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MULTIPRODUCTS PRODUCTION RELATIONS IN
MANUFACTURING PLANTS: AN EXPLORATORY
STUDY ON SIX SELECTIVE MANUFACTURING
ACTIVITIES IN KOREA

presented by

Seung Yoon Rhee

has been accepted towards fulfillment
of the requirements for

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Major professor

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MULTIPRODUCTS PRODUCTION RELATIONS IN MANUFACTURING PLANTS: AN EXPLORATORY STUDY ON SIX SELECTIVE MANUFACTURING ACTIVITIES IN KOREA

By

Seung Yoon Rhee

A DISSERTATION

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ABSTRACT

MULTIPRODUCTS PRODUCTION RELATIONS IN MANUFACTURING PLANTS: AN EXPLORATORY STUDY ON SIX SELECTIVE MANUFACTURING ACTIVITIES IN KOREA

By

Seung Yoon Rhee

The subject of a production technology is one of the areas of economics where the gap between theoretical formulations and empirical knowledges is still quite wide. Furthermore there have been only a few theoretical and empirical studies on the multi-input multi-output production technology until recently.

The purposes of this study are (i) to understand the theory of a multi-input multi-output (and uni-output) production technology, (ii) to investigate the workability of the multi-input multi-output production technology for a cross-section data system of the Korean Manufacturing Census, (iii) to find some knowledges on the first and second order properties of a production technology, estimated by the translog approach at an establishment level in manufacturing activities, and (iv) to collect information on the usefulness of the Korean Manufacturing Census System which is quite a common type of data system in most other countries.

Seung Yoon Rhee

A brief review on theoretical formulations of a multi-input uni-output production technology and its extension to a multi-input multi-output technology are followed by another summary on econometric backgrounds in this empirical estimation of a production technology.

Considerations on exclusion rules in sampling establishments, selection of specific industries to be studied, and quality of sample data are followed by preliminary investigations on the industrial characteristics in terms of factor products, factor use ratios, factor prices, factor shares and their variations in sample establishments by size and by industry.

Results on the first and second order properties in the estimated technology suggest, firstly, a strong objection on the conventional Cobb-Douglas form from denial of the self-duality between the translog production and cost functions and from rejections in the null-hypotheses test of the second order parameter estimates. Secondly, the conventional value added approach in production studies should be reevaluated, not only from the wide variations of the value added ratio to gross output across industries but also from the seemingly, unitary substitution elasticities of raw materials with respect to other factor inputs. Thirdly, many interesting results from the second order properties of the estimated production technology are found in terms of the direct substitution elasticities, the Allen-Uzawa partial substitution elasticities, the McFadden shadow partial substitution elasticities and the demand elasticities with respect to price changes over input and output bundles. Examples of specific findings are the supplementary substitution relation between two heterogeneous labor inputs, the inconclusive but close-to-unity substitution elasticity between labor and capital inputs, etc.

Fourthly, further results are found from the supplementary works, such that the workability of the production theory becomes weaker for the small-sized establishments, such that the inclusion of sample establishments with no capital inputs results in the seemingly, unitary substitution elasticity between labor and capital inputs, and such that gains from alternative explanatory variables of different quality are negligible in the Korean census data.

Finally, we have learned something from our investigations, not the least of which is that just "more data" will not do. If we persist in asking rather complicated questions, we shall need much better and more relevant figures before we can hope to answer them previously.

To my parents,

Sung Rae Kim and Young Jin Rhee

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INTRODUCTION

Whether or not there exists a stable functional relationship between inputs and output(s) in a production activity has long been a subject of economic inquiry. In economic theory the production function is a mathematical statement relating quantitatively the purely technological relationship between the output(s) of a process and the inputs of the factors of production, the chief purpose of which is to understand and explain the reality of a production activity, owing to several useful characteristics of a production function in economic analysis.

Many efforts have been expended by economists in developing and refining the theory of a production, and in formulating and estimating the model of a production technology. For example, great efforts on the functional form of a production function, relating directly to the fitting of econometric production function, have been followed by the Cobb-Douglas production function as a simple form toward more complicated functions, such as the CES (constant elasticity of substitution) function, the VES (variable elasticity of substitution as various generalizations of the CES) function, and the translog (transcendental logarithmic) function. In particular, the formulation of the translog production function does not require any *a priori* assumptions on the functional form to be investigated in empirical works, distinctively different from other functions, such as the CES function requiring a priori assumption of the constant elasticity of factor substitution.

Most of the earlier studies of a production technology have been based either on a rather simple specification of the production function (such as the CD and the CES) to be estimated, or on highly aggregative data for the estimation of aggregate production function at a certain macro (or sector) level. Very few studies have actually dealt with data at the plant level and most of the previous micro production functions have been estimated for selected farming activities,[1] for the electricity generating industry, and for the railroad sector, where abundant plant data had become available through the operation of regulatory agencies. There have been only a few econometric studies of production based on individual plant data in manufacturing.[2] In particular, most of the previous works on the production function

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[1]In addition to early empirical efforts by Senator Paul Douglas with the Cobb-Douglas function, agricultural economists did considerable empirical work with the Cobb-Douglas, Mitscherlich, and several other functions, notable among these efforts are Tintner, Brownlee, Heady and Johnson. This line of work seems to continue to date.

[2]Recent examples are Krishna's (1967) study of combined cross-section time series data for three manufacturing industries in the U.S., Hodgins' (1968) study of economies of scale in Canadian manufacturing, Eisner's (1967) study based on data for individual companies rather than plants, and recently Griliches and Ringstadt's (1971) study for the rather higher level of industry (3-digit ISIC classification level) with simple functional forms such as the CD and the CES functions.

have focused either on the possibilities of substitution between factors (mostly two factors) of production to achieve one given output, or on the possibilities of transformation between products (also mostly two) of production, paying no attention to the input side. There have been only a very few econometric studies of the production possibility frontier with more than two inputs and one output.[3]

The present study purports (i) to understand the theory of a multiproduct production function as an extension of a uniproduct production theory, (ii) to investigate the workability of multi-product joint production theory, using the Korean manufacturing census data, (iii) to find some knowledges on the parameters of a production function in a multiproduct establishment which is closer to the reality of most manufacturing activities, and finally (iv) to collect informations on the usefulness of the Korean manufacturing census data which is quite a common type of data system in most of the other countries also.

More specifically this study investigates the production technology of an industry at the micro establishment level, via estimation of the translog production function and also the translog cost function with five distinct inputs and more than (or equal to)

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[3]Recent examples are Christensen, Jorgenson, and Lau's (1973) works of the transcendental logarithmic production frontiers with two inputs and two outputs for the U.S. economy during 1929-1969, and Brown, Caves, and Christensen's (1975) of the translog cost function for the U.S. railroad industry.

two outputs. Our study is very much conditioned by the availability of a particular body of data: <u>the 1973 Census of Mining and Manufacturing in Korea</u>. These data have several important advantages, not the least of which is their accessibility for research purposes. In addition, their comprehensiveness and the potentially large number of observations may allow the testing of much more detailed hypothesis about the structure of production activities than was hereto possible. On the other hand, these data have also serious limitations.

Some of the data have turned out not to be as good as anticipated. But more importantly, our study is limited to only one year, 1973, and these data only to those items for which questions were asked in the Census. One of the main shortcomings is the poorness of data on the capital stocks and on the characteristic (age, sex, skill) of the labor force in the various establishments. In particular, the absence of time series observations in the study makes impossible the construction and estimation of a complete production system inclusive of technical changes.

Since we can not afford in this study to cover all of the manufacturings, only six sepecific industries are selected randomly, the half of which produce multiproducts and the other half of which do a uniproduct.

The plan of this study is as follows: the first part is devoted to a brief review of the theory of a production function, mainly focusing on various efforts in the formulation of the functional form both for the uniproduct (Chapter I) and the multi-product production function (Chapter II).

The second part consists of the very ingredients of the current empirical investigation. Following the introduction of Chapter I of part B, the data and measurement problems are discussed in Chapter II, in terms of the variables in the record of the Census of Mining and Manufacturing in Korea, 1973 and of the derived variables adopted by this empirical investigations. Also the selection rules of specific industries to be studied and the exclusion rules of sample establishments included in the estimation are followed by the sample properties by industry and the general quality of data used.

Chapter III contains the theoretical and statistical backgrounds in the empirical estimations, such as the choice of the estimating equations, the error specification, the estimation method and some Monte Carlo Experiments for the validity of the estimation methods adopted in this study.

In Chapter IV, the empirical estimates of the production technology for the six selective industries are presented and tested. Also several alterative investigations are covered, such as those of alternative restrictions, separate results by different establishment size, those of alternative exclusion rules in sampling establishments, and those of alternative variables of different quality.

The final chapter summarizes the main findings of this empirical study and suggests further efforts to be done necessarily to understand the reality of the micro production technology in the Korean manufacturings.

PART A

CHAPTER I. UNIPRODUCT PRODUCTION THEORY

A.1.1. What is a Production Function

A production function is a complex analytical tool which describes the maximum output that can be obtained from a given set of inputs in the existing state of technological knowledge.[1] It can also be regarded as the technical relationship between the maximum quantity of output and the volume of inputs required to produce it, and as the technical relationships between inputs themselves. The parameters of the production function thus conceived represent the features of the technology according to which a given set of inputs is transformed into a certain output. In general four useful characteristics of a production function have been discussed in economic analysis.

[1]The engineers of the firm are concerned not only with inputs and outputs but with the properties of the energy sources and other factors of production required to transform materials, such as the feed mechanism of certain equipments, etc. An engineering production function can be transformed into an economists' production function so as to provide for it a physical-technical foundation, by leaving out some non-relevant information. The production function is fabricated by the economist and it is probably foreign to the engineering and business world, because it is not directly measurable. The abstractness of the production function concept is precisely its source of value; it produce highly useful and verifiable hypothesis and it enables economists to analyze a wide variety of problems. See H. Chenery (1953) and W. Salter. For a survey of the literature in which production functions are derived from engineering data, see A. A. Walters (1963; pp. 11-14), and R. Dorfman, P. Samuelson, and R. N. Solow (1953; pp. 130).

They are the efficiency of the technology, the degree of economies of scale that are technically determined, the factor intensity of the technology, and the ease with which one input is substituted for another. Abstract technology[2] is followed by the additional uses that is to be made of the production function. Firstly production functions can be used in measuring technical changes.[3] Secondly, the relationships between production functions and isoquants can be broadened and the production function can be used to derive a more general description of technology. For instance, one can obtain the input requirement set of which the isoquants might be considered as the efficient sets. This more abstract and general view has some important analytical attributes and may yield some good returns in empirical work. It has been supported by a great deal of refinement on the axiomization of technology in the last decade. It deals also with the dual relationships between technology and cost functions, more generally, profit functions. The cost function has an important analytical value and in some circumstances some potential promise for empirical work.[4] The next question of great importance both in

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[2]M. Brown call these four characteristics of a production function, taken together, an abstract technology. See M. Brown, On the Theory and Measurement of Technological Change, Cambridge Press, 1968, pp. 12.

[3]Solow (1967) denoted the major part of his survey to discuss this issue. Some other works on this subject have appeared since. In spite of its importance, the subject is not discussed here.

[4]For instance, the allocation of costs into multiproducts can be dealt with the estimation of a joint cost function. See R. Brown, D. Caves, and L. Christensen (1975).

theory and in practice is that of aggregating quantities and prices. The question is when there is a natural way for such aggregation. This question is associated with the notion of the separability[5] of a function, which is of prime importance by itself in empirical analysis and is closely related to our study.

Various specifications of a production technology in the axiomization of production structure are summarized in terms of the input requirement set, the production function, the cost function, and the duality relationships among them, in the following section. Next, the functional forms in historical trend of most empirical researches are reviewed in terms of a quadratic form and of a combination of several subfunctions. Finally the trans-centdental logarithmic functions are defined for a production function and for a cost function, and its properties are investigated in relation to the theory of production and to its empirical analysis.

A.1.2. Specification of a Technology: Axiomization of Production Structure[6]

2.1. The Input Requirement Set

For the production of output y of a particular product, we need at most n factors. Let $\underline{x}$ be a vector of inputs of these n

[5]"Separability" is discussed in later sections in detail. See the subsection 4.1.2. in this chapter.

[6]Let $\underline{x}$, $\underline{y}$ ε Rn and i=1, . . . n. Then for every i; $\underline{x} > \underline{y} \Rightarrow x_i > y_i$, $\underline{x} \geq \underline{y} \Rightarrow x_i \geq y_i$ and $x \neq \underline{y}$, and $\underline{x} \geqq \underline{y} \Rightarrow X_i \geq y_i$. The inner product is denoted as $\underline{x}\ \underline{y}$. The positive and non-negative orthants of Rn are denoted by Ω_n^* and Ω_n respectively.

factors. The technology specifies the various way of producing y, namely

$$X(y) = \{\underline{x}:\underline{x} \text{ can yield } y\} \quad (1)$$

The properties of X(y) are assumed to be: (a) Location--X(y) is a non-empty subset of the non-negative orthant Rn denoted by Ωn. It is possible that some factors will not be utilized, but the only output that can be obtained with no inputs at all is the zero output, that is, $X(0) = \Omega n$ and if $y > 0 \Rightarrow 0 \notin X(y)$. (b) Closure---X(y) is assumed to be closed. That is, if a sequence of points $\{\underline{x}^n\}$ in X(y) converges, the limiting point also belong to X(y), meaning that X(y) contains all its limiting points. (c) Monotonicity---If a given output can be produced by the input--mix $\underline{x}$ it can also be produced by a larger input. Similarly, the inputs required to produce a given output can certainly produce a smaller output. (d) Convexity---X(y) is convex.

2.2. The Production Function

Using the notion of the input requirement set, the production function can be defined by:

$$f(\underline{x}) = \max_{y} \{y : \underline{x} \in X(y)\} \quad (2)$$

When X(y) has the four properties defined above, $f(\underline{x})$ has the following properties: (a) Domain--f(x) is a real-valued function of $\underline{x}$ defined for every $\underline{x} \in \Omega n$ and it is finite if $\underline{x}$ is finite. (b) Monotonicity--an increase in inputs cannot decrease production:

$\underline{x} \geq \underline{x}' \Rightarrow f(\underline{x}) \geq f(\underline{x}')$ (c) Continuity---$f(\underline{x})$ is continuous. (d) Concavity---$f(x)$ is quasiconcave over Ωn.

2.3. The Duality between the Production Functions and the Input Requirement Sets

The production function was derived from the input requirement set. It is possible to assume a production function $f(x)$ with the four properties above and to derive from it:

$$X^*(y) = \{\underline{x} : f(\underline{x}) \geq y, \underline{x} \in \Omega n\} \tag{3}$$

It turns out that $X^*(y)$ possesses the corresponding four properties of an input requirement set. Furthermore, if $X^*(y)$ is used in (2) to derive a production function, say $f^*(x)$, then $f^* = f$. Similarly, if we start with $X(y)$ to derive $f(x)$ and then in turn use of f in (3) to derive $x^*(y)$ then $x^*(y) = X(y)$. Thus, there is a full duality between the input requirement set and the production function.[7]

2.4. The Cost Function

In general, economic models involving production need rules of behavior, in addition to the production function. In the micro analysis the criterion is profit maximization. The selection of the optimal output can be done in stages, first selecting the

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[7]This is discussed in detail by Diwert, W. E. (1971), "An Application of the Shepard Duality Theorem: A Generalized Leontief Production Function," Journal of Political Economy, 79: 481-507.

input mix which minimizes cost for any output $y \varepsilon Y$ and then selecting that y which maximizes profit. The cost minimization for all $\underline{p} \varepsilon \Omega_n^*$ and $y \varepsilon Y$ is described by:

$$C(y, \underline{P}) = \min \{ \underline{px} : \underline{x} \varepsilon X(y)\} \tag{4}$$

where $\underline{p}$ is the vector of factor prices and $C(y,\underline{p})$ is the cost function. If $X(y)$ possesses the properties defined earlier, then $C(y, \underline{p})$ has the following properties:[8] (a) Domain--$C(y, \underline{p})$ is a positive real-valued function defined for all positive prices $\underline{p}$ and all positive producible outputs. (b) Monotonicity--$C(y, \underline{p})$ is a non-decreasing function in output and in prices. (c) Continuity--$C(y, \underline{p})$ is continuous in y and in $\underline{p}$. (d) Concavity--$C(y, \underline{p})$ is concave function in $\underline{p}$. (e) Homogeneity--$C(y, \underline{p})$ is homogeneous of the first degree in prices.

2.5. The Duality between the Cost Function and the technology[9]

Instead of deriving the cost function from the input requirement set according to (4) it is possible to postulate a cost function with the assumed four properties and to define the following set:

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[8]Shephard (1953), Uzawa (1964), McFadden (1966).

[9]McFadden (1966), Diwert (1969).

$$X^{\circ}(y) = \{\underline{x} : \underline{px} \geq C\ (y, \underline{p}) \text{ for every } \underline{p} \in \Omega_n^* \text{ and } \underline{x} \in \Omega n\} \qquad (5)$$

The set X(y) has the corresponding four properties. Further, we can use (5) in (4) to derive a cost function that will be identical to that used in (5). Alternatively, if we use (4) for C $(y, \underline{p})$ in (5), then $X^{\circ}(y)$ is identical to X(y) in (4). This is the duality between a cost function and an input requirement set. But in view of the duality between input requirement sets and production functions, there also exists a duality between the cost and the production functions.

2.6. Implications of the Production Theory for Empirical Analysis

The axiomization of the production structure has been refined in various ways but such refinement seems to have had, so far, little effect on the implications to be drawn for empirical analysis.[10]

For the empirical analysis also, the technology of the economy can be measured either in terms of the input requirement set or in terms of the production function or in terms of the cost function and from any of the three we can derive the other two. However, most empirical works are concerned with production functions. There is hardly any work on the direct measurement

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[10]See also Lau's recent works; Lau (1976). A revision of "Some Applications of Profit Functions," Memorandum 86A and 68B, Center for Research in Economic Growth, Stanford University, November 1969.

of the input requirement set, except that of Hanoch and Rothschild (1972) which attempts to set the ground for such an empirical analysis. More hope has been expressed in the literature on the appropriateness of the cost function to empirical analysis and some work has been done by Nerlove (1963) and Brown, Caves and Christensen (1975).

From the point of view of empirical analysis, the properties of the production function in the previous section, 2.1., impose rather little. Any function used in such analysis assumes much more. In fact, the very notion of representing the production function by a given algebraic form is rather restrictive and very likely can only yield an approximation.[11] This point is of prime importance and has several repercussions. The algebriac formulation is essential for empirical analysis. However, nowhere is it stated that there should be one algebraic form which will give a good approximation for the whole domain. Yet, implicit or explicit in many works is the idea that the particular function should describe the process of production near the origin as well as for outputs which are many times the observed quantities. Therefore, it is suggested here that a particular function used in an empirical analysis should maintain the usual properties assumed only in the neighborhood of the observations. The

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[11]This is discussed in detail in the following section, the function form of a production function.

relevance of this observation will become evident in subsequent discussions.[12]

A.1.3. Functional Forms in Trend

3.1. Choice of the Functional Form

The choice of functional form should depend on two properties. First, the functional form should be capable of representing a wide range of technologies in order to minimize the a prior assumptions imposed on the estimating equations. Second, it should be tractable within the assumption of the model. That is, the estimating equations should be simple enough to carry out the estimation with minimal computational burden and with ease of interpretation. In reality any choice is a compromise between these two objectives and such a choice must be based upon value judgements in general.

It is with respect to the first criterion that simple functional forms of production and cost functions such as the Cobb-Douglas (CD). Leontief (L), and constant elasticity of substitution (CES) forms are dominated by more general forms such as the flexible functional forms to be illustrated below. For example, it is well known that the CD production function has a Hicks--Allen elasticity of substitution (AES) which is unity for all input pairs, and under cost minimization implies that factor

[12] In particular, our empirical work, using the translog approach, emphasizes it.

shares are constant. Since the substitution elasticities are measures of curvature around an isoquant it is evident that the shape of the isoquant is severely restricted by assuming a CD function. This is highlighted by the implication that factor shares are constant. The apparent success of the CD function in applied work seems to be due to two reasons. First, using aggregated time series the direct estimation of the CD function is reasonable since the substitution effects are not well identified by highly collinear data. Second, Fisher (1969) argues that the constancy of factor shares of labour and capital in aggregate data fits the CD hypothesis.[13]

The CES function permits the AES to deviate from unity but does require it to be constant by construction. The CES function thus generalizes the CD and L functions which assume a common constant elasticity of substitution which is unity or zero respectively. But the CES function is restrictive in the nature of the type of substitution permitted. In particular, all factors are equally substitutable with each other, a restriction which has no theoretical justification but which simplified the empirical work considerably. If there are just 2 inputs then this restriction may not be very hard to accept since it simply means that a single cross elasticity of substitution is constant. But to extend this constancy to a multifactor technology and assume that the AES

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[13]But this does not necessarily mean that there exists an aggregate CD production (or cost) function but that we have yet to explain this constancy.

between electricity and machines is the same as between unskilled labour and materials, for example, seems to be unreasonable until verified empirically. Even with two inputs one may not wish to assume that the AES remains constant around the isoquant. We conclude therefore that a priori the CES function is restrictive in that it restricts the elasticities of substitution (a) to be constant, and (b) to be the same constant for every pair of inputs.[14] Such restrictions should be tested not imposed a priori.

Given the a priori presumption against the CES form, there has been considerable effort made to obtain less restrictive functional forms.

One obvious approach is to make the common AES, σ, a function of some variable such as the level of output or the factor ratio or factor share, etc. Such generalization have been called Variable Elasticity of Substitution (VES) functions and have been discussed by Revankar (1971), Lu and Fletcher (1968), Sato and Beckmann (1968) and Lovell (1973).

A recent spurt of functional forms owes its origin to Diwert (1971) who generated a functional form that is linear in parameters and which provides a second order approximation to any arbitrary twice differentiable function. This Generalized

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[14] Another feature of the CES function is that it is additive in terms of input combinations. This special form of separability is not independent of the constancy of the AES, since Berndt and Christensen (1973c) have shown that a certain separability has something to do with a certain set of AES equal. See also Russell (1975).

Leontief (GL) functional form[15] was quickly followed by the Transcendental Logarithmic (translog) functional form developed by Christensen, Jorgenson and Lau (1971), and Sargan (1971). The Generalized Cobb-Douglas (GCD) developed by Diwert (1973) and a generalization of GL by Denny (1974) and Kadiyala (1972).

In addition Diwert (1973) has developed functional forms for special functions such as revenue and variable profit functions as well as indirect utility functions and indirect production functions. Also recently Lau (1976) has developed a profit function.

3.2. Functional Forms in a Quadratic Form

One interesting aspect in the formulation of a production function can be described in a quadratic form which accommodates various functions when properly interpreted. The form is

$$Yo = [1 \ \underline{y}'] \begin{bmatrix} \alpha_0 & \frac{1}{2}\underline{\alpha}' \\ \frac{1}{2}\underline{\alpha} & B \end{bmatrix} \begin{bmatrix} 1 \\ \underline{y} \end{bmatrix} = \alpha_0 + \underline{y}'\underline{\alpha} + \underline{y}'B\underline{y} \tag{6}$$

The functions to be reviewed use the following transformation:

$$\begin{aligned} T_1 &: y_i = x_i^{\rho_i} \quad, \rho_i \neq 0 \\ T_2 &: y_i = \ln x_i, \ \rho_i = 0 \end{aligned} \tag{7}$$

[15]It is called so because when used as a cost function it yields the Leontief cost function as a special case.

where

$$x_o = \text{output}$$
$$x_i = \text{the i-th input}$$

(a) CD-like (Cobb-Douglas production function): This function is obtained from (6) by imposing

$$(6) \quad (B \equiv 0) \cap T_2 \tag{8}$$

By this notation it is meant that the CD function is obtained from (6) imposing on the function $B \equiv 0$ and the variables are obtained by a logarithmic transformation (T_2). The result is, for the production technology of one output and of five inputs,

$$\ln x_o = \alpha_o + \sum_{i=1}^{5} \alpha_i \ln x_i \tag{9}$$

(b) CES-like (constant elasticities of substitution): This function is obtained from (6) by imposing

$$(6) \cap (B \equiv 0) \cap T_1 \cap \{\rho_j = \rho\},\ j = 1, \ldots 5 \tag{10}$$

By this notation it is meant that the CES function is obtained from (6) imposing on the function $B \equiv 0$ and the variables are obtained by a power transformation (T_1) of a constant exponent (ρ). The result is

$$X_o^\rho = \sum_{i=1}^{5} \alpha_i X_i^\rho \tag{11}$$

(c) CRES-like (constant ratios of elasticities of substitution): The function is obtained from (6) by imposing

$$(6) \cap (B \equiv 0) \cap T_1 \tag{12}$$

The CRES function, developed by Mukerji (1963) and Gorman (1965) is a generalization of the CES-like function, the functional form of which is

$$X_0^{\rho} = \sum_{i=1}^{5} \alpha_i \, x_i^{\rho_i} \tag{13}$$

(d) CRESH (homothetic or homogeneous CRES): Hanoch (1971) defined and analysed a functional form for a one-output, many factors production function, which is homothetic (or homogeneous) and exhibits CRES. Its functional form is

$$1 = \sum_{i=1}^{5} \alpha_i \left(\frac{x_i}{x_0}\right)^{\rho_i} \tag{14}$$

In the CRESH function its AES (Allen-Uzawa partial elasticities of substitution) vary along isoquants and differ as between pairs of factors, but the AES stand in fixed ratios everywhere, while the CRES function; however, is not homogeneous or homothetic, so that individual ES vary with output as well as factor combinations, the expansion lines (for given factor prices) being curved in a predetermined way.

(e) GL-like (generalized Leontief function): The GL function, developed by Diwert (1971), is obtained from (6) by imposing

$$(6) \cap (\alpha_0 = 0) \cap (\underline{\alpha} \equiv 0) \cap T_1 \cap \{\rho j = \tfrac{1}{2}\} \tag{15}$$

Hence the result is

$$x_0 = \sum_{i=1}^{5} \sum_{j=1}^{5} \beta_{ij} \sqrt{x_i} \sqrt{x_j} \tag{16}$$

(f) TL-like (Transcendental Logarithmic function): The TL function is developed by Christensen, Jorgenson and Lau (1972). The function is obtained from (6) by imposing

$$(6) \cap T_2 \tag{17}$$

Hence its function form of a production technology of one output and of five inputs is

$$X_0 = \alpha_0 + \sum_{i=1}^{5} \alpha_i \ln x_i + \sum_{i=1}^{5} \sum_{j=1}^{5} \beta_{ij} \ln x_i \ln x_j \tag{18}$$

The CD function captures two important properties of a production function: monotonicity and concavity. It does so with a small number of parameters. This, in addition to the other two plausible reasons explained in the previous section, may explain its dominance of the field for so many years.

The work by Arrow, Chenery, Minhas and Solow (1961) added a new dimension to the analysis: the ease of factor substitution.

The generalization of this measure to the case of more than two inputs, and the implications of such generalization for the form of the production function have been widely discussed. The functions listed in items (b), (c) and (d) represent the results of this discussion.

The constraint ($B \equiv 0$) on (6) implies that the production function is strongly separable between inputs. This property constitutes a strong constraint which simplifies considerably the empirical work. As indicated by Mundlak and Razin (1971) separability has been imposed without being tested and that raises a question with respect to the proper use of this assumption. It is against such a background that the translog function approach broadens the scope of analysis.[16]

We have thus singled out three major stages in the development of algebraic formulation of the production functions: (1) Cobb-Douglas, (2) broadening the scope for factor substitution, and (3) submitting separability to empirical test.

3.3. Functional Forms in a Combination of Subfunctions

Another aspect in the formulation of a production function can be reviewed as a combination of several subfunctions which

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[16]See Christensen, Jorgenson and Lau (1972). The translog function was also discussed by Griliches and Ringstad (1971) and Sargan (1971), but with no particular emphasis on separability.

accommodates various functions when properly interpreted. The form is

$$f(\underline{x}) = g(\underline{x}) * h(\underline{x}) \tag{19}$$

where $f(\underline{x})$ is the production function, $g(\underline{x})$ and $h(\underline{x})$ are two arbitrary functions and * is an arbitrary operator, such as addition, multiplication, or an exponent. This approach provides a convenient framework for classifying functions which do not fall within the general quadratic form of the previous section.

(a) VES function: Revankar (1971) suggested the following function in order to make the elasticity of substitution a linear function of the capital-labor ratio:

$$y = \alpha_0 \, x_1^{\alpha_1} \, (x_2 + \gamma_1 x_1)^{\rho} \tag{20}$$

If we let $g(\underline{x}) = \alpha_0 x_1^{\alpha_1}$ and $h(\underline{x}) = (x_2 + \gamma_1 x_1)^{\rho}$ then we can write this function as

$$y = g(\underline{x})h(\underline{x}) \tag{21}$$

That is simply the product of the CD form and the CES form.

(b) Constant marginal share: This function was suggested by Bruno (1968);

$$y = \alpha_0 X_1^{\alpha_1} X_2^{\alpha_2} - \gamma X_2 \equiv g(\underline{x}) + h(\underline{x}) \tag{22}$$

Again $g(\underline{x})$ has the CD form where $h(\underline{x})$ is linear.

(c) Transcendental production function: Halter, Carter and Hocking (1957) use a function

$$Y = \alpha_o X_1^{\alpha_1} X_2^{\alpha_2} \; e^{\gamma_1 x_1 + \gamma_2 x_2} \tag{23}$$

which can be immediately decomposed into

$$y = g(\underline{x})h(\underline{x}) \tag{24}$$

where $g(\underline{x})$ is a CD and $h(\underline{x}) = e^{\gamma_1 x_1 + \gamma_2 x_2}$

The same procedure can be followed with more than two subfunctions. Having decomposed a particular algebraic form into its components it is then possible to trace the origins of particular properties and search for ways to achieve the same property with as few parameters as possible.

3.4. Functional Form Flexible in Prices

Attentions on functional forms which are flexible in the sense of providing second order approximations in input prices to an arbitrary continuously differentiable cost function, have been paid since it may be unlikely that the production function approach will be useful at the industry level of disaggregation.

Diwert (1971) generated a functional form, called the Generalized Leontief (GL) function, of

$$C(w) = \sum_{i=1}^{5} \sum_{j=1}^{5} b_{ij} \sqrt{w_i} \; \sqrt{w_j} \text{ , where } b_{ij} = b_{ji} \tag{25}$$

to describe a wild range of substitution possibilities for a multi-input technology. Diwert (1973) also suggested another generalization of Cobb-Douglas form, called the Generalized Cobb-Douglas function, of

$$\ln C(w) = b_0 + \sum_{i=1}^{5} \sum_{j=1}^{5} b_{ij} \ln (w_i + w_j) \tag{26}$$

where

$$b_{ij} = b_{ji} \quad \text{and} \quad \sum_i \sum_j b_{ij} = 1.$$

Following to Diwert, Christensen et al. (1971) developed the transcendental logarithmic or translog (TL) form of

$$\ln C(w) = b_{00} + \sum_{i=1}^{5} b_{0i} \ln w_i + \sum_{i=1}^{5} \sum_{j=1}^{5} b_{ij} \ln w_i \ln w_j \tag{27}$$

where

$$b_{ij} = b_{ji}, \ \sum_i b_{io} = 1 \quad \text{and} \quad \sum_j b_{ij} = 0, \ i=1, \ldots 5.$$

Each unit cost function is linear homogeneous in prices as theory requires. Diwert has shown that GL and GCD are decreasing concave functions if the b_{ij} are non-negative while GL is positive if some $b_{ij} > o$ as well and GCD is positive if $b_o > \infty$. Under certain parameter restrictions the GL and GCD functional forms satisfy all of the conditions required of a cost function. The TL form satisfies all these conditions globally only if $b_{ij} = 0$ in which case it reduces to a Cobb-Douglas function. A more general functional form permitting a wider range of special cases have been provided also by Denny (1972) and Kadiyala (1972);

$$C(w) = \{\Sigma \, \Sigma \, b_{ij} \, w_i^{\gamma/2} \, w_j^{\gamma/2}\}^{1/\gamma} \tag{28}$$

where

$$b_{ij} = b_{ji}$$

This reduces to GL when $r = 1$ and to the CES form when $b_{ij} = 0$, $i \neq j$ which in turn reduces to the CD and L forms as limiting cases.

A.1.4. The Transcendental Logarithmic Function

4.1. The Translog Production Function

4.1.1. Introduction

A new class of production function, named the "Transcendental Logarithmic Production Function," or more briefly the translog production function is defined by the following form:

$$V = \alpha_0 \left[\prod_{i=1}^{5} X_i^{\alpha_i} \right] \left[\prod_{i=1}^{5} X_i^{\frac{1}{2} \left(\sum_{j=1}^{5} \gamma_{ij} \ln x_j \right)} \right] \tag{29}$$

where V = quantity of output

X = quantity of the i-th input

and $\gamma_{ij} = \gamma_{ji}$ for $i, j = 1, \ldots 5$

Equivalentely, the translog function may be written as

$$\ln V = \ln \alpha_0 + \sum_{i=1}^{5} \alpha_i \ln x_i + \tfrac{1}{2} \sum_{i=1}^{5} \sum_{j=1}^{5} \gamma_{ij} \ln x_i \ln x_j \qquad (30)$$

The translog function has many desirable features in both theoretical and empirical applications. In particular, it reduces to the CES and the CD functions as special cases--the former as a second-order approximation. It also reduces to most of the CES-like functions as special cases with appropriate restrictions, such as the Uzawa generalization of the CES production function (1962), the McFadden generalization of the CES function (1963), the Mukerji Generalized SMAC Function (1963), the Sato Two-Level CES production function (1967), the Hildebrand-Liu generalization (1965), the McCarthy generalization (1965), and the Transcendental Generalization of Halter, Carter, and Hocking (1957), etc.[17]

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[17]The Uzawa generalization of the m-factor C.E.S. function is given by $V = \gamma_0 \prod_{s=1}^{S} Z_s$, where

$$Z_s = \gamma_s \quad \sum_{i\varepsilon N_s} \delta_i^{(s)} X_i^{(s)\rho} \quad - \frac{1}{\rho} \quad , \quad \sum_{i\varepsilon N_s} \delta_i^{(s)} = 1$$

and N_s is the set of indices of inputs in set s. Uzawa (1962) has proved that the above function completely characterizes the class of homogeneous m-factor production function with constant Allen-Uzawa partial elasticities of substitutions (AES). McFadden (1963) has derived the class of homogeneous m-factor production functions which possess constant direct elasticities of substitutions (DES)--the block additive linear homogeneous functions. The McFadden generalization of the C.E.S. production function is defined by

$$1 = \gamma \sum_{s=1}^{S} \beta_s \prod_{i\,\varepsilon N_s} \left(\frac{x_i}{V}\right)^{-\rho}, \quad \sum_{s=1}^{S} \beta_s = 1$$

The translog production function provides a second order approximation to any arbitrary production function for values of inputs near unity.

The Mukerji generalized SMAC Function is shown in the equation (13) of the previous section 3.2. in this chapter. The Sato Two-Level C.E.S. production function is given by

$$V = \left[\sum_{s=1}^{S} \alpha_s Z_s^{-\rho} \right]^{-\frac{1}{\rho}},$$

where

$$Z_s = \left[\sum_{i \varepsilon N_s} \beta_i^{(s)} (X_i^{(s)})^{-\rho_s} \right]^{-\frac{1}{\rho_s}}, \quad \alpha_s, \ \beta_i^{(s)} > 0, \ -1 < \rho, \ \rho_s < \infty$$

$$\sum_{s=1}^{S} \alpha_s = \sum_{i \varepsilon N_s} \beta_i^{(s)} = 1$$

Here V is a C.E.S. function in {Z} and Z_s, in turn, is a C.E.S. function in $\{X^{(s)}\}$. Hence V is a "two-level" C.E.S. function in {x}. The last three generalizations are of the C.E.S. production function in the two-factor case. Hence the Hildebrand-Liu generalization has the form of

$$V = \gamma \left[\delta K^{-\rho} + (1-\delta)\ \eta\ (\tfrac{K}{L})^{-c(1+\rho)}\ L^{-\rho} \right]^{-\frac{\mu}{\rho}},$$

and the McCarthy generalization is given by

$$V = \gamma \left[\delta_1 K^{-\rho} + \delta_2 K^{-\eta} L^{\eta-\rho} + \delta_3 L^{-\rho} \right]^{-\frac{\mu}{\rho}},$$

and the Transcendental generalization has the form of

$$V = \gamma\ K^{\alpha\mu}\ L^{(1-\alpha)\mu}\ e^{\beta(K/L)}$$

$$\text{Let } V = F[x_1, x_2, x_3, x_4, x_5] \tag{31}$$

be written as

$$\ln V = G[\ln x_1, \ln x_2, \ln x_3, \ln x_4, \ln x_5] \tag{32}$$

where V is an arbitrary production function. Expanding G in a Taylor's series expansion in $\ln x_i$'s around $x_i = 1$ (or $\ln x_i = 0$), $i = 1, . . . 5$, we have

$$\ln V = G[\underline{0}] + \sum_{i=1}^{5} \left. \frac{\partial G}{\partial \ln x_i} \right|_{\ln \underline{x} = [\underline{0}]} \ln X_i$$

$$+ \tfrac{1}{2} \sum_{i=1}^{5} \sum_{j=1}^{5} \left. \frac{\partial^2 G}{\partial \ln x_i \; \partial \ln x_j} \right|_{\ln \underline{x} = [\underline{0}]} \ln X_i \ln X_j$$

$$+ \text{ the high-order terms,} \tag{33}$$

where $\underline{x}$ and $\ln \underline{x}$ represent the vector of x_i's and $\ln x_i$'s respectively and $[\underline{0}]$ is a vector of zeros. A comparison of Equation (30) with (33) indicates that we may set

$$\ln \alpha_o = G[\underline{0}]$$

$$\alpha_i = \left. \frac{\partial G}{\partial \ln x_i} \right|_{\ln \underline{x} = [\underline{0}]} , \quad i = 1, . . . 5$$

and

$$\gamma_{ij} = \left. \frac{\partial^2 G}{\partial \ln x_i \; \partial \ln x_j} \right|_{\ln \underline{x} = [\underline{0}]} , \quad i, j = 1, . . . 5 \tag{34}$$

Hence the translog function provides a second order approximation to any arbitrary production function around $\ln \underline{x} = [0]$.

4.1.2. Properties of the Translog Production Function

(1) Monotonicity condition.

A neoclassical production function should be increasing in all its arguments, i.e.,

$$\frac{\partial V}{\partial x_i} \geqq 0, \; i = 1, \ldots 5,$$

at least in the region of observed operation. This implies that

$$\frac{\partial \ln V}{\partial \ln x_i} = \frac{x_i}{V} \frac{\partial V}{\partial x_i} \geqq 0, \; i = 1, \ldots 5,$$

because of the strict positivity constraints on V and X_i. Hence the monotonicity constraint becomes

$$\frac{\partial \ln V}{\partial \ln x_i} = \alpha_i + \sum_{j=1}^{5} \gamma_{ij} \ln X_j \geqq 0, \; i = 1, \ldots 5 \tag{35}$$

(2) Convexity condition.

In addition to the monotonicity property, a neoclassical production function must also be concave--i.e. it exhibits decreasing returns to scale. Hence $[F_{ij}]$ must be negative semi-definite. A necessary condition is that $F_{ii} \leqq 0$ or

$$\frac{\partial^2 V}{\partial x_i^2} = \frac{V}{x_i^2} \left[\left(\frac{\partial \ln V}{\partial \ln x_i} - 1\right) \frac{\partial \ln V}{\partial \ln x_i} + \gamma_{ii} \right] \leqq 0 \tag{36}$$

These must be satisfied in particular at the point of approximation. Hence

$$\left.\frac{\partial^2 V}{\partial x_i^2}\right|_{\ln \underline{x} = [\underline{0}]} = V\,[(\alpha_i - 1)\,\alpha_i + \gamma_{ii}] \leqq 0 \tag{37}$$

A set of sufficient conditions given mononicity is $1 \geqq \alpha_i \geqq 0$; $\gamma_{ii} \leqq 0$. Moreover

$$\frac{\partial^2 V}{\partial x_i\,\partial x_j} = \frac{V}{x_i\,x_j}\left[\frac{\partial \ln V}{\partial \ln x_i}\,\frac{\partial \ln V}{\partial \ln x_j} + \gamma_{ij}\right] \tag{38}$$

and also

$$\left.\frac{\partial^2 V}{\partial x_i\,\partial x_j}\right|_{\ln \underline{x} = [\underline{0}]} = V\,[\alpha_i\,\alpha_j + \gamma_{ij}],\ i \neq j = 1, \ldots 5 \tag{39}$$

We note that if V is concave at $\ln \underline{x} = [\underline{0}]$, then by a continuity argument it can be shown that V is locally concave in a neighborhood of $\ln \underline{x} = [\underline{0}]$. This local concavity does not rule out the existence of uneconomic or convex regions and especially increasing returns to scale in certain ranges of inputs. A necessary and sufficient condition for local concavity at $\ln \underline{x} = [\underline{0}]$ is that the matrix

$$[F_{ij}\Big|_{\ln \underline{x} = [\underline{0}]} = \begin{bmatrix} (\alpha_1-1)\alpha_1 + \gamma_{11} & \cdots & \alpha_2\alpha_5 + \gamma_{15} \\ \vdots & \ddots & \vdots \\ \alpha_5\alpha_1 + \gamma_{51} & \cdots & (\alpha_5-1)\alpha_5 + \gamma_{55} \end{bmatrix} \tag{40}$$

is negative semi-definite, which in turn requires that all the principal minors be negative semi-definite, or equivalently all the characteristic values of the matrix are non-positive.

(3) Homogeneity conditions.

For homogeneity of degree k of the translog production function we require that $\ln V[\lambda \underline{x}) = \ln v + k \ln \lambda$. This implies the following set of necessary and sufficient conditions on the translog function;

$$\sum_{i=1}^{5} \alpha_i = k \tag{41}$$

$$\sum_{j=1}^{5} \gamma_{ij} = 0, \ i = 1, \ . \ . \ . \ 5, \tag{42}$$

The equation (38) also implies, with the symmetry of γ_{ij}, that

$$\sum_{j=1}^{5} \gamma_{ij} = 0, \ j = 1, \ . \ . \ . \ 5, \tag{43}$$

i.e. the row sums and column sums of $[\gamma_{ij}]$ are identically zero.

(4) Separability conditions.

To define separability among inputs, first denote the set of n inputs by $N = \{i, \ . \ . \ . \ n\}$. A partition S of N is given by $\{N_1, \ . \ . \ . \ N_S\}$ where $N = N_1 \cup N_2 . \ . \ . \cup N_S$ and $N_r \cap N_t$ is empty for $r \neq t$. Let $\frac{\partial F}{\partial x_i} = f_i$, etc. A basic condition to which we refer below is the independence of the marginal rate of substitution of pairs of inputs from another input:

$$\partial \left(\frac{f_i}{f_j} \right) / \partial X_k = 0 \tag{44}$$

We say that F is strongly separable (SS) with respect to the partition S if (44) exists for all $i \varepsilon N_r$, $j \varepsilon N_t$ and $k \notin N_r \cup N_t$. The function is weakly separable (WS) with respect to the partition S if (44) exists for all i, $j \varepsilon N_r$ and $k \notin N_r$.[18]

By differentiation we immediately obtain that (44) is equivalent to

$$f_j \, f_{ik} - f_i \, f_{jk} = 0 \tag{45}$$

The condition for inputs i and j to be functionally separable from input k is that the first and second derivatives of F satisfy,[19] for the translog function,[20]

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[18]Goldman and Uzawa (1964) showed that a function f(x) is SS with respect to the partition S (s > 2) if and only if f(x) = $F [\sum_t f^t (\underline{x}^t)]$ where F is monotone increasing and $f^t(\underline{x}^t)$ is a function of $\underline{x}$. The function is WS if and only if it is of the form;

$$f(\underline{x}) = G[g^1(\underline{x}^1), \ . \ . \ . \ g^S(\underline{x}^S)]$$

Also Lau (1972) showed that the cost function is WS(SS) with respect to the partition S in input prices and in input quantities if and only if f(x) is homothetic. And Berndt and Christensen (1972) related separability to AES.

[19]For weak separability this condition must hold for inputs i and j in one subset and input k in another subset. For strong separability this condition must hold in addition for inputs i,j, and k all in distinct subsets. See Berndt and Christensen (1973b) for a summary discussion of separability conditions.

[20]This is derived by differentiating (31) and (32), and substituting into (45).

$$\gamma_{jk} (\alpha_i + \sum_{\ell=1}^{5} \gamma_{i\ell} \ln x_\ell) - \gamma_{ik} (\alpha_j + \sum_{\ell=1}^{5} \gamma_{\ell j} \ln x_\ell) = 0 \quad (46)$$

The set of conditions necessary and sufficient for inputs i and j to be globally separable from k are that[21]

$$\alpha_i \gamma_{jk} - \alpha_j \gamma_{ik} = 0,$$

$$\gamma_{i\ell} \gamma_{jk} - \gamma_{j\ell} \gamma_{ik} = 0, \; i,j,k,\ell = 1, \ldots 5 \quad (47)$$

When γ_{jk} and $\gamma_{j\ell}$ are nonzero, we can divide by these parameters and alternatively write the separability conditions as

$$\frac{\alpha_i}{\alpha_j} = \frac{\gamma_{ik}}{\gamma_{jk}} = \frac{\gamma_{i\ell}}{\gamma_{j\ell}}, \quad \ell = 1, \ldots 5 \quad (48)$$

4.1.3. Elasticity of Substitution

There exists a transcendental logarithmic production function of 5 inputs which attains both a given arbitrary set of "Direct Elasticities of Substitution" $\{\delta_{rs} \mid r, s = 1, \ldots 5; \delta_{rs} = \delta_{sr}\}$ and a given set of "Allen-Uzawa Partial Elasticities of Substitution" $\{\sigma_{rs} \mid r,s = 1, \ldots 5; \sigma_{rs} = \sigma_{sr}\}$, at given quantities of output and inputs.[22]

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[21]For the derivation of this condition, see Berndt and Christensen (1973a).

[22]About the proofs of these statements, see Jorgenson, Christensen, and Lau (1971), Part II: The Transcendental Logarithmic Production Function, pp. 21-57.

The Direct Elasticity of Substitution (DES) between inputs r and s is defined as[23]

$$\delta_{rs} = - \frac{F_r\, F_s (F_r\, x_r + F_s\, x_s)}{x_r\, x_s (F_{rr}\, F_s{}^2 - 2F_r\, F_s\, F_{rs} + F_{ss}\, F_r{}^2)} \tag{49}$$

where F_r's and F_{rs}'s are the first and second partial derivatives respectively. For the translog function,

$$F_r = \frac{V}{x_r} \; (\alpha_r + \sum_{i=1}^{5} \gamma_{ri} \ln X_i)$$

$$F_{rr} = \frac{V}{x_r^2} [\gamma_{rr} + (\alpha_r + \sum_{i=1}^{5} \gamma_{ri} \ln X_i - 1)(\alpha_r + \sum_{i=1}^{5} \gamma_{ri} \ln X_i)] \tag{50}$$

$$F_{rs} = \frac{V}{x_r\, x_s} [\gamma_{rs} + (\alpha_r + \sum_{i=1}^{5} \gamma_{ri} \ln X_i)(\alpha_s + \sum_{i=1}^{5} \gamma_{si} \ln X_i)]$$

Hence the D.E.S. is given by, in terms of the parameters of the translog function,

$$\delta_{rs} = - \frac{M_r M_s (M_r + M_s)}{M_s^2(\gamma_{rr} + M_r^2 - M_r) - 2M_r M_s (\gamma_{rs} + M_r M_s) + M_r^2(\gamma_{ss} + M_s^2 - M_s)}$$

---

[23]See Allen (1938), pp. 340-345, 503-505, Frisch (1959), and McFadden (1963).

where

$$M_i = \frac{\partial \ln F}{\partial \ln X_i} = \alpha_i + \sum_{j=1}^{5} \gamma_{ij} \ln x_j, \; i=1, \ldots 5 \tag{51}$$

The Allen-Uzawa Partial Elasticities of Substitution (AES) is defined as[24]

$$\sigma_{rs} = \frac{\sum_{i=1}^{5} F_i X_i}{X_r X_s} \frac{|F_{rs}|}{|F|} \tag{52}$$

where

$$|F| = \begin{vmatrix} 0 & F_1 & \cdots & F_5 \\ F_1 & F_{11} & \cdots & F_{15} \\ \vdots & \vdots & \ddots & \vdots \\ F_5 & F_{51} & \cdots & F_{55} \end{vmatrix}$$

and $|F_{rs}|$ is the cofactor of F_{rs} in $|F|$. and the A.E.S. can be again expressed, in terms of the parameters of the translog function, as

$$\sigma_{rs} = \frac{|G_{rs}|}{|G|} \tag{53}$$

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[24]See Allen (1938) and Uzawa (1962).

where $|G|$ is the determinant of

$$G = \begin{bmatrix} 0 & M_1 & \cdots & M_5 \\ M_1 & \gamma_{11}+M_1{}^2-M_1 & \cdots & \gamma_{15}+M_1M_5 \\ \vdots & \vdots & \ddots & \vdots \\ M_5 & \gamma_{51}+M_5M_1 & \cdots & \gamma_{55}+M_5{}^2-M_5 \end{bmatrix}$$

and $|G_{rs}|$ is the cofactor G_{rs} in G.

The formulae for the D.E.S. and the A.E.S. are functions only of the M_i and the γ_{ij}. Since the regressors are logarithmic, estimates of the γ_{ij} are independent of units of measurement. The fitted values M_i are also invariant to scaling of the regressors. Therefore, the estimates of the σ_{rs} are independent of units of measurement.

In general, neither the DES nor the AES of the translog function is constant for all quantities of inputs--and hence indirectly, for all prices of input--as one can readily verify by computing equations (51) and (53) for the translog function. Hence, the translog function exhibits the property of variable DES and AES. Actually this is to be expected in view of the theorems of McFadden (1963), Uzawa (1962) and Gorman (1965), which characterize completely the various highly restrictive classes of functional forms which exhibit the property of constancy of various definitions of elasticities of substitution.[25]

4.1.4. Profit Maximization

Let P_o = price of the output,

P^* = price of a unit of input i, i=1 . . . 5,

$P_i = P_i^* / P_o$, i=1 . . . 5,

$$M_i = \frac{P_i^* X_i}{P_o V} = \frac{P_i X_i}{V}, \quad i=1 \ldots 5.$$

Then the usual marginal conditions for profit maximization can be written as

$$\frac{\partial \ln V}{\partial \ln X_i} = \frac{P_i X_i}{V} = M_i \tag{54}$$

where M_i is the ratio of expenditure on input i to total sales. Equation (54) results in the following system of share equations.

$$M_i = \alpha_i + \sum_{j=1}^{5} \gamma_{ij} \ln x_j, \quad i=1, \ldots 5 \tag{55}$$

Equation (55) is linear in parameters and in addition, there are equality restrictions from the homogeneity conditions across the individual equations corresponding to the γ_{ij}'s.

[25]See McFadden (1963) and Mundlak (1968).

Other important properties of the translog production function in connection with the technical change are omitted from this discussion, because they are beyond the scope of our studies.

Also further discussions on the translog function regarding empirical implementation and its advantages and disadvantages are postponed to the next chapter, the section 4.3.

4.2. The Translog Cost Function

4.2.1. Introduction

A convenient functional form for the unit cost function is the transcendental logarithmic (or translog) cost function,[26]

$$\ln C = \alpha_0 + \sum_{i=1}^{5} \alpha_i \ln W_i + \sum_{i=1}^{5} \sum_{j=1}^{5} \beta_{ij} \ln W_i \ln W_j \tag{56}$$

where C is the production cost and W_i is the i-th input price. Also the translog form provides a second order approximation to an arbitrary twice continuously differentiable unit cost function.

4.2.2. Properties of the Translog Cost Function

(1) Monotonicity condition

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[26]Dual to the production function is a cost function, $C^* = J(y, W_1, W_2, W_3, W_4, W_5)$ where C^* is total cost of production, y is aggregate output, and W_i is the i-th input price. If the production function is a positive, nondecreasing, positively linear homogeneous, concave function, then the cost function can be written $C^* = y \cdot C(W_1, W_2, W_3, W_4, W_5)$ where C is a unit cost function satisfying the same regularity conditions (Diwert, 1973). See also Christensen, Jorgenson and Lau (1973) on the definition of the price possibility frontier under constant returns to scale, by duality in the theory of production.

The cost function must be an increasing function of the input prices. In terms of the parameters of the translog cost function, this implies,

$$\frac{\partial \ln C}{\partial \ln W_i} = \alpha_i + \sum_{j=1}^{5} \beta_{ij} \ln W_j \geq 0, \; i=1, \ldots 5 \tag{57}$$

(2) Concavity condition

The cost function must be concave in the input prices. This implies that the matrix $\left| \frac{\partial^2 C\,(\underline{W})}{\partial W_i \, \partial W_j} \right|$ must be negative-definite within the range of input prices observed, or equivalently all the characteristic values of the matrix are non-positive.

(3) Homogeneity condition

It is also well known that the cost function for a cost-minimizing firm must be homogeneous of degree one in the input prices. Hence,

$$\sum_{i=1}^{5} \alpha_i = 1 ,$$

and

$$\sum_{j=1}^{5} \beta_{ij} = 0, \quad i=1, \ldots 5 \tag{58}$$

(4) Separability condition

Similarly to the case of the translog production function the separability conditions for inputs i and j to be functionally separable from the input k is that the first and second derivatives of C satisfy, for the translog cost function,[27]

$$\beta_{jk}(\alpha_i + \sum_{\ell=1}^{5} \gamma_{i\ell} \ln W_\ell) - \gamma_{ik}(\alpha_j + \sum_{\ell=1}^{5} \beta_{\ell j} \ln W_\ell) = 0 \tag{59}$$

4.2.3. Elasticity of Substitution

A dualistic concept in the cost function to the direct elasticity of substitution (D.E.S.) in the production function can be defined by applying the two-factor elasticity of substitution formula to each pair of factors, holding fixed the imputed prices of the remaining factors and the imputed total cost. McFadden (1963) named it the shadow partial elasticity of substitution (S.E.S.). The S.E.S. can be defined in terms of the cost function $C = C(y,\underline{W})$ of the producer, which specifies the minimum imputed cost C of producing the output y with the according price vector $\underline{W} = (W_1, W_2, W_3, W_4, W_5)$ such as

$$\delta_{ij}^{*} = \frac{-(C_{ii}/C_i^2) + 2(C_{ij}/C_i\, C_j) - (C_{jj}/C_j^2)}{(1/W_i\, C_i) + (1/W_j C_j)}$$

where $C_i = \frac{\partial C}{\partial W_i}$ and $C_{ij} = \frac{\partial^2 C}{\partial W_i\, \partial W_j}$ are evaluated at $(y, \underline{W})$

[27]See the equation (44) through (48) of the section 2.1.2. in this chapter.

Hence the S.E.S. is defined as, in terms of the parameters of the translog cost function,

$$\delta_{ij}^{*} = \frac{-[M_j^{\,2}(M_i^2-M_i+\gamma_{ii})-2M_iM_j(M_iM_j+\gamma_{ij})+M_i^2(M_j^2-M_j+\gamma_{ij})]}{M_iM_j(M_i+M_j)}$$

On the substitution possibilities among inputs implied by the translog cost function, Uzawa (1962) demonstrated that elasticities of substitution (AES) could be computed directly from the cost function and its derivatives. The formula[28] is

$$\sigma_{ij} = \frac{C_{ij}\ C}{C_i\ C_j} \tag{62}$$

where $C_i = \partial C/\partial W_i$ and $C_{ij} = \partial^2 C/\partial W_i \partial W_j$.

For the translog cost function this becomes

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[28]The A.E.S. formula is shown in (47) as

$$\sigma_{ij} = \frac{\sum_{\ell=1}^{5} F_\ell\ X_\ell}{X_i\ X_j} \ \frac{|F_{ij}|}{|\ F\ |}$$

where $y = F(X_1, X_2, X_3, X_4, X_5)$. Also we have $C_i = \frac{\partial C}{\partial W_i} = X_i$ and $C_{ij} = \frac{\partial X_i}{\partial W_j}$ from the cost function $C = C(W_1, W_2, W_3, W_4, W_5)$ where $C(\underline{W}) = \sum_{i=1}^{5} W_i\ X_i$. Hence $\sigma_{ij} = \frac{C}{X_i\ X_j}\ \frac{\partial\ X_i}{\partial\ W_j} = \frac{C_{ij}}{C_i\ C_j}\ C.$

$$\sigma_{ij} = \frac{\gamma_{ij} + M_i M_j}{M_i M_j}, \quad i \neq j$$
$$\sigma_{ii} = \frac{\gamma_{ii} + M_i(M_i - 1)}{M_i^2} \tag{63}$$

where the M_i's are fitted values of the cost share equations. The elasticities of demand with respect to price changes are closely related to the AES:[29]

$$\eta_{ij} = M_j \sigma_{ij} = \frac{\gamma_{ij} + M_i M_j}{M_i}, \quad i \neq j$$
$$\eta_{ii} = M_i \sigma_{ii} = \frac{\gamma_{ii} + M_i(M_i - 1)}{M_i} \tag{64}$$

4.2.4. Cost Minimization

The system of share equations are also obtained by logarithmic differentiation of the unit cost function,[30]

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[29]See also Uzawa (1962) and Brown, Caves and Christensen (1975), p. 26.

[30]In the total cost function of $C^* = y \cdot C(W_1, W_2, W_3, W_4, W_5)$, $\partial C/\partial W_i = X_i/y$, where X_i is the cost minimizing quantity of the i-th input. Since the cost function is linear homogeneous in prices, $C^* = \sum_{i=1}^{5} W_i X_i$ by Euler's theorem. Therefore, $C = \sum_{i=1}^{5} W_i X_i / y$.

From these relations, we can get,

$$\frac{\partial \ln C}{\partial \ln W_i} = \frac{\partial C}{\partial W_i}\frac{W_i}{C} = M_i = \alpha_i + \sum_{j=1}^{5} \beta_{ij} \ln W_j, \tag{65}$$

where M_i is the cost share of the i-th input.

$$\frac{\partial \ln C}{\partial \ln W_i} = \frac{\partial C}{\partial W_i}\frac{W_i}{C} = \frac{X_i}{Y}\frac{W_i}{C} = \frac{X_i W_i}{Y}\frac{Y}{\sum_{i=1}^{5} W_i X_i}$$

$$= \frac{W_i X_i}{\sum_{i=1}^{5} W_i X_i} = M_i$$

CHAPTER II. MULTIPRODUCT PRODUCTION THEORY

A.2.1. Introduction

Economic speculations on the behavior of multiproduct firms can be traced to Pigou (1932), and Robinson (1933), and more recently, Reder (1941), Gordon (1948) and Bailey (1954). These analyses focussed on the selling behaviors (revenue side) of a multiproduct firm, viewing its pricing process as an extended application of the Pigou-Robinson theory of price discrimination. These were followed by those of Hicks (1929), Dorfman (1951) and Ferguson (1971), who adopted the conventional marginalists' method to analyze the profit--maximization behavior of a firm that produces more than one product by means of several variable inputs and occasionally of fixed inputs.

