OPTIMUM DAIRY COW REPLACEMENT POLICIES TO MAXIMIZE INCOME OVER FEED COST

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ABSTRACT

OPTIMUM DAIRY COW REPLACEMENT POLICIES
TO MAXIMIZE INCOME OVER FEED COST

by Richard W. Rundell

Simulation of a dairy herd over a period of 15 years was used to examine replacement strategies among six operationally practical systems of culling cows which would maximize income over feed costs. The strategies or criteria used to remove the lower ranking cows to a relatively constant herd size were: (1) Mature Equivalent (M.E.) milk production, (2) M.E. gross milk income, (3) actual milk production, (4) actual gross income, (5) income over feed cost, and (6) present value of expected gross income of cow and her subsequent replacements. Two each of prices of milk, fat differential, feed, and operational costs totaled 2^4 x 6 x 2 replications per trial or 192 replications.

The following were treated as stochastic variables: (a) variation in milk production and milkfat percentage between cows and between lactations of the same cow, (b) chance of a calf being a heifer or bull, (c) chance of involuntary death or removal of cows and youngstock, and (d) chance of month of the year of involuntary removals.

The mean and variance of the base herd approximated the average Michigan DHIA Holstein population in 1966. The sire value, approximating the best bulls used in A.I. was identical for any given year through all strategies and replications, but improved over time at the rate of 130 lb milk per year.

Culls or cows removed because of low rank in each respective strategy were removed at the most profitable point to cull in their respective lactations by equating the milk income of the marginal month with the sum of the month's feed costs and monthly operational charges. Practical use of this simple method of determining when to cull cows within a lactation was demonstrated.

A complete factorial design to analyze the generated data showed no significant differences between strategies under alternative combinations of prices in average income including salvage over feed cost per cow discounted to the present. This income was affected by the level of milk price ($P < .01$), feed cost ($P < .01$), operational cost ($P < .01$), and fat differential ($P < .05$).

When salvage value was omitted, average income over feed cost per cow per year for the 15 years was $385.94 under the culling strategy of actual gross income and was ($P < .01$) larger than that of M.E. milk production at $385.56 or M.E. gross income at $383.66.

Average milk production per cow, fat production per cow, genetic value per cow, culling rate and average herd size differed ($P < .01$) among strategies while gross income from milk was not significantly different. By mutually orthogonal contrasts, average milk production over the 15 years (14,247 lb) under the strategy of income over feed cost was significantly less than the average of the other five; average milk production under the M.E. milk strategy (14,357 lb) was higher ($P < .01$) than under the strategy of M.E. gross (14,266 lb) and the strategy of actual milk yielded ($P < .01$) more milk over 15 years than did actual gross. Fat production tended to rank the reverse of milk production among strategies. Both culling rate and herd size were higher under the two M.E. strategies. The average genetic milk value of the cows remaining in the herd at the end of each year was highest for the strategy of M.E. milk when averaged over the 15 years.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF APPENDIX TABLES

INTRODUCTION

Dairy farmers, as profit maximizers, are constantly striving to expand the income producing ability of their dairy herds. As managers of their business, their direct concern is to enhance the average quality of their herd by removal of the unprofitable producers and as such are faced with two major decisions, which cows to cull and when within the lactation to cull.

The objectives of various dairymen may be of different kinds; for example, the goals of a dairy farmer may be to maximize average milk or fat production or increase the genetic merit of his herd, regardless of economic considerations. For the majority of dairymen, however, it is assumed the major objective is maximum profit from their dairy herd. Most current systems of ranking or culling cows within a herd compare the relative merit of cows in question on the basis of their respective mature equivalent (M.E.) milk production records. Furthermore, numerous papers have been written regarding the genetic improvement of dairy herds through selection of both females and sires based on these M.E. records. Few studies, however, have dealt explicitly with the economics of dairy cow replacement policies over time.

The present study was initiated to determine the optimum replacement strategy among several operationally practical systems of culling cows which will maximize income over feed costs. Such a system may subsequently be adaptable to standard DHIA operations. Moreover, this study sought to determine the extent to which alternative culling policies deviate from the optimum as well as to derive the economic loss a dairyman would encounter by using a less than optimum strategy. Finally, this study sought to indicate the change in strategy or economic loss which might result from a change in the price of milk, fat differential, feed, or operational costs. Yet to compare economic consequences of different replacement policies over time, it was necessary to discount future income and costs to the present.

Because of the biological nature of a dairy cow, the cyclic aspect of milk production, and the presence of various stochastic elements, an analysis of the dairy cow replacement problem calls for a much more complicated model than ordinary replacement theory assumes. To study effectively the long term effects of various culling strategies, it was proposed to simulate production and reproduction plus certain other stochastic variables of a dairy herd over a period of years. In turn, the data generated from such a simulation would serve as a basis for a comparison of the respective strategies and related variables.

REVIEW OF LITERATURE

Replacement theory

The recent surge of interest in the economics of replacement of durable assets arises largely from the continued substitution of capital goods for other inputs in agriculture. Though the theory of replacement of durable capital used in production may be considered part of production theory in economics, only that part of this theory which deals with production over time is relevant to the replacement problem. For example, the simplest type of replacement problem, such as a dry lot cattle finishing operation, concerns a short production period where the producer realizes returns only by the sale of the asset. Faris (1960) illustrates this concept in a continuous operation, in which each lot of cattle is immediately replaced by a new lot; by this method the operator strives to maximize his average net revenue over time.

Since we simply cannot efficiently manage general equilibrium solutions, economic models dealing with the replacement problem usually are models for sub-optimization or partial equilibrium. At any time a replacement could be made, we will call this a decision; a sequence of such decisions will be called a policy; and the most profitable policy will be called an <u>optimal policy</u>. Introducing a long production period complicates the problem of determining the optimum replacement pattern due to both uncertainty and time preference; therefore, we must consider two types of discounting. The first type of discounting is due to uncertainty of future prices, yields or costs. Burt (1965) has presented a model in which he incorporates such uncertainty but emphasizes that discounting for risk and uncertainty is economic under the assumption that knowledge of future net returns diminishes with time because the variance of net returns increases proportionally with time.

The second type of discounting arises from time preference: a sum of money received or paid at the present time is worth more than the same sum of money at some time in the future; hence, this discounting is primarily a function of opportunity costs (Faris, 1960). Here the supply and demand for loanable funds determines the appropriate discount rate. We can express this relationship by the following formula (Faris, 1960):

$$Y = \frac{\text{Future Income}}{(1 + i)^n}$$

where i = the interest rate, n = the number of years and Y is the discounted future income to present value.

In general, the optimum replacement pattern results from maximizing net revenue over time. Specifically, we select a replacement age to maximize the present value of the whole future difference between income and outlay streams, where the present value equals the value associated with the existing asset plus the value of all future replacements at the time of first replacement, discounted to today (Giaever, 1966).

Except for minor modifications with different types of assets, the above principle holds. For example, in the replacement of trees which have long production periods, returns come only from the sale of the asset, whereas assets such as dairy cows or breeding stock are dominated by a flow of revenue over the life of the asset. The biological phenomena of these latter capital items, because of growth and random removal due to death and disease, complicate the replacement problem.

For a stand of timber, Faris (1960) computed the optimum replacement time as when the marginal net revenue from the present enterprise equals the highest amortized value of anticipated net revenue from the enterprise immediately following. Winder and Trant (1961) refined Faris' equations by defining opportunity cost of applying another unit of input as the marginal factor cost plus the foregone earning of the time required to apply the unit of input.

Further refining Faris' equations, Chisholm (1966) emphasized that not only does the money sunk in fixed and variable costs have an opportunity cost but so does the money tied up in the appreciating asset, the growing trees.

We could demonstrate a similar analogy to money tied up in the salvage value of a dairy cow. Using the net present value criterion, a dairyman would maximize the net present value of all production periods of a cow plus her subsequent replacements, and not, as has been sometimes advocated, choose the single production period having the maximum net present value or simply the lactation with the highest yield.

Of several approaches to the problem of replacement of producing animals, White (1959) demonstrated the use of multistage decision making applied to replacement of laying hens, a method that considers all decision points simultaneously. Tedford (1964) also used a laying flock to demonstrate equations which maximize net earnings for present and all future replacements.

Under conditions of capital rationing, the criterion of "maximum rate of return" may be more appropriate than that of "present value" since a fixed quantity of capital can be compounded at the highest possible rate by maximizing the internal rate of return in the investment (Burt, 1965). With the dairy cow, for example, we must invest in housing and milking equipment as well as the cow, but the amortization rate of such auxiliary equipment is the internal rate of return which is the unknown being determined.

Replacement theory--dairy cows

Few studies have dealt explicitly with the economics of dairy cow replacement. Jenkins and Halter (1963), however, formulated the problem of dairy cow replacement as a problem of maximization or minimization based on dynamic programming principles and maximization of present value. Economic factors which they considered in the decision relative to the replacement of the present animal were: (1) market value of the present animal or its possible replacement, (2) transaction cost of purchasing an animal, (3) net market value of the milk production of the present animal or its replacement, and (4) maximum net return in subsequent enterprise periods from replacement of the present animal.

Jenkins and Halter considered certain stochastic factors which influence the dairy cow replacement decision: (1) probability of an animal failing, that is, involuntary cull or death, (2) probability of an animal succeeding, and (3) likelihood of finding (or having) a cow in a given lactation. Consequently, the expected net returns equals the sum of the returns from the various outcomes times the probabilities associated with the outcomes.

Under various prices for cows, feed, beef, and milk, these authors determined the optimal replacement policies by the model described. When prices were set at 1961 averages, the optimum policy was to purchase an animal of lactation one, keep her until she completed her sixth lactation, then replace her with another animal of lactation one. Beyond this point, policy was indeterminable. Jenkins and Halter's approach is more an illustration of given principles than an attempt to solve a realistic problem and assumptions of the model render it unrealistic and of doubtful practical value; for instance, they assume the replacement animals will produce at the same butterfat level as the present animals if in the same lactation and assume all cows in the herd produce at herd average.

A slightly different approach, presented by Hutton (1965), simulated the experience of a dairy herd over a specified number of years under two alternative basic policies: raise herd replacements or purchase all replacements. After cows are ranked according to projections of net returns, they are compared to the heifers which are ranked according to their "expected" net returns. Then available heifers from the top of their array are substituted for cows from the bottom of their respective array. Each dairyman interested in this program answers a 50 item questionnaire regarding such things as intentions of herd expansion, feeding rates and costs, milk prices expected, certain culling practices, rate of involuntary losses, purchase price of cows, and the 305 day Mature Equivalent-Fat Corrected Milk (305-M.E.-FCM) record of each cow in the herd.

Applying the model to a representative herd, Hutton projected the net returns and production per cow for both the purchase and raise policies in a five year period. Computing both the estimated net returns and the variability in returns, he contended the decrease in capital funds more than offset the generally higher average income under the policy of purchasing replacements. Assuming milk at $4.60/cwt for a 30 cow herd, Hutton discounted the present value of the two policies to an average annual rate over a 5 year period of $7,259 and $6,370 for the "raise" and "buy" policies respectively.

The most intensive study to determine dairy cow replacement policies was developed by Giaever in 1966. Within the framework of Markovian dynamic programming, the resulting derivation specifies we should replace an asset when rate of income from that asset, plus rate of increase in salvage value, less rate of outlay equals the interest rate times the sum of salvage value and present value (at time of replacement) of the next asset in the chain. Such a Markov process is a stochastic process in which the probability distribution of outcomes at any given stage depends only on the actual outcome at the last preceding stage. Based on the parameters derived from the two herds of 700 and 650 cows respectively, the Markovian process thus defines the lactation number to replace a cow in a finite production-lactation class under a given set of prices for feed, milk, and culls for each of three calving intervals. Though the process was defined to represent a dairy cow and the chain of her successive replacements, the term "replacement" meant any case where one durable capital item, in this instance, a heifer, is substituted for another one, whether output changes or not.

Since different stochastic elements enter the replacement problem, Giaever included three such variables: (a) variation in production both between cows within a herd and between different lactations of the same cow, or repeatability, (b) probability of a given calving interval, and (c) probability of involuntary removal; hence, transition probability equaled the product of these three variables. Based on assumed age-corrected herd averages of 12,000 lb and 11,000 lb 305-FCM from the two herds respectively, the seven production levels were derived as deviations from this base. Giaever assumed in his model that if the decision is to replace, the cow will be sold and replaced with a heifer immediately, while each decision to keep or replace, for simplicity, was made seven months after the last calving.

Independently of his Markovian process, Giaever derived a simple decision rule whereby dairymen can determine the optimal time to replace within a lactation: "if it is decided to replace a cow during the current lactation, it will pay to keep the cow as long as the monthly milk income less feed costs exceeds the monthly interest on (salvage plus present value of a heifer including total expected net income over all lactations discounted to the present)" where the interest on (S + V) equals the opportunity cost of keeping a cow in the herd. Accordingly, to justify keeping a potential cull (another month), her milk income less feed cost must equal or exceed this opportunity cost. If we cannot solve the replacement problem by the above method, the interest on (S + V) should approximate "the average per month (milk sales plus beef sales minus feed costs minus replacement costs minus interest on salvage value) per cow for the herd," which says that the opportunity cost equals the possible profit from the average of the rest of the herd.

Not surprisingly, results call for more intensive culling if the calving interval is longer, when replacement prices are lower, cull beef prices higher, or when milk prices are higher. The effect of following an extreme sub-optimal culling policy reduced annual profit per cow by about $6.00 while moderate deviations from the optimal replacement policy suffered only minor losses.

Decision models

Various types of formal analytical models have been developed to tackle the problems of managers, yet some problem situations cannot be modeled in this manner. For instance, the dairy farmer, in his replacement problems, faces two principal decisions: which cows to cull and when to cull them. Accordingly, techniques such as "decision theory" arise which attempt to simulate a great part of the rational processes of managers. A manager can utilize relatively simple decision models somewhat intuitively but in more complex situations this kind of comparison is beyond the capability of a manager, who must then introduce some decision rule into the formal analysis to substitute for the intuitive evaluation (Hutton, 1965).

In general, the decision maker adopts the best or optimum strategy or ranks the strategies which reflect the given order of consequences for the decision maker as defined by utility, expected utility, expected loss, or some other appropriate measure (Tedford, 1964). Thus the dairy farmer, as a decision maker, wishes to select the best culling strategy to maximize net income, production per cow, or some other standard he chooses to optimize.

