ESTIMATING LEAST COST HUMAN DIETS IN THE NORTHEAST OF BRAZIL WITH STOCHASTIC PROGRAMMING-- THEORETICAL ISSUES AND POLICY IMPLICATIONS

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ABSTRACT

ESTIMATING LEAST COST HUMAN DIETS IN THE NORTHEAST OF BRAZIL WITH STOCHASTIC PROGRAMMING--THEORETICAL ISSUES AND POLICY IMPLICATIONS

By

Paulo Roberto Silva

Malnutrition has been the major concern of many nutritionists, economists and public administrators involved in national planning and economic development in less-developed countries (LDC's). An overall approach to the problem usually requires resources and disciplinary skills probably beyond the stocks available in those countries. The approach has been to attack it under different methodologies and perspectives. This study was designed primarily to investigate the extent to which limited purchasing power has prevented consumers in the northeast of Brazil from meeting their minimum nutritional requirements. Therefore, least cost diets meeting minimum levels of nutritional allowances were computed and compared with the actual food expenditures and income patterns. In addition, attempts were made to relate some populations characteristic with the total cost of nutrition as well as to indicate least cost

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substitutions, the most expensive nutrients, and the marginal efficiency of particular foods.

Deterministic and stochastic linear programming were used to determine least cost diets that would meet the minimum levels of nutritional allowances suggested by the Institue of Nutrition at Federal University of Pernambuco (INUFPe). The eligible commodities for the least cost diets were mostly traditional foods and their prices were average retail prices for November, 1973 in the two cities studied. Standard families in Recife and Fortaleza were the basic consumption units. Basically, all the least cost diets fell into the purely nutritional category, that is: no conventional restraints were required and they were designed to provide a list of the least expensive nutrients and most efficient combination of foods for meeting nutritional needs.

Minimum allowances for animal protein (about 45 percent of the total protein in the diet) were imposed and the caloric level was additionally constrained by lower and upper bounds. In addition, the amounts of B vitamins (thiamin, riboflavin and niacin) were required to be a proportion of the total calories in the diet. Formally, there were not specific required amounts for the eight essential amino acids, but excesses of each were provided and compared with the minimum standards suggested by FAO. Experimentally, constraints were set up to explicitly account for conventional habits and people's tastes and preferences.

It was found that actual incomes are adequate to provide standard families in the cities studied with minimum nutritional requirements. In addition, there are indications that those families are inefficient in the purchase of nutrition and that reallocation of the actual food expenditures would improve both their economical and nutritional status. Actual and least cost diets differ primarily in terms of variety, costs, and relative importance of the individual commodities and food groups to the total costs and caloric/protein content. Economic substitutions between traditional and nontraditional commodities were feasible and lowered the cost of nutrition in one of the cities studied (Fortaleza).

The family composition effect on the total cost and food composition of the diets was found to be quite significant, especially for those concerned with pregnancy and lactancy. The family size effect merely shows that large families are more costly to feed than the small ones and that in such diets, by doubling or reducing the family size by a half, the total costs of nutrition should be doubled or reduced proportionally.

The marginal cost of the nutrients obtained as a routine part of the linear programming solution indicated that although all nutrients are equally important in satisfying minimum nutritional requirements, they are not equally costly. Such marginal cost figures were highly sensitive to the market prices of the commodities in the least cost diets and the computational procedures used (deterministic and stochastic). In the overall, proteins (both animal and vegetable) were costly nutrients while fat, phosphorous, and thiamin (B-vitamin) were costless at the margin in all least cost diets calculated for the two cities studied. In the aggregate, protein accounts for the greatest amount of total expenditure on food and is followed by calories associated with the B vitamins. The other nutrients together (calcium, vitamin A and C and iron) shared a small percentage of the total food expenditure as shown by the least cost diets.

The marginal efficiency of the foods, as given by the aggregate monetary value of the nutrients in the foods divided by their market prices, were found to be 100 percent for milk, manioc-flour and sweet potato-yellow in all least cost diets. Rice (in Fortaleza) and corn meal (in Recife) also had a marginal efficiency of 100 percent.

In another routine part of the linear programming solution was indicated the range over which the price of the efficient foods (that is, foods which have a marginal efficiency of 100 percent) could vary without being forced out from the least cost diets. Such a range of variability was quite sensitive to the computational procedures used. Foods like sweet potato, dried beef, manioc-flour, orange, rice, milk and beans-mulatinho were the least sensitive to market price variations.

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By

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To my wife, Norma, my sons André Felipe, Paulo Henrique, Ana Paula, and my grandmother, Hermelinda for their persistent assistance, encouragement and inspiration.

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CHAPTER I

INTRODUCTION

Widespread malnutrition may adversely affect economic development. First of all, limited life expectancy brought about by inadequate nutrient intakes limits the number of productive years and thus reduces the social return from investments in human skills [51]. Second, improper nutrition diminishes labor productivity, lowers resistance to diseases and hinders initiative. Third and most important, malnutrition has been one of the major causes of childhood deaths; and for a sizeable percentage of survivors, malnutrition will retard physical growth and intellectual development, because inadequate nutrient intakes affects bodily growth and thus physical performance.

The thesis that malnutrition may impair intellectual development and affect human development has been the major concern of many nutritionists, economists and public administrators interested in policy questions. For instance, Berg [8] has stressed its long-run effects as follows:

> If the majority of our children do not have a balanced diet we are producing a generation of intellectually and physically stunted growth. And any talk of equality of opportunity for individuals in (such) a society has no meaning (p. 105).

There are others who have argued that malnourished populations will lead to inferior labor and intelligence. So, regardless of the amount of capital and education that society makes available to them in the future, the survivors will not be able to achieve the levels of productivity that the genetic potential dictated [6]. If this is true and if malnutrition is higher among underprivileged groups, it will lead to lower intellectual and physical capacity for a large percentage of the population, mainly in the less developed countries (LDC's).

Actually, there is already an enormous public and professional concern about the nutritional problem and its socio-economic implications in the process of economic development [9, 11, 27, 30, 44]. However, the factors affecting malnutrition are highly interactive and mutually reinforced, so the nutrition problem becomes extremely complex. There are many different ways to improve the nutritional status of a population. For instance, adjustments in agricultural plans in order to provide an abundant supply of high quality and inexpensive food is one possibility; educating the household toward a more efficient use of its income as well as programs designed to raise the purchasing power of lower income groups are obviously open alternatives. How far those approaches are feasible and operative are clearly open questions since each one will involve different administrative arrangements and social costs and they may have quite different political implications. Finally, it should be emphasized that malnutrition exists even among the developed countries and while it is not the only problem in the LDC's, but it is not the least relevant.

A. The Problem Setting and Its Importance

In the northeast of Brazil, which contains 28.7 million inhabitants (30.3 percent of the country's population) and which amounts to 18.2 percent of the total Brazilian area,[1] a great percentage of the population suffers from malnutrition [2, 10, 12, 30, 33, 60, 62]. Most of the population of the northeast obtains an inadequate diet and shows calorie-protein deficits.[2] Vitamin A, B (Riboflavin), vitamin C and

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[1]For details see: [25, Vol. 1, pp. 54-60].

[2]There are indications that such levels have improved over time in the urban northeast, although calorie-protein intakes are still low among poor people [1]. On the contrary, it has been indicated that in rural areas the average calorie intake has declined in recent years from 1,800 to 1,323 a day (Time Magazine, March 25, 1974, p. 42). Yet, other studies based on either historical or clinical data have supported the arguments above [14, 30].

fat intakes are also below the standard nutritional requirements. Iron intakes are high, while calcium, thiamine and niacin intakes are generally adequate [2, 12, 60]. Infants' diets are particularly inadequate with respect to calories, vitamin A and C, iron, thiamine, and niacin.

Diets in the northeast of Brazil are generally poorly balanced for lack of meat, milk, fruits and vegetables, since they are based on beans, meal, rice, and brown sugar. In some areas, diseases and undernourishment caused by malnutrition are frequent and child mortality rates are quite high. There is still evidence that insufficient diets have reduced worker's life expectancy in that part of the country.[3]

It is clear that malnutrition exists in that part of the country, but the question is: why do people there eat what they do! Chaves [12] argues that malnutrition in the northeast of Brazil is to a great extent an economic problem. He suggests that improvements in the consumer's purchasing power is crucial in order to provide the people with good health, education and high quality food. Lustosa [39] and Melo [41] have also

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[3]Calculations made by Campbell showed that an average person in the northeast produces just one-fifth (1/5) as much as the typical worker of the southeast part of Brazil during his lifetime [9, p. 4].

stressed the causal relationship between per capita incomes and low nutritional levels in the northeast of Brazil, and have suggested an integrated and multi-disciplinary approach to deal with the problem, including agricultural development, improvement in the labor productivity, health facilities, and education.[4] Provided that most of the northeast population suffers from malnutrition because of limited incomes, and given that the most undernourished people are poor people, nutritional needs must be linked with the issues of income distribution and improvement in their economic status may be a desirable political goal.

Some have argued that freedom from malnutrition will come only when enough of the right foods are consumed and suggest improvements in the patterns of food consumption to get better nutrition [19]. Such an approach, of course, ignores the relationship between income levels and the cost of nutrients, which can be quite significant as well.[5] For instance, in the northeast of Brazil, it is not likely that one could induce

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[4] Comments about the effect of incomes to malnutrition on the northeast of Brazil can also be found in [37, pp. 3-4].

[5] Obviously, mere increases in income do not necessarily result in better nutrition unless people are taught principles of good nutrition and the housewife is able to obtain the necessary foods in acceptable form at reasonable prices.

poor people to buy the right kinds of foods if they cannot afford them.[6] Provided they can, it is still worthwhile to know how inexpensively they could obtain nutrients and meet their nutritional requirements at minimum cost. By providing a better use of the existing family food budget through better consumers' knowledge of economic diets, one can improve both the nutritional status of the rich and the economic position of the poor--so it may have some distributive impact over the long run.

A related problem is the population element in relation to malnutrition in the northeast of Brazil.[7] In that part of the country high rates of population growth followed by declining trends in mortality has had a direct impact on family size and age structure.[8]

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[6]In the northeast of Brazil per capita income is far below the national average which is about 420 U$A or Cr$2545.20 [2]. The problem, however, is that a large proportion of the population is within the low income brackets and there is evidence that the patterns of income distribution have deteriorated over time [31].

[7]In a recent press interview the president of the National Institute of Food and Nutrition of Brazil claimed that "malnutrition affects 40 percent of the Brazilian population and given that the rate of growth among undernourished people is by and large superior to the other section of the population, it would require heavy investments in order to push forward the Brazilian development plans," [27, p. 57].

[8]The estimated figure was 2.3 percent a year for the last decade [24].

Provided that nutritional requirements vary with age, differences in family structure and composition will affect the total daily nutrient intake.[9] It is quite obvious that the total cost of nutrition will rise with increased family size, but the effects of composition and age structure have not been examined over time. Therefore, it may be relevant to know the least-cost adjustments to changes in family size and structure. In addition, it might be worthwhile to investigate to what extent a high dependency ratio[10] depresses per capita incomes and prevents the largest and poorest families from adequately meeting their basic nutritional needs.

Finally, it has been recognized that a great percentage of poor people would be better off if adjustments in agricultural plans reduce the cost of nutrition. An assessment of the problem, however, needs to be made in terms of the relevant costs, because the market prices of foods in themselves may not reflect accurately the true cost of scarce nutrients. Smith [50, 51] has emphasized that agricultural development plans aimed to

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[9] This is especially true among families with pregnant and lactant women because they would require nutrient supplementation.

[10] The dependency ratio is measured by the portion of the population at working ages (age group 15-65) in relation to the total number of people in such a population [5].

have a major impact on the reduction of nutritional deficiencies must take into account the marginal efficiencies of particular foods as sources of nutrients. The idea is that the ultimate agricultural plans must be defined and based upon some dietary standards, together with prices and a scientific assessment of the kinds and qualities of the foods needed to promote good health and efficiency. The marginal efficiency criterion assigns high value to nutrients that are expensive to replace through the market, and knowledge about the marginal cost of nutrients gives useful indications of how better plans can be formulated and implemented in order to find sources of economic nutrition.

B. Objectives of the Study

The overall objective of this study is to evaluate the extent to which limited purchasing power has prevented consumers (in the northeast of Brazil) from meeting their minimum nutritional requirements. Therefore, the first step can be accomplished by relating family income levels and the proportion of income spent on food with the minimum cost diet. Also, an attempt will be made to evaluate the effect of family structure on the cost of nutrition. Based on the outlined objectives and results obtained, major policy implications will be summarized and used as a basis for recommendations to both consumers and public administrators.

More precisely, the objectives of this research can be stated as follows:

1. To determine least cost diets for a standard family and compare them with actual patterns of food expenditures and per capita incomes.

2. To determine the most expensive nutrients, given predetermined nutritional requirements and the marginal efficiency of foods in the least cost diets.

3. To determine the effect of family structure on the total minimal cost of nutrition.

4. To determine the extent to which food price patterns can change, without affecting the stability and composition of the minimum cost diets.

5. Based on (1), (2), (3), and (4), to show the major policy implications for meeting nutritional needs in the most economical ways.

C. Study Design and Thesis Organization

1. Scope of the Study

a. Selected Survey Areas

This study is concerned with the urban northeast and more precisely it involves two of the most important cities in that part of the country--Fortaleza and Recife. They are both capitals of states (Ceará and Pernambuco, respectively), and besides their active roles as political-administrative centers, they still have a relatively strong influence on the whole northeast region due to their economic dynamism. Due to their geographical characteristics (they are both coastal cities), they are subject to a widely varying climate throughout the year.

The industrial sector is rapidly growing in both cities, but given the steady, natural rate of population growth and rapid influx of people to the coastal areas (rural-urban migration), it has not been able to provide sufficient employment to absorb the growing labor force. The population and income patterns are also quite similar in these two cities. The 1970 Brazilian Demographic Census has estimated a population of 857,980 and 1,060,611 inhabitants for Fortaleza and Recife, respectively. Their rates of population growth (5.0 and 3.1 percent) are much above the northeast and Brazilian averages (2.3 and 2.7 percent, respectively) and in addition, a large percentage of the population is under the fifteen year age group (about 40 percent).[11]

Income levels are low in both cities and the income distribution is highly skewed. For example, in the city of Fortaleza about one-half of the total population received only twenty-five percent of the total

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[11]Basic Sources: The Getulio Vargas Foundation [60] and Censos Demograficos dos Estados do Ceará e Pernambuco, 1970 (FIBGE).

income, while in Recife only twenty-four percent of the total income was shared with fifty-one percent of the total population [1, 3].[12] In addition, it has been reported that a considerable proportion of income is spent on food, especially among the poor people who spent about seventy-five percent (average for the two cities) of their total income on food consumption.[13] There is also a great deal of similarity in the food consumption patterns for these two cities as shown in the Northeast Bank Consumer Budget Studies [3, 4]. However, there are indications that the diets are more diversified in Recife than in Fortaleza although for both cities, most of the basic nutrients (proteins and calories) come from five or six major sources.

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[12]It is based on 1964-68 data, although there are evidences (at least to the Northeast region as a whole) that such a pattern has not changed or even getting worse over time.

[13]Notice that in Fortaleza, where the people in the lowest income groups spent eighty percent of their income on food consumption, items like beef, cereals, fish, vegetables and dairy products account for eighty percent of such expenditures. Given such a consumption pattern, if one assumes that this segment of the population suffers from malnutrition, it obviously implies that the prevailing incomes have really imposed a low absolute level of food consumption, as has been emphasized [36, 114].

Concerning their nutritional status, historical data and clinical studies[14] have stressed the population nutritional deficiencies in both cities, ranging from gross undernutrition to sub-clinical malnutrition, especially among infants [14, 30, 33, 41]. Surprisingly, in the urban Northeast where the access to social and medical services are supposed to be better than in the rural areas, infant mortality rates shifted from 140.6 and 165.3 to 152.0 and 205.7 for Fortaleza and Recife, respectively, during the period 1969-70.[15] Yet, in the city of Fortaleza, a survey undertaken by the secretary of health has indicated that the percent of infants (aged 1-4 years old) who died showing symptoms of

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[14]In the overall there are three major ways to perform a scientific evaluation of the nutritional status of a population: dietary, biochemical and clinical. The dietary approach is focused on the average nutrient intakes by comparing them with some standard or predetermined levels, and it just indicates the nutrient intake for a specific point of time. The biochemical approach reflects the nutrients stored in the tissues in the relatively recent past and concentrates on analysis of blood and urine. The clinical studies are more general (although not less important) and they are focused on the prevalence of symptoms or physical abnormalities which are close indicators of malnutrition or undernutrition such as: height-weight age relationships, skin lesions and spots, decays, depigmentation, parasitic diseases and other infections. It must be emphasized that all of these methods are really complements rather than substitutes.

[15]Those figures refer to mortality rate per 1,000 infants under one year of age [25, Vol. 1, p. 79 and 26, Vol. 2, p. 64].

malnutrition was considerably high. For example: among two hundred children who died in the four hospitals sampled, about one hundred eighty four (92 percent) has showed symptoms of malnutrition [17].

b. Some Methodological Issues

This study is addressed toward some policy questions pertaining to improving the nutritional status of the population in the urban Northeast. Obviously, any policy prescription beyond the scope and sampled area could be quite arbitrary, although it does not mean that the results obtained in this research cannot be used as a basis for policy recommendations.

Public administrators concerned with agricultural and nutritional plans are usually interested in determining the nutritional needs of their populations. The conventional procedure has been to assume certain minimum nutrient requirements (on a per capita basis) and extrapolate to the population as a whole. Such a procedure is not truly incorrect, but it is imprecise because it ignores the population's demographic pattern which is quite important in determining total nutrient requirements, either to a country, region or even to a family.[16] This is especially true because the population (or family) composition has a considerable influence on the total amounts of each nutrient to be required. Economists and nutritionists working in this field and interested in policy questions, are usually dealing with the problem of setting up minimum nutrient requirements in order to attain some economic-nutritional objectives.[17] Again, the procedure has been quite arbitrary because it does not take into account some of the population characteristics such as age, structure, sex composition, physiological state, etc.

Such a consideration leads to the use of a representative or standard family as a basis to establish minimum nutrient requirements and calculate the least cost diets. Such a representative family is focused on the prevailing demographic pattern and population characteristics in both cities such as size, sex and age structure. The average family sizes were estimated to be 5.1 and 5.4 for Recife and Fortaleza, respectively and the age-sex composition is stratifed as seen in Table 1.1.

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[16]Obviously, one may be interested in specific sections of the population (example: low or high income groups) whose size and structure departs from the general population pattern. Yet, it may be interesting to perform a sensitivity analysis in order to see how changes in the average patterns (age, structure, sex, physiological state) would affect the cost of nutrition.

[17]See, for example [1, 19, 50].

Table 1.1. Age and sex composition pattern of the standard family (Northeast of Brazil).

Age groups	Recife		Fortaleza	
	Males	Females	Males	Females
	%	%	%	%
0 - 4	16	13	16	14
5 - 9	15	12	15	13
10 - 14	12	11	12	11
15 - 49	47	53	48	52
> 49	10	11	9	10

Basic Source: Table A-10 (Appendix A).

Another methodological development in this research involves the use and comparison of two computational procedures, that is: the conventional (deterministic) linear programming technique and its stochostic version. The idea, however, is not merely to show the weakness of the conventional method, but to indicate some other alternatives to deal with the problem, as well as to show how to improve its analytical power (standard method) under special circumstances.

The last concern here is probably philosophical rather than methodological. It is felt that in many LDC's where population and income distribution are sensitive political issues, a common denominator has to be found in order to compare and evaluate different plans and policy strategies. To provide a minimum nutritional level for a family is highly desirable and an acceptable social objective. So, it may be used as a basis to set up a poverty line (subsistence level) and/or a point of departure to improve the socioeconomic status of the poor. In other words, poverty is defined by the economic incapacity of a family to feed itself adequately. Given such an objective basis, governments would be more prepared to provide the right amounts of subsidies needed to satisfy other basic family needs such as education, health, housing and so forth.

2. Types and Sources of Data

a. Commodity List

The commodity list used to compute the minimum cost diets relies to a great extent on the Northeast Bank consumer budget studies as well as on information gathered from nutritional experts in the region [48]. The total commodity list included was subdivided into seven categories: cereals and products, roots and tubers, meat, poultry and fish, eggs and milk products, legumes, fruits and vegetables, and fats and oils and miscellaneous. Such a list is supposed to be representative in each city, since it rested upon actual and observed dietary behavior with respect to habits, taboos and food expenditure patterns. The eligible commodity list used to determine the minimum cost diet in each city is confined to those foods consumed by at least twenty percent of the households in each city.[18] The use of conventional foods as a basis to calculate least cost diets has been suggested [50] and it is used in this research to take into account population tastes and preferences. However, the possibility of least cost substitution, between the traditional and other available foods in the region was also considered in this study.

b. Nutritive Content of Foods

The food composition values (proteins, calories, ..., vitamins), are technical information and they are usually obtained from food composition tables, expecially prepared by nutritionists. Because the Brazilian food composition table is quite incomplete and the Northeast food composition table is still under development at the Institute of Nutrition at Federal University of Pernambuco (INUFPe), a variety of sources were used in order to get the nutrient and aminoacids content of the foods used in this research [48].

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[18] Based on the Northeast Bank Consumer Budget Studies [3, 4].

c. Nutritional Requirements

The nutritional requirements to be met in the computation of the minimum cost diets are based on the recommendations made by INUFPe. Such recommendations are supposed to be minimum levels for health for individuals engaged in a regular work activity in the Northeast of Brazil. Needless to say, that such a standard varies with sex, age, physiological state, climate conditions, etc., and that the minimum requirements may not be applicable to all individuals. In addition, it should be emphasized that the suggested levels are actually adjusted data from many other specialized sources such as Food and Agricultural Organization (FAO), World Health Organization (WHO), and Instituto Nacional de Nutricion (Colombia).

When determining least cost diets, the conventional procedure has been to constrain the model with the minimum dietary allowances, but in this study additional constraints (bounds and ratios) were set up in order to make the nutritional model more realistic and operative. Aggregations and additional adjustments were performed in order to take into account sex-age differences in nutritional requirements as well as allowances for lactation and pregnancy (see Appendix Tables A-1, 2, 3, 4, 5, and 6). The age group pattern used in this research is the following: 0-4, 5-9, 10-14, 15-49 and greater than 49 years old.

d. Commodity Prices

There is already a relatively well established market information system in the Northeast of Brazil, but just now they are presently operating only at wholesale level. Therefore, there are no time series data for food products at retail level in that part of the country.[19] This research used cross-sectional data collected by graduate assistants of the Department of Agricultural Economics at Federal University of Ceará, directly from retailers during the period of November 15-25, 1973. The area sampled included only retailers spread around the several local supermarkets, and to a great extent the data represent average prices paid by local consumers.

3. Plan for the Dissertation

The following chapter contains a brief overview of some of the studies which have been undertaken in recent years. Chapter III deals with the theoretical framework used to approach the diet problem. It contains the conventional and an extended formulation of the problem including a stochastic version.

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[19] There are some market studies and price information at retail level [58, 59]. However, those studies just included cereals as well as fruits and vegetables and the prices refer to 1972.

Chapter IV presents the various results and analysis of this study. Chapter V contains conclusions, limitations of the study and suggestions for needed research.

CHAPTER II

REVIEW OF LITERATURE

Literature on least-cost diets is far less extensive than other topics in the economics of nutrition related to economic development. However, some relevant work has been done and this chapter is concerned with some of the past studies that have been undertaken in this specific area. It is not intended to be an exhaustive review of all past studies on least-cost diets and related topics.[1] Instead, it is a summary of the major work in this field with special emphasis on methodological procedures and research achievements. It also attempts to show the relationship of the privious studies to this research, the ways in which they differ, and their possible contributions to this study.