Until the study of Samuelson's singularity theorem for non-joint production was published in 1966, there had been no extensive studies on the specification of a production technology for a multiproduct firm. Samuelson (1966) established the necessary condition for the production possibility frontier not to involve joint production, and the work has been extended by Hirota and Kuga (1971) and Burmeister and Turnovsky (1971). Burmeister and Turnovsky studied the case where commodities are partitioned into joint groups and obtained necessary conditions for such to occur in terms of the second derivatives of the production possibility frontier. They assumed rather a simple type of partition of commodities into non-overlapping groups. More extensive and useful concepts and analysis in commodity structure of group formation, in general cases followed by Kuga (1973). Kuga extended the concept of Hirota-Kuga's "intrinsic non-jointness" to that of "marginal non-jointness," where it is not advantageous for the producer as a whole to change its output level infinitesimally, at the going factor--and commodity--prices, from suitable changes in the factor input, but without requiring any change in the output level of other commodities. He also used the concepts of a "weak joint group" where a commodity may enter joint relations with more than one joint group in which a number of joint groups are formed, not necessarily of the non-overlapping type, and of a "strong joint group" which roughly corresponds to the non-overlapping joint group of Burmeister and Turnovsky.

Together with the problem of jointness in the theory of the multiproduct firm, the concept of separability[1] between inputs and outputs has become focussed on in the specification of the multi-input, multi-output production technology.

Recent progress in the specification of the multiproduct production technology has been achieved in two distinctive

[1]The definition of separability is already discussed in the previous chapter. See the section 4.1.2. Chapter I.

directions. The first approach deals with the production possibility frontier, originally proposed by Mundlak (1963) under certain restrictive assumptions. The second contribution focusses on the further developments and various applications of duality theory.[2] The applications of duality theory in the theory of the multiproduct firm have been elaborated mainly in the two different ways. The dual relationship between the transformation function and the profit function have been speculated by McFadden (1966), Diwert (1973), and Lau (1972, 1976). Christensen, Jorgenson and Lau (1971, 1973) have also made an empirical application to the U.S. economy. In addition, Hall (1973) has approached the problem from the point of view of the dual relationship between the transformation function and the joint cost function, using a generalization of the Generalized Leontief cost function due to Diwert (1971). Also recently Brown, Caves and Christensen (1975) have made an empirical application of a joint cost function to the U.S. railroad industry. The basic duality concepts which underly all these studies may be traced back to the pioneering work of Shephard (1953).

A.2.2. Specification of a Technology

2.1. The Factor Requirement Function

In the specification of a technology with a multiproduct production process, the simplist procedure is to appregate inputs

[2] A very extensive survey on this topic was done by W. E. Diwert (1972).

of various factors. Suppose we have a single input x which can be used to produce various conbinations of three outputs, $\underline{Y} = (Y_1, Y_2, Y_3)$. The technology of multiple outputs, single input firm may be summarized by a factor requirements function $g(\underline{Y})$ which gives the minimal amount of input x required to produce the vector of outputs $\underline{Y}$. The properties of $g(\underline{Y})$ are assumed to be,[3] (a) Domain--$g(\underline{Y})$ is a real valued function defined for $\underline{Y} \geq \underline{0}$ with $g(\underline{0}) = 0$ and $g(\underline{Y}) > 0$ if $\underline{Y} \geq \underline{0}$ (b) Closure--if $Y^n \geq \underline{0}$ and $\lim_{n\to\infty} 1^T Y^n = +\infty$, then $\lim_{n\to\infty} g(Y^n) = +\infty$, (c) Monotonicity--$g(\underline{Y})$ is a nondecreasing function, (d) Convexity-- $g(\underline{Y})$ is a quasiconvex function, and (e) Continuity-- $g(\underline{Y})$ is continuous from below, i.e., for every $\alpha \geq 0$, the set $\{\underline{Y} : g(\underline{Y}) \leq \alpha\}$ is closed.

Condition (a) states that zero input produces only zero output and that a positive amount of input is required in order to produce a positive amount of any output. Condition (b) states that an infinite amount of input is required to produce an infinite amount of any output. Condition (c) states that if more output is produced, then the minimum amount of input needed will not decrease. Condition (d) is a generalization of the classical condition of increasing marginal rate of substitution between products.[4]

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[3]The generalized C.E.S. form of the function was first introduced by Powell and Gruen (1968). Also their properties are well clarified again by Diwert (1974).

[4]On this classical condition, see Hicks (1946), p. 87.

Condition (e) is a weak mathematical regularity condition. If $g(\underline{Y})$ is a continuous function, then (e) will be satisfied.

But the difficulty in this simplist procedure of aggregating various factors into one input is that the aggregate input requirement function is not a single valued function, and its parameters depend on the composition of inputs, which in turn depends on, among other things, the prices in question. This difficulty can be avoided by working on a lower level of aggregation where inputs are not combined.

2.2. The Transformation Function

A well behaved technology can be described equally well in terms of relations between prices, or relations between quantities and prices, as long as markets are competitive and profits are maximized. The basic relation among quantities for our purposes is the transformation function $t(\underline{Y}, \underline{X}) \geq 0$ if $\underline{Y}$ can be produced with $\underline{X}$. We assume that $t(\underline{Y}, \underline{X})$ is defined and continuous for all non-negative $\underline{Y}$ and $\underline{X}$ and that it is decreasing in $\underline{Y}$ and increasing in $\underline{X}$.

Alternatively speaking on the transformation function, the production function is also defined by:[5]

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[5]For a multiple-input, multi-output firm, there is no natural numeraire commodity, such as the single output, to define the production function representation of technology. Following Jorgenson and Lau (1974), the convention of choosing as the left-hand-side variable for production function a variable input which is nonproducible, is adopted here. See also Lau (1976), pp. 52-53.

$$L = t(\underline{Y}, \underline{X}) \tag{1}$$

the minimum value of L for given values of $\underline{Y}$ and $\underline{X}$ such that the production plan $(\underline{Y}, \underline{X}, - L)$ is feasible, where

L is quantity of the left hand-side variable and nonproducible net input,

Y is the vector of net outputs, and

X is the vector of net inputs.

It is assumed that $t(\underline{Y}, \underline{X})$ possesses certain properties, which parallel similar properties of the single output case. (a) Domain--t is a finite, nonnegative, real-valued function defined on $\bar{R}_+{}^n \times \bar{R}_-{}^m$, where $\bar{R}_+{}^n$ denotes the closed nonnegative orthants of R_n for n outputs and $\bar{R}_-{}^m$ denotes the closed nonpositive orthants of R_n for m inputs. (b) Continuity--t is continuous on $\bar{R}_+{}^n \times \bar{R}_-{}^m$. (c) Monotonicity--t is nondecreasing on $\bar{R}_+{}^n \times \bar{R}_-{}^m$ and strictly increasing on $R_+{}^n \times R_-{}^m$ where $R_+{}^n$ denotes the interior of the nonnegative orthant of R_n and R_-^m denotes the interior of the nonpositive orthant of R_n. (d) Convexity--t is convex on $\bar{R}_+^n \times \bar{R}_-^m$ and locally strongly convex on $R_+^n \times R_-^m$. (e) Twice differentiability--t is twice continuously differentiable on $R_+^n \times R_-^m$. (f) Boundedness--

$$\lim_{\lambda \to \infty} \frac{t(\lambda\underline{Y}, \lambda\underline{X})}{\lambda} = \infty$$

for every $\underline{Y}, \underline{X} \in \bar{R}_+{}^n \times \bar{R}_-{}^m$, $\underline{Y}, \underline{X} \neq \underline{0}$. Alternatively saying, $\underline{Y}$ is finite for all finite $\underline{X}$ and $\underline{X}$ is finite for all finite $\underline{Y}$. Also $\underline{Y}$ becomes unbounded for unbounded $\underline{X}$.

2.2. The Cost Function and the Profit Function

The cost function is defined by the relations among quantities and prices, i.e. the function $C(\underline{W},\underline{Y})$ giving the minimum cost at which outputs $\underline{Y}$ can be produced when factor prices are $\underline{W}$. We assume that $C(\underline{W},\underline{Y})$ is defined for all positive $\underline{Y}$ and $\underline{W}$, that it is a continuous, nondecreasing function in $\underline{Y}$ and $\underline{W}$, and that it is homogeneous of the first degree in $\underline{W}$.

Parallel to the definition of the cost function, the profit function is also defined by:

$$\Pi(\underline{P}_1, \underline{Y}_2) \equiv \max_{Y_1} \{\underline{P}_1\underline{Y}_1 : t(\underline{Y}_1, \underline{Y}_2) = 0\} \tag{2}$$

the maximization of a linear function $\underline{P}_1\underline{Y}_1$ over the set of $\underline{Y}_1$ such that $t(\underline{Y}_1, \underline{Y}_2) = 0$ where $\underline{Y}_1$ is a vector of choice variables, $\underline{P}_1$ is the corresponding price vector and $\underline{Y}_2$ is a vector of fixed variables. Here the profit function $\Pi(\underline{P}_1, \underline{Y}_2)$ is dual to the transformation function $t(\underline{Y}_1, \underline{Y}_2)$ in the sense that each may be completely derived from knowledge of the other. Certain regularity conditions are required, of course, leading to different duality theorems.[6]

In particular, different theorems apply depending upon the nature of $\underline{Y}_1$ and $\underline{Y}_2$. If $\underline{Y}_1$ refers to a set of inputs then we refer to $\Pi(\underline{P}_1, \underline{Y}_2)$ as (the negative of) a cost function which may be a

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[6]For a sampling of the literature on duality see Shephard (1953, 1970), Uzawa (1964), Diewert (1971, 1974) and Lau (1969, 1972).

total cost function if $\underline{Y}_2$ refers to outputs only or a variable cost function if $\underline{Y}_2$ includes some fixed factors. If $\underline{Y}_2$ refers to fixed inputs only then $\Pi(\underline{P}_1, \underline{Y}_2)$ is called a variable profit or gross profit function and if $\underline{Y}_2$ does not exist then it is called a profit function. If $\underline{Y}_2$ includes only primary inputs Π is a value added function and if $\underline{Y}_2$ includes only outputs it is called a revenue function.

The normalized profit function is given by[7]

$$\Pi(\underline{P}, \underline{W}) = \sup_{\underline{Y},\underline{X}} \{\underline{P}'\underline{Y} + \underline{W}'\underline{X} - \lambda t(\underline{Y}, \underline{X}) : \underline{Y}, \underline{X} \in \bar{R}_+^{\,n} \times \bar{R}_-^{\,m}\} \tag{3}$$

where $\underline{P}$ and $\underline{W}$ are respectively the normalized prices of $\underline{Y}$ and $\underline{X}$ in terms of L, the numeraire commodity nonproducible in the production function. The corresponding properties of this function are; (a) Domain--Π is a finite, positive, real-valued function defined on $R_+^n \times R_+^m$. (b) Continuity--Π is continuous on $R_+^n \times R_+^m$. (c) Monotonicity--Π is strictly increasing in $\underline{P}$ and strictly decreasing in $\underline{W}$ on $R_+^n \times R_+^m$. (d) Convexity--Π is locally strongly convex on $R_+^n \times R_+^m$. (e) Twice Differentiability--Π is twice continuously differentiable on $R_+^n \times R_+^m$. (f) Boundedness--

$$\lim_{\lambda \to \infty} \frac{\Pi(\lambda\underline{P}, \lambda\underline{W})}{\lambda} = \infty, \text{ for every } \underline{P}, \underline{W} \in R_+^n \times R_+^m.$$

Π is finite for all finite $\underline{P}$ and $\underline{W}$.

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[7]This specification is due to Lau (1976), pp. 54-55.

The importance of the theory for our purposes is that, under certain regularity conditions, $\Pi(\underline{P}_1, \underline{Y}_2)$ and $t(\underline{Y}_1, \underline{Y}_2) = 0$ are two equivalent representations of the technology. First we provide here an explicit statement of duality between the cost function and its underlying technology.[8]

2.4. Shephard-Uzawa-McFadden Duality Theorem for the Cost Functions[9]

Suppose the transformation function $t(Y, X)$ has a strictly convex input structure; that is, the input requirement set $X(Y) = \{X | t(Y,X) \geq 0\}$ is closed and strictly convex.[10] Then there is a unique cost function $C(Y,W)$, differentiable in W, defined by

$$C(Y,W) = \min_{X \in X(Y)} \{WX\}. \tag{4}$$

Further, $C(Y, W)$ is positive, linear, homogeneous, non-decreasing, and concave in the factor prices, W.[11] Finally it obeys Shepard's Lemma (1953),

$$t(Y, \frac{\partial C(Y,W)}{\partial W}) = 0, \tag{5}$$

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[8] On the duality between the transformation function and the profit function, see Lau (1976).

[9] We already speculated its correspondence in the case of a uniproduct technology earlier. See Shepard (1953, 1970), Uzawa (1964), and McFadden (1973).

[10] This rules out the case of factors that are perfect substitutes or perfect complements. See McFadden (1973).

[11] The concavity of the cost function does not follow from the convexity of the technology. All cost functions are concave, irrespective of the characteristics of the underlying technology.

that is, the vector of cost-minimizing factor inputs is equal to the vector of derivatives of the cost function with respect to the factor prices.[12]

Also when the transformation function t(Y, X) is differentiable in outputs, Y, the following condition holds:

$$\frac{\partial\ C(Y,W)\ /\ \partial Y_i}{\partial\ C(Y,W)\ /\ \partial Y_j} = \frac{\partial\ t(Y,X)\ /\partial Y_i}{\partial\ t(Y,X)\ /\partial Y_j}\ , \tag{6}$$

that is, the ratio of the marginal costs of two goods is equal to the marginal rate of transformation between them. Thus the production possibility frontier is tangent to the isocost surface at the point where production takes place.

2.2. Homogeneity and Almost Homogeneity

In the case of multi-input, multi-output transformation functions, the concept of homogeneity is somewhat imprecise unless it is homogeneity of degree one. Intuitively, one would like to say that a transformation function is homogeneous of degree k, if when all inputs are increased by some proportion λ all outputs are increased by the proportion λ^k. Furthermore the concept of "almost homogeneity" has been introduced to facilitate the analysis of the technology with multi-input and multi-output. A function (Y, X) where Y and X are vectors, is almost homogeneous of degrees k_1, k_2

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[12] The proof of this theorem is given by McFadden (1973).

and k_3, respectively, if and only if $t(\lambda^{k_1}Y, \lambda^{k_2}X) = \lambda^{k_3}t(Y,X)$ for every scalar $\lambda > 0$.[13]

It is straightforward to see that the transformation function exhibits constant returns to scale if and only if C(Y,W) is homogeneous of degree one in Y. The transformation function is homogeneous of degree one if

$$t(\lambda Y, \lambda X) = t(Y,X) = 0 \tag{7}$$

This implies

$$C(\lambda Y, W) = \min \Sigma W_i \lambda X_i = \lambda \min \Sigma W_i X_i = \lambda C(Y,W) \tag{8}$$

Similarly (8) implies (7).

Further meaningful analysis on homogeneity and almost homogeneity among the transformation function, the cost function and the profit function, are beyond the scope of the current study and omitted here.[14]

2.6. Separability Between Inputs and Outputs

Most studies of the structure of production utilize a single variable to represent output, no matter how diverse its

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[13]It is clear that k_1, k_2 and k_3 are in general unique. This more general representation is used to allow the possibility of some k_i being equal to zero identically.

[14]For a sampling of the literature on homogeneity in relation to the transformation function, the cost function, the profit function and the revenue function, see Lau (1972, 1976), Diwert (1974), Brown, Caves and Christensen (1975), and Hall (1973).

actual components. The question here is whether there exists an unambiguous measure of output which is valid independent of the relative factor intensities, i.e. whether the transformation function can be written as

$$t(Y,X) = t[H(Y), X] = H(Y) * G(X) = 0 \tag{9}$$

where H(Y) and G(X) are scalar functions of the Y and X vectors respectively, and * is any arbitrary operator such as addition, multiplication or an exponent, etc. Thus the existence of an output index H(Y) implies the existence of an input index G(X). The existence of these indexes is equivalent to t(Y, X) being separable in outputs and inputs.

Lau (1969) has proved many useful theorems relating the properties of transformation functions and profit functions. He noted that revenue and cost functions can be regarded as special cases of the profit function with inputs and outputs fixed. Thus his Theorem is directly applicable in the present context.[15]

Theorem 1 (Lau): $t(Y,X) = t[H(Y), X] = 0$ (10)

if and only if $C(Y,W) = C[H(Y), W]$.

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[15]Hall (1973) and Burgess (1974) also demonstrated similar theorems on separability between inputs and outputs, confining most of their attention to the case of constant returns to scale--a very reasonable specification for the analysis of aggregate data, but less reasonable for the analysis of microeconomic data.

This intuitively appealing result says that the transformation function is separable in outputs and inputs if and only if the joint cost function is weakly separable in outputs.[16]

Another result can be adapted from Lau (1969,) to illustrate the case where both the input and output indexes exist.

Theorem 2 (Lau): $C(Y,W) = H(Y)\, F(W)$

if and only if $t(Y,X) = H(Y) + G(X) = 0$ (11)

and $G(x)$ is homothetic.

Thus strong separability of outputs and factor prices in the joint cost function is equivalent to separability of the transformation function with the input index being homothetic.

2.7. Non-Jointness in Production

The problem of non-jointness has been investigated by Samuelson (1966) and Kuga (1973) who derived necessary and sufficient conditions for a production function to represent a non-joint technology, using the transformation function.

A production function of five inputs and three outputs $L = t(Y,X)$ is said to be non-joint in inputs if there exist individual production functions

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[16]Since the cost function is an implicit function, separability in outputs does not imply separability in factor prices. Hence "weak" separability must be distinguished from "strong" separability. See Berndt and Christensen (1973b) for a recent discussion of "weak" and "strong" separability.

$$Y_i = f_i(X_{i1}, X_{i2}, X_{i3}, X_{i4}, X_{i5}),\ i=1, \ldots 3 \tag{12}$$

such that $t(Y,X) = 0$, if and only if $Y_i = f_i$ and

$$\sum_{i=1}^{3} X_{ij} = X_j,\ j=1, \ldots 5$$

and the inputs are so allocated amongst the industries that the output of no one industry may be increased without decreasing the output of some one industry and no one input may be decreased without increasing another input. It is said to be non-joint in outputs if there exist individual factor requirements functions

$$X_i = g_i(Y_{i1}, Y_{i2}, Y_{i3}),\ i=1, \ldots 5 \tag{13}$$

such that $t(Y,X) = 0$, if and only if, $X_i = g_i$ and

$$\sum_{i=1}^{5} Y_{ij} = Y_j,\ j=1, \ldots 3,$$

and the outputs are so allocated that no input may be diminished without increasing the input of some one joint production process.

To show that a technology is nonjoint, we must exhibit the individual functions $f_i = g_i$ and show that they meet both of these requirements.[17] Although we have the natural definition of

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[17]Note that nonjointness requires only that the f_i exist as functions: there is no requirement that there be physically separate processes producing the various outputs, Y_i. Thus the observation that more than one output is produced in the same plant is not sufficient to rule out nonjointness.

nonjointness, there is no obvious way to translate this definition into an econometric restriction that can be imposed on a more general specification of the technology. Since necessary and sufficient conditions for a transformation function to represent a non-joint technology are not particularly helpful in the specification of functional forms for econometric analysis, Hall (1972) has approached the problem using the joint cost function and Lau (1972) has approached the problem using the profit function. However, the details of this problem seem to be beyond the scope of the current study and here we briefly review an alternative characterization of non-jointness in terms of the joint cost function, suggested by Hall

Theorem 1. (Hall):

A necessary and sufficient condition for nonjointness is that the total cost of producing all outputs be the sum of the costs of producing each separately:

$$C(Y, W) = Y_1 \, \phi^{(1)}(W) + \, . \, . \, . \, . \, . \, . \, + Y_n \, \phi^{(n)}(W) \qquad (14)$$

where $\phi^{(i)}(W)$ is the cost of producing a unit of output i.

If the technology is nonjoint, the marginal cost of each output is independent of the level of any output.

Lastly in the case of the separability of technology, the ratios of the marginal costs depend only on the output mix, while with nonjointness, marginal costs are independent of the output mix. This subject that the overlap between the two restrictions is very small. Hall (1973) proved the following theorem:

Theorem 2 (Hall):

No multiple-output technology with constant returns to scale can be both separable and nonjoint. That is, the individual production functions in such a technology are identical except for a scalar multiple, implying that there is effectively only a single kind of output.[18]

Hence nontrivial separable technologies are inherently joint, and their use in empirical work forecloses investigation of the hypothesis of nonjointness.

A.2.3. Functional Forms in Trend

3.1. Functional Forms of the Production Possibility Frontier

In the specification of a technology with a multiproduct production process, the simplist procedure is to aggregate outputs of various products. The difficulty in such an approach is that the aggregate production function is not a single valued function, and its parameters depend on the composition of output, which in turn depends on, among other things, the prices in question. This difficulty can be avoided by working on a lower level of aggregation where outputs are not combined.

Recent progress in the specification of this multiproduct production technology has been achieved in two distinct directions.

[18] See Hall (1973) on the proof of this impossibility theorem for separable nonjoint technologies, pp. 885-886.

The first approach deals with the production possibility frontier--under certain restrictive assumptions--and the second contribution focusses on the further developments and various applications of the duality theory.

The first approach was originally proposed by Mundlak (1963). He suggested the estimation of the production possibility frontier giving an implicit relation between a vector of outputs, say Y and a vector of total inputs, say X. In general, a production possibility frontier can be defined in terms of a transformation function:

$$t(Y,\ X) = 0 \tag{15}$$

In the absence of further restrictions, this formulation of a technology permits arbitrary kinds of interaction between total factor intensities and the trade-off between various types of output. Mundlak introduced a substantive restriction on the form of the transformation function: he assumed that it can be written in the following way: i.e.

$$t(Y,\ X) = H(Y) - G(X) = 0 \tag{16}$$

Specifically he suggested a transcendental function which forms a generalization of a Cobb-Douglas production function;

$$t(Y,X) = Y_1^{\alpha_1}Y_2^{\alpha_2}e^{\beta_1 Y_1+\beta_2 Y_2} - \alpha_0\, X_1^{\gamma_1}X_2^{\gamma_2}e^{\delta_1 X_1+\delta_2 X_2} = 0 \tag{17}$$

This separability restriction between inputs and outputs has a number of implications. First, separability almost always means that outputs are produced jointly. The only case in which the production structure of a multiproduct firm can be portrayed by separate production functions for each kind of output is the case where all the production functions are identical. Second, separability implies that output price ratios or marginal rates of transformation are independent of factor intensities or factor prices. This rather undesirable property makes it apparent that a specification of joint production with the separability constraint is no more general, in at least this crucial respect, than the specification of a uniproduct technology.

Following Mundlak's transcendental multiproduct production function, a generalization of a Cobb-Douglas production function, Powell and Gruen (1968) derived the family of constant elasticity of transformation (CET) production possibility schedules which turn out to be algebraically identical to the CES isoquants, apart from one difference of sign determining their concavity. Measuring the basic shape of the frontier of production possibilities by the elasticity of transformation between products 1 and 2 as follows:

$$\tau_{12} = \frac{d\left(\frac{Y_1}{Y_2}\right)}{d\left(\frac{\partial Y_1}{\partial Y_2}\right)} \frac{\left(\frac{\partial Y_1}{\partial Y_2}\right)}{\left(\frac{Y_1}{Y_2}\right)}$$

his derived CET function has the functional form of

$$Y_1^{(1-k)} + A\ Y_2^{(1-k)} = B(1-k) \qquad (19)$$

where $k = \frac{1}{\tau_{12}}$, $A = C^k > 0$, and B and C are constants. This expression is nothing but a general mathematical expression of an ellipse. They further showed that a given, constant elasticity of transformation is compatible with product-neutral and product-biased shifts in the location of the frontier, pointing out that the CET model therefore is of potential value in the analysis of technical change.

Further extension of the CET functional form was done again by Mundlak and Razin (1971), applying a n-factor generalization of the CES function of the form presented by Sato (1967) into the specification of a multiproduct technology, called a nested multi-stage multiproduct production functions. "Let there be A products. The output of the α product is denoted by A_α. In the first stage of aggregation (stage α), the A products are grouped into B disjoint and exhaustive groups. A function b_β is defined on each of these groups. The B function b_β are grouped into Γ disjoint and exhaustive groups, and new functions c_γ are defined on these groups. This process continues until a final aggregate function results. So we get

$$\text{stage Alpha} \qquad b_\beta = \left[\sum_{\alpha \epsilon b_\beta} A_\alpha\ a_\alpha^{\rho_\beta} \right]^{1/\rho_\beta}, \ \beta=1, \ . \ . \ . \ B \qquad (20)$$

stage Beta $$c_\gamma = \left[\sum_{\beta\epsilon c_\gamma} B_\beta \, b_\beta^{\rho_\gamma}\right]^{1/\rho_\gamma}, \gamma=1, \ldots \Gamma \qquad (21)$$

. . . . "[19]

But their main contributions in the production studies concern the problem of index numbers in terms of an appropriate aggregation scheme in the measurement of technical change. Hence either single- or multi-stage multiproduct production functions are still conditioned by a severe restriction, i.e. separability, as in the previous Mundlak specification.

Also a functional form without explicit separability between inputs and outputs was suggested by Mundlak and Razin (1971). A simple representation for the two-output two-input case is

$$[\alpha_1 Y_1^{\rho} + \alpha_2(k)Y_2^{\rho}]^{1/\rho} = [\beta_1 X_1^{\delta} + \beta_2 X_2^{\delta}]^{1/\delta} \qquad (22)$$

where $k = X_1/X_2$. The right-hand side of (22) is the usual C.E.S.--like formulation for the factor side and the left-hand side is a similar formulation for the product side. But since α_2 is written as a function of the factor ratio k, the dependence of the transformation curve on the factor ratio is explicitly introduced, rejecting an explicit separability between inputs and outputs.

But the most comprehensive and general representation of the production possibility frontier, not restricted by any _a priori_

[19]See Mundlak and Razin (1971), pp. 493-494.

assumptions such as separability, homogeneity, etc. was recently developed by Christensen, Jorgenson and Lau (1973). Their transcendental logarithmic production frontier is represented by a function that is quadratic in the logarithms of the quantities of inputs and outputs.[20]

This function provides a local second-order approximation to any production frontier. The resulting frontiers permit a greater variety of substitution and transformation patterns than based on the CET-CES. Its functional form of five inputs and three outputs can be shown as follows:

$$\ln F = \alpha_0 + \sum_{i=1}^{5} \alpha_i \ln X_i + \sum_{j=1}^{3} \beta_j \ln Y_j$$

$$+ \sum_{i=1}^{5} \sum_{j=1}^{5} \gamma_{ij} \ln X_i \ln X_j + \sum_{i=1}^{5} \sum_{j=1}^{3} \varepsilon_{ij} \ln X_i \ln Y_j$$

$$+ \sum_{i=1}^{3} \sum_{j=1}^{3} \delta_{ij} \ln Y_i \ln Y_j \qquad (23)$$

where $F = t(Y, X) = 0$. (24)

---

[20]They also developed the specification of a price possibility frontier, based on a complete model of production with a production possibility frontier and with necessary conditions for producer equilibrium under constant returns to scale with the existence of prices consistent with zero profits.

3.2. Functional Forms Flexible in Prices

The second approach to the specification of production process with several kinds of outputs was contributed by Diwert (1971) who generated a functional form which is linear in parameters and which provides a second-order approximation to any arbitrary twice differentiable function. His Generalized Leontief (GL) functional form was quickly followed by the translog (TL) functional form of the price possibility frontier developed by Christensen, Jorgenson and Lau (1971) and Sargan (1971) for the multi-input, multi-output technology.

$$\begin{aligned}\ln \Pi(W, P) = \alpha_0 &+ \sum_{i=1}^{5} \alpha_i \ln W_i + \sum_{j=1}^{3} \beta_j \ln P_j \\ &+ \sum_{i=1}^{5} \sum_{j=1}^{5} \gamma_{ij} \ln W_i \ln W_j + \sum_{i=1}^{5} \sum_{j=1}^{3} \varepsilon_{ij} \ln W_i \ln P_j \\ &+ \sum_{i=1}^{3} \sum_{j=1}^{3} \delta_{ij} \ln P_i \ln P_j \end{aligned} \tag{25}$$

where

W_i = price of the i-th input

P_j = price of the j-th output

Also a Hybrid Diwert joint cost function (HD) was defined by Hall (1973), the functional form of which is,

$$C(Y, W) = \sum_{i=1}^{5} \sum_{\ell=1}^{5} \sum_{j=1}^{3} \sum_{k=1}^{3} A_{i\ell jk} \sqrt{Y_j\ Y_k} \sqrt{W_i\ W_\ell} \tag{26}$$

And Jorgenson et al. (1970) also defined a translog cost function (TC), later extended more by Brown et al. (1975). The TC form is,

$$\begin{aligned} \ln C(Y, W) = {} & \alpha_0 + \sum_{i=1}^{5} \alpha_i \ln W_i + \sum_{j=1}^{3} \beta_j \ln Y_j \\ & + \sum_{i=1}^{5} \sum_{j=1}^{5} \gamma_{ij} \ln W_i \ln W_j + \sum_{i=1}^{5} \sum_{j=1}^{3} \varepsilon_{ij} \ln W_i \ln Y_j \\ & + \sum_{i=1}^{3} \sum_{j=1}^{3} \delta_{ij} \ln Y_i \ln Y_j \end{aligned} \tag{27}$$

where $\gamma_{ij} = \gamma_{ji}$ and $\delta_{ik} = \delta_{ki}$

A.2.4. The Translog Generalization to the Multiproduct Situation

4.1. Extension of the Translog Transformation Function to Multiple Outputs and Multiple Inputs

4.1.1. Introduction

The translog function has an added advantage that it can be generalized to the case of multiple outputs in a straight forward manner. The general transformation function for a multi-output and multi-input technology may be written as $F(Y, X) = 1$,[21] where Y and

[21]It is clear that F is unique only up to a monotonic transformation f such that f(1) = 1.

X are vectors of outputs and inputs respectively. As usual, we approximate ln F by a second order Taylor series expansion in ln X and ln Y. Thus the translog transformation function for 3 outputs and 5 inputs can be written;

$$\ln F = \alpha_0 + \sum_{i=1}^{5} \alpha_i \ln X_i + \sum_{j=1}^{3} \alpha_j \ln Y_j$$

$$+ \tfrac{1}{2} \sum_{i=1}^{5} \sum_{j=1}^{5} \gamma_{ij} \ln X_i \ln X_j + \sum_{i=1}^{5} \sum_{j=1}^{3} \varepsilon_{ij} \ln X_i \ln Y_j \qquad (28)$$

$$+ \tfrac{1}{2} \sum_{i=1}^{3} \sum_{j=1}^{3} \delta_{ij} \ln Y_i \ln Y_j$$

where $\delta_{ij} = \delta_{ji}$, $\varepsilon_{ij} = \varepsilon_{ji}$ and $\gamma_{ij} = \gamma_{ji}$

Similar to the single output case, the translog function provides a second order approximation to an arbitrary transformation function at a specified set of values of Y and X, particularly near unity.

4.1.2. Properties of the Translog Transformation Function

(1) Monotonicity condition.

The corresponding monotonicity conditions are, subject to the convention that $\frac{\partial F}{\partial X_i} < 0$, i=1, . . . 5, are

$$\alpha_i + \sum_{j=1}^{5} \gamma_{ij} \ln X_j + \sum_{j=1}^{3} \varepsilon_{ij} \ln Y_j \leqq 0, \; i=1, \ldots 5 \qquad (29)$$

$$\beta_i + \sum_{j=1}^{5} \varepsilon_{ji} \ln X_j + \sum_{j=1}^{3} \delta_{ij} \ln Y_j \leqq 0, \; i=1, \ldots 3 \tag{30}$$

In particular,

$$\begin{aligned} \alpha_i &\leqq 0 \;, \; i=1, \ldots 5 \\ \beta_i &\geqq 0 \;, \; i=1, \ldots 3 \end{aligned} \tag{31}$$

As in the single-output case these monotonicity conditions cannot be globally satisfied. Hence we have uneconomic regions for the transformation function.[22]

(2) Convexity condition

Convexity conditions require, with our sign convention, that $[F_{ij}]$ be positive semi-definite. This implies, in particular, at $X = Y = [1]$, that the following matrix be positive semi-definite,[23]

$$\left[\begin{array}{ccc|ccc}
(\alpha_1-1)\alpha_1+\gamma_{11} & \cdots & \alpha_1\alpha_5+\gamma_{15} & \alpha_1\beta_1+\varepsilon_{11} & \cdots & \alpha_1\beta_3+\varepsilon_{13} \\
\vdots & & \vdots & \vdots & & \vdots \\
\alpha_5\alpha_1+\gamma_{51} & \cdots & (\alpha_5-1)\alpha_5+\gamma_{55} & \alpha_5\beta_1+\varepsilon_{51} & \cdots & \alpha_5\beta_3+\varepsilon_{53} \\
\hline
\beta_1\alpha_1+\varepsilon_{11} & \cdots & \beta_1\alpha_5+\varepsilon_{15} & (\beta_1-1)\beta_1+\delta_{11} & \cdots & \beta_1\beta_3+\delta_{13} \\
\vdots & & \vdots & \vdots & & \vdots \\
\beta_3\alpha_1+\varepsilon_{31} & \cdots & \beta_3\alpha_5+\varepsilon_{35} & \beta_3\beta_1+\delta_{31} & \cdots & (\beta_3-1)\beta_3+\delta_{33}
\end{array}\right]$$

---

[22] Moreover, the possibility of an input becoming an output or vice versa is allowed by the translog transformation function, the switch occurring when the monotonicity condition is reversed for that particular commodity. This gives a great deal of flexibility in the analysis of inherently joint production processes.

[23] All these conditions may be tested for each observed value of Y and X and at X = Y = (1) in empirical analysis.

(3) Homogeneity condition

Imposition of the assumption of "almost homogeneity" of degree k of the transformation function leads to another set of restrictions, which in terms of the parameters of the translog function, implies

$$\sum_{i=1}^{5} \alpha_i \ln \lambda + k \sum_{i=1}^{3} \beta_i \ln \lambda + \tfrac{1}{2} \sum_{i=1}^{5} \sum_{j=1}^{5} \gamma_{ij} (\ln X_i \ln\lambda + \ln X_j \ln\lambda)$$
$$+ \sum_{i=1}^{5} \sum_{j=1}^{3} \varepsilon_{ij}(\ln X_i \ln\lambda + \ln Y_j \ln\lambda) + \tfrac{1}{2} k \sum_{i=1}^{3} \sum_{j=1}^{3} \delta_{ij}(\ln Y_i \ln\lambda + \ln Y_j \ln\lambda) = 0 \tag{32}$$

This must hold for all X_i's and Y_i's and λ. Hence we have the following homogeneity restrictions:

$$\begin{aligned} &\sum_{i=1}^{5} \alpha_i + k \sum_{i=1}^{3} \beta_i = 0 \\ &\sum_{j=1}^{5} \gamma_{ij} + k \sum_{j=1}^{3} \varepsilon_{ij} = 0, \; i=1, \ldots 5 \\ &\sum_{j=1}^{5} \varepsilon_{ij} + k \sum_{j=1}^{3} \delta_{ij} = 0, \; i=1, \ldots 3. \end{aligned} \tag{33}$$

Observe that in the case of k = 1, i.e. constant returns to scale, the restrictions become the usual restrictions that the row sums and column sums of the following matrix must be all zero:

$$\begin{bmatrix} \gamma_{ij} & \varepsilon_{i\ell} \\ \varepsilon_{kj} & \delta_{k\ell} \end{bmatrix}, \quad \begin{matrix} i,j = 1, \dots 5 \\ k,\ell = 1, \dots 3. \end{matrix}$$

(4) Separability condition

Another advantage of the translog transformation function is that no arbitrary assumptions are imposed between the set of outputs and the set of inputs. The few transformation functions in use in empirical research all assume separability between outputs and inputs.

Separability implies, for all i, j and k,

$$\frac{\partial}{\partial X_k}\left[\frac{\partial F/\partial Y_i}{\partial F/\partial Y_j}\right] = 0; \quad \frac{\partial}{\partial Y_k}\left[\frac{\partial F/\partial X_i}{\partial F/\partial X_j}\right] = 0 .$$

For the translog functional form this implies the following set of necessary and sufficient conditions for separability:[24]

$$\text{(i)} \quad \beta_j\varepsilon_{ki} - \beta_i\varepsilon_{kj} = 0, \; i,j = 1, \dots 3, \; k = 1, \dots 5 \tag{34}$$

$$\text{(ii)} \quad \varepsilon_{\ell j}\varepsilon_{ki} - \varepsilon_{\ell i}\varepsilon_{kj} = 0, \; i,j = 1, \dots 3, \; k,\ell = 1, \dots 5 \tag{35}$$

$$\text{(iii)} \quad \delta_{j\ell}\varepsilon_{ki} - \delta_{i\ell}\varepsilon_{kj} = 0, \; i,j,\ell = 1, \dots 3, \; k = 1, \dots 5 \tag{36}$$

$$\text{(iv)} \quad \alpha_j\varepsilon_{ik} - \alpha_i\,\varepsilon_{jk} = 0, \; i,j = 1, \dots 3, \; k = 1, \dots 5 \tag{37}$$

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[24] For the derivation of these 5 sets of necessary and sufficient conditions of separability between inputs and outputs, see Jorgenson, Christensen and Lau (1970).

and

$$(v)\ \gamma_{j\ell}\varepsilon_{ik} - \gamma_{i\ell}\varepsilon_{jk} = 0,\ i,j,\ell = 1, \ldots 5,\ k = 1, \ldots 3 \qquad (38)$$

An obvious sufficient condition, however, is $\varepsilon_{ij} = 0$ for all i and j. This multiplicative separability is a very strong condition because it implies that the transformation function may be written as $F(Y)\ G(X) = 1.$[25] (39)

4.1.3. Elasticities of Factor Substitution and Elasticities of Product Transformation

Similar to the single output case, the translog function can provide any arbitrary set of the direct and the Allen-Uzawa partial elasticities of substitution (i.e. DES and AES) among factors and these elasticities of transformation at a specified set of values of X and Y. The expressions on these elasticities in the specification of the multi-input, multi-output technology should be discussed with a warning that the transformation break down when any of the variables are zero or change sign in general.[26]

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[25] Note that multiplicative separability is a necessary condition for the CET-CES transformation functions of Powell and Gruen (1968). And the Mundlak's (1964) transcendental multi-product production function can be approximated by the translog function with the assumptions of (1) multiplicative separability, i.e. $\varepsilon_{ij} = 0$, and (2) zero off-diagonal elements of the matrixes of $[\gamma_{ij}]$ and $[\delta_{ij}]$, i.e. $\gamma_{ij} = 0$ for $i \neq j$, $i,j, = 1, \ldots 5$ and $\delta_{ij} = 0$ for $i \neq j$, $i, j = 1, \ldots 3$.

[26] This would offer no problems were it not for the fact that many of the magnitudes are neither always positive nor always negative, where these variables of negative sign can be converted into logarithms only after a reversal of sign. See Samuelson (1966, p. 129) on the details of this topic.

Since productive services or inputs have been regarded simply as negative outputs in the translog function, those elasticity expressions may, therefore, be defined both for the input side and the output side separately under the corresponding restrictions.[27]

4.1.4. Profit Maximization

Let P_i = the price of the i-th output,

W_j = the price of the j-th input,

M_i = the share of the ith-output in total sales,

M_j = the share of the jth-input in total cost.

Then the usual marginal conditions for profit maximization can be written as[28]

$$\frac{\partial \ln F}{\partial \ln Y_i} = \frac{\partial F}{\partial Y_i} \frac{Y_i}{F} = M_i, \; i=1, \; . \; . \; . \; 3 \tag{40}$$

and

$$\frac{\partial \ln F}{\partial \ln X_j} = \frac{\partial F}{\partial X_j} \frac{X_j}{F} = M_j, \; j=1, \; . \; . \; . \; 5 \tag{41}$$

---

[27]Note that multiplicative separability is a necessary condition for CET-CES transformation functions. The multi-input, multi-output translog function reduces to an approximation of the CET-CES transformation function, if there is multiplicative separability, i.e. $\varepsilon_{ij} = 0$.

[28]The derivation of the system of the share equations for inputs and outputs are shown in Appendix A-I.

Hence the following system of share equations can be obtained;

$$M_i = \beta_i + \sum_{j=1}^{5} \varepsilon_{ij} \ln X_j + \sum_{k=1}^{3} \delta_{ik} \ln Y_k, \; i=1, \ldots 3 \qquad (42)$$

and

$$-M_j = \alpha_j + \sum_{\ell=1}^{5} \gamma_{j\ell} \ln X_\ell + \sum_{i=1}^{3} \varepsilon_{ji} \ln Y_i, \; j=1, \ldots 5. \qquad (43)$$

Equations (42) and (43) is linear in parameters and in addition, there are equality restrictions from the homogeneity conditions across the individual equations corresponding to $\gamma_{j\ell}$, ε_{ij} and δ_{ik}.

Other important properties of the multi-input, multi-output translog function in connection with the technical change are omitted from this discussion, simply because they are beyond the scope of our study.

4.2. The Translog Cost Function

4.2.1. Introduction

Cost function have been estimated by economists for several decades. Only since the introduction of duality theory into economics, however, have economists seriously considered restrictions on the cost function implied by restrictions on production or transformation functions. Nerlove (1963) was certainly one of the first authors to include prices directly in the cost function for empirical work.[29]

The translog cost function for 3 outputs and 5 inputs can be written;

$$\ln C(Y,W) = \alpha_0 + \sum_{i=1}^{5} \alpha_i \ln W_i + \sum_{j=1}^{3} \beta_j \ln Y_j$$
$$+ \tfrac{1}{2} \sum_{i=1}^{5} \sum_{j=1}^{5} \gamma_{ij} \ln W_i \ln W_j + \sum_{i=1}^{5} \sum_{j=1}^{3} \varepsilon_{ij} \ln W_i \ln Y_j \quad (44)$$
$$+ \tfrac{1}{2} \sum_{i=1}^{3} \sum_{j=1}^{3} \delta_{ij} \ln Y_i \ln Y_j,$$

where $\delta_{ij} = \delta_{ji}$ and $\gamma_{ij} = \gamma_{ji}$.

4.2.2. Properties of the Translog Cost Function

(1) Monotonicity condition

C(Y, W) is a non-decreasing function in output and in prices. The corresponding monotonicity conditions are

$$\alpha_i + \sum_{j=1}^{5} \gamma_{ij} \ln W_j + \sum_{j=1}^{3} \varepsilon_{ij} \ln Y_j \geqq 0, \; i=1, \ldots 5 \quad (45)$$

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[29] Nerlove (1963, p. 172) states: "Note that the cost function must include factor prices if the correspondence is to be unique. The problem of changing (over time) or differing (in a cross section) factor prices is an old one in statistical cost analysis, ... It seems strange that no one has taken the obvious step in including factor prices directly in the cost function."

$$\beta_i + \sum_{j=1}^{5} \varepsilon_{ji} \ln W_j + \sum_{j=1}^{3} \delta_{ij} \ln Y_j \geqq 0, \; i=1, \ldots 3 \tag{46}$$

In particular,

$$\begin{aligned} &\alpha_i \geqq 0, \; i=1, \ldots 5 \\ &\beta_i \geqq 0, \; i=1, \ldots 3. \end{aligned} \tag{47}$$

(2) Concavity condition

Here also the concavity of the cost function in input prices requires that the Hessian be negative semi-definite. This condition implies that the matrix

$$\left| \frac{\partial^2 C(Y, W)}{\partial W_i \; \partial W_j} \right|$$

must be negative semi-definite within the range of input prices observed.[30]

(3) Homogeneity condition

Every cost functions must exhibit homogeneity of the degree of plus one in factor prices. The following (5+3+1) linear restrictions are necessary and sufficient for linear homogeneity:[31]

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[30]This condition also implies that the own partial elasticities of each factor's substitution (i.e. σ_{ii}'s) are negative.

[31]In an empirical work, one may want to test the validity of this set of restrictions as a test of the cost minimization hypothesis. Alternatively, one may want to estimate the cost function, imposing these restrictions *a priori*.

$$\sum_{j=1}^{5} \alpha_j = 1, \ \sum_{i=1}^{5} \gamma_{ij} = 0, \ j=1, \ldots 5,$$

and (48)

$$\sum_{i=1}^{5} \varepsilon_{ij} = 0, \ j=1, \ldots 3$$

(4) Separability

The separability[32] in the translog cost function requires that

$$\frac{\partial}{\partial \ln W_k}\left[\frac{\partial \ln C}{\partial \ln Y_i} \Bigg/ \frac{\partial \ln C}{\partial \ln Y_j}\right] = 0, \quad \begin{array}{l} i,j = 1, \ldots 3 \\ k = 1, \ldots 5 \end{array} \tag{49}$$

This differentiation yields:

$$\beta_i \, \varepsilon_{\ell k} - \beta_k \, \varepsilon_{\ell i} = 0, \quad i,k = 1, \ldots 3, \ \ell=1, \ldots 5 \tag{50}$$

$$\varepsilon_{\ell i} \, \delta_{kj} - \varepsilon_{\ell k} \, \delta_{ij} = 0, \ i,j,k,= 1, \ldots 3, \ \ell=1, \ldots 5 \tag{51}$$

All $\varepsilon_{ij} = 0$ is sufficient to satisfy these conditions and this strong separability makes the translog cost function groupwise additive in outputs and prices, entailing the existence of a homothetic index of input, i.e. and isoquant map which is independent of the levels or mix of the outputs.

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[32]We can write an arbitrary joint cost function with weak separability in outputs in the following form: $\ln C = G[\ln g(Y), \ln W]$. And under separability, $\theta_j = \varepsilon_{ij}/\alpha_i = (\partial^2 C/\partial \ln g \, \partial \ln W_j)/(\partial G/\partial \ln g)$ is the same for all i given any j, i=1, . . . 3, j=1, . . . 5. See Brown, Caves and Christensen (1975), pp. 9-11.

4.2.3. Elasticity of Substitution

Similar to the single output case, the S.E.S. can be defined for the multi-input, multi-output cost function of the producer, which specifies the minimum imputed cost C of producing the output vector Y with the accounting price vector W.

Also the A.E.S. can be computed directly from the multi-input, multi-output cost function and its derivatives, as shown in the single output case.

For the translog cost function this becomes

$$\sigma_{ij} = \frac{\gamma_{ij} + M_i M_j}{M_i \, M_j} \, , \; i \neq j$$
$$\sigma_{ii} = \frac{\gamma_{ii} + M_i(M_i - 1)}{M_i^{\,2}} \, , \qquad (52)$$

where the M_i's are fitted values of the cost share equations. The elasticities of demand with respect to price changes are closely related to the AES also:

$$\eta_{ij} = M_j \, \sigma_{ij} = \frac{\gamma_{ij} + M_i \, M_j}{M_i} \, , \; i \neq j$$
$$\eta_{ii} = M_i \, \sigma_{ii} = \frac{\gamma_{ii} + M_i(M_i - 1)}{M_i} \, . \qquad (53)$$

4.2.4. Cost Minimization

Using Shephard's Lemma we can write

$$\frac{\partial \ln C}{\partial \ln W_j} = \frac{\partial C}{\partial W_j}\frac{W_j}{C} = \frac{W_j X_j}{C} = M_j \, , \tag{54}$$

where M_j is the share of the j-th factor input in total cost. For the translog cost function this yields the following 5 equations:

$$M_j = \alpha_j + \sum_{k=1}^{5} \gamma_{jk} \ln W_k + \sum_{i=1}^{3} \varepsilon_{ji} \ln Y_i, \; j=1, \ldots 5. \tag{55}$$

While the above share equations of the input factors hold regardless of the degree of returns to scale, the output share equations hold under constant returns to scale as follows:

$$M_i = \beta_i + \sum_{j=1}^{5} \varepsilon_{ij} \ln W_j + \sum_{k=1}^{3} \delta_{ik} \ln Y_k, \; i=1, \ldots 3. \tag{56}$$

With linear homogeneity in factor prices imposed on the joint cost function there are 5+3 independent linear restrictions which are necessary and sufficient to impose constant returns to scale:

$$\sum_{i=1}^{3} \beta_i = 1, \quad \sum_{j=1}^{3} \varepsilon_{ij} = 0, \; i=1, \ldots 5$$

and

$$\sum_{i=1}^{3} \delta_{ij} = 0, \quad j=1, \ldots 3. \tag{57}$$

4.3. Advantages and Disadvantages of the Translog Approach

In the preceeding sections, the many features of the Transcendental Logarithmic Function have been described. Basically the advantages of using a translog function are:

(i) The translog function provides a second order Taylor's series approximation to any arbitrary function. Hence it is also a powerful vehicle for the testing of specific functional form restrictions such as C.E.S. and C.D. as well as of less well known varieties. Therefore there are no a priori guesses or assumptions on the functional form to be estimated in an empirical investigation.

(ii) Many economically meaningful hypothesis also appear as linear restrictions on the parameters and thus may be readily tested.

(iii) Furthermore, the direct calculations of various elasticities of substitution (such as DES and AES) are possible and in general, neither DES nor AES is constant for all quantities of inputs and of outputs (and hence indirectly for all prices of inputs and outputs), exhibiting the property of variable DES and AES, such as VET-VES production functions.

(iv) Empirically, both the production functions and the marginal conditions are linear in parameters and hence may be easily estimated by standard linear regression methods.

(v) It allows the existence of uneconomic regions in the range of the production function and especially increasing returns to scale in certain ranges of inputs through the condition of local convexity.

(vi) As explained in detail already, the generalization to the multiple-output case is straight forward. Here also it requires only local monotonicity through the existence of the uneconomic regions for the transformation function and the possibility of an input becoming an output or vice versa. Both are allowed by the translog transformation function, the switch occurring when the monotonicity condition is reversed for the particular commodity.

Along with the various advantages of using a translog function as described above, the following defects[33] are noted particularly in the body of the present empirical study:

(i) With the direct estimation of the translog function symmetry has to be part of the maintained hypothesis. Further the problem of multicollinearity can be acute because of the inclusion of linear and quadratic terms in the logarithms of the variables in the regression equation. The share equations, on the other hand, conceal the assumption of optimizing behavior and the endogeneity of variables usually taken as exogeneous in

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[33]These defects have been deliberately reconsidered in the present study and in particular the second and the third problems are discussed, in detail in Chapter III, Section 5, Part B.

the theory of the firm. Further the constant returns to scale,[34] i.e. homogeneity of degree 1, has to be part of the maintained hypothesis, even if the multicollinearity problem can be avoided.

(ii) The translog approach needs to be handled with care because the parameter estimates are biased from the "truncation error" of the translog approximation, which is based on a truncated Taylor series expansion with an excluded and unknown remainder term.

(iii) The use of the translog function depends on the quantities of output(s) and input(s) being strictly positive: otherwise the expression is not well defined. This problem becomes a more serious drawback in adopting the translog approach for the multiple-product production technology.[35]

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[34]The use of share equations does not necessarily exclude the case of non-constant returns to scale in their derivation. But the unknown parameter for the degree of homogeneity becomes unidentifiable in the empirical estimation.

[35]Because this problem occurs when the Taylor expansion is expanded on the logarithmic function concerned, i.e. $\ln V = f(\ln X_i)$, alternative expansion may be suggested on the original function, i.e. $V = g(X_i)$, with the natural number, not their logarithms.

PART B

CHAPTER I. INTRODUCTION

This study grew out of a suggestion that information in <u>the Census of Korea Mining and Manufacturing, 1973</u>[1] appears to provide a potentially valuable data base for an empirical study of a multi-product production technology at the micro-establishment level. Most previous econometric studies on the production structure of manufacturing have relied on more or less aggregated data, particularly due to the specification of a uniproduct technology. Only a few such studies have based their works on the micro-unit of observation, such as a plant or establishment, for which it might be natural to assume that it is generated by or represents a point on a production function. The Korean Census seems a likely source of data for the first study, which will be based on individual manufacturing establishments with a multi-product production technology.

The present study is very much conditioned by the availability of a particular body of data: the 1973 census of mining and manufacturing establishments in Korea. These data have several important advantages, not the least of which is their accessibility

[1]See also, Economic Planning Board, Republic of Korea (1973), <u>The Census of Korea Mining and Manufacturing, 1973</u>.

for research purposes.[2] In addition, their comprehensiveness and the potentially large number of observations may allow the testing of a much more detailed hypothesis about the structure of production activities than was hereto possible. On the other hand, these data also have serious limitations. Some of the data used in the study have turned out not to be as good as anticipated.

But more importantly these data are limited to only one year, 1973,[3] and only to that information about which questions were asked in the census. One of the main shortcomings is the lack of information on the financial structure. In particular the unavailability of time-series observations makes the construction and estimation of a complete production, input demand and output supply system impossible and forced us to rely largely on the rather simple estimation methods which are discussed in the following chapter.

In general, both the richness of the data base and the fact that we could neither go beyond its limitations nor overcome its shortcomings greatly circumscribed the range of alternatives open to us. Therefore, much of what follows has been conditioned

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[2]The difficulties with disclosure rules and manpower constraints regarding the handling of this kind of data in most countries, even when sufficient data are available, have made very little econometric work feasible.

[3]Incidentally, the same data base for the year, 1968, is also available and was examined in earlier stages. But constraints on available manpower and the short research period prohibited us from expanding the empirical estimation to the year, 1968. Consequently, it is left as a major future activity.

by the characteristics of the data base, as well as by some restrictions in the theoretical and statistical approach adopted in this empirical study.

CHAPTER II. DATA, MEASUREMENT PROBLEMS AND SAMPLE PROPERTIES

B.2.1. General Descriptions on the "Census of Korea Mining and Manufacturing, 1973"

All of the observations used in this study are drawn from the "Census of Korea Mining and Manufacturing, 1973," which covered all the mining and manufacturing establishments in Korea that:[1]

(i) were operating with five or more persons engaged as of December 31, 1973;

(ii) were operating with five or more persons engaged in average per work-day during December 1973;

(iii) had operated for more than three months during the year, 1973, with an average of five or more persons engaged, even if they were out of operation as of December 1973.

The establishment was the unit of enumeration in the census. The term "establishment" is defined as a physical unit engaging in

[1]Taking account of the peculiarities of saltern operations, those operated for less than three months during the year with an average of five or more persons engaged were also covered. Establishments, however, were excluded which were: (i) under construction as of December 31, 1973; (ii) operated directly by the armed forces; (iii) workshops operated by public occupation guidance centers; (iv) experimental equipment or laboratories attached to public organizations and schools.

industrial activities such as a factory, workshop, office or mine. This, for the most part, may be similar to an enterprise, but it differs from the latter in that firms doing business in more than one area or conducting more than one enterprise in the same area are shown as two or more separate establishments.

The industrial classification of the establishments enumerated in the census was done in accordance with the industry definitions embodied in the revised Korean Standard Industrial Classification (KSIC),[2] which is very similar to the International Standard Industrial Classification (ISIC) of all economic activities.

The Census counted 24,881 mining and manufacturing establishments operating during all or part of 1973, a decrease of 780 from the total shown in the 1968 census, while the number employed increases from 825,810 persons in 1968 to 1,227,566 in 1973.

B.2.2. Variables in the Record

From about 18 major areas of concern in the census questionnaires[3] for each establishment we shall list and describe only what was used in one way or another in our study or in the

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[2] See Economic Planning Board, Republic of Korea (1973), Series II, about the KSIC Mining, Manufacturing, Electricity, Gas and Water, a third revision on 13th of March 1970, pp. 310-334.

[3] Ibid.

experiments associated with it. These are shown below with our designation for each.[4]

2.1. Number of Workers (L)

This is comprised of the average number of employees during the period of operation and the number of working proprietors and unpaid family workers:

L_o: Operatives--the workers on production line directly or auxiliary to it engaging in essentially manual work, including home production workers.

L_a: Administratives and other workers--all workers, other than operatives, who are engaged in technical, managerial, professional, clerical and routine office workers and their helpers. Salaried managers and directors of corporations are also included here.

L_f: Working proprietors and unpaid family workers--proprietors, partners of the incorporated firms, and family members who work 24 hours or more per week, without any regular remuneration.

2.2. Number of Days Operated (WD)

This represents the number of actual days operated except all closing days of the establishments by month during the reporting

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[4]The maximum total record length for each establishment in the working data base is around 6,250 characters for the 1973 census. This final working tape was put together from three separate, original census files into one consistent file with three 2,400 feet computer tape reels in a uniformly sorted format.

year 1973. The days closed include those when establishments were not operated due to electricity failure, or repair and maintenance of machinery.

2.3. Employees' Remunerations (W_o; W_a)

The gross earnings paid to all employees on the payroll of the establishment covered during the year. It includes all types of compensation such as salaries, wages, bonuses, allowances and subsidies irrespective of payment in cash or in kind. It however, excluded payments to the retired, long-term absentees, members of armed forces, and payments accrued prior to the survey year but not actually paid in the year, while the payments accrued during the year but not paid are included here.

Cash payments are gross payments and include taxes, compulsory savings and union dues, etc. Value of compensation in kind is made by applying the F.O.B. plant prices if the establishment is supplied with its own products and by purchase prices if supplied by products other than their own.

2.4. Power Equipment (HP)

This includes fixed tangible assets such as buildings, structures, machinery, equipment, vehicles, ships and other transport equipment with a lifetime of one year or more, but land is excluded. Also excluded are intangible assets such as goodwill, patent rights, mining rights, etc. Here two different evaluations were done, the first is based on the information recorded in the original census questionnaires such that total value represents the total "book value" as of the end of the year. However, the values are cautiously estimated on the basis of market prices if their book value is not available. The second estimate of the fixed tangible assets of each establishment consists of the replacement values, the evaluation of which was done by the Korea Development Institute (KDI).[5] The replacement value of the capital stock by type in an establishment is estimated from the National Wealth Surveys in 1968 and 1973 as follows:

Step 1. The purchasing price in 1973 prices, was evaluated by using different price indexes for the purhcase year for the following seven types of capital goods, i.e., (i) Buildings, (ii) Indoor equipment such as elevators, air-conditioners, heaters, and ventilators, etc., (iii) Structures, (iv) Machinery and equipment, (v) Tools and utensils worth 10,000 won or with a lifetime of one year or more, (vi) Vehicles and other transport equipment such as motorcars, coaches, etc. and (vii) Ships and other equipment such as cargo-boats, sampans, etc.

Step 2. In order to get the net capital stock for each establishment, the above gross capital stock, evaluated by the purchasing prices in 1973, was devaluated by an appropriate constant depreciation rate for the 211 different 5 digit industries and for the 7 different types of capitals, as shown in the following diagram:

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[1]See Choo and Yoo (1978).

