FACTOR SUBSTITUTABILITY, EFFICIENCY GROWTH, AND RELATIVE WAGE INCOME SHARES IN THE KOREAN AGRICULTURAL AND MANUFACTURING SECTORS, 1955 - 1974

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ABSTRACT

FACTOR SUBSTITUTABILITY, EFFICIENCY GROWTH, AND RELATIVE WAGE INCOME SHARES IN THE KOREAN AGRICULTURAL AND MANUFACTURING SECTORS, 1955-1974

By

Young Sik Kim

This study is based on the distributional problem which has attracted increasing attention since the Korean economy has succeeded in achieving growth objectives beyond expectations in the last decade or so. This study is primarily interested in the changes in the relative position of wage earners and the economic forces underlying such changes for the Korean agricultural and manufacturing sectors during the last two decades.

The analytical framework of the study is based on the neoclassical marginal productivity theory with a specified CES production function. Using this theory, this study attempts to explain the behavior of the actual wage income shares.

The estimate of wage income shares shows a significantly different trend between the sectors: a decreasing trend for the agricultural sector, and an increasing trend for the manufacturing sector.

For an explanation of the actual behavior of the wage income share, we estimated the parameters determining the wage income share with various different assumptions and estimation procedures. The major estimation results are as follows. The elasticity of substitution is greater than unity for the agricultural sector and less than unity for the manufacturing sector. The capital augmenting parameter turned out

to be greater than that of labor for both the sectors. The estimate of the scale parameter was close to unity for both the sectors.

The bias of technical progress was measured. The technical progress was capital-using for the agricultural sector and labor-using for the manufacturing sector. It also showed that the technical progress has been more significantly biased in the manufacturing sector than in the agricultural sector.

The theoretical rate of change in the wage income share was predicted. The general direction of the predicted rates of the change is consistent with the actual rates of the changes for both the sectors. However, the major source of the change seems to be different between the sectors. For the agricultural sector, the capital deepening factor appears to be a main source of the change; but for the manufacturing sector the biased technical progress has been more important in affecting the change.

The estimated value of the marginal product of manufacturing labor was far above the actual wage rate; thus wage earners are paid less than their actual contribution to production. But the value of the marginal product of agricultural labor was far below the wage rate; thus the self-employed agricultural workers receive low returns for their labor. The value of the marginal product of agricultural labor is considerably lower than that of manufacturing labor for the whole period.

The marginal productivity theory seems to be consistent with the actual movement of wage income shares in their general directions, but there are considerable differences in actual magnitudes. This difference may indicate that there are some other factors, or disequilibrium factors, playing an important role in the Korean economy. However, the importance of the disequilibrium factors within and between the sectors seems to decrease over time. Thus the Korean economy may have been in the process of adjusting to an equilibrium during the period covered in this study.

However, the weakness in the data and possible errors in estimation could mean that the economy may not have been in disequilibrium as indicated by the results. More work on this possibility is needed. If such a disequilibrium does in fact exist, there are various possible disequilibrium factors in the process of the rapid growth of the Korean economy in the last decade. The analysis of various possible causes of disequilibrium will be another important subject to be investigated.

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CHAPTER I

INTRODUCTION

Background of the Study

During the last two decades, the Korean economy has drastically changed its output growth and input structure. The direction and rate of change have been significantly different over time and between sectors, particularly in the agricultural and manufacturing sectors, which are covered in this study.

The growth rate of the economy has been phenomenal. The GNP grew at about 5.0 percent annual rate in the first decade (1955-64) and 10.3 percent in the second decade (1965-74). The manufacturing sector output grew at about a 9.8 percent annual rate in the first decade and a 19.8 percent annual rate in the second decade, with an accelerating growth rate since the early 1960's. The agricultural sector output grew at about a 3.5 percent annual rate with no significant trend. Thus the manufacturing sector share of the GNP increased from about 11 percent for 1955-56 to about 27 percent for 1973-74, while that of the agricultural sector was reduced from about 45 percent to 25 percent during the same period.

To achieve this remarkable output growth there were also significant changes in the magnitude and the structure of resource employment. For agricultural production, the labor input showed a decreasing trend at an annual rate of about 2.7 percent since 1965. For manufacturing production, the labor input increased steadily at an annual rate of about 12.0 percent with no significant trend over time. Capital input, measured at 1970 constant prices based on various sources, also showed a significant change over time and between sectors. For agriculture, the growth rate of capital input was 2.4 percent for the first decade and 10.2 percent for the second decade. For the manufacturing sector, it was 7.6 percent and 17.6 percent for the respective periods. In the general trend for both sectors, the growth of labor input was faster than that of capital input for the first decade, but this trend changed to the opposite direction for the second decade. Thus the capital-labor ratio has increased only after the mid-1960's. Before that, the growth in the Korean economy was primarily due to increased labor input.

As a result of these changes in output growth and input employment structure, output per unit of input also changed significantly during the period. Average productivity of labor, defined as output per unit of labor input, showed rather a decreasing trend until the mid-1960's in both sectors. But it changed its trend after the mid-1960's. The growth rate of average labor productivity was about 5.8 percent for agriculture and 8.6 percent for the manufacturing sector since 1965. Considering the growth rate of output and inputs, particularly since the mid-1960's, it seems clear that there were some factors other than increase in physical inputs to sustain the continued high growth rate of output, which may be called technical change or efficiency growth of conventional inputs.

On the other hand, during the same period, the real wage rate also changed, with a significant relationship to the average labor productivity, which is consistent with the concave production function and marginal productivity theory of wages. More detailed analysis of the relationships between the economic variables mentioned above will be dealt with in later chapters.

In the process of these significant changes in output growth, resource employment, and economic structure, the distribution problems of output may be considered in many different dimensions. For example, how has distribution of output been changed between sectors, regions, different resource ownership, and some other economic groups classified by different interests. What have been the major economic forces causing these changes, and how does the distribution itself affect other economic variables. This study is concerned with only one aspect of these many potential distributional problems: namely, an analysis of changes in the distribution of income between wage earners and capital owners in the agricultural and manufacturing sectors. More detailed scope and objectives of the study will be stated in the next section.

Scope and Objectives of the Study

There is a wide variety of distribution problems in economics, each corresponding to a different division of society into social groups, classes, or regions. According to the literature, economists have concentrated their attention on two bases for such divisions.

The major distribution problem has been the so-called functional distribution. By functional distribution is meant the division of income as between income from labor and income from property (nonhuman capital, wealth, or assets). The secondary distribution problem as seen by economists has been personal distribution. By personal distribution is meant division of income or wealth by size brackets of the income or wealth of economic units. There are many other bases for income distribution such as occupational distribution, geographical or regional distribution, international distribution, racial distribution, and so on.

At the macroeconomic level, the distributional question of greatest interest concerns the behavior of aggregate relative income shares or income distribution to each productive resource. The total income to the supplier of any productive input or service depends upon the quantity of that input employed and its return per unit. It is with this functional income distribution that this study is concerned.

Distribution of income between different productive factors alone may have limited meaning to the contemporary question of personal or size distribution. For many individuals or groups, income consists of both labor and capital income, even though there are some differences in the proportions of the sources of income. This will be particularly true in the agricultural sector in which most of the farmers are self-employed.

Thus the distributional question between any groups or individuals, which are not classified as pure wage earner or capitalist, is related to the income distribution between productive resources, and also to the distribution of resource ownership. But assuming that the ownership of some resources is more equitably distributed than others, then a more favorable income share to the first resource will lead to more equitable distribution of income as a whole.

Income is created in production, and factors of production collaborate to bring about a product. In the course of this process, returns are paid out to the productive factors. In other words, income is created and distributed in the same process. This primary income distribution comes about in the process of production.

Another concept of distribution is the so-called secondary income distribution, which ultimately occurs after the government has levied taxes and paid out subsidies and so on. Between the primary and secondary income distribution are the income transfers. Thus criticism of the primary income distribution is criticism of a system, a mechanism, and a production process. But the criticism of the secondary distribution is related to a deliberate policy. This study is basically concerned with the primary income distribution that is more fundamental to the distribution problem.

As mentioned, this study is basically limited to the distribution of income among productive factors which is determined by the production process. The determination of prices and employment levels in the factor markets gives us the functional distribution of income into factor shares. For policy purposes, however, one may be interested in the distribution of personal income. For this one needs to examine the distribution of income-earning factors among individual households or groups. But this falls outside the scope of this study.

With the general directions outlined above, this study is mainly concerned with how the relative positions of wage earners have been changed and with what were the major economic forces for this change in the process of the various changes of output growth, resources employment, and economic structure.

Various theories have been developed to explain the income distribution among different resource owners; these will be reviewed in the next section. Two theories dominate in recent economic literature. The first is the marginal productivity theory with production function, and the second is the so-called neo-Keysian Macro distribution theory with the equilibrium relationship of saving and investment. Both theories are basically long-run equilibrium theories. But a main distinction between the two theories is that one assumes that the wage rate will equilibrate the labor market and the other assumes that the wage rate will clear the product market. Acceptability of both theories is at least subject to empirical test. This study is basically concerned with a test of the marginal productivity theory of distribution for the last two decades in both the agricultural and manufacturing sectors of the Korean economy.

As mentioned, the marginal productivity theory of distribution is a long-run equilibrium theory. Thus we cannot fully rely on the theory in order to explain the behavior of various economic variables which may have been affected by various short-run disequilibrium elements or certain institutional constraints. In the economies which have a relatively abundant labor force, a low wage policy is often suggested as a means of absorbing more labor into industry. It is also a fact that particularly in the development planning economy, the capital market is often more or less rationed rather than perfectly price-regulated. Organized trade unions also are frequently an important determinant of wages. But the fact that we may consider various disequilibrium elements does not imply that we can completely ignore the economic forces of competitive equilibrium. This study, while making use of equilibrium assumptions as a part of the analysis, also attempts to measure the degree of disequilibrium and its behavior over time and between sectors.

More specific objectives of the study can be stated as follows:

1. To estimate the wage income shares for the last two decades in the Korean agricultural and manufacturing sectors

2. To explain the forces affecting the changes in wage income shares under the marginal productivity theory of wages

3. For the purpose of explaining the sources of changes in wage income shares, to estimate the factor substitutability and factor saving bias of technical change

4. To measure the disequilibrium of marginal productivity and wages and examine the behavior of its change over time and between sectors.

Review of Distribution Theories

There are numerous theories dealing with functional income distribution. Although various criteria for classifying these theories exist in the literature, at least four different kinds of theories can be distinguished, some of which contain important sub-groups (Bronfenbrenner, 1971; Jan Pen, 1971; Dobb, 1973). These are the Ricardian or classical theory, the Marxian theory, the neo-Keynsian theory, and the neo-classical or marginalist theory.

The Ricardian theory was based on two separate principles which are termed the marginal principle and the surplus principle, respectively (Kaldor, 1956; Dobb, 1973). Under the assumption of the law of diminishing returns (in agriculture), the marginal principle serves to explain the division of output between rent and the other two shares (wages and profits). The surplus principle serves to divide the residual (the produce minus rent) into wages and profits.

The marginal product of labor is not equal to the wages. It is essential to distinguish between the "natural" price and "market" price for labor in Ricardo's theory. The natural wage is the level of wages toward which the actual or market wage will tend in the long run, and which just maintains the size of the labor force intact. Thus in the long run the wage rate is determined quite independently of marginal productivity by the supply price of labor, which Ricardo assumed to be constant at a "natural" rate of wages.* This assumption implies an infinitely elastic supply curve of labor at the given supply price, which is basically based on the Malthusian theory of population.

The fundamental long run behavior of relative shares can be seen by analysis of what takes place as output expands in the economy's agricultural sector. As the attempt is made to expand agricultural output through the increased employment of labor and capital, the rents will necessarily increase because rent is the difference between average

*Ricardo's "natural rate of wages" is different from the Malthusian "subsistence level of wages" which is determined only biologically. The "natural rate of wages" is the rate at which population could remain stationary and from which wages can only deviate temporarily. This rate may be interpreted as the socio-cultural minimum wage. (Kaldor, 1956).

and marginal product of the combined input of labor and capital (note that agriculture is characterized by diminishing returns because land is limited). This development will be reflected in a growing value for rent per unit of output as the employment is expanded. At the same time, wages as a share of total output will also have to increase because the natural wage rate in real terms will not be changed.

In the process what will happen to profit is readily apparent: as output expands, the share of profit continuously decreases, and finally it disappears completely at the employment level where the marginal product of the combined input of labor and capital is equal to the natural wage rate. This is the "stationary" state of classical theory, a situation in which all capital accumulation, population growth and output growth cease.

This behavior of the relative shares of output takes place in the agricultural sector because of the diminishing returns on land, but the same tendency will be at work in the manufacturing sector of the economy, even though the latter may not be subject to diminishing returns (Peterson, 1967, pp. 461-491). The profit will have to be the same in both sectors as long as mobility of capital is assumed. Thus this classical system would lead to an increase in the relative share of wages in the total output, and this would come at the expense of profit.

The Marxian distribution theory is essentially based on capitalist power and Ricardo's surplus theory. But Marx's analysis reaches very different conclusions about the behavior of relative income shares as the level of income and employment changes. To Marx, capitalist power is identical with ownership of the means of production. The wage is determined by the supply price of labor: a subsistence rate of wages.

The Marxian theory rests on the concept of the "labor theory of value" which asserts that the value of a commodity or service is determined by the labor time necessary for its production. Marx's labor theory of value is basically a cost of production theory, which is sometimes said to be an "objective" theory of value in contrast to modern exchange value theory which is in part "subjective" because it depends upon the consumer's utility as well as upon the cost conditions.

This labor theory of value was applied to the determination of the value of labor itself. That is, the supply price for labor is determined by the amount of labor power necessary to produce the commodities and services that will just permit the labor force to survive and reproduce itself, which is a minimum rate of subsistence. This is crucial because it forms the basis for Marx's theory of "surplus" value which in turn provides the fundamental key to an understanding of the process of income distribution.

Surplus value exists because labor will produce more economic value than the cost of labor itself as measured by the supply price of labor. This surplus value is expropriated by the capitalists who are owners of the physical means of production. Surplus value is the source of all profit, thus the amount of surplus value will determine the relative share of profit.*

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*Note that Marx did not believe in the principle of diminishing returns, and thus he did not distinguish in his analysis between rents and profit (Kaldor, 1956).

Marx believed that the most important force in the capitalistic economy is the effort of the capitalists to increase the rate of exploitation of labor, and that the relative share of profit increases as the labor productivity grows by the accumulation of capital and technical progress. Capital accumulation is a necessary structural feature of the economic system which results from the intensity of competition among capitalists seeking to increase the rate of exploitation (Kaldor, 1956; Dobb, 1973).

The market wage tended to remain at the subsistence rate because a vast industrial "reserve army" of unemployed workers prevents the market wage from rising above the minimum wage level. The fruits of increase in productivity go entirely to the owners of the physical means of production. Accordingly the income share of labor falls and exploitation increases. Thus the economic growth leads to the "increasing misery of the working class." But empirical facts are not reconcilable with Marxian theory. Increasing exploitation in the Marxist sense does not occur. Real wages keep on rising. They are far above any biological minimum.

Neo-Keynesian distribution theory is based on the Keynesian products market equilibrium condition: the equality of investment (I) and savings (S). Kaldor assumed that income is divided into two broad categories, wages (W) and profit (P). The important difference between them is in the corresponding propensities to save. The propensity to save out of wage income (s_w) is smaller than that out of profit income (s_p). This assumption is a necessary condition for both stability in the entire system and an increase in the share of profit in income when the investment-output ratio increases (Kaldor, 1956). Under these assumptions, the relative wage income share (S_L) is uniquely determined. The average propensity to save (s) will be a weighted average of the two groups' saving rates, i.e., $s = S_L s_w + (1 - S_L)s_p$. From this simple identity and the equilibrium assumption of the equality of saving and investment, the relative share of profit income can be derived by simple algebraic manipulation as

$$\frac{P}{Y} = \frac{1}{s_p - s_w} \frac{I}{Y} - \frac{s_w}{s_p - s_w}$$

That is, the share of profit ($\frac{P}{Y}$) is determined by the two partial propensities to save and the ratio of investment to income. This relation merely follows from the accounting identities, given the assumptions of the model. Thus it is often criticized that it is not an analytical conclusion (Ferguson, 1971).

From the above relation, it can be seen that an increase in the investment-output ratio (I/Y) will result in an increase in the profit share ($\frac{P}{Y}$) as long as $s_p > s_w$. Let us examine the underlying economic meaning of this relation. Under full employment conditions an increase in investment expenditure must bring about an increase in both the ratio of investment to output (I/Y) and also an increase in the ratio of saving to output (S/Y). This is a necessary condition if a new equilibrium is to be obtained. If the saving-output ratio did not rise, the result would be a continuous upward movement of the general level of prices.

With a higher absolute level of investment, the continued equilibrium can only be achieved either by a change in the propensity to save itself, or by a shift in the distribution of income from the class with the lower propensity to save to the class with the higher propensity to save. But Kaldor rules out the first possibility by his assumption that both s_w and s_p are constant. Here we can see how important the assumption of differential saving propensities between wage earner and profit earner plays in this theory. This is the essential feature of this macro distribution theory.

Whenever there is a rise in the investment-output ratio, the only way the economy can stay in equilibrium is to increase the saving-output ratio for the whole economy by the redistribution of income in favor of the profit earner. The mechanism to redistribute income for new equilibrium is essentially that of the price level. The increase in investment expenditure under full employment conditions leads initially to a general rise in prices. But in the absence of the economic force to equalize marginal productivity the real wage rate, no mechanism exists to insure that money wages rise at the same rate as prices (Sen, 1963; Kaldor, 1956).

The failure of money wages to keep pace with the rise in the general price level will thus reduce the real income of wage earners. This inflation-induced shift in the distribution of income in favor of profits will raise the overall level of real saving in the economy. This process will continue until the saving-output ratio is once again in equilibrium with the investment-output ratio. But the critical assumption of this theory is that the propensity to save of the profit earner is greater than that of the wage earner. Without this assumption, any change in the distribution of income will not affect the saving-output ratio, and thus the system would be unstable.

One of the logical slips in Kaldor's theory was pointed out and corrected by Pasinetti (Pasinetti, 1962). He pointed out that when any individual saves a part of his income, he must also be allowed to own it, otherwise he would not save at all. Thus it is clear that some part of total profits must accrue to workers as a result of their past savings. Pasinetti reformulated the original Kaldor model so as to reflect this observation. But his basic system of relations is rather similar to that of Kaldor.

The neo-classical distribution theory is the generalization of Ricardo's marginal principle distribution theory, so as to make this principle hold true for any factor. Under competitive conditions, any factor variable in supply will obtain a remuneration which must correspond to its marginal product. Thus this theory requires the assumption that the production function be homogeneous of the first degree for all factors. All factor prices and employment are determined by market forces. Given quantities of each factor employed, the total product is determined by the production function. Total output is distributed to the factors in the production process. The shares of the factors are determined by their relative quantities and their relative prices; the price ratios, in turn, are equal to the respective marginal rate of substitution between the factors.

With a given technology, the factor shares may change as a result of changes in their relative quantities, changesin their relative prices (or marginal rates of substitution), and the relationships between these two kinds of change. This leads to the important concept of elasticity of substitution: the ratio of proportional change of the factor ratio to the proportional change of the marginal rate of substitution, or to the proportional change of the factor price ratio under competitive conditions at a given isoquant. For example, unit elasticity of substitution between capital and labor in two factor production implies constancy of the relative income shares. For another example, if the elasticity of substitution is greater than one, say two, a change of one percent in the factor price ratio will be followed by a two percent change in the ratio of factor employment. Therefore, it will result in a one percent change in the relative factor share ratio. In other words, the competitively imputed share in output of the more rapidly growing factor rises. The opposite is the case, if the elasticity of substitution is less than one.

The second possibility is that in the course of time the technology changes, i.e., the whole production function shifts, or the quality of factors changes. The direction and magnitude of the changes of the marginal rate of substitution at given factor ratios will depend on the characteristics of technical change. This is Hick's criterion of classifying technical progress. At given factor ratios, technical progress is capital-using, neutral, or labor-using, according as the marginal rate of technical substitution of capital for labor decreases, remains unchanged, or increases. At a given input price ratio, capital-using technical change will provide an incentive to substitute capital for labor, i.e., to increase the capital-labor ratio.

From the above discussion, we can easily derive two important neoclassical propositions about the relative income shares. The first holds that a factor-saving technical progress, other things constant, reduces the relative share of the income, or the output elasticity of that factor (Hicks, 1932, p. 122). The second maintains that if one factor increases in supply more rapidly than another, and if the elasticity of substitution is less than unity, then the relative share of the first factor decreases (Hicks, 1932, p. 115).

This study uses the neoclassical framework of income distribution. By specifying the form of production function and the representation of technical change in the production function, the above relations of the marginal productivity theory of distribution will be discussed more specifically in Chapter III.

CHAPTER II

ANALYTICAL FRAMEWORK OF THE STUDY

Primary distribution of income is basically the result of production and consumption decisions. Individual incomes are aggregate rewards for productive services at a given distribution of resource ownership. Given their production functions, production units attempt to employ factor services and produce products in such a way as to maximize their flows of profit. These decisions determine demand functions for factor services and supply functions for products in terms of factor and product prices as parameters. On the other hand, consumption units attempt to offer factor services and purchase products in such a way as to attain maximum utility flows. These decisions determine demand functions for products and supply functions for factor services in terms of factor and product prices as parameters.

Given the demand and supply functions determined by the production and consumption units, the market adjusts toward a set of prices for factor services and products that clear all product and factor markets.

In the process of the above relations of economic forces, output is produced and distributed to the resource owners. Each factor receives its price which is determined in the market based on its marginal productivity. This chapter will review some relationships of production parameters to the distribution.

In the first section we will discuss some distributional aspects of production functions, particularly the CES (Constant Elasticity of Substitution) function which will be used in this study. The second section will discuss some alternative formulations of technical change which are widely used in empirical work, and a factor-augmenting assumption of technical change will be introduced in our production relation. The final section will derive distributional relationships with our specified production function and technical change under the marginal productivity theory of wages.

Aggregate Production Function

A production function is an attempt to describe the physical facts of a given technology, showing the relation by which the services of productive factors are transformed into output. The conceptual basis for believing in the existence of a simple and stable relationship between measures of aggregate inputs and a measure of aggregate output is not very sound. But an aggregate production function is a very convenient concept in many economic areas, and it has served as a basic framework for many empirical studies.

The simplest form of a production function widely used in neoclassical macrodistribution analysis is the Cobb-Douglas function. The constant exponent parameters of this function are direct criteria of distribution between factors. Thus the relative share of labor, or the elasticity of production with regard to labor, is independent of the capital intensity or capital accumulation relative to labor input. This relationship can be seen as follows. From the CD function, $Y = a\, L^{\alpha} K^{\beta}$, we can derive the marginal productivities of labor and capital as $MPL = \alpha(Y/L)$ and $MPK = \beta(Y/K)$, where Y is output, L and K are labor and capital inputs respectively, and a, α and β are parameters.

Assuming competitive markets, $\alpha(Y/L) = W/P$ and $\beta(Y/K) = R/P$, or $\alpha = (L \cdot W)/(Y \cdot P)$ and $\beta = (R \cdot K)/(Y \cdot P)$, where W and R are money wage rate and per unit return to capital, respectively and P is price of output. Thus in equilibrium, the coefficients α and β will directly measure the share of total receipts paid to labor and capital.

Another critical feature of the CD function is the extent to which it permits the substitutability between factors. We can see this by examining the marginal rate of substitution of labor for capital as $MRS_{LK} = MPL/MPK = (\alpha \cdot K)/(\beta \cdot L)$. From this one may easily see that the elasticity of substitution, which is defined as the proportionate change in the ratio of capital to labor divided by the proportionate change in the ratio of marginal productivities, is unitary in the CD function.* Thus for any level of output and inputs of the CD function, one input can always be substituted for another input at a fixed elasticity of unity.

But many production processes may have very low elasticities of substitution, or one may prefer a function with fixed requirements of each input in order to produce a unit of output, which is called the Leontief production function. Or at the other extreme, it may be possible to imagine processes where the extent of substitution is very high indeed. The isoquants may be nearly flat. Unitary elasticity of substitution is required for constancy of relative shares, since it

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*From the expression of MRS_{LK}, $\ln(K/L) = \ln(\beta/\alpha) + \ln(MPL/MPK)$. Thus by definition, $\sigma = \frac{d\ln(K/L)}{d\ln(MPL/MPK)} = 1$.

implies that the relative quantities of inputs vary by the same proportions as their relative prices. Thus the CD function is not able to explain the movements of relative factor shares.

The CES production function does not assume that the elasticity of substitution is unitary, but does assume that it is constant. Therefore, the CES function can allow for, and suggest the direction of, changes in relative shares. The rest of this section will review the basic form and some properties of the CES production function which will be used in this study.

The general two-factor CES production function can be written as:

$$Y = \gamma[\delta K^{-\rho} + (1-\delta)L^{-\rho}]^{-\frac{\nu}{\rho}} \qquad (2\text{-}1)$$

where γ, δ, ρ and ν are respectively the parameters of efficiency, capital intensity or distribution, substitution, and degree of returns to scale. This CES function was derived from an empirical relationship between average productivity of labor and the real wage rate (logarithmic relation) and from the assumptions of competitive equilibrium (Arrow, et al., 1961).

The interpretation of the efficiency parameters, γ, can be easily seen. Clearly this varies according to the units in which output and input are measured. But if we use the same units for measuring inputs and output, and we compare production functions with different γ's, the one with the higher γ will have the more efficient production relation. It will also be easy to observe the meaning of the scale parameter ν. The increase in both inputs by a factor λ will give rise to an expansion of output of λ^{ν}, and so when $\nu > 1$ there will be increasing returns to scale, and when $\nu < 1$ there will be decreasing returns.

If there are constant returns to scale (ν=1), then the marginal product relations can be written as:

$$MPL = (1-\delta)\gamma^{-\rho}(Y/L)^{\frac{1}{\sigma}} \tag{2-2}$$

$$MPK = \delta\gamma^{-\rho}(Y/K)^{\frac{1}{\sigma}} \tag{2-3}$$

where $\sigma = (1+\rho)^{-1}$. σ is the elasticity of substitution between capital and labor, as will be shown below. The Cobb-Douglas (CD) function is the special case with ρ=0, in which case the exponents are δ and $(1-\delta)$ respectively; this accounts for calling δ the distribution parameter.

