

**MATHEMATICS TEACHERS' COLLABORATIONS MEDIATED BY A PROBLEM-
BASED CURRICULUM**

By

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ABSTRACT

MATHEMATICS TEACHERS' COLLABORATIONS MEDIATED BY A PROBLEM-BASED CURRICULUM

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Researchers have stressed that knowing more about how teachers work in self-initiated collaboration can help develop collaborative cultures and provide insight on the characteristics of those cultures (e.g., Darling-Hammond & Richardson, 2009), specifically, how collaborations in teacher groups can lead to generative professional learning opportunities (Coburn, Mata, & Choi, 2013). This study sought to address that gap. I conducted a case study of collaboration among middle school mathematics teachers using the *Connected Mathematics Project* (CMP) curriculum. Field observations, pre and post interviews, teacher logs and informal conversations with teachers supported a close examination of these teachers' collaborative culture, allowing me to investigate how, when, and for what reasons they collaborated. Observation data was the main source of analysis and interview data and teacher-logs were used for triangulation purposes, to support and/or validate claims. Data analysis was carried by either adapting existing frameworks already provided in the literature or by using a grounded theory approach with open and axial coding techniques (Glaser & Strauss, 1967) and following with constant comparative analysis (Miles & Huberman, 1994).

I found that the teachers interacted multiple times each day in spontaneous forms but in particular groupings. There were physical, practical and interpersonal factors that contributed to the frequency and directionality of teachers' collaborative interactions. In addition, I found that the depth of their interactions was related with the interpersonal and disciplinary positioning of these teachers. This study provides evidence that even in a highly collaborative setting teachers'

discussion of mathematics as tied closely to their teaching and students, and interactions gained more depth from the initiation or response of a more knowledgeable peer.

Teachers created a collaborative context where they shared their experience, examined emergent concerns and questions in their practice, and as they interacted within a domain of knowledge, they expanded their understanding of the issues and possible ways to address those issues. Teachers' positioning one another in their interactions and their positioning themselves with respect to mathematics and curriculum provided entry points in deepening their understanding. At the same time, teachers' interpersonal or disciplinary positioning limited the depth of their interactions. Shared knowledge co-constructed through collaborative interactions did not appear in different contexts as frequently, so it might be more realistic to expect for minor changes in teachers' practices rather than dramatic shifts.

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TABLE OF CONTENTS

LIST OF TABLES	xi
LIST OF FIGURES	xii
KEY TO ABBREVIATIONS	xiv
CHAPTER 1: INTRODUCTION	1
Framing the Study in Existing Literature	2
Purpose Statement and Research Questions	4
Introducing the Upcoming Chapters	5
CHAPTER 2: LITERATURE REVIEW	8
How has Collaboration and Teacher Collaboration Been Framed?	9
Structure of Collaboration	11
Structured Collaboration	11
Self-structured collaborations	12
Spontaneous Collaboration	14
Teacher talk	15
Teachers' lounge interactions	17
Teaching partners	18
Describing the Process of Teacher Collaboration	19
Collaborative Process Described by Attending to Individuals' Involvement	19
Collaborative Process Described by Attending to Nature of Interactions	20
Collaborative Process Described by Attending to Content Focus	21
Factors Contributing to Collaboration	23
Physical Factors	23
Practical Factors	24
Curriculum materials in teacher collaboration	24
Personal-Interpersonal Factors	25
Some Other Considerations	26
Evaluating Process and Outcomes of Collaboration	26
Teachers' Professional Growth	27
Change in Teachers' Instructional Practice	28
Affective and Social Gains	28
Focus of the Study	29
Theoretical Perspective	30
CHAPTER 3: METHODS	33
Case Study Setting and Participants	34
Central Participants	35
Kristina Gallen	36
Curriculum history	37

Classroom	37
Routines	37
Kayla Illingworth	38
Curriculum history	38
Classroom	39
Routines	39
Paul Earley	40
Curriculum history	40
Classroom	41
Routines	41
Peripheral Participants	42
Danny	43
Mr. Kullen and Ms. Gina	44
Ms. Patz	44
Mr. Norm and Mr. Vall	45
Ms. Jansen	45
Data Collection	46
Interviews	46
Observations	47
Weekly Teacher Logs	48
Data Analysis	48
Addressing First Research Question	51
Addressing research question 1a	51
Addressing research question 1b	54
Addressing research question 1c	56
Interrater reliability	58
Addressing research question 1d	60
Social dynamics of interactions	60
Depth of interactions	63
Interrater reliability	69
Addressing research question 1e	70
Addressing Second Research Question	70
Addressing Third Research Question	71
CHAPTER 4: HOW TEACHERS COLLABORATE: FREQUENCY, FORMS, FACTORS, AND FOCI	72
Frequency and Forms of Collaborations	73
Structure	74
Groupings	74
Prevalent Activities	75
CIN activities	76
IOI activities	78
JTW activities	81
DCT activities	83
Summary	84
Factors That Shaped Teachers' Collaborations	84

Physical Factors	85
Practical Factors	88
Teaching the same grade levels, content, and students	88
Teaching from the CMP curriculum	90
Common time	93
Year-long intern	94
Personal-Interpersonal Factors	95
Shared goals and roles	95
Personal elements	96
Foci of the Collaborations	97
Knowledge of Content and Curriculum	99
Knowledge of Content and Students	101
Knowledge of Content and Teaching	101
Subject Matter Knowledge	102
Non-Mathematical Knowledge for Teaching	103
Summary	104
Chapter Summary	104
 CHAPTER 5: HOW TEACHERS COLLABORATE: NATURE OF INTERACTIONS	107
The Nature of Teachers' Collaborative Interactions	108
Depth of Interactions	108
Low depth	109
Limited by the issue	109
Limited by context	109
Limited by choice	110
Medium depth	111
High depth	114
Summary	117
Social Dynamics of Interactions	117
Systemic positioning	118
Justification	120
Challenging	122
Summary	124
Semantic positioning	124
Curriculum	124
Mathematics	126
Relating Foci of Collaboration to Nature of Interactions	127
Chapter Summary	132
 CHAPTER 6: COLLABORATIONS RELATING TEACHERS' PRACTICES	134
Teachers' Classroom Practices Informing Their Collaborations	136
Concerns about Students	138
Students' behavior	138
Students' understanding	141
Summary	143
Inspirations from Students	144

Concerns about Teaching	148
Inspirations from Teaching	153
Summary	155
Teachers' Collaborative Interactions Informing Their Classrooms	156
Major Changes	157
Problem replacements and additions	158
Facilitating lessons	160
Minor Adjustments	161
Small details	161
Posing questions	162
Selective examples	163
Additional student strategies	164
Informative additions	164
Physical additions	165
Summary	165
Collaboration Related to Professional Actions Teachers Take	
Outside of the Classroom	166
Guided Planning for the Lesson	166
Taking Leading Roles in Finding a Resolution to Emergent Issues	167
Chapter Summary	168
 CHAPTER 7: PRODUCTIVITY OF TEACHER COLLABORATION	171
Products Situated within Daily Practices	174
Assessments	176
Pre-assessment meetings	176
Post-assessment meetings	180
Instructional Scheduling	185
Lesson Plans	187
Considering Mathematics Problems	192
Summary	194
Products as Comprehensive Summary of Interactions	194
Raising Issues	196
Observation evaluations	196
Purpose of assessment	199
Summary	201
Expanded Circle of Collaboration Participants	201
Promoted Self-Care	203
Informed Decision Making	206
Access to Richer Set of Resources	207
Chapter Summary	208
 CHAPTER 8: CONCLUSIONS, LIMITATIONS, AND IMPLICATIONS	211
Summary of Main Results	212
Extensions and Speculations	216
Shared Knowledge Spaces are Productive	217
Curricular decision-making	220

Individual's learning	221
Consideration of the Depth of Interactions	222
Conceptual agency in shaping the depth of interaction	222
Concerns to let-go and concerns to follow-up on	224
Issues with coding interactions on a three point scale	225
Changes Related to Classroom Practices	227
Lack of Conflict	228
Teachers with Divided Roles and Identities	230
Consideration of Other Contexts	231
Limitations of the Study	232
Implications and Recommendations	233
Further Questions	235
Concluding Remarks	236
 APPENDICES	 238
APPENDIX A: DATA COLLECTION AND PILOT WORK TIMELINE	239
APPENDIX B: TEACHER INITIAL INTERVIEW PROTOCOL	240
APPENDIX C: TEACHER POST INTERVIEW PROTOCOL	242
APPENDIX D: SCHOOL PRINCIPAL INTERVIEW PROTOCOL	249
APPENDIX E: OBSERVATION PROTOCOL	250
APPENDIX F: CLASSROOM OBSERVATION PROTOCOL	251
APPENDIX G: DEFAULT FOR TEACHER LOGS	252
APPENDIX H: THE DEPTH OF INTERACTIONS FRAMEWORK	253
APPENDIX I: LIST OF ISSUES RAISED BY THE TEACHERS	254
 REFERENCES	 258

LIST OF TABLES

Table 1 Comparison of codes between two coders	59
Table 2 Distribution of agreement due to chance	59
Table 3 Depth of interactions comparison of codes between two coders	69
Table 4 Depth of interactions distribution of agreement due to chance	70
Table 5 Distribution of structure of collaborative interactions	73
Table 6 Distribution of teachers' interactions in particular groupings	74
Table 7 Content focus of teachers' collaborative interactions	98
Table 8 The variation in depth of interactions with content focus	127
Table 9 Work students were expected to complete with different characters	146
Table 10 Counts of objective products	175
Table 11 Timeline for data collection and pilot work	239
Table 12 The depth of interactions definitions adapted from Coburn and Russel (2008, p. 230)	253
Table 13 List of issues, grouping and context these issues were raised	254

LIST OF FIGURES

Figure 1 Stonebridge Middle School physical setup	36
Figure 2 Peripheral participants	43
Figure 3 Coding example showing different units of analysis	50
Figure 4 Forms of collaborations along with three dimensions	54
Figure 5 Egg model representing MKT	57
Figure 6 Distribution of prevalent activities in each grouping	75
Figure 7 Kayla's example of (x,y) and (y, x) in table of values	82
Figure 8 Reflection over the $x=y$ line	82
Figure 9 Representation of teachers' positioning each other with respect to content and pedagogy	118
Figure 10 Frequency of KCS and student-focused interactions	128
Figure 11 Analyzing teachers' three actions with respect to each other	135
Figure 12 Kayla's student's solution to correlation question	147
Figure 13 Student work examples for problem 2.1 in the <i>Frogs, Fleas and Painted Cubes</i> unit	158
Figure 14 Relating teachers' professional actions	169
Figure 15 Characterization of teachers' productivity of collaborative interactions	172
Figure 16 Variety in the meaning and purpose of the products	173
Figure 17 Butterflies, Wallpapers and Pinwheels Unit, Check-Up 1	179
Figure 18 Stretching and Shrinking Unit Test, Backpack Question	181
Figure 19 Stretching and Shrinking Unit Test, modified Backpack Question	183
Figure 20 Labsheet for the Problem 2.3 in the <i>Looking for Pythagoras</i> unit	189

Figure 21 Kristina's extra-work suggestions for Kayla	208
Figure 22 Teachers' <i>KCC</i> knowledge space	218

KEY TO ABBREVIATIONS

CCSSO	Council of Chief State School Officers
CIN	Checking- In
CMP	Connected Mathematics Project (Lappan et al., 2006, 2014)
DCT	During Class time
ELA	English and Language Arts
IOI	Interchange of Ideas
JTW	Joint Work
KCC	Knowledge of Content and Curriculum (Ball, Thames and Phelps, 2008)
KCS	Knowledge of Content and Students (Ball, Thames and Phelps, 2008)
KCT	Knowledge of Content and Teaching (Ball, Thames and Phelps, 2008)
MKT	Mathematical Knowledge for Teaching (Ball, Thames and Phelps, 2008)
NCTM	National Council of Teachers of Mathematics
NBPTS	National Board for Professional Teaching Standards
NCTAF	National Commission on Teaching and America's Future
Non-MKT	Non-Mathematical Knowledge for Teaching
PCK	Pedagogical Content Knowledge (Ball, Thames and Phelps, 2008)
SMS	Stonebridge Middle School
SMK	Subject Matter Knowledge (Ball, Thames and Phelps, 2008)

CHAPTER 1

INTRODUCTION

The need for high quality teachers is essential, since teachers play an important role in improving students' learning outcomes (Darling-Hammond, 2000). In addition, there is an increasingly long list of expectations from teachers by different stakeholders (Shulman & Shulman, 2004) such as, increasing students' academic standards, responding to new evolving technologies, implementing effective research-based strategies, satisfying better coordination of multiple resources are only a couple of those. Therefore, teacher development always remains an important issue. In contrast to these increasing demands from teachers, there are not many opportunities for teachers to improve their practices. As Eisner (2000) noted teacher educators and building or district level leaders used to believe "they could 'inservice' teachers every few months, a practice that is eerily similar to having automobiles serviced every 15 000 miles" (p. 347), now it is well-documented that those disconnected professional development sessions are not responsive to teachers' needs, and their effect on teachers' instructional practice is almost invisible (Goe, Biggers, & Croft, 2012). In addition, such approaches look down on teachers and the teaching profession by not giving teachers autonomy in their own learning (Hargreaves, 2000). Cochran-Smith and Lytle spoke to that concern:

This means that throughout their careers, teachers are expected to learn about their own profession not by studying their own experiences but by studying the findings of those who are not themselves school-based teachers... We argue for a different knowledge base; one that is not designed so that teachers function simply as objects of study but also as architects of study and generators of knowledge. (1993, pp.1–2)

Teachers can and need to take leading roles in their professional growth and be able to design learning activities responsive to their needs.

Framing the Study in Existing Literature

The conception of professional development has shifted as research provides more evidence that “the most powerful forms of teacher development are fostered most directly and powerfully by conditions unlikely to be found outside the school” (Leithwood, Jantzi, & Steinbach, 1999, p.150). Moreover, it is documented that teachers’ practices change with professional development experiences where they take more active roles in their learning (Darling-Hammond, 2000). Job-embedded professional development activities were planned in response to these considerations (Goe, Biggers, & Croft, 2012 and for the purposes of changing views of teacher professionalism (Hargreaves, 2000).

There is an increasing number of professional development activities that were co-constructed with input from teachers and designed to be attentive to the needs of teachers. For example, professional learning communities (DuFour, 2004) or communities of practice (Lave & Wenger, 1991) were adopted for creating a context for teachers’ professional development. The aim was to create sustainable and responsive learning opportunities for teachers. Even though the intention was to have teachers’ resources within their immediate access, in the implementation there were still top-down requirements, like professional learning communities with set times, goals, and even sometimes agenda. Giving more autonomy to teachers with the intention of distributing leadership in setting up such groups was noted to be more useful (Kennedy, Deuel, Nelson, & Slavit, 2011). However, the leader still was someone other than teachers.

The concerns about contrived collegiality (Hargreaves, 1994) with imposed structure and goals initiated studies that analyzed teachers' interactions in their workplace. The basic premise in those studies was that teachers could facilitate each other's learning and growth (Little, 2003). Such teacher collaboration is believed to help teachers provide solutions to emerging problems of their practice (Glazer & Hannafin, 2006) and can even initiate "generative" practices (Wilson & Berne, 1999, p. 183). In addition self-initiated collaborative interactions contributed to teachers' professional growth (Sun, Wilhelm, Larson, & Frank, 2014) and can contribute to teacher self-efficacy (Shachar & Shmuelewitz, 1997). Moreover, such structural changes can serve as an indirect reward that may encourage teachers to remain in the profession (Eisner, 2000; Zahorik, 1987).

With the purposes of understanding teachers' collaborative interactions in their workplace groups, researchers took a closer look at self-initiated teacher collaborations. For example, Horn and Little (2010), explored teachers' conversational routines in analyzing the generative nature of teachers' interactions. They noted, how "shared dispositions toward improvement may be necessary but not sufficient for collaboration to yield opportunities for professional learning" (p. 212). Teachers' interactions were complex with multiple factors influencing the nature of teachers' collaborative interactions. These in-depth analyses of teachers' interactions were carried out in large-scale studies as well, with analysis of social networks. In their analysis of teachers' social networks, Coburn and Russell (2008) noted the importance of distributed expertise in a group for in-depth interactions. Since issues around teachers' professional development shifted the concept of professional development from some variations of a singular model into a contextualized phenomenon, research needed to explain

particulars of how collaborations in teacher groups lead to generative professional learning opportunities (Coburn, Mata, & Choi, 2013).

In addition to all these calls for responsive and continuous learning opportunities for teachers, there are also realities of schools providing professional development opportunities. Limited staff development budgets play a factor in infrequent occurrences of high quality professional opportunities for teachers (Little, 1993). These scarcity of school-related resources make self-initiated collaborative interactions a viable means for teachers' professional development.

Purpose Statement and Research Questions

To understand teachers' self-initiated collaborative interactions better, this study aims to describe the nature of mathematics teachers' self-structured and spontaneous collaborations by providing rich descriptions of different forms as well as defining and illustrating productive teacher collaboration with specific examples from teachers' authentic settings. I adopted the definition of collaboration as activities and dialogues that "involve teachers working together" (Hargreaves, 1994, p. 188) in a bilateral interaction (Dillenbourg, 1999) by sharing responsibility, initiation, and leadership in their activities (Little, 1990). With this definition, there needs to be ongoing interaction among teachers where each is responsive to prompts from others but where individual roles and contributions may change over time. This is important research, as Dillenbourg (1999) noted the need for specific attention to types of collaborative interactions in order to better understand and explain the outcomes of those interactions because there is little research describing how teachers form such collaborative groups or how they function and change over time (Coburn, Mata, & Choi, 2013). Knowing more about how teachers work in collaboration with their own initiation might help build such cultures or provide

insight to other teachers of the characteristics of those cultures (Darling-Hammond & Richardson, 2009).

This study posed three research questions aiming to provide a detailed picture of teachers' spontaneous and self-structured collaborative interactions:

1. How do middle school mathematics teachers, using *CMP* materials, collaborate with each other spontaneously or in self-structured ways?
2. How do these collaborations relate to teachers' practices?
3. In what ways are teacher collaboration productive?

Introducing the Upcoming Chapters

In the following chapters, I provide a theoretical framing of the study, methods, results and discussion. Chapter 2 is my synthesis of the framing literature for this study. Before an in-depth review of related literature, I reviewed teacher collaboration literature to compare and contrast different conceptions of teacher collaboration. I provided those different conceptions of teacher collaboration along with other well-known terms describing teacher collaboration. Since this is a study of teacher's self-structured and spontaneous collaborations, I summarized externally structured collaborative activities very briefly and moved into more teacher-initiated collaborative settings. My literature review of teachers' self-initiated collaborations was informed from three major studies. First, I reviewed studies exploring teachers' self-structured professional development activities. Some of these were collaborative activities with a set date and group of teachers, but teachers structured the content of their activities. The second group of studies was teachers' collaborative interactions in their workplaces. For both of those structured and non-structured collaborations, I reviewed the studies to describe how processes of teacher collaboration was attended and described. In addition, I also synthesized how researchers

documented and measured the nature of teachers' interactions. The third set of studies, which overlapped with the first two, was studies analyzing the evaluation of the process and outcomes of teacher collaboration. I summarized the measures used in assessing the quality of teacher' interactions and teachers' professional growth. As I reviewed the relevant literature for the purposes provided above, I also looked at contributing factors in teachers' collaborations as outlined in those studies. I considered curriculum as one of the factors shaping teachers collaborative activities and reviewed related literature providing insights around that. At the end of my literature review section, I provide the theoretical perspective that informed the design and analysis of this study. I will argue for coordination of two perspectives: constructivists (Dewey, 1966) and situative (Putnam & Borko, 2000) perspectives to consider teacher collaboration as a topic of exploration. My data collection and analysis were also informed by those perspectives.

In Chapter 3, I introduce my research questions and describe the design of my study. In that chapter, I provide details about my case and provide rich descriptions of my participants. My descriptions are detailed because, it was important for me to provide a picture of these participants and their setting to help the reader interpret my results. In that chapter, I provide my main data collection tools, data analysis process, and frameworks that I used in my data analysis.

The next four chapters include the results from this study. In chapters 4 and 5, I address my first research questions, in chapter 6 my second research questions and in Chapter 7, I address my third research question. At the beginning of each chapter, I first provide quantitative summary of findings, if any, followed by illustrative examples. In Chapter 4, I describe teacher collaboration by attending to forms of teacher collaboration, factors involved in teachers' collaborations and foci of teacher collaborations. In Chapter 5, I describe the nature of teachers' interactions and how the nature of interactions were related to their foci. In Chapter 6, I

considered the results of how teachers' collaborative interactions related to their individual practices. In chapter 7, I introduce teacher productivity and what data provided in terms of productivity of teacher collaborations.

Chapter 8, is the last chapter where I summarize my results in addressing my research questions. After my summary of results, I consider my results in the broader field of literature and discuss how this study contributes to the existing literature in considering teacher collaborations. I talk about limitations of this study and recommendations at the end.

Lastly, I would like to remind the reader of the interpretive nature of this work. As Geertz (1994), indicating the semiotic nature of studies in understanding a culture, noted "the analysis of it therefore is not an experimental science in search of law but an interpretive one in search of meaning" (p. 214). Therefore, this study was my interpretation of teachers' collaborative culture. Even though I tried to be true to the reality of teachers' spontaneous and self-structured collaborations in this setting, this is my description of teachers' collaborative interactions. The conclusions I drew and all generalizations are limited with the interpretive nature of such work.

CHAPTER 2

LITERATURE REVIEW

The ability to collaborate with peers is now widely recognized as an important teacher competency. In 2002, the National Board for Professional Teaching Standards (NBPTS) highlighted five core propositions in summarizing expected teacher behaviors. The fifth proposition, which was “teachers are members of learning communities”, emphasized teachers working collaboratively to improve student learning, to improve curriculum development, to evaluate school progress, to meet standards, and to learn to engage parents. In *Teaming Up for 21st Century Schools*, the National Commission on Teaching and America’s Future (NCTAF) noted the importance of collaborative work and how teamwork was considered as a key to improve school performance and student achievement (Carrol, Fulton, & Doerr, 2010). In their document, *Model Core Teaching Standards and Learning Progressions for Teachers*, the Council of Chief State School Officers (CCSSO) (2013) noted teacher collaboration as one of the standards for professional responsibility. Since collaboration is widely called for by professional organizations that aim to improve teaching, and since it has been studied as an opportunity for teacher learning (e.g., Sun, Wilhelm, Larson, & Frank, 2014; Horn & Little, 2010), it is important to know what collaboration means, how it looks like and what it involves. In the coming paragraphs, I review teacher collaboration and compare to other related terms used in literature. I continue with synthesis of studies describing the process of teacher collaboration in self-structured and spontaneous forms. After that, I present exemplary ways used in the research literature to evaluate the process and outcomes of teacher collaboration. I will finish this chapter with my summary of theoretical perspectives that shaped the design and analysis of this study.

How has Collaboration and Teacher Collaboration Been Framed?

In the research literature, collaboration is mostly used as an adjective describing the forms of interactive activity involving more than one person, like collaborative dialogue or conversation. Collaborative dialogue has been described as “knowledge building dialogue” (Swain, 2000, p. 97) as well as “mediating joint problem solving” (p. 102). Moreover, collaboration has been used to indicate the self-initiated nature of an activity. This is in contrast to instances when there are roles or positions where one is imposed over others (Dillenbourg, 1999). Roschelle and Teasley (1995) defined collaboration as “a coordinated, synchronous activity that is the result of a continued attempt to construct and maintain shared conception of a problem” (p. 70). They explored students’ collaborative interactions by exploring their *Joint Problem Space* (p. 70). The joint problem space is a place where students negotiate the meaning, consider available problem solving actions and develop actions that will help them solve the problem of interest. Therefore, collaboration was described as an activity involving more than one person, taking an action towards a problem of interest by initiation of one or more people involved in the activity.

Dillenbourg (1999) described collaboration by attending to three types of symmetries. First, *symmetry of action* means each person involved in collaboration is involved in similar activities. According to this principle, a curriculum coordinator without teaching responsibilities may not be considered in a collaborative relationship with teachers in an activity about teaching. *Symmetry of knowledge* means each person brings similar skills and development to the collaboration. This does not mean having identical backgrounds, but it more indicates being able to follow a common goal. Lastly, *symmetry of status* means each person involved in collaboration had similar status in their community (Dillenbourg, 1999, p. 7). Interaction

between an employer and employee is not considered being collaborative because of the differences in status according to this perspective. With this perspective, in a school setting, mentor teacher and intern working together might also not be considered as a collaborative work. Yet Dillenbourg noted room for changes in symmetries over time and he noted subjectivity in the evaluation of these symmetries.

Teacher collaboration has been conceptualized in different but related ways in the literature. Teachers' collegial interactions is often used interchangeably with collaborative interactions indicating teachers' "joint work" with respect to their classroom practices (e.g. Cha and Ham, 2012; Little, 1990; Zahorik, 1987). "Such joint work implies collective action, strong interdependence, shared responsibility, and a great degree of readiness to participate in reflective inquiry into practices (Little, 1990 cited in Cha and Ham, 2012, p. 636). Workplace learning (Hodkinson & Hodkinson, 2004) is used for similar purpose. "Critical colleagueship" is another related term. It involves teachers working together on their teaching practices with specific forms of critical perspective (Lord, 1994). Similarly, "Reciprocal interactions" has been used to describe interactions between teachers who were involved in collective relationship to support their learning and development (Glazer & Hannafin, 2006). All these related terms, captures teachers' informal interactions around their teaching.

Lave and Wenger (1991) considered group of professionals as communities of practice and described learning as a process of participation. Wenger (2000) noted joint enterprise (i.e. a collective understanding of what the community is about), mutuality (i.e. interacting with each other), and shared repertoire (i.e. production of communal resources) as defining elements of communities of practice (p. 229). In their analysis of organizations, Galbraith, Downey and Kates (2002) listed communities of practices as a way to facilitate networks within an

organization. They noted that communities of practices useful if it was based on voluntary participation of members and “formed around topics people care about to which they see some self-interest in contributing (p. 73). Professional learning communities involve teachers who meet regularly and work collaboratively toward shared goals and responsibilities (Richmond & Manokore, 2011).

Teacher collaboration can be observed in each of these specific forms of interactions. However, on one hand, collaboration contains a more general meaning without imposing any kind of framework shaping teachers’ way of interaction. On the other hand, collaboration indicates a more specific form of interaction because it requires the participants to share roles, autonomy, and contributions. Cooperation has been distinguished from collaboration in that the former one involves dividing up the task and completing the task by each person picking up individual pieces where the latter involves doing the whole task together (Dillenbourg, 1999; Roschelle & Teasley, 1995).

Structure of Collaboration

In order to understand and support teacher learning, teacher educators have designed and studied forms or structures that directly engaged teachers in collaborative learning with their peers (e.g. communities of practice, video clubs). In this section, I will discuss collaboration as *structured* and *spontaneous* to distinguish teachers’ interactions in contexts that have been organized in advance from contexts that emerge without any prior arrangements.

Structured Collaboration

Collaboration with a preset structure (introduced to teachers or developed by teachers) is considered as *structured collaboration*. With its dictionary definition structure necessitates an arrangement of a plan of action or an organization (Merriam-Webster Dictionary). An agenda

determined and publicized prior to a meeting, assembling people attending to a meeting, and specifying content in advance of discussion, indicates a preset structure. In these forms, teachers are invited into a form of interaction and activity and supported by a guide or director who may or may not be a classroom teacher. In structured collaboration, forms of activity exist in advance of the teachers' engagement with each other. Structured collaboration has been described in the literature as "structural collaboration" (Williams, Prestage, & Bedward, 2001) or "contrived collegiality" (Hargreaves, 1994) if the structure was imposed from top-down.

Self-structured collaborations. In addition to these externally-structured collaborations another form of structured collaboration could involve a teacher or group of teachers taking substantial roles in giving a structure to collaborative activities. Those are considered as self-structured collaboration. For example, in their professional development sessions, Slavit and Nelson (2010) provided teachers the option to take an active role in giving structure to the meetings they were required to attend. They noted that "the self-directed nature of the work stimulated passion, energy, and buy-in, despite this requirement" (p. 218). This structured collaboration is different from the one described above, because even though the number of meetings, or meeting times were decided beforehand, teachers set their own agenda. In that sense, this collaboration was an example of teachers' self-structured collaboration.

In the research literature many examples of self-structured collaborations were presented with some variations in teachers' collaborative practices. Those were usually initiated or led by a researcher and/or professional development provider. In most of those cases, "routines of interactions" (Coburn, Mata, & Choi, 2013, p.317) were predetermined but teachers decided on the content focus. Lesson Study and Video Clubs are examples of such self-structured collaborations. Lesson Study aims to situate teachers' learning into their own classrooms

through the organized study of teaching practices through careful lesson design (Fernandez & Yoshida, 2004). Teachers choose a research goal to examine a specific aspect of their practice, plan a lesson, teach that lesson, and critically examine the observed lesson by making use of various evidences collected from their observations. Video Clubs use classroom videos of one's own teaching or another teacher's teaching with the purpose of supporting teachers' study of and reflection on their practice (Sherin & van Es, 2005). Videos were found to be powerful in creating a context for a productive discussion on teachers' practices because teachers were not expected to rely on their memories about what happened in their classrooms. Both Lesson Study and Video Clubs were noted as responsive to teachers needs and grounded in their own practices (e.g. Hiebert & Stigler, 2000; Borko, Jacobs, Eiteljorg, & Pittman, 2008).

Another type of self-structured collaboration, the collaborative apprenticeship model, was described by Glazer and Hannafin (2006) as a way for teachers to support their learning. In this model, teachers are expected to go through phases of developing mastery towards certain aspects of teaching by working collaboratively under the leadership of one teacher. Initially, a group of teachers reflect on the experiences of a teacher-leader provides for them. As they think about and learn more about topic, they start designing lessons collaboratively and become teacher leaders. Teachers' interactions during this collaborative apprenticeship model were analyzed through a situated learning perspective with an attention to reciprocal interactions. Reciprocal interactions were defined "as interactions demonstrating and influencing a mutual relationship supporting teacher learning and development" (p. 180). *Storytelling, Backscratching, Discussing and Resolving Conflict, Brainstorming, Giving and Seeking Advice, Modeling, Sharing Ideas, Motivating and Reinforcing, Posing and Responding to Task Based Questions* were noted as ways teachers were interacted in reciprocity in this collaborative apprenticeship model. For

example, when a teacher leader shared her lesson with her peers, those teachers asked questions that helped them adopt the shared lesson into their classroom (*Posing and Responding Task Based Questions*). After teachers taught their lessons, they shared experiences by bringing challenges they experienced in their teaching the lesson (*Discussing and Resolving Conflict*). Teachers were observed to be *Motivating and Reinforcing* when they shared their experiences, expressed their ideas as well as plans for improvement in all phases of this collaborative-apprenticeship model.

Spontaneous Collaboration

In contrast to these different forms of structured collaboration, *spontaneous collaboration* takes place when teachers work together, in response to their needs and without a structure provided by others or by teachers themselves beforehand. In his comparison of collaborative cultures and contrived collaboration, which is top-down structured collaboration, Hargreaves (1994) indicated collaborative cultures to be more spontaneous, voluntary, development oriented, pervasive across time and space, and unpredictable. Schools with primarily contrived collaboration were found to be administratively regulated, compulsory, implementation-oriented, fixed in time and space, and predictable. Teachers are more likely to be involved in divergent type of interactions in schools where there is an established collaborative culture which might affect school success positively in the long term, since there is a correlation between the divergent nature of interactions among teachers and school success (Little, 1982).

Spontaneous collaboration was used by Williams et al (2001) to describe the unplanned nature of activities among pre-service teachers and their mentor teachers during their induction period. This kind of collaboration happened within the natural flow of a teacher's day. The

nature of this collaboration was different from structured collaboration “with its emphasis on the informal, the unplanned and the opportunist” (p. 265). Genuine collaboration (Lam, Yim, & Lam, 2002) and improvisational conversations (Scribner, 2007) are other related terms used in the literature in describing the unplanned nature of collaborations. Spontaneous collaborations are *emergent* (rather than planned in advance) and lack a director (as in structured collaboration). Collaborative engagement unfolds on the basis of participants’ sense of need, timing, and what makes sense to do next. Because teachers must find each other, the size of spontaneous collaboration groups may be small, e.g., dyads. Spontaneous collaboration was very powerful for teachers’ development since they were exposed to ongoing support and development (Williams, et al, 2001, p. 265). Because, teachers has access to affective and practical support when they needed the most (Ben-Peretz & Schonmann, 2000; DiPardo, 1999; Lohman and Woolf, 2001; Williams, et al., 2001). In addition, spontaneous collaboration helped teachers to manage ambiguities associated with teaching as well stimulated change (DiPardo, 1999). Even though, such collaborative interactions did not have any pre-determined structure, there were some commonalities in the way teachers came together.

Teacher talk. Lohman and Woolf (2001) explored self-initiated learning activities in public schools thorough in-depth interviews and multiple school visits. They defined self-initiated learning activities “as experiences that teachers initiated and participated in that, in their perceptions, resulted in the development of their professional knowledge and skills.” (p. 60). Teachers found the practical support when they needed it. They reported sharing experiences or issues they came up with in unplanned moments as positive outcomes of these activities.

Teacher talk was informal and unplanned. Chance encounters during the normal course of the working day occurred in such places as hall corridors or departmental offices.

Teachers used these chance encounters, in the words of one junior-high teacher, to 'bounce ideas off of someone else' right when [a problem is] immediate. [That] has made all the difference in the world. I feel like I'm not making all these decisions [by myself] that are very important to these kids. (p. 65)

Even though teacher talk included pieces of teacher collaboration, Lohman and Woolf distinguished the former from the latter. They considered teacher collaboration more narrowly, to involve teachers who voluntarily participate in the groups formed previously and work with other teachers. Teachers were observed to exchange knowledge during those collaborative activities. Researchers noted reflection as an important feature of teachers' self-initiated learning activities leading to changes in practices. For the purposes of this study, I will consider teacher talk as a form of teacher collaboration because as Greeno (2011) mentioned "constructive processes of understanding and reasoning occur in conversation through joint actions of the participants" (p. 44).

Teacher-talk can serve different purposes like *advice-seeking* (Sun, Wilhelm, Larson and Frank, 2014), *normalizing*, or *transparency* (Horn & Little, 2010). In their analysis of teachers' social networks, Sun et al (2014) noted changes in mathematics teachers' practices and mathematical knowledge as a result of their advice-seeking interactions with more "expert" colleagues. These *advice-seeking* exchanges were unplanned and based on teachers' emergent needs, therefore it was spontaneous. Teachers can assume a "*normalizing*" role in their responses by suggesting that such experiences were normal (Horn & Little, 2010). These normalizing responses can "turn the conversation toward the teaching or away from the teaching as an object of collective attention" (Horn & Little, 2010, p. 192). Teacher talks can also

provide “transparency” of their practice when they made their in-classroom experiences visible to other teachers (Little, 2002, p. 934).

Teachers’ lounge interactions. Ben-Peretz and Schonmann (2000) explored teachers’ interactions in teacher lounges because they considered “the lounge as an embodiment of suitable circumstances for the promotion of teacher collaboration and mutual learning, culminating in the creation of communal knowledge” (p. 131). Teacher lounges were professional space where personal and public identities overlap and provide a room for talking about issues in condensed forms. The content of the conversation was mostly about disciplinary issues or content coverage. These spontaneous sharing sessions during break time helped teachers deal with their emotions at the times when they had the most need. However, those issues were rarely taken up further by teachers. “Catharsis” was the term they used to explain teachers’ emotional share-outs as was in theatrical acts and they argued that “through the process of collective catharsis teachers give meaning to their work by relieving the tensions and/or by looking for partners to share in the triumph and despair connected to their work experience” (p. 51).

Similarly, Hammersley (1984) observed teachers in the staffrooms. He noted the dominance of conversation about specific students or events that happened among students. By hearing other teachers talking about similar students or events in the same way, the staffroom conversations served a way “to protect the teachers’ professional identities in the face of the threat to their sense of their own competence posed by the behavior of the pupils” (p. 212). These teachers’ lounge interactions were similar to teacher-talks, Lohman and Woolf (2001) described above. Teacher lounges were reserved spaces for teachers only and at the same time an arena for teacher talk. Therefore, these reserved spaces automatically called for teacher talk.

Teaching partners. DiPardo (1999) explored teacher collaboration in four purposefully selected settings. Even though there were many teachers in the school and collaboration was happening in different forms in different groups of teachers, the focus was on two teachers for each site. Spontaneous collaboration took place between two teachers. DiPardo described the collaboration she observed as productive in some ways as well as dynamic and varied (p. 156). She related those collaborations to a flowing stream:

Different streams hold certain things in common, but they are also shape-shifting and diverse, varied in terms of depth, width, speed, clarity, purity, and direction. Their course is shaped by the preexisting landscape, but streams can also chisel away at their surroundings, sweeping in new elements, overflowing their banks, transforming the context in large and small ways –usually slowly, starting as slow trickles, gaining force and direction over time. They are also remarkably fragile, vulnerable to a host of dangers, forces that can change the chemistry and direction in sometimes catastrophic ways. (P. 156-157).

In this metaphor, she considered school as a landscape for flowing stream with the humans present in the setting as contributing to the stream. She noted the inevitable presence of countervailing forces for every stream and mentioned how such forces may change from school to school, e.g. administrators, parents or other teachers may be countervailing forces. As true for any flowing stream, DiPardo noted the particularity in any collaboration.

To sum up, spontaneous collaboration was an integral part of some teachers' daily practices that emerged and responded to teachers' immediate needs. Spontaneous collaboration might be as simple as teachers talking to each other, or it might involve teachers working on a specific activity like solving a problem or planning a lesson. The specificity in the nature of

interactions and practices for these unplanned interactions was highlighted in many of the studies but still considered as useful information, since it provides an insight into lives of teachers who find support in their own settings in the moments they needed the most.

Describing the Process of Teacher Collaboration

The process of teacher collaboration was described by attending to teachers' level of engagement or the nature of interactions. In this first one, the focus was on the individual teacher whereas in the second one the focus shifted to interactions. In describing the process, the content focus of teachers' collaborative interactions was also attended.

Collaborative Process Described by Attending to Individuals' Involvement

Teachers' level of engagement in their collaborative interactions was described on a hierarchy changing from minimal involvement to increased contribution. For example, Davison (2006), described five levels of teachers' engagement where he analyzed collaboration among English-as-a-Second-Language teachers and content area teachers during their collaborative work as partners. Teachers who displayed discomfort for collaborating with other teachers and preferred their old way of working were considered at *Pseudocompliance or Passive Resistance* level. At the *Compliance* level, teachers' attitudes became more positive about collaboration but they acted with a limited understanding of collaboration and kept showing signs of frustration and discomfort. Third level was *Accommodation* level where teachers started conveying their understanding of collaborative work into their practices. At the *Convergence* level, teachers aligned their practices and beliefs. The last level was *Creative Co-Construction* where teacher collaboration turned into a seamless relationship and teachers became more creative in their collaborative work. First two levels hints contrived collegiality (Hargreaves, 1994), where last three levels can inform assessment of effectiveness of teacher initiated collaborations.

Building on Little's (1990) work, Cousins, Ross and Maynes (1994) described teachers' collaborative work on a continuous scale starting from storytelling to joint work. The first level was *Information Exchange*, mainly discussions on the hallway not going further than sharing ideas or materials and it was characterized as a one way interaction. *Joint Planning and Participation* was the second level involving teachers working on a project or material together like coursework production. Researchers noted out of 72 interviews teachers noted the most common form of collaboration as in some ways exchange of information and joint planning. *Concurrent Implementation* included sessions where teachers took roles for the different parts of the same lesson, e.g. a joint planning beforehand and at the end teachers share their feedback about their students. *Joint Implementation* was sharing a common teaching time as co-teaching or observing. Joint implementation type of interaction got the least frequency in describing teachers' collaborative work. Further studies exploring teachers' collaborations in their workplaces reported similar findings where teachers were mostly involved in sharing-type activities like sharing ideas, or materials (e.g. Kwakman, 2003). Still, Kwakman noted the contextual nature of teachers' activities because, even though teachers' collaborative interactions were described in simplistic ways further analysis of interactions accounted for differences.

Collaborative Process Described by Attending to Nature of Interactions

Coburn and Russell (2008) used social capital approach in their analysis of teachers' professional relations. They attended to structure of ties, access to expertise, trust and the content of interactions as the sources of social capital (p. 204). Later, these social capital resources were narrowed down to three as tie strength, depth of interactions and access to expertise (Coburn, Russell, Kaufman, & Stein, 2012). A network was considered with "strong ties if teachers, on average, interacted more than twice a month" (p. 147). Tie strength was

important because social networks with strong ties allowed participants to interact around complex tasks whereas the participants in networks with weak ties interacted mainly for the dissemination of information (Coburn and Russell, 2008). Teachers' expertise was assessed on a three-point scale with high expertise represented teachers with four or more intensive professional development experiences (or in combination with a math major). Moderate expertise represented teachers with two or three intensive professional development experiences (or in combination with a math major) (Coburn, Russell, Kaufman, & Stein, 2012, p. 172).

The depth of interactions was assessed on a three-point scale where the low depth represented discussion of issues at the surface level without providing any rationale. In medium depth interactions teachers brought up their reasons into their conversations and high depth interactions involved theory based conversations like conversations around "pedagogical principles underlying instructional approaches, or how students learn" (p. 173). Such an approach by attending to the depth of interactions in describing teachers' colloquial interactions provided a unique approach in analysis of teachers' collaborative interactions, because the previous studies exploring teachers' collaborative interactions attended less to mutual engagement but more from the point of individual teachers' contributions. Coburn et al considered the presence of all these three resources, high depth interactions, access to high expertise and a network with strong ties, as synergy leading to sustained high quality instruction (p. 160).

Collaborative Process Described by Attending to Content Focus

There is a scarcity of research in describing content focus of teachers' collaborative interactions beyond using general descriptors like students or planning. Coburn, Mata and Choi (2013) speak to that concern when they noted how "most social network research tends to be

structuralist, explaining various network processes and outcomes in terms of network structure” (p. 331). The majority of teachers’ spontaneous interactions were focused on student related issues (e.g. Horn & Kane, 2015; Ben-Peretz & Schonmann, 2000; Zahorik, 1987, Hammersley, 1984). Horn and Kane (2015) noted how teachers brought up their experiences and perspective on students into their conversations. In an interview with 52 teachers exploring collegial interactions, teachers reported student-related disciplinary issues, and students’ learning problems as topics that dominated their conversations or prompted them to ask for help from other teachers (Zahorik, 1987). Ben-Peretz and Schonmann (2000) noted how teachers carried students’ disciplinary issues into staff rooms. In addition, teachers’ used student work during their collaborative inquiries, however students’ work provided a context for teachers to talk about instructional methods and teaching or used as a tool to understand their students’ thinking (e.g. Richmond & Manokore, 2011; Slavit and Nelson, 2010).

Some studies have explored teachers’ work around specific curriculum materials (e.g. Doerr & Chandler-Olcott, 2009; McDuffie & Mather, 2009). In these studies the content of the collaboration predominantly concerned teaching and learning the content as provided in the materials. For example, McDuffie and Mather (2009) created a professional development team among teachers’ using Connected Mathematics Project (CMP) (Lappan, Fey, Fitzgerald, Friel, & Philips, 2006) and they facilitated teachers’ work in “cycles of planning, implementing and reflecting on instruction” (p. 306). They found that teachers were involved in four instructional activities. Teachers analyzed curriculum materials from learners’ perspective, completed the main tasks as learners, mapped learning trajectories for their students and revised their plans based on work with their students. The content of these collaborative instructional activities were

mathematics and teaching strategies blended with their students' learning needs and state standards. McDuffie and Mather called this combination of reasoning as *Curricular Reasoning*.

Factors Contributing to Collaboration

Teachers' collaborative interactions are due interaction of many interfering factors (e.g., Di Pardo, 1999; Kwakman, 2003; Hodkinson and Hodkinson, 2004). Even though the research evidence on factors contributing to teacher collaboration is scarce (Cha & Ham, 2012; Gumus, Bulut, & Bellibas, 2013), some studies suggest different factors that influence teacher collaboration, evidenced mostly through teacher self-report (survey or interviews) data. In addition there were some long term studies observing teachers working together. I discuss factors contributing to collaboration as *physical*, *practical*, and *personal-interpersonal* factors. This discussion of the factors in these three categories resembles with "personal factors, task factors and work environment factors" suggested by Kwakman (2003), yet there are some differences. The focus is on the context in the first one, whereas in the latter case the focus is on the individual.

Physical Factors

Physical factors include tangible elements in teachers' professional lives like physical set up of the schools. Teachers reported they interacted more with their peers if their classrooms were located in close proximity to each other (e.g., Coburn, Mata, & Choi, 2013; Coburn & Russell, 2008; Lohman & Woolf, 2001). In their analysis of social networks, Coburn and Russell (2008) noted that 25 out of 48 teachers reported that they interacted around teaching mathematics with teachers in their physical proximity. In addition to physical proximity, there were other physical factors like if they shared a common room like a departmental office or lunch area (Lohman & Woolf, 2001; Zahorik, 1987), and if there were readily available

technological tools such as electronic mailing system (Lohman & Woolf, 2001), that contributed to the frequency of teachers' collaborative interactions. Glazer and Hannafin (2006) noted the importance of such physical resources in affecting reciprocity in teachers' interactions.

Practical Factors

Practical factors include concerns in teachers' lives with respect to their practice. Teachers interacted more with teachers who taught at the same grade level (Zahorik, 1987) and the same subject matter (Lohman & Woolf, 2001). In explaining their collaborative interactions, teachers also noted their need for a different perspective in responding to emergent problems (Cha & Ham, 2012). However, they usually looked for a person holding similar roles, like another teacher teaching the same content area or grade. Access to expertise was also a contributing factor in teachers' collaborations (Coburn et al, 2013). Distributed expertise (Shulman & Shulman, 2004) within a group of teachers helped teachers to be more selective in their interactions (Sun et al, 2013). Teachers looked for the other person that would address their needs the best.

Having common time contributed to teachers' collaborations (e.g., Cha & Ham, 2012), yet lack of sufficient time for teachers' individual planning pushed collaborative activities further down in teachers' priority list (DiPardo, 1999, Lohman & Woolf, 2001). One teacher described this situation as follows, "...So, therefore, sad but true, learning from my colleagues or helping my colleagues sometimes has to go by the wayside because we do not have the time to do it" (Lohman & Woolf, 2001, p. 69). However, a clear focus might let teachers to be able to use their available time efficiently in collaboration with their colleagues (DiPardo, 1999).

Curriculum materials in teacher collaboration. In addition to teaching the same content, teaching from the same materials suggested a common ground for teachers'

collaborative interactions (McDuffie & Mather, 2009). Sherin and van Es noted (2009) the differences in teachers' interpretation of the relevant and important content which will require curricular decision making (Remillard, 2005). Remillard noted how "teachers would benefit from opportunities to read and examine a new curriculum with colleagues, making their interpretations and decisions explicit to themselves and others" (p. 239). Furthermore, Bay, Reys and Reys (1999) noted teacher collaboration as a natural way of dealing with issues arising from a change in the curriculum materials and noted teacher collaboration "as the most critical feature in enabling change; indeed, it was the only supporting element mentioned more frequently than administrative support". (p. 505).