Two general types of decision models have been described, namely, game theoretic and probabilistic. Whereas game theoretic models (Tedford, 1964) emphasize decision making in situations of conflict which involve the behavior of the players, probabilistic models stress the decision maker's expectations and use of probability measures as expressing weights he attaches to possible states of the world. Within the operation of a dairy herd, for instance, such subjective probability distributions as the price of milk, sex ratio of calves born, chance of involuntary deaths, and the future production of cows and their offspring, confront the manager in his decision process. In fact, the biological and environmental nature of livestock require modifications to the common replacement theories which assume fixed output.

Simulation

Through simulation, one of a growing number of quantitative techniques developed to assist management in decision making, Monte Carlo or other methods generate a stream of behavior which allows management to determine the effect of following alternative policies without actually putting these policies into effect (Babb, 1963). With this method, variables such as sales, production, weather, or death losses of livestock are subject to probability.

By simulation, we can formulate more complex and realistic models than possible by conventional mathematical techniques. Such a technique has been used to study a wide range of problems (Shubik, 1960), including genetics (Fraser, 1962; Gill, 1963) and teaching principles of selection (Heidhues and Henderson, 1961; Everett et al., 1967). Monte Carlo techniques developed by Glickstein et al. (1962) duplicated the probabilistic nature of milk arrivals of cheese plants in Indiana. With this information, the authors examined 12 different policies or decision rules which affected the amount of milk purchased, the distribution of milk receipts and prices paid for milk for one year. More directly related to farm managers' activities, Halter and Dean (1965) simulated decision making processes of a large California range feedlot operation where they compared the mean and variance of net income for each of three models of price expectation over a period of 40 years.

Optimal organization and managerial policies under the uncertain conditions of low and unstable rainfall of Israel were inspected by Zusman and Amid (1965) through simulation tools. They defined the optimal decision rules in terms of the present value of the net return flow and the coefficient of variation of the income flows. But these authors emphasized that because of the dynamic nature of crop and livestock farming, any choice among these decision rules and farm organizations should account for the economic performance over a sufficiently long sequence of years.

Dairy cattle selection

Improving the milk-producing abilities of dairy cattle populations is inherently slow because of low reproductive rates, long generation intervals and small heritabilities. Deciding which cows to keep forces the breeder to ponder several different characters, i.e., milk, fat percentage, type and ease of milking. Moreover, natural selection plus the tendency to select for these multiple traits, some of doubtful value, hinder a dairyman's progress. Finding the relative economic value of these characters often proves difficult because it may vary from region to region or according to market demand. Hazel and Lush (1942) compared the three most common methods of selection for more than one trait: (a) tandem, or a sequence of one trait at a time, (b) total score or all traits simultaneously by some index, and (c) independent culling which establishes a certain level of merit for each trait. They professed total score most efficient and tandem least efficient of the three.

Though intensity is maximum when we select strictly on the item under consideration, the degree of intensity depends entirely on how large a fraction must be saved: with the fraction saved at 0.8, 0.7, and 0.6, the intensities are 0.35, 0.50, and 0.64 respectively (Lush, 1960). When dairymen pay attention to less important traits or sell some higher producing cows for dairy purposes, they reduce the actual intensity of selection.

To make selection of dairy cattle or other animals more accurate, we need sound and simple selection indices to place proper relative weights on each of the traits to evaluate. Such weights depend on the heritability of the traits, genetic and phenotypic correlations between traits and their economic importance (Lush, 1960; Hazel, 1943). Lush (1960) emphasizes when economic values are changing radically that one cannot afford to follow economic values closely; instead, one would like to select today in accordance with economic values which will prevail 20 years from now. But this reasoning, because it is inconsistent with the principle of discounted present value, disregards the optimum economic path to these goals.

By comparing the actual selection practiced among Holstein herds with that which might have been possible, Allaire and Henderson (1966) estimated the efficiency of intra-herd-year-season-lactation phenotypic selection for M.E. milk as 42, 34, 27, and 25% for the first four lactations respectively. From another classification by these authors, the efficiency values of phenotypic selection were 27, 30, and 38% for less than \$6,000, \$6,000 to \$8,000, and over \$8,000 income over feed cost per worker respectively.

One of the few attempts to quantify the selection characters of milk production on a purely net economic basis has been presented by Soller et al. (1966) who described A.I. sire selection in Israel on the basis of live weight for age (LFA) of the bull calves as well as the milk production of their daughters. The relative economic value of genetic progress in milk production and of LFA results from comparing the increase in gross income as a result of a unit increase in each of the traits, which they found stable under differing economic conditions.

Genetic progress by selection

Expression of the progress achieved in our dairy herd selection programs usually has been based on genetic change in milk or fat production over time. Possible gains from selection for a trait per unit of time depend on: (1) the selection differential or intensity, which is the difference between the herd or population mean and the mean of the parents selected from this population to produce the next generation, (2) heritability of the trait, and (3) length of the generation interval (Laben, 1965). The expected gain per year in a breeding program for a given trait then equals the selection differential times the heritability divided by the generation interval.

Laben (1965) analyzed the records of 12 herds over a 30 year period involving 3,900 cows and found an average increase of 74 lb of FCM per year, an average yearly increase of 0.7%. From these 12 herds, the respective dairymen culled 23% each generation though they could have progressed as far by culling only 4% had they culled for milk yield alone.

Everett (1966) found phenotypic trends of 201 $\pm$ 5.7 lb of milk per year for the A.I. sired population of Michigan Holsteins enrolled in DHIA, while the genetic increase was 2.4% per year (1953 to 1965) in milk production and 2.6% per year in fat compared to 1.0% and 0.8% genetic increase per year in milk and fat respectively in the naturally sired progeny. Everett et al. (1967) calculated the genetic and environmental trends from simulated records as 135 and -4 lb milk respectively per year.

Replacing 20% of the herd annually with young stock from the top 75% of the cows in the herd, 6% of the herd having been culled for low production, Searle (1961) estimated additive genetic superiority after 15 years as 15, 19, and 38 lb of milk fat for the policies of (a) culling low producers, (b) culling low producers and selecting replacements, and (c) culling low producers, selecting replacements using above-average sires, respectively.

Van Vleck (1966) has compared the genetic progress expected under three different levels of culling. Assuming all herds following the same plan with an assumed 13% involuntary culling, plan one involves all herds using superior proven A.I. sires (+1,500 lb milk) and females culled to the limit of replacement heifers. Such a plan with a 62% survival rate would progress 13,000 lb M.E. milk in 50 years.

Plan two uses top A.I. sires (+1,000 lb milk) but heifers are bred to natural service bulls from the top half of the herd. With an optimum survival rate of 70-76%, genetic progress in 50 years would approximate 8,200 lb M.E. milk.

Plan three, when all breeders mate their cows to bulls from the top half of the herds, the optimum survival rate is at 68% with estimated genetic progress 4,000 lb M.E.

milk in 50 years. The above author raises the question of whether a dairyman should improve his herd genetically as fast as possible or earn as much money as possible right now.

Reasons for disposal

Since the average dairy cow produces fewer lactations than her potential, it becomes economically important to examine the reasons for disposal. When farmers sell large numbers of cows for dairy purposes, sterility, and miscellaneous diseases, they considerably reduce the possible economic progress through elimination of low producers. But the primary reason for decreasing involuntary losses is to increase the culling rate for low production rather than to increase longevity.

Various research studies have listed reasons for disposal of dairy cows. In an extensive study by Seath (1940), total culls were 28.6 and 32.9% respectively of over 8,000 Iowa and 4,000 Kansas cows. In the Beltsville herd where no cows were deliberately culled for low production or type, Parker et al. (1960) found 41.3% of the Holsteins and 21.3% of the Jerseys were removed as non-breeders.

That farmers consider more than one reason for removal of cows is substantiated by a mail survey of New York herds in which dairymen listed up to five reasons for disposing of each cow (O'Bleness and Van Vleck, 1962). From a total of 12 categories for culling and eight for death, these authors expressed the most important reasons for disposal as: low production, 27-32% (percentage of total number of reasons); sterility, 16-19%; udder trouble or mastitis, 14-20%; and dairy purposes, 14-15%. The proportion culled for low production declined with increasing age but problems of fertility and udder trouble increased with increasing age. Further evidence that cows vary by lactation number or age in both voluntary and involuntary culls was confirmed by Specht and McGilliard (1960) with Michigan DHIA data. Economic conditions may influence rates of removal and reasons for disposal. Asdell (1951) maintained this factor caused the largest year to year variation in the number of cows culled for low production. The price of milk and the cost of cows contributed the greater share of these economic factors.

Maintenance of sufficient culling power is dependent upon reduced heifer losses as well as the control of involuntary culls. From a summary of data on heifers available for replacement, Frick and Henry (1956) suggest that with good commercial management, 335 heifers (similar to estimate of Pelisser, 1965) would enter the milking herd per 1,000 cows. But they stress that net reproduction rate or ratio of animals in two consecutive generations more accurately measures the amount of selection available in heifer calves.

Economics of longevity

Value of a cow to a herd depends upon her ability to produce and reproduce for many years, both from an economic and breed improvement standpoint. Such longevity should reduce the number, and thereby the cost of herd replacements and subsequently increase the proportion of mature cows in the herd which produce at near maximum capacity.

Several studies have considered the relationship between early production of a cow and length of time she stays in the herd. This relationship could influence evaluation of lactation records both for sire proofs and culling females. In an analysis of production in 79 Holstein herds, Gaalaas and Plowman (1963) obtained small but highly significant regression and correlation coefficients of final age on first lactation. Van Vleck (1964) and White and Nichols (1965) agree with these data that high-producing first lactation cows produced over a longer life. These results do not substantiate the claim that high producers in the first lactation burn themselves out. But if high producers do not burn out or become more susceptible to mastitis and other diseases, they would automatically live longer in the herd because they could survive more culling decisions at those higher production levels.

To obtain a more direct answer to the relationship of economically important characters of longevity, Evans et al. (1964) calculated the intra-sire phenotype correlation between the first record 2X-305-M.E. milk production and length of productive life in days, actual lifetime milk production, and actual production per day of productive life as 0.217, 0.285, and 0.497 respectively.

The present system now in use in Michigan DHIA reports only current lactations, thus a cow may be subject to culling on the basis of her present lactation. But in an analysis of New York DHIA herds, Van Vleck (1964) noted that the terminal records and to some extent the penultimate lactation records averaged considerably lower than previous records. If the causal factors are non-genetic, we should exclude such records when evaluating either sires or female relatives. If, on the other hand, these factors such as susceptibility to mastitis, feet and leg difficulties do have a genetic origin, we should include these records. Whenever the production ability and hence economic value of the individual cow is hindered by injuries, we can more easily validate our desire to cull on the basis of one lactation, especially if the injury or disease causes permanent damage. One would expect that, as animals grow older, culling for genetic causes other than yield might become more important, but for yield, on the other hand, the separate culling processes all act on more or less the same genetic variance (Robertson, 1966).

Environmental influence

Both heredity and environment affect milk and fat yield of dairy cattle but since environment causes a major portion of variation among production records, the usefulness of records from all cows for sire evaluation and selection of cows within a herd depends in large part on the possibility of removing or adjusting these environmental effects. Pirchner and Lush (1959) using two methods, found environment to account for 90 and 93.5% respectively of the variation among Holstein herds. From A.I. sired Holstein cattle, Van Vleck et al. (1961) calculated the following components of variation in production records: herd, 30%; sire, 6-8%; interactions, 12-14%; and residual, 50%.

A number of experiments have been devised to determine which environmental factors affect production measurements the most. Of 21 environmental influences tested in 47 herds, Bayley and Heizer (1952) selected nine as most important: number of days carrying calf, length of preceding dry period, length of lactation, pounds of TDN fed per 1,000 pounds body weight, herd size, age at calving, and selection rating. Further, though body weight accounted for 11.9% of the variation, Lee et al. (1961) calculated three other environmental factors (breed, stage of gestation, and season of calving) that had significant effects on FCM production. Moreover, Erb and Ashworth (1961) found the age of the cow affected FCM production twice as much as body weight but the influence of weight was not significant if age and breed effects were removed.

Deciding which cows to cull involves deciding what records to use and how to use them most efficiently to measure the profit or producing ability of each cow; thus we might decide on single records or multiples combined in various ways. Deaton and McGilliard (1964) indicated that the first record gives essentially as reliable an estimate of a cow's breeding value measured by the daughter's performance as does an appropriately weighted combination of multiple records. Results from analysis of Holstein records (Barr and Van Vleck, 1963) suggest "repeatability" in the range of 0.40 to 0.45 but first lactations were most unlike all others phenotypically and genetically.

The degree of accuracy of correction factors which standardize production records may in part rely on the bias due to interaction of the environment and the level of production. Lush and Shrode (1950) indicate that under practical conditions, relationships between yield and age cannot be estimated without bias. Since culling removes lower producers at young ages, if only repeated records are used to derive correction factors, such factors overestimate the producing ability of very young cows. Hickman and Henderson (1955) derived a negative correlation between the level of first-lactation production and increase from first to second lactation (comparable to Hickman, 1962), which indicates an interaction between level of herd production and relationship between yield and age.

To compare accurately the milk records of different cows for culling purposes, we may want to consider the differences in reproductive performance affecting the respective records. Investigations of such relationships should account for the influence of herd, year, season, age and length of the previous dry period. Smith and Legates (1962) found intra-herd-year-season phenotypic correlations (0.05 to 0.08) between 90-day production and days open were not significant, suggesting level of production had very little influence on this measure of fertility. Furthermore, length of the previous dry period accounted for less than 0.1% of the variation in 305-day milk production, while days open during the lactation accounted for 6.6 and 4.2% of the variation in yield for first and all lactations respectively. Yet the ratios developed by Sargent et al. (1967) seem to justify a correction for fat percentage when incomplete records are extended.

When we compare cows of varying lactation lengths for culling or ranking purposes, we may wish to consider the relative merits of the lactation lengths. Though Frieze and Corley (1962) stratified 81,226 Holstein records into five different duration groups up to 365 days, they noted no important trends in the distribution of length of lactation over three levels of 2X herds. Neither did significant differences exist in the estimates of 305 vs 365-day records, but they did not study the difference in production between 305 and 365-day records per day of life or differences in long run economic return.

Tucker and Legates (1965), from an analysis of Holstein HIR data consisting of 121,935 lactations, found a significant difference between the variance within the different months of the year, fall values being large. Similarly, from a study of 108,000 observations of Canadian Holstein records, Gravir (1966) noted the effect of season of freshening (fall or spring) has a marked effect on the yield-age relationship for animals of all ages irrespective of lactation number.

Part lactations

Because herdsmen must frequently decide which cows to cull before a cow completes her lactation, part records occupy an important position in dairy cattle selection--both for culling cows from the herd and for sire selection. Furthermore, since they will have cows in various stages of lactation, dairymen must know not only the relative accuracy of various parts of the lactation in predicting the whole but must know the most economical point in the lactation to cull both the voluntary and involuntary culls.