Economists concerned with nutrition and economic development have dealt with problems of least-cost diets

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[1]Although the nature of the problem is essentially the same if one is dealing with animals or human beings, literature on least-cost feed mixes for animals will not be included in this chapter. As a matter of fact, many of the basic procedures and the same programming techniques have been used in solving both types of problems.

since 1941, when Cornfield formulated the problem of a minimum cost diet mathematically.[2] Later on, even before the discovery and development of the simplex method by Dantzig, Stigler computed two diets for an active man weighing 70 kilograms and living in a large city [57]. Stigler's food list included about 70 commodities. The price quotations were averages of many large cities. Stigler did not use mathematical programming to determine the cost of his diet and his daily allowances for nutrients were based on the 1943 version of Recommended Dietary Allowances by the National Academy of Sciences, U.S.A. [42].

Beckman, in 1959, was the first author to consider amino acid requirements (at least indirectly) by computing a subsistence diet as a classroom exercise. His diet was computed on a per-capita basis for a male 45 years old. Both traditional and nontraditional foods were included in the commodity list. Minimum daily requirements were based on the 1958 Recommended Dietary Allowances [43] and the prices were average retail prices for 1959 in two specific cities of the United States.

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[2] The reference to Cornfield's work is found in Smith [50, pp. 12]. It should be emphasized that the studies related with economical diets started even before the 1940's [57, pp. 213].

Three Yugoslavian authors have been quoted as the first ones to consider the minimum cost diet matter in a broader perspective by comparing their physiologically required diets with actual food expenditures.[3] Linear programming was the analytical tool used by the authors in such a study. They concluded that a typical family of four was unable to buy the physiologically required diet at the time of the research. After these pioneer works, the major development in this area of the economics of nutrition has paralleled the gradual expansion of mathematical programming techniques and the growing concern for nutritional problems as related to economic development.

A book written by Smith [50] in 1963 seems to be the first concise and comprehensive statement about the methodology, analytical tools and the importance of determining least-cost diets for human beings. In Chapter II of this book, Smith presents one of his first nutritional models which was devised for a family of three (45 year old male and female and their 18 year old daughter) for four weeks. The model included a list of traditional foods consumed by families in East Lansing. The prices were average prices paid by 176 families who participated in the Michigan State University

[3] The references to these other two pioneer works (Beckman and the Yugoslavians) are also found in Smith [50, pp. 13, 14].

consumer panel. Structurally, such a model still falls within the conventional patterns, but by setting up a range for the caloric content in the diet it departed from the old formulations and represented an improvement in terms of nutritional theory. In the book the author goes beyond a simple least-cost diet formulation by opening the subject towards other theoretical and practical issues such as: diet palatability, costs of habits and preferences, minimum cost substitutions of foods, and marginal efficiencies of foods and the effects of seasonality on the total cost and composition of the diets.

Again, Smith, in a subsequent study [52], improved his basic formulations, especially with respect to the protein allowances to be considered in determining the least-cost diets. He shifted away from the conventional method of setting up minimum allowances for proteins (in quantitative terms only) and started to develop a model which would provide the calculated least-cost diets with proteins of a specific quality or range of qualities.[4] Because the protein quality depends mainly on its amino acid pattern, he

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[4]Protein quality is an attribute of the protein *per se* by itself. The efficiency of utilization will depend on both the quality and the quantity of protein in the diet, along with the adequacy of the total diet, environmental conditions and the physiological state of the recipient [17, p. 65].

included additional constraints in order to assure that the diet would provide an appropriate amount of fully utilizable protein.[5] Still a further extension brought about by Smith, at that time, was to make adjustments in order to make sure that no tryptophan would be diverted from tissue formation for use in the synthesis of nicotinamide.

Finally, recognition that the efficiency of any protein in tissue formation depends on both quantity and quality, together with the adequacy of the diet as a whole, made Smith bring an additional improvement on previous work and come up with a diet model with protein quality variable [56]. At the present time, it seems to be the most advanced and consistent formulation of the problem, because it was devised to determine simultaneously the most economical combination of quality and quantity of protein in the diet. The model defines and starts from the structure of a fully utilizable protein, but in addition, it takes into account the quantitative relationship between protein and calories in the diet.[6] The

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[5] A fully utilizable protein is defined as a protein which contains the essential amino acids in the required proportions [56].

[6] As mentioned before, the efficiency of utilization of the proteins depends not only on its amino acid pattern, but also on other factors present or absent in the total diet (see footnote 4).

model has been applied empirically in Nigeria twice where the nutrition restraint is just a portion of a general model that includes agricultural production activities. More precisely, the author was concerned with determining the set of agricultural activities which provides acceptable levels of nutrients in the most economical way [53, 55].

Out of the United States many of the studies have been referred to in the literature. One of these studies is not published in English, but it is listed in the bibliography as an additional source for those who may have access to it [66]. Two other studies were conducted by Fonseca [22, 23] in India and both were concerned with the cost of an adequate diet for an industrial worker class family consisting of the father, mother and two children below the age of fourteen. The study covered over forty industrial centers in India and its major objective was to provide a basis for determination of a need-based wage for a family unit. Minimum daily requirements were set up by National Nutritional Advisory Committee and linear programming was the technique used to calculate the minimum cost diets. Seventeen food items usually consumed by the industrial workers were initially selected and the diets were calculated at both 1960 and 1967 prices. The author found that calorie and calcium requirements have been fulfilled precisely, while the quantity of the other nutrients were above the minimum requirements. Sensitivity analysis was performed in order to evaluate the seasonality effect on the total cost and composition of the minimum cost diets. In addition, the calculated minimum cost diets were added together with other needs of the worker (clothing, housing and miscellaneous) and used as a basis for determining minimum wages in the studied areas.

Several important studies have been made in Latin America during recent years. Florencio [19], the first nutritionist to work on the problem, used a linear programming nutrition model to determine the efficiency of food expenditures among working-class families in eight cities in Colombia. The model design follows conventional patterns, although it handles the caloric and B vitamin content of the diet more accurately than conventional versions.[7] Another departure from the conventional model was to set up protein allowances on two levels depending upon the biological

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[7]Conventional versions mean the straightforward mathematical statements of the nutritional problem as referred to in other studies and intermediate textbooks [1, 13, 47].

value of the diet. In this research the data about daily nutritional requirements was provided by the Instituto Nacional de Nutricion, Colombia, complemented by the dietary standards formulated by National Research Council in the United States. The least-cost diets were calculated on a family basis, and the nutritional allowances were computed by summing up the individual allowances for the average number of persons eating per day. In computing this average the number of persons eating each meal was weighed by the relative importance of the meal.[8] Among the major findings was that it was found the working-class families spent sufficient money to buy adequate nutrition but failed to obtain the necessary nutrients because a significant portion of their food expenditures was spent for nonnutritional objectives. Some of the foods in these calculated least-cost diets had marginal efficiencies of 100 percent; i.e., the aggregate monetary value of their nutrients was equal to their market price.

In another Colombian study, the IIT (<u>Instituto de Investigaciones Tecnologicas</u>, Bogotá, Colombia) calculated least-cost diets for twenty cities around

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[8]In determining the magnitude of the nonnutritional component of food expenditures, the author used the concept of a standard family made up of six persons including an adult male, a pregnant female and four children with ages ranging from one to nine years.

the country [32]. The diets were calculated on a per capita basis (by specific age groups), constrained by ten nutrients and the essential amino acids. Additional constraints were imposed in order to limit the maximum amounts of specific foods in the least-cost diet, domestic supply of foods and the amount of calories to be provided in the diet. The data on the nutritive value of foods were derived from two major sources: the Colombian Food Composition Table and the FAO Amino-Acid Content of Foods. Nutritional daily requirements were the recommended allowances for the Colombian population made by the Instituto Nacional de Nutricion, and the prices were yearly averages for 1971. The research was primarily designed to serve as a basis of a multidisciplinary approach to malnutrition in that country and many other related points were investigated such as: the effect of changes in food prices in the composition of the least-cost diets, the effect of marketing margins on the total cost of the diets and the possibility of reducing the total cost of the diets obtained by introducing nutritional supplements (synthetic amino acids, enriched foods and new agricultural varieties).

Still in Colombia, two other papers by Florencio and Smith [20, 21] have showed the following: (a) the families failed to obtain an adequate nutrition because given the levels of expenditure about 23 to 63 percent of the money went for nonnutritional objectives; (b) by using three-fifths of the actual amount of money spent on food, the families could have bought a least-cost diet which would have provided the same amount of nutrients; (c) the percentage contribution of the various food groups in the total food expenditures differ considerably in the actual and the calculated least-cost diets; (d) cereals were the major source of calories and proteins for both actual and the least-cost diets; (e) family size and composition had little effect upon the composition of the least-cost diets, while city-to-city differences were quite significant. Prices for foods used in determining the least-cost diets for the 38 families included in these studies were provided by the Colombian Departamento Administrativo Nacional de Estadistica and the food composition data was obtained from a food composition table prepared by the Instituto Nacional de Nutricion (Bogotá).

In Brazil, at least three studies in this field must be mentioned. The first was done by Langier [34] in 1967 and it dealt primarily with a method to measure the nutritional contribution of different foods and obtaining economical diets.[9] It is claimed that linear programming requires mathematics probably beyond the training of nutritionists and that its use in LDC's can be additionally constrained because of the costs and shortage of computers and skilled personnel. Yet, contrary to linear programming, such a method can provide a measure of the nutritional contribution of food, whether or not the resources available permits the full set of nutritional requirements to be met.[10] An empirical application of the method was performed to obtain economical supplementary diets for families with deficient nutritional intake in four villages in the Northeast of Brazil.

The basic data used in Langier's research were derived from a field survey conducted by nutritionists of the Brazilian Ministry of Health through the period 1959-61. Information concerned with the nutrient

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[9]A more concise statement about Langier's method can be found in [35, pp. 13-45].

[10]The author's statements may be very true at the time of his research but they are far beyond being generalized. He even did not show how inexpensive and efficient his method should be if one is dealing with about 100 commodities and 15 to 20 restrictions. In addition, the idea that linear programming is not a feasible technique under resource unavailability because the people cannot buy the least-cost diets, frequently ignores the possibility of getting government subsidies in order to improve the economic status of people in the low income class.

content of foods and the recommended allowances were derived from several Brazilian sources. Langier's method is rather intuitive and it consists of a sequence of decision rules which, carefully followed, will lead to an economical diet close to the least-cost diet obtained by linear programming as shown in Chapter V of his dissertation.

Still in the Northeast of Brazil, the Northeast Development Bank in a consumption budget study including five cities in the urban northeast, used linear programming to determine least-cost diets [1]. The nutritional model used was again of the conventional type[11] and the study was very aggregative, with respect to some of the basic data used.[12] Nutritional daily requirements as well as the nutrient content of foods were derived from several sources, including the ICNND[13]

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[11]See footnote 7.

[12]This was specially true with respect to the price figures since that the data used were extrapolated from one city to all other cities together. Such a procedure of course can completely invalidate the results obtained, provided that a considerable city-to-city price variation exists. In other words, it is quite possible that the calculated diets are not really the least-cost diets for most of the other cities.

[13]Interdepartmental Committee on Nutrition for National Development [33].

research and other Brazilian studies. The food list included about 37 commodities and the prices were averages paid by local consumers in one of the cities at the time of the research. The diets were calculated on a per capita basis and some comparisons between the income levels of the household with the expenditures on a nutritionally and physiologically acceptable diet were performed. They found that riboflavin was the most expensive nutrient for the urban northeast--it costs about Cr $80.00/mg, and it was followed by fat, protein, vitamin C, calories and vitamin A.[14] At least at the time of the research urban income levels were referred to as sufficient to buy enough of the right foods in that part of the country.

Finally, Patrick and Simões [46] conducted a study on least-cost diets in the State of Goias (City of Cristalina) in 1969. Using linear programming the authors calculated least-cost diets for three socio-economic groups and compared them with their food expenditure patterns and incomes.[15] Minimum daily

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[14]The relative importance of fat (high cost) compared with the other nutrients may be explained by the high level of nutrient requirement imposed, which was 77 gm/person/day.

[15]The available reference does not show the mathematical formulation of the problem, but it seems to be of the conventional type (see footnote 7) with additional restrictions to deal with consumers' tastes and preferences.

requirements were based on the recommended dietary allowances given by the National Research Council, U.S.A. [43]. The diets were calculated for a family of six for a one-week period.[16] The total cost of the least-cost nutritional diets including restrictions to reflect the consumer's tastes and preferences were reported to be 50 percent higher than the basic one,[17] and the savings obtained by adopting the least-cost diets were quite substantial for all socioeconomic groups.

In summary, there is a considerable overlapping and similarity in the objectives and methodological procedures used by several authors, especially in the field of applied research.[18] However, this does not mean that these studies are irrelevant as a methodological basis for those interested in the economics of nutrition. At least they have suggested additional explorations in the methodology as well as how the

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[16]There is no indication about family composition and how the total family nutrient intake was computed.

[17]That is, the initial least-cost diets, which were calculated without such restrictions.

[18]Exceptions must be made for Langier's work which is unique and essentially methodological as well as some of Smith's later work (both theoretical and applied) dealing with mathematical nutritional models to calculate least-cost diets.

research results in this field can be used as a basis for policy recommendations.

None of these studies attempted to define a representative or standard family based on demographic patterns.[19] The general procedure was to assume a standard family size and/or given that to assume a certain age structure and family composition as a basis to calculate the least-cost diets.[20] Such a procedure, although not incorrect, is not the most appropriate way to handle the problem, especially if one is interested in answers to questions concerned with the population as a whole.[21] Secondly, in spite of the importance of the demographic factor for many LDC's, the effect of population structure on the total cost of the least-cost diets has either not been emphasized or has been overlooked in all the studies.

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[19]That is, a family which is representative in terms of size, age structure and sex composition.

[20]Some have argued that the standard family should be as representative as possible, but even those never used the population demographic characteristics to determine it [21, p. 225].

[21]According to nutritional theory, dietary allowances varies with age and sex along with other physiological and environmental conditions. So, given the population structure by assuming an identical pattern for a family of a known size, one can determine precisely the average nutrient intake for a family, state, region or country.

Additional comments must be directed to the Brazilian studies where the models used turned out to be very crude simplifications of the nutritional problem involving the determination of least-cost diets; i.e., they just follow the simplified mathematical formulations of the problem with no explicit recognition on the foundations of the nutritional theory.

CHAPTER III

THEORETICAL FRAMEWORK FOR DETERMINING LEAST COST DIETS

Minimum cost diets constrained by nutritional requirements implies the allocation of limited resources in order to meet desired objectives. The problem can be handled adequately by mathematical programming techniques. Needless to say, the widespread use of the linear programming technique to approach the diet problem is not a requirement by itself. There are many practical as well as theoretical reasons to use it, although in some circumstances other programming techniques such as non-linear programming, stochastic programming, etc. may be appropriate.

As mentioned in Chapter II, attempts to formulate a low-cost diet come from the early 1940's, even before the discovery of the mathematical programming technique. At that time Stigler [57] formulated a nutritional diet problem and determined the types and amounts of each food which would satisfy the daily minimum nutrient requirements at very low cost (not a minimum!). More recent attempts at formulating diets for human beings using linear programming have met great acceptance among nutritionists and economists who are interested in nutritional problems [1, 19, 32, 46, 50].

A. The Standard Mathematical Formulation of the Diet Problem

The standard linear programming approaches to the diet problem include both the deterministic and stochastic versions. The first one has been emphasized in many intermediate textbooks [13, 18, 28, 47, 63] as a classical example of a linear programming problem, and it has been used in practical research to determine minimum cost diets [1, 19, 32, 50]. The stochastic[1] approach has been the concern of more advanced papers [15, 16, 40] and some applications have been made to determine animal least-cost feed mixes [7, 65].

In any event, the mathematical statement of this programming problem (low cost feed mixes or human diets) involves a set of simultaneous linear (or non-linear) equations and/or inequalities which represent

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[1]The word "stochastic" is quite general and in a linear programming formulation it can refer to both the objective function and/or the constraints. Here such a term applies specifically to the special case in which the elements of the technology matrix may be subject to random variations.

the restrictions of the problem and an objective to be attained, that is:

$$\text{MIN} \quad C = P_1x_1 + P_2x_2 + \ldots + P_nx_n$$

subject to the conditions:

$$\begin{array}{l}
a_{11}x_1 + a_{12}x_2 + \ldots a_{1n}x_n \geq b_1 \\
a_{21}x_1 + a_{22}x_2 + \ldots a_{2n}x_n \geq b_2 \\
\vdots \qquad \vdots \qquad \vdots \qquad \vdots \\
a_{m1}x_1 + a_{m2}x_2 + \ldots a_{mn}x_n \geq b_m
\end{array}$$

and

$$\begin{array}{llll}
x_1 & & & \geq 0 \\
& x_2 & & \geq 0 \\
& & \ddots & \vdots \\
& & x_n & \geq 0
\end{array}$$

or more concisely:

$$\text{MIN} \quad C = \Sigma P_j x_j \qquad (j=1,\ 2\ldots\ldots,n)$$

subject to:

(1) $x_{j's} \geq 0$

(2) $\sum_j^n a_{ij}x_j \geq b_i \qquad (i=1,\ 2\ldots\ldots,m)$

Where:

C = the total expenditure on foods purchased.

P_j = the unit price of jth food.

x_j = the unknown quantity of the jth food to be purchased or consumed.

m = the number of nutrients.

n = the number of foods.

a_{ij} = the amount of ith nutrient supplied by a unit of the jth food.

b_i = the required level of ith nutrient.

Such a linear programming structure is deterministic in nature, i.e., it is assumed that both the objective function and constraints are not subject to stochastic variations. However, the need for taking into account the random variations in the input-output coefficients (a_{ij}'s) has been recognized, because the nutrient content of the inputs may be subject to significant variation [7, 16, 65]. Technically speaking, the food composition tables are based on estimates of the true mean composition which is not fully known. Nutritionists recognize that food composition varies from sample to sample and, therefore, one may take into account explicitly when trying to meet specified nutritional requirements. Empirically there are also strong reasons for taking into account the a_{ij}'s variability. For instance, most of the LDC's do not have food composition table and to a great extent they have relied on many different sources in order to get the nutrient content of their foods. However, when even the experts in the field do not know what is the best source to use, the choice of the appropriate data becomes just a matter of judgment. Given this fact, one way to handle the problem is perhaps to combine all of the available information (most commonly used in that particular country or region), and get an average as well as the degree of variability (variance).

In any event, the problem can be handled properly, by converting the conventional deterministic constraint into a probabilistic one, by requiring that the probability of a certain nutrient (for example: proteins, calories, . . . etc.) in the final mix to be equal to or greater than a certain minimum. Mathematically, if $\sum_{j}^{n} a_{ij}x_j$ is denoted as the sum of the quantities $a_{ij}x_j$ provided by each food, one should replace the conventional constraint:

$$\sum_{j}^{n} a_{ij}x_j \geq b_i, \qquad (i=1, 2\ldots,m)$$

and rewrite it as follows:

$$(1) \qquad P\left\{\sum_{j}^{n} a_{ij}x_j \geq b_i\right\} = 1 - \varepsilon_i$$

Where:

$P\{\ \}$ = the probability that the sum of the ith nutrient on jth food equals or exceeds b_i.

$1 - \varepsilon_i$ = the required minimum probability.

ε_i = the probability that a given nutrient is below the constraint level, b_i.

and:

a_{ij}, x_j and b_i have the same meaning as before.

By assuming that the a_{ij}'s are independent random variables and that $\sum_j^n a_{ij}x_j$ is normally distributed with mean of $\sum_j^n x_j\mu_{ij}$ and variance $\sum_j^n x_j^2\sigma^2_{ij}$, that is: $\sum_j^n a_{ij}x_j$ is $N\left(\sum_j^n x_j\mu_{ij}, \sum_j^n x_j^2\sigma^2_{ij}\right)$, it can be converted into a standard normal distribution $Z\ N(0,1)$ and the expression (1) will take the equivalent form:

$$(2) \qquad P\left\{ Z \geq \frac{b_i - \sum_j^n x_j\mu_{ij}}{\left(\sum_j^n x_j^2\sigma^2_{ij}\right)^{1/2}} \right\} = 1 - \varepsilon_i$$ [2]

If one takes Z_o to be $1 - \varepsilon_1$ first quantile of Z i.e.: $P\{Z \geq Z_o\} = 1 - \varepsilon_1$ then (2) implies,

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[2]Note that $1 - \varepsilon_i$ is the required minimum probability and for any value of ε_1 there is a corresponding Z value in the standardized normal curve (it will be labeled here as ψ). Such Z values are negative for any confidence level $(1 - \varepsilon_i)$ greater than .5 and positive for any $(1 - \varepsilon_i) < .5$.

$$\frac{b_i - \sum_j^n x_j \mu_{ij}}{\left(\sum_j^n x_j^2\ \sigma_{ij}^2\right)^{1/2}} \leq Z_o$$

Let $Z_o = \psi$ and it follows:

$$b_i - \sum_j^n x_j \mu_{ij} \leq \psi\left(\sum_j^n x_j^2 \sigma_{ij}^2\right)^{1/2}$$

$$- \sum_j^n x_j \mu_{ij} \leq \psi\left(\sum_j^n x_j^2 \sigma_{ij}^2\right)^{1/2} - b_i$$

$$\sum_j^n x_j \mu_{ij} \geq b_i - \psi\left(\sum_j^n x_j^2 \sigma_{ij}^2\right)^{1/2}$$

and it follows that:

(3) $$\sum_j^n \mu_{ij} x_j + \psi\left(\sum_j^n x_j^2 \sigma_{ij}^2\right)^{1/2} \geq b_i$$ [3]

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[3]Note that the chance constraint (3) may also be written as follows: $\sum_j^n x_j \mu_{ij} \geq b_i - \psi\left(\sum_j^n x_j^2 \sigma_{ij}^2\right)^{1/2}$, where ψ is assumed to be negative for any $(1 - \varepsilon_i) > .5$. Also observe that in such an expression the covariances among the nutrients ($i = 1 \ldots m$) contained in the foods j and k ($j \neq k$), i.e.: $\sum_j \sum_k \sigma_{ij} \sigma_{ik} x_j x_k$ is assumed to be zero. In other words, it is assumed that the nutrient content of the foods in the diet are not interdependent. Although such an assumption deserves some qualification, there is in practice other reasons for assuming that, as for example, the lack of the covariance data and the complications which may arise in the computational procedures.

The detailed stochastic programming model would be written as follows:

$$\text{MIN} \quad C = P_1x_1 + P_2x_2 + \dots P_nx_n$$

subject to the conditions:

$$x_1E(a_{11}) + x_2E(a_{12}) + \dots x_nE(a_{1n}) + \psi\left[x_1{}^2\text{Var}(a_{11}) + x_2{}^2\text{Var}(a_{12}) + x_n{}^2\text{Var}(a_{1n})\right]^{1/2} \geq b_1$$

$$x_1E(a_{21}) + x_2E(a_{22}) + \dots x_nE(a_{2n}) + \psi\left[x_1{}^2\text{Var}(a_{21}) + x_2{}^2\text{Var}(a_{22}) + x_n{}^2\text{Var}(a_{2n})\right]^{1/2} \geq b_2$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

$$x_1E(a_{m1}) + x_2E(a_{m2}) + \dots x_nE(a_{mn}) + \psi\left[x_1{}^2\text{Var}(a_{m1}) + x_2{}^2\text{Var}(a_{m2}) + x_n{}^2\text{Var}(a_{mn})\right]^{1/2} \geq b_m$$

and:

$$x_1 \geq 0$$

$$x_2 \geq 0$$

$$\ddots$$

$$x_n \geq 0$$

or

$$\text{MIN} \quad C = \Sigma P_jx_j \qquad (j=1,2.....,n)$$

subject to:

(1) $x_j \geq 0$

(2) $\sum_j^n x_j \mu_{ij} + \psi(\sum_j^n x_j^2 \sigma_{ij}^2)^{1/2} \geq b_i$ (i=1,2.....,m)

where:

$$\sum_j^n x_j \mu_{ij} = \sum_j^n x_j E(a_{ij})$$

$$\sum_j^n x_j^2 \sigma_{ij}^2 = \sum_j^n x_j Var(a_{ij})$$

μ_{ij} = the mean content of ith nutrient in the jth food.