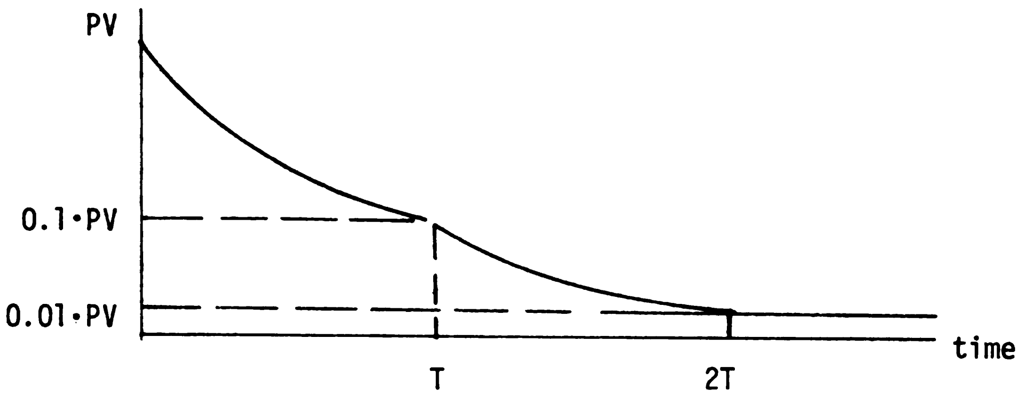

Figure I.--Depreciation Rate of Capital Stock

Note: PV = the present values in 1973 prices of the capital goods owned by an establishment, by industry and by type of capital.

T = the gestation period for capital as determined by the tax law.

In summary:

$$GK^{1973}_{ijk} = \sum_{t=1951}^{1973} A^{t}_{ijk} P^{t}_{i}$$

$$NK^{1973}_{ijk} = GK^{1973}_{ijk} \, (1.0 - d_{ik})$$

where i is the type of capital good,

j is an establishment in 1973,

k is the 211 5-digit industries,

and GK = gross capital,

NK = net capital,

A = fixed tangible assets,

P = price index,

d = depreciation rate.

2.6. Production Costs

This term refers to the direct charges actually paid to or payable for materials and services consumed or put into production during 1973 including freight charges and other direct charges incurred by the establishment in acquiring them. Costs of raw materials, fuel, electricity and water, contract work, repairs and maintenances were obtained under the category or production costs.

Detailed entries for raw materials consumed by each establishment are recorded in separate classifications for quantities and values of raw materials and parts consumed, and for inventories at the beginning of and the end of the year respectively according to the 7-digit item code of the KSIC.

The cost of fuel is the total amount actually paid or payable during the year for all fuel consumed for heat, power or generation of electricity. Here the first type of fuel refers to coal consumption, the second to oil and the third to other miscellaneous fuel where both information on quantity and on value are available. The cost of electricity purchased is the total actually paid or payable for electricity purchased by the establishment for the production of goods during the year. Charges for electric lights in offices are also included but only for the purchased part.

2.7. Values of Shipments and Inventories

Under the general term "value of shipments," the proceeds from shipments of products and wastes, and receipts for processing and repair work during the year are classified separately. They are evaluated in principle at the F.O.B. plant prices and include excise taxes.

The value of inventories is the value of the goods in the possession of the establishments during the year. Classified separately as (i) finished, and (ii) the semi-finished goods and work-in-progress, the inventory is evaluated on the basis of book value or otherwise at approximate current market prices at the beginning and the end of the year.

2.8. Output Produced

The gross output is the total value of all goods produced and services rendered to others by the establishment during the year. In practice the quantity (or value) of output was calculated as the quantity (or value) of shipments plus the net addition to inventories of finished, semi-finished goods and work-in-progress.

Detailed entries for the commodities produced by each establishment are reported in the separate tables on quantities and values of products shipped, and of inventories at the beginning and the end of the year, respectively, by the 7-digit item code of the KSIC.

B.2.3. Derived Variables

The variables discussed below are those on which the results in this empirical study is based. The particular definitions were chosen only after some preliminary experimentation during which we tried different measures of such central variables as the output of each product, the labor force of operatives and administratives, the capital stock, raw materials, and fuels, and after we also investigated the effects of some mixed variables. The more important results of these preliminary runs and also of some experiments appear later in the empirical findings.

3.1. Multiproduct Output

In the body of commodity classifications there are two types of commodities, the classified and the unclassified, in the sense that the former is a type of commodity with a measurable physical unit and the latter is a type without one. Also are faced with the problem of the semi-finished goods and work-in-progress in an establishment during the year in general. In our empirical study we tried to classify the values of the unclassified commodities and the semi-finished goods and work-in-progress both in quantity and value terms. By assuming that the components of the semi-finished goods and work-in-progress in gross output are proportionately distributed according to the structure of final products, we adjusted our first measures of each commodities as follows:

Step 1: allocation of the semi-finished goods and work-in-progress such that:

$$PQ_i^* = PQ_i + \Delta SFW_i, \; i=1, \; . \; . \; . \; N,$$

and

$$\Delta SFW_i = SFW \times PQ_i / \sum_{j=1}^{N} PQ_j,$$

where PQ_i^* = the value of the i-th products adjusted for the semi-finished goods and work-in-progress,

PQ_i = the value of the i-th products produced.

SFW = the value of the semi-finished goods and work-in-progress.

N = the total number of multiproducts.

Step 2: adjustment of the quantity of the classified products, such that:

$$Q_i^* = Q_i + \Delta SFW_i / P_i^* \text{ and } P_i^* = PQ_i / P_i,$$

where P_i^* = the unit price of the i-th classified commodities,

Q_i = the quantity of the i-th product produced.

Q_i^* = the quantity of the i-th product adjusted.

Next we adopt the assumption that the components of the unclassified commodities are distributed, in value terms, according to the structure of the classified products.[6] The reason we have chosen this particular concept of commodity production is that we needed a production measure for each product of a multiproduct establishment that came as close as possible to the value of the work done by "internal" factors at the establishment and, at the same time, conformed with the usual definition of production.

As will be seen later, the census shows that as many as 19 different kinds of products are produced by an establishment and more than 60 different products are produced by establishments in certain industries. In our study, therefore, we restricted for simplicity the number of products analyzed by classifying them into several major and nonmajor group commodities. Major

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[6]The procedures for the adjustments are just the same as before; i.e.,

Step 1: $PQ_i^{**} = PQ_i^{*} + \Delta VUC_i$

Step 2:

$$\Delta VUC_i = \sum_{j=1}^{NUC} PQ_j^{*} + PQ_i^{*} / \sum_{j=1}^{NC} PQ_j^{*}$$

$$Q_i^{**} = Q_i^{*} + \Delta VUC_i / P_i^{*}$$

$$P_i^{**} = PQ_i^{*} / Q_i^{*}$$

where PQ_i^{**} = the values of the i-th commodity adjusted,
Q_i^{**} = the quantity of the i-th commodity adjusted,
P_i^{**} = the unit price of the i-th commodity adjusted,
NUC = the total number of the unclassified commodities for each establishment,
NC = the total number of the classified commodities for each establishment.

commodities are identified as those important commodities which most establishments in a specified industry produce and the weight of which is significantly high in terms of the value of the industry's production. For the nonmajor commodities a quantity index is formulated, according to the usual aggregation method of quantity indexing weighted by its values, such that:

Step 1: evaluation of the averages by the nommajor commodities,[7]

$$Q_{oi} = \sum_{j=1}^{S} Q_{ij}/s_i,$$

$$(P_oQ_o)_i = \sum_{j=1}^{S} (P\ Q)_{ij}/s_i,\ i=1,\ .\ .\ .\ NMC$$

$$P_{oi} = (P_oQ_o)_i/Q_{oi},$$

Step 2: formulation of a quantity index,

$$Q_j^* = \sum_{i=1}^{NMC} P_{oi}\ Q_{ij} \Big/ \sum_{i=1}^{NMC} (P_oQ_o)_i,\ j=1,\ .\ .\ .\ S,$$

where S = the total number of establishments in an industry,

NMC = the total number of nonmajor classified commodities in an industry,

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[7]Since no base exists it seems appropriate to circulate the mean values for P_i and P_iQ_i (i=1, . . . N) and use these to serve as a base in the aggregation. Also it helps us in getting a better approximation of the TRANSLOG function, because the distribution of the values of this quantity index is located around 1.0, resulting $\ln Q_j^* \approx 0.0$.

s_i = the number of establishments producing the i-th nonmajor commodity,

$(PQ)_{ij}$ = the value of the i-th commodity produced by the j-th establishment,

Q_{ij} = the quantities of the i-th commodity produced by the j-th establishment,

Q_{oi} = the average quantity of the i-th commodity,

P_{oi} = the average unit price of the i-th commodity,

$(P_oQ_o)_i$ = the average unit value of the i-th commodity produced by an establishment,

Q_j^* = the quantity index of the j-th establishment for nonmajor commodities.

3.2. Labor Inputs

The simplest measure of labor input is the total number of persons engaged in the establishment's activities, i.e., the sum of the numbers of operatives, administratives, working proprietors, and unpaid family workers. The reliability of this unweighted sum of the number of workers in all three categories assumes that the three types of labor concerned are equally productive. But the census data indicate a significant difference in respective wage rates, suggesting substantial variations in their marginal productivities. In the main body of the study the dichotomy in labor homogeneity between operatives and the administratives is utilized, and the working proprietors and unpaid family workers are adjusted into the second category of labor according to the average salaries of the administratives.[8]

An alternative measure of labor inputs for these two distinct types was also constructed from available data on the number of working days during the year, as an approximate labor services (flow) variable, under the assumption that the average working hours per day are similar over all establishments within the same industry.

3.3. Capital Input

Our first measure of capital input is the unweighted sum of its components, i.e., the sum of values of buildings, structures, machinery, equipment, vehicles, ships and other transport equipment, but it excludes land. All of these are evaluated at their replacement values in 1973 market prices.[9]

An alternative measure of capital input is of power equipment utilized, i.e., the sum of the horsepower capacities of the electric motors, prime movers, and generators which an establishment owns is multiplied by the number of working days.

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[8]It can be stated that the more modern management system an establishment has, the fewer the third type of laborers are. Also only in a very few small establishments, one or two unpaid family workers or working proprietors act as administratives. In this case their remunerations are approximated by the average wages of the operatives in that establishment.

[9]See Choo and Yoo (1978), Vol. I.

3.4. Fuel Input

The value of fuel consumption is the sum of the costs of the oil and coal consumed and of electricity purchases.[10] The quantity measure for this is calculated by the following conversion formulae,[11] i.e.,

$$\underset{\text{(kilocalorie)}}{\text{fuel consumption}} = \text{Coal consumption (M/T)} * 5{,}100{,}000 \text{ (Kcal/MT)}$$
$$+ \text{ oil consumption } (\ell) * 9{,}900 \text{ (Kcal/}\ell)$$
$$+ \text{ electricity purchased (1000KWH)} * 860{,}000 \text{ (Kcal/1000KWH)},$$

where kilocalories are used as a common unit of the heating value (specific calorific values) of each type of energy input.

3.5. Raw Material Input

In the production studies using "value added" (V) as a measure of output, material inputs (M) are treated rather asymptotically. They are subtracted from gross output, (X), i.e.,

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[10] In total fuel consumption, miscellaneous fuel consumption is also included without any quantity measures. We incorporated this amount into the values of our total fuel consumption by adjusting the quantity index measured in kilocalories.

[11] For coal and oil, the respective heating values in units of kilocalories depends crucially on the quality of the fuel itself. The quality of the energy produced in Korea ranges from 3,000 to 6,000 kilocalories per kilogram of coal in general and the official average figures used among Korean energy experts is 5,100 kilocalories. For oil also, kilocalories per liter of oil range from 9,500 to 10,000; we chose the figure of 9,900 which is officially used in Korea. For secondary energy such as electricity, the technical conversion is 860 kilocalories per kilowatt hour.

$V = X - M$ and hence not included explicitly in the list of inputs. This procedure has been justified in the past since (i) it facilitates the comparison of results for different industries with different material-use intensities, and it improves the comparability of data for individual establishments, even within the same industry, as long as they differ in their thickness (the amount of vertical integration). (ii) It facilitates the aggregation of output measures across industries through the reduction of "double counting." When output is measured by value added only, the materials that are embedded in a particular product are not counted each time the product crosses industry lines on its way toward final consumption. (iii) It reduces the problems of estimation and interpretation by the elimination of a variable (M) from both sides of the production relation. (iv) "Materials" are an asymetric input. Often their use is very closely associated with the level of gross output and hence their inclusion as an "independent" variable in a regression analysis could obscure the relationships of interest. (v) Any short run fluctuation in demand may be met without much change in the work forces or machinery in place, but will usually induce a similar fluctuation in the use of raw materials (or energy inputs). In this sense, raw materials are more endogenous, than labor and capital and their use as an independent variable is more likely to lead to simultaneous equation biases if standard least squares estimation procedures are followed. (vi) Finally, the value-added procedure, if possible at all, is also robust in the sense that it is consistent with two polar assumptions about the role of materials in the production function: (a) the elasticity of substitution (σ_{MV}) between the value added and raw materials in $X = g[F(L,K),M] = g[V, M]$ is infinite, allowing one to rewrite it as $X = F(L, K) + M$ or $V = X - M = F(L, K)$; and (b) the elasticity of substitution (σ_{MV}) between them is zero, materials being used in fixed proportion to output: $M = \alpha X$. This model can then be written as $X = F(L, K)$, $M = \alpha X$ which implies that $X - M = V = X(1 - \alpha) = (1 - \alpha)^{-1} F(L, K)$ and that the value added procedure is again appropriate as long as α is either a constant or is uncorrelated with the levels of labor and capital.

The first two justifications for the "value-added" procedure may still be possible even in the "gross production" procedure through alternative specifications of the production function. First, the comparability of the results for different industries may disappear when we form a unified (quantity) index of raw materials, defined as a certain degree of homogeneity comparable over many different industries. Secondly, the double counting problem may only exist in the "vertical" aggregation over different commodities, not in the "horizontal" aggregation at the cross-section level of establishments within the industry concerned. Further different commodities and/or industries can also be aggregated, if necessary in the "gross production" procedure, by applying alternative specification schemes, one of which, for example, could be the multi-product multi-stage production function suggested by Mundlak and Razin (1971). The third point is the desirability of convenience and simplicity in any research. The fourth is the type of empirical question to be studied and specially the degree to which the asymmetric property of raw material inputs vary from industry to industry. Lastly, degree of endogeneity as an independent variable in the time-series model may differ from that in the cross-section model.

In their recent empirical studies, Griliches and Ringstadt (1971) concluded that most of the evidence they examined vindicates the use of the value-added measure of output. They found that variations in material use do account for a large fraction of variations in gross production, but this variation is largely unrelated to the levels of use of the other inputs.

However, as already noted, the production technology of multiproducts, differs from that of uniproducts and may not be separately measured for the respective value added of each product. This is because the subtraction of various production costs from each product value, requires some knowledge of the multiproduct production technology to be studied in order to allocate the cost shares of each product separately.

For the formation of one unified quantity index for raw material inputs as a whole. The following quantity-indexing procedure was followed:

Step 1: assign the unclassified inputs to the classified components proportionally where all classified inputs consist of quantities and values.

Step 2: evaluation of the averages.[12]

$$Q_{oi} = \sum_{j=1}^{S} Q_{ij}/s_i,$$

$$(P_oQ_o)_i = \sum_{j=1}^{S} (PQ)_{ij}/s_i, \; i=1, \; . \; . \; . \; MC,$$

$$P_{oi} = (P_oQ_o)_i \; /Q_{oi},$$

Step 3: formulation of a quantity index.

$$Q_j^* = \sum_{j=1}^{MC} P_{oi} \, Q_{ij} \quad \sum_{i=1}^{MC} (P_o \, Q_o)_i, \; j=1, \; . \; . \; . \; S,$$

where S = the total number of establishments

s_i = the total number of establishments, using the i-th classified raw material.

MC = the total number of classified raw material inputs.

$(PQ)_{ij}$ = the values of the i-th raw material used by the j-th establishment.

Q_{ij} = the quantities of the i-th raw material used by the j-th establishment.

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[12]See footnote 7.

P_{oi} = the average unit price of the i-th raw material input.

Q_{oi} = the average quantity of the i-th raw material input.

$(P_oQ_o)_i$ = the average values of the i-th raw material input used by an establishment.

Q_j^* = the quantity index for the raw material inputs of the j-th establishment.

3.6. The Share Variables

Since the current production study is based on the estimation of the system of share equations, we also derived the value shares of all variables either with respect to the total input values or to the total output values.

The value shares of each product are defined as the values of the i-th product produced, divided by the sum of all products produced by an establishment.

By counting all the cost elements of input variables, we assume accounting identity between the values of outputs and that of inputs.[13] Hence the returns on capital services are calculated as a residual, i.e., the total input values minus the costs of

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[13]It is worthwhile to note here that there were three cost components missing from the production costs, i.e., costs of water purchased, contract and commission work, and repair and maintenance work. Adopting several assumptions, we added the costs of water purchased to fuel costs, without any corresponding quantity adjustment. Also the cost of contract work on materials by others was simply added into the costs of raw materials. Finally, costs of repair and maintenance services for the normal performance of tangible fixed assets for production activities are assumed to be included in the cost shares of capital services. More details will appear in later section on the "quality of data."

labor, of fuel consumptions, and of the raw materials used in the production process.

3.7. The Prices of Input Factors

As an alternative specification for the multiproduct production technology, the share equations system of the TRANSLOG joint cost function is also estimated where the prices of all input factors are used as explanatory variables. The factor prices are identified as the returns on each factor divided by its respective quantity variable. For example, the wage rate is calculated by dividing the total remuneration by the number of workers by type.

B.2.4. Selection and Exclusion Rules in the Sample Establishments

When we first tried to analyze the census data we ran into difficulties with missing or obviously erroneous data. In such a situation, one can either try to estimate the missing observations or exclude the units with incomplete data, and we decided to do the latter. We found also that very small establishments gave us difficulties. The frequency of erroneous or missing data was relatively moderate but the activities of very small establishments (such as those with less than 5 workers) did not seem to fit our idea of "manufacturing." In the experimental runs we attempted some alternative selection procedures and finally settled upon the following exclusionary rules in the study.

All units with one or more of the following characteristics were excluded in addition to the limitations in the coverage of the census itself.[15]

(i) total number of administratives, working proprietors and unpaid family workers less than or equal to zero.

(ii) no quantity data available for any of the major products.

(iii) the derived returns on capital services less than or equal to zero.

(iv) no value and quantity data available for any of input vairables.

The first criteria is designed to check the possibility of missing information on a critical variable-labor input-in our study. The second excludes establishments that produce only the unclassified commodities. The third characterizes the production activity of an establishment as economic and the last limitation was added arbitrarily to gain more meaningful knowledge of the production technology.

The adoption of the exclusionary rules results in a moderate reduction in the number of sample establishments, as shown in Table (I-1). For example, in the canning industry, 35 establishments (about 30%) out of the total of 118 establishments have some deficiencies in their data records such as no production quantity data available, etc. (see the first column of the row with the footnote c). Furthermore, when the capital input data is considered

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[15]This is discussed already in the section (B-2-1).

TABLE I-1.--Number of the Excluded Establishments (Percentages in Parentheses).

Exclusionary Rules	Industry[a]					
	Canning Industry	Leather Footwear Industry	Screw Products Industry	Manufacture of Knitted Underwear	Manufacture of Briquettes	Molding Industry
Total Number of Sample Establishments[b]	118 (100.00)	273 (100.00)	128 (100.00)	152 (100.00)	280 (100.00)	218 (100.00)
No production Quantity Data	21 (17.80)	26 (9.52)	0 (0.00)	0 (0.00)	1 (0.36)	12 (5.50)
Negative or Zero Number of the Administratives	0 (0.00)	3 (1.10)	0 (0.00)	0 (0.00)	1 (0.36)	0 (0.00)
Negative or Zero Quantity of Fuel Consumption	2 (1.69)	14 (5.13)	1 (0.78)	1 (0.66)	3 (1.07)	1 (0.46)
Negative or Zero Quantity of Raw Materials Used	4 (3.39)	16 (5.86)	4 (3.13)	2 (1.32)	0 (0.00)	6 (2.75)
Negative or Zero Returns on Capital Services	17 (14.41)	25 (9.16)	11 (8.59)	8 (5.26)	15 (5.36)	19 (8.72)
Subtotal, Number Excluded[c]	35 (29.66)	65 (23.81)	11 (8.59)	9 (5.92)	15 (5.36)	34 (15.60)
Subtotal Number Included[d]	83 (70.34)	208 (76.19)	117 (91.14)	143 (94.08)	265 (98.64)	184 (84.40)
Negative or Zero Horsepower Capacity[e]	33 (27.97)	247 (90.48)	14 (10.94)	26 (17.11)	30 (10.71)	22 (10.09)
Subtotal Number Included[f]	66 (55.93)	20 (7.33)	94 (73.44)	114 (75.00)	233 (83.21)	131 (60.09)
Negative or Zero Value of the Net Capital Stock[e]	58 (49.15)	255 (93.41)	27 (21.09)	140 (92.11)	247 (88.21)	150 (68.81)
Subtotal Number Included[g]	45 (38.14)	13 (4.76)	15 (11.72)	10 (6.58)	32 (11.43)	55 (25.23)

[a]Industry classification is discussed in detail in the next section.

[b]Total number of establishments in each industry is directly counted from the original data tape of the 1973 Korean census, held by the Bureau of Statistics, Economic Planning Board, Korea.

[c]Subtotal excluded by industry is the sum of establishments excluded by the first 5 exclusionary rules. Here more than one exclusionary rule may be applied to one establishment. Hence, this subtotal is not necessarily equal to the sum of each number corresponding to each exclusion rule.

[d]Subtotal included is the total number of sample establishments (see the above footnote a) subtracted by the subtotal excluded (the footnote c).

[e]Numbers of establishments in these rows include establishments which may also be excluded by other exclusionary rules listed above, such as no production quantity data available, etc.

[f]Subtotal included here is the total number of sample establishments (footnote a) and further subtracted by those excluded establishments which do not have the data on horsepower capacity.

[g]Subtotal included here is the total number of sample establishment (footnote a) subtracted by the subtotal excluded (footnote c) and further subtracted by those excluded which do not have the data on the net capital stock.

by horsepower capacity, almost 44% of total sample (i.e. 52 establishments) has to be excluded in our picture.

In spite of these drastic reduction in the number of sample establishments from the exclusion rules, some moderate sample size by each industry still could be kept for the purpose of the study on production technology, such as 66 establishments (56%) out of 118 samples in the canning industry, 94 establishments (73%) out of 128 in the screw products industry, 114 (75%) out of 152 in the knitted underwear, 233 (83%) out of 280 in the briquettes, and 131 (60%) of 218 in the moulding industry. Most drastic changes in sample size happen from the capital input data unavailable in most establishments of the leather footwear industry (see the third column in the row with the footnote f in Table I-1). Even though we strictly follow these exclusion rules in sampling, further investigation about the implication on the existence of establishments with capital input data unavailable, is implemented separately, followed by the basic empirical findings in the next chapter.

In summary, there seem to exist still some moderate sample establishments to be studied on production technology, even with rather rigid exclusion rules adopted in the sampling, as in most empirical investigations based on the cross-section data base.

B.2.5. Industry Classifications and the Selection of Specific Industries to be Studied

Since our main interests are the identification and understanding of the production structure in the micro-reality of a multiproduct manufacturing unit, the selection criterion for industries to be studied is quite arbitrary. We did not restrict ourselves to any a priori conceptual limitations on it, even if we deliberately adopted several practical frameworks. The first criterion is the sample size, i.e., the number of establishments, the second the size distribution of establishments, the third the number and density of the various products produced by establishments which are classified into a specific 5-digit industry.[14]

The specific industries chosen in this study are shown in Table II-1. In the study of multiproduct production structure, they are the canning industry (canning and preserving of fruits, vegetables, fish, and shellfish), the leather footwear industry (manufacturing of footwear except vulcanized or molded rubber or plastic footwear), and the screw products industry (manufacturing of screw machine products such as bolts and nuts, rivets, screws, nails, clamps, irons, and zippers). Also manufacture of knitted underwear, manufacture of briquettes, and molding industry (molding and casting of iron and steel) are selected in the study of uniproduct production structure.

Total number of commodities produced by establishments within the 5-digit industry classified varies from industry to industry

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[14]The KSIC includes 12 of the 2-digit, 32 of the 3-digit, 93 of the 4-digit and 27 of the 5-digit industry classifications, covering more than 2,000 7-digit commodity codes. For manufacturing it includes 9 of the 2-digit, 29 of the 3-digit, 83 of the 4-digit, and 272 of the 5-digit industries, covering about 1970 7-digit commodities.

TABLE II-1.--Industries Selected.

Industry[a]	No. of Establishments[b]	No. of Commodities Produced[c]	No. of Raw Materials Used[c]
Canning Industry	83	20	134
Leather Footwear Industry	208	16	69
Screw Products Industry	117	33	65
Manufacture of Knitted Underwear	143	10	45
Manufacture of Briquettes	265	5	16
Molding Industry	184	50	122

[a]In the KSIC industry code, canning industry covers more than one 5-digit industry (i.e., 31131, 31132, and 31141), mainly because an establishment produces several commodities which belong to several different 5-digit industries. This is due to the fact that an establishment, for example, producing two different commodities, is classified under the 5-digit industry which their first major commodity belongs to. Therefore, after some deliberate review of the structure of the data base, the grouping of several 5-digit industries was necessary here.

All the other industries are, in this sense, well classified in the census, such as, the leather footwear industry has the 5-digit industry code of 32400, the screw products industry of 38196, manufacture of knitted underwear of 32133, manufacture of briquettes of 35401, and the molding industry of 37102, respectively.

[b]The number of establishments here is from the sample size, excluded by the exclusionary rules of no production data available, no administrative workers, no fuel consumptions, no raw materials used and negative or zero capital returns. See the row of the footnote d, in Table I-1.

[c]All the numbers in this Table are directly counted from the original data file of the 1973 Korean Census, held by the Bureau of Statistics, Economic Planning Board, Korea.

and shows quite a complex structure of multiproduct production activities. For example, 20 different kinds of commodities (at the 7-digit classification level in KSIC) are produced in the canning industry with 134 different raw materials used (at the 7-digit level), as shown in Table II-1.

The structure of multiproduct production at the 5-digit industry level in KSIC cannot be identified in a rather neat way but with a high degree of arbitrariness in the selection of a few of the major commodities and in building a manageable, analytical framework for a study on multiproduct technology. Hence, in this study, the selection of major commodities is mainly based on two criteria, (1) the amounts of each major commodity produced may exceed 20% of total value of production in an industry, and (2) there should exist a certain number of establishments producing more than one major commodity.

Table II-2 shows the name and the KSIC 7-digit item codes of major commodities selected by the first criteria in each industry. For example, the canning industry produces 3 major commodities, i.e., the canned vegetables, the canned fruits, and the canning of fish and shellfish. Also Table II-3 indicates the value composition of major commodities within each industry. The production of the canned vegetables amounts to 48% of total production of the canning industry, that of the canned fruits

TABLE II-2.--Major Commodities Identified.

Industry	KSIC 7-Digit Code	Name
CANNING INDUSTRY: 31131, 31132, 31141		
Major Commodity - 1	3113102	Canned vegetables
Major Commodity - 2	3113101	Canned fruits
Major Commodity - 3	3114101	Canning of fish and shellfish
LEATHER FOOTWEAR INDUSTRY: 32400		
Major Commodity - 1	3240001	Leather footwear for men
Major Commodity - 2	3240002	Leather footwear for women
SCREW PRODUCTS INDUSTRY: 38196		
Major Commodity - 1	3819601	Bolts and nuts
Major Commodity - 2	3819605	Rivets, screws,etc.
MANUFACTURE OF KNITTED UNDERWEAR: 32133		
Major Commodity - 1	3213301	Knitted underwear
MANFACTURE OF BRIQUETTES: 35401		
Major Commodity - 1	3540101	Briquettes
MOLDING INDUSTRY: 37102		
Major Commodity - 1	3710213	Hoops of alloy steel
Major Commodity - 2	3710211	Sheets of alloy steel
Major Commodity - 3	3710202	Steel bars

SOURCE: Korea Standard Industry Classification, Series II, Economic Planning Board, Republic of Korea, 3rd revision on the 13th of March 1870.

TABLE II-3.--Composition of Major Commodities Identified (Unit = %).

Industry	Classified Commodities					Unclassified Commodities	Total Commodities
	Major Commodities			Other Miscellaneous Commodities	Subtotal		
	1	2	3				
Canning Industry	47.92	22.04	22.24	3.72	95.92	4.08	100.0
Leather Footwear	51.33	16.22	--	22.20	90.75	9.25	100.0
Screw Products	42.78	21.82	--	21.89	86.49	13.51	100.0
Knitted Underwear	96.34	--	--	3.66	100.00	0.00	100.0
Briquettes	99.56	--	--	0.42	99.98	0.02	100.0
Molding Industry	16.85	15.35	13.95	52.61	98.76	1.24	100.0

to 22%, and the canning of fish and shellfish to 22% respectively. The sum of these three major commodities covers slightly more than 92% of total production of the industry concerned.

Table II-4 shows the distribution of establishments producing more than one commodity within each industry. In terms of the second criteria, for example, 53 establishments in the total of 83 in the canning industry produce only either one of the three major commodities, consisting of about 64% of total establishments, and 30 establishments (about 36%) produce either two or all three major commodities.

Referring to the Tables II-1 through II-4, in terms of the criteria adopted in the selection of specific industries to be studied, the choice of six industries in Table II-1 seems to be rather reasonable in investigating the realism of a multiproduct production technology.

In the selection of industry for a uniproduct production study, the molding industry is shown such that there are more than one major commodity, but their importance in total production seems to be rather little, as shown in the last row of the Table II-3. Also the distinction of establishments producing multiproducts turns out to be that only 22% of total establishments produce either one or more of these three major commodities in the industry (see Table II-2). Hence this molding industry is intentionally classified into the industry group of producing a uniproduct, through forming one appropriate quantity index out of these various commodities.

TABLE II-4.--Distribution of Establishments Producing Multiproducts (Unit: number of establishments percentage in parenthesis).

Commodities Produced[a]	Canning Industry	Leather Footwear Industry	Screw Products Industry
Major Commodity - 1	25(30.12)	51(24.52)	73(62.39)
Major Commodity - 2	4(4.82)	12(5.77)	27(23.08)
Major Commodity - 3	24(28.92)	--	--
" " 1 & 2	7(8.43)	145(69.71)	17(14.53)
" " 1 & 3	3(3.61)	--	--
" " 2 & 3	16(19.28)	--	--
" " 1, 2 & 3	4(4.82)	--	--
Total Establishments	83(100.00)	208(100.00)	117(100.00)
Uniproduct Establishments	53(63.86)	63(30.29)	100(85.47)
Multiproducts Establishments	30(36.14)	145(69.71)	17(14.53)

[a]The last major commodity in each industry here includes the other miscellaneous commodities among the classified commodities, which was separately shown in Table II-3. For example, 3.72% of total production in the canning industry are classified and added into the 3rd major commodity.

Lastly, Table II-5 shows the total number of establishments and its size distribution by industry. The size distribution of establishments by the number of workers within each industry is rather skewed in either the small scale or the large, where the small establishments are defined as ones with more than five workers and less than 50 workers in their average production activity,[16] while the large are with more than 49 workers. From Table II-5, it is noted that 38 establishments (45.78%) among the total of 83 belong to the small size in the canning industry, 190 establishments (91.35%) out of 208 in the leather footwear industry, 98 (83.76%) out of 117 in the screw products industry, 119 (83.22%) out of 143 in the manufacture of knitted underwear, 226 (85.28%) out of 265 in the manufacture of briquettes, and 125 (67.93%) out of 184 in the molding industry. Hence, in all five industries, except the canning, the size distribution of establishments within each industry are skewed more or less toward the small size of a production unit. The implication of these skewed size distribution on the industry's production technology will be further explored by the alternative empirical estimation, appearing in the next chapter on the empirical findings.

In summary, we deliberately selected those six specific industries out of more than 20 industries at an early experimental

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[16]The criteria distinguishing the size of an establishment seems to be arbitrary, even without considering any a priori information on some intrinsic characteristics of production technology for each industry. But we follow this criteria directly in this study, where separate investigations by different size of establishments are done in our empirical estimations.

TABLE II-5.--The Size Distribution of Establishments (Unit: No. of establishments, percentage in parenthesis).

Size	1	2	3	4	5	6	7	8	
No. of Workers[a]	5-9	10-19	20-49	50-99	100-99	200-299	300-499	500-More	Total
Industry									
Canning Industry	11 (13.25)	8 (9.64)	19 (22.89)	12 (14.46)	16 (19.28)	6 (7.23)	8 (9.64)	3 (3.61)	83 (100.00)
Leather Footwear	156 (75.00)	23 (11.06)	11 (5.29)	7 (3.37)	2 (0.96)	4 (1.92)	3 (1.44)	2 (0.96)	208 (100.00)
Screw Products	42 (35.90)	37 (31.62)	19 (16.24)	9 (7.69)	3 (2.56)	1 (0.85)	2 (1.71)	4 (3.42)	117 (100.00)
Knitted Underwear	18 (12.59)	58 (40.56)	43 (30.07)	10 (6.99)	7 (4.90)	3 (2.10)	4 (2.80)	0 (0.00)	143 (100.00)
Briquettes	115 (43.40)	79 (29.81)	32 (12.08)	20 (7.55)	11 (4.15)	4 (1.51)	4 (1.51)	0 (0.00)	265 (100.00)
Molding Industry	15 (8.15)	46 (25.00)	64 (34.78)	22 (11.96)	15 (8.15)	4 (2.17)	7 (3.80)	11 (5.98)	184 (100.00)

[a]The number of workers means the average persons engaged on average per work-day during the operation period of an establishment.

stage, owing to the three arbitrary but seemingly reasonable and appropriate criteria in the selection of specific industries to be studied on multiproduct production technology.

B.2.6. Some Characteristics of the Industries Selected

Sample properties in this study, which partially reflect the characteristics of an industry's production structure, are analyzed in terms of factor productivity, factor use ratio, factor share, and factor price.

6.1. Factor Products

As shown in Table III-1, average factor products are widely different from industry to industry. The highest average product per man-day operative worker with respect to gross output is shown as 21,300 won in the manufacture of briquettes, with its moderate level for capital input, i.e., either 1,790 won per 1,000 horsepower or 4,000 won per net capital stock in 1,000 won. And the lowest average product per man-day operatives is revealed as 4,970 won in the leather footwear industry, also with quite a low level of average product for capital input, i.e., either 880 won per 1,000 horsepower or 530 won per net capital stock.[17]

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[17]The dispersion of each factor product is found to be so huge among the current sample establishments in each industry that the degree of its relative dispersion, measured by the coefficient of variation as the standard deviation divided by its mean, is greater than 1.0 in some factor products. For example, in the canning industry, the average product per man-day operatives in an establishment 11,100 won with a coefficient of variation of 1.12. Such a wide variation in average factor products over most

TABLE III-1.--Factor Products by Industry[a] (Unit: 1,000 won).

Factor Products / Industry:	Canning Industry	Leather Footwear	Screw Products	Knitted Underwear	Manufacture of Briquettes	Molding Industry
I-WITH RESPECT TO GROSS OUTPUT						
Operatives[b]	11.10	4.97	5.76	6.06	21.30	13.88
	(24.49)	(3.09)	(6.29)	(3.29)	(20.53)	(19.56)
Administratives[b]	67.53	24.34	30.38	57.43	51.98	92.59
	(54.54)	(31.53)	(24.01)	(40.08)	(47.89)	(109.38)
Power Equipment[c]	4.72	0.88	0.83	2.51	1.79	1.69
	(17.72)	(3.54)	(0.70)	(3.28)	(5.35)	(3.89)
Net Capital Stock[d]	7.39	0.53	0.85	0.90	4.00	6.98
	(22.33)	(3.20)	(3.68)	(6.08)	(20.07)	(32.87)
II-WITH RESPECT TO VALUE ADDED						
Operatives[b]	5.61	2.22	2.56	2.42	6.32	5.09
	(13.11)	(1.47)	(2.96)	(3.89)	(11.09)	(10.63)
Administratives[b]	26.74	10.68	13.79	23.41	14.75	35.16
	(25.36)	(12.82)	(11.61)	(44.65)	(22.76)	(88.32)
Power Equipment[c]	2.08	0.38	0.39	0.89	0.62	0.64
	(8.46)	(1.50)	(0.38)	(1.09)	(2.90)	(1.59)
Net Capital Stock[d]	2.67	0.22	0.37	0.40	1.39	2.12
	(7.64)	(1.52)	(1.40)	(3.10)	(7.42)	(8.79)

[a]The figures shown in the table are the monthly averages of factor products measured in 1,000 won in establishments within each industry and those in parentheses are the corresponding standard deviations.

[b]Measured by gross output (or value added) divided by the number of man-day workers. For example, the number of man-day operatives is the number of operatives multiplied by the monthly average number of working days in an establishment.

[c]Calculated by gross output (or value added) divided by horsepower capacity, measured in 1,000 horsepower.

[d]Calculated by gross output (or value added) divided by the amounts of net capital stock, measured in 1,000 won.

In the industrial comparison of average factor product, the relatively higher average products for labor and capital inputs are revealed in the canning industry, manufacture of briquettes, and the molding industry. On the other hand, the leather footwear industry and the screw product industry show relatively lower average products for both inputs. The manufacture of knitted underwear shows relatively lower product for labor input but higher product for capital input of horsepower (see 2.51 in the 4th column of the third row in Table III-1).

Average factor products with respect to the industry's value added reveals, in general, the similar phenomena as mentioned above.

6.2. Factor Use Ratios

The factor use ratio is focused especially upon the relationships between capital and labor inputs by industry. Two proxies are chosen separately for the capital input, power equipment and net capital stock. As shown in Table III-2,[18] the highest capital-labor

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industries seems to be closely related to the low level of multiple correlation coefficients in the input share equations to be estimated. See the discussion on the size of error sums of squares in the next chapter.

[18]The dispersion of these factor use ratios in the distribution of the sample establishments shows the highest coefficient of variation, for example, 6.21 in the horsepower-total worker ratio in the canning industry, while the leather footwear industry also shows a high coefficient of variation, 5.32. On average all the factor use ratios evaluated here, except the horsepower-worker ratio in the screw products industry, have a degree of relative dispersion greater than unity.

TABLE III-2.--Factor Use Ratios by Industry.[a]

Ratios — Industry:	Canning Industry	Leather Footwear	Screw Products	Knitted Underwear	Manufacture of Briquettes	Molding Industry
Horsepower/Total Workers[b]	650.72 (4038.52)	3.07 (16.32)	142.72 (114.85)	63.45 (68.69)	368.58 (1395.76)	353.04 (676.87)
Horsepower/Total Operatives[b]	893.13 (5467.46)	3.36 (17.72)	172.30 (137.75)	73.09 (80.34)	544.48 (1776.99)	426.99 (825.02)
Horsepower/ Administratives[b]	11339.53 (80269.25)	42.47 (223.72)	1071.65 (1077.70)	665.87 (826.06)	1539.12 (7469.71)	2405.58 (4001.54)
Net Capital Stock/ Total Workers[c]	33.32 (62.54)	1.84 (9.66)	18.83 (109.41)	5.59 (25.25)	6.62 (23.19)	49.09 (287.47)
Net Capital Stock/ Operatives[c]	52.89 (173.77)	2.07 (10.80)	25.37 (132.47)	6.45 (29.93)	10.89 (39.18)	56.15 (306.33)
Net Capital Stock/ Administratives[c]	271.09 (457.65)	22.11 (117.95)	182.42 (1182.63)	51.41 (208.22)	32.23 (149.45)	596.90 (5476.76)

[a]The figures shown are the averages of the corresponding factor use ratios of establishments within each industry and those in parentheses are their standard deviations respectively.

[b]Power equipment is measured in 1,000 horsepower (Hp).

[c]Net capital stock is in the value of 1,000 won.

ratio is shown in the canning industry while the lowest is in the leather footwear industry. On average, the canning industry, the manufacture of briquettes, and the molding industry belong to the group of higher capital-labor ratio, while the leather footwear industry, the screw products industry and the manufacture of knitted underwear belong to that of lower capital-labor ratio. Thus, those industries of a higher capital-labor ratio correspond to the industries of higher factor products for labor and capital inputs, as noted in the previous subsection. When the capital-labor ratio is viewed as a criterion for judging the factor intensity of an industry, the canning industry, the manufacture of briquettes, and the molding industry are relatively capital intensive to the other three industries and shows relatively higher factor products for labor and capital inputs in general.[19]

6.3. Factor Prices

Average factor prices paid for each factor within an industry are shown in Table III-3. The highest wage per operative and the highest salary per administrative are respectively shown as 21,760 won and 36,000 won in the molding industry, while the lowest wage level of 15,690 is shown in the manufacture of knitted underwear and the lowest salary level of 20,760 won in the leather footwear

[19]Here one qualification becomes necessary in the case of the manufacture of briquettes, where the capital-labor ratio is relatively low if measured by the net capital stock. Also in the screw products industry, the capital-labor ratio seems to be moderately high, even if their factor products for both inputs were relatively low.

TABLE III-3.--Factor Prices by Industry[a] (Unit: 1,000 won).

Factor	Canning Industry	Leather Footwear	Screw Products	Knitted Underwear	Manufacture of Briquettes	Molding Industry
Operatives[b]	16.36 (13.54)	19.38 (9.45)	18.27 (11.18)	15.69 (6.16)	21.38 (8.57)	21.76 (9.91)
Administratives[b]	30.75 (20.84)	20.76 (16.91)	24.89 (14.82)	22.32 (11.99)	26.38 (17.20)	36.00 (21.58)
Operatives/Man-Days[c]	0.76 (0.62)	0.76 (0.38)	0.74 (0.42)	0.65 (0.26)	0.87 (0.34)	0.91 (0.43)
Administratives/ Man-Days[c]	1.45 (1.06)	0.81 (0.66)	1.01 (0.62)	0.93 (0.48)	1.08 (0.69)	1.49 (0.86)
Power Equipment[d]	1.48 (7.24)	0.24 (1.00)	0.19 (0.26)	0.51 (1.10)	0.51 (2.82)	0.29 (1.58)
Net Capital Stock[d]	1.80 (4.86)	0.16 (1.35)	0.25 (1.04)	0.19 (1.24)	1.27 (7.08)	0.59 (9.32)
Fuel[e]	7.07 (14.36)	24.68 (15.66)	14.71 (11.24)	16.02 (14.12)	20.81 (44.20)	74.57 (212.69)
Raw Materials[f]	23.76 (66.75)	23.05 (58.93)	69.05 (154.88)	2.76 (3.86)	0.45 (0.56)	115.62 (188.83)

[a]The figures shown are the averages of factor price of establishments within each industry and those in parentheses are their corresponding standard deviations.

[b]Calculated by the wages for operative workers and salaries for administrative workers, divided by the number of the corresponding workers in monthly base.

[c]These are measured in daily base.

[d]These are not the prices of power equipment and of net capital stock in their strict definition, but are calculated by the returns on capital input divided by the corresponding inputs, i.e., power equipment in 1,000 Hp and net capital stock in 1,000 won.

[e]The price of fuel consumed is measured for 1,000,000 calory.

[f]The price of raw materials is evaluated by the purchase amounts of raw materials used in production and divided by the quantity index, formulated for each establishment within an industry.

industry. But the salary level in average is not necessarily higher than the wage level across the six industries selected here, where the average monthly salary of 20,760 won in the leather footwear industry is lower than the average wages of 21,380 won in the manufacture of briquettes and of 21,760 won in the molding industry.

Hence, relatively high factor payments for two labor inputs are revealed in the canning industry, the manufacture of briquettes, and the molding industry, while lower payments are shown in the other industries. That is, those industries with higher factor products (in the section 7.1.) tend to reveal also relatively higher factor payments for labor inputs. Here the canning industry shows relatively low wage of 16,360 won for operative but it reveals moderately high wage of 19,760 won, the payment for the man-day operative, which is the daily payment of 760 won multiplied by the average working days of 26 in a month.

Average returns on capital input (either evaluated by power equipment or by net capital stock) also show the similar phenomena, as the above, among the six industries. The price per million calory-equivalent fuel consumed ranges from 74,570 won in the molding industry to 7,070 won in the canning industry. The seemingly wild variations in the price of fuel consumption may reveal significantly different composition of fuel consumed (i.e., among coal, oil, and/or electricity) from the different structure of production technology by industry. The price of raw materials may not be, from the deficiency in its evaluation based on the quantity index formulated by each industry, an appropriate measure for industry comparison but for establishment comparison within each industry.

The distribution of factor prices within each industry is provided by firm size in the appendix B-1, Table I-1 through I-6. A few findings may be worthwhile to note here. First, the wage level in smaller establishments is rather higher than in larger establishments, while the salary level for the administratives in smaller establishments is lower than in larger establishments.[20] Average returns on capital input, either in terms of horsepower or of net capital stock, are getting higher as the size of establishments in all six industries become bigger. On the contrary, the unit prices of fuel consumption and of raw materials used are getting lower as the size of establishments become bigger, where there may exist considerable economies and advantages in the process of factor price bargaining and in more efficient process of production technology.

6.4. Factor Shares

Average factor shares within each industry are shown in Table III-4. In general, the cost share of fuel consumption is

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[20] This phenomena may be, in part, attributed to possible deficiencies in evaluating employees' remuneration, especially allowances and subsidies in kind at smaller size of establishments, where bigger firms probably have better accounting systems and smaller firms may exaggerate those items of subsidies in kind among their factor payments.

TABLE III-4.--Factor Shares by Industry[a] (Unit: %).

Factor Shares / Industry:	Canning Industry	Leather Footwear	Screw Products	Knitted Underwear	Manufacture of Briquettes	Molding Industry
Wages	11.96	17.64	19.20	13.39	7.39	11.34
	(7.45)	(7.46)	(11.44)	(8.31)	(6.58)	(6.60)
Salaries	3.33	5.27	5.03	2.56	3.40	3.11
	(2.96)	(3.66)	(3.78)	(2.45)	(3.01)	(2.47)
Capital Returns	25.38	24.29	27.47	19.97	16.30	22.25
	(17.50)	(12.25)	(20.55)	(11.29)	(14.90)	(14.78)
Total Value Added[b]	40.67	47.20	51.70	35.92	27.09	36.70
	(17.43)	(11.96)	(15.82)	(14.27)	(14.79)	(14.82)
Fuel Consumption	3.72	1.43	4.22	1.50	1.43	7.76
	(3.65)	(1.31)	(3.40)	(1.09)	(1.22)	(5.83)
Raw Materials	55.61	51.37	44.08	62.58	71.48	55.54
	(18.59)	(12.39)	(15.77)	(55.06)	(47.45)	(16.43)
WITH RESPECT TO VALUE ADDED						
Wages	33.34	39.39	40.83	38.35	29.44	33.14
	(21.72)	(14.65)	(21.89)	(18.65)	(19.55)	(17.19)
Salaries	9.34	12.28	10.80	7.53	13.91	9.37
	(7.36)	(9.22)	(7.25)	(6.61)	(10.09)	(6.91)
Capital Returns	57.32	48.33	48.37	54.12	56.65	57.49
	(24.94)	(18.94)	(25.93)	(21.37)	(25.38)	(20.87)

[a]The figures shown are the averages of factor shares, in percentage, of establishments within each industry and those in parentheses are the corresponding standard deviations.

[b]Total value added here is the sum of wages, salaries, and capital returns.

negligible, far less than 10% of total production, where the highest 7.76% is revealed in the molding industry, and the next 4.22% is in the screw products industry (see the fifth row in Table III-4). The share of raw materials in total production varies from industry to industry, the highest 71.48% in the manufacture of briquettes and the lowest 44.08% in the screw products industry. This implies a strong suspicion on one of the conventional hypotheses that the elasticity of substitution between value added and raw material is zero, raw materials being used in fixed proportion to output, i.e., $M = \alpha X$, M is the amounts of raw materials used, X is the amounts of total production, and α is a fixed coefficient.[21] This suspicion becomes more evident when the distribution of the average shares of raw materials used by firm size within each industry, are referred to (see Appendix B-II, Table II-1 through Table II-6). That is, more significant variations in the average share of raw materials used in total production are revealed within each industry, as the size of establishments varies.

The value added ratios, defined as the ratio of value added to gross output, consequently vary from industry to industry with a significant variation, even among these six manufacturings. The highest ratio of 0.517 is revealed in the screw products industry and the lowest of 0.271 is in the manufacture of briquettes. Consequently, this simple observation suggests that the hypotheses on

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[21] See the previous discussions on the role of raw materials in the production theory in section B.2.3.

the elasticity of substitution between value added and raw materials (which are assumed either zero or infinite conventionslly in most production studies of the value added approach) should be a real question of empirical investigation.

The shares of labor and capital also show quite a significant variation among our six industries. And in general, as noted above, the relatively higher capital returns are revealed in those industries of canning, briquettes, and molding, which show relatively higher factor products and have higher capital-labor ratios.

6.5. Summary

In summary, the characteristics of the industries to be studied are such that the canning industry, the screw product industry, the manufacture of briquettes, and the molding industry show, relatively to the other three industries selected here, higher levels of factor products for labor and capital inputs and have higher capital-labor ratios. And relatively high level of factor remunerations for labor and capital inputs are paid up in these industries and also distributive shares in value added are more favorable for the capital input than for the labor inputs.

One important observation on the relation between value added and raw materials is added. That is, there exist quite a significant variations in the cost share of raw materials by the size of establishments within an industry, where there are no sound evidence for assuming any _a priori_ role of raw materials in the study of production technology, based on the cross-section data.

B.2.7. Quality of Data: Some General Considerations

In general, two major measurement problems plague more or less all empirical studies. First, the correspondence of the main variables used in the study to the presumably "correct" measures of these variables and second, the reliability of the information provided by the census about the components from which the variables were actually constructed.

7.1. Aggregation

Most of our variables are still aggregated although correct measurement depends very much on correct aggregation.[22] In the aggregations of miscellaneous products and raw material inputs to form the respective quantity index, we used the conventional quantity indexing method which inevitably results in the so-called index number problem. However, these aggregations in this study exist not because of any theoretical or practical restrictions on constructing a multi-product production function or a "correct" aggregation but simply as a matter of convenience in empirical estimation with a large number of variables.

Regarding remuneration of administratives including working proprietors and unpaid family workers, the use of average salary levels for the third category of labor may bias cost shares either upward or downward, presumably downward when we refer to the common

[22] But that is easier said than done, in most empirical works.

practices in most Korean businesses. Nevertheless, the degree of the bias and its direction cannot be stated a priori unless very specific evidence is available.

The measure of operatives and administratives does not allow for the differences in the quality of labor among establishments. There is no information available about educational, occupational, or skill levels that would make it possible to adjust for such nonhomogeneous factors. There are also variations in efficiency of labor between regions and/or different firm sizes, but in the present study it is simply assumed that all these differentials are equally well reflected in the respective variations of wages and salaries.

Conceptually, the replacement value, not the book value, of capital stock evaluated at market prices should be a good measure to use in our context, since it reflects both the quantity and quality of components. It does not, and should not, reflect the capitalized value of monopoly, location, or other sources of rent. Hence, we may escape Friedman's (1955) criticism against capital accounting measures which imply constant returns to scale by capitalizing all rents into the value of capital.[23] But this capital stock measures is still based on what an establishment owned, not used during the production process. In short, there were no data available to adjust for capital utilization and furthermore there were many establishments with no capital stock

[23]See Grilliches and Ringstadt (1971), pp. 59.

values available in the census, as shown in the previous section under exclusion rules.

As an alternative to capital inputs, power equipment was measured and aggregated in a common physical unit (HP). One potential source of measurement error is that the capacity of power equipment is recorded by the horsepower figures shown on the labels or specifications neglecting any present physical and/or economic obsolescence.

Lastly, various fuels are aggregated into a rather perfect dimensional unit, say kilocalories of heating value, which allows comparison between ingredients of coal, oil and electricity.[24]

7.2. Modifications of Some Unclassified or Miscellaneous Factors

There are several cost factors of which only amounts are available such as water purchased, payments for contract work on raw materials, and the cost of maintenance of the production facilities. In the current works, the costs of water are counted as a part of the cost of fuel consumption, the payments for contract work as raw material costs, and the maintenance costs as capital services.[25] As shown in Table IV-1, although their portions are

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[24] Here the only possible doubt may reside in the significant variations in the conversion coefficients for each type of energy. See footnote 11 in this chapter.

[25] These share adjustments seem to be rather appropriate, even if there may be some other alternative way to handle them more appropriately.

TABLE IV-1.--Measurement Deviations (%) by Industry.[a]

Miscellaneous Factors	Industry: Canning Industry	Leather Footwear	Screw Products	Knitted Underwear	Manufacture of Briquettes	Molding Industry
(Work-in-Progress / Gross Outputs)	0.44 (2.56)	0.03 (0.30)	0.33 (2.00)	0.58 (3.22)	-0.02 (0.31)	0.23 (1.98)
(Other Revenue / Gross Output)	1.68 (7.72	0.80 (3.01)	7.10 (17.40)	2.13 (7.40	0.69 (4.81)	2.09 (6.28)
(Repairs / Net Capital Stock)	5.21 (13.00	2.23 (30.39)	1.34 (10.25)	5.03 (54.36)	3.32 (20.42)	8.38 (40.52)
(Water Purchased / Fuel Consumed)	16.94 (34.28)	8.44 (18.45)	4.47 (9.13)	7.17 (8.72)	15.49 (113.33)	4.46 (34.55)
(Contract Work / Raw Materials)	0.02 (0.15	0.11 (1.11)	0.57 (2.10)	2.52 (6.78)	0.12 (1.00)	0.51 (2.45)
((Repairs + Water + Contract Work) / Gross Output)	2.07 (2.64)	1.14 (1.58)	1.98 (2.41)	3.57 (4.74)	2.24 (3.00)	1.73 (2.83)

[a]The figures are shown in the percentages (%) for each industry and those in parentheses are their standard deviations.

quite negligible, these modifications facilitate the identity of the sum of output values and that of input values.

7.3. Implications of the Exclusion Rules and the Selection of Industries to be Studied

The deliberate exclusion of a few establishments in this empirical work may create another type of error, not an "error of measurement" but rather a "sampling error." The question in this context is: to what extent will the results be valid for the actual industry as a whole? Obviously, they are unlikely to be valid for very small establishments, since all units with less than 5 workers are excluded from the sample spaces of the current empirical estimation. Another significant source of "selection error" may be the exclusion of units reporting neither capital stock nor power equipment which mostly belong to the group of the smallest firms (such as $5 \leq L \leq 10$).[26]

All possible empirical results become in principle applicable neither to any other industries, nor to manufacturing as a whole, and can only be valid for the few very specific industries randomly chosen in the present study.

7.4. General Considerations

Although the quality of the data base has been improved by the adoption of appropriate selection rules, it is certainly not free of errors of measurement. The Bureau of Statistics has,

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[26] See the exclusion status by industry in Table II, Section B.2.5.

however, carried out very detailed control and revision of the original data and corrected some "impossible" combinations of information through thoughtful editing either by contacting and checking directly with the establishments or by imputing information to be consistent at the establishment level. In most cases it was obvious which information was false and the imputed values seem to be at least closer to reality than what was originally reported.

In general, the information on the gross output of each product and the raw materials used is of good quality. The same is true of labor measures and the components of capital input measures.

There is some evidence that the reliability of the information provided by larger establishments is significantly higher than that of smaller units. This has been confirmed by random investigations[27] and it is also reasonable in that bigger firms probably have better accounting systems.

The basic problem is the poor quality of certain information, particularly for capital stock and capacity utilization. This limitation has been dealt with in the current study by excluding the bulk of the sample establishments with no capital stock data

[27] Also some investigations of other information for large firms in a specific industry conform more with the results of other data sources. Financial Statement Analysis for 1973 (Bank of Korea), which surveys Balance Sheets, Profit and Loss Statements, Statements of Manufacturing Costs, and Fund Statements and presents various managerial statistics for average firms by industry at the 4-digit level.

and alternatively by trying to define the capital stock through the proxy of power equipment capacity.

CHAPTER III. THEORETICAL AND STATISTICAL BACKGROUNDS IN THE EMPIRICAL ESTIMATION OF A PRODUCTION TECHNOLOGY

B.3.1. Introduction

The general approach to estimating a production technology is to assume the existence of a production possibility set which contains all feasible input-output points under a given technology--a certain state of knowledge. Where Y is a vector of net outputs (plus if output, minus if input) the boundary of this set may be represented by the transformation frontier, $t(Y) = 0$. (1)
Economic theory imposes certain restrictions on the production possibility set and hence upon the transformation frontier (1). In reality we do not know t but do have observations on Y which refer to different points of time or different production units or both.

Assuming all observations come from a common transformation frontier[1] we can estimate that frontier in two ways. The first is to use a nonparametric method developed by Farrell (1957) and

[1] One crucial problem is that the state of knowledge may not be constant over the observations. We either assume that (i) it is constant over the observations or (ii) that the transformation frontier shifts in a particular way over the observation. In general, the problem of technical change cannot be properly handled in a cross-section study.

further analyzed by Afriat (1972).[2] The second approach is to postulate that the observation were generated from the transformation frontier but subject to random disturbances. Given the density of the disturbances and an assumed functional form t, the parameters of this functional form may be estimated and statistical inferences made. The choice of density and functional form impose restrictions in making the model operational, that is, making it feasible to empirically estimate the frontier. They necessarily force the researcher to trade-off between the generality of functional form and stochastic specification and simplicity of execution.[3]

In this chapter, some theoretical and statistical background in the present empirical estimation of a production technology are presented in terms of the choice of the estimating equation, the specification of the disturbance term in the equation to be estimated, and the estimation procedure perhaps more specific. In addition to these, a Monte Carlo experiment is designed and implemented on how the existence of the zero-valued data in some variables affect on the parameter estimation, since the use of the translog equations depends on the quantities of the explanatory

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[2]This method has been used by Hanoch and Rothschild (1972), Geiss (1971), Timmer (1971) and Aigner and Chu (1968). The linear programming technique has been utilized in searching for optimal points of production activity among observations.

[3]There does not seem to exist any consistent set of rules to guide the researcher in his choice though, as Mundlak (1973) points out, the choice depends to a large extent upon the use to which the results are to be put.

variables being strictly positive, otherwise, the expression is not well defined.

B.3.2. Choice of the Estimating Equations

In practice there are various ways to estimate the parameters of a production frontier. Those differ in two major characteristics: (i) The function from which the parameters are estimated. (ii) The account which is taken of the constraints of a complete model.

The most obvious approach, and the first from the historical point of view, is the direct approach with output being the dependent variable and inputs being the explanatory variables. For the function, $t(Y) = 0$, to be linear in the parameters, the outputs and inputs may be replaced by some known functions. Usually a disturbance term is appended to the production frontier function in either additive or multiplicative form and it is assumed that the inputs are independent of this disturbance. There are several disadvantages to estimating this function. First, it is unlikely that the inputs will be independent of the disturbance, particularly if the inputs are chosen by the firm.[4] Second, multicollinearity is likely to be a problem, especially with time series data where the various inputs vary with time along a similar pattern.[5] Third,

[4]The idea is that the observed quantities are the results of some equilibrium solutions. See Marshak and Andrews (1944).