Several workers have attacked various phases of this problem. To learn the extent to which farmers actually cull before the end of a lactation, Maxley (1966) sampled 18

Holstein herds which had been on test for ten years. Not only did these dairymen cull 67% of the cows before the end of their lactation, but of those culled after 10 months lactation, they sold 60% for dairy. Estimates in this study of the genetic correlations between cumulative parts of the lactation for milk were essentially unity (Madden et al., 1955 obtained over 0.9). Consequently, selecting on the cumulative part record would improve production nearly as much as selecting on the complete record itself.

When Van Vleck and Henderson (1961) compared regression vs ratio factors to determine relationships of part lactation to the whole, they found six and seven cumulative months had a correlation with total yield of 0.93 and 0.95 respectively while corresponding multiple correlations were 0.94 and 0.96. The more practical way, especially for central processing centers, is to extend cumulative monthly test-day records.

Effect of body weight on selection

The primary purpose of selection by dairymen should be to maximize the profit of their cows by methods which weigh the relative economic importance of all traits concerned. What role body size or weight plays in relation to milk production in such dairy cattle selection has been subject to much controversy. Presumably milk represents income, a positive value, whereas body weight reflects maintenance and should be charged against the cow. Despite this relationship, dairymen generally discriminate against smaller animals which produce on a par with their larger herd mates. Further complications arise in assessing relationships of size to production because environmental conditions which are conducive to large size also contribute to higher milk production (Mason et al., 1957, Clark and Touchberry, 1962).

Lush and Shrode (1950) state that presently used age correction factors tend to favor early-maturing individuals. Whereas McDaniel and Legates (1963) found a positive linear regression of milk yield on weight independent of age, results by Miller and McGilliard (1956) indicate influence of weight on production is largely the product of differences among herds and that large heifers have little or no economic advantage in their first lactation if calving is delayed to obtain additional weight. Moreover, Clark and Touchberry (1962) noticed that increases in yield associated with increases in weight among cows of the same age and herd suggest heavier cows possess little if any superiority in production efficiency.

On the other hand, when cows are culled on production regardless of size, small cows which remain in a herd after culling produce more efficiently than large ones (Clark and Touchberry, 1962). Harville and Henderson (1966) indicated the assumed traits for which we select are breeding value for milk or milk fat production but that in practice dairymen should be trying to breed cows which will return maximum profit. A positive genetic correlation between body size and production means that selection for production will result in large cows with increased growth and maintenance costs. Though these authors found body weight a slightly more important source of the intraherd variation in actual first lactation, incorporation of body weight into a selection index for predicting breeding value for Holstein milk production increased the accuracy less than 1%. Further, these authors believe if the economic value or lack of body size relative to economic value of milk production were known and included in an index, these overall indices should more efficiently estimate the breeding value for monetary profit.

To compare two or more cows for selection purposes, a dairyman may need to adjust or correct for cows of different ages or weights to accurately estimate income over feed cost. From first lactation Holstein data, Clark and Touchberry (1962) indicated that for a constant age, milk production increased 134 lb and fat 7.8 lb for each 100 lb increase in body weight. Their comparable genetic correlations between body weight and production ranged from near zero in the first lactation to -0.53 in the second.

McDaniel and Legates (1965) emphasize the relative values of body weight and production rather than their absolute values deserve importance in determining proper selection emphasis to apply to each trait. For example, if feed costs and milk prices were both low (hay @ $25/T; concentrate @ $56/T; milk @ $3.50/cwt) or both high, return over feed costs from about 225 lb of milk would offset the cost of 100 lb of maintenance for one year. Combining medium priced milk ($4.75/cwt) however, with high feed costs would raise the requirement to 300 lb of milk.

Relationships of feed costs to profit

If dairymen expect to cull their cows on the basis of income over feed costs, they must first accurately determine feed consumption. Furthermore, the relationship between feed consumption, the input, and milk production, the output, must be known for varying levels and months of the lactation. Conversely, if the higher producing cows of a herd convert feed more efficiently than the lower yielding cows, we can select cows on the basis of milk production or gross income from milk.

Attempts to measure the economics, relationships, and cause of variation between feed intake and milk output have been approached from several directions. Following pioneer methodological studies estimating milk production functions by Jensen et al. (1942) and by Heady (1951), McDaniel et al. (1961) indicated that the therms of net energy consumed was the most important environmental factor of those studied influencing changes in production over a seven year period. Coffey and Toussaint (1963) compared net returns over feed cost for "optimum rations" with most profitable "stomach capacity rations" at several feed:milk price ratios. Their results suggest (1) there is a fairly wide range of feeding over which returns are not affected much, (2) hay-grain isoquants are essentially linear, (3) optimum rations lie near the stomach-capacity limit for most prices of hay, grain and milk, and (4) significant differences in returns exist between rations.

Contrarily, Hoover et al. (1967) provide evidence indicating nonlinearity of the milk production function and decreasing marginal rates of substitution of grain for hay. These researchers further stressed that the definition of 4% FCM as an energy transformation has no direct relation to the price of milk assumed in the analysis.

From 1,014 herd-month observations, Matherne (1965) developed a regression equation to estimate income over feed cost from independent variables: days in milk and therms ENE from each of concentrates, silage, hay and pasture. From another report, Green (1966) confirmed that the feeding program significantly affected the level of milk production and income over feed cost. He found the program that furnished the most energy per 100 lb body weight yielded most milk and income over feed cost whereas regression of income over feed cost was linear. Of the changes in income over feed cost, 45% were noted as levels of production increased.

Measurement of feed intake of individual cows to compare income over feed cost is further complicated because cows vary widely in their appetite and consequently feed intake. Even after Johnson et al. (1966) corrected for FCM yield, body weight, weight changes, grain intake, age, body condition, and stage of lactation or gestation, they could account for only about one-half of the variation in forage DM intake but stage of lactation appeared the most important.

Fixed costs

The role of fixed costs in replacement theory has been approached from several points of view. In the long run, to progress financially in the business, dairymen must cover their fixed costs, though trouble arises when we attempt to correctly specify all cost elements in a replacement problem.

Since depreciation costs are that portion of original cost of buildings and equipment charged to each year of use, total depreciation chargeable to the milking herd equals the sum of depreciation for all items used by the milking herd: buildings, milk house, feed storage and facilities, paving, milking machine and bulk tank (Schultis et al., 1963). If feed, however, is charged at farm cost, feed storage is not charged to the milking herd but to the cropping enterprise.

As referred to previously, Burt (1965) specified that the present value per cow investment in auxiliary capital (housing and milking equipment) is added to the cost of the cow; hence, it is this total investment that must earn a rate of return as applied to the replacement decision. Chisholm (1966) asserts there is general agreement that fixed and variable costs which are actually incurred should be compounded at an appropriate interest rate to permit comparison of costs and returns occurring at different points in time. Since sunk dollars establish no minimum marginal return that must be met other than salvage value, Breimyer (1966) emphasizes that when these fixed costs are disregarded in making decisions as to variable inputs, a bias, usually upward, enters into the decisions on use of those inputs.

EXPERIMENTAL PROCEDURE

General Procedure and Derivation of Parameters

To study how various strategies of culling dairy cows affect such factors as income over feed cost and average production per cow over time, it is first necessary to establish parameters of such a cattle population which conform to the variables under study. One possible method to examine these factors is simulation. Not only will computer models allow us to look at the effect over an intermediate run of 2 to 3 years but also over the longer run of 10 to 20 years when future generations of those cows selected to remain in the herd come into production. In contrast to actual current production records, by simulation procedure, we can fix certain variables not under study, while those variables subject to uncertainty we can vary randomly. Therefore, this study treated certain elements as stochastic while others were held constant. The stochastic factors were: (a) variation in milk production and milkfat percentage between cows and between lactations of the same cow, (b) chance of a calf being a heifer or bull, (c) chance of involuntary death or removal of cows and youngstock, and (d) chance of month of the year of involuntary removal and death.

To prepare for the simulation, the first procedure established parameters from known research in regard to such data as milk production and its variation, probability distribution of involuntary culls, milk prices, and costs of feed and other inputs. After computing or deriving the parameters necessary for the study, a herd of 80 cows plus their offspring was generated by use of CDC 3600 computer. Their production and reproductive performance was then simulated over a period of 15 years.

Strategies and prices studied

Six different culling strategies were examined; that is, the cows were ranked yearly on each "extended" 305 day record according to a determined strategy which was constant throughout the 15 years of the trial. The bottom cows of the rank were then culled until the herd equalled approximately 80 cows at the end of each year. The following strategies or criteria for culling or ranking the cows were used:

1. Mature Equivalent (M.E.) milk (305 days).
2. Mature Equivalent gross income from milk.
3. Actual milk (305 days).
4. Actual gross milk sales (305 days).
5. Actual income over feed costs (365 days).
6. Present value of expected gross income of cow and her subsequent replacements.

Under each strategy $2^4 = 16$ different trials or combinations of prices were employed with 2 replications per trial totaling 16 x 2 x 6 = 192 replications. The following prices used as parameters represent approximate low and high values respectively on Southern Michigan dairy farms in the 1960's:

			Low	High
a.	Base milk price (cwt)		$ 4.25	$ 5.25
b.	Fat differential		.07	.08
c.	Price of feed (ton)	Grain	60.00	70.00
		Hay	20.00	25.00
		Silage	7.00	8.00
d.	Operational costs/cow/month		21.92	25.38

The price of milk in the above table represents approximate manufacturing and Class I price minus hauling charges respectively, whereas the fat differentials of $0.07 and $0.08 for adjustment of price of milk per 0.1 change from 3.5 test represents two alternatives used in milk markets in Michigan. The alternative feed prices simply represent a low and high for on-farm costs for Southern Michigan dairy farms. A cull price of $16/cwt used throughout all replications approximates the average yearly prices received in southern Michigan country markets from 1958 through 1966.

Though not charged against the cows per se, operating costs from the above table functioned as parameters to help determine the profitable point to cull a cow within her lactation. These operational costs were calculated as follows: From 266 "Southern Michigan Dairy farms" utilizing the Telfarm record keeping system in 1966, 72 farms were selected which had 60 or more cows per herd. From a ranking of these farms on the basis of improvement investment per cow, the records of the top one-third (23) farms were classified as "high" operating costs while the records of the bottom third (23) represented "low." From an average of the 23 farms in each group the following costs were derived as a pro-rated share (in parenthesis) of the entire farm business charged to the dairy herd (Hepp and Brown, 1967):

	Charge per cow per year	
	Low	High
1. Power & machinery expense (30%)	$ 33.86	$ 37.78
2. Improvement costs (80%)	30.41	44.63
3. Livestock expense (100%)	30.19	33.57
4. Utilities (90%)	9.20	11.49
5. Taxes (31.24% & 36.19%)	5.11	7.20
6. Interest paid (31.24% & 36.19%)	10.93	15.86
7. Interest on owned investment (31.24% & 36.19%)	21.11	31.87
8. Other expenses (30%)	1.37	1.35
Total	$142.18	$183.75
Charge per cow per month	$ 11.85	$ 15.31
Labor @ $1.50/hr, 70 hr per cow per year	8.75	8.75
Interest on salvage value	1.32	1.32
Total operational costs/month	$ 21.92	$ 25.38

It is not surprising these operational costs of $21.92 and $25.38 should fall in the range of the $22.21 per month opportunity cost from the example of Giaever (1966) in determining the point to cull within the lactation. To compute the above interest charges allocatable to the cow herd the following figures are relevant:

	Low	High
Improvement invest./cow (80%)	$ 322	$ 568
Machinery invest./cow (30%)	272	360
Livestock invest./cow (100%)	373	373
Total farm invest./cow	$2282	$4285

i.e., (0.80 x 322 + 0.30 x 272 + 373)/2282 = 31.24%. Hence, taxes and interest were allocated from the sum of the livestock investment, 30% of the machinery investment and 80% of the improvement investment as a proportion of the total farm investment or 31.24% for the low and 36.19% for the high farms.

Feed consumption

Profitable operation of a dairy herd involves both the income and costs of each cow in the herd as well as overhead costs charged to the dairy herd. To determine which cows in the herd return the most profit and which a dairyman should cull then becomes a matter of determining which incomes and which costs vary between cows. Of the several incomes, milk sales vary the most between cows, while potential sale as dairy or salvage ranks a poor second.

Conversely, by far the greatest difference in costs between cows is in their feed intake. This difference results largely from such variables as milk production, body size, stage of lactation, age, and individual appetite. But under farm conditions, the individual dairyman has no practical method to measure the appetite differences between cows; consequently he must rely on the other variables mentioned to help determine differences in feed consumption.

For this simulation program, the higher requirements for Total Digestible Nutrients (TDN) established by Morrison (1959) according to body weight, milk production, pregnancy, and growth were the criteria for feed consumption and requirements. TDN requirements per pound of milk below the 3.0% test listed by Morrison were linearly extrapolated down to 2.0% test. An additional 15% was charged for waste due to refusal by the cow, shrinkage, rodents and spillage. Accordingly, such waste was charged on a prorated basis because these are the actual costs incurred for the feed.

All cows in milk were fed 40 lb corn silage per day, grain was fed 1 lb per 3 lb milk, and the balance alfalfa hay to meet TDN requirements. The limiting factor at all levels of production for all ages except 2 and 3 year old cows was TDN. To preserve model simplicity, additional hay was fed to these animals at certain levels of milk production to meet protein requirements. These added amounts are given in Appendix Table 5. Grain was assumed to contain 12%

digestible protein (D.P.) and 75% TDN; alfalfa hay 10.9% D.P. and 50.7% TDN; corn silage 1.3% D.P. and 19.8% TDN. For the 10 month lactation for cows which remained in the herd, feed requirements simply equaled 305 times the daily requirement. During the 60 day dry period cows consumed 40 lb silage per day plus enough hay to meet TDN requirements (see Appendix Table 4).