σ_{ij}^2 = the variance of the ith nutrient contained in the jth food.

ψ = a value of Z corresponding to $P(Z \geq \psi) = 1 - \varepsilon_i$, where Z is N(0,1).[4]

and:

P_j, x_j and b_i have the same meaning as before.

It must be noticed that the sample means and variances are actually used to estimate μ_{ij} and σ_{ij}^2; the true population values. In addition, it must be emphasized that the stochastic linear programming version departs from the deterministic one in two major points. (i) The conventional linear programming (LP) problem is turned into a nonlinear programming (NLP) with a linear objective

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[4]Similarly for small sample sizes and σ_{ij}'s unknown, the student's t distribution can be used in place of the standard normal.

function and nonlinear constraints.[5] (ii) The introduction of a standard deviation in the new constraint, which will cause the average level to be above the minimum level.[6] One practical and economic implication of such a transformation is that it will increase the total cost of the diet, because more of all or some food will be required to meet the new specified level, b_i.

B. An Extended Linear Programming Formulation to Approach the Diet Problem

The stochastic version as shown earlier can bring an improvement in the deterministic LP formulation, mainly because the a_{ij}'s (input-output coefficients) are subject to significant variation. In the overall, however, both are still very crude simplifications of the diet problem from a nutritional theory viewpoint. For instance: in many cases the quantity of a nutrient needed cannot be specified without knowing the quantity of other nutrients: i.e., the b_i's cannot be specified

[5] There are alternative solution techniques for this NLP problem. Some have suggested to keep the solution within the LP framework by linearizing the variance [7]. Panne and Popp [65] have used Zoutendijk's method of feasible directions to solve it, and still another alternative solution has been suggested by Evers [16].

[6] See footnote 3. In addition, note that such a new constraint will increase the LP tableau by one row and one column.

a priori, and in order to determine them one needs to impose additional constraints. A typical example is the B vitamin requirement which depends upon the calorie content in the diet. An additional problem arises with respect to palatability constraints as well as with the calorie/protein allowances in the diet.

The approach to dealing with the palatability constraint in this study was quite pragmatic, mainly because some of the suggested ways to deal with it become increasingly complicated when the commodity list is large.[7] To get a minimum cost reasonably palatable diet we have relied on conventional foods as well as on actual dietary behavior with respect to habits, tastes, and food expenditures in the Northeast region. On the other hand, in this research the protein/calories allowance was approached as follows:

1. Model A

First, the possibility of random variations in the protein/calorie content was considered so the

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[7]One practical way suggested to deal with the palatability in the diet is to impose some maximum or minimum allowances for certain kinds of foods, which are supposed to come in the food mix [61]. As a matter of fact, such a procedure can work relatively well in formulating animal feed mixes, especially when the number of inputs involved is quite small. Still a more theoretical approach to the problem would be to use an "index of desirability" as developed by Wolfe [50, pp. 29-30]. The problem here, of course, is how to get the fatigue functions.

corresponding stochastic constraint was turned into a probabilistic one. Secondly, protein requirements are specified in terms of total and animal protein and minimum levels were set up for each one. Such a procedure was based on the foundations of nutrition theory as well as some empirical evidence about price and food production patterns in the Northeast of Brazil. For instance: protein from plant sources is deficient in certain essential amino acids,[8] specially lysine, tryptophan and methionine, while protein from animal sources is a rich source of all of these. Provided that the cost of plant protein is usually lower than the cost of animal protein, by letting free the required amounts of each kind of protein the results would probably be a diet which is not nutritionally adequate to maintain nitrogen balance and normal growth.[9]

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[8]Protein is a composite of several related substances called amino acids. The amino acids can be broken in two major groups: the essential and the nonessential amino acids. For an adult man, eight amino acids are essential: isoleucine, leucine, lysine, methionine, phenylalamine, threonine, tryptophan and valine. The infant requires all of these plus histidine.

[9]Someone would ask why a similar restriction was not imposed to plant protein. The answer is straightforward, that is: because plant protein is cheaper than animal protein it would come out in the diet anyway, and probably in a greater amount than the animal protein, in order to fulfill the general requirement for total protein.

Finally, the total calories to be provided in the diet by this model are constrained by an upper and lower bound, and the right amount is determined inside the model.[10] In addition, other constrainst were imposed (in a ratio form) in order to deal with the calories and B vitamin minimum allowances, i.e., the amount of thiamin, riboflavin and niacin will be determined simultaneously together with the amount of calories to be provided in the diet. Therefore, the nutritional model can be written as follows:

	MIN $C = \Sigma P_j x_j \quad (j=1,2,\ldots,n)$		Make C the total expenditure on foods as small as possible.
subject to:			
(1)	x_j	≥ 0	No negative quantities of food can occur.
(2)[11]	$\sum\limits_{j}^{n} a_{1j}x_j + \psi(\sum\limits_{j}^{n} x_j^2\sigma_{1j}^2)^{1/2}$	$\geq b_1$	There is a probability (say 95 percent) that the total protein in the diet will exceed the required amount b_1.

[10]Excess of calories will be either wasted or stored as fat--so this can be physiologically harmful. For this reason and other computational advantages, it was established a range within the total calories in the diet can vary.

[11]Restrictions (2) and (3) as well as (6) and (7) were linearized by a proxy method [7, p. 615], in order to keep the solution of the system within the linear programming framework.

(3)[12] $$\sum_{j=1}^{k} a_{2j}x_j + \psi\left(\sum_{j=1}^{k} x_j^2\sigma_{2j}^2\right)^{1/2} \geq k_1 b_1$$

There is a probability of (say 95 percent) that the animal protein in the diet will be at least a proportion k_1[13] of the total required protein in the diet.

(4) $$\sum_{j}^{n} a_{ij}x_j \qquad (i=3,4\ldots8) \qquad \geq b_i$$

The total amount of each of six nutrients shall equal or exceed the required amount of each.[14]

(5) $$\sum_{j}^{n} a_{9j}x_j - x_{(n+1)} = 0$$

(6) $$\sum_{j}^{n} a_{10j}x_j + \psi\left(\sum_{j}^{n} x_j^2\sigma_{10j}^2\right)^{1/2} \geq b_{(i+1)}$$

There is a probability (say 95 percent) that the total calories in the diet shall be equal or greater than the minimum caloric allowance $b_{(i+1)}$.

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[12]Observe that a_{ij} and a_{2j} all refer to the protein nutrient in food j. On the other hand, the subscript j runs from (1...n) for total protein, and it runs from (1...k) for animal protein. In addition, note that in a general matrix form, each variance term will increase the matrix by one row and one column, so the subscript i will take the appropriate value according to its position in a row.

[13]The k values in this model (k=1,2,...4) are given constants and they will be specified according to the nutritional objectives to be attained.

[14]These nutrients include: fat, calcium, phosphorous, iron, vitamin A and vitamin C.

(7)[15]	$\sum_{j}^{n} a_{11j}x_j + \psi(\sum_{j}^{n} x_j^2\sigma_{11}^2)^{1/2}$	$\leq b_{(i+2)}$	There is a probability (say 95 percent) that the total calories in the diet should be equal or less than the maximum caloric allowance $b_{(i+2)}$.
(8)	$\sum_{j}^{n} a_{12j}x_j - x_{(n+2)}$	$= 0$	
(9)	$x_{(n+2)} - k_2\, x_{(n+1)}$	≥ 0	The quantity $x_{(n+2)}$ of the thiamine needed to go with the calories in the diet, shall be equal or greater than a proportion k_2 of the calories in the diet.
(10)	$\sum_{j}^{n} a_{13j}x_j - x_{(n+3)}$	$= 0$	
(11)	$x_{(n+3)} - k_3 x_{(n+1)}$	≥ 0	The quantity $x_{(n+3)}$ of the riboflavin needed to go with the calories in the diet shall be equal or greater than a proportion k_3 of the calories in the diet.
(12)	$\sum a_{14j}x_j - x_{(n+4)}$	$= 0$	

[15]Observe that a_{9j}, a_{10j} and a_{11j} all refer to calories, i.e., $a_{9j} = a_{10j} = a_{11j}$.

(13)	$x_{(n+4)} - k_4 x_{(n+1)}$	≥ 0	The quantity $x_{(n+4)}$ of the niacin needed to go with the calories in the diet shall be equal or greater than a proportion k_4 of the calories in the diet.
(14)	$\sum_{j}^{n} a_{ij} x_j \quad (i=15,16...22)$	≥ 0	The total amount of each of the eight amino acids shall be equal or exceed zero.[16]

Deterministic vs. probabilistic constraints (a theoretical digression)

The mathematical solution of the nonstochastic restrictions (2) and (3) in Model A implies a simultaneous fulfillment of the two conditions or stated requirements. However, by making them as chance constraints, this is not necessarily true, because the probability of fulfilling simultaneously all of those conditions is not implied by the fulfillment of each condition separately. For instance, if one denotes restrictions (2) and (3) in Model A as follows:

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[16]Given that restrictions (1) through (13) are fulfilled, by introducing restriction (14) an excess is left over. Such an excess compared with the suggested pattern of amino acid requirement, make possible to show how adequate is the calculated least cost diet, in providing all of the essential amino acids.

$\sum_{j}^{n} a_{1j}x_j = A + V =$ total protein in the diet.

$\sum_{j}^{k} a_{2j}x_j = A \quad =$ total animal protein in the diet.

The objective is to minimize some objective function $C = \Sigma P_j x_j \quad j = (1, 2 \ldots n)$ subject to:

$$A + V = b_1$$

$$A \quad = k_1 b_1$$

By solving this system simultaneously, mathematically it should get the feasible region; that is, all of the above conditions should be mathematically satisfied.

Now, let A be the event that restriction (2) will be satisfied. The probability of the occurrence of event A can be stated as follows:

$$P\ (A + V \geq b_1) = 1 - \varepsilon_1 \qquad \text{or}$$

$$P\ (\sum_{j}^{n} a_{1j}x_j \geq b_1) = 1 - \varepsilon_1.$$

Also, let B be the event that restriction (3) will be satisfied. The probability of the occurrence of event B can be stated as follows:

$$P\ (A \geq k_1 b_1) = 1 - \varepsilon_1 \qquad \text{or}$$

$$P\ (\sum_{j}^{k} a_{2j}x_j \geq k_1 b_1) = 1 - \varepsilon_1.$$

The probability that both conditions will be simultaneously fulfilled is given by the intersection of A and B[17] and one should denote such event as D.

$$P\ (A + V \geq b_1,\ A \geq k_1 b_1) = 1 - \varepsilon_1.$$

In other words, Event D implies a more stringent side condition than the restrictions represented by events A and B separately. The practical implications of this are that by changing those restrictions to chance constraints we can get quite different results depending upon how we define the nature of our constraints. The important point is that for a given probabilistic level, fulfilling simultaneously those two restrictions (2 and 3) implies normally that one would have met each one separately, but the opposite is not true. For instance: in order to be 95 percent confident that the restrictions (2) and (3) would be fulfilled simultaneously, one should impose a higher confidence level (say x > 95 percent) for each restriction separately.

The burden of proof for this argument is given by a fundamental law in statistics known as DeMorgan's Law [38], which states the following:

> If there are n events, called A_1, A_2 ... A_n the probability that all of them occur simultaneously is greater or equal to one, minus the sum of the probabilities of A_1 complement, plus the A_2 complement + . . . plus A_n complement.

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[17]In statistical notation it implies $P\ (A \cap B)$.

In other words:

$$(4) \quad P\ (A_1 \cap A_2 \ldots \cap A_n) \quad 1 - \left[P\ (A_1{}^c) + P\ (A_2{}^c) + \ldots P\ (A_n{}^c)\right].$$

So, if there is a requirement that all of the previous restrictions represented by events A and B should be fulfilled simultaneously at least 95 percent of the time, it would require each restriction to be satisfied 97.5 percent of the time. For example, given that:

$$P\ (A) \geq 1 - \varepsilon_1 = .975 = P\ (A^C) \leq \varepsilon_1 \leq .025$$

$$P\ (B) \geq 1 - \varepsilon_1 = .975 = P\ (B^C) \leq \varepsilon_1 \leq .025$$

it follows according to expression (4) that:

$$\begin{aligned} P\ (A \cap B) &\geq 1 - \left[P\ (A^C) + P\ (B^C)\right] \\ &\geq 1 - \left[.025 + .025\right] \\ &\geq .95 \end{aligned}$$

Therefore, DeMorgan's Law provides an intuitive and pragmatic way to overcome the problem since it is possible to attain a satisfactory probability that all of the restrictions are fully satisfied by imposing higher confidence levels for each of them.

Similarly by fulfilling separately restrictions (6) and (7) in model A at some probabilistic level (say

$1 - \varepsilon_1 = .95$) does not imply that both restrictions will be fulfilled simultaneously at least for the same confidence level. To simplify, let the total calories provided by the diet ($\sum_j^n a_{10j}x_j = \sum_j^n a_{11j}x_j$) be equal X a standard normally distributed random variable, that is, "X" is N(0, 1). Therefore, it follows that:

(a) $$P(X \geq b_{(i+1)}) = P\left\{\frac{X-\mu}{\sigma} \geq \frac{b_{(i+1)}-\mu}{\sigma}\right\} = P(Z \geq Z_o).$$[18]

Assuming $1 - \varepsilon_1 = .95$, expression (a) implies finding the area to the right of Z_o under the standard normal distribution curve, that is:

$$= P\ (Z \geq -\ 1.645)$$
$$= P\ (-1.645 \leq Z \leq 0) + P\ (Z \geq 0)$$
$$= .45 + .50$$
$$= .95$$

(b) $$P\ (X \leq b_{(i+2)}) = P\left\{\frac{X-\mu}{\sigma} \leq \frac{b_{(i+2)}-\mu}{\sigma}\right\} = P\ (Z \leq Z_o)$$

Assuming $1 - \varepsilon_1 = .95$, expression (b) implies finding the area to the left of Z_o under the standard normal distribution curve.

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[18]Note that Z_o has been labeled ψ along this chapter.

$= P\ (Z \leq 1.645)$

$= P\ (0 \leq Z \leq 1.645) + P\ (Z \leq 0)$

$= P\ (0 \leq Z \leq 1.645) + P\ (Z \geq 0)$

$= .45 + .50$

$= .95$

On the other hand, to satisfy simultaneously restrictions (6) and (7) would correspond to find the area under the normal distribution curve as follows:

(c) $P\ (b_{(i+1)} \leq X \leq b_{(i+2)})$

$$= P \left\{ \frac{b_{(i+1)}-\mu}{\sigma} \leq \frac{X-\mu}{\sigma} \leq \frac{b_{(i+2)}-\mu}{\sigma} \right\}$$

$$= P\ (-Z_o \leq Z \leq Z_o)$$

Again, assuming $1 - \varepsilon_1 = .95$, expression (c) implies finding the area in between $-Z_o(-1.645)$ and $Z_o(1.645)$ under the normal distribution curve, that is:

$= P\ (-1.645 \leq Z \leq 1.645)$

$= P\ (0 \leq Z \leq 1.645) + P\ (-1.645 \leq Z \leq 0)$

$= 2P\ (0 \leq Z \leq 1.645)$

$= .90.$

Therefore, as it has been shown before, the simultaneous fulfillment of conditions (6) and (7) at least 95 percent of the time would require setting up a higher Z_o^* $(Z_o^* > Z_o)$ value, for each of those conditions separately. In other words, in order to make sure that each condition would be satisfied 95 percent of the time one should set up a Z_o* value for each one equal to 1.96.[19]

2. Model B

Original plans were to use both the deterministic and stochastic versions of this model[20] for determining the least cost diets. However, data and other resources constraints made difficult both applications, specially the stochastic version which would require specific computer programming and other basic information probably beyond the actual stage of knowledge of nutrition theory.[21] It has been suggested, however, that it may be worthwhile to present the theoretical developments, mainly because the basic model is supposed to be one of the most advanced and consistent formulations of the problem up to the present time.[22] A detailed exposition of the

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[19]If the student "t" distribution is used, such a value will also depend upon the number of degrees of freedom.

[20]"A Diet Model with Protein Quality Variable," by Smith [56].

[21]For example: the covariance between the amount of utilizable protein and the total protein in the food.

[22]Also it will provide a chance to other people to apply it, as soon as the data becomes available.

deterministic version of this model is given by Smith [56], and so, we shall concentrate on its basic foundations, and use them as a basis to convert the deterministic version into a stochastic one.

As described in Chapter II, the unique characteristic of the basic model (A Diet Model with Protein Quality Variable) is that the protein allowance in the diet is made up as a function of both its quantity and quality. It was grounded on the foundations of nutrition theory which asserts that the efficiency of any protein in tissue formation depends on both attributes (quality and quantity) together with the adequacy of other components (example: calories) in the diet. Therefore, in such a model, the protein allowance is a function of its amino acid pattern (the proportion among essential amino acids and the ratio of essential to nonessential amino acids) as well as the protein/calories concentration of the diet.

It is also argued that for a protein to be completely utilizable in tissue formation it must contain all of the essential amino acids in the right proportions, and must be a part of a diet which has an adequate quantity of calories. Therefore, the basic formulation starts by defining the structure of a fully utilizable protein[23] that economizes the use of the essential amino acids. It is made by stating the proportions in which the essential amino acids (as well as the ratio among essential to nonessential amino acids) should occur in the diet (Table 3.1).

Table 3.1. The amino acid composition of a fully utilizable protein (U).[24]

Amino acid	Proportions	(k_p)
	(grams per 100 grams of protein)	
Lysine	4.000	(k_1)
Total Sulfur-Containing Amino Acid	1.712	(k_2)
Methionine	1.712	(k_3)
Threonine	3.168	(k_4)
Tryptophan	.992	(k_5)
Unspecified Amino Acids	88.416	(k_6)
Total	100.000	

Source: Smith [56].

[23]"That is, all the protein that is absorbed is fully utilizable" [56, p. 973].

[24]"It is assumed that one hundred grams of this fully utilizable protein will contain 32 grams of the essential amino acids. So, if these 32 grams are distributed in the proportions found in hen's eggs, 11.584 grams will consist of the five essential amino acids listed in Table 3.1, and the remaining 80.415 (including 20.416 grams of essential amino acids) are labeled as unspecified amino acids."

However, even if all protein consumed is fully utilizable (to say, its amino acid pattern is perfect) net protein utilization[25] may be less than 100 percent, because part of the protein absorbed may be used for energy production rather than for tissue formation. Therefore, the exact quantity of fully utilizable protein (U) is determined endogeneously and the amount to be provided will be in between the total amount (quantity) of food protein in the diet and a minimum which will depend upon the quality of the total protein and the calories/protein percent in the diet. Mathematically the protein-calories segment of the model can be written as follows:

System of Equations 1

(i)[26]

$\sum_{j}^{n} a_{1j}x_j$	$=$	$\bar{c}$	Provides a predetermined amount of calories.
$\sum_{j}^{n} a_{2j}x_j - U$	$\geq$	0	Provides food protein at least equal to the quantity of fully utilizable protein.
$\sum_{j}^{n} a_{3j}x_j - k_1U$	$\geq$	0	Provides the minimum amounts of each of the following amino acids: lysine, sulfur-containing amino acids, methionine, threonine and tryptophan.
$\sum_{j}^{n} a_{4j}x_j - k_2U$	$\geq$	0	

[25] Net protein utilization (NP_U) is defined as the percentage of actual nitrogen (protein) intake which is retained by the body for maintenance and/or growth [56, p. 972].

[26] The amount of calories to be provided is assumed to be a known constraint.

(ii)[27] $$\sum_{j}^{n} a_{5j}x_j - k_2U \geq 0$$

$$\sum_{j}^{n} a_{6j}x_j - k_3U \geq 0$$

$$\sum_{j}^{n} a_{7j}x_j - k_5U \geq 0$$

So, given a fixed dietary allowance $\bar{r}$,[28] the amount of fully utilizable protein can be determined by a simultaneous solution of equations (iii) and (iv).

(iii)[29] $$\bar{r} = \frac{Zt}{100}$$

(iv)[30] $$Z = \left(\frac{100\ U}{t} + 8\right)\left(1 - \frac{7.6\ t}{\bar{c}}\right)$$

where:

$\bar{r}$ = the amount of reference protein required (in gm per person/day).

t = the total amount of protein (in gm per person/day).

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[27]The k_p values (p=1,2...5) correspond to the essential amino acids ratios given in Table 3.1.

[28]$\bar{r}$ stands for reference protein which is defined as a protein which is completely utilizable for anabolic purposes [56, p. 972].

[29]Expression (iii) requires that the actual amount of total protein with a net protein utilization of 100 percent to be equivalent to $\bar{r}$ grams of reference protein.

[30]"Expressions (iii) and (iv) together relate the reference protein requirement to the total quantity of protein, its quality, and the proportion of the total calories in the diet that the protein provides" [56, p. 976].

Z = the net protein utilization (in percent).

U = a fully utilizable protein (in gm per person/ day).

$\bar{c}$ = the total amount of calories to be provided (in kcal).

Then, by setting $7.6/\bar{c} =$, and solving (iii) and (iv) simultaneously, we get

(v) $$U(1 - \alpha t) + .08t\ (1 - \alpha t) = \bar{r}$$

And finally the deterministic model (protein/ calories segment) will be completed by incorporating expressions (1), (v) and the identity $\Sigma a_{2j} x_j = t$[31] into the system of equations 1, i.e.:

<u>System of Equation 2</u>[32]

(1) $$x_j > 0$$

(2) $$\sum_j^n a_{1j} x_j = \bar{c}$$

(3) $$\sum_j^n a_{2j} x_j = t$$

---

[31]Such expression merely states that the total food protein is equal to the amount of fully utilizable protein (U) and nonutilizable protein (S), i.e., $\Sigma a_{2j} x_j = U + S$. Also note that $a_{2j} = a_{3j}$.

[32]The requirements stated here must be fulfilled along with other requirements as one can see at the end of this chapter.

(4) $\sum_{j}^{n} a_{3j} x_j \quad - \quad U \geq 0$

(5) $(1-\alpha t)U + .08(1-\alpha t) \quad \bar{r}$

(6) $\sum_{j}^{n} a_{4j} x_j - k_1 U \geq 0$

(7) $\sum_{j}^{n} a_{5j} x_j - k_2 U \geq 0$

(8) $\sum_{j}^{n} a_{6j} x_j - k_3 U \geq 0$

(9) $\sum_{j}^{n} a_{7j} x_j - k_4 U \geq 0$

(10) $\sum_{j}^{n} a_{8j} x_j - k_5 U \geq 0$

So far the emphasis has been in the basic Smith's protein quality model, specially its protein/calories segment. As observed throughout this exposition, such a model is deterministic in nature,[33] but from now it will be considered the possibility of extending it, by converting their protein deterministic constraints into a probabilistic one. However, in this nutritional model to make such a conversion throughout constraints (4), (5)...(10) two major difficulties arise:

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[33]This is true with respect both to the a_{ij}'s coefficients and the elements on the right hand side, but the primary concern here will be the first ones, i.e., the elements of the technological matrix.