Personal-Interpersonal Factors

Personal and interpersonal factors include variables concerning teachers' beliefs, personalities as well as the shared culture among teachers and school culture. Glazer and Hannafin (2006) noted affect, beliefs, cognition, culture and personality as domains affecting reciprocity. School culture, and culture among teachers with a trustworthy relationship were important in building up and sustaining collaborative relationships (e.g., DiPardo, 1999; Lam & Yim, 2002). Coburn, Mata and Choi (2013) related frequent occurrence of teachers' interactions due to *homophily*. It was based on the premise that "people are more likely to make contact with others that are similar to them" (p. 314). This search for likeness came out as an important element in explaining particular groupings of collaborations.

Trust was an important factor in contributing to the frequency of collaborations. Borko (2004) noted how trust was important for teachers' critical dialogues because it helped to "maintain a balance between respecting individual community members and critically analyzing issues in their teaching" (p. 7). Therefore, once trust was established, teachers responded

conflicts and disagreements better by holding similar goals (e.g. Broko, 2004; DiPardo, 1999). Coburn and Russell (2008) noted the importance of prior relationship in establishing trust and they noted how repeated interactions supported the trust in between teachers. Shared professional history was another complimentary factor besides established trust, as Lohman and Woolf (2001) reported teachers chose to work with teachers who they worked for a long time. Hence, teachers' repeated interactions created many other commonalties which became reasons for furthering their interactions.

Some Other Considerations

Kilduff and Brass (2010) noted the embeddedness of interactions. In other words, teachers' interactions were influenced by the forms of interactions took place before. Therefore, the culture of the school also played an important factor in teachers' collaborative interactions. In addition, Coburn, Mata and Choi (2013) noted many district level factors that contributed to teachers' collaborative interactions. Those changed from assigning coaches, as content experts, suggesting teachers' different routines of interactions which allowed them to focus more and teaching and students' learning.

Evaluating Process and Outcomes of Collaboration

Some researchers evaluated teachers' collaborative interactions by attending to the process and outcomes of those interaction (e.g., Sun, Wilhelm, Larson, & Frank, 2014; Penuel, Riel, Krause, & Frank, 2009). Vescio, Ross and Adams (2008) noted how their review of literature on professional learning communities provided evidence on positive association between teachers' students learning based interactions and student achievement. Similarly Bryk, et al (2009) noted that teachers' professional interactions were related with student learning, however there was not many studies directly mapping teachers' professional interactions to

students' learning outcomes. Nevertheless, there were many studies documenting how teachers' collaborative activities supported their professional growth, led to changes in their instructional practices and provided affective and social gains.

Teachers' Professional Growth

Teachers' professional interactions supported their understanding of content and pedagogical skills (Sun, et al, 2014). Sun et al used Mathematical Knowledge for Teaching (MKT) (Ball, Thames and Phelps, 2008) to document teacher learning from their professional activities. Researchers noted that subject matter focused interactions led to increases in teachers' understanding of the content (Borko, 2004, p. 5). In addition, teachers' professional development activities supported their repertoire of teaching and students' understanding by creating a venue for teachers to share their experiences (Lampert, Boerst, & Graziani, 2011). When teachers' shared their classroom experiences they made their teaching visible to other person (Little, 2002). Therefore, teachers' collaborative interactions provided access to different perspectives and other teachers' experiences (Penuel, et al, 2009).

Furthermore, teachers' continuous interactions allowed them to channel their conversations to more targeted and focused interactions (Coburn, Mata, Choi, 2013), provided access to solutions of existing problems in their practice, or helped them identify broader problems (Horn & Kane, 2015; Scribner, Sawyer, Watson, & Myers 2007), so in some ways their interactions nurtured their professional identity by letting them take autonomy in their learning (Little, 2002). These shares also helped teachers to expand their "collective zone of proximal development" by "expanding their sense of what is possible" (Horn & Kane, 2015, p. 376).

In contrast to these, collaboration might lead to affirmation of some undesired practices among teachers (e.g., Horn & Little, 2010; Di Pardo, 1999). Teachers' *normalizing* moves might serve as an incentive for such undesirable practices. Horn and Little distinguished "toward" and "away from" moves in a conversation as prompts for teachers' to discuss further the issue of consideration or can become a conversation stopper with confirmation of teacher's existing practice (p. 192). In order to help teachers in changing their undesirable practices, researchers noted the importance of appropriate level of conflict (e.g. Borko, 2004). Davis and Sumara (2001) noted how teachers' disequilibrium (Piaget, 1970) led to changes at the school level. Ben-Peretz (2002) also noted disequilibrium as important for a change in teachers' practices.

Change in Teachers' Instructional Practice

There was a correlation between teachers in schools with culture of collaboration and change in teachers' practices (e.g., Sun, et al, 2014, Newmann, King, & Youngs, 2000). Graham (2007) unpacked this correlational statement by listing particular characteristics of collaborative cultures. She noted that collective participation, content focus, active learning and coherence were important characteristics of collaborative interactions that would potentially lead to changes in teachers' knowledge and practice.

Affective and Social Gains

Teachers' collaborative interactions also provided some affective and social gains. DiPardo (1999) reported that the collaborations she observed stimulated change among teachers, helped them manage ambiguities, and promoted an ethic of care (p. 158). Stimulating change indicated triggering changes in teachers' practices or attitudes by being exposed to another person's practices or just accepting collaboration as a way of practice. In other words, collaboration was "personally transformative" (p. 158). Sometimes teacher pairs were balanced

in attributes and skills; each had different strengths. One learned to be more vocal and the other to listen and watch. In addition teachers created other resources together as well as brought some sensitive topics to the focus since they were not alone. Teacher collaboration provided a context for teachers to move forward together by considering new ways of dealing with emergent issues and ambiguities. In addition, teachers promoted an ethic of care by building a friendship, and by being willing and available to listen or help.

Focus of the Study

The focus of this study is to (1) explore teachers' self-structured and spontaneous collaborations in the context of their work to teach a specific mathematics curriculum, (2) to provide rich examples specific to content and forms of those collaborations, and (3) analyze the mediating factors and outcomes of those collaborations. This study is important because there is a call for teacher collaboration by many organizations (e.g. NBPTS; NCTAF; CCSSO) and research noted importance of teachers' joint work (e.g. Little, 1990; Slavit & Nelson, 2010) but "we know relatively little about the various forms such activities can take, or their implications for school reform." (Dipardo, 1999, p 3).

In this study, I adopted an inclusive definition of collaboration as activities and dialogues that "involve teachers working together" (Hargreaves, 1994, p. 188) overall in a bilateral interaction (Dillenbourg, 1999) by sharing responsibility, initiation, and leadership in their activities (Little, 1990). It is broader than other definitions discussed above in the sense that it does not limit teacher collaboration to certain forms or activities like dialogue or problem solving and does not necessarily require symmetries of actions, status and knowledge to always be present. It only requires symmetry in leadership and responsibility in the long term. With this

definition there must be an ongoing interaction among teachers where each part is responsive to prompts coming from others, but these roles and contributions may change over time.

Consider an example where one teacher brings a mathematics problem to another teacher and asks for different ways of representing the solution. The other teacher responds to this teachers' question by providing her ways of solving the problem and the teacher uses the problem sharing later to demonstrate other ways to represent solutions to her students. This example is considered collaborative interaction because two teachers are working together with a change in leadership in articulating the topic of conversation and both take responsibility of sharing different solutions methods with each other as well as with their students.

Theoretical Perspective

In this study, I applied two coordinated perspectives on learning to describe and understand teacher collaboration as a form of professional growth. As Cobb noted, "learning should be viewed as both a process of active individual construction and a process of enculturation into the practices of wider society" (1994, p. 13). Sfard (1998) considered these two related metaphors for learning (the acquisition metaphor and the participation metaphor) essential for describing and understanding learning. In the first case, learning is defined as something acquired; knowledge is an individual possession. Yet this possession can bring power and status and may create boundaries among people. As Sfard noted, "if people are valued and segregated according to what they have, the metaphor of intellectual property is more likely to feed rivalry than collaboration" (1998, p.8). The latter brings value to the community and to the activities taking place in that community because it frames teacher learning as "a process of increasing participation in the practice of teaching, and through this participation, a process of becoming knowledgeable in and about teaching" (Adler, 2000, p. 37-38). Knowledgeability

meant “how we act” and it was considered as a function of people involved, time and place and social dynamics and background of all participants present in a specific context (Adler, 2000, p.35).

The situative perspective is useful for studying teacher collaboration, because there is a collective practice and work in the possibility of shared knowledge. Anything involved in that context can provide affordances for different learning opportunities. Teachers develop shared practices as they participate in joint activities (Putnam & Borko, 2000) and this continuous participation carries a culture of collaboration (Sfard, 1998).

In this study, groups of teachers who teach mathematics and (or) seek to understand mathematics made up the community of practice. Teachers’ practices can take different forms depending on the participants and social dynamics among those participants (Greeno, 2011). Some common practices and shared meanings can develop in the group, and as they keep working together, those practices might change. The level of complexity of interactions might transform from conversational contributions and building common patterns of practices to negotiating for different understanding and becoming more generative and productive in joint activities (Greeno, 2011, p. 42). It is change in knowledgeability by increasing the extent of transparency of the resources present in teachers’ joint practices (Adler, 2000, p. 35).

From this perspective the focus is not just on teachers as participants by considering the rest as context but “on processes of an activity system composed of the several individuals present (if there are more than one) along with their material and informational resources” (Greeno, 2011, p. 41). Therefore, I consider curriculum materials as contributing to the processes of teachers’ activities (Remillard, 2005).

In this perspective, social dynamics of interactions among teachers are as important as what is involved. Positioning is one way to explain nature of interactions among teachers. Greeno (2011) introduced two ways of positioning: *systemic* and *semantic*. In this specific context systemic positioning is used to explain how much teachers' initiating, leading or contributing to their joint activities. Semantic positioning is about how much teachers interpret or make their own choices in relation to mathematics and their textbooks.

The acquisition metaphor is also important for this study because as individual participants make up the social whole, individual teacher's characteristics becomes important. Apparently shared knowledge is shaped from individual understandings that have some commonalities and some differences. Therefore, it is important to document what teachers build individually to capture the amount of growth for each teacher as well as to describe the contribution of contexts on teacher's growth. In addition, documenting the transfer of actions or ideas from one context or teacher to another requires some focus on individual teachers.

CHAPTER 3

METHODS

The purpose of this study was to describe the characteristics of collaboration initiated among middle school mathematics teachers with their own motivation using a problem based curriculum. In addition, I aimed to describe how teacher collaboration related to teachers' classroom practices and the possible outcomes of such collaboration. My research questions focused on describing the nature of self-structured and/or spontaneous collaboration among teachers, examining relations to teachers' classroom practice, and evaluating the outcomes of the collaboration. I asked the following research questions in exploring middle school mathematics collaborative activities:

1. How do middle school mathematics teachers, using CMP materials, collaborate with each other spontaneously or in self-structured ways?
 - a. What are the frequency and forms of these collaborations?
 - b. What factors are involved in these collaborations? Specifically, in what ways are mathematics and mathematics curriculum materials involved in these collaborations?
 - c. What are the foci of these collaborations? What are the issues they discuss about and the activities they engage in?
 - d. What is the nature of the interactions among teachers during their collaborative activities?
 - e. In what ways is the nature of interactions related to the topics of collaborative activities?
2. How do these collaborations relate to teachers' practices?

- a. In what ways do the collaborations arise from issues or problems in the teachers' practice?
 - b. In what ways do these collaborations relate to subsequent actions by teachers in the classroom?
 - c. Is there evidence that the ways that teachers collaborate relate to professional actions they take outside of the classroom?
3. In what ways are teacher collaborations productive?

In order to answer these research questions, I conducted a case study of collaboration among middle school mathematics teachers' using the *Connected Mathematics Project* (CMP) curriculum. The case study approach provided a detailed view of a small number of mathematics teachers' collaboration in their school. To deepen and sharpen my characterization of teachers' collaborative activities, I was present in the school for a long period of time (about 23% percent of the school days in one school year) and stayed with my focal teachers almost all the time when I was in the school. The extensive data collection period (see Appendix A) allowed me to capture teachers' spontaneous collaborations during the flow of their daily activities. In the coming sections, I describe my case, the setting and the participants, explain my data collection tools, and talk about my data analysis.

Case Study Setting and Participants

The participants in my case were three teachers teaching grades seven and eight at the Stonebridge Middle School (SMS) located in Stonebridge. SMS was the only school in the district serving grades six, seven, and eight. The school has approximately 450 students who are mostly white, and come from middle-class families. This site fulfilled the necessary conditions for my study because grades seven and eight mathematics teachers working in this school

reported that spontaneous collaboration was a regular part of their daily activities during school time and my pilot work confirmed these teachers' reports. This study aimed to describe teachers' spontaneous and self-structured collaborative activities mediated by a problem-based curriculum. CMP is a problem centered curriculum. This school had maintained a long connection with CMP authors, served as a pilot site for many different editions of this curriculum, and had many teachers who later became influential in illustrating the enactment of CMP lessons. CMP3 was used during data collection.

For this study, permissions were taken from individual teachers and the school principal. The study was not disruptive because the data collection happened during the natural school and classroom activity. However additional time was requested from teachers for interviews and teacher self-reports might have created an extra burden on teachers. A small stipend was provided to teachers for their participation in these additional requests.

Central Participants

The three mathematics teachers who were the central participants of this study provided me consent for my observations and made themselves available for interviews. Observations were scheduled for a long amount of time (See Appendix A) and my observations gave them hardly any personal time so I am very thankful for my participants for letting me to be with them for such a long amount of time.

Kristina was a teacher leader who worked with CMP authors in piloting and revising the CMP curriculum materials. Kayla, a former grade six teacher, had just taken the grades seven and eight mathematics teaching position during the time of data collection. The third teacher, Paul, was in charge of technical education class as well teaching mathematics.

In the coming paragraphs, I described each central participant in detail starting with their educational background, their history with CMP followed with my description of their classroom and classroom routines. Teachers reported that they came together informally to talk about teaching, mathematics, or their students. Teachers' classes were physically located close to each other (see Figure 1), and Kristina reported that the physical arrangement as an advantage in “bouncing off” ideas with each other.

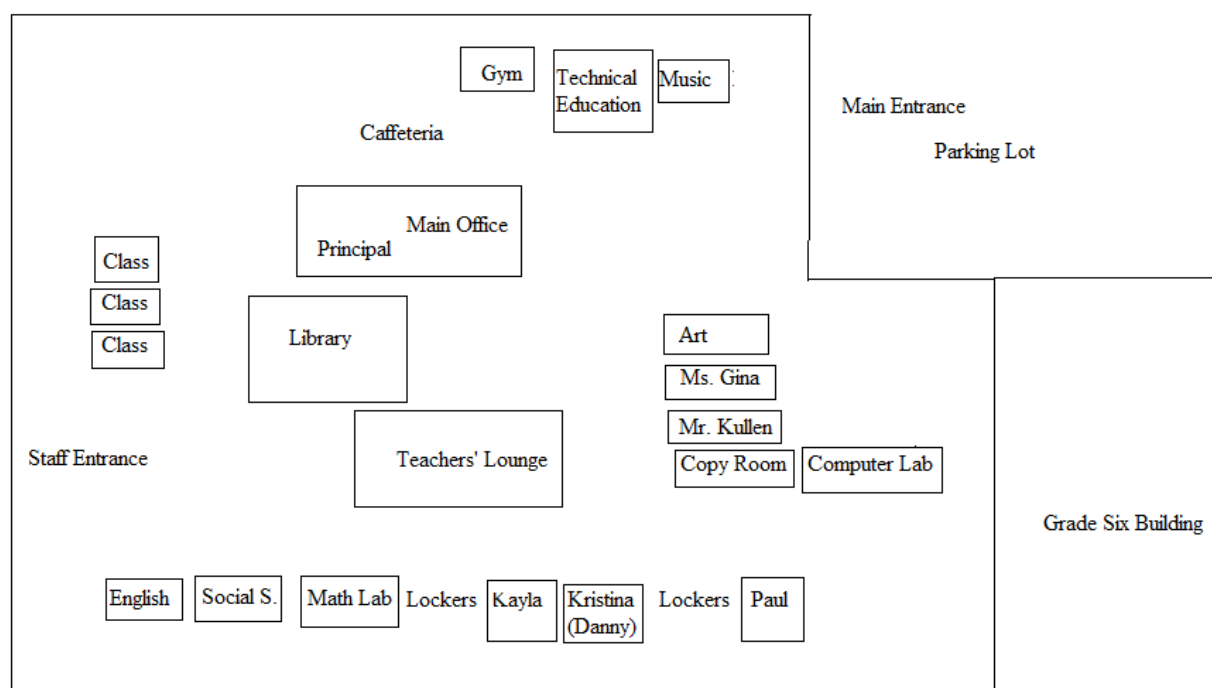


Figure 1

Stonebridge Middle School physical setup

Kristina Gallen. Kristina had worked at the SMS, teaching grades seven and eight mathematics for 13 years. Before SMS, she taught grades seven and nine for two years in a different district in the same state. She had a major in mathematics and Masters in teaching and curriculum. She had established the trust of her peer teachers, as Kayla noted in her interviews. She was also one of the teachers chosen for school improvement team.

Curriculum history. Kristina knew the CMP curriculum and taught it for several years. She had a different relationship with the curriculum and the curriculum authors than the other focal teachers. She was identified as a teacher leader by CMP authors and had worked with CMP authors in the revisions for CMP3. She took active roles in the *CMP Users'* and *Getting to Know* conferences. The *Getting to Know* conferences were held for teachers who were going to teach CMP (or a certain grade level) for the first time. Before Kayla started teaching grades seven and eight that year, she attended *Getting to Know* conferences and Kristina was her session facilitator with another CMP teacher leader.

Classroom. Her classroom contained bulletin boards saved for specific purposes and bookshelves at the back of the class. She had a section for the display of exemplary student work. She used one of the boards for announcement purposes where she displayed students' grades, classroom averages as well as school related announcements like how to respond to bullying. She kept various mathematics books on the bookshelves, including older versions of CMP textbooks and high school algebra books as well as return and pick up trays for homework assignments or exams, and a stack of white paper. There was a birthday section on her board that she updated that section by listing birthday students' names. There were numerous posters in the classroom with different messages; some conveyed messages she valued and wanted to communicate to her students like "Math is Beautiful" or "Being Hard working and Honest is Important."

Routines. Kristina usually stood by her desk during *Launch* (introducing the problem) and *Summarize* (summarizing the problem) segments of her mathematics lessons, and she moved through the classroom during the *Explore* (students explore the problem in groups or individually) segment, either watching students or talking with them. Sometimes she worked

with them to help if they were stuck on something and could not figure out in their groups. She usually used document camera and asked students to share their work during the *Summarize* segment. If Danny (her student intern) was teaching, she either sat at Danny's desk or circulated around the room to observe students. At the end of the year, she spent more time outside of the classroom as Danny assumed more of a full-time teacher's role.

Kayla Illingworth. Kayla had eight years of teaching experience. She taught grade seven mathematics and grades six and eight English and Language Arts (ELA) for four years in another school. At the SMS, she taught grade six mathematics and ELA for three years. The year of the study was her first teaching grades eight mathematics. It was also her first year teaching only mathematics. She considered this new role as a great opportunity. She was certified to teach the elementary grades with an elementary major in mathematics; she also held a Master's degree in teaching and curriculum. She consistently kept a to-do list of school-related items and went through that list before or after school or during her planning hour. She came to school early and left late, so that she could, if possible, get everything done within the boundaries of the school day. She expressed her desire to keep her personal and professional lives separate.

Curriculum history. Kayla had only used CMP materials in her years of mathematics teaching. She indicated she did not know how it might be teaching mathematics with another curriculum. She attended to the *Getting to Know* conference for three times, before teaching grades six, seven, and eight. During those conferences, Ms. Jansen facilitated her grade seven workshop and Kristina facilitated her grade eight workshop. She also attended and made presentations in the *CMP User's* conference the previous year and was planning to do so again in the summer after the school year of the study.

Classroom. Similar to Kristina, Kayla's classroom also contained boards saved for specific purposes and bookshelves at end of the class. She wrote her plans for the whole week down on the white board on the left side of the room from the door. She assigned team numbers to each group, and sometimes she called on tables of students by their numbers during class discussions. She had unit vocabulary charts on the right hand corner for both grade seven and eight classes. When she introduced new terms, she added those to the vocabulary list. On the white board facing to the class, the right hand corner was used to list absent student work. Students knew where they could find the work they missed from the previous classes. She also used that corner for communication purposes (e.g., if she needed to see a student or she found a missing student book or student work without a name on it). The bookshelves at the back of the class had return and pick-up trays, and materials for classroom use (e.g., paper and scissors). The Common Core mathematical practices were listed out on the walls, but I did not see her refer to them. She told me the posters on the wall were Ms. Jansen's.

Routines. Also similar to Kristina, Kayla used *Launch-Explore-Summarize* instructional model associated with CMP. But somewhat differently than Kristina, she had her Teachers' Guide book open before and during the class. As part of her preparation before her classes, she read almost every line in the Teachers' Guide, most of the time highlighted the questions and problems she wanted to use from the list of the recommended questions in the book. She used the focus questions provided in the Teachers' Guide in the *Launch* and *Summarize* segments of her lessons to inform students about lesson objectives and to focus discussions. Also different from Kristina, Kayla used power point slides to organize her class hour and a timer during the *Explore*. During *Explore* time she followed a routine of five minutes of individual work time and about 10 minutes of group time. When the alarm sounded at the end of five minutes, she

asked her students to work in their groups. Similar to Kristina, she talked with students, she listened to their conversations, and sometimes worked with them. She also used document camera and asked students to share their work during *Summarize*.

Paul Earley. Paul graduated with an Industrial Arts degree in wood shop and a minor in mathematics. It was his 24th year of teaching and his 20th year at the SMS. Before SMS, he taught wood shop in a different district for four years. In his first years of teaching seventh and eighth grades at SMS, he only taught wood shop but later he began teaching mathematics as well. He identified himself as being a math and technical education teacher. He indicated to me that he loved teaching at middle school.

During the year of the study, Paul taught two sections of grade seven mathematics and grades seven and eight technical education classes and did so in two different classrooms. He spent the first two hours and the last hour in his mathematics class and during the time in between those hours he was in the other end of the corridor where his technical education classroom was located (See Figure 1).

Curriculum history. During his time at SMS, Paul taught mathematics exclusively with CMP materials. He was aware that CMP presented mathematics differently from more traditional textbooks. He even mentioned that to his students once during my observations. He told them things were not like what they studied in CMP when he was a student. His teachers, he explained, provided formulas and numbers to plug into them. He told them when a story problem came up they all stumbled upon because in order to solve a story problem they needed to understand what was going on. He added, “I learnt once I started teaching this book. I had the college courses and this book. Even a lot of professors do not get why things are like as they are” (Field notes, 03.21.14). As his story indicates, he was aware that he had to teach

mathematics differently from how he learned the subject. Neither Kristina nor Kayla told a similar story. Paul did not attend to any of the CMP conferences.

Classroom. Paul's classroom was more "spare" than Kayla's and Kristina's; there were fewer items displayed on the walls and his boards, and his bookshelves held fewer items than his peers. He had a couple charts with math symbols on the walls and two posters—the one on the classroom door said, "Listen and Silent are spelled the same," and the one in the teacher corner said, "Determine what is right instead of who is right." His board was usually clean; he did not list the topic or the goals of his lessons. His document camera and his computer were located in the back table labeled as Paul's desk. His bookshelves were largely empty except for some copies of books for his technical education class. He did not keep a return tray for student work or a section of the board for absent students. On the materials table, he had angle rulers, rulers in a box, scissors in a separate box, and overhead projector, a box of Kleenex and books, and stack of copies of graph papers and worksheets. On the left hand corner of the board there was a happy birthday section, but I never saw any names written there.

Routines. Paul had all his materials on this materials table which was at the front of the classroom, in the direction where his students would be facing. This materials table was the place where he usually stood when he was teaching. He did not use organization aids, like PowerPoints or a timer, as Kayla did. I did not observe him using his document camera or computer during my observations of his lessons. Though he reported that he liked teaching from CMP materials and valued the content provided in the textbook, he substantially adjusted CMP's instructional model. Numerous times he assigned *Explore* questions as homework because he was behind Kayla and Kristina. There were also times when he rushed through his *Summary* because students used too much time during *Explore* segments. In his *Summaries* he asked students to

share their work like Kayla and Kristina did, but he provided more of his own explanation and commentary than they did to emphasize key mathematical ideas (e.g., that area changes by scale factors times scale factor in similar figures).

Peripheral Participants

In addition to three main participants, some other participants were influential to some degree in the professional lives of these teachers. I called them as peripheral participants of this study. There were three teachers, one professional development provider, one intern and two administrators. I will call them with their last names with one exception in order to show their peripheral participant status to this study. The intern, Danny, had a different status. Even though he was not one of the main participants of this study and because his status determined his roles and responsibilities, he was physically present and actively participating to these teachers' collaborative interactions. There will be times I will call his name in conversation exchanges I quoted from these teachers. Because of that I will call him with his first name. Figure 2 below is

a representation of these other participants' interactions with the three main participants of this study.

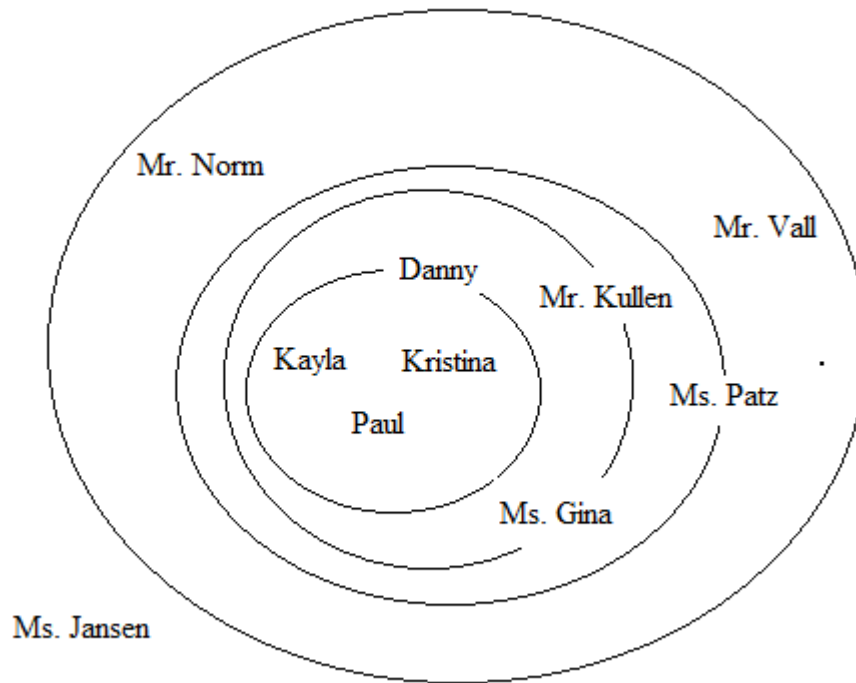


Figure 2

Peripheral participants

The circles represent the physical boundaries in combination with my three central participants' interaction routines. For example, Ms. Jansen is placed outside of the circle because she was not physically present in the school and even though she was influential and had a sustained relationship with teachers (mostly with Kristina via email or phones) she did not have face-to-face interaction with them during school time.

Danny. He was an intern in Kristina's class. By mid-November (as I started my observations) he started taking a lead teacher role. He was initially in charge of one grade eight class and a couple of weeks later grade eight he shifted to two grade eight classes. In the spring semester starting from February, Danny was the lead teacher for grades seven and eight classes

for about seven weeks. Once Danny took over grades seven and eight regular mathematics classes, Kristina was left with only advanced algebra class. Danny was physically present most of time during these teachers' interactions. He participated into the conversations or expressed his view. When Danny was the lead teacher Kayla had quick checking in conversations with him like which problem he was at or whether to spend one day or two days on a problem. During their meetings where they discussed upcoming assessments Kristina preferred staying silent and had Kayla and Danny to discuss about their preference for point distributions. Kristina shared what they did the previous years if Kayla or Danny asked her.

Mr. Kullen and Ms. Gina. They were the two science teachers in physical proximity to these teachers. Mr. Kullen's classroom was located across Kristina and Kayla's classrooms and Ms. Gina's classroom was right next to Mr. Kullen's (see Figure 1). During break times as Mr. Kullen was outside waiting for his students, it was very natural to all three central participants to include him into their conversations or he might have included himself by making a comment on the topic these teachers were talking about. There were many times main teachers and Mr. Kullen formed a big circle of collaboration. Mr. Kullen was also an alternative "go to" person for Paul. For example, Paul visited Mr. Kullen's classroom when he needed stop watches for one of the problems. Even though Ms. Gina's classroom not as close as Mr. Kullen's class, she was a close friend of Kristina so there were many times these two teachers visited each other's classrooms.

Ms. Patz. Ms. Patz was a former mathematics teacher at the SMS and took various administrator roles in the district. Kristina and Ms. Patz knew each other before Kristina started working at the SMS. Kristina told that Ms. Patz informed her about the opening at the SMS and recommended her for the position. Therefore Kristina and Ms. Patz, had a long personal history.

Both Kayla and Kristina respected Ms. Patz and respected her as a mathematician and a teacher. They agreed to work on a project on formative assessment with her for that year. Therefore even though she was not longer working at the SMS, she was in a regular interaction with Kayla and Kristina. Kayla and Kristina taught the lessons Ms. Patz shared with them. After they taught the lessons, they debriefed the lessons with Ms. Patz. In addition, Ms. Patz was the person Kayla and Kristina brought up some school related issues because of her administrator background.

Mr. Norm and Mr. Vall. Both Mr. Norm and Mr. Vall were new administrators. Mr. Norm was the principal and Mr. Vall was the assistant principal. Mr. Norm was a special education teacher before stepping into this administration position. He was new to the school, new to the teachers, and new to being a principal. Even though Mr. Vall was new to SMS, teachers knew him from high school. He was working at the Stonebridge high school the previous year, so the teachers had familiarity with him but that was his first time as SMS assistant principal. He was part time in the middle school and part time in the high school. He spent the first half of the day at the SMS and second half in the high school. Teachers found that problematic in building up a relationship with him both for students and teachers, because they did not see him often.

Ms. Jansen. There was another teacher (Brenda Jansen) in the school the year before data collection period who had used CMP materials and had worked with CMP authors since the start of the CMP project. Ms. Jansen stepped down from teaching to accept a temporary position to work with the CMP authors. There will be times, these teachers will mention her with her first name so I also included her first name here. Ms. Jansen was an important character in these teachers' lives because she was a former teacher at the SMS. Kayla was using Ms. Jansen's class which was decorated with posters reflecting Ms. Jansen's view of teaching and

mathematics. Even though she was not present in the building, there were times Ms. Jansen's voice was heard in their conversations. Sometimes, Kristina shared her excitement of some of her exemplary student work with Ms. Jansen in emails.

Data Collection

There were three main sources of data for this study: Interviews, field observations, and teacher logs. The questions raised in this study required an extensive data collection period that allowed me to richly describe the case of teacher collaboration. The data collection period lasted about a semester and provided a good sampling of collaborations happening in these teachers' activities. I started my observation in November and ended in mid-April. That captured 23% of the school year. During that time, I conducted semi-structured interviews, I took field notes of teachers' spontaneous and self-structured meetings and their classroom teaching. I asked teachers to keep weekly logs keeping track of their collaborative activities. Before, data collection period, I visited school for pilot observations (see Appendix A). Those short observations allowed me to modify my observation protocols.

Interviews

I conducted two an hour-long semi-structured interviews with each teacher both at the beginning and at the end of the data collection period, and an hour long group interview (see Appendix B and C for interview protocols) with the presence of all three teachers. All interviews were audio-taped. In those interviews, I aimed to learn about several things: the frequency and form of collaborative activities, teachers' roles in those collaborative activities, and teachers' view of description of collaborative activities. The final individual interviews included questions asking teachers' perspective on their collaboration (compared to what they noted in the initial interview) and their evaluation of their collaborations with respect to their

practice, and mathematics. The group interview took place after data collection period to eliminate its potential effect on the teachers' collaboration. In this interview, I provided specific examples from teachers' collaboration and I questioned their decisions over what they worked on (or talked about) and why they worked on (or talked about). Interview data was mainly used to validate or elaborate my observations.

In addition to these interviews, I did an interview with the school principal. In this interview, I asked questions about the collaborative culture in the school, frequency and forms of regular teacher meetings, and teachers' involvement in decision-making in planning these meetings. I also had unstructured conversations with a former CMP teacher, Ms. Jansen about these teachers and the culture of collaboration in this school setting.

Observations

I conducted regular field observations as a complete observer, that is, I did my observations with minimal participation (Creswell, 2002). This allowed me to record information as it was revealed (Creswell, 2002, p. 186) and I was able to capture things that did not come out in teacher reports either via interviews or written notes. I observed mathematics lessons taught by each participant as well as their collaborative activities with their teaching peers. I focused on one teacher at a time and I changed my focal teacher on a weekly basis. The observations took place during a regular school day, so was not devoted entirely to the teachers' teaching. Because of this, I was able to capture spontaneous collaborations in those time intervals between classes. I took my field observation notes in a semi-structured way (See Appendix D and E for protocols).

Each week I had one focus teacher and I stayed with that teacher the whole week. I rotated my focus teacher weekly. I was usually at the school three consecutive days like

Monday, Tuesday, and Wednesday. I adjusted my observation days to get a good sampling of days of the week.

Weekly Teacher Logs

In order to capture teacher collaborations as fully as possible, I asked teachers to compose short weekly logs (See Appendix F for prompts). I provided prompts asking teachers to note who they interacted and for what purpose, and brief description of their collaborative activity. In addition, I asked teachers to provide weekly brief reflective write-ups in relating their collaborations to their classroom practices with examples from their practices. Teachers submitted these logs on a bi-weekly basis as a hard copy. Teachers log entries were short and mostly included a short list of their activities. Teachers did not provide reflective write-ups on a regular basis.

Data Analysis

I used my observation data as my main source of analysis and considered interview data and teacher -logs for triangulation purposes, to support and/or validate my claims. For the purpose of addressing my research questions the best, I changed my method of analysis, I either adapted frameworks already provided in the literature or used a grounded theory approach with open and axial coding techniques (Glaser & Strauss, 1967) and following with constant comparative analysis (Miles & Huberman, 1994). Grounded theory approach allowed me to organize my data according to emerging themes and I refined those themes by going back to existing literature and different sources of data.

My field notes was the main source of data. I tried to capture as much of the content and wording of teachers' conversations as possible but I would like to acknowledge that the issue of not being able to capture everything during teachers collaborative interactions or not having

verbatim transcripts of what they said was one of the limitations of this study. All taped interviews were transcribed in verbatim transcripts. I prepared a set of summary of descriptive statements for each recorded interview. I recorded those summary statements to a different page in the same blog. Teachers' statements in their logs were also carried to the blog for later use. Once the digital transfer of data was completed, I used NVivo 7 to analyze the data.

I counted any instance of interaction among these teachers either related with their teaching or not, as collaborative interaction because, these teachers' personal relationships or other school related issues contributed in building up a relationship. One of the assumptions in this study was the importance of context and culture in shaping up relationships and if I did not count non-academic interactions, it might have been not fair to the data in describing why particular pairings of teachers appeared more for particular purposes.

In summarizing the frequencies of collaborations among these teachers, I counted all the instances the teachers interacted with each other in a continuous manner. In these counts, I did not consider the length or variation in the content. For example, Kayla and Paul had a brief conversation starting from 9:46 am to 10:00 am and I counted that as one for Kayla-Paul pairing. Similarly if Kayla and Kristina came together at 2:30 pm and worked together till 3:20 pm, I counted that as one too even though the length of the latter one was significantly longer than the first one.

My unit of analysis was changed for different purposes. Different from frequencies and forms of interactions, in my other analysis like for the foci of interactions or depth of interactions, I used a slice of data with a particular focus. For example Kayla and Kristina was involved in a conversation from 2:32 pm to 2:39 pm (see Figure 3).

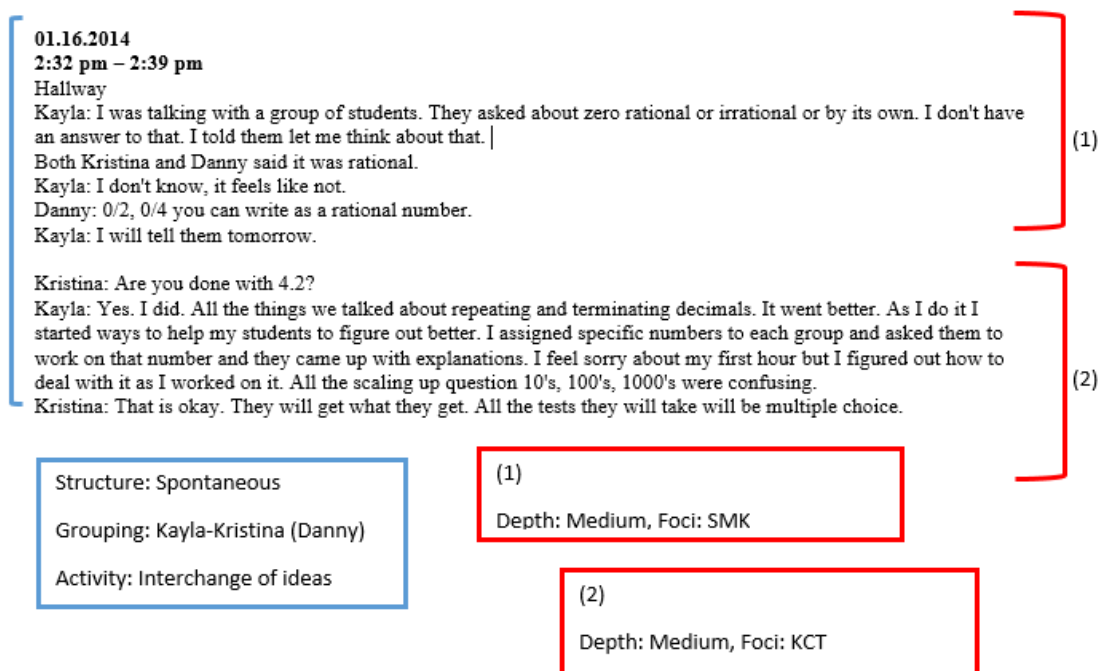


Figure 3

Coding example showing different units of analysis

This single interaction, in the hallway from 2:32 pm to 2:39 pm, provided me different units of analysis for different purposes. I considered this whole interaction when I coded for the forms of interactions, however when I reviewed the data for Depth of Interactions and Foci of Interaction I considered this interaction in two different sets. I will provide more explanation in the analytical frameworks section to explain how forms, depth and foci were operationalized.

Addressing First Research Question

My first question asking how middle school mathematics teachers, using CMP materials, collaborate with each other spontaneously or in self-structured ways was composed of five component questions. In those component questions, I examined the frequency and forms (research question 1a), factors involved in (research question 1b), foci (research question 1c), and the nature of teachers' collaborative interactions (research question 1d). In addition, I considered teachers' interactions in relation to the topics of their collaborative activities (research question 1e). In describing foci of teachers' collaborative interactions, I considered content focus and used Mathematical Knowledge for Teaching (MKT) framework (Ball, Thames, & Phelps, 2008). For the nature of interactions, I used the Depth of Interactions framework (Coburn & Russell, 2008) and Social Dynamics of Interactions framework (Greeno, 2011). This first research question and my response was descriptive in nature and provided me an insight to understand and characterize collaborative relationships in a school setting among mathematics teachers.

Addressing research question 1a. In this question I sought to answer forms and frequencies of teacher collaboration. *Forms of collaborations* were conceptualized by attending to particular *Groupings* of teachers in their collaborative activities, *Structure* and *Prevalent Activities* that took place during the time of collaborations. Prior research has indicated that teachers work together in structured forms where structure was imposed on them (Hargreaves, 1994) or develop structures like deciding on the content they will study (e.g., Slavit & Nelson, 2010). I called the first *Externally-Structured* and the latter *Self-Structured*. Teachers also collaborated in *Spontaneous* forms without having a preset structure or shared agenda (e.g., Ben Peretz & Schonmann, 2000 ; Williams, Prestage, & Bedwards, 2001). Since the focus of this

study was to characterize teachers collaborative interactions with their own initiation, I did not observe or include externally structured collaborative interactions in the data analysis except from counting the ones teachers noted either during interviews or our informal conversations. These different structures appeared in dyads or triads where number of teachers collaborating varied. I referred to those as *Groupings*, below and in Results.

In synthesizing *Prevalent Activities* characterizing teachers' collaborative interactions, I considered how other researchers described teachers' collaborative activities. Even though my data carried some of the similar characteristics some other researchers described (e.g., Little, 1990; Glazer & Hannafin, 2006), those descriptions were only used to inform my analysis. That was because the names carry meaning and the meanings associated with the themes as they were introduced in other studies, did not correspond perfectly to what I observed in my data. My data was clustered around four activities: *Checking-In*, *Interchange of Ideas*, *During Class Time*, and *Joint Work*.

Checking- In (CIN), included activities where one teacher seek out another teacher to pose a specific set of questions or for a simple request and response that did not require deep thinking or take too long time. The other teacher responded to that request without questioning the reasons for a request. It involved activities like sharing materials, providing a written list of practice questions for students, sharing copy of an assessment or lesson plan, or simple question(s) and answer(s).

In the *Interchange of Ideas (IOI)*, all teachers involved in the interaction contributed to the conversation by asking questions or making comments beyond providing simple responses. In the IOI example below, Kristina and Kayla were talking about assigning homework to students:

Kristina: I was very selective in assigning homework for them. I assigned them 24a and 26a and d. I wanted them to work more on intersections because they need to practice that.

Kayla: I assign homework's for more myself them. As Ms. Watrich said [a former CMP teacher] "it is more for me than for them". I can't grade them all. Basically assign two homework each week. One is to see where they are and another to give them more practice. I can't assign more. (Field Notes, 04.04.2014).

This interaction was an example of IOI because both Kristina and Kayla informed each other why and how they assigned homework to their students. Interactions similar to this one, with a contribution from all the teachers, were counted as IOI.

The difference between CIN and IOI is that in CIN one part is simply addressing the other parts' questions without elaborating on more or adding new dimensions to an already existing conversation whereas in IOI both parts contribute in addition to addressing the raised conversations and can see examples and perspectives from all the parts involved in the conversation.

During Class Time (DCT) activities represented teacher activities that took place in one of the teacher's classes during class time. It might be co-teaching the same class or observing each other's class. *Joint Work (JTW)* described teacher activities where teachers worked on something tangible. It could be either working from a worksheet, or a test, or it might involve referrals to specific pages in a book. Assessment preparation meetings or teachers working on a mathematics problem in their book were considered as JTW. See Figure 4 below summarizing *Structure*, specific *Groupings*, *Prevalent Activities* in describing *Forms* of collaborative activities.

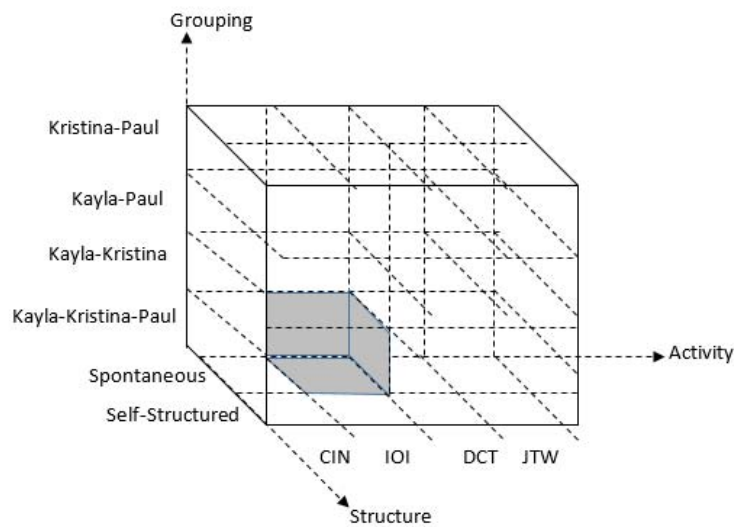


Figure 4

Forms of collaborations along with three dimensions

For example, the cell highlighted in gray represented a *Spontaneous Triad* collaboration among Kayla, Kristina and Paul with *Interchange of Ideas*. In the results section, I presented the distribution of different *Forms* illustrated with exemplary cases for each form.

Addressing research question 1b. In this question I asked factors involved in teachers' collaborative interactions. My factors analysis involved three phases. The first phase of my analysis served as a memo where I reviewed data (field notes, teacher log entries and

interviews) and highlighted possible features of the context possibly contributing or limiting teacher collaboration. In this initial review, I looked for evidence of teachers' personal value statements, like "this is important for me", "I care very much about..." or "I respect knowledge and experience" or expression of feelings like "I like how we get along together" or "this is huge for me..." In addition, I considered my observations of this context holistically and jot-down specific features of this context like "Kayla and Paul had common conference hour" or "Kristina adopted the mentoring role from Ms. Jansen". Once this memo was complete, I reviewed the data for completeness of the memo. The second phase served data reduction purpose. I paired the entries with similar features. For example, in the memo I had "Kristina's advance class is at noon and Kayla's two eighth grade classes are in the afternoon", "Kayla asked for extra work from Kristina for her advanced students", and "Kristina teaches her grade seven classes in her first and second hours, Kayla teaches in her third and fourth hours" all these three were combined into two summary statements. These features suggested how one teachers' class informed the other teachers' class. The same entries also brought convenient class schedules into attention. In my final review of factors data, I turned back to my literature review. In my review of the framing research, I discussed factors as *Physical*, *Practical* and *Personal-Interpersonal* factors. I reconsidered those three factors and mapped context specific features under each category. My data mapped onto these three categories nicely. At the end, I asked a mathematics educator with a doctorate degree in mathematics education to review the list and categories and asked her whether she agreed or not. In addition, I asked her if she noticed any other factors except from the three factors I described here. We did not have any disagreement but she provided some suggestions for more pairings of the data provided in each category.

Different from the other questions, for this question I drew on the interview data the most because there were some specific questions in the interview that prompted teachers to consider their collaboration by attending to the nature of the community in their school and the role of CMP and mathematics in shaping their interactions. This question also asked the role of curriculum and mathematics in teachers' collaborations. I considered mathematics and curriculum materials under practical factors and provided illustrative examples in explaining their roles in teachers' collaborations.

Addressing research question 1c. The foci of collaborations were analyzed from the perspective of Mathematical Knowledge for Teaching (MKT) framework (Ball, Thames and Phelps, 2008). In the initial review of the data I coded content of the collaborations as MKT-related or non-MKT related content. For non-MKT related conversations, I used Shulman's (1986) consideration of knowledge space by attending to *Personal, Context, Teaching, Students, and Values* components. I considered *Personal and Values* conversations together and those involved topics about families, vacations, sports, games, and retirement plans or people's values. *Context* related conversations involved issues with school in general or some specific issues that appeared or possibly might appear in this specific context. Conversations that brought other teachers, administrators, rules and regulations were this kind. In Shulman's major categories of teacher knowledge this was "knowledge of educational contexts, ranging from workings of the group or classroom, the governance and financing of school districts, to the character of communities and cultures" (2008, Ball, Thames and Phelps, p. 391). Conversations on *Teaching* included general pedagogical issues, classroom management, classroom set-up, and materials. This was "general pedagogical knowledge, with special reference to those broad principles and strategies of classroom management and organization that appear to transcend subject matter"

(2008, Ball, Thames and Phelps, p. 391), therefore did not included any consideration of mathematics. *Students* focused conversation were about disciplinary issues, students' personal issues, comparing students across classes. This was “knowledge of learners and their characteristics” (2008, Ball, Thames and Phelps, p. 391).

