: GETTING TO THE FAIR

In this problem your students are to find a way to get a farmer, his goat, a wolf, and a cabbage across a river in a boat which is only large enough for any two of them. Have the students cut out the four rectangles representing the farmer, the goat, the wolf, and the cabbage and try to solve the dilema. We know: Only the farmer can row the boat. The wolf and the goat can never be left alone. The goat and the cabbage can never be left alone.

Your students can work out the problem using contradiction to arrive at the following solution:

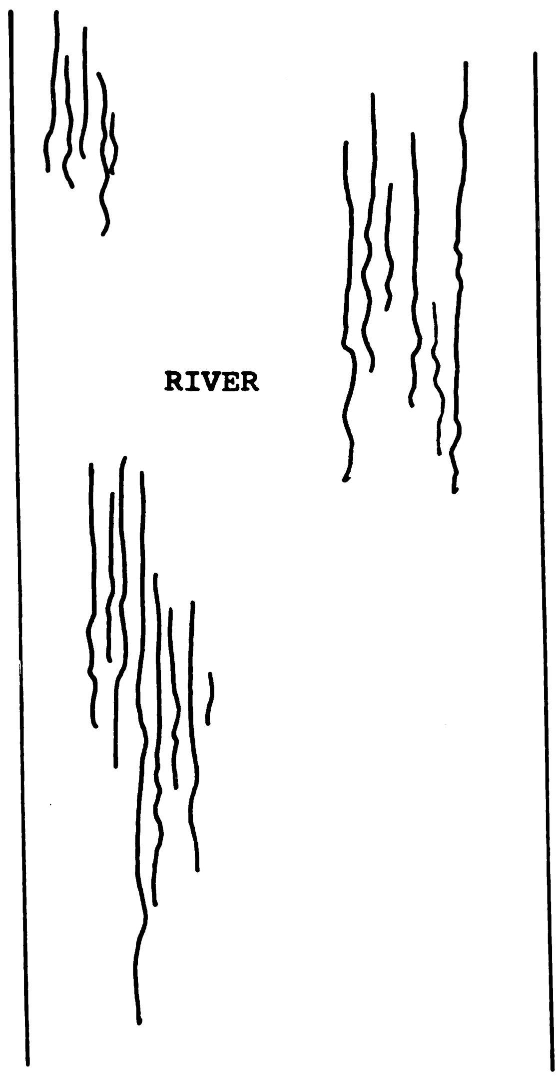
Firstly, row the goat across and return. Farmer Reynolds could not take the cabbage across first since that would leave the goat and wolf together, nor could he row the wolf across since the goat would eat the cabbage. Secondly, row the wolf across the river and row the goat back. Thirdly, row the cabbage across and return. Finally, row the goat across and continue to the county fair.

On his way to the Dutchess County Fair with his pet goat, his tame wolf and his prize head of cabbage, Farmer Reynolds came to a river that had to be crossed.

On the river was a small boat just large enough to transport the farmer and one of his possessions. This presented a serious problem. If the farmer took the cabbage and left the wolf and the goat, the wolf would eat the goat. If he took the wolf and left the goat with the cabbage, the goat would eat the cabbage. For these same reasons, the farmer had to plan ahead so that these unsatisfactory combinations would not be left together on the other side of the river.

Can you help Farmer Reynolds to get his possessions across the river and to the fair?

(Cut out the pictures at the bottom of the page and try to get all four across the river.)



FARMER REYNOLDS

GOAT

WOLF

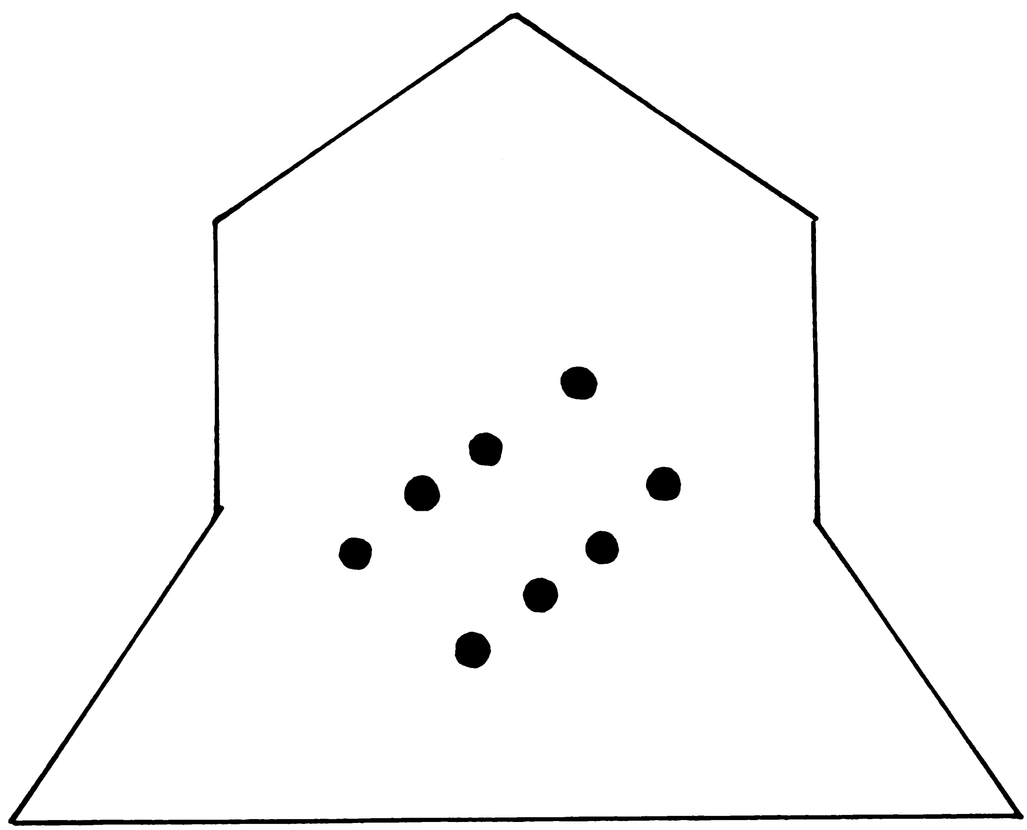
CABBAGE

SUBDIVIDING THE FARM

The next puzzle (page 35) involves dividing an area into four equal parts, each of which contain two circles representing trees. Your students might eliminate various possibilities by trial and error. The most common mistake in working this type of problem is in allowing oneself to assume that the two trees in a region must be in the same straight line. If some of your students have trouble in getting started, you might suggest that they try putting one tree from each line into some region. The answer is indicated on your worksheet.

