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SCHOOL MATHEMATICS LESSONS AS A COLLABORATIVE EFFORT BETWEEN TEACHER AND STUDENTS IN TWO NINTH GRADE MATHEMATICS CLASSES - GENERAL MATH AND ALGEBRA

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ABSTRACT

SCHOOL MATHEMATICS LESSONS AS A COLLABORATIVE EFFORT BETWEEN TEACHER AND STUDENTS IN TWO NINTH GRADE MATHEMATICS CLASSES - GENERAL MATH AND ALGEBRA

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Concern about the degree of mathematical illiteracy present in the American population prompted this study. General math is the last math class taken by half of the ninth graders in the United States. Their mathematics education stops at general math.

The purpose of this study was to describe and compare a ninth grade general math class and a ninth grade algebra class taught by the same teacher. Algebra is the first class in a sequence for college bound students. The two classes were observed for one school year and the data base for the study consisted of 62 sets of field notes, 31 for each class. A method of analysis was developed for the study which used the mathematics content of the lessons as the basis for studying the teacher/student communication during the class. The teacher's logical presentation of content for each lesson was stated in a sequence of steps. These steps structured the lesson and provided a framework for examining the social interaction which occurred around the content. A coding system was devised for the study to analyze the teacher/student interaction during the mathematics lesson.

The findings revealed that mathematics lessons are a collaborative effort between teacher and students. The teacher prepares and presents the content and the students respond to teacher and content. They jointly produce the mathematics experience. During this process content and social organization are mutually influenced by the progress of the collaboration. The algebra class involved more communication by the teacher than the students. The algebra students were cooperative with the teacher in the joint production of the lesson and more content was presented in the lessons. The general math class involved almost the same amount of communication by the teacher and the students. The general math students were adversaries to the teacher in the joint production of lessons and the content presented was simplified and familiar.

The analysis of collaboration in these two classes illuminated some issues regarding what makes general math harder to teach than algebra. This analysis of collaboration also provided a new focus for research in classroom analysis with the subject matter content as a basis for examining the classroom social organization.

Dedicated to

Mother Mary Honora Kroger, R.S.M.
and
Mother Rita Brunner, F.S.E.
the two persons
most influential and supportive
in my achieving
this goal

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INTRODUCTION

Ninth grade general math is the last mathematics class taken by an estimated fifty percent of the ninth grade population in the United States. The other fifty percent usually takes algebra. These two ninth grade classes have acquired over time rather distinctive characteristics. There are interesting contrasts between general math and algebra. What follows is an overview of these contrasts, as they are generally conceived. After the overview a more detailed discussion of the literature will follow.

Overview of General Math and Algebra

Ninth grade general math has been described as an unpleasant experience for both teachers and students. The general view is that the content usually repeats that of late elementary school and consists of innumerable worksheets and exercises. Because general math students are notorious for not doing homework, most of the class time seems to be spent doing seat work in order to complete the assignments. The class has been labeled a "zoo" because of the disruptive, uncooperative behavior of the students. The teacher appears to exert more energy trying to keep some measure of control than in presenting the mathematics content. The teacher, frustrated in any effort to teach mathematics and unable to recognize any significant amount of student achievement, often considers this class unsuccessful.

Ninth grade algebra is considered easy to teach. It is usually the first course in the sequence of mathematics classes for college bound students. The curriculum has been described as integrated and sequential because the content is organized in a progressive plan to present concepts from simple to more complex. The students seem interested in the class and apply themselves in study because they know this is the first important step in their program. They appear to want to be in the class, to listen to the teacher, to answer the questions asked, to ask questions about the content, and to do the homework. Because these students exhibit this interest and cooperate with the teacher during class, most of the class period is usually spent in teacher/student interaction about the content. This interaction is often directed by the teacher as the content is presented to the students. These students have proven their ability to do homework outside of class. Behavior problems have been known to occur in algebra classes but the teacher or other students are usually able to correct the problem. The teacher reports progress and success both in content covered and in student achievement.

Based on the conceptions just presented, it is generally assumed that it is harder to teach general math than algebra. Reasons given for this assumption include the differences in the mathematical content, the way the class is organized, and student behavior. For general math the mathematics content is usually a repeat of various topics from elementary school while for algebra the content is new and organized in a logical sequence for learning. The general math class spends most of its time working problems individually while the algebra class spends most of its time in whole class instruction receiving new content from the teacher. General math students are usually portrayed as disruptive and uncooperative, often working against the teacher. Algebra students are credited with being interested in learning math, able to sit quietly and pay attention, and capable of asking intelligent questions about the content and of doing their work. The classroom experience in general math becomes a struggle between students as adversaries and a frustrated teacher. The classroom experience in algebra becomes one of cooperation between students and teacher in the forward progress of the lesson with the teacher feeling successful about the progress of the class.

Currently there is national concern about the mathematical illiteracy that exists not only in our young people but also within the adult population of America. The general math phenomenon is one reason for this national concern because it is the benchmark for mathematical illiteracy. What the students know mathematically at the completion of this class could be the extent of their mathematical knowledge through their adult life. This concern is being heightened by the increased visibility and influence of technology and scientific advancements within our society. Lack of mathematical expertise inhibits the intelligent and creative use of such innovations as the computer and computerized appliances.

Researchers have addressed the issue of mathematical illiteracy by studying school mathematics education. They have considered several aspects of the classroom experience. The curriculum has received considerable attention as demonstrated by new programs, textbooks, and teaching materials. Student achievement and factors influencing achievement have been studied extensively. Educational anthropologists have focused on the social aspects of the classroom scene creating an awareness of the influence of social organization in the educational process. The teaching of mathematics, historically perceived as a straightforward presentation of content, is being studied more insightfully in the social context of the classroom experience.

Having separated out the mathematics content and the social aspects of the teaching/learning dynamic researchers are only now beginning to inquire into the influence these two aspects have upon one another. What happens to the mathematics content when it is presented in the classroom experience? How is the social organization of the students affected by the mathematics content presentation? What kind of interaction occurs between teacher and students during the lesson and how does this reflect the forward movement or progress of the lesson as successful or unsuccessful according to teacher expectations? To provide possible answers to these questions some basic assumptions must be considered.

Assumptions

This paper began with a presentation of the usual assumptions about what constitutes a general math class or an algebra class. In order to address the questions just suggested a different set of assumptions is required. These assumptions pertain to the how of a general math class or an algebra class, the processes of instruction and interaction by which they end up being like they are.

1. The presentation of mathematical content may be influenced in some way during the actual mathematics class. This is a challenge to the belief that the teaching of mathematics consists of

simple step-by-step explanations and the working of a few examples. The presentation of content involves some kind of interaction with the students. This interaction between teacher and students has an impact on how the presentation actually takes place within the lesson.

2. It matters what content is presented and how it is presented. The students will respond in a certain way to the content itself, from past experience or from their own expectations. The teacher may directly organize the students for a specific presentation of content and the students in turn will respond to that kind of organization. These participation structures create a basis for interaction between the teacher and students about the mathematics content of the lesson.

3. There are qualitatively different kinds of interaction between the teacher and students and these differences in kind profoundly affect differences in the mathematics content to which the students are exposed. From a teaching perspective interaction may be helpful to the lesson (a question to help clarify or a right answer to a question) or interaction may be a distraction (an irrelevant question or nonmath conversation). The students may cooperate with the teacher by paying attention and participating in the class by answering and asking questions about the content. Or the students may be adversaries by providing several interruptions and generally not paying attention or working. Different kinds of interaction result from the teacher's presentation and the attitudes of the students.

These qualitative differences in interaction affect how the lesson actually gets taught in the classroom. Each lesson has an objective formulated by the teacher. The usual plan is to present the mathematics content in order to accomplish the goal which may be to learn a new concept or practice a learned concept or solve a particular problem. As the teacher begins the lesson the response of the students determines if the lesson will reach its goal directly or whether there will be interruptions and sidetracking before the goal is reached. Perhaps the goal is never reached due to some interference. The teacher must cope with the interference and sidetracking, always trying to bring the lesson back to the task to be accomplished. Interaction between teacher and students affects the path to the goal and often regulates how much mathematics content is actually presented during the classroom experience.

4. The school mathematics lesson is a collaborative effort, a joint production between the teacher and the students. Teacher and students have input into the classroom experience and the interaction between them determines if the movement of the lesson is forward to the completion of the task at hand or if there is some interference in the forward movement. The back and forth activity about the content by teacher and student determines what happens to the mathematics of the lesson. The mathematics classroom experience is a collaborative effort, a joint production between teacher as content presenter and student as learner.

5. There is a way to look analytically at the forward movement of the lesson, the process of completing a given objective (solving a problem, teaching a concept). The process or movement to completion includes both the teacher's presentation of content and the interaction that results because of the presentation. The teacher's logical presentation of the mathematics serves as a structure for the lesson. The interaction within the lesson provides a pattern of

cooperation or lack of cooperation between teacher and students. The merging of the mathematics structure and the pattern of interaction shows the progress of the lesson as successful or unsuccessful in completing the goal. By reviewing the lesson as a joint production it is possible to study the contributions of teacher and students as they create the experience of the lesson.

The concept of collaboration as the joint production of lessons is a basis for rethinking possible answers to what makes it harder to teach general math than algebra. By comparing types and amounts of collaboration in the enactment of lessons in both general math and algebra new insights as to the nature of the differences emerge.

Beliefs about the teaching of mathematics are being reconsidered by the awareness of the collaborative aspect of teaching and learning. Mathematics education has a history of step-by-step development which has rested for the most part in the hands of mathematicians and mathematics educators. Only now is the trend to study classroom interaction opening areas of needed examination in mathematics classes. The progress of a lesson is worked out by teacher and students and is not solely a teacher directed effort. The advent of classroom analysis has provided a new perspective for addressing the issues of a general math class.

CHAPTER II

REVIEW OF THE LITERATURE

Introduction

Mathematics is a well-established element of our culture. It has developed into an area of abstract systems and thought besides maintaining a basic and essential role within our daily lives and the progress of society. However, the place of mathematics in education is at a critical point, as expressed by the National Advisory Committee on Mathematical Education (1975).

> School mathematics is in an unusual state today. Long enshrined as a unique and well-supported discipline with a clearcut and almost monolithic identity, it is suddenly beset with many troubles -- an identity crisis brought on by the usual causes: internal confusion and loss of clearcut direction and external changes in familiar support and status structures. (p. 147)

Internal confusion and loss of clearcut direction are exemplified in the transformation of the goals of mathematics education.

Mathematics Education Goals in the United States

The goals and importance of mathematics are influenced by society's growth and the demands of its citizens. In the United States the goals of mathematics education have developed within the historical growth pattern of the country itself. In the colonial period the needs of society were simple and practical. The goals of education, including mathematics education, were those of practical utility and mental discipline. The rapidly developing technology and the demands of engineering, westward expansion, and complex manufactured products called for more advanced mathematical knowledge. The rise of science and its mathematical basis created an emphasis on reasoning rather than rule-learning, on exercising the mind, promoting a deductive structure rather than an inductive one.

At the beginning of the twentieth century there was a new quest for interrelationships of mathematics with science, for lab-teaching techniques, for problem-solving and discovery methods. The concept of mental discipline was expanded by the perspective of the mind as a searching, inquisitive instrument. In 1923 the National Committee on Mathematical Requirements identified three classes of goals for school mathematics: 1. practical or utilitarian, 2. disciplinary, 3. cultural or aesthetic. In summary the Committee made the following statement:

> The primary purposes of the teaching of mathematics should be to develop those powers of understanding and of analyzing relations of quantity and of space which are necessary to an insight into and control over our environment and to an appreciation of the progress of civilization in its various aspects, and to develop those habits of thought and of action which will make these powers effective in the life of the individual. (Schorling, 1936, p. 5)

In 1963 a small number of professional mathematicians came together to discuss and to formulate an ideal curriculum for mathematics education. The report of this Cambridge Conference on School Mathematics was titled "Goals for School Mathematics". With freedom of thought and the search for the ideal mathematics education they set down the following goals, if for no other purpose than to create discussion, debate, and possibly experimentation. There was the hope, also, that other subject areas, concerned about the quality of education as well as its pragmatic value for society in process, would do the same for their own disciplines.

The broad goals of the School Mathematics Curriculum are given as follows:

1. Acquistion of Skills: replacing drill of classical arithmetic by problems which illustrate new mathematical concepts.
2. Familiarity with Mathematics: spiral curriculum repeatedly returning to each topic, expanding it and showing more connections with other topics.
3. Mathematics in Liberal Education: mathematics is a subject of great humanistic value, important to the educated man; modification of many poor patterns of thought common in ordinary life.
4. Building self-confidence: analytical thinking.
5. The role of "Modern" mathematics: useful in "organizing" material to be presented.
6. Technical Vocabulary and Symbolism: need for communication and organization.
7. Pure and Applied Mathematics: approached intuitively.
8. Power of Mathematics: an understanding of what mathematics is and what it is not.
9. Understanding the limitations of mathematics: a distinction between the real world and its various mathematical models.

This statement of goals by the Cambridge Conference (1963) was a product of the rapid growth and development of the field of mathematics during the nineteenth and twentieth centuries.

> So vast has been the increase of mathematical knowledge since the early 1800's that few men would presume to claim more than an amateur's acquaintance with more than one of the four major divisions of modern mathematics. The field of higher arithmetic alone is probably beyond the complete mastery of any two men, while geometry, algebra, and analysis, especially the last, are of even greater extent. (Bell, 1951, pp. 6-7)

This rapid growth brought with it a sharp difference in the viewpoints of mathematicians toward mathematics. The trend toward abstraction and the study of general abstract systems has caused concern for those who maintain that mathematical ideas originate in observations and experience. The controversy resulting from these two trends prevents a consensus in answering the question, What is mathematics?

> On one thing, however, mathematicians would probably agree: that there are and have been, at least since the time of Euclid, two antithetical forces at work in mathematics. These may be viewed in the great periods of mathematical development, one of them moving in the direction of "constructive invention, of directing and motivating intuition" (Courant & Robbins, 1941), the other adhering to the ideal of precision and rigorous proof that made its appearance in Greek mathematics and has been extensively developed during the 19th and 20th centuries. (Rees, 1966, p. 3)

Each group of mathematicians defined mathematics according to their beliefs with respect to the nature and growth of the field. In looking at the forces which impinged upon mathematics education, the forces within mathematics itself played a role in the changing scene.

> For example, the forces derived from mathematics are the rapid growth of its content, the phenomenal expansion of its applications (both in their number and in the fields to which mathematics is applied), and the changing nature of the subject or of mathematicians' view of it. All these forces from mathematics were among the stimuli that produced the "revolution" of the decade from 1952 to 1962. (Jones, 1970, p. 460)

Despite the controversy of viewpoints, the nature of mathematics was characterized by rigorous thinking; sweeping generalizations;

precision of language; symbolism; pure abstractions; and concern with patterns of ideas, structure of forms, and qualities of relationships. Expressions of what mathematics is were both technical and poetic. Mathematics was discussed as an art or a science or both. Rees (1966) made the following statement:

> For mathematics is the servant as well as the queen of the sciences, and she weaves a rich fabric of creative theory, which is often inspired by observations in the phenomenal world but is also inspired often by a creative insight that recognizes identical mathematical structures in dissimilar realizations by stripping the realizations of their substance and concerning itself only with undefined objects and the rules governing their relations. (p. 4)

It was with this level of understanding in the field of mathematics that the Cambridge Conference made their recommendations. It was a combination of these forces which created internal confusion in the field of mathematics, and the loss of clearcut goals which provided a crisis for mathematics education.

The events of the period from 1945 to the present have a profound message for mathematics education as it is developing and must develop for the future. The technological advances with the appearance of computers called for people highly trained in mathematics.

> It is, however, the twentieth century that has witnessed not only a remarkable flowering of mathematical discovery but also an unprecedented penetration of mathematical methods into other fields of human activity. In addition to the physical sciences and engineering, these fields now include the biological and medical sciences, the social and behavioral science, various domains within humanities, and the general field of business operations and management. During the past twenty years, this virtual 'mathematization of culture' has been greatly accelerated and intensified by the evolution of the high speed electronic computer. (Botts, 1970, p. 449)

The goals of mathematics education today are those historically developed: utility, mental discipline, cultural or aesthetic. But society today provides a complex and technological environment in which to establish the meaning of, as well as the means of, attaining these goals. Calculators and computers handle much of the utility mathematics which presently formulates a large portion of the content for school mathematics. The attitude of instant productivity and availability challenges the patience needed to develop mental discipline. The cultural or aesthetic goals of mathematics are becoming lost in the negative attitude and anxiety expressed in widespread mathematical illiteracy. And yet we must face the reality of the society in which we live as expressed by Max Bell (1975):

> For better or worse in the world we have built, countless fields of work and inquiry have come to depend heavily on mathematical methods. Nor is this true only of our science and technology. Social, business, political, and economic decisions increasingly depend both on understanding information given in various mathematical forms and on use of mathematical tools to facilitate decision making. This, of course, makes mathematics helpful to more and more people in their occupations; but the point I want to stress for citizen everyman is that anyone with poor understanding of certain basic mathematical tools and concepts is to that extent unable to exercise responsible citizenship or control over events. (pp. 39-40)

The attempt to define and implement the certain basic mathematical tools spoken of by Bell created the "external changes in familiar support and status structures" (NACOME, 1975, p. 147). This attempt became msot critical during the 1960s. To this point school mathematics consisted of arithmetic in elementary school and algebra and geometry in the high school. Junior-high schools were offering arithmetic, possibly some business applications, and algebra.

E. T. Bell (1951) gave an overview of high school mathematics education at the beginning of the 1950's.

> To get some sort or perspective, let us consider roughly the kind of mathematics acquired by a student who takes all that is offered in a good American high school. The geometry taught is practically that of Euclid and is about 2,200 years old. It is a satisfactory first approximation to the geometry of the physical universe, and it is good enough for some engineers, but it is not that which is of vital interest in modern physics, and its interest for working mathematicians evaporated long ago. Our vision of the universe has swept far beyond the geometry of Euclid.
>
> In algebra the case is a little better. A well-taught student will master the binomial theorem for a positive whole number exponent with Pascal (1623-1662) discovered in 1653. Then he will stop. And yet the really interesting things in algebra are the creation of the nineteenth and twentieth centuries, and began to be developed over a century and a half after Pascal died.
>
> Of the higher arithmetic - Gauss's Queen of Mathematics - the graduate of a good school will learn precisely nothing. (p. 9)

The innovative programs of the 1960's provided a more advanced content for the school mathematics curriculum and a more creative set of strategies for teaching the new content. Discovery learning, learning hierarchies, problem-solving which looked at real problems in the school or community were among the approaches suggested. Programs were funded and training sessions were given for teachers. Researchers entered classrooms to support and encourage the development and survival of the programs, but as the money disappeared and the researchers left the scene, the programs began to disappear. Teachers who tried to implement programs on their own, may or may not have succeeded in capturing the spirit of the program and engaging the students in the desired activities and learning. Parents were unable to help their children as the content and methods were foreign to them. The "modern math" movement, although intended to increase the mathematical understanding of the population, created a new host of problems to be faced by mathematics educators.

The "back to basics" movement was an attempt to undo the confusion and inefficiency of the "modern math" movement. But the "basics" had been defined and redefined in several dimensions of meaning. What should be taught in elementary school mathematics no longer had a simple solution and the choices of possible content increased. Local communities began to establish their own understanding and rationale for what they wanted in their school programs. Each curriculum agent recognized the need to clearly express goals and objectives, complete with philosophy and rationale, for the materials being published. Mathematics educators stated the beliefs and assumptions which formed the basis for their programs in order to communicate to the public the goals and objectives for the mathematics education of the children.

Mathematics educators on the university level maintained that mathematics is an essential part of the curriculum in educating and preparing students for the life they must lead. "Mathematics is more than just a school subject. It is a national resource, a national concern, and, at times, a national issue" (NACOME, 1975, p. 146). Mathematics educators showed concern about the statistics regarding mathematics education in today's schools. The results of a study conducted at Michigan State University showed that half of the 7,300 freshmen who were admitted to the university fall term, 1977, were required to take at least one remedial mathematics course. Of these 3,500+ students, about 1,700 had had more than one year of high school mathematics.

Regarding the National Assessment scores from 1973 to 1978 "Survey findings show that overall mathematics achievement has declined over the five years" (Education Commission of the States, 1979, p. 2). In light of these results mathematics educators urgently defined for teachers the necessity of teaching mathematics and promoted innovative programs, evaluated teacher education programs, and provided inservice workshops.

Faced with the issues of mathematics education the teachers were forced to justify the teaching of mathematics and to decide how it related to daily living. They had to resolve for themselves how the curriculum they taught related to what "should" be taught according to mathematics educators. Involved in this process of resolution was the influence of outside pressures, demands, and accountability. And what about the students? Most of them didn't like math. It didn't make sense to them. As Robert Wirtz (1978) pointed out with respect to mathematics:

> By the middle grades, children are effectively separated into two distinct and easily discernible groups. Teachers refer to them as the "I don't get its" and the"I get its." And, as this outmoded curriculum unfolds year by year, the group of victims grows and the group of survivors dwindles. (p.2)

Wirtz (1978) referred to the "outmoded curriculum" that separated the survivors from the victims in the middle grades. The attempts of the 1960's to update mathematical content according to the trends in the field of mathematics did have an impact. The elementary school curriculum now includes more geometry, some probability, and an introduction to more advanced number ideas such as negative numbers. The high school now offers calculus, linear algebra, matrices, and probability and statistics. But a further concern in the development of the mathematics curriculum is the teaching strategy used by the teacher.

> Some psychologists hold that students will learn their needed skills and routines more easily, for longer retension and simpler extension, if they understand the "why", the reasoning and the structure of the system. In this case an understanding of proof and structure becomes a goal of instruction, not merely because in it one sees the beauties of the art that is mathematics nor because through it one can find the proper structure to use in applying mathematics, but because it creates the "understanding" or meaning that is needed.
>
> All this leads to the questions How do we teach mathematics? and How do we help students to become mathematicians? (Jones, 1970, p. 458)

The consideration of what is being taught and how it is being taught provides a basis for discussing the curriculum of school mathematics.

As shown historically the mathematical content to be taught in school has been an issue of concern for mathematicians, mathematics educators, school administrators, teachers, parents, and students. Mathematicians cannot agree on a definition of mathematics and there is a vast difference in understanding between the mathematicians' mathematics and school mathematics, what is being taught in schools. O'Brien (1973) has stated, "Mathematics, it is commonly believed by elementary school teachers, is concerned chiefly with algorithms, rules, procedures, conventions, and notation" (p. 262). Romberg (1977) supported and extended this statement as he stated, "Unfortunately, what people take to be mathematics - the symbols, statements, propositions, and rules that appear in textbooks - is only a written record of mathematical knowledge. Rarely do students have an opportunity to relate real world facts to mathematical ideas" (p. 161). The choice of content, the method of presentation, and the interaction with the students during the presentation depends to a great extent upon the teacher's background, attitudes, and conception of mathematics. If mathematics is commonly believed to be a set of algorithms, rules, procedures, conventions, and symbols, the task of the mathematics teacher is to efficiently transmit this collection of facts to the individual student. But the evidence of our national tests and the performance of our high school graduates force us to reconsider this statement. Compounded with this reconsideration is the statement by Wadsworth (1978): "If Piaget is correct, failure of students to develop comprehension of mathematics does not imply any lack of intelligence or ability to learn the concepts but results from the type of instruction to which children are exposed in schools" (p. 162).

Classroom Research

Bauersfeld (1979b) in his paper Research related to the mathematical learning process pointed out that research in mathematics education is shifting from the curriculum and/or the student to the teacher. The failure of past research to produce effective and integrative curriculum theories and learning theories has created an interest in the development of teaching-learning theories. This has promoted the interpretation of the word interaction in the social interaction sense rather than the statistical interaction sense. Research has broadened to include descriptive studies and the use of interviews or less formal approaches. As a result of his own work Bauersfeld (1979a) stated:

> The learning of mathematics requires more than the availability of rote knowledge. This "more" can be described as meaning,

> as understanding or insight, as adaptation to reality, etc. Research shows that this "more" is employed in the classroom in at least three forms: as the structure of the mathematical discipline, the "matter meant"; as the content of the teacher process shaped by the teacher's learned structure and routines, the "matter taught"; and as the cognitive structure of the individual student, the "matter learned". These three forms coincide in the ideal case only. (p. 10)

Viewing the learning of mathematics from these three aspects: matter meant, matter taught, and matter learned implies that teaching mathematics may not be as straight forward as many believe. If the mathematics in the textbook is not what actually gets taught or learned then what is taking place in the classroom or in the students that keeps these three from coinciding? In response to the question Bauersfeld (1979b) asserted:

> We have to abandon the image of the unchangeable subject mathematics which is passed onto the student by the teacher. On the contrary, the subject matter varies and changes in the course of the teaching process and as well in the individual learning process. By the way, this also supports the relevance of theories of social interaction for mathematics education. (p. 11)

To go a step further, Bauersfeld (1979a) stated that "Teaching and learning mathematics is realized through human interaction" (p. 19). If this is the case then social relations are essential in the teaching/learning process of mathematics. We cannot ignore the method of presentation or the response patterns of the students. No matter how logical the presentation, the responses of the students help determine how the class progresses and how the material is assimilated.

Davis (1976) reported a study of ninth grade mathematics students. Working with the students, the researchers were concerned that the students were not exhibiting the extent of their cognitive ability. "The limiting factor was classroom social organization. We suspect this may often be the case, and that the limitations of content in school curricula are often concessions to easier ways to maintaining classroom order" (p. 216).

If this is the case, then the interaction in the classroom between teacher and student, and student with student, affects the mathematics presented and learned. Mathematics education becomes a social process which is a new and different concept in light of the present understanding of what school mathematics is. The present aspects of school mathematics, computation, drill, practice, and memorization, are not usually considered the basis for social interaction. And yet students clarify their ideas and concepts through interactions with others. From the perspective of social interaction, the mathematics educator needs to ask what is being taught and learned in the classroom through the school mathematics content and how is it being done?

To address this question Ian Westbury (1978) suggested an entrance point.

> A curriculum only finds its meaning in teaching, in the actions and relationships between teachers and students as they engage mutually in the activities we call education. Clearly, a comprehensive understanding of the tasks of curriculum planning and curriculum development demands an understanding of the classroom. (p. 283)

The classroom is a complex social system influenced by several and diverse factors. These factors may be sorted out through the use of Walter Doyle's (1977) study on preservice teachers and how they coped with the complexity of the classroom. Through the study the classrooms were characterized by multidimensionality, simultaneity, and unpredictability. "Classrooms are multidimensional in the sense that they serve a variety of purposes and contain a variety of events and processes, not all of which are necessarily related or even compatible" (Doyle, 1977, p. 9). Specific factors which influence the interaction from this viewpoint include: informal social groups, teacher conceptions of teaching and learning, teacher expectations, student conceptions of what needs to be learned, student motivation for studying or being in school.

Simultaneity refers to the issue of all these conceptions and expectations coming together on one subject matter. If these factors are complementary or compatible the learning process may proceed as planned. If not, the conflict resulting will affect the teaching/ learning as it progresses.

Unpredictability refers to the fact so many human beings changing at every second are trying to adjust to a social context. "The simultaneous occurrence of multiple dimensions, together with the continuous possibility of internal and external interruption, contributes to an unpredictability in the sequence of classroom events" (Doyle, 1977, p. 10).

In Doyle's study the preservice teachers were observed during their teaching and interviewed with respect to their decisions and behavior during the lesson. Doyle was interested in how the preservice teachers chose to cope with classroom complexity. His research results presented several options including ignoring aspects of the classroom, chunking periods of time and activities, defining a line of action and maintaining it. The results of the study emphasized the variety of contrasts and decisions a teacher must address.

As another approach to viewing the classroom complexity, Joseph Schawb (1973) explained what he calls the four commonplaces: teacher, learner, subject matter, and milieu. These four commonplaces are vital and necessary aspects of the classroom. They demand equal attention in terms of teaching and learning. As presented by Schwab, the teacher commonplace refers to the individual in his/her background, personal interest, training, and beliefs about teaching and learning. The learner commonplace incorporates the needs, characteristics, demands, and interaction of the students. Subject matter is the content and activities of the lessons. Milieu includes all aspects of social influence from the social environment inside the classroom to the school, the family, the community, the state, and circling outward to the world. Whatever may impinge upon the teaching/learning activities in the classroom is part of the milieu. The commonplaces provide a framework for looking at the complex social system of the classroom.

Research in education has attempted to sort out the salient variables in the classroom and establish methods for effective teaching and increased student outcomes. As it became apparent that the quantitative research results were yielding very little noticeable improvement of practice, educational researchers began to engage in qualitative research and to negotiate entry into classrooms, observing the day to day life of teacher and students. These fieldwork researchers viewed the happenings of the classroom from the perspective of the participants, teachers and students. Wilson (1977) explained two sets of hypotheses underlying the rationale for fieldwork research:

> (a). Human behavior is complexly influenced by the context in which it occurs. Any research plan which takes the actors out of the naturalistic setting may negate those forces and hence obscure its own understanding. (b). Human behavior often has more meaning than its observable "facts". A researcher seeking to understand behavior must find ways to learn the manifest and latent meanings for the participants, and must also understand the behavior from the objective perspective. (p. 253)

With a focus on human behavior in a given context, the classroom, and in search of the meaning of the participants, researchers published their findings about classrooms. They suggested ways of looking at classrooms and the complexity of the activity there. Key in these models was the interaction between and among teacher and students.

Philips (1972) talked about "participation structures" in the classroom. She claimed that structural arrangements of interaction could be described within the framework of teacher-controlled interaction. Philips identified four types of participant structures:

1. the teacher interacting with all students,
2. the teacher interacting with only some of the students at once,
3. all students working independently at their desks with the teacher explicitly available, and
4. students being divided into small groups that run themselves with more distant teacher supervision.

With these structures it is possible to talk about the social organization of the classroom in a global way. The teacher decides the kind and extent of the interaction and how the students will relate to him/her and to each other. However, the clarity of this model of participation structures marked the beginning of a more direct study of classroom interaction.

Within the global orientation of the four participation structures, each structure was studied in terms of a learning environment.

> Philips (1972) provides another way of formally describing the social organization of the multiplicity of events in the classroom - a way that permits specification of variation across different structures of participation, different social environments for learning. Those environments may be the key unit of analysis for the study of classroom interaction. (Erickson, 1977, p. 67)

Different structures produced different social and learning contexts. Erickson and Schultz (1977) explained that "contexts are constituted by what people are doing and where and when they are doing it" (p. 6). A context, then, does not consist simply in the physical surroundings or in the persons involved. Aspects of a context include a basic understanding of what is taking place and the behavior required of a member of the event. "Ultimately, social contexts consist of mutually shared and ratified definitions of situation <u>and</u> in the social actions persons take on the basis of those definitions (Mehan et al., 1976)" (Erickson & Schultz, 1977, p. 6).

A study of context, and the moment by moment shifting and changing within a context, directed researchers to the question of how a person knows what behavior is acceptable in a given social context. The constant changing requires that a person be capable of recognizing the changes and adjusting appropriately to the new circumstances. The cues for change may be verbal or nonverbal, obvious or subtle, sequential in nature, or culturally based, just to name a few characteristics.

The classroom is a large set of events and each event is a set of social contexts. Students must learn to behave appropriately within the social organization of the classroom. The demands of any social context are present in the school environment and knowing how to go to school requires the skills of reading the cues and responding according to the message. The inability to do this often results in what is termed classroom misbehavior and produces teacher/student misunderstanding.

Schultz, Florio, and Erickson (1979) conducted a study which observed children in the school and home environments in order to determine if social responses patterned in the family could be seen in school behavior. Although the environments were very different, there were similar cues for interactional behavior in both environments. As the child learns the acceptable social behavior to certain cues in the home do the same behaviors for the same cues apply in school? The study provided examples of transfer behavior, acceptable in the home but not acceptable at school.

The researchers observed and analyzed the sequential development of the family evening meal and school mathematics lessons. In a systematic manner, the two kinds of events were chunked into phases such as preparation, getting started, etc., and each phase was identified in terms of participation structures, who interacts with whom and how. The interactional behavior within each participation structure was analyzed and similar sets of circumstances were found at the dinner table and in the math lesson such as the number of persons speaking and those listening. It was observed that the acceptable behavior at the dinner table when one main speaker was being listened to was not acceptable behavior in the classroom when the teacher was the main speaker. "A child entering school for the first time may make errors relative to the classroom norms for interaction because of the ways in which participation structures and constituent phases are matched up in the classroom as contrasted to the way they are matched at home" (Schultz, Florio, Erickson, 1979, p. 50).

Through this study new insights emerged regarding the classroom environment in a practical dimension, how a lesson is chunked into phases, and in a research dimension, how to study a lesson in a systematic way. The systematic approach of analysis in sequencing of the participation structures within the order of the various phases of the event was a means of looking more closely at interactional behavior in the classroom. With the focus of this kind of research, Mehan (1979) studied interactional competence in the classroom. His discussion involved looking at both content, the subject matter being taught, and form, how to present the content in the classroom interaction.

> To be successful in the classroom, students not only must know the content of academic subjects, they must learn the appropriate form in which to cast their academic knowledge. That is, competent membership in the classroom community involves employing interactional skills and abilities in the display of academic knowledge. They must know with whom, when, and where they can speak and act, and they must provide the speech and behavior that are appropriate for a given classroom situation. (Mehan, 1979, p. 133)

By concentrating on teacher initiated sequences of interaction, such as question-answer-evaluation, Mehan analyzed the student response patterns from the content and social behavior perspective. He found that correct answers given in an unacceptable manner (shouting out rather than raising one's hand) were negatively evaluated by the teacher. Likewise proper procedures which resulted in little or no academic performance (raising one's hand at the appropriate time but not having an answer) received a negative evaluation. "In sum, effective participation in classroom lessons involves the integration of interactional skills and academic knowledge" (Mehan, 1979, p. 139). Due to this attention to content and form, researchers have tried to capture the event of integration or nonintegration of these interactional skills and academic knowledge.

Erickson (1982) referred to the content of the lesson as the academic task structure and the form as the social participation structure.

> Teachers and students engaged in doing a lesson together can be seen as drawing on two sets of procedural knowledge simultaneously; knowledge of the academic task structure and of the social participation structure. The academic task structure (ATS) can be thought of as a patterned set of constraints provided by the logic of sequencing in the subject-matter content of the lesson. The social participation structure (SPS) can be thought of as a patterned set of constraints on the allocation of interactional rights and obligations of various members of the interacting group. (Erickson, 1982, pp. 283-284)

As the teacher presents the content of the lesson, a specific logical sequence is employed as an instructional strategy. This may be, for example, a lecture presentation, a question-answer presentation, an organized activity, or a combination of these. Whatever the teacher decides, the progression of the lesson can be chunked into a sequence of phases. Within this sequence the social participation structures can be identified. These two dynamics, the sequence of phases of the lesson progression and the changing social participation structure, exist at the same time. Erickson was able to give examples in which these two dynamics seemed to influence each other rather than exist in a parallel fashion. He presented the analysis of two school mathematics lessons, a first grade mathematics lesson, in a bilingual

Spanish and English classroom, and a mathematics lesson in a kindergarten-first grade classroom.

By chunking the lesson into its phases and analyzing each phase, a model for the logical presentation of the subject matter emerged. Having established a possible model, the lesson was analyzed for consistency or discrepancy in the use of the model. The discrepancies were any modification or interruption in the established pattern of instruction.

