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thesis entitled

A STATISTICAL MODEL FOR CHARACTERIZING PRICE VARIABILITY WITH APPLICATION TO DAIRY INVESTMENT ANALYSIS

presented by

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has been accepted towards fulfillment of the requirements for

Ph.D. degree in Dairy Science

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Date September 28, 1980

A STATISTICAL MODEL FOR CHARACTERIZING PRICE VARIABILITY WITH APPLICATION TO DAIRY INVESTMENT ANALYSIS

By

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A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Dairy Science

1980

ABSTRACT

A STATISTICAL MODEL FOR CHARACTERIZING
PRICE VARIABILITY WITH APPLICATION TO
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A statistical linear model was developed to estimate variability for price time series data. The quarterly or yearly estimates resulting from the statistical model were used to generate simulated probability distributions for each of the variables considered. A set of five interactive Fortran computer programs was developed to perform the statistical and simulation procedures.

A linear one-way classification model with fixed effects was used to compute quarterly and yearly solutions. The model ($Y_{ij} = A_i + E_{ij}$) is full ranked and unique solutions were computed under the constraint that the overall mean (μ) was zero. The estimable function was $\mu + A_i$, which becomes A_i with μ set equal to zero. Although small sample sizes (one observation per month) did not permit a statistical test for nonhomogeneity of variance between quarters or years, the variances across time periods for the price series were assumed to be different. Covariances between time periods were considered to be non-zero, since the objective was to develop "best estimates" for time series data, error terms were assumed to be correlated across observations. As a result, autocovariances were used to develop the variance-covariance

matrix used in the linear model to calculate quarterly estimates.

A simulation procedure was used to generate probability distributions based on the statistical estimates. A triangular matrix derived from the historical variance-covariance matrix and a random normal deviate generator were used to simulate the price series. The simulation procedure generated a random series of normally distributed and appropriately correlated probability distributions for each variable considered. The procedure assumed that each variable reacts to the changes in the other variable in a way that could be described by the variance-covariance matrix.

The input time series were tested for normally distributed residuals. The results of the Shapiro-Wilk test for normality indicated that for the eleven time series selected, the normality assumption was not always met. The price series were deflated by U.S.D.A. price indices in attempting to normalize these data. While there was some improvement in the normality of the time series when deflated, selection of the appropriate deflator and the interpretation of deflated series were considered major problems.

Statistical tests were also performed to validate the output from the statistical and simulation computer programs. Based on comparisons made between original and simulated sample variances and means the technique appears to generate reasonable probability distributions based on the input time series.

By incorporating probability distributions, such as those generated using the above described procedures for price, income or cost variables into a capital budgeting model, the information generated by the analysis represents a substantial improvement over conventional methods of

incorporating risk into investment decision models. The technique was illustrated with an example. The statistical estimates and simulated probability distributions for two price series were applied to the capital budgeting model.

The example compared the profitability of leasing 100 bred dairy heifers with purchasing the same over a four year period. Michigan cull cow and calf price probability distributions were used in the model. The analysis demonstrated the difference between the net present value that resulted using the expected values for each variable compared to the distribution of net present values using probability analysis. Probaility statements were made based on these results.

A users guide to the computer programs and source program listings were included.

ACKNOWLEDGMENTS

I wish to express my gratitude to those who provided advice and assistance throughout this study and over the course of my graduate education. Special thanks are extended to my major professor, Dr. John A. Speicher, for his continuous encouragement, support and sharing of his knowledge, throughout my graduate experience. I would also like to thank Dr. Ivan L. Mao for his valuable assistance and suggestions during the preparation of this dissertation.

I would like to extend my thanks to Drs. John R. Brake and William G. Bickert for serving on my guidance committee and for their time spent on my behalf, in and out of the classroom.

An appreciative thank you is offered to Dr. Harold D. Hafs for his encouragement and the financial support provided by the Department of Dairy Science during my graduate study.

My deepest appreciation goes to my wife, Cynthia, for her constant support and understanding throughout the course of my graduate education. An a special thanks must go to our son, Ryan, for the added happiness and joy he has brought into our family.

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INTRODUCTION

Price and income instability has been characteristic of American agriculutre. Since the 1930's, price and income variability in agriculture have indirectly been met by various government price, income, trade, resource and inventory programs. However, the price-supporting features of many of these programs has focused more strongly on increasing the level of farm income rather than reducing its variability. As a result, these programs have often stimulated growth in production capacity, thereby compounding problems of supply management and income instability (Barry and Fraser, 1976). The combined effects of investment in larger, more efficient technologies and the expanded production capacity requires high rates of financial growth in order to preserve the economic viability of the farm business.

Producers have little capacity for influencing resource and product prices. The prices paid, prices received and level of productive capacity of a farmer is affected by internal as well as external forces. Comodity price support programs, environmental regulations, trade restrictions, wars, changes in demand and changes in government monetary and fiscal policies, are externalities which have an impact on the farm firm. Adverse weather, pests and disease outbreaks can be disastrous to crop and livestock production. Family health and management ability and continuity are examples of internal sources of uncertainty.

Since farm growth objectives and investment alternatives imply long-term planning horizons for proper economic analysis of all flows, costs, returns and cash, a manager's inability or reluctance to plan over periods of sufficient length can lead to inadequate economic decisions. Current methods designed to aid in the decision making process can be improved upon and new ones need to be developed. The principal objective of this study was to develop a statistically-sound procedure for incorporating yield, price and income variability into a capital budgeting model.

Most dairy farmers had quite favorable returns during the five-year period, 1968 to 1972. Increasing feed costs caused a profit squeeze in 1973, although milk prices were also rising (Knoblauch, 1976; U.S.D.A., 1978). Many dairy farmers experienced negative returns to their labor and management during the period, 1974 to 1977 (Kelsey and Johnson, 1979; U.S.D.A., 1978). The Food and Agriculture Act of 1977 required a milk price support level of not less than 80 percent of parity through March 1979 (then the minimum support level reverted to not less than 75 percent of parity). The act also required that the support level be adjusted semi-annually through March 31, 1981 to reflect changes in the parity index. If Congress does not act again before September 30, 1981, the support level will revert to 75 percent of parity. The difference between 75 and 80 percent parity represents a difference of approximately 75 cents per hundredweight (cwt) of milk. Partially as a result of the higher support prices, the financial conditions of dairy farmers have improved and returns to labor and management were sharply higher in 1978 and 1979 (Kelsey and Johnson, 1979; U.S.D.A., 1978).

While empirical variability estimates are not necessarily identical with the traditional concept of risk or uncertainty, they are objective measures of past variability in income, prices and yields (Carter and Dean, 1960). Knoublauch (1976) showed that the economic environment in which Michigan dairy producers operate has become more variable in terms of product prices and input costs. As yield and price were combined into gross income per acre, more variation existed for all crops during the 1970 through 1974 period than for the previous 10 year period. Base milk prices increased slightly in variability during the 1970 to 1974 period but were the most stable of all farm product prices. Farm input costs also increased in variability with the exception of farm wage rate which remained about the same, while 6-24-24 mixed fertilizer cost variability was less in the 1970 to 1974 period.

Barry and Brake (1) noted that multiperiod planning horizons introduce risk not found, or at least well beyond that found in single period production models. Future coefficient values may be specified as point estimates of expectations or as probability distributions of future values with mean and variance estimates or probability functions which describe the kind of relationships which are not deterministic but more obscure. In the latter case, variables may vary jointly but not in an exact manner, resulting in relationships which are necessarily probabilistic and subject to equation errors.

Revenue and cost information relevant to evaluating capital investment proposals should be expressed in terms of cash flows into and out of the business during its expected economic lifetime. Cash flow analysis aids in evaluating the impact of both alternative investment and financing strategies used in acquiring resources, on liquidity position, income expectations and the present worth of the firm (Barry and Brake, 1971).

To collect and assemble realistic estimates for the key factors which might be expected to impact an investment proposal, means to find out a great deal about them. Hence, the kind of uncertainty that is involved in each estimate can be evaluated ahead of time. Using this knowledge of uncertainty, decision makers can maximize the value of the available information. The value of computer programs in developing clear portrayals of the uncertainty associated with alternative investments has been shown (Hertz, 1964, 1968; Bennett et al., 1970).

To have calculations of the odds on all possible outcomes lends some assurance to the decision-makers that the available information has been used with maximum efficiency. The probabilistic approach has the inherent advantage of simplicity in that it requires only an extension of the input estimates (to the best of our ability) in terms of probabilities. Using this approach, managers can take the various levels of cash flows, present value and other results of a proposed outlay and get estimates of the odds for each potential outcome.

If one assumes that the future variability associated with a particular enterprise or set of enterprises is closely related to past variability, empirical variability estimates should provide a more reasonable basis for making both short and long run management decisions. Instead of quantifying only single point estimates for input and output parameters (as is most commonly done), the uncertainty in these parameters is quantified by computing best estimates of each parameter and then using the associated variance-covariance matrix and a correlation technique similar to that described by Clements et al. (1971) to simulate the outcomes of the correlated events. The resulting simulated distributions provide a measure of risk or uncertainty which could be incorporated into a capital budgeting model and used to generate probability distributions of projected net cash flows. The financial outcome of the combined variables could be calculated each time the set of variables are sampled and the resulting output would simulate the range and distribution of possible outcomes from a proposed investment in terms of the particular "profitability" critereon to be used or tested.

This research effort was designed to develop a method for characterizing the price, yield and income variability facing dairy producers in Michigan. The principal objective was to design a statistical model to describe the variability associated with price variables as well as the covariances between variables over time.

The statistical estimates, based on the input time series are used as input to a simulation program. The simulation procedure generates a probability density function for each variable which has the variance and covariance characteristics of the original input time series. The output from these procedures is examined statistically in attempting to examine the significance of improvements made over existing methodologies.

The generated density functions are then incorporated into a capital budgeting example in which leasing of dairy cattle is compared with purchasing. This approach provides the decision-maker with an estimate of the effect of price variability on the profitability of alternative investments. This approach to investment analysis offers the decision-maker a wealth of information that represents a substantial improvement over more conventional attempts at incorporating risk into investment analysis.

LITERATURE REVIEW

Knight (1931) defined risk as a condition where the probabilities associated with outcomes are known (measurable). This contrasts with uncertainty where probabilities associated with outcomes are not known (not measurable). Johnson (1957) stated that risk exists when prescriptive knowledge is sufficient to make a decision (decision mode). Uncertainty, on the other hand, exists when specifications for prescriptive knowledge are not met (learning mode). Johnson described a decision-maker with sufficient knowledge to derive a frequency probability distribution as being in a risk-knowledge situation. With only partial knowledge he is in a subjective risk-knowledge situation. While recognizing the above distinctions made between risk and uncertainty, the two terms will be used interchangeably in this thesis to describe actions with more than one possible outcome where the likelihood of all possible outcomes is described by a probability distribution or probability density function.

Heady _et al_. (1954) approached economic instability and crop production choices involving risk using a mathematical approach which involved measuring variances and correlation coefficients of income from various enterprises. The mathematical formulae derived stated that the variance for the whole farm is the sum of the variances of the enterprises plus the covariances of the enterprises. This formula was then expanded to account for the proportion of the enterprises making up the total farm. They found that when the number of enterprises combined was three or greater, the formula became complicated making variability calculations difficult.

Carter and Dean (1960) attempted to provide a more objective measurement of the uncertainty or variability associated with various crops and cropping systems in California. Three types of crop variability were considered in their study; price variability, yield variability and income variability which arises from the interaction of product yield per acre and product prices relative to costs. Bartlett's test was used to test the homogeneity of the variance over time. Where variance was not homogeneous the variance based on the most recent (5 year) period was taken as the best estimate of future variance. Little year-to-year correlation between price and yield was evident for California field crops, and yield variability for field crops was relatively low. Therefore the most important factor contributing to gross income variability of field crops was felt to be the variability of prices. Correlation between the "random elements" of pairs of net income time series were estimated using the variate difference method, and were typically lower than those of the original series. The actual net incomes of crops tend to be highly correlated because the major economic influences (inflation, price cycles, wars, level of technology, etc.) affect most enterprises similarly. Their data appeared to support the hypothesis that considerable individual farm yield variability is "averaged out" when state or even county series are used.

Greve *et al*. (1960) analyzed the variabiltiy of production, price and gross returns of wheat, grain sorghum, steer and cow-calf enterprises in Northwestern Oklahoma. A summary of the estimated coefficients of variation for the specified enterprises indicated that grain sorghum was more variable than wheat in production, price, gross income and returns. The cow-calf system, ignoring inventory changes, is the more stable of the two livestock enterprises. With inventory changes included, the cow-calf enterprise shows the greatest relative degree of variation in gross income and returns using actual and deflated prices. The authors pointed out that the pattern, or sequence of favorable and unfavorable years, may be at least as critical as the degree of variation over years. They used the Wallis and Moore non-parametric test to check for the randomnesss of a time series. Each of the three tests for bunchiness (runs of specified duration, 4-year moving averages and non-parametric statistical test) suggest the presence of cycles or "bunches" in each of the series of data tested. Cow-calf enterprise data tended to bunch near the mean with a low (4 percent) coefficient of variation indicating the relative stability of cow-calf production. The small amount of correlation between wheat returns and the returns from each of the other chief farm enterprises implied a stabilizing effect if enterprises were combined.

Day (1965) studied the possible asymmetry or skewness of field crop yield probability distributions. The contrast among the estimated distributions suggests that decisions for maximizing profit and minimizing risk must be based not only on expected yields and variances but upon skewness as well. In the case of cotton and corn, nitrogen application not only increased average yields but concurrently reduced positive skewness. However, in the case of oats, negative skewness was increased by nitrogen application meaning that at higher nitrogen levels above average yields can be expected more than half of the time. These results suggest that a model of a single firm or a small relatively homogenous region that uses average yields at low nutrient levels will overpredict yields more often than under predict them and vice versa at high levels. The author suggests that the mode or median may be preferred for prediction or forecasting purposes. If the objective were to maximize the probability of coming close to the observed values the mode should be used. If the objective were to have nearly equal chance of over or underpredicting, the median should be used.

Luttrell and Gilbert (1976) analyzed average yields for a number of major crops in the nation and the leading producing states. These data provided little evidence that yields were either cyclical or bunchy as a result of weather. However, there was some evidence of positive autocorrelation in yields since 1933. Such results appeared to reflect the uneven rate of application of high yield producing inputs rather than a non-random influence of weather. No attempt was made to anlyze yield patterns in areas smaller than states. The Durbin-Watson statistic was used to test for buchiness of regression analysis in terms of first-order positive autocorrelation. The Wallis-Moore test (nonparametric test of randomness) was used to test the randomness of yields since it is not limited by the assumption of normally distributed observations.

