UNPACKING MATHEMATICS TASKS IN MIDDLE SCHOOL CLASSROOMS

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ABSTRACT

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Mathematical tasks form the basic unit in instruction and learning in K-12 classrooms. Research indicates that different math tasks elicit different levels and types of student thinking. Because this variety is viewed as a plus, curriculum designers have made efforts to develop different kinds of task. One kind of task, called for by researchers and evident in "Common Core" mathematical curricula, is the contextual task, which is designed to get students to connect disciplinary knowledge to real world contexts. Researchers have debated the effectiveness of contextual tasks, often because the problematic contextual tasks and teachers have difficulty in realizing the instructional intent of these tasks.

The purpose of this study is to examine this issue. Specifically, it addresses two questions: What do mathematics tasks look like as written in textbooks? What's the relationship between the context feature and the cognitive demand of contextual math tasks? It draws on a representative sample of math tasks across three different curriculum materials (i.e., textbooks) at three intermediate grade levels--sixth, seventh, and eighth to address these questions. The study focuses on two important categories of written math tasks: the first called "context," the second "cognitive demand.", the third "structure", and the forth "representation". The "context" category, in my

analysis, are further divided into three sub-categories. The set of variables, for context, are labelled "mathematization," "realism," and "necessity."

As reported in the text, I found considerable variation across the three curriculum materials in the way math tasks were written. Although the majority of the math tasks in all three curriculum materials were presented as contextual tasks, the importance of the role that context played in supporting the mathematical knowledge differed in two ways: In the first approach, the task was less complex, and thus afforded students less opportunity to think about the mathematics of the situation (e.g., a shopper adding up items he or she wants to buy). In the second, the context was more complex and thus increased the likelihood that students would connect the mathematics to the context (e.g., shopping with a limited budget). The correlation results suggested that the likelihood to mathematize and the necessity degree of the context are significantly correlated to the cognitive level of the task. The realism, however, is not significantly correlated to the cognitive level of the task.

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KEY TO ABBREVIATIONS

CCSS	Common Core State Standards
CMP	Connected Mathematics Project (Lappan et al., 2014)
CPM	College Preparatory Mathematics (CPM Educational Program)
TIMSS	The Trends in International Mathematics and Science Study
NAEP	National Assessment of Educational Progress
RME	Realistic Mathematics Education

CHAPTER 1

INTRODUCTION

In Merriam-Webster, *problem* is defined as “a proposition in mathematics or physics stating something to be done.” In mathematics education, a mathematics task is a set of problems or a single problem that draws student’s attention to certain mathematical ideas (Stein, Grover, & Henningsen, 1996). Tasks are the basic element in the subject of mathematics. Not only do teachers convey mathematical ideas and skills to students through math tasks, but also students build and practice math knowledge through tasks.

Different tasks require different levels and types of student thinking (Schraw, Dunkle, & Bendixen, 1995; Stein, Smith, Henningsen, & Silver, 2009). Students learn the most in class when the instructional tasks are of high cognitive demand and students have opportunities to explain their thinking and reasoning (Stein, Smith, Henningsen, & Silver, 2009). Empirical evidence also indicates that certain aspects of mathematics problems influence students’ math performance. For example, the external representations, context familiarity, and the match of number in the problem and in the problem content all affect students’ performance on solving these problems (Koedinger, & Nathan, 2004; Janassen, 2000; Baranes, Perry, & Stigler, 1989).

On the other hand, it has been found that teachers have difficulty in maintaining the math task as written when implementing the task (Stein and Smith, 1998; Stigler & Hiebert, 2004). Thus, to select and use suitable math tasks is the basis for making good use of the tasks in math education. Understanding the nature and features of math tasks will provide a foundation for selection and implementation of math tasks. The main purpose of this study is to answer the general question, "What are the features of math tasks as written in textbooks?"

In 1990s, the QUASAR project studied math tasks using the task framework they proposed (Stein and Smith, 1998). The task framework identified math tasks from three phases: written (math tasks initiated in curriculum material or any written form), teacher set up, and enacted. Findings from the QUASAR project suggested that, with the help of reform-based professional development, two-thirds of 144 teacher-selected written math tasks were high-level cognitive, with multiple strategies and multiple presentations. Half of the teachers maintained the features of math task during set up, but teachers are still not able to maintain the features of math task when enacting the task in class.

Today, with the wide adoption of Common Core State Standards (CCSS) in the country, there have been growing efforts in the math-education community to address the CCSS. Math tasks still go through the three phases as in the task framework: written, teacher set up, and enacted. Math tasks, however, involve different features. For example, the NSF-funded curriculums took different approaches to present the math content and support the student math-learning trajectory. A large amount of real-context problems are found in CCSS curriculums. Therefore, a study to investigate math tasks as written in different curriculums will provide information for teachers and educators better to understand the math tasks at the current time.

A Study of Math Tasks

Using the written phase of math tasks in the math task framework as conceptual support, this study will explore multiple aspects of written math tasks in different textbooks. The data includes math tasks from six selected grade 6^{th}-8^{th} textbooks from three curriculums.

I address and distinguish math tasks across the three curriculums: Connected Math Project (CMP3), Glencoe, and College Prep Math (CPM), as well as between two math content topics: linear relationship and proportional reasoning. Four aspects of math task are explored, including the context, structure, cognitive demand, and representation. To study the four aspects of math tasks at the same time, I present a thorough image of the approaches each curriculum took to portray and present the context through math tasks.

Existing studies suggested that the cognitive demand of math tasks are related to and are affected by other aspects, such as context, structure and representation. Several theories, such as Realistic Mathematics Education (RME) and Contextual Teaching and Learning (CTL) argued for situating mathematics in the context. Building upon those findings, this study explored the relationship between the context and the cognitive demand aspect of math task. The goal is to study further the particular aspects of the context that relate to and affect the cognitive demand of math tasks.

Rationale/Significance of the Study

Studying math tasks is important because it provides information to help teacher educators and professional development individuals understand the content to which they have access. It will also enable teacher educators to support teachers in offering students quality math tasks. Math tasks "form the basic treatment unit in classrooms" (Doyle, 1983, p. 163). Students spend the majority of their time in math class not only working on math tasks, but also in learning and practicing mathematical ideas and skills through math tasks.

In addition, this study analyzes math tasks in middle school textbooks, which will provide evidence so as to contribute to the field about textbook analysis at secondary school level. After reviewing studies on textbook analysis in the five major math education journals from before 1980 until 2012, Fan, Zhu, and Miao (2013) reported that the majority of textbook analysis studies are at elementary school level, noticeably fewer studies are at secondary level.

Furthermore, this study focused on math tasks in textbooks at grades 6, 7 and 8, which are the grades studies showed use textbooks mostly and most frequently. The evidence of math-task characteristics in textbooks from this study can enable teacher educators better to understand the textbooks teachers are using. Researchers reported that middle-grades (grades 6-8) math teachers used the most of textbooks and use the most frequently. Two-thirds of middle-grade math teachers were found to use at least three-fourths of the textbooks each year (Weiss et al, 2001). In the 1996 National Assessment of Educational Progress (NAEP), 75 per cent of participating eighth-grade students have math teachers who use textbook material everyday (Grouws and Smith, 2000). In the 2000 NAEP, 72 per cent of participating eighth-grade students did math problems from textbook every day (Braswell et al, 2001). Such evidence indicates that math textbooks were highly relied on in middle-grades math teaching and learning. The quality of math tasks in textbooks plays a vital role in middle-grades students' math learning. To analyze math tasks in textbooks at these grades where textbooks are highly used is necessary and meaningful.

Finally, this study examined different aspects of math tasks at the same time, which I hope will contribute to the knowledge of math tasks. Literature review showed that many studies have examined cognitive level of math tasks. Fewer studies focused on the contextual aspect of math task. Few studies have examined multiple aspects of math tasks at the same time. What will we learn about math tasks were we to study different aspects of math tasks at the same time, such as the context, structure, cognitive demand, and representation. Thus, studying the context and cognitive demand together will provide the field a better image of what math tasks look like in current curriculums.

It is reasonable and necessary to explore the relationship between context and cognitive demand due to the increasing emphasis of contextual and high-level math tasks in the math education field. Both real-life context and cognitive demand have gradually become two essential and common aspects of math tasks in current curriculums as well as in classroom teaching. In the United States, textbooks are found to use real-life context a great deal, especially the reform-based textbooks. In this study, I want to explore whether or not the real-life context and cognitive demand aspect mutually support each other in one math task, or whether they conflict. What are some types of context that are better than others in increasing cognitive demand?

The approach to exploring real-life context in this study also meets the claim from researchers and has significance. Since the use of contextual math tasks became one feature of curriculum materials, it has become less meaningful and helpful to distinguish and compare the amounts of contextual and abstract math tasks, in order to capture the characteristics of curriculum materials and teaching. Rather, researchers argued that “the consideration of more salient aspects of tasks that impact on their effectiveness” is more helpful (Beswick, 2011, p.367). This study took this approach to study different aspects of real-life context so as to capture the manner in which different curriculums embedded and made use of the real-life context.

Overview of Chapters

In the next chapter, I review the literature on math tasks from two perspectives: 1) the nature and the role of math tasks, and 2) math tasks in curriculum. I state the purpose of this study and research questions in a later session of Chapter 2.

Chapter 3 describes the design of the study. I first introduce the curriculum data used in this study. I then outline the analytic framework for analyzing the curriculum data. Because of the nested feature of math questions and math tasks in curriculums, the selection of coding unit and the strategy I used to analyze the curriculum data are presented as the last section of the chapter.

In Chapter 4, I present the results from curriculum data analysis. These results answered the research questions on features of math tasks in textbooks. Descriptive statistical results as well correlation test results are presented in this chapter. The results reveal that there are considerable variation across three curriculums (CMP3, CPM, and Glencoe) in the way each curriculum designs math tasks and utilizes real context with math.

Chapter 5 provides conclusion and discussion. I summarize results on features of math tasks as written in each curriculum. Further, I include in my argument the implications for research, curriculum design, and teacher education, and limitations and directions for future research.

CHAPTER 2

LITERATURE REVIEW

In this chapter, I first review literature on math task from two perspectives: (1) studies on math task, including the nature of math tasks and the role of math tasks; and (2) math tasks in curriculum. Then, I present the conceptual framework used for this study, followed by the purpose of the study and research questions.

Studies of Math Tasks

The Nature of Math Tasks

To investigate math tasks, we first need to identify what math task is. Multiple words and phrases are used to refer to the same activity, e.g., problem, (instructional) task, and activity. "A *problem* is an unknown entity in some situation", and there is some "social, cultural, or intellectual value" in finding the answer (Jonassen, 2000, p.65). An *instructional task* is an academic activity in which teachers and students engage during classroom instruction to develop and perform certain concept, knowledge, and skills (Doyle, 1983). In this study, I will use the word *task*, in particular, *math task* to refer to the instructional tasks presented both in textbook and classroom teaching.

As commonly known, tasks differ in nature. Jonassen (2000) categorized eleven types of problems from six perspectives: learning activity, inputs, success criteria, context, structuredness, and abstractness. Through cognitive task analysis of hundreds of sample problems, he argued that all types of problems vary in terms of the structure, complexity, and situatedness. For example, he distinguished the structure of problem as either ill-structured or well-structured. The various types of tasks support, request, and engage the problem solver differently. For example, a memorization-oriented instructional task engaged the student's knowledge construction less than a constructive modeling-oriented task (Stein & Lane, 1996; Smith, Stein, Arbaugh, Brown, & Mossgrove, 2004). In math education, researchers examined various aspects of math tasks so as to explore the potential relationship between the nature of math task and the interaction of math teaching and learning. Below, I report on my review of studies on the nature of math task and categorize my review into four aspects: real-life context, structure, cognitive demand level, and representation of math task.

Real Context of Math Tasks. In general, math tasks are of two kinds: the abstract math task and the contextual math task. As opposed to an abstract math problem that has no context or situation, a contextual math task is a math problem that is described in words and embedded mathematical concepts and structures in real context (De Book, Verschaffel, Janssens, Van Dooren, & Claes, 2003; Boaler, 2008). Various types of math tasks are embedded in real context, such as word problems, story problems, math modeling and applications. Contextual math tasks are always viewed as connecting mathematics and real world. What counts as real context? What's the role of real context in math teaching and learning? What do we know about the real context? The following section reviews the literature that focuses on issues related to the above three questions.

Define real context. Various aspects have been brought in when defining real context. In modernist paradigms, real context math problems need to have high fidelity to “real-life situations”, which means there exists a “one-to-one matching or mapping models of the relationship between mathematical representations and ‘reality’” (Gerofsky, 2010, p. 63). In Realistic Mathematics Education (RME), mathematics must be connected to the reality. The realistic context can be real in the real world, or real in the student’s mind, which includes both real or “fantasy” situations. A “realistic” context in RME means that the context is “imaginable for the students” (Van den Heuvel-Panhuizen, 2005, p.2) for students. Thus, an imaginary situation or a fairy tale can be “realistic” context as long as they can be imaginable for students.

Bakhtin’s genre theory brought in the language and culture aspect of each individual student. The genre theory pointed out that all representations are generic, and are influenced by the individual’s environment. According to the genre theory, it is impossible to “represent ‘base-level’ reality” (Gerofsky, 2010). Scholars thus argued that the real context doesn’t necessary need to be a reproduction of the real situation. The real context is meaningful and real as long as the students’ social and cultural experiences could be connected when solving the contextual math tasks (Boaler, 1993B; Gerofsky, 2010).

In math education, the general understanding of real context includes connecting mathematics to real-world and students’ experiences. Under the name of real context connection, researchers have identified a range of contextual mathematical activities:

- Simple analogies, e.g., negative number and temperatures

- Word problems
- Real-data analysis
- Discussions of mathematics in a society, e.g., media misuses of statistics to sway public opinion
- Hands-on representations of mathematics concepts
- Real-world phenomena and math modeling (Gainsburg, 2008, p.20; Lee, 2012, p. 430)

Role of real context. Various arguments exist as to why the contextual problem is necessary and important in math learning. A well-known theory on this is the Realistic Mathematics Education (RME) developed by the Freudenthal Institute. "What humans have to learn is not mathematics as a closed system, but rather as an activity, the process of mathematizing reality and if possible even that of mathematizing mathematics" (Freudenthal, 1968, p. 7). In his view, mathematics must be connected to reality. Contextual Teaching and Learning (CTL) is a pedagogical system based on the philosophy that students can learn from what is meaningful to them and can be connected to their prior knowledge and experiences (Johnson, 2002). Berns and Erickson (2001) defined Contextual Teaching and Learning as follows:

> "Contextual Teaching and Learning is a conception of teaching and learning that helps teachers relate subject matter content to real world situations, and motivates students to make connections between knowledge and its applications to their lives as family members, citizens, and workers and engage in the hard work that learning requires."

In CTL, learning is viewed as a circular process wherein the student is familiar with the context, obtains the knowledge (e.g., concepts and skills), and then links the subject matter to the real world (Berns, & Erickson, 2001).

From the learners' perspective, scholars argued that "learners construct mathematical concepts if they are provided with concrete, familiar experiences" (Boaler, 1994, p. 555). Researchers argued that the use of contextual math tasks benefits student learning in various ways: by motivating students, by increasing student interest, by providing students image of the usefulness of school mathematics, by affecting student choice of math procedures, by supporting to build mathematical ideas, by applying math concepts and skills student have learned, and by helping students relate real world activity or phenomena to the use of academic math (Gravemeijer, 1997; Pierce, & Stacey, 2006; Reusser & Stebler, 1997; Boaler, 1993B; Zawojewski, 2013; Lesh & Zawojewski, 2007; Van den Heuvel-Panhuizen, 2005).

Acknowledging the positive influence real context has on student math learning, members of the math education community have made efforts to situate math in real context (e.g., Common Core State Standards-Mathematics). With the increasing amount of contextual math tasks in curriculum materials and classroom teaching, various aspects of the contextual math tasks gained the attention of researchers.

Aspects of real context. Different aspects of real context have been studied, such as types of the context, the authenticity of the context, and role of the context in the contextual task.

Types of the context. As defined in RME, a real context can be either a real or imaginary situation. Within the real situations, scholars categorized context into several types according to the different sources of the context: adult world, student world, community-based, general (Wernet, 2015; Boaler, 1993A, 1993B). The adult-world context involves situations relevant to adult life, such as house bills or business decisions, while the student-world context consists of events relevant to student life, such as school activities. The community-based context relates closer to the students' local community when compared to a general context. For example, a context about local gas prices is closer to students' local community than is general information about cars' fuel economy information.

Authenticity of the context. Scholars used different words to address the issue of the word "real" in real context. "Authenticity" and "realism" are the two most frequently used words in studying this aspect of real context. Palm (2006) defined the authenticity of real context as "the concordance between mathematical school tasks and situations in the real world beyond the mathematics classroom" (p. 43). Acknowledging the fact that context in word problem simulates the real situation but is never completely faithful to the reality, he proposed a framework about different aspects of the real context to be considered when simulating the real situation. The framework covers eight aspects of real context, including event, question, information/data, presentation, solution strategies, circumstances, solution requirements, and purpose. Palm argued that this framework is needed so as to develop realistic contextual math tasks in their "simulations" (Palm, 2006, p.44).

The authenticity of the context is believed to be important for the quality of contextual math tasks. Researchers found that the role of contextual math tasks in student math learning can be either positive or negative, depending on the authenticity of the context. For context that is simulated from real world situations, a well-designed authentic and realistic context has positive potential. A context that is in conflict with reality, however, impacted students' conception toward mathematics negatively.

When trying to solve real-context problems that conflict with reality, students tend to view mathematics as reasonable and useful in the classroom, while useless in real-life. Such "problematic modeling problem, which was 'solvable' only on the basis of problematic mathematical modeling assumptions" (Reusser & Stebler, 1997, p. 311) prevents students from really "using" mathematics. Further, it made students less aware of their existing realistic knowledge (Boaler, 1994, 2008; De Bock, Verschaffel, Janssens, Van Dooren, & Claes, 2003; Reusser & Stebler, 1997). Such phenomenon is called the "peculiar nature" (Greer, 1997, p. 294) of contextual math tasks in school.

Wyndhamn and Saljo (1997) tried to explain what might be the reasons to cause the "peculiar nature" of contextual tasks in school math. They pointed out that the design of a word problem is a process of "decontextualization" (Wyndhamn & Saljo, 1997, p.366) and "recontextualization" (Wyndhamn & Saljo, 1997, p.366). A daily-life situation is decontextualized into pieces of information, and then recontextualized within the mathematical world so as to involve certain mathematical relationships. During the process of de- and re-contextualization, the life situation changed. Such changes sometimes make the situation in word problems different from students' life experiences, which might cause the authentic and realistic aspects of word problems to become problematical (Wyndhamn & Saljo, 1997).

Although empirical evidence supports the importance of the authenticity of context used in contextual math tasks, studies also revealed that low authenticity context can play positive roles, and that high authenticity but unfamiliar context might be a challenge. Empirical evidence showed that imaginative contexts with low authenticity can be engaging for students (Nicol & Crespo, 2005; Weist, 2001, Wernet, 2015).