> If one is not simply to regard these discrepancies as random error (free variation), one has at least two options: to elaborate the formalization of the model by stating an embedded system of optional rules; or to assume that what is happening is adaptive variation, specific to the immediate circumstances of practical action in the moment of enactment. (Erickson, 1982, p. 318)

Erickson opted for the latter explanation, adaptive variation. This adaptation may occur in the logical presentation due to a student response, a question, a comment, or behavior, or it may occur in the social participation structure, how to give answers, due to the presentation of the subject matter. In a given lesson both kinds of adaptation may occur several times. As the logical organization of the subject matter and the social participation structure of the classroom come together, the lesson is enacted with both structures subject to being modified.

> The interpretive analysis of instances from the lesson was done to argue that the discrepancies from the ideal model represent adaptive action taken by, in most instances, the teacher as instructional leader, and in one instance, the student Ernest. Since lesson discourse, like all other face-to-face interaction, is jointly produced as various actors in the event take action in account of the actions of others, the variants chosen by the teacher have consequences for what the students will do and vice versa. Moreover, I have attempted to show how adaptive changes in the academic task structure have consequences for the social participation structure and vice versa. (Erickson, 1982, p. 319)

In Erickson's study the enactment of a lesson showed the logical sequence of presentation interrupted and possibly changed through an interruption from the structuring of the social participation. Likewise, the social participation structure seemed to change as the content sequence changed and developed. Through these lessons he demonstrated the inter-relationship between the academic task structure and the social participation structure. More research needs to be done to consider the nature of this inter-relationship. If the two influence each other then what is the result of a modified logical presentation and a modified social participation structure? What does the enactment of a lesson look like?

Summary

Preparing lessons, teaching lessons, and assigning and supervising appropriate tasks revolved around the subject matter of the task. Because it is assumed that logical and sequential presentations are the most appropriate teaching strategy for mathematics, the social dimension of personal interaction in the learning of mathematics is usually overlooked. However, the classroom social environment may prove to be more significant for mathematics learning than we have been willing to consider in the past. Easley (1978), in a set of case studies in science, mathematics and social studies at eleven sites across the country, concluded that subject matter knowledge is transformed during instruction into means for the attainment of socialization goals, e.g., mathematics is used to teach morals and ethics. Through field observation the social influence appeared to have a dramatic effect on content taught. In an article by James T. Fey

(1979) titled "Mathematics teaching today: perspectives from three national surveys" the statement was made:

> There are certainly effective teachers of mathematics, teachers whose students enjoy and learn mathematics. But at the present time, such effectiveness appears to be the result of classroom activity that is an idiosyncratic product of a constantly changing interaction among the teacher, the student, and the mathematics being taught. (p. 496)

It is in this scope of interaction that the enactment of mathematics takes place, the process that occurs through collaboration between the teacher in the logical presentation of the subject matter and the students in the social participation structure for the lesson. The school mathematics lesson is jointly produced through this collaboration. The content of the lessons and the manner of presentation are influenced by the response patterns of the participants. In turn, the response patterns as determined by the social participation structure for the lesson, are influenced by the school mathematics content and the teacher's logical presentation of the content.

The purpose of this study is to describe and compare this process of collaboration in school mathematics lessons as it was observed in two ninth grade mathematics classes - general math and algebra. The review of the literature has presented the two major aspects of this collaboration: first, the content of school mathematics as formulated through the goals of mathematics education and second, teacher/student interaction as studied through classroom research. What follows is a description of this study and its setting and the methods used for data collection and data analysis.

CHAPTER III

PURPOSE, SETTING, AND PROCEDURES

Introduction

Ninth grade general math and algebra can be quickly described according to the characteristics which have been generally recognized in the majority of such classes. Because of the reputation these classes have developed over time, mathematics educators and educational researchers have begun to inquire into the nature of these classes as revealed in actual classroom experience. This study was designed to respond to this inquiry into the nature of these classes by looking at one general math class in contrast to one algebra class. By studying one class in contrast to the other, the basic issues of difference can be identified and compared within a concrete situation.

To obtain a picture of what is happening in a general math and an algebra class, the method of data collection selected was participant observation. The researcher observed in the classroom for both a general math and an algebra class taught by the same teacher. The classes were taught consecutively with algebra first. By taking notes during the class period and writing the field notes more completely after each session, the data base was collected. It consisted of 62 sets of field notes, 31 for general math and 31 for algebra.

A suburban school was chosen for the research site. Because the study was designed as a comparison, the desired situation was one teacher teaching both general math and algebra. This condition held constant the teacher's views and attitudes about mathematics and about objectives for mathematics education. It happened that this particular year the school administration decided to use a pre-algebra book for the general math classes. This provided an unexpected matching of content between the two classes. Instead of the repeated content of the globally described general math classes there was a planned, sequenced curriculum with some future orientation.

The focus of the study was on the collaboration (working jointly) which existed between teacher and students in the actual enactment of the mathematics lesson. The joint production was based on the teacher's presentation of the mathematics content and decision on how to organize the class, and on student interaction which took place during the class period. The collaboration became evident in the description of the lesson as a process toward the completion of the mathematics tasks being studied for that lesson. The field notes were examined for instances of cooperation and instances of interference which were exhibited in the process or movement of the lesson toward completion of the tasks. The pattern of instances of cooperation and interference provided a description of collaboration between teacher and students in the general math and algebra classes and became the basis for comparing the two classes.

Purpose of the Study

The purpose of this study was to describe and compare school mathematics lessons as a collaborative effort between teacher and students in two ninth grade mathematics classes - general math and algebra. It was hoped that the descriptions and comparison would demonstrate how mathematics lessons are jointly produced by the teacher and students through their interaction about the mathematics content.

Terms

1. Collaboration

Collaboration is working jointly with others especially in an intellectual endeavor. School mathematics lessons as collaboration refers to the joint production of lessons by teacher and students. The teacher formulates a logical presentation of the mathematical content and decides on a particular way to present it which regulates student participation in the lesson. The students respond to the teacher and the content through the mode of participation selected. Through the back and forth response (interaction) of teacher and student the experience of the mathematics class is jointly produced.

2. Logical Organization of Mathematics for Classroom Presentation

School mathematics, what is being taught in mathematics classes in our schools, is considered to be a well-defined, logical structure which is based on simple, elementary facts and, through step-by-step progression, builds into more complex and abstract dimensions. A basic assumption in the teaching of mathematics is that the ideal "plan" of presentation is a logically sequential step-by-step procedure, complete with examples, and followed by a set of problems to give the students some practice. Such a plan, it is believed, will result in the successful learning of mathematics.

There are two levels of logical organization mentioned in the above description, that which is inherent in the mathematics itself and that which exists in the classroom presentation.

The logical organization inherent in the mathematics itself is based in the structure of the mathematical systems. These systems are composed of undefined terms, defined terms, axioms which are statements accepted without proof, and theorems which are proven statements of relationships within the system. It is required that a system be consistent, which means there are no contradictions in the relationships established through the definitions, axioms, and theorems. Through the relationships established in the definitions and axioms, conditional relationships can be formulated and proven as theorems. The logic of the system resides in its relationships and consistency more than in a sequential mode of construction or presentation. The individual teacher formulates a logical method of presentation, based on a sequential building up of a structure, in an attempt to organize and efficiently convey the existing relationships.

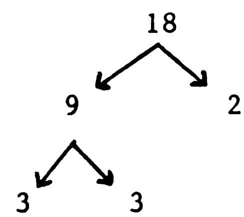
For example, let us consider the concept of a prime number. A branch of mathematics called number theory resulted from the study of the properties of the integers, which includes the positive and negative counting numbers, and zero. Number theory is concerned with classes of objects or numbers rather than single numbers, and the patterns and relationships that exist in and among classes. A class is determined by some common property of its members, such as the class of all even integers or the class of all integers divisible by three.

Looking at the components of elements is a basic objective of mathematics. In working with integers, the factors or numbers which multiply to obtain a given number are usually the most useful components. Some integers can be resolved into factors which are smaller than the number itself and greater than one. Those integers which cannot be broken down in this way are in the class of prime numbers. That is, a prime is an integer greater than one, which has no factors other than itself and one. The importance of the prime numbers rests in the fact that every integer not a prime can be expressed as a product of primes. It can be proven that this product of primes for a given number is unique. Through the use of prime numbers it is possible to work with such concepts as the greatest common divisor, the least common multiple and the lowest common denominator as well as the abstract structures which developed in the field of number theory.

As an example of a logical organization for the presentation of the concept of prime number, consider the following:

1) Definition: A prime number is an integer greater than one, having no positive integral factor other than itself and one.
2) Examples: The first prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, . . .
3) Every number can be factored into prime numbers.
4) To factor 18 into prime factors, do the following.
    a) Find two numbers that multiply to get eighteen. e.g., $18 = 9 \times 2$
    b) If some of the factors are not prime, factor them. 2 is a prime. $9 = 3 \times 3$
    c) Keep factoring until all the factors are prime.

d) The prime factors of 18 are 2 and 3, with 3 occurring twice.



3. Social Participation Structures

Social participation structures are the arrangements selected by the teacher for the presentation of content which determine how the students may acceptably participate or interact during the lesson. For example, one arrangement is to send a group of students to the board and work with that group as a way of working with the class. For this social participation structure the students have the freedom to interact with each other, and talking out or giving help to those at the board is usually acceptable. Another arrangement is teaching the whole class from the front of the room. For this social participation structure student to student interaction is not acceptable and students are usually expected to raise their hands and be called on before answering a question. The social participation structure focused on the whole class is more limiting to student interaction than the structure focused on the group at the board.

Consider the following classroom interaction. This was a ninth grade general math class. The students were doing problems involving finding the prime factorization of numbers. This example was based on an actual classroom observation.

Prior to the following excerpt, the teacher, with the class, had factored the numbers 15 and 24 into their respective prime factorizations.

$$15 = 3.5 \qquad 24 = 3\cdot2\cdot2\cdot2 = 3\cdot2^3$$

The teacher did this exercise by asking for ways to factor the number. When all the ways with two numbers as factors were given, she asked for other ways of factoring with more than two numbers. Eventually the prime factorization, the factorization with all prime factors, was given.

At the point in the lesson where the excerpt begins, the class was looking for the prime factorization of the number 36. The ways of factoring 36 using two numbers were written across the front broad: 4·9, 6·6, 36·1, 18·2, 12·3. The teacher pointed to the factorization 4 times 9.

(1) T: "Question: Is this a prime factorization of 36?"
(2) A group of students answered: "No."
(3) The teacher pointed to 6 times 6.
(4) T: "Is this?"
(5) A group of students answered: "Yes."
(6) T: "What is the prime number?"
(7) S: "Is one a prime number?"
(8) T: "I think one is a prime. It follows the rule of a prime number.
(9) Yes, I think it is a prime. Two is the only even prime number."
(10) Three students are sent to the board. Each one of them continues
(11) to factor to primes one of the sets of two factors for 36
(12) that are written on the board.

There were two social participation structures used in this excerpt.

1. lines 1-9 The teacher was speaking to the whole class using the front board.
2. lines 10-12 The teacher sent three students to the board, using their problems as examples for the class.

In the first one the social interaction consisted of the teacher asking a question to the whole class and accepting whatever answer was given by however many students answered. She did not call on particular students. In the second one the social interaction was open to student comments and help given by the teacher. The teacher later asked the class if they agreed with what was being done and the interaction depended on the work done at the board.

A pattern of interaction in the first participation structure was as follows:

(lines 1-2) question/answer/accepted by the teacher,

(lines 4-5) question/answer/not accepted by the teacher,

(line 6) the teacher changed her line of questioning,

(lines 6-7) question/question,

(lines 8-9) the teacher answered the student's question.

The response by one student in the form of a question (line 7) broke the pattern that had been established. The roles changed here, since prior to this the teacher had been asking the questions. This was an especially interesting exchange because the teacher answered the question incorrectly. One is not a prime number.

At this point (line 10) the teacher changed the social participation structure of the class by sending three students to the board. This put the attention back on the problem of prime factorization of numbers. In lines 10-12 the teacher accomplished the objective of this part of the lesson, determining a prime factorization of 36, by sending three students to the board. This was also evidence that these three students knew how to find the prime factorization of 36.

In this section of classroom social interaction the teacher had established the participation structure of speaking to the whole class. The question/answer exchange of teacher and students was established for two rounds. On the third set of the question/answer pattern, one student responded by asking a question. However, the teacher asked a different question than the previous two questions in the pattern which may have initiated the different response on the part of the student.

As the teacher answered the student's question not only the pattern of response was interrupted but the content of the lesson was changed. Instead of sorting out what a prime factorization was, the teacher told the students that one is a prime number. She then changed the class social participation structure. Through this change she was able to redirect the class to the instructional question of what is a prime factorization of 36.

The logical presentation of the subject matter was interrupted by the question of a student breaking the pattern of interaction. The teacher responded to the variation but in doing so inserted a comment about prime numbers that was incorrect. The social organization of the classroom was changed from question/answer to the whole class, to sending some students to the board which changed the interaction pattern and the logical presentation. The social participation structures regulated the interaction which in turn influenced the mathematics content.

Within a mathematics lesson the teacher may use more than one social participation structure as shown in the above excerpt. The sequencing of these participation structures is usually determined by the teacher but the students' responses may influence the teacher's choices. The social participation structure provides a framework for classroom interaction and the enactment (the process of joint production during the class session) of the mathematics lesson.

Research Questions

Ninth grade general math classes and algebra classes are generally understood to be two very different and contrasting school mathematics experiences. The usual descriptions of what makes these classes differ include mathematics content, classroom organization, and student behavior. This study compared one general math class and one algebra class from the focus of the processes which make the classes different. These processes which make a class come to be what it is are based in the interaction between the teacher and students during the actual mathematics class experience. With this focus the major issues of the study were the presentation of the mathematics content, the social participation structures that were used, and the enactment of the lesson between teacher and student as recorded in classroom interaction about the mathematics content.

The overarching question of this study was: What makes it harder to teach general math than algebra?

The guiding questions of the study were:

1. How did the presentation of school mathematics content differ between general math and algebra?

2. How did the social participation structures differ between general math and algebra?

3. How did the patterns of enactment between teacher and students differ between general math and algebra?

The following sections give a brief sketch of the researcher, the teacher, the classroom, the mathematics program, and the two classes observed.

Research Setting

Background

The proposed study originated from research conducted through the General Math Project under the direction of Dr. Perry Lanier within the Institute for Research on Teaching. The program of research for the General Math Project was designed to investigate ninth grade general mathematics as the terminal mathematics class for approximately half of the ninth grade American population. The General Math Project has studied the factors influencing the decisions which determine the students who take general math, the actual classroom experience of general math for both teachers and students, the differences between the general math experience and the algebra experience, and the improvement of practice for teaching general mathematics. The research from the General Math Project has generated questions of inquiry about the logical organization of the classroom presentation of the subject matter of school mathematics and the social organization of the classroom. Observations as recorded gave evidence that the content being presented in the classroom affected the social organization of the class and the social organization influenced the content that was presented. The mutual influence of classroom presentation of school mathematics and the classroom social organization prompted the investigation of the enactment of school mathematics.

The Researcher

Educationally this researcher's background includes a B.A. in mathematics with a minor in philosophy, an M.A. in mathematics, and an M.A.T. This dissertation is part of the requirement for a Ph.D. in Mathematics Education in the College of Education, Department of Elementary and Special Education. This researcher's education, including her undergraduate work, was in Catholic schools. She became a Catholic Sister at the beginning of her undergraduate work and had five years teaching experience in Catholic elementary schools before she entered graduate studies.

Since her undergraduate degree was in mathematics the emphasis of her teaching career was mathematics, specifically in grades 4-8. Her concern about the present elementary school mathematics curriculum and how it was being taught impelled her into higher studies. The state of mathematical illiteracy today and the irrelevance of a major part of the school mathematics curriculum in elementary schools were two specific motivations for pursuing a degree in research and mathematics education.

This researcher's experience in Catholic schools both as student and teacher gave her a different perspective of education than she found in public schools. Her own bias of expectations in terms of discipline and classroom behavior demanded an immediate practice of the principle of being nonjudgmental as a participant observer. Her elementary school experience was both an advantage and a disadvantage in observing in ninth grade mathematics classes. The advantage was first hand knowledge of the mathematics preparation of these students. The disadvantage was lack of experience working with this age student and the problems they face besides those of an academic nature.

This dissertation is a version from this researcher's perspective of the teaching of mathematics in two ninth grade mathematics classes. As a mathematics educator, she was particularly interested in what was being taught, how it was being taught, and what she could observe was taking place with the content between the teacher and students during the class sessions. Her focus on the teaching of the school mathematics and the interaction among the participants about the content created her interest and concern about the enactment of mathematics. Observing the process by which the school mathematics is worked out between teacher and students had provided insights into the demands of mathematics education.

The Teacher

Susan Hanley was about 40 years old and had been in teaching for 19 years. She has her undergraduate and Master's Degree in Mathematics Education. She has taught mathematics in all grade levels, 9 through 12, and had maintained a position as mathematics teacher for twelve years at Shelton High School. Susan was married and had two children, a son five years old and a daughter eight years old. Her husband had held a position for 12 years as a researcher in plant pathology at a nearby university.

Susan became interested in educational research while working on her Master's Degree in Mathematics Education. Her experience as a teacher, her desire to further her own education (having taken courses at the university beyond her Master's work), and her concern about her professional development proved to be points of motivation for her participation in the research project. Apart from her own teaching schedule, Susan was elected as the high school representative to the Education Association of the area. This required two hours a week. She also participated in a program titled "Equal Opportunities Classroom" (EOC). This program worked with teachers with respect to underprivileged or handicapped students in the classroom. The program involved six three-hour sessions and required six half-day observations in other teachers' classrooms. Techniques for drawing forth the disadvantaged student were taught, implemented, and evaluated.

Susan did not participate directly in any outside school-related activities involving the students. She offered time after school for anyone who wanted extra help which amounted to a couple of hours a week. She also provided what she called a neighborhood tutorial service for the high school students in her neighborhood who wanted extra help in mathematics.

As the school year began, Susan's teaching load included an applications II class for seniors, three algebra I classes, and one geometry class. In addition, one class period per day was designated for planning. When the General Math Project inquired about the possibility of observing a teacher of both applications I and algebra I, Susan was asked to consider the experience. She agreed to participate which necessitated changing her teaching assignment. Rather than switch a class to her two weeks into the year, students were pulled from the existing applications I classes according to possible schedule changes. This created a new class which Susan then taught instead of the applications II class. Her schedule before and after changes appeared as follows.

Hour	Weeks 1 & 2 of school year	Week 3 of school year
1	Geometry	Geometry
2	Algebra I	Algebra I
3	Applications II	Applications I
4	Planning	Planning
5	Algebra I	Algebra I
6	Algebra I	Algebra I

Two researchers from the General Math Project observed Susan's second period algebra I class and third period applications I class during the 1979-80 school year. Applications I is Shelton High School's designation for general math. This class is a step above the foundations or computation class and the 1979-80 school year was the first year a pre-algebra book was used with this level student. Unlike the usual repeated topics curriculum for general math, these same ability students were introduced to a somewhat sequenced and future oriented curriculum in terms of providing a basis for going on to more mathematics.

Susan spoke of mathematics in terms of systems, rules, and patterns. She distinguished between mathematics and teaching mathematics. In the teaching of mathematics she distinguished between learning mathematics (the struggle of understanding) and the mechanical working of problems once the understanding is reached. For Susan learning mathematics was the struggle to understanding. The mechanical was the arithmetic or computation.

When asked, what is the relationship between math and the world around you?, Susan responded: Math is a world of patterns. The world is the puzzle board. In an interview on her conceptions of mathematics Susan expanded on this answer in the following way:

> Math is a world of patterns. The world is a puzzle board. No, math is the abstract patterns and then when you find a piece of the world that will fit the pattern you found a use for math, you know. And until you find a piece of the world that fits math, fits the pattern, that math is useless. But the fact that it is there is good because later on someone else might find such a pattern. (Weisbeck, Interview, 10/8/80, p. 25)

The concept of patterns was a key one for Susan. She used it as a basis for finding examples of what mathematics is like. When asked what other school subject mathematics most resembled she chose history. She spoke of history as studying one segment (the colonial period) at a time and then fitting the segments together. By understanding each segment separately, relationships can be discovered as the periods are then fitted together. In the same way Susan spoke about mathematics.

> Because you establish patterns and procedures over a specific topic and then you fit them all together. . . You learn certain things in algebra I, you learn certain things in geometry and then there is a time when you bring these two together. All right, and throughout your study of math you know you are learning one little thing. You are learning matrix algebra say, ok, but then you have got to fit that in so that it ties in with other things and it becomes a part of the whole. (Weisbeck, Interview, 10/8/80, pp. 5-6)

In Susan's own words, "Math is just a glorified set of rules. The difference is it fits in lots of situations." (Weisbeck, Field Notes, 4/21/80, p. 10.) To Susan everything in math was determined by rules and axioms. It was basic to learn the rules and necessary to go beyond the rules to understanding. The rules were explicit and you couldn't assume anything that was not stated. Susan felt this allows for the objectivity that exists in mathematics.

Susan made a clear distinction between mathematics and working problems. She talked about mathematics as exploring, as getting things and understanding. There was usually a time of confusion and anxiety when things didn't fall together but this was the time when mathematics was being done. Once the understanding came, then working the problems was mechanical. It was just plugging in numbers and doing what you know you had to do to get the solution.

Susan, at this point in time, was sure she could not be a mathematician and do math all day but she did enjoy teaching math.

> So, I don't think that I have the natural curiosity, the quest for understanding math it would take to make a true mathematician. I enjoy math, but without the interplay between students, I don't think I could sit in an office and do math all day and have a good time with it. Whereas I can certainly teach it all day and have a good time with it. I find that regardless of how stressful my day has been that if I can come in class and shut the door and we can really get going on something and I am saying things that make them understand and they are responding, at the end of the period I am perfectly relaxed. Sometimes I am exhausted, but I am perfectly relaxed and that is really where I am at right now.
> (Weisbeck, Interview, 10/8/80, p. 3)

The Classroom

The classroom was toward the rear of the school, not far from the main office or counseling services. The room contained desks set in rows, and two tables. The desks were arranged in five rows, seven desks in a row except for the row closest to the door which had three desks and a table. There was room for about thirty-six students. The second table was set against the wall in the back of the room with the shortest side against the wall providing the most room possible for access by the students. This table was the place for assignment sheets, extra worksheets, or other duplicated material made available for the students. There was a set of baskets for collecting tests during a test period and also for returning tests, especially for those who were absent.

The teacher's desk was at the front of the room to the right as one entered the room. In this same corner was a file cabinet and a two-tiered cart. The front wall was a blackboard with a small bulletin board on both ends. The teacher usually had these decorated with mathematical materials. The back wall also had a full-length blackboard but with only one bulletin board to the right as one faced the wall. The table was to the left.

As one faced the back wall, the wall to the right had windows the full length of it and the wall to the left contained the door and a set of cupboards from the door to the back wall. The cupboards included a full length closet where the teacher kept her personal things, a ledge for books and a set of doors under the ledge for storage, and more closet space on the other side of the ledge. The pencil sharpener was on the wall in the corner by the closet and the second table. The observations were usually made from beside this table near the pencil sharpener, on the same side of the room as the teacher's desk.

The front blackboard was the site of most of the large-group instruction and student "boardwork". The blackboard in the back of the room was used only occasionally for student work. Otherwise this blackboard was not used in the two classes observed.

<u>The School Mathematics Program</u>

Shelton High School had grades nine through twelve. There was a three-year mathematics/science requirement for graduation which specified one year must be mathematics, one year must be science, and the third was optional. The third year may be all mathematics, all science or a combination since half-year mathematics and science courses were offered. At the ninth-grade level there were five entry points: foundations, applications I, algebra I, geometry, and advanced geometry. For the foundations class (the lowest ability class) the students were given a placement test which determined their computation skills. This class was strictly computation and was conducted mostly through the use of worksheets. In computation was included addition, subtraction, multiplication, division of whole numbers and fractions, including decimals and percents. In applications I a pre-algebra text was being used for the curriculum. Pre-algebra does not presume the computation skills required for algebra. The work was less abstract and examples of problems were less complex. It was understood that if you work in pre-algebra you will pass, unlike in algebra where you must not only work but you must also show progress in grasping the concepts as they become more complex.

With the three-year requirement, there were various paths the ninth grader could take beyond the first mathematics course. Outside of taking two years of science, those students who took foundations or applications I go to algebra I or skip to applications II, a ½ year course, offered in their senior year. Algebra I students go on to geometry or skip to applications II. Geometry students go to algebra II and then usually to ½ year of advanced algebra II (trigonometry) and a ½ year of pre-calculus. Advanced geometry students go to advanced algebra II and then either to algebra III and pre-calculus or to analysis and advanced pre-calculus. The school also offered half-year courses in BASIC (for computers) with an algebra I prerequisite, FORTRAN (computers) with BASIC and geometry prerequistites, COBOL (computers) with BASIC prerequisite and probability and statistics with an algebra II prerequisite.

The General Math Class

There were twenty-five students observed at least once in the general math class. Eight of these students were present all year. Of the twenty-five, twelve were males and thirteen were females.

At Shelton High School, those students not tracked into algebra have a choice of classes and sequences of classes through which to satisfy their mathematics credits. Foundations was a course strictly for teaching computation. Applications I was a class which takes specific topics such as fractions or decimals and teaches concepts in units. Both of these classes would be classified under general math, terminal mathematics classes for those not necessarily going into algebra, geometry, and more advanced mathematics courses.

Researchers from the General Math Project observed Susan Hanley teach an applications I class as the general math class to compare with her algebra class. For the school year, 1979-80, it was decided that a pre-algebra book would be used as the text for the class. The use of a pre-algebra text was a first in the mathematics curriculum for a general math class and an experiment for this level student. This approach provided an integrated (concepts sequenced from simple to complex) and future-oriented curriculum with a basis for going on in mathematics which was not necessarily the case in the previous approach to general math. The pre-algebra content was less abstract than the algebra content and the examples of problems were less complex. Computation skills were not presumed for pre-algebra as they were for algebra. Susan explained that it was understood that if you work in pre-algebra, you will pass, which was not the case in algebra where you must show progress as well as work.

The book chosen for this class was the Dolciani Pre-Algebra book. Susan followed the book rather closely as far as content and organization were concerned, with a few minor changes. She did not feel as bound to following this book chapter-by-chapter as she did the algebra book. She often provided worksheets for the students and included aspects of content not in the book. Since these students were weak in fractions and decimals, more time and practice were given to those areas.

Susan began with Chapter 1 and worked through the pre-algebra textbook up to and including Chapter 13. Concepts from Chapters 14, 15 and 16 were inserted into the content flow at various times for further enrichment and understanding. Chapter 9 was taken out of sequence in order to emphasize and enlarge a section involving decimals. Each chapter had from 5 to 14 sections.

Susan Hanley took from 8 to 20 class sessions to cover a chapter of the text. She gave a quiz or test about every week to check the progress of the students. Assignments were specified for each section taken from the book or a worksheet was prepared to give the needed practice. Susan collected homework from the general math students and gave points for the work completed. The use of worksheets insured the student knowing the assignment and Susan would sometimes collect the worksheets at the end of a class period to see how much the student actually completed. Homework was collected at the beginning of the period if it was taken home to be finished. Homework papers were collected and returned by way of a basket on the table in the back of the room. The points for homework were considered in the grade given the student.

On all but test days, the major part of the 50 minutes of class was on the average evenly divided between instruction of the whole class by Susan and seat activity. The instruction itself was usually given by Susan from the front of the room, using the blackboard, and through question/answer interaction with the students. The seat activity period usually consisted of students doing a specific assignment or worksheet to be handed in for grading.

Review of material previously learned was a part of Susan's teaching style. About half of the classes observed contained some time given to review. This may involve going over the homework, doing some problems either together or individually and checking the answers, or going over a test or worksheet. Occasionally, an entire class may be used for review, specifically before a test was to be given.

A typical day for the general math class consisted of essentially four phases: <u>acclimatization</u>, <u>pre-instruction</u>, <u>instruction</u>, and <u>seat activity</u>. Susan sometimes took a few minutes at the end of seat activity before dismissal to talk to the students, explain or clarify an assignment, or collect homework.

Each class period was 50 minutes long, and the general math class began at 9:50 a.m. This was the third period of the day and the class directly following the algebra class taught by Susan which was also observed for the project.

Phase one, acclimatization, was designated as the time when class officially began (9:50) until Susan called the class to order. This phase averaged about four minutes. During this time, Susan took attendance, asked for homework, passed out corrected papers, or settled some business with one of the students. Usually during this time, Susan assessed the grouping arrangement chosen by students and moved at least one student to another location in the room. At the end of January, a senior girl became a teacher aide for the general math class. She corrected papers, kept track of homework, helped answer questions during the seat activity period, and did other jobs for Susan. During this acclimatization phase, Susan often gave the aide her tasks for the class period.

Phase two, pre-instruction, was a section for review or questions about previous work. This section occurred in about half of the observed class sessions and was about 12 minutes long on the average. A review might be that of a homework assignment, either going over the problems worked or doing similar problems at the board, or material for a test. The review could be conducted by the teacher at the board, or by the students individually at their desks, or by the students coming to the board to do their problems.

Phase three, instruction, was the section of the class where new content was presented. Instruction of new material occurred almost every day, excluding test days and occasional days when extra time was given for individual work. The average length of this section was 24 minutes. The instruction itself took several forms. The teacher at the board would present the content, using question/answer dialogue with the students. Practice might then be given by doing the sets of oral problems provided in the book. If this was the case, Susan called on students in a rapid-fire manner and if there was no solution given or if the solution given was incorrect, she called on someone else. Students may be sent to the board during this phase, either in groups of three or four or one at a time to solve a problem being worked by the entire class. Problems may be written on the board, the students work the problems individually and then the class discusses the solution together. The major part of instruction of new material, however, was done by Susan at the front board with interaction with the students.

Phase four, seat activity, was the time set aside during which the students worked at their desks individually, or occasionally together, on problems either from the book or on a worksheet. Susan was always available to answer questions and give additional help. When the aide became part of the class, she also helped during this time. This phase occurred in about 75% of the classes observed and was about 22 minutes long on the average. Phase three and phase four were given almost equal time in the organization of this class. The majority of these sections observed were designated for the students to work on assigned problems from the book. Worksheets were sometimes given as extensions of the content in the book. At times, a review sheet of problems and formulas, was used as an opportunity for the students to go back over previous material, usually before a test. At times, problems were written on the board and worked on individually by the students.

Susan occasionally would take a few minutes at the end of the period to remind the students of an assignment, emphasize a particular concept, or request that the room be put in order. This transition was usually about two minutes long and did not occur in every class.

Since this period was followed by the lunch break, there was time for interaction between teacher and students and teacher and researcher. Susan used detention after class as a disciplinary measure and four instances of this action were recorded in the observations.

Interaction between Susan and the students included getting help with a problem, asking about grades, sharing a family situation, and being reminded to finish the homework. No instance was recorded during this time, or any other time, of a general math student setting up an appointment to meet with Susan after school for extra help.

It was in this interim time that Susan talked with the researcher on an informal basis. During these interactions, she shared various concerns about teaching, about the students, and particularly about the general math class.

The grading system at Shelton High School was structured around four grading periods each consisting of nine weeks. At the end of the first nine weeks the students received a progress report with grades but the grades were not recorded on a permanent file. These grades served as indicators of where the student was at this time. The grade for the first 18 weeks, however, was a final grade and was recorded on the student's permanent record. The student received a ½ credit for work completed at the end of these 18 weeks. The same pattern existed for the second half of the year -- nine weeks was a progress report and the grade not permanently recorded and a final grade at the end of the second 18 weeks period. The grade scale Susan used was as follows:

90 - 100	A
80 - 89	B
70 - 79	C
60 - 69	D
below 60	E

For the final grade at the end of 18 weeks, the nine weeks grade (test averages) counted 40% each and the final exam counted 20%. In the general math class homework was collected and points were given for completed assignments. These points were averaged into the test scores to determine the final grade.

Of the 25 students observed in Susan's applications I class, eight went on to algebra I of which seven remained until the end of the 1980-81 school year. Two students took pre-algebra again but one moved and the other dropped before the year ended. One went into applications II but dropped it. Six students did not take any mathematics during the 1980-81 school year. The remaining eight students had moved and were no longer at Shelton High School. From these figures at least 28% of the class went on to the next class in mathematics and remained the full year.

The Algebra Class

There were thirty-two students observed at least once in the algebra I class. Twenty-two of these students were present all year. Of the thirty-two, fifteen were males and seventeen were females.

The class used the Book One Modern Algebra Structure and Method by Dolciani and Wooton as their textbook.

When asked about what content was taught in the algebra class, Susan replied that she followed the book, which ever one was chosen. She commented that it was necessary to follow the algebra book in order to prepare the students properly for their next mathematics courses. Consequently, Susan began with Chapter 1 and worked through the algebra textbook up to and including Chapter 12, excluding Chapters 13 and 14. Each chapter had from eight to fourteen sections which provided a means for structuring class sessions and homework.

Susan Hanley took from 12 to 16 class sessions to cover each chapter of the text. She scheduled a quiz or test every four to six class sessions in order to check the progress of the students. Although assignments were stipulated for each section of each chapter, Susan did not collect or grade the homework. She allowed time in class for questions over the assigned problems and stated that through this interaction she could tell who was working and who was not. She felt that the frequency of testing would prevent any student from getting too far behind. Some kind of review activity occurred in half of the classes observed. This may have consisted of students asking questions, certain problems being done at the board, or a group of students going to the board to practice problems on material already taught. Occasionally (particularly with the sections on story problems) an entire class may be devoted to review and practice. However, the usual pattern was to have a few minutes of review before the new material was presented.

The tests were corrected and returned as soon as possible. Time was usually given to go over the most commonly made mistakes and answer any questions the students might have on grading or particular problems.

On all but test days, the major part of the 50 minute class was used for whole group instruction and interaction about the lessons. The instruction time itself had several structures but presentation of new material was usually done by Susan Hanley from the front of the room, using the blackboard and asking questions to engage student participation.

On a typical day the algebra class consisted of essentially four phases: acclimatization, pre-instruction, instruction, and seat activity. The phases usually occurred in the order reported above.

Each class period was 50 minutes long and algebra class began at 8:50 a.m. There was a 10 minute break between class periods and algebra was during the second period of the day. Phase one, acclimatization, began as students entered the room and took their respective places for class. Susan had a seating chart for algebra and required the students to remain in their appointed places. There was usually quiet conversation before class began. This time before instruction, calculated from the time the period began at 8:50, averaged about six minutes. During this time Susan took attendance. She stood in front of the room, silently checking off the absent students in her attendance book. Occasionally, she would ask the class about a particular student. An attendance slip was written and placed outside the classroom to be collected. At times Susan would forget to make out the slip and the girl who collected the slips would come into the classroom to get it. On these occasions Susan stopped what she was doing and made out the slip.

Students were allowed to keep talking during attendance taking. This period might also include passing out test papers, which Susan did herself, or passing out assignment papers with the assignments listed for the next chapter. Sometimes Susan would take time to review what the next few days were going to involve and she might write the agenda on the front board. This information did not include assignments but only the sections and the days.

Phase two, pre-instruction, included review of assigned work, going over test questions, a review of a unit of material. This was accomplished through students asking questions, Susan demonstrating problems, or students helping each other with the work. Pre-instruction occurred in about half of the observed class sessions and was about 15 minutes long on the average.

Phase three, instruction, consisted of presentation of new material. Susan usually conducted this section at the blackboard in front of the room. However, there were times when she sent students to the board as part of the instruction and practice aspect of learning the new material. Susan presented new material in such a way that she elicited student participation and interaction. Her lessons were planned to include a method, often a specific set of steps, for doing the problems, and several examples. Examples were a significant part of her teaching strategy. It was through the problems that she often engaged the students. By asking the students for step by step progression through the examples she was able to call forth the already learned material and present the new material at the same time.