The empirical findings of Black and Thompson (1977) were consistent with the existence of drought cycles for corn, soybeans and wheat yields. However, not every year within a drought period exhibited below average yields and vice versa. There was no evidence for a two-year cycle.

Barry and Fraser (1976) evaluated the feasibility and structural implications of relevant risk responses available to producing firms that differ by size and type. A table of annual coefficients of variation for monthly prices of selected commodities over 1959 to 1974 period was presented. The coefficients reflected irregular influences on prices, since the data were modified to account for estimates of seasonal price patterns in the respective commodities. The increasing effects of irregular influences on intrayear price variation are clearly evident for crops and to a lesser degree for livestock. Coefficients of variation for monthly prices of wheat, corn and soybeans and sorghum increased greatly in 1973 and 1974.

Markowitz (1959) argued that if investors were faced with two investment alternatives with the same levels of expected return but with different variance, they would prefer the investment with the smaller variance. Thus, the choice set would solve for investment alternatives which minimized variance at each level of expected wealth. Markowitz referred to this resulting choice set as the expected value-variance (EV) efficient set and used quadratic programming to derive the EV set, which for a given level of expected wealth minimizes the variance.

Webster and Kennedy (1975) estimated sets of indifference curves for five farmers who were willing to forego expected income for increases in a probabilistically defined minimum income. Three of the five farmers showed an increasing marginal utility for less variability in incomes. The other two farmers showed increasing followed by decreasing marginal utility for variability of income. The information obtained was to be used for predictive rather than normative purposes.

Aanderund (1966) estimated the income variability of enterprise combinations for Northwest Oklahoma farm and ranch situations. The study was designed to determine the probably effect on capital accumulation and survival of farm operators using alternative farm plans.

The results were presented in an income opportunity framework allowing the returns and variability estimates of alternative farm organizations to be examined with the farmer deciding on the level and variability of income which meets his preferences.

The objective of a study by Bostwick (1963) was to determine prior wheat yield variability and construct yield probability functions designed to be of assistance to managers in making planning decisions. Probability functions were derived from wheat yield observations (n=5000) over a thirty-five year period using an extreme value statistical distribution approach.

Johnson _et al_. (1967) incorporated a distribution of crop yields in a deterministic linear programming model to demonstrate the impact of random variations in yield on firm growth. Solutions to the stochastic linear programming problem were approximated for three assumed farm situations and compared with nonstochastic solutions in which means of the probability distributions for crop yields were used. In each of the three farm situations the nonstochastic solution was higher than the estimated mean for the stochastic solution. It was pointed out that this was due in part to interactions among household consumption withdrawals, the investment policy and yield variability.

Boehlje and White (1969) applied a multi-period linear programming model to a hypothetical farm firm over a ten-year planning horizon. One of the two objective functions maximized the present value of the annual disposable income, and the other maximized the net worth of the firm at the end of the planning horizon. Technical coefficients and input and product prices were assumed to be constant throughout the 10 years since their principal objectives were to analyze the impact of resource availability and different optimizing criterea on farm firm growth. The empirical results for the corn-hog farm situation indicated that in all cases the farm specializes in the production of the most profitable enterprise, which with the assumed technical and price coefficients was the production of hogs. The major limitation of the multiperiod linear program was felt to be the difficulty of incorporating elements of risk and uncertainty in the model.

How and Hazell (1968) used quadratic programming to incorporate income variance into a linear programming model. While the combination of farm enterprises obtained by the solution of a linear programming model may provide the maximum expected income under given constraints, the year to year degree of uncertainty from one year to the next might not be acceptable to a farmer. The quadratic programming procedure results in a set of solutions for each of which the variances of expected income is at a minimum for the given expected mean income level, with income level varying over the whole feasible range. This is accomplished by setting the linear programming objective function as a constraint to be varied parametrically and substituting the variance-covariance matrix as the objective function to be minimized. The resulting EV efficient pairs trace out a pattern of increasing expected mean income associated with increasing income variance.

Scott and Baker (1972) used a quadratic programming model to generate income variabilities. Their model incorporated income variance and covariance of possible enterprise combinations. The quadratic programming model is the same as a linear programming model except that the quadratic programming model contained a risk aversion coefficient which could be varied. By varying the risk aversion coefficient, points on the efficient frontier were generated. An efficient frontier allows the decision maker to choose the level and variability of income which meets his preferences. Since there has been little if any success in quantifying the correspondence between a risk aversion coefficient and a decision maker's utility function, risk aversion coefficients have had little empirical use.

Hazell (1971) developed an approach which minimizes total absolute deviations rather than variance, which may be solved using a linear programming algorithm and which gives results remarkably similar to those of quadratic programming. The model was solved to determine the set of production activities that minimizes variance of net returns subject to receiving a specified level of income or to develop the efficiency frontier showing trade-offs between expected income and variance.

Thompson and Hazell (1972) later showed that a linear programming approximation using the mean absolute deviation (MAD) could be used to derive efficient EV farm plans when a suitable quadratic programming routine was not available. A Monte Carlo simulation study demonstrated that the MAD estimator of variance was only marginally less efficient than that of the sample variance in ranking equal-income plans by their variance. The percentage of additional loss in utility incurred by employing the MAD exceeded 10 percent only in cases of high correlation and/or large sample sizes.

Lin et al. (1974) compared utility maximization with profit maximization as predictors of farmer behavior. The EV (expectation, variance) frontier for each farm shows a set of alternative production plans, each providing minimum net income variance for specified levels of expected net income. Empirically, the EV frontier from each farm was efficiently derived using a quadratic programming model. The decision maker's subjective probability distributions of price and yields were incorporated, insofar as possible, in the estimation of the expected net returns and the variance-covariance matrix of net returns. While the means and variances of net income for individual crops on each farm were estimated subjectively, it proved impossible to obtain subjective estimates of covariances. They also found that combining historical covariances with subjective variances let to inconsistencies revealed by a variance-covariance matrix that was not positive semi-definite. Thus, to generate estimates of covariances, time series of net incomes for each crop were reconstructed by expressing the historical trend-corrected net incomes for each crop in terms of standard normal deviates about the mean. Then, substituting the standard deviations derived from the subjective net income distributions, calculation of the variance-covariance matrix from this reconstructed set of time series data preserved the subjective net income variances, incorporated the historical relationships among crops and guaranteed a positive semi-definite matrix.

Barry and Willman (1976) used a multiperiod, quadratic linear program to model a firm's growth environment and to evaluate the effects of external credit rationing on optimal levels of foward contracting. The model's basic structure resembled that of other growth models (Barry and Baker, 1971; Hazell and How, 1968) in terms of multiperiod linear programming with the addition of risk information expressed as variances and covariances on selected activities in each period. The program derives an efficient EV solution, called a "growth plan", that minimizes variance for a given net present value of income. The model was designed to provide the decision maker with a set of a priori growth plans that organize the business to accept alternative combinations of risk and returns valued over the planning period. The decision maker then chooses the plan that best satisfies his preferences toward risk and returns.

Robison and Brake (1979) define portfolio theory as "an efficiency critereon that indentifies a set of investment plans that minimize variance (maximize expected returns) for given levels of expected wealth (variance) from which well-defined classes of decision makers can find their expected utility-maximizing solution". This set of investment plans, often referred to as the expected value-variance (EV) set, is efficient because it restricts the search for preferred solutions to those EV efficient plans. As a financial model, the assumptions and restrictions of portfolio theory seem acceptable: production is linear, asset choices are mostly divisible and the variance is on the price side. But as a farm planning tool, portfolio models seem less useful because production is not linear, asset choices are seldom completely divisible and variance on the output side is at least as important as variance of the price side. Still as an empirical tool, it represents an improvement over previously popular linear programming models (Brake and Robison, 1979).

Robison and Barry (1977) used EV analysis to model farm plans by limiting resources with a set of linear constraints and estimated the variances and covariances of returns between production alternatives. This application of portfolio analysis (referred to as risk programming) has been useful in applied decision making problems when the choice set was not predetermined. It has been shown that only if the probability distribution of outcomes for each alternative is normally distributed or if the investors possess quadratic utility functions can we be sure that the EV set will include the expected utility maximizing choice (Barry and Fraser, 1976). However, unless the probability distributions of outcomes are highly skewed, the expected utility maximizing choice will be very close to at least one alternative included in the EV set.

Halter and Dean (1965) used an empirical model to simulate both the uncertain environment facing management and management's decision made in response to that uncertain environment. The model simulated range conditions and price relationships for a 40 year period. Simulation was then used to evaluate the desirability of one specific change in management policy: namely, consideration of alternative price expectations models to be used in making the critical May-June decisions on buying cattle directly for the feedlot. The authors concluded from the results obtained from this study that it was difficult to make marked improvements in either level of income or in reduction of income variability by adjusting the buying decisions since range and price conditions are essentially exogenous to the farm. They further state that management may be forced to accept wide variability as unavoidable and must therefore investigate ways to improve the technical efficiency of the firm (i.e. better feed conversion, faster gains and lower operating costs).

Zusman and Amaid (1965) evaluated the performance characteristics of various decision rules by simulating a beef cattle enterprise over a period of 16 years with weather events constituting the main stochastic input. Optimal decision rules were defined in terms of the net present value and the coefficient of variations of the income flows. Plotting the coefficient of variation against the present value of the income stream showed that it was impossible to increase the present value of annual income without simultaneously raising the coefficient of variation and vice versa.

Patrick and Eisguber (1968) developed a simulation model of farm firm behavior with managerial ability of the operator and farm capital structure as the controlled variables. Managerial ability of the farm operator was expressed in terms of the technical transformation rates (i.e. yield per acre, etc.). Capital structure was divided into three parts: interest rate, long-term loan limits and intermediate-term loan limit. Managerial ability of the farm operator was the major factor, among those considered, determining the rate of growth of the farm firm. Improvement of the technical rates of transformation by 10 percent increased the farmer's net worth about $2,000 per year or about 25 percent at the end of the 20-year period.

Blackie and Dent (1976) incorporated risk and uncertainty into simulation model predictions only to the extent that individual managers defined their price and cost expectations. Although it is possible to select prices and costs stochastically from an appropriate distribution the authors believed that these investigations would be of greater general interest if these effects were not included.

Sadan (1970) describes the opportunities facing the investment decision maker in a farm firm under risk in terms of "efficiency frontiers", the dimensions of which are the expected present value and the variance of the farm's future net returns. A simulation model was developed to trace possible efficiency frontiers for an actual Israel kibbutz. The outcome of each "simulation experiment" included the unit's net worth, its consumption allowance and the corresponding variances or coefficients of variation. The point of operation for the actual farm was located on the simulated efficiency frontier and the observed safety margin (defined as the ratio of the internal rate of return on farm investments, divided by the respective external rate, imputed as the sum of the rate of interest and the depreciation rate) was interpreted as a measure of risk aversion. An arc estimate of the slope of the frontier at the point of operation indicated a marginal rate of substitution of 55 cents per unit variance at that point. It was suggested that this and similar models be used to stimulate the possible effects of alternative policies of the lending agencies upon the expected value and variance of the firm's net returns.

Hertz (1968) described a computer simulation model used to assess the variability associated with various investment policies and alternatives. The technique requires three basic steps: 1) estimating the range of values for each of the factors and, within that range the likelihood of occurence of each value, 2) randomly selecting from a distribution of values for each factor one particular value to be combined with one for each other factor to compute the net present value or rate of return from that combination, and 3) repeating step 2) over and over again to define and evaluate the odds of the occurence of each possible outcome through simulation.

Clements _et al_. (1971) developed a procedure for correlating events in farm firm simulation models. Simulation models frequently incorporate Monte Carlo procedures to represent uncertainty. The Monte Carlo applications typically assume that the correlation between any two events is either non-existent (zero) or perfect (one). This assumption does not realistically represent the covariance between related events and may even introduce artificial and unrealistic variability into an analysis. The procedure developed by Clements (1971) can be incorporated in Monte Carlo simulation models to correlate events at any desired level from minus one to plus one. In general, the procedure involves defining an upper triangular matrix of coefficients, calculating the numerical values of these coefficients using the variance-covariance matrix and combining the estimated coefficients with a series of random normal deviates to generate the correlated outcomes.

Hertz (1964) points out that the controversy and furor associated with the development of ways to improve our ability to discriminate among investment alternatives has largely been resolved in favor of the discounted cash flow method. As these techniques have progressed, the mathematics involved has become more and more precise so that we can now calculate discounted cash flows to the penny or rates of return to a fraction of a percent. However, behind these precise calculations are data which are not that precise. There is something more the decision maker ought to know in addition to the expected net present value or expected rate of return.

Hertz (1968) found in general that risk-based policies consistently gave better results than those using single-point, deterministic decision rules. The program also allowed management to ascertain the sensitivity of the results to each or all of the input factors. Simple by running the program with changes in the distribution of an input factor, it is possible to determine the effect of added or changed information (or the lack of information).

Bennet et al. (1970) developed methods for the financial evaluation of mineral deposits. The objective was to select the best alternative mining and processing methods and production rates. This was done by performing a financial evaluation through computer simulation using probablistic risk analysis and sensitivity analysis. The probabilistic analysis approach, which was based on the premise that the expression of a range and/or distributional characteristics of variables in establishing an estimate for a parameter, is more realistic than the choice of a single point estimate. Numerous simulations were made using randomly selected values of the input parameters resulting in a frequency distribution of the rates of return and their relative probability of occurence. The sensitivity analysis method was combined with the probabilistic analysis to interrelate mineral reserves, capital investments, operating costs and production rates. The evaluation of a mineral deposit was performed using three alternative mining and processing methods at three different quantities of resources and various production rates and prices. The grade, recovery, operating costs and annual production rates were entered as probability density functions, the capital investments as normal distributions and working capital and ore as point estimates. The results were reported for different ore prices for the recovered product. Results were reported as the range and most frequent rate of return (percent).

MATERIALS AND METHODS

A set of five interactive computer programs was developed to perform the statistical analyses and the simulation procedures developed in this study. The first four programs are used to obtain statistical estimates based on the input data. These quarterly or yearly estimates are used in the simulation program to generate a series of normal random vectors with the prescribed correlation between variables. The statistical estimates represent a major improvement over earlier work (Clements *et al.*, 1972) in terms of generating input data for the simulation procedure. The simulation procedure, adapted from Spence (1976), assumes that the variation and covariation of each of the variables considered will react to changes in the others in the way that was observed and described by the variance-covariance matrix, during the time period from which the data were taken. The generated vectors describe probability density functions for each variable that can be used as input to a capital budgeting model. This approach to investment analysis represents a substantial improvement over the more common "point estimate" approach. The procedures developed in this study for incorporating variability into investment analysis go beyond those which are based on the decision-maker deriving optimistic, most likely and pessimistic forecasts for the series of events considered. This technique recognizes that the component cash flows are most often related to the movements of other price series.