Studies raise such questions as, "on whose reality?" and "authentic to whom?" Some studies used the word "familiarity" to describe this challenge. "A constructivist perspective suggests that no one task context can offer a universal application which is familiar and, more importantly meaningful, for all students" (Boaler, 1993A, p. 14). The degree of familiarity of the situation to learners varies due to the learners' background, experiences, gender, age, and so on. Researchers pointed out that there is a relationship between the preference and familiarity of context to learners and learners' performance on the contextual problems. For example, Boaler (1994) explored gender preference when solving contextual math problems. She argued that female underachievement on mathematics is related to the use of contextual problems. In particular, the selection of contexts matters. In her study, Boaler gave the example of a male-preferred context (football) versus a female-preferred context (shopping) that result in different performance levels between male and female students.

Not only is there a variation among different learners, but also a difference between adults' and students' reality. For example, students solving a contextual math task on house bills or salary issues are contexts "extracted from the adult world" (Boaler, 1993A, p. 14). Cooper and Harries (2002) reported that students (second- to fifth-grade English students) were more willing and able to produce realistic responses when given "suitable realistic problems" (p.1).

Role of the context in contextual math task. Due to the different purposes that contextual math tasks have, the role of context in the contextual math tasks varies. In some tasks, the context serves as a cover story that is not "essential" (Van den Heuvel-Panhuizen, 2005, p.4) Which means change of the context would not affect students' ability to solve the contextual math task. Many word problems, especially traditional word problems, contain context that plays this role of cover story. In other tasks, the context ties closely to the mathematical ideas involved. The context plays vital role in the process of solving the contextual math tasks; for example, the task might offer opportunity for mathematizing, constraining math solutions, etc.

According to the different roles that context plays in the contextual math tasks, researchers studied the relationship between the context and the mathematical ideas involved in the contextual task. In De Lange's work (as cited in Van Den Heuvel-Panhuizen, 2005), he categorized three types of context based on the opportunity offered from the context for mathematization: first-order, second-order, and third-order. First-order context tasks only "involve the translation of textually packaged mathematical problems" (p.4), whereas second-order and third-order contexts offered opportunities to mathematize as well as validate. Wernet (2015) used "centrality" (p.16) to describe the degree to which a context is necessary for making sense of the math task.

Structure of math task. The structure of a math problem determines whether the problem is open-ended or closed. In Jonassen's (2000) categories, he defined the problem as well-structured or ill-structured. A well-structured problem is a closed problem that has given information and a close-end question, while an ill-structured problem is an open-ended problem. In textbooks and exams, it is common to see well-structured problems, such as word and story problems. A well-structured problem often presents well-organized information and asks students to use routine mathematical rules. Such well-structured contextual problems have been called "transformation problems (Greeno, 1978)". The well-designed feature of well-structured problems is one main reason that leads to students' stereotype of word problems and story problems. Later, I will review studies that specifically focused on word and story problems, and will discuss students' stereotype on those two types of well-structured problems.

On the other hand, ill-structured problems are problems that individuals encounter in their daily life. The information provided in an ill-structured problem is often less-organized, more or less than adequate, not commonly formatted, has no specific content domain, and might have multiple strategies and solutions (Jonnesson, 2000; Stein and Smith, 1998; Boaler, 2008; Van Dooren, Bock, Vleugels, & Verschaffel, 2011). Researchers also found that the structure of a contextual problem related to the cognitive process involved in the problem.

Representation of Math Task. The term *representation* refers to “both the internal and external manifestations of mathematical concepts” (Pape, & Tchoshanow, 2001, p. 118). An *internal* representation refers to the “abstractions of mathematical ideas or cognitive schemata” (Pape, & Tchoshanov, 2001, p. 119), and the *external* representations include all types of ways to present the math concepts, such as symbols, numerals, graphs, tables, diagrams, etc. In the NCTM (2000) standards, “the term ‘*representation*’ is used to refer both to process and to product… to the act of capturing a mathematical concept or relationship in some form and to the form itself” (p. 67).

Researchers studied the role of representation in math education. Studies showed that learners benefit in various ways from multiple representations in math learning. Learners, when they are provided the opportunities to transfer between different representations, or to discuss across multiple representations, develop the connection between multiple representations (Ainsworth, Bibby, & Wood, 2002; Brenner, et al, 1997). Students are able to improve their mathematical abilities as well as problem-solving and reasoning skills with the use of multiple representations (Presmeg, 1999; Greeno & Hall, 1997). Also, the use of multiple representations supports students’ further understanding of math concepts. For example, the use of external representations helped learners understand the math concepts (Pape, & Tchoshanow, 2001; Janvier, Girardon, & Morand, 1993). The way a math problem is represented also related to the way learners perceive the problem. In Koedinger and Nathan’s (2004) study, students regarded a simple algebra story problem easier than mathematically equivalent equations, because the external representation of a simple algebra story is easier for students to access.

In addition, scholars considered representation as an "inherently social activity" (Pape, & Tchoshanow, 2001, p. 120). They defined some of the students' mathematical abilities as "*representational thinking" (p.120)*. The representational thinking is "the learner's ability to interpret, construct, and operate (communicate) effectively with both forms of representations, external and internal, individually and within social situations" (p. 120).

Cognitive Demand of Mathematical Ideas. No matter for what purpose a math task is used; the mathematical ideas involved in the problem are the primary focus. Various categories and levels are identified to describe the cognitive demand of the mathematical ideas involved in math task. In this section, I review some well-known and widely used frameworks defined by scholars and by national and international assessment organizations. Those frameworks include Bloom's taxonomy, cognitive domains in math (TIMSS, 2015), mathematical complexity framework (NAEP, 2005), mathematics cognitive levels (Porter, 2006), Depth of Knowledge levels (Webb, 1997), and cognitive demand levels (Stein & Smith, 1998). All the frameworks reviewed in this section are intended to describe how students interact cognitively with the mathematics involved in the math tasks. Some frameworks focused on the cognitive complexity, others distinguished the cognitive demand levels.

Bloom's taxonomy was first published in 1956 with six categories and revised in 2001 by a group of cognitive psychologists and curriculum, instruction, and assessment researchers. The original Bloom's taxonomy consisted of "knowledge, comprehension, application, analysis, synthesis, and evaluation" (Paul, 1985, p. 36; Richard, 1985). The revised taxonomy involves six cognitive processes: "remember, understand, apply, analyze, evaluate, and create." Those six cognitive processes cover four types of knowledge: factual, conceptual, procedural, and metacognitive knowledge.

The Trends in International Mathematics and Science Study (TIMSS) identified three major cognitive domains in mathematics: *knowing*, *applying*, and *reasoning* (TIMSS, 2015, p. 25). *Knowing* refers to the familiarity to and fluency with math concepts. Six categories belong to knowing: recall, recognize, classify/order, compute, retrieve, and measure. *Applying* includes the application of mathematics, such as to determine, represent/model, and implement. The *Reasoning* category "mathematically involves logical, systematic thinking" (TIMSS, 2015, p. 27). Six categories under reasoning are termed as follows: analyze, integrate/synthesize, evaluate, draw conclusions, generalize, and justify (TIMSS, 2015).

The National Assessment of Educational Progress (NAEP) defined mathematical complexity framework for math items. The 2005 framework comprises three levels: low, moderate, and high complexity. Low complexity items are mainly "recall and recognition" with specification of what to do. Students can perform the procedure "mechanically" without being offered opportunities for "original method or solution" (NAEP, 2005). Moderate-complexity items involve "flexibility of thinking and choice among alternatives" (NAEP, 2005). The item has no specific direction on what to do and involve multiple steps to solve the problem. High-complexity items required student engagement in “abstract reasoning, planning, analysis, judgment, and creative thought” (NAEP, 2005).

Andrew Porter (2006) distinguished five mathematical cognitive levels in describing the way students interact with math tasks: “memorize; perform procedures; communicate understanding; solve non-routing problems; and conjecture, generalize, prove” (Porter, 2006, p.3). Norman Webb proposed the “Depth of Knowledge” (DOK) levels for four content areas: reading, writing, mathematics, and social studies. The mathematics DOK levels include “recall, skill/concept, strategic thinking, and extended thinking” (Webb, 2002, p.3). *Recall* includes the “recall of information such as fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula” (Webb, 2002, p.3). *Skill/Concept* involves some basic application of concept and skills. *Strategic thinking* requires “reasoning, planning, using evidence, and a higher level of thinking” than the recall and skill/concept (Webb, 2002, p.4). *Extended thinking* requires “complex reasoning, planning, developing, and thinking most likely over an extended period of time” (Webb, 2002, p.4). Stein and Smith (1998) proposed four cognitive demand levels to analyze math tasks, including memorization and procedure without connection as two low levels, and procedure with connection and doing math as two high levels.

The Role of Math Tasks

Math tasks play a role in students’ math learning, both in theory and in practice. Practically, due to the specialty of mathematics as a school discipline, students spent the majority of time in math class working on math tasks. In the TIMSS video study, students in all seven countries spent more than 80 percent of their time in class working on math tasks (Hiebert, 2003). Students learn and practice mathematical ideas and skills through math tasks.

Theoretically, researchers regarded math tasks as the basic element of math teaching and learning. “Tasks form the basic treatment unit in classrooms” for two reasons (Doyle, 1983, p. 163). First, a math task involves students with one or more mathematical concepts as well as mathematical ideas and skills. During the process of understanding and solving a math task, students develop and practice mathematical ideas both procedurally and conceptually. Second, a math task was designed to guide students’ learning by providing settings, in which students can choose approaches to perceive and process information and mathematical ideas. Students learn and practice mathematical concepts as well as ideas through solving math tasks that embedded the concept. Different math tasks create different learning opportunities for students. (Stein, Smith, Henningsen, & Silver, 2009; Hiebert & Wearne, 1993).

In general, empirical evidence indicated that students’ learning gains are the most when they are involved in solving high-level cognitive demand math tasks in classrooms, and the teacher maintain the high-level cognitive demand of math task in teaching (Hiebert & Wearne, 1993; Boston & Smith, 2009). Furthermore, evidence supported the association of original high level cognitive demand math tasks with moderate student learning gains, even when the tasks are enacted at a lower level. In other words, the high-level cognitive demand math task is better than low-level task in relation to student learning gains, no matter whether the task is enacted to maintain the cognitive demand level or not (Boston & Smith, 2009; Stein & Lane, 1996). In particular, Stein and Lane (1996) found that the greatest student learning gains related to the high cognitive level math tasks, “especially those that encouraged non-algorithmic forms of thinking associated with the doing of mathematics” (Stein & Lane, 1996, p. 74). Such findings illustrated the role of math tasks in students’ mathematics learning. Such evidence also responds to the call for high cognitive level math tasks in mathematics standards, such as NCTM principles and standards (2000) and Common Core State Standards (CCSS, 2010).

Across countries, a correlation was also found between student mathematics achievement and the high cognitive level math tasks they encountered. In TIMSS 1999 video study, students from countries that shown student performances higher than American students worked on high cognitive level math tasks in class. The math classes in those countries not only presented students with high cognitive level math tasks, but also maintained the high cognitive level of the math task during the teacher implement activity (Stigler & Hiebert, 2004). In America, teachers were found often to lower the cognitive level of a math task when implementing the task in class (Stein, Smith, Henningsen, & Silver, 2009; Boston & Smith, 2009). In the later section about teacher use of math tasks, I will review more studies about how teachers enact math tasks and what factors are related to teachers’ implementation of math tasks.

In sum, different math tasks provided different learning opportunities for students. Students’ development of mathematical thinking depended on the types of math tasks they worked on (Stein & Smith, 1998). High cognitive level math tasks offer more opportunities for students compare to low-level tasks in terms of thinking mathematically and conceptually.

Although to maintain the high cognitive level of a math task during teacher implementation is the goal for math instruction, empirical evidence indicated that starting with a high cognitive level math task always related to better students' learning gains than starting with a low-level math task, no matter whether teachers maintain the cognitive level of the task or not.

Math Tasks in Curriculum Materials

Curriculum material, in particular the textbook, is one of the main sources teachers use when planning and teaching a math lesson. Due to the different phases curriculums go through in teaching and learning, scholars distinguished the following types: intended, enacted, and assessed curriculum (Porter, 2006). Intended curriculum refers to the activities set by curriculum designers or administrators that are expected to achieve, such as textbooks. Textbook studies have been done for a long time from various perspectives, such as content and opportunity to learn, social justice issues, problem, etc.

Content analysis studies explored these different issues related to math curriculums, such as the alignment of content in textbooks and standards (Porter, 2006); and content analysis in curriculum and tests, etc. Studies focus on opportunities to learn unpack how the curriculum offers students learning opportunities. Those learning opportunities include not only those for various math content topics, such as geometry (Sears, & Chávez, 2014), arithmetic (Levin, 1998), statistics (Pickle, 2012), but also opportunities for mathematical practices, for example, reasoning and proof (Bieda, Ji, Drwencke & Picard, 2014; Otten, Gilbertson, Males, & Clark, 2014; Stylianides, 2009), and application (Wijaya, Heuvel-Panhuizen, Doorman, 2015).

Social-justice issues include culture, equity, gender, ethnicity, etc. In early days, researchers explored issues related to gender and equity in U.S. textbooks, such as the portrait of gender and minority-related topics (Garcia et al, 1990). Some studies focused on the culture characteristics reflected in the textbooks. For example, Lui and Leung (2012) compared the textbooks in Hongkong and Berlin and argued that the Hongkong textbook roots in the Confucian culture.

A third category of textbook studies focused on the math problem. These studies researched the sequence, types, amount, distribution of math problems, as well as various aspects of math problems, such as structure, cognitive demand, representation of the math problems (Zhu & Fan, 2006; Li, 2000; Jones, & Tarr, 2007; Sears, & Chávez, 2014; Stein, Grover, & Henningsen, 1996). Since this study aims to explore the different aspects of math tasks in textbooks from three current and widely used curriculums in U.S., I review in the following part the studies that address math tasks in textbooks in the following part.

Various studies have explored the aspects of math tasks in textbooks. Some studies focused on one aspect, e.g., Baker and his colleagues (2010), who examined the cognitive demand of math tasks in U.S. elementary math textbooks from 1900 to 2000. Other studies investigated multiple aspects of math tasks at the same time, e.g., the QUASAR project that looked at the cognitive demand and features of tasks in textbooks simultaneously (Stein, Grover, & Henningsen, 1996). To review the studies about math tasks in textbooks, I use the following four perspectives: (1) real-life connection in math task, (2) mathematical idea and practices in math task, (3) representation of math task and (4) cross-country comparison of math tasks.

Real-life Connection in Math Tasks

In general, there is an emphasis on real-life situations and the real-life situations are presented frequently in textbooks (Park & Leung, 2006, Alajmi, 2012). Alajmi (2012) analyzed fraction tasks in one U.S. elementary textbook and noticed that real-world problems were presented in textbooks beginning in first grade. But there is lack of clarity of the link between real-life situation and math concept (Park & Leung, 2006). Sears and Chavez (2014), however, found that in two U.S. high school geometry textbooks, the proof tasks are mainly abstract math tasks with no real context. Word problems are a common and widely used type of math task embedded in the real context. Many studies in the field have been specifically focused on word problems in textbooks.

Palm (2006) proposed a framework that described the elements that need to be considered when designing or evaluating a real-context problem, in particular a word problem. There are seven elements in Palm's (2006) framework: event, question, information/data, presentation, solution strategies, circumstances, solution requirements, and purpose. Five elements in the framework related to the context aspect of word problem, including event, question, information/data, solution requirements, and purpose. All of the five elements focused on the features of context mentioned in the previous section, such as the realism of the context, the meaning of context to learners, etc. Three elements focused on the cognitive-demand aspect of word problems: question, information/data, and solution strategies. Question, information/data, and circumstances are the three elements related to the structure aspect of a problem.

Using Palm's (2006) framework, researchers from different countries examined the nature of word problems in mathematics textbooks. Depaepe et al. (2009) examined all word problems in sixth-grade math textbooks in Beligum as well as word problems that teachers actually used in classrooms. Gkoris et al. (2013) examined all word problems in the fifth-grade national math textbooks of Greece. Both studies found that the context of word problems has improved in terms of relating to students' interest and experience, compared to findings of earlier studies.

The purpose and solution requirements of word problems, however, are found to be problematic. The authors claimed that most word problems in textbooks are easy, straightforward, and routine, containing few challenges for students; most of the word problems can be solved by straightforward routine using all numbers given. Therefore, the purpose of solving the word problems in textbook is different from the purpose of solving problems in a corresponding situation in real life. Students can hardly transfer or generate the knowledge and skills from solving word problems to solving real-life problems (Depaepe et al., 2009; Gkoris et al., 2013).

Mathematical Idea and Practices in Math Tasks

The math curriculum in United States was described as "a mile wide and an inch deep" (Cogan & Schmidt, 1999, p. 1). As the major carrier of mathematical ideas and practices, how well does a math task in textbook convey mathematical ideas to students or develop students' mathematical practices skills occupied the main body of studies on math tasks in textbooks. In general, researchers argued that the mathematical ideas conveyed in math tasks in US textbooks are of lower cognitive level, and the textbooks could increase the challenge as well as the depth of math knowledge for students. In the domain of arithmetic and algebra, math problems in textbooks were found to require mainly lower-cognitive level. For example, math problems on complex numbers and decimals in Grade 3 through 6 textbooks were found mainly to require recall and reproduction (Nicely, 1985; Nicely, 1986).

A study on math tasks in textbooks on fractions and divisions showed that fractions and division were not connected, neither were fractions found to be defined as division (Levin, 1998). Li (2000) analyzed math problems on integer addition and subtraction that has no solution or answers presented in textbooks. He found that most problems in textbooks require answers with 19% of them requiring students' explanations and reasoning. He also reported that 63% of the problems require procedural practice while only 26% of the problems involved conceptual understanding.

In geometry domain, Sears and Chavez (2014) analyzed the nature of proof tasks in two US high school geometry textbooks. They found that in one textbook, half of the proof tasks are of high cognitive level (procedure with connections) while in another textbook half of the proof tasks are of low cognitive level (memorization).

In statistics and probability domain, although there is a similar trend as in other math domains to contain a large percentage of lower-cognitive level math tasks, some studies had encouraging results. For example, Jones and Tarr (2007) studied 6-8th grade textbooks from

1957-2004. The results showed that, although lower-cognitive demand math tasks occupied more than 80% of probability tasks in textbooks throughout the history, the high-cognitive level probability tasks in *Connected Mathematics* reached 59%.

Similar results were found about the mathematical practices involved in the math tasks in textbook. Studies showed that the opportunities for students to develop core mathematical practices in solving math tasks in textbook are lower than expected in the standards or called for by educators. By analyzing a total of 4,855 tasks in US math textbooks on reasoning-and-proving, Stylianides (2009) reported that more than 50% of the tasks provided no opportunity for students to use reasoning and proving, and only about 40% of them involved one or more opportunities. Based on Stylianides (2009) study, Bieda and her colleagues (2014) studied reasoning and proof tasks in US upper elementary math textbooks to examine the opportunities for students to learn and develop core mathematical practices: reasoning, argumentation, and proving. They found that the average percentage of tasks that offer reasoning-and-proving opportunities was 3.7%. Math tasks in US middle-grades textbooks were also found to require less steps compare to the Eastern textbooks. Zhu and Fan (2006) suggested that the US textbook could increase the challenge for students to solve math problems, such as require more multi-steps in the math problems.