Instruction of new material occurred almost every day, excluding test days and days when more time was given for individual work. The average length of this section was 30 minutes.

Phase four, seat activity was the time set aside for the students to work individually on the assigned problems. This was included as part of the class period in about half of the classes observed and averaged about 14 minutes of the 50 minute period. The students were allowed to work on their own and Susan would walk among the students answering questions and checking their work. She expected the students to take full responsibility for doing their work and also for getting help if they needed it.

Since the school had no bell system for beginning and ending the class sessions, the students kept a close watch on the time. At 9:40 they were mostly packed up and ready to go. One of the most effective penalties was to be kept after class. This happened on occasion to the whole class but usually to small groups. But the time kept after was not observed to be longer than 5 minutes.

The informal communication which occurred after class revolved around grades and getting help outside of class. Concern for achievement was reflected by both teacher and students.

For the algebra class, the nine weeks grade was an average of all the test scores from that period, and only the test scores. Any assignments included were those considered as take home quizzes (referred to as worksheets) and graded on the point system. Each test was worth a certain number of points and the average was the number of gained points over the total. The grading scale and system have been reported in the previous section on the general math class. However, for the algebra class, Susan did provide an extra aspect to her grading scheme. She told the students that if at the end of the first nine weeks the grade received was low and by the end of the second nine weeks it had been raised significantly and held on the final exam, she would ignore the first nine weeks' grade. Susan reported that a student did occasionally benefit from this special offer.

Susan had a three stage policy for homework. She sent this to the parents of the algebra students in letter form. There were assignment sheets and assigned problems for each chapter covered in the book. The students were expected to do the problems in a notebook or on papers which were to be kept in an ordered fashion to be checked if asked. They were to star the problems they had trouble with and get help with those problems. The papers were not collected or graded by Susan. At some point during the work period she might ask to see the homework and then she would go around and look at the notebooks. If there was a question about a test score, Susan might ask the student about doing the homework and request seeing the notebook. This often produced an explanation for the low grade.

If a worksheet was given to the students, Susan regarded this as a take-home quiz and would collect it and grade it with a point system. This score was averaged along with the test scores to get the nine-week grade.

Susan formulated this system of homework after experiencing the incredible amount of time checking papers demanded and the fact she felt little value was accrued by having the students turn it in. She said so many of them could just as well copy or rewrite the problems and not gain the practice that motivated giving assignments. This way they were on their own and the responsibility was theirs. The check system rested in the number of tests she gave, forcing the student to keep up with the work.

During the course of the year five algebra I students dropped into general math. Two of these five switched into the general math class being observed by the researchers. One of these two then dropped out of mathematics altogether.

Of the thirty-two students observed in Susan's algebra I class, 19 went on to geometry and 17 remained the full year. One student went into advanced geometry, one into algebra I, one into pre-algebra and all three stayed the full year. Four students did not take a mathematics class during the 1980-81 school year, five moved away, and one student graduated. From these figures 56% of the algebra I class went to a higher mathematics class and remained the full year.

Procedures

Mathematics lessons are a collaborative effort between the teacher and the students. This collaboration is about the mathematics content and takes place within a social participation structure which is selected by the teacher. Students respond to the content through the social participation structure with varying degrees of cooperation. The sequential moments of interaction between teacher and students constitute the enactment of the mathematics lesson. The purpose of this study was to describe and compare the collaborative efforts exhibited in the general math and algebra classes observed. This required examination of the presentation of the mathematics content, the social participation structures used, and the enactment that took place during the class period. These issues were the basis for comparing the two classes. The following were the procedures used to accomplish the purpose of the study.

Data Collection

Classroom observation was used in the study in order to gather the data source in the actual classroom settings of the enactment of the mathematics. The same observer watched both classes and gathered all the data analyzed for the study. Observations began October 9, 1979 and ended June 3, 1980. There were 31 observations in each class, about one every week. A set of field notes was recorded for each observation (62 sets in total) which included as accurate an account of the events of that day's class as possible and a set of inferences about the events as the observer witnessed them. Any informal interaction with the teacher was also recorded. Interaction among students or with teacher and students before or after class was recorded as well. Emphasis in both classes was placed on the teacher teaching the mathematics content.

Overview of Data Analysis

The first major decision in the study after the data source was collected was how to use the field notes in an effective way to study the collaborative effort between teacher and students. Since a key element in collaboration is the interaction between teacher and students, particularly about the mathematics content, attention was directed to those sections of the field notes. However, one limitation of field notes is the lack of recorded accurate minute by minute dialogue in the research site. Because of the nature of field observations and the skills of the researcher there are necessarily gaps in the flow of events. Given this limitation it was decided to focus on the mathematics of the lessons and consider connected sequences of interaction about specific content wherever this existed in the field notes. These connected sequences were called episodes. For the purpose of analysis episodes were identified, a series of classifications were conducted, a coding system was determined, and a process for outlining each episode was devised. The following overview provides a framework for these procedures. Each procedure is explained in detail on pages 72-105.

An episode, then, was an excerpt from the field notes which contained specific mathematical content and interaction between teacher and students about this content. The content was specified to one math problem, one math concept, or a set of related problems. The interaction revolved around the content in a sequence of responses between teacher and students which portrayed the progress of the class in accomplishing the task of the lesson. This progress of the lesson as recorded in the interaction was the basis for studying the collaboration between teacher and students as they worked jointly to produce the mathematics experience.

The episodes began with the teacher's presentation of the content and were considered concluded when the teacher changed the subject matter either to a new problem or concept or to another aspect of the class such as a study period. This provided a connected sequence of interaction about specific mathematics content. All 62 sets of field notes were examined for episodes. The result was 125 algebra episodes and 100 general math episodes. These episodes then became the units of analysis for the study, creating a sample of instances of collaboration from the two classes, general math and algebra.

Once the episodes were identified, a series of classifications were conducted. A classification consisted of sorting the episodes according to a specific aspect, such as content, and grouping those episodes which were similar. For example, for a classification about content all episodes on fractions were grouped together. The general math episodes were sorted separately from the algebra episodes. The episodes were classified according to mathematics content, how the teacher organized the class for her presentation, and the conclusion of the episode. After the sorting process each group was identified by a statement or label and a frequency count of the episodes in each group was recorded. The labels and frequency counts served as a means of comparing the two classes.

Analysis of the interaction in the episodes resulted in a coding system. In order to have a means for identifying the enactment of the lesson between teacher and students, the basic unit to be coded was termed a communication act. A communication act could be a verbal communication (statement or question) by one person, a recorded gesture in response to a question (such as nod of the head) or work done at the board. The episodes were read several times in order to get basic categories for the communication which took place between teacher and students. From these categories the specific items to be coded were differentiated.

The episodes were connected sequences of interaction and the focus of the study was the collaborative effort which existed between teacher and students in producing the mathematics experience. The sequence of interaction in solving the problem or problems, or learning a concept, provided a record of progress in the lesson toward accomplishing the goal of the episode. This progress from the content presentation to the conclusion of the episode was regarded as the forward movement of the lesson. Collaboration between teacher and students either moved the lesson forward to completion or involved instances of inhibiting progress and completion. Each coding category was evaluated as to whether it contributed to the forward movement of the lesson or whether it inhibited the forward movement. Those categories which contributed to the forward movement were signed with a plus and those which were inhibitive were signed with a minus.

The communication acts for each episode were coded and the list of coded items formed a script for the interaction of the episode. By assigning the coded items with their respective plus and minus signs, patterns began to emerge among the scripts for each class. The communication acts were studied as categories and within the patterns. By counting the coded items across the episodes for each class, frequencies were tabulated. Percentages were calculated according to the occurrence of the plus and minus signs.

The mathematics content of each episode was studied to determine the logical organization the teacher used in presenting the subject matter. This underlying structure was written down in the form of statements which served as the guide the teacher used in leading the students through the content. These statements were written in the sequence in which they occurred in the episode and served as steps to the completion of the task set in each episode. These steps were another way of classifying and comparing the episodes for the two classes.

Once the steps were identified, the script of interaction was merged within the steps according to the enactment of the episode. This formed an outline of the episode. By examining the outline it was possible to determine from the pattern of plus and minus within the steps if the lesson moved forward to completion without interruption or if there was interference of some kind within the movement of the lesson. The coded items identified the cause of the interference. The collaboration between teacher and students was exemplified by the pattern of plus and minus signs in the outline. The outlines were studied for the most common ones representative of each class and then used as a basis for comparing the general math and algebra classes.

In order to portray more clearly the process of analysis used in this study an excerpt from a set of field notes has been selected which will be used to demonstrate how an episode was determined. The episode from this demonstration will then be used as a model unit of analysis for the process of data analysis.

The names used in the excerpt and throughout the quoted field notes are not the real names of the participants. The communication acts of the quoted field notes have been numbered as a means of identification when referring to the excerpts. Some communication acts are longer than one line and at times two acts are on one line. The numbering is done according to the placement of the communication acts.

Discussion of an Excerpt from Field Notes

The following is an excerpt from one set of field notes (one day's observation in one of the two mathematics classes). This excerpt will provide the basis for explaining the method of analysis used in this study.

The excerpt being presented was taken from the November 6, 1979 set of field notes for the algebra class. The teacher was introducing the mathematical concept known as a function. A function was defined as a set of ordered pairs of numbers. An ordered pair of numbers, e.g. (2,4), consists of two numbers related by a rule, e.g. let the second number be twice as large as the first number. The pair of numbers is written so that one of the numbers is designated as the first and the other as the second, e.g. the second number always equals twice the first number. This creates the order mentioned in the definition of the function. To find the set of ordered pairs which constitute a function, a set of numbers is designated as the domain of the function and these numbers become the first numbers of the ordered pairs. A rule is given which indicates the relationship between the first number and the number to be paired with it. This rule may be written verbally or stated algebraically. For example, the domain of a function may be the set of numbers 1, 2, 3. The rule may be to double each element of the domain or, stated algebraically, the rule for the function is $2x$ where x is an element of the domain.

The second numbers of the ordered pairs are determined by the rule. That is, 1 is paired with 2 ($2(1) = 2$), 2 is paired with 4 ($2(2) = 4$), and 3 is paired with 6 ($2(3) = 6$). The set of ordered pairs or the function is $\{(1,2), (2,4), (3,6)\}$. The second numbers of the pair are designated as the range of the function. The range of the function given above is 2, 4, and 6.

In the excerpt, problems are given to the student with the domain (set of first numbers) and the rule or statement of relationship. The student is to find the function or the set of ordered pairs which results from using the numbers in the domain and applying the rule to find the range (the second members of the ordered pair). The student is often asked to identify the set of numbers which constitutes the range.

Text of an Excerpt

1.) 9:02 Mrs. H: "Let's go to section four-eight which we started yesterday,"

2.) Board

$$D = \{1, 4, 7, 12\}$$

$$f: \quad x \to 2x - 5$$

$$f: \quad 4 \to 2 \cdot 4 - 5$$

3.) Mrs. H: "All right. There is another notation. All right. Which means the same thing."

4.) Board

$$f(x) = 2x - 5$$

$$f(7) = 2 \cdot 7 - 5$$

$$f(7) = 9$$

5.), 6.) Mrs. H: "Is it all right to put a seven in there?"/"Yes. Because seven is in the domain."/

7.) "All right. A function is a set of order pairs."

8.) Board

A function is a set of ordered pairs.

$$\{(7, 9), (4, 3), (1, -3), (12, 19)\}$$

9.) Mrs. H: "All right. You've got to look at the board and find the ordered pairs."

10.) "All right. Some of you are not even looking."

11.)	9:06	Mrs. H:	"All right. On your own find the other two ordered pairs."
12.)			Mrs. Hanley let Martha leave the room. Before Martha left, Mrs. Hanley reprimanded her for whispering and asked her to move.
13.)			<u>Board</u> Range = {9, 3, -3, 19}
14.)	Mrs. H:		"All right. I'd like someone to come to the board and write the range of this function. Luke."
15.), 16.)			Luke made a mistake./Roger and the boys around him said, "No, No."
17.)	Mrs. H:		"What is the domain? Roger?"
18.)			"What is the rule of this function? Brad?"
19.), 20.)			"Is it the same as the range?/What is the range of this function? Frank?"
21.)			(The students are answering the questions.)
22.)	9:10		<u>Board</u> D = {negative integers} $g{:}g \rightarrow 3$ Martha returned. She did not move her place as she was told.
23.)	Mrs. H:		"All right. I want someone to come up and write three elements of the functions. Teresa."
24.)			Teresa wrote {-2, -4, -5}.
25.)	Mrs. H:		"Helen, write the range of the function."
26.)	Helen:		"I don't know."
27.)	Mrs. H:		(to the class) "You are not paying attention. I look at you and you're out the window, up at the ceiling - all over the place."
28.)			"What is a function? Roger."
29.)	Roger:		"A set of ordered pairs."

30.)		<u>Board</u>
		$f: x \to 2x - 5$
		$7 \to 2 \cdot 7 - 5 \quad 9$
		$(7, 9)$
31.)	Mrs. H:	"You are not operating in a vacuum. You would like to think you don't know anything. But I know you know something."
32.)	9:15	There is an announcement over the intercom.
33.)	Mrs. H:	"All right, Teresa. Come back up here." "Write down a negative integer."
34.)		"What is it paired with?"
35.)		Teresa writes
		$-4 \to 3$
36.), 37.)	Mrs. H:	"Now write your ordered pair."/Teresa does.
38.)	Teresa:	"That's all?"
39.)	Mrs. H:	"Once you know what to do it's not hard."
40.)		"All right. Now that you've written one can you write two others?"
41.)		Teresa writes
		$(-5, 3), (-10, 3)$
42.)	Mrs. H:	"All right. Thank you."
43.)	9:19	The attendance girl came in.
44.)	Mrs. H:	"Teresa was a good sport. We won't ask her to come up again."
45.)		"Helen, write the range of this function."
46.)		<u>Board</u>
		$R = \{-4, -5, -10\}$
47.)		Class response, "Nooo."
48.)	Mrs. H:	"All right. We have some knowledgeable young men
49.)		in the back of the room./Frank, you write the range."
50.)	Helen:	"Let Dean do it."
51.)		Frank wrote $R = \{3\}$. (clapping)

52.) **Board**

$D = \{-1, 0, 1\}$

$f(x) = 7x + 5$

53.) 9:23 Mrs. H: "All right. Darin, write the function in less than thirty seconds."

54.) $F\ (1) = 7 \cdot 1 + 5$

$F\ (1) = 12$

(Weisbeck, Field Notes, 11/6/79, pp. 1-3)

Episodes

An episode is a connected sequence of interaction about specific content. The specific mathematical content could be one math problem, one math concept, or a set of problems. Looking at the field notes just presented there were three distinct problems worked on by the class. There were $D = \{1, 4, 7, 12\}$ $f\colon\ x \rightarrow 2x - 5$, $D = \{$negative integers$\}$ $g\colon\ g \rightarrow 3$, and $D = \{-1, 0, 1\}$ $f(x) = 7x + 5$. There are three possible episodes in this part of the field notes according to the mathematical content.

Mrs. Hanley organized the students differently in the presentation of these problems. For the first problem she began the solution process by teaching from the front of the room. The problem was written on the board and she gave information and asked questions. In lines 9 and 10 she reprimanded the class and told them they've got to look at the board. She said that some of the students were not looking. After this statement she changed the structure of the class in line 11 and had the students work individually on the problem. In line 14 she changed the structure once again and sent one student to the board. The interaction then returned to question/ answer as it was at the beginning of the episode.

For the second problem Mrs. Hanley began by sending Teresa to the board. After Helen announced that she didn't know how to do what she was asked to do Mrs. Hanley reprimanded the class for not paying attention. She asked Roger a question for clarification of the concept and then sent Teresa back to the board. Most of the interaction of this episode was through one student at the board while the class watched.

For the third problem Mrs. Hanley began the interaction by sending Darin to the board. The episode began with one student at the board while the class watched.

In each episode there must be teacher/student interaction about the content. In going back over the field notes each problem solution was examined for interaction. For the first problem $D = \{1, 4, 7, 12\}$ $f: x \to 2x - 5$ Mrs. Hanley was the only recorded participant until line 14 when she asked Luke to go to the board. There was indication of interaction after that which satisfied the requirement. For the second problem $D = \{\text{negative integers}\}$ $g: g \to 3$ Mrs. Hanley involved a student almost immediately. There was much interaction throughout the solution of this problem. For the third problem $D = \{-1, 0, 1\}$ $f(x) = 7x + 5$ Mrs. Hanley involved Darin immediately. Although the remainder of this problem solution was not given it had some indication for a possible episode.

The episode concluded just prior to the teacher changing the problem or focus of the class. With this as a guide the first episode from the 11/6/79 field notes quoted above would end at line 21.

Immediately after this line a new problem was introduced. The second episode concluded at line 51. Immediately after this line a new problem was introduced. There was no indication for conclusion for the third episode as given in the excerpt.

From the field notes quoted above there are two complete episodes. The first extends from line 1 to line 21, and the second extends from line 22 to line 51. The second episode, which will be referred to as Episode #22 Alg. will be used throughout this section as a model for a unit of analysis. The episode is presented below.

Example of an Episode

21.)		(The students are answering the questions.)
22.)	9:10	<u>Board</u> D = {negative integers} $g:g \rightarrow 3$ Martha returned. She did not move her place as she was told.
23.)	Mrs. H:	"All right. I want someone to come up and write three elements of the functions. Teresa."
24.)		Teresa wrote {-2, -4, -5}.
25.)	Mrs. H:	"Helen, write the range of the function."
26.)	Helen:	"I don't know."
27.)	Mrs. H:	(to the class) "You are not paying attention. I look at you and you're out the window, up at ceeling - all over the place."
28.)		"What is a function? Roger."
29.)	Roger:	"A set of ordered pairs."
30.)		<u>Board</u> $f:x \rightarrow 2x - 5$ $7 - 2 \cdot 7 - 5 \quad 9$ (7, 9)

31.	Mrs. H:	"You are not operating in a vacuum. You would like to think you don't know anything. But I know you know something."
32.)	9:15	There is an announcement over the intercom.
33.)	Mrs. H:	"All right, Teresa. Come back up here." "Write down a negative integer."
34.)		"What is it paried with?"
35.)		Teresa writes $-4 \rightarrow 3$
36.), 37.)	Mrs. H:	"Now write your ordered pair."/Teresa does.
38.)	Teresa:	"That's all?"
39.	Mrs. H:	"Once you know what to do it's not hard."
40.)		"All right. Now that you've written one can you write two others?
41.)		Teresa writes (-5, 3), (-10, 3)
42.)	Mrs. H:	"All right. Thank you."
43.)	9:19	The attendance girl came in.
44.)	Mrs. H:	"Teresa was a good sport. We won't ask her to come up again."
45.)		"Helen, write the range of this function."
46.)		<u>Board</u> $R = \{-4, -5, -10\}$
47.)		Class response, "Nooo."
48.)	Mrs. H:	"All right. We have some knowledgeable young men in
49.)		the back of the room./Frank, you write the range."
50.)	Helen:	"Let Dean do it."
51.)		Frank wrote $R = \{3\}$. (clapping)
52.)		<u>Board</u> $D = \{-1, 0, 1\}$ $f(x) = 7x + 5$

Mathematical Content

Since the episodes were first selected according to specific mathematical content, the first classification was according to content. The episodes were sorted into groups of like topics and then the topics were listed according to that which most often appeared to that least often in the episodes. The mathematical topics in the algebra episodes are the fourteen topics listed below.

Topics for Algebra Episodes

I. Linear Equations
II. Algebraic Simplification
III. Geometry
IV. Inequalities
V. Functions
VI. Fractional Equations
VII. Variation
VIII. Factoring
IX. Two Variables
X. Exponents
XI. Multiplication of Polynomials
XII. Quadratic Equations
XIII. Signed Numbers
XIV. Decimals

The mathematical topics in the general math episodes are the ten topics listed below.

Topics for General Math Episodes

I. Percents
II. Geometry
III. Flow Charts
IV. Signed Numbers
V. Exponents
VI. Fractions
VII. Whole Numbers
VIII. Decimals
IX. Factoring
X. Functions

Once the topics were identified a frequency count was made of how many episodes dealt with a topic. Each episode belonged to only one topic. Percentages were calculated according to number of episodes on a topic over the total number of episodes. Episode #22 Alg. was classified under the topic Functions (V) for the algebra class.

Social Participation Structures

The social participation structure was the arrangement of the class selected by the teacher as a means of presenting the content. This structuring of the class created the boundaries for classroom interaction and helped to clarify what was acceptable in response behavior and what was not acceptable. The episodes were sorted according to the participation structures that were actually used and these structures were then recorded. Through this initial sorting the same four strategies were identified for both classes. These four were as follows: 1) the teacher directing the presentation from the front of the room, 2) a group of students working problems at the board, and 3) one student working at the board as the class watched, and 4) the class working the problem or problems first before the solution or solutions were discussed. (See Appendix A for examples.)

The first social participation structure was the teacher directing the presentation from the front of the room. The dynamic of interaction was usually question/answer with the teacher asking the questions. This was the most restrictive interaction structure in that the students were expected to listen, keep quiet, and respond to the questions asked. Mrs. Hanley often invited questions from students at this time but she controlled the interaction for the most part.

The second social participation structure was a group of students working problems at the board. This structure was more open than the first and allowed for voluntary responses and some off-task behavior (behavior not directed toward the content or task of the lesson). The students were still required to pay attention to what was happening at the board as well as try the problems themselves. But they could talk to each other (about the math, of course) and could also help those at the board. This situation was still controlled by the teacher who gave the problems and checked the answers.

The third social participation structure was <u>one student working at the board as the class watched</u>. Mrs. Hanley would usually ask a student to go to the board to work a problem for the class. At times she worked through one problem by sending students one at a time to work parts of the problem in succession. This was less open than the second structure with a group at the board. The class was expected to pay attention as if Mrs. Hanley were doing the problem herself. Students could volunteer suggestions but Mrs. Hanley often chose persons in the algebra class who might have trouble with the problem and wanted them to struggle with the process. She used mistakes as a basis for teaching in algebra. This was not the case in general math. She seldom put a student in general math on the spot probably because of the inability to predict student behavior. An algebra student would at least attempt the problem. This structure was one way Mrs. Hanley tried to draw the student into the interaction.

The fourth social participation structure was <u>the class working the problem or problems first before the solution or solutions were discussed</u>. Mrs. Hanley usually wrote the problem or set of problems on the board and then asked the students to work them out first before the answers were given. This may be a quiz or just practice. If the exercise was a quiz then silence was expected. If it was practice then the students could work with each other. This provided a more open environment in which to interact. Mrs. Hanley would walk among the students giving help and trying to keep the class on task (relating to the content and accomplishing the goal of the lesson). Going through the answers took various forms. Mrs. Hanley might do it from the board, a student might put up the solutions, or the answers might be recited. This participation structure had the least control exhibited by Mrs. Hanley.

Episode #22 Alg. was classified under social participation structure number three, one student working at the board as the class watched. During the course of the lesson there were five instances of one student at the board doing a step in the solution process. This structure was interrupted by a question/answer exchange but for the most part was structure three. After all the episodes were classified as one of these four, the frequencies and percentages were calculated and recorded.

Conclusion of an Episode

The episode was considered concluded when the teacher changed the content or focus of the class. Moving to another problem, beginning the study period, or giving the assignment were signals of the change. The interaction just prior to the change was considered the conclusion of the episode. The conclusions for both classes were studied and categorized to give some indication of how closure in the progress of the lesson of each episode was established. Through the categorizing process five types of conclusions were identified. The first four were common to both general math and algebra. The fifth conclusion occurred only in general math. The following are the conclusions: 1) instruction by the teacher, 2) a comment by the teacher on procedure or expectations, 3) a statement on content by a student, 4) feedback information requested by the teacher and given (or not given) by a student or students, and 5) a comment or question by a student which did not pertain to the task of the lesson. (See Appendix B for examples.)

The first conclusion was instruction by the teacher. This was a statement by the teacher about the content or it was work done by the teacher at the board. The second conclusion was a comment by the teacher on procedure or expectations. This conclusion was a statement about study habits, or the next topic to be studied or what the teacher expected them to do. This comment was not directly about content but was related to something about the class. The third conclusion was a statement on content by a student. This statement could be a question, a correct answer, or some work done at the board. The fourth conclusion was feedback information requested by the teacher and given (or not given) by a student or students. Mrs. Hanley frequently asked the students if a problem solution made sense or if they understood the work being done. She would occasionally ask if there were any questions. These were considered feedback questions because she was attempting to elicit where the students were in the process of understanding. Sometimes someone would answer her question and sometimes not. Both answered and unanswered requests for feedback by the teacher were grouped under this conclusion. The fifth conclusion was a comment or question by a student which did not pertain to the task of the lesson. This conclusion occurred in the general math class and not in the algebra class. Grouped under this conclusion were statements by students which were not about content or significantly related to what was happening in the class.

Episode #22 Alg. was classified under conclusion number three, a statement on content by a student. In this episode Frank wrote on the board (line 51) the correct answer for the range of the function being studied. Work done on the board was considered a statement on content. After all the episodes were classified as one of these five, the frequencies and percentages were calculated and recorded.

Coding System

The episodes were read several times in order to identify basic categories for the communication which took place between teacher and students. In this communication (interaction) was embodied the evidence of collaboration between teacher and student for each episode. Success in capturing this interaction was crucial to the study. In order to insure as much as possible that the interaction be accurately preserved in the analysis, the episodes themselves were used for determining a coding system. The basic categories for the coding system which emerged from reading the episodes were the six which follow:

C content question or statement, including boardwork;

A assistance in teaching - questions which lead to reteaching;

I insight given by a student about the subject matter;

F feedback (responses such as yes, no, I don't know);

R reprimand for disruptive behavior or for not knowing the content;

N an irrelevant comment or question (off task, off subject matter, not helpful to teaching).

With these six categories came some basic issues that needed to be faced. What would be the basic unit for a coded item? What was it that a coded item was to express for the study? What modifications needed to be made to the six basic categories?

The basic unit was termed a communication act. A communication act could be a verbal communication, a recorded gesture in response to a question (such as a nod of the head) or work done at the board. If the teacher or a student spoke for a length of time, this communication may be broken into more than one coded item. For communication about the mathematical content a coded item would be determined by statements about different aspects of a concept or problem. There were two recorded instances of a nod of the head indicating feedback to the teacher. Since feedback was a part of this teacher's pattern of interaction, these two instances were included as communication acts. A large part of the episodes was work at the board which had been recorded in the field notes. This activity was considered interaction between student and teacher through the content. It recorded some aspect of student performance which was meaningful to the teacher and served as communication.

Each coded item was an act of communication during the mathematics lesson by the teacher or by a student (or group of students). This communication was either about the focus of the mathematics lesson or it was irrelevant to what was happening in the lesson. Communication which was related to the focus was called on task communication. Irrelevant communication was referred to as off task. The focus of the mathematics lesson for each episode was to solve a given problem or set of problems or to explain a specific concept. The process of solving the problem or explaining the concept was regarded as the forward movement of the lesson. This forward movement could be direct with no interference or there may be instances of interruptions which inhibit progress momentarily, or at times, stop it completely. Off task behavior interrupted the forward movement of the lesson while on task behavior contributed to the forward movement. A coded item, then, should express who is communicating, how it relates to the content (instruction, question, content) and if it contributes to or inhibits the forward movement.

The six coding categories were evaluated in terms of the basic unit for a coded item and the expressed needs of this coding system. After rereading the episodes several variations within the six categories were recognized. The differentiation of the categories occurred as follows.

C content question or statement, including boardwork

<u>Teacher</u>

TC: teacher content, comment, directions

TC_B: teacher at board

TQ: teacher question

TE: teacher exhortation (urging, advising, or warning)

TH: teacher helping an individual student during a general work time

TW: teacher giving an incorrect response about content

TW_B: teacher incorrect at the board

Student

SQ: student question about content

SR: student right answer

SR_B: student correct at the board

SA: student affirmation of another student

SH: student helping another student

SC: student comment or statement

SW: student wrong answer

SW_B: student wrong answer at the board

These communication acts related to the content and focused on the task being studied. Teacher and student have been indicated separately and boardwork has been included. These coding categories were helpful in determining the kind of act as well as the one acting.

A assistance in teaching - questions which lead to reteaching

Teacher

TP: teacher providing reteaching of a concept or problem

Student

SP: student providing an opportunity for reteaching

These were instances in the episodes when a response by a student prompted the teacher to explain something again or to review a concept already taught. On the part of the student this response could be a question, a wrong answer, a feedback response of no or I don't know, or a comment about the content. The teacher used these responses as an opportunity to reinforce her instruction. It seemed important to designate these instances with specific categories because not all such opportunities were acted upon by the teacher. These instances gave the episode the added characteristic of reinforced teaching and teacher response to student needs.

I insight given by a student about the subject matter

Student

SI: student insight about content

At times a student was able to see beyond the problem or concept being studied and make a relationship to something else or see a relationship within the problem that the teacher hadn't mentioned. These instances seemed important enough to be given special recognition.

F feedback (responses such as yes, no, I don't know)

Teacher

TF: teacher request for feedback

Student

SF_+: student positive feedback

SF_-: student negative feedback

SF_o: student response of I don't know

This teacher used various means of staying in touch with where the students were with the content. The most frequent means was asking them individually or in general if something made sense or if they had any questions. Students sometimes answered the general question and sometimes not. At times the answer served as an indication to reteach the problem or to move to the next one.

R reprimand for disruptive behavior or for not knowing the content

Teacher

RB: teacher reprimanding about behavior

RC: teacher reprimanding about content

These kinds of statements occurred so often in both classes it seemed necessary to code them as such. This teacher had expectations for learning and for behavior. She wanted the students in both classes to understand the mathematics and she demanded a certain kind of behavior in order for that to happen.

N an irrelevant comment or question (off task, off subject matter, not helpful to teaching)

Teacher

TOT: teacher about content but off the point of the lesson

TOL: teacher off content, irrelevant remark or question

Student

SOT: student about content but off the point of the lesson

SOL: student off content, irrelevant remark or question

In studying the off task comments made by both teacher and students it became clear that some of the remarks actually referred to the content but were not on the point of the lesson or were spoken out of turn. Other remarks were in no way related to the content. It became necessary to separate these two kinds of remarks. This was done as shown in the coding categories given above.

There were two other types of situations which were not covered by the above coding categories. At times in the field notes it was simply recorded that the class in general was talking or that a group of students were being disruptive. On other occasions it was recorded that the teacher and students discussed a particular point but no specific communication acts were recorded. This was also a limitation of field notes, the reporting of events in a general manner. But these interactions were part of the movement of the lesson and therefore the following coding categories were established.

- GP: generally constructive and on task interaction
- GP_B: generally correct at the board
- GN: generally incorrect or generally disruptive and off task interaction
- GN_B: generally incorrect at the board

The students were the main ones communicating in these coding categories, although the teacher may have been involved in the GP or GN categories. Work at the board was not always completed to the final solution. The coding categories GP_B and GN_B referred to the incomplete work as in the right direction (GP_B) or already in error (GN_B).

One other set of communication acts had to do with interruptions from outside the room. These were not initiated by the teacher or a student. Such interruptions included intercom announcements and the student who collected the attendance slips. This coding category was O.

- O: interruption from neither teacher nor student, other

This differentiation of the six categories specified who was communicating and what kind of communication was given. (See Appendix C for further explanation and examples.) Another need of the study for the coding system was to express if a coded item contributed to or inhibited the forward movement of the lesson.

Each episode had a problem to solve or a set of problems or a concept to be presented. The forward movement was the progress to the solution or explanation. The ideal path would be straight to the answer, with all communication acts focused on the task at hand. However, this straight path was not always the one that occurred in the enactment. There were instances when students asked questions or made statements that were not related to the problem. There were questions or statements that were related but did not help toward the problem solution. There were instances of disruptive behavior. The teacher would stop instruction and reprimand the students for their behavior. The teacher at times made comments that were not related to the subject matter. Each of these instances inhibited the forward movement. There was interference with getting straight to the answer.

Another set of instances was considered in terms of interference. These were instances of wrong answers given by students. Teachers may expect wrong answers because they are part of the learning process. Wrong answers can be helpful in determining why a student is not understanding the material. They are instances that require specific attention in order to correct the misconception which exists. Wrong answers may also mean a lack of atention on the part of the student and lack of interest in studying. Because wrong answers may be a help as well as a hindrance, decisions about forward movement and wrong answers were difficult. After working some time with the episodes and trying to capture the movement of a lesson it was decided that those instances of wrong answers which led to reteaching would be marked accordingly and be considered as contributing to the forward movement. Those wrong answers which were not followed by reteaching would be considered as interference or inhibiting the forward movement. They were jarring, unresolved communication acts which left a need for repair. By marking them as interference it would be possible to see where the lesson needed some repair. The instances coded 0 were also considered inhibiting since they stopped the class for a time.

With this rationale it was decided to sign the coded items which contributed to the forward movement with a plus. Those instances which inhibited the forward movement were signed with a minus.

There were 34 communication acts for this coding system. Of these 34, 14 pertained to the teacher, 19 to the students, and one pertained to neither. Of the 14 communication acts for the teacher, eight were signed as plus (contributing to the forward movement), and six were signed as minus (inhibiting the forward movement). The minus acts were TW, TW_B, RB, RC, TOT and TOL. Of the 19 communication acts for the student, 11 were signed as plus and eight were signed as minus. The minus acts were SW, SW_B, SF_-, SF_O, SOT, SOL, GN, and GN_B. The communication act 0 was signed as minus because it created interference in the forward movement of the lesson. Once the coding system was finalized each of the episodes was coded and each communication act numbered. There was a coded item for each number and the last number for each episode recorded the number of coded items.

This coding system was not tested for inter-rater reliability. However, because of its basis in the forward movement of the lesson and the mathematical content, it is apt for teaching others and then comparing results. Further use of this coding system would warrant such a test.

The following is Episode #22 Alg. with its communication acts coded and numbered. This episode was renumbered beginning with one to better exemplify the process used in analyzing the episodes.