For MKT related interactions I used the egg model (Figure 5). Ball, Thames and Phelps (2008, p. 403) described MKT by attending to Pedagogical Content Knowledge (PCK) and Subject Matter Knowledge (SMK).

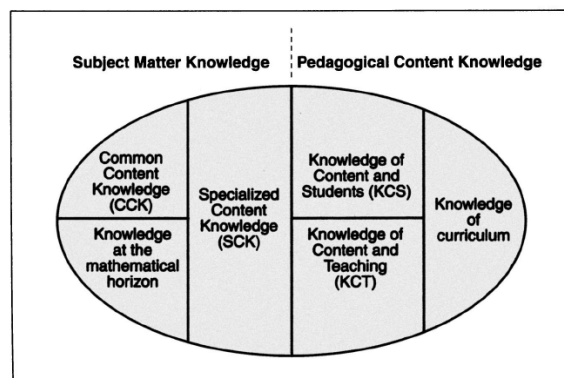


Figure 5

Egg model representing MKT

This egg model representation of MKT brought different components of mathematical knowledge required for teaching. In this study, I mostly focused on the right side of the egg where Ball, Thames and Phelps (2008) considered as PCK more than SMK because, for this study, PCK explained the foci of interactions better in many instances. This observation was also confirmed with these teachers' self-reports. Therefore, in this study I used a simpler version of the egg model to characterize teachers' foci of collaborative interactions. In addition, I did not need that much of detail in the consideration of SMK because there was only a few of those instances.

In my analysis following MKT and not-MKT coding, I coded MKT related conversations either as SMK or PCK. SMK represented any content focused conversation that did not include references to students or teaching and was not grounded in some specific curricular context. If the conversations were purely mathematical, SMK code was used. Instances that were given PCK codes were later coded as KCS, KCT or KCC depending on the focus whether being on teaching or on students or on curriculum. For example, a conversation about students' performance on a math test and likely errors were coded as KCS because the focus was on students' performance in a test that assessed students' knowledge of specific content. These teachers had discussions about better examples to use in the classroom to support students' better conceptualization of certain concepts. I coded those instances as KCT because teachers considered mathematics with respect to their teaching of that content. Curriculum related conversations like skipping a Problem or parts of a Problem or pacing related conversations were coded as KCC. Assessment discussions were also considered as KCC. There were some assessment conversation where teachers brought their consideration of possible student responses or what they did in their classes but those discussions usually stayed within the limitations of the assessment provided by the curriculum materials.

Once I completed coding of the data in each domain like MKT-KCC or not-MKT-Context, I reviewed those coding instances for descriptive purposes and summarized the variation in each category with descriptive notes. For example, teachers' KCS conversations focused on grading student work, issues with student understanding and student strategies.

Interrater reliability. I double coded 17% of the data for interrater reliability purposes. I used excel random formula and selected 48 cases. In coding this data set I asked another person with a doctoral degree in mathematics education. The second coder was experienced in

qualitative coding and had a strong background in mathematics education. She also had background knowledge on the study and particulars of this case. As an initial training about the coding framework I shared my coding framework with illustrative cases for each category. We coded first eight cases separately and compared our codes. We had two disagreements. We came together and talked about our discrepancies and resolved those. She finished the rest of the coding and we ended our coding with five discrepancies in total including the initial discrepancies. All the discrepancies were resolved at the end. I used Cohen's Kappa interrater reliability index (see Table 1 and Table 2).

Table 1

Comparison of codes between two coders

	KCC	KCT	KCS	SMK	Context	Students	Teaching	Personal	Total
KCC	4	0	0	0	0	0	1	0	5
KCT	1	2	0	0	0	0	0	0	3
KCS	0	1	10	0	0	0	0	0	11
SMK	0	0	0	2	0	0	0	0	2
Context	0	0	0	0	7	0	0	0	7
Students	1	0	0	0	0	5	0	0	6
Teaching	1	0	0	0	0	0	4	0	5
Personal	0	0	0	0	0	0	0	9	9
Total	7	3	10	2	7	5	5	9	48

Table 2

Distribution of agreement due to chance

	KCC	KCT	KCS	SMK	Context	Students	Teaching	Personal	Total
Agreement	4	2	10	2	7	5	4	9	43
By Chance	1.52	0.39	4.77	0.17	2.12	1.52	1.09	3.52	15.10

$$\text{Cohen's Kappa} = \frac{n_a - n_e}{n - n_e} = \frac{43 - 15.10}{48 - 15.10} = .848$$

N = number of subjects, Na= number of cases, Ne= number of agreements due to chance

With 85% agreement, the agreement between coder 1 and coder 2 cannot be explained by chance (from <http://www.real-statistics.com/reliability/cohens-kappa/>).

Addressing research question 1d. I described the nature of teachers' interactions by attending to the Depth of Interactions (Coburn and Russell, 2008) and social dynamics of interactions by using Greeno's (2011) consideration of interpersonal positioning and positioning with respect to mathematics and the curriculum.

Social dynamics of interactions. In describing the social dynamics of teachers' collaborative interactions, I wanted to be able characterize symmetries and non-symmetries (Dillenbrough, 1999) in teachers' collaborative interactions. Since, I did not have teachers' conversation excerpts as they were created, a discourse analysis attending to teachers' positioning was not be possible. Moreover, in addition to teachers' positioning each other the context of the interactions was also important, so I considered the positioning as described by Greeno (2011). Therefore, I described teachers' interactions by attending to their positioning relative to (1) their peers and (2) the curriculum materials and/or mathematics. The first one was considered as *Systemic Positioning* and the latter was *Semantic Positioning* (Greeno, 2011). In my analysis of systemic and semantic positioning my purpose was to capture cases illustrating teachers' positioning the best. Therefore, I coded the cases where teachers' positioning each other or their positioning with respect to mathematics or the curriculum was easily identifiable. Therefore this framework provided me a tool to describe teachers' relationships but I did not aim to have cumulative summaries.

Systemic positioning is attending to positioning in relation to other people whereas semantic positioning involved relations to concepts and methods of mathematics. Greeno (2011) described semantic positioning as "the degree to which a student [teacher in this case] is entitled

and expected to initiate contributions, to question or challenge proposals that are made by others, and to be given satisfactory explanations of meanings and methods involved in instructional tasks” (p. 47). As Greeno noted, in an interaction there might be certain people who were considered as to go person to provide an explanation. Similarly depending on the context some people might be more comfortable in questioning the reasoning of others.

In terms of systemic positioning I described cases where the participants were entitled to (1) initiate conversations- *Initiation*, (2) challenge proposals made by others – *Challenging*, and (3) provide explanations – *Justification*. I used *Initiation* to identify the person who started the interaction. *Challenging* (or questioning) was used if there was a person who challenged (or questioned) the other person’s ideas, judgements, or decisions. Finally, I used *Justification* to identify instances where one person provided explanation to issues raised in a conversation. The last two codes were used if data provided evidence of existence. There were some interactions, the *Initiation* was also not clear. Teachers were in the same context and conversation started without necessarily an initiation from one teacher. The following conversation between Kayla and Kristina is used to illustrate how Greeno’s (2011) definition of semantic positioning helped me in considering positioning among these teachers. In this example, Kayla and Kristina were talking about Problem 3.3 in *Comparing and Scaling* Unit.

- 1 Kayla: Do you remember how Problem 3.3 went in your class?
- 2 Kristina: I was in and out for that problem so I really don’t know how it went.
- 3 Kayla: I think it is really hard for my kids especially with B if my students don’t freak out.
- 4 I would be surprised. There is this percent stuff here. When I read TE [Teacher Edition]
- 5 there were pieces of some new understanding they [CMP authors] brought up and may
- 6 be that is what making me uncomfortable. They are not prepared for that.

- 7 *Kristina: They [CMP authors] brought percent staff from 3.1 and ratio and unit rate*
8 *from investigation 2. They wanted to pull all together.*
9 *Kayla: It is confusing and very wordy.*
10 *Kristina [read B1 as an example]: Is that what you consider as wordy? [in a teasing*
 mode]
11 *Kayla: I don't know. I think there is a lot of things going on and I think it will be too*
12 *difficult for my students.*
13 *Kristina: Don't let your judgement influence you teaching*

This conversation was initiated by Kayla (Kayla, line 1). Kristina provided a justification to Kayla's concern about the Problem (Kristina, line 7). In addition, Kristina challenged Kayla's view of problem as being wordy by reading an example question in the Problem (Kristina, line 10). Similarly she questioned her attitude about the complexity of this problem by saying her not to let her judgement influence her teaching (Kristina, line 13). Being entitled to multiple of these roles determined some form of hierarchy among my participants. I will talk about those hierarchies as I discuss my results with respect to these three identifiers in explaining their positioning of each other.

In semantic positioning people may show conceptual agency with respect to object she is interacting with. A person having a conceptual agency will make "choices and judgments involving meaning and appropriateness of methods and interpretations." In the following example, Kayla's approach to consideration of zero provided an evidence of to her lack of conceptual agency.

- 1 *Kayla: I was talking with a group of students. They asked about zero whether it is*
2 *rational or irrational or by its own. I don't have an answer to that. I told them let me*

- 3 *think about that*
- 4 Both Kristina and Danny said it was rational.
- 5 *Kayla: I don't know, it feels like not.*
- 6 *Danny: $\frac{0}{2}, \frac{0}{4}$ you can write as a rational number.*
- 7 *Kayla: I will tell them tomorrow.*

Instead of using her knowledge of rational numbers to consider case of zero, Kayla told her students she would think about their question (Kayla, line 1). She posed this question to Kristina and Danny and as soon as hearing from an explanation from Danny she accepted that as true (Kayla, line 7). Semantic positioning was harder to capture compared to systemic positioning because teachers did not make their reasoning explicit in all the cases. I used the accumulation of evidence for presence or non-presence of teachers' conceptual agencies. Therefore if there was not supporting evidence for teachers' conceptual agency I considered that teacher with low conceptual agency.

Depth of interactions. Coburn and Russell (2008) described the depth of content of interactions on a three-level scale (low, medium, and high). The depth of an interactions was increased as teachers brought reasons to their conversations and started moving from specific case to the consideration of more general issues. Descriptions provided by Coburn and Russell were short and insufficient for some cases, so I modified and expanded their descriptions as I coded (see Appendix G for complete definitions and examples). An interaction with the same focus might have been a low depth, medium depth or high depth interaction depending on how the issue was picked up. For example, a conversation on how a lesson went was considered as low depth if the teacher only said "great" without giving details about why she thought her lesson went great. Whereas if teacher provided the details of what made her consider her lesson

went great, like students were on task and engaged with the problem, that was considered as a medium depth interaction. In addition to these classroom specifics, describing certain characteristics of student behaviors that would give teachers a feeling of satisfaction with their lesson can be an example of high depth interaction.

Once I completed my coding for the content foci of interactions, I coded the same set of data for the depth of interactions. Therefore the total counts for the content was the same as the counts for the total number of interactions coded for the depth. In the coming paragraphs, I provided examples from my data set for low, medium and high depth of interactions. I provided reasons for why I considered those instances as low, medium or high. There were some cases where it was hard for me to make a decision quickly. I considered those cases as borderline cases. At the end, I provided examples of borderline cases and the coding level decisions for those cases. The first example, is a low depth example between Kayla and Kristina:

- 1 *Kristina: I could not jumped into Problem 3.4. I had to do review.*
- 2 *Kayla: I started Problem 3.4 before break so I picked up from there. I did not go all crazy*
- 3 *staff in the CMP3. I felt like students cannot take much.*
- 4 *Kristina: But you did the distributive property, right?*
- 5 *Kayla: Yes. (Field Notes, 04.14.2014)*

In this conversation, teachers provided updates about their lessons after Spring break. This interaction was a low-depth interaction because teachers did not provide reasons for their decisions. They provided updates about where they were at and Kristina checked in with Kayla whether she did distributive property or not but she did not say why she asked about the distributive property or Kayla did not explain when or how she did the distributive property.

In contrast to quick updates like this, teachers' explained reasons for their opinions in medium depth conversations. In another case, Kristina walked into Kayla's class and asked how the test went. Kayla holding up a bunch of papers complained about many students not finishing the test on time. Kayla picked one of the students' test and showed that one to Kristina. He was identified as a student with behavior issues. That student asked a question to Kayla during the test and his question made Kayla question whether he knew what an exponential function was.

1 *Kayla: He even typed $2x$ into his calculator to solve a problem where needed to find*
2 *values of an exponential function 2^x . He deserved what he got because he did not pay*
3 *any attention to what was going to happen in the class. (Field Notes, 11.18.2013)*

This example was considered as medium depth interaction because Kristina and Kayla moved their conversation beyond addressing a simple question of how the test went to consideration of students' performance and reasons for one of the students' low performance. Kayla initiated the conversation by talking about how many students did not finish the test on time. They looked carefully at one student's work and talked about how this student's question in the test showed his lack of engagement in the lesson. Since Kayla and Kristina reviewed students' tests together for the purpose of understanding what students struggled with and since Kayla provided her reasoning about one particular students' low performance in her class this case was considered as an example of a medium depth conversation.

1 *Kayla: It comes back to respect again. Just because teachers have reputations with bad*
2 *management skills. It comes to us again. We were in the interview team. Todd (ex-*
3 *principal) told me if Eryn have management issues she can come to you. Kristina: So you*
4 *were her mentor.*
5 *Kayla: I don't know what a mentor teacher does.*

6 *Kristina: Mentor teacher should be relieved from teaching duty once in a month to go*
7 *and observe. Like coaching. And needs to have time to talk about it. Not for evaluation.*
8 *Kayla: I will admit. I dropped the ball for Eryn. With my personality if I had questions I*
9 *would ask, I would seek out and with Eryn when I asked she said everything was fine.*
10 *Kristina: Not going out for evaluation. It cannot be tied to evaluation in anyway. If you*
11 *were in her room she might have started asking questions. There you develop a*
12 *relationship.*
13 *Kayla: I know I did not develop a relationship so I think I dropped the ball. I don't know*
14 *what to do. That is why....Talking about being a mentor. That is a pain in my stomach.*
15 *didn't do a good job with Eryn.*
16 *Kristina: It isn't your fault. As a building that is something we need to develop. That is a*
17 *weak area in our building. It is not hard to improve on. Supporting new teachers. We end*
18 *up with teachers who don't want to be here. (Field Notes, 03.24.2014)*

In this interchange of ideas about Kayla's role as Eryn's mentor, the conversation moved to a more general issue of what a mentor does (Kayla, line 5 and Kristina lines 6 and 7) and moved back to the specific context of SMS's way of dealing with mentorship (Kayla, line 8). Teachers did not unpack what a mentor does or how a mentorship should be however they named the issue of not having a clear picture of what a mentor does and lack of good support system at the SMS for new teachers.

There were borderline cases where the depth of the interactions fell on both categories like medium-high and low-medium. There were statements coming from one teacher potentially moving the conversation to a more general form or at least bringing up reasons to the conversation in addition to stating just facts however those cases were more complex because

either what one person said was not picked up by the other person or the conversation changed its direction with some other distractions that took place in that time. I coded those instances in the favor of having in-depth conversations because I wanted to value and give credit to these teachers' desire to create a learning environment for each other. In addition, when I have the accumulated summaries independent from specific context and examples, I did not want to make a case of these teachers interactions stayed at the low level because I was physically present in that school setting for a long time and these teachers' interactions was multidimensional and provided room for moving from low depth interactions to high depth interactions. Here is an example of a low-medium depth borderline interaction. Kristina checked with Kayla whether she needed angle rulers for her fifth hour or not and she went back to her classroom and handed her the angle ruler.

- 1 *Kristina: I better give you now so I don't forget.*
- 2 *Kayla: I would not interrupt your class by that way.*
- 3 *Kristina: It is my conference hour, so you would not interrupt my class.*
- 4 *Kayla: They [students] had a hard time in drawing 90 degrees so I told them to use a*
- 5 *piece of paper and they were surprised how it worked. But it is a good to have those*
- 6 *[angle rulers] thought in case they wanted to use it. (Field Notes, 11.21.2013)*

This example is a borderline example of low to medium level but I considered that as medium-depth interaction in the final coding. In this simple exchange of materials (Kristina, line 1), some depth added when Kayla talked about how she used papers instead of angle rulers when students needed to draw 90 degree angles (Kayla, lines 4 and 5). She did not show Kristina how paper served as an angle ruler but both knew how that worked so it was already a shared knowledge for them.

The example of medium to high depth borderline interaction was from Problem 5.1 in the *Growing Growing Growing* unit. In this Problem “students use table of powers to find special relationships among numbers written in exponential form” (*Growing Growing Growing*, Teachers’ Guide, p. 222).

- 1 *Kristina: did you have your students to look for patterns in Problem 5.1*
2 *(she was gesturing tracing same column and row while she was asking this question).*
3 *Kayla: No I did not. I did not enough time even for completing the table for some*
4 *students. There were so many students did mistakes in their tables even with the*
5 *calculators.*
6 *Kristina: Why don’t you keep copies of already filled table and giving those copies for*
7 *struggling students? It is more important to find the patterns than filling the whole table.*
8 *Kayla: It is already too late for making copies and I still have two eight grade classes.*
9 *classes. (Field Notes, 11.18.2013)*

This interaction mostly included a case of medium level interaction where Kayla shared her students’ experience with Problem 5.1. Students needed to fill exponential tables and look for patterns as they fill their table. However there were two places during this interaction, conversation moved beyond Kayla’s classroom. Kristina’s silent contribution where she gestured tracing same column and row (Kristina, line 2) and Kristina’s comment on the main purpose of this activity, how it was more important to find the patterns than filling the whole table (Kristina, line 7). It was a borderline case because Kayla kept the conversation still in a specific context of this Problem even though with the generalities Kristina brought up. I considered this example as a high depth interaction in the final coding.

Interrater reliability. In coding this data set I asked another person with a doctoral degree in mathematics education to code 20% of the data. In order to provide a set with low, medium and high cases I used stratified random sampling. I used excel random formula and selected 20% of low depth cases, 20% of medium depth and 20% of high depth cases. Those cases added up to 48 cases. When I reviewed the data for finer analysis, I parted some of the examples with their content focus. At the end 48 examples made up 17% of the coding.

The second coder was experienced in qualitative coding and had a strong background in mathematics education. She also had background knowledge on the study and particulars of this case. As an initial training about the coding framework I shared my coding framework with illustrative cases for each category. We had half an hour session where we talked about what each category meant. I explained the reasoning behind my coding for each category by referring back to the language used in the framework. The discussion was more focused on the borderline cases. After this initial review of coding framework she coded 17% of the data as first iteration of coding (see Table 3). We compared our codes and there was 100% agreement in this first round of coding. She finished the rest and we had 87.5 % agreement in our coding. The cases we had disagreements were mostly considered as borderline cases. I used Cohen's Kappa interrater reliability index (see Table 4). It was 80%.

Table 3

Depth of interactions comparison of codes between two coders

Coder 1	Coder 2			
	Low	Medium	High	Total
Low	22	0	0	22
Medium	1	13	4	18
High	0	1	7	8
Total	23	14	11	48

Table 4

Depth of interactions distribution of agreement due to chance

Agreement	22	13	7	42
By chance	10.5417	5.2500	1.8333	17.6250

$$\text{Cohen's Kappa} = \frac{n_a - n_e}{n - n_e} = \frac{42 - 17.63}{48 - 17.63} = .802$$

N = number of subjects, Na= number of cases, Ne= number of agreements due to chance

In Cohen's Kappa 80% or higher considered as very good. Therefore the agreement between coder 1 and coder 2 cannot be explained by chance.

Addressing research question 1e. This research question was addressed by cross tabulation of the data coded for foci of interactions and depth of interactions.

Addressing Second Research Question

In my second research question, I asked how teachers' collaborations related to their teaching practices. Specifically, I looked at how the collaborations arose from issues or problems in teachers' practices, the ways these collaborations related to subsequent actions by teachers in the classroom and if there was evidence that the ways that teachers collaborate related to professional actions they took outside of the classroom. The data set for this research question was larger than the data set in the other questions. I analyzed in classroom observations or teachers' individual actions in addition to teachers' collaborative interactions.

I reviewed observation data multiple times. In my first review of the data, I looked at the explicit mention of teachers practices (either in-class practice brought to other teachers' attention or teachers' collaborative interactions were carried to teacher's in-class practice). During data collection, if I noticed an overlap between teachers' two practices (in-class and collaboration), I checked with the teacher the resource(s) she got the idea from if she did not provide any

explanation on her own beforehand. Therefore this issue was resolved by itself for most of the cases. However, in order to capture all possible connections, I reviewed the data in a chronological order in order to locate all possible overlaps in teachers' in-class practices and teachers' collaborative interactions. After locating relevant data, I reviewed data for emergent themes addressing each sub-question the best.

Addressing Third Research Question

The productivity of collaborative activities was judged first by distinguishing *Objective* and *Interpretive* products. Objective products were products that were concrete and tangible (e.g. a lesson plan or computer simulation or worksheet) or were ideas told out loud and agreed upon by the group like a solution provided to a problem, problems identified. Such products were situated within teachers' daily practices. There were also interpretive products without tangible components like teachers' self-reports of changes their practices with respect to their collaboration. Some aspect of a teacher's practice changed or teachers simply reported that they have thought more about some issues in their practice, and those were considered as interpretive products. After this initial consideration of products as objective and interpretive, I noticed that the identified products fell into somewhere in a continuum from objective to interpretive. Even the objective products had some interpretive parts. Therefore, keeping this objective-interpretive nature of products, I reiterated my analysis by considering the focus and the way product was appeared. There were outcomes situated within teachers' daily practices and some other outcomes provided gains in the long term like collaboration helped teachers raise issues. Therefore, my review of data provided evidence of two sets of products. For the products in the second category, I revisited framing literature to compare and contrast themes appeared in my data and outcomes discussed in the research literature.

CHAPTER 4

HOW TEACHERS COLLABORATE: FREQUENCY, FORMS, FACTORS, AND FOCI

In this chapter and the next, I address my first research question in which I aimed to provide descriptions of how collaborative interactions took place at Stonebridge Middle School.

I first present the frequencies of the teachers' collaborative activities (grouping, structure, common activities, etc.). Following those summaries, I present general descriptions and representative examples. My first research question asked:

How do middle school mathematics teachers using CMP materials collaborate with each other in both spontaneously and self-structured ways?

I unpacked that broad question into five component questions:

- a) What are the frequency and forms of collaborations?
- b) What factors are involved in collaborations? Specifically, in what ways are mathematics and mathematics curriculum materials involved in collaborations?
- c) What are the foci of collaborations? What are the issues discussed and the activities teachers engage in?
- d) What is the nature of the interactions among teachers during their collaborative activities?
- e) In what ways is the nature of interaction related to the topics of collaborative activities?

I address questions a, b and c in this chapter, and d and e in Chapter V. I chose to present the first three component questions in this chapter because those three provided a general picture of what happened in this particular case. The next two questions added an analytical perspective to add details of teachers' interactions, such as who initiated the interactions the most, and the relation between those interactions and the content focus of interactions therefore it made more sense to present those results together.

Frequency and Forms of Collaborations

In order to describe how middle school mathematics teachers using CMP materials collaborated with each other, I first counted the instances in which the participants interacted with each other. I will report those frequencies with respect to the different forms. As I described in Chapter III (Methods), I conceptualized forms of collaboration by attending to the *Structure*, particular *Groupings* of teachers in their collaborative activities, and the *Prevalent Activities* that took place during collaborations. The focus of this study was on teachers' spontaneous and self-structured collaborations, so I did not observe any of the externally structured collaborative interactions such as school professional development days. The counts for externally structured collaborations came from teachers' notes of presence of such collaborations. I provided the distribution of *Structure* of collaborative interactions in Table 5.

Table 5

Distribution of structure of collaborative interactions

Structure of Collaborations	Frequency
Spontaneous	183
Self-Structured	13
Externally-Structured	5
Total	201

Table 5 showed that more than 90% of the activities were spontaneous and 6 % of the remaining were self-structured collaboration. I was in the school for 42.5 days so 183 spontaneous interactions meant these teachers interacted spontaneously at least 4 times a day. In the following section, I report specifically about the spontaneous and self-structured collaborative interactions (196 counts).

Structure

Spontaneous interactions happened without any planned time or agenda. For example, teachers discussed mathematics problems while they were waiting for their students in the hallway for their next class. Alternatively, the self-structured collaborations were meetings with a set time and a rough agenda. Department meetings were considered self-structured collaboration because even though the administrators set the date once a month, teachers put the agenda together. In addition, other meetings like assessment preparation meetings, were self-structured if teachers set the time beforehand with a specific goal.

Groupings

Teachers' interactions in spontaneous and self-structured ways took place in particular groupings. Three central participants in this study determined particular groupings. The distribution of teachers' interactions in particular groupings is provided in Table 6.

Table 6

Distribution of teachers' interactions in particular groupings

Groupings		Frequency
Dyads	Kayla-Paul	26
	Kayla-Kristina	146
	Kristina-Paul	7
Triad		7
Total Number		186

The most frequent grouping was between Kayla and Kristina, consisting of 78% of the interactions that I observed. In many of these instances, Danny was a peripheral participant in interactions between Kayla and Kristina. Typically he was silent, but from time to time joined

conversations by offering one or two comments. He took a more active role during self-structured meetings where they discussed assessments. Pairings between Kayla and Paul were the second most frequent grouping. There were ten additional instances of spontaneous and self-structured collaboration that are not represented in the Table 2. These involved one of the central participant teachers and a peripheral participant like interactions between Kayla and Mr. Norm or between Ms. Patz and Kristina.

Prevalent Activities

My data provided evidence of four types of prevalent activities: Checking-In (CIN), Interchange of Ideas (IOI), During Class Time (DCT), and Joint Work (JTW). The most frequent activities were CIN type activities that appeared in 104 of collaborations. There were 69 counts of IOI, 17 JTW and six DCT activities. Figure 6 presents the distribution of prevalent activities in each grouping.

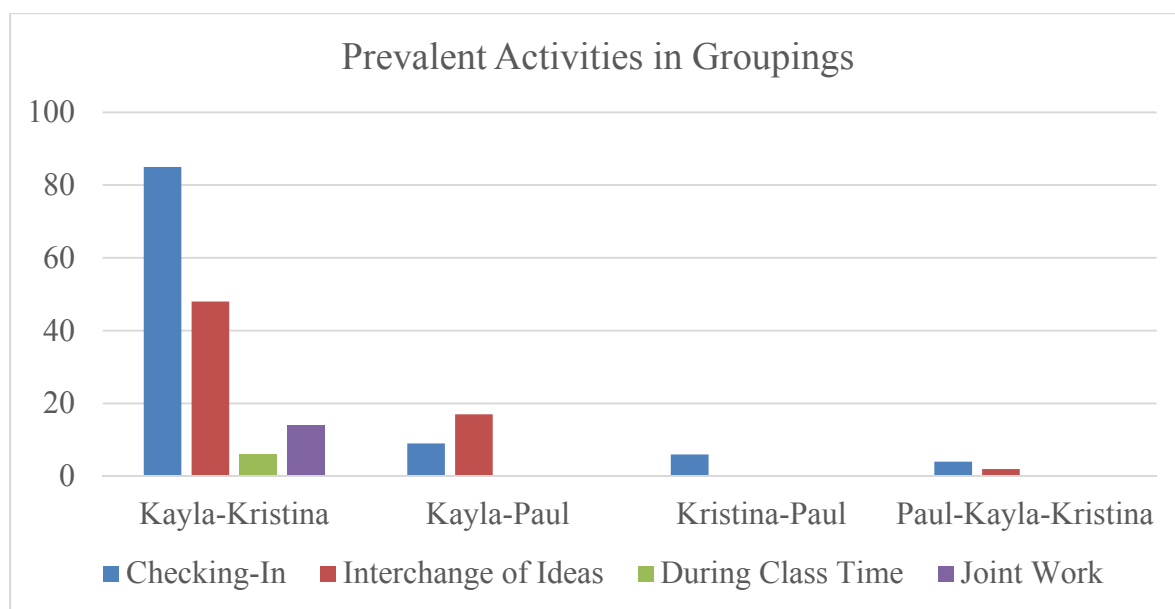


Figure 6

Distribution of prevalent activities in each grouping

The most frequently interacting dyad, Kristina and Kayla, also had the most variety in their interactions. All four types of activities appeared in their interactions. Kayla and Paul had more IOI than CIN; and Kristina and Paul had only CIN activities. I provide illustrating examples for each kind for different pairings.

CIN activities. CIN activities were short interactions where teachers showed interest in each other's lives, professional and/or personal, but did not get into details either because of time considerations, presence of other pressing duties or they simply did not want to have a longer conversation. They were diverse in nature, from asking for a set of materials or briefly telling each other what they did in their classes to what they did over the weekend. IOI activities were relatively longer and included perspectives from all the participants. Both CIN and IOI happened on a regular basis. In most cases they took place in their shared space either in the hallway or in one of the teachers' classroom during break time. Kayla and Kristina also used email exchanges as a collaboration tool that mostly represented CIN or IOI types of interactions.

Kayla and Kristina had CIN conversations that verbalized their emotional state about teaching a topic or certain students, or sometimes more on a personal level. For example, right before starting the *Looking for Pythagoras* unit, Kayla shared her concerns about teaching the Unit. At the beginning of the day she said to Kristina, "I looked at all the *Looking for Pythagoras* stuff but still I am afraid of this book!" Kristina did not respond because their conversation was interrupted when the Special Education teacher came by and they started talking about Christmas decorations. At the end of the same day, Kristina visited Kayla's class before leaving the school building. Kayla provided an update by saying "I feel much better now after working on it more and looking at the assessments you sent to me." Kristina's response

was, “Good!” (Field notes, 12.02.2013). Similar to these examples, CIN emotional shares were quick ways of informing the other person about their feelings.

There were informative CIN conversations in addition to emotionally focused ones. For example, teachers often checked in with each other for scheduling purposes, which materials they used (or planned to use) or what parts of the problems they did or they did not do. In addition to updates from both sides, there were questions about whether they recommended doing a particular Problem or Investigation.

A few CINs were longer and more informative, where one person did the majority of talk. The purpose was to share some information she was excited about and the other person was taking a passive role by listening and confirming what the other person said. Some of those CINs later led to follow up CIN’s or IOI’s. For example, as soon as Kristina finished one of her lessons with her advanced students, she invited Kayla to her class and showed her samples of student work. It was a new Problem she used instead of Problem 2.1 in the *Frogs, Fleas and Painted Cubes* Unit. She was excited because her students were very engaged with the Problem and the visuals they produced helped them to identify how the area changed with a fixed perimeter. After listening to Kristina, Kayla responded by saying: “I don't know what you are talking about. To be honest, I have not looked at the *Frogs* since the workshop and I don't even know what is in it” (Field notes, 01.15.2014). Kayla responded to Kristina’s excitement with voicing her lack of insight on what Kristina shared with her. Later, Kayla followed up this conversation with an email and with another CIN conversation because she wanted to use the lesson Kristina shared with her.

Different from Kayla and Kristina pairing, all of Paul and Kristina’s interactions were short CINs. One time Kristina asked Paul whether she could send two of her students to his class

to take an exam. Two other conversations were about ordering mini-white boards for their classrooms. There was one CIN between Paul and Kristina, which started because Paul was worried about Kayla. Kayla took the day off because her daughter was sick, so Paul came to Kristina's classroom to learn about Kayla's daughter. Kristina told him that Kayla's daughter was fine but Kayla herself got sick (field notes, 12.09. 2013). Those interactions occurred in less than one minute.

IOI activities. A conversation became an IOI if all teachers involved in the interaction contributed to the conversation by asking questions or making comments beyond providing simple responses. The conversation in the following excerpt happened after school. Kayla and Kristina were in the hallway and started talking about random things. It initially concerned one of the students in Kayla's last hour.

Kayla: He always writes to pre-assessments I will know instead of I don't know and I told him that is the best answer. He works hard.

Kristina: That is what you need to do to be better.

Kayla: In the Looking for Pythagoras everything connects so well now. Going on and over really helps making squares. I have seen Problem 3.1 and now it makes sense. I was so freaked out this morning but now it makes sense. That work helped me to make squares.

Kristina: I am glad. That is what matters.

Kristina: I saw that Rachel got an academic excellence. I am very glad. She is such a nice student. Most of our grade seven students are nice.

Kayla: A girl in my class is tutoring one of the sixth grade students. That is very good for her. She even prepared a folder with resources in it and she said she needed to work on stuff before. I told her that is exactly what I do. I prepare a portfolio and relearn.

Kristina: She will practice grade 6. That is what our kids are weak at. Especially fractions. I am curious about how some students will be doing in Looking for Pythagoras because it is geometry. Some students are weak at it and some are strong.

Kayla: I always felt weak about it. That was why I was so freaked out when we were starting Filling and Wrapping and that is why I was freaked out this morning.

Kristina: That is how I feel about Butterflies, Pinwheels and Wallpapers Unit.... (Field notes, 12.02.2013).

In this conversation, both participants were active and the conversation was bi-directional. Both Kayla and Kristina responded to what the other person said by bringing an example from their observations. When Kristina said how most grade seven students were nice Kayla brought up one of her grade seven students to the conversation. The conversation changed its direction because Kayla shared one other detail about this student who was tutoring a grade six student. They followed up by sharing their concerns about students' challenges with fractions. Kristina verbalized students' strengths and weakness in different topics and that led to a discussion of how they felt about certain topics. Even though teachers talked about various topics one after another, conversation involved two active participants with some insight from both.

There were relatively shorter IOIs that communicated updates about teachers' lessons. However, since those updates involved contribution from both sides, I considered those as IOI instead of CIN. Here is a shorter IOI example from Kayla and Paul:

Paul: I am doing Problem 1.2 [Stretching and Shrinking Unit]

Kayla: I did Problem 1.3 and it did not go as I expected. The numbers were ugly.

Kristina said she did not have kids do the measurement but one of the students asked for a ruler and all the students started measuring.

Paul: From previous years, I remember, it is really hard with inches so I preferred metric measurement if I have my students measure. There were so many troubles with fractions and it is hard to see change in fractional numbers. There were also issues with rulers.

Rulers are different too. Some rulers have eighth fractional parts some do not.

Kayla: One student reported a measurement as 1 inch and 3 mm. Can you imagine that?

From a previous IOI conversation with Kristina, Kayla knew doing precise comparisons was not the main goal of this Problem, but Paul's similar experience gave her the occasion to be more vocal about some other details in her lesson. Same was true for Paul. They shared their ideas and experiences with each other around this particular Problem.

Two IOIs took place in triads, among Paul, Kayla and Kristina. One was after the annual CMP Users' Conference, Paul came to Kayla's class to check with them how the conference went. Kristina was there and they all started talking about the conference, and the sessions. The other one took place right after Paul's students took the Partner Quiz in the *Stretching and Shrinking* Unit. I will call this example the Dot Paper example and I will discuss it throughout the rest of the manuscript. Many students struggled with the dot paper provided for students to draw their shapes during the quiz. Paul brought up this issue to Kayla and Kristina and as they talked about it their conversation changed its focus to what they wanted to assess with these students (Field notes, 12.19.2013). The conversation was a good example of IOI because it started with a simple question asking, "why do we have dots?", but expanded to address what

they want to assess in quizzes and how partner quizzes were different from other assessments. In addition everyone in this conversation shared their thoughts.

JTW activities. JTW represented activities in which teachers worked on something tangible. Teachers usually worked from a shared document collectively. They were in a seated position for most of the time, taking notes, sometimes working over something on the white board. JTWs only took place between Kristina and Kayla, and in them, both teachers were active and contributing. They prepared assessments, talked about how to assign points to each questions, and changed the wording in some questions. Here, I will share an example that was not so common but moved teachers' lesson planning activities beyond lesson update CINs or IOI where they shared their insights about their lessons. I will call this example the Reflection over a Line example from now on. The conversation between Kayla and Kristina started with Kayla calling Kristina to her classroom to get her thoughts about something they discussed in the class about similar triangles. That conversation led to another conversation about Kayla's plans for her next class with her grade eight students.

Kayla: I will have the partner quiz tomorrow. I don't think it will take a long time so I can start Problem 3.1. I finished all of Investigation 3 and it did not seem difficult to me.

Kristina: Can you remind me what was in Investigation 3?

Kayla: It is Mugs and plotting coordinate points which they already know.

Kristina skimmed Problem 3.1 quickly. They started talking over the questions in the book.

Kristina: You need to be very careful when they [CMP] gives (x,y) and asks for the opposite they get $(-x, -y)$. However when it is $-x$ it is hard for them to think of the opposite of $-x$. Because they already consider $-x$ as opposite.

Kayla: Some of the things in Problem 3.2 might be confusing, too. They [CMP] give (x,y) and they[students] need to reverse the order and write (y,x) .

She gave an example on the board (Figure 7).

x	y	y	x
0	0	0	0
1	2	2	1
2	4	4	2

Figure 7

Kayla's example of (x, y) and (y, x) in table of values

Kristina drew a graph (Figure 8)

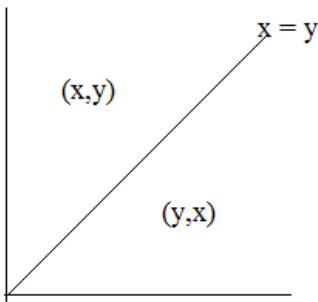


Figure 8

Reflection over the $x=y$ line

Kristina: All they need to think is the reflection over $x = y$ line.

Kayla: I will make sure to mention the reflection over $x = y$ line.

They continued their conversation by talking about some other details with their eighth grade plans and they got into some personal conversation afterwards (Field Notes, 03.17.2014). This was an example of JTW because both teachers were active and they reviewed the Problems by

looking at the materials, followed with their work on the board together to explain their thinking and to provide examples.

DCT activities. DCT represented teacher activities that took place in one of the teacher's classes during class time. Similar to JTW, DCT happened only between Kayla and Kristina and was the least frequent activity overall. There was variation in teachers' roles, mostly taking only an observer role, and a couple of times they shared teaching roles for a short amount of time. During one of our interviews with Kayla, she mentioned how she invited Kristina for a short amount of time (for about 10 minutes) and had her lead the classroom discussion. She wanted her to help students understand one of the topics with which she observed her students were struggling. Here is how she described this experience during our interview:

Students were not getting the mathematics. I walked over there [meaning Kristina's classroom]. It was an inverse problem in the Thinking with Mathematical Models Units. They weren't getting the whole idea that $xy = k$. They were saying x value goes up, y value goes down. That could happen more than inverse. Put my head out the door. She came into my classroom and started asking questions to my kids (Kayla, interview, 11.19.2013).

In this case, Kayla was frustrated with her students because they seemed to have hard time explaining the relationship between two variables in inverse functions. So she asked for help from Kristina.

The other time Kristina invited Kayla to her class to ask questions and remind her students of some of the mathematical ideas they learned in the *Variables and Patterns* Unit in grade six. They had a quick conversation about what they did in the *Variables and Patterns*

(Field Notes, 01.15.2014). These were the only two cases when both teachers were active in their teacher roles. In the other cases of DCT, either Kristina was peeking into Kayla's class by the door or she stayed in the class for a short amount of time, mostly without saying anything.

Summary

In this question, I described different forms of collaboration and their frequencies. Teachers' self-initiated collaborations ranged from spontaneous collaborations without a set time and agenda to self-structured collaborations with a set time and agenda. The self-structured collaborations were usually JTWs (with three exceptions). If teachers had set time and agenda they usually had a document to work from. DCTs were infrequent and included less mutual contribution in this setting. If an outsider came to this school and stayed for a day to observe the interactions among these three teachers, she would possibly observe CIN-type short hallway conversations in between Kayla and Kristina but not the other forms.

Factors That Shaped Teachers' Collaborations

In addressing this question, I tried to identify the specific features of this context that shaped teachers' collaborative interactions. I sought to understand what contributed to the frequency and forms of collaborative interactions and whether some specific features of this context had a positive or negative influence on teachers' collaborative interactions. Specifically, I considered in what ways mathematics and mathematics curriculum materials were involved in these collaborations.

This study confirmed the presence of three factors in shaping teachers' collaborative interactions: Physical factors, practical factors and personal-interpersonal factors. *Physical* factors included tangible elements in a teachers' professional life like physical layout of the schools. *Practical* factors included concerns in teachers' lives with respect to their practice. I

considered mathematics and CMP as practical factors. *Personal-Interpersonal* factors included variables concerning teachers' beliefs and personalities as well as the shared culture among teachers and the school culture. In discussing the specific features of this context, my aim was to explain how the presence of those features influenced teacher collaboration. However, it is important to note that teachers' collaborations were shaped by the interaction of all these three factors together, not only with the presence of one single factor.

Physical Factors

The proximity of teachers' classrooms, use of common communication tools like texting, emails, and shared resources were physical features of this context that influenced teachers' collaborative interactions. Recall the representative sketch of the school building in Chapter III (Figure 1). Kayla's and Kristina's classrooms were next to each other, and the physical proximity of their classrooms contributed to the high frequency of their interactions. When Kayla or Kristina left their classroom, usually the other one was in the hallway, monitoring their students leaving or arriving for the next class. This arrangement was ideal if they wanted to share something that happened in their previous hour or if they had concerns or questions they wanted to voice. Kayla in particular was very vocal and Kristina's presence right next door was good for Kayla's reflective talks. In addition, the breaks were short, only four minutes long, so there would likely not have been time to find each other if their classes were not close by. Since their classes were next to each other, walking down to each other's classes did not eat up all their break time, so they did not mind walking into their classes to ask a question or make a request or just to talk. Similarly, while they were walking down to their classroom from their lunch break, since their classrooms were next to each other, one could just peek into the other's class. Kristina would briefly observe Kayla's class when she left her class for a bathroom break, or

right after her lunch she took a short detour to Kayla's class just to get an update (e.g., Field Notes, 12.16.2013).

Paul's class was relatively distant from Kayla's and Kristina's classes. There was a set of student lockers in between Kristina's and Paul's classrooms. For some spontaneous hallway conversations, those lockers ended up being a physical barrier between Paul, and Kristina and Kayla. On some days, Paul had to walk down to stay closer to Kristina and Kayla. While he was keeping an eye on his students coming into the class he tried to join conversations taking place between Kristina and Kayla. Most of those times, the science teacher also came closer and they expanded the conversation beyond the grades seven and eight mathematics teachers to more students or school-related issues. In addition to this, Paul needed to walk down to his Technical Education class at the end of second hour until the last class hour. So in some ways, he was also not physically present in these teachers' lives half of the day.

Some other common communication tools like texting and emails also influenced the frequency of their interactions. All three teachers had access to an internal email system or had cell phones with a text messaging feature. Kayla and Kristina used emails and text messages to keep their communication ongoing. It was common for Kayla or Kristina to type a short email sometimes with a single question about something that came up in the class or that they would need shortly. The other person provided a short response to the email or chose to talk in person. For example, one time Kayla wrote an email to Kristina asking how to grade some students' responses in a Unit test. Kristina came into Kayla's class and they read students' responses together (Field Notes, 03.05.2014). Therefore, Kayla's short email informed Kristina that Kayla needed her insight on grading particular students' work, though she was in the class at that time. Her students were taking a test. Kristina was available, so she came into Kayla's class and they

reviewed students' work together. Email provided them an opportunity to communicate even during class time.

Paul did not use these extra communication tools as frequently as the other teachers. Kayla and Kristina included Paul in their emails if they wanted to share a document or provide an update about something. For example, after Kristina finalized the grade seven Unit test for the *Stretching and Shrinking* Unit, she emailed the final version both to Paul and Kayla (Field Notes, 01.16.2014). At another time, Kayla emailed Paul, Danny, and Kristina to inform them that she wanted to talk about points for the *Comparing and Scaling* Unit test sometime during their upcoming professional development day (Email exchanges, 02.27.2014). Even though Paul was not using his emails as frequently, Kayla and Kristina were able to share recent updates with him via emails.

There were some shared resources that provided opportunities for teachers' interactions. These quick interactions served as a support to each other. For example, teachers had one set of certain materials such as angle rulers and pattern blocks, so they visited each other's classes to borrow materials for their classroom (Field Notes, 12.09.2013). Teachers had decided to use CMP3 but the school could not afford getting new set of student books. So they needed to print out some new CMP materials from CMP online platform (Dash). Only Kristina had access to this platform, so she shared her account information with Kayla. These interactions sometimes led to more sharing, like when Kayla borrowed pattern blocks from Kristina she stayed for a couple of minutes in her class and observed what students were doing. Kristina also provided a brief description of how she used pattern blocks. Another time, when Kristina asked Kayla if she had rice, she explained why she needed the rice for. She was going to have her students' explore volumes of different three-dimensional shapes (Field Notes, 03.10.2104). Therefore, those

physical issues created extra opportunities for teachers to interact with each other and ended up becoming elements contributing to teachers' collaboration.

Practical Factors

Practical factors included teaching the same grade level and content (even sometimes having same students), teaching the same content (mathematics) and from the same set of materials (CMP), and having (or not having) a common extra-time. Each of these served to expand collaborative interactions among teachers. These practical factors, particularly teaching the same grades, content, and students, provided a common ground for teachers' interactions. Those practical factors were main drivers of teachers' continued interactions. Each will be discussed in greater detail below.

Teaching the same grade levels, content, and students. Kayla, Paul and Kristina, all taught sections of grades seven mathematics. At the same time Kayla and Kristina were teaching grade eight mathematics. Teaching the same grade levels and teaching the same content provided a common context for teachers to share their experiences. Since I explored teachers' collaboration around the same set of materials, I do not have data to make a claim that teaching the same grade level by itself contributed to teachers' collaborative interactions, however, data supported that teaching the same grade levels provided more context and shared challenges for teachers to collaborate. For example, Paul noted in one of their department meetings how they all talked about grade seven content first and after that Kayla and Kristina started talking about grade eight content. He brought up this example, not with a negative connotation, but as a fact that Kayla and Kristina had more to consider (Paul, Post Interview, 04.20.2014).

Kristina taught grades seven during the first and second hours. After she finished her teaching, she usually brought up some of the things she noticed in her classes in her conversation

with Kayla (Kristina, Post Interview, 04.21.2014). Similarly, Kristina had her advanced class with grade eight students before Kayla's afternoon eighth grade classes. Kayla used some of the questions Kristina recommended with her advanced students in her afternoon classes. She said that those were students who did not qualify for the advanced class so the extra work suggestions from Kristina kept them busy (e.g., Field Notes, 11.11.2013). There were also shared concerns and ideas related with teaching the same thing. For example, Paul brought up his students' responses to certain questions on a test and talked with Kayla about how to grade those (Field Notes, 11.19.2013). In addition to these, there were cases where the teachers had the same students in different semesters or years. Kristina asked for Kayla's view of some students who she had the previous year when teaching grade six (Field Notes, 02.24.2014). Similarly, some students changed classes because of scheduling issues so they ended up having each other's students within the same year. They had brief, informative conversations about some of those students (e.g., Field Notes, 02.05.2014).

Mathematics as a content area initiated many of the teachers' interactions as it was linked to their teaching as teachers talked about possible student difficulties and teaching strategies for certain topics. In addition to teaching-focused mathematics conversations, there were particular situations where Kayla and Kristina interacted around mathematics. In almost all of those interactions Kayla had a particular question. Kayla told me multiple times that because it was her first year of teaching grades eight mathematics she did not feel comfortable in that content area, so she was doing all the mathematics in the Units before teaching (Kayla, informal conversations, 12.19.2013). Therefore, Kayla's feeling of insecurity in mathematics contributed to mathematics-focused conversations between Kayla and Kristina.