Generation and use of random numbers

Sequences of numbers generated by an arithmetical process in a computer can be used as random numbers for simulation problems. Though they fulfill many criteria for randomness, such numbers are often called pseudorandom since they are generated by a deterministic process and hence are not actually random. The computer system available for this study, the CONTROL DATA 3600 located at Michigan State University, is a general purpose digital computing system with large storage capacity. A library program, RANF, which generates a sequence of over 68 billion numbers before repeating, was available for generation of these uniformly distributed pseudo-random numbers. To avoid separate trials on runs of this simulation program from starting at the same point in the sequence, RANFSET(TIMEF) was called which started the sequence according to the time on the computer clock. All random numbers called in floating point consequently lie $0 < r_i < 1$. This particular function generated numbers by the mixed congruential method, more specifically by (Conner, 1965):

$$r_{i+1} \equiv (2^{10} + 1)r_i + 101(\text{mod } 2^{36}).$$

Random deviates assume a significant role in most simulation procedures. There are numerous methods available for generation of these deviates which can then be stored in the computer. But because a very large number of deviations were required in this study, such deviates were generated as needed. In the present investigation from a method by Gill (1963), twelve random numbers were generated using the procedure described above. By adding these numbers with a mean of 0.5 and range 0 to 1.0, the variance obtained equals the square of the range divided by 12. Therefore $\sum^{12}$ uniform numbers $\Longrightarrow$ mean 6, variance 1.0 and ($\sum^{12}$ uniform) - 6.0 provided N(0,1) random deviates. Subsequently, several samples of random deviates were generated which conformed closely to the standard normal distribution.

Random numbers again were used to determine the involuntary culls and deaths. A random number was multiplied by 100, producing a number in the range of 0 to 99.999, which by the addition of 1 yielded a number in the range of 1 to 100.999. This number was then truncated to integer value resulting in a number which has the range of 1 to 100. Such a number was drawn each year for each animal. With cows 5 years of age or older, for instance, a "1" signified death, while a number ranging from 2 to 15 inclusive signified an involuntary cull. The chance of culls by age were as follows:

Age	Chance of death	Chance of involuntary cull
5	.01	.14
3 & 4	.01	.08
2	.01	.05
yearlings	.17	...
calves	.17	...

This simulation program maintained distinction between death and involuntary culls in the milking herd in order to credit the culls with a salvage value but since the yearlings and calves did not enter into a return or cost, all deaths and culls for these animals for all causes including stillborn calves and sterility of yearlings were treated as death or simply elimination from the herd. The above probabilities of death and involuntary culls for milking cows were computed from data derived by Dayton (1966) from 30,308 complete Holstein records made from 1957 to 1962 of A.I. offspring in Michigan DHIA. Involuntary culls are here defined as cows removed for all reasons except dairy and low production. Probability of calf and yearling losses, on the other hand, approximate the distribution of such losses of simulated herds in the undergraduate breeding course described by Everett (1966).

The chance of a cow having a calf of a given sex was also considered a stochastic variable so it too was determined by random numbers. Since the distribution of the random numbers has a mean of 0.5, by adding 1.5 to a random number then truncating to an integer value, either a "1" or "2" is obtained at random. Thus arbitrarily, a "1" designated a heifer while a "2" designated a bull calf resulting in a probability of 0.5 or a calf being a heifer.

Probability of a cow dying in any given month of the year once she drew a "1" signifying death was assumed to be 1/12. Similarly, but contrary to intuitive expectation, the probability of an involuntary cull in any given month of the lactation or dry period was also considered 1/12 once the random process has determined a cow as an involuntary cull (Aulerich, 1966). Therefore, since the specific month of removal is a stochastic process, this event was similarly determined by the random number generator.

Age correction factors

For simplicity of design and to eliminate any weight-age interaction, all cows of a given age were assumed to weigh the same. From the regression equation developed by McDaniel and Legates (1965) from heart girth measurements of 1595 Holsteins, the respective weights were determined by the equation:

$$\hat{Y} = 757 + 20.91M - 0.2036M^2 + 0.00066M^3$$

where $\hat{Y}$ is the predicted body weight and M is age in months. The resulting weights by ages are listed in Appendix Table 3 and refer to age at freshening. Such body weights helped determine both feed consumption and salvage value.

The basic program in simulation of the base herd and subsequently the offspring called for generation of the mature equivalent (M.E.) milk production. Therefore, to convert the M.E. to actual milk production for certain culling strategies and to determine actual milk income, correction factors are needed. In such a reverse process, the reciprocal of the standard USDA age correction factors were utilized (McDaniel et al., 1967). Since the model implied no seasonal effect, a weighted average of two seasons based on the number of cows used to determine the regional (including Michigan) Holstein age correlation factors were used. Resulting correction factors and their reciprocal are listed in Appendix Table 1.

Similarly, under the assumptions of the model, the 305 day complete records were simulated for each cow. In reality, 305 day records can be predicted from the extension of partial records with a reasonable degree of accuracy. Madden et al. (1959) obtained correlations of 0.90 and 0.95 between 3 and 4 months cumulative records respectively and 305 days production of cows under 3 years and 0.85 and 0.90 for the corresponding figures of cows over 3 years old, (similar to Van Vleck and Henderson, 1961, and Van Vleck, 1964). But since the potential 305 day record of each cow served as the base, it was necessary to compute a partial record from the whole by the reciprocal of the extension factor to determine any incomplete actual milk production records of involuntary culls and to help determine the most profitable month to remove the voluntary culls. For sake of simplicity, the model assumed identical lactation curves for any given 305 day milk production record. Though this method (rather than generating each month's production independently) introduces some error in monthly variation of the lactation curve, little error will be implied in the mean of the lactation curve for a given 305 day production.

The reciprocal correction factors carried to the nearest 4 places employed to estimate these partial records were derived from data of 15,330 Michigan Holstein records $<$ 36 months old and 32,986 $\geq$ 36 months old (Aulerich, 1966) using the weighted average according to numbers of cows in each of the two seasons and divided into their respective age groups. These correction factors are listed in Appendix Table 2.

<u>Present value strategy</u>

The sixth strategy examined, based on the present discounted value of expected gross income of a cow and her subsequent replacements, employed a simple set of correction or adjustment factors according to the age of the cows. That is, to rank a herd of cows and adjust for the different ages, you mulitply each cow's current gross milk income by the appropriate correction factor. Such a procedure is based on the principle that a sum of money at present is worth more than the same sum in the future (Giaever, 1966; Faris, 1960). This discounting furnishes a method of transferring a stream of future income to a single number called the "present value" of the future stream. From this approach each cow and her subsequent replacements can be thought of as a stanchion or space in the herd. Thus, with a 15 year planning horizon, the income stream develops from the expected relative production of each cow until the year of her expected involuntary loss or death plus the expected production of her future replacements over their lifetime.

To determine these corrections factors, a two dimensional array was made of each age group (2, 3, ... 12) times the 15 year planning horizon. By the standard M.E. age correction factors for milk production used by DHIA, the relative expected production for each year was listed; for example, for a 2 year old with a relative value of 1.0, her 7 year old value, 5 years hence, would simply equal the correction factor of 1.28 times 1.0 or 1.28 while, say her expected 5 year old production would equal the reciprocal age correction factor (0.971) times 1.28.

The potential length of life for each cow was determined from Pearl's formula (1930) for life expectancy, in this case, based on the rate of involuntary culls used in this study:

$$e_x = \frac{1/2\ l_x + l_{x+1} + l_{x+2} \cdots}{l_x}$$

where l_x is the number of cows living in year x, l_{x+1} the number living in year x + 1 etc. The resulting life expectancies were truncated to the nearest one-half year resulting in the following expectations:

Age	Life Expectancy
2	7.0 years
3	6.5 years
4	6.0 years
5-8	5.5 years
9-12	5.0 years

Two year olds coming into the array in the second and following generations were given a genetic value equal to a 2% gain per year from the genetic value of the cow she is replacing, hence assuming that each year's crop of heifers will produce 2% above the previous year's crop. For discounting calculations, the year's values following on the half year basis were pro-rated according to the respective values in the cells. The final year that each animal was expected to produce, a relative salvage value of 0.5 was added to the milk value. This procedure assumes that the salvage income will approximate one-half of the income over feed cost for that year. Each cell was then divided by its respective discount rate which was as follows:

Year	Discount
2	1.03
3-15	$(1.03)(1.06)^{n-2}$

where n = year and the first year was considered the year of production to be compared and the second year the first year of future income. A discount rate of 3% was used the second year on the assumption that milk checks would be received monthly so a monthly discount rate is effective.

Then the respective discounted values were added to obtain the total present value and subsequently reduced to a value of 1.00 assigned to the present value of the 7 year old cow. By multiplying these adjustment values times the corresponding gross income values, the proper weight should be given each animal relative to her present value and all her subsequent replacements in a 15 year planning horizon. Such a planning horizon was simply truncated at the end of 15 years but this does not imply the herd would be sold at that time. By relating to gross milk income rather than milk production, allowances are made for the relative economic value of milk and fat.

These correction or adjustment factors were then adjusted 10% of the difference between this new value toward the standard M.E. milk age correction factors to allow for a discounted genetic value of the offspring of these respective cows on the assumption that half of the calves will be heifers and that heritability is about 25% and therefore only 12.5% of the transmitting ability of a cow can be realized and roughly some of this will be discounted, so an approximate value of 10% of the transmitting ability can be allowed toward the discounted producing ability of the animal in question.

The resulting adjustment factors and their method of derivation are unique but the component parts consist simply of the concepts of milk production converted to income, standard age correction factors, life expectancy, expected genetic progress and standard discounting principles. These factors are as follows:

Age	P.V. Factor	Age	P.V. Factor
2	1.201	8	1.004
3	1.127	9	1.030
4	1.056	10	1.052
5	1.016	11	1.066
6	1.004	12	1.115
7	1.000		

General assumptions

For simplicity, since computer time and storage capacity are finite, and to determine relevant effects of the given strategies and prices, certain variables were held constant. Accordingly, several assumptions in addition to those explained elsewhere are in order:

1. All cows freshen on September 1 and milk for 10 months unless removed by death, involuntary cull or voluntary cull; calendar year in turn commences September 1.
2. All heifers freshen at 2 years of age and if they remain in the herd maintain a calving interval of 12 months.

3. All cows of a given weight and milk production will eat the same amount of feed.
4. All deaths, involuntary and voluntary culls take place at the end of the month, where respective months are determined by methods described above.
5. Cows are ranked once per year on their "potential" 305 day record but this does not imply cows should be ranked only once per year nor simply that they are ranked at the end of their lactation. A dairy cow's 305 day record can be accurately estimated from a 3 or 4 month part record (Madden et al., 1959).
6. Time of paying for feed and time for receiving milk income were assumed identical in that there is no difference in discounting the two accounts; in fact, within a given year, no discounting of feed or milk is assumed when computing income over feed cost; but of course over the 15 year period average income (including salvage) over feed cost per cow is discounted by the respective years to equate all trials to a common present value.
7. All replacements are raised, and it is assumed the dairyman has no restrictions on capital, labor or building to raise the replacements.

Determination of When to Cull Within a Lactation

After he has determined which cows of his herd to cull, the next major decision facing the dairyman is to decide the most profitable point within the lactation to cull these cows. This point will depend on the value of cull beef and value of milk and the costs of production including feed, labor, livestock expense and interest on investment. These in turn are dependent on the weight of the cow, her level of milk production within the lactation, and related prices.

At most levels of production and prices, it will not pay to keep a potential cull until she is dry or due to dry up. By that time, if she is within 60 days of calving, in order to capitalize on her peak producing period in the following lactation, it would pay to keep her until she freshens again. More importantly, a cow nearly dry is losing money for her owner, for under most conditions, she is not paying for her feed, labor and fixed costs. In fact, carrying a potential cull beyond the point where MC = MR will reduce the average net revenue for the herd.

If a dairy herd is to profit in the long run, each cow must pay her share of not only her variable costs but her fixed costs. It can be argued that the proportional share of both interest paid and interest on investment allocated to the milking herd must somehow arise from the milking herd, each cow to pay her share if possible. Of course, a dairyman has no direct control over death losses and certain involuntary losses in regard to paying these interest costs. Though this simulation program specified involuntary losses occurred randomly as to month of lactation, it is conceivable that under actual conditions, a farmer could sell shy breeders at the most profitable time within their lactation.

Thus, to determine the most profitable point to cull within a lactation, a cow must cover her share of feed costs, the interest and taxes mentioned above, labor, and interest on salvage value, plus other expenses of livestock, machinery, and building allocated to the milking herd (consistent with fixed cost allocation proposed by Chisholm, 1966; Burt, 1965; and Breimyer, 1966).

Since all cows milked for 10 months in this simulation program, it was necessary to consider the possibility that potential cull cows from high level herds may still return a profit in their 10th month. This is especially probable when high milk prices fuse with low feed and operational costs. From the potential 305 day actual production, where months of a lactation were defined as 30.5 days, the simulation model computed milk production and milk sales for the 10th month then compared these sales with the sum of the feed costs plus operational costs. Feed costs for the 10th month were similar to the other 9 months except that allowances of the 10th month were increased for pregnancy. Grain was fed according to production, 1 lb grain:3 lb milk; silage 40 lb per day; and enough alfalfa hay to meet TDN and protein requirements.

If milk sales of the cow to be culled exceeded feed costs plus operational costs, she was culled at the end of the 10th month; if not, the potential milk sales allocated to the 9th month were compared to the potential feed costs of this month. Likewise, if the milk sales were greater than these two items, the cow was culled at the end of the 9th month. The program then computed total milk sales, total feed costs and salvage value and added this to the respective herd totals according to the month she left the herd. The operational costs were not charged to the cows as such in this program, but used only as a parameter to help determine the most profitable point within the lactation to remove each voluntary cull.

If the milk sales for the 9th month were less than the feed costs plus operational costs, the computer program compared the 8th and earlier months in sequence in a similar manner, all cows being culled at the end of the month. Such a procedure can be applied to actual dairy herd conditions under the assumption that one can predict the whole lactation or any of its respective parts from 90 or 120 days production. Under the conditions of constant milk prices and feed and operational costs in this simulation program, the actual point of breaking even for profit or month in this case, can be determined with more accuracy.

Generation of Base Herd and Succeeding Generations

Base herd

Forming of a base dairy herd or genetic system for simulation procedure involves the establishment of certain parameters or characteristics needed to identify the herd. Such procedures have been demonstrated by Heidhues and Henderson (1961) and Everett et al. (1967). In this case variance for both milk and fat percentage and their correlation were chosen as the relevant characteristics. The variances of milk production used to generate the base herd were obtained from the same bases used in an undergraduate dairy cattle breeding course at Michigan State University which represented the approximate variation found in the Holstein population of Michigan (Everett, 1966; Everett et al., 1967). Since the present study involved a single herd under constant yearly herd environment, the variances between herd and between year were eliminated, but the residual or temporary environmental variance representing variation between lactations of the same cow was retained. The genetic and permanent environment on the other hand were constant throughout each cow's life, which automatically accounts for some, if not all the repeatibility of milk production.