Data from U.S.D.A. Agricultural Prices, Michigan Agricultural

Reporting Service and Michigan State University Enterprise Budgets were used to illustrate the technique and program output. The raw data and deflated data (deflated by USDA price indices) and output data were tested for the assumption of normally distributed residuals.

General Linear Model

The general linear fixed model $Y = Xb + e$ has expectations $E(e) = 0$ and $E(y) = X_b$, and variance-covariance matrix $Var(y) = Var(e) = V$. According to Searle (1971) least square estimation involves minimizing

$$(Y - Xb)' V'' (Y - X_b)$$

with respect to b where, ' refers to the transpose and " the inverse. This leads to the least square solutions.

$$b = (X' V'' X)'' X' V'' Y$$

For a full rank model, the unique inverse of X'X, (X'X)", exists. Therefore, solutions for the elements in b are unique solutions or estimates (b). For a non-full rank model, those involving fixed effects, (X' V" X)" does not exist, and as a consequence the solutions are not unique. However, unique solutions can be obtained when certain constraints are imposed in the process of solving the normal equation.

The approach taken to compute quarterly estimates from monthly observations on a variable, assumed that the residual random error (E_{ij}) was distributed with mean zero and variance-covariance matrix V. In order to obtain the quarterly estimates, μ was set equal to zero. Since, $\mu + A_i$ is an estimable function, with A_i set to zero, $\mu + A_i$ is equal to A_i. The consequence of this constraint is that the model is full ranked and unique solutions exist. In this particular one-way classification model situation, $\mu + A_i$ is then equivalent to A_i, since A_i is really $\mu + A_i$, which is an estimable function. Thus, the following linear model was used,

$$Y_{ij} = A_i + E_{ij}$$

where, Y_{ij} was the response of the jth observation in the ith quarter (three monthly observations per quarter), and A_i the fixed effect for quarter j.

From the model, the following normal equation is constructed,

$$X' \; V'' \; X \; b = X' \; V'' \; Y$$

where, Y is an observation vector of length n (number of months), X is a known n x q (number of quarters) matrix derived from the data (containing ones and zeros according to the presence or absence of Y's in quarterly classes), b is a q x 1 vector of unknown quarterly constraints, X' is the transpose of X, and V" is the inverse of the variance-covariance matrix.

The normal equation is then solved for

$$b = (X' \; V'' \; X)'' \; X' \; V'' \; Y$$

Although the small sample sizes (one observation per month) do not permit a statistical test for non-homogeneity of variance between quarters or years, the intuitive assumptions made for this analysis were that variances over time are not alike and that the covariances between time periods are non-zero.

Development of the Variance-Covariance Matrix

If observations from a population are random samples drawn in a completely random and uncorrelated manner (drawn with replacements or from an extremely large population), then all covariances would be zero. Further, if all observations are random samples from the same population with a constant variance or from different populations with homogenous variances, the variances would be equal to a constant variance. In practice the sampling scheme is designed to make certain that $V = \sigma^2 I$, or the assumption is made that this is the case when writing the linear model.

However, if for example, one assumes that the price of a commodity this month is related to the price of the same commodity last month and the month prior to that, and so forth, then the error terms can be expected to be correlated across observations. This phenomenon is termed autocorrelation and V does not equal $\sigma^2 I$. Autocorrelation is most often a characteristic of time series data, when data are collected from the same observational unit at successive points in time. An appropriate procedure to use when faced with situations where the error terms are correlated or have unequal variances is refered to as generalized least squares. This method uses the information about the variance and covariances of the error terms to increase the accuracy of the estimators. In the case of observations with error terms of unequal variance, the procedure effectively gives greater weight to those observations whose error terms have smaller variances. In the autocorrelated case, this procedure transforms the variables in such a way that the error terms implicit in the transformed variables are uncorrelated. With autocorrelation it is difficult to state *a priori* the magnitude of the correlation among succesive error terms. Given the values X_1, X_2,... ...,X_n; the (n-1) pairs (X_1,X_2), (X_2,X_3),......, (X_{n-1},X_n) constitute a set of bivariate values which have a correlation coefficient associated with them, as do the (n-2) pairs and so on. The coefficient (k-1) terms apart, i.e. of X_t and X_{t+1}, is called the serial or autocorrelation of order k.

The autocorrelation principle was used to develop the covariances of the variance-covariance matrix (V) needed to calculate the quarterly solutions. In this way autocovariances were used as estimates of covariances across months within quarters, between quarters within years and between quarters across years. For the input time series (Wi), i = 1,......,n, autocorrelations (AC_j), j = 1,....n-1 were computed using maximum likelihood estimates.

$$AC_j = \frac{\frac{1}{n-j}\sum_{i=1}^{n-j}(X_i-\overline{X})\ (X_{i+j}-\overline{X})}{\frac{1}{n}\sum_{i=1}^{n-1}(X_i-\overline{X})^2}\ ,\ j = 1,......k$$

Where, X is the mean and n is the number of observations in the input time series.

A variance-covariance matrix (lower triangle) of generalized form is depicted below to identify the individual elements (Figure 1). Each row or column represents a month of the year. Therefore, a variance-covariance matrix for monthly observations over a five year period would be dimensioned 60 x 60.

Figure 1. Generalized Variance-Covariance Matrix

$$
\begin{array}{llllllll}
V_1 \\
C_{1,1} & V_1 \\
C_{1,2} & C_{1,1} & V_1 \\
C_{2,3} & C_{2,2} & C_{2,1} & V_2 \\
C_{2,4} & C_{2,3} & C_{2,2} & C_{2,1} & V_2 \\
C_{2,5} & C_{2,4} & C_{2,3} & C_{2,2} & C_{2,1} & V_2 \\
\vdots & & & & & & \ddots \\
C_{n,n-3} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & V_n \\
C_{n,n-2} & \cdots & \cdots & \cdots & \cdots & \cdots & C_{n,1} & V_n \\
C_{n,n-1} & \cdots & \cdots & \cdots & \cdots & C_{n,2} & C_{n,1} & V_n
\end{array}
$$

where, $V_1, \ldots\ldots V_n$ is the estimated variances (diagonal) for quarter one through the nth quarter, respectively; C_{ij} is the covariance for the ith quarter for $i = 1, \ldots\ldots, n$ and the jth lag for $j = 1, \ldots\ldots n-1$. The variance within a quarter was the basic unit used to generate the covariances (off-diagonals) for the variance-covariance matrix.

The second Fortran program (PRG2) calculates the estimated variances and covariances for an input time series for each variable (up to 10 variables) for up to five years of monthly observations. The variance-covariance matrix, for each variable is written on a tape from which the third Fortran program (PRG3) calculates (V") the inverse for each variable. The inverted matrices then are entered into the normal equation in the fourth program (PRG4), and used in calculating the quarterly solutions (b) for each variable.

$$b = (X' \; V'' \; X)'' \; X' \; V'' \; Y$$

The IMSL Library (1979), an extensive collection of mathematical and statistical subroutines written in Fortran, was used to perform most of the matrix manipulations and computations. Appendix A, Table 1 contains a complete list and brief description of the subroutines used. The Fortran programs (Appendix A) are well documented with comment cards interspersed throughout to allow one with a basic knowledge of Fortran to follow them.

Simulation Model

A correlation technique adapted from that described by Clements *et al*. (1971) was used to simulate events based on the quarterly solutions computed by Program Four (PRG4). Variability estimates were based on the quarterly estimates, a triangular matrix derived from the historical variance-covariance matrix and the assumption that the input data were normally distributed. The simulation procedure generates for each variable a random series which is normally distributed and appropriately correlated with the series generated for the other variables considered. This procedure assumes that each variable will react to changes in the other variables in the way that was observed during the period (up to 5 years) which provided data for the variance-covariance matrix (calculated in PRG1) for all variables over the entire time period.

According to Anderson (1958), if Z is a normal vector with mean μ and variance V, there exists a unique upper triangular matrix C such that,

$$X = C\,Z + \mu$$

In this case $(X-\mu)$ has a variance-covariance matrix $V = CC'$.

In order to obtain C from V the so-called "square root method" can be used which provides a set of recursive formulaes for the computation of the elements of C.

$$C_{i1} = \frac{\sigma_{i1}}{\sigma_{i1}^{1/2}}, \quad 1 \leq i \leq m$$

$$C_{ij} = (\sigma_{ii} - \sum_{k=1}^{i-1} C_{ij}^{2})^{1/2}, \quad 1 < i \leq m$$

$$C_{ij} = (\sigma_{ij} - \sum_{k=1}^{j-1} C_{ik}C_{jk})/C_{jj}, \quad 1 < j < i \leq m$$

Since C is an upper triangular matrix, ($C_{ij} = 0$ for all $j < i$) after obtaining the elements of C, all components of X can be determined from Z as weighted sums: $X_i = \text{sum } (C_{ij}\ Z_i + \mu_i)$.

The generation of a random vector X with mean μ and variance-covariance matrix V can be programmed in the following steps.

1) Obtain the triangular matrix C from V.

2) Generate m independent random normal variates.

3) Perform the matrix-vector multiplication and vector addition.

The result is a normal vector from the multivariate distribution defined by μ, V and a probability density function of X. In this way two or more (up to 10) correlated random normal variates can be generated from independent normal variates by the transformation process of $X = CZ + \mu$.

Test for Normality

According to Gill (1978) the assumption that the errors are normally distributed is not essential to the partition of variance and development of point estimators but is critical in probability statements about the reliability of estimates (confidence intervals) and decisions based on tests of hypotheses. Two statistical facts lending support to the appropriateness of the assumption of normality for a large proportion of practical cases are; 1) the central limit theorem which establishes approximate normality of means from all but very small samples; and 2) the f test of the hypothesis of treatment effects is known to be robust, i.e. the probabilities of errors of Type I and Type II are little affected by moderate departures from normality.

The analysis of variance test for normality described by Shapiro and Wilk (1965) is designed to provide an index or test statistic to evaluate the assumed normality of even small samples ($n < 20$). For $n < 50$) this test compared to goodness of fit utilizing chi-square distributions or Kolmogorov-Smirnov non-parametric test, is considered a somewhat better approximation for small samples and is quite sensitive to a wide range of departures from normality such as skewness and kurtosis of the distribution (Gill, 1978).

The W statistic used to test for normality is scale and origin invariant and hence supplies a test of the composite null hypothesis of normality. Non-normality is indicated when the calculated "W" statistic is smaller than the appropriate critical value. By studying the distributional characteristics of a body of data one may be encouraged to consider, for example, normalizing transformations, the use of distribution-free techniques, as well as detection of gross peculiarities such as outliers or data errors.

By the central limit theorem, the actual distribution of a variable can sometimes deviate considerably from the normal distribution without significantly affecting the final results. The major consequence of non-normality is that, without knowledge of the distribution of the variables, precise probability statements cannot be made. The Shapiro-Wilk test was used to test the normality assumption for the input data (raw and deflated) as well as the quarterly solutions and generated output.

An application of the statistical procedures and simulation technique

The statistical estimates and simulated probability distributions were used to analyze a practical example, involving a comparison between leasing and purchasing dairy cattle. A computerized dairy herd growth model (TELPLAN 52) was used to project cull cow and bull calf numbers over a four year period for 100 bred heifers. Nott and Sargent (1975) developed the recursive, deterministic growth model capable of calculating dairy livestock inventories for individual farm situations. The model was verified with farm accounting and DHIA summary statistics. The statistical estimates for cull cow and calf prices in Michigan for the period 1975-1979 were used in the example. A discussion of the results of the capital budgeting analysis is included in the Results and Discussion section, to follow.

RESULTS AND DISCUSSION

Evaluating the Normality Assumption

The Shapiro-Wilk method was used to test the assumption of normally distributed residuals, since the simulation procedure assumes a normal distribution. The 11 sample time series (Table 1) were analyzed over four, three year time periods, (1967-1969, 1970-1972, 1973-1975, 1976-1978) and for one year (1979) individually. The results generally indicated that the price series were non-normally distributed. An attempt was made to rid the series of the effects of changes in the general price level by deflating the original data. It was felt that by removing the effects of movements in the general price level, the deflated price series would be a better representation of price variability related to factors other than the movements in the general price level. It was also felt that the deflated series would more closely approximate a normal distribution, in which case the assumptions of the statistical and simulation procedures would be met.

Two USDA monthly indices were used to deflate the selected price series. The index of prices received by farmers for feed grains and hay (IPRFGH) and the index of prices received by farmers for dairy products (IPRDAIRY) were the two indices used. Table 1 shows a comparison with respect to the test for normality between the deflated and undeflated price series for 11 monthly price series for the years 1967 through 1979.

TABLE 1. A Comparison Between Original and Deflated Time Series with Respect to Normality[1]

Price Series	Years				
	67-69	70-72	73-75	76-78	79
Milk	D	D	D	D	D
Milk cows	D	A	D	D	D*
Cull cows	D	D	D	D	D
Calves	F	D	D	F	D
Soybean oil meal	D	F	F*	N	F
16% grain mix	D	N	F	F	F
Alfalfa hay	F	F	F	F	D
Corn	N	D	F	N	D
Wheat	N	F*	N	F	F*
Oats	N	F	F	F	D
Soybeans	N	N	N	N	D

[1]Letter designation in each cell represents form of data most closely approximating a normal distribution.
N = not deflated
D = deflated by index of prices received by farmers for dairy products.
F = deflated by index of prices received by farmers for feed grains and hay.
* Indicates a probability (P<.01) of rejecting the non-normality hypothesis.

The results presented in Table 1 would appear to indicate that deflating generally resulted in a more normally distributed price series for those examined. The IPRDAIRY index seemed to be the appropriate deflator to use for milk, milk cows, cull cows and calf prices across time periods. The choice of a deflator for the remaining price series would not be as obvious. Undeflated soybean price appeared to be more normally distributed than when the series was deflated by IPRFGH. The IPRFGH index was most effective in terms of normalizing the soybean oil meal, 16 percent dairy concentrate, alfalfa hay and oat price series, although was not consistent across time periods.

Tomek and Robinson (1972) point out that if the index selected contains the price series being deflated, then the coefficients relating the deflated variables to other variables are likely to be biased. The amount of bias depends upon the relative weight which the price series being deflated has in the index. In other words, since the index changes in part as a function of the change in the particular commodities price, deflating by the index tends to cancel the influence of the price change. An alternative to deflating would be to include the price indices as separate independent variables. However, since the price indicies often have strong trends, they are often highly intercorrelated with other explanatory variables.