Teaching from the CMP curriculum. Teaching from CMP materials provided an important common context for teachers. The contextual nature of the problems in CMP sometimes required additional guidance from teachers to help students understand the context as well as mathematics. For example, Kayla thought that Problem 3.1 in the *Comparing and Scaling* Unit could be challenging for her students so she shared her worry with Paul. Paul confirmed what Kayla said and told her that buying and selling prices might be a challenging concept for students. Later, Kayla reviewed the Problem with Kristina to address her concerns. (Field Notes, 03.03.2014). Teachers often had conversations similar to this one, either related with the problem context or mathematics where they brainstormed how to provide entry points to support students in solving problems.

Unlike some other curriculum materials with less context around mathematics problems, CMP Problems were rich in context and that placed a different challenge for teachers. Paul even shared this difference with his students in his class. He told his students how his mathematics learning experience was different—his teachers used to give a formula and as students they used to plug in numbers. He said when a story problem came up they all struggled with it because with story problems they needed to understand the concepts. He added how he learned some of the things once he started teaching out of CMP (Field Notes, 03.21.2014). He told this story to explain how as students, they needed to understand what was going on before doing any work, and that can be reiterated for teachers too. Teachers needed know how each context was relevant and important for a particular mathematics idea. Kristina also confirmed that “with Connected Math there are so many opportunities for things that sometimes you don’t see what you are supposed to get out of the Problem.” (Kristina, Initial Interview, 12.03.2013).

CMP as a problem-centered curriculum placed a different demand on teachers compared to traditional way of teaching. Kristina compared the CMP way of teaching to traditional ways of teaching and expressed how much she appreciated the focus on students and their different strategies:

... if you were to teach a traditional curriculum out of the textbook, straight out of the textbook which is most people who teach a traditional curriculum do, you show them the example and then they copy like little monkeys and they do 50 more problems. Just like it. There is nothing to talk about. CMP is problem solving so... Because there are so many entry points into a problem, how a kid might tackle a problem, there is an endless number of things that you can talk about. This kid did it this way, this kid did it this way. I never thought about it that way. I mean it is just... how can I push a kid further. They don't ask that question in a traditional curriculum. Let's do more problems. (Kristina, Initial Interview, 12.03.2013)

As Kristina noted, CMP provided rich problems with multiple entry points, so it was helpful for teachers to talk about different ways students might tackle a problem. In addition, it was a challenge for teachers to decide how they could have students do the work instead of teachers telling them. Kayla came many times with a concern about running her classes in the “right” way, that is, closer to what was suggested in the CMP teachers’ guide. She read the teachers’ guide many times and brought up her concerns and questions to Kristina:

Am I getting, what I mentioned earlier, am I getting out of this problem for my kids what is supposed to be? What they are supposed to be. ... What questions I can ask in the explore to push those kids that are done early? God, my launch sucked today! What did I do? (Kayla, Initial Interview, 11.19.2013)

As Kayla noted, she knew she needed conversations about teaching using CMP materials. For example, Kayla complained about teaching the *Say it with Symbols* Unit traditionally multiple times. It was a challenge for her to teach differently and Kayla voiced her worries (e.g., Field Notes, 02.05.2014). Further, Kayla and Kristina talked about how students' work needed to inform teachers' summaries (Field Notes, 03.03.2014). They talked about how sometimes teachers needed to make curricular decisions on whether to skip a Problem or not. For example, in the *Moving Straight Ahead* Unit, Kristina reminded Kayla that she did not need to do Problem 2.2 if she already had multiple strategies in the Problem 2.1 (Field Notes, 03.24.2014). Therefore, with different teaching demands, there was a need for teachers to talk with each other to improve their instruction with CMP. For that reason, it was natural for Kayla to approach Kristina because she had more experience in teaching CMP and Kayla needed more guidance.

The way teachers interacted with each other was also influenced by CMP materials. In a CMP classroom the teacher's role is usually described as a facilitator of learning and students are considered as resources to each other as well the teacher. Kayla mentioned how this CMP model of teaching showed up itself in their interactions with Kristina.

You just can't show an example and say this is the way we do it. You keep questioning. Sometimes I say to Kristina, I don't want you to play the CMP game with me now, because she keeps questioning. I say I just want an answer. I even use that at home with Eryn [Kayla's 4 years old daughter]. It is all about problem solving. She asks me a question and I ask her a question." (Post interview, Kayla).

Kristina's common response to Kayla involved modeling the CMP way of teaching, and at the same time it was Kristina's way of helping Kayla to deepen her understanding. Kayla, wanting to have the answer right away, as sometimes her students did, was honest enough to express her

lack of patience to go through the whole process of questioning and sense-making. But in the end she knew it was better to learn that way and that was what she wanted it to carry to her interactions with her daughter.

The development of mathematical ideas through CMP as a Problem-centered curriculum was different from other curriculum materials that did not have the same design structure. Both Kayla and Paul noted how Pythagorean Theorem was just “ $a^2 + b^2 = c^2$ ” for them before teaching from CMP, which devotes a full Unit to the development of the concepts underlying the Theorem. Kayla voiced frustration at the beginning of the *Looking for Pythagoras* unit, but as she talked more about it with Kristina and worked more on the Unit she overcame her frustration and was able to see how ideas were connected.

In addition to different ways of developing mathematical ideas, there were also different teaching demands for teachers who follow CMP instructional model as *Launch-Explore-Summarize*. Each phase of the lesson carried a different meaning. Kayla and Kristina had many conversations about each phase especially for *Explore* and *Summarize* phases of their lessons. They talked about what a teacher’s role should be during *Explore* and how they should plan for *Summarize*. Therefore CMP as a problem-centered curriculum placed a particular demand on teachers and that created many of the interactions between Kayla and Kristina.

Common time. Except from the four minutes between class periods, or the time before and after school hours, some teachers shared the same conference hour and/or lunch time. Kayla and Paul had both in common. That common time provided an opportunity for them to have conversations around their teaching. In all those interactions, Paul visited Kayla’s class with a specific goal, such as grading students’ work or talking about an issue about the school. During lunch, Kayla brought up different stories from her personal life, sometimes Paul talked about his

sons or daughter, so they knew more about each other's personal lives. When other teachers were around, Paul and Kayla brought up some other classroom related issues so they learned more about their classrooms. Therefore, that extra-time provided Kayla and Paul an opportunity to interact more and learn more about each other.

In contrast to this common time for Kayla and Paul, Kristina had a different conference hour and lunch time. Therefore, she did not have as much interaction with Paul as Kayla did. Paul brought up that as a concern during our post interview. He indicated that all mathematics teachers used to have the same conference hour and he missed having that time slot providing more time to interact with Kristina (Paul, Post Interview, 04.20.2014). However, as Kristina noted during our interview "if you have the willingness you can find a time to make it work" and she did it work with Kayla. They used their break time or before or after school times more to collaborate. In addition, pairs of teachers (either Kristina & Kayla or Kristina & Paul) used the other teacher's availability during their conference hours to ask for support. Kristina sent a couple of her students to Kayla's class or Paul's class to finish an assignment or take a test during their conference hours. Similarly, Kayla sent a couple of her students to Kristina's class to finish their tests. Kristina or Kayla also used a couple of minutes of their teaching-free time to observe each other's classrooms to get a lesson update because they wanted to see what was going on in the other class.

Year-long intern. Having a year-long intern provided the time Kristina to engage in other teaching-related tasks. After Danny became the lead teacher, Kristina used her additional time to walk out of her classroom and stayed longer with Kayla if needed. For example, there were a couple of times Kristina came to Kayla's classroom during Kayla's conference hour and they had a conversation for lasting 20 to 30 minutes. On the other hand, she had less classroom

experiences to share with Kayla as Danny started teaching more. All of the more inspirational student work that I will share in Chapter VI came from classes where Kristina was the main teacher. Therefore, during Danny's teaching period, Kristina either talked from her experience from the previous years or some potential student challenges she observed while Danny was teaching.

Personal-Interpersonal Factors

There were personal and interpersonal elements that contributed to these teachers' collaborative interactions. These include variables concerning teachers' beliefs, personalities as well as the shared culture among teachers and school culture. I will present the personal-interpersonal factors by attending to teachers' shared goals and roles, personal characteristics, and common personal interests.

Shared goals and roles. First of all, these teachers reported that they collaborated with each other because they shared similar learning- and teaching-related goals. This goes beyond the fact they were teaching from the same curricular materials. They could have taken very different stances toward their teaching, but they did not. During our initial interview, they rated statements asking for their shared interest toward teaching on the high end. Their average score for the statement "we all want us to move in a direction which all of us agreed on" was 4.7 out of 5. Therefore, they considered each other alike in terms of their teaching related goals.

In contrast to their report of their shared goals, they noted differences in their commitment to teaching. Kayla related differences in their commitment to their personal life and lack of experience teaching this content:

I think we do it different ways and I think our lifestyles are different. So Kristina has been doing this for so long she can just wing it whereas I have to sit down and prepare for hours

over these lessons. She can just show up and teach it. And hers probably will go better than mine. But you know, I have a daughter. She does not have a husband or children. So that changes things too. So that is what I meant by that. Lifestyle I guess. (Kayla, Initial Interview, 11.19.2013).

Kayla recognized differences both in the fact that she had to spend more time as she was new to teaching this content and that she had a demanding personal life she needed to plan for.

Kristina thought Paul was less committed to teaching. In the initial interview she was not explicit regarding this fact, but in the post interview she was more open about it and talked about Paul as being less invested in improving himself as a teacher.

In addition, there were differences in these teachers' self-assumed roles. Kristina considered herself more as mentor for Kayla. Kristina said "this year I do feel like I really have a role to be more supportive with Kayla just because it is newer to her" (Kristina, Initial Interview, 12.03.2013). Paul raised a similar concern and talked about how they wanted to support Kayla as much as they can in her transition to her new role. That might be one of the reasons why he was more open to interactions with Kayla. Similarly, Kayla noted how they shared different roles. She said she considered Kristina more of a leader. Their self-accepted roles influenced the way they interacted with each other. The differences in their accepted roles might also explain the lack of interaction between Kristina and Paul.

Personal elements. Some personal characteristics of these teachers contributed to their collaboration. Kayla and Kristina both had a desire to resolve issues they came across. For example, they collaborated on finding a solution to an emergent issue between school administrators and teachers (I will return to this as Evaluation Observations example in Chapter VII). Kristina was a self-described perfectionist, so if she noticed something was not going as

she planned, she brought those up to find a way to deal with the issue. Kayla was very organized and attended to every detail in her planning. Therefore, she always had a list of questions she thought of as she was planning. In order to address those she mostly reviewed her list of questions with Kristina.

Differences in their personal background shaped their interactions as well. Kristina had a long history with the CMP curriculum. Kayla respected Kristina with her long background in teaching this set of materials but also by having teacher leader role in nationwide conferences. She talked about this as “Kristina trained me. Presented for CMP. That was cool.” (Kayla, Initial Interview, 11.19.2013). Therefore, Kristina was a confirmed authority in some ways for Kayla. This way of seeing Kristina shaped the way she responded to Kristina’s justifications. She mostly accepted those as true without much questioning. I will provide examples of these interactions when I addressed research question 1d.

Foci of the Collaborations

I considered the content in describing the foci of collaboration interactions. I used the Mathematical Knowledge for Teaching (MKT) framework (Ball, Thames & Phelps, 2008) for analyzing interactions with MKT focus. Conversations about students with a concern of teaching mathematics such as likely student mistakes in a topic were considered as Knowledge of Content and Students (KCS). Conversations including pieces like mathematics questions to pose in the classroom with a focus on teaching mathematics made up Knowledge of Content and Teaching (KCT). If teachers talked about pacing, assessment, mathematics standards or considered mathematical topics broader than teaching a particular lesson with a consideration of the curriculum over multiple lessons, those were counted as Knowledge of Content and Curriculum

(KCC). There were times teachers talked about mathematics without considering their teaching or students and that was considered as Subject Matter Knowledge (SMK).

For Non-MKT focused interactions, I used Shulman's (1986) categorization of other content areas besides MKT. Those were Context, involving teachers' conversations about school; Teaching, conversations about teaching without necessarily teaching mathematics focused; Students, conversations about students without any mathematics related consideration; and finally Personal conversations where teachers shared their personal lives and values. The results suggest variety in teachers' foci of collaborations (See Table 7).

Table 7

Content focus of teachers' collaborative interactions

Content Domain	Sub-Domain	Frequency
MKT (110)	KCT	23
	KCS	36
	KCC	47
	SMK	4
Non-MKT (172)	Context	69
	Teaching	19
	Students	50
	Personal	34
Total		282

KCT: Knowledge of content and teaching; KCS: Knowledge of content and students, KCC: Knowledge of content and curriculum; SMK: Specialized content knowledge

Overall, there were more non-MKT (61%) events in teachers' collaborative interactions than those with a MKT focus (39%). The non-MKT focus was mostly about the context or about students in general. Forty-three percent of MKT-focused interactions were categorized as KCC, 33% was KCS, and 21% was KCT. There was very little SMK focus in teachers' collaborative

interactions. Next, I will provide an overview of what was attended to with each MKT focus and after that I will provide a brief description of major non-MKT focus interactions.

Knowledge of Content and Curriculum

KCC included teachers' interactions around: (1) assessments, (2) pacing related considerations, and (3) curriculum storyline. The last one included both conversations unpacking development of mathematics ideas over a Unit or multiple Units or specific features of curriculum materials. If the conversations were more focused on teaching and students rather than understanding the development of mathematical ideas better, those were considered as either KCT or KCS. Teachers' KCC interactions were almost equally distributed among pacing (15), assessment (16), and curriculum storyline (16) conversations.

There were examples of conversations including pieces from all three together. In this example, Kayla asked Kristina about the Check-Up quiz for *Frogs, Fleas and Painted Cubes* Unit. Following that conversation, they started talking about time they needed to spend to the Unit and content focus:

Kristina: We still can have three to five weeks for this Unit.

Kayla: All in this Unit is quadratics and its algebra standard, right. So it is not eighth grade, it is extra for my students.

Kristina: It is nailing down the function idea by adding quadratic functions. They had the inverse functions before. Besides, Investigation 2 is all about distributive property and distributive property is completely eighth grade. (Field Notes, 02.05.2014).

In this conversation, Kayla and Kristina considered how much to cover from this Unit.

However, as they were talking about what they needed to do, Kristina explained how this Unit

was important for eight grade students. This overview of what students' would take away from this Unit was a KCC conversation over assessment, pacing and curriculum storylines.

A curriculum storyline conversation sometimes brought up specific features of the curriculum materials, such as in the following KCC example from Kristina and Danny. It was about a question from Problem 3.3 in the *Comparing and Scaling* Unit. The question provided an imaginary student's work, Meng, and asked students to analyze and explain Meng's reasoning in his solution. Danny asked students to solve the problem instead of analyzing Meng's work. During break time Kristina had a conversation with Danny about this question:

Kristina: One of the common core practices is criticizing and analyzing others' work. Remember Ms. Patz's lesson I did with them. They are not good at that. One of the things with CMP3 authors want to have students practice that. In here (3.3- D1) you asked them how they would solve it. It is actually asking to analyze Meng's work. This one is pretty straightforward. But there are more of those kinds of questions in CMP3 and that is why they have those questions. (Field Notes, 2.24.2014).

In this excerpt, Kristina provided the reasoning authors might have had in providing such questions. Even though Kristina noted, how students were not good in criticizing and analyzing others' work, she wanted to let Danny know that providing student work examples was a curricular choice and the intention was having students analyze imaginary students' work. That was why this interaction was counted as KCC. In most of the other curriculum storyline examples, conversations were about understanding the development of mathematical ideas and not about unpacking features of materials. Therefore, this example was a not typical curriculum storyline example.

Knowledge of Content and Students

KCS interactions involved teachers' conversations focused on students' work in mathematics, such as their responses, errors or struggles. The majority of the teachers' KCS interactions were about the issues around their students' understanding (64%). Those issues either came from their classrooms or based on their previous experience. For example, when Kayla told Kristina she was planning to be done with the *Moving Straight Ahead* unit, Kristina brought up potential student difficulties she might come across in doing slopes:

Kristina: Are you expecting to be done with slope that quickly?

Kayla: I don't know. There is all this rate of change idea. We did not do it in this unit but they did for a while. So it should not take long.

Kristina: From my experience it really takes a long time (with a sigh) taking two points and finding slope is hard.

Kayla: They did this kind of work before, in grade six?

Kristina: They looked at the graph increasing by 1 and it was easy to say it goes up by 10 each time. And they did not go beyond that. 10, 20, 30...(Field Notes, 04.14.2014).

In this interaction, Kristina suggested to Kayla that students might struggle in learning slope. There were also other examples Kayla brought up from her classroom. In addition to the issues around student understanding, teachers interacted around making sense of students' work on tests and sharing student strategies that came up in their classrooms. Those two types appeared with about equal frequency.

Knowledge of Content and Teaching

KCT focused interactions involved teachers conversations focused on their teaching mathematics. The majority of conversations coded as KCT were either about voicing teaching

related concerns, before or after teaching (35%), or unpacking how to facilitate (or facilitated) classroom activities (26%). For example, Kayla unpacked what she did for Problem 4.3 in the *Butterflies, Pinwheels and Wallpapers* Unit. She told Kristina that she took students to the cafeteria and had them find their lengths by using mirrors (Field Notes 04.02.2014). Students were supposed to create two similar triangles and make estimates by using their triangles. KCT's also included cases where teachers reminded each other of the important part to emphasize in a Problem during their teaching "you did the distributive property, right?" (Field Notes, 04.04.2014). In these KCTs teachers picked up particular parts of their teaching to converse about it.

Subject Matter Knowledge

SMK conversations focused on mathematics. Those were usually initiated when a mathematics question came up during teachers' lesson preparation or brought by a student. If those questions led to teacher interactions with a focus on mathematics independent from consideration of their teaching or students, I considered those as SMK. For example, the following excerpt was coded as SMK because teachers articulated their weakness in their spatial visualization skills. The conversation started with Kayla's reflection on of the formative assessment lessons she did for Ms. Patz.

Kayla: Ms. Patz asked me whether my students have a clear view of what a cross-section looks like.

Kristina: I would not give a crap to it. I am saying this because I don't like it. It is like butterflies. It is really hard for me to imagine. I can say what will happen to cylinder when slicing horizontally--a circle. I need to think about it but I can see a rectangle

vertically. But still confused about where the circle went. But other cuts are really hard for me.

Kayla: I am glad I hear you saying this. That is that same with me. I am really weak with that spatial visualization thing. (Field Notes, 03.24.2014).

In this conversation, Kayla and Kristina were very specific about what mathematics they felt weak about and that was why it was counted as an example of SMK focused interaction.

Different from this example where teachers only voiced their weakness, in the other SMK instances teachers ended up their conversation with a resolution. I will provide those in the Chapter VII when I describe how teacher collaborations helped teachers consider mathematical problems.

Non-Mathematical Knowledge for Teaching

The majority of teachers' non-MKT conversations were about the Context and Students. Teachers' context related conversations spanned a range of topics from makeup days for snow days to administrators, teaching assignments, decisions over teacher professional development activities and use and share of materials. Administrators got the most attention, with 26% of the non-MKT collaborations. That was because there was a tension between new administrators and teachers and teachers had a need to talk about it.

The second most frequent non-MKT content that appeared in teachers' interactions involved student-related interactions. Those interactions distributed almost equally among exchanging information about particular students, classroom issues with particular students or the entire class, and general student-related concerns. For example, teachers were concerned about eighth grade students' indifferent attitudes towards grades, since students knew that their grades would not count for college (Field Notes, 04.14.2014).

There were also many personal conversations that were not about teaching, school, or students. Teachers' shared some details about their lives and provided updates about their personal plans. Kristina and Paul always checked-in with Kayla about her daughter Eryn and she liked to talk about her. Kayla and Paul talked about Paul's daughter. Kayla had a niece who was about the same age as Paul's daughter and Paul sometimes came to Kayla to get her idea on something about her daughter. Kristina thought these personal conversations were important in building up a safe relationship:

But even to be able to build a personal relationship I think makes it safe...like you can ask questions and feel like we can question each other without feeling threatened."

(Kristina, 12.03.2013).

Summary

The focus of teachers collaborative interactions' foci were split between MKT and Non-MKT related topics. Non-MKT issues captured teachers' attention because there were some emergent issues that might happen in different contexts, like student disciplinary issues or teachers' need to learn more about particular students. There were also some emergent issues that were specific to this context, like having so many snow days in an academic year or tension between teachers and administrators. The majority of the MKT-related interactions concerned teachers' content knowledge and curriculum. If it was about their teaching or students those were mostly either to voice their concerns about something that happened in their classroom or to get an insight around an issue they were concerned before getting into classroom.

Chapter Summary

Teachers' collaborative interactions included externally-structured, self-structured, and spontaneous collaborations, but the majority of were spontaneous. Those interactions took place

in particular groupings, with the highest frequency occurring between one pair of teachers (Kayla & Kristina). There were also self-structured collaborations by teachers with a set date and agenda. These were requests from teachers based on their needs, like meetings to prepare for upcoming assessments. There were also externally-structured collaborations where an agenda was set and a date was established without any input from these teachers. However, those externally-structured collaborations were infrequent and were less connected to teachers' regular daily practices.

The three central participants interacted with each other on a daily basis at least once, and these interactions mostly involved a *Checking-In* or *Interchange of Ideas*. Kayla and Kristina typically interacted more than once. *Checking-In* interactions were as important as the *Interchange of Ideas* because they kept teachers' interactions going; teachers were able to keep each other informed about where they were at or what they just did. The quick updates were the glue for longer and more substantive interactions.

Most of the teachers' interactions were face-to-face, either in the hallway or in one of these teacher's classrooms, though there were also exchanges via email. Since Kayla's and Kristina's classrooms were next to each other, they used the four minutes "passing" time to raise or report issues that came up in their classes. The very frequent interaction between Kayla and Kristina was an important nexus of collaboration because unlike Kristina, Kayla was also in a regular interaction with Paul. Therefore, she was the key "linkage" person in carrying issues from one context to another.

The interaction of physical, practical and personal/interpersonal factors shaped the frequency and form of the teachers' interactions. Physical factors included physical features of the context, such as having classrooms next to each other and using common communication

tools. Practical factors included teaching the same grade level and content and using the same curriculum materials, which provided a common context for teachers to structure their interactions. Personal and interpersonal factors also affected the frequency and foci of teachers' interactions, especially their curriculum-related history, professional values, and commitments to teaching. Teachers showed their care for each other and interest in their personal lives which helped them to establish a trustworthy relationship. Considering all these factors, Kayla and Kristina interacted the most, mainly because their classrooms were next to each other, Kayla had a desire to get better in teaching grades seven and eight mathematics, and because Kristina had the availability and experience to address Kayla's questions.

Overall, with respect to content, teachers' interactions were more focused on non-MKT considerations, especially school-level issues (e.g., responding to administration initiatives) and issues related to interactions with students (e.g. responding to problematic behavior). The most frequent MKT-focused conversations were related to knowledge of content and curriculum (KCC). The focus in these conversations were assessments, pacing-related decisions, and understanding the intended development of mathematical ideas. Knowledge of content and students (KCS) and knowledge of content and teaching (KCT) were the next most frequently focused topics, respectively. Unlike students-focused conversations, in KCS interactions teachers tended more to provide reasons in their conversations and other teachers involved in those conversations engaged in the conversation beyond showing their empathy.

CHAPTER 5

HOW TEACHERS COLLABORATE: NATURE OF INTERACTIONS

Teachers' interactions took various forms in different groupings. They considered topics from MKT (Ball, Thames & Phelps, 2008) or other topics that were not focused on teaching mathematics (non-MKT) (Shulman, 1986). In order to have a better picture of these teachers' collaborative interactions, I will provide more details about teachers' collaborative interactions by attending to their nature of interactions. In addition, I will describe whether teachers' nature of interactions is related to the content focus of those interactions. Therefore in this chapter, I address two component questions pertaining to my first research question:

- What is the nature of interactions among teachers during their collaborative activities?
- In what ways the nature of interaction is related to the topics of collaborative activities?

I considered nature of interactions by attending the depth of teachers' interactions and positioning among teachers in their interactions. In describing teachers' interactions the depth was important, because that would provide in what detail they got into in their conversations, whether they provided any rationale for their thoughts or whether they moved their conversation beyond the consideration of specifics to more fundamental and general issues of students, teaching, and learning. The positioning meant the roles teachers took in their interactions. Positioning allowed me to capture certain roles teachers took that characterized their interactions, like if there was one particular teacher who always initiated interactions or another teacher who was expected to provide justifications. Once I describe nature of interactions, I will discuss teachers' nature interactions in relation to foci of their interactions.

In the coming sections, I will provide results addressing the component questions I listed above, starting with frequencies, followed by some illustrative examples. At the end, I will

summarize how all the sub-questions helped me to answer my first research question about how middle school mathematics teachers using CMP collaborated with each other in spontaneous and self-structured ways.

The Nature of Teachers' Collaborative Interactions

When studying the nature of interactions among teachers, my aim was to describe teachers' interactions as they developed. Specifically, I wanted to know who initiated the interaction, and in what depth they discussed topics in their conversations. With these objectives, I analyzed the data considering depth of interactions. To do so, I adapted the *Depth of Interactions* framework from Coburn and Russell (2008). I also wanted to gain insight into the social dynamics of teachers' interactions, in a way to describe how teachers positioned each other in their interactions. For this, I used positioning descriptions described by Greeno (2011).

Depth of Interactions

I considered the depth of interactions on a three-point scale—low, medium, and high. Low-depth interactions meant teachers only provided brief information, their views without giving any rationale. In medium-depth interactions, teachers provided some rationale for the topics they picked up in a conversation. They talked about reasons for their decisions or provided more explanation about their choices in their practices. In high-depth interactions, they moved their conversations from the specific to more general to consideration of other contexts beyond their classrooms or school. For the 280 cases of collaborative interactions with a particular content focus, there were 151 instances of low depth interactions, 94 medium depth and 35 high depth interactions. This meant 54% of these teacher interactions were low-depth interactions. I will provide details about the context as I provide examples of interactions in low, medium, and high depth.

Low depth. Low depth interactions spanned a variety of topics from very personal issues to short content-focused issues. Even though all the interactions in this set involved conversations where teachers only provided their opinions or factual information without providing any reasons, there were contextual details for why those interactions stayed in the low-depth category. I will describe those effects of context by describing low-depth interactions as (1) limited by the issue, (2) limited by context, and (3) limited by choice.

Limited by the issue. Some of the teachers' interactions were limited in its nature, and there was not a lot of room to move those interactions to a higher depth. These usually involved simple requests and responses that did not need a lot of input from the participants. For example, Kristina handed a quiz to a student who missed the assignment in the previous week. She checked with Kayla whether her class was available and she sent the student to Kayla's class to take the test (Field Notes, 02.12.2013). In another example of this kind, Kristina learned from Kayla how to send an email to multiple people at the same time without any identifier information about the other recipients (Field Notes, 03.10.2014). In the first example, Kristina interacted with Kayla to ask for a support and that support did not involve them working together. In the second example, Kristina's question required a factual answer. Therefore both interactions examples were limited by the way they set up.

Limited by context. Other interactions remained at the low level because of the contextual limitations like presence of other teachers, or students in the context or because of limited time. These contextual factors became a limitation in their interactions, so teachers could not get in more depth in their conversations. For example, during break time, Kayla shared how she felt towards teaching the *Looking for Pythagoras* unit by saying "I looked at all *Looking for Pythagoras* stuff but still I am afraid of this book". Without any response from Kristina, their

conversation was interrupted with special education teacher coming by. All three teachers started talking about Christmas decorations (Field Notes, 12.02.2013). This conversation was limited-by-context because Kayla could have provided reasons for her frustration however, there were other teachers around and one of the teachers interrupted this conversation. There were many instances of personal conversations that were limited-by-context. In those cases, the contextual limitation was not an interruption from another person, but it was because the number of participants involved did not allow to a more depth interaction. For example, right after Spring break Kayla, Mr. Norm, Mr. Kullen, Kristina, and Danny talked about their Spring break. Most of them quickly summarized their Spring break with one or two sentences. Since it was four-minute break and there were many people involved in this conversation, the exchange stayed at the low-level (Field Notes, 04.14.2014). With these examples, I can't know whether teachers would have followed up their conversations in more depth or not, in the absence of these contextual limitations.

Limited by choice. I considered an interaction limited-by-choice if teachers stopped their conversation or moved to another topic without bringing any reasons for the things they said. In those cases, I looked for evidence in the context whether there were any restraining contextual factors like time or presence of other people. For example, right after her eighth grade class, Kayla came across with Kristina in the hallway:

Kristina: The summary took the whole hour.

Kayla: So it is taking 2 hours. That is what I am planning.

Kristina: I was planning it to take 1.5 hours but it took 2 hours (Field Notes, 12.02.2013).

Kristina informed Kayla how long her summary took. With the summary part the whole Problem took 2 hours. Kayla did not ask what made summary longer than Kristina planned for,

and Kristina did not provide any details. *Summarize* is the third phase in the CMP instructional model where teachers help students formalize their ideas (Lappan, Fey, Friel, & Phillips, 2014). Therefore, more details about why the summary took longer could have provided some important details about the lesson like what was challenging for students. In this example, Kayla and Kristina did not take their conversation any further than providing updates, even though there was not any contextual limitation preventing them having a longer conversation. As a result, this was a low depth interaction limited-by-choice. The majority of teachers' low depth interactions were limited by their choice (61%).

Medium depth. In medium-depth interactions teachers provided reasons for the assertions they made in their conversations. Any conversation that included reasons, not necessarily comprehensive rationales, was coded as a medium depth interaction. There were some medium-depth interactions, shorter in-length like low-depth interactions, however unlike low-depth interactions, teachers' provided an explanation to what they said. In contrast to a limited-by-choice example I provided above, Kayla had shared again her emotional state about *Looking for Pythagoras* unit but this time it involved some more explanation:

Kayla: In the Looking for Pythagoras unit everything connects so well now. Going up and over really helps making squares. I have seen Problem 3.1. Now it makes sense. I was so freaked out this morning but now it makes sense. That work done in Investigation 1 helped me to make squares.

Kristina: I am glad. That is what matters (Field Notes, 12.02.2013).

Kayla talked about how she started making sense of the mathematics ideas in the *Looking for Pythagoras* unit. She mentioned the going up and over work and making squares. Kayla shared change in her emotional state with a brief explanation of reasons that helped her feel better. In

her explanation, she might have provided more details like in what ways the work she did was helpful but she did not. Therefore, even though there were reasons in Kayla's update of her emotional state this conversation was only one step beyond a low-depth interaction.

In a slightly different medium depth conversation compared to the previous example, both teachers contributed by providing reasons for their views. After Spring break, Kayla and Kristina talked about how Mr. Norm wanted teachers to be firm at the beginning of the semester. They shared their opinions and practices:

Kristina: We should be firm may be more than we are regularly, because the kids think that it is spring. The end of school is here but we are not way near it. We have still nine more weeks to go so we should really make our students see that.

Kayla: That is what I was trying to do [she emailed parents about particular students]. I am feeling like I need to get things straightened up. I am looking back to my emails and nobody responded back yet...

They continued to cite examples from their students and their parents (Field Notes, 04.17.2014). This was beyond a low-depth interaction because teachers did not state just facts like saying "Mr. Norm asked to be more firm at the beginning of the semester," but they also talked about why they agreed with him and what they did in their classes to address that issue. However, this conversation was not high level because teachers did not move beyond the consideration of their classrooms or students. There were a couple of general considerations teachers could have picked up in this conversation like, reasons and ways of communicating with parents, however, this conversation only allowed them to share their thoughts and provide updates about their practices. Generally, teachers' conversations about their students usually stayed within the context of their classrooms, therefore ended up being a medium-depth interaction.

There were longer conversations where all teachers contributed with providing their rationale for their decisions, like assessment preparation meetings. However, most of those examples stayed at the medium depth because teachers stayed within the limitations of the specific assessment. For example, when Kayla and Kristina reviewed the Check-Up quiz for the *Moving Straight Ahead* unit, they had a long conversation about what points each question should get. For a question asking students to make tables and graphs, Kayla wanted to assign four points and Kristina explained why she wanted to give three points:

Kayla: ... I feel so annoyed when the graphs are not done. They don't really know how to scale up... What about labeling axes?

Danny: One point.

Kayla: Axis, scale, graph...

Kristina: They [students] started making tables and graphs in the Variables and Patterns Unit [it is a six grade Unit]. Those are reviewed so I am not giving points for review. Graphs are harder so it is 2 points. If they don't scale up I am going to take their points or they don't label axis correctly take 1 points off. But for linear relationships this is a review (Field Notes, 03.11.2014).

Kayla thought the question deserved four points because some students still struggled in making graphs. When Kayla noted how students did not know how to scale up, she hinted her students' struggle in changing the scale to match the data set. With similar consideration but with a difference stance to this struggle, Kristina thought the question included some review, so she did not want to give four points to a review question. This kind of conversations where teachers provided their reasoning for their choices were routine in their assessment preparation interactions. Teachers' interactions around their assessments were usually the longest in

duration. For example, Kayla, Kristina and Danny talked about this one single assessment for about forty minutes. They provided their reasoning to rationalize their decisions however they did not move their conversation beyond the scope of this specific assessment.

As I have shown in these examples, teachers' interactions moved beyond low depth because they included their reasons for their actions or feelings, even with one or two sentences. Those additional sentences helped conversations to gain more depth. However, those conversations stayed at medium depth because teachers did not move those conversations beyond the specific context of their teaching or students.

High depth. In high-depth interactions, teachers took conversations beyond considerations of specific contexts and framed them in more general terms. These high-depth conversations provided entry points for teachers for broader perspectives in consideration of general teaching practices, students related issues, or curricular choices beyond their classrooms or their school. However, teachers' consideration of the general did not go any further than naming the general issue of a concern as I will provide in the examples below. For example, when Kayla and Kristina talked over Danny's struggles as a first year teacher and moved into general teaching related issues:

Kristina: Danny was concerned that when I was in the class, about all students were naturally straightened up but he wanted to have himself do it. It is not easy. The things he lets go. Those are the things I would let it go. But you need to have the norms set up first and then you can say I can go those things let go. It is also part of who you are.

Kayla: I remember Brenda telling me at one point you cannot change who you are. If you are not a strict person you can't pretend to be. You just find your way.

Kristina: You are doing great with your students.

Kayla: I need to work more on some things.

Kristina: In this school any teacher cannot say they don't have any behavior issues in their classes. It is part of the job we are doing. (Field Notes, 03.03.2014)

Kristina noted how it was difficult to set the norms up. They both noted how a teacher needed to find his or her own way in addressing behavior issues. Kristina also added how behavior issues were embedded in teaching. Their consideration of establishing norms was a general teaching consideration but they also brought their own classrooms and Danny as a context to elaborate more on those general considerations. Yet, when they brought those specific cases into their conversation, those specific examples served to expand the context of their consideration beyond their own classrooms.

Another conversation, between Paul and Kayla this time, opened a door to consider a more general issue about students' lack of engagement. At the beginning of this conversation, Paul and Kayla talked about possible reasons for their students' low performance. They brought up how students, in general, did not know what they did not know before getting their tests back and this led to a more general concern about students. "I know these kids are growing up in a tech-age but we can't entertain them all the time. We are here to teach. We can't always do fun activities. They just need to care for what is going on (Kayla, Field Notes, 01.15.2014). Kayla noted a general problem about students and the complexity of teachers' role in keeping students engaged. In this example, just a small contribution from Kayla moved their conversation from their classrooms and students to the consideration of the broader issue on how to keep students engaged in this high-tech age.

High-depth interactions also helped teachers to consider their curricular choices at a more general level. Recall the Dot Paper example I provided in Chapter IV as an interchange of ideas

(IOI) type activity involving all three teachers. Paul's students took the Partner Quiz in the *Stretching and Shrinking* Unit. There were many students who struggled with a question where students needed to draw their shapes on a dot paper. Paul showed that question to Kayla and Kristina and asked whether their students struggled with the same question or not.

Kristina: They are counting dots instead of intervals and there were a lot of students doing that this year.

...

Kayla: Some of my students wanted to see coordinates so I told them to go for it. But once they wrote down the coordinates it became more difficult.

Kristina: It would be the same problem. They might still get "7" on grid. They don't know how to measure...

Kayla: Isn't this unit is about Stretching and Shrinking, so why do we want to confuse them?

Kristina: This is a partner quiz. Partner quizzes are for them to elevate their problem solving skills. Otherwise we have asked them to do individually.... (Field Notes, 12.19.2013).

In this conversation, teachers considered what students struggle with in general and how Partner Quizzes were different from other assessment tools provided in the CMP. As Kristina noted, partner quizzes were for students to elevate their thinking. These general considerations moved this issue beyond Paul's or Kayla's students struggle with the dot paper.

As in the examples I provided here, teachers' reference to general issues was very short, and they quickly turned back to the specific context they interacted around. I found no examples where the teachers brought up learning theories or considered generally identified

misconceptions. Their consideration of student learning or teachers' practices were limited with what they experienced in their classrooms or what was provided in their materials.

Summary. I used a three points scale-low, medium and high in considering depth of interactions however, as the examples illustrated, three points scale only provided information about whether the conversation met the minimal baseline or not. There was more to say in explaining the depth of those interactions. On one hand, some medium depth interactions were closer to low-depth because the reasons teachers included in their conversations did not provide all the details about their decisions. On the other hand, there were medium-depth conversations that were closer to high-depth because teachers considered many possible reasons but those explanations were within the consideration of a specific context. The high-depth conversations in this context, moved beyond medium depth, because even though teachers cited a more general issue in their discussion of the specific issue that arose in their teaching. They either only addressed the general issue in one or two comments or moved back to the consideration of the specific case they started with.

Social Dynamics of Interactions

In addition to the depth of interactions, I also considered the social dynamics of teachers' interactions. I attended to positioning as systemic and semantic positioning (Greeno, 2011). The first one was about interpersonal relations whereas the second one was about the relation between teachers and mathematics or CMP. The systemic positioning helped me to describe different roles that teachers took in their interactions. I looked at who in the group initiated interactions, who were more likely to challenge and whether there was one person (or persons) in different groupings that was more entitled to provide justifications. For the last two (challenging and justification), I only reviewed the MKT-focused interactions. In addition to systemic

positioning, I considered teachers' positioning themselves with respect to curriculum and mathematics (sematic positioning). I looked for signs that these teachers provided any conceptual agency in their consideration of mathematics or curriculum materials. People with conceptual agency will make choices and judgements as well as question the underlying reasons for accepted practices (Greeno, 2011)

Systemic positioning. There was a change in the roles teachers took in their interactions and systemic positioning helped me to characterize teachers' roles by attending to initiation, justification, and challenging. The representative sketch provided in Figure 9 provides an overview of interactions among these teachers.

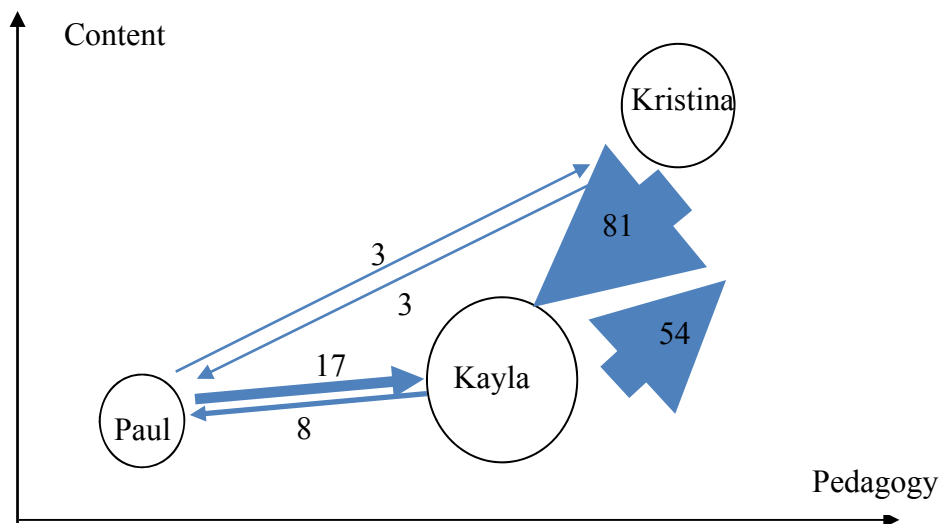


Figure 9

Representation of teachers' positioning each other with respect to content and pedagogy

The weight of the arrows conveys the frequency of the interactions. I represented Kayla with a larger circle because she was involved in interactions with these two teachers the most. The

placement of these teachers over content-by-pedagogy dimensions reflected Kayla's and Kristina's view of themselves and each other. As Kayla noted "I respect for mathematical knowledge for Kristina. Paul... I don't see him pushing himself". (Kayla, Post Interview, 04.18.2014). Kayla considered highly of Kristina's pedagogical skills too. During my observations, Kayla mentioned a couple of times how Kristina did not have as many student-related issues as she had. My observations confirmed Kayla's view of Kristina's expertise in dealing with pedagogical issues and her depth of content knowledge, so Kristina was placed further from the origin on both dimensions. However, it was harder to capture Paul's way of seeing himself and the other teachers, because he was not as vocal and he had lesser interactions than Kristina and Kayla.

The numbers in the arrows showed how many times these teachers started an interaction with one another. Kristina and Kayla both initiated numerous interactions, with Kristina taking slightly more initiation role. In Kayla and Paul pairing, Paul carried the role of initiator more. Paul frequently came into Kayla's class with specific questions either about grading a student work or to share a document with her. During my group interview, Kristina told that she let Kayla drive the meetings with her questions because their priority, for that year, was to address Kayla's needs. Therefore their background and experience also influenced their roles in their interactions (Group Interview, 05.02.2014).

For justification and challenge, I only considered the interactions with MKT focus. There were 54 interactions with a clear justification provided by one of the teachers and 16 interactions where one teacher challenged another. Some interactions included both justification and challenge, so there were some overlaps.

Justification. In 33 of the MKT-focused interactions between Kayla and Kristina, Kristina provided justifications; in the seven, both did so. In five of the MKT-focused interactions between Kayla and Paul, most justifications came from Kayla (one from Paul and one from both). In assessment preparation meetings, as I have provided an example above illustrating medium-depth interactions, teachers typically provided their justifications for their decisions.

In Kayla and Kristina pairing, the justifications from Kristina provided rationales for their curriculum- or teaching-related decisions. For example, an interaction between Kayla and Kristina reviewed one of the students' responses in the Unit test for the *Frogs, Fleas and Painted Cubes* unit. A couple of students incorrectly wrote down 28 as the maximum point.

Kayla: They came up with 28 because they didn't change their scale.

Kristina: I would not give points because they should notice in their table 28 shows up more than once and at least they should know that. (Field notes, 03.05.2014).

Kayla noted how students might have misled by their graphic calculators since they did not change their scales. Kristina, provided a different perspective for why those students did not deserve any points. If those students checked from their tables they would have noticed 28 appeared twice and maximum points needed to appear only once in the table of values for quadratic relations. Kristina provided a mathematical justification for her decision about not giving credits for those students' responses and at the same time her response addressed Kayla's concern about whether it was just a scaling problem or not.

In the next example, Kristina provided justifications to help Kayla consider the development of mathematics over multiple units.

Kayla: All in this Unit [Frogs, Fleas, and Painted Cubes] is quadratics and it is Algebra standard, right? So it is not eighth grade. It is extra for my students.

Kristina: It is nailing down the function idea by adding quadratic functions. They had the inverse function before. Investigation 2 is all about distributive property and distributive property is completely eighth grade (Field Notes, 02.05.2014).

During this interaction, Kayla questioned the need to do all the *Frogs, Fleas, and Painted Cubes* unit in her regular classroom. Kristina addressed Kayla's skepticism by noting how that Unit would help students in developing the function idea. I included this example for a couple of reasons. First, this kind of questioning, especially around curriculum related issues is not a typical of Kayla. In addition, Kristina addressed Kayla's concern by showing her depth of knowledge. She talked about where students were coming from and what they would need. So, in some ways Kristina's response confirmed her role as a person who was entitled to provide justifications.

In the interactions, between Kayla and Paul, three of Kayla's justifications helped Paul understand student work in a particular exam. In one of those, after showing students' poster presentation to Paul, Kayla explained why and how she had students work on posters during *Explore* time. In another case, Kayla and Paul talked about Problem 3.3 in the *Comparing and Scaling* Unit:

Kayla: Do you know Problem 3.3 (grade 7)? I did not like it. It is confusing, tricky.

Paul: It is the baby chimpanzee problem. I know it but I have not done it yet.

Kayla: I compared CMP3 and CMP2 versions. They [authors] threw percent here.

[She read the question in B] I want to skip it [question in B]. It is so confusing. (Field Notes, 03.03.2014).

She also showed her notes to Paul like “too wordy”. In this interaction, Kayla told Paul that she did not like Problem 3.1 and she provided a justification by showing specific parts of the Problem that she found confusing. This interaction is different from the previous interactions I described above. Kristina’s justifications were oriented to understand Kayla’s concern and/or provide a more in-depth look. In contrast, Kayla’s justification provided evidence for her claim that a specific Problem was “problematic”. That was due to her own experience of the Problem.

Challenging. Kayla and Paul did not take on challenging roles in their MKT-focused interactions, but challenge did arise in Kayla and Kristina’s interactions. In the examples given above, Kristina provided justification to address Kayla’s particular concern. In some of those conversations, Kristina brought some new considerations by challenging Kayla’s decisions. In those cases, challenge and justification happened together. In the following example, Kayla and Kristina talked about how the idea of slopes might be more difficult for students than Kayla anticipated:

Kayla: I am gonna start with Problem 4.1 tomorrow and I am thinking of even finishing Problem 4.1 tomorrow. We will start right from B don't we? And I want to have unit test next week.

Kristina: Are you expecting to be done with slope that quickly.

Kayla: I don't know. There is all this rate of change idea. We did not do in this Unit but they did it for a while. So it should not take long.

Kristina: From my experience it really takes a long time. [With a sigh] Taking two points and finding slope is hard.

Kayla: They did this kind of work before, in grade six?

Kristina: They looked at the graph increasing by one and it was easy to say it goes up by 10 each time. And they did not go beyond that. 10, 20, 30... (Field Notes, 04.17.2014).

In this example, Kristina challenged Kayla's grade seven plans. She specifically asked Kayla whether she would be able to finish slopes in her grade seven class as quick as she planned. After that question, she started to explain why she thought developing slope concept could be harder for the students.

There were also cases, Kristina challenged Kayla without any further explanation. For example, Kayla shared her experience with her grade seven class for Problem 3.3 in the *Comparing and Scaling* unit that asked students to solve a problem by using ratios, rates, percent and proportions. The Problem was called "Mixing it Up" and true to its name all important mathematical ideas students learned in that Unit was mixed up in the Problem. Kayla was worried about her students' struggles.

Kayla: They struggled but everything I highlighted based out of TE. They were able to talk about it.

Kristina: Did you need to stress out?

Kayla: No, but they were frustrated.

Kristina: Isn't it okay to be frustrated?

Kayla: I was frustrated because they didn't get what the problem was asking.