Therefore, the following variance components for milk were used:

Milk	Variance	Std. Dev.
Total (lb)	4,100,000	...
Genetic	1,210,000	1100
Permanent envir.	640,000	800
Residual (temp. envir.)	2,250,000	1500

The fat percentages and correlations of milk with fat percent were taken from data by Butcher et al. (1967) from records of Holstein cows in the North Carolina Institutional Breeding Association and are as follows:

Fat percentage	Variance
Total	.0862
Genetic	.0537
Environmental	.0325

From a genetic correlation of -0.61 ± 0.13 and an environmental correlation of 0.16 between milk and fat percent (Butcher et al., 1967), the respective genetic and environmental coefficients were derived using 14,000 lb milk, 3.63% test as approximate Michigan DHIA Holstein M.E. average. Mature Equivalent milk production of base population then equals:

$$Y_{M_{ij}} = \mu_m + g_i + p_i + r_{ij}$$

$$= 14{,}000 + \underline{N}\ \text{Dev.}(1100) + \underline{N}\ \text{Dev.}(800) + \underline{N}\ \text{Dev.}(1500)$$

where $\underline{j}$ in this case is the base year or lactation

$Y_{M_{ij}}$ is the $\underline{j}$th M.E. milk record of $\underline{i}$th cow,

μ_m is the population mean for milk (M.E.)

g_i is the additive genetic effect of *i*th cow,

p_i is the permanent environmental effect of *i*th cow,

r_{ij} is the residual or temporary environmental effect of *j*th record of *i*th cow.

Fat percentage of base population equals:

$$Y_{T_{ij}} = \mu_t + b_1(\mu_m + g_i) + g_i^t + b_2(r_{ij}) + e_{ij}$$

$$= 3.63 = 0.00013(\mu_m + g_i) + \underline{N}\ \text{Dev.}(0.18)$$
$$+ 0.000017(r_{ij}) + \underline{N}\ \text{Dev.}(0.18)$$

where

$Y_{T_{ij}}$ is the *j*th test of the *i*th cow,

μ_t is the population mean test,

b_1 is regression coefficient for correlation of test with genetic milk,

g_i^t is genetic chance effect of test of *i*th cow,

b_2 is regression coefficient for correlation of temporary environmental portion of test with residual (temporary envir.) portion of milk,

e_{ij} is environmental chance effect of test of *j*th record of *i*th cow.

That portion of the variation in test associated with permanent environment was only 0.01 so was considered as zero.

Though none of the culling strategies was based on the "true" genetic value of each cow, this value was preserved internally in the computer so that at any given year or generation, the true genetic value of each animal living in the herd could be examined.

To assess more accurately the effect of the culling strategies on a female population under the fixed prices, the breeding value of the sires were assumed equal and held constant over all strategies and prices for any given year. From a breeding value of 15,000 lb milk for the base year, the sire value advanced 130 lb of milk each year; thus the calves born in year two were sired by bulls with the 15,000 lb breeding value. This procedure was identical for all trials, that is, all strategies and prices. This genetic value and gain per year was assumed to represent the relative value of the best Holstein bulls used in A.I. studs. Likewise, the breeding value of sire fat test was started at the base year at 3.63 - 0.00013(1000) = 3.5% and was decreased each year by 0.00013(130) or the identical regression coefficient used in the cow population, thus consistent with the negative correlation of milk and fat test.

Second and following generations

The genetic ability of an offspring results from averaging the genetic ability of both parents and adding a chance deviate. Mendelian segregation in the parents followed by recombination in the zygote results in half of the additive genetic variation in an offspring accounted for by the parents and half being random. This random part has a standard deviation of approximately 800, therefore the variation due to chance is added to the genetic ability of each individual by multiplying a random normal deviate by 800.

Specifically, the milk production of the second and following generations were derived as follows:

$$Y_{M_{ij}} = 1/2\ V_{m_s} + 1/2\ V_{m_d} + gd_i + p_i + r_{ij}$$

$$= 1/2\ V_{m_s} + 1/2\ V_{m_d} + \underline{N}\ \text{Dev.}(800) + \underline{N}\ \text{Dev.}(800) + \underline{N}\ \text{Dev.}(1500)$$

where

$Y_{M_{ij}}$ is *j*th M.E. milk record of *i*th cow,

V_{m_s} is breeding value of sire for milk,

V_{m_d} is breeding value of dam for milk,

gd_i is genetic chance effect of milk of *i*th cow,

p_i is permanent environment effect of milk for *i*th cow,

r_{ij} is residual effect of *j*th record of *i*th cow.

The first three terms of the above equation define the genetic ability and the first four the producing ability of the animal in question. Thus the genetic and permanent environment effect are determined at birth and the residual determined independently for each lactation.

The fat percentage of the second and following generations were derived as follows:

$$Y_{T_{ij}} = 1/2\ V_{t_s} + 1/2\ V_{t_d} + b_1(gd_i) + gt_i' + b_2(r_{ij}) + e_{ij}$$

$$= 1/2\ V_{t_s} + 1/2\ V_{t_d} - 0.00013(gd_i) + \underline{N}\ \text{Dev.}(0.129)$$

$$+ 0.000017(r_{ij}) + \underline{N}\ \text{Dev.}(0.18)$$

where

$Y_{T_{ij}}$ is the *j*th fat test of the *i*th cow,

V_{t_s} is breeding value of sire for test,

V_{t_d} is breeding value of dam for test,

b_1 is regression coefficient for regression of test on genetic chance of milk,

gt_i' is remaining genetic chance effect of test of *i*th cow,

b_2 is regression coefficient for correlation of temporary environmental portion of test with residual (temporary environmental) portion of milk,

e_{ij} is environmental chance effect of test of *j*th record of *i*th cow.

For model simplicity, fat test did not vary by stage of lactation. The regression coefficients used in the above equations were determined as follows from correlations derived by Butcher et al. (1967). For the base population the components of variance of fat test of a single record were postulated to be:

$$\sigma_t^2 = \sigma_{gt}^2 + \sigma_{et}^2 \quad \text{or}$$

total test variance = genetic variance of test + environmental variance of test where $0.0862 = 0.0537 + \sigma^{t}_{et}$

therefore: $\sigma^{2}_{et} = 0.0862 - 0.0537$

$= 0.0325$

and b_1 = correlation of genetic test with genetic milk $(\sigma_{gt}/\sigma_{gm})$

$= -0.61(\sqrt{0.0537/1,210,000})$

$= -0.00013$

and b_2 = correlation of environmental test with environmental milk $(\sigma_{et}/\sigma_{em})$

$= 0.16(\sqrt{0.325/2,890,000})$

$= 0.000017$

The genetic portion of the variance of test can be further divided into a portion correlated with the genetic milk and a portion due to genetic chance. Thus:

$$\sigma^{2}_{gt} = b^{2}_{1}\sigma_{gm} + \sigma^{2}_{gt'}$$

where $0.0537 = (-0.00013)^{2}(1,210,000) + \sigma^{2}_{gt'}$

therefore: $\sigma^{2}_{gt'} = 0.0341$

$\sigma_{gt'} = \sqrt{0.0341} = 0.18$

Likewise, the environmental portion of test can be further divided into a portion correlated with permanent environment effect of milk, a portion correlated with the residual or temporary environmental portion of milk plus a random environmental effect. Thus:

$$\sigma^2_{et} = b^2_2\sigma^2_{em} + \sigma^2_{et}$$

where $$0.0325 = (0.000017)^2(2{,}890{,}000) + \sigma^2_{et'}$$

therefore: $$\sigma^2_{et'} = 0.0316$$

$$\sigma_{et'} = \sqrt{0.0316} = 0.18.$$

For the second and following generations, the additional coefficients and multipliers for test were derived as follows from correlations of Butcher et al. (1967):

$$\sigma^2_{gt} = 1/4\sigma^2_s + 1/4\sigma^2_d + 1/2\sigma^2_{gt'}$$

therefore: $$\sigma^2_{gt'} = 1/2\sigma^2_{gt} - b^2_1(1/2\sigma^2_{gm})$$

$$= 1/2(0.054) - (-0.00013)^2(605{,}000)$$

$$= 0.0166$$

$$\sigma^2_{gt'} = \sqrt{0.0166} = 0.129.$$

Culling Procedure

First year

The basic procedure in any simulation process involving generation and projection of events over time must start with a given population, herd, firm, or the unit on which you base the simulation. Therefore, to generate an 80 cow dairy herd and accompanying young stock with known mean and variance of milk production involves the use of the random number generator as described previously. The base herd was composed of the following age distribution:

Age	No. of animals
6	8
5	10
4	16
3	22
2	24
1	26
calves	38
Total	144

In the first or base year, after the computer program generated the M.E. milk production and test for each of the cows in milk along with the genetic and permanent environment milk production potential of the young stock, the number and identity of deaths and involuntary culls and their month of removal were derived from the random number process described above; however, no cows were culled voluntarily the first year. Then the actual 305-day milk production of each cow was computed by the reciprocal factor times the generated M.E. milk production. Milk sales for each cow were then determined according to the given price of milk, fat differential and respective test of each cow where the fat differential was added or subtracted for each tenth point in test $\pm$ 3.5%.

If the simulation program removed a cow involuntarily before she completed her lactation (one of first 9 months), her milk production and milk sales were adjusted according to the proper ratio factor to convert whole lactations to their respective cumulative parts.

Feed costs were then computed for each cow according to her body weight and milk production. The 11th and 12th month consisting of 30 days each were considered as dry months where feed consumption was 40 lb silage per day plus enough hay to meet TDN requirements for maintenance and pregnancy according to Morrison's (1959) higher standards. For cows which died or were sold before the end of the 11th month, feed costs were reduced proportionately. Likewise, if a cow was sold in the 12th month she was charged a full year's feed costs.

Yearly herd totals of actual milk production, feed costs, income over feed costs, number of culls, etc. were then recorded for the milking herd and average per cow computed where average number of cows equals total cow months divided by 12.

Second and following years

To start the second year of the simulated herd, the age of each animal advanced one year and her weight advanced according to her respective age. Since all cows were assumed to begin their lactation on the first of the year, all calves were born at that time. To generate the offspring of each cow and determine its sex, the random number generator was again utilized as described previously. If a bull calf was born, it simply was ignored since their records were not utilized for any purpose. As the heifers were born, their genetic and permanent environment portion of their potential milk production which then equals their "producing ability" plus their genetic portion of milk test was generated as described in the previous section.

For the milk cows, their "potential" M.E. milk production was generated each lactation by:

$$(\text{M.E. milk})_i = (\text{producing ability})_i + \underline{N}\ \text{Dev.}(1500)$$

where the last term is the residual (R_i) or temporary environment effect, yet "producing ability" of $\underline{i}$th cow generated at birth is constant throughout her life. Thus, the model generated a new temporary environment each lactation for each cow where the normal deviate ($\underline{N}$ Dev.) N(0,1) created from the random number generator is unique for each lactation of each cow.

Each cow's test was determined by:

$$(\text{Test})_i = (\text{genetic test})_i + 0.000017(R_i) + \underline{N}\ \text{Dev.}(0.18)$$

where the second term of the above equation is the correlation of residual milk production of $\underline{i}$th cow with test and the last term a chance portion of the environmental test (0.18 is the standard deviation of the environmental test). Again, $\underline{N}$ Dev. is unique for each lactation of each cow and is distinct from the $\underline{N}$ Dev. in the milk equation.

The next step was to "estimate" the number of involuntary culls among the milking cows. This was assumed to be consistent with reality in that from past experience, a dairyman can estimate the number of involuntary culls with some intuitive probability distribution. For this problem in year 2, the estimated number was set at 8% of the herd but for the remaining years estimated number equaled average number of involuntary culls of past 2 years.

The number of cows available then for voluntary culls equals: (number of cows in the herd) minus (estimate of involuntary culls) minus 80. This definition assumes a dairyman would have a chance to milk all his two year olds at least two to three months before he determined which of his cows to cull voluntarily and thus would at some times of the year have flexibility to maintain some cow number above the basic herd size of 80. From a total of say, 95 cows that have been in the herd at one time during a twelve month period, not all cows will be in the herd at any given time. For this simulated herd, however, for model simplicity, all cows which are ever in the herd during the year will produce milk at least the first month of the lactation year.

After determining how many cows to cull, the computer ranked the cows from high to low based on the current year's records according to one of the six culling strategies, while each respective strategy was constant for any given run of 14 years beyond the base year. Then the number of voluntary culls specified were culled from the bottom of this rank. Based on the principle of marginal returns where each month's income and expenses are considered as the marginal increments, the actual month to remove the voluntary culls was determined by the point which would yield the highest total net income over feed and operating costs for the lactation, as explained previously on page 52.

The true number of involuntary culls and their month of removal was then determined by the random number generator so that at the end of the year, there should remain approximately 80 milk cows. It is logical this step should succeed ranking, since when a dairyman ranks his cows he does not know exactly which potential voluntary culls or which potential survivors he may cull for involuntary reasons. For instance, from an extension of a three month record, a cow a farmer plans to cull for low production in the seventh month of her lactation, he possibly may cull for disease in the fifth month. For this simulation problem, if a cow pegged to be a voluntary cull was also drawn to be culled involuntarily, her month of involuntary cull was then drawn randomly in the usual procedure and she was subsequently culled on the first of the two specified months.

From the voluntary and involuntary culls, the milk production, sales and feed costs were computed according to month of removal. Then total herd production, milk and salvage sales, feed costs and income over feed costs for all cows were summarized and average per cow computed for the year.

For the third and succeeding years up to 15 years of simulation, the procedure was identical with that of the second year; that is, the animals were advanced one year in age, a new crop of calves was generated, M.E. milk production generated, actual milk and income over feed cost computed, then voluntary and involuntary culls determined and yearly income and costs summarized.

For each set of fixed prices and strategy, two replications were run; that is, for each replication, a new base herd was generated but with the same mean and variance of milk production and fat test. Likewise, for each new strategy or new set of prices, a new base herd was generated and simulated over the 15 year horizon.

Comparison of Strategies

Multiperiod production, or production over a period of years, i.e., 15, is characterized by factors of production employed during one time period which influence levels of output during subsequent time periods. Specifically, different culling strategies may have diverse effects on income at various points in time. It is further assumed that there exists a market for money at which money can be borrowed or lent at a given rate of interest. By using this compounded rate of interest, not only can outlays or income incurred during different time periods be made comparable by discounting them to one and the same time period, but for this problem, such a procedure can reduce each strategy to a single value relative to its future income stream discounted to the present.

Therefore, the basic criterion used to compare each strategy under all combinations of prices was the "present values of income over feed cost per cow over the 15 year horizon." This total figure was then averaged by Hutton's (1966) method to obtain a more conceptual figure. Thus:

$$\text{Present value of income over feed cost per cow} = \sum_{i=1}^{15} \left[\frac{I/FC_i + Salv_i}{(1.03)(1.06)^{i-1}} \right] \Big/ 15$$

where I/FC_i = income over feed cost per cow for year i.