The CES production function has all the properties of the neoclassical production function and includes the Cobb-Douglas and Leontif production functions as special cases. From (2-2) and (2-3) it is obvious that the CES function satisfies the condition that marginal product of each input, MPL and MPK, are positive. It also can be shown that $\partial MPL/\partial L$ and $\partial MPK/\partial K$ are negative for reasonable values of the scale parameter ν, including ν=1. For sufficiently large values of ν, these partial derivatives become positive, but such large values of ν are not likely (Brown, 1966). Also if ρ is positive, the CES function does reach a finite maximum as one factor increases while the other is held constant. Hence limits do exist when σ, the elasticity of substitution, is less than unity. However, when the elasticity of substitution is greater than unity, the function does not have a limit.

We have mentioned above that when ρ=0, i.e., the elasticity of substitution is unity, the CES reduces to a CD function. The relation between the two functions can be shown by means of a Taylor series approximation of the CES function; this approximation may also be used in one method of estimating the parameters of the CES function (see Chapter V). From the CES function (2-1), the following can be defined:

$$Z(\rho) = \delta K^{-\rho} + (1-\delta)\cdot L^{-\rho}, \quad \text{and}$$

$$\phi(\rho) = \ln Z(\rho).$$

The term $\phi(\rho)$ can now be expanded around the value $\rho=0$ in a Taylor Series approximation. Then, disregarding the terms of third and higher orders, the expansion becomes

$$\phi(\rho) = \phi(0) + \rho\phi'(0) + \frac{\rho^2}{2}\phi''(0) + \dots.$$

$$= -\rho[\delta \ln K + (1-\delta)\ln L] + \frac{\rho^2\delta(1-\delta)}{2}(\ln K + \ln L)^2 + \dots.$$

Taking logs of the CES function (2-1), we get

$$\ln Y = \ln \gamma - \frac{\nu}{\rho}\ln Z(\rho), \text{ i.e.,}$$

$$\ln Y = \ln \gamma + \nu\delta \ln K + \nu(1-\delta)\ln L - \frac{\nu\rho\delta(1-\delta)}{2}(\ln K - \ln L)^2 \tag{2-4}$$

This form is linear in the unknown parameters and allows direct estimation of parameters; this estimation method was proposed by Kmenta (Kmenta, 1967). Setting $\rho=0$ in equation (2-4), we get

$$\ln Y = \ln \gamma + \nu\delta \ln K + \nu(1-\delta)\ln L. \tag{2-5}$$

Thus when $\rho=0$ the CES function is a CD function.

The marginal rate of substitution of labor for capital can be derived by taking the ratio of the marginal product of labor to the marginal product of capital. From (2-2) and (2-3), the marginal rate of substitution of labor for capital (MRS_{LK}) can be expressed as:

$$MRS_{LK} = \frac{\partial K}{\partial L} = \frac{MPL}{MPK} = \frac{(1-\delta)}{\delta} \left(\frac{K}{L}\right)^{\frac{1}{\sigma}} \tag{2-6}$$

This equation suggests some important relations. A small value of δ implies that the production process has labor intensive characteristics. For a small value of δ, the marginal product of labor is high relative to that of capital for a given capital-labor ratio. Thus a unit reduction in the labor input has to be compensated for by a larger increase in the rate of capital than if the process were less labor intensive (larger value of δ). In this sense, δ is a measure of capital intensity of the technology.

Another aspect to note in the equation (2-6) is how the elasticity of substitution, σ, affects the marginal rate of substitution. At a given factor ratio, a high value of σ means that capital can be easily substituted for labor. In other words, if we reduce the rate of capital input by one unit we have to increase the rate of labor input by a greater amount when the factors are not easily substituted for each other, other things being constant. A high value of σ means greater similarity between inputs, and vice versa. Equation (2-6) also implies that when the value of σ is low, diminishing returns to labor set in more rapidly than when the value of σ is at a higher level.

To show that σ is the elasticity of substitution, let us take the equation (2-6) in logarithms. We get

$$\ln(MPL/MPK) = \ln\frac{1-\delta}{\delta} + \frac{1}{\sigma} \ln \left(\frac{K}{L}\right). \tag{2-7}$$

Taking the derivative with respect to $\frac{K}{L}$, we get

$$\frac{d(MPL/MPK)}{MPL/MPK} = \frac{1}{\sigma} \frac{d(K/L)}{K/L} .$$

The solution for σ is

$$\sigma = \frac{d(K/L)}{K/L} \Big/ \frac{d(MPL/MPK)}{MPL/MPK} \tag{2-8}$$

which is the definition of the elasticity of substitution.

When there exists equilibrium in the market, we can write Equation (2-6) as

$$\frac{1-\delta}{\delta} \left(\frac{K}{L}\right)^{\frac{1}{\sigma}} = \frac{w}{r} \tag{2-9}$$

where r is the real rental of a unit of capital and w is the real wage rate. Multiplying $\frac{L}{K}$ in both sides of Equation (2-9), we get

$$\frac{w \cdot L}{r \cdot K} = \frac{1-\delta}{\delta} \left(\frac{K}{L}\right)^{\frac{1}{\sigma} - 1} \tag{2-10}$$

The expression $(w \cdot L)/(r \cdot K)$ is the relative share of labor to capital. Clearly, as the value of σ goes to unity, the relative share of labor to capital approaches to the value of $(1-\delta)/\delta$. Hence, in a Cobb-Douglas production function, the ratio of relative factor shares can be represented by the ratio of the exponents, or production elasticities of the factors.

But with the CES production function, δ is not sufficient to determine the distribution of income between factors. We also require a knowledge of the substitution parameter, σ. In the CD function, we found that the distribution of income between factors depends on only the coefficients of the production function, and thus the distribution did not vary with the factor ratio. With the CES production function, however, we see that the distribution is a function of the factor ratio.

The greater the deviation of σ from zero, the greater the effect of the factor ratio on the distribution of income.

So far, our production function has been related to physical output and physical inputs, in which the relation was assumed stable over time. But many empirical works show that there is an important factor which changes the production relation and thus changes output or the productivities of physical inputs at given factor inputs. This intangible factor is named simply technical change. Sometimes it is interpreted as an aggregate effect of missing inputs in the production relation, such as investment in research and extension, education and other social investments which change the environment on which the production relations are based. There are numerous empirical studies which have attempted to incorporate these various input variables in production relations. The next section will discuss some possible alternative hypotheses which represent the variable, so-called technical change, in a production function framework.

Representation of Technical Change

In empirical work, technical progress may be formulated as "embodied" or "disembodied." "Disembodied" technical progress applies equally to all resources in current use. But in "embodiment" assumption, technical progress is embodied only in the new capital equipment of improved design or in new labor of enhanced skill as opposed to existing machines or to labor trained at earlier times. Thus the investment in new equipment or new skills is the essential carrier of new technology.

A general way to represent technical change is to write $Y = F(L, K; T)$, where Y is output, L and K are inputs measured by physical unit, and T is a parameter or a vector of parameters, each value of which corresponds to a different level of technology. If we conceive of technical change altering the production function, then the parameters of that function must change with different levels of T, e.g., in our CES case, δ, ρ, γ and ν. Any one of the parameters can be thought of as a function of T, or simply a smooth or discrete function of time. The discrete concept of technical change was used to measure technical change by Brown who refers to technical epochs (Brown, 1966).

One very restrictive but convenient specification of technical change is that the effects of technical progress are neutral or uniform in the sense that marginal rates of substitution do not change with different levels of technology at a given capital-labor ratio. In that case, we could write the shifting production function as $Y = a(t)\ F(K,L)$. The production function shifts over time simply by a uniform upward displacement of the whole function without disturbing the balance between capital and labor in current production. It implies that the distribution of income remains unchanged at the same capital-labor ratio in a competitive economy. This formulation was used by Solow (Solow, 1957) to estimate the rate of technical change after testing neutrality.

This formulation implies that all technical progress is a way of improving the organization and operation of inputs without reference to the nature of inputs themselves. Technical change is disembodied.

The isoquant contours of the production function shift inward toward the origin as time passes. "It floats down from the outside." (Solow, 1959).

Another assumption to represent technical progress is that technical advance takes the form of making labor and capital input more productive. Formally, we could write $Y = F(a(t)K, b(t)L)$ where $a(t)$ and $b(t)$ are factor augmentation rates (Solow, 1967). Thus $a(t)K$ and $b(t)L$ are interpreted as inputs of K and L in efficiency units. Under this assumption, we can write the production function as stable by adjusting inputs in efficiency units. But if we take input variables as natural units, the production function will shift over time.

Technical change is said to be purely labor augmenting if $\dot{a}(t) = 0$ and $\dot{b}(t) > 0$,* whereas it was purely capital augmenting if $\dot{b}(t) = 0$ and $\dot{a}(t) > 0$. It is equally capital and labor augmenting if $\dot{a}(t) = \dot{b}(t) > 0$. These definitions of technical change are related to the various classifications of technical change, which are based on its effect on the relative factor shares.

The theory of marginal productivity is used to analyze the effects of technical change on factor income. For this reason, the technical change has been classified according to its neutrality in terms of its effect on the relative shares of labor and capital: neutral, capital-saving, or labor-saving technology.

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*The dot over a variable indicates the proportional rate of change (or growth rate) of the variable, for example,

$$\dot{X} = \frac{d \ln X}{dt} = \frac{1}{X} \cdot \frac{dX}{dt}.$$

Hicks classified the factor saving bias of technical change at a constant capital-labor ratio (Hicks, 1932). His definition of neutrality means that the technical change does not affect the marginal rate of substitution of capital for labor at a given capital-labor ratio. Thus in competitive conditions, it implies that the distribution of income remains unchanged at the same capital-labor ratio. This definition of neutrality is equivalent to the above definition of the equally capital and labor augmenting technical change. Technical change is said to be labor-saving (or capital-using) if the relative share of labor is decreasing at a constant capital-labor ratio. Technical change is said to be capital-saving (or labor-using) if the relative share of labor is increasing at a constant capital-labor ratio.

But Harrod measures the bias of technical change along a constant capital-output ratio, and Solow measures it along a constant labor-output ratio. By these classifications, Harrod's neutrality is equivalent to pure labor augmenting, and Solow's neutrality is equivalent to pure capital augmenting technical change (Uzawa, 1961; Allen, 1968). Thus factor augmenting specification of technical change is a generalization of various definitions of neutrality.

It seems usual to think of technical change as occurring through time, perhaps smoothly, particularly at the macro level, as knowledge accumulates. Technical progress proceeds at a proportional rate $\frac{1}{a}\frac{da}{dt}$, generally varying over time. If it is at a constant proportional rate m, then $\frac{1}{a}\frac{da}{dt} = m$, with a=1 at t=0. Hence $a(t) = e^{mt}$. This assumption has been widely used in empirical work. It may be doubtful that inventive and innovative processes within a single firm can be represented by smooth exponential time trends. However, when many discrete and almost random influences are aggregated, it seems reasonable to suppose that aggregate technical change can be represented by a smooth time trend.

With the above specification of factor augmenting technical change, the CES production function may be written as

$$Y = \gamma[\delta(a(t)K)^{-\rho} + (1-\delta)(b(t)L)^{-\rho}]^{-\frac{1}{\rho}} \tag{2-11}$$

where a(t) and b(t) represent efficiency indexes of the conventional inputs of capital and labor. L and K represent conventional measures of the physical flow of labor and capital inputs.

The changes in a(t) and b(t) through time are interpreted as capital-augmenting and labor-augmenting technical changes, although this says nothing about the sources of such efficiency growth. Note that the neutral component of technical progress is also embodied in a(t) and b(t) if we assume that the neutral efficiency parameter γ is constant over time. The increase in a(t) or b(t) has the same effect on output as an equiproportional increase in inputs. Therefore factor augmentation restricts technical change so that it cannot alter the form or the parameters of the production function. It enters by changing the quantity of the effective factor unit.

The course of technical progress is often described by an index of the rate of progress (R) and an index of its bias (B). The rate of technical progress, R, is defined as

$$R = \frac{d \ln F}{dt} = \frac{F_t}{F} = \dot{F}$$

Considering F as homogeneous of degree one in the two inputs, the rate of change can be rewritten as*

$$R = \frac{KF_{Kt} + LF_{Lt}}{KF_K + LF_L} = S_L \cdot \dot{F}_L + (1-S_L) \cdot \dot{F}_K \qquad (2\text{-}12)$$

where F_{Kt} and F_{Lt} are the changes over time of the marginal products of capital and labor respectively, and S_L is the wage income share. Thus the rate of technical progress is the share-weighted sum of the proportional changes in the marginal products of each factor.

Using the Hicksian manner, the bias of technical progress B is defined as the proportional change in the marginal rate of substitution of labor for capital, i.e.,

$$B = \frac{d}{dt} \ln (F_K/F_L) = \dot{F}_K - \dot{F}_L \qquad (2\text{-}13)$$

Note that F_K and F_L are functions of time. It shows what happens to the marginal rate of substitution between L and K for a fixed capital-labor ratio as the level of technology t changes.

With the production function (2-11) the bias of technical progress (B) can be expressed in terms of a and b by using the definition of bias.**

$$B = (1-\frac{1}{\sigma})(\dot{a}-\dot{b}) \qquad (2\text{-}14)$$

From the above expression, it is apparent that the concept of factor augmenting technical progress is related to the Hicksian concept

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*MPL and F_L both refer to the marginal product of labor. We shall use whatever symbol is most convenient in each context. A similar remark applies to the symbols MPK and F_K for the marginal product of capital.

**From production function (2-11), $\dot{F}_K = -\rho\dot{a}$ and $\dot{F}_L = -\rho\dot{b}$, where $\rho = \frac{1}{\sigma} - 1$.

of neutral, labor-using and capital-using technical progress. The equality of the rates of growth of labor and capital efficiency, or unitary elasticity of substitution, is exactly equivalent to neutrality in Hick's sense. Technical progress is labor-using if the elasticity of substitution is less than unity and the rate of capital augmentation exceeds the rate of labor augmentation, or if the elasticity of substitution is greater than unity and the rate of labor augmentation exceeds the rate of capital augmentation. Similar statements apply to capital-using technical progress.

The form of factor augmentation needs to be specified when statistical estimation is attempted. One common specification is to assume that factor augmentation occurs at a constant exponential rate (David, 1965; Ferguson, 1969). But Lianos specified a(t) and b(t) as $a(t) = a_o t^{\alpha}$ and $b(t) = b_o t^{\beta}$ (Lianos, 1971). This formulation implies that the rate of factor augmentation is declining since $\dot{a} = \alpha t^{-1}$ and $\dot{b} = \beta t^{-1}$. Fishelson explained that this formulation is less restrictive than constant exponential specification (Fishelson, 1974).

Marginal Productivity and Distribution Theory

Based on the specified formulation of the production function and technical change, this section will derive the relationships underlying the sources of change in the relative wage income share, which we will apply to explain its behavior. For doing this we need to review the general economic mechanism of distribution of output to the resource owners under the marginal productivity theory. The distribution mechanism in marginal productivity theory is basically the micro-economic problem of the determination of the employment and the prices of the factors of production. Thus the distribution theory has come to mean the process by which factor prices are determined through the interplay of the producer's demand for the factors and the supply conditions for these factors.

We assume there exists an aggregate production function with smooth factor substitutability and marginal productivities as

$$Y = F(K, L) \qquad (2\text{-}15)$$

where F is homogeneous of first degree in the homogeneous inputs, capital (K) and labor (L). By its homogeneity property, the production function may be written in a per capita form as

$$y = f(k) \qquad (2\text{-}16)$$

where $y = \frac{Y}{L}$ and $k = \frac{K}{L}$. By assumption, $f_k > 0$ and $f_{kk} < o$.

The per capita form is specific to the linear and homogeneous function, i.e., to the case of constant returns. It is also assumed that production is based on profit maximization under perfect competition. The necessary conditions for profit maximization imply that production should be pushed to the point where the marginal products are equal to the corresponding factor prices. From Equation (2-16) these conditions may be written as

$$MPL = f(k) - kf_k(k) = w \qquad (2\text{-}17)$$

$$MPK = f_k(k) = r \qquad (2\text{-}18)$$

where w and r denote the real wage rate and the rate of return on capital. These equations are the derived input demand equations.

The above conditions provide for the distribution of output to the factors. From the equations (2-17) and (2-18), we have

$$rk + w = kf_k(k) + f(k) - kf_k(k) = y$$

Hence the output divides in per capita terms as $y = rk + w$. Multiplying by L on both sides, it can be seen that total output is distributed as $Y = rK + wL$. Thus with constant returns to scale under the competitive equilibrium, the total product is just sufficient to pay each input its marginal product.

Under the optimum position, the above relationships of determination of factor prices and distribution of output can be seen more clearly in Figure 1. Given the per capita production function, $y = f(k)$, which describes technical conditions of production, the rate of return on capital (r), wage rate (w) and output per capita (y) are determined by the selection of factor ratio (k). The rate of return on capital is the slope of the tangent at the point P, and the wage rate is the intercept OW which is equal to $f(k) - kf_k(k)$. Thus the share of wage income is also determined by $\frac{OW}{ON}$, and $\frac{NW}{ON}$ is the share of capital income.

For a given stable production function, Figure 1 shows the interrelationships among these variables: wage rate, returns to capital, capital-labor ratio, and output per unit of labor input. Any one of those variables is determined; the rest of the variables are also uniquely determined. Thus, assuming no technical change in the production, the distribution of output is determined purely by the shape of production function and the capital-labor ratio.

It was also shown that, from Figure 1, the wage rate (w) is the distance OW, and the rate of return to capital (r) is the slope of the tangent line Wr in the competitive equilibrium. The slope of the tangent line can be expressed as the ratio of OW to OB. But note that

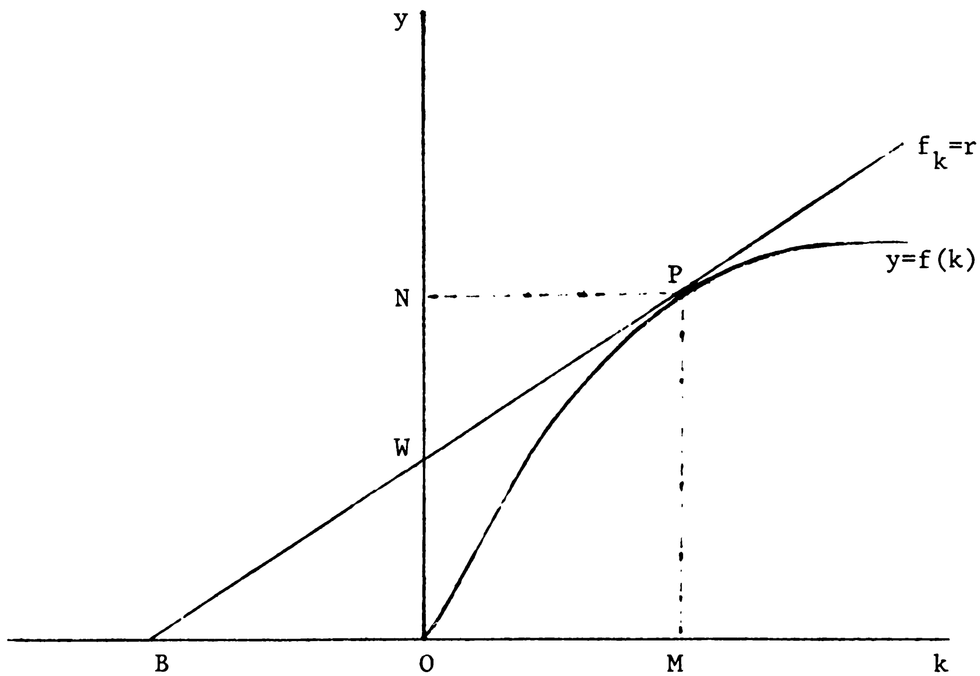

Figure 1. The Distribution of Output

the distance OW is the wage rate. Thus, $r = \frac{w}{OB}$, or $OB = \frac{w}{r}$. From this we can easily see the effect of the capital-labor ratio on the distance OB, which is the ratio of the wage rate and the rate of return to capital. The higher capital-labor ratio is associated with the higher value of $\frac{w}{r}$. Thus the capital-labor ratio has a positive relation with $\frac{w}{r}$ and a negative relation with $\frac{r}{w}$. That is, $k = f(\frac{r}{w})$. The elasticity of this curve, as a usual definition, is the elasticity of substitution which we defined as the responsiveness of the capital-labor ratio (k) to the prices of capital and labor. We can also see the relationship between the elasticity of substitution and the ratio of relative shares by writing the ratio of relative shares as $\frac{rK}{wL} = (K/L)/(w/r)$.

From Figure 1 it also can be seen that higher capital-labor ratios are associated with lower values of r and with higher values of w. This implies an inverse relationship between the wage rate and the rate of return to capital, which is called "Factor price frontier" or "Wage frontier" (Samuelson, 1962). It is important to note that the slope of the w-r curve $(\frac{dw}{dr})$ at any point is equal to the capital-labor ratio (k). From Equation (2-17) $\frac{dw}{dk} = -kf_{kk} > 0$ and from Equation (2-18) $\frac{dr}{dk} = f_{kk} < 0$. Hence $(dw)/(dr) = \frac{dw}{dr} = \frac{dw}{dk} / \frac{dr}{dk} = -k < 0$. It is therefore clear that the elasticity of the wage frontier $(-\frac{r}{w}\frac{dw}{dr})$ must equal $\frac{r}{w}k = \frac{rK}{wL}$, i.e., the ratio of relative shares of capital and labor.

So far we have examined the marginal productivity theory of distribution with a stable production function and the assumption of competitive market. The theory tells us that the distribution of output is changed by the changes in the factor ratio for a given stable production function. This theory also implies that if the economy gorws in the steady state equilibrium path, or at constant capital-labor ratio and constant output-labor ratio, then the distribution of output remains constant.

But when the economy is out of a long-run steady-state equilibrium growth path, there always exists the possibility to change the distribution of output as the capital-labor ratio changes, depending on the nature of the production technology. There is also the possibility that the production technology is changed and biased, which may shift the production function and thus lead to a change in distribution. Under the assumptions we made in the above, the relationship of the relative wage income share can be derived in terms of changes in capital-labor ratio and technical progress.

The share of labor (S_L) is defined simply as $S_L = (wL)/Y$. Assuming the competitive equilibrium conditions, then the real wage rate is equal to the marginal product of labor, and thus the relative share of labor S_L can be written as

$$S_L = \frac{LF_L}{Y} = MPL/APL \tag{2-19}$$

Equation (2-19) says that the wage income share is equal to the ratio of the marginal product of labor to the average product of labor. It also shows how important the law of diminishing returns is to the distribution.

Equation (2-19) may also be written as

$$S_L = \frac{\partial Y}{\partial L} / \frac{Y}{L}$$

This is the definition of the elasticity of production with regard to labor. It says that the relative factor share of income is equal to the elasticity of production with regard to each factor.

From the concepts introduced above, the important relationship of the rate of change in wage income share can be derived by the following procedure. This derivation procedure is based on Ferguson (Ferguson, 1968). First, the proportional time changes in the marginal products of labor and capital can be solved from Equations (2-12) and (2-13) as functions of the rate and bias of technical progress:*

$$\frac{F_{Kt}}{F_K} = R + S_L B \tag{2-20}$$

$$\frac{F_{Lt}}{F_L} = R - (1-S_L)B \tag{2-21}$$

Next, the rates of growth of the marginal products and of output may be expressed as functions of the rate and bias of technical progress, the elasticity of substitution, and the rates of change in the capital and labor:**

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*In what follows, when a subscript is attached to a functional notation, it indicates partial differentiation. Thus, for example, $F_t = \partial F/\partial t$. A superior dot denotes the proportional rate of change (or growth rate) of the variable, for example, $\dot{F} = \frac{d \ln F}{dt} = \frac{1}{F} \cdot \frac{dF}{dt}$. Note that $\frac{dF}{dt} \neq F_t = \frac{\partial F}{\partial t}$. For example, $\frac{dF}{dt} = F_t + F_K \frac{dK}{dt} + F_L \frac{dL}{dt}$.

**From the assumption of linear homogeneity, $-LF_{LL} = KF_{LK}$, $- KF_{kk} = LF_{KL}$, $\sigma = \frac{F_K F_L}{F\,F_{KL}}$. See Allen (1938, pp. 340-343). Note that $\frac{dF_K}{dt} = F_{Kt} + F_{KK}\frac{dK}{dt} + F_{KL}\frac{dL}{dt}$.

$$\dot{F}_K = R + S_L B - \frac{1}{\sigma} S_L(\dot{K}-\dot{L}) \qquad (2\text{-}22)$$

$$\dot{F}_L = R - (1-S_L)B + \frac{1}{\sigma}(1-S_L)(\dot{K}-\dot{L}) \qquad (2\text{-}23)$$

$$\dot{F} = R + (1-S_L)(\dot{K}-\dot{L}) + \dot{L} \qquad (2\text{-}24)$$

Finally, from the definition of the wage income share, the rate of change in the labor share can be expressed as

$$\dot{S}_L + \dot{F}_L + \dot{L} - \dot{F}$$

Substituting Equations (2-23) and (2-24) into above equation, we may get our main expression for the rate of change in labor's relative share.

$$\dot{S}_L = -(1-S_L)[B + (1-\frac{1}{\sigma})(\dot{K}-\dot{L})] \qquad (2\text{-}25)$$

Equation (2-25) shows that the rate of change of the wage income share depends on the bias of technical change (B), on the value of the elasticity of substitution, σ, and on the direction of change in the

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Thus

$$\dot{F}_K = \frac{F_{Kt}}{F_K} - \frac{LF_L}{F}\,\frac{FF_{KL}}{F_K F_L}(\dot{K}-\dot{L}) = \frac{F_{Kt}}{F_K} - \frac{S_L}{\sigma}(\dot{K}-\dot{L}).$$

Substituting Equation (2-20) into the above equation, one may get the Equation (2-22). Equation (2-23) can be obtained in the same way.

To obtain Equation (2-24), note that $\frac{dF}{dt} = F_t + F_K \frac{dK}{dt} + F_L \frac{dL}{dt}$

Thus, $\dot{F}\ \frac{F_t}{F} + \frac{KF_K}{K}\ \dot{K} + \frac{LF_L}{F}\dot{L}$.

Substituting Equations (2-12) and (2-19) into the above equation, one may get the Equation (2-24).

capital-labor ratio. Of course, if the technical change is neutral, that is, B = 0, then the rate of change in the wage income share depends only on the value of the elasticity of substitution and the growth rates of capital and labor.

Thus Equation (2-25) suggests that there are many ways that the relative factor share can be changed over time. For example, if the elasticity of substitution is unity, the relative share of labor will increase, remain constant, or decrease accordingly as the bias of technical change is negative, zero, or positive. If the elasticity of substitution is not unitary, there are many ways the relative share can be changed, depending on the direction and magnitude of technical bias, the elasticity of substitution, and the growth rates of capital and labor.