Example of a Coded Episode

				(The students are answering the questions.)
(+) TC_B	(1)	1.) 9:10		Board $D = \{\text{negative integers}\}$ $g{:}g \rightarrow 3$
				Martha returned. She did not move her place as she was told.
(+) TC	(2)	2.)	Mrs. H:	"All right. I want someone to come up and write three elements of the functions. Teresa."
(-) SW_B	(3)	3.)		Teresa wrote $\{-2, -4, -5\}$.
(+) TC	(4)	4.)	Mrs. H:	"Helen, write the range of the function."
(-) $SF_{\circ}$	(5)	5.)	Helen:	"I don't know."
(-) RB	(6)	6.)	Mrs. H:	(to the class) "You are not paying attention. I look at you and you're out the window, up at ceiling - all over the place."
(+) TQ	(7)	7.)		"What is a function? Roger."
(+) SR	(8)	8.)	Roger:	"A set of ordered pairs."
(+) TC_B	(9)	9.)		Board $f{:}x \rightarrow 2x - 5$ $7 \rightarrow 2 \cdot 7 - 5 \quad 9$ $(7, 9)$

(−) RC	(10)	10.)	Mrs. H:	"You are not operating in a vacuum. You would like to think you don't know anything. But I know you know something."
(−) O	(11)	11.) 9:15		There is an announcement over the intercom.
(+) TC	(12)	12.)	Mrs. H:	"All right, Teresa. Come back up here." "Write down a negative integer."
(+) TQ	(13)	13.)		"What is it paired with?"
(+) SR_B	(14)	14.)		Teresa writes $-4 \to 3$
(+) TC	(15)	15.), 16.)	Mrs. H:	"Now write your ordered pair." Teresa does.
(+) SR_B	(16)			
(+) SQ	(17)	17.)	Teresa:	"That's all?"
(+) TC	(18)	18.)	Mrs. H:	"Once you know what to do it's not hard."
(+) TQ	(19)	19.)		"All right. Now that you've written one can you write two others?"
(+) SR_B	(20)	20.)		Teresa writes $(-5, 3), (-10, 3)$
(+) TA	(21)	21.)	Mrs. H:	"All right. Thank you."
(−) O	(22)	22.) 9:19		The attendance girl came in.
(+) TA	(23)	23.)	Mrs. H:	"Teresa was a good sport. We won't ask her to come up again."
(+) TC	(24)	24.)		"Helen, write the range of this function."
(−) SW_B	(25)	25.)		<u>Board</u> $R = \{-4, -5, -10\}$
(−) SOT	(26)	26.)		Class response, "Nooo."
(−) RB	(27)	27.)	Mrs. H:	"All right. We have some knowledgeable young men in the back of the room. /
(+) TC	(28)	28.)		Frank, you write the range."
(−) SOL	(29)	29.)	Helen:	"Let Dean do it."
(+) SR_B	(30)	30.)		Frank wrote $R = \{3\}$. (clapping)

<u>Board</u>

$$D = \{-1, 0, 1\}$$

$$f(x) = 7x + 5$$

Narrative Description of a Coded Episode

This episode occurred twenty minutes into the class period. Mrs. Hanley had reviewed story problems with the class and was now continuing the work with functions which began in the previous class on November 5, 1979. An example with a function was completed just prior to the problem worked in Episode #22.

Mrs. Hanley wrote the problem on the board. (line 1, (+)TC_B) She indicated the domain for the function as the set of negative integers and the rule as matching every element of the domain with the number three.

Martha had left the room and returned at this point in time. She had been whispering before she left and Mrs. Hanley asked her to change her place when she returned. She did not do this. This was not considered a communication act since no interaction or interference occurred because of it.

After putting the problems on the board, Mrs. Hanley asked Teresa to come up and write three elements of the function. (line 2, (+)TC - directions) Teresa went to the board and wrote in set notation -2, -4, and -5. (line 3, (-)SW_B) These three numbers are members of the domain of the function, not elements of the function which are ordered pairs. This error indicated that Teresa did not yet understand what was meant by the concept function.

Mrs. Hanley apparently disregarded Teresa's error, leaving the incorrect answer unresolved, creating a break in the forward movement that needed repair. She then told Helen to write the range of the function. (line 4, (+)TC) This would be the set of numbers paired to the negative numbers of the domain. Helen simply admitted she didn't know. (line 5, $(-)SF_O$) Mrs. Hanley did not respond to Helen directly which left Helen's response unresolved, creating another break which needed repair.

At this point Mrs. Hanley reprimanded the class. The class was restless and not paying attention. (line 6, (-)RB) The previous example before this episode indicated that some of the students had not caught on to what a function was. Teresa was one of the brighter, quicker students in the class and her incorrect response supported the lack of understanding. When Helen stated that she didn't know the answer the situation climaxed in a class reprimand which was another break in the forward movement of the lesson. There was a sense that the class was not getting anywhere in solving the problem.

In order to restore some kind of progress Mrs. Hanley asked Roger for the definition of a function (line 7, (+)TQ) which he gave correctly. (line 8, (+)SR) Mrs. Hanley reviewed the previous example by finding an ordered pair from the rule $f:x \rightarrow 2x - 5$. (line 9, $(+)TC_B$) The lesson appears to be moving now. She concluded from this exercise that the students knew what was going on because they had done it already. (line 10, (-)RC) However, the problem being worked at the board was a constant function, which pairs every number in the domain to the same number (three in this case). This function was different from the previous problem. The fact there was no arithmetic operation in order to find the second number significantly changed the task.

There was an announcement over the intercom. (line 11, (-)O) The lesson stopped. After the intercom announcement Mrs. Hanley called Teresa back to the board. She began a step by step review of the process of writing the function. Mrs. Hanley told Teresa to write down a negative integer (a number in the domain). (line 12, (+)TC) Then she asked her what number was paired with the number she chose. (line 13, (+)TQ) Teresa wrote it correctly by following the notation of the given rule. (line 14, (+)SR_B) With this completed, Mrs. Hanley then told Teresa to write the ordered pair, (line 15, (+)TC) which she did correctly. (line 16, (+)SR_B) After going through these steps Teresa said with surprise, "That't all?" (line 17, (+)SQ) Mrs. Hanley then commented that once you know what to do it's not hard. (line 18, (+)TC) This proved to be true for Teresa. Mrs. Hanley asked her to write two more ordered pairs. (line 19, (+)TQ) Teresa did so correctly. (line 20, (+)SR_B) Mrs. Hanley thanked Teresa, who sat down. (line 21, (+)TA) This was Mrs. Hanley's way of telling a student they were correct and/or finished. The lesson moved forward during this interaction with Teresa. The girl who collected attendance entered the room. The lesson stopped. (line 22, (-)O) When the attendance was taken care of Mrs. Hanley turned again to the class.

Mrs. Hanley referred to Teresa as a good sport (line 23, (+)TA) and then returned to Helen. She told her a second time to write the range of the function. (line 24, (+)TC) Unlike her previous response of "I don't know," Helen went to the board and wrote the first numbers (elements of the domain) of the ordered pairs instead of the second numbers which are in the range. (line 25, (-)SW_B) The class had begun to pay attention and several students responded to Helen's answer with, "No." (line 26, (-)SOT) This answer was correct because Helen's answer was wrong. However, this kind of spontaneous answering out was not acceptable in this participation structure. This act called Mrs. Hanley's attention to a group of boys in the back who had instigated the response. The lesson was again not moving forward. Leaving Helen's response unattended to, Mrs. Hanley picked up on this class response by remarking that there were some boys who seem to know how to do the problem. (line 26, (-)RB) This comment was an indication of disapproval of the boys' response. She then called on Frank to write the range. (line 28, (+)TC) Helen would have preferred to have Dean put on the spot, however. (line 29, (-)SOL) This seemed to be unrelated to what was happening. Frank correctly wrote the range and there was some clapping from the class. (line 30, (+)SR_B) The problem was completed.

The Script of an Episode

The objective of the lesson was to write three elements for the function with a domain of all the negative integers and the rule $g: g \to 3$ and to indicate the range. The forward movement in writing the three elements met several instances of interference. The path to the solution was recorded in the coded items and the plus and minus signs of the communication acts. The string of coded items as they occurred in sequence formed the script of the episode and told the story of the interaction between the teacher and students.

From the script it was possible to tabulate the kinds of communication acts for each episode and the frequency of each act. The following is the result of tabulating Episode #22 Alg.

Communication Acts

(+) TC	(7)	(+) SR	(1)	(-) 0	(2)
(+) TC_B	(2)	(+) SR_B	(4)		
(+) TQ	(3)	(+) SQ	(1)		
(+) TA	(2)	(-) SW_B	(2)		
(-) RB	(2)	(-) SF_O	(1)		
(-) RC	(1)	(-) SOT	(1)		
		(-) SOL	(1)		

There were thirty communication acts for this episode. About half of these were teacher acts that had a plus sign. Communication acts were tabulated across episodes as well as within episodes. This made it possible to look at the most frequently used coding categories in both classes and to compare between classes.

Another way of looking at the communication acts was through the use of a contingency table. This provided a means of looking at two aspects of the data at the same time and provided a way of seeing how these two aspects influence each other. The two aspects of the communication acts were the performer of the act (teacher or student) and the sign of the act (plus, contributing to the forward movement, or minus, inhibiting the forward movement). The acts marked 0 were not included in the contingency table because the performer was neither teacher or student. The following is the contingency table for Episode #22 Alg.

	T	S
+	14	6
-	3	5

The horizontal rows were the plus and minus signs and the vertical columns were the teacher and student. The contingency table shows that there were 14 teacher plus acts, 6 student plus acts, 3 teacher minus acts, and 5 student minus acts. There were 17 teacher acts to 11 student acts. But there were almost as many student minus acts as student plus acts. The large number of minuses would indicate several instances of interference in the forward movement of the lesson. To get a better picture of what the interferences were, the contingency table can be extended as follows.

		T	S
	+	14	6
(on task)	-	0	3
(reprimand for content)	-	1	0
(reprimand for behavior)	-	2	0
(off task)	-	0	2

The extended contingency table shows that most of the student minus acts were on task (related to the mathematical content) and the teacher minus acts were reprimands. That means that only two of the minus acts were off task (irrelevant to the content of the lesson). For the most part the class appeared to be focused on the problem being solved and was cooperating with the teacher in reaching the solution.

The Mathematics Structure of the Episode

The structure of the lesson is the mathematical content as the teacher has decided to present it. This decision making includes selection of the topic, organization of the essential aspects of the topic, and a format for classroom presentation. So if the teacher is going to teach addition of fractions, one way of organizing the topic may be to review the concept of fraction, give an example with fractions of like denominator, and practice with three more problems. The teacher may decide that the best way to do this would be demonstration from the front of the room using the blackboard. Another presentation of the same topic and organization may be to send a group of students to the board and give an exercise in the concept of fraction, talk through with them a problem of adding fractions of like denominator and then have them work three problems on their own.

In each case given above, the lesson has a structure based on the mathematical content and demonstrated through the teacher's logical presentation of the content. The structure can be depicted by steps taken in the classroom presentation.

Each episode was studied from the perspective of the mathematics involved in the lesson and the format used by the teacher for presenting the mathematics. From the beginning of the episode to its conclusion, each episode was outlined according to the steps used by the teacher in the development of the mathematics presented. These steps provided the underlying mathematics structure of the lesson and were used as representative of the teacher's logical presentation of content.

Episode #22 Alg. had a mathematics structure of three steps. These three steps were 1) Presenting the problem on the board, 2) Identifying the elements of the function, and 3) Identifying the range of the function. These steps were spaced throughout the lesson and the forward movement progressed from step to step of the mathematics structure. The first step was line 1 as Mrs. Hanley put the problem on the board. Step 2 extended from line 2 to line 23. In line 2 Mrs. Hanley named Teresa to put three elements of the function on the board. This was finally accomplished at line 20. In line 24 Helen was told to write the range, which was the beginning of step 3 which extended to line 30. Frank wrote the range correctly in line 30. Within the structure of these three steps the solution to the problem was found.

The following is Episode #22 Alg. with its mathematics structure indicated according to steps. The lines of the episode which pertain to each step are also given.

Example of an Episode with its Mathematics Structure

				(The students are answering the questions.)
1. Presenting the problem on the board. (line 1)	1.)	9:10		Board $D = \{\text{negative integers}\}$ $g: g \rightarrow 3$ Martha returned. She did not move her place as she was told.
2. Identifying the elements of the function. (lines 2-23)	2.)		Mrs. H:	"All right. I want someone to come up and write three elements of the functions. Teresa."
	3.)			Teresa wrote $\{-2, -4, -5\}$.
	4.)		Mrs. H:	"Helen, write the range of the function."
	5.)		Helen:	"I don't know."
	6.)		Mrs. H:	(to the class) "You are not paying attention. I look at you and you're out the window, up at ceiling - all over the place."
	7.)			"What is a function? Roger."
	8.)		Roger:	"A set of ordered pairs."
	9.)			Board $f: x \rightarrow 2x - 5$ $7 \rightarrow 2 \cdot 7 - 5 \quad 9$ $(7, 9)$
	10.)		Mrs. H:	"You are not operating in a vacuum. You would like to think you don't know anything. But I know you know something."
	11.)	9:15		There is an announcement over the intercom.
	12.)		Mrs. H:	"All right, Teresa. Come back up here." "Write down a negative integer."
	13.)			"What is it paired with?"
	14.)			Teresa writes $-4 \rightarrow 3$

	15.), 16.)	Mrs. H:	"Now write your ordered pair." Teresa does.
	17.)	Teresa:	"That's all?"
	18.)	Mrs. H:	"Once you know what to do it's not hard."
	19.)		"All right. Now that you've written one can you write two others?"
	20.)		Teresa writes (-5, 3), (-10, 3)
	21.)	Mrs. H:	"All right. Thank you."
	22.) 9:19		The attendance girl came in.
	23.)	Mrs. H:	"Teresa was a good sport. We won't ask her to come up again."
3. Identifying the range. (lines 24-30)	24.)		"Helen, write the range of this function."
	25.)		**Board** $R = \{-4, -5, -10\}$
	26.)		Class response, "Nooo."
	27.)	Mrs. H:	"All right. We have some knowledgeable young men in
	28.)		the back of the room. / Frank, you write the range."
	29.)	Helen:	"Let Dean do it."
	30.)		Frank wrote $R = \{3\}$. (clapping)

Board

$D = \{-1, 0, 1\}$

$f(x) = 7x + 5$

The Outline of an Episode

The focus of this study was to look at the collaborative efforts of teacher and students in producing lessons. The teacher presents the content and selects a participation structure for interaction. The student interacts with the teacher about the content. In order to get a picture of this joint production of lessons an outline was formed for each episode. The outline consisted of the steps of the logical mathematical structure of the teacher's presentation and the script of interaction with the plus and minus signs. The outline began with a statement of the problem of the episode and then the first step of the mathematical structure. The script followed in sequence until the next step was inserted as it occurred in the interaction, and so on until steps and script were merged.

The outline provided a representation of the episode with the mathematics structure of the teacher's presentation and the interaction of the teacher and students about the content. The plus and minus signs gave an abstract view of the forward movement of the lesson and, therefore, of the collaboration between teacher and students.

The following is the outline of Episode #22 Alg.

Problem: $g: \ g \to 3$, $D = \{\text{negative integers}\}$

1. Presenting the problem on the board.

   $(+)TC_B$

2. Identifying the elements of the function.

   $(+)TC$

   $(-)SW_B$

(+)TC

(−)SF_o

(−)RB

(+)TQ

(+)SR

(+)TC_B

(−)RC

(−)O

(+)TC

(+)TQ

(+)SR_B

(+)TC

(+)SR_B

(+)SQ

(+)TC

(+)SR_B

(+)TA

(−)O

3. Identifying the range of the function.

(+)TA

(+)TC

(−)SW_B

(−)SOT

(−)RB

(+)TC

(−)SOL

(+)SR_B

The outline shows that there were three steps to the episode and that step two involved quite a long sequence of interaction. There were several instances of interference in the forward movement of the lesson as indicated by the minuses. By reading down the outline it can be seen that the second act of the second step was a student wrong answer. The fourth act of step two was a feedback response of I don't know which resulted in a teacher reprimand for behavior. This was not a good start. There was another start which seemed more promising but was followed by a reprimand by the teacher about content. It could be surmised that the reprimand was about not knowing how to do the problem. There was an interruption from outside the classroom. After the interruption there followed a long sequence of plus acts involving both teacher and students which would indicate the students had caught on to the solution. There was another outside interruption just before step three.

Step three began somewhat like step two in that the first student act was a wrong answer. There followed a remark off the point of the class which provoked a reprimand. Before the answer was given an irrelevant comment was made.

This lesson had several interruptions but for the most part about the content. There was a collaborative effort between teacher and students and the students eventually gave the solution to the problem.

An outline was prepared for each of the episodes. These outlines were studied for patterns and similarities. Since the outline presented the forward movement of the lesson, the presence or absence of interference became the main issue. The minus acts, however, separated into two groups according to the nature of the interference created. A mistake about the content due to misunderstanding was a different kind of interference than a comment about something irrelevant to the lesson (such as the weather). A mistake concerning content was still on task with respect to the lesson although it may warrant a minus sign. A comment irrelevant to the lesson was off task but also given a minus sign. Therefore, the minus acts were considered during the analysis as on task minus acts and off task minus acts. This distinction was used in studying the patterns of the outlines. Because on task minus acts were content oriented, a lesson with on task minus acts had different implications for collaboration than a lesson with off task minus acts. On task minus acts indicated that the students were attentive to the content although not producing correct answers. They were attempting to work with the teacher in accomplishing the goal of the lesson. Off task minus acts indicated that the students were not cooperating with the teacher and were working against accomplishing the goal of the lesson. Therefore, the outlines were grouped into four patterns: 1) outlines with all plus signs, 2) outlines with minus acts that were only on task, 3) outlines with minus acts that were only off task, and 4) outlines with on task and off task minus acts. Episode #22 Alg. was a pattern four since it had both kinds of minus acts. By counting the number of episodes in each pattern and finding percentages, the two classes, general math and algebra, could be compared according to the collaborative effort shown in the lessons between teacher and student.

Summary

The described method of analysis used episodes from the field notes as units of analysis. These units as connected sequences of interaction about specific content provided a way to look at the collaborative effort between teacher and students in producing a general math class and an algebra class. By outlining the mathematics in the lesson as the teacher presented it and by coding the teacher and student interaction, an outline of the episode was formed. This outline provided a pattern of collaboration which was used in describing the class it represented as well as used in comparing with the other class.

CHAPTER IV

FINDINGS

Introduction

The results of this study support the assumption that general math is harder to teach than algebra. In analyzing the processes of instruction and interaction in one general math class and one algebra class (both taught by the same teacher), there were significant qualitative and quantitative differences between the two classes in the presentation of mathematics content, the use of social participation structures, and the patterns of enactment of the lesson by teacher and students. The collaborative effort between teacher and students showed more cooperation on the part of the algebra students than the general math students. There was less interference in the forward movement of the lessons in the episodes and less irrelevant interaction in the algebra class than in the general math class. There was more mathematics content presented and of a more complex nature in algebra than in general math. The range of social participation structures was the same for both classes but the frequency of usage varied dramatically.

Each episode, as a unit of analysis for this study, was examined and outlined according to the mathematical structure of the teacher's presentation of content and the interaction between teacher and students. This outline provided patterns of lesson progress or forward movement to the completion of the task set forth in the episode. It was from these outlines that characteristics representative of each class were identified. These characteristics then provided a profile for referring to a typical or representative episode for each class.

The findings are presented through the selection of a representative algebra episode and a representative general math episode as characterized from this study. The major issues of the study are then addressed through the presentation of specific findings and the comparison of the general math and algebra classes.

A Representative Algebra Episode

A representative algebra episode was characterized by at least twice as many teacher communication acts as student communication acts, by the teacher directing instruction from the front of the room, and by student cooperation signified by the lack of interruptions in the progress of the class in solving the problem or problems being studied. Episode #76 Alg. is an example of a representative episode for the algebra class.

Text of the Episode

#76 3/4/80 pp. 2-3 Algebra

Mrs. Hanley: "This is our answer here." "Does that make sense?" "Do you have any questions on the problems that you worked?"

(+) TC_B (1)	1.)		**Board**	
			$y-3/8y-4 \cdot 10y-5/5y-15$	
(+) SR (2)	2.)		Sarah gave the first step for the solution.	
			Board:	
			$y-3/4(2y-1) \cdot 5(2y-1)/5(y-3)$	
(+) TC (3)	3.)	Mrs. H:	"You have to get out the highest common factor or it isn't going to count for a thing."	
(+) TC (4)	4.)		"Now that you have it factored then you can look to see what cancels out."	
(+) SR (5)	5.)		Brenda finished the problem.	
			Board	
			$1 \cdot (\cancel{y-3})/4(\cancel{2y-1}) \cdot \cancel{5}(\cancel{2y-1})/\cancel{5}(\cancel{y-3})$	
			1/4	
(+) TC (6), (+) TF (7)	6.), 7.)	Mrs. H:	"One fourth is the answer. Does that make sense?"	
(+) TC (8), (+) TQ (9)	8.), 9.)		"Dividing out is the easy part. What is the hard part? Alice?"	
(+) SR (10)	10.)	Alice:	"Finding all the factors."	

Luke: "Could you do number twenty-seven?"

Mrs. Hanley: "Yes, I wanted to do some harder ones."

Narrative Description

At the beginning of this class Mrs. Hanley gave the students a few minutes to work among themselves. She signalled the beginning of class instruction by erasing the board. She asked three students what problem they were on and then said she was concerned because some students were cancelling out in their problems before they factored. The class was studying algebraic simplification. In order to cancel out factors the terms must be expressed as multiplication problems. A common mistake was to cancel through addition and subtraction rather than through multiplication. The class instruction for this day consisted of working through problems which required algebraic simplification.

The problem in Episode #76 was the third example taken in class. Mrs. Hanley wrote the problem on the board (line 1) and used the social participation structure of whole class instruction while she directed the class from the front of the room. She asked questions at each step of the problem and performed the operation suggested by the student.

The first step as given by Sarah (line 2) was to factor each of the terms. Mrs. Hanley wrote out the factored form of the problem. She then stated two important points regarding the problem: to factor out the highest common factor (line 3) and to look for factors that cancel (line 4). Brenda named the factors that cancel and stated the answer to the problem (line 5). Mrs. Hanley restated the answer (line 6) and asked if the problem made sense (line 7). In order to reinforce the point of doing these problems (line 8), factoring before cancelling, Mrs. Hanley asked a student what the hardest part of the problem was (line 9). Alice answered correctly that it was finding the factors (line 10). The episode was concluded as a student asked for another problem and the class focused on it.

The outline for this episode is as follows:

Problem: Simplify $\frac{y - 3}{8y - 4} \cdot \frac{10y - 5}{5y - 15}$

1. Presenting the problem on the board.

   $(+)TC_B$ (teacher at board)

2. Factoring each expression.

   (+)SR (student right answer)

   (+)TC (teacher comment)

3. Cancelling out factors.

   (+)TC (teacher content)

   (+)SR (student right answer)

   (+)TC (teacher comment)

   (+)TF (teacher request for feedback)

   (+)TC (teacher comment)

   (+)TQ (teacher question

   (+)SR (student right answer)

(For the complete coding system refer to pgs. 81-87, in Chapter III.)

The underlying mathematics structure consisted of three steps to the solution of the problem: Mrs. Hanley presented the problem, the expressions were factored, and the appropriate factors were cancelled. The coded items of interaction showed that there were seven teacher coded items and three student coded items. The contingency table of communication acts is as follows:

	T	S	
+	7	3	
-	0	0	

The plus sign refers to those communication acts which contributed to the forward movement of the lesson. The minus sign refers to those communication acts which inhibit the forward movement of the lesson. The teacher had the most recorded communication acts and all were signed plus. All student communication acts were signed plus which demonstrated cooperation in the forward movement of the lesson. The pattern of enactment was all plus acts which indicated that the lesson progressed to completion without interference. The joint production of the lesson was accomplished through full cooperation by teacher and students.

A Representative General Math Episode

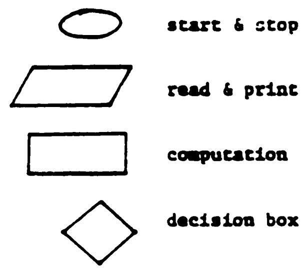
A representative general math episode was characterized by almost the same number of student communication acts as teacher communication acts, by the teacher directing instruction from the front of the room or, almost as frequently, by a group of students working at the board, and by interruptions produced by irrelevant comments of questions during the mathematics lesson inhibiting the progress of the class in solving the problem or problems being studied. Episode #22 G.M. is an example of a representative episode for the general math class.

Text of the Episode

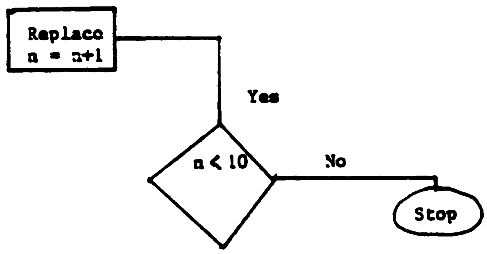
#22 11/5/79 pp. 9-10 General Math

Mrs. Hanley: "You might use this (rounding off numbers) when you are writing a newspaper article."

(+) SQ	(1)	1.)			Adam asked about flow charting.
(-) 0	(2)	2.)	10:11		The girl came in for the attendance slip. Mrs. Hanley did put it out there but it's gone.
				Mrs. H:	"Did you get the ones for first and third hour?"
				Girl:	"No."
				Mrs. H:	"Let me give you that, too. It might have been someone from second hour."
(+) TF	(3)	3.)		Mrs. H:	"Any more questions?"
(+) SF_+	(4)	4.)		Tony:	"No ma'm."
(+) TQ	(5)	5.)	10:13	Mrs. H:	"What about flow charting?"
(+) SF_+	(6)	6.)		Brian:	"We know how to do that."
(-) SF_-	(7)	7.)		Tony:	"No we don't."
(+) TC_B	(8)	8.)			<u>Board</u>



(-) SOT	(9)	9.)	Brent:	"Why don't you just give us the boxes?" (on the test).
(-) TOT	(10)	10.)	Mrs. H:	"I'll give you a problem to work out. It would be silly to give you the boxes and then a problem - you'd have everything right there."
(-) RB	(11)	11.) 10:16	Mrs. H:	"Brian!"
(-) SOL	(12)	12.)	Brian:	"I'm sorry."
(-) RB	(13)	13.)	Mrs. H:	"I know you are. Cut it out."
(+) TC_B	(14)	14.)		<u>Board</u>



(+) TC	(15)	15.)	Mrs. H:	"If you look back on all your worksheets and quizzes, I think you have enough problems to practice."

Mrs. Hanley: "Now, for today."

10:18 S: "Can we go to the board?"

Mrs. Hanley: "Why? You went on Friday."

Keith: "That was two days ago."

Narrative Description

Mrs. Hanley had announced that the next day there would be a test covering everything from the beginning of the year. She wrote nine topics on the board which would be on the test. She asked the class if there was anything they were not sure of at this point. Episode #22 contains the fourth topic to be reviewed with the class.

Adam asked about flow charting (line 1), but Mrs. Hanley's response was interrupted by the attendance girl (line 2). After the attendance was taken care of Mrs. Hanley asked if there were any more questions (line 3) and Tony answered no (line 4). This seems unrelated in the sequence of things because next Mrs. Hanley asked what about flow charting (line 5). Brian announced that the class knew how to do that (line 6) but Tony admitted that at least he didn't (line 7). Mrs. Hanley then reviewed the shapes used in making the flow charts by drawing them on the board and giving the operation each represented (line 8).

After she had done this Brent asked Mrs. Hanley why she didn't just give them the boxes on the test (line 9). Mrs. Hanley expected the students to have the boxes and operations memorized. Notice that this question was off task in that it was not related to the presentation of the content. Asking about the test at this point was off target. Mrs. Hanley chose to address Brent's question. She stated she would give them a problem. She commented that it would be silly to give them the boxes because they would have everything, implying it would not be much of a test (line 10).

The line of teaching was further interrupted as Mrs. Hanley called on Brian (line 11) who responded "I'm sorry" (line 12). Brian was misbehaving and Mrs. Hanley reprimanded him. His response was typical for the way he accepted reprimands. However, the effect never appeared to be long lasting. Mrs. Hanley replied to Brian with "I know you are" but followed it with "cut it out" (line 13). She acknowledged his contrite words but backed up her intentions with a stronger command.

After these two interruptions Mrs. Hanley returned to the content of the lesson and put part of a flow chart on the board (line 14). This demonstrated the use of the boxes and the operations involved. This episode was concluded by a statement by Mrs. Hanley telling the students how to prepare for the test (line 15). She mentioned worksheets and quizzes which were essential aspects of her teaching in the general math class and she stated that they had problems to practice. Practicing was a specific expectation that Mrs. Hanley had for this class. This question on flow charts ended the review section of the class and Mrs. Hanley began the presentation of new content for that day.

The outline for this episode is as follows:

Problem: Flow Charting

1. Presenting flow chart shapes on the board.

    (+)SQ (student question)

    (-)O (interruption from outside)

    (+)TF (teacher request for feedback)

    (+)SF_+ (student positive feedback)

(+)TQ (teacher question)

(+)SF_+ (student positive feedback)

(-)SF_- (student negative feedback)

(+)TC_B (teacher at board)

(-)SOT (student off target)

(-)TOT (teacher off target)

(-)RB (teacher reprimanding about behavior)

(-)SOL (student irrelevant remark)

(-)RB (teacher reprimanding about behavior)

2. Giving an example of flow charting.

(+)TC_B (teacher at board)

(+)TC (teacher comment)

The underlying mathematics structure consisted of two steps in reviewing flow charting. Mrs. Hanley first reviewed the geometric shapes used in this procedure and then gave an example for using the shapes. The coded items of enactment showed that there were eight teacher coded items and six student coded items. The contingency table of communication acts (with + for contributing to the forward movement and - for inhibiting the forward movement) is as follows:

	T	S	
+	5	3	
-	3	3	

The teacher had the most communication acts but the students were not far behind. There was interference in the forward movement of the lesson as indicated by the minus acts. The pattern of enactment as given by the outline showed that the lesson was interrupted at the beginning by outside interference and then was off task for a number of acts prior to the second step. Mrs. Hanley pulled the lesson back on task with the second step and the episode concluded on task.

The following extended contingency table gives an idea of the type of interference which existed in this episode.

		T	S
	+	5	3
(on task)	-	0	1
(reprimand for behavior)	-	2	0
(on task/ off target)	-	1	1
(off task)	-	0	1

Mrs. Hanley reprimanded a student twice and answered an off task question. A student gave negative feedback which was an on task minus communication act. One student asked an on task/off target question (a question related to the content but not to the point of the lesson) and another student made an irrelevant comment.

The interference created an uncooperative atmosphere about the episode. The students seemed to be working against the teacher in terms of bringing the task at hand to completion. There did not exist a sense of hostility in the responses of the students but more of an occupation with their own interests and goals. This occupation was not conducive to cooperation and consequently the lesson jointly produced contained interference in the forward movement and the need to regroup in terms of staying on task.

Collaboration

Collaboration is the joint effort between teacher and students in producing the mathematics lesson. The teacher plans and presents the mathematics content. The presentation includes a specific organization of the class in relationship to the teacher. This organization involves the roles of teacher and student as well as the acceptable rules for participating in the interaction of the lesson. The students, in this joint effort of producing the lesson, respond to the content and the organization of the class. Their responses may be cooperative in that they are paying attention to the teacher and trying to understand the content as well as interacting with the teacher according to the specified rules of participation. Or the responses of the students may be uncooperative in that they do not pay attention to the teacher or the content but engage in off task and disruptive behavior which in turn violates the rules of participation selected by the teacher.

Teacher/Student Interaction

The joint production of the mathematics lesson was recorded in the interaction between the teacher and the students. In this communication the kind of collaboration which existed could be identified. What was said and done by teacher and students with respect to the mathematics content determined the progress of the lesson toward its goal. Therefore, the type of communication was carefully analyzed.

The interaction was categorized in terms of communication acts. These acts were coded according to the coding system derived for this study. (See the discussion in Chapter III, pp. 81-87). The number of communication acts for each class was counted. For the 125 episodes of the algebra class there were 1613 coded communication acts. For the 100 episodes of the general math class there were 1218 coded communication acts. The percentage of coded items for teacher and student was strikingly different between the two classes.

Table 4.1

Communication Acts by Teacher and Students
Across all Episodes in Algebra and General Math

	Algebra N = 1613	General Math N = 1218
Teacher Acts	69%	52%
Student Acts	30%	46%
Total	99%*	98%*

*The difference from 100% in the totals was due to the acts coded 0 which were neither teacher acts or student acts and not included in the calculations given.

Note: Episodes number 125 for algebra and 100 for general math.

Note: Percentages were derived from the number of teacher or student acts over the toal number of acts for the respective class.

The teacher dominated the algebra class being responsible for 69% of the total number of communication acts and the students for 30%. For the general math class the teacher was responsible for 52% of the communication acts and the students for 46%. The students in general math had almost as many communication acts as the teacher and significantly more percentage-wise than the students in the algebra class. This indicated that the general math students interacted more in the classroom than did the algebra students. This aspect of classroom participation was again visible by summing the number of teacher communication acts and the number of student communication acts for a given episode and determining if teacher or students had the most coded items per episode. The following table shows the results.

Table 4.2

Episodes with the Highest Number of Communication Acts by the Teacher or by the Student or by Both

	Algebra N = 125	General Math N = 100
Teacher	86%	50%
Student	8%	44%
Teacher = Student	6%	6%
Total	100%	100%

Note: Percentages were derived from the number of episodes with teacher acts the highest, those with student acts the highest, and those with teacher = student acts, over the total number of episodes for the respective class.

The table shows that 86% of the algebra episodes had a greater number of teacher coded acts than student coded acts. There were only 8% of the episodes with student coded acts the highest and 6% of the episodes had the same number for teacher and students. For the general math class 50% of the episodes had a greater number of teacher coded acts and 44% of the episodes had more student coded acts. There were 6% of the episodes which had the same number of coded acts for teacher and students.

The teacher's dominance of the interaction in the algebra class was even more dramatic in this analysis. She had the greater participation in the vast majority of the algebra episodes making the student participation seem rather minor. In contrast the teacher and students were almost the same for dominance in the general math class. The extent of the student participation projected the possibility of a different kind of atmosphere in the general math class than the algebra class. To determine this difference it was important to look at the kind of communication acts which constituted this participation.

By looking at the categories of communication acts the algebra class had the highest percentages as follows:

26% TC - teacher content, comment, directions;

15% TC_B - teacher at board;

11% TQ - teacher question;

7% SR - student right answer; and

5% SR_B - student correct at the board.

Teacher comments, work at the board, and questions were the coding categories with the highest percentages of coded items. These were categories for instructional communication acts and supported the teacher's role of directing of instruction and guiding the content presentation in the lesson. The last two categories of the five groups, student right answers and student right answers at the board, indicated a response of the students in a cooperative content oriented manner. All five of these categories were related to on task communication and projected an academic nature to the interaction that occurred in the algebra class.

The general math class had the following results:

23% TC - teacher content, comment, directions;

11% TC_B - teacher at board;

10% SR_B - student correct at the board;

7% SOL - student off content, irrelevant remark or question;

5% TQ - teacher question.

Teacher comments, work at the board, and student right answers at the board were the coding categories with the highest percentages of coded items. Student participation was visible in the three highest items. These first three were on task categories and indicated academic orientation. The teacher does have the greatest percentage of items. The 10% for SR_B indicated time spent at the board by the general math students. The fourth highest category for general math was SOL which was student irrelevant comments. These were remarks which did not pertain to the task at hand and were the cause of interruptions in the forward movement of the lesson. By contrast the algebra class had only 1% of its coded items in the category

SOL. This high percentage of off task communication acts for general math indicated uncooperative responses on the part of the students in the joint production of the lesson. Teacher and students were not focused together on the content at these times and the lesson did not progress toward its goal. This inhibited the amount of content which may be presented and created situations which required energy for regrouping by the teacher and trying to regain the forward movement of the lesson. The fifth highest category of coded acts for general math was TQ which was teacher questions and was oriented to instruction. (See Appendix D for a complete table of percentages for coding categories.)

In the highest five categories, the general math class had a higher percentage of student coded acts than the algebra class, which supported the greater student participation in general math. However, a major part of this participation was off task communication which was inhibiting to the successful completion of lessons. Greater student participation in this case was a detriment to the classroom experience and a force working against the teacher in her efforts to reach the goal of each episode.

Each of the coded acts was signed as plus or minus according to the influence of the communication act upon the forward movement of the lesson. A communication act which inhibited the forward movement was signed as a minus (such as an SOL act) and one contributing to the forward movement was signed as a plus (such as an SR_B act). By looking at the plus and minus signs of the communication acts it was possible to further discriminate the nature of teacher and student participation.