Let us suppose a farmer has a piece of land shaped like the drawing below. It's a good, high section of land with eight (8) valuable shade trees. A company offers to buy it from him for a housing development, providing it is possible to divide the property into four equal portions, each to contain two of the shade trees.

Can you show the farmer how to draw the lines so that he can sell the land?



THE GREATEST WOMAN ATHLETE

This problem is a short puzzle which is solved by using the strategy of working backwards. The problem provides some intuitive work with fractions.

We are told that Babe Zaharias' proportions were perfect; waist size in correct relationship to her neck, her neck in proportion to her wrist, and her wrists to her ankles. If we know her ankle measurement was 9 inches, we can work backwards to find the size of her waist.

a) Once around her ankle is equal to one and half times around her wrist. Hence, wrist measurement is 6" (one and a half of 6 is equal to 9).

b) Since her neck was twice around the wrist we know her neck was 12".

c) Finally, since her waist size is twice the size of her neck, she had a 24" waist.

This puzzle provides an excellent example of a problem in which the solver must proceed in steps, that is find subgoals which will lead to the ultimate goal of finding her waist measurement.

Babe Zaharias was voted the greatest woman athlete of the twentieth century. A sport writer once claimed that she had the perfect figure for sports. The proper proportions for a woman are: waist = twice around the neck; neck is twice around the wrist; and once and a half around the wrist is the same as once around the ankle.

If Babe had the perfect figure, and her ankle measurement was 9", can you determine the following?

ANKLE ____________________

WRIST____________________

NECK______________________

WAIST____________________

PATTERNS AGAIN

This worksheet contains ten series of numbers and asks the student to find the next few terms of the sequence. As in the earlier pattern worksheet, the solver must make inferences concerning the data and formulate a hypothesis which will enable him to predict the next few terms. The first several patterns are easily recognized, but series five through nine may require some hint. The solutions are indicated on your worksheet.

Find the patterns, and then write the next four terms of each of the following series:

1. 0 , 3 , 6 , 9 , 0 , 3 , 6 , 9 , 0 , 3 , 6 , 9 , . . .
2. 5 , 4 , 10 , 9 , 15 , 14 , 20 , 19 , . . .
3. 1 , 4 , 9 , 16 , 25 , . . .
4. 2 , 6 , 12 , 20 , 30 , 42 , 56 , . . .
5. 1 , 1 , 2 , 4 , 3 , 9 , 4 , 16 , 5 , 25 , . . .
6. 1 , 1 , 2 , 3 , 5 , 8 , 13 , 21 , . . .
7. 2 , 4 , 8 , 16 , 32 , . . .
8. 1 , 4 , 6 , 7 , 10 , 12 , 13 , 16 , 18 , 19 , . . .
9. 1 , 3 , 2 , 5 , 7 , 4 , 9 , 11 , 6 , . . .
10. 1 , 3 , 5 , 7 , 9 , 11 , . . .

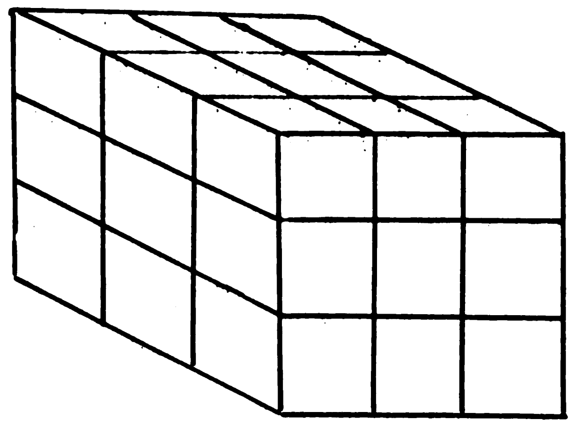
Worksheet Sixteen: PAINTED SOLIDS

Pictured in this worksheet are two painted solids which are cut along the indicated lines. The task is finding the number of sections of the cut up solids which have three, two or one side painted. Using the strategy of making inferences, we note that the only places where three sides will be painted are at the corners. We then conclude that to have two painted sides a section would have to be along an edge, but not at a corner. Finally, we can infer that the only sections with one painted side would have to be the non-edge parts of the solid faces.

For the cube, there are eight sections with three sides painted. Twelve sections have two sides painted, and there are six sections with exactly one side painted. For the triangular block, there are six sections which have paint on three sides, eleven sections have two sides painted, and six sections have exactly one side painted.

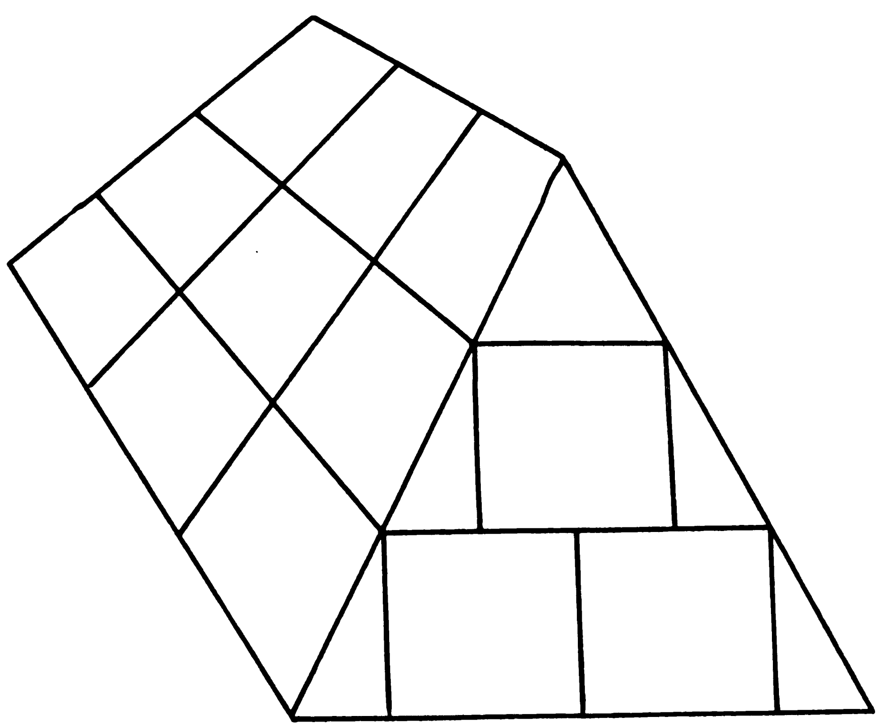
A large cube contains 27 small cubes as shown below. The top and the bottom, and the four sides of the large cube are painted grey.