For another example, the constancy of a relative share may be explained in two ways. The first case is where the technical progress is neutral and the elasticity of substitution is unity. For the second case, suppose that the technical progress is capital-using and that the elasticity of substitution is less than unity and the growth rate of capital-labor ratio is positive. Then the relative share will remain constant if the decrease in relative demand for labor attributable to capital-using technical progress is exactly offset by the decrease in the relative supply of labor attributable to capital deepening and inelastic substitutability. Alternatively, if the elasticity of substitution exceeds unity, shares will remain constant if the increase in the relative demand for labor attributable to labor-using technical progress is precisely offset by the increase in the relative supply of labor. Unless these conditions are exactly satisfied, the relative shares will change over time.

Substituting the expression for the bias of technical change (B) from the Equation (2-14) into Equation (2-25), the rate of change of wage income share can be written as

$$\dot{S}_L = -(1-S_L)\left(\frac{\sigma-1}{\sigma}\right)[(\dot{a} + \dot{K}) - (\dot{b} + \dot{L})] \tag{2-26}$$

From Equation (2-26) it appears that the behavior of the relative share of labor, for a given value of σ, depends not just on the capital-labor ratio in a conventional unit but also on the changes in the productivity of the two factors. The term in brackets on the right-hand side of Equation (2-26) may be viewed as the adjusted rate of change in the capital-labor ratio, that is, physical units of capital and labor converted to effective units by using the differential rates of factor augmentation.

We specified the form of factor augmentation as $a(t) = a_o t^{\alpha}$ and $b(t) = b_o t^{\beta}$, which was discussed in a previous section. Thus $\dot{a} = \alpha t^{-1}$ and $\dot{b} = \beta t^{-1}$.

Now the rate of change of wage income share can be expressed as

$$\dot{S}_L = -(1-S_L)\left(\frac{\sigma-1}{\sigma}\right)[(\dot{K}-\dot{L}) + (\alpha-\beta)t^{-1}] \tag{2-27}$$

Since $0 < S_L < 1$ and $t > 0$, the direction of the rate of change of the labor share depends on the elasticity of substitution, the growth rates of capital and labor, and the difference between α and β. This is the analytical relation underlying the relative wage income share which we will use to explain the behavior of actual wage income shares.

CHAPTER III

ESTIMATION OF WAGE INCOME SHARES AND SOME PRELIMINARY ANALYSIS OF THEIR CHANGES

The main task of this chapter is to estimate the actual movements of the wage income share for both the agricultural and manufacturing sectors. It is the aggregate variable in which the four variables --employment, wage rate, output, and price--are directly involved. The economic forces or relationships which determine the variables were explained in the previous chapter. Using the derived relationships, the behavior of the actual wage income share will be analyzed in subsequent chapters.

The first section will describe the sources and specifications of the data to be used for the estimations. The second section will estimate the wage income shares and observe their movements over time and between the sectors. The last section will examine the behavior of the variables which are involved directly in the movements of the wage income shares.

Specifications of Variables and Sources of the Data

The Measurement of Aggregate Output and Price

The output data used were the Bank of Korea (BOK) time series, which are published from 1953 to 1974 for both the sectors in "National Income in Korea." For the agricultural sector, the output was measured by the value of gross output and also the value added. Both series were used for the estimation of the wage share. According to the descriptions of the data, the value of gross output was calculated by multiplying the annual average price of each product group by the physical quantity of each product group and summing. The basic data were provided by estimation of the Ministry of Agriculture and Fisheries (MAF). The annual average prices were calculated by weighting monthly prices by the monthly marketing volumes of the year. The basic price data have been collected every month since 1956 in the 56 selected rural areas by the National Agricultural Cooperative Federation (NACF). The output measured production in the year in which it was produced on the farm, even though some of the production may have been marketed or self-consumed in subsequent years.

The intermediate products, which were produced and consumed in the production process on the same farm, were not included in the measure of gross output, but they were included to the extent that they enter into marketings. Thus the series may overestimate the gross agricultural output since it includes some double counting such as interfarm sales of such intermediate products as seed and feed. But due to the lack of reliable information, no attempts were made to measure aggregate agricultural output net of intermediate products produced and consumed within the agricultural sector, which might have provided a better definition of gross value of output.

The value added series were defined as the differences between the value of gross output and purchases of intermediate products consumed in the production process. These include feed, seed, fertilizer, insecticides, and other items charged to current expenses. Their exclusion yields the net value added to the national products by the primary agricultural factors of production, such as land, reproduceable capital, and labor. But considering the definition of gross output it is net to the individual farm rather than net to the agricultural sector of the economy.

For the manufacturing sector, the output was measured by the value added. The basic sources of the gross physical output data were the Economic Planning Board (EPB) and various government and private agencies. The price data used for the aggregation of output were the wholesale prices for that portion used in domestic consumption and FOB prices for the portion exported. The major source of price data was BOK's "Wholesale Price Survey" in which data are collected in ten-day intervals for 15 selected areas.

The aggregate real output was measured at 1970 constant prices for both sectors, which are reported in the publication cited above. This publication is the most complete source of output measures of sectoral aggregates. Ban's study (Ban, 1974) also prepared a time series of the aggregate output of the agricultural sector, which was measured at 1965 constant price from 1955 to 1971. But the sources of the basic data were identical, thus the two series were not significantly different.

The Specifications and Measurements of Input Variables

In empirical work in production economics, there are various difficulties in specifying input variables and their measurement, such as grouping of input categories, units of measurement, choice of weighting system to be used for the aggregation of various inputs, derivation of a flow concept of service input from stock measurement, adjustment of input measure of capacity concept to actually utilized input, and so on. Many of the difficulties underlying the measurement of real factor input, particularly in capital input, have not been fully solved at the operational level. Thus various inferences or approximations are used in empirical work. In many cases these approximation techniques are largely dictated by the availability of data.

We need to explain in detail the sources of data and discuss some of the problems in the specification and measurement of our input variables. The source of input data for the agricultural sector is Ban's times series, which are the most complete estimates of national aggregates of input variables. His input series were estimated basically from average per farm household data, which have been collected from 1,200 randomly selected sample farms by MAF. The average input data per farm household have been reported in "Report on the Results of Farm Household Economy Survey and Production Cost Survey of Agricultural Products," published annually by MAF from 1962. Before 1962, Ban used the NACF average per farm household input data, which is also based on farm surveys.

For the manufacturing sector, the major sources of the data were BOK's sample survey on "Business Management" and "Monthly Earnings and Man-days of Regular Employees in Manufacturing Industry." The data have been reported since 1957 in the BOK's annual publication "Economic Statistics Yearbook."

The input data for the two sectors, derived from the above sources, are basically of a private accounting nature based on firm or farm household surveys. Conceptually all input and output data that enter into aggregate production relationships should be based on social accounts (Jorgenson and Griliches, 1967; Griliches, 1964). But as in most empirical work, the measurement of input and output variables are subject to the limitations of social accounting depending on the availability of data.

Various public investments, such as research, extension, education, transportation and other development investments are excluded in the measurement of our aggregate inputs. All prices of inputs and outputs also reflect only private benefits and costs. The productive contributions of the excluded social inputs are costless from the point of view of private decision makers. Since no allowance was made, in the estimation procedures, for the effects of social inputs on productivity, it is likely that these effects are captured by the estimates of technical change. More specific forms and explanations of technical change were discussed in the previous chapter.

Many different classifications of input variables have been used in empirical work depending mainly on the purpose of the study. Following one conventional method, this study classifies all the inputs into two groups: labor and capital. But theoretically some conditions are required for different inputs to be aggregated as a group. The necessary and sufficient conditions are stated as: (a) that the rate of substitution between inputs of different types be independent of the quantities of other inputs used with them, and

(b) that the marginal rate of substitution between different types of input must be constant, i.e., two types of input are perfect substitutes. It will be also possible to aggregate perfectly complementary inputs with afixed ratio. However, these conditions are quite stringent to be satisfied in the real world.

But as mentioned, inputs were calssified simply into two groups for the purpose of this study.* We need to specify more detailed definitions and measurements of the inputs. There were some differences in the definitions of the variables between the sectors due to the availability of data and some conceptual differences.

Measures of Labor Input and Wages

The measurement of labor input is relatively easy compared to capital measurement. But the aggregation of different qualities of labor should be in terms of a standard unit. The marginal productivities of different kinds of labor is probably the best indicator of their quality differences. Since in competitive equilibrium, marginal products are proportional to wage rates, this suggests that labor should be weighted according to its hourly remuneration.

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*Conceptually, it is possible to regard the aggregate quantity index of capital inputs in the following way. One may think that the process of production has two stages such that capital, K, is a manufactured output produced by all the individual capital goods (the capital index function), and then this K is combined with other inputs to produce the final output (production function). That is, $Y = F(L, K_1, K_2) = H(L, K)$, $K = G(K_1, K_2)$. This means that the index of capital quantity is the output of a production process which uses various capital goods to produce capital in general. For more discussion, see Solow (Solow, 1956).

For the agricultural sector, however, the original survey data classified labor only by sex and age rather than by the wage rates actually paid. This may be because the major portion of agricultural labor is family labor, which is not paid or valued at each point of input. For this reason, the labor input was aggregated by the MAF "labor ability weighting system"* which actually considered only the age and sex factors.

Using this weighting system, the labor input was aggregated as that of a 20-54 year aged man equivalent day unit of 8 hours of actual work. This measure consists of labor input actually used for all farmwork, and it includes farmers and unpaid family workers as well as hired labor. However the "ability weighting index" may not accurately reflect the quality of different labor in the sense that labor ability may be different by kinds of job or perhaps individual skills.

Ban's study also pointed out that there are some differences in the labor input estimate between the MAF "ability weighting index" and wage weighting aggregation. But due to the availability of the data, the study aggregated labor inputs by using the MAF weighting index without any adjustments.

Rural wage data was collected from a survey which has been conducted by NACF from 1956. NACF collected rural prices and wage data

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*The weighting index standardized labor ability as, for example, 1.0 unit for a 20-54 years aged man, .8 unit for the same aged female, .8 unit for a 54-59 years aged man, .6 unit for the same aged female and so forth.

every month from the 56 nationwide selected rural market areas. From the data they estimate an annual average wage rate per man-equivalent day unit by weighting monthly labor inputs. The annual average wage data have been reported since 1959 in the "Agricultural Cooperative Yearbook." Before that, wage data were derived from Ban's study.

For the manufacturing sector, the quantity of labor input was calculated from the data on the number of employees and average work days per month, which have been reported since 1957 in the "Economic Statistics Year Book" published by BOK. The measure of labor was not corrected by any quality factors. It was simply aggregated by physical unit of work day. Age and sex may not be important factors for the ability of labor for manufacturing production. Certain specialized skills or experience may be more important ability correction factors, but no such relevant data for this correction was available.

The annual wage series were derived from the average work days per month and the average monthly pay per employee, which are reported in the publication cited above. According to the descriptions of the data, any fringe benefits were not included. Thus the wage series derived may not accurately reflect the production decision price, and also may underestimate the wage income share. From the deriving procedure of wage data, it is also clear that some errors in the quantity measure of labor input may lead to some errors in the wage rate in the opposite direction. As has been seen, there are some possible deficiencies in our data, but any reliable information for making an adjustment was not available.

Measure of Capital Quantity and Its Prices

There are more difficulties in the estimation of the quantity and price of capital inputs which are actually used in a given production period in which output was measured such as the well known index number problem in aggregating various heterogeneous capital goods, the conversion problem from stock to flow services which are actually associated with given production, and so on.

More theoretically, it is often argued that it is impossible to conceive a quantity of "capital in general," the value of which is independent of the rate of interest (Robinson, 1954). The argument may depend on the concept of capital, in other words, whether capital is defined as physical goods itself, such as labor, or some abstract productive power, which may be defined as the discounted value of the future stream of revenue expected from capital goods. But in the latter definition, we need the rate of interest as given to measure the value quantity of capital goods, whereas our main purpose in analyzing the production function is to show how wages and the rate of interest are, in part, determined by the technical conditions and the factor ratio.

Several different aggregation techniques have been suggested for the measurement of the capital input: aggregation in terms of other factors, such as labor time used to produce the capital goods, aggregation by weighting relative expenditure shares, and aggregation by certain functional forms, which can produce "capital-in-general" from different capital goods. But none of these methods avoids the complexities of the problem, and all suffer from some limitation.*

As in many empirical studies, capital was defined as tangible physical productive goods, and it was aggregated in value terms. That is, capital was measured as a value quantity, which is different from a physical quantity by which labor was measured. The value aggregate was converted to real capital goods by deflating to a 1970 base price index. Under the perfect expectation and equilibrium conditions, a price weighted aggregate will measure the physical complex of capital goods in terms of its estimated ability to contribute to production over their life time.

However, in view of the divergent trends in relative prices, the choice of the price-weight base year will affect capital aggregation. Of course this problem will apply equally to output aggregation. Accordingly, in principle, considerable care should be taken in the choice of the price-weight base year. However, because of time constraints, relatively little effort was expended in this study on the selection of the base year. But the choice of 1970 as base year may be at least partly justified in the sense that the 1970 relative price structure of capital goods did not appear to be abnormal.

Using the general guidelines explained above, the capital input for agricultural production was measured by the flow service concept at 1970 constant prices. Capital consisted of the depreciation charge on durable capital goods, irrigation fees, and intermediate inputs. The price of capital was derived by dividing total current expenditure

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*Some detailed discussion of the problem can be seen in Harcourt (Harcourt, 1972) and Kendrick (Kendrick, 1961).

on capital input by the quantity of capital input measured. The total current expenditure was measured by the current cost of depreciation, irrigation charges, intermediate costs and interest at 15 percent on the stock value of durable capital goods.

But for manufacturing production, the capital input was measured at 1970 constant prices of all tangible durable capital goods. For the years 1963 to 1974, there are data published on the ratio of value added to tangible, durable productive assets (i.e., V_t/K_t) and also data on V_t. These data series are reported in the BOK publication. From these we calculated the implied values of K_t. For the years 1955 to 1974, there are data published on the "marginal durable capital-output ratio" (i.e., $\Delta K_t/\Delta V_t$). From these ratio and data on ΔV_t, we calculated ΔK_t, and hence (working backward from the value of K_t in year 1963) the values of K_t in preceeding years.

As seen, the measure of capital input for the manufacturing sector was based on the stock value concept. This was due to the availability of reliable data. But what we want to measure is the annual flow of capital services which is actually associated with current production. However we may be content with the measure of stock value under the assumption that the flow of services is proportional to the stock of capital.

But such measure of capital input may not lead to a good approximation when the average life and age of capital goods are changing. If the average life of capital goods increases, our estimate will be biased upward. And if average age is older, our estimate can be biased downward. This can be seen by pointing out that the value of the stock of capital at any point in time is the current valuation of current and all future services expected from the stock, whereas what we are interested in for current production is the value of current services from this stock.

The price of capital was measured by dividing total return to capital by the quantity of capital input. The total return to capital was derived by the residual concept. That is, the price of capital $r = (V-wL)/K$. This procedure may lead to a good approximation if the economy is close to equilibrium. Conceptually we can impute the capital price more accurately from the data on depreciation charge, interest rate, and capital gain or loss. But no such attempt could be made due to the unavailability of data.

Estimation of Wage Income Shares in the Agricultural and Manufacturing Sectors

This section presents estimates of the wage income shares in both the agricultural and manufacturing sectors for the last two decades. The sources and specifications of the data to be employed for the estimates are described in the previous section. The behavior of their movements will be explained in the subsequent chapters.

Estimation of the wage income shares is based on the simple definition $S_L = (W \cdot L)/(Y \cdot P)$, where W is money wage rate, L is the measure of labor input actually used, Y is total output or value added at constant price, and P is output price. As mentioned in the previous section, for the agricultural sector gross income data are used for the estimate. But for the manufacturing sector the estimate is based on value added data.

Both estimates, one based on gross income and one based on value added, will exhibit the same behavior of movement if the intermediate inputs have a fixed proportion to the gross income. But it is often argued that for agricultural production, the intermediate inputs are substitutable for other inputs. Thus for agricultural production we specified the production relation as between gross output and inputs, which include intermediate inputs, rather than subtracting the intermediate inputs from both sides of the production relation. We will discuss this problem more in the next chapter.

The Agricultural Sector, 1955-1974

The estimate of the wage income share for the agricultural sector is presented in Table 1. For the estimation, the labor input data included hired workers as well as farmers and unpaid family laborers. And the wage rate used for the estimates was the rate paid to hired labor. But for the period, the proportion of hired labor to total labor input was only about 15 to 20 percent, with a slightly declining trend over time. Thus some may argue that applying the market wage rate to the unpaid family workers may not be appropriate for the estimation in the agricultural sector. It seems true that the return to farmer and family labor is determined as residuals rather than as the market wage rates. On this ground, the alternative estimate based on the residual concept was also obtained and is presented in column 7 of Table 1.

Under the equilibrium assumption, the above two alternative estimates, one based on hired wage rate and the other based on the

Table 1. Estimates of Wage Income Share in the Agricultural Sector, Korea, 1955 - 1974

Year	Gross output[1] (Y) (Mil. Won) (1)	Output price[1] (P) (1970=1.) (2)	Labor input[2] (L) (Mil. Man day) (3)	Wage[3] (W) (Won/day) (4)	Wage share[4] S_{L1} (5)	S_{L2} (6)	(%) S_{L3} (7)
1955	469,498	.1420	456.2	76	.5200	.6070	.4414
56	436,075	.1819	481.5	85	.5159	.6083	.4256
57	474,647	.1952	512.7	92	.5091	.6018	.4189
58	504,912	.1804	540.0	92	.5454	.6422	.4007
59	503,079	.1851	571.1	93	.5704	.6774	.3775
60	498,853	.2060	625.8	96	.5846	.6992	.3774
61	557,630	.2355	657.7	106	.5309	.6243	.3897
62	525,236	.2797	610.3	115	.4777	.5724	.3702
63	562,065	.3587	650.9	143	.4616	.5519	.3966
64	650,085	.5090	672.1	199	.4042	.4812	.4013
65	660,066	.4860	654.0	221	.4506	.5608	.3631
66	727,187	.5217	655.4	256	.4422	.5472	.3459
67	693,593	.6497	651.7	307	.4440	.5638	.3582
68	709,653	.7156	607.2	381	.4555	.5807	.3424
69	800,962	.8355	586.9	463	.4060	.5151	.3595
70	793,984	1.0000	562.9	579	.4105	.5283	.3665
71	811,457	1.2104	568.6	695	.4023	.5187	.3817
72	807,878	1.4521	541.6	803	.3707	.4807	.3796
73	840,204	1.5843	536.5	886	.3571	.4674	.3731
74	870,827	2.1267	509.5	1,141	.3139	.4106	.3645

[1]Gross output was measured in 1970 constant prices, from BOK, "National Income in Korea". 1975, pp. 192-193. The price data are from the same source.

[2]Labor input was measured by man-equivalent unit. For 1955-1971, from Ban's study. For 1972-74, calculated from the MAF, "Report on the Results of Farm Household Economy Survey and Production Cost Survey of Agricultural Products," 1975.

[3]The wage rate was measured as per man-equivalent day unit. For 1959-1974, from NACF, "Rural price statistics," 1974. For 1955-1958, calculated from Ban's study.

[4]S_{L1} is the wage share of the gross income, S_{L2} is the share of the value added, and S_{L3} is the share of the gross income estimated by the residual concept where we assumed the land share was 35 percent of the gross output.

residual concept, will be approximately the same. The first estimate basically assumes that all family workers, including the operator, received the same returns on their labor as hired farm workers. Thus if there is some lagged response of wage rates to changes in marginal value productivity, this estimate will tend to underestimate the returns to labor during periods when farm prices are rising and to overestimate during periods when farm prices are declining. The second estimate assigns to labor the residual share remaining after the computed share for capital input has been subtracted from total output. Thus this estimate may result in an opposite bias of wage share estimate during periods when farm prices are rising or declining.

For the simple comparison between sectors, the wage income share, based on the value added data, were also estimated for agriculture. The estimate is presented in column 6 of Table 1. The value added series used for the estimate are BOK data, which were calculated by subtracting the value of the intermediate inputs purchased from outside of agriculture from the value of gross output. But the value of interfarm transactions of intermediate goods was not considered in the estimation of value added. Thus this series may overestimate total agricultural value added since it includes some double counting, to the extent of interfarm transactions of feed and seed. As a result the estimate of the wage income share based on this value added series may be biased downward.

The three alternative estimates are consistent in the general trends. But there are some inconsistencies in the short-term fluctuations between the residual based estimate (S_{L3}) and the hired wage based estimate (S_{L1}). For the period 1958-60, S_{L1} showed an increasing trend while S_{L3} had a slightly decreasing trend. The main reasons for this inconsistency can be seen in that the output decreased in both 1959 and 1960, and the output price also decreased in 1958 due to relatively large imports of surplus food. For the period 1968-71, S_{L1} had a decreasing trend but S_{L3} had an increasing trend. The inconsistency for this period is mainly attributable to a significant decrease in labor input and the higher price of the output (see Table 1).

Assuming an equilibrium, the two estimates S_{L1} and S_{L3} should be nearly equal. Thus the difference between S_{L1} and S_{L3} can be considered the degree of disequilibrium. From Table 1, we can observe the difference has been significantly decreasing over time. It can be calculated that the average difference between the two estimates was about 10 percent for the first decade and about 5 percent for the second decade.

Considering that about 20 percent of the total labor input was hired labor, it also can be calculated that the self-employed workers in agricultural production received only about 73 percent of the market wage rate for the first decade and about 87 percent for the second decade covered in this study. But since 1970, the two estimates are fairly close, which implies that the self-employed farmers receive approximately the market wage rate.

The Manufacturing Sector, 1957-1974

The estimation of wage income share for the manufacturing sector is relatively less complicated than for the agricultural sector, where many resources, particularly family labor and land, do not receive a market return or have a market-determined price. The wage rates used for the estimate are the market rates actually paid to the workers. The estimate for the manufacturing sector is based on the value added series and the results are presented in Table 2.

Table 2 shows that the wage share for the manufacturing sector has an increasing trend for the period with a few exceptional years, particularly the three years 1962-64. The major reason for the low wage income share for the three years was the high inflation rate of the output price. The average annual inflation rate of the three years was about 25 percent, compared with about 12 percent as an average inflation rate for the whole period covered in this study.

One thing to note in the above estimate is that the value added series employed for the estimate is derived as the difference between gross income and intermediate cost. But the intermediate cost did not include capital consumption and business taxes. Thus the residual shares, or the differences between total value added and the wage income share, are not net return to capital. The residual shares include the depreciation cost, business taxes, interest, rent, and business profit. Thus a decrease or increase in the wage income share in our estimate does not necessarily mean an increase or decrease in the net return to capital.

From the estimate, we can observe the significantly different trends of the wage income shares between the two sectors. For the agricultural sector, it has decreased about 15 percentage points during

Table 2. Estimates of Wage Income Share in the Manufacturing Sector, Korea, 1957-1974

Year	Value added[1] (V) (Mil. Won) (1)	Output price[1] (P) (1970=1.) (2)	Labor input[2] (L) (Mil. Man day) (3)	Wage[3] (W) (Won/day) (4)	Wage share ($S_L = \frac{L \cdot W}{V \cdot P}$) (5)
1955	74,568	.1736			
56	87,440	.1974			
57	94,652	.2259	91.2	77	.3284
58	103,254	.2420	98.7	84	.3318
59	112,777	.2651	115.1	86	.3312
60	121,997	.2746	125.7	92	.3451
61	125,875	.3157	131.3	104	.3439
62	142,344	.3560	141.3	112	.3124
63	166,955	.4381	181.5	128	.3177
64	177,862	.6046	198.8	179	.3319
65	213,345	.6744	240.7	205	.3429
66	249,874	.7641	261.9	265	.3635
67	306,773	.7796	307.5	295	.3793
68	389,666	.8404	352.7	347	.3737
69	473,034	.9105	375.5	444	.3871
70	560,011	1.0000	392.9	571	.4006
71	569,206	1.0392	407.2	683	.4060
72	762,785	1.1838	467.4	798	.4132
73	998,750	1.2717	576.4	878	.3984
74	1,173,119	1.6124	628.8	1,194	.4119

[1]Value added was measured in 1970 constant prices. From BOK, "National Income in Korea," 1975 pp. 198-199. The price data are from the same source.

[2]Labor input was measured by the physical labor unit of work day, which is calculated from the number of employees and average work days per month reported in EPB, "Korean Statistical Yearbook," 1975.

[3]The wage rate was calculated by dividing the average monthly pay by average work days per month, which are reported in the above source.

the period. But for the manufacturing sector, it has increased about 8 percentage points during the period. We also observe from short-run fluctuations of the trends in both sectors.

Recent studies indicate that there have been various trends in wage income share in different sectors and in different countries or regions. Some of these studies are listed in the footnote.* Comparing our estimates with those of other countries cited there, the wage income share in Korea is significantly low, particularly in the manufacturing sector, which may contribute to the attraction of a large inflow of foreign capital in the last decade.

Preliminary Analysis of the Wage Income Shares and Their Related Variables

From the previous section we have observed the general trends of the actual wage income shares in both the agricultural and manufacturing

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*For the U. S. national economy, the wage income share changed with a significant increasing trend from 55 percent for 1900-1909 to 67 percent for 1949-1957 (Kravis, 1959). For U. S. agriculture, the wage income share to net agricultural income increased fairly steadily from 58 percent for 1910-14 to 65 percent for 1945-46 (Johnson, 1954). But since 1946 it declined fairly steadily from 55 percent to 44 percent for 1954-57 (Rutan and Stout, 1960). For the U. S. manufacturing sector, the wage share has significantly increased at the rate of 0.4 percent per annum during the period 1948-1962 (Ferguson and Moroney, 1969). For the Canadian manufacturing sector, no significant trend with about 50 percent of wage share from 1926-58 (Goldberger, 1964). For Canadian agriculture, the share of labor in gross agricultural output decreased from about 51 percent for 1941-45 to 25 percent for 1961-65 (Lerohl and Maceachern, 1967). For the Israel agricultural and manufacturing sectors, the wage income share showed a steadily declining trend during the period 1952-69, at the rate of 1.3 percent and 0.8 percent per year, respectively (Fishelson, 1974).

sectors for the last two decades. We also indicated, in the previous chapter, the major economic forces affecting these trends, with an equilibrium assumption and a specified production function.

There are a variety of relationships between the wage income share and its various determinants. This section will analyze the behavior of the major economic variables affecting the trends for the purpose of examining the consistency of our data and the assumptions made in the previous chapter.