Representations of Math Tasks

Visual representations as well as visual information are greatly used in U.S. textbooks (Zhu & Fan, 2006; Mayer et al., 1995). Nathan, Long, and Alibali (2002) analyzed textbooks and found that nine out of ten textbooks presented new topics with symbolic activities and involved story problems quite late in the chapter. The authors argued that to sequence symbolic representation before story problems in textbook, assumes that symbolic representations are more "accessible" to students than story problems (Koedinger & Nathan, 2004, p.130). Such findings aligned with learning science researchers' claim that "word problems are notoriously difficult to solve" (Cummins, Kintsch, Reusser, & Weimer, 1988, p.1).

Cross-country Comparison of Math Tasks in Textbooks

Scholars studied math problems in textbooks all around the world. In Eastern countries, problems in textbooks are found to focus on conveying the mathematical ideas, and are less focused on real-life situations (Park and Leung, 2006). For example, in selected Chinese textbooks for grades 7 and 8 more than 92% of math problems were irrelevant to real-world situations (Zhu & Fan, 2006). A high level of mathematical skill is required to solve the problems in textbook, and a depth of math knowledge is required as well. For example, in Japanese elementary math textbooks, the computational fluency is required at a high level in the tasks (Reys, et al., 1996). Also, the quadratic equations in Japanese textbooks used approaches that were both "algebraically and graphically" based (Whitburn, 1995). In selected grade 7 and 8 Chinese textbooks, challenging multiple-steps problems were found to be a large part of the math problems (Zhu & Fan, 2006). In grade 7 Japanese textbooks on addition and subtraction, 81% of the content was related to the procedure and explanation of the procedure of worked examples. In grade 7 and 8 Shanghai textbooks, which are one kind of Chinese textbooks, math tasks on linear function were found to require a high level of abstract conceptual understanding of the concept of linear function (Wang, Barmby, & Bolden, 2015).

In other Western countries, problems in textbook were found have similar features as in US textbooks. Evidence of lower cognitive-level math tasks were reported from various textbooks in different countries. In Australia, large numbers of problems in grade 8 math textbooks are found to be of low-level procedural complexity that lack reasoning (Vincent and Stacey, 2008). Even when the textbooks contained reasoning, it was mainly for the purpose of justification rather than "thinking tools" (Vincent & Stacey, 2008; Zhu & Fan, 2006, p. 637). In Queensland, Australia, the analysis of two selected grade 8 textbooks suggested that the majority of math tasks in the textbooks were procedural practice, with few tasks that supported reasoning and conceptual understanding (Dole & Shield, 2008). In England and Wales, 30 out of 51 textbooks or textbook series were found to present math problems in a traditional way (Breakell, 2001). The linear-function units in selected middle grades of textbooks in England require lower-level conceptual understanding. The English textbooks "constrained the structural aspect of understanding linear function due to a point-to-point view of function" (Wang, Barmby, & Bolden, 2015, p. 1).

Synthesis

In sum, math tasks have an important and essential role in math teaching and learning. Teachers and students communicate and develop mathematical ideas through math tasks. There is evidence both theoretically and empirically to identify aspects that constitute quality mathematics tasks. Such aspects include context, cognitive demand of mathematical ideas, structure, and representation. Although there are multiple definitions and level identifications for cognitive demand of math tasks, scholars all acknowledged that cognitive demand of math task is the essential and priority aspect when thinking about math tasks. Other aspects, including context, structure, and representation relate to and support the cognitive requirement of math tasks. Scholars argued that conflicts between context problems in school math and learner's life experiences created negative influences not only by influencing the students' image of mathematics as a science, but also by preventing students from thinking mathematically. Real context problems that are worth seeking answers to and have meaning for learners will enhance the transfer of knowledge and allow a deeper understanding of mathematics (Boaler, 1993A, 1993B; Verschaffel, 2002; Van den Heuvel-Panhuizen, Middleton, & Streefland, 1995; Van den Heuvel-Panhuizen, 2005). Open-structured or close-structured math problems are related to students' cognitive process when solving a problem. The weak structure of word problems in textbooks affect the quality of word problems, and lead to the stereotype of word problems. Word problems are viewed as being solved by applying routine procedure and using all numbers mentioned. The representation of math problem affects both the cognitive demand level of the problem and the accessibility when students perceive the problem.

Empirical evidence on math tasks in curriculum materials indicated that the four aspects (context, cognitive demand, structure, and representation) of math tasks vary among different textbooks. For example, the Connect Math Project (CMP) textbooks are found to involve more high cognitive-level task than low-level tasks (Cai, Nie, Moyer, & Wang, 2014), while half of the tasks in one geometry textbook used in U.S. high schools are of low cognitive level (Sears and Chavez, 2014). Additionally, studies also found that among the four aspects of math tasks, some aspects were better designed than others. For example, studies that focused on word problems in elementary textbooks indicate that, even though some of the word-problem contexts are meaningful, the structure of word problems are mostly easy, straightforward, and routine, and contain few challenges for students.

Review of comparison studies on math tasks in textbooks showed that there are similarities and differences of math tasks in Eastern and Western textbooks. The general characteristics of math tasks in U.S. textbooks are:

- Tasks emphasize real-life situations, but the link is weak between mathematical ideas and the real-life connection.
- The mathematical ideas conveyed in math tasks are of low cognitive level. Math tasks in American textbooks have less challenge and require a lower depth of math knowledge for students.
- The opportunity for students to develop mathematical practices from solving the math tasks is insufficient.
- Tasks contain many uses of visual representations and much visual information.

On the other hand, the Eastern math textbooks showed these characteristics of math tasks:

- An emphasis on conveying mathematical ideas from math tasks, rather than focusing on real-life situations.

- A high level of mathematical skills is required, as well as a deeper level of math knowledge covered.

Conceptual Framework

My review of the literature illustrated that researchers not only have identified various features of math tasks, but also have highlighted the role of each aspect of math tasks (e.g., context can either motivate learners' interest or support learners' development of mathematical ideas). Situated this study in the literature, I continue to explore features of math tasks in terms of four aspects: context, cognitive demand, structure, and representation, with a focus on math tasks as written in textbooks.

Many have studied written tasks in curriculum materials. Fan and his colleagues (2013) reviewed math-textbook studies published in five major math education journals from before 1980 until 2012. They categorized studies on textbook analysis into five themes: "content and topics, cognition and pedagogy, gender, ethnicity, equity, culture and value, comparison of different textbooks, and conceptualization and methodological matters" (Fan, Zhu, & Miao, 2013, p. 637). Within the textbook-analysis theme, this study builds upon the existing studies to explore written tasks in textbooks. This study also adds to the literature in the field not only by exploring the context, cognitive demand, structure, and representation aspects at the same time, but also by examining the relationship between context and cognitive demand of written math tasks.

To explore simultaneously the four aspects of the written task, this study adopts the first phase of the math task framework (MTF) proposed by Stein, Grover, and Henningsen (1996) as the conceptual framework (Figure 1). In the math task framework, a math task can be transformed between any two of the three phases in classroom instruction: task as written, task as teacher set up, and task implementation. In this study, I focus on the first phase of a math task: *task as written*, which is a math task appearing in the curriculum or other instruction resources. In this study, the task as written is a math task presented in textbook.

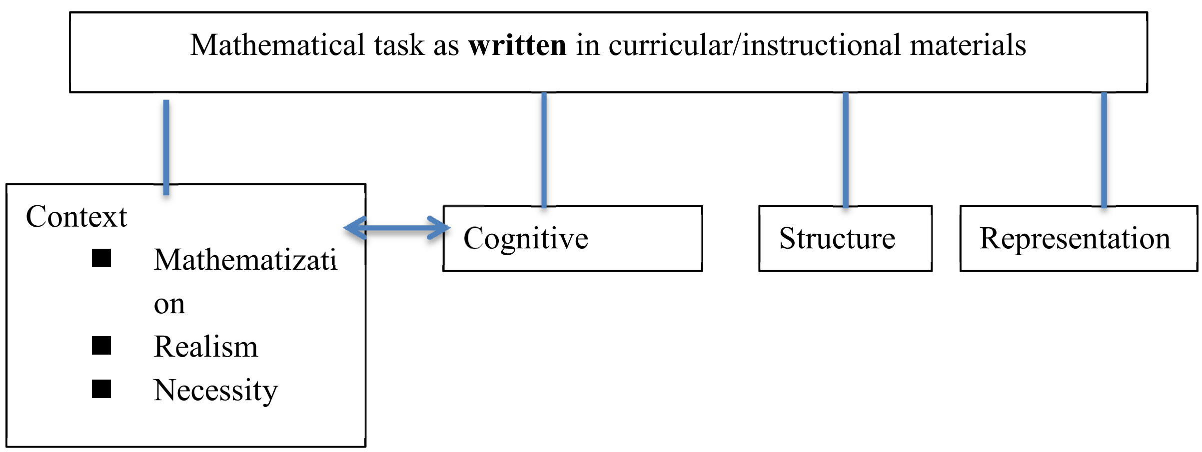

Figure 1. A conceptual framework shows aspects of math task as written and the relationship between context and cognitive demand. This is adopted from the Mathematical Task Framework (Stein, Grover, & Henningsen, 1996)

Regarding written math tasks in textbooks, I examine them from four aspects: context, structure, cognitive demand, and representation. I use the four cognitive demand levels (memorization, procedure without connection, procedure with connection, and doing math) proposed by Stein, Grover, and Henningsen (1996) to examine the cognitive demand aspect of written math tasks. The structure and representation aspects were explored using the analytic rubric develop based on the literatures. Based on literature review, I categorized three sub-aspects under the context aspect. The three sub-aspects are *mathematization*, *realism*, and *necessity*.

In the Merriam-Webster dictionary, *mathematize* is defined as “the process to reduction to mathematical form”. In math teaching and learning, Freudenthal (1968) introduced the idea of learning math should be a process of mathematize context or even mathematizing the math. De Lange (1979) distinguished three types of context according to the opportunities to mathematize offered in the context (as cited in Van Den Heuvel-Panhuizen, 2005). In this study, I use the word “mathematization” refer to the activity of the mathematical question arising from the context. Thus, to explore the mathematizing aspect of context is to examine the degree to which the mathematical question arises naturally (likely) from the context when the context happens in daily life. For example, the math question of shopping with a budget arises naturally in a daily-life shopping context. On the other hand, a math question such as determining whether or not the total cost and the unit price form a linear relationship in a shopping context, is less likely to arise in real life.

Realism is “the quality or fact of representing a person, thing, or situation accurately or in a way that is true to life” (Merriam-Webster). Some scholars used the word “authenticity” to refer to the same issue. Palm (2006) defined authenticity as the “the concordance between mathematical school tasks and situations in the real-world beyond the mathematics classroom” (p. 43). Both authenticity and realism focus on the fidelity issue between the real world “simulations” (Palm, 2006, p. 44) and the real world. Realistic Mathematics Education (RME) used the word “realistic” to define a broad range of being “real”: real in the real world or real in learners’ mind (Van Den

Heuvel-Panhuizen, 2005). This definition of realistic context includes both real and imaginary context. In this study, I use *realism* to include both the dictionary definition and the definition proposed by Palm (2006). *Realism* focused on the "concordance" (Palm, 2006, p. 43) between the context and the real world, as well as "a way that is true to life" (Merriam-Webster). For example, an advertisement in a pizza restaurant is a real context. But if the restaurant advertised 12 large pizzas for $180, the information provided in this context would not align with the real situation. Also, the realism of imaginary context is counted as being in conflict with real life.

Necessity is "the quality or state of being in need" (Merriam-Webster). Wernet (2015) studied the neediness of context and defined "centrality" (p.73) as the degree to which the context is necessary for making sense of math task. According to the different attention needed to the context, Wernet (2015) distinguished three levels: "(a) peripheral—the context was unnecessary in making sense of and solving the task; (b) helpful but not necessary in making sense of the task; or (c) necessary in making sense of the task" (p. 73-74). De Lange's (1979) opportunities to mathematize context involved the idea of the extent to which the context is needed to mathematize (as cited in Van Den Heuvel-Panhuizen, 2005). I use the word "necessity" in this study to identify the extent to which a context is needed to assist the mathematical ideas. A context is needed to assist the mathematical ideas in multiple ways, including support and provoke mathematical model when mathematizing the context, and to understand the math solution when validating and interpreting the math solution in the context. Take shopping context as an example. The context of shopping with a budget is more needed to assist the mathematical ideas than shopping without a budget. Although both contexts support the idea of constructing addition, the context of shopping with a budget not only limits the math solutions, but also involves some extra mathematical ideas (e.g., comparison of the total cost with the budget) as well as the understanding of how to solve the math question.

In addition to examining each aspect of the written task, this study also explores the relationship between sub-aspects of context and the cognitive demand of contextual math tasks. Several existing theories focus on linking the context and teaching and learning. The theoretical root of studying the relationship between context and cognitive demand in this study is the Realistic Mathematics Education (RME).

Realistic Mathematics Education (RME) is a mathematics subject-specific instructional theory (Treffers, 2012). In RME, math must connect to the context that is real to students, either real in the world or real in students' imagination. Six core principles are proposed and revised by Treffers and others, including the "activity, reality, level, intertwinement, interactivity, and guidance principle" (Van den Heuvel-Panhuizen, & Drijvers, 2014, p.522-523). The reality principle refers to two points: (1) math education should develop students' competence to solve contextual problems with mathematics, and (2) math education should "start from problem situations that are meaningful to students, which offers them opportunities to attach meaning to the mathematical constructs they develop while solving problems" (Van den Heuvel-Panhuizen, & Drijvers, 2014, p.523).

The RME provides the theoretical basis for this study to explore the link between context and cognitive demand for two reasons. First, the RME is a mathematics-specific instruction theory. Written math tasks in textbooks serve as the main source of teachers' instruction tasks. Thus, the RME could be used as a basis to support the instructional tasks. Secondly, the RME argued for connecting math knowledge and context. Researchers found that the cognitive demand of a math tasks is related to various other aspects of the math task, such as structure and representation. Context, in particular certain type of context (e.g., a meaningful context and one worth seeking answer to) is also found to have impact on deepen students' understanding of mathematics. Building upon those arguments, this study further explores which aspect of context (mathematization, realism, and necessity) are related to the cognitive demand of the math task.

Purpose of the Study and Research Questions

The purpose of this study is twofold: (1) to investigate in middle school (grades 6^{th}-8^{th}) how math tasks are portrayed and presented as written in different textbooks and (2) how differently designed contexts are related to the cognitive demand of the contextual task. Regardless of the call for uniformity in curriculum elements such as real-life contextual problems, or multiple representations, by researchers and Common Core Standards, many curriculums enacted in schools have taken different approaches to present the content.

This study aims to provide information on how each curriculum embedded those curriculum elements to convey mathematical ideas. Thus, the study will contribute to understanding how different approaches to presenting content affect different types of curriculums. It must be kept in mind that each curriculum will be influenced by the curriculum writer's philosophy of how to support the math-learning trajectory of the student.

This study aims also to explore the relationship between aspects of context (mathematization, realism, and necessity) and the cognitive demand of contextual math tasks. Evidence of such relationship will inform teacher educators and curriculum designers on the ways to utilize context in math tasks.

To meet those purposes, the following research questions are investigated in this study:

1. What are the features of math tasks as written in textbooks in terms of context, cognitive demand, structure, and representation?
    - What are the features of math tasks in textbooks in general?
    - Across three curriculums what are the features of math tasks in textbooks under two focal math topics respectively: proportional reasoning and linear relationship?
    - Across two math topics, what are the features of math tasks in different curriculums?
2. What are the features of sequencing math tasks in different curriculums in terms of context, cognitive demand, structure, and representation?
3. What is the relationship between aspects of context (mathematization, realism, and necessity) and cognitive demand of contextual math task?

CHAPTER 3

METHODS

The purpose of this chapter is to provide the design of the study. In this chapter, I first identified the data selected in this study: textbooks from three curriculums. Then, I described how I determined the unit of analysis to code for the curriculum data, the coding framework for math questions, the coding process for curriculum data, and ways I decided to use to analyze the curriculum data.

Data Collection

The data for this study are curriculum materials - textbooks. Three curriculums are collected in this study: Connected Mathematics 3 (CMP3), College Preparatory Mathematics (CPM), and Glencoe Math. Various resources can be regarded as curriculum materials, such as textbooks, teacher guidance, assessment, etc. In order to explore math tasks as written, the curriculum data in this study will focus solely on textbooks. One reason is that textbooks are the resources of teaching and a bridge to connect intended curriculum and enacted curriculum (Houang & Schmidt, 2008; Foxman, 1999). Teacher guides or other curriculum materials mainly serve the purpose to support teachers' use of the curriculum. To answer the research questions, the data will be derived from math tasks in textbooks from the three curriculums: CMP3, CPM, and Glencoe Math. A total of seven chapters in seven textbooks from the three curriculums will be selected.

The Connected Mathematics 3 (CMP3) (Lappan, Phillips, Fey, &Friel, 2014)

- Comparing and Scaling: Ratios, Rates, Percents, and Proportions

- Moving Straight Ahead: Linear relationships

College Preparatory Mathematics (CPM)

- College Preparatory Mathematics (CPM) Algebra Connection. Chapter 7, Linear relationship
- CPM Core Connection: Course 1. Chapter 7, Rates and Operations
- CPM Core Connection: Course 2. Chapter 7, Proportions and percents

Glencoe Math (McGraw-Hill authors, 2012)

- Glencoe Math Course 3 Volume 1, Common Core. Chapter 3, Equations in Two Variables
- Glencoe Math Algebra 2. Chapter 2 Linear Relations and Functions.

Moreover, researchers classified math problems in textbooks based on their location and purpose as “text problems” and “exercise problems” (Li, 1999). Text problems are mainly for teacher-instruction purposes while exercise problems are those for students to work on (Li, 1999; Love & Pimm, 1996). In this study, the math tasks being analyzed in textbooks are “text problems”, excluding “exercise problems”. Table 1 is a summary of data, research questions, and data size.

Table 1. Data, research questions, and data size

<table>
<tr><th>Research question</th><th>Data</th><th>Data size</th></tr>
<tr><td>Q1: What are the features of math tasks as written in terms of context, cognitive demand level, structure, and representation?</td><td rowspan="2">CMP3: two textbooks
CPM: three chapters in three textbooks
Glencoe: two chapters in two textbooks</td><td>Question level: 741 questions
Task level: 344 tasks</td></tr>
<tr><td>Q2: What’s the feature of sequencing math tasks in each curriculum?</td><td>Lesson level: 59.5 lessons</td></tr>
</table>

Table 1 (cont'd)

Q3: What is the relationship between aspects of context (mathematization, realism, and necessity) and cognitive demand level of contextual math task?		Task level: 248 contextual tasks

Data Analysis

Coding Unit

Due to the structure of the curriculum, I coded at the finest grain size- the question level, but analyzed the curriculum data at three different sizes: question level, task level, and lesson level. A math task is identified as a problem that requires an answer from students (Stein, et al., 2009). In actual textbooks, a math task is not always one problem, but consists of multiple questions. For example, FreshFoods has oranges on sale at 10 for $2. For each part, find the unit rate. Be sure to label your answers with the proper units.

1. What is the cost per orange?

2. How many oranges can you buy for $1?   (CMP3, Comparing and Scaling, p.48)

To gain a detailed picture of the math tasks in each curriculum, the coding unit is each single math question. In other words, I coded question 1 and 2 separately in the above example. Since the three curriculums structured differently, I provide examples and explanations for how each curriculum are coded in the size of single question.