How many small cubes have three grey sides? Two grey sides? One grey side?



A large triangular block consists of 24 small pieces, as shown in the drawing below. Two sides, the bottom, and the front and back of the large triangular block are painted with blue paint.

How many sections have three blue sides? Two blue sides? One blue side?



MADDER'S LADDER

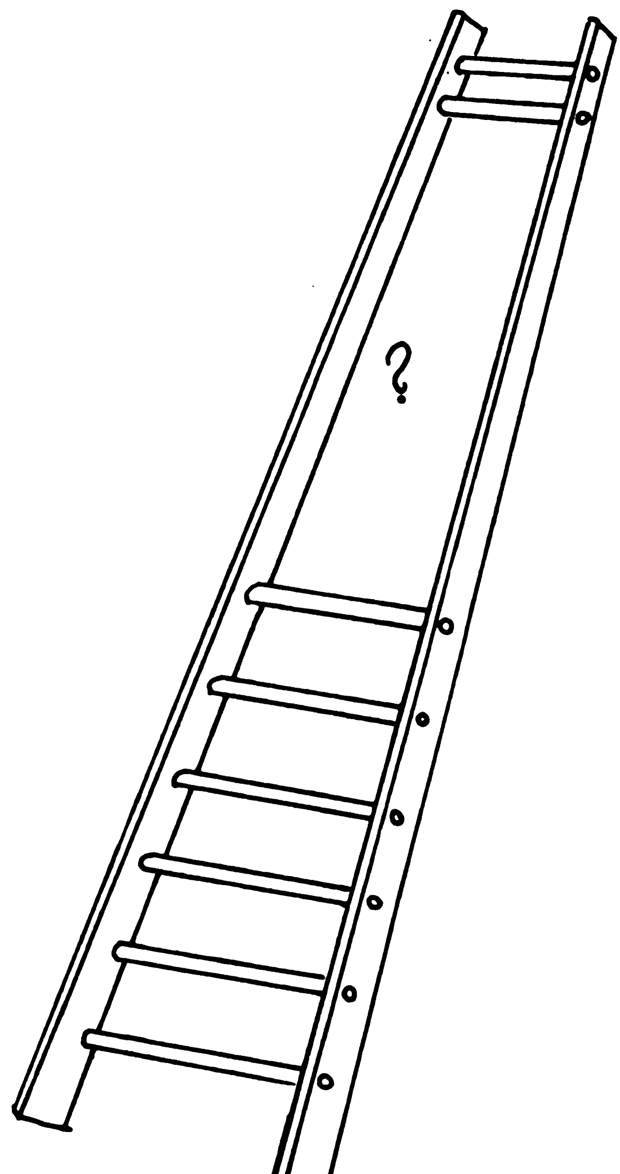
This problem involves finding the number of rungs on a ladder. To solve this question, the student must ignore the first half of the problem and work backwards to answer the first question. That answer can, in turn, be used to solve the second part of the problem. There are 17 rungs on the ladder. Don't forget that you do not count the rung you are standing on when you start up or down.

An excellent way to solve this problem is to draw lines representing rungs on the ladder, carry out the perscribed instructions, and note the total number of lines drawn.

When I first saw Mr. Madder, he was part way up a ladder. He went up four rungs, down seven rungs, and up ten. That put Mr. Madder at the top of the ladder.

Then he went down nine rungs, up three rungs, and down ten. That put him at the bottom of the ladder with his feet on the ground.

How many rungs has Madder's ladder? On what rung was Madder when I first saw him?



Worksheet Eighteen: NO SWIMMING

The problem of getting nine men and two boys across a river in an inflatable boat is similar to the earlier problem of getting the farmer, and his possessions, across the river. Your students can use a similar procedure to the one they utilized in that situation.

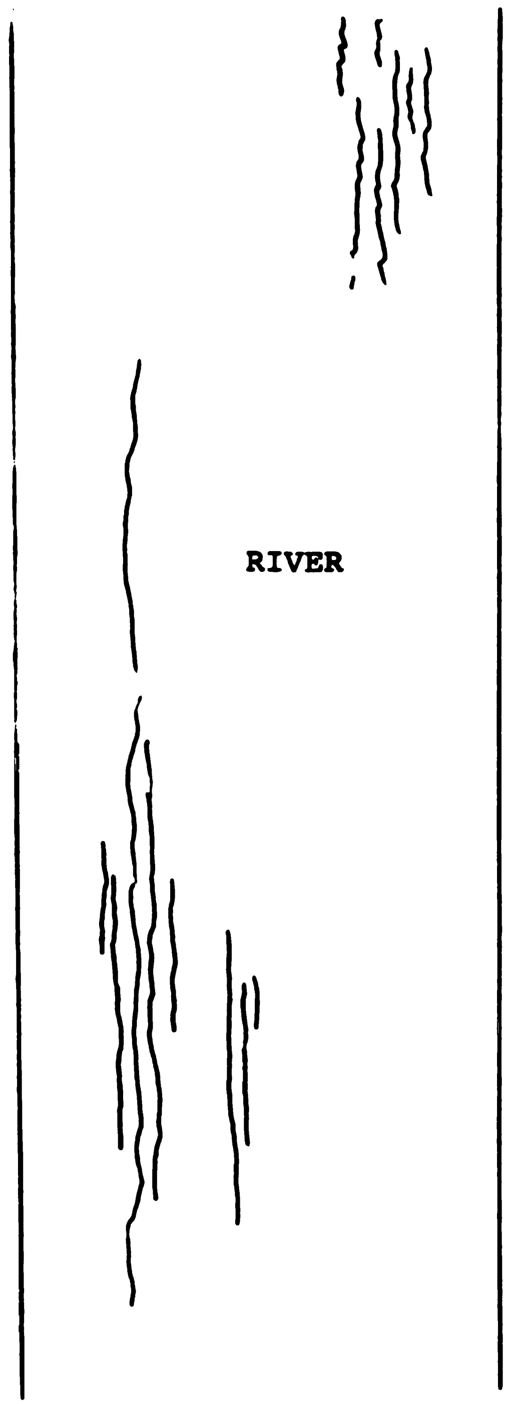
Since only one man could be in the boat at any one time, the number nine just means that there will be a number of trips necessary to complete the crossing. For students who need some direction, you might suggest that they first find a way to get one man across the river and the boys and the boat back to the original side. Once this is accomplished, the same procedure is repeated eight more times.