Real Wage Rate and Average Labor Productivity

From the definition of the wage income share, $S_L = (W/P) \cdot (L/Y) = (W/P)/(Y/L)$, we can separate the change of S_L as the change of the real wage rate (W/P) and the average labor productivity (Y/L). Assuming the marginal productivity theory of wages, then $\dot{S}_L = \dot{MP}_L - \dot{AP}_L$, where the dot over the veriables means the proportional rate of change, or growth rate, of the variables, as used in the previous chapter. Thus the rate of change in the wage share is simply the difference between the rates of change in marginal productivity and average productivity which are basically determined by the nature of the production function.

It is clear that constancy of the wage share requires an equal growth rate of the real wage rate and average labor productivity. If the growth rate of the average labor productivity is faster than that of marginal productivity or the real wage rate, the share will decrease, and vice versa.

Table 3 shows the behavior of both the variables--real wage rate and average labor productivity for the specified periods. In the table the rates of change in output, labor input, output price and wage rate are set forth in terms of averages for the specified periods. Among the important changes during the periods are the following.

The Agricultural Sector

The real wage rate, which is the wage rate deflated by the price of agricultural output, showed a decreasing trend in the first decade, 1955-1964. But it showed a significantly increased rate during the late 1960's due to the relatively low inflation rate of the output price.

The growth rate of output was about a 5.0 percent annual rate in the 1960's, which was relatively high compared to other periods. The labor input tended to decline at about a 2.7 percent annual rate since mid-1960's. The differential growth rate of output and labor input resulted in an increase in the average productivity of labor from 1960. Thus the growth rate of the average productivity of labor was higher than that of the real wage rate except in the first period, which led to the decrease in the relative wage income share for all the other periods.

The Manufacturing Sector

Since the mid-1960's, the real wage rate has increased very rapidly with about an 11 percent average annual rate. This resulted from a relatively lower inflation rate for manufacturing products and a faster increasing rate of money wage rate. The growth rate of average productivity was also significantly higher than that of the agricultural sector, which resulted from the larger differential growth rate of

Table 3. Rate of Changes in Some Variables Related to Wage Shares for Specified Periods in the Agricultural and Manufacturing Sectors, Korea.

	Annual Rate of Change (%)[1]					
Periods	Output (Y)	Labor (L)	Price (P)	Wage (W)	$\frac{W}{P}$	$\frac{Y}{L}$
Agriculture						
1955-59	1.94	5.78	7.61	5.29	- 2.32	-3.84
60-64	5.56	3.48	22.90	19.13	- 3.77	2.08
65-69	4.46	-2.65	11.85	18.49	6.64	7.11
70-74	1.71	-2.76	20.81	19.95	- .86	4.47
55-64	3.95	4.50	16.10	12.98	- 3.12	- .55
65-74	3.08	-2.70	15.83	19.22	3.39	5.78
55-74	3.50	.71	15.96	16.26	.30	2.79
Manufacturing						
57-64	9.51	12.02	15.58	12.30	- 3.28	-2.51
65-69	21.65	13.69	8.60	20.66	12.06	7.96
70-74	20.04	10.77	12.32	22.29	9.97	9.27
65-74	20.85	12.23	10.46	21.48	11.02	8.62
57-74	16.18	12.14	12.57	17.70	5.13	4.04

[1]Calculated from Tables 1 and 2. The rates of change of the variables are calculated as, for example, the rate of change of output $\dot{Y} = (Y_t - Y_{t-1})/Y_{t-1}$.

output and labor input. From the mid-1960's, the growth rate of the real wage rate was higher than that of the average productivity of labor, or opposite to the trend in the agricultural sector. As a result, the wage income share has been increased in the manufacturing sector for the period.

During the whole period the real wage rate grew at an average of .3 percent annually for the agricultural sector and 5.1 percent for the manufacturing sector. The average productivity of labor grew at 2.8 percent in the agricultural sector and 4.0 percent in the manufacturing sector. As a result the growth rate of average labor productivity for the agricultural sector was greater than that of the real wage rate, but for the manufacturing sector the growth rate of average labor productivity was less than that of the real wage rate. Thus the wage income share should show a decreasing trend for the agricultural sector and an increasing trend for the manufacturing sector during the period.

Relative Quantities and Prices of Factors

It is also possible to examine the behavior of the wage income share, S_L, by the relationships between the ratio of wage and property shares and its determinants. The relative share of labor to capital can be regarded as the product of the capital-labor quantity and price ratios, and changes in the division of income resulting from changes in these ratios.* That is, $S_L/S_K = (w/r)\cdot(L/K) = (w/r)/(K/L)$.

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*Once S_L/S_K is known, the wage income share S_L can be calculated since the sum of the wage and property shares must equal 1, $S_L/S_K = S_L/(1-S_L)$. Thus S_L can be found by substituting the numerical value of S_L/S_K and solving for S_L.

Thus the rate of change in the share ratio is the sum of the rates of change in w/r and L/K, or the difference of the rates of changes in w/r and K/L.

Since the ratio of the rate of change in the relative quantity (K/L) to the rate of change in the relative price (w/r) is equal to the elasticity of substitution, we are dealing with the proposition that changes in relative shares depend upon the elasticity of substitution.

Assuming that the relative price (w/r) and quantity (K/L) would move in same direction, the possibility for factor substitution would serve as a stabilizing force for changes in relative shares. When the rates of change in relative quantity (K/L) and price (w/r) are equal--in other words, the elasticity of substitution is unity--relative shares will of course remain unchanged.

The behavior of both variables is represented in Table 4 for both the sectors for specified periods. For both sectors, our data showed the same direction of movement for the relative quantity (K/L) and price (w/r), which are consistent with our assumption. For the agricultural sector, the relative wage rate (w/r) has moved much more slowly than the increase in the relative quantity (K/L). But for the manufacturing sector, it has moved faster than the relative quantities. The negative differential growth rate of the relative price and quantity lead to a decreasing trend of the wage share in the agricultural sector, while the positive differential growth rate leads to an increasing trend of the wage share in the manufacturing sector.

Table 4. Rate of Changes in Relative Quantities and Prices of Factors for Specified Periods in the Agricultural and Manufacturing Sectors, Korea

Periods	Annual Rate of Change (%)[1] Relative quantity $(\dot{\frac{K}{L}})$	Annual Rate of Change (%)[1] Relative price $(\dot{\frac{w}{r}})$	$(\dot{\frac{K}{L}}) / (\dot{\frac{w}{r}})$
Agriculture			
1955-59	- 2.053	- 1.573	1.305
60-64	2.238	1.730	1.294
65-69	12.762	5.362	2.380
70-74	6.698	3.484	1.923
55-64	.331	- .262	1.263
65-74	9.730	4.423	2.200
55-74	5.278	2.452	2.153
Manufacturing			
1957-64	- 2.246	- 2.093	1.073
65-69	4.280	5.838	.733
70-74	3.540	5.312	.666
65-74	3.910	5.575	.701
57-74	1.374	2.418	.568

[1] See the footnote to Table 3.

As explained above, the ratio of the growth rate of the relative quantity and price is interpreted as the elasticity of substitution between labor and capital. The ratios are shown for the specified periods in Table 4. From the results, we can see the ratios are greater than one in the agricultural sector and less than one in the manufacturing sector, which is consistent with the movements of the wage share in the two sectors.

For the explanation of the behavior of the wage share, we need to explain why the relative quantity and price have moved as we observed in the above. Under the equilibrium assumption, the reasons must be found from the nature of supply and demand conditions of the factors. A variety of assumptions can be made to answer the question.

Let us assume that the relative marginal productivities were unchanged for any given ratio of capital to labor, a situation which describes the nature of the demand condition or production relationships. If both factors were perfectly elastic in supply, there would be proportionate increases in both, and no change in relative prices or wage shares would occur. If one were more elastic in supply than the other, the relative quantity of the more elastic factor would increase, and the relative price would change depending on the degree of substitutability between the factors.

We can also consider the case in which the marginal productivity of one factor has improved relative to that of the other at any given factor ratio. In other words, the technical progress is biased. The changes in the relative quantity and price are also related to the supply conditions. If technical progress is biased to the inelastic factor, then even the smaller increase in the demand for it would result in large price increases.

As explained above, the relative quantity and price are determined by the difference in the supply elasticities and the marginal rate of substitution between the factors. If the marginal rate of substitution were constant at an equilibrium position over time, there could be no change in relative prices, and only relative quantities would change. But as we observed, the relative price also changed significantly during the periods, thus it indicates that the changes in the marginal rate of substitution have been an important factor influencing the behavior of the wage share.

With no reliable information on the supply side, this study has put more emphasis on explanation of the demand side. If we could assume that the supply curve for labor had remained fairly stable during the period, the observed behavior of prices and quantities reflect the movement of the demand curve for labor.

As we explained in the previous chapter, there are two sources causing the changes in the relative marginal productivity. One is the relative quantity of the factors, and the other is the technical progress. The responsiveness of the relative marginal productivity to the changes in the relative quantity and technical change is the main parameter to be estimated in the next chapter.

Growth Rate of Output, Input, and Aggregate Productivity

As seen in Table 3, the average productivity of labor has significantly changed during the period. It grew at about a 2.8 percent annual rate for the agricultural sector and 4.0 percent for the manufacturing sector. We also observed that the average productivity of labor has been an important factor causing the changes in the wage share for both sectors.

By definition, the rate of change in the average productivity is simply the difference between the rate of changes in output and labor input. The sources of the changes in output are changes in inputs and technical change. Assuming that factors are paid their marginal products and linear homogeneity of production function, the rate of change in output can be segregated as the effect of input change and technical change.* (Solow, 1957).

Under the assumptions, the rate of change in output is the sum of the share weighted input growth rate and the rate of technical change. In other words, the rate of technical change is just the difference between the growth rate of output and share weighted inputs. Tables 5 and 6 show the growth rate of output and inputs for both sectors.

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*Assume the production function as $Y = A(t)f(K,L)$, where $A(t)$ is a shift factor which reflects the pull of all the forces of technical change. Differentiating the production function with respect to time and dividing by Y, we obtain

$$\dot{Y} = \dot{A} + \frac{\partial Y}{\partial K}\frac{K}{Y}\dot{K} + \frac{\partial Y}{\partial L}\frac{L}{Y}\dot{L} .$$

If factors are paid their marginal products, then $\dot{Y} = \dot{A} + S_k \dot{K} + S_L \dot{L}$. And assuming constant returns to scale, $\dot{Y} - \dot{L} = \dot{A} + S_k(\dot{K}-\dot{L})$ or $(\dot{\frac{Y}{L}}) = \dot{A} + S_k(\dot{\frac{K}{L}})$, where $\dot{A}$ is the rate of productivity growth.

Table 5. The Growth Rate of Output, Inputs, and Aggregate Productivity in the Agricultural Sector, Korea, 1955-1974

Year	Annual Growth rate (%)				Productivity[3] growth index (A(t))
	Output[1] ($\dot{Y}$)	Capital[1] ($\dot{K}$)	Labor[1] ($\dot{L}$)	Aggregate[2] productivity ($\dot{A}$)	
1955					1.0000
56	- 7.12	- 2.40	5.54	-2.615	.9745
57	8.85	9.97	6.49	1.870	.9931
58	6.38	3.84	5.32	1.215	1.0053
59	- .36	3.49	5.76	5.939	1.0687
60	- .84	3.23	9.57	-9.995	.9716
61	11.78	3.70	5.10	6.847	1.0430
62	- 5.81	4.66	-7.21	- .645	1.0363
63	7.01	4.41	6.65	.765	1.0443
64	15.66	12.57	3.27	10.104	1.1617
65	1.54	21.85	-2.69	- .663	1.1540
66	10.17	8.15	.20	8.318	1.2587
67	- 4.62	5.38	- .56	-5.285	1.1955
68	2.32	4.34	-6.84	6.985	1.2853
69	12.87	10.90	-3.34	12.735	1.4729
70	- .87	4.02	-4.08	1.270	1.4918
71	2.20	2.77	1.01	.754	1.5031
72	- .44	2.58	-4.75	2.263	1.5379
73	4.00	6.67	- .94	2.711	1.5808
74	3.64	3.68	-5.03	5.743	1.6771

[1] The growth rates of output and inputs are calculated from Table 1 and the growth rate was calculated as $\dot{Y} = (Y_t - Y_{t-1}) / Y_{t-1}$.

[2] The growth rates of aggregate productivity $\dot{A}$ are calculated by the difference between the growth rate of output and share (S_{L1}) weighted inputs growth rate.

[3] A(t) is calculated from $\dot{A}$ by $A_t = A_{t-1} (1 + \dot{A})$.

Table 6. The Growth Rate of Output, Inputs, and Aggregate Productivity in the Manufacturing Sector, Korea, 1957-1974

Year	Growth rate (%)				Productivity growth index[3] (A(t))
	Output[1] ($\dot{V}$)	Capital[1] ($\dot{K}$)	Labor[1] ($\dot{L}$)	Aggregate productivity[2] ($\dot{A}$)	
57					1.000
58	9.09	7.80	8.23	- 1.15	1.015
59	9.22	10.31	16.70	- 3.21	.982
60	8.18	7.47	9.16	.127	.983
61	3.10	2.19	4.50	.116	.984
62	13.16	9.04	7.62	4.564	1.029
63	17.29	11.20	28.45	.610	1.036
64	6.53	13.04	9.51	- 5.368	.990
65	19.95	29.39	21.06	- 6.584	.925
66	17.12	14.13	8.82	4.920	.970
67	22.77	16.45	17.42	5.952	1.028
68	27.02	19.98	14.68	9.021	1.121
69	21.39	9.90	6.47	12.818	1.264
70	18.39	13.38	4.63	8.515	1.372
71	17.71	14.80	3.64	7.441	1.474
72	15.71	14.44	14.77	1.137	1.491
73	30.93	16.88	23.32	11.484	1.662
74	17.46	13.67	9.11	5.668	1.756

[1]Calculated from Table 2 by the same procedure as in Table 5.

[2]See footnote [2] of Table 5.

[3]See footnote [3] of Table 5.

Using the estimates of the wage income share from Tables 1 and 2, the residuals, or the rates of aggregate productivity change, are calculated and presented in Tables 5 and 6. There may be some possible errors in the specifications of technical change and the underlying assumptions. But the estimate can give us at least some information about the importance of technical change for the output growth and average productivity increase.

One observation from the estimates is that the rate of aggregate productivity growth fluctuated greatly year to year for both sectors. It fluctuated relatively more in the agricultural sector, which may result from the fact that agricultural production has more random elements, such as weather conditions. There may be various sources of errors for the estimates, but from the fluctuations one may question particularly the measurement of capital input. Note that we measured capital input in stock concept for the manufacturing sector. Thus the fluctuations of investment affect largely the measures of capital input, which also affect the estimates of A_t as can be seen in Table 6.

The results show that the average rate of aggregate productivity change during the whole period was about 2.5 percent per year for the agricultural sector and 3.5 percent for the manufacturing sector. There is some evidence that the productivity growth rate may have accelerated from the mid-1960's. And it is also calculated that about 73 percent of the agricultural output growth and 22 percent of the manufacturing output growth* for the period are explained by the

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*Solow estimated the contribution of technical change to the growth rate of average labor productivity in the U. S. nonfarm private sector for the period 1909-1949 as 87.5 percent of the total growth rate (Solow, 1957).

productivity growth. Thus it is clear that productivity growth should be included as an important factor in the explanation of production relationships.

CHAPTER IV

ESTIMATION OF FACTOR SUBSTITUTABILITY AND THE BIAS OF FACTOR EFFICIENCY GROWTH

In estimating the parameters of a specified production function, it is possible to fit either the function directly or the marginal productivity conditions.* This chapter will attempt to estimate the parameters by using the marginal productivity relations. In the next chapter more attention will be given to direct estimation procedures for the specified production function.

The indirect estimation procedure, which is based on marginal productivity relations, is commonly used in many empirical researches because of the difficulty of direct estimation when the production function is nonlinear with respect to its parameters. However, the validity of the indirect procedure will depend on how close the production behaviors of the economy are to the assumption.

Basic Estimating Equations with Marginal Productivity Relations

For the purpose of the study we are mainly interested in estimating the substitution parameter ρ and factor augmenting parameters α and β or

*However, if the error terms of the production function and marginal productivity relations are jointly distributed, the two sets of relations are interdependent, and thus the estimation, based on a separated relation, will suffer from simultaneous equations bias. But it was proved that under certain assumptions, the single estimation procedures are consistent and they are also unbiased if the error terms of the production function and marginal productivity relations are independent (Hobges, 1969; Zellner, et al., 1966).

their difference. For this reason the production function (2-11), specified in Chapter II, can be rewritten as

$$V = [C_1(t^{\alpha}K)^{-\rho} + C_2(t^{\beta}L)^{-\rho}]^{-\frac{1}{\rho}} \tag{4-1}$$

where C_1 and C_2 are constants. From the above production function, the marginal product of each factor can be derived as

$$MPL = C_2 \ t^{-\rho\beta} \ (V/L)^{\rho+1} \tag{4-2}$$

$$MPK = C_1 \ t^{-\rho\alpha} \ (V/K)^{\rho+1} \tag{4-3}$$

Under the assumption of perfect competition and profit maximization, the marginal product of each factor should be set equal to the price of the factor. Thus rearranging the marginal productivity relationships and taking them in logarithms we get

$$\ln (V/L) = \sigma \ln w + (1-\sigma)\beta \ln t - \sigma \ln C_2 \tag{4-4}$$

$$\ln (V/K) = \sigma \ln r + (1-\sigma)\alpha \ln t - \sigma \ln C_1 \tag{4-5}$$

where w and r are real prices of labor and capital, and $\sigma = \frac{1}{1+\rho}$.

The Equation (4-4) is the famous estimation equation which was derived from empirical relationships by Arrow, et al. (Arrow, et al., 1961). Not having capital data, they did not attempt to estimate Equation (4-5). Using Equation (4-4), Lee (Lee, 1974) estimated the elasticity of substitution for the Korean agricultural sector as slightly greater than unity with a neutral assumption of technical change. But if β is positive and the true value of the elasticity of substitution is greater than unity, the neutrality assumption will lead to a downward bias of his estimate.

From Equation (4-2), we can derive an equation slightly different from (4-4). Dividing both sides of Equation (4-2) by w and manipulating the resulting expression, we obtain

$$\ln\left(\frac{L\cdot w}{V}\right) = (1-\sigma)\ln w + \beta(\sigma-1)\ln t + \sigma \ln C_2 \qquad (4\text{-}6)$$

With an assumption of an exponential rate of technical progress, Arrow, et al. fitted this equation to data for the U. S. private nonfarm sector during the period 1909-49 and obtained $\sigma = .569$ and a 1.83 percent annual rate of growth of productivity. Lianos (Lianos, 1971) also applied this equation to U. S. agriculture during the period 1949-68 and obtained $\sigma = 2.44$ and a negative value of β.

Since the marginal rate of technical substitution is equal to the real factor price ratio in competitive equilibrium, we may derive the expansion path by taking the ratio of the Equations (4-2) and (4-3).

$$\frac{w}{r} = \frac{C_2}{C_1}\, t^{-(\beta-\alpha)} \left(\frac{K}{L}\right)^{\rho+1} \qquad (4\text{-}7)$$

From the standpoint of the individual firm the capital intensity (K/L) is a decision variable that is changed by the entrepreneur in response to changes in relative factor prices and improvements in input efficiency. From this viewpoint, it appears reasonable to treat K/L as the dependent variable. Thus rearranging the equation and taking it in logarithm, we obtain

$$\ln (K/L) = \sigma \ln (w/r) + (\beta-\alpha)(1-\sigma) \ln t + \sigma \ln (C_1/C_2) \qquad (4\text{-}8)$$

This equation has been used widely to estimate the elasticity of substitution and bias of technical change (Ferguson and Moroney, 1969; Lianos, 1971).

By dividing both sides of Equation (4-7) by $\frac{K}{L}$ and rearranging the terms in logarithm, we obtain the equation relating the capital-labor ratio to the factor share ratio as

$$\ln \left(\frac{K}{L}\right) = \frac{\sigma}{1-\sigma} \ln \left(\frac{S_L}{1-S_L}\right) + (\beta-\alpha) \ln t + \frac{\alpha}{1-\alpha} \ln \left(\frac{C_1}{C_2}\right) \qquad (4\text{-}9)$$

Equation (4-9) has some advantage in data requirements. It does not involve real factor price data. In most cases, factor share information is easier to get than factor price data, particularly on capital.

Because of the difficulty of direct measurement, capital price is often derived in empirical work as $r = \frac{V-wL}{K}$. Thus r is not independent of K, and an error of measurement in K would also appear in r. If this is the case, Equation (4-9) will result in better estimates than Equation (4-8).

If the elasticity of substitution is estimated separately from each marginal productivity conditions, the resulting estimates are in general not consistent with each other (Dhrymes, 1965). The difficulty may be overcome by estimation from the ratio of the marginal productivity equations. But the profit maximization assumption requires that each marginal condition has to hold by itself. This requirement does not necessarily hold for the ratio Equation (4-8). Thus simultaneous estimation of both marginal equations with the constraint of an equal estimate for σ may improve the estimation (Fishelson, 1974). From Equations (4-4) and (4-5) we get a simultaneous equation system as

$$\begin{bmatrix} \ln (V/L) \\ \ln (V/K) \end{bmatrix} = \begin{bmatrix} \ln w & \ln t & 0 & 1 & 0 \\ \ln r & 0 & \ln t & 0 & 1 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{bmatrix} \qquad (4\text{-}10)$$

where $b_1 = \sigma$

$b_2 = (1-\sigma)\beta$

$b_3 = (1-\sigma)\alpha$

$b_4 = \sigma \ln C_2$

$b_5 = \sigma \ln C_1$

So far, the economy has been assumed to adjust instantaneously to an equilibrium situation at any time. But the adjustment process of the capital intensity of production in response to a change in relative factor price may not be a short-run process.

In order to take account of the short-run inflexibility of factor proportion, the above estimation model may be reformulated to include a partial adjustment process in the capital-labor ratio. To this end we may assume that the adjustment toward the desired capital-labor ratio, $(K/L)^*$, has an exponentially distributed lag of the form

$$(K/L)_t/(K/L)_{t-1} = [(K/L)^*_t/(K/L)_{t-1}]^\lambda \tag{4-11}$$

where λ is an adjustment coefficient and $0 < \lambda \leq 1$.

Under the above lagged adjustment assumption, the capital-labor ratio of Equation (4-8) can be interpreted as the desired capital-labor ratio. Thus by combining the Equations (4-8) and (4-11), we obtain the equation as

$$\begin{aligned} \ln (K/L)_t &= \lambda\sigma \ln(w/r)_t + \lambda(\beta-\alpha)(1-\sigma) \ln t \\ &\quad + (1-\lambda) \ln (K/L)_{t-1} + \lambda\sigma \ln (C_1/C_2) \end{aligned} \tag{4-12}$$

Using the same assumption, the equation system (4-10) also can introduce the lagged adjustment process. Thus it can be modified as

$$\begin{bmatrix} \ln (V/L)_t \\ \ln (V/K)_t \end{bmatrix} = \begin{bmatrix} \ln w_t & \ln t & 0 & \ln(V/L)_{t-1} & 1 & 0 \\ \ln r_t & 0 & \ln t & \ln(V/K)_{t-1} & 0 & 1 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \end{bmatrix} \quad (4\text{-}13)$$

where $b_1 = \lambda\sigma$

$b_2 = \lambda(1-\sigma)\beta$

$b_3 = \lambda(1-\sigma)\alpha$

$b_4 = (1-\lambda)$

$b_5 = \lambda\sigma \ln C_2$

$b_6 = \lambda\sigma \ln C_1$

In the above, we derived various estimating equations which are commonly used in empirical work, except that we have introduced a somewhat different assumption of technical progress. However, the validity of the equations depends on the marginal productivity assumption. And as seen, most of the equations used the factor price or the ratio of the prices as an exogenous variable. This choice of exogenous variable may be reasonable when the regression is performed on the individual firm data. But when the regression is applied at an aggregate national level, the choice of exogenous variable is not clear because of simultaneous determination of factor price and employment. We will discuss this problem further in the next section.

Initial Estimation with a Static Equilibrium Assumption

As an initial estimation for the parameters, three basic estimating equations have been fitted to the data for both the agricultural and manufacturing sectors. They are Equations (4-4), (4-5), and (4-8), which were derived in the preceding section. Equations (4-4) and (4-5) are the equilibrium condition of labor and capital markets, respectively, and Equation (4-8) is the production expansion path equation which is the ratio of the other two equations.

Application of the three different equations is basically aimed at a test of the validity of the assumptions underlying the estimation equations--homogeneity of degree one and perfect competition. Thus we hope to find the general directions of our efforts for a better estimation. If the assumptions were valid, then the estimates of parameters should not be significantly different across the different estimation equations.

The sources and descriptions of the data employed for the estimation were explained in more detail in the first section of the previous chapter. The definitions of the data used for measurement are not identical between the sectors mainly because of the lack of availability. For the agricultural sector, the measure of labor input is an adult-man-day equivalent unit, which was corrected by age and sex. Accordingly, the wage rate was also measured per adult-man-day equivalent unit.

But for the manufacturing sector, the measure of labor input was not corrected, partially due to the lack of data and partially due to the conceptual difficulty. Age and sex may not be important factors for the ability of labor for the manufacturing production. Rather certain specialized skills or experiences may be more important ability correction factors, but no such relevant data for this correction was available. The wage rate is the average rate per uncorrected physical day unit.

Capital inputs were measured in 1970 constant prices. For agriculture, the measure of capital input was the flow service concept, which included the depreciation cost on durable capital goods such as machinery and farm equipment, farming building, perennial trees and livestock, irrigation charges, and all variable costs of intermediate inputs. The price of capital was calculated by dividing the total current expenditures on nonlabor inputs by the measure of capital input. Total expenditures consisted of the current value of all service charges and interest on the stock value of capital goods.

But for the manufacturing sector, the capital input was measured by the stock value of all tangible fixed capital goods. The price of capital was derived by dividing total returns to capital by the measure of capital input. Total returns to capital were calculated by the residual concept, that is, $V - wL$. Thus the deriving procedure of capital prices is different between sectors, but if the equilibrium assumption is satisfied, the two procedures should give similar results.

The measure of output is the value added concept for the manufacturing sector but the gross output concept for the agricultural sector. The different specification of the output variables is not based on data availability, but it is rather based on a theoretical argument.

The value added data are more commonly used as a measure of output in most empirical studies. These data are based on the assumption that there is no substitution between primary inputs and intermediate inputs. Thus it is convenient to subtract the intermediate input from both sides of the production function. But it is often argued that some intermediate inputs, particularly for agricultural production, are substitutes for primary inputs rather than being in a fixed proportion relation (Griliches, 1964). In Korean agriculture, it seems true that some intermediate inputs such as fertilizers, weed killers and other agricultural chemicals can be substituted for labor input more smoothly than machinery.