[5]This results in high correlation and large standard errors in the estimated parameters associates with the curvature of the function, thus making it difficult to allocate correctly changes in outputs in the various inputs.

although price data and behavioral assumptions do not appear to be required, they are needed to construct indices of input quantities since aggregation is usually mandatory.[6] Fourth, if cost minimization takes place, accuracy requires that we model such behavior.

Alternatively these drawbacks can be avoided by considering the production function as a part of a complete economic model. A complete model will consist of a production function and a set of conditions for the rules of behavior. The first order conditions for profit maximization applicable to a competitive firm provide a framework for analysis and for formulation which accord with the conventional theory.

However, this approach has been used in the literature mainly for analytic discussions rather than for actual empirical estimation.[7] Perhaps, the main contribution of these literature to empirical work has been the introduction of the first order conditions. These are in turn used to obtain estimates of the

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[6]See Mundlak (1963).

[7]This factor is indicative of the intrinsic difficulty in the use of the complete model. The construction of a complete model is accompanied by classification and identification of all the endogeneous and exogeneous variables within the period of accounting (usually a year). For example, a simple-minded formulation for a competitive firm assumes prices to be exogeneous and quantities to be endogeneous. From the statistical point of view, the important question is whether the inputs are correlated with the error term of the production function, implying the optimal properties of the least squares estimators against the simultaneous equation bias. See Mundlak (1973) for this issue in detail.

parameters of the production function on a partial basis--that is, by using a single equation and ignoring the remaining ones.[8]

In addition to the direct estimation of a production function or the function of a transformation frontier (1), two approaches have been popular in the econometric world, followed by the discussions on a complete model of production structure.

If the transformation frontier (1) is differentiable in Y then both the factor demand (or product supply) functions can be derived from the partial derivatives of t with respect to the quantities of input and output factors:

$$W_i = t_i(Y) \equiv \partial t(Y)/\partial Y_i, \quad i=1, \ldots 5$$

and (2)

$$P_j = t_j(Y) \equiv \partial t(Y)/\partial Y_j, \quad j=6, \ldots 8$$

where $t(Y) = 0$,

Y = a vector of net outputs, which consist of 5 inputs and 3 outputs,

W_i = the price of i-th input,

P_j = the price of j-th output.

If we define $S_i \equiv W_i Y_i / \sum_{\ell=1}^{5} W_\ell Y_\ell$ as the share of factor i in the cost of production and define $S_j \equiv P_j Y_j / \sum_{\ell=6}^{8} P_\ell Y_\ell$ as the share of product

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[8]Here still we may have difficulties with multicollinearity if there are more inputs and with transforming more complex functional forms into linear forms in the parameters.

j in the output of production respectively, then the system of share equations may be obtained from (2)[9] or, writing the transformation function in log form as

$$S_i = \partial \ln t(Y)/\partial \ln Y_i, \; i=1, \ldots 5$$

and (3)

$$S_j = \partial \ln t(Y)/\partial \ln Y_j, \; j=6, \ldots 8$$

At the same time, if the duality theorem is used under certain regularity conditions[10] there is no loss of information or generality in using the cost function as the point of departure for either theoretical or empirical research. Consequently, given any valid functionsl form for t we can derive in principle, the cost function which yields t as its transformation function. The cost function is

$$C(W, Y) \equiv \min_{X} \{WX : t(Y) \geqq 0\} \tag{4}$$

where X = a subvector of Y, consisting of inputs only.

In the same way as above, we can derive the factor demand functions and the share equations, when C(W, Y) is differentiable in W, as follows:[11]

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[9]The derivation of the share equations system is already discussed in Part A, Chapters I and II.

[10]See also Part A, Chapters I and II about the duality theorem.

[11]See Woodland (1976) about the general and specific approaches in terms of the cost function.

$$X_i \equiv C_i(W, Y) = \partial\ C(W, Y)/\partial\ W_i,\ i=1, \ldots 5 \qquad (5)$$

and

$$S_i \equiv S_i(W, Y) = \partial\ \ln\ C(W, Y)/\partial\ \ln\ W_i,\ i=1, \ldots 5 \qquad (6)$$

The main differences between estimating the cost function and estimating the production function are the data used and the assumptions regarding how the data were generated. In the present study where data on quantities and prices are available we have followed these two lines of estimation.

Choice between the factor demand functions (2) and (5) and the share equations system in our estimation was made mainly on the basis of the various advantages in the TRANSLOG approach discussed in Chapter II, Part A. That is, the TRANSLOG function is basically specified in log form, the first derivatives of which directly generate the share equations.

The TRANSLOG procedure usually estimates the first and second derivatives of the Taylor series expansion from the first order side conditions; the alternative is to estimate them from the production function. One problem in the direct approach is the collinearity between the linear and quadratic terms in the Taylor series expansion; a second is that symmetry has to be part of the maintained hypothesis. The indirect equations, however, conceal the assumption of optimizing behavior as well as some rather peculiar assumptions about the error processes[12] and the

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[12]The peculiarity of error specification will be discussed in detail in the following section.

endogeneity of variables usually taken as exogeneous in the theory of a competitive firm.

Given the difficulty in obtaining analytic results about the effect of the truncation error[13] which arises from the Taylor series expansion of a production function, a recent Monte Carlo experiment, undertaken by R. Byron (1977), on the efficiency of the indirect estimation (i.e., the estimations of the share equations) was compared with that of the direct estimation (i.e., those of the production function) in the TRANSLOG approach, concluding that "Direct estimation of the production function was found to be inferior to indirect estimation based on the first order conditions, ..."

B.3.3. Error Specification and its Properties

The introduction of stochastic disturbance in any econometric model is another very important problem in connection with the choice of the estimation techniques.

Here we discuss first the specification of the disturbance terms and their properties as assumed in the study. Discussion of another type of error, named the "truncation error" in the previous section, will be discussed later since the error is more related to the choice of the estimating equation and the estimation procedure rather than to an approximation error.

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[13]Its mathematical representation will appear in the later section on Monte Carlo experiments. The "truncation error" will be further discussed.

For the purpose of estimation, it is necessary to make some assumptions with respect to the disturbances in the system. From the specification of the disturbances in a complete model of the production structure it may be clear that all inputs and outputs are endogeneous and therefore least-squares estimates of the system of share equations may be expected to be inconsistent. The covariance and instrumental variable estimates[14] have been suggested for this problem.

Aside from the endogeneity problem of the variables concerned, the introduction of disturbances in the first order conditions further complicates the matter. That is, we may have the following error specification on the producers' fulfillment of the first order conditions for profit maximization behavior:

$$W_i = \left[\frac{\partial t(Y)}{\partial Y_i}\right] \cdot E_m + E_a, \; i=1, \; . \; . \; . \; 5 \tag{7}$$

where E_m is a multiplicative disturbance and E_a is an additive one.[15] This is a general specification which allows us to consider special cases when one of the two terms vanishes.[15]

When the errors are included explicitly in the share equations system, it becomes:

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[14]See Mundlak (1964).

[15]See Mundlak (1964), and Mizon (1974).

[16]For instance, in a single product Cobb-Douglas production function the convention has been to ignore E_a.

$$S_i = \frac{\partial \ln t(Y)}{\partial \ln Y_i} = \frac{\partial t(Y)}{\partial Y_i} \frac{Y_i}{t(Y)} = \frac{(W_i - E_a)}{E_m} \frac{Y_i}{t(Y)} \tag{8}$$

Thus, even if we assume either that E_a is identically zero, or that E_m is identically unity, least-squares estimators may not possess any optimum property.

On the other hand, we may approach the problem of specification somewhat differently by asking what assumptions with respect to the first order conditions will lead to the relation

$$S_i = \frac{W_i Y_i}{t(Y)} = \frac{\partial \ln t(Y)}{\partial \ln Y_i} + \mu \tag{9}$$

Working back, we got

$$W_i = \frac{\partial t(Y)}{\partial Y_i} + \frac{t(Y)}{Y_i} \mu \tag{10}$$

which implies that the discrepancy between the marginal productivity and the factor price is directly proportional to the average product of the factor and therefore to the level of the input.[17]

From all this it appears that assumptions which simplify the procedure of estimation imply a peculiar specification of the

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[17]The equation (10) is derived straightforwardly from (9), where

$$\frac{\partial \ln t(Y)}{\partial \ln Y_i} = \frac{\partial t(Y)}{\partial Y_i} \frac{Y_i}{t(Y)} .$$

first order conditions: whereas simple specifications of the first order conditions complicate the method of estimation. In the specific body of the current empirical study we follow the second type of error specification, reserving all these complications for the interpretation of our estimates.

If the assumptions of the standard linear model hold for each of the share equations, the elements of the disturbance vector of each equation have zero mean but they do not have the same variance, since the variance of one equation is, in general, not equal to that of the other. In addition, we have three kinds of covariances. The first is $E\,(\mu_i^k\ \mu_j^k)$ for $i \neq j$ which concerns disturbances of the i-th and the j-th different establishments but of the same k-th factor share equation. These covariances are assumed to vanish in the standard linear model. The second kind is $E(\mu_i^\ell\ \mu_j^k)$ for $i \neq j$ which deals with disturbances both of the i-th and the j-th different establishments and of the k-th and the l-th different factor share equations. It will be assumed that such covariances, too, are zero: this assumption is an extension of the zero correlation condition of the standard linear model. The third kind, finally, is $E(\mu_i^k\ \mu_i^\ell)$, which concerns disturbances of the k-th and the l-th different equations but of the same establishment.[18]

[18]The expression "contemporaneous covariance" refers specifically to time series applications, but it is normally used in a more general context. See Theil (1971), pp. 297-302.

This condition on contemporaneous covariance amounts to an extension of homoscedasticity.[19] It means that the disturbances, μ_i^k and μ_i^ℓ are respectively random drawings from a multivariate population with a zero mean vector and a constant covariance matrix.

In summary, we specify a classical additive disturbance for each of the share equations of the translog production function, (3), and for each of those of the translog cost function, (6). The disturbance for (3) can be interpreted as random errors in achieving profit-maximizing behavior by individual establishments and the disturbances for (6) be interpreted as those in achieving cost-minimizing behavior by individual establishments within each industry. We expect the disturbances for each establishment to be correlated since errors involving one input will affect the shares of the other inputs. However, we assume that the errors made by one establishment are not correlated with the errors by any other establishment.

B.3.4. Estimation Methods

As described so far, we have adopted the following system of the share equations in the estimation.[20]

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[19]Because the link implied in the covariance between the k-th and the l-th equations is rather subtle, the system of equations is called a system of "seemingly unrelated regression equations" in some econometric publications. See Kmenta (1971), pp. 517-530.

[20]This is from the equations (3) and (6) in the section B.3.2. and (42) in the section A.3.2.

For the inputs in the production function and in the cost function,

$$S_i = \alpha_i + \sum_{j=1}^{5} \gamma_{ij} \ln X_j + \sum_{j=6}^{8} \varepsilon_{ij} \ln Y_i + \mu_i, \; i=1, \ldots 5 \quad (11)$$

and

$$S_i^* = \alpha_i^* + \sum_{j=1}^{5} \gamma_{ij}^* \ln W_j + \sum_{j=6}^{8} \varepsilon_{ij}^* \ln Y_j + \mu_i^*, \; i=1, \ldots 5 \quad (12)$$

and for the outputs,

$$S_i = \beta_i + \sum_{j=1}^{5} \xi_{ij} \ln X_j + \sum_{j=6}^{8} \delta_{ij} \ln Y_j + \mu_i, \; i=6, \ldots 8 \quad (13)$$

and

$$S_i^* = \beta_i^* + \sum_{j=1}^{5} \xi_{ij}^* \ln W_j + \sum_{j=6}^{8} \delta_{ij}^* \ln Y_j + \mu_i^*, \; i=6, \ldots 8. \quad (14)$$

In matrix notation, this can be written as:

$$Y = X \, \Pi + \mu, \quad (15)$$

and

$$Y^* = X^* \, \Pi^* + \mu^*, \quad (16)$$

where $Y = \begin{bmatrix} S_i \\ \vdots \\ S_8 \end{bmatrix}$, $X = \begin{bmatrix} \bar{X} & 0 & \cdots & 0 \\ 0 & \bar{X} & & \vdots \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & \bar{X} \end{bmatrix}$, $\bar{X} = \begin{bmatrix} \ln X_1 \\ \vdots \\ \ln Y_8 \end{bmatrix}$, $\bar{X}^* = \begin{bmatrix} \ln W_1 \\ \vdots \\ \ln Y_8 \end{bmatrix}$,

$$= \begin{bmatrix} \alpha_1 & \gamma_{11} \ldots \gamma_{15} & \varepsilon_{16} \cdots \varepsilon_{18} \\ \vdots & \vdots \quad\quad \vdots & \vdots \quad\quad \vdots \\ \beta_8 & \xi_{81} \ldots \xi_{85} & \delta_{86} \cdots \delta_{88} \end{bmatrix}, \mu = \begin{bmatrix} \mu_1 \\ \vdots \\ \mu_8 \end{bmatrix},$$

and Y*, X*, Π* and μ* are defined similarly to Y, X, Π and μ respectively.

Again X and X* can be simplified in terms of Kronecker products:

$$X = I \otimes \bar{X} \quad \text{and} \quad X^* = I \otimes \bar{X}^*. \tag{17}$$

and our contemporaneous covariances can be represented by:

$$V(\mu) = \Sigma \otimes I, \tag{18}$$

where $$\Sigma = \begin{bmatrix} E(\mu_1\mu_1) & \cdots & E(\mu_1\mu_1) \\ \vdots & & \vdots \\ E(\mu_8\mu_1) & \cdots & E(\mu_8\mu_8) \end{bmatrix} = \begin{bmatrix} \sigma_{11} & \cdots & \sigma_{18} \\ \vdots & & \vdots \\ \sigma_{81} & \cdots & \sigma_{88} \end{bmatrix}.$$

4.1. The Aitken Coefficient Estimator

4.1.1 The Unrestricted Model

We proceed to estimate the complete Π (and Π*) matrix(es) of (15) and (16) by the Generalized Least Squares (GLS) method such that:

$$\hat{\Pi} = [X'(\Sigma^{-1} \otimes I)\, X]^{-1}\, X'(\Sigma \otimes I)Y \tag{19}$$

with the following covariance matrix:[21]

$$V(\hat{\Pi}) = [X'(\Sigma^{-1} \otimes I)\, X]^{-1} \tag{20}$$

[21] See Theil (1971), pp. 306-311.

Here we do not have any a priori knowledge of the variance-covariance matrix, Σ . Thus we replaced Σ by a consistent estimator of Σ, S, from ordinary least squares residuals (which we call ρ_{mt}) as suggested by Zellner.[22]

$$S = \begin{bmatrix} s_{11} & \cdots & s_{18} \\ \vdots & & \vdots \\ s_{81} & \cdots & s_{88} \end{bmatrix}, \text{ where } \quad s_{mp} = \frac{1}{T-K} \sum_{t=1}^{T} \rho_{mt}\rho_{pt}$$

$m,p = 1 \ldots 8$,

T = number of observations,

K = number of explanatory variables.

It is well known that s_{mm} is an unbiased and consistent estimator of σ_{mm}, and it can be shown that $s_{mp} (m \neq p)$ is a consistent estimator of σ_{mp}. The resulting estimator of Π becomes:[23]

$$\hat{\Pi} = [X'(S^{-1} \otimes I) X]^{-1} X'(S^{-1} \otimes I)Y \tag{21}$$

with

$$\text{Assympt Var-Cov } (\hat{\Pi}) = [X'(S^{-1} \otimes I) X]^{-1}. \tag{22}$$

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[22]See Kmenta (1971), pp. 524-529. Since we are only concerned with consistency, we could use T instead of (T-K) in calculating the estimates of σ_{mp} without affecting the asymptotic properties of the estimator of $\hat{\Pi}$.

[23]This is sometimes called a two-stage Aitken estimator because its value is calculated in two stages. Some econometricians prefer a method of the so-called "Iterative Zellner Efficient." But it also can be argued that the iterative process is unnecessary because if S is consistent, $\hat{\Pi}$ is also consistent and the efficiency gains from iterating on S, with heavy computational burden, are not great. An equivalent comparison is 3SLS and FIML. 3SLS or a 3SLS type estimator do iterate because 3SLS type and GLS procedures are

4.1.2. The Restricted Model

We also want to test several hypotheses such as separability, homogeneity, symmetry and mixtures of these two that involve coefficients of different equations in the system, that is, we shall be interested in testing the linear constraint:

$$\gamma = R\,\Pi$$

where γ is a known q-element vector and R a known matrix of full row rank and of order $q \cdot K \cdot L$ where L is 8, the number of equations in the system.

This restricted estimator of $\tilde{\Pi}$ can be obtained:[24]

$$\tilde{\Pi} = \hat{\Pi} + CR'\lambda \;, \tag{23}$$

where

$$C = [X'(S^{-1} \otimes I)\; X]^{-1},$$

$$\lambda = [RCR']^{-1}\; (\gamma - R\,\hat{\Pi}),$$

with its covariance matrix as,

$$V(\tilde{\Pi}) = [C - CR'(RCR')^{-1}\; RC]. \tag{24}$$

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asymptotically efficient. See also Kment (1971) on the same asymptotic properties of the two-stage Aitken estimator as the maximum likelihood estimator.

[24]See Theil (1971), p. 308 and also see Wallace and Anderson.

In the actual estimation, a consistent estimation of Σ, S, also is replaced for Σ in (23).

4.2. Some Properties in the System of the Share Equations

4.2.1. The Case of Identical Explanatory Variables

As already noted in (17), we have the system of the share equations with the same explanatory variables. Therefore, the following relations hold;

$$C = [X'(\Sigma \otimes I)^{-1}X]^{-1} = [(I \otimes \bar{X}')(\Sigma^{-1} \otimes I)(I \otimes X)]^{-1}$$

$$= [\Sigma^{-1} \otimes (\bar{X}'\bar{X})]^{-1} = [\Sigma \otimes (\bar{X}'\bar{X})^{-1}], \text{ and}$$

$$\hat{\Pi} = [\Sigma \otimes (\bar{X}'\bar{X})^{-1}(I \otimes \bar{X}')(\Sigma^{-1} \otimes I)Y = [I \otimes (\bar{X}'\bar{X})^{-1} \bar{X}]Y. \qquad (25)$$

That is, the $\hat{\Pi}$ vector becomes identical to the Ordinary Least Squares (OLS) estimates.

4.2.2. Restrictions Implied in the Estimates

An intrinsic property of the share equations system, such that the sum of the factor's share is unity,

$$\sum_{i=1}^{5} S_i = 1.0 \qquad \text{and} \qquad \sum_{i=6}^{8} S_i = 1.0 \qquad (26)$$

implies the equivalent restrictions to linear homogeneity (to be discussed later) on the estimates of the system parameters such that:[25]

$$\sum_{i=1}^{5} \alpha_i = -1.0, \quad \sum_{i=6}^{8} \beta_i = 1.0, \quad \sum_{i=1}^{5} \gamma_{ij} = 0.0,$$

$$\sum_{i=1}^{5} \varepsilon_{ij} = 0.0, \quad \sum_{i=6}^{8} \xi_{ij} = 0.0, \text{ and } \sum_{i=6}^{8} \delta_{ij} = 0.0. \tag{27}$$

Here (26) implies that the disturbance terms in (15) must sum identically to zero. Thus the covariance matrix of the disturbance terms must be singular, and the systems estimation method of the GLS will not be operational. This difficulty has generally been overcome by deleting one of the share equations from the estimation procedure and by iterating the so-called Zellner procedure, provided that the parameter estimates with converging iteration are independent of which share equation is deleted.[26]

But as already discussed in the previous section, 4.2.1., regarding the joint GLS estimation with identical explanatory

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[25] The deviation of these conditions is shown in the Appendix A-II.

[26] Barten (1969, pp. 24-25) has shown that maximum likelihood parameter estimates of a system such as the one being considered are independent of which equation is deleted. Kmenta and Gilbert (1968) have shown that if one iterates the Zellner procedure, the parameter estimates (if they converge) will converge to maximum likelihood values. See Berndt and Christensen (1973) for further discussion of estimation procedures for translog share equations.

variables, the fact that the estimates of the joint GLS are equivalently identical to those of the OLS, allows us to get consistent estimates of the covariance matrix, S, from that of the OLS covariance matrix. This implies that the estimates of parameters in the unrestricted as well as the restricted cases are independent of the equation(s) deleted.[27]

Thus in the present study the two-stage Aitken estimation method is used in the place for the Iterative Zellner Efficient method in order to avoid heavy computational burden at the cost of probably negligible efficiency gains from the iterative procedure.[28]

4.3. Restrictions Considered

In the empirical estimations, we want to test several hypotheses, such as the combination of homogeneity of degree 1 and symmetry conditions, and explicit separability between input and output.

4.3.1. Homogeneity Restrictions

The sufficient conditions for homogeneity of degree 1 in the parameters of the share equations can be written as follows:

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[27]Differently from our case, the Berndt and Christensen result (1973) is specific to systems with autoregressive errors. If the errors are not autocorrelated (as in our cross section data, there is no reason to assume the errors are autocorrelated), the results become invariant to the choice of equation deleted.

[28]No efficiency gains from the iterative procedure are found in the empirical results (see Chapter IV).

$$\sum_{k=1}^{5} \alpha_k = -1.0, \quad \sum_{k=6}^{8} \beta_k = 1.0,$$

$$\sum_{k=1}^{5} \gamma_{ik} = 0.0, \quad \sum_{k=6}^{8} \varepsilon_{ik} = 0.0, \; i=1, \ldots 5, \tag{28}$$

$$\sum_{k=1}^{5} \xi_{jk} = 0.0, \quad \sum_{k=6}^{8} \delta_{jk} = 0.0, \; j=6, \ldots 8.$$

Thus the total number of restrictions become 18. But in the formulation of the matrix R, only the number of 12 restrictions are counted in the actual estimating equations system.[29]

4.3.2. Symmetry Restrictions

The symmetry restrictions can be described straight forwardly:

$$\gamma_{ij} = \gamma_{ji}, \; i,j = 1, \ldots 5,$$

$$\varepsilon_{ij} = \xi_{ji}, \quad i = 1, \ldots 5, \; j = 6, \ldots 8, \tag{29}$$

and

$$\delta_{ij} = \delta_{ji}, \; i,j = 6, \ldots 5.$$

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[29]Since one of the share equations among inputs and among outputs is deleted respectively in the actual estimating equations system, the first two restrictions on the α_i and β_i are not counted. Furthermore, among the rest four sets of conditions, each one of them is not counted. Hence, 6 restrictions as a total become eliminated in the actual estimation from two deleting equations.

The number of the restrictions here becomes 28, but only 15 restrictions are encountered in the matrix R.[30]

4.3.3. Strong Separability Between Inputs and Outputs

The strong separability conditions are equivalent to assuming that:

$$\varepsilon_{ij} = 0.0 \qquad i = 1, \ldots 5,$$

and (30)

$$\xi_{ji} = 0.0 \qquad j = 6, \ldots 8.$$

The estimation of the system with this restriction can be done separately in the following joint GLS estimation without forming any restriction such as $\gamma = R\Pi$;

$$S_i = \alpha_i + \sum_{j=1}^{5} \gamma_{ij} \ln X_j \,, \quad i=1, \ldots 5, \tag{31}$$

and

$$S_i = \beta_i + \sum_{j=6}^{8} \delta_{ij} \ln Y_j \,, \quad i=6, \ldots 8. \tag{32}$$

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[30] When there are M inputs and N outputs, the total number of restrictions are counted by the following relation, i.e., $\frac{1}{2}M(M-1) + \frac{1}{2}N(N-1) + \frac{1}{2}MN$. But in actual estimation with those deleting equations, it becomes $\frac{1}{2}(M-1)(M-2) + \frac{1}{2}(N-1)(N-2) + (M-1)(N-1)$. Therefore in our case it is 15. This is because when we estimate the restricted model we only take (M-1) input equations and (N-1) output equations to estimate jointly. See the section 4.2.2. Also see Berndt & Christensen (1973) and Brown, Caves and Christensen (1975).

4.4.1. Likelihood Ratio Test

The likelihood ratio, $\Lambda = \frac{\max_{\omega} L}{\max_{\Omega} L}$, depends on the maximum value of the likelihood function for the unrestricted (Ω) system and that of the likelihood function of the system subject to the restriction (ω). The test statistic for each set of restrictions is based on minus twice the logarithm of the likelihood ratio

$$-2 \ln \Lambda = T[\ln |\hat{\Sigma}_{\omega}| - \ln |\hat{\Sigma}_{\Omega}|] \tag{33}$$

where T is the number of data points,

$|\hat{\Sigma}_{\omega}|$ is the determinant of the restricted estimate of the covariance matrix, and

$|\hat{\Sigma}_{\Omega}|$ is the determinant of the untrestricted estimate.

Under the null hypothesis $-2 \ln \Lambda$ is distributed asymptotically as a chi-square with the degree of freedom equal to the number of restrictions being tested.[31]

4.4.2. F Test for Small Sample

For the hypothesis $\gamma = R\Pi$, where r and R have q rows, R has rank q, and Π is the parameter vector of the share equations system, two χ^2 variates can be considered with the M+N-normal variates (μ_i), one being $\chi^2_{(q)}$ if the null hypothesis is true:[32]

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[31] Edward (1972) and Seber (1966).

[32] The test statistics is known as the Wald for testing linear restrictions on the coefficients of certain linear models (Wald 1943).

$$(\gamma - R\tilde{\Pi})' \{R[X'(\Sigma^{-1} \otimes I)X]^{-1} R'\}^{-1} (\gamma - R\tilde{\Pi}) \tag{34}$$

and the other being

$$(Y - X\tilde{\Pi})' (\Sigma^{-1} \otimes I) (Y - X\tilde{\Pi}), \tag{35}$$

having $(M+N)\cdot T-K\cdot(M+N) = D$ (say) degrees of freedom, where K is the number of the unknown parameters in each equation. The statistic is equal to the ratio of (34) and (35) multiplied by D/q, and its distribution is F(q,D) if $\gamma = R\Pi$ is true.

We replaced the unknown Σ by S (and hence $\tilde{\Pi}$ by $\hat{\Pi}$) in both (34) and (35). For (34) we thus use:

$$(\gamma - R\hat{\Pi})' \{R[X' (S^{-1} \otimes I) X]^{-1} R'\}^{-1} (\gamma - R\hat{\Pi}). \tag{36}$$

Since the quadratic form (34) is continuous in Σ, and S is a consistent estimator, the substitute form (36) converges in distribution to (34), so that its limiting distribution is $\chi^2_{(q)}$ under the null hypothesis and the normality condition.

By dividing the quadratic form (35) by the number of degrees of freedom (D), we obtain a ratio which converges in probability to 1 as T (and hence also D) increases indefinitely.[33] Since the form (35) is a continuous function of Σ, the corresponding fraction $\frac{1}{D}$ of the substitute form,

[33] See Theil (1971), pp. 143-144, pp. 313-314, pp. 402 - 403.

$$(Y - X\hat{\Pi})' (S^{-1} \otimes I) (Y - X\hat{\Pi}) \tag{37}$$

also converges in probability to 1.

The ratio of (36) and (37) multiplied by D/q has its limiting distribution, $(1/q)\ \chi^2_{(q)}$, under the null hypothesis and the normality condition. Instead of this limiting distribution we use F(q,D). This makes no difference asymptotically, since F(q,D) converges in distribution to $(1/q)\ \chi^2_{(q)}$ as $D \to \infty$. For finite D the F approximation is more cautious than the χ^2 approximation because it gives a negative verdict on the null hypothesis in a smaller number of cases. This cautious attitude is preferable since the procedure implies that the value of the quadratic form (37) may just as well be replaced by its expectation (D).[34]

4.4.3. Other Statistics

One other statistic, R^2, was investigated in the restricted model but in vain. Under the OLS, R^2 can be defined:

$$\hat{R}^2 = 1 - \frac{e'e}{y'y} = \frac{\hat{y}'\hat{y} - T\bar{y}^2}{y'y - T\bar{y}^2} \quad ,$$

[34] Alternative procedures for testing linear restrictions on the coefficients of a linear regression model may produce conflicting decisions. In the cases where conflict among tests arises, the conflict may be resolved if one test can be shown to be more powerful than the others. But, in practice, economic theory almost invariably suggests a range of alternative hypotheses, i.e., tests of composite rather than simple hypothesis. It is well known that when testing a composite hypothesis there may be no uniformly most powerful test. At present, relatively little is known about the comparative power of various alternative criteria. Some recent researches on this topic have been done. See N. E. Savin (1966) and T. S. Breusch (1977).

as far as there exists orthogonality between e'e and y'y. But in the restricted model, we can no longer expect this nice property nor use the above definitional relation in order to get the multiple correlation coefficients.[35] But in the present work, in spite of all these deficiencies we try to measure the portion of the error sum of squares in a total sum of squares by calculating our substitute for R^2 in the restricted model as in (38). Another statistic for testing the significance of Lagrangian multipliers, λ_i in (23), is computed for the standard error of λ_i as follows:

$$SE(\hat{\lambda}_i) = [RCR']^{-1}_{ii}, \tag{39}$$

where

$$V(\hat{\lambda}) = [RCR']^{-1} .$$

Hence the Student-t values for the null hypotheses of λ's are calculated, assuming that the Lagrangians are asymptotically

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[35]Although it was not attempted, there is an interesting definition for the case with no constant term in the regression equation, which also does not have the orthogonality property, such that:

$$\tilde{R}^2 = \frac{\hat{y}'\hat{y} - T\tilde{y}^2}{y'y - T\bar{y}^2} ,$$

where T = the number of observations
$\tilde{y}$ = the sample mean of the estimated values of y.
But this concept, unfortunately, does not have any comprehensive correspondence, in the sense that $\tilde{R}^2$ does not generate R^2 for the case with the constant term. See The Wharton School of Finance and Commerce, University of Pennsylvania (1973).

normal, where the degrees of freedom on the Lagrangians is the large sample number, $N = 30$.[36]

B.3.5. Monte Carlo Experiments

In the connection with the estimating methods it is worthwhile to note the two following Monte Carlo experiments, the first done recently by Byron (1977) and the second more specifically designed and simulated in this study.

5.1. The Truncation Error in the TRANSLOG Approximation

The TRANSLOG approximation is based on a truncated Taylor series expansion with an excluded and unknown remainder term. That is, using production theory the unknown production function is approximated by a second order Taylor series expansion. Since the production function is monotonic with nonnegative inputs it may be expressed logarithmically, and after appropriate scaling the Taylor series expansion may be taken around zero. This step is intended to minimize higher order terms and the convergencies of the series. The log quadratic Taylor series expansion is linear in the unknown parameters and these parameters, which are simply the derivatives evaluated at zero, enable inferences to be made about the characteristics of the underlying production function.

To illustrate, consider any general logarithmic production function with 2 inputs,

[36]The reason is that the small sample theory has not been worked out and we are assuming the Lagrangians are asymptotically normal, consequently $N \geq 30$.

$$\ln y = f[\ln X_1, \ln X_2] \tag{40}$$

The quadratic Taylor series approximation has the general form

$$Z = Z_o + \frac{\partial Z}{\partial \phi}(\phi-\phi_o) + \tfrac{1}{2}(\phi-\phi_o)\frac{\partial^2 Z}{\partial\phi\partial\phi'}(\phi-\phi_o) + \text{HOT},$$

where (41)

$$Z = \ln y,\ Z_o = f[\ln X_o],\ \phi = \ln X,\ \phi_o = \ln X_o\ .$$

HOT is called "the remainder term" consisting of the third and higher order terms. The derivatives are evaluated at ϕ_o corresponding to $X_o = [1]$. Thus

$$Z = \alpha_o + \sum_{i=1}^{2} \alpha_i \ln X_i + \tfrac{1}{2}\sum_{i=1}^{2}\sum_{j=1}^{2} \gamma_{ij} \ln X_i \ln X_j + \text{HOT}. \tag{42}$$

In the above,

$$\alpha_o = Z_o,\ \alpha_i = \frac{\partial \ln y}{\partial \ln X_{io}},\ \gamma_{ij} = \frac{\partial^2 \ln y}{\partial \ln X_{io}\,\partial \ln X_{jo}},$$

and HOT can be expressed, due to Cauchy,[37] as follows:

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[37]Due to Lagrange, the another form of the third order term can be written

$$\text{HOT}_3 = \frac{1}{6}\sum_i\sum_j\sum_k \left.\frac{\partial^3 Z}{\partial \ln X_i\,\partial \ln X_j\,\partial \ln X_k}\right|_{X=X^*} \ln X_i \ln X_j \ln X_k,$$

where lnX* is some number between zero and lnX.

$$\mathrm{HOT}\ (\ln X) = \tfrac{1}{6} \sum_k \sum_j \sum_k \left. \frac{\partial^3 Z}{\partial \ln X_i \ \partial \ln X_j \ \partial \ln X_k} \right|_{X=X_0} \ln X_i \ \ln X_j \ \ln X_k + \overline{\mathrm{HOT}}\ (\ln X), \tag{43}$$

where $\overline{\mathrm{HOT}}$ consists of the fourth and the higher order terms. Here it is not possible to make analytic statements about the TRANSLOG approximation because the remainder term (HOT) cannot be expressed with mathematical exactness even with some additional constraints on the production function concerned, unless we can find some relationship between the first, the second, and the higher order terms.

The statistical bias in the estimation from this specification error can be shown as follows by comparing the estimates of the true equation with the present estimating one:

The true least squares estimates of β in the equation,

$$y = X\beta + R + \mu \tag{44}$$

where R is HOT may be written

$$\hat{\beta} = (X'X)^{-1}\ X'(y - R), \tag{45}$$

and the bias is obviously

$$E(\hat{\beta}) - \beta = (X'X)^{-1}\ X'R \tag{46}$$

What does emerge is that the higher the correlation between X and R the larger will be the bias in the least squares estimates of β. It is true, however, that this will be moderated by the magnitude of R and, in particular, by the moment of $X'R$. Since it is not sufficient to argue that if R is of a small order of magnitude its effect on the bias in the parameter estimates is negligible, the results of the Monte Carlo experiments reported by Byron (1977, p. 18) should be noted, "Direct estimation of the production function was found to be inferior to indirect estimation based on the first order conditions. What emerged in both cases is the indisputable results that the parameter estimates are biased, but not seriously; . . . To the author the translog procedure emerged from these experiments better than anticipated, especially in relation to the estimates of the elasticity of substitution."

5.2. Experiments on the TRANSLOG Approach with the Existence of the Zero-Valued Variables

As noticed in the previous section, the closeness of the TRANSLOG approximation depends on the proximity of the quantities of the inputs (X_i's) to unity. There is no general limit to the approximation error incurred in such power series expansion. However, as one is free to choose the scaling of the measurement of the inputs, one can minimize the approximation error by setting the sample means of the X_i's at unity.[38]

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[38]It is worthwhile to note here that the choice of the sample means in the normalization of the variables, such as between the arithmetic and the geometric means, does not affect the coefficients of the first order terms (i.e., constant terms in the share equations). But does it not affect the second order terms of the approximation from the different dispersion (or deviations) under different sample means?

Note also that the use of the equations (15) and (16) depends on the quantities of the inputs being strictly positive, otherwise the expression is not well defined. Aside from the case of a uni-product production technology, the micro-reality of a multiproduct technology does, almost everywhere, allow a situation where some sample establishments do produce some of the products produced in the industry but not all of them. Thus the reality of the system construction necessarily bring us the model(s) of (15) and (16) with the zero-valued variables in the cases of more than one product. There have been suggested two general methods available to circumvent this problem:

(1) One can insert nonnegative nonhomogeneity parameters and redefine the product such that

$$Z_i = X_i + X_i^o \tag{47}$$

(2) One can construct a new variable as a sub-aggregate of several original variables having the form of

$$Z_i = \sum_{j=1}^{N} \beta_{ij} X_{ij}^{\delta_{ij}} \tag{48}$$

where X_{ij} is the j-th kind of product in the i-th category of products[39] and N is the number of products classifiable into the i-th group of commodity.

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[39] Jorgenson, Christensen, and Lau (1970).

In the first method, the X_i^o's, of course, must be either known a priori or estimated by nonlinear methods and the concept of nonhomogeneity parameters may arise in the theory of consumption as minimum quantities at a subsistence level, measured in the unit (such as calory) of characteristics of commodities. However, in the theory of multiproduct production the corresponding concept seems to be hard to define, except for by-products common to all establishments in the industry.

The second method seems to be very useful particularly in the present study, i.e., in constructing a quantity index for the nonmajor commodities.[40] In general, we do need a priori knowledge of β_{ij} and δ_{ij} in this aggregation, which will in turn be studied through investigation of a technology. Hence, in the present study we assume $\delta_{ij} = 1$ for the nonmajor commodities.[41] Nevertheless, in the reality of a multiproduct technology, this problem is encountered in the variables of the major products for some sample establishments.

Allowing the zero-valued variables in the estimating TRANSLOG share equations, by replacing certain negligible small figures, the Monte Carlo experiments here were set up under the statistical

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[40]Chemical fertilizer is a kind of fertilizer, for instance.

[41]If one tries not to distinguish the major and the nonmajor commodity groups and tries only to avoid zero-valued products for all establishments then almost every industry turns out to be the case of uniproduct technology. See Chapter II on this sub-aggregation of the structure of industry's multiproduct production.

assumptions made by the proponents of the translog procedure to examine its performance under their conditions.

The TRANSLOG production equation below,

$$\ln V = \alpha_o + \sum_{i=1}^{2} \alpha_i \ln X_i + \sum_{i=j}^{2} \sum_{j=1}^{2} \gamma_{ij} \ln X_i \ln X_j + e \quad (49)$$

was based on an underlying CES production function,

$$V = \alpha[\delta_1 X_1^{-\rho} + \delta_2 X_2^{-\rho}]^{-\mu/\rho} \text{ with } \delta_1 + \delta_2 = 1. \quad (50)$$

Since $\alpha_i = \frac{\partial \ln V}{\partial \ln X_i}$ and $\partial_{ij} = \frac{\partial \ln V}{\partial \ln X_i \, \partial \ln X_j}$ the derivatives for the CES function at ln X = [1] are $\alpha_i = \mu\, \delta_i$, $\gamma_{ii} = \mu\rho\delta_i(\delta_i - 1)$ and

$$\gamma_{ij} = \mu\, \rho\, \delta_i\, \delta_j\, . \quad (51)$$

Based on the first order conditions, we have the following share equations to be estimated,

$$P_i = \frac{\partial V}{\partial X_i} \quad (52)$$

$$\frac{P_i X_i}{V} = \frac{\partial \ln V}{\partial \ln X_i} \quad (53)$$

$$S_i = \alpha_i + \sum_{j=1}^{2} \gamma_{ij} \ln X_j + e, \; i=1,2\, . \quad (54)$$

To generate data for this model X_i and V are assumed exogeneous and the shares, S_i's, are endogeneous. In other words, the systematic part of the shares, based on a CES production function, is

$$S_1 = \mu\delta_1 \alpha^{(-\rho/\mu)} V^{\rho/\mu} X_1^{-\rho}, \tag{55}$$

$$S_2 = \mu\delta_2 \alpha^{(-\rho/\mu)} V^{\rho/\mu} X_2^{-\rho}. \tag{56}$$

Given $S_1 + S_2 = 1$, the disturbance has to be introduced with the property $e_1 = -e_2$. X_1 and X_2 were first generated as uniformly distributed variables, secondly the zero-valued observations with various frequencies were inserted in X_1 and X_2 separately and finally they were transformed into random normal variates. Also the generated X_1 and X_2 were held as fixed regressors in repeated samples and experiments. V was generated using (49). The systematic part of the share equations was then generated using (55) and (56) and the disturbance introduced additively with a pre-specified signal-noise ratio. The TRANSLOG estimates were obtained by applying generalized least squares (GLS) to (54) with the usual singular covariance adjustment as described in the previous section.

The characteristics of the CES parameters, the translog parameters at $X_i = 1$, the exogenous variables and the disturbances were as follows:

CES Parameters;

$$\alpha = 1,\ \delta_i = 0.4,\ \delta_2 = 0.6,\ \rho = -0.5,\ \mu = 1.$$

Translog Parameters;

$$\alpha_i = 0.4, \quad \alpha_2 = 0.6, \gamma_{11} = 0.12, \gamma_{12} = -0.12, \gamma_{22} = 0.12 \ .$$

Exogenous Variables;

Means: $\bar{X}_1 = 20000$, $\bar{X}_2 = 32000$.

Covariance structure $(\times \ 10^5)$:

$$\Sigma_{LV} = \begin{bmatrix} 0.9 & 0.5 \\ 0.5 & 1.6 \end{bmatrix}, \Sigma_{MV} = \begin{bmatrix} 9.0 & 5.0 \\ 5.0 & 16.0 \end{bmatrix}, \Sigma_{HV} = \begin{bmatrix} 160. & 70. \\ 70. & 360. \end{bmatrix}.$$

Disturbances:

$$R^2_{LN} = 0.99, R^2_{MN} = 0.95, R^2_{HN} = 0.80 \ .$$

The exogenous variables before scaling were generated with the above means and covariance matrices: LV - low variability, MV = medium variability and HV = high variability. The variance of the disturbances was related to the variance of the systematic part of the dependent variables in order to correspond to the R^2 indicated above: again LN = low noise, MN = medium noise, HN = high noise. All the results were based on 50 replications with a sample size of 50.

The results at the three levels of variability for the exogeneous variables and at the three different noise levels in the nine combinations are given in Table V-1, where the zero-valued

TABLE V-1.--Mean and Standard Errors of Translog Estimates of CES Parameters[a]

BASE (3, 8) and (1.0×10^{-20})[b]

	α_1	γ_{11}	γ_{12}	α_2	γ_{21}	γ_{22}
Population	0.4	0.12	-0.12	0.6	-0.12	0.12
LV/LN	0.400445	0.120027	-0.120007	0.599555	-0.120027	0.120007
	(0.2970-2)	(0.1492-3)	(0.2506-3)	(0.2970-2)	(0.1492-3)	(0.2506-3)
LV/MN	0.400994	0.120059	-0.120015	0.599006	-0.120059	0.120015
	(0.6640-2)	(0.3337-3)	(0.5604-3)	(0.6604-2)	(0.3337-3)	(0.5604-3)
LV/HN	0.401989	0.120119	-0.120030	0.598011	-0.120119	0.120030
	(0.1328-1)	(0.6673-3)	(0.1121-2)	(0.1328-1)	(0.6673-3)	(0.1121-2)
MV/LN	0.400445	0.120027	-0.120007	0.599555	-0.120027	0.120007
	(0.2375-2)	(0.1492-3)	(0.2506-3)	(0.2975-2)	(0.1492-3)	(0.2506-3)
MV/MN	0.400995	0.120059	-0.120015	0.599005	-0.120059	0.120015
	(0.6651-2)	(0.3337-3)	(0.5604-3)	(0.6651-2)	(0.3337-3)	(0.5604-3)
MV/HN	0.401990	0.120119	-0.120030	0.598010	-0.120119	0.120030
	(0.1330-1)	(0.6674-3)	(0.1121-2)	(0.1330-1)	(0.6674-3)	(0.1121-2)
HV/LN	0.400445	0.120027	-0.120007	0.599555	-0.120027	0.120007
	(0.2977-2)	(0.1493-3)	(0.2507-3)	(0.2977-2)	(0.1493-3)	(0.2507-3)
HV/MN	0.400995	0.120059	-0.120015	0.599005	-0.120059	0.120015
	(0.6658-2)	(0.3338-3)	(0.5605-3)	(0.6658-2)	(0.3338-3)	(0.5605-3)
HV/HN	0.401990	0.120119	-0.120030	0.598010	-0.120119	0.120030
	(0.1332-1)	(0.6676-3)	(0.1121-2)	(0.1332-1)	(0.6676-3)	(0.1121-2)

[a]In each column of the Table the mean value of the estimates from 50 replications of the estimation with a sample size of 50 random observations. And the standard error of their distribution is specified in the parenthesis respectively, the specification of which is a string of decimal digits written with a decimal point and with an exponent. For example, the standard error of the first element in the first column of the table, i.e., 0.2970-2, is equivalent to the value of 0.002970 = 0.2970×10^{-2}. In the first column, "population" means the population value of each parameter to be estimated among sample observations. And LV, MV, and HV mean respectively the low, medium, and high variability of the exogeneous variables. And LN, MN, and HN imply the low, medium, and high noise level in disturbance respectively.

[b]Base (3.8) here means that there exist the zero-valued quantities of the first exogeneous variables, X_1, with the frequency of 33.33% among total observations (i.e., 1/3 = 0.3333) and those of the second variable, X_2, with the frequency of 12.50% (i.e., 1/8 = 0.1250). (1.0×10^{-20}) implies that the zero-valued quantities are replaced, in the estimation, by the small number of 1.0×10^{-20}.

quantities of the first exogeneous variable, X_1, with the frequency of 33.33% and those of the second, X_2, with the frequency of 12.50% were replaced by the negligible figure of 1.0×1.0^{-20}.[42] What does emerge from the table is, as can be expected, that various degrees of noise level result in rather different estimates of the parameters where the differences in the first order terms (i.e., α_1 and α_2) are bigger than those in the second order terms (i.e., γ_{11}, γ_{12}, γ_{21}, and γ_{22}) but they are not significant in average. On the other hand, higher degree of variability in the exogeneous variables does not have any significant effects on the parameter estimates but a bigger stand error, even if they still seem to be negligible. For example, in the estimate of α_1, its population value is 0.40000. Its estimate in the case of low variability and of low noise level is 0.40045 with the standard error of 0.00297, while its estimate in the case of high variability and of high noise level is 0.40199 with the standard error of 0.01332.

The results of a sensitivity test with the replacements of alternative small figures for the zero-valued quantities do not show any meaningful significance both in the means and the standard error of parameters to be estimated. As shown in Table V-2. the estimates of α_1 in the case of high variability and of high noise

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[42] In the computer simulation program, the base number set (3.8) was used for generating the zero-valued variables, that is, 33.33% was based on the number 3, i.e., 1/3.

TABLE V-2.--Mean and Standard Errors of Translog Estimates of CES parameters:[a] BASE (3, 8) and $(1.0 \times 10^{-20}, 1.0 \times 10^{-60})$.[b]

	α_1	γ_{11}	γ_{12}	α_2	γ_{21}	γ_{22}
Population	0.4	0.12	-0.12	0.6	-0.12	0.12
LV/LN/ - 20	0.400445 (0.2970-2)	0.120027 (0.1492-3)	-0.120007 (0.2506-3)	0.599555 (0.2970-2)	-0.120027 (0.1492-3)	0.120007 (0.2506-3)
LV/LN/ -60	0.400445 (0.2968-2)	0.120010 (0.5939-4)	-0.120003 (0.9520-4)	0.599555 (0.2968-2)	-0.120010 (0.5630-4)	0.120003 (0.9520-4)
LV/HN/ -20	0.401989 (0.1328-1)	0.120119 (0.6673-3)	-0.120030 (0.1121-2)	0.598011 (0.1328-1)	-0.120119 (0.6673-3)	0.120030 (0.1121-2)
LV/HN/ -60	0.401988 (0.1327-1)	0.120045 (0.2522-3)	-0.120011 (0.4258-3)	0.598012 (0.1327-1)	-0.120045 (0.2522-3)	0.120011 (0.4258-3)
HV/LN/ -20	0.400445 (0.2977-2)	0.120027 (0.1493-3)	-0.120007 (0.2507-3)	0.599555 (0.2977-2)	-0.120027 (0.1493-3)	0.120007 (0.2507-3)
HV/LN/ -60	0.400445 (0.2971-2)	0.120010 (0.5640-4)	-0.120003 (0.9521-4)	0.599555 (0.2971-2)	-0.120010 (0.5640-4)	0.120003 (0.9521-4)
HV/HN/ -20	0.401990 (0.1332-1)	0.120119 (0.6676-3)	-0.120030 (0.1121-2)	0.598010 (0.1332-1)	-0.120119 (0.6676-3)	0.120030 (0.1121-2)
HV/HN/ -60	0.401989 (0.1329-1)	0.120045 (0.2522-3)	-0.120011 (0.4258-3)	0.598011 (0.1329-1)	-0.120045 (0.2522-3)	0.120011 (0.4258-3)

[a]See the footnote (a) in the Table V-1. Also the integers appearing in the first column (i.e., -20 and -60) designating the exponents of the small numbers replaced, in these experiments, for the zero-valued quantities.

[b]See the footnote (b) in the Table V-1.

level are 0.401990 and 0.401989, with the small figures of 1.0^{-20} and 1.0^{-60} respectively.

Table V-3 contains those of a sensitivity test with various frequencies of the zero-valued quantities in X_1, such as 12.50%, 16.67%, 25.00% and 50.00% when that in X_2 is fixed at the 10.00% level. The existence of the zero-valued quantities with various frequencies results in quite a significant difference both in the means and the standard errors of the parameter estimates. In particular, if there exist the zero-valued quantities in more than 50% of total observation, then the standard error of the estimates becomes bigger than the mean of the parameter estimates. In general the higher level of noise has quite a significant effect on the standard error of the estimates, while the higher level of variability may offset the effects of the high frequency of the existing zero-valued quantities in the observations. For example, the average of the estimates of γ_{12} in the Table V-3, is measured as -0.11712 with the standard error of 0.2121 in the case of low variability and of low noise level, while it is -0.10712 with that of 0.9487 in the case of high noise level and of the same low variability. But it is -0.11976 with the standard error of 0.0415 in the case of high variability and of low noise level, and further, it is improved as -0.11891 with the standard error of 0.0186 in the case of high variability and of high noise level (see the figures with the asterisk, *, in the Table V-3).

Lastly, the effects of the existence of the zero-valued quantities on the estimated elasticities of factor substitution

TABLE V-3.--Mean and Standard Errors of Translog Estimates of CES Parameters:[a] Base (8, 10), (6, 10), (4, 10), (2, 10), and (1.0×10^{-70}).[b]

Population	α_1 0.4	γ_{11} 0.12	γ_{12} -0.12	α_2 0.6
LV/LN/(8, 10)	0.400012 (0.1537-2)	0.120000 (0.5793-4)	-0.120000 (0.7268-4)	0.599988 (0.1537-2)
LV/LN/(6, 10)	0.400089 (0.1710-2)	0.120003 (0.4877-4)	-0.120000 (0.7868-4)	0.599911 (0.1710-2)
LV/LN/(4, 10)	0.399779 (0.2177-2)	0.119997 (0.4472-4)	-0.120008 (0.8744-4)	0.600221 (0.2177-2)
LV/LN/(2, 10)	0.399231 (0.2646-2)	0.119991 (0.3253-4)	-0.117120 (0.2121-0) (*)	0.600769 (0.2646-2)
LV/HN/(8, 10)	0.400053 (0.6873-2)	0.120001 (0.2591-3)	-0.119998 (0.3250-3)	0.599947 (0.6873-2)
LV/HN/(6, 10)	0.400397 (0.7649-2)	0.120015 (0.2181-3)	-0.120001 (0.3519-3)	0.599603 (0.7649-2)
LV/HN/(4, 10)	0.399012 (0.9736-2)	0.119985 (0.1200-3)	-0.120036 (0.3911-3)	0.600988 (0.9736-2)
LV/HN/(2, 10)	0.396560 (0.1183-1)	0.119959 (0.1455-3)	-0.107120 (0.9487-0) (*)	0.603440 (0.1183-1)
HV/LN/(8, 10)	0.400012 (0.1540-2)	0.120000 (0.5798-4)	-0.120000 (0.7271-4)	0.599988 (0.1540-2)
HV/LN/(6, 10)	0.400089 (0.1713-2)	0.120003 (0.4880-4)	-0.120000 (0.7870-4)	0.599911 (0.1713-2)
HV/LN/(4, 10)	0.399779 (0.2179-2)	0.119997 (0.4472-4)	-0.120008 (0.8742-4)	0.600221 (0.2179-2)
HV/LN/(2, 10)	0.399226 (0.2640-2)	0.119991 (0.3231-4)	-0.119757 (0.4152-1) (*)	0.600774 (0.2640-3)
HV/HN/(8, 10)	0.400053 (0.6887-2)	0.120001 (0.2593-3)	-0.119998 (0.3252-3)	0.599947 (0.6887-2)
HV/HN/(6, 10)	0.400398 (0.7661-2)	0.120015 (0.2182-3)	-0.120001 (0.3520-3)	0.599602 (0.7661-2)
HV/HN/(4, 10)	0.399011 (0.9745-2)	0.119985 (0.1200-3)	-0.120036 (0.3910-3)	0.600989 (0.9745-2)
HV/HN/(2, 10)	0.396540 (0.1181-1)	0.119959 (0.1445-3)	-0.118914 (0.1857-0) (*)	0.603460 (0.1181-1)

[a]See the footnote (a) in Table V-1. And the parentheses in the first column of this table is the Base set applied in this experiment.

[b]See the footnote (b) in Table V-1.

are negligible in most cases, except in special cases where there exist significantly high frequency of the zero-valued quantities among total observations, say 50%, in more than half of all explanatory variables.[43] Here also what really matters is not much of the degree of variability in the exogeneous variables, but more of the variance of disturbance, in estimating elasticities of substitution, as shown in Table V-4.

In summary, these Monte Carlo experiments are very informative in making more correct inferences on the parameter estimates and the estimated elasticities of factor substitution. First, the degree of variability in the variables tends to affect the standard errors of the parameter estimates, while the noise level affects the size of the parameter estimates. But both their effects seem to be negligible in our experiments. Second, the replacement of alternative small figures for the zero-valued quantity in the observations does not have any significant effects on neither the parameter estimates nor their standard errors. Finally, the existence of the zero-valued quantities in the observations does not have, in most cases, significant effects on the parameter estimates and the

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[43]In this experimentation, the function with more than two exogeneous variables was not intended at the beginning, so that there seems to be no evidence on how many variables in total exogeneous variables do affect significantly the parameter estimates and the estimated elasticities of substitution. But here since one out of two variables had more than 50% of zero-valued quantities in its total observation, a simple statement is made on the number of variables with zero-valued quantities, i.e., more than half of total exogeneous variables.

TABLE V-4.--Estimated Elasticities of Substitution.[a]

	σ_{11}	σ_{12}	σ_{22}
LV/LN/Population	-4.04673	2.13212	-1.12336
(8, 10)	-4.04673	2.13212	-1.12336
(6, 10)	-4.04682	2.13216	-1.12338
(4, 10)	-4.04675	2.13213	-1.12337
(2, 10)	-7.24654(*)	3.81802(*)	-2.01162(*)
LV/HN/Population	-4.04673	2.13212	-1.12336
(8, 10)	-4.04674	2.13213	-1.12336
(6, 10)	-4.04713	2.13233	-1.12347
(4, 10)	-4.04684	2.13218	-1.12339
(2, 10)	-6.41383(*)	3.37929(*)	-1.78046(*)
HV/LN/Population	-4.08140	2.13687	-1.11879
(8, 10)	-4.08140	2.13687	-1.11879
(6, 10)	-4.08149	2.13692	-1.11881
(4, 10)	-4.08143	2.13688	-1.11879
(2, 10)	-4.15790	2.17692	-1.13976
HV/HN/Population	-4.08140	2.13687	-1.11879
(8, 10)	-4.01842	2.13688	-1.11879
(6, 10)	-4.08181	2.13708	-1.11890
(4, 10)	-4.08152	2.13693	-1.11882
(2, 10)	-11.5526 (*)	6.04853(*)	-3.16679(*)

[a]See the footnote (a) in Table V-1 and also the footnote (a) in Table V-3.

estimated elasticities of factor substitution, unless their frequencies are extraordinarily high in many of the exogeneous variables.

CHAPTER IV. EMPIRICAL ESTIMATION OF THE TRANSCENDENTAL LOGARITHMIC PRODUCTION FUNCTION AND THE TRANSCENDENTAL LOGARITHMIC COST FUNCTION

B.4.1. Introduction

The results of empirical estimation are presented for the six industries selected in this chapter. Main results are discussed in terms of the parameter estimates, the significances of current estimations such as the multiple correlation coefficients (R^2) and test statistic for the restrictions imposed, the properties of the translog production and the translog cost function such as the monotonicity and convexity conditions, the output elasticities of factor inputs, and the estimated elasticities of substitution among inputs and outputs.

Supplementary results contain the comparison of separate estimations by the size of establishments within each industry, the comparison of alternative estimations under different inclusion rules applied on sample establishments within each industry, and the comparison of alternative estimations with the explanatory variables of different quality. But only few of the conclusions from each experimental estimation results are included in this volume, disregarding the details of the parameter estimates, significance of estimation and other analyses such as ES, etc.

The production structures of six manufacturing industries at the 5-digit industry classification level are investigated in terms of five inputs, such as the man-day operative workers, the man-day administrative workers, the power equipments utilized, the fuel consumptions and the raw materials and in terms of a number of products, which varies with the corresponding industries.

Industries chosen in empirical estimations are the canning industry, the leather footwear industry, the screw products industry, the manufacture of knitted underwear, the manufacture of briquettes, and the molding industry, where some of their industrial characteristics are already speculated in the earlier chapter II. Among these industries, the first three industries produce more than one commodity (i.e., three commodities in the canning industry, and two commodities in the leather footwear and in the screw products industry) and the rest three industries produce only one commodity.

In order to facilitate interpretation of our results, we adopt subscripts which represent the three outputs, denoted by Y, and the five inputs, denoted by X. We use 1 for the first major commodity, 2 for the second, and 3 for the third major commodity in each industry. The inputs are represented by P for the operative worker, A for the administrative worker, K for the capital input, F for the fuel (or energy), and R for the raw material input.