Analytical Design and Methods

To compare the effects of the six strategies and the effect of various levels of fixed prices upon the results generated by the simulation program, a 6 x 2^5 complete factorial design as described by Steel and Torrie (1960) was utilized. Six strategies times two levels of each of four prices times two replications each yielded 192 replications. Logical mutually orthogonal contrasts were predetermined before the data were generated to compare the strategies under various categories. Since the major criterion of comparison was income over feed cost, the culling strategy based on income over feed cost was compared with the average of the other five; present value, because it was unique, was compared against the average of strategies 1 to 4; the average of 1 and 2, the M.E. strategies was compared against strategies 3 and 4, those based on actual milk and gross income. The remaining contrasts compared 1 vs 2 and 3 vs 4, or milk compared to gross income within each group.

If there were no significant interactions between strategies and prices, if there was significant differences between strategies, and if orthogonal contrasts were not utilized, Duncan's new multiple range test described by Steel and Torrie (1960) was used to test for significant differences between all combinations of the ranked values.

If interactions existed, we can assume the importance of each strategy or price is affected by the particular combinations of prices in effect in those replications. On all values but the discounted income over feed cost including salvage, a simple average of the 15 years was used as the basis to test for significant difference among strategies and prices for: income over feed cost without salvage, gross income, milk production, fat production, genetic milk value, herd size, average month of culling, and culling rate.

Computer time for the main program or simulation of the data was approximately one minute per replication or 192 minutes.

RESULTS AND DISCUSSION

Comparison of Strategies

The present value of income over feed cost per cow averaged over the 15 year horizon, used as the major criterion to compare strategies, showed no significant differences between the six culling strategies examined. From Table 1, the overall mean was $295.85 while the mean value for culling strategies based on M.E. milk, M.E. gross income, actual milk, actual gross income, income over feed cost, and present value was $295.68, $296.70, $294.91, $296.21, $295.72, and $295.87 respectively.

TABLE 1. Present value of income over feed cost (including salvage)

Strategy	Present value of income over feed cost[a]
1. M.E. milk	$295.68
2. M.E. gross income	296.70
3. Actual milk	294.91
4. Actual gross income	296.21
5. Income over feed cost	295.72
6. Present value	295.87
Overall Mean	$295.85

[a]No significant difference between strategies.

From this data, we can surmize that the choice of one of these strategies by a dairyman is not critical in the generation of income. Results from the 6 x 2^5 complete factorial design related to the above income values showed no significant interaction between strategies and levels of prices used in this study. Consequently, a change in prices does not indicate a change in strategy to maximize income over feed cost. This income was affected by the levels of milk price, feed cost, and operational cost ($P < .01$) and fat price differential ($P < .01$).

TABLE 2. Effect of levels of prices on present value of income (including salvage) over feed cost

	Prices			
Level	Milk Price	Fat Differential	Feed Costs	Operational Costs
Low	$252.19	$295.37	$310.82	$295.14
High	339.51	296.32	280.88	296.56
Difference	$ 87.32**	$ 0.95*	$ 29.94**	$ 1.42**

*Significant ($P < .05$)

**Significant ($P < .01$)

This model does not imply these prices are the expected prices but that a dairyman facing a similar range of prices can expect similar changes in income.

Average milk income over feed cost when salvage value was omitted was different ($P < .01$) among strategies. From Table 3 and Figures 1.1 and 1.2, we can see that the strategy of actual gross ($385.94) was higher ($P < .01$) than either M.E. milk ($383.56) or M.E. gross ($383.66) when tested by Duncan's multiple range test. An examination of Table 3 shows that the actual gross strategy ranks highest in income over feed cost more often than any other strategy. The slight advantage of actual gross is the most obvious after eight years.

Dairymen whose feeding practices or costs vary greatly from those assumed in this model may face a different relationship between strategies than this study indicates. The wide range in grain feeding rates among dairy farmers and the deviation from the model in actual feed efficiency by individual cows could lead to discrepancies.

Several factors may account for the similarity between strategies, especially when based on income over feed cost plus salvage. First, many of the cows which are culls under one stratey will also rank as potential culls under the other strategies. For example, cows ranking in the lower 10% of the herd on milk production will likely rank close to the bottom 10% on gross income or income over feed cost. Only certain cows with extreme fat tests will become culls under one strategy yet not the others.

TABLE 3. Average income over feed cost per cow

	Strategies					
Year	M.E. Milk	M.E. Gross	Actual Milk	Actual Gross	Actual Inc. Over Feed Cost	Present Value
1	$329.46	$328.32	$327.11	$327.62	$327.52	$326.14
2	343.87	349.01	343.81	344.50	346.15	348.08
3	363.68	366.26	363.83	363.82	360.27	363.52
4	369.09	370.50	370.73	369.70	371.79	372.44
5	375.90	370.12	376.71	379.87	377.66	376.33
6	379.59	382.20	385.34	383.84	382.36	381.37
7	384.53	385.48	388.59	388.16	385.47	385.49
8	389.73	387.28	391.69	392.37	391.30	392.96
9	392.34	394.24	393.43	396.99	393.99	390.60
10	397.76	393.29	397.14	399.08	397.65	395.95
11	401.22	398.02	401.35	404.43	400.63	401.68
12	402.93	402.71	402.49	403.77	405.94	401.03
13	404.14	405.62	403.95	410.04	409.07	407.36
14	407.12	405.80	410.93	412.37	409.16	409.79
15	412.03	409.98	412.27	412.62	413.95	413.07
Ave.	$383.56[b]	$383.66[b]	$384.62[ab]	$385.94[a]	$384.86[ab]	$384.39[ab]

[ab]Values with the same superscript indicate no significant difference (P> .01).

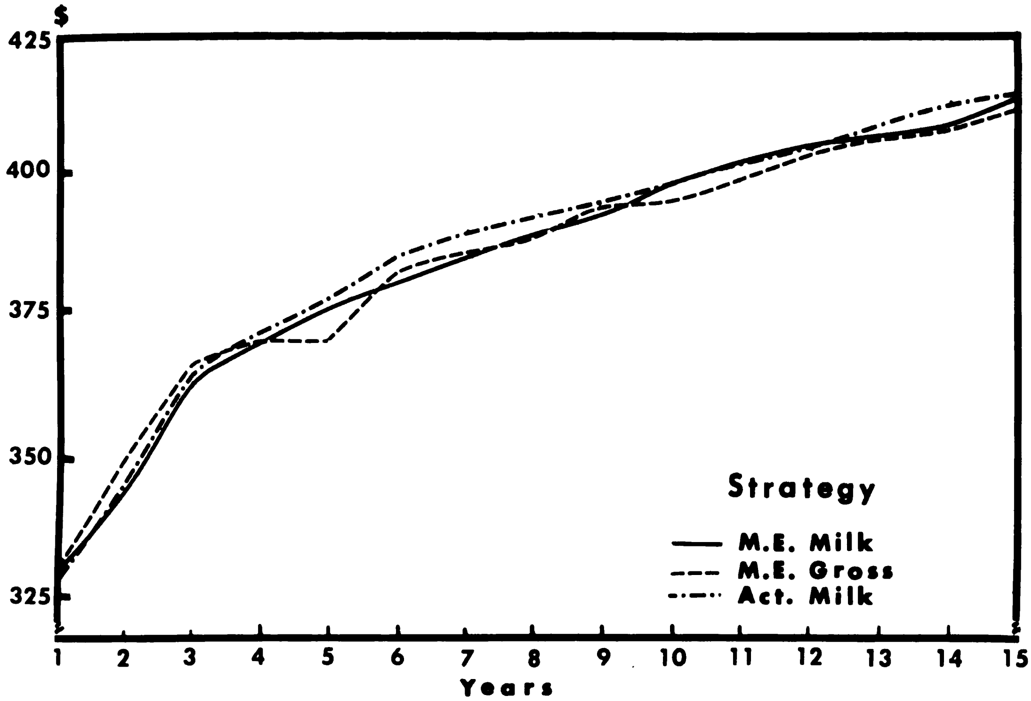

Fig. 1.1 Average income over feed cost per cow.

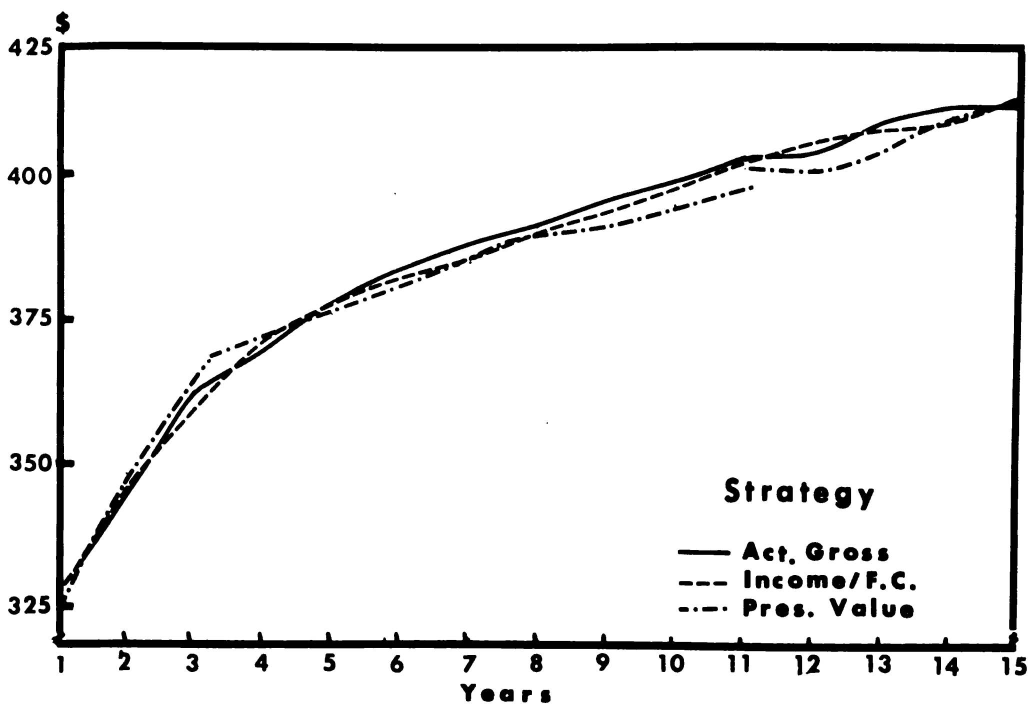

Fig. 1.2 Average income over feed cost per cow.

Secondly, though the genetic milk value may be highest at the end of 15 years when culling on M.E. milk as demonstrated in Table 4 and Figures 2.1 and 2.2, the profit may be little different than that realized by culling on actual milk, actual gross, or actual income over feed cost. Even in the 15th year when the difference in genetic value was the greatest, the differences in returns over feed cost were minor, due in part to the presence of young animals in the herd which did not produce at their mature value.

Thirdly, strategies which rank higher in milk production also tend to rank lower in fat and vice versa. Average fat production per cow listed by years and strategies in Table 5 was different ($P < .01$) among strategies though the magnitude of the differences are not great. The strategy with the highest fat production was actual gross with 505.6 lb while the lowest was with M.E. milk at 500.8 lb.

Fourthly, the time of culling within the lactation is similar among strategies, which subsequently affects total herd production. An example may show why we obtain similar milk production and income over feed cost under the two culling strategies of either M.E. milk or actual milk. If we compare the records of two cows, one a three year old, the other seven years old, and the consequences of culling one of them under the two strategies, we can see how the advantages of each strategy balance each other. From Table 6

TABLE 4. Average genetic milk value per cow (lb)[d]

	Strategies					
Year	M.E. Milk	M.E. Gross	Actual Milk	Actual Gross	Actual Inc. Over Feed Cost	Present Value
1	13,988	14,029	14,028	13,991	13,985	14,000
2	14,075	14,087	14,099	14,066	14,025	14,069
3	14,199	14,192	14,197	14,148	14,105	14,166
4	14,400	14,406	14,394	14,303	14,264	14,376
5	14,638	14,583	14,558	14,477	14,404	14,558
6	14,845	14,756	14,745	14,646	14,568	14,740
7	15,055	14,938	14,911	14,802	14,735	14,892
8	15,237	15,104	15,086	14,957	14,900	15,047
9	15,402	15,254	15,227	15,110	15,029	15,219
10	15,581	15,393	15,397	15,250	15,177	15,382
11	15,753	15,559	15,561	15,392	15,336	15,526
12	15,923	15,700	15,714	15,524	15,469	15,662
13	16,078	15,860	15,882	15,672	15,617	15,804
14	16,231	15,989	16,022	15,810	15,751	15,941
15	16,376	16,136	16,139	15,963	15,888	16,085
Ave.	15,188[a]	15,065[b]	15,064[b]	14,941[c]	14,884[c]	15,031[b]

[abc]Values with the same superscript indicate no significant difference ($P > .01$).

[d]Computed as average genetic value of the cows remaining in the herd at the end of each year.

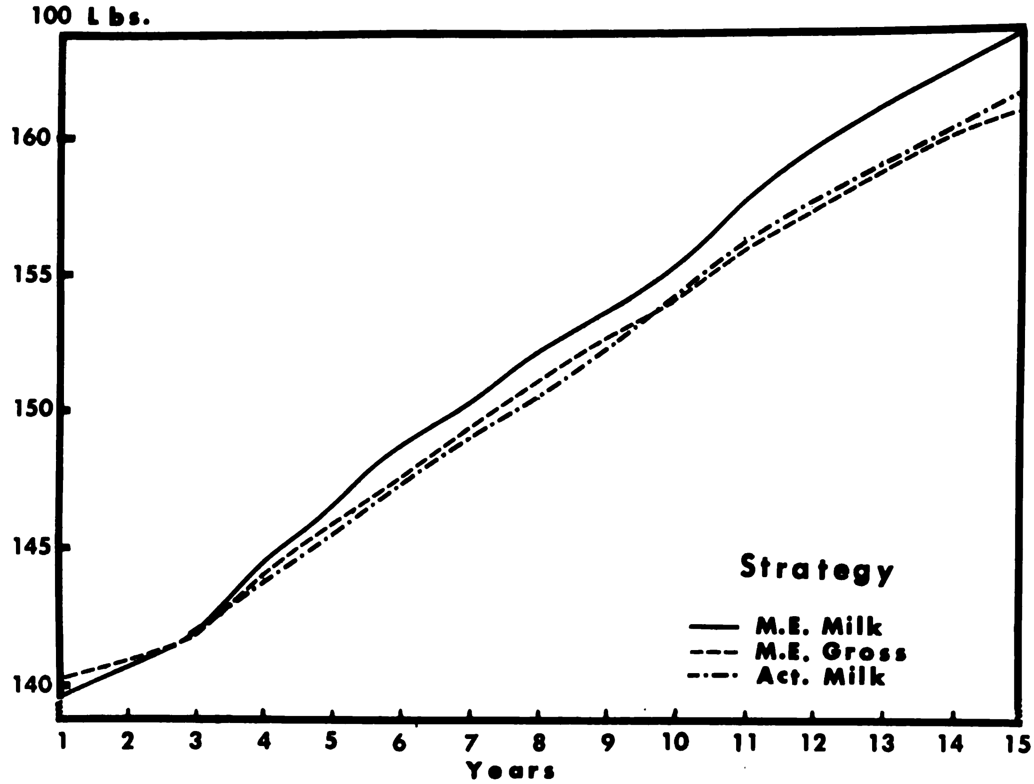

Fig. 2.1 Average genetic milk value per cow.