It should be also pointed out that the process of technical change must be viewed in the context not only of capital and labor inputs, but also of intermediate inputs as a whole. Thus for agriculture, the output variable was measured by gross output, and the intermediate input was included in the measure of capital input.

Before fitting the data prepared with the above concepts to the estimating equation, we need to discuss some of the characteristics of the estimating equations. Equations (4-4) and (4-5) assumed profit maximization and constant returns to scale. But Equation (4-8) assumed only cost minimization for a given rate of output and does not require the assumption of constant returns to scale. However, if one is not interested in estimating the constant term, or the logarithm value of the ratio of distribution parameter, the above equations are also valid for the estimation even under the weaker assumption that each factor price is just proportional to its marginal product.

Another point we need to discuss is the choice of exogenous variables. It may be argued that both variables--employment level or factor input ratio and the prices of factors or the ratio of the prices--may be endogenous or determined simultaneously in some larger system. And it was also pointed out that, based on empirical study, the estimates of the parameters of the production function are very sensitive to the classification of the variables into exogeneous and endogenous groups (Nerlove, 1967). The argument is basically saying that both variables are subject to random error, or possibly the variable we choose as exogenous is correlated with the disturbance term in the regression equations. If this is the case, the simple ordinary least squares(OLS) estimation procedure will not provide even the desirable large sample properties.

As can be seen in the equations, we choose the factor prices as exogenous variables. Since the regression equations pertain to sectors, it may be plausible to assume that the factor prices, or their ratios, are exogenous variables, which may be determined at the more aggregate level. Thus the factor employment level is the decision variable that is changed by the production decision units of each sector in response to changes in factor prices. But this may not be quite true if we consider that both sectors are relatively large compared to the total economy. Thus any change in the factor demand in one of the sectors would likely affect the factor price.

There are also some other reasons for our choice of exogenous variables. Considering the fact that the price of capital has been largely controlled by the government development policy for the periods observed, it is desirable to consider the factor price variable as exogenous.

Another justification for the choice of capital-labor ratio or capital input as a dependent variable is that considering the nature of the data, the capital input data are more likely to contain substantial errors of measurement. This being the case, econometric considerations indicate the desirability of having the errors of measurement occur in the dependent variable and not among the explanatory variables of the regression, which will lead to inconsistent estimates of our parameters.

With the above specifications, the three basic marginal productivity relations are fitted to the data for both sectors. Assuming that the error term, u, is normally and independently distributed with zero mean and constant variance and is independent of all the explanatory variables, the ordinary least squares estimation method will give at least consistent estimates. But the first results of the OLS application to the regressions showed positive autocorrelation in most of the equations, which is one of the general problems of time series data. Thus the regression equations, except Equations (4-5) and (4-8) for the agricultural sector in which the D-W statistics are in the inconclusive region, were recomputed by the iterative two-stage procedure.

The first stage computes the OLS residuals ignoring all complications of the covariance matrix. Next, compute the ratio of the mean product of the successive residuals to the OLS variance estimator which is to be regarded as an estimator of autocorrelation coefficient, ρ.

The second stage uses the estimated value of ρ, constructs new variables $(y_t - \hat{\rho}\, y_{t-1})$ and $(X_t - \hat{\rho}\, X_{t-1})$, and applies OLS to the new variables. Again compute the second round residuals and estimate the second round $\hat{\beta}$. These procedures are to be iterated until the values of the estimators converge. The procedure is convergent and the final round estimates coincide with the maximum likelihood estimates (Kmenta, 1971, p. 288).* But actually, the iterative procedure was reduced by stopping after obtaining the second round estimates.

The results estimated by the procedure are presented in Table 7. Various observations can be made from the results of our initial estimates. For agriculture, the estimates are largely inconsistent across different estimation equations, but Equation (4-8) is relatively better than the other two equations in a statistical sense. There may be various sources which lead the estimates to be biased, such as errors of observation in the explanatory variables, the simultaneity problem in the more complete system, some factors causing market imperfections, and so on.

From the results shown in Table 7, however, an important question can be raised as to the assumption of profit maximization and/or constant returns to scale for agricultural production, which was made for the derivation of Equations (4-4) and (4-5). Considering the small scale production, with a low marketing rate for their products, the self-food supply objective may be more important for Korean agriculture

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*It was pointed out that there is some possibility that the likelihood function may have multiple local maxima. Hence the iterative procedure is subject to the risk that the local maximum obtained may not be a general maximum. But the empirical examples showed that it is very rare case (Hildreth and Lu, 1960).

during the period, perhaps implying that the cost minimization assumption is better than profit maximization.

The assumption of constant returns to scale is also suspect when we consider the fact that land, the most important factor for agriculture, is a very limiting input, and that it was fairly constant for the time periods. Thus decreasing returns to scale may be more likely than constant returns to scale. If this is the case, the estimate of the elasticity of substitution from Equations (4-4) and (4-5) will be biased. The directions of the bias will be discussed in a later part of this section.

From the above discussion, for the estimation of the parameters for the agricultural production it seems relatively reasonable to rely more on the estimates of Equation (4-8) than on Equations (4-4) and (4-5). If this is acceptable, it is possible to argue that the estimates of the elasticity of substitution parameter provided by Equations (4-4) and (4-5) are significantly low. There may be various possibilities by which the estimates from both equations are biased, particularly negative bias, for the explanation of low value of the estimate.

One possibility comes from the lagged behavior of factor price to product price change, or the degree of market imperfection. For example, from Equation (4-4), assuming that the money wage rates are lagged to price change, which means that the price increase is negatively correlated with the real wage rates, then the estimate of the substitution parameter will be downward biased if the price increase is positively related with the output (V), which agrees with our data.

Table 7. Initial Estimates of Elasticities of Substitution and Factor Efficiency Growth Parameters, 1955-1974

Estimation Equation	Parameters				$\bar{R}^2$**
	α	β	$\beta-\alpha$	σ	
Agriculture					
(4-4)		-.1516 (.1621)*		1.2706 (.5204)	.844
(4-5)	.0406 (.1052)			.6592 (.4986)	.786
(4-8)			-.1129 (.1818)	1.7738 (.4718)	.914
Manufacturing					
(4-4)		.0153 (.1310)		.6719 (.1362)	.9387
(4-5)	.4537 (.3142)			.7024 (.3842)	.8224
(4-8)			-.5243 (.1092)	.7872 (.1562)	.9012

*The numbers in the parentheses are the standard errors of the regression coefficients. The standard errors of the parameters, α, β and $\beta-\alpha$, which are nonlinear functions of random variables are calculated by a Taylor's approximation procedure. Let $y = y(x)$ be a differentiable function of x where x is a vector of random variables. Then the approximated variance of y can be expressed as Var $(y) = j'\Sigma j$ where $j = \partial y/\partial x$ and Σ is the covariance matrix of x (Goldberger, 1964, p. 125). Thus for example, since $\hat{\beta-\alpha} = \frac{\hat{C}_1}{1-\hat{C}_2}$ in Equ. (4-8),

$$V(\beta-\alpha) = \left(\frac{1}{1-\hat{C}_2}\right)^2 V(\hat{C}_1) + \left[\frac{\hat{C}_1}{(1-\hat{C}_2)^2}\right]^2 V(\hat{C}_2) + 2\frac{\hat{C}_1}{(1-\hat{C}_2)^3} \operatorname{COV}(\hat{C}_1, \hat{C}_2).$$

**The $\bar{R}^2$ denotes the coefficient of determination after adjusting for degree of freedom.

Another important possibility is the misspecification of the scale parameter in which we assumed constant returns to scale. The directions of the bias can be seen from the following expression. From the example of the marginal productivity relation of labor, we can derive the relation of Equation (4-4) without any a priori assumption of the value of the scale parameter. With a little manipulation of Equation (4-2), we can get the expression (4-4') as seen in the footnote.* By assuming the scale parameter, $\nu=1$, the expression will be identical with Equation (4-4).

From Equation (4-4'), it is clear that the estimate based on Equation (4-4) has two sources of bias if the true relation of production is not constant returns to scale. One is from ignoring the factor of the coefficient of ln w in the true relation, $\frac{1}{\nu}(1-\sigma)+\sigma$. And the other source of bias may come from ignoring term ln L which may be correlated with other explanatory variables which are included. Thus the directions of the bias due to the assumption of constant returns to scale will depend on the true values of ν, σ and the degree of correlation between ln w and ln L.

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*From Equation (4-2),

$$C_2\nu t^{-\rho\beta}\left(V^{\frac{1}{\nu}+1}/L^{\rho+1}\right) = C_2\nu t^{-\rho\beta}\left(V^{\frac{1}{\nu}(\rho+1)}/L^{\rho+1}\right)V^{1-\frac{1}{\nu}} = w.$$

Taking the logarithm and substituting $\rho+1 = \frac{1}{\sigma}$ and $\rho = (1-\sigma)/\sigma$, we get the expression as

$$\ln(V^{\frac{1}{\nu}}/L) = (1-\sigma)\ln(C_2\nu) + (1-\sigma)\beta\ln t + \sigma\ln w + \sigma\left(\frac{1}{\nu}-1\right)\ln V.$$

With some more manipulations, we get

$$\ln(V/L) = \frac{(1-\sigma)\nu}{1-\sigma+\nu\sigma}\ln(C_2\nu) + \frac{(1-\sigma)\beta}{\nu^{-1}(1-\sigma)+\sigma}\ln t + \frac{\sigma}{\nu^{-1}(1-\sigma)+\sigma}\ln w + \frac{\sigma+\nu(1-\sigma)-1}{1-\sigma+\nu\sigma}\ln L \qquad (4\text{-}4')$$

The reliability of the data used can also be questioned, but there are very little intuitive insights into the nature of the errors of observations and thus no insight into the direction of the bias.

The estimates for the manufacturing sector are relatively better than for agriculture. The estimates of the substitution parameter are fairly consistent between the different estimation equations. The estimates of the factor efficiency parameter also showed consistent trends. This may be explained by the more appropriateness of the assumptions that were made for the derivation of the estimation equations. However, one point to note for the manufacturing sector is the derivation procedure of capital price data, in which we used the residual concept, that is $r = \frac{V-wL}{K}$. The possible influence of this derivation procedure of capital input price on the estimate of the parameter is discussed by Dhrymes (Dhrymes, 1965).

Assuming that the capital series is substantially adulterated by errors of observation and the data series of V, w and L are free of observational errors, then the correlation between V/K, and r may be biased in an upwards direction.* This bias will lead to a higher R^2 and higher value of the elasticity of substitution in the capital marginal productivity relation than in the labor marginal productivity relation. But this argument was not consistent with our initial estimation results. Possibly the assumption made for deriving the

*This argument was proved by Dhrymes. Let the observation error of capital input be multiplicative so that we can write $\ln K = k + u$. Put $\ln V = x$, $\ln (V-wL) = y$ and assume $Cov(x, u) = Cov(k, u) = Cov(y, u) = 0$. Then we have $Cov(\ln(V/K), \ln r) = Cov(x-k-u, y-k-u) = Cov(x-k, y-k) + Var(u)$. From this Dhrymes showed that the correlation between ln (V/K) and ln r is higher with observational errors in capital than without the errors, that is $Corr(x-k-u, y-k-u) - Corr(x-k, y-k) > 0$. (For the details of the proof, see the original paper.)

argument may not be satisfied in our data, or there may be some other reasons for which we have no insight.

From Table 7, it may be observed that the estimate of the elasticity of substitution is greater than one for the agricultural production and less than one for the manufacturing production, and the factor efficiency growth is biased to capital augmenting for both sectors. Thus in Hicksian terminology the nature of the technical progress is capital-using for agriculture and labor-using for manufacturing during the periods covered in this study. However, the results are very tentative considering the various possible sources of the bias which are discussed above. Thus more detailed discussions about the magnitudes of the estimated values of the parameters are reserved until we get more reliable evidence.

With the above very tentative observations from the initial estimation results, more efforts will be directed in the remaining part of this chapter to the improvement of the estimates by introducing variuos assumptions and estimation methods. In the following section we will concentrate our efforts on estimating the parameters of agricultural production; a partial adjustment assumption in the expansion path equation will be introduced. In the final section the efforts will be toward the improvement of the estimates for the manufacturing production. In this discussion we will maintain the profit maximization assumption and will apply different estimation methods with different specifications of the error term in the hope of increasing the efficiency of the estimates. Application of the direct estimation method will be discussed in the next chapter. This method requires only the acceptance of a specified production function. Particularly for agriculture, we may suspect the equilibrium assumption underlying the indirect estimation procedure as pointed out above.

Estimation from the Expansion Path Equation for the Agricultural Sector

The main effort of this section is to improve the estimates of the parameters for agricultural production. Based on the results drawn from the simple basic regressions, attention will be given to the production expansion path equation. On the same ground, the equilibrium relations of each marginal productivity equation will not be given any further attention for agricultural production.

Equation (4-8) related the actual level of capital intensity to the relative factor price ratio. But it may be interpreted as indicating the relationship between the desired or optimal capital intensity and relative factor prices. There may be various reasons causing the lagged behavior of the actual input adjustment to optimum combinations. They may include technical constraints, institutional rigidities, the fixity of some factors, persistence of habit, and so on (Griliches, 1967). Thus it should not be assumed that such adjustments are completed instantly so that the desired capital intensity is always identical to the actual capital intensity of production in each period of time.

In order to take account of the lagged behavior of the input adjustment or the short-run inflexibility of factor proportions, Equation (4-8) was reformulated by replacing the optimal input combination, $(K/L)_t{}^*$, for the actual inputs ratio, $(K/L)_t$. The modified estimation equation was derived in Equation (4-12) by specifying the adjustment process of $(K/L)_t$ as an exponentially distributed lag of a Nerlove type.

But assuming that the optimum combination of factors is not known in advance, the specification of the adjustment process described in Equation (4-11) may not be appropriate. Thus the adjustment process may be revised by assuming that the current input ratio $(K/L)_t$ is adjusted by comparing $(K/L)_{t-1}$ with $(K/L)^*_{t-1}$ rather than with $(K/L)^*_t$. Thus, the adjustment process can be expressed as

$$(K/L)_t/(K/L)_{t-1} = [(K/L)^*_{t-1}/(K/L)_{t-1}]^{\lambda} \qquad (4\text{-}11')$$

With this formulation of the adjustment process, estimation Equation (4-12) was slightly changed and written with an error term as

$$\ln(K/L)_t = b_o + b_1 \ln(t-1) + b_2 \ln(w/r)_{t-1} + b_3 \ln (K/L)_{t-1} + e_t \qquad (4\text{-}14)$$

where $b_1 = \lambda(1-\sigma) \quad (\beta-\alpha)$,

$b_2 = \lambda\sigma$

$b_3 = 1-\lambda$.

In the above equation, λ is the actual input adjustment coefficient to the optimum input combination, while the other parameters are identical with the same notation in the previous equation.

One thing to note in Equation (4-14) is that the current factor price ratio was excluded from the explanatory variable. The equation is now an ex-ante input decision-making equation, and thus precludes a possible complication due to a simultaneous relation between the current factor ratio and the factor price ratio, which was discussed in the previous section.

One possible problem in the above equation is that the disturbance term e_t may be serially correlated and hence the lagged dependent variable may be correlated with e_t. Thus it is pointed out that the serial correlation problem in the disturbances is much more serious in the distributed lag model than in the classical regression case. There it may result in somewhat inefficient estimates. But here it is likely to lead to inconsistent ones. Moreover, one also cannot guard against serial correlation in this case by observing the Durbin-Watson statistic.*

From the above possible problem, some alternative estimation procedures are suggested by Griliches (Griliches, 1967). The suggested procedures are the use of an instrumental variable and the use of a two-stage estimation procedure, substituting for $(K/L)_{t-1}$, $\widehat{(K/L)}_{t-1}$ predicted by a finite term approximation to the true reduced form

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*For this case several new test statistics have recently been proposed by Durbin (Durbin, 1970). 1) $h = \hat{\rho}\sqrt{T/(1-T\cdot V(\hat{a}))}$ where $\hat{\rho}$ is the first order serial correlation coefficient estimated by OLS residuals, $V(\hat{a})$ is the variance of OLS estimate of coefficient of lagged dependent variable and T is the sample size. Use h as a standard normal deviate to test the H_0: $\rho = 0$. 2) From OLS compute the residuals $\hat{e}_t$. Then regress $\hat{e}_t$ on $\hat{e}_{t-1}$, y_{t-1} and X_t. The test for $\rho=0$ is carried out by testing the significance of the coefficient of $\hat{e}_{t-1}$ by OLS.

equation of the system.* However, no such effort was made for the estimation because of some other problems involved in the alternative procedures, such as the loss of data point and the choice of an instrumental variable.

Instead, this study applied the Hildrethe-Lu procedure (Hildreth and Lu, 1960). Assuming that the disturbance term e_t in the Equation (4-14) follows a first order autoregressive scheme as

$$e_t = \rho e_{t-1} + v_t$$

where v_t is an independently and normally distributed random variable with mean zero and a constant variance, from Equation (4-14), we can obtain the expression as

$$\begin{aligned}[\ln (K/L)_t - \rho \ln (K/L)_{t-1}] &= b_o(1-\rho) + b_1 [\ln(t-1)-\rho\ln(t-2)] \\ &+ b_2[\ln(w/r)_{t-1} - \rho \ln(w/r)_{t-2}] + b_3[\ln(K/L)_{t-1} - \\ &\rho \ln(K/L)_{t-2}] + v_t \qquad (4\text{-}15)\end{aligned}$$

The least squares method was applied to the equation by searching over ρ with a specified grid value of .05. The resulting estimate is equivalent to a maximum likelihood estimate, thus optimal properties for large samples are assured.**

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*For the detailed procedure, see Z. Griliches, "Distributed Lags: A Survey" (Griliches, 1967, p. 41).

**But in this case, the usual theorems on optimal properties of MLE were questioned because the likelihood function is not the product of independent identical distributions (Hildreth and Lu, 1960). Nevertheless, it was proved that the MLE of Equation (4-15) is consistent and asymptotically equivalent to BLUE. Consistency was proved in Appendix B of (Hildreth and Lu, 1960, pp. 52-54) and asymptotic equivalence to BLUE was proved by Malinvaud (Malinvaud, 1966, pp. 439-445).

The estimation results for the agricultural data are

$\hat{b}_1 = .0281$ $\qquad\qquad$ $S_{\hat{b}_1} = .0195$

$\hat{b}_2 = .5697$ $\qquad\qquad$ $S_{\hat{b}_2} = .0612$

$\hat{b}_3 = .5907$ $\qquad\qquad$ $S_{\hat{b}_3} = .1063$

$\hat{\rho} = .65$ $\qquad\qquad$ $S_{\hat{\rho}} = .2208$

$\bar{R}^2 = .9648$

From the estimated reqression coefficients we obtain

$\hat{\lambda} = 1 - \hat{b}_3 = .4093$ $\qquad\qquad$ $S_{\hat{\lambda}} = .1063$

$\hat{\sigma} = \dfrac{\hat{b}_2}{1 - \hat{b}_3} = 1.3919$ $\qquad\qquad$ $S_{\hat{\sigma}} = .3960$

$\widehat{\beta-\alpha} = \dfrac{\hat{b}_1}{1-\hat{b}_2-\hat{b}_3} = -.1752$ $\qquad\qquad$ $S^{*}_{\widehat{\beta-\alpha}} = .1852$

The regression results have improved compared to the initial estimates of Equation (4-8) in terms of explanatory power and standard errors of the estimates.

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*Following the same approximation procedure as in the footnote to Table 7,

$$V(\widehat{\beta-\alpha}) = (1/1-\hat{b}_2-\hat{b}_3)^2\ V\ (\hat{b}_1) + (\hat{b}_1/(1-\hat{b}_2-\hat{b}_3)^2)^2\ (V(\hat{b}_2) + V(\hat{b}_3))$$

$$+\ 2\hat{b}_1/(1-\hat{b}_2-\hat{b}_3)^3\ (Cov(\hat{b}_1\hat{b}_2 + Cov(\hat{b}_1\hat{b}_3)) + 2\hat{b}_1{}^2/(1-\hat{b}_2-\hat{b}_3)^4$$

$$Cov(\hat{b}_2\hat{b}_3).$$

And the estimated variance-covariance matrix of regression coefficients ($\hat{\Omega} = S^2(X'X)^{-1}$) is

$$\hat{\Omega} = 10^{-5} \begin{bmatrix} 38 & 14 & 8 \\ 14 & 375 & -45 \\ 8 & -45 & 1130 \end{bmatrix}$$

The point estimate of the adjustment coefficient λ, which is the elasticity of the input adjustment with respect to the ratio of the optimal and actual capital-labor ratio in previous years is about .41. The uaual t-statistic does not accept the null hypothesis of full adjustment assumption; $H_o : \lambda = 1$, or $b_3 = 0$ at any degree of acceptable significance level. Thus we may conclude that the input adjustment process to the price change has been fairly slow for Korean agricultural production during the period.

The point estimate of elasticity of substitution was about 1.39, which is greater than unitary as assumed by Cobb-Douglas specification. No comparable study is available except Lee's estimate of about 1.1 (Lee, 1974), in which Lee used the simple relation of average productivity and wage rate without allowing any technical change. The t-test on a linear combination of regression coefficients was performed on the null hypothesis $H_o : b_2+b_3 = 1$ or $\sigma=1$. The result shows that the estimate of σ is not significantly different from unitary at a 5 percent significance level, but it is significantly different at a 10 percent level.*

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*This is the test of a linear combination of regression coefficients. The statistic, $(w'b - w_o)/s\sqrt{w'(X'X)^{-1}w}$ is distributed as $t(n-k)$, where w is a known vector, and w_o a known scalar. In our case $w'=(0\ 1\ 1)$ and and $w_o=1$. The numerator of the ratio was calculated as .5697 + .5907 - 1 = .1604, while the square of the denominator can be calculated from the matrix $(S^2(X'X)')$ in the above footnote: .00375 + .01130 - 2 (.00045) = .01415. So that we obtain the t-value as .1604/.01415 = 1.3484.

The differential of the factor efficiency growth rate of labor and capital shows a slightly negative value, but it may not be significant considering the relatively large value of its approximated standard error.

However, the usual significance test of the ratio of regression coefficients presents a difficulty. The difficulty is that the distribution of the ratio of two normal variables with non-zero means is unknown.* This difficulty also applies to the construction of the confidence interval of the ratio of regression coefficients. Thus in an empirical study, the confidence interval of the ratio has been constructed by calculating the upper and lower bound of the value of the ratio, which is based on the separate confidence intervals of each parameter or on the rectangular joint confidence region.

However, it is not true that if $P(b_1 \varepsilon\ I_1)=1-\alpha$ and $P\ (b_2 \varepsilon\ I_2) = 1-\alpha$, then $P(b_1 \varepsilon\ I_1,\ b_2 \varepsilon I_2)$ is equal to $(1-\alpha)^2$, because of the lack of independence. The construction of the rectangular joint confidence region is explained in Theil (Theil, 1971, pp. 132-134). If $P(b_1 \varepsilon I_1) = 1-\alpha_1$ and $P(b_2 \varepsilon I_2)=1-\alpha_2$, then $P(b_1 \varepsilon I_1,\ b_2 \varepsilon I_2) \geq 1-\alpha_1 - \alpha_2$. This amounts to a confidence region for the two parameters jointly with a confidence coefficient at least equal to $1-\alpha_1 - \alpha_2$. Thus the confidence coefficient will be biased downward.

The above difficulty of the significance test and the construction of the confidence interval of the ratio of two normally distributed

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*If the means of the two variables were zero, the ratio would follow a Cauchy-distribution that has no moments, and thus the testing would be also impossible (Lianos, 1971).

random variables with different means, different variances, and covariance may be solved with the following procedures.

First, derive the joint confidence region of two normal variables. The confidence region has the form of an ellipse with the center at their respective means. The axes of the ellipse are not parallel to the axes of the coordinate system if two variables are correlated.

Next, for the construction of the confidence interval of the ratio, we need to find the maximum and minimum value of the ratio within the joint confidence region of the specified probability.

The analytical solutions of two coordinates which give the maximum and minimum value of the ratio can be derived by the method of Lagrange multipliers, as can be seen in the Appendix.

Using the derived formula, the confidence intervals of σ and $\beta-\alpha$ are constructed below. The values of a_{ij} were calculated by using the variance-covariance matrix ($\hat{\Omega}$) of regression coefficients in the previous footnote.* The calculated values of a_{ij} of b_2 and $d_1 = 1-b_3$ are $a_{11} = 134$, $a_{22} = 44.5$ and $a_{12} = 5.5$.

Thus with the 95 percent confidence coefficient the joint confidence interval of b_2 and $d_1 = 1-b_3$ is stated as

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*The statistic, $\frac{1}{2s^2}(b_1 - \beta_1 \; b_2 - \beta_2)\, C^{-1} \begin{bmatrix} b_1 - \beta_1 \\ b_2 - \beta_2 \end{bmatrix}$ is distributed as $F(2, n-k)$, where b_1 and b_2 are normal variables with the respective means of β_1 and β_2, and s^2C is the variance-covariance matrix of the two random variables. Thus the joint confidence region for b_1 and b_2 has the form: $P\{a_{11}(b_1-\beta_1)^2 + a_{22}(b_2-\beta_2)^2 + 2a_{12}(b_1-\beta_1)(b_2-\beta_2) \leq F_\alpha\} = 1-\alpha.$

where $a_{ij} = c^{ij}/2\,s^2$, c^{ij} being the (ij)th elements of C^{-1}. The values of a_{ij} are calculated from the inverse of the lower right-side 2 x 2 submatrix of $\hat{\Omega}$, $\begin{bmatrix} 268 & 11 \\ 11 & 89 \end{bmatrix}$.

$$P\{134(b_2 - .5697)^2 + 44.5(d_1 - .4093)^2 + 11(b_2 - .5697)$$
$$(d_1 - .4093) \leq 3.68\} = .95$$

The two points which give maximum and minimum values of the ratio $b_2/1-b_3$ within the above joint confidence region are calculated by the derived formula. The calculated coordinate points are

the maximum point ($1-b_3 = .2230$, $b_2 = .5735$) and

the minimum point ($1-b_3 = .5393$, $b_2 = .4876$).