Based on the results of this portion of the study, deflating the price series appears to have some merit in terms of normalizing the series. It should be noted that the results of the statistical analysis using the Shapiro-Wilk test of normality indicate that the undeflated price series (of those analyzed) do not approximate the characteristics of a normal distribution as well as the deflated series do. This becomes an important consideration with respect to the simulation procedure, which is based on the normality assumption in terms of generating the simulated data. Assuming the appropriate deflator were chosen, the question of how the output from the deflated model is to be interpreted remains an important one. If one were to develop a model to predict future indices then there may be some value in deflating the price series and using the index prediction model to predict future prices based on the model of changes in the general price level. This predictive capability could certainly add a great deal to the techniques developed in this study.

A statistical comparison between the original and simulated data

The means and variances computed for the simulated data are considered "best estimates" of the population parameters, μ and σ^2, respectively. Statistical tests were performed to compare the means and variances of the simulated population with those of the original input time series.

In testing the difference between the input data sample variance and the generated sample variance, the ratio of the two sample variances was used to calculate the f statistic. The function (S_1^2/σ_1^2) / (S_2^2/σ_2^2) is the F distribution with r_{1-1} and r_{2-1} degrees of freedom for independent samples of size r_1 and r_2 for two normally distributed variables. Therefore, the hypothesis that $\sigma_1^2 = \sigma_2^2$ is tested against the hypothesis that the population variances are not equal. The value of the test statistic is the ratio of the sample population variances, $f = S_1^2/S_2^2$ and is compared with respective critical values, $f_{1-\alpha/2}$, r_1-1, r_2-1 found in an F distribution table. Thus, if the test statistic, $f = S_1^2/S_2^2$ is larger that the upper-tail critical value, one may reject the hypothesis that the population variances are equal and conclude, (for the selected significance level, $1-\alpha$) that the S_1 sample data shows less variability than the S_2 sample.

The results of comparing the sample variance (S_1^2) associated with the original input time series (n = 60, i.e. five years) with the sample variance (S_2^2) for the simulated data (n = 1000, e.e. five years) were as might be expected. The test statistic was in general consistently less than the upper-tail critical value ($f_{0.025,999,59} = 1.48$) for cases in which $S_2^2 > S_1^2$, and $f_{0.025,59,99} = 1.39$ for cases in which $S_1^2 > S_2^2$. The values for the sample population standard deviations are shown in Table 2, along with the results of the statistical test used to compare sample variances. Since this test should be applied to normally distributed variables. the results of the non-deflated data were compared with the deflated data, since the non-deflated data were shown to be more normally distributed. While deflating the data obviously removed a large portion of the variation associated with movements in the general price level, the simulated sample variances were statistically different for the input sample in six of the 54 cases tested, compared with two incidences of dissimilar variances for the non-deflated data. The simulated sample variance appeared to be consistently larger, though not necessarily statistically different than the input sample variance for most cases tested. No explanation was made for this trend in the data.

The simulated and input data were also compared with respect to their respective sample means. The test of difference between the two means was made using the Wilcoxon's rank sum test. This test is used for comparing the means of two populations having the same but unspecified distributions. The IMSL Library (1979) subroutine NRWRST was used to perform the test. The results showed that the hypothesis of equal means (simulated vs actual) would be rejected only in those few cases indicated in Table 3, for the selected time series.

These analyses were performed as an attempt at verifying the validity of the statistical and simulations procedures carried out by the computer programs. Based on these analyses one should have some confidence in concluding that the sample variances and sample means for the original

Table 2. A comparison Between Input Sample Variance[1] and Simulated Sample Variance for Selected Deflated and Undeflated Time Series

		1967 - 1971		1972 - 1976		1975 - 1979	
Item		N	D	N	D	N	D
Corn ($/bu)	A	.152	.129	.693	.264*	.321	.197
	S	.181	.153	.774	.327	.361	.162
Wheat ($/bu)	A	.155	.149	.658	.463*	.585*	.230
	S	.164	.171	.781	.347	.725	.253
Oats ($/bu)	A	.079	.086	.339	.123	.176	.136
	S	.094	.104	.336	.127	.203	.157
Soybeans ($/bu)	A	.228*	.193	1.492	.695	1.081	.541*
	S	.292	.226	1.788	.762	1.317	.767
Hay ($/ton)	A	2.171	1.755	7.679	4.383	3.208	1.974*
	S	2.382	1.991	7.006	4.352	3.223	1.470
Milk ($/cwt)	A	.272	.184*	1.386	.152	1.251	.131
	S	.316	.231	1.475	.180	1.523	.157
Milk cow ($/hd)	A	39.00	22.46	47.02	46.16	236.58	75.63
	S	36.25	26.37	47.56	53.58	222.01	76.32
Cull cow ($/cwt)	A	2.031	1.771	4.993	4.829	11.086	3.732*
	S	2.263	2.122	6.045	5.150	10.712	2.904
Calf ($/hd)	A	5.173	3.323	13.50	12.44	22.28	7.974
	S	4.563	3.346	14.06	12.03	20.85	7.533

1 - Values are standard deviations
A - actual input data
S - simulated data
N - undeflated
D - deflated
* - this sample variance is significantly different (P<.05) than corresponding sample variance.

Table 3. Results of Wilcoxon's Rank Sum Test Comparing Simulated Mean with Actual Input Data Mean

	Years		
Price series	1967 - 71	1972 - 76	1975 - 79
Corn			*
Wheat		*	
Oats			
Soybeans	*	*	
Hay			*
Milk			
Milk cow		*	
Cull cow			
Calf			

*Indicates hypothesis of equal means was rejected ($\alpha > .10$)

data are quite similar to those generated by the simulation procedure.

The major shortcoming of the computer programs developed to perform the statistical analysis are with respect to the design of their data handling capabilities. Each data point is in effect handled only as a monthly observation. In other words, variables which generally call for annual observations would be treated as if the observations were monthly. As a result, three annual crop yields would be treated as a "quarter" (i.e. three months). The output data must be used and interpreted assuming the input data represents monthly observations. The statistical programs and simulation procedure are designed to handle the computations for from two to ten variables. However, each variable must necessarily have the same number of observations as all other variables considered during one pass through the program sequence.

Thus, for example, annual crop yield and monthly crop price data are not compatible in the same model. It would however, be possible to run the program with the monthly data initially and then use the yearly data.

Application of Simulation Technique to Capital Budgeting

The statistical and simulation procedures can be used to incorporate the variability of a wide variety of events into an investment analysis. In addition to the leasing versus purchasing example described below, ones imagination is the only limit to the application potential of this technique. Events such as seasonal and year-to-year variations in milk production levels, and yearly crop yields would be expected to have a major impact on various managerial decisions. The variability in the level of milk production and milk price, for example, over the course of expanding the size of a dairy enterprise should certainly be incorporated into a complete farm budget analysis. Analysis of variations in estimated gross income per acre or per animal unit are examples of variables which include the variability owing to both physical production and prices.

A relatively simple example was developed to illustrate the use of the statistical and simulation procedures within a capital budgeting framework. The example used compares the relative profitability of leasing 100 bred dairy heifers versus purchasing the same. The example is designed to demonstrate the kinds of information the capital budgeting output provides the decision maker with when the simulation technique is incorporated into the analysis. The results of the "point estimate" approach are compared with those generated when the statistical solutions and simulation technique is used.

Leasing versus purchasing dairy cattle - an example

In general, a lease is a contract by which the lessee acquires sole use of an asset in return for lease payments. The lease payments reflect the value of the leased asset and the lessor's (owner of the asset) carrying costs and his profits. Thus, a lease is similar to a conventionally financed installment loan. A dairyman entering into a dairy cattle leasing agreement generally agrees to maintain a specified number of cows for the duration of the agreement (usually for 3 to 7 years).

Leasing agreements differ greatly with respect to culling leased cows, replacing culled cows, disposition of calves born to leased cows and disposition of leased cows at the time the lease expires. The dairyman typically receives the milk income from the leased cows. Therefore, when comparing leasing with purchasing one may assume that milk receipts will be the same, if the dairyman is comparing purchasing a group of cows with leasing the same group of cows. Thus, a comparative analysis need only include differences in costs and returns due to leasing or purchasing.

Determining whether a change in the farm business will be profitable or not does not require a complete budget. A partial budget is a plan that lists only the receipts and expenses which are expected to change with the proposed change. In order to evaluate and compare the costs and returns over time, for any method of acquiring control of an asset, one must recognize the time value of money. Net present value (NPV) analysis or capital budgeting is a technique used to evaluate the projected cash flows for an investment or to compare investment alternatives. This technique discounts the projected cash flows (outflows or costs and inflows or returns) to their present values (their equivalent worth today).

There are seven basic types of information required for an NPV analysis. Namely, 1) the initial investment of equity (owned) capital; 2) the annual cash flows (cost and returns) attributed to the investment; 3) the length of time over which the analysis is being made; 4) the salvage value (if any) of the investment; 5) the interest or discount rate or required rate of return; 6) the applicable marginal income tax rate for generating results on an after-tax basis; and 7) the depreciation method used for old or new assets.

In the example chosen, the only costs associated with leasing are the monthly lease fees since only those costs and returns that differ with respect to leasing versus purchasing need to be considered. In this particular example the leasing company would receive all cull leased cows and pay the dairyman a fixed rate ($30/head) for all bull calves born from leased cows. The cost side of the comparison for the purchasing example includes only the purchase price. The returns associated with the purchase alternative include, the market value of culled cows and the actual market value for the sale of bull calves. In addition to these costs and returns there are important tax considerations that need to be included in the analysis.

In the case of the leased animals, the lease fees are the only deductible expenses associated with leasing. The example assumes that the lessor chooses to pass the investment tax credit to the lessee, although it does not enter the analysis since it is assumed that the same tax credit would be available under the purchase alternative. In the case of the purchased animals, depreciation, and the interest paid on borrowed capital are included because of the effect they have as tax shields on cash flows.

The acceptability and ranking of investments based on discounted cash flows depends on the sign (positive or negative) and the relative magnitude of the resulting NPV figures. In this example, since it was assumed that actual cash receipts from milk sales would be the same whether the animals were purchased or leased, the resulting NPV figures would be expected to be negative as the costs (negative NPV) outweigh the tax credits and income from the sale of calves (leasing and purchasing) and the sale of cull cows (purchasing). Therefore, the smallest negative NPV would be the favored investment.

The dairy herd growth model (TELPLAN 52) was run for 100 purchased bred heifers to project the number of culls and bull calves to be expected over a four year planning horizon. Average death losses and cull rates were used to generate the results. Death losses for purchased bred heifers were assumed to be 2, 4, 3 and 2 percent for years one through four, respectively. The average assumed cull rates for purchased bred heifers were assumed to be 35, 33, 28 and 26 percent for years one through four, respectively. Death losses for bull calves from birth to sale as newborn calves was assumed to be five percent. The average age at which heifers freshened was set at 24 months and an average calving interval of 13 months was used. The heifer raising strategy used to generate the desired results was one in which no heifers were raised as replacements. The results from the animal inventory analysis are presented in Table 4.

Table 4. Animal Numbers from 100 Bred Heifer Over Four Years

	Year			
	1	2	3	4
Cull cows	34	21	10	1
Bull calves	47	31	21	15

Table 5 presents a net present value analysis of the costs and returns associated with leasing versus purchasing 100 bred heifers using the conventional "point estimate" approach and the simulation technique developed in this thesis.

Since -\$29,702 (NPV purchase) is less negative than -\$31,189 (NPV lease) based on expected values, one would conclude that purchasing is the preferred alternative. However, the range and distribution of NPV's around the expected value resulting from the simulation procedure indicate there are times when the leasing option should be the preferred option in this example.

According to Gill (1978), for any normally distributed random variable Y, the transformation, $Z = (Y - \mu_y/\sigma$, always leads to a normal distribution with mean 0 and variance 1, where μ_y is the population mean and σ is its' standard deviation. The probability density function for the standard normal form is:

Table 5. A Comparison of Approaches to Net Present Value (NPV) Analysis of Costs and Returns for Leasing versus Purchasing 100 Bred Heifers[1]

		Present Value
LEASE		
Costs:		
Lease fees ($10.50/head/mo)		$41,400
	Total costs	$41,400
Returns:		
Lease fees (tax credit)[2]		$ 7,988
Sale of bull calves ($30/head/ml)		$ 2,223
	Total returns:	$10,211
Net Present Value = (-$41,400 + $10,211) =		$-31,189
PURCHASE		
Costs:		
Purchase price ($600/head)		$60,000
	Total costs:	$60,000
Returns:		
Interest (tax credit)[2,3]		$ 1,744
Depreciation (tax credit)[2,4]		$ 4,064
Cull cows[5]: Expected P.V.		$20,115
	Range	$16,513 - $24,083
	Standard deviation	$ 1,708
Bull calves: Expected value		$ 4,375
	Range	$ 3,686 - $ 4,963
	Standard deviation	
Total Returns: Expected value		$30,298
	Range	$26,817 - $34,024
	Standard deviation	$ 1,742
Net Present Value: Expected value		-$29,702
	Range	-$33,183 - $-25,976
	Standard deviation	$ 1,742

[1] 10% discount rate over 48 months

[2] 20% marginal tax rate

[3] 10% annual rate of interest

[4] straight-line depreciation: salvage value of $200/head

[5] assumes average of 12.5 cwt/head

$$F_Z(z) = (1/\sqrt{2\pi}) \exp (z^2/2)$$

and the cumulative density function is:

$$P(Z < z) = F_Z (z) = \int_{-\infty}^{Z_i} f(z) \, dz$$

Initially, one must establish with some degree of confidence that the distribution approximates a normal distribution, prior to making standard normal transformations and probability statements.

The third and fourth moments for the 1000 simulated net present values were 0.080 and -0.643, respectively. The moments of a sample can indicate a great deal about the shape of the parent population's distribution. The third moment is used to determine whether a distribution is symmetric or skewed about the mean. Skewness is estimated by calculating the ratio of the third central moment to the standard deviation cubed. The third moment is equal to zero when the distribution is symmetric about the mean. A distribution is said to be skewed to the right or positively skewed when the third moment is positive and left or negatively skewed when the third moment is negative. It is generally accepted that a distribution is symmetric about the mean when the value of the third moment lies between -0.5 and 0.5. The fourth moment (kurtosis) is used to interpret the flatness or peakedness of a distribution. Kurtosis is estimated by dividing the fourth central sample moment by the standard deviation raised to the fourth power and then subtracting three. The fourth central moment is also zero for a random variable distributed exactly normal (Gill, 1978). This would indicate that this sample population has a distribution which is skewed slightly to the right and is somewhat peaked. However, the sample appears to approach normality. A normal distribution would be expected since the simulation procedure assumed normality in that random normal deviates were used to generate the simulated values. The Shapiro-Wilk test for normality was not used because the sample sizes were greater than 50 which is the limit for this procedure.