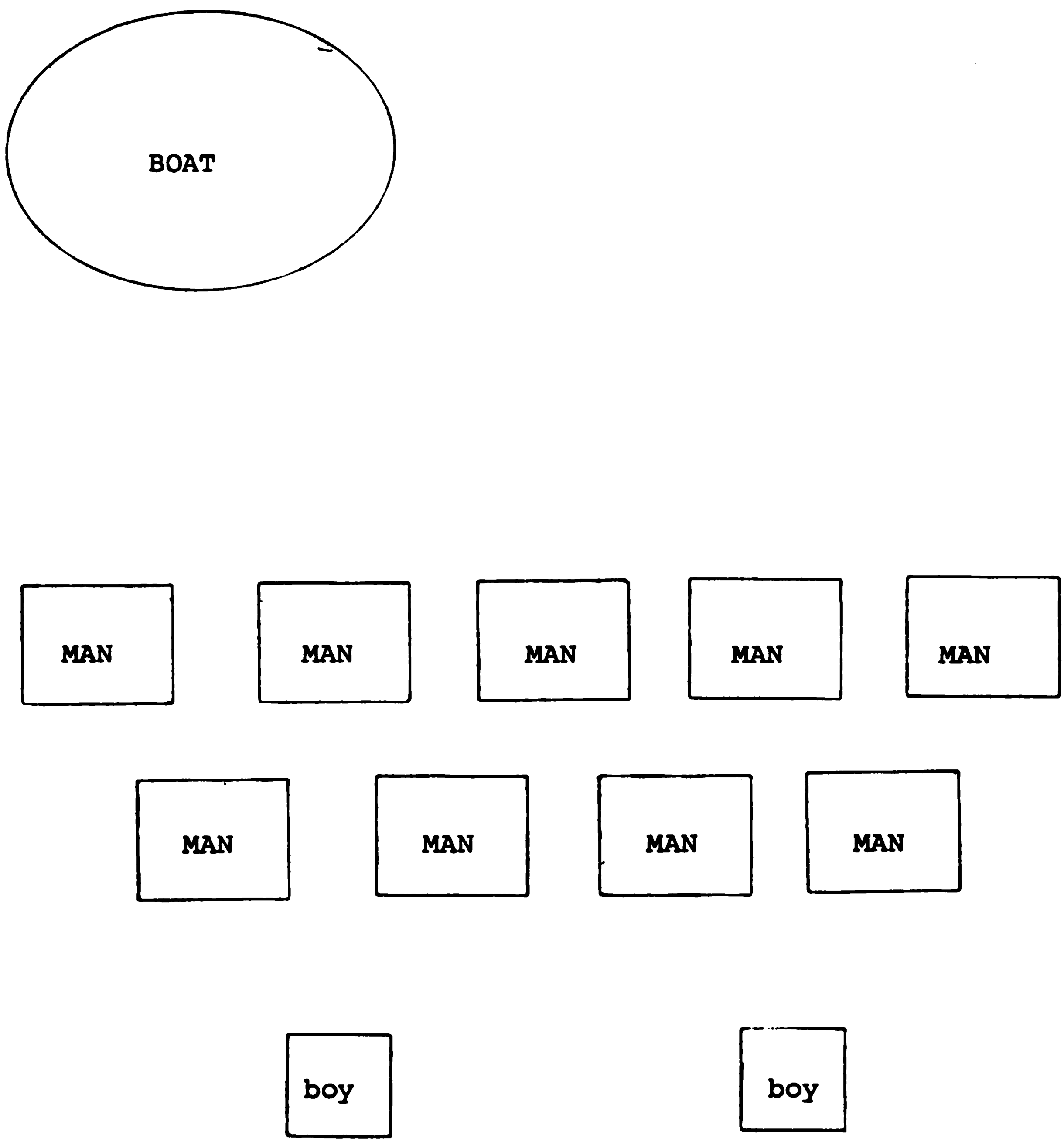
To get one man across the river and the boys back to the starting point: Two boys cross the river and one boy rows back (that is two crossings). Now a man alone rows across and the other boy rows back to the original shore. There are now eight crossings.

Repeat this procedure eight additional times and the two boys will be on the original shore (after 36 crossings). One more trip will get the two boys across, and we have therefore found that it takes 37 times across the river to get all nine men and the two boys to the far shore.

Nine men and two boys want to cross a river using an inflatable raft. The raft will carry one man or the two boys.

How many times must the boat cross the river to accomplish this goal?
(A round trip equals two crossing.)




BOAT
MAN
MAN
MAN
MAN
MAN
MAN
MAN
MAN
MAN
boy
boy

Worksheet Nineteen: WHO PLAYS WHERE

Worksheet nineteen contains two puzzles which can be solved using contradiction. The first puzzle involves matching three people to three different positions on a basketball team. The second one requires that your students play detective to determine who broke a vase.

Give each of your students a worksheet, and ask them to solve the first problem without talking to their friends. After they have worked on it for a while, you might ask all students with the correct answer to give clues to those who have not arrived at a complete solution. The usual approach to this type of problem is to draw a diagram showing the people and their jobs. Assume one person plays one position, for example suppose Bill plays guard, then check to see if this assumption will contradict some given piece of information. If it does, then try Lew at guard and see what happens.

	Bill	Lew	Tom
Center			
Guard			X
Forward	X		X

Note that Tom can't be the guard because of statement four. Likewise, Bill is not an only child and therefore he can't be the forward (statement three). Finally, Tom can't be the forward since he has a higher free-throw average than the guard and the forward has the lowest free-throw average.

If no one gets the correct answer, show the class who the center is (Tom) and see if they can get the other two boys in the correct positions.

WHO PLAYS WHERE?

On a certain class basketball team, the positions of center, guard, and forward are held by Bill, Lew, and Tom (though not necessarily in that order).

The forward is an only child.

The forward has the lowest free-throw average.

Tom's best friend is Bill's brother.

Tom has a higher free throw average than the guard.

What position does each student play?

WHO BROKE THE VASE?

The Foremans have gone out for the evening, leaving their four children with a babysitter, Nancy Wiggens. Among the many instructions the Foremans gave Nancy before they left was that three of their children were consistent liars and only one of them consistently told the truth, but they didn't tell her which one of them was truthful. As she was preparing dinner for the children, one of them broke a vase in the next room. Nancy rushed in and asked who broke the vase. These were the children's statements:

Betty: "Steve broke the vase."

Steve: "John broke it."

Laura: "I didn't break it."

John: "Steve lied when he said that I broke it."

Knowing that only one of these statements was true, Nancy quickly determined which child broke the vase.