Thus the 95 percent confidence interval estimate of σ is

$$.9041 \leq \sigma \leq 2.5717.$$

Using the same procedure, the confidence interval of $\beta-\alpha$ is also constructed. The factor augmenting bias of labor and capital was derived by the ratio of b_1 and $d_2 = 1-b_2-b_3$. The calculated values of a_{ij} are $a_{11} = 1327$, $a_{22} = 31.5$, and $a_{12} = 18.5$.* Thus the 95 percent joint confidence region of b_1 and d_2 is stated as

$$P\{1327(b_1 - .0281)^2 + 31.5\ (d_2 + .1604)^2 + 37(b_1 - .0281)$$
$$(d_2 + .1604) \leq 3.68\} = .95.$$

Two coordinate points which give the maximum and minimum values of $\beta-\alpha(=b_1/1-b_2-b_3)$ are solved, but the solutions turned out an imaginary value, which means that the joint confidence region includes the point (0, 0). Thus any meaningful confidence interval of $\beta-\alpha$ could not be constructed. The problem can be seen in the diagram in the Appendix.

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*$V(d_2) = V(b_2) + V(b_3) + 2\,Cov(b_2\ b_3)$ and $Cov(b_1\ d_2) = -Cov(b_1\ b_2) - Cov(b_1\ b_3)$. Thus the variance-covariance matrix of b_1 and d_2 is

$$\Omega = 10^{-5}\begin{bmatrix} 38 & -22 \\ -22 & 1595 \end{bmatrix} \text{ and } \Omega^{-1} = \begin{bmatrix} 2653 & 37 \\ 37 & 63 \end{bmatrix}.$$

Efficient Estimation with an Equilibrium Condition for the Manufacturing Sector

Following the results in the second section of this chapter, more effort will be given to the improvement in estimation for the manufacturing sector. In order to maximize profits, each marginal productivity condition has to hold by itself, i.e., the value of the marginal product of each input should equal its price.

However, the separate estimation from each marginal productivity relation may lose some information in the sense of a seemingly unrelated regression procedure. The disturbance terms of each marginal productivity relation may be mutually correlated. If this is the case, the separate estimations by the ordinary least squares method to each marginal productivity equation would be unbiased and consistent but might not be efficient.

For this reason, the simultaneous estimation procedure for both equations was applied by the two-stage Zellner-Aitken method. Here we assumed the disturbance terms have zero mean but do not have the same variance for both equations, that is, $E(u_{1t}^2) = \sigma_{11}$ and $E(u_{2t}^2) = \sigma_{22}$. It was also assumed that $E(u_{1t}\ u_{1i}) = E(u_{2t}\ u_{2i}) = E(u_{1t}\ u_{2i}) = 0$ for $t \neq i$, and $E(u_{1t}\ u_{2t}) = \sigma_{12}$. That is, the usual assumption of standard linear models hold for each equation and the disturbances of different equations are contemporaneously correlated.

With the above specifications of errors, the first stage obtain the OLS estimates for each equation and use the resulting residuals to estimate the variances and covariances of the disturbances.* The second stage is to use the estimated variances and covariances and estimate both equations simultaneously.

This two-stage estimation procedure is asymtotically equivalent to Aitken's GLS and, therefore, to the maximum likelihood estimation which is shown in Theil (Theil, 1971, pp. 399-402). Thus the estimator is asymtotically efficient and its asymtotic distribution is normal. There is also some evidence that the small sample properties of the two-stage estimates are fairly similar to the MLE (Kmenta and Gilbert, 1968).

The above estimation procedure was applied to the following estimating Equations (4-16), which was slightly changed from Equation (4-13) by changing the distributed lag assumption as the Equation (4-11'). The estimating equations are

$$\ln(V/L)_t = b_o+b_1 \ln w_{t-1} + b_2 \ln (t-1) + b_3 \ln (V/L)_{t-1} + u_{1t}$$

$$\ln(V/K)_t = a_o+a_1 \ln r_{t-1} + a_2 \ln (t-1) + a_3 \ln (V/K)_{t-1} + u_{2t} \tag{4-16}$$

where
$$b_1 = \lambda_1\sigma$$
$$b_2 = \lambda_1(1-\sigma)\beta$$
$$b_3 = (1-\lambda_1)$$
$$a_1 = \lambda_2 \sigma$$
$$a_2 = \lambda_2(1-\sigma)\alpha$$
$$a_3 = (1-\lambda_2)$$

*With the assumptions made, the variance-covariance matrix has the form $\hat{\Sigma} = S \otimes I$, where S is the matrix of mean squares and products of the OLS residuals, I is n x n unit matrix and $\otimes$ is the notation of Kronecker product.

In order to constrain the estimation of σ to be equal in both marginal productivity relations, it is necessary to apply a nonlinear constraint to the regression parameters.* But no computing routine was available at hand for the estimation with such a nonlinear constraint. Thus the above equations were tried for the estimation without the constraint. However the results provided a negative value for the labor-augmenting parameter β, which may not be justified. And also the adjustment coefficient was not significantly different across the equations.

After this experiment, alternative linear constraints were applied for the estimation. The imposed constraints on regression parameters are $b_1 = a_1$ and $b_3 = a_3$, which require the equal value of σ as well as the equal value of the adjustment coefficient for both factors.

The regression results are

$$\ln (V/L)_t = 2.0352 + \underset{(.1034)}{.5788} \ln w_{t-1} + \underset{(.0291)}{.0171} \ln(t-1) + \underset{(.0652)}{.1524} \ln (V/L)_{t-1}$$

$$R^2 = .9809$$

$$\ln (V/K)_t = -.1762 + \underset{(.1034)}{.5788} \ln r_{t-1} + \underset{(.0663)}{.1609} \ln(t-1) + \underset{(.0652)}{.1524} \; V/K)_{t-1}$$

$$R^2 = .8671$$

---

*For the equal value of σ, it is necessary to constrain the regression parameters as $a_1 - b_1 + b_1 a_3 - a_1 b_3 = 0$, where b_1, b_3, a_1 and a_3 are regression parameters of (4-16).

From the estimated regression coefficients we obtain

$\hat{\lambda}_1 = \hat{\lambda}_2 = 1 - \hat{b}_3 = .8476$ $\qquad S_{\hat{\lambda}} = .2252$

$\hat{\sigma} = \hat{b}_1/(1-\hat{b}_3) = .6829$ $\qquad S_{\hat{\sigma}} = .1634$

$\hat{\beta} = \hat{b}_2/(1-\hat{b}_1-\hat{b}_3) = .0636$ $\qquad S_{\hat{\beta}} = .1249$

$\hat{\alpha} = \hat{a}_2/(1-\hat{a}_1-\hat{a}_3) = .5987$ $\qquad S_{\hat{\alpha}} = .2177$

As seen, the regression results correct the sign of the estimate $\hat{\beta}$, and most of estimates are somewhat improved in terms of standard errors.

The interval estimates of the parameters σ, α, and β are also made by the same procedure as used in the previous section. The joint confidence regions of the regression coefficient are based on the following statistic, which is distributed as F (2, LN-K).*

$$\frac{(b_i - \beta_i \; b_j - \beta_j)\; \Omega^{-1} \begin{bmatrix} b_i - \beta_i \\ b_j - \beta_j \end{bmatrix}}{(y-xb)'(\Sigma^{-1} x\, I)(y-xb)} \cdot \frac{LN - K}{2}$$

where L is the number of equations, N is the number of observations, K is the number of explanatory variables for all equations, Σ is the variance covariance of disturbance terms, and $\Omega = [X'(\Sigma^{-1} \otimes I)X]^{-1}$ which is the variance-covariance matrix of b_i and b_j. For our case, L=2, K=6 and N=19.

Based on the above statistic, the joint confidence retion for the regression coefficients b_i and b_j is constructed as

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*$(y-xb)'(\Sigma^{-1} \otimes I)(y-xb)$ is distributed as $\chi^2(LN-K)$. And the numerator of the left side is distributed as $\chi^2(2)$ (Theil, 1971, p. 133 and pp. 313-316).

$$P\{a_{ii}(b_i-\beta_i)^2 + a_{jj}(b_2-\beta_2)^2 + 2a_{ij}(b_i-\beta_i)(b_j-\beta_j) \leq F_\alpha\} = 1-\alpha$$

where $a_{ij} = C^{ij}\, k$, C^{ij} being the (ij) element of Ω^{-1} and k being the quadratic form:

$$k = [(y-xb)'(\Sigma^{-1} \otimes I)(y-xb)]^{-1} \cdot \frac{LN - K}{2}.$$

By replacing Σ^{-1} with its estimated value of S^{-1}, the value of k is calculated as .5576.*

For the interval estimate of σ, we need to construct the joint confidence region of b_1 and $d_1 = 1-b_3$. Using the calculated value of k and the variance-covariance matrix of the regression parameters, the values of a_{ij} are calculated, and the joint confidence region of b_1 and d_1 constructed as

$$P\{305.56(b_1 - .5788)^2 + 54.53(d_1 - .8476)^2 - 12.26(b_1 - .5788)(d_1 - .8476) \leq 3.30\} = .95$$

Using the derived formula (see the Appendix), the 95 percent confidence interval estimate of σ is derived as

$$.3849 \leq \sigma \leq .9409$$

Using the same procedure, the interval estimates of α and β are also derived. The results are

---

*Σ^{-1} was replaced with the estimated value of S^{-1},

$$S = 10^{-6} \begin{bmatrix} 1796 & -900 \\ -900 & 1523 \end{bmatrix} \text{ and } S^{-1} = \begin{bmatrix} 791 & 467 \\ 467 & 932 \end{bmatrix}$$

and for the calculation: $(y-xb)'(S^{-1} \otimes I)(y-xb) = S^{11}\, e_1^2 + S^{22}\, e_2^2 + 2\, S^{12}\, e_1 e_2$, where S^{ij} is the (ij)th elements of the matrix S^{-1} and e_i^2 is sum of squares of residuals of equation i.

$$.0662 \leq \alpha \leq .8418$$

$$-.1354 \leq \beta \leq .4551$$

However, note that the above interval estimates will be only approximations because the procedures are based on the estimated variances and covariances of the disturbances which have only asymptotic properties.

The above estimation results show that the elasticity of substitution is significantly below unity for the manufacturing sector which may imply that the CD function would lead to a specification bias for the sector. The point estimate of the labor-autmenting parameter was slightly positive, but it is not significant. Thus during the period, the productivity growth of manufacturing production has been mostly contributed by the efficiency growth of capital input which was rapidly expanded, particularly in the last decade.

CHAPTER V

ESTIMATION (Continued)

The indirect estimation procedures, used in the previous chapter, assumed the marginal productivity conditions, which in turn require that the returns to scale parameter is unity. This chapter uses direct estimation procedures which do not rely on these assumptions.

Two alternative methods are used; one is a linear approximation procedure suggested by Kmenta (Kmenta, 1967), and the other one is a nonlinear least squares estimation method. The direct estimation procedures require only the specification of the production function form.

Kmenta's Approximation Procedure

Following Kmenta's linear approximation procedure, we expand the logarithm of the CES function by a Taylor series expansion around ρ=o which corresponds to the value σ=1.

It has been pointed out that this method is consistent not only with the CES function, but also with a quite general class of production functions (Macarthy, 1967). We explained the approximation procedure in Chapter II.

Disregarding the terms of third and higher orders in the expansion, we obtain an approximation as given in Equation (5-1). The equation is linear in the unknown parameters and allows direct estimation of the parameters. However, the accuracy of the approximation decreases as the elasticity of substitution departs from unity since the expansion occurs around σ=1.

Due to the identification problem, technical progress is assumed neutral in the Hicksian sense, which means equal values of the efficiency growth parameters of capital and labor, α and β, in our notation. But the possibility of nonconstant returns to scale is allowed for in the estimation.

The approximation equation* is

$$\ln V_t = a_o + a_1 \ln K_t + a_2 \ln L_t - a_3 (\ln \frac{K}{L})_t^2 + a_4 \ln t + u_t \qquad (5\text{-}1)$$

where $a_o = \ln\gamma$

$a_1 = \nu\delta$

$a_2 = \nu(1-\delta)$

$a_3 = \frac{1}{2}\nu\rho\delta(1-\delta)$

$a_4 = \alpha\nu$

The error of approximating the CES function by the above equation depends on the extent to which ρ departs from zero, on the ratio of the two inputs and on the value of the remaining parameters. Numerical calculations of Kmenta show that the error resulting from neglect of the higher order terms is not serious unless both the capital-labor ratio and the elasticity of substitution are either very large or very small.

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*It has been stated that Kmenta's approximation is in the space of parameters rather than the space of variables. An approximation to the CES function that is quadratic in the logarithms of the variables is identical to Kmenta's approximation. Kmenta's approximation equation is considered as a production function in its own right, and it is a homogeneous translog production function (Christenson, et. al., 1973).

The approximation equation was fitted to output and input data by the OLS procedure. These estimates are consistent if the input variables are nonstochastic or independent of the disturbance in the production function.

But the simple OLS results showed some negative autocorrelation for the agricultural data. Thus an attempt to correct the regression was made by the Hildreth-Lu procedure as used in the previous chapter.

Following a first order autoregressive scheme, the regression Equation (5-1) is changed as

$$(\ln V_t - \rho \ln V_{t-1}) = a_o(1-\rho) + a_1(\ln K_t - \rho \ln K_{t-1}) + a_2 (\ln L_t - \rho \ln L_{t-1}) - a_3 \left[(\ln \tfrac{K}{L})^2_t - \rho (\ln \tfrac{K}{L})^2_{t-1} \right] + a_4 [(\ln t - \rho \ln(t-1)] + e_t \qquad (5\text{-}2)$$

The least squares method was applied to Equation (5-2) by searching over ρ. The estimated regression coefficients for the agricultural sector are presented below.

$\hat{a}_o = .7647$	$S_{\hat{a}_o} = .8154$
$\hat{a}_1 = .1922$	$S_{\hat{a}_1} = .1032$
$\hat{a}_2 = .7692$	$S_{\hat{a}_2} = .3140$
$\hat{a}_3 = .0091$	$S_{\hat{a}_3} = .0173$
$\hat{a}_4 = .1772$	$S_{\hat{a}_4} = .1934$
$\hat{\rho} = -.45$	$S_{\hat{\rho}} = .2049$
$\bar{R}^2 = .8788$	$DW = 2.0175$

As seen, the estimates show relatively large standard errors. Only a_1 and a_2 are significant at 10 and 5 percent significance levels respectively.

From the estimated regression coefficients we obtain

$\hat{\nu} = .9614$ $\qquad$ $S_{\hat{\nu}} = .5106$

$\hat{\rho} = -.1178$ $\qquad$ $S_{\hat{\rho}} = .1925$

$\hat{\alpha} = \hat{\beta} = .1843$ $\qquad$ $S_{\hat{\alpha}} = .2135$

$\hat{\delta} = .1999$ $\qquad$ $S_{\hat{\delta}} = .2471$

$l_n\gamma = \hat{a}_o = .7647$

The standard errors of the parameters are calculated by the same approximation procedure as in the footnote to Table 7. The implied value of the elasticity of substitution, 1.1335, is not significantly different from unity.

The estimated results for the manufacturing sector are as follows*

$$\begin{aligned} \ln V_t = \underset{(1.1435)}{1.3510} + \underset{(.3041)}{.6745} \ln K + \underset{(.1814)}{.3994} \ln L + \underset{(.0119)}{.0076} [\ln(K/L)]^2 \\ + \underset{(.1013)}{.1941} \ln t \qquad (5\text{-}3) \end{aligned}$$

$\bar{R}^2 = .9091$

$DW = 1.5701$

From the above regression coefficients we obtain

---

*No adjustment was made for autocorrelation in this case, because the Durbin Watson statistic was inconclusive.

$\hat{\nu} = 1.0739$ $\qquad S_{\hat{\nu}} = .4391$

$\hat{\rho} = -.0606$ $\qquad S_{\hat{\rho}} = .0863$

$\hat{\alpha} = \hat{\beta} = .1941$ $\qquad S_{\hat{\alpha}} = .3846$

$\hat{\delta} = .6281$ $\qquad S_{\hat{\delta}} = .4723$

$l_n\gamma = \hat{a}_o = 1.3510$

One of the problems in the above estimating equation is the presence of multicollinearity which contributes to relatively large standard errors of the estimates. Thus, as seen, many of the estimates are not statistically significant. To attempt to improve the efficiency of the estimates, we took the distribution parameter δ as predetermined based on the previous estimate of the parameter, but the resulting estimates are not improved.

The Equation (5-1) can be separated into two parts, one corresponding to the CD form and one representing a "correction" due to the departure of ρ from zero.

Thus the regression coefficient a_3, the square of the logarithmic capital-labor ratio term, is important for the CES specification. The significance test of a_3 is often used for the test of the hypothesis of CD form. As seen, our regression result showed a positive value of the coefficient for both the sectors even though both of the coefficients are not significant. The positive value of the coefficient implies that the elasticity of substitution is greater than unity, and the negative value of the coefficient implies that the elasticity of substitution is less than unity.

The point estimates of elasticity of substitution were slightly greater than unity for both the sectors. For the agricultural sector

it was about 1.13, which is consistent with the previous estimate at least in its direction. But for the manufacturing sector the estimate was largely inconsistent with the previous estimate, and the estimate turned out to be about 1.06.

The estimate of the scale parameter turns out to be .9614 for the agricultural sector and 1.0739 for the manufacturing sector, both of which are not significantly different from unity. Thus it is consistent with our assumption of constant returns to scale used in previous estimating equations. It is also fairly consistent with Ban's estimate (Ban, 1974). Ban estimated the scale parameter for the agricultural production as 1.0223 with a CD specification. But no comparable estimate is available for the manufacturing sector.

The estimate of neutral technical progress turned out to be .2135 for the agricultural sector which corresponds to an average annual rate of growth of aggregate productivity of 2.05 percent. For the manufacturing sector it was estimated as .1807 which corresponds to the annual growth rate of 1.74 percent. But as seen, the estimates for both sectors are subject to a large standard error, thus both of the estimates are not statistically significant.

Nonlinear Least Squares Estimation

Another attempt at a direct estimation of the CES function was made using the nonlinear least squares method. We obtain our estimates by minimizing the sum of squared deviations of the observed values of the dependent variable from the fitted values.

The estimates obtained by the nonlinear least squares method are equivalent to maximum likelihood estimates if, in addition to the other standard assumptions, the disturbances are normally distributed. It has been also pointed out that even without the assumption of normality the asymptotic distribution of the nonlinear least squares estimate may be (under suitable assumptions) normal with the same asymptotic means and variances as the maximum likelihood estimates (Malinvaud, 1966, pp. 277-299).

Several alternative methods are available for the solution of the nonlinear estimation problem. This study used an iterative linearization method (Bard's computer version of the Gauss-Newton Method), in which the nonlinear equation is linearized by using a Taylor series expansion around some initial set of coefficient values. Then ordinary least squares was performed on this linear equation generating a new set of coefficient values. The nonlinear equation is relinearized around these new coefficient values. This iterative process is repeated until convergence is attained.

Let us have a nonlinear model:

$$V_t = f\ (X_t\ ;\ \theta) + u_t \qquad t = 1 \ldots n$$

where X_t denotes a vector of independent variables for t^{th} observation giving rise to V_t, while θ is a vector of unknown parameters, and u_t is an independent error term. The estimation of θ by nonlinear least squares is to minimize

$$S\ (\hat{\theta}) = \Sigma_t (V_t - f(X_t : \hat{\theta}))^2$$

The multiple first order Taylor expansion at θ_o at certain initial values for θ will be given by

$$V_t = \hat{V}_t + (\theta - \theta_o) \left.\frac{\partial V_t}{\partial \theta}\right|_{\theta = \theta_o} + R_t + u_t$$

where $\hat{V}_t = f(X_t; \theta_o)$ and R_t is the remainder terms of the expansion. Using OLS we estimate $D = \theta - \theta_o$ as

$$D = (A'A)^{-1} \; A' \; (V_t - \hat{V}_t)$$

where the matrix A consists of the vectors which are the evaluation of gradient vectors at initial parameter values. Then we correct initial values θ_o by the estimate D. Thus our new estimate $\theta_1 = \theta_o + D$. The process is repeated until the k^{th} iteration converges to the criterion given by

$$\frac{\theta_k - \theta_{k-1}}{\theta_k} = C$$

where C is a specified rate of change given a priori. The detailed algorithm for the estimation is explained in Kuester and Mize (Kuester and Mize, 1973, pp. 218-239).

We applied the above nonlinear least squares method to the estimation of the CES production function with various forms of technical change. After some experiments, we chose the form of technical change as a capital augmenting specification. In most of the cases, the labor efficiency growth parameter yields a nonsense result of a negative value with a large standard error. The specification of the exponential rate of neutral technical progress also provides a relatively reasonable estimate, but the capital augmenting specification was slightly better in a statistical sense. The specification of capital augmenting form is also consistent with our previous estimation results.

Thus the specified form of the production function is

$$V_t = \gamma[\delta(t^{\alpha} K_t)^{-\rho} + (1-\delta) L_t^{-\rho}]^{-\frac{\nu}{\rho}} + u_t \tag{5-4}$$

$$= f\,(K_t, L_t;\theta) + u_t$$

where all the parameters and variables are the same as in our previous notation, and θ denotes the vector of unknown parameters to be estimated.

The multiple first order Taylor series expansion at initial values of parameters θ_o will be given by

$$\begin{aligned} V_t - f(K_t, L_t;\theta_o) &= (\gamma-\gamma_o)\frac{\partial f}{\partial \gamma}\Big|_{\theta=\theta_o} + (\alpha-\alpha_o)\frac{\partial f}{\partial \alpha}\Big|_{\theta=\theta_o} \\ &+ (\rho-\rho_o)\frac{\partial f}{\partial \rho}\Big|_{\theta=\theta_o} + (\delta-\delta_o)\frac{\partial f}{\partial \delta}\Big|_{\theta=\theta_o} + (\nu-\nu_o)\frac{\partial f}{\partial \nu}\Big|_{\theta=\theta_o} \\ &+ e_t \end{aligned} \tag{5-5}$$

where $e_t = u_t + R_t$, R_t being the remainder of the Taylor expansion. Note that this is a first order Taylor series expansion of the untransformed CES function as opposed to Kmenta's approach of the second order, expanding only one term of the logarithm of the CES function.

Using the least squares principle, we may solve for $D = \theta - \theta_o$ in Equation (5-5), and we can correct the initial values of the parameters by $\theta_1 = \theta_o + D$ and repeat the process until the k^{th} iteration converges to the criterion specified a priori. It has been proven that with the usual kinds of assumptions about the disturbances, the nonlinear least squares estimates are consistent and asymptotically efficient if the exogenous variables are nonstochastic (Hartley, 1965).

As a set of initial values for the parameters, we used the values estimated in the previous section, and as a convergency criteria we used one percent of the rate of change in the value of the parameters, up to a maximum of twenty iterations.

With the procedure explained above, we fitted the data to the specified CES function (5-4) for both the sectors. However, the agricultural data did not achieve the specified convergency criteria within 20 iterations. Most of the parameter estimates produced at the end of the 20th iteration had unexpected magnitudes and signs (excepting only the scale parameter which was 1.087), and also showed large standard errors and low R^2. Thus the estimation results for the agricultural sector are not included here.

The estimation results for the manufacturing sector also showed relatively low R^2 compared to other estimation results, but most of the estimates are fairly consistent with the estimates in the previous section. The resulting estimates are:

$$\begin{aligned} \hat{\nu} &= 1.1204 & S_{\hat{\nu}} &= .3116 \\ \hat{\rho} &= .2038 & S_{\hat{\rho}} &= .1435 \\ \hat{\alpha} &= .2996 & S_{\hat{\alpha}} &= .3704 \qquad\qquad (5\text{-}6) \\ \hat{\delta} &= .7815 & S_{\hat{\delta}} &= .6578 \\ \hat{\gamma} &= 4.0834 & S_{\hat{\gamma}} &= 5.8673 \\ R^2 &= .7867 \end{aligned}$$

As discussed in Hartley and Booker (Hartley and Booker, 1965), the variances and covariances of the least squares estimates are given approximately by

$$\text{Cov }(\theta) = S^2(A'A)^{-1}$$

where A is the matrix consisting of the column vectors which are evaluations of partial derivatives, $\frac{\partial f}{\partial \theta}$, at the final round estimates of the parameters. And variance S^2 is estimated by $S(\hat{\theta})/N\text{-}K$, where

$S(\hat{\theta})$ is the sum of the squares of residuals, N is the number of observations, and K is the number of parameters to be estimated.

The estimate of elasticity of substitution is calculated as .8307, which is somewhat lower than the value estimated by Kmenta's method. The estimate of the scale parameter turns out to be 1.1204, which is slightly greater than unity, but it is not significantly different from constant returns to scale. The capital efficiency growth parameter is estimated as .2996, which corresponds to the average annual growth rate of 3.59 percent, but it is statistically nonsignificant due to the large standard error.

As seen, the estimation results showed some consistent trends in their directions, but there are considerable differences in their actual magnitudes compared to the estimates based on marginal productivity relationships in the previous chapter. As we discussed, there are various possible sources of errors in both of the estimates, such as measurement errors associated with the data, the various specification errors of the production function, technical progress and disturbance term, and approximation error in the estimation procedure.

Considering the facts, it is difficult to select one rather than the others as having greater credibility. Thus we will use both of the estimates for the analysis in the next chapter. However if one accepts the marginal productivity theory of distribution, the analysis of changes in the wage income share, which will be in the next chapter, will provide some evidence on the estimates.

CHAPTER VI

ANALYSIS OF CHANGES IN WAGE INCOME SHARES, MARGINAL PRODUCTS, AND ACTUAL RETURNS TO LABOR

In Chapter III, we observed the actual movements of the wage income shares for both the agricultural and manufacturing sectors during the last two decades. The behavior of the movement was significantly different between the sectors. It showed a decreasing trend for the agricultural sector and an increasing trend for the manufacturing sector.

Consider the theoretical relationship of the rate of change in the wage income share and its related parameters and variables which was derived in Chapter II, under the marginal productivity theory. For convenience, we rewrite the relationship expressed in Equation (2-27) of Chapter II.

$$\dot{S}_L = -(1-S_L)\left(\frac{\sigma-1}{\sigma}\right)[(\dot{K}-\dot{L}) + (\alpha-\beta)t^{-1}] \qquad (6\text{-}1)$$

where σ is the elasticity of substitution, and α and β are factor augmenting parameters of capital and labor. The notation, a dot over a variable, denotes the proportional rate of change, or growth rate, of the variable as previously used.

From the equation, the sources of changes in the wage income share can be separated as the changes in the capital-labor ratio and the differential growth rate of factor efficiency at a given technical condition of substitutability between factors.

We are now equipped with the information about the parameters determining the wage income share, which we estimated in the previous two chapters. Using the information obtained, this chapter will attempt to explain the sources of the changes. The first section will explain the sources of the decreasing trend of the wage income share for the agricultural sector. The second section will explain the increasing trend of the share for the manufacturing sector. Using the estimates of production parameters, the final section will measure the marginal product of labor and examine the extent of differences, between the estimated marginal value product and the actual wage rate over time and between sectors.