In CMP3, the textbook author labeled three to four problems in each unit with numerical names, such as problem 1.1, problem 1.2 in Unit 1. Within each problem, there are several sub-problems labeled with letters, such as A, B. Some sub-problems are one single question that students need to solve; other sub-problems contain multiple questions. All questions under the same numerically named problem are labeled consecutively (see Table 2 for example). Also, all single questions in the entire numerically named problem shared the same context or have situations that are present throughout the problem.

Table 2. Format of one unit in CMP3 textbook

Unit 1 Problem 1.1 A (1), (2) B (3), (4), (5) C (6), (7), (8), (9) D (10) E	 Problem 1.2 A (1) B (2), (3) C (4), (5), (6) D

From Table 2, one can count that there are a total of 11 questions in problem 1.1. According to my coding rule, the coding units for problem 1.1 are the 11 questions. Same rule applies to problem 1.2 ends with seven questions being coded. The same coding unit rule applies to CPM and Glencoe.

Secondly, as mentioned in the data collection section, the math questions being coded in textbooks are math questions designed for teachers to teach in class, excluding exercises and homework. Based on these two coding rules, a total of 741 math questions were coded. Table 3 shows a summary of questions coded in each curriculum.

Table 3. Total amount of questions coded in three curriculums

Curriculum	Textbook name	Math context topic	Amount of questions coded
CMP3	Comparing and Scaling	Proportional reasoning	118
	Moving Straight Ahead	Linear relationship	178

Table 3 (cont'd)

			Sub-total: 296
CPM	Core Connection: Course 1, Chapter 7	Proportional reasoning	126
	Core Connection: Course 2, Chapter 7	Proportional reasoning	84
	Algebra Connection Chapter 7	Linear relationship	57
			Sub-total: 267
Glencoe Math	Course 3, Volume 1, Chapter 3	Proportional reasoning	88
	Algebra 2, Chapter 2	Linear relationship	90
			Sub-total: 178
			Total: 741

Whether real context is used in a math question or not determines if a math question is a non-contextual question or a contextual question. Any math question that doesn't involve a real context is coded as a non-contextual question. A contextual question is a math question embedded in real context.

Among the 741 math questions in three curriculums, 30% are non-contextual questions while 70% are contextual math questions. The percentage of real-life contextual math questions in each curriculum various a little, from 64% to 77% (Table 4).

Table 4. Distribution of contextual and non-contextual math questions in each curriculum

	CMP3	CPM	Glencoe	Total
Non-contextual	68 (23%)	96 (36%)	55 (31%)	219 (30%)
Contextual	228 (77%)	171 (64%)	123 (69%)	522 (70%)
Total	296	267	178	741

Analytic Framework

As shown in the conceptual framework earlier, the analytic framework for math tasks includes two categories: context and cognitive demand.

The majority (70%) of math questions in these three curriculums are embedded in real-life context. The differences between the amount of real-life contextual math problems and abstract math problems in textbooks are not a significant factor to distinguish different types of curriculums. “The consideration of more salient aspects of tasks that impact on their effectiveness” is more helpful (Beswick, 2011, p.1). Thus, I developed three sub-categories under the context aim to capture features that reflects the different approaches each curriculum took when utilize real world context with mathematics.

Coding Framework for Context. Each curriculum took certain approach to design and utilize real-life context to support student math learning. One of the goals of this study is to analyze how real-life context are used and integrated in different curriculum. Thus, I developed the coding framework for real-life context aim to capture to what extent the real-life context and mathematical idea are related. Table 5 shows the outline of the code for real-life context.

Table 5. Outline of the codes for real-life context

Mathematization	Realism	Necessity
How likely will the problem in the math task rise from the situation if it in real life.	The extent to which the context situation is likely to happen in real life, the information provided in the context is aligned or conflicted with the real life experiences.	The extent to which the real-life context support, provoke, or integrate with the expected math idea or math problem.

Mathematization In realistic approach, contextual problems are used as a source for the learning process. Contextual problems support students both to construct and apply math concepts (Freudenthal, 1968, 1991; Marja van den Heuvel Panhuizen, 2001). In the 2009 NCTM standards, context problems are stated for learning purpose, such as access to the mathematics.

Scholars also proposed that students would learn math effectively through tasks that involve real world experience, especially students' familiar experiences (Boaler, 2008). Thus, to which degree a math question will raise naturally from the real world context is important for serving the purpose of contextual task help student learning. The mathematization category aims to examine the likelihood a math question will raise from the real context. Two likelihood levels are coded:

- Likely: the math question is likely to rise from the real context.
- Less likely: the math question is less likely to rise from the real context.

Realism Realism, or authenticity in some studies, is the most studied and debated features of real-life context in existing studies. In this study, *realism* focused on the "concordance" (Palm, 2006, p. 43) between the context and the real world, as well as "a way that is true to life" (Merriam-Webster dictionary). Two parts of a context are examined: the source of the real-context and the information provided in the real-context.

- Real: The real-life situation and data are aligned with the real-world situation, data, or reality.
- Mix: Either the real-life situation or the data provided in the situation are in conflict with the real-world.
- Conflict: The real-life situation and data are in conflict with the real-world situation, data, or reality.

Necessity. In this study, necessity is used to examine the role of real-context in the contextual math problem: how much is the context needed to assist the mathematical ideas. To distinguish the different degree to which the context is needed to assist the mathematical ideas in contextual math task, I code the *necessity* either higher-necessary or lower-necessary. When a real-context is needed to build mathematical model/ideas, or to select a suitable strategy to solve the math problem, or if the change of the real-context affects the selection of strategies or mathematical models/ideas, or the context helps or contains the math solutions, then the real-context is the higher-necessary. When a real-context is helpful, but not connected and needed to build mathematical model/ideas or to select strategy, or the real-context can be neglected or changed, then the real-context is lower-necessary.

The role of real-context differs based on the purpose and types of contextual problems. It is reasonable to assume that, although all curriculums involve a large amount of real-world problems, different curriculums use the real-context for different purposes.

To explore the necessity of real-context in a contextual problem can provide further information on the degree to which a real-context is designed for teacher and students to include or pay attention to in different curriculums for different purposes.

The table below shows the code, description of code, and examples for the three aspects of characterizing real-life context (Table 6).

Table 6. Code, description and example for characterizing real-life context

Code	Description	Example
Mathematization (focus on the question, how likely the question will rise in the situation)		

Table 6 (cont'd)

likely	The math question is likely to rise from the real context.	To determine your walking rate: line up ten meter sticks, end to end, in the hall of your school. Have a partner time your walk. Start at one end and walk the length of the ten meter sticks using your normal walking pace. What is your walking rate in meters per second? (CMP3, moving straight ahead, p. 9)
less likely	The math question is less likely to rise from the real context.	A theater company sells child and adult tickets. For every 1 child ticket it sells, 3 adult tickets are sold. Fill in the ratio table and graph according to this scenario. (Glencoe, XX, P.XXX)
Realism (authenticity of the context and realism of the information given in the context)		
Real	• The real-life situation and data are aligned with the real-world situation, data, or reality. • The situation is indeed from real-life situations or collected by students themselves from their daily lives (Zhu & Fan, 2006, p. 614). Or the situation is possible in real-life. • The situation can be simplified for the grade level.	Water freezes at 0°C, which is 32°F. Water boils at 100°C, which is 212°F. (CMP3, Moving straight ahead, p. 96) Notes: examples for "situation be simplified for the grade level": constant rate for human running, driving, and interest rate.
Mix	• The context is likely to happen in real life, but the information provided in the context is conflict with the reality. • Or, the context is not likely to happen in real life, although the information provided in the context align with the reality.	The pizza problem in CMP3.
Conflict	• The real-life situation and data are in conflict with the real-world situation, data, or reality. • The situations are fictitiously made by textbook authors. (Zhu & Fan, 2006, p. 614) • The situation will not likely to happen in real life.	In Ms. Chang's class, Emile found out that his walking rate is 2.5 meters per second. (CMP3, Moving straight ahead, p. 30)

Table 6 (cont'd)

Necessity		
Higher-necessary	• The real-life context is highly needed in order to solve the math problem, or to make sense of the math problem, or to build the mathematical mode/idea. The change of the real-context will affect the strategy to solve the math problem or the mathematical model/idea involved in the math task. • Students need to situate or interpret the math solution in the context. Or the context constrains the math solutions.	To determine your walking rate: line up ten meter sticks, end to end, in the hall of your school. Have a partner time your walk. Start at one end and walk the length of the ten meter sticks using your normal walking pace. A. What is your walking rate in meters per second? (CMP3, moving straight ahead, p. 9)
Lower-necessary	• The real-life context is helpful, but is less needed for solving the math problem or building the mathematical idea. The math problem can be solved without the real-life context in the problem. Or, the change of the real-life context will not affect the strategy to solve the math problem or the mathematical model/ideas involved in the math problem. • The context is merely serve as a "cover story". Students don't need to situate or relate the context after they build the math model.	Eg.1, A grocery store sells 6 oranges for $2. Assume that the cost of the oranges varies directly with the number of oranges. This situation can be represented by y=(1/3)x. Graph the equation. What is the cost per orange? (Glencoe Math) Eg. 2, If I went to Wegmans and three bags of pretzels cost $12.00, what is the unit price for a bag of pretzels? (this example is also low-necessary, because when students grab the math, they can use ratio to find the answer and don't need to situate the solution back to the context)

Characterizing the structure of the math task. I coded the structure of math tasks as open-structured and close-structured (Table 7). The structure of the math task is one basic element of a math task. Different structures allow students access to the math task differently and are related to the possibility of single strategy/solution or multiple strategies/solutions. It also scaffolds or supports students' construct, practice, or application of mathematical concept and skills differently, such as applying routine algorithms or inviting alternative strategies.

Table 7. Code, description and example of structure of the math task

Code	Description	Example
Open-structured	• No specific strategy is explicitly stated in the problem • The problems allow for multiple strategies and solutions	Consider the following pledge plans. In each equation, y is the amount pledged in dollars by each sponsor, and x is the distance walked in kilometers. Plan 1: y=5x-3 Plan 2: y=-x+6 Plan 3: y=2 A for each pledge plan: 1. What information does the equation give about the pledge plan? Does the plan make sense? (CMP3, moving straight ahead, p. 37)
Close-structured	• Presents well-organized information • Ask students to use routine mathematical rules or procedure. • Only one strategy, no multiple strategies and solutions	If I went to Wegmans and three bags of pretzels cost $12.00, what is the unit price for a bag of pretzels?

Characterizing the representation of the math task. I distinguished representation of the math task either as a single representation or as multiple representations. Multiple representations are claimed to support different learners' needs and preferences. Studies also show that one main feature of American math textbooks is the use of a large amount of visual representation. I examined representations in different curriculum in this study aim to see if the use of visual representation is still one main feature of American math textbooks.

Based on the trial coding, I found that in using multiple representations, there are different approaches to design math tasks. (See figure 2)

Example 1. A grocery store sells two varieties of granola trail mix: Tree Bark's Grossola granola and Nature Hugger's Oldee granola. Look at the table and graph; which brand is the better buy?

Cost of Tree Bark's Grossola Granola	
Ounces	Cost
5	$2
10	$4

Example 2. A theater company sells child and adult tickets. For every 1 child ticket it sells, 3 adult tickets are sold. Fill in the ratio table and graph according to this scenario.

Child Tickets Sold	Adult Tickets Sold

Figure 2 Examples of using multiple representations

In Example 1, the math task used multiple representations and the table is used as part of the problem that provides the given information for students to solve the problem. In Example 2, the table is used differently. The problem and given information are presented in single representation, but students are asked to solve the problem with the given information using a table and a graph. Both examples are coded as multiple representations. In order to capture these two different approaches of using multiple representations, I added a sub-code to each example. Example 1 is coded as multiple representations; problem and question; Example 2 is coded as multiple representations – text problem with question request for visual solutions. Table 8 shows the codes, description, and examples for characterizing the representation of math tasks.

Table 8. Codes, description, and examples of the representation of math tasks

<table>
<tr><th>Code</th><th>Description</th><th>Example</th></tr>
<tr><td>Single representation</td><td>The math task is presented with only one representation: words, numerical, math formulas, etc.
*If a table is presented in the problem, but the question asked students to fill the table without write an equation or anything else, then it counts as single representation.</td><td>If I went to Wegmans and three bags of pretzels cost $12.00, what is the unit price for a bag of pretzels?</td></tr>
<tr><td>Single with non-math picture</td><td>The math task is presented with only one representation. There is a non-math picture in the problem.</td><td>Problem in CPM: a math task about the speed of a rabbit and have a rabbit picture next to the task.</td></tr>
<tr><td>Multiple representations -Problem and question</td><td>The math task is presented with more than one representation.
The math task is presented with visual support, such as pictures, graph, table, etc</td><td>A grocery store sells two varieties of granola trail mix: Tree Bark’s Grossola Granola and Nature Hugger’s Oldee Granolee. Look at the table and graph; which brand is the better buy?
<table>
<tr><th colspan="2">Cost of Tree Bark’s Grossola Granola</th></tr>
<tr><td>Onces</td><td>Cost</td></tr>
<tr><td>5</td><td>$2.00</td></tr>
<tr><td>10</td><td>$4.00</td></tr>
</table></td></tr>
<tr><td>Multiple representations-text problem, question request for visual solution</td><td>The math task is presented with more than one representation
The problem is presented only with words, numerical, math formulas, etc.
Visual representations (table, graph, picture, etc.) are used only in the question part that students need to answer.</td><td>A theater company sells child and adult tickets. For every 1 child ticket it sells, 3 adult tickets are sold. Fill in the ratio table and graph according to this scenario. (Glencoe)
<table>
<tr><th>Child Tickets sold</th><th>Adult Tickets Sold</th></tr>
<tr><td></td><td></td></tr>
<tr><td></td><td></td></tr>
</table></td></tr>
</table>

Sorting cognitive demand of math task. Cognitive demand has been studies from various perspectives. In this study, I used the widely known cognitive demand level framework (Smith and Stein, 1998) to code the cognitive demand of math tasks. The cognitive demand level framework can be found in the appendix. Below I provided some examples coded at each cognitive demand level with some explanations.

Memorization. Example 3. Use the reasoning you applied in parts A to C to solve these proportions for the variable x. $\frac{8}{5} = \frac{32}{x}$ (CMP3, PR, problem1.4d1)

This is a memorization example because students are asked to practice what they memorized from previous learning. From the previous problems, students learned to set up ratios and use scaling to solve the proportion. There are worked examples and practices before this problem taught students how to set up ratios and scaling. In this problem, the ratio is already set up for student. Students are able to use the scaling to solve this proportion mentally (e.g., the scaling from 8 to 32, and 5 to x). There is no connection to the concept or the meaning of the concept when solving the problem.

Procedure without connection. Example 5. Solve the system y=-2x-2 and y=2x+5 by graphing? (Glencoe, LR, Lesson 7, p. 234)

Example 5 is coded as procedure without connection because students are basically applying routine procedure to find the answer. It is algorithmic and requires limited cognitive effort. There is no connection to the concept or the meaning of the concept involved in completing the problem.

Procedure with connection. Example 6. Max thinks that Mix A and Mix C are the same. Max says "They are both the most 'orangey' since the difference between the number of cups of water and the number of cups of concentrate is 1." Is Max's thinking correct? Explain. (CMP3, PR, 1.2b2, p. 11)

This is a procedure with connection problem. It requires students to connect to the context and thus understand the meaning of the most "orangey". It requires the understanding of the meaning of ratio comparison. It also requires students to explain their reasoning and justification.

Doing math. Example 7. In Mr. Chang's class, Emile found out that his walking rate is 2.5 meters per second. That is, Emile walks 2.5 meters every 1 second. When he gets home from school, he times his little brother Henri as Henri walks 100 meters. He figures out that Henri's walking rate is 1 meter per second. Henri walks 1 meter every second. Henri challenges Emile to a walking race. Because Emile's walking rate is faster, Emile gives Henri a 45-meter head start. Emile knows his brother would enjoy winning the race, but he does not want to make the race so short that it is obvious his brother will win.

Question A. How long should the race be so that Henri will win in a close race?

Example 7 is a doing math problem. It is non-algorithmic, not predictable, and no direction given for ways to solve the problem. It is open-structured with the possibility to use multiple strategies and have multiple solutions. The problem requires students to understand the meaning of various concepts, such as unit rate, linear relationship, and make connections between these math concepts. The problem also requires students to construct math from the real context and connect back to the real context.

In the cognitive demand level framework (Smith and Stein, 1998) for doing math, the last bullet states as “Require considerable cognitive effort and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required.” In this study, this rubric is not considered as how long the math task will take student to solve. In other words, I didn’t take into consideration about how long a math task will take students to solve as one dimension for coding doing math. A math task that is open, no direction or guidance for how to solve the problem, involve conceptual practice and connection, etc. For example, the example 7 might not take a long time for students to solve the math and get the math answer as soon as students construct the math from the real context. It is still a high cognitive level, doing math problem.

Analysis Process

The analysis process includes reliability coding, individual coding, and data analysis at three levels of analysis unit size. As mentioned in the previous part, there are different types of math tasks in terms of the structure of the math task. One type of math task is one single problem. Another type of math tasks comprise of various single questions nested under on task. To gain the most detailed information of math tasks, all the coding units in this study are the finest grain size, the math question. Both the reliability coding and individual coding coded at the question level. The following part describes the reliability coding as well as some challenges that results into some revises of the coding rubric during the reliability coding process.

Reliability Coding. To obtain reliability, a second-rater double coded 17% (N=126) of the total tasks (N=741). The second-rater is a math education doctoral student in the same university with experience in textbook analysis. In addition to the training session, the double coding process included two rounds before my individual coding of tasks and a mid-time check in after I coded half of the tasks (N=380). The goal of the two rounds of double coding before I started my individual coding is to obtain reliability. Thus, the second-rater and I double coded two completed lessons from each textbook on the two math context topics respectively. The IRR for the two rounds are both close to 0.8 (K_1=0.799, K_2=0.78). After I coded half of the tasks, we doubled coded 17 tasks to make sure I am still coding with the reliability. The value of Kappa for the IRR mid-time check in is 0.76. Since it was not significantly different from the first IRR value, I identified as my coding is still maintain reliability. Table 9 showed the amount of tasks and math topics that were double coded for the whole double coding process.

Table 9. Description of double coded tasks

Double code rounds	**Amount of tasks double coded**		
Training	10		
Round 1 (Proportional reasoning)	CMP3: 21	CPM:35	Glencoe: 7
Round 2 (Linear relationship)	CMP3: 22	CPM: 13	Glencoe: 11
Mid-time check in	CMP3: 5	CPM: 6	Glencoe: 6

Notes on Coding. There are several common patter tasks and contexts used in all three textbooks that were discussed and reached agreement on the codes.

First, in the linear relationship lessons, there are quite a lot tasks in all three textbooks asked students to “write an equation of the relationship between XXX and XXX”. For example, “write an equation of the relationship between the speed of the car and the distance it travels.” We agreed that such question is not likely to rise in a real situation. When in a real situation of discussion a car speed and how far it travels, people are less likely to write an equation of the speed and the distance, but likely to calculate or estimate the distance the car travels. One exception we thought people will likely to write an equation for the situation is when they want to make more than one predictions. Therefore, we decided to code questions like “write an equation of the relationship between XXX and XXX” as “less likely” to rise the math question in real situation, unless the situation is make more than one predictions.