Table 4.3

Communication Acts by Teacher and Students Across Episodes in Algebra or General Math which Contributed to the Forward Movement or which Inhibited the Forward Movement

Algebra N = 1605

	Teacher Acts	Student Acts
+	63%	22%
-	6%	9%
Total	69%	31%

General Math N = 1205

	Teacher Acts	Student Acts
+	47%	28%
-	6%	18%
Total	53%	46%

Note: + = communication acts that contributed to the forward movement
- = communication acts that inhibited the forward movement

Note: The acts in coding category 0 were not included in these calculations.

Note: Percentages were derived from the plus acts over the total acts and the minus acts over the total acts for the teacher and for the students.

Of the 1605 communication acts for the algebra episodes (the 8 items coded 0 were not included because the percentages were to be for teacher or student and these items were for neither) 63% were teacher plus acts, 22% were student plus acts, 6% were teacher minus acts, and 9% were student minus acts. The algebra class was engaged for the most part in plus communication acts which indicated that the interaction between teacher and students was task oriented and promoted the forward movement of the lesson. The percentage of minus acts indicated there were some interruptions at times.

Of the 1205 communication acts for the general math class (the 13 items coded 0 were not included) 47% were teacher plus acts, 28% were student plus acts, 6% were teacher minus acts, and 18% were student minus acts. The general math class showed a majority of plus communication acts which indicated on task interaction between teacher and students. However, the percentages again showed the greater extent of student participation in general math than in algebra. This set of percentages indicated that about two fifths of the student participation for general math were minus acts. There was a much greater possibility for interruption in the forward movement of the lesson for general math than algebra. The percentages showed there were twice as many minus acts on the part of the students for general math than for algebra.

The minus coded acts included both wrong answers or reprimands about content and off task comments or questions. Wrong answers and reprimands about content were still on task communication acts although they created interruptions in the forward movement of the lesson. Because of this distinction it was of considerable interest to study the minus acts in terms of on task and off task responses. The following was the set of results.

Of the 138 student minus acts for the algebra class 69% were about content, that is wrong answers, negative feedback or an I-don't-know response. For the algebra students 31% were off task communication acts which included SOT (student off task), SOL (student off content), and GN (student generally off content) acts. Twice as many minus acts were on task than off task in algebra.

Of the 225 student minus acts for the general math class 39% were about content and 61% were off task communication acts. There were significantly more off task acts than on task acts for the general math class. There were almost twice as many off task acts

Table 4.4.

Communication Acts which were Inhibitive to the Forward Movement of the lesson by On Task Interference or Off Task Interference

Algebra

	Teacher N = 101	Student N = 138
On Task	46%	69%
Off Task	54%	31%
Total	100%	100%

General Math

	Teacher N = 69	Student N = 225
On Task	30%	39%
Off Task	70%	61%
Total	100%	100%

Note: Percentages were derived from the number of on task minus acts (acts related to content) over the total number of minus acts for teacher and for student and the number of off task minus acts (irrelevant to the lesson) over the total number of minus acts for teacher and for student.

in general math than in algebra. This again reflected the type of communication included in the increased student participation in the general math class and accounted for the interruptions in the progress of the mathematics lessons.

Of the 101 teacher minus acts for the algebra class, 46% were about content which included reprimands to students about content and teacher wrong answers. For algebra, 54% were off task communication acts which included reprimands to the students for behavior as well as teacher off task remarks. Student reprimands amounted to 26% of the teacher off task acts and teacher off task remarks (TOT, TOL) were 29% of the acts. Teacher off task remarks in the algebra class were usually comments about the book, joking with a student, or responding to a student's off task comment. The teacher would sometimes express her opinion about something which was not related to the progress of the lesson.

Of the 69 teacher minus acts for the general math class, 30% were about content and 70% were off task communication acts. Student reprimands amounted to 30% of the teacher off task acts and teacher off task remarks (TOT, TOL) were 39% of the acts. Teacher off task remarks in the general math class were usually responses to student off task remarks. The teacher seldom joked with a student or offered her opinion about something to the general math class.

The teacher minus communication acts were a response to the student minus acts. This relationship was a good example of the kind of interaction involved in collaboration. The teacher presented the content, the students responded, and the teacher responded to the students. If student responses were content oriented (cooperative in the plan of the lesson), as in algebra, the teacher continued the presentation in response to the students and the lesson moved forward. Although there was a break due to a wrong answer, the content was still the focus of the lesson. Interestingly when the students were on task there was the freedom on the part of the teacher to insert an opinion or unrelated comment without losing the class. However, for this study, such comments were considered an interruption in the progress of the lesson and signed as a minus.

If the students were not focused on the content as in general math, then the teacher was pulled off task more frequently and spent time trying to repair the continuous interruptions from the students. The teacher still presented the content but was pulled off the task with each interruption and was constantly trying to keep the class focused on the subject matter. The large percentage of off task communication acts indicated this kind of interaction between teacher and students in the general math class. Collaboration was without cooperation and teacher and students struggled to maintain adverse agendas for the mathematics lesson.

Patterns of Enactment

The patterns of enactment were displayed in the outlines of the episodes. These patterns demonstrated the joint production of the mathematics lessons. The outlines showed the mathematical steps (the teacher's presentation of content) and the script (the coded items in the interaction about the content by teacher and students) of the episode. The pattern of plus and minus signs for the coded items showed the process of collaboration between teacher and students and gave an indication of the forward movement of the lesson with the interference which was encountered.

The outlines were studied for categories of similar patterns. Because the focus of the study was the collaboration between teacher and students in producing the lesson, the patterns were classified according to the forward movement of the lesson. This classification developed into those patterns without interruptions (outlines with all plus signs), those patterns with interruptions concerning on task acts, those patterns with interruptions concerning off task acts, and those patterns which contained both types of interruptions (on task and off task). (See the discussion in Chapter III pp. 104-105.)

Table 4.5

Episodes According to the Patterns of Enactment Determined by the Minus Acts in the Forward Movement of the Lesson

	Algebra N=125	General Math N=100
1. No Interruptions	34%	20%
2. On Task Interruptions	27%	22%
3. Off Task Interruptions	16%	31%
4. On Task and Off Task Interruptions	22%	27%
Total	99%*	100%

*Due to rounding off to the second place the percentage does not equal 100%.

Note: Percentages were derived from the number of episodes of a pattern of enactment over the total number of episodes for the respective class.

Of the 125 algebra episodes, 34% had no interruptions. The forward movement was direct and the goal was accomplished. Of the 100 general math episodes, 20% were without interruptions. This indicated that the general math class was capable of cooperating with the teacher but did so to a far lesser degree than the algebra class.

Episodes which had only interruptions concerning content were episodes focused on the subject matter but without a direct movement to the goal. For the algebra class this involved 27% of the episodes and for general math 22% of the episodes. From the perspective of on task lessons the percentages of these two patterns of no interruptions and just on task interruptions indicated that 61% of the algebra episodes and 42% of the general math episodes were focused on content without interruption by off task acts. Over half of the algebra episodes were in this focus and less than half for general math. This would indicate that the algebra students were more academically oriented and were more in agreement with the teacher's presentation of the mathematics.

On the other hand, 38% of the algebra episodes and 58% of the general math episodes had patterns of enactment which included interruptions due to off task communication acts. For algebra, 16% of the episodes had just off task minus acts while general math had 31% of the episodes with just off task minus acts. This was almost twice as many for general math as for algebra. This showed that the amount of off task communication acts was distributed throughout the mathematics lessons for general math. The off task acts were a characteristic of the general math class and pervaded the episodes. This was supported by the 27% general math episodes with both on task and off task interruptions.

Collaboration for algebra was a cooperative, positive joint effort. For general math the students seemed to be working against the teacher more than with the teacher. The collaboration for general math was more of an adversary role (although without hostility) than a cooperative role. The students were capable of collaboration but they opted not to sustain this role of cooperation in the majority of instances analyzed for this study.

A Nonrepresentative Episode for General Math

Because of this option of not cooperating, an episode without interruptions in general math was a nonrepresentative episode for the class. However, it is thought provoking to note the contrast of the usual with the sometimes possible. Episode #25 G.M. is an example of a nonrepresentative episode for the general math class. The students were cooperative and collaborated with the teacher in creating a forward moving, on task lesson which reached its completion without interference. The fact such episodes did exist for general math focused more emphasis on the adversary role of the general math students which was mentioned earlier. The students were capable of cooperating but opted not to do so most of the time. Episode #25 G.M. shows teacher and students working together on the content to complete the task of the lesson.

Text of the Episode

#25 11/13/79 pp. 7-8 General Math

Mrs. Hanley: "If you start with smaller ones and write the larger ones then you'll meet in the middle and be all done."

9:57 Mrs. Hanley: "These are two things that we have learned" (factoring to primes and finding the set of all factors).

(+) TC (1) 1.) Mrs. H: "Another thing we have learned is the lowest common multiple."

(+) TC_B (2) 2.) Board

Find the lowest common multiple for 27 and 45.

(+) TC (3) 3.) Mrs. H: "You always find the lowest common multiple for two numbers."

(+) TC_B (4) 4.) Board

27	45
$9 \quad 3$	$9 \cdot 5$
$3 \cdot 3 \cdot 3$	$3^2 \cdot 5$
3^3	

(+) TC_B (5) 5.) $3^3 \cdot 5 = 135$

$$\begin{array}{r} 27 \\ \underline{5} \\ 135 \end{array}$$

(+) SF_+ (6) 6.) Chad: "I gotcha now."

(+) SQ (7) 7.) Brent asked about twenty-seven.

(+) SP (8) 8.) Keith asked why it wasn't three to the fourth power.

(+) TP (9) 9.) Mrs. H: "We are in the business of finding the lowest common multiple."

(+) TC (10) 10.) "If we are looking for the lowest common denominator of 8/27 + 13/45, then the lowest common denominator is the lowest common multiple."

(+) TC_B (11) 11.) Board

8/27 + 13/45

40/135 + 39/135 = 79/135

(+) TE (12) 12.) Mrs. H: "All right. We've had quite a bit of practice with these already. You are expected to know this as of today."

10:03 Mrs. Hanley: "We are going to do something new today. We are going to look for the highest common factor."

Narrative Description

The new content for the class this day was how to find the highest common factor for two numbers. In order to teach that procedure Mrs. Hanley reviewed some concepts and problems she had already taught to the class. The first part of the review was factoring a number to prime numbers and then listing all of the factors. She used thirty-six as an example. The second part of the review was how to find the lowest common multiple. This was the content for Episode #25.

Mrs. Hanley began the episode by stating that the class had learned how to find the lowest common multiple (line 1). She then wrote a problem on the board (line 2). She commented that two numbers were needed (line 3). On the board were two numbers factored to primes which was reviewed just previously in the class (line 4). Unfortunately, it was not noted as to whether the students helped do the factoring or whether Mrs. Hanley put it on the board. It would be more like Mrs. Hanley to have interacted with the students while working out the factoring. After the prime factorization was found (the product in primes), then the two numbers were looked at together. The lowest common multiple was the smallest number which both twenty-seven and forty-five will divide evenly. The smallest set of prime factors that contained the prime factorization of the two numbers gave the answer (line 5). The solution $3^3 \cdot 5$ contains 3^3 (=27) and $3^2 \cdot 5$ (=45) and equals 135. The lowest common multiple for 27 and 45 is 135.

Chad commented that he got it now (line 6), implying he had some question before this. Brent asked a question about 27 (line 7).

Keith wanted to know why the answer wasn't $3^4 \cdot 5$ (line 8). It's not obvious why he asked this question unless he wanted to multiply another three to the prime factorization of forty-five ($3^2 \cdot 5$) in order to make it $3^3 \cdot 5$. This extra three would then have to be included in the final answer. Mrs. Hanley responded by stating that the lowest common multiple was the solution (line 9), which left the student to deduce that $3^4 \cdot 5$ was too big to be the answer.

Mrs. Hanley then applied the concept of lowest common multiple in a familiar problem situation, addition of fractions. The lowest common denominator is the same as the lowest common multiple (line 10). The example Mrs. Hanley used (8/27 + 13/45) has the two numbers just worked with previously as the denominators of the fractions. She used the lowest common multiple as the lowest common denominator, changed the numerators accordingly and added the two fractions. The application of the concept was completed (line 11).

Mrs. Hanley then exhorted the students about knowing how to find the lowest common multiple (line 12). She stated that they had had quite a bit of practice already and told the class as of today they were expected to know this process. This exhortation concluded the episode. Mrs. Hanley then announced they were going to learn something new which was the highest common factor of two numbers.

The outline for this episode is as follows:

Problem: Find the Lowest Common Multiple for 27 and 45

1. Presenting the problem on the board.

   (+)TC

   (+)TC_B

   (+)TC

2. Factoring 27 and 45 to primes.

   $(+)TC_B$

3. Finding the lowest common multiple for 27 and 45.

   $(+)TC_B$

   $(+)SF_+$

   (+)SQ

   (+)SP

   (+)TP

4. Applying the lowest common multiple to 8/27 and 13/45.

   (+)TC

   $(+)TC_B$

   (+)TE

The underlying mathematics structure consisted of four steps in the presentation of the procedures. Mrs. Hanley wrote the problem on the board, factored each number, found the solution and applied the process to a familiar problem. The coded items of enactment showed that there were nine teacher coded items and three student coded items. The contingency table of communication acts is as follows:

	T	S	
+	9	3	
-	0	0	

The teacher dominated the lesson with all plus communication acts. All students acts were plus. This was atypical of the general math class. The teacher seldom had this many more communication acts than the students, nine to three, and the students seldom had only plus communication acts. However, the cooperation given by the students and the pattern of enactment of all pluses, indicating forward movement without interference, was possible in the general math class. The students were capable of being on task and helpful in the collaboration of the lesson.

A Nonrepresentative Algebra Episode

On the other hand, an algebra episode with off task interruptions was a nonrepresentative episode for that class. Episode #19 Alg. is an example of such a nonrepresentative episode. The students exhibited disruptive behavior and interference showing a lack of cooperation in the forward movement of the lesson. The fact such episodes existed for algebra focused emphasis on the cooperation which was exhibited most of the time. The students were capable of being disruptive but they opted for the most part to cooperate with the teacher. Episode #19 Alg. shows the teacher and students at odds with each other in the joint production of the mathematics lesson.

Text of the Episode

#19 11/5/79 p. 4 Algebra

9:29 Mrs. Hanley: "When numbers are always paired with the same number we say it is constant. This is called the constant function."

Code		Line	Time	Speaker	Text
(+) TC_B	(1)	1.)			**Board** $\{(1, 5), (3, 9), (5, 13)\}$ $D = \{1, 3, 5\}$ $R = \{5, 9, 13\}$
(+) TC	(2)	2.)		Mrs. H:	"The domain is the set of first numbers." "The range is the set of second numbers."
(+) TC_B	(3)	3.)			**Board** $g: y \rightarrow y+2$ $D = \{1, 2, 3\}$
(+) TC	(4)	4.)		Mrs. H:	"Darin."
(−) SOL	(5)	5.)		Darin:	"He took my pencil."
(+) TC	(6)	6.)		Mrs. H:	"Darin."
(−) SOL	(7)	7.)		Darin:	"He took my pencil."
(+) TQ	(8)	8.)		Mrs. H:	"What do you do here?" (referring to the problem).
(−) SF_o	(9)	9.)		Darin:	"I don't know."
(−) RB	(10)	10.)		Mrs. H:	"I don't like the way you are conducting yourself in this class. That goes for you, too, Kevin."
(+) TC_B	(11)	11.)			**Board** $g: y \rightarrow y+2$ $D = \{1, 2, 3\}$ $1 \rightarrow 3$ $\{(1, 3), (2, 4), (3, 5)\}$ $2 \rightarrow 4$ $R = \{3, 4, 5\}$ $3 \rightarrow 5$
(+) TE	(12)	12.)	9:34	Mrs. H:	"Some of you are probably thinking this is so easy. It is not hard to do. What you have to figure out is what to do."

Mrs. Hanley: "Your written exercises are on pages one forty eight and one forty nine."
"We'll finish this up tomorrow. Your test is on Wednesday."

Narrative Description

At the beginning of this class Mrs. Hanley returned a test. She took some time to answer questions and as a result worked four problems for the class. After this session she began new material which was the introduction to the concept of function. She explained by giving a familiar example (a mother relationship - matching student with mother) and then transferring the concept to matching numbers with a rule or relationship.

She worked through matching a number with its double and taught the appropriate notation. Eventually she gave the mathematical definition and went through some oral exercises in the book.

After the oral exercises Episode #19 begins. Mrs. Hanley put a function (a set of ordered pairs) on the board (line 1). She explained that the domain was the set of first numbers in the ordered pairs and listed those numbers in a set. She also explained that the range is the set of second numbers in the ordered pairs and listed those numbers in a set (line 2). She gave a rule for finding a function and gave a domain to use as the first numbers in the ordered pairs (line 3). The students were to take a number in the domain, apply the rule (in this case add two) and get the second member of the ordered pair. By writing these two numbers in the proper notation an element of the function would be named.

Mrs. Hanley called on Darin to find the first element (line 4). However, Darin was distracted by the fact that someone (probably Kevin) took his pencil. He stated this (line 5). Mrs. Hanley repeated his name (line 6) and he repeated his first statement (line 7). Mrs. Hanley didn't respond to his remark but asked what to do in the math problem (line 8). Darin (usually honest when put on the spot) answered that he didn't know (line 9). Mrs. Hanley had given him an out but since he didn't respond to it she reprimanded him and Kevin for their off task behavior (line 10). This was a rather crucial point in the class to lose the students attention. It was the first problem in writing a function after having been given the introduction.

Mrs. Hanley proceeded to write out the ordered pairs and to indicate the range (the set of second numbers) (line 11). It was not possible to know if student help was elicited during this time. It probably was since Mrs. Hanley generally instructed through interaction with the students. Mrs. Hanley commented on the simplicity of the problem and the fact the students probably thought this was easy (line 12). She admitted it wasn't hard and added they had to figure out what to do. Having been given the rule and a set of numbers, the hard part was knowing how to start. This advice concluded the episode. Mrs. Hanley then gave the assignment and reminded the students about a test coming up that week.

The outline for this episode is as follows:

Problem: $g: \ y \to y + 2$

1. Giving an example of a function.

   $(+)TC_B$

   $(+)TC$

2. Presenting the problem on the board.

   $(+)TC_B$

3. Identifying the ordered pairs.

   $(+)TC$

   $(-)SOL$

(+)TC

(-)SOL

(+)TQ

(-)SF_o

(-)RB

(+)TC_B

(+)TE

The underlying mathematics structure consisted of three steps. Mrs. Hanley gave an introduction by showing an example, presented the problem to be solved and identified the elements of the function. The coded items of enactment showed that there were nine teacher coded items and three student coded items. The contingency table of communication acts is as follows:

	T	S	
+	8	0	
-	1	3	

The teacher dominated the lesson with eight plus coded items and one minus. However, there were no plus coded items for the students and all students acts were minus. This was most unusual for the algebra class. By looking more closely at the minus acts the following extended contingency table resulted.

		T	S
	+	8	0
(on task)	-	0	1
(reprimand behavior)	-	1	0
(off task)	-	0	2

Only one of the three student minus acts was on task and this was Darin's I don't know. The other two responses were off task. The teacher minus coded item was the reprimand for behavior that she gave the two boys. There was no recorded cooperation from the students during this episode and two out of the three recorded acts were off task communication. This produced a pattern of enactment which showed much interference in the forward movement of the lesson. The joint production of the lesson was not a cooperative collaboration but an effort by the teacher to keep the students on task which was not the usual case for the algebra class. (See Appendix E for further examples of Patterns of Enactment.)

The Mathematics Structure of an Episode

In the joint production of lessons the teacher prepares and presents the mathematics content. The purpose of teacher and students coming together is to interact about the subject matter with teacher as authority and students as learners. Therefore each lesson must be structured to some extent on the content being studied.

Each episode was studied from the perspective of the mathematics involved in the lesson and the format used by the teacher for presenting the mathematics. From the beginning to its conclusion each episode was outlined according to the procedure used by the teacher in the classroom presentation of the content. The procedure was written in the form of steps taken in a logical development. These steps provided the underlying mathematics structure of the lesson and were used as representative of the teacher's logical presentation of content. The results were as follows.

Table 4.6

Episodes According to the Mathematical Structure (Steps) in the Teacher's Logical Presentation of Content

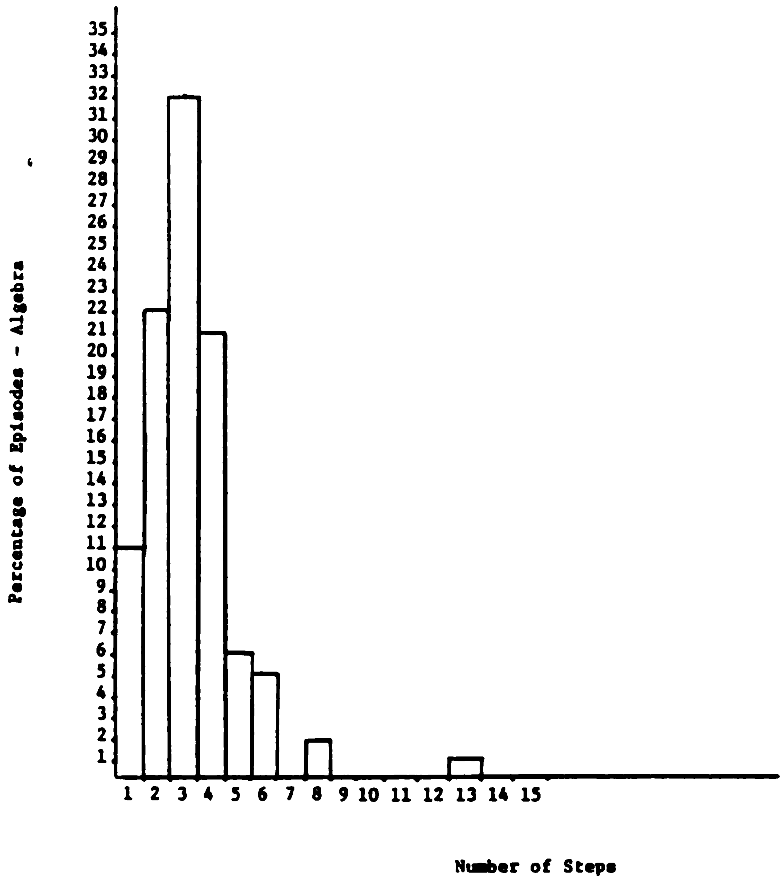
STEPS	Algebra N=125 episodes	General Math N=100 episodes
1	11% (14)	38% (38)
2	22% (28)	24% (24)
3	32% (40)	16% (16)
4	21% (26)	15% (15)
5	6% (8)	4% (4)
6	5% (6)	1% (1)
7	0% (0)	1% (1)
8	2% (2)	1% (1)
13	1% (1)	0% (0)
Total	100% (125)	100% (100)

NOTE: Percentages were derived from the number of episodes with a certain number of steps over the total number of episodes for the respective class.

For the algebra class the number of steps ranged from one to thirteen. The mode for this class was three steps, involving 32% of the episodes. Within a mathematical structure of three steps the teacher presented the problem and then directed the solution of the problem by means of the subsequent steps. The complexity of the content and the teacher's orientation to procedures and demonstration of procedures was indicated by these steps. Figure 4.1 shows the variability of the steps used by the teacher. Attention to demonstration and procedure was extended to four steps for 21% of the episodes. The 22% of the episodes for two step lessons involved problems done at the board, problems worked orally from the book, and the presentation of new concepts. Many of these two step lessons were not demonstration lessons involving a given procedure for problem solution.

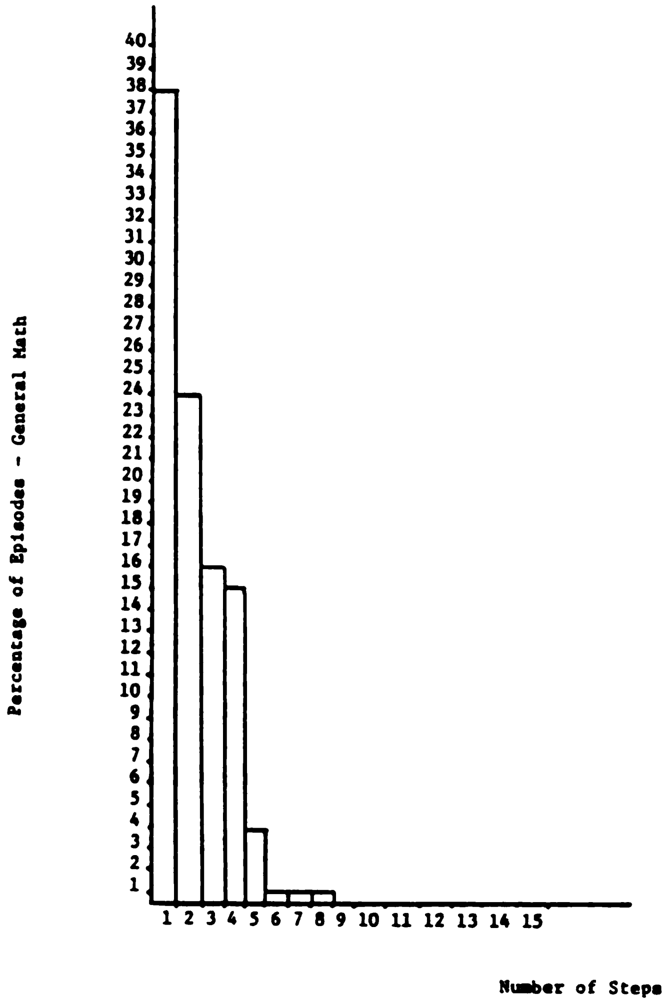
For the general math class the number of steps ranged from one to eight. The mode for this class was one step. The one step episodes consisted of the presentation of the problem by the teacher and then no further guidance as the problem was solved by a student or group of students at the board. The one step structure was indicative of the orientation for this class to practice doing problems. Figure 4.2 shows the variability of the steps used by the teacher for the general math class. The 24% of the episodes for two step lessons involved problems worked orally, problems worked by the class first, and other lessons at the board. This use of one or two steps in a lesson indicated a lack of direction on the part of the teacher and simple or familiar content which did not require demonstration or procedures in solving the problems.

The teacher gave more direction in how to work the problems and gave more steps to follow in solution procedures for algebra than she did for general math. This was partly because the content was more complex for algebra and needed more steps as a guide. It could be also that the teacher felt that the general math students wouldn't remember nor could they follow steps or procedures. There were more



Note: Percentages were derived from the number of episodes with a certain number of steps over the total number of episodes. N = 125

Figure 4.1 Distribution of Algebra Episodes According to the Number of Steps in the Teacher's Logical Presentation of Content



Note: Percentages were derived from the number of episodes with a certain number of steps over the total number of episodes. N = 100

Figure 4.2 Distribution of General Math Episodes According to the Number of Steps in the Teacher's Logical Presentation of Content three step episodes in algebra than any other number of steps with the majority of episodes having 2 to 4 steps. The general math class had the majority of episodes with one or two steps. For the general math class this was due to the work at the board which usually had the step of presenting the problem and then no further direction in solving it. There was a sense that most of the content presented was of the nature of practice for the general math class. Problems were relatively simple and given as a means of review or clarification. The algebra content was usually presented as demonstration, using the steps and pointing out errors and misunderstandings. In the algebra class the teacher used mistakes as a basis for teaching. She seldom drew attention to errors in the general math class.

The teaching of mathematics content for algebra was the focus of the classroom experience. The teacher demonstrated problems and walked the students through the procedures. The general math class practiced doing problems. The teacher did not emphasize procedures and did not spend that much time in demonstration.

The teacher taught the two classes differently. The two classes responded differently to the teacher and the content. A good example of these differences can be shown by looking at episodes in which the teacher taught the same content to both classes. The content of the following episodes was the mathematical theorem that any integer can be expressed as a unique product of prime integers. The teacher began both classes by teaching or reviewing what it means to factor a number and then to factor a number to primes. The theorem was to be the conclusion from these exercises.

For the algebra class this content was taught on January 29, 1980 and Episode #63 Alg. contains the actual lesson.

Text of an Algebra Episode

#63 1/29/80 pp. 1-2 Algebra

Mrs. Hanley pointed out the times for a quiz. She said to the class, "The pace will probably be uncomfortable for you at times."

(+) TC (1) — 1.) 9:00 Mrs. H: "The first thing we have to do is distinguish between addition and multiplication."

(+) TC_B (2) — 2.) Board

Distinguish between factoring and multiplying

	factors		product
Multiply:	4 · 6	=	24

	verb	product		factors	noun
To Factor:		42	=	7 · 6	

(+) TC (3) — 3.) Mrs. H: "You start out with a product and come up with factors, numbers that you multiply to get the product."

(+) TC_B (4) — 4.) 9:04 Board

3	12	19	24		
	3 · 4		6 · 4	=	2 · 3 · 2 · 2
	6 · 2		3 · 8	=	3 · 4 · 2
	3 · 2 · 2			=	3 · 2 · 2 · 2
	3 · 2 · 2		12 · 2	=	6 · 2 · 2

(+) TC (5) — 5.) Mrs. H: "You will notice that although we started out with different factors we came up with the same answer."

(+) TQ (6) — 6.)
(+) SR (7) — 7.)
(-) SOL (8) — 8.)
Mrs. Hanley asked Luke for a different factorization of twenty-four than six times four. He answered twelve times two. Roger said, "Wow."

(-) RB (9) — 9.) Mrs. H: "Roger, was that you? I think that is an unnecessary distraction."

(+) TC (1) — 10.) "The fact that you can start out with different factors and come up with the same answer in the end we summarize in this statement." (Mrs. Hanley wrote the following on the board.)

(+) TC_B (11) — 11.) Board

Any integer can be expressed as a unique product of prime integers.

(+) TC (12) — 12.) In reference to this statement Mrs. Hanley said, "That means take it down as far as it can go."

(+) TC (13) — 13.)
(+) TC (14) — 14.)
Mrs. Hanley explained that unique means when you get to the bottom of the factoring everyone gets the same answer. She commented that this will be important later on.

(+) TQ (15) — 15.),16.) 9:10 Mrs. H: "Seven times what is three?" No one answered. She asked them if three over seven worked.
(+) TQ (16)

(+) TC (17) — 17.)
(+) TC (18) — 18.)
"When I said factor over here you all assumed I meant integers." She explained why integers had to be in the definition, referring back to the example of seven times what equals three.

9:13 Mrs. Hanley talked about factors and where they existed. She said that if we think of monomials then the factors are obvious to us.

Narrative Description

Mrs. Hanley had passed out assignment sheets and spoken to the class about the amount of work yet to be covered during the year. She mentioned they would have to push to get finished and told the class, "The pace will probably be uncomfortable for you at times." Having said this the episode began.

Mrs. Hanley directed the instruction from the front of the room. She stated that first they had to distinguish between addition and multiplication (line 1). It is unclear what she meant by that and the rest of the episode gives no evidence of how she did it. Mathematically addition has addends (numbers added together to get a sum) and multiplication has factors (numbers multiplied to get a product). Whether this was her distinction is unknown. Her next step was to distinguish between factoring and multiplying, which she did on the board (line 2). Multiplying starts out with factors and produces a product. Factoring starts out with the product and produces the factors (line 3).

With this distinction, Mrs. Hanley put four numbers on the board to be factored (line 4). Two of the numbers 3 and 19 could not be factored except by themselves and one, which Mrs. Hanley did not record. She completed factoring 12 to primes by using two different sets of factors ($3 \cdot 4$ and $6 \cdot 2$). She pointed out that although the two sets of factors at first were different the factoring to primes was the same (line 5).

Up to this point there had not been any recorded student participation. It can be reasonably assumed that Mrs. Hanley was interacting with the students up to this point as well as after.

Twenty-four was the other number to be factored. Two sets of two factors ($6 \cdot 4$ and $3 \cdot 8$) had already been factored to primes. Mrs. Hanley asked Luke for another set of two factors (line 6) and he said twelve times two (line 7). Roger, who usually tried to get Mrs. Hanley's attention whenever he could, said Wow (line 8). Mrs. Hanley addressed Roger and told him that was unnecessary (line 9). The incident was dropped and Roger did not again appear in the episode. Mrs. Hanley again called attention to getting the same set of primes for each factorization for a given number (line 10) and wrote the theorem on the board (line 11). She simplified the statement by saying, "That means take it down as far as it can go" (line 12). She explained what unique means (line 13) and commented on this being important (line 14). Notice how much direct teaching Mrs. Hanley was doing. There was no indication of lack of attention or off task communication from the students. The subject matter presentation was straightforward and clearly organized.

She then addressed the reason for stating that the product be prime <u>integers</u>. She did this rather indirectly by establishing the following relationship. She asked for a number such that seven times the number is three (line 15). No one answered. Mrs. Hanley suggested the fraction three sevenths, which does work (line 16). She then explained why integers were required by the definition (line 17) using her example of seven times what equals three (line 18). Without integers in the definition fractions would be allowed and the final factorization need not then be unique. For example, $12 = 3 \cdot 2 \cdot 2/3 \cdot 3/2 \cdot 2$. A prime integer could then be expressed as a product other than itself and one. For example $3 = 3 \cdot 1$ but also $3 = 7 \cdot 3/7 \cdot 1$

or $3 = 4 \cdot 3/4 \cdot 1$.

Mrs. Hanley concluded the episode with her explanation about integers. The class continued as she talked about factors in algebraic expressions.

For the general math class this content was taught on November 5, 1979. The complete teaching of the theorem was contained in two episodes, Episode #23 G.M. and Episode #24 G.M. which were consecutive in the November 5 class. Both episodes will be presented here.

Text of Two General Math Episodes

#23 11/5/79 pp. 10-11 General Math

Mrs. Hanley: "What if I said book four. You don't know what to do."

There is a continuous stream of comments.

Code			Speaker	Text
(+) TC	(1)	1.)	Mrs. H:	"Let's take these numbers."
(+) TC_B	(2)	2.)		**Board**
				15
				$3 \cdot 5$ $1 \cdot 15$
				24
(+) TC_B	(3)	3.)		$8 \cdot 3$ $6 \cdot 4$ $12 \cdot 2$ $24 \cdot 1$ $3 \cdot 4 \cdot 2$
				$2 \cdot 4 \cdot 3$ $2 \cdot 3 \cdot 4$ $3 \cdot 2 \cdot 2 \cdot 2 = 3 \cdot 2^3$
				$2 \cdot 2 \cdot 2 \cdot 3$ $2 \cdot 3 \cdot 2 \cdot 2$
(+) TC	(4)	4.)	Mrs. H:	"All right. Factor twenty-four, Leah."
(+) SR	(5)	5.)		(She does.)
(+) TC	(6)	6.)		"Factor twenty-four, Sadie."
(-) SW	(7)	7.)	Sadie:	"That's all."
(+) TC	(8)	8.)	Mrs. H:	"That's all using two numbers."
(+) TC_B	(9)	9.)		"When you get the numbers down to prime numbers they are all the same. There is only one way to write the number
(+) TC^B	(10)	10.)		in prime factors."

10:24 Mrs. Hanley: "All right let's try another number."
"Question: Is this a prime factorization of thirty-six?"

#24 11/5/79 pp. 10-11 General Math

Mrs. Hanley: "When you get the numbers down to prime numbers they are all the same. There is only one way to write the number in prime factors."