The Decreasing Trend of the Wage Income Share in the Agricultural Sector

During the last two decades, actual wage income as a share of total gross income in the agricultural sector has decreased from about 51 percent for 1955-1957 to 35 percent for 1972-1974, some 16 percentage points, or about one-third. From Table 1 of Chapter III, we observed some short-term fluctuations of the share, which we explained as attributable to output price fluctuations due to the irregular imports of surplus food, and some production fluctuations together with a lagged adjustment behavior.*

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*As can be seen in Table 8, the actual wage income share decreased more sharply in two specified 5-year periods, i.e., at an annual rate of 5.0 percent for the period 1960-64 and 3.1 percent for 1970-74. The annual inflation rates of these periods are 20.8 and 22.9 percent respectively. These inflation rates are compared with 7.6 and 11.9 percent annual inflation rates for the other two periods.

In this chapter we are mainly interested in the explanation of the more long-term trend of the share which may be attributed to the basic nature of the production relation. For this purpose, we calculate the average annual rates of change of the actual wage income share for the specified five-year periods and the overlapping ten-year periods from Table 1 of Chapter III. The results are represented in column 7 of Table 8.

During the whole period. 1955-1974, the wage income share for the agricultural sector has decreased at an average annual rate of about 2.0 percent. However the rate of the change has been fairly unstable compared to the manufacturing sector, even though it shows consistently negative trends except before 1960. For the first ten-year period, 1955-1964, the average annual rate of change was about -1.3 percent, but it was about -2.6 percent for the second ten-year period, 1965-1974.

For the explanation of the change, we now estimate the theoretical rate of the change by the marginal productivity relation derived in Chapter II. As explained above, the rate of change in the wage income share can be separated into two parts.

The first part, we call it A, is

$$A = \frac{\sigma-1}{\sigma} (\dot{K}-\dot{L}) \qquad (6\text{-}2)$$

Using the definition of σ, the expression A can be written as

$$A = [1 - (\dot{\frac{MPL}{MPK}}) \;/\; (\dot{\frac{K}{L}})] \; (\dot{\frac{K}{L}}) = (\dot{\frac{K}{L}}) - (\dot{\frac{MPL}{MPK}}) = \dot{S}_K - \dot{S}_L$$

With this expression it is clear that the term, $-(1-S_L)\,A$, can be interpreted as the rate of change in wage income share ($\dot{S}_L$)

due to the changes in capital-labor ratio. That is, if $\alpha-\beta$ were zero, we would have

$$-(1-S_L)\ A = -S_K \cdot \dot{S}_K + (1-S_L)\cdot \dot{S}_L = \left(\frac{\partial S_K}{\partial t} + \frac{\partial S_L}{\partial t}\right) + \dot{S}_L = \dot{S}_L \ .$$

Note that assuming constant returns to scale, the expression in parentheses in the above equation is equal to zero.

The second part, we call it B, is

$$B = \frac{\sigma-1}{\sigma}(\alpha-\beta)t^{-1} \tag{6-3}$$

where α and β are the capital and labor augmenting parameters, respectively. With the form of production function and technical progress specified in Chapter II, we can find the relations as

$$\frac{\sigma-1}{\sigma}\ \alpha t^{-1} = \frac{1}{MPK}\ \frac{\partial MPK}{\partial t} \quad \text{and}$$

$$\frac{\sigma-1}{\sigma}\ \beta t^{-1} = \frac{1}{MPL}\ \frac{\partial MPL}{\partial t} \ .$$

Using these relationships, the term B can be expressed as

$$B = M\dot{P}K - M\dot{P}L$$

From this expression, it is clear that B is the difference between the proportional rates of change in marginal productivity of capital and labor due to technical change at a given capital-labor ratio, which we defined in Chapter II as the bias of technical progress.

From Equation (6-3), the bias of technical change B is calculated by using the estimation results of the previous chapters. The estimate of σ was about 1.39 in the estimation Equation (4-15), and the estimate of $\alpha-\beta$ was about .18 in the same equation. Thus at t=10, the bias of technical progress B for the agricultural sector is calculated as about

.51 percent with the estimation results, which means that in the middle of our sample period, the marginal productivity of capital was growing at about a .5 percent higher proportional rate than was the marginal productivity of labor. In other words, the technical change was contributing to the change in the marginal rate of substitution between capital and labor, $\frac{MPK}{MPL}$, at an annual rate of .5 percent for the agricultural sector. Thus in the terminology used in Chapter II, the technical progress has been capital using for the agricultural sector.

Given a factor price ratio, the biased technical progress would give an incentive to producers to substitute capital for labor, that is, to increase the capital-labor ratio. But during the period, the relative factor price $\frac{W}{R}$ also increased at the annual rate of about 2.5 percent (see Table 4). Thus the capital-labor ratio has increased more rapidly with an annual rate of about 5.3 percent during the period. The actual rates of change in the capital-labor ratio are also presented in terms of average rates for the specified periods in column 8 of Table 8.

As explained, the change in the wage income share depends not only on the change in the capital-labor ratio but also on the direction of technical change at the given nature of production relationships. Using the terms A and B explained in the above Equations (6-2) and (6-3), the theoretical rate of change in the wage income share was predicted by the two sources--capital deepening (A) and biased technical change (B). The results are presented in Table 8, and they are compared with the actual rates of the change.

For the prediction of the rate of change in the wage income share, we used two sets of estimates of production parameters. One was the

Table 8. Rates of Change in Actual and Predicted Wage Income Shares for Specified Periods in the Agricultural Sector, 1955 - 1974

Periods	Proportional Annual Rate of Change in Wage Share (S_L) (Percent)								
	Predicted								
	by capital[1] deepening (1)		by biased[2] technical change (2)		1 + 2		Actual[3]	$\dot{K}/L$[4]	S_L[3]
	I	II	I	II	I	II			
1955-74	- .805	-.337	-.229		-1.034	-.337	-1.970	5.278	.4583
1955-64	- .046	-.019	-.440		- .486	-.019	-1.299	.331	.5113
1965-74	-1.629	-.683	-.201		-1.830	-.683	-2.641	9.730	.4053
1955-59	.271	.114	-.766		- .495	.114	2.425	-2.053	.5307
1960-64	- .320	-.134	-.329		- .649	-.134	-5.022	2.238	.4918
1965-69	-2.014	-.844	-.221		-2.235	-.844	-2.180	12.762	.4396
1970-74	-1.187	-.497	-.177		-1.364	-.497	-3.102	6.698	.3709

[1]Calculated by $-(1-S_L)\frac{\sigma-1}{\sigma}\ \frac{\dot{K}}{L}$, where $\sigma = 1.3929$ for I and $\sigma = 1.134$ for II.

[2]Calculated by $-(1-S_L)\frac{\sigma-1}{\sigma}(\alpha-\beta)t^{-1}$, where $\alpha-\beta = .175$ for I and $\alpha-\beta = 0$ for II. The value of t was taken as of the midpoint of each of the respective periods (with t=1 for 1955).

[3]From Table 1.

[4]From Table 3.

result of the estimation Equation (4-15) which was based on the production expansion path (estimate I), and the other was the result of Equation (5-2) which was based on Kmenta's approximation procedure (estimate II). The theoretical relation predicts the wage income share to be decreased at an average annual rate of 1.03 percent from estimate I and .34 percent from estimate II, which are compared with the 1.97 percent of the actual rate of change during the whole period.

Based on estimate I, the rate of the change in wage income share is separated into two sources as explained above. The results show that, of the 1.03 percent of the annual rate of the change, the capital deepening or the term A contributes .80 percent and the biased technical change or the term B contributes .23 percent. Thus the capital deepening factor appears to be the major source of the change with an elastic substitutability between capital and labor for the period.

For other specified five- or ten-year periods, the theoretical rates of the change were also calculated. The results show that the actual rates of the changes were consistently faster than the predicted rates except for the period 1965-1969. And the predicted rates of change which were based on estimate II were slower than those based on estimate I.

As can be seen from Table 8, the actual and predicted rates of change are at least in the same direction, but there are large differences in actual magnitudes of the rates. In the average term for the whole period, it is calculated that our theoretical relation can explain only about 52 percent of the decreasing trend of the actual share based on estimate I and only 17 percent based on estimate II.*

Using the estimated rates of the change in five-year average term, the wage income share index is derived for both the actual and predicted shares. The base of the index is 1974 as 1.0 for actual share and .91 for predicted shares, which is the average ratio of the estimated marginal product to the actual wage rate for the period 1970-1974. (See Table 12.) The derived indexes are presented in Table 9. The difference between the actual and predicted indexes will show the degree of deviation from the marginal productivity theory, assuming no logical errors in our analysis, no specification errors in our models, and no sampling errors in our estimates.

From Table 9 we can observe that the actual wage income share index is considerably above that of the predicted share for the earlier period. But the difference between the two indexes has been significantly decreased over time as can be seen in the last two columns of Table 9. Before 1960, the predicted share was about 75 percent of the actual share from estimate I and only 65 percent from estimate II. But after 1970, both of the predicted indexes are about 88 percent of the actual

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*In our specification, the bias of technical change B is a function of t. In Tables 8 and 10, we calculated B as an average concept for the specified periods by taking t in each case at the middle point of the respective period. For comparison, we also calculated the values of B for the specified periods by calculating B for each year using the value of t for that year, and then averaging these annual B values for the respective periods. The results were not substantially different from the values calculated by the procedure used in Tables 8 and 10.

Table 9. Actual and Predicted Wage Income Share Indexes in the Agricultural Sector, 1955 - 1974.

Year	Actual[1] (A)	Predicted[2] I	Predicted[2] II	I/A	II/A
1955	1.422	1.132	.964	.796	.678
56	1.457	1.126	.966	.773	.663
57	1.494	1.120	.967	.750	.647
58	1.531	1.115	.968	.728	.632
59	1.569	1.109	.969	.707	.618
1960	1.608	1.104	.970	.687	.603
61	1.531	1.097	.969	.717	.633
62	1.458	1.090	.968	.748	.664
63	1.388	1.083	.967	.780	.697
64	1.322	1.076	.965	.814	.730
1965	1.259	1.069	.964	.849	.766
66	1.232	1.045	.956	.848	.776
67	1.205	1.023	.948	.849	.787
68	1.180	1.000	.940	.847	.797
69	1.155	.978	.932	.847	.807
1970	1.130	.957	.924	.847	.818
71	1.096	.944	.920	.861	.839
72	1.063	.931	.915	.876	.861
73	1.031	.918	.911	.890	.884
74	1.000	.910	.910	.910	.910

[1]The base of the index is 1974 as 1.0. The indexes are derived by using the 5 year average rates of the change which are presented in column 7 of Table 8. The calculation procedure is $I_{t-1} = (1+\dot{S}_L)^{-1} I_t$, where I_t is the share index at time t.

[2]Derived with the same procedure as above. The base of the index is 1974 as .91 which is the average ratio of the marginal product and actual wage rate for 1970-1974. (from Table 12).

index. This trend in the difference between the actual and predicted indexes may imply that there were some other factors playing an important role in the change of the actual wage share for the period.

One explanation for the trend may be that, particularly in the earlier period, there was a relatively large amount of unemployed or underemployed labor in Korean agriculture, which made possible the overemployment in the agricultural sector by self-employed workers. Thus the marginal product of self-employed workers may be fairly low compared to the actual wage rate. But as the economy expanded, or the amount of unemployed labor decreased, the agricultural labor market moved toward equilibrium. However, this explanation may be risky for various reasons which we will discuss later in this chapter.

The Increasing Trend of the Wage Income Share in the Manufacturing Sector

The manufacturing sector showed an increasing trend in its actual wage income as a share of the total value added for the sample period, 1957-1974, which is quite an opposite trend to that for the agricultural sector. The share has increased from 33 percent for 1957-1959 to 41 percent for 1972-1974, some 8 percentage points, or about one-fourth.

Following the same procedure used in the previous section, this section will analyze the sources of the increasing trend of the actual wage share in the manufacturing sector. It has increased at an average annual rate of about 1.5 percent during the whole sample period.

The trend was accelerated over the period 1965-69 with an annual growth of 3.1 percent. For the other specified five-or-ten year periods, the average rates of change were also calculated and are presented in column 7 of Table 10.

As seen in Table 4 of Chapter III, the capital-labor ratio has also significantly changed with an average annual rate of increase of 1.4 percent during the whole sample period. But this rate of growth of the capital-labor ratio was much slower than that in the agricultural sector. For the same period, the capital-labor ratio in the agricultural sector grew about 5.3 percent annually.

For the manufacturing sector, we estimated the elasticity of substitution σ as .68 from the estimation Equation (4-16), which is based on the marginal productivity relation (estimate I) and .83 from the Equation (5-6), which is based on the nonlinear estimation directly from the specified production function (estimate II). The estimate of the difference in the factor augmenting parameters between capital and labor, $\alpha-\beta$, was about .53 from Equation (4-16) and .30 from Equation (5-6).

With the above estimation results, the bias of technical change, B, was calculated by the Equation (6-3) used in the previous section. The value of B (at the sample period midpoint, i.e., t=10) turned out to be -2.49 percent based on estimate I and -1.09 percent based on estimate II. Thus the bias of technical progress has been labor-using and it appears to be more significantly biased compared with the agricultural sector. Thus, at a given capital-labor ratio, the technical change increases the marginal product of labor much faster than that of capital, and the difference in the annual growth rate of marginal productivity between labor and capital is about 1.1-2.5 percent.

As seen above, for both the sectors, technical change was characterized by a capital augmenting bias, which means that the capital augmenting rate exceeds that of labor. But technical change is capital-using in the agricultural sector and labor-using in the manufacturing sector. In other words, at a given capital-labor ratio, technical change increased marginal productivity of capital relative to that of labor in the agricultural sector, thus leading to a decrease in wage income share in the sector. But for the manufacturing sector, technical change increased the marginal productivity of labor relative to that of capital, thus leading to increase in the relative wage income share for the manufacturing sector.

The underlying relations of the above explanation can be seen more clearly as follows. The capital augmenting biased technical change increased the quantity of efficiency units of capital relative to those of labor, thus increasing the capital-labor ratio in efficiency unit terms. But the inelastic nature of factor substitutability resulted more in increase in relative marginal productivity of labor than in the decrease in the relative quantity of labor. In other words, the increase in the relative quantity of capital in efficiency units is not enough to offset the decrease in the relative marginal productivity or the relative price of capital. For this reason, the technical change reduced the relative marginal productivity of capital in physical unit terms at a given capital-labor ratio, and thus decreased the relative

share of capital and correspondingly increased the relative share of labor.

The underlying economic relationships may be interpreted as follows. In the labor market, the labor-using technical change will shift the demand for labor to the right at a given volume of output and factor price ratio. If there were no change in the supply of labor, the result would be an increase in the wage rate and employment, and accordingly an increase in the wage share.

But as we observed in Chapter III, the total output continually increased during the whole sample period, and the supply curve of labor may also shift to the right with a significant movement of labor from the agricultural sector to the manufacturing sector. The result would be a decrease in wage rates and an increase in employment. In fact, actual data show a rapid increase in employment with an annual growth of about 12 percent. And the real wage rate also significantly increased, with an annual rate of about 5 percent.

Using the relation expressed in Equation (6-1), the theoretical rate of change in the wage income share was predicted in average terms for the specified periods. The rate of the change was also separated into two sources--capital deepening and biased technical change. The results are presented in Table 10.

The results show that the predicted growth rate of the wage income share based on the marginal productivity theory is about 1.7 percent annually from estimate I, which is fairly close to the 1.5 percent actual rate of change for the whole sample period. But estimate II

Table 10. Rates of Changes in Actual and Predicted Wage Income Shares for Specified Periods in the Manufacturing Sector, 1957 - 1974

Periods	Proportional Annual Rate of Change in Wage Share ($\dot{S}_L$) (percent)								
	by capital deepening (1)[1]		by biased technical change (2)[2]		1 + 2		Actual[3]	$\dot{K}/L$[4]	S_L[3]
	I	II	I	II	I	II			
1957-74	.407	.178	1.319	.325	1.726	.503	1.544	1.374	.3618
1965-74	1.112	.486	.950	.233	2.062	.719	2.171	3.910	.3877
1957-64	-.699	-.306	2.079	.763	1.380	.457	.649	-2.246	.3295
1965-69	1.253	.548	1.204	.296	2.457	.844	3.058	4.280	.3693
1970-74	.976	.427	.819	.200	1.795	.627	1.284	3.540	.4060

[1] Calculated by $-(1-S_L)\ \frac{\sigma-1}{\sigma}\ \frac{\dot{K}}{L}$, where σ = .6839 for I and σ = .831 for II.

[2] Calculated by $-(1-S_L)\ \frac{\sigma-1}{\sigma}\ (\alpha-\beta)\ t^{-1}$, where $\alpha - \beta$ = .4351 for I and $\alpha - \beta$ = .300 for II.

[3] From Table 2.

[4] From Table 4.

predicted the rate of change as only .50 percent annually for the period, which was much lower than the rate of actual change.

For other specified periods, the rates of the change were also calculated, and they are compared with the actual rates of the change. In most of the periods, the rates of changes predicted by estimate I are fairly consistent with the actual rate of the change. But the results predicted based on estimate II are significantly lower than the actual rates, even though the direction of the changes is consistent for all the specified periods. As a whole one may conclude that the marginal productivity theory can explain the behavior of the wage income share in the manufacturing sector relatively better than in the agricultural sector.

The predicted rate of the change is separated by the two sources--capital deepening and biased technical progress. With estimate I, it is calculated that the capital deepening factor has contributed to the growth of the wage share at an annual rate of .41 percent, while the labor-using biased technical progress has increased the share at an annual rate of 1.32 percent. Using estimate II, about .18 percent of the annual rate was contributed by capital deepening and .35 percent of the annual rate by biased technical progress. Thus for the manufacturing sector, the biased technical change has been more important in changing the wage income share than was the capital deepening.

Following the same procedure used in the previous section, the wage income share indexes were derived for both the actual and predicted shares with the 5 year average rates in the changes. The base of the index is 1.0 for the actual share in 1974. The base index for the predicted share is set as 1.537 in 1974, which is the ratio of measured marginal product and the actual wage rate for the period 1970-74. The derived indexes are presented in Table 11.

The results show that both of the predicted indexes are far above the actual share indexes as can be seen from Table 11. The indexes predicted by estimate I are about 48 percent higher than the actual indexes for the period 1957-1969 and about 52 percent higher than the actual indexes for 1970-1974. Thus the difference between the two indexes has slightly increased over time. And the indexes predicted by estimate II are about 79 percent higher than the actual indexes for 1957-1959 and 56 percent higher than the actual share indexes for 1970-1974. The difference between the two indexes has a significantly decreasing trend based on estimate II.

The general results of the above two sections show that the predicted and actual shares have moved in the same directions but that there are large differences in the actual magnitudes. But the explanation for the differences in the actual magnitudes is not clear concerning whether our estimates are biased in certain directions or whether some other vairables, which were ignored in the marginal productivity theory of distribution, have played an important role in the determination of wage income shares for the period.

One more complication is that one set of our estimates of parameters used for the analysis is based on the marginal productivity relationship. Assume for a moment that our estimate of parameters reflects the true

Table 11. Actual and Predicted Wage Income Share Indexes in the Manufacturing Sector, 1957-1974

Year	Actual[1] (A)	Predicted[1] I	Predicted[1] II	I / A	II / A
1957	.772	1.134	1.383	1.469	1.791
58	.777	1.149	1.389	1.479	1.788
59	.782	1.165	1.395	1.490	1.784
60	.787	1.181	1.402	1.501	1.781
61	.792	1.198	1.408	1.513	1.778
62	.797	1.214	1.415	1.523	1.775
63	.802	1.231	1.421	1.535	1.772
64	.808	1.248	1.427	1.545	1.766
65	.813	1.269	1.434	1.561	1.764
66	.839	1.290	1.446	1.538	1.723
67	.865	1.322	1.458	1.528	1.686
68	.893	1.355	1.470	1.517	1.646
69	.921	1.388	1.483	1.507	1.610
70	.950	1.422	1.495	1.497	1.574
71	.962	1.457	1.508	1.515	1.568
72	.974	1.483	1.518	1.523	1.559
73	.987	1.510	1.527	1.530	1.547
74	1.000	1.537	1.537	1.537	1.537

[1]Used the same procedure as in Table 9 (see footnote to Table 9).

value of the production relationship. Then the results of the analyses imply that we could not rely on the marginal productivity theory to explain the behavior of the wage income share in the Korean economy. Then the validity of the estimation procedure which is based on the marginal productivity relationship is also questionable. Thus estimate I, which is based on the relationship has a problem in its reliability. In another way, if we assume that the marginal productivity theory has been truely valid for the period, then both of our estimates are biased by some other sources of errors. Thus in any ways, the reliability of our estimate I is questionable.

However, remember that, as pointed out in Chapter IV, we could still obtain unbiased parameter estimates with the marginal productivity relationships if the disequilibrium factors are not correlated with the regression variables under the given assumptions. But as seen, the gap between the actual and predicted shares derived from both of the estimates, or the degree of disequilibrium, has been changed significantly over time. This fact makes a more serious problem for the reliability of estimate I. For this reason, one may give more credit to estimate II, which is not based on the marginal productivity relations. Accordingly one comes to the conclusion that for this period the disequilibrium factors played an important role in the Korean economy with a different direction between sectors, but with the degree of disequilibrium being significantly reduced over time for both the sectors.

Marginal Products and Actual Returns to Labor

In the previous two sections, we analyzed the behaviors of the relative wage income shares with the estimation results of production parameters for both the agricultural and manufacturing sectors. The analyses showed that the general directions of the predicted rates of change in the wage shares, based on the marginal productivity theory, are consistent with the actual rates of the changes for both the sectors, but there were large differences in actual magnitudes. Thus the results may indicate that the theory we used could not be a complete explanation of the changes in the wage income shares for the period under study in the Korean economy.

With this tentative observation, this section will attempt to estimate the marginal product of labor by using the estimates of production parameters and to examine the extent of variations between the estimated marginal product and the actual wage rate over time and between sectors.

The marginal product of labor is found simply by differentiating the specified production function with respect to labor. The variables that determine the marginal product are the same as those in the production function. If the economy is in equilibrium and there are no economies or diseconomies of scale, the value of the marginal product and actual returns to labor should be equal.*

*The statement that, in equilibrium, value of marginal product is equal to the wage rate is, strictly speaking, applicable to the individual firm in purely competitive markets. The discussion which follows assumes that taking the partial derivative of our aggregate production function gives a marginal product which is equal to the individual firms' marginal products.

From the specified production relation, we can derive the marginal product of labor as

$$MPL = \gamma^{-\frac{\rho}{\nu}}(1-\delta)\ t^{-\beta\rho}\ V^{(\frac{\rho}{\nu}+1)}\ L^{-(\rho+1)} \tag{6-4}$$

With some manipulations of the above equation, we can get the equation as*

$$MPL = \nu[1 + (\frac{\delta}{1-\delta})t^{-(\alpha-\beta)\rho}(\frac{K}{L})^{-\rho}]^{-1}\ \frac{V}{L} \tag{6-5}$$

where all notations are the same as previously.

From Equation (6-5), we can see more clearly how the production parameters and variables affect the marginal product of labor. At a given capital-labor ratio, a larger value of ν and smaller value of δ will give the higher marginal product of labor. And the larger value of ρ will yield a larger marginal product of labor. When the value of ν

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*From the production function

$$V = [\delta(t^{\alpha}K)^{-\rho} + (1-\delta)(t^{\beta}L)^{-\rho}]^{-\frac{\nu}{\rho}} \tag{6-6}$$

we can derive the expression as

$$V^{\frac{\rho}{\nu}}/L^{\rho} = \gamma^{\frac{\rho}{\nu}}[(1-\delta)t^{-\beta\rho}+\delta t^{-\alpha\rho}(\frac{K}{L})^{-\rho}]^{-1} \tag{6-7}$$

and the Equation (6-4) can be written as

$$MPL = \gamma^{-\frac{\rho}{\nu}}(1-\delta)\nu t^{-\beta\rho}(V^{\frac{\rho}{\nu}}/L^{\rho})\frac{V}{L} \tag{6-8}$$

Substituting (6-7) into (6-8), we obtain

$$MPL = \nu[1+(\frac{\delta}{1-\delta})t^{-(\alpha-\beta)\rho}(\frac{K}{L})^{-\rho}]^{-1}\ \frac{V}{L}$$

Multiplying $\frac{L}{V}$ in both sides of above equation we also obtain the expression as

$$\frac{MPL\cdot L}{V} = S_L = \nu[1+(\frac{\delta}{1-\delta})t^{-(\alpha-\beta)\rho}(\frac{K}{L})^{-\rho}]^{-1}$$

is unity and the value of ρ is zero, Equation (6-5) reduces to the expression MPL = $(1-\delta)\ \frac{V}{K}$ which is a Cobb-Douglas case.

With Equation (6-4), empirical estimates of the marginal productivity of labor are calculated by using the estimates of production parameters and data of the production function. As explained in the section on data description, the output was measured in units of million won at 1970 constant price, and labor input was measured in units of million days of work. Thus the unit of calculated marginal product of labor is won per day of work (or 8 hours of work). The empirically estimated marginal product of labor for both the sectors is presented in Tables 12 and 13.

For the agricultural sector, the results show that the actual wage rate was far above the estimated marginal product of labor particularly until the early 1960's. However, the difference between the marginal product and actual wage rate decreased over time.

The actual wage rate was about 50 percent higher than the estimated value of the marginal product of labor for the first five-year period 1955-59, but it was fairly close to the actual wage rate for the last five-year period, 1970-74. In other words, it is calculated that the values of the marginal product were only about 67 percent and 93 percent of the actual wage rate for the respective periods. Thus the returns to the self-employed labor in agricultural production appear to have been far below the actual wage rate, but have largely improved over time.

The marginal product of labor for 8 hours of work in the agricultural sector rose from 316 won in the period 1955-56 to 558 won in the period

Table 12. Marginal Products and Actual Returns to Labor for the Agricultural Sector, 1955-74

Year	MPL[1]	VMPL[2] (Won)	W[3] (Won)	W/VMPL
1955	316	51	76	1.481
56	315	59	85	1.438
57	332	65	92	1.420
58	336	61	92	1.518
59	319	59	93	1.574
60	329	60	96	1.589
61	310	73	106	1.452
62	324	88	115	1.308
63	336	113	143	1.263
64	349	178	199	1.121
65	363	176	221	1.254
66	394	206	256	1.245
67	381	247	307	1.242
68	413	296	381	1.288
69	474	396	463	1.169
70	483	483	579	1.169
71	494	597	695	1.164
72	514	746	803	1.070
73	536	850	886	1.042
74	580	1,232	1,141	.926

[1]The unit is 1970-constant won.