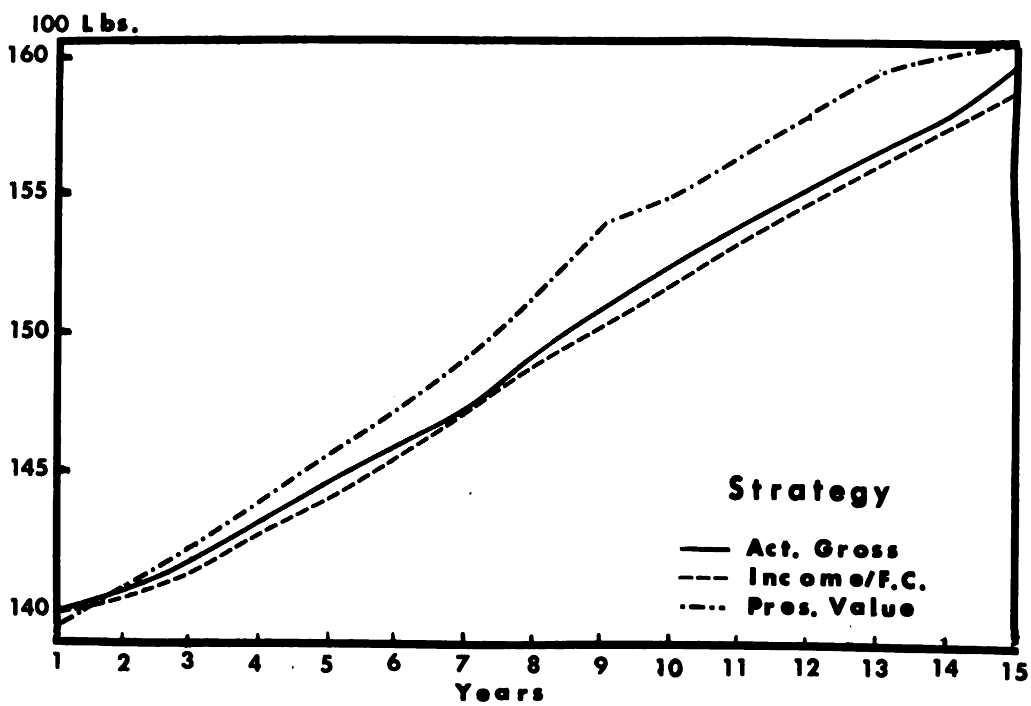

Fig. 2.2 Average genetic milk value per cow.

TABLE 5. Average fat production per cow (lb)

Year	Strategies: M.E. Milk	M.E. Gross	Actual Milk	Actual Gross	Actual Inc. Over Feed Cost	Present Value
1	450	449	448	449	449	448
2	466	471	465	467	468	471
3	486	489	486	487	482	486
4	491	492	492	492	495	495
5	496	497	498	502	500	498
6	499	503	505	506	505	503
7	503	505	508	509	507	506
8	507	506	510	512	512	513
9	509	512	511	517	514	509
10	513	511	514	517	517	514
11	515	515	517	523	519	519
12	516	519	517	521	524	518
13	517	521	517	526	526	523
14	518	520	523	529	525	525
15	523	523	525	528	530	527
Ave.	500.8	502.2	502.6	505.6	504.8	503.6
Ave. Test	3.49%	3.53%	3.50%	3.54%	3.54%	3.52%
Ave. Test Yr. 15	3.35%	3.40%	3.37%	3.43%	3.44%	3.41%

TABLE 6. Comparison of production records of two strategies

	Production Record	
Cow	Actual 305-day	M.E.
3 yr. old	12,000 lb	14,160 lb
7 yr. old	13,000 lb	13,000 lb

are listed the predicted 305-day actual and adjusted age corrected M.E. production of these two cows computed from say, a 60 or 90 day record. The 3 year old at 12,000 lb would have an M.E. record of 14,160 while the 7 year old at 13,000 lb actual would have an M.E. of 13,000 lb milk.

A comparison of the results of culling under the two strategies is presented in Table 7. Assuming we want to cull one of these cows, under actual milk the 3 year old would be culled, while under the M.E. milk strategy, the 7 year old would be the one culled. Yet either cow would be culled at the most profitable time to cull within their respective lactations. Under a given price structure then, each would be culled at the approximate same level of production although at different points in their lactations. If their total production to date before culling was say, 8,000 lb milk, then the remaining cow would produce at her predicted actual 305-day production; thus when culling is based on actual milk production, total production for the

TABLE 7. Results of two alternative strategies

Cow	Culling Strategy	
	Actual Milk	M.E. Milk
3 year old	8,000 lb	12,000 lb
7 year old	13,000 lb	8,000 lb
Total for year	21,000 lb	20,000 lb

two cows would yield 8,000 lb for the 3 year old plus 13,000 lb for the 7 year old or 21,000 lb milk, while under the M.E. strategy, production would total 20,000 lb. However, we are sacrificing some future producing ability and some genetic value by removing the 3 year old.

Average milk production per cow was affected ($P < .01$) by the strategy used. No interaction between strategies and prices was significant from the factorial design. Since the means were significantly different, orthogonal contrasts were utilized to test the more logical comparisons. They are listed in Table 8.

TABLE 8. Comparison of mean milk production among strategies

Strategy	Mean Milk	Orthogonal Contrasts
1. M.E. milk	14,357 lb	5 vs 1,2,3,4,6**
2. M.E. gross income	14,266	6 vs 1,2,3,4
3. Actual milk	14,374	1,2 vs 3,4
4. Actual gross income	14,301	1 vs 2**
5. Income over F.C.	14,247	3 vs 4**
6. Present value	14,288	

**Significant ($P < .01$).

The strategy based on income over feed cost was lower ($P < .01$) in average milk production than the average of the other five; M.E. milk at 14,357 lb was higher ($P < .01$) than M.E. gross at 14,266 lb milk per cow while the culling strategy based on actual milk (14,374 lb) was higher ($P < .01$) than the one based on actual gross with 14,301 lb. From Table 9 and Figures 3.1 and 3.2, average milk production under the strategy of actual milk was higher than under M.E. milk up through the 10th year when M.E. strategy gained a slight advantage.

Average gross income per cow per year for the 15 year span did not differ significantly by strategies. These ranged from a high of \$682.72 under the strategy of actual gross to a low of \$679.52 per cow under M.E. gross.

TABLE 9. Average milk production per cow (lb)

	Strategies					
Year	M.E. Milk	M.E. Gross	Actual Milk	Actual Gross	Actual Inc. Over Feed Cost	Present Value
1	12,441	12,417	12,373	12,403	12,403	12,354
2	12,861	13,026	12,876	12,894	12,926	12,981
3	13,441	13,495	13,458	13,442	13,328	13,463
4	13,659	13,693	13,726	13,653	13,704	13,739
5	13,915	13,878	13,950	13,997	13,910	13,898
6	14,113	14,088	14,264	14,130	14,058	14,079
7	14,316	14,248	14,408	14,314	14,206	14,255
8	14,519	14,370	14,552	14,461	14,388	14,498
9	14,649	14,603	14,674	14,653	14,528	14,526
10	14,882	14,636	14,846	14,746	14,676	14,724
11	15,043	14,809	15,046	14,937	14,806	14,917
12	15,163	15,001	15,125	14,995	15,023	14,958
13	15,303	15,143	15,231	15,205	15,140	15,197
14	15,432	15,201	15,503	15,325	15,221	15,299
15	15,625	15,381	15,587	15,392	15,386	15,464
Ave.	14,357	14,266	14,374	14,301	14,247	14,288

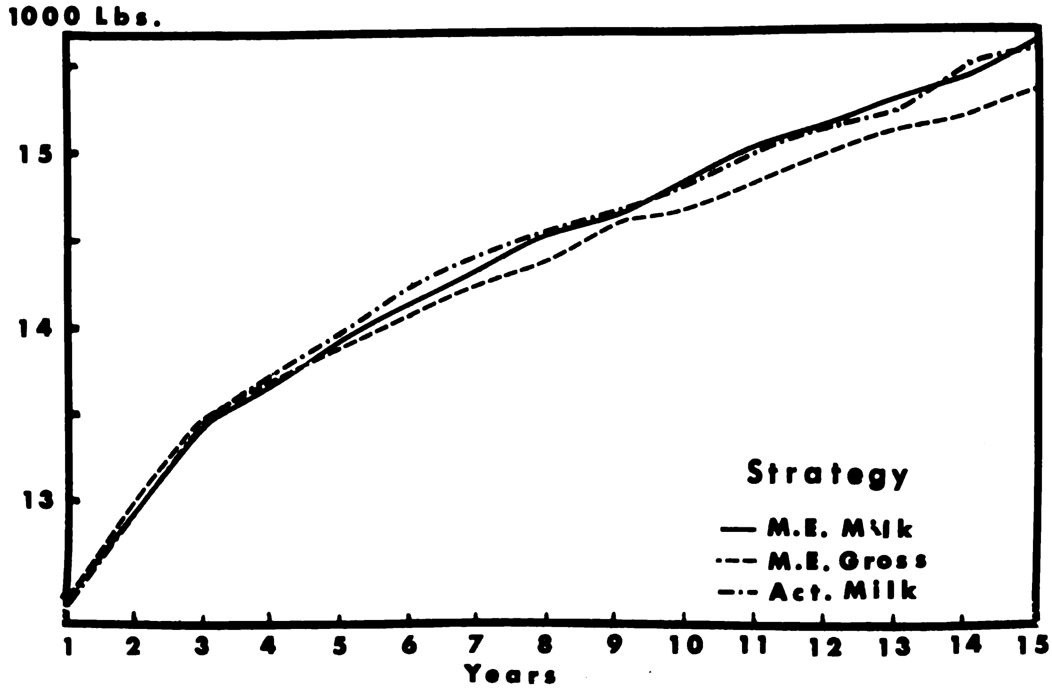

Fig. 3.1 Average milk production per cow.

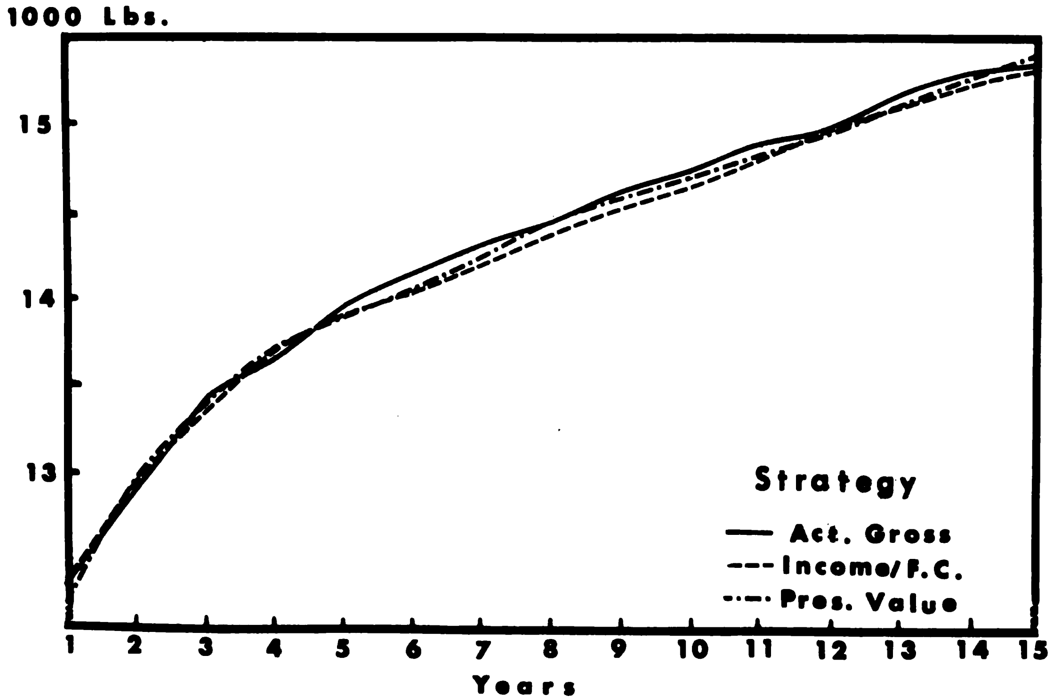

Fig. 3.2 Average milk production per cow.

The mean percentage of voluntary culls also varied ($P < .01$) by strategies. The mean rate of culling computed as the percentage of the herd at the beginning of the year which were subsequently removed voluntarily was 29.7%. The difference in percentage of voluntary culls among strategies is listed in Table 10. The culling rate of 30.93% under the strategy of M.E. gross was significantly higher than those under actual milk (28.66%) or income over feed cost (28.98%) while M.E. milk strategy at 30.58% voluntary culls was significantly higher than actual milk.

TABLE 10. Percentage of voluntary culls

Strategy	Culling Rate (%)	
M.E. milk	30.58^{ac}	ab
M.E. gross	30.93^{a}	a
Actual milk	28.66^{b}	c
Actual gross	29.43^{ab}	bc
Income over F.C.	28.98^{bc}	c
Present value	29.61^{ab}	abc
Level of sig.	.01	.05

[abc]Values with the same superscript indicate no significant difference.

Since all strategies culled on the basis of available replacements, the only explanation for these differences is that under the strategy of actual milk, actual gross income and income over feed cost, there would tend to be a higher proportion of the younger cows culled voluntarily than for strategies based on some mature equivalent factor. But these younger cows have a lower involuntary removal rate; thus the herd will tend to average a higher age under culling on actual records because a higher proportion of younger cows will be culled voluntarily. Consequently, more cows will be culled voluntarily under the M.E. strategies than under the actual strategies. The culling rate, however, in any herd will depend on both the available replacements and rate of involuntary culls.