Kristina: Isn't that what we want to improve? Improve their reading. (Field Notes, 03.05.2014)

Kristina challenged the way Kayla thought about the Problem. Kayla did not provide any response to the last question Kristina asked and in some ways it was less a question but more a general suggestion for Kayla to consider in her teaching.

Summary. I found evidence that these teachers positioned themselves with certain assumed roles. In the Kayla and Kristina dyad, both shared initiation role, but Kristina was entitled to provide justifications and challenge the other person. In Paul and Kayla dyad, Paul took more initiation role and Kayla provided justifications if there were any, but neither took on a role of challenging the other. In addition to these roles, in these three teachers' interactions, Kayla was entitled to bring questions as the new person in this group. Their self-structured meeting agendas were shaped with Kayla's questions.

Semantic positioning. I also examined how the teachers' positioned themselves with respect to mathematics, and the curriculum. In all of teachers' *Knowledge of Content and Curriculum (KCC)*-focused conversations, there were signs of conceptual agency with respect to curriculum. In some of their interactions, teachers also showed signs of presence of or lack of agency with respect to mathematics. Kristina had high conceptual agency with respect to curriculum but her agency in mathematics was content-dependent. Kayla showed signs of low conceptual agency in both.

Curriculum. Kristina showed her agency in her curricular decisions in all *KCC* situations where Kayla generally showed her lack of agency by expressing her intention to follow exactly what was suggested in the materials. Kayla liked following suggestions in the Teacher Edition as they were provided and she was less willing to make any changes. According to Kayla, she was showing her trust into the materials:

I feel like I do trust this thing. It is not just hodge-podge thing that put together. CMP wants CMP to be taught in certain way. Going to summer training and having the opportunity to talk with authors gives another perspective. It is not just a book. It is a

bible (laughing). If I trust something I usually go after it! (Kayla, Post Interview, 04.18.2014)

As evident from this quote, CMP was the authority for Kayla so she was very careful to enact the lessons as written. She made some minor revisions such as providing a shorter version of a focus question provided in the Teachers' Guide, because she thought the original version gave away mathematics (Field Notes, 03.21.2014). In another case, she made a one-page version of a worksheet which was two pages in its original form (Field Notes, 12.02.2013).

Kristina also trusted the materials, but she also had confidence in her skills for minor or major revisions if needed. That confidence gave her flexibility in her planning. Therefore, she used her agency towards minor changes and for a couple major changes. For example, she combined two Problems or used a different Problem with the same mathematical goal with a different context. I will talk about those minor and major changes in Chapter VI in relating collaboration to classroom practice.

The way Kristina positioned herself with respect to materials was consistent with her interactions with other teachers. She was pretty confident in addressing curriculum-related issues. For examples, in the Dot Paper example I provided above, she was the one who brought up the difference between partner quizzes and other tests: "This is a partner quiz. Partner quizzes are for them to elevate their [students] problem solving skills." There were also times, she addressed Kayla's frustrations or concerns by providing an overview of the development of ideas in a Unit or Investigation, as in the example above where she explained the role of the quadratic functions unit in developing the idea of function.

In contrast, there was not enough evidence for Paul for me to be able to describe how he positioned himself either towards the curriculum materials or mathematics. He noted how he

liked CMP materials with its rich contextual Problems, but he did not explain how he saw his role with respect to CMP. Once Kristina described him finding his own way of teaching CMP traditionally. She also noted how students coming from Paul's class were different (Kristina, post interview, 04.18.2014). Therefore, Paul used his agency in facilitating lessons different from what was suggested in the materials (like assigning Explore questions as homework), but he was not open about his changes and I am not sure even he was aware of those changes.

Mathematics. Kayla's and Kristina's positioning with respect to mathematics was not as evident as the way they positioned themselves with respect to the curriculum. There were two cases where both Kristina and Kayla showed signs of lack of agency; both came from geometry. One was about finding cross-sections of three dimensional shapes (Field Notes, 02.24.2014). The other one concerned transformational geometry. In this Center of Rotation example, Kayla was not sure how to find the center of rotation and she brought her question to Kristina:

Kayla: In TE [Teacher Edition] there is rotation about a point other than the origin or a vertex of a shape. How are they [students] supposed to determine where the center of rotation is?

Kristina: I would not do anything except from around a point [Kristina had a hesitancy in her response, and she started turning the pages of the book].

Kayla: That teaching aid [the one talking about this topic] was really confusing.

Kristina: I am trying to think how kids did that. They did guess and check. When it is vertices it is easy, but... We don't ask them to locate any point of rotation.

Kayla: That is good!

Kristina's response indicated that she was not completely comfortable addressing the issue that Kayla raised. Instead of directly answering Kayla's question, she turned to the curriculum to find

support for her stance that they did not need to worry about that topic. Kristina usually liked to be challenged, but in this case she did not consider Kayla's specific question about locating center of rotation. Kayla accepted Kristina's response without further questions.

Kristina provided evidence of conceptual agency in other topics with respect to mathematics. For example, when Kayla asked Kristina how to test for constant of proportionality Kristina immediately told her to consider checking the y-values for $x=0$ case (Field Notes, 03.10.2014). Therefore, evidence of agency in mathematics changed for Kayla and Kristina depending on the topic. The interactions presented above provided evidence of agency for Kristina in certain topics (e.g., proportionality, functions) but the lack of agency for both Kayla and Kristina in other (e.g., geometry) topics. The lack of agency in mathematics influenced teachers' interactions—that is, in leaving some questions unanswered as in the Center of Rotation example.

Relating Foci of Collaboration to Nature of Interactions

In order to describe how teachers' foci of collaborations related to their nature of interactions, I reviewed my analysis of teachers' depth of interactions by attending to teachers' content focus. I looked at cross tabulations of depth of interactions and teachers' content focus. Teachers' interactions varied in its depth depending on the content focus (Table 8).

Table 8

The variation in depth of interactions with content focus

		Low Depth	Medium Depth	High Depth	Total
MKT	KCT	9	11	3	23
	KCC	22	20	5	47
	KCS	4	23	9	36
	SMK	0	2	2	4
Total		35	56	19	110

Table 8 (cont'd)

Non-MKT	Personal	32	1	1	34
	Context	40	20	8	68
	Teaching	13	3	2	18
	Students	31	17	2	50
Total		116	41	13	170

KCT: Knowledge of content and teaching; KCS: Knowledge of content and students, KCC: Knowledge of content and curriculum; SMK: Specialized content knowledge

66% of teachers' MKT focused interactions (n=110) were high-or medium-depth whereas 69% of teachers' non-MKT focused interactions (n=170) were low-depth interactions. When teachers addressed MKT, their interactions got in more depth, at least by providing reasons into their judgments. The most medium-depth non-MKT focused interactions concerned Students or Context. The teachers' context-focused interactions had the most high-depth interactions (8 of 13 total).

Teachers' student-focused interactions gained more depth as teachers related students to issues of mathematical content (KCS) (See Figure 10).

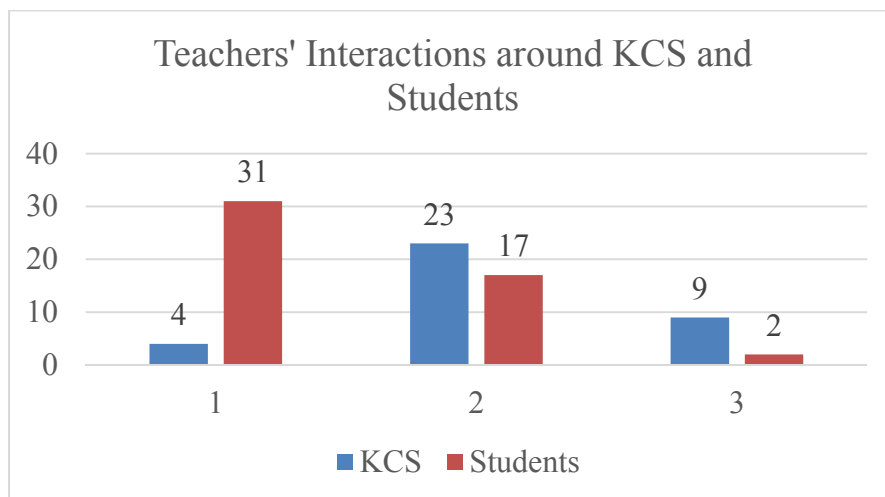


Figure 10

Frequency of KCS and student-focused interactions

Most KCS-focused interactions were medium-depth, whereas the student-focused interactions were more frequently low-depth. This meant teachers brought less explanation to their conversations about their students when they did not focus on mathematics. I will provide two examples to illustrate differences in teachers' interactions. One KCS-focused example concerned students' understanding of the change in area in the *Stretching and Shrinking* unit.

Kayla: How did Problem 1.3 go in your class? Did they figure out area change as scale factor times scale factor?

Kristina: They compared areas by putting one onto another one and finally a couple of students said "I don't know what is happening but it is different". That is what they need to hear at this point. (Field notes, 12.03.2014).

In this example, Kayla's and Kristina's focused on students' understanding and Kristina explained how her students figured out the area change. Since she provided an explanation for how her students' reasoned, this interaction was coded as a medium-depth interaction. It was medium-depth because both teachers addressed how students did or might have reasoned on the problem.

The second example concerned a couple of particular students in Kayla's class. This example is different from the previous one because teachers' focus in this one is on their students' behavior issues.

Kayla: Bill [a student in Kayla's class] thinks himself as very popular. He wears expensive clothes and has this high self-confidence and he is the only child. I hope Eryn [Kayla's daughter] does not turn out to be like him. Sarah [a student with special needs] is sitting on a Pilates ball [an intervention to address that student's needs] during class. She is a mat- lab student. Trying hard, really hard. She is a modest kid. Bill raised his

hand and asked “how come she had a ball and we don’t have. It is very distracting”.

Another student agreed. I was very mad. I told him if he knew how to sit in the class maybe he would not even notice Sarah was sitting on a ball. He is getting enough love at home.

Kristina: So you started punching him on face. Just kidding [both laughed].(Field notes, 04.02.2014).

In this conversation, Kayla explained what happened in her class, bringing details from her own perspective and reasons for her actions. So I coded it as a medium-depth interaction. It is less dialogic and the associated reasoning is less informed from professional considerations. Kristina supportive response, suggested how she empathized with Kayla’s frustration.

Even though both interactions involved students and were medium depth, they were different in important ways. In the KCS-focused example, Kristina provided a justification for why she was satisfied with her students’ response. There were also signs of her conceptual agency with respect to curriculum because she said “that is what they need to hear at this point”. So she sounded confident about what students needed to understand from that Problem. In the student-focused example, Kayla provided a justification for her actions. Her justification was not about solving the issue she just described but it was more to justify her actions in response to Bill. During this interaction, Kristina was mostly listening except from her empathetic response at the very end. Both teachers’ interactions rated as medium depth but the nature of their interactions were still different considering teachers’ positioning each other and the motives for teachers’ justifications.

In non-MKT focused conversations, the teachers’ interaction achieved greater depth when their focus was context. Context-focused interactions generally involved school issues. For

example, in one medium-depth interaction, Kayla and Kristina talked about a question Mr. Norm's (the school principal) asked to Kayla:

Kayla: Mr. Norm came by and asked my perspective on submitting grades. I am not sure if he really trusts me or he is checking on me. I know he comes and gets your opinion.

Kristina: He really trusts you and he wants to get your perspective. He looks at our classes and knows how much we care about our teaching and respects us. There are some people who he trusts like us and some who he does not (Field Notes, 04.17.2014).

Kayla was skeptical about Mr. Norm's question. Kristina did not respond to Kayla's question by just saying "yes, he trusts you" or "no, he does not trust you". She elaborated why she thought Mr. Norm trusted them both. The actions of administrators were the most frequent focus of teachers' context-related interactions (26%). Most were medium-depth (44%), but there were also low-depth (28%) and high (28%). The teachers' context-focused conversations gained more depth in the issues where teachers talked about their administrators. This finding might be specific to this site because there was rising tension between administrators and teachers around issues of observation evaluations and long discussions between teachers about administrators and their practices.

Overall teachers' KCC-focused conversations stayed more on the low-to-medium depth. The high frequency of low level KCC conversations can be explained with quick pacing related check-in (CIN) type interactions among teachers. One feature of KCC-focused interactions was that they were diverse in content. All four activity types were captured. Joint work (JTW) activities predominately addressed KCC-related topics. In addition, there was one example of during class time (DCT) activity with KCC focus. The majority of KCC-focused interactions involved Kayla and Kristina, in varying depth; only two involved Kayla and Paul.

Chapter Summary

In this chapter, I described the nature of teachers' interactions and in what ways the nature of their interactions related to the foci of those interactions. Specifically, I analyzed the depth of teachers' interactions (Coburn and Russell, 2008) on a three-point scale (low, medium and high) and teachers' positioning of each other and their positioning themselves with respect to mathematics and the curriculum (Greeno, 2011). Positioning captured the roles teachers took that characterized their interactions, (e.g., who typically initiated interaction and who was expected to respond).

The majority of teachers' interactions were low-depth because their interaction was limited by the issue, by context, or by choice. Some interactions were limited by the issue; they were simple requests and responses. Other interactions stayed at the low level due to contextual limitations during the interactions like another teacher joining to conversations. The majority of teachers' interactions appeared limited-by-choice: Teachers stopped their conversation or moved to another topic without explanation, even though they had time to continue their conversation and there were no obvious contextual limitations.

Medium-depth interactions were also frequent and showed more variety in terms of length and rationales the teachers provided. There were times teachers only explained reasons for their decisions with one or more sentences without attending to all the details like when they provided updates on how particular problems went in their classes. They usually gave a report by saying it went well or not and added one or two sentences explaining their perception of the lesson without talking about what students did or which parts of the problem they worried the most. There were times teachers' considered issues from multiple perspectives where they provided reasons, attended to details and illustrated the topic of their conversations with

examples from their classroom or teaching. Teachers' high-depth interactions mostly addressed a general issue in one or two comments and moved back to the consideration of the specific case they started with like when they talked about what a mentor does.

Justification and challenging roles were mostly taken by one teacher (Kristina). She had more agency with respect to both mathematics and the curriculum. She was expected to provide justifications to and challenges for Kayla, while Kayla was very comfortable to bring any of her questions from simple pacing questions to more sophisticated questions aiming to understand the development of mathematics over a unit. Since Kayla had low conceptual agency with respect to both mathematics and curriculum, she rarely question Kristina's decisions and suggestions.

The depth of teachers' interactions varied depending on their focus. Overall more than half of teachers' interactions were low depth, however MKT-focused interactions typically shifted to medium or high depth. Teachers provided examples from their classrooms and talked about reasons for their concerns or decisions in their MKT-focused interactions. Therefore, even when teachers' activities were the same, mostly *Checking-In* or *Interchange of Ideas*, depending on content focus, the depth of their interactions differed. This difference was most evident in teachers' students focused conversations, where the majority of student-focused conversations were low-depth (e.g., when they talked about their students outside of teaching mathematics). Yet, teachers' MKT-focused conversations included reasons and picked up in more detail therefore mostly medium or high depth.

CHAPTER 6

COLLABORATIONS RELATING TEACHERS' PRACTICES

In this chapter, I address my second research question in which I describe how teachers' collaborative interactions related to their practices. One of the motivations for this study was documenting ways of teacher collaboration when there was not an imposed upon structure or set goals for teachers because literature noted how externally structured collaborations were not responsive to teacher's needs (e.g., Borko, 2004). In order to make a claim that teachers' collaborations in spontaneous and self-structured ways were responsive to their needs, I needed to document if teachers' self-initiated collaborations were related to their practices in some ways. With this motivation I posed the following questions:

How do self-structured and spontaneous collaborations relate to teachers' practices?

- a. In what ways do the collaborations arise from concerns or inspirations in the teachers' practice?
- b. In what ways do these collaborations relate to subsequent actions by teachers in the classroom?
- c. Is there evidence that the ways that teachers collaborate relate to professional actions they take outside of the classroom?

I was interested in three actions regarding these teachers' professional lives: classroom practices, collaborative interactions, and professional actions outside of the classroom (see Figure 11).

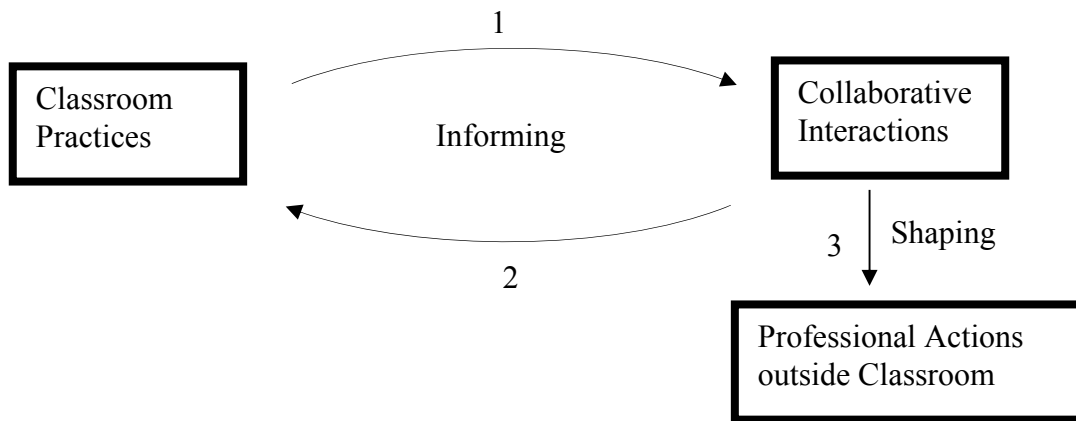


Figure 11

Analyzing teachers' three actions with respect to each other

Specifically, in the first question, I was interested in how teachers' classroom practices informed collaborative interactions (labeled as linking arrow 1 in Figure 11); in the second question, I wanted to know how teachers' collaborative interactions informed classroom practices (labeled as linking arrow 2 in Figure 11); and in the third question I looked for evidence for whether teachers' collaborative interactions shaped their professional actions outside their classroom (labeled as linking arrow 3 in Figure 11). In analyzing teachers' classroom practices and professional actions, the focus was on the individual teachers. The presence of these three arrows suggests that teachers' collaborative interactions were instructive for teachers' professional practices.

Linking arrow 1 represented cases where teachers shared from their in-class practices with their colleagues. For example, if a teacher shared how one of her students did not follow directions, that counted as an evidence of classroom-related teacher collaborations. Similarly, if a teacher provided a quick update of which Problem she did in her class that was also counted as

classroom-related collaboration because she was specifically referring to her in-class practice even though it did not include details of her specific activities.

For linking arrow 2, I looked for evidence of teachers' collaborative interactions leading to change in classroom practices. I looked at teachers' spontaneous and self-structured collaborations. However, I also reviewed lessons before or after collaborative interactions to see those connections. Changes included teachers telling students how they decided to skip a Problem together to more substantial changes like replacing a Problem in the Unit with a new one if that was informed from teachers' collaborations.

Linking arrow 3 represented any of the teachers' collaborative interactions that related to other individual professional practices. One of the teacher's extra-work on a set of materials recommended by another teacher was an example of this kind of individual professional action. Some professional actions took place outside of the school boundaries and I only became aware of these when teachers informed me about those practices. Therefore, there were limitations in collaborations relating to professional activities arising from the data collection process.

Out of 280 interactions coded for depth of interactions, 124 instances were about classroom practices informing collaborative actions (linking arrow 1). Teachers' 44 collaborative interactions provided evidence for informing their classroom practices (linking arrow 2). For collaborations relating to professional activities (linking arrow 3), there were only 12 places identified for presence of such a relation. In the following sections, I examine each of the questions more closely.

Teachers' Classroom Practices Informing Their Collaborations

In this section, I share results addressing the first sub-question relating to linking arrow 1. To describe how teachers' classroom practices informed their collaborations, I related what

happened (concerns or inspirations) in the classroom to the content of teachers' collaborative interactions. Concern is defined as "an uneasy state of blended interest, uncertainty, and apprehension" and "a matter of consideration" (Merriam-Webster dictionary). Inspiration is defined as "something that makes someone want to do something or that gives someone an idea about what to do or create: force or influence that inspires someone" and "a good idea". I mapped concerns and inspirations to teachers' collaborative activities when I noticed teachers explicitly bringing up "matter of considerations" or "ideas" about their classroom practices. I also analyzed if there was an overlap in the topics taken up by teachers and "matter of considerations" or "ideas" that I observed in class situations and if two events happened close to each other (e.g. an issue arose in the classroom and conversation took place during the break following that class hour).

Teachers carried their concerns or inspirations into their conversations, however those conversations were more than simple complaints and idle conversations. For most of the cases, teachers brought up classroom issues trying to find ways to engage every student in their classrooms. As I will illustrate in the examples below, those conversations happened with a desire not to overlook any student in their class. Yet, there were times they voiced their concerns not necessarily to find a solution but with a desire to let other teachers know that issues they brought up were common teacher problems. For example, Kayla talked about a need to communicate such issues openly when she said:

I worked with teachers in the past where it just sounds like they don't have any classroom management issues. Like everything is just so perfect. And I kind of like, not that I want to admit that I screw up, but I do. I want people to know that this is real and if you have something that is not going on your way I want to know it. I guess I kind of know.

Because I want to know that we all screw up together” (Kayla, Post interview, 04.18.2014).

As Kayla noted, teachers’ conversations about issues in their classrooms, whether about their students or teaching, also served to bring real problems of teaching to the surface.

In this specific context, the majority of teachers’ concerns or inspirations from classroom situations either arose from students or related to particular aspects of their teaching. There were more attention to concerns than inspirations. Teachers’ concerns about their students captured 39% of the data and teachers concerns about their teaching captured 35% of the data. In the coming sections, I unpack concerns and inspirations separately as: 1. Concerns about students 2. Inspirations from students 3. Concerns about teaching 4. Inspirations from teaching.

Concerns about Students

It is commonplace for teachers to bring up student related issues since their interactions with students was a big portion of what they did (Ben-Peretz & Schonmann (2000). Teachers verbalized issues they worried about in their classes for a variety of reasons: in some ways, to feel better about it, sometimes to get a different perspective, or to have more information on a particular student. In these conversations, they brought up concerns about students’ behavior or understanding to their colleagues.

Students’ behavior. Issues related to student behavior usually came up once or twice each day. Problem students were the subject of quick shares as soon as teachers got out of their classrooms. Kayla usually provided updates about her students’ engagement with statements like, “some were on task finishing all the questions and some wrote nothing” (Field notes, 12.19.13). She mostly got a response suggesting other teacher’s understanding of the situation or a response providing a similar example from the other teacher’s classroom.

There were times when teachers were more elaborate about their concerns if it was about a particular student. Verbalizing their concerns about particular student's behavior also helped teachers to think through the problem. For example, Kayla brought up Isla, one of her high achieving grade seven students. She did not like how Isla told Kayla that she was not happy the way mathematics class was run. Isla said "you should give examples and ask us to do something, but you have us do all the work by ourselves". As Kayla was quoting from Isla she raised her hands joyfully and told Kristina that "this is what I want to hear, so I do things right!" Kristina agreed with her by saying that was more of a compliment than a complaint (Field notes, 12.03.2013). On one hand, Kayla brought up Isla in her conversation because she did not like her attitude in expressing her thoughts about Kayla's teaching. On the other hand, she wanted to bring the issue of conflict between students' perception of experience of learning and the way mathematics classes were run in CMP classrooms of these teachers. In addition, Isla was keeping Kayla busy for some time, Kayla even asked her parents for a conference, so it was time for Kayla to verbalize what she experienced with her and how she was going to address this issue.

Kayla brought up Isla in another conversation when Kristina was talking about how she kept her advanced students busy instead of letting them read their books when they were done early. Isla was one of early finishers in Kayla's class and once she was done, she read books. They talked about Isla's attitude again and her lack of desire to do extra work (Field Notes, 03.03.2014). Kayla wanted to find a way to address Isla's needs but she had not come up with one so far. Therefore, Isla was a student that kept Kayla concerned because of her desire to address the needs of all her students.

It was rare to hear from Kristina about her students' behavior issues. If she brought one up, though, she usually provided her way of dealing with the issue. For example, in the hallway, Kayla and Kristina were talking about some of the girls who liked talking. Kristina showed a couple of girls and said "those girls are really chatty boxes". She continued with "I had a couple in my fourth hour and asked them whether they wanted homework tonight. They stopped talking" (Field Notes, 01.09.2014). In this case, Kristina brought up a potential behavior issue from her classroom by explaining how she resolved the issue. Kristina's motive to bring this issue to the conversation might also be a way to give a colleague advice without explicitly stating it that way.

Teachers also brought up their concerns about their students to make some classroom problems more visible to other teachers. For example, Kayla usually brought up Carson's behavior issues to her colleagues. Even though Kayla tried many different strategies with him, like communicating with his mother and using behavior cards, she could not resolve the issue. All the teachers knew the case of Carson in Kayla's classroom. Thus, either Kristina checked with Kayla or Kayla provided an update about Carson on a regular basis. One day, as Kayla was talking about some behavior issues with some of her grade eight students, she brought up Carson again. Kristina told her not to worry about him. She recommended she send him to the copy room or library, or if it was her 5th hour [Kristina's planning hour] she could send him to her class. She said she would send him to the corner and would not pay any attention to him (Field Notes, 03.03.2014). Like Kristina used Kayla as a support, she provided her availability to become a support if Kayla needed. However, Carson and other problem students were not necessarily brought up in conversations to find a solution. Teachers sometimes just wanted to communicate their frustrations as Kayla told in our interview:

I also think we are just humans. You just got to release. I can go home and talk to my husband about it but sometimes I tell him I just need teacher talk because teachers understand teachers. May be that just helps because then you know you are not the only one like “oh good he is a jerk for you too!”, “Thank goodness, I am not the only one.” I don’t know, I know I keep talking about Carson but I think he is my biggest struggle. I don’t think that there is a solution for him... I think I want to know that it is happening in other classrooms too so I am not the only one. Like I am not screwing it all up. I want to know it happened in all other classrooms too (Kayla, post-interview, 04.26.2014).

Therefore, the purpose of bringing up discipline issues was sometimes just to hear each other’s concerns. Bringing students’ discipline issues to conversations may not always bring solutions, but it might help teachers make teaching-related issues more transparent.

Students’ understanding. Teachers brought up their concerns about students’ understanding to their peers’ for the purpose of addressing the issue better. For example, Kristina brought up a student in her conversation with Kayla to learn about Kayla’s observations of that student. Kayla had that student in her class the previous year.

Kayla: He communicates really well verbally in the class but he just makes silly mistakes in the test. You think he got 100 % but in the test he can’t.

Kristina: Same this year. He messed up on his test a little bit (Field notes, 02.24.2014).

This particular student was good in the class but Kristina was concerned about him because he did not do well on the test. Since Kayla had him last year, Kristina wanted to learn her observations of him. This exchange about one particular student provided additional perspective to a student in Kristina’s class.

Paul brought up his experiences with particular students more frequently as a response to what the other person said. One time, Kayla and Paul were in a conversation where they talked about students' low performance on a test.

Kayla: I know I always complain about my first hour, eighth graders. But we were doing this wheel of Theodorous and they were looking at me without knowing what they needed to do. Legs and hypotenuse. This is what they have been doing. After check-up and unit test they start asking questions and start to realize they did not know.

Paul: I have Tony who missed the class Thursday, Friday and Monday. He was in the class and he was not even paying attention. I told him to listen. It was five or six minutes and he lost his focus again. I cannot stand next to him all the time and tell him pay attention.

Kayla: I know these kids are growing up in a tech-age but we can't entertain them all the time. We are here to teach. We can't always do fun activities. They just need to care for what is going on (from field notes, 01.15.2013).

Sometimes teachers' student-related conversations allowed more general issues to surface. In this one, Kayla and Paul noted how students were not aware of their needs. This example also illustrates the overlap in their conversations between students' behaviors and their understanding. Students with behavioral issues were usually academically challenged students. Once teachers got into details about these students, they usually brought up issues about those students' behavior issues in addition to sharing their concerns about their learning. There were many other cases where Kayla and Paul or Kayla and Kristina reviewed student mistakes from an assessment and talked about particular students who did not do what they needed to do during class and did poorly on the test.

In another example, teachers moved their concerns about students' understanding to cross-disciplinary considerations. Paul, Kayla and Mr. Kullen shared how students could not make connections in their learning. The conversation started when Mr. Kullen shared how his students did poorly when he asked them to find percent change. Kayla was surprised when Mr. Kullen gave this example because students were doing *Stretching and Shrinking* Unit in their mathematics classes and that Unit was all about scaling up and down. These teachers started bringing examples from their classroom where students needed to make connections but could not. Kayla brought up her eighth graders' struggles in finding the cube root of negatives numbers. It was a topic students did in the previous year. Paul gave an example from his class:

I was talking about change in area. You know how scale-factor changes. They were not getting it and there was a pizza flyer. I started talking about ordering pizzas in two different sizes and asked which one they would order and why. You know what they thought, I gave up and started talking about a different thing (Field Notes, 12.09.2013).

These specific examples from their classrooms helped teachers to identify a common issue about students' understanding. They were all bringing their anecdotal examples, but those examples provided a stronger ground in their claim about students' struggle in making connections within the discipline, across disciplines, and with mathematics and real life situations. This conversation also helped the mathematics and science teachers be more knowledgeable about what students were doing in both classes.

Summary. Student-related concerns were almost equally divided as concerns about students' understanding and concerns about students' behaviors. Both came up during teachers' interactions on a regular basis. Each teacher had their own way of expressing those concerns. For example, Kayla was always vocal about what happened in her class, whereas Paul usually

brought up cases from his class as a response to Kayla's concerns, and Kristina typically shared issues with her solutions. Teachers voiced their concerns, sometimes to get a different perspective or to find a way to deal with the situation. Voicing their concerns about students' also served as a place-holder which helped them revisit the same problem from different aspects for the purpose of finding a better solution. In addition, Paul noted how those shares made teachers accountable to each other by publicizing their concerns (Paul, Post Interview, 04.20.2014).

Inspirations from Students

Inspirations came from students' different approaches or questions. Teachers were very excited to share their students' exemplary work or different approaches to the same problem, however there were very few of those. They all made up only 7% of the cases of the linking arrow 1. Inspirational students' work provided an opportunity for teachers to introduce additional work which would deepen their students' understanding of the content, or provided a different perspective to the topic of exploration, or increased teachers' repertoire of anticipated student responses. So these unexpected student questions or work, made their way into teachers collaborative interactions.

I will share four of these cases, two from Kristina and two from Kayla's classrooms. The examples, I will provide here are different not only because they come from different classrooms but also they served different purposes. The first two examples were from Kristina's classroom and those increased teachers' repertoire of student responses and teaching materials. The last two examples were from Kayla's classroom and those examples provided an opportunity for teachers to have a closer look at the topic of exploration.

In the first example from Kristina's class, she prepared a worksheet as a follow-up to an interesting student question. In the *Stretching and Shrinking* unit, Problem 2.1, students were expected to draw various shapes by using the rules and coordinate points provided in the book. There was a main character Mug Wump with (x,y) rule and some other characters with different rules. For example, character Zug had the $(2x, 2y)$ rule. First, students needed to find some other points not given and represent characters on a graph by using those points. One of the students in Kristina's class asked about the case of negatives. She asked whether they could have negative coordinates or side lengths. Kristina responded to that student by noting the possibility of having negative points. She told the class that she would come back to that question later. As a follow-up to that question, she prepared extension work with negative numbers so characters would end up in different quadrants. Once she had the worksheet ready, she gave a copy of it to Kayla and shared what happened in the class with her. Later, Kayla used that worksheet with her advanced students as extra-work (Field notes, 12.02.13). One student's question about the case of negative numbers provided an opportunity for Kristina to expand the work to four quadrants and also provided an opportunity to make connection to one of the prior units, the *Accentuate the Negatives* unit where students worked with negative numbers. Therefore, Kristina was excited about this unique opportunity and she shared the details of this experience with Kayla. Kayla knew why Kristina prepared this extension worksheet and she had a copy of the worksheet if she wanted to use with her students.

Another inspirational student work collaboration came from Kristina's classroom. It was also from the *Stretching and Shrinking* unit, Problem 2.1 that I described above. After students drew their characters, they needed to compare characters to identify which characters were in the Wump family. The characters were counted as in the Wump family if they were mathematically

similar to the Mug character. Kristina left the question open and let students bring different strategies in making their decisions. During the *Summarize*, Kristina also asked students to create some additional characters. For example, Blug was a character students created in the Table 9.

Table 9

Work students were expected to complete with different characters

	Mug	Zug	Blug
Rule	(x,y)	(2x,2y)	(2.5x,4y)
Point	(2,1)	(4,2)	

One student, Gavin, told Kristina that he wanted to think of points as fractions and multiply by the rule.

For Lug

$$\frac{2}{1} \times \frac{2}{2} = \frac{4}{2} \text{ these were equivalent fractions.}$$

For Blug

$$\frac{2}{1} \times \frac{2.5}{4} = \frac{10}{4} \text{ these were not equivalent fractions. So Blug was not from the Wump family.}$$

Gavin's approach was different from typical student responses. Students typically compared sizes of the characters whether they were enlarged or reduced the same or not, and some compared rules like Zug was in the Wump family because both x and y were multiplied by two. Kristina was very excited for this strategy and she could not wait to share that with Kayla. She jotted down what Gavin said and waited in the hallway for Kayla to come out of her class. As soon as she saw Kayla, she started explaining what Gavin did. She added how other students listened to him and were not confused at all. Kristina said she found Gavin's work very impressive because he used the idea of ratios and equivalent fractions which would come up in

the last Investigation partially and in the next unit. Kayla looked at Gavin's work carefully and told Kristina that it was very impressive (Field notes, 12.03.2013). This inspirational work from Kristina's classroom informed Kayla about an unusual student approach in considering a question provided in the book.

The next two examples from Kayla's class, provided a more in-depth look at a topic of consideration in mathematics. In this first one, Kayla asked her students to consider a statement saying "if there is not a correlation between two variables it means there is no association between two variables". This statement was from a formative assessment lesson Kayla and Kristina needed to teach for a project led by Ms. Patz. When Kayla and Kristina reviewed the lesson together they had to think about this statement for a while. They were challenged with the question and were very curious about how students would do in the class. After teaching this lesson in her two eighth grade classes, Kayla came to Kristina's classroom to share one of her students' explanations to this question. This student responded to the question by bringing other relationship examples like exponentials and quadratic relations (see Figure 12).

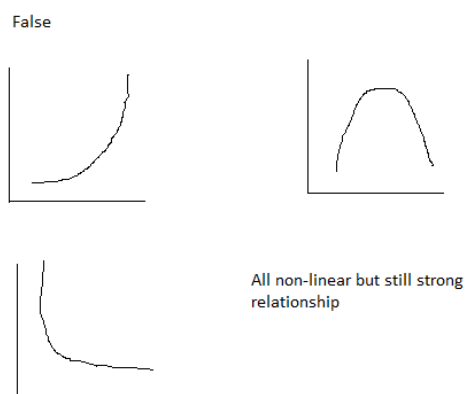


Figure 12

Kayla's student's solution to correlation question

Kayla found this student's answer impressive not only because both Kayla and Kristina considered the question possibly challenging for their students, but also because no students in Kayla's previous hour came up with such an explanation. In addition, even Kayla and Kristina were challenged initially, but this student seemed pretty comfortable in addressing the question and provided other examples that Kayla and Kristina did not consider initially.

In another example, Kayla's students challenged her by asking whether zero was a rational number, an irrational number, or both. Kayla brought up this question to Danny and Kristina. Both Kristina and Danny told her that zero was a rational number. Kayla was still not entirely convinced. Danny explained by giving examples of cases like $\frac{0}{2}, \frac{0}{4}$ and said they could write zero as rational numbers (field notes, 01.15.14). Students' in her class made Kayla question one of the basic things she might not have questioned before. However she knew that she could get help addressing this question. Kristina and Danny helped Kayla answer this question not only for her students, but for her own learning too.

As it was in all four examples, some unusual student performances excited teachers and created a desire to share those inspirational students' work with each other. With their spontaneous share-outs of unexpected episodes, these teachers informed each other about their classrooms, sometimes fine-tuning their own understanding. In addition, they enriched their repertoire of strategies and students' responses.

Concerns about Teaching

Teachers shared their concerns about teaching, especially if there were some components of their lessons they were dissatisfied with or if there was an issue they noticed and wanted to share. Similar to the concerns about their students, these shares provided room for teachers to

voice their concerns and in some ways helped them to get over frustrations, but more evidently their shares were for the purpose of seeking feedback. Except from a couple of more general teaching considerations voiced by Kristina, like teachers' role during group work, concerns about teaching were overwhelmingly initiated by Kayla's concerns and most of the time Kristina provided feedback. In the examples below, I aimed to depict teachers' overarching concerns with specific cases from particular lessons. These teachers wanted to improve their teaching by moving from "spoon feeding" their students but at the same time making sure learning was taking place (Kayla, Log Entry, 12.19.2013).

Kayla and Kristina regularly used the CMP Instructional Model, Launch-Explore-Summarize, which was recommended in the CMP Teachers' Guide. *Launch* is an introduction to the Problem, *Explore* is time for students' interactions with the problem, and mostly students work in groups. *Summarize* is the final phase where teacher helps students put all mathematical ideas together. During the *Explore* phase, Kayla and Kristina were quite active. They sometimes quietly observed students' work, sometimes took notes, and responded to student questions or posed additional questions. *Explore* and *Summarize* phases of their lessons came up in their conversations a couple of times. They wanted to articulate clearly the expectations from each of those phases as recommended by CMP.

One student in Kristina's Algebra track classroom told her how she made her nervous when Kristina looked at what they were doing over their shoulders. Kristina asked about her experience in other classes. Other teachers, as the student responded, were at their desks during students' group work time. Kristina talked about this conversation with Kayla. Kristina noted, for her, walking around the students was the best way of knowing her students. She brought up some other teachers' complaints about not knowing their students well and added, "that is how

you know your students. You walk around, you talk to them, joke with them.” Kayla agreed and told her how she felt awkward if she ever sat at her desk. In addition to knowing what students were doing, Kayla was also worried about what other people would think of her if they saw her sitting at her desk (from field notes, 02.24.2014). This discussion confirmed to each other that their definition of group time was different from other teachers’ way of using group time. They identified the similarities in their practices. In addition, they both voiced how it was not a good practice to sit at their desks if they wanted to know their students better.

The *Explore* phase of a lesson came up again in a follow-up conversation that started with Kayla’s worry about her students’ struggle in one of the Problems. For the same Problem, Danny told that his students did great. Kristina and Kayla followed up this conversation one day after, during Kayla’s planning hour. Kayla brought up her concern about students’ struggle and questioned whether she did something wrong because Danny noted that his students did great when working on that Problem. Kristina reminded Kayla that, as a novice teacher, Danny might have had different expectations for the students. She voiced her concern about Danny’s use of Explore time:

He is not really looking at students' work in the Explore or using Explores to plan for summarize. He just walks around or he sits on one table and tutors a group of students for 15 minutes” (Field notes, 03.03.2014).

Kristina noted different expectations and roles Danny and Kayla took on during Explore phase. Kayla walked around and looked at students’ work during the *Explore* so she might have a better view of what students were and were not doing. Following this conversation, Kristina told Kayla how Danny spent too much time on Explore, leaving less time to *Summarize* (from field notes, 03.24.2014). These short and contextual conversations, where they brought their concerns and

experiences from their classrooms, helped Kayla and Kristina question what should be a teacher's role during the *Explore* and *Summarize* phases of instruction.

Along these lines, Kayla brought up classroom examples to voice her concern about acting too much like a traditional teacher. During the *Say it with Symbols* unit, there were many times Kayla complained about how she felt like she was in her own high school class by having her students work on simple questions over and over again. In her log entry she noted "Kristina and I discussed *Frogs, Fleas and Painted Cubes* launch since I felt so traditional, but I think it will improve as we go further into the unit" (Kayla, log entry, 02.07.2014). As it was clear from this statement from Kayla, she was not happy with the way she led some of the classroom discussions. She voiced this concern again when Kristina shared a variety of strategies her students came up with in the Problem 3.5 in the *Moving Straight Ahead* unit. They reviewed the strategies students came up with.

In this Problem context, P represented the profit, I was for the income and E was for the expenses. Students needed to find the profit. Kristina was happy with the variety of strategies that came up in her class, so she shared those with Kayla. After seeing the variety in students' equations, Kayla reflected back to her own class. With a worried tone of voice:

Kayla: I don't know. I am holding my students' hands too much, I think. This meant they did not come up with a variety.

Kristina: You have special need students.

Kayla: Still.... They all came up with $P = I.E$ and we talked about different ways to express I. I asked them we can express 6 as 2×3 and as 6. They all started saying maybe he should not be selling cakes but PIE's. It came to we should all get PIEs. [They both smiled] It would be fabulous to have pies. (Field notes, 04.16.2014).

Kayla questioned whether it was her teaching that led students into one direction. Kristina acknowledged Kayla's special needs students; however, as a reflective teacher, Kayla was aware that she still needed to provide more room for explorations. The conversation stopped when they moved from PIE as an equation to PIE as a food and Kayla's concern about how to help her students without guiding them too much was left unaddressed.

In addition to these concerns hinting a more general consideration about teaching, there were times Kayla's concerns focused on specific Problems in a Unit. In those cases, Kristina with her diagnostic approach helped Kayla better articulate which part of her lesson did not go as planned compared to the expectation as outlined in CMP. For example, after teaching Problem 1.3 in the *Stretching and Shrinking* unit, Kristina asked Kayla how her lesson went. Kayla responded in a frustration:

Kayla: Awful! Numbers were awful and I could not go where I want to go.

Kristina: They don't know what to do or how to compare. They just say 90% 100% without actually measuring.

Kayla: I did not have rulers or angle measures out and they were just sitting. One student asked "can we measure" and I said go for it (Field Notes, 12.02.2013).

In response to Kayla's expression of her frustration with this Problem, Kristina told Kayla that the expectation from that Problem was not to have students do accurate comparisons. As a response to Kristina, Kayla explained how students got into measurement even though she did not intend it. After Kristina's diagnostic response, Kayla brought up this issue to Paul in a conversation with him and told him how Kristina did not have students do measurements (Field Notes, 12.03.2013). Therefore, this conversation helped Kayla identify the issue better and informed her about the goals of this Problem.

With these conversations expressing concerns about teaching, teachers articulated better what their role should be as a teacher in their classrooms. When they talked about their teaching, they compared their classrooms to each other which informed them what was alike and different in their classes. Some of those differences were related to having different groups of students but some were not. As they talked more about it, general concerns like “what I can do to be a less traditional teacher” stayed with them longer and they revisited them. However, if it was an issue about a specific part of their lessons, sometimes it was easier for them to provide an accurate diagnosis and a solution as it was in the example of Problem 1.3 in the *Stretching and Shrinking* unit.

Inspirations from Teaching

Teachers shared examples from their teaching if they noticed that their students were more engaged with the lesson than was typical or if the lesson involved non-routine components. In addition, Kristina brought up her teaching in her conversation with Kayla to show her that there was room for change in teaching CMP because she knew Kayla usually followed what CMP recommended. These teaching inspirations captured 15% of the data. Therefore, there were more instances of this type of sharing compared to student inspirations but still a lot less than concerns.

Teaching-related conversations happened more between Kayla and Kristina but there were a couple of cases when Paul and Kayla shared teaching episodes with each other. One example was from a lesson Paul taught differently because of an inspiration from Kayla’s teaching. Kayla asked her students to work on two questions in a Problem and asked them to share their work with their friends by preparing poster presentations. Kayla shared her students’ work with Paul. After this share, Paul had his student work on posters too. He assigned

different questions to each group and had each group summarize their work on a poster. Right after the class he went into Kayla's classroom with all his students' posters. He was very excited because all of his students were engaged in the lesson so he shared with Kayla right after his class (Field Notes, 0.305.2014). Paul had a lot of management issues in this class, but in this lesson they did not arise. This level of students' engagement was a big change for Paul, and he experienced it very positively.

Another teaching related conversation between Kayla and Paul was from Problem 2.1 or the Mixing Juices Problem in the *Comparing and Scaling* unit. Kayla shared her concern about her summary. Paul had a different experience and shared details of his lesson. He brought up an actual can and showed his students how to prepare orange juice from concentrate. He said they had a really good discussion at the end of the problem. All of the students were writing, doing the math and coming up with different comparisons. Kayla said she only showed a picture of a can (Field Notes, 02.07.2014). There might be many other reasons why Kayla's summary of the lesson did not go as she planned but as they talked about the lesson Kayla remembered a question (either came from students or from Kayla herself) with the lesson. She had a question about liquid conservation. She asked Paul when the concentrate was mixed with the water whether it would be the same amount or not, since the concentrate dissolved in the water. Paul told Kayla it would be the same amount since both of them were liquid.

Kristina and Kayla also shared their class work to inform each other if they did something different from the book. Those sharing moments were also opportunities for them to voice their excitement about their students' engagement or learning. Some of those provided better access to particular mathematics ideas for students. For example, in the *Moving Straight Ahead* unit students are formally introduced to linear equations in the general equation, $y=mx+b$.

Kayla told Kristina that even though her students seemed ready, she wanted to wait till Problem 2.3 to introduce this formal equation. As a response to that, Kristina talked about what she did differently in her class from Kayla. Kristina said that at the end of Problem 2.2 she had her students find all the linear equations that came up to that point and asked them to talk about commonalities. Students talked about unit rate as something always multiplied by an independent variable. When she finally introduced “ $y=mx+b$ ” students provided their own explanations to this equation. They described m as always multiplied with and b as a base where you start from (03.24.2014). Kristina indicated she was happy with how her students were able to make sense of the general form of expressing linear relationships. Kristina brought this example from her teaching not only to share her happiness in their guided-discovery but also to challenge Kayla’s deterministic attitude to follow everything in CMP as it was suggested.

Sharing different strategies teachers used in their classroom provided an opportunity for other teachers to access a variety of strategies. Paul was inspired from Kayla and changed his summaries for once. Also, because Kayla shared what she did in her class, he tried a different approach and had his students do poster presentations too. Therefore teachers brought up their classroom examples not only to share their feeling of accomplishment as a teacher but also to inform other teachers about different ways of facilitating student work.

Summary

In this section, I addressed the first question in which I asked in what ways the collaborations came out of concerns or inspirations in the teachers’ practice (linking arrow 1). Teachers brought up their concerns about their students’ behavior or understanding into their conversations, but also shared their exemplary students’ work. They talked about their concerns about their teaching, different teaching strategies, and provided lesson updates to keep each other

informed about what they did and what their short term plans were. Conversations were a way of keeping each other updated about their concerns but also provided opportunities to get different perspectives. Teachers' conversations about students helped them to consider different approaches to tackle various issues. Moreover, those conversations helped them build a culture where teachers welcomed and understood each other's concerns. In addition to conversations, teachers shared artifacts produced in their classes. They talked about those artifacts by explaining what motivated students or the teacher for such a work so that other teacher might adopt a similar approach in their classrooms.

Teachers' Collaborative Interactions Informing Their Classrooms

In this research question, I analyzed the ways teachers' collaborations related to subsequent actions in their classrooms. To address this research question, I considered actions by teachers in their classrooms in relation to their collaboration (linking arrow 2). I mapped teachers' collaboration to subsequent actions if I noticed a teacher explicitly referred back to a collaboration with other teachers. I considered those *explicit* cases of collaboration relating to practice. If there was not an explicit mention of teachers' collaborative activities, I took into account similarity between teachers' subsequent actions and the content of collaborations. For most of the cases, subsequent actions that were considered here were actions taken up in the same day or within a day or two of the collaboration. Because of the interpretive nature of this causal relationship the link between teachers' collaboration and subsequent actions by teachers in the classroom may be weak for some instances.