Once the values of the random variable have been transformed to units of 2 (standard deviations of Y) probabilities can be evaluated. In order for the leasing option to be equivalent or preferred over the purchase option, the NPV of the purchase option must be -$31,189 or less. The NPV for the lease (-$31,189) can be transformed to standard normal form in order to evaluate the probability of leasing being the more profitable alternative. The transformation procedure for this example is shown below.

$$A = (Y - \mu)/\sigma$$

$$Z_i = ((-31{,}189) - (-29{,}702))/1742$$

$$Z_i = -0.8536$$

$$\text{Probability } (Y < -31{,}189) = \text{Probability } (Z < -0{,}8536)$$

$$\text{Probability } (Z < -0.8536) = 0.1967$$

This result indicates that although the "point estimate" approach using expected values would result in a decision maker accepting the purchase option as the most favorable option the results of the simulation procedure indicates there would be times in which the leasing option would be preferred. For this specific example, the calculated probability would indicate that leasing would have been the more profitable alternative 20 percent of the time.

However, Gill (1978) warns that probabilities cannot be evaluated without knowing the values of μ_y and σ for the defined population of interest. In cases where large amounts of empirical evidence about the values of μy and σ have been accumulated, one may assume the values of the parameters and proceed directly with questions of probability. It can be shown that with information from 1000 subjects from the relevant population, one may be approximately 95 percent confident that the population variance lies in the interval 0.95 S_y to 1.05 S_y i.e., with 1000 observations the sample variance still is subject to an error of 5 percent (or more). However, somewhat fewer observations are required for equivalent reliability of estimates of the oppulation mean, unless the coefficient of variation is relatively large.

Therefore, even with 1000 observations (simulations) on Y, in the example, in which S_y = 1742., one can be 95 percent confident that the population variance lies within 0.95 and 1.05 times the sample variance. Thus S_y would lie (with 95 percent confidence) between 1698 and 1785. Therefore, the probability that leasing would have been the preferred option would lie between 19.1 and 20.2 percent, as shown below.

$$Z_j = ((-31{,}189) - (-29{,}702)/1968 = -0.8757$$

$$\text{Probability } (Z < -0.8757) = .1906$$

$$Z_k = ((-31{,}189) - (-29{,}702))/1785 = -0.8331$$

$$\text{Probability } (Z < -0.8331) = 0.2024$$

A guide to the use of the statistical and simulation programs

The techniques developed in this study were designed to provide an objective measurement of the uncertainty or variability associated with prices, cost and income, based on historical price series. If the desire for income stability is strong, farm managers may consider combining enterprises that could be expected (based on historical data) to reduce the variability of annual incomes even at the cost of some reduction in average income over a period of time. The resulting variance of income from a combination of more than one enterprise is dependent upon the variability (variance) of the individual enterprises to be considered, and the degree of association or correlation (covariance) of the returns of these enterprises. If for example, farm resources were divided equally among two or more enterprises, total variance for the farm business would be reduced provided the variances (measure of variability) of the individual enterprises were approximately equal and there was less than a perfect correlation (price movements in the same direction for inputs and outputs) between enterprises.

Decisions made with respect to capital expenditures are among the most difficult managerial problems. Most investments occur over a considerable time in the future, and therefore considerable effort is needed to predict probable costs and returns of each alternative. Secondly, often times most, if not all, of the capital must be laid out immediately, while benefits or returns occur over time. Thus, a decision-maker must balance added returns that will occur in future years against an expenditure that will be made immediately. Understanding that the value of money is influenced by time is important in evaluating the profitability of investment opportunities. A dollar received or spent some time in the future is not worth a dollar today. Discounting is the process used to find the present value of a given amount of money to be received or paid in the future. Net present value is simply the difference between the present value of benefits and the present value of costs.

Because there are few cases in which prices, costs and income levels are known precisely beforehand, an analysis of alternative outcomes is essential to making a good decision based on the information available. One common way in which risk or uncertainty is incorporated into an investment analysis, is to estimate different levels of costs and benefits and compute the net present value for each combination. These estimates are typically considered: 1) the best or most reasonable; 2) pessimistic; 3) optimistic. Such estimates provide a basis for taking into account the consequences of unexpected or unforseen situations associated with the investment. Calculating the net present value for each estimate represents only a few points on a continuous distribution of possible combinations of future events. Since every factor that enters into the evaluation of a specific decision is subject to some uncertainty, the decision maker needs a portrayal of the effects that the uncertainty surrounding each of the significant factors has on the returns he is likely to achieve. The overall objective of this study was to develop a procedure which combines the variabilities inherent in the factors considered. Historical price data is used to compute quarterly or yearly estimates for each variable entered into the procedure. These estimates are then used as input to the simulation model. The purpose of the simulation model is to generate a distribution (1000 generated values) for each input price series. This goes beyond representing the distribution of prices with only a few points. The simulation procedure also recognizes that prices for different variables are interrelated or correlated. The simulation technique generates distributions for each variable, that are correlated with the other variables as dictated by the historical input price series. A probability distribution represents the odds of achieving a particular value or range of values based on the input time series.

The following discussion describes, for the user's benefit, the set of five interactive computer programs developed in this study. The first four programs perform the statistical procedures described in detail in the materials and methods section of this thesis. Monthly data can either be read from a computer disk file or input directly by the user typing it into the computer terminal. The programs will accept monthly observations for from two to ten variables over a time period of up to five years for each variable. The statistical programs (PRG1 thru PRG4) compute both quarterly and yearly best, linear, unbiased estimates (B.L.U.E.) from the input series. A variance-covariance matrix (which is a representation of the variability associated with each variable over time, the "co-variability" between variables and across time periods) is used to generate another matrix (unique triangular matrix) which is used in the fifth program to generate the simulated data. If the user is interested, he can elect to have the variance-covariance matrix, correlation matrix and unique triangular matrix printed out.

After the statistical computations are completed by the first four programs the simulation program (PRG5) uses the statistical solutions to generate the simulated data. These simulated data are normally distributed about the statistical estimates and appropriately correlated across variables and across time periods. These simulated data represent a "best estimate" of a sample probability distribution for each variable, thereby providing an estimate of the distribution of values for each variable.

By incorporating these distributions into a capital budgeting model, a computer program can be used to carry out the discounted cash flow calculations a large number of times to generate an output to which probabilities can be attached. In other words, instead of outputting a single value as a measure of an investment's worth (as is most commonly done), a decision maker would be presented with a distribution of net present values or rates of return. Sensitivity analysis could then be used to determine how sensitive the results are to a change in the tax rate or discount rate, by rerunning the program for each change. This thesis contains an example in which the generated distributions for dairy calf and cull cow prices were incorporated into a capital budgeting model to illustrate this technique.

Listings of the source programs (PRG1 thru PRG5) are included in Appendix A, Tables 3 through 7. A sample output is also presented in Appendix A, Table 8.

<u>Procedure for running the program</u>

1) Sign onto the computer (See CDC Interactive Terminal User's Manual).

2) Type: ATTACH,EX,TRSEXEC. (Hit RETURN after each input line.)

3) Type: EX.

4) The program will ask the user if he would like a brief description of the programs. ALL yes/no questions should be answered with: 1=YES; 0=NO.

5) The programs now begin executing in sequence.

6) Each program will ask the user several questions relative to the data input during execution of program 1 (PRG1). When the correct response has been entered the program will continue to execute.

7) Program 1 will ask whether the data is written on a disk file or whether it will be entered by the user. If the data resides on a disk file (data for one variable followed by data for the next) the program will not execute properly until the data file has been attached as TAPE1 (i.e., ATTACH,TAPE1, user's data file).

8) The user can receive a copy of the full matrix output should he desire it, by indicating this in response to a question asked in the beginning of program 1. (Certain statistical results will always be printed during execution).

9) After having completed the series of five programs, TAPE11 will contain the simulated data, with one variable followed by another in the order in which they were input. This file may be saved and cataloged for future use by typing: CATALOG,TAPE11,your data file name.

If at any time the user wants to stop execution of the programs he can do so by pressing the ESC key. This will immediately terminate the program sequence. To rerun the program follow the instructions outlined previously.

For those interested in altering the programs in any way, the following steps should be followed:

ATTACH,P*,TRSPRG*. (where, * is the program number)

SYSTEM,FORTRAN.

OLD,P*,FR,1,BY,1.

If you want a program listing, type: LIST.

SAVE,XX,NS.

RETURN,P*.

CATALOG,P*,your file name.

The user's copy of the program can now be altered without affecting the existing program.

SUMMARY AND CONCLUSIONS

Conclusions

The statistical procedure used in this study to compute unique solutions for price time series data represents a major improvement with respect to the input to a simulation model designed to generate probability functions for each variable considered. These quarterly or yearly solutions (best linear unbiased estimates) were incorporated into a simulation procedure designed to generate correlated time series. Although, the simulated sample variances appeared consistently larger than the original series, a statistical test used to compare the variances indicated they were not significantly different. It was demonstrated that the deflated time series were more normally distributed than the non-deflated series. It was concluded that there is a need for further study in the area of characterizing the distributional characteristics of time series data. An example was provided illustrating how the simulated distributions could be incorporated into an investment analysis. The output from the capital budgeting analysis provides a decision-maker with both the expected return based on the probabilities of all possible returns and more importantly, the expected variability in returns. Decision-makers can take the various levels of possible cash flows, and get estimates of the odds for each potential outcome from a net present value analysis. Such a procedure could also be used to produce valuable information about the sensitivity of possible outcomes to the variability of input factors and to the likelihood of achieving various net present values or rates of return. To have calculations of the odds on all possible outcomes (based on statistical estimates) lends some assurance to the decision-maker that the available information has been used with maximum efficiency.

The model, $Y_{ij} = A_i + E_{ij}$, is a one-way classification model with fixed effects. Unique solutions are obtained from what would typically be a non-full ranked model by constraining the overall mean ($\mu = 0$). Only in this way is the model a full-ranked model with a unique inverse $(X' V'' X)''$, and therefore unique solutions. The interactive computer programs compute the quarterly estimates for monthly time series data for up to ten variables over a time period of up to five years.

Recognizing that the error terms for time series data are typically correlated across observations, the autocorrelations were used in the development of the variance-covariance matrix. This procedure gives greater weight to those error terms with smaller variances. Autocovariances were used as estimates of covariances across months within quarters, between quarters within years and between quarters across years. In this way, the variance-covariance matrix V used in the normal equation to compute unique solutions incorporated trend into the linear model.

If one assumes that the future variability associated with a particular set of conditions is closely related to past variability, empirical estimates do provide a reasonable basis for making short and long run decisions. Therefore, the problem becomes one of selecting a length and time period most representative of expected variability in the future. Five years were selected as the maximum period over which the statistical analysis can be performed. This decision was based principally on computer memory limitations and computing costs. The approximate cost of inverting a variance-covariance matrix for each input time series represents the major cost associated with executing these programs. The approximate cost of inverting a variance-covariance matrix representing five years of monthly observations (60 x 60) is $1.00 per variable. As the size of the matrix increase the rate of increase in the cost of computing the inverse also increases. Thus, it was felt that a five year time period was a reasonable period of time in terms of characterizing price variability and at the same time holding down the computer memory and computational requirements.

The simulation model used to generate the probability distributions is based on the assumption that the input variables are normally distributed. Statistical tests were conducted on 11 price series (milk, milk cow, cull cow, calf, corn, wheat, oats, soybeans, hay, soybean oil meal, 16 percent concentrate) for three year time periods from 1967 through 1979. The results from the Shapiro-Wilk test for normality indicated that removing the effects of changes in the general price level by deflating, improved the distributional characteristics of the input time series. The index of prices received by farmers for dairy products seemed to be an appropriate deflator for milk, milk cow, cull cow and calf prices across those time periods used. The index of prices received for feed grains and hay was effective as a deflator in terms of normalization, for soybean oil meal, 16 percent dairy concentrate, alfalfa hay and oat price series. Although these results were not consistent across time periods. The basic problem associated with deflating time series is the bias introduced as a result of the series itself being a part of the index. The choice of an appropriate index and the interpretation of the output from a deflated model are difficult. Further research needs to be done in the area of describing the distributional characteristics of price series. All too often, researchers assume normal distributions without making any attempt at veryifying their assumption.

The tests used to compare the original input price series with the simulated series proved to be encouraging. The variances and means did not appear to be greatly different for those time series tested, thus, validating the performance of the statistical and simulation procedures. Although the simulated sample variance appeared consistently larger, this difference was not shown to be significant.

The value of computer programs in developing clear portrayals of the risk or uncertainty associated with alternative investments has been demonstrated. An investment analysis using discounted cash flows (capital budgeting) should provide the decision maker with more information than the expected net present value or internal rate of return. By incorporating probability distributions for price, income or cost variables into a capital budgeting model, the information generated by the analysis represents a substantial improvement over conventional methods of incorporating uncertainty into an investment decision model.

An illustration of the risk analysis technique using capital budgeting was presented using the correlated price series for two variables. Michigan cull cow and bull calf price data for the four-year period 1976-1979 were used to compare leasing dairy heifers with purchasing the same. The procedure used the statistical estimates and resultant simulated probability distributions for the input price series, to generate a random normal series of net present values for the purchasing option. Values of the random series were transformed to standard normal form and probabilities evaluated. The numerical results using expected values (point estimates) would have caused one to select the purchase option. However, the results differed when the probability distributions for the two correlated price series were incorporated into the model. These results would offer the decision maker a probability distribution of net present values to which the probability that a value will occur can be computed. In the example, the leasing option should have been the preferred option approximately 20 percent of the time. However, it should be recognized that the generated results will necessarily be indicative of conditions during the historical price series for the four-year period selected. If one assumes that the variability of the recent past is a relatively good indicator or predictor of future variability, then this approach to investment analysis offers some improvement over other attempts at incorporating measures of risk.

The major advantage of a probability analysis of investment alternatives is that it results in a distribution of values to base decisions on. Whether the results are in terms of net present value, internal rate of return or other criteria, probability statements can be made.

There are limitations to the techniques and procedures developed and examined in this study and a great deal of room for improvement. The key to improving decision making aids or models is using the available data and information, be it historical series or predictions, to the fullest extent possible. A good decision is one in which the resource allocations are most likely, in a probabilistic sense, to produce favorable outcomes. Good decisions only improve the chance of favorable outcomes, they do not guarantee them!