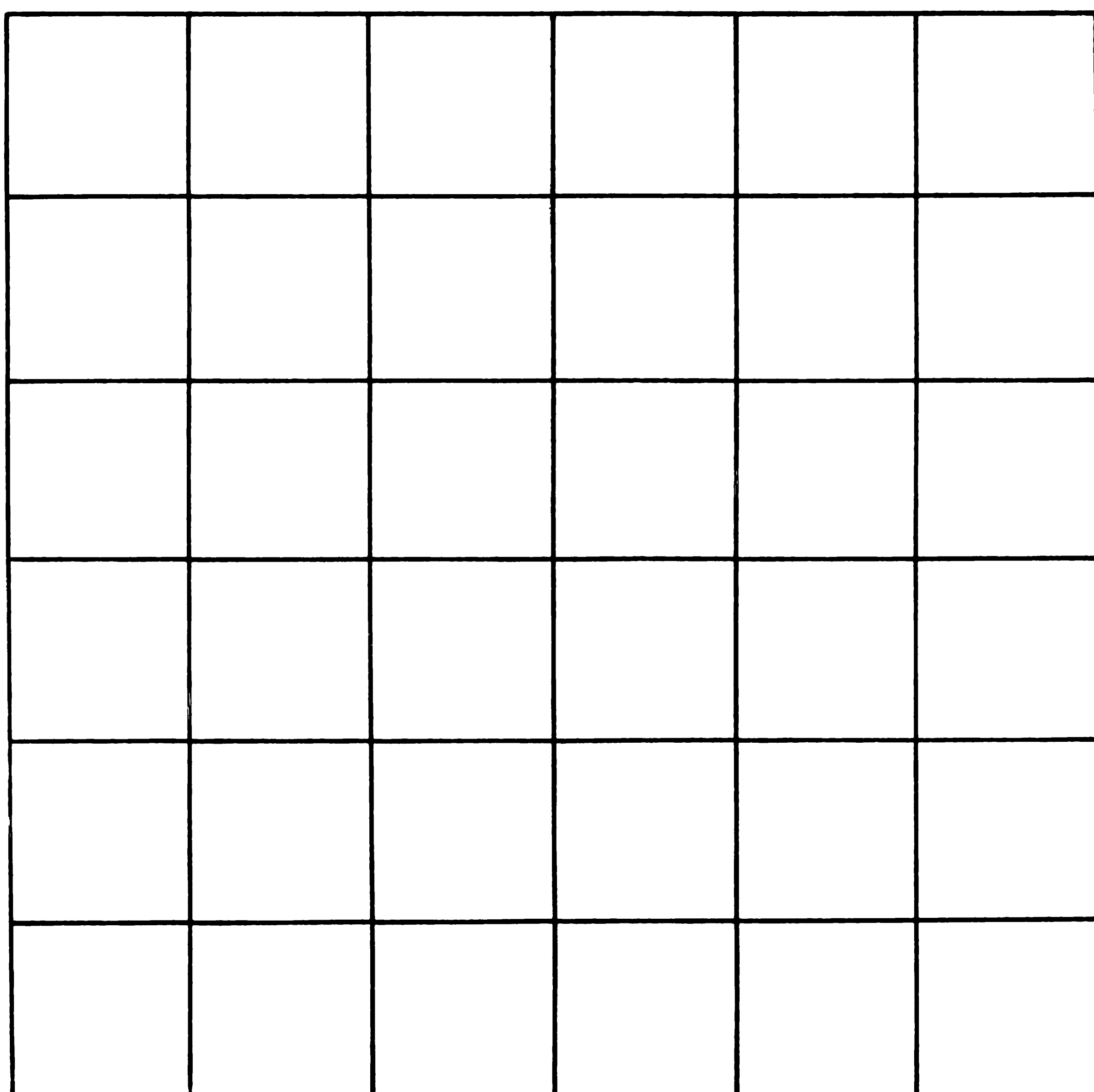
Who was it?

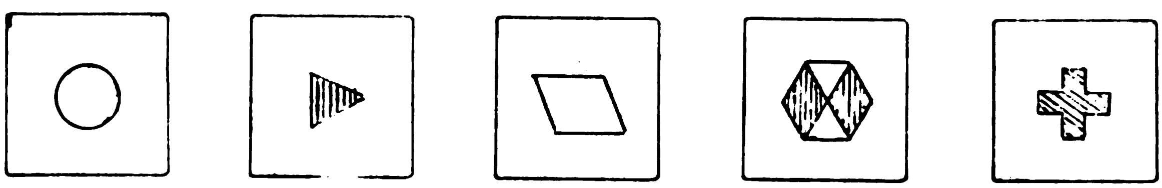
COIN COLLECTION PUZZLE

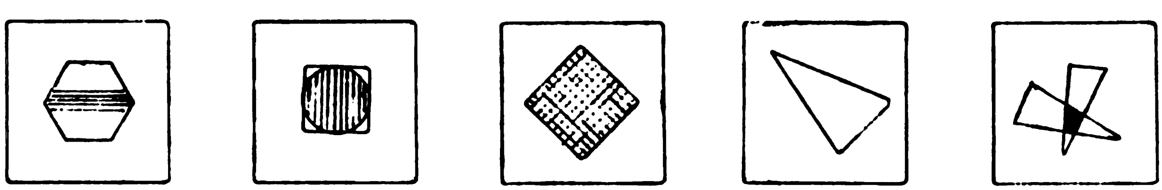
This puzzle requires your students to cut out twelve squares, each representing an ancient coin, and arrange them in a six by six grid. It can be solved by simply trying various possibilities and eliminating those which lead to a contradiction of the constraint that exactly two coins be in any one row. To aid some students in getting started, you might put the first one in for them, later adding a second, then a third, and so on until they can finish the problem themselves. The solution is indicated on your worksheet.

Mr. Clifford Bradshaw, a wealthy coin collector, owned twelve ancient coins. At a coin collectors meeting, Mr. Bradshaw wanted to display his twelve coins in a velvet lined case. The case had thirty-six (36) compartments, as shown below. He ordered his assistant to arrange the display so that there would be two coins in each row, column, and diagonal, and not more than two coins in the same straight line.

How did the assistant carry out his employer's instructions? (Cut out the coins and try to find the answer.)