B.4.2. Main Results of the Empirical Estimation

2.1. The Parameter Estimates

The parameter estimates of the multi-input multi-output share equations systems derived from the translog production function are presented in Tables VI-1-a through VI-3-f, according to the following specification of the estimating equations,[1]

$$-S_P = \alpha_P + \gamma_{PP}\ln X_P + \gamma_{PA}\ln X_A + \gamma_{PK}\ln X_K + \gamma_{PF}\ln X_F + \gamma_{PR}\ln X_R$$
$$+ \varepsilon_{P1}\ln Y_1 + \varepsilon_{P2}\ln Y_2 + \varepsilon_{P3}\ln Y_3 \qquad (1)$$

$$-S_A = \alpha_A + \gamma_{AP}\ln X_P + \gamma_{AA}\ln X_A + \gamma_{AK}\ln X_K + \gamma_{AF}\ln X_F + \gamma_{AR}\ln X_R$$
$$+ \varepsilon_{A1}\ln Y_1 + \varepsilon_{A2}\ln Y_2 + \varepsilon_{A3}\ln Y_3 \qquad (2)$$

$$-S_K = \alpha_K + \gamma_{KP}\ln X_P + \gamma_{KA}\ln X_A + \gamma_{KK}\ln X_K + \gamma_{KF}\ln X_F + \gamma_{KR}\ln X_R$$
$$+ \varepsilon_{K1}\ln Y_1 + \varepsilon_{K2}\ln Y_2 + \varepsilon_{K3}\ln Y_3 \qquad (3)$$

$$-S_F = \alpha_F + \gamma_{FP}\ln X_P + \gamma_{FA}\ln X_A + \gamma_{FK}\ln X_K + \gamma_{FF}\ln X_F + \gamma_{FR}\ln X_R$$
$$+ \varepsilon_{F1}\ln Y_1 + \varepsilon_{F2}\ln Y_2 + \varepsilon_{F3}\ln Y_3 \qquad (4)$$

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[1] See the equations (11) and (13) in the section B.3.4.

$$-S_R = \alpha_R + \gamma_{RP}\ln X_P + \gamma_{RA}\ln X_A + \gamma_{RK}\ln X_K + \gamma_{RF}\ln X_F + \gamma_{RR}\ln X_R$$

$$+ \varepsilon_{R1}\ln Y_1 + \varepsilon_{R2}\ln Y_2 + \varepsilon_{R3}\ln Y_3 \qquad (5)$$

$$S_1 = \beta_1 + \xi_{1P}\ln X_P + \xi_{1A}\ln X_A + \xi_{1K}\ln X_K + \xi_{1F}\ln X_F + \xi_{1R}\ln X_R$$

$$+ \delta_{11}\ln Y_1 + \delta_{12}\ln Y_2 + \delta_{13}\ln Y_3 \qquad (6)$$

$$S_2 = \beta_2 + \xi_{2P}\ln X_P + \xi_{2A}\ln X_A + \xi_{2K}\ln X_K + \xi_{2F}\ln X_F + \xi_{2R}\ln X_R$$

$$+ \delta_{21}\ln Y_1 + \delta_{22}\ln Y_2 + \delta_{23}\ln Y_3 \qquad (7)$$

$$S_3 = \beta_3 + \xi_{3P}\ln X_P + \xi_{3A}\ln X_A + \xi_{2K}\ln X_K + \xi_{2F}\ln X_F + \xi_{2R}\ln X_R$$

$$+ \delta_{31}\ln Y_1 + \delta_{32}\ln Y_2 + \delta_{33}\ln Y_3 \qquad (8)$$

Similarly the parameter estimates of the cost share equations system are presented in the same way, where the input variables in the cost function become their input prices, denoted by W, instead of their quantity, denoted by X, in the production function. Hence, for example, the cost share equation for operative worker becomes;[2]

$$S_P = \alpha_P^* + \gamma_{PP}^*\ln W_P + \gamma_{PA}^*\ln W_A + \gamma_{PK}^*\ln X_K + \gamma_{PF}^*\ln W_F + \gamma_{PR}^*\ln W_R$$

$$+ \varepsilon_{P1}^*\ln Y_1 + \varepsilon_{P2}^*\ln Y_2 + \varepsilon_{P3}^*\ln Y_3 \qquad (9)$$

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[2]See the equations (12) and (14) in the section 4, Chapter III, Part B.

And the cost share equation for the first major commodity becomes:

$$S_1 = \beta_1^* + \xi_{1P}^* \ln W_P + \xi_{1A}^* \ln W_A + \xi_{1K}^* \ln W_K + \xi_{1F}^* \ln W_F + \xi_{1R}^* \ln W_R$$

$$+ \delta_{11}^* \ln Y_1 + \delta_{12}^* \ln Y_2 + \delta_{13}^* \ln Y_3 \qquad (10)$$

Also, the parameter estimates of the uniproduct share equations system are presented in Tables VI-4-a through VI-6-b, in the following specification of the estimating equations;

$$S_P = \alpha_P + \gamma_{PP} \ln X_P + \gamma_{PA} \ln X_A + \gamma_{PK} \ln X_K + \gamma_{PF} \ln X_F + \gamma_{PR} \ln X_R \qquad (11)$$

$$S_A = \alpha_A + \gamma_{AP} \ln X_P + \gamma_{AA} \ln X_A + \gamma_{AK} \ln X_K + \gamma_{AF} \ln X_F + \gamma_{AR} \ln X_R \qquad (12)$$

$$S_K = \alpha_K + \gamma_{KP} \ln X_P + \gamma_{KA} \ln X_A + \gamma_{KK} \ln X_K + \gamma_{KF} \ln X_F + \gamma_{KR} \ln X_R \qquad (13)$$

$$S_F = \alpha_F + \gamma_{FP} \ln X_P + \gamma_{FA} \ln X_A + \gamma_{FK} \ln X_K + \gamma_{FF} \ln X_F + \gamma_{FR} \ln X_R \qquad (14)$$

$$S_R = \alpha_R + \gamma_{RP} \ln X_P + \gamma_{RA} \ln X_A + \gamma_{RK} \ln X_K + \gamma_{RF} \ln X_F + \gamma_{RR} \ln X_R \qquad (15)$$

Similarly those of the cost share equation are in the similar specification as above. For example, the cost share equation of the operative worker becomes;

$$S_P = \alpha_P^* + \gamma_{PP}^* \ln W_P + \gamma_{PA}^* \ln W_A + \gamma_{PK}^* \ln W_F + \gamma_{PF}^* \ln W_F + \gamma_{PR}^* \ln W_R \qquad (16)$$

Each table for an industry contains two parts of the parameter estimates, i.e., those of the production function and of the cost function. In the first column of each part, we present estimates of the parameters of the corresponding translog share equations: these estimates were obtained with no restrictions imposed on the parameters.[3] The second column of each part contains estimates of the translog parameters constrained to impose linear homogeneity and symmetry conditions.[4] Finally, the third column contains estimates constrained to impose explicit separability between input and output imposed on the corresponding translog function.[5]

Note that in a purely Cobb-Douglas technology, only the constants α_i's and β_i's would be significant. As revealed by the standard errors in parentheses of the tables, all of the own variable's coefficients are significant as well as numerous cross variables' coefficients of the restricted estimations, while they are not always significant in the unrestricted estimations in the six industries selected.

In the canning industry, for example, not only the constant term α_A of the administrative labor input equation and the own variable's coefficient γ_{KK} of the capital input are insignificant in the unrestricted estimation of the translog production function, but also most of other cross variables' coefficients in the eight share equations are shown to be insignificant. Meanwhile, all the

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[3]See the section 4.2.2. and section 4.2.1., Chapter III, Part B.

[4]See the section 4.3.3., Chapter III, Part B.

[5]See the section 4.3.3., Chapter III, Part B.

TABLE VI-1-a.--Parameter Estimates for the Translog Functions[a]--Canning Industry.

Parameter	Translog Production Function			Translog Cost Function		
	Unrestricted	Symmetry	Explicit Separability	Unrestricted	Symmetry	Explicit Separability
α_P	-0.05329 (0.03173)	-0.11044 (0.00101)	-0.11340 (0.00101)	0.04709 (0.03154)	0.12378 (0.00099)	0.12685 (0.00099)
α_A	-0.00498 (0.01077)	-0.01386 (0.00012)	-0.01477 (0.00012)	0.01299 (0.01391)	0.02324 (0.00019)	0.02460 (0.00019)
α_K	-0.29474 (0.08101)	-0.34838 (0.00656)	-0.35118 (0.00656)	0.35780 (0.08070)	0.29145 (0100651)	0.29130 (0.00651)
α_F	-0.02160 (0.01410)	-0.04813 (0.00020)	-0.04930 (0.00020)	0.00871 (0.01499)	0.03094 (0.00022)	0.03296 (0.00022)
α_R	-0.62540 (0.07793)	-0.47919 (0.0)	-0.47135 (0.0)	-0.57342 (0.08859)	0.53059 (0.0)	0.52429 (0.0)
β_1	0.51400 (0.07670)	0.41452 (0.00500)	0.41627 (0.00500)	0.57911 (0.07581)	0.41385 (0.00575)	0.41609 (0.00575)
β_2	0.30672 (0.08731)	0.28911 (0.00763)	0.28830 (0.00763)	0.27240 (0.09322)	0.29016 (0.00869)	0.28883 (0.00869)
β_3	0.18843 (0.09196)	0.29637 (0.0)	0.29544 (0.0)	0.15818 (0.10119)	0.29599 (0.0)	0.29508 (0.0)
γ_{PP}	-0.03990 (0.01055)	-0.03155 (0.00011)	-0.03180 (0.00011)	0.00251 (0.01245)	0.01479 (0.00015)	0.01399 (0.00015)
γ_{PA}	0.02322 (0.01084)	0.00988 (0.00012)	0.00885 (0.00012)	-0.06483 (0.01302)	-0.01503 (0.00017)	-0.01439 (0.00017)

TABLE VI-1-b.--<u>Continued</u>.

Parameter	Translog Production Function Unrestricted	Translog Production Function Symmetry	Translog Production Function Explicit Separability	Translog Cost Function Unrestricted	Translog Cost Function Symmetry	Translog Cost Function Explicit Separability
γ_{PK}	0.00251 (0.00494)	0.00140 (0.00002)	0.00142 (0.00002)	-0.00682 (0.00426)	-0.00820 (0.00002)	-0.00860 (0.00002)
γ_{PF}	0.00279 (0.00596)	0.00133 (0.00004)	0.00158 (0.00004)	-0.00816 (0.00792)	0.00487 (0.00006)	0.00521 (0100006)
γ_{PR}	0.01523 (0.00482)	0.01894 (0.00002)	0.01995 (0.00002)	-0.00879 (0.00587)	0.00356 (0.00003)	0.00379 (0.00003)
ε_{P1}	0.00012 (0.00017)	-0.00014 (0.00000)	0.0 (0.00000)	0.00006 (0.00015)	0.00017 (0.00000)	0.0 (0.00000)
ε_{P2}	0.00034 (0.00011)	0.00024 (0.00000)	-0.0 (0.00000)	-0.00026 (0.00011)	-0.00016 (0.00000)	0.0 (0.00000)
ε_{P3}	0.00017 (0.00016)	-0.00010 (0.00021)	0.0 (0.00023)	-0.00006 (0.00015)	-0.00001 (0.00020)	-0.0 (0.00021)
γ_{AP}	0.00052 (0.00358)	0.00988 (0.00001)	0.00885 (0.00001)	-0.01041 (0.00549)	-0.01503 (0.00003)	-0.01439 (0.00003)
γ_{AA}	-0.02124 (0.00368)	-0.01985 (0.00017)	-0.01878 (0.00001)	0.01576 (0.00574)	0.01981 (0.00003)	0.01933 (0.00003)
γ_{AK}	0.00347 (0.00168)	0.00420 (0.00000)	0.00423 (0.00000)	-0.00116 (0.00188)	-0.00294 (0.00000)	-0.00307 (0.00001)
γ_{AF}	0.00397 (0.00202)	0.00330 (0.00000)	0.00247 (0.00000)	0.00220 (0.00349)	-0.00111 (0.00001)	-0.00121 (0.00001)

TABLE VI-1-c.--Continued.

Parameter	Translog Production Function			Translog Cost Function		
	Unrestricted	Symmetry	Explicit Separability	Unrestricted	Symmetry	Explicit Separability
γ_{AR}	0.00038 (0.00164)	0.00247 (0.00000)	0.00324 (0.00000)	-0.00044 (0.00259)	-0.00074 (0.00000)	-0.00067 (0.00000)
ε_{A1}	0.00007 (0.00006)	-0.00006 (0.00000)	0.0 (0.00000)	-0.00003 (0.00007)	0.00006 (0.00000)	0.0 (0.00000)
ε_{A2}	0.00008 (0.00004)	0.00003 (0.00000)	0.0 (0.00000)	-0.00009 (0.00005)	-0.00009 (0.00007)	-0.0 (0.00000)
ε_{A3}	0.00010 (0.00005)	0.00003 (0.00007)	-0.0 (0.00008)	-0.00008 (0.00006)	0.00002 (0.00008)	-0.0 (0.00009)
γ_{KP}	0.00071 (0.02693)	0.00140 (0.00073)	0.00142 (0.00073)	0.02693 (0.03185)	-0.00820 (0.00101)	-0.00860 (0.00101)
γ_{KA}	-0.04104 (0.02768)	0.00420 (0.00077)	0.00423 (0.00077)	-0.01933 (0.03332)	-0.00294 (0.00111)	-0.00367 (0.00111)
γ_{KK}	-0.00157 (0.01261)	-0.01894 (0.00016)	-0.02052 (0.00016)	0.04643 (0.01091)	0.03111 (0.00012)	0.03207 (0.00012)
γ_{KF}	0.00233 (0.01522)	-0.00219 (0.00023)	-0.00200 (0.00023)	0.03417 (0.02026)	-0.00391 (0.00041)	-0.00401 (0.00041)
γ_{KR}	0.03308 (0.01231)	0.01553 (0.00015)	0.01638 (0.00015)	0.00883 (0.01501)	-0.01606 (0.00023)	-0.01640 (0.00023)
ε_{K1}	-0.00013 (0.00044)	-0.00018 (0.00000)	-0.0 (0.00000)	-0.00017 (0.00039)	0.00021 (0.00000)	0.0 (0.00000)

TABLE VI-1-d.--Continued.

	Translog Production Function			Translog Cost Function		
Parameter	Unrestricted	Symmetry	Explicit Separability	Unrestricted	Symmetry	Explicit Separability
ε_{K2}	-0.00027 (0.00024)	-0.00018 (0.00000)	0.0 (0.00000)	0.00022 (0.00027)	0.00013 (0.00000)	-0.0 (0.00000)
ε_{K3}	0.00038 (0.00041)	0.00036 (0.00054)	0.0 (0.00058)	-0.00082 (0.00038)	-0.00034 (0.00050)	0.0 (0.00054)
γ_{FP}	-0.00015 (0.00469)	0.00133 (0.00002)	0.00158 (0.00002)	0.00452 (0.00592)	0.00087 (0.00003)	0.00521 (0.00004)
γ_{FA}	0.00056 (0.00482)	0.00330 (0.00002)	0.00247 (0.00002)	-0.01759 (0.00619)	-0.00111 (0.00004)	-0.00121 (0.00004)
γ_{FK}	-0.00073 (0.00220)	-0.00219 (0.00000)	-0.00200 (0.00000)	-0.00225 (0.00203)	-0.00391 (0.00000)	-0.00401 (0.00000)
γ_{FF}	-0.00631 (0.00265)	-0.00753 (0.00000)	-0.00786 (0.00000)	0.00008 (0.00376)	0.00129 (0.00001)	0.00100 (0.00001)
γ_{FR}	0.00401 (0.00214)	0.00508 (0.00000)	0.00579 (0.00000)	-0.00307 (0.00279)	0.00115 (0.00000)	-0.00099 (0.00000)
ε_{F1}	0.00006 (0.00008)	-0.00008 (0.00000)	0.0 (0.00000)	0.00003 (0.00007)	0.00011 (0.00000)	-0.0 (0.00000)
ε_{F2}	0.00016 (0.00005)	0.00011 (0.00000)	0.0 (0.00000)	-0.00014 (0.00005)	-0.00011 (0.00000)	0.0 (0.00000)
ε_{F3}	0.00009 (0.00007)	-0.00003 (0.00009)	0.0 (0.00000)	-0.00007 (0.00007)	0.00000 (0.00009)	-0.0 (0.00010)

TABLE VI-1-e.--Continued.

	Translog Production Function			Translog Cost Function		
Parameter	Unrestricted	Symmetry	Explicit Separability	Unrestricted	Symmetry	Explicit Separability
γ_{RP}	0.03419 (0.02591)	0.01894 (0.0)	0.01995 (0.0)	-0.02335 (0.3496)	0.00356 (0.0)	0.00379 (0.0)
γ_{RA}	0.03850 (0.02663)	0.00247 (0.0)	0.00324 (0.0)	0.08599 (0.03658)	-0.00074 (0.0)	-0.00067 (0.0)
γ_{RK}	-0.00368 (0.01213)	0.01553 (0.0)	0.01688 (0.0)	-0.03619 (0.01197)	-0.01606 (0.0)	-0.01640 (0.0)
γ_{RF}	-0.00278 (0.01465)	0.00503 (0.0)	0.00579 (0.0)	-0.02829 (0.02225)	-0.00115 (0.0)	-0.00099 (0.0)
γ_{RR}	-0.05270 (0.01184)	-0.04203 (0.0)	-0.04585 (0.0)	-0.00346 (0.01648)	0.01438 (0.0)	0.01426 (0.0)
ε_{R1}	-0.00011 (0.00043)	0.00046 (0.0)	0.0 (0.0)	0.00012 (0.00042)	-0.00056 (0.0)	0.0 (0.0)
ε_{R2}	-0.00031 (0.00028)	-0.00020 (0.0)	0.0 (0.0)	0.00027 (0.00030)	0.00023 (0.0)	-0.0 (0.0)
ε_{R3}	-0.00073 (0.00039)	-0.00026 (0.0)	0.0 (0.0)	0.00103 (0.00042)	0.00032 (0.0)	-0.0 (0.0)
ξ_{1P}	0.00153 (0.02351)	-0.00014 (0.00055)	0.0 (0.00055)	-0.02079 (0.02992)	0.00017 (0.00090)	-0.0 (0.00090)
ξ_{1A}	0.03224 (0.02416)	-0.00006 (0.00058)	0.0 (0.00058)	-0.03197 (0.03131)	0.00006 (0.00100)	0.0 (0.00098)

TABLE VI-1-f.--Continued.

Parameter	Translog Production Function			Translog Cost Function		
	Unrestricted	Symmetry	Explicit Separability	Unrestricted	Symmetry	Explicit Separability
ξ_{1k}	-0.01212 (0.01101)	-0.00018 (0.00012)	0.0 (0.00012)	0.01482 (0.01025)	0.00021 (0.00010)	-0.0 (0.00010)
ξ_{1F}	-0.00138 (0.01329)	-0.00008 (0.00018)	-0.0 (0.00018)	0.01786 (0.01904)	0.00011 (0.00036)	0.0 (0.00036)
ξ_{1R}	-0.01592 (0.01074)	0.00046 (0.00012)	0.0 (0.00012)	0.00399 (0.01410)	-0.00056 (0.00020)	0.0 (0.00020)
δ_{11}	0.04078 (0.00039)	0.00340 (0.00000)	0.00329 (0.00000)	0.00408 (0.00036)	0.00341 (0.00000)	0.00328 (0.00000)
δ_{12}	-0.00129 (0.00025)	-0.00176 (0.00000)	-0.00170 (0.00000)	-0.00135 (0.00025)	-0.00176 (0.00000)	-0.00172 (0.00000)
δ_{13}	-0.00111 (0.00036)	-0.00164 (0.00049)	-0.00158 (0.00049)	-0.00115 (0.00036)	-0.00164 (0.00049)	-0.00156 (0.00049)
ξ_{2P}	-0.00356 (0.02905)	0.00024 (0.00084)	-0.0 (0.00084)	0.01684 (0.03679)	-0.00016 (0.00135)	0.0 (0.00135)
ξ_{2A}	-0.00756 (0.02985)	0.00003 (0.00089)	0.0 (0.00089)	0.01803 (0.03849)	-0.00009 (0.00148)	-0.0 (0.00148)
ξ_{2K}	0.00742 (0.01360)	-0.00018 (0.00019)	-0.0 (0.00019)	-0.00200 (0.01260)	0.00013 (0.00016)	-0.0 (0.00016)
ξ_{2F}	0.01889 (0.01642)	0.00011 (0.00027)	0.0 (0.00027)	-0.03001 (0.02341)	-0.00011 (0.00055)	-0.0 (0.00055)

TABLE VI-1-g.--Continued.

Parameter	Translog Production Function			Translog Cost Function		
	Unrestricted	Symmetry	Explicit Separability	Unrestricted	Symmetry	Explicit Separability
ξ_{2R}	-0.00922 (0.01327)	-0.00020 (0.00018)	0.0 (0.00018)	0.00980 (0.01734)	0.00023 (0.00030)	-0.0 (0.00030)
δ_{21}	-0.00209 (0.00048)	-0.00176 (0.00000)	-0.00170 (0.00000)	-0.00207 (0.00045)	-0.00176 (0.00000)	-0.00172 (0.00000)
δ_{22}	0.00335 (0.00031)	0.00336 (0.00000)	0.00327 (0.00000)	0.00335 (0.00031)	0.00335 (0.00000)	0.00331 (0.00000)
δ_{23}	-0.00154 (0.00044)	-0.00159 (0.00058)	-0.00157 (0.00059)	-0.00162 (0.00044)	-0.00158 (0.00059)	-0.00159 (0.00059)
ξ_{3P}	0.00451 (0.03058)	-0.00010 (0.0)	-0.0 (0.0)	-0.00085 (0.03994)	-0.00001 (0.0)	0.0 (0.0)
ξ_{3A}	-0.02482 (0.03142)	0.00003 (0.0)	-0.0 (0.0)	0.01692 (0.04178)	0.00002 (0.0)	0.0 (0.0)
ξ_{3K}	0.00392 (0.01432)	0.00036 (0.0)	0.0 (0.0)	-0.01241 (0.01367)	-0.00034 (0.0)	0.0 (0.0)
ξ_{3F}	-0.01738 (0.01728)	-0.00003 (0.0)	0.0 (0.0)	0.01080 (0.02541)	0.00000 (0.0)	-0.0 (0.0)
ξ_{3R}	0.02500 (0.01397)	0.00026 (0.0)	-0.0 (0.0)	0.01381 (0.01882)	0.00032 (0.0)	-0.0 (0.0)
δ_{31}	-0.00193 (0.00050)	-0.00164 (0.0)	-0.00158 (0.0)	-0.00194 (0.00049)	-0.00164 (0.0)	-0.00156 (0.0)

TABLE VI-1-h.--Continued.

	Translog Production Function			Translog Cost Function		
Parameter	Unrestricted	Symmetry	Explicit Separability	Unrestricted	Symmetry	Explicit Separability
δ_{32}	-0.00208 (0.00033)	-0.00159 (0.0)	-0.00157 (0.0)	-0.00201 (0.00034)	-0.00158 (0.0)	-0.00159 (0.0)
δ_{33}	0.00272 (0.00047)	0.00323 (0.0)	0.00315 (0.0)	0.00284 (0.00048)	0.00322 (0.0)	0.00315 (0.0)

[a](i) The figures in the table are the parameter estimates and those in parentheses are the corresponding standard errors. (ii) The symbols for the parameter concerned are only specified for those in translog production function. Hence, those in translog cost function should be interpreted correspondingly. For example, α_P in the first row of the table (VI-1-a) should be viewed as α_P^* for the parameter in translog cost function. (iii) It is also worthwhile to note that in the 0.0 is interpreted as different from 0.00000 either in the parameter estimate or in its standard error in parenthesis. For example, the standard error of 0.0 for the parameter, α_R, in the restricted estimation of symmetry (i.e., the figures shown in the fifth row of the second column in Table VI-1-a) is given a priori constraints in the estimation. Similarly the estimates of ε_{P1} in the estimation of explicit separability (the number in the 14th row of the third column in Table VI-1-a) is constrained as 0.0 in the estimation.

TABLE VI-2-a.--Parameter Estimates for the Translog Functions[a]--Leather Footwear Industry.

Parameter	Translog Production Function			Translog Cost Function		
	Unrestricted	Symmetry	Explicit Separability	Unrestricted	Symmetry	Explicit Separability
α_P	-0.13522 (0.02927)	-0.12496 (0.00086)	-0.12523 (0.00086)	0.12054 (0.02349)	0.15414 (0.00055)	0.15117 (0.00055)
α_A	-0.02836 (0.01748)	-0.03884 (0.00031)	-0.03827 (0.00031)	0.02880 (0.00924)	0.03220 (0.00009)	0.03117 (0.00009)
α_K	-0.19289 (0.00491)	-0.20404 (0.00241)	-0.20153 (0.00241)	0.28135 (0.3008)	0.29262 (0.00090)	0.29073 (0.00090)
α_F	-0.01794 (0.00240)	-0.01759 (0.00001)	-0.01763 (0.00001)	0.00785 (0.00325)	0.01316 (0.00001)	0.01322 (0.00001)
α_R	-0.62610 (0.04994)	-0.61458 (0.0)	-0.61734 (0.0)	0.56147 (0.04231)	0.50788 (0.0)	0.51372 (0.0)
β_1	0.52732 (0.04278)	0.52581 (0.00183)	0.52674 (0.00183)	0.56176 (0.03715)	0.52607 (0.00138)	0.52545 (0.00138)
β_2	0.47753 (0.04290)	0.47419 (0.0)	0.47326 (0.0)	0.43560 (0.03663)	0.47393 (0.0)	0.47455 (0.0)
γ_{PP}	0.00686 (0.02395)	-0.03141 (0.00057)	-0.02940 (0.00057)	0.02268 (0.04243)	0.06913 (0.00180)	0.07009 (0.00180)
γ_{PA}	0.01936 (0.01983)	0.01648 (0.00039)	0.02081 (0.00039)	-0.03287 (0.03931)	-0.03957 (0.00155)	-0.03870 (0.00155)
γ_{PK}	-0.00487 (0.01655)	-0.00853 (0.00027)	-0.01415 (0.00027)	-0.02633 (0.01234)	-0.03124 (0.00015)	-0.03171 (0.00015)

TABLE VI-2-b.--Continued.

Parameter	Translog Production Function			Translog Cost Function		
	Unrestricted	Symmetry	Explicit Separability	Unrestricted	Symmetry	Explicit Separability
γ_{PF}	-0.01516 (0.01527)	0.00318 (0.00023)	0.00348 (0.00023)	-0.01567 (0.02139)	-0.00098 (0.00046)	-0.00094 (0.00046)
γ_{PR}	0.01108 (0.01383)	0.02029 (0.00019)	0.01926 (0.00019)	0.00559 (0.01309)	0.00266 (0.00020)	0.00126 (0.00000)
ϵ_{P1}	0.00022 (0.00023)	0.00011 (0.00000)	0.0 (0.00000)	-0.00031 (0.00022)	-0.00005 (0.00000)	0.0 (0.00000)
ϵ_{P2}	-0.00004 (0.00019)	-0.00011 (0.00025)	0.0 (0.00027)	-0.00025 (0.00018)	0.00005 (0.00024)	0.0 (0.00026)
γ_{AP}	0.01044 (0.01430)	0.01648 (0.00020)	0.02081 (0.00020)	-0.06741 (0.01669)	-0.03957 (0.00028)	-0.03870 (0.00028)
γ_{AA}	-0.02449 (0.01184)	-0.01306 (0.00014)	-0.01215 (0.00014)	0.04265 (0.01547)	0.04256 (0.00024)	0.04237 (0.00024)
γ_{AK}	-0.00188 (0.00988)	-0.00668 (0.00010)	-0.01176 (0.00010)	-0.00262 (0.00486)	-0.00641 (0.00002)	-0.00647 (0.00002)
γ_{AF}	-0.00317 (0.00912)	-0.00033 (0.00008)	-0.00040 (0.00008)	-0.00921 (0.00842)	0.00260 (0.00007)	0.00242 (0.00007)
γ_{AR}	0.01064 (0.00826)	0.00359 (0.00007)	0.00350 (0.00007)	0.00457 (0.00551)	0.00082 (0.00003)	0.00038 (0.00003)
ϵ_{A1}	0.00017 (0.00014)	0.00011 (0.00000)	0.0 (0.00000)	-0.00001 (0.00009)	-0.00002 (0.00000)	0.0 (0.00000)

TABLE VI-2-c.--Continued.

Parameter	Translog Production Function			Translog Cost Function		
	Unrestricted	Symmetry	Explicit Separability	Unrestricted	Symmetry	Explicit Separability
ε_{A2}	-0.00003 (0.00012)	-0.00011 (0.00015)	0.0 (0.00016)	-0.00003 (0.00007)	0.00002 (0.00009)	0.0 (0.00010)
γ_{KP}	0.01011 (0.04018)	-0.00853 (0.00161)	-0.01415 (0.00161)	0.05556 (0.05433)	-0.03124 (0.00295)	-0.03171 (0.00295)
γ_{KA}	-0.05612 (0.03328)	-0.00668 (0.00111)	-0.01176 (0.00111)	0.08102 (0.05034)	-0.00641 (0.00253)	-0.00647 (0.00253)
γ_{KK}	0.00880 (0.02776)	-0.02122 (0.00077)	-0.01190 (0.00077)	0.04665 (0.01581)	0.03510 (0.00025)	0.03423 (0.00025)
γ_{KF}	-0.03656 (0.02562)	0.00381 (0.00066)	0.00362 (0.00066)	-0.05925 (0.02740)	-0.00301 (0.00075)	-0.00297 (0.00075)
γ_{KR}	0.05327 (0.02320)	0.03261 (0.00054)	0.03420 (0.00054)	0.04407 (0.01792)	0.00555 (0.00032)	0.00692 (0.00032)
ε_{K1}	0.00005 (0.00039)	-0.00016 (0.00000)	0.0 (0.00000)	-0.00026 (0.00028)	-0.00004 (0.00000)	0.0 (0.00000)
ε_{K2}	0.00050 (0.00032)	0.00016 (0.00043)	0.0 (0.00046)	-0.00030 (0.00024)	0.00004 (0.00031)	0.0 (0.00033)
γ_{FP}	0.00758 (0.00196)	0.00318 (0.00000)	0.00348 (0.00000)	0.00977 (0.00587)	-0.00098 (0.00003)	-0.00094 (0.00003)
γ_{FA}	0.00025 (0.00162)	-0.00033 (0.00000)	-0.00040 (0.00000)	0.00455 (0.00544)	0.00260 (0.00003)	0.00242 (0.00003)

TABLE VI-2-d.--Continued.

Parameter	Translog Production Function			Translog Cost Function		
	Unrestricted	Symmetry	Explicit Separability	Unrestricted	Symmetry	Explicit Separability
γ_{FK}	0.00183 (0.00135)	0.00381 (0.00000)	0.00362 (0.00000)	-0.00197 (0.00171)	-0.00301 (0.00000)	-0.00297 (0.00000)
γ_{FF}	-0.00818 (0.00125)	-0.00940 (0.00000)	-0.00946 (0.00000)	-0.00051 (0.00296)	0.00255 (0.00001)	0.00267 (0.00001)
γ_{FR}	0.00058 (0.00113)	0.00274 (0.00000)	0.00276 (0.00000)	-0.00051 (0.00194)	-0.00117 (0.00000)	-0.00118 (0.00000)
ε_{F1}	0.00003 (0.00002)	0.00000 (0.00000)	0.0 (0.00000)	-0.00007 (0.00003)	0.00000 (0.00000)	0.0 (0.00000)
ε_{F2}	0.00001 (0.00000)	-0.00000 (0.00002)	0.0 (0.00002)	-0.00005 (0.00003)	-0.00000 (0.00003)	0.0 (0.00003)
γ_{RP}	-0.03450 (0.04085)	0.02029 (0.0)	0.01926 (0.0)	-0.02060 (0.07642)	0.00266 (0.0)	0.00126 (0.0)
γ_{RA}	0.06100 (0.03383)	0.00359 (0.0)	0.00350 (0.0)	-0.09525 (0.07082)	0.00082 (0.0)	0.00038 (0.0)
γ_{RK}	-0.00388 (0.02822)	0.03261 (0.0)	0.03420 (0.0)	-0.01574 (0.02223)	0.00555 (0.0)	0.00692 (0.0)
γ_{RF}	0.06307 (0.02605)	0.00274 (0.0)	0.00276 (0.0)	0.08484 (0.03854)	-0.00117 (0.0)	-0.00118 (0.0)
γ_{RR}	-0.07557 (0.02359)	-0.05922 (0.0)	-0.05971 (0.0)	-0.05372 (0.02521)	-0.00786 (0.0)	-0.00738 (0.0)

TABLE VI-2-e.--Continued.

Parameter	Translog Production Function: Unrestricted	Translog Production Function: Symmetry	Translog Production Function: Explicit Separability	Translog Cost Function: Unrestricted	Translog Cost Function: Symmetry	Translog Cost Function: Explicit Separability
ε_{R1}	-0.00046 (0.00040)	-0.00007 (0.0)	0.0 (0.0)	0.00064 (0.00039)	0.00011 (0.0)	0.0 (0.0)
ε_{R2}	-0.00044 (0.00033)	0.00007 (0.0)	0.0 (0.0)	0.00062 (0.00033)	-0.00011 (0.0)	0.0 (0.0)
ξ_{1P}	0.01329 (0.03499)	0.00011 (0.00122)	0.0 (0.00122)	0.01657 (0.06710)	-0.00005 (0.00450)	0.0 (0.00450)
ξ_{1A}	0.00146 (0.02898)	0.00011 (0.00084)	0.0 (0.00084)	0.09318 (0.06218)	-0.00002 (0.00389)	0.0 (0.00387)
ξ_{1K}	0.05437 (0.02418)	-0.00016 (0.00058)	0.0 (0.00058)	-0.03014 (0.01952)	-0.00004 (0.00038)	0.0 (0.00038)
ξ_{1F}	-0.06249 (0.02231)	0.00000 (0.00050)	0.0 (0.00050)	0.07109 (0.03383)	0.00000 (0.00114)	0.0 (0.00114)
ξ_{1R}	0.00409 (0.02021)	-0.00007 (0.00041)	0.0 (0.00041)	0.00695 (0.02213)	0.00011 (0.00049)	0.0 (0.00049)
δ_{11}	0.00329 (0.00034)	0.00309 (0.00000)	0.00308 (0.00000)	0.00339 (0.00034)	0.00309 (0.00000)	0.00310 (0.00000)
δ_{12}	-0.00303 (0.00028)	-0.00309 (0.00038)	-0.00308 (0.00038)	-0.00297 (0.00029)	-0.00309 (0.00039)	-0.00310 (0.00039)
ξ_{2P}	-0.01482 (0.03509)	-0.00011 (0.0)	0.0 (0.0)	-0.01481 (0.06615)	0.00005 (0.0)	0.0 (0.0)

TABLE VI-2-f.--Continued.

	Translog Production Function			Translog Cost Function		
Parameter	Unrestricted	Symmetry	Explicit Separability	Unrestricted	Symmetry	Explicit Separability
ξ_{2A}	0.00418 (0.02906)	-0.00011 (0.0)	0.0 (0.0)	-0.09210 (0.06130)	0.00002 (0.0)	0.0 (0.0)
ξ_{2K}	-0.05694 (0.02425)	0.00016 (0.0)	0.0 (0.0)	0.03172 (0.01924)	0.00004 (0.0)	0.0 (0.0)
ξ_{2F}	0.06164 (0.02238)	-0.00000 (0.0)	0.0 (0.0)	-0.07200 (0.03336)	-0.00000 (0.0)	0.0 (0.0)
ξ_{2R}	-0.00167 (0.02027)	0.00000 (0.0)	0.0 (0.0)	-0.01204 (0.02182)	-0.00011 (0.0)	0.0 (0.0)
δ_{21}	-0.00333 (0.00034)	-0.00309 (0.0)	-0.00308 (0.0)	-0.00339 (0.00034)	-0.00309 (0.2)	-0.00310 (0.0)
δ_{22}	0.00296 (0.00028)	0.00309 (0.0)	0.00308 (0.0)	0.00293 (0.00029)	0.00309 (0.0)	0.00310 (0.0)

[a]See the footnote (a) in Table VI-1.

TABLE VI-3-a.--Parameter Estimates for the Translog Functions[a]--Screw Products Industry.

	Translog Production Function			Translog Cost Function		
Parameter	Unrestricted	Symmetry	Explicit Separability	Unrestricted	Symmetry	Explicit Separability
α_P	-0.14153 (0.02900)	-0.15454 (0.00084)	-0.15533 (0.00084)	0.12046 (0.02748)	0.14613 (0.00076)	0.14797 (0.00076)
α_A	-0.04155 (0.00936)	0.04154 (0.00009)	-0.04145 (0.00009)	0.04110 (0.00980)	0.04133 (0.00010)	0.04531 (0.00010)
α_K	-0.26532 (0.04298)	-0.27204 (0.00185)	-0.26922 (0.00185)	0.23044 (0.02886)	0.24414 (0.00083)	0.25767 (0.00083)
α_F	-0.06064 (0.00948)	-0.06417 (0.00009)	-0.06369 (0.00009)	0.04025 (0.01040)	0.04525 (0.00011)	0.03789 (0.00011)
α_R	-0.49096 (0.04439)	-0.46722 (0.0)	-0.047031 (0.0)	0.56776 (0.04565)	0.52316 (0.0)	0.51116 (0.0)
β_1	0.62576 (0.02502)	0.52421 (0.00063)	0.52521 (0.00063)	0.63083 (0.02495)	0.52383 (0.00062)	0.52487 (0.00062)
β_2	0.38021 (0.02484)	0.47579 (0.0)	0.47479 (0.0)	0.37549 (0.02461)	0.47617 (0.0)	0.47513 (0.0)
γ_{PP}	-0.06396 (0.02257)	-0.05162 (0.00051)	-0.05135 (0.00051)	0.03874 (0.02914)	0.06657 (0.00085)	0.06605 (0.00085)
γ_{PA}	0.04392 (0.01996)	0.02491 (0.00040)	0.02509 (0.00040)	-0.01156 (0.02183)	-0.02138 (0.00048)	-0.02094 (0.00048)
γ_{PK}	0.02631 (0.01502)	0.00977 (0.00023)	0.00954 (0.00023)	-0.04483 (0.00783)	-0.04794 (0.00006)	-0.04861 (0.00006)

TABLE VI-3-b.--Continued.

Parameter	Translog Production Function			Translog Cost Function		
	Unrestricted	Symmetry	Explicit Separability	Unrestricted	Symmetry	Explicit Separability
γ_{PF}	0.00715 (0.00717)	0.00981 (0.00005)	0.00943 (0.00005)	0.01122 (0.00889)	0.00812 (0.00008)	0.00849 (0.00008)
γ_{PR}	0.00451 (0.00452)	0.00712 (0.00002)	0.00730 (0.00002)	-0.00569 (0.00389)	-0.00536 (0.00002)	-0.00499 (0.00002)
ε_{P1}	0.00002 (0.00021)	-0.00001 (0.00000)	0.0 (0.00000)	-0.00018 (0.00019)	0.00002 (0.00000)	0.0 (0.00000)
ε_{P2}	0.00003 (0.00019)	0.00001 (0.00025)	0.0 (0.00026)	-0.00021 (0.00017)	-0.00002 (0.00022)	0.0 (0.00024)
γ_{AP}	0.01848 (0.00729)	0.02491 (0.00005)	0.02509 (0.00005)	-0.03088 (0.01038)	-0.02138 (0.00011)	-0.02094 (0.00011)
γ_{AA}	-0.03508 (0.00644)	-0.03634 (0.00004)	-0.03645 (0.00004)	0.03117 (0.00778)	0.02974 (0.00006)	0.02919 (0.00006)
γ_{AK}	0.01538 (0.00485)	0.01305 (0.00002)	0.01275 (0.00002)	-0.01338 (0.00279)	-0.01460 (0.00001)	-0.01508 (0.00001)
γ_{AF}	-0.00031 (0.00231)	-0.00161 (0.00001)	-0.00154 (0.00001)	0.00696 (0.00317)	0.00731 (0.00001)	0.00714 (0.00001)
γ_{AR}	-0.00013 (0.00146)	-0.00001 (0.00000)	0.00014 (0.00000)	-0.00114 (0.00139)	-0.00108 (0.00000)	-0.00030 (0.00000)
ε_{A1}	-0.00002 (0.00007)	-0.00001 (0.00000)	0.0 (0.00000)	0.00004 (0.00007)	0.00004 (0.00000)	0.0 (0.00000)

TABLE VI-3-c.--Continued.

	Translog Production Function			Translog Cost Function		
Parameter	Unrestricted	Symmetry	Explicit Separability	Unrestricted	Symmetry	Explicit Separability
ε_{A2}	0.00001 (0.00006)	0.00001 (0.00008)	0.0 (0.00008)	-0.00004 (0.00006)	-0.00004 (0.00008)	0.0 (0.00008)
γ_{KP}	-0.05375 (0.03345)	0.00977 (0.00112)	0.00954 (0.00112)	-0.00669 (0.03060)	-0.04794 (0.00094)	-0.04861 (0.00094)
γ_{KA}	0.02386 (0.02958)	0.01305 (0.00088)	0.01275 (0.00088)	-0.05316 (0.02292)	-0.01460 (0.00053)	-0.01508 (0.00053)
γ_{KK}	0.01634 (0.02226)	-0.01376 (0.00050)	-0.01628 (0.00050)	0.09539 (0.00822)	0.08870 (0.00007)	0.08679 (0.00007)
γ_{KF}	0.00111 (0.01062)	0.00174 (0.00011)	0.00391 (0.00011)	-0.02808 (0.00933)	-0.01260 (0.00009)	-0.01210 (0.00009)
γ_{KR}	-0.00786 (0.00670)	-0.01080 (0.00004)	-0.00992 (0.00004)	-0.01322 (0.00408)	-0.01356 (0.00002)	-0.01100 (0.00002)
ε_{K1}	0.00005 (0.00031)	0.00004 (0.00000)	0.0 (0.00000)	0.00001 (0.00020)	0.00012 (0.00000)	0.0 (0.00000)
ε_{K2}	0.00023 (0.00028)	0.00004 (0.00037)	0.0 (0.00038)	-0.00017 (0.00018)	-0.00012 (0.00024)	0.0 (0.00025)
γ_{FP}	0.00658 (0.00738)	0.00981 (0.00005)	0.00943 (0.00005)	-0.00912 (0.01103)	0.00812 (0.00012)	0.00849 (0.00012)
γ_{FA}	0.00113 (0.00653)	-0.00161 (0.00004)	-0.00154 (0.00004)	0.01561 (0.00826)	0.00731 (0.00007)	0.00714 (0.00007)

TABLE VI-3-d.--Continued.

Parameter	Translog Production Function: Unrestricted	Translog Production Function: Symmetry	Translog Production Function: Explicit Separability	Translog Cost Function: Unrestricted	Translog Cost Function: Symmetry	Translog Cost Function: Explicit Separability
γ_{FK}	0.00525 (0.00491)	0.00174 (0.00002)	0.00391 (0.00002)	-0.01146 (0.00296)	-0.01260 (0.00001)	-0.01210 (0.00001)
γ_{FF}	-0.01270 (0.00234)	-0.01240 (0.00001)	-0.01277 (0.00001)	-0.00438 (0.00336)	-0.00455 (0.00001)	-0.00386 (0.00001)
γ_{FR}	0.00216 (0.00148)	0.00246 (0.00000)	0.00097 (0.00000)	0.00178 (0.00147)	0.00172 (0.00000)	0.00033 (0.00000)
ε_{F1}	0.00008 (0.00007)	0.00007 (0.00000)	0.0 (0.00000)	-0.00011 (0.00007)	-0.00007 (0.00000)	0.0 (0.00000)
ε_{F2}	-0.00005 (0.00006)	-0.00007 (0.00008)	0.0 (0.00009)	0.00003 (0.00006)	0.00007 (0.00008)	0.0 (0.00009)
γ_{RP}	0.09245 (0.03455)	0.00712 (0.0)	0.00730 (0.0)	0.00795 (0.04839)	-0.00536 (0.0)	-0.00499 (0.0)
γ_{RA}	-0.03383 (0.03055)	-0.00001 (0.0)	0.00014 (0.0)	0.01795 (0.03625)	-0.00108 (0.0)	-0.00030 (0.0)
γ_{RK}	-0.06328 (0.02299)	-0.01080 (0.0)	-0.00992 (0.0)	-0.02572 (0.01300)	-0.01356 (0.0)	-0.01100 (0.0)
γ_{RF}	0.00476 (0.01097)	0.00246 (0.0)	0.00097 (0.0)	0.01428 (0.01476)	0.00172 (0.0)	0.00033 (0.0)
γ_{RR}	0.00133 (0.00692)	0.00123 (0.0)	0.00151 (0.0)	0.01827 (0.00645)	0.01828 (0.0)	0.01596 (0.0)

TABLE VI-3-e.--Continued.

Parameter	Translog Production Function			Translog Cost Function		
	Unrestricted	Symmetry	Explicit Separability	Unrestricted	Symmetry	Explicit Separability
ε_{R1}	-0.00013 (0.00032)	-0.00002 (0.0)	0.0 (0.0)	0.00023 (0.00031)	-0.00011 (0.0)	0.0 (0.0)
ε_{R2}	-0.00022 (0.00028)	0.00002 (0.0)	0.0 (0.0)	0.00039 (0.00028)	0.00011 (0.0)	0.0 (0.0)
ξ_{1P}	-0.00504 (0.01947)	-0.00001 (0.00038)	0.0 (0.00038)	0.00003 (0.02645)	0.00002 (0.00070)	0.0 (0.00070)
ξ_{1A}	0.02105 (0.01722)	-0.00001 (0.00030)	0.0 (0.00030)	-0.03155 (0.01981)	0.00004 (0.00039)	0.0 (0.00039)
ξ_{1K}	-0.01733 (0.01296)	-0.00004 (0.00017)	0.0 (0.00017)	0.01934 (0.00710)	0.00012 (0.00005)	0.0 (0.00005)
ξ_{1F}	0.00306 (0.00618)	0.00007 (0.00004)	0.0 (0.00004)	-0.01132 (0.00806)	-0.00007 (0.00006)	0.0 (0.00007)
ξ_{1R}	0.00080 (0.00390)	-0.00002 (0.00002)	0.0 (0.00002)	0.00018 (0.00353)	-0.00011 (0.00001)	0.0 (0.00001)
δ_{11}	0.00389 (0.00018)	0.00309 (0.00000)	0.00307 (0.00000)	0.00394 (0.00017)	0.00309 (0.00000)	0.00308 (0.00000)
δ_{12}	-0.00242 (0.00016)	-0.00309 (0.00022)	-0.00307 (0.00022)	-0.00237 (0.00015)	-0.00309 (0.00021)	-0.00308 (0.00021)
ξ_{2P}	0.01345 (0.01933)	0.00001 (0.0)	0.0 (0.0)	0.00120 (0.02609)	-0.00002 (0.0)	0.0 (0.0)

TABLE VI-3-f.--Continued.

	Translog Production Function			Translog Cost Function		
Parameter	Unrestricted	Symmetry	Explicit Separability	Unrestricted	Symmetry	Explicit Separability
ξ_{2A}	-0.01908 (0.01710)	0.00001 (0.0)	0.0 (0.0)	0.04086 (0.01954)	-0.00004 (0.0)	0.0 (0.0)
ξ_{2K}	0.01279 (0.01287)	0.00004 (0.0)	0.0 (0.0)	-0.01697 (0.00701)	-0.00012 (0.0)	0.0 (0.0)
ξ_{2F}	-0.00247 (0.00614)	-0.00007 (0.0)	0.0 (0.0)	0.01325 (0.00796)	0.00007 (0.0)	0.0 (0.0)
ξ_{2R}	-0.00139 (0.00387)	0.00002 (0.0)	0.0 (0.0)	0.00041 (0.00348)	0.00011 (0.0)	0.0 (0.0)
δ_{21}	-0.00393 (0.00018)	-0.00309 (0.0)	-0.00307 (0.0)	-0.00400 (0.00017)	-0.00309 (0.0)	-0.00308 (0.0)
δ_{22}	0.00241 (0.00016)	0.00309 (0.0)	0.00307 (0.0)	0.00237 (0.00015)	0.00309 (0.0)	0.00308 (0.0)

[a]See the footnote (a) in Table VI-1.

TABLE VI-4-a.--Parameter Estimates for the Translog Functions[a]--
Manufacture of Knitted Underwear.

	Translog Production Function		Translog Cost Function	
Parameter	Unrestricted	Symmetry	Unrestricted	Symmetry
α_P	0.12064 (0.01235)	0.11602 (0.00811)	0.12446 (0.00942)	0.11980 (0.00809)
α_A	0.01617 (0.00262)	0.01624 (0.00207)	0.02431 (0.00236)	0.02273 (0.00206)
α_K	0.21565 (0.02028)	0.22754 (0.01253)	0.26835 (0.01308)	0.25476 (0.01052)
α_F	0.02064 (0.00136)	0.02157 (0.00127)	0.01267 (0.00119)	0.01471 (0.00105)
α_R	0.62691 (0.02219)	0.61862 (0.0)	0.57020 (0.01778)	0.58801 (0.0)
γ_{PP}	0.07416 (0.01430)	0.06069 (0.00928)	0.08612 (0.02205)	0.05682 (0.00962)
γ_{PA}	-0.03284 (0.01423)	-0.01617 (0.00248)	0.00413 (0.01896)	-0.01275 (0.00417)
γ_{PK}	-0.00066 (0.00996)	-0.00398 (0.00834)	-0.02264 (0.00697)	-0.03137 (0.00583)
γ_{PF}	-0.00254 (0.00602)	-0.00319 (0.00127)	-0.01869 (0.00814)	-0.00196 (0.00180)
γ_{PR}	-0.03744 (0.00636)	-0.03735 (0.00621)	-0.00529 (0.00858)	-0.01074 (0.00770)
γ_{AP}	-0.01310 (0.00303)	-0.01617 (0.00248)	-0.00860 (0.00553)	-0.01275 (0.00417)
γ_{AA}	0.02628 (0.00302)	0.02532 (0.00203)	0.02071 (0.00476)	0.01762 (0.00370)
γ_{AK}	-0.00168 (0.00211)	-0.00290 (0.00200)	-0.00721 (0.00175)	-0.00830 (0.00164)
γ_{AF}	-0.00221 (0.00128)	-0.00070 (0.00077)	0.00254 (0.00204)	0.00302 (0.00127)
γ_{AR}	-0.00593 (0.00135)	-0.00556 (0.00132)	0.00187 (0.00215)	0.00042 (0.00193)
γ_{KP}	0.00199 (0.02349)	-0.00398 (0.00834)	-0.01246 (0.03061)	-0.03137 (0.00583)

TABLE VI-4-b.--Continued.

	Translog Production Function		Translog Cost Function	
Parameter	Unrestricted	Symmetry	Unrestricted	Symmetry
γ_{KA}	0.02540 (0.02337)	-0.00290 (0.00200)	0.00493 (0.02633)	-0.00830 (0.00164)
γ_{KK}	0.02984 (0.01636)	0.02617 (0.01168)	0.06515 (0.00968)	0.05605 (0.00766)
γ_{KF}	-0.01280 (0.00989)	0.00106 (0.00104)	-0.00298 (0.01130)	-0.00233 (0.00085)
γ_{KR}	-0.02355 (0.01045)	-0.02036 (0.00898)	0.00235 (0.01191)	-0.01405 (0.00848)
γ_{FP}	-0.00458 (0.00158)	-0.00319 (0.00127)	-0.00360 (0.00279)	-0.00196 (0.00180)
γ_{FA}	-0.00091 (0.00157)	-0.00070 (0.00077)	-0.00235 (0.00240)	0.00302 (0.00127)
γ_{FK}	0.00033 (0.00110)	0.00106 (0.00104)	-0.00209 (0.00088)	-0.00233 (0.00085)
γ_{FF}	0.00521 (0.00066)	0.00490 (0.00061)	-0.00336 (0.00103)	-0.00108 (0.00089)
γ_{FR}	-0.00201 (0.00070)	-0.00207 (0.00070)	0.00057 (0.00109)	0.00235 (0.00097)
γ_{RP}	-0.05847 (0.02570)	-0.03735 (0.0)	-0.06146 (0.04161)	-0.01074 (0.0)
γ_{RA}	-0.01793 (0.02557)	-0.00556 (0.0)	-0.02781 (0.03579)	0.00042 (0.0)
γ_{RK}	-0.02783 (0.01791)	-0.02036 (0.0)	-0.03321 (0.01316)	-0.01405 (0.0)
γ_{RF}	0.01234 (0.01082)	-0.00207 (0.0)	0.02249 (0.01537)	0.00235 (0.0)
γ_{RR}	0.06892 (0.01144)	0.06534 (0.0)	0.00050 (0.01618)	0.02202 (0.0)

[a]See the footnote (a) in Table VI-1.

TABLE VI-5-a.--Parameter Estimates for the Translog Functions[a]--Manufacture of Briquettes.

Parameter	Translog Production Function		Translog Cost Function	
	Unrestricted	Symmetry	Unrestricted	Symmetry
α_P	0.03747 (0.00440)	0.04134 (0.00357)	0.03385 (0.00532)	0.03956 (0.00438)
α_A	0.03096 (0.00216)	0.03307 (0.00175)	0.02281 (0.00284)	0.02271 (0.00241)
α_K	0.21490 (0.01174)	0.17562 (0.00810)	0.25783 (0.00976)	0.24605 (0.00701)
α_F	0.01344 (0.00092)	0.01322 (0.00076)	0.00842 (0.00111)	0.01015 (0.00094)
α_R	0.70323 (0.01160)	0.73673 (0.0)	0.67709 (0.01379)	0.68154 (0.0)
γ_{PP}	0.03109 (0.00580)	0.04208 (0.00348)	0.02652 (0.01090)	0.02470 (0.00492)
γ_{PA}	-0.00856 (0.00457)	-0.00870 (0.00175)	-0.01411 (0.00885)	-0.00334 (0.00336)
γ_{PK}	0.00837 (0.00384)	0.00948 (0.00368)	-0.02563 (0.00227)	-0.02700 (0.00209)
γ_{PF}	0.00173 (0.00245)	0.00039 (0.00078)	-0.01130 (0.00378)	0.00137 (0.00118)
γ_{PR}	-0.04108 (0.00318)	-0.04325 (0.00291)	0.00053 (0.00584)	0.00426 (0.00491)
γ_{AP}	-0.01233 (0.00285)	-0.00890 (0.00175)	0.00506 (0.00581)	-0.00334 (0.00336)
γ_{AA}	0.02488 (0.00225)	0.02522 (0.00164)	0.01021 (0.00472)	0.01360 (0.00347)
γ_{AK}	-0.00000 (0.00189)	0.00050 (0.00185)	-0.00835 (0.00121)	-0.00885 (0.00117)
γ_{AF}	0.00019 (0.00121)	0.00034 (0.00059)	-0.00191 (0.00201)	0.00104 (0.00104)
γ_{AR}	-0.01622 (0.00156)	-0.01736 (0.00146)	-0.00160 (0.00311)	-0.00246 (0.00266)
γ_{KP}	0.02304 (0.01549)	0.00948 (0.00368)	-0.01121 (0.02000)	-0.02700 (0.00209)

TABLE VI-5-b.--Continued.

Parameter	Translog Production Function		Translog Cost Function	
	Unrestricted	Symmetry	Unrestricted	Symmetry
γ_{KA}	0.02847 (0.01221)	0.00050 (0.00185)	-0.00521 (0.01625)	-0.00885 (0.00117)
γ_{KK}	-0.01150 (0.01026)	-0.01697 (0.00794)	0.05950 (0.00416)	0.05689 (0.00380)
γ_{KF}	0.01043 (0.00655)	0.00135 (0.00079)	-0.00780 (0.00693)	-0.00337 (0.00046)
γ_{KR}	-0.00963 (0.00849)	0.00564 (0.00652)	0.00158 (0.01072)	-0.01767 (0.00509)
γ_{FP}	0.00006 (0.00122)	0.00039 (0.00078)	0.00065 (0.00228)	0.00137 (0.00118)
γ_{FA}	0.00071 (0.00096)	0.00034 (0.00059)	-0.00103 (0.00185)	0.00104 (0.00104)
γ_{FK}	0.00134 (0.00081)	0.00135 (0.00079)	-0.00329 (0.00047)	-0.00337 (0.00046)
γ_{FF}	0.00403 (0.00051)	0.00397 (0.00042)	-0.00263 (0.00079)	-0.00065 (0.00060)
γ_{FR}	-0.00608 (0.00067)	-0.00604 (0.00061)	-0.00023 (0.00122)	0.00160 (0.00104)
γ_{RP}	-0.04186 (0.01530)	-0.04325 (0.0)	-0.02102 (0.02827)	0.00426 (0.0)
γ_{RA}	-0.04549 (0.01206)	-0.01736 (0.0)	0.01014 (0.02297)	-0.00246 (0.0)
γ_{RK}	0.00179 (0.01013)	0.00564 (0.0)	-0.02224 (0.00588)	-0.01767 (0.0)
γ_{RF}	-0.01638 (0.00647)	-0.00604 (0.0)	0.02363 (0.00980)	0.00160 (0.0)
γ_{RR}	0.07301 (0.00839)	0.06100 (0.0)	-0.00028 (0.01515)	0.01426 (0.0)

[a]See the footnote (a) in Table VI-1.

TABLE VI-6-a.--Parameter Estimates for the Translog Functions[a]--
Molding Industry.

Parameter	Translog Production Function		Translog Cost Function	
	Unrestricted	Symmetry	Unrestricted	Symmetry
α_P	0.06467 (0.00983)	0.08149 (0.00888)	0.10325 (0.1188)	0.10587 (0.01075)
α_A	0.01937 (0.00323)	0.02178 (0.00305)	0.02470 (0.00417)	0.02416 (0.00382)
α_K	0.24182 (0.02103)	0.24213 (0.01452)	0.23530 (0.01915)	0.19254 (0.01258)
α_F	0.06395 (0.00881)	0.08153 (0.00698)	0.08283 (0.00940)	0.07010 (0.00704)
α_R	0.61019 (0.02294)	0.57307 (0.0)	0.55392 (0.02434)	0.60733 (0.0)
γ_{PP}	0.02433 (0.00890)	0.03598 (0.00418)	0.02430 (0.01582)	0.02190 (0.00526)
γ_{PA}	-0.01438 (0.00953)	-0.01499 (0.00206)	-0.04535 (0.01279)	-0.01240 (0.00327)
γ_{PK}	-0.00894 (0.00561)	-0.00944 (0.00484)	-0.00873 (0.00385)	-0.01149 (0.00337)
γ_{PF}	-0.00885 (0.00336)	-0.00523 (0.00236)	0.00729 (0.00335)	0.00563 (0.00230)
γ_{PR}	-0.00316 (0.00345)	-0.00633 (0.00323)	-0.00524 (0.00352)	-0.00364 (0.00348)
γ_{AP}	-0.01668 (0.00293)	-0.01499 (0.00206)	-0.00490 (0.00556)	-0.01240 (0.00327)
γ_{AA}	0.02221 (0.00313)	0.02035 (0.00208)	0.01189 (0.00449)	0.01769 (0.00299)
γ_{AK}	-0.00203 (0.00184)	-0.00246 (0.00171)	-0.00384 (0.00135)	-0.00447 (0.00129)
γ_{AF}	-0.00231 (0.00111)	-0.00094 (0.00092)	0.00191 (0.00118)	0.00109 (0.00097)
γ_{AR}	-0.00156 (0.00113)	-0.00197 (0.00109)	-0.00250 (0.00124)	-0.00191 (0.00122)
γ_{KP}	0.05326 (0.01905)	-0.00944 (0.00484)	-0.02373 (0.02552)	-0.01149 (0.00337)

TABLE VI-6-b.--Continued.