APPENDIX A TABLES

Appendix Table 1. International Mathematical and Statistical Laboratory (IMSL)

BECORI-	Estimates of means, standard deviations, correlation coefficients
BECOVM-	Estimates means and variance-covariance matrix
BEMMI-	Estimates of means, standard deviations, third and fourth moments
FTAUTO-	Estimates variance, autocorrelation, autocovariance
GGMML-	Normal or Gaussian random deviate generator
LINV2F-	Matrix inversion, full storage mode, high accuracy version
USMMMX-	Determines minimum and maximum values in a vector
USWFM-	Prints a matrix stored in full storage mode
VCVTFS-	Matrix storage mode conversion-full to symmetric
VCVTFS	Matrix storage mode conversion-symmetric to full
VMULFF-	Matrix multiplication-full by full
VMULFM-	Matrix multiplication-transpose of A by B
VMULFP-	Matrix multiplication-A by transpose of B

Appendix Table 2. Computer program variable list

A	MATRIX (NMONTHS X NMONTHS) FULL STORED VBLOWUP (TAPES) GOES INTO INVERSION PROGRAM
A	MATRIX (NVARS X NVARS) UNIQUE TRIANGULAR FROM TRSPRG1 (AMAT)
AMAT	MATRIX (NVARS X NVARS) UNIQUE TRIANGULAR MATRIX (UPPER TRIANGLE) OF A- GOES INTO TRSPRG4
AMINPUT	MATRIX (NQOBS,NVARS) OF INPUT VARIABLES
AMQRTS	MATRIX (3 X NQRTS) OF MONTHLY OBSERVATIONS WITHIN QUARTERS
AP	MATRIX (NVARS X NVARS) UNIQUE TRIANGULAR MATRIX (LOWER TRIANGLE)
C	MATRIX (NQRTS X NMONTHS) X-PRIME*V-INVERSE MATRIX PRODUCT S*A
CV	COEFFICIENT OF VARIATION
D	MATRIX (NQRTS,1) X-PRIME*V-INVERSE*Y
DENOM	DENOMINATOR FOR WEIGHTED MEAN CALCULATION SUBROUTINE: WEIGHT
E	MATRIX (NQRTS,NQRTS) X-PRIME*V-INVERSE*X
EINV	MATRIX (NQRTS,NQRTS) INVERSE (X-PRIME*V-INVERSE*X)
F	MATRIX (NQRTS,1) SOLUTIONS
FVCVU	FULL STORAGE OF VAR-COV MATRIX (ALL VARIABLE-UNADJUSTED FOR TIME
IDGT	NUMBER OF SIGNIFICANT DIGITS TO BE USED IN ACCURACY TEST SUBROUTINE: LINV1F (IMSL)
IER	ERROR PARAMETER FROM IMSL SUBROUTINES
INCD	VECTOR (NVARS*(NVARS+1)/2 WORKSPACE SUBROUTINE: BEMMI
IVNBR	VECTOR (6) INPUT TO VAR-COV SUBROUTINE
NOBSSM	NUMBER OF OBSERVATIONS PER VARIABLE IN SUBMATRIX ENTERED INTO VAR-COV SUBROUTINE
NQOBS	NUMBER OF OBSERVATIONS PER QUARTER (NQOBS 3)
NQRTS	NUMBER OF QUARTERS (N YEARS*4)

Table 2. (continued)

NSUBM	NUMBER OF SUBMATRICES ENTERED INTO VAR-COV SUBROUTINE
NTOBS	TOTAL NUMBER OF OBSERVATIONS (NYEARS*12+1) INCLUDING ONE FROM PREVIOUS YEAR FOR SERIAL CORRELATION
NVARS	NUMBER OF VARIABLES
NVCVU	NUMBER OF VALUES IN SYMMETRICALLY STORED VAR-COV MATRIX (NVARS*(NVARS+1)/2
NYEARS	NUMBER OF YEARS CONSIDERED FOR EACH VARIABLE
PROD	MATRIX (1,NQRTS) MATRIX MULTIPLICATION PRODUCT SUBROUTINE: WEIGHT
R	MATRIX CORRELATION COEFFICIENTS SUBROUTINE: SECOR VECTOR (NVARS*IN) RAMDOM DEVIATES
S	VECTOR CONTAINING STD. DEV. FOR VARIABLES SUBROUTINE: SECOR
SCORR	VECTOR CORRELATION COEFFICIENTS SUBROUTINE: SECOR
V	MATRIX (3,NVARS) STD. DEV., 3RD, 4TH MOMENTS
VAL	MATRIX LAGGED DATA BY QUARTERS SUBROUTINE: SECOR
VBLOWUP	MATRIX (NMONTHS,NMONTHS) FULL STORED EXPANDED VAR-COV MATRIX WITH DERIVED COVARIANCES INSERTED
D	
VCOVT	COVARIANCES DERIVED FROM VARIANCES AND SERIAL CORRELATIONS SUBROUTINE SECOR
VCVT	VECTOR (NQRTS*(NQRTS+1)/2 SYMMETRIC STORAGE OF VAR-COV MATRIX FOR NQRTS PERIODS
VCVU	VECTOR (NVARS*(NVARS+1)/2 SYMMETRIC STORAGE OF VAR-COV MATRIX UNADJUSTED FOR TIME
VMEANS	VECTOR (NVARS) OF MEANS FOR EACH VARIABLE
VTEMP	VECTOR (NVARS) FOR WORKING STORAGE
WKAREA	VECTOR (NQRTS**2+3*NQRTS) WORKSPACE SUBROUTINE: LINV2F (IMSL)
X	VECTOR RAW DATA- TRSFRG2

Table 2. (continued)

XM	VECTOR CONTAINS MEAN FOR VARIABLES- SUBROUTINE: SECOR
XMAX	VECTOR (NVARS) MAXIMUM VALUE FOR EACH VARIABLE
XMEAN	VECTOR (NVARS) MEAN FOR EACH VARIABLE
XMIN	VECTOR (NVARS) MINIMUM VALUE FOR EACH VARIABLE
XPRIME	MATRIX (NQRTS X NMONTHS) X-PRIME FOR NORMAL EQUATION
Y	VECTOR RAW DATA LAGGED ONE PERIOD- TRSPRG2
Y	MATRIX (NMONTHS,1) Y MATRIX FOR NORMAL EQUATION
ZNUM	NUMERATOR FOR WEIGHTED MEAN CALCULATIONS SUBROUTINE: WEIGHT

Appendix Table 3. Source listing - Program 1 (PRG1)

```
      PROGRAM TRSPRG1(INPUT,OUTPUT,TAPE1,TAPE2,TAPE3)
*
* THIS PROGRAM READS INPUT DATA (TAPE1) IN FREE FORMAT AND CALCULATES
* A VARIANCE-COVARIANCE MATRIX AND UNIQUE TRIANGULAR MATRIX
* FOR UP TO TEN VARIABLES.
*
* TAPE1 = INPUT DATA READ IN FREE-FORMAT ONE VARIABLE AFTER ANOTHER
*
* TAPE2 = OUTPUT UNIQUE TRIANGULAR MATRIX (AMAT)
*
      DIMENSION AMINPUT(62,50),VTEMP(50),VMEANS(50),VCVU(1275),
     +IVNBR(6),FVCVU(50,50),AMAT(50,50),INCO(1275),
     +XMOMENT(3,50)
      REWIND1
      REWIND2
      REWIND3
      IBUG=0
      PRINT*," "
      PRINT*," ****EXECUTING TRSPRG1****"
      PRINT*," "
      IFLAG=0
      PRINT*," ENTER NUMBER OF VARIABLES TO BE READ IN"
      READ*,NVARS
      PRINT*," ENTER NUMBER OF YEARS"
      READ*,NYEARS
      NN=NVARS*NYEARS
      PRINT*," ENTER NUMBER OF OBSERVATIONS PER YEAR"
      READ*,NYROBS
      IF(NVARS.LE.1)PRINT*," CANNOT CALCULATE VAR-COV MATRIX"
      PRINT*," ARE THE OBSERVATIONS FOR EACH VARIABLE WRITTEN "
      PRINT*," TO A TAPE? (1=YES:0=NO)"
      READ*,NY
      PRINT*," ENTER NUMBER OF OBSERVATIONS TO BE READ IN FOR EACH"
      PRINT*," VARIABLE INCLUDING TWO OBSERVATION FROM LAST TWO PERIODS"
      PRINT*," OF PREVIOUS YEAR."
      READ*,NTOBS
      PRINT*," DO YOU WANT FULL OUTPUT OF MATRICES PRINTED ON TERMINAL?"
      READ*,YY
      IF(YY.EQ.1)IFLAG=1
      IF(NY.EQ.1)GO TO 310
* DATA READING LOOP
22    DO 30 I=1,NVARS
      DO 25 J=1,NTOBS
      READ*,AMINPUT(J,I)
25    CONTINUE
      PRINT*," WERE ALL VALUES ENTERED CORRECTLY?"
      READ*,NY
      IF(NY.EQ.0)GO TO 31
      WRITE(1,*)(AMINPUT(NJ,I),NJ=1,NTOBS)
      GO TO 30
```

Table 3. (continued)

```
* DATA CORRECTION LOOP
31    PRINT*," COUNT DOWN FROM FIRST VALUE ENTERED TO INCORRECT VALUE?"
      PRINT*," TYPE IN POSITION OF  INCORRECT VALUE AND CORRECT VALUE"
      PRINT*," ON SAME LINE SEPARATED BY A COMMA"
      READ*,INCOR,OKCOR
      PRINT 35,INCOR,OKCOR
35    FORMAT(* OBSERVATION *,I3,* HAS BEEN CHANGED TO *,F10.5)
      PRINT*," IS THIS THE CORRECT CHANGE?"
      READ*,NY
      IF(NY.EQ.C)GO TO 31
      PRINT*," ARE THERE ANY OTHER CHANGES?"
      READ*,NY
      IF(NY.EQ.1)GO TO 31
      WRITE(1,*)(AMINPUT(NJ,I),NJ=1,NTOBS)
30    CONTINUE
      GO TO 32
310   READ(1,*)((AMINPUT(J,I),J=1,NTOBS),I=1,NVARS)
32    PRINT 33,NVARS,NTOBS
33    FORMAT(*0DATA HAS BEEN READ IN FOR *,I2,* VARIABLES,*,I3,
     +*  OBSERVATIONS PER VARIABLE*)
      J=1
      DO 10 I=1,NVARS
      PRINT 5,I,AMINPUT(J,I)
5     FORMAT(*0FIRST OBSERVATION VARIABLE  *,I2,* =*,F10.2)
      PRINT 6,I,AMINPUT(NTOBS,I)
6     FORMAT(* LAST OBSERVATION VARIABLE   *,I2,* =*,F10.2)
      J=1
10    CONTINUE
* ESTIMATES OF MEAN, STD. DEV., 3RD, 4TH MOMENTS  & CORRELATIONS
      REWIND1
      N=NTOBS-2
      DO 1 I=1,NVARS
      READ(1,*)(VTEMP(II),II=1,2)
      DO 2 J=1,N
      READ(1,*)AMINPUT(J,I)
2     CONTINUE
1     CONTINUE
      CALL BEMMI(AMINPUT,N,NVARS,62,VTEMP,XMOMENT,VCVU,INCD,IER)
      PRINT*,"  "
      CALL USWSM(18HCORRELATION MATRIX,18,VCVU,NVARS,2)
      IF(IBUG.EQ.1)PRINT*,IER,"    SUBROUTINE: BEMMI"
      DO 15 I=1,NVARS
      PRINT 16,I,VTEMP(I)
16    FORMAT(*0VARIABLE*,I2,*  ARITHMETIC MEAN =*,T35,F8.3)
      PRINT 17,XMOMENT(1,I)
17    FORMAT(* *,12X,*STANDARD DEV. =*,T35,F10.5)
      CV=XMOMENT(1,I)/VTEMP(I)*100.
      PRINT 18,CV
18    FORMAT(* *,12X,*COEFF. VAR. (PCT) =*,T35,F8.3)
      PRINT 19,XMOMENT(2,I)
19    FORMAT(* *,12X,*THIRD MOMENT =*,T35,F10.5)
```

Table 3. (continued)

```
      PRINT 20,XMOMENT(3,I)
20    FORMAT(* *,12X,*FOURTH MOMENT =*,T35,F10.5)
15    CONTINUE
      WRITE(3,*)((AMINPUT(J,I),J=1,N),I=1,NVARS)
      REWIND3
      READ(3,*)((AMINPUT(J,I),J=1,NYROBS),I=1,NN)
      CALL BEMMI(AMINPUT,NYROBS,NN,62,VTEMP,XMOMENT,VCVU,INCD,IER)
      IF(IBUG.EQ.1)PRINT*,IER,"    SUBROUTINE: BEMMI"
      PRINT*," "
      IF(IFLAG.EQ.1)CALL USWSM(18HCORRELATION MATRIX,18,VCVU,NN,2)
      NVCVU=(NN*(NN+1)/2)
      IVNBR(1)=NN
      IVNBR(2)=NYROBS
      IVNBR(3)=NYROBS
      IVNBR(4)=1
      IVNBR(6)=0
      CALL BECOVM(AMINPUT,62,IVNBR,VTEMP,VMEANS,VCVU,IER)
      IF(IBUG.EQ.1)PRINT*,IER,"   SUBROUTINE: BECOVM-1"
      CALL VCVTSF(VCVU,NN,FVCVU,50)
      IF(IBUG.EQ.1)PRINT*,"    SUBROUTINE: VCVTSF-1"
      PRINT*," "
* PRINT OUT VAR-COV MATRIX (UNADJUSTED)
      PRINT*," "
      IF(IFLAG.EQ.1)CALL USWFM(16HVAR - COV MATRIX,16,FVCVU,50,NN,NN,2)
* CALCULATE UNIQUE TRIANGULAR MATRIX "AMAT" FROM VAR-COV MATRIX
      CALL AMATRIX(FVCVU,NN,AMAT)
      PRINT*," "
* PRINT OUT UNIQUE TRIANGULAR "AMAT" MATRIX
      IF(IFLAG.EQ.1)CALL USWFM(11HAMAT MATRIX,11,AMAT,50,NN,NN,2)
      WRITE(2,*)((AMAT(I,J),J=1,NN),I=1,NN)
      PRINT*," "
      PRINT*," BE SURE TO CATALOG TAPE2 - UNIQUE TRIANGULAR MATRIX"
      PRINT*," TO BE USED LATER IN THE SIMULATION"
      PRINT*," "
      STOP
      END
      SUBROUTINE AMATRIX(FVCVU,NVARS,A)
      DIMENSION FVCVU(50,50),A(50,50)
      ICOL=NVARS
      IROW=NVARS
      ISTOP=NVARS-1
      CHEK11=0.
      DO 10 I=1,NVARS
      DO 10 J=1,NVARS
      A(I,J)=0.
10    CONTINUE
*
* CALCULATE THE NVARS TH COLUMN
*
      A(NVARS,NVARS)=SQRT(FVCVU(NVARS,NVARS))
      DO 50 I=1,ISTOP
```