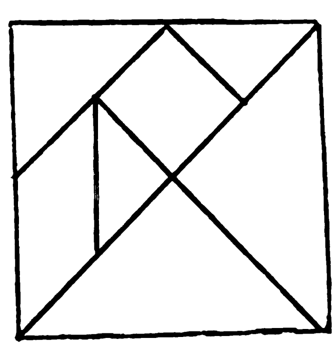


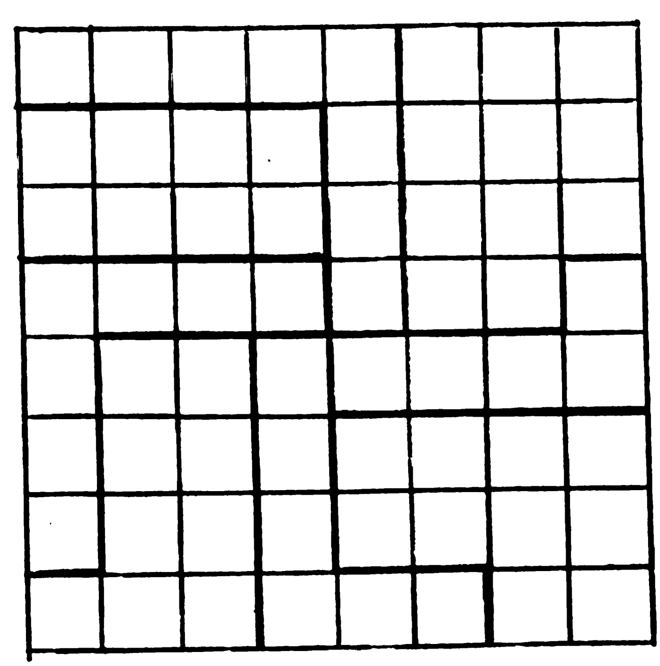


RECONSTRUCTING THE SQUARE-REVISITED

The last two worksheets are similar and both involve some cutting out of pieces and fitting them together to form the shape of a square. The seven piece Tangram is a popular commercially produced puzzle which may require a great deal of time to complete. Adults may spend an hour working on the problem so it is probably wise to give your students a hint after they have it cut out and are familiar with the pieces. After a time, tell them that the two large triangles fit together to form half of the square. The solution is given below in Figure 1.

Similarly, the reconstruction of the checker board is a difficult problem and many of your students will lose interest quickly if you don't give continued assistance. It is included to challenge your best student.