Though the base herd size was set at 80 cows and the approximate size of the herd at the end of the year was 80, the average herd size as defined by total cow months divided by 12 resulted in an average herd size of 98.8 for all replications. There was a difference ($P < .01$) among strategies in herd size as listed in Table 11. Both the culling strategies based on M.E. milk (99.453) and the one based on M.E. gross income (99.447) had larger ($P < .05$) average herd size than any of the other strategies.

TABLE 11. Average herd size by strategies

Strategy	Average Herd Size
M.E. milk	99.453[a]
M.E. gross	99.447[a]
Actual milk	98.397[b]
Actual gross	98.597[b]
Income over F.C.	98.359[b]
Present value	98.562[b]

[ab]Values with the same superscript indicate no significant difference ($P > .05$)

Mean age of voluntary culls was not recorded but we could assume a herd culled on the basis of actual milk production would lose a higher proportion of young cows than one culled on some basis of mature equivalent. Both herd size and culling rate are thus affected by this age relationship. One would wonder why culling rate is higher under the M.E. strategies yet herd size is larger. Since younger cows have a somewhat lower lactation curve, when culling at the most profitable month, the younger cows would on the average be culled earlier in their lactation than older cows, thus reducing the number of cow months and subsequently herd size. From the factorial design, however, in analysis of average month culled there is a four way interaction of strategies with milk prices, operational costs, and feed prices.

The assumed discount rate of 6% compounded and applied to the income over feed cost (including salvage) indicates a dairyman can either earn that much interest with the money he possesses or he can borrow at that rate to expand his business. The 15 year planning horizon does not imply the farmer will disperse his herd at the end of the 15 years or that potential income of year 16 is sacrificed to maximize early revenue, but it simply means truncating at the end of 15 years. Yet a farmer planning to disperse his herd, say in one or two years, may wish to discount at a different rate or he may even choose to cull his less profitable cows on a different strategy. Conversely, beyond 15 years, planning and goals lead to more uncertainty; thus different planning horizons may indicate various discount rates.

Farmers whose herds deviate markedly from the involuntary losses of either cows or young stock assumed in this study may find comparison of strategies different than these results indicate. For instance, a dairyman who has high death losses among his calves and consequently has fewer replacements, would have a higher percentage of mature cows than a farmer with few calf losses. Since the strategies based on M.E. production or income tend to cause a higher proportion of older cows culled than under actual strategies, this difference could cause a change in profit among strategies.

Simple strategies such as those based on actual milk, or actual gross income should appeal to most dairymen in that they are easy to use and understand and need no special adjustment factors. Such strategies he can apply from simplified DHIA reports or simply from milk weight records. If dairymen know they would not sacrifice profit by using such strategies, they may wish to employ these simpler culling policies. Importantly as well, errors in age adjustment factors which admittedly differ from herd to herd and region to region are eliminated when using actual records.

Effect of Prices

Average milk income over feed cost over the total of 15 years was analyzed in a complete factorial design. Results show there was a three way interaction ($P < .01$) of milk price, feed, and operational costs.

Average milk production per cow under low milk prices was 14,392 lb while that of high milk price was 14,220 lb per cow. The difference was significant ($P < .01$) and apparently existed because potential culls are removed sooner in the lactation under low than under high milk prices. Consequently, cows will be producing at a relatively higher point in the lactation curve under a low milk price thus contributing to a higher average herd production.

TABLE 12. Average milk production by levels of prices

Level	Prices: Milk Price	Fat Differential	Feed Costs	Operational Costs
Low	14,392 lb	14,302 lb	14,275 lb	14,281 lb
High	14,220 lb	14,310 lb	14,337 lb	14,331 lb
Difference	172 lb	8 lb	62 lb	50 lb

**Significant (P< .01).

As shown in Table 12, average milk production under low feed cost was 14,275 lb while under high feed costs the average was 14,337 lb. Here higher feed costs would cause a farmer to remove culls earlier in the lactation and thus the higher point in their curve subsequently contributing to higher average herd production since number of cows are defined as cow months divided by 12. The same factors contribute to the difference in average milk production by the two levels of operational costs.

Average fat production per cow was different (P <.01) by milk prices, feed costs, and operational costs but the two levels of differential price were not significantly different. Table 13 shows fat production under the two levels for each of the prices. The reasons for the difference in fat due to different levels of prices are identical to those of milk production per cow.

TABLE 13. Average fat production by levels of prices

Level	Prices			
	Milk Price	Fat Differential	Feed Costs	Operational Costs
Low	506.7 lb	503.0 lb	502.2 lb	502.2 lb
High	499.7 lb	503.4 lb	504.2 lb	504.2 lb
Difference	7.0 lb**	0.4 lb	2.0 lb**	2.0 lb**

**Significant ($P < .01$).

As expected, gross income was affected ($P < .01$) by the level of milk price. It was also affected by feed prices and level of operational costs. Gross milk income according to the levels of prices are given in Table 14.

Higher milk prices resulted in higher gross income for obvious reasons but not in direct proportion to the difference, because of the lower average milk production prevalent under high prices. The lower feed price resulted in an average gross income of $679.56 while under high feed price, gross income averaged $682.45. This difference results from keeping potential culls later in the lactation before removal and thus in a lower portion of their lactation curve, hence lower gross income per cow. Though operational costs are not charged against the cows per se, the higher operational costs result in a higher gross income because of removing culls higher on their lactation curves.

TABLE 14. Average gross milk income by levels of prices

	Prices			
Level	Milk Price	Fat Differential	Feed Costs	Operational Costs
Low	$613.93	$680.60	$679.56	$679.74
High	748.08	681.40	682.45	682.27
Difference	$134.15**	$ 0.81	$ 2.89**	$ 2.53**

**Significant ($P < .01$).

The price of milk also affected ($P< .01$) the percentage of cows culled voluntarily though the difference was small. Under low milk price ($4.25) mean voluntary culling rate was 30.2% while under high prices ($5.25) mean rate was 29.2%. No other main effects or interactions were significant from the complete factorial design.

When to Cull Within the Lactation

To maximize profit from a dairy herd it is important for the farmer to know at which point in the lactation to cull. A potential cull can return a profit if she is removed from the herd before her marginal costs exceed her marginal revenue. Economic theory states that a cow should be retained in the herd until such time as her value of product equals variable costs. In other words, if her milk sales exceed her feed and labor costs and a small amount of livestock expense, the dairyman should keep her. Under conditions of fixed overhead charges along with available space, a dairyman should consider only the variable costs and not the operational costs as computed in this simulation program. With average herd production most potential culls would simply not decline below the point where returns equal variable costs prior to a time when it becomes more profitable to keep her for the next lactation.

Under conditions where heifers are available to replace the cull cows, the most profitable point to cull within the lactation involves opportunity cost. Though this simulation program did not compute opportunity cost per se, when heifers are immediately available for replacement as implied in the model, the principle of opportunity cost becomes the objective. By removing the potential culls at the point where returns equal total costs and applying the feed and labor to their replacement, opportunity costs are minimized.

Operation in the long run requires that all costs be covered. If a dairy herd is to achieve maximum profit over time, cows must be removed at the point where returns equal total cost. Therefore, each potential cull must cover both her feed and operational costs.

Results of this study show the most profitable point to remove voluntary culls was affected by the level of milk prices, feed and operational costs, and by the level of milk production of each respective cow. A complete factorial design analyzing the average month of voluntary culling showed a four way interaction between the strategy used, the price of milk, feed costs and operational costs. No significant difference in month of cull resulted from a change in the level of fat differential.

Table 15 presents the break even point in terms of pounds of milk under selected levels and combinations of these prices applied to a cow weighing 1472 lb with a 3.6% test. For example, under conditions of high milk and differential prices, along with low feed and operational costs, the break even point was only 703 lb of milk per month or approximately 23 lb per day. Milk income for this marginal month would total $37.47. With operational costs of $21.92 subtracted from this total, feed costs approximate $15.55 for the month or $2.21/cwt milk. The feeding rates used were those outlined in this study. The average month of voluntary culling under these particular levels of prices was 8.86. On the other hand, with low milk differential prices and high feed and operational costs, the break even point was at 1175 lb milk per month or 38.5 lb per day. Under these latter combinations, cows were removed at an average of 3.32 months. The overall average month of culling in

TABLE 15. When to cull within the lactation

Level	Relevant Prices: Milk $/cwt	Fat Differential	Feed $/T	Operational Costs $/cow/mo
Low	$4.25	$0.07	Grain $60 Hay 20 Silage 7	$21.92
High	$5.25	$0.08	Grain $70 Hay 25 Silage 8	$25.38

Level of Prices: Milk & Diff.	Feed	Oper.	Lb Milk to Break Even[a]: Per Mo.	Per Day	Milk Income	Approx Feed Cost[b]	Feed Cost /cwt Milk	Ave. Mo. Cull[c]
High	Low	Low	703	23.0	$37.47	$15.55	$2.21	8.86
High	High	Low	775	25.4	41.30	19.38	2.50	8.28
High	Low	High	788	25.8	42.00	16.62	2.11	8.18
High	High	High	865	28.4	46.10	20.72	2.40	7.39
Low	Low	Low	935	30.7	40.39	18.47	1.98	6.38
Low	Low	High	1048	34.4	45.27	19.89	1.90	4.12
Low	High	Low	1052	34.5	45.45	25.53	2.43	4.77
Low	High	High	1175	38.5	50.76	25.38	2.16	3.32

[a]Averange month = 30.5 days.

[b]1472 lb cow, 3.6% test fed according to those in study.

[c]Results from simulation program.

this study was 6.5 months. This system carries a cow to the point where marginal cost equals marginal revenue, yet if there is a limitation in space or labor she may be culled prior to this time.

The assumed lactation curves generated in this study were taken from the records of complete lactations. Aulerich (1966), however, found the terminal voluntary culls to possess a steeper decline in their lactation curves than either involuntary culls or cows which completed their lactation. Consequently, under actual herd conditions with similar prices as used in this study, cows may be culled at an earlier point in the lactation than this study would indicate. In theory, the shape of the lactation curve for a given 305 day record would greatly affect the profit that could be realized from a potential cull. For instance, a cow starting at a high level of production with a steep decline would return more profit up to a given production level than one with a lower more persistent curve. Farmers will generally cull according to available replacements but the exact time of the year or point in the lactation for each cow will depend on the most profitable point in the lactation to cull and not necessarily the exact month the replacement heifer freshens. Here we must assume a certain flexibility in herd size.

Implications for Further Research

The results of this study indicate that various factors related to income over feed cost tend to balance each other when comparing the respective strategies. Other variables important to dairymen in relation to culling cows within a herd may be worth studying. If a farmer is in a herd expansion program or if he sells some bred heifers for dairy purposes, certain culling strategies may prove more advantageous than others. Moreover, dairymen who buy all their replacements may find one strategy more profitable. As found in actual herds, relaxation of the assumption that heifers freshen at two years of age and that cows maintain 12 month calving intervals could change the results of this study. Lengthening of either or both of these two would cause a slower rate of herd turn over and consequently less genetic progress. This in turn may affect the relative advantage of say, M.E. gross vs actual gross strategies. Cows in a 13 month calving interval would return a lower income per year on the average than those in a 12 month interval. Likewise, with fewer replacements available due to these two changes, culling rate would be reduced which would subsequently reduce the income. These factors may affect the advantage of a given strategy.

This study did not determine if one should enforce one strategy for a few years, then use another the latter half of the 15 year period to maximize the discounted income for the 15 years.

Unusual price relationships which deviate markedly from those tested may indicate a difference in strategies. For instance, allowances for seasonal differences in milk production and milk prices may indicate a different type of culling program. Though equating marginal cost with marginal revenue is still a valid criterion for determining when to cull within a lactation, a rapidly changing price structure may complicate the problem. Whereas previous research has demonstrated the additional accuarcy of predicting milk production when basing culling on more than one record of a cow or on records of her relatives, little or no research has examined the effect on income over feed cost.

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APPENDIX

Appendix Table 1. Age correction factors

Age	Factor	Reciprocal
2	1.28	0.781
3	1.18	0.847
4	1.08	0.926
5	1.03	0.971
6	1.01	0.990
7	1.00	1.000
8	1.00	1.000
9	1.01	0.990
10	1.03	0.971
11	1.04	0.962
12	1.05	0.952
13	1.08	0.926
≥ 14	1.12	0.893

Appendix Table 2. Part lactation factors

	< 3 yrs. old			≥ 3 yrs. old		
Mo.	Cum.part to whole	Whole to part cum.	Single mo.	Cum.part to whole	Whole to part cum.	Single mo.
1	8.35	.1198	.1198	7.40	.1351	.1351
2	4.17	.2399	.1201	3.71	.2695	.1344
3	2.83	.3533	.1134	2.54	.3937	.1242
4	2.16	.4630	.1097	1.97	.5076	.1139
5	1.77	.5650	.1020	1.63	.6135	.1059
6	1.51	.6623	.0973	1.40	.7143	.1008
7	1.32	.7576	.0953	1.25	.8000	.0857
8	1.18	.8475	.0899	1.14	.8772	.0772
9	1.08	.9260	.0785	1.06	.9434	.0662
10	....	.....	.0740	....	.....	.0566

Appendix Table 3. Body weights by age of cows

Age	Body Weight
2	1151 lb
3	1277 lb
4	1365 lb
5	1421 lb
6	1454 lb
7	1467 lb
8	1472 lb
9	1472 lb
$\geq$ 10	1474 lb

Appendix Table 4. Feed requirements for dry period

Age	Silage (per mo.)	Hay (per mo.)
2	1200 lb	510 lb
3	1200 lb	558 lb
4	1200 lb	537 lb
5	1200 lb	555 lb
6	1200 lb	567 lb
$\geq$ 7	1200 lb	576 lb

Appendix Table 5. Extra hay added to ration of young cows to meet protein needs

Age	Production (lb)	Extra Hay (for 10 mo.)
2	≥ 9150, < 12,200	120 lb
	≥ 12,200, < 15,250	220 lb
	≥ 15,250	320 lb
3	≥ 12,200, < 15,250	55 lb
	≥ 15,250	175 lb

For lactations less than 10 months

Age	Production (per mo.)	Extra Hay (per mo.)
2	≥ 610, < 915	18 lb
	≥ 915, < 1220	30 lb
	≥ 1220, < 1525	40 lb
	≥ 1525	48 lb
3	≥ 915, < 1220	12 lb
	≥ 1220, < 1525	24 lb
	≥ 1525	30 lb