Table 3. (continued)

```
50      A(I,NVARS)=FVCVU(I,NVARS)/A(NVARS,NVARS)
*
* NEXT DIAGONAL ELEMENT
*
90      ICOL=ICOL-1
        IROW=ICOL
        IROWP1=IROW+1
        SUM=0.
        DO 100 K=IROWP1,NVARS
100     SUM=SUM+A(IROW,K)**2
        IF(SUM.GT.A(IROW,ICOL))SUM=A(IROW,ICOL)
        A(IROW,ICOL)=SQRT(FVCVU(IROW,ICOL)-SUM)
        IF(ICOL.EQ.1)GO TO 220
*
* COMPLETE THE COLUMN
*
        ICOLP1=ICOL+1
        ISTOP=ISTOP-1
        DO 200 J=1,ISTOP
        IROW=IROW-1
        IF(IROW.EQ.0)GO TO 210
        SUM=0.
        DO 150 K=ICOLP1,NVARS
150     SUM=SUM+A(IROW,K)*A(ICOL,K)
        A(IROW,ICOL)=(FVCVU(IROW,ICOL)-SUM)/A(ICOL,ICOL)
200     CONTINUE
210     GO TO 90
220     CONTINUE
        RETURN
        END
```

Appendix Table 4. Source listing - Program 2 (PRG2)

```
C
      PROGRAM TRSPRG2(INPUT,OUTPUT,TAPE1,TAPE4,TAPE5,TAPE20)
      DIMENSION X(133),VMEANS(3),R(3),SCORR(44),S(44),
     +AMQRTS(3,44),VTEMP(44),VCVT(1830),VCOVT(44),VAL(3,2),IVNBR(6),
     +XM(2),VBLOWUP(60,60),VCVTF(20,20),W(5),AC(60),ACV(60),PACV(60),
     +WKAREA(12),VARS(60)
*
* TAPE1  = INPUT DATA READ IN FREE FORMAT ONE VARIABLE AFTER ANOTHER
*
* TAPE4  = OUTPUT- FULL STORED VAR-COV MATRIX- TIME ADJUSTED
*
* TAPE5  = EXPANDED VAR-COV MATRIX- TO GO INTO INVERSION PROGRAM
*
* TAPE20 = AUTO-CORRELATIONS TO BE USED TO CALCULATE AUTO-COVARIANCES
*
      REWIND1
      REWIND4
      REWIND5
      REWIND20
*
      IBUG=0
      PRINT*," "
      PRINT*," ****EXECUTING TRSPRG2****"
      PRINT*," "
      PRINT*," ENTER NUMBER OF VARIABLES TO BE READ IN"
      READ*,NVARS
      PRINT*," ENTER NUMBER OF YEARS DATA REPRESENTS"
      READ*,NYEARS
      PRINT*," ENTER NUMBER OF OBSERVATIONS TO BE READ IN FOR EACH"
      PRINT*," VARIABLE INCLUDING TWO OBSERVATIONS FROM LAST TWO PERIODS
     +"
      PRINT*," OF PREVIOUS YEAR."
      READ*,NTOBS
      NMONTHS=NYEARS*12
      NQOBS=3
      NQRTS=NYEARS*4
*
      DO 19 I=1,NVARS
      PRINT*," "
      PRINT*," VARIABLE: ",I
      READ(1,*)(X(L),L=1,NTOBS)
*
      NVCVT=(NQRTS*(NQRTS+1))/2
*
* CALCULATE AUTO-CORRELATIONS AND AUTO-COVARIANCES
      CALL SECOR(X,NTOBS,NQRTS,NQOBS,VMEANS,VCVT,VCOVT,NVCVT)
*
* INSERT DERIVED COVARIANCES INTO VBLOWUP
      CALL INSERT(VCVT,VCOVT,NQRTS,NMONTHS,VBLOWUP)
*
* WRITE EXPANDED VAR-COV MATRIX TO GO INTO INVERSION PROGRAM
```

Table 4. (continued)

```
      WRITE(5,*)((VBLOWUP(IX,JX),JX=1,NMONTHS),IX=1,NMONTHS)
19    CONTINUE
*
      PRINT*," "
      PRINT*," CATALOG TAPES TO GO INTO THE MATRIX INVERSION PROGRAM"
      STOP
      END
*
*
      SUBROUTINE SECOR(X,NTOBS,NQRTS,NQOBS,VMEANS,VCVT,VCOVT,NVCVT)
      DIMENSION X(133),VMEANS(3),R(3),S(44),W(5),AC(60),ACV(60),P
     +ACV(60),
     +WKAREA(12),VARS(60),VBLOWUP(60,60),
     +AMQRTS(3,44),VTEMP(44),VCVT(1830),VCOVT(44),VAL(3,2),IVNBR(6),XM(2
     +)
*
      N=1
      NMONTHS=NQRTS*3
* CALCULATE AUTO COVARIANCES
      LA=1
      DO 99 IS=1,NQRTS
      DO 88 J=1,5
      W(J)=X(N)
      N=N+1
88    CONTINUE
      LW=5
      K=L=2
      ISW=7
      CALL FTAUTO(W,LW,K,L,ISW,AMEAN,VAR,ACV,AC,PACV,WKAREA)
      DO 55 IA=1,2
* PUT ONE AND TWO LAG AUTO-CORRELATIONS INTO VCOVT
      VCOVT(LA)=AC(IA)
      LA=LA+1
55    CONTINUE
      N=N-2
99    CONTINUE
      NVCVT=(NQRTS*(NQRTS+1)/2)
      IVNBR(1)=NQRTS
      IVNBR(2)=NQOBS
      IVNBR(3)=NQOBS
      IVNBR(4)=1
      IVNBR(6)=C
* CALCULATE VAR-COV MATRIX FOR THE I-TH VARIABLE BY QUARTERS
* WRITE DATA INTO THREE OBSERVATIONS PER QUARTER
      N=2
      DO 90 IS=1,NQRTS
      DO 80 J=1,3
      N=N+1
      AMQRTS(J,IS)=X(N)
80    CONTINUE
90    CONTINUE
```

Table 4. (continued)

```
      CALL BECOVM(AMQRTS,NQOBS,IVNBR,VTEMP,VMEANS,VCVT,IER)
      PRINT*," OK"
      N=IK=M=1
* VCCVT: COVARIANCES DERIVED FROM VARIANCES AND SERIAL CORRELATIONS
      DO 21 K=1,NQRTS
      VCOVT(M)=VCOVT(M)*VCVT(IK)
      M=M+1
      VCOVT(M)=VCCVT(M)*VCVT(IK)
      M=M+1
      N=N+1
      IK=IK+N
21    CONTINUE
* CONVERT SYMMETRIC VCV MATRIX TO FULL STORAGE MOE
      CALL VCVTSF(VCVT,NQRTS,VBLOWUP,60)
      WRITE(4,*)((VBLOWUP(IV,JV),JV=1,NQRTS),IV=1,NQRTS)
      JX=1
* EXPAND DIAGONAL OF VCV (3X): TO GO INTO VCV-EXPANDED MATRIX
      DO 50C IT=1,NQRTS
      DO 60C JT=1,NQOBS
      VARS(JX)=VBLOWUP(IT,IT)
      JX=JX+1
600   CONTINUE
500   CONTINUE
      KK=1
      NAC=NTOBS-3
      CALL FTAUTO(X,NTOBS,NAC,0,5,AMEAN,VAR,ACV,AC,PACV,WKAREA)
      WRITE(20,*)(AC(LL),LL=1,NAC)
*FULL MATRIX
      DO 100 IO=1,NMONTHS
      DO 200 JO=1,NMONTHS
      LAG=IABS(JO-IO)
      IF(LAG.EQ.0)GO TO 150
      VBLOWUP(IO,JO)=AC(LAG)*VARS(IO)
      GO TO 200
150   VBLOWUP(IO,JO)=VARS(IO)
200   CONTINUE
100   CONTINUE
      CALL VCVTFS(VBLOWUP,NMONTHS,60,VCVT)
      RETURN
      END
      SUBROUTINE INSERT(VCVT,VCOVT,NQRTS,NMONTHS,VBLOWUP)
      DIMENSION VCVT(1830),VCOVT(40),VBLOWUP(60,60)
      NVCVT=NMONTHS*(NMONTHS+1)/2
      K=9
      J=2
      I=2
      N=1
1     VCVT(I)=VCCVT(N)
      I=I+J
      NEXT=N+1
      VCVT(I)=VCCVT(NEXT)
```

Table 4. (continued)

```
      I=I+1
      VCVT(I)=VCCVT(N)
      IF(I.EQ.NVCVT-1)GO TO 99
      I=I+K
      J=J+3
      K=K+6
      N=N+2
      GO TO 1
99    CONTINUE
      CALL VCVTSF(VCVT,NMONTHS,VBLOWUP,60)
      RETURN
      END
```

Appendix Table 5. Source listing - Program 3 (PRG3)

```
0
      PROGRAM TRSPRG3(INPUT,OUTPUT,TAPE5,TAPE6)
      DIMENSION A(60,60),AINV(60,60),WKAREA(3780)
*
* TAPE5 = EXPANDED VAR-COV MATRIX- TO 60 INTO INVERSION PROGRAM
*
* TAPE6 = INVERSE OF EXPANDED VAR-COV MATRIX
*
      REWIND5
      REWIND6
      IBUG=0
      PRINT*," "
      PRINT*," ****EXECUTING TRSPRG3****"
      PRINT*," "
      PRINT*," ENTER NUMBER OF VARIABLES"
      READ*,NVARS
      PRINT*," ENTER NUMBER OF YEARS DATA REPRESENTS"
      READ*,NYEARS
      PRINT*," ENTER NUMBER OF OBSERVATIONS PER YEAR"
      READ*,NOBS
      NMONTHS=NYEARS*NOBS
      IDGT=0
      DO 77 I=1,NVARS
      READ(5,*)((A(IN,JN),JN=1,NMONTHS),IN=1,NMONTHS)
* INVERSION OF "A" USING HIGH ACCURACY INVERSION ROUTINE
* INVERSE OF "A" = "AINV"
      IF(IBUG.EQ.0)CALL UERSET(1,4)
      CALL LINV2F(A,NMONTHS,60,AINV,IDGT,WKAREA,IER)
      IF(IBUG.EQ.1)PRINT*,IER,"    SUBROUTINE: LINV2F    VARIABLE: ",I
      IF(IER.EQ.0)PRINT*," INVERSE COMPUTED- VARIABLE ",I
      IF(IER.EQ.129)PRINT*," MATRIX IS ALGORITHMICALLY SINGULAR"
      IF(IER.EQ.131)PRINT*," MATRIX TOO ILL-CONDITIONED FOR ITERATIVE IM
     +PROVEMENT"
      IF(IER.EQ.34)PRINT*," INVERSE COMPUTED- ACCURACY TEST FAILED"
      WRITE(6,*)((AINV(IZ,JZ),JZ=1,NMONTHS),IZ=1,NMONTHS)
77    CONTINUE
      PRINT*," "
      PRINT*," CATALOG TAPE6: WHICH CONTAINS INVERSES FOR ",NVARS," VARI
     +ABLE(S)"
      PRINT*," "
      END
```

Appendix Table 6. Source listing - Program four (PGR4)

```
C
      PROGRAM TRSPRG4(INPUT,OUTPUT,TAPE1,TAPE4,TAPE6,TAPE7,TAPE8,TAPE9,
     +TAPE10,TAPE11)
      DIMENSION XPRIME(20,60),C(20,60),Y(60,1),D(20,1),E(20,20),
     +EINV(20,20),F(20,1),WKAREA(460),AINV(60,60)
      IBUG=0
*
* TAPE1 = TRSSYMATRIX:Y-MATRIX
*
* TAPE4 = FULL STORAGE OF VAR-COV MATRICES-EACH VARIABLE
*
* TAPE6 = INVERSE OF BLOWN-UP VAR-COV MATRIX: V-INV
*
* TAPE7 = WEIGHTED MEAN TO GO INTO SIMULATION
*
* TAPE8 = RIGHT-HAND SIDE: (X-PRIME V-INV Y)
*
* TAPE9 = (X-PRIME V-INV X)
*
* TAPE10 = SOLUTIONS:  (X-PRIME V-INV X)INV *Y
*
* TAPE11 = SIMULATED DATA VECTORS
*
      REWIND1
      REWIND4
      REWIND6
      REWIND8
      REWIND9
      REWIND10
      L=1
      PRINT*," "
      PRINT*," ****EXECUTING TRSPRG4****"
      PRINT*," "
      PRINT*," ENTER NUMBER OF VARIABLES"
      READ*,NVARS
      PRINT*," ENTER NUMBER OF YEARS DATA REPRESENTS"
      READ*,NYEARS
      PRINT*," ENTER NUMBER OF OBSERVATIONS PER YEAR"
      READ*,NOBS
      NMONTHS=NYEARS*NOBS
      M=NMONTHS/3
      MQRTS=M
      NYEARS=NMONTHS/12
* N AND IA (NMONTHS) ARE EQUAL TO SIZE OF BLOWN-UP VAR-COV MATRIX
      N=NMONTHS
      IA=NMONTHS
      IDGT=0
* READ IN THE INVERSE OF THE BLOWN-UP VARIANCE COVARIANCE MATRIX
      DO 800 IN=1,NVARS
      PRINT*," VARIABLE: ",IN
      READ(6,*)((AINV(I,J),J=1,N),I=1,N)
```