Code			Time	Speaker	
(+) TC	(1)	1.)	10:24	Mrs. H:	"All right let's try another number."
(+) TQ	(2)	2.)			"Question: Is this a prime factorization of thirty-six?" (4 x 9)
(+) SR	(3)	3.)		Ss:	"No."
(+) TQ	(4)	4.)		Mrs. H:	"Is this?" (6 x 6)
(-) SW	(5)	5.)		Ss:	"Yes."
(+) TQ	(6)	6.)		Mrs. H:	"What is a prime number?"
(+) SQ	(7)	7.)		Brent:	"Is one a prime number?"
(-) TW	(8)	8.)		Mrs. H:	"I think one is a prime. It follows the rule of a prime number. Yes - I think it is a prime. Two is the only even prime number."
(+) TC	(9)	9.)			
			10:26		Students are at the board. (Sadie, Leah, Brian).
(-) RB	(10)	10.)		Mrs. H:	"Renea, keep your mouth shut, darling.
(-) SOL	(11)	11.)		Class:	"Yeah, yeah."
(-) SOL	(12)	12.)		Tony:	"Factor them babies."
(+) SR_B	(13)	13.)			Board
(+) SR_B	(14)	14.)			36
(+) SR_B	(15)	15.)			(see board work below)

36

4 · 9 → 2 2 3 3 → $2^2 \cdot 3^2$ (Leah)

6 · 6 → 3 2 3 2 → $3^2 \cdot 2^2$ (Sadie)

36 · 1   18 · 2 → 9 2 2 → 3 3 2 2 → $3^2 \cdot 2^2$ (Brian)   12 · 3

Code			Time	Speaker	
(+) TF	(16)	16.)	10:28	Mrs. H:	"Do you see when they factored it to prime they all got the same answer?"
(+) SF_+	(17)	17.)		Ss:	"Yeah."
(+) TQ	(18)	18.)		Mrs. H:	"Do you know what unique means?"
(+) GP	(19)	19.)			(Several answers.)
(+) TQ	(20)	20.)			"Is there a unique way to factor thirty-six?"
(+) SR	(21)	21.)		Ss:	"No."
(+) TQ	(22)	22.)		Mrs. H:	"Is there a unique way to factor thirty-six to prime?"
(+) SR	(23)	23.)		Ss:	"Yes."

Mrs. Hanley: "Incidentally we are in chapter five now."
"Now listen. I am getting impatient with both of you" (Chad and Brent).

Narrative Description

At the beginning of this class Mrs. Hanley announced that there would be a test the next day. She wrote nine topics for review on the board and asked if there was anything the students wanted reviewed. She went through four of the nine topics before she started the new material for the day. She had written on the board the words Factors - noun, and factor - verb. It was at this point that Episode #23 began. Mrs. Hanley had introduced factoring so she began by doing some examples (line 1). She directed instruction from the front. She factored 15 into 3 · 5 and 1 · 15 (line 2). It was interesting to note that the factors 15 · 1 were written in general math but not in the algebra class. The next number she took was 24 (line 3). She asked a student to factor 24 (line 4). After all the pairs of factors were mentioned she asked for another set of factors (line 6). Sadie said that was all the factors (line 7) and Mrs. Hanley replied that that was all using two numbers (line 8). From there the prime factors were written out (line 9). At this point Mrs. Hanley pointed out that all the prime factorizations were the same (line 10). She stated the theorem simply by saying there was only one way to write the number in prime factors. This statement ended this episode. Mrs. Hanley then suggested taking another number. The interaction about this new number determined Episode #24.

The new number was 36 (line 1). On the board were written five pairs of factors for 36 (4 · 9, 6 · 6, 36 · 1, 18 · 2, 12 · 3). Mrs. Hanley directed instruction from the front. She pointed to 4 · 9 and asked if it was a prime factorization of 36 (line 2). Some students answered no (line 3), which was correct. She then asked if $6 \cdot 6$ was a prime factorization of 36 (line 4). Some students answered yes (line 5), which was incorrect. Mrs. Hanley tried to remedy the situation by asking the definition for a prime number (line 6). With the definition they could see that six was not a prime number so $6 \cdot 6$ could not be a prime factorization.

What happened here was a good example of general math diversion. Instead of answering the question Brent asked a question. He asked if one was a prime (line 7). This was a tricky question to ask a math teacher because one is a special case with respect to primes and therefore ambiguous. The answer to the question is no, one is not a prime number. Two is the smallest prime. However, the question struck ambiguity for Mrs. Hanley and she tried to use the rule for primes, found it worked, and said she thought it was a prime (line 8). The rule states that any number which has only itself and one for its factors is a prime. But one is a special case and not considered a prime. Her statement that two is the only even prime was true but didn't seem to fit here (line 9). Maybe she said it for her own benefit to confirm her statement about one. Anyway, this interruption caused the question about a prime number to go unanswered. In order to regain the thread of the lesson Mrs. Hanley sent three students to the board. Their exercise was to factor a pair of factors for 36 to prime numbers.

This change in the social participation structure from teacher directed from the front to three students at the board provided an opportunity for volunteer student interaction. Renea was offering assistance as usual and Mrs. Hanley told her to be quiet (line 10).

The class agreed with this reprimand (line 11) as Renea often tried to dominate other students in their work or responses. Tony gave some encouragement to those at the board which was a little out of line (line 12). While these disruptions were going on the three at the board finished their problems (lines 13, 14, 15). Mrs. Hanley again employed her strategy of asking questions in general to the class. She asked if they noticed all the answers in prime numbers were the same (line 16). Some students answered yes (line 17). Then she asked in general if they knew what unique means (line 18). Unfortunately none of the responses were individually recorded and it was not noted if someone gave an adequate response to the question (line 19).

As almost an indirect way of testing this knowledge Mrs. Hanley asked two more questions. She asked if there was a unique way to factor thirty-six (line 20), to which was answered no (line 21). The second question was if there was a unique way to factor thirty-six to primes (line 22), to which was answered yes (line 23). This was a high level of discrimination and if the answers were from a knowledge base rather than guessing, the answers signified that the students understood the distinction. It is hard to know from what was recorded whether there was knowledge or guessing. This concluded the episode. Mrs. Hanley gave the assignment after settling an argument which occurred between two of the boys.

The Outlines of the Three Episodes

The general math lesson involved more student participation than the algebra lesson. There were more interruptions in the forward movement of the general math lesson as well. Mrs. Hanley changed the social participation structure part way through the second general math episode which allowed the students more freedom for interaction. She was less formal in the content presentation in general math than in algebra and tried to draw information from the students. In algebra she told them directly most of the content.

The outlines of the three episodes tell the story of the lessons in a more graphic manner.

Episode #63 Alg.

1. Distinguishing between factoring and multiplying

   $(+)TC$
   $(+)TC_B$
   $(+)TC^B$

2. Giving examples of factoring

   $(+)TC_B$
   $(+)TC^B$
   $(+)TQ$
   $(+)SR$
   $(-)SOL$
   $(-)RB$

3. Giving the theorem

   $(+)TC$
   $(+)TC_B$
   $(+)TC^B$
   $(+)TC$
   $(+)TC$
   $(+)TQ$
   $(+)TQ$
   $(+)TC$
   $(+)TC$

Episode #23 G.M.

1. Factoring 15

   $(+)TC$
   $(+)TC_B$

2. Factoring 24

   $(+)TC_B$
   $(+)TC^B$
   $(+)SR$
   $(+)TC$
   $(-)SW$
   $(+)TC$

3. Computing prime factors for 24

   $(+)TC_B$
   $(+)TC^B$

Episode #24 G,M.

1. Factoring 36

   $(+)TC$
   $(+)TQ$
   $(+)SR$
   $(+)TQ$
   $(-)SW$
   $(+)TQ$
   $(+)SQ$
   $(-)TW$
   $(+)TC$

2. Factoring 36 to primes

   $(-)RB$
   $(-)SOL$
   $(-)SOL$
   $(+)SR_B$
   $(+)SR_B$
   $(+)SR_B$

3. Establishing uniqueness of prime factors

   $(+)TF$
   $(+)SF$
   $(+)TQ^+$
   $(+)GP$
   $(+)TQ$
   $(+)SR$
   $(+)TQ$
   $(+)SR$

The algebra episode was structured around three steps while it took six steps to present the same content to the general math class. There was one disruption in the algebra class by a student making an irrelevant comment and being reprimanded. There were four instances of interference in the general math class. There were two wrong answers by students, one wrong answer by the teacher, and one incident of disruptive behavior by a student which prompted a reprimand from the teacher and was followed by two irrelevant comments from other students. The patterns of enactment demonstrated a more direct forward movement in the algebra class while the general math class had more interference in movement to completion of the lesson. A look at the contingency tables for these episodes may clarify some of the differences.

Episode #63 Alg.

		T	S
	+	15	1
(on task)	-	0	0
(reprimand behavior)	-	1	0
(off task)	-	0	1

Episode #23 G.M.

		T	S
	+	8	1
(on task)	-	0	1
(reprimand behavior)		0	0
(off task)	-	0	0

Episode #24 G.M.

		T	S
	+	9	9
(on task)	-	1	1
(reprimand behavior)	-	1	0
(off task)	-	0	2

The forward movement of the algebra class was due to the extreme domination by the teacher (15 acts) since there were only two student communication acts. There were nineteen teacher communication acts for general math and fourteen student communication acts. There was much more student interaction in the general math class than in the algebra class.

The teacher provided opportunities for interaction in the general math class both in her asking questions in general and in sending students to the board. She was less formal in general math than in algebra and it took longer to present the content in general math.

Because of the lack of student interaction in the algebra class it is difficult to make a statement about collaboration. There was one plus student act and one minus. In the joint production of this lesson it is undetermined what the actual role of the students was except that the attention and silence of the students allowed the teacher to continue with the content presentation without more than one interruption. The greater participation in the general math class provided a basis for commenting on collaboration. Looking at the episodes separately, Episode #23 G.M. was teacher dominated with two student acts, one plus and one minus. The minus act was on task which means the episode was on task. The minus act was an incorrect answer which created an interruption in the forward movement but was corrected by the teacher's subsequent response. Episode #24 G.M. was almost evenly divided in communication acts with the students having one more than the teacher. There was the same number of plus acts for both and two off task comments by the students. The joint production of this lesson showed instances of cooperation and instances of interference in the forward movement of the lesson.

The teacher presented content differently to the two classes. By studying the underlying mathematics structure as embodied in the steps of her logical presentation of content and by examining the enactment between teacher and students it is possible to determine those differences. There was more content for algebra and presented in a more formal manner than in general math. General math had more examples and less complexity than algebra.

Mathematical Content

There was a wider range of mathematical topics in algebra than in general math.

Table 4.7

Episodes According to the Mathematical Content Topics for the Mathematics Lesson

Topics of Algebra Episodes N = 125		
I.	Linear Equations	26%
II.	Algebraic Simplification	22%
III.	Geometry	11%
IV.	Inequalities	8%
V.	Functions	6%
VI.	Fractional Equations	6%
VII.	Variation	5%
VIII.	Factoring	4%
IX.	Two Variables	4%
X.	Exponents	3%
XI.	Multiplication of Polynomials	2%
XII.	Quadratic Equations	2%
XIII.	Signed Numbers	1%
XIV.	Decimals	1%
Total		101%*

Topics of General Math Episodes N = 100		
I.	Percents	23%
II.	Geometry	19%
III.	Flow Charts	13%
IV.	Signed Numbers	10%
V.	Exponents	9%
VI.	Fractions	8%
VII.	Whole Numbers	7%
VIII.	Decimals	6%
IX.	Factoring	4%
X.	Functions	1%
Total		100%

*Due to rounding off to the second place the percentage does not equal 100%

NOTE: Percentages were derived from the number of episodes on a mathematical topic over the total number of episodes for the respective class.

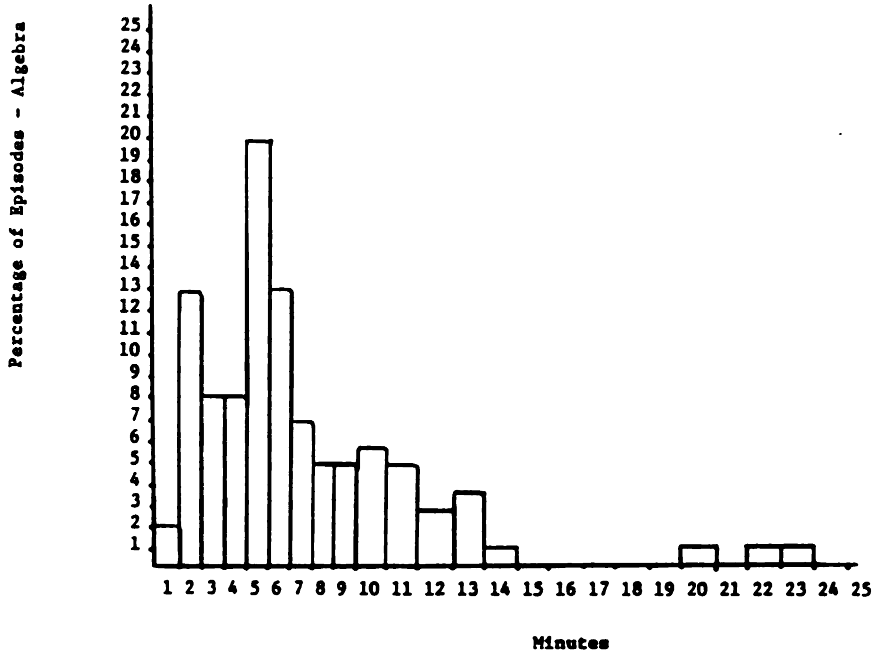
The subject matter contained in the algebra episode involved fourteen topics. The two topics which most frequently occurred were major topics of the first year of algebra, linear equations and algebraic simplification. These topics are examples of subject matter which can begin with a simple problem and progress to quite complex problems. There is a need by the student for the teacher to be clear in the presentation of the concepts and in the logical steps in the procedures for solving the problems.

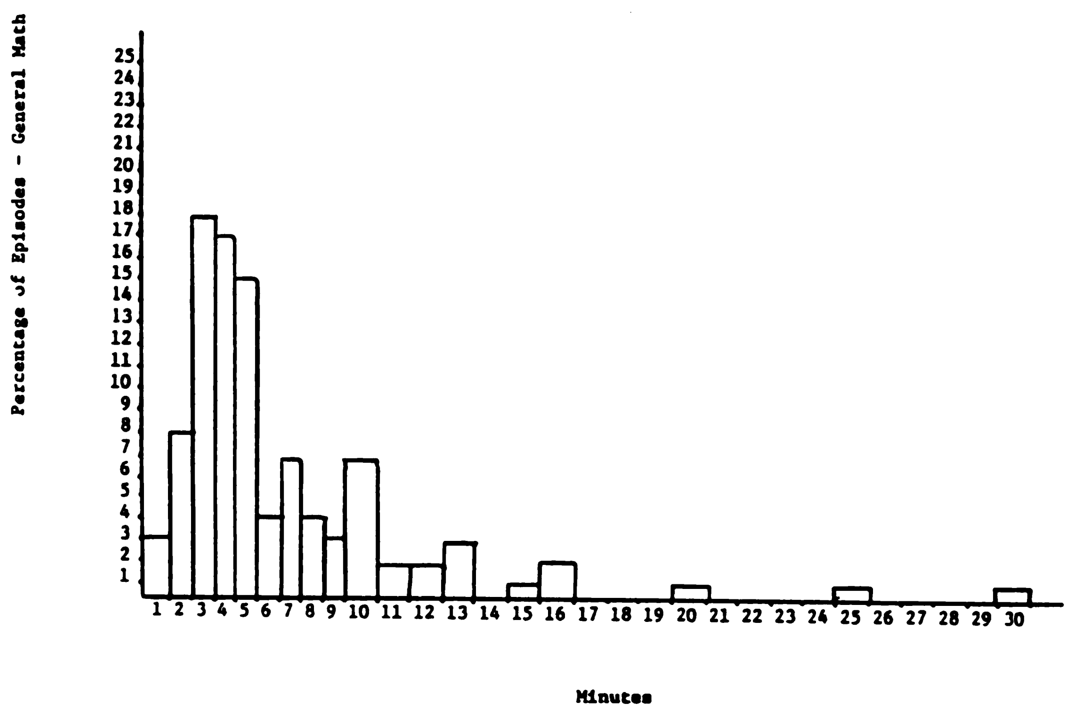
The subject matter contained in the general math episodes involved ten topics. The two topics which most frequently occurred were percents and geometry, two topics these students would have already met in elementary school. Although a pre-algebra book was used in the general math class, several topics already familiar to the students were presented. The third and fourth topics in the order of percentages, flow charts and signed numbers, would probably be new for most of the students. The flow charting was significantly successful in capturing the attention of the students.

The content for the two classes varied in topics and complexity. Most of the algebra content was new to the students while most of the general math content would have already been presented in elementary school. This could account in part for the use of more steps or direction in the algebra lessons and more practice in the general math lessons.

More content was taught to the algebra class than the general math class. This was supported by the fact there were 125 algebra episodes and only 100 general math episodes. It was possible to find more instances of the teacher teaching content in the algebra class than in the general math class.

The modal time for an algebra episode was five minutes while for a general math episode it was three minutes. (See Figure 4.3.) The teacher held the algebra class in longer experiences of instruction than she did the general math class. This could indicate more content being taught.





Note: Percentages were derived from the number of episodes of a given length of time over the total number of episodes. (Algebra N = 125, General Math N = 100)

Figure 4.3 Distribution of Episodes for Each Class According to the Length of Time of the Episode.

Social Participation Structures

In the joint production of the lesson the teacher selected a social participation structure for the presentation of content for a given lesson. This structure provided the rules for interaction between the teacher and students. The teacher would select the structure according to the type of content being presented as well as the set of students being taught. It happened at times that the teacher would change the participation structure during the lesson either to accommodate the content and/or the response patternings of the students. Each episode was categorized in one of four participation structures according to the one used most consistently in the episode. The four social participation structures were: 1) the teacher directing the presentation from the front of the room, 2) a group of students working problems at the board, 3) one student working at the board as the class watched, and 4) the class working the problem or problems first before the solution or solutions were discussed. The following are the results of the categorizing.

Table 4.8

Episodes According to the Social Participation Structure Selected by the Teacher for the Lesson

	Algebra N = 125	General Math N = 100
1. Teacher directing	68%	44%
2. Group working	17%	34%
3. One student working	10%	8%
4. Class working	5%	14%
Total	100%	100%

NOTE: Percentages were derived from the number of episodes selected for a certain social participation structure over the total number of episodes for the respective class.

Of the 125 algebra episodes 68% of them were organized with the teacher directing the presentation from the front of the room. The instructional technique used was a question/answer pattern of interaction with the teacher at the front of the room asking the questions. This social participation structure required the most disciplined cooperation from the students. They were expected to be quiet, pay attention, only answer when called on, and raise their hand if they wanted to recite in class. Because the teacher used this structure so often with the algebra class she must have found it an effective structure for content presentation. This would be the most appropriate structure for demonstrations and for teaching procedures.

Of the 100 general math episodes, 44% of them were organized with the teacher directing the presentation from the front of the room. This structure appeared to be less effective in the general math class. The general math students were more apt to answer out of turn and make irrelevant or distracting statements which inhibited the forward movement of the lesson and was frustrating for this teacher. Almost as if to provide a structure that could accommodate such responses, the teacher selected for 34% of the general math episodes the social participation structure of a group of students working at the board. This participation structure allowed voluntary interaction and could accommodate off task communication without interrupting the forward movement of the lesson. The teacher gave less direction, and less content was transmitted. Only 17% of the algebra episodes were organized with a group of students working at the board. Even so, the instances in algebra of a group of students at the board were usually more structured than for general math and the teacher did succeed in transmitting content to the algebra classes in these episodes as well.

The need to provide a more informal environment for interaction in the general math class may also account for the fact 14% of the general math episodes involved the class working a problem or set of problems first and then discussing the answers. The students were usually able to interact with each other during the working time and the teacher directed the class for the checking of solutions. Through this participation structure there was more of a possibility that someone would be able to respond to the questions the teacher asked about the problems, a concern always present in the general math class. Only 5% of the algebra episodes were of this participation structure. The algebra class did not require time to work problems first or the practice this participation structure provided. This was the participation structure least used in the algebra episodes.

The social participation structure of one student working at the board while the class watched was used in 10% of the algebra episodes and 8% of the general math episodes. This participation structure required attention on the part of the class and direction from the teacher. In the algebra class the student was often pressured to figure out the problem and usually selected as an example for the class. If the content was new a brighter student was chosen to work it through. If the content was more of a review a student who might make a useful mistake was chosen. In the general math class the student was not pressured and those who worked at the board usually volunteered. The instances were for practice rather than instruction and the teacher did not attempt to hold the class in strict attention with no interaction. Because of the unpredictable behavior of the general math students it was a risk to put someone on display in front of the class. Selecting volunteers to go up was generally a safety measure for using this participation structure. It was the participation structure least used in the general math episodes.

The Conclusions of the Episodes

The conclusions of the episodes reflected the student interaction of the general math class which seemed to influence the teacher in her selection of social participation structures. The conclusions for the algebra episodes reflected the teacher as directing this class.

The conclusion of the episode was that interaction which occurred just prior to the teacher changing the subject matter or the class activity. The conclusions were categorized according to the following five types of conclusions: 1) instruction by the teacher, 2) a statement about procedures or expectations by the teacher, 3) a statement about content by a student, 4) feedback requested by the teacher, and 5) an off task comment by a student. The following are the results of the categorizing.

Table 4.9

Episodes According to the Type of Conclusion to the Mathematics Lesson

	Algebra N = 125	General Math N = 100
1. Instruction by the teacher	33%	24%
2. Procedure or expectations by the teacher	34%	21%
3. Content by a student	20%	39%
4. Feedback	13%	7%
5. Off task by a student	0%	9%
Total	100%	100%

NOTE: Percentages were derived from the number of episodes with a certain conclusion over the total number of episodes for the respective class.

The conclusions were of particular interest in terms of who closed the episode, the teacher or a student. This information was easily calculated since the first, second, and part of the fourth conclusions were by the teacher. The fourth conclusion was the request for feedback by the teacher and sometimes was answered by a student and sometimes not. Those unanswered requests made the teacher the last speaker and the answered requests made a student the last speaker. The third, part of the fourth, and the fifth conclusions were by a student. The following are the results of who gave the final communication act in an episode.

Table 4.10

Episodes Concluded by the Teacher or by a Student

	Algebra N = 125	General Math N = 100
Teacher	75%	47%
Student	25%	53%
Total	100%	100%

NOTE: Percentages were derived from the number of episodes concluded by the teacher over the total number of episodes and the number of episodes concluded by a student over the total number of episodes for the respective class.

The teacher generally had the final say in the algebra class. She ended the lesson with a statement about the mathematical content or some advice to the students. With 8% of the feedback conclusions given by the teacher, she gave 75% of the conclusions. The algebra students were responsible for only 25% of the conclusions and 20% of those were statements about content. The teacher as instructor of the class was exemplified in these endings.

In the general math class, however, the students had the final say slightly more often than the teacher. With 5% of the feedback conclusions given by the students, they gave 53% of the conclusions. The teacher was responsible for 47% of the conclusions. The students had the highest percentage in the conclusion of a statement of content by a student. This included work at the board as the last thing recorded before a change and this board work accounts for several of the third type of conclusion in general math. The teacher gave significantly more statements on procedure and expectations to the algebra class than to the general math class. These statements included advice on study habits, expectations about performance and goals, and warnings about certain mistakes for specific problems. The lesser number of these kind of statements to the general math class indicated a reluctance by the teacher in trying to give advice or warnings to these students. The warnings that were given to the general math class were usually related to grades or immediate punishment rather than the academic warnings of learning something now so you can use it later, which were given to the algebra class. The 9% of the episodes for general math which ended with an off task statement by a student contrasts strikingly with no such conclusions for the algebra class. This again indicated that increased student interaction by the general math students involved a large percentage of off task communication. The off task communication reflected the informal nature of the general math class in contrast to the academic nature of the algebra class.

Summary

The findings of this study support the assumption that general math is harder to teach than algebra. In the collaborative effort between teacher and students as observed in these two classes, the algebra students cooperated with the teacher in the joint production of the lesson while the general math students were adversaries in the joint production. The algebra students paid attention and observed the rules of a chosen participation structure which allowed the teacher to use the participation structure of teaching from the front of the room. The general math students for the most part did not pay attention or observe the rules of interaction which influenced the teacher to select less formal participation structures than teaching from the front of the room. The mathematical content taught influenced the teacher in her choice of social participation structures and the student interaction influenced how much content actually was taught. The logical presentation of mathematical content and the social participation structures were influenced by and influenced each other in the enactment of the lesson.

CHAPTER V

CONCLUSIONS AND IMPLICATIONS

Introduction

This study involved a general math class and an algebra class taught by the same teacher. The motivation for this study was the present state of mathematics education and the fact that general math is the last math class taken by about half of the ninth grade population in the United States--their last chance to learn some mathematics. In order to see what a general math class looks like, an algebra class was also observed as a contrasting classroom mathematics experience. Algebra is the math class most ninth graders want to take because it is the first class in the mathematics sequence for college bound students. Both classes were taught by the same teacher so that the same philosophy for teaching would be functional in both classes. Classroom observations were made in both classes for one school year and field notes recorded for each observation. The focus was how the teacher taught mathematics and how she might do it differently in the two classes.

The teacher liked math, knew her content, and was interested in continuing her own education. She was always prepared for class and was very well organized. Over her years of teaching she had formulated some techniques and presentations for certain topics, especially story problems. She was a strict teacher, demanded work from the students and pushed them to take responsibility for their work. She was apt to raise her voice at a student on occasion and to be demanding when correcting a wrong answer. She spoke of personal concern and desire for success for the students and often appeared in the classroom to be quite strict and somewhat aggressive with the students. Some students exhibited fear of her at times. But she was untiring in her own efforts to transmit the mathematics content. She knew these classes were first steps in learning high school math, and she was determined to provide a good foundation. This was the attitude which prevailed in both classes in the beginning of the year.

Over the year of observation, each class took on a specific set of characteristics. The algebra students were attentive, seemed to want to learn, did their work, were obedient and cooperative. The teacher could teach them. She was the authority, and they responded as if she was.

The general math students were interested in doing math but were more interested in what else could be done or talked about during the class. They were not hostile in their behavior but immature and, for the most part, not caring about learning math. They appeared to be somewhat alert to what was going on because at times their off task comments related to the subject matter presentation. They were more interested in each other than the class as a whole. There were some loners as well. It was as if everyone was for himself or herself and it didn't matter that there might be a bigger picture. Some listened and tried to learn but not consistently. The class had more of an informal nature about it than an academic nature. It wasn't always obvious that the teacher was the authority. These students were not afraid of her. For the most part they did not do their work but usually got upset about their grades.

The teacher gradually compensated for the lack of attention and lack of completed assignments in general math. She began to use more class time to work on assignments and more assignments came in worksheet form. She organized the class instruction to be more open and informal rather than try to hold the class in strict order. She gave practice problems and review material rather than new concepts and complex problems which required attention and concentration for longer periods of time than practice and review. The teacher often appeared frustrated and discouraged in the general math class. A good general math class occurred when one of the more disruptive students was absent and she could actually talk to the class about something without being interrupted or challenged by simultaneous conversation.

It was through the observance of these two classrooms that the concept of mathematics lessons as a joint effort between teacher and students emerged. The cooperation of the algebra students allowed the teacher to present her plan and work through the content in a sequential manner. Within this cooperation there was still modification of the teacher's plan whenever a wrong answer was given or a question was asked. This required some backtracking and some reteaching which were not in the forward movement of the plan. There were some behavior interruptions but not to the point of losing the goal of the lesson. Teacher and students together worked on the content to produce an experience in mathematics. The teacher brought her plan and expertise. The students brought their desire to learn and their study habits. The algebra class as a group provided an environment for learning by agreeing on why they were together and as a group focusing on teacher and content. Response patterns were matched to the way the teacher oganized the class. If she wanted question/answer, they raised their hands in response to her questions. If she sent a group to the board, they participated at their desks by working and watching. They asked questions when they didn't understand and they attempted to follow the steps as the teacher outlined them for a given set of problems. Together they made a math lesson which progressed through a certain amount of content and created a sense of accomplishment for both teacher and students.

There was also a joint production of a math lesson in the general math class. The outcome of each class was a collaborative effort between teacher and students. The teacher came with her plan and format for classroom organization. The students came with their expectations and social demands. However, these two lines of approach were usually not compatible in the general math class. The teacher's plan usually became modified with interruptions. The students did not often try to match their response patterns to the social participation structure that the teacher selected. The students were not that interested in what the teacher had to say but more interested in each other. Math lessons were a struggle between teacher and students. Agreement consisted in being together in the classroom for a general math class. From then on it was a struggle as to who was in charge and what agenda was going to get preference. At times teacher and student would come together on the content, and cooperative progress would be made. On the whole, each class was about mathematics and some content made it into the interaction but the content presentation was usually quite simple and the goal of the lesson was often to practice or review. Seldom was there significant progress in content covered and a sense of accomplishment by teacher and/or students. The lesson jointly produced was often interrupted with off task comments creating interference in the forward movement of the lesson toward the goal.

Summary of Findings by Research Questions

The overarching question of the study was What makes it harder to teach general math than algebra? This question was based on the general conception that general math is harder to teach than algebra. In order to address the overarching question three guiding questions were posed. The answers to these questions follow.

1. How did the presentation of school mathematics content differ between general math and algebra?

Analysis of the underlying mathematical structure of the episodes showed that the teacher typically covered a longer sequence of logical steps in her classroom presentation for the algebra class than for the general math class. Her classroom instruction was a demonstration of content and problem solving for algebra. She emphasized procedures and methods for the students and worked through several examples with the class. She reviewed simple concepts prior to their use in more complex problems and worked with the students in recognizing the patterns that existed in progressing from simple to complex exercises.

For the general math class, the teacher typically covered fewer steps in her logical presentation. Classroom instruction was a practice of problems already learned. Very few procedures or methods were presented to this class. Although the teacher did present new content with examples, there were not many instances of learning complex material which required progressing from simpler concepts. For the general math class, the mathematics activity was mostly review and practice.

More mathematics was taught in the algebra class and there was typically a longer sequence of logical steps in the teacher's classroom presentation in the algebra episodes than in the general math episodes. This could be the result of the observed fact that the algebra students were more cooperative during the classroom instruction which allowed the teacher to complete the planned presentation for the lesson. The interference encountered in the general math class prompted the teacher to make simpler presentations and, therefore, have less content and fewer steps in her logical presentation.

2. How did the social participation structures differ between general math and algebra?

The teacher in this study opted to use the less formal social participation structures of a group of students working at the board and the class working the problems first, more often in the general math class than in the algebra class. The more formal social participation structure of the teacher directing the instruction from the front of the room seemed to invite the disruptive behavior and irrelevant remarks of the general math students as they chose not to cooperate with the teacher in the forward movement of the lesson. The teacher was then placed in a position to maintain the order that the social participation structure required. In a less formal social participation structure the teacher could ignore some of the interference because the rules for interaction were more open and the forward movement could continue in spite of some irrelevant communication.

The algebra students cooperated with the rules for interaction for the more formal social participation structure of the teacher directing instruction from the front of the room. The teacher chose to use this classroom organization most of the time in the algebra class. This allowed for the presentation of complex subject matter and the opportunity to ask questions and find trouble spots early. This participation structure created a formal and academic atmosphere within the classroom with the teacher as authority in front of the room and the students interacting about the content. The teacher was able to expound on the content because the students listened to her explanations and illustrations and asked questions for further clarification.

The choice and frequency of the social participation structures used in the general math class created an informal open atmosphere within the classroom. Students were often at the board and the teacher mingled with the rest of the class. There was more opportunity for student to student interaction and for off task communication. Disruptive behavior, which occurred within all four participation structures, added to the informal atmosphere and detracted from the academic focus of the lessons. The content at times was lost in the struggle for authority which often occurred between teacher and students.

This teacher focused on the mathematics content of the lessons and appeared most at ease in exercising her authority through the more formal social participation structure, with herself teaching from the front of the room. It was a source of frustration to her that she could not successfully organize and maintain the general math class in this participation structure. The lack of cooperation from the students undermined her preferred manner for exercising her role as teacher.

3. How did the patterns of enactment between teacher and students differ between general math and algebra?

The patterns of enactment demonstrated the interaction between the teacher and students. The pattern of plus and minus signs of the coded items gave an abstract picture of the forward movement of the lesson. The abstract patterns demonstrated that the algebra students were more cooperative with the teacher than the general math students. The algebra students stayed on task by following the rules of the social participation structures, asking questions about the content, and answering the content questions asked by the teacher. The forward movement of the lesson was at times interrupted but usually by content related mistakes or teacher reprimands about content. Disruptive behavior occurred at times but not frequently.

The general math students were, for the most part, uncooperative. They exhibited frequent off task communication by ignoring the rules of the social participation structures, making irrelevant comments, asking unrelated questions during the lesson and by competing with the teacher for attention during instruction. The forward movement of the lesson was frequently interrupted by disruptive and irrelevant communication acts followed by the teacher reprimands about student behavior. There were instances of patterns of enactment which showed the general math students cooperating with the teacher in producing a lesson that progressed forward to completion. There were instances when the general math students were interested in the subject matter and struggled with the problems, even working together on the problems. But the frequency of the minus acts often overshadowed the instances of patterns of all plus acts for general math.

The patterns of enactment also indicated the degree of teacher interaction during the mathematics lesson. For the algebra class the teacher had more communication acts than the students. She taught for the most part from the front of the room and used a question/answer method to draw the students into participation in the content presentation. She talked about the content and demonstrated how to do the problems. She gave procedures to aid in problem solution and tried to find mistakes by certain students and help them correct their misconceptions.

For the general math class the teacher and students had almost the same number of communication acts. The teacher did not try to hold the class in whole group instruction more than was necessary. She sent students to the board almost as much as she stayed in front to teach. She tried to control interaction rather than evoke it as in the algebra class. She did not spend much time talking about the content and preferred letting the students practice doing problems rather than demonstrating problems for them. She did not use mistakes as a basis for teaching a concept.

Collaboration came to have two distinct patterns for general math and algebra. For algebra the collaborative effort between teacher and students was a cooperative venture in which the lesson was jointly produced in the forward movement of the lesson to completion. For the general math class the collaborative effort was the struggle between two adverse purposes trying to produce a mathematics lesson which resulted in an interrupted movement toward the completion of the lesson. Although the goal of the lesson may have been attained in most instances, the frustration of the struggle was usually the prevailing experience of the lesson in general math.

Implications for Practice

The major findings of this study support the assumption that mathematics lessons are a collaborative effort between teacher and students. The teacher prepares and presents the content and the students respond according to their own perceptions about the purpose of the class. The set of students for a given class provides a specific environment of attitudes, expectations, and response patterns. These may be in line with or at least compatible with the teacher's attitudes, expectations, and response patterns, or in conflict. If in conflict, a struggle for authority becomes the underlying theme for the process of enactment. Such a struggle was evident in the general math class. There did not seem to be a basis for compromise between teacher and students regarding their concepts of what the general math class was to be. This suggests that perhaps such a homogeneous set of individuals may be inhibitive of a cooperative academic environment. One implication of this study is to reconsider the present tracking system in our schools which determines which students take which mathematics class. By eliminating general mathematics classes and integrating these students into other classes, a set of students can be selected which provide the needed student responses that successful collaborative lessons require. Both teacher and students will have a wider range of abilities and learning experiences to work with in the classroom.

The general math students in this observed class were actively engaged in the content when the topic of flow charting was being presented. This was a new topic for most if not all of the students. It was a skill which had a broad content including symbols and drawings and a relationship to computers which some of the boys mentioned. The arithmetic skills used in the process were essential but not the focus of the lesson. Perhaps more attention needs to be placed on the context within which the basic skills are presented to the general math students. Providing a larger scope of problem solving (preferably a scope of interest like computers or even equations) would capture the students' attention. Once the interest was involved the skills could be reviewed without the usual drill and practice which seems to be the essence of most general math classes. This requires more work on the part of the teacher but may alleviate some of the frustration experienced in these classes.