	Translog Production Function		Translog Cost Function	
Parameter	Unrestricted	Symmetry	Unrestricted	Symmetry
γ_{KA}	-0.03197 (0.02040)	-0.00246 (0.00171)	0.01368 (0.02062)	-0.00447 (0.00129)
γ_{KK}	-0.00411 (0.01200)	-0.00385 (0.00958)	0.05884 (0.00621)	0.05154 (0.00522)
γ_{KF}	-0.00223 (0.00720)	0.00846 (0.00398)	-0.00547 (0.00540)	-0.01403 (0.00240)
γ_{KR}	0.00603 (0.00738)	0.00728 (0.00663)	-0.01465 (0.00567)	-0.02155 (0.00475)
γ_{FP}	-0.00426 (0.00798)	-0.00523 (0.00236)	-0.00924 (0.01252)	0.00563 (0.00230)
γ_{FA}	-0.00431 (0.00855)	-0.00094 (0.00092)	0.00054 (0.01012)	0.00109 (0.00097)
γ_{FK}	0.00532 (0.00503)	0.00846 (0.00398)	-0.01145 (0.00305)	-0.01403 (0.00240)
γ_{FF}	0.00247 (0.00312)	0.00241 (0.00286)	0.01085 (0.00265)	0.00899 (0.00231)
γ_{FR}	-0.00665 (0.00309)	-0.00472 (0.00280)	0.00146 (0.00278)	-0.00169 (0.00251)
γ_{RP}	-0.05665 (0.02078)	-0.00633 (0.0)	0.01357 (0.03244)	-0.00364 (0.0)
γ_{RA}	0.02845 (0.02225)	-0.00197 (0.0)	0.01925 (0.02621)	-0.00191 (0.0)
γ_{RK}	0.00976 (0.01309)	0.00728 (0.0)	-0.03481 (0.00789)	-0.02155 (0.0)
γ_{RF}	0.01091 (0.00785)	-0.00472 (0.0)	-0.01458 (0.00686)	-0.00169 (0.0)
γ_{RR}	0.00534 (0.00806)	0.00574 (0.0)	0.02094 (0.00721)	0.02879 (0.0)

[a]See the footnote (a) in Table VI-1.

constant terms, all the own variable's coefficients and most of the cross variables' coefficients in each share equation of the restricted estimation of the translog production function are significant, except few of the cross variables' coefficients between inputs and outputs (such as ε_{K3}, ξ_{1P}, ξ_{1A}, ξ_{1F}, ξ_{2P}, ξ_{2A}, ξ_{2K}, and ξ_{2F} etc.). These insignificant coefficients may be attributed to the existence of the weak separability between inputs and outputs in the multi-input, multi-output production technology. The validity of this weak separability is again discussed in the next section 2.2.2. in the significance of alternative estimations with respect to different hypothesis.

The significance of the parameters estimated in the translog cost function is also similar to that in the translog production function over all six industries, in general.

2.2. Significance of the Estimation

In this subsection to investigate statistical significance of the current empirical estimation, two different measures are considered, i.e., the first for the goodness of fit and the second for testing the validity of the restrictions imposed.

2.2.1. The Goodness of Fit

As a measure for the goodness of fit in each share equation, the R^2 is calculated as one minus the ratio of the error sum of squares (about its mean) to the total sum of squares (about its mean). Although only four of the input shares are included in the estimation procedure, the R^2 for the deleted input share can be inferred using the linear homogeneity constraints. The R^2 for the deleted output share also can be inferred in a similar way. Since we have more than one equation to be estimated, no definite inferences can be made on the goodness of fit for the equations system as a whole. Hence, two arbitrary measures to the goodness of fit for the whole system are considered, i.e., the simple average of the R^2's of each share equations and their weighted average by each factor share. And as already noted in the previous section, the interpretation of the R^2's in the case of estimation with restrictions imposed on the parameters should be different, in the sense that the sum of the unexplained variation in the explained variable (i.e. the error sum of aquares) and the explained variance is no more equal to the total variance of the explained variable (i.e., the total sum of squares).[6] Tables VII-1 through VII-3 contain, for six industries, the R^2's of each share equations and their averages by row, and by column those in the unrestricted and the restricted estimations, separated by the translog production and cost functions.

Measured by the multiple correlation coefficient, R^2, the fit is very poor in the input share equations, averaging under 0.5, and is much better in the output equations, averaging above 0.8, for all of six industries.

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[6]See the section 4.4.3., Chapter III, Part B.

TABLE VII-1.--R^2 and Relative Size of the Error Sum of Squares[a]--Canning Industry.

	Translog Production Function			Translog Cost Function		
Equations[b]	Unrestricted	Symmetry	Explicit Separability	Unrestricted	Symmetry	Explicit Separability
S_P	0.3318	0.2645	0.1874	0.4252	0.1950	0.1217
S_A	0.4381	0.2962	0.2302	0.1838	0.1184	0.0446
S_K	0.1872	0.1044	0.0560	0.2976	0.1973	0.1519
S_F	0.3152	0.2673	0.1692	0.3260	0.2058	0.0566
S_R	0.3895	0.3157	0.2381	0.3130	0.1652	0.0474
Simple Avg.	(0.3324)	(0.2496)	(0.1762)	(0.3091)	(0.1763)	(0.0844)
Weighted Avg.	(0.3431)	(0.2338)	(0.1649)	(0.3537)	(0.1784)	(0.0875)
S_1	0.9017	0.8844	0.8837	0.9015	0.8846	0.8832
S_2	0.7339	0.7210	0.7188	0.7362	0.7212	0.7194
S_3	0.7768	0.7486	0.7492	0.7646	0.7486	0.7497
Simple Avg.	(0.8041)	(0.7847)	(0.7839)	(0.8008)	(0.7848)	(0.7841)
Weighted Avg.	(0.8349)	(0.7975)	(0.7964)	(0.8436)	(0.7969)	(0.7964)

[a]Only for the unrestricted case, the usual concept of R^2 is valid and for the other cases with specific restrictions imposed on the parameters to be estimated, one minus the relative size of the error sum of squares are presented here. The unrestricted case means that *a priori* assumption of the linear homogeneity of the function is implied in the estimating equations, but no explicit restrictions are imposed on the parameter estimated. In the other cases, for example, the case with symmetry restrictions contains the restrictions of linear homogeneity and symmetry conditions, imposed explicitly on the parameters estimated.

[b]The equations referred to here are, for example, such that S_P indicates the share equation of operative worker input and S_1 refers to that of the first major commodity in the canning industry. Simple average is the arithmetic mean of R^2's for all factors of inputs and outputs and the weighted average is the average weighted by each factor's elasticity with respect to total output.

TABLE VII-2.--R^2 and Relative Size of the Error Sum of Squares[a]--Leather Footwear and Screw Products Industries.

	Translog Production Function			Translog Cost Function		
Equations[b]	Unrestricted	Symmetry	Explicit Separability	Unrestricted	Symmetry	Explicit Separability
Leather Footwear Industry:						
S_P	0.3960	0.2387	0.1949	0.4041	0.2252	0.2194
S_A	0.3901	0.3094	0.1994	0.7387	0.6715	0.6681
S_K	0.3353	0.1307	0.0920	0.6180	0.2054	0.2022
S_F	0.7900	0.6896	0.6900	0.4078	-0.0213	-0.0260
S_R	0.5541	0.3097	0.3070	0.5095	0.0076	-0.0026
Simple Avg.	(0.4931)	(0.3356)	(0.1967)	(0.5356)	(0.2177)	(0.2122)
Weighted Avg.	(0.4902)	(0.2710)	(0.2523)	(0.5331)	(0.1200)	(0.1110)
S_1	0.9674	0.9475	0.9475	0.9623	0.9475	0.9475
S_2	0.9669	0.9467	0.9468	0.9630	0.9468	0.9467
Simple Avg.	(0.9672)	(0.9471)	(0.9472)	(0.9627)	(0.9472)	(0.9471)
Weighted Avg.	(0.9718)	(0.9471)	(0.9472)	(0.9601)	(0.9472)	(0.9471)
Screw Product Industry:						
S_P	0.1779	0.1274	0.1259	0.3092	0.2897	0.2890
S_A	0.3136	0.3044	0.3034	0.2967	0.2876	0.2696

TABLE VII-2.--Continued.

	Translog Production Function			Translog Cost Function		
Equations[b]	Unrestricted	Symmetry	Explicit Separability	Unrestricted	Symmetry	Explicit Separability
S_K	0.0693	0.0075	0.0035	0.6072	0.5837	0.5732
S_F	0.3043	0.2925	0.2504	0.2165	0.1910	0.1422
S_R	0.1151	0.0249	0.0240	0.1244	0.0931	0.0822
Simple Avg.	(0.1960)	(0.1513)	(0.1414)	(0.3108)	(0.2890)	(0.2712)
Weighted Avg.	(0.1322)	(0.0649)	(0.0603)	(0.2687)	(0.2541)	(0.2501)
S_1	0.9636	0.9528	0.9528	0.9661	0.9529	0.9528
S_2	0.9647	0.9535	0.9533	0.9676	0.9535	0.9534
Simple Avg.	(0.9642)	(0.9532)	(0.9531)	(0.9669)	(0.9532)	(0.9531)
Weighted Avg.	(0.9698)	(0.9531)	(0.9530)	(0.9728)	(0.9532)	(0.9530)

[a]See the footnote (a) in Table VII-1.

[b]See the footnote (b) in Table VII-1.

TABLE VII-3.--R^2 and Relative Size of the Error Sum of Squares[a]--Manufacture of Knitted Underwear, Briquettes, and Molding Industry.

	Translog Production Function		Translog Cost Function	
Equations	Unrestricted	Symmetry	Unrestricted	Symmetry
Manufacture of Knitted Underwear				
S_P	0.5326	0.5147	0.3978	0.3601
S_A	0.4658	0.4522	0.1897	0.1721
S_K	0.1332	0.0075	0.4722	0.4589
S_F	0.3931	0.3926	0.2194	0.1809
S_R	0.2635	0.1937	0.0820	0.0497
Simple Avg.	(0.3576)	(0.3121)	(0.2722)	(0.2443)
Weighted Avg.	(0.2536)	(0.1877)	(0.1969)	(0.1667)
Manufacture of Briquettes				
S_P	0.2865	0.2743	0.2373	0.1524
S_A	0.4759	0.4551	0.2160	0.1948
S_K	0.0781	0.0492	0.2959	0.2688
S_F	0.4266	0.3991	0.1928	0.1048
S_R	0.2511	0.2293	0.1175	0.0322
Simple Avg.	(0.3036)	(0.2814)	(0.2119)	(0.1506)
Weighted Avg.	(0.2253)	(0.2009)	(0.1836)	(0.1116)
Molding Industry				
S_P	0.2891	0.2247	0.1965	0.1272
S_A	0.3232	0.3105	0.1266	0.1008
S_K	0.0903	-0.0121	0.4165	0.3840
S_F	0.0795	0.0237	0.1904	0.1610
S_R	0.0911	0.0100	0.2085	0.1345
Simple Avg.	(0.1746)	(0.1096)	(0.2277)	(0.1815)
Weighted Avg.	(0.1070)	(0.0298)	(0.2527)	(0.1828)

[a]See the footnote (a) in Table VII-1. Assumption of explicit separability between input and output is *a priori* given in the case of a uniproduct production technology. See also the footnote (b) in Table VII-1.

For example, in the canning industry, the multiple correlation coefficient in the share equation of operative worker in 0.3318 in the unrestricted estimation of the translog production function, 0.2645 in the restricted one of linear homogeneity and symmetry condition, and 0.1874 in the one of explicit separability between input and output. Also the explanatory power of the restricted share equations decline markedly relative to that of the unrestricted ones, i.e., the simple average of the R^2's in the input share equations of translog production function changes from 0.3324 of the unrestricted estimation to 0.2486 of the restricted one by symmetry condition, and to 0.1762 of the restricted one by the explicit separability. On the other hand, the simple average of the R^2's in the output equations has very little changes from 0.8041 of the unrestricted one to 0.7839 of the restricted one of the explicit separability.

The relative size of the error sum of squares becomes, in general, more drastically large in the input share equations than in the output share equations, both in translog production and cost function, as more rigid restrictions are imposed on the parameters to be estimated, while it seems to be indeterminate whether the relative effects of various restrictions on the goodness of fit are more serious in translog production function or in translog cost function. For example in the canning industry, and in the leather footwear industry, greater changes in the R^2's of the production function than in the cost function, while in the screw products industry the reverse is true, but of negligible degree, as shown in Tables VII-1 and VII-2.

In the comparison of the translog production function approach with the translog cost function approach, the goodness of fit in the translog production function (measured by one minus the relative size of the error sum of squares) is better, judged by either the simple or the weighted averages of R^2's in the canning industry, the manufacture of knitted underwear and the manufacture of briquettes, while that in the translog cost function is better in the leather footwear industry, the screw products industry and the molding industry.[7]

The tendency of the low R^2 in the input equations seems to be in common either to alternative restrictions imposed, to different establishment size, or to alternative choices of variables of different quality, as will be seen in the following sections.

This result seems to be attributed to our earlier findings in "the sample properties," section 6, Chapter II, such that there exist quite a wild variations in factor productivity and factor price of the sample establishments. Alternatively speaking, the degree of market imperfection in the factor markets is so high that the actual decisions on factor shares (or factor demand) deviate very widely from the first-order conditions for either

[7]The comparison of the goodness of fit by each share equation is not attempted here, since their wild variations in the equations system seem to force us to make their averages for the inputs and the outputs.

profit maximization or cost minimization. In reality there may exist quite a high degree of input hoardings such that considerably high portions of factor demands are not price-responsive and cannot be adjusted appropriately during given production period.

2.2.2. Significance of Alternative Hypotheses

The likelihood ratio test statistics (i.e., $-2 \ln \Lambda$)[8] are shown in Table VIII-1 for the validity of the hypotheses under consideration (i.e., those of linear homogeneity, symmetry and explicit separability between input and output) both in the translog production and cost functions by industry. The hypotheses of linear homogeneity and symmetry conditions are accepted both in the estimations of the translog production and cost functions of the leather footwear industry, and also accepted in that of the translog production function of the manufacture of knitted underwear, while it is either mildly or decisively rejected all in the other cases. The hypothesis of explicit separability between input and output is also accepted in the estimations of both functions of the leather footwear industry.[9]

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[8] See the section 4.4.1., Chapter III, Part B.

[9] As mentioned earlier, the hypothesis of the explicit separability between input and output are included as an a priori given assumption in the estimation of share equations system for the industries where only one commodity is produced, i.e., the manufacture of knitted underwear, of briquettes, and the molding industry. Hence for these industries, once the hypothesis of symmetry conditions is accepted in the test, then the hypothesis of the explicit separability is subsequently identified as an accepted one. This is the case of production function in the manufacture of knitted underwear.

TABLE VIII-1.--Likelihood Ratio Test for Alternative Restrictions.[a]

Industry & Restrictions	Degree of Freedom (γ)	Critical Level of $\chi^2(\gamma)$ at $\alpha = 0.01$	Test Statistics	
			Production Function	Cost Function
I. Canning Industry				
(a) Symmetry	27	45.642	47.522	57.696
(b) Explicit Separability	35	57.292	66.160	75.232
II. Leather Footwear Industry				
(a) Symmetry	20	37.566	35.471	31.756
(b) Explicit Separability	24	42.980	39.357	32.018
III. Screw Products Industry				
(a) Symmetry	20	37.566	47.400	42.036
(b) Explicit Separability	24	42.980	55.240	53.222

TABLE VIII-1.--Continued.

Industry & Restrictions	Degree of Freedom (γ)	Critical Level of $\chi^2(\gamma)$ at $\alpha = 0.01$	Test Statistics	
			Production Function	Cost Function
IV. Knitted Underwear				
(a) Symmetry	10	23.209	19.901	31.526
V. Briquettes				
(a) Symmetry	10	23.209	42.521	29.906
VI. Molding Industry				
(a) Symmetry	10	23.209	32.824	26.064

[a]Here the estimation with symmetry restrictions indicates the case with restrictions of linear homogeneity and symmetry conditions explicitly imposed on the parameter estimated. And that with explicit separability also includes the case with restrictions of linear homogeneity, symmetry, and explicit separability between input and output. But in those uniproduct industries (i.e., manufactures of knitted underwear and briquettes, and molding industry) the case with symmetry restrictions imply that the explicit separability between input and output is _a priori_ given in the estimations.

On the other hand, Table VIII-2 contains the alternative F-statistics for testing the validity of the same hypotheses under consideration. Owing to the F-test suggested by Theil,[10] both the hypotheses of symmetry and explicit separability between input and output are accepted in the estimation of either the translog production or cost functions in the canning industry and the leather footwear industry, while the hypothesis of symmetry conditions in the translog production function in the manufacture of briquettes is decisively rejected. All the other cases are either accepted or rejected rather mildly, as shown in Table VIII-2. For example, the symmetry conditions in the production function are accepted, but those in the cost function are rejected in the manufacture of knitted underwear, while the explicit separability restriction in the cost function is accepted but rejected in the production function of the screw products industry. Hence, in the comparison of two alternative test statistics, the findings seem to be consistent in the sense that the accepted hypothesis in the likelihood ratio test are also accepted in the F-test, but not necessarily true in the reverse. Hence here we find no serious conflicts from alternative procedures for testing restrictions on the coefficients of a linear regression model, suggested by Savin.[11]

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[10]See Theil (1971), pp. 143-144, pp. 313-314, and pp. 402-403. Also see section 4.4.1. and section 4.4.2, Chapter III, Part B.

[11]See Savin (1966). Also see the footnote 34 in section 4.4.2., Chapter III, Part B.

TABLE VIII-2.--F-Test for Alternative Restrictions.[a]

Industry & Restrictions	Degree of Freedom v_1	Degree of Freedom v_2	Critical Level of F at $\alpha = 0.01$[b]	Test Statistics: Production Function	Test Statistics: Cost Function
I. Canning Industry					
(a) Symmetry	27	342	1.832	1.434	1.752
(b) Explicit Separability	35	342	1.705	1.496	1.703
II. Leather Footwear Industry					
(a) Symmetry	20	60	2.200	1.060	0.952
(b) Explicit Separability	24	60	2.120	0.742	0.640
III. Screw Products Industry					
(a) Symmetry	20	430	1.919	2.128	2.002
(b) Explicit Separability	24	430	1.839	1.958	1.833

TABLE VIII-2.--Continued.

Industry & Restrictions	Degree of Freedom v_1	v_2	Critical Level of F at $\alpha = 0.01$[b]	Test Statistics: Production Function	Cost Function
IV. Knitted Underwear					
(a) Symmetry	10	432	2.368	1.904	2.998
V. Briquettes					
(a) Symmetry	10	908	2.345	4.228	2.894
VI. Molding Industry					
(a) Symmetry	10	500	2.365	3.103	2.461

[a] See the footnote (a) in Table VII-1.

[b] The critical value of F-distribution at significance level of 0.01 is calculated by interpolation between two values for different degrees of freedom specified in the table of F-distribution, when the corresponding values are not appeared for the degrees of freedom which we have here.

In particular, since the explicit separability between input and output in translog cost function means homotheticity of the production in input (implying that there exists an isoquant map which is independent of the levels or mix of outputs)[12], this homotheticity property holds, judged by the F-test in Table VIII-2, in all of three multiproduct industries. But only for the screw products industry, the explicit (or multiplicative) separability in translog production function is rejected by the F-test, implying that the functional form of transformation may not be written as that of F(X)G(Y)=1.

2.2.3. Significance of Each Restriction Imposed[13]

The null hypotheses of each restriction are also investigated by the significance test of t-values for the Lagrangian multipliers corresponding to each restriction, shown in Table VIII-3. The acceptance of each null hypothesis varies from the type of restriction to industry concerned. Where the Lagrangians are assumed to be asymptotically normal and hence the critical value of 2.750 in absolute value at the significance level of 0.01 is adopted. For example, in the canning industry, the linear homogeneity condition for the input factors in the share equation

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[12] See the section 3.2.6., Chapter III, Part A, on the discussion of homotheticity of the cost function.

[13] Significance of separate restriction is investigated only for the case of linear homogeneity and symmetry conditions and the case of explicit separability between input and output is omitted here.

TABLE VIII-3-1.--Test Statistics for Separate Restriction (t-value of the Lagrangian Multiplier).[a]

Restriction[b]	Canning Industry		Leather Footwear Industry		Screw Products Industry	
	Production Function	Cost Function	Production Function	Cost Function	Production Function	Cost Function
$\sum_{j=1}^{5} \gamma_{ij}=0; i=P$	2.288	-3.356*	2.502	-2.448	2.537	-0.497
$i=A$	-4.233*	2.200	-3.240*	-0.865	-2.642	-0.416
$i=K$	0.667	1.449	0.446	1.076	0.073	0.981
$i=F$	-0.878	0.631	2.852*	0.698	0.400	-0.272
$\sum_{k=1}^{3} \varepsilon_{ik}=0; i=P$	1.469	-1.075	-1.435	-1.981	-1.161	-1.237
$i=A$	0.305	-1.361	0.356	1.123	1.183	0.211
$i=K$	1.261	-1.607	-1.713	-1.986	-1.556	-0.440
$i=F$	0.911	-1.322	-1.332	-0.670	0.063	-0.069
$\sum_{j=1}^{5} \varepsilon_{kj}=0; k=1$	-0.420	0.110	-1.683	0.143	-0.661	-0.067
$k=2$	-0.651	0.687	--	--	--	--
$\sum_{\ell=1}^{3} \delta_{k\ell}=0; k=1$	1.874	2.332	2.242	1.443	5.296*	5.140*
$k=2$	1.247	1.400	--	--	--	--
$\gamma_{PA} = \gamma_{AP}$	1.142	-1.972	-0.176	1.540	0.845	0.283
$\gamma_{PK} = \gamma_{KP}$	1.891	0.658	-0.404	0.203	2.689	0.631
$\gamma_{PF} = \gamma_{FP}$	0.414	0.112	-3.142*	-1.985	0.951	0.871

TABLE VIII-3-1-.--Continued.

Restriction[b]	Canning Industry		Leather Footwear Industry		Screw Products Industry	
	Production Function	Cost Function	Production Function	Cost Function	Production Function	Cost Function
$\gamma_{AK} = \gamma_{KA}$	2.043	1.679	-7.192*	-0.244	0.920	1.286
$\gamma_{AF} = \gamma_{FA}$	0.957	1.535	1.493	1.210	0.531	-0.417
$\gamma_{KF} = \gamma_{FK}$	-0.896	0.055	-2.819*	-4.369*	-0.764	-1.794
$\varepsilon_{P1} = \varepsilon_{1P}$	-1.442	0.889	-0.453	0.168	-0.627	0.072
$\varepsilon_{P2} = \varepsilon_{2P}$	-1.025	0.634	--	--	--	--
$\varepsilon_{A1} = \varepsilon_{1A}$	-1.512	0.171	0.009	0.067	-0.161	0.121
$\varepsilon_{A2} = \varepsilon_{2A}$	-1.101	0.460	--	--	--	--
$\varepsilon_{K1} = \varepsilon_{1K}$	-0.509	-1.162	0.238	0.620	-0.811	-1.186
$\varepsilon_{K2} = \varepsilon_{2K}$	-0.995	0.281	--	--	--	--
$\varepsilon_{F1} = \varepsilon_{1F}$	-1.223	0.102	-2.321	-1.227	-0.556	0.122
$\varepsilon_{F2} = \varepsilon_{2F}$	-1.461	1.256	--	--	--	--
$\delta_{12} = \delta_{21}$	1.396	1.378	--	--	--	--

[a]The Lagrangian Multiplier is explained in section 4.1.2. and 4.4.3., Chapter III, Part B. The numbers with asterisk (*) in the table indicates that the corresponding t-value for each restriction greater than the critical level of 2.750 in absolute value has a 0.01 probability.

[b]Restrictions considered here is explained in detail in section 4.3., Chapter III, Part B. For example, the linear homogeneity condition of $\sum_{j=1}^{5} \gamma_{ij} = 0$, where i=P, indicates that $(\gamma_{PP} + \gamma_{PA} + \gamma_{PK} + \gamma_{PF} + \gamma_{PR})$ is equal to zero.

TABLE VIII-3-2.--Test Statistics for Separate Restriction (t-value of the Lagrangian Multiplier).[a]

Restriction	Manufacture of Knitted Underwear		Manufacture of Briquettes		Molding Industry	
	Production Function	Cost Function	Production Function	Cost Function	Production Function	Cost Function
$\sum_{j=1}^{5} \gamma_{ij}=0; i=P$	-3.790*	2.289	-2.328	-1.006	-3.078*	-2.311
$i=A$	2.584	1.413	-0.244	1.672	1.528	1.151
$i=K$	1.010	2.077	4.421*	2.466	0.176	2.532
$i=F$	-3.062*	-4.131*	2.819*	-3.055*	-3.202*	2.257
$\gamma_{PA} = \gamma_{AP}$	-2.474	-0.521	-0.624	2.272*	-2.100	-2.256
$\gamma_{PK} = \gamma_{KP}$	-0.650	1.103	-3.649*	-1.465	-3.677*	-2.188
$\gamma_{PF} = \gamma_{FP}$	3.016*	3.207*	-2.864*	1.318	-0.944	-0.633
$\gamma_{AK} = \gamma_{KA}$	2.440	1.094	-1.243	0.937	1.457	1.047
$\gamma_{AF} = \gamma_{FA}$	3.171*	3.940*	-2.573	2.264	2.175	1.440
$\gamma_{KF} = \gamma_{FK}$	3.052*	3.931*	-2.355	2.915*	2.687	-0.211

[a]See the footnote (a) in Table VIII-3-1.

[b]See the footnote (b) in Table VIII-3-1.

of administrative worker (i.e., $\gamma_{AP} + \gamma_{AA} + \gamma_{AK} + \gamma_{AF} + \gamma_{AR} = 0$) is found to be a significant constraints in the estimation of translog production function, where the null hypothesis of the Lagrangian multiplier for this condition is rejected, judging from its t-value of -4.233 which is far greater than the critical value Of 2.750 in absolute value at the significance level of 0.01.[14]

Among the multiproduct industries, the leather footwear industry has a total of five significant restrictions in its estimation of translog production function, two of which are the linear homogeneity conditions and three of which are the symmetry conditions, as shown in the third column of the Table VIII-3-1. Also the manufacture of knitted underwear and of briquettes has four or five significant restrictions in their estimation of either production or cost function. In average among our six industries, the symmetry conditions are found to be more significantly binding in the estimation of either production or cost function.[15]

The acceptance of null hypothesis on each restriction imposed on the parameter estimated seems to have no direct and consistent implications on the validity of those hypotheses of linear homogeneity and symmetry conditions as a whole, compared with the case

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[14]See the number with asterisk, *, in the first column of the Table VIII-3-1.

[15]For example, in the production of manufacture of knitted underwear the significant restrictions are found to be two of linear homogeneity and three of symmetry conditions, shown as the t-values with asterisk in the first column of the Table VIII-3-2.

of unrestricted, where the restricted estimation of linear homogeneity and symmetry conditions in the production function of the leather footwear industry was accepted as a meaningful result both by the likelihood ratio test and the F-test, even if five Lagrangian multipliers out of twenty restrictions are significantly different from zero.

2.3. Property of the Functions Estimated

In this subsection some properties of the production and cost functions estimated are investigated in terms of the monotonicity and convexity conditions of the functions concerned.

2.3.1. Monotonicity Condition[16]

It is clear that in general these monotonicity conditions cannot be globally satisfied for all quantity configurations.[17] However, we are not so much interested in global monotonicity, because it is conceivable that at a certain input (and/or output) combination the marginal product of some one or more inputs may be negative. In general, though, we do expect these monotonicity conditions to be satisfied locally, and in particular, at the

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[16] This is already explained in the section 4.3., Chapter I, Part A, for the uniproduct technology. Also for the multiproduct technology, see the section 3.1.3., Chapter II, Part A, for the production function and the section 3.2.3., Chapter III, Part A, for the cost function.

[17] Other examples of production functions which do not satisfy the monotonicity conditions globally are the quadratic production function and the Generalized Leontief function introduced by Diwert (1971).

point of approximation, namely X = [1](and/or Y = [1]). Hence this local monotonicity conditions imply that in the specification of multiproduct production technology, the constant term of each input share equation (i.e., α_i) is negative and that of output share equation (i.e., β_i) is positive, while in the uniproduct production technology the constant term of input share equation is positive, corresponding to the specification of the estimating equations in this study.[18] In the specification of cost function, the monotonicity condition implies that the constant term of each share equation for both input and output is positive respectively.[19]

As shown in Table VI-1-a through VI-6-b, these local monotonicity conditions are satisfied well, judged by the sign of constant term in each share equation estimated, in translog production and cost functions for all six industries. Also the Tables IX-1-1 and IX-1-2 show the number (and percentage) of sample establishments (in the total establishments), not satisfying this monotonicity condition in any one of their share equations. In average, the monotonicity condition is well satisfied over the whole sample space in the five industries out of the six industries selected in this study, where the condition is not satisfied in about one fourth of total establishments in the canning industry.

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[18]See, for example, the equations (1) and (6) of the section 2.1. in this chapter for the sepcification of multiproduct technology and the equation (11) for that of uniproduct technology. See also the section 4.3., Chapter I, Part A.

[19]See the equations (9) and (10) of the section 2.1. in this chapter. See also the section 3.2.3., Chapter III, Part A.

TABLE IX-1-1.--Number of Establishments, Not Satisfying Monotonicity Conditions (percentage in parentheses).

Restricted Estimation	Canning Industry		Leather Footwear Industry		Screw Products Industry	
	Production Function	Cost Function	Production Function	Cost Function	Production Function	Cost Function
Total No. of Establishments	66 (100.00)	66 (100.00)	20 (100.00)	20 (100.00)	94 (100.00)	94 (100.00)
Symmetry	17 (25.76)	15 (22.73)	2 (10.00)	0 (0.00)	1 (1.06)	2 (2.13)
Explicit Separability	17 (25.76)	16 (24.24)	3 (15.00)	1 (5.00)	1 (1.06)	2 (2.13)

TABLE IX-1-2.--Number of Establishments, Not Satisfying Monotonicity Conditions (percentage in parentheses).

	Manufacture of Knitted Underwear		Manufacture of Briquettes		Molding Industry	
Restricted Estimation	Production Function	Cost Function	Production Function	Cost Function	Production Function	Cost Function
Total No. of Establishments	114 (100.00)	114 (100.00)	233 (100.00)	233 (100.00)	131 (100.00)	131 (100.00)
Symmetry	0 (0.00)	0 (0.00)	9 (3.86)	6 (2.58)	0 (0.00)	0 (0.00)

2.3.2. Convexity Condition

In addition to the monotonicity property, the local concavity of the production function in the input quantities and that of the cost function in the input prices are further assessed by the corresponding Hessians of the functions concerned. Since a necessary and sufficient condition for the local concavity of a function is the negative semi-definiteness of the Hessian, this property of a matrix is evaluated by the eigenvalues of all non-positive.[20]

Since there is no need to assume global concavity, it is found in the tables IX-2-1 and IX-2-2 that the local concavity of the production and the cost function are well satisfied in the canning industry and the screw products industry.[21] But the cost function of the leather footwear industry and the production function of the manufacture of knitted underwear, of briquettes, and of the molding industry are found to have no such a concavity property. The existence of some positive eigenvalues in these industries implies also that there exist uneconomic or convex (not

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[20] The meaning of eigenvalues (or characteristic values) of a matrix can be usually found in most books on the linear algebra (or theory of matrix). And also the relation between a negative semi-definite matrix and its eigenvalues can be referenced similarly.

[21] Since significantly heavy computational burdens are involved in calculating the eigenvalue of the Hessian matrix for each observed establishments, the local concavity of the functions in a neighborhood of X=Y=(1), is encountered here. See also the section 4.4., Chapter I, Part A, the sections 3.1.4. and 3.2.4., Chapter II, Part A, about the discussion on the Hessians.

TABLE IX-2-1.--Eigenvalues of the Hessian Matrix.

Restricted Estimation	Canning Industry		Leather Footwear Industry		Screw Products Industry	
	Production Function	Cost Function	Production Function	Cost Function	Production Function	Cost Function
Symmetry	-0.0000	-0.0000	-0.0000	0.0823	-0.0000	-0.0000
	-0.0518	-0.0014	-0.0284	0.0243	-0.0618	-0.0055
	-0.0719	-0.0382	-0.0647	-0.0636	-0.1194	-0.0500
	-0.1974	-0.1432	-0.2205	-0.1674	-0.2568	-0.0700
	-0.4344	-0.3526	-0.4556	-0.4180	-0.3499	-0.3173
Explicit Separability	-0.0000	-0.0000	-0.0000	0.0290	-0.0000	-0.0000
	-0.0522	-0.0037	-0.0282	-0.0000	-0.0623	-0.0107
	-0.0738	-0.0413	-0.0593	-0.0129	-0.1196	-0.0447
	-0.1982	-0.1462	-0.2124	-0.1147	-0.2534	-0.0720
	-0.4380	-0.3507	-0.4564	-0.3870	-0.3511	-0.3255

TABLE IX-2-2.--Eigenvalues of the Hessian Matrix.

	Manufacture of Knitted Underwear		Manufacture of Briquettes		Molding Industry	
Restricted Estimation	Production Function	Cost Function	Production Function	Cost Function	Production Function	Cost Function
Symmetry	0.0149	-0.0000	0.0082	-0.0000	0.0035	-0.0000
	-0.0000	-0.0020	-0.0000	-0.0075	-0.0000	-0.0052
	-0.0193	-0.0190	-0.0104	-0.0130	-0.0548	-0.0665
	-0.0830	-0.0730	-0.0212	-0.0186	-0.1232	-0.0942
	-0.2815	-0.3289	-0.2847	-0.3249	-0.3642	-0.2826

concave) regions and have increasing returns to scale in certain ranges of the inputs. In particular, this concavity condition seems to be satisfied better in the separate estimations by the size of sample establishments, even in the cost function of the leather footwear industry and in the production functions of the manufacture of knitted underwear, of briquettes, and of the molding industry.[22]

On the other hand, none of the output-transformation curves in all 3 industries satisfy the convexity conditions. That is, the eigenvalues of the Hessians for the multioutputs are -0.0000, -0.2596 and -0.3731 in the canning industry, -0.0000 and -0.4586 in the leather footwear, and -0.0000 and -0.4078 in the screw products industry. These results seem to be very confusing at first where the conventional textbooks show a transformation curve expressing a technical relation in transforming one output into another given fixed input bundles, which is convex to the origin, not concave to the origin as in this result.

Two possible explanations on these non-convex output-transformation curves may be investigated, i.e., the heterogeneity of the output with different quality and the existence of technical changes, between two groups of sample establishments, one of which

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[22]This results are shown in the later section of the supplementary results on the production and cost function, separately estimated by the size of sample establishments. The existence of increasing returns to scale may explain the non-concavity of the estimated isoquants in these industries.

produce only a uniproduct (group A) and the other of which produce multiproducts (group B).

Corporating with the distribution of sample establishments producing a uniproduct and multiproducts by industry, shown in Table II-4 of the section 5, Chapter II, the estimated technology implies such a shape of the transformation curve from its concavity as shown in the Figure II, in the case of two outputs.

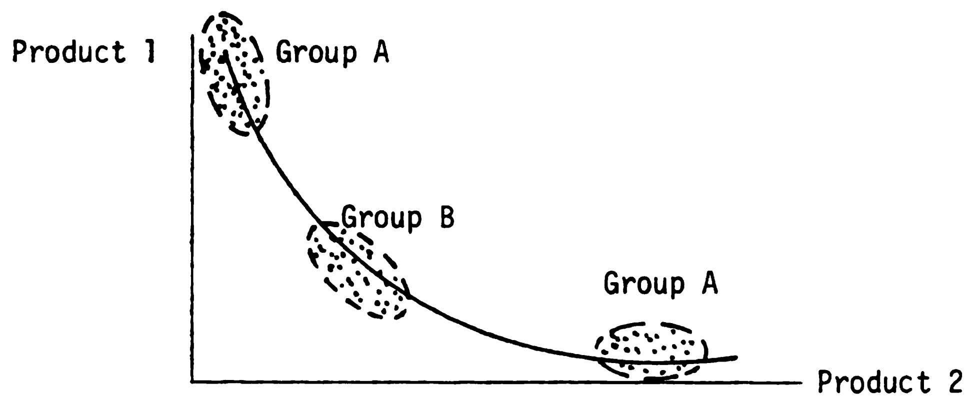

Figure II.--Estimated Transformation Curve

The first testable argument can be stated as follows: The quality of the output produced by the uniproduct establishments may be different from that of the output produced by the multiproduct establishments, even if these products are classified into the same commodity category in the census data base (i.e. KSIC 7-digit commodity code). When it can be assumed safely that the degree of the heterogeneity is well reflected in their individual output prices, it is possible that the actual transformation curve is convex to the origin in terms of values if the output price of the uniproduct establishments is significantly different from that of the multiproduct establishments. That is, the possibility of the actual transformation curves, against the estimated curve, from the existence of a heterogeneous output can be shown as in the Figure III.

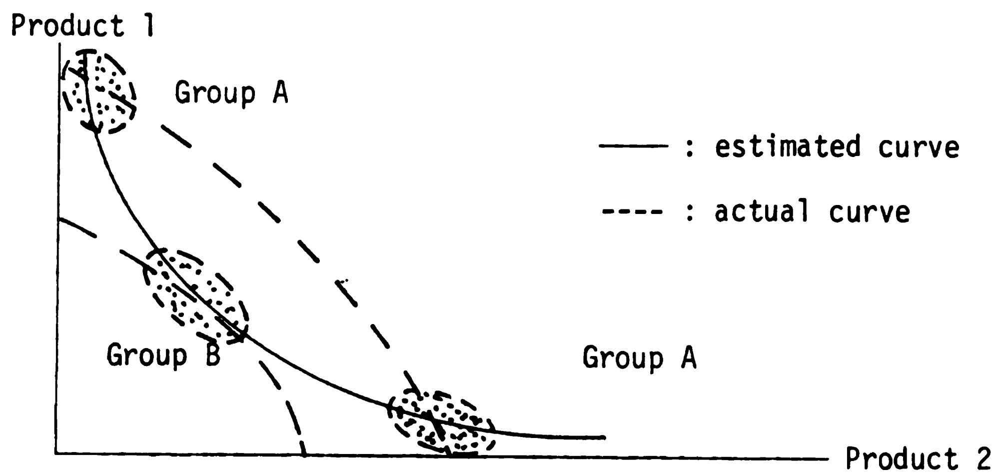

Figure III.--Possible Transformation Curve With Heterogeneous Output

In general, two different measures for the average price of each classified output can be defined separately for the group of sample establishments producing one output only and the other producing more than one output. The first measure is a quantity-weighted average price, assuming that each classified output is homogeneous whether they are produced by uniproduct establishments or not. The second is a value-weighted average price, assuming that the classified outputs are of different quality from whether they are produced by uniproduct firms or not.

The Table IX-2-3 contains these two different measures of the average prices for three multioutput industries, representing the differentials in average prices by the average price ratio between the uniproduct and the multiproduct establishments. In the canning industry the quantity-weighted average price differential is 28.90% in the output 1, 46.54% in the output 2 and 1.90% in the output 3, while the value-weighted average price differential is 21.20% in the output 1, 21.01% in the output 2 and 4,24% in the output 3. Hence it may well be said that the outputs 1 and 2 are significantly heterogeneous in the sense that the output price differentials between two types of establishments are above 30% (see the column C) and also the differentials in the value-weighted average price are above 20% in the outputs 1 and 2 (see the column F), showing the 6% changes in average price differentials between two alternative measures for the output 1 and the 17.42% for the output 2 (see the column G). This result may imply that the heterogeneity of these classified outputs between two groups of establishments not only results from a significant degree of price differentials but all results from some other factors such as managerial efficiency in marketing, or possibly technical change, etc.

The leather footwear industry shows also 90,86% average price differentials in the output 1. And the screw products industry shows 68.86% in the output 1 and 27.88% in the output 2. The common phenomena in all three industries are that the output

TABLE IX-2-3.--Average Prices of Outputs in the Uniproduct and the Multiproduct Firms.

	Quantity-Weighted Average Price[a]			Value-Weighted Average Price[b]			
	Uniproduct Firm (A)	Multiproduct Firm (B)	Ratio (C)=(A/B)	Uniproduct Firm (D)	Multiproduct Firm (E)	Ratio (F)=(D/E)	Ratio (G)=(F/C)
Canning Industry							
Output-1	0.1399	0.1086	1.2890	0.2673	0.2206	1.2120	0.9403
Output-2	0.2066	0.1410	1.4654	0.7423	0.6134	1.2101	0.8258
Output-3	0.2337	0.2293	1.0190	0.3442	0.3302	1.0424	1.0229
Leather Footwear							
Output-1	2.2421	1.1748	1.9086	2.3111	1.4481	1.5959	0.8362
Output-2	1.0504	1.1430	0.9190	1.1201	1.2913	0.8674	0.9439
Screw Products							
Output-1	0.1145	0.0678	1.6886	0.5413	0.1129	4.7960	2.8402
Output-2	0.5285	0.4133	1.2788	0.8571	0.5322	1.6106	1.2595

[a]The quantity-weighted average price in the uniproduct establishments is calculated as follows; $\sum_i \frac{Q_i}{\Sigma Q_i} P_i = \frac{\Sigma P_i Q_i}{\Sigma Q_i}$, i's are only for the establishments producing a uniproduct. The same formula is used for the multiproduct establishments

[b]The value-weighted average price in the multiproduct establishments is calculated by the following formula; $\sum_i \frac{P_i Q_i}{\Sigma P_i Q_i} P_i$, where i's are only for the establishments producing more than one output. The same formula is used for the uniproduct establishments.

price of the uniproduct establishments is always higher than that of the multiproduct establishments for each classified output. This supports that a significant quality differentials in one or two outputs of each industry may result in the non-convex transformation curves in our empirical estimations.

The next possibility is related to the existence of technical changes between the two establishment groups from the different year of an establishment in which the firm began to produce the same type of output as it produced in 1973, without regard to change in ownership or other factors concerning ownership. In other words, technical change may explain this non-convexity of the estimated output-transformation curve if average age is significantly different between the uniproduct and the multiproduct firms. Therefore the possible, actual transformation curves can be shown again as in the Figure IV, when technical change is assumed to be reflected by the business-starting year of sample establishments:

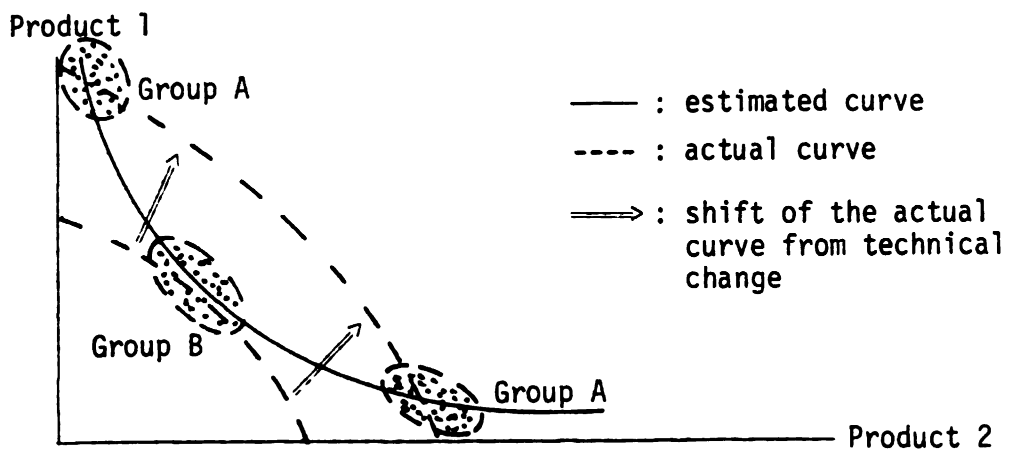

Figure IV.--Possible Transformation Curve With Technical Change

Table IX-2-4 contains the age structure of the uniproduct firms and the multiproduct firms by industry. The calculated average age of the uniproduct establishments in the canning industry is 5.58 years while that of the multiproduct establishments is 8.61 years, showing the age difference of about 3 years. Also the age difference in the leather footwear industry is 2.5 years and that in the screw products industry is about 4 years. It is also in common for all three industries that the age of the uniproduct establishments is always lower than that of the multiproduct establishments. This implies that the possibility of technical changes in the uniproduct firms become greater when 2- to 4-year of age difference may be sufficient for introducing new technical innovations, supporting the possibility of non-convex transformation curve estimated in the three industries.

In summary, the true transformation curve must not be convex to the origin even if the estimated curve is shown to be so. Furthermore we can not deny strongly the possibility on the concavity of true transformation curve, as far as the above two explanations are both convincing and unless we can strongly reject the existence of product heterogeneity and of technical change between the uniproduct establishments and multiproduct establishments in each industry.

2.4. Parameter Estimates Once Again

Some meaningful inferences on the production technology of the six industries selected in this study can be made again from

TABLE IX-2-4.--Age Structure of Uniproduct and Multiproduct Firms.[a]

	Canning Industry		Leather Footwear		Screw Products	
Age Code	Uniproduct Firms	Multi-product Firms	Uniproduct Firms	Multi-product Firms	Uniproduct Firms	Multi-product Firms
1) before 1945	1	1	0	0	0	0
2) 1946-1950	1	1	0	0	3	0
3) 1951-1955	0	0	1	1	2	1
4) 1956-1960	1	5	0	0	9	3
5) 1961-1965	2	1	0	0	13	6
6) 1966-1970	17	14	7	4	20	3
7) 1971	7	4	2	1	7	0
8) 1972	9	1	4	0	13	1
9) 1973	1	0	0	0	13	0
Number of Firms	39	27	14	6	80	14
Total Ages	217.5	232.5	17.0	45.0	547.0	148.0
Avg. Ages(Yrs.)	5.58	8.61	5.00	7.50	6.84	10.57

[a]Total ages are calculated by using the average years for the beginning-year before 1971 such as 30.5-year for the year before 1945, 25.5-year for the period of 1946-1950, etc. Also 2.5-year for the year of 1971 and 0.5-year for the year of 1973 are used.

the parameter estimation of the first-order terms (i.e., the constant terms in the share equations such as α_i and β_i) and of the second order terms (γ_{ij}, ε_{ij}, ξ_{ij} and δ_{ij}) in the specification of the transcendental logarithmic functions.

2.4.1. Output Elasticity of Factor[23] and Factor Share

In the implicit production function of $t(X,Y) = 1$,[24] where X is a vector of inputs and Y is a vector of output, the output elasticity of the i-th factor, denoted by Ω_i, is defined as the proportionate rate of change of t with respect to X_i:

$$\Omega_i = \frac{\partial \ln t(X,Y)}{\partial \ln X_i} = \frac{X_i}{t(X,Y)} \frac{\partial t(X,Y)}{\partial X_i}^{25} \tag{13}$$

This elasticity again becomes the i-th factor share in total output, when the marginal productivity theory of distribution is employed

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[23]This is often referred to as "the elasticity of a function." See Allen (1938), pp. 251-2.

[24]Compare the specification of the functional form, $t(Y) = 1$, in the section 1, Chapter II, Part B. Here the input and output are separated.

[25]This can be shown, in particular, in the production function of explicit (or multiplicative) separability between input and output. When we have the function form of $t(X,Y) = F(X)G(Y) = 1$, then

$$\frac{\partial \ln t(X,Y)}{\partial \ln X_i} = \frac{X_i}{t(X,Y)} \frac{\partial t(X,Y)}{\partial X_i} = \frac{X_i}{F(X)G(Y)} \left[G(Y) \frac{\partial F(X)}{\partial X_i} \right] = \frac{X_i}{F(X)} \frac{\partial F(X)}{\partial X_i}$$

This is the conventional definition of the output elasticity of the i-th factor, shown in most economic textbooks. For example, see Henderson and Quandt (1971), p. 57.

in factor pricing.[26] In relation to the specification of translog production function, the output elasticity of the i-th factor can be identified as the constant term of each share equation also, evaluated at X = Y = [1].[27] In the same way as in the production function, the cost elasticity of the i-th factor price can be defined, denoted by Ω_i^* as the proportionate rate of change of total cost with respect to the i-th factor price:

$$\Omega_i^* = \frac{\partial \ln C(W,Y)}{\partial \ln W_i} = \frac{w_i}{C(W,Y)} \frac{\partial C(W,Y)}{\partial W_i} \tag{14}$$

The cost elasticity of the i-th factor price also becomes the i-th factor's cost share and hence is identified as the constant term of the i-th cost share equation, evaluated at W = Y [1].[28]

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[26]Again this can be shown for a homogeneous production function with the multiplicative separability as before:

$$\frac{\partial \ln t(X,Y)}{\partial \ln X_i} = \frac{X_i}{F(X)} \frac{\partial F(X)}{\partial X_i} = \frac{X_i}{F(X)} W_i = S_i,$$

where W_i is the i-th factor price and S_i is the i-th factor share in total output.

[27]See the equations (1) through (5) in the section 2.1. of this chapter.

[28]Since the cost function for a cost-minimizing firms must be homogeneous of degree one in the input prices, the cost elasticity of the i-th factor price becomes:

$$\Omega_i^* = \frac{W_i}{C(W,Y)} \frac{\partial C(W,Y)}{\partial W_i} = \frac{W_i}{C(W,Y)} X_i = S_i,$$

from the cost equation of $C(W,Y) = \sum_{i=1}^{M} W_i X_i$, where M = total number

As shown in the Tables VI-1-a through VI-6-a, the estimates of these output elasticities are drastically different in the unrestricted estimation from those in the restricted estimations of both functions. But there is no significant differences found in the estimations of different restrictions. For example, in the canning industry, the output elasticity of the operative worker in the production function is estimated as 0.053 in the unrestricted case, while it is 0.110 in the restricted case of symmetry and is 0.113 in that of explicit separability.[29] Also its share in the cost function is 0.047, while it is 0.124 in the restricted case of symmetry and is 0.127 in that of explicit separability. But it is verified that the estimates of its output elasticity in the restricted estimation are very close to the average factor shares which are directly calculated from the observations on sample establishments, over all six industries, relative to its estimates in the unrestricted estimations. For example in the canning industry, the average factor share of fuel input in the observations is calculated as 0.047,[30] while its output elasticity is estimated as 0.022 in the unrestricted estimation of the production function

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of input, and X_i = quantity of the i-th factor input. Also see the equation (9) in the section 2.1 of this chapter.

[29]The parameter estimates of 0.053, 0.110, and 0.113 can be found in the first, second, and third column of the first row of the Table VI-1-a, the section 2.1. of this chapter.

[30]Data synthesis on factor shares is explained in the section 6.3., Chapter II, Part B.

and as 0.009 in that of the cost function. On the other hand, this elasticity is estimated as 0.048 (or 0.049) in the restricted estimations of the production function, and estimated as 0.031 (or 0.033) in that of the cost function. These verifications are found as a consistent phenomena in both the production function and the cost function of all six industries selected in this study. Hence the estimates of the output elasticities of factor inputs are found to be good and stable both in the restricted estimations of the production function and the cost function over all six industries.

The importance of each factor input in the production activity is also found significantly different from industry to industry. The output elasticity of the operative worker ranges from 0.10 to 0.15 in average among the industries, except it is only about 0.04 in the manufacture of briquettes.[31] And the output elasticity of the administrative worker is less than 0.05 in average. Hence for labor inputs as a whole they range from 0.15 to 0.20 in all five industries, except about 0.07 in the manufacture of briquettes again.[32]

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[31]This lowest elasticity of operative workers is quite understandable, where the industry was found to have quite a high capital-labor ratio, high average products of that input and hence high wage level, relative to the other industries. See the section 6, Chapter II, Part B, on "Some characteristics of the Industries selected."

[32]Compared with the average labor shares calculated directly from the observations, their drastic drops are found in some industries. For example, the output elasticity of labor inputs in gross outputs are reduced from about 0.28 to 0.18 in the leather footwear industry, and from 0.24 to 0.19 in the screw products industry.

The output elasticity of the capital input has a large variation in the different industries and also between the estimates of the production function and the cost function. For example, in the manufacture of briquettes they are estimated as 0.18 in the production function and as 0.25 in the cost function. And in the molding industry they are 0.24 in the production function and 0.19 in the cost function. Again we find that the output elasticity of capital input estimated in the production function is more reliable and close to the factor share directly calculated from the original data base, relative to its estimates in the cost function.[33] Based on the estimates of the production function, we also find the relatively high output elasticities of capital input in the industries which have relatively high level of capital-labor ratio. For example, the canning industry, the screw products industry, and the molding industry are shown to have significantly high capital-labor ratio in the Table III-2, where the Tables of VI-1-a, VI-3-a, and VI-6-a show quite a high output elasticity of capital input in the estimations of their production functions.

The output elasticity of fuel input is estimated around 0.05 in average of all six industries and its estimates are rather stable both in the production function and in the cost function,

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[33]One exceptional case is found in the canning industry. The estimated capital share in the production function is 0.348 and that in the cost function is 0.291, while the calculated share in the data is 0.254. See the Tables III-4 of the section 6.4., Chapter II, Part B and the Table VI-1-a in this chapter.

relative to the average share in gross output, calculated directly from the observations. But this dependency of energy input in gross outputs varies very much among different industries. In general, the industries of higher capital-labor ratio show relatively high level of energy consumptions. For example in the canning industry, the screw products industry, and in the molding industry, the factor share of fuel consumption ranges from 0.05 to 0.08, while in the other three industries it ranges from 0.01 to 0.02.

The output elasticity of raw material input ranges between 0.5 and 0.6 in the estimations of the production and the cost function of the five industries selected, except the highest elasticity of about 0.7 is found in the manufacture of briquettes.[34] Here also some significant differences in the size of the elasticity estimated are noticed between the production function and the cost function of the leather footwear industry and the molding industry. But still the estimated elasticities in the production function are found close to the calculated average factor shares previously in the four industries out of the six selected ones, except the canning industry and the leather footwear industry. Based on these empirical results it becomes noteworthy that the output elasticity of raw material input usually varies from industry to industry so widely that the conventional hypothesis on the role of raw materials as an fixed proportion to gross outputs may well be rejected.

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[34]In the manufacture of briquettes, the lowest elasticities of labor and capital inputs seem also to be attributed to this extremely high elasticity of the raw materials.

2.4.2. Elasticity of Factor Substitution and Elasticity of Product Transformation

The most common quantitative indices of production factor substitutability are forms of the elasticity of substitution (E.S.).[35] The defining formulae for these indices have the disadvantage of not allowing direct empirical evaluation. But the translog function in general exhibits the property of variable elasticities of substution (V.E.S.) in each observed establishment.[36] However, the assumption of constant E.S. leads to simple estimation methods, and has been widely used.[37] On the other hand, in the current empirical study on production technology with a larger number of factors (and products), there is no traditional definition of the E.S., but three forms have been suggested in the literature:[38]

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[35]For two factors of production, the E.S. is defined along an equal-product curve as the elasticity of the factor input ratio with respect to the marginal rate of substitution. See Allen (1938), pp. 340-3.

[36]But the estimation of these V.E.S. for each establishment contains really heavy computational burdens. Hence it is left for the later study subject in this study.

[37]The references in Arrow, Chenery, Minhas, and Solow (1961), and Morrissett (1953) include many of the empirical studies of the E.S. which make this assumption.

[38]The definitions of the A.E.S. and the D.E.S. appear in the section 4.2.2., Chapter I, Part A. And the definition of the S.E.S. also appear in the section 3.2.2., Chapter II, Part A. The Allen E.S. and the D.E.S. were introduced in Allen and Hicks (1934), pp. 202-6, 211-14, in the terminology "elasticity of complimentary" and "elasticity of substitution between Y and Z in the YZ indifference direction," respectively. The Allen E.S. is developed further in Arrow, Chenery, Minhas, and Solow (1961), p. 503. Also Uzawa (1962) has reformulated the definition of the A.E.S., and has characterized

(a) the Allen partial elasticity of substitution (A.E.S.),

(b) the Direct partial elasticity of substitution (D.E.S.),

(c) the Shadow partial elasticity of substitution (S.E.S.).

Owing to the two theorems,[39] presented in Jorgenson, Christensen, and Lau (1970) we measure the corresponding constant E.S.'s in this empirical works, mainly because of heavy computational burdens involved in measuring various E.S. for sample establishments.[40]

Again, the defining formulae for these indices of the D.E.S. and the A.E.S. have been worked out in Mundlak and Razin (1973) in terms of parameters of the translog production function, and in Brown, Caves, and Christensen (1975) in terms of the translog cost function. Also the defining formulae for the S.E.S. in terms of parameters of the translog cost function is worked out at the section 3.2.2., Chapter II, Part A in this study.

Tables X-1-1 through X-1-6 contain the estimates of these various elasticities of substitution for the translog production

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its class of constant E.S. production functions. The D.E.S. has been used in Morrissett (1953), pp. 42, 49-52, and Meade (1961), pp. 77-82. Also McFadden (1963) has characterized its class of constant E.S. production functions. The S.E.S. is originally reformulated by McFadden (1963) and was characterized its class of constant E.S. cost functions. But no estimations of the S.E.S. seem to be tried in any empirical works, as far as we have surveyed.

[39]See the section 4.2.2., Chapter I, Part A, about the existence of a transcental logarithmic production function which attains a given arbitrary set of either the D.E.S. or the A.E.S. at given quantities of outputs and inputs. See also Jorgenson, Christensen, and Lau (1970), pp. 24-27.

[40]See the footnote 36 in this section.

TABLE X-1-1.--Estimated Elasticities of Substitution[a]--Canning Industry.

	Translog Prod. Funct.		Translog Cost Funct.		Demand
Factors	D.E.S.	A.E.S.	S.E.S.	A.E.S.	Elasticity
(P : P)	-1.0000	-5.2265	-1.0000	-6.2075	-0.8299
(P : A)	2.5982	-0.1482	0.3563	-2.1768	-0.0770
(P : K)	1.2382	0.9485	0.8542	0.7974	0.2414
(P : F)	1.2498	0.7516	1.0050	1.7939	0.0824
(P : R)	1.3669	0.7669	0.9185	1.0552	0.5090
(A : A)	-1.0000	-17.4930	-1.0000	-28.1500	-0.9959
(A : K)	2.1636	0.7548	0.4704	0.7253	0.2195
(A : F)	1.8980	-0.1879	0.6441	0.3184	0.0146
(A : R)	2.1854	0.8533	0.4734	0.9568	0.4615
(K : K)	-1.0000	-2.1164	-1.0000	-2.3464	-0.7102
(K : F)	1.1627	1.1031	0.9386	0.7187	0.0330
(K : R)	1.1252	0.9061	0.8844	0.8900	0.4293
(F : F)	-1.0000	-18.0010	-1.0000	-20.1600	-0.9259
(F : R)	1.2170	0.8053	0.9673	0.9481	0.4573
(R : R)	-1.0000	-0.9111	-1.0000	-1.0783	-0.5201
(1 : 1)	1.0000	0.6021	1.0000	0.5997	0.3748
(1 : 2)	-1.0246	-1.0195	-0.9753	-0.9802	-0.1397
(1 : 3)	-1.0161	-1.0117	-0.9845	-0.9887	-0.2298
(2 : 2)	1.0000	5.9396	1.0000	6.0215	0.8582
(2 : 3)	-1.0288	-1.0497	0.9717	-0.9523	-0.2214
(3 : 3)	1.0000	3.5114	1.0000	3.3019	0.7675

[a](i) Various elasticities substitution (E.S.) are referred by their abbreviations respectively. Hence the D.E.S. indicates the Direct partial elasticity of substitution. The A.E.S. indicates that the Allen partial elasticity of substitution, the S.E.S. indicates the shadow partial elasticity of substitution. The demand elasticity indicates the elasticity of factor demand with respect to their price changes. (ii) The column of "factors" indicates the factors interacted directly in the elasticity concerned, where p indicates the operative worker, A indicates the administrative worker, K does the capital input, F does the fuel input, and R indicates the raw material input. Also for the output commodities, the numeric number is used for the multiproduct industries, such that 1 indicates the first major-commodity produced in each industry and 2 does the second major-commodity, etc. (iii) For example, the estimated E.S. of -0.1482 in the second row of the second column indicates that the Allen partial elasticity of substitution between the inputs of operative worker and of administrative worker is measured as -0.1482. (iv) Since the elasticity of substitution between the i-th factor and the j-th factor is, by definition, equal to that between the j-th factor and the i-th factor (that is, $\sigma_{ij} = \sigma_{ji}$ for example), the table only contains one of these two same measures.