Table 6. (continued)

```
* GENERATE THE X-PRIME MATRIX
      CALL XMATRIX(M,NMONTHS,XPRIME)
*PREMULTIPLY V INVERSE BY X PRIME
370   CALL VMULFF(XPRIME,AINV,M,M,N,20,60,C,20,IER)
      IF(IBUG.EQ.1)PRINT*,IER,"    SUBROUTINE: VMULFF-1"
      IF(IER.EQ.129)PRINT 129
129   FORMAT(* *,*TERMINAL ERROR (129) INDICATES MATRICES BEING*/,
     +* MULTIPLIED WERE DIMENSIONED INCORRECTLY*)
*READ Y MATRIX AND PREMULTIPLY BY X PRIME V INVERSE
      J=1
* DELETE FIRST TWO VALUES FROM RAW DATA (USED FOR SERIAL CORRELATION)
      DO 33 II=1,2
      READ(1,*)DELETE
33    CONTINUE
* READ IN Y-MATRIX
      READ(1,*)(Y(I,1),I=1,N)
      CALL VMULFF(C,Y,M,N,L,20,60,D,20,IER)
      IF(IBUG.EQ.1)PRINT*,IER,"    SUBROUTINE: VMULFF-2"
      IF(IER.EQ.129)PRINT 129
* MULTIPLICATION OF X PRIME V INVERSE BY TRANSPOSE OF X PRIME IE. X
      CALL VMULFP(C,XPRIME,M,N,M,20,20,E,20,IER)
      IF(IBUG.EQ.1)PRINT*,IER,"    SUBROUTINE: VMULFP"
      IF(IER.EQ.129)PRINT 129
* CALCULATE INVERSE; X PRIME V INVERSE X
      IF(IBUG.EQ.0)CALL UERSET(1,4)
      CALL LINV2F(E,M,20,EINV,IDGT,WKAREA,IER)
      IF(IBUG.EQ.1)PRINT*,IER,"    SUBROUTINE: LINV2F"
      IF(IER.EQ.129)PRINT*," MATRIX ALGORITHMICALLY SINGULAR"
* MULTIPLY RHS BY X PRIME V INVERSE X INVERSE TO GET ESTIMATES
      CALL VMULFF(EINV,D,M,M,L,20,20,F,20,IER)
      IF(IBUG.EQ.1)PRINT*,IER,"    SUBROUTINE: VMULFF-3"
      IF(IER.EQ.129)PRINT 129
      J=1
      WRITE(10,*)(F(I,L),I=1,M)
      IL=1
      PRINT*," VARIABLE  ",IN,"QUARTERLY SOLUTIONS:"
      DO 1 I=1,NYEARS
      PRINT*,"    YEAR: ",I
      DO 2 IJ=1,4
      PRINT*,"       QUARTER: ",F(IL,L)
      IL=IL+1
2     CONTINUE
1     CONTINUE
800   CONTINUE
      STOP
      END
      SUBROUTINE XMATRIX(ROW,COLUMN,XPRIME)
      DIMENSION XPRIME(20,60)
      INTEGER ROW,COLUMN
      L=1
      N=COLUMN/ROW
```

Table 6. (continued)

```
      NN=N
      DO 10 I=1,ROW
      DO 8 J=1,COLUMN
      IF(J.EQ.L.AND.J.LE.N)GO TO 7
      XPRIME(I,J)=0
      GO TO 6
7     XPRIME(I,J)=1
6     IF(J.EQ.L)L=L+1
8     CONTINUE
      N=N+NN
      L=N-(NN-1)
10    CONTINUE
      RETURN
      END
```

Appendix Table 7. Source listing - Program 5 (PRG5)

```
0
      PROGRAM TRSPRG5(INPUT,OUTPUT,TAPE2,TAPE10,TAPE11)
      DIMENSION R(1000),A(50,50),XMEAN(50),XMIN(50),XMAX(50),
     +V(3,50),WTMEANS(50),INCD(55),SMEAN(4),UA(1000,10),C(55)
      IBUG=0
*
* TAPE2 = UNIQUE TRIANGULAR "A" AMATRIX (AMAT) FROM PRG1
* TAPE10 = QUARTERLY SOLUTIONS FROM PRG4.
* TAPE11 = SIMULATED DATA
*
      REWIND2
      REWIND10
      REWIND11
* NVARS = NUMBER OF VARIABLES - DIMENSION OF VAR-COV MATRIX
* IN = NUMBER OF DEVIATES TO BE GENERATED
* NYEARS = NUMBER OF YEARS
      PRINT*," "
      PRINT*," ****EXECUTING TRSPRG5****"
      PRINT*," "
      PRINT*," ENTER NUMBER OF VARIABLES"
      READ*,NVARS
      PRINT*," ENTER NUMBER OF YEARS"
      READ*,NYEARS
      NN=NVARS*NYEARS
      IN=1000/NYEARS
      READ(2,*)((A(I,J),J=1,NN),I=1,NN)
*
* CALCULATE YEARLY MEANS FROM QUARTERLY SOLUTIONS
      ZSUM=0.0
      DO 10 I=1,NN
      DO 11 J=1,4
      READ(10,*)SMEAN(J)
      ZSUM=ZSUM+SMEAN(J)
11    CONTINUE
      WTMEANS(I)=ZSUM/4.
      ZSUM=0.0
10    CONTINUE
*
      DSEED=10.00
* WARM-UP RANDOM NUMBER GENERATOR
      N=100
      CALL  GGNML(DSEED,N,R)
      CONTINUE
*
* RANDOM OR GAUSSIAN RANDOM DEVIATE GENERATOR
      CALL GGNML(DSEED,IN,R)
      ZCUM=0.0
      K=1
      DO 5 I=1,NN
      DO 4 M=1,IN
      DO 3 J=1,NN
```

Table 7. (continued)

```
      Z=A(I,J)*R(K)
      ZCUM=ZCUM+Z
      K=K+1
      IF(K.GE.IN)K=1
3     CONTINUE
      U=ZCUM+WTMEANS(I)
      WRITE(11,*)U
      ZCUM=Z=0.0
4     CONTINUE
      CALL GGNML(DSEED,IN,R)
5     CONTINUE
*
* FIND MINIMUM AND MAXIMUM VALUES GENERATED FOR EACH VARIABLE
      NIN=NYEARS*IN
      REWIND11
      DO 12 I=1,NVARS
      READ(11,*)(R(J),J=1,NIN)
      CALL USMNMX(R,NIN,1,ZMIN,ZMAX)
      XMIN(I)=ZMIN
      XMAX(I)=ZMAX
12    CONTINUE
*
* ESTIMATES OF SIMULATED MEANS, STD. DEV.,3RD, 4TH MOMENTS,CORRELATIONS
      REWIND11
      READ(11,*)((UA(J,I),J=1,NIN),I=1,NVARS)
      CALL BEMMI(UA,NIN,NVARS,1000,XMEAN,V,C,INCD,IER)
      IF(IBUG.EQ.1)PRINT*,IER,"    SUBROUTINE: BEMMI"
      PRINT*," "
      CALL USWSM(18HCORRELATION MATRIX,18,C,NVARS,2)
      DO 90 I=1,NVARS
      PRINT 91,I,XMEAN(I)
91    FORMAT(*0VARIABLE *,I2,* SIMULATED MEAN =*,T35,F8.3)
      PRINT 92,V(1,I)
92    FORMAT(* *,12X,*STANDARD DEV. =*,T35,F10.5)
      CV=V(1,I)/XMEAN(I)*100.
      PRINT 97,CV
97    FORMAT(* *,12X,*COEFF. VAR. (PCT) =*,T35,F8.3)
      PRINT 93,V(2,I)
93    FORMAT(* *,12X,*THIRD MOMENT =*,T35,F10.5)
      PRINT 94,V(3,I)
94    FORMAT(* *,12X,*FOURTH MOMENT =*,T35,F10.5)
      PRINT 95,XMIN(I)
95    FORMAT(* *,12X,*MINIMUM VALUE =*,T35,F8.3)
      PRINT 96,XMAX(I)
96    FORMAT(* *,12X,*MAXIMUM VALUE =*,T35,F8.3)
90    CONTINUE
      PRINT*," "
      PRINT*," **** END OF SIMULATION ****"
      PRINT*," "
      STOP
      END
```

Appendix Table 8. Sample output from interactive computer programs

```
                              -----
 ****EXECUTING TPSPRG1****

 ENTER NUMBER OF VARIABLES TO BE READ IN
*4
 ENTER NUMBER OF YEARS
*5
 ENTER NUMBER OF OBSERVATIONS PER YEAR
*12
 ARE THE OBSERVATIONS FOR EACH VARIABLE WRITTEN
 TO A TAPE? (1=YES;0=NO)
*1
 ENTER NUMBER OF OBSERVATIONS TO BE READ IN FOR EACH
 VARIABLE INCLUDING TWO OBSERVATION FROM LAST TWO PERIODS
 OF PREVIOUS YEAR.
*62
 DO YOU WANT FULL OUTPUT OF MATRICES PRINTED ON TERMINAL?
*0

DATA HAS BEEN READ IN FOR  4 VARIABLES, 62  OBSERVATIONS PER VARIABLE

FIRST OBSERVATION VARIABLE    1 =       5.60
LAST OBSERVATION VARIABLE     1 =       6.30

FIRST OBSERVATION VARIABLE    2 =     270.00
LAST OBSERVATION VARIABLE     2 =     395.00

FIRST OBSERVATION VARIABLE    3 =      16.50
LAST OBSERVATION VARIABLE     3 =      20.60

FIRST OBSERVATION VARIABLE    4 =      26.50
LAST OBSERVATION VARIABLE     4 =      42.00

CORRELATION MATRIX
                  1                  2                  3
                  4

  1             1.00000

  2              .76613            1.00000

  3              .37733             .31711            1.00000

  4              .73135             .93762             .32517
                1.00000

VARIABLE 1   ARITHMETIC MEAN =        5.311
             STANDARD DEV. =           .27248
             COEFF. VAR. (PCT) =      4.689
             THIRD MOMENT =          -.01569
             FOURTH MOMENT =         -.58423

VARIABLE 2   ARITHMETIC MEAN =      336.000
             STANDARD DEV. =         39.00022
             COEFF. VAR. (PCT) =     11.607
             THIRD MOMENT =           -.05792
             FOURTH MOMENT =        -1.39855

VARIABLE 3   ARITHMETIC MEAN =       19.012
             STANDARD DEV. =          2.03054
             COEFF. VAR. (PCT) =     10.680
             THIRD MOMENT =           -.02663
             FOURTH MOMENT =        -1.20391

VARIABLE 4   ARITHMETIC MEAN =       33.185
             STANDARD DEV. =          5.17254
             COEFF. VAR. (PCT) =     15.587
             THIRD MOMENT =            .24772
             FOURTH MOMENT =        -1.50126


 BE SURE TO CATALOG TAPE2 - UNIQUE TRIANGULAR MATRIX
 TO BE USED LATER IN THE SIMULATION
```

Table 8. (continued)

```
****EXECUTING TRSPP62****

ENTER NUMBER OF VARIABLES TO BE READ IN
•4
ENTER NUMBER OF YEARS DATA REPRESENTS
•5
ENTER NUMBER OF OBSERVATIONS TO BE READ IN FOR EACH
VARIABLE INCLUDING TWO OBSERVATIONS FROM LAST TWO PERIODS
OF PREVIOUS YEAR.
•62

VARIABLE: 1
OK

VARIABLE: 2
OK

VARIABLE: 3
OK

VARIABLE: 4
OK

CATALOG TAPES TO GO INTO THE MATRIX INVERSION PROGRAM
      STOP
       1.740 CP SECONDS EXECUTION TIME
OK-ATT,P3,TRSPP63LGO.HAL.P3.
    ATTACHING-TRSPR63LGO
HAL 5.77
EXEC BEGUN.13.05.02.

****EXECUTING TRSPP63****

ENTER NUMBER OF VARIABLES
•4
ENTER NUMBER OF YEARS DATA REPRESENTS
•5
ENTER NUMBER OF OBSERVATIONS PER YEAR
•12
INVERSE COMPUTED- VARIABLE 1
INVERSE COMPUTED- ACCURACY TEST FAILED
INVERSE COMPUTED- ACCURACY TEST FAILED
```

Table 8. (continued)

```
****EXECUTING TRSPRG4*****

 ENTER NUMBER OF VARIABLES
*3
 ENTER NUMBER OF YEARS DATA REPRESENTS
*5
 ENTER NUMBER OF OBSERVATIONS PER YEAR
*12
  VARIABLE   1 QUARTERLY SOLUTIONS:
    YEAR: 1
      QUARTER: 1   5.540551653562
      QUARTER: 2   5.418719844265
      QUARTER: 3   5.476568749422
      QUARTER: 4   5.600820262087
    YEAR: 2
      QUARTER: 1   5.755995284612
      QUARTER: 2   5.745949929322
      QUARTER: 3   5.651458258452
      QUARTER: 4   5.765312975663
    YEAR: 3
      QUARTER: 1   5.825643068983
      QUARTER: 2   5.970776486342
      QUARTER: 3   5.75297581644
      QUARTER: 4   5.948924730206
    YEAR: 4
      QUARTER: 1   5.895694409969
      QUARTER: 2   5.982557471951
      QUARTER: 3   5.864737655905
      QUARTER: 4   6.165929819315
    YEAR: 5
      QUARTER: 1   6.218632674302
      QUARTER: 2   6.108844205485
      QUARTER: 3   6.039902573676
      QUARTER: 4   6.139346852044
  VARIABLE   2 QUARTERLY SOLUTIONS:
    YEAR: 1
      QUARTER: 1   273.2039117329
      QUARTER: 2   284.1910949706
      QUARTER: 3   287.687673585
      QUARTER: 4   287.0158431972
    YEAR: 2
      QUARTER: 1   296.1126400849
      QUARTER: 2   301.7239480258
      QUARTER: 3   311.4423672112
      QUARTER: 4   310.9100735176
    YEAR: 3
      QUARTER: 1   304.3247712147
      QUARTER: 2   321.582035075
      QUARTER: 3   345.4979878522
      QUARTER: 4   353.7149076164
    YEAR: 4
      QUARTER: 1   359.2207956999
      QUARTER: 2   348.3338520727
      QUARTER: 3   342.839937549
      QUARTER: 4   363.6053399717
    YEAR: 5
      QUARTER: 1   354.7968331681
      QUARTER: 2   389.4066265196
      QUARTER: 3   381.3597948303
      QUARTER: 4   395.711952453
  VARIABLE   3 QUARTERLY SOLUTIONS:
    YEAR: 1
      QUARTER: 1   16.17570072738
```

Table 8. (continued)

```
 ****EXECUTING TRSPPG5****

 ENTER NUMBER OF VARIABLES
*3
 ENTER NUMBER OF YEARS
*5

CORRELATION MATRIX
                  1                  2                  3

  1            1.00000

  2             .70392           1.00000

  3             .08123            .09779            1.00000

VARIABLE   1 SIMULATED MEAN =        5.349
             STANDARD DEV. =          .26334
             COEFF. VAR. (PCT) =     4.930
             THIRD MOMENT =          -.07992
             FOURTH MOMENT =         -.13494
             MINIMUM VALUE =         5.012
             MAXIMUM VALUE =         6.658

VARIABLE   2 SIMULATED MEAN =      330.417
             STANDARD DEV. =        34.68460
             COEFF. VAR. (PCT) =    10.497
             THIRD MOMENT =           .26325
             FOURTH MOMENT =         -.90967
             MINIMUM VALUE =       283.727
             MAXIMUM VALUE =       422.397

VARIABLE   3 SIMULATED MEAN =       18.124
             STANDARD DEV. =        10.95501
             COEFF. VAR. (PCT) =    60.444
             THIRD MOMENT =          -.32791
             FOURTH MOMENT =         1.31560
             MINIMUM VALUE =       -27.055
             MAXIMUM VALUE =        64.665

 **** END OF SIMULATION ****

     STOP
      1.727 CP SECONDS EXECUTION TIME
OK-
```

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