Collaborative interactions showed in teachers' practices as (1) major changes, or (2) minor adjustments to the lessons. Major changes were mostly *explicit* cases of collaboration relating to teachers' practice. With major changes, it was easier for me to notice changes in

teachers' lessons (CMP lessons as they were provided in the book), and I was able to relate those changes to observations of teachers' collaborations leading to particular changes. For other cases, I followed up with questions about what made teachers consider such changes in their lessons. Minor adjustments reflected some changes to the lesson as a consequence of teachers' interactions. These were explicit in some cases (e.g., teacher informed students that she got the idea from another teacher) but not in other cases. Such adjustments were more obscure because sometimes it was harder to capture change in the lesson or teachers were not explicit about changes to various Problems.

There were few major changes beyond cuts on the content due to time constraints and alignment to the CCSSM. That was because teachers indicated their trust in the materials and valued the content in the materials as it was. Therefore they did not consider Problem replacements or radical changes. Kristina noted how "authors make all these connections and it is great to see it comes alive." (Field notes, 01.20.2014). She noted her happiness and willingness to make in-print content come alive. Kayla also indicated how she liked the content as it was provided in the materials. She thought "with CMP everything unfolds very nicely" (Field notes, 11.19.2013). Therefore, I predicted I would not see major changes. However, there were important minor adjustments that were shaped by collaborative engagement.

Major changes

The *Major Changes* appeared as (1) problem replacements (or additions), (2) changes in the way teachers facilitated their lessons, and (3) cuts to the content. The last category appeared the most and it was more about cuts to the content because of time constraints. For example, teachers decided to skip Investigation 5 in the *Looking for Pythagoras* unit because they had so many snow days and needed to be done with the unit before the end of the semester (Field notes,

01.09.2014). Similarly, one time they decided to skip a Check-Up test given in-class and instead, Kayla assigned it as a homework because she thought it was a good review of the Investigation (Field notes, 03.24.2014). These general planning level decisions were made collaboratively and implemented in all the classrooms.

Problem replacements and additions. One of the Problem replacements was initiated by Kristina's desire to use a Problem designed by another CMP teacher. Kristina told me, she talked about the Problem as an alternative Problem in CMP *Getting to Know* conference and that was her first time using it in her classroom. It was Problem 2.1 in the *Frogs, Fleas and Painted Cubes* unit. Right after the class, Kristina invited Kayla to her class and shared her excitement about her students' work. She shared the visuals students prepared (Figure 13).

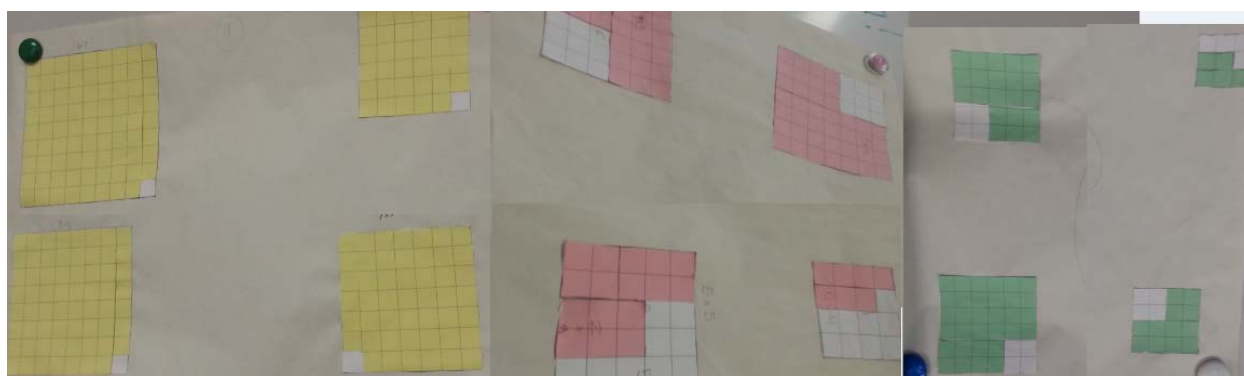


Figure 13

Student work examples for problem 2.1 in the *Frogs, Fleas and Painted Cubes* unit

In the original Problem, students were asked to consider a land swap in which a square piece of land is swapped for a rectangular one with the same perimeter. In this version, each group were assigned a number and they used their assigned numbers to transform their squares into new rectangles. In the original version of the Problem students make a table to consider change in the area with the fixed perimeter. In this version, each group produced a different area with the

same perimeter (Field notes, 01.17.2013). Kristina found this version of the Problem more visual. The *Summarize* part of the lesson was also more constructive so she shared this new version with Kayla. Kayla did not teach the original version so both versions were new to her but she sensed Kristina's excitement and later told me that she liked the visual display of change in the areas. Kayla used this new Problem with some adjustments recommended by Kristina (Field notes, 02.12.2014).

Another example was an additional lesson that took place as an end result of collaboration between Kristina and Ms. Gina (science teacher). Remember in the section above, on teachers' concerns about students, I provided an example of an interaction from Paul, Kayla and Mr. Kullen. It was about students' struggle in making connections. A similar conversation took place between Ms. Gina and Kristina. As Kristina reported, they talked about Boyle's law, setting up proportions and students' struggles in making connections for half an hour. She was curious if her students would be able to make any connection. She said "they get amazed when they see those connections." Students were doing *Comparing and Scaling* unit in mathematics and Boyle's and Charles' laws in science. Both of those topics were about proportional reasoning. She said being consistent in both classes might help students to see those connections. So she used the worksheet Ms. Gina gave in science class. At the beginning students did some computations, as they did in their science classes. Later Kristina asked students what the formula of density represented without using the word density. She connected students' work to unit rate. She told them "when you are calculating density in your science class you are finding unit rate. Which is another thing we are doing in math class. Math and science are like bodies" (Field notes, 02.27.2014). Before teaching this class Kayla saw Kristina getting prepared and she told her "look, how excited you are!" Kristina was willing to use some

of her class time for that additional lesson and she was excited about it. Later Kayla and Kristina had a quick debrief of the lesson. Kristina told her that students had a hard time connecting density to unit rate but when they figured out density was actually a unit rate it became easier. “They were fine and liked writing proportions” (Field Notes, 03.03.2014). This was an additional lesson that came as a follow-up to a conversation between Kristina and Ms. Gina about their concerns of their students’ understanding.

Facilitating lessons. Besides Problem replacements and additions, there were changes in the way teachers facilitated their lessons. For example, Kristina had her students work on the Mathematical Reflections by preparing poster presentations. At the end of her class she excitedly shared her students’ work with Kayla. Kayla used that idea in the *Comparing and Scaling* Unit and had her students work on the posters for Mathematical Reflections. She asked students to consider themselves as preparing commercials and asked them to come up with some catchy ideas like in the commercials. She even shared a couple commercial examples from a newspaper. Students addressed the questions in the Mathematical Reflections and tried to make it visually attractive too. Kayla extended the use of poster presentations in facilitating classroom discussions too. She shared students’ poster presentations with Paul and Paul used that idea with his students. This new way of student exploration started with Kristina, carried by Kayla, and from Kayla to Paul.

The changes in teachers’ lessons were due to their needs. Sometimes they were open to try a new Problem like in the Problem 2.1 example I provided above, but most of the time they did not want to change the Problems because they thought those Problems were rich enough as they were. That was why there were only so few of these major changes.

Minor Adjustments

Minor Adjustments included when teachers changed a small portion of their lesson, like adding a couple extra questions to the lesson, introducing a new strategy or assigning extra homework questions. I observed this kind of adjustments the most in this specific context. Even though the numbers were relatively high, there might have been much more of these that I was not able to capture because the changes were subtle and so ingrained in teachers' lessons.

Most of these minor adjustments were from Kayla's classes. She either made small tweaks in the way she facilitated a classroom activity, or she added targeted questions to her *Launch* or *Summarize* serving a specific purpose that was brought to her attention by Kristina. In addition, she incorporated different strategies to her lessons as Kristina shared them.

Small details. There were some very small adjustments Kayla made to her lessons with suggestions from Kristina like changing the way students work in the groups, or whether to use the projector instead of board. For example, students needed to use triangular pattern blocks to create larger triangles when they worked on an activity in the Problem 3.2, "Are all triangles reptiles?" in the *Stretching and Shrinking* Unit. Kayla borrowed pattern blocks from Kristina. Kristina told Kayla that since there were not enough triangles she made her students work in pairs so Kayla asked her students work in pairs (Field notes, 12.09.2013). Similarly, there were other very minor adjustments like using a projector instead of student posters during summary (field notes, 12.18.2013) or instead of using six different squares only using squares with an area of one, two and three square units (Field notes, 02.12.2014). I provide these examples to show a range of possibilities for a teacher to improve her lessons. These examples show that not every change needed to be substantial.

Posing questions. In other cases, Kayla adjusted the way she posed questions in her lessons. For example, in the *Say it with Symbols* Units, when helping students consider area of

rectangles with a fixed perimeter, she used an example with decimal numbers to help students consider subtraction. Later, she told me that Kristina suggested using such an example. Kayla started her lesson by reminding students what they did when in Problem 1.1 where they considered a rectangle with a fixed perimeter of 20 units. She asked them to think about the dimensions of this rectangle. One student said that the two adjacent dimensions added up to 10. Students gave examples with different integer values like 6 and 4. Kayla asked them what happened if one of the dimensions was “7.236” then “ $10 - 7.236$ ” came as an answer (Field notes, 02.05. 2014). Adding one extra example helped her students see the connection between the sides of the rectangle and the perimeter. This was an important piece of the lesson, because this work with specific numbers was going to prepare students to write symbolic expressions. Instead of Kayla telling students, she helped them consider subtraction by using a very specific number with a suggestion from Kristina.

In another case, when Kayla shared her plans for her grade seven class with Kristina, Kristina brought up some potential student difficulties. Kayla planned to assign different linear equations to the groups and have students explain what each equation meant. Kristina told Kayla that it might be harder for students to make sense of “ $-x$ ” in the equation “ $y = -x + 6$ ”. She said it might be much harder for students see “ -1 ” as the unit rate. Kayla did not change her plans for the lesson or replace “ $y = -x + 6$ ”. However, during *Summarize* she spent more time on the meaning of “ $-x$ ” in terms of the Problem context. Students were able to identify “ -1 ” as a unit rate. Kayla asked students to explain what “ $-x$ ” meant in that particular Problem context. One student said that “for every meter they walked, they lose money”. Kayla added by saying “the further you walk, the less money you make” (field notes, 03.24.2014). In this case, the connection was not explicit but Kayla’s questions pushed student thinking further in making

sense of “ $-x$ ” in the Problem context. This might be related to Kristina’s note about possible student struggle considering “ -1 ” as a unit rate.

Selective examples. Another *Minor Adjustment* came for Problem 4.2 in the *Looking for Pythagoras* Unit. In this Problem, students tried to justify whether rational numbers could be terminating or repeating decimals. Kayla noted a big difference in her two classes (morning class and afternoon class) after changing the questions she asked students to work on. The previous day Danny taught the same lesson and the students in the class had a lot of issues about long division and had hard time figuring out whether repeated or terminating decimal numbers were rational numbers. In debriefing this lesson, Kristina told Danny that the numbers he chose did not work well for repeating and terminating decimals. Since she knew Kayla was going to teach this lesson the next morning she went to Kayla’s class to give a heads up about the Problem. She suggested using questions B1, 2 and 3 and warned her about possible struggle in doing the long division. Kayla already had her first hour and she had a similar experience as Danny. She worked on the Problem again during her planning hour, considering Kristina’s suggestion as well. Kristina’s suggestions and experience with the challenge in her first hour made Kayla more careful. Her afternoon lessons went much better and she even admitted that she felt sorry for her first hour (field notes, 01.15.2014). This change was due to both Kristina’s suggestions and Kayla’s effort to make her lesson better for her other classes.

Additional student strategies. Kayla brought up different ways of solving problems that came up in Kristina’s class, like finding similar figures using Gavin’s strategy described above in the Inspirational Student Work section. She introduced the strategy by telling students that another student came up with this strategy and explained what Gavin did (Field notes, 12.05.2013). In addition, Kayla had her students work on specific practice questions (called

ACE questions), as suggested by Kristina, to help her students have a chance to practice and see the big picture in some Units. For example, at the end of Investigation 4 in the *Frogs, Fleas and Painted Cubes* unit, Kayla asked students to work on ACE questions four, five and six as Kristina suggested (field notes, 03.03.2014). Similarly, with suggestions from Kristina, Kayla used some ACE questions as extensions for her advanced students (e.g., Field Notes, 11.11.2013).

Informative additions. In addition to *Minor Adjustments* to the general flow of lessons, teachers brought up collaborations with other teachers to inform students of other teachers' involvement in the decision making process. For example, when one of the students in Kayla's class asked her why they skipped Problems 5.3 and 5.4 in the *Growing Growing Growing* unit, she brought up her conversation with Kristina. She said "because Ms. Gallen [Kristina] and I decided that it will be better if we jump to 5.5" (Kayla, Field Notes, 11.18.2013). The decision to skip Problems 5.3 and 5.4 was coded as a *Major Change* to the lesson but this short conversation between one student and Kayla was an addition to her flow of lesson to provide a justification for their decision.

Another *Minor Adjustment* example showed up in Kayla's class as an added note when she ran a review session for her grade seven students before the Unit test at the end of the *Stretching and Shrinking* Unit. She added an emphasis to Problem 2.2 in her note on the board. Kayla's review by connecting Problems to big ideas in the *Stretching and Shrinking* Unit. This Problem came up during a hallway conversation with Kayla, Kristina and Paul. Kayla articulated one of her concerns in the Unit test: "Did we ever do something with a shape that is similar to itself?" (Kayla, field notes, 01.16.2014). Both Paul and Kristina reminded Kayla the Hats Problem [Problem 2.2] where students considered such cases of similarity when they

considered how addition and subtraction affected similar shapes. The following week, as she was helping students organize their folders in preparation for the Unit test, she unintentionally circled the Hats Problem (Field Notes, 01.20.2014).

Physical additions. There were also some *minor adjustments* in the physical set-up of the classrooms as a result of teachers' collaborative interactions. For example, in Kayla's classroom there was a pencil sharpener attached to the wall with a note. The same note appeared in both Kristina's and Kayla's classrooms, asking "is this a good time to make a noise?" Kayla told that she got the idea from Kristina. Both noted how it worked for their students (field notes, 02.05.2014). There were many other similarities between the organization in Kayla's and Kristina's classrooms, like return trays for homework and exams, writing focus questions to inform students about the objective of the lesson, but I did not have explicit evidence if one of the teachers inspired the other one, or if it was a good example of "great minds think alike".

Summary. I aimed to provide a variety in the scope of *Minor Adjustments*, sometimes from very small changes like having students work in pairs to more substantial ones like adding a couple extra questions in facilitating classroom discussions or engaging students to work on some specific set of questions that would support students' understanding. These *Minor Adjustments* took place because one of the participants had specific questions and their collaborative work in order to address those questions led to some adjustments in lessons. In most of those cases, Kristina's justifications or challenging questions helped Kayla consider other details about the lesson she did not consider herself. There were times, when those modifications resulted from their accumulated experience. Kristina and Kayla continuously provided updates about their lessons, talked about possible student strategies or challenges in the Units and these conversations resulted in Kayla's lessons as minor adjustments.

Collaboration Related to Professional Actions Teachers Take Outside of the Classroom

In this question, I looked for evidence to relate teachers' collaborative interactions to their individual professional actions different from teaching. Teachers' collaborative interactions provided a guidance for teachers' individual work in planning their lessons. Therefore, teacher collaboration informed their individual planning sessions. In addition, I found that teachers stepped-up to leading roles in case of emergent issues. Because they knew they had each other to support in case something unexpected happened, they took those roles individually. In addition, because they talked more about the issue, they came to an agreement that somebody needed to step-up in putting what they agreed on into an action.

Guided Planning for the Lesson

The conversations between Kayla and Kristina about their lessons informed Kayla's individual planning time. For example, after her conversation with Kristina about the development of mathematics in the *Looking for Pythagoras* unit, she used her conference hour to review specific Problems suggested by Kristina. At the end of the day with a confident tone of voice she said "in *Looking for Pythagoras*, everything connects so well now. Going up and over really helps making squares. I saw Problem 3.1 and now it makes sense. I was so freaked out this morning but now it makes sense. That work helped me to make squares." (Kayla, field notes, 12.02.2013). Kayla referred to two sets of resources in guiding her work. She referred to Kristina's guidance "going up and over helping making squares" and her individual work as a follow-up to this guidance. There were some other examples to this guided planning sessions. In other time Kayla reviewed additional ACE questions that might help her students to see the big picture in the *Frogs, Fleas and Painted Cubes* unit (Field Notes, 03.03.2014).

Taking Leading Roles in Finding a Resolution to Emergent Issues

Throughout my observations, there was ongoing tension between administrators and teachers. I will provide more details about this issue in the next chapter however, both Kayla and Kristina did not stay quiet while all this was happening. As they talked about it they decided that they needed to do something to resolve the issue. Kayla wanted to have all teachers sit-down and talk together. With Kristina's support Kayla sent an email to all the staff asking for a meeting. Kayla told me how Kristina supported her by continuously saying "you can be a leader" in making a decision to step-up for a leading role in resolving this emergent school drama (Kayla, post-interview, 04.26.2014). Kayla prepared an agenda and a power point presentation to organize their meeting with input from Kristina. As Kayla prepared for this meeting with continuous check-ins with Kristina, Kristina took up a different role and tried to address one of the issues that increased the tension between administrators and teachers. She worked on the language in the school's detention policy. As she worked on the document, she kept Kayla updated about what she prepared so far, however she prepared this document individually. She shared her text with teachers and later with Mr. Norm [the school principal].

Kristina took leadership in CMP related issues. She was a regular participant of CMP users' conference, but Kayla attended to the conference as she needed. Kayla told Kristina many times how much she benefitted from annual the Getting to Know conferences. They both talked about how it might have been useful to have their new administrator attend to the conference. They also talked about possibility of having some other teachers attend to the conference. As a follow-up to these conversations, Kristina took a leadership role and got in touch with the CMP office to see if they could get any free spots for the conference in the summer.

The evidence supported the presence of a few instances in relating teachers' collaborative interactions to their professional actions outside of their classrooms. However, the collaborative

culture in this school helped teachers describe their role as CMP mathematics teachers. During interviews, teachers mentioned the big role collaboration had in identifying themselves as teachers. Kristina and Kayla discussed how other teachers helped Kristina or Ms. Jansen to become good teachers. It was like passing the baton from one hand to another. It was now Kristina's role to support Kayla in teaching grades seven and eight mathematics for the first time. For example, even accepting me as a researcher was a reflection of the collaborative culture in this school to their professional actions. When I thanked Kristina and Kayla at the very beginning of my observations, Kristina told me "we are a family!" They were CMP teachers and part of the CMP family. CMP teachers talk, collaborate and try to support each other in their work.

Chapter Summary

In this chapter, I addressed the question of how teachers' collaborative interactions related to their teaching practice. I looked for evidence that related teachers' collaborative interactions to individual teachers' practice both inside their classrooms and outside of their classrooms as professionals. Figure 14 framed and summarized my findings.

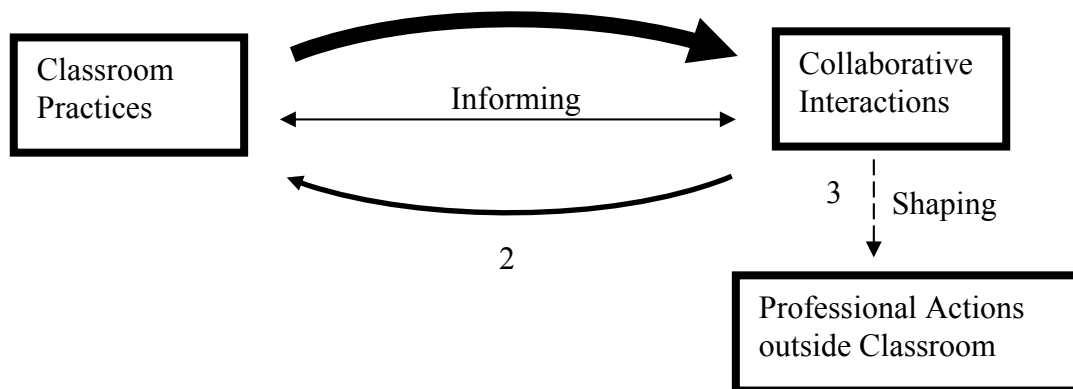


Figure 14

Relating teachers' professional actions

Concerns and inspirations that surfaced in teachers' classrooms frequently became topics of conversation (hence the bold thickness of linking arrow 1). They provided an outlet for teachers to share their in-class experiences and bring artifacts from their classes. There were fewer cases where collaborative interactions influenced classroom practices (linking arrow 2). But in some cases, based on collaborative interactions, teachers adjusted their teaching practices. Those were mostly minor adjustments, mostly taking place in one teachers' classroom (Kayla). These adjustments included but not limited to posing additional questions in the class or bringing some other student work into class discussions. Linking arrow 3 is a dotted arrow to indicate that strong influences from collaborative interactions to teaching activities with students were not commonly observed—though there could have been non-observable effects.

When classroom practices and events shaped interactions, the teachers voiced concerns more than inspirations. These concerns focused both on teaching and students. Concerns about their students stemmed from the teachers' desire to address behavior issues or find help in supporting student's understanding. Those conversations helped teachers identify issues better,

made their teaching visible to each other, and sometimes provided different ways to tackle the issue. In addition, when teachers raised specific classroom-related concerns to another teacher with a higher expertise, those interactions provided a context for teachers to voice more general concerns (Coburn & Russell, 2008; Horn & Little, 2010). For example, when Kristina shared a variety of student work from her class, Kayla asked “why so many strategies did not come up in my class.” This concern later voiced as a worry about “holding students’ hands too much”. In this case, it was “facilitating learning without holding students’ hands”.

The teachers, mostly Kayla and Kristina, also shared inspirations, but much less frequently. In those instances, they were excited to share exemplary student work examples or teaching strategies they found successful. Their intentions appeared not only to share their feelings of accomplishment but also to inform their peers about different ways of facilitating student work (Penuel et al, 2009).

CHAPTER 7

PRODUCTIVITY OF TEACHER COLLABORATION

This chapter addresses my third research question, which focuses on the ways that the teachers' collaboration was productive. Teacher collaboration was considered productive if the collaborative activities stimulated positive outcomes in teachers' professional lives. In the consideration of productivity, I looked for outcomes that became visible after teacher-to-teacher interactions that were not apparent before the interactions. Even though it is not possible to claim an explicit causal relationship, data supported an association between teachers' interactions and the claimed products.

My focus was on positive outcomes with respect to these three teachers' professional lives, because the term productivity has a positive connotation and my overall focus in this study was to be able to capture "the good" in these teachers' professional activities. There could have been negative outcomes, like developing a negative attitude or flawed teaching of mathematics, however, I did not observe such negative outcomes. There were a couple of instances where there were signs of conflict but those conflicts were resolved as the teachers kept their interaction ongoing. An example of this is the *Observation Evaluations* that I will discuss in the coming paragraphs.

In consideration of the productivity of collaborative interactions I attended to the *Objective* and *Interpretive* nature of the products. Objective products were products that were concrete and tangible whereas interpretive products were inferred from teachers' self-reports of their practices and/or my judgment of intangible results of teachers' collaborative activities. Even though I made the distinction between objective and interpretive very clearly, the data

provided evidence to the presence of products along a continuum from all-objective to all-interpretive. (See Figure 15).



Figure 15

Characterization of teachers' productivity of collaborative interactions

Teachers' collaborative activities created products tied to specific places in the curriculum or situated within the context of their teaching with tangible artifacts like solving a mathematics problem or producing assessments. Solving a mathematical problem was an all-objective product because both the problem and the solution were concrete and did not include any interpretation on my side. As a product moved along the continuum it became less tangible. The products that were closer to the all-interpretive end were not only intangible or situated less within the context of a curriculum, but they also represented my synthesis of characterization of outcomes of teachers' ongoing interactions, such as promoting self-care. Therefore, products as they appeared in this study, were more complex than it was framed at the beginning (see Figure 16).

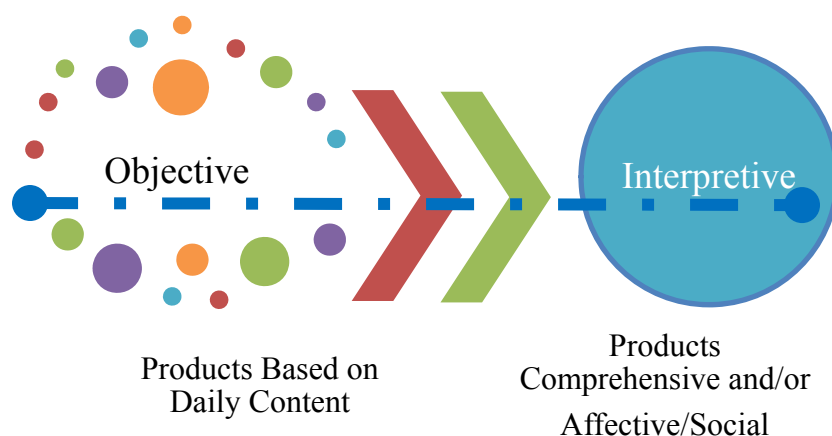


Figure 16

Variety in the meaning and purpose of the products

There were individual products—some tangible some less tangible—that were produced as a part of teachers’ daily activities and tended to have more of a content-based focus. Those products fell into the left-hand side along the continuum from all-objective to all-interpretive. These individual products were represented as small circles on the left-hand side in Figure 16. I represented those in different colors and different sizes because they represent different tangible products with a different scope like produced assessments, or a question changed in the lesson. In addition to individual products, teachers’ collaborative interactions served to build a larger set of products. Those were interpretive in nature and reflected my comprehensive summary of teachers’ gain from their overall interactions.

The products discussed in this section were documentation of positive outcomes with respect to teachers’ collaborative interactions in this specific context. In addition, these products contributed to a larger product in that they helped nurture the teachers’ Mathematical Knowledge for Teaching (MKT). As I discuss examples in the following sections, there was not always

strong evidence to claim, for example, how discussions over a lesson improved or changed their lesson, but teachers noted many times that their ongoing conversations over a lesson or a mathematical topic helped them become better teachers. Those ongoing conversations enriched their shared knowledge space and improved their MKT. Moreover, these products helped teachers with the affective and social aspects of teaching (not-MKT). As the product moved to right on the continuum, it became a broader gain rather than being an artifact by itself and had an impact on teachers' negotiating who they were as teachers and person(s) in this school context.

In the coming sections, I will present the products in two sets. First, I will present products situated within teachers' daily activities. Following that I will talk about products that were a comprehensive summary of teachers' overall interactions. In each section, I will provide a description of each product with an overview of frequency of observations with respect to products discussed. I provide typical and/or prominent examples for illustration purposes. A typical example represented most of the cases within the set of the product whereas a prominent example was an example that illustrated the product the best. In order to prepare the reader for the examples I will present here, I want to note that the products I discuss in the first set, products situated within daily practices, captured the outcomes of interactions between Kayla and Kristina. The interactions between Kayla and Paul contributed to the second set of products rather than the first one. Therefore their collaboration led to less tangible and more affective and social outcomes.

Products Situated within Daily Practices

I observed the presence of five types of products situated within teachers' daily practice. These were *Assessments*, *Instructional Schedules*, *Lesson Plans*, and *Considering Mathematics Problems*. These products came out of teachers' interactions as checking-in (CIN), interchange

of ideas (IOI) or joint work (JTW). I provide frequency distribution of these products in Table 10.

Table 10

Counts of objective products

	Checking-In	Interchange of Ideas	Joint Work
Assessments	2	6	6
Instructional Schedules	5	7	0
Lesson Plans	2	7	4
Considering Mathematics Problems	1	1	2

The counts in Table 1, were not count of all interactions leading to these products. I only counted, instances where there was a tangible product at the end. For example, Kayla and Kristina talked about *Stretching and Shrinking* unit test more than once but I counted it once, because there was one single product at the end of those multiple interactions. First, the teachers worked on and prepared upcoming *Assessments* as well as had short and quick assessment reflection conversations. Those were mostly JTW where teachers had a document to work from or IOI where they had a conversation from all teachers. Second, the teachers decided on *Instructional Schedules* like what to teach, when, and for how long. Those were either CIN, one teacher checked with the other teacher about plans or IOIs teachers shared their ideas about scheduling decisions. These teachers prepared for their lessons and produced their *Lesson Plans*. This outcome appeared in various activities, like when a teacher reminded other teacher about one specific part of the lesson like emphasizing distributive property or reviewing the lesson over their book. Therefore, Lesson Plans had some tangible parts like pieces of a lesson plan written on the board or annotations in the textbook. In addition to these, teachers *Considered Mathematics Problems* as they worked together or had a conversation as IOI. *Assessments*,

Instructional Schedules and *Lesson Plans* were the most prevalent objective products accounting for 91% of the products situated within daily practices.

Assessments

Teachers interacted around assessments leading to products like modified assessment tools or resolutions over assessing specific pieces of work. These interactions happened both before and after assessments were given to students. During pre-assessment meetings, teachers modified the assessment materials (e.g., Unit Tests, Check-Up Quizzes, etc.). Post-assessment conversations involved teachers' reflections over assessment materials with more focus on making sense of students' work. Post-assessment conversations were considered a product if there was a decision made at the end of teachers' conversations like changing the wording or point distribution of a question for the coming year or deciding how many points to give to an unanticipated student response. Overall, pre- and post-assessment conversations improved teachers' assessment materials in addition to enriching their repertoire of possible student responses (Knowledge of Content and Students –KCS).

Pre-assessment meetings. In a typical pre-assessment meeting, if there were different versions of the same assessment tool, like CMP3 or CMP2 versions, teachers quickly went over all different versions and made a decision about which one to use before getting into a specific conversation about the assessment. Following that, they talked about the content coverage, the context and wording of the questions, overall points, and point distribution of each question. First, they reviewed the assessment content coverage as it was reflected in the assessment tool to make sure what they did in their class was reflected in the assessment. Later, they moved into discussion of readability and design related issues. They considered ambiguity in the wording of the questions, familiarity of the problem contexts, or formatting issues like not having enough

space for students to explain their reasoning. The majority of time was given to the discussion of point distribution of the questions. They talked about how many points they wanted to assign for each question and their plans on how to grade their anticipated student responses.

I will provide an excerpt from teachers' interactions around the Check-Up quiz (see Figure 3) from *Butterflies, Wallpapers and Pinwheels* unit to illustrate their typical assessment preparation meetings leading to modified assessments. Kayla, Kristina and Danny participated in this discussion in an afterschool meeting in Kristina's classroom. The initial conversation was similar to the one I provided above: they compared different versions of the test then looked at wording and general design. The test had all the content they covered so far in their class. After this initial conversation they moved into a discussion about point values for questions.

...Kayla: [Talking about the second question on the test] I know this is going to be a lot of work but I put only two points because I did not want to put a lot of weight. (Kayla turned to Kristina and asked her) How did you do it last year?

Kristina: I'll tell you once you're done.

Danny: Seems two or three. One for labeling. I would like to see lines drawn with angles.

Kristina: This was a nine points check-up last year.

Kayla: So did you do three points? Are the points "Same distance? Cross 90 degrees angles? Labeled correctly"? Realistically would be nice to have 4 points. Not gonna make it 4 points. For same distance, angle, label. 3 points

Kayla: Five points for rotation question. It should have been worth more points. It is more difficult.

Kristina: Knowing it is difficult. It is also the first^t investigation, do we want to make it more difficult?

Kayla: Isn't it 11 points then?

Kristina: Our points were different than yours. We just went three for reflection, three for rotation, three for translation. Gave .5 for explanation. Not saying you have to do that way. I am just sharing.

Kayla: I like the big picture talk. We are spending equal amount of time in each of these ideas so we are assessing with equal points. I am good with equal points. But I am going to change it to A' , B' , C' .

Kayla changed the points in the test.

Kayla: I am good with that. Did you have anything to talk with us Danny? I came with all these questions...

I have provided parts of the quiz including the second question teachers talked about in Figure 17.

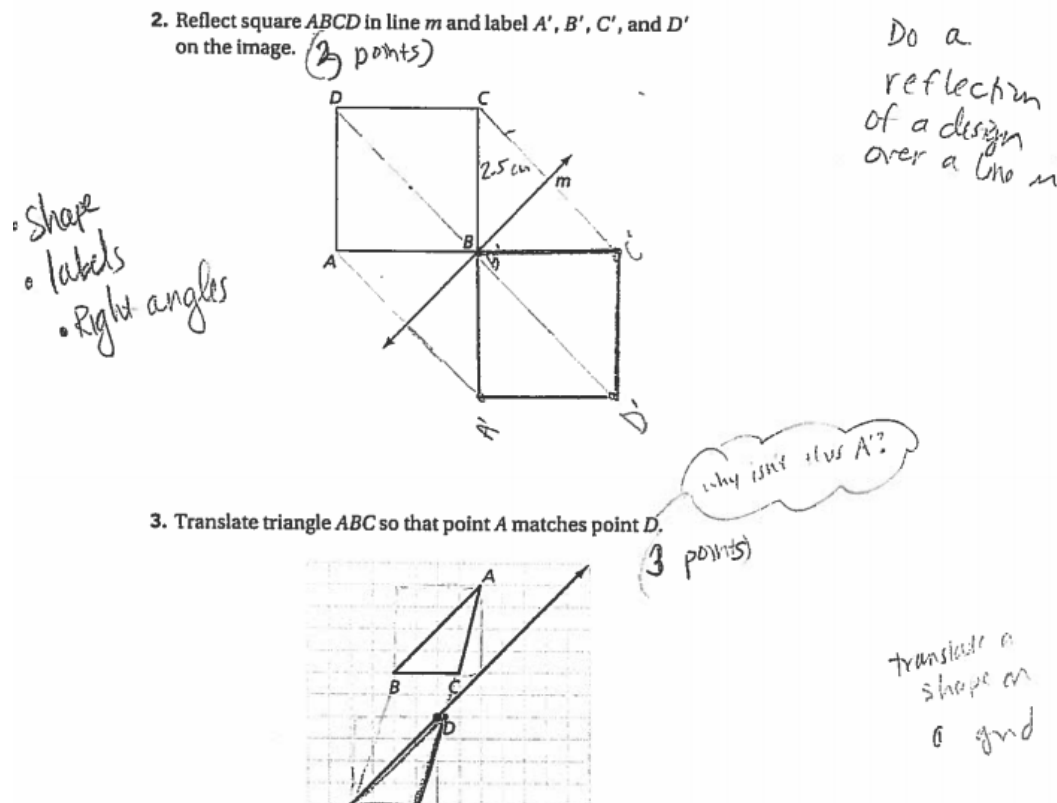


Figure 17

Butterflies, Wallpapers and Pinwheels Unit, Check-Up 1

The version I provided in Figure 18 was taken from the copy Kayla brought to their department meeting with changes and her notes from the meeting. During this interaction, Kayla and Kristina provided their reasoning for the points they wanted to assign for each question. They decided this Check-Up quiz would be over nine points and they would assign equal points to each type of symmetry. Point distributions reflected their concern about how much weight they wanted to give this assessment (it was the first test in the Unit), and their decision on how to assess students' work. In Figure 17, on the left hand column, Kayla's note says "shape, labels,

right angle” meaning each should receive 1 point. I would like to make a note of Danny’s peripheral participation status in these meetings. Even though he was physically present in most of these meetings he was mostly listening and accepting decisions made by Kayla and Kristina.

This *Butterflies, Pinwheels and Wallpapers* unit Check-Up test was a product with slight modifications. Teachers determined the point-values for each question and changed the labeling to reflect the conventions they used in the class. For labeling they usually represented the transformed objects by using letters like A' , B' , C' instead of introducing new letters. There were some intangible gains from this conversation for these teachers too. Some assessment-related issues surfaced in their conversation. They talked about the weight of this test in the Unit, how much emphasis to give a single topic, especially if it was relatively difficult topic for students, such as rotation symmetry. They talked about possible tools students might use in solving the questions (they might use rulers or even a corner of a paper might help them to decide whether an angle is 90 degrees or not) and all these aspects improved their consideration of assessments specific to this unit and assessments in general.

Post-assessment meetings. Teachers’ interactions around their assessment provided them assessment tools responsive to their concerns and needs. There were also post-assessment conversations where teachers reflected on the assessment tool by considering students’ work. In these cases, they made some changes addressing their concerns. For example, the Backpack Question (Figure 18) was modified based on both pre-assessment and post-assessment conversations.

5. The following rules for drawing backpacks for the Wumps are given below:

Backpack 1: (x, y)

Backpack 2: $(2x, 2y)$

Backpack 3: $(x + 8, y - 2)$

Backpack 4: $(x, 2y)$

- a. Backpack 1 is plotted on the grid below. Match the remaining Backpacks 2–4 with graphs A–C on the next page. Explain your reasoning.

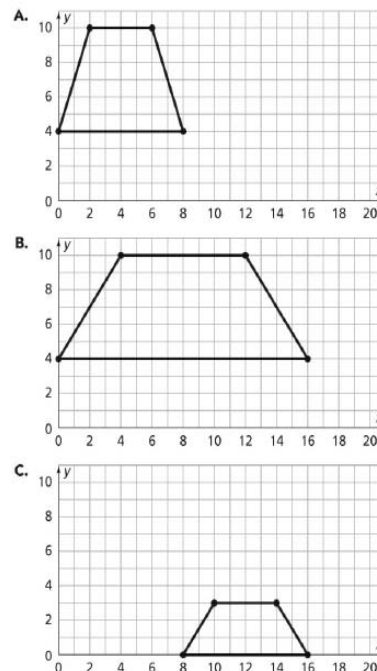
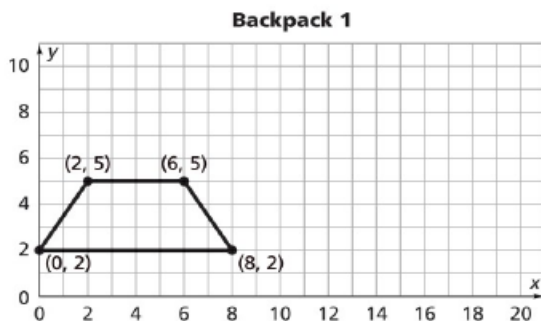


Figure 18

Stretching and Shrinking Unit Test, Backpack Question

This Backpack Questions were in the Unit Test for the *Stretching and Shrinking* unit. When Kayla and Kristina reviewed the Unit test, Kayla shared her concern about this Backpack Question. She was worried that the question was presented in a confusing way to the students. Kayla asked if it was possible to get rid of part “a” because there was much labeling in the question. Kristina and Kayla could not come up with a better suggestion for wording but they worked on some formatting issues to make the question look better before giving the Unit Test to their students. This wording issue stayed with them for a while. Kristina was the first one who gave the test to her students so she was able to observe her students’ struggle before Kayla experienced a similar issue. After her first hour she brought up her concerns about students’ confusion with this Backpack Question to Kayla. It was similar in her second hour—there were

many students struggling with the question. This time, after the test, she asked for suggestions from her students. She asked them how they would want the question to be worded to make it less confusing. She got a couple of suggestions. She shared those with Kayla and they decided the suggestion where they put A, B, C inside the shape was the best. They changed this question in the Unit Test reflecting the feedback they got from their students. I have provided the new version of this question in Figure 19.

5. The following rules for drawing backpacks for the Wumps are given below. Fill in each blank with the letter of the graph that matches the rule.

Backpack 1: (x, y) shown below

Backpack 2: $(x, 2y)$ _____

Backpack 3: $(2x, 2y)$ _____

Backpack 4: $(x + 8, y - 2)$ _____

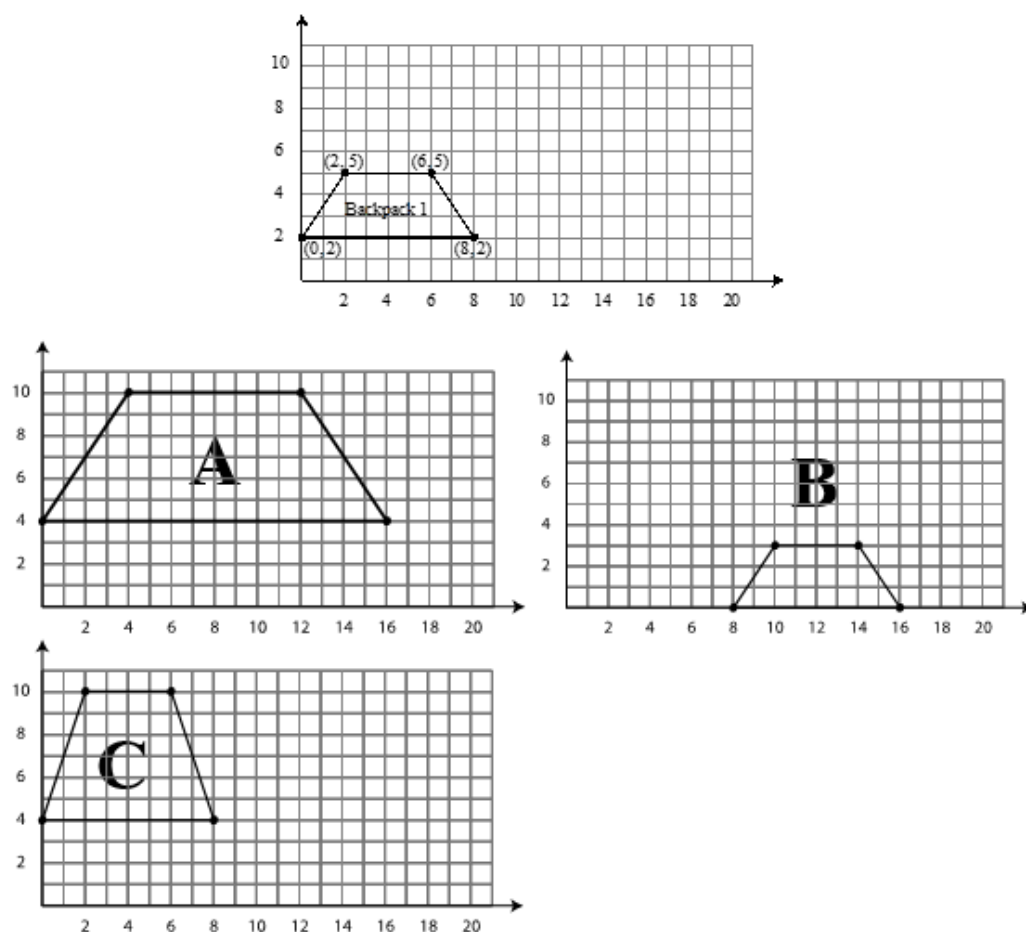


Figure 19

Stretching and Shrinking Unit Test, modified Backpack Question

This new version came as an end result of teachers' concerns about a question in the test.

Teachers' were concerned about the many different components of the question. Kayla and Kristina thought about a better framing before giving it to students, but did not change it except for addressing formatting issues. Following this conversation, Kristina watched her students' as they worked on the test. Their conversation about this Backpack Question served as a warning

for them and Kristina confirmed with her classroom observations that the question was confusing to students. While students were taking their tests, she observed students while responding to that question. She even asked a couple of students why they left the question unanswered and they noted they did not understand what the question asked. After students got their tests back, Kristina asked students how they would prefer the question to be worded to be less confusing for them. With the suggestions they got from Kristina's students they revised the question. This single question was a product of teachers' continuous interactions around the same issue.

In addition to assessment preparation meetings, there were post assessment conversations mostly helping these teachers to make decisions over grading unanticipated student work. For example, after a check-up test in the *Moving Straight Ahead* unit, Kayla and Kristina talked about how to grade a student's work who used a calculator to solve equations. As an explanation, the student wrote down "I did it in my calculator and checked for the equilibrium" (field notes, 04.14.2014). Kayla could not decide how to grade this student's work. She thought it was correct but since he did not use any formal mathematics to reach a solution she was not sure how to grade. This uncertainty was also related with the question prompt. The prompt did not specify any strategy or did not limit use of any other tool. After reading students' answer together with Kristina, Kayla gave the full credit to student's response. However, they both agreed to change the wording of the question for the coming year to prevent such cases. Similar to the Backpack Question example above, there was a wording-related issue in a slightly different way. This time the question was not confusing but it did not reflect teachers' anticipated method of solution completely. Therefore they made a note next to the question to consider modification for next year. In addition, teachers made a collective decision on how to grade a student's work that did not include all the components they anticipated in a typical response.

Lastly, there was something in the student's answer that they thought would make this question memorable to Kayla. This student used the term "equilibrium" instead of "equivalence".

Kristina recommended that Kayla make a copy of that answer for her own records because they both agreed that those kind of things were the things they would remember in the future when they looked back.

I provided an illustration of typical tangible assessment products that resulted from either JTW or IOIs including working over a document with accompanied conversations. Those conversations provided a glimpse of how teachers worked together that led to some changes in the assessment materials. In addition, those conversations enriched their way of considering questions in a broader context, the meaning of a question relative to the main goals of the unit, possible sticking points for students or variety of responses to expect from students. Post-assessment conversations typically focused on how to grade non-typical student responses, but in some cases those conversations also helped teachers reconsider some of the issues in the assessment by either changing it for further use or making a note to themselves to remind themselves of an issue with the question. In addition, as teachers talked about different solution strategies they increased their knowledge of content and students.

Instructional Scheduling

One of the decisions teachers had to make on a regular basis was when to teach, what to teach, and for how long. Conversations where teachers talked about what topics they covered in their classes or about their near future or long term plans were counted as *Instructional Scheduling* products. Even though such conversations were informative exchanges, as CINs or IOIs, I considered those as products because those were collective decisions on what to teach or not to teach.

Both Kayla and Kristina had their weekly plans written on their boards. They knew their plans were there, and if one of them needed to learn where the other one was, they quickly checked their planning corner. There were conversations over their plans on their boards, but, most of the time, their *Instructional Scheduling* conversations were not based on anything written. Typical *Instructional Scheduling* conversations included a quick share of their plans for the week or day. If something needed to be changed, they informed each other. For example, in the *Looking for Pythagoras* unit they decided to skip Investigation 5 completely because they did not have enough time to cover the whole book. This decision, as in most of the scheduling decisions, was based on Kristina's planning considerations. As Kayla and Kristina were reviewing their plans, Kristina talked about possibility of skipping Investigation 5. She told Kayla "if you have time you can do it. It is all good stuff. But Investigation 4 has four problems and they are all important stuff. Investigation 5 is good but we don't need it at this time." (Field Notes, 03.24.14). After this interaction with Kristina, Kayla reviewed the content by herself and followed up this conversation with an email: "You are skipping Investigation 5 of eighth grade, right? I don't even think I will have time for Investigation 5 before the half days schedule. I will plan on skipping that as well." (Kayla, Email, 01.09.2014). Therefore, both Kristina and Kayla skipped Investigation 5 in the *Looking for Pythagoras* unit. This example illustrates how scheduling plans were not final, however, they kept each other informed about their decisions, and in some cases they changed their decisions based on suggestions coming from the other person.