(a) The first one is that U being defined as a function of t one must determine the mean and variance of U as a function of the mean and variance of t; that is, given:

$$U = f(t) = \frac{\bar{r}}{1-\alpha t} - .08t \tag{5}$$

one can get:

$$E(U) = E\left(\frac{\bar{r}}{1 - \alpha t}\right) - .08E(t)$$

Taylor's expansion of f(t) at point $t=t_o$ will lead to:

$$U = f(t) = f(t_o)+(t-t_o)f'(t_o)+(t-t_o)f''\frac{(t_o)}{2!} + \ldots$$

$$= \frac{\bar{r}}{1-\alpha t_o} -.08t_o+\left\{\left[\frac{\alpha\bar{r}}{(1-\alpha t_o)^2} -.08\right]\left[t-t_o\right]\right\}+ \ldots$$

$$= \frac{\bar{r}}{1-\alpha t_o} -.08t_o+\left[\frac{\alpha\bar{r}t}{(1-\alpha t_o)^2} - \frac{\alpha\bar{r}t_o}{(1-\alpha t_o)^2} -.08t+.08t_o\right]$$

$$= \frac{\bar{r}}{1-\alpha t_o} + \frac{\alpha\bar{r}t}{(1-\alpha t_o)^2} - \frac{\alpha\bar{r}t_o}{(1-\alpha t_o)^2} - .08t$$

$$= \frac{\bar{r}(1-\alpha t_o) + \alpha\bar{r}t - \alpha\bar{r}t_o - .08t(1-\alpha t_o)^2}{(1-\alpha t_o)^2}$$

$$= \frac{\bar{r} - \bar{r}\alpha t_o + \alpha\bar{r}t - \alpha\bar{r}t_o - .08t(1-\alpha t_o)^2}{(1-\alpha t_o)^2}$$

$$= \frac{\bar{r} - 2\alpha\bar{r}t_o}{(1-\alpha t_o)^2} + \frac{\alpha\bar{r}}{(1-\alpha t_o)^2}\ t - .08t$$

Now: Let

$$\frac{\bar{r} - 2\alpha\bar{r}t_o}{(1-\alpha t_o)^2} = \phi_1 \quad \text{and} \quad \frac{\alpha\bar{r}}{(1-\alpha t_o)^2} = \phi_2.$$

So, the expectation and variance of $U = f(t)$ can be written as follows:

$$\begin{aligned} E[f(t)] &= E(\phi_1 + \phi_2 t - .08t) \\ &= E(\phi_1) + \phi_2 E(t) - .08E(t) \\ &= \phi_1 + \phi_2\mu_t - .08\mu_t \\ &= \phi_1 + (\phi_2 - .08)\mu_t = E(U) \qquad (6) \end{aligned}$$

And:

$$\begin{aligned} Var[f(t)] &= E\left[\phi_1 - E(\phi_1)\right]^2 + \phi_2^2 E\left[t-E(t)\right]^2 + .08^2 E\left[t-E(t)\right]^2 \\ &= \phi_2^2\ \sigma_t^2 + .08^2\ \sigma_t^2 \\ &= (\phi_2^2 + .08^2)\sigma_t^2 = \sigma_U^2 \qquad (7) \end{aligned}$$

Now, restriction (4) on the deterministic protein quality model is written as follows:

(4b) $\sum_j^n a_{3j}x_j - U \geq 0.$

The expression 4b is a linear combination of two independent random variables, and by assuming that both $\sum_{j}^{n} a_{3_j} x_j$ and U are normally distributed with means $\sum_{j}^{n} x_j \mu_{3_j}$, μ_U and variances $\sum_{j}^{n} x_j^{\,2} \sigma_{3_j}^{\,2}, \sigma^2_U$, one can construct a third variable which is also $N(\mu_\phi, \sigma^2\phi)$. In other words, the restriction (4b) would appear as a chance constraint equivalent to expression (3) on page 7 that is:

$$\text{(4c)} \qquad \mu_\phi + \psi(\sigma^2_{\ \phi})^{1/2} \geq 0$$

where:

$$\begin{aligned}
\mu_\phi = E(\phi) &= E(\sum_{j}^{n} a_{3_j} x_j - U) \\
&= E(\sum_{j}^{n} a_{3_j} x_j) - E(U) \\
&= \sum_{j}^{n} x_j E(a_{3_j}) - E(U) \\
&= \sum_{j}^{n} x_j \mu_{3_j} - \left[\phi_1 + (\phi_2 - .08)\mu_t\right] \\
&= \mu_t - \mu_U
\end{aligned}$$

and:

$$\sigma^2_\phi = E\,[\phi - E(\phi)]^2$$

$$= \mathrm{Var}(\phi)$$

$$= \mathrm{Var}\ (\sum_j^n a_{3_j} x_j - U)$$

$$= \mathrm{Var}\ (\sum_j^n a_{3_j} x_j) + \mathrm{Var}(U) - 2\ \mathrm{Cov}\ (U, \sum_j^n a_{3_j} x_j)$$

$$= \sum_j^n x^2{}_{3_j} \sigma^2{}_{3_j} + (\phi_3^2 + .08^2)\sigma_t^2 - 2\rho\sigma_U\sigma_t$$

$$= \sigma_t^2 + \pi\sigma_t^2 - 2\rho(\sigma^2{}_U \sigma^2{}_t)^{1/2} \quad [34]$$

And finally expression (4c) will become:

$$\text{(4d)} \quad \mu_t - \mu_U + \psi\ \sigma^2{}_t + \sigma^2{}_U - 2\rho(\sigma^2{}_U \sigma^2{}_t)^{1/2} \geq 0 \quad [35]$$

(b) The second difficulty arises because restriction (5) in the deterministic protein quality model is nonlinear in both the variables U and t. Such a nonlinearity would make the problem extremely complicated mainly because U and t are not independent random variables. To overcome this problem, Taylor's expansion can be used to linearize such an expression in both U and t as follows:

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[34] In this expression $\pi = (\phi^2{}_2 + .08^2)$ and ρ stands for the correlation coefficient between U and t.

[35] Remember that in such expression $\sigma^2{}_U = \pi\sigma^2{}_t = (\phi_2^2 + .08^2)\sigma^2{}_t$.

Let's denote the condition (5) as a F(U,t), and make:

(5a) $F(U,t) = (U + .08t)(1 - \alpha t)$ to be linearized at the points $U = U_0$ and $t = t_0$. So it follows:

$$F(U,t) = F(U_o,t_0) + \left[(U - U_0)\frac{\partial F}{\partial U}\right]U_0,t_0 + \left[(t-t_0)\frac{\partial F}{\partial t}\right]U_0,t_0$$

$$\frac{\partial F}{\partial U} = 1 - \alpha t$$

$$\frac{\partial F}{\partial t} = 0.08(1 - \alpha t) - (U + 0.08t)\alpha$$

$$F(U,t) = \left[U_0 + 0.08t_0\right](1-\alpha t_0) + (U-U_0)\left[1-\alpha t_0\right] +$$

$$+ (t-t_0)\left[0.08(1-\alpha t_0) - (U_0+0.08t_0)\alpha\right] =$$

$$= U_0 - U_0\alpha t_0 + 0.08t_0 - 0.08\alpha t_0^{\,2}$$

$$+ U - U_0 - \alpha U t_0 + U_0\alpha t_0 +$$

$$+ (t-t_0)\left[0.08 - \alpha 0.08t_0 - U_0\alpha - 0.08t_0\alpha\right] =$$

$$= U_0 + 0.08t_0 - 0.08\alpha t_0^{\,2} + U - U_0 -$$

$$- \alpha U t_0 + \left[0.08 - U_0\alpha - 0.16t_0\alpha\right]t$$

$$- 0.08t_0 + 0.16\alpha t_0^{\,2} + U_0\,\alpha\,t_0 =$$

$$= 0.08\alpha t_0^{\,2} + U_0\alpha t_0 + U(1-\alpha t_0) + (0.08 -$$

$$- U_0\alpha - 0.16t_0\alpha)t$$

Let:

$$(.08t_0 + U_0)\alpha t_0 \qquad = \quad A$$

$$(1-\alpha t_0)U \qquad = \quad B$$

$$(.08-U_o\alpha-.16t_0\alpha)t \qquad = \quad C$$

And it follows that:

(5b) $\quad F(U,t) \;=\; A \;+\; BU \;+\; Dt$

Given that F(U,t) is linear in both U and t, by taking the exprectations and variances of F(U,t) one will convert it into a change constraint, that is:

$$P\left[F(U,t) \geq \bar{r}\right] = 1 - \varepsilon_1$$

Since expression (5b) is a linear combination of independent and normally distributed random variables, it can be replaced by another random variable F which will be also $N(\mu_F,\sigma^2{}_F)$. Then the new function would appear in its chance constrained form as follows:

(5c) $\quad \mu_F \;+\; \psi(\sigma^2{}_F)^{1/2} \geq \bar{r}$

where:

$$\mu(F) = E(F)$$

$$= E(A + BU + Dt)$$

$$= A + BE(U) + DE(t)$$

$$= A + B\left[\phi_1 + (\phi_2 - .08)\mu_t\right] + D\left(\sum_j^n x_j \mu_{2j}\right)$$

$$= A + B\mu_U + D\mu_t$$

and

$$\sigma^2_F = E[F - E(F)]^2$$

$$= Var(F)$$

$$= Var(A) + B^2\ Var(U) + D^2\ Var\ (t) + 2\ Cov\ (Ut)$$

$$= B^2\left[(\phi^2_2 + .08^2)\sigma^2_t\right] + D^2\ (\sigma^2_t) + 2\rho(\sigma^2_U\sigma^2_t)^{1/2}$$

$$\sigma^2_F = B^2(\sigma^2_U) + D^2(\sigma^2_t) + 2\rho\ (\sigma^2_U\sigma_t)^{1/2}$$

By substituting E(F) and Var(F) in expression (5c) we will finally get the chance constraint written as follows:

$$\text{(5d)}\quad A + B\mu_U + D\mu_t + \psi\left[B^2(\sigma^2_U) + D^2(\sigma^2_t) + 2\rho(\sigma^2_U\sigma^2_t)^{1/2}\right]^{1/2} \geq \bar{r}$$

Expressions (6), (7) ... (10) in the basic deterministic model can be handled in the same way as expression (4):[36] i.e., provided that they are linear

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[36]See development of the expression (4) on pages 30-32.

combinations of independent random variables, a third function can be constructed expressing them in chance constraint form as follows:

$$\sum_{i=r}^{n} x_j\mu_{ij} - k_p\mu_U + \psi\left[\sum_j^n x_j^2\sigma_{ij}^2 + k_p^2\sigma_U^2 - 2\ \rho(\sum_j^n x_j^2\sigma_{ij}^2\sigma_U^2)^{1/2}\right]^{1/2} \geq 0$$

And finally the complete stochastic version of the protein quality model can be written (along with the other requirements) as follows:

MIN	$C = \Sigma P_x x_j$ $(j=1,2...,n)$		Make C the total expenditures on food as small as possible.

subject to:

(1)	x_j	≥ 0	No negative quantities of food will be purchased.
(2)	$\sum_j^n a_{1j}x_j$	$= \bar{c}$	The total calories in the diet shall be equal to some specified minimum caloric allowance.
(3)	$\sum_j^n a_{2j}x_j$	$= t$	The total food protein in the diet must be equal to the utilizable plus nonutilizable protein, i.e., $t = U + S = \Sigma a_{2j}x_j$.
(4)[37]	$\mu_t - \mu_U + \psi\left[\sigma^2_t + \sigma^2_U - 2\rho(\sigma^2_t\sigma^2_U)^{1/2}\right]^{1/2}$	≥ 0	There is a probability (say 95 percent) that the diet will provide food protein at least equal to the quantity of fully utilizable protein in such a diet.

[37]Remember that $\mu_t = \sum_j^n x_j\mu_{2j}$ and $\sigma^2_t = \sum_j^n x_j{}^2\sigma^2{}_{2j}$.

(5) $\mu_a + B\mu_U + D\mu_t + \psi\left[B^2(\sigma^2_U) + D^2(\sigma^2_t) + 2\rho(\sigma^2_U\sigma^2_t)^{1/2}\right]^{1/2} \geq \bar{r}$

There is a probability (say 95 percent) that the diet will provide the fully utilizable protein required to attain a reference protein level of $\bar{r}$ or more.

(6)[38] $\sum_j^n \mu_{ij}x_j - k_p\mu_U + \psi\left[\sum_j^n x_j^2\sigma_{ij}^2 + k_p^2\,\sigma^2_U - 2\rho(\Sigma x_j^2\sigma_{ij}^2\sigma^2_U)^{1/2}\right]^{1/2}$ $(i=4,5\ldots7) \geq 0$

There is a probability (say 95 percent) that the diet will provide the required minimum ratio of lysine, the sulfur-containing amino acids, methionine and threonine.

(7) $\sum_j^n \mu_{8j}x_j - k_5\mu_U + \psi\left[\sum_j^n x_j^2\sigma^2_{8j} + k_5^2\sigma^2_U - 2\rho(\sum_j^n x_j^2\sigma_{8j}^2\sigma^2_U)^{1/2}\right]^{1/2} - 6\,n \geq 0$

There is a probability (say 95 percent) that the diet will provide the minimum ratio of tryptophan for use in fully utilizable protein or for conversion to nicotinamide.

(8) $\sum_j^n a_{9j}x_j + 100n$[39] $\geq b_8$

The total amount of nicotinamide shall be equal to or exceed the required amount b_9.

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[38] Remember that the k_p values (p=1,2...5) correspond to optimum ratios among the essential amino acids as defined by the structure of a fully utilizable protein in Table 3.1

[39] It is based on the conversion ratio of 60 mg. of tryptophan to 1 mg. of nicotinamide.

(9) $\sum_{j}^{n} a_{ij} x_j$ (i = 10, 11 ... 15) b_i The total amount of each of the other six nutrients shall equal or exceed the required amount of each (b_i).[40]

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[40]These nutrients include: calcium, phosphorous, iron, fat, vitamin A and vitamin C.

CHAPTER IV

PRESENTATION AND ANALYSIS OF THE RESULTS

This chapter focuses on the presentation and analysis of the results obtained in this research. Inferences and conclusions are drawn from results which were evaluated under two different methods: deterministic and stochastic. This was considered to be relevant because some of the results obtained by the two methods differ significantly and lead to different policy implications.

The mathematical models (either the deterministic or stochastic) from which the results were generated were designed to provide one with a list of the least expensive and most efficient combination of foods for meeting nutritional needs. However, by limiting the number of eligible commodities to only those commonly eaten in the cities studied, palatability and people's tastes and preferences have been indirectly considered.[1] The basic results and policy implications to be derived from this study must be confined to the areas and population sampled, i.e., standard families in Recife and Fortaleza.

The subsequent sections describe and analyze more specifically the results obtained through the computer programming routines.

A. Least Cost Diet vs Actual Diets

1. Total Costs and Patterns of Food Expenditures

Comparisons of the actual food expenditures with the costs of the least cost diets using both the deterministic and stochastic models are provided in Table 4.1. Actually, these are total monthly costs adjusted for family size and composition in each of the cities. Except for one case (Fortaleza-stochastic), the actual food expenditures by the people in the lowest income group exceeded the amount needed to obtain the minimum required nutrients intake. The cost differential is more significant in Recife (ranging from Cr $147.18 to Cr $236.99 per month), than in Fortaleza where the total cost of the least cost diet happened

[1]As a matter of fact, even the purely nutritional models have some nonnutritional (or cultural) components on it. In this study conventional restraints (requirements dealing with specific amounts of certain foods) experimentally were set up in order to evaluate their effects on the total cost of nutrition.

Table 4.1. Actual food expenditures in the lowest income group, compared with the least cost monthly diet calculated for standard families in Recife and Fortaleza (Northeast of Brazil).

Diet patterns	City 1 (Recife)	City 2 (Fortaleza)	Cost differential	
	Diet cost (Cr $/month)	Diet cost (Cr $/month)	Cr $/month	%
A_1 Actual diets (A.D.)[a]	478.65	343.87	134.78	28.1
B_1 Least cost diets (L.C.D.)				
B_{11}: Method I (Deterministic)	241.66	265.28	-23.62	-9.7
B_{12}: Method II (Stochastic)	331.47	390.55	-59.08	-17.8
C_1 Cost differential				
$(A_1 - B_{11})$	236.99	78.59	--	--
$(A_1 - B_{12})$	147.18	-46.68	--	--
$(B_{12} - B_{11})$	89.81	125.27	--	--

[a]Basic Source: BNB-ETENE [3, 4].

to be sometimes higher than the actual food expenditures (Example: Cr $390.55 against Cr $343.87/month). Such city-to-city variations may be explained by the amounts of nutrients actually provided relative to the minimum dietary allowances required in the least cost diets for each city. For example: Table 4.2 shows that actual diets in Fortaleza are providing most of the nutrients below the minimum required levels, while in Recife at least six of those nutrients (proteins, phosphorous, iron, thiamin, vitamin C and fat) are exceeding the minimum required amounts.[2] Then, actual food expenditures in Fortaleza are relatively lower to Recife and the calculated least cost diets merely because the families are eating less of each of the essential nutrients.[3]

However, if one defined efficiency in the purchase of nutrition as the ability to obtain the necessary nutrients in the least costly way, low-income families in both cities were inefficient, that is, they are either buying more or less than the minimum required amounts of each of the nutrients--so they are spending more than they should. The calculated least cost diets based on purely nutritional objectives have shown that

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[2]Actually, to provide nutrients above the minimum required may be harmful and uneconomical, although it is a more acceptable social standard than to prevent people from satisfying their minimum needs.

[3]This is true even for those nutrients which have been provided in excess such as phosphorous and fat.

Table 4.2. Monthly nutrient amounts provided by the actual diets compared with the minimum required in the calculated least cost diets (Northeast of Brazil).

Nutrients	Actual diets (A.D.)[a]		Least cost diets (L.C.D.)[b]		Difference[c]	
	Recife (R_1)	Fortaleza (F_1)	Recife (R_2)	Fortaleza (F_2)	(R_1-R_2)	(F_1-F_2)
Calories (kcal)	315216.4	258515.4	330776.8	349071.1	-15560.4	-90555.7
Proteins (gm)	9612.1	7081.7	9063.7	9504.5	+548.4	-2422.8
Calcium (mg)	68551.4	53939.3	82255.9	87147.9	-13704.5	-33208.6
Phosphorous (mg)	132376.8	112753.9	75753.4	80262.9	+56623.4	+32493.0
Iron (mg)	2169.1	1726.8	1921.7	2029.9	+247.4	-303.1
Thiamin (mg)	164.3	136.2	136.3	143.0	+28.0	-6.8
Riboflavin (mg)	117.3	92.8	197.7	208.7	-80.4	-115.9
Niacin (mg)	1905.6	1171.9	2204.9	2324.2	-299.3	-1152.3
Vitamin A (IU)[d]	239885.6	54654.3	445462.6	464381.1	-205577.0	-409726.8
Vitamin C (gm)	13834.1	4574.2	6900.3	7273.8	+6933.8	-2699.6
Fat (gm)	5976.9	4522.8	3171.7	3337.2	+2805.2	+1351.1

[a] Basic source: BNB-ETENE [3, 4]. Nutrient intake at the lowest income level.

[b] Minimum dietary allowance for standard families in the two cities.

[c] A plus sign means a nutrient intake above the required level, while a minus sign indicates that the nutrient intake is below the required levels.

[d] IU stands for international units (1 IU = .6 micrograms beta-carotene).

reallocation of the actual food expenditures would improve actual dietary patterns to meet family nutritional needs at low cost.

Tables B-1 and B-2 in the Appendix show the percentage contribution of the various commodities groups to the total costs in the actual and in the least cost diets calculated for both cities. Total food expenditures among the lowest income families in Recife and Fortaleza were allocated in seven major categories as follows: cereals and products (16 and 21 percent for Recife and Fortaleza, respectively), meat, fish and poultry (45 and 42 percent), eggs and milk products (4 percent in both cities), roots and tubers (6 and 4 percent), fats and oils (2 and 4 percent), vegetables, and fruits (18 and 13 percent) and finally the miscellaneous group which apportioned 9 and 12 percent of the total expenditures in each of the cities.[4]

Total expenditure allocation in the least cost diets varies depending upon the methodological procedure used. For instance, in Recife the highest amount of the total expenditure was allocated to the cereal group, and it ranges from 34 percent (under method I) to 41 percent (under method II). Contrary to the actual diets in which the proportion of incomes allocated for legumes, fruits,

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[4]The miscellaneous group includes sugar, coffee, and doce (jelly).

and vegetables is quite significant (18 percent), in the least cost diet for Recife such a percentage was relatively small (a maximum of one percent in the deterministic model). In Recife, one of the primary differences between the two types of diets is that no money was allocated for the purchase of fats and oils or to those items included in the miscellaneous group such as sugar, coffee, and doce (jelly). There are also considerable differences in the kinds of foods as well as in the proportion of the total expenditures allocated with individual foods in those diets.

In Fortaleza the divergence in the patterns of food expenditures between the actual and the calculated least cost diets is still great. For example, the commodity group including meat, fish, and poultry, which accounts for 42 percent of the money allocated in the actual diets, was excluded in the least cost diet. The same is true for fats-oils and miscellaneous items which all together accounted for 16 percent of the total expenditures in the actual diets. Among other differences between the two types of diets, are the proportion of money allocated among the several commodities groups and the percentage of income devoted to individual foods within each food group. Foods like beef, lard and coffee, which accounted for a high proportion of the total expenditures within their respective groups, turn out to be uneconomical foods in the least cost diets.

Table B-3 in the Appendix shows the percentage contribution of the several commodities groups to the total calories and proteins in both actual and least cost diets computed for standard families in Recife. One of the primary differences between the two diets was the exclusion of the commodities group including fats-oils and miscellaneous items, which together provided 20 percent of the total calories in the actual diets. The cereal group is still the most important source of calories (more than 45 percent of the total calories provided), although the single largest source of that nutrient shifted from bread (19 percent) to cornmeal-white (49 percent under method I and 69 percent under method II). In both diets, the roots and tubers group was the second major source of calories, and manioc flour is the single largest component accounting for 14 percent in the actual diets against 28 and 12 percent in the calculated least cost diets. In Recife, the percentage contribution of the several commodities groups to the total protein in the two diets is quite divergent and it can be summarized as follows: the meat-fish-poultry group is no longer the most important source of protein, and is replaced by the cereal group in the least cost diets (calculated by both methods I and II). Eggs and milk products also turn out to be an important source of protein in the least cost diets (22 and 25 percent of the total, respectively). On the other hand, the legumes-vegetable-fruit group decreased sharply in importance as a source of protein in the least cost diet, making up a maximum of 2 percent of the total proteins required.

Table B-4 in the Appendix indicates the percentage contribution of the several commodities groups to the total calories and proteins of the diets, which would provide all the essential nutrients for a standard family in Fortaleza. Again, one of the primary differences between the two types of diets was the exclusion of meat-fish-poultry, fats-oils, and miscellaneous groups which together provide 31 and 25 percent of the total calories and protein in the actual diets. In addition, the legume-vegetable-fruit commodity group contribution to the total calories and protein in the least cost diets decreased sharply.

The cereal group improved its relative position as a source of both calories and proteins. That is, it shifted from 33 percent (calories) and 29 percent (protein) in the actual diets to a minimum of 55 percent (calories) and 42 percent (protein) in the least cost diets. Also, the eggs-milk commodity group which accounts for a small percentage of the total protein in the actual diets, sharply increased its relative position by shifting from 8 percent to 42 and 46 percent, respectively (methods I and II) in the least cost diets. The patterns showing the changes in the individual's commodity contribution to the total calories and protein within each group seems to remain stable except for the food group roots and tubers in which potato has been replaced by sweet potato (white).