Another common type of task is given one or multiple student solutions and reasoning to a contextual math problem. The question is “Is student X’s solution correct or not, explain.” Or “do you agree with student A?” Or “which one do you agree with? Student A or B?” In such cases, there are two situations involved: (1) the situation in the given contextual math problem, and (2) the situation in which asking the problem solver to decide which given student solution to agree with. We decided to code the second situation, not the first situation. Our codes is “it is likely to make decision on whether agree with someone’s solution or not.

Secondly, in the initial codes for “realism,” a situation can be coded as “real” if it is “simplified for the grade level.” In tasks in textbooks, this rubric applied to tasks that involved “a constant rate of running, driving, bank interest.” As we know it is hard to keep a constant rate when running, driving in real situation. The tasks examined in this study, however, are for intermediate grade level students (6th-8th grade) who haven't learned about changing rate. Therefore, the "constant rate" was simplified for the grade level and coded as "real."

Thirdly, there are some challenges to code the cognitive demand level for tasks that asked for explanation. Various types of language used in tasks to ask for explanation, such as: explain, explain your work, explain your answer, explain how you get the answer, and explain your reasoning. It is challenge to decide if the task ask for explanation that is beyond only procedure or not. We coded the tasks that are likely to ask for procedural explanation as "procedure without connection," while the tasks are likely to ask beyond procedural explanation as "procedure with connection." The following two examples show the different types of asking for explanation.

> Example 1: "Suppose each student walks 8 kilometers in the walkathon. How much money does each sponsor donate? Explain how you found your answer." (CMP3, Moving Straight Ahead, 1.3B1, p.13)
>
> Example 2: Is this relationship linear? Explain. (CMP3, Moving Straight Ahead, 1.3C3, p.13)

Example 1 asked for only procedural explanation, while example 2 asked for more than procedural explanation. In example 2, to explain if the relationship is linear or not needs to explain what counts as a linear relationship. In another words, the explanation involves conceptual explanation.

Last but not least, we added one sub-code for the code, “single,” in representation category. There are many tasks in all three textbooks only have single representation in the problem, neither do the question request students to use any other representations, such as table or graph. Students, however, need to reference information from multiple representations to solve the problem. In such cases, the task is coded as “single representation” with a sub-code “need information from multiple representations (NIFM)”. Below is one example of a task coded as “single representation – NIFM.”

Example 3: How can you determine if a relationship is linear from a table, a graph, or an equation? (CMP3, Move Straight Ahead, 1.3A4c, p.13)

Analysis at the Question Level. After coding all math questions, I analyzed at three different size levels from small to large: question level, task level, and lesson level. At the question level, quantitative analysis was conducted. Descriptive statistics and compare mean tests were used to capture detailed information of both context and cognitive demand features of math questions contained in each curriculum.

Analysis at the Task Level. The second level of analysis is at the task level. I aggregated all question level codes by the unit of “*task*”. Math task is identified as listed in the textbook. In each textbook, there is clear bound of a complete math task. At the task level, I conducted correlation test, ANOVA, and Tukey post-hot tests. The analysis at the task level mainly focused on the difference across three curriculums in terms of the context features and cognitive demand features. Also, the correlation between any aspect of the context feature and cognitive demand level of the math tasks was explored.

Analysis at the Lesson Level.

Characterize Math Tasks by Lesson. Last but not least, I analyzed math tasks at the lesson level. I aggregated the codes at the question level by the unit of "*lesson*". In this study, a "*lesson*" is defined according to the textbook designer's identification of one lesson in curriculum materials, not the actual teaching unit. The "*lesson*" in CMP3 was identified based on the block pacing chart in the teachers' guide. In CPM and Glencoe, the textbook has explicit title for each lesson. Using this aggregate rule, a total of 59 lessons were identified in three curricula. Table 10 shows the amount of lessons in each textbook.

Table 10. Amount of lesson in each textbook

Curriculum and math topic	Amount of lesson	Note
CMP3 LR	13.5	90-minutes per lesson
CMP3 PR	7	90-minutes per lesson
CPM LR	12	
CPM PR	10	
Glencoe LR	8	
Glencoe PR	9	

Within each lesson, I explored the patter of math tasks in terms of both context feature and cognitive demand feature. The goal is to provide features of math tasks within each lesson and across lessons in each curriculum. For example, I examined within and across lessons in CMP3, the pattern of context use.

Language Use. As a supplementary to the task feature analysis, I looked at the language use in each curriculum. Although all textbooks are designed for intermediate grade students (6^{th}-8^{th}), the language use in each textbook has some unique features. The purpose of exploring the language use in each curriculum is to provide extra evidence to the first research question about “the features of math tasks as written in textbooks”.

CHAPTER 4

CAPTURE FEATURES OF MATH TASKS AS WRITTEN IN TEXTBOOKS

The purpose of this study is to unpack math tasks as written in textbooks and as enacted by teachers in classrooms. In this chapter, I am reporting the findings that answer my first research question. The first research question is, "What are the features of math tasks as written in textbooks in terms of context, cognitive demand, structure, and representaiton?" In particular, "What are the features of math tasks as written across two math-content topics?" and "across three different curriculums?"

To gain a detailed picture of math tasks as written in textbooks, I coded each math task at the finest grain size: the question level. I then aggregated the codes from question level to task level. Last, I aggregated the codes by lesson and explored the pattern of math tasks within lesson and across lessons in each curriculum. I have organized the findings in this chapter according to the three grain-size levels: question level, task level, and lesson level.

The first section of this chapter presents results on the features of math questions in three curriculums: CMP3, CPM, and Glencoe. At the question level, a total of 741 math questions in three curriculums are coded from all categories: real-life context (mathematization, realism, necessity, completion) and cognitive demand (cognitive level, structure, representation). Quantitative analysis was conducted at the question level. The findings in the first section aim to answer the question: what are the context and cognitive demand features of math questions in each curriculum. In specific, three questions are explored focus on the context features of math questions.

- How likely each math question will rise from the real context in each curriculum, likely or less likely?
- How real is the real context used in each curriculum for each question? In other word, to what degree each real context aligns with the real world, real, mix, or conflict?
- To what degree is the real context needed to support the development and understanding of the mathematical ideas involved in the question, highly needed or low-level needed?

Three question focused on the cognitive demand features are answer as well, including

- What is the cognitive level of each math question according to the four-level: memorization, procedure without connection, procedure with connection, and doing math?
- What is the structure of each question in each curriculum, open-structured or close-structured?
- What representation does each question used in each curriculum, single representation, single representation with non-math picture, multiple representations?

As the answers to the above six questions, I first present the distribution of contextual and non-contextual questions across two math-content topics and across three curriculums. Following that, I present the features of contextual and non-contextual math questions. The results presented in the first section of this chapter not only answered the above six specific questions, but also provided information about the comparison across the three curriculums: CMP3, CPM, and Glencoe, together with the comparison between the two math content topics: linear relationship and proportional reasoning. The comparison of three curriculums mainly expected to answer the question: what types of math questions does each curriculum have in terms of the context and cognitive demand features. In particular, what approach does each curriculum take when utilize real context in math question? What are the cognitive level, structure, and representations does each curriculum tend to use in the lens of math question?

The comparison between the two math content topics focused on the similarities and differences on the way utilizing real context, structure, cognitive level, and representations. In particular, the results focused on the preferred way real context are utilized in linear relationship and proportional reasoning.

The second part of this chapter presents results on the context and cognitive demand features at the task level. A total of 344 tasks in three curriculums were analyzed, including 87 in CMP3, 151 in Glencoe, and 106 in CPM. Quantitative analysis was conducted as well at the task level to explore the ways each curriculum takes to design tasks in order to present the math content. The specific questions answered in this section include,

- What are the differences of the ways each curriculum utilizing real context with math tasks? In specific, what are the differences across three curriculums in the real context they use in terms of the mathematization likelihood, realism, and necessity?

- What are the differences across three curriculums in terms of the cognitive level, structure, and representations of the math tasks they use?
- What is the relationship between the context features and the cognitive demand features? In particular, is there any correlation between any of the context feature (mathematization, realism, and necessity) to the cognitive level of math task?

A synthesis is presented at the end of the first and the second part to offer a summary of the quantitative results.

The third part of this chapter reports the qualitative results at the lesson level, including the pattern of math task within one lesson and across lessons and the language use. At the lesson level, the results answered the second research question: what are the features of sequencing math tasks in difference curriculums in terms of context, cognitive demand, structure, and representation. In particular, the findings focus on what are the context use pattern and cognitive level pattern of math tasks within one lesson as well as across lessons in each curriculum. What are some features of language use within one lesson or across lessons in each curriculum?

At the end of this chapter is a summary of the results to all the questions mentioned above. This whole chapter aims to capture features of math tasks as written in different textbooks at question level, task level, and lesson level.

Findings at Question Level

Distribution of Contextual and Non-contextual Questions

As mentioned in the previous chapter, most of the math questions in all three curriculums are contextual questions (70%). Before presenting the findings of distribution of contextual and non-contextual questions across textbooks and math topics, I will first present the number of questions in each textbook under each math topic (see Table 11).

Table 11. Number of questions in each textbook under each math topic

	CPM3	CPM	Glencoe	Total
Linear Relationship (LR)	178	126	90	394
Proportional Reasoning (PR)	118	141	88	347
Total	296	267	178	741

No major difference is found in the distribution of contextual and non-contextual questions among different curriculum. There is, however, significant difference in the distribution of contextual and non-contextual questions between two math-context topics across three curricula ($p=0.000$). Overall, the linear-relationship questions comprise 44% non-contextual questions, while proportional-reasoning questions comprise only 13% (see Figure 2).

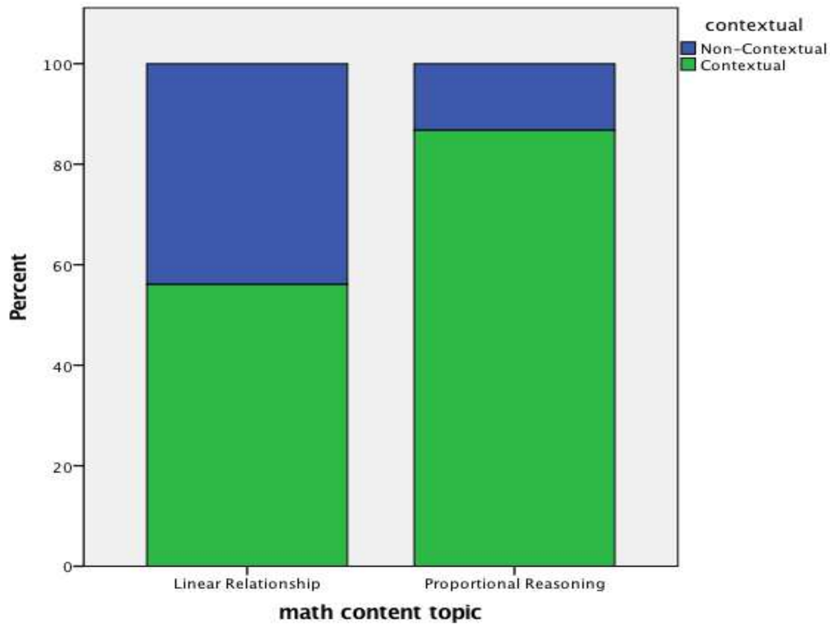

Figure 3.Percentage of contextual and non-contextual questions in all three curricula

Each curriculum contains more non-contextual linear relationship questions than proportional-reasoning questions. In another words, the distribution of contextual and non-contextual questions between linear relationship and proportional reasoning is consistent across three curricula (see Table 12).

Table 12. Percentage of non-contextual and contextual questions between two math topics

	Linear Relationship (LR)			Proportional Reasoning (PR)		
	CMP3	CPM	Glencoe	CMP3	CPM	Glencoe
Non-contextual	33%	58%	47%	8.5%	16%	15%
Contextual	67%	42%	53%	91.5%	84%	85%

In sum, the math-content topic plays a vital role in the distribution of non-contextual and contextual questions in textbooks. Of those questions about half are non-contextual linear-relationship questions and only about 13% are non-contextual proportional-reasoning questions.

Contextual and Non-contextual Math Questions

Both contextual and non-contextual math questions consist of the cognitive demand feature (cognitive level, structure, and representation). This section reports the findings of the three aspects of all questions from two perspectives: (1) across three curriculums, and (2) between two math topics. A comparison of the three aspects of contextual and non-contextual math questions follows.

Features of Math Questions Across Three Curricula: CMP3, CPM, and Glencoe.

Cognitive level. Overall, 60% of math questions in textbooks are low cognitive level questions and 40% are high cognitive level questions. Most of the questions are either procedure without connection (N=362 out of 741, 49%), or procedure with connection (N=247 out of 741, 33%). The memorization and doing-math questions are 10% (77 out of 741) and 7% (55 out of 741), respectively.

A big difference is evident between low-level versus high cognitive level questions across three curriculums (see Table 13).

Table 13. Percentage of cognitive level of questions in three curriculums

		CMP3	CPM	Glencoe
Low-level	Memorization	6 (2%)	12 (5%)	40 (22.5%)
	Procedure without connection	110 (37.2%)	153 (57%)	115 (64.5%)
High-level	Procedure with connection	141 (47.6%)	86 (32%)	23 (13%)
	Doing math	39 (13.2%)	16 (6%)	0 (0%)
	Total	296	267	178

From the above table, one can see that CMP3 has the most high-level questions (60.8%), while Glencoe has the least high-level questions (13%). CPM has 38% high-level questions and 63% low-level questions. For the highest level doing-math question, CMP3 has 13%, which is twice as many as CPM. No doing-math questions are found in Glencoe. In CMP3, the most common cognitive-level question is procedure with connection (high level, 47.6%), followed by the low-level question, procedure without connection (37.2%). Both CPM and Glencoe have more than half low-level questions: procedure without connection. The percentages are close, 57% and 59.5% respectively. The second most cognitive level question in CPM and Glencoe differed. The CPM contains high-level questions (32%), while the Glencoe has low-level questions (27.5%).

In sum, most of the questions in CMP3 and CPM are either procedure without connection or procedure with connection. The difference between CMP3 and CPM is that CMP3 has more high-level questions than low-level questions, while CPM is the opposite. Glencoe tends to involve low-level questions that are either procedure without connection or memorization. Only 13% high-level questions are found in Glencoe, and none is a doing-math question.

Structure. The structure of questions in CMP3 and CPM is similar and is significantly different from questions in Glencoe. Only four open-structured questions are found in Glencoe. Nearly all the other questions in Glencoe are close-structured (N=174 out of 178). In CMP3 and CPM, each contains approximately one-third of questions that are open-structured (see Table 14).

Table 14. Amount of open- or close-structured questions in textbooks

	CMP3	CPM	Glencoe
Open-structured	102	72	4
Close-structured	194	195	174
Total	296	267	178

Representation. The distribution of representation is similar across the three curriculums. All three curriculums use single representation the most. Both CPM and Glencoe use non-math pictures as support for certain single-representation questions. For example, in a question relating to a bicycle race in CPM, there is a picture of people racing bicycles with no mathematically related information in it. In CMP3, however, no non-math pictures are used. All pictures used in CMP3 to support questions involve mathematically related information. In each curriculum, about one-third of math questions use multiple representations to present the questions, such as table, graph, picture, etc. Two types of multiple representations are used in questions: (1) single-representation problem (text only) that asks for a multiple-representation solution, and (2) multiple-representation problem.

Features of Math Questions between Two Math Content Topics.

Cognitive level. The distribution of cognitive level between two math-content topics is similar. For both linear relationship and proportional reasoning, about half of the questions are procedure without connection and about one-third of the questions are procedure with connection. The percentages of memorization and doing-math questions are close, with a few more memorization questions than doing-math questions. Table 15 shows the amount of cognitive-level questions under the two math-content topics.

Table 15. Amount of cognitive level questions under the two math content topics

		LR	PR
Low-level	Memorization	40	33
	Procedure without connection	185	181
High-level	Procedure with connection	134	113
	Doing math	35	20
	Total	394	347

Structure. A significant difference of structure of questions exists between the two math-content topics (p=0.004). Among the math-content topics of LR and PR, there are more open-structured questions in linear relationship than in proportional reasoning. Some open-structured linear relationship examples are: (1) Describe two situations in Question A for which you could write more than one equation to represent the situation (CMP3, LR, 3.3A5, p.63), and (2) What do you notice about the graphs of each pair of equations? (CMP3, LR, 4.3C2, p. 94).

Representation. Representation of questions between the two math-content topics is significantly different (p=0.004). The use of text problem with question requesting multiple representations in linear relationship (N=63) is four times as many as the amount in proportional reasoning (N=16).

Comparison between Contextual and Non-contextual Questions.

Cognitive level. Overall, a significant difference (P=0.000) exists in cognitive level between contextual and non-contextual questions, there being more low-level non-contextual questions than contextual questions. The percentage of high-level contextual questions (48%) is double the non-contextual questions (23%). Figure 3 shows the percentage of cognitive level between contextual and non-contextual questions across three curricula.

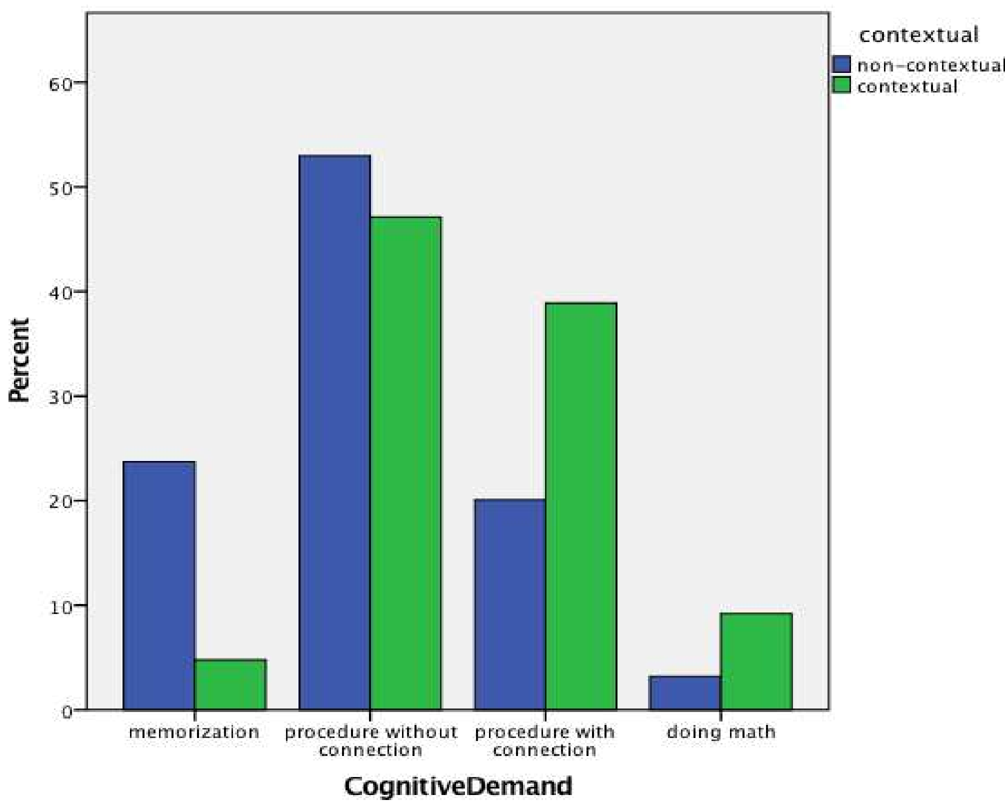

Figure 4. Percentage of cognitive demand level between contextual and non-contextual questions

When looking at the distribution of cognitive level within each curriculum, the distribution in CMP3 is different from CPM and Glencoe. In CPM3, two-thirds of contextual questions are high-level questions and two-thirds of non-contextual questions are low-level questions. In CPM and Glencoe, however, there are more low-level questions than high-level questions in both contextual and non-contextual questions. The percentage of low-level questions is close to the percentage of high-level questions in CPM, while in Glencoe, the percentage of low-level questions is much higher than high-level questions. The following table shows the percentage of cognitive level between contextual and non-contextual questions within each curriculum (see Table 16).