The teacher and students jointly produce the mathematics lesson. The more aware the teacher becomes of the students' role in this joint production, the more possible it will be to organize the class for the advantage of teaching and learning. The teacher assumes a role of influencing and changing the students. This study has shown that the students in turn influence and change the teacher. The more a teacher reflects on how this dynamic works in the classroom, the more insight may be applied to what is happening and why it is happening. A general math class provokes certain behavior patterns from teachers and from students. Each class has its own twist to the manner by which these responses are provoked. Teacher awareness may be able to change and/or redirect some of these patterns.

This relfective awareness is essential in any teaching situation. The teacher is always working with the students and needs to know what forces are influencing decisions and responses. Collaboration in lessons implies an openness on the part of teachers to listen to the students and be able to interpret their needs and responses in the forward movement of the lesson.

Implications for Research

The method of analysis used in this study has shown that the teacher's structure for content presentation can serve as a basis for studying classroom interaction and the enactment of mathematics lessons. The interaction of the classroom is governed by the content and the on task/off task orientation of the communication provides the enactment process of the lesson. The content of a lesson is essential to understanding what is happening in the classroom. Researchers need to begin to include this aspect of content in their work on classroom analysis, both as a structure for studying interaction and as a means of examining classroom curriculum development.

This study has shown that content was affected by the enactment process both in kind and complexity. Although mathematics is regarded as a stable, logical subject matter, unchanging over time and an easier subject to research than reading or social studies, it too was influenced by the interaction in the classroom. Through the interaction of teacher and students during the class session the school mathematics content for a given lesson was determined. The curriculum was developed through the classroom interaction.

Of interest would be observing other mathematics areas, such as geometry or calculus, to see if such influences on content are existent there as well. Is the interaction between teacher and students such that the planned structure is modified and to what extent? What type of classes are more susceptible to these changes than others? How are decisions about content influenced by classroom interaction in the more advanced mathematics classes? Is this same phenomenon found in other subject matter areas?

The collaboration between teacher and students in jointly producing lessons needs to be further studied. To get a more accurate picture of the teacher/student interaction audiotapes and videotapes could be used. The coding system for each class would need to be modified to the content being studied and the response patterns of teacher and students. Inter-rater reliability tests could be used to improve the coding systems. The complete sequence of interaction with tapes will provide the possibility for identifying specific interaction patterns for teacher and students and a more complete outline for a lesson. With tapes a whole class could be analyzed and transitions to different activities could be studied. A sequence of classes over time would give a broader picture of curriculum development and collaboration between teacher and students.

Within the concept of collaboration, cooperation from the students was the essential factor in completing the goal of the lesson. Yet to be studied is whether the cooperation needs to be with all the students or some of the students. If the cooperation is with some of the students, must it always be the same ones and what type of student fits into this group? There was no observable teacher support group in the general math class in this study. There were several instances of support or cooperation in the algebra class. Does the teacher depend on a certain group when teaching? Could the lack of such a group be one of the major problems in a general math class? Can a teacher form such a group if it doesn't exist? The answers to these questions may provide another entrance into the problems of low ability classes such as general math.

APPENDICES

APPENDIX A

APPENDIX A

SOCIAL PARTICIPATION STRUCTURES

The following episodes demonstrate the four social participation structures used by the teacher in this study.

In Episode #32 G.M. the teacher was presenting the concept of parallel lines. She did this by putting two drawings on the board, one of parallel lines and one not parallel lines, and asked the class a series of questions about the concept. Through interaction with the students a definition of parallel lines was formulated. This episode gave an example of the first social participation structure, the teacher directing the presentation of content from the front of the room.

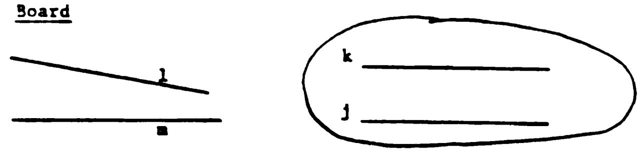
#32 12/10/79 pp. 7-8 General Math

Mrs. Hanley: "Knowing these definitions is important." "All right. The other thing we have to talk about today is parallel and perpendicular."

10:05 Mrs. H: "All right. If I can have your attention again."

Brian: "Yes."

<u>Board</u>



Mrs. Hanley asked the class what parallel lines are.

Renea: "Lines that run on forever and never hit each other."

Adam: "Like railroad tracks."

Mrs. Hanley showed them skew lines. With this example she said. "Your definition does not hold." Eventually the discussion got around to the fact parallel lines have to be in a plane.

Brian: "That's true. Very true."

Mrs. H: "Do you know what a plane is?"

Tony: "Ah. Let's see."

Mrs. Hanley described a plane as a surface that goes on forever.

Mrs. H: "Lines in different planes that don't intersect are called skew lines. That is not in your book."

10:09 <u>Board</u>

1. Parallel lines are lines in a plane that do not intersect.

2. Skew lines are lines that don't intersect because they are in different planes.

Craig: "Should we write this down?"

Mrs. H: "Yes."

Brian: "Do we write true or false?"

Mrs. H: "No. These are statements you are supposed to know."

Brian: "Oh."

Mrs. Hanley: "After parallel we have to talk about perpendicular." "It's got to be lines that are perpendicular."

In Episode #34 G.M. the teacher called on four students to go to the board. She then gave the problem to the four students and they were to find the solution. The rest of the class was expected to work the problem at the same time as those at the board. This episode gave an example of the second social participation structure, a group of students working problems at the board.

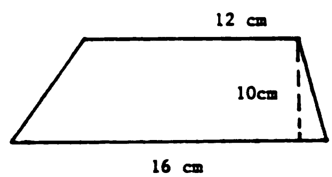
#34 12/18/79 pp. 7-8 General Math

Eddie is working along. Craig asked about extra credit. Renea asked the researcher if Mrs. Hanley saw the researcher's notes.

10:34 Mrs. Hanley sent Tony, Brian, Craig and Brent to the board.

Mrs. Hanley asked the four at the board to draw a trapezoid, draw its height, and then she gave them measurements for the figure. With this information they were to find the area.

<u>Board</u>



Brian: "Eighty centimeters."

After Brian gave an answer everyone else at the board repeated it.

The students at the board are yelling out answers. Tony got it right.(140 sq. cm.).

10:38 Mrs. Hanley: "O.K. Draw a rectangle."
Renea is giving the formulas.

In Episode #16 Alg. the teacher was going over a problem on a test that the students had just gotten back. The teacher had made a comment about one of the problems and a student asked the teacher to do that problem for the class. Rather than work it out herself the teacher called on two students to come to the board one at a time and work the problem while the class watched. This episode gave an example of the third social participation structure, one student working at the board as the class watched.

#16 11/5/79 p. 2 Algebra

Mrs. Hanley: "I can tell you it is advisable to write this stuff out. But I can't make you do it."

9:04 Mrs. H: (Referring to number three on the test)

"Most of you added three x and nine x and came up with something you should not have had there."

Helen: "Could you do number three?"

Mrs. H: "I think I'll let someone else do that one."

Board

$6 - 3x = 9(16 - x)$

Mrs. H: "Cecile, why don't you come up and do something to this problem."

Board

$6 - 3x = 144 - 9x$

$6 - 3x + 9x = 144$

rs. H: "Julie, how about coming up and finishing up from here."

Board

$6 - 12x = 144$

Mrs. H: "She did it."

Roger: "All right!"

Mrs. H: "Julie, honey, you made a mistake."

9:08 Brenda banged into the door.
Martha let her in.
Brenda is in a wheel chair. No one even paid attention when she hit the door and Martha let her in.

Board

$6 + 6x = 144$

$1/6 \cdot 6x = 1/6 \cdot 138$ $\quad 6\overline{)138}$

$x = 23$

9:10 Mrs. Hanley: "Any other questions on this quiz?"
"Any other questions on story problems?"

In Episode #24 Alg. the teacher presented a problem for the class to solve. She had previously in this lesson introduced the concept of function to the class and three problems had already been completed. The problem presented in Episode #24 was the fourth problem for this class period and she instructed the class to work it first and then the solution was given by one of the students on the board. This episode gave an example of the fourth social participation structure, the class working the problem first before the solution was discussed.

#24 11/6/79 p. 4 Algebra

Mrs. Hanley: "Some of you are writing so slow. O.K. you are buying time. You won't have that kind of time on a test. Some of you are getting slower and slower."

9:28 Mrs. H: "All right. I'm going to give you a problem that I want all of you to do on a piece of paper."

Board

$$D = \{-2, -1, 0, 4\}$$

$$g(x) = 3x - 5$$

Mrs. H: "All right. Write the function. Write the range."

9:31 Julie: "Ta da!"

Mrs. H: (to Julie) "All right. Write it on the board."

Julie: "Can I take my paper up?" Mrs. Hanley told her she could.

Board

$$\{(-2, -11), (-1, -8), (0, -5), (4, 7)\}$$

$$R = \{-11, -8, -5, 7\}$$

9:34 Board: $D = \{\text{All real } \#\}$

$f(x) = 2x \quad g(x) = x + 5$

APPENDIX B

APPENDIX B

CONCLUSIONS OF AN EPISODE

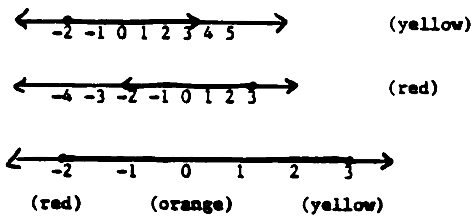
The following episodes demonstrate the five types of conclusions that occurred in the episodes.

In Episode #28 Alg. the teacher introduced the concept of a double inequality. She used a set of line graphs to clarify the union and intersection of sets and then stated an inequality for which the solution was depicted by the graphs. After working through the problem with the students she stated a set of steps they could follow for solving this kind of problem. This episode gave an example of the first conclusion which was instruction by the teacher. After the steps were given a new problem was begun.

#28 11/13/79 p. 5 Algebra

Mrs. Hanley: "All right. I thought about this a long time (colors). If you don't get this you are lost."
"This is a lot of work. I hate doing all those dots. It's stupid. But sometimes you have to do it."

9:29 Board



Union is the set of all real nos.

Intersection - orange port.

9:32 $5 < x + 7 < 10$

$5 < x + 7$ and $x + 7 < 10$

$x + 7 > 5$

Mrs. H: "All right. How would you solve this? Darin."

Darin: "X is greater than negative two."

Roger answered out.

Mrs. H: "How would you solve this one, Brenda?"

Brenda: "X is less than three."

Mrs. H: "The answer is the numbers that are orange. Does that make sense?"

Darin: "Yeah."

9:35 Board

WTIWC
SI_s
GI_s
GRC

Mrs. H: "You're going to write this on your paper first before anything else. This is your first step."
"Write two inequalities with a connector." (WTIWC)
"Second step is SI_s: Solve inequalities."
"The next step is GI_s: Graph inequalities."
"The last step is GRC: Graph reflecting the connector."

9:39 Board: $+1 \leq y + 7 < 6$

In Episode #19 Alg. the teacher was working through examples of functions. She had a function written on the board and stated the definition for the domain and range. She then gave a problem to be solved with the class. Having solved the problem she gave the class some advice as to the procedure for solving this kind of problem. This episode gave an example of the second conclusion which was a comment by the teacher on procedure or expectations. The conclusion was determined by the teacher giving the assignment.

#19 11/5/79 p. 4 Algebra

9:29 Mrs. Hanley: "When numbers are always paired with the same number we say it is constant. This is called the constant function."

Board

$\{(1, 5), (3, 9), (5, 13)\}$

$D = \{1, 3, 5\}$

$R = \{5, 9, 13\}$

Mrs. H: "The domain is the set of first numbers."
"The range is the set of second numbers."

Board

$g: y \rightarrow y+2 \qquad D = \{1, 2, 3\}$

Mrs. H: "Darin"

Darin: "He took my pencil"

Mrs. H: "Darin"

Darin: "He took my pencil."

Mrs. H: "What do you do here?" (referring to the problem).

Darin: "I don't know."

Mrs. H: "I don't like the way you are conducting yourself in this class. That goes for you, too, Kevin."

Board

$g: y \rightarrow y+2$

$1 \rightarrow 3$

$2 \rightarrow 4$

$3 \rightarrow 5$

$D = \{1, 2, 3\}$

$\{(1, 3), (2, 4), (3, 5)\}$

$R = \{3, 4, 5\}$

9:34 Mrs. H: "Some of you are probably thinking this is so easy. It is not hard to do. What you have to figure out is what to do."

Mrs. Hanley: "Your written exercises are on pages one forty eight and one forty nine."
"We'll finish this up tomorrow. Your test is on Wednesday."

In Episode #57 G.M. one student is at the board working a problem given to her by the teacher. The episode ends with the correct solution of the problem written on the board by the teacher. This is one example of the third conclusion which was a statement on content by the student. Work at the board was considered interaction with the teacher through the content for this study. Another problem was given directly after the solution was written.

#57 3/25/80 p. 7 General Math

Board: 12. 35/100 35%

Mrs. H: "Sandy, why don't you go up and do number eleven?"

<u>Board</u>

11. 1/4 .25%

Sandy: "No. Wait a minute." (Sandy changed her answer.)

<u>Board</u>

11. 4/25 = 16/100 = .16%

Mrs. H: "Now wait a minute. You're doing two things at once."

<u>Board</u>

11. 4/25 = 16/100 = 16% = .16

Mrs. Hanley asked for someone to do problem number fifteen.

Another example of the third conclusion was given by Episode #43 Alg. In this episode the teacher was solving a problem at the board through question/answer interaction with the students. The episode concludes with a student giving the correct solution. Another problem was given immediately.

#43 12/10/79 pp. 2-3 Algebra

Mrs. Hanley: "Again they give you some problems with a verticle arrangement which I think is bad news. I don't think they ought to do that."

9:10 Sarah returned.

<u>Board</u>

(4a + 4) - (2a + 1)

4a + 4 - 2a - 1

Clinton gave the answer.(the second line above).

Mrs. H: "Now you're ready to combine terms. And when you combine terms what do you get here, Martha?"

Martha: "Two a plus three."

Board: (ax + by) - (-2ax + by)

The fourth conclusion had to do with feedback information requested by the teacher. There were episodes in which the request was answered by a student or students and episodes for which no response was recorded. Both of these cases were included in this category.

In Episode #12 Alg. the class was doing a story problem. The problem was taken from the book and the class read it first silently before the teacher asked the series of questions she used to set up the problem and solve it. At the completion of the problem the teacher asked if the problem solution made sense. She was requesting feedback from the students. In this episode two students answered. One said yes and the other said no. The next line went into the next problem, concluding the episode.

#12 10/30/79 p. 2 Algebra

Roger: "x plus x equals two x."

Mrs. Hanley: "You're not following your steps."

Mrs. Hanley gave another problem.

9:02 Mrs. H: "Go on your reading." There was silence in the room.

9:03 Mrs. H: "All right. Close your books."
"What are the relationships?"

<u>Board</u>

1st piece of cable: $2x - 10 = 62$ ft.

2nd piece of cable: $x = 36$ ft.

Mrs. H: "What is one relationship between these two pieces of cable?"
"How can we use that relationship to write an equation?"

<u>Board</u>

$x + 2x - 10 = 98$

$3x - 10^{+10} = 98^{+10}$

$3x/3 = 108/3$

$x = 36$ ft.

9:07 Mrs. H: "Does that make sense?"

One student said yes and another said no.

Mrs. Hanley: "Now this is very hard for me to do. I'm just going to call on people. I'm not going to say anything. The whole problem is coming from you."

In Episode #31 G.M. a student asked for an explanation of adjacent angles. After the teacher gave the explanation she asked if there were any other questions. No response was recorded. This episode gave an example of the fourth conclusion in which the request for feedback was unanswered. This was followed by directions to the students concerning the test about to be given.

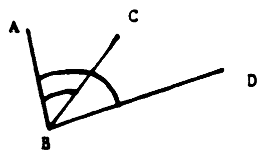
#31 12/4/79 p. 5 General Math

Leah: "I wasn't here yesterday. Can you show me what we did yesterday?"

Mrs. Hanley told her she would have to take the test anyway and she would mark out the ones she didn't have to do.

Chad asked about adjacent angles.

Board



$\angle$ ABC and $\angle$ CBD are adjacent.

Mrs. H: "Does that make sense?"

Brent: "I don't get it."

9:58 Mrs. H: "Adjacent angles don't overlap. They must be side by side."
Mrs. Hanley tore paper triangles and showed how adjacent must be side by side but not overlapping.

Mrs. H: "All right any other questions?"

Mrs. Hanley: "Those of you who just transferred in and there are a lot of you, just take the test and at the end of the term we'll look at all your scores before we give you a grade."

The fifth conclusion was an off task comment or question by a student. An example of this was Episode #18 G.M. In this episode the teacher gave the students the task of writing a flow chart that would print the larger of two numbers. The class had worked through a set of six problems in the book and completed the printout for a flow chart that printed the smaller of two numbers. The class worked the problem individually and then the teacher asked at random for someone to put the answer on the board. As one student got up to go to the board another student commented that she told him the answer and she would put it on the board. This constituted a remark not of feedback nor on content and was therefore a basis for putting this episode in the category of the fifth conclusion. The teacher gave another problem after this conclusion.

#18 10/30/79 p. 9 General Math

Mrs. Hanley: "All right. Getting that data over there you get solutions three, four, seven, two, five."
"It's always the smallest one that's printed."

Mrs. H: "I'd like for you to write a program that will print the larger of two numbers and then we'll put it on the board."

10:18 Tony and Chad are keeping up a running conversation. Mrs. Hanley came back to see Craig. "All right. Did you get a start?"

10:21 Mrs. H: "Anyone who has the answer, put it on the board."
Brent got up.
Renea said, "Hey, I was the one who told you. I'll write the answers."

10:22 Mrs. Hanley: "I want you to write a flow chart to find all the multiples of three less than three thousand."

APPENDIX C

APPENDIX C

EXPLANATION OF CODING CATEGORIES

The communication acts of the teacher, which were signed plus, numbered eight.

TC This coding category indicated a statement by the teacher which referred to content or directions for the lesson.

TC_B This coding category indicated work by the teacher at the board which was not stated verbally.

TQ This coding category indicated a question by the teacher.

TF This coding category indicated the teacher's request for feedback from the students. This request was for a yes, no, or I don't know response. Examples of such requests were: (the number refers to the episode.)

Does that make sense? (#4 Alg., #4 G.M.)

Any questions? (#1 Alg.)

Does everyone understand that? (#51, Alg.)

Do you see the process in that? (#98, Alg.)

TE This coding category indicated a statement by the teacher which served as an exhortation to the students. It consisted of urging, advising, or warning students about some aspect of learning the mathematics content. Examples of such statements were:

"If you don't get your negative signs right or your absolute value signs right, there's not much I can do for you." (#8 Alg.)

Mrs. Hanley told the class that sometimes if you do a problem one day that seems very hard it is helpful to go back the next day and do it again. It helps to remember how a certain kind of problem is done." (#11 Alg.)

Mrs. Hanley: "You will need it (converting metric measurements) for physics and chemistry. You will need it later on in high school. Get used to it now." (#28 G.M.)

The discussion about story problems continued. Mrs. Hanley pointed out that sometimes things were implied in the problems and it was important that the students read the problems carefully. She warned that there may be irrelevant numbers that have nothing to do with the problem. She said these were put in to confuse them. (#51 G.M.)

TA This coding category indicated a statement of teacher affirmation to a student response. Affirmation usually followed a correct answer to a question, correct work on the board, or a statement by a student with regard to the content being studied. Examples of affirmation are the following:

Mrs. Hanley: "Let's see who knows the answer to number seven."
Diane: "Transitive."
(TA) Mrs. Hanley: "Transitive it is!" (#6 Alg.)

Mrs. Hanley: "Suppose that 't' is point two. What would 't' be paired with?"
Student: "Point 0 four."
(TA) Mrs. Hanley: "Right." (#18 Alg.)

(On the board) Brent wrote seven and one fourth as the decimal seven and twenty-five hundredths.
(TA) Mrs. Hanley: "Good, Brent." (#35 G.M.)

Mrs. Hanley: "Thirty percent is what fraction?"
Laurie: "Thirty over a hundred."
(TA) Mrs. Hanley: "Thank you." (#61 G.M.)

TH This coding category indicated an instance when the teacher was helping a student on an individual basis while a group was at the board or during a time when the class was working a problem or problems. These instances may have been simply stated in the field notes such as, Mrs. Hanley is helping Mark (#36 G.M.) or dialogue between teacher and student may have been recorded. An example of an instance of the teacher helping an individual and recorded dialogue follows:

Brian asked Mrs. Hanley how you would write number three. Brian: "Then I cross multiply?" Mrs. Hanley: "That's correct." (#68 G.M.)

The instances with dialogue were coded TH and recorded as one communication act since the individual communication acts were not part of the whole class movement of the lesson.

TP This coding category indicated instances where the teacher responded to a student by reteaching or repeating something that had already been taught. This kind of response by the teacher prompted by a student question, comment, or feedback communication act, provided the class with further clarification of the content in the lesson. Examples of such a communication act were as follows;

Mrs. Hanley is talking about the sum of two angles being ninety degrees. Hans said he didn't get it. Mrs. Hanley put numbers in the picture on the board and explained it again. Hans said he got it. (#30 Alg.)

(Mrs. Hanley had written on the board .005 (60,000) = $300) Sandy asked about the decimal places in the multiplication problem in finding the commission. Mrs. Hanley worked it out for her. (#69 G.M.)

The communication acts of the teacher which were signed minus, numbered six.

TW This coding cateogry indicated a mistake in content made by the teacher. This occurred once in the entire 62 sets of field notes. It was during a general math class. A student asked if the number one was a prime and the teacher said that it was. (#24 G.M.) Two is the smallest prime number.

TW_B This coding cateogry indicated a mistake at the board by the teacher. This occurred once in the 62 sets of field notes. The instance involved a student reading a problem to the teacher and the teacher writing it incorrectly on the board. Another student went to the board and wrote it correctly for the teacher. (#117 Alg.)

RB This coding category indicated a statement by the teacher reprimanding a student or students or the class for some unacceptable behavior. Unacceptable behavior was usually talking or not paying attention. The following are examples of reprimands.

Mrs. Hanley: "All right. There's too much noise, people." (#7 Alg.)

The boys in the back have maintained their conversation.
Mrs. Hanley: "Darin, come up here and sit."
Darin moves to the first desk in row one. (#32 Alg.)

Mrs. Hanley: "O.K. All right. There are five boys who are not going to finish out the period." (#9 G.M.)

Mrs. Hanley: "All right, people. We're not going to have this today. We are not going to have this murmur of conversation." (#53 G.M.)

RC This coding cateogry indicated a statement by the teacher reprimanding a student or students or the class for lack of understanding about the content being studied. This could involve a mistake that was made or not remembering something already taught. Examples of reprimands for content were as follows:

(Three students were at the board)
Mrs. Hanley: "All right. Kevin, you blew it. A little thing he should have remembered but he forgot." (#25 Alg.)

Mrs. Hanley: "To tell you the truth, Alice, you're not doing what you need to do first and what you've written down there doesn't make sense." (#39 Alg.)

Mrs. Hanley: "Think now. You're not thinking. You're guessing. Brian, you're guessing." (#11 G.M.)

(The problem to be solved was written on the board.)
Mrs. Hanley: "How do we start?"
Keith: "Division."
Mrs. Hanley: "No. We want to multiply." (#86 G.M.)

TOT This coding cateogry indicated a statement or question by the teacher which was related to the content being studied but was not to the point of the lesson. This kind of remark or statement was not helpful in forwarding the flow or movement of the mathematics being taught in the episode. The following were examples of this coding category.

(The lesson was on functions and the aspect being discussed was what letters are used to signify the function and what letters for the elements of the function.)
Mrs. Hanley: "I don't really know how they set this thing up. It's just something you learn after studying math for fifteen or twenty years." (#17 Alg.)

In referring to the oral exercises for the lesson Mrs. Hanley made the following remark: "We aren't going to do all of them. Some of them are kind of stupid, really." (#18 Alg.)

(The lesson was on making flow charts. The class was in the process of constructing a flow chart.) One of the students asked what Fortran is. Mrs. Hanley talked about Fortran and reading by electrical impulses. (#10 G.M.)

(The class is doing problems with percentage, particularly working with original price and reduced price.) Mrs. Hanley told the class the reduction is twelve dollars. She then posed the question of whether it was reduction or deduction. (#63 G.M.)

TOL This coding category indicated a statement or question by the teacher which was irrelevant to the content being studied.

This could be a response to an irrelevant question posed by a student or a self-initiated communication act. The following were examples of this communication act.

(Mrs. Hanley was explaining how to do subtraction on the number line.)
Student: "Can we put the shades down?"
Mrs. Hanley: "You may adjust them if you need to."
(#1 G.M.)

A group of students were at the board. Renea is telling one of the students how to do the problem. Vicki tells Renea to stop telling Ruth how to do it.
Mrs. Hanley: "Renea can't keep her mouth shut. If she learns something we all know it." (#40 G.M.)

(The class is studying proportions and ratios.)
Mrs. Hanley: "What can you do to a proportion that you can't do to any other thing?"
Craig: "Erase it."
Mrs. Hanley: "You can erase a lot of things." (#60 G.M.)

The communication acts of the students which were signed plus, numbered eleven.

SQ This coding category indicated a question by a student which pertained to content.

SR This coding category indicated a right answer that was given to a question about the content or a correct step given in the working of a problem.

SR_B This coding category indicated a correct answer given at the board. This response would have been written on the board and not stated in the verbal acts recorded.

SF_+ This coding category indicated positive feedback to a feedback question by the teacher. Positive feedback consisted of a "yes" to understanding the problem, a "no" to the inquiry of any more questions.

SA This coding category indicated a statement of affirmation by one student to another student. An example of such an instance occurred during a general math episode in which students were being selected one at a time to go to the board. As Esther was asked to go to the board Chad said: "Esther will get it right." (#29 G.M.)

SH This coding category indicated an instance of a student helping another student while a group was at the board or during a time when the class was working a problem or problems. These instances may have been simply stated in the field notes such as, Renea is answering Chad's questions (#36 G.M.) or some dialogue may have been recorded or indicated. The instances with dialogue were coded SH and recorded as one communication act since the individual communication acts were not part of the whole class movement of the lesson.

SI The coding category indicated a particular insight about the content by a student which was beyond what the teacher had presented. Examples of student insight were as follows.

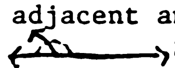
(The algebra class was working with the property of transitivity. This property is stated if a = b and b = c then a = c. In the problem being studied the result was if a = b and b = 0 then a = 0. The student making the comment referred to the 0 as blank.)

(SI) Sam: "Then when they leave the left side blank that's a dead give away for transitive."
Mrs. Hanley: "That's right."
(SI) Student: "Except for a few exceptions."
Mrs. Hanley: "Right. Except for a few exceptions." (#6 Alg.)

(The general math class was studying perpendicular lines. Mrs. Hanley was demonstrating perpendicular lines using three pencils and showing the pattern  .)

(SI) Renea: "You can't have an S and two straight lines." (Here she drew what she meant  .)
Mrs. Hanley: "No. The third line must also be straight." (#33 G.M.)

SP This coding category indicated instances of a student providing an opportunity for the teacher to explain or clarify the mathematics being taught. Only those communication acts for which the explanation or clarification actually occurred were coded SP. The following were examples of this kind of communication act.

(The algebra class was studying how to measure angles. Mrs. Hanley explained that a straight angle has 180°. She drew a diagram of two adjacent angles with the sum of their degrees equaling 180°.  She wanted the students to look at the diagram and be able to tell her the sum was 180°).
Mrs. Hanley: "Does that make sense? Yes or no. Cathy?"
Cathy: "Yes."
Mrs. Hanley: "Roger?"
(SP) Roger: "No."
Mrs. Hanley explained once again how the sum of the degrees of the two angles equaled one hundred eighty degrees. (#29 Alg.)

The general math class had been working with place value. There were four students at the board and Mrs. Hanley had given them a set of digits to be written in the order she gave them in a horizontal line. Then the students were asked to place the decimal point to make a number of a certain amount. Given the digits 2305 the students were asked to place the decimal point to make a number between twenty and two hundred (23.05). After about five such exercises the following occurred.
(SP) Laurie: "How does it go? I don't remember this stuff."
Mrs. Hanley stated that maybe she should go through the sequence of place values. She then listed off the place values from ones upward. (#38 G.M.)

SC This coding category indicated a student comment which was relevant to the movement of the lesson but did not fit into any of the above categories. These comments may have been about the content or responses to comments by the teacher. The following were examples of this communication.

(The algebra class was studying how to solve linear equations.)
(SC) Student: "It looks easy."
Mrs. Hanley: "It is easy. That's the problem. Learn the transformations as we go along." (#5 Alg.)

(A student was working at the board while the class watched. He completed part of the problem and stopped.)
(SC) Alice: "He's not finished yet." (#23 Alg.)

(The general math class was being introduced to the Euclidean Algorithm, a method for finding the highest common factor between two numbers. In the course of introducing the process the following occurred.)
Mrs. Hanley: "Algorithim is a glorified word for rule."
(SC) Tony: "I thought it stood for arithmetic." (#26 G.M.)

(Four students were at the board doing a problem in the addition of decimals. All four had the wrong answer.)
(SC) Chad: "You have to keep your decimals lined up." (#43 G.M.)

GP This coding category indicated an instance in the episode where more than one communication act had occurred but the acts were not recorded individually. This was usually noted as a discussion or together the teacher and students come to a conclusion. If this discussion contributed to the forward movement in the lesson it was marked GP for generally positive. The following were examples of such instances.

(The algebra class was practicing the solving of linear equations. Squad leaders were appointed, one from each row, to check the work of the other students in their row and to help them solve the equation.) After the problem was given they (the squad leaders) walked up and down their row of students and watched them work. They answered their questions and corrected their mistakes. (#7 Alg.)

(The general math class was working on a set of problems for review. The class is doing the problems individually at first). Boys are asking questions. Mrs. Hanley is among them answering them. (#3 G.M.)

GP_B This coding category indicated an instance at the board of a student having begun a problem correctly but the solution was not recorded.

The communication acts of the student which were signed minus, numbered eight.

SW This coding category indicated a wrong answer that was given to a question about the content or an incorrect suggestion for how to solve a problem.

SW_B This coding category indicated an incorrect answer given at the board. This response would have been written on the board and not stated in the verbal acts recorded.

SF_- This coding category indicated negative feedback to a feedback question by the teacher. Negative feedback consisted of a "no" to understanding the problem or a "yes" to the inquiry of any more questions.

SF_o This coding category indicated a feedback response of "I don't know" to a question by the teacher.

SOT This coding category indicated a statement or question by a student which was related to the content being studied but was not to the point of the lesson. This kind of remark or statement was not helpful in the forward movement of the lesson. Instances where the student shouted out the answer, having been asked to wait to be called on, were included in this category. The following were examples of this communication act.

(The algebra class was solving story problems. They had just attempted a problem that did not have sufficient information to be solved.)
Dean: "They wouldn't put a problem in there that you couldn't solve." "That's dumb to put something like that in there." (#36 Alg.)

(Four students were working at the board. They were solving linear equations that had decimals and fractions in them. Mrs. Hanley had just shown the class how one problem could be done three different ways.)
Roger to Mrs. Hanley: "Do you know where this started in the book?"
Mrs. Hanley answered that there really weren't examples of this kind in the book. (#60 Alg.)

(The general math class was constructing flow charts.)
Mrs. Hanley: "We need to put something in our program to turn the machine off."
Chad: "Push the stop button." (#8 G.M.)

(The general math class was working problems out of the book.)
Mrs. Hanley: "Get the answer for five and I'll call on someone to give it."
(SOT) Brian just calls it out. (#15 G.M.)

SOL This coding category indicated a statement or question by a student which was irrelevant to the content being studied. The following were examples of this communication act.

(The algebra class was solving story problems. Most of the problem was on the board.)
Mrs. Hanley: "Brad, will you hop up there and finish the problem?" Brad hopped up.
(SOL) Darin: "That Brad, he is such a card."
(SOL) Roger to Brad: "Go jump off a bridge." (#37 Alg.)

(The algebra class was graphing inequalities on the board. The class was working on a problem as Mrs. Hanley drew the graphs.)
Roger: "Mrs. Hanley, can we go back to story problems? Digit story problems?" (#114 Alg.)

(In the general math class the lesson was on expanded notation. One student was at the board working a problem.) Tony got up and checked out the temperature. "It's almost eighty degrees in here." (#4 G.M.)

(Mrs. Hanley was teaching the general math class about lateral area of a prism. She began teaching about volume by asking the following question.)
Mrs. Hanley: "Suppose I asked you to fill this prism up."
(SOL) Brian: "I would."
(SOL) Chad: "With what?" (#37 G.M.)

GN This coding category indicated an instance in the episode where more than one communication act had occurred but the acts were not recorded individually. This situation or occurrence did not contribute to the forward movement of the lesson and was marked GN for generally negative or inhibiting. The following were examples of these kind of instances.

> Dean and Darin are keeping up a constant conversation all during class. Frank enters in frequently. Hans comments occasionally. (#9 Alg.)

> (Mrs. Hanley was introducing the multiplication of fractions.) The boys are yelling out so Mrs. Hanley asked Brent to come to the front desk, row one. (#48 G.M.)

GN_B This coding category indicated an instance at the board of a student beginning a problem incorrectly and the final solution not being recorded. There was only one example of this act in Episode #94 G.M.

O This coding category indicated an instance of the class being interrupted by neither the teacher nor a student in the class but by another source. An intercom announcement or a student coming in the room to get the attendance slip were two examples of this act.