B. Deterministic vs. Stochastic Formulation (Method I vs. Method II)

Under method I (deterministic linear programming) whether or not a specific food or set of foods A, B, ... , C will enter in a least cost diet will depend basically on two things: the food prices relative to its nutrient content are relative to the minimum required in the diet. Thus, at the extreme case, if there are only two eligible foods (A and B) for a particular diet and one (say, food A) is cheaper than the other and has more of all of the nutrients, the cheapest diet would consist solely of food A. Food B would be inefficient and it will not enter in the diet under those conditions, no matter what the proportions of the dietary requirements may be.

Under the stochastic formulation (method II), variability is allowed and the cheapest diet would not necessarily consist solely of food A. Actually, it should be the case if besides the two previous conditions stated (low price and high nutrient content), the nutrients contained in Food A are also subject to lower variability than Food B. Provided that it is not true, a trade-off will occur and the cheapest diet may consist solely of Food A, or Food B, or some combination of the two. The argument, of course, holds for any set of foods, and overall such a variability component may change the composition and the total cost of the least cost diets.[5] For example: Table 4.3 shows the changes in the total cost and food composition when the caloric/protein vector is subject to random variations. Thus, in Recife it would cost Cr $89.81 more for a standard family to get all nutrients and the required amount of calories and proteins at least 95 percent of the time. In addition, amounts of milk and manoic-flour are reduced by more than a half while chicken practically replaced dried beef as one of the sources of animal protein. Also, tomatoes, macaroni, and oranges become efficient

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[5]In this study only the protein/calorie variability has been taken into account. Both were assumed to be normally distributed random variables, with a probability of meeting the requirement of 95 percent.

Table 4.3. Foods in the least cost monthly diets, providing the standard families in Recife and Fortaleza with the minimum nutrient intake.

Consumption activities (foods in and L.C.D.)	City 1 (Recife)		City 2 (Fortaleza)	
	Method I	Method II	Method I	Method II
	Kg/month	Kg/month	Kg/month	kg/month
Chicken	--	8.95	--	--
Milk	57.74	28.86	33.52	42.41
Eggs	--	17.34	25.79	28.29
Dried beef	4.47	--	--	--
Rice	--	--	53.48	77.98
Macaroni	--	9.83	--	3.03
Sweet potato (white)	10.03	--	55.29	--
Sweet potato (yellow)	13.47	12.69	12.10	7.12
Cornmeal (white)	46.37	60.57	--	--
Manioc flour	27.27	12.92	6.79	--
Beans (mulatinho)	--	--	--	.32
Beans (macassar)	.99	--	1.12	--
Tomato	--	.10	--	52.26
Orange	--	1.44	--	--
Total monthly cost of the L.C.D. (in Cr $)	241.66	331.47	265.28	290.55

foods when random variations in both protein and calories are allowed to occur. Such a pattern of substitution combined with the information given in Table 4.4 seems to suggest the following: due to its high variability (50.3 and 27.9 percent), dried beef becomes an inefficient source of both animal protein and calories, while chicken, eggs, and macaroni now turn out to be an inexpensive source of both nutrients. Manioc-flour is an inexpensive food, but because it provides essential calories and shows a high coefficient of variation (43 percent), other potential sources of this nutrient (Example: macaroni and cornmeal-white) partially replace it. Tomatoes and oranges, although subject to higher caloric/protein variability than beans (macassar) turn out to be economical foods under method II, probably to offset the sharp decrease in the amount of sweet potatoes (yellow), the richest source of vitamins A in that diet.

In Fortaleza, the change in total costs and substitution patterns also seem to be closely related to the random variations in the protein/caloric vector. For instance, the total cost differential between the two methods is Cr $105.27/month. The amount of rice increased by almost 50 percent and macaroni turns out to be an efficient food under the stochastic formulation.

Table 4.4. Means and proteins/calories variability of the foods in the least cost monthly diets providing the standard families in Recife and Fortaleza with the minimum nutrient intake.[a]

Consumption activities (foods in the L.C.D.)	Proteins (x_1)			Calories (x_2)		
	$\bar{x}_1$	s_{x_1}	c.v. = $\frac{\bar{x}_1}{s_{x_1}} \times 100$	$\bar{x}_2$	s_{x_2}	c.v. = $\frac{\bar{x}_2}{s_{x_2}} \times 100$
	gm/100 gm of food	gm/100 gm of food	%	cal/100 gm of food	cal/100 gm of food	%
Chicken	20.56	1.78	8.7	164.33	54.98	33.5
Milk	3.38	.20	6.0	62.16	3.43	5.5
Eggs	12.18	.52	4.3	155.33	5.40	3.5
Dried beef	47.44	23.90	50.3	304.80	85.12	27.9
Rice	7.33	.55	7.5	356.00	8.09	2.3
Macaroni	11.58	2.00	17.2	355.00	11.28	.03
Sweet potato (white)	1.43	.46	32.3	105.83	19.87	18.8
Sweet potato (yellow)	1.42	.25	17.5	104.50	17.52	16.8
Cornmeal (white)	1.43	.46	17.5	355.60	2.88	.8
Manioc flour	1.33	.52	39.3	337.00	14.02	4.2
Beans (mulatinho)	22.01	1.60	7.3	326.67	20.77	6.4
Beans (macassar)	22.25	2.18	9.8	328.50	20.17	6.1
Tomato	.90	.14	15.7	21.00	1.22	5.8
Orange	.75	.13	18.3	46.16	3.54	7.6

[a]Basic source: Silva, P. R. [48].

Needless to say, rice and macaroni have comparatively lower coefficients of variations for both calories and protein than other close competitors foods (example: sweet potato-white). Both manioc-flour and sweet potato (white) were omitted in the optimal solution, and a trade-off between the two varieties of beans (mulatinho and macassar) has occurred.

Table 4.5 is an extention of Table 4.4, and shows the effect of different probability constraint patterns on the total costs and composition of the least cost diets, calculated for a standard family in Recife. The least cost diets so far, fall into two major categories, i.e., those with no random or stochastic component (deterministic), and others in which both calories and protein are assumed to be random variables (stochastic). Such cases may be labeled as PC and $\overline{PC}$ to indicate the presence or absence of random elements, and the differences in costs and food composition associated with each one were quite significant. The next step was to investigate the effect of each probability constraint (calories or protein) on the total cost and composition of the least cost diets. To begin with, the cost difference is still significant when the probability constraint is taken one at a time, i.e.,; it would cost Cr $20.69/ month more for a standard family in Recife to purchase an adequate diet that, on the average, would provide all

Table 4.5. Foods in the least cost monthly diets under different patterns of probability constraints (standard family in Recife).

Consumption activities (foods in the L.C.D.)	Probability constraint patterns				Coefficients of variation (C.V.)	
	$(\overline{PC})$[a]	$(C\overline{P})$[b]	$(P\overline{C})$[c]	(PC)[d]	C.V. (calories)	C.V. (protein)
	Kg/month	Kg/month	Kg/month	Kg/month	%	%
Chicken	--	--	--	8.95	33.5	8.7
Milk	57.74	62.13	23.74	28.86	5.5	6.0
Eggs	--	--	31.39	17.34	3.5	4.3
Dried beef	4.47	4.16	--	--	27.9	50.3
Macaroni	--	12.74	--	9.83	3.3	17.2
Cornmeal (white)	45.37	32.18	38.36	60.57	.8	8.4
Sweet potato (white)	10.03	--	82.74	--	18.7	32.3
Sweet potato (yellow)	13.47	--	10.67	12.69	16.7	17.5
Manioc flour	27.27	45.20	--	12.92	4.2	39.3
Beans (mulatinho)	--	--	9.82	--	6.3	7.3
Beans (macassar)	.99	--	--	--	6.1	9.8
Tomato	--	--	--	.10	5.8	15.7
Carrots	--	7.05	--	--	20.3	26.1
Orange	--	--	--	1.44	7.6	18.3
Total monthly cost of the L.C.D. (in Cr $)	241.66	262.35	295.97	331.47	--	--

[a] $\overline{PC}$ = a L.C.D. wihtout any probability constraint.

[b] $C\overline{P}$ = a L.C.D. with calories probability constraint.

[c] $P\overline{C}$ = a L.C.D. with protein probability constraint.

[d] PC = a L.C.D. with both calories and protein probability constraints.

nutrients, plus the required amount of calories at least 95 percent of the time. On the other hand, if protein rather than calories is the nutrient required by the diet at least 95 percent of the time the associated costs would be still higher, that is, Cr $54.31/month. In this study the total costs of the least cost diets were more sensitive to random variations in proteins than to calories. It is to be expected that the higher the number of nutrients subject to random variations, the higher should be the cost of the least cost diets. Total costs also must be higher when the variability of the nutrients of the foods in the solution is high. In any event, decisions about what nutrients are allowed to be random variables and how much should be paid are probably a matter of personal preference and will depend on the families' utility risk preference functions. A conservative family perhaps will be willing to pay a high price for an adequate diet in which the important nutrients are allowed to be below the minimum required only five percent of the time. On the other extreme, there will be families which are not willing to pay any additional cruzeiros for such a risk factor.

As shown before, the food composition of the least cost diets varies significantly when the deterministic constraints are converted into a probabilistic one.

Because the extreme cases (PC and $\overline{PC}$) were already discussed, two other intermediate situations should be emphasized. Label $C\overline{P}$ is chosen to indicate a least cost diet in which only the caloric vector is allowed to be stochastic. And $P\overline{C}$ stands for a least cost diet in which the protein content of the food is subject to some variability (see Table 4.5). As the patterns have indicated, a diet in which both calories and protein are probability constraints, should be more costly and selective with respect to these sources of nutrients than one which does not impose any kind of restrictions. It is obvious that the patterns should hold true, for those cases in which one or another nutrient variability is allowed to occur, i.e., the least cost diet labeled $C\overline{P}$ on Table 4.5 should be more selective with respect to the sources of calories, and to a lesser extent with respect to the sources of proteins because no additional costs are charged against it. The reverse must hold true for the least cost diet labeled $P\overline{C}$, probably because the variability component will increase the per-unit cost of such a nutrient relative to the other.

The figures on Table 4.5 (column $P\overline{C}$ and $C\overline{P}$) support the arguments above. For example, in the least cost diet with the caloric probability constraint, dried beef appears to be an economical food, but it is excluded from the optional solution when the protein probability constraint was imposed. In spite of its high caloric variability, it is relatively low compared with the protein variability (27.9 vs. 50.3 percent) which makes such a food a more costly source of protein than calories. Yet, in the least cost diets with caloric probability constraint, five main sources of calories are included (macaroni, manioc-flour, cornmeal-white, sweet potatoes-yellow), most of the calories have been provided by those foods with the lowest coefficient of variation (the first three commodities). Cornmeal-white continues in the optimum solution even if calorie probability constraint is imposed and manioc-flour, although with a low coefficient of variation, was excluded probably because of its high relative price. Such patterns also can explain the high amounts of sweet potato (yellow and white) in the $P\overline{C}$ diets when the variability component seems to be offset by the low prices and protein content. In addition, sweet potato (yellow) seems to be the major substitute for carrots which is a good source of vitamin A, but with high calories/protein coefficients of variation (20.3 and 26.1 percent).

Finally, it must be emphasized that even in the stochastic formulation, the relative prices of the foods continue to play a major role in the selection of the foods that will be in the optional solution. For example, rice which is a rich source of calories, turns out to be an economical food in Fortaleza but it does not enter in the least cost diets for Recife where its market price is 40 percent higher. The nutritive content and variability are the same in both cities, but the difference in relative prices between rice and other competitor foods makes it an inefficient food in one of the cities. Therefore, whether or not a specific food enters in the final solution depends on the price of the food, its nutrient content, and the variability of nutrient content.

C. Least Cost Substitutions in the Least Cost Diets

This section deals with the most economical way of introducing a new set of foods excluded from the initial commodity list. Because all other characteristics of the diet remain the same, such a minimum cost substitution will indicate what foods or set of foods could be economically replaced. For a country or region experiencing shortages of those foods already in the least cost diets, such a kind of adjustment is obviously an open alternative, i.e., knowing some of these efficient foods will be useful to agricultural planners and administrators because they indicate where the emphasis must be placed in order to meet nutritional needs in the most economical way. To be realistic, the substitutions should not drastically affect actual consumption patterns. Therefore, the potential foods to be considered should be confined to those commodities already (or potentially) available in that specific place or region.

The initial commodity list included about 51 items in Recife and 41 in Fortaleza. Such a list was extended to include 15 more items in Recife and 9 in Fortaleza.

Table 4.6 shows the least cost substitutions when a new set of foods is included along with the basic list provided to Recife and Fortaleza. The new least cost diets were computed by methods I and II, and both are supposed to provide standard families in both cities with the minimum nutrient intake. In Recife little comment is needed. Under method I no economical substitutions were performed, and none of the potential foods considered were efficient enough to replace those commodities already into the optimal solution.[6] Under method II, minor adjustments were performed in the amounts of the foods already in the basic least cost diet, but the magnitude of the cost reductions is quite insignificant, i.e., Cr $.04/month.

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[6]This is not to say that such a food can not be more efficient than those appearing in the commodity list but not included in the least cost diets.

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[6]This is not to say that such a food can not be more efficient than those appearing in the commodity list but not included in the least cost diets.

Table 4.6. Economic substitutions in the least cost diets calculated for the standard families in Recife and Fortaleza (Northeast of Brazil).

Consumption activities (foods in the L.C.D.)	City 1 (Recife)				City 2 (Fortaleza)			
	Method I		Method II		Method I		Method II	
	A_1[a]	B_1[b]	A_2	B_2	A_3	B_3	A_4	B_4
	Kg/month	Kg/month	Kg/month	Kg/mongh	Kg/month	Kg/month	Kg/month	Kg/month
Chicken	--	--	8.95	8.98	--	--	--	--
Milk	57.74	57.74	28.86	26.88	33.52	58.11	42.41	28.11
Eggs	--	--	17.34	17.27	25.79	--	28.29	22.28
Dried beef	4.47	4.47	--	--	--	--	--	--
Beef liver	--	--	--	.02	--	.93	--	1.30
Visceras	--	--	--	--	--	13.47	--	--
Sardines	--	--	--	--	--	--	--	5.08
Rice	--	--	--	--	53.48	55.51	77.98	76.35
Macaroni	--	--	9.83	9.60	--	--	3.03	--
Sweet potato (white)	10.03	10.03	--	--	55.29	--	--	--
Sweet potato (yellow)	13.47	13.47	12.69	12.71	12.10	9.89	7.12	3.21
Cornmeal (white)	45.37	45.37	60.57	60.78	--	--	--	--
Manioc flour	27.27	27.27	12.92	12.94	6.79	21.65	--	4.43
Beans (mulatinho)	--	--	--	--	--	--	.32	--
Beans (macassar)	.99	.99	--	--	1.12	3.26	--	3.00
Tomato	--	--	.10	--	--	--	52.26	--
Squash	--	--	--	--	--	--	--	7.34
Cabbage	--	--	--	--	--	--	--	9.18
Orange	--	--	1.44	1.45	--	--	--	--
Total monthly cost of the L.C.D. (in Cr $)	241.66	241.66	331.47	331.43	265.28	252.14	390.55	307.45

[a] $A_{1,2,3,4}$ = L.C.D. in which the eligible commodities include basically conventional foods.

[b] $B_{1,2,3,4}$ = L.C.D. in which the eligible commodities include both conventional and nonconventional (potential) commodities.

In Fortaleza, the adjustments were quite significant, especially under method II, where the total cost of the least cost diets was reduced by 21 percent. Under method I the cost differential resulting from the least cost substitutions was much lower (Cr $13.14/month) but it can be considered significant if compared with the results for Recife where no minimum cost substitutions were performed. Overall, there were significant adjustments in the quantities and kinds of the commodities involved. However, the computer routine used in this study for computing the least cost diets does not allow a precise evaluation of the qualitative and quantitative nature of the adjustments undertaken. So, in spite of the indication that item-for-item replacements occurred, it is not plausible to assume that the substitution pattern follows such a straight fashion. Instead, it is more likely to be a result of an overall adjustment in the quantities and kinds of all foods included in the optimal solution.

The city-to-city variations may be explained by the number and kinds of foods involved, as well as by difference in relative prices between foods previously included in the optimal solution and those added to the basic commodity list. Within cities relative price differences are probably the most significant factor. Under method II, the least cost substitutions are additionally affected by the magnitude of the variability component of the foods included in the final solution.

D. The Cost of Satisfying Additional Requirements

Many least cost diets computations require additional restraints besides the minimum physiological requirements. Examples include constraints of the conventional types (maximum and minimum amounts of certain foods, complementary restraints, etc.), and they are primarily designed to force the diet into a pattern which corresponds to the conventional consumption habits. Still others have used additional constraints which may well fall into the purely nutritional category. Such constraints include imposing minimum amounts of the essential amino acids in the least cost diets. Economists and nutritionists have emphasized the need for introducing this kind of restraint and the arguments rely on both empirical and theoretical grounds, i.e., it is argued that an adequate diet must provide all of the eight essential amino acids. Restrictions imposed on the minimum amount of total protein is no guarantee that the amino acid requirement will be fulfilled. The amount of protein in a least cost diet can be derived either from animal or vegetable sources, but because they have a different amino acid pattern an inexpensive but inadequate diet may result.[7]

Two of the least cost diets had conventional restraints besides the purely nutritional requirements. The conventional restraints were set up as follows: The least cost diets should include minimum amounts (and sometimes maximum) of ten commodities mostly consumed by the households in the two cities studied. The two least cost diets for standard families in Recife and Fortaleza were computed by method I, and the effect of such restraint was to increase the total cost of the least cost diet in Recife and Fortaleza (relatively to the least cost diet with only purely nutritional objectives) by 32 and 24 percent, respectively.[8]

In Fortaleza, even when conventional restraints are applied, milk continues to be an efficient food along with eggs, rice, sardines and squash. Manioc flour, sweet potato (yellow and white) come out at the maximum amounts allowed, and they also can be considered as economical

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[7]This is especially true if the protein requirement is not broken into the two sources (vegetable and animal) and one happens to be much cheaper than the other one.

[8]The restraint types as well as the foods included in such least cost diets are shown in Tables C-1, C-2 in the Appendix C.

foods.[9] The fact that beef, bread, crackers, biscuits, beans (multatinho and macassar), sugar and coffee were forced into the least cost diets at the minimum amounts required indicates that they are not economical foods.

In Recife, where the same kind of constraints were imposed, milk, dried beef, macaroni, carrots, manioc flour, corn meal and sweet potato (white and yellow) are among the most efficient foods. On the contrary, coffee, bread, rice, beef, tomato, lard, onion, and corn meal turn out to be uneconomical foods. It is obvious, though, that variety to satisfy conventional consumption habits can be provided, but it will impose additional costs. How far one can go will depend upon the marginal benefit (utility) and the marginal costs (desutility) imposed by both nutritional and nonnutritional requirements, i.e., for some people the benefits and costs accruing from certain conventional constraints in the model are subjectively weighted in terms of the utilities provided (satisfaction) and the costs incurred (desutility)

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[9]If the restraints are purely nutritional, any food in the optimal solution is considered to be efficient. If conventional restraints are imposed some of the foods in the optimal solution may or may not be economical, in a sense that the aggregate value of the nutrients is not equal to their market price. Strictly speaking, if bounded restraints are imposed and the amounts of foods provided happen to be over the minimum level they are economical foods. On the other hand, if the provided amounts just correspond to the minimum required, they are not efficient foods.

The amino acid requirement was handled indirectly in this research, i.e., instead of requiring minimum amounts of each one, they were required to be equal or greater than zero. However, provided that other nutrient requirements must be satisfied, an excess (slack) of all were obtained. Table 4.7 shows the amino acid and pattern obtained in the least cost diets compared with the minimum allowance suggested by FAO [17]. In Recife the amounts of amino acids provided by the least cost diet calculated by method I are below the required levels, except for Isoleucine and Leucine. Actually, because the Phenylalanine allowances suggested by FAO includes tyrosine, and the data used in this study does not, the figures provided are not really comparable.[10] In Recife, the amounts of amino acids provided by the least cost diets computed through method II have fulfilled all the requirements, including Phenylalanine if the I.I.T. minimum allowances are taken into account.

In Fortaleza, under method I, provided that tyrosine is excluded, the only underfulfilled requirement is lysine. On the other hand, under method II, all of the suggested levels have been overfulfilled. The

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[10] In the I.I.T. study [32], minimum allowances for Phenylalanine without Tyrosine were set up. Based on the allowances suggested, the minimum amounts of such amino acids should be 231541 and 245160 mg, for Recife and Fortaleza, respectively.

Table 4.7. The essential amino acid monthly requirements compared with the amounts provided by the least cost monthly diets (calculated for the standard families in Recife and Fortaleza--Northeast of Brazil).

The essential amino acids	City I (Recife)			City II (Fortaleza)		
	Minimum requirements[a]	Provided by the L.C.D.		Minimum requirements[a]	Provided by the L.C.D.	
		Method I	Method II		Method I	Method II
1. Phenylalanine (mg)	543822[b]	357513*	526336*	570270	488119*	596316
2. Isoleucine (mg)	362548	426068	560869	380180	524333	609570
3. Leucine (mg)	634459	828091	1094316	665315	735251	917668
4. Methionine (mg)[c]	317230	220937*	385530	332658	373423	458777
5. Threonine (mg)	362548	324357*	447758	330180	389204	466856
6. Tryptophan (mg)	90637	78250*	110838	95045	133449	156151
7. Lysine (mg)	498504	415664*	532347	552748	482675*	558622
8. Valine (mg)	453185	439986*	623445	475225	612453	742092

[a]Based on FAO/WHO [17, p. 63] per unit requirement (mg per g of total protein).

[b]Includes tyrosine.

[c]Includes cystine.

*Below the required level.

fact that Model II has provided a least cost diet in which most of the requirements were satisfied, while method I does not is not conclusive, but it raises some interesting questions, that is, perhaps both the relative and absolute level of total protein in the diet are crucial in providing all of the essential amino acids required by nutritional standards.

E. Least Cost Diets and Income Patterns

Per capita annual incomes in the Northeast of Brazil is very low. It is about half of the Brazilian average which is U$A 420.00. In addition, there is a considerable range of variation between states, and within states, between rural and urban areas, and also economic groups. Needless to say, the income distribution issue, although very important, should not be overemphasized, because the absolute size of the incomes may be as important as well. Actually, this is a very fundamental issue but not always understood. That is, it is quite possible to improve the population nutritional status by changing one or another, although improvements in both has been unusually accepted as being the most desirable social political goal.

If the objective is to make it possible for the standard family to consume the recommended nutritional requirements, then the family must have a minimum income equal to the least cost diet. If pure nutrition is the only objective, average families in Recife and Fortaleza would be required to spend Cr $241.66/month (Cr $331.47) and Cr $265.28/month (Cr $390.55), respectively, in the purchasing of all essential nutrients that they need.[11] Or, Cr $47.38/month (Cr $66.95) and Cr $49.12/month (Cr $76.57) in each city would be required for an individual to feed adequately himself.[12] Those figures imply that in spite of being low, the actual levels of income (Cr $106.05/month) are more than enough to buy the minimum nutrient requirements.[13] It also implies that reallocations in the actual patterns of food expenditures to buy the least cost diets in Recife would represent an annual net savings of Cr $551.64 per person (or Cr $316.80 if method II is used in the computation of the diet). In Fortaleza, reallocation of actual food monthly expenditures to buy the least cost diets would be costly at least in one of the cases (under method II) since the total cost of the calculated least

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[11]The figures in parentheses refer to the cost of the least cost diets calculated by method II.

[12]The per capita figures were based on the average family size of 5.1 and 5.4 in Recife and Fortaleza, respectively.