Table 16. Percentage of cognitive level between contextual and non-contextual questions within each curriculum

CMP3 (296 total questions)					
		Non-contextual		Contextual	
Low-level	Memorization	8.8%	**61.8%**	0	32.5%
	Procedure without connection	53%		32.5%	
High-level	Procedure with connection	33.8%	38.2%	51.8%	**67.5%**
	Doing math	**4.4%**		**15.7%**	
CPM (267 total questions)					
Low-level	Memorization	**10%**	**72.9%**	0.5%	**55.5%**
	Procedure without connection	**62.9%**		**55%**	
High-level	Procedure with connection	**22.9%**	**27.1%**	**37.4%**	**44.4%**
	Doing math	**4.2%**		**7%**	
Glencoe (178 total questions)					
Low-level	Memorization	**56.2%**	**100%**	12.8%	**81.3%**
	Procedure without connection	**43.8%**		**69.5%**	
High-level	Procedure with connection	**0**	**0**	**18.7%**	**18.7%**
	Doing math	**0**		**0**	

From the table shown above, one can see that the context aspect of a math question plays **a** role in the cognitive level of the question. Although there is very low correlation between contextual and cognitive level, there are slightly more high-level questions among contextual questions than among non-contextual questions. In Glencoe, all high-level questions are contextual questions. Below are some examples of questions that (1) use the same context, but involve different cognitive-level questions, or (2) are within the same context, but involve non-contextual and different cognitive-level questions. I use the examples to show how textbooks use different cognitive levels and context to support student math learning (see figure 4 and 5).

The same context and different cognitive-level questions. (The context of the following example is that two T-shirt companies sell T-shirts at a different rate and in different ways.)

Example 1.

A. Explain why the relationship between the cost and the number of T-shirts for each company is linear.

B. In each equation, what is the pattern of change between the two variables? That is, by how much does *C* change for every 1 unit THAT *n* increases? (CMP3, LR, 2.3A5a, 5b, p. 34)

Figure 5. Example of context use

In this example, both questions use the context of two T-shirt companies operating differently. To answer Question A, a high-level procedure-with-connection question is needed because students need to connect to the concept of "linear relationship" so as to determine whether or not the relationship is linear. To answer Question B, however, the student may use a low-level procedure-without-connection question because of the hint in the question. The hint provides information about how to find the pattern of change procedurally. Students can simply follow the direction given in the hint to find the answer.

Same lesson context and non-context questions. (The context of the following example is an imaginary situation, in which there is a specific way to pack coins in a pouch.)

Example 2.

Question A contains four situations of different combinations of pouch and coins pictures and ask students to figure out how many coins in a pouch.

5. Describe two situations in Question A for which you could write more than one equation to represent the situation.

B. Use your strategies from Question A to solve the equation and check your answer.

(1). $30=6+4x$

C. Describe a general method for solving equations using what you know about equality.

(CMP3, LR, 3.3 p 62-63)

Figure 6. Example of context use in the same lesson

These are examples of questions that are in the same lesson, involving both contextual and non-contextual questions. In the first part of the lesson, there is an imaginary situation of coins in the pouch, and students are expected to find how many coins are in each pouch. In the second part of the lesson, the context is moved. The non-contextual question is a mathematical abstract version of the context. Question 5 requires a high-level procedure doing a math question, while other questions need only low-level procedures without connection questions.

The above two examples from CMP3 are one way of designing questions with or without context to provide a variety of cognitive levels within one lesson. Across three curricula there are many other ways the curriculum designers used to design questions via the different approaches of using context to provide a variety of cognitive levels when presenting math content to students. I will discuss the topic in more detail in the later part of this chapter, which focuses on the qualitative analysis of textbooks.

Structure. The percentages of open- and close-structured questions are almost the same between contextual and non-contextual questions across three curricula. There are 24.5% and 22.8% open-structured questions in contextual and non-contextual questions, respectively. Within CMP3, however, there is a significant difference in the amount of open- and close-structured questions between contextual and non-contextual questions ($p=0.008$).

Table 17. Percentage of structure of questions in each curriculum

	CMP3		CPM		Glencoe	
	Non-contextual	Contextual	Non-contextual	Contextual	Non-contextual	Contextual
Open	22.1%	**38.2%**	33.3%	23.4%	5.5%	0.8%
Close	77.9%	61.8%	66.7%	76.6	94.5%	99.2%

Data in Table 17 indicate that the distribution of open- and close-structured questions in CMP3 is different from the amount in CPM and Glencoe. In CMP3, there are 15% more open-structured questions that are contextual problems than are non-contextual problems. The percentage, however, is the opposite in CPM and Glencoe. In particular, only 6.3% OF questions in Glencoe are open-structured, including both contextual and non-contextual problems. The percentage is almost one-third of the average percentage across three curriculums.

Various types of open-structured non-contextual questions are used. The student may be asked to solve the problem in another way, or to find two more points on the line, or to state what he or she notices about the XXX, or to look for approaches or write equations for given data, etc. One example in CPM is: Using the portions web shown at right, work with your team to find two other ways to write the equation $25 \times \frac{3}{5} = 15$. For example, one way might be $25 \times \frac{6}{10}$=15 (CPM, PR, 7.1.2, 7-18a). (CPM, PR, 7.1.2, 7-18a).

Representation. The majority of questions use only single representation, no matter in which curriculum (Table 18). In CMP3 and Glencoe, there is significant difference in distribution of representation between contextual and non-contextual questions. In both CMP3 and Glencoe, there are 13% more single-representation non-contextual questions than contextual questions, and 13% more multiple-representation contextual questions than non-contextual questions.

Table 18. Percentage of representation in each curriculum

	CMP3		CPM		Glencoe	
	Non-contextual	Contextual	Non-contextual	Contextual	Non-contextual	Contextual
Single	86.8%	73.2%	60.4%	58.5%	72.7%	49.6%
Single-non math picture	0	0	1%	4.1%	0	8.9%
Multiple-verbal problem	7.4%	7.9%	21.9%	10.5%	9.1%	9.8%
Multiple	5.9%	18.9%	16.7%	26.9%	18.2%	31.7%

The only non-contextual question that has a non-math picture in CPM is a math question linear-relationship problem. The problem asked students to use a graph calculator to find a trend line and equation from the previous question data. The non-math picture is a graph calculator.

In conclusion, a significant difference is found between contextual and non-contextual questions in CMP3 in terms of all the cognitive demand aspects: cognitive demand level, structure, and representation. The contextual questions in CMP3 are more likely than the non-contextual questions to be high-level, open, and use multiple representations.

In CPM, the contextual and non-contextual questions are significantly different in terms of cognitive demand level. For example, the percentage of high-level procedure-with-connection questions increased when there is a context within the question. Meanwhile, the percentage of low-level questions, such as procedure without connection, decreased when a context was added in the question.

As for Glencoe, the significant difference appears in two aspects: cognitive demand level and representation. Although the majority of questions in Glencoe--no matter contextual or non-contextual—are low-level questions, the majority of low-level contextual questions are procedure without connection, while more than half of the low-level non-contextual questions are memorization. When there is a context involved in the question, there is a 13% increase in the use of multiple representations and a 13% decrease in single-representation questions.

The distribution of questions is almost equal in terms of the cognitive demand aspect (cognitive level, structure, and representation) between the two math-context topics in each curriculum. Thus, no comparison results ARE reported here between contextual and non-contextual questions under the two math-context topics.

Contextual Questions

As mentioned in Chapter 3, the majority of math questions in all three curriculums embedded context. Thus, the difference between the amount of contextual and non-contextual questions is not as helpful and meaningful as comparing the "more salient aspect of questions that impact on their effectiveness" (Beswick, 2011, p.1). In this section, I will present the findings on the real-context aspect analysis.

Mathematization. Overall, 38.9% questions in all curricula are likely to have a mathematical question arise from the situation. The percentage in Glencoe, however, is only half of the average percentage (see Table 19).

Table 19. Distribution of questions in terms of mathematization in each curriculum

	CMP3	CPM	Glencoe
Likely	46%	43%	20%
Less likely	54%	57%	80%

Within CMP3 and CPM, the math context topic plays a significant role (p=0.000). In CMP3, significantly more proportional-reasoning questions contain a math question that is likely to arise from the situation. The percentage in CPM, however, is the reverse. Below is the table showing the distribution.

Table 20. Distribution of questions in terms of mathematization separated by math topics

	LR (Linear relationship)		PR (proportional reasoning)	
	Likely	Less likely	Likely	Less likely
CMP3	29%	71%	65%	35%
CPM	64%	36%	37%	63%
Glencoe	17%	83%	20%	80%

Although both CMP3 and CPM contain significantly different percentages of likely and less likely mathematization of contextual questions between the two math- context topics, the two curriculums differ. For the linear-relationship topic, IN CMP3 there is less than one-third likely-to-mathematize contextual questions. In CPM, the percentage of the same category is almost two-thirds (64%). The distribution of questions under the proportional reasoning, however, is the reverse. The CMP3 curriculum is 65% more likely to mathematize contextual questions in contrast to only 37% in CPM. In Glencoe, about 20% questions for both math topics are likely to raise the math question from the situation.

In the following section, I'll provide examples from each curriculum to show how a likely mathematization and a less-likely mathematization question would look. Three groups of examples are arranged in most-likely to least-likely order. Each group consists of three examples, one from each curriculum. Although the first two groups of examples are both coded as likely to have the math question arise from the situation, there are some subtle differences between them in terms of the degree of likelihood. I present the examples, followed by an explanation of what I observed in the example related to "How likely will the math question arise in the situation?"

Likely – high degree

Example 3. A customer wants to buy A minivan. Her budget is $23,000. The selling price plus 5% sales tax goes over the customer's budget. What maximum selling price can the customer afford? Explain. (CMP3, PR, 3.1A3d, p. 65)

Example 4. Find the unit price it costs $2 for eight juice boxes. (Glencoe, LR, Lesson 1, example 2, p. 11)

Example 5. Today is the final event of "The Big Race"! Your teacher will give you each a card that describes how you travel in the race. … Use your results from "The Big Race" to answer the following questions…. Be sure to justify each response. Who won "The Big Race"? Who came in last place? (CPM, LR, 7.2.3, 7-72a)

Figure 7. High degree likely to be mathematized examples

Examples 3-5 are to a high degree likely to be mathematized questions. Calculating the maximum price within a budget, finding the unit price, and determining who won the race are questions we will try to solve in real-life situations. Thus, it is comfortable for students to think about and want to solve the mathematical questions from reading the contexts. The use of real context in this approach plays the role of integrating mathematical ideas so as to support students' thinking mathematically. It aligns with the belief the math education community holds about the role of real-context in students' math learning, that is, "learners construct mathematical concepts if they are provided with concrete, familiar experiences" (Bolar, 1994, p. 555).

Likely – low degree

Example 6. Assume you continue to walk at this constant rate, how long would it take you to walk 500 meters? (CMP3, LR, 1.1B1, p. 9)

Example 7. Congratulations! The president of the Line Factory has presented your class with a special challenge: She now wants a way to find the equation of a line generated when a customer walks in front of a motion detector. …To impress the president, you have decided to reverse the process: Write instructions for a client on how to walk in front of the motion detector in order to create a graph for a given rule. (CPM, LR, 7.2.1, 7-54)

Example 8. Olivia bought 6 containers of yogurt for $7.68. Write an equation relating the cost *c* to the number of yogurts *y*. How much would Olivia pay for 10 yogurts at this same rate? (Glencoe, PR, Lesson 6, example 3, p.57)

Figure 8. Low degree likely to be mathematized examples

Examples 6-8 are low-degree likely-to-mathematize problems. On one hand, it is likely in reality that people will want to know how long would it take to walk a certain distance, how to walk so as to get a graph for a given rule, and how much a certain amount of yogurt would cost. On the other hand, it is less likely that people will think about and want to solve the mathematical question as posed in the examples. In example 6, it is more likely that in real life, people would estimate how long it will take to walk from one place to another place using common knowledge from experience, rather than knowing or using a constant walking rate.

For Example 7, if we disregarded the fact that a Line Factory doesn't exist in the real world and simply look at the situation and figure out how to walk a graph for a given rule, it is more likely that in that situation people will try to solve it by guess and check, rather than by using rigorous math knowledge and multiple representations.

Example 8 is similar to Example 7. It is likely that people would figure out the cost of 10 yogurts in real life via mental math or estimation, rather than via an equation. Thus, mathematical questions are likely to arise in such contexts, but chances are small that people will think about or solve it in real life in the way the question is presented. The mathematical questions or approaches presented in these contextual questions serve to target a certain mathematical concept or procedure. Therefore, the degree to which the mathematical questions will arise from the real situation is lower than from the first three examples.

Less likely

Example 9. Here are the walking rates that Gilberto, Alana, and Leanne found in their experiment. Graph the times and distances for the three students on the same coordinate axes. Use a different color for each student's data. How does the walking rate affect the graph? (CMP3, LR, 1.2A2, p. 10)

Example 10. Andrew earns $18 per hour for mowing lawns. Is the amount of money he earns proportional to the number of hours he spends mowing? Explain. (Glencoe, PR, Lesson 4, example 1, p. 34)

Figure 8 (cont'd)

Example 11. Nicolette decided to see what the class could earn from each activity in the same number of weeks. She decided to see how much they could earn in weeks.

a. Why do you think Nicolette chose 12 weeks? **(likely)**

b. How much could they earn from each activity in 12 weeks?

c. Write a pair of equivalent ratios (as fractions) for each of the relationships in part (a) and (b) above.

d. How can this help Nicolette decide which way will earn more money? (CPM, PR, 7.1.1, 7-5)

Figure 9. Less likely to be mathematized examples

Examples 9-11 are less likely to raise mathematical questions from the situation. In Example 9, it is less likely a person would graph and think about "how the walking rate affects the graph in a real situation" knowing the walking rate of three different persons' and then trying to learn about their times and distances based on their walking rate. Rather, people might talk about how much time it will take to walk from one place to another in order to get a general picture of their individual times and distances of walking.

In Example 10, it is unlikely for someone in a real situation to wonder if an hourly paid rate is proportional to the working hours. In Example 11, for Question A, it is likely for people to wonder why Nicolette chose 12 weeks to figure out which activity might earn more money. But for the remaining three questions, it is less likely that people will think about and try to find answers to those questions. The common feature of Examples 9-11 is that the mathematical questions in the contextual question are explicitly and purposely designed to encourage students routinely to use certain math ideas, concepts and procedures.

Realism. The degree to which the context is aligned with real-world situations is similar in each curriculum. (see Table 21).

Table 21. Percentage of realism in each curriculum

	Real	Mix	Conflict
CMP3	55.7%	37.3%	7%
CPM	69.9%	20.3%	9.8%
Glencoe	64.3%	26.9%	8.8%

In CPM and Glencoe, there are 64.3% and 69.9%, respectively, of the context aligned with real situation. The aligned context in CMP3 is 55.7%, about 10% lower than the other two. Some "real" contextual-question examples are situations in real life, such as: A grocery store sells six oranges for $2 (Glencoe, LR, Lesson 3, b, p.191), and Noralie's car uses 20 gallons of gasoline to go 600 miles (CMP3, PR, 2.3B, p. 49).

Another type of context that aligns with the real situation is a teaching scenario that will most likely happen in a classroom setting.

Example 12. Michaela was trying to find the slope of the line shown at right, so she selected two lattice points (locations where the grid lines intersect) and then drew a slope triangle. Her teammate, Cynthia, believes that $\Delta y=3$ because the triangle is three units tall, while her other teammate, Essie, thinks that $\Delta y=-3$ because the triangle is three units tall and the line is pointing downward.

Question a. With whom do you agree and why? (CPM, LR, 7.1.3, 7-24a).

Figure 10. Example of a classroom oriented context

Example 12 is an interaction moment that aligns with real classroom moment. The students' solutions in the question are possible in the classroom.

A "mix" realism context in a contextual question is defined as a context where either the situation or the information/data in the situation is in conflict with the real situation. Below is an example from CMP3 that is coded as a "mix" realism of the context.

Example 13. Two posters for Pizza restaurant advertisements:

Poster 1: Family owned and operated, 15 large pizzas for $195.

Poster 2: Royal Pizza, pizza, sandwiches, calzones, salads, 10 large pizzas for $120. (CMP3, PR, 2.2, p. 44)

Figure 11. Example of a "mix" realism context

In Example 13, the information on the advertisement posters are the main information for solving the problem related to pizza prices at two stores. The situation in this context, that is, of pizza prices at two stores and comparison of the prices, is aligned with a real-life situation. Also, the context of pizza restaurant use posters for advertisement is likely to occur in real life. The information in the posters, however, is in conflict with the real-life situation. A pizza restaurant poster is unlikely to advertise the price for 10 or 15 pizzas. Most advertising posters for a pizza restaurant would list the price for one or two pizzas. Therefore, in this context, the situation is "real," but the information is conflicted. Such context is coded as a "mix" realism context.

On the other hand, CMP3 contains slightly fewer "conflict-context" contextual questions than CPM and Glencoe. There are two types of "conflict context": a situation that conflicts with the real-life situation or an imaginary situation, or a teaching scenario that conflicts with real classroom teaching. The "Line Factory" in Example 5 is an imaginary situation that was considered a "conflict context."

A conflicted-with-real-situation context is this: "Carla buys a minivan for $20,500. She writes a proportion to find the selling price S. $\frac{S}{110} = \frac{20{,}500}{100}$ (CMP3, 3.1A3a, p. 64). In A real situation, when someone buys a car, he or she notes the selling price first and then calculates the price after taxes and fees.

A conflicted teaching scenario would be: " 'I see a Giant One!,' exclaimed Lee." Where is the Giant One? Help rewrite the left side of the equation. $\frac{\frac{2}{5}x}{\frac{2}{5}} = \frac{100}{\frac{2}{5}}$" (CPM, 7.1.4, 7-42a). It is likely that Lee will notice the $\frac{\frac{2}{5}}{\frac{2}{5}}$ part in the equation, but how likely that Lee will call it "a Giant One" is questionable. By looking at the lessons following this one in CPM textbook, one sees that the "Giant One" is used repeatedly in the following lessons. Thus, I coded this scenario as a "conflict context" because the "Giant One" could be understood to come from the textbook author, rather than as a potential real scenario.

Furthermore, there is significant difference (p=0.000) of the distribution of realism between the two math-content topics in CMP3, but not in CPM and Glencoe. In CMP3, the linear-relationship questions involved more than half "mix" realism context, while the majority of proportional-relationship questions have "real" context (see Table 22).