APPENDIX D

APPENDIX D

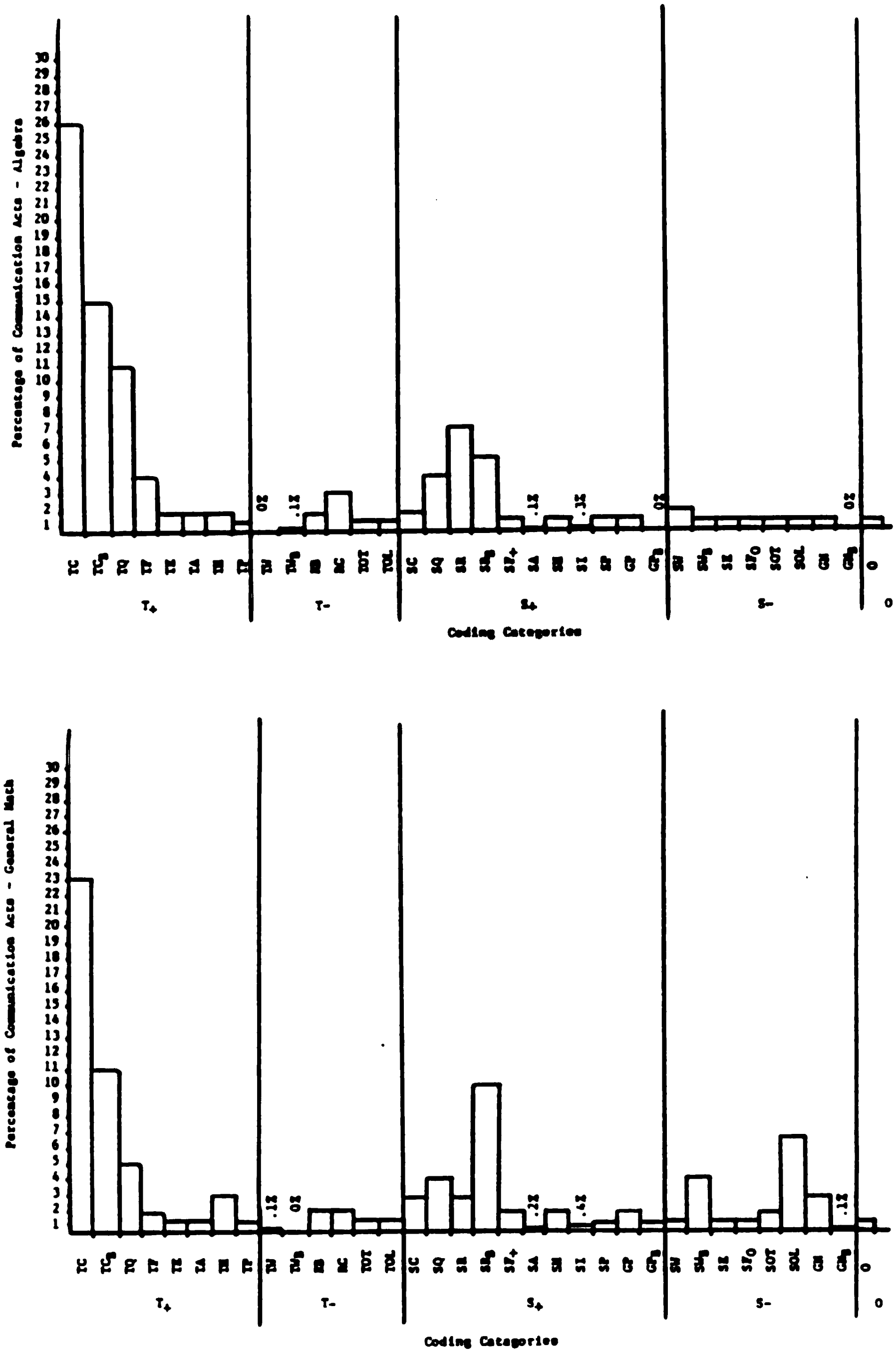
COMMUNICATION ACTS ACCORDING TO THE FREQUENCY OF A CODING CATEGORY

Coding Category	Algebra (N=1613)	General Math (N=1218)
TC	418 (26%)	276 (23%)
TC_B	248 (15%)	128 (11%)
TQ	175 (11%)	66 (5%)
TF	59 (4%)	29 (2%)
TE	37 (2%)	15 (1%)
TA	32 (2%)	8 (1%)
TH	28 (2%)	35 (3%)
TP	20 (1%)	13 (1%)
TW	0 (0%)	1 (.1%)
TW_B	1 (.1%)	0 (0%)
RB	26 (2%)	21 (2%)
RC	45 (3%)	20 (2%)
TOT	13 (1%)	15 (1%)
TOL	16 (1%)	12 (1%)
SC	32 (2%)	39 (3%)
SQ	58 (4%)	44 (4%)
SR	108 (7%)	39 (3%)
SR_B	81 (5%)	123 (10%)
SF_+	17 (1%)	25 (2%)
SA	2 (.1%)	2 (.2%)
SH	11 (1%)	20 (2%)
SI	5 (.3%)	5 (.4%)
SP	20 (1%)	13 (1%)
GP	15 (1%)	19 (2%)
GP_B	0 (0%)	12 (1%)
SW	37 (2%)	11 (1%)
SW_B	32 (2%)	52 (4%)
SF_-	9 (1%)	12 (1%)
$SF_\circ$	16 (1%)	9 (1%)
SOT	17 (1%)	25 (2%)
SOL	11 (1%)	83 (7%)
GN	16 (1%)	32 (3%)
GN_B	0 (0%)	1 (.1%)
O	8 (1%)	13 (1%)
Total	102.5%*	101.8%*

* Due to rounding off to the second place the percentages do not sum to 100%

NOTE: Percentages were derived from the number of communication acts for a coding category over the total number of communication acts.

Communication Acts According to the Frequency
of a Coding Category



Note: Percentages were derived from the number of communication acts in a coding category over the total number of communication acts. (Algebra N = 1613, General Math N = 1218)

APPENDIX E

APPENDIX E

PATTERNS OF ENACTMENT

Algebra

The 43 algebra episodes which had outlines of all plus acts were generally teacher dominated. That is, the social participation structure of the teacher directing from the front was most often used, a teacher coding category had the highest frequency, the teacher more often had the highest number of communication acts on the contingency table, and the teacher was most often the one who concluded the episode. The teacher instructed for the most part through a question/answer dynamic and the student communication acts consisted of correct answers to questions, feedback statements, and questions about the content.

Episode #92 Alg. was an example of an episode with an enactment pattern of all plus acts. The following is the outline for Episode #92 Alg.

Problem: Story Problem

1. Reading the problem.
   (+) TC (1)
2. Identifying the variables.
   (+) TQ (2)
   (+) SR (3)
   (+) TQ (4)
   (+) SR (5)
   (+) TC (6)
   (+) TQ (7)
   (+) TC (8)
   (+) TC_B (9)
   (+) TQ (10)
   (+) SR (11)

3. Setting up the equation.
   (+) SR (12)
   (+) TE (13)
4. Solving the equation.
   (+) TC_B (14)

The outline shows that this episode was structured on the teacher's logical presentation consisting of four steps. The content was a short problem and the steps indicated the teacher's approach to solving story problems. First of all the problem was read to the class. The problem was then analyzed to determine the variables or what was being looked for in the problem. After identifying the variables, the relationships among the variables were stated in an equation. The proper procedures were followed to solve the equation and reach the solution.

The script of the episode, the string of coding categories, and the plus/minus signs gave some indication of the flow of the lesson. All plus acts indicated that in the lesson the problem was solved without interruption or interference. The coded items indicated that the teacher read the problem and the variables were identified through a series of teacher questions and student answers. The student answers were correct each time and the teacher commented and wrote on the board during this step in the lesson. A student gave the equation for step three and the teacher gave some kind of exhortation to the class. The teacher put the solution to the equation on the board. This can be generated from the outline. The following is Episode #92 Alg. as it was recorded.

#92 3/31/80 pp. 2-3 Algebra

(Referring to the previous problem) If you need thirty-five seconds you would adjust the fuel supplies of the engines to meet your needs.

(+) TC (1) — 1.) 9:15 — Mrs. Hanley reads problem number six. The problem is: One pipe can fill a small reservoir in fifteen hours, while with a second pipe also in operation, the reservoir can be filled in six hours. How long would it take the second pipe alone to fill the reservoir?

(+) TQ (2)
(+) SR (3) — 2.), 3.) Mrs. H: "What is the job in this problem?" Some in the class answer filling the reservoir.

(+) TQ (4)
(+) SR (5) — 4.) 5.) Mrs. H: "And the participants are?" Someone names the pipes.

As Mrs. Hanley is writing the problem on the board she asks several questions.

(+) TC (6) — 6.) Mrs. H: "I want to know how long it will take pipe number one to do it along."

(+) TQ (7) — 7.) The answer is fifteen hours. Mrs. Hanley asked how
(+) TC (8) — 8.) long it would take to fill the reservoir. She made sure it was understood that the six hours applied to both pipes working together.

(+) TC_B (9) — 9.) Board

	Alone	Time worked	Fractions
Pipe #1	15 hours	6	6/15
Pipe #2	x 10 hours	6	6/x

(+) TQ (10) — 10.)
(+) SR (11)
(+) SR (12)
(+) TE (13) — 11.), 12.), 13.) Mrs. Hanley asked Alia what the fractions and equation were. He got everything right. Mrs. Hanley commented that if you just read the problem you won't get the equation.

Mrs. H: "If you go step by step you will get there." Mrs. Hanley was referring to the fact most students just read the problem and say they don't get it. If they follow the steps she is showing them, they can get it.

Mrs. Hanley solved the equation and indicated the answer up by the x in the first chart of the problem.

(+) TC_B (14) — 14.) Board

$15x \cdot 6/15 + {}^{15}\cancel{x} \cdot 6/\cancel{x} = 1 \cdot 15x$

$6x + 90 = 15x$

$90 = 9x$

$10 = x$

$x - 10$

9:20 The next problem to be worked is number twelve on page three hundred forty.

There were 34 algebra episodes which had patterns of enactment containing minus acts pertaining to interruptions related to on task or content communication acts. Such interference included teacher errors, reprimands by the teacher, student errors, and negative feedback. These episodes for the most part had content about linear equations or algebraic simplification. The episodes were generally teacher dominated. The social participation structure most often used was that of the teacher directing from the front of the room and teacher coding categories were more frequent than student coding categories. The teacher gave most of the conclusions.

Episode #51 algebra was an example of an episode with an enactment pattern containing interference related to on task acts. The following is the outline for Episode #51 Alg.

<u>Problem</u>: Simplify $5h^2(-hk^4) - (-3h)(2h^2k)\ (k^3)$

1. Giving the problem to those at the board.
   (+) TC (1)
   (+) TC (2)
2. Asking for the order of operations.
   (+) TQ (3)
   (+) SR (4)
   (+) TC (5)
   (+) SC (6)
   (+) TQ (7)
   (-) SW_B (8)
   (-) RC^B (9)
   (+) TF (10)
   (-) SF_- (11)
   (+) TQ^- (12)
   (+) TQ (13)
   (+) SR_B (14)

This episode was structured on the teacher's logical presentation of content consisting of two steps. The content was an algebraic simplification problem and a group of students were working the problem at the board. The problem was given and before the students began the problem the teacher asked one of the students how the problem was to be worked. With this social participation structure of students working at the board the teacher seldom structured the problem solving activity.

The script of the episode and the plus/minus signs gave some indication of the flow of the lesson. The three minus acts indicated there were interruptions in the flow of the lesson and the coding categories indicated there were acts of a student wrong answer, a teacher reprimand for content, and negative feedback by a student. The teacher gave the problem to those at the board. The student she asked about the order of operations answered correctly and the work at the board began. As the script showed there was teacher/student interaction during this part of the enactment. There was a teacher comment and a student comment followed by a teacher question. A student made a mistake at the board which prompted a reprimand by the teacher about the mistake made. This constituted an interruption in the flow of the lesson. The teacher requested feedback at this point and received a negative response. This prompted two questions and the episode concluded with the correct solution on the board by one of the students. The following is Episode #51 Alg. as it was recorded.

#51 12/18/79 p. 4 Algebra

Mrs. Hanley: "The exponent tells you how many times it is used as a factor." "In terms of common usage you can always go back to simple problems."

(+) TC (1)	1.)	9:30	Mrs. H:	"We didn't get nearly as many people to the board as possible but we are going to do one more round with people who haven't been up for a while."
(+) TC (2)	2.)			This round included Frank, Roger, Helen, Cathy, and Martha. There was a sigh of relief from some. At the board everyone is getting the same problem.

Board

$$5h^2\ (-h\ k^4) - (-3h)\ (2h^2\ k)\ (k^3)$$

(+) TQ (3) (+) SR (4)	3.), 4.)			Mrs. Hanley asked Roger about order of operations. He rattled it off correctly. (That is, where to start and what to do next.)
		9:35		Cathy just erased her whole answer. Roger hasn't done anything. Martha erased her whole answer.
(+) TC (5)	5.)		Mrs. H:	"Helen, you look like you are having decision problems."
(+) SC (6)	6.)		Helen:	"I am."
(+) TQ (7)	7.)			Mrs. Hanley helped her by asking about h's and k's.
(-) SW_B (8)	8.)			Board

Cathy

$5hk^4 + -6h^3\ k \cdot k^3$

$\rightarrow -5h^3k^4 + 6h^3k^4$

$1\ h^6\ k^8$

(-) RC (9)	9.)		Mrs. H:	"Cathy, you did the same thing. When you're combining terms what do you do with exponents? Nothing!"
(+) TF (10)	10.)			"Does everyone understand that?"
(-) SF_- (11)	11.)			There were some no's in response to the question.
(+) TQ (12)	12.)	9:40	Mrs. H:	"Who said no?"
(+) TQ (13)	13.)			"What don't you understand about it?"
(+) SR_B (14)	14.)			Board

Roger

$5h^2 - hk^4 + 3h \cdot 2H^2\ k \cdot k^3$

$-5H^3k^4 + 6H^3k^4$

H^3k^4

Mrs. Hanley explained there would be a substitute tomorrow.

There were 20 algebra episodes which had patterns of enactment containing minus acts pertaining to interruptions related to off task communication acts. This interference included teacher reprimands for behavior, students talking during the teaching of the lesson, and teacher and student comments that were either directed toward the topic being studied but not helpful to the solution of the task or irrelevant comments or questions. About one third of the off task minus acts for these algebra episodes were directed toward the content.

These episodes for the most part had content about linear equations or algebraic simplification. The episodes were generally teacher dominated The social participation structure most often used was that of the teacher directing from the front of the room and teacher coding categories were more frequent than student coding categories. The teacher gave most of the conclusions.

Episode #46 Alg. was an example of an episode with an enactment pattern containing interference related to off task acts. The following is the outline for Episode #46 Alg.

Problem: Solve $(p^2-2p+1) - (2p^2+3p+5) = 2-3p-p^2$

1. Giving the problem on the board.
   (+) TC_B (1)
   (-) TOL (2)
   (+) SC (3)
   (-) GN (4)
   (-) RB (5)
2. Removing parentheses.
   (+) SR (6)
3. Combining terms.
   (+) SR (7)
   (+) SR (8)
   (+) TC (9)
   (+) SR (10)
4. Solving for p.
   (+) SR (11)
   (+) TE (12)
   (-) O (13)
   (+) SQ (14)
   (+) SC (15)

This episode was structured on the teacher's logical presentation of content consisting of four steps. The content was a linear equation and the steps indicated the teacher's approach to solving linear equations. The equation was written on the board and the first step was to remove the parentheses. This involved knowing that in removing parentheses preceded by a minus sign all the signs inside the parentheses are changed. Once the parentheses were removed and signs were changed the next step was to combine terms. This involved transferring like terms to the same side of the equals sign, another place where signs were important. After like terms were combined then the value for p was determined. This involved having all the terms with p in them on one side of the equals sign and the remaining terms on the other side. Then using the multiplication transformation the value for p was determined. This completed the solving of the linear equation.

The script of the episode, the string of coding categories, and the plus/minus signs gave some indication of the flow of the lesson. The four minus acts indicated interruptions in the flow of the lesson with three of the minus acts occurring between the first and second step. The coded items indicated that the first minus act was an irrelevant comment by the teacher. The next two minus acts were general negative behavior on the part of the students followed by a reprimand for behavior by the teacher. The fourth minus act was a zero coded item which indicated an interruption from outside the classroom.

Reading through the script the teacher put the problem on the board and made some comment irrelevant to the task at hand. A student made a comment and then the minus behavior on the part of students and teacher took place. A student successfully completed the second step of removing the parentheses. The terms were combined through student/teacher interaction. The fourth step of solving for p was completed by a student. The teacher gave some advice or warning which was followed by an interruption from outside the classroom. A student asked a question pertaining to the problem and a statement was made by a student about the problem. The following is Episode #46 Alg. as it was recorded.

#46 12/10/79 pp. 4–5 Algebra

Mrs. Hanley: "The rest of it is putting it together correctly."

Code	No.	Item	Speaker	Text
(+) TC_B	(1)	1.)		**Board** $(p^2 - 2p + 1) - (2p^2 + 3p + 5) = 2 - 3p - p^2$
(-) TOL	(2)	2.)	Mrs. H:	"That's a nice bunch of junk."
(+) SC	(3)	3.)		Dean began explaining how to work the problem. Mrs. Hanley stopped him by saying:
(-) GN	(4)	4.)		"Excuse me, Dean, but somebody is talking." She went on to say she probably shouldn't be insulted.
(-) RB	(5)	5.)	Mrs. H:	"That shouldn't bother me but it does. Now would that person please be quiet?" This was prompted by the fact that Teresa and Sarah had been carrying on a rather consistent conversation during class. They stopped after the above remarks were made.
(+) SR	(6)	6.)		Dean explained the following steps to the problem. **Board** $p^2 - 2p + 1 - 2p^2 - 3p - 5 = 2 - 3p - p^2$
(+) SR	(7)	7.)		$- p^2 - 5P - 4 = 2 - 3p - p^2$ $+p^2+3p \quad +p^2+3p$
(+) SR	(8)	8.)		Thelma suggested that p squared be added to both sides.
(+) TC	(9)	9.)	Mrs. H:	"There's something else you may not want over here, Thelma." **Board** $-2p - 4^{+4} = 2^{+4}$ $-2p = 6$
(+) SR	(10)	10.)	Thelma:	"Add four."
(+) SR	(11)	11.)		Martha contributed the step of multiplying by negative one half. **Board** $-1/2\ (-2p) = (6) - 1/2$ $p = -3$
(+) TE	(12)	12.)	Mrs. H:	"It's not that hard. It's just so you don't get lost while you're doing it."
(-) O	(13)	13.) 9:24		A boy came in to give Mrs. Hanley a note.
(+) SQ (+) SC	(14) (15)	14.), 15.)		Sarah asked a question and then said, "Oh, I didn't see that minus."

Mrs. Hanley gave the assignment. It is long and there is not much time left in class.

There were 28 algebra episodes which had patterns of enactment containing minus acts pertaining to interruptions related to on task acts and minus acts related to off task acts. Of the 127 minus acts coded in these episodes 64 were on task acts and 63 were off task acts. These episodes varied in social participation structures used and whether the teacher or a student concluded the episode. The teacher had the most coded items.

Episode #47 Alg. was an example of an episode with an enactment pattern containing interference related to on task acts and interference related to off task acts. The following is the outline for Episode #47 Alg.

Problem: Solve 2(x+5) - 3(x-4) = 7

1. Presenting the problem to a student at the board.
   (+) TC (1)
   (+) SR_B (2)
   (+) TF (3)
   (+) TF (4)
   (-) GN (5)
   (-) RB (6)
2. Requesting the property for the last step.
   (+) TQ (7)
   (+) TQ (8)
   (-) SW (9)
   (-) RC (10)
   (-) SW (11)
   (+) TQ (12)
   (+) SR (13)

This episode was structured on the teacher's logical presentation of content consisting of two steps. The content was a linear equation and the problem was worked by a student at the board. There were four students at the board and each worked a different problem. Only one student's work was recorded in this episode, although another student's problem was referred to by the teacher. The problem was presented to the student by the teacher. After the student had solved the problem the teacher asked the class for the mathematical property used to complete the last step of the problem. The script of the episode and the plus/minus signs gave some indication of the flow of the lesson. The five minus acts indicated there were breaks in the flow of the lesson, two just before step two and three during the interaction about step two. The coding categories indicated that the two minus acts before step two were off task acts, generally negative behavior by the students followed by a reprimand for behavior by the teacher. The three minus acts during the interaction about step two were on task minus acts in a series with a student wrong answer, a teacher reprimand for content and another student wrong answer.

Reading through the script the teacher gave the problem of the episode to a student at the board. The student worked the problem correctly. The teacher asked for feedback but none was recorded. There followed the negative student behavior and the reprimand by the teacher. The teacher asked two questions in a row related to the student's work on the board. One student gave a wrong answer and the teacher stated it was incorrect. A second incorrect answer was given and the teacher asked a question. A student gave the correct answer to this question. The following is Episode #47 Alg. as it was recorded.

#47 • 12/18/79 pp. 1-2 **Algebra**

Mrs. Hanley sent the first person in each row to the board. She is reading off a problem for each one. Mrs. Hanley is sitting in the last desk of the fourth row. At the board are Sam, Brad, Matt, and Cecile.

(+) TC	(1)	1.)			**<u>Board</u>**
					<u>Brad</u>
					$2(x+5)-3(x-4)=7$
(+) SR_B	(2)	2.)			$2x+10-3x+12=7$
					$-x+22=7$
					$-x+22-22=7-22$
					$-x=-15$
					$x=15$
(+) TF	(3)	3.)	8:58	Mrs. H:	**"All right. Does anyone have any questions about Cecile's problem? Any question about where anything came from, powers or signs?"**
(+) TF	(4)	4.)			
(−) GN	(5)	5.)			**During class Joyce and Edna are eating and talking. They don't appear to be paying any attention to the board at all. At one point Mrs. Hanley came over to the table and asked them to stop eating.**
(−) RB	(6)	6.)			
(+) TQ	(7)	7.)			**Referring to Brad's problem Mrs. Hanley said, "How did he get from the second to the last line to the last line? Alice, do you know?"**
(+) TQ	(8)	8.)			
(−) SW	(9)	9.)		Alice:	**"Divide by negative x."**
(−) RC	(10)	10.)		Mrs. H:	**"No."**
(−) SW	(11)	11.)			**Edna gave an answer about looking for x and changing the signes.**
(+) TQ	(12)	12.)			**Mrs. Hanley asked for algebraic properties.**
(+) SR	(13)	13.)			**Kevin gave the right answer, dividing by negative one.**

Sam got stuck on his problem and Mrs. Hanley helped him out.

General Math

The 20 general math episodes which had outlines of all plus acts were generally teacher dominated. The social participation structure of the teacher directing from the front was most often used, a teacher coding category had the highest frequency, the teacher more often had the highest number of communication acts on the contingency table, and the teacher was most often the one who concluded the episode. The content centered around percents and signed numbers. The teacher instructed for the most part through a question/answer dynamic and the student communication acts consisted of questions, feedback, and correct answers to questions.

Episode #25 G.M. was an example of an episode with an enactment pattern of all plus acts. The following is the outline for Episode #25 G.M.

Problem Lowest Common Multiple of 27 and 45

1. Giving the problem on the board
   (+) TC (1)
   (+) TC_B (2)
   (+) TC_B (3)
2. Factoring 27 and 45 to primes.
   (+) TC_B (4)
3. Finding the lowest common multiple of 27 and 45.
   (+) TC_B (5)
   (+) SF_+ (6)
   (+) SQ^+ (7)
   (+) SP (8)
   (+) TP (9)
4. Solving the problem 8/27 + 13/45
   (+) TC (10)
   (+) TC_B (11)
   (+) TE^B (12)

This episode was structured on the teacher's logical presentation of content consisting of four steps. The content was finding the lowest common multiple of 27 and 45. The teacher presented the problem to the class by writing it on the board. The next step was to factor each of the two numbers to primes. The combination of primes was selected which when multiplied contained both the numbers and was the smallest number which contained both. This gave the lowest common multiple. For the fourth step the teacher demonstrated that when finding the lowest common denominator, as in the problem 8/27 + 13/45, what was actually being formed was the lowest common multiple. These two concepts are the same. This provided an application for the concept and made it more familiar to the students.

The script of the episode and the plus/minus signs gave some indication of the flow of the lesson. All plus acts indicated the lesson went directly through the problem with no interference along the way. The coding categories indicated that the teacher gave the problem on the board and worked out the factoring on the board without any recorded student interaction. During the third step when the primes were selected to form the lowest common multiple, one student gave positive feedback (indicating an understanding of the process), another student asked a question. A third student asked a question which provided an opportunity to restate the purpose of the problem which the teacher did. Step four provided an application of the concept demonstrated by the teacher and the episode was concluded by some advice or a warning by the teacher. The following is Episode #25 G.M. as it was recorded.

#25 11/13/79 pp. 7-8 General Math

Mrs. Hanley: "If you start with smaller ones and write the larger ones then you'll meet in the middle and be all done."

9:57 Mrs. Hanley: "These are two things that we have learned" (factoring to primes and finding the set of all factors).

1.) Mrs. H: "Another thing we have learned is the lowest common multiple."

2.) Board

Find the lowest common multiple for 27 and 45.

3.) Mrs. H: "You always find the lowest common multiple for two numbers."

4.) Board

27	45
9 3	$9 \cdot 5$
$3 \cdot 3 \cdot 3$	$3^2 \cdot 5$
3^3	

5.) $3^3 \cdot 5 = 135$

$$\begin{array}{r} 27 \\ \underline{5} \\ 135 \end{array}$$

6.) Chad: "I gotcha now."

7.) Brent asked about twenty-seven.
8.) Keith asked why it wasn't three to the fourth power.

9.) Mrs. H: "We are in the business of finding the lowest common multiple."
10.) "If we are looking for the lowest common denominator of 8/27 + 13/45, then the lowest common denominator is the lowest common multiple."

11.) Board

8/27 + 13/45

40/135 + 39/135 = 79/135

12.) Mrs. H: "All right. We've had quite a bit of practice with these already. You are expected to know this as of today."

10:03 Mrs. Hanley: "We are going to do something new today. We are going to look for the highest common factor."

There were 22 general math episodes which had patterns of enactment containing minus acts pertaining to interruptions related to on task or content communication acts. Such interference included reprimands by the teacher for content, student errors, and negative feedback. These episodes for the most part had content about percents. The social participation structure most often used was that of a group of students working at the board. Consequently most of these episodes had only one step in the teacher's logical presentation of content. The episodes were student dominated. That is, the student coding categories were more frequent than the teacher coding categories and the students gave most of the conclusions.

Episode #88 G.M. was an example of an episode with an enactment pattern containing interference related to on task acts. The following is the outline for Episode #88 G.M.

Problem: Draw a Picture of a Budget

1. Giving the problem to 3 students at the board.

(+)	TC	(1)
(+)	GP	(2)
(-)	SW_B	(3)
(-)	SW_B	(4)
(+)	SR_B	(5)
(+)	SC	(6)
(+)	TC	(7)
(+)	SH	(8)
(+)	SR_B	(9)

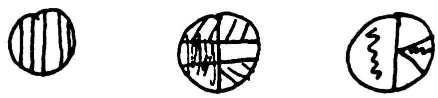
The episode was structured on one step, the teacher's presentation of the problem to three students at the board. Once the problem was given the students were allowed to work on their own until the solution was produced.

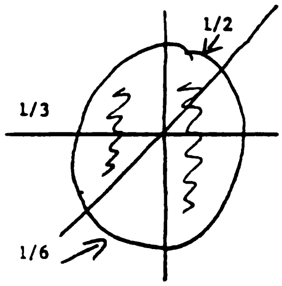
The script of the episode and the plus/minus signs gave some indication of the flow of activity. The two minus acts indicated there were interruptions in the flow of the lesson and the coding categories indicated these interruptions were due to student wrong answers at the board. The teacher gave the problem to the three students at the board and there was some generally positive help from the class as the students began working. However, two of the students worked the problem incorrectly while the third seemed to have the solution. A student made a comment to which the teacher responded and some student helped another work the problem. The episode ends with the correct answer on the board by one of the students. The following is Episode #88 G.M. as it was recorded.

#88 6/3/80 pp. 8-9 **General Math**

Laurie came in and told Brian he was in her seat. Brian wouldn't move.

9:55 Barbra arrived.

(+) TC	(1)	1.)		Mrs. Hanley asked those at the board to draw a picture of a budget such that one half is spent for salaries, one third for overhead, and one sixth is for profit.
(+) GP	(2)	2.)		Since no one at the board knew what to draw (Chad had started with drawing a smiling face) several students gave suggestions from their desks. To make a pie and divide it up was the most common suggestion. The following drawings were produced.
(−) SW_B	(3)	3.)		<u>Board</u>
(−) SW_B	(4)	4.)		<u>Keith</u> <u>Tony</u> <u>Chad</u>
(+) SR_B	(5)	5.)		
(+) SC	(6)	6.), 7.)	Sandy:	"I know how to do it." Mrs. Hanley tells her to go to the board and draw it. Chad got the idea and is helping Tony. Sandy drew the following on the board.
(+) TC	(7)			
(+) SH	(8)	8.)		
(+) SR_B	(9)	9.)		<u>Board</u>



10:00 Mrs. Hanley gives a similar problem in which the students have to draw a budget which is divided into one half for overhead, one fourth for salaries, and one eighth for profit.

There were 31 general math episodes which had patterns of enactment containing minus acts pertaining to interruptions related to off task communication acts. This interference included teacher reprimands for behavior, students talking during the teaching of the lesson, and teacher and student comments that were either directed toward the topic being studied but not helpful to the solution of the task or irrelevant comments or questions. About one sixth of the off task minus acts for these general math episodes were directed toward the content.

These episodes for the most part had content about geometry or flow charting. The episodes were generally teacher dominated. The social participation structure most often used was that of the teacher directing from the front of the room and teacher coding categories were more frequent than student coding categories. The teacher gave most of the conclusions.

Episode #6 G.M. was an example of an episode with an enactment pattern containing interference related to off task acts. The following is the outline for Episode #6 G.M.

Problem: Exponents and Powers of 10

1. Giving problems on the board.
   (+) TC_B (1)
2. Determining the powers for each problem.
   (-) GN (2)
   (+) TQ (3)
   (-) GN (4)
   (-) SOT (5)
3. Explaining ten to the zero power.
   (+) TC (6)
   (+) TC (7)
4. Giving an example of expanded notation using powers of ten.
   (+) TC_B (8)
   (+) SP^B (9)
   (+) TP (10)

This episode was structured on the teacher's logical presentation of content consisting of four steps. The concept was the concept of powers of ten and the steps indicated the teacher's approach to presenting this concept. A series of problems was written on the board which gave indication for the use of powers of ten. Through interaction with the students the teacher worked out the exponents for each problem on the board. The zero power was treated separately in the third step because of its place in the pattern the teacher was establishing. In the fourth step the teacher tied this new lesson into what had been studied before in expanded notation. Previously just places where there were numbers were written in the expansion. Now with all the powers of ten, including ten to the zero power, every place value can be represented. The fourth step was a presentation of the completed expansion of a given number.

The script of the episode and the plus/minus signs gave some indication of the flow of the lesson. The three minus acts indicated interruptions in the flow of the lesson with all three minus acts occurring between the second and third step. The coded items indicated that two of the minus acts were representative of generally negative instances which probably involved more than one student. The third minus act was a comment by a student on the concept being presented but off the task being addressed.

Reading through the script the teacher put some problems on the board. During the process of determining the powers for each problem there was an instance of generally negative behavior followed by a question by the teacher. This provoked another instance of generally negative behavior. Then a student made a comment about the content but not directed at the task at hand. The teacher then directed the class to the situation of ten to the zero power. She explained the concept without recorded interaction with students. For the fourth step an example was given by the teacher in which the consecutive powers of ten were used in the expanded notation for a given number. A student made a comment which provided the teacher with an opportunity to reteach or clarify the concept being presented. The episode ended with the teacher making this clarification. The following is Episode #6 G.M. as it was recorded.

#6 10/22/79 p. 9 General Math

Mrs. Hanley: "That's how I feel about this whole class." "I think Leah feels like I do sometimes." "Let's work on that."

Code		Line	Time	Speaker	Transcript
(+) TC_B	(1)	1.)	10:23		Board
					$10^0 = 10 \cdot 10 \cdot 10$
					$10 = 10 \cdot 10 \cdot 10 \cdot 10$
					$10 = 10 \cdot 10$
					$10 = 10$
(-) GN	(2)	2.)			Mrs. Hanley wanted to get across the exponent concept. Tony uses every opportunity to express himself loudly. If he makes a mistake he corrects for it later by being louder.
(+) TQ	(3)	3.)		Mrs. H:	"What number do you put here?" (Mrs. Hanley points to the place for the exponent.) There follows a slight yelling session. Tony and Craig are going at it. In the problems they worked, Craig put six times one hundred instead of six times ten squared. (643.52 =)
(-) GN	(4)	4.)			
(-) SOT	(5)	5.)		Craig:	"It comes out to the same thing so I don't care."
(+) TC	(6)	6.)	10:26	Mrs. H:	"Ten to the zero power fits in to the expression over there." "This is where we make a definition in math. This doesn't make sense but it fits into the pattern." "Just as your mother calls you what she does we call ten to the zero one." "Some things that we do later on will make this seem more reasonable to you."
(+) TC	(7)	7.)			
(+) TC_B	(8)	8.)			Board
					$489.63 = 4 \cdot 10^2 + 8 \cdot 10' + 9 \cdot 10^0 + 6 \cdot 10^{-1} + 3 \cdot 10^{-2}$
(+) SP	(9)	9.)		Keith:	"I still don't understand why we write ten squared instead of one hundred."
(+) TP	(10)	10.)		Mrs. H:	"Because we are studying exponents. We are studying numbers a little and exponents a lot."

10:30 Mrs. Hanley told the class to do the written exercises twenty-one to twenty-four on page one hundred six.

There were 27 general math episodes which had patterns of enactment containing minus acts pertaining to interruptions related to on task acts and minus acts related to off task acts. Of the 140 minus acts coded in these episodes 63 were on task acts and 77 were off task acts. These episodes varied in social participation structures between the teacher directing from the front and a group of students working at the board. The students dominated the lessons, having the most coding categories and giving most of the conclusions.

Episode #37 G.M. was an example of an episode with an enactment pattern containing interference related to on task acts and interference related to off task acts. The following is the outline for Episode #37 G.M.

<u>Problem</u>: Volume of a prism

1. Discussing the properties of a prism.
   (+) TC (1)
   (+) TQ (2)
   (-) SW (3)
   (-) RC (4)
   (-) SOL (5)
2. Explaining volume and how to find it.
   (+) TQ (6)
   (-) SOL (7)
   (-) SOL (8)
   (+) TC (9)
   (+) TC (10)
   (+) TC (11)

This episode was structured on the teacher's logical presentation of content consisting of two steps. The content was geometry, the volume of a prism, and the concept was presented by the teacher from the front of the room. The teacher introduced the concept by reviewing the properties of a prism. Having done this she explained the concept of volume and told the class the procedure for determining the volume of a prism.

The script of the episode and the plus/minus signs gave some indication of the flow of the lesson. The five minus acts indicated there were interruptions in the flow of the lesson, three just before step two and two during the interaction about step two. The coding categories indicated that the first two minus acts prior to step two were related to content, a student wrong answer and a teacher reprimand about content. The third minus act, as well as the two minus acts during step two, were statements by students which were irrelevant to the task at hand.

Reading through the script, the teacher introduced the topic, asked a question and was given a wrong answer. The teacher corrected the student and another student made an irrelevant statement. At this point the teacher began the explanation of what volume is by asking a question. Two statements were made in response to the question but irrelevant to the lesson. The teacher then instructed the class on the topic and the episode was concluded by the teacher's instruction. The following is Episode #37 G.M. as it was recorded.

#37 1/8/80 p. 7 General Marh

Renea keeps calling out the answers. She laughs whenever Mrs. Hanley hesitates or makes a mistake.

Chad: "I think we did it wrong."

10:17 Mrs. Hanley brought out the boxes - prisms made of colored construction paper.

(+) TC (1) (+) TQ (2) (−) SW (3) (−) RC (4)	1.) 2.) 3.), 4.)		Mrs. Hanley talked about the lateral area of the prisms. She discussed the bases and what it meant to change bases to faces and faces to bases. In the course of the discussion Mrs. Hanley asked Keith what the green box was. He gave an incorrect answer and Mrs. Hanley said, "Be careful about squares here."
(−) SOL (5)	5.)	Craig:	"Yeah. That's a prison."
(+) TQ (6)	6.)	Mrs. H:	"Suppose. I asked you to fill this prism up."
(−) SOL (7)	7.)	Brian:	"I would."
(−) SOL (8)	8.)	Chad:	"With what?"
(+) TC (9) (+) TC (10) (+) TC (11)	9.) 10.) 11.)		Mrs. Hanley talked about what volume is with respect to the prisms. She told them how to find the volume. She then told the class the name of each prism by calling it whatever color it was. Mrs. Hanley asked the class to measure and find the lateral area and the volume of each prism.

She passed the prisms out so one or two could work at a time measuring.

10:21 Mrs. Hanley to Craig, Chad, Tony: "Where's Brent today?"

Chad: "He's sick."

Craig: "He died."

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