[13]Based on the exchange rate: Cr $1.00 - .1650 U$A.

cost diet is greater than the actual amounts of money actually spent.[14]

The results presented here should not be overemphasized for several reasons: first, per capita incomes are average figures and they may cancel vital differences between various sections of the population. In other words, although such incomes may be representative for the region as a whole, it may be plausible that the lowest income group in those two cities is getting incomes below the estimated average level. Secondly, people buy foods not only to satisfy minimum physiological requirements--the desire for variety, and other cultural factors are also important and may push such costs upward quite significantly. Third, the data used to represent actual food expenditures in both cities were derived from a consumer survey conducted several years ago, and adjusted to 1973 prices. It is likely that changes in the patterns of consumption have occurred throughout this period. Fourth, and more important, although good nutrition is one of the basic human needs, there are others like good health, education, housing which may be desirable goals as well. Provided that income levels are low and because people

[14]As a matter of fact, although more costly, such a diet would be nutritionally more efficient, because it has been providing all of the essential nutrients at the required amounts. In this sense, changes in the actual expenditure patterns would be still economical.

are more likely to maximize satisfactions by the purchasing of all those basic needs (nutrition, education, health, etc.) together, some trade-offs must occur, i.e., instead of purchasing them according to some ordering criteria (for example, good nutrition first, education second, and so forth) people may be induced to get all of them simultaneously and end up having each of these below the acceptable levels.

Perhaps governments rather than the individuals themselves must take the major responsibility in solving this ordering problem, but ranking all people's needs accordingly. If good nutrition happens to be ranked first, the least cost diets may be an appropriate basis to define the amounts of government subsidies needed to fulfill other basic human needs.

F. The Marginal Cost of the Nutrients in the Least Cost Diets

Some of the methods used to determine the cost of the nutrients contained in a food are quite arbitrary.[15] The failure is mainly because the weights attached to evaluate the value of the nutrients are inaccurate and to a great extent do not reflect man's

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[15]For a detailed exposition, see [34, 50].

need for each of them. Another crucial and related problem has been the measure of the most economical sources of particular nutrients. It comes out because foods usually provide much more than a single nutrient and not always the same food is an inexpensive source of all of them. Fortunately, linear programming allows one to evaluate what nutrients or set of nutrients are more costly in providing some nutritional objective. It also provides a meaningful measure about the most economical food[16] to be included in a least cost diet.

Table 4.8 shows the marginal cost of the scarce nutrients in the least cost monthly diets providing standard families in Recife and Fortaleza with the minimum nutrient intake. Actually, they represent the costs of adding one unit of a particular nutrient in those diets, holding the other nutrients' level constant. In satisfying the minimum dietary allowances, the least cost diets usually provide an excess of some of the required nutrients. Those are the "costless" nutrients because they exist in excess. There are no additional costs in providing extra units of each of them (the marginal or opportunity cost is zero).[17]

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[16]By definition, foods included in the optimal solution are more economical than those excluded ones. In addition, among the excluded foods there are some considered more economical than the others (for details, see section on marginal sufficiency of foods).

[17]This is true as far as one keeps the allowances increasing by less than the amount of the nutrient excess.

Table 4.8. Marginal cost of the scarce nutrients in the least cost monthly diets for standard families in Recife and Fortaleza (Northeast of Brazil).

Nutrients	Marginal cost (Recife)		Marginal cost (Fortaleza)	
	Method I	Method II	Method I	Method II
Proteins[a]				
Vegetable (gm)	.0104	.0067	.0099	.0427
Animal (gm)	.0112	.0341	.0261	.0519
Calcium (mg)	--	.0001	.0001	.0006
Iron (mg)[b]	.0003	--	.0088	--
Vitamin A (100IU)	.0530	--	--	.0200
Vitamin C (mg)	--	.0021	--	--
Calories (100 kcal)				
Gross calories[c]	.0300	.0400	.0215	-.0500
B vitamins (required for 100 kcal)	.0100	.0500	.0223	.1300
Net calories (100 kcal)	.0200	-.0100	-.0008	-.1800

[a]The marginal cost of the total or unspecified protein is the marginal cost of the vegetable protein plus the marginal cost of the animal protein.

[b]I.U. = stands for international units (1 I.U. = 6 micrograms beta-carotene).

[c]This refers to the marginal cost of calories plus the B vitamins associated with them.

On the other hand, there will be nutrients which come out at the minimum level because they are scarce and difficult to obtain at given market prices. In Recife, for example, proteins (animal and vegetable), vitamin A, iron, calories and B vitamins (riboflavin and niacin) are costly nutrients.[18] In order to provide extra units of each of them the total cost of the least cost diets should be increased by their respective marginal costs. Holding other things constant, by increasing the animal protein allowance in one gm/day the total cost of the least cost diet would go up by approximately Cr $.010 /day. However, there will be no marginal costs in providing extra units of calcium, phosphorous, fat, vitamin C and thiamin (under method I) because these are costless nutrients. Because the amount of B vitamins in these least cost diets is tied to the caloric level, two marginal costs actually have been provided. One is the cost of adding extra units of calories alone (net calories), and the other is the marginal cost of calories and B vitamins (riboflavin and niacin) together. If the ties between calories and B vitamins are broken, it would cost Cr $.010 less to provide an extra unit of calories in the diet calculated for Recife. It would cost Cr $.030 if the B vitamins have to be provided.

[18]Under method II iron and vitamin A were dropped out while calcium and vitamin C became costly nutrients.

The marginal cost of the scarce nutrients to be provided for a standard family in Fortaleza is also shown in Table 4.8. Under method I proteins (animal and vegetable), calcium, iron, calories and one of the B vitamins (niacin) are the costly nutrients. Here, if calories have to be provided alone (without the B vitamins) it would be a costless nutrient.[19] However, being provided with the B vitamins under the required proportions turns out to be costly. Nutrients like phosophorous, fat, vitamin A, vitamin C and the two other B vitamins (thiamin and riboflavin) are costless nutrients. At the margin, the total costs of the least cost diets could be decreased if the initial amount of proteins, iron, calcium, and calories associated with the B vitamins (each one at a time) is reduced by one unit.

The marginal cost of the nutrients calculated for both cities under method II closely follows the patterns shown by method I although some major points should be mentioned. First, some of the nutrients with a positive marginal cost now turns out to be costless (Example, iron in both cities and vitamin A in Recife). There are also cases in which the nutrients

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[19]Actually, it implies that additional units of calories would lower the cost of the diet.

were costless under method I but not under method II (Example, calcium in Recife and vitamin A in Fortaleza). Second, the marginal costs of the nutrients computed for both cities are initially higher under method II. Third, the figures for both proteins and calories actually represent an adjusted marginal cost, after allowing for nutrient variability. Finally, in Recife, net marginal cost of calories was always negative, although they are positive if the B vitamins must be provided along with the calories.

Although the marginal cost figures from a purely nutritional model provide a consistent measure of the relatively scarcity of the nutrients required in a diet, the units in which they are ordinarily measured are arbitrary and not related to the quantities required by the body [50]. It has been suggested that a more meaningful way to show the relative importance of each of the nutrients in a diet is by evaluating them as an aggregate, i.e., the way in which total expenditures are allocated among the several scarce nutrients.[20] Thus, for a standard family in Recife and Fortaleza, the most costly requirement to be fulfilled under method I is protein which accounts for

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[20]This is easily obtained, if one multiplies the marginal cost of the nutrients by the amount of the nutrients required in the diet.

57.9 and 61.7 percent of the total expenditures in two cities (see Table 4.9). Calories alone apportioned 28 percent of the total expenditures in Recife, but in Fortaleza it turns out to be a costless nutrient (Cr $-2.8) unless it is required to come along with the B vitamins (Cr $77.9 or 29 percent). A relatively high amount of the total expenditures has been allocated to B vitamins in both cities (Cr $33.00 in Recife and Cr $77.9 in Fortaleza). Calcium, vitamin A, vitamin C, and iron apportioned no more than 1.1 and 10 percent, respectively, of the total expenditures in the least cost diets calculated for both cities.

The amount of the total expenditures allocated with the scarce nutrients under method II is also shown in Table 4.9. As has been indicated, proteins continue to share the highest amount of the total expenditures, followed by vitamin C (in Recife) and vitamin A (in Fortaleza). On the other hand, net calories turn out to have a negative net contribution to total expenditures, and in Recife, it is true even when it is associated with the B vitamins. Allowing more calories and more variability would be less expensive and the net

Table 4.9. Aggregate value of the scarce nutrients in the least cost monthly diets for standard families in Recife and Fortaleza (Northeast of Brazil).

Nutrients	Aggregate value (Recife)		Aggregate value (Fortaleza)	
	Method I	Method II	Method I	Method II
	Cr $	Cr $	Cr $	Cr $
Proteins				
Animal	51.8	33.4	51.9	223.0
Vegetable	87.7	139.1	111.6	221.5
Calcium	--	8.2	8.7	52.0
Iron	.6	--	17.9	--
Vitamin A	2.3	--	--	92.4
Vitamin C	--	14.5	--	--
Calories				
Gross calories[a]	99.2	136.2	75.1	-198.4
B vitamins	33.0	191.8	77.9	179.6
Net calories	66.2	-55.6[b]	-2.8	-378.0
Total monthly cost of the L.C.D. (in Cr $)	241.6	331.4	265.2	390.5

[a]As referred on food note b, Table 4.8.

[b]Obtained by difference because rounding off errors on the significant figures.

contribution of calories is to reduce the cost of the diet in Fortaleza.[21]

G. The Marginal Efficiency of Foods

It has been established that the marginal cost measures of the nutrients in the least cost diets properly represent the true cost value of the nutrients contained in a specific food, because it is really an opportunity cost. Thus, the scarce nutrients have a positive opportunity cost but there is no opportunity cost at all for those abundant nutrients. Such an opportunity cost can be multiplied by the amounts of each nutrient required in the diets and added together in order to get an aggregate monetary value of the nutrients in a food dividing by its market price, it turns out to be a number (or a percentage if multiplied by 100) and it has been referred to in the literature as an adequate index to measure the marginal efficiency of the foods in a least cost diet [50]. Thus, a food in which the aggregate value of their nutrients is

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[21]Actually, there is a trade-off between the cost of meeting a minimum requirement for calories with B vitamins subject to low variability and the burden imposed by calories consumed because they use up valuable opportunity for calorie consumption. The net result turns out to be negative in this case, implying that relaxing the upper constraint imposed to calories would lower the cost of the diet.

equal to its market price has an efficiency at the margin equal to one (or 100 percent).[22] On the other hand, a food in which the aggregate value of its nutrients is smaller or greater than its market price will have a marginal efficiency smaller or greater than one. Foods with marginal efficiency equal to zero merely means that the aggregate value of its nutrients is worthless and if such a food is forced into the diet, it would increase the total cost by the right amount of its market price.

A more unusual, although plausible, situation is that one in which foods are having a negative marginal efficiency. The interpretation in such a case, although more difficult, can be easily understood at the extreme cases. For example, assume that a food (say lard) has a marginal efficiency equal to zero. Thus, if such a food is free and is forced into the diet, no additional cost would be incurred. However, if lard happens to have a negative marginal efficiency (instead of zero) it would increase the total cost of the diet even though it can be provided freely. As a matter of fact, by forcing lard into the diet, it will require that other foods are also added, and provided that those foods are

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[22]Theoretically, the marginal efficiency of a food is a real number ranging from minus infinity to plus infinity.

costly, the net effect will be an increase in the cost because the substitute foods are more expensive than the originally selected foods.

Table 4.10 lists the marginal efficiency of a set of foods in both cities, computed through methods I and II. Without going into the details, it should be mentioned that any food included in a least cost diet with purely nutritional objectives[23] has a marginal efficiency of 100 percent. That is, the aggregate monetary value of the nutrients contained in those foods is just equal to their market prices.[24] This means that all foods shown in Table 4.10 have a marginal efficiency less than one (or less than 100 percent), or none turns out to be economical because the aggregate monetary value of their nutrients was smaller than either their market price or zero. More precisely, foods like beans, eggs, coffee (in Recife) and chicken, beans (mulatinho) and butter (in Fortaleza) were not included in the diet because the expenditures required to buy any of these would be greater than the monetary value of the nutrients that they contain. Yet, some

[23]Purely nutritional diets, as referred to in this study, are those designed to satisfy purely nutritional objectives, that is, minimum nutrient physiological requirements. This is not to say that other factors like bulk, total weight or even palatability can not be considered nutritional objectives as well.

[24]For more detailed explanations and formal proofs of this statement see [19, p. 113 and 50].

Table 4.10. Marginal efficiency of some foods included in the commodity list for determining the least cost monthly diets of the standard families in Recife and Fortaleza (Northeast of Brazil).[a]

Order	City 1 (Recife)				City 2 (Fortaleza)			
	Method I		Method II		Method I		Method II	
	Foods	ME (%)[b]	Foods	ME (%)	Foods	ME (%)	Foods	ME (%)
1	Beans (mulatinho)	90.0	Carrots	97.0	Chicken	97.0	Fish (salt water)	89.0
2	Eggs	89.0	Rice	92.0	Beans (mulatinho)	93.0	Lettuce	85.0
3	Macaroni	87.0	Cabbage	82.0	Macaroni	82.0	Beans (macassar)	67.0
4	Corn meal (yellow)	81.0	Pepper	79.0	Codfish	79.0	Chicken	66.0
5	Bread	79.0	Fish (salt water)	78.0	Fish (salt water)	79.0	Potato	41.0
1	Coffee	1.0	Bacon	-105.0	Butter	-3.0	Doce (jelly)	-681.0
2	Butter	13.0	Butter	-86.0	Coffee	-2.0	Sweet potato (white)	-680.0
3	Pepper	13.0	Doce (jelly)	-82.0	Lard	-.1	Papaya	-437.0
4	Lettuce	14.0	Bread	-66.0	Sugar	.5	Sugar	-401.0
5	Chayotte	14.0	Yam	-12.0	Veg. oils	1.0	Butter	-360.0

[a]Such a list includes the five most efficient and the five most inefficient foods in both cities.

[b]ME = marginal efficiency, and it is calculated by dividing the aggregate monetary value of the nutrients in a food by its market price.

foods like bacon, butter, and yam (in Recife) and butter, coffee . . . and vegetable oils (in Fortaleza) would not be economical even though amounts of each of them were to be provided freely.

As indicated in Table 4.10, negative marginal efficiency is more frequent in those least cost diets computed by method II. This should be the pattern, however, because in this case the process of obtaining the aggregate monetary value of the nutrients makes allowances for the variability component. Actually, the mean nutrient content of a food can be interpreted as a benefit, while its variability is truly a cost. If a food shows a positive, zero or negative marginal efficiency, it will depend upon the magnitude or sizes of these two effects together (the mean nutrient content and its variability).[25] If variability is assumed to be zero as in the deterministic formulation (method I), the second effect drops out and the only remaining term is that accounting for the mean. There are occasions, however, in which the benefits become a cost (or a negative benefit) such as occurred with butter,

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[25]It can be shown that any food with a mean nutrient content (say, $\bar{x}$), will have a negative marginal efficiency as long as its variability (s_x) times the allowed confidence level (z or t value) is greater than such a mean.

coffee and lard in Recife.[26] In this specific case the three mentioned foods just provide one or two nutrients (example: calories and fat) and they may force other foods out of the diet. So even if they are free, the combined or aggregate costs of all nutrients contained in the complementary foods would be positive and increase the total cost of the diet. Also of interest are the patterns showing the marginal efficiency of the foods (on Table 4.10) calculated by methods I and II. For instance, in Recife, none of the five most efficient foods under method I is included in the list when method II is used. Yet bread, which is the fifth most efficient food at the margin under method I (79 percent) turns out to be a very inefficient food under method II (-66 percent).[27] In Fortaleza the patterns are more stable, where at least two of the most efficient foods under method I (chicken and salt water fish) are also efficient ones, when variability is allowed to occur. In addition, none of the most efficient foods under method I were found to be among the most inefficient under method II and vice versa.

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[26]Obviously, if such a benefit is negative under method II, the overall effect would still be great, since the variability effect is always negative.

[27]As pointed out before, this is probably due to its high protein/calorie variability.

Smith [7] has used the term Stigler's gap to indicate the difference between the price of the commodity and the value of the nutrients that it contains. According to him, this is a measure of the contribution of the nutritional and nonnutritional components of the total expenditure and it is paid to satisfy people's preferences and other nonnutritional objectives rather than the nutritional ones. Thus, the purchasing of the foods listed in Table 4.10 can be interpreted roughly as a measure of the nonnutritional component of the food expenditure since none were economical enough to be in the calculated least cost diets. In other words, if only the nutritional component of the total expenditure on food matters, none of those foods should be purchased. And the proportion of family incomes spent on the purchasing of such foods may be interpreted as an index of inefficiency because part of that is not being used to meet nutritional needs.

Finally, it is important to notice that some of the most inefficient foods listed in Table 4.10 (Example, coffee, sugar, doce, etc.), are ranked among the first items actually consumed by the households in both cities.

H. Effects of Family Size and Composition on the Total Cost of the Least Cost Diets

Table 4.11 indicates how changes in family structure will effect the total cost and food composition of the least cost diets. The family structure effect was evaluated by holding family size constant and stratifying it into three major categories as follows: a family with no pregnant or lactant women, and two others including either a pregnant or a lactant woman. The age effect on the total cost and composition of the least cost diets was computed by taking the standard family and dividing it in two major groups. The young (with 75 percent of the persons below the age of 15 and 25 percent over 15 years old) and the old in which at least 75 percent of the people are over 15 years old. The pregnancy and lactancy effect on the total cost of the diets were confined to the city of Recife and it was evaluated under both methods I and II.

As shown in Table 4.11, both pregnancy and lactancy effects on the total costs are quite significant, ranging from Cr $250.70 to Cr $367.70/month and Cr $259.04 to Cr $428.04/month for methods I and II, respectively. Although the effect is more significant under method II, the overall pattern is very consistent in both cases. That is, in the Northeast of Brazil it

Table 4.11. Total costs and food patterns in the least cost monthly diets providing different family patterns in Recife with the minimum nutritional intake.[a]

Consumption activities (foods in the L.C.D.)	Family patterns[b]					Coefficient of variations (c.v.)	
	$\overline{PL}$	P_w	L_w	Y_f	O_f	Protein	Calories
	Kg/month	Kg/month	Kg/month	Kg/month	Kg/month	%	%
Chicken	10.587	10.858	11.957	6.751	12.159	8.7	33.5
Milk	43.940	39.947	43.668	37.805	32.564	6.0	5.5
	(57.137)	(59.435)	(62.925)	(55.023)	(60.967)		
Eggs	10.118	13.186	13.786	13.115	19.019	4.3	3.5
Dried beef	--	--	--	--	--	50.3	27.9
	(4.449)	(4.690)	(4.611)	(3.622)	(5.484)		
Bacon	--	1.370	6.601	--	5.301	102.2	9.3
Corn meal (white)	65.302	69.842	68.390	60.787	70.727	8.4	.8
	(44.896)	(48.384)	(49.441)	(38.726)	(52.958)		
Sweet potato (white)	1.308	--	--	37.118	--	32.2	18.8
	(9.907)	(8.717)	(14.667)	(19.926)			
Sweet potato (yellow)	11.775	--	--	8.522	--	17.5	16.7
	(12.282)	(16.666)	(17.512)	(9.555)	(17.801)		
Manioc flour	15.868	14.276	9.102	11.384	6.026	39.3	4.2
	(27.002)	(29.951)	(29.003)	(27.827)	(26.558)		
Beans (mulatinho)	1.322	.929	1.598	--	3.071	7.3	6.3
Beans (macassar)	--	--	--	--	--	9.8	6.1
	(1.093)	(.540)		(.508)	(1.565)		
Tomato	--	16.576	27.617	--	18.876	15.7	5.8
Orange	--	.599	1.531	--	3.094	18.3	7.6
Total monthly costs of the L.C.D. (in Cr $)	325.61 (239.19)	367.70 (250.70)	428.04 (259.04)	293.50 (218.00)	425.05 (269.69)	--	--

[a]The figures in parenthesis refer to the foods in the least cost diets obtained by Method I.

[b]For a detailed exposition about family patterns and their nutritional daily allowances, see Table A-7 in Appendix A.

is more costly (Cr \$11.51 and Cr \$19.85/month under method I)[28] to provide adequate diets to families which have either a pregnant or a lactant woman. It also shows that to provide an adequate diet to a family with a pregnant woman would cost Cr \$8.34/month (Cr \$60.34) less than to feed the same family with a lactant woman. Some of the cost differentials between families with either a pregnant or lactant and those who show this pattern are quite obvious. The differences between pregnancy and lactancy nutritional costs can be additionally explained by the quantity and quality of the nutrient supplementation as well as the extension period involved in both cases. For example, lactancy requirements with respect to calories, proteins, vitamins A, B and C are usually higher than those to pregnancy. In addition, lactancy is assumed to extend by a period of six months, while pregnancy nutrient supplementation was confined just to a half of that period (4 1/2 months). The cost differentials between methods I and II, is due to the variability component which is assumed to be zero under the deterministic formulation.

The pregnancy and lactancy effects on the patterns of food composition seem to be quite significant

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[28]Or Cr \$42.09/month and Cr \$102.43/month under method II.

as shown in Table 4.11. It is important to notice, however, that under method I the composition patterns concerning the kinds of foods involved is quite stable. The major adjustments performed involve the amounts rather than the quality of the foods in the least cost diets. With minor exceptions, the quantities of milk, dried beef, cornmeal-white, sweet potatoes (white and yellow) and manioc flour are usually higher in the diets calculated for families with either pregnant or lactant women.

Under method II the lactancy and pregnancy effects on the food composition of the least cost diets were much more significant. Major adjustments were performed either in the quantities or in the qualities of the commodities involved. Allowing variability in the nutrients of the foods usually required not only high amounts of each food, but also different kinds and combinations of the foods in the least cost diets. For example, bacon for the first time is brought into the least cost diets, probably to satisfy additional fat and caloric requirements of pregnancy and lactancy. The major structural change in both lactants and pregnants diets is the high amounts of tomato and oranges and the complete exclusion of both varieties of sweet potato.

In Table 4.11, the total cost and food composition of least cost diets for a young and old family are shown. The cost differential between these two diets is quite significant. That is, in the Northeast of Brazil, old families would be more costly to feed than young families (23 and 44 percent higher, under methods I and II, respectively). Again, the quantitative differences on the dietary allowances accounts for the major portion of the cost differential, although under method II, the allowances for variability can be as significant as well. Therefore, as one should expect, the change patterns involving either the amounts and kinds of foods were much more significant under method II.

The procedure used to categorize the families in this research (young and old), along with the basic age structure used, may ignore some real effects on the total cost of nutrition for these two particular types of families. For instance, the first age group (0-4) could be broken into two other categories so that the category 0-4 years is less than one, and between one and four years old. Infants aged 0-1 usually require very special nutrient supplementation and because of their age, they cannot be exposed to the same commodities and dietary patterns as other people in the family unit. Therefore, although such a cost differential was quite significant, it is not conclusive and its implications cannot be evaluated without a careful assessment of the families capacity to feed itself. Given the structure of the young family defined here (75 percent below age 15 years old) it will impose a definite constraint in its capacity to generate enough income and getting all the essential nutrients that they need. In other words, in spite of the higher costs involved, it is very likely that the old families would be much better off than the young ones.

Due to the nature of the linear programming solution and the assumptions made about family structure, the effects of family size on the total cost of the least cost diets are quite obvious. That is, given the linearity assumption of the mathematical model used, doubling or reducing the size of the family by a half, one should either increase or reduce the cost of the diets proportionately. Therefore, provided that family age structure and sex composition do not change, the size effect will be reflected entirely on the amounts of the nutrients needed, and the costs of the diets will change proportionately. As one should expect, the food composition pattern is also the same among all the least cost diets, with their amounts changing proportionally.