[2]VMPL = P * MPL.

[3]From Table 1.

Table 13. Marginal Products and Actual Returns to Labor for the Manufacturing Sector, 1957-74

Year	MPL[1]	VMPL[2] (Won)	W[3] (Won)	W/VMPL
57	560	138	77	.558
58	561	148	84	.568
59	566	150	86	.573
60	558	154	92	.597
61	550	174	104	.598
62	582	207	112	.541
63	520	228	128	.561
64	532	304	179	.589
65	549	333	205	.616
66	538	411	265	.645
67	566	441	295	.669
68	636	534	347	.650
69	742	676	444	.657
70	858	858	571	.666
71	837	870	683	.785
72	1,003	1,187	798	.672
73	1,096	1,394	878	.630
74	1,167	1,881	1,194	.635

[1]The unit is 1970-constant won.

[2]VMPL = P * MPL.

[3]From Table 2.

1973-74 in 1970 constant prices. Thus in the last two decades, the marginal product of labor has increased about 77 percent. However the growth of marginal productivity of labor was not significant until the early 1960's, but it has accelerated since the mid-1960's as can be seen in Table 12.

Quite unlike the agricultural sector, the data for the manufacturing sector shows that the estimated marginal product of labor was far above the actual wage rate for the whole sample period. The actual wage rate was only about 56 percent of the value of the marginal product in 1957-58 and 63 percent in 1973-74. But recall that the measure of the actual wage rate did not include any side benefits as noted in the data explanation. Even considering this fact, it seems clear that there was also a large gap between the actual wage rate and the marginal product. However the wage rate is approaching the value of the marginal product over time.

The marginal product of manufacturing labor rose from 611 won in 1957-1958 to 1,132 won in 1973-1974 in 1970 constant prices. Thus in the last two decades the marginal product of labor has about doubled compared with the 77 percent increase for the agricultural sector. As in the agricultural sector, the marginal product of manufacturing labor also remained fairly stable until the mid-1960's, but it has rapidly grown in the ten-year period, 1965-1974.

As seen in the above, the data indicate that there were significant differences between the actual returns and marginal products of labor in both the sectors. But the directions of the differences or disequilibriums were not the same between the sectors. For the manufacturing setor, the value of marginal product of labor was far above the actual wage rate, thus the wage earners were paid too little compared to their actual contributions to the production. But the value of the marginal product of agricultural labor was lower than the wage rate. Thus the self-employed agricultural workers were left with low returns to their labor. However, the disequilibrium or the gap between the value of the marginal product and the actual wage rate has been decreased significantly over time for both sectors.

The above results may partially explain the fact that the growth rate of the capital-labor ratio was significantly lower in the manufacturing sector than in the agricultural sector for the whole sample period. As seen in Table 4 of Chapter III, the annual growth rate of the capital-labor ratio was about 1.4 percent in the manufacturing sector and 5.3 percent in the agricultural sector. A different rate of substitution between factors is basically determined by the nature of production. But with the wage rate far below the marginal product, the optimal amount of capital in the manufacturing sector is less than it would be if labor were paid its marginal product. Thus the pressure to substitute capital for labor is relatively reduced. With a reverse situation, the agricultural sector has had more incentive to substitute capital for labor than it would be in an equilibrium case.

It is also interesting to compare the estimated results between the sectors. In Table 14, we have calculated the ratios of the estimated marginal products, the values of the marginal product, and the actual wage rates between the sectors for specified periods.

Table 14. Comparisons of Actual Wage Rates, Marginal Products, and Value of Marginal Products of Labor Between the Agricultural and Manufacturing Sectors for Specified Periods

Periods	MPL_A/MPL_M [1]	$VMPL_A/VMPL_M$	W_A/W_M
1955-59	.583	.423	1.123
60-64	.604	.466	1.064
65-69	.670	.546	1.048
70-74	.580	.628	1.002
1955-74	.597	.516	1.059

[1] Calculated from Tables 12 and 13. Subscripts A and M stand for the agricultural sector and the manufacturing sector respectively. Marginal products are in value units at 1970 prices.

The estimated marginal product of agricultural labor was only about 60 percent of that of manufacturing labor. The relative marginal product of agricultural labor has been improved from 58 percent in 1957-1959 to 67 percent in 1965-1969, but it went back down to 58 percent for 1970-1974.

The relative value of the marginal product of agricultural labor to manufacturing labor was also calculated for the specified periods. It shows that the value of the marginal product of agricultural labor was about 52 percent of that of manufacturing labor as an average for the whole period, which implies that there was also a large disequilibrium between the sectors. However, the disequilibrium between the sectors has been significantly lessened over time. The relative value of the marginal product of agricultural labor rose from 42 percent in 1957-1959 to 63 percent in 1970-1974.

The improvement in the relative value of the marginal product of the agricultural sector is attributed partially to the increase in the relative price of the agricultural output to manufacturing output and partially to the increase in the relative physical marginal product of agricultural labor, as seen in the above.

From the above results, or the presence of a large disequilibrium between the sectors, one would expect that the large amount of migration which was actively taking place in the last decade from rural to urban areas will continue at least in the near future until the disequilibrium is adjusted.

Even though there were large differences in the value of the marginal product of labor in the two sectors, the actual wage levels were fairly close.* The data indicate that the actual wage rates of both the sectors fall within the range between the values of marginal products of both sectors during the whole period. Thus manufacturing workers were paid less than the values of their marginal products. And self-employed agricultural workers earned less return than they could earn in other hired work.

As a summary, the results of this section seem to lead to a conclusion that disequilibrium within and between the sectors has been significant in the Korean economy for the last two decades. But both of disequilibriums appear to be adjusted over time.

However, the conclusion is very tentative and it is valid only when the data and estimates used for the analyses reflect true values. As explained in previous chapters, there may be various possibilities that the data and estimates are biased. The weakness in the data and possible errors in estimation could mean that the economy may not be in disequilibrium as indicated by the above results. More work on this possibility is needed.

If such a disequilibrium does in fact exist, there have been various possible causes in the process of rapid growth of Korean economy in the last decade. The analysis of these causes will be another important subject for investigation.

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*Note that the measurement units of wage rate are not the same between sectors. As explained in the section of data description, W_A was measured by adjusted man-equivalent unit and W_M was measured by unadjusted physical labor unit.

However, our analysis could make at least one clear conclusion. If one believes the assumption that a competitive equilibrium or marginal productivity theory has truly operated in the Korean economy during the period, most of our estimates of production parameters are biased. For the agricultural sector, the directions of the bias will be downward in either one or both of the estimates, σ and $\alpha - \beta$, for both sets of our estimates. (See Table 8.) For the manufacturing sector, estimate II will give an upward bias in the estimate of σ and/or a downward bias in the estimate of $\alpha - \beta$, but estimate I is fairly consistent with the marginal productivity theory. (See Table 10.)

But if one believes the opposite assumption, then there is no ground to justify the validity of estimate I which is based on the marginal productivity relations. The production parameters should be estimated directly from the specified production function. Thus more reliability will be given to estimate II, accordingly one prefers to believe the results of the analysis based on that estimate. As a consequence, one may conclude that there have been large disequilibriums within and between the sectors for the period, but these disequilibriums have been decreased over time. Thus the Korean economy is in the process of adjustment to an equilibrium. (See Tables 12, 13, and 14.)

CHAPTER VII

SUMMARY AND CONCLUSIONS

The study is based on the distributional problem which has been increasingly called into question with many different dimensions since the Korean economy has succeeded in achieving growth objectives beyond expectation in the last decade or so.

The distributional questions are broad--for example, how has the distribution of output been changed between sectors, regions, different resource ownership, and some other economic groups classified as having different interests. What have been the major economic forces causing these changes, and how does the distribution itself affect other economic variables, thus interacting between distribution and other economic variables in the dynamic growth process. However, this study is mainly concerned with how the relative positions of wage earners have been changed and with what were the major economic forces for this change in the process of the various changes of output growth, resources employment, and economic structure. For the purpose, this study analyzed the behavior of wage income shares and the sources of the changes in the Korean agricultural and manufacturing sectors during the last two decades.

The analytical framework used for the analysis is based on the neoclassical marginal productivity theory with a specified CES production function. The distribution mechanism of the theory is basically the process of determining factor prices and employment. At a given factor ratio, the relative share of each factor is determined by the price ratio, which is equal to the marginal rate of substitution between factors under the assumption of the theory.

The marginal rate of substitution between factors can be changed, and as a result the distribution of output to each of the factors changed. There are basically two possibilities for the change: the changes in factor ratio and biased technical change.

We specified the aggregate production relation as a CES function with a factor augmenting form of technical progress. The augmenting form of factor efficiency was specified as $K_t^* = t^{\alpha}K_t$ and $L_t^* = t^{\beta}L_t$, where K_t^* and L_t^* are capital and labor inputs in efficiency units, K_t and L_t are conventional units of respective input, and α and β are factor augmenting indexes of the respective factors due to technical progress.

With such a specified production relation, we can derive the theoretical relationships of the rate of change in the wage income share ($\dot{S}_L$) to related variables and parameters as

$$\dot{S}_L = -(1-S_L)\left(\frac{\sigma-1}{\sigma}\right)[(\dot{K}-\dot{L}) + (\alpha-\beta)t^{-1}]$$

where σ is the elasticity of substitution between factors. From the expression, it is clear that the rate of change in wage income share depends on the changes in the capital-labor ratio and the differential growth rate of factor efficiency at a given technical condition of factor substitutability. Using the theoretical relationships described above, this study attempted to explain the behavior of the actual wage income share for both the sectors.

Using various sources of data, the actual wage income share was estimated during the last two decades. The results showed that the behavior of the share has been significantly different between the sectors.

For the agricultural sector, three different estimates of the actual wage income share were obtained; two were based on a hired wage rate and the other one was based on the residual concept. All of the estimates showed a consistently decreasing trend during the period as a whole.

Agricultural wage income as a share of total gross agricultural income (where the wage share calculation was based on the hired wage rate) has decreased from 51 percent for 1955-1957 to 35 percent for 1972-1974, some 16 percentage points, or about one-third. It was also calculated that the wage income share has decreased at an average annual rate of about 2.0 percent. But the rate of change in the agricultural sector fluctuated widely compared to the rate in the manufacturing sector. For the first ten-year period, 1955-1964, the annual rate of change was -1.3 percent, but it accelerated in the second ten-year period to a -2.6 percent annual rate of change.

For the manufacturing sector, the estimate of the actual wage income as a share of the total value added showed an increasing trend during the period, 1957-1974, or contrary to the trend in the agricultural sector. The share has increased, from 33 percent for 1957-1959 to 41 percent for 1972-1974, some 8 percentage points, or about one-fourth. The average annual rate of change is calculated as 1.5 percent during the whole period.

For the explanation of the actual behavior of the wage income share, we estimated the parameters determining the wage share.

One of the major difficulties for the estimation was the availability and reliability of basic data. The definitions of data used for the measurement are not identical between the sectors partially because of the lack of availability and partially because of the conceptual differences.

For the agricultural sector, output was measured by the total gross output concept. The measure of the labor input was an adult-man-day equivalent unit adjusted by age and sex, and the capital input was measured by the flow service concept.

For the manufacturing sector, output was measured by the total value added concept. The measure of the labor input was unadjusted physical units of labor, and the capital input was measured by the stock concept.

After experimenting with various different estimation procedures, we derived two sets of estimates for each sector. For both sectors, one set of the estimates (estimate I) was based on the marginal productivity relationships and the other one (estimate II) was the result estimated directly from the specified production function by a linear approximation or nonlinear least squares procedure.

For the agricultural sector, estimate I was based on the estimating equation which was derived from the production expansion path with a partial adjustment assumption. Estimate II was based on a linear approximation of a specified CES function by expanding the logarithm of the function. The Hildreth-Lu estimation procedure was applied for both the estimating equations with a first order autoregressive scheme.

For the manufacturing sector, estimate I was based on the two-stage Zellner-Aitken's efficient estimation method, with the assumption that the disturbance terms of each marginal productivity relation were mutually correlated. Estimate II was based on the nonlinear least squares method, and Bard's version of the Gauss-Newton method was applied for the solution.

The major estimation results are summarized as follows. First, the elasticity of substitution is greater than unity for the agricultural sector and less than unity for the manufacturing sector. For the agricultural sector, the point estimates of the elasticity of substitution were 1.392 and 1.134 in estimates I and II respectively. But for the manufacturing sector, it was .683 in estimate I and .831 in estimate II.

Second, for both the sectors, the estimated capital augmenting parameter α turned out to be greater than that of labor β. For the agricultural sector, the estimate of $\alpha-\beta$ was .175 (estimate I), and for the manufacturing sector, the estimates of α and β were .599 and .064 respectively (estimate I). Thus, the results seem to indicate that during the sample period the growth in productivity has been mainly the result of the efficiency growth of the capital input, which has been rapidly expanded in the last decade.

Third, the point estimate of the scale parameter was consistently close to unity for both the sectors, which agreed with the assumption used in deriving the marginal productivity theory of distribution. For the agricultural sector, the estimate of the adjustment coefficient,

$\lambda = .409$, was significantly below unity, which implies that the input adjustment process to the price change has been fairly slow for Korean agricultural production during the period. But for the manufacturing sector, the adjustment coefficient, $\lambda = .848$, was relatively close to unity.

The confidence interval estimates, or significance tests, for the estimated parameters were also made. However, the usual significance test of the ratio of regression coefficients presented a difficulty because the distribution of the ratio of two normal variables with non-zero means is unknown. For this reason, we derived a formula to find the maximum and minimum values of the ratio of two normal variables within the joint confidence region of the variables with a specified probability.

Using the above procedure, we derived the 95 percent confidence interval estimate of the elasticity of substitution as $.9041 \leq \sigma \leq 2.5717$ for the agricultural sector and $.3849 \leq \sigma \leq .9409$ for the manufacturing sector. This result implies that the usual CD function will have a specification bias at least for the manufacturing sector.

The significance test based on the same procedure showed that the factor augmenting bias $\alpha-\beta$ for agriculture and the labor augmenting parameter β for manufacturing are not significantly different from zero, but the capital augmenting parameter α for manufacturing was significantly positive at the 95 percent significance level.

Using the estimation results, the bias of technical progress was calculated for both the sectors. As in usual terminology, the bias of technical change was defined as a differential growth rate of marginal productivity between factors at a given capital-labor ratio. In our notation, the bias (B) can be calculated as

$$B = \frac{\sigma-1}{\sigma} (\alpha-\beta)\ t^{-1} = \dot{MPK} - \dot{MPL}.$$

At the year 1965 or t=10, the calculated B turned out to be .51 percent for the agricultural sector and -2.49 percent for the manufacturing sector, which means that the technical progress was capital-using for the agricultural sector and labor-using for the manufacturing sector. Thus at a given capital-labor ratio, technical progress results in a decrease in the wage income share for the agricultural sector, but it causes an increase in the wage income share for the manufacturing sector. It also appears that the technical progress has been more significantly biased in the manufacturing sector than in the agricultural sector during the period.

Using the theoretical relationship derived from the marginal productivity theory, we attempted to explain the behavior and sources of changes in wage income shares. For this purpose, the theoretical rate of change in the wage income share was predicted and it was also separated into two sources--capital deepening and biased technical change.

For the agricultural sector, the theoretical relationship predicts the wage income share to decrease at an average annual rate of 1.03 percent with estimate I and .34 percent with estimate II, which are compared with the 1.97 percent of the actual rate of the change during the whole period.

Based on estimate I, the rate of the change was separated into two sources. The results shows that, of the 1.03 percent of the annual rate of change, the capital deepening factor contributed .23 percent. Thus the capital deepening factor appears to be a major source of the change with an elastic substitutability between capital and labor.

The actual and predicted rates of the change are in the same direction, but there are large differences in the actual magnitudes of the rates. In average terms, it is calculated that our theoretical relationship can explain only about 52 percent of the decreasing trend of the actual share with estimate I and only 17 percent with estimate II.

Using the estimated rate of the change, the wage income share index was derived for both the actual and predicted shares. The derived index showed that the actual wage income share index was substantially above that of the predicted share for the earlier period. But the differences between the two indexes have significantly decreased over time.

For the manufacturing sector, the annual predicted rate of increase in the wage income share is about 1.7 percent with estimate I, which is fairly close to the 1.5 percent actual rate of the change during the whole period. But estimate II predicts the rate of change as only .50 percent, which is much lower than the rate of actual change. Based on estimate I, the marginal productivity theory seems to better explain the behavior of the wage income share in the manufacturing sector than in the agricultural sector.

With estimate I, it is also calculated that the capital deepening factor has contributed to the growth of the wage share at an annual rate of .41 percent, while the labor-using biased technical progress has increased the share at an annual rate of 1.32 percent. Using estimate II, about .18 percent of the annual rate was contributed by capital deepening and .35 percent of the annual rate by biased technical progress. Thus for the manufacturing sector, the biased technical progress has been more significant in changing the wage income share than has been the capital deepening.

The analysis showed that the general direction of the predicted rates of change in the wage shares, based on the marginal productivity theory, were consistent with the actual rates of the changes for both the sectors, but there are large differences in actual magnitudes. Thus the results may indicate that the theory we used could not be a complete explanation of the changes in wage income shares for the sample period in the Korean economy. However, any conclusions are tentative due to the various possible sources of bias in the estimates of the parameters.

With the same estimates, this study also attempts to estimate the marginal product of labor, and examines the extent of variations between the estimated value of the marginal product and the actual wage rate over time and between sectors.

For the agricultural sector, the estimated value of the marginal product of labor is far below the actual wage rate particularly until the early 1960's. However, the difference between the value of the marginal product and the actual wage rate has been considerably reduced over time. It is calculated that the value of the marginal product was only about 67 percent of the actual wage rate in 1955-1959 and 93 percent in 1970-1974.

The marginal product of labor for 8 hours of work in the agricultural sector rose from 316 won in 1955-1956 to 558 won in 1973-1974 (in 1970 constant prices). Thus during the last two decades, the marginal product of labor has increased about 77 percent.

Quite unlike the agricultural sector, the data for the manufacturing sector show that the estimated value of the marginal product of labor is far above the actual wage rate for the whole period. The actual wage rate was only 56 percent of the value of marginal product in 1957-1958 and 63 percent in 1973-1974.

The marginal product of manufacturing labor rose from 611 won in 1957-1958 to 1132 won in 1973-1974 (also at 1970 constant prices). Thus during the period, the marginal product of manufacturing labor has about doubled, which is compared with the 77 percent increase in the agricultural sector.

The data indicate that there were significant differences between the actual returns and values of marginal products of labor in both the sectors. But the directions of the differences were not the same between the sectors. The value of marginal product of manufacturing labor was far above the actual wage rate, implying that wage earners have been paid too little compared to their actual contributions to the production. But the value of marginal product of agricultural labor was lower than the wage rate, implying that the self-employed agricultural workers received low returns for their labor. However, the actual wage

rate has tended to approach the value of the marginal product over time for both the sectors.

The data also show that the value of the marginal product of agricultural labor is only about 52 percent of that of manufacturing labor on the average for the whole period, which implies that there was also a large disequilibrium between the sectors. However, the disequilibrium between the sectors has been significantly reduced over time. The relative value of the marginal product of agricultural labor to the manufacturing labor rose from 42 percent in 1957-1959 to 63 percent in 1970-1974.

While it is hard to judge the validity of our analysis, one may draw a conclusion that the marginal productivity theory seems to be consistent with the actual movements of wage income shares in their general directions, but there are considerable differences in actual magnitudes. This may indicate that there are some other factors, or disequilibrium factors, paying an important role in the Korean economy during the period.

However, the importance of the disequilibrium factors within and between the sectors has apparently decreased over time. Thus the Korean economy may have been in the process of adjusting to an equilibrium during the period. But the conclusion is very tentative, and it is only valid when the data and the estimates of parameters used for the analysis reflect true values.

There may be various possibilities the data and estimates were biased. The weakness in the data and possible errors in estimation could mean that the economy may not have been in the disequilibrium

indicated by the results. More work on this possibility is needed.

If such a disequilibrium does in fact exist, there are various possible disequilibrium factors due to the process of rapid growth of the Korean economy in the last decade. The analysis of various possible factors or the causes of disequilibrium will be another important subject to be investigated.

There may be basically two sources of differences in factor substitutability between different sectors. One is the difference in the basic nature of the production technology, and the other one will be the change in the commodity structure of each sector. However, the sources which have caused the differences in the degree of substitutability between the sectors are not clear in our aggregated sector study. Thus more disaggregated study is needed to provide more evidences for the estimate when the data become available.

APPENDIX

APPENDIX

In order to find the maximum and minimum values of the ratio of two normally distributed random variables b_1 and b_2 within the joint confidence region of the two variables, we need to find the slope of T_1 and T_2, or the coordinates of two points, 1 and 3, in Figure 2.

To do this we formulate the standard maximization and minimization problem subject to a constraint which is the ellipse equation of the confidence region:

Max. and Min. $\frac{b_1}{b_2}$

S.T. $C_1 b_1^2 + C_2 b_2^2 + C_3 b_1 b_2 + C_4 b_1 + C_5 b_2 + C_6 = 0,$

where $C_1 = a_{11}$, $C_2 = a_{22}$, $C_3 = 2\, a_{12}$, $C_4 = -2(a_{11}\beta_1 + a_{12}\beta_2)$,

$C_5 = -2(a_{22}\beta_2 + a_{12}\beta_1)$, $C_6 = a_{11}\beta_1^2 + a_{22}\beta_2^2 + 2a_{12}\beta_1\beta_2 - F\alpha$.

The values of the a_{ij} are explained on page 98, β_1 and β_2 are the expected values of b_1 and b_2.

Construct the auxiliary function by the method of Lagrange multipliers as

$$L = \frac{b_1}{b_2} + \lambda(C_1\, b_1^2 + C_2 b_2^2 + C_3 b_1 b_2 + C_4 b_1 + C_5 b_2 + C_6).$$

To maximize and minimize the ratio, find the values of b_1 and b_2 for which the partial derivatives of L are all zero. The partial derivatives are

$$L_{b1} = \frac{1}{b_2} + \lambda(2C_1 b_1 + C_3 b_2 + C_4) \qquad (1)$$

$$L_{b2} = -\frac{b_1}{b_2^2} + \lambda(2C_2 b_2 + C_3 b_1 + C_5) \qquad (2)$$

$$L_\lambda = C_1b_1^2 + C_2b_2^2 + C_3b_1b_2 + C_4b_1 + C_5b_2 + C_6 \tag{3}$$

Taking the ratio of Equations (1) and (2) and rearranging the relation we get the expression as

$$2C_1b_1^2 + 2C_2\ b_2^2 + 2C_3b_1b_2 + C_4b_1 + C_5b_2 = 0 \tag{4}$$

Multiplying Equation (3) by 2 and subtracting it from the above Equation (4), we get a simple straight line equation as

$$C_4b_1 + C_5b_2 + 2C_6 = 0 \tag{5}$$

Equation (5) is the straight line equation connecting the two points which give the maximum and minimum values of the ratio. Equation (5) is M in Figure 2. Then the two equations--the straight line Equation (5) and the ellipse Equation (3)--are solves simultaneously.

The solutions for b_2 which give the maximum and minimum values of the ratio are

$$b_2 = -B \pm \sqrt{B^2 - 4AC}/2A,$$

where $A = C_5(C_1C_5 - C_3C_4) + C_2C_4^2,$

$B = 2C_6(2C_1C_5 - C_3C_4),$

$C = C_6(4C_1C_6 - C_4^2).$

Substituting the values of b_2 into the ellipse equation, we can find the values of b_1. But for each value of b_2, two values of b_1 can be calculated. Thus the four coordinate points 1, 2, 3, and 4 can be found as shown in the diagram. Among the four points, we have to select two points such as 1 and 3 by comparing the values of the ratio b_1/b_2. More easily, substituting the values of b_2 into Equation (5), we can get directly points 1 and 3 which give the maximum and minimum values of the ratio of the two variables.

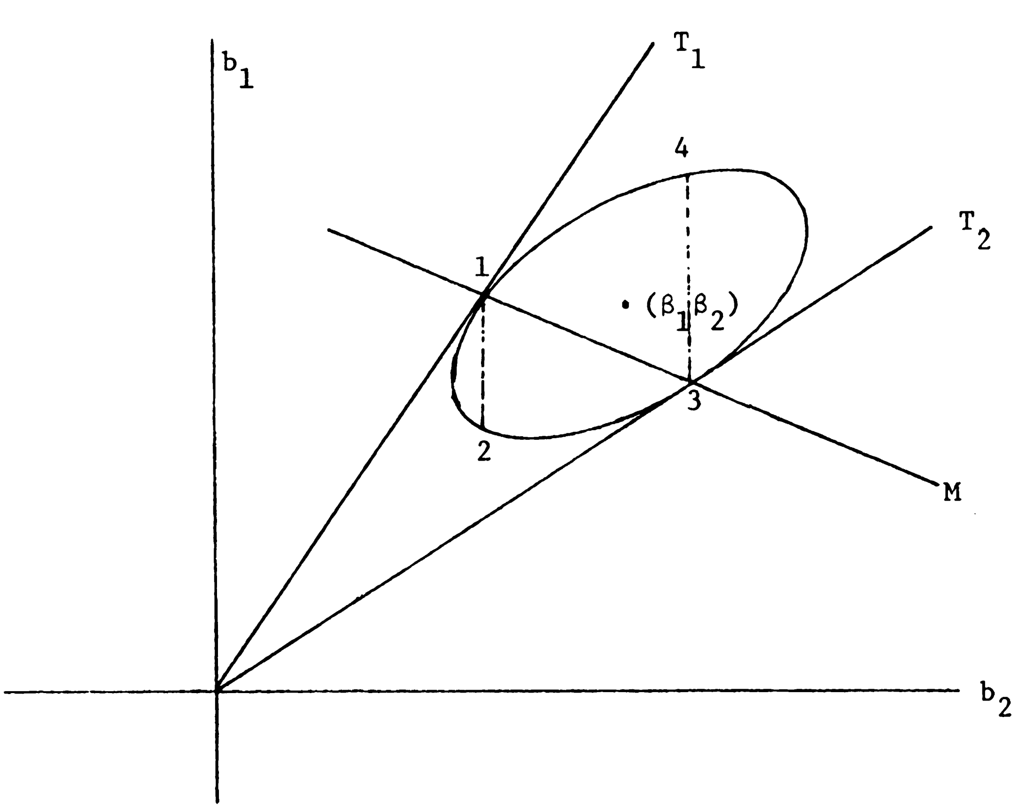

Figure 2

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