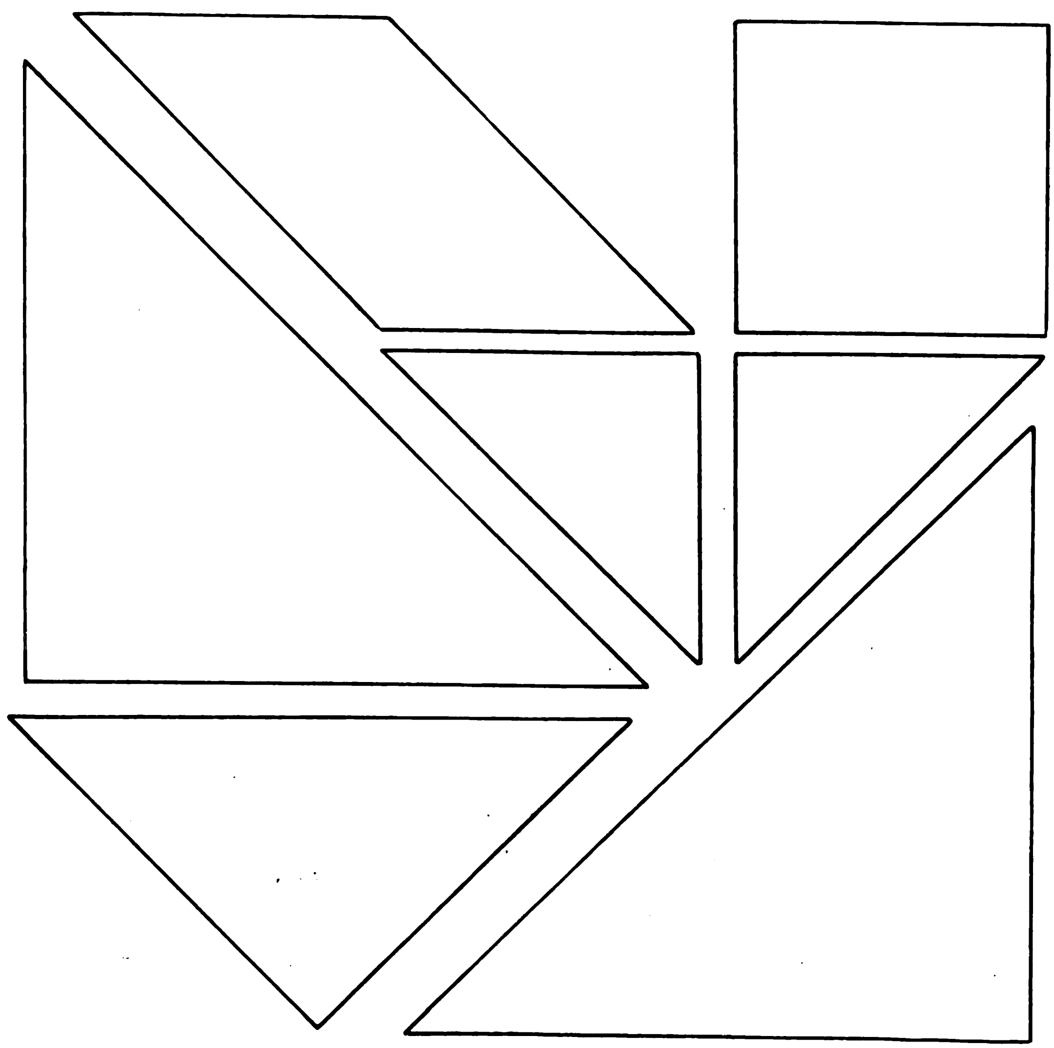




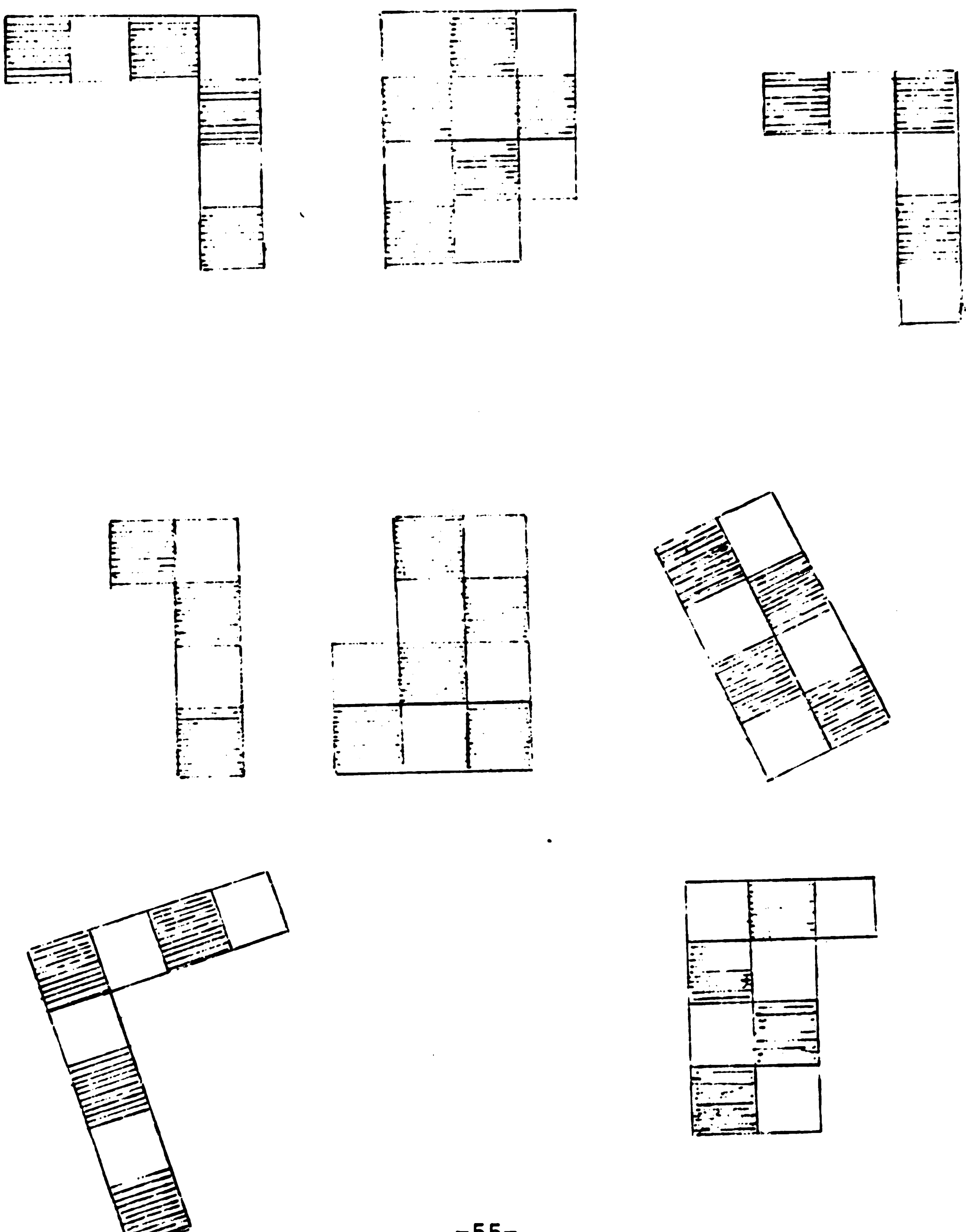
Tangrams

Wo Fung, a very rich man, had a beautiful square mosaic tile which he always carried with him. Upon waking one day he picked up the tile to admire it. Alas! the tile slipped from his hands and fell to the marble floor, breaking into seven (7) pieces. He tried and tried, but he couldn't put it back together again.

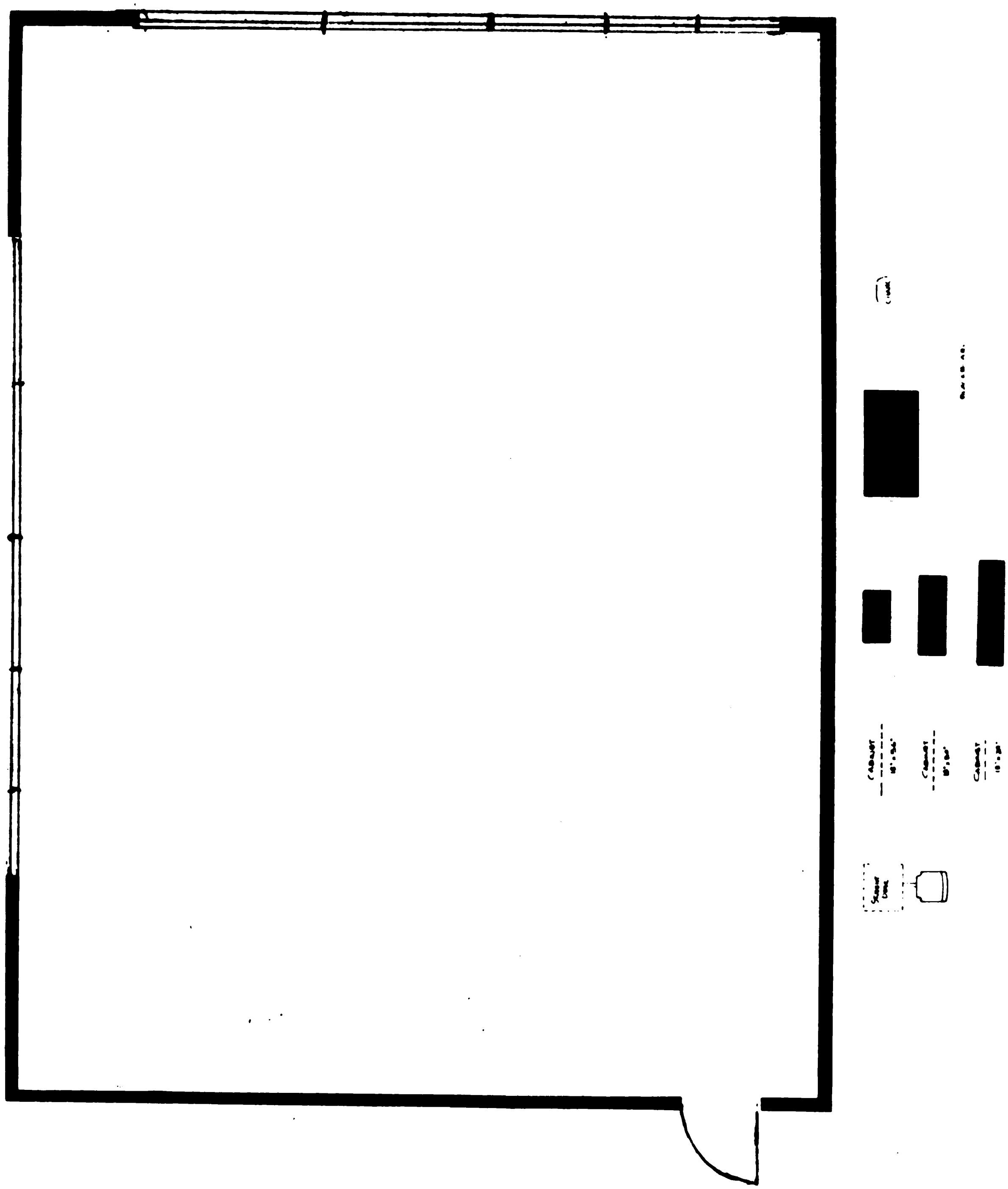
Can you help Wo Fung by reconstructing the square? (Cut out the pieces and try to find the answer.)