Table 22. Percentage of realism in CPM3

	LR	PR
Real	37.5%	75.9%
Mix	52.5%	20.4%
Conflict	10%	3.7%

Necessity. When comparing the distribution of necessity of context in questions, CMP3 (48.7%) was found to have double the percentage of high-necessity questions as CPM (22.8%) and Glencoe (20.3%). Nearly half of the contextual questions in CMP3 are embedded in a context that is needed for supporting students' mathematical understanding. Below are some examples from each curriculum. I provide examples followed by explanations of high-necessity and low-necessity contexts.

Example 14. The campers consider their budget. How many pizzas can they buy from Royal with $400? What if they only have $96? Explain. (CMP3, PR, 2.2A4, p. 45)

Example 15. Eliza is saving her allowance to buy a new computer so she can email her pen pals around the world. She currently saves $45 every 4 weeks.

a. If her brother saves $39 every 3 weeks, who saves at a faster rate? Explain your reasoning. (CPM, PR, 7.1.1, 7-8a)

Figure 12. Examples of high-necessity contexts

Examples 14 and 15 are questions from CMP3 and CPM that contain high-necessity contexts. As defined in the coding rubric in Chapter 3, a high-necessity context is one that supports students' mathematical-idea development and understanding. The real-life context is highly needed in order to solve the math problem, or to make sense of the math problem, or to build the mathematical mode/idea. Any change of the real-context will affect the strategy to solve the math problem or the mathematical model/idea involved in the math problem. Students need to situate the math solution back to the context, or the context will constrain the math solutions.

In Example 14, the context of buying pizza within a budget is highly needed to solve the math problem. Also, the context of pizza and budget constrains the math solutions. Students need to situated the math solution back to the context to get a reasonable math answer that makes sense in the context, because it is not possible to buy pizzas beyond budget, nor is it possible to buy one-half or one-third pizza to fit the budget. Students need to adjust their math solutions based on this context.

In Example 15, in order to compare who saves at a faster rate, students not only need to find the saving rate, but also need to match each saving rate to the person in order to answer "who saves faster." Thus, the context in both examples not only serves as the cover story, but is also integrated with and supports the mathematical ideas.

Example 16. Student Council is selling T-shirts during spirit week. It costs \$20 for the design and \$5 to print each shirt. The cost y to print x shirts is given by y=5x+20. Graph $y=5x+20$ using the slope and y-intercept. **(low-necessity)** (Glencoe, LR, Lesson 4, example 4, p. 201)

Interpret the slope and the y-intercept. **(high-necessity)** (Glencoe, LR, Lesson 4, example 5, p. 201)

Figure 13. Examples of different necessity level contexts

Example 16 contains two questions under the same context. This is an example of a combination of both low-necessity and high-necessity context. The first question "graph $y=5x+20$ using the slope and y-intercept" requests low-necessity of the context of selling T-shirts. Students can graph the rule based on the procedure they learned. The second question, however, needs the context at a high level. In order to interpret the slope and the y-intercept, students have to relate the slope and the y-intercept they found to the context of selling-T-shirts.

Example 17. A set of stairs is being built for the front of the new Arch Middle School. The ration of rise to run is 3 to 5.

1. Is this ratio within the carpenters' guidelines? (CMP3, LR, 4.1B1, p. 89)

Example 18. Review the recent orders and decide if there is anything wrong with each customer's order. If the order is correct, then pass it on to your production department with a rule, a table, and a graph (on graph paper). However, if the order is incorrect, explain to the customer how you know the order is incorrect and suggest corrections.

Figure 13 (cont'd)

Customer A wants a line that has y-intercept at (0, -3) and grows by 4. She ordered the line y=-3x+4 (CPM, LR, 7.1.1, 7-2a)

Figure 14. Examples of low-necessity contexts

Examples 17 and 18 contain low-necessity contexts for solving the math problems. In Example 17, the context serves as a cover story. As soon as students grasp the math in the problem, they can answer the question and don't need the context at all at any point in their mathematical problem solving.

Example 18 is a contextual question that involves an imaginary context: line factory and customer orders. Ignoring the imaginary feature of the context, the factors of reviewing customer orders and explaining to the customer form a low-necessity situation for mathematical problem-solving. With the mathematical information in the problem, students can determine whether or not the rule matches the given y-intercept and rate of change. From the above examples, one can see that a low-necessity context is not needed after students grasp the mathematical information from the contextual question, and thus the low-necessity context does not play a role in supporting students to build and understand math ideas.

Findings at Task Level

Define Contextual and Non-contextual Task

In this part, I present findings at task level. The three curricula designed tasks differently when I aggregated by task. The 228 CMP3 questions aggregated to 87 tasks, while 171 Glencoe questions formed 151 tasks and 123 CPM questions formed 106 tasks. There are contextual and non-contextual questions. When aggregate the questions to tasks, some tasks comprise all contextual questions, some tasks involve all non-contextual questions, and some tasks contain a combination of both contextual and non-contextual questions. At task level, I identified contextual task as a task contains contextual questions, including contains all contextual questions and contains a combination of both contextual and non-contextual questions. The non-contextual task is a task that only contains all non-contextual questions. Table 23 shows the amount of contextual and non-contextual tasks in each curriculum.

Table 23. The amount of contextual and non-contextual tasks in each curriculum

	Contextual task	Non-contextual task	Total
CMP3	71	16	87
CPM	74	32	106
Glencoe	103	48	151

Comparison of Non-contextual Tasks Across Three Curriculums

The comparison of non-contextual tasks across three curricula compared the structure, cognitive level, and representation of math tasks. The results of one-way ANOVA revealed that there are statistically significantly differences between the three curriculums in terms of the structure and cognitive level. Tukey post-hoc tests further reveled that there are significantly differences of structure and cognitive level between CMP3, CPM, and Glencoe. The structure of non-contextual math tasks in CMP3 and CPM are statistically significantly opener than tasks in Glencoe ($F(2, 91)=20.319$, $p=0.000$). Also, the cognitive level of non-contextual tasks in CMP3 and CPM are statistically significantly higher than tasks in Glencoe ($F(2, 91)=18.992$, $p=0.000$). No statistically significantly difference between CMP3 and CPM. See table 14 for the information of non-contextual tasks in each curriculum. A worth note result is that all non-contextual tasks in Glencoe are close-structured tasks.

Table 24. Mean of structure and cognitive level of non-contextual tasks in each curriculum

	CMP3	CPM	Glencoe
Structure (the closer to 1, the opener)	1.61±0.38	1.6±0.44	2±0 (close-structured)
Cognitive level (the higher number, the higher cognitive level)	2.33±0.72	2.23±0.76	1.46±0.5

Features of Contextual Tasks

Context Features Across Curricula. Three categories of context features were examined at task level across three curricula: mathematization, realism, and necessity. There is a statistically significant difference on mathematization and necessity between the three curriculums (CMP3, Glencoe, and CPM) as determined by one-way ANOVA (Mathematization, $F(2, 245)=14.629$, $p=0.000$; Necessity, $F(2, 245)=13.698$, $p=0.000$). No statistically significant difference was found of the realism between the three curricula.

A Tukey post-hoc test revealed that the likelihood to mathematize a contextual problem is statistically significantly more likely in the CMP3 and CPM compared to the Glencoe. No statistically significant difference of mathematizes likelihood between CMP3 and CPM.

Similar result was found in the necessity category. The Tukey post-hoc test result showed that the degree to which a context is needed to understand mathematics is statistically significantly higher in CMP3 compared to the Glencoe and CPM. No statistically significant difference of necessity between Glencoe and CPM. Table 25 shows the mean of mathematize likelihood and necessity level across three curricula.

Table 25. Mean of mathematization and necessity across three curricula

	CMP3	CPM	Glencoe
Mathematization (the closer to 1, the more likely)	1.40±0.48	1.46±0.48	1.75±0.45
Necessity (the closer to 1, the higher needed)	1.42±0.44	1.68±0.49	1.78±0.44

Cognitive Features Across Curricula. Three categories of cognitive features were tested at task level across three curricula: structure, cognitive level, and representation. A statistically significant difference was found on the structure, cognitive level, and representation across three curricula. The one-way ANOVA results are:

- $F(2, 245)=33.721$, $p=0.000$ for the structure category
- $F(2, 245)=36.330$, $p=0.000$ for the cognitive level category,
- $F(2, 245)=4.835$, $p=0.009$ for the representation category.

A Tukey post-hoc test further revealed that the structure at task level is statistically significantly opener in CMP3 and CPM than Glencoe. Similar results were found at the cognitive level and representation as computed by the Tukey post-hoc tests. The cognitive level at task level is statistically significantly higher in CMP3 and CPM than Glencoe. CMP3 used statistically significantly less multiple representations than Glencoe. No statistically significantly difference of structure and cognitive level between CMP3 and CPM. Table 26 shows the mean of structure, cognitive level, and representation across three curricula.

Table 26. Mean of structure, cognitive level, and representation across three curricula

	CMP3	CPM	Glencoe
Structure (the closer to 1, the opener)	1.60±0.39	1.71±0.41	198±0.12
Cognitive level (the higher number, the higher cognitive level)	2.84±0.57	2.6±0.68	2.1±0.55
Representation (the higher number, the more multiple representations)	1.72±0.98	2.13±1.16	2.28±1.32

Correlation between Context Features and Cognitive Features. The correlation test showed that there is statistically significant correlations between mathematization and cognitive level ($p=0.000<0.01$). Also, a statistically significant correlation exists between necessity and cognitive level ($p=0.000<0.01$). The two correlations are both weak negative ($r= -0.23$). In other words, the mathematize likelihood and necessity level of a real context are negatively weak correlated to the cognitive level of the contextual task in all three curriculums. The code for mathematize likelihood is the higher number, the less likely to be mathematized. Similar code rubric applied to the necessity level, the higher number, the less needed. Thus, the correlation results indicated that in all three curriculums, the more likely the context could be mathematized, the higher cognitive level the contextual task will be. Also, the higher the context is needed for understanding mathematics, the higher cognitive level the contextual task will be. The realism of the real context, however, doesn't have statistically significant correlations with the cognitive level of the contextual task ($p=0.32>0.01$).

Synthesis

Summary of Features at Question and Task Level Across Three Curricula

The quantitative results at both question and task level showed that each curriculum has a special way utilizing real context with mathematics, as well as design math questions and tasks. In the synthesis, I summarized the cognitive demand features and context features at question and task level respectively for each curriculum. The following two tables show a summary of cognitive demand features and context features of questions and tasks in each curriculum.

Table 27. Summary of cognitive demand features in each curriculum

	CMP3	CPM	Glencoe
Cognitive level			
Question level	Mainly procedure without connection and procedure with connection questions. More **high-level** than low-level questions Among the **high-level** questions, more **contextual** questions than non-contextual questions.	Mainly procedure without connection and procedure with connection questions. More **low-level** than high-level questions Among the **high-level** questions, more **contextual** questions than non-contextual questions.	Mainly memorization and procedure without connection questions **Low-level questions** Among the **high-level** questions, more **contextual** questions than non-contextual questions.
Task level	Both contextual and non-contextual tasks are **significantly higher** level cognitive level	Both contextual and non-contextual tasks are **significantly higher** level cognitive level	Both contextual and non-contextual tasks are **significantly lower** level cognitive level

Table 27 (cont'd)

Structure			
Question level	More **open-structured** than close-structured. Among the **open-structured** tasks, more **contextual** than non-contextual.	More **open-structured** than close-structured. Among the **open-structured** tasks, more **non-contextual** than contextual.	**Close-structured** Among the **open-structured** tasks, more **non-contextual** than contextual.
Task level	Both contextual and non-contextual tasks are **significantly opener structured**	Both contextual and non-contextual tasks are **significantly opener structured**	**All non-contextual tasks are close-structured** **Contextual tasks are significantly closer structured**
Representation			
Question level	Mainly **single** representation. **No** non-math picture	Mainly **single** representation.	Mainly **single** representation.
Task level	Contextual tasks use significantly **less** representations		Contextual tasks use significantly **more** representations

Table 28. Summary of context features in each curriculum

	CMP3	CPM	Glencoe
Mathematize			
Question level	**Likely** (46%) and less likely (54%) mathematized questions are about equally distributed. Among the **likely** questions, more PR than LR	**Likely** (43%)and less likely (57%) mathematized questions are about equally distributed Among the **likely** questions, more LR than PR	Mainly are **less likely** to have the math question Arise from the situation (80%)

Table 28 (cont'd)

Task level	The context are significantly **more likely** to be mathematized	The context are significantly **more likely** to be mathematized	The context are significantly **less likely** to be mathematized
Realism			
Question level	Real context 55.7 Mix realism 37.3% Conflict 7%	Real context 69.9% Mix realism 20.3% Conflict 9.8%	Real context 64.3% Mix realism 26.9% Conflict 8.8%
Task level			
Necessity			
Question level	Almost **half are high-necessity** 48.7%	Mainly **low-necessity** 77.2%	Mainly **low-necessity** 79.7%
Task level	The context are significantly **higher needed** to support math learning	The context are significantly **less needed** to support math learning	The context are significantly **lower level needed** to support math learning
Completion	Incomplete context N=2 out of 228	Incomplete context N=8 out of 171	Incomplete context N=9 out of 123

Summary of Features at Question Level between the Two Math Topics

Furthermore, the quantitative results indicated differences between the two math content topics: linear relationship and proportional reasoning exists at the question level. Table 29 is the summary of differences of questions in the three curriculums between the two math-content topics.

Table 29. Summary of features in three curriculums between the two math topics at question level

	LR (linear relationship)	PR (proportional reasoning)
Context	Contextual questions (56%)	Contextual questions (87%)
Structure	Open-structured questions (28%)	Open-structured questions (19%)
Representation	Text question request multiple representation responses (16%)	Text question request multiple representation responses (5%)

Findings at Lesson Level

Pattern of Math Tasks by Lesson

In this part, I present findings of qualitative analysis of math tasks that use "lesson" as the unit of analysis. As mentioned earlier, the "lesson" used here is identified according to the textbook designer's identification of one lesson in curriculum materials, not the actual teaching unit.

Context Use by Lesson. The three curricula used contexts differently when tasks were aggregated by lesson. In CMP3, math tasks within one lesson are designed around the same real context, or around a series of real-context tasks. In Glencoe, math tasks within one lesson are independent from each other. There is neither overarching context nor series of real context. Each contextual task in Glencoe within one lesson has an independent context, which has no relationship with any other tasks in the same lesson. CPM, however, is a combination of CMP3 and Glencoe. In CPM, some lessons developed math tasks around the same real context within and across lessons. Other tasks use independent real contexts within one lesson.

Here is one example of contexts used by CMP3 to illustrate the way CMP3 uses one real-context task as overarching and makes a series of contexts from it.

Table 30. Example of context use in CMP3 linear relationship by lesson

Lesson	Context
Lessons 1-3 (90-mins per class period)	1.1-1.4 walking rates 1.1 walking marathons 1.2 walking rates 1.3 raising money 1.4 USING the walkathon money

Table 30 (cont'd)

Lessons 3-5.5	2.1-2.2 Henri and Emile's race 2.3 Comparing T-shirts costs 2.4 Pledge plan from walking rates context in Lesson 1-3
Lessons 5.5-10	3.1Walking rates from walking rates context in Lesson 1-3 3.2-3.3 Mystery pouches 3.4 Teaching scenario 3.5 Bakery income
Lessons 10-13.5	4.1 Climbing stairs 4.4 (1) Money saving (2) Temperature

From Table 30, one can see that within one lesson, only one real context is used. For the first three lessons, the "walking rate" is used as an overarching context. A series of contexts designed around the walkathon is used for lessons one to three, such as walking marathons, raising money, and using the money. Also, the walking-rate and raising-money contexts are re-used in the later lessons. Altogether 178 math questions are found in the CMP3 linear relationship textbook using the above nine contexts.

In CPM, contexts are used not only within the lesson, but also across lessons. Below is an example from the CPM linear-relationship textbook that uses contexts within and across lessons.

Example 19. Contexts used in CPM linear relationship

Lesson 1 Line Factory

Lesson 2 Roller coaster

Lesson 3 Climbing stairs

Lesson 4 Teaching scenario

Lesson 5 Line Factory: slope walk

Figure 14 (cont'd)

Lesson 6	The big race
	Line Factory: take a walk
Lesson 7	The big race
Lesson 8	On the farm
Lesson 9	Dizzyland
Lesson 10	Line Factory logo
Lesson 11	Save the earth

Figure 15. Example of lesson titles

In this example, one can see that there are 11 lessons using eight contexts. Two contexts are used multiple times across lessons. The "Line Factory" context is used in four lessons discretely. "The big race" context is used in two consecutive lessons. A total of 57 math questions are designed around the above eight contexts.

Some lessons in CPM used multiple contexts within one lesson. Table 31 is an example of using multiple contexts in one lesson.

Table 31. Example of using multiple contexts in one lesson from CPM

CPM PR	Context
Lesson 1	1. Making money from selling 2. SAVING money
Lesson 2	1. Athlete of the week 2. Triathlon 3. Travel distance and time
Lesson 3	1. Car travel: distance and gas use 2. Knitting

Table 31 (cont'd)

	3. Wheel of Winning game
Lesson 4	Teaching scenario

In this example, there are multiple contexts used within one lesson. Either in each lesson or across lessons, no relationship exists between the contexts. One or more than one math questions appear in each context. To use contexts independently within one lesson is the typical way presenting context in Glencoe.

Math Task Cognitive Feature by Lesson. There is a repetition of math tasks within one lesson in Glencoe. Lesson 7, for example, has altogether eight math tasks in the lesson. The eight math tasks can be grouped into three sets based on the similarities of the task. Example 20 is the first set in Lesson 7.

Example 20 Sample tasks from Glencoe, PR, Lesson 7, p. 66

Note: The examples recorded below are from the textbook excluding solutions, tables, graphs, and formulas; only the texts of the math problem are recorded verbatim.

Example 1. The table shows the amount of money a booster club makes washing cars for a fundraiser. Use the information to find the constant rate of change in dollars per car.

a. The table shows the number of miles a plane traveled while in flight. Use the information to find the approximate constant rate of change in miles per minute.

b. The table shows the number of students that buses can transport. Use the table to find the constant rate of change in students per school bus.

Figure 16. Example of a set of questions in one lesson in Glencoe

In Example 20, the three tasks are the same in cognitive-demand feature. They have the same cognitive level, same structure, and same representation. The only difference between the three tasks is the context. However, the context is also the same in terms of the context features: mathematize, realism, necessity, and completion. The contexts differ only in the situation and information facts. Such repetition of tasks appears in every lesson in Glencoe. Each lesson in Glencoe can be divided into several sets of tasks. Within each set, the math tasks share the same cognitive feature as shown in Example 20.

No repetition of math tasks is found in CMP3 and CPM. They both contain a variety of math tasks in one lesson and across lessons. Students experience math tasks with various cognitive features in one lesson.

Language Use

I observed two features of language use in each curriculum. First, more texts exist in CMP3 and CPM than Glencoe. Regardless whether math tasks use single representation or multiple representations, the tasks in CMP3 and CPM are written in much longer text than in Glencoe.

Example 21

7-10. Today you will use your new knowledge of y=mx+b to solve "Newton's Revenge," problem 1-15, which is summarized below.

Newton's Revenge, the new roller coaster, has a tunnel that thrills riders with its very low celling. The closest the celling of the tunnel ever comes to the seat of the roller-coaster car is 200cm. Although no accidents have yet been reported, rumors have been spreading that very tall riders have been injured as they went through the tunnel with their arms raised over their heads. The management needs your help in convincing the public that the roller coaster is safe.