Instructional Scheduling was a dynamic product because teachers modified their plans as they got further in the Unit. It was mostly Kayla who changed her plans with Kristina's suggestions. Those conversations were important because it was a way of "keeping each other

on track with pacing and assessment” (Kayla log entry, 03.07.14). Making *Instructional Scheduling* related decisions transparent was especially important for Kayla since she was new to teaching grades seven and eight, she needed a person to help her make decisions if something needed to change. As she said “*if it was just me and my own I would probably would have done the whole thing. Because I would just have felt like, better I stick to this since CMP did this.*” (Kayla, Initial Interview, 11.19.2013). Those instructional scheduling interactions also helped Kayla to understand the curriculum with respect to her teaching needs, which helped her build Knowledge of Content and Curriculum (KCC) in the long term.

Lesson Plans

The teachers’ generated detailed lesson plans as they interacted more around their lessons. The teachers valued CMP lessons and Problems in the CMP units, so their interactions were situated within CMP Problems with conversations to understand the content better for their teaching. Those conversations sometimes led to minor adjustments to meet their needs or sometimes became annotations reminding teachers (mostly Kayla) some details about the lesson without any changes. See Chapter VI for examples of minor adjustments and major changes relating to teachers’ in-class practices. Those changes were tangible end products of teachers’ collaborative interactions.

In this section, I will talk about the outcomes of the process of “collaborative thinking-through a lesson” as intangible gains that helped teachers address their questions on particular Problems in a Unit and helped them to understand the storyline of the curriculum, so those conversations nurtured their MKT. These teachers’ ongoing conversations over lessons became unwritten annotations to lessons provided in elaborated Launch-Explore –Summarize write-ups in the Teachers’ Guide. For example, right after her eighth grade class, Kayla came into

Kristina's class with her *Frogs, Fleas and Painted Cubes* unit. Problem 2.3 was open in her book. They started having a conversation about the Problem:

Kayla: After done this (last 2 questions D and E) and this. Are we expecting our kids to learn everything?

Kristina: Yes, but not subtraction.

Kayla: Okay, that makes me feel better.

Kristina: We throw a couple of subtractions here and there and let them see what that looks like but we do not expect them to get it all and we do not assess them.

.....

Kristina: They get used to doing when you change the side they forget what happens if one side stays the same (showing Problem 2.2. example of $x(x+2)$). They get into that mind set so easily they forget about doing without any addition. [she pointed $2x(5-x)$] ...yeah like this one. This is a crazy question. $2x(x-5)$ is okay but $(5-x)$. It is really hard for them... (Field Notes, 02.12.2014)

Kayla was uncomfortable because she thought the examples provided in the book would not be enough for her students to practice distribution of subtraction when changing from factored form to expanded form. Once Kristina clarified that the expectation was not to have students master subtraction cases, she was relieved. However, Kristina did not stop there. She continued sharing her own experience with her students. She noted how it might be harder for students to consider cases of distribution for simpler cases like $x(x+2)$. She also talked about other potential difficulties in cases with subtraction. In this example, Kristina and Kayla revisited the lesson Kayla had just taught and they both clarified their goal for the Unit (mastery in distributing addition, an introduction to distributing subtraction). In addition, this conversation exchange

helped Kayla understand what to anticipate as more difficult for the students. Therefore, it increased her knowledge of content and students.

Conversations over the lessons they planned to teach also helped Kayla to see the big picture (I called this curriculum storyline) to the eighth grade mathematics content. For example, Kayla had many questions as she was planning for teaching *Looking for Pythagoras* unit. In the example below, before the actual conversation, Kayla sent an email to Kristina asking how students were supposed to use slope in Problem 2.3. As soon as it was break time Kayla went to the hallway with her *Looking for Pythagoras* book and a copy of the accompanied labsheet (see Figure 20) and waited for Kristina. There were notes on her book (it looked like she did the mathematics required in the book) and she appeared to be in a panic.

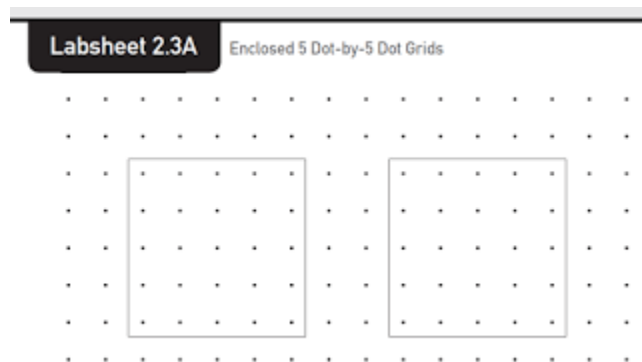


Figure 20

Labsheet for the Problem 2.3 in the *Looking for Pythagoras* unit

Kristina: Lots of questions? Don't stress out!

Kayla: How are kids expected to draw lengths of $\sqrt{32}$ and $\sqrt{17}$ (showing the labsheet in Figure 20)

She showed her way of measuring it with a piece of paper in her hand. She was using the paper like a ruler.

Kristina: All they wanted kids to do is to come up with strategies for creating squares.

Kristina opened Investigation 1 and showed Kayla questions asking for squares.

Kristina: A lot of students will not bring in slope. We will bring it up. They will probably not use it again but we will talk about it so that we can bring it back in Investigation 3.

Kayla: I don't see how these are connected.

Kristina: The first thing they do is to compare flying distance to driving distance. They start thinking about hypotenuse. All we want is for them to be able to create squares over triangles. Squares on the hypotenuse are important.

They continued looking at different pages in the unit as Kristina explained how all the ideas were *connected to each other*.

Kayla: From math perspective $a^2 + b^2 = c^2$ all I was thinking. This square thing is all new to me. (Field Notes, 12.02.2013)

The conversation started with a very specific question Kayla had about a Problem she was going to teach. She wanted to learn how students were expected to draw squares with non-measurable side lengths and if there was any expectation in considering slope. However this conversation helped her voice her struggle in seeing the role of the Problems in the book in building an understanding of Pythagorean Theorem. After this conversation, Kayla did some additional work. At the end of the day, Kayla shared how she was able to overcome her frustration by saying “in the *Looking for Pythagoras* unit everything connects so well now. Going up and over really helps making squares. I saw Problem 3.1 and now it makes sense. I was

so freaked out this morning but now it makes sense. That work helped me to make squares.” (Kayla, Field Notes, 12.02.2013). She had a better expectation from her students in drawing lengths such as $\sqrt{32}$ and $\sqrt{17}$ however their conversation helped her beyond answering this specific question. She was able to see how ideas connected to each other and how previous work would support students’ future work.

Some of their lesson planning conversations did not have an initiator rising out of a concern. Kristina verbalized her decision making process by providing justifications. For example, there were a couple of times Kristina combined two problems and shared her decision with her. In the CIN type interaction example below after Kristina told Kayla that she started the *Say it with Symbols* unit in her algebra track class, she started explaining what the Problem was about and what she did in the class:

Kristina: It is the place where they start making sense of symbols. I combined the first two problems. The first problem is the Pool problem. They need to come up with at least two different ways of expressing how they can tile the pool. They [students] came up with a list of expressions. In the second problem they [authors] give four expressions and ask them to compare those to theirs. If they already come up with 10 different ways I don't use those. I put up all different ways they come up with and had them discuss those. When they talk about it since it is theirs. So I combined those two problems and made one.

Kayla: It is good to know. (Field Notes, 03.03.2014)

Kristina explained to Kayla what Problem she did and why she combined two Problems. In this example, Kristina made her decision-making process transparent to Kayla, so Kayla knew

what to look for in her class to combine those two Problems or not. In addition, to gaining an insight on enactment of these two Problems, Kristina also provided an example of flexibility in curricular decision making. For her, Problem 1.2 was optional and used only when students could not come up with many examples. Later, they had a similar conversation for one of Kayla's lessons. Kayla was planning to do Problem 2.2 in the Moving Straight Ahead unit and Kristina challenged her by asking whether she needed to do it or not. For her, that Problem was a summary of Problem 2.1, brining different representations in expressing linear relationships together, and since students' in Kayla's class understood it already, she did not need to do the Problem (Field Notes, 03.24.2014). Kayla did not change her plans because she already planned for the Problem and this conversation took place right before the lesson. But this conversation added to their previous conversation in making curricular decisions.

There were lessons, as products of collaborative interactions, with minor adjustments or major changes. Those were tangible outcomes of teachers' collaborative interactions. In addition to those tangible outcomes, teachers' understanding of the lessons improved over time. With continuous conversation with Kristina, Kayla improved her repertoire of anticipated student strategies and possible student challenges. She had a better understanding of the development of the mathematics in the Unit. Those interactions also modeled how to use agency in curricular decision making for Kayla. Therefore, those conversations gave Kayla additional insights during her lesson planning activities.

Considering Mathematics Problems

Teachers considered many mathematics problems however their interactions were situated within the context of their teaching. There were not as many cases teachers involved in interactions where they talked about the problem from a mathematical point of view with a

resolution at the end of their interactions. I will provide two illustrative examples where this collective thinking helped one or both teachers consider solutions to mathematics problems. In one of these cases, Kayla and Kristina got into a conversation on mathematics as they reviewed a formative assessment lesson Kayla was going to teach for Ms. Patz. There was a question in the lesson asking students to consider the truth value of a statement saying “if there is not a correlation between two variables it means there is no association between two variables”. They talked about what correlation and association meant. They considered correlation in explaining linear relationships. As they talked about it, they considered cases like quadratic relations. For example, in the case of quadratic relations, there was some association between variables but the relationship was not linear. At the end of this interaction, Kristina left the class by saying “I learned something today. Thank you for making me think!” (Field Notes, 02.12.2014). Kristina liked to be challenged mathematically and one single statement made them question basic definitions like relationships, linear relations, quadratic relations, and association between variables. Therefore, this was a collective thinking of mathematics ended with a resolution.

In another example, Kayla came to Kristina’s class with a question from Problem 1.3 in the *Moving Straight Ahead* unit. She had just done the Problem in her class and she had a question. In this Problem, students were provided three different scenarios, where two of them led to equations without a constant and one with a constant. Since this problem was early in the Unit, students did not practice representing relationships symbolically so the expectation was to have them make a decision either from a table or graphs. However, Kayla was not sure herself about how to use either graph or table to make a decision over proportionality. Kristina told her to look at zero. Kayla responded, “I don’t get it” with a confused look on her face. Kristina provided her another example showing that if there was zero for one variable they needed to get

zero for the other variable. Kayla, with a sigh, “ooh I got it now. We were stuck in my last hour. I understand now.” (Field notes, 03.10.16).

These two cases were learning moments for Kayla and Kristina. They reviewed a mathematics problem together and as they thought about it, they came to a resolution addressing their concerns about the problem. In addition, those considerations improved their understanding of mathematics.

Summary

Teachers’ collaborative interactions produced outcomes situated within their daily practices. These collaborative interactions were motivated by an assessment they needed to prepare or a question that came out in a lesson one of the teachers taught and led to tangible gains like, modified assessment tools, answered questions, or lesson plans with minor adjustments. In addition to those objective products, teachers’ interactions created opportunities for these teachers to improve their mathematical knowledge for teaching. As teachers’ interacted around issues that appeared in their daily practices, they also thought more about teaching mathematics and students, and curriculum or their teaching.

Products as Comprehensive Summary of Interactions

Teachers’ collaborative interactions with a wide range of prospects led to affective and social gains. Those were captured in the data as comprehensive summary of teachers’ interactions. First, collaboration helped these teachers in *Raising Issues*. While these teachers worked together on an assessment or had a conversation about what just happened in their classrooms, those conversations sometimes helped them to raise issues that moved their thinking beyond the specific context of those conversations. Second, teachers’ collaboration *Expanded the Circle of Collaboration Participants*. As these three main teachers’ collaborated with each

other, some other peripheral participants (like the teacher across the hallway) became involved in their collaborative activities. In addition, teachers who collaborated the most supported and fostered a collaborative culture in their classrooms. Therefore teachers' collaborative activities expanded the circle of collaboration participants. Third, teachers' collaborative interactions *Promoted Self-Care*. The collaborative culture among teachers created a positive atmosphere where teachers showed their care and concern for each other, for their personal lives in addition to their professional lives. In that sense collaborative culture promoted self-care among these teachers where they showed their respect for their choices, care for their feelings, and how they were attentive to their individual needs. Moreover, their collaboration *Informed their Decision Making*. During informal conversations with teachers and in their log entries they noted how they felt strong about their decisions because it was not only one person making all the decisions but a product of collective thinking. Lastly, the collaborative culture provided *Access to Richer Set of Resources*. Since teachers relied on each other, they had access to set of materials, and because they all had different experiences on the use of tools, they shared their experiences on using them. Therefore, they had access to different set of resources than their own set and they had access to the experience to use them.

In my discussion of the results for these comprehensive products, I will not provide any numbers because the products in this part were synthesis of the broader themes instead of collection of individual products and evidence was used for illustration purposes. Overall there were more instances highlighted as supporting evidence for collaboration provided an *access to richer resources* (all the individual products discussed in the previous part in addition to expertise and questions these teachers brought to their conversations). There was less evidence supporting collaboration *expanding the circle of collaboration* participants.

Raising Issues

Teachers' conversations usually included their perspective on things, sometimes shifting back and forth between general and specifics. They raised many issues during those conversations. These issues spanned a range of topics from school related issues to issues about their teaching, students, or assessment. See Appendix F for full list of issues raised by teachers. Some of those issues were revisited multiple times and discussed in detail, but most of the times issues were hinted or brought up explicitly in the flow of conversation, although teachers did not follow through with solutions to all issues. I will provide two examples of issues (one with MKT focus and one with not-MKT focus) that were picked up multiple times in order to illustrate how ongoing conversations helped teachers deepen their perspective on the issue they tackled.

Observation evaluations. With new administrators in the school, communication issues developed. The *Observation Evaluations* created a common talking point for teachers to voice their concerns with each other. These *Observation Evaluations* were performance assessment visits to teachers' classes, without letting them know beforehand, by the assistant principal (Mr. Vall). This was a common topic in teacher dialogue as they shared their evaluation experiences. Therefore, there were many other issues nested under this big issue.

This issue was a common topic of teachers' lunch conversations for some time. I provided some excerpts from one of the conversations between Kayla and Paul to highlight the issues they raised as they shared their concerns:

Kayla: I don't want to freak out about this but I am. They just come in for an hour and do all the evaluation based on one hour? I just want to ask one question. Will they be happy if we did the same thing to our students?

...

Kayla: It is not like I don't like or trust my administrators, but I feel like I don't know them enough.

Paul: In the staff meetings last year there was more communication.

Kayla: Yes it was more teamy.

...

Kayla: My administrators do not know me well enough, they don't know my class well enough. To be honest Funda can do my evaluation much better because she is here all the time. She sees things that don't go well or when things go well.

Paul: I told her yesterday my tech-ed class might look like chaos but that is what I wanted, I wanted them to practice what they know, play with the tools before starting working on their projects. Something you plan really well and think that it is everything will be perfect but something happens and you bump.

Kayla: Sometimes you don't plan well and have a great class.

Paul: Even one student might change things.

....

Kayla: You know what. I work my butt off and I am still learning. We are not perfect. I don't think any teacher here thinks that she is perfect. Otherwise you need to quit teaching. I don't know. Most teachers are nervous about this. (Field Notes, 02.12.2014).

Paul and Kayla questioned how one time observation can provide a good picture of what was happening in their classrooms. In addition to being skeptical about one time observation's value, they also shared their concerns about new administrators. Partially this uncomfortable feeling was due to not being so sure about how much their administrators knew them.

In another conversation, Kayla and Kristina shared their concern about how the principals used rubrics with very narrow definitions of how classes should run, were limiting, and did not provide fair assessment decisions.

Kristina: He marked me down because I did not have an objective on the board. And the way how to measure it. He said I needed to have measurable objectives.... I don't think that it is fair to assess somebody based on a couple of criteria they came up with.

Education turned into this checklist. They need to see the big picture.

Kayla: One time observation will tell nothing.

Kristina: I have my focus question up and kids know that is where we want to go. How do I assess? I have check-ups, partner quizzes, observations, classroom discussions. I know how to assess my students' learning.

....

Kayla: I would get offended and be defensive like you. You can say the whole math department does it that way. Focus questions are really good. It gives me a direction.

Kristina: Yes, It summarized the problem. I don't care if there is a question mark at the end or not. Students and I know that we are going to answer the question at the end. With shapes and design it was so much different. Last year, I know how it is different with focus question this year. There are so many things you might get into and but with focus question you know where you wanna go.

(Field Notes, 02.15.2014).

This conversation between Kayla and Kristina brought up additional issues that Kayla and Paul noted in the previous conversation. Kayla and Kristina questioned observers' narrowed perspective on how things should run. Kristina told Kayla she had an objective and her own way

of assessing students but Mr. Vall wanted to see an objective written explicitly as well as a more traditional form of assessment. So they were both not happy about turning education in to this checklist instead of interpreting what happened in a class in its own context.

Teachers raised the ambiguity of the process of observation evaluations as the main issue. There were many other issues nested under this big issue. As teachers talked about this issue other issues surfaced. There was a lack of trust and communication between administrators and teachers. Teachers did not know what to expect from this *Observation Evaluations* and how their administrators would use them. In addition to this concern, teachers complained about not being included in decision making about things concerning them. Teachers also questioned the value and use of performance assessments based on narrow definitions. Therefore this *Observation Evaluations* as an issue was an outlet for teachers to bring other contextual issues to the surface.

Purpose of assessment. Teachers picked up assessments in their MKT-focused conversations too. Kayla, Kristina and Paul were involved in a conversation where they questioned the purpose of their assessments. Similar to the *Observation Evaluations* issue, this issue came up multiple times, however the conversations were shorter and more about the purpose of specific types of assessments they were planning to give their students.

Teachers' interactions provided them a context to consider assessments from a variety of perspectives. For example, recall the Dot Paper example I provided in Chapters IV and V, teachers had a discussion about a partner quiz in the *Stretching and Shrinking* unit. Kayla voiced her concern about providing dot paper for students to draw their shapes on. Dot paper was confusing to the students and in their conversation Kayla questioned why they wanted to challenge students on something beyond the goals of the Unit. As a follow-up to this concern,

Kristina raised an issue of consideration. She told them: “Partner quizzes are for them to elevate their problem solving skills. Otherwise we have asked them to do individually” (Field Notes, 12.03.2013). Therefore in this conversation Paul, Kayla and Kristina raised issues over assessments and provided their perspectives on those issues.

In another exchange between Kayla and Kristina, they picked up this issue of purpose of assessments in their consideration of a different CMP assessment tool. This time the topic of conversation was check-up quizzes.

Kristina: ... so I wanted to spend more time because it is such an important topic and also I took over from Danny. It is not that what he was doing was not right. He did what he needed to do but still I felt I like I needed to go over some stuff.

Kayla: That made me question the decision I made with my eighth graders. I was going to give check-up tomorrow and today there were so many kids who were struggling with distributive property. I took Kara's Unit test from Frogs, Fleas and Painted Cubes and literally showed them how many questions on expanded form, factored form and distributive property. I told them they already knew this. ...

Kristina: I think you made a right decision because distributive property is so important and they need to know how to move from expanded form to factored form in Investigation 3.

Kayla: I did not! That is what I am saying. I want to give the test tomorrow. They should already know this.

Kristina: That is a question about why you want to give check-up. Is that because are you checking up whether they learned it or not? If you already know without giving check-up that they don't why give check-up?

Kayla: Not all students.

Kristina: I know similar to mine. There are some students who will do fine but there are some who won't. You might want to give extra homework to the ones who are not trying and let them know that needed extra practice. (Field Notes, 04.14.2014)

Kayla planned on giving her check-up quizzes the next day, but from their conversation it was clear that she knew what her students were missing. Since Kayla already knew what her students did and did not know, Kristina questioned Kayla's decision of giving check-up quizzes the next day. Kristina considered check-up quizzes as informative to teachers about what student knew and did not know and she raised the issue of why Kayla still wanted to give the check-up quiz. As it was in these two examples, teachers' conversations around specific assessments opened up conversations for reconsidering the purpose of different types of assessments.

Summary. Three teachers in this study came together and had conversations over single and multiple topics at the same time. These conversations helped them name issues with respect to their context, their teaching, students' learning, assessment, and curricular decisions they made or they were about to make. These issues were raised or it was brought up explicitly because they had each other to talk to. These ongoing conversations helped them to identify issues but not necessarily helped them provide solutions to the issues raised.

Expanded Circle of Collaboration Participants

The teachers in this study mentioned many times that their collaborative activities were part of their regular activities. Especially for Kayla and Kristina it was a routine for them to seek out each other and keep short conversations ongoing over one single topic. Paul was involved in these conversations at the times when he was physically present. There were other teachers who were involved in teachers' collaborative activities like Mr. Kullen and Ms. Gina. They were

science teachers who participated in these teachers' conversation about students or school related issues. Mr. Kullen's classroom was right across the hallway from Kayla and Kristina's classroom so sometimes he joined the conversation by walking closer to Kristina's and Kayla's classrooms. Paul liked to have conversations with Mr. Kullen so sometimes he stayed longer as he was walking down from technical education class to mathematics class to have a conversation with him. For example, when Paul needed stop watches for one of the activities in the *Moving Straight Ahead* unit he borrowed them from Mr. Kullen's class (Field Notes, 03.17.2014). In chapter VI, I described how Ms. Gina and Kristina had a long conversation about similar topics they did in their classes that led to a new lesson in Kristina's class. Therefore, these teachers' willingness to interact with each other included other teachers too. In addition, by expanding their circle of collaboration, Kayla and Kristina created collaborative opportunities to involve other teachers and Mr. Norm, the school principal.

As it was described partially in the *Observation Evaluations* example above, there was an uprising tension between teachers and administrators. This tension started showing up as teachers learned they were going to have *Observation Evaluations* in February and reached its peak in March in one of the Staff meetings. As I listened to Kayla and Kristina, teachers and Mr. Norm had a rough time during the staff meeting over a conversation about student detention policy. After this incident many teachers wanted to talk about the issue with Mr. Norm and they wanted to show their willingness to find a common solution with him. At the same time, Mr. Norm started pulling teachers' out of their classrooms and having side conversations which usually resulted in tears from teachers. Unlike other teachers' individual initiations, Kayla and Kristina thought this was an issue they needed to solve collectively. With Kayla's leadership and Kristina's support, Kayla and Kristina sent an email to the staff calling for a meeting. They

wanted to move away from those side conversations and help teachers and their administrators to establish a better communication with each other. As she was talking with Kristina she said “we are acting like middle school kids. It is like a marriage. We need to learn how to get along” (Field Notes, 03.21.2014). Kayla and Kristina had teachers come together in Kayla’s classroom. In this meeting, with eight teachers participating including Kayla and Kristina, teachers talked about their experience with their administrators and what they wanted to see improved (Field Notes, 03.24. 2014). Later, Kristina and Kayla shared a summary report of this meeting with Mr. Norm. Kayla said as he read the list for suggestions he was very positive and told them that he could do all of those (Field Notes, 04.02.2014). Therefore, Kayla’s and Kristina’s collective problem solving orientation created an opportunity for teachers to address one of the issues of concern for all the teachers.

Promoted Self-Care

Teachers showed their care for each other by attending to their professional needs and each other’s personal lives as they opened up to each other. Kristina had regular check-ins with Kayla to make sure her needs were attended (Kristina, Initial Interview, 12.03.2013). Kristina showed her care for Kayla as she was going through emotional ups and down when she was preparing to teach *Looking for Pythagoras* unit. Kristina made herself available during Thanksgiving break and responded to Kayla’s messages about the Unit, however, Kayla had more questions and some concerns. The day after Thanksgiving break started with Kayla’s expression of her frustration by saying “I looked at all the *Looking for Pythagoras* stuff but still I am afraid of this book” (Field notes, 12.02.13). Kayla made her need for help explicit for this Unit by verbalizing how she felt. She and Kristina talked about the Unit, how the ideas were connected throughout the day as the time allowed. At the end of the day before leaving, Kristina

went into Kayla's classroom. Before getting into Kayla's classroom Kristina told me that she just wanted to make sure that Kayla was good and felt ready.

Kristina: How was your day today?

Kayla: I am okay now. I feel much better now!

Kristina: Just to make sure you are good. That is what matters.

Kayla: How are you doing (emphasized you)?

Kristina: Third hour really helped me this morning (8th graders were on a field trip and she had an extra planning hour).

They started talking about Christmas shopping, family, gifts, party plans. Kristina asked Kayla for suggestions for getting a present for her grandfather who was 96 years old.

Kristina talked about Saturday's girls' night out and she said it was nice to have that time and have little bit more wine than the usual.

Before leaving Kayla's class Kristina turned to Kayla and asked her "So everything is good?" (Field notes, 12.02.13).

As it was evident in this exchange, even though it was Kristina who wanted to make sure Kayla was okay and her needs were attended, it was not a one-way checking-in conversation. Kayla did not end the conversation by saying I feel much better now but she asked how Kristina was and by emphasizing "you" in her question she made herself clear that she really wanted to hear from her how she was doing. That opened the room for more personal conversation where Kayla provided some Christmas shopping suggestions to Kristina. Right before leaving the room Kristina checked-in with Kayla one more time by saying "so everything is good!" This quick visit that involved more personal conversation than Kayla's concerns about *Looking for Pythagoras* unit reflected their care for each other.

Since these teachers disclosed their lives professionally, it was a routine part of their interactions to make themselves available if someone needed to talk. That was the intention for their quick Checking In type of activities. A conversation started with Kristina's initiation such as "how was their focus today?" was at the same time a message saying that "I am here to listen if you need to talk". Their collaboration also meant to show their availability and care for each other. Kayla, Kristina and Paul knew which classes were more problematic or who were the students they struggled the most. For example Kayla's first hour, she had some management issues. In her third hour with seventh grade students there were many special education students so they were struggling with the topics more than her other seventh grade class. Kristina knew that and when she shared something from her classes she always noted "you have so many special needs students". Kristina sometimes complained about her students in her first hour. She said they were still sleeping so their participation was minimal compared to her second hour. Paul had usually management related issues with his last hour. It was part of their regular conversation to check-in with each other how those classes went. Those conversations, were like "a mental break" as Kayla noted and provided them a fresh transition opportunity to their next hour (Kayla, post-interview, 04.18.2014).

In addition to caring for each other professionally, Kayla, Paul and Kristina listened to each other's personal stories as they brought up to conversations. Kayla talked about her 4 year old daughter. Both Kristina and Paul knew Kayla was giving smiley faces to her daughter if she did not cry during morning drop-offs. One day Paul was in his technical education class and it was only a couple minutes left to the bell. He walked down all the way from his technical-education class to Kayla's class just to check with her whether her daughter got five smiley faces or not (Field notes, 02.02.2014). Therefore, they knew what was going on in their personal lives

and they showed their desire to get updates about those during their conversations. Even though this self-care came as an outcome of teachers' continuous collaborative interactions, at the same time this self-care became a factor contributing to teachers' further professional interactions.

Informed Decision Making

Ongoing interactions among teachers provided a warrant to their decisions. This collective decision making made them feel stronger about their decisions, because their collaborative interactions provided an opportunity to think through issues from different perspectives. Kristina said Kayla's presence and questions made her question the reasons for some of their established practices. Kristina liked having collaborative interactions with Kayla because "I think sometimes I get caught up in making sure every detail is covered on the unit test and having a fresh perspective helps me focus on what is important." (Kristina log entry, 01.13.14). Collective thinking also helped them consider all possible scenarios, likely student struggles, good examples to pose, ideas to look for during group work therefore when they make decision overs their lessons or questions to have or not to have in the assessment there were reasons behind those decisions. Having multiple opportunities to talk about an assessment or a lesson gave these teachers satisfaction about their decisions. Kayla noted how she "feel satisfied with tests for end of unit in both 7th & 8th grade since we were able to talk all week about it. (Kayla log entry, 01.16.13).

In addition, Kayla liked collective decision making because she wanted to be able to say "all mathematics department makes this way" when she needed to provide an explanation to other partners who question their actions. It was important for her because as she said "it feels like someone has my back. If I get called out by a parent or administrator, it is not like I made that decision. The idea of two minds makes better. Bounce ideas by each other." (Kayla, Post

interview, 04.18.2014). Therefore, involving multiple people in making professional decisions, helped teachers consider an issue from multiple perspectives as well as made them feel confident in their practices.

Access to Richer Set of Resources

The ongoing collaboration among these three teachers provided an access to richer set of resources because the materials in their classrooms were made available to each other, as well as their time, perspective and experience. Kayla borrowed pattern blocks from Kristina and Kristina told her which pattern blocks worked better for the specific purpose. Kristina borrowed angle rulers from Kayla's class. Kristina asked Paul for mini-white boards and he said he could easily cut mini white boards in his technical education class. If Kayla wanted to have her students take an exam she sometimes asked Kristina to watch her students and this was true for Kristina and Paul. If their students needed additional time to finish their exam, even though they did not have their classroom available or fully occupied for the rest of the day, they could ask each other to watch their students while taking the test. For all these things, they even did not need to check with each other beforehand. It was part of the culture they could use each other's classroom if it was available or send a student to finish an assignment or take test during each other's planning time.

In addition to sharing physical resources, they also shared extra-materials they created for their classes. For example, Paul shared one of his extra worksheets for students in *Stretching and Shrinking* unit with both Kayla and Kristina (Field Notes, 12.09.2103). He had his students draw a Raider Bird by placing coordinate points on a coordinate grid. The Raider Bird was the school's mascot. Kayla had some advanced students in one of her eight grade classes and she

got suggestions from Kristina for extra-student work Kristina used with her algebra track students (Figure 21).

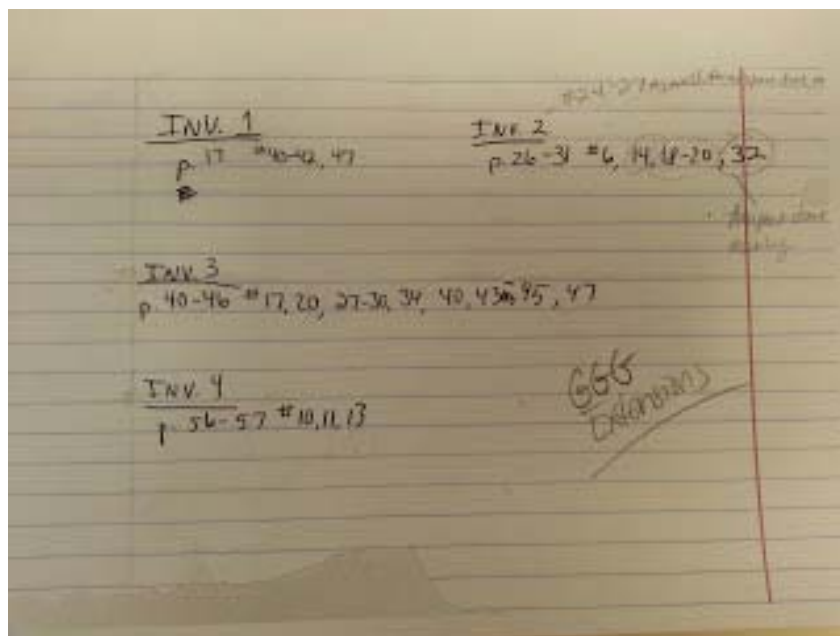


Figure 21

Kristina's extra-work suggestions for Kayla

During Explore time Kayla assigned those questions to her advanced students. Kristina also shared other student materials she created for her students. In addition to extra student work coming from each other's' classes, they also had access to student issues or exemplary students work in other classrooms, because they talked to each other. Kayla and Kristina provided updated to each other, so they knew what was going on and if there were some strategies or examples that worked better or that did not work.

Chapter Summary

This chapter concerned the outcomes of collaboration. Some outcomes were more objective, with some tangible components, and other were more interpretive, requiring my

judgment as a researcher. Thus, the outcomes of teachers' collaborative interactions were located along a continuum from *all objective products* to *all interpretive products*. Products situated within teachers' daily activities had more tangible components and some intangible aspects. *Assessments, Instructional Schedules, Lesson Plans, and Considering Mathematics Problems* were examples of this first category. Teachers produced assessments or provided solutions to mathematics problems as a product of their collaborative interactions. In addition to tangible products, teachers' interactions enriched their shared knowledge space and likely improved their MKT. For example, as they interacted about their shared curriculum, they improved their understanding of content and curriculum (KCC). When Kristina brought some outstanding student work from the *Stretching and Shrinking* unit, it not only informed Kayla's lesson planning, but also her understanding of knowledge of content and students (KCS).

Those individual products and teachers' continuous interactions served to build a broader set of products. The collaborations helped teachers in *Raising Issues, Expanding the Circle of Collaboration Participants, promoting Self-Care, Supporting Informed Decision Making, and providing Access to Richer Set of Resources*. While teachers worked together on an assessment or discussed what just happened in their classrooms, conversations helped them to *Raise Issues* that moved their thinking beyond the specific context of those conversations. In addition, other peripheral participants (e.g., the science teacher across the hallway) became involved and, therefore, *Expanded their Circle of Collaboration Participants*. Teachers' collaborative interactions also *Promoted Self-Care* as they showed their care and concern for each other, for their personal lives in addition to their professional lives. Furthermore, their collaboration *Informed their Decision Making* in the sense that it was not only one person making all the decisions but it was a product of collective thinking. Kayla noted this collective decision-

making as one of the positive aspects of their collaborative interactions. Lastly, their collaborations provided *Access to Richer Set of Resources*, because they had access to materials in each other's classrooms and to experience using those materials. For example, Kristina shared how she created an extra worksheet for her students in *Stretching and Shrinking* unit involving negative numbers. She shared a copy of the worksheet with Kayla as well as her rationale for creating it and how she would use it.

These products provided a wide range of examples on how teachers addressed some of their concerns about relationships at the school level, how those interactions led to naming issues, and how teachers' collaborative effort provided some resolutions. They helped the teachers with the affective and social aspects of their work (non-MKT); they helped teachers negotiate who they were as teachers and person(s) in this school context. For example, when Kayla questioned Mr. Norm's visit to his classroom in asking for her opinion for a school level issue, the conversations between Kayla and Kristina provided Kayla an outsider view of her teaching. In addition, these products contributed to their bonding as a community because as they noted they had each other for support and to think through their school- or teaching-related concerns. Lastly, these products helped teachers to eliminate potential issues that might have distracted from their teaching focus.

CHAPTER 8

CONCLUSIONS, LIMITATIONS, AND IMPLICATIONS

In this study, I aimed to describe one set of middle school mathematics teachers' self-structured and spontaneous collaborations as they taught from the *Connected Mathematics* (CMP) curriculum . I adopted an inclusive definition of collaboration as activities and dialogues that “involve teachers working together” (Hargreaves, 1994, p. 188) in a bilateral interaction (Dillenbourg, 1999) by sharing responsibility, initiation, and leadership in their activities (Little, 1990). I used this definition because it did not limit teacher collaboration to certain forms or activities like dialogue or problem solving and did not necessarily require symmetries of actions, status, and knowledge among those collaborating.

The case study posed three research questions aiming to provide a detailed picture of teachers' spontaneous and self-structured collaborative interactions:

1. How do middle school mathematics teachers, using *CMP* materials, collaborate with each other spontaneously or in self-structured ways?
2. How do these collaborations relate to teachers' practices?
3. In what ways are teachers' collaborations productive?

I addressed the first question by examining the forms, foci, and nature of interactions as well as by describing the factors shaping the teachers' collaborations. For the second, I analyzed teachers' collaborative interactions in relation to their individual teaching practices (in-class or outside of class). For the third, I examined the evidence that teachers' collaborative interactions led to different types of positive outcomes for them and their students, both directly and indirectly.

In this chapter, I will first summarize my main results. In this first summary section, I will highlight connections to related literature when they are relevant. Following that I will discuss some of the issues that appeared in my results for further consideration. This discussion will move beyond the results pertained to this study and will involve some speculations concerning what might happen in other contexts with similar characteristics or how teachers' interactions might appear in an even longer term than my data collection process. Following this extensions and speculations section, I consider what this study means in a broader context. I will end this chapter with limitations, implications and further questions to consider.

Summary of Main Results

The majority of teachers' collaborative interactions were spontaneous; they took place in particular groupings, with the highest frequency occurring between a particular pair (Kayla & Kristina). Teachers interacted often in check-ins that provided quick updates or responded to simple requests from one teacher. Almost as frequently, they exchanged ideas that extended or deepened the scope of their interaction. These findings confirmed reports from other researchers that the majority of teachers self-initiated activities involved sharing information (e.g., Cousins, Ross, & Maynes, 1994; Davison, 2006). This study provided more insight into the concerns raised by other researchers (e.g., Coburn, Mata, & Choi, 2013; Kwakman, 2003) that teacher collaboration has not been described as clearly as needed to understand it. Coburn et al (2013) noted how viewing teacher activities from a structural perspective (that is, "explaining various network processes and outcomes in terms of network structure" by looking at who is interacting with who and what they are doing) provided only a limited view of teacher collaboration (p. 331). For example, in my study teacher activities characterized as interchange of ideas showed a variety in purpose and depth depending on the content focus. Therefore, conclusions such as the

majority of teacher collaborations were either check-ins or interchange of ideas would have oversimplified and under-explained what happened in those interactions.

In contrast to other studies where researchers used Mathematical Knowledge for Teaching (MKT) (Ball, Thames and Phelps, 2008) to document teacher learning in professional activities, I used the MKT framework to describe teachers' interactions which were focused on mathematics teaching. Overall, in all groupings, teachers' interactions were more often focused on non-MKT issues, such as the focus on student disciplinary issues or school related issues. The most frequent MKT-focused conversations were over *Knowledge of Content and Curriculum* (KCC). As Coburn et al (2013) noted, most studies on teachers' spontaneous collaborative interactions have attended on the process of collaboration and focused less on their content (e.g., Little 2003) whereas this study's detailed analysis of content, provided insight into the content of teachers' collaborative interactions—that is, what the teachers interacted about.

This study also characterized the depth of the interactions and the positioning among teachers. I considered depth of interactions over a three point scale—low, medium and high (Coburn and Russell, 2008). The majority of teachers' interactions were low-depth. However, when teachers had MKT-focused interactions, their interactions gained depth; MKT-focused interactions were generally medium-depth. Interactions around student-related issues gained depth as the attention shifted to teaching mathematics from more general teaching-related considerations, such as how it was important to set the norms at the beginning. The frequency of medium-depth interactions focused on curriculum, students or teaching in mathematics provides a counterexample to what Coburn et al (2013) argued about the role of district policies in shaping high-depth interactions in contrast to teachers' spontaneous interactions which were less focused

and took on story-telling forms. Depth in teachers' spontaneous collaborations need not arise from top-down district policies and initiatives.

This study also examined teachers' positioning each other (systemic) and positioning themselves with respect to mathematics and curriculum (semantic). I determined systemic positioning in interaction by considering teachers' initiation, justification, and challenging roles (Greeno, 2011). The role of initiation was distributed among teachers. Justification and challenging roles were mostly taken by the one teacher (Kristina) who had the most curriculum-related experience. She had more agency with respect to both mathematics and curriculum, and therefore was expected to provide justifications and challenges. This pattern confirmed the role of more knowledgeable participant in moving teachers' collaborative interactions to a higher depth (Coburn et al, 2013). This study provided numerous examples of one teacher raising specific classroom-related concerns to another teacher with higher expertise, where those interactions provided a context for teachers to voice more general concerns.

This study also established that physical, practical and personal-interpersonal factors play important roles in shaping teachers' collaborative interactions. Within these factors I identified some particular elements that had more direct impact in shaping teachers' interactions than others: (a) teaching a demanding curriculum, in this case, *CMP*; (b) the differential expertise in the group; (c) the desire to improve practice through sharing teaching-related concerns and inspirations; (d) continuous interactions where teachers considered MKT-related issues and (e) trust as necessary precondition for serious concerns to be raised. The first factor (a) was practical; the rest were personal-interpersonal factors. Therefore, this study concurred what other researchers have found: Teacher collaboration is due to interaction of different factors (e.g.,

Hodkinson & Hodkinson, 2004; Kwakman, 2003) but personal and interpersonal factors can play an especially important role (Kwakman, 2003).

There were many examples of teachers' classroom concerns or inspirations arising in their collaborative interactions. The majority were informed by teachers' concerns about their students or teaching. These detailed accounts of teachers' concerns about their students and teaching, informed by their in-class practices, made their teaching visible to their peers. This is important finding because this supports what Little (2002) noted about the role of transparency in supporting teacher learning from collaborative interactions. Specifically, such conversations can help teachers to voice their concerns to colleagues in order to gain insight from multiple perspectives (Penuel, Riel, Krause, & Frank, 2009). In addition, they can become venues for sharing their experiences and artifacts from their class (Lampert, Boerst, & Graziani, 2011).

Teachers brought up classroom examples not only to share their feelings of accomplishment but also to inform other teachers about different ways of facilitating student work. However, unlike the frequent occurrences of in-class concerns and inspirations in teachers' collaborative interactions, there were fewer instances where there was clear evidence that teachers' collaborative interactions influenced their classroom practices. Adjustments to practice were mostly minor, and most took place in one teacher's classroom (Kayla's).

In examining the productivity of teachers' collaboration, I looked for outcomes that became visible after teacher-to-teacher interactions that were not apparent before those interactions. Those outcomes were partly objective, with some tangible components, and partly interpretive, where I (the researcher) imputed some intangible gains to the teachers. Teachers' redesigned assessments or "new" solutions to mathematics problems were important tangible products. In terms of interpretive products, teachers' collaborative interactions nurtured their

MKT. As they interacted around their curricula materials, they improved their understanding of content and curriculum (KCC). Those individual products and teachers' continuous interactions served to build a broader set of "products," such as the willingness to raise issues or expand the circle of collaboration. They also provided social and affective support to the teachers. In addition, these products helped teachers to eliminate some potential issues that might have distracted from their teaching focus, such as the classroom observation and evaluation of their teaching by building administrators.

This analysis of productivity of teacher collaboration contributed to the research on teacher collaboration by relating teachers' collaborative interactions to teachers' tangible gains and contributions to teacher learning. The latter was documented by Sun et al (2014) through assessment of teachers' knowledge, where in this study teachers' MKT-related interactions were counted as evidence supporting teacher learning. Since this study considered both objective and interpretive products, the results present a more comprehensive description of implications of teachers' collaborative interactions in contrast to some other studies focusing on individual gains (e.g., Penuel, Sun, Frank, & Gallagher, 2012).

Extensions and Speculations

This case study focused on a particular group of teachers in a particular school, but it raises important issues for further consideration. I consider and discuss five such issues that arise from the study. Exploring them would take me beyond the bounds of this study. First, teachers' collaborative interactions created a shared knowledge space for teachers where they became more knowledgeable in and about teaching (Adler, 2000). Second, teachers' positioning one another in their interactions and their positioning themselves with respect to mathematics and curriculum provided entry points and also became a limitation in deepening their understanding.

Third, teachers' collaborative interactions did not appear in their practices as frequently, and where changes were observed in their practices, those were minimal. Fourth, this study suggests that a lack of conflict around central issues in teaching mathematics may limit teacher growth. In this study, observed challenges were mostly one directional and if there was case of conflict teachers often decided not to pursue it. Fifth, teachers with divided roles and identities (such as Paul in this study) can benefit from the presence of a newcomer (Kayla) in becoming a more central participant in their interactions. In the coming paragraphs, I elaborate more on these points by bringing related literature when appropriate.

Shared Knowledge Spaces are Productive

In this study, I used the coordination of constructivist (Dewey, 1966) and situative (Putnam & Borko, 2000) perspectives on teacher learning to describe and understand teacher collaboration as a practice of professional growth. With that purpose, I observed teacher activities in their shared space as well as during their individual practices. By shared space I mean physical space that are inherently social, as hallways or teacher lounges. In this study, teachers' classrooms also became shared spaces during break times. Teachers' shared spaces can also become intellectual spaces as teachers make their thinking visible to each other.

Constructivist and situative perspectives allowed me to identify the value of the community in providing opportunities to participate in teaching related activities and "through this participation, a process of becoming knowledgeable in and about teaching" (Adler, 2000, p. 37-38). I analyzed teacher practices in their shared space by its content focus and depth of the interactions. The MKT framework allowed me to characterize the content foci in their shared knowledge space with continuous involvement in various collaborative activities. The depth of interactions suggested ways teachers' interactions can move them further in their practices.

Viewing knowledge as interaction (Greeno, 2011), I would argue that teachers in this study built a knowledge space of *KCC* through focused interactions in varying depth (Figure 22).

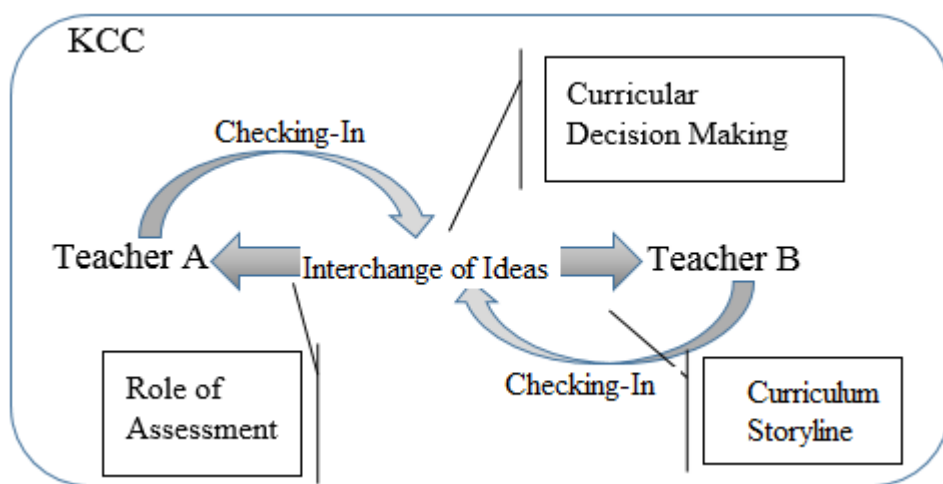


Figure 22

Teachers' *KCC* knowledge space

Their interactions were sometimes only check-ins where one teacher simply responded to the other teacher's questions, or provided updates. But some checking-in conversations became interchanges of ideas and as teachers interacted in a particular domain, those interactions provided them access to in-depth considerations of some topics. Teachers in this study became more knowledgeable about curricular decision making, role of assessments and content specific curriculum storylines in their *KCC* knowledge space.

This study provided evidence that instead of length or form of interaction, it was the continuity of interactions helped teachers to co-construct their shared knowledge space. Moreover, *KCC* was a significant domain of knowledge teachers needed to and did interact around. For example, teachers talked about how mathematics ideas were developed over a unit, how the mathematics students were expected to complete over a class period would contribute to students' understanding in the long term. This finding expands consideration of the role of

curriculum materials in shaping teachers' practices. Researchers (e.g. Remillard, 2005) have noted how curriculum played a substantial role in shaping teachers' classroom practices, and this study provided evidence that curriculum also informed and shaped teachers' self-initiated collaborative activities.

Steps toward building a co-constructed shared knowledge space are important for stimulating further collaborative interactions. Teachers' can view each other as resources in seeking answers to emergent questions, exploring uncertainties appeared during their daily practices and in-depth consideration of some issues. Similar to Ben Peretz and Schonman's (2000) analysis of teacher lounges as a place to let out teachers' concerns about their teaching or students, teachers can create a new meaning to their shared space. A shared physical space can become a shared knowledge space as it happened in this case. Teachers' shared spaces can even serve as an impulse triggering further learning opportunities. Once the shared space becomes an outlet where teachers' voice their teaching related concerns and inspirations, with their continuing interchange of ideas, they will likely explore the topic of concern in more depth from multiple perspective hence become more knowledgeable.