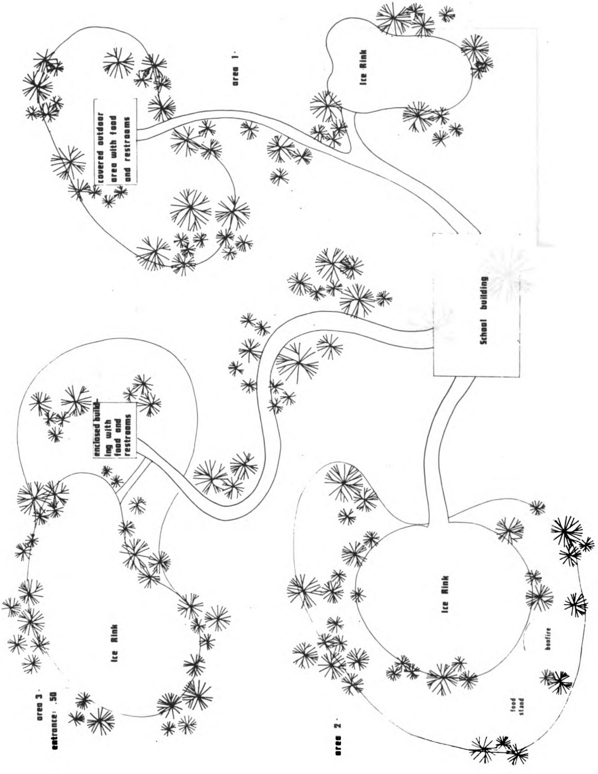
Cut out the eight (8) pieces below and arrange them so that they form an 8 x 8 checkerboard.



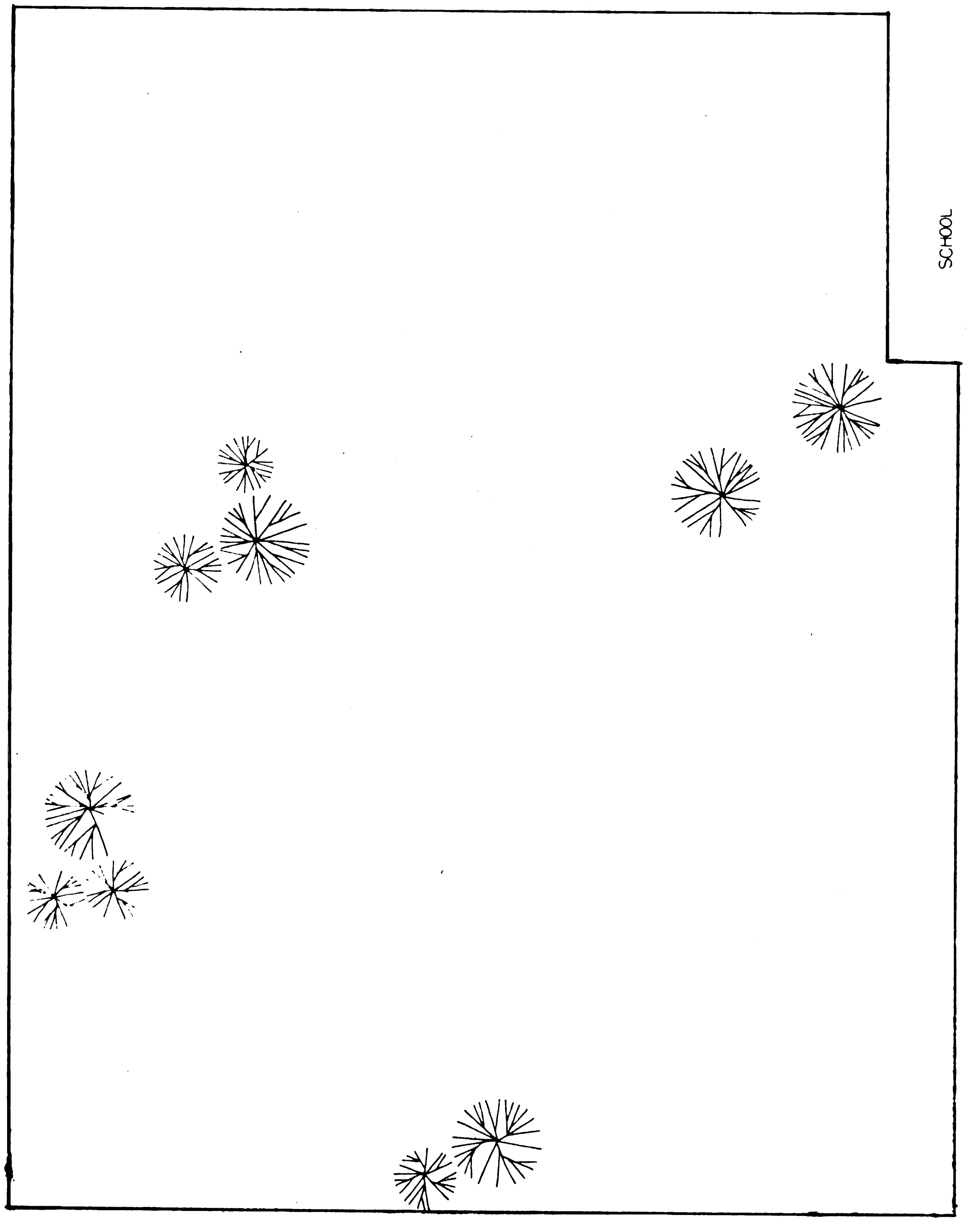
APPENDIX C



APPENDIX D


area 1
Ice Rink
covered outdoor area with food and restrooms
School building
enclosed build-ing with food and restrooms
Ice Rink
bonfire
food stand
area 2
Ice Rink
area 3
entrance: .50

APPENDIX E



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