Figure 16 (cont'd)

Your Task: To help determine whether the tunnel is safe for any rider, no matter how tall, plot the data collected in problem 1-15 into a grapher, such as a graphing calculator or the Newton's Revenge Student eTool. The height and reach should both be measured in centimeters. If you do not have the data from Chapter 1, your teacher may instruct you to use the data provided at right. As you enter the data into the grapher, answer the questions below.

a. What window should you use to be able to see all of your data in scatter plot? Set up the appropriate window and make a scatter plot with your grapher.

b. is this plot useful for making predictions? Why or why not? If not, how could you change the plot to make it more useful? (CPM, LR, 7.1.2)

Example 22.

2. The cost of renting video games from Games Inc. is shown in the table. Determine whether the cost is proportional to the number of games rented by graphing on the coordinate plane. Explain your reasoning. (Glencoe, PR, Lesson 5, p. 47)

Figure 17. Examples of language use in CPM and Glencoe

Example 21 and 22 are two examples of math tasks in CPM and Glencoe. From the examples, we can see that both tasks use multiple representations. More text is used in CPM tasks than in Glencoe. On one hand, the use of more text material, especially in a contextual problem to describe the context**,** will potentially help provide more information for the context. More information of the context might increase the clarity of the problem. On the other hand, scholars argued that the use of text has the potential to challenge students' reading comprehension. In other words, more text in a math task will potentially require higher reading comprehension ability. Moreover, the text used in Glencoe differs from CPM and CMP3 in terms of the tone. In Glencoe, the text material is precise and direct about the math question. In many cases, Glencoe tasks are explicit about the math questions and they direct students to which potential math routine to use.

Secondly, the language used in Glencoe Math shares common vocabularies within one lesson. Below is one sample lesson from Glencoe to explain the language use.

Sample lesson from Glencoe PR, Lesson 7, p. 66-68

Note: Below is the summary of the math tasks in Lesson 7 before the practice section. By summary, I mean the examples recorded below are from the textbook, excluding solutions, tables, graphs, and formulas. ONLY the texts of the math problem are recorded verbatim.

Lesson 7

Example 1. The table shows the amount of money a booster club makes washing cars for a fundraiser. Use the information to find the constant rate of change in dollars per car.

a. The table shows the number of miles of a plane traveled while in flight. Use the information to find the approximate constant rate of change in miles per minute.

b. The table shows the number of students that buses can transport. Use the table to find the constant rate of change in students per school bus.

Example 2. The graph represents the distance traveled while driving on a highway. Find the constant rate of change.

Example 3. Explain what the points (0, 0) and (1, 60) represent.

c. Use the graph to find the constant rate of change in miles per hour while driving in the city.

d. On the lines below, explain what the points (0, 0) and (1, 30) represent.

Figure 17 (cont'd)

Example 4. The table and graph below show the hourly charge to rent a bicycle at two different stores. Which store charges more per bicycle? Explain.

Figure 18. Sample lesson from Glencoe

In this sample lesson, there are a total of eight math tasks. They can be grouped into three sets: Set1: Example 1, a, b; Set 2: Example 2, 3, c, d; and Set 3: Example 4. Within each set, the tasks use the same sentence structure and share common vocabularies. For tasks in Set 1, the same language is used: "The table shows the XXX of XXX. Use the information to find the constant rate of change in XXX per XXX." The only difference between the three tasks in Set 1 is the context information. The mathematical ideas and procedural steps involved in the three tasks in Set 1 are the same. The manner of organizing the context information and asking the mathematical question in the three tasks in Set 1 are the same, too. Three tasks in set 1 used the same context information and presented the math questions in the same way, offering three opportunities for practicing the same math ideas. That repetition pattern is also observed in Sets 2 and 3. Therefore, the Glencoe lessons offer via the use of language repeated tasks that allows for the processing of math ideas.

Summary

In sum, the results suggested that the three curriculums differ not only in the way they utilize real context with mathematics, but also the cognitive demand features they design questions and tasks. Also, there is correlation between the context features and cognitive level at the task level. Only the realism of the real context used in three curriculums has no significantly difference from each other. Nor the realism of the real context has significant correlation with the cognitive level of the math task.

CMP3 structured the lesson in the way that multiple questions nested within one task, and then multiple tasks formed one lesson. By analyzing pattern of math tasks at the lesson level, I found that CMP3 lessons tend to use the same real context or develop a series of real contexts. The same real context or a series of real contexts serve as an overarching context within one lesson or across lessons. Math questions and tasks in one lesson are at various cognitive level, structure, and representation, which provide students a learning experience of solving all types of math questions and tasks.

At the task level, CMP3 tasks are significantly opener-structured, higher cognitive level, and use more single representations, compare to the other curriculums. The real contexts utilized in CMP3 are significantly more likely to be mathematized and higher needed to support the mathematical ideas.

Breaking the tasks into questions, a big difference of cognitive level exists between non-contextual and contextual questions. About half (48.5%) of the non-contextual questions are low cognitive level: procedure without connection questions, while 32.4% are high level procedure with connection questions. The contextual questions, on the other hand, comprise 50.9% high-level procedure with connection questions and 15.7% high-level doing math questions. CMP3 consists of more high cognitive level questions than low-level questions and tasks matches the findings from existing studies (Cai, Nie, Moyer, & Want, 2014).

Lessons in CPM also comprise multiple tasks nested with multiple questions. At the lesson level, real context are used in two ways. In some lessons, the real contexts are the same or belong to a series. In other lessons, the real contexts are independent from each other. A variety of math questions and tasks are selected within one lesson.

At the task level, CPM tasks are also significantly opener-structured and higher cognitive level. The real contexts used in CPM are significantly more likely to be mathematized, but less needed to support the development of mathematical ideas. Breaking the tasks into questions, the majority of questions in CPM are either procedure without connection or procedure with connection. For the non-contextual questions, 62.5% are low-level procedure without connection questions, and 22.9% procedure with connection questions. Similar distribution exists among the contextual questions. Slightly above half (52%) of the contextual questions are procedure-without-connection questions, and 37.4% are procedure with connection questions.

In Glencoe, however, the features at questions, tasks, and lesson level are quite different from CMP3 and CPM. The real contexts used in Glencoe are independent from each other. There is no relationship between any real contexts either within one lesson or across lessons. At the task level, Glencoe tasks are significantly closer-structured, lower cognitive level, and use more multiple representations. One worth notice finding is that all non-contextual tasks in Glencoe are close-structured. Real contexts used in Glencoe are significantly less likely to be mathematized and less needed to support mathematical ideas. At the question level, all non-contextual questions in Glencoe are low cognitive level question, including 58.2% memorization questions and 41.8% procedure without connection questions. The majority of contextual questions are low-level questions as well, including 67.5% procedure without connection and 13.8% memorization questions. Only 18.7% contextual questions are high-level procedure with connection questions.

Correlation tests also indicate that the mathematize likelihood and the necessity degree are significantly correlated to the cognitive level of the math task. Although the correlation is a weak one, the more likely the real context could be mathematized, or the higher needed the real context is to support mathematical ideas, the higher level the cognitive level will be.

Last but not least, I found that math-content topic affects the features of questions. More contextual questions are found in proportional reasoning (PR) than linear relationship (LR). There are less open-structured PR questions compared to the LR questions. The percentage of single representation (text only) questions is lower in the PR tasks than in the LR questions.

CHAPTER 5

CONCLUSIONS AND DISCUSSION

This study explored the features of math tasks as written in textbooks from three curriculums. In particular, I focused on unpacking the context features and cognitive demand features of math tasks. Since curriculums are attempting to incorporate the increasing call to connect mathematics and the real world, I intended to study the question via two avenues. One was to explore the unique characteristics of different curriculums when utilizing real context with mathematics. Another was to uncover the relationship between context and cognitive demand of contextual tasks. In this chapter, I will first summarize the results from the previous chapter, then I will provide my arguments and interpretation based on the results I drew from this study. The value of my conclusions may potentially benefit those in the math education and teacher education field.

Math Tasks As Written

In this study, I asked the following three research questions:

1. What are the features of math tasks as written in textbooks in terms of context, cognitive demand, structure, and representation?
2. What are the features of sequencing math tasks in different curriculums in terms of context, cognitive demand, structure, and representation?
3. What is the relationship between aspects of context (mathematization, realism, and necessity) and cognitive demand of contextual math task?

For the first and second research questions, I analyzed each curriculum from three levels of grain sizes: question, task, and lesson level. The results in Chapter 4 revealed that the three curriculums differed not only in the way they utilize real context with mathematics, but also in the way they design the cognitive demand features, of the task, including structure, cognitive level, and representation. Also, I found that the math-content topic affects the features of tasks.

For the context features, more than 70% of the math tasks in all three curriculums are contextual math tasks. This result confirms the findings in existing studies that math tasks in U.S. textbooks emphasize real-life situations (Zhu, & Fan, 2006; Cai, Nie, Moyer, & Wang, 2014; Park & Leung, 2006; Alajmi, 2012). Although all three curriculums utilized the real contexts that are mostly aligned with the real world, the three curriculums differed largely, especially in the way a math question is likely to arise from the context, and to what degree a real context supports the mathematical ideas. In respect to the cognitive demand features, the three curriculums differed greatly in the way each curriculum designed the structure, the cognitive level, and representation of the math task.

For the third research question, a correlation analysis indicates that two categories of the context feature are significantly correlated with the cognitive level of math tasks. The more likely a math question will arise from the real context, or the more a real context is needed to support mathematical ideas, the higher cognitive level the contextual math task will be. The realism, however, has no significant correlation with the cognitive level of math tasks. This result hasn’t been reported in any existing studies, but it has the potential to contribute to the existing research as well as the practice of mathematics. I will discuss this result in further detail later in this chapter.

What Kind of Contextual Tasks Are of High Cognitive Level?

Tasks are the basic element in the subject of mathematics. Students learned the most in class when the instructional tasks are of high cognitive demand level (Stein & Smith, 2000). In the Common Core era, when more than 70% of math tasks in curriculum and classroom teaching are contextual math tasks, to select and implement a high cognitive demand level contextual task is essential for productive math teaching and learning.

Those results suggest that the three curriculums (CMP3, CMP, and Glencoe) analyzed in this study differ in the way they select, and thus are able to utilize real-world context in the mathematical tasks they present to students. Although most of the tasks in all three curriculums are intended to be contextual tasks--which is to say, they are intended to be tasks that connect math with the real world--there is considerable variation in the success with which they carry out that mission. Some attempts to make that connection fail to achieve the goal. This shortcoming, as my analysis shows, is due to the fact that the real-world contexts chosen to be the occasions for mathematizing clearly appear not to lend themselves to that goal. In other words, some of the real-world contexts used in the mathematization tasks are insufficiently complex or rich enough to serve as occasions for learning the mathematical ideas that students are expected to learn as part of the task.

Embedding math problems in real-world contexts does not, therefore, necessarily increase the quality of learning that is achieved from mathematizing the context. Even utilizing math problems in realistic context is not enough, if it doesn't lead to engagement with the mathematics. This analysis suggests that real-world contexts differ in the mathematical complexity they afford the learner and, like any pedagogical tool, they thus must be carefully chosen and designed with that goal in mind. Unfortunately, even in a very good mathematical curriculum like CPM3, the tasks presented for mathematizing differ in the complexity of the content that is embedded in the real-world context. Thus, it needs also to be emphasized that this affordance factor interacts with the complexity of the two math-content topics studied in this dissertation: linear relationship and proportional reasoning. Based on these findings, therefore, the author argues that three factors have a role in math learning and are related in the ways they influence written assignments in mathematical curriculum. Those factors are: mathematical demands of tasks, the ways the tasks are contextualized as real-world situations, and the math content topics they are intended to teach.

The correlation results between context feature and cognitive demand feature suggested that the mathematize likelihood and the necessity degree of the real context are significantly correlated to the cognitive level of the task. The realism, however, is not significantly correlates to the cognitive level of the task. That result provides new insight for the field on how to utilize real context with mathematics in math tasks so as to support student math learning.

The debate in the math education field on the role of real context in math teaching and learning mainly involved two arguments. On one hand, researchers argued that real context helps student math learning. "Students learn math the best if there is any familiar experience involved." On the other hand, critics pointed out that the problematic contextual tasks have a negative influence on student math learning as well as on the students' impression about mathematics as a subject. The problematic contextual tasks are mainly regarded as contextual tasks that involved contexts conflicted with student real world experience and are only 'solvable' on the basis of problematic mathematical modeling assumptions" (Reusser & Stebler, 1997, p. 311). Therefore, efforts have been made in the field to increase the realism of the real context.

Results in this study confirmed that curriculums have been designed to increase the realism of the real context used in the textbooks. The real contexts used in three curriculums are mainly real (62.5%), with only 8.4% of the contexts in conflict with the real world. Increasing the realism of the real context, however, doesn't necessarily relate to an increase in the cognitive demand level of the contextual tasks. I argue that the efforts to minimize problematic contextual tasks should be not only to increase the realism of the real context, but also to increase the likelihood the real context could be mathematized, and the degree a real context is needed to support the mathematical ideas.

Increase the Mathematize Likelihood of the Real Context

First I argue that to increase the mathematize likelihood of the real context will help to increase the cognitive level of the contextual math task. There are examples in this study that involve real context aligned with the real world, but the math questions are less likely to be asked in a real world situation. Here is a sample question: Andrew earns $18 per hour for mowing lawns. Is the amount of money he earns proportional to the number of hours he spends mowing? Explain. The hourly rate of mowing lawns and the situation of mowing lawns to earn money in this contextual task aligned with the real experience. The math question, however, is less likely asked in the real situation. Imagining the real world situation of mowing lawns to earn money, a question likely to be asked is "How much could be earned?" If the mowing lawns context has to be used to teach the proportional relationship, an open question such as "What's the relationship between the hourly rate and the hours he spends mowing?" might be more likely to arise in this real world situation.

One can also see that question has the potential to increase the cognitive level of the contextual task. One reason is that the new question is more open than the original question. The original question explicitly guides students to apply the rule of proportional relationship for the two variables, while the new question doesn't reveal what mathematical relationship students should apply for the two variables. Students are provided with the chance to explore the relationship between the two variables in the hope of understanding the definition of the proportional relationship through this investigation.

One explanation for the correlation of a contextual task likely to raise a math question with the high cognitive level of the math task lies in the process used to solve the contextual task. Solving contextual tasks involves the mathematical modeling process, in which problem solvers mathematize the real world problem into the math world. When a math question arises from a real context, it connects the real world with the math world. Therefore, a likely to be mathematized real contextual task may offer opportunities for problem solvers to develop mathematical ideas. It would also correlate to a high cognitive level of the math task.

Increase the Necessity Level of the Real Context

I also argue that the more the real context is needed to support mathematical ideas, the higher the cognitive level the math task could be. Real context plays various roles in student math learning, such as motivating student interest, supporting student math learning, and enabling the student to apply the math he has learned. Realistic mathematics claimed that the real context should be the source to support student math learning. The results of my study confirmed this claim. The degree to which a real context is needed as a source for the development of mathematical ideas is significantly correlated with the cognitive level of the contextual math task. One of the criticism of traditional word problems is that students view them as a simple two-step process: first use the given numbers and then apply a routine math procedure they already know. Students, however, neglect the story or the context involved in the word problem because the context serves only as a cover story without being closely tied to the mathematics. In other words, the context is less needed or not needed to develop mathematical ideas.

Examples of such less needed contextual math tasks are “A swordfish can swim at a rate of 60 miles per hour. How many feet per hour is this?” and “Review the recent orders and decide if there is anything wrong with each customer’s order. Custom A wants line that has y-intercept at

(0, -3) and grows by 4. She ordered the line y=-3x+4." Ignoring the realism of the context in those two examples, the context is less needed for students to solve the math problem and to develop the mathematical ideas involved in the task. What students do to solve the two tasks will be simply to apply the procedural steps they learned. But a more needed context will have the potential to increase the cognitive level of the task.

In conclusion, contextual math tasks have the potential to benefit student math learning in many ways. The efforts to connect math and real world should be continued not only to emphasize on the realism of the context, but also to enhance the link between math and the context. As pointed out in the literatures, math tasks in U.S. textbooks emphasize real life situations, but the link is weak between mathematical ideas and the real-life connection (Park & Leung, 2006; Alajmi, 2012). Results in this study revealed that the mathematization and necessity of the context correlate to the cognitive level of the math problem. Therefore, one way to improve the quality of contextual math tasks is to increase the likelihood the real context could be mathematized, and the degree a real context is needed to support the mathematical ideas.

Implications for Research and Practice

Research

This study developed a set of rubrics to analyze the context features of contextual math tasks. The rubric is developed and are discussed, revised, and used with other researchers during the process of inter rater reliability check. Moreover, the rubric was not developed to target a specific grade level, so it provides a wide range of potential usage for other researchers. In sum, the rubric have the potential for future research related to contextual math tasks. Because of the increased use of contextual math tasks, the goal to study issues related to contextual math tasks has the potential to grow into an important research area.

Further, the process of determining the unit of analysis when analyzing math tasks as written in textbooks I experienced in this study can inform other researchers who intend to conduct curriculum analysis studies about the unit of analysis selection. Since each curriculum has a unique approach to structure math tasks, the selection of unit of analysis might reveal or dismiss features of the curriculum. For example, in this study, both CMP3 and CPM have math tasks that consist of multiple levels of multiple sub-questions. Selecting different grain size as a unit of analysis might provide different results. In this study, I coded at the finest grain size – the question level, and then analyzed at these three different levels, the question level, aggregated to task level, and then further aggregated to lesson level. Each grain size provides some results worth noticing and are of different perspectives. In this study, the question and task level provide detailed evidence on task features as written in different curriculum, while the lesson level results reflect a general pattern of each curriculums. Thus, my experience suggests that the selection of unit of analysis in curriculum studies plays a role in the types of results one will get. Researchers should attempt to select the most reasonable unit of analysis so as to get results that would best achieve the research goals.

Curriculum Design

On one hand, the results in this study confirmed the efforts curriculum designers had made to incorporate the calls from Common Core and the research field. For example, the majority of math task in textbooks, both in NSF-funded curriculum like CMP3 and in other curriculums like Glencoe, are contextual math tasks, in which a large percentage of the real contexts aligned with real world situations.

On the other hand, the correlation between the mathematize likelihood and necessity in the context feature, and the cognitive level of the contextual math task highlighted the claim that certain types of real context would potentially increase the cognitive level of the task. This result provides evidence and information to curriculum designers on directions they could potentially take to design and further refine contextual tasks.

Teacher Education

Teacher educators who support both pre-service and in-service teachers could benefit from this study. I argue that to increase the likelihood the real context could be mathematized and the degree to which a real context is needed to support the mathematical ideas relate to the increasing of the cognitive level of the contextual math task. This result provides insight for teacher educators to design, adapt, and select contextual math tasks for the purpose of productive math learning.

Limitations and Directions for Future Research

As the results indicated, certain types of contextual tasks are of high cognitive demand. Student work and learning outcome could be one direction for future research. Future research could collect data on student work and learning outcomes to explore the productivity of student math learning through different types of contextual tasks.

Secondly, teachers' enactment of written tasks could be another direction for future research. Scholars found that teachers have challenges to maintain the cognitive demand level of math tasks from written in curriculum materials to enact in classrooms. What about the interaction with context? What will teacher do about the context when enact contextual tasks in teaching? Future research on this question could collect data from teachers, such as teaching videos, teacher interviews and surveys, etc.

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