Although straightforward, such results have some implications mainly to policy makers in less developed countries concerned with population issues and family planning programs. For instance, it shows that if only minimum physiological requirements must be satisfied, there are no economies of scale in providing families of different sizes with all of the nutrients that they need.[29] More precisely, economies of scale may be possible if wastes in consumption or non-nutrition factors are explicitly considered, but it would never come out if physiological requirements are the only thing to be taken into account. Obviously, there is a reason to fully accept such no economies of scale argument in the real world. First of all, family structure is very likely to change when family size changes; second, there is enough evidence that people are willing to pay for both the nutritional and nonnutritional components contained in a food or set of foods. Third, and more important, one can not ignore real economies of scale which may accrue from low wastes in consumption as well as the pecuniary economies derived

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[29]Obviously, such a conclusion deserves some theoretical qualifications, i.e., first, such no economies of scale is actually an assumption, not a fact; and second, the total cost curves, although assumed to be linear, can experience discontinuous decreases under certain ranges or limits for family size.

from the high amounts of food purchased when a family gets larger and larger.

I. Price Range over Which the Optimal Solution Holds

As mentioned earlier, the marginal efficiency of foods which are included in the least cost diet is 100 percent. This is because the aggregate monetary value of their nutrients is equal to the market price. It has also been indicated that the marginal efficiency of any food will vary as the price of the food varies. That is, foods which are sufficiently economical to be included in the optimal solution may not remain under a different set of market prices. Then, it may be worthwhile to know under what range of prices those economical foods will remain in the least cost diets.

Such information may provide interesting insights for agricultural and nonagricultural planning agencies concerned with questions about food price policies and its implications for both consumers and producers. The price range over which the economical foods will remain in the optimal solution shown in Table 4.12 was obtained by a routine part of the linear programming solution of the least cost diet problem.

Within cities, the price range differentials observed for the same commodity is by and large due to

Table 4.12. Price range over which the optimal solution holds (least cost monthly diet for standard families in Recife and Forteleza--Northeast of Brazil).

Consumption activities (foods in the L.C.D.)	City I (Recife)			City 2 (Fortaleza)		
	M.P.[a]	Method I	Method II	M.P.	Method I	Method II
	Cr $/Kg	Cr $/Kg	Cr $/Kg	Cr $/Kg	Cr $/Kg	Cr $/Kg
Chicken	9.00	--	9.28 (3.0)	--	--	--
Milk	1.10	1.20 (9.0)[b]	1.21 (10.0)	1.10	1.29 (17.0)	2.24 (221.8)
Eggs	3.75	--	3.91 (4.0)	3.50	3.68 (5.0)	3.92 (12.0)
Dried beef	12.40	15.32 (23.5)	--	--	--	--
Rice	--	--	--	1.50	1.83 (22.0)	2.09 (39.3)
Macaroni	2.70	--	2.82 (4.0)	2.98	--	3.14 (5.3)
Sweet potato (white)	.50	.52 (4.0)	--	.70	.72 (2.0)	--
Sweet potato (yellow)	.50	.85 (70.0)	.55 (10.0)	.70	.72 (2.0)	1.05 (.45)
Cornmeal (white)	1.80	1.81 (.5)	--	--	--	--
Manioc flour	.95	1.08 (13.6)	1.31 (37.0)	1.00	1.93 (93.0)	
Beans (mulatinho)	--	--	--	4.00	--	5.24 (31.0)
Beans (macassar)	3.30	3.64 (10.3)	--	3.50	3.75 (7.1)	--
Tomato	1.50	--	1.52 (1.0)	2.00	--	2.30 (15.0)
Orange	1.30	--	1.61 (23.0)	--	--	--

[a]M.P. = the food market prices at the time of the survey (November, 1973).
[b]The number in parentheses show the upper bounds for prices on a percental basis.

differences in the computational methods used (deterministic and stochastic). Between cities, they reflect differences in market prices along with their prices relative to other commodities included and excluded from the optimal solution. Some foods are more stable to price variations than others, and the patterns seem to be quite different between the two methods used. Within cities, the patterns still differ considerably between commodities or even for the same commodity. For example, in Recife under method I, milk would be driven from the optimal solution if its market price goes up by more than 9 percent (or Cr $1.20/Kg), but under method II, it is an efficient food as long as its market price does not increase more than 10 percent (or Cr $1.21/Kg). Still, in Recife, the most stable commodities are: sweet potato (yellow)--70 percent; dried beef--23.5 percent and manioc flour--13.6 percent. Manioc flour is still among the most stable commodities when method II is used, and it is followed by orange--23.0 percent, and sweet potato (yellow)--10 percent.

The figures for Fortaleza closely follows the overall patterns observed in Recife, where a considerable range of variation is observed between methods and the commodities involved. Then, under method I, manioc flour together with rice and milk, are among the most stable commodities, and they would remain as economical foods as long as their market prices do not go up more than 93, 22 and 17 percent, respectively. Under method II, however, milk is a highly stable commodity, and it is followed by rice and beans (mulatinho) which would remain in the optimal solution if their market price does not increase by more than 34.3 and 31.0 percent, respectively. In summary, the information provided by this computer routine can be very useful for policy makers concerned with both agricultural and nonagricultural commodities. In other words, the knowledge about the most economical foods and how market price variations will affect the stability of the least cost diets, may provide an objective basis for decision makers setting up and implementing price policies more consistent with other development goals.

CHAPTER V

CONCLUSIONS, LIMITATIONS OF THE STUDY AND RECOMMENDATIONS FOR NEEDED RESEARCH

This chapter is divided into two major sections. The first summarizes the results obtained and their major policy implications. Section two presents the limitations of this study and recommendations for improving future research by both economists and nutritionists.

A. Summary and Conclusions

Most of the purely nutritional diets provided in this study have indicated that it is possible to improve the actual nutrient intake levels, even among the lowest income group. Actual food expenditures were shown to be inefficient when compared with the least cost diets. There are also indications that actual incomes can be better allocated to meet family nutritional needs at lower cost. This leads to the conclusion that families in both cities are not efficient in the purchase of nutrition, that is, they have not been able to provide themselves with all essential nutrients in the most economical way.

The economical waste as reflected by the difference in the patterns of food expenditures for actual and least cost diets, seem to have imposed a high economic burden, especially for those families in the lowest income levels. Computations of least cost diets allowing conventional constraints to take into account tastes and preferences support this argument, that is, although those factors affect one's expenditure on food, they cannot account for the total inefficiency in the system. Ignorance about most efficient foods and the knowledge about adequate dietary allowances seems equally important. Yet, changes in the conventional consumption patterns were found to be economical in at least one of the cities studied. Economical substitutions between traditional and nontraditional commodites may lower the cost of nutrition in that part of the country.[1] In Fortaleza, for example, where the qualitative and quantitative nature of the adjustments were more significant, beef liver, sardines, squash and cabbage turn out to be very efficient foods.

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[1]To be precise, the word "nontraditional" should not be applied fully, because the commodities involved have been usually consumed by the families in the two cities.

Although very low (about 20 U$A or Cr $1272.12/ year on the average), actual levels of per capita incomes are adequate to provide standard families in Recife and Fortaleza with their minimum physiological requirements. To buy adequate nutrition, it would cost Cr $47.38 (Cr $66.95) per person per month in Recife and Cr $49.12 (76.51) in Recife.[2] Compared with the actual levels of per capita incomes (Cr $106.50/month), such a figure implies that 46 to 63 percent of the actual income in Recife and 46 to 71 percent in Fortaleza should be allocated to the purchasing of nutrition. There are no major indications that the actual levels of income would prevent consumers in both cities from buying all the essential nutrients that they need. Obviously, for those below the average income levels or those willing to pay for other components of the total food expenditure (e.g., cultural), a much higher proportion of their incomes should be spent on food. For instance, it was shown that the costs of least cost diets allowing conventional restrictions rather than the purely nutritional ones, would be 32 and 42 percent higher than those calculated to satisfy purely nutritional objectives. Policy makers using minimum nutritional standards as a basis for establishing poverty

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[2]The figures in parenthesis refer to the total costs computed by method II.

lines, have to decide whether or not to satisfy peoples' conventional habits and preferences must be explicitly accounted among their socioeconomic goals.

Overall, all of the least cost diets obtained in this study are poor in variety. This pattern is quite consistent with other studies in which linear programming has been used to determine the least cost diets based on purely nutritional objectives. The shifts, however, would not drastically affect the conventional consumption patterns in both cities because they involve merely changes in the proportions of the foods already eaten. In Recife, for example, it means a shift from foods like bread, beef, butter, vegetable oils and coffee to cornmeal (white), dried beef, chicken, milk, eggs, beans (macassar) and tomatoes. In Fortalezo the families would spend relatively more on rice, eggs, sweet potatoes (white and yellow), tomatoes and less on bread, beef, milk, manioc flour, coffee and both varieties of beans.

Obviously, the least cost diets in this study would never be acceptable to all families in Recife and Fortaleza, at least in the short run. However, knowing the most economical foods would provide guidelines to public administrators by indicating where emphasis must be placed with regard to agricultural production plans and nutrition education. For example, milk turns out to be a highly efficient food, entering in all least cost diets computed in the two cities studied, including those allowing conventional constraints to take into account explicitly traditional consumption habits. Cornmeal (white) and manioc flour (in Recife) and chicken, rice and sweet potato (yellow) (in Fortaleza) also are economical foods in most of the calculated least cost diets. Such results have both agricultural and non-agricultural policy implications, that is, nutrition educational programs indicating the kinds and right amounts of foods to be purchased in both cities would improve the allocation of the scarce resources in a sense that consumers would pay less for better nutrition. Yet adjusting agricultural production plans toward those patterns may bring an efficient allocation of the scarce resources to advance both agricultural and nutritional targets. Obviously, it should not be ignored that the macro effects on agricultural output and prices resulting from simultaneous production of certain food stuffs. For example: adjusting agricultural production forward the suggested patterns may be unfeasible because it is beyond the region or country's production possibility. Yet, given that the adjustment lags in both demand and supply, such an adjustment will result in low or high prices, which either desertimulate producers or make the foods more expensive and uneconomical to be included in the least cost diets.

In Recife the least cost substitutions indicating which set of foods could be economically replaced, showed that none of the potential commodities considered (lobster, shrimp, beef liver, visceras, sardines, margarine, bananas and garlic) were efficient enough to replace milk, chicken, dried beef, sweet potatoes (white and yellow), etc., present in the basic least cost diets. In Fortaleza, however, some of the potential commodities considered (beef liver, visceras, sardines, squash and cabbage) entered in the new optimal solution either by replacing or altering the quantities of the other items in the diet. The least cost adjustment was quite significant, and it ranged from Cr $13.14 per month under method I to Cr $83.10 when the computations were performed by method II.

The family composition effects on the total cost and food pattern of the least cost diets were significant, especially those concerned with pregnancy and lactancy. It would be costly to provide the essential nutrients to families which have either a pregnant or a lactant woman. Yet, the lactancy effect which reflects a long period and relatively higher nutrient daily supplementation seems to be most significant. The age effect on the total cost of nutrition, although significant, cannot be conclusive for the purpose in view, mainly because it cannot be evaluated without a careful assessment of the family's economic ability to purchase the right foods. In addition, the basic age structure pattern used in this study ignores some of the real age effects on the total cost of nutrition, such as the infant group (aged 0-1) which deserves special attention with respect to both quantity of the daily supplementation and the kinds of the commodities to be provided in the least cost diet.

Because of the assumptions and the nature of the linear programming solution, the family size effect on the total cost of nutrition is obvious. The total cost and amounts of each food in the diet will increase or decrease proportionally, when family size is increased or decreased accordingly. To put it another way, there are no economies of scale in providing the right foods to families of different sizes, when the diets have purely nutritional objectives. Given the linearity assumption of the mathematical model used, doubling or reducing the family size by a half, one would double or reduce proportionally the total cost of the least cost diets. Obviously, such a conclusion deserves some qualification, because it is not a fact but a result of the assumptions (linearity of the total cost function and constant family structure) implicit in the computation of the least cost diets. Policy makers concerned with population issues may be interested in using this information in order to determine the right amounts of subsidies required to feed large families or when there are persons under a special physiological state (pregnancy or lactancy). They may also be interested in constructing indexes relating the families capacity to generate income and the total cost of nutrition. Yet, by relating family size with the amounts and the total cost of obtaining all the essential nutrients, one can fairly well indicate an optimum family size given the country or region production possibilities.

The marginal cost of the nutrients calculated in this study indicated that although all nutrients are important in satisfying minimum dietary standards, they are not equally costly. The marginal figures computed were sensitive to the market prices of the foods in the least cost diets as well as to the computational procedure used. Because these marginal cost figures reflect opportunity costs, they can provide guidelines for an efficient allocation of the scarce resources. That is, efforts should be devoted to supplying additional amounts of the nutrients which have an opportunity cost greater than zero. In Recife, the proteins (animal and vegetable), vitamin A, iron, calories, plus the B vitamins (riboflavin and niacin) were among the costly nutrients. Those patterns changed a little under method II, in which calcium became an expensive nutrient at the margin, while vitamin A and iron turned out to be costless. In Fortaleza the costly nutrients were: proteins (animal and vegetable) calcium, iron and calories associated with niacin (under method I). Under method II, vitamin A became a costly nutrient and iron turned out to be costless. Nutrients like fat, phosophorous and thiamin (B vitamin) were costless nutrients in all least cost diets calculated for Recife and Fortaleza.

In the aggregate protein accounted for more than 50 percent of the total expenditures in both cities. Calories associated with the B vitamins are usually sharing the second greatest amount of the total expenditures on food in both cities, except in Fortaleza (under method II) where such a nutrient component turns out to have a negative contribution. That is, by allowing more calories in such a diet, one would actually lower the total cost of nutrition. The other nutrients together like calcium, vitamin A, vitamin C and iron are sharing a small part of the total food expenditures as shown by the least cost diet (a maximum of 10 percent).

As mentioned before, foods like milk, manioc flour and sweet potato-yellow (for both Recife and Fortaleza) were found to be highly efficient foods (marginal efficiency equal to 100 percent), by entering in the optimal solution for almost all of the least cost diets calculated in this study. Other foods like beans--mulatinho, eggs, carrots, pepper . . . fish--salt water . . . beans--macassar (in Fortaleza) also were found to have a high marginal efficiency (although less than 100 percent) in the basic diets calculated for standard families in those two cities. Policy makers may find it worthwhile to identify which foods are most efficient in meeting the population's nutritional goals and use them as guidelines for production and nutritional plans. They may need to know which foods excluded from the optimal solution are the most efficient in order to estimulate both production and consumption. On the contrary, they may know what are the less efficient commodities which provide the population with all the essential nutrients. For instance, it was found in this study that some of the foods largely consumed by families in these two cities, such as sugar, coffee and doce (jelly) are highly inefficient with marginal efficiencies ranging from 1 to -681.0 percent.

The range over which the price of the efficient foods could vary without driving them out of the least cost diets was sensitive to market prices as well as to the computational procedures used in this study (methods I and II). In Recife, sweet potato (yellow), dried beef and manioc flour (under method I) and orange and sweet potato--yellow (under method II) are among the most stable commodities. They would remain economical foods unless their market price variations are excessively high. On the other hand, in Fortaleza, manioc flour, together with rice, milk and beans--mulatinho are the commodities least sensitive to market price variations. The potential use of such information is obvious. Knowledge about the most economical foods along with their degree of stability to price variations will indicate the best price policy strategies to achieve both targets of production and population nutritional goals.

Finally, it should be pointed out that the results obtained in this study differ quite substantially depending upon the computational procedures used (deterministic or stochastic). Allowance for variability in the nutrient content of the foods (calories and proteins) has been reflected not only on the total cost, but also in the qualities and quantities of foods in the least cost diets. The patterns of least cost substitutions, the marginal cost of nutrients, the marginal efficiency of the foods and the commodity stability to market price variations are also sensitive to changes in the methodological procedures used to compute the least cost diets. The effect of probability constraints on the total cost and food pattern of the least cost diets is still quite significant even when it was broken into its two major components: calories and proteins. In other words, allowing either calorie or protein variability to be random variables would be more costly than diets in which the variances of the nutrients contained in a food is assumed to be zero. The effects, however, are not additive and one would expect that the higher the number of the nutrients subject to random variations, the higher should be the effect on the total cost and composition of the least cost diets. Given that food nutrients variability is consistent with both theory and empirical evidence, a mathematical model designed to take this into account should be more operative in the real world and probably more relevant to serve as a basis for policy recommendations.

B. Limitations of the Study and Suggestions for Needed Research

Although a number of conclusions have been drawn from this study, both the limitations and recommendations for future research should be emphasized. The limitations and suggestions for needed research are concerned with both the methodology and the basic data used in this study. The following paragraphs present the limitations and suggestions that researchers in this field should consider improvement of analytical power and the basis for policy recommendations. Limitations and recommendations are as follows:

1. The sampled area in this study was confined to the urban northeast, where habits, foodstuff supplies, commodity prices, consumption and income patterns may differ quite significantly from rural areas. Therefore, that the policy prescriptions should be confined to the population and sampled areas. Extending the sampled area to include both the urban and rural northeast could provide a wider basis for policy prescriptions and recommendations.

2. The commodity list and the actual consumption patterns considered in this study were confined to a Northeast Bank Consumers Budget Survey, undertaken several years ago. Thus, it is very likely that the patterns have changed appreciably, either because of price rises of certain foodstuffs or changes in consumer's habits and preferences. Because the purely nutritional diets should be consistent with actual consumers habits, such a consumer's budget survey must be up-dated and extended to both urban and rural consumers.

3. The commodity prices used in this study are cross-sectional data for November, 1973. Therefore, the commodity bundle on the least cost diets calculated in

this study reflects a relative price structure for that particular point of time. Efforts to improve the actual market information system (at the retail level) and providing time series data of all commodities seem to be feasible and need to be considered. This would permit an identification of the change in total costs and food composition of the diets due to seasonal adjustments in commodity prices.

4. In this study it was implicitly assumed that all commodities included in the matrix would be available for purchase by consumers in both cities. Eventually, some of the commodities would be in shortages or may be completely absent from the market at certain times of the year. To be realistic, one must try to select a commodity list which fits in both consumption and production patterns.

5. The income patterns used in this study are per capita averages and are assumed to be representative for the northeast region as a whole. As such they may ignore vital differences in income levels between states, rural/urban areas or even among the various sections of the population. Accurate estimates of such income levels to compare with the costs of least cost diets, would provide policy makers with a more solid basis to set up poverty lines and deciding whether or not subsidies must be provided in order to fulfill other basic human needs.

6. Some nutritional factors, like digestibility, volume eaten per meal, as well as wastes in cooking were not explicitly considered in this study. Other materials needed by the cook such as salt, spices, vinegar, etc., although essential to life and needed to improve palatability, were also omitted in this research. Actually, it is very unlikely that salt and condiments would come out in the optimal solution of the purely nutritional diets, mainly because they do not provide many of the nutritional elements taken into account in the model. However, if one considers salt and condiments to be essential to palatability, the model can be easily extended to include specific requirements for them.

7. The proxy method used in this study to linearize the variances in the stochastic formulation have introduced a bias in the results. As Bender [7] pointed out, such a bias is upward, but its magnitude is not known. The immediate effect of such a bias is to increase upward the total cost of the least cost diets, since such a safety margin (the variance term) is actually higher than it should be.

8. It comes to the limitations of the study. In this study the minimum dietary allowances for vitamin A, phosphorous and fat were derived from a secondary source [19]. Provided that minimum dietary allowances varies with sex, age, climate, working conditions, etc., such standards may not be the most appropriate,

if one decides to use them as a basis for policy recommendations. In some cases (e.g., vitamin A) the gap was quite significant and, therefore, a word of caution must be called for before extrapolating and/or using it as a basis for policy prescriptions. As a matter of fact, the usual procedures by nutritionists in setting up such standards should be qualified as well. It has been recognized that such standards vary considerably among countries or regions in the world, and even within countries (or regions), they are changing over time. In other words, a least cost diet calculated at a time period t, may not be the least expensive at time period t + 1. In addition, most of the literature in this specific field does not show any explicit statement of how such standards should be interpreted (minimum substance levels!) as well or the existence and basis for introducing safety margins in the suggested dietary allowances.

In summary, nutritional research designed to improve the stock and quality of the physical data available should be encouraged. The areas requiring more attention are those concerned with the minimum dietary allowances and the nutrient content of the foods. The suggested dietary allowances for the Northeast population by INUFPe is actually an adaptation from many specialized sources like Food and Agriculture

Organization (FAO), World Health Organization (WHO) and Instituto Nacional de Nutricion (Colombia). Also, there is no effort indicating amino acid daily requirements by sex and age categories in that part of the country.

Nutritional research involving food composition should continue and improvements are needed in at least three areas (i) and food composition analysis should include not only the conventional commodities, but also other foods (natural or synthetic) potentially available to consumers; (ii) an increase in sample size and explicit recognition of the ranges of variability of nutrient content of foods; and (iii) provide aminograms (amino acid composition) of foods.

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APPENDIX A

NUTRITIONAL REQUIREMENTS AND THE DEMOGRAPHIC PATTERNS IN THE NORTHEAST OF BRAZIL

Table A-1. Recommended dietary allowances by the Institute of Nutrition at Federal University of Pernambuco (INUFPe) - Northeast, Brazil.[1]

	Calories (kcal)	Protein (gm)	Calcium (mg)	Phosphorous[2] (mg)	Iron (mg)	Thiamin (mg)	Ribo-flavin (mg)	Niacin (mg)	Vit. A[2] (IU)	Vit. C (mg)	Fat[2] (gm)
Infants and children											
0 - 1 both sexes	880.0	27.0	550.0	500.0	7.0	.4	.5	5.8	1300.0	40.0	9.0
1 - 3 "	1270.0	31.0	450.0	400.0	8.0	.5	.7	8.3	800.0	40.0	12.0
4 - 6 "	1570.0	36.0	450.0	400.0	10.0	.6	.9	10.3	1100.0	40.0	14.0
7 - 9 "	2050.0	43.0	450.0	400.0	12.0	.8	1.2	13.5	1500.0	40.0	16.0
10 - 12 males	2345.0	61.0	650.0	600.0	15.0	.9	1.4	15.4	2000.0	40.0	18.0
10 - 12 females	2250.0	60.0	650.0	600.0	15.0	.9	1.3	15.0	2000.0	40.0	17.0
Adolescents											
13 - 15 males	3030.0	78.0	650.0	600.0	15.0	1.2	1.8	19.9	2700.0	40.0	22.0
16 - 19 "	3220.0	78.0	550.0	500.0	15.0	1.3	1.9	21.2	3600.0	40.0	27.0
13 - 15 females	2640.0	73.0	650.0	600.0	15.0	1.1	1.6	17.4	2900.0	40.0	21.0
16 - 19 "	2340.0	65.0	550.0	500.0	15.0	.9	1.4	15.4	3300.0	40.0	20.0
Adults											
20 - 29 males	2790.0	68.0	450.0	400.0	10.0	1.1	1.7	18.4	3900.0	50.0	29.0
30 - 39 "	2700.0	68.0	450.0	400.0	10.0	1.1	1.6	17.8	3900.0	50.0	28.0
40 - 49 "	2570.0	68.0	450.0	400.0	10.0	1.0	1.5	16.9	3900.0	50.0	26.0
50 - 59 "	2420.0	68.0	450.0	400.0	10.0	1.0	1.4	15.9	3900.0	50.0	25.0
60 - 69 "	2200.0	68.0	450.0	400.0	10.0	.9	1.3	14.5	3300.0	50.0	22.0
Adults											
20 - 29 females	1860.0	60.0	450.0	400.0	15.0	.8	1.1	12.2	3300.0	50.0	19.0
30 - 39 "	1800.0	60.0	450.0	400.0	15.0	.8	1.0	11.8	3300.0	50.0	18.0
40 - 49 "	1710.0	60.0	450.0	400.0	15.0	.8	1.0	11.2	3300.0	50.0	18.0
50 - 59 "	1620.0	60.0	450.0	400.0	12.0	.8	1.0	10.6	3300.0	50.0	17.0
60 - 69 "	1480.0	60.0	450.0	400.0	12.0	.8	.9	9.8	3300.0	50.0	16.0
Pregnant Women (Δ_{PW})	+200.0	+12.0	+650.0	+800.0	+3.0	+.2	+.3	+2.0	+4280.0	+15.0	+21.0
Lactant Women (Δ_{LW})	+800.0	+18.0	+650.0	+900.0	+3.0	+.4	+.5	+6.0	+5280.0	+15.0	+27.0

[1]Basic Sources: Food and Agriculture Organization (FAO), World Health Organization (WHO) and Instituto Nacional de Nutricion (Colombia).