TABLE X-1-2.--Estimated Elasticities of Substitution[a]--Leather Footwear Industry.

Factors	Translog Prod. Funct. D.E.S.	Translog Prod. Funct. A.E.S.	Translog Cost Funct. S.E.S.	Translog Cost Funct. A.E.S.	Demand Elasticity
(P : P)	-1.0000	-4.2228	-1.0000	-4.9295	-0.8361
(P : A)	2.0094	-0.8166	-0.6245	-6.5607	-0.2024
(P : K)	1.1199	1.1646	0.5577	0.3209	0.0870
(P : F)	2.7025	0.3455	0.7953	0.6032	0.0088
(P : R)	1.2983	0.8096	0.7052	1.0306	0.5295
(A : A)	-1.0000	-20.4400	-1.0000	-28.6790	-0.8849
(A : K)	1.4163	1.5272	-0.2940	0.2347	0.0636
(A : F)	2.1781	1.1163	0.5535	6.8011	0.0988
(A : R)	1.5825	0.8316	0.2973	1.0515	0.5402
(K : K)	-1.0000	-2.4786	-1.0000	-2.7278	-0.7399
(K : F)	2.6968	0.4467	0.8055	0.2365	0.0034
(K : R)	1.2116	0.8082	0.9348	1.0398	0.5342
(F : F)	-1.0000	-40.3870	-1.0000	-55.7320	-0.8097
(F : R)	2.6820	0.7480	0.8250	0.8435	0.4334
(R : R)	-1.0000	-0.7685	-1.0000	-0.9508	-0.4885
(1 : 1)	1.0000	0.5884	1.0000	0.4899	0.3288
(1 : 2)	-1.0135	-1.0135	-0.9860	-0.9860	-0.3242
(2 : 2)	1.0000	1.7457	1.0000	2.0414	0.6712

[A]See the footnote (a) in the Table X-1-1.

TABLE X-1-3.--Estimated Elasticities of Substitution[a]--Screw Products Industry.

	Translog Prod. Funct.		Translog Cost Funct.		Demand
Factors	D.E.S.	A.E.S.	S.E.S.	A.E.S.	Elasticity
(P : P)	-1.0000	-3.4723	-1.0000	-4.3295	-0.7756
(P : A)	7.2532	-0.3284	0.2414	-1.5003	-0.0716
(P : K)	1.3039	0.7691	0.4220	-0.0310	-0.0080
(P : F)	1.5947	0.2412	1.0891	2.0695	0.0877
(P : R)	1.2926	0.9338	0.7036	0.9365	0.4412
(A : A)	-1.0000	-11.2220	-1.0000	-16.7410	-0.7990
(A : K)	3.5491	0.3546	0.3257	-0.1779	-0.0462
(A : F)	1.8731	1.0988	0.9263	4.6157	0.1956
(A : R)	3.0625	0.9955	0.4266	0.9521	0.4486
(K : K)	-1.0000	-2.7922	-1.0000	-3.0391	-0.7890
(K : F)	1.3586	0.8632	0.9610	-0.1456	-0.0062
(K : R)	1.0048	1.0826	0.7288	0.8891	0.4189
(F : F)	-1.0000	-17.0810	-1.0000	-25.1320	-1.0651
(F : R)	1.3725	0.8974	1.1021	1.0863	0.5118
(R : R)	-1.0000	-1.1229	-1.0000	-1.1147	-0.5252
(1 : 1)	1.0000	0.4199	1.0000	0.4204	0.2960
(1 : 2)	-1.0151	-1.0151	-0.9852	-0.9852	-0.2915
(2 : 2)	1.0000	2.4543	1.0000	2.3789	0.7039

[a]See the footnote (a) in the Table X-1-1.

TABLE X-1-4.--Estimates Elasticities of Substitution[a]--Manufacture of Knitted Underwear.

	Translog Prod. Funct.		Translog Cost Funct.		Demand
Factors	D.E.S.	A.E.S.	S.E.S.	A.E.S.	Elasticity
(P : P)	-1.0000	-0.6540	-1.0000	-6.3139	-0.8755
(P : A)	-8.1768	-55.4660	0.1847	-2.6777	-0.0670
(P : K)	1.5101	1.7747	0.4852	0.0333	0.0078
(P : F)	1.5973	5.2598	0.9978	0.0762	0.0012
(P : R)	1.9167	1.6732	0.6318	0.8681	0.5096
(A : A)	-1.0000	271.3300	-1.0000	-34.1600	-0.8543
(A : K)	19.2350	-1.8209	0.2764	-0.4174	0.0977
(A : F)	2.6161	-0.3817	0.9269	8.9067	0.1360
(A : R)	164.6800	2.2774	0.3242	1.0282	0.6036
(K : K)	-1.0000	-3.8042	-1.0000	-3.3146	-0.7759
(K : F)	1.4277	0.4265	1.0332	0.3473	0.0053
(K : R)	1.1922	1.1643	0.7839	0.8978	0.5270
(F : F)	-1.0000	-96.7760	-1.0000	-69.1560	-1.0556
(F : R)	1.4760	1.1203	1.0758	1.2623	0.7410
(R : R)	-1.0000	-0.9857	-1.0000	-0.6968	-0.4090

[a]See the footnote (a) in the Table X-1-1.

TABLE X-1-5.--Estimated Elasticities of Substitution[a]--Manufacture of Briquettes.

Factors	Translog Prod. Funct. D.E.S.	Translog Prod. Funct. A.E.S.	Translog Cost Funct. S.E.S.	Translog Cost Funct. A.E.S.	Demand Elasticity
(P : P)	-1.0000	-37.2550	-1.0000	-11.7880	-0.9053
(P : A)	5.6468	32.5300	0.5749	-0.2317	-0.0082
(P : K)	1.3752	-0.7432	0.4584	-1.0452	-0.1796
(P : F)	1.4362	-0.5766	1.0163	2.2256	0.0324
(P : R)	2.5454	2.7462	0.7191	1.0791	0.7570
(A : A)	-1.0000	-135.4900	-1.0000	-26.5230	-0.9351
(A : K)	2.4175	2.2060	0.5381	-0.4609	-0.0792
(A : F)	1.6429	-2.8721	0.9607	3.0308	0.0442
(A : R)	3.8845	2.6336	0.6251	0.9008	0.6319
(K : K)	-1.0000	-4.2213	-1.0000	-4.9326	-0.8477
(K : F)	1.2929	0.2974	0.8791	-0.3448	-0.0050
(K : R)	0.9336	1.0461	0.6897	0.8534	0.5987
(F : F)	-1.0000	-92.5390	-1.0000	-70.6660	-1.0301
(F : R)	1.3924	2.0970	1.0478	1.1569	0.8116
(R : R)	-1.0000	-0.7515	-1.0000	-0.4223	-0.2962

[a]See the footnote (a) in the Table X-1-1.

TABLE X-1-6.--Estimated Elasticities of Substitution[a]--Molding Industry.

	Translog Prod. Funct.		Translog Cost Funct.		Demand
Factors	D.E.S.	A.E.S.	S.E.S.	A.E.S.	Elasticity
(P : P)	-1.0000	-15.6390	-1.0000	-7.0575	-0.8342
(P : A)	4.7683	24.8740	0.3379	-2.4180	-0.0742
(P : K)	1.3267	1.4944	0.7250	0.5434	0.1157
(P : F)	1.2371	1.9551	0.9125	1.6438	0.1216
(P : R)	1.3735	1.1026	0.8273	0.9455	0.5334
(A : A)	-1.0000	-130.3600	-1.0000	-30.4150	-0.9337
(A : K)	2.4829	1.4562	0.4293	0.3168	0.0675
(A : F)	1.9839	1.0130	0.5781	1.4818	0.1096
(A : R)	2.7461	1.1993	0.4446	0.8896	0.5019
(K : K)	-1.0000	-3.6647	-1.0000	-4.0058	-0.8530
(K : F)	0.9620	0.4104	0.7496	0.1094	0.0081
(K : R)	0.9718	0.9370	0.7548	0.8206	0.4630
(F : F)	-1.0000	-13.1180	-1.0000	-10.8730	-0.8045
(F : R)	1.0469	1.1007	0.8814	0.9596	0.5414
(R : R)	-1.0000	-0.7942	-1.0000	-0.7778	-0.4388

[a]See the footnote (a) in the Table X-1-1.

and cost functions by industry. Each table consists of three parts. The first part in the production function contains the D.E.S. in their first column and the A.E.S. in their second column. The second part of the table contains the S.E.S. in their first column and the A.E.S. in their second column estimated from the parameters of the cost function. In the last column of the Table X-1, the elasticity of demand with respect to price changes is presented, which relates, denoted by η_{ij}, the proportionate change in the i-th factor quantity to the proportionate change in the j-th factor price:[41]

$$\eta_{ij} = \frac{\partial \ln X_i}{\partial \ln W_j} = \frac{W_j}{X_i} \frac{X_i}{W_j} \tag{15}$$

The index of short-run responsiveness in factor substitution between the operative and the administrative workers, measured by the D.E.S.(P:A), is a positive value of 2.5982 in the production function of the canning industry, drastically different from 0.3563 of the S.E.S.(P:A) in its cost function. The differences between the D.E.S.(P:A) and the S.E.S.(P:A) may be interpreted as the different reaction of the producers in the canning industry upon the different situations, where the D.E.S. measures the producers' responsiveness in their optimizing behavior given the fixed levels of all the other inputs in given production technology and the

[41]For example, see Brown, Caves, and Christensen (1975), p. 26.

S.E.S. is measured given the fixed prices of all the other inputs on the same given production technology.

On the other hand, the index of long-run responsiveness in factor substitution between these two factor inputs, measured by the A.E.S. (P:A) is -0.1482 in its production function, also significantly different from -2.1768 in its cost function. However, one thing in common in the production technology of the canning industry is that a proportionate increase of the operative workers reduces the input level of the administrative worker in the short-run, but it increases also the input level of the administrative worker eventually. This complementary relationship between two different labor inputs in the long-run responsiveness of factor substitution[42] is identified by the negative sign of the A.E.S. (P:A) in the production function and in the cost function, not only of the canning industry but also all the other five industries selected in this study.[43] Hence the complementarity in these two labor inputs is well reflected in the measure of the demand elasticity of the operative worker with respect to the salary change of the administrative worker. That is, the demand elasticity of the operatives with respect to the administratives' salary

[42]The meaningful interpretations on the sign of the A.E.S. are well explained in Ferguson (1971), pp. 107-100.

[43]Two exceptions are found in the production functions of the manufacture of briquettes and the molding industry, but still the cost function of these industry show the complementary relationship between the two labor inputs.

increase is measured as -0.0770 in the canning industry, as -0.2024 in the leather footwear industry, and -0.0716 in the screw products industry, etc.

The short-run responsiveness in factor substitution between the labor inputs and the capital input, denoted by the D.E.S. (P:K) and the D.E.S. (A:K) in the production function of the Table X-1-1 for the canning industry, are shown also quite different from those of the S.E.S. in the cost function. For example, the D.E.S. (P:K) is shown as of 1.2382 in Table X-1-1, while the S.E.S. (P:K) is of 0.8542. On the other hand, the long-run measures of the A.E.S. (P:K) and the A.E.S. (A:K) are relatively stable in the production and the cost functions, where the A.E,S.(P:K) is 0.9485 in the production function and the A.E.S. (P:K) is 0.7974 in the cost function.[44] At the same time, we find that the short-run responsiveness in factor substitution between labor and capital inputs is more elastic than its long-run responsiveness, where the D.E.S. (P:K) is 1.2382, the D.E,S. (A:K) is 2.1636, the A,E.S. (P:K) is 0.9485, and the A.E.S. (A:K) is 0.7548, in the canning industry. The D.E.S. between labor inputs and capital input, greater than 1.0 and the A.E.S. between labor inputs and capital input, less than 1.0, are found not only in the canning industry but also in the

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[44]But the stable measures of the A.E.S. (P:K) and the A.E.S. (A:K) between the production function and the cost function do not seem to hold in the other industries any more, For example, see the A.E.S. (P:K) of 1.1646 in the third row of the second column in the Table X-1-2 for the leather footwear industry and compare the A.E.S. (P:K) of 0.3209 in the third row of the fourth column on the same table.

other industries.[45] Also the A.E.S. (P:K) is greater than the A.E.S. (A:K), while the D.E.S. (P:K) is smaller than the D.E.S. (A:K) in the canning industry. This implies that the input of administrative workers is more elastic in factor substitution to the capital input than the operative worker in the short-run, but in the long-run the input of the operative worker becomes more elastic to the capital input in factor substitution. Again it is found not only in the canning industry but also in the most other industries.[46]

The demand elasticities of two labor inputs with respect to the price changes in capital input is estimated as 0.2414 and 0.2195 respectively in the canning industry. These labor demand elasticities with respect to capital price changes are found to be the second largest, next to that with respect to the price change in the raw materials, not only in the canning industry but also all the other five industries.

The D.E.S.'s between the labor inputs and the fuel input are found to be greater than 1.0 and their A.E.S.'s are less than 1.0 in the canning industry. But the A.E.S.'s between these labor and fuel inputs seem to vary very wildly from the function estimated to the industry selected. Also the elasticity of substitution

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[45]Two exceptions are found in the leather footwear industry and the manufacture of knitted underwear. Also the A.E.S. (P:K) is smaller than the D.E.S. (P:K) in the molding industry.

[46]Only exception in this phenomena is found in the leather footwear industry with the A.E.S. (P:K) of 1.1646 and the A.E.S. (A:K) of 1.5272, and in the manufacture of briquettes with the A.E.S. (P:K) of -0.7432 and the A.E.S. (A:K) of 2.2060.

between capital and fuel inputs varies from industry to industry, reflecting its different production technology. Also the elasticity of substitution between fuel and raw material inputs is estimated as 0.8053 and 0.9481 in the production and the cost functions respectively, in the canning industry. The substitution elasticities of fuel input with respect to the other inputs seem to vary differently from industry to industry. For example, the elasticity of fuel substitution with respect to the capital and the raw material inputs are greater than the substitution elasticity of fuel with respect to the labor inputs in the screw products industry, while those with respect to labor inputs are greater than those with respect to capital and raw material inputs in the molding industry.

The direct elasticity of substitution of the raw materials for the operative worker is 1.3369, for the administrative is 2.1854, for the capital input is 1.1252, and for the fuel input is 1.2170 in the canning industry, which are all higher than their Allen partial elasticities respectively. The distribution of these A.E.S. between the raw materials and the other factors ranges from 0.7669 for the operatives to 0.9061 for the capital input, showing rather small variations among them seemingly. These ranges are slightly different from industry to industry[47] but they seem to be quite stable relative to the other substitution elasticities of

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[47]For example, the leather footwear industry shows their ranges from 0.7480 for the fuel to 0.8316 for the administrative worker. And also the molding industry shows their ranges from 0.9370 for the capital input to 1.1993 for the administrative worker.

factors other than raw material input. This seemingly constant, around 1.0, if we can say, substitution elasticities of the raw material input may suggest the separability between the raw material input and the other inputs,[48] implying again the usual separability between the raw material input and the value added to justify the conventional value added approach[49] in the study of a production function. Even if the value added procedure can be justified under the assumption of the separability between the value added and the raw material input, still it is worthwhile to note that the A.E.S. of raw materials with respect to the other factor inputs are neither zero nor infinity, but around unity, as found in this study on the

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[48]Berndt and Christensen (1973b) have established that separability restrictions are equivalent to certain equality restrictions on the Allen partial elasticities of substitution (A.E.S.). To illustrate, we note that the following are equivalent restrictions for a production function of three inputs, i.e., $y = F(X_1, X_2, X_3)$:

(i) inputs X_1 and X_2 are functionally separable from X_3, i.e.,

$$F(X_1, X_2, X_3) = H(J[X_1, X_2], X_3);$$

(ii) equality of the A.E.S., i.e.,

the A.E.S. (X_1, X_3) = the A.E.S. (X_2, X_3)

(iii) the existence of a consistent aggregate price index P^* and a consistent aggregate index X^* with components P_1 and P_2, X_1 and X_2, respectively.

See also Berndt and Christensen (1973a) on the application in their empirical works.

[49]The conventional value added approach in the production study has been discussed in detail, in the section 3.5, Chapter II, Part B.

six industries selected. Another interesting finding in this empirical estimation is that the demand elasticities of all the factor inputs with respect to the price change in the raw material input are also constant seemingly, around 0.5, not only in the canning industry but also in all other five industries. For example, the canning industry shows its range from 0.4293 for the capital input to 0.5201 for the raw material input itself.

The estimated own-substitution elasticity of each factor is more or less stable between the production function and the cost function and it is drastically different from factor to factor. But in average the highest own-elasticity of substitution is shown for the fuel input, the second for the input of administrative worker, the third for the operative worker, the next for the capital input and the smallest for the raw material input, in all six industries. On the other hand, the own elasticity of factor demand is shown as the highest for the administratives, as the second for the fuel input, the third for the operative worker, the next for the capital input, and finally the lowest for the raw material input in the canning industry. And this seems to be more or less in common in the other five industries, except that the fuel and the labor inputs are reversed in their order in some industries.[50]

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[50]The leather footwear industry shows the reversed order between the fuel input and the operative workers. But the screw products industry, the manufacture of briquettes, and the manufacture of knitted underwear show the highest own elasticity of demand for the fuel input, less than -1.0.

Finally, the cross-elasticities of products transformation in the canning industry are measured around unity, i.e., slightly greater than 1.0 in the production function, and slightly less than 1.0 in the cost function. The bigger figures in the production function than in the cost function are in common in the three multi-products industries selected in this study. Also the cross-price elasticities of demand for the major commodities range from 0.1387 to 0.2298 in the canning industry and around 0.3 in the leather footwear and the screw products industry, indicating quite a stable responsiveness.

On the other hand, the own-transformation elasticities of each product in the long-run show that the nonmajor commodities are more elastic than the first major commodities not only in the canning industry but also in the other two industries. Hence the own-price elasticities of commodity demand also show the lower (around 0.4) for the first major-commodity produced and the higher (around 0.8) for the less-major-commodities in the canning industry. This holds also in the other two industries. Hence the first major-commodity in these industries is less price-elastic then the less-major-commodities are.

In summary, what we find from the evaluation of various E.S. estimated over the six industries selected in this study are the followings:

(1) The D.E.S. between factor inputs are, in general, greater than the A.E.S. where the former reflects the short-run

responsiveness in factor substitution and the latter shows its long-run responsiveness.

(2) The D.E.S. measured under the fixed levels of all the other factors, other than two factors concerned are found much bigger than the S.E.S. measured under the fixed prices of all the other factors, other than two factors concerned. This implies that the elasticity of factor substitution in general becomes much smaller when there exist a strong stability (or rigidity) in all factor prices.

(3) The A.E.S. between the operative worker and the administrative worker is of a negative value, implying that the complementary relationship between these two labor inputs hold in the production activities of all the six industries.

(4) The A.E.S. between the operative worker and the capital input is quite different from the A.E.S. between the administrative worker and the capital input, implying that the hypothesis on the existence of a proper aggregate quantity (or price) index for the labor input as a whole should be rejected, based on the strong agreement on the separability between the operative worker and the administrative worker.[51]

(5) The conventional agreement on the unitary elasticity of capital-labor substitution may or may not be accepted, i.e., inconclusive in this study, since they vary from industry to industry but by and large they seem to range around unity.

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[51]See the footnote (48) on the separability restrictions.

(6) The D.E.S. between the operative worker and the capital input is smaller than that between the administrative worker and the capital input, while the A.E.S. between the operative worker and the capital input is greater than that between the administrative worker and the capital input. This implies that the factor substitution between the administrative worker and the capital input may happen strongly in the short run, but eventually after producers's full adjustment in the production process is done the factor substitution between the operative worker and the capital input become significant.

(7) The factor substitution of the fuel input with respect to other input factors happen differently from industry to industry. For example, in some industries the substitutions become more elastic with respect to the labor inputs, and in some other industries with respect to the capital input and the raw material input.

(8) The seemingly constant elasticities of substitution of raw material input with respect to all the other factors are found around 1.0 in all the six industries, implying that there may exist the separability between the value added and the raw material input to justify the conventional value added procedure in the empirical study on a production function, but still nothing is found to support the elasticity between the value added and the raw material input be either zero or infinity. Our findings on this elasticity seem to be more or less unity, instead of either zero or infinity.

(9) The own-factor substitution elasticities are found the highest for the fuel, the next for the administrative worker, the operative worker, the capital input, and the lowest for the raw material input.

(10) The own-price elasticities of factor demand are shown the highest for either the administrative worker or the fuel input, the next for the operative, the capital input, and the lowest for the raw material input.

(11) The cross-price elasticities of factor demand are found the highest (around 0.5) from the price changes in the raw material input, the next from the price changes in the capital input, those in the labor inputs, and the lowest from the price changes in the fuel input.

(12) The own-product transformation elasticities are found to be higher for the nonmajor commodities produced in an industry than for the first major-commodity.

(13) The own-price elasticities of each product demand are also found to be higher for the nonmajor commodities produced in an industry than for the first major-commodity.

(14) The cross-product transformation elasticities are found quite stable, ranging around unity, in the three multiproducts industries.

(15) The cross-price elasticities of product demand are found stable, ranging around 0.3, in the three multiproducts industries.

2.4.3. Share Elasticities with Respect to Quantity Changes and to Price Changes

The estimates of the γ_{ij}, ε_{ij} and δ_{ij} parameters in the translog production function can also be interpreted as estimated share elasticities with respect to quantity changes. The cost share of input i is equal to $\frac{\partial \ln F}{\partial \ln X_i}$. The cross partial derivative $\frac{\partial^2 F}{\partial \ln X_i \, \partial \ln X_j} = \gamma_{ij}$ can be defined as a constant share elasticity summarizing the response of cost share S_i to a change in $\ln X_j$. Alternatively the share elasticity can be defined as

$$\frac{\partial \ln S_i}{\partial \ln X_j} = \frac{\gamma_{ij}}{S_i} \tag{16}$$

In the latter case, the estimated share elasticities at the means of the data will be equal to the estimates of γ_{ij}/α_i. In the same way, the estimates of the same parameters in the translog cost function can be interpreted as estimated share elasticities with respect to price changes and the alternative definition of the share elasticity with respect to price changes can also be defined similarly.

Table X-2 contains only the alternative measures of the estimated share elasticities with respect to the own quantity changes and with respect to the own price changes respectively.

TABLE X-2.--Share Elasticities with Respect to the Own Quantity Changes and the Own Price Changes.

	Canning Industry	Leather Footwear	Screw Products	Knitted Underwear	Briquettes	Molding Industry
With Respect to the Own Quantity Changes						
$\dot{S}_P$	.2856	.2514	.3340	.5231	1.0180	.4415
$\dot{S}_A$	1.4325	.3363	.8748	1.3881	.7627	.9342
$\dot{S}_K$	.0544	.1040	.0506	.1150	- .0966	- .0159
$\dot{S}_F$	.1565	.5343	.1918	.2270	.3003	.0296
$\dot{S}_R$	.0877	.0964	- .0026	.1056	.0828	.0100
$\dot{S}_1$	.0082	.0059	.0059	--	--	--
$\dot{S}_2$	.0116	.0065	.0065	--	--	--
$\dot{S}_3$	.0109	--	--	--	--	--
With Respect to the Own Price Changes						
$\dot{S}_P$	.1195	.4485	.4555	.4743	.6244	.2069
$\dot{S}_A$	.8523	1.3215	.7195	.7751	.5989	.7320
$\dot{S}_K$	.1067	.1200	.3633	.2200	.2312	.2677
$\dot{S}_F$	.0418	.1942	- .1006	- .0735	- .0641	.1283
$\dot{S}_R$	.0271	.0155	.0349	.0374	.0209	.0474

[a]The symbols in the table denote the corresponding share elasticities to each factor inputs and outputs. For example, $\dot{S}_P$ means the operative labor's share elasticity with respect to its quantity changes and to its price changes respectively and $\dot{S}_1$ means the first product's share elasticity with respect to quantity changes in each industry.

In the canning industry, the operative labor input's share elasticity with respect to its own quantity change (0.2856) is greater than the share elasticity with respect to its price change (0.1195). Also the share elasticities of the administrative labor, fuel and raw materials are more sensitive from their corresponding quantity changes than from their corresponding price changes, except the capital input's share is less elastic with respect to quantity change than to price change. In particular, the capital input's shares are less elastic to their quantity changes than to their price changes in all the six industries, while the labors' shares seem to be more elastic to their quantity changes than to their price changes in most of the six industries except in the leather footwear and the screw products industries. Also the shares of the fuel and the raw materials are more elastic to their quantity changes than to their price changes in most of the six industries except the screw products and the molding industries. When the share elasticity of a factor is more elastic to quantity change than to price change, it may well be said that changes in the factor price become greater than changes in factor quantity if the factor share remains unchanged, i.e., the price flexibility should be greater than quantity adjustments. This phenomena seem to be rather consistent to our common sense such that the price adjustments in capital input are mostly difficult in all the six industries, while the price adjustments in labor inputs, fuel and raw materials inputs are mostly easier.

Also note that some labor shares' elasticities are even greater than unity in most industries, i.e., the administratives' share elasticities with respect to quantity change are 1.4325 in the canning industry and 1.3881 in the manufacture of knitted underwear, the administratives' share elasticity to price change is 1.3215 in the leather footwear industry, and the operatives' share elasticity to quantity change is 1.0180 in the manufacture of briquettes. On the other hand, the capital input's share elasticities to quantity change are negative, i.e., -0.0966 in the manufacture of briquettes and -0.0159 in the molding industry. And the fuel's share elasticities to price change are also negative, -0.1006 in the screw products, -0.0735 in the manufacture of knitted underwears and -0.0641 in the manufacture of briquettes. The negative share elasticities to price change imply that the demand elasticities with respect to its own price change should be smaller than -1.0.[52]

B.4.3. Supplementary Results

Followed by the main empirical estimations for the six industries, three supplementary speculations are designed in this study, covering separate estimations of production technologies for different size of establishments within each industry,

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[52]This is well verified in the previous section. That is, the demand elasticities of fuel input with respect to its own price change are -1.0651, -1.0556 and -1.0301 in the screw products, the knitted underwear and the briquettes respectively, as shown in the tables from X-1-3 through X-1-5.

alternative estimations with different exclusion rules in sampling establishments, and alternative estimations with the explanatory variables of different quality. The details of the parameter estimates, test statistics and other results such as share elasticity, etc., are not included in this volume, but only a few, significant results are briefly discussed in this section.

3.1. Estimations by Establishment Size

Separate estimation results of the share equations system of two establishment groups, classified by the number of total workers (i.e., the large size of establishments with more than 49 workers and the small size with less than 50 workers), are compared for the five industries, excluding the leather footwear industry due to the inappropriateness of the number of sample units in estimation.

As shown already in the Table II-5 of the section 2.5, Chapter II, Part B, the size distribution of sample establishments are biased toward the small size in all five industries except the canning industry. That is, 78 establishments out of 94 in the screw product industry, 96 out of 114 in the manufacture of knitted underwear, 202 out of 233 in the manufacture of briquettes and 89 out of 131 in the molding industry belong to the small-sized establighment group, while 37 out of 66 establishments in the canning industry are the large-sized establishments.[53]

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[53]The total sample units here are different from those in the earlier table (II-5), which include such establishments that

3.1.1. Goodness of Fit

The goodness of fit is found, measured by the weighted average of the quasi-R^2's of the five input share equations (defined in the previous chapter), to be much better in the estimations for the large-sized establishment group and slightly poorer in those for the small-sized group than in those for the total establishments in the most industries.[54] The calculated average of the quasi-R^2's are shown in the Table XI-1. For example in the canning industry, the weighted average in the input equations is 0.1694 for the small-sized group and 0.4647 for the large-sized group in the production function, while it was 0.2338 for the total establishments. Also in the cost function, it is 0.1507 for the small-sized group and 0.2770 for the large-sized group, while it was 0.1784 for the total establishments.

On the other hand, the weighted average in the output share equations shows the reverse in the sence that the fit becomes slightly better for the small-sized group and moderately poorer for the large-sized group, both in the canning and the screw product industries. For example, the weighted average is 0.8217 for the

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the informations on capital input are not available for in the census data file, i.e., the exclusion rules in sampling was not applied yet.

[54] Two exceptions are found here. The weighted average of the quasi-R^2's in the input share equations of the manufacture of briquettes is 0.2629 for the small-sized group and 0.1241 for the large-sized group for the production function, while it was 0.2009 for the total samples. Second, it is 0.2500 for the small- and 0.2480 for the large-sized groups in the cost function of the screw product industry, where it was 0.2541 for the total establishments.

TABLE XI-1.--The Weighted Average of the Quasi-R^2's.[a]

Industry	Production Function			Cost Function		
	Small -Sized Group	Large -Sized Group	Total Samples	Small -Sized Group	Large -Sized Group	Total Samples
Canning Industry						
(i) Input	0.1694	0.4647	0.2338	0.1507	0.2770	0.1784
(ii) Output	0.8217	0.7354	0.7975	0.8191	0.7357	0.7969
Screw Products						
(i) Input	0.0603	0.1351	0.0649	0.2500	0.2480	0.2541
(ii) Output	0.9668	0.9018	0.9531	0.9668	0.9019	0.9532
Knitted Underwear						
(i) Input	0.1653	0.4238	0.1877	0.1276	0.2322	0.1667
Briquettes						
(i) Input	0.2629	0.1241	0.2009	0.1117	0.4825	0.1116
Molding Industry						
(i) Input	0.0207	0.1030	0.0298	0.1658	0.2569	0.1828

[a]The weighted average of the quasi-R^2's for the input (and the output) share equations are the average of the quasi-R^2's in the input (and output) share equations weighted by each factor's (and output's) share, where the quasi-R^2 is defined as the relative size of error sum of squares, subtracted from unity.

small-sized and 0.7354 for the large-sized groups in the production function of the canning industry, while it is 0.8191 and 0.7357 respectively in the cost function. As shown earlier, it was 0.7975 in the production function and 0.7969 in the cost function for the total establishments of the canning industry.

As a result, the poor fit shown in the estimations of the input share equations for the total establishments in each industry seems to come mostly from the poorer fit for the small-sized establishment group, where the differences of the weighted average of the quasi-R^2's in the output share equations between the small- and the large-sized groups of the multi-output industries are found to be very negligible. This may imply that the very concept of either a production function or a cost function become poorer when it is applied for a moderately small size of establishments.[55]

3.1.2. Properties of the Functions Estimated

The hypotheses of linear homogeneity and symmetry are more specifically accepted in the different size groups of establishments within each industry, compared with those in its total establishments. For example, judged by the likelihood ratio test, those hypotheses are accepted at the significance level of 1% in the estimations of both the production and the cost functions for the large-sized group of the canning industry. The details of these test statistics are shown in the Table XI-2. In addition, these hypotheses are accepted

[55] This may become more convincing when the significance test of the function estimated is discussed.

TABLE XI-2.--Likelihood Ratio Test Statistics.[a]

	Degree of Freedom (γ)	Critical Value of $\chi^2(\gamma)$ at $\alpha = 0.01$	Test Statistics		
			Small -Sized Group	Large -Sized Group	Total Samples
Canning Industry					
Production Function	27	45.642	70.575	35.349	47.522
Cost Function	27	45.642	95.645	38.936	57.696
Screw Products					
Production Function	20	37.566	45.568	32.409	47.400
Cost Function	20	37.566	40.359	47.359	42.036
Knitted Underwear					
Production Function	10	23.209	7.482	24.066	19.901
Cost Function	10	23.209	16.765	27.081	31.526
Briquettes					
Production Function	10	23.209	18.433	24.857	42.521
Cost Function	10	23.209	31.952	21.992	29.906
Molding Industry					
Production Function	10	23.209	17.210	15.053	32.824
Cost Function	10	23.209	17.615	13.348	26.064

[a]The test statistics are for the restrictions of linear homogeneity and symmetry imposed on the separate estimations over three different samples, i.e., the small-sized, the large-sized and the total establishments of each industry.

in the estimations of the production function for the large-sized establishment group of the screw product industry and the molding industry, and also are accepted in those of the cost function for the large-sized group of the briquettes and the molding industries.[56]

On the other hand, the convexity conditions of the isoquants are also well satisfied in the production and the cost functions estimated for two groups of different establishment size in the most industries, particularly for the large-sized group in all five industries,[57] while the monotonicity conditions seem not to be improved in terms of the absolute number of sample units in any industry.

As a result, a production technology in the industries selected in this study may be significantly different from the different size of establishments included, when the results are combined from the goodness of fit, significance test of the restrictions imposed and from the properties of the technology estimated by two different establishment groups.[58] But not

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[56]One exception is found in the manufacture of knitted underwear. Both in the production and the cost functions for the small-sized groups of the industry are these restrictions accepted significantly at $\alpha = 0.01$, while in both functions for the large-sized group are these rejected.

[57]The non-convex isoquants are found in both functions estimated for the small-sized group of the three uniproduct industries, i.e., the knitted underwear, briquettes and molding industries.

[58]This statement can be verified when an additional test is employed which is not covered here. That is the so-called Chou-test, defined as $F^{k}_{n+m-2k} = [(SSE_t - SSE_n - SSE_m)/k]/[(SSE_n + SSE_m)/(n+m-2k)]$, where SSE is the square sum of errers, n, m, and t are the sample size of the small, the large-sized and the total establishment groups respectively. And k is the number of explanatory variables. Also see Kmenta (1971), p. 373.

necessarily all these differentials are attributable to the effects of returns to scale naively, since these results in this study are very restricted by the constant returns to scale in the technology estimated.

3.1.3. Parameter Estimates

The scale effect on output elasticities of factor inputs is shown in the Table XI-3, in terms of the sign in direction which changes as the firm size gets bigger. Output elasticities of two labor inputs as a whole decrease as the size of establishments becomes larger in most industries, but the manufacture of knitted underwear, the characteristics of which was revealed by a low capital-labor use ratio and relatively lowest wage rates for workers.[59] On the other hand, the scale effect on the capital input share is found to be positive in all industries. Together with the negative scale effect in the labor shares, the share ratio between capital labor inputs are increasing in all industries, in general.[60] This may imply that most industries selected in this study reveal themselves to be capital-using and labor share-decreasing as the firm size becomes bigger.

The scale effect on the share of raw materials in total input are all negative, and that of the fuel consumption is also

[59] About the detail characteristics of this industry, see the subsection 2.6., Chapter II, Part B.

[60] One exception is found in the cost function of the knitted underwear.

TABLE XI-3.--Scale Effects in Output Elasticities.[a]

Factor	Canning Industry		Screw Products		Knitted Underwear		Briquettes		Molding Industry	
	p.f.	c.f.	p.f.	c.f.	p.f.	c.f.	p.f.	c.f.	p.f.	c.f.
Operatives	-	-	-	-	+	+	-	-	-	-
Administratives	-	+	+	+	+	+	-	-	-	-
Total Labors	-	-	-	-	+	+	-	-	-	-
Capitals	+	+	+	+	+	+	+	+	+	+
Fuels	+	+	+	+	-	-	-	-	-	-
Raw Materials	-	-	-	-	-	-	-	-	+	-
Share Ratio of Capital to Labor	+	+	+	+	+	-	+	+	+	+

[a]In the table, "p.f." and "c.f." denote the production and the cost functions respectively. Also the plus (+) sign means that the factor share concerned are increasing as the establishment size get larger, and the negative (-) sign means the reverse.

all negative, except in the canning and the screw product industries which have relatively low capital-fuel use ratio and cheaper fuel prices in common.[61]

On the other hand, there does not seem to be any uniform tendency in the scale effects on the partial elasticities of factor substitutions (or output transformations), as shown in the Tables III-1 through III-5 of the Appendix B. In particular, the complementary relationship between two heterogeneous labor inputs does not hold for some of the large-sized establishments either in the production or the cost functions, i.e., of the canning, knitted underwear and the briquettes industries. Factor substitution between operatives and capital input becomes slightly more elastic for the large size of establishments than for the small size, either in the production function of most industries (except the molding industry) or in the cost function of most industries (except the manufacture of knitted underwear). The fact that factor substitution elasticity between labor and capital inputs becomes higher as the firm size gets bigger, implies that the shape of isoquants reflected in the labor-capital subspace becomes steeper as the firm size (or alternatively speaking, the level of output) becomes bigger, when it can be said safely that the level of output in establishment becomes higher as the firm size measured by the number of workers gets bigger.

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[61] See the footnote (59).

Further analyses on the relationships between the elasticities of factor substitution and the output level (or the factor use ratio) could be done more deliberately in the sense that the substitution elasticity may be significantly influenced and directly determined either by the output level or by the capital-labor ratio.[62] But these experiments are postponed to later researches and not covered in this study.

3.2. Alternative Exclusion Rules in Sampling

The current empirical study on the micro-reality of production technology in the Korean manufacturing industries is very much conditioned by the exclusion rules in the sense that the exclusion criteria from data availability of capital input reduced sharply the number of sample establishments in each industry.[63] The absence of informations on capital input (such as the horsepower equipment or the net capital stock) for certain establishments in the manufacturing census can be interpreted in three different ways. First, the establishments have significant level of capital input in their production process but not reported in the census

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[62]The hypotheses that the common, constant elasticity of factor substitution is a function of either the level of output or the capital-labor ratio, have been suggested by Revankar (1971), Lu and Fletcher (1968), Sato and Beckmann (1968) and Lovell (1973). The detailed discussion on this topic was already discussed in the section 3.1., Chapter I, Part A.

[63]Refer the section 2.4., Chapter II, Part B.

questionnaires, i.e., the case of the missing data. Second, the establishments have some but very neligible amounts of capital input hence recorded as if they have zero capital input. Third, the establishments have no capital input in their production process. The first two possibilities bring us to the problem of errors in measurement, i.e., "underestimation" of capital input.

The inclusion of such an establishment into the empirical estimation[64] also implies that the results would be consistent with those of aggregate production (or cost) function at a certain macro (or sector) level based on highly aggregative data.[65]

The number of total sample units covered in this inclusive case is 79 (i.e., 11 more than before) establishments in the canning industry, 206 (i.e., 186 more) establishments in the leather footwear industry, 104 (i.e., 20 more) in the screw product industry, 139 (i.e., 25 more) in the manufacture of knitted underwear, 262 (i.e.,

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[64]The inclusion of such an establishment in the estimation of the translog function again involves the problem of the log-transformation of the zero-valued variable, discussed already in details in the section 3.5., Chapter III, Part B. Hence the actual estimation in this section was done after replacing this zero-valued variable by certain neglibibly small figure, i.e., 1.0×10^{-70}.

[65]This can be valid in the sense that many empirical studies on aggregate production function are based on the capital data, which again come from the same census data file. Hence the aggregative capital data based on the census file are usually measured by the simple summation of capital input of all establishments recorded in the file.

29 more) in the manufacture of briquettes and 152 (i.e., 21 more) establishments in the molding industry. Hence the increment of the sample size due to the different exclusion rules is rather minor in most industries, except the leather footwear industry.

3.2.1. Goodness of Fit

On average, the quasi-R^2 of each share equation, their simple average and their weighted average in the inclusive cases are worse than those in the previous exclusive cases for most industries. In particular, the quasi-R^2 in the capital share equation changes as drastically as expected in each industry. Table XII-1 includes only the quasi-R^2 of the capital share equation and the weighted average of the quasi-R^2's of input- and output-share equations.

3.2.2. Properties of the Functions Estimated

The imposed restrictions of linear homogeneity and symmetry on the functions estimated are more strongly rejected by the likelihood ratio test, for the inclusive cases than for the exclusive cases in most industries. On the other hand, the number of sample establishments, not satisfying the monotonicity conditions in any share equations, is increasing slightly but decreasing as apercentage for the inclusive cases, relative to the exclusive cases in most industries. And the convexity conditions are satisfied in the inclusive cases similarly as in the exclusive cases in most industries. Hence the properties of the functions estimated do not seem

TABLE XII-1.--The Quasi-R^2's by Industry.[a]

	Production Function			Cost Function		
	Capital Share Equation	Weighted Average		Capital Share Equation	Weighted Average	
		Input	Output		Input	Output
Canning Industry						
Inclusive Case	0.0523	0.2120	0.8044	0.0544	0.1067	0.8080
Exclusive Case	0.1044	0.2338	0.7975	0.1973	0.1784	0.7969
Leather Footwear						
Inclusive Case	0.0087	0.0393	0.7497	0.0081	0.0237	0.7495
Exclusive Case	0.1307	0.2710	0.9471	0.2054	0.1200	0.9472
Screw Products						
Inclusive Case	0.0168	0.0459	0.9592	0.0096	0.0534	0.9590
Exclusive Case	0.0075	0.0649	0.9531	0.5837	0.2541	0.9532
Knitted Underwear						
Inclusive Case	0.0090	0.1414	--	0.0095	0.0191	--
Exclusive Case	0.0492	0.2009	--	0.2688	0.1116	--
Briquettes						
Inclusive Case	0.0460	0.1991	--	0.0309	0.0273	--
Exclusive Case	0.0075	0.1871	--	0.4589	0.1667	--
Molding Industry						
Inclusive Case	0.0532	0.0732	--	0.0466	0.0697	--
Exclusive Case	-0.0121	0.0298	--	0.3840	0.1828	--

[a]See the footnote (a) in the Table XI-1.

to have much difference between the inclusive and the exclusive cases, except that the goodness of fit and the significance of the restrictions imposed are influenced significantly from the inclusion of sample establishments with zero capital input.

3.2.3. Parameter Estimates

The effects of the inclusion of establishments with zero capital input on the parameter estimates of a production technology seem to be so variant over the six industries, as shown in the Table XII-2. Particularly in the leather footwear industry where the most drastic change occurs in the sample size with alternative exclusion rules, the increased share of capital input in the estimated production function is accompanied, together with the increased labor share, with the drastically decreased share of raw materials, compared to those in the exclusive case. On the other hand, the decreased capital shares in the cost function is accompanied with the increased share of raw materials, leaving labor shares as same as in the exclusive case. Apparently, the inclusion of those establishments with zero capital input into the estimation samples results in the lower average level of capital input in the production function and higher average price of capital input in the cost function than the previous exclusion rules result in. But the effects of these possible errors in measurements of capital input on the capital share seem to be inconclusive across the industries selected in this study.

TABLE XII-2.--Factor Shares Estimated.

	Operatives	Adminis-tratives	Capital Input	Fuel	Raw Materials
Canning Industry					
Production Function	0.1167	0.0268	0.2929	0.0444	0.5193
Cost Function	0.1246	0.0259	0.2949	0.0343	0.5203
Leather Footwear					
Production Function	0.1522	0.0492	0.2711	0.0178	0.5098
Cost Function	0.1707	0.0197	0.2706	0.0148	0.5242
Screw Products					
Production Function	0.1578	0.0468	0.2592	0.0640	0.4723
Cost Function	0.1476	0.0386	0.2559	0.0480	0.5099
Knitted Underwear					
Production Function	0.0975	0.0150	0.2340	0.0200	0.6334
Cost Function	0.1335	0.0254	0.2346	0.0158	0.5906
Briquettes					
Production Function	0.0358	0.0310	0.1780	0.0127	0.7425
Cost Function	0.0733	0.0342	0.1781	0.0143	0.7002
Molding Industry					
Production Function	0.0904	0.0224	0.2131	0.0892	0.5849
Cost Function	0.1093	0.0258	0.2142	0.0667	0.5840

More interesting results are found in the estimated elasticities of factor substitutions. The complementary relationship between operatives and administratives, as shown in the Tables IV-1 and IV-2 in the Appendix B, are also identified by the negative sign of the A.E.S. between two factors in each function of most industries. Secondly, the elasticity of substitution between either of two labors and capital input approaches to unity more closely in the inclusive cases than in the exclusive cases for most industries, leaving its own substitution elasticity almost the same as before.[66] This finding may have some implication for the popular hypothesis of unitary elasticity of substitution between labor and capital inputs in most empirical studies on aggregate production functions of manufacturing industry. Since the inclusive cases here have the same data base as have most studies done at the aggregate level from the census file, the above result may not be ignored as trivial. Hence the conventionally accepted, unitary elasticity of substitution between labor and capital may be partly attributed to the use of the aggregate data base where problems of errors in records, particularly the undermeasurement of the capital variable, prevail.

The elasticities of substitution of raw material with respect to other inputs in the inclusive cases are again found to be within some stable regions around unity. In summary, the

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[66]Two exceptions are found in the cost functions of the manufacture of briquettes and the molding industry. See the Table IV-2 of the Appendix B.

empirical estimations in the inclusive cases are not particularly preferable unless there exist significant improvements either in the goodness of fit or in the significance of the restrictions acceptable conventionally, or in eliminating possible errors in measurements, in addition to a simple merit of using a large sample size.

3.3. Alternative Variables of Different Quality

The estimations with alternative variable sets of different quality are investigated in this section, which contains 4 sets: the set A of the man-day workers for labor inputs and of the power equipment for capital input, the set B of the man-day workers and of the net capital stock, the set C of the number of workers and of the power equipment, and the set D of the number of workers and of the net capital stock. The resulting sample size is 66 establishments in the sets A and C while 45 establishments in the sets B and D of the canning industry, 20 in the sets A and C while 13 in the sets B and D of the leather footwear industry, 94 in the sets A and C while 15 in the sets B and D of the screw product industry, 144 in the sets A and C while 10 in the sets B and D of the manufacture of knitted underwear, 233 in the sets A and C while 32 in B and D of the manufacture of briquettes, and 131 establishments in the sets A and C while 55 establishments in the sets B and D of the molding industry. Hence the reduction of the sample size occurs very drastically, depending on the choice of capital variable between the power equipment and the net capital stock.

3.3.1. Goodness of Fit

The estimated quasi-R^2's vary from the share equations to the function considered in the industries. Measured by the simple (and the weighted) average(s) of the quasi-R^2's of each share equation, the estimations of both the production and the cost functions with the net capital stock (i.e., the sets B and D) are found to fit better than those with the power equipment (i.e., the sets A and C) in most industries. On the other hand, no discriminations between the man-day workers and the number of workers (i.e., the sets A and C v.s. B and D) are found at all.

3.3.2. Properties of the Functions Estimated

The significance test of the hypotheses of linear homogeneity and symmetry are found to be different from the industry concerned to the function estimated. For example in the canning industry, these hypotheses in the sets B and D are more strongly rejected in the production function, but well accepted in the cost function where the likelihood ratio statistics are 33.492 and 36.615 respectively, compared to the critical value of 45.642 at the significance level of 1%. On the other hand, in the leather footwear industry the hypotheses in all four sets are accepted in the production function but rejected in the sets B and D of the cost function.[67]

The monotonicity conditions in both the production and the cost functions are satisfied better in the sets A and C in terms of the percentages of such establishments among total sample units.

3.3.3. Parameter Estimates

The output elasticities of two labor inputs are estimated relatively bigger in the sets A and C than in the sets B and D, in general. Hence the capital shares are found to be smaller in the sets A and C than in the sets B and D of the screw product, knitted underwear, briquettes and molding industries.[68] Also this alternative choice for capital data have shown more significant changes in the estimated cost function than in the estimated production function, implying that the choice of capital variable here results in more drastic changes in factor price than in the level of capital input. These results may indicate the direction of a bias in the parameter estimates, if the level of net capital stock recorded in the data file is a true measure of capital input used in the production process, instead of power equipment.

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[67] Here the hypotheses are accepted in the sets A and C of the cost function where the test statistics are 31.756 and 31.083 respectively, compared to the critical value of 37.566 at the significance level of 1%.

[68] One exception is the leather footwear industry where the higher shares of labor and capital inputs in the set A are shifted into that of raw materials in the set C.

On the other hand, the discrimination in the choise of two labor inputs (i.e., utilization rate of labors) does not result in any significant differences in the estimated A.E.S.'s in general, as shown in the Tables V-1 through V-6 of the Appendix B. This implicates that the utilization of labor forces, measured by working hours, may be quite similar across most establishments within each industry. But alternative choice for capital data results in some differences in the estimated A.E.S.'s. First, the complementary relationship between two heterogeneous labor inputs does not hold for the sets B and D in both the production and the cost functions of the manufacture of knitted underwear. Second, the A.E.S.'s between operatives and capital input are higher, above unity, in the sets B and D than in the sets A and C for the production function of most industries. Finally, the A.E.S.'s of raw materials with respect to other inputs are also stable in all four alternative sets.

In summary, no meaningful gains are found from the selection of net capital stock for capital variable, where no decisive improvements are noticed either in the goodness of fit, or in the validity of the popularly acceptable hypotheses, or in the satisfaction of certain properties of a production technology, which may well compensate its clear disadvantage of drastic reductions in sample establishments to be used in empirical estimations.

CHAPTER V. CONCLUSIONS AND RECOMMENDATIONS

The subject of a production technology is one of the areas of economics where the gap between theoretical formulations and empirical knowledges is still quite wide. This is why the nature and magnitude of scale, share and substitution parameters continue to attract research interest. Furthermore, there seem to have been few theoretical and empirical studies on the multi-input multi-output production technology until recently. Hence the very concept of the multi-input multi-output production function, together with the multi-input multi-output cost function via their duality relationship, have become popular in most of recent theoretical studies in the theory of production and a few empirical knowledges are being accumulated in its beginning stage, mostly limited to a very aggregate (or sector) level.

The present study has purported (i) to understand the theory of a multi-input multi-output production technology as an extension of a theory of two-input uni-output production technology, (ii) to investigate the workability of the multi-input multi-output production theory, using a cross-section data system of the Korean manufacturing census, (iii) to find some knowledges on the first and second order properties of a production technology, using the translog approach, at an establishment level which is close to the reality of most manufacturing activities, and finally (iv) to collect informations on the usefulness of the Korean manufacturing census data system which is quite a common type of data system in most of the other countries.

It is hard to summarize and evaluate such a study as ours succinctly, particularly when it occurs for only a few, specific industries, which have seemingly few substantive characteristics in their production technologies in common. If one judges by one of the major purposes of this study, the first and second order properties of the multi-input multi-output (and also uni-output) production technology, the returns have been moderately high from using the translog approach. Also another of our purposes was to enter into and analyze a body of data at a level of disaggregation rarely encountered before. Micro-data at the establishment level are largely *terra incognita* for economists. While the promise of great discoveries was not met, we did learn something about the structure of production in a few of Korean manufacturing industries, and much more about the structure of such data and problems that they pose for the analyst. This knowledge should be helpful to other research workers who will undoubtedly want to continue exploring such data.

We shall divide our concluding comments into two parts: a review of the empirical findings and lessons for further research. The validity of the empirical findings should also be limited to the six industries selected in this study in principle.

In the estimation of the share equations system in each industry, the degree of factor market imperfection is found to be so high that the producers' decisions on factor demand deviate very widely from the first order conditions for either profit maximization or cost minimization. This implies that there exists quite a high degree of input hoardings in reality in the sense that considerably high portions of factor demands are not price-responsive and can not be adjusted appropriately during given production period. This phenomena seem to be more serious in the small-sized establishments, as recognized from their poorer quasi-R^2's in the input share equations appeared in our supplementary investigations.

On the other hand, most properties considered here seem to be acceptable in the estimated production technologies of most industries, even if the exact acceptances of the hypotheses of linear homogeneity, symmetry and separability between inputs and outputs are rather inconclusive such that it partly depends on the choice of test statistics between the likelihood ratio test and the F-test discussed earlier and it also varies from the industry concerned to the function estimated. Further, the monotonicity and convexity conditions are found to be well satisfied in general, particularly better in the estimations of the translog cost function than in those of the production functions. But the shape of the output transformation curves in three multi-output industries is not found as usual in the sense that they are all convex to the origin. Two possible explanations are also investigated in terms of the existence of output heterogeneity and of technical changes between the uni- and multi-output establishment groups in each industry.

The first order properties of the multi-input multi-output (and uni-output) production technologies are again synthesized in terms of the output elasticity of factor input and output share. And the second order properties of the technologies are investigated in terms of various elasticities of factor substitution, of elasticities of output transformation and of share elasticities.

First, the wide differences in the estimates of factor shares between the production function and the cost function in some industries made us to reject the self duality between two functions. This also implies that the validity of the naive Cobb-Douglas form should be rejected for the specification of a technology proper. This is again supported by the null hypothesis test on the second order terms of the translog functions where most of them are significantly rejected from being zero.

Second, the labor share ranges from 0.15 to 0.20 in most industries, and the capital share ranges from 0.18 to 0.25. Hence the labor share in value added ranges from 0.38 to 0.53 and the capital share from 0.47 to 0.62.

Third, the share of raw materials varies wildly from industry to industry, ranging from 0.4 to 0.7 at most. This implies that the conventional hypotheses on the role of raw materials should be rejected and hence the so-called value-added approach should also be tested, not a priori given assumption that the value added can be dealt properly as a good proxy for output measure in the empirical study of a production technology. This suspecion is well supported again from the findings such that the substitution elasticities of raw materials with respect to other factor inputs are close to neither zero nor infinity but unity.

Fourth, many interesting results are found in the close looking at the estimated, various elasticities of substitutions, the least of which are as follows:

(1) All the D.E.S.'s are greater than the A.E.S.'s.

(2) There exists a supplementary relationship between operatives and administrative workers, holding a strong separability condition between them.

(3) The elasticity of substitution between labor and capital inputs are around unity, but not very conclusive.

(4) The substitution elasticities of raw materials with respect to other inputs are around unity.

(5) The own substitution elasticities of factors are the greatest in fuel, the second in administratives, operatives, capital and the least in raw materials. And this ordering holds also for their demand elasticities with respect to their own price changes.

(6) The cross-demand elasticities with respect to other factors's price changes are the highest for raw materials, the second for capital input, labors, and the lowest for fuel input.

(7) The higher own transformation elasticities are found for the non-major output(s) of the industries than for the major output. This ordering also holds for the own-supply elasticities of outputs with respect to their own price changes.

(8) The cross-output transformation elasticities are around unity, while the cross-output supply elasticities with respect to other outputs's price changes are around 0.3 in three multi-output industries.

Fifth, several results from the analyses on the share elasticities of factor inputs are also shown in this study. The least of them is that the share elasticity of capital input is greater with respect to the level of capital input changes than to its price changes.

Further results are included in this volume from the supplementary estimations, supporting the main findings above mentioned.

First, much poorer fits are found in the estimation of two translog functions for the small-sized establishment group within each industry, even bringing us a suspecion on the very concept of the production or the cost functions for this group of establishments in manufacturing activities. On average, the share ratio between capital and labor inputs are increasing from the increasing capital share and the decreasing labor share, as the firm size gets bigger. Also the substitution elasticities between two factors are getting bigger in most industries. Like the labor share, the shares of fuel and raw materials are decreasing as the firm size gets bigger in most industries.

Second, no significant gains are found from the close investigations on the estimation results inclusive of such establishments with zero capital input, but much poorer fits in general in all six industries. Again one interesting result is noticed such that the estimated substitution elasticity between labor and capital is very close to unity in both the production and the cost functions of all six industries. This may implicate that the most popular finding of unitary substitution elasticity between labor and capital in an aggregate production function may be partly attributed to the use of such an aggregate data that includes establishments of zero capital input in samples.

Third, the alternative estimations covering several variables of different quality do not seem to result in any significant differences in the properties of the functions estimated in general and suggest no specific preference in choosing alternative labor and capital variables in this empirical estimation. Specifically, the choice of the number of workers for labor input does not seem to be preferable not only from any improvements found in the actual estimations but also from the very concept of labor input in a production process. The choice between the horsepower equipment and the net capital stock also does not show any significant, discriminating results, except such a drastic reduction in the estimation sample size in this study.

The suggestive directions for further researches are not only restricted to the improvement toward more profound economic analyses and to the specification of production technology for empirical works, but also to the refinements in data problems.

First of all, several interesting analyses can be designed, based on the results from this study.

First, the relationships between the factor substitution elasticity (in particular, with respect to labor and capital inputs) and either the level of output or the factor use ratio (i.e., capital-labor ratio) have been questioned in many production studies. This can be focused rather easily here but with heavy computational burdens involved, since the translog functional forms assume the variable elasticities of factor substitutions over different sample establishments in the industry concerned. One immediate suggestive work can be formulated for testing a function of capital-labor substitution elasticity with explanatory variable of either the level of output[1] or the capital-labor ratio.

Second, the complete system of a production technology which consists of the production function and certain conditional equations derived from the first order conditions for profit maximization, can be estimated for further knowledges on returns to scale, based on the same data system.

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[1]The level of output can indicate either the quantity data or the amounts in the unioutput case, but only the amount data can be used for the level of output in the multioutput case.

Third, certain investigations on technological changes can also be possible by introducing directly the business-beginning year for time variable into the translog functions. As already shown in the previous section, for example, a significant evidence was noticed in our three multioutput industries that the average age of single output-producing establishments is moderately smaller than that of multiproducts-producing establishments.

Fourth, the validity of the functional forms such as CD, CES and many variants of VES, can be identifiable by either checking the relationships among the parameter estimates of the translog functions, or by testing the significances of alternative estimations under various restrictions implied by those specific functional forms.

Fifth, in this study the translog production function and the translog cost function are estimated and compared each other due to their dual relationship. But also the translog profit function and the translog revenue function can be estimated and verified together, since the duality among these four functions are well specified already in some recent theoretical works and there exist many data available for quantity and price of various inputs and outputs in the Korean manufacturing census system.

Sixth, the specification of disturbance terms are defined for a deviations in producers' decision from the optimal factor share decision, since we adopt the estimation method of linear regression for the share equations system. More exact specification of errors in the first order conditions of profit maximization, will certainly introduce some nonlinearity in the function to be estimated, as already clearified. Hence an introduction of a proper, nonlinear estimation method can be helpful for estimating further the new translog share equations system which is consistent with the specification of errors in the first order conditions.

Seventh, since one of the most weak data point in this study is capital input, it may be also worthwhile to formulate and estimate a production technology which does not have capital input specifically, definable for the short-run technology. That is, capital input can be viewed as the nonproducible input L in the earlier specification of a multi-input multi-output technology, and hence the following relation can be shown; $X_k = F(X_p, X_A, X_F, X_R, Y_1, Y_2, Y_3)$.

Eighth, all five production inputs are defined, in this study, in a horizontal way such that they are all variable with respect either to establishment or to product. But in the reality of a production technology, this may not be valid always. In general we can classify three categories for factor inputs, i.e., the factors variable for firms such as labors and fuels, the factors fixed for establishment such as capital stock and the factors fixed for each output such as raw materials different from one output to another. Different characteristics of each factor input may be specified differently in the specification of a production technology. Some further efforts on the specification of a technology should be worthwhile along this line. Earlier studies on this issue can be traced back to Bradford and Johnson (1953), Beringer (1955) and Baquet (1976), mostly in the farm management analyses.[2] Hence the following specification for the input bundle, in the case of weak separability between input and output bundles, can be shown;

$$Y_1 = f(X_1 \ . \ . \ . \ X_g \ \vdots \ X_{g+1} \ . \ . \ . \ X_k \ \Big| \ X_{k+1} \ . \ . \ . \ X_\ell)$$

and

$$Y_2 = g(X_1 \ . \ . \ . \ X_g \ \vdots \ X_{g+1} \ . \ . \ . \ X_k \ \Big| \ X_{\ell+1} \ . \ . \ . \ X_n)$$

where (Y_1, Y_2) are outputs,

$(X_1 \ . \ . \ . \ X_g)$ are factors variable for firm,

$(X_{g+1} \ . \ . \ . \ X_k)$ are factors variable for establishments,

and

$(X_{k+1} \ . \ . \ . \ X_\ell)$ and $(X_{\ell+1} \ . \ . \ . \ X_n)$ are factors fixed for each output.