In the coming paragraphs, I elaborate more on *curricular decision making* as an arena for becoming more knowledgeable and for *individual teacher's learning* as change in patterns of interactions. These two topics are discussed as part of "shared knowledge space" because both speak to what was evident in their shared space and what was missing. For the first, the teachers' interactions examined in this study were predominantly shaped by curriculum-related considerations. Curricular decision making was one component of the KCC knowledge domain. For the second, I will talk about how teachers' roles in their interactions did not change over time.

Curricular decision-making. Planning for teaching requires decision making that involves various teaching-related considerations. Especially in reform-oriented curricula, with many details to attend to, teachers need to identify what is important, what to pay attention in students' work, among many other challenges (Remillard, 2000). Curricular decision-making requires the coordination of teachers' knowledge of teaching, students, curriculum, and mathematics in a dynamic relationship with curriculum materials (Remillard, 2005). Remillard (2005) called "for opportunities to read and examine a new curriculum with colleagues, making their interpretations and decisions explicit to themselves and others" (p. 239). This study provided evidence that teachers' interactions around their curriculum materials provided opportunities for understanding and improving curricular decision making.

Greeno (2011) described "negotiating different interpretations for mutual understanding" as a sign of achievement in building emergent understanding (p. 42). Teachers in this study considered making changes to the content provided in the materials. With their discrete but connected interactions, they became more knowledgeable about curriculum-related considerations, such as how ideas were connected, what to expect when from students, or what to consider in skipping a Problem or not. The interactions illustrating curricular decision-making were mostly facilitated with pivotal contributions from one teacher (Kristina) (Coburn & Russell, 2008). The teacher with more experience in using curriculum materials directed conversations and illustrated teachers' use of autonomy in making decisions (Little, 2002).

Remillard (2005) noted how curriculum-related experience and knowledge of the content and the materials were important in curricular decision-making. Engaging in the process with access to the reasons behind certain decisions instead of blindly following what is provided in the materials or having another person making all the decisions can make this process more

transparent. It can also help teachers become more knowledgeable about this process. This study provided evidence of how presence of more informed other person can model the process and help teaching peers move from blind followers of the materials and prevent them from making uninformed decisions. It also showed how hard it can be for beginning teachers to make curriculum-related decisions partly because they do not know where students are coming from and where they are going. They also lack a complete picture of how mathematics is developed, so presence of another person helping out in making such decisions can make this process more transparent and can also provide the appropriate support in becoming more knowledgeable about curriculum related issues.

Individual's learning. Change in an individual participants' discourse patterns has been taken as an indicator of learning (Lave & Wenger, 1991). One main way that discourse patterns change is through shifts in the roles that interacting parties take on. Therefore in speculating about teacher's individual learning, I considered the roles that teachers took on in their interactions over the course of my observations. I analyzed teachers' social interactions by examining how they were positioned in their MKT-focused interactions (Greeno, 2011). Contrary to my expectations, there were no dramatic changes in those patterns. The teachers' interpersonal interactions were not as dynamic as I expected. I want to acknowledge the complexity in shifting teachers' self-assumed roles, since teachers' interpersonal positioning was not the only factor shaping their interactions. Hodkinson and Hodkinson (2004) noted presence of many interfering factors in shaping individual's learning when they analyzed workplace learning experiences of two teachers. In this context, teachers' roles were determined by interaction of their systemic and semantic positioning (Greeno, 2011). Changes in Kayla's role in her relationship with Kristina were also related with changes in her agency with respect to

curriculum and mathematics. Even though there was an extensive data collection period, teachers might have needed more time for visible changes in their roles during their interactions. In contrast to this expectation that the teachers' roles could have changed over the longer term, it is also possible that teachers' roles might not. Since Coburn, Mata, and Choi (2013) reported change in teachers' interaction routines as they continued their interactions, in a context with frequent collaborative interactions change in teachers' interaction routines is a more likely possibility. As teachers evolve and expand their shared knowledge space, they might gain more agency with respect to curriculum and mathematics leading to changes in their patterns of their interactions.

Consideration of the Depth of Interactions

The depth of interactions can vary in the way teachers interacted around topics. Teachers' interactions gain more depth when they provide their rationale for their decisions or if they consider underlying principles or theories (Coburn & Russell, 2008). Depth of interactions, as analyzed in this study by using a three point scale –low, medium and high- contributed to the existing literature introducing and using this framework (e.g., Coburn & Russell, 2008) because this study detailed descriptions of low, medium and high-depth interactions and provided a large set of examples for the interactions falling in each category. In addition, this study raised some issues for further consideration related to the analysis of the depth of interactions. First, teachers' systemic and semantic positioning explained presence or non-presence of some of the medium and high-depth interactions. Second, findings in this study suggested some limitations in considering teachers' interactions on a three-point scale.

Conceptual agency in shaping the depth of interaction. This community had strong ties via their frequent interactions, and there was variety in teachers' expertise (Coburn et al,

2012). This differential expertise (Shulman & Shulman, 2004) brought variety in the depth of their interactions. The “synergy” came alive between Kayla and Kristina with the presence of both strong ties and differential expertise (Coburn et al, 2012, p. 160) and led to many medium-depth interactions. Moreover, it was important to have Kayla, engaged in interactions with Paul. She became the “go-to” colleague for Paul and brought up justifications in their interactions.

Interactions categorized as interchange of ideas provided room for teachers to take turns in moving their conversations further, so that all three teachers contributed to the variety in the depth of their interactions. In MKT-focused interactions, there was a repeated pattern of teachers providing justification and challenging each other. The interactions gained more depth when the challenges were picked up by Kristina. This finding confirmed what some other researchers noted about the role of a more knowledgeable other person (e.g., Sun et al, 2014; Coburn et al, 2012). Therefore, when someone has conceptual agency teachers’ collaborative interactions interaction can gain depth.

This study provided insight into the analysis of teachers’ collaborative interactions by showing how teachers’ existing roles influenced the topics they chose to address more and how they left some questions unaddressed. For example, Kayla and Kristina did not consider Kayla’s center of rotation question due to Kristina’s low conceptual agency in transformational geometry. Even with her overall “high expertise,” her “low conceptual agency” on this topic was very influential in shaping the outcome. So, even though there was synergy in Kayla’s and Kristina’s interactions as a frequently interacting pair, and there were many examples of at least medium-depth interactions, the teachers’ expertise and willingness to come together were not the sole determining factors shaping their interactions. Teachers’ positioning themselves with respect to curriculum and discipline also played a role in shaping the depth of interactions. This

finding highlights the importance of surrounding context in shaping teachers' interactions. That context includes (but is not limited to) participants' backgrounds and their disciplinary positioning themselves with respect to the topic of conversation. Even in highly collaborative environments, some issues are not articulated in-depth or left unaddressed. It is important to know what issues teachers leave behind besides knowing more about issues they pay more attention.

Concerns to let-go and concerns to follow-up on. Teachers' conceptual agency was not the sole factor in determining whether or not to follow up on a concern. There were unspoken social norms in teachers' lives that also influenced the routines of their interactions (Kilduff, Brass, 2010). Ben-Peretz and Schonmann (2000) described the role teachers' lounges play as settings for Catharsis, where teachers are able to voice their concerns and frustrations, and more importantly, to overcome them. In those professional spaces, teachers' personal and public identities overlapped and provided a room for talking about issues in condensed forms. In this study, teachers' professional spaces were their hallways. In hallway conversations they voiced their concerns and frustrations. For some issues, the purpose was not necessarily to find a solution. This was apparent the ways teachers addressed student-related issues outside of MKT domain. These were mostly low-depth interactions. As Kayla noted, letting those concerns out was the "mental break" from those issues. Letting-go of some issues, without bringing too many details, might have been teachers' choice. In contrast to this catharsis-focused interactions, when teachers brought up students with a consideration of teaching mathematics, there were more details provided around the issue with reasons for opinions, and they followed up many of these KCS-related concerns. Conversations situated within the context of teaching mathematics were like a safe arena in letting teachers to get in more-depth interactions. When teachers elaborate

more on issues MKT related issues it is not about one teachers' classroom or students so it is less personal and less threatening in that way. This result is important because it suggests less-threatening ways to initiate collaborative cultures where teachers develop bonding. As teachers keep their interactions ongoing they may establish the trust where they pick up some other issues. With established trust they could expect that their concerns will not be downplayed or be seen as personal weaknesses.

Issues with coding interactions on a three point scale. Teachers considered issues from their practice in order to address their specific needs. Typically, their interactions remained within the limitations of the specific context of their school, or students, or materials. From these results, it would be unfair to draw the conclusion that teachers cannot hold high-depth interactions in self-initiated professional development settings. The teachers in this study wanted to improve their practice by attending to demands coming from their professional lives. If those demands required them to attend to particulars in the context of their school, that was what they did. For example, in parallel to what Slavit and Nelson (2010) found, when teachers looked closely at specific students' work, they had more discussion with contributions from all teachers. Addressing their students' needs were their immediate concerns. When they attended to those, their interactions often moved from low-depth to medium-depth. Their interactions might have become high-depth if they considered more general learning theories or commonly documented student misconceptions in their conversations, but they generally did not. One reason may be that these ideas were not within the immediate access of teachers. Alternatively, the teachers may not have felt a need to bring such general considerations into their discussion of specific issues, because the medium-depth interactions addressed their concerns. This finding speaks to the concerns about teachers not holding in-depth interactions. Arguably, teachers' medium-

depth interactions are as valuable as high-depth interactions with being responsive to their “local” needs.

Assessing the depth of interactions over a three-point scale provided general insight into the characteristics of teachers’ interactions, however the variety within each level was obscured by that scale. The pool of medium-depth interactions provided examples of such variety. Within the pool of medium-depth interactions, some interactions were scarcely above low-depth interactions where others were very close to high-depth interactions. This raises the question of whether this three-point scale was useful in understanding the depth of teachers’ interactions. In addition, if medium-depth interactions made up the majority of spontaneous interactions, what kind of reasoning teachers should bring into their conversations? For example, is it enough to say, “we don’t have time for this test” when Kristina provided as a justification for her suggestion to skip a check-up quiz for the *Frogs, Fleas and Painted Cubes* unit? This instance was different from others where teachers brought more explanation to matter of consideration. Therefore, there were differences in the scope of justification provided for decisions. Considering these results, working with a continuum may be preferable to a discrete three level scale.

Furthermore, when teacher raised more general issues of teaching and learning, their perspective was limited by their experience. When the interaction moved from the specific context of their classrooms or school to consideration of more general contexts, it was very most frequently Kristina, the teacher with the high expertise, who did so. The generalities the teachers brought up were generalities from their own teaching practice, rather than generalities inferred from any learning principle or a research-based resource. For that reason, most of teachers’ high-depth interactions stayed at the boundary of medium-to-high depth interactions.

The dilemma between teachers' involvement in more medium-depth interactions (and high-depth interactions that were informed from their own experiences) and the teacher educators' desire to involve teachers in higher-depth interactions, speak to the concerns about connecting research and practice. On one hand, teacher educators want to help teachers in moving their thinking beyond particulars. On the other hand, teachers want to address context-specific issues. These two separate groups (teacher educators and teachers) need to interact solely for the purpose of understanding needs, goals, and expectations. This speaks to what Horn and Kane (2015) noted when they analyzed teachers' learning opportunities in teacher workgroups, those "are only one facet in a broader system of teacher development, and the expertise of an instructional coach or professional development provider can press teacher conversations toward the consideration of other facets of teaching" (p. 415). Teachers' will want to see particular value in discussions of general issues. At the same time, they will need help in moving from their specific context of teaching in order to understand issues better.

Changes Related to Classroom Practices

The context of the study was different from some other settings in that the teachers trusted the curriculum materials they were using. Their interactions mostly aimed to better understand the materials and get closer to "CMP way of teaching." So teacher collaboration in this setting was likely different from other school settings where teachers have major issues with their curriculum materials. In this setting, there were few major changes to the existing lessons, but there were many minor adjustments to lessons. But change is an ongoing process. This study confirmed that teachers' collaborative interactions can lead to changes in teachers' practices (e.g. Sun, Wilhelm, Larson, & Frank, 2014; Slavit & Nelson, 2010). Since most of those changes were content-based, this study also provided evidence that teachers' content-

focused activities with their collective participation will more likely change their practices (Graham, 2007). As teachers bring concerns from their in-class practices, they consider issues from different perspectives (Little, 2002) and those reviews and continuing minor adjustments can create a path to major improvements in teachers' practices.

The frequency of the relatively minor changes in teachers' in-class practices in this highly collaborative environment raises the question of what is reasonable to expect from teachers in learning and changes in practice. Most of the professional development activities look for evidence in substantial changes in teachers' practices but change in teachers' practices can take longer time than anticipated and minor but ongoing change in teachers' practice can be good enough to count as impact of teachers' collaborations.

Lack of Conflict

The interactions observed in this context were mostly free of observable conflict. There were some issues that could have created conflict among teachers, but they typically ended those conversations by accepting the other person's view and did not push their conversations further in such cases. Piaget (1970) notion of disequilibrium, that describes the confusion between a person's existing knowledge and new knowledge, was a major factor in his theory of cognitive development. At the organizational level, disequilibrium moments have been seen as effective for transformative learning experience (Davis & Sumara, 2001). In this context, interactions with potential to cause disequilibrium moments were brought in several ways by Kristina or Kayla as challenging moves. Those challenges were usually addressed by the same person (Kristina) without leaving any room for a real conflict. There were also times when challenging questions remained unresolved without further conversations. For example, when Kristina asked Kayla "isn't it what we want to see" as a response to Kayla's anxiety about her students'

frustration in her lesson, they did not talk further about what frustration meant and Kayla did not indicate whether she was okay with the frustration or not. Therefore, there were not many opportunities for a real disequilibrium even between the most interacting pair.

This lack of conflict might suggest uncertainty among the teachers about when to push and how to push. In describing the characteristics of effective professional development settings Borko (2004) noted that importance of critical dialogues by maintaining “a balance between respecting individual community members and critically analyzing issues in their teaching” (p. 7). Therefore, professional conflict requires coordination of multiple considerations including trust and respect. Conflict can create frustration. Teachers’ uncertainty about the level of frustration peers may experience and their willingness to be supportive of each other might be reasons for absence of conflict in this context. Desire to see commonalties in contrast to differences may also be another reason for these teachers’ unwillingness to experience professional conflict in their interactions. As Kayla noted in one of our interviews, she wanted to see “we all screw up” for sometimes. Teachers may want to see uniformity in their actions rather than diversity, as the latter may bring conflict. Coburn, Mata and Choi (2013) noted related this issue to desire for homophily, interacting with people who some kind of resemblance. The lack of conflict in this context might be because of teachers’ desire for similarities rather than differences.

Lack of major changes in teachers’ practices may be related to the relative lack of conflict in this context. Transformational learning experiences with a potential to lead to dramatic changes in teachers’ practices can occur with true confrontations. Yet, teachers will need some other teachers who mirror flaws in their teaching. Those other teachers need to be persons whom they trust and who are familiar with their routines. There needs to be a balance in

between those confrontational interactions and shared similarities. The latter will serve as a glue in keeping the community together.

Teachers with Divided Roles and Identities

Paul was an example of a teacher with a divided identity and a divided schedule. His connections to the curriculum and subject could have been different if his role was the same as Kristina and Kayla. With his dual role of teaching mathematics and woodshop and split time and attention that came with that role, he is an example of teachers that can be found in many contexts. But his case also illustrates how a teacher can move more to the center from being a peripheral participant. In this case, the relation between Kayla and Paul was crucial. Kayla was a new addition to the grades seven and eight mathematics team, and Paul described one of his roles as supporting Kayla. At the same time, however, their conversations were useful for him, and even led him to change one of his lessons. Paul's interactions with Kayla were more focused on students and context, but if they had continued, those interactions might have come to focus more on MKT-related issues.

The frequent interactions between Paul and Kayla showed it is important for teachers to find another they can relate to. These teachers' self-assumed roles helped them to keep their interaction frequent and substantive. In addition, as teachers keep their interactions ongoing, their initial normalizing approaches such as showing each other that they have similar disciplinary issues in their classrooms, can turn their conversations to content-based in-depth interactions. Therefore, this study suggested ways to help teachers to find a peer they can relate to and illustrated how teachers can help each other in becoming more central participant in their interactions.

Consideration of Other Contexts

This study provided insight into the teacher collaboration in a very specific context, but there are arguably some generalities represented in this specific context as well. This study showed that teachers can sustain collaborative interactions focused on different aspect of their teaching. Those interactions need not be long, and most may be responsive to teachers' immediate needs. Having teachers' classroom in close proximity can allow teachers use short break time and provide opportunities to keep their interactions ongoing. As teachers' will be more focused on MKT, their interactions can turn into deeper conversations when they bring reasons into their conversations. However, the presence of "more knowledgeable other person" can be crucial in those interactions. Therefore, in addition to having a colleague in close proximity, the distributed expertise within the group of teachers can play a key factor in teachers' interactions.

Scope and focus of change is related to the nature of teachers' interactions and some other factors. In this context, teachers carefully aligned the pacing of their teaching and that provided a common context for them to talk about. When teachers become more deliberate in increasing commonalities like teaching the same content around the same time, giving the same assessments, those commonalties will likely increase the frequency of their interactions and will provide a content focus to their interactions.

Teachers' willingness to make changes in their practices, as well as their reflective considerations of their teaching, will likely initiate conversations where they voice concerns and share their expertise. Yet, teachers' collaborative interactions may not lead to sudden dramatic changes in their practices. So it might be unrealistic to expect large shifts in teachers' instructional practices. In addition, even in a collaborative culture, if teachers are less willing to

change and not as reflective about their practices, collaborative interactions might help teachers to identify some problems, but will less likely to provide resolutions.

Limitations of the Study

Some limitations arose from the data collection process and the nature of the study. The generality of the results should be considered within these limitations. My main data analysis was based on field notes. There was a time lag between observations and recording of observations. I intentionally chose not record teachers' routine interactions with a recorder and only the interviews were audiotaped. The bulk of the data came from my field notes. The notes of teachers' spontaneous interactions were captured right after their interaction was ended. Because of that, my notes may not have captured completely what happened in this setting. Therefore, the issue of not being able to capture everything during teachers' collaborative interactions or not having verbatim transcripts of what they said is one of the limitations of this study.

Another limitation of this study was my background both as a second language speaker of English. In recording my observations, there was always some interpretation and there might have been cases my notes were not 100% true to observations. In addition, I am biased in terms of the materials, as a person involved in developing the next generation of CMP materials, and that might have influenced what I saw and I did not see.

There were also limitations in my data analysis. The frameworks used in the analysis helped me to describe my findings, but they also limited how I could describe my data. For example, as I noted in questioning coding the depth of interactions using a three point scale, presenting teachers' interactions over a continuum might have shed a light to differences in the

range of medium depth interactions. In addition, except from partial double coding all coding is carried out by one researcher and there might be some issues associated with the coding.

This was a study of one particular school setting. Teachers are dynamic communities therefore this community will evolve over time and the dynamics in this community might be different than what I observed during the data collection period. For example, Kristina noted how she intentionally let Kayla to be the main driver of their departmental meetings with all her questions. Other teachers tried to be responsive to Kayla's needs because she was the one who was in need of support at that time. Therefore, some results of the study were true for this specific context within the time frame of data collection. In addition, in a community with more than three teachers or with different background there will be different patterns of interactions.

One initial expectation I had when I entered this study was to capture symmetry in teachers' interactions over the long term. Even though teachers switched taking roles in initiating their interactions, there were more constant asymmetry in their expertise and roles in providing justifications and challenging each other. Kristina's background provided Kayla opportunities to engage in MKT-focused interactions, but how that collaboration might look like in a more balanced group of teachers is still not addressed. Therefore, even though this study provided a detailed examination of teachers' collaborative interactions, the generalizations were limited with the special characteristics of this group of teachers.

Implications and Recommendations

This study has the most implications to building leaders and teacher leaders. Many factors contributed to making teachers' collaborations happen. Teachers have limited time and needs that call for their immediate attention. Therefore, having a colleague teaching the same content in close proximity provides access to another person to share materials or bring questions

or concerns without any time delay or worrying for an extra trip to other teacher's classroom. Teachers had a shared space right in front of their classrooms and that helped to maintain their interactions ongoing. Coburn and Russell (2008) noted the importance of repeated interactions in building trust. Therefore, creating extra opportunities for teachers to engage, even at low-depth, will serve to help them build trust, and with that established trust, they may deepen their interactions. The distributed expertise in this group, helped teachers to gain more in-depth perspective in their interactions. Therefore, in teaching arrangements, building leaders might pay more attention in teaching assignments, like not having all novice teachers teach the same grade level.

This study has implications for professional development providers, in providing effective and responsive professional development opportunities. This study confirmed the calls for need for authentic professional development activities. These teachers' collaborative activities were grounded into their own teaching practices. In addition, the specific features of teachers' collaborative interactions described here illustrated the specific details of teachers working in collaboration with their own initiation. Teacher leaders or professional development providers, might make use of information on features of collaborative cultures and collaborative activities in working to build collaborative cultures in schools.

Building leaders and professional development providers need to consider multiple factors in helping teachers build a collaborative culture. Building a collaborative culture is a long-term investment. This school had a history of collaboration, and when a new teacher came, she accepted the role already defined for her. In addition, building personal relationships is important for not taking disagreements personal. Even though there were not that many conflicts, there were times Kristina challenged Kayla, and Kayla always responded to those

challenges very positively. Di Pardo (2009) noted how collaborative cultures promoted self-care. This self-care was the outcome of these teachers' collaborative interactions but at the same time was the catalyzer of their interactions. For a true collaborative culture, the aim should not be only improving teachers' MKT. Social and affective gains are as important as content based gains and in some ways carry the burden over time in keeping teachers' conversations ongoing.

This study may also inform researchers who want to study teachers' collaborative interactions. I attended to multiple factors in describing teacher collaboration and adjusted definitions Coburn and Russell (2008) used in describing depth of interactions with rich examples for each. In addition, I used MKT framework in describing content foci of teachers' interactions. This study was unique in that sense, because where the MKT framework was introduced for and used with individual teachers' assessment, I considered knowledge as interactions I operationalized a new way of using this framework. In addition, this study can inform researchers by providing an evidence-based descriptions of productive teacher collaboration.

Further Questions

This study raised questions for further consideration. I would like to acknowledge the limitation of this case with very particular characteristics. This was a case study of a setting where collaboration was part of the culture. In addition, teachers used CMP for years with the presence of a teacher with high expertise. So the findings might change by changing those determining characteristics of this case. One of the questions to follow up in contrast to this case is: What would collaboration look like when teachers were more balanced in terms of their experience? Will they be able to get their interactions into more depth? Will those interactions take longer? How different will be teachers' gains from that interaction? The outcomes of such a

collaboration might be different than this one because there will not be a person guiding their thinking.

Similarly, another study might be conducted where teachers' use a traditional curriculum. The foci of those interactions might be very different than the foci of interactions in this case. Will KCC be still the focus or whether teachers would have more KCT- and KCS-related considerations in planning for their lessons? In such a context, how different would be the nature of teachers' in-class practices informed from their collaborative interactions? Teacher collaboration with a focus on recrafting existing curriculum would change the case completely and teachers' interactions might be different than teachers' interactions in this context.

Last, there can be cases of Paul in every school. There might be even more Pauls than Kaylas and Kristinas. So my question is how can we help teachers like Paul? What does it take for teachers to notice flaws in their practice and be attentive and willing to get in more in depth, MKT-focused interactions? What is the appropriate level of conflict and main motivators for moving Paul forward? Along the same line with this question another research might explore how teacher educators can help teachers to build a culture where professional conflict is okay and productive in changing teachers' practice.

Concluding Remarks

This case study of a particular group of teachers in a particular school provided some important issues that will inform further research. Teachers' can create a collaborative context where they share their experience, examine emergent concerns and questions in their practice and as they interact within a domain of knowledge, they expand their understanding of the issues and possible ways to address those issues. Checking-ins can serve as an initiator and can become a glue for further activities. Teachers' positioning one another in their interactions and

their positioning themselves with respect to mathematics and curriculum can provide entry points in deepening their understanding. At the same time, teachers' positioning of themselves at the interpersonal or disciplinary level can limit in furthering their interactions. Shared knowledge co-constructed through collaborative interactions may not appear in different contexts as frequently, and it might be more realistic to expect for minor changes in teachers' practices rather than dramatic shifts.

APPENDICES

APPENDIX A

DATA COLLECTION AND PILOT WORK TIMELINE

Table 11

Timeline for data collection and pilot work

	Months	Pilot Work		Data Collection		CMP Units Math Focus
		Meet	Obs.	Int.	Obs.	
Spring 2013	May	√	√			Symmetry and Transformations, Making Sense of Symbols (Grade 8)
Fall 2013	Oct	√	√			Negative numbers (Grade 7) Exponential functions (Grade 8)
	Nov			√	√	Integers and rational numbers (Grade 7) Exponential functions (Grade 8)
	Dec				√	Understanding Similarity (Grade 7) The Pythagorean Theorem (Grade 8)
Spring 2014	Jan				√	Ratios, Rates, Percent and proportions (Grade 7) The Pythagorean Theorem Symmetry and Transformations (Grade 8)
	Feb				√	Ratios, Rates, Percent and Proportions (Grade 7) Symmetry and Transformations (Grade 8)
	Mar				√	Linear Relationships (Grade 7) Symmetry and Transformations Making sense of Symbols
	Apr			√	√	Linear Relationships (Grade 7) Making sense of Symbols (Grade 8)
	May			√		

Obs.: Observation, Int.: Interview, √: Presence of the activity

APPENDIX B

TEACHER INITIAL INTERVIEW PROTOCOL

Goal: Daily routines and learning opportunities

Describe a typical day in your work life? How do those affect your opportunities to learn?

Describe any defining moments in your daily routines where you learned important knowledge about what you do.

What role other teachers play in your professional development?

Goal: Nature of community in the school

Can you indicate how would you characterize your relationship with other mathematics teachers teaching grades 7 and 8 by attending to the following items?

(Never:1, Rarely:2, Sometimes:3, Often:4, Always:5) (Starting from Shared Interests: Strongly Disagree:1, Disagree:2, Undecided:3, Agree:4, Strongly Agree:5)

	1	2	3	4	5
Collaborative Relationships					
We have conversations structured around our teaching practices where I am able to share my own experiences.					
We have conversations structured around our teaching practices where I get to listen to other teachers' experiences.					
We have discussions over our practices where we look for things that help us to characterize best practices.					
We have discussions over a specific mathematics topic where we deepen our understanding.					
We have discussions about our students and their learning.					
I receive warm and sincere attitude from other mathematics teachers.					
I do not see a major distinction in between our roles in mathematics department.					
Shared Interests					
We all want us to move to a direction which all of us agreed on.					
We are able to discuss and share our philosophies.					
We do not see major differences when we share our beliefs.					
We all have a desire to learn more together.					
We have similar commitments to teaching mathematics and or student's learning.					
Voice					
We consider us as equals in voicing our thoughts.					
I receive respect when I express my thoughts.					
I show respect in listening others express their thoughts.					
We have a shared language in communicating our beliefs and ideas.					
Trust					
We know that our voices are heard in our community.					
There is a climate for asking questions and expressing conflict.					

I have learned to trust myself and my colleagues.						
Joy						
We celebrate each other's success.						
I feel lucky to be a part of this community.						
I am proud of what we are as a learning community.						
We have many joyful moments.						
There is an excitement about what we are doing and learning.						
There is an excitement in talking about our program.						
Support						
Everyone is welcoming.						
There is a special bond among us.						
We listen, question and encourage others' work.						
We get support from each other.						

Goal: Focus of collaboration

What is your definition of collaboration?

Can you tell me how would you characterize content of your collaborative activities with other mathematics teachers? In other words what do you do and what does get your most attention?

I would like to hear more about specifics of your collaboration. Please give some examples with respect to mathematics/teaching/learning and students.

Are there other things you collaborate on? If so, please give examples.

Goal: Frequency and forms of collaborative activities

Can you describe your regular meetings with other teachers and with mathematics teachers in your school? Who arranges those meetings and in what frequencies?

During a typical week in this school, how often do you interact with other teachers in one of the following ways? (on a scale from 1 (never), 5 (very often)). Please give examples.

Informal conversations?

Solving student problems?

Mathematical ideas?

Teaching?

Offering/receiving advice and assistance on a one-on-one basis?

Routine sharing of materials, Methods, and ideas?

Joint work involving shared responsibility for teaching and cooperative organization of tasks, time and resources?

Which of those happened more informally/formally?

Goal: Role of content and curriculum materials and other factors

In what ways do you think CMP shape the collaborative activities you are involved in with other teachers?

Do you recall moments where something in mathematics initiated your collaboration with other teachers? If so, can you explain?

Do you recall moments where something in CMP initiated your collaboration with other teachers? If so, can you explain?

Are there other things you can list contributing your collaboration with other teachers?

APPENDIX C

TEACHER POST INTERVIEW PROTOCOL

Group Interview Questions

Content, frequency and forms of collaborations

How would you define collaboration [teacher collaboration]?

How do you see yourself collaborating with each other?

What do you think that you end up talking about most frequently with each other?

I would like to hear more about specifics of your collaboration. This year what kind of issues in teaching mathematics and students' work in mathematics got your attention?

How did you interact with respect to those? Please give specific examples.

Were there other things you collaborated on? If so, please give examples.

Are there patterns of work that influence your collaboration?

You have department meetings regularly once in a month. How structured are those meetings from your perspective? What kind of agenda you usually have? Do you think any of you is more influential in setting up the agenda for these meetings? Who do you think more leading the conversations?

Role of Curriculum

Are there ways that you think CMP shapes your collaborative activities?

Are there specific features of CMP that lead you to work as a group or to ask for help from each other?

In what ways do you think that being a CMP pilot school shapes your collaborative activities?

Nature of community

If you had to use one word to describe your three-teacher group, what would that word be?

Can you talk about what do you think is unique about your community? What is more common?

Are there things you would like to see more of? And less of?

What are things you wanted to change?

What are some moments in your day in the school you enjoy? In what ways do you see each other contributing to those brief moments?

Conflict management

I know from what I have observed so far you don't have big disagreements. With respect to the minor how do you think you resolve those?

There was a partner quiz in Stretching and Shrinking and you have interacted before and after the quiz. At one point three of you were standing in front of D's classroom talking about the quiz.

There were some level of disagreement about a couple of questions or how they were provided.

Here is the partner quiz. (I will hand the Partner Quiz to them). Can you say more about that?

Would you consider that as a conflict or not? Do you all think that you resolved the issues came up with that partner quiz?

Were there other examples you remember like this and you either resolved or not?

Productivity of collaborations

What do you gain out of this interaction personally?

In addition to your personal gains do you think are there other things coming out of your interactions? Any tangible products? Any gains for other parties?

In what ways do you think your interactions with each other shaping up your classroom practices? Can you give some specific examples?

In what ways do you think your interactions with each other shaping up your practices outside of classroom? Can you give some specific examples?

Final Words

There are so many things unique about the community here but there are also some things that might be carried over to other settings. From your perspective what are a couple of things that you would recommend other teachers to build professional interactions as it is present here?

[personal and group wise]

Do's and Do not's, some personal characteristics

Kayla

In general, how do you see yourself interacting with Kristina and Paul?

Do you feel that you learn from your interactions with them? [Does she speak in terms of increasing her "confidence" or "learning" from her peers?]

From my observations I noticed there were some topics you continued talking about and others that were named as an issue but you did not follow through? Like point of rotations or using slopes in rotation. Can you say more about how you make those decisions (to pursue or not)?

Other collaborations: Ms. Patz and ICT?

All teachers share things happened in the class with respect to some problem students. From what I observed there was more listening and less suggestions. Was that a choice? Why.

Do you remember instances not-supporting my observation?

Personal Characteristics

What is your view of [teachers'/personal] weaknesses and strengths?

How do you see your strengths and weaknesses in teaching mathematics/8th grade mathematics?

Role of Curriculum

Since you have been teaching grades 6 and now 7 and 8 how would you assess students' learning? What is the role of the curriculum in their learning?

You are using teachers guide a lot. How do you find it useful? In what ways teachers' guide initiating or limiting your interactions with others?

Are there times you feel lost or confused? Do you recall any of those moments? What do you do in those cases? Are you satisfied with the support you get from others in those moments?

What is the role of the curriculum (CMP) in your learning? In what ways CMP is shaping up your interactions with other teachers?

Conflict:

What is your view of conflict?

Personal Conflict:

Have you ever see yourself going through a conflict personally? How frequently? Were there times more frequent than others? Less? Can you talk about a little bit about those moments? What caused that? How did you feel?

How do you (prefer to) resolve those? Do you feel like you get any help from other teachers in the building? If yes, can you say more on that. Who are those and in what ways they support you?

Conflict with others?

Reflecting back to your time here, are there things that you remember that can be considered as conflict involving other bodies? How did you resolve those? What kind of support did you get from others? Were you happy with the resolution?

Learning from conflict

In what ways those experiences contributed to you as a person and as a teacher? Would you identify any learning on your side related to those experiences?

Assessment of this Year

If we compare this year to previous years how would you consider this year as regular and not-regular? Explain.

Reflecting back over the time that I have been here (since November), what you do you consider high points and low points in your teaching work?

Collaboration relating practices

In what ways do you think your interactions with D and M shaping up your classroom practices? Your practices outside of classroom?

Personal Care

For example once you described your lunch times as mental break. Can you say more on that?

I remember you were saying you were lucky to have each other? Why are you lucky to have Kristina? Why are you lucky to have Paul? Do you feel the same for any other teacher? If yes, what makes you to say that?

How are you supporting each other in general? Are there things you personally need but you don't find?

Tag, (Map) and Rate

Reflect on your activities with other mathematics teachers. Can you list the activities and content, note names next to each showing who you are interacting the most with respect to that activity and content and can you rate those activities/content from 0 to 10, 0 meaning none 10 meaning happening more than twice per day on a regular basis.

Activities	Content
Informal conversations	Mathematics
solving students' problems	Teaching
solving mathematics problems	Students
offering advice	Assessment
receiving advice	Pacing,
assisting someone else	Homework
receiving assistance	Teachers
planning for teaching	School related issues
preparing test, reading	
observing another teacher teaching	
sharing materials	
joint work	

Kristina

In general, how do you see yourself interacting with Kayla and Paul?

How do you see your strengths and weaknesses as shaping or influencing those interactions?

If you reflected back to your learning experience this year, how would you summarize that? Did you have more or less of it? If you consider yourself having less learning on your side, what might be the reason for that? Did you ever feel you were challenged?

From my observations I noticed there were some topics you kept talking about it but there were some topics that were named as an issue but you did not follow through? Like point of rotations or using slopes in rotation. Can you say more about that? What made you to make a choice one way or another?

All teachers share things happened in the class with respect to some problem students. From what I observed there was more listening and less suggestions. Was that a choice? Why.

Do you remember instances not-supporting my observation?

Personal Characteristics

What is your view of [teachers'/personal] weaknesses and strengths?

I remember you were saying multiple times trying hard to give Andy more control. With respect to your personality – desire to keep things in control – how do you think that is affecting your relationship with other teachers?

In what ways it is contributing to the interactions you are having with Danny, Kayla and Paul?

Do you see any downside of things associate with that?

Conflict:

What is your view of conflict?

Personal Conflict:

Have you ever see yourself going through a conflict personally? How frequently? Were there times more frequent than others? Less? Can you talk about a little bit about those moments?

What caused that? How did you feel?

How do you (prefer to) resolve those? Do you feel like you get any help from other teachers in the building? If yes , can you say more on that. Who are those and in what ways they support you?

Asking M to CMP Getting to Know conference. Why do they think he needed to attend and what was their hesitation for not asking to him? Are there things you see he is missing? What is your thinking about what you can do and cannot do?

Conflict with others?

Reflecting back to your time here, are there things that you remember that can be considered as conflict involving other bodies? How did you resolve those? What kind of support did you get from others? Were you happy with the resolution?

Learning from conflict

In what ways those experiences contributed to you as a person and as a teacher? Would you identify any learning on your side related to those experiences?

Personal Care

I remember you were saying you were lucky to have each other? Why are you lucky to have Kayla? Why are you lucky to have Paul? Do you feel the same for any other teacher? If yes, what makes you to say that?

How are you supporting each other in general? Are there things you personally need but you don't find?

Collaboration relating practices

In what ways do you think your interactions with Kayla and Paul shaping up your classroom practices? You practices outside of classroom?

Assessment of this Year

If we compare this year to previous years how would you consider this year as regular and not-regular? Explain.

Reflecting back over the time that I have been here (since November), what you do you consider high points and low points in your teaching work?

Can you compare this year to last year. Since you had Yvonne last year do you think there are things you are missing?

Paul

In general, how do you see yourself interacting with Kristina and Kayla?

I realize that you don't teach mathematics all day. Do you ever feel left out in the group? Do you ever feel that you choose to be not included?

If you reflected back to your learning experience this year, how would you summarize that? Did you have more or less of it? If you consider yourself having less learning on your side, what might be the reason for that? Did you ever feel you were challenged?

All teachers share things happened in the class with respect to some problem students. From what I observed there was more listening and less suggestions. Was that a choice? Why.

Do you remember instances not-supporting my observation?

I have seen you have a good and consistent relationship with Mr. Kullen. Can you say more about that?

With respect to emergent issues how do you prefer to resolve those more often?

For example, ...

Collaboration relating practices

In what ways do you think your interactions with Kristina and Kayla shaping up your classroom practices? You practices outside of classroom?

Conflict:

What is your view of conflict?

Personal Conflict:

Have you ever see yourself going through a conflict personally? How frequently? Were there times more frequent than others? Less? Can you talk about a little bit about those moments?

What caused that? How did you feel?

How do you (prefer to) resolve those? Do you feel like you get any help from other teachers in the building? If yes , can you say more on that. Who are those and in what ways they support you?

Conflict with others?

Reflecting back to your time here, are there things that you remember that can be considered as conflict involving other bodies? How did you resolve those? What kind of support did you get from others? Were you happy with the resolution?

Learning from conflict

In what ways those experiences contributed to you as a person and as a teacher? Would you identify any learning on your side related to those experiences?

Personal Care

Would you consider having D and F with you as two other math teachers as an advantage. If yes, explain why. Are there other teachers you would consider the same way? If yes, who and what makes you to say that?

How are you supporting each other in general? Are there things you personally need but you don't find?

List, Tag, Map and Rate

Reflect on your activities with other mathematics teachers. Can you list the activities and content, note names next to each showing who you are interacting the most with respect to that activity and content and can you rate those activities/content from 0 to 10, 0 meaning none 10 meaning happening more than twice per day on a regular basis.

Activities: Informal conversations, solving students' problems, solving mathematics problems, offering advice, receiving advice, assisting someone else, receiving assistance, planning for teaching, preparing test, reading, observing another teacher teaching, sharing materials, joint work

Content: Mathematics, teaching, students, assessment, pacing, homework, teachers, teaching related issues, school related issues, ...

Activities	Content

Assessment of this Year

If we compare this year to previous years how would you consider this year as regular and not-regular? Explain.

Reflecting back over the time that I have been here (since November), what you do you consider high points and low points in your teaching work?

APPENDIX D

SCHOOL PRINCIPAL INTERVIEW PROTOCOL

Goal: Background

How long have you been school principal at the Portland Middle School?

Before being a school principal, what other duties did you perform for the school?

Thinking about a typical work day here, could you describe the interactions you have with teachers during the day?

How often do you attend common planning period meetings?

Goal: Professional development opportunities within the school

Describe the type and range of professional development activities that occur at this school?

How decisions are made concerning the content and organization of professional development activities?

How does the school identify and address teachers' needs?

Goal: Reciprocity of interactions among teachers

How would you characterize interaction among teachers in this school?

How does your group of teachers work together? Please describe and give examples of specific things.

Are teachers rewarded for working together?

Goal: Collaborative activities taking place in the school

What is your definition of collaboration?

Can you give examples of collaborative activities you observe or participate in this school?

How frequently do those activities take place?

APPENDIX E
OBSERVATION PROTOCOL

Date:

Time observation started:

Time Observation ended

Teacher Activity:

(Circle one that best describes the form of activity teacher is involved in)

Teacher alone --- Teacher is talking with students --- Teacher is talking with other teachers --- teacher is talking with another adult (administrator or parents or else)

Teacher collaborating with other teachers --- Classroom Teaching -- Structured Meeting
Nature of Interactions

Describe briefly who is leading the activity and for how long.

Content

What is the content of the activity?

Are there any issues raised? Explain.

Are there any strategies agreed upon? Explain.

Are there any products? Explain.

Are there any mathematics problems solved? Explain.

Is there a plan concerning future activities? Explain.

Referrals

Is there anything they refer to their previous-future collaborative activities? Explain.

Is there anything that looks as addition or adaptation of what is available? Explain.

APPENDIX F

CLASSROOM OBSERVATION PROTOCOL

Launch:

Teacher explicitly mentioned collaboration....

Might be related with collaboration..

Interesting...

Explore

Teacher explicitly mentioned collaboration....

Might be related with collaboration..

Interesting...

Summarize

Teacher explicitly mentioned collaboration....

Might be related with collaboration..

Interesting...

APPENDIX G

DEFAULT FOR TEACHER LOGS

Please consider other mathematics teachers in your school and respond to questions the following questions.

Who did you meet this week?

Why did you meet?

What did you do?

What do you consider as changed in your practice in relation to your collaborative activities? Please provide one or two specific examples.

APPENDIX H

THE DEPTH OF INTERACTIONS FRAMEWORK

Table 12

The depth of interactions definitions adapted from Coburn and Russel (2008, p. 230)

Depth	Description	Example
Low	Stating facts without talking about rationale; naming issues; surface level discussion of topics	How to use materials; how to coordinate text, standards, assessments, and pacing guides; how to organize the classroom; sharing materials or activities; general discussions of how a lesson went or whether students were getting it
Medium	Bringing how and/or why into the conversation, still talking referring to the specific context like a particular example in the textbook or a specific student in the class.	Discussions of how lesson went, including a discussion of why; detailed planning for lessons; including a discussion of why; specific and detailed discussion of instructional strategies in the context of observation; doing mathematics problems together with discussion
High	Moving from discussion of specific example to consider more general issues and even bringing underlying principles, theories and decisions.	Addressing pedagogical principles underlying instructional approaches; how students learn, or the nature of students' mathematical thinking; mathematical principles or concepts

APPENDIX I

LIST OF ISSUES RAISED BY THE TEACHERS

Table 13

List of issues, grouping and context these issues were raised

List of Issues	Grouping	Context
Reflecting on teaching		
When is it more effective to debrief a lesson?	Kayla and Kristina	Formative lesson debriefings with Ms. Patz
What a teacher can/should do in responding to conflict between students	Kayla and Kristina	Student with special needs sitting on a Pilates ball during the math class and one other student- who had a lot of confidence is not happy about it.
How can we help our struggling students?	Kayla, Kristina and art teacher	One student was not doing any of the assignments- Kristina wanted to help that student because she thought he could do much better than he was doing
	Kayla and Paul	It is a different time – students sued to see things so how we can keep them engaged in the class – some students do not even try
What are homework assignments for? Why and how many homework a teacher should give to students?	Kayla and Kristina	Being intentional with questions they wanted to assign in the homework. Kayla told about her policy of two homework assignments for each week- not more because she can't grade more. One assessment – see where they are at the other for practice.
What is the purpose of teaching math?	Kayla and Kristina	Assessment, focusing what students know to don't know instead of what they need to know or how they know, attending students individual needs
Teacher's role during group work	Kayla and Kristina	One student complained about how Kristina always looks at what they were doing unlike other teachers
		Danny not really looking at what students were doing during explore- explore needs to inform your summarize – explore room for differentiation
Setting up norms as a teacher	Kayla and Kristina	What to let go and what not.
Why become a teacher	Kayla, Paul and Kristina	Low salary, more investment less gain, lack of respect.

Table 13 (cont'd)

Curricular Decisions		
Why CMP Data Unit is the last Unit at Grade 7?	Kayla and Kristina	There were so many snow days and some units took longer than planned. They talked about skipping Data Unit which was the last Unit.
	Kayla and Paul	Paul gave students a chart where they would keep track of their weight he wanted to bring that chart up in the Data unit
How long summaries should take	Kayla and Kristina	Kristina critique Danny planning less time for summaries
Why CCSSM standards did not mention linear relationships but only proportionality at Grade 7?	Kayla and Kristina	Debriefing hair on fire meeting
Assessment		
What is the purpose of assessments?	Kayla and Kristina	Check-up: is it informative for teacher or is it for grading? Partner quiz: isn't it to elevate students' learning? Partner quiz: converting inch and feet- out of place
Who to pair up in partner quizzes	Paul and other teachers	Complain from a parent about her kid's pair – he did not know anything so why pair him up with that kids
Students		
What can be challenging for students with Rate of change? Isn't this they already did in grade 6? Is there more to that?	Kayla and Kristina	Planning for Grade 7 Moving Straight Unit. How long they anticipate Problem 4.1 would take.
Why finding the patterns are hard for students?		Debriefing lesson (problem 3.1). Noted what students struggled with. 12.16.13
What to do with 8 grade students' attitude toward grades?		Your high school counts for college so why they needed to worry about Grade 8. T
Is retention good or bad for students	Kayla and Kristina	Reviewing students' exams together they talked about what do to with failing students and what to expect from parents.
Is it acceptable to interact mostly with students who are more expressive and positive about the teacher or the course?	Kayla and Kristina	Talked about teachers who mostly interacted with students who were vocal about their interest in their courses.

Table 13 (cont'd)

Will students' performance change depending on the focus of the unit whether it is geometry or algebra?	Kayla and Kristina	They were about the start Looking for Pythagoras Unit and there was more geometry involved in that unit. Both Kristina and Kayla mentioned how they felt weak about geometry units.
Students' struggling with fractions so how to expect they make decisions over fractional quantities	Kayla and Paul	Stretching and Shrinking Unit students were expected to compare change in area and perimeter and they ended up measurements in decimals or fractions
Why students' can't transfer what they learnt in other contexts?	Kayla, Paul and Science teacher	Talking about reasoning in percent change in science and math classes. They were doing the same thing but students were still having hard time.
Students' making sense of what they are doing		Students picked up algorithm multiply and add but were they able to make sense of what they were doing. Problem 2.4 (02.19.2014)
For students "isn't it okay to be frustrated?"	Kayla and Kristina	Kayla was reflecting on her students' struggle in the lesson
Students' having issues with visualizing what a cross section look like	Kayla and Kristina	Debriefing one of the formative assessment lessons
Teacher Evaluations		
How to move education from being a checklist and help administrators see the big picture	Kayla and Kristina	Debriefing their Evaluative observations e Debriefing their Evaluative observations experience
Who can truly assess what is going on in a lesson?	Kayla and Kristina	Is one or two visits enough to see what is going on in the lesson? Are periodic visits enough?
	Kayla and Paul	
Communicating with Parents		
Why do we have Parent - Teacher conferences right at the beginning of the new semester?	Kayla and Paul	They complained about they did not have a lot to share with parents- even not knowing some of the student because they were completely new.
How to communicate with parents and what should expect from them?	Kayla and Kristina	Sending emails to parents about good and bad behaviors
Access to resources		
Dash – CMP online platform not very user friendly	Kayla and Kristina	Access to CMP materials on an online platform. It was not a user friendly platform.

Table 13 (cont'd)

How can they have Paul participate CMP related Professional Development activities?	Kayla and Kristina	Planning for having people participate summer CMP Getting to Know conference
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