[2]Source: Florencio, C. A. The "Efficiency of Food Expenditures Among Working-Class Families in Colombia," Unpublished Ph.D. Dissertation, Michigan State University, 1967, pp. 13-14. The daily nutrient supplementation of fat, phosphorous and vitamin A are averages for women in this category (pregnant and lactant).

Table A-2. Nutrient daily requirement by sex and age categories (abridged data).[1]

Age Groups		Calories (kcal)	Protein (gm)	Calcium (mg)	Phosph. (mg)	Iron (mg)	Thiamin (mg)	Riboflav. (mg)	Niacin (mg)	Vit. A (IU)	Vit. C (mg)	Fat (gm)
0-4	(both sexes)	1240.0	31.0	483.0	433.0	8.0	.50	.70	8.1	1067.0	40.0	12.0
5-9	(both sexes)	1810.0	40.0	450.0	400.0	11.0	.70	1.10	11.9	1300.0	40.0	15.0
10-14	(M)	2687.5	69.5	650.0	600.0	15.0	1.05	1.60	17.7	2350.0	40.0	20.0
	(F)	2445.0	66.5	650.0	550.0	15.0	1.00	1.45	16.2	2450.0	40.0	19.0
15-49	(M)	2862.0	72.0	510.0	460.0	12.0	1.10	1.70	18.8	3600.0	46.0	26.0
	(F)	2070.0	64.0	510.0	460.0	15.0	.90	1.20	13.6	3200.0	46.0	19.0
>49	(M)	2310.0	68.0	450.0	400.0	10.0	.95	1.40	15.2	3600.0	50.0	24.0
	(F)	1550.0	60.0	450.0	400.0	12.0	.80	.95	10.2	3300.0	50.0	17.0

[1] Basic Source: Table A-1.

[2] M = Male; F = Female

Table A-3. Nutrient daily requirement by age categories (data adjusted for sex composition).[1]

Age Groups	Calories (kcal)	Protein (gm)	Calcium (mg)	Phosph. (mg)	Iron (mg)	Thiamin (mg)	Riboflav. (mg)	Niacin (mg)	Vit. A (IU)	Vit. C (mg)	Fat (gm)
0 - 4	1240.0	31.0	483.0	433.0	8.0	.5	.7	8.1	1067.0	40.0	12.0
5 - 9	1810.0	40.0	450.0	400.0	11.0	.7	1.1	11.9	1300.0	40.0	15.0
10 - 14	2565.0	68.0	650.0	575.0	15.0	1.0	1.5	16.9	2401.0	40.0	20.0
15 - 49	2422.0	68.0	510.0	460.0	13.7	1.0	1.4	16.1	3378.0	46.0	22.0
> 49	1884.0	64.0	450.0	400.0	11.0	1.0	1.1	13.2	3432.0	50.0	20.0

[1]Basic Source: Table A-2. The adjustment for age was performed according to formula given by Silva, P. R., Staff Papers No. 74-20, Department of Agricultural Economics, Michigan State University, 1974.

Table A-4. Nutrient daily requirement by age categories (data adjusted for sex composition and pregnancy).[1]

Age Groups		Calories (kcal)	Protein (gm)	Calcium (mg)	Phosph. (mg)	Iron (mg)	Thiamin (mg)	Riboflav. (mg)	Niacin (mg)	Vit. A (IU)	Vit. C (mg)	Fat (gm)
0 - 4		1240.0	31.0	483.0	433.0	8.0	.5	.7	8.1	1067.0	40.0	12.0
5 - 9		1810.0	40.0	450.0	400.0	11.0	.7	1.1	11.9	1300.0	40.0	15.0
10 - 14		2565.0	68.0	650.0	575.0	15.0	1.0	1.5	16.9	2401.0	40.0	20.0
15 - 49	(M)[2]	2862.0	72.0	510.0	460.0	12.0	1.0	1.7	18.8	3600.0	46.0	26.0
	(F)	2270.0	76.0	1160.0	1280.0	18.0	1.1	1.5	15.6	7480.0	61.0	40.0
> 49		1884.0	64.0	450.0	400.0	11.0	1.0	1.1	13.2	3432.0	50.0	20.0

[1]Basic Source: Table A-3.

[2]The male dietary allowance is according to recommendations on Table A-2. The adjustments for pregnancy were performed according to recommendations on Table A-1.

Table A-5. Nutrient daily requirement by age categories (data adjusted for sex composition and lactancy).[1]

Age Groups	Calories (kcal)	Protein (gm)	Calcium (mg)	Phosph. (mg)	Iron (mg)	Thiamin (mg)	Riboflav. (mg)	Niacin (mg)	Vit. A (IU)	Vit. C (mg)	Fat (gm)
0 - 4	1240.0	31.0	483.0	433.0	8.0	.5	.7	8.1	1067.0	40.0	12.0
5 - 9	1810.0	40.0	450.0	400.0	11.0	.7	1.1	11.9	1300.0	40.0	15.0
10 - 14	2565.0	68.0	650.0	515.0	15.0	1.0	1.5	16.9	2401.0	40.0	20.0
15 - 49[2] (M)	2862.0	72.0	510.0	460.0	12.0	1.1	1.7	18.8	3600.0	46.0	26.0
15 - 49[2] (F)	2870.0	82.0	1160.0	1380.0	18.0	1.3	1.7	19.6	8480.0	46.0	61.0
> 49	1884.0	64.0	450.0	400.0	11.0	1.0	1.1	13.2	2432.0	50.0	20.0

[1]Basic Source: Table A-3.

[2]The male dietary allowance is according to recommendations on Table A-2. The adjustments for lactancy were performed according to recommendations on Table A-1.

Table A-6. Nutrient daily requirement by age categories (data adjusted for sex composition, pregancy and lactancy).[1]

Age Groups	Calories (kcal)	Protein (gm)	Calcium (mg)	Phosph. (mg)	Iron (mg)	Thiamin (mg)	Riboflav. (mg)	Niacin (mg)	Vit. A (IU)	Vit. C (mg)	Fat (gm)
0 - 4	1240.0	31.0	483.0	433.0	8.0	.5	.7	8.1	1067.0	40.0	12.0
5 - 9	1810.0	40.0	450.0	400.0	11.0	.7	1.1	11.9	1300.0	40.0	15.0
10 - 14	2565.0	68.0	650.0	575.0	15.0	1.0	1.5	16.9	2401.0	40.0	20.0
15 - 49[2]	2476.0	69.0	568.0	539.0	14.0	1.0	1.5	16.5	3855.0	48.0	25.0
> 49	1884.0	64.0	450.0	400.0	11.0	1.0	1.1	13.2	3432.0	50.0	20.0

[1]Basic Source: Table A-3.

[2]Special adjustments were performed to this age group in order to allow the computation of the expected number of lactant and pregnant women in such age groups. For details see: Silva, P. R., Staff Papers No. 74-20, Department of Agricultural Economics, Michigan State University, 1974.

Table A-7. Monthly family nutrient requirements to Recife-Pe (Northeast of Brazil).[1]

Nutrients	Family Group (standard or representative-R)	Family Group with a pregnant woman-Pw)	Family Group with a lactant woman-Lw)	Family Group (without pregnant or lactant women - PL)	Family Group Young[2] Y_f	Family Group Old[2] O_f
Calories (kcal)	330776.8	342545.8	360545.8	326645.8	309480.7	356184.0
Proteins (gm)	9063.7	9413.2	9593.2	8987.2	7956.0	10365.7
Calcium (mg)	82255.9	97318.9	97318.9	72818.9	82275.7	82390.5
Phosphorus (mg)	75753.4	94309.9	97309.9	69709.9	74472.7	77150.2
Iron (mg)	1921.7	1948.7	1948.7	1898.7	1836.0	2027.2
Thiamin (mg)	136.3	144.0	150.0	136.3	122.4	153.0
Riboflavin (mg)	197.7	207.0	213.0	190.8	183.6	214.2
Niacin (mg)	2204.9	2284.8	2404.8	2174.3	2042.5	2398.3
Vit. A (IU)	445462.6	542355.1	572355.1	408972.1	329829.7	573635.2
Vit. C (mg)	6900.3	7197.3	9197.3	6747.3	6426.0	7420.5
Fat (gm)	3171.7	3668.2	3848.2	2942.2	2754.0	3635.7

[1]Based on average family size, of 5.1 as given on Table A-9, and a 30-days period. The family nutrient requirements are based on the dietary allowances showed on Table A-2, 3, 4, 5, and 6, weighted by the relative frequency of each age group within the family (number of people in each age category).

[2]In the young group 75% of the family is assumed to be under age 15 and 25% above it. In the old family group 75% of the family is assumed to be within age group 15-49, and the remaining 25% above 49 years old.

Table A-8. Monthly family nutrient requirements to Fortaleza-Ce (Northeast of Brazil).[1]

Nutrients	Family Group (standard or representative-R)	Family Group with a pregnant woman-Pw)	Family Group with a lactant woman-Lw)	Family Group (without pregnant or lactant women - PL)	Family Group Young[2] Y_f	Family Group Old[2] O_f
Calories (kcal)	349071.1	362577.1	380557.1	344697.1	327685.5	377136.0
Proteins (gm)	9504.5	9867.5	10047.5	9423.5	8424.0	10975.5
Calcium (mg)	87147.9	101949.9	101949.9	82449.9	87115.5	87237.0
Phosphorus (mg)	80262.9	98463.9	101463.9	73863.9	78853.5	81688.5
Iron (mg)	2029.9	2047.9	2047.9	2005.6	1944.0	2146.5
Thiamin (mg)	143.0	151.1	157.1	143.0	129.6	162.0
Riboflavin (mg)	208.7	218.9	224.9	200.6	194.4	226.8
Niacin (mg)	2324.2	2414.5	2534.5	2291.8	2162.7	2539.3
Vitamin A (IU)	464381.1	560126.1	590126.1	425744.1	349231.5	607378.5
Vitamin C (mg)	7273.8	7561.8	7561.8	7111.8	6804.0	7857.0
Fat (gm)	3337.2	3838.2	4018.2	3094.2	2916.0	3847.5

[1]Based on an average family size of 5.4 as given on Table A-9, and a 30-days period.

NOTE: For the computational procedures and description of the young and old family, see footnotes (1) and (2) on Table A-7.

Table A-9. Average family sizes in Recife and Fortaleza (Northeast of Brazil).[1]

Classes (No. of People)	Frequency (No. of families)		f_1X	f_2X
X	Fortaleza (f_1)	Recife (f_2)		
1	5,618.0	11,487.0	5,618.0	11,487.0
2	21,218.0	30,185.0	42.436.0	30,185.0
3	24,444.0	33,738.0	73,332.0	33,738.0
4	24,013.0	31,021.0	96,052.0	31,021.0
5	21,992.0	27,975.0	109,960.0	27,975.0
8*	57,541.0	67,770.0	460,328.0	67,770.0
12.5*	6,599.0	7,789.0	82,488.0	7,789.0
15*	477.0	590.0	7,155.0	590.0
Totals	161,902.0	210,555.0	877,369.0	1,081,403.0

[1]Basic Sources: Censos Demográficos des Estados do Ceará e Pernambuco (Fundacão IBGE-1970). The estimated averages are 5.1 and 5.4 to Recife and Fortaleza, respectively.

[2]These figures represent class middle points for the following age groups: 6-10, 11-14 and greater than 15.

Table A-10. Population by age and sex in Recife and Fortaleza (Northeast of Brazil).[1]

Age groups	Males (A)	Females (B)	Total (A + B = C)		A/C	B/C
	No.	No.	No.	%		
0-4	63,573.0 (74,984.0)	62,798.0 (72,922.0)	126,371.0 (147,906.0)	15 (14)	.50 (.51)	.50 (.49)
5-9	60,018.0 (71,843.0)	59,203.0 (71,220.0)	119,221.0 (143,063.0)	14 (13)	.50 (.50)	.50 (.50)
10-14	47,865.0 (62,032.0)	52.963.0 (65,621.0)	100,838.0 (127,653.0)	12 (12)	.47 (.52)	.53 (.48)
15-49	192,286.0 (230,382.0)	235,777.0 (293,409.0)	428.063.0 (523,791.0)	50 (50)	.45 (.44)	.55 (.56)
>49	36,845.0 (51,462.0)	46,652.0 (66,736.0)	83,497.0 (118,198.0)	9 (11)	.44 (.44)	.56 (.56)
Totals	400,587.0 (450,703.0)	457,393.0 (569.908.0)	857,980.0 (1,060,611.0)	100	.47 (.46)	.53 (.54)

[1]Basic Sources: Censos Demográficos dos Estados do Ceará and Pernambuco (Fundacão IBGE-1970).

NOTES:
(1) The figures in parentheses refer to Recife.
(2) The number of children aged 0-1 is 26,823 in Fortaleza and 30,081 in Recife.
(3) The child-women ratio was computed as follows: population under 1 year of age divided by the female population in the 15-49 age group. Such a percentage was 11.3% to Fortaleza and 10.2% to Recife, which gives an average of 10.25% for the two cities.

APPENDIX B

PATTERNS OF FOOD EXPENDITURES AND PERCENT CONTRIBUTIONS OF THE SEVERAL FOOD GROUPS TO THE TOTAL PROTEIN AND CALORIES IN THE DIETS

Table B-1. Total monthly food expenditures of the actual diets, compared with the least cost diets (Recife, Northeast of Brazil).

Commodity groups	Actual diets (A.D.)[1]		Least cost diets (L.C.D.)			
			Method I		Method II	
	Cr $	%	Cr $	%	Cr $	%
I. Cereals and Products	78.12 (bread)[2]	16 (9)	81.67 (corn meal white)	34 (34)	135.58 (corn meal white)	41 (32)
II. Meat, Fish and Poultry	214.33 (beef)	45 (23)	55.55 (dried beef)	23 (23)	80.61 (chicken)	24 (24)
III. Eggs, Milk Products	19.84 (butter)	4 (0)	63.52 (milk)	27 (27)	94.64 (eggs)	29 (20)
IV. Roots and Tubers	29.45 (manioc flour)	6 (3)	37.66 (manioc flour)	15 (11)	18.62 (manioc flour)	6 (4)
V. Fats and Oils	9.95 (veg. oils)	2 (1)	--	--	--	--
VI. Legumes, Veg. and Fruits	83.14 (orange)	18 (3)	3.26 (beans macassar)	1 (1)	2.02 (tomato)	0 (0)
VII. Miscellaneous	43.82 (coffee)	9 (4)	--	--	--	--
Totals	478.65	100	241.66	100	331.47	100

[1]Basic source: Banco do Nordeste do Brasil. "Abastecimento de Generos Alimenticios do Recife." BNB-ETENE, Fortaleza-Ce, dez 1962. The calculations were based on an average family size of 5.1 and excluded the following commodities: coconut, cabbage, shallot, cucumber, coentro and vargens.

[2]The figures in parenthesis show the single largest contribution to the total food expenditure.

Table B-2. Total monthly food expenditures of the actual diets compared with the least cost diets (Fortaleza, Nostheast of Brazil).

Commodity group	Actual diets (A.D.)[1]		Least cost diets (L.C.D.)			
			Method I		Method II	
	Cr $	%	Cr $	%	Cr $	%
I. Cereal and Products	71.93 (bread)	21 (13)[2]	80.23 (rice)	31 (31)	126.02 (rice)	32 (30)
II. Meat, Fruit and Poultry	144.23 (beef)	42 (34)	--	--	--	--
III. Eggs and Milk Products	14.63 (milk)	4 (2)	127.16 (eggs)	48 (35)	145.71 (eggs)	38 (26)
IV. Roots and Tubers	14.76 (manioc flour)	4 (3)	53.97 (sweet potato/ white)	20 (15)	4.98 (sweet potato/ yellow)	1 (1)
V. Fats and Oils	13.93 (lard)	4 (2)	--	--	--	--
VI. Legumes, Veg. and Fruits	43.99 (beans/macassar and mulatinho)	13 (7)	3.92 (beans/macassar)	1 (1)	113.84 (tomato)	29 (29)
VII. Miscella-neous	40.40 (coffee)	12 (7)	--	--	--	--
Totals	343.87	100	265.28	100	390.55	100

[1]Basic source: Banco do Nordeste do Brasil. "Suprimento de Generos Alimenticios da Cidade de Fortaleza." BNB-ETENE, Fortaleza-Ce. Dez 1964. The calculations were based on an average family size of 5.4 and the only excluded commodity was mango.

[2]The figures in parentheses show the single largest contribution to the total food expenditure.

Table B-3. The percent contributions of the foods to the total calories and proteins in the diets (Recife, Northeast of Brazil).

Group of commodities	Actual diets (A.D.)[1]		Least cost diets (L.C.D.)			
			Method I		Method II	
	Calories	Proteins	Calories	Proteins	Calories	Proteins
	%	%	%	%	%	%
I. Cereals and Products	35 (bread-19)[2]	31 (bread-19)	49 (corn meal/ white-49)	45 (corn meal/ white-45)	69 (corn meal/ white-59)	57 (corn meal/ white-57)
II. Meat, Fish and Poultry	9 (beef-4)	39 (dried beef 17)	4 (dried beef 4)	24 (dried beef 24)	4 (chicken-4)	16 (chicken-16)
III. Eggs and Milk Products	5 (butter-2)	7 (powd. milk 2)	11 (milk-11)	22 (milk-22)	12 (eggs-8)	25 (milk-18)
IV. Roots and Tubers	19 (manioc flour-14)	4 (manioc flour-2)	35 (manioc flour-28)	7 (manioc flour-4)	15 (manioc flour-12)	2 (manioc flour and sweet potato/yellow-1)
V. Fats and Oils	5 (veg. oils-4)	3 (bacon)	--	--	--	--
VI. Legumes, Veg.	12 (beans/ macassar-4)	19 (beans/ macassar-8)	1 (beans/ macassar-1)	2 (beans/ macassar-2)	0 (beans/ mulatinho)	0 (beans/ mulatinho)
VII. Miscellaneous	15 (sugar-15)	0 (coffee)	--	--	--	--
Total	100	100	100	100	100	100

[1]Basic source: Banco do Nordeste do Brasil. Abastecimento de Generos Alimenticios do Recife. BNB-ETENE, Fortaleza-Ce, De3, c 1962.

[2]The figures in parentheses show the single largest source of calories and protein in the diets.

[3]A dash indicates absence of that specific food group, while a zero indicates a small percent contribution (less than 1 percent).

Table B-4. The percent contributions of foods to the total calories and proteins in the actual and the least cost diets (Fortaleza, Northeast of Brazil).

Group of commodities	Actual diets (A.D.)[1]		Least cost diets (L.C.D.)			
			Method I		Method II	
	Calories	Proteins	Calories	Proteins	Calories	Proteins
	%	%	%	%	%	%
I. Cereals and Products	33 (rice-18)[2]	29 (bread-14)	55 (rice-55)	42 (rice-42)	76 (rice-74)	53 (rice-50)
II. Meat, Fish & Poultry	6 (beef-5)	25 (beef-17)	--	--	--	--
III. Eggs & Milk Products	4 (milk-1)	8 (eggs-3)	17 (eggs-12)	46 (eggs-34)	19 (eggs-12)	42 (eggs-30)
IV. Roots & Tubers	15 (manioc flour-15)	0 (potato)	27 (sweet potato/ white-24)	9 (sweet potato/ white-8)	1 (sweet potato/ yellow-1)	1 (sweet potato/ yellow-1)
V. Fats and Oils	8 (veg. oils-4)	0 (bacon)	--	--	--	--
VI. Legumes, Veg. and Fruits	17 (beans/ macassar-7)	38 (beans/ macassar-18)	1 (beans/ macassar-1)	3 (beans/ macassar-3)	4 (tomato-4)	4 (tomato-4)
VII. Miscellaneous	17 (sugar-16)	0 (coffee)	--	--	--	--
Total	100	100	100	100	100	100

[1]Basic source: Banco do Nordeste do Brasil. Suprimento de Generos Alimenticios da Cidade de Fortaleza. BNB-ETENE, Fortaleza-Ce. Dez 1964.

[2]The figures in parentheses show the single largest source of calories and proteins in the diets.

[3]A dash indicates absence of that specific food group, while a zero indicates a small percent contribution (less than 1 percent).

APPENDIX C

LEAST COST MONTHLY DIETS ALLOWING EXPLICIT CONVENTIONAL CONSTRAINTS

Table C-1. Least cost monthly diets for a standard family in Recife, Northeast of Brazil.[1]

Consumption activities	Amounts required (Kg/month)		Amounts in the L.C.D.	
	Minimum	Maximum	Obtained (Kg/month)	Rank
Beef	8.1	--	8.8	= Min.
Lard	.4	--	.4	= Min.
Milk*	--	--	--	--
Dried beef*	--	--	--	--
Rice	5.5	--	5.5	= Min.
Bread	18.6	--	18.6	= Min.
Macaroni*	--	--	--	--
Cornmeal (white)**	1.0	3.3	3.3	= Max.
Cornmeal (yellow)	1.0	3.3	1.0	= Min.
Sweet potato (white**	2.3	3.2	3.2	= Max.
Sweet potato (yellow)**	2.3	3.2	3.2	= Max.
Manioc flour**	5.5	12.8	12.8	= Max.
Beans (mulatinho)	1.5	--	6.7	> Min.
Beans (macassar)	1.5	--	1.5	= Min.
Onion	1.7	--	1.7	= Min.
Tomato	4.3	--	4.3	= Min.
Carrots*	--	--	--	--
Sugar	11.7	--	24.2	> Min.
Coffee	2.7	--	2.7	= Min.

[1]The total cost of the least cost monthly diet is Cr. $351.47.

*Foods with a marginal efficiency equal to 100 percent.

**Foods with a marginal efficiency greater than 100 percent.

Table C-2. Least cost monthly diets for a standard family in Fortaleza, Northeast of Brazil.[1]

Consumption activities	Amounts required (Kg/month)		Amounts in the L.C.D.	
	Minimum	Maximum	Obtained (Kg/month)	Rank
Beef	7.9	--	7.9	= Min.
Milk*	--	--	17.2	--
Eggs	1.8	--	5.5	= Min.
Sardines*	--	--	6.3	--
Rice	12.7	--	21.8	> Min.
Bread	10.5	--	10.5	= Min.
Crackers	.3	--	.3	= Min.
Biscuits	.3	--	.3	= Min.
Sweet potato (white)**	.8	1.8	1.8	= Max.
Sweet potato (yellow)**	.8	1.8	1.8	= Max.
Manioc flour**	2.2	12.2	12.2	= Max.
Beans (mulatinho)	5.3	--	5.3	= Min.
Beans (macassar)	5.3	--	5.3	= Min.
Bananas (prata)	3.5	--	3.5	= Min.
Bananas (comprida)	3.5	--	3.5	= Min.
Sugar	10.7	--	10.7	= Min.
Coffee	2.7	--	2.7	= Min.
Squash*	--	--	24.2	--

[1]The total cost of the least cost monthly diet is: Cr $371.10.

*Foods with a marginal efficiency equal to 100 percent.

**Foods with a marginal efficiency greater than 100 percent.