Because the data we used are a bit unusual, there are also a few lessons to be learned from our experience with them. The Korean census includes establishments with more than 5 workers. This is a very low limit indeed. While for many purposes dispersion

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[2] I am very grateful to professor Glenn L. Johnson, Michigan State University, on his remarks. In the agricultural production studies, this topic seems to have been very popular so long time and discussed by the problem of "horizontally versus vertically combined production technology."

along the scale dimension is desirable, the data for very small units seem to be less complete and accurate than those of larger units. Also in some industries, the production structure of such very small units is very different from the larger units, even though both types belong to the same industry group. In general, there is much more "noise" in the smaller units. Hence in studies of this type, the very small units might well be either excluded or subjected to some other special treatment.

The other important missing ingredient in our data is information on variation in labor quality across establishments. "Quality" is a many dimensional concept, the most important being education, occupation and other indices of skill levels. No data are available in Korea on the education and skill level of the labor force at the establishment level but only for relatively crude industrial groupings. It should not be too difficult to expand the present operative/administrative workers questions and inquire about the education, sex and skill composition of the establishments workforce.

Data that are collected could be also significantly improved. For example, the only wage rate derivable from the figures is an annual average which may be quite a far removed from any relevant concept of the marginal cost of labor input. Significant improvement could be achieved if overtime hours and payrolls were segregated from the total.

Similarly, the capital data should come closer to the concept of capital used rather than capital owned. Either one should ask about the value of capital used irrespective of ownership or one should inquire about the rental costs of leased equipment as well as about rental receipts from rented out capital.

The final important missing ingredient in the census data is information on variations in output quality across establishments. Even a naive way of specifying output quality by several grades which an establishment sells at different prices should help a lot for this type of study. The heterogeneity of output was briefly evidenced such that the average price of a certain output, produced by one product-producing establishments, is significantly different from the average price of that product, produced by more than one product-producing establishment in each industry, where the product concerned is classified into the same 7-digit KSIC commodity code in the census.

We have learned something from our investigations, not the least of which is that just "more data" will not do. If we persist in asking rather complicated questions, we shall need much better and more relevant figures before we can hope to answer them precisely.

APPENDICES

APPENDIX A

APPENDIX A-I

DERIVATION OF THE SHARE EQUATIONS SYSTEM IN THE MULTIPRODUCT TRANSFORMATION FUNCTION

The Lagrangian function for the profit maximization is;

$$L = \sum_{i=1}^{n} P_i Y_i - \sum_{j=1}^{m} w_j x_j + \lambda F(\underline{x}, \underline{y}). \tag{1}$$

And the first-order conditions are;

$$\frac{\partial L}{\partial y_i} = P_i + \lambda \frac{\partial F}{\partial y_i} = 0 \Rightarrow \frac{\partial F}{\partial y_i} = - \frac{P_i}{\lambda}, \; i=1, \ldots n, \tag{2}$$

$$\frac{\partial L}{\partial w_j} = - w_j + \lambda \frac{\partial F}{\partial x_j} = 0 \Rightarrow \frac{\partial F}{\partial x_j} = \frac{w_j}{\lambda}, \; j=1, \ldots m, \tag{3}$$

$$\frac{\partial L}{\partial \lambda} = F(\underline{x}, \underline{y}) = 0.$$

In the Translog function, the first-derivatives become

$$\frac{\partial \ln F}{\partial \ln y_i} = \frac{\partial F}{\partial y_i} \frac{y_i}{F} = - \frac{P_i}{\lambda} \frac{y_i}{F} = P_i y_i \frac{-1}{\lambda F} \Rightarrow P_i y_i = (-\lambda F) \frac{\partial \ln F}{\partial \ln y_i}, \tag{4}$$

$$\frac{\partial \ln F}{\partial \ln x_j} = \frac{\partial F}{\partial x_j} \frac{x_j}{F} = \frac{w_j}{\lambda} \frac{x_j}{F} = w_j x_j \frac{1}{\lambda F} \Rightarrow w_j x_j = (\lambda F) \frac{\partial \ln F}{\partial \ln x_j} \cdot \tag{5}$$

From (4) and (5), we can have;

$$\sum_{i=1}^{n} P_i y_i = -\lambda F \sum_{i=1}^{n} \frac{\partial \ln F}{\partial \ln y_i} ; \Rightarrow -\lambda F = \sum_i p_i y_i / \sum_i \frac{\partial \ln F}{\partial \ln y_i} , \tag{6}$$

and

$$\sum_{j=1}^{m} w_j x_j = \lambda F \sum_{j=1}^{m} \frac{\partial \ln F}{\partial \ln x_j} ; \Rightarrow \lambda F = \sum_j w_j x_j / \sum_j \frac{\partial \ln F}{\partial \ln x_j} . \tag{7}$$

Hence from (4) and (5);

$$\frac{\partial \ln F}{\partial \ln y_i} = P_i y_i \left(\frac{-1}{\lambda F}\right) = \frac{P_i y_i}{\sum_i P_i y_i} \sum_i \frac{\partial \ln F}{\partial \ln y_i} , \tag{8}$$

and

$$\frac{\partial \ln F}{\partial \ln x_j} = w_j x_j \frac{1}{\lambda F} = \frac{w_j x_j}{w_j x_j} \sum_j \frac{\partial \ln F}{\partial \ln x_j} . \tag{9}$$

The derived equations (8) and (9) can be written as;

$$S_i = \frac{\partial \ln F}{\partial \ln y_i} = \beta_i + \sum_j \varepsilon_{ij} \ln x_j + \sum_k \delta_{ik} \ln y_k , \tag{10}$$

and

$$-S_j = \frac{\partial \ln F}{\partial \ln x_j} = \alpha_j + \sum_\ell \gamma_{j\ell} \ln x_\ell + \sum_i \varepsilon_{ji} \ln y_i , \tag{11}$$

where

$$\sum_i \frac{\partial \ln F}{\partial \ln y_i} = \sum_i \beta_i + \sum_i \sum_j \varepsilon_{ij} \ln x_j + \sum_i \sum_k \delta_{ik} \ln y_k = 1.0, \ i,k=1, \ldots n, \tag{12}$$

and

$$\sum_j \frac{\partial \ln F}{\partial \ln x_j} = \sum_j \alpha_j + \sum_j \sum_\ell \gamma_{j\ell} \ln x_\ell + \sum_j \sum_i \varepsilon_{ji} \ln y_i = -1.0, \; j,l=1, \; . \; . \; . \; m \; . \tag{13}$$

Here the relations, (12) and (13), hold under the sufficient conditions for linear homogeneity, that is, $\sum_j \alpha_j = -1.0$, $\sum_i \beta_i = 1.0$,

$\sum_i \varepsilon_{ij} = \sum_j \varepsilon_{ij} = 0.0$, $\sum_i \delta_{ik} = 0.0$, and $\sum_j \gamma_{j\ell} = 0.0$.

APPENDIX A-II

INTRINSIC PROPERTY OF THE SHARE EQUATIONS SYSTEM

$$y_i = f(\underline{x}) + e_i$$

Here the GLS estimator (β_i^*) is identical to the OLS estimator (β_i). Therefore, $\beta_i^* = \beta_i = (x'x)^{-1}\ x'y_i$, (1)

where

$$\begin{bmatrix} \beta_i = \begin{bmatrix} \beta_{oi} \\ \beta_{1i} \\ \vdots \\ \beta_{mi} \end{bmatrix} \end{bmatrix} , \qquad \begin{bmatrix} y_i = \begin{bmatrix} y_{1i} \\ y_{2i} \\ \vdots \\ y_{ti} \end{bmatrix} \end{bmatrix}$$

Since $\sum_i y_{ji} = 1.0$ for $j = 1, \ldots T$, the sum of the coefficients; becomes;

$$\sum_i \beta_i = \sum_i (x'x)^{-1}\ x'y_i = (x'x)^{-1}\ x' \sum_i y_i = (x'x)^{-1}\ x'I, \qquad (2)$$

where $I = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$.

On the other hand, the vector I can be expressed in our estimating system as

$$I = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix} = X \cdot \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} .$$

Hence the relation, (2), can be represented as

$$\sum_i \beta_i = \begin{bmatrix} \Sigma\, \beta_{oi} \\ \Sigma\, \beta_{1i} \\ \vdots \\ \Sigma\, \beta_{mi} \end{bmatrix} = (x'x)^{-1}\, x'x \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} .$$

APPENDIX B

APPENDIX B-I

TABLE I-1.--Factor Prices by the Size of Establishments: Canning Industry.[a] (Unit: 1,000 won)

Size[b]	1	2	3	4	5	6	7	8	Total
Operatives	28.09 (18.77)	17.46 (20.18)	12.75 (6.58)	21.66 (17.33)	13.27 (5.61)	11.75 (4.26)	11.40 (6.28)	10.90 (4.61)	16.36 (13.54)
Administratives	23.75 (15.39)	16.86 (10.40)	23.93 (19.13)	36.96 (18.22)	33.14 (16.40)	50.82 (37.29)	42.03 (8.88)	28.73 (21.58)	30.75 (20.84)
Operatives/ Man-Days	1.33 (0.98)	1.12 (0.81)	0.64 (0.35)	0.92 (0.58)	0.57 (0.28)	0.43 (0.15)	0.46 (0.29)	0.44 (0.20)	0.76 (0.62)
Administratives/ Man-Days	1.15 (0.99)	1.23 (0.71)	1.13 (0.75)	1.94 (1.68)	1.51 (0.81)	1.91 (1.41)	1.72 (0.37)	1.18 (0.93)	1.45 (1.06)
Power Equipment	0.08 (0.10)	0.55 (1.70)	1.03 (1.84)	0.74 (0.74)	5.94 (19.97)	1.20 (1.35)	0.58 (0.84)	2.77 (2.18)	1.48 (7.24)
Net Capital Stock	0.45 (1.30)	1.96 (3.32)	0.63 (1.58)	4.28 (9.21)	2.77 (6.08)	1.04 (1.04)	1.52 (2.51)	0.87 (0.70)	1.80 (4.86)
Fuel	24.21 (31.19)	7.76 (8.55)	3.19 (4.03)	2.90 (1.22)	4.88 (6.39)	5.02 (6.59)	6.30 (9.10)	1.51 (0.51)	7.07 (14.36)
Raw Materials	22.84 (66.95)	1.57 (1.28)	1.22 (4.46)	1.28 (0.44)	2.57 (4.68)	3.72 (3.81)	4.90 (7.31)	15.68 (20.60)	23.76 (137.45)

[a]See the footnotes (a) and (f) in the Table III-3, Chapter II, Part B.

[b]See the footnote (a) in the Table II-5, Chapter II, Part B.

TABLE I-2.--Factor Prices by the Size of Establishments: Leather Footwear Industry.[a]

Size	1	2	3	4	5	6	7	8	Total
Operatives	19.02 (9.44)	23.39 (9.31)	17.86 (6.98)	19.19 (6.25)	13.73 (4.88)	21.21 (16.91)	17.63 (6.55)	15.50 (3.35)	19.38 (9.45)
Administratives	17.74 (10.98)	23.24 (8.21)	21.03 (8.78)	28.06 (6.58)	33.81 (7.15)	74.09 (71.71)	50.61 (12.67)	36.45 (1.24)	20.76 (16.91)
Operatives/ Man-Days	0.74 (0.37)	0.90 (0.36)	0.73 (0.33)	0.78 (0.31)	0.49 (0.17)	0.83 (0.66)	0.68 (0.24)	0.61 (0.14)	0.76 (0.38)
Administratives/ Man-Days	0.69 (0.42)	0.90 (0.33)	0.85 (0.39)	1.14 (0.33)	1.22 (0.28)	2.90 (2.79)	1.95 (0.47)	1.43 (0.06)	0.81 (0.66)
Power Equipment	0.04 (0.24)	0.39 (0.84)	1.57 (2.56)	1.79 (2.87)	0.98 (0.98)	0.07 (0.10)	0.46 (0.45)	1.23 (1.23)	0.24 (1.00)
Net Capital Stock	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)	2.78 (6.45)	2.37 (2.10)	0.95 (1.20)	1.41 (0.23)	0.85 (0.06)	0.16 (1.35)
Fuel	26.64 (16.25)	21.98 (9.58)	23.61 (15.22)	16.47 (8.99)	10.73 (5.54)	6.70 (6.96)	9.80 (9.54)	10.24 (3.67)	24.68 (15.66)
Raw Materials	25.91 (66.68)	10.68 (9.32)	18.37 (26.04)	30.17 (33.40)	34.42 (30.54)	3.96 (3.68)	2.71 (1.08)	1.03 (0.16)	23.05 (58.93)

[a]See the footnotes (a) and (b) in the Table I-1, Appendix B.

TABLE I-3.--Factor Prices by the Size of Establishments: Screw Products Industry.[a]

Size	1	2	3	4	5	6	7	8	Total
Operatives	17.39	17.57	17.83	16.08	18.02	97.74	23.55	18.83	18.27
	(9.67)	(7.82)	(8.06)	(7.88)	(7.37)	(0.0)	(0.10)	(1.80)	(11.18)
Administratives	18.12	22.62	31.54	35.25	42.18	5.30	55.92	38.45	24.89
	(10.21)	(11.69)	(42.28)	(22.70)	(18.07)	(0.00)	(6.33)	(5.05)	(14.82)
Operatives/ Man-Days	0.71	0.70	0.71	0.64	0.82	3.65	0.94	0.74	0.74
	(0.36)	(0.31)	(21.15)	(0.33)	(6.52)	(0.00)	(0.00)	(0.08)	(0.42)
Administratives/ Man-Days	0.74	0.92	1.29	1.39	1.96	0.20	2.23	1.51	1.01
	(0.39)	(0.54)	(7.18)	(0.84)	(6.57)	(0.00)	(0.26)	(0.22)	(0.62)
Power Equipment	0.13	0.23	0.19	0.19	0.48	0.0	0.10	0.37	0.19
	(0.27)	(0.30)	(0.16)	(0.14)	(0.11)	(0.0)	(0.08)	(0.24)	(0.26)
Net Capital Stock	0.0	0.01	-0.00	1.44	0.58	1.22	0.41	3.05	0.25
	(0.0)	(0.08)	(0.00)	(2.32)	(0.71)	(0.0)	(0.34)	(2.54)	(1.04)
Fuel	18.65	14.42	12.88	8.08	9.72	3.98	13.79	6.51	14.71
	(11.85)	(10.20)	(18.34)	(7.99)	(11.64)	(0.00)	(5.06)	(4.67)	(11.24)
Raw Materials	110.78	44.83	48.86	74.75	52.86	1.77	0.70	1.26	69.05
	(218.52)	(81.34)	(98.22)	(156.74)	(73.71)	(0.00)	(0.07)	(0.36)	(154.88)

[a]See the footnotes (a) and (b) in the Table I-1, Appendix B.

TABLE I-4.--Factor Prices by the Size of Establishments: Manufacture of Knitted Underwear.[a]

Size	1	2	3	4	5	6	7	8	Total
Operatives	16.40 (3.89)	16.50 (6.22)	14.01 (4.95)	16.93 (9.40)	16.75 (7.17)	21.59 (6.43)	9.26 (2.67)	0.0 (0.0)	15.69 (6.16)
Administratives	16.40 (3.89)	19.85 (9.33)	20.12 (7.56)	36.86 (13.15)	39.80 (21.29)	42.79 (12.80)	26.22 (12.65)	0.0 (0.0)	22.32 (11.99)
Operatives/ Man-Days	0.71 (0.20)	0.68 (0.27)	0.59 (0.25)	0.69 (0.35)	0.69 (0.22)	0.79 (0.22)	0.44 (0.09)	0.0 (0.0)	0.65 (0.26)
Administratives/ Man-Days	0.71 (0.20)	0.82 (0.39)	0.83 (0.31)	1.57 (0.65)	1.59 (0.64)	1.60 (0.56)	1.16 (0.38)	0.0 (0.0)	0.93 (0.48)
Power Equipment	0.42 (0.45)	0.34 (1.26)	0.56 (0.66)	0.68 (0.63)	0.66 (0.80)	2.73 (3.29)	0.28 (0.36)	0.0 (0.0)	0.51 (1.10)
Net Capital Stock	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)	0.02 (0.16)	1.97 (3.65)	3.47 (4.68)	0.60 (0.66)	0.0 (0.0)	0.19 (1.24)
Fuel	21.49 (10.02)	17.97 (17.85)	14.71 (10.28)	11.61 (8.43)	8.70 (7.92)	5.88 (4.68)	8.43 (11.19)	0.0 (0.0)	16.02 (14.12)
Raw Materials	2.71 (2.02)	2.65 (3.93)	2.75 (4.22)	2.22 (1.75)	5.13 (6.55)	2.98 (2.24)	1.80 (0.77)	0.0 (0.0)	2.76 (3.86)

[a]See the footnotes (a) and (b) in the Table I-1, Appendix B.

TABLE I-5.--Factor Prices by the Size of Establishments: Manufacture of Briquettes.[a]

Size	1	2	3	4	5	6	7	8	Total
Operatives	19.59 (7.06)	21.40 (7.05)	22.52 (12.47)	25.82 (11.64)	25.54 (8.33)	25.20 (5.42)	26.04 (4.56)	0.0 (0.0)	21.38 (8.57)
Administratives	21.59 (11.51)	25.15 (12.18)	27.66 (14.54)	36.80 (19.13)	54.65 (40.09)	36.33 (11.62)	38.68 (33.09)	0.0 (0.0)	26.38 (17.20)
Operatives/ Man-Days	0.85 (0.33)	0.87 (0.31)	0.87 (0.45)	0.95 (0.41)	0.92 (0.30)	0.92 (0.21)	0.97 (0.13)	0.0 (0.0)	0.87 (0.34)
Administrative/ Man-Days	0.95 (0.53)	1.03 (0.61)	1.09 (0.59)	1.35 (0.70)	1.97 (1.44)	1.33 (0.44)	1.49 (1.35)	0.0 (0.0)	1.08 (0.69)
Power Equipment	0.15 (0.22)	0.42 (2.43)	0.18 (0.18)	0.20 (0.30)	3.78 (9.45)	6.85 (9.32)	1.48 (0.70)	0.0 (0.0)	0.51 (2.82)
Net Capital Stock	0.00 (0.02)	0.04 (0.19)	0.36 (1.18)	7.62 (18.50)	10.43 (17.22)	8.35 (13.58)	5.10 (3.88)	0.0 (0.0)	1.27 (7.08)
Fuel	16.14 (9.17)	21.40 (19.81)	17.04 (8.16)	18.69 (9.69)	19.41 (6.25)	20.01 (4.38)	16.26 (8.33)	0.0 (0.0)	20.81 (44.20)
Raw Materials	0.37 (0.54)	0.67 (3.03)	0.38 (0.56)	0.35 (0.32)	0.26 (0.06)	0.28 (0.01)	0.27 (0.03)	0.0 (0.0)	0.45 (1.71)

[a]See the footnotes (a) and (b) in the Table I-1, Appendix B.

TABLE I-6.--Factor Prices by the Size of Establishments: Molding Industry.[a]

Size	1	2	3	4	5	6	7	8	Total
Operatives	23.06 (10.26)	21.21 (11.78)	19.77 (7.04)	18.81 (5.85)	21.64 (5.77)	34.62 (7.56)	24.31 (11.59)	33.60 (12.78)	21.76 (9.91)
Administratives	20.83 (10.70)	27.26 (16.47)	30.91 (12.65)	42.47 (19.15)	43.12 (17.27)	68.15 (23.77)	59.99 (33.87)	73.24 (20.65)	36.00 (21.58)
Operatives/ Man-Days	1.06 (0.44)	0.91 (0.50)	0.84 (0.39)	0.78 (0.29)	0.88 (0.22)	1.36 (0.29)	0.94 (0.43)	1.23 (0.43)	0.91 (0.43)
Administratives/ Man-Days	0.96 (0.47)	1.18 (0.69)	1.31 (0.62)	1.78 (0.91)	1.81 (0.80)	2.66 (0.89)	2.29 (1.14)	2.70 (0.77)	1.49 (0.86)
Power Equipment	0.32 (0.46)	0.12 (0.32)	0.22 (1.36)	0.10 (0.35)	0.14 (2.68)	3.61 (6.02)	0.31 (0.23)	0.83 (1.63)	0.29 (1.58)
Net Capital Stock	0.0 (0.0)	0.08 (0.36)	0.59 (2.22)	3.59 (11.04)	3.95 (8.59)	12.92 (21.78)	1.46 (0.89)	3.69 (7.22)	1.52 (6.23)
Fuel	161.47 (377.92)	113.72 (318.44)	64.71 (122.69)	72.40 (94.33)	6.67 (6190)	4.12 (3.00)	19.11 (30.95)	7.65 (6.23)	74.57 (212.69)
Raw Materials	115.62 (188.63)	172.59 (561.64)	908.18 (6003.17)	61.95 (52.10)	41.67 (20.39)	39.59 (8.44)	46.60 (19.36)	36.94 (9.01)	384.10 (3572.92)

[a]See the footnotes (a) and (b) in the Table I-1, Appendix B.

APPENDIX B-II

TABLE II-1.--Factor Shares (%) by the Size of Establishments: Canning Industry.[a]

Size	1	2	3	4	5	6	7	8	Total
Wages	12.39	8.09	14.12	10.20	14.36	8.51	8.44	17.54	11.96
	(6.59)	(7.10)	(8.05)	(6.66)	(8.31)	(3.27)	(3.40)	(5.72)	(7.45)
Salaries	2.73	2.94	2.38	3.70	3.62	4.72	2.84	8.18	3.33
	(1.62)	(3.09)	(1.52)	(2.45)	(2.84)	(3.30)	(1.34)	(7.92)	(2.96)
Capital Returns	23.22	36.17	26.76	26.43	34.79	35.55	30.75	28.43	25.38
	(11.59)	(27.72)	(16.29)	(15.10)	(18.92)	(20.15)	(19.75)	(6.38)	(17.50)
Fuel	5.11	2.05	2.83	5.25	4.18	2.83	3.33	2.94	3.72
	(5.37)	(0.63)	(2.26)	(4.94)	(3.68)	(1.45)	(2.84)	(0.57)	(3.65)
Raw Materials	56.55	50.75	53.91	54.42	43.05	48.39	54.64	42.91	55.61
	(13.50)	(16.56)	(17.58)	(26.39)	(20.67)	(18.55)	(17.91)	(5.78)	(18.59)
Value Added	38.34	47.20	43.26	40.33	52.77	48.78	42.03	54.15	40.67
	(10.76)	(25.79)	(14.80)	(16.54)	(18.50)	(15.43)	(14.02)	(3.66)	(17.43)

[a]See the footnotes (a) and (b) in the Table III-4, Chapter II, Part B.

TABLE II-2.--Factor Shares (%) by the Size of Establishments: Leather Footwear Industry.[a]

Size	1	2	3	4	5	6	7	8	Total
Wages	17.49	20.60	18.92	14.91	12.60	18.74	10.41	11.93	17.64
	(6.50)	(10.44)	(11.08)	(4.85)	(0.82)	(7.45)	(7.68)	(2.52)	(7.46)
Salaries	5.63	3.59	3.97	5.79	1.61	7.43	2.65	4.40	5.27
	(3.66)	(2.40)	(3.17)	(4.29)	(0.90)	(6.16)	(0.46)	(1.27)	(3.66)
Capital Returns	24.43	24.79	27.11	19.10	22.82	17.96	30.10	16.74	24.29
	(12.24)	(12.53)	(8.68)	(16.98)	(8.65)	(8.34)	(10.30)	(1.56)	(12.25)
Fuel	1.49	1.30	1.39	0.92	0.79	1.09	1.91	0.45	1.43
	(1.41)	(0.63)	(1.22)	(0.94)	(0.24)	(0.75)	(1.54)	(0.01)	(1.31)
Raw Materials	50.96	49.72	48.61	59.28	62.18	54.78	54.93	66.48	51.37
	(11.93)	(12.07)	(7.95)	(16.52)	(7.05)	(18.55)	(17.47)	(0.34)	(12.39)
Value Added	47.55	48.98	50.00	39.80	37.03	44.13	43.16	33.07	47.20
	(11.61)	(11.32)	(8.19)	(14.79)	(6.81)	(17.70)	(16.69)	(0.14)	(11.96)

[a]See the footnote (a) in the Table II-1, Appendix B.

TABLE II-3.--Factor Shares (%) by the Size of Establishments: Screw Products Industry.[a]

Size	1	2	3	4	5	6	7	8	Total
Wages	22.25 (11.20)	20.23 (13.63)	15.36 (8.07)	12.46 (7.16)	18.90 (1.88)	7.14 (0.0)	19.81 (4.37)	14.01 (3.10)	19.20 (11.44)
Salaries	6.37 (4.84)	3.98 (2.27)	4.32 (3.06)	5.41 (4.11)	5.74 (2.13)	2.71 (0.0)	5.28 (1.08)	2.95 (0.32)	5.03 (3.78)
Capital Returns	24.78 (23.32)	27.10 (19.04)	24.53 (14.32)	36.91 (9.18)	39.07 (15.33)	50.83 (0.0)	18.53 (6.10)	42.04 (13.34)	27.47 (20.55)
Fuel	4.33 (3.37)	3.69 (2.66)	4.31 (3.84)	4.33 (3.40)	2.56 (1.59)	2.08 (0.0)	11.98 (0.64)	5.14 (4.24)	4.22 (3.40)
Raw Materials	42.27 (16.30)	45.00 (15.90)	51.48 (16.15)	40.89 (8.97)	33.73 (11.07)	37.24 (0.0)	44.40 (4.33)	35.86 (11.98)	44.08 (15.77)
Value Added	53.40 (15.71)	51.31 (16.21)	44.21 (13.85)	54.78 (18.68)	63.71 (6.17)	60.68 (0.0)	43.62 (3.17)	59.00 (12.56)	51.70 (15.82)

[a]See the footnote (a) in the Table II-1, Appendix B.

TABLE II-4.--Factor Shares (%) by the Size of Establishments: Manufacture of Knitted Underwear.[a]

Size	1	2	3	4	5	6	7	8	Total
Wages	13.32	12.94	12.38	18.95	18.28	15.41	6.97	0.0	13.39
	(5.73)	(5.95)	(6.26)	(18.95)	(11.48)	(9.89)	(2.94)	(0.0)	(8.31)
Salaries	3.56	2.73	1.82	2.80	3.36	2.33	1.92	0.0	2.56
	(3.05)	(3.01)	(1.18)	(1.20)	(2.19)	(1.14)	(0.86)	(0.0)	(2.45)
Capital Returns	22.82	11.61	24.62	20.75	34.86	27.26	44.64	0.0	19.97
	(10.92)	(9.34)	(11.59)	(13.59)	(16.84)	(5.29)	(21.66)	(0.0)	(11.29)
Fuel	1.62	1.49	1.64	1.27	1.02	0.95	1.60	0.0	1.50
	(1.20)	(1.17)	(0.99)	(1.05)	(0.69)	(0.61)	(0.60)	(0.0)	(1.09)
Raw Materials	58.68	71.23	59.54	56.23	42.48	54.05	44.87	0.0	62.58
	(13.84)	(70.58)	(14.07)	(21.14)	(12.54)	(17.03)	(7.57)	(0.0)	(55.06)
Value Added	39.70	27.28	38.82	42.50	56.50	45.00	53.53	0.0	35.92
	(12.09)	(14.18)	(15.02)	(22.75)	(15.02)	(14.62)	(8.81)	(0.0)	(14.27)

[a]See the footnote (a) in the Table II-1, Appendix B.

TABLE II-5.--Factor Shares (%) by the Size of Establishments: Manufacture of Briquettes.[a]

Size	1	2	3	4	5	6	7	8	Total
Wages	9.64 (7.92)	7.02 (4.97)	5.82 (4.41)	3.38 (2.82)	2.40 (1.54)	1.91 (0.31)	1.67 (0.50)	0.0 (0.0)	7.39 (6.58)
Salaries	3.93 (3.51)	3.44 (3.04)	2.78 (1.70)	2.68 (1.56)	1.52 (0.90)	2.18 (1.12)	2.72 (0.37)	0.0 (0.0)	3.40 (3.01)
Capital Returns	17.82 (12.07)	6.99 (8.97)	16.19 (9.72)	21.31 (17.48)	40.32 (29.23)	52.18 (16.05)	30.45 (14.11)	0.0 (0.0)	16.30 (14.90)
Fuel	1.62 (1.30)	1.47 (1.31)	1.09 (0.73)	1.38 (1.14)	0.81 (0.80)	0.97 (0.68)	0.47 (0.26)	0.0 (0.0)	1.43 (1.23)
Raw Materials	67.00 (15.15)	81.08 (82.21)	74.12 (10.39)	71.25 (18.02)	54.95 (27.66)	42.76 (14.62)	64.69 (13.61)	0.0 (0.0)	71.48 (47.45)
Value Added	31.38 (14.29)	17.45 (9.22)	24.79 (9.94)	27.37 (17.92)	44.24 (29.27)	56.27 (14.99)	34.84 (13.06)	0.0 (0.0)	27.09 (14.79)

[a]See the footnote (a) in the Table II-1, Appendix B.

TABLE II-6.--Factor Shares (%) by the Size of Establishments: Molding Industry.[a]

Size	1	2	3	4	5	6	7	8	Total
Wages	14.81	13.88	11.67	9.41	9.33	7.07	6.65	5.52	11.34
	(5.11)	(7.00)	(6.16)	(5.07)	(5.92)	(3.96)	(4.52)	(5.92)	(6.60)
Salaries	3.18	3.92	3.02	2.92	3.14	2.18	2.08	1.48	3.11
	(2.02)	(3.67)	(1.96)	(1.60)	(1.74)	(0.70)	(1.00)	(0.95)	(2.47)
Capital Returns	20.74	22.72	20.30	26.56	34.31	21.04	26.41	28.36	22.25
	(9.05)	(16.26)	(12.59)	(22.10)	(16.53)	(14.10)	(19.04)	(16.83)	(14.78)
Fuel	8.71	7.41	8.76	7.12	6.44	7.47	9.60	4.14	7.76
	(6.23)	(4.12)	(6.58)	(5.04)	(5.65)	(7.65)	(7.96)	(3.32)	(5.83)
Raw Materials	52.56	52.07	56.25	53.99	46.78	62.24	55.26	60.80	55.54
	(12.67)	(14.08)	(16.68)	(14.69)	(11.62)	(17.91)	(15.48)	(23.46)	(16.43)
Value Added	38.73	40.52	34.99	38.89	46.78	30.29	35.14	35.06	36.70
	(9.71)	(15.04)	(12.43)	(14.29)	(16.97)	(14.00)	(19.50)	(19.01)	(14.82)

[a]See the footnote (a) in the Table II-1, Appendix B.

APPENDIX B-III

TABLE III-1.--Partial Elasticity of Substitution Estimated: Canning Industry.[a]

A.E.S.	Production Function		Cost Function		Demand Elasticity of Substitution	
	Small	Large	Small	Large	Small	Large
(P : P)	- 4.8700	- 4.7691	- 5.3601	- 7.1259	- 0.8068	- 0.9096
(P : A)	- 0.4889	0.7561	- 1.5223	- 1.3510	- 0.0411	- 0.0541
(P : K)	0.6776	0.9933	0.6599	0.8662	0.1815	0.2856
(P : F)	0.4602	1.3855	1.9867	0.2335	0.0860	0.0113
(P : R)	0.9643	0.4633	1.0278	1.3562	0.5181	0.6158
(A : A)	-25.9490	-13.3200	-29.8140	-29.3210	- 0.8055	- 1.1739
(A : K)	0.8012	0.7890	0.4523	1.0062	0.1244	0.3317
(A : F)	- 1.8512	0.3898	4.8703	- 3.4014	0.2107	- 0.1654
(A : R)	1.1193	0.5456	0.8715	1.4588	0.4393	0.6624
(K : K)	- 1.9287	- 2.0810	- 2.7853	- 2.0283	- 0.7661	- 0.6687
(K : F)	1.1899	1.0148	0.0483	1.0341	0.0021	0.0523
(K : R)	0.8180	1.0173	0.8261	0.8791	0.4165	0.3992
(F : F)	-22.6790	-15.1210	-20.8550	-19.5570	- 0.9024	- 0.9507
(F : R)	1.0561	0.5491	0.9094	1.5773	0.4585	0.7162
(R : R)	- 0.8987	- 0.9982	- 0.9914	- 1.1407	- 0.4998	- 0.5179
(1 : 1)	1.445	0.3072	1.4318	0.3050	0.5886	0.2337
(1 : 2)	- 1.0192	- 1.0210	- 0.9814	- 0.9810	- 0.1708	- 0.1261
(1 : 3)	- 1.0128	- 1.0162	- 0.9874	- 0.9828	- 0.4096	- 0.1035
(2 : 2)	4.8645	7.6475	4.7490	6.7889	0.8266	0.8724
(2 : 3)	- 1.0238	- 1.0854	- 0.9762	- 0.9191	- 0.4050	- 0.0968
(3 : 3)	1.4391	8.0448	1.4106	8.4949	0.5852	0.8948

[a]See the footnote (a) in the Table X-1-1, Chapter IV, Part B.

TABLE III-2.--Partial Elasticity of Substitution Estimated: Screw Product Industry.[a]

A.E.S.	Production Function		Cost Function		Demand Elasticity of Substitution	
	Small	Large	Small	Large	Small	Large
(P : P)	- 3.3030	- 4.0473	- 4.3063	- 4.5397	- 0.7822	- 0.6977
(P : A)	- 0.4037	- 1.0250	- 1.0181	- 5.7214	- 0.0466	- 0.2834
(P : K)	0.7415	0.7640	- 0.1136	0.4210	- 0.0285	0.1311
(P : F)	0.3065	0.0770	1.9078	3.4419	0.0760	0.2094
(P : R)	0.9215	1.0252	0.9474	0.8624	0.4568	0.3661
(A : A)	-11.1380	-10.5190	-17.1270	-14.0150	- 0.7838	- 0.6942
(A : K)	0.4435	0.5396	- 0.4252	0.5295	- 0.1066	0.1649
(A : F)	1.0101	0.6981	5.2763	5.2111	0.2103	0.3171
(A : R)	0.9485	1.1998	0.9952	0.9322	0.4798	0.3958
(K : K)	- 3.0411	- 1.8532	- 3.1809	- 2.4752	- 0.7972	- 0.7707
(K : F)	0.9232	0.5155	- 0.2005	- 0.3492	- 0.0080	- 0.0212
(K : R)	1.0947	1.0052	0.8889	0.8771	0.4285	0.3724
(F : F)	-18.1970	-11.4150	-26.5330	-18.4390	- 1.0576	- 1.1219
(F : R)	0.8760	1.1290	1.0784	1.0448	0.5199	0.4436
(R : R)	- 1.0647	- 1.4700	- 1.0677	- 1.3490	- 0.5147	- 0.5727
(1 : 1)	0.3925	0.7250	0.3766	0.7125	0.2736	0.4161
(1 : 2)	- 1.0155	- 1.0131	- 0.9846	- 0.9871	- 0.2693	- 0.4106
(2 : 2)	2.6272	1.4156	2.6551	1.4036	0.7263	0.5838

[a]See the footnote (a) in the Table X-1-1, Chapter IV, Part B.

TABLE III-3.--Partial Elasticity of Substitution Estimated: Manufacture of Knitted Underwear.[a]

A.E.S.	Production Function		Cost Function		Demand Elasticity of Substitution	
	Small	Large	Small	Large	Small	Large
(P : P)	- 3.1304	- 9.3684	- 6.9101	- 4.1928	- 0.8852	- 0.8171
(P : A)	-36.3650	1.4227	- 3.7929	- 1.9422	- 0.0921	- 0.0561
(P : F)	1.2232	1.3771	0.1165	- 0.4579	0.0265	- 0.1230
(P : K)	8.8592	- 0.5049	0.1466	0.0931	0.0023	0.0012
(P : R)	1.4749	2.8752	0.9337	0.9262	0.5643	0.4579
(A : A)	189.8600	-47.5750	-35.3940	-27.3710	- 0.8595	- 0.7904
(A : K)	- 1.4433	0.4402	- 0.5243	0.3329	- 0.1193	0.0894
(A : F)	-21.4790	9.5229	8.4284	14.6680	0.1319	0.1939
(A : R)	1.3914	1.7244	1.0063	1.1849	0.6082	0.5858
(K : K)	- 3.8846	- 3.1726	- 3.4381	- 2.7261	- 0.7825	- 0.7324
(K : F)	0.7449	1.6306	0.3549	0.9184	0.0056	0.0121
(K : R)	1.2423	1.1120	0.8467	1.2850	0.5117	0.6352
(F : F)	-95.2120	-109.6500	-67.3490	-95.1870	- 1.0538	- 1.2583
(F : R)	1.1696	1.6889	1.2403	1.1526	0.7496	0.5698
(R : R)	- 0.8667	- 1.8837	- 0.6484	- 1.0188	- 0.3919	- 0.5036

[a]See the footnote (a) in the Table X-1-1, Chapter IV, Part B.

TABLE III-4.--Partial Elasticity of Substitution Estimated: Manufacture of Briquettes.[a]

A.E.S.	Production Function		Cost Function		Demand Elasticity of Substitution	
	Small	Large	Small	Large	Small	Large
(P : P)	-85.8580	-107.6300	-10.9400	-34.0100	- 0.8921	- 1.0796
(P : A)	150.2200	- 5.6466	- 1.5115	7.0758	- 0.0547	0.1794
(P : K)	- 2.4704	0.0941	- 1.2885	0.2526	- 0.2056	0.0721
(P : F)	-11.0950	-27.4670	2.7898	- 8.3509	0.0410	- 0.0994
(P : R)	3.1526	5.9771	1.0535	2.2355	0.7458	1.4436
(A : A)	-411.9500	-48.6320	-26.2550	-36.5470	- 0.9508	- 0.9266
(A : K)	6.8031	0.4776	- 0.5490	0.2465	- 0.0876	0.0703
(A : F)	23.0500	- 8.6608	1.8826	5.0400	0.0277	0.0600
(A : R)	1.5473	2.1356	0.8846	0.8508	0.6263	0.5494
(K : K)	- 5.0786	- 2.3127	- 5.4048	- 2.5407	- 0.8624	- 0.7247
(K : F)	- 0.2673	0.6769	- 0.4959	0.1643	- 0.0073	0.0020
(K : R)	1.1098	0.9857	0.9729	0.5763	0.6887	0.3721
(F : F)	-92.3970	-138.7800	-68.5380	-103.5800	- 1.0086	- 1.2331
(F : R)	2.1393	3.9499	1.1186	2.0497	0.7919	1.3236
(R : R)	- 0.7514	- 0.8859	- 0.4101	- 0.5292	- 0.2903	- 0.3417

[a]See the footnote (a) in the Table X-1-1, Chapter IV, Part B.

TABLE III-5.--Partial Elasticity of Substitution Estimated: Molding Industry.[a]

	Production Function		Cost Function		Demand Elasticity of Substitution	
A.E.S.	Small	Large	Small	Large	Small	Large
(P : P)	-16.0040	-20.0920	- 6.2233	- 9.9209	- 0.8349	- 0.8370
(P : A)	33.7450	14.2240	- 4.9525	- 0.7756	- 0.1651	- 0.0195
(P : K)	1.7880	1.4044	0.3370	0.8796	0.0663	0.2173
(P : F)	1.4446	1.0917	1.3902	2.2520	0.1102	0.1414
(P : R)	0.9984	1.5890	0.9394	1.0229	0.5227	0.5941
(A : A)	-165.9100	-90.9420	-28.3030	-36.2110	- 0.9437	- 0.9086
(A : K)	0.9723	0.9579	0.2890	0.7199	0.0569	0.1778
(A : F)	2.0940	1.1033	1.2897	2.0567	0.1023	0.1291
(A : R)	1.1640	1.3360	0.8848	0.8386	0.4923	0.4870
(K : K)	- 4.4531	- 2.7701	- 4.4026	- 3.2760	- 0.8667	- 0.8092
(K : F)	0.8049	0.2522	0.1991	0.1042	0.0158	0.0065
(K : R)	0.9716	0.9055	0.8296	0.7733	0.4616	0.4491
(F : F)	-12.6120	-16.4500	-10.3220	-11.7270	- 0.8184	- 0.7361
(F : R)	1.0386	1.4643	0.9880	0.8071	0.5497	0.4687
(R : R)	- 0.8023	- 0.8320	- 0.7991	- 0.7427	- 0.4446	- 0.4314

[a]See the footnote (a) in the Table X-1-1, Chapter IV, Part B.

APPENDIX B-IV

TABLE IV-1.--Partial Elasticity of Substitution Estimated.[a]

A.E.S.	Canning Industry: Production Function	Canning Industry: Cost Function	Leather Footwear: Production Function	Leather Footwear: Cost Function	Screw Products: Production Function	Screw Products: Cost Function
(P : P)	- 5.4464	- 6.7146	- 4.0479	- 4.6572	- 3.8572	- 4.4787
(P : A)	- 0.1855	- 1.8227	- 0.6141	- 1.1290	- 0.4755	- 0.8476
(P : K)	1.0066	1.0031	0.9958	1.0007	1.0004	1.0019
(P : F)	0.6114	1.2411	- 0.0573	0.6960	0.2231	1.2190
(P : R)	0.7566	1.0140	0.9886	0.8652	0.9608	0.8672
(A : A)	-19.8700	-27.9120	-12.1600	-18.0470	-13.7610	-18.4640
(A : K)	1.0008	1.0003	1.0017	0.9843	1.0058	0.9983
(A : F)	0.2095	0.6478	- 0.2388	1.4159	0.4119	2.7149
(A : R)	0.8086	0.9098	1.0043	0.8123	0.9859	0.8662
(K : K)	- 2.3326	- 2.3143	- 3.1870	- 3.1847	- 2.8849	- 2.8798
(K : F)	1.0019	1.0035	0.9934	1.0033	1.0051	1.0062
(K : R)	0.9987	0.9992	0.9996	0.9995	1.0011	1.0010
(F : F)	-18.5140	-21.6130	-51.3020	-64.4680	-17.6010	-25.9490
(F : R)	0.8373	0.9321	0.9565	0.8745	0.9131	1.0517
(R : R)	- 0.9317	- 1.0413	- 0.9273	- 0.9356	- 1.0981	- 1.1171
(1 : 1)	0.7837	0.8017	0.5986	0.5825	0.4351	0.4288
(1 : 2)	- 1.0248	- 0.9743	- 1.0129	- 0.9872	- 1.0149	- 0.9852
(1 : 3)	- 1.0103	- 0.9903	--	--	--	--
(2 : 2)	7.2989	7.2976	1.7139	1.7167	2.3678	2.3322
(2 : 3)	- 1.0440	- 0.9594	--	--	--	--
(3 : 3)	2.2412	2.0820	--	--	--	--

[a]See the footnote (a) in the Table X-1-1, Chapter IV, Part B.

TABLE IV-2.--Partial Elasticity of Factor Substitution Estimated.[a]

A.E.S.	Knitted Underwears		Briquettes		Molding Industry	
	Production Function	Cost Function	Production Function	Cost Function	Production Function	Cost Function
(P : P)	1.0339	- 6.4498	-40.1170	- 0.9325	-14.2740	- 0.0722
(P : A)	-48.9560	- 3.2167	27.5310	- 0.0878	20.2420	- 0.0948
(P : K)	1.0006	1.0010	0.9908	1.8897	0.9916	0.2282
(P : F)	3.2290	- 0.4053	- 2.8720	0.0088	1.9751	0.0894
(P : R)	1.4827	0.8838	2.9719	0.7059	1.1527	0.5124
(A : A)	209.8600	-35.0280	-111.1400	- 0.9484	-106.0500	- 0.9788
(A : K)	0.9857	0.9924	0.9930	0.1900	1.0012	0.2268
(A : F)	1.5990	4.4876	- 1.3585	0.0324	0.7759	0.0665
(A : R)	1.6522	0.9208	2.2041	0.6010	1.1615	0.4762
(K : K)	- 3.3724	- 3.3725	- 4.3396	- 0.8115	- 3.3999	- 0.7738
(K : F)	1.0058	0.9983	0.9940	0.0142	1.0049	0.0754
(K : R)	1.0018	0.9987	0.9968	0.6935	0.9961	0.5550
(F : F)	-91.8780	-67.0230	-97.3260	- 1.0865	-13.9220	- 0.8445
(F : R)	1.2044	1.2537	2.2189	0.7764	1.0465	0.4577
(R : R)	- 0.8428	- 0.6812	- 0.7674	- 0.3073	- 0.8542	- 0.4603

[a]See the footnote (a) in the Table X-1-1, Chapter IV, Part B.

APPENDIX B-V

TABLE V-1.--Elasticity of Substitution Estimated: Canning Industry.[a]

	Production Function			
	A	B	C	D
(P : P)	- 5.2265	- 5.9661	- 5.1913	- 5.9653
(P : A)	- 0.1482	0.2393	- 0.1493	0.1396
(P : K)	0.9485	0.9402	0.9467	0.9678
(P : F)	0.7516	0.1148	0.7026	0.0474
(P : R)	0.7669	0.8793	0.7665	0.8781
(A : A)	-17.4930	-20.0070	-17.4500	-19.7030
(A : K)	0.7548	1.0259	0.7258	1.0506
(A : F)	- 0.1879	- 0.9732	- 0.0157	- 0.9248
(A : R)	0.8533	0.8512	0.8607	0.8615
(K : K)	- 2.1164	- 1.8139	- 2.1246	- 1.8133
(K : F)	1.1031	1.2799	1.0620	1.2962
(K : R)	0.9061	0.8040	0.9194	0.8011
(F : F)	-18.0010	-17.0420	-17.8890	-16.7680
(F : R)	0.8053	0.8379	0.8247	0.8515
(R : R)	- 0.9111	- 0.9330	- 0.9247	- 0.9458
	Cost Function			
(P : P)	- 6.2075	- 6.3487	- 6.1989	- 6.3667
(P : A)	- 2.1768	- 1.1662	- 2.0014	- 1.1506
(P : K)	0.7974	0.5422	0.8097	0.5230
(P : F)	1.7939	3.1030	1.7842	3.0736
(P : R)	1.0552	1.0186	1.0178	1.0118
(A : A)	-28.1500	-22.6630	-27.5350	-22.5740
(A : K)	0.7253	0.3404	0.7740	0.3345
(A : F)	0.3184	2.7251	0.8774	2.8923
(A : R)	0.9568	0.9654	0.9449	0.9707
(K : K)	- 2.3464	- 2.1006	- 2.3381	- 2.0933
(K : F)	0.7187	0.3852	0.7697	0.4038
(K : R)	0.8900	0.9754	0.8695	0.9771
(F : F)	-20.1600	-22.8070	-20.4790	-22.8720
(F : R)	0.9481	0.9936	0.9065	0.9729
(R : R)	- 1.0783	- 1.1758	- 1.0824	- 1.1782

[a]Set A (man-day workers : horsepower equipment); Set B (man-day workers : net capital stock); Set C (No. of workers : horsepower equipment); Set D (No. of workers : net capital stock); See the footnote (a) in the Table X-1-1, Chapter IV, Part B.

TABLE V-2.--Elasticity of Substitution Estimated: Leather Footwear Industry.[a]

	A	B	C	D
	Production Function			
(P : P)	- 4.2228	- 5.8606	- 4.1898	- 5.9699
(P : A)	- 0.8166	- 2.2873	- 0.9180	- 2.4500
(P : K)	1.1646	1.9197	1.1917	1.9897
(P : F)	0.3455	1.0485	0.2974	0.9745
(P : R)	0.8096	0.6865	0.8009	0.6937
(A : A)	-20.4400	-15.9750	-22.3200	-16.3200
(A : K)	1.5272	3.6744	1.5375	3.7796
(A : F)	1.1163	- 0.1459	1.0727	- 0.1022
(A : R)	0.8316	0.3770	0.8014	0.3918
(K : K)	- 2.4786	- 5.7787	- 2.4813	- 5.7769
(K : F)	0.4467	- 0.5880	0.5427	- 0.5701
(K : R)	0.8082	1.2477	0.8088	1.2262
(F : F)	-40.3870	-60.7050	-42.4200	-60.3890
(F : R)	0.7480	0.9413	0.7504	0.9415
(R : R)	- 0.7685	- 0.5828	- 0.7607	- 0.5772
	Cost Function			
(P : P)	- 4.9295	- 4.3092	- 4.9429	- 4.2616
(P : A)	- 6.5607	-12.9270	- 6.6702	-13.1930
(P : K)	0.3209	- 1.4373	0.3128	- 1.4337
(P : F)	0.6032	14.4470	0.4465	14.5510
(P : R)	1.0306	0.9274	1.0392	0.9256
(A : A)	-28.6790	-41.2250	-28.2560	-42.3390
(A : K)	0.2347	1.9161	0.1377	2.0893
(A : F)	6.8011	-30.2620	7.7002	-31.4920
(A : R)	1.0515	1.6330	1.0846	1.6290
(K : K)	- 2.7278	- 5.2466	- 2.7251	- 5.2692
(K : F)	0.2365	- 1.5883	0.2872	- 1.7114
(K : R)	1.0398	0.8766	1.0327	0.8644
(F : F)	-55.7320	-88.7540	-57.5150	-86.2190
(F : R)	0.8435	0.7047	0.8649	0.7294
(R : R)	- 0.9508	- 0.5855	- 0.9502	- 0.5855

[a]See the footnote in Table V-1, Appendix B.

TABLE V-3.--Elasticity of Substitution Estimated: Screw Product Industry.[a]

	Production Function			
	A	B	C	D
(P : P)	- 3.4723	- 4.7685	- 3.4174	- 4.7602
(P : A)	- 0.3284	- 0.6814	0.3491	- 0.7757
(P : K)	0.7691	1.2943	0.7074	1.2770
(P : F)	0.2412	- 0.9874	0.3284	- 1.1575
(P : R)	0.9338	1.1136	0.9399	1.1073
(A : A)	-11.2220	-16.5530	-11.3220	-17.6660
(A : K)	0.3546	1.6234	0.4135	1.6861
(A : F)	1.0988	- 1.2560	1.0192	- 1.5598
(A : R)	0.9955	1.3899	0.9886	1.4379
(K : K)	- 2.7922	- 2.1921	- 2.7115	- 2.1548
(K : F)	0.8632	1.4086	0.7870	1.4613
(K : R)	1.0826	0.8164	1.0641	0.8185
(F : F)	-17.0810	-13.6730	-16.9720	-14.7270
(F : R)	0.8974	1.3614	0.9040	1.3999
(R : R)	- 1.1229	- 1.4809	- 1.1151	- 1.4635
	Cost Function			
(P : P)	- 4.3295	- 4.5618	- 4.3066	- 4.5387
(P : A)	- 1.5003	1.3427	- 1.4549	1.4984
(P : K)	- 0.0310	0.7130	- 0.0566	0.7206
(P : F)	2.0695	2.9893	2.0044	3.0594
(P : R)	0.9365	0.9377	0.9403	0.9469
(A : A)	-16.7410	-16.4090	-16.7280	-16.9830
(A : K)	- 0.1779	0.3784	- 0.1932	0.3140
(A : F)	4.6157	2.6018	4.4706	2.0312
(A : R)	0.9521	0.7374	0.9533	0.7109
(K : K)	- 3.0391	- 2.1274	- 3.0744	- 2.1244
(K : F)	- 0.1456	0.5790	- 0.0776	0.5978
(K : R)	0.8891	0.9683	0.8929	0.9688
(F : F)	-25.1320	-22.3000	-25.0550	-22.0170
(F : R)	1.0863	0.9155	1.0881	0.9097
(R : R)	- 1.1147	- 1.4693	- 1.1100	- 1.4700

[a]See the footnote in Table V-1, Appendix B.

TABLE V-4.--Elasticity of Substitution Estimated: Manufacture of Knitted Underwear.[a]

	Production Function			
	A	B	C	D
(P : P)	- 0.6540	-15.4760	1.8864	-14.9310
(P : A)	-55.4660	2.1888	-69.3430	6.8133
(P : K)	1.7747	3.3659	1.6541	3.7110
(P : F)	5.2598	7.5327	6.4901	11.0890
(P : R)	1.6732	3.6025	1.6804	2.8064
(A : A)	271.3300	-68.2990	337.0000	-77.3110
(A : K)	- 1.8209	1.7248	- 0.5644	1.3651
(A : F)	- 0.3817	- 5.4368	- 6.2888	-26.4190
(A : R)	2.2774	3.1576	2.4098	2.8584
(K : K)	- 3.8042	- 3.1778	- 3.8651	- 3.4564
(K : F)	0.4265	- 0.1246	0.3664	- 0.4765
(K : R)	1.1643	0.4403	1.1651	0.4768
(F : F)	-96.7760	-116.3200	-96.2110	-128.3600
(F : R)	1.1203	0.6417	1.0906	1.2856
(R : R)	- 0.9857	- 1.8726	- 0.9926	- 1.5869
	Cost Function			
(P : P)	- 6.3139	- 4.7331	- 6.2990	- 4.6459
(P : A)	- 2.6777	0.7884	- 2.5255	1.1806
(P : K)	0.0333	0.4845	0.0418	0.4176
(P : F)	0.0762	5.8002	0.2123	6.9653
(P : R)	0.8681	0.2090	0.8375	0.3938
(A : A)	-34.1600	-33.3320	-34.4600	-36.7950
(A : K)	- 0.4174	0.2236	- 0.4330	0.1785
(A : F)	8.9067	- 9.1790	8.4156	-18.4200
(A : R)	1.0282	0.9452	1.0276	1.2975
(K : K)	- 3.3146	- 2.4775	- 3.3067	- 2.4783
(K : F)	0.3473	0.5195	0.4672	0.4996
(K : R)	0.8978	1.1632	0.9351	1.1572
(F : F)	-69.1560	-122.1900	-70.7260	-122.4000
(F : R)	1.2623	1.3931	1.2441	1.6321
(R : R)	- 0.6968	- 0.9842	- 0.6973	- 0.9784

[a]See the footnote in Table V-1, Appendix B.

TABLE V-5.--Elasticity of Substitution Estimated: Manufacture of Briquettes.[a]

	Production Function			
	A	B	C	D
(P : P)	-37.2550	-85.1360	-36.2660	-79.4500
(P : A)	32.5300	- 8.9121	32.7820	-11.4560
(P : K)	- 0.7423	1.3426	- 0.8289	1.4961
(P : F)	- 0.5766	-10.4710	- 0.7154	-10.5580
(P : R)	2.7462	4.5347	2.6494	4.4236
(A : A)	-135.4900	-72.6910	-140.7500	-74.5320
(A : K)	2.2060	1.8974	2.6325	2.0971
(A : F)	- 2.8721	-17.6200	- 3.5722	-17.4980
(A : R)	2.6336	2.8157	2.7770	3.0118
(K : K)	- 4.2213	- 2.9283	- 4.2105	- 2.9379
(K : F)	0.2974	0.9239	0.4831	0.9893
(K : R)	1.0461	1.1500	1.0268	1.1379
(F : F)	-92.3590	-139.8700	-93.1240	-136.2000
(F : R)	2.0970	3.1485	2.1170	3.1440
(R : R)	- 0.7515	- 0.9254	- 0.7434	- 0.9355
	Cost Function			
(P : P)	-11.7880	-26.8720	-11.8470	-26.8830
(P : A)	- 0.2317	5.0023	- 0.5029	4.2466
(P : K)	- 1.0452	- 0.2867	- 0.9862	- 0.2634
(P : F)	2.2256	4.1361	2.2371	4.1811
(P : R)	1.0791	1.5135	1.1102	1.5755
(A : A)	-26.5230	-38.5340	-26.5730	-38.4210
(A : K)	- 0.4609	0.0372	- 0.3995	0.0447
(A : F)	3.0308	0.1037	3.1769	0.3962
(A : R)	0.9008	1.0134	0.9212	1.0674
(K : K)	- 4.9326	- 2.6136	- 4.8978	- 2.6142
(K : F)	- 0.3448	0.1565	- 0.2587	0.1532
(K : R)	0.8534	0.7321	0.8560	0.7292
(F : F)	-70.6660	-122.8900	-71.6140	-123.6200
(F : R)	1.1569	1.6846	1.1429	1.6847
(R : R)	- 0.4223	- 0.5271	- 0.4230	- 0.5270

[a]See the footnote in Table V-1, Appendix B.

TABLE V-6.--Elasticity of Substitution Estimated: Molding Industry.[a]

	Production Function			
	A	B	C	D
(P : P)	-15.6390	-24.3890	-15.7890	-24.3400
(P : A)	24.8740	19.5130	25.1900	20.8510
(P : K)	1.4944	1.1750	1.5464	1.1148
(P : F)	1.9551	0.6025	1.9786	0.5240
(P : R)	1.1026	1.7042	1.0940	1.6570
(A : A)	-130.3600	-109.9300	-132.3100	-110.0000
(A : K)	1.4562	1.1189	1.5721	0.9488
(A : F)	1.0130	4.7376	0.9429	4.6474
(A : R)	1.1993	2.4523	1.2048	2.3529
(K : K)	- 3.6647	- 3.6754	- 3.6408	- 3.6650
(K : F)	0.4104	0.9045	0.3184	0.9169
(K : R)	0.9370	1.1464	0.9228	1.1588
(F : F)	-13.1180	-15.3890	-12.9580	-15.4010
(F : R)	1.1007	1.0614	1.1133	1.0736
(R : R)	- 0.7942	- 0.9503	- 0.7890	- 0.9449
	Cost Function			
(P : P)	- 7.0575	-11.3050	- 7.0997	-11.3770
(P : A)	- 2.4180	- 0.6748	- 2.5011	- 0.6597
(P : K)	0.5434	0.9782	0.5726	0.9943
(P : F)	1.6438	1.5224	1.5764	1.4385
(P : R)	0.9455	0.9576	0.9339	0.9635
(A : A)	-30.4150	-26.8180	-30.3500	-27.0470
(A : K)	0.3168	0.8998	0.3498	0.8987
(A : F)	1.4818	2.6690	1.5088	2.5595
(A : R)	0.8896	0.9179	0.8894	0.9047
(K : K)	- 4.0058	- 3.3610	- 4.0088	- 3.3576
(K : F)	0.1094	0.7355	0.1006	0.7475
(K : R)	0.8206	0.7444	0.8280	0.7401
(F : F)	-10.8730	-10.7060	-10.7940	-10.7430
(F : R)	0.9596	0.5789	0.9653	0.5955
(R : R)	- 0.7778	- 0.7449	- 0.7770	- 0.7430

[a]See the footnote in Table V-1, Appendix B.

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